Assignment 3

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Abstract—**This document solves question based on triangle.**

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1 PROBLEM

Line L is the bisector of $\angle A$ and B is any point on L.BP and BQ are perpendiculars from B to the arms of $\angle A$.Show that:-

$$a) \quad \triangle APB \cong \triangle AQB \qquad (1.0.1)$$
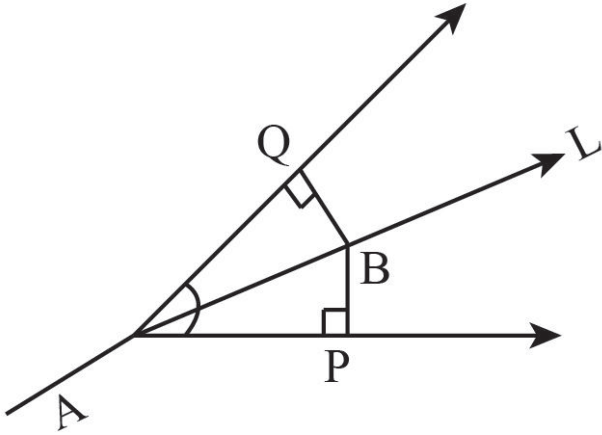$$b) \quad BP = BQ \qquad (1.0.2)$$

Fig. 0

2 EXPLANATION

Given:-

$$\angle BAP = \angle BAQ = \alpha \qquad (2.0.1)$$
$$\angle AQB = \angle APB \qquad (2.0.2)$$

In $\triangle ABQ$

$$\angle ABQ + \angle AQB + \angle BAQ = 180° \qquad (2.0.3)$$

In $\triangle ABP$

$$\angle ABP + \angle APB + \angle BAP = 180° \qquad (2.0.4)$$

Subtracting (2.0.3) and (2.0.4) and using (2.0.1) and (2.0.2) we get,

$$\angle ABQ = \angle ABP \qquad (2.0.5)$$

Since,line BP and BQ are perpendicular to AP and AQ respectively..So,their respective dot product will be zero.We get,

$$(\mathbf{B} - \mathbf{Q})^T(\mathbf{A} - \mathbf{Q}) = 0 \qquad (2.0.6)$$
$$(\mathbf{B} - \mathbf{P})^T(\mathbf{A} - \mathbf{P}) = 0 \qquad (2.0.7)$$

We know that,$(\mathbf{B} - \mathbf{P})^T(\mathbf{B} - \mathbf{P}) = \|\mathbf{B} - \mathbf{P}\|^2$
Also let

$$\|\mathbf{B} - \mathbf{A}\|^2 = k^2 \qquad (2.0.8)$$

$$(\mathbf{B} - \mathbf{P})^T(\mathbf{B} - \mathbf{P}) = (\mathbf{B} - \mathbf{A} + \mathbf{A} - \mathbf{P})^T(\mathbf{B} - \mathbf{A} + \mathbf{A} - \mathbf{P}) \qquad (2.0.9)$$

$$\begin{aligned}
\|\mathbf{B} - \mathbf{P}\|^2 &= (\mathbf{B} - \mathbf{A})^T(\mathbf{B} - \mathbf{A}) \\
&\quad +(\mathbf{A} - \mathbf{P})^T(\mathbf{A} - \mathbf{P}) \\
&\quad +(\mathbf{A} - \mathbf{P})^T(\mathbf{B} - \mathbf{A}) \\
&\quad +(\mathbf{B} - \mathbf{A})^T(\mathbf{A} - \mathbf{P}) \\
&= \|\mathbf{B} - \mathbf{A}\|^2 +\|\mathbf{A} - \mathbf{P}\|^2 \\
&\quad +2\|\mathbf{A} - \mathbf{P}\|\|\mathbf{B} - \mathbf{A}\|
\end{aligned} \qquad (2.0.10)$$

Substituting (2.0.8) in (2.0.10) we get,
And $(\mathbf{A} - \mathbf{P})^T(\mathbf{B} - \mathbf{A})=(\mathbf{B} - \mathbf{A})^T(\mathbf{A} - \mathbf{P})$
$= \|\mathbf{A} - \mathbf{P}\| \|\mathbf{B} - \mathbf{A}\| \cos\alpha$

$$\|\mathbf{B} - \mathbf{P}\|^2 = k^2 +\|\mathbf{A} - \mathbf{P}\|^2 + 2k\|\mathbf{A} - \mathbf{P}\| \cos\alpha \qquad (2.0.11)$$

Similarly,we get

$$\left\| \mathbf{B} - \mathbf{Q} \right\|^2 = k^2 + \left\| \mathbf{A} - \mathbf{Q} \right\|^2 + 2k \left\| \mathbf{A} - \mathbf{Q} \right\| \cos \alpha \tag{2.0.12}$$

$$\cos \alpha = \frac{(\mathbf{B} - \mathbf{A})^T (\mathbf{P} - \mathbf{A})}{k \left\| \mathbf{P} - \mathbf{A} \right\|} = \frac{(\mathbf{B} - \mathbf{A})^T (\mathbf{Q} - \mathbf{A})}{k \left\| \mathbf{Q} - \mathbf{A} \right\|} \tag{2.0.13}$$

$$(\mathbf{B} - \mathbf{A})^T (\mathbf{P} - \mathbf{A}) = (\mathbf{B} - \mathbf{P} + \mathbf{P} - \mathbf{A})^T (\mathbf{P} - \mathbf{A}) \tag{2.0.14}$$

$$\implies (\mathbf{B} - \mathbf{P})^T (\mathbf{P} - \mathbf{A}) + \|\mathbf{P} - \mathbf{A}\|^2 \tag{2.0.15}$$

$$(\mathbf{B} - \mathbf{A})^T (\mathbf{Q} - \mathbf{A}) = (\mathbf{B} - \mathbf{Q} + \mathbf{Q} - \mathbf{A})^T (\mathbf{Q} - \mathbf{A}) \tag{2.0.16}$$

$$\implies (\mathbf{B} - \mathbf{Q})^T (\mathbf{Q} - \mathbf{A}) + \left\| \mathbf{Q} - \mathbf{A} \right\|^2 \tag{2.0.17}$$

Substituting (2.0.6) and (2.0.7) in (2.0.17) and (2.0.15) respectively we get,

$$(\mathbf{B} - \mathbf{A})^T (\mathbf{P} - \mathbf{A}) = \|\mathbf{P} - \mathbf{A}\|^2 \tag{2.0.18}$$

$$(\mathbf{B} - \mathbf{A})^T (\mathbf{Q} - \mathbf{A}) = \left\| \mathbf{Q} - \mathbf{A} \right\|^2 \tag{2.0.19}$$

Substituting (2.0.18) and (2.0.19) in (2.0.13) we get,

$$\cos \alpha = \|\mathbf{P} - \mathbf{A}\| = \left\| \mathbf{Q} - \mathbf{A} \right\| \tag{2.0.20}$$

Substituting (2.0.20) in (2.0.11) and (2.0.12) we get,

$$\|\mathbf{B} - \mathbf{P}\| = \left\| \mathbf{B} - \mathbf{Q} \right\| \tag{2.0.21}$$

From (2.0.21) we can say that $\mathbf{B}$ is equidistant from the arms of $\angle A$,where $\mathbf{P}$ and $\mathbf{Q}$ are the points on the arms of $\angle A$ Using (2.0.1),(2.0.2),(2.0.21) and by AAS(Angle Angle Side) property of congruency we can say that:-

$$\triangle APB \cong \triangle AQB \tag{2.0.22}$$