

EE5609 Assignment 2

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Abstract—This assignment involves finding the matrix \mathbf{X} by solving the equation. From (2.0.1),

The python code solution can be downloaded from

https://github.com/Vaibhav11002/EE5609/blob/master/Assignment_2/Codes/assignment_2.py

1 PROBLEM

Find \mathbf{X} if $\mathbf{Y} = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$ and $2\mathbf{X} + \mathbf{Y} = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix}$. Express $2\mathbf{X} + \mathbf{Y} = \mathbf{AB}$, where \mathbf{B} is a block matrix comprising of \mathbf{X} and \mathbf{Y} and find the matrix \mathbf{A} .

2 SOLUTION

We have,

$$2\mathbf{X} + \mathbf{Y} = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix} \quad (2.0.1)$$

$$\Rightarrow 2\mathbf{X} = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix} - \mathbf{Y} \quad (2.0.2)$$

$$\begin{aligned} 2\mathbf{X} &= \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix} - \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} \\ &= \begin{pmatrix} -2 & -2 \\ -4 & -2 \end{pmatrix} \end{aligned} \quad (2.0.3)$$

Now,

$$\mathbf{X} = \frac{1}{2} \begin{pmatrix} -2 & -2 \\ -4 & -2 \end{pmatrix} \quad (2.0.4)$$

$$\begin{aligned} &= \begin{pmatrix} -2/2 & -2/2 \\ -4/2 & -2/2 \end{pmatrix} \\ &= \begin{pmatrix} -1 & -1 \\ -2 & -1 \end{pmatrix} \end{aligned} \quad (2.0.5)$$

Thus from (2.0.5) we get,

$$\mathbf{X} = \begin{pmatrix} -1 & -1 \\ -2 & -1 \end{pmatrix}$$

$$2\mathbf{X} + \mathbf{Y} = \mathbf{AB} \quad (2.0.6)$$

$$= \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix} \quad (2.0.7)$$

Where \mathbf{B} is a block matrix comprising \mathbf{X} and \mathbf{Y} . So,

$$\mathbf{B} = \begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix} \quad (2.0.8)$$

Now,

$$\begin{aligned} \mathbf{AB} &= \mathbf{A} \begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix} \end{aligned} \quad (2.0.9)$$

Since \mathbf{B} is a 4×2 matrix, \mathbf{A} should be 2×4 so that the product \mathbf{AB} is a 2×2 matrix.

Let \mathbf{A} be a block matrix comprising of \mathbf{A}_1 and \mathbf{A}_2 .

$$\mathbf{A} = (\mathbf{A}_1 \quad \mathbf{A}_2) \quad (2.0.10)$$

Now,

$$\begin{aligned} \mathbf{AB} &= (\mathbf{A}_1 \quad \mathbf{A}_2) \begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix} \\ &= \mathbf{A}_1\mathbf{X} + \mathbf{A}_2\mathbf{Y} \end{aligned} \quad (2.0.11)$$

From (2.0.6) we get,

$$\begin{aligned} 2\mathbf{X} + \mathbf{Y} &= (2\mathbf{I})\mathbf{X} + \mathbf{I}\mathbf{Y} \\ &= \mathbf{A}_1\mathbf{X} + \mathbf{A}_2\mathbf{Y} \end{aligned} \quad (2.0.12)$$

Thus, $\mathbf{A}_1 = 2\mathbf{I}$ and $\mathbf{A}_2 = \mathbf{I}$

Hence we get the matrix,

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{pmatrix}$$