ECON 4360: Empirical Finance

Term Structure Models

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Empirics Lecture #11

What are we doing today?

- Term Structure Models

Term Structure Models

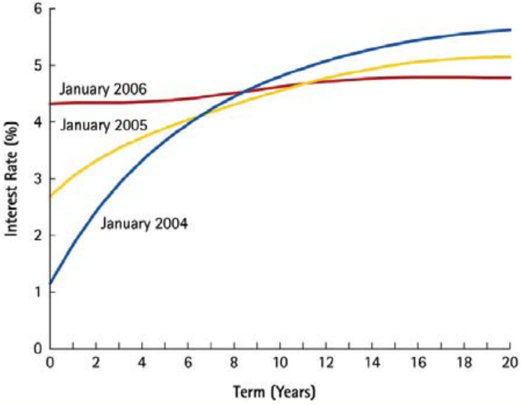
- Three basic types
  - Macro Models
    - Expectations Hypothesis (EH) - The most rejected hypothesis in economics!
  - Finance Models
    - No deep theory - Find 'factors' that minimize pricing errors
  - Macro-Finance Models
    - Lots of recent papers in this area

First, Some Definitions...

- Spot rate: the interest rate on a T-year loan that is to be made today
  - e.g., $R_t^1 = 5\%$ means that the current rate for a one-year loan is 5%
  - e.g., $R_t^2 = 6\%$ means that the current rate for a two-year loan is 6%
- Term Structure of Interest Rates: the series of spot rates R_1, R_2, R_3, ... relating *interest rates* to the *term* of the investment
  - Yield Curve: just a plot of the term structure: interest rates plotted against investment term (i.e., maturity)
- Forward Rate: a rate agreed upon today for loan to be made in the future
  - Not necessarily equal to the future spot rate!

Sample Yield Curves

Term (years)	Date Jan. 2004	Jan. 2005	Jan. 2006
1	1.15%	2.69%	4.32%
2	1.87%	3.06%	4.34%
3	2.48%	3.34%	4.34%
4	2.98%	3.57%	4.34%
5	3.40%	3.76%	4.36%
6	3.75%	3.93%	4.38%
7	4.05%	4.08%	4.42%
8	4.31%	4.22%	4.48%
9	4.53%	4.36%	4.53%
10	4.72%	4.49%	4.59%
11	4.88%	4.61%	4.65%
12	5.02%	4.73%	4.70%
13	5.15%	4.83%	4.73%
14	5.25%	4.91%	4.76%
15	5.35%	4.99%	4.78%
16	5.43%	5.05%	4.79%
17	5.49%	5.09%	4.79%
18	5.55%	5.12%	4.79%
19	5.59%	5.14%	4.78%
20	5.62%	5.15%	4.78%



Macro Models

- Rational Expectations Hypothesis of the Term Structure (EH)
  - Essentially says that yields (long-term) will be the expected average short-term rate over the life of the bond.
  - Equivalently, the forward rate equals the expected future spot rate
  - EH: There should be no difference in returns to holding a long-term bond or rolling over a sequence of short-term bonds, except for maybe a term premium.
- Standard reference: Campbell and Shiller (1991), ReStud.
- There are a number of tests of the EH...

Test #1 of EH

- One approach to testing the EH due to Campbell and Shiller (1991) does the following:
- The EH implies the n-period yield is the expected average m-period interest rate over the next n period:

$$R_t^{(n)} = \frac{1}{K} \sum_{i=0}^{K-1} E_t R_{t+mi}^{(m)}$$

  - $R_t^{(n)}$ is an n-period bond
  - $R_{t+mi}^{(m)}$ is an m-period bond at time $t + mi$
  - $K = n/m$ (needs to be an integer)

Test #1 of EH

- From the previous equation implies we can write

$$R_t^{(n)} - R_t^{(m)} = \frac{1}{K} \sum_{i=0}^{K-1} E_t \left[R_{t+mi}^{(m)} - R_t^{(m)} \right]$$

- Take a concrete example - $m = 3$, $n = 6$, $(K = 2)$, then we can consider a regression like

$$\frac{1}{2} \left[R_{t+3}^{(3)} - R_t^{(3)} \right] = \alpha + \beta \left(R_t^{(6)} - R_t^{(3)} \right) + \varepsilon_t$$

- If the EH holds, we should get a coefficient $\beta = 1$, if there is no time-varying term premia

Test #1 of EH

- For our example, derivation from Mankiw and Miron (1986) QJE shows

$$p\lim\left(\widehat{\beta}\right) = \frac{\sigma_{e3}^2 + 2\rho\sigma_{e3}\sigma_{K_t t}}{\sigma_{e3}^2 + 4\sigma_{K_t}^2 + 4\rho\sigma_{e3}\sigma_{K_t}}$$

  - σ_{e3}^2 is the 3 month forecast error and $\sigma_{K_t}^2$ is the variance in the term premium.
- Messy, but gives the intuition that the $p\lim\left(\widehat{\beta}\right) = 1$ if there is no time-varying term premia.
- What is there is a time-varying term premia?
  - It gets wrapped up in the ε_t term, but it should be mean zero just like the forecast errors
  - If the variance of the time-varying term premia is large, it will decrease β

Test #1 of EH

- Testing if $\widehat{\beta} = 1$ here is considered the 'strong' version of the EH Hypothesis
  - It is always rejected
- In fact, we see in the data that when *m* is short (say 3 months) and *n* is long (say 10 years), the estimated slope coefficients are negative.
- According to the EH, when the yield curve is steep long-term interest rates should be rising, but instead they are falling.

Coefficient Estimates from the Literature

Table 1 Coefficient Estimates from Literature

Source	Short (m) Long (n)	1 period 2 period	1 period 3 period	1 period 4 period	1 period 6 period	2 period 4 period	3 period 6 period
Campbell & Shiller Table 2, T-bills, 1952–87	coefficient	0.5010	0.4460	0.4360	0.2370	0.1950	−0.1470
	standard error	0.1190	0.1990	0.2380	0.1670	0.2810	0.2000
Roberds, Runkle & Whiteman, Table 6 F Fund, 1984–91	coefficient	0.5925	0.3935	na	0.2121	na	−0.1411
	standard error	0.0983	0.1437	na	0.2822	na	0.6079
Roberds, Runkle & Whiteman, Table 9 F Fund, 1984–91, SW*	coefficient	0.7596	0.2953	na	0.1557	na	−0.2971
	standard error	0.1359	0.1399	na	0.1861	na	0.3675
Roberds, Runkle & Whiteman, Table 11 F Fund, 1984–91, FOMC†	coefficient	0.7119	0.4104	na	0.0869	na	−0.3149
	standard error	0.1720	0.1688	na	0.1878	na	0.4553

* Settlement Wednesday.
† FOMC meeting date.

Note: Roberds, Runkle, and Whiteman use daily data in their regressions.

Test #2 of EH

- Campbell and Shiller (1987) JPE also consider a second test of the EH.
- They compare actual spreads with calculated 'theoretical spreads' to see if they are close.
  - The idea is that the theoretical spread uses the econometrician's expectation of the spread

$$s_t' = \sum_{j=1}^{N} E\left[\triangle R_{t+j} | w_t\right]$$

  - where s_t' is the theoretical spread, $\triangle R_{t+j}$ are changes in the short rate, and w_t is the econometrician's information set
- How is the forecast made?

Test #2 of EH: Not as Bad

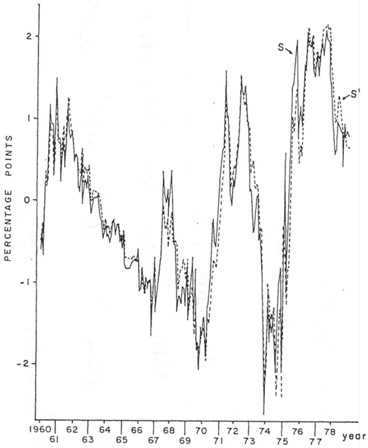


FIG. 1.—Term structure: deviations from means of long-short spread S_t and theoretical spread S'_t.

Finance Models

- This literature focuses on multifactor, or k-factor, models for bond returns.
  - Finance models still model (bond) returns as functions of factors
  - And the goal is to find factors that price assets well
- We've already seen a famous example: FF (1993) Common Risk Factors in Returns of Stocks and Bonds, JFE.

FF 1993

- Recall, that for the three factor model

$$R_t - R_t^f = a + b\left[R_t^m - R_t^f\right] + sSMB_t + hHML_t + e\left(t\right)$$

- This model did a horrible job pricing government securities
- (Looked at short-term - 1-5G is 1-5 year; long-term - 6-10G is 6-10 years)
    - Coefficient estimates show we don't have priced factors here for bonds
    - R^2 really low

FF 1993: Table 6

	1–5G	6–10G
b	0.10	0.18
$t(b)$	6.45	6.75
s	−0.06	−0.14
$t(s)$	−2.70	−3.65
h	0.07	0.08
$t(h)$	2.66	1.83
R^2	0.10	0.12
$s(e)$	1.19	1.91

FF 1993

- The factors TERM and DEF work best to explain the variation in bond returns (Table 7b)
- And their regression intercepts are close to zero (Table 9b)
    - This is not surprising since the average excess returns on bond portfolios are already close to zero.
- However, the formal tests *rejects* that the intercepts are actually equal to zero! (F-Stat of GRS (1989))
    - Just means that low average TERM and DEF returns cannot explain the cross-section of stocks and bond returns
        - (FF argue important for time-series variability!)

Regressions of excess stock returns on government and corporate bonds (in percent) on the stock-market returns, $RM-RF$, SMB, and HML, and the bond-market returns, $TERM$ and DEF: July 1963 to December 1991, 342 months.[a]

$$R(t) - RF(t) = a + b[RM(t) - RF(t)] + sSMB(t) + hHML(t) + mTERM(t) + dDEF(t) + e(t)$$

	Bond portfolio						
	1–5G	6–10G	Aaa	Aa	A	Baa	LG
b	−0.02	−0.04	−0.02	0.00	0.00	0.02	0.18
$t(b)$	−2.84	−3.14	−2.96	0.06	1.05	1.99	7.39
s	0.00	−0.02	−0.02	−0.01	0.00	0.05	0.08
$t(s)$	0.30	−1.12	−2.28	−2.42	0.40	3.20	2.34
h	0.00	−0.02	−0.02	−0.00	0.00	0.04	0.12
$t(h)$	0.44	−1.29	−2.46	−0.40	0.90	2.39	3.13
m slope of term factor	0.47	0.75	1.03	0.99	1.00	0.99	0.64
$t(m)$	30.01	36.84	93.30	117.30	124.19	50.50	14.25
d → coeff. on value of defaults.	0.27	0.32	0.97	0.97	1.02	1.05	0.80
$t(d)$	9.87	8.77	49.25	65.04	71.51	30.33	9.92
R^2	0.80	0.87	0.97	0.98	0.98	0.91	0.58
$s(e)$	0.56	0.73	0.40	0.30	0.29	0.70	1.63

FF 1993: Table 9b - Intercepts

Intercepts form excess bond return regressions for two government and five corporate bond portfolios: July 1963 to December 1991, 342 months.[a]

	Bond portfolio						
	1–5G	6–10G	Aaa	Aa	A	Baa	LG
	(i) $R(t) - RF(t) = a + mTERM(t) + dDEF(t) + e(t)$						
a	0.08	0.09	− 0.02	− 0.00	− 0.00	0.06	0.06
$t(a)$	2.70	2.16	− 1.10	− 0.55	− 0.29	1.42	0.67
	(ii) $R(t) - RF(t) = a + b[RM(t) - RF(t)] + e(t)$						
a	0.08	0.08	− 0.03	− 0.02	− 0.01	0.04	0.00
$t(a)$	1.27	0.76	− 0.24	− 0.15	− 0.11	0.37	0.03
	(iii) $R(t) - RF(t) = a + sSMB(t) + hHML(t) + e(t)$						
a	0.12	0.16	0.07	0.07	0.07	0.11	0.08
$t(a)$	1.70	1.47	0.52	0.58	0.55	0.82	0.58
	(iv) $R(t) - RF(t) = a + b[RM(t) - RF(t)] + sSMB(t) + hHML(t) + e(t)$						
a	0.06	0.07	− 0.07	− 0.07	− 0.08	− 0.05	− 0.11
$t(a)$	0.89	0.62	− 0.62	− 0.64	− 0.69	− 0.41	− 1.00
	(v) $R(t) - RF(t) = a + b[RM(t) - RF(t)] + sSMB(t) + hHML(t) + mTERM(t) + dDEF(t) + e(t)$						
a	0.09	0.11	− 0.00	− 0.00	− 0.00	0.02	− 0.07
$t(a)$	2.84	2.77	− 0.17	− 0.25	− 0.57	0.52	− 0.77

[a]See footnote under table 9c

More Recent Fiance Literature

- To look at changes in the yield curve...
  - Fit a model to the observed yield curve with a limited number of parameters
  - Typical models use polynomial functions or Nelsen-Siegel functions.
- Diebold and Li (2006) adaptation of Nelsen-Siegel (1987)

$$y_t\left(\tau\right) = L_t + S_t\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) + C_t\left(\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) - e^{-\lambda_t\tau}\right)$$

  - where $y_t\left(\tau\right)$ is yield at time t for maturity τ
  - DL set $\lambda = 0.0609$ (parameter)
  - L_t, S_t, and C_t are parameters (i.e., the factors) to be estimated via OLS

Implementation

- To estimate this, just regress yields of different maturities on date t factors

$$\begin{bmatrix} y_t(1) \\ \vdots \\ y_t(n) \end{bmatrix} = L_t \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} + S_t \begin{bmatrix} \frac{1-e^{-\lambda_t}}{\lambda_t} \\ \vdots \\ \frac{1-e^{-\lambda_t n}}{\lambda_t n} \end{bmatrix} + C_t \begin{bmatrix} \frac{1-e^{-\lambda_t}}{\lambda_t} - e^{-\lambda_t} \\ \vdots \\ \frac{1-e^{-\lambda_t n}}{\lambda_t n} - e^{-\lambda_t n} \end{bmatrix} + \varepsilon$$

- The coefficients in the regression at period t are the factor in period t
- Then just repeat at each date.
- The measure of fit is going to be the R^2
  - The size of the pricing errors relative to variation in the data

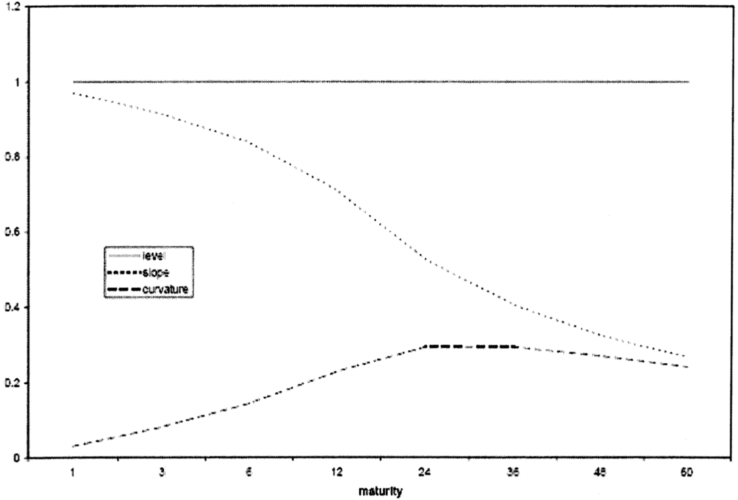
Diebold and Li (2006): Results

- From

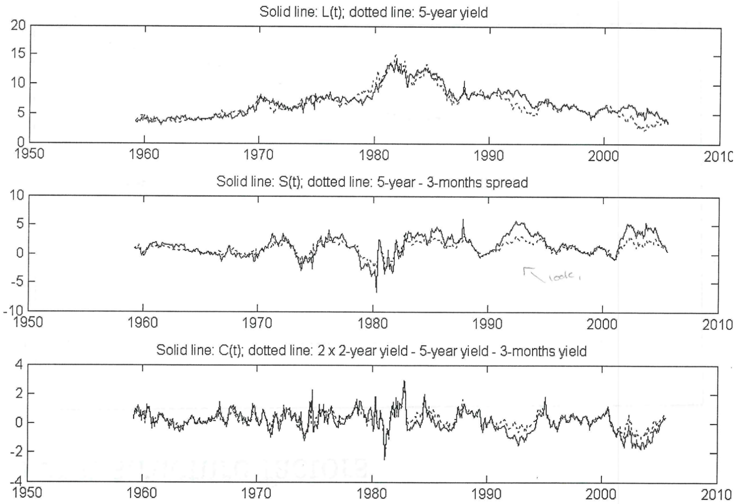
$$y_t(\tau) = L_t + S_t\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) + C_t\left(\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) - e^{-\lambda_t\tau}\right)$$

- Estimated factors have a clear interpretation
  - L_t is Level: determines mostly movements in long rates; factor loading set at 1, a constant
  - S_t is Slope: associated with the spread between long and short rates
  - C_t is Curvature: approximates the different between short and long spreads
- Results: Level and Slope factors explain more than 90 percent of yield fluctuations across the entire term structure

Term Structure Factors



Model-Based v Data-Based LSC



Fit of Diebold-Li Factors

Maturity	$R^2(L)$	$R^2(L,S)$	$R^2(L,S,C)$
3	0.3823	0.9953	0.9982
4	0.4037	0.9925	0.9989
5	0.4333	0.9862	0.9990
6	0.4312	0.9727	0.9912
12	0.5272	0.9497	0.9983
24	0.6642	0.9351	0.9991
36	0.7489	0.9316	0.9992
48	0.8108	0.9353	0.9995
60	0.8470	0.9392	0.9994

Diebold-Li (2006) Summary

- The model is 'good' because we know how to price these interest rates; and we can extrapolate, i.e., forecast from this model
- From a finance perspective, the model is successful because it prices asset well
    - I.e., the errors are small
- From a macro-finance perspective, we want to understand the interaction between these variables and interest rates
    - E.g., a structural model with some kind of reasonable story would be nice
    - The objective is to find risk factors that price bonds accurately - e.g., to write down a stochastic process for risk
- Recent macro-finance literature has been trying to bridge the gap between traditional macro and finance...

End of Today's Lecture.

- That's all for today. There are a few pages on the term structure in a section of Chapter 20 of Cochrane (2005) that you should read; and there is additional material in Chapter 19 if you're feeling adventurous!