Método de Euler

$$\begin{cases} x_n = x_{n-1} + h\, f(t_{n-1}, x_{n-1}) \\ x_0 = x(t_0) \text{ dato} \quad (f(t,x) \text{ dato}) \\ t_n = t_{n-1} + h \end{cases}$$

En gral $\quad x_n = x_{n-1} + h\, \Phi(t_{n-1}, x_{n-1}, h)$

método de un paso

$$\epsilon_n = x(t_n) - (x(t_{n-1}) + h\, f(t_{n-1}, x(t_{n-1})))$$

sol. exacta

ϵ_n : error local de truncamiento

$\epsilon_n = O(h^2)$ — acotaciones de f, f_t, f_x

Obs: $x(t) \in \mathbb{R}^n \quad f(t,x) \in \mathbb{R}^n \quad (t,x) \in \mathbb{R} \times \mathbb{R}^n$

$$\frac{\partial f}{\partial x} = f_x \longrightarrow D_x f \in \mathbb{R}^{n \times n}$$

$$D_x f = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \\ \frac{\partial f_n}{\partial x_1} & & \frac{\partial f_n}{\partial x_n} \end{pmatrix}$$

Pregunta: ¿qué entendemos por acotar una matriz?

$$E_n = x(t_n) - x_n$$

$$|E_n| \leq \frac{e^{Lnh(t_n - t_0)} - 1}{Lh} \underbrace{\max_{1 \leq j \leq n} |\epsilon_j|}_{O(h^2)} = O(h) \qquad nh = t_n - t_0$$

Método de Taylor

$$x_n = x_{n-1} + h\, f(t_{n-1}, x_{n-1}) + \frac{h^2}{2}\left(f_t(\ ) + f_x(\ )f(\ )\right)$$

t_{n-1}, x_{n-1}

$$\epsilon_n = O(h^3)$$

acot. $f, f_t, f_x, f_{tt}, f_{tx}, f_{xx}$

$$|E_n| = O(h^2)$$

Tenemos que conocer f ✓ y también f_t, f_x

Obs: $\dot{x}(t) = f(t, x(t))$

$$\ddot{x}(t) = f_t + f_x(t, x(t))\, f(t, x(t))$$

en el caso $\mathbb{R}^n$

$$f_t(t,x) \in \mathbb{R}^n \quad D_x f(t, x(t)) \in \mathbb{R}^{n\times n} \quad f(t, x(t)) \in \mathbb{R}^n$$

$$\ddot{x}(t) = f_t + D_x f \cdot f$$

producto matriz por vector

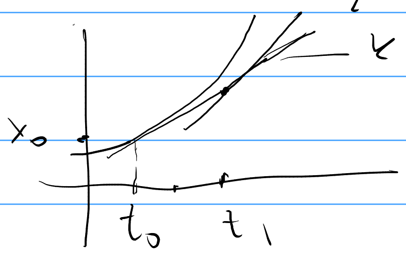
$$x_n = x_{n-1} + h\, f(t_{n-1}, x_{n-1}) + \frac{h^2}{2}\Big[f_t(\ ) + \underbrace{D_x f(\ )}_{n\times n} \cdot \underbrace{f(\ )}_{n\times 1}\Big]$$

Métodos Runge-Kutta

m. Euler

$k_2 = f(t_0 + h, x_0 + h k_1)$

$k_1 = f(t_0, x_0)$



$$k = \frac{k_1 + k_2}{2}$$

$$x_1 = x_0 + h\, k \qquad \text{R-K 2}$$

$t = t_0$
$x = x_0$
for $j = 1$ to n
  $k_1 = f(t, x)$
  $k_2 = f(t+h, x + h k_1)$
  $t = t + h$
  $x = x + \frac{h}{2}(k_1 + k_2)$

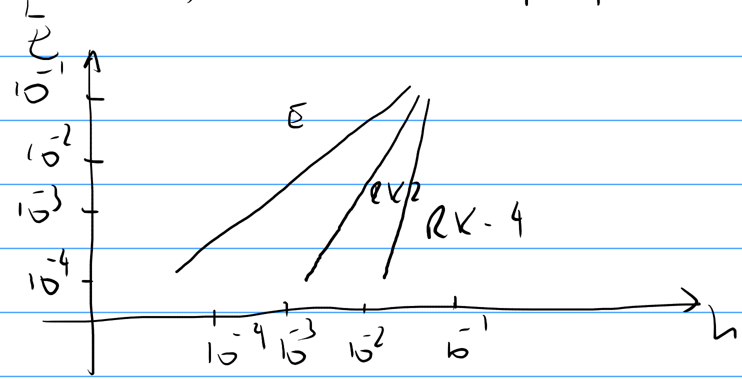
No guardamos los valores intermedios (t_j, x_j)

$f(t_0+h,\ x_0+y) =$

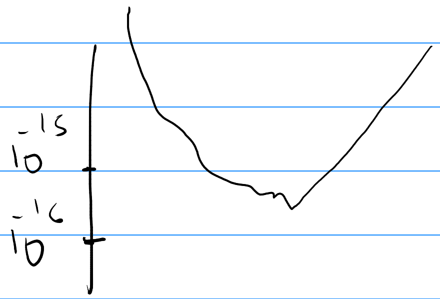
$f(t_0,x_0) + f_t(t_0,x_0)\,h + f_x(t_0,x_0)\,y + \underbrace{O(h^2+y^2)}_{O(h^2)}$ (Taylor)

$y = hk_1 \qquad y^2 = k_1^2 h^2$

M Euler R-K, etc h pequeño



$|E| = Ch^p \qquad \ln|E| = \ln C + \underset{p}{p}\ln(h)$



ε
doble pre.

ode23$(t_0, x_0, f, t_f) \longrightarrow (t_0, \ldots, t_n), (x_0, \ldots, x_n)$
ode45 $\longrightarrow$ orden 2 y 3
ode ~~?~~

$f(t_0,x_0),\ f(t_0+\frac{h}{2},\ x_0+\frac{h}{2}k_1),$