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MAXIMALLY PERMISSIVE PETRI NET SUPERVISORS FOR FLEXIBLE MANUFACTURING SYSTEMS WITH UNCONTROLLABLE AND UNOBSERVABLE TRANSITIONS

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ABSTRACT

This paper presents a deadlock prevention policy to obtain behaviorally optimal supervisors for flexible manufacturing systems with uncontrollable and unobservable transitions. The conditions of uncontrollability and unobservability of transitions are revealed in the sense of the implementation of a Petri net supervisor. Then, integer linear programming models are designed to obtain a Petri net supervisor such that all legal markings are reachable and the number of control places is reduced. We also show that a controllable transition can be unobservable and self-loops can be used to disable the transition but do not observe its firing. Finally, examples are provided to illustrate the proposed approach.

Key Words: Petri net, flexible manufacturing system (FMS), deadlock prevention, supervisory control, uncontrollable transition, unobservable transition.

I. INTRODUCTION

As a typical abstraction of automatically running systems controlled by computers, flexible manufacturing systems (FMSs) can continuously produce various types of products twenty-four hours a day. However, deadlocks [11] are a constant threat to the continuous run of an FMS. They have the ability to block a system and can even lead to catastrophic results in highly automated production systems. Thus, deadlock resolution methods have received much attention from academic and industrial communities.

Petri nets are a mathematical tool to detect and control deadlocks in FMSs [28]. They have the advantages of compact structures and matrix representation. Many researchers have developed a number of deadlock resolution methods such as deadlock detection and recovery [23,45], deadlock avoidance [1,18,19,33,42–44], and deadlock prevention [5,12,14,17,20,22,24–26,29,30,47,49]. A deadlock prevention approach prevents a system from reaching deadlocks by adding external monitors (control places).

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Based on Petri nets, behavioral permissiveness plays an important role in evaluating the performance of a liveness-enforcing supervisor. A maximally permissive, *i.e.*, optimal, supervisor means that all legal markings are kept in the controlled system. In this case, the utilization of system resources has the minimal limitation. Therefore, much effort has been made on the design of optimal Petri net supervisors [4,6–9,15,21,27,34,35]. In this paper, "optimal" is used as a synonym for "maximally permissive".

A representative work on the design of optimal Petri net supervisors is the theory of regions [15,39]. It deals with deadlocks by considering the marking/transition separation instances (MTSIs). An MTSI is a pair of a marking and a transition (M, t), where M is a legal marking and t is enabled at M. Once t fires, it leads to a deadlock marking or a bad marking that inevitably leads to deadlock markings. Hence, t must be prohibited at M. In [15], Ghaffari *et al*. develop a linear programming model to obtain an optimal control place to implement an MTSI. The process is repeated until all MTSIs are implemented. Then, an optimal supervisor can be obtained. The study considers the controllability of transitions, but ignores the observability. In [34,35], Piroddi *et al*. propose an approach to design a supervisor by considering the relation between siphons and markings. A set covering technique is utilized to design an optimal supervisor for every example in their work with a simple structure. However, the work does not consider the controllability and observability of transitions.

In the previous work [6], we presented an iterative approach to find an optimal supervisor by solving integer linear programming problems (ILPPs). This study is based on the analysis of reachability graphs of Petri nets. A reachability graph is divided into two parts: a live-zone (LZ) and a deadlock-zone (DZ). The LZ includes all the legal markings and the DZ contains all illegal markings (bad markings and dead markings). Deadlocks are prevented by forbidding all first-met bad markings (FBMs) that represent the first entry from the LZ to the DZ. At each iteration, an FBM is singled out and a control place is designed to forbid it. The process does not terminate until all FBMs are forbidden. The optimal control purpose is ensured by keeping all legal markings reachable and all FBMs unreachable. In [7], we proposed a non-iterative approach to design an optimal supervisor by solving only one ILPP. The obtained supervisor is maximally permissive with the minimal number of control places. The work in both [6] and [7] can find an optimal supervisor for a Petri net model if there exists an optimal pure net supervisor. However, there are some net models without optimal pure net supervisors. In [10] we develop a nonpure supervisory structure that can optimally control the net models without optimal pure net supervisors. All the previous work [6,7,10] aims to obtain optimal supervisors under the assumption that all transitions are controllable and observable. Thus, they cannot be applied to FMSs with uncontrollable and unobservable transitions. In this work, we aim to design optimal Petri net supervisors for FMSs by considering the controllability and observability of the transitions. That is to say, we focus on the design of optimal supervisors for FMSs with uncontrollable and unobservable transitions.

Controllability and observability are important properties of events in a real system. An uncontrollable event indicates that one cannot prohibit its occurrence by external monitors and an unobservable event implies that its occurrence cannot be detected. Uncontrollable (unobservable) events appear when some events are impossible or expensive to control (observe), or when actuators (sensors) fail. In [31], Moody and Antsaklis propose an approach to transform a given generalized mutual exclusion constraint (GMEC) to an admissible one by considering uncontrollable and unobservable transitions. It is effective and efficient to obtain an admissible GMEC but it is usually not maximally permissive. Since then, a lot of work has been done to derive admissible GMECs from given GMECs [3,16,41] .

Previous work mainly focuses on obtaining admissible GMECs for given inadmissible GMECs. Instead of dealing with GMECs, this paper tries to design optimal supervisors for given FMSs with uncontrollable and unobservable transitions. Behavioral permissiveness and structural complexity are taken into account, aiming to design an optimal Petri net supervisor with a small number of control places. The obtained supervisor satisfies that no uncontrollable transition is disabled and no unobservable transition is observed by it at any legal marking. Furthermore, the concept of controllable but unobservable transitions is developed and a control place is designed to control but not observe it. In summary, we reach the following contributions in this paper.

1. Meanings of uncontrollability and unobservability of transitions are clearly revealed from the viewpoint of the implementation of a supervisor. An unobservable transition can be controllable in this work. Unobservable transitions are traditionally considered as uncontrollable transitions in the sense that the control of a transition is considered as an arc from a supervisor to the transition. In this case, once an unobservable transition fires, it cannot be observed, which means that the tokens in the supervisor cannot be changed. Thus, there should be no control arc from the supervisor to the unobservable transition, which indicates traditional uncontrollability. However, in this work, we show that an unobservable transition can be controlled by a Petri net supervisor even if its occurrence cannot be observed. Meanwhile, controllability and observability of transitions are studied in detail from the viewpoint of the implementation of a supervisor.
2. For an MTSI, we develop an ILPP to design a control place to implement it. Meanwhile, no uncontrollable transition is disabled and no unobservable transition is observed by the control place. Self-loops are used to design a control place that can disable an unobservable transition but does not observe it. The objective function is utilized to implement as many MTSIs as possible. Thus, the obtained supervisor is behaviorally optimal and structurally simple.
3. The proposed method is general since it can be applied to all FMS-oriented Petri net models, which include PPN [18,46], S^3PR [12], ES^3PR [37], S^4PR [38], S*PR [13], S^2LSPR [32], S^3PGR^2 [33], and S^3PMR [20].

The remainder of the paper is organized as follows. Section II outlines some basics of Petri nets and a PI-based control place synthesis method in [48]. Section III provides detailed analysis of the controllability and observability of transitions and reveals their properties in the sense of control places. The design of optimal control places by considering uncontrollable and unobservable transitions is proposed in Section IV. Section V develops a deadlock prevention policy and provides an illustrative example. A widely used example from the literature is provided in Section VI. Finally, some conclusions are made in Section VII.

II. PRELIMINARIES

2.1 Petri nets

A Petri net is a four-tuple $N = (P, T, F, W)$ where P and T are finite and non-empty sets. P is a set of places and T is a set of transitions with $P \cap T = \emptyset$. $F \subseteq (P \times T) \cup (T \times P)$ is called a flow relation of the net, represented by arcs with arrows from places to transitions or from transitions to places. $W:(P\times T)\bigcup(T\times P)\rightarrow\mathbb{N}$ is a mapping that assigns a weight to an arc: $W(x, y) > 0$ if $(x, y) \in F$, and $W(x, y) = 0$, otherwise, where $x, y \in P \bigcup T$ and $\mathbb{N}$ is the set of non-negative integers. ${}^{\bullet}x = \{y \in P \bigcup T|(y, x) \in F\}$ and $x^{\bullet} = \{y \in P \bigcup T|(x, y) \in F\}$ are called the preset and the postset of x, respectively. A marking is a mapping $M: P\rightarrow\mathbb{N}$ $M(p)$ denotes the number of tokens in place p. The pair (N, M_0) is called a marked Petri net or a net system. A net is pure (self-loop free) if $\forall(x, y) \in (P \times T) \bigcup (T \times P)$, $W(x, y) > 0$ implies $W(y, x) = 0$. Incidence matrix $[N]$ of pure net N is a $|P| \times |T|$ integer matrix with $[N](p, t) = W(t, p) - W(p, t)$.

A transition $t \in T$ is enabled at marking M if $\forall p \in {}^{\bullet}t$, $M(p) \geq W(p,t)$. This fact is denoted as $M[t\rangle$. Once an enabled transition t fires, it yields a new marking M', denoted as $M[t\rangle M'$, where $M'(p) = M(p) - W(p, t) + W(t, p)$. $M_0[\rangle$ is called the set of reachable markings of net N with initial marking M_0, often denoted by $R(N, M_0)$. The reachability graph of a net (N, M_0), denoted as $G(N, M_0)$, is a directed graph whose nodes are markings in $R(N, M_0)$ and arcs are labeled by the fired transitions.

Let (N, M_0) be a net system with $N = (P, T, F, W)$. A transition $t \in T$ is live at M_0 if $\forall M \in R(N, M_0)$, $\exists M' \in R(N, M)$, $M'[t\rangle$. (N, M_0) is live if $\forall t \in T$, t is live at M_0. A marking M is dead if $\nexists t \in T$ such that $M[t\rangle$.

A P-vector is a column vector $I: P\rightarrow\mathbb{Z}$ indexed by P and a T-vector is a column vector $J: T\rightarrow\mathbb{Z}$ indexed by T, where $\mathbb{Z}$ is the set of integers. P-vector I is called a P-invariant (place invariant, PI for short) if $I \neq \mathbf{0}$ and $I^T[N] = \mathbf{0}^T$. P-invariant I is said to be a P-semiflow if $I \geq 0$. Let I be a PI of (N, M_0) and M a reachable marking from M_0. Then, $I^TM = I^TM_0$.

2.2 Analysis of reachability graphs

A reachability graph can be partitioned into a deadlock-zone (DZ) and a live-zone (LZ) [40]. The DZ contains deadlocks and bad markings that inevitably lead to deadlocks. The LZ contains all the legal markings. The set of legal markings $\mathcal{M}_L$ is the maximal set of reachable markings, from which initial marking M_0 is reachable without leaving $\mathcal{M}_L$. It can be defined as:

$$\mathcal{M}_L = \{M|M \in R(N, M_0) \wedge M_0 \in R(N, M)\}. \tag{1}$$

In this work, a control place is said to be optimal if it can forbid markings in DZ but no legal marking. A net is said to be optimally controlled if the controlled net is live with all legal markings.

An MTSI [15] is a pair of a marking M and a transition t such that the firing of t at M leads to an illegal marking in DZ. Hence, the set of MTSIs is defined as follows:

$$\Omega = \{(M, t)|M[t\rangle M' \wedge M \in LZ \wedge M' \in DZ\} \tag{2}$$

where M is called a dangerous marking. The set of dangerous markings is denoted as $\mathcal{M}_D$. Removing all dangerous markings from $\mathcal{M}_L$, the rest are called good markings. Therefore, the set of good markings $\mathcal{M}_G$ can be defined as follows:

$$\mathcal{M}_G = \mathcal{M}_L - \mathcal{M}_D. \tag{3}$$

From a reachability graph, it can be seen that the liveness can be guaranteed by disabling the firing of the transition at the dangerous marking for each MTSI. In this case, the controlled system cannot leave LZ, *i.e.*, it keeps running in LZ and is live.

2.3 Control place computation by place invariants

This section briefly recalls the control place synthesis method by a PI [48]. Let $[N]$ be the incidence matrix of a plant with n places and m transitions and $[N_c]$ the incidence matrix of the control places. Suppose that the control goal is to enforce the plant to satisfy the following constraint:

$$\sum_{i=1}^{n} l_i \cdot \mu_i \leq \beta \tag{4}$$

where μ_i denotes the marking of place p_i, and l_i and β are non-negative integers. A non-negative slack variable μ_s is introduced to transform the inequality constraint into an equality, as presented below:

$$\sum_{i=1}^{n} l_i \cdot \mu_i + \mu_s = \beta \tag{5}$$

where μ_s represents the marking of control place p_s, also called a monitor. All constraints in the form of (4) can be grouped into a matrix form as follows:

$$[L]\cdot\mu_p \leq b \tag{6}$$

where μ_p is the marking vector of the Petri net model, $[L]$ is an $n_c \times n$ nonnegative integer matrix, b is an $n_c \times 1$ nonnegative integer vector, and n_c is the number of constraints. By introducing a non-negative slack variable vector μ_c, these inequality constraints can be transformed into equalities:

$$[L]\cdot\mu_p + \mu_c = b \tag{7}$$

where μ_c is an $n_c \times 1$ vector that represents the marking of the control places. According to PI equation $I^T[N] = \mathbf{0}^T$, the supervisor $[N_c]$ can be computed as follows:

$$[N_c] = -[L]\cdot[N]. \tag{8}$$

Equation (7) is also true at the initial marking μ_0 of a net. Thus, the initial marking μ_{c_0} of the supervisor can be calculated as follows:

$$\mu_{c_0} = b - [L]\cdot\mu_0. \tag{9}$$

III CONTROLLABILITY AND OBSERVABILITY OF TRANSITIONS

This section provides detailed analysis of the controllability and observability of transitions and studies their relations in the sense of control places. Based on the controllability and observability of transitions, transitions in a Petri net are classified into four groups: controllable and observable transitions, controllable and unobservable transitions, observable and uncontrollable transitions, and unobservable and uncontrollable transitions, whose sets are denoted as T_{co}, $T_{c\bar{o}}$, $T_{\bar{c}o}$, and $T_{\bar{c}\bar{o}}$, respectively.

Definition 1. A transition is said to be controllable if it can be disabled by a control place. Otherwise, it is uncontrollable.

Definition 2. Let (N, M_0) be a net system with $N = (P, T, F, W)$, $p_s \notin P$ a control place, and $M \in R(N, M_0)$ a reachable marking. A transition $t \in T$ is said to be disabled at M by p_s if: 1) $\forall p \in {}^{\bullet}t \cap P$, $M(p) \geq W(p, t)$, and 2) $M(p_s) < W(p_s, t)$.

Definition 2 means that a transition t is disabled at a marking M by a control place if $M(p_s) < W(p_s, t)$ is the unique condition that forbids the firing of t. That is to say, if the transition is disabled by both a place $p \in P$ and p_s at M, p_s does not try to disable its firing since its firing has been disabled by the place p in the original net. In this case, even though p_s is not added to the net system, t cannot be fired at M. Hence, we claim that p_s does not disable t at M. Fig. 1 shows a simple situation to illustrate the above statements. In Fig. 1a, p_s disables t since the other input place p does not disable t. However, in Fig. 1b, p_s does not disable t since it has been disabled by p. In this case, even though we remove p_s, t still cannot fire since p disables it.

Definition 3. Let t be a transition and p_s a control place. t is said to be controlled by p_s if $\exists M \in R(N, M_0)$, t is disabled at M by p_s. Otherwise, t is not controlled by p_s.

Clearly deciding whether a control place controls a transition is interesting for both the design and the implementation of a control policy. If a control place does not control a transition, there is no need to add an actuator to the transition even if there is an arc from the control place to the transition.

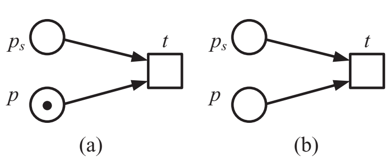

Fig. 1. (a) p_s disables t and (b) p_s does not disable t.

On the other hand, if a transition is uncontrollable, we still can have an arc from a control place to the uncontrollable transition if the control place does not disable it at any reachable marking.

Definition 4. A transition is said to be observable if its firing can be detected by a control place, *i.e.*, its firing changes the tokens in the control places. Otherwise, it is unobservable.

Definition 5. Let t be a transition and p_s a control place. t is said to be observed by p_s if $W(p_s, t) \neq W(t, p_s)$. p_s does not observe t if $W(p_s, t) = W(t, p_s)$.

Note that Moody and Antsaklis propose a seminal work [31] on uncontrollable and unobservable transitions. In their study, they claim that an unobservable transition is implicitly uncontrollable since a Petri net supervisor cannot have any connection to an unobservable transition. They also state that: "one can imagine a situation in which the occurrence of some event in a plant could be blocked without the controller ever receiving any feedback relating directly to that event, but, in practical situations, the ability to inhibit an event is usually coupled with the ability to detect occurrences of that event. For this reason, this limitation on Petri-net-based controllers is not too severe" [31]. The limitation arises since their work designs pure Petri net supervisors by PIs. An arc must be added from a control place p_s to a transition t if p_s tries to inhibit the firing of t. In this case, the tokens in p_s are changed once t fires. Thus, they claim that a controllable transition is implicitly observable. Just as stated in Moody and Antsaklis' work [31], there may still exist the situation that the occurrence of some event in a plant can be blocked while the supervisor has never received any feedback relating directly to the event. This means that a transition in a Petri net model can be prohibited but cannot be observed, *i.e.*, a controllable and unobservable transition.

This work aims to release the limitation of [31] where a controllable transition must be observable. We consider the fact that a controllable transition can be unobservable. In fact, controllable and unobservable transitions are possible in the real world. An unobservable transition represents the occurrence of a real event but the event is either impossible or too expensive to detect directly. It is also possible that a sensor fails, *i.e.*, the corresponding transition may suddenly become unobservable. However, the event still can keep its controllability. In this case, the transition can be prohibited by a control place while its firing cannot be detected. Self-loops and inhibitor arcs are two special structures of Petri nets, which can be used to prohibit the firing of a transition but does not change the tokens in a control place. In the following, self-loops are considered to deal with a controllable and unobservable transition.

IV. OPTIMAL CONTROL PLACE SYNTHESIS

In this section, we discuss the design of optimal control places for FMSs with uncontrollable and unobservable transitions. Also, the vector covering approach proposed in [6] and [10] is briefly reviewed to reduce the computational burdens of the proposed method.

4.1 Computation of optimal control places for unobservable transitions

According to the reachability graph of a Petri net model, transitions are classified into two groups: critical and good, whose sets are denoted as T_c and T_g, respectively, as defined below [10]:

$$T_c = \{t \in T | \exists M \in R(N, M_0), s.t. (M, t) \text{ is an MTSI}\} \quad (10)$$

$$T_g = \{t \in T | \nexists M \in R(N, M_0), s.t. (M, t) \text{ is an MTSI}\} \quad (11)$$

The firing of a critical transition may lead the system from the LZ into the DZ. For the optimal control purposes, a critical transition t must be prohibited at the marking M if (M, t) is an MTSI. Thus, for critical transitions, we have the following results on non-existence of an optimal control place.

Theorem 1. Let (N, M_0) be a Petri net system and T_c the set of critical transitions. (N, M_0) cannot be optimally controlled if $\exists t \in T_c$, t is uncontrollable.

Proof 1. Let (M, t) be an MTSI. Once t fires at M, the system enters the DZ. For the optimal control purposes, M is reachable in the controlled system since M is a legal marking. Therefore, t must be disabled at M. If t is uncontrollable, it cannot be disabled, *i.e.*, one cannot prevent the system from entering the DZ by firing t at M. Thus, (N, M_0) has no optimal supervisor.

Theorem 1 indicates that we cannot find an optimal supervisor for a Petri net if there exists a critical transition that is uncontrollable. Next, we show that an optimal control place does not intend to control any good transition.

Theorem 2. Let (N, M_0) be a Petri net system, T_g the set of the good transitions, and p_s an optimal control place. $\forall t \in T_g$, p_s does not disable t at any legal marking.

Proof 2. Let M be a legal marking and t a good transition that is enabled at M. Since t is a good transition, (M, t) is not an MTSI. That is to say, firing t yields a new marking M' and M' is a legal marking. Thus, for optimal control purposes, p_s should not disable t at M. Therefore, the conclusion holds.

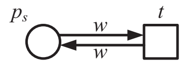

Fig. 2. A control place p_s that can prohibit but does not observe t.

Theorem 2 indicates that we do not need to consider the controllability of a good transition when an optimal control place is designed. Next, controllability and observability of transitions are considered in the design of an optimal control place.

For an unobservable transition t, an optimal control place p_s should not observe its firing. According to Definition 5, we have $W(p_s, t) = W(t, p_s)$. Next, two subcases are considered: (i) t is uncontrollable; and (ii) t is controllable.

First, we consider Case (i), *i.e.*, t is an uncontrollable and unobservable transition in $T_{\bar{c}\bar{o}}$. In this case, we should not add any arc between p_s and t, *i.e.*, $[N_c](p_s, t) = 0$. Since $[N_c](p_s, t) = -\sum_{i=1}^{n} l_i \cdot [N](p_i, t)$, we have

$$\sum_{i=1}^{n} l_i \cdot [N](p_i, t) = 0. \quad (12)$$

Equation (12) is called the unobservability condition of t.

Second, we consider Case (i), *i.e.*, t is a controllable and unobservable transition. Since t is unobservable, according to Definition 5, we have $W(p_s, t) = W(t, p_s)$. Thus, we cannot add a control arc from a control place p_s to t. In this case, self-loops are taken into account to prohibit t where $W(p_s, t) = W(t, p_s)$ can also be ensured, as shown in Fig. 2. In the figure, t can be disabled by p_s at a marking M if $M(p_s) < w$. On the other hand, p_s does not observe t since $W(p_s, t) = W(t, p_s)$.

In [10], we propose an approach to design an optimal control place by solving an ILPP, where self-loops are allowed. Next, we briefly review the approach and show its application to design an optimal control place with a self-loop for a controllable and unobservable transition. Let (M, t) be an MTSI, t a controllable and unobservable transition, and p_s a control place with a self-loop associated with t and also involved in a PI defined by (5), where the weight on the self-loop is w. First, p_s does not forbid any legal marking. Hence, we have

$$\sum_{i=1}^{n} l_i \cdot M_l(p_i) \le \beta, \forall M_l \in \mathcal{M}_L. \quad (13)$$

The PI defined by (13) should not add a new arc to t. Thus, p_s must satisfy the unobservability condition of t, *i.e.*, (12). For (M, t), t should be disabled by p_s at M. Thus, we have $M(p_s) < w$. Since $M(p_s)$ and β are integers, $M(p_s) < w$ can be written as $M(p_s) \le w - 1$. According to (5), we have $M(p_s) = \beta - \sum_{i=1}^{n} l_i \cdot M(p_i)$. Thus, ($M$, t) can be implemented by p_s if

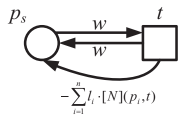

Fig. 3. A self-loop associated with transition t and an arc generated by a PI.

$$\sum_{i=1}^{n} l_i \cdot M(p_i) \geq \beta - w + 1. \tag{14}$$

Equation (14) is called the disabled condition.

For transition t, all legal markings can be classified into two parts [10], as shown in the following.

Definition 6 [10]. Let M be a legal marking and t a transition. M is called a t-good marking if t is not enabled at M, or M' with $M[t\rangle M'$ is a legal marking. M is called a t-dangerous marking if M' with $M[t\rangle M'$ is an illegal marking. The sets of t-good and t-dangerous markings are denoted by $\mathcal{G}_t$ and $\mathcal{D}_t$, respectively.

In fact, t may be enabled at some t-good markings. For the optimal deadlock control purposes, p_s should not disable t at any t-good marking. In this case, for transition t, all good markings are classified into two sets: t-enabled markings and t-disabled markings, as presented below.

Definition 7 [10]. Let t be a transition and M a t-good marking. M is called a t-enabled good marking if $M[t\rangle$. Otherwise, M is called a t-disabled good marking. The sets of t-enabled and t-disabled good markings are denoted by $\mathcal{E}_t$ and $\mathcal{E}_{\bar{t}}$, respectively.

Control place p_s should not disable transition t at every marking M_k in $\mathcal{E}_t$, *i.e.*, $M_k(p_s) \geq w$. Since $M_k(p_s) = \beta - \sum_{i=1}^{n} l_i \cdot M_k(p_i)$, we have

$$\sum_{i=1}^{n} l_i \cdot M_k(p_i) \leq \beta - w, \forall M_k \in \mathcal{E}_t. \tag{15}$$

Equation (15) is called the enabled condition of t.

Equations (13), (14), and (15) can be used to determine l_i's ($i = 1, 2, \ldots, n$), β, and w. The previous work [10] does not consider the controllability and observability of transition t. Thus, the PI determined by l_i's ($i = 1, 2, \ldots, n$) and β may generate an additional arc on t if $-\sum_{i=1}^{n} l_i \cdot [N](p_i, t) \neq 0$, as shown in Fig. 3. In this case, the additional arc may change the tokens in the obtained control place when t fires. That is to say, the control place observes t. Hence, we should add the following condition to ensure that no additional arc on t can be generated:

$$-\sum_{i=1}^{n} l_i \cdot [N](p_i, t) = 0. \tag{16}$$

In fact, (16) is the same as the unobservability condition, *i.e.*, (12). Equations (13), (14), (15), and (16) can be combined into an ILPP to determine l_i's ($i = 1, 2, \ldots, n$), β, and w. Then, p_s can be computed by (5) with an additional self-loop associated with t but does not observe t.

4.2 Reduction of considered markings

For the Petri net model of an FMS, its places can be classified into three categories: idle, operation, and resource places, whose sets are denoted as P^0, P_A, and P_R, respectively [12,33,36,50]. Tokens in an idle place represent the maximal number of concurrent operations that may happen in a production sequence. An operation place represents an operation to be processed for a part in a production sequence and initially has no token. Resource places represent the resources in an FMS and their initial tokens represent the number of available resource units.

In [40], only the tokens in operation places are considered to construct a PI. This section recalls the marking reduction technique to reduce the number of the considered markings in $\mathcal{M}_L$ and $\mathcal{E}_t$, and reduce the number of the considered MTSIs, which is originally presented in [6] and [10]. In the following, $\mathbb{N}_A$ denotes $\{i | p_i \in P_A\}$.

Definition 8 [6]. Let M and M' be two markings in $R(N, M_0)$. M A-covers M' (or M' is A-covered by M) if $\forall p \in P_A$, $M(p) \geq M'(p)$, which is denoted as $M \geq_A M'$(or $M' \leq_A M$).

Definition 9 [6]. Let $\mathcal{M}_L^\star$ be a subset of legal markings. $\mathcal{M}_L^\star$ is called the minimal covering set of legal markings if the following two conditions are satisfied:

1. $\forall M \in \mathcal{M}_L, \exists M' \in \mathcal{M}_L^\star$ $s.t.$ $M' \geq_A M$;
2. $\forall M \in \mathcal{M}_L^\star, \nexists M'' \in \mathcal{M}_L^\star$ $s.t.$ $M'' \geq_A M$ and $M \neq M''$.

Definitions 8 and 9 were originally proposed in our previous work [6]. If a PI does not forbid any marking in $\mathcal{M}_L^\star$, no legal marking is forbidden by the PI. Hence, we consider markings in $\mathcal{M}_L^\star$ only to guarantee the optimality of the computed control places. Therefore, (13) can be reduced as:

$$\sum_{i \in \mathbb{N}_A} l_i \cdot M_l(p_i) \leq \beta, \forall M_l \in \mathcal{M}_L^\star \tag{17}$$

For an MTSI (M, t), an approach to reduce the considered t-enabled good markings is presented in the following, which was originally proposed in [10].

Definition 10 [10]. Let $\mathcal{E}_t^\star$ be a subset of t-good markings. $\mathcal{E}_t^\star$ is called the minimal covering set of t-enabled good markings if the following two conditions are satisfied:

1. $\forall M \in \mathcal{E}_t, \exists M' \in \mathcal{E}_t^\star \ s.t. \ M' \geq_A M$;
2. $\forall M \in \mathcal{E}_t^\star, \nexists M'' \in \mathcal{E}_t^\star \ s.t. \ M'' \geq_A M$ and $M \neq M''$.

If a transition t is enabled at every marking in $\mathcal{E}_t^\star$, t is enabled at all markings in $\mathcal{E}_t$. Thus, when we design a control place with a self-loop associated with transition t, markings in $\mathcal{E}_t^\star$ are required to be considered only. Therefore, the enabled condition, *i.e.*, (15) can be reduced as:

$$\sum_{i \in \mathbb{N}_A} l_i \cdot M_k(p_i) \leq \beta - w, \forall M_k \in \mathcal{E}_t^\star \tag{18}$$

Accordingly, (16) becomes

$$-\sum_{i \in \mathbb{N}_A} l_i \cdot [N](p_i, t) = 0 \tag{19}$$

Similarly, by considering operation places only, (14) is modified as:

$$\sum_{i \in \mathbb{N}_A} l_i \cdot M(p_i) \geq \beta - w + 1 \tag{20}$$

Definition 11 [10]**.** Let t be a critical transition and $\mathcal{D}_t$ the set of t-dangerous markings. (M, t) is called a t-critical MTSI. The set of t-critical MTSIs is denoted as Ω_t, *i.e.*, $\Omega_t = \{(M, t) | M \in \mathcal{D}_t\}$.

A control place with a self-loop associated with t may implement more than one MTSI in Ω_t. Thus, we do not need to compute a control place with a self-loop associated with t for each MTSI in Ω_t.

Theorem 3 [10]**.** Let (M_1, t) and (M_2, t) be two MTSIs in Ω_t with $M_1 \leq_A M_2$. If (M_1, t) is implemented by a control place with a self-loop associated with t, then (M_2, t) is implemented by the control place.

Definition 12 [10]**.** Let $\Omega_t^\star$ be a subset of t-critical MTSIs. $\Omega_t^\star$ is called the minimal covered set of t-critical MTSIs if the following two conditions are satisfied:

1. $\forall (M, t) \in \Omega_t, \exists (M', t) \in \Omega_t^\star \ s.t. \ M' \leq_A M$;
2. $\forall (M, t) \in \Omega_t^\star, \nexists (M'', t) \in \Omega_t^\star \ s.t. \ M'' \leq_A M$ and $M \neq M''$.

If all MTSIs in $\Omega_t^\star$ are implemented by control places with self-loops associated with t, then all MTSIs in Ω_t are implemented by the control places. Hence, for a transition t, MTSIs in $\Omega_t^\star$ need to be considered only, which can greatly reduce the computational overhead and the number of resulting control places. A control place with a self-loop associated with a transition t may implement more than one MTSI in $\Omega_t^\star$, as defined below.

Definition 13 [10]**.** Let t be a transition, $\Omega_t^\star$ the minimal covered set of t-critical MTSIs, and p_s a control place with a self-loop associated with t. The set of MTSIs that are implemented by p_s is defined as $\Omega_{p_s} = \{(M, t) \in \Omega_t^\star | \sum_{i \in \mathbb{N}_A} l_i \cdot (M(p_i) + [N](p_i, t)) \geq \beta - w + 1\}$.

For an MTSI (M, t) in $\Omega_t^\star$, we can use (17), (18), (19), and (20) to obtain a solution for variables l_i's $(i \in \mathbb{N}_A)$, β, and w. Then, we can compute a control place with a self-loop associated with t by (5) to implement (M, t) but does not observe t. In general, $\mathcal{M}_L^\star$, $\mathcal{E}_t^\star$, and $\Omega_t^\star$ are much smaller than $\mathcal{M}_L$, $\mathcal{E}_t$, and Ω_t, respectively. Therefore, the reduction approach can greatly reduce the computational overhead of each control place and the total number of the computed control places.

V. OPTIMAL DEADLOCK PREVENTION POLICY

This section develops an optimal deadlock prevention policy for the Petri net models of FMSs with uncontrollable and unobservable transitions. First, we propose an ILPP to design an optimal control place to implement as many MTSIs as possible, aiming to reduce the obtained supervisory structure.

Let t be a critical transition that is controllable and unobservable and $\Omega_t^\star$ the minimal covered set of its critical MTSIs. Let $\mathbb{N}_t^\star = \{j | (M_j, t) \in \Omega_t^\star\}$. We introduce a set of variables f_j's $(j \in \mathbb{N}_t^\star)$ to represent whether an MTSI $(M_j, t) \in \Omega_t^\star$ is implemented by a control place p_s with a self-loop associated with t. The disabled condition for (M_j, t) is rewritten as follows:

$$\sum_{i \in \mathbb{N}_A} l_i \cdot M_j(p_i) \geq -Q \cdot (1 - f_j) + \beta - w + 1, \quad \forall (M_j, t) \in \Omega_t^\star \tag{21}$$

where Q is a positive integer constant that must be big enough, $f_j \in \{0, 1\}$ $(j \in \mathbb{N}_t^\star)$, and $\mathbb{N}_t^\star = \{j | (M_j, t) \in \Omega_t^\star\}$. In (21), $f_j = 1$ indicates that (M_j, t) is implemented by p_s and $f_j = 0$ implies that (M_j, t) is not implemented.

In fact, we should consider all unobservable transitions. Thus, (19) should be rewritten as:

$$\sum_{i \in \mathbb{N}_A} l_i \cdot [N](p_i, t) = 0, \forall t \in T_{c\bar{o}} \cup T_{\bar{c}\bar{o}}. \tag{22}$$

For a transition $t \in T_c$ that is controllable and unobservable, we have (17), (18), (21), and (22). Therefore, the following ILPP is developed to design an optimal control place p_s with a self-loop associated with t, which is denoted as the maximal number of MTSIs problem (MNMP).

MNMP(t):

$$max\ f = \sum_{j \in \mathbb{N}_t^\star} f_j$$

subject to

$$\sum_{i \in \mathbb{N}_A} l_i \cdot M_l(p_i) \le \beta, \forall M_l \in \mathcal{M}_L^\star \tag{23}$$

$$\sum_{i \in \mathbb{N}_A} l_i \cdot M_k(p_i) \le \beta - w, \forall M_k \in \mathcal{E}_t^\star \tag{24}$$

$$\sum_{i \in \mathbb{N}_A} l_i \cdot M_j(p_i) \ge -Q \cdot (1 - f_j) + \beta - w + 1, \quad \forall (M_j, t) \in \Omega_t^\star \tag{25}$$

$$\sum_{i \in \mathbb{N}_A} l_i \cdot [N](p_i, t) = 0, \forall t \in T_{c\bar{o}} \cup T_{\bar{c}\bar{o}} \tag{26}$$

$$l_i \in \{0, 1, 2, \ldots\}, i \in \mathbb{N}_A$$

$$\beta, w \in \{1, 2, 3, \ldots\}$$

$$f_j \in \{0, 1\}, j \in \mathbb{N}_t^\star$$

The objective function f is used to maximize the number of MTSIs that are implemented by control place p_s. Denote its optimal value by f^*. If $f^* = 0$, we have $f_j = 0$, $\forall j \in \mathbb{N}_t^\star$, implying that no MTSI in $\Omega_t^\star$ can be implemented by p_s. In this case, the proposed method cannot obtain an optimal control place with a self-loop associated with t. If $f^* = |\Omega_t^\star|$, we have $f_j = 1$, $\forall j \in \mathbb{N}_t^\star$, implying that only one control place with a self-loop associated with t is required to implement all MTSIs in $\Omega_t^\star$.

Algorithm 1. Optimal Deadlock Prevention Policy

Input: Petri net model (N, M_0) of an FMS with $N = (P^0 \cup P_A \cup P_R, T_{co} \cup T_{\bar{c}o} \cup T_{\bar{c}\bar{o}} \cup T_{c\bar{o}}, F, W)$.

Output: an optimally controlled Petri net system (N^α, M_0^α) or the fact that (N, M_0) has no optimal Petri net supervisor or the method fails.

1. Find the set of legal markings $\mathcal{M}_L$ and the set of MTSIs Ω for (N, M_0).
2. Find the set of critical transitions T_c and the minimal covering set of legal markings $\mathcal{M}_L^\star$ for (N, M_0).
3. **if** $\{ T_c \cap (T_{\bar{c}o} \cup T_{\bar{c}\bar{o}}) \neq \emptyset \}$ **then**
   Exit, as (N, M_0) has no optimal Petri net supervisor.
   endif
4. $V_M := \emptyset$. /* V_M is used to denote the set of control places to be computed.*/
5. **foreach** $\{t \in T_c\}$ **do**
   Derive the set $\mathcal{G}_t$ of t-good markings and the set $\mathcal{D}_t$ of t-dangerous markings from the set of legal markings.
   Derive the set $\mathcal{E}_t$ of t-enabled good markings from $\mathcal{G}_t$ and the minimal covering set $\mathcal{E}_t^\star$ of t-enabled good markings.
   Compute the set Ω_t of t-critical MTSIs according to $\mathcal{D}_t$ and the minimal covered set $\Omega_t^\star$ of t-critical MTSIs.
   while $\{\Omega_t^\star \neq \emptyset\}$ **do**
   Solve MNMP(t). If $f^* = 0$, exit, as the method fails to find an optimal supervisor.
   Let l_i's $(i \in \mathbb{N}_A)$, β, and w be the solution.
   Compute control place p_s with a self-loop associated with t.
   $V_M := V_M \cup \{p_s\}$ and $\Omega_t^\star := \Omega_t^\star - \Omega_{p_s}$.
   endwhile
6. Add all control places in V_M to (N, M_0) and denote the resulting net system as (N^α, M_0^α).
7. Output (N^α, M_0^α).
8. End.

Algorithm 1 computes the reachability graph of a Petri net only once. By using the vector covering approach, only markings in $\mathcal{M}_L^\star$ are considered to keep all legal markings reachable. For a transition t in T_c, only markings in $\mathcal{E}_t^\star$ are required to satisfy the enabled conditions. Meanwhile, only the MTSIs in $\Omega_t^\star$ are considered to implement all critical MTSIs of t. Finally, we can obtain a set of control places to implement all MTSIs but no legal marking is removed. The marking reduction technique can greatly lessen the number of constraints in the ILPPs to compute control places, which reduces the computational burden.

Next, the computational complexity of Algorithm 1 is discussed. First, the algorithm is based on reachability graph analysis of a Petri net model, which suffers from the state explosion problem since the number of reachable markings increases exponentially with respect to the size of a Petri net model. Second, ILPPs are required to be solved, which are NP-hard. The computational time to find an optimal solution of an ILPP greatly depends on the number of its constraints and variables. Thus, a marking reduction technique is introduced to reduce the number of constraints in the ILPPs, aiming to reduce the computational burden of the proposed method. In summary, the proposed method is still NP-hard in the sense of its computational complexity.

Theorem 4. Algorithm 1 can obtain a maximally permissive net supervisor for a Petri net model of an FMS if there exists a solution that satisfies (17), (18), (20), and (22) for each MTSI in $\Omega_t^\star$, where $t \in T_c$.

Proof 3. Equation (17) can ensure that each computed control place does not forbid any legal marking. If a transition $t \in T_c$ is enabled at a good marking, (18) is used to ensure that the additional self-loop does not disable its firing. Equation

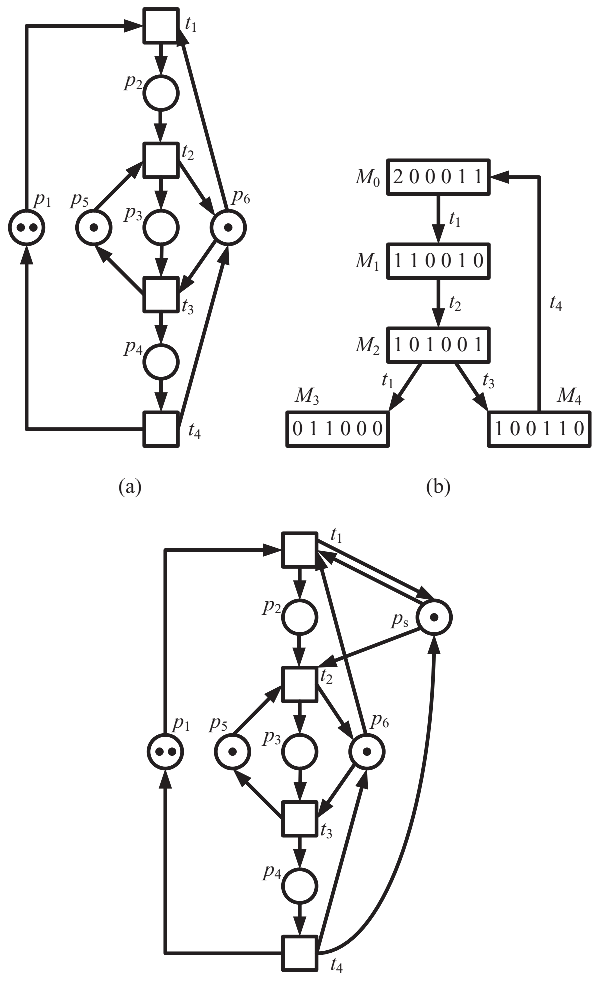

Fig. 4. (a) A Petri net example, (b) its reachability graph, and (c) an optimal controlled net.

(20) can ensure that each MTSI in $\Omega_t^\star$ has an optimal control place to implement it. Equation (22) indicates that the obtained control place does not observe any unobservable transitions. Thus, the proposed algorithm can find control places to implement all MTSIs in $\Omega_t^\star$, implying that all MTSIs in Ω_t can be implemented. Finally, for each $t \in T_c$, its critical MTSIs are implemented. Then, all MTSIs are implemented. That is to say, the final controlled system is live and all legal markings are kept, *i.e.*, the conclusion holds.

Next, a simple example in Fig. 4a is used to illustrate the proposed method. The Petri net has six places and four transitions, where the places are classified into the following groups: $P^0 = \{p_1\}$, $P_R = \{p_5, p_6\}$, and $P_A = \{p_2, p_3, p_4\}$. Its reachability graph is shown in Fig. 4b. It has five reachable markings, in which four are the legal markings and one is the deadlock marking. Using the marking reduction method, we have $\mathcal{M}_L^\star = \{M_1, M_2, M_4\}$. Suppose that t_1 and t_3 is unobservable, t_2 and t_4 is uncontrollable, *i.e.*, $T_{c\bar{o}} = \{t_1, t_3\}$ and $T_{\bar{c}o} = \{t_2, t_4\}$. Next, we apply the proposed method to the example.

The net has only one MTSI, *i.e.*, $\Omega = \Omega_{t_1} = \Omega_{t_1}^\star = \{(M_2, t_1)\}$. Thus, we only need to design a control place for the MTSI. Let p_s be a control place that satisfies a constraint: $l_2 \cdot \mu_2 + l_3 \cdot \mu_3 + l_4 \cdot \mu_4 \le \beta$. Since t_1 is controllable and unobservable, we add a self-loop between p_s and t_1 with a weight w. There is one t_1-enabled good marking only, *i.e.*, $\mathcal{E}_{t_1}^\star = \{M_0\}$. First, p_s should not disable t_1 at M_0. According to (18), we have

$$l_2 \cdot 0 + l_3 \cdot 0 + l_4 \cdot 0 \le \beta - w.$$

Second, every marking in $\mathcal{M}_L^\star$ should be reachable. According to (17), we have

$$l_2 \cdot 1 + l_3 \cdot 0 + l_4 \cdot 0 \le \beta,$$
$$l_2 \cdot 0 + l_3 \cdot 1 + l_4 \cdot 0 \le \beta, \quad \text{and}$$
$$l_2 \cdot 0 + l_3 \cdot 0 + l_4 \cdot 1 \le \beta.$$

Third, p_s should disable t_1 at M_2. According to (14), we have

$$l_2 \cdot 0 + l_3 \cdot 1 + l_4 \cdot 0 \le \beta - w + 1.$$

Finally, p_s does not observe t_1 and t_3. According to (22), we have

$$l_2 = 0 \quad \text{and}$$
$$l_3 - l_4 = 0.$$

Combining all the above constraints, we obtain an ILPP, namely MNMP(t_1). It has an optimal solution with $l_3 = l_4 = \beta = w = 1$. Then, we can design a control place p_s with a self-loop associated with t_1, where p_s satisfies $\mu_3 + \mu_4 \le 1$ and the weight of the self-loop is 1. That is to say, ${}^\bullet p_s = \{t_1, t_4\}$, $p_s^\bullet = \{t_1, t_2\}$, and $M_0(p_s) = 1$. The controlled net is shown in Fig. 4c.

It can be verified that the controlled net is live with all the four legal markings. And, the added control place p_s does not observe any unobservable transition. Note that traditional methods cannot find an optimal supervisor for this example since they cannot design a control place that can disable a transition but does not observe it.

VI. AN FMS EXAMPLE

This section provides a well-known example that is studied in several papers [27,34,35,39]. The example is shown in Fig. 5, where there are 19 places and 14 transitions. The places can be classified into the following sets: $P^0 = \{p_1, p_8\}$, $P_R = \{p_{14} - p_{19}\}$, and $P_A = \{p_2 - p_7, p_9 - p_{13}\}$. The model

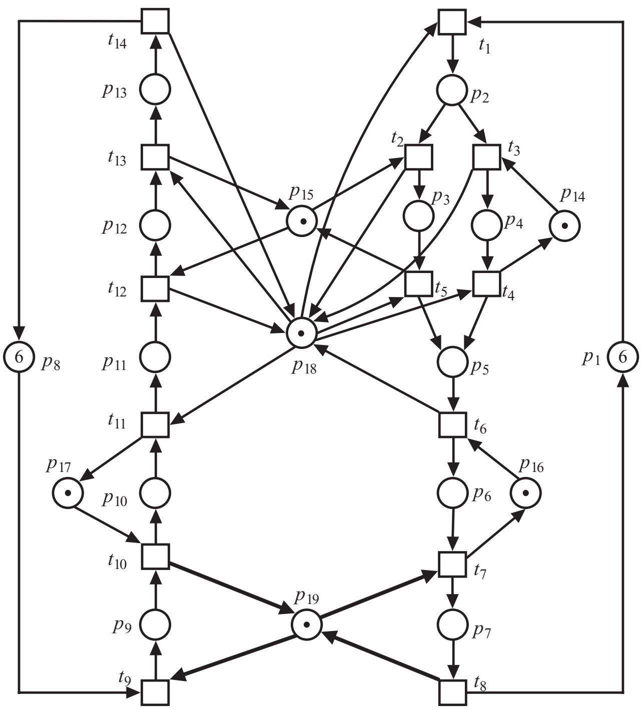

Fig. 5. The Petri net model of an FMS in [39].

has 282 reachable markings, where 205 are legal ones. By using the marking reduction approach, $\mathcal{M}_L^\star$ has only 26 markings. It has 59 MTSIs and five critical transitions, *i.e.*, $T_c = \{t_1, t_2, t_4, t_9, t_{11}\}$. Suppose that the set of uncontrollable transitions is $\{t_3, t_8, t_{10}\}$ and the set of unobservable transitions is $\{t_6, t_{11}\}$. In the following, we show the application of the proposed method.

According to Theorem 1, if there exists a critical transition that is uncontrollable, the net cannot be optimally controlled. On the other hand, if all critical transitions are controllable, we do not need to consider the controllability of transitions since an optimal supervisor prohibits the critical transitions only. Thus, we need to consider the observability of the transitions only. Table I shows the outcomes of the proposed method, where t_i is a critical transition, $|\mathcal{E}_{t_i}|$, $|\mathcal{E}_{t_i}^\star|$, $|\Omega_{t_i}|$, and $|\Omega_{t_i}^\star|$ indicate the numbers of markings in $\mathcal{E}_{t_i}$, $\mathcal{E}_{t_i}^\star$, Ω_{t_i}, and $\Omega_{t_i}^\star$, respectively, I_i implies the computed PI, w indicates the weight of the self-loop associated with t_i, and ${}^\bullet p_s$, $p_s^\bullet$, and $M_0(p_s)$ are the preset, postset, and initial marking of the computed control place p_s, respectively. Note that there are two control places computed for transition t_9 since we cannot find a control place to implement all MTSIs in $\Omega_{t_9}^\star$.

It can be verified that the supervisor shown in Table I can make the net live with all 205 legal markings. It can also be seen that the obtained supervisor does not prohibit any uncontrollable transition and observe any unobservable transition. In fact, this example has been widely used in a number of papers [6,7,10,27,34,35,39]. All these papers can find an optimal supervisor to make all 205 legal markings reachable. However, none of them considers the controllability and observability of the transitions.

Note that t_{11} is controllable and unobservable, and is a critical transition. The proposed approach can lead to an optimal supervisor that can prohibit its firing but does not observe it. As far as the authors know, no existing work (such as [6,7,10,27,34,35,39]) can find an optimal supervisor for this example if t_{11} is unobservable.

Next, we do more simulations on this example by considering different sets of uncontrollable transitions and unobservable transitions. We consider that the set of uncontrollable transitions is $\{t_2, t_3, t_6, t_7, t_8, t_9, t_{10}, t_{11}, t_{12}, t_{13}, t_{14}\}$ and the set of unobservable transitions is $\{t_6, t_{12}\}$. Since there are three uncontrollable transitions t_2, t_9, and t_{11} that are also critical transitions. The firing of a critical transition leads to the DZ at some particular markings. For the optimal control purposes, a critical transition should be disabled at some particular markings. Therefore, there is no optimal supervisor if a critical transition is uncontrollable. This fact can also be verified by Theorem 1 since two of the critical transitions, *i.e.*, t_2 and t_9, are uncontrollable.

Next, we consider that all critical transitions are controllable, *i.e.*, the set of uncontrollable transitions is $\{t_3, t_6, t_7, t_8, t_{10}, t_{13}, t_{14}\}$ and the set of unobservable transitions is $\{t_6, t_{12}\}$. In this case, the proposed method is applied to the net model in Fig. 5. The obtained results are shown in Table II. Adding the supervisor to the original net model, we can obtain a controlled system that is live with all 205 legal markings. Note that t_6 is uncontrollable and unobservable. It can be seen that there is no arc between the supervisor and t_6.

VII. CONCLUSIONS

This paper presents an optimal deadlock prevention policy for FMSs with uncontrollable and unobservable transitions. Its main difference from the previous work is that a controllable transition can be unobservable. Then, a non-pure Petri net structure, self-loop, is introduced to control a transition but does not observe the transition. The proposed method is general since it is applicable to all FMS-oriented classes of Petri net models. The disadvantages of the method is that it requires a complete generation of the reachability graph that always suffers from the state explosion problem. Future work will focus on the reduction of the computational complexity. The first one is that we can focus on some classes of Petri nets that have very special structures and properties that can make it possible to avoid the complete enumeration of reachable markings to obtain an optimal supervisor. Another solution is to develop efficient approaches to compute the reachability graph of a Petri net model.

Table I. Experimental results for the net shown in Fig. 5 with $T_{\bar{c}o} = \{t_3, t_8, t_{10}\}$ and $T_{c\bar{o}} = \{t_6, t_{11}\}$.

t_i	$\lvert\mathcal{E}_{t_i}\rvert/\lvert\mathcal{E}^{\star}_{t_i}\rvert$	$\lvert\Omega_{t_i}\rvert/\lvert\Omega^{\star}_{t_i}\rvert$	I_i	w	$^{\bullet}p_s$	$p_s^{\bullet}$	$M_0(p_s)$
t_1	45/8	24/3	$3\mu_3 + 4\mu_4 + \mu_5 + \mu_6 + \mu_9 + \mu_{10} + \mu_{11} + 3\mu_{12} \le 10$	4	$4t_1, 2t_5, 3t_4, t_7, 3t_{13}$	$4t_1, 3t_2, 4t_3, t_9, 2t_{12}$	10
t_2	22/3	1/1	$\mu_5 + \mu_6 + \mu_9 + \mu_{10} + \mu_{11} \le 4$	2	$2t_2, t_7, t_{12}$	$2t_2, t_4, t_5, t_9$	4
t_4	32/5	3/2	$\mu_3 + \mu_5 + \mu_6 + 2\mu_9 + 2\mu_{10} + 2\mu_{11} \le 7$	3	$2t_4, t_7, 2t_{12}$	$t_2, 3t_4, 2t_9$	7
t_9	62/15	9/4	$3\mu_3 + \mu_5 + \mu_6 + 2\mu_{10} + 2\mu_{11} \le 7$	2	$2t_5, t_7, 2t_9, 2t_{12}$	$3t_2, t_4, 2t_9, 2t_{10}$	7
			$3\mu_2 + \mu_4 + 4\mu_5 + 4\mu_6 + 2\mu_{10} + 2\mu_{11} \le 11$	2	$3t_2, 2t_3, 4t_7, 2t_9, 2t_{12}$	$3t_1, 3t_4, 4t_5, 2t_9, 2t_{10}$	11
t_{11}	12/2	22/2	$\mu_3 + \mu_{12} \le 1$	1	t_5, t_{11}, t_{13}	t_2, t_{11}, t_{12}	1

Table II. Experimental results for the net shown in Fig. 5 with $T_{\bar{c}o} = \{t_3, t_7, t_8, t_{10}, t_{11}, t_{13}, t_{14}\}$, $T_{c\bar{o}} = \{t_{12}\}$, and $T_{\bar{c}\bar{o}} = \{t_6\}$.

t_i	$\lvert\mathcal{E}_{t_i}\rvert/\lvert\mathcal{E}^{\star}_{t_i}\rvert$	$\lvert\Omega_{t_i}\rvert/\lvert\Omega^{\star}_{t_i}\rvert$	I_i	w	$^{\bullet}p_s$	$p_s^{\bullet}$	$M_0(p_s)$
t_1	45/8	24/3	$3\mu_3 + 4\mu_4 + \mu_5 + \mu_6 + \mu_9 + \mu_{10} + 3\mu_{11} + 3\mu_{12} \le 10$	4	$4t_1, 2t_5, 3t_4, t_7, 3t_{13}$	$4t_1, 3t_2, 4t_3, t_9, 2t_{11}$	10
t_2	22/3	1/1	$\mu_5 + \mu_6 + \mu_9 + \mu_{10} \le 3$	1	t_2, t_7, t_{11}	t_2, t_4, t_5, t_9	3
t_4	32/5	3/2	$\mu_3 + \mu_5 + \mu_6 + 2\mu_9 + 2\mu_{10} \le 5$	1	$t_7, 2t_{11}$	$t_2, t_4, 2t_9$	5
t_9	62/15	9/4	$2\mu_2 + 3\mu_3 + \mu_4 + 2\mu_5 + 2\mu_6 + 2\mu_{10} \le 9$	3	$2t_7, 3t_9, 3t_{11}$	$t_1, t_2, t_4, 3t_9, 3t_{10}$	9
t_{11}	12/2	22/2	$\mu_3 + \mu_{11} + \mu_{12} \le 1$	1	t_5, t_{13}	t_2, t_{11}	1

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