Matemática 3 - Resultados

Práctica 4

1. a) 0,68 b) 0,375

2. a) 0,5 b) 0,6875 c) 0,6328 d) $F(x) = \begin{cases} 0 & \text{si x} < -1 \\ \frac{3}{4}x - \frac{x^3}{4} + \frac{1}{2} & si - 1 \leq x < 1 \\ 1 & si\ x \geq 1 \end{cases}$

3. a) k = 3/2

   b) $F(x) = \begin{cases} 0 & \text{si x} < 0 \\ x^{\frac{3}{2}} & si\ 0 \leq x < 1 \\ 1 & si\ x \geq 1 \end{cases}$

   c) 0,3004

4. Respecto al ejercicio 1: E(X) = 1 E(X^2) = 7/6 V(X) = 1/6 dt(X) = 0,4082
   Respecto al ejercicio 2: E(X) = 0 E(X^2) = 1/5 V(X) = 1/5 dt(X) = 0,4472
   Respecto al ejercicio 3: E(X) = 0,6 E(X^2) =3/7 V(X) = 0,0686 dt(X) = 0,2619

5. 109

6. a) $F(x) = \begin{cases} 0 & \text{si x} < 0 \\ \frac{x^2}{48} - \frac{x^3}{864} & si\ 0 \leq x < 12 \\ 1 & si\ x \geq 12 \end{cases}$

   b) P(X ≤ 4)= $\frac{7}{27}$ =0,2593 P(X > 6)= 0,5 P(4 < X < 6) = $\frac{13}{54}$=0,2407

   c) 6

   d) $\frac{7}{27}$ = 0,2593

7. a) 0,6 b) 0,7 c) 0,5 d) E(X) = $\frac{17}{2}$ V(X) = $\frac{3}{4}$

8. a) P(Z ≤ 2,24) = 0.9875
   P(Z > 1,36) = 0.0869
   P(0 < Z < 1,5) = 0.4332
   P(0,3 < Z < 1,56) = 0.3227
   P(−0,51 < Z < 1,54) = 0.6332

   b) z = 0
   z = 1,03
   z = -2,55
   z = 1,645

9. a) 0,7257 b) 0,6753 c) 0,3783

10. a) 0,3085 b) 0,1336 c) 0,3942

11. a) 0,1889 b) 0,0357

12. a) 0,0498 b) 0,9502

13. a) 0,1353 b) 0,8647 c) 0,1733 horas (≈ 10,4 minutos)

14. No

PRÁCTICA 4

1) X: "tiempo en cientos de horas que el adolescente utiliza el celular"

a) $P(X<1,2)=\int_{-\infty}^{1,2} f(x)\,dx = \int_{-\infty}^{0} 0\,dx + \int_{0}^{1} x\,dx + \int_{1}^{1,2} (2-x)\,dx$

$= \frac{x^2}{2}\Big|_0^1 + \left(2x-\frac{x^2}{2}\right)\Big|_1^{2}$

$= \frac{1}{2} - 0 + \left[2\cdot 1,2 - \frac{1,2^2}{2} - \left(2-\frac{1}{2}\right)\right] = \boxed{0,68}$

b) $P(0,5 \le X \le 1)=\int_{0,5}^{1} f(x)\,dx = \int_{0,5}^{1} x\,dx = \frac{x^2}{2}\Big|_{0,5}^{1} =$

a) $P(x>0)=$

2) $\int_0^1 f(x)\,dx = \int_0^1 0,75(1-x^2)\,dx = \int_0^1 \frac{3}{4} - \frac{3}{4}x^2\,dx = \frac{3}{4}x\Big|_0^1 - \frac{1}{4}x^3\Big|_0^1 =$

$= \frac{3}{4}1 - \cancel{\frac{3}{4}0} - \left(\frac{1}{4}1^3 - \cancel{\frac{1}{4}0^3}\right) = \frac{3}{4} - \frac{1}{4} = \frac{1}{2} = \boxed{0,5}$

b) $P(-0,5 \le X \le 0,5) = \int_{-0,5}^{0,5} f(x)\,dx = \int_{-0,5}^{0,5} 0,75(1-x^2)\,dx = \int_{-0,5}^{0,5} \frac{3}{4} - \frac{3}{4}x^2\,dx =$

$\frac{3}{4}x\Big|_{-0,5}^{0,5} - \frac{1}{4}x^3\Big|_{-0,5}^{0,5} = \frac{3}{4}\cdot 0,5 - \frac{3}{4}\cdot -0,5 - \left(\frac{1}{4}\cdot 0,5^3 - \frac{1}{4}\cdot -0,5^3\right)$

$= \frac{3}{8} + \frac{3}{8} - \left(\frac{1}{32} + \frac{3}{32}\right) = \frac{6}{8} - \frac{1}{16} = 0,6875$

c) $P(X<-0,25 \text{ o } X>0,25) = P(X<-0,25) + P(X>0,25)$

$= \int_{-1}^{-0,25} f(x)\,dx + \int_{0,25}^{1} f(x)\,dx$

$= \int_{-1}^{-0,25} 0,75(1-x^2)\,dx + \int_{0,25}^{1} 0,75(1-x^2)\,dx = (*)$

$(*)\ \frac{3}{4}x\Big|_{-1}^{-0,25} - \frac{1}{4}x^3\Big|_{-1}^{-0,25} = \frac{9}{16} - \frac{63}{256} = 81/256$

$\frac{3}{4}x\Big|_{0,25}^{1} - \frac{1}{4}x^3\Big|_{0,25}^{1} = 81/256$

$81/256 \times 2 = \boxed{0,6328}$

$$\frac{3}{4}x\Big|_{-1}^{-0,25} = \frac{3}{4}\cdot\left(\frac{-1}{4}\right) - \left(\frac{3}{4}\cdot(-1)\right) = \frac{-3}{16} + \frac{3}{4} = \frac{9}{16}$$

$$\frac{1}{4}x^3\Big|_{-1}^{-0,25} = \frac{1}{4}\cdot\left(\frac{-1}{4}\right)^3 - \left(\frac{1}{4}\cdot(-1)^3\right) = \frac{1}{4}\cdot\left(\frac{-1}{64}\right) + \frac{1}{4} = \frac{63}{256}$$

$$\frac{3}{4}x\Big|_{0,25}^{1} = \frac{3}{4}\cdot 1 - \left(\frac{3}{4}\cdot\frac{1}{4}\right) = \frac{3}{4} - \frac{3}{16} = \frac{9}{16}$$

$$\frac{1}{4}x^3\Big|_{0,25}^{1} = \frac{1}{4}\cdot 1 - \left(\frac{1}{4}\cdot\frac{1}{64}\right) = \frac{1}{4} - \frac{1}{256} = \frac{63}{256}$$

D) $F(X) = \begin{cases} 0 & \text{si } x < -1 \\ \frac{3}{4}x - \frac{1}{4}x^3 & \text{si } -1 \leq x \leq 1 \\ 1 & \text{si } x > 1 \end{cases}$

3) $f(x) = \begin{cases} K\sqrt{x} & 0 < x < 1 \\ 0 & \text{c.c} \end{cases}$

a) $1 = P(-\infty < X < \infty) = \int_{-\infty}^{\infty} f(x)\,dx$

$\int_0^1 K\sqrt{x}\,dx = 1$ $\quad\rightarrow \frac{2}{3}K\left(1^{3/2} - \cancel{0^{3/2}}\right) = 1$

$K\int_0^x \sqrt{x}\,dx = 1$ $\quad \frac{2K}{3} = 1$

$K\left.\frac{x^{3/2}}{3/2}\right|_0^1 = 1$ $\quad \boxed{K = 3/2}$ $\quad f(x) = \begin{cases} 3/2\sqrt{x} & 0 < x < 1 \\ 0 & \text{c.c} \end{cases}$

b) $F(x) = P(X \leq x) = \int_{-\infty}^{x} 3/2\sqrt{t}\,dt$

$= \frac{3}{2}\int_0^x \sqrt{t}\,dt = \frac{3}{2}\cdot\left.\frac{t^{3/2}}{3/2}\right|_0^x = x^{3/2} - 0^{3/2} = \boxed{x^{3/2}}$

$F(x) = \begin{cases} 0 & \text{si } x < 0 \\ x^{3/2} & \text{si } 0 \leq x < 1 \\ 1 & \text{si } x \geq 1 \end{cases}$

c) $P(0,3 < X < 0,6) = F(0,6) - F(0,3) = 0,6^{3/2} - 0,3^{3/2} = 0,4648 - 0,1643 = \boxed{0,3004}$

4) Ej 1: $E(X)=\int_0^1 x\cdot x\,dx+\int_1^2 x\cdot(2-x)\,dx=\int_0^1 x^2\,dx+\int_1^2 2x-x^2\,dx=$

$=\frac{x^3}{3}\Big|_0^1+\left(x^2-\frac{x^3}{3}\right)\Big|_1^2=\frac{1}{3}+\left(4-\frac{8}{3}\right)-\left(1-\frac{1}{3}\right)=\boxed{1}$

$E(X^2)=\int_0^1 x^2\cdot x\,dx+\int_1^2 x^2(2-x)\,dx=\int_0^1 x^3\,dx+\int_1^2 2x^2-x^3\,dx=\frac{x^4}{4}\Big|_0^1+\left(\frac{2}{3}x^3-\frac{x^4}{4}\right)\Big|_1^2=$

$\frac{1}{4}+\left(\frac{2}{3}\cdot 2^3-\frac{2^4}{4}\right)-\left(\frac{2}{3}\cdot 1^3-\frac{1^4}{4}\right)=\frac{1}{4}+\frac{4}{3}-\frac{5}{12}=\frac{7}{6}$

$V(X)=\frac{7}{6}-1^2=1/6$

$dt(X)=\sqrt{1/6}=\boxed{0{,}4082}$

Ej 2: $E(X)=\int_{-1}^1 x\cdot(0{,}75(1-x^2))\,dx=\int_{-1}^1 0{,}75x-0{,}75x^3\,dx=0{,}75x\Big|_{-1}^1-0{,}75x^3\Big|_{-1}^1=$

$0{,}75+0{,}75-(0{,}75\cdot 1^3-0{,}75\cdot(-1)^3)=1{,}5-0{,}75-0{,}75=\boxed{0}$

$E(X^2)=0{,}75\int_{-1}^1 x^2(1-x^2)\,dx=0{,}75\int_{-1}^1 x^2-x^4\,dx=0{,}75\cdot\left(\frac{1}{3}x^3\Big|_{-1}^1-\frac{1}{5}x^5\Big|_{-1}^1\right)=$

$0{,}75\left(\frac{1}{3}+\frac{1}{3}-\left(\frac{1}{5}+\frac{1}{5}\right)\right)=0{,}75\cdot(4/15)=1/5$

$V(X)=\frac{1}{5}-0^2=\boxed{1/5}$

$dt(X)=\sqrt{1/5}=0{,}4472$

Ej 3: $E(X)=\int_0^1 x\cdot 3/2\sqrt{x}\,dx=\frac{3}{2}\int_0^1 \underbrace{x\cdot\sqrt{x}}_{x^{3/2}}\,dx=\frac{3}{2}\cdot\left(\frac{x^{5/2}}{5/2}\Big|_0^1\right)=\frac{3}{2}\cdot\left(\frac{1^{5/2}}{5/2}\right)=\frac{3}{5}\cdot 1=\boxed{3/5}$

$E(X^2)=3/2\int_0^1 x^2\sqrt{x}\,dx=\frac{3}{2}\int_0^1 x^{5/2}\,dx=\frac{3}{2}\left(\frac{x^{7/2}}{7/2}\Big|_0^1\right)=3/7$

$V(X)=\frac{3}{7}-\left(\frac{3}{5}\right)^2=\frac{3}{7}-\left(\frac{9}{25}\right)=0{,}0686$

$dt(X)=\sqrt{0{,}0686}=\boxed{0{,}2619}$

5) $E(Y) = E(Y(X)) = \int_{-\infty}^{\infty} Y(x) f(x) dx = \int_0^1 (60x^2+39x) \cdot x \, dx + \int_1^2 (60x^2+39x) \cdot (2-x) dx =$

$= \int_0^1 60x^3 + 39x^2 + \int_1^2 120x^2 - 60x^3 + 78x - 39x^2 \, dx = 60\int_0^1 x^3 dx + 39\int_0^1 x^2 dx + 81\int_1^2 x^2 dx -$

$60\int_1^2 x^3 dx + 78\int_1^2 x \, dx = 15 \cdot x^4 \Big|_0^1 + 13 \cdot x^3 \Big|_0^1 + 27x^3 \Big|_1^2 - 15 \cdot x^4 \Big|_1^2 + 39x^2 \Big|_1^2 =$

$= 15 + 13 + (27 \cdot 8 - 27) - (15 \cdot 16 - 15) + (39 \cdot 4 - 39) = 15 + 13 + 189 - 225 + 117 = \boxed{109}$

Teorema: Si X es una v.a continua con f.d.p. $f(x)$ y $h(X)$ es cualquier función de X, entonces

$$E(h(X)) = \int_{-\infty}^{\infty} h(x) \cdot f(x) dx$$

6) a) $F(x) = P(X \leq x)$

$\int_0^x 1/24 \, t(1 - t/12) dt = \frac{1}{24}\int_0^x T dt - \frac{1}{288}\int_0^x T^2 dt = \frac{1}{24} \cdot \frac{T^2}{2}\Big|_0^x - \frac{1}{288} \cdot \frac{T^3}{3}\Big|_0^x =$

$\frac{1}{48} \cdot (x^2) - \frac{1}{864} \cdot (x^3) =$

$$F(x) = \begin{cases} 0 & \text{Si } x < 0 \\ -\frac{x^3}{864} + \frac{1}{48}x^2 & \text{Si } 0 \leq x < 12 \\ 1 & \text{Si } x \geq 12 \end{cases}$$

b) $P(X \leq 4) = F(4) = -\frac{4^3}{864} + \frac{1}{48} \cdot 4^2 = \frac{-2}{27} + \frac{1}{3} = \boxed{0{,}2593}$

c) $E(X) = \int_0^{12} x \frac{1}{24} \cdot x \left(1 - \frac{x}{12}\right) dx = \int_0^{12} \frac{1}{24} x^2 - \int_0^{12} \frac{1}{288} x^3 dx =$

$= \frac{1}{24} \cdot \left(\frac{x^3}{3}\Big|_0^{12}\right) - \frac{1}{288} \cdot \left(\frac{x^4}{4}\Big|_0^{12}\right) = \frac{1}{24} \cdot 576 - \frac{1}{288} \cdot 5184 = \boxed{6}$

D) $P(X > E(X) + 2) = P(X > 8) = 1 - P(X \leq 8) = 1 - F(8) = 1 - \left(-\frac{8^3}{864} + \frac{1}{48} \cdot 8^2\right) =$

$= 1 - \left(\frac{-16}{27} + \frac{4}{3}\right) = 1 - \frac{20}{27} = 7/27 = \boxed{0{,}2593}$

$X \sim N(\overset{\text{media}}{0}, \underset{\text{varianza}}{1})$

8) a) a1) $P(Z \leq 2{,}24) = \Phi(2{,}24) \overset{\text{App}}{=} \boxed{0{,}9875}$

a2) $P(Z > 1{,}36) = 1 - P(Z \leq 1{,}36) = 1 - \Phi(1{,}36) \overset{\text{App}}{=} 1 - 0{,}9131 = \boxed{0{,}0869}$

a3) $P(0 < Z < 1{,}5) = \Phi(1{,}5) - \Phi(0) \overset{\text{App}}{=} 0{,}9331 - 0{,}5 = \boxed{0{,}4331}$

a4) $P(0{,}3 < Z < 1{,}56) = \Phi(1{,}56) - \Phi(0{,}3) \overset{\text{App}}{=} 0{,}9406 - 0{,}6179 = \boxed{0{,}3227}$

a5) $P(-0{,}51 < Z < 1{,}54) = \Phi(1{,}54) - \Phi(-0{,}51) \overset{\text{App}}{=} 0{,}9382 - 0{,}305 = \boxed{0{,}6332}$

b) b1) $P(Z \leq z) = 0{,}5 \overset{\text{App}}{=} P(Z < 0) = 0{,}5$ ⊛ $\hookrightarrow \boxed{z = 0}$

9) $X \sim N(\overset{\text{Media}}{10}, \overset{\text{Varianza}}{36}) \quad X \sim N(\mu, \sigma^2)$

$Z = \dfrac{X - \mu}{\sigma} \sim N(0,1)$

a) $P(X > 6{,}4) = 1 - P(X \leq 6{,}4) = 1 - P\left(\overset{\sim N(0,1)}{\dfrac{X-10}{6}} \leq \dfrac{6{,}4 - 10}{6}\right)$

$= 1 - \Phi(-0{,}6) \overset{\text{App}}{=} 1 - 0{,}27425 = \boxed{0{,}72575}$

b) $P(4{,}2 < X < 16) = P(X < 16) - P(4{,}2)$

$= P\left(\dfrac{X-10}{6} < \dfrac{16-10}{6}\right) - P\left(\dfrac{X-10}{6} < \dfrac{4{,}2-10}{6}\right)$

$= \Phi(1) - \Phi(-0{,}96) \overset{\text{App}}{=} 0{,}8413 - 0{,}1685 = \boxed{0{,}6728}$

c) $P(X \leq 8{,}14) = P\left(\dfrac{X-10}{6} \leq \dfrac{8{,}14-10}{6}\right) = \Phi(-0{,}31) \overset{\text{App}}{=} \boxed{0{,}3783}$

$\circledast$ b2) $P(Z < z) = 0{,}8485$ $\to z = 1{,}03$

$P(Z < 1{,}03) = 0{,}8485$

b3) $P(Z < z) = 0{,}0054$ $\to z = -2{,}55$

$P(Z < -2{,}55) = 0{,}0054$

b4) $P(-z < Z < z) = 0{,}90$

$\Phi(z) - \Phi(-z) = 0{,}90$

$\Phi(z) - (1 - \Phi(z)) = 0{,}90$

$-1 + 2\Phi(z) = 0{,}90$

$2\Phi(z) = 0{,}90 + 1$

$\Phi(z) = 1{,}9 : 2$

$\Phi(z) = 0{,}95 \Longrightarrow \boxed{z = 1{,}6448}$

7) X: "Cantidad de café diaria en litros que sirve una máquina que se localiza en el vestíbulo de un aeropuerto"

$X \sim U(7, 10)$

a) $F(x) = \begin{cases} 0 & \text{si } x < 7 \\ \frac{x-7}{3} & \text{si } 7 \leq x < 10 \\ 1 & \text{si } x \geq 10 \end{cases}$

a) $P(X \leq 8{,}8) = F(8{,}8) = \frac{8{,}8-7}{3} = \frac{1{,}8}{3} = \boxed{0{,}6}$

b) $P(7{,}4 < X < 9{,}5) = P(X < 9{,}5) - P(X < 7{,}4)$ $\to$ por ser V.A. continuas

$= F(9{,}5) - F(7{,}4)$

$= \frac{2{,}5}{3} - \frac{0{,}4}{3} = \boxed{0{,}7}$

c) $P(X \geq 8{,}5) = 1 - P(X < 8{,}5)$

$= 1 - F(8{,}5)$

$= 1 - \frac{1{,}5}{3} = \boxed{0{,}5}$

D) $E(X) = \frac{7+10}{2} = \boxed{\frac{17}{2}}$ $V(X) = \frac{(10-7)^2}{12} = \frac{9}{12} = \boxed{\frac{3}{4}}$

10) $E(X) = 3 \quad \sigma = 0{,}05$

a) $P(X > 3{,}025) = 1 - P(X \leq 3{,}025)$ $\sim N(0,1)$

$= 1 - P\left(\frac{X - \mu}{\sigma} \leq \frac{3{,}025 - 3}{0{,}05}\right)$

$= 1 - P(Z \leq 0{,}5)$

$= 1 - \Phi(0{,}5) \underset{\text{App}}{=} 1 - 0{,}6915 = \boxed{0{,}3085}$

b) $P(X < 2{,}925) + P(X > 3{,}075) = 1 - P(X \leq 2{,}925) + 1 - P(X \leq 3{,}075)$

$= 1 - P\left(\frac{X-\mu}{\sigma} \leq \frac{2{,}925 - 3}{0{,}05}\right) + 1 - P\left(\frac{X-\mu}{\sigma} \leq \frac{3{,}075 - 3}{0{,}05}\right)$

$= 1 - P(Z \leq -1{,}5) + 1 - P(Z \leq 1{,}5)$

$= 1 - \Phi(-1{,}5) + 1 - \Phi(1{,}5)$

$= 1 - 0{,}0668 + 1 - 0{,}9332 = \boxed{0{,}1336}$

c) X: "nº de comprimidos defectuosos en una caja"

$X \sim B(10,\ 0{,}1336) \qquad P(X = k) = \binom{n}{k} p^k \cdot (1-p)^{n-k}$

$P(X \geq 2) = 1 - P(X < 2) = 1 - P(X = 0) + P(X = 1) = 1 - 0{,}2383 - 0{,}3675 = \boxed{0{,}3942}$

$P(X = 0) = \binom{10}{0} 0{,}1336^0 \cdot (1 - 0{,}1336)^{10} \underset{\text{App}}{=} 0{,}2383$

$P(X = 1) = \binom{10}{1} \cdot 0{,}1336^1 \cdot (1 - 0{,}1336)^9 \underset{\text{App}}{=} 0{,}3675$

11) X: "tiempo de respuesta (en segundos) de cierto sistema de computadoras"

$X \sim Exp(1/3) \qquad E(X) = 3 = 1/\lambda \rightarrow \lambda = 1/3$

$F(x) = \begin{cases} 1 - e^{-\lambda x} & \text{si } x \geq 0 \\ 0 & \text{c.c} \end{cases}$

a) $P(X > 5) = 1 - P(X \leq 5)$

$= 1 - F(5)$

$= 1 - \left(1 - e^{-\frac{1}{3} \cdot 5}\right) \underset{\text{App}}{=} \boxed{0{,}1889}$

b) $P(X>10) = 1-P(X\leq 10)$

$= 1-F(10)$

$= 1-(1-e^{-\frac{1}{3}\cdot 10}) \underset{\uparrow}{=} \boxed{0{,}0357}$

App

=

12) $\lambda = 3$ por minuto $\quad X_1 \sim P(\lambda=3)$

X_1 = "n° de visitas a un sitio web en un minuto"

a) $P(X=0) = e^{-3} \cdot \frac{3^0}{0!} = \boxed{0{,}0498}$

b) X_i = "n° de visitas en un sitio web en i minutos", $X_i \sim P(\lambda = 3i)$

T = "tiempo en minutos entre dos visitas a la web consecutivas" $\quad T \sim Exp(\lambda=3)$

$P(T<3 \mid T>2) = \frac{P(T<3 \cap T>2)}{P(T>2)} = \frac{P(2<T<3)}{1-P(T\leq 2)} \underset{\uparrow}{=} \frac{F(3)-F(2)}{1-F(2)} =$

V.a. cont

App

$= \frac{0{,}9999-0{,}9975}{1-0{,}9975} = \boxed{0{,}96}$

13) X: "tiempo, en horas, empleado diariamente en transporte"

$E(X)=0{,}25 \quad 1/\lambda = 0{,}25 \quad \lambda = 4$

$X \sim Exp(4)$

a) $P(X>0{,}5) \underset{\uparrow}{=} 1-P(X\leq 0{,}5) = 1-(1-e^{-4\cdot 0{,}5}) = \boxed{0{,}1353}$

prop. complemento

b) $P(X\leq 1{,}5 \mid X\geq 1) = \frac{P(X\leq 1{,}5 \cap X\geq 1)}{P(X\geq 1)} \underset{\uparrow}{=} \frac{P(1\leq X\leq 1{,}5)}{1-P(X<1)} = \frac{F(1{,}5)-F(1)}{1-F(1)} =$

complemento

App

$= \frac{0{,}9975-0{,}9817}{1-0{,}9817} = \boxed{0{,}8634}$

c) $P(X>t) = 0{,}5$

$1-P(X\leq t) = 0{,}5$

$0{,}5 = F(t)$

$1-e^{-4t} = 0{,}5$

$e^{-4t} = 0{,}5$

$\ln(e^{-4t}) = \ln(0{,}5)$

$-4t = \ln(0{,}5)$

$t = \ln(0{,}5) / -4 = \boxed{0{,}1732 \text{ hs}}$

14) Las duraciones de los comp. no cumplen con la prop. de falta de memoria para la distrib. exponencial $\therefore$ No es posible que las duraciones de los comp se distribuyan exponencialmente