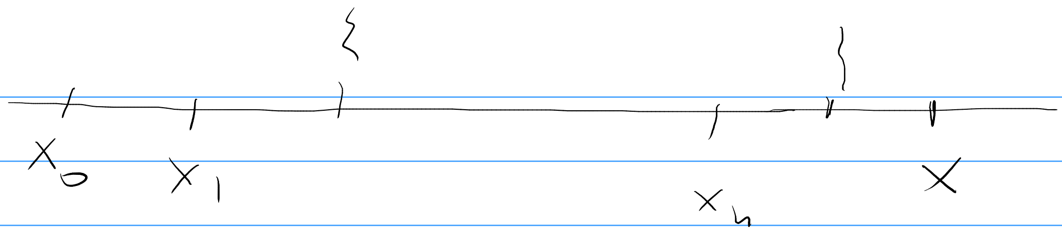


$\xi \in$ intervalo que contiene

a $x_0, \ldots, x_n, x$

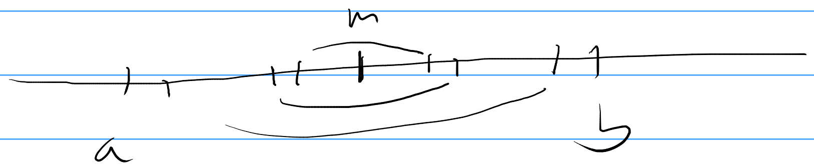
$W(x) = \prod_{j=0}^{n} (x - x_j)$ $\qquad$ $gr(W) = n+1$

$W(x)$ es mónico

$W(x) = x^{n+1} + \ldots$

coef $= 1$

$n!$ formula Stirling



$\dfrac{(b-a)^n}{n^{n+1}}$ $\qquad$ $n!$

Formula del error

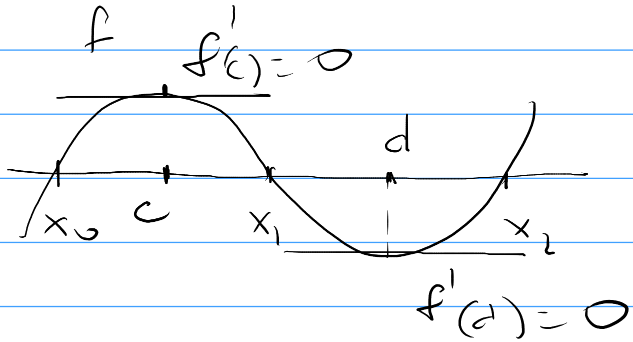
$p_n(x)$ pol. interp. $x_0, \ldots, x_n$

$$r_n(x) = f(x) - p_n(x)$$

Teorema de Rolle (generalizado)

T. Rolle: $f(x_0) = f(x_1) \Rightarrow c \in (x_0, x_1)$ $f'(c) = 0$

$n = 2$ $\quad f(x_0) = f(x_1) = f(x_2)$ $(= 0)$



$$\left.\begin{array}{l} f'(c) = 0 \\ f'(d) = 0 \end{array}\right\} f''(y) = 0$$

$f(x_0) = f(x_1) = \quad = f(x_n) \Rightarrow$ existe $c \in [x_0, x_n]$

$$f^{(n)}(c) = 0$$

$x \neq x_0, x_1, \ldots, x_n \Rightarrow W(x) \neq 0$

$$h(t) = f(t) - p_n(t) - \frac{W(t)}{W(x)}\left(f(x) - p_n(x)\right)$$

$$h(x_j) = \underbrace{f(x_j) - p_n(x_j)}_{0} - \frac{\overbrace{W(x_j)}^{0}}{W(x)}\left(f(x) - p_n(x)\right) = 0$$

$$h(x) = \cancel{f(x) - p_n(x)} - \cancel{\frac{W(x)}{W(x)}}\left(\cancel{f(x) - p_n(x)}\right) = 0$$

$h(t)$ se anula en $\underbrace{x_0, x_1, \ldots, x_n, x}_{n+2 \text{ ptos}}$

$\Rightarrow \xi$ en el intervalo /

$$h^{(n+1)}(\xi) = 0$$

$$0 = f^{(n+1)}(\xi) - 0 - \frac{\overbrace{W^{(n+1)}(\xi)}}{W(x)}\left(f(x) - p_n(x)\right)$$

$$W(t) = t^{n+1} + w_n t^n + \ldots + w_0$$

$$W^{(n+1)}(t) = (n+1)!$$

$$0 = f^{(n+1)}(\xi) - \frac{(n+1)!}{W(x)} \underbrace{\left(f(x) - P_n(x) \right)}$$

$$f(x) - P_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} W(x)$$

¿Cómo elegir $W(x)$ / $\max_{x \in [a,b]} |W(x)|$ lo menor posible?

$$W_T(x) = \prod_{k=0}^{n} \Big[\underbrace{x - \cos\left((2k+1)\theta\right)}_{x_k} \Big] \qquad \theta = \frac{\pi}{(2n+2)}$$

$$|W(x)| \leq \frac{1}{2^n}$$

$P(x)$ polinomio de grado $n+1$ mónico

$$W(x) = \prod_{j=0}^{n} (x - x_j) \qquad \max_{[-1,1]} |P(x)| \geq \frac{1}{2^n}$$

$$W_J(x) = T_{n+1}(x)$$

$$T_n(x) = \cos(n \overbrace{\arccos(x)}^{\theta}) \qquad x \in [-1,1]$$

$$\left(n = 0 \quad T_0(x) = \cos(0) = 1 \right)$$

$$x = \cos(\theta)$$

$$T_1(x) = \cos(\arccos(x)) = x \qquad \theta \in [0, \pi]$$

$$T_n(x) = \cos(n\theta)$$

$$n\theta + \theta$$

$$T_{n+1}(x) = \cos((n+1)\theta) = \cos(n\theta)\cos(\theta) - \cancel{\sin(n\theta)\sin(\theta)}$$

$$+$$

$$T_{n-1}(x) = \cos(n\theta - \theta) = \cos(n\theta)\cos(\theta) + \cancel{\sin(n\theta)\sin(\theta)}$$

$$T_{n+1}(x) + T_{n-1}(x) = 2T_n(x)\,x$$

$$T_{n+1}(x) = 2x\,T_n(x) - T_{n-1}(x)$$

$$gr(T_{n+1}) = n+1 \qquad T_2(x) = 2x^2 - 1$$

$$T_3(x) = 4x^3 - 2x - x = 4x^3 - 3x$$

$$T_{n+1}(x) = 2^n x^{n+1} + \dots$$

$$W_T(x) = \frac{1}{2^n} T_{n+1}(x)$$

$x = \cos\theta$

Raices de $T_{n+1}(x) = \cos((n+1)\theta)$

$\cos(\alpha) = 0 \qquad \alpha = \frac{\pi}{2} + k\pi \qquad k \in \mathbb{Z}$

$$(n+1)\theta = \frac{\pi}{2} + k\pi = \frac{(2k+1)\pi}{2}$$

$$\theta_k = \frac{2k+1}{2n+2}\pi \qquad k \in \mathbb{Z} \qquad \theta \in [0, \pi]$$

$k = 0, 1, \dots, n$

$$x_k = \cos\left(\frac{2k+1}{2n+2}\pi\right)$$

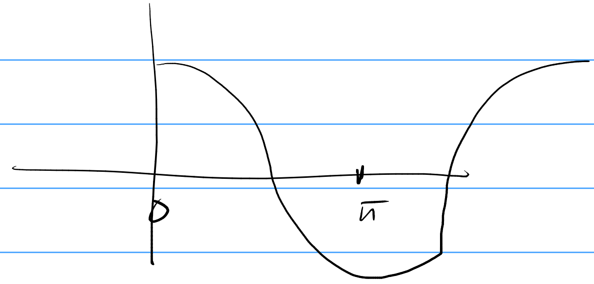
$$W_T = \prod_{k=0}^{n}(x - x_k) \quad \& \quad T_{n+1}(x) = 2^n \prod_{k=0}^{n}(x - x_k)$$

$$W_T(x) = \frac{1}{2^n} T_{n+1}(x)$$

$x \in [-1,1]$ $\qquad T_{n+1}(x) = \cos\left((n+1)\arccos(x)\right)$

$$|T_{n+1}(x)| \leq 1$$

$$|W_T(x)| = \left|\frac{1}{2^n} T_{n+1}(x)\right| \leq 2^{-n}$$



Obs: $|T_{n+1}(x)| = 1$ $\qquad (n+1)\theta = k\pi$

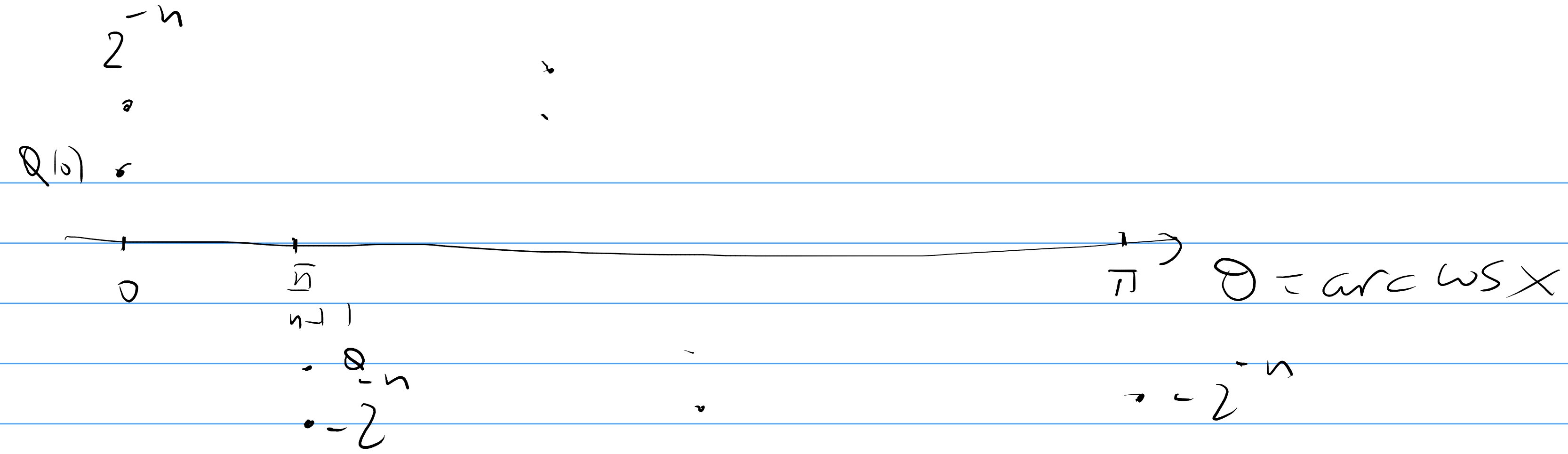
$$\theta = \frac{k\pi}{n+1} \in [0,\pi] \qquad k = 0, \ldots, n+1 \Rightarrow$$

hay $n+2$ ptos en $[-1,1]$ donde $|T_{n+1}(x)| = 1$

$P(x)$ polinomio de grado $= n+1$ mónico

y $|P(x)| < 2^{-n}$ <u>esto no existe</u>

$W_T(x) - P(x) = Q(x) \qquad gr(Q) \leq n$



$Q\left(\frac{k\pi}{n+1}\right)$ tiene el mismo signo que $W_T\left(\frac{k\pi}{n+1}\right)$

$Q(x)$ cambia de signo $n+1$ veces

tiene $(n+1)$ raíces abs

$$P(x) = a_0 + a_1(x+1) + a_2(x+1)^2 + a_3(x+1)^2 x$$
$$+ a_4(x+1)^2 x^2 + a_5(x+1)^2 x^3$$

	x_0	x_1	x_2
y	y_0	y_1	y_2
y'		—	
y''		y_1''	

4 cond.

$p(x)$ grado 3

valor arbitrario

$p(x)$ grado 4

Existe el polinomio de Hermite

x	-1	0	1
y	y_{-1}	y_0	y_1
y'	y'_{-1}	y'_0	
y''		y''_0	

$H : \Pi_5 \longrightarrow \mathbb{R}^6$ trans. lineal

$$HP = \left(P(-1), P'(-1), P(0), P'(0), P''(0), P(1) \right)$$

$\dim(\bar{n}_{\bar{s}}) = 6$ $\dim(\mathbb{R}^6) = 6$

$HP = (0, 0, 0, 0, 0, 0)$ $gr(P) \leq 5$

-1 raiz doble 2 $P = 0$

0 raiz triple 3

1 raiz simple 1

6 raices