

Exercises:

Supplementary Problems.

Exercise 1:

Determine the duals of the given programs.

$$\text{Minimize: } Z = 12x_1 + 26x_2 + 80x_3$$

Subject to the constraints:

$$2x_1 + 6x_2 + 5x_3 \geq 4$$

$$4x_1 + 2x_2 + x_3 \geq 10$$

$$x_1 + x_2 + 2x_3 \geq 6$$

with: all variables nonnegative

Exercise 2:

Determine the duals of the given programs.

$$\text{Maximize: } Z = 4x_1 + 5x_2 + 7x_3$$

Subject to the constraints:

$$x_1 + x_2 + x_3 \leq 10$$

$$4x_1 - x_2 + 2x_3 \geq 15$$

$$-x_1 + 4x_2 + 7x_3 \geq 35$$

with: all variables nonnegative

Exercise 3:

Determine the duals of the given programs.

$$\text{Maximize: } Z = 51x_1 + 52x_2 - 53x_3$$

Subject to the constraints:

$$11x_1 + 12x_2 + 13x_3 \leq 10$$

$$21x_1 + 22x_2 + 23x_3 \leq 20$$

with: all variables nonnegative

Exercise 4:

Determine the duals of the given programs.

$$\text{Minimize: } Z = 5x_1 - 6x_2 + 4x_3$$

Subject to the constraints:

$$3x_1 + 4x_2 + 6x_3 \geq 9$$

$$x_1 + 3x_2 + 2x_3 \geq 5$$

$$7x_1 - 4x_2 - x_3 \geq 10$$

$$x_1 - 2x_2 + 4x_3 \geq 4$$

$$2x_1 + 5x_2 - 3x_3 = 3$$

with: all variables nonnegative

Exercise 5:

A company wishes to get at least 160 million 'audience exposures' the number of times one of the advertisements is seen or heard by a person. Because of the nature of the product the company wants at least 60 million of these exposures to involve persons with family income of over Rs 10,000 a month and at least 80 million of the exposures to involve persons between 18 and 40 years of age.

The relevant information pertaining to the two advertising media under consideration-magazine and television is given below:

	Magazine	Television
Cost per advertisement (Rs thousand)	40	200
Audience per advertisements (million)	4	40
Audience per advertisement with monthly income over Rs 10000 (million)	3	10
Audience (per advertisement) in the age group 18-40 (million)	8	10

The company wishes to determine the number of advertisements to be released each in magazine and television so as to keep the advertisement expenditure to the minimum. Formulate it as a linear programming problem. Write 'dual' of this problem. **Solve the DUAL problem to find the minimum expenditure and its allocation among the two media.**

Exercise 6:

A firm produces three types of biscuits: A, B and C. It packs them in assortment of two sizes I and II. Size I contains 20 biscuits of type A, 50 biscuits of type B and 10 biscuits of type C. The Size II contains 10 biscuits of type A, 80 biscuits of type B and 60 biscuits of type C. A buyer intends to buy 120 biscuits of type A, 740 biscuits of type B and 240 biscuits of type C.

Determine the least number of packets he should buy. **Write the associated Dual.**
Use simplex method to solve dual and read solution of the Primal.

Exercise 7:

A diet is to contain at least 20 ounces of protein and 15 ounces of carbohydrates. There are three foods: A, B and C available in the market, costing Rs 2, Rs 1 and Rs 3 per unit respectively. Each unit of A contains 2 ounces of protein and 4 ounces of carbohydrates. Each unit of B contains 3 ounces of protein and 2 ounces of carbohydrates. Each unit of C contains 4 ounces of protein and 2 ounces of carbohydrates.

- I. Formulate LPP so as to minimize the cost of diet.
- II. Find its dual.
- III. Solve the dual by simplex method and from the dual solution read the solution to the primal problem.

Exercise 8:

Determine the duals of the given programs.

Minimize: $Z = 3x_1 + 2x_2 + x_3 + 2x_4 + 3x_5$

Subject to the constraints:

$$2x_1 + 5x_2 + x_4 + x_5 \geq 6$$

$$4x_2 - 2x_3 + 2x_4 + 3x_5 \geq 5$$

$$x_1 - 6x_2 + 3x_3 + 7x_4 + 3x_5 \leq 7$$

with: all variables are nonnegative

Exercise 9:

Use the results of **Exercise 8** to verify the complementary slackness principle.

Exercise 10:

Use the simplex or two-phase method to solve the following problems.

Minimize: $Z = x_1 + 2x_2$

Subject to the constraints:

$$x_1 + 3x_2 \geq 11$$

$$2x_1 + x_2 \geq 9$$

with: x_1 and x_2 nonnegative

