Assignment 4

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Abstract—**This document solves the isosceles triangle problem.**

Download all latex-tikz codes from

https://github.com/Vaibhav11002/EE5609/tree/master/Assignment_4

1 Problem

Prove that sides opposite to equal angles of a triangle are equal.

2 Solution

Let's consider $\triangle ABC$ where $\angle ABC = \angle ACB = \theta$,

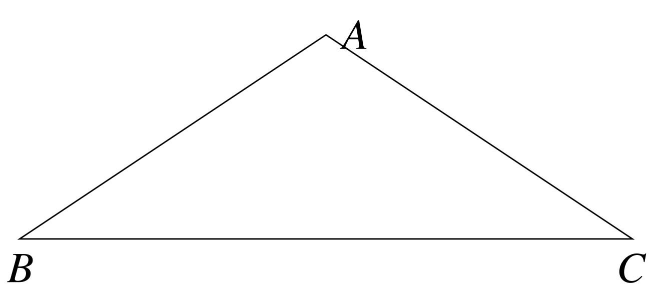

Fig. 2: Triangle by Latex-Tikz

Let the points $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ be

$$\mathbf{A} = \begin{pmatrix} x \\ y \end{pmatrix} \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \mathbf{C} = \begin{pmatrix} c \\ 0 \end{pmatrix} \tag{2.0.1}$$

The direction vector of sides is given as,

$$\mathbf{m}_{AB} = \mathbf{A} - \mathbf{B} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{m}_{CB} = \mathbf{C} - \mathbf{B} = \begin{pmatrix} c \\ 0 \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{m}_{AC} = \mathbf{A} - \mathbf{C} = \begin{pmatrix} x - c \\ y \end{pmatrix} \tag{2.0.4}$$

Taking the inner product of sides AB,CB and sides AC and BC.

$$\mathbf{m_{AB}}^T \mathbf{m}_{CB} = \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} c \\ 0 \end{pmatrix} = xc \tag{2.0.5}$$

$$= \|\mathbf{A} - \mathbf{B}\| \, \|\mathbf{C} - \mathbf{B}\| \cos\theta \tag{2.0.6}$$

$$\mathbf{m_{AC}}^T \mathbf{m}_{BC} = \begin{pmatrix} x - c & y \end{pmatrix} \begin{pmatrix} -c \\ 0 \end{pmatrix} = (c - x)c \tag{2.0.7}$$

$$= \|\mathbf{A} - \mathbf{C}\| \, \|\mathbf{B} - \mathbf{C}\| \cos\theta \tag{2.0.8}$$

We know that,

$$\|\mathbf{A} - \mathbf{B}\| = \sqrt{x^2 + y^2} \tag{2.0.9}$$

$$\|\mathbf{B} - \mathbf{C}\| = c \tag{2.0.10}$$

$$\|\mathbf{A} - \mathbf{C}\| = \sqrt{(x - c)^2 + y^2} \tag{2.0.11}$$

From (2.0.6),

$$\cos\theta = \frac{xc}{\sqrt{x^2 + y^2}c} = \frac{x}{\sqrt{x^2 + y^2}} \tag{2.0.12}$$

From (2.0.8),

$$\cos\theta = \frac{(c - x)c}{\sqrt{(x - c)^2 + y^2}c} = \frac{(c - x)}{\sqrt{(x - c)^2 + y^2}} \tag{2.0.13}$$

squaring and equating (2.0.12) and (2.0.13),

$$x^2((x - c)^2 + y^2) = (c - x)^2(x^2 + y^2) \tag{2.0.14}$$

$$x^2(x - c)^2 + x^2y^2 = x^2(x - c)^2 + y^2(x - c)^2 \tag{2.0.15}$$

$$x^2y^2 = y^2(x - c)^2 \tag{2.0.16}$$

$$x^2 = (x - c)^2 \tag{2.0.17}$$

$$x = \pm(x - c) \tag{2.0.18}$$

$$\implies x = c/2 \tag{2.0.19}$$

Thus,

$$\|\mathbf{A} - \mathbf{B}\| = \sqrt{(c/2)^2 + y^2} \tag{2.0.20}$$

$$\|\mathbf{A} - \mathbf{C}\| = \sqrt{(c/2)^2 + y^2} \tag{2.0.21}$$

Thus it is proved that $AB = AC$