Question

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***Abstract*—This document solves the system of linear equations using Gaussian Elimination**

Download all latex codes from

https://github.com/Matish007/Matrix−Theory−EE5609−/tree/master/Question

1 Problem

Find all binary solutions using Gaussian Elimination of:-

$$\mathbf{x} \oplus \mathbf{z} = 1 \tag{1}$$
$$\mathbf{x} \oplus \mathbf{y} \oplus \mathbf{z} = 1 \tag{2}$$
$$\mathbf{y} \oplus \mathbf{z} = 0 \tag{3}$$

2 Explanation

According to Gauss Elimination we will apply operations on coefficient matrix **A** such that it is converted to Upper triangular matrix **U**.It will be a success if we get all 3 pivots otherwise failure.If success,then by Back Substitution,we will find the values of **x**,**y**,**z**. We will merge **b** also in **A** , so it will become a Augmented Matrix.

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \tag{4}$$

$$\mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \tag{5}$$

$$\mathbf{A}|\mathbf{b} = \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right) \tag{6}$$

where equation (4) is our coefficient matrix and equation (6) is our Augmented Matrix
Now,applying operations on the augmented matrix:-

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right) \xleftrightarrow{R2 \leftarrow R2-R1} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right) \xleftrightarrow{R3 \leftarrow R3-R2} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \tag{7}$$

Equation (7) final result is **U**|**c** Where **U** is Upper triangular Matrix,Coefficient Matrix **A** got converted to **U** and **b** got converted to **c**. We converted (1),(2),(3) equations in a simplified way which is represented by **U**|**c** matrix.Let's see what we got

$$\mathbf{x} \oplus \mathbf{z} = 1 \tag{8}$$
$$\mathbf{y} = 0 \tag{9}$$
$$\mathbf{z} = 0 \tag{10}$$

From equation (9) and (10) it is clear that **y**=0 and **z**=0
Substituting (10) in (8) we get

$$\mathbf{x} = 1 \tag{11}$$

Equation (9),(10),(11) combinely is our required solution of the linear system of equations.