ECON 4360: Empirical Finance

Finite Sample Statistics

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Empirics Lecture #09

What are we doing today?

- Finite Sample Distributions

Motivation: Looking at Finite Sample Distributions

- What's the motivation?
- Formal tests of asset pricing models generally reject the models
    - One response is to toss out the model.
    - A second is to *not even test* the model, but look for consistency with the data along some other dimension
        - E.g., using bounds such as the H-J Bounds.
    - A third is to re-assess the econometric tests.
    - For most asset pricing models, the asymptotic distributions for standard errors and J-Tests are *far* from the finite sample distributions.

Finite Sample Distributions

- Since all tests of our models result from the asymptotics, but we have "small" samples, we want to look at the finite sample properties.
- Differences between finite samples and the asymptotic distributions can lead to
  - Type I Errors: Over-rejection of a true model
  - Type II Errors: Inability to reject false models.
- We can examine these issues using Monte Carlo methods

Finite Sample Distributions

- A 'good' test will reject false models and will not reject true models *too often*.
- Size: Probability of rejecting a true model
  - This is what we choose in a test
- Power: Probability of rejecting a false model
  - We hope it's big... but power against what?

Monte Carlo Methods

- General Procedures:
  - Simulate the model under the null hypothesis
  - Using the simulated time-series, calculate the statistics of interest
  - Repeat (many times).
- We can draw a histogram of the empirical distribution and tabulate rejections.
  - We can check if the model is rejected or not (we know if it should be!)
- Note that testing power requires another simulation: simulating the model under some alternative hypothesis.

Questions about Statistical Tests

- Do asymptotic distributions work in small samples?
  - Does the test have the proper size?
  - Is it a powerful test? (Power against what?)
  - Do the answers to these questions depend on sample size?
  - Is our test 'robust' to unmodelled autocorrelation?
  - Do additional assets yield more powerful tests?
  - Is Fama-Macbeth better than using one large cross-section?
  - How do OLS and GMM compare?

Simple Example: One Factor Excess Return Model

- In finance, we want to identify the 'factors' that price assets accurately.
    - The emphasis is on fit, not economics (E.g., Fama-French Factors)
- For a simple example, take

$$f_t = R_t^m - R_t^f$$

so the theory says that

$$E\left(R_t^{ei}\right) = \beta_i E\left(f_t\right)$$

- I.e., the expected excess return on asset i is proportional to the market return (it should be a linear function); and β_i is the covariance of asset i with the market return.

Empirical Counterpart of the Theory

- Empirical Test: For

$$R_t^{ei} = \alpha_i + \beta_i f_t + \varepsilon_t^i$$

If the theory is correct,

$$\alpha_i = 0$$

where again, α_i is interpreted as a pricing error.

Procedure

- Run OLS for each excess return on the market factor.
    - Let $E(f_t)$ and σ be the sample mean and standard deviation of the factor.
    - Let Σ be the residual VCV matrix, and α be the vector of constant terms.
- A test of the model (just for one factor) is that the intercepts are jointly zero is given by

$$T\left[1+\left(\frac{E(f_t)}{\sigma}\right)^2\right]^{-1}\alpha'\Sigma^{-1}\alpha \sim \chi^2_n$$

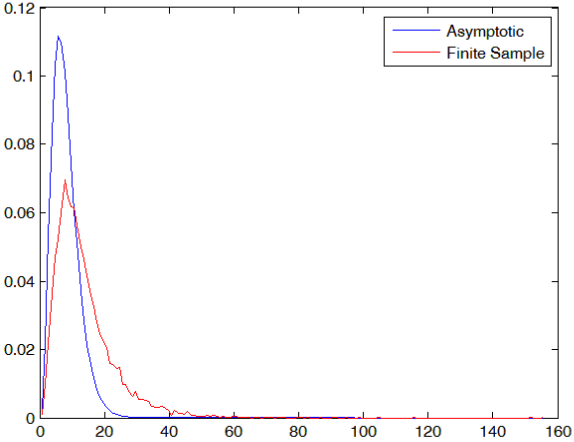
Procedure

- We're going to do a Monte Carlo experiment to look at size as a function of sample size.
- By construction, the model is true.
- We pick a critical value with size of 5 percent.
    - (So we should reject the true model 5 percent of the time!)
- The Matlab file for today's class exercise is under the Resources tab on the Collab site, and is named "monte_class.m".
    - Here we go...

Results

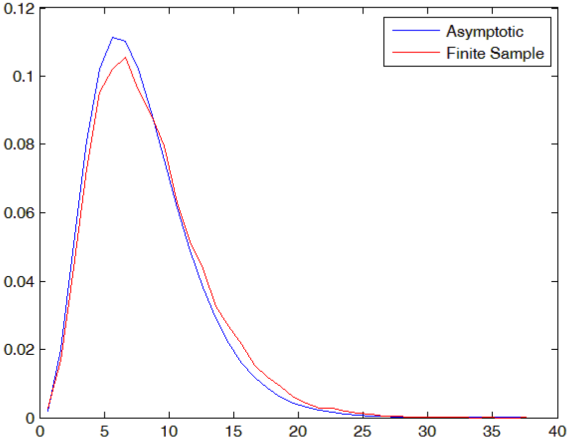
- Monte Carlo Results:
  - For $T = 25$, the model is rejected 0.318 percent of the time.
  - For $T = 50$, the model is rejected 0.154 percent of the time.
  - For $T = 100$, the model is rejected 0.089 percent of the time.
  - For $T = 200$, the model is rejected 0.066 percent of the time.
  - For $T = 500$, the model is rejected 0.056 percent of the time.
  - For $T = 1000$, the model is rejected 0.054 percent of the time.

Results: T = 25



0.12
0.1
0.08
0.06
0.04
0.02
0
0
20
40
60
80
100
120
140
160
Asymptotic
Finite Sample

Results: $T = 200$



What do we do?

- If the sample size is small, be wary of model rejections.
- Calculate your own finite sample critical values.
  - Only works if you know the true DGP!
- Solution: Pick one that seems reasonable for your data.
  - E.g., try to match mean, variance, autocorrelation (and perhaps cross-correlation) in actual data.

What about Power?

- Choose an alternative model you are concerned about.
  - E.g., one or two 'large' alphas (Fama-French claim)
  - E.g., Autocorrelation in the Fama-Macbeth procedure.
- Note: The histogram of the test statistic should be difference than the asymptotic one under the null!!!
- Count rejections of the false model to check power.
  - Note that it's often hard to reject models that are very 'close'
  - Power, as a concept, is not nearly as well-defined as size.

End of Today's Lecture.

- That's all for today. There is some additional reading in your book from Chapter 15 of Cochrane (2005).