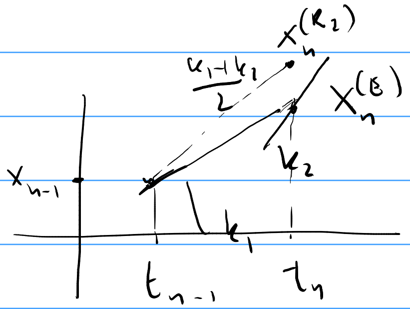
$$x_n = x_{n-1} + h\,\frac{k_1+k_2}{2}$$

$$k_1 = f(t_{n-1}, x_{n-1})$$

$$k_2 = f(\underbrace{t_{n-1}+h}_{t_n}, \underbrace{x_{n-1} + \overbrace{h f(t_{n-1}, x_{n-1})}^{y}}_{x_n \text{ de método de Euler}})$$



$$x(t_n) = x(t_{n-1}+h) = \overbrace{x(t_{n-1}) + \dot{x}(t_{n-1})h + \frac{1}{2}\ddot{x}(t_{n-1})h^2}^{A}$$

$$+ \frac{1}{6}\dddot{x}(\xi)h^3$$

$$x_n = x_{n-1} + \frac{h}{2}(k_1+k_2) = A + O(h^3)$$

$$k_1 = f(t_{n-1}, x(t_{n-1}))$$

$$k_2 = f(t_{n-1}+h, x(t_{n-1}) + h f(t_{n-1}, x(t_{n-1})))$$

$$\dot{x}(t) = f(t, x(t))$$

$$\ddot{x}(t) = f_t(t, x(t)) + f_x(t, x(t))\,\dot{x}(t) = f_t(t, x(t)) + f_x(t, x(t)) \times f(t, x(t))$$

$$f(t_{n-1}+h, x(t_{n-1})+y) = f(t_{n-1}, x(t_{n-1})) +$$

$$+ f_t(t_{n-1}, x(t_{n-1}))h + f_x(t_{n-1}, x(t_{n-1}))\,y + O(h^2+y^2)$$

Si $y = h f(t_{n-1}, x(t_{n-1}))$

$$f(t_{n-1}+h, x(t_{n-1}) + h f(\;)) = f(t_{n-1}, x(t_{n-1})) +$$

$$f_t(\;)h + f_x(\;)h f(t\;) + O(h^2+h^2)$$

$$h\frac{k_1+k_2}{2} = hf(t_{n-1}, x(t_{n-1})) + \frac{h^2}{2}\left[f_t(\ ) + f_x(\ )f(\ )\right] + O(h^3)$$

$$x(t_{n-1}) + h\frac{k_1+k_2}{2} = x(t_{n-1}) + h\dot{x}(t_{n-1}) + \frac{h^2}{2}\ddot{x}(t_{n-1}) + O(h^3)$$

Jacobi

$$x_i^{(k)} = \frac{1}{a_{ii}}\left(b_i - \sum_{\substack{j=1\\ j\neq i}}^{n} a_{ij}\, x_j^{(k-1)}\right)$$

G-S

$$x_i^{(k)} = \frac{1}{a_{ii}}\left(b_i - \sum_{j=1}^{i-1} a_{ij}\, x_j^{(k)} - \sum_{j=i+1}^{n} a_{ij}\, x_j^{(k-1)}\right)$$

Método intermedio $t\in[0,1]$

$$x_i^{(k)} = \frac{1}{a_{ii}}\left(b_i - \sum a_{ij}\left((1-t)x_j^{(k-1)} + t x_j^{(k)}\right) - \sum_{j=i+1}^{n} a_{ij}\, x_j^{(k-1)}\right)$$

$$y = Bx = (D+tL)^{-1}((1-t)L+U)x$$

$$(D+tL)y = ((1-t)L+U)x$$

$$|a_{ii}||y_i| - t\sum_{j=1}^{i-1}|a_{ij}||y_j| \leq (1-t)\sum_{j=1}^{i-1}|a_{ij}|\overbrace{|x_j|}^{\leq\|x\|_\infty} + \sum_{j=i+1}^{n}|a_{ij}|\overbrace{|x_j|}^{\|x\|_\infty}$$

$|y_i| = \|y\|_\infty$

$|y_j| \leq \|y\|_\infty$

$$\leq \left[(1-t)\sum_{j=1}^{n}|a_{ij}| + \sum_{j=i+1}^{n}|a_{ij}|\right]\|x\|_\infty$$

$$|a_{ii}|\|y\|_\infty - t\sum|a_{ij}|\|y\|_\infty$$

$$\left(|a_{ii}| - t\sum_{j=1}^{i-1}|a_{ij}|\right)\|y\|_\infty \leq [\qquad]\,\|x\|_\infty$$

$$\|y\|_\infty \leq \frac{[\qquad]}{[\qquad]}\|x\|_\infty \leq \underbrace{\beta}_{<1}\|x\|_\infty$$

$A \in \mathbb{R}^{n\times n}$

$$P_\alpha = \begin{pmatrix} 1 & & & 0 \\ & \alpha & & \\ & & \ddots & \\ 0 & & & \alpha^{n-1} \end{pmatrix} \qquad P_\alpha^{-1} = \begin{pmatrix} 1 & & & 0 \\ & \alpha^{-1} & & \\ & & \ddots & \\ 0 & & & \alpha^{-n+1} \end{pmatrix}$$

$$P_\alpha^{-1} \underbrace{A P_\alpha} = P_\alpha^{-1} \begin{pmatrix} a_{11} & \alpha a_{12} & & \alpha^{n-1} a_{12} \\ a_{21} & \alpha a_{22} & & \\ & & & \\ a_{1n} & \alpha a_{1n} & & \alpha^{n-1} a_{1n} \end{pmatrix}$$

$$= \begin{pmatrix} a_{11} & \alpha a_{12} & & \alpha^{n-1} a_{12} \\ \alpha^{-1} a_{21} & a_{22} & & \alpha^{n-2} a_{2n} \\ & & & \\ \alpha^{-n+1} a_{n1} & \alpha^{-n+2} a_{n2} & & a_{nn} \end{pmatrix}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & & 0 \\ a_{21} & a_{22} & \ddots & \\ & \ddots & \ddots & \\ 0 & & & \end{pmatrix} \qquad P_\alpha^{-1} A P_\alpha = \begin{pmatrix} a_{11} & \alpha a_{12} & & 0 \\ \alpha a_{21} & \ddots & \ddots & \\ & \ddots & & \alpha a_{n-1,n} \\ 0 & & \alpha^{-1} a_{n,n-1} & a_{nn} \end{pmatrix}$$

αU

$\alpha^{-1} L$

$$\det\left(-D^{-1}(L+U) - \mu I\right) = \det\left(-D^{-1}\left[L+U+\mu D\right]\right) =$$

$$= \underbrace{\det(-D^{-1})}_{\neq 0} \det(L+U+\mu D) = 0$$

$$\Updownarrow$$

$$\det(L+U+\mu D) = 0$$

En la dem A simetrica def + $\Rightarrow$ G-S converge

$x^* = \overline{x^T}$ $\qquad x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{C}^n$

$x^* = (\bar{x}_1\ \bar{x}_2\ \cdots\ \bar{x}_n)$ $\qquad |x\rangle$

$\langle x|$