

$$L = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 0 & 3 & 3 \end{bmatrix} \xrightarrow{F_3 - \frac{1}{3}F_2} \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 0 & 3 & 3 \end{bmatrix} \xrightarrow{F_2/3} \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\dim(L) = \mathbb{R}^3 \in \mathbb{R}^4$$

$$\text{NULL}(L) \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 0 & 3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 - x_2 + x_3 - x_4 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} x_1 - x_2 + x_3 - x_4 = 0 \\ x_3 = -x_4 \end{cases}$$

$$\begin{cases} x_1 - x_2 + 0x_3 = 0 \\ x_3 = \frac{x_2 - x_1}{2} \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ \frac{x_2 - x_1}{2} \\ \frac{x_1 - x_2}{2} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ -1 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix} \xrightarrow{\begin{matrix} F_3 + F_1 \\ F_4 - F_1 \end{matrix}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow{F_4 + F_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\begin{matrix} -F_2 + F_4 \\ -F_2 \end{matrix}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\dim(L) = \mathbb{R}^3 \in \mathbb{R}^3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ -1 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} x_1 = 0 \\ -x_2 + x_3 = 0 \\ -x_1 = 0 \\ x_1 + x_2 + x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = x_3 \end{cases}$$