Assignment 13

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***Abstract*—This document solves a problem of Linear Algebra.**

Download all latex-tikz codes from

https://github.com/Matish007/Matrix−Theory−EE5609−/tree/master/Assignment_13

1 Problem

Let $\mathbf{A}$ be a $m \times n$ matrix and $\mathbf{B}$ be a $n \times m$ matrix over real numbers,with

$$m < n \tag{1.0.1}$$

Then,

1) $\mathbf{AB}$ is always nonsingular
2) $\mathbf{AB}$ is always singular
3) $\mathbf{BA}$ is always nonsingular
4) $\mathbf{BA}$ is always singular

2 SOLUTION

$$rank(\mathbf{A}) \leq \min(m, n) \tag{2.0.1}$$
$$\implies \leq m, \because m < n \tag{2.0.2}$$
$$rank(\mathbf{B}) \leq \min(n, m) \tag{2.0.3}$$
$$\implies \leq m, \because m < n \tag{2.0.4}$$

We also know that $\mathbf{AB}$ will be a $m \times m$ matrix and $\mathbf{BA}$ will be a $n \times n$ matrix.

$$rank(\mathbf{AB}) \leq \min(rank(\mathbf{A}), rank(\mathbf{B})) \tag{2.0.5}$$
$$\implies \leq m \tag{2.0.6}$$
$$rank(\mathbf{BA}) \leq \min(rank(\mathbf{B}), rank(\mathbf{A})) \tag{2.0.7}$$
$$\implies \leq m \tag{2.0.8}$$

Options	Explanation
AB is always nonsingular	$rank(\mathbf{AB}) \leq m$ Let,$rank(\mathbf{AB}) = k, k < m$. So,there are $m - k$ linearly dependent columns or rows So,**AB** will be singular Hence,incorrect
Example	$\mathbf{A}=\begin{pmatrix}1 & 2 & 3\\2 & 4 & 6\end{pmatrix},\mathbf{B}=\begin{pmatrix}1 & 3\\2 & 6\\5 & 6\end{pmatrix}$ $\mathbf{AB}=\begin{pmatrix}20 & 33\\40 & 66\end{pmatrix}, rank(\mathbf{AB}) = 1$ 2^{nd} row is linearly dependent on 1^{st} row. **AB** is singular
AB is always singular	$rank(\mathbf{AB}) \leq m$ Let,$rank(\mathbf{AB}) = m$ So,there are 0 linearly dependent columns or rows So,**AB** will be nonsingular Hence,incorrect
Example	$\mathbf{A}=\begin{pmatrix}1 & 2 & 3\\2 & 4 & 5\end{pmatrix},\mathbf{B}=\begin{pmatrix}1 & 3\\2 & 4\\5 & 6\end{pmatrix}$ $\mathbf{AB}=\begin{pmatrix}20 & 29\\35 & 52\end{pmatrix}, rank(\mathbf{AB}) = 2$ **AB** is nonsingular
BA is always nonsingular	$rank(\mathbf{BA}) \leq m. rank(\mathbf{BA})$ can be atmost m **BA** is $n \times n$ matrix.$n > m$. So,there are atleast $n - m$ linearly dependent columns or rows. So,**BA** will be singular always. Hence,incorrect
Example	$\mathbf{A}=\begin{pmatrix}1 & 2 & 3\\2 & 4 & 5\end{pmatrix},\mathbf{B}=\begin{pmatrix}1 & 3\\2 & 4\\5 & 6\end{pmatrix}$ $\mathbf{BA}=\begin{pmatrix}7 & 14 & 18\\10 & 20 & 26\\17 & 34 & 45\end{pmatrix}, rank(\mathbf{BA}) = 2$ 2^{nd} column is linearly dependent on 1^{st} column **BA** is singular
BA is always singular	$rank(\mathbf{BA}) \leq m. rank(\mathbf{BA})$ can be atmost m **BA** is $n \times n$ matrix.$n > m$. So,there are atleast $n - m$ linearly dependent columns or rows. So,**BA** will be singular always. Hence,correct
Example	Same example as above. **BA** is always singular.

TABLE 1: Finding Correct Option