

# EE5609 Assignment 2

Gaydhane Vaibhav Digraj  
RollNo : AI20MTECH11002

**Abstract**—This assignment involves finding the matrix  $\mathbf{X}$  by solving the equation. From (2.0.1),

The python code solution can be downloaded from

[https://github.com/Vaibhav11002/EE5609/blob/master/Assignment\\_2/Codes/assignment\\_2.py](https://github.com/Vaibhav11002/EE5609/blob/master/Assignment_2/Codes/assignment_2.py)

## 1 PROBLEM

Find  $\mathbf{X}$  if  $\mathbf{Y} = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$  and  $2\mathbf{X} + \mathbf{Y} = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix}$ . Express  $2\mathbf{X} + \mathbf{Y} = \mathbf{AB}$ , where  $\mathbf{B}$  is a block matrix comprising of  $\mathbf{X}$  and  $\mathbf{Y}$  and find the matrix  $\mathbf{A}$ .

## 2 SOLUTION

We have,

$$2\mathbf{X} + \mathbf{Y} = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix} \quad (2.0.1)$$

$$\Rightarrow 2\mathbf{X} = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix} - \mathbf{Y} \quad (2.0.2)$$

$$\begin{aligned} 2\mathbf{X} &= \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix} - \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} \\ &= \begin{pmatrix} -2 & -2 \\ -4 & -2 \end{pmatrix} \end{aligned} \quad (2.0.3)$$

Now,

$$\mathbf{X} = \frac{1}{2} \begin{pmatrix} -2 & -2 \\ -4 & -2 \end{pmatrix} \quad (2.0.4)$$

$$\begin{aligned} &= \begin{pmatrix} -2/2 & -2/2 \\ -4/2 & -2/2 \end{pmatrix} \\ &= \begin{pmatrix} -1 & -1 \\ -2 & -1 \end{pmatrix} \end{aligned} \quad (2.0.5)$$

Thus from (2.0.5) we get,

$$\mathbf{X} = \begin{pmatrix} -1 & -1 \\ -2 & -1 \end{pmatrix}$$

$$2\mathbf{X} + \mathbf{Y} = \mathbf{AB} \quad (2.0.6)$$

$$= \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix} \quad (2.0.7)$$

Where  $\mathbf{B}$  is a block matrix comprising  $\mathbf{X}$  and  $\mathbf{Y}$ . So,

$$\mathbf{B} = \begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix} \quad (2.0.8)$$

Now,

$$\begin{aligned} \mathbf{AB} &= \mathbf{A} \begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix} \end{aligned} \quad (2.0.9)$$

Since  $\mathbf{B}$  is a  $4 \times 2$  matrix,  $\mathbf{A}$  should be  $2 \times 4$  so that the product  $\mathbf{AB}$  is a  $2 \times 2$  matrix.

Let  $\mathbf{A}$  be a block matrix comprising of  $\mathbf{A}_1$  and  $\mathbf{A}_2$ .

$$\mathbf{A} = (\mathbf{A}_1 \quad \mathbf{A}_2) \quad (2.0.10)$$

Now,

$$\begin{aligned} \mathbf{AB} &= (\mathbf{A}_1 \quad \mathbf{A}_2) \begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix} \\ &= \mathbf{A}_1\mathbf{X} + \mathbf{A}_2\mathbf{Y} \end{aligned} \quad (2.0.11)$$

From (2.0.6) we get,

$$\begin{aligned} 2\mathbf{X} + \mathbf{Y} &= (2\mathbf{I})\mathbf{X} + \mathbf{I}\mathbf{Y} \\ &= \mathbf{A}_1\mathbf{X} + \mathbf{A}_2\mathbf{Y} \end{aligned} \quad (2.0.12)$$

Thus,  $\mathbf{A}_1 = 2\mathbf{I}$  and  $\mathbf{A}_2 = \mathbf{I}$

Hence we get the matrix,

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{pmatrix}$$

**Method 2**(To find  $\mathbf{X}$ ):

The first equation is,

$$\begin{aligned} \mathbf{Y} &= \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} \\ \Rightarrow (\mathbf{O} \ \mathbf{I}) \begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix} &= \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} \end{aligned} \quad (2.0.13)$$

The second equation is,

$$\begin{aligned} 2\mathbf{X} + \mathbf{Y} &= \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix} \\ \Rightarrow (2\mathbf{I} \ \mathbf{I}) \begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix} \end{aligned} \quad (2.0.14)$$

Subtracting (2.0.14) from (2.0.13),

$$\begin{aligned} \Rightarrow (2\mathbf{I} \ \mathbf{O}) \begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix} &= \begin{pmatrix} -2 & -2 \\ -4 & -2 \end{pmatrix} \\ \Rightarrow (\mathbf{I} \ \mathbf{O}) \begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix} &= \begin{pmatrix} -1 & -1 \\ -2 & -1 \end{pmatrix} \end{aligned} \quad (2.0.15)$$

Thus we get,

$$\mathbf{X} = \begin{pmatrix} -1 & -1 \\ -2 & -1 \end{pmatrix}$$