Matthias NADAL

Randomized Algorithms (RA-MIRI): Assignment #1

14/10/2024

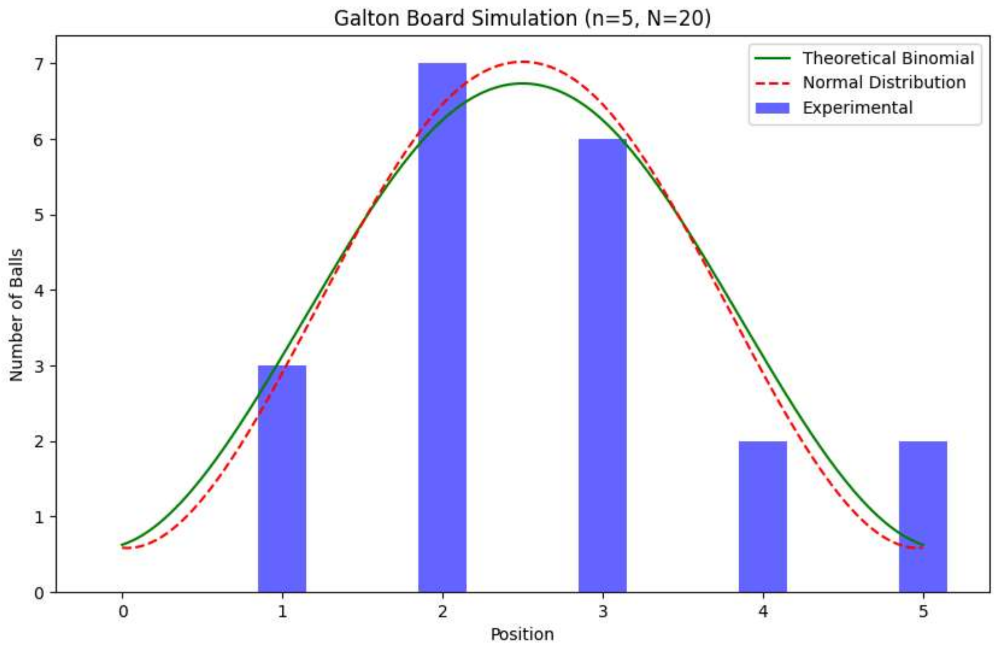
MIRI - Advanced Computing - Semester 1

Graphical Representations

I ran the simulation for different values of n and N, producing graphical representations for each. Below are the key experiments and results:

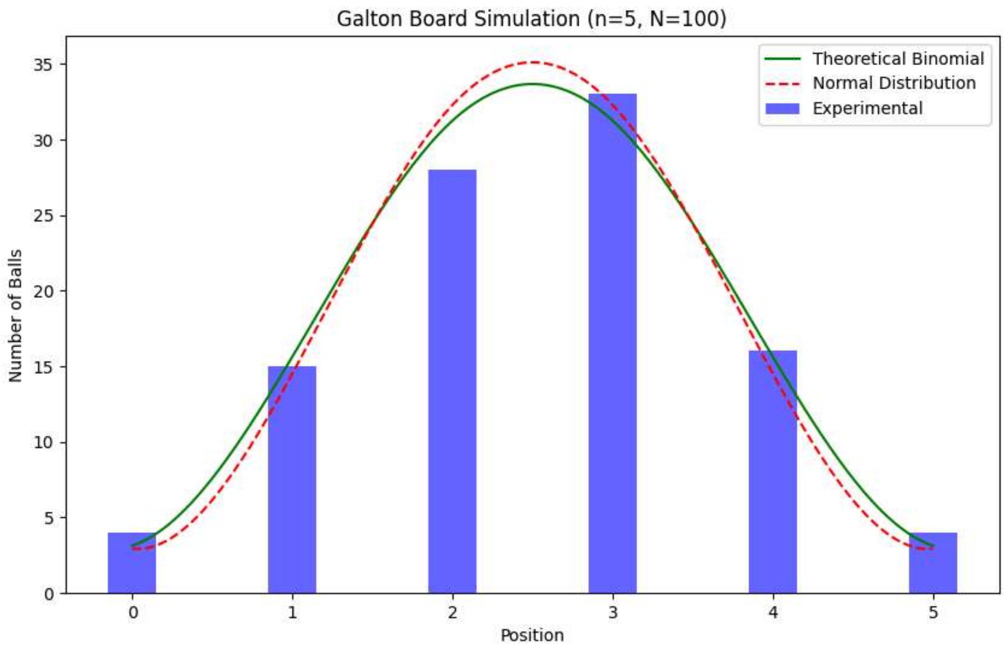
1. 5 Steps and 20 Balls

The ball distribution with small n and N, showing the early emergence of the binomial shape.

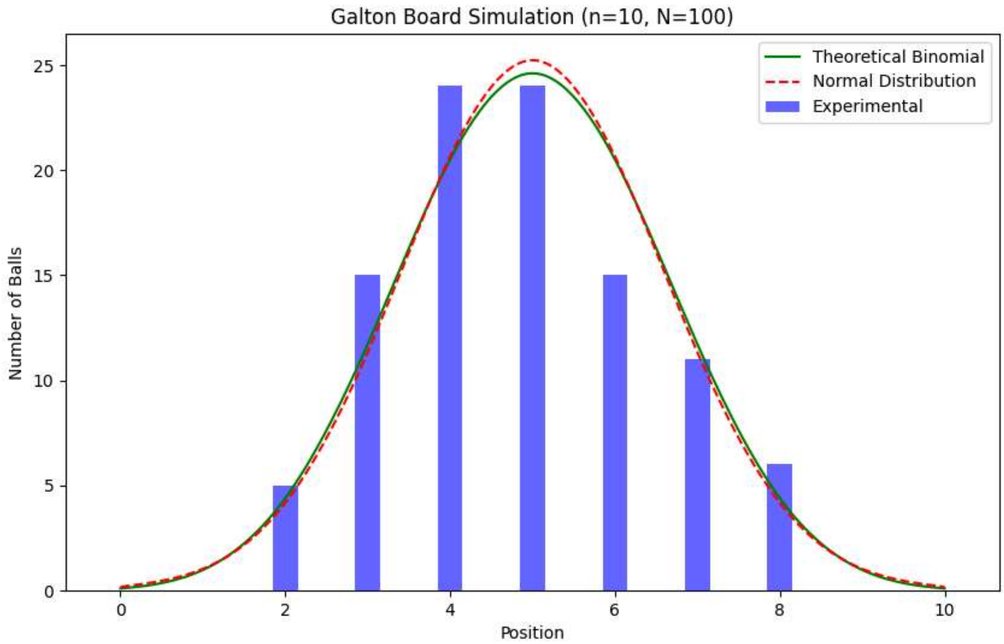


2. 5 Steps and 100 Balls

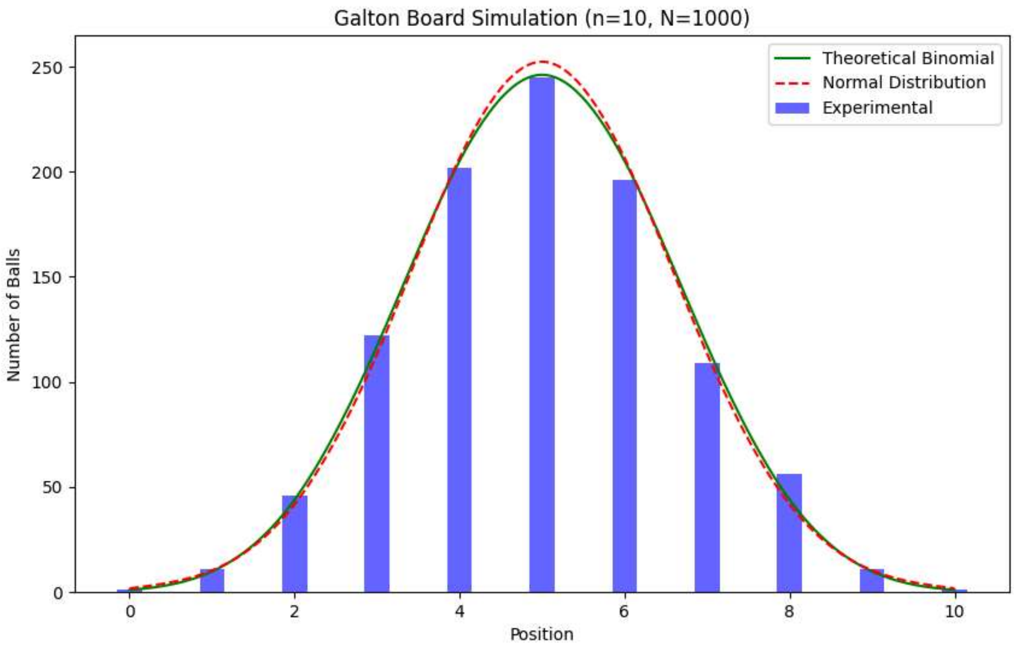
A larger sample that shows better adherence to the binomial distribution.



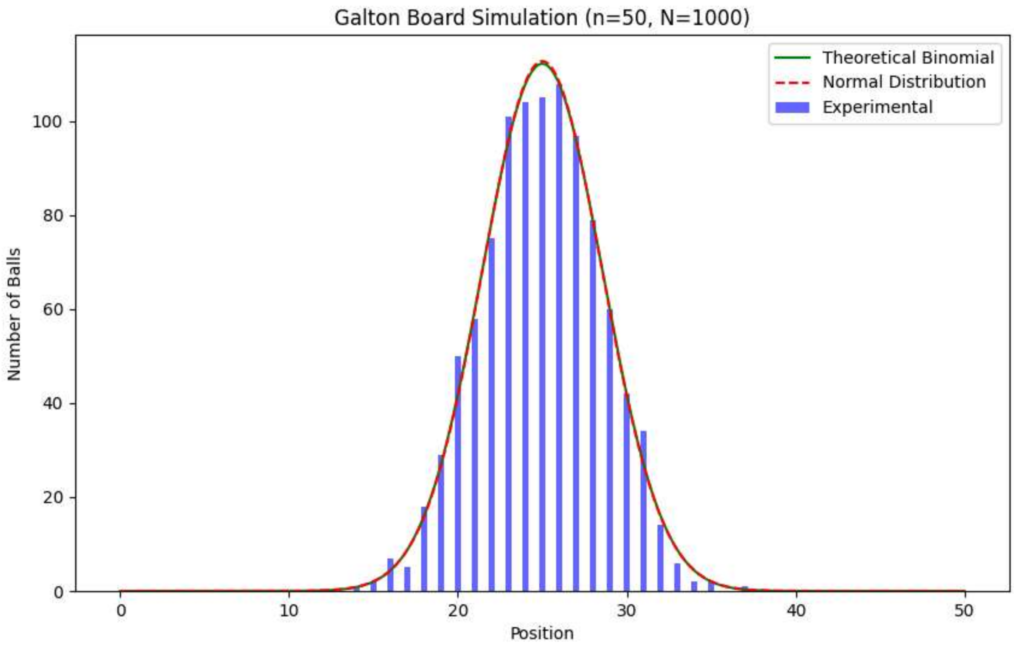
3. 10 Steps and 100 Balls



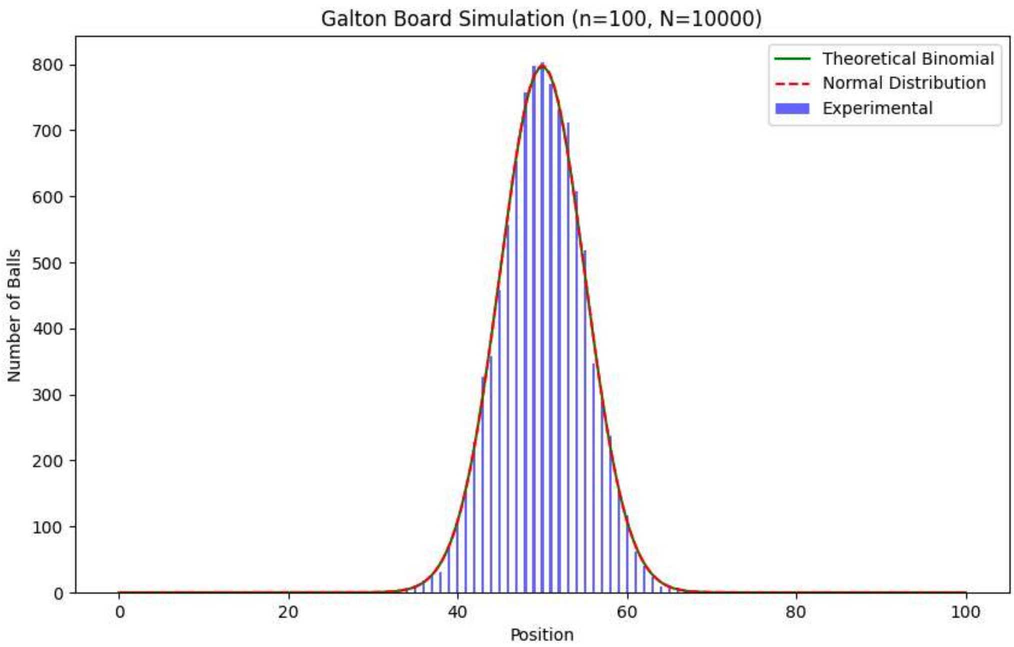
4. 10 Steps and 1000 Balls



5. 50 Steps and 1000 Balls



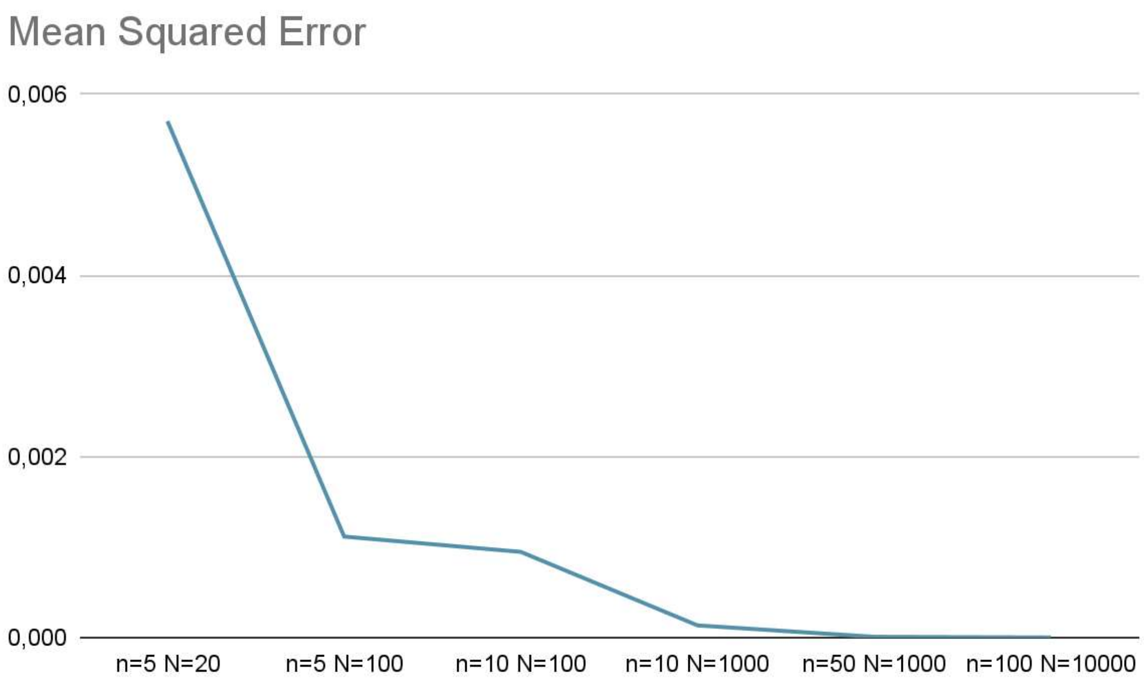
6. 100 Steps and 10000 Balls



As n and N increase, the Galton board simulation shows a clearer approximation of the binomial distribution, which increasingly resembles a normal distribution. Larger simulations result in smoother distributions, confirming the central limit theorem and highlighting the importance of large sample sizes for accurate probabilistic outcomes.

Mean Squared error

The mean squared error between the empirical binomial distribution and the theoretical normal distribution was computed for each experiment. The error decreases as the number of steps n and the number of balls N increase, which aligns with theoretical expectations.



Conclusion

The simulation confirmed the central limit theorem, showing that as n increases, the binomial distribution approaches the normal distribution, particularly with a large number of balls. The results are consistent with the theoretical model, with the error between the empirical and theoretical distributions diminishing as we increase both n and N.

The graphical analysis and error quantification underscore the importance of large sample sizes in achieving accurate distribution approximations.