1 New Models

1.1 Rejection of the CRRA Model

- The power utility model with T-bill and S&P500 Returns
  - Estimates an unreasonable discount factor and parameter for risk aversion
- The power utility model with size-sorted portfolios and T-bill returns
  - Estimates a discount factor closer to 1, but estimated risk aversion is still too high
  - The J-Test rejects the model
- But these are rejections of a specific SDF, *not* of the SDF methodology, rationality, or market efficiency.
  - There is an SDF that accurately prices the market, just not this one.

1.2 Some Notes...

- GMM picks the parameters that give the model the best chance of passing the J Test
  - Therefore, no other set of parameters can do better
  - The data is fixed - it is the data we want to understand and explain
- The J Test does not tell us *why*, in economic terms, we reject the model
  - The J Test does tell us that the pricing errors are large
  - Later on, we will turn to the HJ Bound to get more of a 'why' answer
- But for now, let's consider another SDF...

1.3 Habit

1.3.1 What's wrong with power utility?

- Power utility assumes that utility over consumption is separable across periods.
  - We call this class of functions 'time-separable' utility functions.
- An alternative assumption is that consumption is habit-forming.
  - We get 'used to' consuming at a certain level.
  - Declines in consumption are more costly as we approach the 'habit' level of consumption.
  - We call this class of functions 'non-time-separable' utility functions.

1.3.2 Campbell and Cochrane (1999)

- People develop 'habits' for consumption
  - The 'habits' form a 'trend' in consumption
- The idea is that what you consume yesterday has an impact on how you feel about more consumption today.
  - If we get used to a certain standard of living, a fall in our consumption hurts more following a few years of 'good times' than that same level feels after a few years of 'bad times.'
  - Idea is to not tie risk aversion to the level of consumption/wealth (since this has increased over time, but the equity premium has not declined), but to the level relative to some 'trend' or recent past

1.4 Internal Habit

- Internal Habit Formation

$$u\left(c_t\right) = \frac{\left(c_t - \delta c_{t-1}\right)^{1-\gamma}}{1-\gamma}$$

with corresponding IMRS

$$\beta \frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)} = \beta E_t \left[\frac{\left(c_{t+1} - \delta c_t\right)^{-\gamma} - \beta\delta\left(c_{t+2} - \delta c_{t+1}\right)^{-\gamma}}{\left(c_t - \delta c_{t-1}\right)^{-\gamma} - \beta\delta\left(c_{t+1} - \delta c_t\right)^{-\gamma}} \right]$$

- We can also write this in terms of consumption growth rates

$$\beta u'\left(\triangle c_{t+1}\right) = \beta E_t \left[\frac{\left(\triangle c_{t+1} - \delta\right)^{-\gamma} - \beta\delta\left(\triangle c_{t+1}\right)^{-\gamma}\left(\triangle c_{t+2} - \delta\right)^{-\gamma}}{\left(1 - \delta\triangle\left[c_{t-1}\right]^{-1}\right)^{-\gamma} - \beta\delta\left(\triangle c_{t+1} - \delta\right)^{-\gamma}} \right]$$

- How did we get here? (Divide the top and bottom of the fraction on the RHS by $1/c_t$)
- It's "like" you don't care about consumption; but instead, the first-difference of consumption.
  - Usually use 0.7 as habit parameter, but there are different numbers you can use as well (you can estimate it!).
- Note that our specification for habit is for a "one-period" habit
  - This is the simplest version of the specification, and makes it easier to understand what's going on...
- However, note that when we described habit, we really described it as some standard that we've gotten used to over a number of years.
  - We really want something that is slow moving...
  - A lot of macro is now using a one-period habit model... This is a very bad idea!

1.5 External Habit

- External Habit Formation (Keeping up with the Joneses)

$$u\left(c_t\right) = \frac{\left(c_t - \delta \widetilde{c}_{t-1}\right)^{1-\gamma}}{1-\gamma}$$

- Care about your consumption relative to your neighbors.
- This version of the model has many of the same economic features of internal habit model; but taking derivatives in this model is much simpler!
  - You are small, relative to the world; so if you change your consumption today, you don't change the aggregate stock of consumption in the US
  - For internal habit, if you change your consumption today, it matters for your habit stock in the future.

1.6 Why does habit help?

- For any model to explain the high equity premium, it has to come from either a large *risk* or from a large *risk aversion*...
  - Now, not just the fear of a loss of wealth from a bad market return *per se*; but the loss of wealth because it tends to come in recessions
- Mechanically, makes you more risk averse
  - Gets the equity premium in the model
  - Introduces a time-varying risk aversion
    - (The crash in the Fall of 2008 sort of looks like habit-induced rising risk aversion!)
  - Increases the volatility of the SDF

2 Equity Premium

2.1 Recall, the Equity Premium Puzzle (Again)

- Either a large risk aversion coefficient or counterfactually large consumption variability is required to explain the means and variances of asset returns...
- We think reasonable measures for risk aversion are about 1 or 2.
  - The highest possible reasonable measure 10 doesn't get us anywhere close to matching the equity premium.
- Can explain the equity premium by continuing to increase the risk aversion coefficient, but you get a risk-free rate puzzle.

  - High values of γ near 50 or 60 can get an equity premium of about 7 percent (matching the differences between bond and stock returns)
  - But yield ridiculously high levels, e.g., risk-free rates around 60 percent!

2.2 Risk Aversion and the IMRS

- One of the reasons we don't like to raise risk-aversion in our models is because risk aversion is the inverse of intertemporal substitution
  - So if people are very risk-averse, it means people are not very willing to substitute consumption over time
- But all the micro panel data says that intertemporal substitution is not that far from 1, maybe down as low as 0.7
  - If you start raising risk aversion to 10 or 15, it makes people very unwilling to substitute over time
- What that means in a macro model is that consumption is not going to fluctuate very much over time, which is clearly counterfactual.
  - Aggregate consumption data is volatile. Rules out the possibility where people are unwilling to substitute over time.

2.3 Why does habit help?

- Gets around the risk-free rate puzzle through 'precautionary savings'
- If we are in bad times (where consumption is low, relative to habit)
  - We want to borrow against the future, so this should drive up interest rates
- But people are also *more risk-averse* when consumption is low
  - This makes you save more to build up assets against the possibility that tomorrow might be worse
  - This precautionary motive to save drives interest rates back down
- The habit model allows high risk aversion with low aversion to intertemporal substitution that is consistent with consumption and interest rate data

2.3.1 So are we done?

- Note that the habit model is quite ad hoc
  - Not derived from any deep principles
  - What is the mechanism?
    - Habit, leverage, and irreversible durable goods models all behave the same way
  - Many argue that a 'solution' to the equity premium should feature a low risk aversion

3 More Exotic Preferences

3.1 Epstein-Zin

- Whole literature on exotic preferences
  - The habit model is just one of these
- Epstein-Zin preferences are also popular
  - Way to separate risk aversion and intertemporal substitution
  - Can keep intertemporal substitution around 1, which we like; then can start raising risk aversion to see what it does for asset prices
- Preferences are represented recursively (see Weil, 1989)
  - Values current consumption and your expectation of (all) future utilities
  - So, your risk-aversion is over your wealth, which is like future utility (and you're risk-averse over that)
  - See these preferences a lot in finance; also those trying to relate macro models to asset prices
- Preferences are $V_t = U\left[C_t, E_t V_{t+1}\right]$ where $U\left[\cdot\right]$ is an aggregator function

$$U = \frac{\left\{(1-\beta)\, C_t^{1-\rho} + \beta\left[1 + (1-\beta)\,(1-\gamma)\, E_t V_{t+1}\right]^{\frac{1-\rho}{1-\gamma}}\right\}^{\frac{1-\gamma}{1-\rho}} - 1}{(1-\beta)\,(1-\gamma)}$$

  - Risk-aversion: γ
  - Intertemporal substitution: $1/\rho$

4 Other Solutions to the Equity Premium Puzzle?

4.1 Catastrophe Literature

- After Mehra and Prescott's (1985) paper...
  - (Puzzle described in context of a two-state Markov model for consumption growth)
- Introducing a small probability of a big crisis (Reitz, JME)
  - Some 'crash' probability (larger than the Great Depression!)
  - Finds - with risk aversion of 10 - gets equity premium and lowers the risk-free rate to match the data.
  - The (small) probability of a (big) crash increases the demand for a risk-less asset (so the price goes up, the return goes down)
- Mehra and Prescott immediately dismiss the paper: Claim that Reitz is explaining the puzzle by something never observed.

  - If we can put things in our models that we never observe, is there anything we can't explain?

- Barro has recently re-introduced the idea; saying - 25 years later - Reitz was correct (minus some empirical work).
  - Introduced a new literature, the 'catastrophe literature'.
  - Estimated (using data over different countries), that the probability of a big crisis is actually much bigger than Reitz initially claimed.
    * So the model doesn't require as big of a drop in output as Reitz initially assumed.
  - Probability of a big crisis for a $15-20$ percent drop in output is about 1 percent.

- Relatively new (and active) literature.
  - Want to also match distributions of returns, serial correlations of returns, time-variation in expected returns, etc.
  - Criticism 1: Taking too many degrees of freedom, e.g., ideas of time-varying probabilities of crisis, to match additional moments.
  - Criticism 2: Why is it the case that investors in the US are considering the probability of a crisis in South America?