Assignment 16

Matish Singh Tanwar

***Abstract*—This document solves a problem of Linear Algebra.**

Download all latex-tikz codes from

https://github.com/Matish007/Matrix−Theory−EE5609−/tree/master/Assignment_16

1 Problem

Let

$$\mathbf{A} = \begin{pmatrix} x & y \\ -y & x \end{pmatrix} \tag{1.0.1}$$

where x,y $\in \mathbb{R}$ such that

$$x^2 + y^2 = 1 \tag{1.0.2}$$

Then,we must have:

1) $\mathbf{A^n} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \forall n \geq 1$
   where x=cos($\frac{\theta}{n}$),y=sin($\frac{\theta}{n}$)
2) $trace(\mathbf{A}) \neq 0$
3) $\mathbf{A^T} = \mathbf{A^{-1}}$
4) $\mathbf{A}$ is similar to a diagonal matrix over $\mathbb{C}$

2 SOLUTION

Options	Explanation
$\mathbf{A^n} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \forall n \geq 1$ where x=$\cos(\frac{\theta}{n})$,y=$\sin(\frac{\theta}{n})$	$\mathbf{A} = \begin{pmatrix} x & y \\ -y & x \end{pmatrix}$ $\mathbf{A} = \begin{pmatrix} \cos(\frac{\theta}{n}) & \sin(\frac{\theta}{n}) \\ -\sin(\frac{\theta}{n}) & \cos(\frac{\theta}{n}) \end{pmatrix}$ $\mathbf{A^2} = \mathbf{A.A} = \begin{pmatrix} \cos(\frac{\theta}{n}) & \sin(\frac{\theta}{n}) \\ -\sin(\frac{\theta}{n}) & \cos(\frac{\theta}{n}) \end{pmatrix}\begin{pmatrix} \cos(\frac{\theta}{n}) & \sin(\frac{\theta}{n}) \\ -\sin(\frac{\theta}{n}) & \cos(\frac{\theta}{n}) \end{pmatrix}$ $\mathbf{A^2} = \begin{pmatrix} \cos(\frac{2\theta}{n}) & \sin(\frac{2\theta}{n}) \\ -\sin(\frac{2\theta}{n}) & \cos(\frac{2\theta}{n}) \end{pmatrix}$ $\mathbf{A^3} = \mathbf{A^2.A} = \begin{pmatrix} \cos(\frac{2\theta}{n}) & \sin(\frac{2\theta}{n}) \\ -\sin(\frac{2\theta}{n}) & \cos(\frac{2\theta}{n}) \end{pmatrix}\begin{pmatrix} \cos(\frac{\theta}{n}) & \sin(\frac{\theta}{n}) \\ -\sin(\frac{\theta}{n}) & \cos(\frac{\theta}{n}) \end{pmatrix}$ $\mathbf{A^3} = \begin{pmatrix} \cos(\frac{3\theta}{n}) & \sin(\frac{3\theta}{n}) \\ -\sin(\frac{3\theta}{n}) & \cos(\frac{3\theta}{n}) \end{pmatrix}$ .. .. .. $\mathbf{A^n} = \begin{pmatrix} \cos(\frac{n\theta}{n}) & \sin(\frac{n\theta}{n}) \\ -\sin(\frac{n\theta}{n}) & \cos(\frac{n\theta}{n}) \end{pmatrix}$ $\mathbf{A^n} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \qquad \forall n \geq 1$ Hence,correct
$trace(\mathbf{A}) \neq 0$	Let,$x = 0, y = 1$,Substitute in (1.0.1) $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ $trace(\mathbf{A}) = 0$ Hence,incorrect
$\mathbf{A^T} = \mathbf{A^{-1}}$ $\mathbf{AA^T}$	$\mathbf{A} = \begin{pmatrix} x & y \\ -y & x \end{pmatrix}$ $\mathbf{A^T} = \begin{pmatrix} x & -y \\ y & x \end{pmatrix}$ $\begin{pmatrix} x & y \\ -y & x \end{pmatrix}\begin{pmatrix} x & -y \\ y & x \end{pmatrix}$ $\begin{pmatrix} x^2 + y^2 & -xy + xy \\ -xy + xy & x^2 + y^2 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\mathbf{A}A^T = \mathbf{I} = \mathbf{A^T}A$ $\implies \mathbf{A} = \mathbf{A^{-1}}$ $\implies \mathbf{A}$ is an orthogonal matrix. Hence,correct.

1

Options	Explanation
A is similar to a diagonal matrix over $\mathbb{C}$ Using Spectral Theorem	Every real orthogonal matrix is diagonalizable over $\mathbb{C}$ **A** is orthogonal from above. Since,$x, y \in \mathbb{R}$ .So,**A** is a real orthogonal matrix.
$\mathbf{A} = \begin{pmatrix} x & y \\ -y & x \end{pmatrix}$	$det(\mathbf{A} - \lambda\mathbf{I})) = 0$ $(x-\lambda)^2 + y^2 = 0$ $\lambda_1 = x - iy \qquad \lambda_2 = x + iy$ For two eigen values λ_1, λ_2 let heir corresponding eigen vectors be $\mathbf{V_1}, \mathbf{V_2}$
Finding $\mathbf{V_1}$	$(\mathbf{A} - \lambda_1\mathbf{I})\mathbf{V_1} = 0$ $(\mathbf{A} - \lambda_1\mathbf{I}) = \begin{pmatrix} iy & y \\ -y & iy \end{pmatrix}$ By Elementary row operations we get, $(\mathbf{A} - \lambda_1\mathbf{I}) = \begin{pmatrix} iy & y \\ 0 & 0 \end{pmatrix}$ $\mathbf{V_1} = \begin{pmatrix} i \\ 1 \end{pmatrix}$
Finding $\mathbf{V_2}$	$(\mathbf{A} - \lambda_2\mathbf{I})\mathbf{V_2} = 0$ $(\mathbf{A} - \lambda_2\mathbf{I}) = \begin{pmatrix} -iy & y \\ -y & -iy \end{pmatrix}$ By Elementary row operations we get, $(\mathbf{A} - \lambda_2\mathbf{I}) = \begin{pmatrix} -iy & y \\ 0 & 0 \end{pmatrix}$ $\mathbf{V_2} = \begin{pmatrix} -i \\ 1 \end{pmatrix}$
$\mathbf{A} = \mathbf{PDP^{-1}}$	**P** is a matrix containing eigen vectors of **A** ,**D** is the diagonal matrix where diagonals are the eigen values of **A** $\mathbf{P^{-1}} = \frac{1}{2i}\begin{pmatrix} 1 & i \\ -1 & i \end{pmatrix}$ $\mathbf{A} = \frac{1}{2i}\begin{pmatrix} i & -i \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x-iy & 0 \\ 0 & x+iy \end{pmatrix}\begin{pmatrix} 1 & i \\ -1 & i \end{pmatrix}$ Hence,**A** is similar to a diagonal matrix over $\mathbb{C}$ Hence,correct.

TABLE : Finding Correct Option