Assignment 11

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***Abstract*—This document contains a solution for a problem related to linear transformation.**

Download all latex-tikz codes from

https://github.com/Matish007/Matrix−Theory−EE5609−/tree/master/Assignment_11

1 Problem

Let V be a finite-dimensional vector space and let T be a linear operator on V. Suppose that rank (T^2) = rank (T). Prove that the range and null space of T are disjoint, i.e., have only the zero vector in common.

2 SOLUTION

Given	$\mathbf{T} : \mathbf{V} \to \mathbf{V}$ be a linear operator. Rank(T^2)=Rank(T)
Rank of T and T^2	Let $\left(e_1, e_2, ..., e_n\right)$ be a basis for T,then Rank(T) is linearly independent vectors in the set $\left(Te_1, Te_2, ..., Te_n\right)$ Let,Rank(T)=r=Rank(T^2)
Rank Nullity Theorem	If Rank(T)=r then $\left(Te_1, Te_2, ..., Te_r\right)$ is the basis of range T. Similarly for T^2,$\left(T^2e_1, T^2e_2, ..., T^2e_r\right)$ is the basis of range T^2
v∈ range(T) **v**∈ nullspace(T)	$\mathbf{v}$=$c_1Te_1 + c_2Te_2 + ... + c_rTe_r$ T(**v**)=0 T($c_1Te_1 + c_2Te_2 + ... + c_rTe_r$)=0 $c_1T^2e_1 + c_2T^2e_2 + ... + c_rT^2e_r$=0 But,$\left(T^2e_1, T^2e_2, ..., T^2e_r\right)$ is the basis of range T^2 So,c_1=c_2=......=c_r=0 Substituting these in **v** we get **v**=0
Conclusion	Hence from above it can be seen that when **v** belongs to both range(T) and nullspace(T) then **v** is a zero vector. Hence,range(T) and nullspace(T) are disjoint.