

# Lisy odsylaczowe

## Jednokierunkowa lista odsylaczowa

### Implementacja struktury #1

#### (Implementacja obiektowa)

```
In [1]: class Node:
    def __init__(self, val=None):
        self.val = val
        self.next = None

class LinkedList:
    def __init__(self, values: 'Iterable' = None):
        self.head = self.tail = None
        self.length = 0
        values and self.extend(values) # The same as 'if values: self.extend(values)'

    def iter(self):
        curr = self.head
        while curr.val:
            yield curr.val
            curr = curr.next

    def str(self):
        return ' -> '.join(map(str, self))

    def len(self):
        return self.length

    def append(self, val: object):
        node = Node(val)
        if self.head == self.tail == node:
            self.head = self.tail = node
        else:
            self.tail.next = node
            self.tail = node
            self.length += 1

    def extend(self, values: 'Iterable'):
        if values:
            iterator = iter(values)
            if not self:
                self.head = self.tail = Node(next(iterator))
            for val in iterator:
                self.tail.next = Node(val)
                self.tail = self.tail.next
                self.length += len(values)

    def appendleft(self, val: object):
        node = Node(val)
        if not self:
            self.head = self.tail = node
        else:
            node.next = self.head
            self.head = node
            self.length += 1

    def extendleft(self, values: 'Iterable'):
        # Note that extendleft adds values in a reversed order so the first value of a linked
        # list will be the last value of the 'values' iterable (the same rule applies to the Python's
        # deque data structure, which is, in fact, a doubly linked list)
        if values:
            iterator = iter(values)
            if not self:
                self.head = self.tail = Node(next(iterator))
            for val in iterator:
                node = Node(val)
                node.next = self.head
                self.head = node
                self.length += len(values)

    def popleft(self) -> object:
        if not self:
            raise IndexError('pop from an empty (self._class.__name__)')
        removed = self.head.val
        if len(self) == 1:
            self.head = self.tail = None
        else:
            self.head = self.head.next
            self.length -= 1
        return removed

    def clear(self):
        self.length = 0
        self.head = self.tail = None

    def copy(self) -> 'LinkedList':
        return self._class_(self)

    def count(self, val: object) -> int:
        total = 0
        for curr_val in self:
            if curr_val == val:
                total += 1
        return total

    def index(self, val: object) -> int:
        for curr_val in enumerate(self):
            if curr_val == val:
                return i
        raise ValueError(f'{val} not in {self._class.__name__}')

    def insert(self, idx: int, val: object):
        # Depending on the idx variable value, insert a new node in a correct position
        if idx <= 0:
            self.appendleft(val)
        elif idx == len(self):
            self.append(val)
        else:
            # Look for the previous node after which a new node will be inserted
            prev_node = self._traverse_to_index(idx)
            self._insert_after(prev_node, Node(val))

    def remove(self, val: object):
        if self:
            if self.head.val == val:
                self.popleft()
            else:
                prev_node = self.head
                while prev_node.next:
                    if prev_node.next.val == val:
                        self._remove_node_after(prev_node)
                        return
                    prev_node = prev_node.next
            raise ValueError(f'{self._class.__name__}.remove(val): val not in {self._class.__name__}')

    def reverse(self):
        # Modify a Linked List only if its length is greater than 1 as empty or single-element
        # Linked List will remain unchanged after having been reversed
        if len(self) > 1:
            curr_node = self.head
            next_node = curr_node.next
            while next_node:
                temp = next_node.next
                next_node.next = curr_node
                curr_node = next_node
                next_node = temp
            self.head = curr_node

    def traverse_to_index(self, idx: int) -> 'previous node object':
        if idx <= 0: # We cannot return a node previous to the first one
            return None
        curr_node = self.head
        curr_idx = 1
        while curr_idx < idx:
            curr_node = curr_node.next
            curr_idx += 1
        return curr_node

    def insert_node_after(self, prev_node: Node, curr_node: Node):
        curr_node.next = prev_node.next
        prev_node.next = curr_node
        self.length += 1

    def remove_node_after(self, prev_node: Node):
        if prev_node.next is self.tail:
            self.tail = prev_node
        prev_node.next = prev_node.next.next
        self.length -= 1
```

Kilka testów

```
In [2]: a = LinkedList(range(5))
print(a)
a.append(5)
print(a)
a.appendleft(-1)
print(a)
a.clear()
print(a)
b = a.copy()
b.extend(range(0, -15, -4))
print(b, a, sep='\t')
a.insert(-1, 5)
insert_in_linked_list(a, 12345, 5)
a.insert(12345, 5)
print(a)
a.insert(4, 5)
print(a)
print(a.count(5))
print(a.index(10), a.index(5), a.index(8), sep='\t')
print(*(a.popleft() for _ in range(4)))
print(a)
a.remove(5)
print(a)
a.reverse()
print(a)
reverse_linked_list(a)
print(b)
print(a.popleft()) # this is to check if an exception is properly raised

0 -> 1 -> 2 -> 3 -> 4
0 -> 1 -> 2 -> 3 -> 4 -> 5
0 -> -1 -> 0 -> 1 -> 2 -> 3 -> 4 -> 5

5 -> 8 -> 11 -> 14
-12 -> -8 -> -4 -> 0 -> 5 -> 8 -> 11 -> 14      5 -> 8 -> 11 -> 14
5 -> 5 -> 8 -> 11 -> 14
5 -> 5 -> 8 -> 11 -> 14 -> 5
5 -> 5 -> 8 -> 11 -> 5 -> 14 -> 5
4
5
5 5 8 11
5 -> 14 -> 5
14 -> 4
14
-12 -> -8 -> -4 -> 0 -> 5 -> 8 -> 11 -> 14
14 -> 11 -> 8 -> 5 -> 0 -> -4 -> -8 -> -12
14
```

## Implementacja struktury #2

### (Implementacja funkcyjna)

```
In [3]: class Node:
    def __init__(self, val=None):
        self.val = val
        self.next = None

def create_linked_list(values: 'Iterable' = None) -> 'linked list head (sentinel)':
    if not values: return head
    for val in values:
        a.insert(-1, val)
    return head

def print_linked_list(ll_head: 'linked list head (sentinel)':
    print(ll_head.val, end=" ")
    while curr:
        print(curr.val, end=" ")
        curr = curr.next
    print()

# This operation below is very inefficient (O(n)) compared to the same operation
# implemented using a LinkedList class above (which is O(1) in such a case)
def append_to_linked_list(ll_head: 'linked list head (sentinel)', val: object):
    # Store a tail node
    tail = _get_last_node(ll_head)
    # Append a node to the last one node
    tail.next = Node(val)

# Also is inefficient but traverses to the last node only once as then appends
# values to the last node without traversing there again
def extend_linked_list(ll_head: 'linked list head (sentinel)', values: 'Iterable'):
    tail = _get_last_node(ll_head)
    for val in values:
        # Appended values one by one to the last node
        tail.next = Node(val)
        tail = tail.next

def appendleft_to_linked_list(ll_head: 'linked list head (sentinel)', val: object):
    node = Node(val)
    node.next = ll_head.next
    ll_head.next = node

def extendleft_linked_list(ll_head: 'linked list head (sentinel)', values: 'Iterable'):
    Create a new linked list of the values specified
    new_head = new_curr = Node() # Create a sentinel node
    for val in values:
        new_curr.next = Node(val)
        new_curr = new_curr.next
    # Link a new linked list to the one given as a function argument
    new_curr.next = ll_head.next
    ll_head.next = new_head.next

def popleft_from_linked_list(ll_head: 'linked list head (sentinel)' -> 'removed value':
    if not ll_head.next:
        raise IndexError('pop from an empty linked list')
    removed = ll_head.next.val
    ll_head.next = ll_head.next.next
    return removed

def clear_linked_list(ll_head: 'linked list head (sentinel)':
    ll_head.next = None

def copy_linked_list(ll_head: 'linked list head (sentinel)' -> 'linked list copy head (sentinel)':
    copy_head = copy_curr = Node() # A sentinel node
    curr = ll_head.next
    while curr:
        copy_curr.next = Node(curr.val)
        copy_curr = copy_curr.next
        curr = curr.next
    return copy_head

def count_value_in_linked_list(ll_head: 'linked list head (sentinel)', val: object) -> int:
    curr = ll_head.next
    total = 0
    while curr:
        if curr.val == val:
            total += 1
        curr = curr.next
    return total

def index_of_value_in_linked_list(ll_head: 'linked list head (sentinel)', val: object) -> int:
    curr = ll_head.next
    while curr:
        if curr.val == val:
            return curr
        curr = curr.next
        idx += 1
    raise ValueError(f'{val} not in linked list')

def insert_in_linked_list(ll_head: 'linked list head (sentinel)', idx: int, val: object):
    if idx <= 0:
        appendleft_to_linked_list(ll_head, val)
    else:
        prev_node = _traverse_to_index(ll_head, idx)
        _insert_node_after(prev_node, Node(val))

def remove_from_linked_list(ll_head: 'linked list head (sentinel)', val: object):
    if ll_head.next:
        prev_node = ll_head
        while prev_node.next:
            if prev_node.next.val == val:
                _remove_node_after(prev_node)
                return
            prev_node = prev_node.next
        raise ValueError('linkedlist.remove(val): val not in linkedlist')

def reverse_linked_list(ll_head: 'linked list head (sentinel)':
    # Reverse a linked list only if there are at least 2 values
    # (in other cases reversing is pointless)
    if ll_head.next and ll_head.next.next:
        curr_node = ll_head.next
        next_node = curr_node.next
        while next_node:
            temp = next_node.next
            next_node.next = curr_node
            curr_node = next_node
            next_node = temp
        tail_node = curr_node
        # Link a reversed linked list to a sentinel node
        ll_head.next.next = None
        ll_head.next = tail_node

def linked_list_to_list(ll_head: 'linked list head (sentinel)' -> list:
    values = []
    curr = ll_head.next
    while curr:
        values.append(curr.val)
        curr = curr.next
    return values

# If an index is too big (exceeds a number of elements in a linked list), the last node will be returned
def _traverse_to_index(ll_head: 'linked list head (sentinel)', idx: int) -> 'previous node object':
    curr = ll_head
    i = 0
    while curr.next:
        if i == idx:
            break
        curr = curr.next
        i += 1
    return curr

def _get_last_node(ll_head: 'linked list head (sentinel)' -> 'linked list tail':
    curr = ll_head
    while curr.next:
        curr = curr.next
    return curr

def _insert_node_after(prev_node: 'node after which a curr_node will be inserted', curr_node):
    curr_node.next = prev_node.next
    prev_node.next = curr_node

def _remove_node_after(prev_node: 'node after which a curr_node will be inserted'):
    prev_node.next = prev_node.next.next
```

Kilka testów

```
In [4]: a = create_linked_list(range(5))
print(linked_list(a))
append_to_linked_list(a, 5)
print(linked_list(a))
appendleft_to_linked_list(a, -1)
print(linked_list(a))
clear_linked_list(a)
print(linked_list(a))
extend_linked_list(a, range(5, 15, 3))
print(linked_list(a))
b = a.copy_linked_list(a)
extendleft_linked_list(b, range(0, -15, -4))
print(linked_list(b))
print(linked_list(a))
print(linked_list(b))
insert_in_linked_list(a, -1, 5)
print(linked_list(a))
insert_in_linked_list(a, 4, 5)
print(linked_list(a))
print(count_value_in_linked_list(a, 5))
print(index_of_value_in_linked_list(a, 14),
      index_of_value_in_linked_list(a, 5),
      index_of_value_in_linked_list(a, 8),
      sep='\t')
print(*(popleft_from_linked_list(a) for _ in range(4)))
remove_from_linked_list(a, 5)
print(linked_list(a))
reverse_linked_list(a)
print(linked_list(a))
reverse_linked_list(b)
print(linked_list(b))
print(popleft_from_linked_list(a))
# print(popleft_from_linked_list(a)) # this is to check if an exception is properly raised
print(linked_list_to_list(b))

None -> 0 -> 1 -> 2 -> 3 -> 4
None -> 0 -> 1 -> 2 -> 3 -> 4 -> 5
None -> -1 -> 0 -> 1 -> 2 -> 3 -> 4 -> 5
None
None -> 5 -> 8 -> 11 -> 14
None -> 0 -> -4 -> -8 -> -12 -> 5 -> 8 -> 11 -> 14
None -> 5 -> 8 -> 11 -> 14
None -> 5 -> 8 -> 11 -> 14
None -> 5 -> 8 -> 11 -> 14 -> 5
None -> 5 -> 5 -> 8 -> 11 -> 5 -> 14 -> 5
5
0
5 5 8 11
5 -> 14 -> 5
None -> 14 -> 5
None -> 14
None -> 14
None -> 0 -> -4 -> -8 -> -12 -> 5 -> 8 -> 11 -> 14
None -> 14 -> 11 -> 8 -> 5 -> -12 -> -8 -> -4 -> 0
14
[14, 11, 8, 5, -12, -8, -4, 0]
```

## Dwukierunkowa lista odsylaczowa

Poniższa implementacja jest bardzo podobna do implementacji listy jednokierunkowej (wersja obiektowa). Wynika to z faktu, że lista dwukierunkowa to taka lista jednokierunkowa na steroidsach, w której poza połączeniem węzłów z następnymi węzłami, występują również wskaźniki do poprzednich węzłów. Pozwala to na przeszukiwanie struktury "od tyłu", jeżeli nie, wsumy, że dana wartość znajduje się bliżej końca lub musimy "się cofnąć" do poprzednich węzłów. Jedyną większą różnicą jest to, że używamy ostatniej wartości z listy dwukierunkowej (oraz także niekiedy ustawianiu z użyciem metody .insert()) jest dużo szybsze (O(1)) niż w zwykłej jednokierunkowej liście odsylaczowej (O(n)), ponieważ po ustaniu ostatniego węzła, możemy przypisać bez problemów poprzedstani węzeł do atrybutu tail bez konieczności iteracji przez całą listę od początku (dla tego też w implementacji listy jednokierunkowej zapożyczyliśmy od implementowania metody .pop(), bo jest ona niewydajna). Również metoda .reverse() jest dużo łatwiejsza do zaimplementowania, ponieważ wystarczy zamienić wskaźniki, a także można w wydajny sposób zaimplementować metodę .rotate(), która powoduje "przesunięcie" wszystkich wartości listy odsylaczowej o 1 miejsce w prawo (dokł. przerzucenie wartości końcowej na początek).

### Implementacja struktury #1

#### (Implementacja obiektowa)

```
In [5]: class Node:
    def __init__(self, val=None):
        self.val = val
        self.next = None
        self.prev = None

class DoublyLinkedList:
    def __init__(self, values: 'Iterable' = None):
        self.head = self.tail = None
        self.length = 0
        values and self.extend(values) # The same as 'if values: self.extend(values)'

    def iter(self):
        curr = self.head
        while curr.val:
            yield curr.val
            curr = curr.next

    def str(self):
        return ' <-> '.join(map(str, self))

    def len(self):
        return self.length

    def append(self, val: object):
        node = Node(val)
        if self.head == self.tail == node:
            self.head = self.tail = node
        else:
            node.next = self.tail
            self.tail = node
            self.length += 1

    def extend(self, values: 'Iterable'):
        if values:
            iterator = iter(values)
            if not self:
                node = Node(val)
                self.head = self.tail = node
            for val in values:
                node.next = self.tail
                node.prev = self.tail
                self.tail = node
                self.length += len(values)

    def appendleft(self, val: object):
        node = Node(val)
        if not self:
            self.head = self.tail = node
        else:
            node.next = self.head
            self.head.prev = node
            self.head = node
            self.length += 1

    def extend(self, values: 'Iterable'):
        # Note that extendleft adds values in a reversed order so the first value of a linked
        # list will be the last value of the 'values' iterable (the same rule applies to the Python's
        # deque data structure, which is, in fact, a doubly linked list)
        if values:
            iterator = iter(values)
            if not self:
                self.head = self.tail = Node(next(iterator))
            for val in iterator:
                node = Node(val)
                node.next = self.head
                node.prev = self.head
                self.head = node
                self.length += len(values)

    def pop(self) -> object:
        if not self:
            raise IndexError('pop from an empty (self._class.__name__)')
        removed = self.tail.val
        if len(self) == 1:
            self.head = self.tail = None
        else:
            self.tail = self.tail.prev
            self.tail.next = None
            self.length -= 1
        return removed

    def popleft(self) -> object:
        if not self:
            raise IndexError('pop from an empty (self._class.__name__)')
        removed = self.head.val
        if len(self) == 1:
            self.head = self.tail = None
        else:
            self.head = self.head.next
            self.head.prev = None
            self.length -= 1
        return removed

    def clear(self):
        self.length = 0
        self.head = self.tail = None

    def copy(self) -> 'LinkedList':
        return self._class_(self)

    def count(self, val: object) -> int:
        total = 0
        for curr_val in self:
            if curr_val == val:
                total += 1
        return total

    def index(self, val: object) -> int:
        for i, curr_val in enumerate(self):
            if curr_val == val:
                return i
        raise ValueError(f'{val} not in {self._class.__name__}')

    def insert(self, idx: int, val: object):
        # Depending on the idx variable value, insert a new node in a correct position
        if idx <= 0:
            self.appendleft(val)
        elif idx == len(self):
            self.append(val)
        else:
            # If an index is too big (exceeds a number of elements in a linked list), the last node will be returned
            node = Node(val)
            if idx < center:
                prev_node = self._traverse_from_left(idx)
                self._insert_node_after(prev_node, node)
            else:
                # Look for the next node before which a new node will be inserted
                next_node = self._traverse_from_right(idx)
                self._insert_node_before(next_node, node)

    def remove(self, val: object):
        if self:
            if self.head.val == val:
                self.popleft()
            else:
                prev_node = self.head
                while prev_node.next:
                    if prev_node.next.val == val:
                        self._remove_node_after(prev_node)
                        return
                    prev_node = prev_node.next
            raise ValueError(f'{self._class.__name__}.remove(val): val not in {self._class.__name__}')

    def reverse(self):
        # Modify a Linked List only if its length is greater than 1 as empty or single-element
        # Linked List will remain unchanged after having been reversed
        if len(self) > 1:
            curr_node = self.head
            while curr_node:
                next_node = curr_node.next
                curr_node.next, curr_node.prev = curr_node.prev, curr_node.next
                curr_node = next_node
            # Swap a tail pointer with a head pointer
            self.head, self.tail = self.tail, self.head

    def rotate(self):
        if self:
            self.appendleft(self.pop())

    def traverse_from_left(self, idx: int) -> 'previous Node object':
        if idx <= 0: # We cannot return a node previous to the first one
            return None
        curr_node = self.head
        curr_idx = 1
        while curr_idx < idx:
            curr_node = curr_node.next
            curr_idx += 1
        return curr_node

    def traverse_from_right(self, idx: int) -> 'next Node object':
        if idx == len(self)-1: # We cannot return a node next to the last one
            return None
        curr_node = self.tail
        curr_idx = len(self)-1
        while curr_idx > idx:
            curr_node = curr_node.prev
            curr_idx -= 1
        return curr_node

    def insert_node_after(self, prev_node: Node, curr_node: Node):
        curr_node.next = prev_node.next
        curr_node.prev = prev_node.next
        prev_node.next = curr_node
        curr_node.next.prev = curr_node
        self.length += 1

    def insert_node_before(self, next_node: Node, curr_node: Node):
        curr_node.next = next_node
        curr_node.prev = next_node
        next_node.prev = curr_node
        curr_node.prev.next = curr_node
        self.length += 1

    def remove_node_after(self, prev_node: Node):
        if prev_node.next is self.tail:
            self.tail = prev_node
        prev_node.next = prev_node.next.next
        if prev_node.next:
            prev_node.next.next = prev_node.next.next
            self.length -= 1
```

Kilka testów

```
In [6]: a = DoublyLinkedList(range(5))
print(a)
a.append(5)
print(a)
a.appendleft(-1)
print(a)
print(a)
a.extend(range(5, 15, 3))
print(a)
b = a.copy()
b.extendleft(range(0, -15, -4))
print(b, a, sep='\t')
a.insert(-1, 5)
print(a)
a.insert(4, 5)
print(a)
print(a.count(5))
print(a.index(14), a.index(5), a.index(8), sep='\t')
print(*(a.popleft() for _ in range(4)))
print(a)
a.remove(5)
print(a)
a.reverse()
print(a)
print(b)
reverse_linked_list(a)
print(b)
print(a.popleft()) # this is to check if an exception is properly raised
print(b)
print(b)

0 <-> 0 <-> 1 <-> 2 <-> 3 <-> 4
0 <-> 0 <-> 1 <-> 2 <-> 3 <-> 4 <-> 5
-1 <-> 0 <-> 0 <-> 1 <-> 2 <-> 3 <-> 4 <-> 5

5 <-> 5 <-> 8 <-> 11 <-> 14
-12 <-> -8 <-> -4 <-> 0 <-> 5 <-> 8 <-> 11 <-> 14      5 <-> 5 <-> 8 <-> 11 <-> 14
5 <-> 5 <-> 8 <-> 11 <-> 14
5 <-> 5 <-> 8 <-> 11 <-> 14 -> 5
5 <-> 5 <-> 5 <-> 8 <-> 11 <-> 5 <-> 14 <-> 5
5
0
5 5 8 11
11 <-> 5 <-> 14 <-> 5
11 <-> 14 <-> 5
11 <-> 14
-12 <-> -8 <-> -4 <-> 0 <-> 5 <-> 8 <-> 11 <-> 14
14 <-> 11 <-> 8 <-> 5 <-> 5 <-> 5 <-> 0 <-> -4 <-> -8 <-> -12
11 <-> 14
-12 <-> 11 <-> 8 <-> 5 <-> 5 <-> 5 <-> 0 <-> -4 <-> -8 <-> -12
-12 <-> 14 <-> 11 <-> 8 <-> 5 <-> 5 <-> 5 <-> 0 <-> -4 <-> -8 <-> -12
```

## Implementacja struktury #2

### (Implementacja funkcyjna)

Warto mieć na uwadze, że w tym przypadku lista jest reprezentowana, przy pomocy dwóch wskaźników (jeden na początek i drugi na koniec)



```
None <-> 14 <-> 11 <-> 8 <-> 5 <-> 0 <-> -4 <-> -8 <-> -12 <-> None
None <-> -12 <-> -8 <-> -4 <-> 0 <-> 4 <-> 8 <-> 11 <-> 14 <-> None
[-12, 14, 11, 8, 5, 0, -4, -8]
```

# Kolejka

UWAGA:

W poniższych przykładach znajduje się tylko implementacja zwykłej kolejki. Kolejka priorytetowa została umieszczona w pliku ze strukturami drzewiastymi, ponieważ opiera się ona na kopkach binarnych, które reprezentują kompletne drzewa binarne.

## Implementacja struktury #1 (w oparciu o listę jednokierunkową)

### (Implementacja obiektowa)

```
In [9]: class Node:
def __init__(self, val=None):
    self.val = val
    self.next = None

class Queue:
def __init__(self, values: 'Iterable' = None):
    self.head = self.tail = None
    self.length = 0
    values and self.enqueue_many(values)

def __iter__(self):
    curr = self.head
    while curr:
        yield curr.val
        curr = curr.next

def _str(self):
    # An arrow indicates in which direction a queue moves
    return ' < '.join(map(str, self))

def __len__(self):
    return self.length

def is_empty(self):
    return not bool(self)

def peek(self):
    if self:
        return self.head.val

def enqueue(self, val: object):
    node = Node(val)
    if not self:
        self.head = self.tail = node
    else:
        self.tail.next = node
        self.tail = node
    self.length += 1

def dequeue(self) -> object:
    if not self:
        raise IndexError('dequeue from an empty {self.__class__.__name__}')
    removed = self.head.val
    if len(self) == 1:
        self.head = self.tail = None
    else:
        self.head = self.head.next
        self.length -= 1
    return removed

def enqueue_many(self, values: 'Iterable'):
    if values:
        iterator = iter(values)
        if not self:
            self.head = self.tail = Node(next(iterator))
        for val in iterator:
            self.tail.next = Node(val)
            self.tail = self.tail.next
        self.length += len(values)
```

Kilka testów

```
In [10]: q = Queue()
for n in range(5):
    q.enqueue(n)
print(q)

for i in range(3):
    print('Removed:', q.dequeue())
print(q)

for i in range(10, 20, 3):
    q.enqueue(i)
print(q)

print(q.dequeue())
print(q)
print(q.is_empty())
print(q.peek())
q.enqueue(20)
print(q.peek())
print(q)

while q:
    print(q.dequeue(), end='\t')
print()

print(q, q.is_empty(), q.peek())

0 <- 1 <- 2 <- 3 <- 4
Removed: 0
Removed: 1
Removed: 2
3 <- 4
3 <- 4 <- 10 <- 13 <- 16 <- 19
4
4 <- 10 <- 13 <- 16 <- 19
False
4
4 <- 10 <- 13 <- 16 <- 19 <- 20
4      10      13      16      19      20
True None
```

## Implementacja struktury #2 (w oparciu o listę dwukierunkową)

### (Implementacja funkcyjna)

W przypadku funkcyjnej implementacji kolejki konieczne jest skorzystanie z listy dwukierunkowej, w celu umożliwienia dodawania elementów na koniec kolejki oraz zdejmowania z początku w czasie O(1). Oczywiście można by za każdym razem po zaktualizowaniu kolejki zwracać w funkcji wskaźnik do nowego ostatniego lub nowego pierwszego elementu, lecz aktualizowanie zmiennych wskaźnikowych uważam za niewygodne i podatne na błędy. Z tego powodu preferuję powyższą implementację obiektową, lub, jeżeli jest konieczna funkcyjna, poniższą implementację.

```
In [11]: class Node:
def __init__(self, val=None):
    self.val = val
    self.next = None
    self.prev = None

def create_queue(values: 'Iterable' = None) -> ('queue head (sentinel)', 'queue tail (sentinel)':
    head = Node() # A head sentinel node
    tail = Node() # A tail sentinel node
    if values:
        head.next = curr = Node(values[0])
        curr.prev = head
        for i in range(1, len(values)):
            node = Node(values[i])
            curr.next = node
            node.prev = curr
            curr = curr.next
    else:
        curr = head
        curr.next = tail
        tail.prev = curr
    return head, tail

def print_queue(head: 'queue head (sentinel)':
    curr = head.next
    print(head.val, end= ' ')
    while curr:
        print('<<>>', curr.val, end= ' ')
        curr = curr.next
    print()

def enqueue(tail: 'queue tail (sentinel)', val: object):
    node = Node(val)
    node.prev = tail.prev
    node.prev.next = node
    node.next = tail
    tail.prev = node

def dequeue(head: 'queue head (sentinel)' -> 'removed value':
    if not head.next.next:
        raise IndexError('dequeue from an empty queue')
    removed = head.next.val
    head.next = head.next.next
```

```
head.next = head.next.next
head.next.prev = head
return removed
```

```
def enqueue_many(tail: 'queue tail (sentinel)', values: 'Iterable'):  
    for val in values:  
        enqueue(tail, val)  
  
def is_empty(head: 'queue head (sentinel)') -> bool:  
    return not head.next.next
```

Kilka testów

```
In [121]: q = create_queue()  
          q_head, q_tail = q  
          enqueue_many(q_head, [1, 2, 3, 4, 5])
```

```
print_queue(q_head)
enqueue_many(q_tail, range(0, 10, 2))
print_queue(q_head)
print(dequeue(q_head), dequeue(q_head), dequeue(q_head))
enqueue(q_tail, 12)
```

```
print(dequeue(q_head))
print_queue(q_head)
while not is_empty(q_head):
    dequeue(q_head)
print_queue(q_head)

None <> None
None <> 0 <> 2 <> 4 <> 6 <> 8 <> None
0 2 4
```

None <-> 8 <-> 12 <-> None  
None <-> None

## Implementacja struktury #3 (z użyciem stosów)

### (Implementacja obiektowa)

Przykładowe implementacje stosów znajdują się niżej. Ponieważ ta implementacja jest raczej przedstawiona tutaj jako ciekawostka a nie użyteczna wersja struktury, zdecydowałem się ją umieścić przed przedstawieniem wariantów implementacji stosu.

### Omówienie działania struktury

Szczegółowe wyjaśnienie działania struktury znajduje się w poniższym video:

<https://www.youtube.com/watch?v=Wg8iYtLbLI>

### Właściwa implementacja

```
[13]: class CrayQueue:
    def __init__(self, values: 'Iterable' = None):
        self.first_stack = [] # A stack for elements that are enqueued (first on a top)
        self.last_stack = [] # A stack for elements that are dequeued (first on a top)
        if values: self.first_stack = values[:]

    def len(self):
        # The length of a queue will always be a sum of stacks' lengths
        return len(self.first_stack) + len(self.last_stack)

    def iter(self):
        # In the printed representation we assume that elements on the right side are added
        # before elements on the left side. Arrow show how a queue moves, and therefore, they
        # are pointing from the last to the first element (they are directed from the last to
        # the first element in a queue)
        yield from self.first_stack[::-1]
        yield from self.last_stack

    def __str__(self):
        # An arrow indicates in which direction a queue moves
        return ' < '.join(map(str, self))

    def is_empty(self):
        return not bool(self)

    def peek(self):
        # Returns the first element of a queue
        if self:
            return self.first_stack[-1] if self.first_stack else self.last_stack[0]

    def enqueue(self, val: object): # O(1)
        # This stack is only receiving pushes so we don't have to do any
        # extra work here
        self.first_stack.append(val)

    def dequeue(self): # O(1)
        # This stack is only receiving pushes so we don't have to do any
        # extra work here
```

```

    # If there is no element in a last_stack, we have to move all values
    # to the first_stack
    if not self.first_stack:
        self._move_stack(self.last_stack, self.first_stack)

```

```

        # If there is still no first_stack, our queue must be empty
        # If not self.first_stack:
        #     raise IndexError('dequeue from an empty {self.__class__.__name__}')
        #     raise IndexError('dequeue from an empty {self.__class__.__name__}')
        # else:
        #     return self.first_stack.pop()
    def move_stack(self, initial_stack, target_stack):
        while initial_stack:
            target_stack.append(initial_stack.pop())

```

```
for _ in range(3):
    cq.enqueue(n)
print(cq)

for _ in range(3):
    print('Removed:', cq.dequeue())
print(cq)

for i in range(10, 20, 3):
    print(i)
```

```

    print(cq.dequeue())
    print(cq)

    print(cq.dequeue())
    print(cq)

    print(cq.is_empty())
    print(cq.peak())
    cq.enqueue(cq)
    print(cq.peak())

```

```
print(cq)

while cq:
    print(cq.dequeue(), end='t')
    print()

print(cq, cq.is_empty(), cq.peak())

0 <- 1 <- 2 <- 3 <- 4
Reserve: 0
```

```
Removed: 1  
Removed: 2  
3 <- 4  
3 <- 4 <- 10 <- 13 <- 16 <- 19  
3  
4 <- 10 <- 13 <- 16 <- 19  
False  
4  
4  
4 + 15 + 12 + 16 + 18 + 20
```

```

3         <- 10      13      16      19      20
4     True None

```

zadanie ze stron. W gruncie rzeczy, w poniższej implementacji tworzymy kolejkę, która ma stały rozmiar (ale możliwe jest zamplementowanie amortyzacji tablicy, a więc przepięszeniowa wartości do większej tablicy, gdy jest to potrzebne). Za początek kolejkę przyjmujemy element, który znajduje się pod indeksem zapisanym jako początkowy. Warto zauważyć, że może zdarzyć się (nawet często) sytuacja, gdy koniec kolejki będzie wcześniej w tablicy niż początek. Z tego powodu całą kolejkę nazywamy się często przebiegającą kolejką, ponieważ jej początek i koniec według pod tablicy (tak naprawdę nie przemieszczamy elementów, a jedynie ponuszamy indeksami).

```

In [15]: class CrawlQueue:

```

```
def __init__(self, size):
    self.arr = [None] * size
    self.max_length = size # A maximum length of a queue
    self.length = 0 # A current length of a queue
    self.first_idx = 0

def __len__(self):
    return self.length
```

```
def __repr__(self):
    return f'({self.__class__.__name__}({self.max_length})'

def __iter__(self):
    for i in range(self.first_idx, self.first_idx + self.length):
        yield self.arr[i % self.max_length]

def __str__(self):
    # An arrow indicates a direction in which a queue moves
```

```

        return ' <- '.join(map(str, self))

def is_empty(self):
    return not bool(self)

def peek(self):
    # Returns the first element of a queue
    if self:
        return self.arr[self.first_idx]

```

```
def enqueue(self, val):
    if self.length == self.max_length:
        raise OverflowError(f'{self.__class__.__name__} has no space left.')
    new_idx = (self.first_idx + self.length) % self.max_length
    self.ar[new_idx] = val
    self.length += 1

def dequeue(self):
```

```
# If a queue is empty, raise an Exception
if not self:
    raise IndexError('dequeue from an empty (self._class.__name__)')
removed = self.arr[self.first_idx]
self.first_idx = (self.first_idx + 1) % self.max_length
self.length -= 1
return removed
```

```
In [16]: cq = CrawlingQueue(10)
for n in range(5):
    cq.enqueue(n)
print(cq)

for _ in range(3):
    print('Removed:', cq.dequeue())
print(cq)
```

```
for i in range(10, 20, 3):
    cq.enqueue(i)
print(cq)

print('How an array looks like:')
print(cq.arr)

print(cq.dequeue())
```

```
print(cq)
print(cq.is_empty())
print(cq.peak())
cq.enqueue(20)
print(cq.peak())
print(cq)

while cq:
    print(cq.dequeue(), end='\t')
```

```
print()

print(cq, cq.is_empty(), cq.peek())
print('How an array looks like:')
print(cq.arr)

for i in range(10):
    cq.enqueue(i)

print(cq)
```

```
print('How an array looks like:')
print(cq.arry)

cq.dequeue()
cq.dequeue()
cq.dequeue()
cq.enqueue('new')
cq.enqueue('another new')
print(cq.arry)
```

```
while cqi
  print([q.dequeue(), end= ' ' )
0 <- 1 <- 2 <- 3 <- 4
Removed: 0
Removed: 1
Removed: 2
3 <- 4
3 <- 4 <- 10 <- 13 <- 16 <- 19
Now an array looks like
```

```

3 {0, 1, 2, 3, 4, 10, 13, 16, 19, None}
4 3
4 <- 10 <- 13 <- 16 <- 19
4 False
4
4
4 <- 10 <- 13 <- 16 <- 19 <- 20
4 10 13 16 19 20
4 True None

```

```
How an array looks like:
[0, 1, 2, 3, 4, 10, 13, 16, 19, 20]
0 <= 1 <= 2 <= 3 <= 4 <= 5 <= 6 <= 7 <= 8 <= 9
How an array looks like:
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
['new', 'another new', 2, 3, 4, 5, 6, 7, 8, 9]
3 4 5 6 7 8 9 new another new
```

```
(Implementacja obiektowa) (Amortyzowana pelzajazą kolejka - modyfikacja powyższej)
```

```
In [17]: class AmortizedQueue:
          # amort_factor is an amortization factor value which indicates how many times will
          # a size of an array be increased (while creating a new one) if its length was exceeded
          def __init__(self, amortization_factor=2):
              self.arr = None # other languages we don't have to initialize an array
              self.length = 0 # A current length of a queue
```

```

        self.first_idx = 0
        self.amort_factor = self.amortization_factor

    def __len__(self):
        return self.length

    def __repr__(self):
        return f'({self.__class__.__name__}({len(self.arr)})'

```

```
def _iter_(self):
    for i in range(self.first_idx, self.first_idx + self.length):
        yield self.arr[i % len(self.arr)]

def _str_(self):
    # An arrow indicates a direction in which a queue moves
    return ' <- '.join(map(str, self))

def __eq__(self):
```

```

        return not bool(self)

    def peek(self):
        # Returns the first element of a queue
        if self:
            return self.arr[self.first_idx]

    def enqueue(self, val):
        # Increase the size of an array if we exceeded its length

```

```

    if self.length == len(self.arr):
        self._expand_array()

        new_idx = [self.first_idx + self.length % len(self.arr)
                    self.arr[new_idx] = val
                    self.length += 1

def dequeue(self):
    # If a queue is empty, raise an Exception
    if not self:

```

```

        raise IndexError('dequeue from an empty (self._class, _name)')
    # Decrease a length of an array only if much empty space is left:
    if self._length <= len(self.arr) // (self._amort_factor * 2):
        self._shrink_array()

    removed = self.arr[self.first_idx]
    self.first_idx = self.first_idx + 1
    self.length -= 1
    return removed

```

```
def expand_array(self):
    self.arr = self._new_array(len(self.arr) * self._amort_factor)
    # Reset the first_idx value as all the values are now placed at the beginning
    # of an array
    self.first_idx = 0

def shrink_array(self):
    # The same rule as above, we also assume it works in constant time
```

```
self.arr = self._new_array(len(self.arr) // self.smolt_factor)
# Reset the first_idx value as well
self.first_idx = 0

def _new_array(self, length):
    # We assume that initialization of a new array costs us constant time
    # (In Python it's impossible without extra modules which has arrays implemented)
    new_arr = (None) * length
    # Rewrite values from the first array to a new one
```

```

    for i in range(self.length):
        new_arr[i] = self.arr[self.first_idx + i] % len(self.arr)
    return new_arr

```

Kilka testów

```

In [18]: aq = AmortizedQueue()
         for n in range(5):
             aq.enqueue(n)

```

```
print(aq)

for _ in range(3):
    print('Removed:', aq.dequeue())
print(aq)

for i in range(10, 20, 3):
    aq.enqueue(i)
print(aq)
```

```
print('How an array looks like:')
print(aq.arr)

print(aq.dequeue())
print(aq)
print(aq.is_empty())
print(aq.peek())
aq.enqueue(20)
```

```
print(aq.peak())
print(aq)

while aq:
    print(aq.dequeue(), end='\\t')
    print()

print(aq, aq.is_empty(), aq.peak())
print("Now an array looks like:")
```

```
print(aq.arr)

for i in range(9):
    aq.enqueue(i)
print(aq)
print('Now an array looks like:')
print(aq.arr)

while aq:
```



```
0 <= 1 <= 2 <= 3 <=
Removed: 0
Removed: 1
Removed: 2
3 <= 4 <= 10 <= 13 <= 16 <= 19 <= 20
How an array looks like:
[19, 1, 2, 3, 4, 10, 13, 16]
3
4 <= 10 <= 13 <= 16 <= 19
False
4
4 <= 10 <= 13 <= 16 <= 19 <= 20
4      10      13      16      19      20
True None
How an array looks like:
[20, None]
0 <= 1 <= 2 <= 3 <= 4 <= 5 <= 6 <= 7 <= 8
How an array looks like:
[0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None, None]
0 1 <= 2 <= 3 <= 4 <= 5 <= 6 <= 7 <= 8
[0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None, None]
1 2 <= 3 <= 4 <= 5 <= 6 <= 7 <= 8
[0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None, None]
[0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None, None]
2 3 <= 4 <= 5 <= 6 <= 7 <= 8
[0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None, None]
3 4 <= 5 <= 6 <= 7 <= 8
[0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None, None]
4 5 <= 6 <= 7 <= 8
[5, 6, 7, 8, None, None, None, None]
5 6 <= 7 <= 8
[5, 6, 7, 8, None, None, None, None]
6 7 <= 8
[5, 6, 7, 8, None, None, None, None]
7 8
[7, 8, None, None]
8
[8, None]
```

## Stos

W Pythonie (i wielu innych językach, w których występują dynamiczne tablice) warto stos zaimplementować z użyciem takiej tablicy (w Pythonie listy), ponieważ większość operacji jest już zaimplementowanych za nas i nie trzeba ich tworzyć na nowo.

## Implementacja struktury #1 (z użyciem listy Pythonowej)

### (Implementacja obiektowa)

```
In [19]: class Stack:
def __init__(self, values: 'Iterable' = None):
    self.values = [] if not values else list(values) # Create a copy as we don't want to modify the entire
    # iterable

def iter(self):
    yield from self.values

def __str__(self):
    return ' '.join(map(str, self))

def __bool__(self):
    return bool(self.values)

@property
def size(self):
    return len(self.values)

def push(self, val: object):
    self.values.append(val)

def pop(self) -> object:
    if not self.values:
        raise IndexError('pop from an empty {self.__class__.__name__}')
    return self.values.pop()

def peek(self) -> object:
    return None if not self.values else self.values[-1]
```

### Kilka testów

```
In [20]: s = Stack()
print(s)
for v in range(3, 30, 4):
    s.push(v)
print(s)
print(s.size)
print(s.pop())
print(s.peak())
while s:
    print(s.pop(), end='\t')
print('\n', s)
print(s.size)

3 7 11 15 19 23 27
27
23
23      19      15      11      7      3
0
```

## Implementacja struktury #2 (w oparciu o listę jednokierunkową)

### (Implementacja obiektowa)

```
In [21]: class Node:
def __init__(self, val=None):
    self.val = val
    self.next = None

class Stack:
def __init__(self, values: 'Iterable' = None):
    self.top = None
    self.size = 0
    values and self.__create_stack(values)

def iter(self):
    curr = self.top
    while curr:
        yield curr.val
        curr = curr.next

def __str__(self):
    return ' '.join(map(str, self))

def __bool__(self):
    return self.size > 0

def push(self, val: object):
    node = Node(val)
    if not self:
        self.top = node
        self.size += 1
    else:
        node.next = self.top
        self.top = node
        self.size += 1

def pop(self) -> object:
    if not self:
        raise IndexError('pop from an empty {self.__class__.__name__}')
    removed = self.top.val
    self.top = self.top.next
    self.size -= 1
    return removed

def peek(self) -> object:
    return None if not self else self.top.val

def __create_stack(self, values: 'Iterable'):
    # First values of the 'values' iterable will be placed first on the stack
    if values:
        # Create a linked list
        iterator = iter(values[::-1])
        curr = head = Node(next(iterator))
        for val in iterator:
            curr.next = Node(val)
            curr = curr.next
        self.size = len(values)
        self.top = head
```

### Kilka testów

```
In [22]: s = Stack()
print(s)
for v in range(3, 30, 4):
    s.push(v)
print(s)
print(s.size)
print(s.pop())
print(s.peak())
while s:
    print(s.pop(), end='\t')
print('\n', s)
print(s.size)

27 23 19 15 11 7 3
7
27
23
23      19      15      11      7      3
0
```

## Implementacja struktury #3 (w oparciu o listę jednokierunkową)

### (Implementacja funkcyjna)

```
In [23]: class Node:
def __init__(self, val=None):
    self.val = val
    self.next = None

def stack_create(values: 'Iterable' = None) -> 'stack_top (sentinel)':
    if not values: return head
    head.next = curr = Node(values[0])
    for i in range(1, len(values)):
        curr.next = Node(values[i])
        curr = curr.next
    return head

def stack_print(stack_top: 'stack_top (sentinel)':
    curr = stack_top.next
    while curr:
        print(' ', curr.val, end=' ')
        curr = curr.next
    print()

def stack_push(stack_top: 'stack_top (sentinel)', val: object):
    node = Node(val)
    node.next = stack_top.next
    stack_top.next = node

def stack_pop(stack_top: 'stack_top (sentinel)') -> object:
    if not stack_top.next:
        raise IndexError('pop from an empty stack')
    removed = stack_top.next.val
    stack_top.next = stack_top.next.next
    return removed

def stack_peek(stack_top: 'stack_top (sentinel)') -> object:
    return stack_top.next and stack_top.next.val

def stack_is_empty(stack_top: 'stack_top (sentinel)') -> bool:
    return not stack_top.next
```

### Kilka testów

```
In [24]: s = stack_create()
stack_print(s)
for v in range(3, 30, 4):
    stack_push(s, v)
print(s)
print(stack_pop(s))
print(stack_peek(s))
while not stack_is_empty(s):
    print(stack_pop(s), end='\t')
print()
stack_print(s)
print(stack_peek(s))

27 23 19 15 11 7 3
27
23
23      19      15      11      7      3
None
```

## Implementacja struktury #4 (z użyciem tablicy)

### (Implementacja obiektowa) (Amortyzowany stos - modyfikacja powyższej)

```
In [25]: class AmortizedStack:
def __init__(self, _amortization_factor=2):
    self.arr = (None)
    self.top_idx = -1
    self.amort_factor = _amortization_factor

def iter(self):
    for i in range(self.top_idx, -1, -1):
        yield self.arr[i]

def __str__(self):
    # This error points towards values which are closer to the top of a stack
    return '< '.join(map(str, self))

def __bool__(self):
    return bool(self.size)

@property
def size(self):
    return self.top_idx + 1

def push(self, val: object):
    if self.size == len(self.arr):
        self._expand_array()
    self.arr[self.top_idx + 1] = val
    self.top_idx += 1

def pop(self) -> object:
    if not self:
        raise IndexError('pop from an empty {self.__class__.__name__}')
    if self.size <= len(self.arr) // (self.amort_factor ** 2):
        self._shrink_array()
    removed = self.arr[self.top_idx]
    self.top_idx -= 1
    return removed

def peek(self) -> object:
    if self: return self.arr[self.top_idx]

def _expand_array(self):
    self.arr = self._new_array(len(self.arr) * self.amort_factor)

def _shrink_array(self):
    self.arr = self._new_array(len(self.arr) // self.amort_factor)

def _new_array(self, length):
    # We assume that initialization of a new array costs us constant time
    # (in Python it's impossible without extra modules which has arrays implemented)
    new_arr = (None) * length
    # Rewrite values from the first array to a new one
    for i in range(self.size):
        new_arr[i] = self.arr[i]
    return new_arr
```

### Kilka testów

```
In [26]: sa = AmortizedStack()
print(sa)
for v in range(3, 30, 4):
    sa.push(v)
print(sa)
print(sa.size)
print(sa.pop())
print(sa.peak())
while sa:
    print(sa.pop(), end='\t')
print('\n', sa)
print(sa.size)

for i in range(9):
    sa.push(i)
print(sa.size, sa.arr)
while sa:
    print(sa.pop())
    print(sa.size, sa.arr)

27 <= 23 <= 19 <= 15 <= 11 <= 7 <= 3
27
23
23      19      15      11      7      3
0
1 [0, None]
2 [0, 1]
3 [0, 1, 2, None]
4 [0, 1, 2, 3]
5 [0, 1, 2, 3, 4, None, None, None]
6 [0, 1, 2, 3, 4, 5, None, None]
7 [0, 1, 2, 3, 4, 5, 6, None]
8 [0, 1, 2, 3, 4, 5, 6, 7]
9 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
10 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
11 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
12 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
13 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
14 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
15 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
16 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
17 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
18 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
19 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
20 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
21 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
22 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
23 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
24 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
25 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
26 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
27 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
28 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
29 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
30 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
31 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
32 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
33 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
34 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
35 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
36 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
37 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
38 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
39 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
40 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
41 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
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43 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
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45 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
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47 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
48 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
49 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
50 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
51 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
52 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
53 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
54 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
55 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
56 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
57 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
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64 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
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98 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
99 [0, 1, 2, 3, 4, 5, 6, 7, 8, None, None, None, None, None, None]
```

In [ ] :