Computer Architecture: Computer Arithmetic: Fixed-Point & FP IEEE 754

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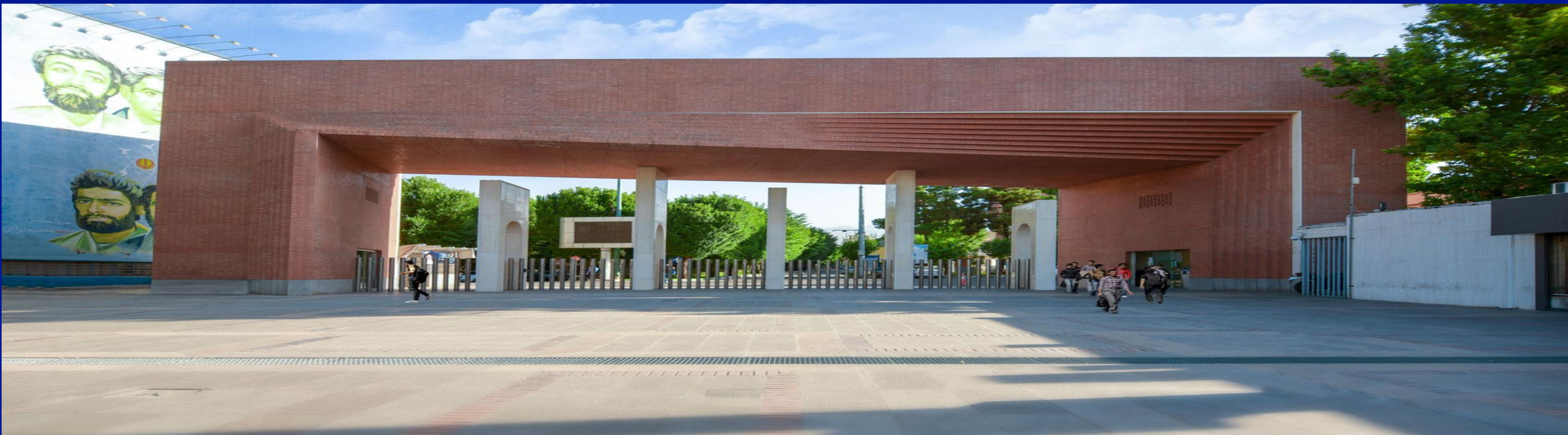

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- Some Parts (text & figures) of this Lecture adopted from following:
  - D.A. Patterson and J.L. Hennessy, "Computer Organization and Design: the Hardware/Software Interface" (MIPS), 6th Edition, 2020.
  - J.L. Hennessy and D.A. Patterson, "Computer Architecture: A Quantitative Approach", 6th Edition, Nov. 2017.
  - "Intro to Computer Architecture" handouts, by Prof. Hoe, CMU, Spring 2009.
  - "Computer Architecture & Engineering" handouts, by Prof. Kubiatowicz, UC Berkeley, Spring 2004.
  - "Intro to Computer Architecture" handouts, by Prof. Hoe, UWisc, Spring 2021.
  - "Computer Arch I" handouts, by Prof. Garzarán, UIUC, Spring 2009.

Topics Covered in This Lecture

- Fixed Point
- Floating Point

Real Numbers in Computers

- Fixed-Point Representation
  - Example: $d_{23}d_{22}\ldots d_1d_0.f_0f_1f_2f_3f_4f_5f_6f_7$
  - 24-bit: integer bits
  - 8-bit: fraction bits
- Application
  - Used in CPUs with no floating-point unit
    - Embedded microprocessors and microcontrollers
  - Digital Signal Processing (DSP) applications

Real Numbers in Computers

- Fixed-Point Representation
  - Pros
    - Simple hardware
    - Fast computation
  - Cons
    - Low precision
    - Small range

Real Numbers in Computers

- Floating-Point Representation
  - Scientific notation in base 2
  - $1.xxxxxx_{two} * 2^{yyyy}$

Floating-Point Notation

- FP Notation Consists of:
  - Fraction (F): 23 bits
  - Exponent (E): 8 bits
  - Sign bit (S)
  - Also called, single precision floating-point
- $N = (-1)^S * F * 2^E$

31	30	…	24	23	22	21	…	1	0
S	Exponent				Fraction				

Floating-Point Notation (cont.)

- Pros (compared to fixed-point)
  - Very Wide Range
  - More precision bits
- Cons (compared to fixed-point)
  - Arithmetic operation more complicated
  - HW more complicated
  - More time-consuming

31	30	…	24	23	22	21	…	1	0
S	Exponent				Fraction				

Floating-Point Notation (cont.)

- Precision versus Range
  - Wider range ➔ less precision?
  - More precision ➔ smaller range?

31	30	…	24	23	22	21	…	1	0
S	Exponent				Fraction				

Floating-Point Notation (cont.)

- IEEE 754 FP Standard
  - $N = (-1)^S * (1 + F) * 2^E$
  - Significand: 1 + F
  - Fraction: F
  - Used in MIPS and most microprocessors

31	30	…	24	23	22	21	…	1	0
S	Exponent				Fraction				

Floating-Point Notation (cont.)

- Overflow:
  - Can we have overflow in FP notation?
    - Exponent too large to fit in “Exponent” field
- Underflow:
  - Non-zero fraction so small to represent
    - Negative exponent too large to fit

31	30	…	24	23	22	21	…	1	0
S	Exponent				Fraction				

Floating-Point Notation (cont.)

- Biased-Notation in Exponent Field
  - Used in IEEE 754 FP Standard
    - In order to compare FP numbers faster
  - Uses a bias of 127 in single-precision FP
    - $N = (-1)^S * (1 + F) * 2^{(E\text{-}bias)}$

Floating-Point Notation (cont.)

- Biased-Notation in Exponent Field
  - Uses a bias of 127 in single-precision FP
    - $N = (-1)^S * (1 + F) * 2^{(E\text{-}bias)}$
    - 0 reserved
    - (-126) represented by -126+127 = 1
    - (-1) represented by -1+127 = 126
    - (0) represented by 0+127 = 127
    - (+1) represented by 1+127 = 128
    - (+127) represented by 127+127 = 254
    - 255 reserved

Floating-Point Notation (cont.)

- Double-Precision Floating-Point
  - Uses two words
  - Reduces chances of overflow & underflow
  - Format
    - Fraction (F): 52 bits
    - Exponent (E): 11 bits
    - Sign bit (S)
  - Uses a bias of 1023 in double-precision FP

Floating-Point Notation (cont.)

- Double-Precision Floating-Point
  - Fraction (F): 52 bits
  - Exponent (E): 11 bits
  - Sign bit (S)

<table>
<tr><td>31</td><td>30</td><td>…</td><td>21</td><td>20</td><td>19</td><td>18</td><td>…</td><td>1</td><td>0</td></tr>
<tr><td>S</td><td colspan="4">Exponent</td><td colspan="5">Fraction</td></tr>
</table>

<table>
<tr><td>31</td><td>30</td><td>…</td><td>24</td><td>23</td><td>22</td><td>21</td><td>…</td><td>1</td><td>0</td></tr>
<tr><td colspan="10">Fraction (continued)</td></tr>
</table>

Floating-Point Notation (cont.)

Single Precision		Double Precision		Object Represented
Exponent	Fraction	Exponent	Fraction	
0	0	0	0	0
0	Nonzero	0	Nonzero	Denormalized
1-254	Anything	1-2046	Anything	FP No
255	0	2047	0	Infinity
255	Nonzero	2047	Nonzero	NaN

Floating-Point Notation (cont.)

- $N = (-1)^S * (1 + F) * 2^E$
- Questions on Single Precision FP:
  - Smallest positive number?
    - $1.0000\ 0000\ 0000\ 0000\ 0000\ 000_{two} * 2^{-126}$
  - Smallest absolute negative number?
    - $-1.0000\ 0000\ 0000\ 0000\ 0000\ 000_{two} * 2^{-126}$

31	30	…	24	23	22	21	…	1	0
S	Exponent				Fraction				

Floating-Point Notation (cont.)

- $N = (-1)^S * (1 + F) * 2^E$
- Questions on Single Precision FP:
  - Largest positive number?
    - $1.1111\ 1111\ 1111\ 1111\ 1111\ 111_{two} * 2^{+127}$
  - Largest absolute negative number?
    - $-1.1111\ 1111\ 1111\ 1111\ 1111\ 111_{two} * 2^{+127}$

31	30	…	24	23	22	21	…	1	0
S	Exponent				Fraction				

Floating-Point Notation (cont.)

- Denormalized Numbers
  - Smallest positive normalized number

    $= 1.0000\ 0000\ 0000\ 0000\ 0000\ 000_{two} * 2^{-126}$

    $= 1._{two} * 2^{-126}$
  - Smaller positive numbers using exponent 0

    $= 0.0000\ 0000\ 0000\ 0000\ 0000\ 001_{two} * 2^{-126}$

    $= 1._{two} * 2^{-149}$

Floating-Point Notation (cont.)

- Practice:
  - Represent following number in IEEE 754 single-precision FP
    - (-0.75)

$$= -\tfrac{3}{4} = -3 * 2^{-2} = -11_{two} * 2^{-2} = -0.11_{two}$$

$$= -1.1_{two} * 2^{-1} = -1.1_{two} * 2^{127-1} = -1.1_{two} * 2^{126}$$

31	30	…	24	23	22	21	…	1	0
S	Exponent				Fraction				
1	01111110				10000000000000000000000				

Floating-Point Notation (cont.)

- FP Addition
  - Example:
    - $1.000_{two} * 2^{-1} + -1.110_{two} * 2^{-2}$

$$
\begin{aligned}
& \quad 1.0000_{two} * 2^{-1} \\
& + -0.1110_{two} * 2^{-1} \\
& = \ 0.0010 * 2^{-1} \\
& = \ 1.0 * 2^{-4}
\end{aligned}
$$

Floating-Point Notation (cont.)

- Another Practice:
  - Convert (7.75) in IEEE 754 single-precision FP

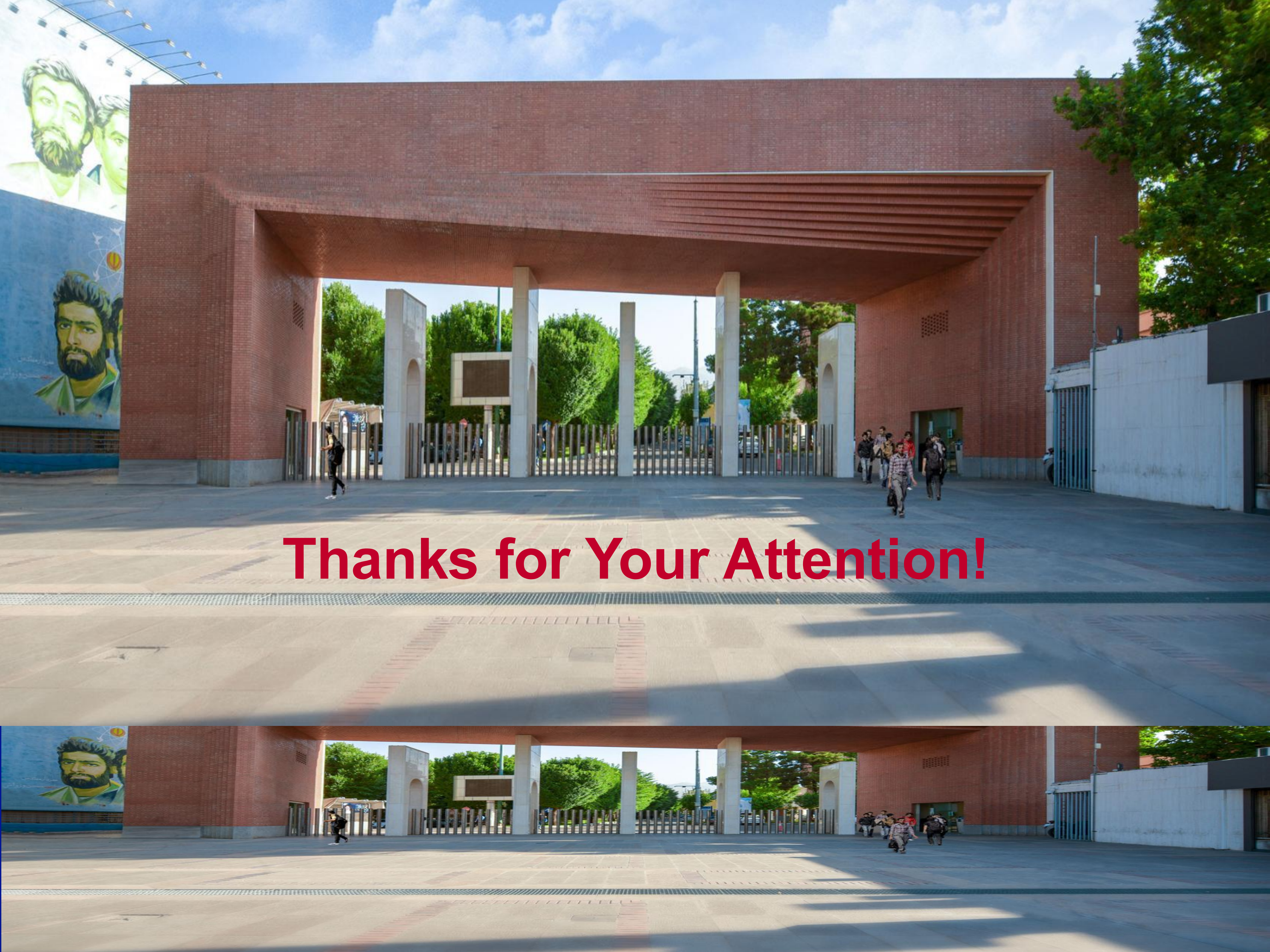
$$= 7 + ¾ = 111_{two} * 2^{0} + 11_{two} * 2^{-2} =$$

$$= 1.11_{two} * 2^{2} + 0.0011_{two} * 2^{2}$$

$$= 1.1111_{two} * 2^{2}$$

$$= 1.1111_{two} * 2^{2+127} = 1.1111_{two} * 2^{129}$$

31	30	…	24	23	22	21	…	1	0
S	Exponent				Fraction				
0	10000001				11110000000000000000000				



Thanks for Your Attention!