Assignment 11

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***Abstract*—This document contains a solution for a problem related to linear transformation.**

Download all latex-tikz codes from

https://github.com/Matish007/Matrix−Theory−EE5609−/tree/master/Assignment_11

1 Problem

Let V be a finite-dimensional vector space and let T be a linear operator on V. Suppose that rank (T^2) = rank (T). Prove that the range and null space of T are disjoint, i.e., have only the zero vector in common.

2 Solution

Given,

$$T : V \rightarrow V \tag{2.0.1}$$

Let basis of V be:

$$\{\alpha_1, \alpha_2, .., \alpha_n\} \tag{2.0.2}$$

Rank of T is number of linearly independent vectors in the set

$$\{T\alpha_1, T\alpha_2, ..., T\alpha_n\} \tag{2.0.3}$$

Let Rank of T=r,then by rank nullity theorem $\{T\alpha_1, T\alpha_2, ..., T\alpha_r\}$ is basis of range T

$$rank(T^2) = rank(T) = r \tag{2.0.4}$$

So,$\{T^2\alpha_1, T^2\alpha_2, ..., T^2\alpha_r\}$ is basis of range T^2
Let,

$$v \in range(T) \tag{2.0.5}$$
$$v = c_1T\alpha_1 + c_2T\alpha_2 + ... + c_rT\alpha_r \tag{2.0.6}$$
$$v \in nullspace(T) \tag{2.0.7}$$
$$T(v) = 0 \tag{2.0.8}$$

Substituting (2.0.6) in (2.0.8) we get,

$$T(c_1T\alpha_1 + c_2T\alpha_2 + ... + c_rT\alpha_r) = 0 \tag{2.0.9}$$
$$c_1T^2\alpha_1 + c_2T^2\alpha_2 + ... + c_rT^2\alpha_r = 0 \tag{2.0.10}$$

We know that,$\{T^2\alpha_1, T^2\alpha_2, ..., T^2\alpha_r\}$ is basis of range T^2

$$\implies c_1 = c_2 = ... = c_r = 0 \tag{2.0.11}$$

Substituting (2.0.11) in (2.0.6) we get,

$$v = 0 \tag{2.0.12}$$

From (2.0.12) it can be seen that when v belongs to both range(T) and nullspace(T) then v is a zero vector. Hemce,range(T) and nullspace(T) are disjoint.