$(0,0,1,0) \in N_U \Rightarrow \quad \alpha.(0,0,1,0) \in N_U$

$$\Rightarrow f(\alpha.(0,0,1,0)) = (0,0,0)$$

$$\alpha . f(0,0,1,0) = (0,0,0)$$

Veo que

$$\begin{cases} f(1,1,0,1) = (a,1,a) \\ f(1,1,1,1) = (a^2,1,a) \end{cases}$$

$$f(0,0,1,0) = (a^2-a,0,0) \overset{(0,0,1,0)\in N_U}{=} (0,0,0)$$

$$a^2 - a = 0$$

$$a(a-1) = 0$$

$$a_1 = 0$$
$$a_2 = 1$$

Si $a = 0$:

$$\left.\begin{array}{l} f(1,0,0,0) = (1,0,0) \\ \left.\begin{array}{l} f(1,1,0,1) = (0,1,0) \\ f(1,1,1,1) = (0,1,0) \end{array}\right\} \text{mismas imagenes} \end{array}\right\} \Rightarrow f(x_1,x_2,x_3,x_4) = (x_1,x_2,0)$$

Si $a = 1$:

$$\left.\begin{array}{l} f(1,0,0,0) = (1,0,0) \\ \left.\begin{array}{l} f(1,1,0,1) = (1,1,1) \\ f(1,1,1,1) = (1,1,1) \end{array}\right\} \text{mismas imágenes} \end{array}\right\} \Rightarrow f(x_1, x_2, x_3, x_4) = (x_1, x_2, x_2)$$