Assignment 14

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***Abstract*—This document solves a problem of Linear Algebra.**

Download all latex-tikz codes from

https://github.com/Matish007/Matrix−Theory−EE5609−/tree/master/Assignment_14

1 Problem

Let $B : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be the function $B(a, b) = ab$. Which of the following is true-

1) B is a linear transformation
2) B is a positive definite bilinear form
3) B is symmetric but not positive definite
4) B neither linear nor bilinear

2 SOLUTION

Let

$$\mathbf{x} = \begin{pmatrix} x & y \end{pmatrix}^T \tag{2.0.1}$$

Then

$$B(x, y) = \mathbf{x}^T \frac{\mathbf{R}}{2} \mathbf{x} \tag{2.0.2}$$

where $\mathbf{R}$ is the reflection matrix defined as:-

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2.0.3}$$

(2.0.2) represent Quadratic form of B(x,y)

Options	**Explanation**
B is a linear transformation	Let the transformation be $B : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ such that, $B(\mathbf{x}) = xy$ where $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ Now $B(\mathbf{e}) = ab$ where $\mathbf{e} = \begin{pmatrix} a \\ b \end{pmatrix}$ Hence, $B(c\mathbf{e}) = c^2 B(\mathbf{e})$ Hence B is not a linear transformation. Hence incorrect.
B is a positive definite bilinear form Bilinear Form Symmetric Positive Definite	$f : \mathbb{V} \times \mathbb{V} \rightarrow \mathbb{F}$ where $\mathbb{V}$ is a vector space and $\mathbb{F}$ is a field f is a bilinear if the following holds true - 1.$f(c\alpha_1 + \alpha_2, \beta) = cf(\alpha_1, \beta) + f(\alpha_2, \beta)$ 2. $f(\alpha_1, c\beta_1 + \beta_2) = cf(\alpha_1, \beta_1) + f(\alpha_1, \beta_2)$ Now, $B(ca_1 + a_2, b) = (ca_1 + a_2)b = cB(a_1, b) + B(a_2, b)$ And, $B(a_1, cb_1 + b_2) = a_1(cb_1 + b_2) = cB(a_1, b_1) + B(a_1, b_2)$ Hence B is a bilinear form. Again a bilinear form f is symmetric if $f(\alpha, \beta) = f(\beta, \alpha)$ Here, $B(a, b) = ab = ba = B(b, a)$ hence B is symmetric. A symmetric bilinear f is positive definite if $f(\alpha, \alpha) > 0 \ \forall \alpha \neq 0$ Here, $B(a, a) = a^2 > 0 \ \forall a \neq 0$ **Conclusion:** B is symmetric and positive definite bilinear form. Hence Correct.
B is symmetric but not positive definite	From previous proof it is obvious that B is both symmetric as well as positive definite Hence incorrect
B neither linear nor bilinear	From previous proofs it is obvious that B is bilinear. Hence incorrect.
Result	B is symmetric and positive definite bilinear form

TABLE 1: Finding Correct Option