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Deadlock Avoidance in Petri Nets with Uncontrollable Transitions*

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Abstract

Recent results in the literature have provided efficient control synthesis techniques for the problem of deadlock avoidance in Petri nets. These results are shown to fit within an established framework for the enforcement of linear constraints on the marking behavior of a net. Framing the problem in this way allows uncontrollable and/or unobservable transitions to be included in the plant model when deadlock avoidance is performed.

1. Introduction

Supervisory control techniques for deadlock avoidance in Petri nets (PN's) have been proposed by several researchers [1, 2, 4]. The techniques seek to identify the problematic structures in the plant and then to restrict the plant behavior relating to these structures so as to prevent deadlock. An important result relating to the problem is *Commoner's liveness theorem*, which states that a free choice Petri net is live if and only if all of its siphons contain a marked trap. The techniques mentioned above assume that the plant transitions are all controllable. The existence of liveness-enforcing supervisors for plants with uncontrollable transitions is studied in [8].

Techniques for enforcing general linear constraints on Petri nets with uncontrollable transitions do exist [5–7]. These controllers enforce linear inequalities on the reachable markings of the plant while avoiding the inhibition of uncontrollable transitions. Unfortunately these controllers have not, in the past, accounted for the deadlock problem. In fact, the supervisors generated by these techniques may actually be the cause of plant deadlocks! In this paper, results for the synthesis of deadlock avoiding controllers are placed within the framework for constraint enforcement in the face of uncontrollable transitions.

2. Deadlock Avoidance

A supervisory control technique is introduced in [1] for handling the problem when not all of the siphons in a given Petri net are controlled, either by a marked trap or a place invariant. The method involves adding a place for each uncontrolled siphon in the net such that they become controlled, i.e., each control place insures that its siphon will never be emptied of all of its tokens. An analysis of the synthesis technique in [1] shows that this is done by creating place invariants in the closed loop PN system:

$$\Big(\sum_{p_i \in S} \mu_i\Big) - \mu_c = 1 \tag{1}$$

where S is an uncontrolled siphon, μ_i is the marking of plant place p_i, and μ_c is the marking of the controller.

Controlling all of the formerly uncontrolled siphons in a net is sufficient for insuring liveness for a wide variety of Petri nets. Liveness is not guaranteed for nets outside this class. In fact, Ezpeleta *et al.* [2] have shown that the act of creating a supervisor to control the siphons of a plant may actually result in the creation of new siphons that are not controlled. Of course some systems simply can not be made live by any supervisor (see [8] for existence theorems).

3. Handling Uncontrollable Transitions

Invariant (1) is equivalent to enforcing a linear inequality on the reachable marking of the plant where the controller place plays the part of a nonnegative excess variable. Techniques for creating Petri net supervisors for enforcing general linear inequalities on the markings of Petri nets appear in [3, 6, 9]. Methods for modifying the inequality such that the resulting controller accounts for uncontrollable transitions have been presented in [5–7].

The control method of [9] indicates that the constraint $l^T \mu_p \leq b$, where μ_p is the plant's marking vector, can be enforced by the following maximally permissive controller

$$D_c = -l^T D_p \qquad \mu_{c_0} = b - l^T \mu_{p_0} \tag{2}$$

where (D_p, μ_{p_0}) and (D_c, μ_{c_0}) are the incidence matrices and initial markings of the plant and controller respectively. The closed loop system is then

$$D = \begin{bmatrix} D_p \\ D_c \end{bmatrix} \qquad \mu = \begin{bmatrix} \mu_p \\ \mu_c \end{bmatrix} \tag{3}$$

Let D_{uc} be an incidence matrix composed of the columns of D_p that correspond to uncontrollable transitions. It is shown in [7] that if

$$l^T D_{uc} \leq 0 \tag{4}$$

then the constraint is *admissible* and may be directly imposed on the given plant using the technique described above. If the inequality is not met, then analytical and computational techniques are given in [7] for obtaining a new constraint that satisfies both the conditions of the original constraint and (4).

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4. Example – The Unreliable Machine

The Petri net of Figure 1 models the operation of a plant that contains an "unreliable machine." The machine is used to process parts from an input queue, completed parts are moved to an output queue by an automated guided vehicle (AGV). The machine is considered unreliable because it is possible that it may break down and damage a part during operation. Damaged parts are moved to a separate queue by a second AGV. Tokens in p_1 represent parts being worked on by the unreliable machine. These parts are either completed, through the *uncontrollable firing of* t_2, or the unreliable machine breaks down and the part is damaged through the *uncontrollable firing of* t_6. Places c_1 and c_2 form the liveness-enforcing supervisory controller, the design of which is covered here.

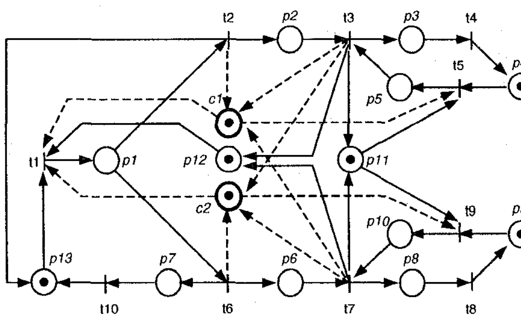

Figure 1: The closed loop live unreliable machine model.

The plant (without places c_1 and c_2) contains two uncontrolled siphons:

$$
\begin{aligned}
S_1 &= \{p_1, p_2, p_{10}, p_{11}, p_{12}\} \\
S_2 &= \{p_1, p_5, p_6, p_{11}, p_{12}\}
\end{aligned}
$$

For each siphon, a control place is created that insures that the sum of the tokens in the siphon remains greater than or equal to one. Before proceeding to create the control structure using (2), we must check to see if the constraint meets condition (4). Unfortunately both siphon-controlling inequalities fail the test. If the supervisor were created using these initial inequalities, then it would attempt to achieve its goal by inhibiting t_2 as well as t_6, which corresponds to machine break down. Transformed constraints that eliminate the influence of the controller on t_2 and t_6 are constructed following the technique of [7]. The controls for siphons S_1 and S_2 are shown as c_1 and c_2 in Figure 1. Note that neither control place will ever attempt to inhibit t_2 or t_6. A final analysis of the siphons of the closed loop system shows that all of the net's siphons are controlled, thus the system is live.

5. Conclusions

A method for deadlock avoidance has been combined with results for enforcing constraints on Petri nets in the presence of uncontrollable transitions. The results expand the applicability and utility of the linear constraint inequality used in [3,5,7,9]. Furthermore, they introduce a useful method for dealing with the deadlock problem into the area of PN DES control with its concept of uncontrollable plant transitions. For a more detailed look at this topic, see [6].

References

[1] K. Barkaoui and I. B. Abdallah, "Deadlock avoidance in FMS based on structural theory of petri nets", In *IEEE Symposium on Emerging Technologies and Factory Automation*, volume 2, pp. 499–510, Piscataway, NJ, 1995. IEEE.

[2] J. Ezpeleta, J. M. Colom, and J. Martínez, "A Petri net based deadlock prevention policy for flexible manufacturing systems", *IEEE Transactions on Robotics and Automation*, vol. 11, no. 2, pp. 173–184, April 1995.

[3] A. Giua, F. DiCesare, and M. Silva, "Generalized mutual exclusion constraints on nets with uncontrollable transitions", In *Proceedings of the 1992 IEEE International Conference on Systems, Man, and Cybernetics*, pp. 974–979, Chicago, IL, October 1992.

[4] H.-H. Huang, F. L. Lewis, and D. A. Tacconi, "Deadlock analysis using a new matrix-based controller for reentrant flow line design", In *IECON Proceedings (Industrial Electronics Conference)*, volume 1, pp. 463–468. IEEE, Los Alamitos, CA, 1996.

[5] Y. Li and W. M. Wonham, "Control of vector discrete event systems II – controller synthesis", *IEEE Transactions on Automatic Control*, vol. 39, no. 3, pp. 512–530, March 1994.

[6] J. O. Moody, *Petri Net Supervisors for Discrete Event Systems*, PhD thesis, Department of Electrical Engineering, University of Notre Dame, Notre Dame, IN., 1998.

[7] J. O. Moody and P. J. Antsaklis, "Supervisory control of Petri nets with uncontrollable/unobservable transitions", In *Proceedings of the 35th IEEE Conference on Decision and Control*, pp. 4433–4438, Kobe, Japan, December 1996.

[8] R. S. Sreenivas, "On the existence of supervisory policies that enforce liveness in discrete-event dynanic systems modeled by controlled Petri nets", *IEEE Transactions on Automatic Control*, vol. 42, no. 7, pp. 928–945, July 1997.

[9] K. Yamalidou, J. O. Moody, M. D. Lemmon, and P. J. Antsaklis, "Feedback control of Petri nets based on place invariants", *Automatica*, vol. 32, no. 1, pp. 15–28, January 1996.