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DESIGN OF LIVENESS-ENFORCING SUPERVISORS VIA TRANSFORMING PLANT PETRI NET MODELS OF FMS

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ABSTRACT

This paper focuses on the deadlock prevention problems in a class of Petri nets, systems of simple sequential process with resources, S^3PR for short. By structure analysis, we propose an approach that can transform a plant net model into a weighted S^3PR (WS^3PR) that is behaviorally equivalent to the plant model. The WS^3PR is made to be live by properly reconfiguring its weight distribution such that its all strict minimal siphons are self-max'-controlled. The resulting WS^3PR can serve as a liveness-enforcing Petri net supervisor for the plant model after removing some idle and operation places. A live controlled system can be accordingly obtained by synchronizing a plant model and the places whose weights are regulated. This research shows that a small number of monitors is obtained, leading to more permissive behavior of the controlled system. Examples are used to demonstrate the proposed concepts and methods.

Key Words: Petri net, flexible manufacturing system, deadlock prevention, siphon.

I. INTRODUCTION

As an effective and efficient innovative production technique, flexible manufacturing systems (FMS) have been well accepted by manufacturers to adapt to the intense market competition and quick requirement changes. In an FMS, to fully use the resources of the system, they are always shared by a number of jobs. This means that different types of raw parts are executed concurrently and therefore have to compete for the limited resources such as machine tools, automated guided vehicles, robots, buffers, and fixtures. This competition can cause deadlocks that are a highly undesirable situation, where each of a set of two or more jobs keeps waiting indefinitely for the other jobs in the set to release resources [1]. Hence it is important to develop efficient methods to deal with deadlocks.

As a mathematical tool, Petri nets have been widely used to model and deal with deadlock problems in FMS [2–5]. As in other resource allocation system context, deadlock control in an FMS are categorized into three strategies: deadlock detection and recovery [6, 7], deadlock avoidance [8–14], and deadlock prevention [15–19]. Deadlock prevention is a well-defined problem in resource allocation systems.

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It is usually achieved by using an off-line mechanism to deal with the requests of resources to ensure that deadlocks never occur. Monitors and related arcs are often added to the original Petri net model to realize such a mechanism [2, 15–17].

A siphon is a set of places in a Petri net. As a structural object, siphons are widely used to characterize and analyze deadlock situations in an FMS that is modeled with Petri net [20, 21]. An insufficiently marked siphon can cause dead transitions. In an ordinary Petri net, a siphon that can be unmarked can cause dead transitions. Deadlock prevention in an ordinary Petri net can be achieved by preventing all its strict minimal siphons from being emptied via adding monitors, as done in [15, 16]. The case in a generalized Petri net is much more complex than that in an ordinary net. On one hand, the existence of tokens in a siphon at any reachable marking is not sufficient for the absence of dead transitions. On the other hand, the existence of a strict minimal siphon does not necessarily imply the occurrences of deadlocks [22]. In [23], Zhong and Li develop a sufficient condition with respect to weight distribution and an initial marking under which a WS^{3}PR is live without external control agents, *i.e.*, self-live, even if there exist strict minimal siphons.

Based on the concept of self-max'-controlled siphons, we focus on the deadlock prevention for S^{3}PR. The concept of equivalent P-invariants is developed. An S^{3}PR is transformed into a WS^{3}PR. Both nets have the same behavior. Then we reconfigure the weights of some arcs of the WS^{3}PR such that its siphons are self-max'-controlled. Finally, a liveness-enforcing Petri net supervisor can be obtained. This means that the number of monitors is no more than that of resources in the plant net model.

The rest of this paper is organized as follows. Section II briefly reviews the basics of Petri nets used throughout this paper. The major results are developed in Section III. An example is given in Section IV to demonstrate the proposed method. We discuss the performance of the proposed method in Section V. Finally, Section VI concludes this paper.

II. PRELIMINARIES

2.1 Basics of petri nets

A Petri net [24] is a four-tuple $N=(P,T,F,W)$ where P and T are finite, non-empty, and disjoint sets. P is a set of places and T is a set of transitions. $F\subseteq(P\times T)\cup(T\times P)$ is called the flow relation or the set of directed arcs. $W:(P\times T)\cup(T\times P)\rightarrow\mathbb{N}$ is a mapping that assigns a weight to any arc: $W(x,y)>0$ if $(x,y)\in F$ and otherwise $W(x,y)=0$, where $\mathbb{N}=\{0,1,2,\ldots\}$. $N=(P,T,F,W)$ is said to be ordinary, denoted as $N=(P,T,F)$, if $\forall f\in F$, $W(f)=1$. A net is pure (self-loop free) iff $\nexists x,\ y\in P\cup T,\ f(x,y)\in F\wedge f(y,x)\in F$. A pure Petri net $N=(P,T,F,W)$ can be alternatively represented by its flow matrix or incidence matrix $[N]$, where $[N]$ is a $|P|\times|T|$ integer matrix with $[N](p,t)=W(t,p)-W(p,t)$.

A marking is a mapping $M:P\rightarrow\mathbb{N}$. The set of all possible markings reachable from M_0 in (N,M_0) is denoted by $[M_0\rangle$ or $R(N,M_0)$. $M(p)$ indicates the number of tokens in p under M. p is marked by M iff $M(p)>0$. A subset $D\subseteq P$ is marked by M iff at least one place in D is marked by M. The sum of tokens in all places in D is denoted by $M(D)$ where $M(D)=\sum_{p\in D}M(p)p$.

Let (N,M_0) be a marked net. Transition $t\in T$ is enabled at M iff $\forall p\in{}^\bullet t$, $M(p)\geq W(p,t)$. Marking M' reached by firing t from M is defined by $M'(p)=M(p)-W(p,t)+W(t,p)$, which is denoted by $M[t\rangle M'$. Marking M' is said reachable from marking M if there exists a sequence of transitions $\sigma=t_0t_1\ldots t_n$ and markings M_1, $M_2,\ldots$, and M_n such that $M[t_0\rangle M_1[t_1\rangle\ldots M_n[t_n\rangle M'$ holds. $M'=M+[N]\boldsymbol{\sigma}$ is called the state equation of the net N, where $\boldsymbol{\sigma}$, called firing count vector, is a vector whose ith entry denotes the number of occurrences of t_i in σ. A transition $t\in T$ is live under M_0 iff $\forall M\in R(N,M_0)$, $\exists M'\in R(N,M)$, $M'[t\rangle$ holds. (N,M_0) is deadlock-free iff $\forall M\in R(N,M_0)$, $\exists t\in T$, $M[t\rangle$ holds. (N,M_0) is quasi-live iff $\forall t\in T$, $\exists M\in R(N,M_0)$, $M[t\rangle$ holds. (N,M_0) is live iff $\forall t\in T$, t is live under M_0. (N,M_0) is bounded iff $\exists k\in\mathbb{N}$, $\forall M\in R(N,M_0)$, $\forall p\in P$, $M(p)\leq k$ holds.

A $P(T)$-vector is a column vector $I(J):P(T)\rightarrow\mathbb{Z}$ indexed by $P(T)$, where $\mathbb{Z}$ is the set of integers. I is a P-invariant (place invariant) iff $I\neq\mathbf{0}$ and $I^T[N]=\mathbf{0}^T$ hold. A P-semiflow I is a P-invariant satisfying $I\geq\mathbf{0}$. A P-semiflow I is said to be minimal if $\nexists$ a P-semiflow I' such that $\|I'\|\subset\|I\|$ where $\|I\|=\{p\in P|I(p)>0\}$ is called the support of P-invariant I. If I is a P-invariant, $\exists M\in\mathbb{N}^{|P|}$, $I^TM=I^TM_0$. Let $I_r(N,M_0)$ denote the set of invariant markings. We have $I_r(N,M_0)=\{M\in\mathbb{N}^{|P|}|I_r^TM=I_r^TM_0\}$.

Given a node $x\in P\cup T$, ${}^\bullet x=\{y\in P\cup T|(y,x)\in F\}$ is called the preset of node x, while $x^\bullet=\{y\in P\cup T|(x,y)\in F\}$ is called the postset of node x. This notation can be extended to a set of nodes as follows: given $X\subseteq P\cup T$, ${}^\bullet X=\cup_{x\in X}{}^\bullet x$ and $X^\bullet=\cup_{x\in X}x^\bullet$. A string $x_1\ldots x_n$ is called a path of N iff $\forall i\in\{1,2,\ldots,n-1\}$: $x_{i+1}\in x_i^\bullet$, where $\forall x\in\{x_1,\ldots,x_n\},x\in P\cup T$. A simple path is a path whose nodes are all different (except, perhaps, x_1 and x_n). A simple path is denoted by $SP(x_1, x_n)$. A circuit is a simple path with $x_1 = x_n$.

A nonempty set $S \subseteq P$ is a siphon (trap) iff ${}^\bullet S \subseteq S^\bullet$ ($S^\bullet \subseteq {}^\bullet S$) holds. A siphon is minimal iff there is no siphon contained in it as a proper subset. A siphon is called strict minimal if it is minimal and does not contain a marked trap. A strict minimal siphon is denoted as SMS for short.

Let $N = (P, T, F, W)$ be a net and $X \subseteq P \cup T$ a subset of nodes. Then, X generates the subnet $N_X = (P_X, T_X, F_X, W_X)$ where $P_X = P \cap X$, $T_X = T \cap X$, $F_X = F \cap (X \times X)$, and $W_X : (P_X \times T_X) \cup (T_X \times P_X) \to \mathbb{N}$ is a mapping that assigns a weight to any arc: $W_X(x, y) > 0$ if $(x, y) \in F_X$ and otherwise $W_X(x, y) = 0$.

2.2 Max-controlled siphons

The following notations and properties of generalized Petri nets are from [25]. Given a place p, we denote $\max_{t \in p^\bullet}\{W(p, t)\}$ by $\max_{p^\bullet}$.

Definition 1 ([*25*]). Let S be a siphon of a marked net (N, M_0). S is said to be max-marked at a marking $M \in R(N, M_0)$ iff $\exists p \in S$ such that $M(p) \geq \max_{p^\bullet}$.

Definition 2 ([*25*]). Let S be a siphon of a marked net (N, M_0). S is said to be max-controlled iff S is max-marked at any reachable marking.

Definition 3 ([*25*]). A net (N, M_0) is said to be satisfying the max cs-property (controlled-siphon property) iff each minimal siphon of N is max-controlled.

2.3 S^{3}PR nets

The definition of S^{3}PR nets is first proposed in [15], which can model a large class of real-world FMS.

Definition 4 ([*15*]). A System of Simple Sequential Process with Resources (S^{3}PR) is defined as the union of a set of nets $N_i = (P_{A_i} \cup \{p_i^0\} \cup P_{R_i}, T_i, F_i)$ sharing common places, where the following statements are true:

1. p_i^0 is called the process idle place of N_i. Elements in P_i and P_{R_i} are called operation and resource places, respectively;
2. $P_{R_i} \neq \emptyset$; $P_{A_i} \neq \emptyset$; $p_i^0 \notin P_{A_i}$; $(P_{A_i} \cup \{p_i^0\}) \cap P_{R_i} = \emptyset$; $\forall p \in P_{A_i}$, $\forall t \in {}^\bullet p$, $\forall t' \in p^\bullet$, $\exists r_p \in P_{R_i}$, ${}^\bullet t \cap P_{R_i} = t'^\bullet \cap P_{R_i} = \{r_p\}$; $\forall r \in P_{R_i}$, ${}^{\bullet\bullet} r \cap P_{A_i} = r^{\bullet\bullet} \cap P_{A_i} \neq \emptyset$; $\forall r \in P_{R_i}$, ${}^\bullet r \cap r^\bullet = \emptyset$; and ${}^{\bullet\bullet}(p_i^0) \cap P_{R_i} = \emptyset$;
3. N_i' is a strongly connected state machine, where $N_i' = (P_{A_i} \cup \{p_i^0\}, T_i, F_i)$ is the resulting net after the places in P_{R_i} and related arcs are removed from N_i;
4. Every circuit of N_i' contains place p_i^0;
5. Any two N_i are composable when they share a set of common places. Every shared place must be a resource place;
6. Transitions in $(p_i^0)^\bullet$ and ${}^\bullet(p_i^0)$ are called source and sink transitions of an S^{3}PR, respectively.

Definition 5 ([*15*]). (N, M_0) is called an S^{3}PR (net system) iff (i) N is an S^{3}PR net structure; (ii) $\forall p \in P$, $M_0(p) = 0$, $\forall p \in (P^0 \cup P_R)$, $M_0(p) > 0$.

Proposition 1 ([*15*]). Let (N^a, M_0^a) be the controlled system of an S^{3}PR (N, M_0). $\forall S \in \Pi, \forall M^a \in R(N^a, M_0^a)$, $M^a(S) \neq 0$, (N^a, M_0^a) is live.

Let S be a strict minimal siphon where $S = S^P \cup S^R$, $S^R = S \cap P_R$ and $S^P = S \backslash S^R$. For $r \in P_R$, $H(r) = {}^{\bullet\bullet} r \cap P_A$, the operation places that use r, is called the set of holders of r. $[S] = (\bigcup_{r \in S^R} H(r)) \backslash S$ is called the complementary set of S.

2.4 WS^{3}PR nets

A WS^{3}PR is a weighted version of an S^{3}PR, that is, the weight of an arc in a WS^{3}PR can be greater than one. A WS^{3}PR becomes an S^{3}PR if the weight of each arc is changed to be one.

Definition 6 ([*15*]). A Simple Sequential Process (S^{2}P) is a Petri net $N = (P_A \cup \{p^0\}, T, F)$, where: i) $P_A \neq \emptyset$ ($p \in P_A$ is called an operation place), $p^0 \notin P_A$ (p^0 is called a process idle place); ii) N is a strongly connected state machine; and iii) every circuit of N contains place p^0.

Definition 7 ([*23*]). A Weighted Simple Sequential Process with Resources (WS^{2}PR) is a Petri net $N = (P_A \cup \{p^0\} \cup P_R, T, F, W)$ such that:

1. The subnet generated by $X = P_A \cup \{p^0\} \cup T$ is an S^{2}P;
2. $P_R \neq \emptyset$ ($r \in P_R$ is called a resource or resource place) and $(P_A \cup \{p^0\}) \cap P_R = \emptyset$;
3. $W = W_S \cup W_R$, where W_S: $((P_A \cup P^0) \times T) \cup (T \times (P_A \cup P^0)) \to \{0, 1\}$ and W_R: $(P_R \times T) \cup (T \times P_R) \to \mathbb{N}$, where $P^0 = \{p^0\}$;
4. $\forall p \in P_A, \forall t \in {}^\bullet p, \forall t' \in p^\bullet, {}^\bullet t \cap P_R = t'^\bullet \cap P_R = \{r_p\}$ and $W(r_p, t) = W(t', r_p) \geq 1$; $H(r) = {}^{\bullet\bullet} r \cap P_A$ denotes the set of holders of r (operation places that use r). $\forall r \in P_R$, $\exists$ a unique minimal P-semiflow I_r s.t. $\|I_r\| \cap P_R = \{r\}$, $\|I_r\| \cap P^0 = \emptyset$ and $I_r(r) = 1$. Furthermore, $P_A = \bigcup_{r \in P_R}(\|I_r\| \backslash P_R)$;
5. The two following statements are verified:
   (a) $\forall r \in P_R$, ${}^{\bullet\bullet} r \cap P_A = r^{\bullet\bullet} \cap P_A \neq \emptyset$;
   (b) $\forall r \in P_R$, ${}^\bullet r \cap r^\bullet = \emptyset$;
6. ${}^{\bullet\bullet}(p^0) \cap P_R = (p^0)^{\bullet\bullet} \cap P_R = \emptyset$.

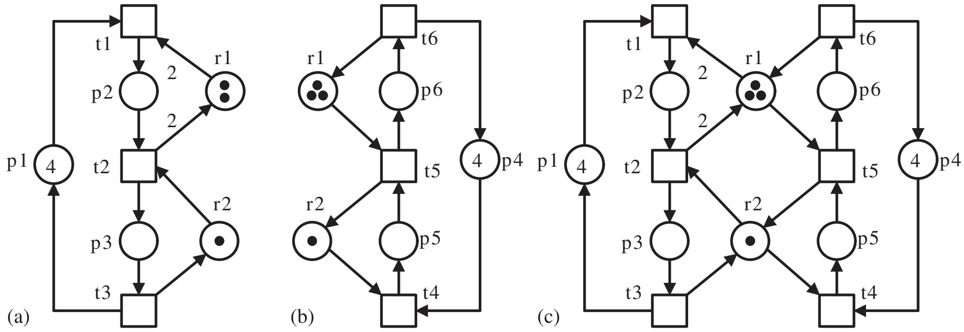

Fig. 1. (a) and (b) are two WS^{2}PR nets: (N_1, M_1) and (N_2, M_2). (c) The composed WS^{3}PR net (N, M).

In the net shown in Fig. 1(a), let $\{p^0\} = \{p_1\}$, $P_A = \{p_2, p_3\}$ and $P_R = \{r_1, r_2\}$. It is a WS^{2}PR. The net has three minimal P-semiflows: $I_1 = 2p_2 + r_1$, $I_2 = p_3 + r_2$, and $I_3 = p_1 + p_2 + p_3$. Similarly, the net shown in Fig. 1(b) is a WS^{2}PR.

Definition 8 ([*23*]). Let $N = (P_A \cup \{p^0\} \cup P_R, T, F, W)$ be a WS^{2}PR. An initial marking M_0 is called an acceptable initial marking for N iff $\forall p \in P_A$, $M_0(p) = 0$; $\forall r \in P_R$, $M_0(r) \geq \max_{p \in \|I_r\|} I_r(p)$; and $M_0(p^0) \geq 1$.

The couple (N, M_0) with N being a WS^{2}PR and M_0 being an acceptable initial marking is called a well-marked WS^{2}PR. In a well-marked WS^{2}PR, each transition is potentially fireable. As shown in Fig. 1(a), $\exists \sigma = t_1 t_2 t_3$, each transition in it can fire in sequence at the initial marking.

Definition 9 ([*23*]). A system of WS^{2}PR, WS^{3}PR, is recursively defined as follows:

1. A WS^{2}PR is a WS^{3}PR;
2. Let $N_i = (P_{A_i} \cup P_i^0 \cup P_{R_i}, T_i, F_i, W_i)$, $i \in \{1, 2\}$, be two WS^3PR such that $(P_{A_1} \cup P_1^0) \cap (P_{A_2} \cup P_2^0) = \emptyset$, $P_{R_1} \cap P_{R_2} = P_C (\neq \emptyset)$, and $T_1 \cap T_2 = \emptyset$ (in this case we say that N_1 and N_2 are two composable WS^{3}PR). Then, the net $N = (P_A \cup P^0 \cup P_R, T, F, W)$ resulting of the composition of N_1 and N_2 via P_C (denoted as $N = N_1 \circ N_2$) defined as follows: i) $P_A = P_{A_1} \cup P_{A_2}$, ii) $P^0 = P_1^0 \cup P_2^0$, iii) $P_R = P_{R_1} \cup P_{R_2}$, iv) $T = T_1 \cup T_2$, v) $F = F_1 \cup F_2$, and vi) $W = W_1 \cup W_2$ is also a WS^{3}PR.

Definition 10 ([*23*]). Let N be a WS^{3}PR. (N, M_0) is a well-marked WS^{3}PR iff one of the two following statements is true:

1. (N, M_0) is an acceptably marked WS^{2}PR;
2. $N = N_1 \circ N_2$ such that (N, M_0) is a well-marked WS^{3}PR:
   (a) $\forall i \in \{1, 2\}$, $\forall p \in P_{A_i} \cup P_i^0$, $M_0(p) = M_{0_i}(p)$;
   (b) $\forall i \in \{1, 2\}$, $\forall r \in P_{R_i} \setminus P_C$, $M_0(r) = M_{0_i}(r)$;
   (c) $\forall r \in P_C$, $M_0(r) = \max\{M_{0_1}(r), M_{0_2}(r)\}$.

In Fig. 1(c), let $P^0 = \{p_1, p_4\}$, $P_{A_1} = \{p_2, p_3\}$, $P_{A_2} = \{p_5, p_6\}$ and $P_R = \{r_1, r_2\}$. It is a WS^{3}PR and has four minimal P-semiflows: $I_1 = 2p_2 + p_6 + r_1$, $I_2 = p_3 + p_5 + r_2$, $I_3 = p_1 + p_2 + p_3$, and $I_4 = p_4 + p_5 + p_6$. $M_0(r_1) = \max\{M_{0_1}(r_1), M_{0_2}(r_1)\} = \max\{2, 3\} = 3$. If the weight of any arc in the net in Fig. 1(c) is unit, it is an S^{3}PR.

Let S be a siphon in a well-marked WS^{3}PR (N, M_0). $\forall r \in S$, We denote $\{t | t \in r^\bullet \setminus {}^\bullet S\}$ by T_r and $\sum_{t_i \in T_r} a_i W(r, t_i) > 0$ by W_r, where a_i denotes the times t_i fires and $a_i \in \mathbb{N}$. If t_i fires a_i times, r will loses $a_i W(r, t_i)$ tokens. Take siphon $S = \{p_3, p_6, r_1, r_2\}$ in Fig. 1(c) as an example. We have $T_{r_1} = \{t_1\}$ and $W_{r_1} = a_1 W(r_1, t_1) = 2a_1$ ($a_1 \in \mathbb{N}$). If t_1 fires a_i times, r will loses $2a_i$ tokens.

Definition 11 ([*23*]). Let S be a siphon in a well-marked S^{4}R (N, M_0). S is said to be max'-marked at marking $M \in R(N, M_0)$ iff $\exists p \in S^P$ such that $M(p) \geq 1$ or $\exists p \in S^R$ such that $M(p) \geq \max_{t \in (p^\bullet \cap [S]^\bullet)} \{W(p, t)\}$.

Definition 12 ([*23*]). Let S be a siphon in a well-marked S^{4}R (N, M_0). S is said to be max'-controlled iff S is max'-marked at any reachable marking, *i.e.*, $\forall M \in R(N, M_0)$, $\exists p \in S^P$ such that $M(p) \geq 1$ or $\exists p \in S^R$ such that $M(p) \geq \max_{t \in (p^\bullet \cap [S]^\bullet)} \{W(p, t)\}$.

Note that an S^{4}R is more general than a WS^{3}PR.

Definition 13 ([*23*]). Let S be a siphon in a well-marked WS^{3}PR (N, M_0). If it contains a marked trap or $\exists r \in S, min\{M_0(r) \% W_r\} \geq \max_{t \in (r^\bullet \cap [S]^\bullet)} \{W(r, t)\}$, it is called self-max'-controlled, where '%' is the modulus operator.

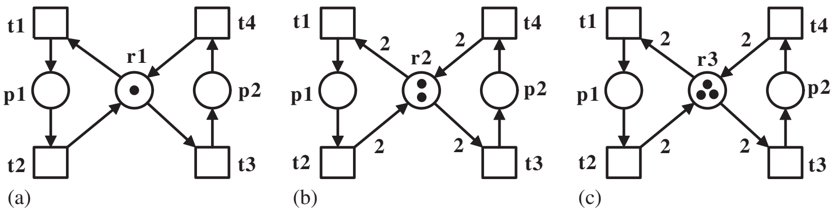

Fig. 2. A minimal basic P-semiflow and its two equivalent P-semiflows.

For example, $S=\{p_3, p_6, r_1, r_2\}$ is an SMS in Fig. 1(c) with $[S]=\{p_2, p_5\}$, $[S]^\bullet=\{t_2, t_5\}$, and $r_1^\bullet \cap [S]^\bullet=\{t_5\}$. At the initial marking M_0, $\exists r_1 \in S^R$, $M_0(r_1)=3>\max_{t\in(r_1^\bullet\cap[S]^\bullet)}\{W(r_1,t)\}=W(r_1,t_5)=1$. S is max'-marked at the initial marking. We have $W_{r_1}=2a_1$, where $a_1 \in \mathbb{N}$. Let $\rho=M_0(r_1)\%W_{r_1}$. Then $\rho=3\%(2a_1)$. It is clear that $\rho=3$ or 1, depending on a_1. Therefore $min\{M_0(r_1)\%W_{r_1}\}=min\{3\%2a_1\}=1$. When $min\{3\%2a_1\}=1$, we have $a_1=1$. Since $\max_{t\in(r_1^\bullet\cap[S]^\bullet)}\{W(r_1,t)\}=1$, $min\{M_0(r_1)\%W_{r_1}\}=\max_{t\in(r_1^\bullet\cap[S]^\bullet)}\{W(r_1,t)\}$, S is hence self-max'-controlled.

Proposition 2 ([23]). Let S be a minimal siphon in a well-marked WS^{3}PR (N, M_0). It is max'-controlled if it is self-max'-controlled.

For example, $S=\{p_3, p_6, r_1, r_2\}$ is an SMS in Fig. 1(c). Since $\min\{M_0(r_1)\%W_{r_1}\}=1$, there is at least one token left in the siphon. The token is either in place p_6 or r_1. We have $M(p_6)=1$ or $M(r_1)=1$. By $\max_{t\in(r_1^\bullet\cap[S]^\bullet)}\{W(r_1,t)\}=1$, S is max'-controlled.

Theorem 1 ([22]). Let (N, M_0) be a well-marked S^{4}R and $t'\in T$ be a dead transition under $M\in R(N, M_0)$. Then there exist $M'\in R(N, M)$ and siphon S such that S is not max'-marked under M'.

Note that $S=S^R\cup S^P$, where $S^R=\{r\in P_R|\exists t\in r^\bullet, M'(r)<W(r,t), p\in P_A\cap {}^\bullet t, M'(p)>0\}$ and $S^P=\{p\in H(r)|r\in S^R, M'(p)=0\}$.

Theorem 2 ([23]). Let (N, M_0) be a well-marked WS^{3}PR. If all the SMS of the net are max'-controlled, it is live.

Corollary 1 ([23]). Let (N, M_0) be a well-marked WS^{3}PR. If all the SMS of (N, M_0) are self-max'-controlled, it is live.

Max'-controllability of a siphon is a weak condition under which the transitions in the postset of a siphon are not dead. For example, it is easy to verify that the net in Fig. 1(c) is live. However S is not max-controlled.

III. TRANSFORMING AN S^{3}PR NET INTO A WS^{3}PR

Let I be a minimal P-semiflow. (N_I, M_0) is a subnet generated by $X=\|I\|\cup^\bullet\|I\|\cup\|I\|^\bullet$, where $N_I=(\|I\|, {}^\bullet\|I\|\cup\|I\|^\bullet, F_X, W_X)$.

Definition 14 ([23]). Let (N_1, M_{01}) and (N_2, M_{02}) be two Petri nets with $N_1=(P_1, T_1, F_1, W_1)$ and $N=(P_2, T_2, F_2, W_2)$. (N_1, M_{01}) is called a subnet of (N_2, M_{02}) if (1) $P_1\subseteq P_2$, (2) $T_1\subseteq T_2$, (3) $F_1\subseteq F_2$, (4) $\forall f\in F_1$, $W_1(f)=W_2(f)$, and (5) $\forall p\in P_1$, $M_{01}(p)=M_{02}(p)$.

Let $\lfloor x\rfloor$ be an integer, where $\lfloor x\rfloor=\max\{a\in\mathbb{Z}|a\leq x\}$. For example $\lfloor 2.3\rfloor=2$ and $\lfloor 5.78\rfloor=5$. It is obvious that $x\geq\lfloor x\rfloor$.

Definition 15. A minimal P-semiflow I is called a minimal basic P-semiflow if $\forall p\in\|I\|$, $I(p)=1$.

It is obvious that all the minimal P-semiflows in S^{3}PR net are basic.

Definition 16. Let I_{r_1} and I_{r_2} be two minimal P-semiflows associated with resource places r_1 and r_2, respectively. Suppose that I_{r_1} is basic. I_{r_2} is said to be equivalent to I_{r_1} if $I_{r_1}(r_1)=I_{r_2}(r_2)=1$, $H(r_1)=H(r_2)$, and $\forall p\in H(r_2)$, $\lfloor M_0(r_2)/I_{r_2}(p)\rfloor=M_0(r_1)/I_{r_1}(p)$.

In Fig. 2, $I_{r_1}=r_1+p_1+p_2$ is a minimal basic P-semiflow, where $M_0(r_1)=1$. $I_{r_2}=r_2+2p_1+2p_2$ with $M_0(r_2)=2$ and $I_{r_3}=r_3+2p_1+2p_2$ with $M_0(r_3)=3$ are its two equivalent P-semiflows.

Lemma 1. Let $(N_{I_{r_1}}, M_0)$ and $(N^e_{I^e_{r_2}}, M^e_0)$ be two subnets corresponding to a basic minimal P-semiflow I_{r_1} and its equivalent minimal P-semiflow $I^e_{r_2}$, respectively. $N_{I_{r_1}}$ and $N^e_{I^e_{r_2}}$ have the same topological structures without considering the weights of their arcs.

Proof. It follows immediately from Definition 16. □

Definition 17. Let (N, M_0) be a Petri net and L be the set of finite sequences of transitions of the net. L is called the behavior of N if $\forall \sigma \in L$, $\exists M \in R(N, M_0)$, such that $M_0[\sigma\rangle M$ holds and $\forall M' \in R(N, M_0)$, $\exists \sigma \in L$, such that $M_0[\sigma\rangle M'$ holds.

Theorem 3. Let $(N_{I_{r_1}}, M_0)$ and $(N^e_{I^e_{r_2}}, M^e_0)$ be two subnets corresponding to a basic minimal P-semiflow I_{r_1} and its equivalent minimal P-semiflow $I^e_{r_2}$, respectively. $N_{I_{r_1}}$ and $N_{I^e_{r_2}}$ have equivalent behavior.

Proof. Two cases should be considered.

1. For $(N_{I_{r_1}}, M_0)$, suppose that there exist a sequence of transitions $\sigma = t_0t_1 \dots t_n$ and markings M_1, $M_2, \dots, M_n$ and M_{n+1} such that $M_0[t_0\rangle M_1[t_1\rangle \dots M_n[t_n\rangle$ M_{n+1} holds. For $(N^e_{I^e_{r_2}}, M^e_0)$, suppose that there do not exist a corresponding sequence of transitions $\sigma = t_0t_1 \dots t_n$ and markings M^e_1, $M^e_2, \dots, M^e_n$ and M^e_{n+1} such that $M^e_0[t_0\rangle M^e_1[t_1\rangle \dots M^e_n[t_n\rangle M^e_{n+1}$ holds. Suppose that $M^e_i[t_i\rangle M^e_{i+1}$ $(i \in \{0, 1, \dots, n\})$ is not true. Then both nets have a sequence of transitions $\sigma_i = t_0t_1 \dots t_{i-1}$ and corresponding markings. Since they have the same structures without considering the weights of their arcs, $\forall p \in H(r_1)$, we have $M_i(p) = M^e_i(p)$. If $t_i \in p^\bullet$, it is obvious that $M^e_i[t_i\rangle M^e_{i+1}$ holds. Hence we have $t_i \in r_2^\bullet$ and $M^e_i(r_2) < W(r_2, t_i)$. Let $p_x \in t_i^\bullet$. Then $I^e_{r_2}(p_x) = W(r_2, t_i)$ $(I_{r_1}(p_x) = W(r_1, t_i))$. $\forall p \in H(r_1)$, $\lfloor M_0(r_2)^e / I^e_{r_2}(p) \rfloor = M_0(r_1)/I_{r_1}(p)$. Furthermore, we have $I^e_{r_2}(p) \le M^e_0(r_2)/M_0(r_1)$. Due to $M_i(r_1) = M_0(r_1)$ $-\sum_{p \in H(r_1)} M_i(p)$, we have $M^e_i(r_2) = M^e_0(r_2) - \sum_{p \in H(r_2)} I^e_{r_2}(p) M^e_i(p) \ge M^e_0(r_2) - \sum_{p \in H(r_2)} M^e_i(p) M^e_0$ $(r_2)/M_0(r_1) = (M_0(r_1) - \sum_{p \in H(r_1)} M_i(p)) M^e_0(r_2)/M_0$ $(r_1) = M_i(r_1) M^e_0(r_2)/M_0(r_1) \ge M^e_0(r_2)/M_0(r_1) \ge I^e_{r_2}(p)$. Hence $M^e_i[t_i\rangle M^e_{i+1}$ holds.

2. Suppose that $M_i[t_i\rangle M_{i+1}$ does not hold. From 1), $t_i \in r_1^\bullet$ is true. Therefore, $M_i(r_1) = M_0(r_1) - \sum_{p \in H(r_1)} M_i(p) = 0$, i.e., $M_0(r_1) = \sum_{p \in H(r_1)} M_i(p)$. $\forall p \in H(r_2)$, let $I^e_{r_2}(p_x) \le I^e_{r_2}(p)$. $\sum_{p \in H(r_2)} I^e_{r_2}(p) M^e_i(p)$ $\ge \sum_{p \in H(r_2)} I^e_{r_2}(p_x) M^e_i(p)$. Since $t_i \in r_2^\bullet$, then $\sum_{p \in H(r_2)}$ $I^e_{r_2}(p_x) M^e_{i+1}(p) > \sum_{p \in H(r_2)} I^e_{r_2}(p_x) M^e_i(p)$, i.e., $\sum_{p \in H(r_2)} M^e_{i+1}(p) > \sum_{p \in H(r_2)} M^e_i(p)$. Note that $I^e_{r_2}$ is a P-invariant. We have $M^e_0(r_2) \ge \sum_{p \in H(r_2)} I^e_{r_2}(p_x) M^e_{i+1}$ (p). Accordingly, $M^e_0(r_2)/I^e_{r_2}(p_x) \ge \sum_{p \in H(r_2)} M^e_{i+1}(p)$. Since $\sum_{p \in H(r_2)} M^e_{i+1}(p)$ is an integer, $\lfloor M^e_0(r_2)/I^e_{r_2}(p_x) \rfloor$ $\ge \sum_{p \in H(r_2)} M^e_{i+1}(p)$. As a result, we have $\lfloor M^e_0(r_2)/I^e_{r_2}$ $(p_x) \rfloor > \sum_{p \in H(r_2)} M^e_i(p) = M_0(r_1)$. This contradicts the fact that $\forall p \in H(r_2)$, $\lfloor M^e_0(r_2)/I^e_{r_2}(p) \rfloor = M_0(r_1)/I_{r_1}(p)$. We conclude that $M_i[t_i\rangle M_{i+1}$ holds.

From the two cases, the two subnets have equivalent behavior. □

Proposition 3. Let I_{r_1} be a minimal basic P-semiflow associated with a resource place in a well initially marked WS^{3}PR. $I^e_{r_2}$ is equivalent to I_{r_1} if $I_{r_1}(r_1) = I^e_{r_2}(r_2) = 1$, $H(r_1) = H(r_2)$, $\forall p \in H(r_2)$, $I^e_{r_2}(p) = m$, and $M^e_0(r_2) = mM_0(r_1) + n$, where $m, n \in \mathbb{N}$, $n < m$.

Proof. $\forall p \in H(r_2)$, $\lfloor M^e_0(r_2)/I^e_{r_2}(p) \rfloor = \lfloor [mM_0(r_1) + n]/m \rfloor = M_0(r_1)$. $M_0(r_1)/I_{r_1}(p) = M_0(r_1)$. Hence $\lfloor M^e_0(r_2)/I^e_{r_2}(p) \rfloor = M_0(r_1)/I_{r_1}(p)$. From Definition 16, $I^e_{r_2}$ is equivalent to I_{r_1}. □

Definition 18. Let (N, M_0) be an S^{3}PR and (N^e, M^e_0) a WS^{3}PR. The WS^{3}PR is said to be an equivalent net of the S^{3}PR if any P-semiflow in one net has an equivalent P-semiflow in the other.

As shown in Fig. 3, (N, M_0) is an S^{3}PR net and (N^e, M^e_0) is a WS^{3}PR net that is equivalent to (N, M_0).

Proposition 4. Let WS^{3}PR (N^e, M^e_0) be an equivalent net of an S^{3}PR (N, M_0). (N, M_0) and (N^e, M^e_0) have equivalent behavior.

Proof. From Theorem 3, the proposition holds. □

In this section we first re-construct an S^{3}PR net, leading to an equivalent WS^{3}PR from the behavior point of view. Then we reconfigure the weight of some arcs of the WS^{3}PR such that it is self-live. The resulting net can work as a liveness-enforcing supervisor for the S^{3}PR net.

Theorem 4. Let S be an SMS in an S^{3}PR (N, M_0). If $\exists r \in S$, $M_0(r) \ge 2$, $r^\bullet \cap [S]^\bullet \ne \emptyset$ and $(r^\bullet \cap [S]^\bullet)^\bullet \cap [S] = \emptyset$, an equivalent WS^{3}PR (N^e, M^e_0) of (N, M_0) can be constructed and a corresponding SMS S^e can be found. S^e can be made self-max'-controlled by properly configuring the weight of its arcs.

Proof. From Definition 13, an SMS is self-max'-controlled if $\exists r \in S$, $min\{M^e_0(r) \% W_r\} \ge \max_{t \in (r^\bullet \cap [S]^\bullet)}$ $\{W(r, t)\}$. Note that $r^\bullet \cap [S]^\bullet \ne \emptyset$. If $(r^\bullet \cap [S]^\bullet)^\bullet \cap [S] \ne \emptyset$, $min\{M^e_0(r) \% W_r\} < \max_{t \in (r^\bullet \cap [S]^\bullet)} \{W(r, t)\}$ is true. It is obvious that $M^e_0(r) > 2 \max_{t \in (r^\bullet \cap [S]^\bullet)} \{W(r, t)\}$ and furthermore, $\lfloor M^e_0(r)/I^e_r(p) \rfloor = M_0(r)/I_r(p) = M_0(r) \ge 2$. Therefore, properly reconfiguring the weight of the arcs of S^e, it can be made self-max'-controlled. □

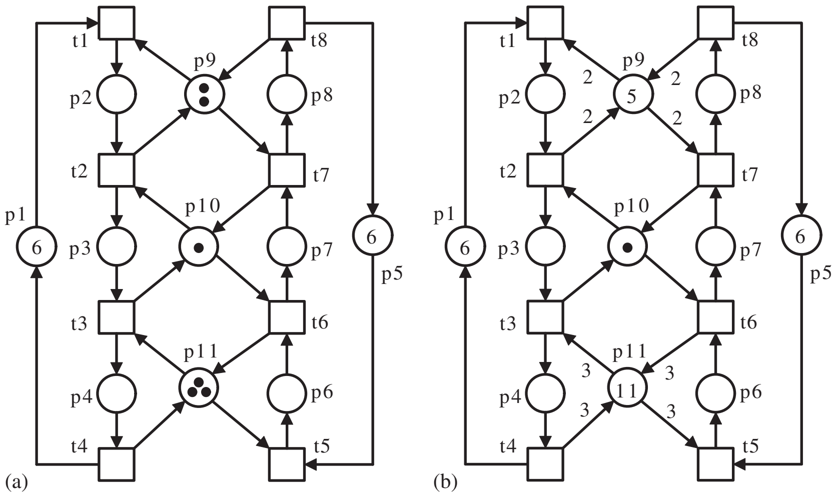

Fig. 3. (a) an S^3PR net (N, M_0), (b) a WS^3PR net (N^e, M_0^e) that is equivalent to the S^3PR net.

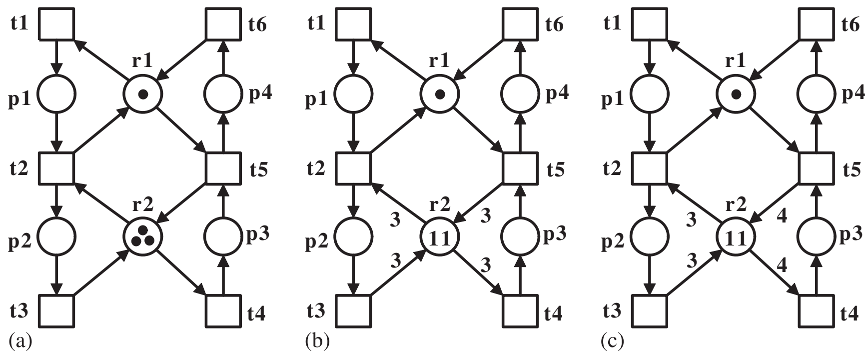

Fig. 4. (a) A part of an S^3PR net, (b) A part of a WS^3PR net that is equivalent to (a), (c) A parts of a WS^3PR net.

Let S be a siphon in an S^3PR net (N, M_0). If $\exists r \in S$, $M_0(r) \geq 2$, $r^\bullet \cap [S]^\bullet \neq \emptyset$ and $(r^\bullet \cap [S]^\bullet)^\bullet \cap [S] = \emptyset$. Algorithm 1 describes how to get a self-max'-controlled siphon.

Algorithm 1. *Self-max'-controlled Siphon*
Input: a siphon S in an S^3PR net (N, M_0)
Output: a self-max'-controlled siphon in a WS^3PR net (N^1, M_0^1)
Step 1. Find $r \in S$, where $M_0(r) \geq 2$, $r^\bullet \cap [S]^\bullet \neq \emptyset$ and $(r^\bullet \cap [S]^\bullet)^\bullet \cap [S] = \emptyset$
Step 2. Construct an equivalent P-invariant I_r^e of I_r. $\forall p \in H(r)$, $I_r^e(p) = m$, $M_0^1(r) = mM_0(r) + n$ $(m, n \in \mathbb{N}, n < m)$
Step 3. $\forall p \in [S]$, let $I_r^e(p) = m+1$
Step 4. $min\{M_0^1(r)\%W_r\} = min\{M_0^1(r)\%a(m+1)\} = (mM_0(r)+n)\%(m+1)$
Step 5. Let $[(mM_0(r)+n)\%(m+1)] \geq m$, *i.e.* $[(mM_0(r)+n)\%(m+1)] = m$
Step 6. Find a set of m and n
Step 7. Output a self-max'-controlled siphon

Proposition 5. Algorithm 1 generates a self-max'-controlled siphon.

Proof. Note that $\min\{M_0^1(r)\%W_r\} = [(mM_0(r)+n)\%(m+1)] = m$. From Definition 13, the output siphon is self-max'-controlled. $\square$

To avoid the restriction of the firings of the transitions in the postset of a siphon, $M(H(r) \cap [S])$ should be as large as possible. Hence we have $\lfloor (mM_0(r)+n)/(m+1) \rfloor = M_0(r) - 1$. From Algorithm 1, $[(mM_0(r)+n)\%(m+1)] = m$ is true. It is obvious that $mM_0(r)+n = (m+1)(M_0(r)-1)+m = mM_0(r) + M_0(r) - 1$. Therefore, we have $n = M_0(r) - 1$.

In Fig. 4(a) there is a siphon $S = \{p_2, p_4, r_1, r_2\}$. $[S] = \{p_1, p_3\}$, $[S]^\bullet = \{t_2, t_5\}$, $M_0(r_2) = 3$ and $r_2^\bullet = \{t_2, t_4\}$. We have $r_2^\bullet \cap [S]^\bullet = \{t_2\} \neq \emptyset$ and $(r_2^\bullet \cap [S]^\bullet)^\bullet \cap [S] = \emptyset$.

From Proposition 3, $I_{r_2}^e$ is equivalent to I_{r_2}, where $M_0^e(r) = mM_0(r) + n = 3m + n$. Note that $[(mM_0(r) + n)\%(m+1)] = m$. In Fig. 4(b) we have $m = 3$ and $n = 2$.

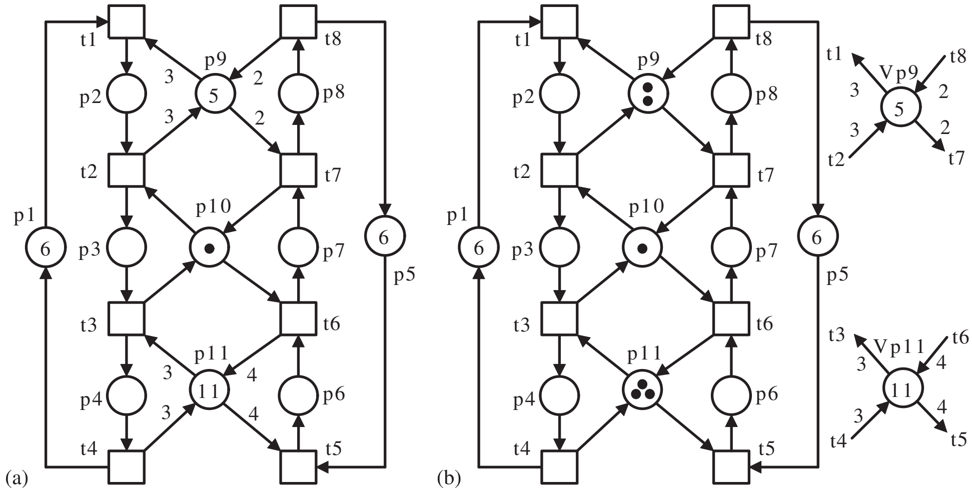

Fig. 5. (a) a self-live WS^{3}PR net (N^1, M_0^1), (b) a liveness-enforcing supervisor of the S^{3}PR net (N^c, M_0^c) shown in Fig. 3(a).

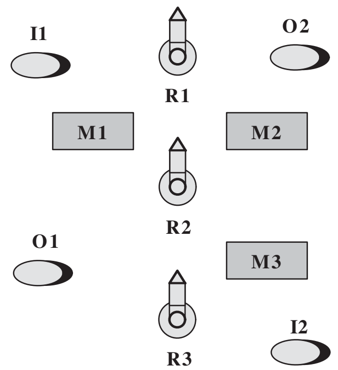

Fig. 6. An FMS layout.

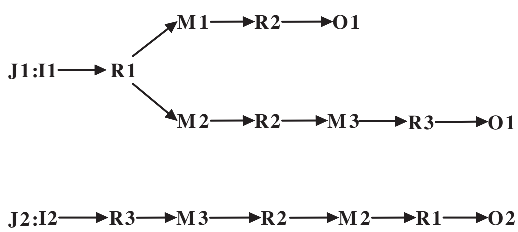

Fig. 7. Production routings.

In Fig. 4(c) we get a self-max'-controlled siphon $S = \{p_2, p_4, r_1, r_2\}$.

Definition 19. Let Π_r be a subset of the SMS in an S^{3}PR (N, M_0) and r be a resource place of the net, where $M_0(r) \geq 2$. Π_r is called the set of r-controllable-siphon, if $\forall S \in \Pi_r,\ r \in S,\ r^\bullet \cap [S]^\bullet \neq \emptyset$ and $(r^\bullet \cap [S]^\bullet)^\bullet \cap [S] = \emptyset$, and $\forall S_i, S_j \in \Pi_r,\ H(r) \cap [S_i] = H(r) \cap [S_j]$ $(i, j \in \{1, 2, \ldots, |\Pi_r|\})$.

Theorem 5. Let Π be the set of all the SMS in an S^{3}PR (N, M_0) with $N = (P_A \cup P^0 \cup P_R, T, F)$. If $\Pi = \Pi_{r_{k1}} \cup \Pi_{r_{k2}} \cup \cdots \cup \Pi_{r_{kn}}$ $(r_{k1}, r_{k2}, \ldots, r_{kn} \in P_R)$, then an equivalent WS^{3}PR (N^e, M_0^e) of (N, M_0) can be made self-live from resetting the weight of its arcs, where Π_{r_x} is the set of r-controllable-siphon, $r_x \neq r_y$ $(x, y \in \{k1, k2, \ldots, kn\})$.

Proof. From Definition 19 we have $\forall S \in \Pi_{r_x}$, $M_0(r_x) \geq 2$, $r_x^\bullet \cap [S]^\bullet \neq \emptyset$ and $(r_x^\bullet \cap [S]^\bullet)^\bullet \cap [S] = \emptyset$. Then from Theorem 4 we conclude that S^e can be made self-max'-controlled by resetting the weight of the arcs related to r_x. $\forall S_i, S_j \in \Pi_{r_x}$, $H(r_x) \cap [S_i] = H(r_x) \cap [S_j]$. Then all the siphons in $\Pi_{r_x}^e$ can be made self-max'-controlled at the same time. By $\Pi = \Pi_{r_{k1}} \cup \Pi_{r_{k2}} \cup \cdots \cup \Pi_{r_{kn}}$, we can conclude that all the SMS in (N^e, M_0^e) can be made self-max'-controlled at the same time. From Corollary 1, (N^e, M_0^e) can be made self-live by resetting the weight of its arcs. □

For an S^{3}PR net (N, M_0), we compute its all SMS and the equivalent net (N^e, M_0^e). First, we make the siphons max'-controlled if each of them has only one resource place such that $M_0(r) \geq 2$, $r^\bullet \cap [S]^\bullet \neq \emptyset$ and $(r^\bullet \cap [S]^\bullet)^\bullet \cap [S] = \emptyset$. Then we can make the other siphons max'-controlled. Finally, we get a self-live WS^{3}PR net (N^1, M_0^1).

Definition 20. Let (N, M_0) be an S^{3}PR and (N^1, M_0^1) be a self-live WS^{3}PR that is derived from an equivalent net (N^e, M_0^e) of (N, M_0). The resource places in N^1,

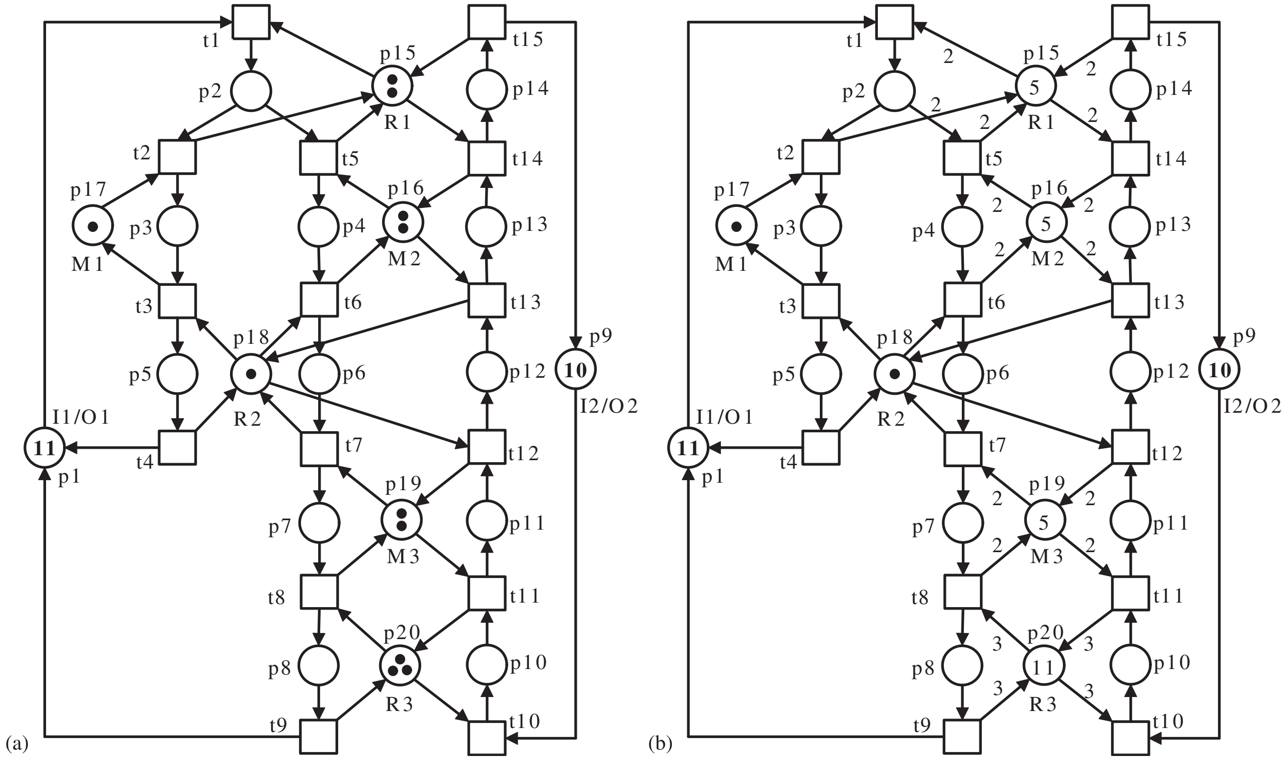

Fig. 8. (a) A Petri net model of an S^{3}PR net (N, M_0) and (b) its equivalent WS^{3}PR net (N^e, M_0^e).

which are different from those in N, are called monitor places of N.

As shown in Fig. 5(a), the resource places p_9 and p_{11} are the monitor places of the net shown in Fig. 3(a).

Theorem 6. Let (N^c, M_0^c) be the resulting net of an S^{3}PR (N, M_0) after all the monitor places and their relate arcs by Definition 20 are added to N. Then, (N^c, M_0^c) is live.

Proof. If the resource places of N are added to N^1, they will not change the behavior of N^1. For (N^1, M_0^1) is self-live, we conclude that (N^c, M_0^c) is live. □

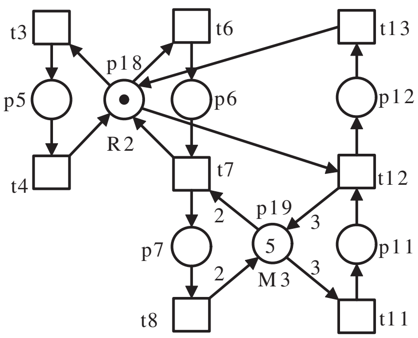

Fig. 9. A self-max'-controlled siphon of S_1.

IV. AN EXAMPLE

Consider a hypothetical FMS with its layout shown in Fig. 6 and production routings in Fig. 7. It consists of three robots R1-R3, each of which can hold one, two or three products every time, and three machines M1-M3, each of which can process one or two products every time. There are two loading buffers I1 and I2, and two unloading buffers O1 and O2 to load and unload the FMS. There are two raw product types, namely J1 and J2, to be processed. For these raw product types the production cycles are as shown in Fig. 7. According to the production cycles, a raw product J1 is taken from I1 by R1 and put in M1 or M2. After being processed by M1 it is then moved to O1 by R2 or after being processed by M2, it is moved to M3 by R2 and then moved to O1 by R3. A raw product J2 is taken from I2 to M3 by R3. After being processed by M3 it is then moved from M3 to M2 by R2. Finally,

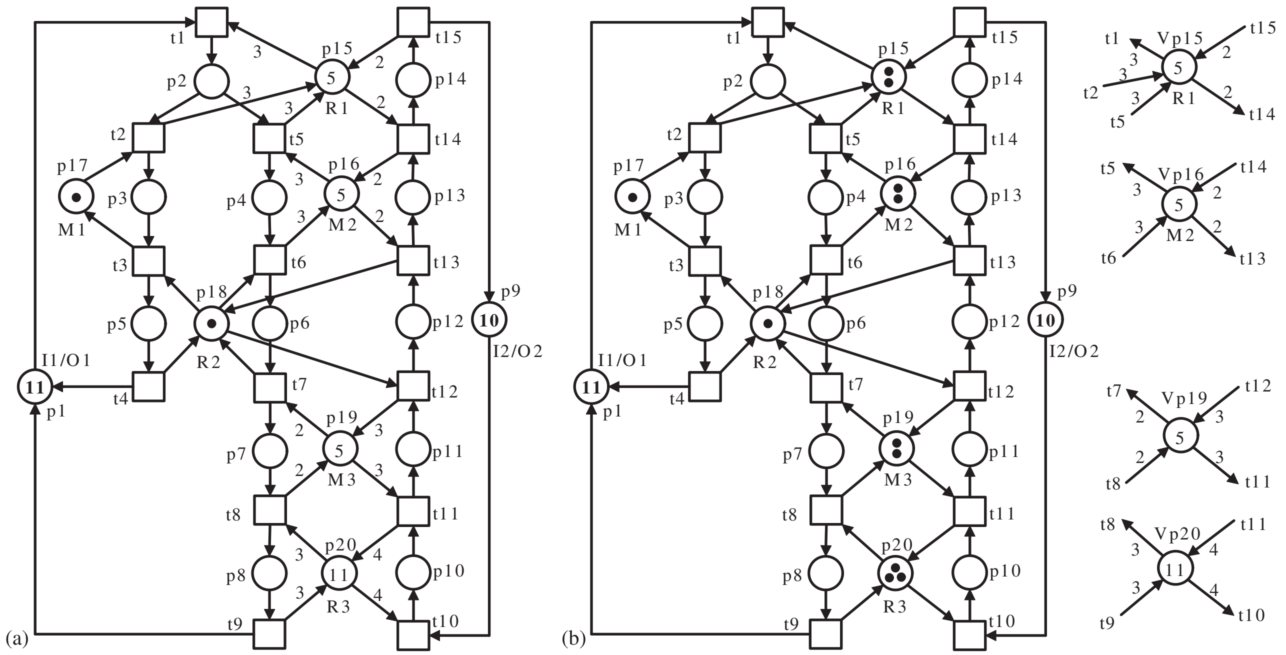

Fig. 10. (a) A self-live WS^{3}PR net (N^1, M_0^1) and (b) a liveness-enforcing supervisor (N^c, M_0^c).

after being processed by M2 it is then moved from M2 to O2 by R1.

The net model shown in Fig. 8 is an S^{3}PR, where $P^0=\{p_1, p_9\}$, $P_{A_1}=\{p_i|i=2,3,\ldots,8\}$, $P_{A_2}=\{p_i|i=10,11,\ldots,14\}$, $P_{R_1}=\{p_{15}, p_{16}, p_{17}, p_{18}, p_{19}, p_{20}\}$, and $P_{R_2}=\{p_{15}, p_{16}, p_{18}, p_{19}, p_{20}\}$. Places p_{15}, p_{16}, p_{17}, p_{18}, p_{19} and p_{20} denote R1, M2, M1, R2, M3, and R3, respectively. Initially, it is assumed that there are no parts in the system. $M_0(p_1)=11$ and $M_0(p_9)=9$ represents the maximal number of concurrent activities that can take place for part types P1 $\rightarrow$ J1 and P2 $\rightarrow$ J2, respectively.

The net has nine SMS: $S_1=\{p_5, p_7, p_{12}, p_{18}, p_{19}\}$, $S_2=\{p_5, p_7, p_{14}, p_{15}, p_{16}, p_{17}, p_{18}, p_{19}\}$, $S_3=\{p_5, p_8, p_{12}, p_{18}, p_{19}, p_{20}\}$, $S_4=\{p_5, p_6, p_{13}, p_{16}, p_{18}\}$, $S_5=\{p_5, p_8, p_{13}, p_{16}, p_{18}, p_{19}, p_{20}\}$, $S_6=\{p_5, p_6, p_{14}, p_{15}, p_{16}, p_{17}, p_{18}\}$, $S_7=\{p_5, p_7, p_{13}, p_{16}, p_{18}, p_{19}\}$, $S_8=\{p_8, p_{11}, p_{19}, p_{20}\}$, and $S_9=\{p_5, p_8, p_{14}, p_{15}, p_{16}, p_{17}, p_{18}, p_{19}, p_{20}\}$.

For S_1 and S_4, each of them has only one resource place that satisfies $M_0(r)\geq 2$, $r^\bullet\cap[S]^\bullet\neq\emptyset$, and $(r^\bullet\cap[S]^\bullet)^\bullet\cap[S]=\emptyset$. Hence we first get self-max'-controlled siphons for them.

For S_1, $[S]=\{p_6, p_{11}\}$, $[S]^\bullet=\{t_7, t_{12}\}$, and $M_0(p_{19})=2$. Note that $n=M_0(p_{19})-1=1$ and $m>n$. If $m=2$, then we can get a self-max'-controlled siphon as shown in Fig. 9. Note that $T_{p_{19}}=\{t|t\in p_{19}^\bullet\setminus{}^\bullet S_1\}=\{t_{11}\}$, $W_{p_{19}}=a_{11}W(p_{19}, t_{11})=3a_{11}$, and min $\{M_0(r)\%W_r\}=\min\{5/3a_{11}\}=2$. Since $\max_{t\in(p_{19}^\bullet\cap[S_1]^\bullet)}\{W(p_{19}, t)\}=2$, S_1 is self-max'-controlled by Definition 13. At the same time, self-max'-controlled siphons for S_2 and S_7 can be obtained as shown in Fig. 10.

In the same way, we can get a self-max'-controlled siphon for S_4 and a self-max'-controlled siphon for S_5 at the same time by properly configuring the weight of the arcs related to p_{16}.

Since p_{16} and p_{19} have been processed, they cannot be considered any more. In S_3, only p_{20} can be considered. Since $n=M_0(p_{20})-1=2$, $m>n$, we have $m=3$. Furthermore, we get a self-max'-controlled siphon for it as shown in Fig. 10. Meanwhile, self-max'-controlled siphons for S_8 and S_9 can be obtained. Finally, we find a self-max'-controlled siphon for S_6.

In Fig. 10(a) we get a self-live WS^{3}PR net (N^1, M_0^1). Fig. 10(b) shows a liveness-enforcing supervisor (N^c, M_0^c). We add four monitors to the original net (N, M_0) and get a live net. By INA [26] we can test that (N^c, M_0^c) has 7608 live states.

V. DISCUSSION

As is well known, the major criteria for a liveness-enforcing Petri net supervisor for a plant model are its computational complexity, behavior permissiveness and structural complexity. The research in [17] compares 14

Table I. Computed monitors for the net system in Fig. 8(a) due to [15].

Monitor	Preset	Postset	M_0
C_1	t_2, t_7, t_{12}	t_1, t_{10}	2
C_2	t_2, t_7, t_{13}	t_1, t_{10}	4
C_3	t_2, t_8, t_{12}	t_1, t_{10}	5
C_4	t_2, t_6, t_{13}	t_1, t_{10}	2
C_5	t_2, t_8, t_{13}	t_1, t_{10}	7
C_6	t_3, t_6, t_{14}	t_1, t_{10}	5
C_7	t_3, t_7, t_{14}	t_1, t_{10}	7
C_8	t_3, t_8, t_{14}	t_1, t_{10}	10
C_9	t_2, t_8, t_{11}	t_1, t_{10}	4

Table II. Computed monitors for the net system in Fig. 8(a) due to [16].

Monitor	Preset	Postset	M_0
C_1	t_2, t_7, t_{12}	t_1, t_{10}	2
C_4	t_2, t_6, t_{13}	t_1, t_{10}	2
C_6	t_3, t_6, t_{14}	t_1, t_{10}	5
C_9	t_2, t_8, t_{11}	t_1, t_{10}	4

Table III. The performance of supervisors due to different deadlock prevention policies.

Policy	No. reachable states	No. monitors	No. arcs
[15]-policy	4217	9	35
[16]-policy	4217	4	20
Our method	7608	4	16

policies in terms of the three aspects. Here we choose two of them: the seminal work in [15] and the work based on elementary siphons in [16].

For the plant model in Fig. 8(a), we use the policies in [15] and [16] and the method proposed in this paper to design a liveness-enforcing Petri net supervisor, respectively. Table I shows the results of the method in [15]. Table II shows the results due to the method in [16]. In Table III we compare the three methods.

All the three methods need the complete enumeration of strict minimal siphons of a plant model. Since the number of siphons in a plant model is in theory exponential with respect to its size, the computational complexity is in theory exponential. As shown in Table III, the supervisor by the proposed method in this paper has more reachable states with less monitors and arcs.

To make a siphon self-max'-controlled, we restrict the use of some resources by some processes. Hence the method proposed in this study cannot obtain a maximally permissive liveness-enforcing Petri net supervisor.

VI. CONCLUSIONS

This paper proposes a novel approach to deal with the deadlock problems in a typical class of Petri nets. The concept of equivalent P-invariants is developed. Then a sufficient condition that a siphon has an equivalent siphon that can be made self-max'-controlled is established. We can get an equivalent WS^{3}PR for an S^{3}PR plant model. Under some conditions the WS^{3}PR can be made self-live by reconfiguring the weights of some arcs. The resulting net can serve as a liveness-enforcing Petri net supervisor for the original plant net model. Future work includes extending the work to more general classes of Petri nets.

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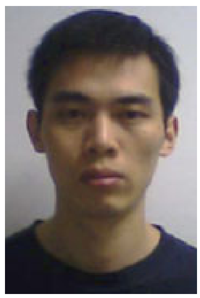

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