1 Selected Schedule

We have constructed a template for the schedule with the following constraints:

1. A team plays at most once against each other team.
2. Each team plays exactly one match per day.
3. Each team plays exactly against two teams from each pot, one home match and one away match.
4. Perfect home-away alternation for the 32 teams other than A1, B1, C1, and D1.
5. A single break for teams A1, B1, C1, and D1, which does not occur during the first two or last two matchdays.
6. Even distribution over the 8 matchdays for intra-pot matches (1 day with 2 matches, and 7 days with 1 match).
7. Even distribution over the 8 matchdays for inter-pot matches AB, AC, and BC (2 days with 3 matches, and 6 days with 2 matches).
8. The number of inter-pot matches AD, BD, and CD per matchday ranges between 1 and 3.
9. Inter-pot matches form a cycle of maximum length 9, meaning X1 hosts X2, X2 hosts X3, ..., X8 hosts X9, and X9 hosts X1, for any pot X.
10. No team can face teams from pot A or B twice over 3 consecutive matchdays.
11. No team can face 2 teams from pot C or 2 teams from pot D in the first two or last two matches.

We propose two tables: the first detailing the match schedule day by day, and the second providing the schedule for each of the 36 teams individually.

Matchday 1

Home	Away
A1	B9
A2	D4
A4	A5
A6	B7
A8	C5
B1	A3
B2	B3
B4	C1
B6	D6
B8	D2
C2	A7
C4	D8
C6	B5
C8	C9
D3	C7
D5	C3
D7	A9
D9	D1

Matchday 2

Home	Away
A3	D5
A5	B2
A7	A8
A9	A1
B3	D9
B5	A4
B7	B8
B9	C4
C1	C2
C3	A2
C5	D7
C7	B6
C9	A6
D1	B4
D2	D3
D4	C6
D6	C8
D8	B1

Matchday 3

Home	Away
A2	A3
A4	D2
A6	D6
A8	B3
B1	C3
B2	C9
B4	B5
B6	A7
B8	A9
C1	A1
C2	B9
C4	A5
C6	C7
C8	D1
D3	D4
D5	B7
D7	D8
D9	C5

Matchday 4

Home	Away
A1	D9
A3	B4
A5	A6
A7	C6
A9	C2
B3	C8
B5	D3
B7	A2
B9	B1
C3	C4
C5	B2
C7	A8
C9	D5
D1	C1
D2	B6
D4	B8
D6	D7
D8	A4

Matchday 5

Home	Away
A2	C9
A4	B1
A6	C3
A8	A9
B2	A1
B4	D8
B6	B7
B8	C7
C1	B3
C2	D4
C4	C5
C6	D2
C8	A3
D1	A7
D3	A5
D5	D6
D7	B5
D9	B9

Matchday 6

Home	Away
A1	A2
A3	C4
A5	D7
A7	B8
A9	B6
B1	D5
B3	B4
B5	C2
B7	C6
B9	D1
C3	D3
C5	A4
C7	C8
C9	C1
D2	A6
D4	A8
D6	B2
D8	D9

Matchday 7

Home	Away
A2	B5
A4	C7
A6	A7
A8	D8
B2	D4
B4	A5
B6	C5
B8	B9
C1	D6
C2	C3
C4	B1
C6	A9
C8	B7
D1	D2
D3	B3
D5	A1
D7	C9
D9	A3

Matchday 8

Home	Away
A1	C8
A3	A4
A5	C1
A7	D3
A9	D1
B1	B2
B3	A6
B5	B6
B7	D7
B9	A8
C3	B4
C5	C6
C7	D9
C9	B8
D2	C4
D4	D5
D6	A2
D8	C2

Table 1: Match schedule Day by Day

Team	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
A1	B9 (H)	A9 (A)	C1 (A)	D9 (H)	B2 (A)	A2 (H)	D5 (A)	C8 (H)
A2	D4 (H)	C3 (A)	A3 (H)	B7 (A)	C9 (H)	A1 (A)	B5 (H)	D6 (A)
A3	B1 (A)	D5 (H)	A2 (A)	B4 (H)	C8 (A)	C4 (H)	D9 (A)	A4 (H)
A4	A5 (H)	B5 (A)	D2 (H)	D8 (A)	B1 (H)	C5 (A)	C7 (H)	A3 (A)
A5	A4 (A)	B2 (H)	C4 (A)	A6 (H)	D3 (A)	D7 (H)	B4 (A)	C1 (H)
A6	B7 (H)	C9 (A)	D6 (H)	A5 (A)	C3 (H)	D2 (A)	A7 (H)	B3 (A)
A7	C2 (A)	A8 (H)	B6 (A)	C6 (H)	D1 (A)	B8 (H)	A6 (A)	D3 (H)
A8	C5 (H)	A7 (A)	B3 (H)	C7 (A)	A9 (H)	D4 (A)	D8 (H)	B9 (A)
A9	D7 (A)	A1 (H)	B8 (A)	C2 (H)	A8 (A)	B6 (H)	C6 (A)	D1 (H)
B1	A3 (H)	D8 (A)	C3 (H)	B9 (A)	A4 (A)	D5 (H)	C4 (A)	B2 (H)
B2	B3 (H)	A5 (A)	C9 (H)	C5 (A)	A1 (H)	D6 (A)	D4 (H)	B1 (A)
B3	B2 (A)	D9 (H)	A8 (A)	C8 (H)	C1 (A)	B4 (H)	D3 (A)	A6 (H)
B4	C1 (H)	D1 (A)	B5 (H)	A3 (A)	D8 (H)	B3 (A)	A5 (H)	C3 (A)
B5	C6 (A)	A4 (H)	B4 (A)	D3 (H)	D7 (A)	C2 (H)	A2 (A)	B6 (H)
B6	D6 (H)	C7 (A)	A7 (H)	D2 (A)	B7 (H)	A9 (A)	C5 (H)	B5 (A)
B7	A6 (A)	B8 (H)	D5 (A)	A2 (H)	B6 (A)	C6 (H)	C8 (A)	D7 (H)
B8	D2 (H)	B7 (A)	A9 (H)	D4 (A)	C7 (H)	A7 (A)	B9 (H)	C9 (A)
B9	A1 (A)	C4 (H)	C2 (A)	B1 (H)	D9 (A)	D1 (H)	B8 (A)	A8 (H)
C1	B4 (A)	C2 (H)	A1 (H)	D1 (A)	B3 (H)	C9 (A)	D6 (H)	A5 (A)
C2	A7 (H)	C1 (A)	B9 (H)	A9 (A)	D4 (H)	B5 (A)	C3 (H)	D8 (A)
C3	D5 (A)	A2 (H)	B1 (A)	C4 (H)	A6 (A)	D3 (H)	C2 (A)	B4 (H)
C4	D8 (H)	B9 (A)	A5 (H)	C3 (A)	C5 (H)	A3 (A)	B1 (H)	D2 (A)
C5	A8 (A)	D7 (H)	D9 (A)	B2 (H)	C4 (A)	A4 (H)	B6 (A)	C6 (H)
C6	B5 (H)	D4 (A)	C7 (H)	A7 (A)	D2 (H)	B7 (A)	A9 (H)	C5 (A)
C7	D3 (A)	B6 (H)	C6 (A)	A8 (H)	B8 (A)	C8 (H)	A4 (A)	D9 (H)
C8	C9 (H)	D6 (A)	D1 (H)	B3 (A)	A3 (H)	C7 (A)	B7 (H)	A1 (A)
C9	C8 (A)	A6 (H)	B2 (A)	D5 (H)	A2 (A)	C1 (H)	D7 (A)	B8 (H)
D1	D9 (A)	B4 (H)	C8 (A)	C1 (H)	A7 (H)	B9 (A)	D2 (H)	A9 (A)
D2	B8 (A)	D3 (H)	A4 (A)	B6 (H)	C6 (A)	A6 (H)	D1 (A)	C4 (H)
D3	C7 (H)	D2 (A)	D4 (H)	B5 (A)	A5 (H)	C3 (A)	B3 (H)	A7 (A)
D4	A2 (A)	C6 (H)	D3 (A)	B8 (H)	C2 (A)	A8 (H)	B2 (A)	D5 (H)
D5	C3 (H)	A3 (A)	B7 (H)	C9 (A)	D6 (H)	B1 (A)	A1 (H)	D4 (A)
D6	B6 (A)	C8 (H)	A6 (A)	D7 (H)	D5 (A)	B2 (H)	C1 (A)	A2 (H)
D7	A9 (H)	C5 (A)	D8 (H)	D6 (A)	B5 (H)	A5 (A)	C9 (H)	B7 (A)
D8	C4 (A)	B1 (H)	D7 (A)	A4 (H)	B4 (A)	D9 (H)	A8 (A)	C2 (H)
D9	D1 (H)	B3 (A)	C5 (H)	A1 (A)	B9 (H)	D8 (A)	A3 (H)	C7 (A)

Table 2: Match schedule Team by Team

2 Mathematical Modeling and Solver Implementation Details

2.1 Problem Notations

For all $i \in \{1, \dots, 36\}$, team i is in pot:

- A if $1 \leq i \leq 9$
- B if $10 \leq i \leq 18$
- C if $19 \leq i \leq 27$
- D if $28 \leq i \leq 36$

For all $i \in \{1, \dots, 36\}, j \in \{1, \dots, 36\}, t \in \{1, \dots, 8\}$:
$x_{\mathrm{ijt}} = 1$ if on day t, i plays against j at home, and $x_{ijt} = 0$ otherwise.

2.2 Necessary Constraints

Let's write the constraints:

$$\forall t \in \{1, \dots, 8\}, \forall i \in \{1, \dots, 36\}, \quad x_{iit} = 0 \tag{1}$$

→ A team cannot play against itself.

$$\forall (i,j) \in \{1, \dots, 36\}^2, \quad \sum_{t=1}^{8} (x_{ijt} + x_{jit}) \leq 1 \tag{2}$$

→ A team plays at most once against each other team.

$$\forall t \in \{1, \dots, 8\}, \forall i \in \{1, \dots, 36\}, \quad \sum_{j=1}^{36} (x_{ijt} + x_{jit}) = 1 \tag{3}$$

→ Each team plays exactly one match per day.

For all $i \in \{1, \dots, 36\}$:

$$\begin{aligned}
\sum_{t=1}^{8}\sum_{j=1}^{9} x_{ijt} = 1 &\quad \sum_{t=1}^{8}\sum_{j=1}^{9} x_{jit} = 1 \\
\sum_{t=1}^{8}\sum_{j=10}^{18} x_{ijt} = 1 &\quad \sum_{t=1}^{8}\sum_{j=10}^{18} x_{jit} = 1 \\
\sum_{t=1}^{8}\sum_{j=19}^{27} x_{ijt} = 1 &\quad \sum_{t=1}^{8}\sum_{j=19}^{27} x_{jit} = 1 \\
\sum_{t=1}^{8}\sum_{j=28}^{36} x_{ijt} = 1 &\quad \sum_{t=1}^{8}\sum_{j=28}^{36} x_{jit} = 1
\end{aligned} \tag{4}$$

→ Each team plays exactly against two teams from each pot, one home match and one away match.

The solver unsurprisingly finds a solution to this problem, giving us a first functional template. We will now try to add constraints to optimize the schedule.

2.3 Home-Away Alternation

We now attempt to add a constraint to alternate home and away matches for each team.

$$\forall t \in \{1, \dots, 7\}, \forall i \in \{1, \dots, 36\}, \quad \sum_{j=1}^{36} (x_{ijt} + x_{ij(t+1)}) = 1 \tag{5}$$

$\rightarrow$ Each team plays once at home and once away, with perfect alternation.

This new constraint is very strong. By integrating it into the solver, we do not get a solution, so no template can fulfill this constraint.

Let's attempt to relax this constraint slightly, allowing at most one break per team, meaning a team can have two consecutive home or away days, but only once, and neither in the first two nor the last two days. Formally, it is written:

$$\forall t \in \{2, \ldots, 6\}, \forall i \in \{1, \ldots, 36\}, \quad \begin{aligned} \sum_{j=1}^{36} x_{ijt} + x_{ij(t+1)} \leq 1 + b_{it} \\ \sum_{j=1}^{36} x_{jit} + x_{ji(t+1)} \leq 1 + b_{it} \end{aligned} \tag{6}$$

where $b_{it} \in \{0, 1\}$ is a binary variable indicating if team i has a "break" between days t and $t + 1$.

Moreover, for each team i:

$$\sum_{t=2}^{6} b_{it} \leq 1 \tag{7}$$

This ensures that at most one "break" is allowed for each team.

The strict constraints for the first two and last two matches remain unchanged:

For all $i \in \{1, \ldots, 36\}$:

$$\begin{aligned} \sum_{j=1}^{36} x_{ij1} + x_{ij2} = 1 \\ \sum_{j=1}^{36} x_{ij7} + x_{ij8} = 1 \end{aligned} \tag{8}$$

These constraints ensure a strict home-away alternation for the first two and last two days.

Now, the solver finds a solution. Initially, we aimed to establish an optimal objective function aimed at minimizing the number of breaks, given the constraint that each team was limited to at most one break. However, we encountered significant computational time challenges when attempting to solve with this objective function. Empirically, we discovered a scenario in which only 4 teams—specifically A1, B1, C1, and D1—experience a single break. This result demonstrates a considerable improvement over our initial constraint, significantly enhancing the schedule's balance and fairness by ensuring that the vast majority of teams enjoy a perfect alternation of home and away matches.

2.4 Even Distribution of Matches of the Same Interest Over the 8 Days

To maximize audience engagement over 8 days, UEFA would benefit from not scheduling top teams to play against each other on the same day, and similarly for the lower-ranked teams. Thus, it is advisable to distribute the matches from each pot evenly across the 8 days. For a given pot, there are 9 matches that feature two teams from that pot. Therefore, we set a constraint that on one of the 8 days, 2 such matches will occur, and exactly one match of this type will occur on each of the other 7 days.

Let the binary variables $b_{\mathrm{A}t}$, $b_{\mathrm{B}t}$, $b_{\mathrm{C}t}$, $b_{\mathrm{D}t}$ for each pot (A, B, C, D) and each day t indicate whether that day has 2 matches for the corresponding pot. The constraints are as follows:

- For each pot (A, B, C, D), there is exactly one day with 2 matches:

$$\begin{aligned}\sum_{t=1}^{8} b_{\mathrm{A},t} = 1 \\ \sum_{t=1}^{8} b_{\mathrm{B},t} = 1 \\ \sum_{t=1}^{8} b_{\mathrm{C},t} = 1 \\ \sum_{t=1}^{8} b_{\mathrm{D},t} = 1\end{aligned} \tag{9}$$

- For each day t and each pot (for example, pot A with teams 1 to 9):

$$\begin{aligned}\sum_{i=1}^{9}\sum_{j=1}^{9} x_{ijt} = 1 + b_{\mathrm{A},t} \\ \sum_{i=10}^{18}\sum_{j=10}^{18} x_{ijt} = 1 + b_{\mathrm{B},t} \\ \sum_{i=19}^{27}\sum_{j=19}^{27} x_{ijt} = 1 + b_{\mathrm{C},t} \\ \sum_{i=28}^{36}\sum_{j=28}^{36} x_{ijt} = 1 + b_{\mathrm{D},t}\end{aligned} \tag{10}$$

These constraints ensure that matches between teams from the same pot are evenly distributed over the 8 days.

A similar setup can be applied for matches between teams from two different pots. In this scenario, there are 18 matches to be scheduled in total. An ideal distribution would be 2 days with 3 such matches and 6 days with 2 such matches. We present the version for pots A and B. We define the binary variable $b_{\mathrm{AB}t}$, equal to 1 if on day t, there are 3 matches between a team from pot A and a team from pot B. We have:

- Since there are 2 days where 3 matches of A against B take place, the sum equals 2:

$$\sum_{t=1}^{8} b_{\mathrm{AB},t} = 2 \tag{11}$$

- To ensure that every day there are either two matches of a team A against a team B or three such matches:

$$\sum_{i=1}^{9}\sum_{j=10}^{18} x_{ijt} + x_{jit} = 2 + b_{\mathrm{AB},t} \tag{12}$$

Applying this constraint to all pot pairs showed no solution. However, by applying it to the pairs (A,B), (A,C) and (B,C), the solver was able to find a solution. This decision is made considering these to be respectively the most interesting types of inter-pot matches. However, for the other pairs, we mandate that there should be between 1 and 3 matches per day.

2.5 Sequential Match Ordering within Pots

In an effort to streamline the scheduling of intra-pot matches, we adopted a strategy of implementing a single cycle for matches within each pot. This approach dictates that matches are organized in a sequential manner, such as $X1$ vs. $X2$, $X2$ vs. $X3$, and so forth, culminating in a match between $X9$

and $X1$. This method ensures that every team within a pot plays against its immediate predecessor and successor, thereby preventing the formation of mini-groups within the championship phase.

Such a configuration was chosen to avoid scenarios where teams might end up playing in a small loop, for example, $X1$ vs. $X2$, $X2$ vs. $X3$, and $X3$ vs. $X1$, which could inadvertently create a subgroup effect within the broader tournament structure. By enforcing this sequential match order, we aim to maintain the integrity of the championship's competitive balance and fairness, ensuring that all teams are treated equitably and that the schedule reflects a coherent and logical progression of matches.

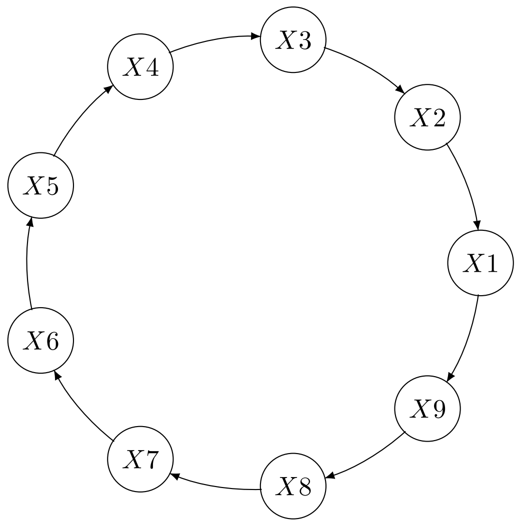

Figure 1: Distribution of Intra-Pot Matches for Pot X

2.6 Balancing the Schedule of Matches for Each Team

We also aim for each team's schedule to be well-balanced over the tournament duration. Specifically, we prioritize distributing encounters with strong teams (from pots A and B) evenly across the 8 matchdays. This approach ensures that no team faces a concentration of matches against top-tier opponents in a short span, promoting a fairer and more equitable competition structure.

For each team i and for each days $t \in \{2, \ldots, 6\}$:

$$\begin{aligned} \sum_{j \in \text{Pot A}} \left(x_{ijt} + x_{jit} + x_{ij(t+1)} + x_{ji(t+1)} + x_{ij(t+2)} + x_{ji(t+2)}\right) \leq 1 \\ \sum_{j \in \text{Pot B}} \left(x_{ijt} + x_{jit} + x_{ij(t+1)} + x_{ji(t+1)} + x_{ij(t+2)} + x_{ji(t+2)}\right) \leq 1 \end{aligned} \tag{13}$$

Finally, we added one last constraint, which is that no team can face 2 teams from pot C or 2 teams from pot D in the first two or last two matches.

We contemplated adding further constraints to enhance the match distribution for the UEFA Champions League's championship phase. However, we encountered limitations with our solver, which made it impossible to incorporate these additional constraints. This realization led us to believe that the constraints currently in place are the most suitable and effective for optimizing the match schedule, striking an ideal balance between logistical feasibility and competitive fairness.