ECON 4360: Empirical Finance

Classic Issues in Finance I

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Theory Lecture #05

What are we doing today?

- Overview of Classic Issues in Finance

Recall: Returns in General

- Last time, we talked about returns
  - The gross return is the dollar payoff per dollar invested
- We divide the payoff x_{t+1} by the price p_t to get a gross return

$$R_{t+1} = \frac{x_{t+1}}{p_t}$$

- So you can think of a return as a payoff with a price of one

$$1 = E_t \left(m_{t+1} R_{t+1} \right)$$

Recall: Returns for a Risk-Free Bond

- Last time, we also saw that a risk-free bond is a claim to a \$1 unit payoff in every state.

$$p_t = E_t\left[m_{t+1} 1\right]$$

- The payoff of a risk-free \$1 par bond is $x_{t+1} = 1$
- The return is

$$R^f_{t+1} = \frac{1}{p_t}$$

Risk-Free Rate

- We can use the asset pricing equation for bonds as follows

$$p_t = E_t\left[m_{t+1}1\right] = \frac{1}{R^f}$$

- This tells us something important about the expected value of the SDF

$$E_t\left[m_{t+1}\right] = \frac{1}{R^f}$$

or equivalently

$$R^f = \frac{1}{E_t\left[m_{t+1}\right]}$$

Shadow Risk-Free Rate

- If there is no risk-free security, we can call

$$R^f := \frac{1}{E_t\left[m_{t+1}\right]}$$

  the "shadow" risk-free rate, or zero-beta rate.
  - An asset that has a zero-beta has a covariance of zero with consumption
- Note that investors would be just indifferent to buying or selling a risk-free security with return R^f precisely because $R^f = \frac{1}{E_t[m_{t+1}]}$

Risk-Free Rate and Risk Aversion

- Let's use the fact that $R^f_{t+1} = \frac{1}{E_t[m_{t+1}]}$ and think about how this relates to risk-aversion
- Suppose there is no uncertainty. Let investors have power utility

$$u'(c) = c^{-\gamma}$$

so

$$m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} = \beta \left(\frac{c_t}{c_{t+1}} \right)^{\gamma}$$

and so with no uncertainty,

$$R^f_{t+1} = \frac{1}{m_{t+1}} = \frac{1}{\beta} \left(\frac{c_{t+1}}{c_t} \right)^{\gamma}$$

- What can this equation tell us about interest rates? ...

Rf and Time Discounting

- From $R^f_{t+1} = \frac{1}{m_{t+1}} = \frac{1}{\beta}\left(\frac{c_{t+1}}{c_t}\right)^{\gamma}$...
- As β increases, what happens to R^f?

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  - As β increases, people become less impatient - i.e., they discount the future at a lower rate. This makes them willing to accept a lower interest rate.

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  - As β increases, people become less impatient - i.e., they discount the future at a lower rate. This makes them willing to accept a lower interest rate.
  - Similarly - as β decreases, people become more impatient - i.e., they discount the future at a higher rate. This means they must have a higher interest rate to convince them to save.

Rf and Consumption Growth

- From $R^f_{t+1} = \frac{1}{m_{t+1}} = \frac{1}{\beta}\left(\frac{c_{t+1}}{c_t}\right)^{\gamma}$...
- What happens to R^f as consumption growth increases - i.e., (c_{t+1}/c_t) goes up?

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    - The second way interprets this the other way around. In times of high interest rates, it pays to consume less now to consume even more in the future. High interest rates lower consumption today, while raising it tomorrow.

Risk-Free Rate

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- What about γ? How does R^f respond to changes in γ?

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  - Recall that the parameter controlling IMRS is $1/\gamma$. As γ goes up, $1/\gamma$ goes down - so investors are less willing to substitute over time in response to interest rate changes.

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  - Recall that the parameter controlling IMRS is $1/\gamma$. As γ goes up, $1/\gamma$ goes down - so investors are less willing to substitute over time in response to interest rate changes.
  - It then takes a larger interest rate to do so.

Pricing Risk Corrections

- We've already demonstrated how the SDF generalizes standard present-value ideas...
- Recall, that when we put the discount factor inside the expectation,

$$p_t = E_t \left[m_{t+1} x_{t+1} \right]$$

- We can re-write this expression using the definition of a covariance

$$cov\left(m_{t+1} x_{t+1}\right) = E\left(m_{t+1} x_{t+1}\right) - E\left(m_{t+1}\right) E_t\left(x_{t+1}\right)$$

to get

$$p_t = E_t\left(m_{t+1}\right) E_t\left(x_{t+1}\right) + cov\left(m_{t+1}, x_{t+1}\right).$$

Pricing Risk Corrections

- From

$$p_t = E_t\left(m_{t+1}\right) E_t\left(x_{t+1}\right) + cov\left(m_{t+1}, x_{t+1}\right)$$

use the fact that $R^f = 1/E_t\left(m_{t+1}\right)$ to write

$$p_t = \frac{E_t\left(x_{t+1}\right)}{R^f} + cov\left(m_{t+1}, x_{t+1}\right)$$

and we see that

  - the first term is the expected payoff discounted at the risk-free rate (standard PV formula)
  - the second term is the risk correction

Pricing Risk Corrections

- Using

$$p_t = \frac{E_t\left(x_{t+1}\right)}{R^f} + cov\left(m_{t+1}, x_{t+1}\right)$$

  we see that an asset whose payoff covaries positively with the discount factor has its price raised.

- How does that asset covary with consumption?

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- How does that asset covary with consumption?
  - Recall that we can write

$$p_t = \frac{E_t\left(x_{t+1}\right)}{R^f} + \beta \frac{cov\left(u'\left(c_{t+1}\right), x_{t+1}\right)}{u'\left(c_t\right)}$$

    and remember that marginal utility rises as c falls, so

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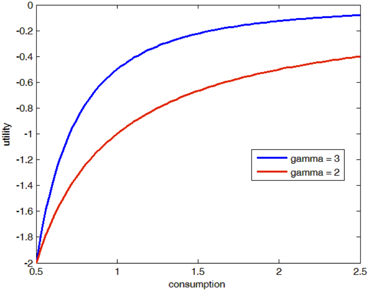
    and remember that marginal utility rises as c falls, so
  - If the payoff covaries positively with the discount factor, it covaries negatively with consumption

Pricing Risk Corrections

- So what securities are "risky"?
  - Not necessarily the ones with the highest variance or volatility! Why?
  - According to our central asset pricing equation, the "risky" securities are the ones that have low payoffs in bad states of nature.
- Take careful note that when we refer to "risk", this is what we mean. Risk and volatility are different.

- To think about "risky" securities...
- Below is a plot of power utility for $\gamma = 3$ and $\gamma = 2$. Using this graph, what intuition can you get about a security's "riskiness" using the example that follows on the next slide?



Pricing Risk Corrections

- You can see the general intuition as follows:
- Suppose that you have a 50 percent probability of $c_{t+1} = 1$ or $c_{t+1} = 2$ (bad state and good state, respectively)
- Suppose that there are two different kinds of risky assets:
  - The first asset correlates perfectly positively with consumption: you get 0.5 units of consumption if $c_{t+1} = 2$, or -0.5 units of consumption if $c_{t+1} = 1$.
  - The second asset has the same potential payoffs, but reversed.
- What asset has higher volatility? Which asset is more "risky"? Which asset helps to lessen volatility of consumption?

Math: Risk Pricing for Returns

- Now, we're going to talk about that same intuition for returns
- Write

$$1 = E_t \left[m_{t+1} R^i_{t+1} \right]$$

and use the definition of a covariance to write

$$1 = E_t \left(m_{t+1} \right) E_t \left(R^i_{t+1} \right) + cov \left(m_{t+1}, R^i_{t+1} \right)$$

- Now use $R^f = 1/E\left(m_{t+1}\right)$

$$1 = \left(1/R^f \right) E_t \left(R^i_{t+1} \right) + cov \left(m_{t+1}, R^i_{t+1} \right)$$

and write

$$E_t \left(R^i \right) = R^f - R^f cov \left(m_{t+1}, R^i_{t+1} \right).$$

Math: Risk Pricing for Returns

- From

$$
\begin{aligned}
E_t\left(R^i\right) &= R^f - R^f cov\left(m_{t+1}, R^i_{t+1}\right) \\
&= R^f - R^f \frac{\beta cov\left(u'\left(c_{t+1}\right), R^i_{t+1}\right)}{u'\left(c_t\right)}
\end{aligned}
$$

- We get

$$
\begin{aligned}
E_t\left(R^i\right) - R^f &= -R^f \frac{\beta cov\left(u'\left(c_{t+1}\right), R^i_{t+1}\right)}{u'\left(c_t\right)} \\
&= -\frac{u'\left(c_t\right)}{\beta E_t\left[u'\left(c_{t+1}\right)\right]} \frac{\beta cov\left(u'\left(c_{t+1}\right), R^i_{t+1}\right)}{u'\left(c_t\right)} \\
&= -\frac{cov\left(u'\left(c_{t+1}\right), R^i_{t+1}\right)}{E_t\left[u'\left(c_{t+1}\right)\right]}
\end{aligned}
$$

So what does this say?

- Now we have

$$E_t\left(R^i\right) - R^f = -\frac{cov\left(u'\left(c_{t+1}\right), R^i_{t+1}\right)}{E_t\left[u'\left(c_{t+1}\right)\right]}$$

- So assets that covary positively with consumption...
  - i.e., R^i is low when c is low (so $u'\left(c_{t+1}\right)$ is high)
- Make consumption more volatile, and therefore have to have a higher expected return
- Note: The two relations for prices and returns are the same -
  - A low price today is the same as a higher expected return, for a given random payoff

What about Idiosyncratic Risk?

- At the end of the day, investors really only care about consumption (and its volatility)
  - This is why it's the covariance with consumption that determines an asset's riskiness, not an asset's individual variance
- From

$$p_t = \frac{E_t\left(x_{t+1}\right)}{R^f} + \beta \frac{cov\left(u'\left(c_{t+1}\right), x_{t+1}\right)}{u'\left(c_t\right)}$$

  if

$$cov\left(u'\left(c_{t+1}\right), x_{t+1}\right) = 0$$

  then

$$p_t = E_t\left(x_{t+1}\right) / R^f$$

- So there would be no risk correction even if the asset is very volatile.

Idiosyncratic Risk and Overall Volatility?

- It's easy to see why idiosyncratic risk shouldn't matter if we use portfolio theory.
- Suppose you are concerned about the volatility of your consumption and you consider adding a "small" - i.e., epsilon - amount of a security x to your portfolio. How does this affect overall volatility?
- The volatility of your new portfolio is

$$\sigma^2\left(c+\varepsilon x\right)=\sigma^2\left(c\right)+2\varepsilon cov\left(c,x\right)+\varepsilon^2\sigma^2\left(x\right)$$

- This just shows the portfolio theory result of the benefits of diversification for portfolio volatility. Idiosyncratic asset volatility is less important for overall portfolio volatility than is its covariance/correlation with the rest of the portfolio.

ER - Beta: Derivation

- Let's go back to the expression

$$E_t\left(R^i\right) = R^f - R^f\,cov\left(m_{t+1}, R^i_{t+1}\right)$$

- Now, we're going to do another derivation that gets us to a familiar expected return - beta representation.
- Use $R^f = 1/E_t\left[m_{t+1}\right]$ again to get

$$E_t\left(R^i\right) = R^f - \frac{cov\left(m_{t+1}, R^i_{t+1}\right)}{E_t\left[m_{t+1}\right]}$$

- And then multiply and divide by one $\left(\frac{var(m_{t+1})}{var(m_{t+1})}\right)$ to get

$$= R^f + \frac{cov\left(m_{t+1}, R^i_{t+1}\right)}{var\left[m_{t+1}\right]}\left(-\frac{var\left[m_{t+1}\right]}{E_t\left[m_{t+1}\right]}\right)$$

ER - Beta: Derivation and Interpretation

- Now we can define β and λ in such a way that

$$\begin{aligned} E_t\left(R^i\right) &= R^f + \frac{cov\left(m_{t+1}, R^i_{t+1}\right)}{var\left[m_{t+1}\right]}\left(-\frac{var\left[m_{t+1}\right]}{E_t\left[m_{t+1}\right]}\right) \\ &= R^f + \beta_{i,m}\lambda_m \end{aligned}$$

  - $\beta_{i,m}$ is the "quantity of risk", the regression coefficient of R^i on m - different for each asset
  - λ_m is the "price of risk" - the same across assets
- Note that the "price of risk" depends on the volatility of the discount factor...

ER - Beta: What does it say?

- From

$$\begin{aligned} E_t\left(R^i\right) &= R^f - \frac{cov\left(m_{t+1}, R^i_{t+1}\right)}{E_t\left[m_{t+1}\right]} \\ &= R^f + \beta_{i,m}\lambda_m \end{aligned}$$

- Assets with payoffs that have positive covariance with consumption $\Rightarrow$ high (negative) beta
  - These assets make consumption more volatile, so must have a higher expected return
- The higher risk aversion or the more volatile consumption, the larger λ_m
  - People require a higher return to hold risky assets

End of Today's Lecture.

- That's all for today. Today's material corresponds roughly to parts of Chapter 1 in Cochrane (2005).