

Assignment 12

Matish Singh Tanwar

Abstract—This document finds the eigen values of a special matrix.

Download all latex-tikz codes from

https://github.com/Matish007/Matrix-Theory-EE5609-/tree/master/Assignment_12

1 PROBLEM

Let n be an odd number ≥ 7 . Let $\mathbf{A} = (a_{ij})$ be an $n \times n$ matrix with $a_{i,i+1} = 1$ for all $i=1,2,\dots,n-1$ and $a_{n,1}=1$. Let $a_{i,j}=0$ for all other pairs (i,j) . Find eigen values.

2 SOLUTION

We can represent our matrix as:

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 & \dots & 1 \\ 0 & 0 & 1 & \dots & 1 \\ & & \cdot & & \\ & & \cdot & & \\ 0 & 0 & 0 & \dots & 1 \\ 1 & 0 & 0 & \dots & 0 \end{pmatrix} \quad (2.0.1)$$

$$\mathbf{A}^T = \begin{pmatrix} 0 & 0 & 0 & \dots & 1 \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ & & \cdot & & \\ & & \cdot & & \\ 0 & 0 & 0 & \dots & 1, 0 \end{pmatrix} \quad (2.0.2)$$

$$\mathbf{C} = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 & -a_0 \\ 1 & 0 & 0 & \dots & 0 & -a_1 \\ 0 & 1 & 0 & \dots & 0 & -a_2 \\ & & \cdot & & \\ & & \cdot & & \\ 0 & 0 & 0 & \dots & 1 & -a_{n-1} \end{pmatrix} \quad (2.0.3)$$

\mathbf{A} is our given matrix. We know that Characteristic Equation of \mathbf{A} and \mathbf{A}^T is same. \mathbf{C} is the companion matrix. We also know characteristic equation of \mathbf{C} is

$$x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_0 \quad (2.0.4)$$

Comparing (2.0.2) with (2.0.3) we get:

$$a_0 = -1, a_1 = a_2 = a_3 = a_4 = \dots = a_{n-1} = 0 \quad (2.0.5)$$

Substituting (2.0.5) into (2.0.4) we get:

$$x^n - 1 = 0 \quad (2.0.6)$$

By Cayley-Hamilton Theorem:

$$\lambda^n - 1 = 0 \quad (2.0.7)$$

$$(2.0.8)$$

$\lambda = n^{th}$ roots of unity.

Since n is odd so one of the eigen values cannot be -1. So $n-1$ imaginary eigen values and one real eigen value i.e. 1