Arbitrage

Arbitrage is the use of discrepancies in currency exchange rates to transform one unit of a currency into more than one unit of the same currency. For example, suppose that 1 U.S. dollar buys 49 Indian rupees, 1 Indian rupee buys 2 Japanese yen, and 1 Japanese yen buys 0:0107 U.S. dollars. Then, by converting currencies, a trader can start with 1 U.S.dollar and buy 49*2*0.0107 = 1:0486 U.S.dollars, thus turning a profit of 4:86 percent. Suppose that we are given n currencies $c_1, c_2, \ldots, c_n$ and an $n \times n$ table R of exchange rates, such that one unit of currency c_i buys $R[i, j]$ units of currency c_j.

a. Give an efficient algorithm to determine whether or not there exists a sequence of currencies $\langle c_{i1}, c_{i2}, \ldots, c_{ik} \rangle$ such that $R[i_1, i_2].R[i_2, i_3]\ldots R[i_{k-1}, i_k].R[i_k, i_1] > 1$.

Analyze the running time of your algorithm.

b. Give an efficient algorithm to print out such a sequence if one exists. Analyze the running time of your algorithm.