Model Following Control Applied to a Three Dimensional Overhead Crane

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The crane System

Real Crane

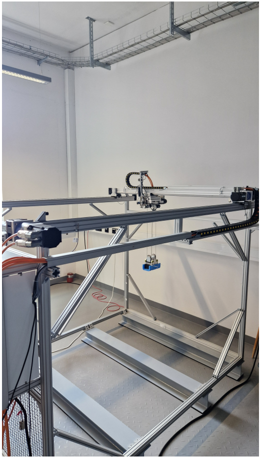

Figure 1: Real 3-dim overhead Crane

The crane System

Scheme

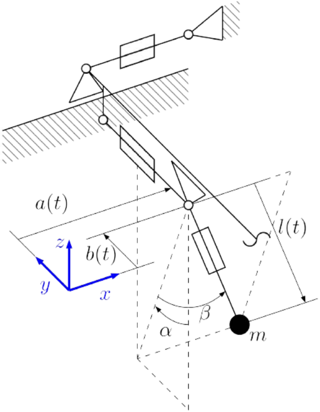

Figure 2: Schematics of three dimensional overhead crane [1]

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Control Goals

The goals of this work are :

- Implement a Model Following Control (MFC) to make the crane's load track set points and trajectories
- Highlight the attenuation of the peaking phenomenon by the MFC

Outline

The Trolley

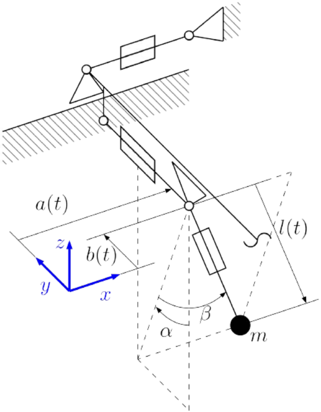

Figure 3: Schematics of three dimensional overhead crane [1]

First, the crane dynamic equations are computed as in [1]. The trolley position $\boldsymbol{r}_t(t)$ is considered as a time-dependent function given by:

$$\mathbf{r}_t(t) = \begin{bmatrix} a(t) & b(t) & 0 \end{bmatrix}^T \quad (1)$$

where $a(t)$ and $b(t)$ are the positions of the trolley in the x and y directions.

System output

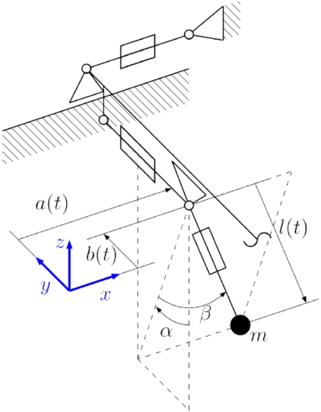

Figure 4: Schematics of three dimensional overhead crane [1]

The position of the load, which will be the output of the system as $\mathbf{y}(t)$ given by :

$$\mathbf{y}(t) = \begin{bmatrix} a(t) + l(t)S_\beta \\ b(t) + l(t)C_\beta S_\alpha \\ -l(t)C_\beta C_\alpha \end{bmatrix} \qquad (2)$$

With the notations $S_\gamma = sin(\gamma)$ and $C_\gamma = cos(\gamma)$.

Dynamic Equation

We define the generalized coordinates vector $\boldsymbol{\theta} = [\alpha\ \beta]^T$ to get the Newton-Euler equation of the load system.
First let's note $\boldsymbol{J}_l(t)$, the jacobian of the crane over the generalized coordinates :

$$\boldsymbol{J}_l(t) = \frac{\partial \boldsymbol{y}(t)}{\partial \boldsymbol{\theta}} = \begin{bmatrix} 0 & l(t)C_\beta \\ l(t)C_\beta C_\alpha & -l(t)S_\beta S_\alpha \\ l(t)C_\beta S_\alpha & l(t)S_\beta C_\alpha \end{bmatrix} \tag{3}$$

Dynamic Equation

The velocity and acceleration of the load are given by :

$$\dot{\boldsymbol{y}} = \boldsymbol{J}_l(t)\dot{\theta} + \underbrace{\left.\frac{\partial \boldsymbol{y}}{\partial t}\right\|_{\theta=cst}}_{\bar{\dot{\boldsymbol{y}}}(t)} \tag{4}$$

$$\ddot{\boldsymbol{y}} = \boldsymbol{J}_l(t)\ddot{\theta} + \underbrace{\frac{d\boldsymbol{J}_l(t)}{dt} \cdot \dot{\boldsymbol{\theta}} + \frac{d\bar{\dot{\boldsymbol{y}}}(t)}{dt}}_{\bar{\ddot{\boldsymbol{y}}}(t)} \tag{5}$$

Crane Model Overview

After some steps and by applying the Newton-Euler equation, the system reads :

$$\boldsymbol{J}_l(t)^T \boldsymbol{J}_l(t) \cdot \ddot{\boldsymbol{\theta}} = -\boldsymbol{J}_l(t)^T \, \ddot{\bar{\boldsymbol{y}}}(t) \; + \; \boldsymbol{J}_l(t)^T \boldsymbol{g} \tag{6}$$

Where $\boldsymbol{g} = \begin{bmatrix} 0 & 0 & -g \end{bmatrix}^T$ denotes the gravity vector.

Dynamic Equation

The input is be the same as in the simulation and the real crane experimentation, they will directly drive the $\left[\ddot{a}(t) \quad \ddot{b}(t) \quad \ddot{l}(t)\right]$ coordinates. Which gives us the complete dynamics of the crane plant system in the form of a non-linear vectorial second order differential equation [1] :

$$\begin{cases} \ddot{a}(t) &= u_1 \\ \ddot{b}(t) &= u_2 \\ \ddot{l}(t) &= u_3 \\ \ddot{\alpha}(t) &= \frac{1}{l(t)C_\beta}\left(2\dot{\alpha}\dot{\beta}S_\beta l(t) - 2\dot{l}(t)\dot{\alpha}C_\beta - S_\alpha g - C_\alpha u_2\right) \\ \ddot{\beta}(t) &= \frac{1}{l(t)}\left(-C_\alpha S_\beta g - S_\beta C_\beta \dot{\alpha}^2 l(t) - 2\dot{l}(t)\dot{\beta} + S_\beta S_\alpha u_2 - C_\beta u_1\right) \end{cases} \tag{7}$$

and the output : $\mathbf{y}(t)$

Dynamic Equation

We can now write this system into the normal form :

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u}$$
$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x})$$

where :

$$\begin{aligned}\boldsymbol{x}(t) &= (x_i(t))_{1\leq i\leq 10} \\ &= \begin{pmatrix} a(t) & \dot{a}(t) & b(t) & \dot{b}(t) & l(t) & \dot{l}(t) & \alpha(t) & \dot{\alpha}(t) & \beta(t) & \dot{\beta}(t)\end{pmatrix}^T\end{aligned}$$

Denotes the state vector
It is required for a system to be in Byrnes-Isidori to apply MFC. Exact feedback linearization is used in this MIMO system.

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Dynamic Extension

The Feedback linearization applied directly to the crane equation has internal dynamics. One solution is to artificially increase the relative degree of y_3 by implementing a dynamic extension as in [2] by introducing a virtual control input that reads :

$$\ddot{\nu} = w_3 \tag{8}$$

$$u_3 = \psi(\boldsymbol{x}, \nu) \tag{9}$$

u_3 is computed so that $\forall t \geq 0 \ , \quad \ddot{y}_3(t) - \nu(t) = 0$

Feedback Linearization

- With our crane system, the relative degrees of our outputs $[y_1, y_2, y_3]^T$ are respectively $[2, 2, 2]^T$, which leads to internal dynamics.
- One way to address this issue is to artificially increase the relative degree of y_3 by applying the previous dynamical extension that has in our case the propriety ti increase the relative degrees to $[4, 4, 4]^T$.

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Feedback Linearization

Now it can be shown that by applying the following transformation to each output y_i for $i \in \{1, 2, 3\}$ of the system,

$$\boldsymbol{\xi}_i = \boldsymbol{T}_i(\boldsymbol{x}) = \begin{bmatrix} y_i \\ \dot{y}_i \\ \ddot{y}_i \\ y_i^{(3)} \end{bmatrix} \tag{10}$$

The system is divided into 3 decoupled subsystems in Byrnes-Isidori form that read :

$$\forall i \in \{1, 2, 3\} : \begin{cases} \dot{\boldsymbol{\xi}}_i = \boldsymbol{A}\boldsymbol{\xi}_i + \boldsymbol{B}\big(a_i(\boldsymbol{\xi}) + b_i(\boldsymbol{\xi})u\big) \\ \boldsymbol{y}_i = \boldsymbol{C}\boldsymbol{\xi}_i \end{cases} \tag{11}$$

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Outline

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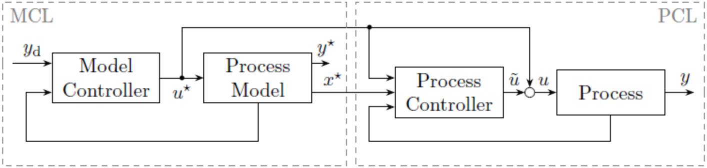
Control Loop

Figure 5: Model Following control block diagram with model control loop process and control loop [4]



General Equation

Consider the flat system :

$$\begin{cases} \dot{\xi} = A\xi + B\big(a(\xi) + b(\xi)u + \Delta(\xi, t)\big) \\ y = C\xi \end{cases} \tag{12}$$

where and $\xi(t) \in \mathbb{D}_\xi \subseteq \mathbb{R}^n$ denote the states. $\Delta : \mathbb{D}_\xi \times \mathbb{R}^+ \to \mathbb{R}$ are the perturbations.
The output is $y(t) \in \mathbb{R}$ and $u(t) \in \mathbb{R}$ denotes the input. The relative degree is $n \geq 1$.

Remark

Here we consider a SISO system. That doesn't matter as the crane's 3 outputs are decoupled. The control loop can be separated into 3 distinct SISO control loops of dimension 4.

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General Equation

The matrices A, B, and C are given by:

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \in \mathbb{R}^{n\times n}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in \mathbb{R}^{n},$$

$$C = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \in \mathbb{R}^{1\times n}$$

Outline

MCL Dynamics

An analysis of the control loop dynamics can be carried out in two stages. First, we consider the Model Control Loop (MCL), which is a replica of system 12, excluding the perturbations Δ, and is described by:

$$\dot{\xi}^* = A\xi^* + B\big(a(\xi^*) + b(\xi^*)u^*\big) \tag{13}$$

Open Loop dynamics

We write : $u = \tilde{u} + u^*$, where u^* : the output of the MCL and $\tilde{u}$: the output of the process control loop
The error states are defined as $\tilde{\xi} = \xi - \xi^*$, and the open-loop dynamics of the MFC are given by:

$$\dot{\xi}^* = A\xi^* + B\left(a(\xi^*) + b(\xi^*)u^*\right) \tag{14}$$

$$\dot{\tilde{\xi}} = A\tilde{\xi} + B\left(\tilde{a}(\xi^*, \tilde{\xi}, u^*) + b(\xi^* + \tilde{\xi})\tilde{u} + \Delta(\xi^* + \tilde{\xi}, t)\right) \tag{15}$$

where

$$\tilde{a}(\xi^*, \tilde{\xi}, u^*) = a(\xi^* + \tilde{\xi}) - a(\xi^*) + \left(b(\xi^* + \tilde{\xi}) - b(\xi^*)\right) u^* \tag{16}$$

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Feedback Linearization

Ensure closed-loop stability : a feedback linearization law is applied to the MCL defined by:

$$u^* = -\frac{a(\xi^*) + v^*}{b(\xi^*)} \tag{17}$$

The design of v^* is based on the error dynamics within the MCL : $\tilde{\xi}^* = \xi^* - \xi_d$ representing the deviation between the model and desired states. The associated error dynamics are given by:

$$\dot{\tilde{\xi}}^* = A\tilde{\xi}^* + B(v^* - y_d^{(r)}) \tag{18}$$

Closed Loop Dynamics

The new input v^* is chosen as:

$$v^* = y_d^{(r)} + K\tilde{\xi}^* \tag{19}$$

where $K = [-\alpha_0, -\alpha_1, \ldots, -\alpha_{r-1}] \in \mathbb{R}^{1\times r}$ is selected such that the matrix $A + BK$ is Hurwitz. The closed-loop dynamics of the MCL become:

$$\dot{\tilde{\xi}}^* = (A + BK)\tilde{\xi}^* \tag{20}$$

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Introduction

The process control loop is design to counter the perturbations and model uncertainties. The design method to assure robustness is the high-control gain approach. The desired output trajectory $y_d(t) \in \mathbb{R}$ is assumed to be at least n-times continuously differentiable. The desired external states $\xi_d(t) \in \mathbb{D}_{\xi_d} \subseteq \mathbb{R}^n$ are generated by

$$\dot{\xi}_d = A\xi_d + By_d^{(n)} \tag{21}$$

Open Loop Dynamics

The open-loop error dynamics :

$$\dot{\tilde{\xi}} = A\tilde{\xi} + B\left(\tilde{a}(\xi^*, \tilde{\xi}, u^*) + b(\tilde{\xi}^* + \tilde{\xi})\tilde{u} + \Delta(\tilde{\xi}^* + \tilde{\xi}, t)\right) \tag{22}$$

The design of the process controller will also be a feedback linearizing control law :

$$\tilde{u} = \frac{-\tilde{a}(\xi^*, \tilde{\xi}, u^*) + \tilde{K}\tilde{\xi}}{b(\xi^* + \xi)} \tag{23}$$

where : $\tilde{K} = KD^{-1}\varepsilon^{-1}$ with $D = \text{diag}(\varepsilon^n, \varepsilon^{n-1}, \ldots, 1)$ and $0 < \varepsilon < 1$ is a time scaling parameter.

Overall Dynamics

We proceed the time scaling change of variable :

$$\zeta = D^{-1}\tilde{\xi} \tag{24}$$

The closed-loop overall dynamics of the MFC scheme for set-point and trajectory tracking are given by

$$\dot{\tilde{\xi}}^* = (A + BK)\tilde{\xi}^* \tag{25}$$

$$\varepsilon\dot{\zeta} = (A + BK)\zeta + \varepsilon B\left(\Delta(\xi_d + \tilde{\xi}^* + D\zeta, t)\right). \tag{26}$$

Process Control Loop

- If $A + BK$ is Hurwitz, the eigenvalues of the error dynamics (without time-scaling) can be shifted arbitrarily far to the left of the imaginary axis as $\epsilon \to 0$.
- Theoretically, the error dynamics can be made to converge arbitrarily quickly.
- Achieving this may necessitate a significantly large control effort.
- A small ϵ enhances robustness against perturbations $\Delta(\xi, t)$. One can with this assumption :

$$|\Delta(\xi, t)| \leq \delta + L_\Delta \|\xi\|_2$$

  Find a condition on ϵ to encounter the perturbations.
- Additionally, a small ϵ accelerates the error dynamics of the PCL.

A Simpler Design

[3] showed a simpler architecture of the MFC, which includes the following key points:

- The feedback linearization control law is given by: $u = \frac{-a(\xi)+y_d^{(n)}+v}{b(\xi)}$ where $v = v^* + \tilde{v}$.
- The dynamics of the open loop are described by:

$$\dot{\xi} = A\xi + B(y_d^{(n)} + v) + \Delta(\xi, t)$$

- The nominal model of the external dynamics is given by the integrator chain:

$$\dot{\xi}^* = A\xi^* + B(y_d^{(n_\xi)} + v^*)$$

  with the initial state $\xi^*(0) = \xi_0^*$, and the error dynamics:

$$\dot{\tilde{\xi}} = A\tilde{\xi} + B(\tilde{v} + \Delta(\xi, \eta, t))$$

A Simpler Design

[3] continued:

- The dynamics of the open-loop MFC system include:

$$\dot{\tilde{\xi}}^* = A\tilde{\xi}^* + Bv^* \tag{27}$$

$$\dot{\tilde{\xi}} = A\tilde{\xi} + B(\tilde{v} + \Delta(\xi, t)) \tag{28}$$

- One can rewrite the control loop as follows :

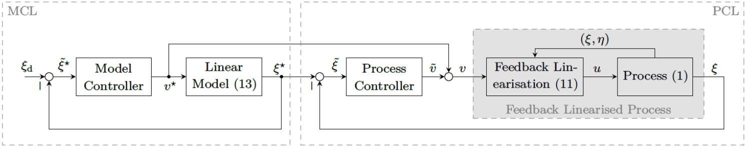
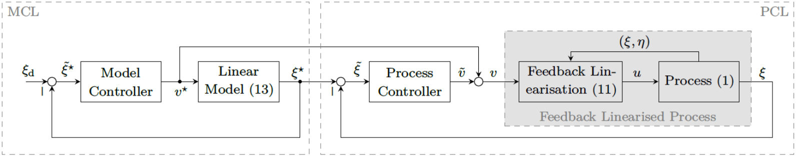
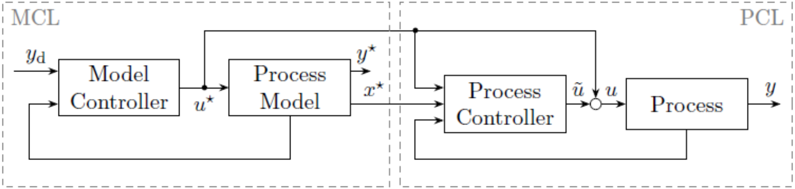


Figure 6: Block-diagram of the MFC architecture with linear model of the feedback linearized process [3]

Control Loops Comparison





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High Gain Control

The High Gain Control law is formulated as:

$$u = \frac{-a(\xi) + K_\epsilon \xi}{b(\xi)} \tag{29}$$

- With only one loop in the control. The slower motion of the MCL are not present and the PCL alone try to stabilize the dynamics.
- This control law is designed to aggressively reduce the error, leading to rapid convergence to the desired state.

Peaking Phenomenon

However, it is clear that we have peaking with this method, which is characterized by:

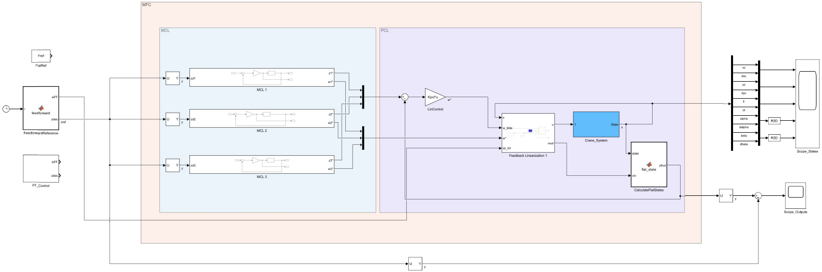
Peaking Phenomenon

Peaking refers to large transient peaks in control signals or state variables, caused by high gain amplifying initial errors or disturbances. This can lead to:

- Large temporary deviations in system response.
- Potential saturation of actuators, resulting in performance degradation or instability.
- Challenges in systems with fast dynamics or constrained states.

Outline

Implementation on Matlab



Set-Point Tracking in the Experiment

Here is the desired trajectory :

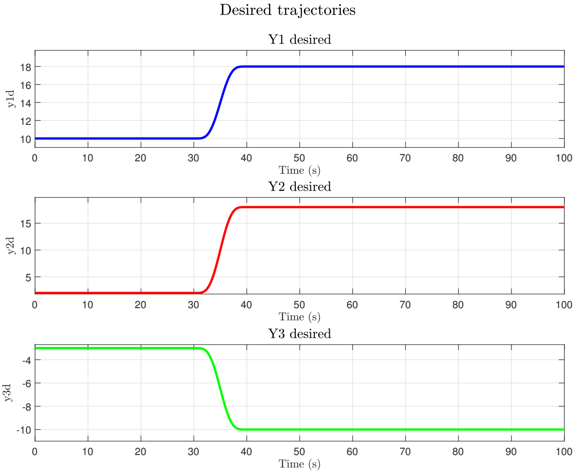


Figure 8: Desired Trajectories of the Load

Simulation Results

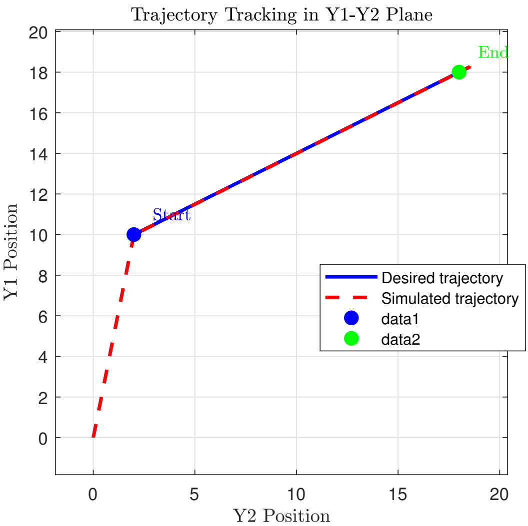


Figure 9: Difference between CI desired states and Model States

Simulation Results

High Gain

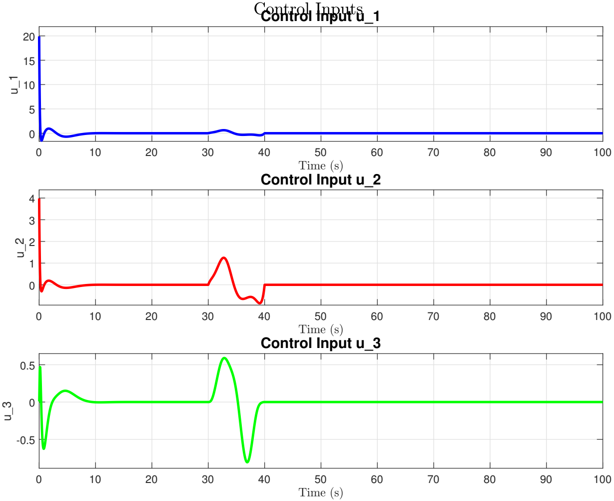


Figure 10: Input for Set-point tracking High gain control

Simulation Results

MFC

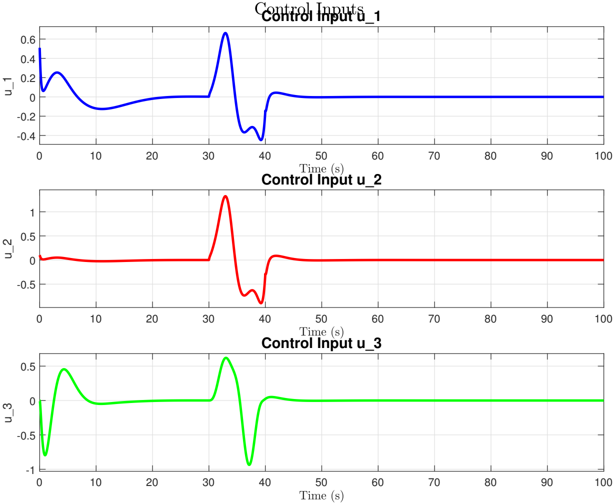


Figure 11: Input for Set-point tracking MFC

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Presentation

The Trolley and the Load

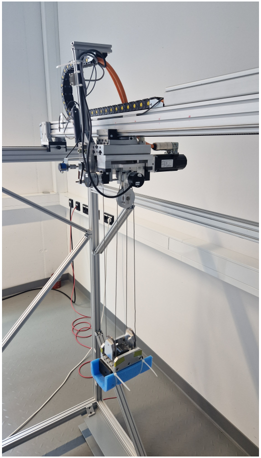

Figure 12: Zoom on the crane's trolley

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X and Y motors

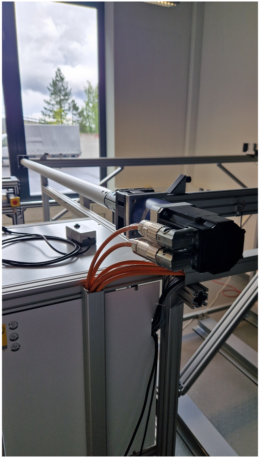

Figure 13: Zoom on X motor

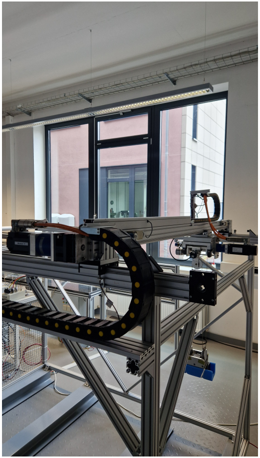

Figure 14: Zoom on Y motor

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Results

Conclusion

The main points to remember about Model Following Control (MFC) are:

- This control strategy can be applied to a wide range of real-world systems with input-to-state stable dynamics.
- The design proposed by [3] provides a simple implementation compared to other techniques, which are less direct.
- MFC can mitigate the peaking phenomenon, providing the advantages of High-Gain control without its drawbacks.
- Perturbations can be easily counteracted by tuning ϵ, provided their bounds are known.

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Thank you!

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