Assignment 4

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***Abstract*—This document involves solving geometry concepts using linear algebra.**

Download all latex-tikz codes from

https://github.com/Vaibhav11002/EE5609/tree/master/Assignment_4

1 Problem

AB is a line segment and line l is its perpendicular bisector. If a point P lies on l, show that P is equidistant from A and B.

2 Solution

We have to prove that P is equidistant from A and B i.e. length of lines AP and BP are equal.

Let DP be the perpendicular bisector of line AB. So,

$$\mathbf{A}-\mathbf{D}=\mathbf{D}-\mathbf{B} \tag{2.0.1}$$
$$\|\mathbf{A}-\mathbf{D}\|=\|\mathbf{D}-\mathbf{B}\|=k \tag{2.0.2}$$
$$\|\mathbf{D}-\mathbf{P}\|=l \tag{2.0.3}$$

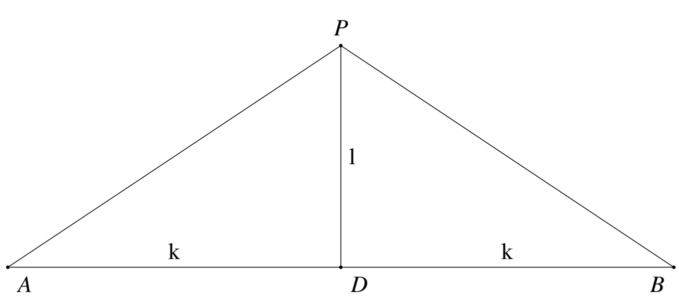

Fig. 1: $PD \perp AB$ by Latex-Tikz

Finding the length of line AP,

$$\begin{aligned}(\mathbf{A}-\mathbf{P})^T(\mathbf{A}-\mathbf{P}) &= (\mathbf{A}-\mathbf{D}+\mathbf{D}-\mathbf{P})^T(\mathbf{A}-\mathbf{D}+\mathbf{D}-\mathbf{P})\\ &= [(\mathbf{A}-\mathbf{D})^T+(\mathbf{D}-\mathbf{P})^T][(\mathbf{A}-\mathbf{D})+(\mathbf{D}-\mathbf{P})]\\ &= (\mathbf{A}-\mathbf{D})^T(\mathbf{A}-\mathbf{D})+(\mathbf{A}-\mathbf{D})^T(\mathbf{D}-\mathbf{P})+\\ &\quad (\mathbf{D}-\mathbf{P})^T(\mathbf{A}-\mathbf{D})+(\mathbf{D}-\mathbf{P})^T(\mathbf{D}-\mathbf{P}) \end{aligned} \tag{2.0.4}$$

Since, line AB is perpendicular to line DP the inner product is zero.

$$(\mathbf{A}-\mathbf{D})^T(\mathbf{D}-\mathbf{P})=(\mathbf{D}-\mathbf{P})^T(\mathbf{A}-\mathbf{D})=0 \tag{2.0.5}$$

Thus,

$$\begin{aligned}(\mathbf{A}-\mathbf{P})^T(\mathbf{A}-\mathbf{P}) &= (\mathbf{A}-\mathbf{D})^T(\mathbf{A}-\mathbf{D})+(\mathbf{D}-\mathbf{P})^T(\mathbf{D}-\mathbf{P})\\ \implies \|\mathbf{A}-\mathbf{P}\|^2 &= \|\mathbf{A}-\mathbf{D}\|^2+\|\mathbf{D}-\mathbf{P}\|^2\\ \implies \|\mathbf{A}-\mathbf{P}\| &= \sqrt{k^2+l^2} \end{aligned} \tag{2.0.6}$$

Next finding the length of line BP,

$$\begin{aligned}(\mathbf{B}-\mathbf{P})^T(\mathbf{B}-\mathbf{P}) &= (\mathbf{B}-\mathbf{D}+\mathbf{D}-\mathbf{P})^T(\mathbf{B}-\mathbf{D}+\mathbf{D}-\mathbf{P})\\ &= [(\mathbf{B}-\mathbf{D})^T+(\mathbf{D}-\mathbf{P})^T][(\mathbf{B}-\mathbf{D})+(\mathbf{D}-\mathbf{P})]\\ &= (\mathbf{B}-\mathbf{D})^T(\mathbf{B}-\mathbf{D})+(\mathbf{B}-\mathbf{D})^T(\mathbf{D}-\mathbf{P})+\\ &\quad (\mathbf{D}-\mathbf{P})^T(\mathbf{B}-\mathbf{D})+(\mathbf{D}-\mathbf{P})^T(\mathbf{D}-\mathbf{P}) \end{aligned} \tag{2.0.7}$$

Again since the inner product of lines AB and DP is zero,

$$\begin{aligned}\implies (\mathbf{B}-\mathbf{P})^T(\mathbf{B}-\mathbf{P}) &= (\mathbf{B}-\mathbf{D})^T(\mathbf{B}-\mathbf{D})+(\mathbf{D}-\mathbf{P})^T(\mathbf{D}-\mathbf{P})\\ \implies \|\mathbf{B}-\mathbf{P}\|^2 &= \|\mathbf{B}-\mathbf{D}\|^2+\|\mathbf{D}-\mathbf{P}\|^2\\ \implies \|\mathbf{B}-\mathbf{P}\| &= \sqrt{k^2+l^2} \end{aligned} \tag{2.0.8}$$

From equations (2.0.6) and (2.0.8) we get,

$$\|\mathbf{A}-\mathbf{P}\|=\|\mathbf{B}-\mathbf{P}\| \tag{2.0.9}$$

Lengths of line AP and BP are equal. Hence, P is equidistant from A and B.