GUAYMAS

EJERCICIO 13

$\boxed{1.}$ $T(n) = \begin{cases} 1 & , n \leq 1 \\ T(n-1)+c & , n \leq 2 \end{cases}$

Paso 1: $T(n) = T(n-1)+c$

Paso 2: $T(n) = T(n-2)+c+c$
$T(n) = T(n-2)+2c$

Paso 3: $T(n) = T(n-3)+c+2c$
$T(n) = T(n-3)+3c$

Paso i: $T(n) = T(n-i)+i.c$

$n-i=1$
$i=n-1$

Reemplazo i: $T(n) = T(n-n+1)+(n-1).c$

$T(n) = T(1)+nc-c$

$T(n) = 1+nc-c \therefore O(n)$

$\boxed{2.}$ $T(n) = \begin{cases} 1 & , n=1 \\ T(n/2)+c & , n \geq 2 \end{cases}$

Paso 1: $T(n) = T(n/2)+c$

Paso 2: $T(n) = T(n/4)+c+c$
$T(n) = T(n/4)+2c$

Paso 3: $T(n) = T(n/8)+2c+c$
$T(n) = T(n/8)+3c$

Paso i: $T(n) = T(n/2^i)+i.c$

$n/2^i = 1$
$n = 2^i$
$i = \log_2 n$

Reemplazo i: $T(n) = T(n/2^{\log_2 n})+\log_2 n.c$

$T(n) = T(1)+\log_2 n.c$

$T(n) = 1+\log_2 n.c \therefore O(\log_2(n))$

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3. $T(n) = \begin{cases} 1 & n=1 \\ 2T(n/2)+c & ,n \geq 2 \end{cases}$

Paso 1: $T(n) = 2T(n/2)+c$

Paso 2: $T(n) = 2\left[2T(n/4)+c\right]+c$

$T(n) = 4T(n/4)+3c$

Paso 3: $T(n) = 4\left[2T(n/8)+c\right]+3c$

$T(n) = 8T(n/8)+7c$

Paso i: $T(n) = 2^i T(n/2^i) + (2^i-1).c$

$n/2^i = 1$

$i = \log_2 n$

Reemplazo i: $T(n) = 2^{\log_2 n}.T(n/2^{\log_2 n}) + (2^{\log_2 n}-1).c =$

$T(n) = n.T(1) + (n-1).c =$

$T(n) = n + n.c - c \therefore O(n)$

4. $T(n) = \begin{cases} 1 & ,n \leq 5 \\ T(n-5)+c & ,n \geq 6 \end{cases}$

Paso 1: $T(n) = T(n-5)+c$

Paso 2: $T(n) = T(n-10)+c+c$

$T(n) = T(n-10)+2c$

Paso 3: $T(n) = T(n-15)+3c$

Paso i: $T(n) = T(n-5i)+ic$

$n-5i = 5$

$n = 5+5i$

$n-5 = 5i$

$\frac{n-5}{5} = i$

$\frac{n}{5}-1 = i$

Reemplazo i: $T(n) = T\left(n-5.\left(\frac{n}{5}-1\right)\right) + \left(\frac{n}{5}-1\right).c$

$T(n) = T\left(n - \frac{5n}{5} + 5\right) + \frac{nc}{5} - c$

$T(n) = T(5) + \frac{n}{5}c - c$

$T(n) = 1 + \frac{n}{5}.c - c \therefore O(n)$

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$\boxed{5.}$ $T(n) = \begin{cases} 1 & , n = 1 \\ 2T(n-1) + c & , n \geq 2 \end{cases}$

Paso 1: $T(n) = 2T(n-1) + c$

Paso 2: $T(n) = 2[2T(n-2) + c] + c$

$T(n) = 4T(n-2) + 3c$

Paso 3: $T(n) = 4[2T(n-3) + c] + 3c$

$T(n) = 8T(n-3) + 7c$

Paso i: $T(n) = 2^i\, T(n-i) + (2^i - 1).c$

$n - i = 1$

$i = n - 1$

Reemplazo i: $T(n) = 2^{n-1} . T(n-n+1) + (2^{n-1} - 1).c$

$T(n) = 2^{n-1} . T(1) + (2^{n-1} - 1).c$

$T(n) = \frac{2^n}{2} . 1 + \left(\frac{2^n}{2} - 1\right).c$

$T(n) = 2^n . \frac{1}{2} + \left(2^n . \frac{1}{2} - 1\right).c$

$T(n) = 2^n . 1/2 + 2^n . c/2 - c \therefore O(2^n)$

$\boxed{6.}$ $T(n) = \begin{cases} 1 & , n \leq 7 \\ T(n/8) + c & , n \geq 8 \end{cases}$

Paso 1: $T(n/8) + c$

Paso 2: $T(n) = T(n/64) + c + c$

$T(n) = T(n/64) + 2c$

Paso 3: $T(n) = T(n/512) + 3c$

Paso i: $T(n) = T(n/8^i) + i.c$

$n/8^i = 7$

$\log_8(n/7) = i$

Reemplazo i: $T(n) = T(\cancel{n}/8^{\log_8(n/7)}) + \log_8(n/7).c$

$T(n) = T(7) + (\log_8(n) - \log_8(7)).c$

$T(n) = 1 + \log_8(n).c - \log_8(7).c \therefore O(\log_8(n))$