Assignment 17

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***Abstract*—This document solves a problem of Linear Algebra.**

Download all latex-tikz codes from

https://github.com/Matish007/Matrix−Theory−EE5609−/tree/master/Assignment_17

1 Problem

Let $\mathbf{P}$ be a 2×2 complex matrix such that

$$\mathbf{P}^{\theta}\mathbf{P} = \mathbf{I} \tag{1.0.1}$$

where $\mathbf{P}^{\theta}$ is the conjugate transpose of $\mathbf{P}$.Then the eigen values of $\mathbf{P}$ are

1) real
2) complex conjugates of each othe
3) reciprocals of each other
4) of modulus 1

2 SOLUTION

Options	**Explanation**
REAL Counter Example	$\mathbf{P} = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$ $\mathbf{P}^{\theta} = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}$ $\mathbf{P}^{\theta}\mathbf{P} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I}$ Eigen values of $\mathbf{P}$ are i, i which are not real Hence,incorrect.
Complex Conjugates of each other.	From above,(i, i) are not complex conjugate of each other Hence,incorrect.
Reciprocals of each other	Reciprocal of $i = \frac{1}{i} = \frac{i^4}{i} = i^3 \neq i$ Hence,incorrect.
of modulus 1 Proof	$\mathbf{PV} = \lambda\mathbf{V}$ where,$\mathbf{V}$ is eigen vector of $\mathbf{P}$ and λ is eigen value of $\mathbf{P}$ Taking conjugate transpose on both sides,we get $\mathbf{V}^{\theta}\mathbf{P}^{\theta} = \lambda^{\theta}\mathbf{V}^{\theta}$ $\mathbf{V}^{\theta}\mathbf{P}^{\theta}\mathbf{PV} = \lambda^{\theta}\mathbf{V}^{\theta}\lambda\mathbf{V} \qquad ,\because \mathbf{PV} = \lambda\mathbf{V}$ $\mathbf{V}^{\theta}\mathbf{IV} = \lambda^{\theta}\lambda\mathbf{V}^{\theta}\mathbf{V} \qquad ,\because \mathbf{P}^{\theta}\mathbf{P} = \mathbf{I}$ $(1 - \lambda^{\theta}\lambda)\mathbf{V}^{\theta}\mathbf{V} = 0$ Since,$\mathbf{V}$ is not zero. $(1 - \lambda^{\theta}\lambda) = 0$ $\lambda^{\theta}\lambda = 1$ $\lVert\lambda\rVert^2 = 1$ $\lambda = 1$ Hence,correct.

TABLE 1: Finding Correct Option