Assignment 4

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***Abstract*—This document solves the isosceles triangle problem.**

Download all latex-tikz codes from

https://github.com/Vaibhav11002/EE5609/tree/master/Assignment_4

1 Problem

Prove that sides opposite to equal angles of a triangle are equal.

2 Solution

Let's consider $\triangle ABC$ where $\angle ABC = \angle ACB = \theta$,

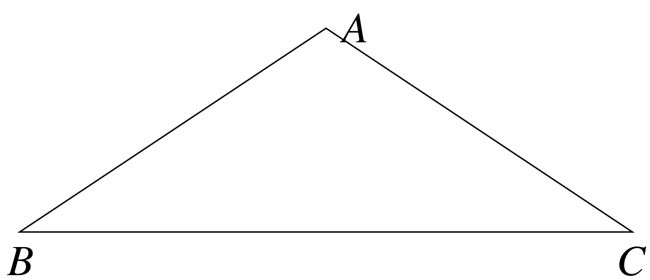

Fig. 2: Triangle by Latex-Tikz

Taking the inner product of sides AB,BC and sides CA,BC.

$$(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C}) = \|\mathbf{A}-\mathbf{B}\|\,\|\mathbf{B}-\mathbf{C}\|\cos\theta \tag{2.0.1}$$

$$(\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) = \|\mathbf{A}-\mathbf{C}\|\,\|\mathbf{B}-\mathbf{C}\|\cos\theta \tag{2.0.2}$$

The cosine from the both the equations is,

$$\implies \cos\theta = \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C})}{\|\mathbf{A}-\mathbf{B}\|\,\|\mathbf{B}-\mathbf{C}\|} \tag{2.0.3}$$

$$\implies \cos\theta = \frac{(\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{C})}{\|\mathbf{A}-\mathbf{C}\|\,\|\mathbf{B}-\mathbf{C}\|} \tag{2.0.4}$$

Equating (2.0.3) and (2.0.4)

$$\frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C})}{\|\mathbf{A}-\mathbf{B}\|} = \frac{(\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{C})}{\|\mathbf{A}-\mathbf{C}\|} \tag{2.0.5}$$

Let, $k_1 = \|\mathbf{A}-\mathbf{B}\|$, $k_2 = \|\mathbf{A}-\mathbf{C}\|$

Now,

$$\frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C})}{k_1} = \frac{(\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{C})}{k_2} \tag{2.0.6}$$

$$k_2(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C}) = k_1(\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) \tag{2.0.7}$$

$$(\mathbf{B}-\mathbf{C})[k_2(\mathbf{A}-\mathbf{B})^T - k_1(\mathbf{A}-\mathbf{C})^T] = 0 \tag{2.0.8}$$

Thus,

$$k_2(\mathbf{A}-\mathbf{B})^T - k_1(\mathbf{A}-\mathbf{C})^T = 0 \tag{2.0.9}$$

$$k_2(\mathbf{A}-\mathbf{B})^T = k_1(\mathbf{A}-\mathbf{C})^T \tag{2.0.10}$$

Taking norm on both sides,

$$k_2\left\|(\mathbf{A}-\mathbf{B})^T\right\| = k_1\left\|(\mathbf{A}-\mathbf{C})^T\right\| \tag{2.0.11}$$

$$\implies k_2k_1 = k_2k_1 \tag{2.0.12}$$