EJERCICIO 10

a. b. $T(n) = \begin{cases} 2 & n = 1 \\ T(n-1)+n & , n \geq 2 \end{cases}$

Suponiendo que $n \geq 2$:

Paso 1: $T(n) = T(n-1)+n$

Paso 2: $T(n) = T(n-2)+(n-1)+n$

Paso 3: $T(n) = T(n-3)+(n-2)+(n-1)+n$

Paso i: $T(n) = T(n-i)+(n-i+1)+(n-i+2)+\dots+\underbrace{(n-i+i)}_{N}$

$T(n) = \overset{T(1)}{2} + \sum_{i=2}^{n} i$

$$T(n) = 2 + \sum_{i=1}^{n} i - 1 = 2 + \frac{n(n+1)}{2} - 1 = 1 + \frac{n^2+n}{2} \in O(n^2)$$

Existen constantes $c>0$ y N_0 tales que:

$1 + \frac{N^2+n}{2} \leq c.n^2$ para todo $n \geq n_0$ $n \in \mathbb{N}$

$1 \leq c_1.n^2$	$(n^2+n):2 \leq c_2 n^2$
$1/n^2 \leq c_1$	$\frac{n+1}{2n} \leq c_2$
$n_1 = 1$	
$c_1 = 1$	$n_2 = 1$
	$c_2 = 1$

$1 + \frac{n^2+n}{2} \leq (c_1+c_2).n^2$

$T(n) \leq (1+1).n^2$

$T(n) \leq c.n^2$

$T(n) \leq O(n^2)$, con $c=2$ para todo $n \geq n_0$, con $n_0 = 1$

GUAYNAS

$$T(n)\begin{cases} 2 & , n=1 \\ T(n-1)+n/2 & , n\geq 2 \end{cases}$$

Paso 1: $T(n-1)+n/2$

Paso 2: $T(n)=T(n-2)+\frac{n-1}{2}+n/2$

Paso 3: $T(n)=T(n-3)+\frac{n-2}{2}+\frac{n-1}{2}+n/2$

Paso i: $T(n)=T(n-i)+\frac{(n-i+1)}{2}+\frac{(n-i+2)}{2}+\dots+\frac{n}{2}$

$n-i=1$
$i=n-1$

Reemplazo i:

$$T(n)=T(1)+\frac{(n-(n-1)+1)}{2}+\frac{(n-(n-1)+2)}{2}+\dots+\frac{n}{2}$$

$$T(n)=2+\frac{1}{2}+\frac{3}{2}+2+\frac{5}{2}+\dots+\frac{n}{2}$$

$$T(n)=2+\sum_{i=2}^{n} i\cdot\frac{1}{2}$$

$$T(n)=2+\frac{1}{2}\cdot\left(\frac{n\cdot(n+1)}{2}-1\right)=2+\frac{1}{2}\left(\frac{n^2+n}{2}-1\right)=2+\frac{n^2}{4}+\frac{n}{4}-\frac{1}{2}=\frac{n^2}{4}+\frac{n}{4}+\frac{3}{4} \therefore O(n^2)$$

Existen constantes $c>0$ y N_0 tales que:

$\frac{n^2}{4}+\frac{n}{4}+\frac{3}{4}\leq c\cdot n^2$ para todo $n\geq n_0$ $n\in\mathbb{N}$

PRIMER TÉRMINO	SEGUNDO TÉRMINO	TERCER TÉRMINO:
$\frac{n^2}{4}\leq c_1 n^2$	$\frac{n}{4}\leq c_2\cdot n^2$	$\frac{3}{4}\leq c_3\cdot n_3$
$\frac{n^2}{4}:n^2\leq c_1$	$1/4n\leq c_2$	$\frac{3}{4n}\leq c_3$
$\frac{1}{4}\leq c_1$	$n_2=1$ $c_2=1$	$n_3=1$ $c_3=1$
$n_1=0$ $c_1=1$		

$$\frac{n^2}{4}+\frac{n}{4}+\frac{3}{4} \leq (c_1+c_2+c_3).n^2$$

$$T(n) \leq (1+1+1).n^2$$

$$T(n) \leq c.n^2$$

$T(n) \leq O(n^2)$ con $c=3$ para todo $n \geq n_0$, con $n_0=1$

$$T(n)=\begin{cases} 1 & n=1 \\ 2T\left(\frac{n}{4}\right)+\sqrt{n} & , n\geq 2 \end{cases}$$

Paso 1: $T(n)=2T(n/4)+\sqrt{n}$

Paso 2: $T(n)=2\left[2T(n/16)+\sqrt{(n/4)}\right]+\sqrt{n}$

$$T(n)=4T(n/16)+\cancel{2}\frac{\sqrt{n}}{\cancel{2}}+\sqrt{n}=4T(n/16)+2\sqrt{n}$$

Paso 3: $T(n)=4\left[2T(n/48)+\sqrt{(n/16)}\right]+2\sqrt{n}$

$$T(n)=8T(n/48)+\cancel{4}\frac{\sqrt{n}}{\cancel{4}}+2\sqrt{n}=8T(n/48)+3\sqrt{n}$$

Paso i: $T(n)=2^i\,T(n/4^i)+i.\sqrt{n} \quad \Rightarrow n/4^i=1 \rightarrow \log_4 n=i$

Reemplazo i: $T(n)=2^{\log_4 n}.T(n/4^{\log_4 n})+\log_4 n.\sqrt{n}$

$$T(n)=\sqrt{2^{\log_2 n}}.T(1)+\log_4 n.\sqrt{n}=\sqrt{n}.1+\log_4(n).\sqrt{n} \therefore O(\log_4(n).\sqrt{n})$$

Existen constantes $c>0$ y N_0 tales que:

$\sqrt{n}+\log_4(n).\sqrt{n} \leq c.\log_4(n).\sqrt{n}$ para todo $n \geq n_0 \quad n \in \mathbb{N}$

$\sqrt{n} \leq c_1.\log_4(n).\sqrt{n}$	$\log_4(n).\sqrt{n} \leq c_2.\log_4(n).\sqrt{n}$	$\sqrt{n}+\log_4(n).\sqrt{n} \leq (c_1+c_2).\sqrt{n}.\log_4(n)$
$\sqrt{n}:\sqrt{n} \leq c_1.\log_4(n)$	$(\log_4(n).\sqrt{n}):(\log_4(n).\sqrt{n}) \leq c_2$	$T(n) \leq (1+1).f(n)$
$1 \leq c_1.\log_4(n)$	$1 \leq c_2$	$T(n) \leq c.f(n)$
$n_1=4$ $c_1=1$	$n_2=1$ $c_2=1$	$T(n) \leq O(\log_4(n).\sqrt{n})$ para todo $n \geq n_0$, con $n_0=4$; y con $c=2$

GUAYMAS

$$T(n) = \begin{cases} 1 & , n=1 \\ 4T(n/2)+n^2 & , n \geq 2 \end{cases}$$

Paso 1: $4T(n/2)+n^2$

Paso 2: $T(n) = 4[4T(n/4)+(n/2)^2]+n^2$

$$T(n) = 16T(n/4) + \cancel{4}(n^2/\cancel{4}) + n^2 = 16T(n/4)+2n^2$$

Paso 3: $T(n) = 16[4T(n/8)+(n/4)^2]+2n^2$

$$T(n) = 64T(n/8) + \cancel{16}(n^2/\cancel{16}) + 2n^2 = 64T(n/8)+3n^2$$

Paso i: $T(n) = 4^i T(n/2^i) + i.n^2$

$n/2^i = 1$

$i = \log_2 n$

Reemplazo i: $T(n) = 4^{\log_2 n} . T(n/2^{\log_2 n}) + \log_2 n . n^2$

$$T(n) = (2.2)^{\log_2 n} . T(1) + \log_2 n . n^2$$

$$T(n) = n.n.1 + \log_2 n . n^2 \therefore \log_2(n) n^2$$

Existen constantes $c>0$ y N_0 tales que:

$n^2+\log_2(n).n^2 \leq c.\log_2(n).n^2$ para todo $n>n_0$ $n \in \mathbb{N}$

$n^2 \leq c_1.\log_2(n).n^2$	$\log_2(n).n^2 \leq c_2.\log_2(n).n^2$
$n^2/n^2 \leq c_1.\log_2(n)$	$(\log_2(n).n^2)/(\log_2(n).n^2) \leq c_2$
$1 \leq c_1.\log_2(n)$	$1 \leq c_2$
$n_1=2$	$n_2=1$
$c_1=1$	$c_2=1$

↳ $n^2+\log_2(n).n^2 \leq (c_1+c_2).(\log_2(n).n^2)$

$$T(n) \leq (1+1).(\log_2(n).n^2)$$

$$T(n) \leq c.(\log_2(n).n^2)$$

$T(n) \leq O(\log_2(n).n^2)$, con $c=2$ para todo $n \geq n_0$, con $n_0=2$