Homework 1

Introduction to Astrobiology - ASP5022

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1. The Sun radiates light with the luminosity $L_\odot = 3.828 \times 10^{26}$ W. Let's compute the average temperature on the surfaces of the planets in the Solar System. For this you need to consider the following:

   (a) Find an expression for the flux (L/Area) absorbed by the planet disk (πR^2) at the average D of the planet from the Sun.
   However, consider that some fraction of the incident light will be reflected back into space, something that is called albedo ($a =$ reflected light/incident light) and depends on the surface composition. The albedo values for the planets in our Solar system and Pluto are listed in the table below; use them for your calculations.

   Answer: To get the flux absorbed, first we need the luminosity flux at a distance D away from the Sun, which obeys the **inverse-square law**. This is:

   $$\frac{L_\odot}{4\pi D^2}$$

   Then, we multiply it by the area reached by the photons, in this case, the planetary disk:

   $$\frac{L_\odot}{4\pi D^2} \times \pi R^2$$

   Since we are considering albedo, which corresponds to the fraction of light **reflected**, we must multiply the expression by its complement:

   $$L_{\text{absorbed}} = \frac{L_\odot}{4\pi D^2} \times \pi R^2 \times (1-a) = \frac{L_\odot R^2 (1-a)}{4D^2}$$

   where $L_\odot$ is the luminosity of the Sun, D is the distance from the sun to the planet, R is the radius of the planet disk, and a is the albedo.

   (b) Now consider that, in the equilibrium state, the planet absorbs the same amount of energy per second (in Watts) as it radiates per second back into space across the planet surface ($4\pi R^2$), again in Watts. Assume that the planet redistributes the absorbed energy instantaneously across its entire surface and is homogeneously "warm" as it radiates. This allows you to simply use Stefan-Boltzmann's Law ($L = \text{Surface} \times \sigma T^4$) to compute the radiated luminosity of the planet. Find the expression for the equilibrium temperature.

   Answer: Since we are considering an equilibrium state, we have that the luminosity absorbed by the planet and the luminosity emitted cancel out:

   $$0 = \frac{L_\odot R^2 (1-a)}{4D^2} - \text{Surface} \times \sigma T^4$$

   If we approximate the planetary surface to a sphere, we get the following parameters:

   $$\frac{L_\odot R^2 (1-a)}{4D^2} = 4\pi R^2 \times \sigma T^4$$

   Next, we just need to solve for T:

   $$T = \frac{1}{2}\sqrt[4]{(1-a)\frac{L_\odot}{\pi \sigma D^2}}$$

where $L_\odot$ is the luminosity of the Sun, D is the distance from the sun to the planet, a is the albedo, and σ is the Stefan-Boltzmann constant.

(c) Compute the average surface temperatures and compare them to the measured average surface temperatures summarized in the table. Discuss possible reasons for the differences.

Planet	Albedo	Distance [$10^6 km$]	Min/Max Surf. Temp [°C]
Mercury	0.106	57.9	-173/427
Venus	0.650	108.2	464
Earth	0.367	149.6	-88/58
Mars	0.150	227.9	-153/20
Jupiter	0.520	778.6	-110
Saturn	0.470	1433.5	-140
Uranus	0.510	2872.5	-195
Neptune	0.410	4495.1	-200
Pluto	0.300	5906.4	-225

Table 1: Solar System Data

Answer: We can calculate the surface temperature (in Kelvin) of planets using the equation found in **b)**. However, we need to subtract 273.15 from the result to get the Temperature in Celsius degrees. The results are summarized in the following table:

Planet	Albedo	Distance [$10^6 km$]	Theoretical Surf. Temp [°C]	Min/Max Surf. Temp [°C]
Mercury	0.106	57.9	161.878	-173/427
Venus	0.650	108.2	-21.426	464
Earth	0.367	149.6	-24.890	-88/58
Mars	0.150	227.9	-56.627	-153/20
Jupiter	0.520	778.6	-171.601	-110
Saturn	0.470	1433.5	-196.433	-140
Uranus	0.510	2872.5	-220.008	-195
Neptune	0.410	4495.1	-228.649	-200
Pluto	0.300	5906.4	-232.633	-225

Table 2: Solar System Data and their theoretical surface temperatures.

Analyzing further, it can be pointed out that the results obtained are either inside the Min/Max range, or lower than the empirical values. This may be consequence of the following assumptions we made to get the values:

- We approximated the planets to perfect black bodies by using Stefan-Boltzmann law.
- We assumed the planets redistribute energy evenly across the surface (which in practice is not possible). This accounts for a significant error in bodies with a thin or no atmosphere, more specifically, Mercury and Mars, in which the Min. and

Max. temperatures differ greatly, mostly because these planets can't distribute solar energy efficiently across their entire surface. This causes the side facing the sun to be much hotter than the dark side.

- We assumed planets are at energy equilibrium in the first place, and did not consider energy produced by the planet itself (e.g. volcanic activity, Jupiter's gravitational compression).
- Planets are not perfect spheres, so the surface area may differ from the real value.
- Our calculations did not account for greenhouse effect. Pay special attention to Venus, which has a temperature way higher than the one our model predicts. This will be discussed in the following sections.

2. Let's talk about the general planetary greenhouse effect.

(a) How does it work? What is required? Discuss with as much detail as you deem necessary

Answer: Greenhouse effect is a natural process that occurs in planets with a dense enough atmosphere. It consists in the "trapping" of solar radiation between a planet's atmosphere and its surface. More specifically, part of the radiation that enters the atmosphere gets reflected by the surface, and then back into the planet by the atmosphere itself, causing some sort of "bouncing" motion of radiation between the sky and surface. This causes the planet to absorb the trapped energy, instead of reflecting it back into space, heating the surface and rising its average temperature. Greenhouse effect is not necessarily a bad thing. In fact, life on Earth would not be possible without this process, as the temperatures would be too low to support life (-24.89°C), according to our results from part 1).

As said before, for a planet to produce greenhouse effect, it needs a surface to reflect the photons, and a dense enough atmosphere that contains greenhouse gases. These are water vapor, carbon dioxide and methane. High concentrations of these gases in the atmosphere strengthen the magnitude of the greenhouse effect, therefore causing global warming, breaking the temperature stability and climate regulation.

(b) Where do you find greenhouse effect operating in the Solar System?

Answer: Any object with an atmosphere and a solid surface should bear greenhouse effect to some degree. The objects that have a relevant greenhouse effect in the Solar System are **Venus, Titan (Saturn moon), Earth, and Mars** (although almost negligible in this one).

Venus atmosphere is 96.5% CO_2, which "traps" a great amount of the Sun's radiation, causing a very drastic greenhouse effect.

Titan atmosphere is mostly composed of **nitrogen**, but it has a significant percentage of **methane** (1.6%), also causing greenhouse effect.

Earth's atmosphere has a variable amount of **water vapor** ($[H_2O] < 1\%$), **carbon dioxide** ($[CO_2] < 0.05\%$) and methane, which cause an important but less strong greenhouse effect.

Lastly, Mars has a large fraction of CO_2 in its atmosphere, but its way too thin compared to the objects listed before to have a significant effect in temperature and climate.

3. With the Kepler spacecraft now switched off and TESS scanning the sky for new exoplanet candidates we have a good sample to explore exoplanet systems. Go to NASA Exoplanet Archive and download the catalog of confirmed exoplanet parameters.

   (a) Construct your own Hertzsprung-Russell diagram for all stars with confirmed exoplanets.

Answer: The Hertzsprung-Russell diagram plots a star's luminosity against its temperature. NASA's catalog of confirmed exoplanets has a total of 4152 items (as of April 16, 2020), but only 632 entries had enough parameters to make this plot. Those parameters are **Effective Temperature** (*st_teff*) and **Stellar Luminosity** (*st_lum*). Note that this diagram does not take the data error into consideration. The following figure shows a Hertzsprung-Russell diagram built with this data.

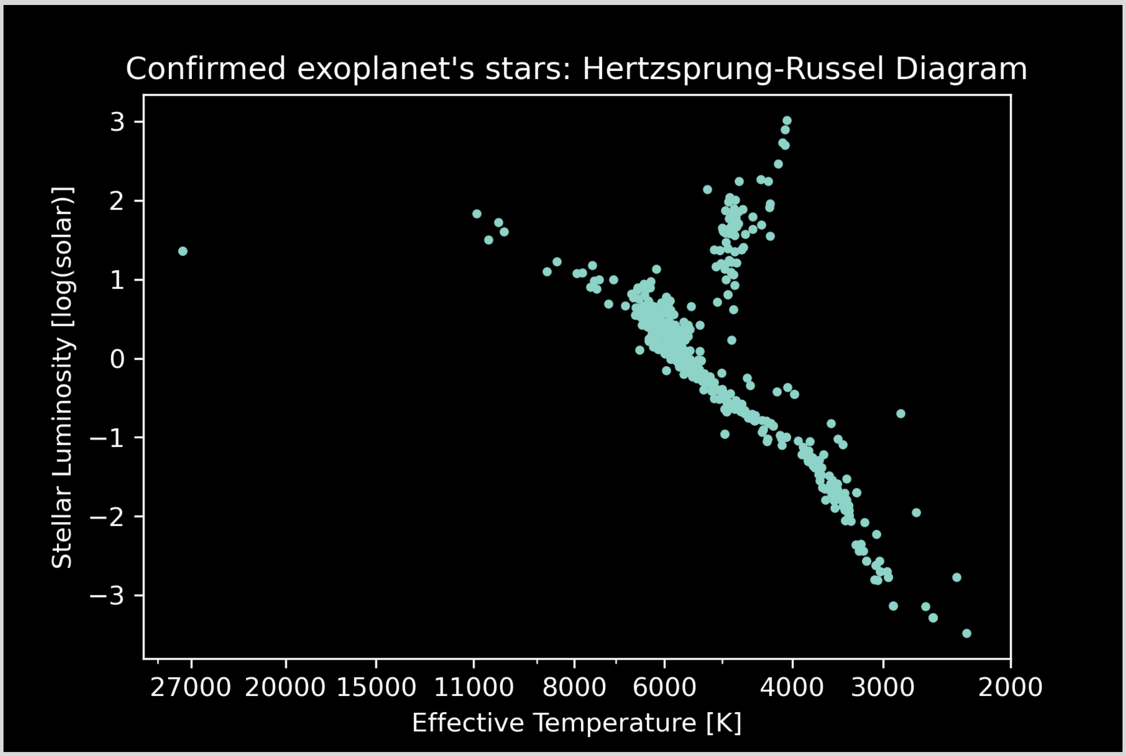

Figure 1: HR diagram of confirmed exoplanet's stars.

It is possible to observe the main sequence and giant/supergiant starts. Similarities are clear when compared side by side with a more populated H-R diagram. It is also interesting to note that there is no white dwarfs present in this diagram.

(b) Now color the star symbols in the HR diagram according to the orbit semi-major axis distance of their exoplanet. What systematics do you see in the plot? Why?

Answer:

The following diagram contains most of the stars of the previous one. Some entries had to be dropped because they didn't have the **Orbit Semi-Major Axis** parameter (*pl_orbsmax*). This leaves us with 603 exoplanets.

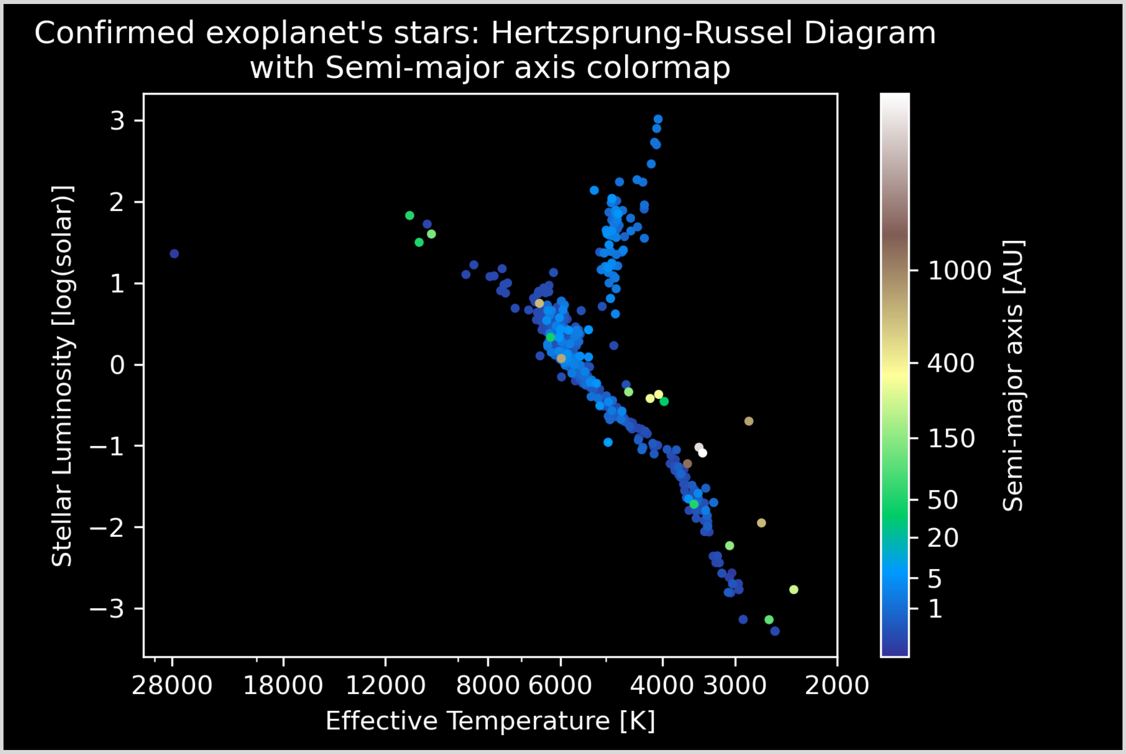

Figure 2: HR diagram of confirmed exoplanet's stars with Semi-major axis colormap.

Analyzing this plot further, it is possible to see that most of the dots in this diagram have a blue-ish tone, which corresponds to a relatively low **semi-major axis** (< 5 AU). This is related to the methods astronomers use, and because this database only contains data from **confirmed** exoplanets.

Most exoplanets in this database were discovered using the **Transit method** (around 76.5%), and for a planet to be catalogued as confirmed, several transits must be recorded. Kepler's third law states that *"the square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit"*. If we generalize this law for exoplanetary systems, it's safe to assume that astronomers will register more transits from exoplanets closer to its star, and therefore, exoplanets with these characteristics will be catalogued as confirmed more often. Actually, if we take a look at the data, we'll notice that the exoplanets with the largest semi-major axes were discovered via the **Imaging** and **Radial Velocity** methods, not the previously stated **Transit method**. Another argument may be that this plot only contains entries with a **Semi-major axis** parameter, and its easier to correctly measure this parameter for planets with a low **Semi-major axis**.

(c) Determine now the exoplanets in the habitable zone of their host star. What is the overall fraction of exoplanets in the habitable zone in the entire dataset? Mark these stars in the HR diagram. Discuss what you observe.

Answer:

Its important to note that not every entry in the original dataset could be used for the next plot, because some of them were lacking the necessary parameters to calculate the **CHZ**, and at the same time, place them in the H-R diagram. Of the remaining 269 rows, a total of **19 exoplanets** are in its star's habitable zone. This corresponds to 7.06% of all exoplanets tested. On a side note, if we also consider the exoplanets without the parameters necessary for plotting (a total of 700), the number grows to **32 exoplanets**, but the fraction drops to 4.57%. The following figure shows the resulting H-R diagram.

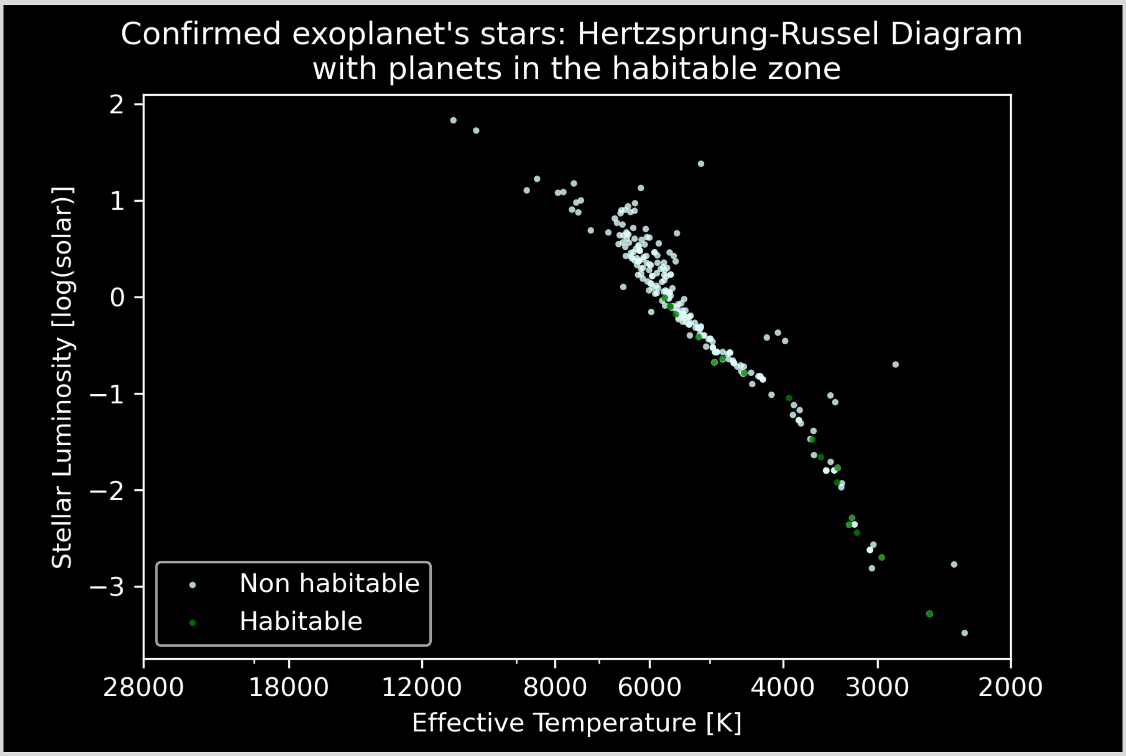

Figure 3: HR diagram of confirmed exoplanet's stars with planets in the habitable zone.

For the sake of simplicity, the **Equilibrium Temperature** (*pl_eqt*) parameter was the one who determined if the exoplanet belonged to its star's CHZ. Professional models should use a more complex algorithm and a larger quantity of parameters, for example, stellar radius, spectral type, stellar age, etc.

In the resulting diagram, the planets are scattered across the main sequence. Of the stars that have a luminosity greater than the Sun's ($y = 0$), none of them have orbiting exoplanets in its habitable zone. It would seem like this is the upper bound, at least for main sequence stars.

(d) Compute your own Earth Similarity Index (ESI) from the parameters available in the NASA Exoplanet Archive. Explain where you would like to send your next interstellar spaceship.

Answer: The following ESI is based off this source. It uses the following parameters:

Mean radius	$i = 1$	$w_i = 0.57$
Bulk Density	$i = 2$	$w_i = 1.07$
Escape Velocity	$i = 3$	$w_i = 0.70$
Surface Temperature	$i = 4$	$w_i = 5.58$

Note: Escape Velocity is not in the database, but can be calculated with mass and radius.

A total of **344 exoplanets** have the previously listed parameters. With this data, the ESI can be calculated using this formula:

$$\prod_{i=1}^{4}\left(1 - \left|\frac{x_i - x_{i0}}{x_i + x_{i0}}\right|\right)^{\frac{w_i}{4}}$$

where x_i are the properties of the exoplanet, x_{i0} are the properties of Earth, and w_i is the weighted exponent of each property.
The following table shows the top 10 results, after this formula is applied to the dataset:

Planet name	**ESI**
TRAPPIST-1 d	0.9364
TRAPPIST-1 e	0.8562
TRAPPIST-1 c	0.8496
K2-18 b	0.8239
TRAPPIST-1 f	0.7376
TRAPPIST-1 g	0.7365
LHS 1140 b	0.7340
TRAPPIST-1 b	0.7205
Kepler-538 b	0.6986
LHS 1140 c	0.6842

Table 3: Top 10 habitable planets sorted by **Earth Similarity Index**.

With these results in consideration, the next interstellar spaceship should definitely go to the **TRAPPIST-1** system, more specifically to **TRAPPIST-1 d** exoplanet.

(e) Plot the orbital period of planets vs. planet size and discuss the distribution of exoplanet data and the limits of this diagram.

Answer: A total of 3170 exoplanets had the necessary parameters to make the next plot. It shows exoplanet **Planet Radius** (*pl_rade*) and **Orbital Period** (*pl_orbper*). It also shows each exoplanet's **Discovery Method**, which helps to understand the limits/frontiers of this plot. The grey lines represent said limits.

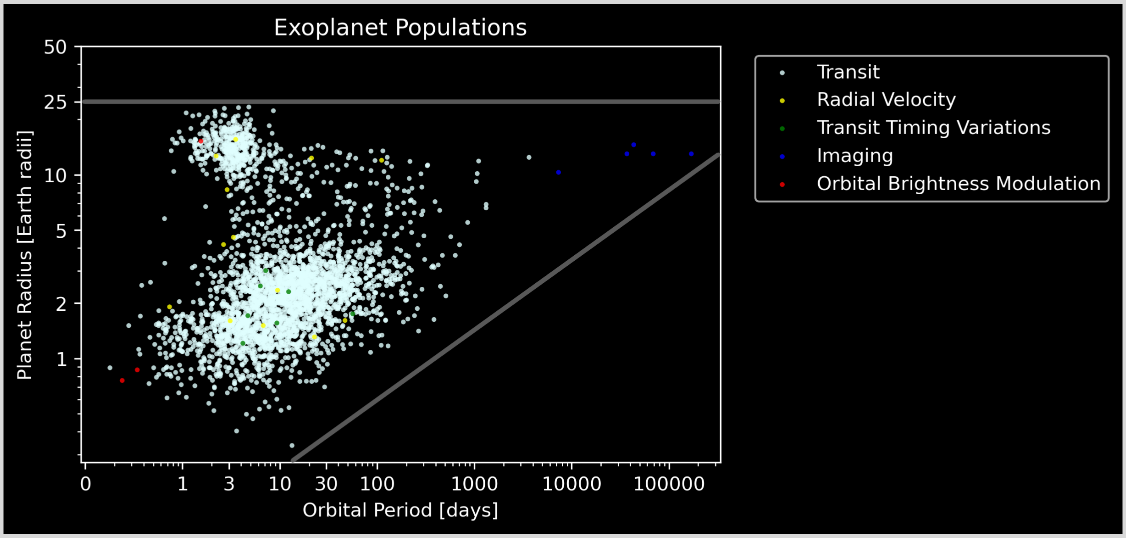

Figure 4: Discovered exoplanet populations and their limits: Orbital period, radius, and discovery method.

The following observations can be drawn from this diagram:

- Most of the exoplanets in this diagram have relatively low orbital period, and a high planetary radius (when compared to Earth).
- Most of the exoplanets in the diagram were discovered via **transit method**.
- The ones discovered by **direct imaging**, have large radii (around Jupiter's size) and long orbital periods (which, by Kepler's third law, means large orbits). This characteristics and other specific conditions allow them to be directly observed by astronomers.
- There are few to no planets over 25 Earth radii (note the upper grey line), because having a larger size would imply having a more mass, which would eventually form a star (brown dwarf).
- There are no exoplanets to the right of the bottom right limit line, mainly because we can't detect them (or haven't detected them yet). Several transits need to be registered for an exoplanet to be marked as confirmed, and longer orbital periods mean longer wait times for the next transit to occur. Also, we can't use other existing methods to observe exoplanets with these characteristics (except for possibly microlensing). We can't use **direct imaging** because they are too small, and we can't use **radial velocity** because low mass planets don't have a strong

enough effect over their host star. That being said, it is extremely difficult to detect low-mass and high period exoplanets with the current methods and technology.

4. BONUS QUESTION: As we discussed in the lecture, the stellar masses are not randomly distributed, but follow a mass function that astronomers refer to as the initial mass function. The number density of stars of a given mass can be written as $\frac{dN}{dM} = cM^{-2.35}$.
Because the central temperatures of stars with stellar masses $m \lesssim 0.1M_\odot$ are too low to ignite hydrogen fusion and stars with masses $m \gtrsim 100M_\odot$ are unstable against their own radiation pressure, the IMF is typically evaluated in the mass range between 0.1 and $100M_\odot$.
   (a) Compute what fraction of stars is at sub-solar masses $(0.1 - 1M_\odot)$ and in the super-solar mass range $(1-100M_\odot)$, and do the same for the stellar masses and luminosities, assuming $L \propto M^{3.5}$

Answer: Using integral calculus we obtain:

$$\frac{dN}{dM} = cM^{-2.35}$$
$$N = \int_{0.1M_\odot}^{100M_\odot} cM^{-2.35}\,dM$$
$$N = c\left(\frac{M^{-1.35}}{-1.35}\right)\Bigg|_{0.1M_\odot}^{100M_\odot}$$
$$N = \frac{c}{-1.35}((100M_\odot)^{-1.35} - (0.1M_\odot)^{-1.35})$$

If we want to obtain a fraction out of the IMF, we must normalize the number of stars N between the range of mass $0.1M_\odot - 100M_\odot$ to 1. Then, we'll be able to obtain a c constant to use in our calculations.

$$1 = \frac{c}{-1.35}((100M_\odot)^{-1.35} - (0.1M_\odot)^{-1.35})$$
$$c = \frac{-1.35}{(100M_\odot)^{-1.35} - (0.1M_\odot)^{-1.35}}$$
$$c = \frac{1.35}{(0.1M_\odot)^{-1.35} - (100M_\odot)^{-1.35}}$$
$$c \approx 0.0603M_\odot^{1.35}$$

Then, the fraction of stars with sub-solar masses $(0.1 - 1M_\odot)$ is:

$$N = \frac{0.0603M_\odot^{1.35}}{-1.35} \times ((1M_\odot)^{-1.35} - (0.1M_\odot)^{-1.35})$$
$$N = 0.9554172$$

And the fraction of stars with super-solar masses $(1 - 100M_\odot)$ is:

$$N = \frac{0.0603M_\odot^{1.35}}{-1.35} \times ((100M_\odot)^{-1.35} - (1M_\odot)^{-1.35})$$
$$N = 1 - 0.9554172 = 0.0445832$$

Similarly for **mass**, we can use the following function:

$$M = \int_{0.1M_\odot}^{100M_\odot} \alpha M^{-1.35}\, dM$$
$$M = \frac{\alpha}{-0.35}((100M_\odot)^{-0.35} - (0.1M_\odot)^{-0.35})$$

Normalizing and finding the value of the α constant:

$$1 = \frac{\alpha}{-0.35}((100M_\odot)^{-0.35} - (0.1M_\odot)^{-0.35})$$
$$\alpha = \frac{0.35}{(0.1M_\odot)^{-0.35} - (100M_\odot)^{-0.35}}$$
$$\alpha \approx 0.1716 M_\odot^{0.35}$$

Then, the fraction of mass within sub-solar stars ($0.1 - 1M_\odot$) is:

$$M = \frac{0.1716 M_\odot^{0.35}}{-0.35} \times ((1M_\odot)^{-0.35} - (0.1M_\odot)^{-0.35})$$
$$M = 0.607456$$

And the fraction of mass within super-solar masses ($1 - 100M_\odot$) is:

$$M = \frac{0.1716 M_\odot^{0.35}}{-0.35} \times ((100M_\odot)^{-0.35} - (1M_\odot)^{-0.35})$$
$$M = 1 - 0.607456 = 0.392544$$

Lastly, for **luminosity**, assuming $L \propto M^{3.5}$, we can use:

$$L = \int_{0.1M_\odot}^{100M_\odot} \beta M^{3.5}\, dM$$
$$L = \frac{\beta}{4.5}((100M_\odot)^{4.5} - (0.1M_\odot)^{4.5})$$

Normalizing and finding the value of the β constant:

$$1 = \frac{\beta}{4.5}((100M_\odot)^{4.5} - (0.1M_\odot)^{4.5})$$
$$\beta = \frac{4.5}{(100M_\odot)^{4.5} - (0.1M_\odot)^{4.5}}$$
$$\beta \approx 4.5 \times 10^{-9} M_\odot^{-4.5}$$

Then, the fraction of luminosity of sub-solar stars ($0.1 - 1M_\odot$) is:

$$L = \frac{4.5 \times 10^{-9} M_\odot^{-4.5}}{4.5} \times ((1M_\odot)^{4.5} - (0.1M_\odot)^{4.5})$$
$$L = 0.00000000999968 \approx 0$$

And the fraction of luminosity of super-solar masses ($1 - 100M_\odot$) is:

$$L = \frac{4.5 \times 10^{-9} M_\odot^{-4.5}}{4.5} \times ((100M_\odot)^{4.5} - (1M_\odot)^{4.5})$$
$$L = 0.999999999000032 \approx 1$$

(b) Given your knowledge about the equilibrium temperature from exercise I, discuss at what stellar masses you would find most of the "habitable zone real estate" (i.e. area of the habitable zone).

Answer:

Final notes and considerations

- I used as reference the lectures and knowledge I had beforehand, specially from AST0112 course "Introduction to Astronomy". I extracted data from the both courses slides (e.g. atmosphere characteristics in question 2). Any other references should be linked in place.
- I used **Python** libraries `pandas` and `matplotlib` to make the scatter plots. If you need the original figures (in higher resolution) and/or the python script I used to generate them feel free to ask!