EE5609 Assignment 2

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***Abstract*—This assignment involves finding the matrix X by solving the equation.**

The python code solution can be downloaded from

https://github.com/Vaibhav11002/EE5609/blob/master/Assignment_2/Codes/assignment_2.py

1 Problem

Find $\mathbf{X}$ if $\mathbf{Y} = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$ and $2\mathbf{X} + \mathbf{Y} = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix}$. Express $2\mathbf{X} + \mathbf{Y} = \mathbf{AB}$, where B is a block matrix comprising of $\mathbf{X}$ and $\mathbf{Y}$ and find the matrix $\mathbf{A}$.

2 Solution

We have,

$$2\mathbf{X} + \mathbf{Y} = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix} \tag{2.0.1}$$

$$\implies 2\mathbf{X} = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix} - \mathbf{Y} \tag{2.0.2}$$

$$2\mathbf{X} = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix} - \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & -2 \\ -4 & -2 \end{pmatrix} \tag{2.0.3}$$

Now,

$$\mathbf{X} = \frac{1}{2}\begin{pmatrix} -2 & -2 \\ -4 & -2 \end{pmatrix} \tag{2.0.4}$$

$$= \begin{pmatrix} -2/2 & -2/2 \\ -4/2 & -2/2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -1 \\ -2 & -1 \end{pmatrix} \tag{2.0.5}$$

Thus from (2.0.5) we get,

$$\boxed{\mathbf{X} = \begin{pmatrix} -1 & -1 \\ -2 & -1 \end{pmatrix}}$$

From (2.0.1),

$$2\mathbf{X} + \mathbf{Y} = \mathbf{AB} \tag{2.0.6}$$

$$= \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix} \tag{2.0.7}$$

Where B is a block matrix comprising $\mathbf{X}$ and $\mathbf{Y}$. So,

$$\mathbf{B} = \begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix} \tag{2.0.8}$$

Now,

$$\mathbf{AB} = \mathbf{A}\begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix} \tag{2.0.9}$$

Since $\mathbf{B}$ is a 4x2 matrix, $\mathbf{A}$ should be 2x4 so that the product $\mathbf{AB}$ is a 2x2 matrix.
Let $\mathbf{A}$ be a block matrix comprising of $\mathbf{A_1}$ and $\mathbf{A_2}$.

$$\mathbf{A} = \begin{pmatrix} \mathbf{A_1} & \mathbf{A_2} \end{pmatrix} \tag{2.0.10}$$

Now,

$$\mathbf{AB} = \begin{pmatrix} \mathbf{A_1} & \mathbf{A_2} \end{pmatrix}\begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix}$$

$$= \mathbf{A_1X} + \mathbf{A_2Y} \tag{2.0.11}$$

From (2.0.6) we get,

$$2\mathbf{X} + \mathbf{Y} = (2\mathbf{I})\mathbf{X} + \mathbf{IY}$$

$$= \mathbf{A_1X} + \mathbf{A_2Y} \tag{2.0.12}$$

Thus, $\mathbf{A_1} = 2\mathbf{I}$ and $\mathbf{A_2} = \mathbf{I}$
Hence we get the matrix,

$$\boxed{\mathbf{A} = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{pmatrix}}$$

Method 2(To find $\mathbf{X}$):

The first equation is,

$$\mathbf{Y} = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$$

$$\implies \begin{pmatrix} \mathbf{O} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} \quad = \mathbf{M} \tag{2.0.13}$$

The second equation is,

$$2\mathbf{X} + \mathbf{Y} = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix}$$

$$\implies \begin{pmatrix} 2\mathbf{I} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix} \quad = \mathbf{N} \tag{2.0.14}$$

Combining equations (2.0.13) and (2.0.14) into a single matrix equation and constructing the augmented matrix,

$$\begin{pmatrix} 0 & I & M \\ 2I & I & N \end{pmatrix} \tag{2.0.15}$$

Transforming (2.0.15) using row reduction,

$$\begin{pmatrix} 0 & I & M \\ 2I & I & N \end{pmatrix} \xleftrightarrow{R1 \leftrightarrow R2} \begin{pmatrix} 2I & I & N \\ 0 & I & M \end{pmatrix} \xleftrightarrow{R1 \leftarrow R1 - R2}$$

$$\begin{pmatrix} 2I & 0 & N - M \\ 0 & I & M \end{pmatrix} \xleftrightarrow{R1 \leftarrow \frac{R1}{2}} \begin{pmatrix} I & 0 & \frac{N-M}{2} \\ 0 & I & M \end{pmatrix} \tag{2.0.16}$$

From (2.0.16),

$$\mathbf{X} = \frac{\mathbf{N} - \mathbf{M}}{2} = \begin{pmatrix} -1 & -1 \\ -2 & -1 \end{pmatrix}$$