Error de Hermite

	x	y		
1	x_1	y_1		
2	x_2	y_2	y_2'	
+				
3	x_n	y_n	y_n'	y_n''
$m+1$				

$gr(P) = m$

$$P(x) = a_0 + a_1(x-x_1) + a_2(x-x_1)(x-x_2) + a_3(x-x_1)(x-x_2)^2 + a_m(x-x_1)(x-x_2)^2 \ldots (x-x_n)^2$$

$$f(x) - P(x) = \frac{f^{(m+1)}(\xi)}{(m+1)!} W(x)$$

$$W(x) = (x-x_1)(x-x_2)^2 \ldots (x-x_n)^3 \qquad gr(W) = m+1$$

x	y	y'	y''
0	1	—	—
1	1	2	-4
2	0	-2	

1
3
2

$6 = m+1 \qquad m = 5$

x	y					
0	1					
		0				
1	1		2			
		2		-4		
1	1		-2		3/2	
		2		-1		3/4
1	1		-3		3	
		-1		+2		
2	0		-1			
		-2				
2	0					

$$P_5(x) = 1 + 0(x-0) + 2(x-0)(x-1) - 4(x-0)(x-1)^2 + \tfrac{3}{2}(x-0)(x-1)^3 + \tfrac{3}{4}(x-0)(x-1)^3(x-2)$$

$$f(x) - P_5(x) = \frac{f^{(6)}(\xi)}{6!}\underbrace{(x-0)(x-1)^3(x-2)^2}_{w(x)}$$

$$y = \alpha + \beta \ln(1+x) + \gamma x^4$$

$$\phi_0(x) = 1$$

$$\phi_1(x) = \ln(1+x)$$

$$\phi_2(x) = x^4$$

$$Y = \begin{pmatrix} y_1 \\ \vdots \\ y_N \end{pmatrix} \quad N \times 1$$

$$\Phi = \begin{pmatrix} 1 & \ln(1+x_1) & x_1^4 \\ 1 & \ln(1+x_2) & x_2^4 \\ \vdots & & \\ 1 & \ln(1+x_N) & x_N^4 \end{pmatrix}$$

$$N \times 3$$

$$\underset{3\times N}{\Phi^T} \cdot \underset{N\times 1}{Y} = \underbrace{\underset{3\times N}{\Phi^T} \, \underset{N\times 3}{\Phi}}_{3\times 3} \underset{3\times 1}{\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}}$$

Ec. normales

$$Y = a_0 + a_1 X_1 - - + a_p X_p$$

$$\vec{Y} = \begin{pmatrix} Y_1 \\ \\ \\ Y_N \end{pmatrix} \qquad \Phi = \begin{pmatrix} 1 & X_{1,1} & X_{1,2} & X_{1,p} \\ 1 & X_{2,1} & X_{2,2} & X_{2,p} \\ \\ \\ 1 & X_{N,1} & X_{N,2} & X_{N,p} \end{pmatrix}$$

$$Y = \alpha + \beta X$$

X	Y
X_1	Y_1
X_N	Y_N

$$\Phi = \begin{pmatrix} 1 & X_1 \\ 1 & X_2 \\ 1 & X_N \end{pmatrix} \qquad \Phi^T \Phi = \begin{pmatrix} N & \Sigma X_j \\ \Sigma X_j & \Sigma X_j^2 \end{pmatrix}$$

$\vec{x} \not\parallel \vec{1} \Rightarrow \Phi^T \Phi$ es inversible

$\sum x_j = \vec{x} \cdot \vec{1}$ $\qquad \|\vec{1}\|^2 \|\vec{x}\|^2 - (\vec{x} \cdot \vec{1})^2 = \det(\Phi^T \cdot \Phi)$

$\vec{x} = k \vec{1}$

x	y
k	y_1
k	y_2
k	
k	y_N

$$\sum y_j = N\alpha + \sum x_j \beta$$

$$\sum x_j y_j = \sum x_j \alpha + \sum x_j^2 \beta$$

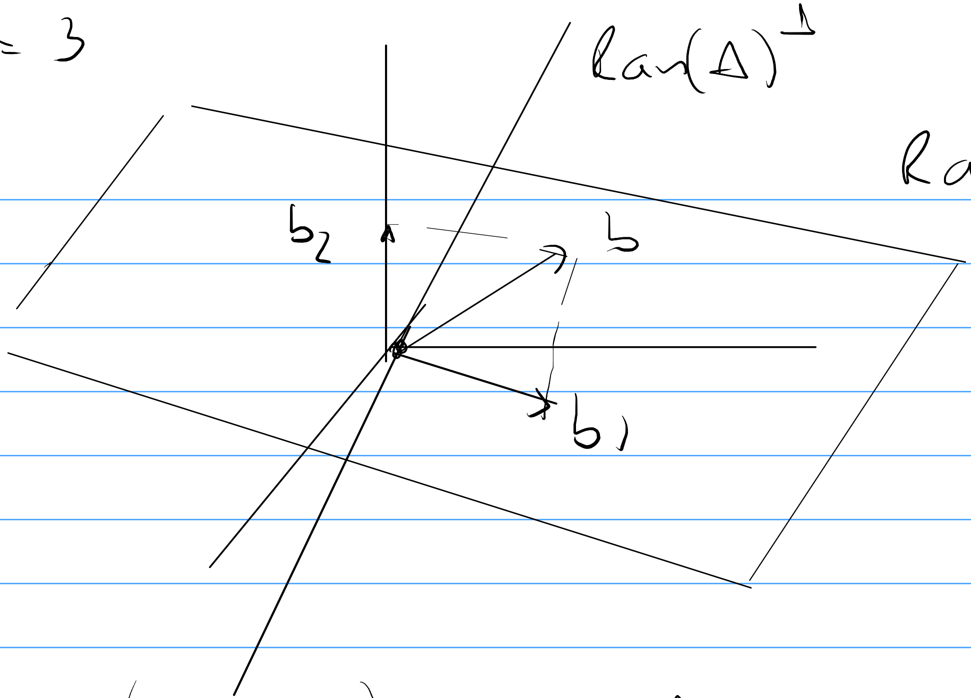
$$\begin{cases} \sum y_j = N\alpha + Nk\beta \\ k\sum y_j = kN\alpha + k^2 N \beta \end{cases} \cdot k$$

∞ soluciones

$$\boxed{\alpha = \frac{\sum y_j}{N} - k\beta}$$

Hay razones para elegir α, β / $\alpha^2 + \beta^2$ sea mínimo

$m = 3$



$A = \begin{pmatrix} & \\ & \end{pmatrix}$

$A \quad 3 \times 2$

$V \Rightarrow 3 \times 3 \qquad U \to 2 \times 2$

$\Sigma \quad 3 \times 2 \qquad \Sigma = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \end{pmatrix}$

B simétrica $\quad \lambda_1, \ldots, \lambda_n \in \mathbb{R}$

Existen n autovectores $\quad B v_k = \lambda_k v_k$

$\lambda_k \neq \lambda_j \Rightarrow v_k \perp v_j$

B semidef + $\Rightarrow \lambda_j \geq 0$

$B v_j = \lambda_j v_j \rightarrow A^T A v_j = \lambda_j v_j$

$\lambda_1, \_\_, \lambda_r > 0 \quad \lambda_{r+1} = 0, \_\_, \lambda_n = 0$

$\sigma_j = \frac{1}{\sqrt{\lambda_j}} A v_j \quad j = 1, \_, k$

$\lambda_l v_l$

$\sigma_j^T \cdot \sigma_l = \frac{1}{\sqrt{\lambda_j}} v_j^T A^T \frac{1}{\sqrt{\lambda_l}} A v_l = \frac{1}{\sqrt{\lambda_j \lambda_j}} v_j^T B v_l$

$= \frac{\lambda_l}{\sqrt{\lambda_j \lambda_l}} v_j^T v_l = \begin{cases} 1 & j = l \\ 0 & j \neq l \end{cases}$

$m \times n$

$4 \times 3 \quad k = 2$

$\underbrace{\begin{pmatrix} v_1 & v_2 & v_3 & v_4 \end{pmatrix}}_{V}$

$\Sigma = \begin{pmatrix} \lambda_1^{1/2} & 0 & 0 \\ 0 & \lambda_2^{1/2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\underbrace{\begin{pmatrix} u_1 & u_2 & u_3 \end{pmatrix}^T}_{U^T}$

$$U = (u_1 \; u_2 \; \dots \; u_n)$$

$$A \cdot U = (A \cdot u_1 \;\; A \cdot u_2 \;\; \dots \;\; A \cdot u_n) = (\lambda_1^{1/2} v_1 \;\; \dots \;\; \lambda_k^{1/2} v_k \;\; 0 \; \dots \; 0)$$

$$\underbrace{(v_1 \; v_2 \; \dots \; v_k \; v_{k+1} \; \dots \; v_m)}_{V} \underbrace{\begin{pmatrix} \lambda_1^{1/2} & & & & \\ & \lambda_2^{1/2} & & & \\ & & \lambda_k^{1/2} & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix}}_{\Sigma}$$

$$V\Sigma = AU \longrightarrow \boxed{V\Sigma U^T = A}$$

$$A = V\Sigma U^T \qquad A^T A x = A^T b$$

$$A^T = U \Sigma^T V^T$$

$$A^T A = U \Sigma^T \underbrace{\cancel{V^T V}}_{I_m} \Sigma U^T = U \Sigma^T \Sigma U^T = U \begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & 0 \end{pmatrix} U^T = U \Sigma^T V^T$$

$$\Sigma = \begin{pmatrix} \lambda_1^{1/2} & & \\ & \lambda_2^{1/2} & \\ & & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \Sigma^T = \begin{pmatrix} \lambda_1^{1/2} & 0 & 0 & 0 \\ 0 & \lambda_1^{1/2} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\Sigma^T \Sigma = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$U = A \overbrace{U^T x}^{y}$$

$$A = V \Sigma U^T \qquad A^{\dagger} = U \Sigma^{\dagger} V b$$

$$\cancel{U} \Sigma^T \Sigma \underbrace{U^T x}_{y} = \cancel{U} \Sigma^T V b$$

$$\Sigma y = V b$$

$$\Sigma = \begin{array}{c} \overbrace{\quad y_1 \quad \cdots \quad y_k \quad} \\ \begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \lambda_k & \\ & & & 0 \\ 0 & \cdots & \cdots & 0 \end{pmatrix} \end{array}$$

$\underbrace{y_1 \; \cdots \; y_k}$ fijos

$\underline{y_{k+1} \; \cdots \; y_m \text{ arbitrarios}}$

$$\|y\|^2 = \underbrace{y_1^2 + \cdots + y_k^2}_{} + \underbrace{y_{k+1}^2 + \cdots + y_n^2}_{}$$

A^+ pseudo-inverso (Moore-Penrose)

$A \in \mathbb{R}^{m \times n}$ $\quad \min_{x \in \mathbb{R}^n} \|Ax - b\|_2 \;/\; \|x\|_2 \min$

$x = A^+ b \qquad b \longrightarrow x$

Si $m = n$ $\quad A$ inversible $\Rightarrow A^+ = A^{-1}$

$m > n \quad rg(A) = n$

$$A^+ = \underset{n \times n}{(A^T A)^{-1}} \underset{n \times m}{A^T}$$