MATRIZ DE CAMBIO DE BASE

POLINOMIO INTERPOLADOR DE LAGRANGE

SEAN $a_0, \dots, a_m \in K$, DISTINTOS DOS A DOS. SEAN

$$P_i = \prod_{\substack{j=0 \\ j \neq i}}^{m} (X - a_j) \in \underbrace{K_m[X]}_{\text{POL. DE GRADO } \leq m}, \quad 0 \leq i \leq m$$

ASÍ SI $\lambda_{ij} = P_i(a_j)$, TENEMOS QUE $\lambda_{ij} = 0$ SII $i \neq j$.

AFIRMO: $\mathcal{B} = \{P_0, \dots, P_m\}$ ES BASE DE $K_m[X]$

DEM: BVQ SON LI;

$$0 = \sum_{i=0}^{m} \alpha_i P_i \underset{\text{EV. EN } a_j}{\Longrightarrow} 0 = \sum_{i=0}^{m} \alpha_i \lambda_{ij} = \alpha_j \overbrace{\lambda_{jj}}^{\neq 0}$$

$$\Rightarrow \alpha_j = 0 \quad \square$$

EJEMPLO: $K = \mathbb{Q}$, $m = 2$, $a_0 = 1$, $a_1 = 2$, $a_2 = 3$.

SEA $P = b_0 + b_1 X + b_2 X^2$;

$$P = \sum_{i=0}^{2} \alpha_i P_i \longleftrightarrow (\alpha_0, \alpha_1, \alpha_2) = [P]_{\mathcal{B}}$$

$$P(\lambda_j) = \alpha_j \lambda_{jj}$$

VIMOS: • $[P]_B = C(E,B) \cdot \begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix}$,

SIENDO $E = \{1, X, X^2\}$

• $C(E,B) = \underbrace{C(B,E)}_{\text{FÁCIL}}{}^{-1}$

TENEMOS
$$P_1 = (X-2)(X-3) = X^2 - 5X + 6$$
$$P_2 = (X-1)(X-3) = X^2 - 4X + 3$$
$$P_3 = (X-1)(X-2) = X^2 - 3X + 2$$

$$\Rightarrow C(B,E) = \begin{pmatrix} 6 & 3 & 2 \\ -5 & -4 & -3 \\ 1 & 1 & 1 \end{pmatrix} \underset{\text{INVIERTO}}{\Rightarrow} C(E,B) = 1/2 \begin{pmatrix} 1 & 1 & 1 \\ -2 & -4 & -8 \\ 1 & 3 & 9 \end{pmatrix}$$

VOLVIENDO,
$$(\alpha_0, \alpha_1, \alpha_2)^t = 1/2 \begin{pmatrix} 1 & 1 & 1 \\ -2 & -4 & -8 \\ 1 & 3 & 9 \end{pmatrix} \begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix} = 1/2 \begin{pmatrix} b_0 + b_1 + b_2 \\ -2b_0 - 4b_1 - 8b_2 \\ b_0 + 3b_1 + 9b_2 \end{pmatrix};$$

i.e.,

$$P = 1/2 \Big((b_0 + b_1 + b_2)(X-2)(X-3) + (-2b_0 - 4b_1 - 8b_2)(X-1)(X-3) + (b_0 + 3b_1 + 9b_2)(X-1)(X-2) \Big)$$

NOTAR:
$$\alpha_0 = 1/2(b_0 + b_1 + b_2) = 1/2 \cdot P(\lambda_0)$$
$$\alpha_1 = 1/2(-2b_0 - 4b_1 - 8b_2) = (-1) \cdot P(\lambda_1)$$
$$\alpha_2 = 1/2(b_0 + 3b_1 + 9b_2) = 1/2 \cdot P(\lambda_2)$$

¡LO SABÍAMOS! $P(\lambda_j) = \alpha_j \lambda_{jj}$