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SPECIALIZAREA INFORMATICĂ

Lucrare de licență

DIGITAL IMAGE WATERMARK

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Abstract

In this paper a potential scheme for embedding binary image watermarks in RGB color images using the Fast Quaternion Fourier Transform will be presented by recreating the results of Wang et al. [4]. Firstly, the binary watermark image will be scrambled with Arnold's Cat Map and divided into 2x2 blocks. Secondly, the host image will be divided into 8x8 blocks, peform Quaternion Fourier Transform on each block and embed each watermark block into the real part of the transformed host block. Lastly, perform the Inverse Transform of each block and reconstruct the image.

Contents

Chapter 1

Introduction

As multimedia content has increased over the last few decades, the digital security of data and intellectual property has become a serious issue. Digital watermarking is a technique used for embedding information into digital content such as images, audio, video, etc. to ensure traceability in various scenarios such as healthcare data protection, digital assets, document authentication, etc.

For better data consistency, robust embedding schemes are a great method, the data becoming invariant to geometrical transformations such as rotation, scaling and reflection.

Chapter 2

Preliminary

In this chapter, we discuss the mathematical concepts necessary for describing the operations that will be performed on the data.

2.1 Watermark Scrambling

In this embedding scheme, the pixels of the binary image W of shape $N \times N$ are displaced by the chaotic transform *Arnold's Cat Map* also named *Cat Face Transfer* given by $\Gamma : \mathbb{N}^2 \times \mathbb{N} \to \mathbb{N}^2$

$$\Gamma(\begin{bmatrix} x \\ y \end{bmatrix}, k) = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}^k \begin{bmatrix} x \\ y \end{bmatrix} \ (mod\ N)$$

where $\begin{bmatrix} x \\ y \end{bmatrix}$ coresponds to the pixel on line x and column y, and result in a binary image W'.

Arnold's Cat Map is a periodic transform, and so the number of scramblings k can represent the *security key* of the transform.

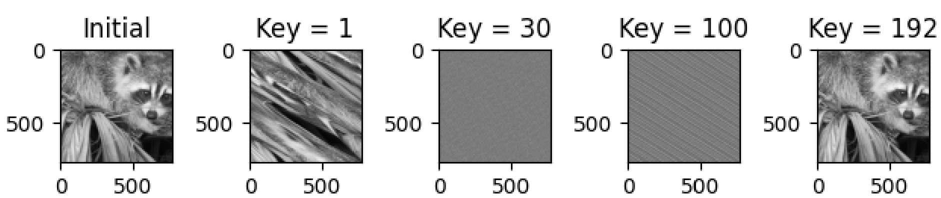

Figure 2.1: Visualization of a binary image after scrambling using Arnold's Cat Map.

2.2 Fast Quaternion Fourier Transform on RGB Image

2.2.1 Quaternions

Quaternions are four-dimensional hyper-complex numbers defined in the space $\mathbb{H}$, applied to mechanics in three-dimensional space defined as

$$a + bi + cj + dk$$

where $a, b, c, d \in \mathbb{R}$ and $1, i, j, k$ are vector basis on $\mathbb{H}$ with the following multiplicative laws:

$$ij = -ji = k,\ jk = -kj = i,\ ki = -ik = j,\ i^2 = j^2 = k^2 = -1$$

Every quaternion

$$q = a + bi + cj + dk \in \mathbb{H},\ a, b, c, d \in \mathbb{R}$$

has a corresponding *conjugate*

$$\overline{q} = a - bi - cj - dk \in \mathbb{H}$$

and a *norm*

$$\mid q \mid = \sqrt{q\overline{q}} = \sqrt{a^2 + b^2 + c^2 + d^2}$$

As shown by Moxey et al. [3], quaternions can be used for representing color images for multiple color spaces such as *RGB, YCbCr, CIE 1931* and so forth. For example an RBG pixel will be described as follows:

$$Image(x, y) = R(x, y)i + G(x, y)j + B(x, y)k$$

where *Image(x,y)* represents the pixel at row x and column y and *R/G/B(x,y)* is the R, G or B value of pixel (x, y).

2.2.2 Fourier Transform

The Fourier Transform is an analysis method of decomposing a function $f(x)$ from the *time domain* into it's corresponding energy levels in the *frequency domain* also referred as *the spectrum* of the function, given by:

$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi\xi x} dx$$

with the inverse transform:

$$f(x) = \int_{-\infty}^{\infty} \hat{f}(\xi) e^{-i2\pi\xi x} d\xi$$

Due to computers only being capable of representing finite, discrete values, more suitable for digital computation, *Discrete Fourier Transform (DFT)* is usually used, given by:

$$X_k = \sum_{n=0}^{N-1} x_n e^{-i2\pi nk/N}$$

and the corresponding inverse transform:

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{i2\pi nk/N}$$

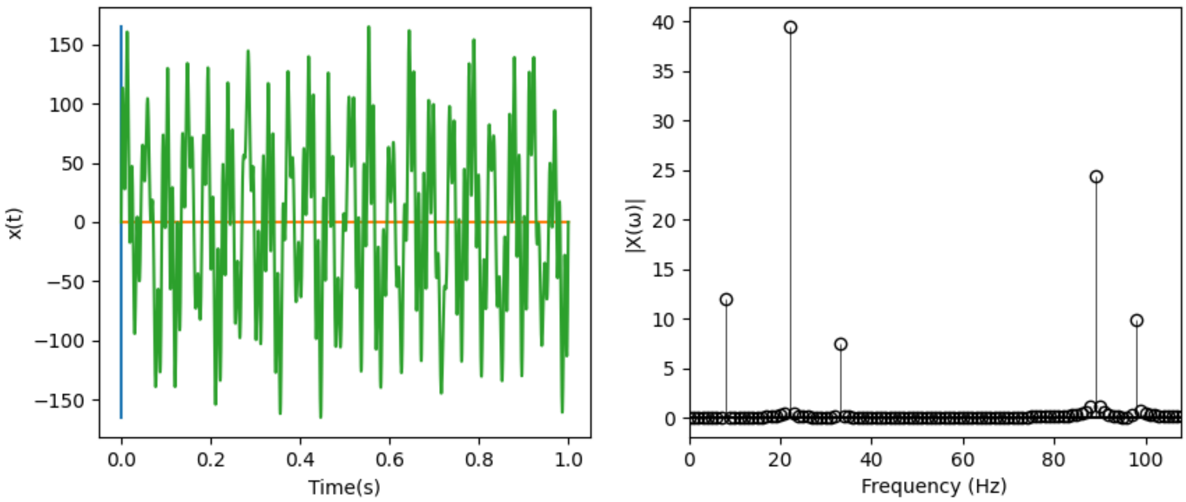

Figure 2.2: Visualization of 1D Discrete Fourier Transform.

This notion can also be extended for 2-dimensional functions such as images, resulting in 2-dimensional spectrum, by the formula:

$$F(m,n) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n) e^{-i2\pi(\frac{xm}{M} + \frac{yn}{N})}$$

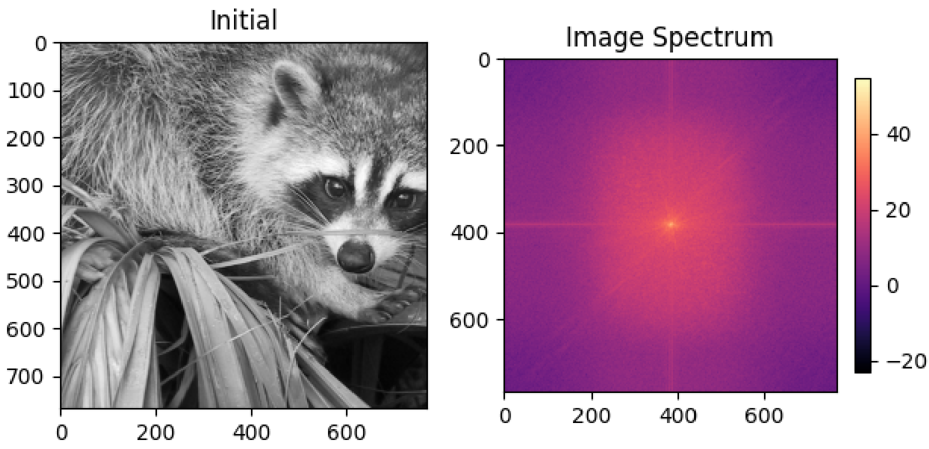

Figure 2.3: Visualization of 2D Discrete Fourier Transform on a gray scale image.

2.2.3 Fast Quaternion Fourier Transform

In 1996, S. J. Sangwine has discovered an applicable formulation for a Discrete Quaternion Fourier Transform and it's inverse after previous methods have failed.

$$F(u,v) = \frac{1}{\sqrt{MN}} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} e^{-\mu 2\pi (\frac{mu}{M} + \frac{nu}{N})} f(m,n)$$

where μ is a unit pure quaternion $\mu^2 = -1$. Choosing $\mu = i$ is a special case determined by the regular Fourier Transform.

As shown in the work of S. Jiang et al. [2] there is a fast algorithm for computing the 2D Quaternion Fourier Transform using the regular 2D *Fast Fourier Transform(FFT)*

$$\begin{aligned} F(u,v) = {} & i(\mathfrak{Re}(R_{RFT}) + \mu\mathfrak{Im}(R_{RFT})) \\ & + j(\mathfrak{Re}(G_{RFT}) + \mu\mathfrak{Im}(G_{RFT})) \\ & + k(\mathfrak{Re}(B_{RFT}) + \mu\mathfrak{Im}(B_{RFT})) \end{aligned}$$

where $\mathfrak{Re}$ denotes the real part, $\mathfrak{Im}$ the imaginary part and $R/G/B_{RFT}$ the regular Fourier Transform for each color channel.

Let $A(u,v)$ be the real part of the Quaternion Fourier Transform and $B(u,v),\ C(u,v),\ D(u,v)$ the three imaginary parts. We can rewrite the formula as such:

$$F(u,v) = A(u,v) + iB(u,v) + jC(u,v) + kD(u,v)$$

with the inverse:

$$
\begin{aligned}
f(m,n) = &(\mathrm{Real}(A_{IRFT}) + \mu \cdot \mathrm{Imag}(A_{IRFT})) \\
&+ i(\mathrm{Real}(B_{IRFT}) + \mu \cdot \mathrm{Imag}(B_{IRFT})) \\
&+ j(\mathrm{Real}(C_{IRFT}) + \mu \cdot \mathrm{Imag}(C_{IRFT})) \\
&+ k(\mathrm{Real}(D_{IRFT}) + \mu \cdot \mathrm{Imag}(D_{IRFT}))
\end{aligned}
$$

where $A/B/C/D_{IRFT}$ is the regular Inverse Fourier Transform for each component.

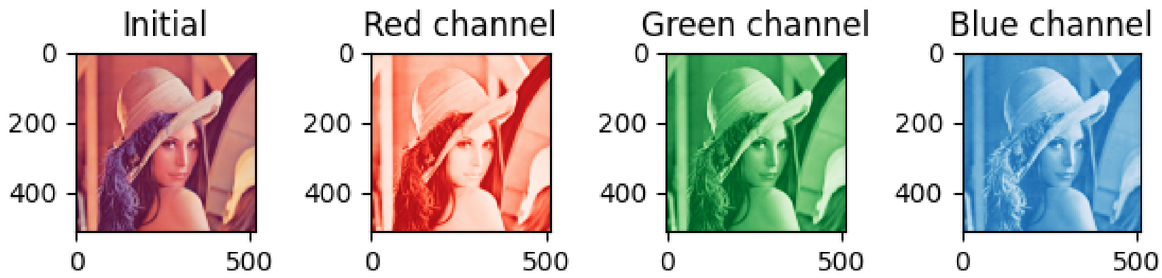

Figure 2.4: RGB Color channels separation for Lena.

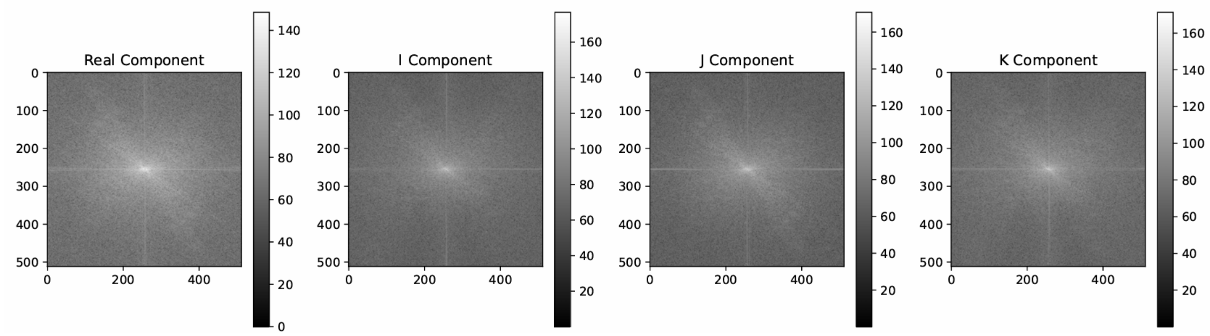

Figure 2.5: QFT spectrum components of Lena

Chapter 3

Methodology

In this chapter, the embedding scheme will be described step by step under the following conditions.

1. The watermarked binary image W must have a shape $N \times N$, with $N = 2k$, where $k \in \mathbb{N}$.
2. The host RGB image I must have a shape $M \times M$, with $M = 8k$, where $k \in \mathbb{N}$.
3. $M \geq 4N$.

3.0.1 Watermark Image Processing

Firstly, the pixels of W will be displaced by applying Arnold's Cat Map with an arbitrary $k \in \mathbb{N}$ that will be considered the *security key* of W, giving the new binary image W'.

Secondly, W' will be broken down into a block of size 2×2 resulting in the set of blocks $W_k = \{w_k(i,j) |\ i,j \in \{0,1\},\ k\ \in \{1, 2 \ldots, \frac{N^2}{4}\}\}$

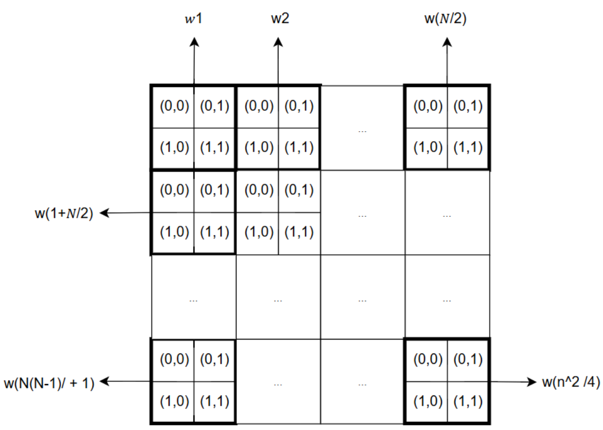

Figure 3.1: Scrambled watermark image blocks structure

3.0.2 Host Image Processing

The host image I will be broken down into blocks of size 8×8 in the same way as the watermark resulting in a set of blocks $H_k = \{h_k(i,j)|\ i,j \in \{0,1,\dots 8\},\ k \in \{1,2\dots,\frac{M^2}{64}\}\}$.

The Fast Quaternion Fourier Transform will be applied on each block of H_k, each resulting in one real part A_k and three imaginary parts B_k, C_k, D_k.

3.0.3 Embedding Scheme

Each block of W_k corresponds to at least one block from H_k. The block W_k will be embedded in A_k as follows:

$$\begin{cases} a'_k(1,1) = 2\omega \ * \ \text{round}\left(\dfrac{a_k(1,1)}{2\omega}\right) + (-1)^{w_k(0,0)+1}\dfrac{\omega}{2} \\ a'_k(1,2) = 2\omega \ * \ \text{round}\left(\dfrac{a_k(1,2)}{2\omega}\right) + (-1)^{w_k(0,1)+1}\dfrac{\omega}{2} \\ a'_k(2,1) = 2\omega \ * \ \text{round}\left(\dfrac{a_k(2,1)}{2\omega}\right) \\ a'_k(2,2) = 2\omega \ * \ \text{round}\left(\dfrac{a_k(2,2)}{2\omega}\right) \end{cases}$$

where ω is the embedding strength of the watermark.

To maintain symmetry and ensure the pure quaternion form is correct before applying the inverse, we must also adjust the coefficients symmetrically corresponding to those modified in the previous step.

$$\begin{cases} a'_k(6,6) = -a'_k(2,2) \\ a'_k(6,7) = -a'_k(2,1) \\ a'_k(7,6) = -a'_k(1,2) \\ a'_k(7,7) = -a'_k(1,1) \end{cases}$$

3.0.4 Reconstruct the Embedded Image

Take each real coefficient components A_k with its corresponding imaginary components B_k, C_k, D_k and perform the Fast Quaternion Inverse Transform.

Lastly, join back all the new blocks generated by the inverse transform, in the shape of the initial image W ($M \times M$) and convert the new quaternion values into RGB.

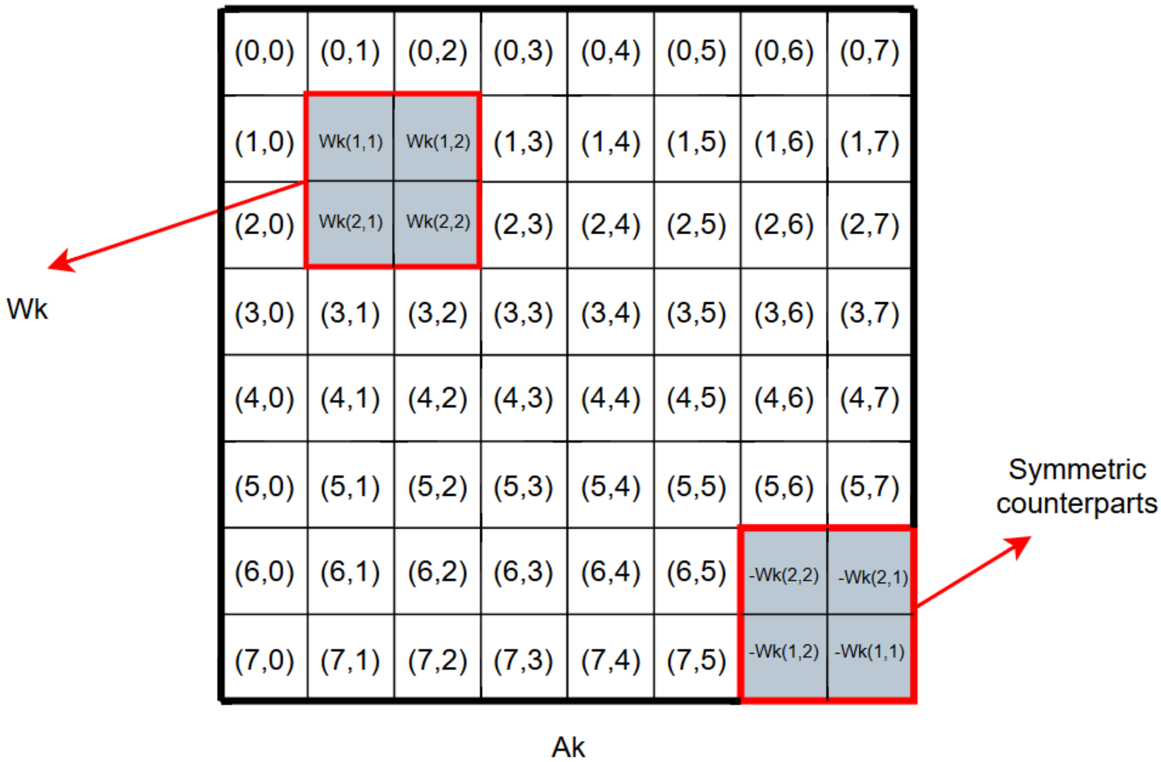

Figure 3.2: Embedding of Wk into Ak

Chapter 4

Experiments

In this chapter, we see the results of applying the embedding scheme on a few color images with different watermarks of various sizes:

Figure 4.1: Catface watemark of size 178x153

Figure 4.2: Football watermark of size 200x200

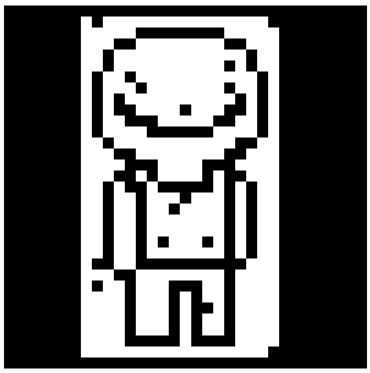

Figure 4.3: A man scatch watermark of size 33x33

Figure 4.4: Watermarked cat with catface

Figure 4.5: Watermarked cat with football

Figure 4.6: Watermarked cats with littleman

Figure 4.7: Watermarked lena with catface

Figure 4.8: Watermarked lena with football

Figure 4.9: Watermarked lena with littleman

Figure 4.10: Watermarked cats with catface

Figure 4.11: Watermarked cats with football

Figure 4.12: Watermarked cats with littleman

Chapter 5

Conclusions

Some of the existing digital watermark schemes are sensible to geometric transformations and the data can easily be corrupted leading to an inability of extract the original embedded watermark. As shown above, the watermarking scheme presented is a secure method for preserving a binary image data into color images.

This implementation is limited to certain sizes for both the host image and the watermark image.

Bibliography

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