1 Finite Sample Distributions

1.1 Looking at Finite Samples

- What's the motivation?
- Formal tests of asset pricing models generally reject the models
  - One response is to toss out the model.
  - A second is to *not even test* the model, but look for consistency with the data along some other dimension
    * E.g., using bounds such as the H-J Bounds.
  - A third is to re-assess the econometric tests.
  - For most asset pricing models, the asymptotic distributions for standard errors and J-Tests are *far* from the finite sample distributions.
- Since all tests of our models result from the asymptotics, but we have "small" samples, we want to look at the finite sample properties.
- Differences between finite samples and the asymptotic distributions can lead to
  - Type I Errors: Over-rejection of a true model
  - Type II Errors: Inability to reject false models.
- We can examine these issues using Monte Carlo methods
- A 'good' test will reject false models and will not reject true models *too often*.
- Size: Probability of rejecting a true model
  - This is what we choose in a test
- Power: Probability of rejecting a false model
  - We hope it's big... but power against what?

1.2 Monte Carlo Methods

- General Procedures:
  - Simulate the model under the null hypothesis
  - Using the simulated time-series, calculate the statistics of interest
  - Repeat (many times).
- We can draw a histogram of the empirical distribution and tabulate rejections.
  - We can check if the model is rejected or not (we know if it should be!)
- Note that testing power requires another simulation: simulating the model under some alternative hypothesis.

1.3 Example Questions about Statistical Tests

- Do asymptotic distributions work in small samples?
  - Does the test have the proper size?
  - Is it a powerful test? (Power against what?)
  - Do the answers to these questions depend on sample size?
  - Is our test 'robust' to unmodelled autocorrelation?
  - Do additional assets yield more powerful tests?
  - Is Fama-Macbeth better than using one large cross-section?
  - How do OLS and GMM compare?

1.4 Simple Example: One Factor Excess Return Model

1.4.1 Model

- In finance, we want to identify the 'factors' that price assets accurately.
  - The emphasis is on fit, not economics (E.g., Fama-French Factors)
- For a simple example, take
$$f_t = R_t^m - R_t^f$$
so the theory says that
$$E\left(R_t^{ei}\right) = \beta_i E\left(f_t\right)$$
- I.e., the expected excess return on asset i is proportional to the market return (it should be a linear function); and β_i is the covariance of asset i with the market return.

1.4.2 Empirical Counterpart

- Empirical Test: For
$$R_t^{ei} = \alpha_i + \beta_i f_t + \varepsilon_t^i$$
If the theory is correct,
$$\alpha_i = 0$$
where again, α_i is interpreted as a pricing error.

1.4.3 Procedure

- Run OLS for each excess return on the market factor.
  - Let $E\left(f_t\right)$ and σ be the sample mean and standard deviation of the factor.
  - Let Σ be the residual VCV matrix, and α be the vector of constant terms.

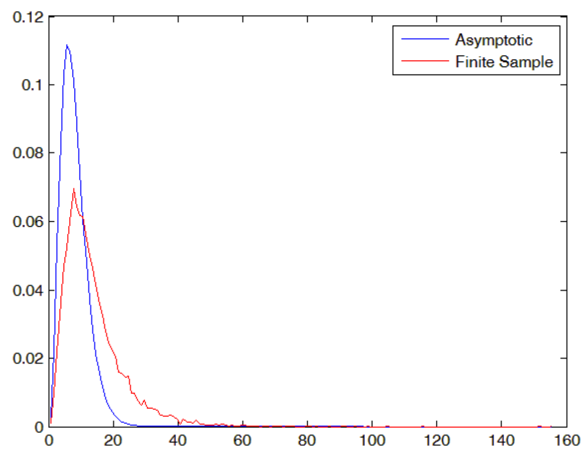
- A test of the model (just for one factor) is that the intercepts are jointly zero is given by

$$T\left[1+\left(\frac{E\left(f_t\right)}{\sigma}\right)^2\right]^{-1}\alpha'\Sigma^{-1}\alpha \;\sim\; \chi_n^2$$

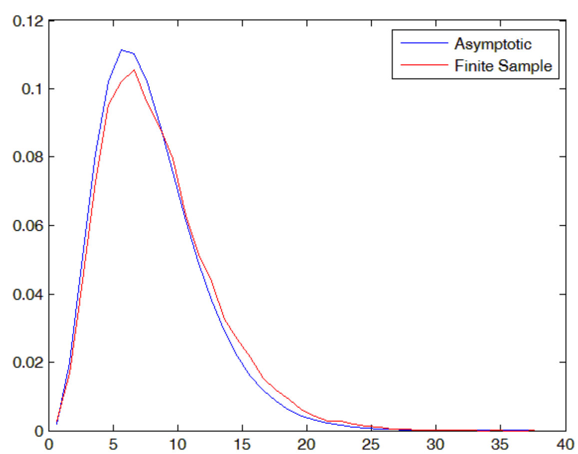
- We're going to do a Monte Carlo experiment to look at size as a function of sample size.
- By construction, the model is true.
- We pick a critical value with size of 5 percent.
  - (So we should reject the true model 5 percent of the time!)
- The Matlab file for today's class exercise is under the Resources tab on the Collab site, and is named "monte_class.m".
  - First, we'll sketch out our coding algorithm. There is space here ↓ for your notes...

1.4.4 Results

- Monte Carlo Results:
  - For $T = 25$, the model is rejected 0.318 percent of the time.
  - For $T = 50$, the model is rejected 0.154 percent of the time.
  - For $T = 100$, the model is rejected 0.089 percent of the time.
  - For $T = 200$, the model is rejected 0.066 percent of the time.
  - For $T = 500$, the model is rejected 0.056 percent of the time.
  - For $T = 1000$, the model is rejected 0.054 percent of the time.
- Results: $T = 25$



- Results: $T = 200$



1.5 So What Do We Do? What Have We Learned?

- If the sample size is small, be wary of model rejections.

- Calculate your own finite sample critical values.
  - Only works if you know the true DGP!
- Solution: Pick one that seems reasonable for your data.
  - E.g., try to match mean, variance, autocorrelation (and perhaps cross-correlation) in actual data.
- What about Power?
  - Choose an alternative model you are concerned about.
    * E.g., one or two 'large' alphas (Fama-French claim)
    * E.g., Autocorrelation in the Fama-Macbeth procedure.
  - Note: The histogram of the test statistic should be difference than the asymptotic one under the null!!!
  - Count rejections of the false model to check power.
    * Note that it's often hard to reject models that are very 'close'
    * Power, as a concept, is not nearly as well-defined as size.