Mixture models and hidden Markov models

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Objectives and scope

There are many situations in which one may encounter distinct types of entities, such as different animal species, and different states in which these entities may exist, for example motivational states like hunger. Cognition is sometimes also best understood in terms of discrete types and states. For example, forms of cognitive development can be characterized as the acquisition of increasingly complex rules [2]. Effectively, these rules constitute different types of reasoning and associated response patterns in reasoning tasks. And rather than a gradually shifting trade-off, people may switch rapidly between distinct decision-making modes favoring either speed or accuracy [1]. The idea that cognitive processes are guided by qualitatively different strategies underlies a wide range of theories concerned with topics such as word recognition, cognitive development, categorization, and decision making, to name but a few [4].

As types and states are generally not explicitly labeled, appropriate statistical techniques are required to identify them. This tutorial will focus on mixture and hidden Markov models, which are the basis of such techniques. In the context of MMs, a type or state (e.g., a motivational state or cognitive strategy) is formalized as a probability distribution over observables. Because a dataset may contain different types, the overall distribution is a mixture of such individual component distributions. As the component distributions need not be of the same parametric family (e.g., Gaussian distributions can be mixed with other distributions), MMs allow for considerable flexibility in the definition of types and states. HMMs are a natural extension of MMs, allowing switches between states over time. For example, these models are useful when people can switch between cognitive strategies during a task. In addition to identifying the different states, HMMs allow one to also focus on the process underlying state transitions.

While mixture models (MMs) and hidden Markov models (HMMs) are widely used in fields such as computational biology (e.g., for DNA sequence analysis) and machine learning (e.g., for speech recognition and estimation of topic models), their use in the analysis of cognition and behavior is relatively rare. This is unfortunate, as MMs and HMMs are ideally suited to test and explore important theoretical ideas in psychology. The objective of this tutorial is to provide researchers with an accessible introduction to MMs and HMMs.

Outline of the tutorial

The half-day tutorial will be divided into two parts. The first part introduces the theory behind mixture and hidden Markov models. The second part will be more practical, using a number of examples to show (a) how to apply MMs and HMMs with user-friendly and freely available software, (b) how to interpret these models, and (c) how they can reveal aspects which remain hidden with more traditional analyses. The first part of the tutorial will be delivered as a classroom style lecture. The second part will use a more hands-on approach with practical computer-based examples and exercises. The audience are encouraged to bring their own laptop. All necessary software and material will be made available in advance.

Part I: Theory

Introduction to mixture models.

This part will introduce the basic structure of mixture models and the use of graphical and other techniques to determine whether MMs might be applicable.

Estimation
This part will provide an intuitive treatment of maximum likelihood estimation and introduce numerical optimization and Expectation-Maximization (EM), the two main methods for this type of estimation of MMs and HMMs. Practical issues such as starting values and local maxima will also be discussed.

Inference
This part will discuss methods for model selection and how to determine the number of components (i.e., types, or latent classes). We also discuss methods to test parameters for significance and the use of posterior probabilities to determine the type/component a data point belongs to.

Hidden Markov models
This part will introduce hidden Markov models as a direct extension of mixture models. We will then discuss how to generalize the previously discussed methods of estimation and inference to these models.

Part II: Practice

Introduction to depmixS4
This part will introduce depmixS4 [8], a flexible package to estimate MMs and HMMs in the R environment for statistical computing [3]. The examples in the remainder of the tutorial will use this package.

Examples of mixture models
Examples will include the use of MMs to detect developmental stages in the liquid conservation task and the use of MMs to detect multiple learning strategies in a category learning task.

Examples of hidden Markov models
Examples will include the use of HHMs to analyse speed-accuracy trade-offs and the use of HMMs to model response strategies in multiple cue learning.

Extensions
This part will briefly discuss some extensions to basic MMs and HMMs, including the use of covariates to predict the identity of mixture components and states. We may also briefly discuss the use of Bayesian methods to estimate MMs and HMMs.

About the organizers
The organizers are the developers of depmixS4 [7], a popular R package to estimate mixture and hidden Markov models. They are also the authors of an upcoming book on this topic (commissioned by Springer for their “UseR” series) and a recent tutorial on hidden Markov models [6]. The organizers have extensive experience in the application of MMs and HMMs to research in developmental and cognitive science [5, 7]. They can draw upon this experience to provide the audience with real examples and practical advice.

Justification
Theories which propose the existence of distinct types and states are widespread in the cognitive sciences. Traditional statistical analysis, such as t-tests and ANOVAs, or not applicable to test such ideas. MMs have been successfully used to test “toolbox models” of cognition [4] and HMMs to test discrete strategy switches [5, 2]. This tutorial will provide cognitive scientists with the intuitive understanding and practical knowledge of these models necessary to help them apply them to their own research.

Intended audience
This tutorial will mainly be introductory and no specific prior background knowledge is required. While basic knowledge of probability and statistics will be helpful, treatment of the theoretical concepts will largely be conceptual. While familiarity with the R environment will be helpful in general, by making the commands available, previous experience is no requirement to follow and replicate the results of the practical examples.

Requirements
Participants would ideally bring a laptop to the tutorial. The required software (R and depmixS4) is open source and freely and easily obtainable. R is available for all major platforms and can be downloaded from http://www.r-project.org. The depmixS4 package can be downloaded from http://cran.r-project.org/web/packages/depmixS4/ or installed from within R through the command install.packages('depmixS4').

References

[1] Gilles Dutilh, Eric-Jan Wagenmakers, Ingmar Visser, and Han L. J. van der Maas. A phase transition model for the speed– accuracy trade–off in response time experiments. Cognitive Science, 35:211–250, 2011.
[2] Brenda R. J. Jansen, Maartje E. J. Raijmakers, and Ingmar Visser. Rule transition on the balance scale task: A case study in belief change. Synthese, 155(2):211–236, 2007.
[3] R Development Core Team. R: A Language and Environment for Statistical Computing. R Foundation for Statistical Computing, Vienna, Austria, 2010. ISBN 3-900051-07-0.
[4] Benjamin Scheibehenne, Jorg Rieskamp, and E.-J. Wagenmakers. Testing adaptive toolbox models: A Bayesian hierarchical approach. Psychological Review, in press.
[5] Maarten Speekenbrink, David A. Lagnado, Leonora Wilkinson, Marjan Jahanshahi, and David R. Shanks. Models of probabilistic category learning in parkinson's disease: Strategy use and the effects of L-dopa. Journal of Mathematical Psychology, 54:123–136, 2010.
[6] Ingmar Visser. Seven things to remember about hidden markov models: A tutorial on markovian models for time series. Journal of Mathematical Psychology, 55:403—415, 2011.
[7] Ingmar Visser, Brenda R. J. Jansen, and Maarten Speekenbrink. A framework for discrete change. In Peter C. M. Molenaar and Karl M. Newell, editors, Individual Pathways of Change: Statistical Models for Analyzing Learning and Development, APA Science, chapter 7, pages 109–123. APA, Washington, 2010.
[8] Ingmar Visser and Maarten Speekenbrink. depmixS4: An Rpackage for hidden Markov models. Journal of Statistical Software, 36(7):1– 21, 2010. R package, current version available from CRAN.