TRILL Manual

SWI-Prolog Version

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1 Introduction

TRILL ("Tableau Reasoner for descrIption Logics in Prolog", [19, 18]) implements a tableau algorithm in Prolog to compute the set of all the explanations of a query. After generating the explanations, TRILL can computes the probability of the query. The management of the tableau rules' non-determinism is delegated to the Prolog language.

TRILL is available in two versions, one for Yap Prolog and one for SWI-Prolog. They differ slightly in the features offered. The Yap version differs principally in the absence of the translation module form OWL/RDF to TRILL syntax.

2 Installation

TRILL is distributed as a pack of SWI-Prolog. To install it, use

```
?- pack_install(trill).
```

Moreover, in order to make sure you have a foreign library that matches your architecture, run

```
?- pack_rebuild(trill).
```

3 Syntax

Description Logics (DLs) are knowledge representation formalisms that are at the basis of the Semantic Web [1, 2] and are used for modeling ontologies. They are represented using a syntax based on concepts, basically sets of individuals of the domain, and roles, sets of pairs of individuals of the domain. In this section, we recall the expressive description logic $\mathcal{ALC}$ [15]. We refer to [8] for a detailed description of $\mathcal{SHOIN}(\mathbf{D})$ DL, that is at the basis of OWL DL.

Let $\mathbf{A}$, $\mathbf{R}$ and $\mathbf{I}$ be sets of *atomic concepts*, *roles* and *individuals*. A *role* is an atomic role $R \in \mathbf{R}$. *Concepts* are defined by induction as follows. Each $C \in \mathbf{A}$, $\perp$ and $\top$ are concepts. If C, C_1 and C_2 are concepts and $R \in \mathbf{R}$, then $(C_1 \sqcap C_2)$, $(C_1 \sqcup C_2)$, $\neg C$, $\exists R.C$, and $\forall R.C$ are concepts. Let C, D be concepts, $R \in \mathbf{R}$ and $a, b \in \mathbf{I}$. An *ABox* $\mathcal{A}$ is a finite set of *concept membership axioms* $a : C$ and *role membership axioms* $(a, b) : R$, while a *TBox* $\mathcal{T}$ is a finite set of *concept inclusion axioms* $C \sqsubseteq D$. $C \equiv D$ abbreviates $C \sqsubseteq D$ and $D \sqsubseteq C$.

A *knowledge base* $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ consists of a TBox $\mathcal{T}$ and an ABox $\mathcal{A}$. A KB $\mathcal{K}$ is assigned a semantics in terms of set-theoretic interpretations $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where $\Delta^{\mathcal{I}}$ is a non-empty *domain* and $\cdot^{\mathcal{I}}$ is the *interpretation function* that assigns an element in $\Delta^{\mathcal{I}}$ to each $a \in \mathbf{I}$, a subset of $\Delta^{\mathcal{I}}$ to each $C \in \mathbf{A}$ and a subset of $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ to each $R \in \mathbf{R}$.

TRILL allows the use of two different syntaxes used together or individually:

- RDF/XML
- TRILL syntax

RDF/XML syntax can be used by exploiting the predicate `owl_rdf/1`. For example:

```
owl_rdf('
<?xml version="1.0"?>

<!DOCTYPE rdf:RDF [
    <!ENTITY owl "http://www.w3.org/2002/07/owl#" >
    <!ENTITY xsd "http://www.w3.org/2001/XMLSchema#" >
    <!ENTITY rdfs "http://www.w3.org/2000/01/rdf-schema#" >
    <!ENTITY rdf "http://www.w3.org/1999/02/22-rdf-syntax-ns#" >
]>

<rdf:RDF xmlns="http://here.the.IRI.of.your.ontology#"
     xml:base="http://here.the.IRI.of.your.ontology"
     xmlns:rdf="http://www.w3.org/1999/02/22-rdf-syntax-ns#"
     xmlns:owl="http://www.w3.org/2002/07/owl#"
     xmlns:xsd="http://www.w3.org/2001/XMLSchema#"
     xmlns:rdfs="http://www.w3.org/2000/01/rdf-schema#">
    <owl:Ontology rdf:about="http://here.the.IRI.of.your.ontology"/>

    <!--
    Axioms
    -->

</rdf:RDF>
').
```

For a brief introduction on RDF/XML syntax see *RDF/XML syntax and tools* section below (Sec. 3.2).

Note that each single `owl_rdf/1` must be self contained and well formatted, it must start and end with `rdf:RDF` tag and contain all necessary declarations (namespaces, entities, ...).

An example of the combination of both syntaxes is shown the example `johnEmployee.pl`. It models that *john* is an *employee* and that employees are *workers*, which are in turn people (modeled by the concept *person*).

```
owl_rdf('<?xml version="1.0"?>
<rdf:RDF xmlns="http://example.foo#"
     xml:base="http://example.foo"
     xmlns:rdf="http://www.w3.org/1999/02/22-rdf-syntax-ns#"
     xmlns:owl="http://www.w3.org/2002/07/owl#"
     xmlns:xml="http://www.w3.org/XML/1998/namespace"
     xmlns:xsd="http://www.w3.org/2001/XMLSchema#"
     xmlns:rdfs="http://www.w3.org/2000/01/rdf-schema#">
    <owl:Ontology rdf:about="http://example.foo"/>

    <!-- Classes -->
    <owl:Class rdf:about="http://example.foo#worker">
        <rdfs:subClassOf rdf:resource="http://example.foo#person"/>
    </owl:Class>

</rdf:RDF>').

subClassOf('employee','worker').

owl_rdf('<?xml version="1.0"?>
<rdf:RDF xmlns="http://example.foo#"
     xml:base="http://example.foo"
     xmlns:rdf="http://www.w3.org/1999/02/22-rdf-syntax-ns#"
     xmlns:owl="http://www.w3.org/2002/07/owl#"
     xmlns:xml="http://www.w3.org/XML/1998/namespace"
     xmlns:xsd="http://www.w3.org/2001/XMLSchema#"
     xmlns:rdfs="http://www.w3.org/2000/01/rdf-schema#">
    <owl:Ontology rdf:about="http://example.foo"/>

    <!-- Individuals -->
    <owl:NamedIndividual rdf:about="http://example.foo#john">
        <rdf:type rdf:resource="http://example.foo#employee"/>
    </owl:NamedIndividual>
</rdf:RDF>').
```

3.1 TRILL Syntax

3.1.1 Declarations

TRILL syntax allows, as in standard OWL, the declaration of classes, properties, etc.

```
class("classIRI").
datatype("datatypeIRI").
objectProperty("objectPropertyIRI").
dataProperty("dataPropertyIRI").
annotationProperty("annotationPropertyIRI").
namedIndividual("individualIRI").
```

However, TRILL properly works also in their absence.

TRILL syntax allows also the declaration of aliases for namespaces by using the `kb_prefix/2` predicate.

```
kb_prefix("foo","http://example.foo#").
```

After this declaration, the prefix `foo` is available, thus, instead of `http://example.foo#john`, one can write `foo:john`. It is possible to define also an empty prefix as

```
kb_prefix("","http://example.foo#").
```

or as

```
kb_prefix([],"http://example.foo#").
```

In this way `http://example.foo#john` can be written only as `john`.

Note: Only one prefix per alias is allowed. Aliases defined in OWL/RDF part have the precedence, in case more than one prefix was assigned to the same alias, TRILL keeps only the first assignment.

3.1.2 Axioms

Axioms are modeled using the following predicates

```
subClassOf("subClass","superClass").
equivalentClasses([list,of,classes]).
disjointClasses([list,of,classes]).
disjointUnion([list,of,classes]).

subPropertyOf("subProperty","superProperty").
equivalentProperties([list,of,properties]).
propertyDomain("propertyIRI","domainIRI").
propertyRange("propertyIRI","rangeIRI").
transitiveProperty("propertyIRI").

sameIndividual([list,of,individuals]).
```

```
differentIndividuals([list,of,individuals]).

classAssertion("classIRI","individualIRI").
propertyAssertion("propertyIRI","subjectIRI","objectIRI").
annotationAssertion("annotationIRI",axiom,literal('value')).
```

For example, for asserting that *employee* is subclass of *worker* one can use

```
subClassOf(employee,worker).
```

while the assertion *worker* is equal to *workingman* can be defined as

```
equivalentClasses([worker,workingman]).
```

Annotation assertions can be defined, for example, as

```
annotationAssertion(foo:myAnnotation,
    subClassOf(employee,worker),'myValue').
```

In particular, an axiom can be annotated with a probability which defines the degree of belief in the truth of the axiom. See Section 4 for details.

Below, an example of an probabilistic axiom, following the TRILL syntax.

```
annotationAssertion('disponte:probability',
    subClassOf(employee,worker),literal('0.6')).
```

3.1.3 Concepts descriptions

Complex concepts can be defined using different operators.

Existential and universal quantifiers

```
someValuesFrom("propertyIRI","classIRI").
allValuesFrom("propertyIRI","classIRI").
```

Union and intersection of concepts

```
unionOf([list,of,classes]).
intersectionOf([list,of,classes]).
```

Cardinality descriptions

```
exactCardinality(cardinality,"propertyIRI").
exactCardinality(cardinality,"propertyIRI","classIRI").
maxCardinality(cardinality,"propertyIRI").
maxCardinality(cardinality,"propertyIRI","classIRI").
minCardinality(cardinality,"propertyIRI").
minCardinality(cardinality,"propertyIRI","classIRI").
```

Complement of a concept

```
complementOf("classIRI").
```

Nominal concept

```
oneOf([list,of,classes]).
```

For example, the class *workingman* is the intersection of *worker* with the union of *man* and *woman*. It can be defined as:

```
equivalentClasses([workingman,
    intersectionOf([worker,unionOf([man,woman])])]).
```

3.2 RDF/XML syntax and tools

As said before, TRILL is able to automatically translate RDF/XML knowledge bases when passed as a string using the preticate `owl_rdf/1`.

Consider the following axioms

```
classAssertion(Cat,fluffy)
subClassOf(Cat,Pet)
propertyAssertion(hasAnimal,kevin,fluffy)
```

The first axiom states that *fluffy* is a *Cat*. The second states that every *Cat* is also a *Pet*. The third states that the role *hasAnimal* links together *kevin* and *fluffy*.

RDF (Resource Descritpion Framework) is a standard W3C. See the syntax specification for more details. RDF is a standard XML-based used for representing knowledge by means of triples. A representations of the three axioms seen above is shown below.

```
<owl:NamedIndividual rdf:about="fluffy">
  <rdf:type rdf:resource="Cat"/>
</owl:NamedIndividual>

<owl:Class rdf:about="Cat">
  <rdfs:subClassOf rdf:resource="Pet"/>
</owl:Class>

<owl:ObjectProperty rdf:about="hasAnimal"/>
<owl:NamedIndividual rdf:about="kevin">
 <hasAnimal rdf:resource="fluffy"/>
</owl:NamedIndividual>
```

Annotations are assertable using an extension of RDF/XML. For example the annotated axiom below, defined using the TRILL sintax

```
annotationAssertion('disponte:probability',
    subClassOf('Cat','Pet'),literal('0.6')).
```

is modeled using RDF/XML syntax as

```
<owl:Class rdf:about="Cat">
 <rdfs:subClassOf rdf:resource="Pet"/>
</owl:Class>
<owl:Axiom>
 <disponte:probability rdf:datatype="&amp;xsd;decimal">
     0.6
 </disponte:probability>
 <owl:annotatedSource rdf:resource="Cat"/>
 <owl:annotatedTarget rdf:resource="Pet"/>
 <owl:annotatedProperty rdf:resource="&amp;rdfs;subClassOf"/>
</owl:Axiom>
```

If you define the annotated axiom in the RDF/XML part, the annotation must be declared in the knowledge base as follow

```
<!DOCTYPE rdf:RDF [
 ...
 <!ENTITY disponte "https://sites.google.com/a/unife.it/ml/disponte#" >
]>

<rdf:RDF
 ...
 xmlns:disponte="https://sites.google.com/a/unife.it/ml/disponte#"
 ...>

 ...
 <owl:AnnotationProperty rdf:about="&amp;disponte;probability"/>
 ...
</rdf:RDF>
```

There are many editors for developing knowledge bases.

4 Semantics

In the field of Probabilistic Logic Programming (PLP for short) many proposals have been presented. An effective and popular approach is the Distribution Semantics [12], which underlies many PLP languages such as PRISM [12, 13], Independent Choice Logic [10], Logic Programs with Annotated Disjunctions [17] and ProbLog [3]. Along this line, many reserchers proposed to combine probability theory with Description Logics (DLs for short) [8, 16]. DLs are at the basis of the Web Ontology Language (OWL for short), a family of knowledge representation formalisms used for modeling information of the Semantic Web

TRILL follows the DISPONTE [11, 18] semantics to compute the probability of queries. DISPONTE applies the distribution semantics [12] of probabilistic logic programming to DLs. A program following this semantics defines a probability distribution over normal logic programs called *worlds*. Then the distribution is extended to queries and the probability of a query is obtained by marginalizing the joint distribution of the query and the programs.

In DISPONTE, a *probabilistic knowledge base* $\mathcal{K}$ is a set of *certain axioms* or *probabilistic axioms* in which each axiom is independent evidence. Certain axioms take the form of regular DL axioms while probabilistic axioms are $p :: E$ where p is a real number in $[0, 1]$ and E is a DL axiom.

The idea of DISPONTE is to associate independent Boolean random variables to the probabilistic axioms. To obtain a *world*, we include every formula obtained from a certain axiom. For each probabilistic axiom, we decide whether to include it or not in w. A world therefore is a non probabilistic KB that can be assigned a semantics in the usual way. A query is entailed by a world if it is true in every model of the world.

The probability p can be interpreted as an *epistemic probability*, i.e., as the degree of our belief in axiom E. For example, a probabilistic concept membership axiom $p :: a : C$ means that we have degree of belief p in $C(a)$. A probabilistic concept inclusion axiom of the form $p :: C \sqsubseteq D$ represents our belief in the truth of $C \sqsubseteq D$ with probability p.

Formally, an *atomic choice* is a couple (E_i, k) where E_i is the ith probabilistic axiom and $k \in \{0, 1\}$. k indicates whether E_i is chosen to be included in a world ($k = 1$) or not ($k = 0$). A *composite choice* κ is a consistent set of atomic choices, i.e., $(E_i, k) \in \kappa, (E_i, m) \in \kappa$ implies $k = m$ (only one decision is taken for each formula). The probability of a composite choice κ is $P(\kappa) = \prod_{(E_i,1)\in\kappa} p_i \prod_{(E_i,0)\in\kappa}(1-p_i)$, where p_i is the probability associated with axiom E_i. A *selection* σ is a total composite choice, i.e., it contains an atomic choice (E_i, k) for every probabilistic axiom of the probabilistic KB. A selection σ identifies a theory w_σ called a *world* in this way: $w_\sigma = \mathcal{C} \cup \{E_i | (E_i, 1) \in \sigma\}$ where $\mathcal{C}$ is the set of certain axioms. Let us indicate with $\mathcal{S}_\mathcal{K}$ the set of all selections and with $\mathcal{W}_\mathcal{K}$ the set of all worlds. The probability of a world w_σ is $P(w_\sigma) = P(\sigma) = \prod_{(E_i,1)\in\sigma} p_i \prod_{(E_i,0)\in\sigma}(1 - p_i)$. $P(w_\sigma)$ is a probability distribution over worlds, i.e., $\sum_{w\in\mathcal{W}_\mathcal{K}} P(w) = 1$.

We can now assign probabilities to queries. Given a world w, the probability of a query Q is defined as $P(Q|w) = 1$ if $w \models Q$ and 0 otherwise. The probability of a query can be defined by marginalizing the joint probability of the query and the worlds, i.e. $P(Q) = \sum_{w\in\mathcal{W}_\mathcal{K}} P(Q, w) = \sum_{w\in\mathcal{W}_\mathcal{K}} P(Q|w)p(w) = \sum_{w\in\mathcal{W}_\mathcal{K}:w\models Q} P(w)$.

Consider the following KB, inspired by the `people+pets` ontology [9]:

$$0.5 \ :: \ \exists hasAnimal.Pet \sqsubseteq NatureLover \qquad 0.6 \ :: \ Cat \sqsubseteq Pet$$
$$(kevin, tom) : hasAnimal \qquad (kevin, fluffy) : hasAnimal \qquad tom : Cat \qquad fluffy : Cat$$

The KB indicates that the individuals that own an animal which is a pet are nature lovers with a 50% probability and that *kevin* has the animals *fluffy* and *tom*. Fluffy and *tom* are cats and cats are pets with probability 60%. We associate a Boolean variable to each axiom as follow $F_1 = \exists hasAnimal.Pet \sqsubseteq NatureLover$, $F_2 = (kevin, fluffy) : hasAnimal$, $F_3 = (kevin, tom) : hasAnimal$, $F_4 = fluffy : Cat$, $F_5 = tom : Cat$ and $F_6 = Cat \sqsubseteq Pet$.

The KB has four worlds and the query axiom $Q = kevin : NatureLover$ is true in one of them, the one corresponding to the selection $\{(F_1, 1), (F_2, 1)\}$. The probability of the query is $P(Q) = 0.5 \cdot 0.6 = 0.3$.

Sometimes we have to combine knowledge from multiple, untrusted sources, each one with a different reliability. Consider a KB similar to the one of Example 4 but where we have a single cat, *fluffy*.

$$\exists hasAnimal.Pet \sqsubseteq NatureLover \qquad (kevin, fluffy) : hasAnimal \qquad Cat \sqsubseteq Pet$$

and there are two sources of information with different reliability that provide the information that *fluffy* is a cat. On one source the user has a degree of belief of 0.4, i.e., he thinks it is correct with a 40% probability, while on the other source he has a degree of belief 0.3. The user can reason on this knowledge by adding the following statements to his KB:

$$0.4 \ :: \ fluffy : Cat \qquad 0.3 \ :: \ fluffy : Cat$$

The two statements represent independent evidence on *fluffy* being a cat. We associate F_1 (F_2) to the first (second) probabilistic axiom.

The query axiom $Q = kevin : NatureLover$ is true in 3 out of the 4 worlds, those corresponding to the selections $\{\{(F_1, 1), (F_2, 1)\}, \{(F_1, 1), (F_2, 0)\}, \{(F_1, 0), (F_2, 1)\}\}$. So $P(Q) = 0.4 \cdot 0.3 + 0.4 \cdot 0.7 + 0.6 \cdot 0.3 = 0.58$. This is reasonable if the two sources can be considered as independent. In fact, the probability comes from the disjunction of two independent Boolean random variables with probabilities respectively 0.4 and 0.3: $P(Q) = P(X_1 \vee X_2) = P(X_1) + P(X_2) - P(X_1 \wedge X_2) = P(X_1) + P(X_2) - P(X_1)P(X_2) = 0.4 + 0.3 - 0.4 \cdot 0.3 = 0.58$

5 Inference

Traditionally, a reasoning algorithm decides whether an axiom is entailed or not by a KB by refutation: the axiom E is entailed if $\neg E$ has no model in the KB. Besides deciding whether an axiom is entailed by a KB, we want to find also explanations for the axiom.

The problem of finding explanations for a query has been investigated by various authors [14, 7, 6, 5, 4, 18]. It was called *axiom pinpointing* in [14] and considered as a non-standard reasoning service useful for tracing derivations and debugging ontologies. In particular, in [14] the authors define *minimal axiom sets* (*MinAs* for short). [MinA] Let $\mathcal{K}$ be a knowledge base and Q an axiom that follows from it, i.e., $\mathcal{K} \models Q$. We call a set $M \subseteq \mathcal{K}$ a *minimal axiom set* or *MinA* for Q in $\mathcal{K}$ if $M \models Q$ and it is minimal w.r.t. set inclusion. The problem of enumerating all MinAs is called MIN-A-ENUM. ALL-MINAS$(Q, \mathcal{K})$ is the set of all MinAs for query Q in knowledge base $\mathcal{K}$.

A *tableau* is a graph where each node represents an individual a and is labeled with the set of concepts $\mathcal{L}(a)$ it belongs to. Each edge $\langle a, b\rangle$ in the graph is labeled with the set of roles to which the couple (a, b) belongs. Then, a set of consistency preserving tableau expansion rules are repeatedly applied until a clash (i.e., a contradiction) is detected or a clash-free graph is found to which no more rules are applicable. A clash is for example a couple (C, a) where C and $\neg C$ are present in the label of a node, i.e. $C, \neg C \subseteq \mathcal{L}(a)$.

Some expansion rules are non-deterministic, i.e., they generate a finite set of tableaux. Thus the algorithm keeps a set of tableaux that is consistent if there is any tableau in it that is consistent, i.e., that is clash-free. Each time a clash is detected in a tableau G, the algorithm stops applying rules to G. Once every tableau in T contains a clash or no more expansion rules can be applied to it, the algorithm terminates. If all the tableaux in the final set T contain a clash, the algorithm returns unsatisfiable as no model can be found. Otherwise, any one clash-free completion graph in T represents a possible model for the concept and the algorithm returns satisfiable.

MIN-A-ENUM is required to answer queries to KBs following the DISPONTE semantics. To compute the probability of a query, the explanations must be made mutually exclusive, so that the probability of each individual explanation is computed and summed with the others. To do that we assign independent Boolean random variables to the axioms contained in the explanations and defining the Disjunctive Normal Form (DNF) Boolean formula f_K which models the set of explanations. Thus $f_K(\mathbf{X}) = \bigvee_{\kappa \in K} \bigwedge_{(E_i,1)} X_i \bigwedge_{(E_i,0)} \overline{X_i}$ where $\mathbf{X} = \{X_i|(E_i,k) \in \kappa, \kappa \in K\}$ is the set of Boolean random variables. We can now translate f_K to a Binary Decision Diagram (BDD), from which we can compute the probability of the query with a dynamic programming algorithm that is linear in the size of the BDD.

To run a query, you can simply load the Prolog file, for example `peoplePets.pl`, as

```
?- [peoplePets].
```

5.1 Files

The `pack/trill/prolog/examples` folder in SWI-Prolog home contains some example programs. The `pack/trill/doc` folder in SWI-Prolog home contains this manual in latex, html and pdf.

6 License

TRILL follows the Artistic License 2.0 that you can find in TRILL root folder. The copyright is by Riccardo Zese.

The library Thea at the basis of the translation module is available under the GNU/GPL license.

The library CUDD for manipulating BDDs has the following license:

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