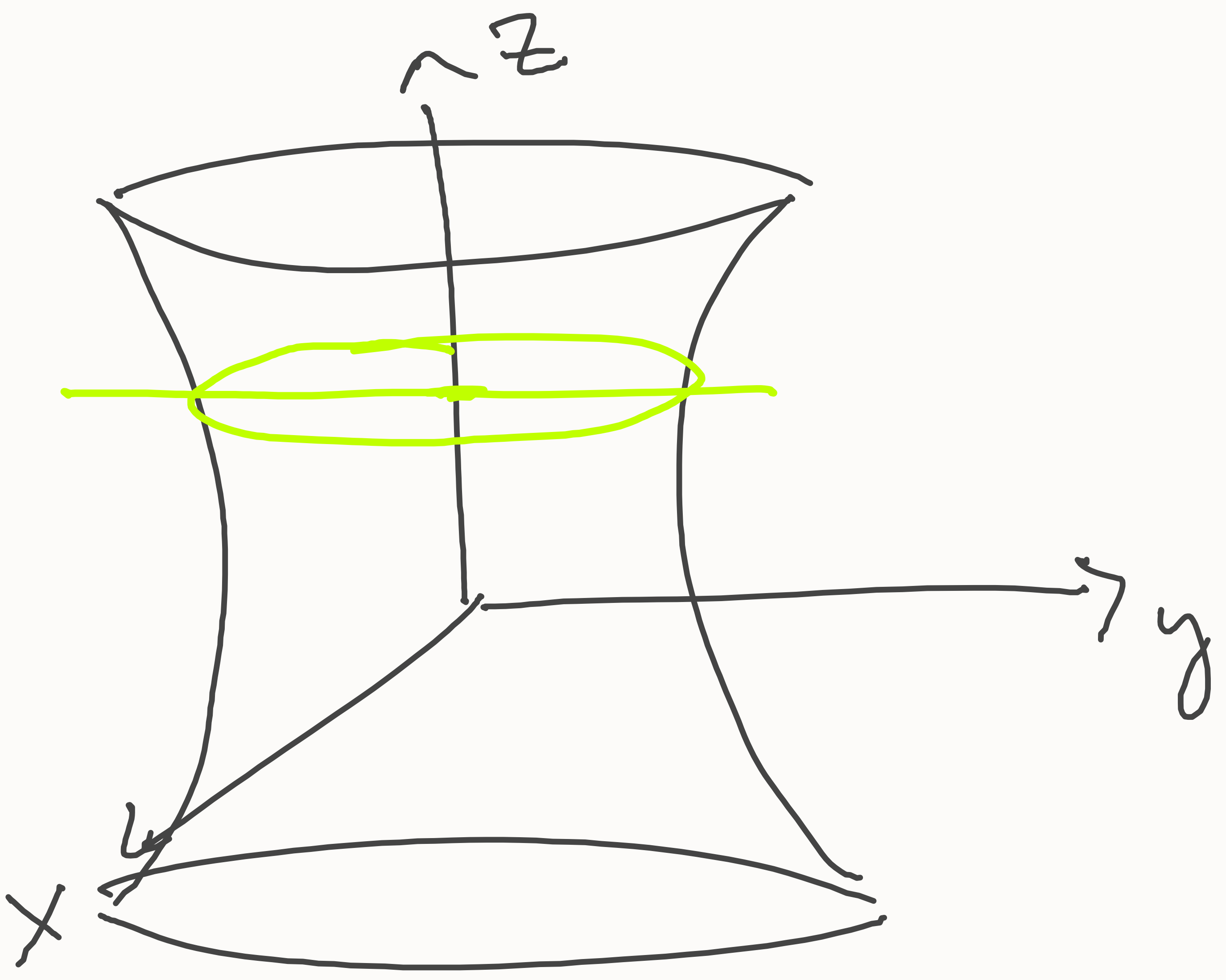
$S = \{ (x,y,z) : x^2+y^2-z^2 = 1 \quad |z| \leq 1 \}$



$T(u,v) = \left(\cos(u)\sqrt{v^2+1},\ \sin(u)\sqrt{v^2+1},\ v \right)$

$x^2+y^2 = 1+z^2$

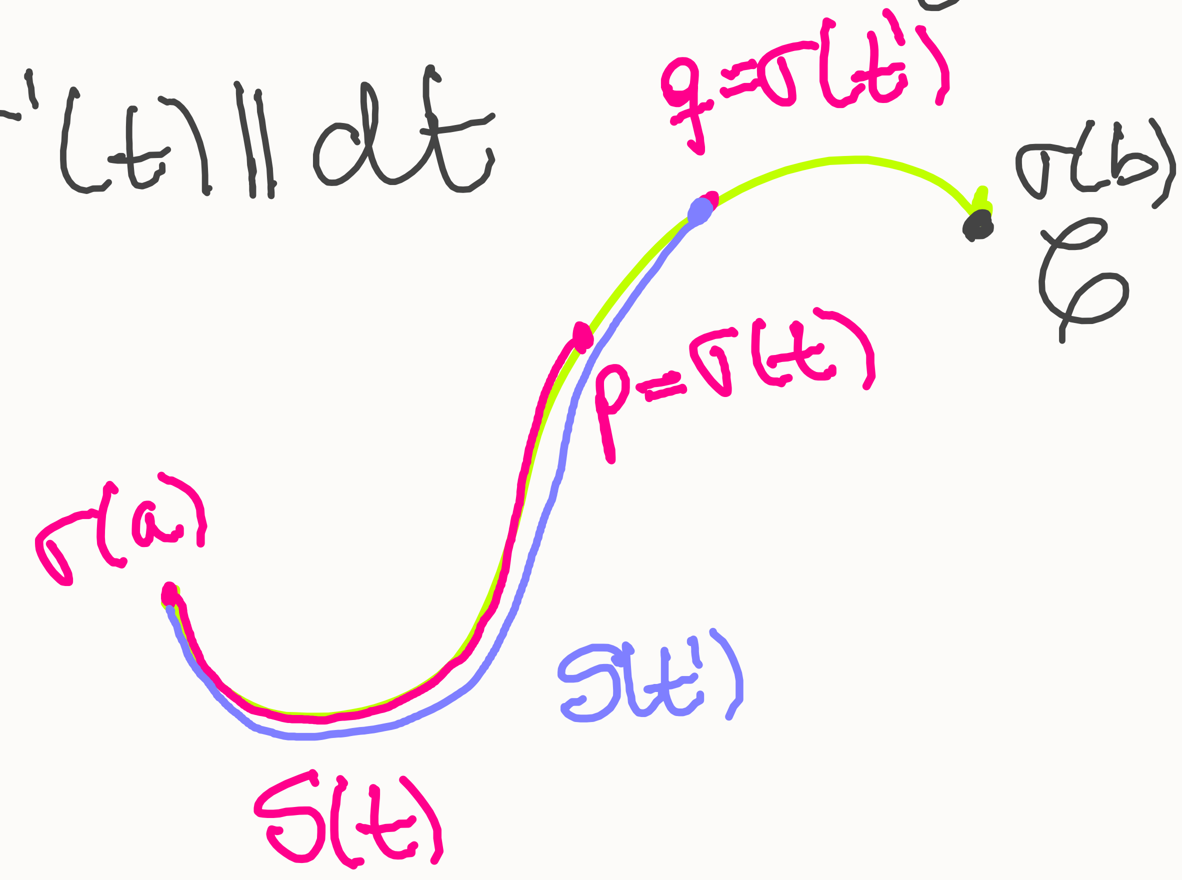
$\sqrt{x^2+y^2} = \sqrt{1+z^2}$

$T(\theta,z) = \left(\cos(\theta)\sqrt{z^2+1},\ \sin(\theta)\sqrt{z^2+1},\ z \right)$

—— o ——

· $\mathcal{C}$ curva $\wedge$ $\sigma : [a,b] \longrightarrow \mathcal{C}$ param. reg.

· long $(\mathcal{C}) = \mathcal{L}(\mathcal{C}) = \int_a^b \|\sigma'(t)\|\, dt$



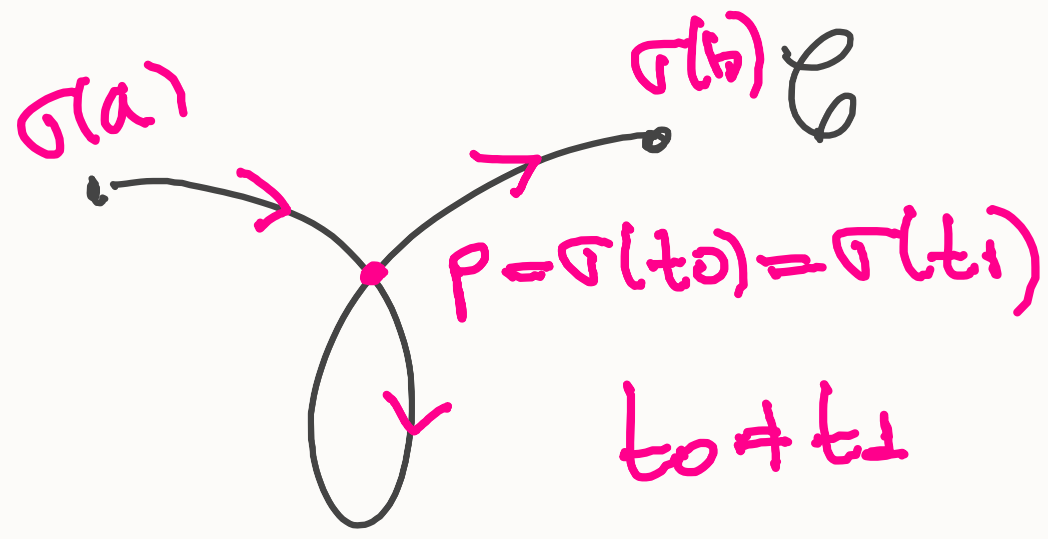
· función long. de arco.

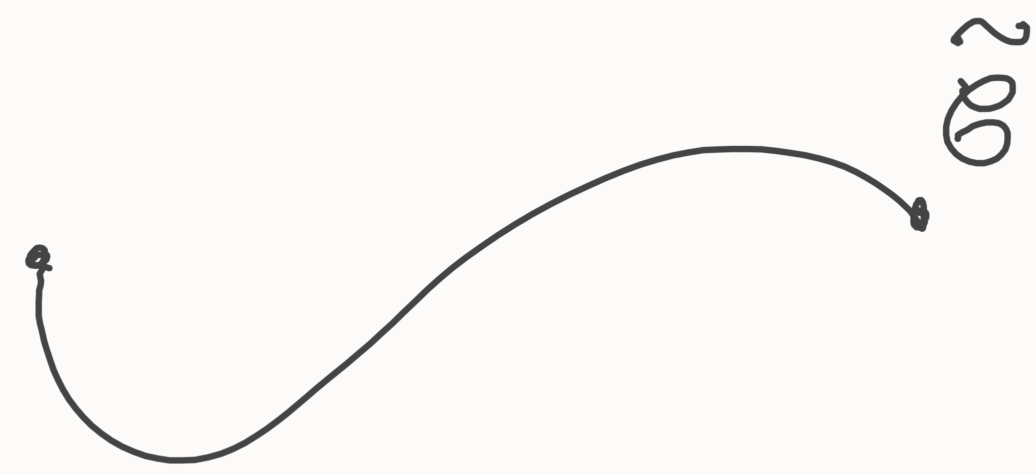
$S(t) = \int_a^t \|\sigma'(r)\|\, dr$ = long de $\mathcal{C}$ entre $\sigma(a)$ y $\sigma(t)$

$\boxed{S(b) = \mathcal{L}(\mathcal{C})}$

—— o ——

Inyectividad:





$\tilde{\sigma}: [c,d] \longrightarrow \tilde{\mathcal{C}}$

$\tilde{\mathcal{C}}$ es simple.

$\sigma: [a,b] \longrightarrow \mathcal{C}$

$\hookrightarrow$ σ no es inyectiva

pues $\sigma(t_0) = \sigma(t_1)$

$\wedge$ $t_0 \neq t_1 \in (a,b)$

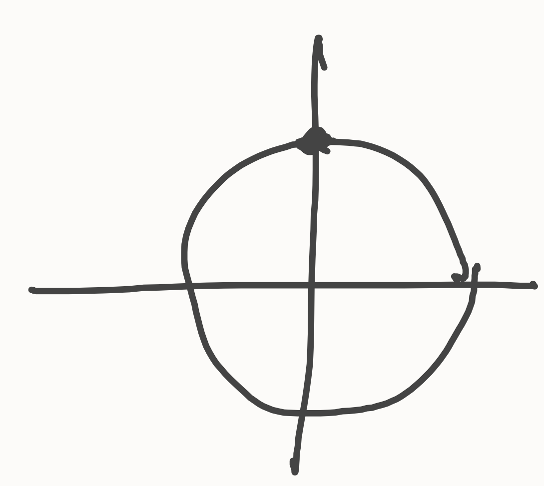
$\mathcal{C}$ no es simple

Ejemplo: $\sigma(t) = (\cos(4\pi t), \operatorname{sen}(4\pi t))$

$\sigma: [0,1] \longrightarrow \mathcal{C}$ $\qquad \mathcal{C} = \{x^2 + y^2 = 1\}$

- $\mathcal{C}$ es simple $\rightarrow$ $\tilde{\sigma}(\theta) = (\cos\theta, \operatorname{sen}\theta)$ $\theta \in [0, 2\pi]$.

- σ no es inyectiva: $(0,1)$



$\sigma\left(\frac{1}{8}\right) = (0,1)$

$(\cos(\pi/2), \operatorname{sen}(\pi/2))$ $\qquad 4\pi t = \frac{5\pi}{2}$

$\sigma(5/8) = (\cos(5/2\pi), \operatorname{sen}(5/2\pi)) = (0,1)$

$\sigma(\underset{t_0}{1/8}) = \sigma(\underset{t_1}{5/8})$ No es inyectiva!..

$$\int_\sigma F\cdot ds \quad \text{ó} \quad \int_{\mathcal{C}} F\, ds \quad \text{ó} \quad \int_{\mathcal{C}} \underline{P\,dx} + \underline{Q\,dy} + \underline{R\,dz}$$

$F=(P,Q,R)$

$\sigma:[a,b] \longrightarrow \mathcal{C}$ param. $\quad \sigma(t)=(x(t),y(t),z(t))$

$\rightarrow \sigma'(t)=(x'(t),y'(t),z'(t))$

$$\int_{\mathcal{C}} F\,ds = \int_a^b \langle \underbrace{F(\sigma(t))}, \underbrace{\sigma'(t)} \rangle\, dt .$$

$$= \int_a^b P(x(t),y(t),z(t)) \cdot x'(t) + Q(x(t),y(t),z(t))\, y'(t)$$

$$+ R(x(t),y(t),z(t))\, z'(t)\; dt .$$

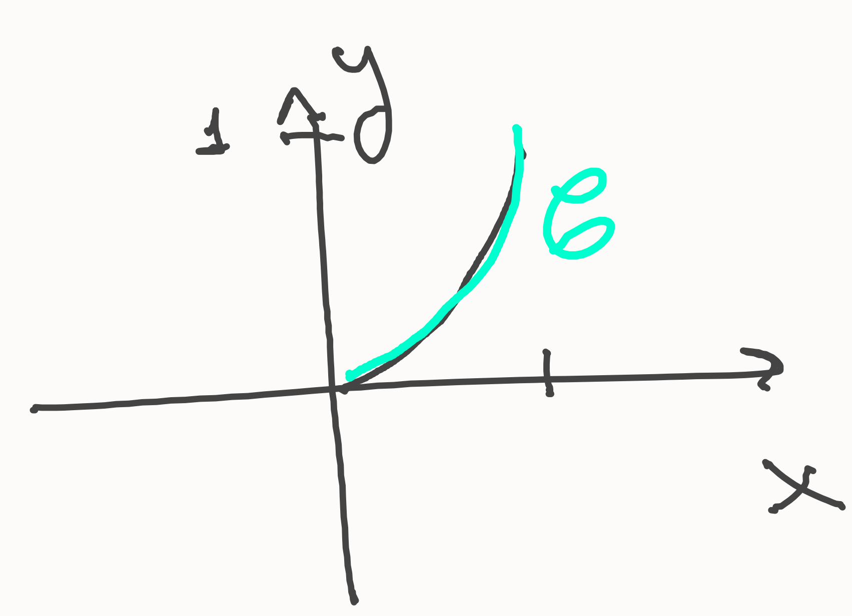
—o—

$\mathcal{C}$ curva $\qquad \sigma:[a,b] \longrightarrow \mathbb{R}^{2 \text{ ó } 3} \qquad \mathcal{C} \in \mathbb{R}^2 \text{ ó } \mathbb{R}^3$

σ es param de $\mathcal{C}$ si $\underline{\mathrm{Im}(\sigma)} = \underline{\mathcal{C}}$.

<u>Ejemplo:</u> $\mathcal{C}=\{(x,y)\in\mathbb{R}^2 : \underline{\sqrt{y}=x},\ \underline{y\in[0,1]}\}$

$\sigma(t)=(t,t^2)\quad t\in[0,1]$



qvq σ es param. de $\mathcal{C}$.

$\mathrm{Im}(\sigma)=\mathcal{C}$

$\subseteq)$ si $t\in[0,1]$, $(t,t^2)\in \mathrm{Im}(\sigma)$

¿ $(t,t^2)\in\mathcal{C}$?

está en $\mathcal{C}$ $\Leftrightarrow$ $y=t^2$, $x=t$

$\sqrt{y}=x \ \wedge\ y\in[0,1]$.

como $t^2=y\in[0,1]$ ✓

$\sqrt{y}=\sqrt{t^2}=|t|=t=x \Rightarrow \sqrt{y}=x \Rightarrow (t,t^2)\in\mathcal{C}.$

$t\in[0,1]$

$\supseteq)$ $(x,y)\in\mathcal{C} \Rightarrow x=\sqrt{y} \wedge y\in[0,1]$

qvq $\exists\ \underline{t\in[0,1]}$ / $\underbrace{\sigma(t)=(x,y)}$

$\underline{(t,t^2)}=\underline{(x,y)}$

tomo $t=x$: veamos que funcion

$\boxed{\begin{array}{c}(\sqrt{x})^2=x \\ \hline \sqrt{x^2}=|x| \\ \hline \end{array}}$

. $t\in[0,1]$: si pues $x=\sqrt{y}\in[0,1]$ xq' $y\in[0,1]$

. $(t,t^2)=(x,x^2)=(x,(\sqrt{y})^2)=(x,y)$

$(x,y)\in\mathcal{C}$

$\Rightarrow (x,y)\in \mathrm{Im}(\sigma).$

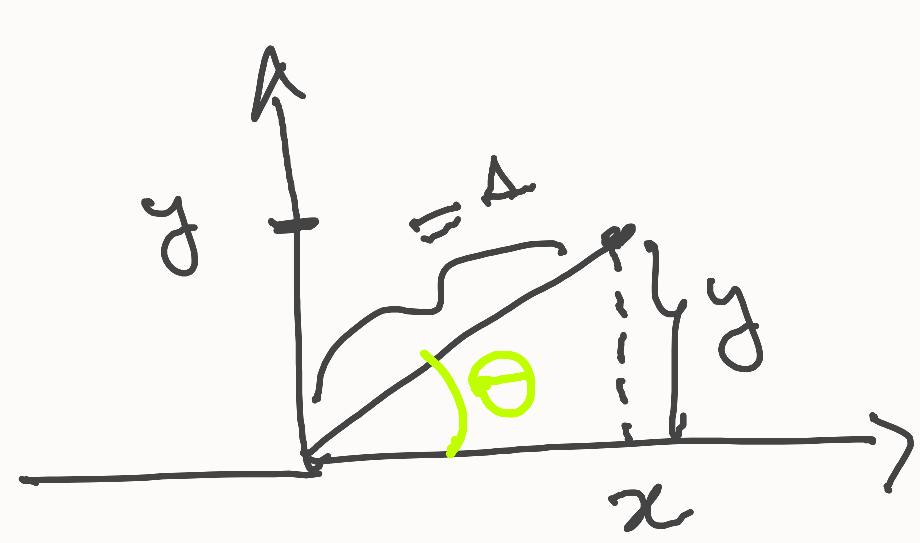
—o—

$\mathcal{C}: x^2+y^2=1.$ $\sigma(\theta)=(\cos(\theta),\operatorname{sen}(\theta))$ $\theta\in[0,2\pi].$

$\mathrm{Im}(\sigma)=\mathcal{C}.$

$\subseteq)$ $\cos(\theta)^2+\operatorname{sen}\theta^2=1 \Rightarrow (\cos\theta,\operatorname{sen}\theta)\in\mathcal{C}$

$\supseteq)$ (x,y) / $x^2+y^2=1$



$x=r\cos\theta$

$y=r\operatorname{sen}\theta.$

$x=\frac{x}{1r}=\cos(\theta)$

$y=\frac{y}{1r}=\operatorname{sen}(\theta).$