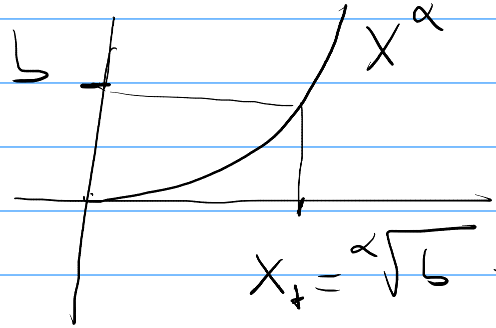
$f'(x) > 0$, $f''(x) > 0$ $\quad$ (Ej: $f(x) = e^{x} - 45$)

en $[x_*, x_0]$

Ej: $x^{\alpha} - b$ $\quad \alpha > 1$, $b > 0$



$x_* = \sqrt[\alpha]{b} = b^{1/\alpha}$

$$f(x) = x^{\alpha} - b$$

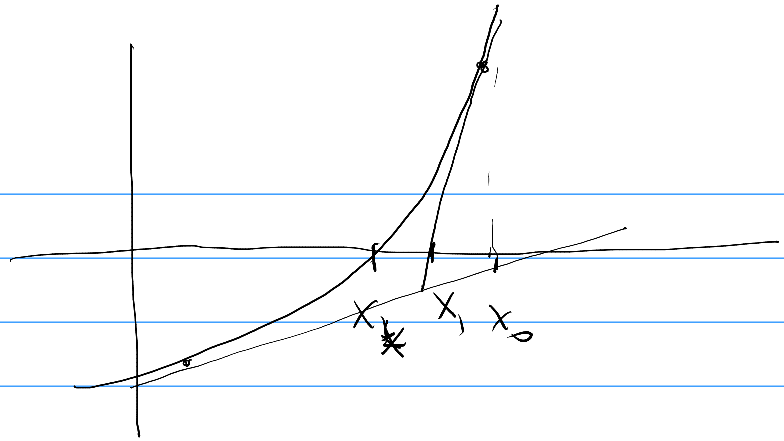
$$f'(x) = \alpha x^{\alpha-1} > 0$$

$$f''(x) = \alpha(\alpha-1) x^{\alpha-2} > 0$$

en $(0, +\infty)$

$x_0 > x_* \Rightarrow x_1 \in [x_*, x_0]$

$x_n \longrightarrow x_*$

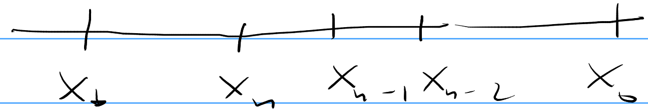


$e_n = x_* - x_n$ $\qquad$ $e_n = -\dfrac{\overset{+}{f''(\xi_n)}}{\underset{+}{2f'(x_{n-1})}}\,\overset{+}{e_{n-1}^{2}} < 0$

$x_* < x_n$

$x_{n-1} > x_*$ $\quad$ f creciente $\Rightarrow f(x_{n-1}) > f(x_*) = 0$
$(f' > 0)$

$$x_n = x_{n-1} - \frac{f(x_{n-1}) \to > 0}{f'(x_n) \to > 0} < x_{n-1}$$



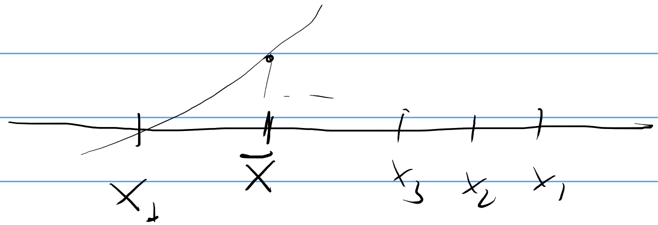
La sucesión que obtenemos aplicando M de Newton es monótona decreciente y $x_n > x_*$

Por lo tanto $x_n \longrightarrow \bar{x} \geq x_*$

Vamos a ver $\bar{X} = X_*$

$$\left.\begin{aligned} X_n &= X_{n-1} - \frac{f(X_{n-1})}{f'(X_{n-1})} \\ f, f' &\text{ continuas } \quad X_n \longrightarrow \bar{X} \end{aligned}\right\} \Rightarrow \bar{X} = \bar{X} - \frac{f(\bar{x})}{f'(\bar{x})}$$

$$\Rightarrow f(\bar{x}) = 0 \Rightarrow \bar{X} = X_*$$



$$\xi_n \in [X_*, X_{n-1}]$$

$$0 = f(X_*) = \cancel{f(X_{n-1})} + f'(X_{n-1})(X_* - X_{n-1}) + \frac{1}{2} f''(\xi_n)(X_* - X_{n-1})^2$$

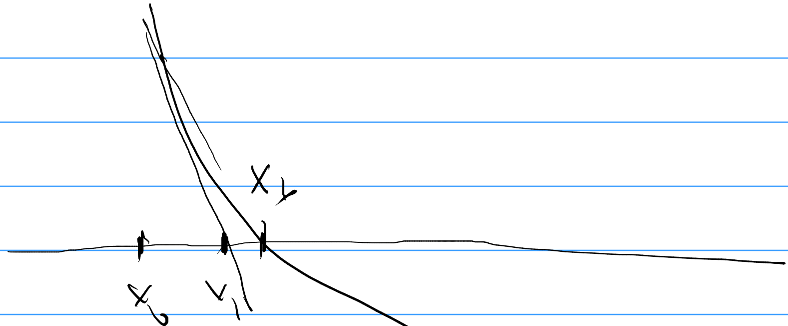
$$0 = \cancel{f(X_{n-1})} + f'(X_{n-1})(X_n - X_{n-1})$$

$$0 = f'(x_{n-1}) \underbrace{\left[X_* - X_n\right]}_{e_n} + \frac{1}{2} f''(\xi_n) \underbrace{(X_* - X_{n-1})^2}_{e_{n-1}^2}$$

$$e_n = - \frac{f''(\xi_n)}{2 f'(x_{n-1})} e_{n-1}^2$$

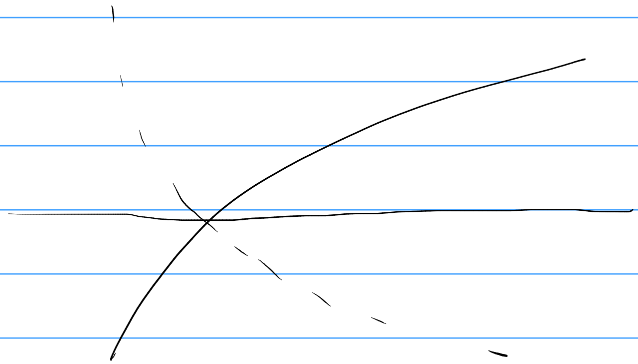
$$\left.\begin{array}{l} f''(\xi_n) > 0 \\ f'(x_{n-1}) > 0 \end{array}\right\} \Rightarrow e_n < 0 \Rightarrow x_* - x_n < 0$$

$$x_* < x_n$$



$f' < 0$ $\quad x_{n-1} < x_n <$ $\quad x_*$

$f'' > 0$ $\quad x_n \longrightarrow x_*$

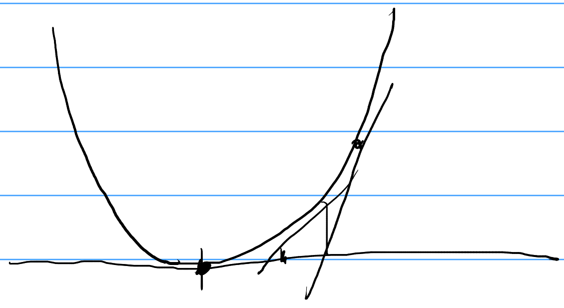


$f \longrightarrow -f$

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

Si $f'(x_*) \neq 0$ $\quad \dfrac{|e_n|}{|e_{n-1}|^2} \longrightarrow \dfrac{|f''(x_*)|}{2|f'(x_*)|}$ convergencia cuadrática

Si $f'(x_*) = 0$ $\quad (f''(x) > 0)$



Sigue valiendo $x_n > x_{n-1} > \cdots x_*$

$x_n \longrightarrow x_*$

x_*

Ej:

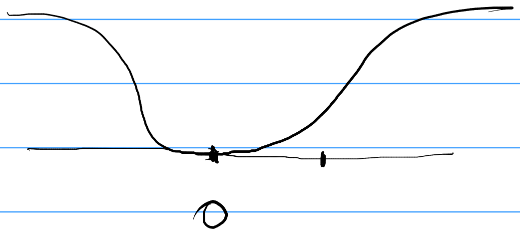
$$f(x) = x^2 \qquad x_n = x_{n-1} - \frac{x_{n-1}^{\cancel{2}}}{2\cancel{x_{n-1}}} = x_{n-1} - \frac{x_{n-1}}{2} = \frac{x_{n-1}}{2}$$

$$x_n = 2^{-n} x_0 \Rightarrow x_n \longrightarrow 0$$

$$e_n = x_* - x_n = 0 - x_n = -2^{-n} x_0 \qquad \frac{|e_n|}{|e_{n-1}|} = \frac{2^{-n}}{2^{-(n-1)}} = 2^{-1} = \frac{1}{2}$$

$$\frac{|e_n|}{|e_{n-1}^2|} = \frac{2^{-n}}{(2^{-(n-1)})^2} \frac{\cancel{x_0}}{\cancel{x_0}} = \frac{2^{2n-2}}{2^n} = 2^{n-2} \longrightarrow +\infty$$

Ejercicio: Analizar $f(x) = \begin{cases} e^{-\frac{1}{x^2}} & x \neq 0 \\ 0 & x = 0 \end{cases}$



el comportamiento del método Newton (x_0 cercano a 0)

$$Ax - b = 0 \longrightarrow X = BX + C$$

Jacobi

G-S

$$X_n = BX_{n-1} + C$$

$$X_n \longrightarrow X_* \,/\, X_* = BX_* + C$$

$$\Updownarrow$$

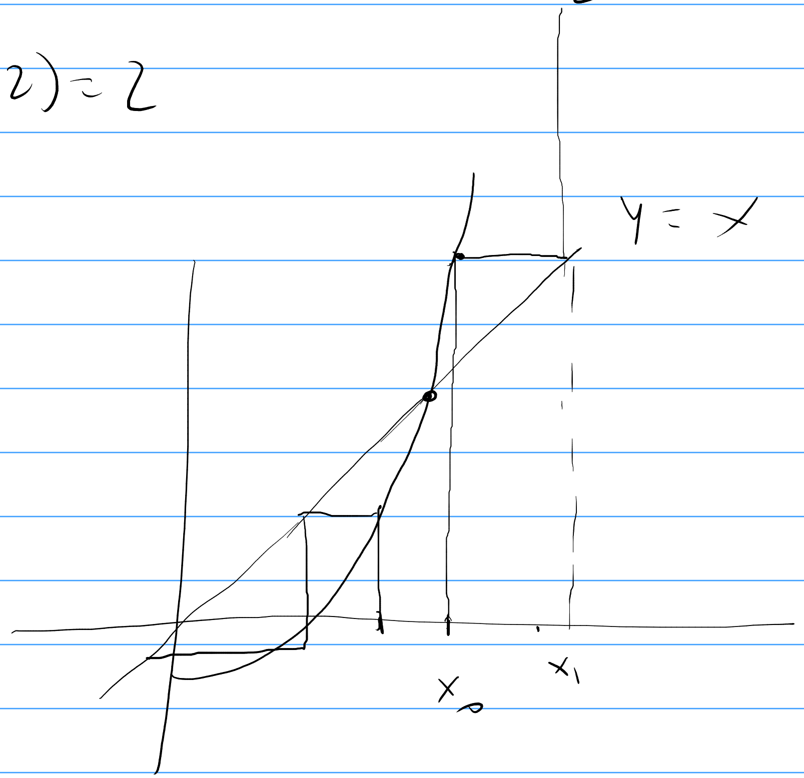
$$AX_* - b = 0$$

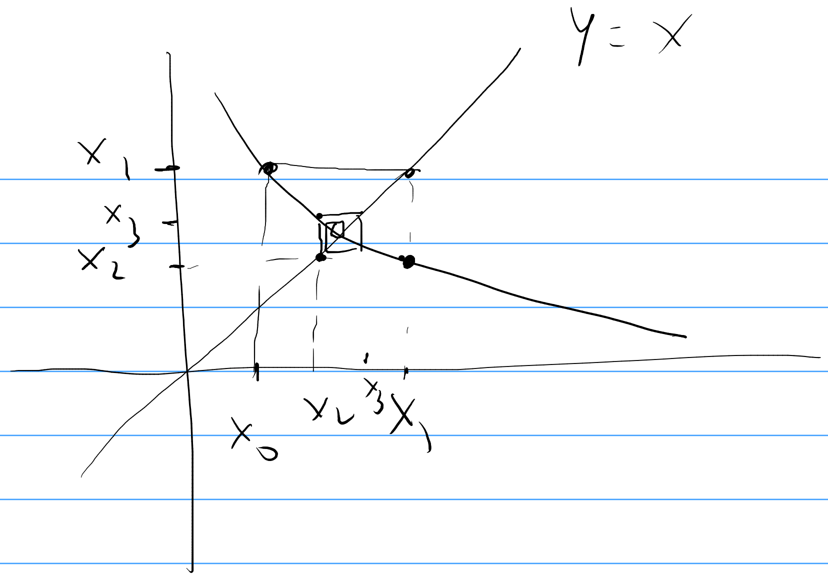
$x > 0$

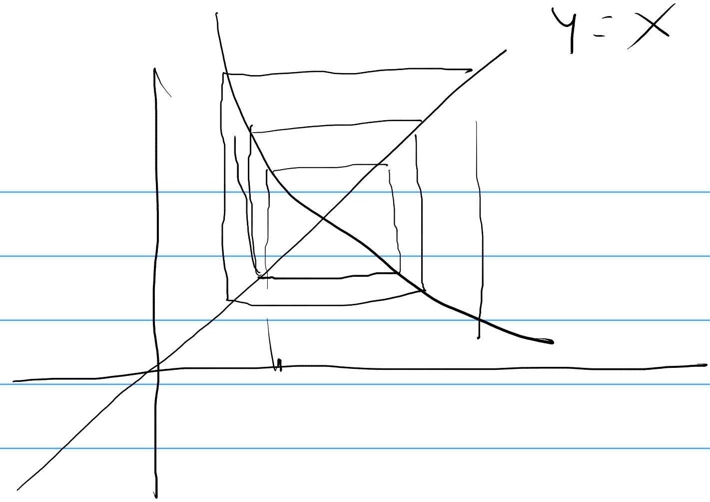
$$0 < x = \underbrace{\sqrt{\frac{x^2}{2} + 2}}_{\phi_2(x)} \iff x^2 = \frac{x^2}{2} + 2$$

$$\frac{x^2}{2} = 2 \iff x^2 = 4$$

$\phi_2(2) = 2$







$$|\phi'(x)| < 1 \Rightarrow x_n \longrightarrow x_*$$

$$|\phi(x) - \phi(x')| = |\phi'(\xi)|\,|x - x'| < |x - x'|$$

ϕ tiene cte de Lipschitz < 1

$$\phi(x) = x - \frac{f(x)}{f'(x)} \qquad x_n = \phi(x_{n-1})$$

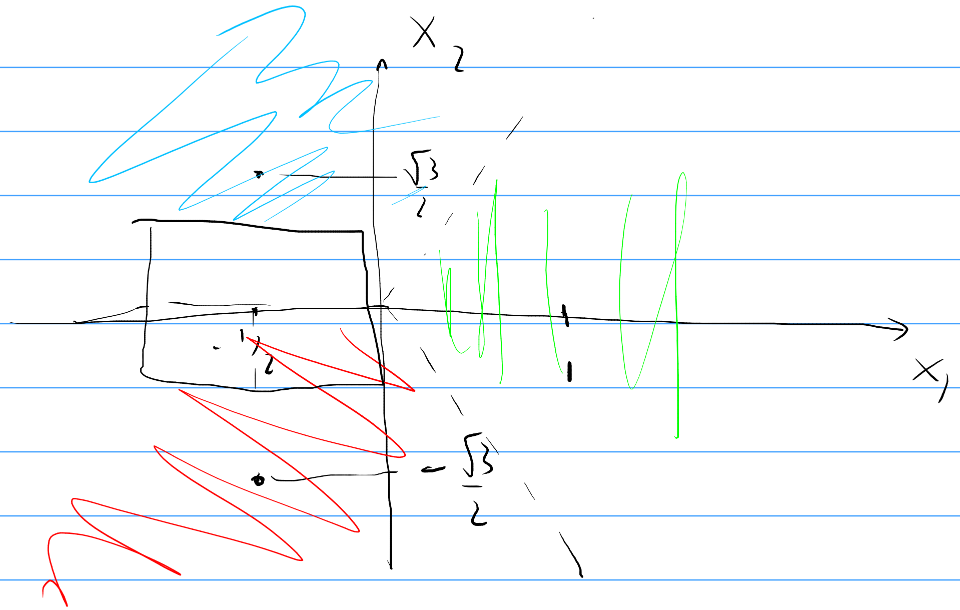
Es el método de Newton

$$\phi(x) = x \iff f(x) = 0$$

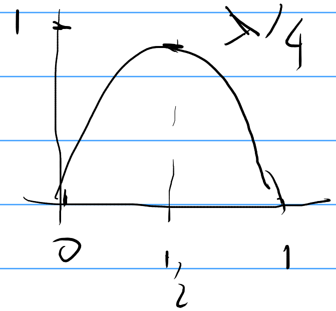
$$\phi'(x) = 1 - \frac{\overbrace{f'(x)\,f'(x)}^{f'(x)^2} - f(x)\,f''(x)}{f'(x)^2} = \cancel{1} - \cancel{1} + \frac{f(x)\,f''(x)}{f'(x)^2}$$

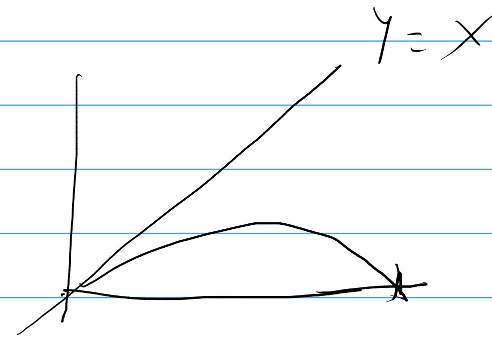
$$\phi'(x) = \frac{f(x)\,f''(x)}{f'(x)^2} \qquad \phi'(x_*) = \frac{\overbrace{f(x_*)}^{0}\,f''(x_*)}{f'(x_*)^2} = 0$$

$$|\phi'(x)| < \varepsilon \quad \text{si} \quad |x - x_*| < \delta$$



$\phi_\lambda(x) = \lambda x(1-x)$ $\quad \lambda \in [0,4]$



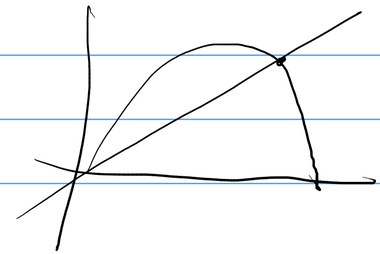


$\phi_\lambda : [0,1] \to [0,1]$

$0 < \lambda < \lambda_0 \qquad x_n \to 0$

$\lambda_0 < \lambda < \lambda_1 \qquad x_n \to x_* \neq 0$

$x_n = \phi_\lambda(x_{n-1})$



$\lambda = 3{,}96\ldots$

constante Fei