# Assignment 13

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***Abstract*—This document solves a problem of Linear Algebra.**

Download all latex-tikz codes from

https://github.com/Matish007/Matrix−Theory−EE5609−/tree/master/Assignment_13

## 1 Problem

Let $\mathbf{A}$ be a $m \times n$ matrix and $\mathbf{B}$ be a $n \times m$ matrix over real numbers,with

$$m < n \tag{1.0.1}$$

Then,

1) $\mathbf{AB}$ is always nonsingular
2) $\mathbf{AB}$ is always singular
3) $\mathbf{BA}$ is always nonsingular
4) $\mathbf{BA}$ is always singular

## 2 SOLUTION

$$rank(\mathbf{A}) \leq \min(m, n) \tag{2.0.1}$$

$$\implies \leq m, \because m < n \tag{2.0.2}$$

$$rank(\mathbf{B}) \leq \min(n, m) \tag{2.0.3}$$

$$\implies \leq m, \because m < n \tag{2.0.4}$$

We also know that $\mathbf{AB}$ will be a $m \times m$ matrix and $\mathbf{BA}$ will be a $n \times n$ matrix.

$$rank(\mathbf{AB}) \leq \min(rank(\mathbf{A}), rank(\mathbf{B})) \tag{2.0.5}$$

$$\implies \leq m \tag{2.0.6}$$

$$rank(\mathbf{BA}) \leq \min(rank(\mathbf{B}), rank(\mathbf{A})) \tag{2.0.7}$$

$$\implies \leq m \tag{2.0.8}$$

| Options | Explanation |
|---|---|
| **AB** is always nonsingular | $rank(\mathbf{AB}) \leq m$<br>Let,$rank(\mathbf{AB}) = k$,$k < m$.<br>So,there are $m - k$ linearly dependent columns.<br>So,**AB** will be singular<br>Hence,incorrect |
| **AB** is always singular | $rank(\mathbf{AB}) \leq m$<br>Let,$rank(\mathbf{AB}) = m$<br>So,there are 0 linearly dependent columns.<br>So,**AB** will be nonsingular<br>Hence,incorrect |
| **BA** is always nonsingular | $rank(\mathbf{BA}) \leq m$.$rank(\mathbf{BA})$ can be atmost $m$<br>**BA** is $n \times n$ matrix.$n > m$.<br>So,there are atleast $n - m$ linearly dependent columns.<br>So,**BA** will be singular always.<br>Hence,incorrect |
| **BA** is always singular | $rank(\mathbf{BA}) \leq m$.$rank(\mathbf{BA})$ can be atmost $m$<br>**BA** is $n \times n$ matrix.$n > m$.<br>So,there are atleast $n - m$ linearly dependent columns.<br>So,**BA** will be singular always.<br>Hence,correct |

TABLE 1: Finding Correct Option