Assignment 4

Matish Singh Tanwar

***Abstract*—This document solves question based on triangle.**

Download all latex-tikz codes from

https://github.com/Matish007/Matrix−Theory−EE5609−/tree/master/Assignment_4

1 Problem

Line L is the bisector of $\angle A$ and B is any point on L.BP and BQ are perpendiculars from B to the arms of $\angle A$.Show that:-

$$a)\quad \triangle APB \cong \triangle AQB \tag{1.0.1}$$
$$b)\quad BP = BQ \tag{1.0.2}$$

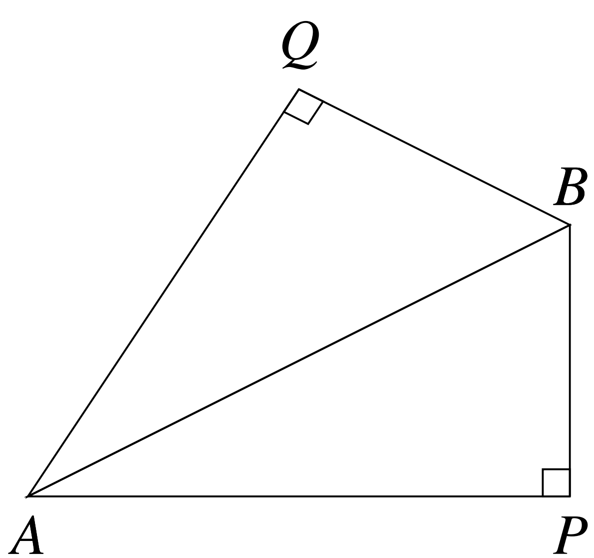

Fig. 1: figure

2 Explanation

Given:-

$$\angle BAP = \angle BAQ = \alpha \tag{2.0.1}$$
$$\angle AQB = \angle APB \tag{2.0.2}$$

In $\triangle ABQ$

$$\angle ABQ + \angle AQB + \angle BAQ = 180° \tag{2.0.3}$$

In $\triangle ABP$

$$\angle ABP + \angle APB + \angle BAP = 180° \tag{2.0.4}$$

Subtracting (2.0.3) and (2.0.4) and using (2.0.1) and (2.0.2) we get,

$$\angle ABQ = \angle ABP \tag{2.0.5}$$

Since,line BP and BQ are perpendicular to AP and AQ respectively..So,their respective dot product will be zero.We get,

$$(\mathbf{B}-\mathbf{Q})^T(\mathbf{A}-\mathbf{Q}) = 0 \tag{2.0.6}$$
$$(\mathbf{B}-\mathbf{P})^T(\mathbf{A}-\mathbf{P}) = 0 \tag{2.0.7}$$

We know that,$(\mathbf{B}-\mathbf{P})^T(\mathbf{B}-\mathbf{P}) = ||\mathbf{B}-\mathbf{P}||^2$
Also let

$$||\mathbf{B}-\mathbf{A}||^2 = k^2 \tag{2.0.8}$$

$$(\mathbf{B}-\mathbf{P})^T(\mathbf{B}-\mathbf{P}) = (\mathbf{B}-\mathbf{A}+\mathbf{A}-\mathbf{P})^T(\mathbf{B}-\mathbf{A}+\mathbf{A}-\mathbf{P}) \tag{2.0.9}$$

$$\begin{aligned} ||\mathbf{B}-\mathbf{P}||^2 &= (\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{A}) \\ &+(\mathbf{A}-\mathbf{P})^T(\mathbf{A}-\mathbf{P}) \\ &+(\mathbf{A}-\mathbf{P})^T(\mathbf{B}-\mathbf{A}) \\ &+(\mathbf{B}-\mathbf{A})^T(\mathbf{A}-\mathbf{P}) \\ &= ||\mathbf{B}-\mathbf{A}||^2 + ||\mathbf{A}-\mathbf{P}||^2 \\ &+2\,||\mathbf{A}-\mathbf{P}||\,||\mathbf{B}-\mathbf{A}||\cos\alpha \end{aligned} \tag{2.0.10}$$

$$\begin{aligned} (\mathbf{A}-\mathbf{P})^T(\mathbf{B}-\mathbf{A}) &= (\mathbf{B}-\mathbf{A})^T(\mathbf{A}-\mathbf{P}) \\ &= ||\mathbf{A}-\mathbf{P}||\,||\mathbf{B}-\mathbf{A}||\cos\alpha \end{aligned} \tag{2.0.11}$$

Substituting (2.0.11), (2.0.8) in (2.0.10) we get,

$$||\mathbf{B}-\mathbf{P}||^2 = k^2 + ||\mathbf{A}-\mathbf{P}||^2 + 2k\,||\mathbf{A}-\mathbf{P}||\cos\alpha \tag{2.0.12}$$

Similarly,we get

$$\|\mathbf{B}-\mathbf{Q}\|^2 = k^2 + \|\mathbf{A}-\mathbf{Q}\|^2 + 2k\|\mathbf{A}-\mathbf{Q}\|\cos\alpha \tag{2.0.13}$$

$$\cos\alpha = \frac{(\mathbf{B}-\mathbf{A})^T(\mathbf{P}-\mathbf{A})}{k\|\mathbf{P}-\mathbf{A}\|} = \frac{(\mathbf{B}-\mathbf{A})^T(\mathbf{Q}-\mathbf{A})}{k\|\mathbf{Q}-\mathbf{A}\|} \tag{2.0.14}$$

$$(\mathbf{B}-\mathbf{A})^T(\mathbf{P}-\mathbf{A}) = (\mathbf{B}-\mathbf{P}+\mathbf{P}-\mathbf{A})^T(\mathbf{P}-\mathbf{A}) \tag{2.0.15}$$

$$\implies (\mathbf{B}-\mathbf{P})^T(\mathbf{P}-\mathbf{A}) + \|\mathbf{P}-\mathbf{A}\|^2 \tag{2.0.16}$$

$$(\mathbf{B}-\mathbf{A})^T(\mathbf{Q}-\mathbf{A}) = (\mathbf{B}-\mathbf{Q}+\mathbf{Q}-\mathbf{A})^T(\mathbf{Q}-\mathbf{A}) \tag{2.0.17}$$

$$\implies (\mathbf{B}-\mathbf{Q})^T(\mathbf{Q}-\mathbf{A}) + \|\mathbf{Q}-\mathbf{A}\|^2 \tag{2.0.18}$$

Substituting (2.0.6) and (2.0.7) in (2.0.18) and (2.0.16) respectively we get,

$$(\mathbf{B}-\mathbf{A})^T(\mathbf{P}-\mathbf{A}) = \|\mathbf{P}-\mathbf{A}\|^2 \tag{2.0.19}$$

$$(\mathbf{B}-\mathbf{A})^T(\mathbf{Q}-\mathbf{A}) = \|\mathbf{Q}-\mathbf{A}\|^2 \tag{2.0.20}$$

Substituting (2.0.19) and (2.0.20) in (2.0.14) we get,

$$\|\mathbf{P}-\mathbf{A}\| = \|\mathbf{Q}-\mathbf{A}\| \tag{2.0.21}$$

Substituting (2.0.21) in (2.0.12) and (2.0.13) we get,

$$\|\mathbf{B}-\mathbf{P}\| = \|\mathbf{B}-\mathbf{Q}\| \tag{2.0.22}$$

From (2.0.22) we can say that **B** is equidistant from the arms of $\angle A$,where **P** and **Q** are the points on the arms of $\angle A$ Using (2.0.1),(2.0.2),(2.0.22) and by AAS(Angle Angle Side) property of congruency we can say that:-

$$\triangle APB \cong \triangle AQB \tag{2.0.23}$$