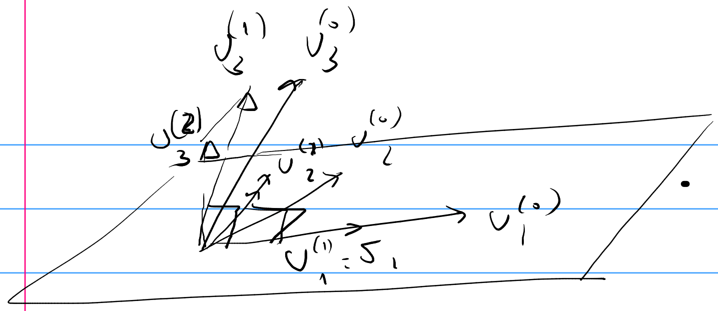


$A \in \mathbb{R}^{m\times n} \quad m > n$

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ a_{21} & & a_{2n} \\ & & \\ a_{m1} & & a_{mn} \end{pmatrix} \longrightarrow \overbrace{\begin{pmatrix} v_{11} & & v_{1n} \\ v_{21} & & v_{2n} \\ & & \vdots \\ v_{m1} & & v_{mn} \end{pmatrix}}^{\tilde{Q}} \underbrace{\overbrace{\begin{pmatrix} r_{11} & r_{12} & r_{1n} \\ 0 & r_{22} & r_{2n} \\ 0 & 0 & \vdots \\ & \vdots & \\ 0 & 0 & r_{nn} \end{pmatrix}}^{R}}_{n\times n}$$

$\uparrow a_1, \ldots, a_n \in \mathbb{R}^m \qquad \uparrow v_1 \quad \uparrow v_n$

$a_1 = r_{11} v_1$

$\tilde{Q}$ sus columnas son vectores ortonormales (no forman base de $\mathbb{R}^m$)

Si completamos $v_1, \ldots, v_n$ a una base ortonormal de $\mathbb{R}^m$

$$\underset{m\times n}{A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ a_{21} & & a_{2n} \\ & & \vdots \\ a_{m1} & & a_{mn} \end{pmatrix}} = \underset{m\times m}{\overset{Q}{\begin{pmatrix} v_{11} & \cdots & v_{1n} & v_{1\,n+1} & \cdots & v_{1m} \\ & & & & & \\ v_{m1} & & v_{mn} & v_{m\,n+1} & & v_{mm} \end{pmatrix}}} \underset{m\times n}{\overset{R}{\begin{pmatrix} r_{11} & r_{12} & r_{1n} \\ 0 & r_{22} & \vdots \\ 0 & 0 & r_{nn} \\ \hline 0 & 0 & 0 \\ \vdots & & \\ 0 & 0 & 0 \end{pmatrix}}}$$

$\qquad\qquad v_1 \quad v_n \quad v_{n+1} \quad v_m$

$A \in \mathbb{R}^{m\times n} \quad A = Q \cdot R \quad Q \in \mathbb{R}^{m\times m} \quad R \in \mathbb{R}^{m\times n}$

las columnas de Q forman una base ort. de $\mathbb{R}^m$ y R tiene estructura triang. superior

$$R = \begin{pmatrix} \diagdown\!\!\overline{}| \\ 0 \\ 0 \end{pmatrix} \begin{array}{l} \big] n\times n \\ \big] (m-n)\times n \end{array}$$

Obs. Podemos usar QR para $m=n$

$$Ax = b \iff QRx = b$$
$$Rx = Q^T b$$

¿Porqué existe LU?

Conociendo b

$$Ax = b \longrightarrow Ux = Ub = b'$$

Golub (Matrix Computation) Sec. 6.7

$LU \longrightarrow \frac{1}{3} n^3$ flops

$GS, QR \quad n^3$ flops

$HH, QR \quad \frac{2}{3} n^3$ flops

Obs. $k(A) \gg 1$ QR es más estable que LU

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ \vdots & \\ a_{n1} & a_{n2} \end{pmatrix} \longrightarrow \begin{matrix} a_{11} & a'_{12} \\ 0 & a'_{22} \\ \vdots & \\ 0 & \boxed{a_{n2}} \end{matrix}$$

$\rho \leq 2^n$

$A \in \mathbb{R}^{m\times n}$ $m > n$ $b \in \mathbb{R}^m$

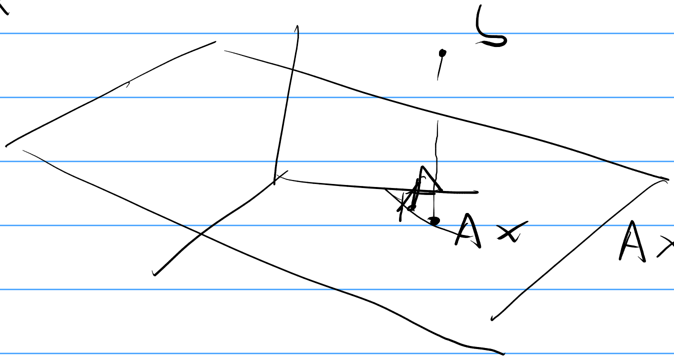
$x \in \mathbb{R}^n$

$Ax = b$

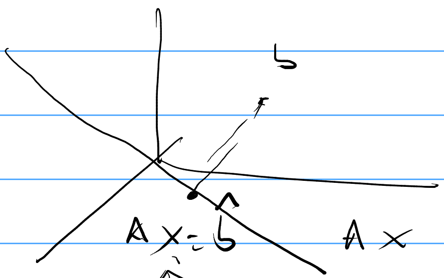
más ec que incógnitas

En gral incompatible

$\min_x \|Ax - b\|_2$ (mínimos cuadrados)

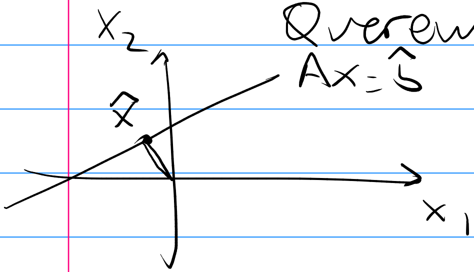


Ax $x \in \mathbb{R}^2$ $A \in \mathbb{R}^{3\times 2}$



$Ax = b$

No hay único $x \in \mathbb{R}^2$



Queremos x de norma mínima

$\|Ax - b\|_2$ mínimo

x de mínima norma

$b \longrightarrow \hat{x}$ es una transf lineal

(Ejercicio*)

$$Q^T = Q^{-1}$$

$$Q^{-1}Q = I \Rightarrow \underbrace{\det(Q^{-1})}\det(Q) = \det(I) = 1$$

$$\underbrace{\det(Q^T)\det(Q)}\ \det(Q)^2 = 1$$

Q ortogonal $\boxed{\det(Q)=1}$

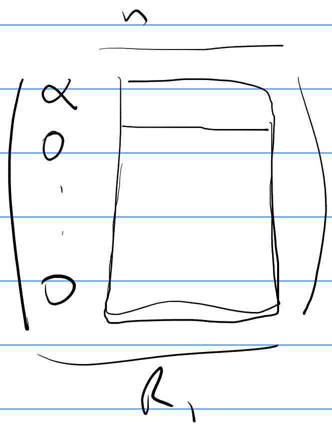
Obs: si n es impar $\Rightarrow$ $\lambda = 1$ es autovalor

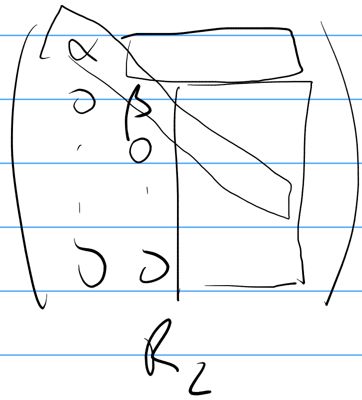
No si n es par $\begin{pmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{pmatrix} = Q \qquad \theta \neq 2k\pi$

House holder

$$P_1 A = R_1$$

$$P_2 P_1 A = R_2$$



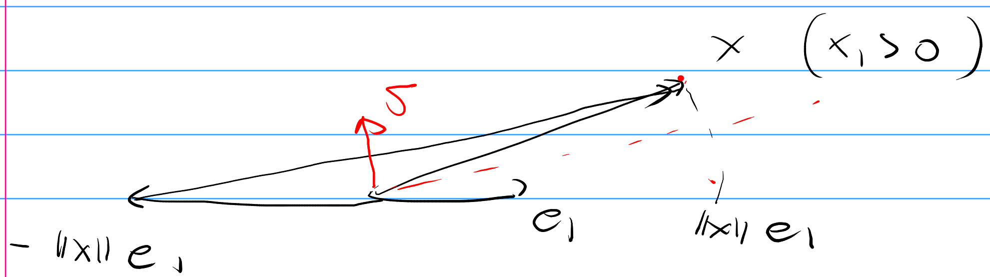


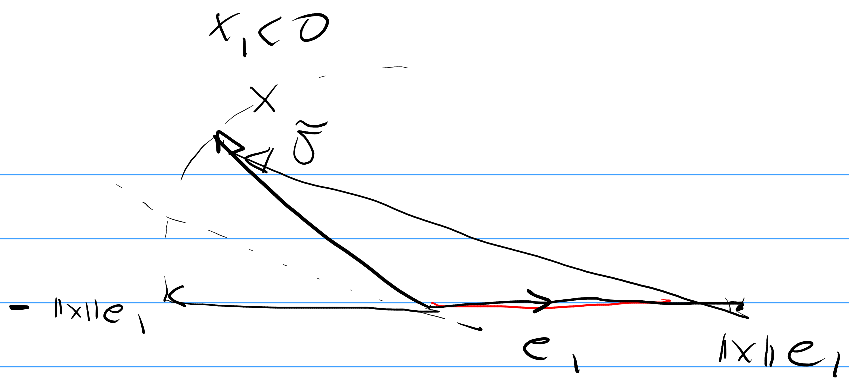
$$\underbrace{P_n P_{n-1} \cdots P_2 P_1}_{Q^T} A = R$$

$$A = P_1^T P_2^T \cdots P_n^T R$$

$$P_1(a_1\ a_2\ \cdots\ a_n) = (P_1 a_1\ \ P_1 a_2\ \cdots\ P_1 a_n)$$

$$\begin{pmatrix}\alpha \\ 0 \\ \vdots\end{pmatrix} = \alpha \begin{pmatrix}1 \\ 0 \\ \vdots \\ 0\end{pmatrix}$$





$$P_1 x = x - 2(v \cdot x)v \qquad \|v\| = 1$$

$$P_1 x = x - \frac{2(\tilde{v} \cdot x)}{\tilde{v} \cdot \tilde{v}} \tilde{v} \qquad \tilde{v} \neq 0$$

$$P_1 = I - 2 v v^T \qquad \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} (v_1 \cdots v_n)$$

$$P_1 = \begin{pmatrix} 1 - 2v_1^2 & -2v_1 v_2 & \cdots & -2v_1 v_n \\ -2v_2 v_1 & 1 - 2v_2^2 & & \\ & & & \end{pmatrix}$$

<u>Obs</u>: $P_1^T = P_1$