Assignment 12

Matish Singh Tanwar

Abstract—**This document finds the eigen values of a special matrix.**

Download all latex-tikz codes from

https://github.com/Matish007/Matrix−Theory−EE5609−/tree/master/Assignment_12

1 Problem

Let n be an odd number ≥ 7.Let,

$$\mathbf{A} = [a_{ij}] \tag{1.0.1}$$

be and $n \times n$ matrix with,

$$a_{i,i+1} = 1, \forall (i = 1, 2, ...n - 1) \tag{1.0.2}$$

and $a_{n,1} = 1$. Let $a_{ij} = 0$ for all the other pairs (i, j). Then we can conclude that,

1) $\mathbf{A}$ has 1 as an eigenvalue
2) $\mathbf{A}$ has -1 as an eigenvalue
3) $\mathbf{A}$ has at least one eigenvalue with multiplicity ≥ 2
4) $\mathbf{A}$ has no real eigenvalues

2 SOLUTION

We can represent our matrix as:

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & ..... & 0 & 0 \\ 0 & 0 & 1 & ..... & 0 & 0 \\ . \\ . \\ 0 & 0 & 0 & ..... & 0 & 1 \\ 1 & 0 & 0 & ..... & 0 & 0 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{A^T} = \begin{pmatrix} 0 & 0 & 0 & ..... & 0 & 1 \\ 1 & 0 & 0 & ..... & 0 & 0 \\ 0 & 1 & 0 & ..... & 0 & 0 \\ . \\ . \\ 0 & 0 & 0 & .... & 1 & 0 \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{C} = \begin{pmatrix} 0 & 0 & 0 & .... & 0 & -a_0 \\ 1 & 0 & 0 & .... & 0 & -a_1 \\ 0 & 1 & 0 & .... & 0 & -a_2 \\ . \\ . \\ 0 & 0 & 0 & .... & 1 & -a_{n-1} \end{pmatrix} \tag{2.0.3}$$

$\mathbf{A}$ is our given matrix.We know that Characteristic Equation of $\mathbf{A}$ and $\mathbf{A^T}$ is same.$\mathbf{C}$ is the companion matrix.We also know characteristic equation of $\mathbf{C}$ is

$$x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_0 \tag{2.0.4}$$

Comparing (2.0.2) with (2.0.3) we get:

$$a_0 = -1, a_1 = a_2 = a_3 = a_4 = ... = a_{n-1} = 0 \tag{2.0.5}$$

Substituting (2.0.5) into (2.0.4) we get:

$$x^n - 1 = 0 \tag{2.0.6}$$

By Cayley-Hamilton Theorem:

$$\lambda^n - 1 = 0 \tag{2.0.7}$$

λ=n^{th} roots of unity.
Since n is odd so one of the eigen values cannot be -1. So $n-1$ imaginary eigen values and one real eigen value i.e. 1