Synchronization Competitive Processes of Flexible Manufacturing System Using Siphons Petri Net

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Abstract*– The synchronization competitive processes of the modeling and design of Flexible Manufacturing System (FMS) is propose to solve the deadlock problem for a class of Petri nets, which is used Systems of Simple Sequential Processes with Resources (S^3PR) net. In this paper presents a deadlock prevention method of a class of FMS, where the unmarked siphon in their Petri net models causes the deadlocks. Siphons are dividing into elementary and dependent ones. An FMS example is using to represent our application of the proposed deadlock prevention policy, also the competitive processes is provided in S^3PRs to show its excellent efficiency. The monitors are using to prevent the presence of unmarked siphons that are direct causes of deadlocks in such a Petri net. The mathematical computations siphon of Petri net is leading to liveness-enforcing Petri net supervisors with behavior that is more permissive. Petri nets are a modeling and simulation formalism for describing concurrency and synchronization in distributed of FMS.

***Keywords* – Petri net, structural analysis, minimal siphons, S^3PR , reachability analysis, FMSs, PN-Tool 2.3**

I. INTRODUCTION

A flexible manufacturing system (FMS) is a set of automated machines and material handling systems, which are linked together with a central computer system. Highly sophisticated machine tools enable the system for a range of manufacturing operations and allowed simultaneous production of multiple part types maintaining a high degree of machine utilization. A FMS usually has a limited number of shared resources. These resources may include machines tools, AGVs, robots, load/unload, buffers and fixtures. Raw parts in a FMS can be processed concurrently, synchronization in the system with respect to time. A dynamic modeling tool should represent these aspects to analyze the conflicts and deadlock during the system execution. In addition to all these requirements, a dynamic modeling tool should support the system designer for system performance evaluation and assist control engineer to control and monitor the FMS. Petri nets have all these capabilities and hence are suitable as dynamic modeling tool irrespective of the various methods used for object model.

The competitive processes of shared resources can cause deadlocks, where each of a set of two or more jobs keeps waiting indefinitely for the other jobs in the set to release resources [7]. A deadlock occurs to a FMS when raw parts are blocked waiting for shared resources held by others that will never be granted. Petri Nets (PNs) constituted a well-known paradigm for the design and operation of many systems is very useful in the deadlock solution of a FMS. Deadlock prevention is considered to be one of the most effective methods in deadlock control [2-7], which is usually implemented by added monitors for a net model to ensure that deadlocks never occur. Especially, the siphon-based methods play an important role in the development of liveness-enforcing supervisors for Petri net models [5]. A Special of Petri net models siphon is defining that allows to capture resource allocation conditions used to synchronize processes that have to share a set of reusable system resources.

Petri nets have been recognized as one of the most powerful tools to describe and analyze the behavior of discrete event systems, including (FMS) [9], because it can describe resource sharing, conflict, mutual exclusion, concurrency, synchronization among objects and performance analysis of FMS, and uncertainty successfully. Besides, due to its brief and normative presentation, Petri net has been applied more broadly and developed further in modeling, analyzing, and controlling the manufacturing systems. In this paper, a deadlock prevention policy is proposed to a class of Petri nets called S^3PR. Also, the structural analysis and reachability graph analysis are using for analysis and control of Petri net.

Ezpeleta et al. [2] pioneer a class of Petri nets called systems of simple sequential processes with resources (S^3PR). Deadlocks can be avoided by adding a control place, and associated arcs, to each emptiable siphon S to prevent it from being unmarked. However, generally too many control places and arcs are required. They developed an approach where liveness is enforced by added monitors, to prevent strict minimal siphons (SMS) in an S^3PR based on resource circuits from being emptied. Almost all the deadlock prevention policies subsequently monitor adding to SMS to enforce liveness, which is a result of a deadlock-free Petri Net.

Many algorithms, for the computation of minimal siphons, have been proposed in the past years [1-7], all specifically designed for the class S^3PR. This means that they profit from the particular structure of S^3PR to obtain good performance in the computation of the set of minimal siphons. Chu et al. [1] have developed a fast deadlock detection approach based on mixed integer programming (MIP) for structurally bounded nets whose deadlocks tied to unmarked siphons. Since no explicit enumeration of siphons is required, this formulation opens a new avenue for checking deadlock-freeness of large

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systems. The MIP method is able to find a maximal siphon unmarked at a reachable marking. A feasible solution corresponds to a maximal unmarked siphon when there are to exists a siphon that can be emptied at a marking that is reachable from the initial marking. Based on this, they can formalize an algorithm that can efficiently obtain a minimal siphon from the result of the MIP method. Deadlock control is usually concerned with minimal siphons. Huang et al. [3, 4] proposed a minimal siphon extraction algorithm such that the complete siphon enumeration was successful avoided. Also, Huang et al. [3] proposed some iterative deadlock prevention policies based on mixed integer programming (MIP). In their methods, two kinds of control places called ordinary control places and weighted control places were added to the original model to prevent siphons from being unmarked.

The elementary siphons are a novel concept of Petri nets, which are first proposed in [6, 7]. They proposed deadlock prevention policy of using the so-called elementary siphon. Require a new way so that is reducing the control of the places in the net by finding out for the siphons, which become or divided into elementary siphons and dependent. During the elementary siphon the control places are being small than accomplishing the same control objective, the control policy is similar to [2]. First, they compute SMS in S^3PR based on resource circuits. Second, separate the set of siphons and traps in the net, than they choice the SMS, do not contain any tarp. Third, compute the dependent siphon from the elementary siphons. The advantage is the possibility to make interactive presentations of the results. This approach reduces the number of control places because it is not necessary to add control places in the redundant siphons.

Organization. In section II, briefly reviews preliminaries provide the necessary background on Petri nets. A method of computing all the concept of elementary siphons in S^3PR is developed in Section III. Section IV, presents a FMS example to demonstrate the method. Finally, we conclude and give perspectives to this work in section V.

II. PRELIMINARIES [9]

Definitions 1. A Petri net is a four-tuple $\Psi = (P, T, E, W)$, where P and T are a finite nonempty, and disjoint sets. P is the set of places, and T is the set of transitions with $(P \cup T) \neq \varnothing$ and $(P \cap T) = \varnothing$. The set of $E \subseteq (P \times T) \cup (T \times P)$ is the flow relation or a set of directed arcs. $W:(P \times T) \cup (T \times P) \longrightarrow Z^+$ is a weight function attached to the arcs, where $Z^+ \longrightarrow (1, 2, \ldots, Z^+)$.

Definition 2. A marking Petri net is $\Psi = (\Psi, M_0)$ where, Ψ is a Petri net $\Psi = (P, T, E, W)$, and M_0 is initial marking, M_0: $P \longrightarrow \{1, 2, \ldots, Z^+\}$. A net (Ψ, M_0) is called a marked net. $M_0(p)$ indicates the number of tokens on p under M. P is marked by M iff $M(p) > 0$. The set of input (resp., output) transitions to a place p are denoted by $\bullet p$ (resp., $p\bullet$). Similarly, the set of input (resp., output) places of a transition t are denoted by $\bullet t$ (resp., $t\bullet$). A Petri net structure (P, T, E, W) without any specific initial marking is denoted by Ψ. A Petri net with the given initial marking is denoted by (Ψ, M_0).

Definition 3. An ordinary Petri net is called a state machine (SM, for short) if $\forall t \in T$, $|t^\bullet| = |{}^\bullet t| = 1$. A Petri net $\Psi = (P, T, E, W)$ is called: (1) a state machine (SM) if and only if (iff) $|{}^\bullet t| = |t^\bullet| \leq 1$ for any $t \in T$; (2) a mark graph (MG) iff $|{}^\bullet p| = |p^\bullet| \leq 1$ for any $p \in P$; (3) when Weights W, of the arcs: $(W) = 1$, the net Ψ is called ordinary Petri net.

Definition 4. A sequence of transitions $\sigma = \{t_1, t_2, \ldots, t_n\}$ is a firing sequence of (Ψ, M_0) iff there exists a sequence of markings such that $M_0[t_1\rangle\ M_1[t_2\rangle\ M_2, \ldots, [t_n\rangle M_n$. Moreover, marking M_n is said to be reachable from M_0 by firing σ, and this is denoted by $M_0[\sigma\rangle M_n$. The firing sequence is a marking $(M_1, M_2, M_3, \ldots, M_{n+1})$ such that:$(\forall\, i, 1 \leq i \leq n)$, and $(M_i[t_i\rangle M_{i+1})$, We can also write its by $[M_1[\sigma\rangle M_{n+1}]$. The set of all markings reachable from M_0 is denoted by reachability set $R(M_0)$. The function σ': $T \longrightarrow N^+$ is the firing count vector of the firable sequence σ, i.e. $\sigma'[t]$ represents the number of occurrences of $t \in T$ in σ. If $M_0[\sigma\rangle M'$, then we can write in vector form $M' = M_0 + C.\sigma'$, which is referred to as the linear state equation of the net. A marking M_0 is said to be potentially reachable iff $\exists X \geq 0$ such that: $M' = M_0 + C.\sigma \geq 0$, where σ is a firing sequence, a vector whose i-th denotes the number of occurrences of in σ.

Definition 5. P-invariant (resp. T-invariant) of a net $\Psi = (\Psi, M_0)$ is a non-negative row integer $|P|$-vector x (resp., $|T|$-vector y) satisfying the equation $x^T. C = 0$, (respectively, $C.\ y^T = 0$), where C is the incidence matrix of Ψ. A non-zero integer vector $y \neq 0$, (resp. $x \neq 0$).

Definition 6. Let S is a non-empty sub-set of places. $S \subseteq P$ is a siphon (trap) iff ${}^\bullet S \subseteq S^\bullet$ and trap $(Q^\bullet \subseteq {}^\bullet Q)$. A marked trap can never be emptied. A siphon is said to be minimal iff contain no other siphons as its proper sub-sets. A minimal siphon is strict if it contains no marked trap. A siphon is said to be controlled in an ordinary net system iff it can never be emptied. A siphon S is said to be invariant- controlled by P-invariant I if $I^T . M_0 > 0$, and $\| I \|^+ \subseteq S$.

Example 1: Consider the Petri net of Fig. 1, show an example of a Petri net. The Petri net consists of the five places, and four transitions is shown in Fig. 1, which has the two minimal siphons as: $S_1 = \{p_1, p_3\}$, $S_2 = \{p_2, p_3\}$, and the two minimal trap as $Q_1 = \{p_3, p_4, p_5\}$, $Q_2 = \{p_2, p_3\}$. Siphons are very important in the analysis structure and control of deadlocks in a Petri net. The Petri net model in Fig. 1 is not live.

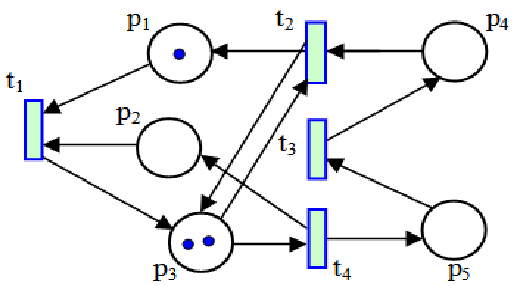

Figure 1 shows a simple example of a Petri net.

In the Fig. 1, the deadlock is occurring at the marking M_{17} = [0, 2, 0, 3, 0]. The minimal siphon is empty marking to leads the deadlock state of the net. To control the net, we can prevent forms being unmarked, a place V_1 is added with ${}^{\bullet}V_1 = \{t_2\}$ and $V_1^{\bullet} = \{t_4\}$, as shown that Fig. 2, to the control is a Petri net. An additional place V_1, called control place to controlled a siphon. We have predicted a deadlock with a look-ahead Petri net controller.

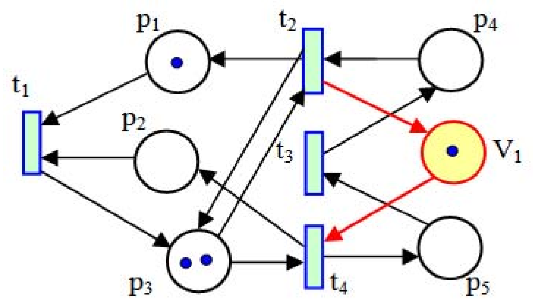

Figure 2 control Petri net of Fig. 1.

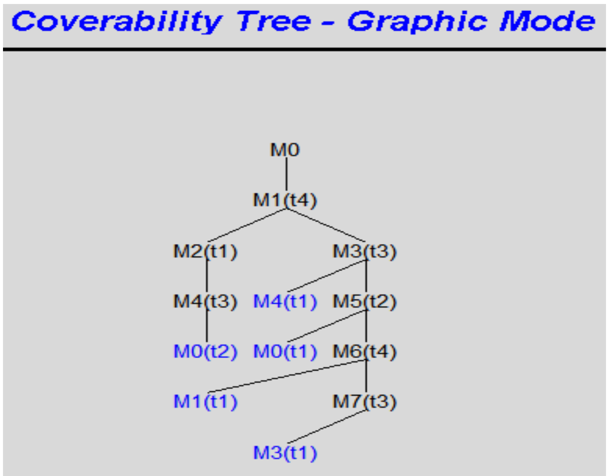

Figure 3. The coverability tree is live of Fig. 2, that runs in PN Toolbox with the MATLAB [8].

This is example motivates one to discover the synchronization mechanism to make a siphon controlled by adding control place. The effecting controlled place in Petri net is shown in Fig. 3. It can verify that deadlock does not occur to this Petri net.

III. DEADLOCK PREVENT POLICY

Deadlock prevention policy is dealing with a special class of Petri nets, which is a subclass of ordinary and conservative Petri nets called S^3PR. In this the section, we introduced some definitions have been needed for our application for a deadlock prevention policy which can be keeping the system in liveness for a class of Petri nets, that is called S^3PR nets.

A. The class of the S^3PR nets

The class of Petri nets investigated in this research is an S^3PR that is first proposed in Ezpeleta et al. [2]. Before the representation of its formal definition is to need for our application. The following results are mainly from [2, 7, 11].

Definition 7. A simple sequential process (S^2P) is a Petri net $\Psi = (P_A \cup \{p_0\}, T, E)$, where the following statements are true: (1) $P_A \neq \varnothing$ is called a set of operation places;
(2) $p^0 \notin P_A$ is called the process idle place;
(3) A net Ψ is a strongly connected state machine;
(4) Every circuit of Ψ contains place p^0.

Definition 8. A system of simple sequential processes with resources (S^3PR): $\Psi = \mathrm{O}_{i=1}^{k} \Psi_i = (p_A \cup p^0 \cup p_R, T, E)$, is defined as the union of a set of nets:
$\Psi_i = (p_i \cup \{p_i^0\} \cup p_{R_i}, T_i, E_i)$, sharing common places,
where the following statements are true: (1) p_i^0 is called the process idle places of net Ψ_i. The elements in p_A^i and p_R^i are called operation places and resource places respectively;
(2) $p_{R_i} \neq \varnothing; p_i \neq \varnothing; p_i^0 \notin p_i; (p_A^i \cup \{p_i^0\}) \cap p_R^i = \varnothing$;
$\forall p \in p_i, \forall t \in {}^{\bullet}p, ; \forall t' \in p^{\bullet}$,
$\exists r_p \in p_{R_i}, {}^{\bullet}t \cap p_{R_i} = t'^{\bullet} \cap p_{R_i} = \{r_p\}$;
$\forall r \in p_r^i, {}^{\bullet\bullet}r \cap p_A^i = r^{\bullet\bullet} \cap p_A^i \neq \varnothing$, and
${}^{\bullet}r \cap r^{\bullet} = \varnothing, {}^{\bullet\bullet}(p_i^0) \cap (p_R^i) = (p_i^0)^{\bullet\bullet} \cap p_R^i = \varnothing$;
(3) $\forall i \neq j, T_i \cap T_j = \varnothing$,
(4) Ψ_i' is a strongly connected state machine, where
$\Psi_i' = (p_A^i \cup \{p_0^i\}, T_i, E_i)$, is the resulting net after the places in p_R^i and related arcs are removed from Ψ_i.
(5) Every circuit of Ψ_i' contains place p_0^i;
(6) Any two Ψ_i' are composable when they share a set of common places. Every shared place must be a resource one.
(7) Transitions in $(p_i^0)^{\bullet}$ and ${}^{\bullet}(p_i^0)$ are called source and sink transitions of an S^3PR, respectively. Ψ is often used to denote an $S^3PR\,(p \cup p^0 \cup p_R, T, E)$, and S a strict minimal siphon in Ψ in case of no confusion.

Definition 9. Let $\Psi_i = (p_A \cup p_0 \cup p_R, T, E)$, be an S^3PR. An initial marking M_0 is called an acceptable one if:
1) $\forall p \in p_0$, $M_0(p) \geq 1$; 2) $\forall p \in p_A$, $M_0(p) = 0$; and
3) $\forall p \in p_R$ R, $M_0(p) \geq 1$.

B. Elementary siphons in Petri nets

The concepts of elementary and dependent siphons are original work by Li et al. [6, 7]. They are development the Petri nets theory of computation and powerful mathematics. We are introduced the concept of elementary and dependent siphons as well as used in this paper.

Definition 10. Let $S \subseteq P$ be a subset of places of Petri net Ψ = (P, T, F, W). P-vector λ_S is called the characteristic P-vector of S iff $\forall p \in S$, $\lambda_S(p) = 1$; otherwise $\lambda_s(p) = 0$.

Definition 11. Let $S \subseteq P$ be a subset of places of Petri net Ψ. η_S is called the characteristic T-vector of S if and only if $\eta_S = \lambda_S^T$. [C], where C is incidence matrix of the net Ψ.

Definition 12. Let $\Psi = (P, T, F, W)$ be a net with $|P| = m$, $|T| = n$, and k siphons, $S_1, S_2, \ldots, S_k$, $m, n, k \in Z^+$. Let λ_{Si} (η_{Si}) be the characteristic P(T)-vector of siphon S_i , where $i \in \{1, 2, \ldots, k\}$. We define $[\lambda]_{k \times m} = [\lambda_{S1} | \lambda_{S2} | \ldots | \lambda_{Sk}]^T$, and $[\eta]_{k \times n} = [\lambda]_{k \times m} \times [C]_{m \times n} = [\eta_{S1} | \eta_{S2} | \ldots | \eta_{Sk}]^T$. Where $[\lambda]([\eta])$ is called the characteristic P(T)-vector matrix of the siphons in net Ψ.

Definition 13. Let $\eta_{S\alpha}$, η_{β}, … , and $\eta_{S\gamma}$ ($\{\alpha, \beta, \ldots, \gamma\} \subseteq \Psi_k$) a linearly independent maximal set of matrix $[\eta]$. Then $\Pi_E = \{S_\alpha, S_\beta, \ldots, S_\gamma\}$ is called a set of elementary siphons in net Ψ.

Definition 14. Let $S \notin \Pi_E$ be a siphon in net Ψ. Then S is called a strongly dependent siphon if $\eta_S = \sum_{S_i \in \Pi_E} a_i \eta_{S_i}$ holds, where $a_i \geq 0$.

IV. AN FMS EXAMPLE

A flexible manufacturing system is considered. The system has three machines M1, M2, and M3, each of which can hold one or two parts. The cell contains two robots, R1 and R2; each robot can hold one part. The robots deal with the movements towards parts. Two load buffers I1/I2, and two unloading buffers O1/O2. The FMS is planning with two production lines (PL_i) requires two operations. Two part types are considered in this FMS. The PN is shown in Fig. 4, its net model (Ψ, M_0) is an S^3PR.

A Petri net based deadlock prevention policy of (FMS) through a special class of Petri Nets that we call S^3PR. We can apply definition 8, which are representing (S^3PR). For the process idles $p^0 = \{p_9, p_{10}\}$, the resource places $P_R = \{p_{11}, p_{12}, p_{13}, p_{14}, p_{15}\}$, $p_A = p_A^1 \cup p_A^2 = \{p_1, p_2, p_3, p_4\} \cup \{p_5, p_6, p_7, p_8\} = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8\}$, holders of resource $H(p_{14}) = (p_4)$, $H(p_{15}) = (p_5)$, $H(p_{11}) = \{p_1, p_8\}$, $H(p_{12}) = \{p_3, p_6\}$, and $H(p_{13}) = \{p_3, p_8\}$. The strict minimal siphon is: $S_1 = \{p_3, p_7, p_{12}, p_{13}\}$ in the net of Fig. 4. Then $S_R \cap p_R = \{p_{12}, p_{13}\}$. Holders of resource $H(p_{13}) = p_{13}^{\bullet\bullet} \cap p = \{p_3, p_6, p_{12}, p_{14}\} \cap \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8\} = \{p_3, p_6\}$. The net shown in Fig. 4, is the Petri net model of an FMS that consists of 3 machine tools and 2 robots. The system Petri net model depicted in Fig. 4 is an S^3PR that contains deadlocks. The structural analysis of Petri nets such as minimal siphons and traps deadlock and (P-T-invariants). Simulation and analysis of Petri net in PN-toolbox in MATLAB [8] are visualization distinguished for siphons and traps, places of deadlocks, and show the reachability tree of Petri net. In Fig. 4, the reachability tree of Petri net can be seeing that is the deadlock marking can occur at the marking: M_{142} = [2, 1, 0, 0, 1, 2, 0, 0, 2, 2, 0, 0, 0, 1, 0], and M_{190} = [2, 0, 0, 0, 1, 2, 1, 0, 3, 1, 0, 0, 0, 1, 0], are real deadlocks, when the reachability tree has (376) states with initial marking.

The Petri net shown in Fig. 4, has 10 minimal siphons:

$S_1 = \{p_3, p_7, p_{12}, p_{13}\}$, $S_2 = \{p_2, p_8, p_{11}, p_{12}\}$,
$S_3 = \{p_3, p_8, p_{11}, p_{12}, p_{13}\}$, $S_4 = \{p_4, p_{14}\}$, $S_5 = \{p_5, p_{15}\}$,
$S_6 = \{p_1, p_8, p_{11}\}$, $S_7 = \{p_3, p_6, p_{13}\}$, $S_8 = \{p_2, p_7, p_{12}\}$,
$S_9 = \{p_5, p_6, p_7, p_8, p_{10}\}$, $S_{10} = \{p_2, p_3, p_7, p_{12}\}$.

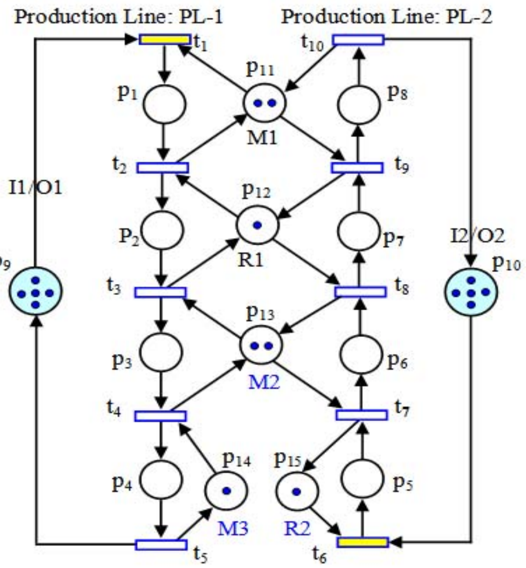

Fig. 4. An S^3PR of net (Ψ, M_0)

The set of siphons from S_4–S_{10}, is both a siphon and trap. We are the choice strict minimal siphons S_1, S_2, and S_3 , which do not contain any traps. The strict minimal siphons are as: $S_1 = \{p_3, p_7, p_{12}, p_{13}\}$, $S_2 = \{p_2, p_8, p_{11}, p_{12}\}$, and $S_3 = \{p_3, p_8, p_{11}, p_{12}, p_{13}\}$, from the ten minimal siphons. We can apply the concept of elementary and dependent siphons are proposed in [6-7]. According the algorithm to elementary siphon theorems which is found in [6-7], one can obtain that:

$\lambda S_1 = (0,0,1,0,0,0,1,0,0,0,0,1,1,0,0)^T$,
$\lambda S_2 = (0,1,0,0,0,0,0,1,0,0,1,1,0,0,0)^T$,
$\lambda S_3 = (0,0,1,0,0,0,0,1,0,0,1,1,1,0,0)^T$. The strict minimal siphons are S_1, S_2, and S_3. We have:

$\lambda S_1 = p_3 + p_7 + p_{12} + p_{13}$,
$\lambda S_2 = p_2 + p_8 + p_{11} + p_{12}$, and
$\lambda S_3 = p_3 + p_8 + p_{11} + p_{12} + p_{13}$.

In addition, the linearly independent T-vectors can be constructed in $[\eta]$ shown as follows:

$$\eta_{S1} = -t_1 + t_2 - t_6 + t_9$$
$$\eta_{S2} = -t_1 + t_3 - t_6 + t_8$$
$$\eta_{S3} = -2t_1 + t_2 + t_3 - 2t_6 + t_8 + t_9$$

The analysis T-vector matrix is as follows. Accordingly, $[\lambda]$ and $[\eta]$ is shown as follows.

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10
η_{S1} =	-1	1	0	0	0	-1	0	0	1	0
η_{S2} =	-1	0	1	0	0	-1	0	1	0	0
η_{S3} =	-2	1	1	0	0	-2	0	1	1	0

Obviously, the rank of $[\eta]$ is 2 since the third row can be linearly represented by the first and second rows. Therefore, the siphons that correspond to the first and second rows are elementary siphons. Next, we can see that $\eta_{S3} = \eta_{S1} + \eta_{S2}$, and $R(|\eta|) = \Psi_{ES} = 2$, which mean that there are two elementary siphons. If S_1, S_2 are elementary siphons and S_3 is a strict redundant siphon. The fact, two are selected as elementary siphon ones, and the third becomes dependent.

S_i	η-T-vector	Set of places
S_1=	$-t_1 + t_2 - t_6 + t_9$	$S_1 = p_3 + p_7 + p_{12} + p_{13}$
S_2=	$-t_1 + t_3 - t_6 + t_8$	$S_2 = p_2 + p_8 + p_{11} + p_{12}$
S_3=	$-2t_1 + t_2 + t_3 - 2t_6 + t_8 + t_9$	$S_3 = p_3 + p_8 + p_{11} + p_{12} + p_{13}$

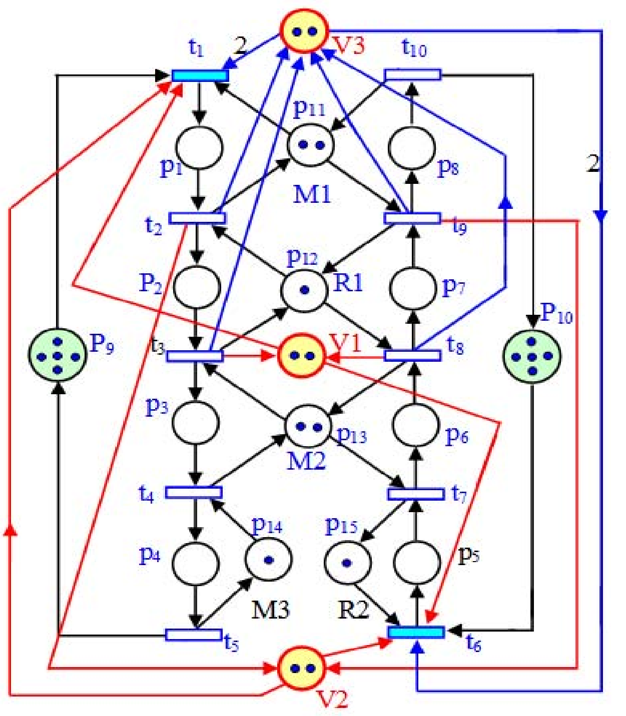

Fig. 5. Deadlock-free supervisor net (Ψ_1, M_1)

The results of the structural analysis of the Petri net in Fig. 4, for the FMS models are controlled through add the "monitors" on Fig. 5. The coverability tree shown that the Petri net model is live, and guaranteeing the deadlock-freeness, i. e. the execution of a plant concurrently, and leading to significant results that theory of Petri net is successful industrial application. By adding a "monitor" place for every elementary siphon S of net Ψ, the liveness behavior of any given S^3PR, $\Psi = (P \cup P^0 \cup P_R, T, E)$ could be a possibility guarantee's if it exists. An elementary siphon S of net Ψ cannot be emptied after the addition of it is corresponding to monitor place.

By considering the possible controlled-siphon property, that is added control place can be connected arcs to the necessary places in order to become deadlock-free. Two optimal controls places V1, and V2 can be control of Petri net. In addition, the controlled net of Fig. 5, is live and there are (188) reachable states, with initial marking, so that the Petri net is deadlock-free actually lives.

In order to reduce the reachability tree, we can add third control places V3 the Petri net on Fig. 5. We can see, that the Petri net is also deadlock-free, and there are reduced to (80) reachable states, with initial marking. Note that the Petri net is deadlock-free. Controlling the three essential siphons with the P-invariant based method results in the additional places V1 –V3, are necessary to added. We can see that the reachability graph has (80) states, and the resultant's net is live as show in Fig. 5, and coverability tree in Fig. 6. The resulting controlled Petri net is shown in Fig. 6. It is being verified that deadlock does not occur to Petri net.

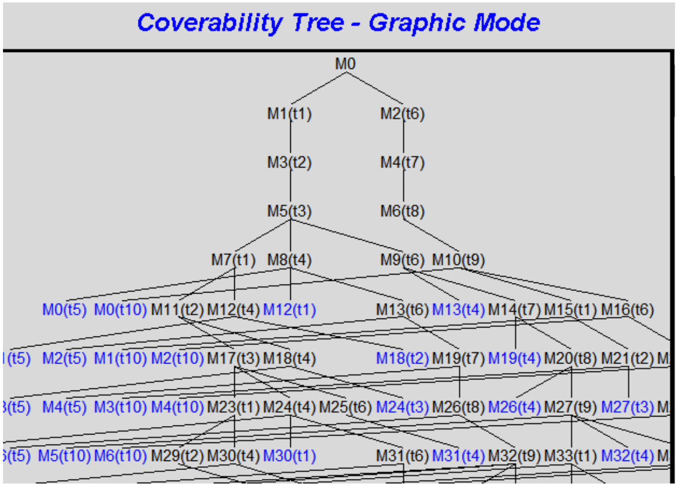

Fig. 6. The segment of coverability tree is live automatically constructed by the PN toolbox with the MATLAB environment [8].

Definition 15. [1]. Siphon S in a net system (Ψ, M_0) is invariant controlled by P-invariant I under M_0 iff $I^T . M_0 > 0$, and $\forall p \in P \backslash S$, $I(p) \leq 0$, or equivalently, $I^T . M_0 > 0$, and that $\|I\|^+ \subseteq S$, such a siphon is also called an invariant- controlled one. If S is controlled by P-invariant I under M_0, S cannot be emptied, i.e., $\forall M \in R(\Psi, M_0)$, S is marked under M.

Definition 16. A P-vector is a column vector I: □P□ → □Z indexed by P, where Z is the set of integers. I is a P-invariant (place invariant) iff (if and only if) I □≠□ 0 and $I^T . [\Psi]$□= 0^T holds. P-invariant I is said to be a P-semiflow if every element of I is non-negative. $\|I\|^+$□= {p□∈ □P |□ I(p)□≠□0} is called the support of I. If I is a P-invariant of (Ψ, M_0) then ∀M□∈ □R(Ψ, M_0): I^T . M□ =□ I^T . M_0. In an ordinary net, siphon S is controlled by P-invariant I under M_0 if and only if (I^T□.□M_0□>□0) and {p□∈ P□|□I(p)□>□0}□ ⊆ □S}. Such a siphon is called invariant-controlled siphons.

A siphon in a net system can be controlled by a P-invariant or a marked trap in it. For an uncontrolled but controllable siphon, we can use an active control method to make it invariant-controlled by adding a monitor. Let us find P–invariant of the Petri net model from the figure 5.

According to definition 5, and definition 16, there are three strict minimal siphons that can be emptied in Ψ. There are $S_1 = \{p_3, p_7, p_{12}, p_{13}\}$, $S_2 = \{p_2, p_8, p_{11}, p_{12}\}$, and $S_3 = \{P_3, P_8, P_{11}, P_{12}, P_{13}\}$. A control place V1 is added such that:

I_1 = (1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1V1) is a P–invariant of (Ψ_1, M_1). Therefore, $I_1 \cdot M_1 = 0$, by definition 16 and it is easy to compute that: [Ψ1](V1, t) = $-t_1 + t_2 - t_6 + t_9$ for siphon S_1. Similarly, I_2 = (1, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1V2) is a P–invariant of (Ψ_1, M_1). Therefore, ($I_2 \cdot M_1$) = 0, and net [Ψ_1] (V2, t) = $- t_1 + t_3 - t_6 + t_8$ for siphon S_2. Where M_1(V1) = $M_0(S_1)$ –1 = 2, and M_1(V2) = $M_0(S_2)$ – 1 = 2. Similarly, I_3 = (2, 1, 0, 0, 2, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1V3) is a P–invariant of (Ψ_1, M_1). Hence, ($I_3 \cdot M_1$) = 0, and [Ψ_1] (V3, t) = $-2t_1 + t_2 + t_3 - 2t_6 + t_8 + t_9$ for siphon S_3, M_1(V3) = $M_0(S_3)$ – 1 = 2. After adding control places V1, V2 and V3 to (Ψ, M_0), the new model (Ψ_1, M_1) is deadlock-free (liveness) as shown in Fig. 5, and in Fig. 6.

For instance, Fig. 5 is showing an S^3PR net. The main purposed is to perform all strict minimal siphons controlled through making elementary siphons invariant-controlled. Petri nets are a modelling formalism for describing concurrency and synchronization in distributed systems. The PIPE is the (Platform Independent Petri Net Editor), write in Java for building, a Java based, open source, graphical tool for drawing and analyzing Petri nets. The main advantage is used tool PIPE v4.3.0 [10] software in the analysis structure of Petri net because this tool has ability to analysis P- and T- invariant, incidence matrix and finds a minimal siphon and trap of Petri net.

For elementary siphons S_1, S_2, and S_3, three monitor V1, V2, and V3 are added respectively to the net. We can apply definition 16, to find a P-invariant controlled of the Petri net depending of the incidence matrix of Fig. 5, implementation manually. For example $S_1 = \{p_3, p_7, p_{12}, p_{13}\}$, is a siphon of the net. A P-invariant of the net in Fig. 5 is : I_1 = (–1, 0, 1, 0, –1, 0, 1, 0,0, 0,0, 1, 1,0,0, –1V1), $I_1 = (p_3 + p_7 + p_{12} + p_{13} - p_1 - p_5 -$ V1), where $I_1 \cdot M_0 = \{M_0(p_3) + M_0(p_7) + M_0(p_{12}) + M_0(p_{13}) - M_0(p_1) - M_0(p_5) - M_0(V1)\}$ =1> 0. $\| I_1 \|^+ = \{p_3, p_7, p_{12}, p_{13}\} \subseteq$ S. So that $S_1 = \{p_3, p_7, p_{12}, p_{13}\}$ is an invariant-controlled siphon and it can never be emptied. Similarly, we can compute P-invariant of the siphon $S_2 = \{p_2, p_8, p_{11}, p_{12}\}$, is a siphon of the net. Siphon S_2 is controlled by P-invariant by:
I_2 = (0, 1, 0, 0, –1, –1, 0, 1, 0, 0 , 1, 1,0, 0,0, –1V2),
$I_2 = (p_2 + p_8 + p_{11} + p_{12} - p_5 - p_6 -$V2). According definition 16, $I_2 \cdot M_0 = \{M_0(p_2) + M_0(p_8) + M_0(p_{11}) + M_0(p_{12}) - M_0(p_5) - M_0(p_6) - M_0(V2)\}$ =1> 0. Where $\|I_2\|^+ = \{p_2, p_8, p_{11}, p_{12}\} \subseteq S$. So that $S_2 = \{p_2, p_8, p_{11}, p_{12}\}$ is an invariant-controlled siphon and it can never be emptied. Similarly, for siphon $S_3 = \{P_3, P_8, P_{11}, P_{12}, P_{13}\}$, is a siphon of the net. I_3 = (–1, 0, 1, 0, –2, –1, 0 , 1, 0, 0, 1, 1, 1, 0, 0, –1V3), $I_3 = (p_3 + p_8 + p_{11} + p_{12} + p_{13} - p_1 - 2p_5 - p_6 -$V3) , where $I_3 \cdot M_0 = \{M_0(p_3) + M_0(p_8) + M_0(p_{11}) + M_0(p_{12}) + M_0(p_{13}) - M_0(p_1) - 2M_0(p_5) - M_0(p_6) - M_0(V3)\}$ =1> 0. $\| I_3 \|^+ = \{p_3, p_8, p_{11}, p_{12}, p_{13}\} \subseteq$ S. So that $S_3 = \{p_3, p_8, p_{11}, p_{12}, p_{13}\}$ is an invariant-controlled siphon and it can never be emptied. Consequently, a possible approach to estimating specific properties in systems is to automatically deduce the invariants from the Petri net structure and then to manually prove that these net P-invariants implies the specific ones given by the designer, which can be proving to the liveness of the Petri net models for FMS.

V. CONCLUSION

The paper deals with structure analysis of Petri net, where 'siphons' is a main utilities used in the development of deadlock control policy of (FMSs), which has been exploited successfully for the design of supervisors of some supervisory control problems. We propose to allocate the tokens in the control places reasonably to guarantee with absence of deadlock states, and monitor added is to each elementary siphon to make ensure that all elementary siphons in the S^3PR net are invariant-controlled. The siphon is successfully controlled and the resultant net system is live (i.e. deadlock-free). A computationally efficient method to compute all the resource allocation system of FMS represented in Petri net model, which is shown in the experimental results, used to test Petri net tool in [8] MATLAB, and tool PIPE v4.3.0 software [10]. Future work includes extending the method to produce class more powerful of Petri nets.

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