Assignment 18

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***Abstract*—This document solves question based on Linear Algebra.**

Download all latex-tikz codes from

https://github.com/Matish007/Matrix−Theory−EE5609−/tree/master/Assignment_18

1 Problem

Let $\mathbf{V}$ be $\mathbb{R}^3$, with the standard inner product. Let $\mathbf{W}$ be the plane spanned by $\alpha=\begin{pmatrix}1 & 1 & 1\end{pmatrix}$ and $\beta=\begin{pmatrix}1 & 1 & -2\end{pmatrix}$. Let $\mathbf{U}$ be the linear operator defined, geometrically, as follows, $\mathbf{U}$ is rotation through the angle θ, about the straight line through the origin which is orthogonal to $\mathbf{W}$. There are actually two such rotations- choose one. Find the matrix of $\mathbf{U}$ in the standard ordered basis.

2 Solution

We use Gram-Schmidt process to the given vector α, β, γ.

$$\mathbf{v}_1 = \alpha = \begin{pmatrix}1 & 1 & 1\end{pmatrix} \tag{2.0.1}$$

$$\mathbf{v}_2 = \beta - \frac{\beta \mathbf{v}_1^T}{\mathbf{v}_1 \mathbf{v}_1^T}\mathbf{v}_1 \tag{2.0.2}$$

$$\beta \mathbf{v}_1^T = \begin{pmatrix}1 & 1 & -2\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix} = 0 \tag{2.0.3}$$

$$\mathbf{v}_1 \mathbf{v}_1^T = \begin{pmatrix}1 & 1 & -2\end{pmatrix} - 0 = \begin{pmatrix}1 & 1 & -2\end{pmatrix} \tag{2.0.4}$$

$$\mathbf{v}_3 = \gamma - \frac{\gamma \mathbf{v}_1^T}{\mathbf{v}_1 \mathbf{v}_1^T}\mathbf{v}_1 - \frac{\gamma \mathbf{v}_2^T}{\mathbf{v}_2 \mathbf{v}_2^T}\mathbf{v}_2 \tag{2.0.5}$$

$$\gamma \mathbf{v}_1^T = \begin{pmatrix}1 & -1 & 0\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix} = 0 \tag{2.0.6}$$

$$\mathbf{v}_1 \mathbf{v}_1^T = \begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix} = 3 \tag{2.0.7}$$

$$\gamma \mathbf{v}_2^T = \begin{pmatrix}1 & -1 & 0\end{pmatrix}\begin{pmatrix}1\\1\\-2\end{pmatrix} = 0 \tag{2.0.8}$$

$$\mathbf{v}_2 \mathbf{v}_2^T = \begin{pmatrix}1 & 1 & -2\end{pmatrix}\begin{pmatrix}1\\1\\-2\end{pmatrix} = 6 \tag{2.0.9}$$

$$\mathbf{v}_3 = \begin{pmatrix}1 & -1 & 0\end{pmatrix} - 0 - 0 \tag{2.0.10}$$

$$= \begin{pmatrix}1 & -1 & 0\end{pmatrix} \tag{2.0.11}$$

The find the orthonormal by dividing the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ by their norm

$$\mathbf{v}_1 = \frac{\mathbf{v}_1}{\|\mathbf{v}_1\|} \tag{2.0.12}$$

$$\frac{1}{\sqrt{3}}\begin{pmatrix}1 & 1 & 1\end{pmatrix} \tag{2.0.13}$$

$$\mathbf{v}_2 = \frac{\mathbf{v}_2}{\|\mathbf{v}_2\|} \tag{2.0.14}$$

$$\frac{1}{\sqrt{6}}\begin{pmatrix}1 & 1 & -2\end{pmatrix} \tag{2.0.15}$$

$$\mathbf{v}_3 = \frac{\mathbf{v}_3}{\|\mathbf{v}_3\|} \tag{2.0.16}$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1 & 0\end{pmatrix} \tag{2.0.17}$$

$$\mathbf{U} = \begin{pmatrix}\mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3\end{pmatrix} \tag{2.0.18}$$

Hence, $\mathbf{U} = \begin{pmatrix}\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}} & 0\end{pmatrix}$