Assignment 5

Matish Singh Tanwar

Abstract—This document solves question based on circle.

Download all python codes from

and latex-tikz codes from

1 Problem

Find the equation of circle that passes through the points $\begin{pmatrix}1\\2\end{pmatrix}, \begin{pmatrix}2\\1\end{pmatrix}, \begin{pmatrix}0\\0\end{pmatrix}$

2 Explanation

The equation of circle can be expressed as

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{c}^T\mathbf{x} + f = 0 \tag{2.0.1}$$

$\mathbf{c}$ is the centre and substituting the points in the equation of circle we get

$$2\begin{pmatrix}1 & 2\end{pmatrix}\mathbf{c} - f = 5 \tag{2.0.2}$$
$$2\begin{pmatrix}2 & 1\end{pmatrix}\mathbf{c} - f = 5 \tag{2.0.3}$$
$$2\begin{pmatrix}0 & 0\end{pmatrix}\mathbf{c} - f = 0 \tag{2.0.4}$$

can be expressed in matrix form

$$\begin{pmatrix}2 & 4 & -1\\4 & 2 & -1\\0 & 0 & -1\end{pmatrix}\begin{pmatrix}\mathbf{c}\\f\end{pmatrix} = \begin{pmatrix}5\\5\\0\end{pmatrix} \tag{2.0.5}$$

Row reducing the augmented matrix

$$\begin{pmatrix}2 & 4 & -1 & 5\\4 & 2 & -1 & 5\\0 & 0 & -1 & 0\end{pmatrix} \xleftarrow{R_2 \leftarrow 2R_1 - R_2} \begin{pmatrix}2 & 4 & -1 & 5\\0 & 6 & -1 & 5\\0 & 0 & -1 & 0\end{pmatrix} \tag{2.0.6}$$

$$\xleftarrow[R_1 \leftarrow R_1 - R_3]{R_2 \leftarrow R_2 - R_3} \begin{pmatrix}2 & 4 & 0 & 5\\0 & 6 & 0 & 5\\0 & 0 & -1 & 0\end{pmatrix} \tag{2.0.7}$$

$$\xleftarrow{R_1 \leftarrow 3R_1 - 2R_2} \begin{pmatrix}6 & 0 & 0 & 5\\0 & 6 & 0 & 5\\0 & 0 & -1 & 0\end{pmatrix} \tag{2.0.8}$$

$$\mathbf{c} = \begin{pmatrix}\frac{5}{6}\\\frac{5}{6}\end{pmatrix} \tag{2.0.9}$$

$$f = 0 \tag{2.0.10}$$

$$r = \sqrt{\|\mathbf{c}\|^2 - f} = \sqrt{\frac{50}{36}} \tag{2.0.11}$$

The required equation of circle is

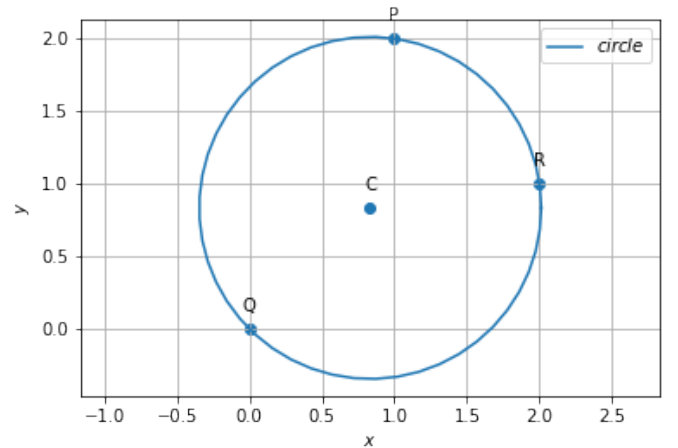$$\mathbf{x}^T\mathbf{x} - 2\begin{pmatrix}\frac{5}{6} & \frac{5}{6}\end{pmatrix}\mathbf{x} = 0 \tag{2.0.12}$$

Fig. 0: Circle passing through point P,Q,R with centre C.