

# Assignment 14

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**Abstract**—This document solves a problem of Linear Algebra.

Download all latex-tikz codes from

[https://github.com/Matish007/Matrix-Theory-EE5609-/tree/master/Assignment\\_14](https://github.com/Matish007/Matrix-Theory-EE5609-/tree/master/Assignment_14)

## 1 PROBLEM

Let  $B : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$  be the function  $B(a, b) = ab$ .

Which of the following is true-

- 1)  $B$  is a linear transformation
- 2)  $B$  is a positive definite bilinear form
- 3)  $B$  is symmetric but not positive definite
- 4)  $B$  neither linear nor bilinear

## 2 SOLUTION

Quadratic form is given by:-

$$a(1)b \quad (2.0.1)$$

| Options   | Explanation  |
|---|--|
| $B$ is a linear transformation  | <p>Let the transformation be <math>B : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}</math> such that,</p> $B(\mathbf{x}) = xy \text{ where } \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ <p>Now <math>B(\mathbf{e}) = ab</math> where <math>\mathbf{e} = \begin{pmatrix} a \\ b \end{pmatrix}</math></p> <p>Hence, <math>B(c\mathbf{e}) = c^2 B(\mathbf{e})</math></p> <p>Hence <math>B</math> is not a linear transformation.</p> <p>Hence incorrect.</p>   |
| $B$ is a positive definite bilinear form<br>Bilinear Form<br><br>Symmetric<br><br>Positive Definite | <p><math>f : \mathbb{V} \times \mathbb{V} \rightarrow \mathbb{F}</math> where <math>\mathbb{V}</math> is a vector space and <math>\mathbb{F}</math> is a field</p> <p><math>f</math> is a bilinear if the following holds true -</p> <ol style="list-style-type: none"> <li>1. <math>f(c\alpha_1 + \alpha_2, \beta) = cf(\alpha_1, \beta) + f(\alpha_2, \beta)</math></li> <li>2. <math>f(\alpha_1, c\beta_1 + \beta_2) = cf(\alpha_1, \beta_1) + f(\alpha_1, \beta_2)</math></li> </ol> <p>Now, <math>B(ca_1 + a_2, b) = (ca_1 + a_2)b = cB(a_1, b) + B(a_2, b)</math></p> <p>And, <math>B(a_1, cb_1 + b_2) = a_1(cb_1 + b_2) = cB(a_1, b_1) + B(a_1, b_2)</math></p> <p>Hence <math>B</math> is a bilinear form.</p> <p>Again a bilinear form <math>f</math> is symmetric if <math>f(\alpha, \beta) = f(\beta, \alpha)</math></p> <p>Here, <math>B(a, b) = ab = ba = B(b, a)</math> hence <math>B</math> is symmetric.</p> <p>A symmetric bilinear <math>f</math> is positive definite if</p> $f(\alpha, \alpha) > 0 \quad \forall \alpha \neq 0$ <p>Here, <math>B(a, a) = a^2 &gt; 0 \quad \forall a \neq 0</math></p> <p><b>Conclusion:</b> <math>B</math> is symmetric and positive definite bilinear form.</p> <p>Hence Correct.</p> |
| $B$ is symmetric but not positive definite  | <p>From previous proof it is obvious that</p> <p><math>B</math> is both symmetric as well as positive definite</p> <p>Hence incorrect</p>  |
| $B$ neither linear nor bilinear   | <p>From previous proofs it is obvious that</p> <p><math>B</math> is bilinear.</p> <p>Hence incorrect.</p>  |
| Result  | $B$ is symmetric and positive definite bilinear form   |

TABLE 1: Finding Correct Option