Assignment 7

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***Abstract*—This document solves question based on QR decomposition.**

Download all python codes from

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https://github.com/Matish007/Matrix-Theory-EE5609-/tree/master/Assignment_7/Codes
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and latex-tikz codes from

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https://github.com/Matish007/Matrix-Theory-EE5609-/tree/master/Assignment_7
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1 Problem

Find the QR decomposition of

$$\mathbf{A} = \begin{pmatrix} 2 & 5 \\ 1 & 4 \end{pmatrix} \tag{1.0.1}$$

2 Solution

The matrix $\mathbf{A}$ can be written as,

$$\mathbf{A} = \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} \tag{2.0.1}$$

where $\mathbf{a}$ and $\mathbf{b}$ are column vectors. From (1.0.1)

$$\mathbf{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{b} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \tag{2.0.3}$$

The QR decomposition of the given matrix is given by

$$\mathbf{A} = \mathbf{QR} \tag{2.0.4}$$

here $\mathbf{R}$ is a upper triangular matrix and Q is a orthogonal matrix.

$$\mathbf{Q^T Q} = \mathbf{I} \tag{2.0.5}$$

where

$$\mathbf{Q} = \begin{pmatrix} \mathbf{q_1} & \mathbf{q_2} \end{pmatrix} \quad \mathbf{R} = \begin{pmatrix} r_1 & r_2 \\ 0 & r_3 \end{pmatrix} \tag{2.0.6}$$

The above values are given by,

$$r_1 = \|\mathbf{a}\| \tag{2.0.7}$$

$$\mathbf{q_1} = \frac{\mathbf{a}}{r_1} \tag{2.0.8}$$

$$r_2 = \frac{\mathbf{q_1^T b}}{\|q_1\|^2} \tag{2.0.9}$$

$$\mathbf{q_2} = \frac{\mathbf{b} - r_2\mathbf{q_1}}{\|\mathbf{b} - r_2\mathbf{q_1}\|} \tag{2.0.10}$$

$$r_3 = \mathbf{q_2^T b} \tag{2.0.11}$$

Substituting (2.0.2) and (2.0.3) we get

$$r_1 = \sqrt{2^2 + 1^2} = \sqrt{5} \tag{2.0.12}$$

$$\mathbf{q_1} = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix} \tag{2.0.13}$$

$$r_2 = \frac{1}{\left(\sqrt{\frac{4}{5} + \frac{1}{5}}\right)^2} \begin{pmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} 5 \\ 4 \end{pmatrix} = \frac{14}{\sqrt{5}} \tag{2.0.14}$$

$$\mathbf{q_2} = \frac{\sqrt{5}}{3} \left(\begin{pmatrix} 5 \\ 4 \end{pmatrix} - \frac{14}{\sqrt{5}} \begin{pmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix} \right) = \begin{pmatrix} \frac{-1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix} \tag{2.0.15}$$

$$r_3 = \begin{pmatrix} \frac{-1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} 5 \\ 4 \end{pmatrix} = \frac{3}{\sqrt{5}} \tag{2.0.16}$$

Hence substituting these values in (2.0.6) and then back in (2.0.4) we get,

$$\mathbf{A} = \begin{pmatrix} \frac{2}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} \sqrt{5} & \frac{14}{\sqrt{5}} \\ 0 & \frac{3}{\sqrt{5}} \end{pmatrix} \tag{2.0.17}$$

Hence QR decomposition is,

$$\begin{pmatrix} 2 & 5 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} \sqrt{5} & \frac{14}{\sqrt{5}} \\ 0 & \frac{3}{\sqrt{5}} \end{pmatrix} \tag{2.0.18}$$