Assignment: Galaxies and their Mass

1 Rotation Curves: Milky Way

The data for Milky Way can be downloaded from `https://github.com/galkintool/galkin/`. Let's first try to predict the model rotation curve. The basic equation is quite trivial: it tells us how the mass $m(r)$ of a sphere of radius r increases with radius, given the local mass density $\rho(r)$:

$$dm/dr = 4\pi r^2 \rho \tag{1}$$

How does the density ρ depend on radius in a galaxy? The matter is not distributed like a uniformly filled sphere, because the shape into which stars assemble is determined by the gravity of the stellar population itself, and by its geometrical shape ... We shall use some reasonable recipes. The main visible component in the milky way is the disk (and the bulge), we shall use the approximation of a thin disk, whose projected density $\Sigma(r)$ drops of exponentially with radius

$$\Sigma(r) = M/(2\pi a^2) \times exp(-r/a) \tag{2}$$

with M is the total mass of this component and a is the radial scale length, which determines how fast the density drops off. Its contribution to the mass $m(< r)$ inside radius r is computed via the gradient

$$dm/dr = 2\pi r \Sigma \tag{3}$$

We start at the centre r=0, where we know the mass m(0) = 0. Then we make a step and we compute the mass inside that larger radius by:

$$m(< r + \Delta r) = m(< r) + dm/dr * \Delta r \tag{4}$$

Having this mass, we can now calculate circular velocity (assuming a circular orbit).

(a) Read in the data using `PANDAS`.

(b) Plot the data with the errorbars.

(c) Plot model rotation curve for the disk. For the Milky Way, we know the mass and the scale lengths of the visible components:

- The disk : $M_D = 6.5 \times 10^{10} M_\odot$, $a = 4kpc$

(d) Does the total velocity fit the data? What do you need to add?

2 Rotation Curves: NGC753

NGC 753 is an inclined spiral galaxy for which we will try to determine the rotation curve. For the disk, we shall again use the approximation of a thin disk, whose projected density $\Sigma(r)$ drops of exponentially with radius

$$\Sigma(r) = M/(2\pi a^2) \times exp(-r/a) \tag{5}$$

again, M is the total mass of this component and a is the radial scale length, which determines how fast the density drops off. Its contribution to the mass $m(< r)$ inside radius r is computed via the gradient

$$dm/dr = 2\pi r \Sigma \tag{6}$$

A disk model was fit to the stellar surface brightness profile. The disk has a disk scale length of 5.8kpc. The total mass of the disk is $1 \times 10^{10} M\odot$ (Courteau et al. 1997). In the previous exercise you have realized that rotation curves are not well fit by assuming visible component only, hence a dark component needs to be added. It was shown from simulations, that the dark matter profile can be well characterized by the the NFW profile as described by Navarro, Frenk, & White 1996. The profile is characterized by

$$\rho(r) = \rho_0 \frac{\delta_c}{\frac{r}{R_s}(1 + \frac{r}{R_s})^2} \tag{7}$$

where ρ_0 is the characteristic density and R_S is the scale radius, both are the parameters of the profile. The rotation curve for the NFW profile can be parametrized as

$$v(r)^2 = v_{200}^2 * \left[\frac{1}{x} \frac{ln(1 + cx) - cx/(1 + cx)}{ln(1 + c) - c/(1 + c)} \right] \tag{8}$$

where $c = r_{200}/R_s$ is the concentration and $x = r/R_{200}$. v_{200} is the circular velocity at R_{200} and R_{200} is the radius where the density is 200 times the critical density, also called the Virial radius as the density is the one required for a spherical collapse.

$$v_{200}^2 = \frac{GM_{200}}{R_{200}}. \tag{9}$$

Mass within the virial radius (M_{200}) and scale radius are related by

$$rs = \frac{R_{200}}{c} = \frac{1.63 \times 10^{-2}}{c} \left(\frac{M_{200}}{M_\odot} \right)^{1/2} h^{-2/3}\text{kpc} \tag{10}$$

where h is the Hubble parameter (0.7 for current cosmology, more on that later). By determining c and M_{200} we can thus fully parametrize the profile.
From http://leda.univ-lyon1.fr// download kinematics (1-D) data of UGC1437 (NGC 753) which is one of the galaxies from Vera Rubin's study. Distance to the galaxy is $67.2 \pm 4.6 Mpc$

(a) Find a galaxy with Aladdin Lite and attach a screenshot to your notebook.

(b) Read in the velocity data and plot the rotation curve

(c) Plot the rotation curve to the data using different components as well as the total rotation curve. Remember they don't simply add. Try to estimate the parameters for the dark matter by eye.

(d) In reality we never estimate parameters by eye. Use your favorite fitting method to fit the data. Visible components are measured, but the unknown is the dark matter component.

(e) Determine the mass and mass to light ratio of this galaxy. How does inclination of the galaxy affect your result? Inclination of this galaxy is $i = 50°$.

(f) Compare the rotation curve data to Rubin et al. 1980. Find the paper on ADS.