1 Factor Pricing Models

1.1 Overview

- While good in theory, the consumption-based model doesn't work too well in practice.
- This motivates the approach of tying m directly to the data. Linear factor pricing models are the most popular way to do this.
  - Linear factor models dominate discrete-time empirical work, so we'll talk about them a little more today to prepare for the second half of the course.
- Factor pricing models replace the IMRS representation of m with a linear model of the form:

$$m_{t+1} = a + b' f_{t+1}$$

  where a is a constant, b is a $k \times 1$ vector of constants, and f_{t+1} is a $k \times 1$ vector of factors.
- We previously saw that this is equivalent to a multiple beta model:

$$E\left(R_{t+1}\right) = \gamma + \beta' \lambda$$

  where the β's are regression coefficients of the returns R on the factors f.
- The big question is what should one use for the factors? Factor pricing models look for factors that are good proxies for aggregate marginal utility growth, i.e.

$$\beta \frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)} \approx a + b' f_{t+1}.$$

- The key to asset pricing is that a sensible and economically interpretable approximation will reflect the fact that there are special states of the world in which investors are particularly concerned that their portfolios do not do badly - i.e., they are willing to trade off some overall performance (i.e., average return) to make sure their portfolios do not do badly in these particular states of nature.
- The factors should be a proxy for the bad states of the world. They should indicate "bad" states where marginal utility is high and "good" states where marginal utility is low.
- Examples include:
  - Returns on broad-based portfolios like the S&P500
  - Interest rates
  - Growth in GDP
  - Growth in investment
- These factors can proxy for the "state" of the economy.
- We can also use forecasting variables that may not directly indicate good times today, but indicate good times to come. Examples include:

  - Term premium (difference between long term and short term interest rates)
  - Dividend to price ratio

- Why can we do this?
  - We know that consumption and marginal utility respond to **news**: if a change in some variable today signals high income in the future, then consumption rises **now**, by permanent income logic.

- To derive a factor pricing model:
  1. Write down a general equilibrium model and solve it to get $c_t = g(f_t)$.
  2. Substitute $g(f_t)$ for c_t in the first-order condition.

- The derivation does two things:
  - First, it starts with a theory and points to a list of factors that one can use to proxy for the IMRS.
  - Second, it shows that the SDF is linear in the factors.

- **Important: All factor models are derived as specializations of the consumption-based model.**
  - Many authors berate the consumption-based model in favor of factor models, but they fail to realize that their factor model IS the consumption-based model plus some extra assumptions that allow one to use a factor to proxy for marginal utility of consumption.

- This economic foundation is important since it helps us to guard against fishing for factors.
  - Remember, there IS an ex-post mean-variance efficient return that will price all assets exactly in sample.

- We need better theories that link the true macro-economic sources of risk to the factors, but the best minds in finance have been working on this since the 1960s and a solution is not in sight.

- The text spends a great deal of time deriving the models in different ways. We will derive the CAPM as an example of how the setup works. These exercises are important, since they allow us to see what kind of unrealistic assumptions we need to arrive at popular models such as the CAPM.