# ECON 4360: Empirical Finance

## ICAPM v. APT and CRR 1986

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Empirics Lecture #07

# What are we doing today?

- ICAPM v. APT?
- Discussion of Chen, Roll, and Ross (1986)

# Factor Pricing Models

- Motivation for looking at factor pricing models:
  - The consumption-based model works well in theory, but not so well in practice.
  - So, much empirical work tried to tie the discount factor to other data - in particular, variables that are good proxies for
- I.e., we try to find $f$ such that

$$\beta \frac{u'(c_{t+1})}{u'(c_t)} \approx a + b' f_{t+1}$$

so we can write

$$m_{t+1} = a + b' f_{t+1}$$

and then use the model (recall: why can we use this model?)

$$E(R_{t+1}) = \gamma + \beta' \lambda$$

# What are the Factors?

- The idea is that there are some particular "states of nature" where investors would *really* like that
  - Variables that are indicative of these particular states are good candidates for our factors
- Examples include consumption (of course), returns on broad-based portfolios (of course), interest rates, GDP growth, and other like **macroeconomic variables** because they
- Other important variables are **forecasting variable**s, since consumption (and hence marginal utilities) respond *today* to news about *tomorrow*.
  - E.g., a variable like the term premium is

# Can I Have my Fishing License, Please?

- No, we can't fish for factors (in theory).
  - Economic foundations are still (should be?) important here.
- Our theoretical models do give us some discipline that we should ideally impose on our empirical work. E.g., ...
  - The CAPM and ICAPM make
  - In the ICAPM, "state variables" should

# What else?

- Remembering that all factor models are just derived as specializations (i.e., some extra assumptions) of the consumption-based model is what allow us to proxy marginal utility growth from other variables.
- And the model also tells us something about our factors...
- E.g., we can re-write our FOC as follows:

$$\frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)} = \frac{1}{\beta R^f} + \varepsilon_{t+1},\ E_t\left(\varepsilon_{t+1}\right) = 0$$

- This expression tells us that if we had a constant risk-free rate, marginal utility growth should be
- So, factors we choose shouldn't be highly predictable, or else we will (counterfactually) predict
- What does this mean in practice?
  - Choose the right units - e.g., GDP expressed through growth rates rather than levels, portfolio returns rather than prices, etc.

# CAPM and ICAPM

- Derivations of the CAPM and ICAPM both determine:
  - Factors that can proxy for marginal utility growth
  - Prove that the relationship is linear
- There are several sets of assumptions that will yield the CAPM:
  - Two-period quadratic utility
  - Exponential utility with normally distributed returns
  - Quadratic value functions and dynamic programming
  - Log utility

# Derivation of the ICAPM

- Merton's (1973) portfolio theory and ICAPM comes from
  - Stating the consumer's problem
  - Proving that the value function depends on $W$ (current wealth) and $z$ (state variables for future investment opportunities)
  - Showing that the optimal portfolio holds the market and hedge portfolios for the "investment opportunity variables".
- The idea is that the state variables determine how well an investor can do in his maximization problem
  - Current wealth $W$ is an obvious one
  - The others should describe the conditional distribution of asset returns the investor will face in the future - or

# What are the ICAPM's state variables???

- The ICAPM
  - It was Fama (1991) that claimed the ICAPM a "fishing license."
- Factor-mimicking portfolios should be the projections of some kind of (identifiable) state variables on the space of returns and they should forecast something
- Empirical successes with ICAPM-like models, but still theoretically artificial / ad hoc
  - E.g., the same intuition is captured by the consumption-based CAPM.
  - Plus, many of the macro variables suffer the same measurement problems as aggregate consumption data and are

# So What is the APT?

- Arbitrage Pricing Theory, Ross (1976).
  - Starts from a statistical characterization of **realized** returns, and tries to derive something about **expected** returns.
- The idea is that there is a big common component to stock returns (i.e., most move with the market) and also that groups of stocks also move together.
  - Asset returns also have idiosyncratic components, but these shouldn't carry any risk prices since this component should be diversifiable.
- Expected returns should then only be related to covariance with

# The APT: How do we do it?

- The original appeal of the APT was that it didn't impose all the economic structure required of the CAPM and ICAPM.
  - We're really just using the LOP
- If you know the factors you want to use, say size and book-to-market portfolios, you can just estimate a factor structure by
- If you don't know the factors, you can use "factor analysis" using statistical techniques to find them - e.g., by

# ICAPM v. APT

- Biggest difference is the inspiration for factors
  - APT starts with statistical analysis of the covariance matrix of returns to find portfolios that characterize the common movements
  - ICAPM starts by thinking about state variables the describe the conditional distributions of future asset returns
- For the APT, the factors should be orthogonal and describe the covariance matrix of returns. High $R^2$'s in time-series regressions can be indicative of factor pricing with APT.
- For the ICAPM, high $R^2$'s aren't necessary and nothing says that the factors describe the covariance matrix of returns or that they have to be orthogonal and i.i.d.

# Lots of Confusion...

- You've already read Fama and French (1993).
  - They describe their paper in their intro as an ICAPM, but their factors are portfolios of assets sorted on size and book-to-market just like the test assets. They talk of the explanation involving "comment movement" in test assets captured by the factors and note the high $R^2$'s in their time-series regressions.
  - Is this an APT?
- Today, we'll talk about CRR (1986).
  - There is no factor decomposition of asset returns or time-series regressions. Industrial production and inflation are some of the main factors.
  - Is this an ICAPM?

# Chen, Roll, and Ross (1986)

- Important to consider the economic insights about how factors relate to uncertainties about consumption and investment opportunities.
  - CRR (1986)
- CRR (1986) look for economic variables correlated with stock returns and then test to see if the loadings of returns on those economic factors describe the cross-section of expected returns.
  - Influential multifactor model paper.

# CRR (1986)

- Motivation
  - Use simple theoretic arguments to select a set of economic "state variables" as candidate sources of
  - Test for significance in explaining expected stock returns.
- Rationale
  - Asset prices react to
  - Possibilities for diversification means that
  - Returns are influenced by
- Main results are that

# Which Macro Variables?

- What macroeconomic variables are the "state variables"?
  - CRR (1986) argue that candidates are anything that might affect
  - Why? Basic argument:

# What changes expected cash flows?

- Expected cash flows change with changes in both
- Candidate influences?
  - Changes in the
    - Influences
  - Unanticipated changes in the
    - Systematic effect if
  - Changes in
    - Uncertainty about

# What changes discount rates?

- Discount rates change with changes in
  - The
- Candidate influences?
  - Unanticipated changes in the
    - Influence the
  - Unanticipated changes in the
  - Changes in

# CRR (1986) Data Descriptions

**TABLE 1 Glossary and Definitions of Variables**

| Symbol | Variable | Definition or Source |
|---|---|---|
| | **Basic Series** | |
| I | Inflation | Log relative of U.S. Consumer Price Index |
| TB | Treasury-bill rate | End-of-period return on 1-month bills |
| LGB | Long-term government bonds | Return on long-term government bonds (1958–78: Ibbotson and Sinquefield [1982]; 1979–83: CRSP) |
| IP | Industrial production | Industrial production during month (*Survey of Current Business*) |
| Baa | Low-grade bonds | Return on bonds rated Baa and under (1953–77: Ibbotson [1979], constructed for 1978–83) |

# CRR (1986) Data Descriptions

| | | |
|---|---|---|
| EWNY | Equally weighted equities | Return on equally weighted portfolio of NYSE-listed stocks (CRSP) |
| VWNY | Value-weighted equities | Return on a value-weighted portfolio of NYSE-listed stocks (CRSP) |
| CG | Consumption | Growth rate in real per capita consumption (Hansen and Singleton [1982]; *Survey of Current Business*) |
| OG | Oil prices | Log relative of Producer Price Index/Crude Petroleum series (Bureau of Labor Statistics) |

# CRR (1986) Derived Series

| | Derived Series | |
|---|---|---|
| MP($t$) | Monthly growth, industrial production | $\log_e[IP(t)/IP(t-1)]$ |
| YP($t$) | Annual growth, industrial production | $\log_e[IP(t)/IP(t-12)]$ |
| E[I($t$)] | Expected inflation | Fama and Gibbons (1984) |
| UI($t$) | Unexpected inflation | $I(t) - E[I(t)\|t-1]$ |
| RHO($t$) | Real interest (ex post) | $TB(t-1) - I(t)$ |
| DEI($t$) | Change in expected inflation | $E[I(t+1)\|t] - E[I(t)\|t-1]$ |
| URP($t$) | Risk premium | $Baa(t) - LGB(t)$ |
| UTS($t$) | Term structure | $LGB(t) - TB(t-1)$ |

# CRR (1986) Model

- Linear Factor Model

$$R = a + b_{MP}\,MP + b_{DEI}\,DEI + b_{UI}\,UI + b_{UPR}\,UPR + b_{UTS}\,UTS + e$$

- The state variables are
- The $b$'s are the

# CRR (1986) Methodology

- Cross-sectional regressions; a *version* of Fama-MacBeth (1973):

1. CRR chose a sample of assets.
2. For any given year:
    a. Each asset's exposure (the $b$'s) to the economic state variables ($MP$, $DEI$, $UI$, $UPR$, $UTS$) was estimated by
    b. The estimates of exposure (the $\widehat{b}$'s) were then used as RHS variables for

1. Repeat 2*a*-2*b* for all years in the sample.

# CRR (1986) Methodology, Continued...

- What do you get?
  - A time-series of estimates for each macro variable's
  - The time-series means of these estimates are then tested for
- Note: The estimated coefficients from the cross-sectional regressions provide estimates of
- Note: Securities are
  - Argument:
  - Recall, why didn't FF (1992) do this?

# CRR (1986) Methodology, Continued...

- Multivariate Approach
  - Tested a five or six factor model with a constant, main results in Table 4
  - Data range: 1958-1984
  - Again for subperiods: 1958-1967, 1968-1977, 1978-1984
    - Why?

# Risk Premia Significance

- Multivariate Approach
  - MP, UI, UPR are
  - UTS is
  - DEI and UI significant 1968-1977; but insignificant in both earlier and later periods.
  - YP not significant.
- Multivariate and Other Approaches (when replacing YP with a market index)
  - EWNY and VWNY

# Results: Table 4-A

**TABLE 4** **Economic Variables and Pricing (Percent per Month × 10), Multivariate Approach**

**A**

| Years | YP | MP | DEI | UI | UPR | UTS | Constant |
|---|---|---|---|---|---|---|---|
| 1958–84 | 4.341 | 13.984 | −.111 | −.672 | 7.941 | −5.87 | 4.112 |
| | (.538) | (3.727) | (−1.499) | (−2.052) | (2.807) | (−1.844) | (1.334) |
| 1958–67 | .417 | 15.760 | .014 | −.133 | 5.584 | .535 | 4.868 |
| | (.032) | (2.270) | (.191) | (−.259) | (1.923) | (.240) | (1.156) |
| 1968–77 | 1.819 | 15.645 | −.264 | −1.420 | 14.352 | −14.329 | −2.544 |
| | (.145) | (2.504) | (−3.397) | (−3.470) | (3.161) | (−2.672) | (−.464) |
| 1978–84 | 13.549 | 8.937 | −.070 | −.373 | 2.150 | −2.941 | 12.541 |
| | (.774) | (1.602) | (−.289) | (−.442) | (.279) | (−.327) | (1.911) |

# Results: Table 4-B

| | B | | | | | |
|---|---|---|---|---|---|---|
| | MP | DEI | UI | UPR | UTS | Constant |
| 1958–84 | 13.589 | −.125 | −.629 | 7.205 | −5.211 | 4.124 |
| | (3.561) | (−1.640) | (−1.979) | (2.590) | (−1.690) | (1.361) |
| 1958–67 | 13.155 | .006 | −.191 | 5.560 | −.008 | 4.989 |
| | (1.897) | (.092) | (−.382) | (1.935) | (−.004) | (1.271) |
| 1968–77 | 16.966 | −.245 | −1.353 | 12.717 | −13.142 | −1.889 |
| | (2.638) | (−3.215) | (−3.320) | (2.852) | (−2.554) | (−.334) |
| 1978–84 | 9.383 | −.140 | −.221 | 1.679 | −1.312 | 11.477 |
| | (1.588) | (−.552) | (−.274) | (.221) | (−.149) | (1.747) |

# Results: Table 4-C

C

| | EWNY | MP | DEI | UI | UPR | UTS | Constant |
|---|---|---|---|---|---|---|---|
| 1958–84 | 5.021 | 14.009 | −.128 | −.848 | 8.130 | −5.017 | 6.409 |
| | (1.218) | (3.774) | (−1.666) | (−2.541) | (2.855) | (−1.576) | (1.848) |
| 1958–67 | 6.575 | 14.936 | −.005 | −.279 | 5.747 | −.146 | 7.349 |
| | (1.199) | (2.336) | (−.060) | (−.558) | (2.070) | (−.067) | (1.591) |
| 1968–77 | 2.334 | 17.593 | −.248 | −1.501 | 12.512 | −9.904 | 3.542 |
| | (.283) | (2.715) | (−3.039) | (−3.366) | (2.758) | (−2.015) | (.558) |
| 1978–84 | 6.638 | 7.563 | −.132 | −.729 | 5.273 | −4.993 | 9.164 |
| | (.906) | (1.253) | (−.529) | (−.847) | (.663) | (−.520) | (1.245) |

# Results: Table 4-D

| | D | | | | | | |
|---|---|---|---|---|---|---|---|
| | VWNY | MP | DEI | UI | UPR | UTS | Constant |
| 1958–84 | −2.403 | 11.756 | −.123 | −.795 | 8.274 | −5.905 | 10.713 |
| | (−.633) | (3.054) | (−1.600) | (−2.376) | (2.972) | (−1.879) | (2.755) |
| 1958–67 | 1.359 | 12.394 | .005 | −.209 | 5.204 | −.086 | 9.527 |
| | (.277) | (1.789) | (.064) | (−.415) | (1.815) | (−.040) | (1.984) |
| 1968–77 | −5.269 | 13.466 | −.255 | −1.421 | 12.897 | −11.708 | 8.582 |
| | (−.717) | (2.038) | (−3.237) | (−3.106) | (2.955) | (−2.299) | (1.167) |
| 1978–84 | −3.683 | 8.402 | −.116 | −.739 | 6.056 | −5.928 | 15.452 |
| | (−.491) | (1.432) | (−.458) | (−.869) | (.782) | (−.644) | (1.867) |

# Risk Premia Sign Stories

- Most significant:
- Also weakly significant: unanticipated inflation and changes in expected inflation d
- MP $(+)$
  - Reflects the value of
- UPR $(+)$
  - Desire to hedge against
- UI and DEI $(-)$
  - Stock market assets assumed to hedge against

# Risk Premia Sign Stories, Continued...

- UTS $(-)$
  - Returns of stocks that are inversely related to increases in long rates over short rates are more valuable
    - Why? If long-term rates decrease, there is a lower return on any form of capital...
    - A relatively higher value is placed on assets whose
    - Returns that are correlated with long-term bond returns

# Stock Market Indices and Other Ideas?

- Basic results for stock market indices
  - Time-series regressions
  - Cross-sectional regressions
- Consumption growth betas and oil prices
  - Neither significant
  - Estimated risk premium for Cg has the wrong sign (should be positive, not negative!)

# CRR (1986) Results Summary

- Returns are
- News can be measured through
- Again, main results are that
  - Their "macroeconomic" variables
  - Stock market indices, consumption growth betas, oil prices are all

# CRR (1986) Take-Aways

- Approach is a useful way to use multifactor models to improve our understanding of asset pricing models.
  - May be able to develop a
- Caution: Multifactor models offer few predictions about which variables are important
  - The power of the CRR (1986) factors are
  - Flexible approach; but not a license to go fishing!
  - Measured relationships between returns and economic factors can be
  - Robustness checks warranted.

# End of Today's Lecture.

- That's all for today. Today's material corresponds to parts of Chapter 9 in Cochrane and CRR (1986).