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Hyperelastic Mooney-Rivlin Model: Determination and Physical Interpretation of Material Constants

Nomesh Kumar
Advanced Systems Laboratory
Defence R & D Organisation
Hyderabad, India

V. Venkateswara Rao
Advanced Systems Laboratory
Defence R & D Organisation
Hyderabad, India

ABSTRACT

Rubber like material, which are characterised by a relative low elastic modulus and high bulk modulus are used in a wide variety of a structural applications. A material is said to be hyperelastic if there exists an elastic strain density function (W) that is a scalar function of strain deformation tensors, whose derivatives with respect to strain components determines the corresponding stress components. Mooney-Rivlin strain energy function with two, three, five or nine parameters is described along with stress strain curve and stability criteria. The crosslinking density is also determined through Mooney Rivlin model.

Keywords: *Hyperelasticity, Mooney Rivlin model, cross linking density.*

I. INTRODUCTION

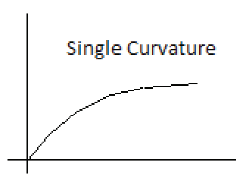
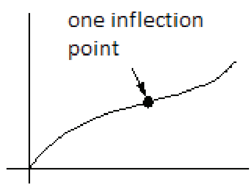
Rubber like material, which are characterised by a relative low elastic modulus and high bulk modulus are used in a wide variety of a structural applications. These materials are commonly experienced with large elastic strains and deformation with small volume change (nearly compressible material) and termed as 'Hyperelastic material'. A material is said to be hyperelastic if there exists an elastic strain density function (W) that is a scalar function of strain deformation tensors, whose derivatives with respect to strain components determines the corresponding stress components. Hence hyperelastic constitutive model have both material nonlinearity and large deformation. A considerable amount of literature has been published on modeling of hyperelastic material [1].

The choice of model depends on its applications, corresponding variables and available data to determine the material parameters. Some of the available hyperelastic models are Neo-Hookean (working strain range 30%), Mooney-Rivlin (30% in compression and 200% in tension depending on order), Arruda-Boyce (300%) and Ogden (upto 700%). Generally, hyperelastic model should satisfy the Ducker stability criterion. Some hyperelastic models satisfy that the stain energy function can be separated into the sum of separate function of the principal stretches [2].

II. MOONEY -RIVLIN MODEL

In this study, solid propellant is assumed as incompressible hyperelastic solid since the elastic effect is predominant compared to visco-elastic one. The stress state in the hyperelastic material is determined by taking the derivatives of the strain energy density with respect to the strain components.

Table 1: Type of Mooney Rivlin models and their uses

S.No.	Type of stress-strain curve	Type of Mooney-Rivlin model
1.		2-parameters/3-parameters model
2.		3-parameters or 5-parameters model

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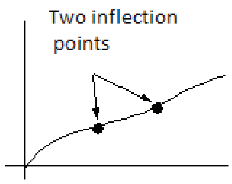
3.		5 –parameters or 9-parameters model

Mooney Rivlin model is one of the famous model of phenomenological type. For a given strain, the stress state is determined as the derivatives of the strain energy density with respect to the strain components. The strain energy density function is given as:

$$W = W(\bar{I}_1, I_2, J) = W(\bar{I}_1, \bar{I}_2) + U(J)$$

and corresponding Second Piola Kirchhoff stress is given as

$$\bar{S} = \frac{\partial \bar{W}(\bar{I}_1, \bar{I}_2)}{\partial E}$$

To describe the material behavior, Mooney-Rivlin model as a function of strain invariants with different parameters such as two, three, five or nine parameters model are available in literature and the suitable model can be selected depending on the type of the stress strain curve [3-4] as shown in Table 1. The curve fitting with 2-parameters, 3- parameters and 5 parameters Mooney Rivlin models for different types of propellants are carried out as shown in Figures 1 to 4 and Table 2.

The strain energy function of Mooney-Rivlin hyperelastic constitutive model is expressed as a function of strain invariants I_1, I_2, $I_3 = J^2$. The form of strain energy density function for two, three, five and nine terms Mooney-Rivlin models are given as:

2- Parameters

$$W_{(2)} = C_{10}(\bar{I}_1 - 1) + C_{01}(\bar{I}_2 - 1) + \frac{1}{d}(J - 1) \text{ K} = \frac{2}{d} \text{ and}$$

$$m = 2(C_{10} + C_{01}d) = (1 - 2 * v) / (C_{10} + C_{01})$$

3- Parameters

$$W_{(3)} = C_{10}(\bar{I}_1 - 1) + C_{01}(\bar{I}_2 - 1) + C_{11}(\bar{I}_1 - 1)(\bar{I}_2 - 1) + \frac{1}{d}(J - 1)^2$$

5- Parameters

$$W_{(5)} = C_{10}(\bar{I}_1 - 1) + C_{01}(\bar{I}_2 - 1) + C_{20}(\bar{I}_1 - 1)^2 + C_{01}(\bar{I}_2 - 1)^2 + C_{11}(\bar{I}_1 - 1)(\bar{I}_2 - 1) + \frac{1}{d}(J - 1)^2$$

9- Parameters

$$W_{(9)} = C_{10}(\bar{I}_1 - 1) + C_{01}(\bar{I}_2 - 1) + C_{20}(\bar{I}_1 - 1)^2 + C_{02}(\bar{I}_2 - 1)^2 + C_{11}(\bar{I}_1 - 1)(\bar{I}_2 - 1) + C_{30}(\bar{I}_2 - 1)^2 + C_{03}(\bar{I}_2 - 1)^3 + C_{21}(\bar{I}_1 - 1)^2(\bar{I}_2 - 1) + C_{12}(\bar{I}_1 - 1)(\bar{I}_2 - 1) + \frac{1}{d}(J - 1)^2$$

The corresponding uniaxial stress for incompressible Mooney-Rivlin models are given as [5]:

2- Parameters

$$\bar{S}_{2p} = 2C_{10}\left(\lambda - \frac{1}{\lambda}\right) + 2C_{01}\left(1 - \frac{1}{\lambda^3}\right)$$

3- Parameters

$$\bar{S}_{3p} = 2C_{10}\left(\lambda - \frac{1}{\lambda}\right) + 2C_{01}\left(1 - \frac{1}{\lambda^3}\right) + 6C_{11}\left(\lambda^2 - \lambda - 1 + \frac{1}{\lambda^2} + \frac{1}{\lambda^3} - \frac{1}{\lambda^4}\right)$$

5- Parameters

$$\bar{S}_{5p} = 2C_{10}\left(\lambda - \frac{1}{\lambda}\right) + 2C_{01}\left(1 - \frac{1}{\lambda^3}\right) + 6C_{11}\left(\lambda^2 - \lambda - 1 + \frac{1}{\lambda^2} + \frac{1}{\lambda^3} - \frac{1}{\lambda^4}\right) + 4C_{20}\lambda\left(1 - \frac{1}{\lambda^3}\right)\left(\lambda^2 + \frac{2}{\lambda} - 3\right) + 4C_{02}\left(2\lambda + \frac{1}{\lambda^2} - 3\right)\left(1 - \frac{1}{\lambda^3}\right)$$

9- Parameters

$$\bar{S}_{9p} = 2C_{10}\left(\lambda - \frac{1}{\lambda}\right) + 2C_{01}\left(1 - \frac{1}{\lambda^3}\right) + 6C_{11}\left(\lambda^2 - \lambda - 1 + \frac{1}{\lambda^2} + \frac{1}{\lambda^3} - \frac{1}{\lambda^4}\right) + 4C_{20}\lambda\left(1 - \frac{1}{\lambda^3}\right)\left(\lambda^2 + \frac{2}{\lambda} - 3\right) + 4C_{02}\left(2\lambda + \frac{1}{\lambda^2} - 3\right)\left(1 - \frac{1}{\lambda^3}\right) + 2C_{21}\left(1 - \frac{1}{\lambda^3}\right)\left(2\lambda + \frac{1}{\lambda^2} - 3\right) + \left(2\lambda^3 - 4\lambda + \frac{1}{\lambda^2} + 1\right) + 2C_{12}\left(1 - \frac{1}{\lambda^3}\right)\left(2\lambda + \frac{1}{\lambda^2} - 3\right) \left(4\lambda^2 - \frac{5}{\lambda} - 3\lambda - 6\right) + 6C_{30}\left(\lambda^2 + \frac{2}{\lambda} - 3\right)^2 \left(\lambda - \frac{1}{\lambda^2}\right) + 6C_{03}\left(2\lambda + \frac{1}{\lambda^2} - 3\right)^2\left(1 - \frac{1}{\lambda^3}\right)$$

Mooney Rivlin model should satisfy stability criterion in order to produce real behavior of the material. The stability criterion are as:

$$\frac{\partial \sigma_{ij}}{\partial \varepsilon_{ij}} \geq 0 \text{ and } \partial \sigma_{ij} \partial \varepsilon_{ij} \geq 0$$

The above mentioned conditions provided certain restrictions on the Mooney-Rivlin parameters as mentioned below:

2- Parameters

$$C_{10} + C_{01} \geq \text{ and } C_{01} \geq 0$$

3- Parameters

$$C_{10} + C_{01} \geq \text{ and } C_{11} \geq 0$$

5- Parameters

$$C_{10} + C_{01} \geq 0, \quad C_{20} \geq 0,$$
$$C_{02} < 0 \text{ and } C_{20} + C_{02} + C_{11} \geq 0$$

9- Parameters

$$C_{10} + C_{01} \geq 0, \quad C_{30} \geq 0,$$
$$C_{03} < 0, \; C_{20} + C_{02} + C_{11}$$
$$\geq 0 \text{ and } C_{30} + C_{03} + C_{12} + C_{21} \geq 0$$

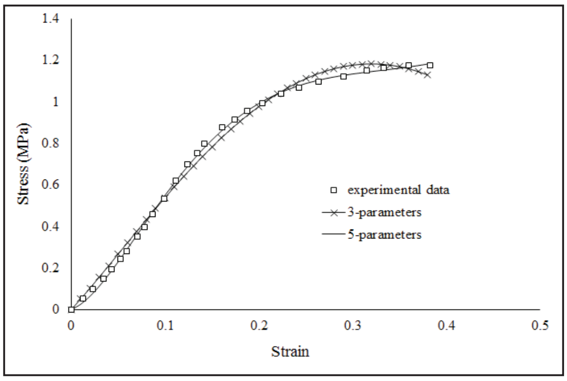


Fig. 1. Stress strain curve for rubber-a at strain rate of 0.2/sec

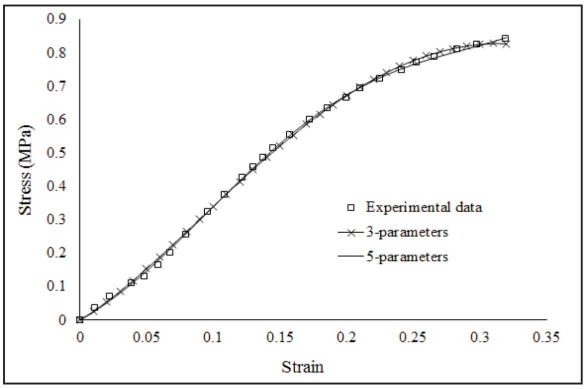


Fig. 2. Stress strain curve for rubber-a at strain rate of 0.002/sec

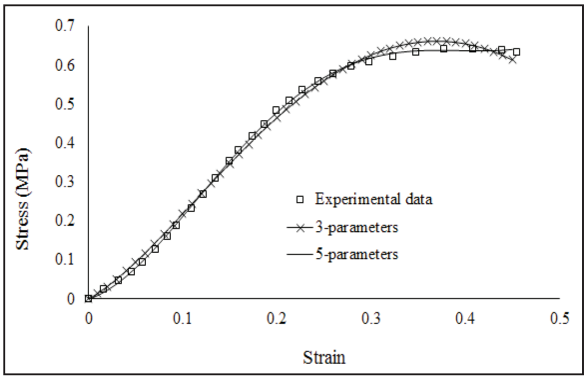


Fig. 3. Stress strain curve for rubber-b at strain rate of 0.2/sec

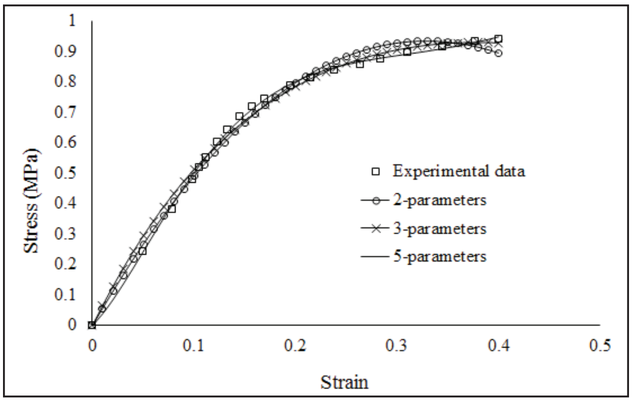


Fig. 4. Stress strain curve for rubber-a type strain rate of 0.2/sec

Table 2: Mooney Rivlin constants for different propellant at different strain rates

S. No.	Material	Strain rates	Properties	values
1.	HTPB propellant (Without Nitramines)	strain rate of 0.002/sec	Mooney-Rivlin constants	C1= 1.8347 C2= -1.2825 C3= -1.0448
		strain rate of 0.2/sec	Mooney-Rivlin constants	C1= 0.58457 C2= -0.37701 C3= -0.59604
2.	HTPB propellant with 10% HMX	strain rate of 0.002/sec	Mooney-Rivlin constants	C1= 3.7686 C2= -3.3689 C3= -1.6176
		strain rate of 0.2/sec	Mooney-Rivlin constants	C1= 3.4490 C2= -2.6145 C3= -1.6508
3.	HTPB propellant with 40% RDX	strain rate of 0.002/sec	Mooney-Rivlin constants	C1= 3.0593 C2= -3.0341 C3= -1.0215
		strain rate of 0.2/sec	Mooney-Rivlin constants	C1= 2.6517 C2= -2.4230 C3= -0.9406

III. Physical Interpretation

The higher crosslinking of polymer represents better physical/ mechanical properties. The cross-linked density of polymer is given as

$$n = \frac{\|c_{10}\|}{RT}$$

The cross-linked density cannot be negative even as C_{10} can be negative. For given typical solid propellant, the cross linked density is 0.0112. The C_{10} represents elastic behavior and C_{01} parameter divert from elasticity. As C_{01} increase nonlinearity in stress-strain curve increases. More parameters lead the curve to have more inflection point.

IV. CONCLUSION

The 2, 3, 5 and 9 parameters Mooney Rivlin strain energy densities are described. Based upon the curve as nonlinear curve or one/two inflection points on curve, corresponding model can be selected. The mechanical tests results shown that the nitramines based propellant have better strain capability as compared to without nitramines.

As number of Mooney-Rivlin parameters increase in curve fitting, matching of simulations with experimental data improve, which may provide more accurate results. However, correct selection of number of parameters is necessary, as more parameters may lead to unconvergency.

Furthermore crosslinking density of polymer can be determined from the Mooney Rivlin constants as well as C_{10} increases, polymer becomes harder.

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