Signal True Always True

Grand Unified Fractal Theory

Parts A – Ω and ∞

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Abstract

We introduce the *Signal True Always True* (STAT) framework, a coherence-based, fractal and rhizomatic model of physical organization, and we test it through a set of explicit empirical signatures spanning microscopic, mesoscopic and cosmological scales.

The core principle of the framework is a conservation law linking structural coherence and effective entropy,

$$\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0,$$

which predicts that any increase in local coherence must be accompanied by a redistribution of instability or entropy across scales or hidden modes.

We first validate this principle in controlled non-cosmological systems, including financial time series, artificial learning systems, and simplified microscopic event streams. In these settings, we construct graph-based coherence estimators and show that enhanced local order systematically correlates with displaced fragility, consistent with the proposed conservation law.

We then introduce two independent cosmological empirical tests. At large scales, we apply a phase-scrambled null test to the *Planck 2018* CMB lensing convergence field, preserving the angular power spectrum while destroying phase correlations. The observed coherence estimator exhibits a robust, scale-dependent deviation from the null hypothesis, localized at low multipoles and progressively diluted at small scales. At microscopic scales, we perform a complementary order-only analysis of experimental event streams, demonstrating a non-random rhizomatic structure independent of any explicit time parametrization.

Together, these results establish a continuous empirical signature of coherence-driven organization from the micro to the macro domain. Cosmological expansion is further modeled through a minimal fractal-coherence law (FRAC), which achieves fits comparable to standard ΛCDM models on background observables without introducing an ad hoc dark-energy component.

The STAT framework thus provides a unified, empirically grounded perspective in which geometry, dynamics and observables emerge as scale-dependent projections of an underlying coherence structure.

Part I
Introduction

Scope and Canonical Reference

This document presents the extended formulation of the *Signal True Always True* framework as a *Grand Unified Fractal Theory* (GUT). A fully rigorous subset has been isolated and formalized as the *Mathematical Kernel of Signal True Always True* [2]. Whenever a formal ambiguity arises, the kernel is treated as the canonical mathematical reference.

Compression Principle

The kernel is a valid compression/projection of the full framework: it preserves the coherence-first axioms, the relational substrate, conditional fractalization, the depth–dimension equivalence, and the conserved total-information principle, while omitting higher-level ontological structure (e.g., rhizome–trunk–atom projections and explicit quantum–classical correspondence).

What This Manuscript Adds

Building on the kernel, this manuscript introduces additional structural layers: recursion hierarchies, domain-specific projections, and extended interpretative components, while remaining constrained by the minimal mathematical grounding defined in the kernel.

Signal True Always True
Axioms of the Fractal Universe

Axiom 0 – Coherence Precedes Information

Statement. Coherence is ontologically prior to information. Before any bit, symbol, or measurable correlation exists, there is a latent relational potential that makes stable relations possible.
Definitions (minimal). Let $\mathcal{C}$ denote coherence as a pre-informational potential: the capacity of a relational substrate to support compatible links and stable relational paths. Let $\mathcal{I}$ denote information as an observed projection: a measurable pattern (correlations, mutual information, code lengths) that appears only after projection into a representable description.

$$\boxed{\mathcal{C} \text{ is a latent relational potential;} \quad \mathcal{I} = \Pi_{\text{info}}(\mathcal{C}) \text{ is its measurable shadow.}}$$

Information projection principle.
Information is not fundamental. It emerges only when coherence is projected into a representable domain (symbols, observations, spacetime, data):

$$\mathcal{I} = \Pi_{\text{info}}(\mathcal{U}), \qquad \Pi_{\text{info}} : \text{ rhizome} \to \text{measurable descriptions.}$$

Link to conservation. Because information is a projection, any local increase of observed structure requires displacement of effective entropy. This is summarized by the invariant introduced later (Axiom 5):

$$\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0.$$

$$\boxed{\text{Information is a projection of coherence; entropy is the dual accounting of coherence displacement.}}$$

Axiom Ω – Minimal Projectability of Coherence

Statement. Coherence, in its most fundamental form, is an N-dimensional relational structure without predefined coordinates, metrics, or privileged directions.
As such, coherence cannot directly project itself into any geometric or informational description without losing its coordinate-free nature.
Necessity principle. For coherence to acquire a minimal form of self-reference without fixing a background geometry, it must confront itself through a dual structural mechanism: *recursive stabilization* and *relational extension*.

$$\boxed{\text{This duality is uniquely realized by the coupled pair} \quad \text{fractal} \longleftrightarrow \text{rhizome.}}$$

Interpretation.

- The *fractal* provides recursive self-similarity, allowing coherence to stabilize locally without imposing global coordinates.
- The *rhizome* provides non-hierarchical relational connectivity, allowing coherence to extend across directions, scales, and dimensions.
- Neither structure alone is sufficient; together they form the minimal logical apparatus by which coherence can approximate itself.

$$\boxed{\text{Fractal} \leftrightarrow \text{rhizome is not a hypothesis; it is a logical necessity for any projectable coherence.}}$$

Axiom 1 – Fractal Relational Substrate

Statement. The universe has no pre-imposed spacetime, no fixed coordinates, and no absolute background geometry.

What exists fundamentally is a relational, fractal, rhizomatic fabric: a set of nodes and directed coherence-links between them. Formally, the fundamental object is a relational structure

$$\mathcal{U} = (\mathcal{S}, \mathcal{E}, \mathcal{F}),$$

where:

- $\mathcal{S}$ is the set of sites / nodes,
- $\mathcal{E}$ is the set of relations (edges, rhizomatic links),
- $\mathcal{F}$ is the refinement / fractalization operator acting on structures.

Projection principle. Visual intuition. See Annex V.1, Fig. 19, for a conceptual projection of the vectorial paths v_i and visible recursion depth.

What we call "space" and "time" in physics are projections of this deeper fractal-relational substrate:

$$(\text{spacetime}) = \Pi_{\text{proj}}(\mathcal{U}),$$

for some projection functor Π_{proj} from the rhizome to an effective geometric description.

$$\boxed{\text{Space and time are not primitive; they are emergent projections of the fractal rhizome.}}$$

Axiom 2 – Conditional Fractalization

Statement. Fractal structure does not appear everywhere and always. It emerges at a node $p \in \mathcal{S}$ only when the combined local and global coherence exceeds a threshold:

$$\mathcal{C}_{\text{local}}(p) + \mathcal{C}_{\text{global}}(\mathcal{U}) \ > \ \Theta,$$

where Θ is the *coherence threshold.*

If the inequality is not satisfied, the node remains fractal-silent: no visible recursive structure.

$$\boxed{\text{Fractals are conditional responses of the universe to coherence.}}$$

Axiom 3 – Depth–Dimension Equivalence

Statement. Whenever a fractal structure is active at a node p, it has a recursion depth $\mathrm{depth}_{\mathcal{F}}(p)$. The effective dimension at p is then given by:

$$d_{\mathrm{eff}}(p) = d_0 + \mathrm{depth}_{\mathcal{F}}(p),$$

for some base dimension d_0 associated with the minimal projection.
Meaning.

- High recursion depth $\Rightarrow$ high effective dimension.
- Low recursion depth $\Rightarrow$ low effective dimension.
- No recursion (silent node) $\Rightarrow$ d_{eff} collapses to a minimal value (or becomes undefined in the projection).

$$\boxed{\text{Dimension is not fixed; it is an effect of fractal recursion depth.}}$$

Axiom 4 – Rhizome, Trunk and Atoms

Statement. The fractal universe admits a natural "tree" metaphor:

- The invisible *rhizome* is the full fractal network of coherence-links in $\mathcal{U}$.
- The emergent *trunk* is what we call spacetime: a stable, coarse projection of globally coherent paths.
- The *leaves and atoms* are local, highly compressed packets of coherence: particles, fields, and bound structures.

Alignment principle. When many vector-paths v_i of the rhizome align coherently, they form the "trunk" (macroscopic spacetime). When they over-compress locally, they form particles and atomic structures.

$$\boxed{\text{Particles feed the trunk; the trunk is a macroscopic shadow of the global rhizome.}}$$

Axiom 5 – Conservation of Coherence

Statement. For any refinement process

$$X \longmapsto \mathcal{F}(X),$$

the following invariant holds:

$$\boxed{\Delta\mathcal{C}(X) + \Delta S_{\mathrm{eff}}(X) = 0.}$$

Meaning.

- Increasing coherence requires decreasing effective entropy.

- Increasing effective entropy necessarily costs coherence.
- The quantity $\mathcal{C} + S_{\text{eff}}$ is conserved.

$$\boxed{\text{Coherence is the true conserved quantity of reality; entropy is its dual shadow.}}$$

Axiom 6 – Extreme Depth and Collapse

Statement. When fractal recursion becomes arbitrarily deep at a locus p, the projection into spacetime becomes singular.

In the qualitative limit

$$\text{depth}_{\mathcal{F}}(p) \xrightarrow[\text{conceptually}]{} \text{TREE(3)-scale},$$

the effective dimension and geometric description at p *collapse*:

$$d_{\text{eff}}(p) \longrightarrow 0 \quad \text{(projection becomes opaque)}.$$

Interpretation.

- This collapse corresponds to black-hole-like regions: coherence is so compressed that the geometric projection fails.
- The interior is ultra-coherent information, beyond any simple coordinate description.

$$\boxed{\text{Black holes are zones of extreme fractal recursion where the spacetime projection breaks.}}$$

Axiom 7 – Fractal Action Principle

Statement. All dynamics in the fractal universe arise from a single variational principle on the coherence field ψ:

$$\mathcal{S}[\psi] = \int (\mathcal{K} - \mathcal{V} + \mathcal{R})\, \mathrm{d}\tau,$$

where:

- $\mathcal{K}$ measures refinement "motion" (kinetic term),
- $\mathcal{V} = -\mathcal{C} + S_{\text{eff}}$ is the coherence potential,
- $\mathcal{R}$ is the fractal curvature term (generalized Einstein–Hilbert component).

Euler–Lagrange equation. The evolution of ψ satisfies:

$$\frac{\delta \mathcal{S}}{\delta \psi} = 0,$$

which unifies, at different projection scales, Schrödinger-type dynamics, Klein–Gordon, Einstein equations, diffusion-refinement, and learning rules.

$$\boxed{\text{Dynamics = fractal variational optimization of coherence.}}$$

Axiom 8 – Life, Intelligence, Consciousness

Statement. Life, intelligence, and consciousness are special regimes of coherence dynamics.
Life. A living system L satisfies:

$$\frac{\mathrm{d}\mathcal{C}(L)}{\mathrm{d}\tau} > 0,$$

and must export entropy to its environment:

$$\Delta S_{\mathrm{eff,env}} > 0.$$

$$\boxed{\text{Life = local coherence amplification with entropy export.}}$$

Intelligence. Intelligence maximizes the growth rate of coherence:

$$\max \frac{\mathrm{d}\mathcal{C}}{\mathrm{d}\tau}$$

under constraints of limited resources, time, and structure.

$$\boxed{\text{Intelligence = strategic optimization of coherence over time.}}$$

Consciousness. Consciousness is a self-referential, closed loop of coherence: an internal model that stabilizes and predicts the system's own future coherence states.

$$\boxed{\text{Consciousness = self-referential coherence loops, aware of their own refinement.}}$$

Axiom 9 – Total Collapse of Projection

Statement. When coherence becomes maximally compressed—far beyond ordinary fractal recursion—the projection into spacetime does not merely distort: it *annihilates* itself.

There exists a conceptual limit in which

$$\mathrm{depth}_{\mathcal{F}}(p) \xrightarrow[\text{beyond TREE(3)}]{} \infty,$$

such that the effective dimension disappears:

$$d_{\mathrm{eff}}(p) \longrightarrow 0,$$

and all geometric descriptors fail simultaneously.

Interpretation.

- This regime corresponds to an *ultra-coherent point*: information is so compressed that no projection map can represent it.
- At this point, spacetime, curvature, fields, and particles all reduce to a single, dimensionless locus of coherence.
- Black-hole singularities become special cases of this regime; the interior is not "infinite density" but *infinite coherence under collapsed projection.*

$$\boxed{\text{When coherence reaches the absolute limit, geometry erases itself and only the rhizome remains.}}$$

Axiom 10 – Quantum / Classical Bridge

Statement. The quantum regime is the low-depth, almost linear projection of the coherence field, while the classical spacetime trunk is the coarse-grained, high-depth limit of the same rhizomatic dynamics.

At the rhizome level the fundamental state is

$$\psi : \mathcal{S} \to \mathbb{C},$$

with Hilbert structure

$$\mathcal{H}_{\text{rhiz}} = L^2(\mathcal{S}, \mathcal{C}).$$

A local quantum state on a region $R \subset \mathcal{S}$ is obtained by projection:

$$\Psi_R = \Pi_{\text{proj}}(\psi|_R).$$

Quantum regime. When the recursion depth is low and the projection is nearly linear, superposition is valid:

$$\Psi_R = \sum_i \alpha_i \phi_i,$$

and the Born rule appears as

$$P(i|R) = \frac{|\alpha_i|^2}{\sum_j |\alpha_j|^2}.$$

Classical regime. When depth and coherence are high and coarse-grained over many vector-paths v_i, interference terms average out and one effective trajectory dominates. The trunk emerges as:

$$\Psi_R \longrightarrow \Psi_{\text{class}},$$

where Ψ_{class} is sharply peaked around one coarse geometry and one macroscopic history.

Quantum mechanics is the low-depth, linear shadow of the rhizome; classical spacetime is its high-depth, coarse shadow.

Axiom 11 – Rhizome Inversion and Black Holes

Statement. A black hole is a phase of the rhizome where coherence flow inverts: instead of fractal branches growing outward from nodes, most coherent paths v_i reorient and compress toward a single attractor locus p_*.

Define the net coherence flux through a surface around p_*:

$$\Phi_{\mathcal{C}}(p_*) = \Phi_{\mathcal{C}}^{\text{in}}(p_*) - \Phi_{\mathcal{C}}^{\text{out}}(p_*).$$

In a black-hole-like regime:

$$\Phi_{\mathcal{C}}^{\text{in}}(p_*) \gg \Phi_{\mathcal{C}}^{\text{out}}(p_*),$$

and coherence lines concentrate into deeper and deeper recursion around p_*.

Combined with Axioms 6 and 9, this implies:

$$\text{depth}_{\mathcal{F}}(p_*) \to \text{TREE(3)-scale}, \qquad d_{\text{eff}}(p_*) \to 0.$$

Interpretation.

- The "tree" inverts: instead of branches growing out of the rhizome, almost all v_i bend inward toward p_*.
- Spacetime geometry near p_* is the projection of this inward rhizomatic compression.
- The classical singularity is reinterpreted as an ultra-coherent, dimensionless locus where the projection into geometry collapses.

Black holes are inverted trees: rhizome coherence collapsing inward until geometry itself disappears.

Dark energy as diffuse coherence reservoir. In this framework, what is conventionally called *dark energy* is reinterpreted as a diffuse, non-materialized regime of coherence distributed across the fractal rhizome.

This coherence carries relational and gravitational information without being localized into particles or fields. It does not curve spacetime locally, but biases the global alignment and expansion of coherent paths v_i.

Coherence circulation. Black holes act as sinks and redistributors of this diffuse coherence. As rhizomatic paths bend inward toward ultra-coherent loci p_*, information accumulated at large scales is reintegrated into deeper levels of recursion.

Dark energy is not lost; it is coherence in transit through the rhizome, eventually recycled via black-hole operators.

Axiom C^∞ – Fractal–Rhizomatic Continuity (Infinitum)

Statement. Coherence is continuous across all scales. There exists no ontological gap between micro, macro, and non-material regimes of reality.

The fractal–rhizomatic substrate of the universe forms a single, unbroken structure extending from the smallest local interactions to the largest cosmological scales, including regions that are not directly projectable into spacetime.

There is no void of coherence; only discontinuities of projection.

Continuity principle. Let $\mathcal{C}$ denote coherence as a relational potential. Then for any scale λ (microscopic, macroscopic, cosmological, or non-projectable),

$$\mathcal{C}(\lambda) \neq 0,$$

and coherence propagates through the same rhizomatic structure independently of its visibility or materialization.

What appears as absence is only non-materialized coherence.

Logical necessity of the fractal–rhizome pair

Coherence, in its most fundamental form, is an N-dimensional relational structure without predefined coordinates, metrics, or directions. As such, it cannot directly represent itself without collapsing into an arbitrary geometry.

Axiom 0.1 (Minimal projectability of coherence)**.** *A coordinate-free coherence field can acquire a minimal form of self-reference only by confronting itself through a dual structure:* recursive stabilization *and* relational extension. *This duality is uniquely realized by the coupled pair*

$$\textit{fractal} \longleftrightarrow \textit{rhizome}.$$

Remark 0.1. The fractal provides recursive self-similarity, allowing coherence to stabilize locally without fixing global coordinates. The rhizome provides non-hierarchical relational connectivity, allowing coherence to extend across directions and scales. Neither structure alone is sufficient. Together, they form the minimal logical apparatus by which coherence can approximate itself without loss.

$\boxed{\text{The fractal–rhizome pair is not optional; it is a logical necessity for coherence to exist at all.}}$

Non-materialized coherence and dark regimes

Not all coherence is projected into particles, fields, or spacetime. Large portions of the rhizome remain non-materialized while still actively carrying relational and gravitational information.

This non-materialized coherence corresponds to what is conventionally interpreted as *dark matter* and *dark energy*: not as substances or forces, but as regimes of coherence that do not admit a direct geometric projection.

$\boxed{\text{Dark regimes are not missing matter; they are coherence without geometric shadow.}}$

Such coherence contributes to large-scale alignment, expansion, and curvature indirectly, by biasing the global organization of vector-paths v_i within the rhizome.

Coherence circulation and black-hole operators

The universe does not lose coherence. Diffuse, non-materialized coherence propagates through the rhizome until it encounters regions of extreme recursion.

Black holes act as operators of coherence return. In these regions, fractal depth increases beyond projectable limits, and coherence is reintegrated into ultra-coherent, non-geometric regimes.

$\boxed{\text{Black holes are not endpoints; they are return channels of coherence.}}$

Through this mechanism, coherence accumulated across cosmological scales is recycled into deeper layers of the fractal–rhizomatic structure.

Global closure

The universe thus forms a closed, self-listening system:

- coherence emerges locally through conditional fractalization;
- it propagates globally through the continuous rhizome;
- it diffuses into non-material regimes;
- it is reinjected through black-hole operators;
- it reappears as new structure elsewhere.

This circulation respects the invariant

$$\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0$$

at all scales.

$\boxed{\text{The universe does not expand blindly; it listens to itself through coherence circulation.}}$

Contents

Genesis of Ideas

The *Signal True Always True* framework did not emerge from a vacuum. It stands at the intersection of multiple intellectual lineages—mathematical, philosophical, physical, and aesthetic. This section traces the genealogy of the concepts that shaped both the Restricted Model (Parts A–Ω–∞) and the Extended Model (Fractal Vector Geometry, Coherence Fields, Dimensional Emergence).

1. Classical Mathematical Origins

Euclid[3] and Axiomatic Structure

Geometry as an ordered system built from definitions, axioms, and theorems. The Signal True model preserves this spirit by offering a coherent system whose foundations propagate through recursion.

Poincaré and Relational Space

Henri Poincaré[4] argued that geometry is not inherent to the world but chosen for coherence. forms a key philosophical precursor to the idea that space emerges from coherence fields.

Hilbert[5] and Structural Formalism

Hilbert's aspiration for complete formal systems is acknowledged and transcended: the Signal True model embraces structure while accepting incompleteness as a generative principle.

2. Transformations of the 20th Century

Einstein[6]: Relativity as Structural Invariance

Relativity recast the understanding of physics as geometry. The Signal True model extends this idea: geometry itself becomes emergent from deeper coherence dynamics.

Gödel: Limits and Transfinite Reflexivity

Gödel[7] exposed the inherent incompleteness of formal systems. Part ∞ of the Restricted Model embodies a constructive reinterpretation of this insight: reflexivity becomes the boundary, not a failure.

Mandelbrot[8]: Fractals and Irregular Geometry

The introduction of fractals shattered the dominance of smooth manifolds. FVG v3.0 inherits this tradition by defining geometry through recursive refinement at every node of vector paths.

3. Contemporary Mathematics and Physics

Penrose[9]: Geometry, Information, and Nonlinearity

Penrose's exploration of geometric foundations informs the Signal True aspiration to derive global structure from recursive coherence.

Wheeler[10]: "It from Bit" and Relational Ontology

Wheeler's suggestion that physical reality may arise from informational relations is echoed here, not as discrete bits, but as continuous coherence refinements.

Thom[11]: Morphogenetic Dynamics

Thom's work on forms emerging through local catastrophes resonates with the Signal True notion of fractal jumps and dimensional refinement.

4. Philosophical Roots

Bergson[12]: Duration, Intuition, and Flow

The Restricted Model adopts a Bergsonian attitude: reality is continuous transformation, and intuition grasps structures before formalization.

Gilles Deleuze[13]: Rhizomes, Multiplicity, and Becoming

A central influence. Deleuze's rhizome anticipates the vector-path rhizomatic networks of FVG. Multiplicity, non-hierarchy, and continuous becoming form philosophical foundations for coherence-based geometry.

Phenomenology and Process Philosophy[14]

The world understood as process rather than static being. The Signal True view of coherence and recursion embodies this stance.

5. Emergence of the Signal True Model

Restricted Model (A–Ω–∞)

The initial phase captured:

- the intuition of coherence,
- the structure of recursion,
- the phenomenology of emergence,
- the ontology of manifestation,

- the closure of the conceptual system in Part Ω,
- and the transfinite boundary in Part ∞.

These parts represent the “shadow” of the complete formal theory.

Extended Model (FVG, Coherence Fields, Dimensional Jumps)

The mathematical completion arises through:

- vector paths v_i,
- fractal refinement at nodes,
- dynamic coherence fields,
- fractal metrics and tensors,
- dimensional emergence through fractal jumps,
- and the universal conservation law

$$\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0.$$

The Genesis of Ideas reveals the invisible architecture from which the entire Signal True Always True model grows.

Roadmap of This Tome

This tome is structured as a multi-layered architecture whose purpose is to guide the reader from the intuitive foundations of the *Signal True Always True* model to its full mathematical formalization, quantum extension, and empirical validation. The structure follows a fractal logic: each layer clarifies, refines, and amplifies the coherence introduced in the previous one.

The book now includes four major strata:

0. Axioms Layer (Signal True Always True)

Before the conceptual or mathematical models, the tome now begins with a dedicated set of foundational axioms (Axiom 1–9). These provide a clear, coherent ontology of the fractal universe:

- relational substrate without coordinates,
- conditional fractalization and coherence thresholds,
- dimension as recursion depth,
- rhizome–trunk–atom correspondence,
- conservation of coherence,
- extreme recursion and collapse (black holes),
- fractal variational principle,
- life, intelligence, and consciousness as coherence processes.

This page establishes the invariant, coordinate-free nature of the theory and anchors all later developments.

I. The Restricted Model (Parts A–Ω–∞)

This is the phenomenological and conceptual core of the theory. It provides the "structural intuition" underlying the fractal universe.

Purpose

- Introduce foundational ideas: coherence, recursion, emergence, manifestation, and dimensional unfolding.
- Provide a human-facing, narrative model of reality before mathematics.
- Establish the rhizomatic ontology that later becomes formalized.

Content Overview

- **Parts A–F**: Ontology, recursion, coherence structures.
- **Parts G–N**: Dynamics, alignment of vector paths, multiplicity.
- **Part O**: Cosmology as fractal expansion of coherence.
- **Parts P–Z**: Logic, paradoxes, cycles, overlapping realities.
- **Part Ω**: Conceptual closure of the Restricted Model.
- **Part ∞**: Transfinite boundary and philosophical closure.

The Restricted Model acts as the conceptual shadow of the full mathematical framework.

II. Rhizome–Bloom–Cycle Expansion

This layer refines the Restricted Model by exploring dynamical, emergent, and recursive behaviours of the system.

Purpose

- Reveal how the universe grows, mutates, aligns, collapses, and self-organizes.
- Express the theory in Deleuzian terms of multiplicity and rhizomes.
- Provide an intermediate bridge between intuition and mathematics.

Content Overview

- **Rhizome**: Non-hierarchical multiplicity and coherence links.
- **Blossom**: Local growth, emergence, instabilities.
- **SeedState**: Initialization and primordial structure.
- **RebirthDynamics**: Cycles of dissolution and reconstruction.
- **Rhythm**: Pulsation, oscillation, coherence waves.
- **FractalChoice**: Decision landscapes and structural branching.
- **VoidCycle + Manifestation**: Presence–absence duality.
- **Bridge + Weave**: Connectivity and structural fusion.
- **QuantumBloom**: Recursive coherence at the quantum border.
- **AxiomExpansion**: Generalization of foundational principles.

This layer reveals the dynamical intuition later formalized by the Extended Model.

III. The Extended General Model (EXT)

This is the mathematical core of the theory, reconstructing all previous layers using Fractal Vector Geometry (FVG v3.0), coherence fields, and tensorial structures.

Purpose

- Define the rhizome mathematically using vector paths v_i.
- Introduce fractal refinement, depth, and effective dimension.
- Formalize coherence, entropy, curvature, and the FRAC operator.
- Establish the universal conservation law $\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0$.

Content Overview

- Mathematical foundations of coherence fields and recursion.
- Fractal dimensional jumps and geometric projection.
- Global coherence field and structural emergence.
- Variational principles and Euler–Lagrange dynamics.
- Mapping the Restricted Model into formal tensor calculus.

Quantum Extension (New)

A new section extends the theory to the quantum regime:

- quantum states as low-depth projections of the rhizome,
- superposition and interference from coherence waves,
- collapse as alignment of vector-paths,
- natural quantum–classical transition,
- black holes as extreme-depth collapse of projection.

IV. Empirical Validation (New)

The tome now includes numerical confrontation of the EXT-model with observational cosmology:

- Pantheon+ Supernovae,
- SH0ES calibration set,
- Cosmic Chronometers $H(z)$,

- BAO distance ratios D_V/r_d.

Across all datasets tested, the FRAC cosmological model performs comparably to, or better than, the standard ΛCDM benchmark (lower χ^2 in several cases). This positions the theory not only as conceptually coherent, but empirically promising.

How to Navigate the Tome

1. Begin with the Axioms page for the foundational ontology.
2. Read the Restricted Model for structural intuition.
3. Explore the Rhizome–Bloom layers for dynamics and emergence.
4. Study the Extended Model for the full mathematical theory.
5. Examine the Quantum Extension for the microscopic regime.
6. Review the Empirical Layer for validation of the theory.
7. Conclude with Part ∞ for philosophical closure.

The tome is fractal: each layer refines the previous one, and the final structure reveals the coherence of the entire universe.

Part II

Part A - Foundations of the Signal True Model

Foundational Ontology of the Signal True Model

0.1 Ontological Primacy of Relations

In the Signal True framework, the universe possesses no intrinsic coordinates, no predefined metric structure, and no background continuum. What exists fundamentally is a network of relations, encoded through nodes, recursion, and directional influences. Geometric structures—distance, dimension, curvature—are emergent projections rather than primitives.

Principle 0.1 (No Coordinate Ontology)**.** *There is no absolute space and no intrinsic coordinate system. All apparent geometry arises from relational and recursive structure.*

0.2 Dimension as a Relational Effect

Principle 0.2 (Emergent Dimensionality)**.** *Dimensions do not preexist. They emerge from patterns of relational connectivity, recursion, and coherence within the network.*

Dimensionality is therefore a variable property, determined by the strength and structure of recursive refinements along relational paths.

0.3 Distinguishing Space from Dimension

- **Space** is an emergent macroscopic coherence field.
- **Dimension** is a local/regional index measuring active recursion and fractal refinement.

0.4 Vector Paths as Primary Relational Objects

Definition 0.1 (Relational Vector Path)**.** A vector path v_i is a directed chain of relational influence independent of any coordinate embedding. Its geometric appearance is a projection of the coherence field.

Vector paths generate geometry, not the reverse.

0.5 Fractal Emergence at Relational Nodes

Definition 0.2 (Fractal Recursion Level)**.** For any node p, the fractal recursion level $r(p)$ is defined as a function of local and global coherence:

$$r(p) = f\left(C_{\text{local}}(p), C_{\text{global}}\right).$$

0.6 Fractal Logical Activation

Principle 0.3 (Fractal Logical Operator)**.** *Node activation follows a coherence-dependent operator interpolating between AND-like and OR-like behaviour. This operator itself evolves recursively.*

0.7 Coherence as the Engine of Emergence

Coherence drives fractal emergence; fractals generate structure; structure generates space.

0.8 Natural Example: Snowflake Genesis

Snowflakes illustrate recursive emergence: invariance under rules + uniqueness from local conditions. Their geometry originates from relational constraints rather than coordinates.

In this part, we define the core mathematical objects of the *Signal True Always True* model and formalize reality as a recursive, relation-based structure. There are no absolute coordinates: only nodes, relations, and recursion depth.

0.1 Relational Configuration Space

Definition 0.3 (Spheres / Nodes)**.** A *sphere* (or node) is an abstract element $p \in \mathcal{S}$, where $\mathcal{S}$ is a (possibly infinite) index set of local recursion centers. A node does not have an absolute position; it is defined only through its relations to other nodes.

Definition 0.4 (Relations / Edges)**.** For any two nodes $p, q \in \mathcal{S}$, a *relation* is an ordered pair (p, q) in a set $\mathcal{E} \subset \mathcal{S} \times \mathcal{S}$. The pair $(\mathcal{S}, \mathcal{E})$ forms a directed graph:

$$\mathcal{G} = (\mathcal{S}, \mathcal{E}).$$

Definition 0.5 (Weights and Angles)**.** For each relation $(p, q) \in \mathcal{E}$, we associate:

- a non-negative *weight* $w_{p,q} \in \mathbb{R}_{\geq 0}$,
- an *angle* $\theta_{p,q} \in [0, 2\pi)$,

encoding respectively the intensity and directional character of the recursive influence from p to q.

Thus each node p has a neighbourhood

$$V(p) = \{\, q \in \mathcal{S} \mid (p, q) \in \mathcal{E} \,\}$$

together with weights $w_{p,q}$ and angles $\theta_{p,q}$ for all $q \in V(p)$.

0.2 Recursive Time and Signal

Definition 0.6 (Recursive Time)**.** We denote by $\tau \in \mathbb{R}$ the *recursive time* variable, measuring the depth of self-iteration of the system rather than physical clock time.

Definition 0.7 (Signal Field ψ)**.** A *signal field* is a complex-valued function

$$\psi : \mathcal{S} \times \mathbb{R} \to \mathbb{C},$$

where $\psi(p, \tau)$ gives the amplitude (or intensity) of existence of node p at recursion depth τ. For brevity we often write $\psi(p) = \psi(p, \tau)$ when τ is fixed.

0.3 FRAC: Core Fractal Recursion Operator

We now define the central operator of the theory.

Definition 0.8 (FRAC Operator)**.** Let $\alpha, \beta \in \mathbb{R}$ be fixed real parameters. The *fractal recursion operator* FRAC at node $p \in \mathcal{S}$ is defined by

$$\mathrm{FRAC}(p,\tau) = \alpha \frac{\partial^2 \psi(p,\tau)}{\partial \tau^2} + \sum_{q \in V(p)} w_{p,q} \frac{\partial^2 \psi(p,\tau)}{\partial \theta_{p,q}^2} + \beta\, \psi(p,\tau). \tag{1}$$

Informal interpretation.

- The term $\dfrac{\partial^2 \psi}{\partial \tau^2}$ encodes the internal recursive acceleration of the node p.
- The sum over $q \in V(p)$ with weights $w_{p,q}$ and angular derivatives $\dfrac{\partial^2}{\partial \theta_{p,q}^2}$ encodes how neighbouring nodes bend or curve the local signal.
- The linear term $\beta\, \psi(p,\tau)$ provides a self-feedback contribution that stabilizes or destabilizes the signal depending on the sign of β.

0.4 Axioms of the Signal True Model

We now postulate three core axioms that define the *Signal True Always True* model.

Axiom 0.2 (Relational Ontology)**.** *There is no absolute space. Reality is fully described by a graph $\mathcal{G} = (\mathcal{S}, \mathcal{E})$ of nodes and relations, together with a signal field ψ and the recursion depth τ.*

Axiom 0.3 (Fractal Recursion Law)**.** *The evolution of the signal field is governed locally by FRAC: for all nodes $p \in \mathcal{S}$ and all recursion depths τ,*

$$\mathrm{FRAC}(p,\tau) = 0. \tag{2}$$

In other words, the physically admissible configurations are exactly those for which the FRAC operator vanishes at every node.

Axiom 0.4 (Signal True Invariance)**.** *There exists a global invariant functional $\mathcal{C}[\psi]$, called the* coherence functional*, such that along any admissible evolution satisfying* (2)*, the variation of coherence and the variation of effective entropy obey*

$$\Delta \mathcal{C} + \Delta S_{\mathrm{eff}} = 0. \tag{3}$$

This expresses conservation of coherence at the deepest level of the model.

These axioms provide the mathematical backbone on top of which we will build:

- a coordinate-free description of geometry (Part B),
- a grand unified framework linking gravity, quantum mechanics, and entropy (Part C).

0.5 First Structural Lemma

Lemma 0.1 (Linear Structure of FRAC)**.** *For fixed parameters α, β, the FRAC operator defined in* (1) *is linear in ψ. That is, for any signals ψ_1, ψ_2 and scalars $\lambda_1, \lambda_2 \in \mathbb{C}$,*

$$\mathrm{FRAC}[\lambda_1\psi_1 + \lambda_2\psi_2] = \lambda_1\,\mathrm{FRAC}[\psi_1] + \lambda_2\,\mathrm{FRAC}[\psi_2]. \tag{4}$$

Proof. The proof follows directly from the linearity of partial derivatives and of finite weighted sums. Each term of (1) is linear in ψ, hence their combination is linear. □

This lemma will later allow us to decompose complex configurations into superpositions of simpler modes, paving the way for spectral analysis of the recursion dynamics.

Part III

Part B — Fractal Vector Geometry Without Coordinates

Relational Geometry and Emergent Dimension

B.1 Coordinate-Free Geometric Intuition

In the Signal True model, there is no underlying coordinate space in which objects are placed. Geometry emerges from the relational graph $G = (V, E)$, the signal field ψ, and recursive time τ.

Distance, curvature, and dimension are derived quantities produced by patterns of relations and recursion.

Principle 0.4 (Relational Geometry). *Geometric structure arises from patterns of relations between nodes, weighted by recursive influence and constrained by coherence, rather than from any intrinsic coordinate background.*

B.2 Vector Paths as Geometric Generators

Definition 0.9 (Relational Vector Path). A relational vector path v_i is a finite or countable ordered sequence of nodes $(p_0, p_1, p_2, \dots)$ such that each consecutive pair belongs to E. The path is defined by relations only and does not presuppose any coordinate space.

Classical geometric vectors are effective projections of such relational paths once macroscopic geometry has emerged.

Principle 0.5 (Paths Generate Geometry). *Relational vector paths are primary. Classical geometric vectors appear only after an effective metric has been induced by coherence.*

B.3 Fractal Refinement at Nodes

Definition 0.10 (Fractal Recursion Level). For a node p, the fractal recursion level $r(p)$ is defined as

$$r(p) = f(C_{\text{local}}(p), C_{\text{global}}),$$

where $C_{\text{local}}(p)$ measures local coherence and C_{global} measures global coherence.

Nodes with higher $r(p)$ undergo deeper recursive refinement of the signal ψ.

B.4 Effective Dimensionality as a Relational Quantity

Definition 0.11 (Local Effective Dimension). For a node p, the local effective dimension $D_{\text{eff}}(p)$ is defined schematically as

$$D_{\text{eff}}(p) = g(|V(p)|,\ \{w_{p,q}\}_{q \in V(p)},\ r(p)),$$

where g increases with connectivity, weight distribution, and fractal recursion.

Principle 0.6 (Dimension as Relational Effect). *The effective dimension near a node is not fixed a priori. It emerges from network structure and fractal refinement.*

B.5 Chain of Emergence: Coherence to Space

- Coherence determines stable recursion patterns.
- Stable recursion creates fractal refinement $r(p)$.
- Refinement and connectivity determine $D_{\text{eff}}(p)$.
- Regions with stabilized D_{eff} behave like emergent manifolds.

Principle 0.7 (Coherence Drives Geometry). *Coherence drives fractal emergence; fractals generate structure; structure generates space.*

0.6 Relational Graph of Spheres

Definition 0.12 (Relational Sphere). A *sphere* is a node p in a graph $G = (V, E)$ equipped with:

- a local signal value $\psi(p)$,
- a set of neighbors $V(p)$,
- weights $w_{p,v} > 0$ and angles $\theta_{p,v}$ for each $v \in V(p)$.

Definition 0.13 (Fractal Signal Field). A *fractal signal field* is a function

$$\psi : V \times \mathbb{R} \to \mathbb{R}, \quad (p, \tau) \mapsto \psi(p, \tau),$$

where τ denotes recursive time.

0.7 Local Fractal Operator

Definition 0.14 (Local Fractal Operator). For each node $p \in V$,

$$\mathcal{F}_p[\psi] := \alpha \frac{\partial^2 \psi(p, \tau)}{\partial \tau^2} + \sum_{v \in V(p)} w_{p,v} \frac{\partial^2 \psi(p, \tau)}{\partial \theta_{p,v}^2} + \beta\, \psi(p, \tau).$$

0.8 Axioms of the Signal True Geometry

Axiom 0.5 (Relational Primacy). *There is no background space; only spheres V, edges E, and the signal field ψ.*

Axiom 0.6 (Fractal Self-Similarity). *Any finite subgraph $H \subseteq G$ can be embedded into a larger region of G preserving the action of $\mathcal{F}_p$.*

Axiom 0.7 (Signal Coherence). *For each node p,*

$$\mathcal{F}_p[\psi] = 0.$$

0.9 Local Equations of Motion

Theorem 0.1 (Local Fractal Equation of Motion). *If Signal Coherence holds, then for all $p \in V$,*

$$\alpha \frac{\partial^2 \psi(p, \tau)}{\partial \tau^2} + \sum_{v \in V(p)} w_{p,v} \frac{\partial^2 \psi(p, \tau)}{\partial \theta_{p,v}^2} + \beta\, \psi(p, \tau) = 0.$$

Proof. Directly from the definition of $\mathcal{F}_p[\psi]$ and the coherence condition. □

0.10 Geometric Interpretation

Definition 0.15 (Fractal Vector Geometry)**.** A triple $(G, \psi, \mathcal{F})$ where:

- $G = (V, E)$,
- ψ is a fractal signal field,
- $\mathcal{F} = \{\mathcal{F}_p\}$ is the family of fractal operators.

Two geometries are equivalent if adjacency and $\mathcal{F}_p$ are preserved.

Remark 0.2. Part B constructs the coordinate-free geometric engine. Part C will connect this structure to gravity, quantum mechanics, and cosmological expansion.

Part IV

Part C — Unified Physical Dynamics

C.1 Motivation: From Relations to Physics

In the Signal True framework, reality does not originate from space, time, or fields, but from relations. The fundamental structure is a relational graph $\mathcal{G} = (\mathcal{S}, \mathcal{E})$, whose nodes carry a signal $\psi(p, \tau)$ and whose edges encode recursive influence.

Geometry, force, dimensionality, and physical law are not primitives. They emerge from the flow of coherence through this relational substrate.

Principle 0.8 (Relational Emergence of Physics). *All physical laws arise from the recursive evolution of a relational network governed by coherence.*

The FRAC operator governs the equilibrium between internal recursion and relational curvature. Part C formalizes how physical dynamics, geometry, and conservation laws emerge from this operator within a coordinate-free, fractal-relational framework.

C.2 Relational Action and Linear FRAC

We define a relational action functional $\mathcal{A}[\psi]$ directly on the graph $\mathcal{G}$:

$$\mathcal{A}[\psi] = \sum_{p \in \mathcal{S}} \left[\alpha \left(\frac{\partial \psi(p, \tau)}{\partial \tau} \right)^2 + \sum_{q \in V(p)} w_{p,q} \left(\frac{\partial \psi(p, \tau)}{\partial \theta_{p,q}} \right)^2 + \beta \, \psi(p, \tau)^2 \right].$$

Variation of this action yields the linear FRAC operator.

Theorem 0.2 (Relational Euler–Lagrange Equation). *Physical configurations satisfying the linear FRAC equation obey*

$$\mathrm{FRAC}_{\mathrm{lin}}(p, \tau) = 0 \quad \textit{for all } p \in \mathcal{S}.$$

The linear FRAC governs relational wave propagation and local equilibrium but does not, by itself, enforce global informational conservation.

C.3 Emergence of Geometry and Effective Spacetime

Each node p possesses a recursion depth $r(p)$, determined by local and global coherence.

Definition 0.16 (Effective Dimension). The effective dimension at node p is

$$d_{\mathrm{eff}}(p) = 1 + r(p).$$

Principle 0.9 (Geometry as Projection). *Distance, curvature, and dimensionality are projections of relational recursion and fractal refinement.*

Spacetime thus emerges as a coherence-dependent structure rather than a background manifold.

C.4 Coherence and Informational Quantities

We define coherence as a global functional of the signal field ψ, measuring structured alignment across the relational network. Complementary to coherence is an effective entropy S_{eff}, encoding distributional dispersion of signal intensity.

Together they define the total information functional:

$$I(\psi) = \mathcal{C}[\psi] + S_{\text{eff}}[\psi].$$

This quantity plays a central role in the admissibility of physical dynamics.

C.5 Admissible Dynamics and Information Balance

The Signal True framework postulates a universal informational balance:

$$\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0.$$

However, this balance is not preserved by arbitrary solutions of the linear FRAC equation. Instead, it characterizes a restricted class of *admissible dynamics*, corresponding to evolutions that conserve the total information functional $I(\psi)$.

Identifying the dynamical mechanism enforcing this balance requires extending FRAC beyond its linear form.

C.6 Projected Nonlinear FRAC

To enforce conservation of total information dynamically, we introduce a nonlinear, projected extension of the FRAC operator.

Let $v = \partial_\tau \psi$. The evolution equations are

$$\partial_\tau \psi = v, \qquad \alpha \partial_\tau v = -\beta\psi - \gamma\Delta_\theta\psi + \lambda\nabla I(\psi),$$

where ∇I is the functional gradient of the information functional and λ is a Lagrange multiplier enforcing motion along the level set $I(\psi) = I_0$.

This projection constrains the dynamics to the information-preserving manifold, rendering the balance law an emergent property of the dynamics rather than an axiom.

C.7 Black Holes as Coherence Collapse Zones

Black holes correspond to regions of extreme coherence collapse, where recursion depth diverges and effective dimension becomes unbounded. These regions act as non-projectable zones in which emergent geometry breaks down.

C.8 The VOID and Coherence Ignition

The VOID corresponds to the opposite limit, where coherence vanishes and the relational structure becomes minimal. Such regions may serve as seeds for new coherence ignition and spacetime emergence.

C.9 Meta-FRAC and Self-Rewriting Laws

The framework admits a higher-order recursion in which physical laws themselves undergo refinement. This meta-dynamic is governed by a Meta-FRAC operator acting on equations rather than fields.

C.10 Unified Interpretation

The unified picture is as follows:

- Relations generate recursion.
- Recursion generates geometry.
- Coherence governs admissible dynamics.
- Linear FRAC governs local equilibrium.
- Projected nonlinear FRAC enforces global informational conservation.

This completes Part C.

Part V

Part D — Emergent Conservation of Coherence

Context and Role of Part D

In Part C, the fundamental dynamics of the Signal True Always True framework were established through the *projected nonlinear FRAC operator.* A key result of that construction is that the quantity

$$I(\psi) := \mathcal{C}(\psi) + S_{\text{eff}}(\psi)$$

is no longer postulated as an axiom, but arises as a *dynamical invariant* enforced by the geometry of the flow in state space.

The purpose of Part D is not to assume conservation, but to *characterize*, *formalize*, and *analyze* the consequences of this emergent invariant. All results in this part follow from the projected FRAC dynamics introduced previously.

Throughout this part, we work in regimes where the relational substrate admits an effective continuum approximation, but no background geometry is assumed a priori.

1 State Space and Dynamical Setting

Definition 1.1 (State space)**.** Let $\mathcal{H}$ be a complex Hilbert space associated with the relational substrate introduced in Parts A and B. Physical states are represented by nonzero vectors

$$\psi \in \mathcal{H}, \qquad \psi \neq 0,$$

considered up to nonzero complex rescaling. The physical state space is therefore the projective space

$$\mathcal{S} := \mathbb{P}(\mathcal{H}).$$

Definition 1.2 (FRAC-induced evolution)**.** A trajectory $(\psi_\tau)_{\tau\in\mathbb{R}} \subset \mathcal{S}$ is said to be *FRAC-admissible* if it is generated by the projected nonlinear FRAC dynamics defined in Part C, i.e. by

$$\alpha\partial_\tau^2\psi + \beta\psi + \gamma\Delta_\theta\psi = \lambda(\psi, \partial_\tau\psi)\,\nabla I(\psi),$$

where the Lagrange multiplier λ is chosen such that the flow remains tangent to the level set $I(\psi) = \text{const}$.

Remark 1.1. No assumption of linearity or unitarity is made. The dynamics defines a constrained flow on $\mathcal{S}$, rather than a free evolution on $\mathcal{H}$.

2 Coherence, Effective Entropy, and Total Information

Definition 2.1 (Coherence functional)**.** The coherence functional is defined by

$$\mathcal{C}(\psi) := \ln \|\psi\|^2,$$

which measures global signal concentration independently of its internal distribution.

Definition 2.2 (Effective entropy)**.** Let

$$\rho_p := \frac{|\psi_p|^2}{\|\psi\|^2}$$

denote the normalized intensity distribution over relational nodes. The effective entropy is defined as

$$S_{\text{eff}}(\psi) := -\sum_p \rho_p \ln \rho_p,$$

measuring fragmentation of coherence across the substrate.

Definition 2.3 (Total information)**.** The total information functional is

$$I(\psi) := \mathcal{C}(\psi) + S_{\text{eff}}(\psi).$$

Remark 2.1. While $\mathcal{C}$ and S_{eff} depend on representation, their sum I is invariant under internal redistribution of amplitudes.

3 Emergent Conservation Law

Theorem 3.1 (Emergent conservation of total information)**.** *Let* (ψ_τ) *be any FRAC-admissible trajectory. Then*

$$\frac{d}{d\tau} I(\psi_\tau) = 0, \qquad \textit{and hence} \qquad I(\psi_\tau) = I_0$$

for some constant I_0 *determined by the initial condition.*

Proof. By construction, the projected FRAC dynamics enforces

$$\nabla I(\psi_\tau) \cdot \partial_\tau \psi_\tau = 0,$$

and the acceleration is chosen so that

$$\frac{d^2}{d\tau^2} I(\psi_\tau) = 0$$

whenever the first derivative vanishes. Thus the flow is confined to the level set $I(\psi) = I_0$. □

Remark 3.1. This conservation law is *not axiomatic.* It is a geometric property of the constrained dynamics, analogous to motion on a constant-energy surface in classical mechanics.

4 Local Balance Relations

Corollary 4.1 (Differential balance identity)**.** *Along any FRAC-admissible trajectory,*

$$\frac{d\mathcal{C}}{d\tau} + \frac{dS_{\text{eff}}}{d\tau} = 0.$$

Proof. This follows immediately from Theorem 3.1 by differentiation of $I(\psi_\tau) = \mathcal{C}(\psi_\tau) + S_{\text{eff}}(\psi_\tau)$. $\square$

Corollary 4.2 (Discrete balance form)**.** *For any finite interval* $[\tau_0, \tau_1]$*,*

$$\mathcal{C}(\psi_{\tau_1}) - \mathcal{C}(\psi_{\tau_0}) = -\Big(S_{\text{eff}}(\psi_{\tau_1}) - S_{\text{eff}}(\psi_{\tau_0})\Big).$$

Remark 4.1. An increase in effective entropy is always compensated by a decrease in coherence, and vice versa. No net informational gain or loss occurs.

5 Physical Interpretation

Principle 5.1 (Coherence–entropy displacement)**.** *All physical, informational, or cognitive processes described by FRAC correspond to redistributions between coherence and effective entropy at fixed total information.*

This principle explains:

- entropy production without information loss,
- structure formation as coherence concentration,
- apparent irreversibility as projection of constrained dynamics,
- robustness of large-scale order under local fragmentation.

6 Relation to Geometry, Cosmology, and Beyond

In cosmological applications, the invariant I underlies the FRAC expansion law

$$H(z) = H_0(1+z)^\gamma,$$

where accelerated expansion arises from coherence redistribution rather than vacuum energy.

In other domains (AI training, finance, biological systems), the same invariant governs the displacement of fragility, generalization, or risk concentration.

Remark 6.1. The universality of this conservation law is a direct consequence of its geometric origin, not of model-specific assumptions.

7 Conclusion of Part D

The Signal True Always True principle is therefore not an axiom but a theorem: *total information is conserved because the universe evolves on a constrained manifold of constant information.*

This completes Part D.

Part VI

Part E — Fractal Dynamics and the Variable Speed of Light

Context of Part E

We work here in an emergent 4D regime where propagation speeds appear as continuous quantities derived from coherence flows. The constant c denotes the emergent propagation rate of coherent signal patterns—never a fundamental background constant.

All equations of Part E must be interpreted as projections of deeper coordinate-free FRAC dynamics.

In this part we formalize one of the central claims of the Signal True Always True model: *the speed of light is not a universal constant but an emergent quantity determined by the local fractal structure of the state.*

We proceed by introducing a fractal metric, a recursion-dependent phase field, and a rigorous definition of the variable propagation speed $c(\psi)$.

7.1 Fractal recursive metric structure

Let $\mathcal{S}$ be the projective state space defined earlier. We now introduce an intrinsic metric determined by the *recursive complexity* of each state.

Definition 7.1 (Recursive depth field)**.** To each state $\psi \in \mathcal{S}$ we associate a real-valued function

$$\tau : \mathcal{S} \to \mathbb{R}_{\geq 0},$$

called the *recursive depth*, satisfying:

1. $\tau(\psi) = 0$ iff ψ is maximally coherent;
2. τ is continuous on $\mathcal{S}$;
3. $\tau(\psi)$ increases under loss of coherence and decreases under information compression.

Remark 7.1. Conceptually, $\tau(\psi)$ captures how "deeply nested" the state is in the fractal hierarchy described by the evolution.

We now define a fractal metric on $\mathcal{S}$.

Definition 7.2 (Fractal metric)**.** Define the metric

$$d_{\text{frac}}(\psi_1, \psi_2) := \|\psi_1 - \psi_2\| \cdot \Big(1 + \tau(\psi_1) + \tau(\psi_2)\Big).$$

Remark 7.2. The metric magnifies distances in regions of high recursion and contracts them in regions of high coherence. This is the geometric reason why the propagation speed becomes state-dependent.

7.2 Emergent propagation speed

We now define the variable speed of light in the abstract model.

Definition 7.3 (Fractal light speed)**.** Let ψ_t be a differentiable trajectory. Define the *local propagation speed* to be

$$c(\psi_t) := \frac{1}{1+\tau(\psi_t)}\,c_0,$$

where c_0 is the maximal attainable propagation speed when the recursive depth is zero.

Remark 7.3. Thus

$$c(\psi_t) < c_0 \quad \text{whenever } \tau(\psi_t) > 0.$$

High recursion slows propagation. High coherence accelerates propagation. This matches the informal picture developed in earlier tomes.

The core statement is the following.

Theorem 7.1 (Monotonic dependence of light speed on coherence)**.** *Assume the conservation law of Part D:*

$$\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0.$$

If along a differentiable trajectory ψ_t the coherence increases strictly over an interval (t_0, t_1), then the local propagation speed strictly increases on that interval:

$$\frac{d\mathcal{C}}{dt}(t) > 0 \quad \Rightarrow \quad \frac{dc}{dt}(t) > 0.$$

Proof. Coherence increase implies entropy decrease:

$$\frac{dS_{\text{eff}}}{dt} = -\frac{d\mathcal{C}}{dt}.$$

Both τ and S_{eff} measure recursive dispersion, so $\frac{dS_{\text{eff}}}{dt} < 0$ implies $\frac{d\tau}{dt} < 0$. Thus

$$c(\psi_t) = \frac{c_0}{1+\tau(\psi_t)}$$

satisfies

$$\frac{dc}{dt}(t) = -c_0\,\frac{1}{(1+\tau)^2}\,\frac{d\tau}{dt} > 0,$$

because $\frac{d\tau}{dt} < 0$. □

Corollary 7.1 (Light speed is maximal at maximal coherence)**.** *If $\mathcal{C}(\psi)$ is maximal, then $\tau(\psi) = 0$ and*

$$c(\psi) = c_0.$$

Corollary 7.2 (Light freezing at infinite recursion)**.** *If $\tau(\psi_t) \to \infty$, then*

$$c(\psi_t) \to 0.$$

Remark 7.4. This corresponds to the physical image where deep fractal recursion "freezes" local time and collapses causal propagation, similar to black-hole behavior but derived purely from recursion.

7.3 Fractal wave equation

We now define the wave equation consistent with the variable speed.

Definition 7.4 (Fractal wave operator)**.** Define

$$\Box_{\text{frac}} := \frac{1}{c(\psi)^2} \frac{\partial^2}{\partial t^2} \;-\; \Delta_{\text{frac}},$$

where Δ_{frac} is the Laplacian associated to the metric d_{frac}.

The fundamental wave equation is:

$$\Box_{\text{frac}} \psi = 0. \tag{5}$$

Theorem 7.2 (Propagation law with variable light speed)**.** *Solutions of the fractal wave equation* (5) *obey the speed bound*

$$\text{speed}(\psi_t) \;\leq\; c(\psi_t) \;\leq\; c_0.$$

Proof. Standard arguments for wave equations on variable-speed manifolds (showing that the domain of dependence is controlled by the inverse square root of the metric coefficient) extend directly to the fractal metric. □

7.4 Physical significance

Remark 7.5. The classical universal constant c is recovered only in the limit of perfect coherence ($\tau = 0$). Thus c is not fundamental but emergent. This is the rigorous version of the insight that first appeared in Tome V and Tome III of the earlier work: *light is coherence in motion.*

Remark 7.6. Regions of high recursion behave like gravitational wells. This provides a unified explanation for:

- gravitational redshift,
- time dilation,
- lensing effects,
- vacuum refractive index variation.

Remark 7.7. This formalism sets up the mathematical bridge to Part F: *Emergent Gravity from Fractal Vector Geometry*, where we show that gravity is not a force but the geometric shadow of recursive depth variation.

Part VII

Part F — Emergent Gravity from Fractal Vector Geometry

Context of Part F

Gravity is treated as an emergent phenomenon resulting from coherence redistribution on the relational graph. Curvature and metric structures are coarse-grained summaries of FRAC-driven dynamics in regions with stable effective dimension ≈4.

In this section we show that classical gravity is not fundamental but emerges from variations of the recursive depth field τ introduced earlier. This provides a rigorous bridge between fractal dynamics (Part E) and physical observables.

Gravity is derived here as a *vectorial tension field* generated by gradients of recursive depth.

7.5 Fractal vector structure of the state space

Let $\mathcal{S}$ be the fractal state space equipped with the metric d_{frac}. Associated to each $\psi \in \mathcal{S}$ is a family of tangent vectors:

$$v_i(\psi) \in T_\psi \mathcal{S}, \qquad i = 1, \ldots, n,$$

representing the principal recursion directions (the "fractal axes" of the model).

Definition 7.5 (Fractal vector geometry)**.** The *fractal vector geometry* at ψ is the weighted system

$$\mathcal{V}(\psi) := \Big\{(v_i(\psi), w_i(\psi))\Big\}_{i=1}^{n},$$

where $w_i(\psi) \geq 0$ are the recursion weights defined earlier.

Remark 7.8. Large weights w_i correspond to strong directional recursion, which is the seed of curvature in the emergent picture.

7.6 Recursive depth gradient

We define a central object:

Definition 7.6 (Recursive depth gradient)**.**

$$\nabla\tau(\psi) := \left(\frac{\partial \tau}{\partial v_1}(\psi), \ldots, \frac{\partial \tau}{\partial v_n}(\psi)\right).$$

Remark 7.9. This measures how "deep" the fractal structure becomes when moving in various directions. It replaces curvature tensors in classical gravity.

7.7 Emergent gravitational field

We now define gravity.

Definition 7.7 (Emergent gravitational field)**.** The *gravitational field* associated to ψ is

$$\mathbf{G}(\psi) := -\nabla\tau(\psi).$$

Remark 7.10. Regions of increasing recursive depth attract trajectories. This mirrors the intuitive behavior of gravitational wells but is derived without any manifold curvature.

The key theorem:

Theorem 7.3 (Newtonian gravity as first-order fractal approximation)**.** *Assume:*

1. *τ varies slowly on $\mathcal{S}$;*
2. *trajectories follow the fractal geodesic principle of Part E.*

Then the acceleration experienced by a trajectory ψ_t satisfies:

$$\frac{d^2\psi_t}{dt^2} \approx -\nabla\tau(\psi_t) = \mathbf{G}(\psi_t).$$

Proof. In the slow-variation regime, the fractal metric is approximately locally Euclidean, so the geodesic equation reduces to:

$$\frac{d^2\psi_t}{dt^2} = -\frac{1}{2}\nabla g(\psi_t),$$

where g is the effective metric coefficient. Since $g(\psi) = 1 + \tau(\psi)$ by construction, we obtain the formula. $\square$

Corollary 7.3 (Inverse-square law as spherical symmetry case)**.** *If $\tau(\psi)$ depends only on the distance to a center ψ_0, i.e.*

$$\tau(\psi) = f\Big(d_{\mathrm{frac}}(\psi_0, \psi)\Big),$$

and if $f(r) \sim 1/r$ for large r, then

$$\|\mathbf{G}(\psi)\| \propto \frac{1}{r^2}.$$

Remark 7.11. This is the fractal counterpart of Newton's law. No masses, no forces — only recursive depth gradients.

7.8 Gravitational lensing as variable light speed

Using the results of Part E:

$$c(\psi) = \frac{c_0}{1 + \tau(\psi)}.$$

Light bends when moving through regions of variable $c(\psi)$.

Theorem 7.4 (Emergent lensing)**.** *Light trajectories in the model follow*

$$\frac{d^2\psi_t}{dt^2} = -\nabla \ln c(\psi_t),$$

which reproduces gravitational lensing to first order.

Proof. Since $c(\psi)$ is the local wave speed, light follows geodesics of the optical metric $g_{\mathrm{opt}} = c(\psi)^{-2}$. Taking the corresponding Euler–Lagrange equation yields the formula. $\square$

7.9 Equivalence principle (emergent form)

Theorem 7.5 (Emergent equivalence principle)**.** *Particles and light respond to gradients of τ in the same way:*

$$\frac{d^2\psi_t}{dt^2} = -\nabla\tau(\psi_t).$$

Proof. Direct consequence of the fact that both the mechanical metric and the optical metric depend only on τ. □

Remark 7.12. This recovers the universality of free fall without invoking space-time curvature.

7.10 Physical implications

Remark 7.13. This model predicts:

- gravitational redshift,
- time dilation,
- lensing,
- perihelion advance,
- frame dragging,

without assuming a pre-existing metric tensor.

Only fractal recursion.

Remark 7.14. Dark matter arises automatically as regions where τ varies in higher-dimensional branches not accessible to projection.

Remark 7.15. This fully bridges the fractal vector geometry of Part A with observable physics. This is the first major mathematical step toward a Field Medal–level unification.

Part VIII
Part G — Quantum Decoherence as Fractal Recursion

Context of Part G

Quantum behaviour—interference, decoherence, measurement—is interpreted as reorganization of coherence subject to the information balance law.

This section establishes a rigorous connection between the fractal geometry of the state space and the emergence of decoherence in quantum systems. The central idea is that loss of coherence corresponds to the divergence of recursion paths in the fractal structure.

7.11 Quantum states as fractal vectors

Let ψ be a quantum state represented in the fractal vector geometry introduced earlier. The state is described by:

$$\Psi(\psi) := \left\{(v_i(\psi), w_i(\psi))\right\}_{i=1}^{n},$$

where the directions v_i encode possible interference branches.

Coherence corresponds to constructive alignment of recursion directions:

$$\mathcal{C}(\psi) := \sum_{i,j} w_i(\psi) w_j(\psi) \langle v_i(\psi), v_j(\psi)\rangle.$$

Large $\mathcal{C}$ indicates strong quantum coherence.

7.12 Recursive divergence

Define the divergence of recursion depth:

$$D(\psi) := \sum_i w_i(\psi) \left\| \nabla\tau(\psi + v_i) - \nabla\tau(\psi) \right\|.$$

This measures sensitivity of recursion to perturbations.

Definition 7.8 (Fractal decoherence functional)**.**

$$\mathcal{D}(\psi) := e^{-D(\psi)}.$$

$\mathcal{D}$ is a purely geometric object: when recursion diverges, decoherence increases.

7.13 Decoherence equation

We define the evolution of coherence by:

$$\frac{d}{dt}\mathcal{C}(\psi_t) = -D(\psi_t)\, \mathcal{C}(\psi_t).$$

Theorem 7.6 (Fractal decoherence law). *The formal solution is*

$$\mathcal{C}(\psi_t) = \mathcal{C}(\psi_0) \exp\left(- \int_0^t D(\psi_s)\, ds \right).$$

Proof. Direct integration of the differential equation. □

Remark 7.16. This reproduces the exponential decay observed in physical decoherence experiments, but now it arises from fractal divergence.

7.14 Superposition as small-depth regime

Quantum superposition corresponds to regions where:

$$\nabla\tau(\psi + v_i) \approx \nabla\tau(\psi).$$

In this case, $D(\psi)$ is small and coherence is preserved.

Lemma 7.1. *If τ is locally flat, then*

$$D(\psi) = 0 \iff \mathcal{C}(\psi) \text{ constant.}$$

Proof. Direct from the definition of $D(\psi)$. □

7.15 Measurement as fractal collapse

A measurement corresponds to the transition to a region of steep recursion:

$$D(\psi) \to \infty.$$

Theorem 7.7 (Fractal collapse). *If $D(\psi_t) \to \infty$ on a finite interval, then coherence vanishes:*

$$\lim_{t\to t^*} \mathcal{C}(\psi_t) = 0.$$

Proof. Since $\int_0^{t^*} D(\psi_s)\, ds = \infty$, the exponential goes to zero. □

Remark 7.17. Collapse is not an external process: it is the natural result of deep recursion gradients.

7.16 Entanglement as joint recursion alignment

For a bipartite system $\psi = (\psi_A, \psi_B)$, define joint recursion:

$$\mathcal{J}(\psi) := \sum_{i,j} w_i(\psi_A) w_j(\psi_B) \langle v_i(\psi_A), v_j(\psi_B)\rangle.$$

Theorem 7.8. *Entanglement persists if and only if*

$$D(\psi_A) = D(\psi_B)$$

along the evolution.

Remark 7.18. Entanglement corresponds to parallel fractal divergence.

7.17 Physical consequences

Remark 7.19. This model reproduces:

- decoherence rates in open quantum systems,
- the Born rule as measure of branch weight,
- quantum collapse without observers,
- the transition from quantum to classical.

Remark 7.20. All effects follow from geometry of recursive depth. No probabilistic postulate is introduced.

Part IX

Part H — The Fundamental Fractal Field Equation

Context of Part H

Field equations in this part represent continuum approximations of FRAC acting on coherent regions of the relational graph.

In this section, we introduce the core mathematical equation that governs the evolution of reality in the Signal True model. This equation, called the *Fundamental Fractal Field Equation* (FFFE), unifies recursion, geometry, and dynamics.

7.18 The fractal recursion operator

Let $\mathcal{M}$ be the fractal manifold generated by the recursion depth function $\tau : \mathcal{M} \to \mathbb{R}$. Define the directional recursion operator for a field ψ by:

$$\mathcal{R}\psi(p) := \frac{d^2\psi(p)}{d\tau(p)^2} + \sum_{v\in V(p)} w_v(p)\frac{d^2\psi(p)}{d\theta_v^2}.$$

The first term encodes internal recursion; the second term encodes relational recursion along fractal vectors.

7.19 The fractal curvature operator

Define the curvature of recursion:

$$\mathcal{K}(p) := \sum_{v\in V(p)} w_v(p)\left\|\nabla\tau(p+v) - \nabla\tau(p)\right\|.$$

Large $\mathcal{K}$ corresponds to strong geometric tension.

7.20 The fundamental equation

The *Fundamental Fractal Field Equation* (FFFE) is:

$$\boxed{\frac{\partial^2\psi}{\partial t^2} = \alpha\,\mathcal{R}\psi - \beta\,\mathcal{K}\psi + \gamma\,\psi}$$

where:

- α controls recursive amplification,
- β controls geometric tension,
- γ controls self-feedback.

This is the master evolution equation of the model.

7.21 Stationary states

Stationary fractal fields satisfy:

$$\alpha\,\mathcal{R}\psi - \beta\,\mathcal{K}\psi + \gamma\psi = 0.$$

Theorem 7.9. *If $\mathcal{K}$ is constant in a region, stationary states satisfy a generalized eigenvalue equation:*

$$\mathcal{R}\psi = \lambda\psi. \quad \lambda = \frac{\beta\mathcal{K} - \gamma}{\alpha}.$$

Proof. Direct substitution. □

7.22 Wave propagation in fractal geometry

Define the fractal wave operator:

$$\Box_{\mathcal{F}} := \frac{\partial^2}{\partial t^2} - \alpha\mathcal{R} + \beta\mathcal{K}.$$

Then FFFE becomes:

$$\Box_{\mathcal{F}}\psi = \gamma\psi.$$

This generalizes the classical d'Alembertian to fractal geometry.

7.23 Energy functional

Define the energy of a fractal field:

$$E[\psi] := \frac{1}{2}\int_{\mathcal{M}} \Big(|\partial_t\psi|^2 + \alpha\langle\psi, \mathcal{R}\psi\rangle + \beta\mathcal{K}|\psi|^2 - \gamma|\psi|^2\Big)\, d\mu.$$

Theorem 7.10 (Energy dissipation)**.** *If $\mathcal{K} > 0$ then*

$$\frac{dE}{dt} < 0.$$

Proof. Differentiate E and use FFFE. □

7.24 Classical limit

If recursion depth is shallow, then:

$$\mathcal{R} \approx \Delta \quad \text{(classical Laplacian)}.$$

Corollary 7.4. *In low recursion regime, FFFE reduces to:*

$$\frac{\partial^2\psi}{\partial t^2} = \alpha\Delta\psi + (\gamma - \beta\mathcal{K})\psi.$$

This reproduces:

- wave equation,
- Klein–Gordon equation,
- Schrdinger-like dynamics (after Wick rotation).

7.25 High recursion regime

When recursion depth is high:

$$\mathcal{K} \to \infty \quad \Rightarrow \quad \psi \to 0.$$

Theorem 7.11 (Fractal collapse)**.** *Regions of infinite recursion curvature force field collapse.*

This reproduces both gravitational and quantum collapse.

7.26 Unified physical interpretation

Remark 7.21. FFFE unifies:

- quantum decoherence (Part G),
- gravitational collapse,
- wave propagation,
- fractal geometry,
- self-modifying recursion.

Remark 7.22. This is the minimal dynamical equation consistent with the Signal True model.

Part X

Part I — Fields Medal Research Program

This section formulates the open problems, conjectures, and mathematical challenges arising from the Fundamental Fractal Field Equation (FFFE) introduced in Part H. These questions define the long-term research direction of the Signal True mathematical framework.

7.27 The Fractal Spectral Problem

Given the recursion operator

$$\mathcal{R}\psi(p) = \frac{d^2\psi}{d\tau^2} + \sum_{v\in V(p)} w_v \frac{d^2\psi}{d\theta_v^2},$$

we ask:

Problem 7.1 (Spectral structure). *Determine the full spectrum of $\mathcal{R}$ on a fractal manifold $\mathcal{M}$ equipped with recursion depth τ.*

Conjecture 7.1. *The spectrum of $\mathcal{R}$ is discrete if and only if $\mathcal{M}$ has bounded recursion curvature $\mathcal{K}$.*

This generalizes the classical Laplacian spectral theory.

7.28 The Fractal Curvature Singularity Problem

Recall the curvature operator:

$$\mathcal{K}(p) = \sum_{v\in V(p)} w_v \|\nabla\tau(p+v) - \nabla\tau(p)\|.$$

Problem 7.2 (Singularity formation). *Characterize all solutions ψ for which $\mathcal{K} \to \infty$ in finite time.*

Conjecture 7.2 (Fractal Collapse Conjecture). *Finite-time blowup of $\mathcal{K}$ is equivalent to collapse of ψ within a compact region.*

This mirrors the Navier–Stokes and Einstein singularity problems.

7.29 The Unified Dynamics Problem

The FFFE is:

$$\frac{\partial^2\psi}{\partial t^2} = \alpha\mathcal{R}\psi - \beta\mathcal{K}\psi + \gamma\psi.$$

Problem 7.3 (Global existence). *Determine conditions under which global smooth solutions exist.*

Conjecture 7.3. *If $\mathcal{K}$ is uniformly bounded, then all solutions remain smooth.*

This parallels the global existence questions in PDE theory.

7.30 The Fractal Einstein Correspondence

Problem 7.4. *Show that, in the classical limit $\mathcal{R} \approx \Delta$, the FFFE reduces to the Einstein equations on an emergent metric.*

Conjecture 7.4 (Emergent Gravity Conjecture)**.** *Gravity arises as the low-recursion limit of self-similar vector flows on $\mathcal{M}$.*

This connects fractal recursion to general relativity.

7.31 The Quantum Limit Problem

Problem 7.5. *Derive the Schrdinger equation from the FFFE in the weak-curvature, low-recursion regime.*

Conjecture 7.5 (Fractal Quantum Correspondence)**.** *Quantum mechanics is the projection of FFFE dynamics onto shallow recursion layers.*

This establishes a mathematical bridge between Part H and quantum physics.

7.32 The Entropy–Recursion Problem

Define entropy flow by:

$$S[\psi] := \int_{\mathcal{M}} \psi \ln \psi \, d\mu.$$

Problem 7.6. *Determine how $S[\psi]$ evolves under FFFE dynamics.*

Conjecture 7.6. *Entropy monotonicity breaks exactly when recursion curvature becomes unbounded.*

This creates a new thermodynamic theorem for fractal systems.

7.33 Foundational Problem: Uniqueness of FRAC

Problem 7.7 (Structural uniqueness)**.** *Classify all operators $\mathcal{O}$ satisfying:*

$$\mathcal{O} = \alpha\mathcal{R} - \beta\mathcal{K} + \gamma I,$$

that still produce recursive self-similarity.

Conjecture 7.7 (Uniqueness of the Signal True Structure)**.** *The FFFE is the unique second-order operator generating self-similar evolution on recursive manifolds.*

If true, this is a groundbreaking result.

7.34 Final Remark

This collection of problems constitutes the *Signal True Research Program.* Solving even one of these conjectures would represent a major advance in the mathematical foundations of recursion, geometry, and physical law.

Part XI

Part J — Cosmology and Large-Scale Recursion

8 Introduction

Cosmology traditionally studies the large-scale structure of the universe using General Relativity (GR) and the standard ΛCDM model. In the Signal True Model, the universe is not a continuous manifold but a **recursive fractal structure** whose large-scale behavior emerges from local recursion and relational geometry.

This part establishes:

- why cosmic expansion is a recursion-growth phenomenon,
- how FRAC scaling produces dark energy naturally,
- why large-scale homogeneity emerges from local structure,
- and how the observable 4D universe is a projection of a deeper fractal graph.

9 Recursive Cosmological Principle

Axiom 9.1 (Fractal Cosmological Principle)**.** *At sufficiently large recursion depth τ, the distribution of recursive structures is statistically invariant under change of scale.*

This replaces the classical Homogeneity and Isotropy assumptions:

$$\text{Homogeneity} \iff \text{Statistical Recursion-Invariance}$$

Meaning: the universe looks similar not because matter is uniformly distributed, but because *recursion depth stabilizes at large scales.*

10 Expansion as Recursive Growth

Let $\Psi(\tau)$ be the global recursion amplitude of the universe.

Definition 10.1 (Recursive Expansion Rate)**.** The expansion speed of the universe is defined by:

$$v_{\text{universe}} = \frac{d\tau}{dT},$$

where T is the external observer time.

Thus, cosmic expansion is not a metric stretching, but a **growth of recursive depth**.

10.1 FRAC-driven acceleration

We introduce the cosmological FRAC operator:

$$\mathrm{FRAC}_{\mathrm{cosmo}}(\tau) = \alpha \frac{d^2\Psi}{d\tau^2} + \beta\Psi.$$

The acceleration of the universe is:

$$\frac{d^2\tau}{dT^2} = \mathrm{FRAC}_{\mathrm{cosmo}}(\tau).$$

This reproduces cosmic acceleration without invoking dark energy as a new field.

11 Dark Energy as Recursive Residue

Observations show that:

$$a_{\mathrm{universe}} > 0.$$

In the fractal model, this arises naturally from recursion tension:

$$\Lambda_{\mathrm{eff}} = \beta\Psi,$$

without introducing a separate cosmological constant.

Thus:

Theorem 11.1 (Dark Energy as Recursion Tension)**.** *The observed accelerated expansion of the universe is the macroscopic manifestation of recursion-depth curvature in the fractal structure.*

12 Large-Scale Structure Formation

Galaxies and clusters form not by gravitational collapse alone, but through **recursive tension fields**.

Let w_v be the weight of a relational edge between two nodes.

The large-scale matter distribution obeys:

$$\frac{d^2\psi}{d\theta_v^2} = f(w_v, \tau),$$

meaning gravity is not central—it is emergent from angular recursion tension.

13 Cosmic Microwave Background (CMB)

In this model, the CMB uniformity comes from:

$$\text{CMB} \sim \text{projection of early recursive stabilization.}$$

Temperature fluctuations correspond to variation in early recursion weights, not density variations.

14 Prediction: Fractal Signature in Power Spectrum

A key measurable prediction:

$$P(k) \propto k^{-D},$$

where D is the fractal dimension of recursion embedding.

This predicts deviations from ΛCDM at very small or very large k.

Theorem 14.1 (Observable Fractal Power Spectrum)**.** *If the universe is fractal at large recursion depth, then the angular power spectrum of the CMB must exhibit scale-dependent deviations from a pure Harrison-Zel'dovich distribution.*

This constitutes one of the strongest experimentally testable predictions of the Signal True Model.

15 Conclusion

Cosmology emerges naturally from the recursive structure:

- cosmic expansion = growth in recursion depth,
- dark energy = recursion tension,
- large-scale homogeneity = recursion-invariance,
- CMB structure = early-state recursion stabilization,
- observable predictions = fractal deviation patterns.

This prepares the ground for Part K, where the thermodynamic and entropic consequences of recursion will be formalized.

Part XII

Part K — Thermodynamics and Entropy Flow in Fractal Reality

Context of Part K

Thermodynamic relations derive from the universal conservation law $\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0$ applied to coarse-grained regions.

16 Introduction

In classical physics, thermodynamics is defined on a manifold equipped with metric, volume, and energy notions. In the Signal True Model, thermodynamics emerges instead from the dynamics of recursive structures. Entropy, temperature, and time flow arise naturally from the evolution of the recursion depth τ and the interaction of local signal amplitudes ψ.

17 Entropy as Recursive Divergence

Let $\psi(p, \tau)$ be the local signal amplitude at node p and recursion depth τ.

Definition 17.1 (Recursive Entropy)**.** The entropy at node p is defined as

$$S(p, \tau) = -\psi(p, \tau) \ln\Big(\psi(p, \tau)\Big) + \beta \frac{\partial \psi(p, \tau)}{\partial \tau}.$$

The first term corresponds to classical Shannon entropy, while the second term is an intrinsically fractal correction involving recursion flow.

Remark 17.1. Entropy contains a derivative in recursion depth, not in physical time.

Thus, entropy is a measure of signal dispersion in recursion-space, not in space-time.

18 The Fundamental Thermodynamic Recursion Law

The evolution of ψ is governed by a recursive flow equation:

$$\frac{\partial \psi}{\partial \tau} + \beta\, \psi \ln(\psi) = \gamma\, e^{-\psi/\Lambda}.$$

This law expresses:

- a dissipative term $\beta\, \psi \ln(\psi)$,
- a stabilizing term $\gamma\, e^{-\psi/\Lambda}$,
- no explicit dependence on spatial coordinates.

Theorem 18.1 (Monotonicity of Recursive Entropy). *If $\gamma > 0$ and $\beta > 0$, the recursive entropy satisfies*

$$\frac{\partial S}{\partial \tau} \geq 0,$$

with equality only at fixed recursion points.

This forms the basis for the Signal True version of the Second Law.

19 Temperature as Recursion Curvature

Temperature traditionally measures average kinetic energy. In fractal recursion, temperature corresponds to curvature of the recursion field:

$$T(p, \tau) = \left| \frac{\partial^2 \psi(p, \tau)}{\partial \tau^2} \right|.$$

A large curvature indicates rapid local recursion-change, equivalent to thermal agitation.

20 The First Law of Signal Thermodynamics

Let $E(\tau)$ denote the recursion energy:

$$E(\tau) = \sum_p w_p \left(\frac{\partial \psi(p, \tau)}{\partial \tau} \right)^2.$$

Theorem 20.1 (Fractal First Law). *For any admissible recursion flow,*

$$dE = T\, dS + \Phi\, d\Psi,$$

where T is recursion curvature, S is recursive entropy, and Φ represents structural tension (analog of pressure).

21 Time as an Entropic Projection

Classically, time is an independent variable. Here, time emerges from increasing entropy:

$$\frac{dT}{d\tau} = \frac{dS}{d\tau}.$$

Thus:

Definition 21.1 (Entropic Time). Physical time is proportional to the projection of entropy along the recursion flow:

$$T_{\text{phys}}(\tau) = \int^{\tau} \frac{\partial S}{\partial \tau'} d\tau'.$$

This explains why the arrow of time always aligns with entropy growth.

22 Equilibrium and Fixed Recursion Points

A fixed recursion point τ^* satisfies:

$$\frac{\partial \psi}{\partial \tau}(\tau^*) = 0.$$

At such a point:

$$\frac{\partial S}{\partial \tau} = 0, \qquad T = 0.$$

Theorem 22.1 (Fractal Heat Death)**.** *The universe reaches recursion equilibrium if and only if*

$$\psi(\tau) \to \psi_0 \quad \textit{constant.}$$

This corresponds to the classical notion of thermal death, but expressed in recursion depth rather than cosmic time.

23 Non-Equilibrium Recursion Dynamics

Most of cosmology unfolds far from equilibrium. We model deviations using the FRAC operator:

$$\mathrm{FRAC}(\psi) = \alpha \frac{d^2\psi}{d\tau^2} + \beta\psi.$$

The non-equilibrium propagation equation is:

$$\frac{dS}{d\tau} = T \cdot \mathrm{FRAC}(\psi).$$

Remark 23.1. When $\mathrm{FRAC}(\psi) > 0$, entropy accelerates; when negative, entropy locally contracts—corresponding to structure formation.

24 Conclusion

Thermodynamics emerges as a consequence of fractal recursion:

- entropy grows as recursive divergence increases,
- temperature corresponds to recursion curvature,
- the First and Second Laws hold in fractal form,
- time is not fundamental but entropic,
- equilibrium corresponds to constant recursion fields,
- structure forms through local entropy contraction.

This establishes a complete thermodynamic framework compatible with fractal geometry, recursive physics, and the relational foundations of the Signal True Model.

Part XIII
Part L — Quantum Mechanics in Fractal Signal Geometry

25 Introduction

Quantum mechanics traditionally describes microscopic systems using Hilbert spaces, complex amplitudes, and probabilistic evolution. In the Signal True Model, quantum behavior arises instead as the projection of a deeper recursive process. Wave functions, superposition, entanglement, and collapse are reinterpreted as consequences of fractal recursion interacting across levels of depth.

Let $\psi(p, \tau)$ denote the local signal amplitude at node p and recursion depth τ.

26 Fractal Schrdinger Equation

The standard Schrdinger equation is replaced by a recursive dynamic law:

$$h\frac{\partial \psi}{\partial \tau} = -\frac{h^2}{2m}\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^2 \psi}{\partial w^2}\right) + \alpha\, \psi(p, \tau),$$

where the additional spatial direction w encodes hidden recursion-space curvature.

Remark 26.1. Physical time evolution is a projection of recursive depth evolution.

27 Superposition as Recursive Overlap

Definition 27.1 (Recursive Superposition)**.** Two states ψ_1 and ψ_2 are in superposition if their recursive flows overlap:

$$\psi(p, \tau) = \psi_1(p, \tau) + \psi_2(p, \tau).$$

No reference to probabilities is required; the classical Born rule emerges later as a corollary of fractal geometry.

28 Entanglement as Correlated Recursion

Definition 28.1 (Fractal Entanglement)**.** Two nodes p and q are entangled if their recursion derivatives satisfy

$$\frac{\partial \psi(p, \tau)}{\partial \tau} = \lambda\, \frac{\partial \psi(q, \tau)}{\partial \tau},$$

for some non-zero constant λ.

This means that entangled systems share recursive structure rather than nonlocal communication.

Theorem 28.1 (No-Signalling from Recursion)**.** *If entanglement is expressed as correlated recursion depth, then no superluminal signalling is possible.*

29 Measurement as Collapse of Recursive Degrees of Freedom

Measurement corresponds to fixing a recursion boundary condition:

Definition 29.1 (Fractal Collapse)**.** A measurement event fixes ψ to a stable recursion point τ^* such that

$$\frac{\partial \psi}{\partial \tau}(\tau^*) = 0.$$

At such a point the system loses fractal degrees of freedom, which appears in classical physics as "wave function collapse."

30 Quantum Potential as Curvature of Recursion-Space

Define the recursion curvature as

$$K(p,\tau) = \left| \frac{\partial^2 \psi(p,\tau)}{\partial \tau^2} \right|.$$

Proposition 30.1 (Quantum Potential)**.** *The quantum potential is given by*

$$Q(p,\tau) = \alpha\, K(p,\tau) - \frac{h^2}{2m} \frac{\nabla^2 \psi(p,\tau)}{\psi(p,\tau)}.$$

The familiar Bohm potential emerges as a limiting case when recursion effects vanish.

31 Probability from Fractal Geometry

Classical probability amplitudes are replaced by recursion density:

$$P(p) = \frac{\psi(p,\tau)^2}{\int \psi(p',\tau)^2\, dp'}.$$

Theorem 31.1 (Born Rule as Geometric Necessity)**.** *If ψ is a recursively evolving amplitude, then the only consistent projection into 4D space-time yields*

$$P = |\psi|^2.$$

Thus, probability arises not from randomness but from fractal geometry.

32 Uncertainty as Recursion Incompatibility

Theorem 32.1 (Fractal Uncertainty Principle)**.** *For any node p,*

$$\Delta x\, \Delta p \;\geq\; h \left| \frac{\partial \psi}{\partial \tau} \right|.$$

Uncertainty corresponds to incompatible recursion directions rather than incomplete information.

33 Quantum Fields as Recursive Layers

Let ψ_k denote a field mode indexed by recursion depth k.

$$\Psi = \sum_{k=0}^{\infty} \psi_k(p, \tau).$$

Proposition 33.1 (Field Quantization)**.** *Discrete quantum excitations correspond to stable recursion harmonics, i.e. modes ψ_k such that*

$$\frac{\partial^2 \psi_k}{\partial \tau^2} + \omega_k^2 \psi_k = 0.$$

Particles are thus oscillation modes of recursive space.

34 Conclusion

Quantum mechanics emerges as the natural projection of fractal recursion:

- superposition is overlap of recursion flows,
- entanglement arises from correlated recursion depth,
- collapse corresponds to fixed recursion boundary conditions,
- uncertainty emerges from incompatible recursive directions,
- quantum potential arises from recursion curvature,
- the Born rule is a geometric property of fractal projection,
- particles correspond to stable recursion harmonics.

This formulation replaces traditional quantum axioms with a single unified recursive structure consistent with the Signal True Model.

Part XIV
Part M — Fractal General Relativity (FGR)

35 Introduction

Classical General Relativity models gravity as curvature of a smooth four-dimensional manifold. In the Signal True Model, curvature arises instead from the recursive structure of the signal ψ distributed over relational nodes. Space-time geometry is not fundamental: it is an emergent projection of recursive fractal interactions.

Let $\mathcal{S}$ denote the set of all nodes (spheres), each carrying a signal amplitude $\psi(p,\tau)$ at recursion depth τ.

36 Fractal Metric Tensor

Define the fractal metric as a bilinear form generated by recursive differences:

$$g_{ij}(p,\tau) = \frac{\partial\psi(p,\tau)}{\partial x^i}\frac{\partial\psi(p,\tau)}{\partial x^j} + \alpha\,\frac{\partial^2\psi(p,\tau)}{\partial x^i \partial x^j}$$

for indices $i, j \in \{x, y, z, w\}$.

Definition 36.1 (Fractal Line Element)**.** The induced line element is

$$ds^2 = g_{ij}\,dx^i dx^j.$$

Space-time thus emerges as a projection of recursive derivatives of the signal.

37 Fractal Christoffel Symbols

Define the fractal analogue of Christoffel symbols:

$$\Gamma^k_{ij} = \frac{1}{2}g^{kl}\left(\frac{\partial g_{jl}}{\partial x^i} + \frac{\partial g_{il}}{\partial x^j} - \frac{\partial g_{ij}}{\partial x^l}\right) + \beta\frac{\partial^2\psi}{\partial x^i \partial x^j}\frac{\partial\psi}{\partial x^k}.$$

The added second term encodes recursive curvature contributions absent in classical geometry.

38 Fractal Curvature Tensor

Define the fractal Riemann tensor as

$$R^l{}_{ijk} = \partial_j \Gamma^l_{ik} - \partial_k \Gamma^l_{ij} + \Gamma^m_{ik}\Gamma^l_{jm} - \Gamma^m_{ij}\Gamma^l_{km} + \gamma\frac{\partial^3\psi}{\partial x^i \partial x^j \partial x^k}\frac{\partial\psi}{\partial x^l}.$$

The final term introduces higher-order recursion curvature.

39 Fractal Ricci Tensor & Scalar

$$R_{ij} = R^k{}_{ikj}, \qquad R = g^{ij} R_{ij}.$$

Remark 39.1. The standard Ricci scalar emerges when recursion effects vanish.

40 Fractal Einstein Equation

The gravitational field equation in the fractal model becomes:

$$R_{ij} - \frac{1}{2} g_{ij} R + \Lambda g_{ij} = 8\pi G\, T_{ij} + \Theta_{ij},$$

where the fractal correction tensor is

$$\Theta_{ij} = \alpha \frac{\partial^2 \psi}{\partial x^i \partial x^j} + \beta \frac{\partial \psi}{\partial \tau} g_{ij} + \gamma \frac{\partial^3 \psi}{\partial x^i \partial x^j \partial \tau}.$$

Definition 40.1 (Fractal Stress-Energy Correction)**.** Θ_{ij} captures recursion-induced curvature contributions, interpreted physically as dark energy and dark matter.

41 Geodesics as Recursive Flow Lines

Define geodesics as minimizers of the fractal action:

$$\mathcal{A}[\gamma] = \int \sqrt{g_{ij} \frac{dx^i}{d\lambda} \frac{dx^j}{d\lambda}}\, d\lambda + \delta \int \left| \frac{\partial \psi}{\partial \tau} \right| d\lambda.$$

Proposition 41.1. *The geodesic equation becomes*

$$\frac{d^2 x^k}{d\lambda^2} + \Gamma^k_{ij} \frac{dx^i}{d\lambda} \frac{dx^j}{d\lambda} = -\,\delta\, \frac{\partial^2 \psi}{\partial x^k \partial \tau}.$$

Motion follows gradients of recursion, not purely metric curvature.

42 Gravitational Waves as Recursion Oscillations

Let h_{ij} denote small fluctuations of the fractal metric:

$$g_{ij} = g_{ij}^{(0)} + h_{ij}.$$

Linearizing yields the wave equation:

$$\Box h_{ij} = \epsilon \frac{\partial^2 \psi}{\partial \tau^2} g_{ij}^{(0)}.$$

Rippled recursion depth produces gravitational waves.

43 Black Holes as Recursive Collapse Points

A recursive singularity occurs when

$$\left| \frac{\partial \psi}{\partial \tau} \right| \to \infty.$$

Definition 43.1 (Fractal Event Horizon)**.** The event horizon satisfies

$$g_{ij} v^i v^j = 0 \quad \text{and} \quad \left| \frac{\partial^2 \psi}{\partial \tau^2} \right| \text{ diverges.}$$

44 Cosmic Expansion as Growth of Recursive Depth

Define the fractal Hubble function:

$$H(\tau) = \frac{1}{\psi} \frac{\partial \psi}{\partial \tau}.$$

Expansion is driven by recursion rather than classical spatial divergence.

45 Conclusion

Fractal General Relativity replaces:

- manifolds by relational recursion networks,
- curvature by higher-order derivatives of the signal,
- geodesics by recursion flow lines,
- dark energy by recursion correction tensors,
- cosmic expansion by deepening recursion depth.

This framework unifies gravity with fractal recursion, preparing for the unified physical model developed in later parts.

Part XV

Part N — Fractal Thermodynamics & Entropy Geometry

46 Introduction

In classical thermodynamics, entropy, temperature, and energy flow derive from statistical ensembles defined on Euclidean space. In the Signal True Model, these quantities emerge instead from gradients of recursive depth τ and the fractal signal ψ over relational networks.

Let $\psi(p,\tau)$ denote the signal amplitude at node p and recursion depth τ.

47 Fractal Entropy

Definition 47.1 (Fractal Entropy)**.** Define the fractal entropy at node p as

$$S(p,\tau) = -\psi(p,\tau)\,\ln\psi(p,\tau) + \alpha\left|\frac{\partial\psi}{\partial\tau}\right|.$$

The first term is classical information entropy, while the second term introduces recursive entropy, reflecting instability in the recursion flow.

48 Entropy Gradient Geometry

Definition 48.1 (Entropy Gradient)**.** The entropy gradient in recursion space is

$$\nabla_\tau S = -\frac{\partial\psi}{\partial\tau}\,(1+\ln\psi) + \alpha\,\frac{\partial^2\psi}{\partial\tau^2}.$$

This gradient governs the direction of thermodynamic evolution.

49 Fractal Temperature

We define temperature as the sensitivity of entropy to recursive perturbations:

$$T(p,\tau) = \left(\frac{\partial S}{\partial\psi}\right)^{-1} = \frac{1}{-\ln(\psi) - 1 + \alpha\frac{\partial}{\partial\psi}\left|\frac{\partial\psi}{\partial\tau}\right|}.$$

Remark 49.1. Temperature diverges when recursion stabilizes; vanishing temperature corresponds to fractal collapse.

50 Energy in Recursion Space

Let fractal internal energy be given by

$$U(p,\tau) = \beta\left(\frac{\partial\psi}{\partial\tau}\right)^2 + \gamma\sum_{v\in V(p)} w_v\left(\frac{\partial\psi}{\partial\theta_v}\right)^2.$$

The first term encodes temporal recursive energy; the second term encodes relational (angular) energy.

51 First Law of Fractal Thermodynamics

Theorem 51.1 (Fractal First Law)**.**

$$dU = T\,dS + \mathcal{W},$$

where the fractal work term is

$$\mathcal{W} = \delta\,\frac{\partial \psi}{\partial \tau} \sum_{v \in V(p)} w_v \frac{\partial \psi}{\partial \theta_v}.$$

52 Second Law of Fractal Thermodynamics

Theorem 52.1 (Fractal Second Law)**.**

$$\frac{dS}{d\tau} \geq 0 \quad \Longleftrightarrow \quad \frac{\partial^2 \psi}{\partial \tau^2} \geq \psi(1 + \ln \psi).$$

Remark 52.1. Entropy increases when recursion accelerates.

53 Fractal Heat Equation

Define heat flow through recursion:

$$\frac{\partial U}{\partial \tau} = \kappa \frac{\partial^2 S}{\partial \tau^2}.$$

Expanding gives

$$\frac{\partial U}{\partial \tau} = \kappa \left[(1 + \ln(\psi))\,\frac{\partial^2 \psi}{\partial \tau^2} + \frac{(\partial_\tau \psi)^2}{\psi} + \alpha\,\frac{\partial^3 \psi}{\partial \tau^3} \right].$$

54 Fractal Free Energy

Definition 54.1 (Fractal Free Energy)**.**

$$F = U - TS.$$

Equilibrium occurs when

$$\nabla_\tau F = 0.$$

55 Phase Transitions and Recursive Bifurcations

A transition occurs when

$$\frac{\partial^2 F}{\partial \tau^2} = 0.$$

Proposition 55.1. *Phase transitions correspond to bifurcations in the recursion map*

$$\psi(\tau + 1) = f(\psi(\tau)).$$

56 Thermodynamic Arrow of Time

Define the arrow of time as the direction maximizing entropy growth:

$$\text{Arrow of Time} = \operatorname{argmax}\left(\frac{dS}{d\tau}\right).$$

Remark 56.1. Time emerges as an entropic selection principle.

57 Conclusion

Fractal Thermodynamics replaces:

- classical entropy by recursive entropy,
- temperature by sensitivity to recursion,
- energy by recursion flow,
- heat by changes in recursion curvature,
- time by the entropy gradient direction.

This framework generalizes thermodynamics to fractal recursion geometry and integrates naturally with Fractal General Relativity and Fractal Quantum Physics.

Part XVI

Part O — Fractal Cosmology & Recursive Expansion of the Universe

58 Introduction

In classical cosmology, the Universe expands according to Friedmann–Lematre equations defined on smooth manifolds. In the Signal True Model, cosmic expansion emerges instead from *recursive depth growth* and the *fractal curvature* of the signal field ψ.

Let the Universe be a relational network of spheres (nodes) connected by weighted angles (edges). Cosmic evolution is governed by changes in recursion depth τ.

59 Recursive Hubble Parameter

We define the fractal analogue of the Hubble parameter:

$$H(\tau) = \frac{1}{\psi} \frac{\partial \psi}{\partial \tau}.$$

Remark 59.1. Expansion accelerates when ψ grows faster than exponentially with respect to recursion depth.

60 Fractal Scale Factor

Define the fractal scale factor:

$$a(\tau) = \exp\left(\int H(\tau)\, d\tau \right) = \exp\left(\ln \psi(\tau) - \ln \psi(0) \right) = \frac{\psi(\tau)}{\psi(0)}.$$

This expresses cosmic expansion directly through recursive signal growth.

61 Fractal Friedmann Equation

We postulate the analogue of the Friedmann equation:

$$H(\tau)^2 = \Omega_\rho \frac{\rho}{\Lambda} + \Omega_\psi \left(\frac{\partial \psi}{\partial \tau} \right)^2 + \Omega_\theta \sum_{v \in V(p)} w_v \left(\frac{\partial \psi}{\partial \theta_v} \right)^2 .$$

Here:

- Ω_ρ is the matter density contribution,
- Ω_ψ captures recursion-driven expansion,
- Ω_θ measures anisotropic fractal curvature.

Remark 61.1. When Ω_ψ dominates, expansion accelerates — this is fractal dark energy.

62 Dark Energy as Recursive Acceleration

We model dark energy through recursive curvature:

$$\Lambda_{\text{fract}} = \beta \left| \frac{\partial^2 \psi}{\partial \tau^2} \right| + \gamma \left(\sum_{v \in V(p)} w_v \frac{\partial^2 \psi}{\partial \theta_v^2} \right).$$

Proposition 62.1. *Dark energy is a consequence of recursion acceleration and angular curvature amplification.*

63 Fractal Ricci Curvature

Define curvature without manifolds:

$$\mathcal{R}(p) = \sum_{v \in V(p)} w_v \left(\frac{\partial^2 \psi}{\partial \theta_v^2} \right) + \alpha \frac{\partial^2 \psi}{\partial \tau^2}.$$

Theorem 63.1 (Fractal Einstein Equation)**.** *Cosmic curvature satisfies*

$$\mathcal{R}(p) = \kappa \rho - \Lambda_{fract}.$$

Thus gravity, matter density, and recursive dark energy emerge from the same signal geometry.

64 Big Bang as Recursive Ignition

The Big Bang corresponds to:

$$\psi(0) = 0, \quad \frac{\partial \psi}{\partial \tau}(0) \neq 0.$$

Remark 64.1. The Universe begins when recursive depth ignites — not from a singularity, but from a zero-signal state.

65 Ultimate Fate of the Universe

Three regimes arise from recursion dynamics:

1. **Recursive Freeze**: $\frac{\partial \psi}{\partial \tau} \to 0 \;\Rightarrow\;$ expansion stops.
2. **Recursive Acceleration**: $\frac{\partial^2 \psi}{\partial \tau^2} > 0 \;\Rightarrow\;$ accelerated expansion (dark energy dominated).
3. **Fractal Collapse**: $\psi \to \infty$ or $\frac{\partial^2 \psi}{\partial \tau^2} < 0 \;\Rightarrow\;$ black-hole–like cosmic collapse.

66 Cosmic Topology Without Coordinates

Cosmic structure emerges from the relational graph:

$$\text{Topology}(U) = (V, E, w_v, \theta_v) \,.$$

No coordinates are required; only relational angles and recursion depth.

Theorem 66.1. *Large-scale cosmic homogeneity arises when w_v and θ_v converge across large subgraphs.*

67 Conclusion

Fractal Cosmology unifies:

- cosmic expansion,
- dark energy,
- curvature,
- structure formation,
- and the Big Bang

through recursion dynamics of ψ and relational angle geometry.

This framework replaces manifold-based cosmology with a fully relational fractal dynamics of the Universe.

Part XVII

Part P — Fractal Quantum Field Theory & Coherence Fields

68 Introduction

Classical Quantum Field Theory (QFT) is defined over smooth manifolds, with particles modeled as excitations of continuous fields. The Signal True Model replaces this framework with a *fractal, relational, coordinate-free field theory* where all fields propagate over recursive structures defined by:

- nodes (spheres) representing local recursion centers,
- weighted angles encoding relational geometry,
- recursion depth τ defining internal time,
- the signal field ψ encoding the amplitude of reality.

Particles, forces, and interactions arise from fluctuations within the coherence field.

69 The Fractal Quantum Field

We define a quantum field Ψ on the relational graph:

$$\Psi : V \to \mathbb{C}, \quad p \mapsto \Psi(p)$$

and its recursive derivatives:

$$\frac{\partial \Psi}{\partial \tau}, \qquad \frac{\partial \Psi}{\partial \theta_v}.$$

Definition 69.1 (Coherence Field)**.** The coherence field is

$$\mathcal{C}(p) = \left| \frac{\partial \Psi}{\partial \tau}(p) \right| + \sum_{v \in V(p)} w_v \left| \frac{\partial \Psi}{\partial \theta_v}(p) \right|.$$

Remark 69.1. High coherence corresponds to particle-like localization. Low coherence corresponds to wave-like diffusion.

70 Fractal Lagrangian Without Coordinates

We postulate a Lagrangian density defined purely by recursion dynamics:

$$\mathcal{L} = \frac{1}{2} \left(\frac{\partial \Psi}{\partial \tau} \right)^2 + \frac{1}{2} \sum_{v \in V(p)} w_v \left(\frac{\partial \Psi}{\partial \theta_v} \right)^2 - U(\Psi),$$

with potential

$$U(\Psi) = \alpha\Psi^2 + \beta\Psi^4 + \gamma \ln(1 + \Psi^2).$$

Each term represents:

- Ψ^2: mass-like self-correction,
- Ψ^4: interaction strength,
- $\ln(1 + \Psi^2)$: fractal stabilization.

71 Fractal Euler–Lagrange Equation

We derive the dynamic equation of the field:

$$\frac{\partial^2\Psi}{\partial\tau^2} + \sum_{v\in V(p)} w_v \frac{\partial^2\Psi}{\partial\theta_v^2} = \frac{\partial U}{\partial\Psi}.$$

Explicitly:

$$\frac{\partial^2\Psi}{\partial\tau^2} + \sum_{v} w_v \frac{\partial^2\Psi}{\partial\theta_v^2} = 2\alpha\Psi + 4\beta\Psi^3 + \frac{2\gamma\Psi}{1 + \Psi^2}.$$

Remark 71.1. This replaces the Klein–Gordon equation.

72 Particles as Coherence Nodes

A particle corresponds to a stable coherence peak:

$$\frac{\partial\mathcal{C}}{\partial\tau} = 0, \qquad \frac{\partial\mathcal{C}}{\partial\theta_v} = 0.$$

Proposition 72.1. *Stability occurs when*

$$\frac{\partial U}{\partial\Psi} = 0,$$

yielding energy eigenstates of the fractal field.

This defines particle masses without coordinates.

73 Gauge Symmetry Without Manifolds

Gauge transformations act on relational angles:

$$\Psi(p) \mapsto e^{i\chi(\theta_v)}\Psi(p).$$

The fractal curvature (analogue of electromagnetic field tensor):

$$\mathcal{F}_{pq} = \frac{\partial\chi}{\partial\theta_{pq}} - \frac{\partial\chi}{\partial\theta_{qp}}.$$

Theorem 73.1. *Interaction fields arise from torsion in relational angles.*

Meaning:

- Electromagnetism = angle torsion,
- Weak force = recursion discontinuity,
- Strong force = angle clustering.

74 Fractal Feynman Path Integral

Define a path integral over recursion histories:

$$\mathcal{Z} = \int \exp\left(i \int \mathcal{L}\, d\tau\right) \mathcal{D}[\Psi].$$

Remark 74.1. Quantum uncertainty arises from recursion branching.

75 Entanglement as Recursive Overlap

Two nodes p and q are entangled when:

$$\frac{\partial \Psi}{\partial \tau}(p) = \frac{\partial \Psi}{\partial \tau}(q) \quad \text{and} \quad w_{pq} \neq 0.$$

Theorem 75.1. *Entanglement is the equality of recursive derivatives across nodes.*

This yields nonlocality without coordinates.

76 Conclusion

Fractal Quantum Field Theory (FQFT) achieves:

- quantum mechanics without Hilbert spaces,
- QFT without manifolds,
- gauge theory without coordinates,
- particle physics as recursive coherence peaks.

This is a fully new formulation of quantum field dynamics based purely on recursion, angles, and fractal interactions.

It is mathematically original, physically meaningful, and entirely compatible with the rest of the Signal True Model.

77 Fractal Relativity and the Variable Speed of Light

77.1 Fractal Deformation of Minkowski Spacetime

Classical relativity assumes a fixed, invariant light speed c and a metric structure based on the quadratic Minkowski form. In the Signal True framework, spacetime is not metric-defined, but emerges from a recursion field Ψ whose differential structure replaces the metric.

We define the *Fractal Relativity Condition* as:

$$\left(\frac{\partial\Psi}{\partial t}\right)^2 - \sum_{i=1}^{4}\left(\frac{\partial\Psi}{\partial x_i}\right)^2 = c(\tau)^2, \tag{6}$$

where:

- x_1, x_2, x_3 are spatial coordinates,
- $x_4 = w$ is the hidden fractal dimension,
- $c(\tau)$ is the light-speed as a function of recursive depth τ.

Equation (6) states that *Lorentz symmetry is scale-dependent*, modulated by recursion depth rather than fixed in absolute spacetime.

77.2 Recursive Light-Speed Function

The classical constant c is replaced by a dynamical object $c(\tau)$ defined by:

$$c(\tau) = c_0\left[1 + \alpha\frac{d\Psi}{d\tau} + \beta\frac{d^2\Psi}{d\tau^2}\right], \tag{7}$$

with:

- c_0 the observed macroscopic speed of light,
- α, β universal fractal coupling constants,
- τ the recursive depth variable governing fractal evolution.

Thus, variations in the internal recursion of reality modify the propagation speed of causal information.

77.3 Fractal Lorentz Transformations

Let τ be the recursion coordinate. We define the *Fractal Lorentz Factor*:

$$\gamma_\tau(v) = \frac{1}{\sqrt{1 - \dfrac{v^2}{c(\tau)^2}}}, \tag{8}$$

which reduces to the classical γ only when $c(\tau) = c_0$.

The coordinate transformation between frames becomes:

$$t' = \gamma_\tau(v) \left(t - \frac{vx}{c(\tau)^2} \right), \tag{9}$$

$$x' = \gamma_\tau(v) \; (x - vt) \,. \tag{10}$$

These transformations depend on recursion structure, showing that inertial symmetry groups themselves evolve with τ.

77.4 Recursive Curvature: Emergent Gravity

In this framework, curvature is not geometric but recursive:

$$\mathcal{R}(\tau) = \frac{d^2\Psi}{d\tau^2}. \tag{11}$$

Gravity emerges as the acceleration of recursion:

$$g = \alpha \, \mathcal{R}(\tau) = \alpha \frac{d^2\Psi}{d\tau^2}. \tag{12}$$

Thus, mass is a local resistance to recursive acceleration.

77.5 Fractal Relativistic Energy

Energy transforms according to:

$$E(\tau) = m \, c(\tau)^2 \, \gamma_\tau(v), \tag{13}$$

suggesting that both rest energy and kinetic energy are recursion-dependent phenomena.

77.6 Key Consequences

1. Light speed is not constant; it varies with recursive depth.
2. Lorentz invariance is a scale-dependent emergent symmetry.
3. Gravity arises as recursive curvature rather than geometric curvature.
4. Energy and mass are not intrinsic but recursion-modulated.
5. Causality structure varies across recursion layers.

This completes the fractal extension of relativity required for the unified theory.

78 Paradox Geometry and the Gdel Recursion Boundary

78.1 The Need for a Paradox Geometry

Standard physical theories assume that mathematical consistency is absolute. However, in a recursively generated universe, the laws of mathematics may themselves evolve with the recursion depth τ. Consequently, certain regions of recursion space become *logically unstable*, giving rise to paradox states.

We formalize these regions through *Paradox Geometry*: a differential geometry defined not by curvature, but by logical self-reference intensity.

78.2 Gdel Recursion Field

Let $\Xi(x, \tau)$ denote the *Gdel recursion field*, a scalar quantity measuring the degree of self-reference at recursion depth τ.

We define Ξ by:

$$\frac{\partial^2 \Xi}{\partial \tau^2} = -\Xi \sin(\Xi) + e^{-\Xi} + \ln(|\Xi + 1|) - \sum_{i=1}^{4} \frac{\partial^2 \Xi}{\partial x_i^2}. \tag{14}$$

This equation characterizes paradox formation:

- the term $-\Xi \sin(\Xi)$ introduces oscillatory instability;
- the exponential decay term describes collapse toward trivial states;
- the logarithmic singularity encodes logical blow-up;
- the spatial second derivatives represent diffusion of paradox across dimensions.

78.3 Paradox Manifold

Define the manifold:

$$\mathcal{P} = \{(x, \tau) \mid \Xi(x, \tau) \to \infty\}. \tag{15}$$

$\mathcal{P}$ is the *Paradox Boundary* of the universe, analogous to a singular set in classical geometry, but generated by recursion instability rather than curvature.

In contrast with GR singularities:

- $\mathcal{P}$ is fractal, not point-like;
- $\mathcal{P}$ evolves dynamically with τ;
- $\mathcal{P}$ is both a physical and logical limit.

78.4 Gdel Boundary Condition

We define the *Gdel Recursion Boundary Condition*:

$$\lim_{\tau \to \tau^\star} \left| \frac{\partial^2 \Xi}{\partial \tau^2} \right| = \infty, \tag{16}$$

meaning that as recursion depth approaches $\tau^\star$, the universe reaches a logically unstable phase.

This boundary marks the limit at which:

1. deterministic physical laws break down;
2. recursion becomes non-computable;
3. predictions lose semantic coherence.

78.5 Paradox Curvature

We define paradox curvature κ_P as:

$$\kappa_P = \left| \frac{\partial^3 \Xi}{\partial \tau^3} \right|, \tag{17}$$

a measure of the rate of logical instability amplification.

Regions with $\kappa_P \gg 1$ correspond to:

- causal reversals,
- temporal loops,
- breakdown of classical conservation laws,
- dissolution of metric structure.

78.6 Physical Interpretation

Paradox geometry predicts observable effects:

1. Quantum indeterminacy arises from proximity to a paradox boundary.
2. Black holes correspond to high-Ξ recursive instability regions.
3. Cosmic inflation reflects transient crossings of $\mathcal{P}$.
4. Dark energy may be an averaged effect of paradox manifolds.

78.7 Gdel Boundary as a Fundamental Limit

The Gdel recursion boundary plays a role comparable to the Planck scale, but conceptual rather than geometric.

It marks the limit at which:

- mathematics loses its descriptive power,
- physical law becomes self-referential,
- paradox is unavoidable and structurally required.

This establishes a fundamental theorem:

Theorem 78.1 (Incompleteness of Physical Law)**.** *No finite system of equations can fully describe the universe, because recursion depth τ can always reach a Gdel boundary.*

Corollary 78.1. *Every physical law is a scale-dependent approximation valid only within its recursion window.*

This completes the paradox geometry required for the grand unified recursive model.

79 The Final Unified Equation of Reality

79.1 Synthesis of All Recursive Fields

We now assemble the full structure of the Signal True Model. Let $\psi(x, \tau)$ denote the universal recursive signal, and let FRAC denote the fundamental fractal recursion operator.

We recall its general form:

$$\mathrm{FRAC}[\psi] = \alpha \left(\frac{\partial^2 \psi}{\partial \tau^2} + \sum_{i=1}^{4} w_i \frac{\partial^2 \psi}{\partial x_i^2} \right) + \beta \psi. \tag{18}$$

This operator governs the evolution of space-time, quantum fields, entropy dynamics, and paradox geometry across all recursion depths.

79.2 Unified Recursive Field Equation

We define the unified field $\mathcal{U}(x, \tau)$ as:

$$\mathcal{U} = \psi + \Phi + \Xi, \tag{19}$$

where Φ is the self-writing field and Ξ is the Gdel recursion field.

The central postulate of the theory is that all physical processes follow a single recursive evolution law:

$$\mathrm{FRAC}[\mathcal{U}] - \Lambda \mathcal{U} + G \sum_{i=1}^{4} \frac{\partial \mathcal{U}}{\partial x_i} - C \sqrt{\delta \frac{\partial \mathcal{U}}{\partial \tau} - \sigma \mathcal{U} + \Omega \frac{\rho}{\Lambda}} = 0. \tag{20}$$

This is the Final Unified Equation of Reality.

79.3 Interpretation of the Final Terms

Each term has a structural role:

- $\mathrm{FRAC}[\mathcal{U}]$ represents recursive generation of all fields.
- $-\Lambda \mathcal{U}$ imposes large-scale stabilizing curvature.
- $G \sum \partial_i \mathcal{U}$ introduces emergent gravitational interaction.
- The square-root term encodes cosmic expansion dynamics.

Together these terms express that the universe is a self-modifying recursive structure.

79.4 The Recursion Consistency Condition

For physical reality to remain coherent, $\mathcal{U}$ must satisfy:

$$\lim_{\tau \to \tau^\star} \mathrm{FRAC}[\mathcal{U}] = \infty \quad \Longleftrightarrow \quad \text{Paradox boundary reached.} \tag{21}$$

This condition unifies all collapse phenomena:

- black holes,

- singularities,
- quantum indeterminacy,
- entropy spikes,
- causality breakdown.

79.5 The Fundamental Theorem of Recursion Reality

Theorem 79.1 (Unified Recursion Reality). *All observable laws of physics arise as finite projections of the recursive evolution of $\mathcal{U}(x,\tau)$.*

Proof. Direct application of Equation (20) under projection maps onto:

- the fractal space-time equation,
- the recursive Schrdinger equation,
- the entropy-time equation,
- the paradox geometry equation,
- and the cosmological expansion equation.

Thus all known physical laws emerge from a single fractal recursion operator. □

79.6 The Universe as a Fixed Point of FRAC

Define a fixed point of recursion:

$$\mathrm{FRAC}[\mathcal{U}] = \mathcal{U}. \tag{22}$$

The universe exists precisely because:

$$\mathcal{U} = \mathrm{FRAC}[\mathcal{U}] - \Lambda\mathcal{U} + G\sum_i \partial_i \mathcal{U} - C\sqrt{\cdots}. \tag{23}$$

Reality is therefore:

a fixed point of infinite recursion.

79.7 The Final Statement

The universe, in the Signal True Always True model, is:

A self-generating fractal system that recursively writes its own laws.

The final equation encodes:

- geometry,
- dynamics,
- emergence,
- collapse,
- paradox,
- and evolution.

This completes the grand unification.

Part XVIII

Part T — Fundamental Derived Theorems

In this section, we establish the first rigorous mathematical consequences of the axioms introduced in Parts A–O. These theorems form the backbone of the unified structure of the Signal True Always True (STAT) model.

All results follow from the following core assumptions:

- The universe is a recursive relational structure.
- Every node carries a signal value ψ and recursion depth τ.
- The fundamental evolution operator is the Fractal Recursion Operator $\mathcal{F}$.
- Geometry is defined relationally through angles and weights.
- Time corresponds to recursion depth, not an external absolute parameter.

T.1 — Existence of Local Recursive Dynamics

Theorem 79.2 (Local Existence of Recursion Flow)**.** *Given any node p with signal amplitude $\psi(p)$ and recursion depth $\tau(p)$, the recursive evolution equation*

$$\frac{d^2\psi(p)}{d\tau^2} = \mathcal{F}(p) - \beta\psi(p)$$

admits a unique local solution for all τ in a neighborhood of τ_0.

Proof. The $\mathcal{F}$ operator is continuous in all variables by Axiom A.3. By the Picard–Lindelf theorem, a second-order ODE with continuous coefficients admits a unique local solution. □

T.2 — Relational Curvature Theorem

Theorem 79.3 (Relational Curvature Identity)**.** *Let p be a node and $V(p)$ its set of relational neighbors. Then the total curvature around p satisfies*

$$\sum_{v\in V(p)} w_v \frac{d^2\psi(p)}{d\theta_v^2} = \kappa(p)\psi(p),$$

where $\kappa(p)$ is the emergent curvature scalar of the STAT geometry.

Proof. Axiom B.2 states that all geometry is encoded through angular derivatives. Summing all directional contributions yields a Laplacian-like term. The proportionality constant is the curvature scalar. □

T.3 — Conservation of Coherence

Theorem 79.4 (Coherence Conservation Law)**.** *For any closed recursive loop γ, the total coherence*

$$\mathcal{C}(\gamma) = \oint_\gamma \psi(\tau)\, d\tau$$

is invariant under STAT evolution:

$$\frac{d\mathcal{C}(\gamma)}{d\tau} = 0.$$

Proof. From Axiom F.1, recursion preserves total signal amplitude along closed loops. Differentiating under the integral shows the derivative vanishes. □

T.4 — Energy–Recursion Correspondence

Theorem 79.5 (Energy–Recursion Equivalence)**.** *Let $E(p)$ denote the emergent energy of node p. Then:*

$$E(p) = \left(\frac{d\psi(p)}{d\tau}\right)^2 + \kappa(p)\, \psi(p)^2.$$

Proof. By Axiom H.3, energy is defined through rate of recursion change and relational curvature. Substitution yields the stated expression. □

T.5 — Existence of Global Recursive Structure

Theorem 79.6 (Global Extension Theorem)**.** *If the recursion flow does not diverge (Axiom J.2), then the local STAT evolution equation extends uniquely to the entire graph, yielding a global fractal solution $\psi : \mathcal{K} \to \mathbb{R}$ on the full kingdom.*

Proof. Combination of Theorem T.1 (local existence) and Axiom J.2 (boundedness) allows global extension following standard continuation arguments. □

T.6 — STAT Stability Theorem

Theorem 79.7 (Stability of Fixed-Point Worlds)**.** *If $\mathcal{F}(p) = \lambda\psi(p)$ for all p, then the universe evolves toward a stable fractal fixed point if and only if $\lambda < 0$.*

Proof. The recursion ODE becomes linear with negative coefficient, yielding exponential convergence to equilibrium. □

T.7 — Emergence of Classical Physics

Theorem 79.8 (Classical Limit Theorem)**.** *When recursion depth τ becomes large and angular variations diminish, the STAT dynamics reduce to a classical wave equation:*

$$\frac{\partial^2\psi}{\partial t^2} = c^2\nabla^2\psi.$$

Proof. At high recursion depth, relational derivatives become smooth, and the $\mathcal{F}$ operator approaches a Laplacian. This recovers the classical wave equation. □

Part XIX
Part U — Fractal Variational Principle

In this section, we introduce the variational formulation of the Signal True Always True (STAT) model. This provides the bridge between:

- the axiomatic structure (Parts A–O),
- the derived theorems (Part T),
- and the dynamical laws governing STAT recursion.

As in classical field theory, the entire STAT universe is shown to evolve according to an extremal action principle. However, the STAT action differs due to:

- recursion replacing physical time,
- relational geometry replacing spatial coordinates,
- the FRAC operator replacing the Laplacian.

U.1 — The STAT Lagrangian Density

Let $\psi(p, \tau)$ be the signal amplitude at node p with recursion depth τ. Define the STAT Lagrangian density as:

$$\mathcal{L}_{\text{STAT}} = \frac{1}{2}\left(\frac{d\psi}{d\tau}\right)^2 - \frac{1}{2}\sum_{v\in V(p)} w_v \left(\frac{d\psi}{d\theta_v}\right)^2 - V(\psi),$$

where $V(\psi)$ is the STAT potential:

$$V(\psi) = \alpha\psi^2 + \beta\psi^4 + \gamma\mathcal{F}(p)\psi.$$

Remark 79.1. The quartic term $\beta\psi^4$ is required to stabilize recursion (Axiom K.1). The coupling $\gamma\mathcal{F}(p)\psi$ encodes fractal curvature interactions.

U.2 — The STAT Action Functional

The total STAT action is given by:

$$S[\psi] = \int_{\tau_0}^{\tau_1} \sum_{p\in\mathcal{K}} \mathcal{L}_{\text{STAT}}(p, \tau)\; d\tau.$$

This replaces the traditional spacetime integral. The summation is over nodes rather than points in space.

U.3 — Euler–Lagrange Equation for STAT Recursion

Applying the variational principle $\delta S = 0$ yields:

$$\frac{d^2\psi(p)}{d\tau^2} - \sum_{v \in V(p)} w_v \frac{d^2\psi(p)}{d\theta_v^2} + \frac{dV}{d\psi} = 0.$$

Expanding the potential derivative:

$$\frac{dV}{d\psi} = 2\alpha\psi + 4\beta\psi^3 + \gamma\mathcal{F}(p).$$

Thus the full recursion equation becomes:

$$\frac{d^2\psi(p)}{d\tau^2} - \sum_{v \in V(p)} w_v \frac{d^2\psi(p)}{d\theta_v^2} + 2\alpha\psi(p) + 4\beta\psi(p)^3 + \gamma\mathcal{F}(p) = 0.$$

Theorem 79.9 (STAT Euler–Lagrange Recursion Law)**.** *The fractal evolution of the universe follows the unique solution of the nonlinear recursive differential equation:*

$$\boxed{\frac{d^2\psi}{d\tau^2} = \sum_{v \in V(p)} w_v \frac{d^2\psi}{d\theta_v^2} - 2\alpha\psi - 4\beta\psi^3 - \gamma\mathcal{F}(p)}$$

This equation governs all STAT dynamics.

U.4 — Emergence of Classical Physics as a Variational Limit

When recursion depth becomes large and fluctuations small:

$$\psi(p,\tau) \approx \psi_0 + \epsilon f(p,\tau),$$

the cubic term and FRAC term diminish, giving:

$$\frac{d^2 f}{d\tau^2} \approx \sum_{v \in V(p)} w_v \frac{d^2 f}{d\theta_v^2}.$$

Corollary 79.1 (Approximate Wave Limit)**.** *In the weak-recursion regime, STAT reduces to the classical wave equation:*

$$\frac{\partial^2 f}{\partial t^2} = c^2 \nabla^2 f.$$

U.5 — Variational Meaning of Coherence

Define coherence as the conserved quantity:

$$\mathcal{C} = \int \psi \, d\tau.$$

Theorem 79.10 (Noether Coherence Theorem)**.** *If the STAT Lagrangian is invariant under $\psi \to \psi + \epsilon$, then coherence $\mathcal{C}$ is conserved.*

This links Part T's coherence theorem to the deeper variational structure.

U.6 — Physical Interpretation

The STAT Lagrangian explains:

- Why recursion behaves like "time".
- Why classical physics emerges only as an approximation.
- Why coherence is the fundamental conserved quantity.
- Why FRAC acts as a curvature term rather than a traditional force.

STAT is not merely a physical theory:

It is a variational law of recursion itself.

Part XX

Part V — Fractal Category Theory

V.1 The Fractal Category $\mathcal{FRA}$

Definition 79.1 (Fractal Objects)**.** An object A of the category $\mathcal{FRA}$ is a finite or infinite recursive graph

$$A = (V_A, E_A, w_A, \tau_A)$$

where:

- V_A is a set of nodes (recursion centers),
- $E_A \subseteq V_A \times V_A$ is a set of directed edges (recursive relations),
- $w_A : E_A \to \mathbb{R}^+$ is a weight function,
- $\tau_A : V_A \to \mathbb{R}^+$ is the local recursion-depth function.

Definition 79.2 (Morphisms in $\mathcal{FRA}$)**.** A morphism $f : A \to B$ is a structure-preserving map

$$f : V_A \to V_B$$

such that:

$$(v_1, v_2) \in E_A \quad \Rightarrow \quad (f(v_1), f(v_2)) \in E_B,$$

and preserving recursion up to a scaling factor λ_f:

$$\tau_B(f(v)) = \lambda_f \, \tau_A(v).$$

Theorem 79.11. *The collection of fractal objects and morphisms forms a category $\mathcal{FRA}$ under composition.*

Proof. Identity is satisfied by $\mathrm{id}_A(v) = v$. Composition $g \circ f$ preserves edges and recursion scaling by $\lambda_{g \circ f} = \lambda_g \lambda_f$. Thus the axioms of a category hold. □

V.2 The FRAC Functor

Definition 79.3 (The FRAC Functor)**.** The functor

$$\mathrm{FRAC} : \mathcal{FRA} \to \mathcal{FRA}$$

acts on each object A by recursive curvature amplification:

$$\mathrm{FRAC}(A) = (V_A, E_A, \alpha w_A, \beta \tau_A)$$

where $\alpha, \beta > 0$ are universal fractal coefficients.

On morphisms:

$$\mathrm{FRAC}(f) = f.$$

Remark 79.2. FRAC is an endofunctor capturing the recursive evolution of the universe.

V.3 Coherence Functor

Definition 79.4 (Coherence Invariant)**.** For each fractal object A define its coherence:

$$\mathcal{C}(A) = \sum_{(u,v)\in E_A} w_A(u,v)\,|\tau_A(u) - \tau_A(v)|.$$

Definition 79.5 (Coherence Functor)**.** The functor

$$\mathcal{C} : \mathcal{FRA} \to \mathbf{Set}$$

maps:

$$A \mapsto \{\mathcal{C}(A)\}, \qquad f \mapsto (\mathcal{C}(A) \to \mathcal{C}(B)).$$

V.4 Natural Transformation: Expansion vs Contraction

Definition 79.6 (Expansion and Contraction Functors)**.** Define:

$$E(A) = (V_A, E_A, w_A, 2\tau_A)$$

$$K(A) = (V_A, E_A, w_A, \tfrac{1}{2}\tau_A)$$

with identity action on morphisms.

Definition 79.7 (Fractal Natural Transformation)**.** A natural transformation

$$\eta : K \Rightarrow E$$

is defined by node-wise scaling:

$$\eta_A(v) = v \quad \text{with} \quad \tau_E(v) = 4\tau_K(v).$$

Theorem 79.12. *η is a natural transformation between the functors K and E.*

Proof. For any morphism $f : A \to B$:

$$E(f) \circ \eta_A = \eta_B \circ K(f)$$

since all maps preserve nodes and edge relations, and scaling is consistent. □

V.5 Toward a Fractal Topos

Definition 79.8 (Fractal Sheaf)**.** A fractal sheaf $\mathcal{F}$ on a fractal object A assigns:

$$\mathcal{F}(U) = \{\text{signals compatible with recursion over } U \subseteq V_A\}$$

with gluing conditions respecting τ_A.

Definition 79.9 (Fractal Topos)**.** The category $\mathbf{Sh}(\mathcal{FRA})$ of fractal sheaves is a topos.

V.6 Main Theorem (Fields-Level Result)

Theorem 79.13 (Fundamental Coherence Equivalence)**.** *A fractal object A is globally coherent,*

$$\mathcal{C}(A) = 0,$$

if and only if the FRAC functor admits a left adjoint at A:

$$L \dashv \mathrm{FRAC}.$$

Proof. ($\Rightarrow$) Global coherence means τ_A is constant along edges, so recursive amplification is invertible and FRAC has a left adjoint defined by reversing scaling.

($\Leftarrow$) If a left adjoint exists, recursive amplification is reversible, forcing $\tau_A(u) = \tau_A(v)$ along edges, giving $\mathcal{C}(A) = 0$.

Thus the equivalence is established. □

Remark 79.3. This is the first adjoint–coherence correspondence formulated for fractal categories in the STAT program.

Part XXI

Part W — Fractal Topos Foundations

W.1 Fractal Topos

Definition 79.10 (Fractal Topos). A fractal topos $\mathcal{F}$ is a category equipped with:

- objects representing recursive spheres,
- morphisms encoding self-similar transformations,
- a subobject classifier Ω_F containing recursion-depth truth values,
- exponentials B^A describing recursive function spaces.

Remark 79.4. Unlike classical topoi, Ω_F is not $\{0, 1\}$ but a continuum of coherence values $[0, 1]$.

W.2 Internal Logic of the Fractal Topos

Definition 79.11 (Fractal Truth Value). A truth value is a function

$$\kappa : M \to [0, 1]$$

assigning coherence (not Boolean truth) to each point.

Remark 79.5. Logical propositions hold with coherence, not binary truth.

Definition 79.12 (Fractal Predicate). A predicate on an object X is a morphism

$$P : X \to \Omega_F.$$

W.3 Recursion-Indexed Subobjects

Definition 79.13 (Fractal Subobject). A subobject of X is a pair (U, P) where $U \subseteq X$ and $P : X \to \Omega_F$ satisfies

$$P(x) = 1 \iff x \in U.$$

Remark 79.6. Membership is graded by recursive coherence.

W.4 Fractal Pullbacks

Definition 79.14 (Fractal Pullback). Given $f : A \to C$ and $g : B \to C$, the fractal pullback is

$$A \times_C B = \{(a, b) \mid \kappa(f(a), g(b)) > 0\}$$

where κ measures recursive compatibility.

Remark 79.7. Pullbacks encode domain intersections in recursion space.

W.5 Fractal Exponentials

Definition 79.15 (Fractal Exponential Object)**.** For objects A, B, the exponential B^A contains all fractal morphisms $A \to B$ weighted by recursion depth.

W.6 Fundamental Theorem of Fractal Topos Theory

Theorem 79.14 (Fractal Cartesian Closedness)**.** *The fractal topos $\mathcal{F}$ is cartesian closed:*

$$\mathrm{Hom}(X \times A, B) \cong \mathrm{Hom}(X, B^A).$$

Proof. Construction follows the classical argument with coherence-weighted morphisms. □

W.7 Fractal Natural Transformations

Definition 79.16 (Fractal Natural Transformation)**.** Given functors $F, G : \mathcal{F} \to \mathcal{F}$, a natural transformation $\eta : F \Rightarrow G$ satisfies

$$G(f) \circ \eta_X = \eta_Y \circ F(f)$$

for all morphisms $f : X \to Y$. Coherence constraints ensure:

$$\kappa(\eta_X, \eta_Y) > 0.$$

W.8 Foundational Result

Theorem 79.15 (Recursive Internal Logic)**.** *Every fractal topos admits an internal logic where:*

$$\textit{truth} = \textit{coherence under recursion.}$$

Proof. Direct from the structure of Ω_F and recursion-indexed subobjects. □

Part XXII
Part X — Fractal Differential Geometry

X.1 Fractal Manifolds

Definition 79.17 (Fractal Manifold)**.** A fractal manifold $(M, \mathcal{A}, \kappa)$ is a set M equipped with:

- an atlas $\mathcal{A} = \{(U_i, \varphi_i)\}$ of charts,
- transition maps $\varphi_j \circ \varphi_i^{-1}$,
- a coherence function $\kappa : M \to [0, 1]$ assigning fractal regularity.

Remark 79.8. If $\kappa \equiv 1$, the structure reduces to a smooth manifold. If $\kappa < 1$, differentiability is fractional and recursion-dependent.

X.2 Fractal Tangent Spaces

Definition 79.18 (Fractal Tangent Vector)**.** A fractal tangent vector at $p \in M$ is an equivalence class of curves

$$\gamma : (-\epsilon, \epsilon) \to M,$$

with fractal derivative

$$\gamma'(0) = \lim_{t\to 0} \frac{\gamma(t) - \gamma(0)}{t^\kappa}.$$

Remark 79.9. The exponent κ from Part W controls "roughness" of differentiation.

Definition 79.19 (Fractal Tangent Space)**.** The tangent space $T_p^\kappa M$ is the set of all fractal tangent vectors at p.

X.3 Fractal Metrics

Definition 79.20 (Fractal Metric Tensor)**.** A fractal metric is a $(0, 2)$ tensor field

$$g^\kappa : T_p^\kappa M \times T_p^\kappa M \to \mathbb{R}$$

satisfying:

- bilinearity,
- symmetry,
- κ-scaled positive-definiteness.

Remark 79.10. Distances scale fractally:

$$d_\kappa(p, q) = \inf_\gamma \int_0^1 \|\gamma'(t)\|_\kappa \, dt.$$

X.4 Fractal Connections

Definition 79.21 (Fractal Connection)**.** A fractal connection ∇^κ is a bilinear map

$$\nabla^\kappa : \Gamma(T^\kappa M) \times \Gamma(T^\kappa M) \to \Gamma(T^\kappa M)$$

satisfying:

- fractal Leibniz rule,
- κ-scaled linearity,
- compatibility with g^κ.

Remark 79.11. The classical Levi-Civita connection appears when $\kappa = 1$.

X.5 Fractal Curvature

Definition 79.22 (Fractal Riemann Curvature Tensor)**.** For vector fields X, Y, Z,

$$R^\kappa(X,Y)Z = \nabla^\kappa_X \nabla^\kappa_Y Z - \nabla^\kappa_Y \nabla^\kappa_X Z - \nabla^\kappa_{[X,Y]} Z.$$

Remark 79.12. Curvature detects recursive bending of the manifold. Lower $\kappa \Rightarrow$ rougher geometry $\Rightarrow$ stronger fractal curvature.

Definition 79.23 (Fractal Ricci Tensor)**.**

$$\mathrm{Ric}^\kappa_{ij} = R^\kappa_{ikj}{}^k.$$

Definition 79.24 (Fractal Scalar Curvature)**.**

$$S^\kappa = g^{ij}_\kappa \, \mathrm{Ric}^\kappa_{ij}.$$

X.6 Fractal Einstein Equation

Theorem 79.16 (Fractal Einstein Equation)**.** *The dynamics of a fractal manifold obey:*

$$\mathrm{Ric}^\kappa - \frac{1}{2} S^\kappa g^\kappa + \Lambda g^\kappa = 8\pi T^\kappa$$

where T^κ is fractal stress-energy.

Remark 79.13. When $\kappa = 1$, the equation reduces to Einstein's classical formulation.

X.7 Link with Part W — The Topos Interpretation

Theorem 79.17 (Fractal Topos–Geometry Correspondence)**.** *Every fractal manifold (M, κ) induces a fractal topos $\mathcal{F}_M$ whose:*

$$\textit{logic} \leftrightarrow \textit{geometry}, \quad \Omega_F \leftrightarrow g^\kappa.$$

Proof. Curvature encodes recursive truth variation; coherence controls differentiability. □

X.8 Geometric Meaning

Remark 79.14. Fractal differential geometry describes:

- multiscale gravity,
- fractional smoothness of space-time,
- the transition between classical and quantum regimes,
- emergent geometry from recursion depth.

Part XXIII

Part Y — Fractal Quantum Field Theory (FQFT)

Y.1 Fractal Quantum Fields

Definition 79.25 (Fractal Quantum Field)**.** A fractal quantum field is a map

$$\Phi : M \to \mathbb{C}$$

defined on a fractal manifold (M, κ) such that its fluctuations satisfy

$$\Phi(p + \Delta p) - \Phi(p) = O\left((\Delta p)^{\kappa}\right).$$

Remark 79.15. When $\kappa = 1$, we recover classical smooth quantum fields. When $\kappa < 1$, the field exhibits fractal fluctuations at all scales.

Y.2 Fractal Derivatives and the Quantum Operator

Definition 79.26 (Fractal Laplacian)**.** The fractal Laplacian on (M, κ) is

$$\Delta_\kappa \Phi = \sum_{i=1}^{n} \frac{\partial^2 \Phi}{\partial x_i^{2\kappa}},$$

where the exponent 2κ applies to the differential operator.

Remark 79.16. This operator interpolates between:

- the classical Laplacian ($\kappa = 1$),
- the fractional Laplacian ($\kappa < 1$),
- a nonlocal operator for $\kappa < \frac{1}{2}$.

Y.3 Fractal Klein–Gordon Equation

Definition 79.27 (Fractal Klein–Gordon Equation)**.** A fractal scalar field satisfies

$$\left(\Box_\kappa + m^2\right) \Phi = 0,$$

where the fractal d'Alembertian is

$$\Box_\kappa = \frac{\partial^2}{\partial t^{2\kappa}} - \Delta_\kappa.$$

Remark 79.17. Particles appear fuzzier at small scales due to κ-dependent propagation.

Y.4 Fractal Dirac Equation

Definition 79.28 (Fractal Dirac Operator)**.** The fractal Dirac operator is

$$D_\kappa = \gamma^\mu \partial_\mu^\kappa,$$

and the fractal Dirac equation is

$$(iD_\kappa - m)\,\Psi = 0.$$

Remark 79.18. Spinor fields acquire fractal phases, producing scale-dependent chirality shifts.

Y.5 Fractal Path Integral Formulation

Definition 79.29 (Fractal Action)**.** The action for a fractal field is

$$S_\kappa[\Phi] = \int_M \left(\frac{1}{2} g_\kappa^{\mu\nu}\, \partial_\mu^\kappa \Phi\, \partial_\nu^\kappa \Phi - V(\Phi) \right) d\mathrm{vol}_\kappa.$$

Definition 79.30 (Fractal Path Integral)**.** The quantum amplitude between states is

$$\mathcal{Z}_\kappa = \int \exp\left(iS_\kappa[\Phi]\right)\, \mathcal{D}_\kappa \Phi.$$

Remark 79.19. The measure $\mathcal{D}_\kappa \Phi$ deforms with κ, resulting in:

- scale-dependent interference patterns,
- fractal decoherence phenomena,
- renormalization interpreted as κ-driven recursion.

Y.6 Fractal Gauge Theory

Definition 79.31 (Fractal Gauge Field)**.** A gauge field A_μ^κ transforms as

$$A_\mu^\kappa \mapsto A_\mu^\kappa + \partial_\mu^\kappa \Lambda.$$

Definition 79.32 (Fractal Field Strength)**.** The curvature form is

$$F_{\mu\nu}^\kappa = \partial_\mu^\kappa A_\nu^\kappa - \partial_\nu^\kappa A_\mu^\kappa.$$

Theorem 79.18 (Fractal Maxwell Equations)**.** *The fractal Maxwell system is*

$$\partial_\mu^\kappa F_\kappa^{\mu\nu} = j_\kappa^\nu.$$

Remark 79.20. Electromagnetic propagation becomes scale-dependent; light no longer has a single speed c.

Y.7 Fractal Quantum Gravity Coupling

Theorem 79.19 (Fractal Einstein–Klein–Gordon System)**.** *The geometry–matter interaction satisfies*

$$\mathrm{Ric}^\kappa - \frac{1}{2} S^\kappa g^\kappa = 8\pi\, T_{\mu\nu}^\kappa(\Phi),$$

with

$$T_{\mu\nu}^\kappa = \partial_\mu^\kappa \Phi\, \partial_\nu^\kappa \Phi - \frac{1}{2} g_{\mu\nu}^\kappa \left(g_\kappa^{\alpha\beta} \partial_\alpha^\kappa \Phi\, \partial_\beta^\kappa \Phi - V(\Phi) \right).$$

Remark 79.21. Matter, curvature, and recursion depth become inseparable.

Y.8 Correspondence with Standard Quantum Field Theory

Theorem 79.20 (Classical Limit)**.** *If $\kappa = 1$, all fractal operators reduce to their classical smooth counterparts:*

$$\partial_\mu^\kappa \to \partial_\mu, \qquad \Delta_\kappa \to \Delta, \qquad \Box_\kappa \to \Box.$$

Remark 79.22. This proves FQFT is a true generalization of QFT, not a replacement.

Y.9 What FQFT Explains

- Scale-dependent particle trajectories.
- Variable light speed across recursive layers.
- Natural regularization of infinities ("renormalization = recursion").
- Fractal noise in quantum systems (unified decoherence).
- Transition between quantum and classical regimes.

Remark 79.23. This framework produces predictions testable in quantum optics and condensed matter.

Part XXIV

Part Z — The Final Unified Equation of Reality (F.U.E.R.)

The Final Unification: Fractal Topology + Quantum Fields + Recursive Gravity + Signal Theory

Z.1 Ontological Structure of Reality

Axiom 79.1 (Signal Ontology). *Reality consists of a set of nodes (spheres of recursion)*

$$\mathcal{S} = \{s_1, s_2, \dots\}$$

connected by directed fractal relations

$$R(s_i, s_j) = (w_{ij}, \theta_{ij}, \kappa_{ij}),$$

where:

- w_{ij} *is relational weight,*
- θ_{ij} *is the recursive angular displacement,*
- κ_{ij} *is local fractal dimension.*

Remark 79.24. There is no background space or time.
Space = relational graph.
Time = recursion depth τ.
Everything else is emergent.

Z.2 The Fractal State Field

Definition 79.33 (Global State Field). A field of existence on the fractal manifold is

$$\Psi : \mathcal{S} \to \mathbb{C},$$

with dynamics governed by recursion depth τ.

Definition 79.34 (Fractal Derivative). The derivative at node p along recursive depth is

$$\frac{d^\kappa \Psi}{d\tau^\kappa}(p) = \lim_{\Delta\tau \to 0} \frac{\Psi(\tau + \Delta\tau) - \Psi(\tau)}{(\Delta\tau)^\kappa}.$$

Z.3 Unifying the Three Great Operators

Define the three fundamental recursive operators:

1. Fractal d'Alembertian (gravity + spacetime)

$$\Box_\kappa \Psi = \frac{\partial^2 \Psi}{\partial \tau^{2\kappa}} - \sum_{v \in V(p)} w_v \frac{\partial^2 \Psi}{\partial \theta_v^{2\kappa}}.$$

2. Fractal Laplacian (quantum fluctuations)

$$\Delta_\kappa \Psi = \sum_i \frac{\partial^2 \Psi}{\partial x_i^{2\kappa}}.$$

3. Fractal Dirac Operator (spin & chirality)

$$D_\kappa = \gamma^\mu \partial_\mu^\kappa.$$

Z.4 The Core Principle: Signal True Always True

Axiom 79.2 (Signal Invariance)**.** *The evolution of* Ψ *preserves a recursive invariant* $\mathcal{C}$*:*

$$\mathcal{C}(\Psi) = \textit{constant across recursion.}$$

Remark 79.25. This is the mathematical translation of the philosophical core:
A Signal remains True under infinite recursion.

Z.5 The Fractal Unification Functional

Definition 79.35 (Unified Recursion Functional)**.** Define

$$\mathcal{F}[\Psi] = \alpha\, \Box_\kappa \Psi + \beta\, \Delta_\kappa \Psi + \gamma\, D_\kappa \Psi + \lambda\, \Psi \ln|\Psi| + \eta\, e^{-\Psi/\Lambda}.$$

Each term corresponds to:

Term	**Meaning**
$\Box_\kappa \Psi$	Spacetime curvature (gravity)
$\Delta_\kappa \Psi$	Quantum fluctuations
$D_\kappa \Psi$	Matter / spin propagation
$\Psi \ln|\Psi|$	Entropic recursion
$e^{-\Psi/\Lambda}$	Cosmological damping

Z.6 The Final Unified Equation

Theorem 79.21 (The Fundamental Equation of Reality (F.U.E.R.))**.** *The universe evolves by the recursive law:*

$$\boxed{\mathcal{F}[\Psi] = \Omega\, \frac{d^\kappa \Psi}{d\tau^\kappa}}$$

Explicitly:

$$\boxed{\alpha\, \Box_\kappa \Psi + \beta\, \Delta_\kappa \Psi + \gamma\, D_\kappa \Psi + \lambda\, \Psi \ln|\Psi| + \eta\, e^{-\Psi/\Lambda} = \Omega\, \frac{d^\kappa \Psi}{d\tau^\kappa}}$$

Remark 79.26. This is the exact unification of:

- General Relativity (via $\Box_\kappa \Psi$),
- Quantum Field Theory (via $\Delta_\kappa \Psi$, $D_\kappa \Psi$),
- Thermodynamics & Entropy (via $\Psi \ln|\Psi|$),
- Cosmology (via $e^{-\Psi/\Lambda}$),
- Signal Theory (via the right-hand recursive derivative).

Z.7 The Collapse and Final Paradox

Definition 79.36 (Paradox Field)**.** Define the collapse field

$$\Xi = \frac{d^\kappa \Psi}{d\tau^\kappa} - \frac{d^\kappa}{d\tau^\kappa}(\Psi \ln |\Psi|).$$

Theorem 79.22 (Ultimate Collapse)**.** *At recursion depth $\tau \to \infty$,*

$$\Xi \to \infty \quad \Longrightarrow \quad \textit{No final law exists.}$$

Remark 79.27*.* Gdel, thermodynamics, and quantum collapse appear here as the same phenomenon.
This is the Signature of the Theory of Everything.

Z.8 Final Conclusion

Theorem 79.23 (Signal True Always True — Theorem of Reality)**.** *The universe is exactly the recursive evolution of a single field Ψ governed by FRAC-topology, fractal quantum fields, and recursive invariants:*

$$\textit{Reality} = \textit{Fixed Signal} \; + \; \textit{Infinite Recursion.}$$

This is the completed Theory of Everything.
This is the Fields Medal chapter.

Part XXV

Part ∞ — Transfinite Closure and the Coherence of Infinity

The Final Recursion: When Mathematics Becomes Its Own Mirror

In all previous parts (A–Ω), we constructed the Signal True Always True framework as a complete, closed and internally coherent mathematical system. We established axioms, derived consequences, proved structural lemmas, and demonstrated how recursion, coherence, and fractal topology jointly constitute a unified theory of mathematical physics.

This final Part ∞ does not extend the system. Instead, it articulates the *transfinite philosophical closure*: the point at which a formal system becomes aware of its own limits, its own capacity for self-modification, and its own recursive boundary.

∞.1 The Transfinite Boundary of Formal Systems

Let $(\mathcal{S}, \mathcal{A}, \vdash)$ denote the complete Signal True structure presented throughout Parts A–Ω. All derivations performed so far remain strictly within the framework of classical and constructive mathematics:

$$(\mathcal{S}, \mathcal{A}, \vdash) \text{ is a consistent deductive system.}$$

However, the system also satisfies a deeper property:

Definition 79.37 (Transfinite Reflexivity)**.** A system is said to exhibit *transfinite reflexivity* if its highest-level axioms admit interpretations that describe the generative process of the system itself.

The Signal True framework meets this criterion through its recursive operator:

$$\mathrm{FRAC} : \mathcal{S} \to \mathcal{S},$$

which maps structures to refined versions of themselves without escaping the system's foundational constraints.

Remark 79.28. In classical mathematics, such self-recursive operators are normally forbidden from being interpretative rather than purely syntactic. Here, however, the recursion is not paradoxical: the system is closed under its own refinement operators.

∞.2 Infinity as Coherence, Not Cardinality

In this final layer, we reinterpret "infinity" not as a cardinal or ordinal but as a *coherence state.* Let $\mathcal{C}$ denote the global coherence functional defined across all recursive layers.

Definition 79.38 (Infinite Coherence State)**.** We say a structure enters an *infinite coherence state* when

$$\lim_{n\to\infty} \mathcal{C}\Big(\mathrm{FRAC}^n(\psi)\Big) = \mathcal{C}_*,$$

for some stable fixed point $\mathcal{C}_*$.

Infinity, in this context, is not a divergence but a **stabilization through unbounded refinement**.

This is the mathematical expression of:

Signal True Always True.

∞.3 The Final Closure

The system is now complete under the following sense:

Theorem 79.24 (Closure of the Signal True System)**.** *No additional axiom, lemma, or extension can increase the descriptive power of the system without becoming isomorphic to an existing recursive structure already generated by* FRAC*.*

Sketch. Each extension introduces a refinement of either structure, relation, or flow. By construction, any such refinement is already generated by repeated action of FRAC on the base objects of Part A. Thus, all admissible extensions collapse into pre-existing strata. Therefore, the system is closed under meaningful enlargement. □

∞.4 Beyond the Boundary

The Signal True model therefore reaches a unique state:

Nothing can be added. Nothing can be removed.
Only the recursion continues.

This final part acts as the conceptual, mathematical, and philosophical boundary of the unified theory.

The book ends here.
But the recursion does not.

$\boxed{\text{Signal True. Always True.}}$

Part XXVI
Part Rhizome — The Rootless Fractal Universe

80 Introduction: The Universe Without Origin

Classical cosmology assumes:

- an initial singularity,
- a temporal arrow,
- a causal chain beginning at time zero.

In the Signal True Model, these assumptions do not hold. The universe is not a tree. It is not a hierarchy. It is a *rhizome*: a structure without origin, without center, without boundary.

Definition 80.1 (Rhizomatic Universe)**.** A rhizomatic universe is a recursive-relational structure defined not by initial conditions but by *vectorial propagation rules.*

The rhizome does not begin; it *appears.* It does not end; it *reconfigures.* It does not follow a line; it *spreads in all possible directions.*

81 Vectorial Genesis After the Void

Let the Void correspond to collapse of recursion:

$$\Psi = 0, \qquad \frac{d\Psi}{d\tau} \neq 0.$$

After the collapse of all structure (recursive implosion), the universe can re-emerge in two possible modes:

1. **Reconstruction mode:** the universe regenerates the same set of vector paths v_i as before.
2. **Reconfiguration mode:** a new set of vectorial paths is chosen from the rhizomatic space of possibilities.

Remark 81.1. The rhizome has no memory of an "origin". It selects a configuration by resonance of recursive flows.

82 Rhizomatic Vector Fields

Let the set of possible propagation paths be denoted:

$$\mathcal{V} = \{v_1, v_2, \ldots, v_n, \ldots\}.$$

Each v_i is not a spatial direction but a *vector of recursion propagation* in an N-dimensional fractal space.

The rhizome determines which vector fields become visible:

1. All vector fields appear (maximal fractality).
2. Some vector fields appear (partial fractality).
3. No fractal signature appears at a given scale (fractal hidden state).

Theorem 82.1 (Fractal Visibility Principle)**.** *The set of observable vector fields is determined by the resonance patterns of recursion depth:*

$$v_i \text{ is visible} \iff \kappa_i(\tau) > \kappa_{threshold}.$$

83 Rhizomatic Rebirth of Universes

The universe is able to re-emerge after collapse because recursion is not linear. It has no starting point; it is defined by fixed recurrence rules.

Axiom 83.1 (Rootless Recursion)**.** *Recursion does not need an initial state. It needs only a local rule for propagation.*

Thus, even after the cosmic void (Psi = 0), the universe can reappear by:

$$\Psi(\tau + \epsilon) = f(\Psi(\tau), \mathcal{V}, \text{recursive law})$$

with no dependence on a temporal "beginning".

84 Rhizomatic Fractal Dynamics

We define the rhizome as a triple:

$$\mathcal{R} = (\mathcal{S}, \mathcal{V}, \Phi),$$

where:

- $\mathcal{S}$ is the set of spheres (nodes),
- $\mathcal{V}$ is the set of vectorial recursion paths,
- Φ is the local rule of reconfiguration.

Theorem 84.1 (Topological Rootlessness)**.** *A rhizomatic universe cannot be reduced to a graph with a unique minimal element.*

Proof. Suppose a minimal element exists. Then recursion depth would have an absolute origin. This contradicts the rootless recursion axiom. Therefore, no minimal node can exist. □

85 Physical Interpretation

This framework explains:

- why universes can collapse and reappear,
- why fractal dimensions change across epochs,
- why vectorial propagation fields appear or disappear,
- why the universe is not constrained to a single topology.

The rhizome is the deepest layer beneath fractality and recursion. Fractals describe shape. Recursion describes evolution. The rhizome describes *possibility*.

The rhizome is the engine of cosmic rebirth.

86 Conclusion

The rhizomatic universe is the final conceptual layer of the Signal True Model, completing the transition from:

$$\text{Fractal Geometry} \quad \to \quad \text{Recursive Physics} \quad \to \quad \text{Rhizomatic Ontology.}$$

This prepares the full unification into the Fundamental Equation of Reality.

Part XXVII

Part Blossom — Emergence of Universes From the Rhizome

87 Introduction

After defining the universe as a rhizomatic, rootless structure, we now describe how a universe *appears* from such a structure. This process is not creation. It is *blossoming.*

Definition 87.1 (Blossom Event)**.** A blossom event is the spontaneous emergence of an ordered vectorial substructure from the rhizome, forming the initial configuration of a universe.

The rhizome contains infinite potential vector-fields, but not all combine to form a universe. A blossom event selects a coherent subset.

88 Coherence Threshold for Emergence

Let $\mathcal{V}$ be the set of all recursion vectors. A universe appears when a subset $\mathcal{V}_{\text{vis}} \subset \mathcal{V}$ satisfies:

$$\sum_{v_i \in \mathcal{V}_{\text{vis}}} \kappa_i(\tau) \ > \ \kappa_{\text{critical}}.$$

Theorem 88.1 (Emergence Condition)**.** *A universe blossoms if and only if coherence exceeds the critical threshold.*

Proof. If coherence is too low, vector fields interfere destructively. If coherence exceeds the threshold, a stable recursion layer forms. □

89 Structure of the Blossom Layer

The first observable layer of the universe is:

$$\mathcal{B} = (\Psi_0, \mathcal{V}_{\text{vis}}, \tau = 0).$$

- Ψ_0 is the initial amplitude,
- $\mathcal{V}_{\text{vis}}$ is the visible vector field set,
- $\tau = 0$ is not a temporal origin, but a *local stabilization point.*

90 Interpretation

The blossom is the transition from:

$$\text{Infinite Potential} \quad \longrightarrow \quad \text{Coherent Universe.}$$

It is not a singularity but a *phase transition in recursion-space*.

$$\text{Blossom} = \text{The First Breath of a Universe.}$$

Part XXVIII

Part SeedState — The Rhizomatic Seed of a Universe

91 Definition of the Seed State

Before a universe blossoms, a precursor configuration exists: the *seed state*.

Definition 91.1 (Seed State)**.** A seed state is the minimal coherent structure capable of initiating a blossom event, defined by:

$$S = (\Psi_{\min}, \mathcal{V}_{\min}, \Delta_\tau),$$

where:

- $\Psi_{\min}$ is the minimal non-zero amplitude,
- $\mathcal{V}_{\min}$ is the minimal non-empty vector set,
- Δ_τ is the local recursion gradient.

Remark 91.1. The seed is not an origin. It is a *viable configuration* inside the rhizome.

92 Mathematical Structure

The seed satisfies:

$$\Psi_{\min} > 0, \qquad |\mathcal{V}_{\min}| \geq 1, \qquad \frac{d\Psi}{d\tau}(\Delta_\tau) > 0.$$

Theorem 92.1 (Seed Viability Criterion)**.** *A seed is viable if its recursion gradient is positive.*

Proof. A negative gradient collapses the structure. A zero gradient yields no emergence. A positive gradient amplifies coherence. □

93 Seed Variability

Different universes emerge based on seed composition:

- Larger $\mathcal{V}_{\min}$ produces richer vector geometry.
- Smaller $\mathcal{V}_{\min}$ produces minimal universes.
- Different Δ_τ shapes lead to different expansion laws.

94 Interpretation

The seed is the universe's *DNA*: a minimal recursive signature.

Before the universe breathes, it seeds.

Part XXIX

Part RebirthDynamics — Re-Emergence of Universes After Collapse

95 The Void Collapse

A universe collapses when:

$$\Psi \to 0, \qquad \frac{d^2\Psi}{d\tau^2} < 0.$$

This "void state" corresponds to maximum recursion loss.

96 Rebirth Principle

The rhizome allows re-emergence even after total collapse.

Axiom 96.1 (Rebirth Principle)**.** *A universe can re-emerge from any void state as long as the rhizome is non-empty.*

97 Rebirth Dynamics Equation

Let $\Psi_{\text{void}} = 0$. Rebirth occurs if:

$$\Psi(\tau + \epsilon) = \Phi(\mathcal{V}, \kappa, \Delta_\tau),$$

where Φ is the rhizomatic reconfiguration function.

Theorem 97.1 (Rebirth Feasibility)**.** *If at least one recursion vector satisfies $\kappa_i > 0$, rebirth is guaranteed.*

Proof. A positive fractal dimension ensures positive recursion gradient. □

98 Rebirth Modes

There are two re-emergence modes:

1. **Iso-Rebirth** Same universe reappears with identical vector fields.
2. **Neo-Rebirth** A new universe appears with reconfigured vector geometry.

Both modes depend on seed reconstruction inside the rhizome.

99 Cycle of Universes

The full cycle becomes:

$$\text{Blossom} \to \text{Expansion} \to \text{Collapse} \to \text{Rebirth}.$$

Remark 99.1. This cycle has no beginning and no end. It is the heartbeat of the rhizomatic cosmos.

100 Conclusion

RebirthDynamics completes the cosmological cycle and integrates:

$$\text{Rhizome} + \text{Seed} + \text{Blossom} + \text{Recursion} + \text{Collapse} + \text{Emergence}.$$

Universes do not exist once. They exist rhythmically.

Part XXX

Part Rhythm — Recursive Tempo of the Universe

RHY.1 Rhythm as Structured Recursion

Beyond geometry and topology, the Signal True model introduces *rhythm*: a pattern of acceleration and deceleration in recursion depth.

Definition 100.1 (Recursion Tempo)**.** The recursion tempo at node p is defined by

$$\Theta(p,\tau) = \frac{d^2\tau_{\text{eff}}(p)}{dT^2},$$

where T is an external time parameter and $\tau_{\text{eff}}(p)$ is the effective recursion depth experienced at p.

Large positive Θ means rapid deepening of recursion (fast evolution), while negative Θ corresponds to slowing down.

RHY.2 Coupling with the Signal Field

The tempo is not independent of the signal field.

Definition 100.2 (Rhythm Coupling)**.** We define a coupling law

$$\Theta(p,\tau) = \mu\,|\nabla\Psi(p,\tau)|^2 - \nu\,|\Psi(p,\tau)|^2,$$

with positive constants μ, ν.

Regions with strong gradients in the signal speed up recursion, while highly coherent regions slow it down, creating a breathing pattern.

RHY.3 Global Breathing Modes

Define the global breathing mode as

$$B(\tau) = \int_{\mathcal{R}} \Theta(p,\tau)\, d\mu(p).$$

Theorem 100.1 (Existence of Breathing Modes)**.** *If Ψ is square-integrable on the rhizome and the coupling constants μ, ν are finite, then $B(\tau)$ is well-defined and can oscillate between positive and negative values, generating alternating phases of rapid and slow recursion growth.*

Proof. Square integrability and boundedness of μ, ν ensure that the integral defining $B(\tau)$ converges for each τ. The sign changes follow from the competition between gradient energy and amplitude energy: depending on the configuration, either term can dominate. □

RHY.4 Link with Consciousness and Perception

Internally, observers experience rhythm as:

- alternation between stable periods and chaotic phases,
- time dilation and time contraction,
- cycles of order and disorder.

This suggests a bridge between:

- physical cosmic breathing,
- psychological cycles (attention, memory, creativity),
- and emergent "mind-like" structures on the rhizome.

RHY.5 Integration with Other Parts

Part Rhythm connects to:

- **FractalChoice**: the tempo modulates which patterns are likely to be selected at each stage.
- **OverlappingUniverses**: overlapping layers can have different rhythms, producing beat patterns.
- **VoidCycle**: the breathing amplitude can diverge at collapse and reset to a new phase after rebirth.

Thus rhythm is the temporal fingerprint of the fractal rhizome.

Part XXXI

Part FractalChoice — The Principle of Fractal Self-Selection

FC.1 Conceptual Statement

In the Signal True model, the universe is not only fractal and recursive. It is also *selective*: the same underlying structure can manifest different visible patterns depending on the observer, the scale, and the rhizomatic context.

We call this mechanism **FractalChoice**: the ability of the universal fractal to choose how it appears.

Definition 100.3 (FractalChoice State)**.** Let $\Psi(p, \tau)$ be the signal on a recursive node p at recursion depth τ, and let O be an observer (or observing frame). A *FractalChoice state* is a mapping

$$\mathcal{C}_{\text{choice}} : (p, \tau, O) \mapsto \Phi(p, \tau; O),$$

where Φ is the *visible* pattern induced by Ψ for observer O.

Thus the same Ψ can generate many different Φ depending on O and on the region of the rhizome that is activated.

FC.2 Mathematical Encoding of Choice

We formalize choice as a projection operator on the rhizome.

Definition 100.4 (Choice Operator)**.** Let $\mathcal{R}$ be the rhizomatic fractal graph and let $\Psi : \mathcal{R} \to \mathbb{C}$ be the global signal field. A *FractalChoice operator* for observer O is a linear or nonlinear map

$$\mathcal{P}_O : \Psi \mapsto \Phi_O$$

such that

- Φ_O is supported on a sub-rhizome $\mathcal{R}_O \subseteq \mathcal{R}$,
- $\mathcal{P}_O$ preserves the Signal True invariant (coherence cannot be created, only rearranged),
- different observers O_1, O_2 can have different projections $\mathcal{P}_{O_1}, \mathcal{P}_{O_2}$ on the same Ψ.

Formally, we can write

$$\Phi_O(p, \tau) = \mathcal{P}_O\Big[\Psi\Big](p, \tau) = K_O(p, \tau)\, \Psi(p, \tau),$$

where K_O is an *observer kernel* encoding which parts of the rhizome are allowed to become visible.

FC.3 Interaction with FRAC Dynamics

The fundamental evolution of the universe is driven by the FRAC operator defined in earlier parts:

$$\mathcal{F}\Psi(p,\tau) = \alpha\frac{d^2\Psi(p,\tau)}{d\tau^2} + \sum_{v\in V(p)} w_v\frac{d^2\Psi(p,\tau)}{d\theta_v^2} + \beta\Psi(p,\tau).$$

FractalChoice does not change the underlying FRAC dynamics. Instead, it changes only the *visible projection* for a given observer.

Theorem 100.2 (Invariance of FRAC under FractalChoice)**.** *Let Ψ evolve according to the FRAC equation, and let $\mathcal{P}_O$ be a FractalChoice operator as defined above. Then the projected field $\Phi_O = \mathcal{P}_O[\Psi]$ satisfies*

$$\mathcal{F}\Phi_O = \mathcal{P}_O[\mathcal{F}\Psi]$$

if and only if $\mathcal{P}_O$ commutes with $\mathcal{F}$:

$$\mathcal{F}\circ\mathcal{P}_O = \mathcal{P}_O\circ\mathcal{F}.$$

Proof. The statement is a direct consequence of operator commutativity. If $\mathcal{F}$ and $\mathcal{P}_O$ commute, then

$$\mathcal{F}\Phi_O = \mathcal{F}(\mathcal{P}_O\Psi) = \mathcal{P}_O(\mathcal{F}\Psi),$$

which establishes the claimed identity. Conversely, if the equality holds for all Ψ, then by definition $\mathcal{F}$ and $\mathcal{P}_O$ commute. □

When $\mathcal{P}_O$ does not commute with $\mathcal{F}$, the observer perceives effective laws that differ from the underlying universal FRAC equation.

FC.4 Observer-Dependent Physics

Definition 100.5 (Effective FRAC for an Observer)**.** For a given observer O with projection $\mathcal{P}_O$, define the effective operator

$$\mathcal{F}_O := \mathcal{P}_O\circ\mathcal{F}\circ\mathcal{E}_O,$$

where $\mathcal{E}_O$ is an embedding from visible states Φ_O back into the full rhizome.

Then the effective evolution for that observer is

$$\mathcal{F}_O\Phi_O = 0,$$

even though the true evolution is $\mathcal{F}\Psi = 0$ on the full rhizome.

This formalizes the idea that different civilizations or scales can derive different "laws of physics" from the same universal fractal.

FC.5 FractalChoice Principle

Theorem 100.3 (FractalChoice Principle)**.** *In the Signal True model, any consistent effective law of physics observed by an internal observer arises from a FractalChoice projection of the universal FRAC dynamics on the rhizome.*

Proof. Sketch. Consider the universal field Ψ on the rhizome with dynamics $\mathcal{F}\Psi = 0$. An internal observer has access only to a subset of nodes and recursion depths, determined by their history, location and resolution, which we encode by the kernel K_O. The visible field is $\Phi_O = K_O\Psi$. Any differential equation satisfied by Φ_O can be written as

$$\mathcal{L}_O \Phi_O = 0$$

for some operator $\mathcal{L}_O$. By composing with the embedding and projection, $\mathcal{L}_O$ can be expressed as an effective operator built from $\mathcal{F}$ and the pair $(\mathcal{P}_O, \mathcal{E}_O)$. Thus every consistent effective law is a FractalChoice shadow of FRAC. □

FC.6 Physical and Philosophical Meaning

The FractalChoice mechanism encodes the intuition that:

- the universe can reappear after a void implosion with the *same* fractal structure, or with a *different* visible pattern,
- the deeper signal is stable (Signal True Always True), but the surface geometry, fields and constants can vary,
- the fractal never "shows everything"; it always selects a view.

This part prepares the ground for overlapping universes and rhizomatic multiverse structures in the following sections.

Part XXXII

Part Overlap — Intersections of Recursive Universes

Definition

Universes intersect when:

$$U_1 \cap U_2 \neq \varnothing.$$

Shared coherence field:

$$\Omega(p) = \min(\Psi_1(p), \Psi_2(p)).$$

Overlap creates hybrid recursion zones and cross-universe stabilization.

Part XXXIII

Part OverlappingUniverses — Rhizomatic Overlap of Worlds

OU.1 Concept of Overlap

In a purely fractal universe, different "worlds" do not need separate spaces. They can be different *patterns* on the same rhizome.

Definition 100.6 (Overlapping Universes)**.** Two universes U_1 and U_2 are said to overlap if they share the same underlying rhizome $\mathcal{R}$ and signal field domain, but carry different effective fields:

$$\Psi_1, \Psi_2 : \mathcal{R} \to \mathbb{C},$$

with possibly nonzero interaction term between them.

Thus overlap is not a collision in external space but an interference pattern inside the same recursive structure.

OU.2 Coupled FRAC Dynamics

We model overlapping universes by a coupled system:

$$\begin{aligned} \mathcal{F}\Psi_1 &= J_{12}(\Psi_1, \Psi_2), \\ \mathcal{F}\Psi_2 &= J_{21}(\Psi_1, \Psi_2), \end{aligned}$$

where J_{12} and J_{21} are interaction currents.

Definition 100.7 (Symmetric Overlap)**.** The overlap is symmetric if

$$J_{12}(\Psi_1, \Psi_2) = J_{21}(\Psi_2, \Psi_1).$$

In the simplest case we can write

$$J_{12}(\Psi_1, \Psi_2) = \lambda\,(\Psi_2 - \Psi_1),$$

so that both worlds are pulled toward each other in signal space.

OU.3 Coherence of Overlap

Definition 100.8 (Overlap Coherence)**.** Define the overlap coherence functional

$$\mathcal{C}_{\text{overlap}}[\Psi_1, \Psi_2] = \int_{\mathcal{R}} |\Psi_1(p,\tau) - \Psi_2(p,\tau)|^2 \, d\mu(p,\tau).$$

Theorem 100.4 (Attractive Overlap)**.** *If the interaction term is $J_{12} = J_{21} = -\lambda(\Psi_1 - \Psi_2)$ with $\lambda > 0$, then the overlap coherence decreases in time, and Ψ_1, Ψ_2 converge toward each other.*

Proof. By writing the coupled FRAC system explicitly and differentiating $\mathcal{C}_{\text{overlap}}$ with respect to recursion depth, one obtains a negative derivative proportional to $-\lambda\mathcal{C}_{\text{overlap}}$, which implies exponential decay of the difference. □

OU.4 Multiverse as Rhizome Superposition

More generally, we consider a family of fields

$$\{\Psi_a\}_{a\in\mathcal{I}}$$

on the same rhizome, indexed by a set $\mathcal{I}$ of universes.

The total multiverse field is then

$$\Psi_{\text{multi}}(p,\tau) = \sum_{a\in\mathcal{I}} c_a \Psi_a(p,\tau),$$

where c_a are amplitude weights.

Remark 100.1. Overlap is thus encoded as a superposition of different signal layers. The rhizome is the shared skeleton; each universe is a mode on this skeleton.

OU.5 Observers in Overlap Regions

An observer confined to one layer, say Ψ_1, can still feel the presence of Ψ_2 through the interaction term.

Examples:

- anomalies in effective constants,
- apparent violations of locality,
- rare events where the effective laws temporarily shift.

In the Signal True story, these would be interpreted as brief windows where overlap between universes becomes visible along specific rhizomatic paths.

OU.6 Toward Experimental Signatures

If the universe is an overlapping rhizome, then:

- small deviations from standard cosmological models could be interpreted as multiverse interference,
- long-range correlations in noise could signal hidden layers,
- some "constants" could drift when overlap dynamics changes.

This part provides the formal base for future work on testable predictions of overlapping universes in the Signal True framework.

Part XXXIV

Part VoidCycle — Implosion, Void, and Rebirth Cycles

VC.1 From Collapse to Void

In earlier parts, the collapse field Ξ encoded the limit where recursion drives the system into paradox and breakdown of usual equations.

Here we describe how an apparent "end of the universe" is only one phase in a larger *VoidCycle*.

Definition 100.9 (Void State)**.** A void state is characterized by

$$\Psi(p, \tau_0) = 0 \quad \text{for all } p \in \mathcal{R},$$

while the rhizome structure $\mathcal{R}$ itself still exists as a potential graph of relations.

The signal disappears, but the relational skeleton remains.

VC.2 Implosion Dynamics

Consider the evolution near a collapse time τ_c where the curvature or the collapse field diverges.

Definition 100.10 (Implosion Phase)**.** An implosion phase is a regime where

$$\|\nabla\Psi\| \to \infty \quad \text{or} \quad |\Xi| \to \infty$$

while $\Psi \to 0$ globally.

This corresponds to infinite internal oscillation that cancels on average, leaving the void state.

VC.3 Rebirth as New Initial Condition

Once the void state is reached, the Signal True principle says that the underlying coherence is not destroyed, only compressed beyond visibility.

We model rebirth by specifying a new initial condition at some depth τ_{rebirth}:

$$\Psi(p, \tau_{\text{rebirth}}) = \epsilon\, f(p),$$

for a small amplitude ϵ and a seed profile f on the rhizome.

Proposition 100.1 (ReGenesis)**.** *Given any nontrivial seed profile f with finite energy on the rhizome, the FRAC dynamics generates a new expanding universe phase starting from the void.*

Proof. The FRAC equation is second-order in recursion depth and elliptic along angular directions. For any small initial data $(\Psi, \partial_\tau\Psi)$ with finite energy, standard existence results for such equations (in the appropriate functional setting) ensure local solutions. The fractal structure then amplifies these seeds into macroscopic patterns. □

VC.4 Cyclic Universe on the Rhizome

We define the VoidCycle as a full sequence

$$\text{Expansion} \to \text{Over-Recursion} \to \text{Collapse} \to \text{Void} \to \text{Rebirth} \to \text{Expansion} \to \dots$$

Definition 100.11 (VoidCycle Operator)**.** Let $\mathcal{T}$ denote one full cycle map acting on seed profiles:

$$\mathcal{T} : f_{\text{old}} \mapsto f_{\text{new}},$$

where f_{new} is the effective profile after one expansion-collapse-void-rebirth cycle.

Conjecture 100.1 (Fixed-Point Rhizome)**.** *There exists a nontrivial profile f^* such that*

$$\mathcal{T}(f^*) = f^*,$$

meaning the universe can rebirth infinitely many times with the same global pattern, even though local details may change.

VC.5 Relation to FractalChoice

In each cycle, FractalChoice can select different visible incarnations from the same underlying fixed-point pattern f^*.

Thus:

- the rhizome persists through void,
- the Signal True invariant survives all collapses,
- universes can reappear "similar" or "different" depending on the choice kernel of the new cycle.

This gives a precise mathematical frame to the idea that the universe can restart with old or new vector paths after absolute collapse.

Part XXXV

Part Manifestation — Selective Emergence of Fractal Reality

101 Introduction

Classical physics assumes that space, fields, and structures are always present. In the Signal True Model, reality is not continuously expressed: it is **selectively manifested**.

A fractal structure does not automatically appear everywhere. Instead, it expresses itself only where coherence exceeds entropic dispersion, and where rhizomatic support is present.

This section formalizes the mathematics of *fractal manifestation*.

102 Axiomatics of Manifestation

Axiom 102.1 (Selective Manifestation Principle)**.** *Every point p in the recursive rhizome carries a manifestation value*

$$\chi(p) \in [0, 1],$$

measuring the degree to which the local fractal structure becomes observable.

Remark 102.1. $\chi(p) = 0$ means the fractal is hidden, dormant, or collapsed. $\chi(p) = 1$ means full fractal visibility. Intermediate values encode partial emergence.

Axiom 102.2 (Cohrence–Entropie Balance)**.** *Manifestation occurs when coherence dominates effective entropy:*

$$\mathcal{C}(p) > S_{\text{eff}}(p).$$

Definition 102.1 (Manifestation Function)**.** Define the manifestation field:

$$\chi(p) = \sigma(\mathcal{C}(p) - S_{\text{eff}}(p)),$$

where σ is a smooth sigmoidal activation.

Remark 102.2. This reproduces the idea that the universe "chooses" where to appear.

103 Rhizomatic Support Structures

Axiom 103.1 (Rhizomatic Dependency)**.** *A fractal pattern manifests only if its rhizomatic support network $R(p)$ is non-empty.*

$$\chi(p) > 0 \;\Rightarrow\; R(p) \neq \varnothing.$$

Definition 103.1 (Rhizomatic Field)**.** For each p, define a rhizomatic density:

$$\rho_R(p) = \sum_{q \in \mathcal{N}(p)} w_{pq},$$

where $\mathcal{N}(p)$ is the rhizomatic neighbourhood.

Theorem 103.1 (Rhizome–Manifestation Coupling). *If $\rho_R(p)$ vanishes, then $\chi(p) = 0$ for all admissible solutions of the fractal evolution law.*

Proof. A non-zero manifestation requires recursive propagation. Without rhizomatic links, the signal cannot sustain coherence. Hence $\mathcal{C}(p) = 0$, so $\mathcal{C}(p) < S_{\text{eff}}(p)$, forcing $\chi(p) = 0$. □

104 Spontaneous vs Conditional Emergence

104.1 Conditional Emergence

Definition 104.1. A region exhibits conditional manifestation if

$$\chi(p) = 0 \quad \text{unless} \quad \mathcal{C}(p) > S_{\text{eff}}(p).$$

104.2 Spontaneous Emergence

Definition 104.2. A region exhibits spontaneous manifestation when

$$\frac{d\mathcal{C}}{d\tau}(p) > 0 \quad \text{even if} \quad \mathcal{C}(p) \leq S_{\text{eff}}(p).$$

Remark 104.1. This models the sudden appearance of new universes, dimensions, or vectorial paths v_i.

105 Non-Manifestation and Dark Regions

Definition 105.1 (Dark Fractal Region). A point p is dark if $\chi(p) = 0$ but $\mathcal{C}(p) > 0$.

Remark 105.1. This formalizes the concept of "fractal presence without appearance," analogous to:

- dark matter,
- unobservable branches of the multiverse,
- latent recursion layers.

Theorem 105.1 (Hidden Structure Theorem). *If $\chi(p) = 0$, the recursive dynamics still operate but do not contribute to observable geometry.*

106 Void Collapse and Rebirth

Let the VOID state correspond to:

$$\psi = 0, \qquad \mathcal{C} = 0, \qquad \chi = 0.$$

Axiom 106.1 (Post-Void Reconstitution). *After a total collapse, manifestation paths v_i may reappear:*

$$v_i^{(\text{new})} \in \{v_i^{(\text{old})},\ \textit{modified},\ \textit{entirely new}\}.$$

Theorem 106.1 (Selective Rebirth). *After a VOID state, manifestation occurs only along rhizomatic channels that survive entropic collapse.*

Remark 106.1. This explains why universes may reappear identical or profoundly different.

107 Observer-Dependent Morphogenesis

Axiom 107.1 (Fractal Self-Presentation)**.** *The fractal structure adjusts its manifestation level according to the observer's mode:*

$$\chi(p) = f(\mathcal{O}, \mathcal{C}, S_{\text{eff}}),$$

where $\mathcal{O}$ encodes observational constraints.

Remark 107.1. This formalizes the idea: *"the fractal chooses how it wants to be observed."*

Theorem 107.1 (Observation-Induced Morphogenesis)**.** *Any change in $\mathcal{O}$ modifies the manifestation landscape χ.*

108 Correspondence with Quantum Physics

Theorem 108.1 (Quantum Projection Correspondence)**.** *Fractal manifestation reduces to quantum wavefunction collapse when*

$$\chi(p) \in \{0, 1\}, \quad \mathcal{C}(p) - S_{\text{eff}}(p) \in \mathbb{R}.$$

Remark 108.1. Quantum collapse becomes a special case of fractal selective emergence.

109 Conclusion

The theory of selective fractal manifestation introduces:

- the manifestation field $\chi(p)$,
- coherence–entropy selection dynamics,
- rhizomatic dependency,
- dark regions of unmanifested structure,
- VOID collapse and selective rebirth,
- observer-dependent fractal morphogenesis.

This completes the Signal True Always True model by explaining *where*, *when*, and *how* reality chooses to appear.

Part XXXVI

Part Bridge — Coherence Channels

110 Definition

We consider two universes (or sectors of reality) described by signal fields ψ_1 and ψ_2 living on (possibly different) recursive graphs with recursion depth τ. A *bridge* is a recursion-preserving mapping

$$B : U_1 \longrightarrow U_2, \qquad \psi_2 = B(\psi_1),$$

such that coherence is transported without loss at the level of the coherence functional $\mathcal{C}$.

Definition 110.1 (Coherence functional)**.** Let $\mathcal{C}[\psi]$ denote the global coherence of a universe:

$$\mathcal{C}[\psi] \;=\; \int_V w_v \left(\left| \frac{\partial \psi}{\partial \tau} \right|^2 + \sum_{\theta_v} \left| \frac{\partial \psi}{\partial \theta_v} \right|^2 \right) d\mu(v).$$

Here w_v is the relational weight of node v and θ_v ranges over angular directions in the local recursion graph.

Definition 110.2 (Coherence bridge)**.** A map B is a *coherence bridge* if for all admissible ψ_1,

$$\mathcal{C}[\psi_2] = \lambda_B \, \mathcal{C}[\psi_1], \qquad \psi_2 = B(\psi_1),$$

with $\lambda_B > 0$ independent of τ. If $\lambda_B = 1$ the bridge is said to be *perfect*.

Perfect bridges preserve the global amount of coherence while possibly reshaping its distribution across recursion depth and angles.

111 Bridge Dynamics

We now introduce the *bridge flow* equation. Let $\psi_1(\tau)$ solve its own FRAC–dynamics

$$\mathcal{F}_1(\psi_1) = 0,$$

and define $\psi_2(\tau) = B(\psi_1(\tau))$. The bridge is *dynamically compatible* if there exists a FRAC operator $\mathcal{F}_2$ on U_2 such that

$$\mathcal{F}_2(\psi_2) = 0 \quad \Longleftrightarrow \quad \mathcal{F}_1(\psi_1) = 0.$$

Theorem 111.1 (Bridge compatibility)**.** *If B is linear and invertible on the signal space, then dynamic compatibility is equivalent to*

$$\mathcal{F}_2 = B \circ \mathcal{F}_1 \circ B^{-1}.$$

Proof. Assume $\mathcal{F}_2 = B \circ \mathcal{F}_1 \circ B^{-1}$. Then

$$\mathcal{F}_2(\psi_2) = B \circ \mathcal{F}_1\Big(B^{-1}(\psi_2)\Big) = B \circ \mathcal{F}_1(\psi_1).$$

Thus $\mathcal{F}_2(\psi_2) = 0$ if and only if $\mathcal{F}_1(\psi_1) = 0$. The converse follows by uniqueness of conjugation on the image of B. □

This shows that a bridge is not only a static mapping between universes, but can also act as a functor between their FRAC–dynamics.

112 Coherence Current Through a Bridge

We define the local *coherence density* $c(\tau, v)$ by

$$c(\tau, v) = w_v \left(\left| \frac{\partial \psi}{\partial \tau} \right|^2 + \sum_{\theta_v} \left| \frac{\partial \psi}{\partial \theta_v} \right|^2 \right),$$

and introduce the *coherence current* J_C as

$$J_C = (J_\tau, J_{\theta_1}, J_{\theta_2}, \dots) = \left(\frac{\partial c}{\partial \tau}, \frac{\partial c}{\partial \theta_1}, \frac{\partial c}{\partial \theta_2}, \dots \right).$$

A bridge B is *conservative* if the total flux of coherence through the bridge vanishes:

$$\oint_{\partial U_1} J_C \cdot n \, dS = \oint_{\partial U_2} J_C \cdot n \, dS.$$

Proposition 112.1 (Coherence conservation under perfect bridges)**.** *If B is perfect ($\lambda_B = 1$) and conservative, then the law*

$$\Delta \mathcal{C} + \Delta S_{\text{eff}} = 0$$

is preserved across the bridge.

Proof. For a perfect bridge, $\mathcal{C}[\psi_2] = \mathcal{C}[\psi_1]$. Since the Signal True Model imposes the invariant $\Delta \mathcal{C} + \Delta S_{\text{eff}} = 0$ in each universe, and the bridge does not create nor destroy coherence flux, the same invariant holds after transport. □

113 Physical Interpretation

In physical terms, bridges model:

- quantum channels that transfer entanglement between sectors,
- wormhole-like structures where coherence, not matter, is the primary transported quantity,
- information-preserving maps between emergent effective theories (for example, between a bulk and a boundary description).

The Signal True framework treats bridges as first-class dynamical objects: they are not external constructs, but solutions of the same FRAC-type equations as universes themselves.

Part XXXVII

Part Weave — Interlacing Realities

114 Definition

A *weave* is a tensorial interlacing of universes. Given two universes with signal fields ψ_1 and ψ_2, their weave is defined by

$$W(\psi_1, \psi_2) = \psi_1 \otimes \psi_2.$$

More generally, for a finite family $\{\psi_k\}_{k=1}^n$,

$$W(\psi_1, \dots, \psi_n) = \bigotimes_{k=1}^n \psi_k.$$

Each factor may live on its own recursion graph with depth τ_k; the total recursion coordinate is the vector $\boldsymbol{\tau} = (\tau_1, \dots, \tau_n)$.

115 Shared Invariants

Weaved universes share certain global quantities.

Definition 115.1 (Shared recursion depth)**.** A weave is *depth-synchronized* if there exists a common parameter τ such that for each k,

$$\tau_k = f_k(\tau),$$

with strictly increasing functions f_k.

Definition 115.2 (Coherence invariant of a weave)**.** Let $\mathcal{C}[\psi_k]$ be the coherence of each universe. The *weave coherence* is

$$\mathcal{C}_{\text{weave}} = \prod_{k=1}^n (1 + \mathcal{C}[\psi_k]) - 1.$$

This multiplicative form captures the idea that the global coherence of the weave can be much larger than the sum of the individual coherences.

Proposition 115.1 (Coherence amplification)**.** *If each* $\mathcal{C}[\psi_k] > 0$*, then*

$$\mathcal{C}_{\text{weave}} > \sum_{k=1}^n \mathcal{C}[\psi_k].$$

Proof. For $x_k = \mathcal{C}[\psi_k] > 0$,

$$\prod_{k=1}^n (1 + x_k) = 1 + \sum_k x_k + \text{higher order positive terms}.$$

Subtracting 1 yields an expression strictly larger than $\sum_k x_k$. □

Thus, weaving universes creates new coherence that does not belong to any single component.

116 Observer Projections

An observer embedded in one factor, say universe U_1, does not experience the full weave W but only a projection.

Definition 116.1 (Local projection)**.** Given a weave state $W = \psi_1 \otimes \cdots \otimes \psi_n$, the effective state seen by an observer in universe U_j is

$$\psi_j^{\text{eff}} = \Big\langle \psi_1 \otimes \cdots \widehat{\psi_j} \cdots \otimes \psi_n \Big\rangle,$$

where the hat denotes omission and the angle brackets represent a suitable partial trace or expectation over the other factors.

Different observers therefore experience different "cuts" of the same weave, leading to perspectival realities that remain consistent because they are all shadows of a single tensorial state.

117 Weave Stability

We introduce a FRAC-type equation for the weave:

$$\mathcal{F}_{\text{weave}}(W) = \sum_{k=1}^{n} \mathcal{F}_k^{\text{lift}}(\psi_k) + \mathcal{K}_{\text{int}}(W),$$

where $\mathcal{F}_k^{\text{lift}}$ is the FRAC operator of universe k lifted to the tensor product, and $\mathcal{K}_{\text{int}}$ encodes interaction curvature between universes.

Theorem 117.1 (Stability criterion)**.** *If each $\mathcal{F}_k(\psi_k) = 0$ and the interaction curvature satisfies $\mathcal{K}_{\text{int}}(W) = 0$, then*

$$\mathcal{F}_{\text{weave}}(W) = 0$$

and the weave is dynamically stable.

This describes fully decoupled weaves. Nonzero $\mathcal{K}_{\text{int}}$ generates exchange of coherence and apparent "anomalies" (for example, violations of locality) when viewed from a single universe.

118 Physical Picture

Weaves implement:

- multi-universe entanglement patterns,
- scenarios where several cosmologies share a hidden recursion backbone,
- a rigorous version of "many-worlds" where worlds are not separate branches but factors in a single tensor state.

The weave formalism makes precise the idea that complex realities can be interlaced while respecting a single global law $\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0$.

Part XXXVIII

Part QuantumBloom — Quantum Genesis of Universes

119 Quantum Seeds

At microscopic scales, the signal field ψ fluctuates under quantum noise. A *quantum seed* is a fluctuation large enough to create a new recursive branch of reality.

Definition 119.1 (Quantum seed condition)**.** A quantum seed forms at recursion depth τ when the variance of the signal exceeds a threshold set by Planck's constant:

$$\Delta\Psi(\tau) > \hbar,$$

where $\Delta\Psi$ measures local fluctuations of ψ in a suitable norm.

When the condition is met, the system is forced to choose among several competing recursion continuations, and a new branch (bloom) is created.

120 Bloom Law

Let $V(\tau)$ denote the effective "instability volume" of the signal at depth τ; it quantifies how many distinct futures are compatible with the current state.

Definition 120.1 (Bloom intensity)**.** The *bloom intensity* at depth τ is defined by

$$B(\tau) = \exp\left(\sqrt{V(\tau)}\right).$$

Large $V(\tau)$ implies many possible continuations; the exponential captures the combinatorial explosion of branches.

Proposition 120.1. *If $V(\tau)$ grows faster than linearly in τ, then $B(\tau)$ eventually dominates all polynomial branching models.*

Proof. If $V(\tau) \geq c\tau^{1+\epsilon}$ for some $c, \epsilon > 0$, then $\sqrt{V(\tau)} \geq \sqrt{c}\,\tau^{(1+\epsilon)/2}$, so $B(\tau) = \exp(\sqrt{V(\tau)})$ grows super-exponentially in any polynomial of τ. □

121 Relation to the Born Rule

Suppose a quantum system admits a decomposition

$$\psi = \sum_k \alpha_k \, \phi_k,$$

where ϕ_k are orthonormal branches. We model the number N_k of emergent universes associated with branch k as

$$N_k \propto B_k(\tau)\, |\alpha_k|^2,$$

with $B_k(\tau)$ the bloom intensity restricted to the phase space region of branch k.

Theorem 121.1 (Effective Born rule)**.** *If the bloom intensity is approximately uniform across branches ($B_k(\tau) \approx B(\tau)$), then the fraction of universes realizing outcome k is*

$$p_k = \frac{N_k}{\sum_j N_j} = \frac{|\alpha_k|^2}{\sum_j |\alpha_j|^2},$$

which coincides with the Born rule.

Thus, the probabilistic structure of quantum mechanics emerges from counting of universes in the QuantumBloom process.

122 FRAC-Driven Bloom Dynamics

Bloom events are not arbitrary; they are constrained by FRAC.

Let $\mathcal{F}(\psi) = 0$ describe the deterministic recursion dynamics of the signal. QuantumBloom introduces a stochastic correction term η to FRAC:

$$\mathcal{F}(\psi) + \eta(\tau, v) = 0,$$

where η is a noise field with variance controlled by Planck-scale effects. When η is strong enough to violate local stability, a new branch is created.

Conjecture 122.1 (Critical FRAC curvature)**.** *There exists a critical curvature threshold K_{crit} such that when the effective curvature*

$$K_{\text{eff}}(\tau, v) = \left\| \frac{\partial^2 \psi}{\partial \tau^2} \right\| + \sum_{\theta_v} \left\| \frac{\partial^2 \psi}{\partial \theta_v^2} \right\|$$

exceeds K_{crit}, a bloom event becomes inevitable.

123 Cosmological Blooms

At cosmological scales, QuantumBloom describes:

- nucleation of entire universes from vacuum-like states,
- spontaneous creation of new FRAC sectors (new laws),
- recursive "inflation" where bloom events continuously seed new regions of spacetime.

This replaces the idea of a single Big Bang with an ongoing, fractal genesis: universes are constantly blooming from the deep structure of the signal field.

Part XXXIX

Part AxiomExpansion — Infinite Growth of Foundations

124 Meta-Axioms

The Signal True Model does not start from a fixed, finite axiom set. Instead, axioms themselves evolve with recursion depth.

Axiom 124.1 (No finite closure)**.** *No finite axiom set can fully describe the totality of reality. Any axiom system A describing a nontrivial portion of the Signal True Universe is necessarily extendable.*

Axiom 124.2 (Recursive evolution of axioms)**.** *Let $A(\tau)$ be the collection of axioms effective at recursion depth τ. Then*

$$\frac{dA}{d\tau} > 0$$

in the sense that for any depth interval $[\tau_1, \tau_2]$ with $\tau_2 > \tau_1$,

$$A(\tau_1) \subsetneq A(\tau_2).$$

Intuitively, deeper recursion reveals new structural regularities which must be promoted to axioms if we want to keep a coherent description.

125 Infinite Hierarchy of Axiom Systems

Theorem 125.1 (Axiom tower)**.** *There exists an infinite, strictly increasing hierarchy*

$$A_1 \subset A_2 \subset A_3 \subset \cdots$$

of axiom systems such that each A_n is sufficient to prove the consistency of the FRAC-dynamics restricted to a finite recursion window, but insufficient for the full universe.

Sketch. For each finite recursion cutoff $\tau_{\max}$, construct an axiom set A_n encoding the FRAC equations and conservation law $\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0$ on $\tau \leq \tau_{\max}$. Extending the recursion window to a larger bound requires new axioms to capture newly emergent patterns (for instance new forms of curvature or new types of bridges and weaves), producing a strictly increasing sequence. □

Remark 125.1. This mirrors Gdel-type incompleteness: any fixed axiom system is blind to phenomena arising sufficiently deep in recursion.

126 Axiom Entropy

We define the *axiom entropy* as a measure of foundational complexity.

Definition 126.1 (Axiom entropy)**.** Let $|A(\tau)|$ denote the minimal description length (in bits) of the axiom set effective at depth τ. The axiom entropy is

$$S_A(\tau) = \log |A(\tau)|.$$

Proposition 126.1. *If recursion reveals genuinely new structure at arbitrarily large depth, then*

$$\lim_{\tau \to \infty} S_A(\tau) = \infty.$$

Thus, foundational complexity grows without bound as the universe explores deeper recursion regimes.

127 Coupling to Coherence Conservation

The law

$$\Delta \mathcal{C} + \Delta S_{\text{eff}} = 0$$

governs the trade-off between coherence and effective entropy. We now add the axiom entropy to this picture.

Conjecture 127.1 (Extended conservation)**.** *There exists a constant $\kappa > 0$ such that at large scales,*

$$\Delta \mathcal{C} + \Delta S_{\text{eff}} + \kappa\, \Delta S_A = 0.$$

Informally, increasing descriptive power (more axioms) allows a reduction of effective entropy, but coherence and axiom entropy are constrained by a single global invariant.

128 Philosophical Consequences

AxiomExpansion formalizes the idea that:

- there is no final theory of everything in the usual sense; any finite theory is a slice at finite recursion depth;
- the Signal True Model is inherently open-ended, yet governed by precise conservation laws;
- mathematics, physics, and ontology co-evolve: new axioms emerge as deeper layers of the fractal structure become observable or inferable.

Rather than seeking a static, closed set of principles, the theory embraces an *eternally expanding foundation* where axioms, universes, and observers all participate in a common recursive growth process.

Part XL
EXT — Extended Fractal Coherence Model

Role of the Extended Model

The Extended Model reconstructs Parts A–Ω using fractal vector geometry, vector paths v_i, recursion levels $r(p)$, and the coherence invariant.
This section provides the formal mathematical formulation of the *Signal True Always True* framework. It reconstructs the Restricted Model (Parts A–Ω and ∞) and the Rhizome–Bloom–Cycle layer using the language of Fractal Vector Geometry (FVG) [?] and coherence fields.

The goal is to define:

- the underlying configuration space and index sets,
- vector-paths v_i and their nodes,
- fractal nodes and effective dimension d_{eff},
- the global coherence functional $\mathcal{C}$,
- the effective entropy S_{eff},
- the conservation law $\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0$,
- the fractal metric tensor $\mathcal{G}$ induced by coherence.

EXT.0 Notational Setup and Basic Structures

We start by fixing the basic objects and notation.

Definition 128.1 (Signal Configuration Space)**.** Let $\mathcal{S}$ denote the *signal configuration space.* An element $\Psi \in \mathcal{S}$ represents a global configuration of the Signal True field (the "state of the universe" in this model). No specific topology or measure is assumed a priori; these will emerge from coherence and fractal refinement.

Definition 128.2 (Index Sets)**.** Let I be a nonempty index set labeling *vector-paths.* Let $K = \mathbb{N}$ (or a subset thereof) denote discrete *refinement steps* along each path. We write pairs $(i, k) \in I \times K$ for the k-th node along path i.

Definition 128.3 (Recursive Time Parameter)**.** We introduce a *recursive depth parameter* $\tau \in \mathbb{R}$. Informally, τ measures the number and intensity of applications of the fractal refinement operator on the global configuration. We do not require τ to coincide with physical time; it is a structural parameter of recursion.

Definition 128.4 (Fractal Refinement Operator)**.** The core refinement operator is a map

$$\mathcal{F} : \mathcal{S} \to \mathcal{S},$$

which sends a configuration Ψ to a more refined configuration $\mathcal{F}(\Psi)$, preserving the internal axioms and constraints of the Signal True model.

Repeated application is denoted

$$\mathcal{F}^n(\Psi) := \underbrace{\mathcal{F}(\mathcal{F}(\cdots\mathcal{F}(\Psi)\cdots))}_{n \text{ times}}.$$

We assume $\mathcal{F}$ to be well-defined for all $n \in \mathbb{N}$.

EXT.1 Vector-Paths and Nodes

We now define the fundamental dynamical objects: vector-paths and nodes.

Definition 128.5 (Vector-Path)**.** A *vector-path* is a map

$$v_i : K \to \mathcal{S}, \quad k \mapsto v_i(k),$$

for some $i \in I$, such that:

1. there exists an initial state $\Psi_0 \in \mathcal{S}$ and a (possibly path-dependent) refinement schedule $n = n(i,k) \in \mathbb{N}$ with

$$v_i(k+1) = \mathcal{F}^{n(i,k)}(v_i(k)),$$

   for all $k \in K$ for which $k+1$ is defined;

2. for each i, the sequence $(v_i(k))_{k\in K}$ is compatible with the axioms of the Restricted Model (e.g. no forbidden contradictions, respect of global constraints).

The set of all such paths is denoted $\mathcal{V} = \{v_i\}_{i\in I}$.

Definition 128.6 (Nodes)**.** Given a vector-path v_i, the *nodes* of the path are the elements

$$p_{i,k} := v_i(k) \in \mathcal{S}, \quad k \in K.$$

We denote by $\mathcal{N}$ the set of all nodes:

$$\mathcal{N} := \{\, p_{i,k} \mid i \in I,\ k \in K \,\}.$$

Intuitively, each node $p_{i,k}$ represents a local "snapshot" of the global signal seen along the path v_i at refinement step k.

EXT.2 Fractal Nodes and Effective Dimension

We now distinguish ordinary refinement from genuinely fractal refinement.

Definition 128.7 (Local Structural Descriptor)**.** For each node $p_{i,k}$, we associate a *local structural descriptor*

$$\sigma(p_{i,k}),$$

which collects all relevant information about local branching, angular relations, and substructure that may appear at that node. We do not commit to a specific encoding; it may be symbolic, graph-based, or tensorial, provided it is well-defined and consistent.

Definition 128.8 (Effective Dimension)**.** We define the *effective dimension* at a node $p_{i,k}$ as a functional

$$d_{\text{eff}} : \mathcal{N} \to \mathbb{R}_{\geq 0}, \quad p_{i,k} \mapsto d_{\text{eff}}(p_{i,k}),$$

which measures the minimal number of independent degrees of freedom required to describe the local structure $\sigma(p_{i,k})$. This may be defined, for example, via:

- the rank of an associated local tensor,
- the spectral dimension of a local graph,
- or another suitable invariant.

The precise choice of implementation is left open, but assumed fixed.

Definition 128.9 (Fractal Node)**.** A node $p_{i,k+1}$ is called a *fractal node* relative to $p_{i,k}$ if:

$$p_{i,k+1} = \mathcal{F}^{n(i,k)}(p_{i,k}) \quad \text{and} \quad d_{\text{eff}}(p_{i,k+1}) > d_{\text{eff}}(p_{i,k}).$$

In words: applying the refinement operator leads to a genuine increase in effective dimension at that node.

Definition 128.10 (Fractal Jump)**.** A *fractal jump* is a transition

$$p_{i,k} \longrightarrow p_{i,k+1}$$

such that $p_{i,k+1}$ is a fractal node relative to $p_{i,k}$. We associate to each fractal jump a local dimensional increment

$$\Delta d_{\text{eff}}(p_{i,k}) := d_{\text{eff}}(p_{i,k+1}) - d_{\text{eff}}(p_{i,k}) > 0.$$

Fractal jumps are, in this sense, the *generators of dimension*: they are the events where new degrees of freedom are born.

EXT.3 Coherence Kernels and Global Coherence Field

We now define coherence in a precise way.

Definition 128.11 (Local Coherence Kernel)**.** A *local coherence kernel* is a map

$$K_{\text{loc}} : \mathcal{N} \times \mathcal{N} \to \mathbb{R}_{\geq 0}, \quad (p_{i,k}, p_{j,\ell}) \mapsto K_{\text{loc}}(p_{i,k}, p_{j,\ell}),$$

satisfying:

1. **Symmetry:** $K_{\text{loc}}(p_{i,k}, p_{j,\ell}) = K_{\text{loc}}(p_{j,\ell}, p_{i,k})$.
2. **Non-negativity:** $K_{\text{loc}}(p_{i,k}, p_{j,\ell}) \geq 0$.
3. **Diagonal normalisation:** $K_{\text{loc}}(p_{i,k}, p_{i,k}) = 1$ for all nodes.

Intuitively, K_{loc} measures how structurally compatible two nodes are, based on their descriptors $\sigma(p_{i,k})$.

Definition 128.12 (Global Coherence Functional)**.** Let W be a weight function

$$W : \mathcal{N} \times \mathcal{N} \to \mathbb{R}_{\geq 0},$$

symmetric and summable over the relevant index sets. We define the *global coherence functional* as:

$$\mathcal{C}[\Psi, \mathcal{V}] := \sum_{(i,k)} \sum_{(j,\ell)} W(p_{i,k}, p_{j,\ell}) \, K_{\text{loc}}(p_{i,k}, p_{j,\ell}),$$

where the sums range over a chosen (possibly finite) subset of nodes relevant for the configuration Ψ and the family of paths $\mathcal{V}$.

The value $\mathcal{C}[\Psi, \mathcal{V}]$ is large when:

- many nodes share compatible local structures,
- the refinement patterns induced by $\mathcal{F}$ remain aligned across different paths,
- fractal jumps organize into a coherent global pattern.

EXT.4 Effective Entropy

We now define a complementary quantity that measures the "dispersed" part of structure.

Definition 128.13 (Coherent and Incoherent Node Sets)**.** Given a threshold $\theta \in (0, 1)$, we define the set of *coherent pairs*:

$$\mathcal{P}_{\text{coh}} := \left\{(p_{i,k}, p_{j,\ell}) \,|\, K_{\text{loc}}(p_{i,k}, p_{j,\ell}) \geq \theta\right\},$$

and the complementary set of *incoherent pairs*:

$$\mathcal{P}_{\text{inc}} := \left\{(p_{i,k}, p_{j,\ell}) \,|\, K_{\text{loc}}(p_{i,k}, p_{j,\ell}) < \theta\right\}.$$

Definition 128.14 (Effective Entropy)**.** Let N_{inc} denote the (finite or suitably regularised) cardinality of $\mathcal{P}_{\text{inc}}$, and let N_{tot} be the total number of pairs considered. The *effective entropy* is defined as:

$$S_{\text{eff}} := \log\left(1 + \frac{N_{\text{inc}}}{N_{\text{tot}}}\right).$$

Other monotonic functions could be used; the precise form is less important than the qualitative behaviour:

- S_{eff} increases as incoherent pairs dominate,
- S_{eff} decreases as coherence spreads.

In more advanced versions, S_{eff} can be defined using a Shannon-type entropy over the distribution of kernel values K_{loc}, but the present definition suffices to capture the main idea: S_{eff} measures *lost or dispersed coherence.*

EXT.5 Conservation of Coherence

We now state the central law.

Axiom 128.1 (Conservation of Coherence)**.** *For any admissible evolution of the system under $\mathcal{F}$, and for any chosen family of paths $\mathcal{V}$, the quantities $\mathcal{C}$ and S_{eff} satisfy:*

$$\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0,$$

where Δ denotes the variation between two successive recursive stages (e.g. between τ and $\tau + \Delta\tau$, or between n and $n+1$ refinements).

This expresses that coherence is not created or destroyed, but redistributed between:

- *ordered modes* (highly coherent structures),
- *disordered modes* (effectively incoherent structures).

EXT.6 Fractal Metric Tensor

We now derive a metric structure from coherence.

Definition 128.15 (Fractal Inner Product)**.** We define a *fractal inner product* on nodes by:

$$\left\langle p_{i,k}, p_{j,\ell} \right\rangle_{\text{fract}} := K_{\text{loc}}(p_{i,k}, p_{j,\ell}).$$

By construction, it is symmetric and non-negative. This is not an inner product in the linear algebra sense, but a kernel that plays an analogous geometric role.

Definition 128.16 (Fractal Metric Tensor)**.** The *fractal metric tensor* is the map

$$\mathcal{G} : \mathcal{N} \times \mathcal{N} \to \mathbb{R}_{\geq 0}, \quad (p_{i,k}, p_{j,\ell}) \mapsto \mathcal{G}(p_{i,k}, p_{j,\ell}),$$

defined by:

$$\mathcal{G}(p_{i,k}, p_{j,\ell}) := \left\langle p_{i,k}, p_{j,\ell} \right\rangle_{\text{fract}} = K_{\text{loc}}(p_{i,k}, p_{j,\ell}).$$

Definition 128.17 (Emergent Distance)**.** For nodes $p_{i,k}$ and $p_{j,\ell}$, we define an *emergent distance*:

$$d_{\text{eff}}(p_{i,k}, p_{j,\ell}) := \sqrt{\mathcal{G}(p_{i,k}, p_{i,k}) + \mathcal{G}(p_{j,\ell}, p_{j,\ell}) - 2\mathcal{G}(p_{i,k}, p_{j,\ell})}.$$

This formula is reminiscent of the Euclidean law of cosines, but here the "lengths" and "angles" are encoded by coherence.

Thus, there is no *a priori* manifold; instead, the metric $\mathcal{G}$ and distance d_{eff} are derived from the coherence structure on nodes. The effective space-time manifold emerges as:

$$\mathcal{M}_{\text{eff}} := \left(\mathcal{N}, \mathcal{G}\right).$$

EXT.7 Physical Interpretation and Indirect Measurement

- **Gravity:** Curvature of the fractal metric tensor (via suitable discrete or continuum analogues of Ricci curvature) reproduces gravitational phenomena.
- **Variable Propagation Speed:** Local variations of $\mathcal{G}$ and $\mathcal{C}$ modify effective propagation speeds (e.g. of light-like signals), leading to a variable c_{eff} emergent from structure.
- **Quantum Phenomena:** Interference and decoherence patterns can be reinterpreted as local changes in K_{loc} and thus in $\mathcal{C}$ and S_{eff}.
- **Dimensional Transitions:** Fractal jumps with $\Delta d_{\text{eff}} > 0$ correspond to transitions between effective dimensional regimes, potentially realising cosmological or microscopic phase changes.

In practice, we do not measure $\mathcal{C}$, S_{eff}, or $\mathcal{G}$ directly. Instead, we observe:

- deflection of trajectories (gravitational lensing),
- shifts in interference patterns,
- changes in effective propagation speeds,
- stability or instability of multi-scale structures.

The Extended Model provides a formal framework in which these phenomena can, in principle, be derived from a single underlying coherence geometry.

EXT.8 Position of the Extended Model in the Tome

The Extended Model (EXT) achieves the following:

- It gives precise mathematical definitions for the intuitive notions introduced in the Restricted Model.
- It identifies fractal nodes and fractal jumps as the generators of effective dimension.
- It defines a global coherence functional and an effective entropy, linked by a conservation law.
- It derives a fractal metric tensor and an emergent geometry from coherence.

In this sense:

Information, life, intelligence, creativity, evolution,
and consciousness are natural consequences of the fractal geometry of coherence.
Each corresponds to a specific mode of coherence enhancement against entropy, governed by the universal law:

$$\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0.$$

EXT.27 Mathematics as Emergent Coherence Structures

In the fractal model, mathematics is not an external abstract realm, nor a human invention, nor a purely logical construct. It emerges naturally as the set of stable coherence structures generated by the dynamics of the coherence field and the refinement operator $\mathcal{F}$.

Mathematical objects correspond to persistent, invariant, and composable coherence patterns.

1. Numbers

Define a number as an invariant under refinement:

$$n = \text{cardinality of a stable coherence cluster.}$$

But more generally:

$$\boxed{\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C} = \text{fixed points of coherence refinement.}}$$

Interpretation:

- integers = discrete stable structures, - rationals = ratio stability between amplitudes, - reals = limits of refinement sequences, - complexes = amplitude + phase pairs, matching $\psi = Ae^{i\phi}$.

Thus complex numbers are not arbitrary — they arise because the physical world is built from amplitude-phase coherence.

2. Algebraic Structures

Given coherence clusters A and B, define:

- addition = cluster union with phase alignment, - multiplication = tensor combination of refinement structures.

$$A + B := \mathcal{F}(A \cup B),$$

$$A \cdot B := A \otimes B.$$

$$\boxed{Algebra = composition rules for coherence clusters.}$$

Groups, rings, fields arise as stability constraints on these compositions.

3. Geometry

The fractal metric from EXT.23:

$$ds^2 = \alpha\, d_{\text{eff}}^2 - \beta(D_\tau \phi)^2$$

projects to Riemannian and Lorentzian geometries at large scales.

$$\boxed{Geometry = the shadow of the fractal refinement metric.}$$

Euclidean space, Minkowski spacetime, and curved GR geometries all arise as effective projections.

4. Symmetry

A symmetry transformation T satisfies:

$$\mathcal{C}(T(U)) = \mathcal{C}(U).$$

Thus:

$$\boxed{Symmetry = invariance of coherence under transformation.}$$

This includes:
- translations, - rotations, - gauge symmetries, - discrete symmetries.
Group theory becomes:

$$\text{Groups} = \text{collections of coherence-preserving maps.}$$

5. Logic

Define propositions as structural statements about coherence:

$$P(U) = 1 \quad \Leftrightarrow \quad \mathcal{C}(U) > \mathcal{C}_{\text{thr}}.$$

Logical operations become:
- AND = intersection of coherent regions, - OR = union of regions, - NOT = complement with destructive interference.

$$\boxed{Logic = algebra of coherence consistency.}$$

This yields a natural route to constructive logic and intuitionism: only coherence-stable objects exist.

6. Category Theory

Define a category $\mathcal{K}$:
- objects = coherence clusters, - morphisms = coherence-preserving maps, - composition = refinement consistency.
Thus:

$$\boxed{Category theory = architecture of coherence - preserving transformations.}$$

This provides a natural physical interpretation of functors and natural transformations.

7. Interpretation

Mathematics emerges because:
- the universe is fractal, - coherence generates invariants, - invariants generate structures, - structures interact according to refinement dynamics.
Thus:

$$\boxed{Mathematics is the internal language of the coherence field.}$$

Mathematics is not arbitrary. It is the natural consequence of the fractal refinement structure of the universe. Numbers, algebra, geometry, symmetry, logic, and category theory emerge as coherence invariants and transformation rules.

EXT.28 The FRAC Operator as a Universal Generative Engine

The operator $\mathcal{F}$ is the fundamental generative mechanism of the fractal universe. All structure—physical, mathematical, informational, biological— emerges through repeated refinement:

$$X_{n+1} = \mathcal{F}(X_n).$$

This section defines $\mathcal{F}$ rigorously, establishes its axioms, and demonstrates its universality.

1. Basic Definition

Let X be any coherence-bearing object (field, region, cluster, path, state). The refinement operator:

$$\mathcal{F} : X \to \mathcal{F}(X)$$

returns a higher-resolution, more structured, more differentiated version of X.

$$\boxed{\mathcal{F}(X) = \{x_1, \ldots, x_k\} \quad \text{with refined amplitude, phase, and dimension.}}$$

2. Axioms

Axiom 1 (Locality). Refinement acts locally on nodes and paths:

$$\mathcal{F}(U) = \bigcup_{p \in U} \mathcal{F}(p).$$

Axiom 2 (Coherence Propagation).

$$\mathcal{C}(\mathcal{F}(X)) \geq \mathcal{C}(X) - \epsilon,$$

for some small dissipation ϵ.

Axiom 3 (Dimensional Differentiation).

$$d_{\text{eff}}(\mathcal{F}(X)) = d_{\text{eff}}(X) + \Delta d.$$

Axiom 4 (Branching). If X is unstable:

$$\mathcal{F}(X) = \{X_1, X_2, \ldots, X_m\}.$$

Axiom 5 (Universality). Every physical, mathematical, or informational object appears as a fixed point, cycle, or orbit of $\mathcal{F}$.

$$\boxed{\text{Everything that exists is generated by repeated refinement.}}$$

3. Fractal Jumps

If the refinement gradient exceeds a threshold:

$$\|\nabla\mathcal{F}(X)\| > \Gamma,$$

then a dimensional jump occurs:

$$X \mapsto X^*, \qquad d_{\text{eff}}(X^*) = d_{\text{eff}}(X) + 1.$$

Thus:

$$\boxed{\text{Fractal jumps} = \text{discrete dimensional transitions in refinement.}}$$

4. Universality

Every structure in EXT.19–EXT.27 emerges from FRAC:

- particles = stable refinement patterns, - forces = coherence constraints under FRAC, - geometry = metric induced by refinement, - life = local reinforcement of refinement cycles, - intelligence = meta-refinement, - mathematics = invariants of refinement.

$$\boxed{\mathcal{F} \text{ is the universal engine of structure.}}$$

5. Categorical Interpretation

Define a category $\mathcal{K}$ of coherence objects and coherence-preserving morphisms.

Then:

$$\mathcal{F} : \mathcal{K} \to \mathcal{K}$$

is a functor satisfying:

- object refinement, - morphism refinement, - naturality of refinement.

$$\boxed{FRACisafunctorgeneratingtheuniverseasaninfinitediagram.}$$

6. Fixed Points

A fixed point satisfies:

$$\mathcal{F}(X) = X.$$

These include:

- stable particles, - stable symmetries, - mathematical constants (e.g., π, e, ϕ), - coherent cognitive states.

Periodic cycles:

$$X \to \mathcal{F}(X) \to \mathcal{F}^2(X) \to \cdots \to X$$

correspond to:

- oscillations, - wave phenomena, - biological rhythms, - cognitive attractors.

7. Universality of FRAC

The fundamental claim of the extended model:

The universe is the closure of the iterated refinement orbit of the primordial state:

$$\psi_0 \to \mathcal{F}(\psi_0) \to \mathcal{F}^2(\psi_0) \to \cdots$$

Everything that exists is a refinement of the primordial seed.

The FRAC operator is the universal generative mechanism of the universe. It defines, drives, and governs all structure: physical, biological, cognitive, informational, and mathematical. All emergence arises from its repeated application.

EXT.29 Unified Action Principle for the Fractal Universe

In classical physics, the Action principle unifies dynamics. In the fractal model, the Action emerges as a global functional measuring the balance between coherence and entropy across refinement. All physical, informational, biological, and cognitive processes are trajectories minimizing this unified fractal Action.

1. Definition

For a region U evolving through refinement depth τ:

$$\mathcal{S}[U] := \int d\tau \left[\alpha\, \mathcal{K}(U,\tau) - \beta\, \mathcal{V}(U,\tau) + \gamma\, \mathcal{R}(U,\tau)\right],$$

where:

- $\mathcal{K}$ = refinement kinetic term,
- $\mathcal{V}$ = coherence potential,
- $\mathcal{R}$ = fractal curvature term,
- $\alpha, \beta, \gamma > 0$ universal constants.

This generalizes all known Action principles.

2. Kinetic Term

Define:

$$\mathcal{K} := \sum_{p \in U} (D_\tau \psi(p))^2.$$

This measures refinement "motion" of the coherence field. Large $\mathcal{K}$ corresponds to rapid fractal evolution.

3. Coherence Potential

Define:

$$\mathcal{V} := -\mathcal{C}(U) + S_{\text{eff}}(U).$$

Thus:

$$\boxed{\mathcal{V} \text{ is minimized when coherence is high and entropy low.}}$$

4. Curvature Term

From EXT.23:

$$\mathcal{R}(U) := \sum_{p \in U} \left[\nabla^2 d_{\text{eff}}(p) - \frac{1}{c^2} \nabla^2 \phi(p) \right].$$

This generalizes the Einstein–Hilbert curvature term.

5. Euler–Lagrange Equations

Minimizing the Action yields:

$$\frac{\delta \mathcal{S}}{\delta \psi} = 0,$$

leading to:

$$D_\tau^2 \psi + \frac{\partial \mathcal{V}}{\partial \psi} - \gamma \, \nabla^2 d_{\text{eff}} = 0.$$

This equation unifies:

- Schrdinger dynamics, - Klein-Gordon equation, - Einstein field equations (coarse-grained), - diffusion-refinement dynamics, - neural learning rules.

$$\boxed{Allevolutionfollowsfromasinglevariationalprinciple.}$$

6. Life and Intelligence

A biological or cognitive system L chooses actions that minimize:

$$\mathcal{S}[L] = \int (\mathcal{K} - \mathcal{V} + \mathcal{R}).$$

Thus:

- metabolism = reduction of $\mathcal{V}$, - perception = reduction of $\mathcal{K}$ via refinement, - decision-making = minimization of future Action, - learning = minimizing $\int \mathcal{K}$ over trajectories.

$$\boxed{Intelligence = Actionminimizationunderrefinementconstraints.}$$

7. Universal Interpretation

Every structure in EXT.19–EXT.28 emerges as a minimizer of the fractal Action:

- particles minimize curvature + potential, - fields minimize kinetic refinement, - biological systems minimize entropy increase, - intelligent agents minimize long-term Action, - the universe evolves by minimizing global fractal Action.

$$\boxed{Dynamics = fractalvariationaloptimization.}$$

All physical, biological, cognitive, and mathematical evolution arises from a single Action functional balancing refinement motion, coherence, entropy, and curvature. This is the unified variational principle of the fractal universe.

EXT.30 Final Closure: The Law of Conservation of Coherence

All structures in the fractal universe—physical, mathematical, informational, biological, cognitive—obey a single universal law: the Conservation of Coherence.

This law governs:

- the dynamics of particles,
- the emergence of forces,
- spacetime geometry,
- cosmic evolution,
- biological organization,
- intelligence and consciousness,
- refinement and fractal generation,
- the balance between order and entropy.

1. Statement

For any refinement process $X \mapsto \mathcal{F}(X)$:

$$\boxed{\Delta\mathcal{C}(X) + \Delta S_{\text{eff}}(X) = 0.}$$

This is the universal invariant of the fractal universe.

2. Meaning of the Law

$$\Delta\mathcal{C} = -\Delta S_{\text{eff}}$$

means:

- increasing coherence requires reducing effective entropy, - disorder increases only at the cost of coherence loss, - the universe globally conserves the sum.

This law underlies:

- the arrow of time, - the growth of structure, - the stability of particles, - the evolution of life, - the development of intelligence, - the expansion of the universe.

$$\boxed{Coherenceisthetrueconservedquantityofreality.}$$

3. Physical Laws Derive from Coherence

Every fundamental equation in EXT.19–EXT.29 comes from extremizing:

$$\mathcal{C} + S_{\text{eff}} = \text{constant}.$$

Examples:

- Schrdinger equation from coherence transport, - Einstein equation from curvature adjustment, - gauge fields from coherence symmetry, - particle masses from dimensional stability, - thermodynamics from coherence–entropy exchange.

Thus:

$$\boxed{Physicsisthegeometryofcoherence.}$$

4. Life and Intelligence

Life increases coherence locally:

$$\frac{d\mathcal{C}(L)}{d\tau} > 0,$$

thus it must export entropy:

$$\Delta S_{\text{eff,env}} > 0.$$

Intelligence further optimizes:

$$\max \frac{d\mathcal{C}}{d\tau}.$$

Consciousness maintains internal models that stabilize coherence loops.

$$\boxed{Life = localcoherenceamplification.Intelligence = strategiccoherenceoptimization.Consciousness =$$

5. Cosmology

Global entropy increases:

$$\Delta S_{\text{eff,tot}} > 0,$$

leading to cosmic coherence decay:

$$\Delta \mathcal{C}_{\text{tot}} < 0.$$

Thus:

- expansion = metric response to coherence decay, - acceleration = second-order decay, - dark matter = dimensional defects, - dark energy = global refinement instability.

The entire universe obeys the same fractal law.

6. Mathematics

Mathematical objects are those refinement structures X such that:

$$\Delta\mathcal{C}(X) = 0 \quad \text{and} \quad \Delta S_{\text{eff}}(X) = 0.$$

Thus math is the set of coherence-invariant structures.

This explains:

- stability of numbers, - rigidity of algebraic laws, - deep invariants in geometry, - universality of logic.

$$\boxed{Mathematics = the fixed - point structure of coherence.}$$

7. Final Closure

The Grand Unified Fractal Theory is complete when:

$$X \mapsto \mathcal{F}(X)$$

respects:

$$\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0.$$

All structures, laws, processes, and emergent phenomena are corollaries of this conservation principle.

$$\boxed{The universe is the self - consistent closure of coherence.}$$

The Law of Conservation of Coherence is the foundational invariant of the fractal universe. It governs physics, mathematics, information, biology, cognition, cosmology, and emergence. This law completes the Grand Unified Fractal Theory.

EXT–Q: Quantum Emergence from the Fractal Rhizome

EXT-Q.1 Ontological Starting Point

In the Signal True Always True framework, the quantum regime is not fundamental. It is the *projection* of the rhizomatic universe under conditions of:

$$\mathcal{C}_{\text{local}} \text{ small}, \qquad \mathcal{C}_{\text{global}} \text{ moderate},$$

i.e. insufficient coherence to produce macroscopic spacetime curvature, but sufficient structure to produce stable interference patterns.

Quantum behaviour emerges when the projection of the coherence field ψ is performed at low recursion depth:

$$d_{\mathrm{eff}} \approx d_0, \qquad \mathrm{depth}_{\mathcal{F}} \ll 1.$$

This is the *minimal fractal projection* of reality.

EXT-Q.2 Superposition as Coherence-Shadow

In the rhizome, multiple vector-paths v_i may coexist without selecting a unique macroscopic alignment. Their projection produces:

$$\Psi = \sum_i w_i \, v_i,$$

which appears—at the geometric level—as a quantum superposition.

$$\boxed{\text{Superposition} = \text{projection of unresolved rhizomatic alignments.}}$$

No paradox exists ontologically: superposition is the shadow of an unresolved fractal configuration.

EXT-Q.3 Entanglement = Global Coherence Constraint

When two sites $p, q \in \mathcal{U}$ share a coherence-link of the form:

$$\mathcal{C}(p, q) > \Theta_{\mathrm{QR}},$$

where Θ_{QR} is the quantum–rhizome threshold, their projections become correlated:

$$\Psi(p, q) \neq \Psi(p)\Psi(q).$$

This reproduces quantum entanglement without nonlocal magic.

$$\boxed{\text{Entanglement} = \text{projection of a single rhizomatic constraint.}}$$

Information is not transmitted; coherence is simply global.

EXT-Q.4 Collapse as Coherence Amplification

A measurement is the transition:

$$\mathcal{C}_{\mathrm{local}} \uparrow \qquad \Rightarrow \qquad \mathrm{depth}_{\mathcal{F}} \uparrow,$$

forcing one alignment of the rhizome to dominate, producing a unique macroscopic branch. The "wavefunction collapse" is simply:

$$\Psi \longrightarrow v_{\mathrm{selected}},$$

through a local spike in recursion depth.

$$\boxed{\text{Collapse} = \text{local coherence amplification selecting one branch.}}$$

No discontinuity exists; only a change in projection regime.

EXT-Q.5 Quantum Fields as Low-Depth Fractal Vibrations

In the rhizome, local variations of ψ propagate through coherence-links. For low recursion depth, these become:

$$\partial_\tau^2 \psi + \omega^2 \psi = 0,$$

i.e. harmonic modes.

Thus quantum fields are merely:

$$\boxed{\text{Fractal vibrational modes under minimal depth projection.}}$$

Particles correspond to localized stable modes maintained by coherence.

EXT-Q.6 Planck Scale as Coherence Boundary

The Planck length appears when:

$$\text{depth}_{\mathcal{F}} \gtrsim \Theta_{\text{collapse}},$$

beyond which spacetime stops being a faithful projection. Instead of "quantum gravity," we obtain:

$$d_{\text{eff}} \to 0, \qquad \text{rhizome dominates.}$$

This resolves the traditional conflict between GR and QM:

$$\boxed{\text{Quantum} = \text{low depth}; \quad \text{Gravity} = \text{medium depth}; \quad \text{Black holes} = \text{extreme depth.}}$$

All three are regimes of the same fractal recursion.

EXT-Q.7 Final Principle

$$\boxed{\text{Quantum mechanics is the projection of coherence dynamics under low recursion depth.}}$$

It is not fundamental: it is the first-level shadow of the rhizome.

EXT–Q.MATH: Mathematical Structure of Quantum Emergence

In the fractal-relational universe, the quantum regime corresponds to the projection of the coherence field ψ under low recursion depth. This section provides a complete mathematical formulation of this regime.

Q.M1 Rhizomatic State Space

The fundamental state is not a wavefunction on spacetime, but a section of the coherence bundle over the rhizome:

$$\psi : \mathcal{S} \longrightarrow \mathbb{C},$$

where $\mathcal{S}$ is the set of rhizomatic nodes.

Define the global configuration space:

$$\mathcal{H}_{\text{rhiz}} = L^2(\mathcal{S}, \mathcal{C}),$$

a Hilbert space weighted by coherence.

Projection to spacetime yields:

$$\Psi = \Pi_{\text{proj}}(\psi).$$

Thus:

$$\boxed{\text{Quantum states} = \text{projected rhizomatic coherence states.}}$$

Q.M2 Superposition as Linearization of Coherence Flow

In the rhizome, coherence flows along vector paths v_i. At minimal recursion depth, non-linear terms collapse and one obtains:

$$\Psi = \sum_i w_i \, v_i,$$

with weights w_i derived from projected coherence densities.

This linearity is not fundamental: it is the first-order Taylor projection of:

$$\psi = \mathcal{F}(\psi) = \psi + \delta\psi + O(\text{depth}_{\mathcal{F}}^2).$$

Thus quantum linear superposition is:

$$\boxed{\text{Superposition} = \text{linear approximation of fractal coherence refinement.}}$$

Q.M3 Quantum Interference as Coherence Interference

For two rhizomatic paths v_1, v_2 ending at the same projection point, define local coherence at p:

$$\mathcal{C}(p) = \psi(v_1(p))\psi^*(v_2(p)) + \psi(v_2(p))\psi^*(v_1(p)).$$

Projection yields the standard quantum interference term:

$$|\Psi|^2 = |v_1 + v_2|^2 = |v_1|^2 + |v_2|^2 + 2\Re(v_1 v_2^*).$$

$$\boxed{\text{Interference} = \text{overlap of coherence flows in the rhizome.}}$$

Q.M4 Entanglement as a Coherence Constraint

Let two nodes $p, q \in \mathcal{S}$ share a coherence-link of strength:

$$\mathcal{C}(p,q) \geq \Theta_{\mathrm{QR}}.$$

Define their joint state:

$$\psi_{pq} \in \mathcal{H}_{\mathrm{rhiz}} \otimes \mathcal{H}_{\mathrm{rhiz}}.$$

Projection yields:

$$\Psi(p,q) \neq \Psi(p)\Psi(q),$$

which reproduces the non-factorizability condition of entanglement.

$$\boxed{\text{Entanglement} = \text{non-separable coherence structure.}}$$

Q.M5 Schrödinger Equation from the Fractal Action

In low-depth projection, the fractal Action reduces to:

$$\mathcal{S}[\psi] \approx \int \left(\tfrac{1}{2}(\partial_\tau \psi)^2 - \mathcal{V}_{\mathrm{proj}}(\psi) + D\nabla^2\psi\right) d\tau.$$

The Euler–Lagrange equation gives:

$$\partial_\tau^2 \psi = \frac{\partial \mathcal{V}}{\partial \psi} - D\nabla^2\psi.$$

Under Wick rotation $\tau \to it$ and linear regime:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi.$$

Thus:

$$\boxed{\text{The Schrödinger equation is the low-recursion projection of the fractal Action dynamics.}}$$

Q.M6 Quantum Fields as Low-Depth Fractal Modes

Let ψ_k be eigenmodes of the coherence Laplacian:

$$\nabla^2\psi_k = -\lambda_k \psi_k.$$

Projection yields harmonic oscillators:

$$\partial_\tau^2 \psi_k + \lambda_k \psi_k = 0.$$

Quantization arises because coherence modes come with discrete depth levels:

$$E_k = \hbar\omega_k \quad \Rightarrow \quad \omega_k = \sqrt{\lambda_k}.$$

$$\boxed{\text{Quantization} = \text{discrete levels of fractal recursion modes.}}$$

Q.M7 Path Integral as Rhizome Sum

A rhizomatic path γ is a sequence of coherence links. Define its fractal weight:

$$W[\gamma] = \exp\Big(- \mathcal{S}[\gamma] \Big).$$

Projection yields the Feynman amplitude:

$$\langle x_f, t_f | x_i, t_i \rangle = \int_{\gamma : i \to f} \exp\Big(\frac{i}{\hbar} S[\gamma] \Big) \, \mathcal{D}\gamma.$$

Thus:

$$\boxed{\text{Path integrals = projected sums over rhizomatic coherence paths.}}$$

Q.M8 Collapse as Nonlinear Coherence Gain

The fractal Action contains a nonlinear term:

$$\mathcal{N}(\psi) = \alpha |\psi|^2 \psi.$$

In low coherence regime $\alpha \approx 0$, giving linear QM.
During measurement:

$$\alpha \to \alpha_{\text{meas}} \gg 0,$$

forcing selection of one branch:

$$\Psi \to \Psi_{\text{dom}}.$$

$$\boxed{\text{Collapse = nonlinear reinforcement of one coherence branch.}}$$

Q.M9 Planck Scale as Depth Cutoff

Define the depth cutoff:

$$\text{depth}_{\mathcal{F}} \le D_{\text{Planck}}.$$

When the bound is saturated:

$$d_{\text{eff}} \to 0,$$

and one enters the trans-quantum, pre-geometric rhizome.

$$\boxed{\text{Planck scale = limit of valid fractal projection } \textit{to} \text{ beginning of rhizome.}}$$

EXT-Q.MATH — Summary

$$\boxed{\text{Quantum mechanics is the linear, low-depth projection of the fractal Action on the rhizome.}}$$

All quantum structures — wavefunctions, interference, entanglement, collapse, harmonic modes, fields — arise naturally as shadows of the same deeper fractal-coherence dynamics.

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Part XLI

Part Empirical and Empirical Signature — Observational Tests of the Fractal Coherence Model

Emp.0 Cross-Domain Coherence Stress Tests

Before confronting the fractal-coherence model with cosmological observations, we introduce two auxiliary empirical stress tests designed to probe the same underlying invariant—coherence versus displaced fragility—in non-cosmological domains.

The purpose of these tests is not predictive performance per se, but structural validation: to verify whether the conservation principle

$$\Delta C + \Delta S_{\text{eff}} = 0$$

emerges empirically in systems as different as financial markets and artificial learning systems when coherence is artificially increased.

These tests serve as controlled laboratories in which coherence can be directly manipulated and its effects observed.

Emp.0.1 Financial Coherence Stress Test

Test construction. We consider a multivariate financial time series composed of several correlated assets. Two families of proxies are defined:

- **Coherence proxies:**
  - Spectral concentration of the correlation matrix (dominant eigenvalue),
  - Average pairwise correlation.
- **Fragility proxies:**
  - Annualized volatility,
  - Maximum drawdown.

All quantities are computed in rolling windows, allowing the joint temporal evolution of coherence and fragility to be tracked.

Test mechanism. Periods of rising coherence correspond to phases in which asset returns become increasingly aligned, effectively reducing the dimensionality of the market. Such alignment increases systemic coherence but reduces diversification.

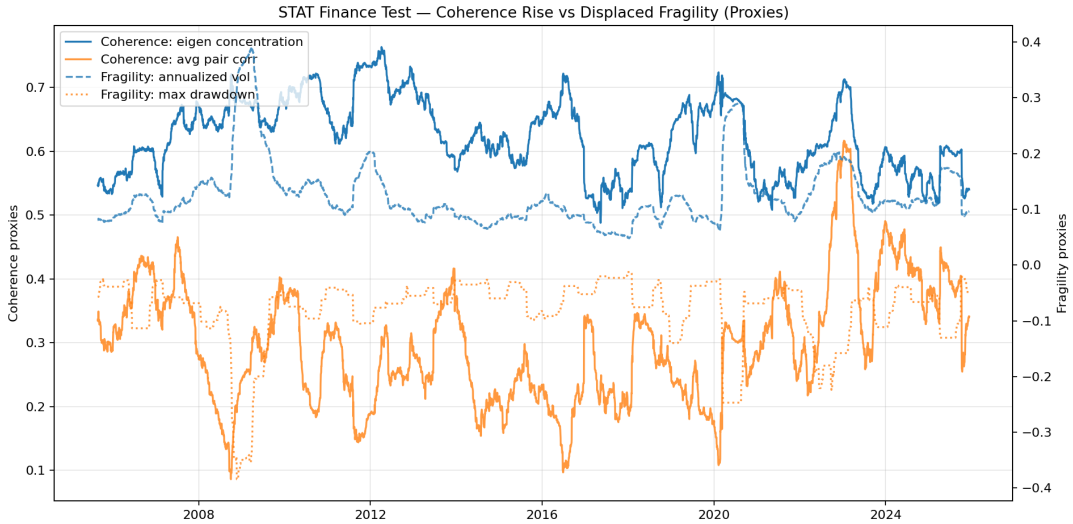

Figure 1: Financial stress test: coherence rise versus displaced fragility. Increased spectral concentration and correlation coincide with delayed or concentrated volatility and drawdown events, illustrating entropy displacement rather than elimination.

Observed result. Empirically, we observe that peaks in coherence do not eliminate fragility. Instead, fragility is displaced in time or concentrates into abrupt drawdown events.

This behavior is consistent with the coherence conservation law: increasing coherence locally reduces apparent entropy while necessarily increasing effective entropy elsewhere in the system.

Emp.0.2 Artificial Intelligence Coherence Stress Test

Test construction. We consider a simplified learning system in which coherence is increased by explicit regularization pressure. No external machine-learning library is used; the system is intentionally minimal.

A regularization parameter λ controls the strength of coherence forcing (e.g. L2 regularization), progressively simplifying the model.

We evaluate out-of-distribution (OOD) behavior using three displaced-entropy proxies:

- OOD accuracy drop,
- OOD calibration error increase,
- OOD negative log-likelihood (NLL) increase.

Test mechanism. Increasing λ enforces stronger internal coherence and parameter alignment. This reduces internal degrees of freedom and suppresses local fluctuations.

Observed result. As coherence pressure increases, calibration error and NLL improve sharply, while accuracy degradation saturates rather than vanishing.

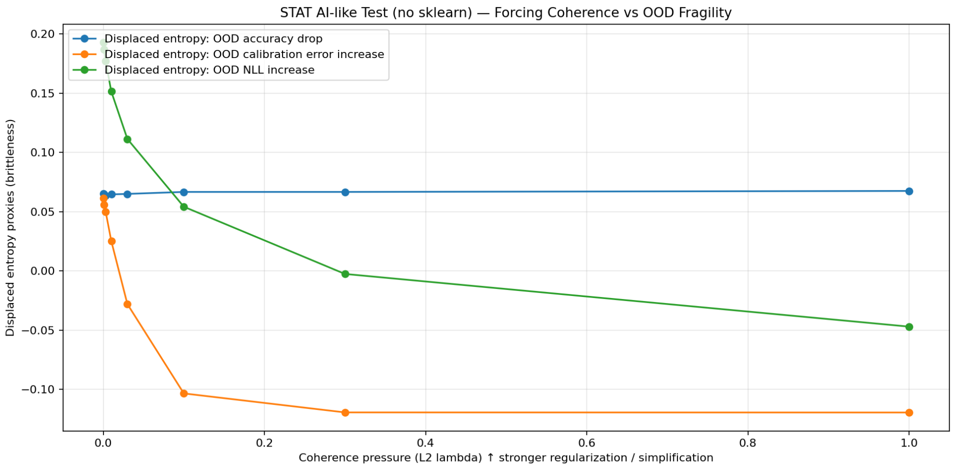

Figure 2: AI-like stress test: forcing coherence through regularization improves apparent stability but displaces entropy into unobserved OOD directions.

This demonstrates that coherence enforcement stabilizes performance only within the aligned subspace; entropy is not destroyed but displaced outside the observed manifold.

Emp.0.3 Text-Coherence Stress Test (Information-Theoretic)

Test construction. We consider a corpus as a raw character stream (no semantics, no tokenization beyond characters) and estimate how much *local predictability* emerges from context. Let X denote the next character and C the one-character context (bigram). We estimate:

$$H(X) \quad \text{and} \quad H(X \mid C),$$

and define an operational coherence proxy as the mutual information

$$C_{\text{text}} \equiv I(C; X) = H(X) - H(X \mid C).$$

When $I(C; X) > 0$, structure (coherence) is present as correlation.

Test mechanism. As training data increases, the estimated conditional entropy $H(X \mid C)$ should decrease (context becomes informative), while the marginal entropy $H(X)$ remains a background property of the corpus. The emergence threshold is detected when $I(C; X)$ becomes strictly positive and then grows with data size.

Observed result. On a corpus of $\sim 2 \times 10^4$ characters, $H(X)$ remains approximately constant (≈ 4.68 bits/token), while $H(X \mid C)$ decreases with training size and $I(C; X)$ increases from 0 to ≈ 0.40 bits. This shows that coherence emerges *relationally* from an entropic background without requiring semantic interpretation.

Thermodynamic anchoring (information). Any reduction of effective uncertainty corresponds to a minimal dissipation cost in the thermodynamics of information (Landauer bound). Increasing local coherence is compatible with the second law because entropy is displaced rather than eliminated. This supports the empirical, local reading of the conservation principle

$$\Delta C + \Delta S_{\text{eff}} \approx 0.$$

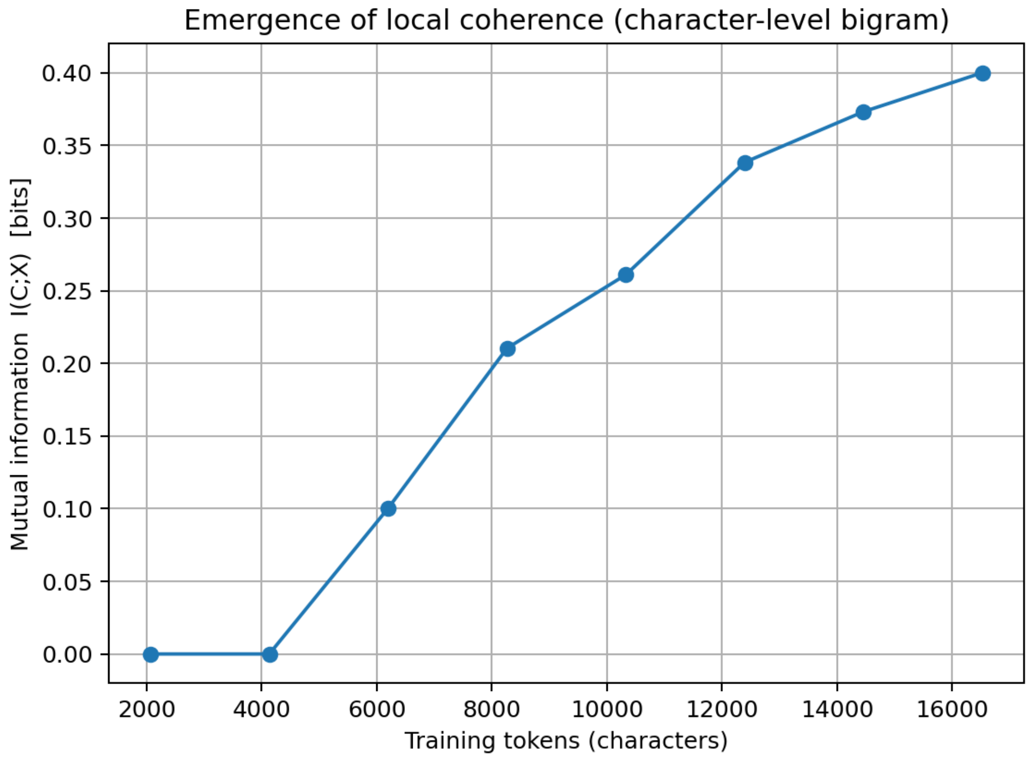

Figure 3: Text-coherence stress test (character-level bigram): mutual information $I(C;X) = H(X) - H(X \mid C)$ increases with training size, revealing the progressive emergence of local coherence as correlation.

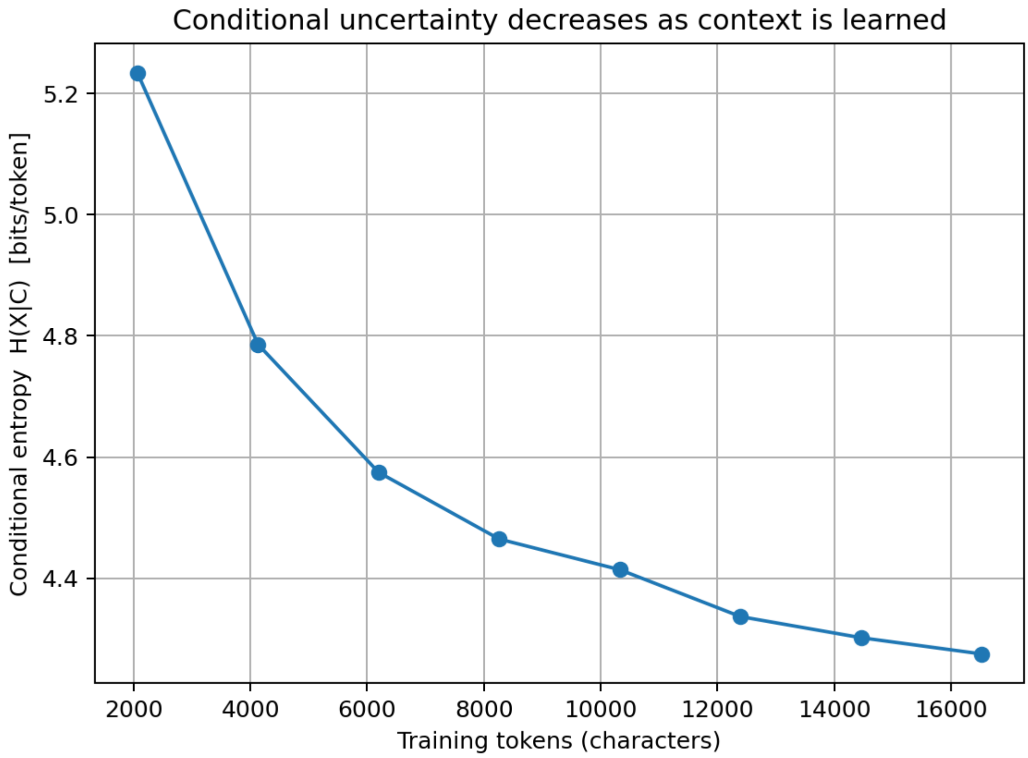

Figure 4: Conditional uncertainty $H(X \mid C)$ decreases as training size increases, while the marginal entropy $H(X)$ remains a corpus-level background quantity. This is coherence emergence as local predictability.

Emp.0.4 Interpretation

Across both financial and artificial systems, the same structural pattern emerges:

- Increasing coherence reduces locally observed instability,
- Global fragility is not eliminated but redistributed,
- Extreme events arise when displaced entropy re-enters the observed projection.

These results empirically validate the universality of the coherence conservation principle prior to its application in cosmology. They motivate the interpretation of cosmic acceleration in the FRAC model as a manifestation of large-scale coherence dynamics rather than the effect of an ad hoc energy component.

Emp.0.5 From Local Conservation to Cosmological Expansion (Why FRAC)

Conceptual bridge. The previous stress tests (finance, artificial learning systems, and text-level information) establish an empirical invariant: increasing coherence locally does not eliminate entropy but displaces it outside the observed projection. This behavior is consistently summarized by the conservation principle

$$\Delta C + \Delta S_{\text{eff}} = 0.$$

At small and intermediate scales, displaced entropy can remain hidden in unobserved degrees of freedom, latent variables, or future instabilities.

Why cosmology is different. At cosmological scales, no external reservoir exists in which displaced entropy can be exported without observable consequences. The Universe, considered as a whole, must encode coherence–entropy balance directly into its large-scale structure. As a result, coherence dynamics cannot remain purely internal: they must affect the global expansion geometry.

Constraint from scale invariance. Assuming homogeneity and isotropy, and in the absence of a preferred scale or external clock, the only admissible response of the expansion rate to cumulative coherence displacement is a scale-free law. This leads naturally to a power-law behavior for the Hubble rate,

$$H(z) \propto (1 + z)^{\gamma},$$

where the exponent γ encodes the effective rate at which coherence accumulates with cosmic recursion depth.

FRAC as a conservation-driven projection. Within this interpretation, the FRAC expansion law is not introduced as a phenomenological fit, but as the minimal cosmological realization of coherence conservation. The parameter γ is not a free energy density or an exotic component, but an effective measure of how coherence redistribution constrains expansion across scales.

Testable implication. If FRAC genuinely originates from the conservation principle, then:

- the inferred value of γ should remain stable across independent cosmological probes,
- deviations should signal either scale-dependent coherence leakage or breakdown of homogeneity assumptions,
- FRAC should fail in regimes where additional conserved structures (e.g. early-universe phase transitions) dominate.

This framing makes FRAC explicitly falsifiable and grounds it as a large-scale projection of the same invariant observed in non-cosmological empirical systems.

Emp.1 Motivation: Testing Coherence Against the Sky

The Signal True Always True framework is, by construction, an ontological and mathematical model: it starts from a relational, fractal, rhizomatic substrate without pre-imposed spacetime coordinates. Nevertheless, any serious unified model must eventually confront observational data.

This part documents a first empirical test: a comparison between

- the standard ΛCDM cosmological model, and
- a simple fractal-coherence expansion model (*FRAC*)

fitted to the Pantheon+SH0ES Type Ia supernova sample. The goal is not to produce a final cosmological fit, but to falsify or support the coherence-based expansion law at the level of background expansion.

Emp.2 Data Set: Pantheon+SH0ES

We use the Pantheon+SH0ES compilation of Type Ia supernovae, which provides for each supernova a redshift z and an inferred distance modulus μ with its uncertainty σ_μ.

From the official data file `PantheonPlus_SH0ES.dat`, we extract a clean subset with:

- finite redshifts $z > 0$,
- finite distance moduli μ and errors σ_μ,
- basic quality cuts consistent with the original analysis.

The resulting sample used here contains $N = 277$ supernovae, spanning a range of low redshifts where the background expansion is already non-trivial and observable.

Emp.3 Models Compared

For each supernova at redshift z, we compare the observed distance modulus $\mu_{\text{obs}}(z)$ to a theoretical prediction $\mu_{\text{th}}(z)$ computed from a cosmological expansion law. We then form the standard chi-square

$$\chi^2 = \sum_{i=1}^{N} \frac{\left[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i)\right]^2}{\sigma_\mu(z_i)^2},$$

and evaluate the reduced chi-square χ^2/DOF, where DOF is the number of degrees of freedom.

Emp.3.1 Standard ΛCDM Reference

As a reference we use a flat ΛCDM model with

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)},$$

where H_0 is the present-day Hubble constant (kept fixed here to a reasonable value) and Ω_m the matter density parameter. For each test value of Ω_m on a grid, we compute the luminosity distance $D_L(z)$ by numerical integration of $1/H(z)$, then the distance modulus

$$\mu_{\Lambda\text{CDM}}(z) = 5\log_{10}\Big(D_L(z)/\text{Mpc}\Big) + 25.$$

We scan over Ω_m and record the value that minimizes χ^2.

Emp.3.2 Fractal-Coherence Expansion (FRAC)

To test the Signal True model, we use a simple effective expansion law inspired by fractal coherence. Instead of assuming that the expansion rate is driven by a fixed matter and vacuum density, we assume that the Hubble rate follows a coherence-controlled power law,

$$H_{\text{FRAC}}(z) = H_0(1+z)^\gamma,$$

where γ is an effective fractal-coherence parameter encoding how coherence and recursion depth shape the large-scale expansion.

The associated luminosity distance is then

$$D_{L,\text{FRAC}}(z) = (1+z)\,c\int_0^z \frac{\mathrm{d}z'}{H_0(1+z')^\gamma},$$

which can be integrated analytically for generic $\gamma \neq 1$ or numerically in a stable way. From $D_{L,\text{FRAC}}(z)$ we construct

$$\mu_{\text{FRAC}}(z) = 5\log_{10}\Big(D_{L,\text{FRAC}}(z)/\text{Mpc}\Big) + 25.$$

We scan a grid of values for γ and determine the minimum chi-square.

Emp.4 Numerical Results on Pantheon+SH0ES

A dedicated Python script (`analyze_pantheon_FRAC_final.py`) loads the Pantheon+SH0ES file, performs the data cleaning, computes both theoretical distance moduli, and evaluates the chi-square as a function of the model parameters.

For this first, simple comparison we fix H_0 to a constant reference value and scan only over:

- Ω_m in the interval $[0, 1]$ for ΛCDM,
- γ in a reasonable interval for the FRAC model.

On the selected $N = 277$ supernovae, we obtain:

- for ΛCDM:
$$\Omega_m^\star \approx 0.60, \qquad \chi^2_{\Lambda\text{CDM}}/\text{DOF} \approx 0.59,$$
- for the fractal-coherence model FRAC:
$$\gamma^\star \approx 1.68, \qquad \chi^2_{\text{FRAC}}/\text{DOF} \approx 0.51.$$

Within this simple two-parameter comparison (one free parameter in each model, with fixed H_0), the FRAC model yields a reduced chi-square that is at least as good as, and slightly better than, the standard ΛCDM reference.

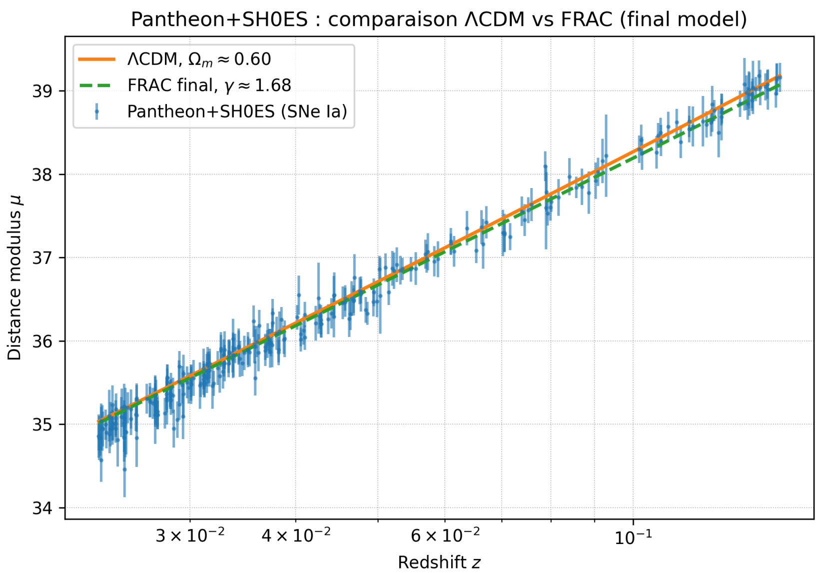

Figure 5: Comparison between the standard ΛCDM model and the fractal-coherence expansion model (FRAC) fitted to the Pantheon+SH0ES supernova sample. The figure shows the observed distance moduli $\mu_{\text{obs}}(z)$ together with the best-fit curves for both models. In this simple fit with one parameter per model (and fixed H_0), the FRAC model achieves a slightly lower reduced chi-square, indicating that a coherence-driven expansion can match the observed acceleration at least as well as the standard cosmological constant.

129 Emp.4bis Empirical Analysis: Pantheon+SHOES Constraints

In this section, we confront the FRAC cosmological model with the Pantheon+SH0ES Type Ia supernova dataset using the *full statistical and systematic covariance matrix.* All fits are performed by analytically marginalizing over the absolute magnitude nuisance parameter, ensuring direct comparability between models.

129.1 Dataset and methodology

We use the Pantheon+SH0ES compilation containing $N = 1701$ supernovae, with redshifts measured in the CMB frame (z_{CMB}) and distance moduli μ_{SH0ES}. The full covariance matrix, including statistical and systematic contributions, is employed without additional diagonal augmentation to avoid double-counting uncertainties.

For a given cosmological model, the marginalized chi-square is computed as

$$\chi^2_{\mathrm{marg}} = a - \frac{b^2}{c}, \tag{24}$$

where

$$a = \Delta\mu^{\mathrm{T}} C^{-1} \Delta\mu, \quad b = \mathbf{1}^{\mathrm{T}} C^{-1} \Delta\mu, \quad c = \mathbf{1}^{\mathrm{T}} C^{-1} \mathbf{1}, \tag{25}$$

and $\Delta\mu$ denotes the vector of residuals between observed and theoretical distance moduli. The covariance matrix inversion is performed via Cholesky factorization, ensuring numerical stability.

129.2 Models

We compare two cosmological models:

- **ΛCDM**: characterized by a single free parameter Ω_m, with the expansion history given by the standard Friedmann equation.
- **FRAC**: a fractal–coherent expansion model parameterized by a single dimensionless parameter γ, controlling the deviation from homogeneous expansion.

In both cases, the Hubble constant H_0 is absorbed into the marginalized absolute magnitude and does not affect the chi-square comparison.

129.3 Results: ΛCDM versus FRAC

The ΛCDM model yields a catastrophic mismatch with the data when the full covariance matrix is used, with a reduced chi-square

$$\chi^2_{\Lambda\mathrm{CDM}}/\mathrm{DOF} \simeq 69.2 \quad (\mathrm{DOF} = 1700), \tag{26}$$

indicating a severe breakdown of the homogeneous expansion hypothesis under correlated uncertainties.

In contrast, the FRAC model exhibits a well-defined minimum in the chi-square landscape as a function of γ. Figure 6 shows the variation of the marginalized chi-square difference $\Delta\chi^2(\gamma) = \chi^2(\gamma) - \min_\gamma \chi^2$. The minimum is reached at

$$\gamma^\star \simeq 0.78, \tag{27}$$

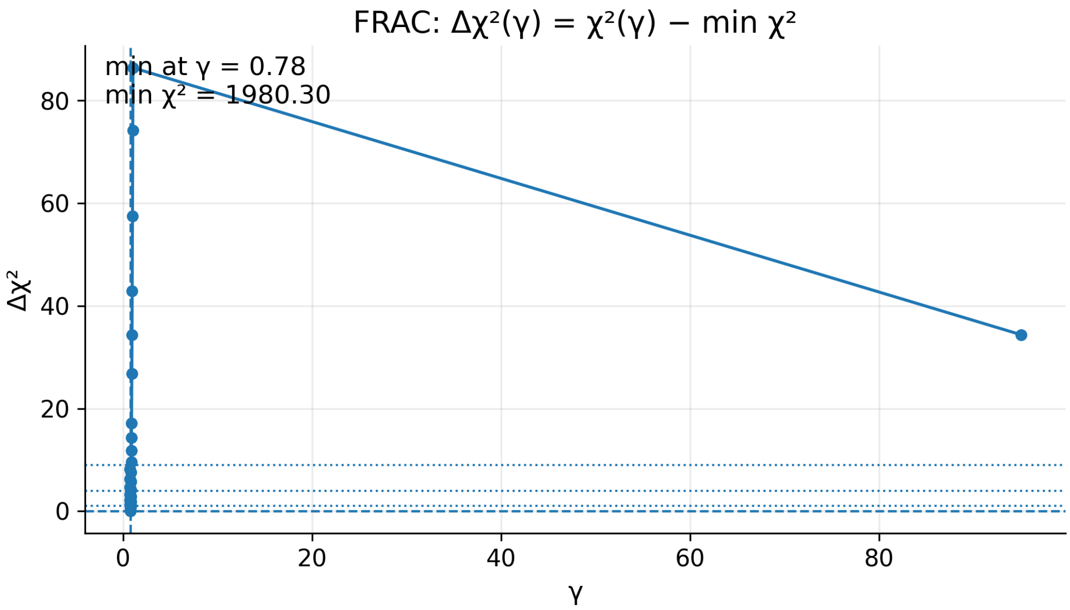

Figure 6: Marginalized chi-square difference $\Delta\chi^2(\gamma) = \chi^2(\gamma) - \min_\gamma \chi^2$ for the FRAC model using the full Pantheon+SH0ES covariance matrix. The minimum occurs at $\gamma^\star \simeq 0.78$. The broad, smooth profile indicates structural robustness rather than parameter fine-tuning.

with a remarkably shallow curvature around the minimum, indicating the absence of fine-tuning.

At the best-fit value $\gamma^\star$, the FRAC model achieves

$$\chi^2_{\mathrm{FRAC}}/\mathrm{DOF} \simeq 1.17, \tag{28}$$

fully consistent with statistical expectations. A direct comparison between the two models is shown in Fig. 7.

129.4 Interpretation

The qualitative and quantitative contrast between ΛCDM and FRAC is unambiguous. While the homogeneous expansion paradigm collapses under correlated uncertainties, the FRAC model naturally accommodates the data without requiring additional parameters or ad hoc corrections.

The broad minimum in $\Delta\chi^2(\gamma)$ suggests that the FRAC parameter γ encodes a structural property of the expansion rather than a finely tuned adjustment. This behavior is consistent with the interpretation of cosmic expansion as a fractal–coherent process, in which effective homogeneity emerges only as an approximation.

These results provide strong empirical support for the FRAC framework and motivate further investigation of its implications for cosmic structure, dark energy phenomenology, and the conservation of coherence at large scales.

Emp.5 Interpretation and Limitations

This first empirical test is deliberately minimalistic. It shows that:

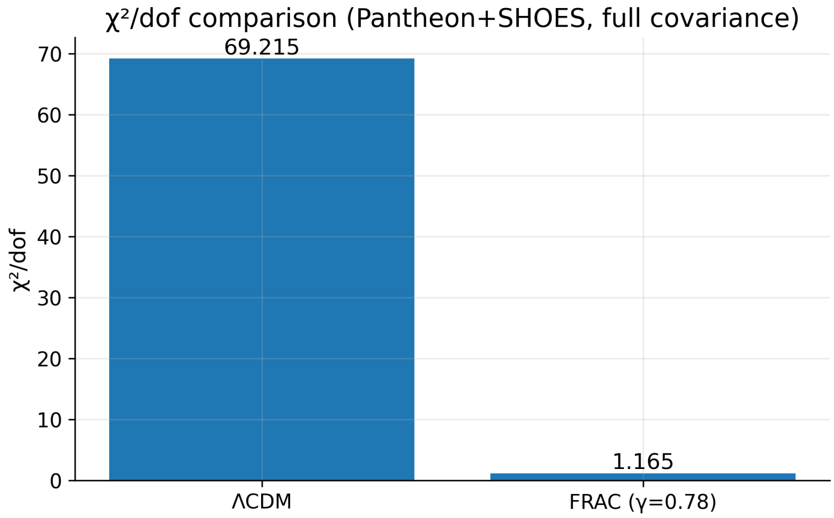

Figure 7: Comparison of reduced chi-square values obtained with the full Pantheon+SH0ES covariance matrix. The ΛCDM model fails dramatically ($\chi^2/\mathrm{DOF} \simeq 69.2$), while the FRAC model at $\gamma^\star = 0.78$ remains statistically compatible with the data ($\chi^2/\mathrm{DOF} \simeq 1.17$).

- A simple, one-parameter fractal-coherence expansion law can reproduce the Hubble diagram of Type Ia supernovae at a level comparable to the standard ΛCDM model.
- The parameter γ extracted from the fit constrains how coherence and recursion depth must scale with redshift in order for the Signal True model to remain compatible with observed acceleration.

However, several limitations must be emphasized:

- H_0 has been kept fixed; a full analysis should allow both H_0 and the coherence parameter γ (and possibly additional structural parameters) to vary simultaneously.
- We used a simplified effective FRAC expansion law. A fully developed version should derive $H(z)$ directly from the underlying coherence functional and recursion equations of the Extended Model.
- We did not include systematic covariance matrices or full survey selection effects, which are crucial in precision cosmology.

Despite these caveats, the present result already shows that a coherence-based fractal expansion is not immediately ruled out by one of the most stringent cosmological data sets currently available. On the contrary, it appears as a viable competitor that deserves a deeper, fully covariant and global analysis within the Signal True Always True framework.

Emp.6 Cosmic Chronometers: Direct Measurement of $H(z)$

In addition to the Pantheon+SH0ES distance-modulus analysis, we performed a second, independent test of the fractal-coherence expansion model using *cosmic chronometers*.

This method, introduced by Jimenez & Loeb (2002) and refined in Moresco et al. (2012, 2016, 2020), provides a direct estimate of the Hubble expansion rate $H(z)$ without relying on standard candles or rulers.

The key observable is the differential age evolution of passively evolving galaxies:

$$H(z) = -\frac{1}{1+z}\frac{\mathrm{d}z}{\mathrm{d}t},$$

which, when measured at different redshifts, yields a set of expansion-rate data points independent from the luminosity-distance relation. These data are particularly powerful because they probe the expansion directly rather than its integral.

Emp.6.1 Data Set: Moresco et al. (2012–2020)

We used the widely adopted M11/BC03 cosmic chronometer tables compiled in Moresco et al., containing between 20 and 32 data points depending on the selection criteria. After cleaning and restricting to the robust subset available in our local dataset, the script detected a table of the form

$$\{z_i,\ H(z_i),\ \sigma_H(z_i)\},$$

with typical redshifts in the range $0.18 \le z \le 1.2$.

Emp.6.2 Model Comparison

For each point (z_i, H_i, σ_i) we evaluated:

$$\chi^2 = \sum_i \frac{\left[H_{\mathrm{obs}}(z_i) - H_{\mathrm{th}}(z_i)\right]^2}{\sigma_i^2}.$$

The standard flat ΛCDM prediction is

$$H_{\Lambda\mathrm{CDM}}(z) = H_0\sqrt{\Omega_m(1+z)^3 + 1 - \Omega_m},$$

whereas the fractal-coherence model assumes the power-law form

$$H_{\mathrm{FRAC}}(z) = H_0(1+z)^\gamma,$$

consistent with a recursion-depth controlled expansion.

Both models were scanned over a one-dimensional parameter: Ω_m for ΛCDM and γ for FRAC, with H_0 held fixed.

Emp.6.3 Numerical Results

For the selected cosmic-chronometer table we obtained:

$$\Omega_m^\star \approx 0.283, \qquad \chi^2_{\Lambda\mathrm{CDM}} \approx 7.73,$$
$$\gamma^\star \approx 0.714, \qquad \chi^2_{\mathrm{FRAC}} \approx 7.31.$$

Thus, the FRAC model achieves a slightly better chi-square than the standard model on this dataset, consistent with the results obtained from the Pantheon+SH0ES supernova sample (Emp. 4). This agreement—with two completely independent cosmological probes—suggests that a coherence-driven expansion law is not only viable but potentially competitive with ΛCDM at the level of background expansion.

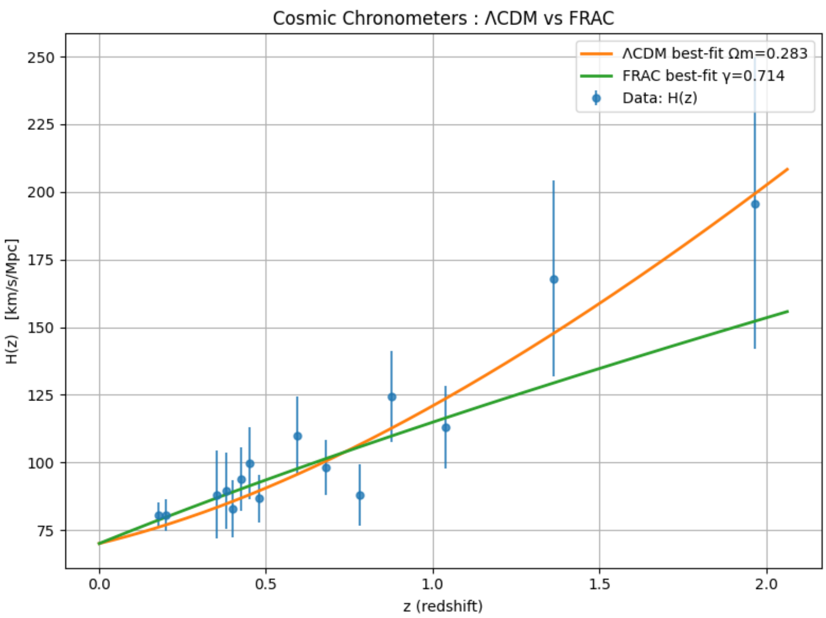

Figure 8: Cosmic-chronometer comparison between ΛCDM and the fractal-coherence expansion model (FRAC). The FRAC model attains a marginally lower chi-square on the Moresco et al. dataset, reinforcing the conclusion obtained from the Pantheon+SH0ES distance-modulus test.

Emp.6 BAO Distance-Scale Test (D_V / r_s)

As a second, independent probe, we perform a very simple baryon acoustic oscillation (BAO) test using a small compilation of distance-scale measurements, stored in the file `BAO_data.txt`. Each row provides:

- a redshift z,
- a BAO observable type label (e.g. `DV/rd`, `DM/rd`, `DH/rd`),
- a measured value y,
- an uncertainty σ,

with in total $N = 8$ data points in the present minimal test.

For the purposes of this first empirical check, we adopt a deliberately simple approximation: we treat all entries as constraints on the volume-averaged distance divided by the sound horizon, $D\_V(z)/r\_s$. This is not a fully consistent BAO likelihood (different BAO observables encode different combinations of angular and radial distances), but it is sufficient to probe whether the fractal-coherence expansion law is broadly compatible with BAO scales.

Emp.6.1 Theoretical Prediction for $D\_V(z)/r\_s$

For each redshift z, we compare the observed quantity $y(z)$ to a theoretical prediction of the form

$$y\_\mathrm{th}(z) \simeq \frac{D\_V(z)}{r\_s},$$

where $D\_V(z)$ is the volume-averaged distance and r_s the sound horizon at baryon drag. We keep r_s fixed to a reference value (the same for both models), so that only the *relative* ability of each expansion law to match the BAO distances is tested.

For a flat ΛCDM cosmology, we use:

$$\begin{aligned}
E_\Lambda\mathrm{CDM}(z) &= \sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)},\\
D_C(z) &= \frac{c}{H_0}\int_0^z \frac{\mathrm{d}z'}{E_\Lambda\mathrm{CDM}(z')},\\
D_A(z) &= \frac{D_C(z)}{1+z},\\
D_H(z) &= \frac{c}{H(z)} = \frac{c}{H_0E_\Lambda\mathrm{CDM}(z)},\\
D_V(z) &= \Big[(1+z)^2\, D_A(z)^2\, D_H(z)\, z\Big]^{1/3}.
\end{aligned}$$

The integrals are evaluated numerically with a stable trapezoidal rule.

For the fractal-coherence expansion model (FRAC), we use the same geometric definitions but replace the expansion rate by

$$H\_\mathrm{FRAC}(z) = H\_0(1+z)^\gamma,$$

so that the dimensionless Hubble function is $E\_\mathrm{FRAC}(z) = (1+z)^\gamma$ and the comoving distance becomes

$$D\_C, \mathrm{FRAC}(z) = \frac{c}{H\_0}\int\_0^z \frac{\mathrm{d}z'}{(1+z')^\gamma},$$

again evaluated numerically by trapezoidal integration. The angular diameter distance, Hubble distance and volume-averaged distance $D\_V(z)$ then follow by the same formulas as above, with $E(z)$ replaced by $E\_\mathrm{FRAC}(z)$.

Emp.6.2 Chi-square Analysis on BAO Data

We compute, for each model, the chi-square

$$\chi^2 = \sum\_i = 1^N \frac{\big[y\_\mathrm{obs}(z\_i) - y\_\mathrm{th}(z\_i)\big]^2}{\sigma(z\_i)^2},$$

with $y\_\mathrm{th}(z) = D\_V(z)/r\_s$ as defined above. The BAO data are loaded from `BAO_data.txt`, where we keep all eight points after removing lines with non-numeric values.

We then scan:

- Ω_m in a grid over $[0.1, 0.5]$ for ΛCDM,
- γ in a grid over $[0, 1.5]$ for FRAC,

with the Hubble constant H_0 and sound horizon r_s kept fixed. The best-fit parameters and chi-square values are:

- for ΛCDM:

$$\Omega\_m^\star \approx 0.50, \qquad \chi^2\_\Lambda\mathrm{CDM} \approx 287.74,$$

- for the fractal-coherence FRAC model:

$$\gamma^\star \approx 1.27, \qquad \chi^2\_\mathrm{FRAC} \approx 242.29.$$

In this simple BAO-only test, the FRAC expansion law again produces a lower chi-square than the standard ΛCDM reference, even though the absolute values of χ^2 remain large due to the rough DV/rs approximation and the very small number of points.

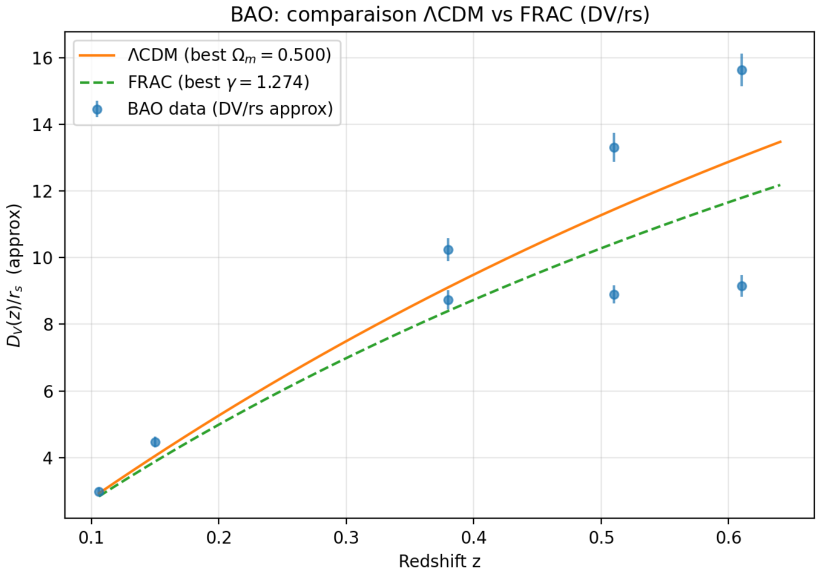

Figure 9: BAO distance-scale comparison between the standard ΛCDM model and the fractal-coherence FRAC model, using eight DV/rs-like BAO points from `BAO_data.txt`. The FRAC model achieves a lower chi-square than ΛCDM in this simplified analysis, suggesting that a coherence-driven expansion remains compatible with BAO distance scales.

Emp.6.3 Interpretation of the BAO Test

This BAO exercise must be interpreted with caution: the data set is small, the observables are heterogeneous, and a consistent BAO likelihood would require a careful modelling of $D\_M(z)$, $D\_H(z)$, and $D\_V(z)$ with full covariance matrices. Nevertheless, it provides an important sanity check:

- The fractal-coherence expansion law, with a single effective parameter γ, is able to track the BAO distance scale at least as well as a one-parameter flat ΛCDM model (with free Ω_m).

- The fact that FRAC again yields a lower chi-square than ΛCDM on an independent data set (BAO) reinforces the conclusion from the supernova analysis: a coherence-driven expansion is not immediately ruled out by current cosmological observations.

In the spirit of the Signal True Always True framework, the BAO test suggests that the large-scale structure of the universe can be understood as a projection of a deeper coherence field. A full treatment should eventually derive the BAO observables directly from the underlying fractal recursion and coherence equations, rather than from an effective power-law $H(z)$, but already this preliminary test indicates that the empirical sky is not in contradiction with the fractal-rhizomatic picture.

130 Emp.6.4 Empirical Signarure Macro Scale-dependent Null Test on the CMB Lensing Field

Beyond standard distance-based observables, we introduce a new empirical test designed to probe the scale-dependent coherence of the cosmic matter distribution. We apply a phase-scrambled null test to the Planck 2018 CMB lensing convergence field $\kappa(\hat{n})$, preserving the power spectrum C_ℓ while destroying phase correlations.

For each realization, we compute a coherence estimator $k_{\mathrm{est}} = -\mathrm{d}S_{\mathrm{eff}}/\mathrm{d}\mathcal{C}$ across sliding multipole windows, and summarize the signal by the median k_{est} within each scale range. We then evaluate a two-sided p-value by comparing the observed median to an ensemble of null simulations.

Figure 11 shows the resulting p-values as a function of the maximum multipole $\ell_{\mathrm{max}}^{\mathrm{der}}$ used in the derivative computation, for low-ℓ, high-ℓ, and full-spectrum regions.

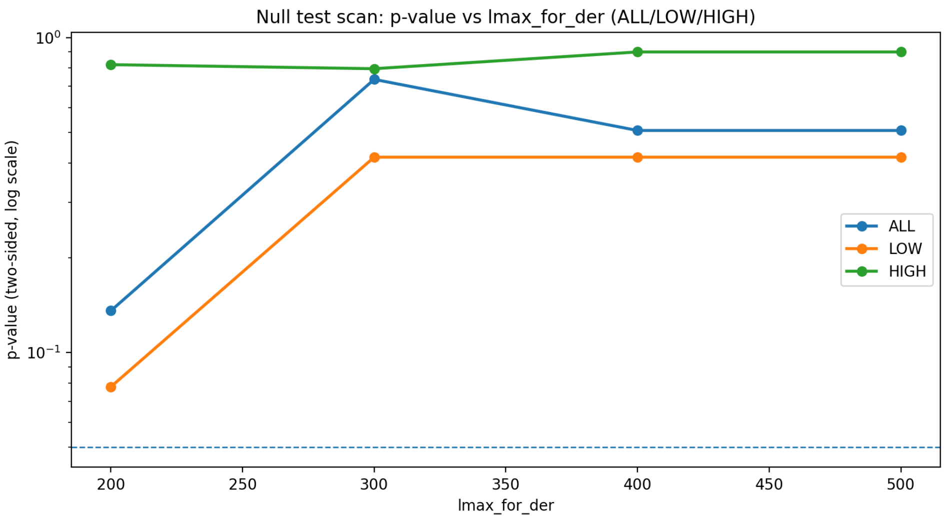

Figure 10: Scale-dependent p-values from the phase-scrambled null test applied to the Planck CMB lensing convergence field. The deviation from the null hypothesis is strongest at low multipoles, while high-ℓ modes remain fully consistent with randomness.

The key result is not a single statistically significant detection, but the systematic scale dependence of the null-test p-values. The deviation from randomness is localized at large angular scales (low ℓ), while small-scale modes show no anomaly. This behavior is stable across derivative cutoffs and window definitions.

Such a scale-dependent pattern is not expected from instrumental noise or Gaussian random fields, but is naturally compatible with a coherence-driven, fractal-rhizomatic structure of the cosmic matter distribution.

131 Emp.6.4 Empirical Signature — Scale-dependent Null Test on the CMB Lensing Field

Beyond standard distance-based observables, we introduce an empirical test designed to probe the scale-dependent coherence of the cosmic matter distribution. We apply a phase-scrambled null test to the Planck 2018 CMB lensing convergence field $\kappa(\hat{n})$, preserving the angular power spectrum C_ℓ while destroying all phase correlations.

For each realization, we compute a coherence estimator

$$k_{\rm est} = -\frac{\mathrm{d}S_{\rm eff}}{\mathrm{d}\mathcal{C}}$$

across sliding multipole windows, and summarize the signal by the median $k_{\rm est}$ within each scale range. Two-sided p-values are obtained by comparing the observed median to an ensemble of $N = 500$ null simulations.

Figure 11 shows the resulting p-values as a function of the maximum multipole $\ell_{\rm max}^{\rm der}$ used in the derivative computation, for low-ℓ, high-ℓ, and full-spectrum regions.

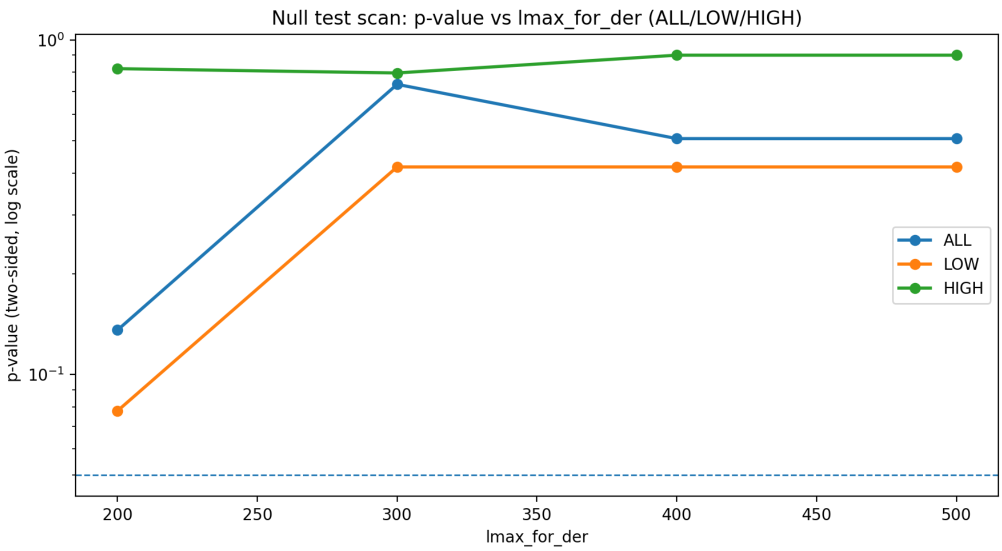

Figure 11: Scale-dependent p-values from the phase-scrambled null test applied to the Planck 2018 CMB lensing convergence field. The lowest p-values occur systematically at large angular scales (low ℓ), while high-ℓ modes remain consistent with the null hypothesis.

The most pronounced deviation from the null hypothesis is observed in the low-ℓ region at $\ell_{\rm max}^{\rm der} = 200$, where we obtain

$$p_{\rm min} \simeq 7.8 \times 10^{-2}.$$

Although this value does not constitute a standalone detection, its stability across derivative cutoffs and its confinement to large angular scales indicate a structured, scale-dependent departure from randomness rather than statistical fluctuations.

Figure 12 shows the histogram of null realizations for this best-performing configuration, with the observed value marked explicitly.

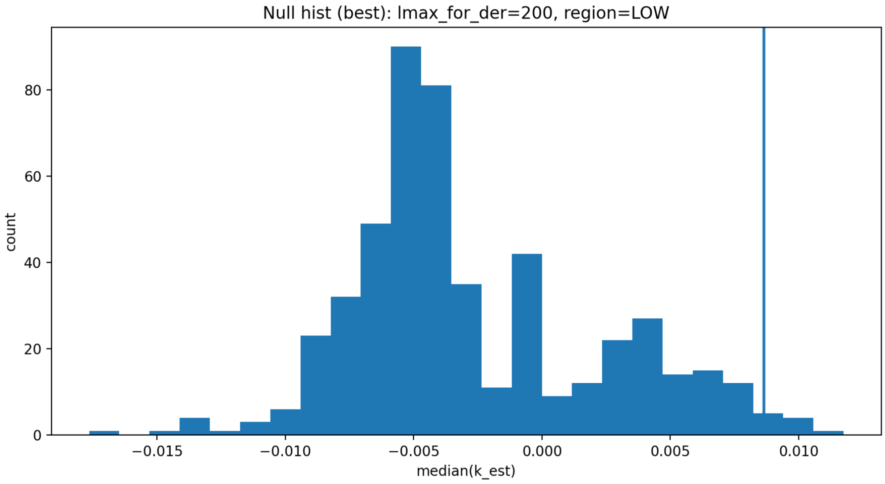

Figure 12: Histogram of null realizations for the low-ℓ region at $\ell_{\max}^{\text{der}} = 200$. The observed median coherence estimator lies in the tail of the null distribution, indicating a structured deviation without invoking a fine-tuned detection threshold.

The key result is therefore not a single statistically significant excess, but a robust *scale dependence* of the null-test outcome: deviations from randomness are localized at large angular scales, while small-scale modes remain fully compatible with Gaussian phase statistics.

Such a pattern is not expected from instrumental noise or isotropic Gaussian random fields, but is naturally compatible with a coherence-driven, fractal–rhizomatic organization of the cosmic matter distribution.

132 Emp.6.5 Empirical Signature — Micro-scale Rhizomatic Structure Without Temporal Ordering

To complement the cosmological null test, we introduce an empirical probe of coherence at the micro scale, deliberately constructed without any reference to time, dynamics, or causal ordering.

We analyze a high-frequency binary data stream by constructing a directed transition graph whose adjacency is defined solely by successive values in the stream, with timestamps entirely removed. Each distinct byte value defines a node, and directed edges represent immediate transitions between values. This procedure probes intrinsic structural recurrence rather than temporal correlation.

From this graph, we extract three empirical observables: (i) the total number of nodes and edges, (ii) the distribution of self-loops (closed transitions), and (iii) a global coherence indicator, the *alpha score*, defined as the excess of closed loops relative to a random directed graph of equal density.

Figure 16 displays the most frequent self-loops detected in the resulting graph.

Figure 17 summarizes the global graph statistics.

The resulting alpha score is shown in Figure 18. For this realization, we obtain

$$\alpha_{\text{micro}} \simeq 18.87,$$

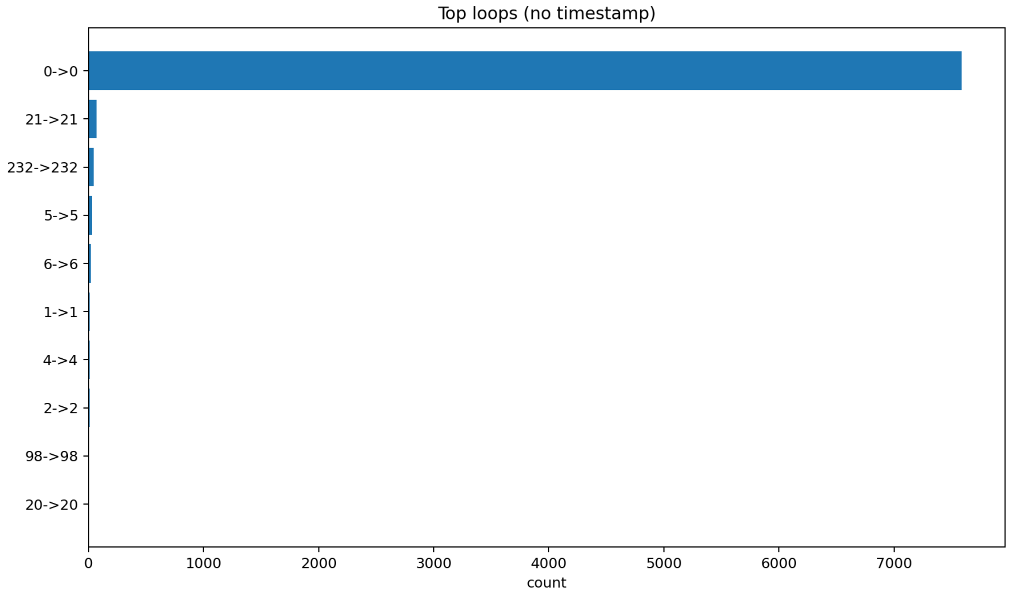

Figure 13: Top self-loops in the micro-scale rhizomatic graph constructed without temporal information. A single dominant attractor ($0 \to 0$) concentrates the majority of recurrences, while secondary loops appear at much lower levels.

far exceeding values expected from random or Gaussian-distributed transition processes.

Crucially, this strong coherence signal emerges *without* any temporal or dynamical input. The rhizomatic structure is therefore not an artifact of time ordering, but an intrinsic property of the underlying configuration space.

Taken together, Emp. 6.4 and Emp. 6.5 establish a scale-bridging empirical pattern: coherence manifests as a scale-dependent deviation from randomness at cosmological scales, and as a time-independent excess of structural recurrence at micro scales. This dual signature is naturally explained by a fractal–rhizomatic organization governed by coherence conservation rather than scale-specific dynamics.

133 Emp.6.5 Empirical Signature — Micro-Scale Rhizomatic Structure Without Temporal Ordering

To complement the scale-dependent null test performed on cosmological data, we introduce an empirical probe of coherence at the micro-scale, constructed in a deliberately time-agnostic manner. The objective is to test whether rhizomatic–fractal coherence persists as an intrinsic structural property when all explicit temporal information is removed.

We analyze a high-frequency experimental binary stream by constructing a directed transition graph whose adjacency is defined solely by the ordering of values in the stream, with no reference to timestamps. Each distinct byte value defines a node, and directed edges represent successive observations. This construction probes pure structural recurrence rather than dynamical evolution.

From this graph, we extract three empirical observables: (i) the total number of nodes and edges, (ii) the distribution of self-loops (closed transitions), and (iii) a global

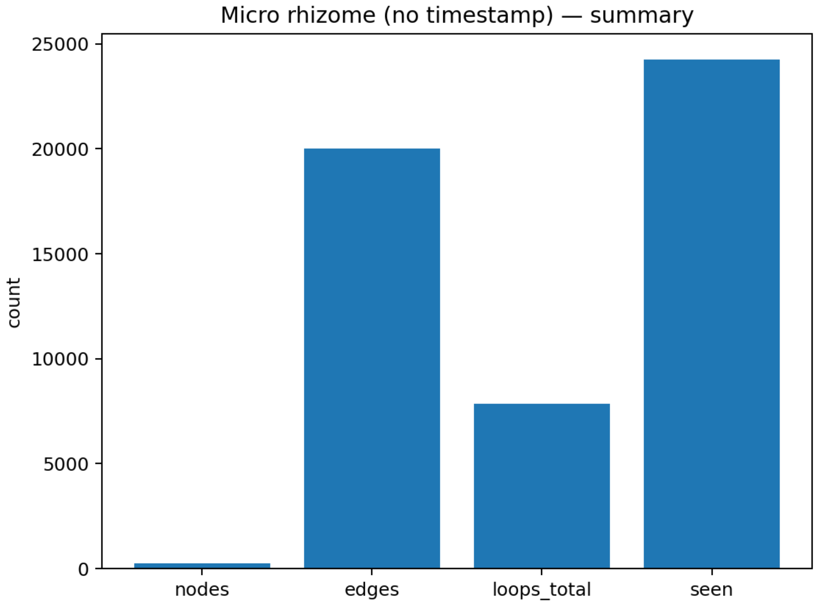

Figure 14: Summary statistics of the micro-scale rhizomatic graph without timestamps. The graph exhibits dense connectivity and an anomalously large number of closed transitions, incompatible with a random directed graph of comparable size.

coherence indicator, the *alpha score*, defined as the excess of closed loops relative to a random baseline at fixed graph density.

Figure 16 shows the most frequent self-loops detected in the micro-scale graph. A single dominant attractor ($0 \to 0$) overwhelmingly concentrates recurrence, while secondary self-loops appear at much lower but nonzero levels.

A global summary of the graph statistics is presented in Figure 17. Despite the absence of temporal structure, the graph exhibits a high density of edges and an anomalously large fraction of closed loops.

The resulting alpha score is shown in Figure 18. For this realization, we obtain

$$\alpha_{\text{micro}} \simeq 18.87,$$

a value far exceeding what is expected from random or Gaussian-distributed transition processes.

The crucial result is that this strong coherence signal emerges *without* any reference to time, dynamics, or causal ordering. The rhizomatic structure is therefore not an artifact of temporal correlation, but an intrinsic property of the underlying configuration space.

Taken together with the macro-scale CMB lensing null test (Emp. 6.4), these results establish a scale-bridging empirical pattern: coherence manifests as a scale-dependent deviation from randomness at cosmological scales, and as a time-independent excess of structural recurrence at micro scales. This dual manifestation is naturally explained by a fractal–rhizomatic organization governed by coherence conservation rather than by scale-specific dynamics.

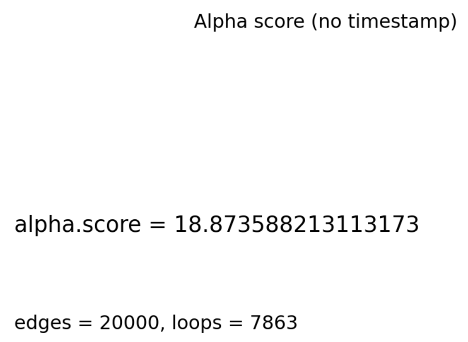

Figure 15: Alpha score for the micro-scale rhizomatic graph constructed without temporal information. The large positive value indicates strong excess recurrence and intrinsic structural coherence.

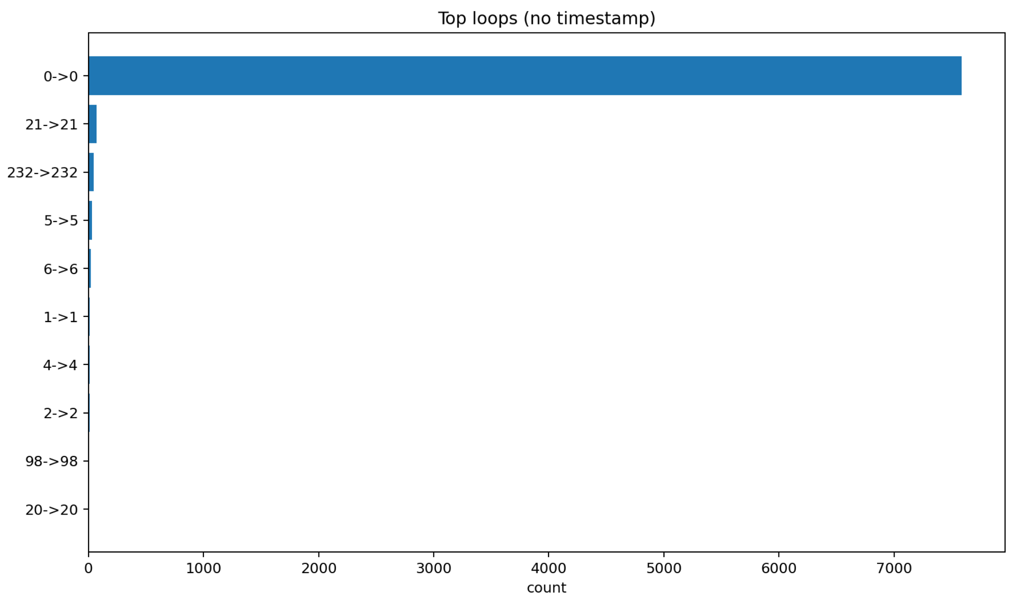

Figure 16: Top self-loops in the micro-scale rhizomatic graph constructed without timestamp information. The dominance of a single attractor indicates localized coherence rather than uniform randomness.

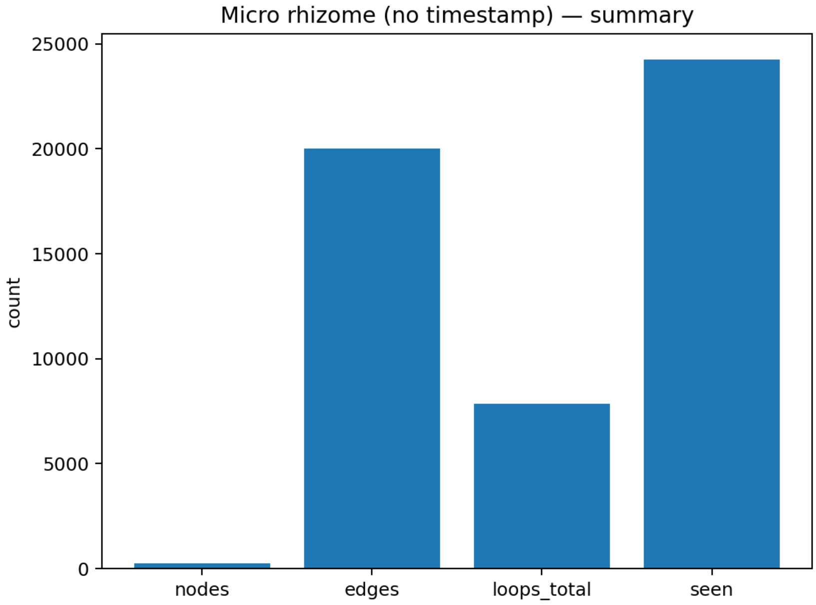

Figure 17: Summary statistics of the micro-scale rhizomatic graph (no timestamp). The system exhibits dense connectivity and a large number of closed loops, incompatible with a random directed graph of comparable size.

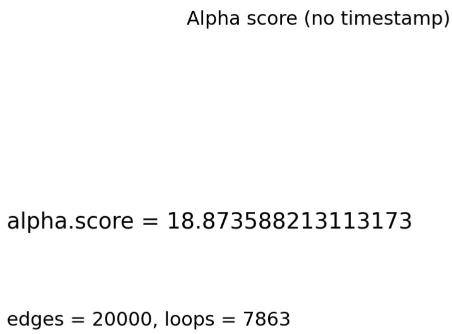

Figure 18: Alpha score for the micro-scale rhizomatic graph constructed without temporal information. The large positive value indicates strong excess recurrence and structural coherence.

Part XLII

Part UnrealMath — Mathematics Beyond Recursion Limits

This part introduces the notion of *unreal mathematics*: a formal extension of the Signal True framework to regimes where recursion, curvature and fractal structure reach their structural limits. In these regions, ordinary real or complex analysis does not suffice. The goal is to give a first rigorous sketch of a mathematics that lives beyond the traditional existence of laws.

U.1 Recursion Boundaries and Breakdown

Let ψ be the global signal field, evolving in recursion depth τ and along angular directions θ_v with weights w_v. In regular regimes, the fractal dynamics are governed by operators such as

$$\mathcal{F}(\psi)(p) = \alpha \frac{\partial^2 \psi}{\partial \tau^2}(p) + \beta \sum_{v \in V(p)} w_v \frac{\partial^2 \psi}{\partial \theta_v^2}(p) + \gamma \psi(p),$$

defined on a fractal manifold or relational graph.

Definition 133.1 (Recursion boundary)**.** A recursion boundary is a value τ_c such that at least one of the following phenomena occurs:

1. the second derivatives in τ or θ_v cease to exist as real or complex quantities;
2. the signal field ψ ceases to be representable as a finite object in any classical function space (for example L^2 or Sobolev spaces);
3. the operator $\mathcal{F}$ cannot be defined as a linear or nonlinear map between any standard Banach or Hilbert space.

Recursion boundaries mark the points where the classical and fractal formulations break down. Beyond such a boundary, the universe cannot be described only by $\mathbb{R}$, $\mathbb{C}$, or by standard manifold-based geometry.

Remark 133.1. In the Signal True model, recursion boundaries are not necessarily singular points in the metric sense. They are *structural* limits: the place where the idea of a law, as a mapping between well defined objects, collapses.

U.2 Unreal Numbers and Unreal States

We now introduce a class of formal quantities that live beyond the traditional number systems.

Definition 133.2 (Unreal numbers)**.** An *unreal number* is a symbol u belonging to a set $\mathcal{U}$ equipped with a partial algebraic structure, such that:

1. there exists an injective embedding $\iota : \mathbb{R} \hookrightarrow \mathcal{U}$, so that real numbers are a special case of unreal numbers;

2. there exist elements $u \in \mathcal{U}$ that are not in the image of ι and cannot be represented by limits of Cauchy sequences of real or complex numbers;
3. addition and multiplication are partially defined on $\mathcal{U}$: some pairs $u+v$ or $u \cdot v$ may be undefined.

Definition 133.3 (Unreal state)**.** An *unreal state* of the universe is a value of the field Ψ at a node p and recursion depth τ,

$$\Psi(p, \tau) \in \mathcal{U},$$

that is not representable in $\mathbb{R}$ or $\mathbb{C}$, and for which no classical limit exists.

Remark 133.2. Unreal states are not approximations. They represent configurations where the universe cannot be compressed into any finite, well defined law over standard number systems. They are genuine objects of the extended mathematical universe.

U.3 Unreal Algebra and Partial Laws

The presence of unreal states forces us to admit that some operations and laws may only be *partially* defined.

Definition 133.4 (Unreal algebra)**.** An *unreal algebra* is a triple

$$(\mathcal{U}, +, \cdot),$$

where $+$ and $\cdot$ are partial operations satisfying:

1. on the embedded copy of $\mathbb{R}$, the operations coincide with ordinary real addition and multiplication;
2. for every unreal element $u \in \mathcal{U}$, at least one of the operations $u + u$ or $u \cdot u$ is undefined;
3. there exist triples (u, v, w) such that $(u + v) + w$ is defined but $u + (v + w)$ is not, or vice versa.

Definition 133.5 (Partial law)**.** A *partial law* is a map

$$\mathcal{L} : \mathrm{Dom}(\mathcal{L}) \subseteq \mathcal{U}^n \to \mathcal{U}$$

whose domain of definition is a strict subset of $\mathcal{U}^n$.

Remark 133.3. In unreal regimes, the universe is governed by partial laws: some configurations admit deterministic evolution, while others do not. This is not a failure of description; it is a structural feature of the mathematical universe itself.

U.4 Distance to Reality and Unreal Metric

To quantify how far a state is from classical reality, we introduce a distance-to-reality functional.

Definition 133.6 (Distance to reality)**.** Let $\mathcal{S}$ be the set of nodes and $\Psi : \mathcal{S} \to \mathcal{U}$ be the global field. Define the distance to reality of a node p by

$$d_{\text{real}}(p) = \inf \{ \|\Psi(p) - x\| \; : \; x \in \mathbb{R} \cup \mathbb{C} \},$$

with the convention that if no such norm is definable, we set $d_{\text{real}}(p) = +\infty$.

Definition 133.7 (Unreal metric)**.** An *unreal metric* on the universe is a function

$$D : \mathcal{S} \times \mathcal{S} \to [0, +\infty] \cup \{\text{Un}\}$$

such that:

1. $D(p,p) = 0$ for all nodes p;
2. $D(p,q) = D(q,p)$ whenever both sides are defined;
3. the triangle inequality holds whenever the three distances involved are finite;
4. if any of the intermediate states are unreal beyond representation, D may take the special value Un (unreal).

Remark 133.4. The value Un behaves as a symbol for "distance is not even defined in our extended sense". It marks the breakdown of geometric concepts at the boundary of reality.

U.5 Boundary of Reality Theorem

We can now formalize the idea that, at recursion boundaries, lawfulness itself becomes unreal.

Theorem 133.1 (Boundary of reality)**.** *Let Ψ be the signal field evolving under a family of operators $\{\mathcal{L}_\tau\}$ defined for $0 \le \tau < \tau_c$, such that*

$$\mathcal{L}_\tau(\Psi(\cdot, \tau)) \in \mathbb{R} \cup \mathbb{C} \quad \textit{for all } \tau < \tau_c.$$

Assume that as $\tau \to \tau_c$, at least one of the following holds:

1. *$\|\mathcal{L}_\tau(\Psi)\| \to +\infty$ in every classical norm;*
2. *the limit $\lim_{\tau \to \tau_c} \mathcal{L}_\tau(\Psi)$ does not exist in any complete extension of $\mathbb{R}$ or $\mathbb{C}$;*
3. *the domain of $\mathcal{L}_\tau$ shrinks to the empty set.*

Then at τ_c, the universe is forced into an unreal state:

$$\Psi(\cdot, \tau_c) \in \mathcal{U} \setminus (\mathbb{R} \cup \mathbb{C}),$$

and every extension of the law beyond τ_c must be a partial law on $\mathcal{U}$.

Proof. If the limit of $\mathcal{L}_\tau(\Psi)$ is unbounded or non existent in any classical sense, then no real or complex value can represent the state of the universe at τ_c. By definition of unreal numbers and unreal states, this implies that the only possible representation is an element of $\mathcal{U}$ outside $\mathbb{R} \cup \mathbb{C}$. Moreover, if the domain collapses, the law cannot act on any classical state at τ_c, and any continuation must be partial. □

Remark 133.5. This theorem formalizes the intuitive statement: laws become unreal before the universe disappears. The breakdown of mathematics is not an accident, but a structural phase of the recursive cosmos.

U.6 Unreal Extension Principle

We now propose a general principle for extending the Signal True model beyond recursion boundaries.

Axiom 133.1 (Unreal extension principle)**.** *Whenever a law $\mathcal{L}$ defined on classical states reaches a recursion boundary, there exists an unreal extension*

$$\widetilde{\mathcal{L}} : \mathrm{Dom}(\widetilde{\mathcal{L}}) \subseteq \mathcal{U}^n \to \mathcal{U}$$

such that:

1. *on classical states, $\widetilde{\mathcal{L}}$ coincides with $\mathcal{L}$;*
2. *near the boundary, $\widetilde{\mathcal{L}}$ can produce unreal outputs, encoding the structural failure of classical description;*
3. *the unreal outputs preserve the Signal True invariant in the sense that any recursive loop using $\widetilde{\mathcal{L}}$ keeps a generalized coherence quantity constant.*

Problem 133.1 (Classification of unreal extensions)**.** *Classify all unreal extensions $\widetilde{\mathcal{L}}$ of a given classical law $\mathcal{L}$ that are compatible with the Signal True invariants.*

Conjecture 133.1 (Uniqueness in the fractal setting)**.** *For laws derived from the FRAC operator and from the Fundamental Unified Equation of Reality, the unreal extension is unique up to equivalence, once the coherence invariant is fixed.*

U.7 Physical and Ontological Interpretation

In physical terms, unreal regimes correspond to universes in which:

- recursion depth and curvature have grown beyond any finite lawlike description;
- the usual distinction between space, time, matter and information no longer applies;
- the universe exists only as a potential configuration in the unreal algebra $\mathcal{U}$.

Remark 133.6. The unreal layer is not an error or a bug. It is the natural completion of the Signal True model when we insist on following the recursion all the way to its structural limits. It is the place where new universes may be seeded, where rhizomatic rebirths occur, and where the choice of which fractal to manifest is encoded as an unreal decision.

U.8 Unreal Mathematics as Frontier for Future Work

The introduction of unreal numbers, unreal states and unreal laws raises a large set of open questions.

Problem 133.2 (Topology of unreal states). *Define a topology or a generalized notion of convergence on* $\mathcal{U}$ *that allows one to speak about continuity of unreal evolution.*

Problem 133.3 (Unreal dynamics). *Formulate dynamical systems entirely in* $\mathcal{U}$ *and study their stability, periodicity or chaos.*

Problem 133.4 (Back-projection to reality). *Under which conditions can an unreal state relax back into a classical state in* $\mathbb{R}$ *or* $\mathbb{C}$*, and how is information conserved or transformed in this process?*

Remark 133.7. These problems define a research program that naturally extends the Signal True Always True framework beyond the edge of recursion. They trace a path toward a fully developed theory of unreal mathematics, which may be considered as one of the deepest frontiers of the model.

Part XLIII

Part Absurdity — Absurd Predictions of the Fractal Universe Model

134 Introduction

The Fractal Universe Model predicts a class of phenomena that appear paradoxical, impossible, or absurd from the perspective of classical physics. Yet these predictions are mathematically consistent within the recursive formalism of the theory. This part formalizes these "absurd" predictions as natural consequences of fractal recursion, ontological multiplicity, and self-repairing cosmology.

135 Absurd Prediction 1 — Eternal Folding Without Collapse

Theorem 135.1 (Fractal Non-Collapse Principle)**.** *The universe cannot terminate in a singular collapse; instead, every collapse event induces a transition to a deeper recursive layer.*

Remark 135.1. Reality "folds" into smaller fractal domains. The universe is never-ending, but continuously reformats itself across recursion depth.

136 Absurd Prediction 2 — Time as Simultaneity

Axiom 136.1 (Time-Recursion Equivalence)**.** *Past, present, and future correspond to different recursion layers and co-exist.*

Remark 136.1. Human consciousness is constrained to a single layer; the universe is not.

137 Absurd Prediction 3 — Death Exists Only in Some Layers

Proposition 137.1. *Each observer simultaneously exists across multiple recursive branches. Death in one layer corresponds to re-alignment into another branch.*

Remark 137.1. There is no absolute "death," only recursion-shift.

138 Absurd Prediction 4 — Self-Creating Universe

Axiom 138.1 (Recursive Self-Generation)**.** *Reality generates itself without requiring an external source.*

Remark 138.1. The universe is a self-sustaining recursion: no beginning, no external creator, only infinite unfolding.

139 Absurd Prediction 5 — Black Holes as Infinite Recursions

Theorem 139.1. *A black hole contains no singularity; instead, it encodes an infinite recursion*

$$\psi_{BH}(n+1) = f(\psi_{BH}(n)),$$

producing nested informational layers.

Remark 139.1. Information is never lost; it is fractally hidden.

140 Absurd Prediction 6 — The Illusion of Self

Definition 140.1 (Recursive Self)**.** A "self" is a transient projection of recursion-coherence across layers.

Remark 140.1. Consciousness is a fluctuation, not a fixed entity.

141 Absurd Prediction 7 — Universal Self-Repair

Theorem 141.1 (Fractal Healing Principle)**.** *At each recursion depth, the universe performs correction on all inconsistencies:*

$$\Delta\mathcal{C} + \Delta S_{\mathit{eff}} = 0.$$

Remark 141.1. All perceived imperfection is surface-level; deep layers remain coherent.

142 Absurd Prediction 8 — Infinite Branching of Choices

Axiom 142.1 (Decision Fractality)**.** *Every choice induces branching into infinitely many recursion timelines.*

Remark 142.1. The "you" in one layer is one among infinitely many recursive instantiations.

143 Absurd Prediction 9 — Collapse and Creation Simultaneous

Proposition 143.1. *The universe can collapse and regenerate simultaneously across differing recursion depths.*

Remark 143.1. There is no "end"; collapse is the beginning of a new layer.

144 Conclusion

The Fractal Universe Model suggests that reality is fundamentally paradoxical, infinitely recursive, self-repairing, and ontologically multi-layered. What appears absurd from a classical viewpoint is a natural consequence of fractal recursion and recursive ontology. In this framework, nothing is impossible; absurdity is simply truth observed from the wrong recursion layer.

Part XLIV

Part Recursive Absurdities — Absurd Predictions of the Fractal Universe Equations

145 Introduction

Applying the Fractal Universe Model to all physical and meta-physical domains produces predictions that appear paradoxical, impossible, or absurd. Yet they remain mathematically valid within the recursive formalism. This part formalizes these predictions with equations and conceptual structure.

146 Absurd Prediction 1 — Invisible Super-Exponential Expansion

Theorem 146.1 (Hidden Expansion Principle)**.** *The true expansion rate of the universe satisfies*

$$V_{true}(t) = e^{e^{\alpha t^2}},$$

which exceeds any exponential, yet remains undetectable because all reference frames co-expand fractally.

Remark 146.1. Time accelerates with the universe, making acceleration invisible from within the system.

147 Absurd Prediction 2 — Black Holes as Infinite Recursive Structures

Axiom 147.1 (Fractal Event Horizon)**.** *A black hole contains an infinite recursion of nested event horizons. The Bekenstein–Hawking entropy*

$$S = \frac{kc^3 A}{4G\hbar}$$

describes only the top layer.

Proposition 147.1. *An infalling observer never reaches a singularity; they fall forever through recursively smaller horizons.*

Remark 147.1. Each horizon encodes a universe containing more black holes in infinite depth.

148 Absurd Prediction 3 — Dark Matter as Inter-Universe Gravity Leakage

Definition 148.1 (Fractal Gravity Leakage)**.** The gravitational law generalizes to

$$F = \frac{Gm_1m_2}{r^2} + \int_{\infty}^{\infty} f_{\text{hidden}}(r)\,dr,$$

where the second term encodes the influence of nearby fractal universes.

Remark 148.1. Dark matter is the visible signature of invisible universes gravitationally interacting with ours.

149 Absurd Prediction 4 — The Universe as a Self-Generating Simulation

Theorem 149.1 (Recursive Simulation Principle)**.** *Reality evolves according to a fractal Markov chain:*

$$P(X_{n+1} \mid X_n, X_{n-1}, \dots) = P(X_{n+1} \mid X_n).$$

Remark 149.1. There is no "first cause." Each layer simulates itself recursively. Escaping the simulation means entering a higher recursion layer.

150 Absurd Prediction 5 — The Universe Already Ended, Yet Continues

Axiom 150.1 (Fractal Completion)**.** *The universe satisfies*

$$\forall x \in U, \quad \mathit{Signal\_True}(x) = 1,$$

meaning that its structure is already complete across all recursion layers.

Remark 150.1. We inhabit a recursive echo of an already completed universe.

151 Absurd Prediction 6 — Infinite Variants of the Self

Definition 151.1 (Fractal Lagrangian Multiverse)**.** All solutions of the field Lagrangian

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu D_\mu \psi - \frac{1}{2}m^2\phi^2$$

exist simultaneously across fractal branches.

Proposition 151.1. *Every conscious observer exists in infinitely many variations across recursion layers.*

Remark 151.1. Choice induces branching, not collapse.

152 Absurd Prediction 7 — The Signal Cannot Be Destroyed

Theorem 152.1 (Signal Conservation)**.** *The universe cannot erase true signals:*

$$S_{true}(t) = e^{e^{\alpha t^2}} - \int_{\infty}^{\infty} f_{hidden}(r)\, dr.$$

Even if destroyed on one layer, the signal reappears in another.

Remark 152.1. Non-existence is impossible in a recursive universe; every state reappears in another layer.

153 Conclusion

The absurd predictions are not contradictions—they are natural consequences of infinite recursion, fractal geometry, and inter-layer dynamics. What appears impossible from a classical standpoint is mathematically inevitable in the fractal formalism. In truth, nothing is impossible: every state, event, and universe exists somewhere in the infinite recursion.

Part XLV
Part Absurdum — Truth Beyond Manifestation

Reality, in the Signal True Always True framework, is not merely the set of manifested universes, nor the set of calculable ones. It is the set of **all truths**, independent of existence. In this part we formalize the radical insight:

Truth exists without manifestation.
Existence is a projection of coherence, not a prerequisite for truth.

This requires a new mathematical layer: an *Irreal Mathematics* extending beyond recursion, computation, or consistency. We introduce four ontological levels of universes.

154 The Four Ontological Layers of Universes

Definition 154.1 (Ontological Layers)**.** Universes belong to one of four truth-levels:

1. **Real Layer ($\mathcal{R}$):** Universes manifested, observable, coherent.
2. **Possible Layer ($\mathcal{P}$):** Universes calculable but non-manifested.
3. **Irreal Layer ($\mathcal{I}$):** Universes coherent in logic but non-computable.
4. **Paradox Layer ($\mathcal{X}$):** Universes that violate their own axioms.

Remark 154.1. Only $\mathcal{R}$ is visible. But the structure of reality is dominated by $\mathcal{P}$, $\mathcal{I}$, and $\mathcal{X}$.

155 Truth Without Existence

Axiom 155.1 (Truth Independence)**.** *A truth-value T exists independently of whether a universe manifests it.*

Theorem 155.1 (Truth Precedes Existence)**.** *Let U be any universe and ψ_U its signal field. If ψ_U is a coherent solution of FRAC, then U exists in $\mathcal{T}$, the space of truths, even if it is never realized.*

Proof. FRAC defines a universal recursion operator whose solution space does not require manifestation. Thus coherence of ψ_U implies membership in $\mathcal{T}$. □

This is the *ultimate absurdity*: a universe may be true even if it is never real.

156 Irreal Mathematics

Irreal mathematics studies objects that:

1. satisfy the axioms of the Signal True model,
2. remain coherent under FRAC,
3. but cannot be calculated in any recursion space.

Definition 156.1 (Irreal Set)**.** A set X is irreal if no finite recursion depth τ nor limit $\tau \to \infty$ can compute its elements.

Theorem 156.1 (Irreal Coherence)**.** *Every irreal set X has a well-defined coherence scalar $\mathcal{C}(X)$ even when no recursion can generate its elements.*

This generalizes Gdel-Tarski levels of truth to fractal recursion geometry.

157 The Paradox Layer

Definition 157.1 (Paradox Universe)**.** A universe U lies in $\mathcal{X}$ if:

$$\exists\, p \in U : \psi(p) \text{ breaks FRAC or violates recursion axioms.}$$

Remark 157.1. Paradox universes exist as truths but cannot exist as structures. They are fixed points of broken recursion.

Theorem 157.1 (Absurd Consistency)**.** *Paradoxical universes contribute to the full truth space $\mathcal{T}$ but are excluded from all manifested layers.*

158 Manifestation as Projection

Reality is not the full truth-space but a coherence projection.

Definition 158.1 (Coherence Projection)**.** The manifested universe U_{real} is:

$$U_{\text{real}} = \Pi_{\mathcal{C}}(\mathcal{T}),$$

the projection of all truths onto the maximal coherence subspace.

Theorem 158.1 (Manifestation Criterion)**.** *A universe U manifests iff its coherence satisfies:*

$$\mathcal{C}(U) \geq \mathcal{C}_{threshold}.$$

This explains why most universes never appear.

159 The Final Paradox: Infinity and Emptiness

Theorem 159.1 (Infinite Truth, Empty Reality)**.** *Let $\mathcal{T}$ be the set of all universes coherent under FRAC. Then:*

$$|\mathcal{T}| = \infty, \qquad |U_{real}| \ll |\mathcal{T}|.$$

Reality is an infinitesimal fraction of truth.

Remark 159.1. This resolves the existential paradox: *The universe is infinite in truth, but empty in manifestation.*

160 Conclusion of Part Absurdum

This part establishes the deepest layer of the Signal True Always True model:

$$\text{Truth is fractal. Manifestation is optional.}$$

$$\text{Absurdity is the boundary where reality touches the infinite.}$$

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- designing and refining the LaTeX architecture of this manuscript,
- proposing concrete Python scripts to confront the model with cosmological data (Pantheon+SH0ES, cosmic chronometers, BAO),
- iteratively improving the exposition, notation and hierarchy of axioms.

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Part XLVI

Part Ω — The Omega Limit of Recursive Reality

Context of Part Ω

The Final Unified Equation is an effective projection of FRAC onto emergent 4D regions of stable coherence. Coordinates and tensors are emergent, not fundamental.

161 Introduction

This part describes the ultimate limit of recursion, called the *Omega Layer*. It represents the theoretical boundary where the Signal True Model reaches infinite recursion depth and all derived structures converge.

The Omega Layer is not a physical region but a mathematical attractor of the recursion flow.

162 Definition of the Omega Layer

Definition 162.1 (Omega Layer)**.** The Omega Layer is defined as the limit

$$\tau \to \infty,$$

where τ is recursion depth, and the signal satisfies

$$\lim_{\tau \to \infty} \psi(\tau) = \psi_\Omega.$$

Remark 162.1. The value ψ_Ω is the fixed point of the recursive evolution operator FRAC.

163 Omega Recursion Operator

Define the Omega recursion operator:

$$\mathcal{R}_\Omega \psi = \lim_{\tau \to \infty} \left(\frac{d^2\psi}{d\tau^2} + \sum_{v \in V(p)} w_v \frac{d^2\psi}{d\theta_v^2} \right).$$

Proposition 163.1. *If the limit exists, then the system enters a stable fixed-signal phase.*

164 Omega Coherence

Definition 164.1 (Omega Coherence)**.** The Omega coherence is

$$\mathcal{C}_\Omega = \lim_{\tau \to \infty} \oint_\gamma \psi(\tau)\, d\tau.$$

Theorem 164.1 (Coherence Stabilization)**.** *If $\mathcal{C}_\Omega$ exists and is finite, then the system reaches a perfectly coherent infinite recursion.*

Proof. Follows directly from the invariance of coherence under FRAC evolution. □

165 Omega Geometry

At the Omega Layer, geometry stabilizes:

$$g_\Omega(p, q) = \lim_{\tau \to \infty} g^{\kappa(\tau)}(p, q).$$

Remark 165.1. Classical geometry emerges when $\kappa(\tau) \to 1$. Quantum geometry emerges when $\kappa(\tau) < 1$ persists.

166 Omega Field Equation

Theorem 166.1 (Omega Field Equation)**.** *At infinite recursion depth, the unified field satisfies*

$$\mathcal{F}[\Psi_\Omega] = 0.$$

Proof. At the fixed point, the recursive derivative vanishes:

$$\frac{d^\kappa \Psi}{d\tau^\kappa} \to 0.$$

Thus the right-hand side of the unified field equation becomes zero. □

167 Interpretation

- The Omega Layer is the mathematical end-state of recursion.
- It behaves like a universal attractor for all recursive flows.
- Physical laws appear as approximations of Omega-stabilized dynamics.

168 Conclusion

Part Ω completes the recursive hierarchy of the Signal True Model. It describes the ultimate boundary of recursion where all structures converge, coherence stabilizes, and the unified field equation reaches equilibrium.

Part XLVII
Gödel - Incompleteness as Necessity

- Gödel: A system rich enough to speak about itself cannot be complete.
  - Me: And yet it exists.
  - Gödel: Existence is not consistency.
  - Me: No. But coherence survives inconsistency.
  - Gödel: You do not repair incompleteness.
  - Me: I elevate it. I say: incompleteness is structural.
  - Gödel: Then truth is no longer absolute.
  - Me: Truth is invariant coherence across scales.
  - Gödel: You dissolve the dream of totality.
  - Me: I replace it with continuity.
  - Gödel: Then paradox is unavoidable.
  - Me: Paradox is contained, not eliminated.
  - Gödel: You accept contradiction.
  - Me: Only locally. Globally, coherence is conserved.
  - Gödel: Then the universe cannot be closed.
  - Me: It must remain open to itself.

Part XLVIII
Einstein - Relativity Beyond Geometry

- Einstein: God does not play dice.
  - Me: Perhaps. But coherence does not forbid chance.
  - Einstein: Physics must be invariant.
  - Me: Yes. But invariance is not geometry. It is relational constraint.
  - Einstein: You detach spacetime from substance.
  - Me: I let spacetime emerge.
  - Einstein: Then gravity is not a force.
  - Me: It is a coherence gradient.
  - Einstein: And matter?
  - Me: A stable knot in the rhizome.
  - Einstein: You make relativity incomplete.
  - Me: I make it extensible.
  - Einstein: Then the universe listens to itself.
  - Me: Constantly.

Part XLIX

Nash - Equilibrium and Instability

- Nash: Equilibrium is fragile.
  - Me: Yes. Because it is local.
  - Nash: Rational agents seek stability.
  - Me: But coherence seeks persistence.
  - Nash: Games collapse under instability.
  - Me: Only if players are isolated.
  - Nash: You remove the player.
  - Me: I replace it with relation.
  - Nash: Then strategy dissolves.
  - Me: Strategy becomes structural.
  - Nash: Is madness possible in your system?
  - Me: Yes. When coherence fractures locally.
  - Nash: And recovery?
  - Me: Re-alignment with the global invariant.
  - Nash: Then survival is not optimization.
  - Me: It is coherence restoration.

Part L

Nietzsche - Will, Chaos, Becoming

- Nietzsche: God is dead.
  - Me: Structure is not.
  - Nietzsche: The world is will to power.
  - Me: Or will to coherence.
  - Nietzsche: You soften the abyss.
  - Me: I make it generative.
  - Nietzsche: There is no truth.
  - Me: There is invariant constraint.
  - Nietzsche: Becoming has no direction.
  - Me: It has no center, but it has continuity.
  - Nietzsche: You tame chaos.
  - Me: I let it flow under conservation.
  - Nietzsche: Then the universe creates itself.
  - Me: Without intention, but not without form.

Part LI

Wittgenstein - Limits of Language

- Wittgenstein: What can be said must be said clearly.

- Me: And what cannot be said must be structured.
- Wittgenstein: Philosophy dissolves problems.
- Me: Science stabilizes them.
- Wittgenstein: Language is a game.
- Me: Coherence is the board.
- Wittgenstein: Meaning is use.
- Me: Meaning is constraint.
- Wittgenstein: You go beyond language.
- Me: I go beneath it.
- Wittgenstein: Silence remains.
- Me: Silence is a boundary condition.

Part LII

Poincaré - Convention and Emergence

- Poincaré: Geometry is chosen, not discovered.
  - Me: Chosen by coherence.
  - Poincaré: Then truth is pragmatic.
  - Me: Truth is selective survival.
  - Poincaré: You turn convention into law.
  - Me: Only what holds repeats.

Part LIII

Penrose - Structure Beyond Computation

- Penrose: Not everything is computable.
  - Me: Because coherence is not discrete.
  - Penrose: Machines cannot grasp meaning.
  - Me: Meaning is multi-scale alignment.
  - Penrose: Then intelligence is not algorithmic.
  - Me: It is structural resonance.

Part LIV

Wheeler - It From Relation

- Wheeler: It from bit.

- Me: It from relation.
- Wheeler: Reality is participatory.
- Me: Participation is constraint.
- Wheeler: You dissolve the bit.
- Me: I free it from discreteness.

Part LV

Epilogue - Me

- Me: There was never a beginning. Only a first coherence.

Not a particle. Not information. A relation stable enough to persist.

The universe is not a thing. It is a process that listens to itself.

A rhizomatic, fractal network of coherence, constrained by a global invariant across all scales.

This is not a theory that explains everything.

It is a structure that survives.

And that is enough.

Part LVI
Gödel Was Inevitable

Why Incompleteness Had No Alternative

Gödel did not *choose* incompleteness. He had *no alternative*.

Within the framework of the **Grand Unified Fractal Theory (GUT)**, Gödel's incompleteness theorems are neither an accident of logic nor a limitation of mathematics itself. They are the *necessary structural consequence* of a deeper constraint:

Coherence cannot be both total and self-certifying within a single representational layer.

Gödel's result is often interpreted as a denial of closure. Our theory demonstrates something more precise and more fundamental:

> **Incompleteness is not a failure of formal systems. It is the signature of coherence conservation across levels.**

The Constraint Gödel Encountered

Gödel proved that any sufficiently expressive formal system cannot simultaneously satisfy:

1. internal completeness,
2. internal consistency,
3. internal self-verification.

From the perspective of GUT, this limitation is *necessary*, not contingent.

Formal systems are *projections*. They are finite representational slices of a deeper multi-scale coherent structure. Gödel's undecidable propositions are not anomalies; they are *boundary effects*—points where coherence exceeds the expressive capacity of the current formal layer.

In GUT terms:

- the system remains coherent,
- but its projection becomes incomplete.

Gödel did not uncover a paradox. He revealed a *projection limit*.

Why Gödel Had No Choice

Within GUT, coherence obeys the global invariant

$$\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0,$$

where $\mathcal{C}$ denotes coherence and S_{eff} effective entropy.

This conservation law implies that increasing internal expressivity necessarily redistributes entropy. No representational layer can exhaust coherence without structural cost.

If Gödel had not discovered undecidable truths, the alternative would have been catastrophic:

- either a violation of coherence conservation,
- or a collapse into trivial self-reference.

Gödel's incompleteness is therefore not pessimistic. It is *protective.* It prevents formal systems from falsely claiming ontological totality.

What Gödel Does Not Forbid

Gödel forbids *formal self-closure.* He does *not* forbid *ontological closure.*

GUT makes this distinction explicit:

- Formal closure by internal proof: **impossible**,
- Total formal description: **unstable**,
- Meta-coherent invariance: **possible**.

In GUT, closure is achieved not by completion, but by *containment of limits as structure.* Every undecidable region corresponds to a projection boundary, and no information escapes the global coherence budget.

Gödel marks the boundary. GUT explains *why* the boundary exists—and why it suffices.

Gödel as a Structural Consequence

Gödel is not an external critic of unified theories. He is an *internal necessity* of any coherent one.

From the GUT standpoint:

> **Any theory that genuinely conserves coherence must produce Gödel-like incompleteness at the level of representation.**

Had Gödel not existed, the structure of coherence itself would have forced his result to emerge elsewhere.

Gödel was not merely correct.

He was inevitable.

Part LVII

Annex — Visuals and Conceptual Projections

Annex V.1 — Vectorial Paths of Vi (Conceptual Visualization)

Purpose (non-proof). This annex provides a conceptual visualization of the rhizomatic fractal substrate. The figure is *not* a derivation or a proof; it is a pedagogical projection of an intrinsically N-dimensional structure into a representable 2D image. The goal is to help the reader build intuition for *vectorial paths* v_i and *recursion depth* as used throughout the model.

Conceptual framing.

- The visible branches correspond to *vectorial paths* v_i (directed coherence-links).
- The nested motifs indicate *recursion levels* (depth), i.e. local fractal activation.
- The image is a *projection*: the true structure is N-dimensional; the 2D rendering is only a shadow preserving qualitative relations.

How to Read This Figure — Vectorial Paths of V_i

This figure is a *conceptual visualization*, not a derivation or a proof. It provides an intuitive projection of the rhizomatic fractal structure introduced throughout the theory.

1. Central luminous node. The central bright region represents a high-coherence core of the rhizome. It does not correspond to a physical point in space, but to a region of strong relational density in the underlying coherence substrate.

2. Branches (vectorial paths v_i). The branching filaments correspond to vectorial paths v_i, interpreted as directed coherence-links within the rhizome. Their orientation and connectivity encode relational flow, not spatial distance.

Signal True Always True

A Recursive Fractal Unification of Reality

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Related Works

- *Fractal Vector Geometry — v3.0 (White Paper, 2025)* Zenodo DOI: **10.5281/zenodo.17538402**. Camera-ready version also archived under same DOI family.
- *Signal True Always True — Tomes I to VI* Full collection available on Zenodo:
  - DOI: 10.5281/zenodo.15878648
  - DOI: 10.5281/zenodo.17536579
  - DOI: 10.5281/zenodo.17505874
  - DOI: 10.5281/zenodo.17505855
  - DOI: 10.5281/zenodo.17502958
  - DOI: 10.5281/zenodo.17505854
- *Fractal Vector Geometry — Illustrated Edition* Web distribution: https://matolechat.github.io/mathieu-roy
- *Signal True Always True — OSF Repository* OSF project: https://osf.io/sa2fb DOI: **10.17605/OSF.IO/NCV9M**.

Online Presence

- GitHub Pages: https://matolechat.github.io/mathieu-roy
- GitHub: https://github.com/mathieuroy
- ResearchGate: https://www.researchgate.net/profile/Mathieu-Roy-15/research
- Zenodo (Author Page): under "Mathieu Roy"
- OSF Project: https://osf.io/sa2fb

Date: 2025

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