On Structural Conditions of S^3PR Based Siphons to Prevent Deadlocks in Manufacturing Systems

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***Abstract* — S^3PR, Systems of Simple Sequential Processes with Resources, is a class of Petri Nets to describe concurrent systems with analyses to include model checking capabilities. The structural object of siphons is extensively used to characterize and analyze deadlock situations in flexible manufacturing system (FMS) that are modeled on Petri Nets (PN). The purpose of this work is to structure a model of Petri nets analyzed to extract minimal siphons which are deduced in the establishment of a static resource allocation policy such that deadlocks cannot occur in the system again. For structural properties formulated in terms of siphons which can be resolved, the problems of deadlock occur to Petri nets which are representative of FMS. The elementary siphons are used to remove deadlock from the control policy as an objective of the system. A Petri net simulation to perform the structural analysis of the model is based on the reachability graph and is employed to synthesize and control FMS. An example is used to illustrate this method.**

***Keywords* — simulation; structure analysis; Petri net; siphon; FMS; S^3PR**

I. INTRODUCTION

Flexible manufacturing system (FMS) is a set of workstations sharing a number of resources such as robots, machines, automatic guided vehicles (AGVs), fixtures and buffers that executive operations under supervisory computer control [4], [7]. Petri nets (PN) have been using for the modeling of FMS [23] - [32]. In order to integrate computers into the automatic manufacturing systems, a process substantial new skill is required. These skills are related not only to the hardware, structured integration of computers and machine tools but also needed to computers, software, systems engineering, production planning, and other organizational aspects, and include a true understanding of the manufacturing process, which the system is supposed to map. The digital computerized manufacturing technology makes it possible for the automatic not only a material processing system but also the information processing between machines concurrently, synchronization in the system with respect to time. While, the mechanization had been mainly concerned with the decrease of labor cost per produced unit. A computer-controlled automation is aimed at reducing all items that make up the total production cost as follow:

1) Reduction of labor cost per unit of production.

2) Higher capital is utilized during a higher degree of the machine as well as plant's utilization.

3) Reduction of capital cost of reduction of in-process as well as finished goods inventory.

4) Faster processes are environments and product development.

5) Higher and more even quantity and quality of products are developed. Petri nets have been utilized extensively to model a variety of synchronizations of process environments, especially on the FMS, because the scope, structures of the nets can be in the design, operation, and management of manufacturing systems.

The work in [6] pioneered a class of Petri Nets (PN) called Systems of Simple Sequential Processes with Resources (S^3PR). For a class of Petri nets, S3PR, a deadlock prevention policy is sophisticated. They compute all minimal siphons without used traps (called strict minimal siphons SMS) of the given model of FMS and find the maximum number of tokens at each idle state followed by control places called (monitors) of adding arcs and places with tokens. The control place can be binding to each (SMS) to become liveness. The monitors (or control places) can be binding to emptiable siphon S to prevent it from being unmarked. However, generally too many control places and arcs are required.

Li and Zhou [7], [8] first utilize a novel concept of siphons to strict minimal siphons $SMSs$ in a Petri net into two classes: elementary and dependent siphons. The former marking is invariant controlled in S^3PR net, they prove that under some status, only used the control places can be obtained liveness by controlling elementary siphons, which is avoiding the redundant monitors and hence significantly improves the computational efficiency. This is method requires a much smaller number of control places. They have been executed by adding monitors (also called control places) to an original plant model, most of which are based on siphons to make sure that it is marked, deadlocks in the system can successfully prevent.

A class of Petri nets called (S^3PR) [6], [12] is a kind of well-accepted process oriented Petri net models for modeling and analyzing deadlock control and liveness-enforcing problems of FMSs. In order to cope with increasingly complex resource usage, a series of extensions to S^3PR are reported, such as systems of simple sequential processes with multiple resources (S^3PMR) [9], [11] system of simple sequential processes with weighted resource allocation (WS^3PR) [14], general S^3PR (GS^3PR) [11] and systems of sequential systems with shared resources (S^4R) [15], and Systems of Simple Sequential Processes with General Resource (S^3PGR^2) [5], and linear S^3PR (L^3SPR) [13].

Abdul-Hussin [1], [2] used deadlock prevention based on siphon control is a typical application to analyses structural of Petri nets. Siphons and traps are analysis structures of deadlock states can be characterized in terms of empty siphons. A Strict Minimal Siphons $SMSs$ and Elementary Siphons were used for deadlocks prevention with to control siphon. To make an S^3PR deadlock-free, one needs to control the elementary siphons. Based on this finding, techniques are presented to synthesize a supervisor with a minimal number of control places and, at the same time, the computational efficiency is improved. The simulation work will be conducted by using software (PN Tool-box with MATLAB) and simulate model choices the un-time Petri net, where it is a kind of tool to simulate and analyze a system effectively. In this work, we propose a method of detection elementary siphons for S^3PR.

Banaszak et al. [3] applied a control strategy, which guarantees the deadlock avoidance. They proposed a general-purpose algorithm that is used in the synthesis of control procedures ensuring the appropriate processes cooperation. The algorithm aimed at the deadlock-free cooperation of concurrent, pipeline-like flowing processes as well as for its direct implementation, in assembler level languages, in real-time systems supervision programs for FMS. Banaszak et. al [4] represented a new algorithm to distributed deadlock avoidance of FMS by measuring process related variables such as flows process. They focused on deadlock avoidance rules for exploiting information on the current resource requirements; to avoid the system before the deadlock states occurred.

Organization. In section II, review briefly preliminaries to Petri nets that are used in this paper. The notion of a controllable siphon basis in an S^3PR is introduced into Section III. Section IV presents structural of a Petri net controller and controls a deadlock prevention application which is presenting of an FMS example. Section V, concludes this paper.

II. PRELIMINARIES

A Petri net is a 4-tuple $\Psi=(P,T,E,W)$ where P is a set of places, T is a set of transitions, $E\subseteq(P\times T)\cup(T\times P)$ is a set of directed arcs, and $W:(P\times T)\cup(T\times P)\to \mathrm{N}=\{0,1,2,...\}$ such that $P\cup T\neq\varnothing, P\cap T=\varnothing$, and $W(x,y)=0$ if $(x,y)\notin E$. The preset of $x\in P\cup T$, is defined as ${}^\bullet x=\{y\in P\cup T\mid(y,x)\in E\}$, and the post set of x is defined as $x^\bullet=\{y\in P\cup T\mid(y,x)\in E\}$. Ordinary Petri net Ψ is named a state machine if $W:E\to\{1\}$ and $\forall t\in T,|{}^\bullet t|=|t^\bullet|=1$. The notation can be extended to a set. For example, let $X\subseteq(P\cup T)$, then ${}^\bullet X=\bigcup_{x\in X}{}^\bullet x$ and $X^\bullet=\bigcup_{x\in X}x^\bullet$. A state machine is a Petri net in which each transition has exactly one input and one output place.

A marking of Ψ is a mapping $M:P\to Z^+$, where Z^+ is the nonnegative integer set. Given a place $p\in P$ and a marking $M, M(p)$ denotes the number of tokens in p at M, and we use $\sum_{p\in P}M(p)p$ to denote vector M. Let $S\subseteq P$ be a set of places. The sum of tokens in its places at M is denoted by $M(S)=\sum_{p\in S}M(p)$. A Petri net Ψ with an initial marking M_0 is called a marked Petri net or net, denoted as (Ψ,M_0). S is marked (unmarked) at M if $M(S)>0,(M(S)=0)$.

A transition $t\in T$ is enabled at a marking *M*, denoted by $M[t\rangle$, if $\forall p\in{}^\bullet t, M(p)>0$. An enabled transition t at M can be fired, resulting in a new marking M', denoted by $M[t\rangle M'$, where $M'(p)=M(p)-1,\forall p\in{}^\bullet t\setminus t^\bullet; M'(p)=M(p)+1,\forall p\in t^\bullet\setminus{}^\bullet t$; and otherwise $M'(p)=M(p)$, $\forall p\in P[({}^\bullet t\setminus t^\bullet)\cup(t^\bullet\setminus{}^\bullet t)]$. A sequence of transitions $\sigma=\{t_1,t_2,...,t_k\}, t_i\in T, i\in\mathrm{N}_k=\{1,2,...,k\}$ is feasible from M, if there exists $M_i[t_i>M_i+1, i\in\mathrm{N}_k$, where $M_1=M$, and M_i is

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called a reachable marking from M, there exists a sequence of markings such that: $M_0[t_1\rangle M_1[t_2\rangle M_2, \ldots, [t\rangle M_n$. Let $R(\Psi, M_0)$ denote the set of all reachable markings of Ψ from M_0.

A P-vector is a column vector $I: P \rightarrow Z$ indexed by P, where Z is the set of integers. P-vector I is a *P*-invariant if $I \neq 0$ and $I^T \bullet [\Psi] = 0^T$. Where, 0 is a zero vector. We conclude that: $\| I \| = \{p \in P \mid I(p) \neq 0\}$ is called the support of I. $\| I \|^+ = \{p \in P \mid I(p) > 0\}$ denotes the positive support of *P*-invariant I and $\| I \|^- = \{p \in P \mid I(p) < 0\}$ denotes the negative one. I is called a minimal *P*-invariant if $\| I \|$ is not a proper superset of the support of any other and the greatest common divisor of its elements is one. If I is a *P*-invariant of (Ψ, M_0), then:

$\forall M \in R(\Psi, M_0): I^T \bullet M = I^T \bullet M_0$.

A marked Petri net (Ψ, M_0), a transition $t \in T$ is live at M_0 if $\forall M \in R(\Psi, M_0), \exists M' \in R(\Psi, M_0), M'[t\rangle$ holds. A transition $t \in T$ is said to be dead at marking $M \in R(\Psi, M_0)$ if $\nexists\, M' \in R(\Psi, M_0),\, M'[t\rangle$. (Ψ, M_0) is live at M_0 if $\forall t \in T$, and t is live at M_0. Otherwise, (Ψ, M_0) is non-live. (Ψ, M_0) is deadlocked at M if $\nexists\, t \in T, M[t\rangle$, where $M \in R(\Psi, M_0)$ and M is named a dead marking. (Ψ, M_0) is $deadlock-free$ if $\forall M \in R(\Psi, M_0), \exists t \in T, M[t\rangle$.

Definition 1. Let S be a non-empty subset of places. $S, Q \subseteq P$ is a siphon (trap) if ${}^\bullet S \subseteq S^\bullet$, $(Q^\bullet \subseteq {}^\bullet Q)$, holds. A strict minimal siphon SMS is a siphon containing neither other siphon nor trap. If $M_0(S) = \sum_{p \in S} M_0(p) = 0$, S is called an empty siphon at M_0. A minimal siphon does not contain a siphon as a proper subset. It is called a strict minimal siphon (SMS), denoted S, if it does not contain a trap. A siphon is said to be controlled if it is marked undergo all reachable markings. I_S is the $I-subnet$, where the subnet derived from $(S, {}^\bullet S)$. Note that $S = P(I_S)$; S is the set of places in I_S. A siphon is said to be controlled by P-invariant I if $I^T \bullet M_0 > 0$, and $\| I \|^+ \subseteq S$.

III. A SIPHONS BASED DEADLOCK PREVENTION

A siphon based control deadlock is concerned with the siphons that get empty of the analysis and control of deadlocks at resources requested (e.g. a robot or a workstation) can be allocated to execute only one operation at a time) is reasonable with Petri nets. In order to thoroughly investigate these properties of the entire system modeled with Petri nets, a special structure object, which is called siphons, is frequently and extensively used. Siphon-based description of deadlocks dominates the methodologies that deal with the deadlock control and liveness enforcement of FMS. We propose a method to compute a SMS in an S^3PR based on resource subsets. The essence of this method is to allocate the resource so that no SMS can be emptied. First presents some definitions and theorems have been necessary to our application.

A. S^3PR Net Models

A structural of siphons is firstly introduced in [6] and then development to divide elementary siphon and dependants in [7], [8], [12] with the aim of selecting the essential siphons for control net. We shall apply our experiment deadlock control method which is firstly developed in [6], and used by classes of S^3PR.

Definition 2. A simple sequential process S^2P is a Petri net $\Psi = (P_A \cup \{P^0\}, T, E)$, where the following statements are true:

(i) $P_A \neq \varnothing$ is called a set of operation places;

(ii) $P^0 \notin P_A$ is called the process idle place;

(iii) A net Ψ is a strongly connected state machine;

(iv) Every circuit of Ψ contains place p^0.

Definition 3. A system of simple sequential processes with resources $(S^3PR): \Psi = \text{O}_{i-1}^k$

$\Psi_i = (P \cup P^0 \cup P_R, T, E)$ is defined in [8] as the union of a set of nets: $\Psi_i = (P_i \cup \{p_i^0\} \cup P_R, T_i, E_i)$, sharing common places, where the following statements are true:

(1) p_i^0 is called the process idle places of net Ψ_i. The elements in p_A^i and p_R^i are called operation places and resource places respectively.

(2) $p_R^i \neq \varnothing$; $p_A^i \neq \varnothing$; $(p_i \cup \{p_i^0\}) \cap p_R^i = \varnothing$; $\forall p \in p_A^i, \forall t \in {}^\bullet p, \forall t' \in p^\bullet, \exists r_p \in p_R^i, {}^\bullet t \cap p_R^i = t'^\bullet \cap p_R^i = \{r_p\}; \forall r \in p_R^i, {}^{\bullet\bullet} r \cap p_A^i = r^{\bullet\bullet} \cap p_A^i \neq \varnothing;$ $\forall r \in p_R^i, {}^\bullet r \cap r^\bullet = \varnothing; and\ {}^{\bullet\bullet}(p_i^0) \cap p_R^i (p_i^0)^{\bullet\bullet} \cap$

$p_R^i = (p_i^0)^{\bullet\bullet} \cap p_R^i = \emptyset.$

(3) Ψ'_i is a strongly connected state machine, where $\Psi'_i = (p_A^i \cup \{p_i^0\}, T_i, E_i)$, is the resultant net after the places in p_R^i and related arcs are removed from Ψ_i.

(4) Every circuit of Ψ'_i contains place p_0^i;

(5) Any two Ψ'_i are composable when they share a set of common places. Every shared place must be a resource.

(6) Transitions in $(p_i^0)^\bullet$ *and* $^\bullet(p_i^0)$ are named source and sink transitions of the net Ψ respectively.

Definition 4. Let $\Psi_i = (p_A \cup p^0 \cup p_R, T, E)$, be an S^3PR. An initial marking M_0 is called an acceptable one if:
i) $\forall p \in p_i^0$, $M_0(p) \geq 1$; ii) $\forall p \in P_A$, $M_0(p) = 0$; and iii) $\forall p \in p_R$, $M_0(p) \geq 1$.

B. Elementary and Dependent Siphons

The concept of elementary and dependent siphons is firstly developed by Li and Zhou [7], [8].

Definition 5. Let $S \subseteq P$ be a subset of places of Petri net $\Psi = (P, T, E, W)$. P-vector λ_S is called the characteristic P-vector of S if and only if $\forall p \in S$, $\lambda_S(p) = 1$; otherwise $\lambda_S(p) = 0$. η_S is called the characteristic T-vector of S if $\eta_S^T = \eta_S^T \bullet [\Psi]$.

Definition 6. Let $S \subseteq P$ be a subset of places of Petri net Ψ. η_S is named the characteristic T-vector of S if $\eta_S = \lambda_S^T \bullet [C]$, where C is incidence matrix of Ψ.

Definition 7: Let $\Psi = (P, T, E, W)$ be an S^3PR with $|P| = m, |T| = n$, and let $\Pi = \{S_1, S_2, \ldots, S_k\}$ be a set of siphons of Ψ, where $S_k, m, n, k \in \mathrm{Z}^+ \setminus \{0\}$. Let $\lambda_{S_i}(\eta_{S_i})$ be the augmented characteristic $P(T)-$ vector of siphon S_i, where $i \in \mathrm{N}_k$. Subsequently, $[\lambda]_{k\,xm} = [\lambda_{S_1} | \lambda_{S_2} | \ldots | \lambda_{S_k}]^T$ and $[\eta]_{k\,xn} = [\eta]_{k\,xm}\; x\; [G]_{m\;xn} = [\eta_{S_1} | \eta_{S_2} | \ldots | \eta_{S_k}]^T$, are called the characteristic $P(T)$ -vector matrices of the siphons in G, respectively.

Definition 8. Let $\eta_{S_\alpha}, \ldots, \eta_{S_\gamma}$ $(\{\alpha, \beta, \ldots, \gamma\} \subseteq I_k)$ be a linearly independent maximal set of matrix [η]. Then $\Pi_E = \{S_\alpha, S_\beta, \ldots, S_\gamma\}$ is called a set of elementary siphons in G, denoted as Π_E [14]. Let $S \notin \Pi_E$ be a siphon in G. Then S is called a strongly dependent siphon [7], if: $\eta_S = \sum_{S_i \in \Pi_E} a_i \eta_{S_i}$, where $a_i \geq 0$. $S \notin \Pi_E$ is called a weakly dependent siphon if $\exists$ non-empty $A, B \subseteq \Pi_E$ such that $A \neq \emptyset, B \neq \emptyset$, $A \cap B = \emptyset$ and $\eta_S = \sum_{S_i \in A} a_i \eta_{S_i} - \sum_{S_i \in B} a_i \eta_{S_i}$, where $a_i > 0$.

Theorem 1. Let Ψ_{ES} be the number of elementary siphons in $\Psi = (P, T, E, W)$. Then we have:
$\Psi_{ES} < \min\{|P|, |T|\}$.
The resultant contains the smallest possible number of places and transitions are needed to compute of elementary siphons.

IV. AN APPLICATION OF PETRI NET

In this section, in the central example of this paper the manufacturing cell of Figure 1 is studied. An FMS that produces two product types, i.e., Parts-1, and Parts-2, with two raw parts, is considered. Fig. 1, shows the block diagram of an FMS where two product types, i.e., J1 − J2, are manufactured. The system consists of finite sets of component, e.g. four machines $M1 - M4$, (each can process two products at a time) and three robots $R1, R2$, and $R3$ (each one can hold one product at a time). Each machine can deal with two products at a time. There are two loading buffers $I_1 - I_2$ and two unloading buffers $O_1 - O_2$ to load and unload the FMS. The sequence of production line of type $part-1$ is: $I_1 \rightarrow R1 \rightarrow M1 \rightarrow R2 \rightarrow M3 \rightarrow R2 \rightarrow M2 \rightarrow R3 \rightarrow O1$, and of production line of type $part-2$ is: $I_2 \rightarrow R3 \rightarrow M2 \rightarrow R2 \rightarrow M4 \rightarrow R2 \rightarrow M1 \rightarrow R1 \rightarrow O2$.

A Petri net model of the FMS is shown in Fig. 2, where places $p_1 - p_5$, and $p_6 - p_{10}$, are denoted to operation places for production Line_1, and production Line_2. Places $p_{11} - p_{15}$, and p_{18}, p_{19} are operation places $(R1, M1, R2, M2, R3, M4, M3)$ respectively.

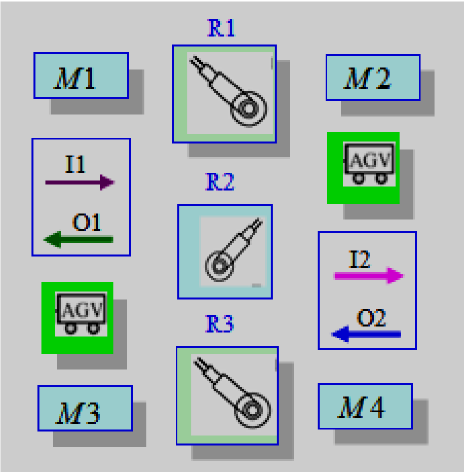

Figure 1. Layout of a manufacturing cell

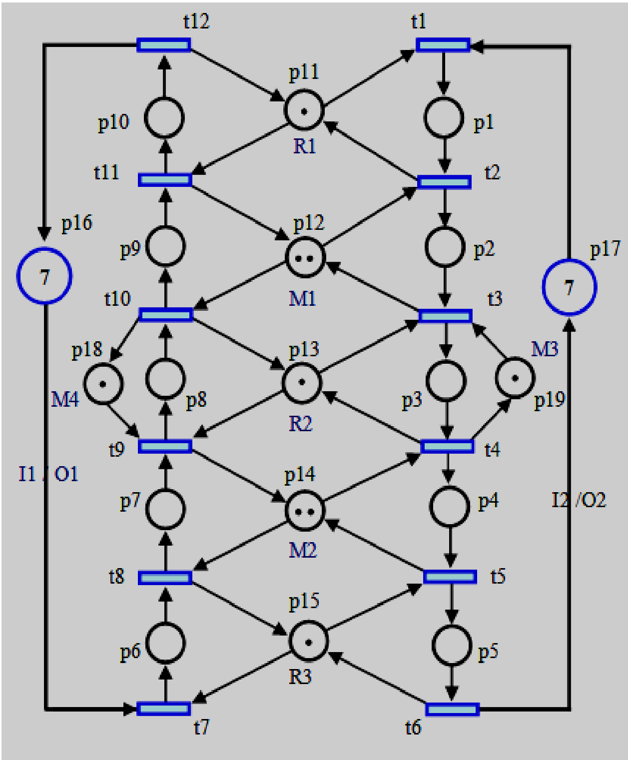

Fig. 2. An S³PR (Ψ, M₀) for example FMS

Places p_{16} and p_{17} are idle places represented Input/Output stores. A Petri net shows in Figure 2 is exemplify of an S^3PR by this place specification. The deadlock situation, which may occur to the manufacture cell, is reflective of marking M such that: tokens in Fig. 2, M_0 is an acceptable initial marking if $M_0(p_{16}) = M_0(p_{17}) = 7, M_0(p_{11}) =$

$M_0(p_{13}) = M_0(p_{15}) = M_0(p_{18}) = M_0(p_{19}) = 1,$ and

$M_0(p_{18}) = M_0(p_{19}) = 1,\ M_0(p_{12}) = M_0(p_{14}) = 2,$ and others places are equal zero. For the Petri net of Fig. 2 is an $S^3PR.$ The elements of this net are defined as:

$$P_A = P_A^1 \cup P_A^2 = \{p_1 - p_5\} \cup \{p_6 - p_{10}\} = \{p_1 - p_{10}\},$$

and for the process idle places $P^0 = \{p_{16}, p_{17}\}$, the resource places are:

$$P_R = \{p_{11} - p_{15}, and\ p_{18}, p_{19}\},$$

and holders of resource:

$H(p_{18}) = \{p_8\},\ H(p_{19}) = \{p_3\}$, and

$$H(p_{11}) \cup H(p_{12}) \cup H(p_{13}) \cup H(p_{14}) \cup H(p_{15}) = \{p_1 - p_{10}\}.$$

In a Petri net formalism, we first model an FMS with Petri nets shown in Fig. 2 is an S³PR that contain deadlocks. We can see the original net system Ψ has 707 reachable states, among which there are 12-deadlock states. Simulation and structure analysis of the Petri net model used MATLAB [10] and the reachability tree has the deadlock marking occurred at marking:

$$M_{375}, M_{389}, M_{490}, M_{496}, M_{521}, M_{523}, M_{585}, M_{597}, M_{600}, M_{620}, M_{662}, and\ M_{669}.$$

The reachability graph has (707) states with the initial marking. A particulars important application of our approach used an S³PR to solve deadlock in FMS, where processes are executed concurrently and sharing resources. We consider Petri nets, which are obtained by asynchronous composition and are important to identify the minimal siphons. The net system is an S³PR, and contains deadlocks.

There are 15 strict minimal siphons as listed below. For the purpose of finding the elementary siphon according to above discussion is algorithm [7, 8] to obtain a set of elementary siphons in an S³PR. The elementary siphons $S_1 - S_3$ can be chosen as a set of elementary siphons, while the others $S_4 - S_{15}$ are (strict) dependent ones that are marked by *.

$S_1 = \{p_2, p_{10}, p_{11}, p_{12}\}$, $S_2 = \{p_5, p_7, p_{14}, p_{15}\}$, $S_3 = \{p_3, p_{10}, p_{11}, p_{12}, p_{13}\}$, $S_4^* = \{p_5, p_{10}, p_{11}, p_{12}, p_{13}, p_{14}, p_{15}\}$, $S_5^* = \{p_5, p_{10}, p_{11}, p_{12}, p_{14}, p_{15}, p_{18}, p_{19}\}$, $S_6^* = \{p_4, p_{10}, p_{11}, p_{12}, p_{14}, p_{18}, p_{19}\}$, $S_7^* = \{p_1, p_2, p_{10}, p_{11}\}$, $S_8^* = \{p_3, p_9, p_{12}, p_{13}\}$, $S_9^* = \{p_4, p_9, p_{12}, p_{14}, p_{18}, p_{19}\}$, $S_{10}^* = \{p_4, p_8, p_{13}, p_{14}\}$, $S_{11}^* = \{p_4, p_9, p_{12}, p_{13}, p_{14}\}$, $S_{12}^* = \{p_4, p_{10}, p_{11}, p_{12}, p_{13}, p_{14}\}$,

$S_{13}^{*}=\{p_5,p_9,p_{12},p_{14},p_{15},p_{18},p_{19}\}$,
$S_{14}^{*}=\{p_5,p_8,p_{13},$
$p_{14},p_{15}\}$, and $S_{15}^{*}=\{p_5,p_9,p_{12},p_{13},p_{14},p_{15}\}$.

As mentioned above, there are four SMS in Fig. 2, S_1-S_3. The elementary siphon theorem was development by [7, 8] we can obtained from siphons above such that; we have three the strict minimal siphons SMS of the elementary theorems is finding as:

$\lambda_{S_1}=\{p_2+p_{10}+p_{11}+p_{12}\}$,

$\lambda_{S_2}=\{p_5+p_7+p_{14}+p_{15}\}$, and

$\lambda_{S_3}=\{p_3+p_{10}+p_{11}+p_{12}+p_{13}\}$.

The linearly independent of T-vectors transition can be structured in [η] shown as follows.

$\eta_{S1}=-t_1+t_2-t_7+t_{11}$,

$\eta_{S2}=-t_1+t_5-t_7+t_8$, and

$\eta_{S3}=-t_2+t_6-t_8+t_{12}$.

This is set of elementary siphons, we can calculate the dependent siphons as: $\eta_{S4}=\eta_{S1}+\eta_{S3}$, and $\eta_{S5}=\eta_{S2}+\eta_{S3}$. The transitions T-vector is described structurally in [η].

From matrix $[\eta]$, we can calculate as:

$\eta_{S_4}=\eta_{S_2}+\eta_{S_3}=-t_1-t_2+t_5+t_6-t_7+t_{12}$,

$\eta_{S_5}=\eta_{S_1}+\eta_{S_3}=-t_1+t_6-t_7-t_8+t_{11}+t_{12}$.

Thus, the SMS S_1-S_3 are elementary siphons and strict dependent siphons are S_4-S_{15}. From the above computation, we can add to two control places VS_1 and VS_2 to control S_1 and S_2, respectively. The controlled net system is shows in Figure 3, that has:

$M_0(VS_1)=M_0(VS_2)=1$. It is obvious that S_1 and S_2, can never be emptied. Moreover, the controlled net is live and there are 19 reachability states liveness.

Lemma 1: Let (Ψ,M_0) be a marked net, and S be a siphon in Ψ. S is controlled by *P*-invariant I under M_0 iff $I^T\bullet M_0>0$, and $\|I\|^+\subseteq S$. Such a siphon is called an invariant controlled one and S can not be emptied if it is invariant-controlled. Proof: See [8] for its details.

The liveness is described by adding two control places (monitors) VS_1, and VS_2. The controlled net model for FMS of Figure 3, is live and obtain to (19) states marking, in such a case, the Petri net is deadlock-free. Where there are no siphons that can be emptied. Moreover, the controlled net Ψ is liveness see in Figure 4. This example motivates one to discover the mechanism to make a siphon controlled by adding monitors. The motivated by the approved Petri net analysis techniques, deadlock prevention can be developed by reachability graph and analysis structure. The important aspect of a monitoring system is how it deals with distributed systems.

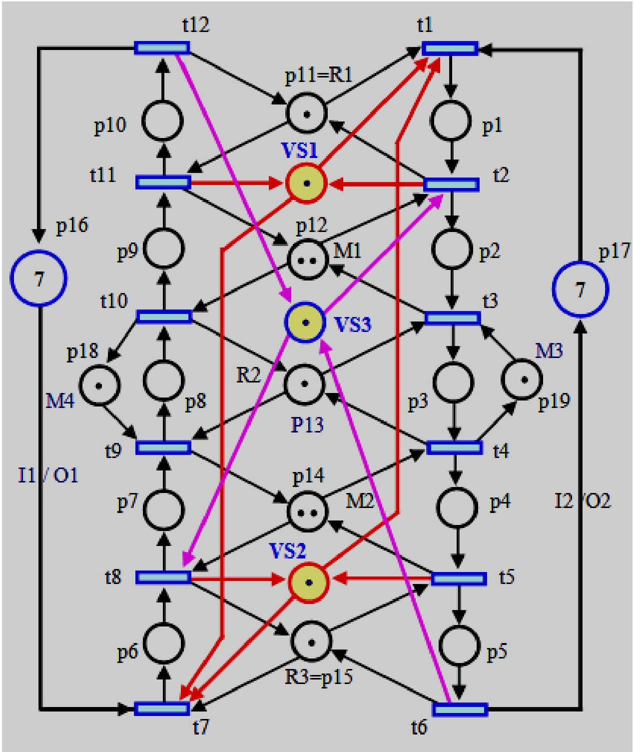

Fig. 3. Added control place VS_1– VS_3 called control places

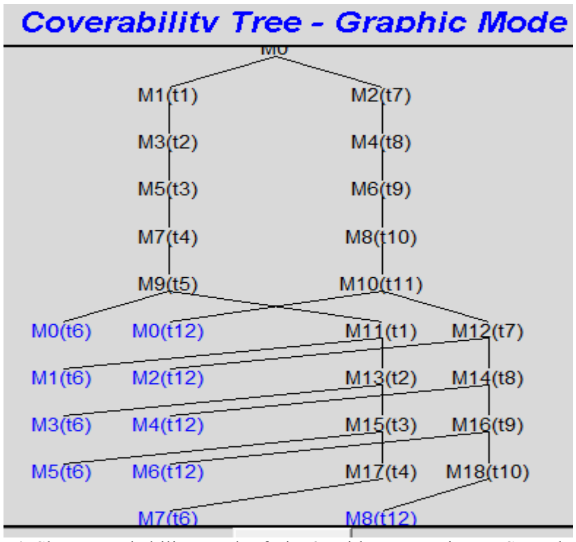

Fig. 4. Shows reachability graph of Fig. 3, with two monitors VS_1, and VS_2

Three control places are necessary adding for elementary siphons, and not necessary to add more control places for dependent siphons because the net has small size tree. Some authors had added control places for all dependent siphons when the Petri net is a large size. We can see, that the Petri net is also deadlock-free, and there are reduced to (12) reachable states, with initial marking. So that, the third control places VS_3 is added important is to reduce the states of the reachability graph has states 12, and the resultant's net is live as show in analyses for that is the third element in siphons with the P-invariant based method results from the additional places. The resulting controlled Petri net is shown in Figure 5. It is being verified that deadlock does not occur again.

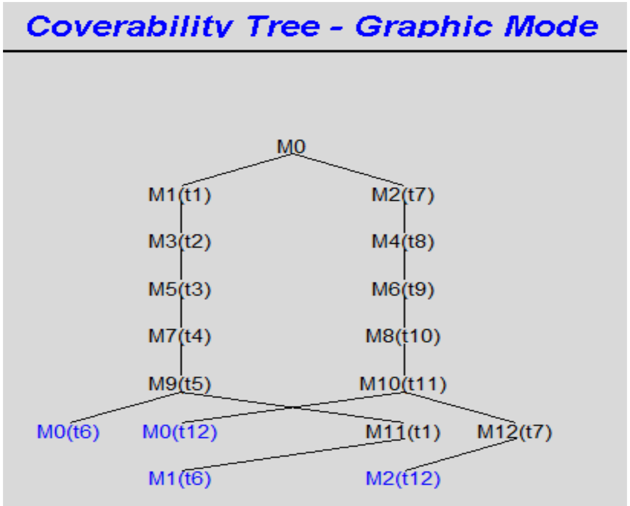

Fig. 5 shows reachability graph of Figure 3, with three monitors VS_1– VS_3

With the regarding of the P-invariant, we can find the incidence matrix of Petri net of Fig. 3. The P-invariant structure analysis is concerned with relationships satisfied with any reachable marking, thus based on the net structure rather than on the initial marking. A control place VS_1 is added such that: $I_1 = \{1,0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0,0,0,1_{VS1}\}$ is a P–invariant of (Ψ_1, M_1). It means: $I_1 = \{p_1 + p_6 + p_7 + p_8 + p_9 + VS_1\}$. Therefore,

$I_1 \bullet M_1 = 0$, by definition 1, the computation of places control that is: $[\Psi_1](VS_1, t) = -t_1 + t_2 - t_7 + t_{11}$. Similarly,

$I_2 = \{1,1,1,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,1_{VS2})$ is also a P–invariant of (Ψ_1, M_1). That is $I_2 = \{p_1 + p_2 + p_3 + p_4 + p_6 + VS_2\}$. Hence $I_2 \bullet M_1 = 0$, $I_2 \bullet M_1 = 0$, and $[\Psi_1](VS_2, t) = -t_1 + t_5 - t_7 + t_8$. Similarly,

$I_3 = \{0,1, 1,1,1,0,1,1,1,1,0,0,0,0,0,0,0,0,0,1_{VS3}\}$, that is

$I_2 = \{p_2 + p_3 + p_4 + p_5 + p_7 + p_8 + p_9 + p_{10} + VS_3\}$.

Then, $I_3 \bullet M_1 = 0$, by definition 1 and the computation of places control that is: $[\Psi_1]$(VS_3, t) = $-t_2 + t_6 - t_8 + t_{12}$. Where $M_1(VS_1) = M_0(S_1) - 1 = 1$, $M_1(VS_2) = M_0(S_2) - 1 = 1$, and $M_1(VS_3) = M_0(S_3) - 1 = 1$.

After adding $VS_1 - VS_3$ to the net (Ψ, M_0), the new model (Ψ_1, M_1), is deadlock free as shown in Figure 5.

The liveness of the controlled system can be assured by adding three monitors' places regardless of their actual possibility of getting unmarked siphons. We are computing P-invariant to find manually executives the elementary siphons are static and dynamic Petri net structural liveness, conservativeness, repetitiveness, consistency, P-invariant, where is stated from the incidence matrix of PN.

For example, elementary siphons $S_1 - S_3$, three monitors $VS_1 - VS_3$ are added respectively. Lemma 1, and definition 1, is used to find a P-invariant controlled of the Petri net depending of the incidence matrix of Figure 3, which is implementation manually executive to find P-invariant. $S_1 = \{p_2, p_{10}, p_{11}, p_{12}\}$, is a siphon of the net. A P-invariant of the net in Fig. 3 is: $I_4 = \{0, 1,0,0,0,-1,-1, -1,0,1,1,1,0,0,0,0,0,0,0,-1_{VS1}\}$. It is mean:

$I_4 = \{p_2 + p_{10} + p_{11} + p_{12} - p_6 - p_7 - p_8 - VS_1\}$, where:

$I_1 \bullet M_0 = M_0(p_2) + M_0(p_{10}) + M_0(p_{11}) + M_0(p_{12}) - M_0(p_6) - M_0(p_7) - M_0(p_8) - M_0(VS_1)\} = 1 > 0$. That is $\| I_4 \|^{+} = S_1 \subseteq S$. Note that S_1 is an invariant-controlled siphon and it can never be emptied. Similarly, we can compute P-invariant of the siphon $S_2 = \{p_5, p_7, p_{14}, p_{15}\}$, is a siphon of the net. Siphon S_2 is controlling by P-invariant:

$I_5 = \{-1,-1,-1,0,1,0,1,0,0,0,0,0,0,1,1,0,0,0,0,-1_{VS2}\}$. It means:

$I_5 = \{p_5 + p_7 + p_{14} + p_{15} - p_1 - p_2 - p_3 - VS_2\}$,

where $I_5 \bullet M_0 = \{M_0(p_5) + M_0(p_7) + M_0(p_{14}) + M_0(p_{15}) - M_0(p_1) - M_0(p_2) - M_0(p_3) - M(VS_2)\} = 1 > 0$. So that $\| I_5 \|^{+} = S_2 \subseteq S$. A siphon S_2 is guaranteeing marking to the controlled system live and no emptied siphon.

Similarly, for siphon $S_3 = \{p_3, p_{10}, p_{11}, p_{12}, p_{13}\}$, is a siphon of the net. Siphon S_3 is controlling by P-invariant:

$I_6 = \{1,0,0,-1,-1,0,-1,0,0,0,1,1,1,0,0,0,0,0,0,-1_{VS3}\}$.
It means:

$$I_6 = \{p_1 + p_{11} + p_{12} + p_{13} - p_4 - p_5 - p_7 - VS_3\},$$

where $I_6 \bullet M_0 = \{M_0(p_1) + M_0(p_{11}) + M_0(p_{12}) + M_0(p_{13}) - M_0(p_4) - M_0(p_5) - M_0(p_7) - M_0(VS_3)\} = 1 > 0$. So that $\| I_6 \|^{+} = S_3 \subseteq S$. Where siphon S_3 is guaranteeing marking to the controlled system live and no emptied siphon.

V. CONCLSION

The Petri net concept of siphon provides excellent additional tools for the control of FMS using structural analysis and reachability graph to avoid deadlock. The approach of *P*–invariant is used to add a monitor to every minimal siphon that can be emptied to ensure that any outgoing arc of the monitors will point to the source transition. A siphon based deadlock prevention policy is found in an S^3PR and the obtained controlled net. The content of this approach is more suitable for applications in manufacturing systems where models are revised several times and also improve reusability. The results have been simulated in the Petri net with MATLAB [10] which help us to illustrate the visualization of Petri nets in showing the reachability graph, and effective when adding monitors to systems where the system events occur. The main application is implied in having efficient siphons for control structure analysis of Petri nets. The reachability graph is also reduced to explaining the substantial conditions structure of PN regarding liveness.

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