Grid Simulation Analysis

1 Seed–Length Grid Diagnostics

1.1 Purpose and Setup

For each episode length T and burn–in fraction $b \in (0, 1)$, and for multiple random seeds s, we simulate an episode and drop the first $\lfloor bT \rfloor$ periods. The simulator returns states s_t and policies p_t as log–deviations from their deterministic steady states. Aggregates are then computed with *fixed* steady–state price weights by passing zero log–deviation weights for (P, P_k, P_m) so that the level weights equal their steady–state levels. Let

$$\left(C_t, L_t, K_t, Y_t, M_t, I_t, u_t, u_t\right)$$

denote the outputs of the aggregation mapping. Here u_t is the (level) period utility and u_t is the corresponding utility log–deviation.

All diagnostics below are computed on the post–burn sample of length $n = T - \lfloor bT \rfloor$.

1.2 Per–Seed Diagnostics

Welfare. Welfare is computed using the same methodology as the main analysis: random trajectory sampling with proper discounting. From the utility series u_t, we sample N random trajectories of length L and compute their discounted present values with terminal continuation value $W_{ss} = \mathbb{E}[u_t]/(1 - \beta)$:

$$W \;=\; \frac{1}{N} \sum_{j=1}^{N} \left(\sum_{\ell=0}^{L-1} \beta^{\ell} u_{t_j+\ell} + \beta^{L} W_{ss} \right).$$

This provides a more accurate welfare measure than simple permanent utility approximations.

Mean sectoral capital (log deviations). Let n_S be the number of sectors and denote by $k_{t,i}$ the i-th sector's capital log–deviation (contained in s_t). The diagnostic stores

$$\bar{k}_i \;=\; \frac{1}{n}\sum_{t=1}^{n} k_{t,i}, \qquad i = 1, \dots, n_S.$$

Large $|\bar{k}_i|$ suggests drift or insufficient burn–in.

IACT for K and u. For a scalar series x_t (either K_t or u_t), define the batch–means estimator with B batches (default $B = 20$). Let the batch size be $m = \lfloor n/B \rfloor$ and $n_{\text{eff}} = mB$. Split $x_{1:n_{\text{eff}}}$ into B contiguous batches and let $\bar{x}_b$ be batch means. With $\text{Var}(x)$ the sample variance,

$$\widehat{\tau}(x) \;=\; \max\Bigl\{1,\; m\,\frac{\text{Var}(\bar{x}_b)}{\text{Var}(x_{1:n_{\text{eff}}})}\Bigr\}.$$

Values $\widehat{\tau} \gg 1$ indicate strong autocorrelation and fewer effective samples; smaller values indicate better mixing.

Linear drift (OLS slope) for K and u. Regress y_t on time $t \in \{0, \dots, n-1\}$. The reported slope per period is

$$b(y) \;=\; \frac{\text{Cov}(t, y_t)}{\text{Var}(t)}.$$

Slopes near zero indicate stationarity; sizable positive/negative slopes indicate trending aggregates.

Out–of–distribution (OOD) fraction. For thresholds $\theta \in \{3, 4, 5\}$, with $z_{t,d}$ each element of the state vector (log deviations),

$$\text{OOD}(\theta) \;=\; \frac{1}{nD}\sum_{t=1}^{n}\sum_{d=1}^{D} \mathbf{1}\{\, |z_{t,d}| > \theta \,\}.$$

Larger fractions signal excursions far from the normalization range (potentially beyond the training distribution).

Shock diagnostics. Denormalize states via $s_t^{\text{lev}} = s_t \odot \sigma_s + \mu_s$, and let a_t be the shock state block. With AR(1) law $a_{t+1} = \rho a_t + \varepsilon_t$ and target covariance Σ_A, we compute

$$\|\mathbb{E}[\varepsilon_t]\|_2, \qquad \frac{\|\text{Cov}(\varepsilon_t) - \Sigma_A\|_F}{\|\Sigma_A\|_F}.$$

Small values indicate that simulated shocks match the intended statistical model.

1.3 Across–Seed Aggregation

Cross–seed dispersion. With per–seed welfare $W^{(s)}$, the welfare dispersion is reported as a fraction of the mean:

$$\frac{\mathrm{sd}(W^{(s)})}{|\overline{W}|}, \qquad \overline{W} \;=\; \frac{1}{S}\sum_{s=1}^{S} W^{(s)}.$$

The absolute value accounts for the fact that welfare (and utilities) are typically negative. For sectoral capital means, form $\mathrm{sd}_s(\bar{k}_i^{(s)})$ across seeds for each sector i, then report the average across sectors.

Average diagnostics. For IACTs, slopes, OOD fractions, and shock diagnostics, we report simple averages across seeds.

1.4 Scaling with Episode Length

For a fixed burn–in fraction, compute standard deviations across seeds at each T, say sd_T, and regress $\log \mathrm{sd}_T$ on $\log T$ via the slope

$$b\big(\log T, \log \mathrm{sd}\big) \;=\; \frac{\mathrm{Cov}(\log T, \log \mathrm{sd}_T)}{\mathrm{Var}(\log T)}.$$

Under pure sampling error with weak dependence, one expects roughly $\mathrm{sd} \propto T^{-1/2}$ so that the slope is near $-\frac{1}{2}$. Substantially different slopes suggest drift or other nonstationarities.

1.5 Interpretation Guide

- **Welfare**: higher is better; compare to steady–state benchmark when relevant.
- **IACT**: high values indicate strong persistence; consider longer episodes or diagnostics of policy mixing.
- **OLS slope**: significant nonzero slopes indicate trending aggregates (potential drift or misspecified aggregation).
- **OOD fraction**: large values indicate states frequently outside typical range; adjust volatility or training coverage.

- **Shock diagnostics**: large mean or covariance errors indicate mismatch with the intended AR(1) process.
- **Cross–seed SD vs** T: slope near $-1/2$ supports sampling–error dominance; flatter slopes point to structural drift.

1.6 Sanity Tests Outside the Grid

Two additional checks are used in the main analysis:

- **Terminal state magnitude**: $\max_d |s_{T-1,d}| < 10$ to rule out explosive trajectories.
- **Stochastic steady state tightness**: the maximal standard deviation of the stochastic steady state observation is required to be < 0.01, enforcing a well–defined stationary fixed point under the learned policy.