Assignment 4

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***Abstract*—This document solves question based on triangle.**

Download all latex-tikz codes from

https://github.com/Matish007/Matrix−Theory−EE5609−/tree/master/Assignment_4

1 Problem

Line L is the bisector of $\angle A$ and B is any point on L.BP and BQ are perpendiculars from B to the arms of $\angle A$.Show that:-

$$a) \quad \triangle APB \cong \triangle AQB \tag{1.0.1}$$

$$b) \quad BP = BQ \tag{1.0.2}$$

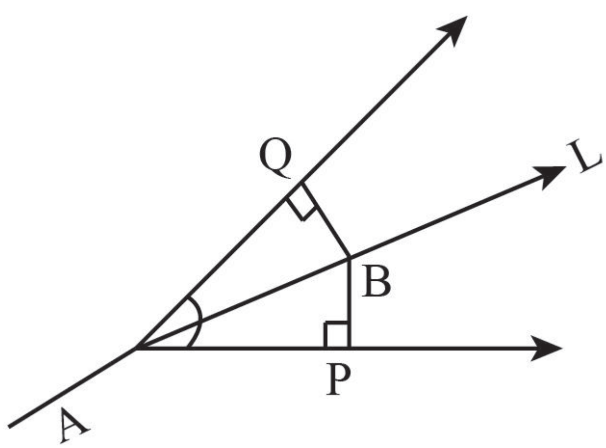

Fig. 0

2 Explanation

Given:-

$$\angle BAP = \angle BAQ = \alpha \tag{2.0.1}$$

$$\angle AQB = \angle APB \tag{2.0.2}$$

In $\triangle ABQ$

$$\angle ABQ + \angle AQB + \angle BAQ = 180° \tag{2.0.3}$$

In $\triangle ABP$

$$\angle ABP + \angle APB + \angle BAP = 180° \tag{2.0.4}$$

Subtracting (2.0.3) and (2.0.4) and using (2.0.1) and (2.0.2) we get,

$$\angle ABQ = \angle ABP \tag{2.0.5}$$

Since,line BP and BQ are perpendicular to AP and AQ respectively..So,their respective dot product will be zero.We get,

$$(\mathbf{B} - \mathbf{Q})^T(\mathbf{A} - \mathbf{Q}) = 0 \tag{2.0.6}$$

$$(\mathbf{B} - \mathbf{P})^T(\mathbf{A} - \mathbf{P}) = 0 \tag{2.0.7}$$

We know that,$(\mathbf{B} - \mathbf{P})^T(\mathbf{B} - \mathbf{P}) = ||\mathbf{B} - \mathbf{P}||^2$
Also let

$$||\mathbf{B} - \mathbf{A}||^2 = k^2 \tag{2.0.8}$$

$$(\mathbf{B} - \mathbf{P})^T(\mathbf{B} - \mathbf{P}) = (\mathbf{B} - \mathbf{A} + \mathbf{A} - \mathbf{P})^T(\mathbf{B} - \mathbf{A} + \mathbf{A} - \mathbf{P}) \tag{2.0.9}$$

$$\begin{aligned} ||\mathbf{B} - \mathbf{P}||^2 = (\mathbf{B} - \mathbf{A})^T&(\mathbf{B} - \mathbf{A}) \\ +(\mathbf{A} - \mathbf{P})^T&(\mathbf{A} - \mathbf{P}) \\ +(\mathbf{A} - \mathbf{P})^T&(\mathbf{B} - \mathbf{A}) \\ +(\mathbf{B} - \mathbf{A})^T&(\mathbf{A} - \mathbf{P}) \\ = ||\mathbf{B} - \mathbf{A}||^2 &+ ||\mathbf{A} - \mathbf{P}||^2 \\ +2||\mathbf{A} - \mathbf{P}||&||\mathbf{B} - \mathbf{A}|| \cos\alpha \end{aligned} \tag{2.0.10}$$

Substituting (2.0.8) in (2.0.10) we get,
And $(\mathbf{A} - \mathbf{P})^T(\mathbf{B} - \mathbf{A})$=$(\mathbf{B} - \mathbf{A})^T(\mathbf{A} - \mathbf{P})$
$= ||\mathbf{A} - \mathbf{P}||\ ||\mathbf{B} - \mathbf{A}||\ \cos\alpha$

$$||\mathbf{B} - \mathbf{P}||^2 = k^2 + ||\mathbf{A} - \mathbf{P}||^2 + 2k||\mathbf{A} - \mathbf{P}|| \cos\alpha \tag{2.0.11}$$

Similarly,we get

$$\left\|\mathbf{B}-\mathbf{Q}\right\|^2 = k^2 + \left\|\mathbf{A}-\mathbf{Q}\right\|^2 + 2k\left\|\mathbf{A}-\mathbf{Q}\right\|\cos\alpha \tag{2.0.12}$$

$$\cos\alpha = \frac{(\mathbf{B}-\mathbf{A})^T(\mathbf{P}-\mathbf{A})}{k\|\mathbf{P}-\mathbf{A}\|} = \frac{(\mathbf{B}-\mathbf{A})^T(\mathbf{Q}-\mathbf{A})}{k\left\|\mathbf{Q}-\mathbf{A}\right\|} \tag{2.0.13}$$

$$(\mathbf{B}-\mathbf{A})^T(\mathbf{P}-\mathbf{A}) = (\mathbf{B}-\mathbf{P}+\mathbf{P}-\mathbf{A})^T(\mathbf{P}-\mathbf{A}) \tag{2.0.14}$$

$$\implies (\mathbf{B}-\mathbf{P})^T(\mathbf{P}-\mathbf{A}) + \|\mathbf{P}-\mathbf{A}\|^2 \tag{2.0.15}$$

$$(\mathbf{B}-\mathbf{A})^T(\mathbf{Q}-\mathbf{A}) = (\mathbf{B}-\mathbf{Q}+\mathbf{Q}-\mathbf{A})^T(\mathbf{Q}-\mathbf{A}) \tag{2.0.16}$$

$$\implies (\mathbf{B}-\mathbf{Q})^T(\mathbf{Q}-\mathbf{A}) + \left\|\mathbf{Q}-\mathbf{A}\right\|^2 \tag{2.0.17}$$

Substituting (2.0.6) and (2.0.7) in (2.0.17) and (2.0.15) respectively we get,

$$(\mathbf{B}-\mathbf{A})^T(\mathbf{P}-\mathbf{A}) = \|\mathbf{P}-\mathbf{A}\|^2 \tag{2.0.18}$$

$$(\mathbf{B}-\mathbf{A})^T(\mathbf{Q}-\mathbf{A}) = \left\|\mathbf{Q}-\mathbf{A}\right\|^2 \tag{2.0.19}$$

Substituting (2.0.18) and (2.0.19) in (2.0.13) we get,

$$\cos\alpha = \|\mathbf{P}-\mathbf{A}\| = \left\|\mathbf{Q}-\mathbf{A}\right\| \tag{2.0.20}$$

Substituting (2.0.20) in (2.0.11) and (2.0.12) we get,

$$\|\mathbf{B}-\mathbf{P}\| = \left\|\mathbf{B}-\mathbf{Q}\right\| \tag{2.0.21}$$

From (2.0.21) we can say that **B** is equidistant from the arms of $\angle A$,where **P** and **Q** are the points on the arms of $\angle A$ Using (2.0.1),(2.0.2),(2.0.21) and by AAS(Angle Angle Side) property of congruency we can say that:-

$$\triangle APB \cong \triangle AQB \tag{2.0.22}$$