Practice Question – Optimization

The data set used in the following exercise is included in the file *Practice exercise optimization model.xlxs*. The questions here can however be answered solely based on the analysis output provided in this text.

A Sustainability Question

Korus, one of the major steel producers in Europe, is under pressure to reduce the air pollution caused by its furnaces. The board of directors has recently launched a new corporate social responsibility agenda, which includes new pollution standards and proposals to reduce pollution.

The three main types of pollutants are Particulate matter, Sulphur Oxides and Hydrocarbons. The new standards require that the company reduce its annual emission of these pollutants by the amounts shown in the following table.

Pollutant	Required Reduction in Annual Emission Rate (million pounds)
Particulates	60
Sulphur Oxides	150
Hydrocarbons	125

The board of directors has set up a team of people from different departments to determine how to achieve those goals in the most economical way. The steelworks have two primary sources of pollution, namely the blast furnaces that produce pig iron and the open-hearth furnaces that transform iron into steel. In both cases, engineers have determined that the most effective methods for reducing pollution are:

(a) increasing the height of the smokestacks

(b) using filter devices

(c) including cleaner, high-grade materials among the fuels for the furnaces

Each of these methods can be implemented to a variable degree, up to a certain limit (e.g. up to a maximum height for smokestacks).

The following table, taken from an environmental impact report, shows how many emissions (in millions of pounds per year) can be eliminated from each type of furnace by using each method to its fullest degree.

Pollutant	Taller Smokestacks		Filters		Better Fuels	
	Blast Furnaces	Open-Hearth Furnaces	Blast Furnaces	Open-Hearth Furnaces	Blast Furnaces	Open-Hearth Furnaces
Particulates	12	9	25	20	17	13
Sulphur Oxides	35	42	18	31	56	49
Hydrocarbons	37	53	28	24	29	20

The benefits of implementing each method are proportional to the degree of implementation. For instance, if smokestacks heights are increased with half the maximum possible increase, the effect will be half of the predicted effect.

The board of directors was interested in a solution that would enable the company to meet the standards at minimum cost. The estimated annual cost of implementing each of the measures is given in the table below. If a method is not used to its fullest extent, the cost will be proportional to the degree of implementation.

Method	Blast Furnaces	Open Hearth-Furnaces
Taller Smokestacks	€8 million	€10 million
Filters	€7 million	€6 million
Better Fuels	€11 million	€9 million

A model was developed to determine the optimal implementation levels for each of the methods, for both the blast furnaces and the open-hearth furnaces. The purpose of the model is to determine the following optimal levels in order to minimize the associated cost:

- **Tb**, taller smokestacks for blast furnaces
- **To**, taller smokestacks for open-hearth furnaces
- **Fb**, filters for blast furnaces
- **Fo**, filters for open-hearth furnaces
- **Bb**, better fuels for blast furnaces
- **Bo**, better fuels for open-hearth furnaces

An algebraic model is shown below.

Minimise

$8Tb + 10To + 7Fb + 6Fo + 11Bb + 9Bo$

Subject to

$12Tb + 9To + 25Fb + 20Fo + 17Bb + 13Bo \geq 60$ Particulates Requirement

$35Tb + 42To + 18Fb + 31Fo + 56Bb + 49Bo \geq 150$ Sulphur Oxides Requirement

$37Tb + 53To + 28Fb + 24Fo + 29Bb + 20Bo \geq 125$	Hydrocarbons Requirement
$0 \leq Tb \leq 1$	Degree of implementation taller smokestacks for blast furnaces
$0 \leq To \leq 1$	Degree of implementation taller smokestacks for open-hearth furnaces
$0 \leq Fb \leq 1$	Degree of implementation filters for blast furnaces
$0 \leq Fo \leq 1$	Degree of implementation filters for open-hearth furnaces
$0 \leq Bb \leq 1$	Degree of implementation better fuels for blast furnaces
$0 \leq Bo \leq 1$	Degree of implementation better fuels for open-hearth furnaces

An Excel model was also developed, and it is shown below:

• **Table 2.1:** The Excel model with the optimal solution (obtained using Solver) and the same spreadsheet, but with the formulas made visible;

• **Table 2.2:** The Solver dialog box (with missing parts);

• **Table 2.3:** The sensitivity analysis report.

Table 2.1: Excel spreadsheet with optimal solution

	A	B	C	D	E	F	G	H
1		Taller Smokestacks		Filters		Better Fuels		Requirements
2		Blast	Open-Hearth	Blast	Open-Hearth	Blast	Open-Hearth	Required
3	Max Effect	Furnaces	Furnaces	Furnaces	Furnaces	Furnaces	Furnaces	Reduction
4	Particulates	12	9	25	20	17	13	60
5	Sulfur Oxides	35	42	18	31	56	49	150
6	Hydrocarbons	37	53	28	24	29	20	125
7								
8	Maximum Cost	€ 8	€ 10	€ 7	€ 6	€ 11	€ 9	
9								
10	Selection	1.00	0.62	0.34	1.00	0.05	1.00	
11								
12	Effect							Result
13	Particulates	12.00	5.60	8.59	20.00	0.81	13.00	60.00
14	Sulfur Oxides	35.00	26.15	6.18	31.00	2.66	49.00	150.00
15	Hydrocarbons	37.00	33.00	9.62	24.00	1.38	20.00	125.00
16								
17	Cost	€ 8.00	€ 6.23	€ 2.40	€ 6.00	€ 0.52	€ 9.00	€ 32.15

	A	B	C	D	E	F	G	H
1		**Taller Smokestacks**		**Filters**		**Better Fuels**		**Requirements**
2		**Blast**	**Open-Hearth**	**Blast**	**Open-Hearth**	**Blast**	**Open-Hearth**	**Required**
3	**Max Effect**	**Furnaces**	**Furnaces**	**Furnaces**	**Furnaces**	**Furnaces**	**Furnaces**	**Reduction**
4	Particulates	12	9	25	20	17	13	60
5	Sulfur Oxides	35	42	18	31	56	49	150
6	Hydrocarbons	37	53	28	24	29	20	125
7								
8	**Maximum Cost**	8	10	7	6	11	9	
9								
10	**Selection**	1	0.62	0.34	1	0.05	1	
11								
12	**Effect**							**Result**
13	Particulates	=B$10*B4	=C$10*C4	=D$10*D4	=E$10*E4	=F$10*F4	=G$10*G4	=SUM(B13:G13)
14	Sulfur Oxides	=B$10*B5	=C$10*C5	=D$10*D5	=E$10*E5	=F$10*F5	=G$10*G5	=SUM(B14:G14)
15	Hydrocarbons	=B$10*B6	=C$10*C6	=D$10*D6	=E$10*E6	=F$10*F6	=G$10*G6	=SUM(B15:G15)
16								
17	**Cost**	=B8*B10	=C8*C10	=D8*D10	=E8*E10	=F8*F10	=G8*G10	=SUM(B17:G17)

Table 2.2: Solver dialog box (with missing parts)

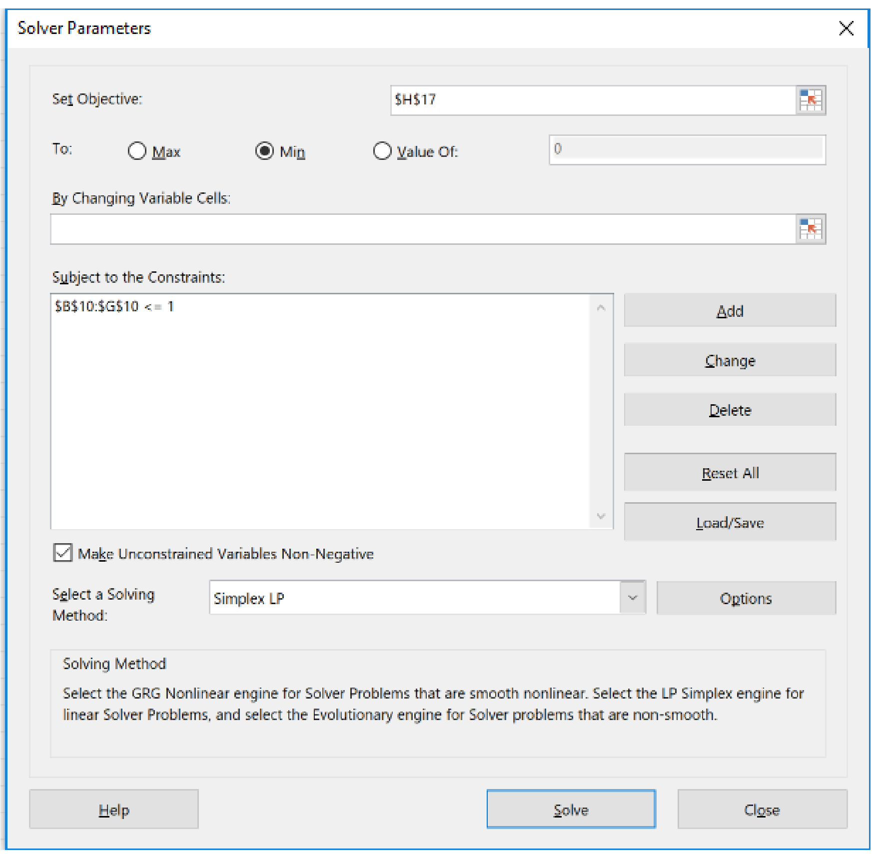

Table 2.3: Sensitivity Analysis report

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B10	Selection Furnaces	1.00	-0.34	8	0.34	1E+30
C10	Selection Furnaces	0.62	0.00	10	0.43	0.67
D10	Selection Furnaces	0.34	0.00	7	0.38	2.01
E10	Selection Furnaces	1.00	-1.82	6	1.82	1E+30
F10	Selection Furnaces	0.05	0.00	11	2.98	0.04
G10	Selection Furnaces	1.00	-0.04	9	0.04	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
H13	Particulates Reduction	60	0.11	60	14.30	7.48
H14	Sulfur Oxides Reduction	150	0.13	150	20.45	1.69
H15	Hydrocarbons Reduction	125	0.07	125	2.04	21.69

A. Complete the Solver dialog box.
B. Which Excel cell contains the objective function?
C. Which Excel cells contain the decision variables?
D. Interpret the solution. Which methods are to be implemented to which degree?
E. Interpret the solution. What it the cost of this policy?
F. Interpret the solution. What is the effect on pollution?

After looking at the solution suggested by the model, David spots a mistake in the model. He claims that implementing both the filter and better fuel options would not generate the combined effect of implementing both separately. Better fuels, to some extent, eliminate the effects of filters, since the fuels are already much cleaner. He suggests changing the model to prevent both filters and better fuels being chosen simultaneously, for both the blast furnaces and open-hearth furnaces.

G. How would you modify the model to incorporate David's suggestion to prevent selecting both better fuels and filters?