Assignment 12

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***Abstract*—This document finds the eigen values of a special matrix.**

Download all latex-tikz codes from

https://github.com/Matish007/Matrix−Theory−EE5609−/tree/master/Assignment_12

1 Problem

Let n be an odd number ≥ 7.Let,

$$\mathbf{A} = [a_{ij}] \tag{1.0.1}$$

be and $n \times n$ matrix with,

$$a_{i,i+1} = 1, \forall (i = 1, 2, ...n-1) \tag{1.0.2}$$

and $a_{n,1} = 1$. Let $a_{ij} = 0$ for all the other pairs (i, j). Then we can conclude that,

1) $\mathbf{A}$ has 1 as an eigenvalue
2) $\mathbf{A}$ has -1 as an eigenvalue
3) $\mathbf{A}$ has at least one eigenvalue with multiplicity ≥ 2
4) $\mathbf{A}$ has no real eigenvalues

2 SOLUTION

We can represent our matrix as:

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & ..... & 0 & 0 \\ 0 & 0 & 1 & ..... & 0 & 0 \\ . & & & & & \\ . & & & & & \\ 0 & 0 & 0 & ..... & 0 & 1 \\ 1 & 0 & 0 & ..... & 0 & 0 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{A^T} = \begin{pmatrix} 0 & 0 & 0 & ..... & 0 & 1 \\ 1 & 0 & 0 & ..... & 0 & 0 \\ 0 & 1 & 0 & ..... & 0 & 0 \\ . & & & & & \\ . & & & & & \\ 0 & 0 & 0 & .... & 1 & 0 \end{pmatrix} \tag{2.0.2}$$

$\mathbf{A}$ is our given matrix.We know that Characteristic Equation of $\mathbf{A}$ and $\mathbf{A^T}$ is same.Consider the minimal polynomial

$$x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_0 \tag{2.0.3}$$

We can represent it in $n \times n$ matrix with 1's on sub-diagonals and in last column it has negative of the coefficient,and rest all 0.We represent it using $\mathbf{C}$.It is known as the companion matrix.

$$\mathbf{C} = \begin{pmatrix} 0 & 0 & 0 & .... & 0 & -a_0 \\ 1 & 0 & 0 & .... & 0 & -a_1 \\ 0 & 1 & 0 & .... & 0 & -a_2 \\ . & & & & & \\ . & & & & & \\ 0 & 0 & 0 & .... & 1 & -a_{n-1} \end{pmatrix} \tag{2.0.4}$$

(2.0.3) is also the characteristic equation of $\mathbf{C}$ Comparing (2.0.2) with (2.0.4) we get:

$$a_0 = -1, a_1 = a_2 = a_3 = a_4 = ... = a_{n-1} = 0 \tag{2.0.5}$$

Substituting (2.0.5) into (2.0.3) we get:

$$x^n - 1 = 0 \tag{2.0.6}$$

By Cayley-Hamilton Theorem:

$$\lambda^n - 1 = 0 \tag{2.0.7}$$

$$\tag{2.0.8}$$

λ=n^{th} roots of unity.

Options	Explanation
A has 1 as an eigen value	One value out of the n^{th} roots of unity is 1.So,correct
A has -1 as an eigen value	Since,n is odd.So,-1 cannot be one of the value of n^{th} roots of unity. Hence,incorrect
A has atleast one eigenvalue with multiplicity ≥ 2	All values of n^{th} roots of unity are distinct. So there is no eigenvalue with multiplicity ≥ 2. Hence,incorrect.
A has no real eigen values	One of the value is 1,which is real. Hence,incorrect.

TABLE 1: Finding Correct Option