ECON 4360: Empirical Finance

Classic Issues II and Applications

Sherry Forbes

University of Virginia

Theory Lecture #06

What are we doing today?

- Finishing Overview of Classic Issues in Finance
- Addressing Assumptions of the Consumption-Based Model

Review Derivation: ER-Beta

- Recall, that we can start with

$$1 = E_t\left[m_{t+1} R^i_{t+1}\right]$$

- To derive

$$\begin{aligned} 1 &= E_t\left(m_{t+1}\right) E_t\left(R^i_{t+1}\right) + cov\left(m_{t+1}, R^i_{t+1}\right) \\ 1 &= \left(1/R^f\right) E_t\left(R^i_{t+1}\right) + cov\left(m_{t+1}, R^i_{t+1}\right) \end{aligned}$$

- So we can write

$$E_t\left(R^i\right) = R^f - R^f cov\left(m_{t+1}, R^i_{t+1}\right)$$

Review Derivation: ER-Beta

- Continuing...

$$E_t\left(R^i\right) = R^f - \frac{cov\left(m_{t+1}, R^i_{t+1}\right)}{E_t\left[m_{t+1}\right]}$$
$$= R^f + \frac{cov\left(m_{t+1}, R^i_{t+1}\right)}{var\left[m_{t+1}\right]}\left(-\frac{var\left[m_{t+1}\right]}{E_t\left[m_{t+1}\right]}\right)$$

- Now we have

$$E_t\left(R^i\right) = R^f + \beta_{i,m}\lambda_m$$

  - where we can define $\beta_{i,m} := \frac{cov\left(m_{t+1}, R^i_{t+1}\right)}{var\left[m_{t+1}\right]}$ as the "quantity of risk"
  - and $\lambda_m := \left(-\frac{var\left[m_{t+1}\right]}{E_t\left[m_{t+1}\right]}\right)$ as the "price of risk"

- Which is the same across assets? Which varies across assets?

Review Derivation: ER-Beta

- What would a graphical representation of $E_t\left(R^i\right) = R^f + \beta_{i,m}\lambda_m$ show you?

Review Derivation: ER-Beta

- What would a graphical representation of $E_t\left(R^i\right) = R^f + \beta_{i,m}\lambda_m$ show you?
- Note that this is a cross-sectional relationship we're trying to explain...

Review Derivation: ER-Beta

- What would a graphical representation of $E_t\left(R^i\right) = R^f + \beta_{i,m}\lambda_m$ show you?
- Note that this is a cross-sectional relationship we're trying to explain...
  - If we plotted $E\left[R^{ei}\right]$ against β's, these things should line up on a line with slope $= \lambda$

Review Derivation: ER-Beta

- What would a graphical representation of $E_t\left(R^i\right) = R^f + \beta_{i,m}\lambda_m$ show you?
- Note that this is a cross-sectional relationship we're trying to explain...
  - If we plotted $E\left[R^{ei}\right]$ against β's, these things should line up on a line with slope $= \lambda$
  - Q: What would it mean if the line got steeper? (What would cause that?)

Review Derivation: ER-Beta

- What would a graphical representation of $E_t\left(R^i\right) = R^f + \beta_{i,m}\lambda_m$ show you?
- Note that this is a cross-sectional relationship we're trying to explain...
  - If we plotted $E\left[R^{ei}\right]$ against β's, these things should line up on a line with slope $= \lambda$
  - Q: What would it mean if the line got steeper? (What would cause that?)
  - A: A riskier economy (higher variance in the SDF) or higher risk aversion

Recall: Expected Return - Beta Representation

- We're going to use this expression again today

$$E_t\left(R^i\right) = R^f + \frac{cov\left(m_{t+1}, R^i_{t+1}\right)}{var\left[m_{t+1}\right]}\left(-\frac{var\left[m_{t+1}\right]}{E_t\left[m_{t+1}\right]}\right)$$
$$E_t\left(R^i\right) = R^f + \beta_{i,m}\lambda_m$$

- So keep in mind some of the results that we got last time:

Recall: Expected Return - Beta Representation

- We're going to use this expression again today

$$
\begin{aligned}
E_t\left(R^i\right) &= R^f + \frac{cov\left(m_{t+1}, R^i_{t+1}\right)}{var\left[m_{t+1}\right]}\left(-\frac{var\left[m_{t+1}\right]}{E_t\left[m_{t+1}\right]}\right) \\
E_t\left(R^i\right) &= R^f + \beta_{i,m}\lambda_m
\end{aligned}
$$

- So keep in mind some of the results that we got last time:
  - Assets with payoffs that have positive covariance with consumption $\Rightarrow$ high (negative) beta

Recall: Expected Return - Beta Representation

- We're going to use this expression again today

$$
\begin{aligned}
E_t\left(R^i\right) &= R^f + \frac{cov\left(m_{t+1}, R^i_{t+1}\right)}{var\left[m_{t+1}\right]}\left(-\frac{var\left[m_{t+1}\right]}{E_t\left[m_{t+1}\right]}\right) \\
E_t\left(R^i\right) &= R^f + \beta_{i,m}\lambda_m
\end{aligned}
$$

- So keep in mind some of the results that we got last time:
  - Assets with payoffs that have positive covariance with consumption $\Rightarrow$ high (negative) beta
  - These assets make consumption more volatile, so must have a higher expected return

Recall: Expected Return - Beta Representation

- We're going to use this expression again today

$$
\begin{aligned}
E_t\left(R^i\right) &= R^f + \frac{cov\left(m_{t+1}, R^i_{t+1}\right)}{var\left[m_{t+1}\right]}\left(-\frac{var\left[m_{t+1}\right]}{E_t\left[m_{t+1}\right]}\right) \\
E_t\left(R^i\right) &= R^f + \beta_{i,m}\lambda_m
\end{aligned}
$$

- So keep in mind some of the results that we got last time:
  - Assets with payoffs that have positive covariance with consumption $\Rightarrow$ high (negative) beta
  - These assets make consumption more volatile, so must have a higher expected return
  - People require a higher return to hold risky assets

Recall: Expected Return - Beta Representation

- Also recall that we found last time that only systematic risk is "priced" - i.e., idiosyncratic volatility doesn't matter.
- To see this another way, we could run a regression:

$$R_t^{ei} = \beta_{i,m} m_{t+1} + \varepsilon_{t+1}$$

- This essentially breaks up the volatility of the return into two parts:

$$\sigma^2 \left(R_t^{ei} \right) = \beta_{i,m}^2 \sigma^2 \left(m_{t+1} \right) + \sigma^2 \left(\varepsilon_{t+1} \right)$$

  a systematic part (that is correlated with the discount factor) and an idiosyncratic (diversifiable) part.
- So only the covariance with m generates a risk premium!

Introduction to Mean-Variance Analysis

- Now, we're going to relate the SDF to traditional mean-variance analysis
- Start with
$$1 = E_t\left[m_{t+1} R^i_{t+1}\right]$$
- And manipulate it with basic definitions from probability...

Introduction to Mean-Variance Analysis

- Now, we're going to relate the SDF to traditional mean-variance analysis
- Start with

$$1 = E_t\left[m_{t+1}R^i_{t+1}\right]$$

- And manipulate it with basic definitions from probability...
  - use $cov\left(x,y\right) = E\left(x,y\right) - E\left(x\right)E\left(y\right)$

$$\begin{aligned} 1 &= E_t\left[m_{t+1}R^i_{t+1}\right] \\ &= cov\left(m_{t+1}, R^i_{t+1}\right) + E_t\left[m_{t+1}\right]E_t\left[R^i_{t+1}\right] \end{aligned}$$

Introduction to Mean-Variance Analysis

- Now, we're going to relate the SDF to traditional mean-variance analysis
- Start with

$$1 = E_t\left[m_{t+1} R^i_{t+1}\right]$$

- And manipulate it with basic definitions from probability...
  - use $cov(x, y) = E(x, y) - E(x) E(y)$

$$\begin{aligned} 1 &= E_t\left[m_{t+1} R^i_{t+1}\right] \\ &= cov\left(m_{t+1}, R^i_{t+1}\right) + E_t\left[m_{t+1}\right] E_t\left[R^i_{t+1}\right] \end{aligned}$$

  - use $cov(x, y) = \rho\sigma(x)\sigma(y)$

$$1 = \rho\sigma(m_{t+1})\sigma\left(R^i_{t+1}\right) + E_t\left[m_{t+1}\right] E_t\left[R^i_{t+1}\right]$$

Mean-Variance Analysis

- Continuing...

$$1 = \rho\sigma\left(m_{t+1}\right)\sigma\left(R^i_{t+1}\right) + E_t\left[m_{t+1}\right]E_t\left[R^i_{t+1}\right]$$

Mean-Variance Analysis

- Continuing...

$$1 = \rho\sigma\left(m_{t+1}\right)\sigma\left(R^i_{t+1}\right) + E_t\left[m_{t+1}\right]E_t\left[R^i_{t+1}\right]$$

- Divide by $E_t\left[m_{t+1}\right]$ and use $E_t\left[m_{t+1}\right] = 1/R^f$

$$R^f = \frac{\rho\sigma\left(m_{t+1}\right)\sigma\left(R^i_{t+1}\right)}{E_t\left[m_{t+1}\right]} + E_t\left[R^i_{t+1}\right]$$

- Continuing...

$$1 = \rho\sigma\left(m_{t+1}\right)\sigma\left(R^i_{t+1}\right) + E_t\left[m_{t+1}\right]E_t\left[R^i_{t+1}\right]$$

- Divide by $E_t\left[m_{t+1}\right]$ and use $E_t\left[m_{t+1}\right] = 1/R^f$

$$R^f = \frac{\rho\sigma\left(m_{t+1}\right)\sigma\left(R^i_{t+1}\right)}{E_t\left[m_{t+1}\right]} + E_t\left[R^i_{t+1}\right]$$

- So

$$E_t\left[R^i_{t+1}\right] = R^f - \frac{\rho\sigma\left(m_{t+1}\right)\sigma\left(R^i_{t+1}\right)}{E_t\left[m_{t+1}\right]}$$

Example: Finding Expected Returns

Example

- Given the payoffs and prices for assets A an B we used previously, use $E_t\left[R^i_{t+1}\right] = R^f - \frac{\rho\sigma(m_{t+1})\sigma\left(R^i_{t+1}\right)}{E_t[m_{t+1}]}$ to find the expected returns.
- Recall that

$$\begin{aligned} \sigma(x) &= \sqrt{\Sigma_s \pi^s (x^s - \overline{x})^2} \\ cov(x,y) &= \Sigma_s \pi^s (x^s - \overline{x})(y^s - \overline{y}) \\ \rho &= \frac{cov(x,y)}{\sigma(x)\sigma(y)} \end{aligned}$$

- (MATLAB Exercise)

Example: Finding Expected Returns

Example

- What is the expected return of asset A? Of asset B?

Example: Finding Expected Returns

Example

- What is the expected return of asset A? Of asset B?
  - $E\left(R^A\right) = 1.55$; $E\left(R^B\right) = 1.18$

Example: Finding Expected Returns

Example

- What is the expected return of asset A? Of asset B?
  - $E\left(R^A\right) = 1.55$; $E\left(R^B\right) = 1.18$
- What intution can you get about expected returns (and prices) from ρ?

Example: Finding Expected Returns

Example

- What is the expected return of asset A? Of asset B?
  - $E\left(R^A\right) = 1.55$; $E\left(R^B\right) = 1.18$
- What intution can you get about expected returns (and prices) from ρ?
  - Asset A is **negatively correlated** with the discount factor... (This implies it is positively correlated with consumption...)

Example: Finding Expected Returns

Example

- What is the expected return of asset A? Of asset B?
  - $E\left(R^A\right) = 1.55$; $E\left(R^B\right) = 1.18$
- What intution can you get about expected returns (and prices) from ρ?
  - Asset A is **negatively correlated** with the discount factor... (This implies it is positively correlated with consumption...)
    - So Asset A pays off well in good states of nature and badly in bad states of nature.

Example: Finding Expected Returns

Example

- What is the expected return of asset A? Of asset B?
  - $E\left(R^A\right) = 1.55$; $E\left(R^B\right) = 1.18$
- What intution can you get about expected returns (and prices) from ρ?
  - Asset A is **negatively correlated** with the discount factor... (This implies it is positively correlated with consumption...)
    - So Asset A pays off well in good states of nature and badly in bad states of nature.
  - This kind of asset is "risky"...

Example: Finding Expected Returns

Example

- What is the expected return of asset A? Of asset B?
    - $E\left(R^A\right) = 1.55$; $E\left(R^B\right) = 1.18$
- What intution can you get about expected returns (and prices) from ρ?
    - Asset A is **negatively correlated** with the discount factor... (This implies it is positively correlated with consumption...)
        - So Asset A pays off well in good states of nature and badly in bad states of nature.
    - This kind of asset is "risky"...
    - So this risk must be compensated for: It must have a higher expected return (i.e., a lower price).

Mean-Variance Analysis

- Mean-Variance Analysis starts from our equation

$$E_t\left[R^i_{t+1}\right] = R^f - \frac{\rho\sigma\left(m_{t+1}\right)\sigma\left(R^i_{t+1}\right)}{E_t\left[m_{t+1}\right]}$$

and uses something that must be true about ρ.

Mean-Variance Analysis

- Mean-Variance Analysis starts from our equation

$$E_t\left[R^i_{t+1}\right] = R^f - \frac{\rho\sigma\left(m_{t+1}\right)\sigma\left(R^i_{t+1}\right)}{E_t\left[m_{t+1}\right]}$$

  and uses something that must be true about ρ.
- What do we know about ρ?

Mean-Variance Analysis

- Mean-Variance Analysis starts from our equation

$$E_t\left[R^i_{t+1}\right] = R^f - \frac{\rho\sigma\left(m_{t+1}\right)\sigma\left(R^i_{t+1}\right)}{E_t\left[m_{t+1}\right]}$$

  and uses something that must be true about ρ.
- What do we know about ρ?
  - We know it must be the case that

$$-1 \leq \rho \leq +1$$

Mean-Variance Analysis

- Now, we can relate the expected return of any asset to its correlation with the SDF:
  - All returns must lie below the line

$$E_t\left[R^i_{t+1}\right] = R^f + \frac{\rho\sigma\left(m_{t+1}\right)\sigma\left(R^i_{t+1}\right)}{E_t\left[m_{t+1}\right]}$$

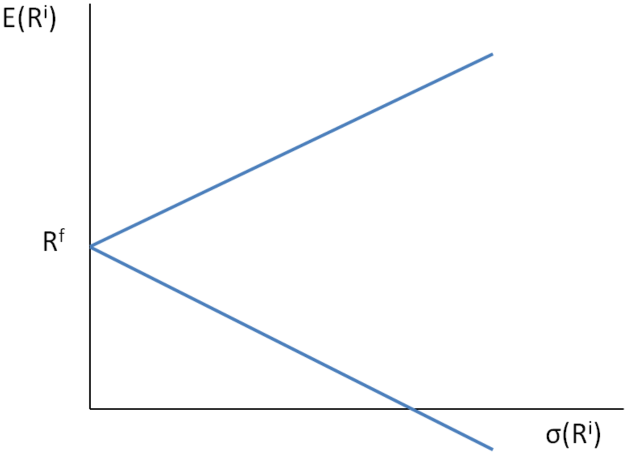
  - and above the line

$$E_t\left[R^i_{t+1}\right] = R^f - \frac{\rho\sigma\left(m_{t+1}\right)\sigma\left(R^i_{t+1}\right)}{E_t\left[m_{t+1}\right]}$$

- So we can graph our "possibilities"...

M-V Frontier - Characterization of Equilibrium Returns

- Top line: $\rho = -1$, slope $= \frac{\sigma(m)}{E(m)}$. Highest risk assets. Why?
- Bottom line: $\rho = 1$, slope $= -\frac{\sigma(m)}{E(m)}$. Lowest risk assets. Why?



Mean-Variance Frontier

- A couple of points to keep in mind for later...
  - Any return can be decomposed into the systematic and an idiosyncratic part.
    - The systematic part is the priced part, perfectly correlated with m
    - The idiosyncratic part generates no expected return

Mean-Variance Frontier

- A couple of points to keep in mind for later...
  - All frontier returns are perfectly correlated with each other.
    - Any two frontier returns can be used to span the frontier.
    - For example, any other frontier return can be expressed as

$$R^{mv} = R^f + a\left(R^m - R^f\right)$$

    - (This gives us the "two-fund theorem" that we'll use later...)

Mean-Variance Frontier

- A couple of points to keep in mind for later...
  - Roll's Theorem also pops out of the MV Frontier...

$$E\left[R^{ei}\right] = \beta_{R^{ei},R^{mv}}\lambda_{R^{mv}} \Leftrightarrow R^{mv} \text{ is on the } MVF$$

    - If $\rho = 1$, then R^{mv} is on the MVF, so $m = a + bR^{mv}$
    - Any asset pricing model is simply positing some "R" on the MVF - e.g., the CAPM uses the market return.

The Equity Premium

- Let's start with $E_t\left[R^i_{t+1}\right] = R^f - \frac{\rho\sigma(m_{t+1})\sigma\left(R^i_{t+1}\right)}{E_t[m_{t+1}]}$

The Equity Premium

- Let's start with $E_t\left[R^i_{t+1}\right] = R^f - \frac{\rho\sigma(m_{t+1})\sigma\left(R^i_{t+1}\right)}{E_t[m_{t+1}]}$
- Now suppose that an asset's return is perfectly correlated with m, then $|\rho| = 1$ and the payoff is on the mean-variance frontier.

The Equity Premium

- Let's start with $E_t\left[R^i_{t+1}\right] = R^f - \frac{\rho\sigma(m_{t+1})\sigma\left(R^i_{t+1}\right)}{E_t[m_{t+1}]}$
- Now suppose that an asset's return is perfectly correlated with m, then $|\rho| = 1$ and the payoff is on the mean-variance frontier.
  - Call this return R^{mv} so

$$\left|\frac{E_t\left[R^{mv} - R^f\right]}{\sigma\left(R^{mv}\right)}\right| = \frac{\sigma(m)}{E_t(m)}$$

The Equity Premium

- Let's start with $E_t\left[R^i_{t+1}\right] = R^f - \frac{\rho\sigma(m_{t+1})\sigma\left(R^i_{t+1}\right)}{E_t[m_{t+1}]}$
- Now suppose that an asset's return is perfectly correlated with m, then $|\rho| = 1$ and the payoff is on the mean-variance frontier.
  - Call this return R^{mv} so

$$\left|\frac{E_t\left[R^{mv} - R^f\right]}{\sigma\left(R^{mv}\right)}\right| = \frac{\sigma\left(m\right)}{E_t\left(m\right)}$$

- The LHS is the Sharpe ratio, the excess return per unit volatility

- Let's start with $E_t\left[R^i_{t+1}\right] = R^f - \frac{\rho\sigma(m_{t+1})\sigma\left(R^i_{t+1}\right)}{E_t[m_{t+1}]}$
- Now suppose that an asset's return is perfectly correlated with m, then $|\rho| = 1$ and the payoff is on the mean-variance frontier.
    - Call this return R^{mv} so

$$\left|\frac{E_t\left[R^{mv} - R^f\right]}{\sigma\left(R^{mv}\right)}\right| = \frac{\sigma\left(m\right)}{E_t\left(m\right)}$$

- The LHS is the Sharpe ratio, the excess return per unit volatility
    - Graphically, it's the slope of the M-V frontier

The Equity Premium

- With power utility, $u'(c) = c^{-\gamma}$ and $m = \beta \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}$ so that

$$\left| \frac{E_t \left[R^{mv} - R^f \right]}{\sigma \left(R^{mv} \right)} \right| = \frac{\sigma \left[\left(c_{t+1} / c_t \right)^{-\gamma} \right]}{E_t \left[\left(c_{t+1} / c_t \right)^{-\gamma} \right]} \approx \gamma \sigma \left(\triangle \ln \left(c \right) \right)$$

- So the Sharpe ratio is higher
  - If consumption is more volatile (the economy is riskier)
  - Or if γ is larger (consumers are more risk-averse)
- In both cases, investors demand a higher return for holding risky assets.

The Equity Premium Puzzle

- The puzzle is that over the past 50 years, the Sharpe ratio has been too high...
  - Stocks have earned too much return, given the riskiness of the economy and the magnitude of consumer risk aversion
- Real stock returns average about 9% with a standard deviation of 16%, while real returns on T-bills are about 1%
  - This gives a Sharpe ratio of about 0.5
- Consumption growth has mean and standard deviation of about 1%
  - $0.5 = \gamma * 0.01$ only if $\gamma = 50$!
- All this assumes that consumption is perfectly correlated with market returns (i.e., $\rho = 1$)
  - Consumption actually has a correlation of $\rho = 0.2$, which just makes it worse

Experimental Analysis

- What does a $\gamma = 50$ mean?
  - Basically, it means that you are so risk averse, that you are afraid to cross the street!
- Experimental analyses on investor behavior typically estimate γ at around 2 or 3.
  - How risk averse do you think you are? (Or are you risk-loving?!)

Equity Premium Puzzle

- So either:

Equity Premium Puzzle

- So either:
  - Consumers are WAY more risk averse than we think.

Equity Premium Puzzle

- So either:
  - Consumers are WAY more risk averse than we think.
  - Stock returns are WAY too high.

Equity Premium Puzzle

- So either:
  - Consumers are WAY more risk averse than we think.
  - Stock returns are WAY too high.
  - We are mis-measuring consumption.

Equity Premium Puzzle

- So either:
  - Consumers are WAY more risk averse than we think.
  - Stock returns are WAY too high.
  - We are mis-measuring consumption.
  - Power utility does a horrible job at capturing consumer behavior.

Equity Premium Puzzle

- So either:
  - Consumers are WAY more risk averse than we think.
  - Stock returns are WAY too high.
  - We are mis-measuring consumption.
  - Power utility does a horrible job at capturing consumer behavior.
- This is the equity premium puzzle.

Assumptions?

- So maybe the assumptions we made in forming the model were wrong...
  - Let's take a look back at them and see if anything stands out...
- So what about our assumptions in deriving $p_t = E_t\left[m_{t+1}x_{t+1}\right]$?
- Maybe we have assumed something we shouldn't have?
  - Let's take a look at these...

Assumptions?

- In deriving $p_t = E_t\left[m_{t+1}x_{t+1}\right]$, we have NOT assumed:

Assumptions?

- In deriving $p_t = E_t\left[m_{t+1}x_{t+1}\right]$, we have NOT assumed:
- Complete markets or a "representative" investor-consumer

Assumptions?

- In deriving $p_t = E_t\left[m_{t+1}x_{t+1}\right]$, we have NOT assumed:
- Complete markets or a "representative" investor-consumer
    - The FOCs we use must hold for *each* investor, and for *any asset that is available*

Assumptions?

- In deriving $p_t = E_t\left[m_{t+1}x_{t+1}\right]$, we have NOT assumed:

Assumptions?

- In deriving $p_t = E_t \left[m_{t+1} x_{t+1} \right]$, we have NOT assumed:
- A distribution for the payoffs

Assumptions?

- In deriving $p_t = E_t\left[m_{t+1}x_{t+1}\right]$, we have NOT assumed:
- A distribution for the payoffs
  - Payoffs do not have to be distributed normally, log-normally, etc.

Assumptions?

- In deriving $p_t = E_t\left[m_{t+1}x_{t+1}\right]$, we have NOT assumed:
- A distribution for the payoffs
    - Payoffs do not have to be distributed normally, log-normally, etc.
    - AND - The basic pricing equation should hold for ANY asset, including stocks, bonds, options, etc.

Assumptions?

- In deriving $p_t = E_t\left[m_{t+1}x_{t+1}\right]$, we have NOT assumed:

Assumptions?

- In deriving $p_t = E_t\left[m_{t+1}x_{t+1}\right]$, we have NOT assumed:
- A 2-Period World

Assumptions?

- In deriving $p_t = E_t\left[m_{t+1}x_{t+1}\right]$, we have NOT assumed:
- A 2-Period World
    - The equation $p_t = E_t\left[m_{t+1}x_{t+1}\right]$ should hold for any two periods in a multi-period model

Assumptions?

- In deriving $p_t = E_t\left[m_{t+1}x_{t+1}\right]$, we have NOT assumed:

Assumptions?

- In deriving $p_t = E_t\left[m_{t+1}x_{t+1}\right]$, we have NOT assumed:
- A separable utility function

Assumptions?

- In deriving $p_t = E_t\left[m_{t+1}x_{t+1}\right]$, we have NOT assumed:
- A separable utility function
  - We do not have to have a time- and state-separable utility function

Assumptions?

- In deriving $p_t = E_t\left[m_{t+1}x_{t+1}\right]$, we have NOT assumed:
- A separable utility function
    - We do not have to have a time- and state-separable utility function
    - As long as the marginal utility is defined, we have m_{t+1}

Assumptions?

- In deriving $p_t = E_t\left[m_{t+1}x_{t+1}\right]$, we have NOT assumed:

Assumptions?

- In deriving $p_t = E_t\left[m_{t+1}x_{t+1}\right]$, we have NOT assumed:
- That other dimensions can't be included

Assumptions?

- In deriving $p_t = E_t\left[m_{t+1}x_{t+1}\right]$, we have NOT assumed:
- That other dimensions can't be included
    - We can include outside income and human capital. We can also include leisure by defining $u = u\left(c, l\right)$.

Assumptions?

- In deriving $p_t = E_t\left[m_{t+1}x_{t+1}\right]$, we have NOT assumed:
- That other dimensions can't be included
  - We can include outside income and human capital. We can also include leisure by defining $u = u\left(c, l\right)$.
  - Again, as long as we can define marginal utility, $u'\left(c, l\right)$ we have m_{t+1}

The Consumption-Based Model in Practice

- We have the equation $E_t\left(R^i\right) = R^f - R^f \frac{\beta cov\left(u'(c_{t+1}), R^i_{t+1}\right)}{u'(c_t)}$
- Now write this in terms of excess returns: since

$$E_t\left(R^e\right) = E_t\left(R^i\right) - R^f$$

so

$$\begin{aligned} E_t\left(R^i\right) - R^f &= -R^f \frac{\beta cov\left(u'\left(c_{t+1}\right), E_t\left(R^i\right) - R^f\right)}{u'\left(c_t\right)} \\ E_t\left(R^e\right) &= -R^f \frac{\beta cov\left(u'\left(c_{t+1}\right), R^e_{t+1}\right)}{u'\left(c_t\right)} \end{aligned}$$

- Note that we can do this because: $cov\left(x, y+a\right) = cov\left(x, y\right)$ if a is a constant

And if we use Power Utility

- With power utility, we get

$$E_t\left(R^e\right) = -R^f cov\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}, R^e_{t+1}\right]$$

  - Cochrane (1996) uses data on 10 portfolios of stocks sorted by size and estimates this model by finding the β and γ that provide the best fit
  - He finds $\beta = 0.98$ and $\gamma = 241$
  - Again, this is the equity premium puzzle

So where does this lead us?

- Poor empirical performance of the **consumption-based** model motivates looking at alternative asset pricing models
- What are the alternatives?
- We will see that all the different alternative models just amount to **different functions for** m

Examples of Some Alternatives?

- Nonseparabilities

Examples of Some Alternatives?

- Nonseparabilities
  - Might require different utility functions or different consumption data

Examples of Some Alternatives?

- Nonseparabilities
  - Might require different utility functions or different consumption data
- GE Models

Examples of Some Alternatives?

- Nonseparabilities
  - Might require different utility functions or different consumption data
- GE Models
  - Link c to fundamental macro variables; *explains* covariances (doesn't have to taken them as givens)

Examples of Some Alternatives?

- Nonseparabilities
  - Might require different utility functions or different consumption data
- GE Models
  - Link c to fundamental macro variables; *explains* covariances (doesn't have to taken them as givens)
- Factor Pricing Models

Examples of Some Alternatives?

- Nonseparabilities
  - Might require different utility functions or different consumption data
- GE Models
  - Link c to fundamental macro variables; *explains* covariances (doesn't have to taken them as givens)
- Factor Pricing Models
  - Use different factors to proxy for m_{t+1} that capture the economy's state: $m_{t+1} = a + \ b_A f_{t+1}^A + b_B f_{t+1}^B \ldots$

Examples of Some Alternatives?

- Nonseparabilities
  - Might require different utility functions or different consumption data
- GE Models
  - Link c to fundamental macro variables; *explains* covariances (doesn't have to taken them as givens)
- Factor Pricing Models
  - Use different factors to proxy for m_{t+1} that capture the economy's state: $m_{t+1} = a + \ b_A f_{t+1}^A + b_B f_{t+1}^B \ldots$
  - CAPM and ICAPM are examples

Examples of Some Alternatives?

- Nonseparabilities
  - Might require different utility functions or different consumption data
- GE Models
  - Link c to fundamental macro variables; *explains* covariances (doesn't have to taken them as givens)
- Factor Pricing Models
  - Use different factors to proxy for m_{t+1} that capture the economy's state: $m_{t+1} = a + \ b_A f^A_{t+1} + b_B f^B_{t+1} \ldots$
  - CAPM and ICAPM are examples
- Arbitrage or Near-Arbitrage Pricing

Examples of Some Alternatives?

- Nonseparabilities
  - Might require different utility functions or different consumption data
- GE Models
  - Link c to fundamental macro variables; *explains* covariances (doesn't have to taken them as givens)
- Factor Pricing Models
  - Use different factors to proxy for m_{t+1} that capture the economy's state: $m_{t+1} = a + \ b_A f_{t+1}^A + b_B f_{t+1}^B \ldots$
  - CAPM and ICAPM are examples
- Arbitrage or Near-Arbitrage Pricing
  - Expresses payoffs in terms of payoffs of other assets to infer pricing information - e.g., Black-Scholes Option Pricing

End of Today's Lecture and Overview.

- That's all for today. Today's material wraps up our overview and what we are covering from Chapter 1 and 2 in Cochrane (2005).