ECON 4360: Empirical Finance

ICAPM v. APT and CRR 1986

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Empirics Lecture #07

What are we doing today?

- ICAPM v. APT?
- Discussion of Chen, Roll, and Ross (1986)

Factor Pricing Models

- Motivation for looking at factor pricing models:
  - The consumption-based model works well in theory, but not so well in practice.
  - So, much empirical work tried to tie the discount factor to other data - in particular, variables that are good proxies for aggregate marginal utility growth
- I.e., we try to find f such that

$$\beta \frac{u'(c_{t+1})}{u'(c_t)} \approx a + b' f_{t+1}$$

so we can write

$$m_{t+1} = a + b' f_{t+1}$$

and then use the model (recall: why can we use this model?)

$$E(R_{t+1}) = \gamma + \beta' \lambda$$

What are the Factors?

- The idea is that there are some particular "states of nature" where investors would *really* like that their portfolios do not do badly
  - Variables that are indicative of these particular states are good candidates for our factors
- Examples include consumption (of course), returns on broad-based portfolios (of course), interest rates, GDP growth, and other like **macroeconomic variables** because they indicate or measure the state of the economy
- Other important variables are **forecasting variable**s, since consumption (and hence marginal utilities) respond *today* to news about *tomorrow*.
  - E.g., a variable like the term premium is not necessarily an indicator of good or bad times today, but it does give us a forecast for these things tomorrow.

Can I Have my Fishing License, Please?

- No, we can't fish for factors (in theory).
  - Economic foundations are still (should be?) important here.
- Our theoretical models do give us some discipline that we should ideally impose on our empirical work. E.g., ...
  - The CAPM and ICAPM make predictions for the risk-free rate and factor risk premia.
  - In the ICAPM, "state variables" should forecast something.

What else?

- Remembering that all factor models are just derived as specializations (i.e., some extra assumptions) of the consumption-based model is what allow us to proxy marginal utility growth from other variables.
- And the model also tells us something about our factors...
- E.g., we can re-write our FOC as follows:

$$\frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)} = \frac{1}{\beta R^f} + \varepsilon_{t+1},\ E_t\left(\varepsilon_{t+1}\right) = 0$$

- This expression tells us that if we had a constant risk-free rate, marginal utility growth should be unpredictable.
- So, factors we choose shouldn't be highly predictable, or else we will (counterfactually) predict large interest rate variation.
- What does this mean in practice?
  - Choose the right units - e.g., GDP expressed through growth rates rather than levels, portfolio returns rather than prices, etc.

CAPM and ICAPM

- Derivations of the CAPM and ICAPM both determine:
  - Factors that can proxy for marginal utility growth
  - Prove that the relationship is linear
- There are several sets of assumptions that will yield the CAPM:
  - Two-period quadratic utility
  - Exponential utility with normally distributed returns
  - Quadratic value functions and dynamic programming
  - Log utility

Derivation of the ICAPM

- Merton's (1973) portfolio theory and ICAPM comes from
  - Stating the consumer's problem
  - Proving that the value function depends on W (current wealth) and z (state variables for future investment opportunities)
  - Showing that the optimal portfolio holds the market and hedge portfolios for the "investment opportunity variables".
- The idea is that the state variables determine how well an investor can do in his maximization problem
  - Current wealth W is an obvious one
  - The others should describe the conditional distribution of asset returns the investor will face in the future - or shifts in the investment opportunity set

What are the ICAPM's state variables???

- The ICAPM doesn't tell us.
  - It was Fama (1991) that claimed the ICAPM a "fishing license."
- Factor-mimicking portfolios should be the projections of some kind of (identifiable) state variables on the space of returns and they should forecast something
- Empirical successes with ICAPM-like models, but still theoretically artificial / ad hoc
  - E.g., the same intuition is captured by the consumption-based CAPM.
  - Plus, many of the macro variables suffer the same measurement problems as aggregate consumption data and are as well only vague proxies of what they are intended to measure.

So What is the APT?

- Arbitrage Pricing Theory, Ross (1976).
  - Starts from a statistical characterization of **realized** returns, and tries to derive something about **expected** returns.
- The idea is that there is a big common component to stock returns (i.e., most move with the market) and also that groups of stocks also move together.
  - Asset returns also have idiosyncratic components, but these shouldn't carry any risk prices since this component should be diversifiable.
- Expected returns should then only be related to covariance with the common components or "factors".

The APT: How do we do it?

- The original appeal of the APT was that it didn't impose all the economic structure required of the CAPM and ICAPM.
  - We're really just using the LOP
- If you know the factors you want to use, say size and book-to-market portfolios, you can just estimate a factor structure by running regressions.
- If you don't know the factors, you can use "factor analysis" using statistical techniques to find them - e.g., by taking an eigenvalue decomposition of the covariance matrix of returns.

ICAPM v. APT

- Biggest difference is the inspiration for factors
  - APT starts with statistical analysis of the covariance matrix of returns to find portfolios that characterize the common movements
  - ICAPM starts by thinking about state variables the describe the conditional distributions of future asset returns
- For the APT, the factors should be orthogonal and describe the covariance matrix of returns. High R^2's in time-series regressions can be indicative of factor pricing with APT.
- For the ICAPM, high R^2's aren't necessary and nothing says that the factors describe the covariance matrix of returns or that they have to be orthogonal and i.i.d.

Lots of Confusion...

- You've already read Fama and French (1993).
  - They describe their paper in their intro as an ICAPM, but their factors are portfolios of assets sorted on size and book-to-market just like the test assets. They talk of the explanation involving "comment movement" in test assets captured by the factors and note the high R^2's in their time-series regressions.
  - Is this an APT?
- Today, we'll talk about CRR (1986).
  - There is no factor decomposition of asset returns or time-series regressions. Industrial production and inflation are some of the main factors.
  - Is this an ICAPM?

Chen, Roll, and Ross (1986)

- Important to consider the economic insights about how factors relate to uncertainties about consumption and investment opportunities.
  - CRR (1986) search for factors with an economic motivation
- CRR (1986) look for economic variables correlated with stock returns and then test to see if the loadings of returns on those economic factors describe the cross-section of expected returns.
  - Influential multifactor model paper.

CRR (1986)

- Motivation
  - Use simple theoretic arguments to select a set of economic "state variables" as candidate sources of systematic asset risk.
  - Test for significance in explaining expected stock returns.
- Rationale
  - Asset prices react to economic news; some unanticipated events are more important than others
  - Possibilities for diversification means that only systematic risks should matter; there should be no extra return earned for bearing risk that could easily be diversified away.
  - Returns are influenced by systematic economic news; they should be priced according to their exposure.
- Main results are that their "macroeconomic" variables matter.

Which Macro Variables?

- What macroeconomic variables are the "state variables"?
  - CRR (1986) argue that candidates are anything that might affect discount factors and/or expected cash flows.
  - Why? Basic argument: Stock prices can be written as expected discounted dividends

What changes expected cash flows?

- Expected cash flows change with changes in both real and nominal forces.
- Candidate influences?
  - Changes in the expected rate of inflation
    - Influences nominal expected cash flows and the nominal rate of interest.
  - Unanticipated changes in the price level
    - Systematic effect if relative (real) asset prices change with changes in the average inflation rate
  - Changes in expected level of production
    - Uncertainty about industrial production affects the current real value of cash flows.

What changes discount rates?

- Discount rates change with changes in
  - The level of rates over time
  - Term-structure spreads across different maturities
- Candidate influences?
  - Unanticipated changes in the riskless interest rate
    - Influence the time value of future cash flows, so they influence returns
  - Unanticipated changes in the risk premium
  - Changes in marginal utility of wealth / consumption

CRR (1986) Data Descriptions

TABLE 1 Glossary and Definitions of Variables

Symbol	Variable	Definition or Source
	Basic Series	
I	Inflation	Log relative of U.S. Consumer Price Index
TB	Treasury-bill rate	End-of-period return on 1-month bills
LGB	Long-term government bonds	Return on long-term government bonds (1958–78: Ibbotson and Sinquefield [1982]; 1979–83: CRSP)
IP	Industrial production	Industrial production during month (*Survey of Current Business*)
Baa	Low-grade bonds	Return on bonds rated Baa and under (1953–77: Ibbotson [1979], constructed for 1978–83)

CRR (1986) Data Descriptions

EWNY	Equally weighted equities	Return on equally weighted portfolio of NYSE-listed stocks (CRSP)
VWNY	Value-weighted equities	Return on a value-weighted portfolio of NYSE-listed stocks (CRSP)
CG	Consumption	Growth rate in real per capita consumption (Hansen and Singleton [1982]; *Survey of Current Business*)
OG	Oil prices	Log relative of Producer Price Index/Crude Petroleum series (Bureau of Labor Statistics)

CRR (1986) Derived Series

	Derived Series	
MP(t)	Monthly growth, industrial production	$\log_e[IP(t)/IP(t-1)]$
YP(t)	Annual growth, industrial production	$\log_e[IP(t)/IP(t-12)]$
E[I(t)]	Expected inflation	Fama and Gibbons (1984)
UI(t)	Unexpected inflation	$I(t) - E[I(t)\|t-1]$
RHO(t)	Real interest (ex post)	$TB(t-1) - I(t)$
DEI(t)	Change in expected inflation	$E[I(t+1)\|t] - E[I(t)\|t-1]$
URP(t)	Risk premium	$Baa(t) - LGB(t)$
UTS(t)	Term structure	$LGB(t) - TB(t-1)$

CRR (1986) Model

- Linear Factor Model

$$R = a + b_{MP}\,MP + b_{DEI}\,DEI + b_{UI}\,UI + b_{UPR}\,UPR + b_{UTS}\,UTS + e$$

- The state variables are MP, DEI, UI, UPR, UTS
- The b's are the "loadings" on the state variables

CRR (1986) Methodology

- Cross-sectional regressions; a *version* of Fama-MacBeth (1973):

1. CRR chose a sample of assets.
2. For any given year:
    a. Each asset's exposure (the b's) to the economic state variables (MP, DEI, UI, UPR, UTS) was estimated by time-series regressions over an estimation period of the previous 5 years
    b. The estimates of exposure (the $\widehat{b}$'s) were then used as RHS variables for 12 cross-sectional regressions, one regression for each of the next 12 months.

1. Repeat 2*a*-2*b* for all (possible) years in the sample.

CRR (1986) Methodology, Continued...

- What do you get?
  - A time-series of estimates for each macro variable's associated risk premium, e.g., $\sum_{t=1}^{T} \widehat{MP}_t$
  - The time-series means of these estimates are then tested for significant difference from zero using a *t-test*.
- Note: The estimated coefficients from the cross-sectional regressions provide estimates of the sum of the risk premium and the unanticipated movement in the state variable for that month.
- Note: Securities are grouped into portfolios based on firm size.
  - Argument: Reduces idiosyncratic volatility and allow factor loadings and risk premia to be estimated more precisely.
  - Recall, why didn't FF (1992) do this?

CRR (1986) Methodology, Continued...

- Multivariate Approach
  - Tested a five or six factor model with a constant, main results in Table 4
  - Data range: 1958-1984
  - Again for subperiods: 1958-1967, 1968-1977, 1978-1984
    - Why? Possible structural shifts in macro variables (e.g., oil price jump in 1973, OPEC becomes important in period 1968-77, etc.)

Risk Premia Significance

- Multivariate Approach
  - MP, UI, UPR are significant over entire sample period
  - UTS is only marginally significant
  - DEI and UI significant 1968-1977; but insignificant in both earlier and later periods.
  - YP not significant.
- Multivariate and Other Approaches (when replacing YP with a market index)
  - EWNY and VWNY not significant

Results: Table 4-A

TABLE 4 **Economic Variables and Pricing (Percent per Month × 10), Multivariate Approach**

A

Years	YP	MP	DEI	UI	UPR	UTS	Constant
1958–84	4.341	13.984	−.111	−.672	7.941	−5.87	4.112
	(.538)	(3.727)	(−1.499)	(−2.052)	(2.807)	(−1.844)	(1.334)
1958–67	.417	15.760	.014	−.133	5.584	.535	4.868
	(.032)	(2.270)	(.191)	(−.259)	(1.923)	(.240)	(1.156)
1968–77	1.819	15.645	−.264	−1.420	14.352	−14.329	−2.544
	(.145)	(2.504)	(−3.397)	(−3.470)	(3.161)	(−2.672)	(−.464)
1978–84	13.549	8.937	−.070	−.373	2.150	−2.941	12.541
	(.774)	(1.602)	(−.289)	(−.442)	(.279)	(−.327)	(1.911)

Results: Table 4-B

	B					
	MP	DEI	UI	UPR	UTS	Constant
1958–84	13.589	−.125	−.629	7.205	−5.211	4.124
	(3.561)	(−1.640)	(−1.979)	(2.590)	(−1.690)	(1.361)
1958–67	13.155	.006	−.191	5.560	−.008	4.989
	(1.897)	(.092)	(−.382)	(1.935)	(−.004)	(1.271)
1968–77	16.966	−.245	−1.353	12.717	−13.142	−1.889
	(2.638)	(−3.215)	(−3.320)	(2.852)	(−2.554)	(−.334)
1978–84	9.383	−.140	−.221	1.679	−1.312	11.477
	(1.588)	(−.552)	(−.274)	(.221)	(−.149)	(1.747)

Results: Table 4-C

C

	EWNY	MP	DEI	UI	UPR	UTS	Constant
1958–84	5.021	14.009	−.128	−.848	8.130	−5.017	6.409
	(1.218)	(3.774)	(−1.666)	(−2.541)	(2.855)	(−1.576)	(1.848)
1958–67	6.575	14.936	−.005	−.279	5.747	−.146	7.349
	(1.199)	(2.336)	(−.060)	(−.558)	(2.070)	(−.067)	(1.591)
1968–77	2.334	17.593	−.248	−1.501	12.512	−9.904	3.542
	(.283)	(2.715)	(−3.039)	(−3.366)	(2.758)	(−2.015)	(.558)
1978–84	6.638	7.563	−.132	−.729	5.273	−4.993	9.164
	(.906)	(1.253)	(−.529)	(−.847)	(.663)	(−.520)	(1.245)

Results: Table 4-D

	D						
	VWNY	MP	DEI	UI	UPR	UTS	Constant
1958–84	−2.403	11.756	−.123	−.795	8.274	−5.905	10.713
	(−.633)	(3.054)	(−1.600)	(−2.376)	(2.972)	(−1.879)	(2.755)
1958–67	1.359	12.394	.005	−.209	5.204	−.086	9.527
	(.277)	(1.789)	(.064)	(−.415)	(1.815)	(−.040)	(1.984)
1968–77	−5.269	13.466	−.255	−1.421	12.897	−11.708	8.582
	(−.717)	(2.038)	(−3.237)	(−3.106)	(2.955)	(−2.299)	(1.167)
1978–84	−3.683	8.402	−.116	−.739	6.056	−5.928	15.452
	(−.491)	(1.432)	(−.458)	(−.869)	(.782)	(−.644)	(1.867)

Risk Premia Sign Stories

- Most significant: industrial production, changes in the risk premium, changes in the yield curve
- Also weakly significant: unanticipated inflation and changes in expected inflation during periods these variables were volatile
- MP $(+)$
  - Reflects the value of insuring against real systematic risks
- UPR $(+)$
  - Desire to hedge against unanticipated increases in aggregate risk premium due to increases in uncertainty
- UI and DEI $(-)$
  - Stock market assets assumed to hedge against adverse effects of changes in inflation on other assets that are relatively more fixed in nominal terms

Risk Premia Sign Stories, Continued...

- UTS $(-)$
  - Returns of stocks that are inversely related to increases in long rates over short rates are more valuable
    - Why? If long-term rates decrease, there is a lower return on any form of capital...
    - A relatively higher value is placed on assets whose price increases when long-term rates decline (they carry a negative risk premium)
    - Returns that are correlated with long-term bond returns will be more valuable than those uncorrelated or negatively correlated.

Stock Market Indices and Other Ideas?

- Basic results for stock market indices
  - Time-series regressions
    - Most significant; "Explain" much of the intertemporal movements in other stock portfolios
  - Cross-sectional regressions
    - Estimated betas for stock market indices (i.e., measures exposure to stock market risk) do not explain cross-sectional differences in average returns
- Consumption growth betas and oil prices
  - Neither significant
  - Estimated risk premium for Cg has the wrong sign (should be positive, not negative!)

CRR (1986) Results Summary

- Returns are exposed to systematic economic news; priced according to exposure.
- News can be measured through changes in "state variables", identified through simple theoretic arguments
- Again, main results are that
  - Their "macroeconomic" variables matter
  - Stock market indices, consumption growth betas, oil prices are all insignificant to the asset pricing story
  - I.e., these betas have little effect on the power of their economic factors to explain the cross-section, but they have no marginal explanatory power.

CRR (1986) Take-Aways

- Approach is a useful way to use multifactor models to improve our understanding of asset pricing models.
  - May be able to develop a unified story for asset pricing and the links between expected returns and the real economy.
- Caution: Multifactor models offer few predictions about which variables are important
  - The power of the CRR (1986) factors are sensitive to assets used and the way factor loadings are estimated, see Shanken and Weinstein (1990)
  - Flexible approach; but not a license to go fishing!
  - Measured relationships between returns and economic factors can be spurious, i.e., the result of special features of a particular sample.
  - Robustness checks warranted.

End of Today's Lecture.

- That's all for today. Today's material corresponds to parts of Chapter 9 in Cochrane and CRR (1986).