

Assignment 4

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Abstract—This document solves question based on triangle.

Download all latex-tikz codes from

https://github.com/Matish007/Matrix-Theory-EE5609-/tree/master/Assignment_4

1 PROBLEM

Line L is the bisector of $\angle A$ and B is any point on L. BP and BQ are perpendiculars from B to the arms of $\angle A$. Show that:-

$$a) \quad \triangle APB \cong \triangle AQB \quad (1.0.1)$$

$$b) \quad BP = BQ \quad (1.0.2)$$

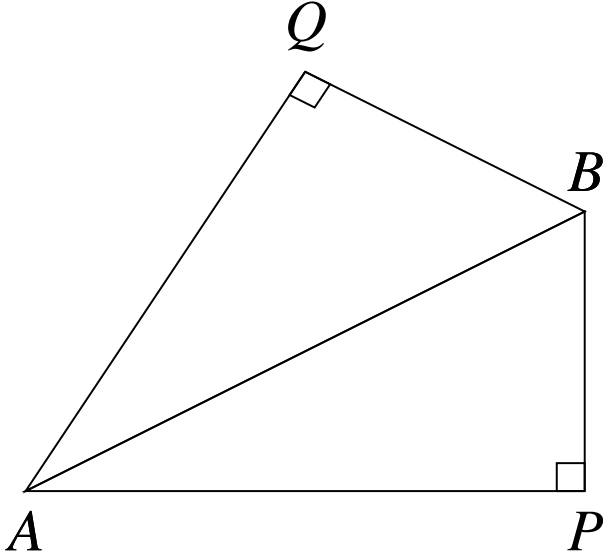


Fig. 1: figure

2 EXPLANATION

Given:-

$$\angle BAP = \angle BAQ = \alpha \quad (2.0.1)$$

$$\angle AQB = \angle APB \quad (2.0.2)$$

In $\triangle ABQ$

$$\angle ABQ + \angle AQB + \angle BAQ = 180^\circ \quad (2.0.3)$$

In $\triangle ABP$

$$\angle ABP + \angle APB + \angle BAP = 180^\circ \quad (2.0.4)$$

Subtracting (2.0.3) and (2.0.4) and using (2.0.1) and (2.0.2) we get,

$$\angle ABQ = \angle ABP \quad (2.0.5)$$

Since, line BP and BQ are perpendicular to AP and AQ respectively..So, their respective dot product will be zero. We get,

$$(\mathbf{B} - \mathbf{Q})^T (\mathbf{A} - \mathbf{Q}) = 0 \quad (2.0.6)$$

$$(\mathbf{B} - \mathbf{P})^T (\mathbf{A} - \mathbf{P}) = 0 \quad (2.0.7)$$

We know that, $(\mathbf{B} - \mathbf{P})^T (\mathbf{B} - \mathbf{P}) = \|\mathbf{B} - \mathbf{P}\|^2$
Also let

$$\|\mathbf{B} - \mathbf{A}\|^2 = k^2 \quad (2.0.8)$$

$$(\mathbf{B} - \mathbf{P})^T (\mathbf{B} - \mathbf{P}) = (\mathbf{B} - \mathbf{A} + \mathbf{A} - \mathbf{P})^T (\mathbf{B} - \mathbf{A} + \mathbf{A} - \mathbf{P}) \quad (2.0.9)$$

$$\begin{aligned} \|\mathbf{B} - \mathbf{P}\|^2 &= (\mathbf{B} - \mathbf{A})^T (\mathbf{B} - \mathbf{A}) \\ &\quad + (\mathbf{A} - \mathbf{P})^T (\mathbf{A} - \mathbf{P}) \\ &\quad + (\mathbf{A} - \mathbf{P})^T (\mathbf{B} - \mathbf{A}) \\ &\quad + (\mathbf{B} - \mathbf{A})^T (\mathbf{A} - \mathbf{P}) \\ &= \|\mathbf{B} - \mathbf{A}\|^2 + \|\mathbf{A} - \mathbf{P}\|^2 \\ &\quad + 2 \|\mathbf{A} - \mathbf{P}\| \|\mathbf{B} - \mathbf{A}\| \cos \alpha \end{aligned} \quad (2.0.10)$$

$$\begin{aligned} (\mathbf{A} - \mathbf{P})^T (\mathbf{B} - \mathbf{A}) &= (\mathbf{B} - \mathbf{A})^T (\mathbf{A} - \mathbf{P}) \\ &= \|\mathbf{A} - \mathbf{P}\| \|\mathbf{B} - \mathbf{A}\| \cos \alpha \end{aligned} \quad (2.0.11)$$

Substituting (2.0.11), (2.0.8) in (2.0.10) we get,

$$\|\mathbf{B} - \mathbf{P}\|^2 = k^2 + \|\mathbf{A} - \mathbf{P}\|^2 + 2k \|\mathbf{A} - \mathbf{P}\| \cos \alpha \quad (2.0.12)$$

Similarly, we get

$$\|\mathbf{B} - \mathbf{Q}\|^2 = k^2 + \|\mathbf{A} - \mathbf{Q}\|^2 + 2k \|\mathbf{A} - \mathbf{Q}\| \cos \alpha \quad (2.0.13)$$

$$\cos \alpha = \frac{(\mathbf{B} - \mathbf{A})^T (\mathbf{P} - \mathbf{A})}{k \|\mathbf{P} - \mathbf{A}\|} = \frac{(\mathbf{B} - \mathbf{A})^T (\mathbf{Q} - \mathbf{A})}{k \|\mathbf{Q} - \mathbf{A}\|} \quad (2.0.14)$$

$$(\mathbf{B} - \mathbf{A})^T (\mathbf{P} - \mathbf{A}) = (\mathbf{B} - \mathbf{P} + \mathbf{P} - \mathbf{A})^T (\mathbf{P} - \mathbf{A}) \quad (2.0.15)$$

$$\implies (\mathbf{B} - \mathbf{P})^T (\mathbf{P} - \mathbf{A}) + \|\mathbf{P} - \mathbf{A}\|^2 \quad (2.0.16)$$

$$(\mathbf{B} - \mathbf{A})^T (\mathbf{Q} - \mathbf{A}) = (\mathbf{B} - \mathbf{Q} + \mathbf{Q} - \mathbf{A})^T (\mathbf{Q} - \mathbf{A}) \quad (2.0.17)$$

$$\implies (\mathbf{B} - \mathbf{Q})^T (\mathbf{Q} - \mathbf{A}) + \|\mathbf{Q} - \mathbf{A}\|^2 \quad (2.0.18)$$

Substituting (2.0.6) and (2.0.7) in (2.0.18) and (2.0.16) respectively we get,

$$(\mathbf{B} - \mathbf{A})^T (\mathbf{P} - \mathbf{A}) = \|\mathbf{P} - \mathbf{A}\|^2 \quad (2.0.19)$$

$$(\mathbf{B} - \mathbf{A})^T (\mathbf{Q} - \mathbf{A}) = \|\mathbf{Q} - \mathbf{A}\|^2 \quad (2.0.20)$$

Substituting (2.0.19) and (2.0.20) in (2.0.14) we get,

$$\cos \alpha = \|\mathbf{P} - \mathbf{A}\| = \|\mathbf{Q} - \mathbf{A}\| \quad (2.0.21)$$

Substituting (2.0.21) in (2.0.12) and (2.0.13) we get,

$$\|\mathbf{B} - \mathbf{P}\| = \|\mathbf{B} - \mathbf{Q}\| \quad (2.0.22)$$

From (2.0.22) we can say that \mathbf{B} is equidistant from the arms of $\angle A$, where \mathbf{P} and \mathbf{Q} are the points on the arms of $\angle A$. Using (2.0.1), (2.0.2), (2.0.22) and by AAS (Angle Angle Side) property of congruency we can say that:-

$$\triangle APB \cong \triangle AQB \quad (2.0.23)$$