1 Classic Issues in Finance, Continued...

1.1 Review: Expected Return-Beta Rep

- Recall, that we can start with

$$1 = E_t\left[m_{t+1}R^i_{t+1}\right]$$

- To derive

$$\begin{aligned} 1 &= E_t\left(m_{t+1}\right)E_t\left(R^i_{t+1}\right) + cov\left(m_{t+1}, R^i_{t+1}\right) \\ 1 &= \left(1/R^f\right)E_t\left(R^i_{t+1}\right) + cov\left(m_{t+1}, R^i_{t+1}\right) \end{aligned}$$

- So we can write

$$\begin{aligned} E_t\left(R^i\right) &= R^f - R^f cov\left(m_{t+1}, R^i_{t+1}\right) \\ &= R^f - \frac{cov\left(m_{t+1}, R^i_{t+1}\right)}{E_t\left[m_{t+1}\right]} \\ &= R^f + \frac{cov\left(m_{t+1}, R^i_{t+1}\right)}{var\left[m_{t+1}\right]}\left(-\frac{var\left[m_{t+1}\right]}{E_t\left[m_{t+1}\right]}\right) \end{aligned}$$

- Now we have

$$E_t\left(R^i\right) = R^f + \beta_{i,m}\lambda_m$$

  - where we can define $\beta_{i,m} := \frac{cov\left(m_{t+1}, R^i_{t+1}\right)}{var\left[m_{t+1}\right]}$ as the "quantity of risk"
  - and $\lambda_m := \left(-\frac{var\left[m_{t+1}\right]}{E_t\left[m_{t+1}\right]}\right)$ as the "price of risk"

- Which is the same across assets? Which varies across assets?

- What would a graphical representation of $E_t\left(R^i\right) = R^f + \beta_{i,m}\lambda_m$ show you?

- We're going to use this expression again today

$$
\begin{aligned}
E_t\left(R^i\right) &= R^f + \frac{cov\left(m_{t+1}, R^i_{t+1}\right)}{var\left[m_{t+1}\right]}\left(-\frac{var\left[m_{t+1}\right]}{E_t\left[m_{t+1}\right]}\right) \\
E_t\left(R^i\right) &= R^f + \beta_{i,m}\lambda_m
\end{aligned}
$$

- So keep in mind some of the results that we got last time:
  - Assets with payoffs that have positive covariance with consumption $\Rightarrow$ high (negative) beta
  - These assets make consumption more volatile, so must have a higher expected return
  - People require a higher return to hold risky assets
- Also recall that we found last time that only systematic risk is "priced" - i.e., idiosyncratic volatility doesn't matter...

1.2 Introduction to Mean-Variance Analysis

- Now, we're going to relate the SDF to traditional mean-variance analysis
- Start with

$$1 = E_t\left[m_{t+1}R^i_{t+1}\right]$$

- And manipulate it with basic definitions from probability...
  - use $cov\left(x, y\right) = E\left(x, y\right) - E\left(x\right)E\left(y\right)$

$$
\begin{aligned}
1 &= E_t\left[m_{t+1}R^i_{t+1}\right] \\
&= cov\left(m_{t+1}, R^i_{t+1}\right) + E_t\left[m_{t+1}\right]E_t\left[R^i_{t+1}\right]
\end{aligned}
$$

  - use $cov\left(x, y\right) = \rho\sigma\left(x\right)\sigma\left(y\right)$

$$1 = \rho\sigma\left(m_{t+1}\right)\sigma\left(R^i_{t+1}\right) + E_t\left[m_{t+1}\right]E_t\left[R^i_{t+1}\right]$$

- Continuing... Divide by $E_t\left[m_{t+1}\right]$ and use $E_t\left[m_{t+1}\right] = 1/R^f$

$$R^f = \frac{\rho\sigma\left(m_{t+1}\right)\sigma\left(R^i_{t+1}\right)}{E_t\left[m_{t+1}\right]} + E_t\left[R^i_{t+1}\right]$$

- So

$$E_t\left[R_{t+1}^i\right] = R^f - \frac{\rho\sigma\left(m_{t+1}\right)\sigma\left(R_{t+1}^i\right)}{E_t\left[m_{t+1}\right]}$$

  - Example: Finding Expected Returns
    - Given the payoffs and prices for assets A an B we used previously, use $E_t\left[R_{t+1}^i\right] = R^f - \frac{\rho\sigma(m_{t+1})\sigma\left(R_{t+1}^i\right)}{E_t[m_{t+1}]}$ to find the expected returns.
    - Recall that

$$\begin{aligned}\sigma(x) &= \sqrt{\Sigma_s \pi^s \left(x^s - \overline{x}\right)^2} \\ cov(x,y) &= \Sigma_s \pi^s \left(x^s - \overline{x}\right)\left(y^s - \overline{y}\right) \\ \rho &= \frac{cov(x,y)}{\sigma(x)\sigma(y)}\end{aligned}$$

    - (MATLAB Exercise)
      - What is the expected return of asset A? Of asset B? What intution can you get about expected returns (and prices) from ρ?

- Mean-Variance Analysis starts from our equation

$$E_t\left[R_{t+1}^i\right] = R^f - \frac{\rho\sigma\left(m_{t+1}\right)\sigma\left(R_{t+1}^i\right)}{E_t\left[m_{t+1}\right]}$$

and uses something that must be true about ρ.

- What do we know about ρ?

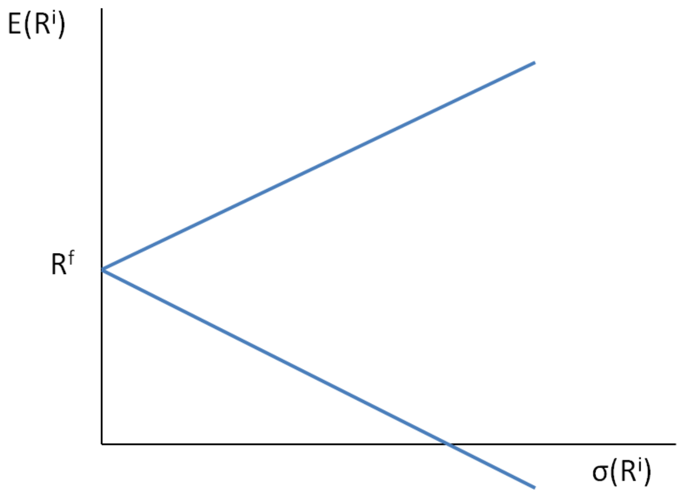
- Now, we can relate the expected return of any asset to its correlation with the SDF:
  - All returns must lie below the line

$$E_t\left[R_{t+1}^i\right] = R^f + \frac{\rho\sigma\left(m_{t+1}\right)\sigma\left(R_{t+1}^i\right)}{E_t\left[m_{t+1}\right]}$$

- and above the line

$$E_t\left[R_{t+1}^i\right] = R^f - \frac{\rho\sigma\left(m_{t+1}\right)\sigma\left(R_{t+1}^i\right)}{E_t\left[m_{t+1}\right]}$$

- So we can graph our "possibilities"...
- The MV Frontier is a graphical characterization of equilibrium returns...



- Top line: $\rho = -1$, slope $= \frac{\sigma(m)}{E(m)}$. Highest risk assets. Why?

- Bottom line: $\rho = 1$, slope $= -\frac{\sigma(m)}{E(m)}$. Lowest risk assets. Why?

- A couple of points to keep in mind for later...
  - Any return can be decomposed into the systematic and an idiosyncratic part.
    - The systematic part is the priced part, perfectly correlated with m
    - The idiosyncratic part generates no expected return
  - All frontier returns are perfectly correlated with each other.
    - Any two frontier returns can be used to span the frontier.
    - For example, any other frontier return can be expressed as

$$R^{mv} = R^f + a\left(R^m - R^f\right)$$

    * (This gives us the "two-fund theorem" that we'll use later...)

  – Roll's Theorem also pops out of the MV Frontier...

$$E\left[R^{ei}\right] = \beta_{R^{ei},R^{mv}}\lambda_{R^{mv}} \Leftrightarrow R^{mv} \text{ is on the } MVF$$

    * If $\rho = 1$, then R^{mv} is on the MVF, so $m = a + bR^{mv}$
    * Any asset pricing model is simply positing some "R" on the MVF - e.g., the CAPM uses the market return.

1.3 Equity Premium Puzzle

- Start with $E_t\left[R^i_{t+1}\right] = R^f - \frac{\rho\sigma(m_{t+1})\sigma\left(R^i_{t+1}\right)}{E_t[m_{t+1}]}$
- If an asset's returns are perfectly correlated with m, then $|\rho| = 1$ and the payoff is on the mean-variance frontier.
  - Call this return R^{mv} so
$$\left|\frac{E_t\left[R^{mv} - R^f\right]}{\sigma\left(R^{mv}\right)}\right| = \frac{\sigma\left(m\right)}{E_t\left(m\right)}$$
- The LHS is the Sharpe ratio, the excess return per unit volatility
  - Graphically, it's the slope of the M-V frontier
- With power utility, $u'\left(c\right) = c^{-\gamma}$ and $m = \beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}$ so that
$$\left|\frac{E_t\left[R^{mv} - R^f\right]}{\sigma\left(R^{mv}\right)}\right| = \frac{\sigma\left[\left(c_{t+1}/c_t\right)^{-\gamma}\right]}{E_t\left[\left(c_{t+1}/c_t\right)^{-\gamma}\right]} \approx \gamma\sigma\left(\triangle\ln\left(c\right)\right)$$
- So the Sharpe ratio is higher
  - If consumption is more volatile (the economy is riskier)
  - Or if γ is larger (consumers are more risk-averse)
- In both cases, investors demand a higher return for holding risky assets.
- The puzzle is that over the past 50 years, the Sharpe ratio has been too high...
  - Stocks have earned too much return, given the riskiness of the economy and the magnitude of consumer risk aversion
- Real stock returns average about 9% with a standard deviation of 16%, while real returns on T-bills are about 1%
  - This gives a Sharpe ratio of about 0.5
- Consumption growth has mean and standard deviation of about 1%
  - $0.5 = \gamma * 0.01$ only if $\gamma = 50$!

- All this assumes that consumption is perfectly correlated with market returns (i.e., $\rho = 1$)
  - Consumption actually has a correlation of $\rho = 0.2$, which just makes it worse
- What does a $\gamma = 50$ mean?
  - Basically, it means that you are so risk averse, that you are afraid to cross the street!
  - How risk averse do you think you are? (Or are you risk-loving?!)
- So either:
  - Consumers are WAY more risk averse than we think
  - Stock returns are WAY too high
  - We are mis-measuring consumption
  - Power utility does a horrible job at capturing consumer behavior
- This is the equity premium puzzle

2 Applying the Basic Model

2.1 Assumptions?

- So maybe the assumptions we made in forming the model were wrong...
  - Let's take a look back at them and see if anything stands out...
- So what about our assumptions in deriving $p_t = E_t[m_{t+1}x_{t+1}]$?
- Maybe we have assumed something we shouldn't have?
  - Let's take a look at these...
- In deriving $p_t = E_t[m_{t+1}x_{t+1}]$, we have NOT assumed:
  - Complete markets or a "representative" investor-consumer
    * The FOCs we use must hold for *each* investor, and for *any asset that is available*
  - A distribution for the payoffs
    * Payoffs do not have to be distributed normally, log-normally, etc.
    * The basic pricing equation should hold for ANY asset, including stocks, bonds, options, etc.
  - A 2-Period World
    * The equation $p_t = E_t[m_{t+1}x_{t+1}]$ should hold for any two periods in a multi-period model
  - A separable utility function
    * We do not have to have a time- and state-separable utility function
    * As long as the marginal utility is defined, we have m_{t+1}

- That other dimensions can't be included
  - We can include outside income and human capital. We can also include leisure by defining $u = u(c, l)$.
  - Again, as long as we can define marginal utility, $u'(c, l)$ we have m_{t+1}

2.2 Consumption-Based Model in Practice

- We have the equation $E_t\left(R^i\right) = R^f - R^f \frac{\beta cov\left(u'(c_{t+1}), R^i_{t+1}\right)}{u'(c_t)}$
- Now write this in terms of excess returns: since

$$E_t\left(R^e\right) = E_t\left(R^i\right) - R^f$$

so

$$\begin{aligned} E_t\left(R^i\right) - R^f &= -R^f \frac{\beta cov\left(u'\left(c_{t+1}\right), E_t\left(R^i\right) - R^f\right)}{u'\left(c_t\right)} \\ E_t\left(R^e\right) &= -R^f \frac{\beta cov\left(u'\left(c_{t+1}\right), R^e_{t+1}\right)}{u'\left(c_t\right)} \end{aligned}$$

- Note that we can do this because: $cov(x, y + a) = cov(x, y)$ if a is a constant
- With power utility, we get

$$E_t\left(R^e\right) = -R^f cov\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}, R^e_{t+1}\right]$$

  - Cochrane (1996) uses data on 10 portfolios of stocks sorted by size and estimates this model by finding the β and γ that provide the best fit
  - He finds $\beta = 0.98$ and $\gamma = 241$
  - Again, this is the equity premium puzzle

- So where does this lead us?
  - Poor empirical performance of the **consumption-based** model motivates looking at alternative asset pricing models
  - What are the alternatives?
  - We will see that all the different alternative models just amount to **different functions for** m

- Examples of Some Alternatives?
  - Nonseparabilities
    - Might require different utility functions or different consumption data
  - GE Models
    - Link c to fundamental macro variables; *explains* covariances (doesn't have to taken them as givens)

- Factor Pricing Models
  - Use different factors to proxy for m_{t+1} that capture the economy's state: $m_{t+1} = a + b_A f_{t+1}^A + b_B f_{t+1}^B ...$
  - CAPM and ICAPM are examples
- Arbitrage or Near-Arbitrage Pricing
  - Expresses payoffs in terms of payoffs of other assets to infer pricing information - e.g., Black-Scholes Option Pricing