Applied Stochastics With Applications in Security and Privacy - Exercises, part 3

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Exercise 1. The National Student Loan Survey collects data to examine questions related to the amount of money that borrowers owe. The survey selected a sample of 1280 borrowers who began repayment of their loans between four and six months prior to the study. The mean of the debt for undergraduate study was \$18 900 and the standard deviation was about \$49 000. This distribution is clearly skewed but because our sample size is quite large, we can rely on the central limit theorem to assure us that the confidence interval based on the Normal distribution will be a good approximation. Compute a 95% confidence interval for the true mean debt for all borrowers. Suppose then that the sample size is only 320 and compute a 95% confidence interval for that case. Compare these results and draw conclusions.

Solution:
Data:

$$\bar{X} = \$18900, \sigma = \$49000, z^* = 1.96, n_1 = 1280, n_2 = 320$$

Calculations:

$$z^* * \frac{\sigma}{\sqrt{n_1}} \approx 2684.4$$
$$z^* * \frac{\sigma}{\sqrt{n_2}}, \approx 5368.8$$

Conclusion:
To assume, we can say that z^* is taken for 95% confidence level under Normal Distribution. It is because the confidence interval for $n = 1280$ is $\bar{X} \pm 2684.4$ and for $n = 360$ it is $\bar{X} \pm 5368.8$. It could be expected, because $5368.8/2684.4 = 2$, and the change in sample size was by a factor of 4 ($\sqrt{4} = 2$). Changes in interval are inversely proportional to the square root of change in the sample size.

Exercise 2. You want to rent an unfurnished one-bedroom apartment in Boston next year. The mean monthly rent for a random sample of 10 apartments advertised in the local newspaper is \$1400. Assume that the standard deviation is \$220.

1. Find a 95% confidence interval for the mean monthly rent for unfurnished one-bedroom apartments available for rent in this community.
2. How large a sample of one-bedroom apartments would be needed to estimate the mean μ with a margin of error of \$50 with 90% confidence? What should be the sample size if we wanted 99% confidence?

Solution:
Data:

$$n = 10, \bar{X} = \$1400, \sigma = \$220, z^* = 1.96$$

Calculations:

$$z^* * \frac{\sigma}{\sqrt{n}} \approx 136.4$$

The confidence interval would be $\bar{X} \pm 136.4$ in this case. The z-score values are as follows:

$$z^* = 1.645 \text{ for confidence level } 90\%$$
$$z^* = 2.575 \text{ for confidence level } 99\%$$

In the case of 90% confidence level:

$$n = \left(1.645 \cdot \frac{220}{50}\right)^2 \approx 53$$

and in the case of 99% confidence level:

$$n = \left(2.575 \cdot \frac{220}{50}\right)^2 \approx 129$$

Conclusion:
Case (90%) confidence - 53 apartments would be required. Case (99%), the sample size required would be greater - 129.

Exercise 3. To assess the accuracy of a laboratory scale, a standard weight known to weigh 10 grams is weighed repeatedly. The scale readings are Normally distributed with unknown mean (this mean is 10 grams if the scale has no bias). The standard deviation of the scale readings is known to be 0.0002 gram.

1. The weight is measured five times. The mean result is 10.0023 grams. Give a 98% confidence interval for the mean of repeated measurements of the weight.
2. How many measurements must be averaged to get a margin of error of 0.0001 with 98% confidence?

Solution:
Data:

$$n = 5, \bar{X} = 10.0023g, \sigma = 0.0002g, z^* = 2.326$$

The z^* is taken for 98% confidence
Calculations:

$$z^* * \frac{\sigma}{\sqrt{n}} \approx 2.08 \times 10^{-4}$$

The confidence interval would be $\bar{X} \pm 2.08 \times 10^{-4} g$

$$m = 1 \times 10^{-4}$$
$$n = (z^* * \frac{\sigma}{m})^2 \approx 22$$

Conclusion:

To sum up: 22 readings would need to be averaged in order to get the desired margin of error with 98% confidence.

Exercise 4. Statistics can help decide the authorship of literary works. Sonnets by a certain Elizabethan poet are known to contain an average of $\mu = 8.9$ new words (words not used in the poet's other works). The standard deviation of the number of new words is $\sigma = 2.5$ Now a manuscript with 6 new sonnets has come to light, and scholars are debating whether it is the poet's work. The new sonnets contain an average of $\bar{x} = 10.2$ words not used in the poet's known works. We expect poems by another author to contain more new words, so we would like to see if we have evidence that the new sonnets are not by our pet.

1. State H_0 and H_1.
2. Give the z test statistic and its p-value. Using the 5% level of significance, what do you conclude about the authorship of the new poems?

Solution:
Hypotheses:

$$H_0 : \mu = 8.9$$
$$H_1 : \mu > 8.9$$

Calculations:

$$z_{stat} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$
$$z_{stat} = \frac{10.2 - 8.9}{\frac{2.5}{\sqrt{6}}} = 1.274$$
$$p \approx 0.2027$$
$$\frac{p}{2} \approx 0.1014$$

Conclusion:

Since the p-value (10%) is greater than the level of significance (5%), there is no sufficient evidence to reject the null-hypothesis.

Exercise 5. The level of calcium in the blood in healthy young adults varies with mean about 9.5 milligrams per deciliter and standard deviation about $\sigma = 0.4$. A clinic in rural Guatemala measures the blood calcium level of 160 healthy pregnant women at their first visit for prenatal care. The mean is $\bar{x} = 9.57$. Is this an indication that the mean calcium level in the population from which these women come differs from 9.5?

1. State H_0 and H_1.
2. Carry out the test and give the p-value, assuming that $\sigma = 0.4$ in this population. Could H_0 be rejected at the 5% level of significance? What about at the 1% level? Report your conclusion.
3. Give a 95% confidence interval for the mean calcium level μ in this population. We are confident that μ lies quite close to 9.5. This illustrates the fact that a test based on a large sample (n = 160 here) will often declare even a small deviation from H_0 to be statistically significant.

Solution:
Hypotheses:

$$H_0 : \mu = 9.5$$
$$H_1 : \mu \neq 9.5$$

Calculations for 2:

$$z_{stat} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$
$$z_{stat} = \frac{9.57 - 9.5}{\frac{0.4}{\sqrt{160}}} \approx 2.214$$
$$p \approx 0.0268$$

Conclusion for 2:

The p-value of about 2.68% so H_0 should be rejected when assuming a 5% level of significance. For a level of significance equal to 1%, there is no sufficient evidence to reject H_0
Calculations for 3:

$$z^* = 1.96$$
$$m = z^* * \frac{\sigma}{\sqrt{n}}$$
$$m = 1.96 * \frac{0.4}{\sqrt{160}} \approx 0.062$$

Conclusion for 3:

With a confidence interval of $\bar{X} \pm 0.062\ mg$, the measured mean of 9.57 mg is outside of the bounds for a confidence of 95%,

Exercise 6. A farmer claims to be able to produce larger tomatoes. To test this claim, a tomato variety that has a mean diameter size of 8.2 centimeters with a standard deviation of 2.4 centimeters is used. If a sample of 36 tomatoes yielded a sample mean of 9.1 centimeters, does this prove that the mean size is indeed larger? Assume that the population standard deviation remains equal to 2.4, and use the 5 percent level of significance.

1. What are H_0 and H_1 ?
2. Calculate the z test statistic and the corresponding p-value. What do you conclude about the farmer's claim?

Solution:
Hypotheses:

$$H_0 : \mu \leq 8.2$$
$$H_1 : \mu > 8.2$$

Calculations:

$$z_{stat} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$
$$z_{stat} = \frac{9.1 - 8.2}{\frac{2.4}{\sqrt{36}}} \approx 2.25$$
$$p \approx 0.0244$$
$$\frac{p}{2} = 0.0122$$

Conclusion:

P-value is much lower than α, which leads to the rejection of H_0. It means that the data supports the farmer's claim.

Exercise 7. You want to rent an unfurnished one-bedroom apartment for next semester. You take a random sample of 10 apartments advertised in the local newspaper and record the rental rates. Here are the rents (in dollars per month):

$$500, 650, 600, 505, 450, 550, 515, 495, 650, 395$$

1. Find a 95% confidence interval for the mean monthly rent for unfurnished one- bedroom apartments available for rent in this community.
2. If you were to use 90% confidence, rather than 95% confidence, would the margin of error be larger or smaller? What about 99% confidence? Explain your answer.
3. Do these data give good reason to believe that the mean rent of all advertised apartments is greater than \$500 per month? State the hypotheses, find the t statistic and the corresponding p-value. Based on the calculated results state your conclusion.

Solution:
Data:

$$n = 10, \bar{X} = 531, S \approx 82.792, t^* = t_{9,0.025} = 2.262, \bar{X} \pm t^* * \frac{s}{\sqrt{n}} \approx 531 \pm 59.22$$

Value t^* comes from table D2
Calculations for 2:

$$\text{Formula: } \bar{X} \pm t^* * \frac{s}{\sqrt{n}}$$

Confidence level for:

$$90\%\ t^* = 1.83$$
$$95\%\ t^* = 2.262$$
$$99\%\ t^* = 3.25$$

Conclusion for 2:

If we use a bigger confidence level, the interval will increase cause it will cover more values in calculation. This results from the formula has been used for the calculations.
Hypotheses for 3:

$$H_0 : \mu \leq \$500$$
$$H_1 : \mu > \$500$$

Calculations for 3:

$$t = \sqrt{n} * \frac{\bar{X} - \mu_0}{s} = \sqrt{10} * \frac{531 - 500}{82.792} = \frac{98.03060747}{82.792} \approx 1,1841$$

Conclusion for 3:

Taking values from table D2, we get:

$$0.1 < P < 0.15$$

Since P-value is greater than $\alpha > 0.05$, we can't reject H_0. That the mean rent of all advertised apartments is not greater than \$500 per month.

Exercise 8. You will have complete sales information for last month in a week but right now you have data from a random sample of 50 stores. The mean change in sales in this sample is +4.8% and the standard deviation of the change is 15%. Are average sales for all stores different from last month?

1. State appropriate null and alternative hypotheses. Explain how you decided between the one- and two-sided alternatives.
2. Find the t statistic and the corresponding p-value. Does this indicate a strong evidence against the null hypothesis? At what significance level you can reject H_0 ? State your conclusions.
3. If the test gives strong evidence against the null hypothesis, would you conclude that sales are up in every one of your stores? Explain your answer.

Solution:
Hypotheses:

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu \neq \mu_0$$

Since the problem is to state whether the average sales are different from last month, the natural choice is the two-sided alternative.
Calculations for 2:

$$t = \sqrt{n}\frac{\bar{X} - \mu_0}{S} = \sqrt{50}\frac{4.8}{15} \approx 2.26$$
$$t_{49,0.01} > t > t_{49,0.02}\text{(upper tail only)}$$

Conclusion for 2:

Therefore it can be assumed that the p-value lies somewhere in the range $(0.01, 0.02)$. With a level of significance equal to 2%, the null hypothesis can be rejected, which means the evidence strongly opposes the hypothesis.
Conclusion for 3:

One of the assumptions in this exercises is that S is a good estimator of σ, and therefore if the mean change in sales differs, then it is expected that the change is similar across all the stores (so that the standard deviation remains similar). The test gives strong evidence against the null hypothesis, so the suggested conclusion seems natural.

Exercise 9. A car is advertised as getting at least 31 miles per gallon in highway driving on trips of at least 100 miles. Suppose the miles per gallon obtained in 8 independent experiments (each consisting of a nonstop highway trip of 100 miles) are:

$$28, 29, 31, 27, 30, 35, 25, 29.$$

1. If we want to check if these data disprove the advertising claim what should we take as the null and alternative hypotheses?

2. Find the t statistic and the corresponding p-value. Is the claim disproved at the 5% level of significance? What about at the 1% level?

Solution:
Hypotheses:

$$H_0 : \mu \geq 31$$

$$H_1 : \mu < 31$$

Data:

$$n = 8, \bar{X} = 29.25, S = 2.964, t_{7,0.05} = 1.895, t_{7,0.01} = 2.998$$

Values $t_{7,0.05}$ and $t_{7,0.01}$ comes from table D2
Calculations

$$T = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}}$$

$$T = \frac{29.25 - 31}{\frac{2.964}{\sqrt{8}}} \approx -1.67$$

Conclusion

Since $t_{7,0.05} > |T|$, $p > 0.05$. ($T < 0$ means only that the measured mean is lower than expected). Therefore for any level of significance that is not higher than 5%, the null hypothesis will not be rejected.
Since z_{stat} = -2.81 is not less than $-t_{7,0.01} = -1.895$, H_0 is not rejected at the 1% level of significance. That is, the data are not inconsistent with the null

Exercise 10. Let p denote the proportion of voters in a large city who are in favor of restructuring the city government, and consider a test of the hypothesis

$$H_0 : p \geq 0.60 \text{ against } H_1 : p < 0.60$$

A random sample of n voters indicated that x are in favor of restructuring. In each of the following cases, would a significance-level-α test result in rejection of H_0 ?

1. $n = 100,\ x = 50,\ \alpha = 0.10$
2. $n = 100,\ x = 50,\ \alpha = 0.05$
3. $n = 100,\ x = 50,\ \alpha = 0.01$
4. $n = 200,\ x = 100,\ \alpha = 0.01$

Solution:

Hypotheses for all cases (1-4):

$$H_0 : \mu \geq 0.6$$
$$H_1 : \mu < 0.6$$

Data for 1-3 cases:

$$n = 100, X = 50, p_0 = 0.6, np_0 = 60, \sigma = \sqrt{np_0(1 - p_0)} \approx 4.8990$$

Calculations for 1-3

$$p - value = Pr[X \leq 50] = Pr[X \leq 50.5] \approx Pr[Z \leq \frac{50.5 - 60}{4.8990}] = Pr[Z \leq -1,9392] = 0.0262$$

Conclusion for 1

Since p-value $= 0.0262 < \alpha = 0.1$, H_0 is rejected at the 10% level of significance. That is, the data are consistent with the alternative hypothesis that the proportion of voters in favor of restructuring the city government is less than 60%.

Conclusion for 2

Since p-value $= 0.0262 < \alpha = 0.05$, H_0 is rejected at the 5% level of significance. That is, the data are consistent with the alternative hypothesis that the proportion of voters in favor of restructuring the city government is less than 60%.

Conclusion for 3

Conclusion: Since p value $= 0.0262 > \alpha = 0.01$, H_0 is not rejected at the 1% level of significance. That is, the data are not inconsistent with the null hypothesis that the proportion of voters in favor of restructuring the city government is less than 60%

Data for 4th case:

$$n = 200, X = 10, p_0 = 0.6, np_0 = 120, \sigma = \sqrt{np_0(1 - p_0)} \approx= 6.9282, \alpha = 0.01$$

Calculations for 4th case

$$p - value = Pr[X \leq 100] = Pr[X \leq 100.5] \approx Pr[Z \leq \frac{100.5 - 120}{6.9282}] = Pr[Z \leq -2.8146] = 0.0025$$

Conclusion for 4

Conclusion: Since p-value $= 0.0025 < \alpha = 0.1$, H_0 is rejected at the 1% level of significance. That is, the data are consistent with the alternative hypothesis that the proportion of voters in favor of restructuring the city government is less than 60%.

Exercise 11. A statistics student wants to test the hypothesis that a certain coin is equally likely to land on either heads or tails when it is flipped. The student flips the coin 200 times, obtaining 116 heads and 84 tails.

1. What are the null and the alternative hypotheses?
2. What is the p-value?
3. For the 5% level of significance, what conclusion should be drawn?

Solution:

Hypotheses:

$$H_0 : \mu = 0.5$$
$$H_1 : \mu \neq 0.5$$

Data:

$$n = 200, X = 116, p_0 = 0.5, np_0 = 100, \sigma = \sqrt{np_0(1 - p_0)} \approx 7,071, \alpha = 0.05$$

Calculations:

$$Pr[X \leq 116] = Pr[X \leq 116.5] \approx Pr[Z \leq \frac{116.5 - 100}{7,071}] = Pr[Z \leq 2.3334] = 0.9901$$
$$P\{X \geq 116\} = 0.014$$
$$\text{p-value} = 2 * (Min(P[X \leq 116], P(X \geq 116))) \approx 0.028$$

Conclusion:

Assuming 5% level of significance, $\alpha >$ p-value, so H_0 is rejected. Therefore the data suggests that the coin is not fair.

Exercise 12. A politician claims that over 50% of the population is in favor of her candidacy. To prove this claim, she has commissioned a polling organization to do a study. This organization chose a random sample of individuals in the population and asked each member of the sample if he or she was in favor of the politician's candidacy.

1. To prove the politician's claim, what should be the null and alternative hypotheses? Consider the following three alternative results, and give the relevant p-values for each one.
2. A random sample of 100 voters indicated that 56 (56%) are in favor of her candidacy.
3. A random sample of 200 voters indicated that 112 (56%) are in favor of her candidacy.
4. A random sample of 500 voters indicated that 280 (56%) are in favor of her candidacy.

Give an intuitive explanation for the discrepancy in results, if there are any, even though in each of cases b), c), and d) the same percentage of the sample was in favor.

Solution:
Hypotheses:

$$H_0 : \mu \leq 50\%$$
$$H_1 : \mu > 50\%$$

Data for 2:

$$n = 100, X = 56, p_0 = 0.5, np_0 = 50, \sigma = \sqrt{np_0(1-p_0)} = 5$$

Calculations for 2:

$$Pr[X \leq 56] = Pr[X \leq 56.5] \approx Pr[Z \leq \frac{56.5-50}{5}] = Pr[Z \leq 1.36] = 0.9131$$

$$Pr[X \geq 56] \approx 0.14$$

Result: p-value $= 0.14$
Data for 3:

$$n = 200, X = 112, p_0 = 0.5, np_0 = 100, \sigma = \sqrt{np_0(1-p_0)} \approx 7,071$$

Calculations for 3:

$$Pr[X \leq 112] = Pr[X \leq 112.5] \approx Pr[Z \leq \frac{112.5-100}{7,071}] = Pr[Z \leq 1,77] = 0.9616$$

$$Pr[X \geq 112] \approx 0.05$$

Result: p-value $= 0.05$
Calculations for 3:

$$n = 500, X = 280, p_0 = 0.5, np_0 = 250, \sigma = \sqrt{np_0(1-p_0)} \approx 11,180$$

Data for 3:

$$Pr[X \leq 280] = Pr[X \leq 280.5] \approx Pr[Z \leq \frac{280.5-250}{11,180}] = Pr[Z \leq 2,73] = 0.9968$$

$$Pr[X \geq 280] \approx 0.004$$

Result: p-value $= 0.004$
Conclusion:
As we can see, the more people are queried, the more reliable the results appear. The null hypothesis is strongly supported by the data.

Tables used in exercises:

Table D.2 Percentiles $t_{n,\alpha}$ of t Distributions

	α									
n	**0.40**	**0.25**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.0025**	**0.001**	**0.0005**
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792

Table D.2 *(Continued)*

23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

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Table 6.1 Standard Normal Probabilities

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Data value in table is $P\{Z < x\}$.