



Brief paper

Synthesis of Petri net supervisors for FMS via redundant constraint elimination[☆]



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ABSTRACT

The Minimal number of Control Places Problem (MCP), which is formulated to obtain optimal and structurally minimal supervisors, needs extensive computation. The current methods to reduce the computational burden have mainly focused on revision of the original formulation of MCP. Instead, this paper presents methods to accelerate its solution by eliminating its redundant reachability constraints. The optimization problem scale required for supervisor synthesis is thus drastically reduced. First, a sufficient and necessary condition for a reachability constraint to be redundant is established in the form of an integer linear program (ILP), based on a newly proposed concept called feasible region of supervisors. Then, two kinds of redundancy elimination methods are proposed: an ILP one and a non-ILP one. Most of the redundant reachability constraints can be eliminated by our methods in a short time. The computational time to solve MCP is greatly reduced after the elimination, especially for large-scale systems. The obtained supervisors are still optimal and structurally minimal. Finally, numerical tests are conducted to show the efficiency and effectiveness of the proposed methods.

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1. Introduction

Various types of jobs concurrently compete for a limited number of resources such as numerically controlled machines, robots, buffers, sensors, and inspection stations in a flexible manufacturing system (FMS). Improper resource allocation may cause deadlocks, which may block and stall all activities in FMS.

Petri nets (PNs) play an important role in modeling and analyzing the behavior of FMS (Huang, Jiang, & Zhang, 2014; Huang, Shi, & Xu, 2012; Murata, 1989; Wu, 1999; Zhou & DiCesare, 1993; Zhou, DiCesare, & Desrochers, 1992; Zhou & Wu, 2010) and addressing

the deadlock problems (Ezpeleta, Colom, & Martinez, 1995; Hu, Zhou, Li, & Tang, 2013; Li, Hu, & Wang, 2007; Li & Zhou, 2006). To prevent deadlocks in FMSs, we mainly have two kinds of PN-based analysis methods: structural analysis (Huang, Jeng, Xie, & Chung, 2001; Li & Zhou, 2006; Wang, Wang, & Yu, 2013; Wang, Wang, Yu, & Zhao, 2012b; Wang, Wang, Zhou, & Li, 2012a; Wang, Wu, & Yang, 2015; Xing, Zhou, Shi, & Ren, 2008) and reachability graph analysis (Chen & Li, 2012; Chen, Li, Khalgui, & Mosbahi, 2011; Ghaffari, Rezg, & Xie, 2003; Uzam & Zhou, 2006). The former often obtains a control policy by controlling special structures of a PN model, e.g., resource-transition circuits (Xing et al., 2008; Xing, Zhou, Wang, Liu, & Tian, 2011) and siphons (Huang et al., 2001; Li & Zhou, 2006; Liu, Li, & Zhou, 2013). The control law of this method is usually simple but the resultant model is not optimal in general. The latter can obtain a controlled model with optimal or nearly-optimal behavior. Note that optimality in this work means the maximal permissiveness in terms of reachable states excluding those deadlocks and states that inevitably evolve into deadlocks.

In Chen and Li (2011), a reachability graph-based method is proposed to obtain an optimal liveness-enforcing supervisor with the fewest control places for FMS modeled by PN. The reachability graph of the PN is divided into a live zone (LZ) and a deadlock zone (DZ). The markings in LZ are the legal ones of an FMS, and

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those in DZ are deadlocks or will inevitably lead to deadlocks. First-met bad markings (FBMs) first proposed by Uzam and Zhou (2006) are those in DZ that are immediately reachable from some in LZ. A vector covering method (Chen et al., 2011) is used to reduce the number of markings to be considered in supervisor synthesis from the set of legal markings and the set of FBMs to a minimal covering set \mathcal{M}_L^* of legal markings and a minimal covered set \mathcal{M}_F^* of FBMs, respectively. An optimal supervisor with the fewest control places can be obtained by solving a Minimal number of Control Places Problem (MCP) to forbid all markings in \mathcal{M}_F^* and permit all markings in \mathcal{M}_L^* when MCP has an optimal solution. The supervisor obtained by the method is optimal and structurally minimal in terms of the number of control places. However, it has a limitation: the computational burden is extremely heavy, especially for large models.

Existing speedup techniques have mainly centered on revising the formulation of MCP. Chen, Li, and Zhou (2012) propose an iterative method to design an optimal supervisor via place invariants (PIs). At each iteration, a Maximal number of Forbidding FBM Problem (MFFP) is solved to forbid as many FBMs as possible while permitting all legal markings. This method can reduce the computational time greatly, but it cannot guarantee the structural minimality of its derived supervisor. To find an optimal and structurally minimal supervisor quickly, Chen and Li (2012) propose a Minimal number of P-semiflows Problem (MPP) that has fewer constraints and variables than MCP. However, MPP has an initially undecidable parameter, i.e., n_l , the number of PIs to be computed, and its efficiency greatly depends on the initial selection of n_l . Moreover, if n_l is set to be less than the unknown minimal number of control places of the problem, MPP fails to generate any solution.

Inevitably, an integer linear program (ILP) should be solved in MCP. Yet, the methods able to accelerate MCP solution by eliminating its redundant constraints have not been investigated. Our previous work (Huang, Zhu, Zhang, & Lu, 2015) proposed an inspiring method to eliminate redundant constraints of an ILP in supervisor synthesis. However, it is conducted in the context of designing an optimal PN supervisor with self-loops. This paper sheds new light on MCP simplification by eliminating redundant reachability constraints. As a matter of fact, most of the constraints in MCP are reachability ones, and many of them are redundant in supervisor synthesis. If they were eliminated, the scale of MCP would be reduced, thereby speeding up the synthesis process.

In order to define a redundant reachability constraint, the concept of a feasible region of supervisors is introduced. It is defined as a set of all feasible combinations of control places for which all constraints in MCP are satisfied. Based on this definition, a reachability constraint is said to be redundant if it can be eliminated without changing our concerned feasible region. A redundant constraint is inactive for all feasible supervisors. Therefore, its elimination does not change the solution to MCP.

The rest of this paper is organized as follows. Section 2 gives the MCP problem formulation. The definition of a redundant constraint and two kinds of redundant constraint elimination methods are proposed in Section 3. Section 4 provides experimental results. Finally, conclusions are given in Section 5.

2. Minimal number of control places problem

The structure optimization of an optimal supervisor is formulated as an MCP (Chen & Li, 2011) with an objective function to minimize the number of control places in the supervisor. To aid supervisor synthesis, minimal covering set \mathcal{M}_L^* of legal markings and minimal covered set \mathcal{M}_F^* of FBMs are used. They can be calculated by a vector covering method, which can be found in Chen and Li (2011). Some notations are introduced as follows and MCP is described in (1).

- a number of activity places in the PN model;
- b number of markings in \mathcal{M}_F^* ;
- c number of markings in \mathcal{M}_L^* ;
- D $(b \cdot c) \times a$ integer matrix of marking differences between $M_l \in \mathcal{M}_L^*$ and $M_j \in \mathcal{M}_F^*$, whose element is $d_{l,j}(p_i) = M_l(p_i) - M_j(p_i)$ with $i \in \{1, 2, \dots, a\}$;
- $d_{l,j}$ a row of D with $l \in \{1, 2, \dots, c\}$ and $j \in \{1, 2, \dots, b\}$;
- an a integer vector of marking differences between $M_l \in \mathcal{M}_L^*$ and $M_j \in \mathcal{M}_F^*$;
- $D^{(*,j)}$ a matrix obtained by including the rows related to $M_j \in \mathcal{M}_F^*$ in D ;
- $\tilde{D}^{(l,j)}$ a matrix obtained by eliminating $d_{l,j}$ from D with $l \in \{1, 2, \dots, c\}$ and $j \in \{1, 2, \dots, b\}$;
- $\tilde{D}^{(l,*j)}$ a matrix obtained by eliminating $d_{l,j}$ and the rows related to the found redundant constraints from $D^{(*,j)}$;
- $\tilde{D}_u^{(l,*j)}$ the u th row of $\tilde{D}^{(l,*j)}$;
- E $(b \cdot (b-1)) \times a$ integer matrix of marking differences between $M_k \in \mathcal{M}_F^*$ and $M_j \in \mathcal{M}_F^*$ ($k \neq j$), whose element is $e_{k,j}(p_i) = M_k(p_i) - M_j(p_i)$ with $i \in \{1, 2, \dots, a\}$;
- $e_{k,j}$ a row of E with $k, j \in \{1, 2, \dots, b\}$ and $k \neq j$; an a vector of marking differences between M_k and M_j in \mathcal{M}_F^* ;
- $f_{j,k}$ a binary variable: $f_{j,k} = 1$ if $M_k \in \mathcal{M}_F^*$ is forbidden by the PI related to $M_j \in \mathcal{M}_F^*$, otherwise $f_{j,k} = 0$;
- g_j an a nonnegative integer vector of the coefficients of the PI corresponding to $M_j \in \mathcal{M}_F^*$;
- h_j a binary variable: $h_j = 1$ if the PI related to $M_j \in \mathcal{M}_F^*$ is selected to compute a control place, otherwise $h_j = 0$;
- Γ a positive integer that should be chosen big enough;
- $\gamma_{l,j}$ a reachability constraint: $d_{l,j} \cdot g_j^T \leq -1$ with $l \in \{1, 2, \dots, c\}$ and $j \in \{1, 2, \dots, b\}$;
- $S(D, E)$ the system defined by constraints in MCP;
- $\Omega(D, E)$ the feasible region of supervisors in $S(D, E)$.

$$\begin{aligned}
 & \min \sum_{j \in \{1, 2, \dots, b\}} h_j \\
 & \text{subject to} \\
 & d_{l,j} \cdot g_j^T \leq -1, \quad \forall l \in \{1, 2, \dots, c\} \text{ and } \forall j \in \{1, 2, \dots, b\} \\
 & e_{k,j} \cdot g_j^T \geq \Gamma \cdot (f_{j,k} - 1), \quad \forall j, k \in \{1, 2, \dots, b\} \text{ and } j \neq k \\
 & f_{j,k} \leq h_j, \quad \forall j, k \in \{1, 2, \dots, b\} \text{ and } j \neq k \\
 & h_j + \sum_{k \in \{1, 2, \dots, b\}, k \neq j} f_{k,j} \geq 1, \quad \forall j \in \{1, 2, \dots, b\}. \quad (1)
 \end{aligned}$$

At the first sight, the above formulation may seem different from the one in Chen and Li (2011). However, they are in essence the same. In this problem, the reachability constraints are expressed by the inequalities containing vector $d_{l,j}$.

3. Eliminating redundant constraints

First, the definition of a redundant reachability constraint is given. If most of the reachability constraints are redundant and eliminated efficiently, the calculation of MCP and the whole supervisor synthesis may be considerably facilitated. Then, two kinds of elimination methods are proposed. Finally, a method to integrate constraint elimination steps into the supervisor synthesis is given.

3.1. Definition of a redundant reachability constraint

The constraints in (1) determine the feasible region of optimal PN supervisors. For generalization, remove the objective function

in (1) and consider the following system denoted by $S(D, E)$:

System $S(D, E)$:

$$\begin{aligned} d_{l,j} \cdot g_j^T &\leq -1, \quad \forall l \in \{1, 2, \dots, c\} \text{ and } \forall j \in \{1, 2, \dots, b\} \\ e_{k,j} \cdot g_j^T &\geq \Gamma \cdot (f_{j,k} - 1), \quad \forall j, k \in \{1, 2, \dots, b\} \text{ and } j \neq k \\ f_{j,k} &\leq h_j, \quad \forall j, k \in \{1, 2, \dots, b\} \text{ and } j \neq k \\ h_j + \sum_{k \in \{1, 2, \dots, b\}, k \neq j} f_{k,j} &\geq 1, \quad \forall j \in \{1, 2, \dots, b\}, \end{aligned} \quad (2)$$

where D represents a $(b \cdot c) \times a$ integer matrix whose element is $d_{l,j}(i)$ that denotes the marking difference in the i th activity place between $M_l \in \mathcal{M}_L^*$ and $M_j \in \mathcal{M}_F^*$, and E represents a $(b \cdot (b-1)) \times a$ integer matrix whose element is $e_{k,j}(i)$ that denotes the marking difference in the i th activity place between two different markings $M_k, M_j \in \mathcal{M}_F^*$. For different plant PN models, D and E may vary due to \mathcal{M}_L^* and \mathcal{M}_F^* of the models. Therefore, a model is denoted by $S(D, E)$. To define a redundant constraint, $\Omega(D, E)$, the feasible region of supervisors in $S(D, E)$, is used to represent all possible supervisors satisfying (2), and $\tilde{D}^{(m,n)}$ is used to denote the matrix obtained by deleting $d_{m,n}$ ($m \in \{1, 2, \dots, c\}$, $n \in \{1, 2, \dots, b\}$) from D .

Definition 1. A reachability constraint

$$d_{m,n} \cdot g_n^T \leq -1 \quad (3)$$

is redundant in $S(D, E)$ if

$$\Omega(D, E) = \Omega(\tilde{D}^{(m,n)}, E). \quad (4)$$

Condition (4) means that the following equations should be satisfied:

$$\Omega(D, E) \subseteq \Omega(\tilde{D}^{(m,n)}, E) \quad (5)$$

$$\Omega(D, E) \supseteq \Omega(\tilde{D}^{(m,n)}, E). \quad (6)$$

Condition (5) is obviously satisfied since the removal of a reachability constraint can only expand the feasible region. Thus, (6) is the center of our focus.

Therefore, a reachability constraint is redundant if the feasible region of a system remains unchanged after the constraint is eliminated. In addition, the redundancy is not defined for any specific objective function or any specific solution, but for all feasible optimal supervisors which satisfy $S(D, E)$.

3.2. ILP elimination

First, a sufficient and necessary condition for a reachability constraint to be redundant is given based on an ILP problem, and then two simpler sufficient conditions are derived. Based on the simplest sufficient condition, a redundancy elimination algorithm for MCPP is presented.

Theorem 2. Consider the following ILP problem:

$$\begin{aligned} \mathbb{P}_{m,n}^1 &= \max d_{m,n} \cdot g_n^T \\ \text{subject to} \\ d_{l,j} \cdot g_j^T &\leq -1, \quad \forall l \in \{1, 2, \dots, c\}, \forall j \in \{1, 2, \dots, b\} \\ \text{and } (l, j) &\neq (m, n) \\ e_{k,j} \cdot g_j^T &\geq \Gamma \cdot (f_{j,k} - 1), \quad \forall j, k \in \{1, 2, \dots, b\} \text{ and } j \neq k \\ f_{j,k} &\leq h_j, \quad \forall j, k \in \{1, 2, \dots, b\} \text{ and } j \neq k \\ h_j + \sum_{k \in \{1, 2, \dots, b\}, k \neq j} f_{k,j} &\geq 1, \quad \forall j \in \{1, 2, \dots, b\}. \end{aligned} \quad (7)$$

The constraint expressed in (3) is redundant in $S(D, E)$ iff $\mathbb{P}_{m,n}^1 \leq -1$.

Proof. First, we prove that if $\mathbb{P}_{m,n}^1 \leq -1$, (3) is redundant in $S(D, E)$. In (7), all feasible solutions to this system constitute $\Omega(\tilde{D}^{(m,n)}, E)$. If $\mathbb{P}_{m,n}^1 \leq -1$, then for any solution within $\Omega(\tilde{D}^{(m,n)}, E)$, we have:

$$d_{m,n} \cdot g_n^T \leq \mathbb{P}_{m,n}^1 \leq -1 \quad (8)$$

which is the same as the eliminated constraint, i.e., $d_{m,n} \cdot g_n^T \leq -1$. It also means that any feasible solution to system $S(\tilde{D}^{(m,n)}, E)$ is also feasible to system $S(D, E)$. Therefore, (6) holds. According to the Definition 1, (3) is redundant in $S(D, E)$.

Then, we aim to prove that if (3) is a redundant constraint, $\mathbb{P}_{m,n}^1 \leq -1$ holds. If (3) is redundant, (6) should be true. So, any solution within $\Omega(\tilde{D}^{(m,n)}, E)$ is also a solution to $S(D, E)$. Since (3) should be satisfied for any solution to $S(D, E)$, the maximum value of the objective function, which is the same as the left side of (3), should be less than -1 . Therefore, $\mathbb{P}_{m,n}^1 \leq -1$ holds. \square

Theorem 2 gives a sufficient and necessary condition for a reachability constraint to be redundant. However, the ILP problem $\mathbb{P}_{m,n}^1$ has $b \cdot (c + 2b - 1) - 1$ constraints and $b \cdot (a + b)$ variables. It has little practical use due to the formidable amounts of time consumed by a series of such ILPs for redundancy elimination in MCPP.

Theorem 3. Consider the following ILP problem:

$$\begin{aligned} \mathbb{P}_{m,n}^2 &= \max d_{m,n} \cdot g_n^T \\ \text{subject to} \\ d_{l,j} \cdot g_j^T &\leq -1, \quad \forall l \in \{1, 2, \dots, c\}, \forall j \in \{1, 2, \dots, b\} \\ \text{and } (l, j) &\neq (m, n). \end{aligned} \quad (9)$$

A reachability constraint expressed in (3) is redundant in $S(D, E)$ if $\mathbb{P}_{m,n}^2 \leq -1$.

Proof. In (9), all constraints except the reachability constraints with $(l, j) \neq (m, n)$ are taken away from (7). So, the feasible region of problem (9) is a superset of that of problem (7), which implies that $\mathbb{P}_{m,n}^2 \geq \mathbb{P}_{m,n}^1$. Therefore, if $\mathbb{P}_{m,n}^2 \leq -1$, then $\mathbb{P}_{m,n}^1 \leq -1$ and (3) is thus redundant in $S(D, E)$ according to Theorem 2. \square

Theorem 3 provides a sufficient condition for a constraint to be redundant. Now, we give the PN interpretation for it. For an FBM, only the tokens in activity places are considered to construct a PI to prevent it from being reached (Uzam & Zhou, 2006). To forbid an FBM by using a PI, the weighted sum of tokens in all activity places of the PI is defined as one less than that of the FBM and greater than or equal to that of any legal marking (Chen & Li, 2011). The objective function in (9) could be interpreted as the maximum possible difference between the weighted sum of tokens in activity places of a legal marking and that of an FBM in any reachability constraints except (3). If the maximum possible difference is still less than or equal to the limit, i.e., -1 , the reachability constraint expressed in (3) is undoubtedly redundant.

The ILP problem $\mathbb{P}_{m,n}^2$ in Theorem 3 has $b \cdot c - 1$ constraints and $b \cdot a$ variables, which are fewer than those in $\mathbb{P}_{m,n}^1$. In Algorithm 1, we propose an MCPP redundancy elimination method based on an ILP problem $\mathbb{P}_{m,n}^3$, which is revised from $\mathbb{P}_{m,n}^2$ and has the fewest constraints and variables to judge the redundancy of each reachability constraint. In this algorithm, $D^{(*,n)}$ denotes a matrix including all rows related to $M_n \in \mathcal{M}_F^*$ in D , and $\tilde{D}^{(m,*,n)}$ represents a matrix obtained by removing $d_{m,n}$ and the rows related to the redundant reachability constraints found by the algorithm from $D^{(*,n)}$.

Algorithm 1 can identify and eliminate redundant reachability constraints from MCPP by using $\mathbb{P}_{m,n}^3$. For each reachability constraint $\gamma_{m,n}$, we first select the reachability constraints related to $M_n \in \mathcal{M}_F^*$ as the candidates for the constraints in $\mathbb{P}_{m,n}^3$. Next, $\gamma_{m,n}$ and all M_n -related redundant constraints found so far are removed

Algorithm 1 Redundant constraint elimination by using an ILP method.

Input: MCPP of the PN model for an FMS.
Output: Reduced MCPP.
1: $R := \emptyset$; $\triangleright R$ is used to denote the set of redundant constraints found in MCPP.
2: **for** each constraint $\gamma_{m,n} : d_{m,n} \cdot g_n^T \leq -1$ **do**
3: $Q := \mathcal{D}^{<*,n>}$; $\triangleright Q$ is used to obtain $\tilde{D}^{<m,*,n>}$.
4: $Q := \tilde{Q}^{<m,n>}$; \triangleright Delete $d_{m,n}$ from Q .
5: **for** each $\gamma_{l,n} \in R$ **do**
6: $Q := \tilde{Q}^{<l,n>}$; \triangleright Delete $d_{l,n}$ from Q .
7: **end for**
8: $\tilde{D}^{<m,*,n>} := Q$;
9: Solve the following ILP problem:
10: $\mathbb{P}_{m,n}^3 = \max d_{m,n} \cdot g_n^T$
11: subject to
12: $\tilde{D}^{<m,*,n>} \cdot g_n^T \leq -1$
13: **if** $\mathbb{P}_{m,n}^3 \leq -1$ **then**
14: $R := R \cup \{\gamma_{m,n}\}$;
15: **end if**
16: **end for**
17: MCPP := MCPP – R ;
18: Output MCPP.

from them. Then, we solve $\mathbb{P}_{m,n}^3$ with the remaining constraints. If the result is less than or equal to -1 , $\gamma_{m,n}$ is redundant. At the end of the algorithm, all redundant constraints found are deleted from MCPP. The advantages of the method are that the numbers of constraints and variables needed to judge the redundancy of a constraint are further reduced to at most $c - 1$ (since some redundant rows may be removed from $\tilde{D}^{(m,*,n)}$) and a respectively, which can be seen in Table 1, and all the constraints eliminated by Algorithm 1 are definitely redundant.

Theorem 4. All constraints eliminated by Algorithm 1 are redundant in $S(D, E)$.

Proof. In Algorithm 1, the constraints in R are the ones to be eliminated. Now, we prove that $\forall \gamma_{m,n} \in R$, it is redundant in $S(D, E)$. $\forall \gamma_{m,n} \in R$, we have that $\mathbb{P}_{m,n}^3 \leq -1$. In the problem $\mathbb{P}_{m,n}^3$, the reachability constraints not related to $M_n \in \mathcal{M}_F^*$ and the redundant constraints found by then are taken away from $\mathbb{P}_{m,n}^2$. Therefore, the feasible region of $\mathbb{P}_{m,n}^3$ is a superset of that of $\mathbb{P}_{m,n}^2$. Thus, we have that $\mathbb{P}_{m,n}^3 \geq \mathbb{P}_{m,n}^2$, which implies that $\mathbb{P}_{m,n}^2 \leq -1$. Based on Theorem 3, $\gamma_{m,n}$ is redundant in $S(D, E)$. \square

3.3. Non-ILP elimination

In Algorithm 1, $\mathbb{P}_{m,n}^3$ is still an ILP problem, only with fewer constraints and variables. It would be desirable to develop a method that identifies redundant constraints without solving any optimization problem.

Theorem 5. A reachability constraint expressed in (3) is redundant in system $S(D, E)$ if there exists at least one other constraint

$$\begin{aligned} d_{l_1,n} \cdot g_n^T &\leq -1, \\ d_{l_2,n} \cdot g_n^T &\leq -1, \\ &\dots\dots\dots \\ d_{l_q,n} \cdot g_n^T &\leq -1 \end{aligned} \quad (10)$$

which satisfies $\forall i \in \{1, 2, \dots, a\}$, $d_{m,n}(p_i) \leq \sum_{k=1}^q d_{l_k,n}(p_i)$.

Proof. By adding up both sides of the constraints in (10), we have $\sum_{k=1}^q d_{l_k,n} \cdot g_n^T \leq -q$ where q is the number of those constraints. Since $\forall i \in \{1, 2, \dots, a\}$, $d_{m,n}(p_i) \leq \sum_{k=1}^q d_{l_k,n}(p_i)$ and all elements in g_n^T are nonnegative, we have that $d_{m,n} \cdot g_n^T \leq \sum_{k=1}^q d_{l_k,n} \cdot g_n^T$. Therefore, $d_{m,n} \cdot g_n^T \leq -q \leq -1$ can be derived, which means that the reachability constraint $d_{m,n} \cdot g_n^T \leq -1$ is satisfied in $S(\tilde{D}^{(m,n)}, E)$. It also indicates that any feasible solution within $\Omega(\tilde{D}^{(m,n)}, E)$ is feasible to system $S(D, E)$. Therefore, (6) is satisfied. According to Definition 1, (3) is redundant in $S(D, E)$. \square

Table 1

Numbers of constraints and variables in ILPs.

ILP	No. constraints	No. variables
$\mathbb{P}_{m,n}^1$	$b \cdot (c + 2b - 1) - 1$	$b \cdot (a + b)$
$\mathbb{P}_{m,n}^2$	$b \cdot c - 1$	$b \cdot a$
$\mathbb{P}_{m,n}^3$	$\leq c - 1$	a

Based on the heuristic rule expressed in Theorem 5, we present an MCPP redundancy elimination method as shown in Algorithm 2, where $\tilde{D}_u^{(m,*,n)}$ denotes the u th row of $\tilde{D}^{(m,*,n)}$, and c is the number of constraints in $\tilde{D}^{(m,*,n)}$ that is also equal to the number of markings in \mathcal{M}_L^* .

Algorithm 2 Redundant constraint elimination by using a non-ILP method.

Input: MCPP of the PN model for an FMS.
Output: Reduced MCPP.
1: $R := \emptyset$; $\triangleright R$ is used to denote the set of redundant constraints found in MCPP.
2: **for** each constraint $\gamma_{m,n} : d_{m,n} \cdot g_n^T \leq -1$ **do**
3: $Q := \mathcal{D}^{<*,n>}$; $\triangleright Q$ is used to obtain $\tilde{D}^{(m,*,n)}$.
4: $Q := \tilde{Q}^{<m,n>}$; \triangleright Delete $d_{m,n}$ from Q .
5: **for** each $\gamma_{l,n} \in R$ **do**
6: $Q := \tilde{Q}^{<l,n>}$; \triangleright Delete $d_{l,n}$ from Q .
7: **end for**
8: $\tilde{D}^{(m,*,n)} := Q$;
9: **for** $u := 1$ to $c - |R| - 1$ **do**
10: **for** $v := u + 1$ to $c - |R| - 1$ **do**
11: **if** $\forall i \in \{1, 2, \dots, a\}$, $d_{m,n}(p_i) \leq \tilde{D}_u^{(m,*,n)}(p_i) + \tilde{D}_v^{(m,*,n)}(p_i)$ **then**
12: $R := R \cup \{\gamma_{m,n}\}$;
13: Go to Step 2 for the next constraint;
14: **end if**
15: **end for**
16: **end for**
17: **end for**
18: MCPP := MCPP – R ;
19: Output MCPP.

Algorithm 2 can identify and eliminate redundant reachability constraints from MCPP by using the non-ILP method. For each reachability constraint $\gamma_{m,n}$, we only consider some constraints having the same variables as that of $\gamma_{m,n}$ to judge its redundancy. First, we select the reachability constraints related to $M_n \in \mathcal{M}_F^*$ as the candidates to be searched. Next, $\gamma_{m,n}$ and all redundant constraints found so far are removed from the candidates. Then, we search the remaining constraints to find whether there exist two constraints which can imply $\gamma_{m,n}$. If yes, $\gamma_{m,n}$ is redundant. At the end of the algorithm, all redundant constraints found are deleted from MCPP. The advantages of this method are that it need not solve any optimization problem, and all the constraints eliminated are definitely redundant.

Theorem 6. All constraints eliminated by Algorithm 2 are redundant in system $S(D, E)$.

Proof. In Algorithm 2, the constraints in R are the ones to be eliminated. $\forall \gamma_{m,n} \in R$, there exist two constraints $\tilde{D}_u^{(m,*,n)} \cdot g_n^T \leq -1$ and $\tilde{D}_v^{(m,*,n)} \cdot g_n^T \leq -1$ which satisfy $\forall i \in \{1, 2, \dots, a\}$, $d_{m,n}(p_i) \leq \tilde{D}_u^{(m,*,n)}(p_i) + \tilde{D}_v^{(m,*,n)}(p_i)$. Since $\tilde{D}^{(m,*,n)} \subseteq \tilde{D}^{(m,n)}$, there exist two constraints $d_{l_1,n} \cdot g_n^T \leq -1$ and $d_{l_2,n} \cdot g_n^T \leq -1$ with $d_{l_1,n} = \tilde{D}_u^{(m,*,n)} \in \tilde{D}^{(m,n)}$ and $d_{l_2,n} = \tilde{D}_v^{(m,*,n)} \in \tilde{D}^{(m,n)}$ which satisfy $\forall i \in \{1, 2, \dots, a\}$, $d_{m,n}(p_i) \leq \sum_{k=1}^2 d_{l_k,n}(p_i)$. According to Theorem 5, $\gamma_{m,n}$ is redundant in $S(D, E)$. \square

3.4. Integration into supervisor synthesis

This section presents a deadlock prevention method by using MCPP with redundancy elimination to obtain an optimal supervisor with the fewest control places.

Table 3

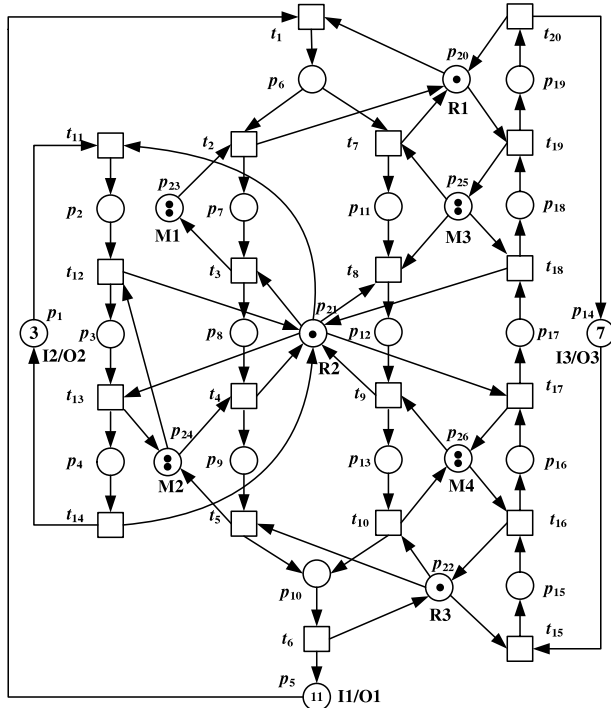
Performance comparison for the model in Fig. 1.

	Uzam (2002)	Li et al. (2008)	Piroddi et al. (2008)	Chen et al. (2011)	Chen and Li (2012)	Chen and Li (2011)	Ours
Places	6	9	5	8	2	2	2
States	205	205	205	205	205	205	205

Table 4

Results for the model in Fig. 2.

	Case 1	Case 2	Case 3
N_{Eli}	/	8939	8461
\mathcal{T}_{Eli}	/	2 m 7 s	2 m 8 s
\mathcal{T}_{MCP}	30 h 29 m 7 s	13 h 53 m 58 s	10 h 41 m 42 s

**Fig. 2.** PN model of another FMS.

the minimality of the supervisory structure of this work can be guaranteed.

To further demonstrate our methods, the manufacturing example shown in Fig. 2 is parameterized to generate some other problem instances. In each instance, we vary the number of parts of each part type (tokens in p_5 , p_{14} and p_{23}) and the number of available units of some resource types (tokens in p_{24} , p_{25} and p_{26}). Table 6 summarizes results of the quintuplet $(M_0(p_5), M_0(p_{14}), M_0(p_{23}), M_0(p_{24}), M_0(p_{25}), M_0(p_{26}))$ in terms of the number of markings in $\mathcal{M}_f^*(b)$, the number of markings in $\mathcal{M}_f^*(c)$, the number of legal markings in the plant net (N_L), the number of control places in the supervisor (N_{CP}), the number of reachable states of the controlled net (N_{Sta}), the number of total constraints in MCP (N_{Con}), the number of reachability constraints (N_{Rec}), the number of eliminated constraints (N_{Eli}), the elimination time (\mathcal{T}_{Eli}), and the solution time of MCP (\mathcal{T}_{MCP}). The following phenomena are observed.

Table 5

Performance comparison for the model in Fig. 2.

	Ezpeleta et al. (1995)	Huang et al. (2001)	Uzam and Zhou (2006)	Chen et al. (2011)	Piroddi et al. (2008)	Chen and Li (2011)	Ours
Places	18	16	19	17	13	5	5
States	6287	12 656	21 562	21 581	21 581	21 581	21 581

(1) The ILP elimination method can eliminate slightly more constraints than the non-ILP one, but the solution time of MCP is not always a monotonically decreasing function of the number of eliminated constraints in the problem. This phenomenon is to be expected by considering the fact that although the number of constraints has a significant impact on the computational complexity of any linear programming problems, it is not the only impact factor in general. For instance, the primal-dual potential-reduction algorithm (Luenberger & Ye, 2008) requires $O(mn^2 \log(n/\epsilon))$ arithmetic calculations in the linear programming model, where m and n are the number of variables and constraints, and ϵ is the maximal acceptable duality gap. Therefore, it is reasonable for some reduced MCP with slightly more constraints to be solved faster than the one with fewer constraints.

(2) The supervisor obtained by our work is optimal since the number of reachable states of the controlled net is the same as the number of the legal markings of the plant net. In addition, the structure of the obtained supervisors can be guaranteed to be minimal via our methods.

(3) The results suggest that the methods presented in this paper are more beneficial for application to large systems. In Table 6, we can see that as the size of model increases, more redundant constraints are eliminated and more MCP solution time is saved. Meanwhile, the redundancy elimination time still remains little.

5. Conclusion

In MCP, many reachability constraints are redundant. The computation of MCP and the whole supervisor synthesis can be accelerated if they can be efficiently eliminated. This paper develops two kinds of such techniques: ILP based methods and non-ILP based ones. First, most of the redundant reachability constraints can be identified and removed by either ILP or non-ILP elimination methods in a short time. In most cases, the ILP methods can eliminate slightly more constraints than the non-ILP one, but the latter does not need to solve any optimization problem. Second, the solution process of MCP is accelerated due to the elimination of redundant constraints, especially for large-scale systems. Third, the supervisors obtained are maximally permissive, while the structural minimality of the supervisors can be guaranteed.

This work has successfully accelerated the synthesis process to obtain an optimal PN supervisor with the minimal structure. In future work, we will focus on combining the redundant constraint elimination with other speedup techniques, such as MFFP in Chen et al. (2012) and MPP in Chen and Li (2012), to further improve the efficiency of supervisor synthesis. In addition, how to integrate the siphon analysis techniques (Li & Zhao, 2008; Li & Zhou, 2008, 2009) with the reachability graph analysis method (Chen, Li, & Zhou, 2014; Huang, Pan, & Zhou, 2012; Wang, Gan, & Zhou, 2015) is also an interesting research topic for us.

Table 6

Solutions of parameterized problem instances.

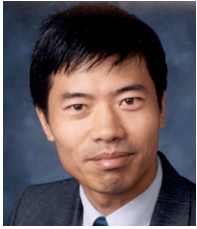
	b	c	N_L	N_{CP}	N_{Sta}	N_{Con}	N_{Rec}		N_{Eli}	\mathcal{T}_{Eli}	\mathcal{T}_{MCP}
(4 3 2 1 1)	13	129	883	4	883	2002	1677	Case 1	/	/	1 m 41 s
								Case 2	534	6.12 s	1 m 25 s
								Case 3	531	3.24 s	1 m 35 s
(6 4 2 1 1)	13	71	990	4	990	1248	923	Case 1	/	/	1 m 14 s
								Case 2	370	2.67 s	0 m 54 s
								Case 3	364	0.59 s	1 m 11 s
(6 6 3 2 1)	17	199	3789	5	3789	3944	3383	Case 1	/	/	47 m 22 s
								Case 2	1654	17.46 s	29 m 23 s
								Case 3	1633	11.77 s	23 m 38 s
(8 6 3 2 1)	17	126	3887	5	3887	2703	2142	Case 1	/	/	28 m 22 s
								Case 2	1157	8.50 s	16 m 34 s
								Case 3	1130	3.18 s	24 m 47 s
(10 6 3 2 2)	34	403	21561	5	21561	15980	13702	Case 1	/	/	30 h 59 m 38 s
								Case 2	9176	2 m 17 s	15 h 39 m 38 s
								Case 3	8671	2 m 19 s	18 h 51 m 10 s
(12 7 4 2 2)	34	393	21581	5	21581	15640	13362	Case 1	/	/	30 h 29 m 07 s
								Case 2	8939	2 m 07 s	13 h 53 m 58 s
								Case 3	8461	2 m 08 s	10 h 41 m 42 s

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