Hamiltonian Monte Carlo

Juan Quintero, Santiago Eliges

$$\mathbb{E}_{\pi}[f] = \int_{Q} \mathrm{d}q \, \pi(q) \, f(q)$$

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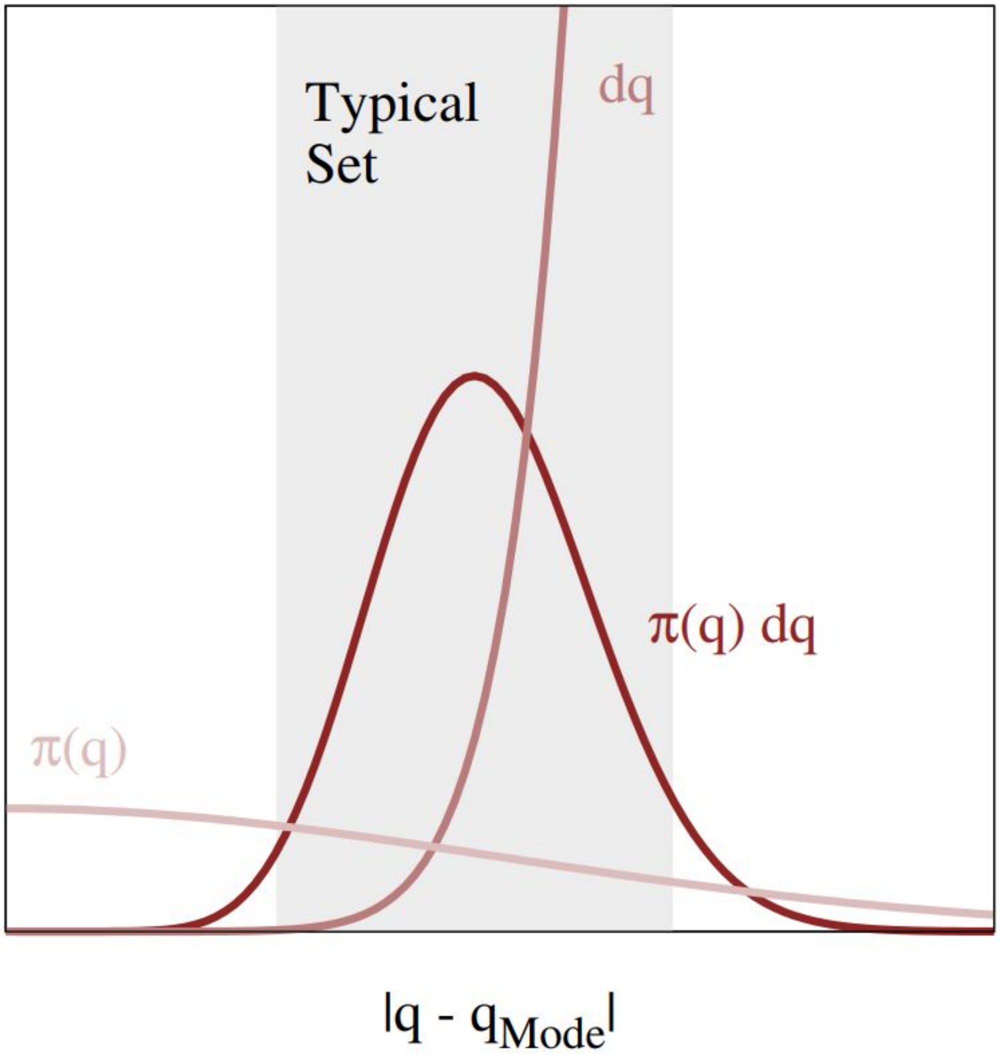
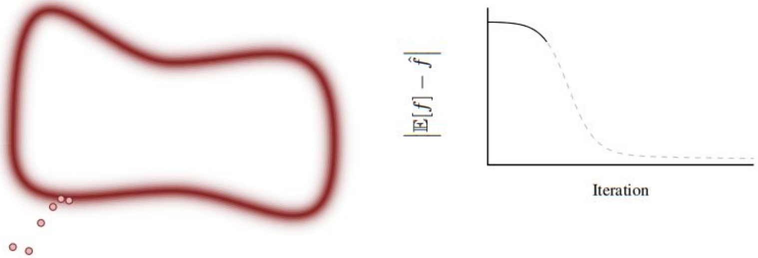
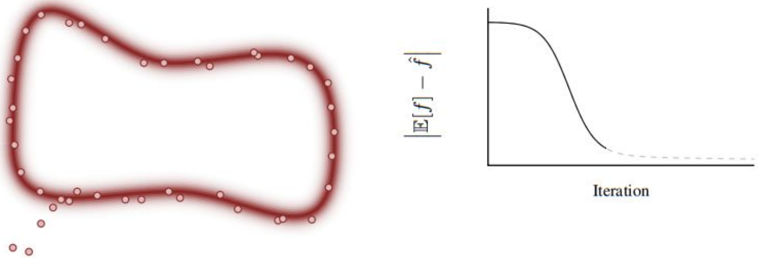
Typical Set





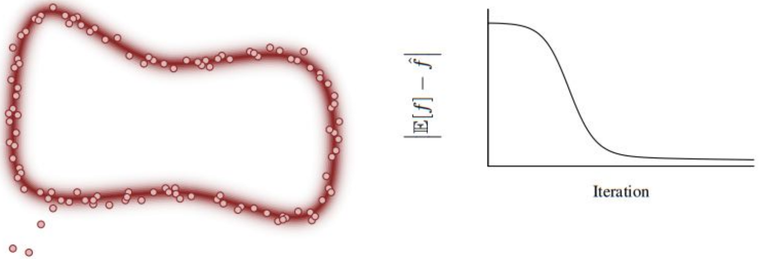
$|\mathbb{E}[f] - \hat{f}|$
Iteration

(a)



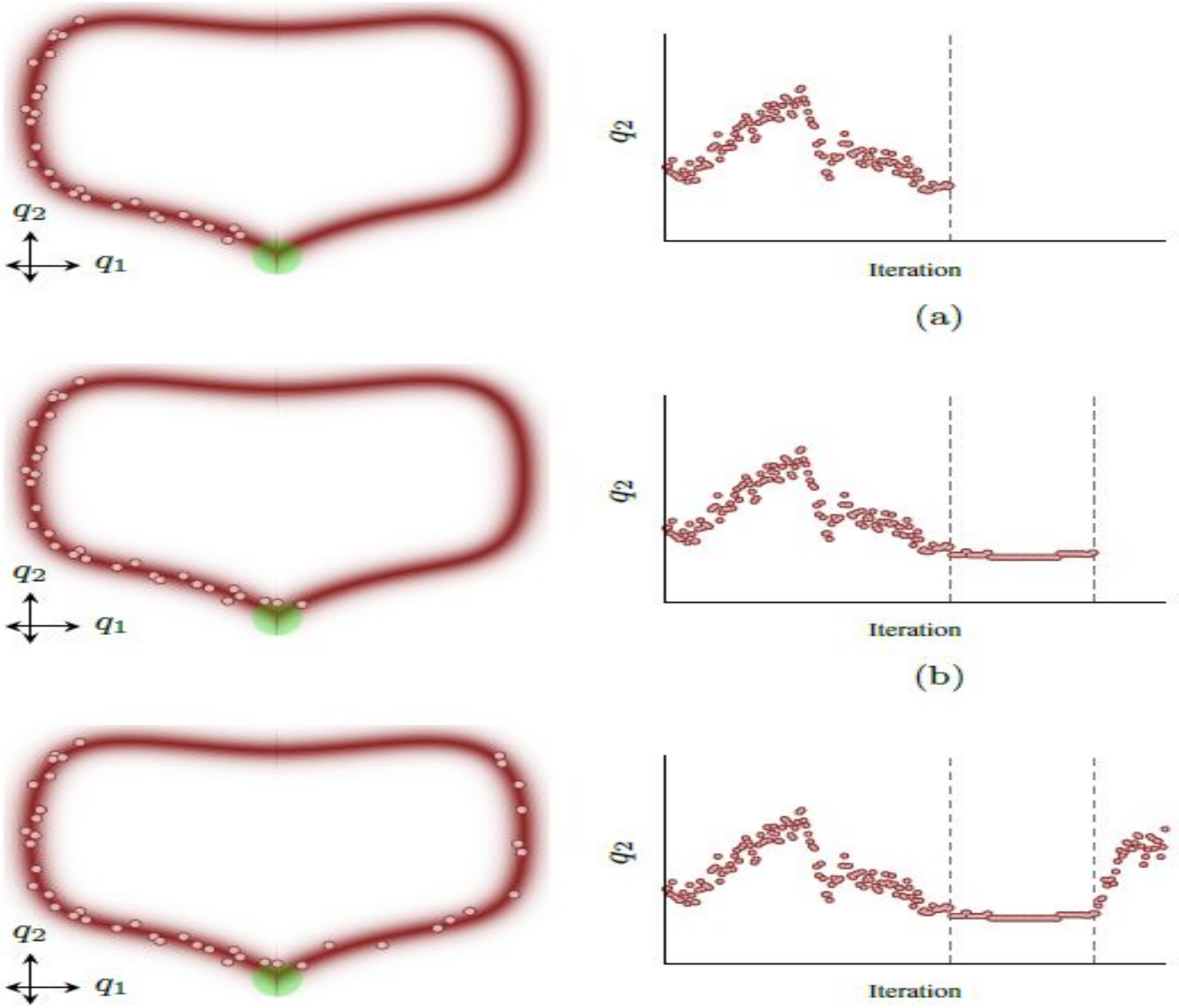
$|\mathbb{E}[f] - \hat{f}|$
Iteration

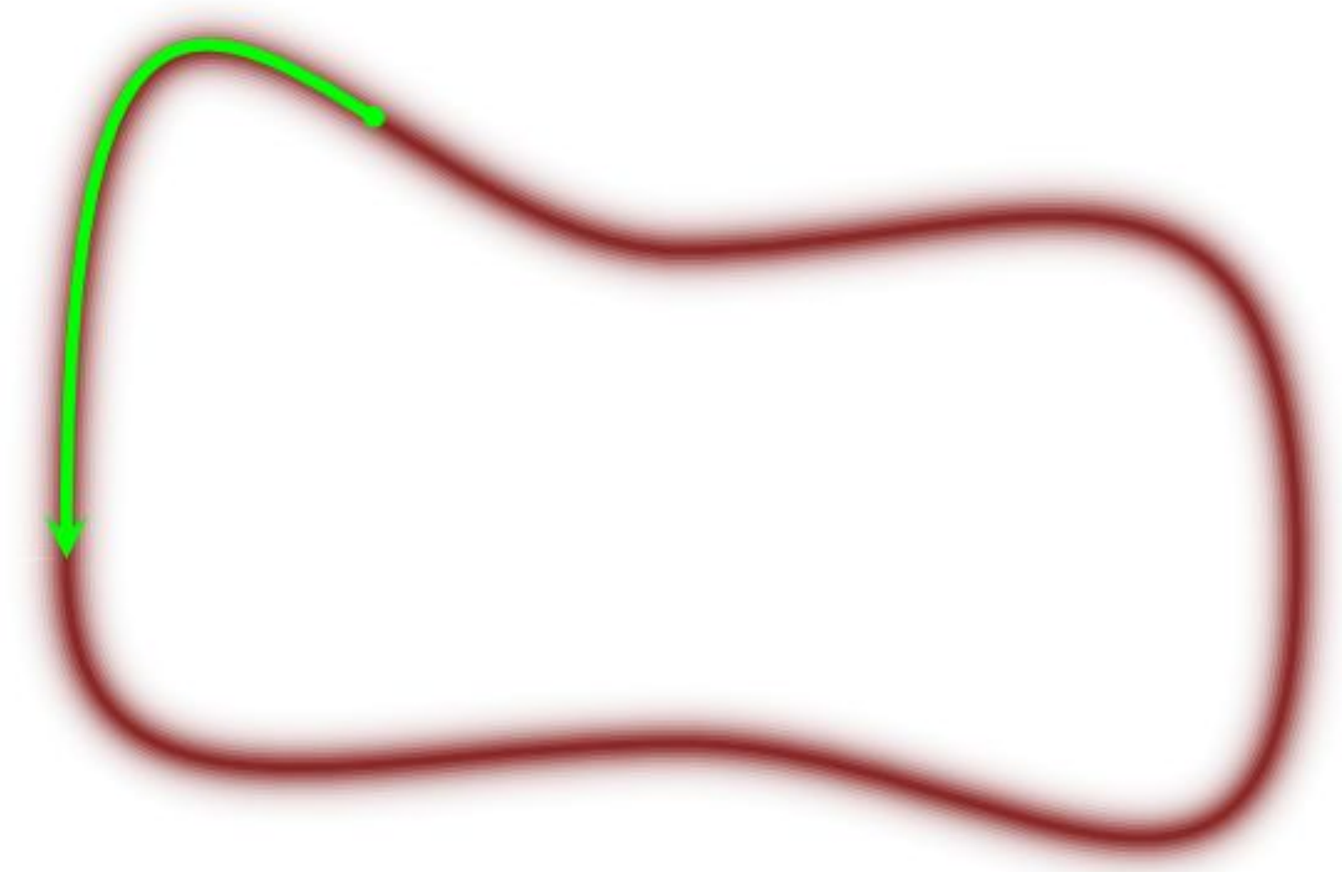
(b)

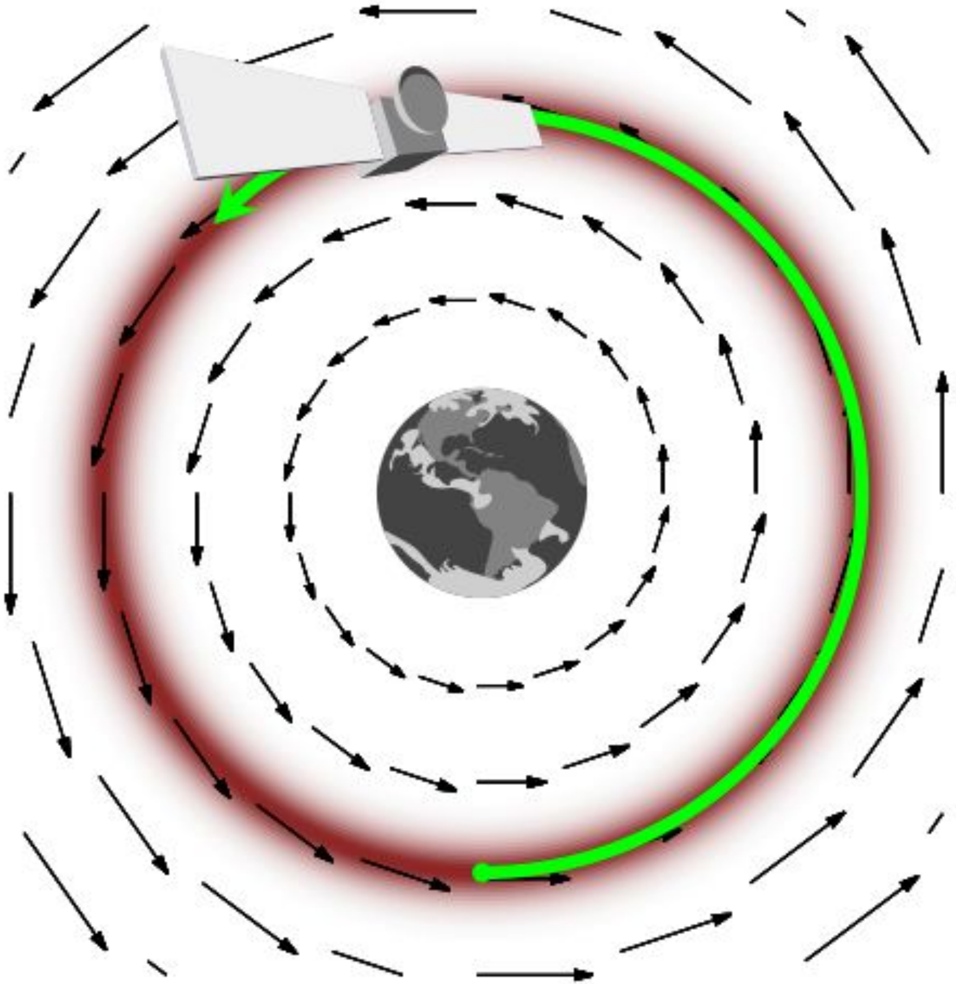


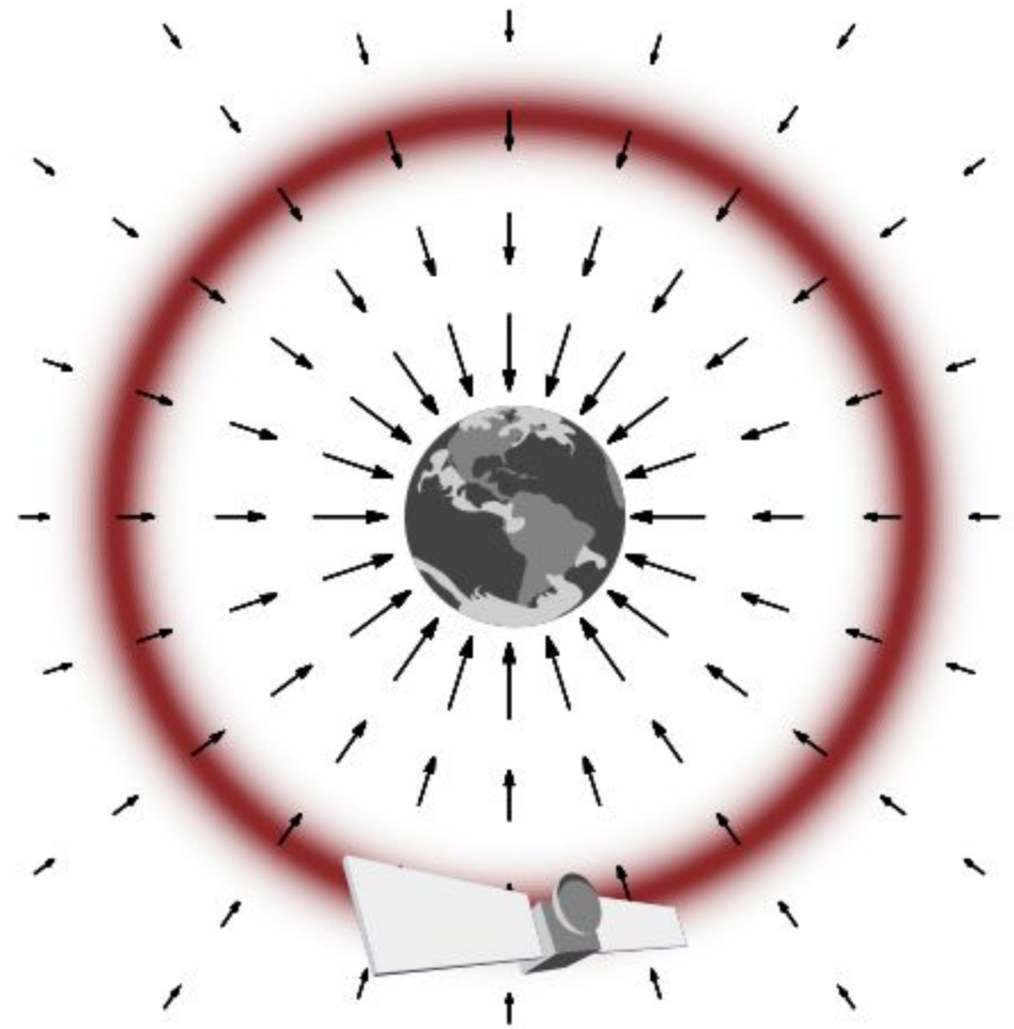
$|\mathbb{E}[f] - \hat{f}|$
Iteration

A veces falla…









$$q_n \to (q_n, p_n)$$

$$\pi(q,p) = \pi(p \mid q)\,\pi(q)$$

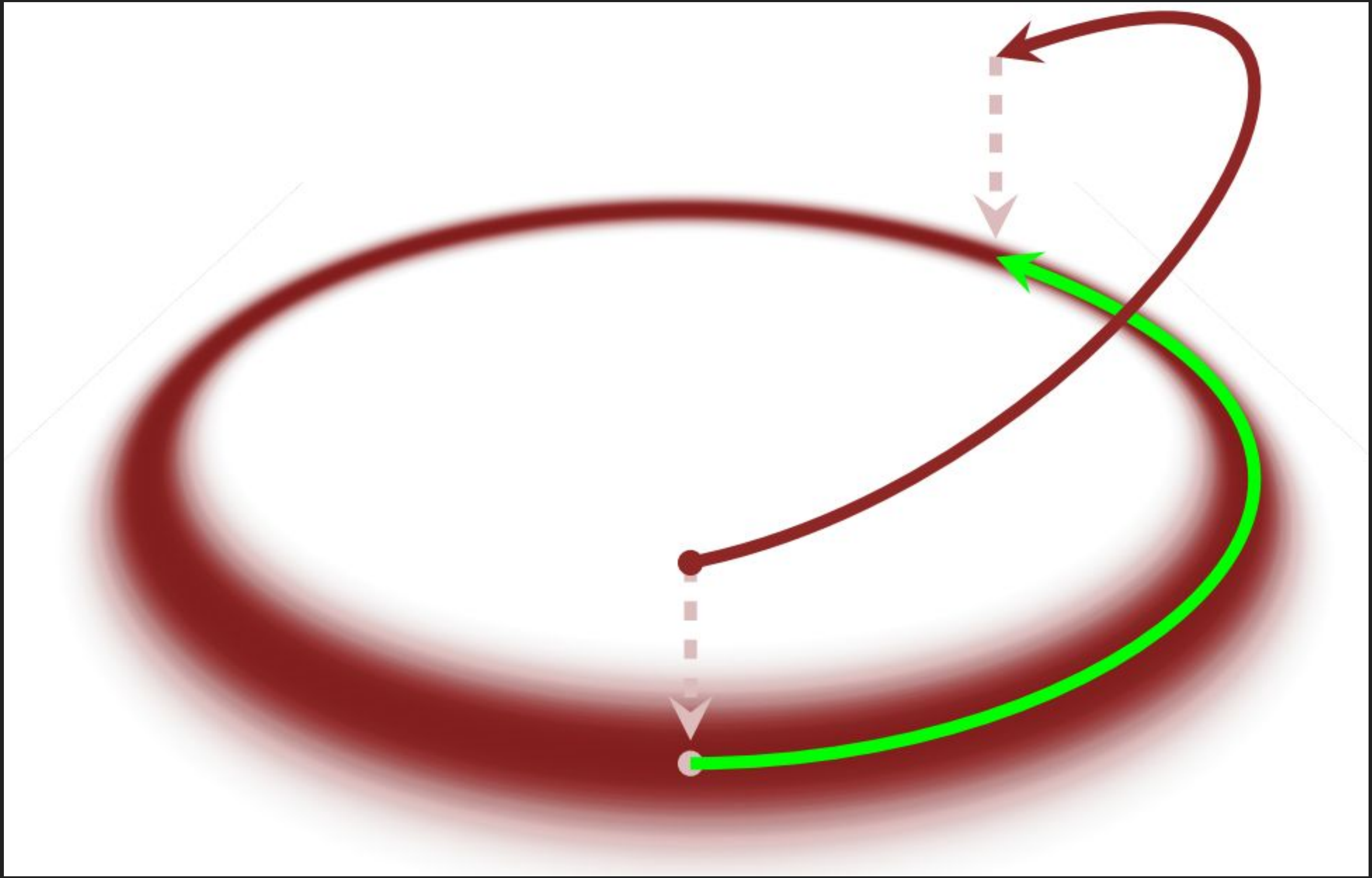
Marginalizado volvemos a nuestro mundo paramétrico

$$H(q,p) \equiv -\log \pi(q,p) \quad \pi(q,p) = e^{-H(q,p)}$$

$$\begin{aligned} H(q,p) &= -\log \pi(p \mid q) - \log \pi(q) \\ &\equiv \quad K(p,q) \quad + \; V(q) \, . \end{aligned}$$

$$\frac{\mathrm{d}q}{\mathrm{d}t} = +\frac{\partial H}{\partial p} = \frac{\partial K}{\partial p}$$

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -\frac{\partial H}{\partial q} = -\frac{\partial K}{\partial q} - \frac{\partial V}{\partial q}$$

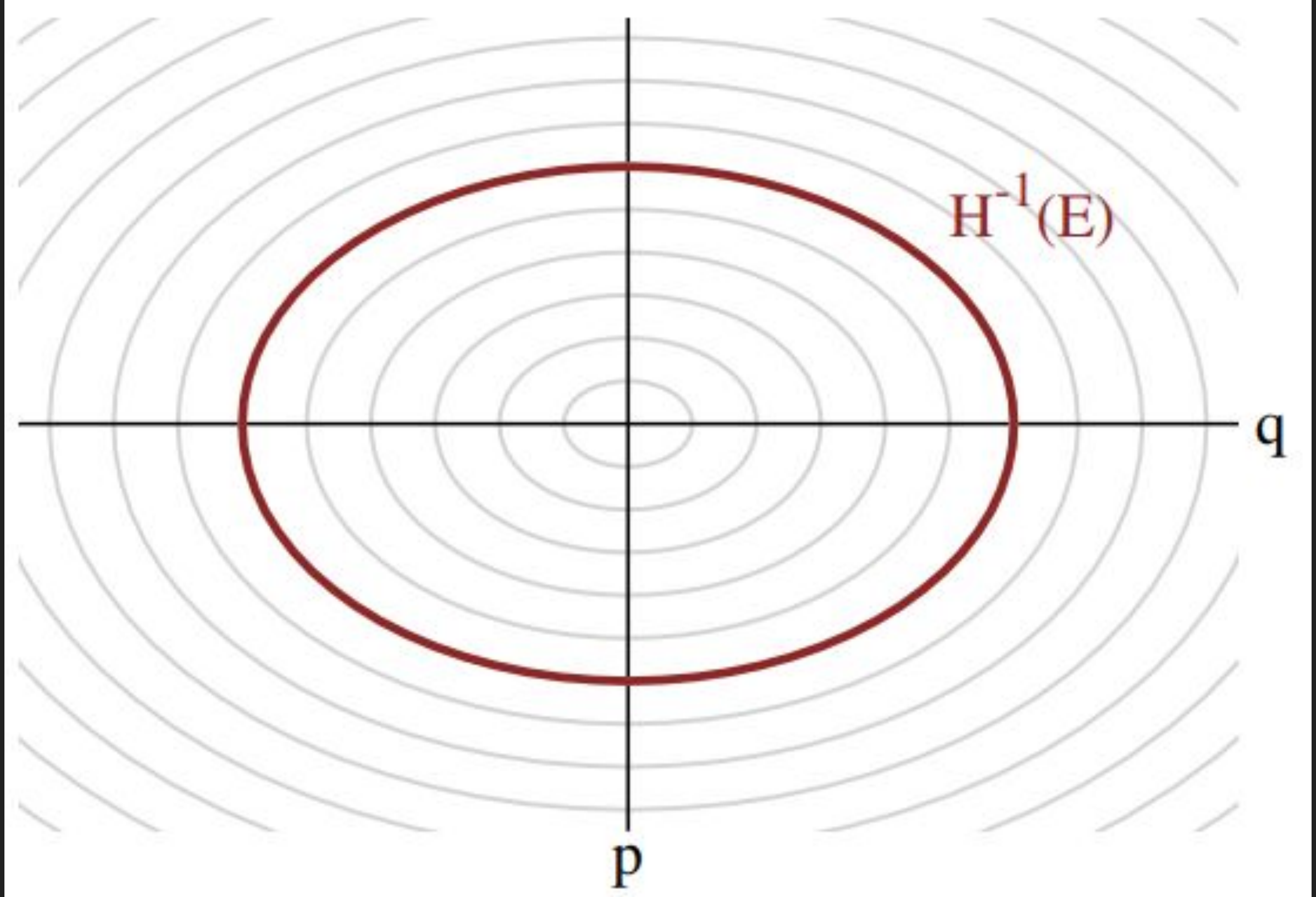


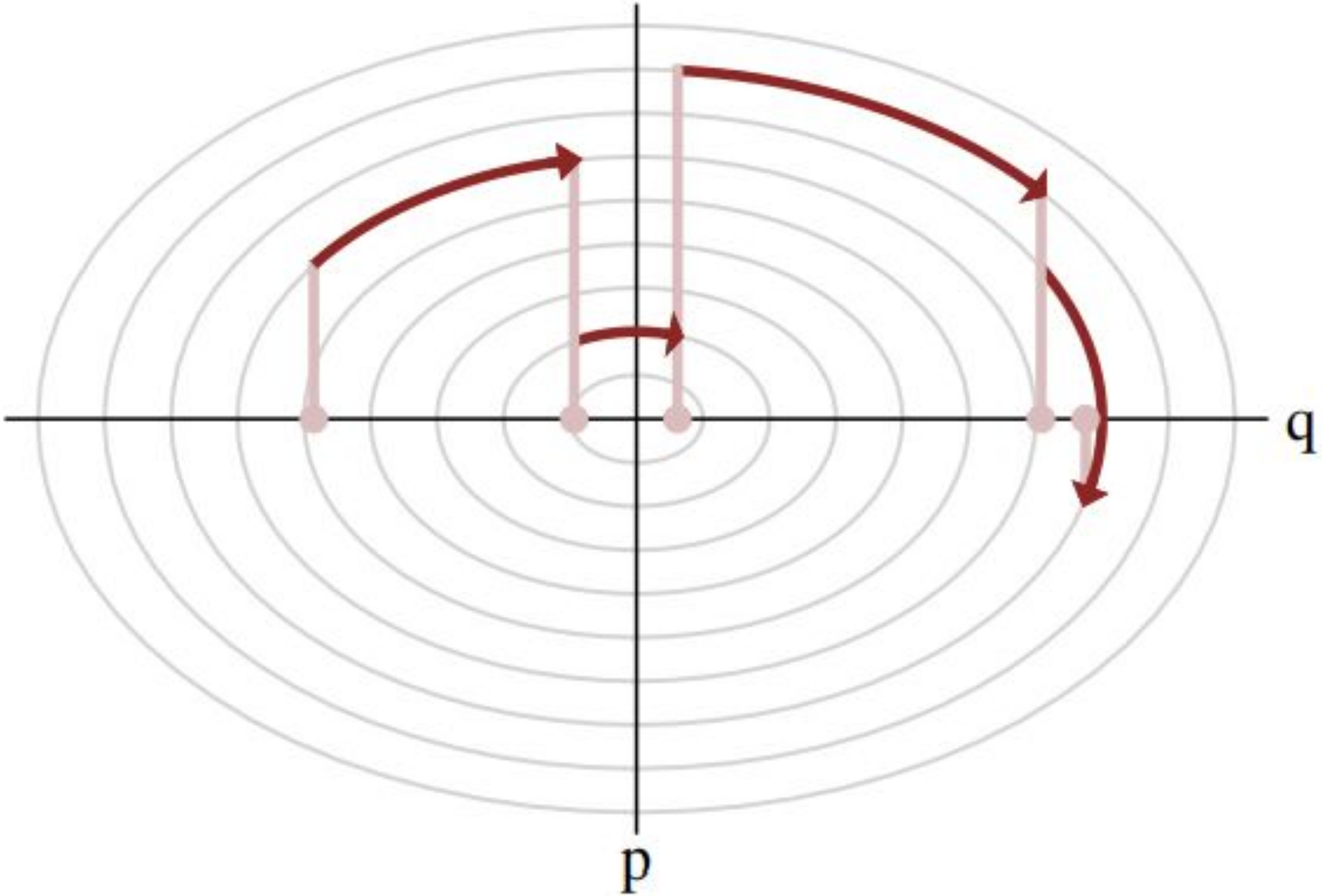
$$p \sim \pi(p \mid q)$$

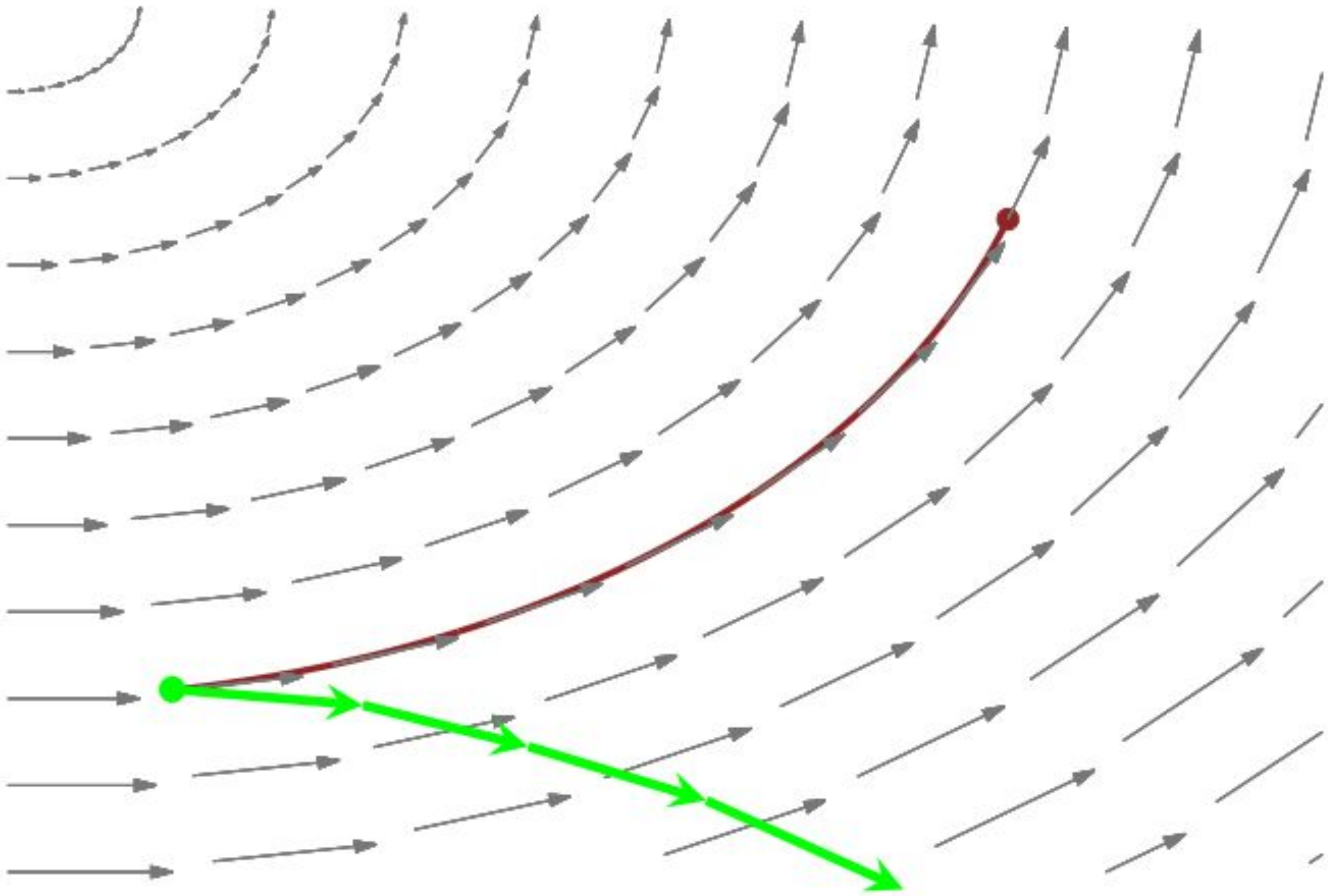
$$(q, p) \to \phi_t(q, p)$$

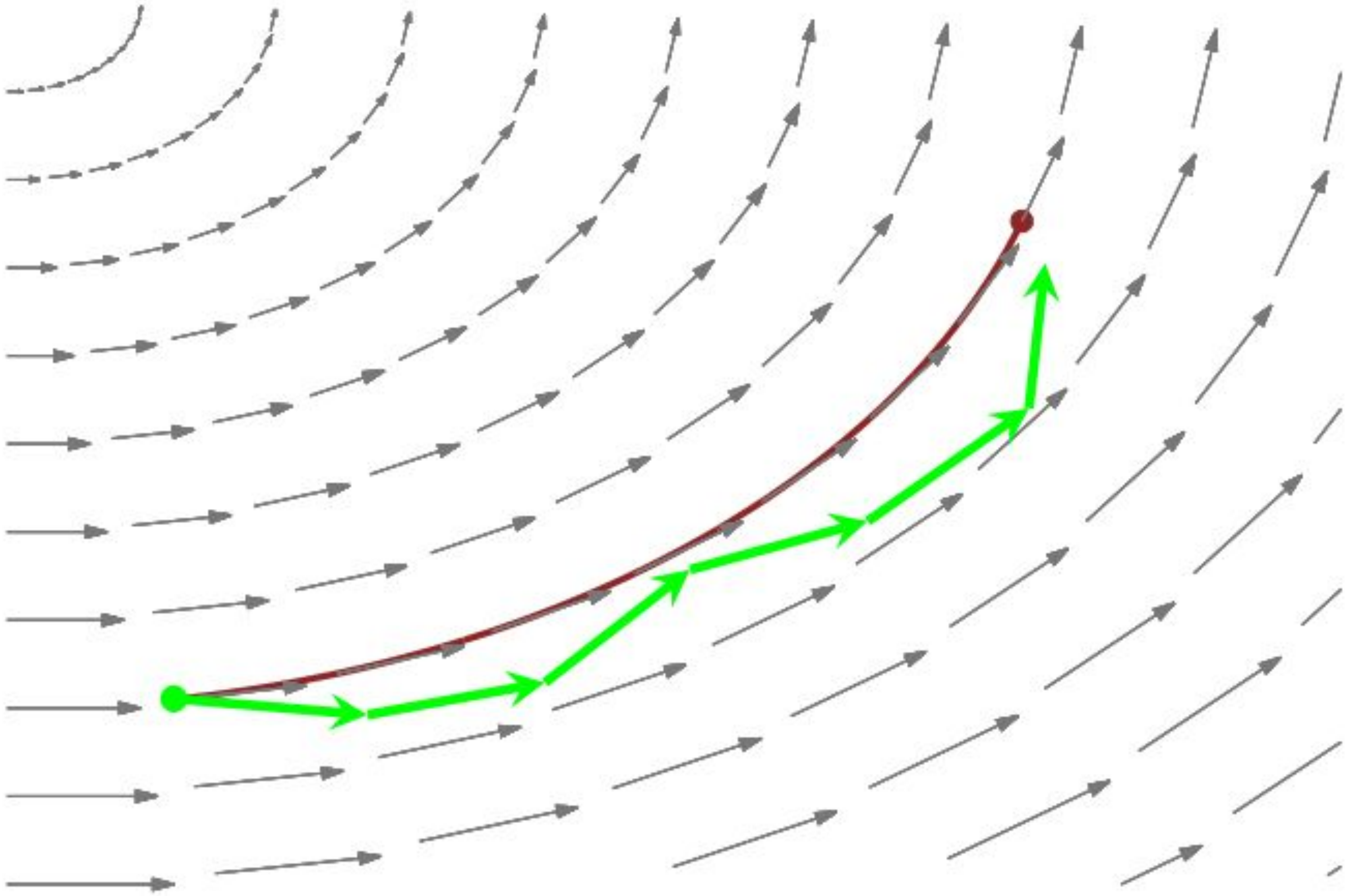
$$(q, p) \to q$$

$$H^{-1}(E) = \{q, p \mid H(q,p) = E\}$$




q
p





Leapfrog integrator

$q_0 \leftarrow q, p_0 \leftarrow p$

for $0 \leq n < \lfloor T/\epsilon \rfloor$ **do**

$$p_{n+\frac{1}{2}} \leftarrow p_n - \frac{\epsilon}{2} \frac{\partial V}{\partial q}(q_n)$$

$$q_{n+1} \leftarrow q_n + \epsilon \, p_{n+\frac{1}{2}}$$

$$p_{n+1} \leftarrow p_{n+\frac{1}{2}} - \frac{\epsilon}{2} \frac{\partial V}{\partial q}(q_{n+1})$$

end for.

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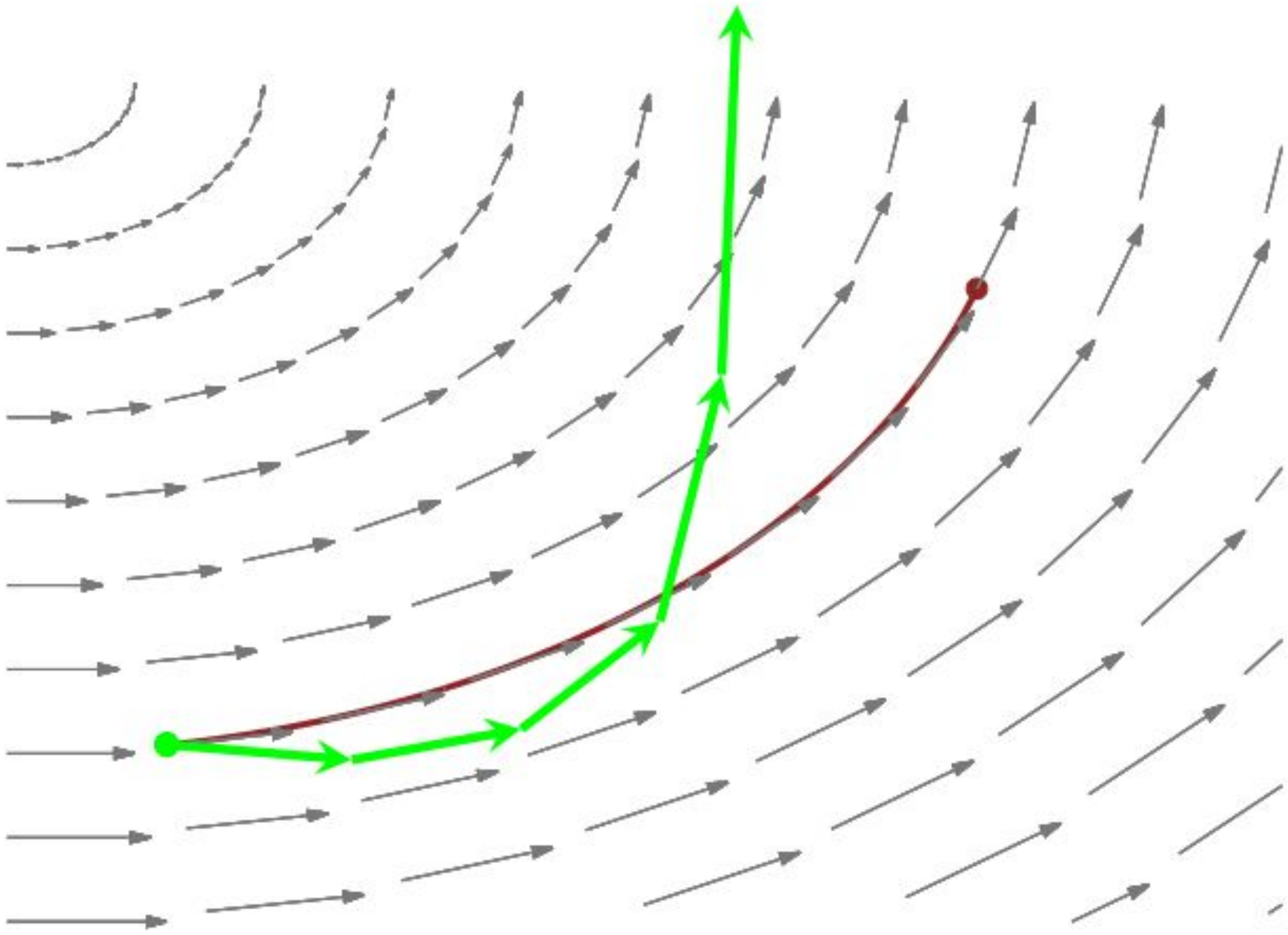
end for.



GRACIAS