1 Overview of Classic Issues in Finance

1.1 A Reminder about Returns...

- Last time, we talked about returns...
  - The gross return is the dollar payoff per dollar invested.
- We divide the payoff x_{t+1} by the price p_t to get a gross return

$$R_{t+1} = \frac{x_{t+1}}{p_t}$$

- So you can think of a return as a payoff with a price of one

$$1 = E_t\left(m_{t+1}R_{t+1}\right)$$

1.2 The Risk-Free Rate

- Last time, we also saw that a risk-free bond is a claim to a \$1 unit payoff in every state.

$$p_t = E_t\left[m_{t+1}1\right]$$

- The payoff of a risk-free \$1 par bond is $x_{t+1} = 1$
- The return is

$$R^f_{t+1} = \frac{1}{p_t}$$

- We can use the asset pricing equation for bonds as follows

$$p_t = E_t\left[m_{t+1}1\right] = \frac{1}{R^f}$$

- This tells us something important about the expected value of the SDF

$$E_t\left[m_{t+1}\right] = \frac{1}{R^f}$$

  or equivalently

$$R^f = \frac{1}{E_t\left[m_{t+1}\right]}$$

- If there is no risk-free security, we can call

$$R^f := \frac{1}{E_t\left[m_{t+1}\right]}$$

  the "shadow" risk-free rate, or zero-beta rate.
  - An asset that has a zero-beta has a covariance of zero with consumption
- Note that investors would be just indifferent to buying or selling a risk-free security with return R^f precisely because $R^f = \frac{1}{E_t[m_{t+1}]}$

1.3 Risk-Free Rate and Risk Aversion

- Let's use the fact that $R^f_{t+1} = \frac{1}{E_t[m_{t+1}]}$ and think about, e.g., how this relates to risk-aversion...
- Suppose there is no uncertainty. Let investors have power utility

$$u'(c) = c^{-\gamma}$$

so

$$m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} = \beta \left(\frac{c_t}{c_{t+1}} \right)^{\gamma}$$

and so with no uncertainty,

$$R^f_{t+1} = \frac{1}{m_{t+1}} = \frac{1}{\beta} \left(\frac{c_{t+1}}{c_t} \right)^{\gamma}$$

- What can this equation tell us about interest rates?
  - As β increases, what happens to R^f?
  - What happens to R^f as consumption growth increases - i.e., (c_{t+1}/c_t) goes up?

  - What about γ? How does R^f respond to changes in γ?

1.4 Pricing Risk Corrections

- We've already demonstrated how the SDF generalizes standard present-value ideas...
- Recall, that when we put the discount factor inside the expectation,

$$p_t = E_t\left[m_{t+1}x_{t+1}\right]$$

- We can re-write this expression using the definition of a covariance

$$cov\left(m_{t+1}x_{t+1}\right) = E\left(m_{t+1}x_{t+1}\right) - E\left(m_{t+1}\right)E_t\left(x_{t+1}\right)$$

  to get

$$p_t = E_t\left(m_{t+1}\right)E_t\left(x_{t+1}\right) + cov\left(m_{t+1}, x_{t+1}\right).$$

- Now use the fact that $R^f = 1/E_t\left(m_{t+1}\right)$ to write

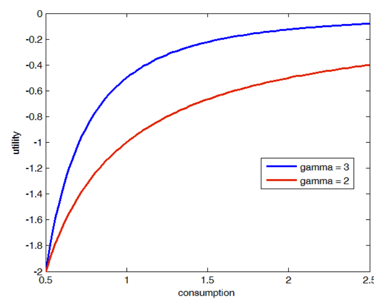
$$p_t = \frac{E_t\left(x_{t+1}\right)}{R^f} + cov\left(m_{t+1}, x_{t+1}\right)$$

  and we see that

  - the first term is the expected payoff discounted at the risk-free rate (standard PV formula)
  - the second term is the risk correction

- We also see that an asset whose payoff covaries positively with the discount factor has its price raised.
  - How does that asset covary with consumption?

- So what securities are "risky"?
  - Not necessarily the ones with the highest variance or volatility! Why?
  - According to our central asset pricing equation, the "risky" securities are the ones that have low payoffs in bad states of nature.
- Take careful note that when we refer to "risk", this is what we mean. Risk and volatility are different.
- To think about "risky" securities...
- Below is a plot of power utility for $\gamma = 3$ and $\gamma = 2$. Using this graph, what intuition can you get about a security's "riskiness" using the example that follows?



- You can see the general intuition as follows:
- Suppose that you have a 50 percent probability of $c_{t+1} = 1$ or $c_{t+1} = 2$ (bad state and good state, respectively)
- Suppose that there are two different kinds of risky assets:
  - The first asset correlates perfectly positively with consumption: you get 0.5 units of consumption if $c_{t+1} = 2$, or -0.5 units of consumption if $c_{t+1} = 1$.
  - The second asset has the same potential payoffs, but reversed.
- What asset has higher volatility? Which asset is more "risky"? Which asset helps to lessen volatility of consumption?

1.5 Pricing Risk Corrections: Returns

- Now, we're going to talk about that same intuition for returns (and do some math).
- Write

$$1 = E_t \left[m_{t+1} R_{t+1}^i \right]$$

and use the definition of a covariance to write

$$1 = E_t \left(m_{t+1} \right) E_t \left(R_{t+1}^i \right) + cov \left(m_{t+1}, R_{t+1}^i \right)$$

- Now use $R^f = 1/E(m_{t+1})$

$$1 = \left(1/R^f\right) E_t\left(R^i_{t+1}\right) + cov\left(m_{t+1}, R^i_{t+1}\right)$$

and write

$$E_t\left(R^i\right) = R^f - R^f cov\left(m_{t+1}, R^i_{t+1}\right).$$

- Next

$$E_t\left(R^i\right) = R^f - R^f \frac{\beta cov\left(u'(c_{t+1}), R^i_{t+1}\right)}{u'(c_t)}$$

- And we get

$$\begin{aligned} E_t\left(R^i\right) - R^f &= -R^f \frac{\beta cov\left(u'(c_{t+1}), R^i_{t+1}\right)}{u'(c_t)} \\ &= -\frac{u'(c_t)}{\beta E_t[u'(c_{t+1})]} \frac{\beta cov\left(u'(c_{t+1}), R^i_{t+1}\right)}{u'(c_t)} \\ &= -\frac{cov\left(u'(c_{t+1}), R^i_{t+1}\right)}{E_t[u'(c_{t+1})]} \end{aligned}$$

- So what does this say?
- Now we have

$$E_t\left(R^i\right) - R^f = -\frac{cov\left(u'(c_{t+1}), R^i_{t+1}\right)}{E_t[u'(c_{t+1})]}$$

- So assets that covary positively with consumption...
  - i.e., R^i is low when c is low (so $u'(c_{t+1})$ is high)
- ...make consumption more volatile, and therefore have to have a higher expected return.
- Note: The two relations for prices and returns are the same -
  - A low price today is the same as a higher expected return, for a given random payoff.

1.6 Idiosyncratic Risk?

- What about idiosyncratic risk?
- At the end of the day, investors really only care about consumption (and its volatility)
  - This is why it's the covariance with consumption that determines an asset's riskiness, not an asset's individual variance
- From

$$p_t = \frac{E_t(x_{t+1})}{R^f} + \beta \frac{cov(u'(c_{t+1}), x_{t+1})}{u'(c_t)}$$

if

$$cov(u'(c_{t+1}), x_{t+1}) = 0$$

then

$$p_t = E_t(x_{t+1})/R^f$$

- So there would be no risk correction even if the asset is very volatile.
- It's easy to see why idiosyncratic risk shouldn't matter if we use portfolio theory.
- Suppose you are concerned about the volatility of your consumption and you consider adding a "small" - i.e., epsilon - amount of a security x to your portfolio. How does this affect overall volatility?
- The volatility of your new portfolio is

$$\sigma^2 (c + \varepsilon x) = \sigma^2 (c) + 2\varepsilon cov (c, x) + \varepsilon^2 \sigma^2 (x)$$

- This just shows the portfolio theory result of the benefits of diversification for portfolio volatility. Idiosyncratic asset volatility is less important for overall portfolio volatility than is its covariance/correlation with the rest of the portfolio.

1.7 Expected Return-Beta Representation

- Let's go back to the expression

$$E_t \left(R^i\right) = R^f - R^f cov \left(m_{t+1}, R^i_{t+1}\right)$$

- Now, we're going to do another derivation that gets us to a familiar expected return - beta representation.
- Use $R^f = 1/E_t [m_{t+1}]$ again to get

$$E_t \left(R^i\right) = R^f - \frac{cov \left(m_{t+1}, R^i_{t+1}\right)}{E_t [m_{t+1}]}$$

- And then multiply and divide by one $\left(\frac{var(m_{t+1})}{var(m_{t+1})}\right)$ to get

$$= R^f + \frac{cov \left(m_{t+1}, R^i_{t+1}\right)}{var [m_{t+1}]} \left(-\frac{var [m_{t+1}]}{E_t [m_{t+1}]}\right)$$

- Now we can define β and λ in such a way that

$$\begin{aligned} E_t \left(R^i\right) &= R^f + \frac{cov \left(m_{t+1}, R^i_{t+1}\right)}{var [m_{t+1}]} \left(-\frac{var [m_{t+1}]}{E_t [m_{t+1}]}\right) \\ &= R^f + \beta_{i,m} \lambda_m \end{aligned}$$

  - $\beta_{i,m}$ is the "quantity of risk", the regression coefficient of R^i on m - different for each asset
  - λ_m is the "price of risk" - the same across assets

- Note that the "price of risk" depends on the volatility of the discount factor...
- What does it say?
- From

$$\begin{aligned} E_t \left(R^i\right) &= R^f - \frac{cov \left(m_{t+1}, R^i_{t+1}\right)}{E_t [m_{t+1}]} \\ &= R^f + \beta_{i,m} \lambda_m \end{aligned}$$

- Assets with payoffs that have positive covariance with consumption $\Rightarrow$ high (negative) beta
  - These assets make consumption more volatile, so must have a higher expected return.
- The higher risk aversion or the more volatile consumption, the larger λ_m
  - People require a higher return to hold risky assets.