Signal True Always True

Mathematical Kernel of the Grand Unified Fractal Theory

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Abstract

Signal True Always True (STAT) is the compressed mathematical kernel of the **Signal True Always True: Grand Unified Fractal Theory (GUT)**, introduced in https://doi.org/10.5281/zenodo.17832587. STAT proposes an ontological priority of *coherence* over *information*, models the universe as a relational rhizome of directed vectorial paths, and states a conservation constraint linking coherence and effective entropy. This note fixes notation, gives a rigorous functional-analytic grounding for the core operators, and isolates an empirical protocol (cosmology and stress-tests) into a clean, reproducible section.

Contents

1 Introduction and Compression Principle

1.1 Why a mathematical kernel?

The full GUT document is intentionally maximal: it behaves like a high-coherence "singularity" that contains ontology, axioms, extended formalism, and cross-domain stress tests in one place. However, arXiv readers (and reviewers) usually require a *small, rigorous entry point*:

- clear mathematical objects and spaces,
- well-defined operators,
- a minimal set of axioms stated in a testable manner,
- a reproducible empirical protocol separated from philosophical discussion.

This note is that entry point.

1.2 Compression principle (GUT → STAT)

We adopt the following compression rule:

- **GUT (Zenodo)** is the canonical, maximal reference: it contains the complete architecture.
- **STAT (this paper)** is the compressed kernel: it fixes notation, defines the core operator layer, and isolates falsifiable signatures.

Operationally, we keep only what is needed to *reconstruct* the model: (i) relational substrate; (ii) coherence field; (iii) angular Laplacian; (iv) FRAC operator and well-posedness; (v) the conservation constraint; (vi) an empirical protocol. Everything else is referenced to the full GUT.

1.3 What this paper is (and is not)

This paper is: a rigorous mathematical grounding of the core kernel, plus a reproducible empirical pipeline (cosmology + stress-tests).

This paper is not: a complete philosophical treatise, nor a full unified field derivation. Those materials remain in the canonical GUT reference.

2 Canonical Axioms (Short Form)

We state a minimal axiom set sufficient to define the kernel operator layer.

Axiom 1 (Ontological priority of coherence). *There exists a conserved coherence potential $\mathcal{C}$ such that measurable information $\mathcal{I}$ is a projection of $\mathcal{C}$ rather than a primitive entity.* Consequence: *the fundamental state variable is the coherence field ψ, not spacetime coordinates.*

Axiom 2 (Relational substrate (no background manifold)). *The universe is modeled at the base level by a directed weighted relational substrate $(\mathcal{S}, \mathcal{E}, w, \vartheta)$ without assuming a differentiable manifold.* Consequence: *geometry is emergent from relational dynamics.*

Axiom 3 (Conditional fractalization). *Local refinement ("fractalization") occurs only when a coherence threshold is exceeded.* Consequence: *the substrate can evolve (graph refinement) as a function of ψ.*

Axiom 4 (Depth–dimension link). *Effective dimension is not fixed; it emerges as a function of recursion/refinement depth.* Consequence: *dimensional transitions are admissible dynamical outcomes.*

Axiom 5 (Conservation constraint). *There exists a constant I_0 such that for an admissible evolution (ψ_τ),*

$$\mathcal{C}(\psi_\tau) + S_{\text{eff}}(\psi_\tau) = I_0.$$

Consequence: *increasing order (coherence) requires displacement of effective entropy.*

3 Mathematical Grounding (Rigorous Kernel)

3.1 Relational substrate

We work on a countable directed weighted substrate. Angles are not background coordinates; they encode *intrinsic cyclic order* of outgoing relations, which will be used to build an angular operator.

Definition 1 (Relational substrate). *A relational substrate is a quadruple* $(\mathcal{S}, \mathcal{E}, w, \vartheta)$ *where:*

- $\mathcal{S}$ *is a countable set (nodes),*
- $\mathcal{E} \subseteq \mathcal{S} \times \mathcal{S}$ *is a set of directed edges,*
- $w : \mathcal{E} \to \mathbb{R}^+$ *is a weight function,*
- $\vartheta = \{\theta_p\}_{p \in \mathcal{S}}$ *with* $\theta_p : V(p) \to \mathbb{T}$ *inducing a strict cyclic order on* $V(p) = \{q \in \mathcal{S} : (p, q) \in \mathcal{E}\}$*. For each* $q \in V(p)$*,* $\sigma_p(q)$ *denotes its cyclic successor.*

Here $\mathbb{T} = [0, 2\pi)/\sim$ *is the unit circle.*

3.2 State space and coherence field

Definition 2 (State space). *Let* $\mathcal{H} = \ell^2(\mathcal{S})$ *with inner product* $\langle \psi, \phi \rangle = \sum_{p \in \mathcal{S}} \overline{\psi(p)} \phi(p)$*.*

Definition 3 (Coherence field). *A coherence field is a map* $\psi : \mathbb{R} \to \mathcal{H}$*,* $\tau \mapsto \psi(\cdot, \tau)$*, with* $\psi \in C^2(\mathbb{R}; \mathcal{H})$*.*

3.3 Angular Laplacian (corrected definition)

We define an angular operator using a local cyclic successor, avoiding any "opposite neighbor" assumption.

Definition 4 (Angular Laplacian). *Let* $\kappa : \mathbb{T} \to \mathbb{R}^+$ *be a continuous kernel and* $d_{\mathbb{T}}$ *the arc-length distance on* $\mathbb{T}$*. Define*

$$a_{p,q} = w(p, q) \cdot \kappa\Big(d_{\mathbb{T}}(\theta_p(q), \theta_p(\sigma_p(q)))\Big).$$

Then $\Delta_\theta : \mathcal{H} \to \mathcal{H}$ *is*

$$(\Delta_\theta \psi)(p) = \sum_{q \in V(p)} a_{p,q} \, [\psi(q) - \psi(p)].$$

Proposition 1 (Boundedness). *If* $\sup_{p \in \mathcal{S}} \sum_{q \in V(p)} |a_{p,q}| < \infty$*, then* Δ_θ *is bounded on* $\mathcal{H}$*.*

3.4 FRAC operator and well-posedness

Boundedness is the minimal technical condition ensuring the second-order evolution problem is well posed in $\mathcal{H}$.

Definition 5 (FRAC operator). *Fix* $\alpha > 0$ *and* $\beta, \gamma \in \mathbb{R}$*. Define* $\mathcal{F} : C^2(\mathbb{R}; \mathcal{H}) \to C(\mathbb{R}; \mathcal{H})$ *by*

$$(\mathcal{F}\psi)(\cdot, \tau) = \alpha \, \partial_\tau^2 \psi(\cdot, \tau) + \beta \, \psi(\cdot, \tau) + \gamma \, (\Delta_\theta \psi)(\cdot, \tau).$$

Theorem 1 (Well-posed Cauchy problem). *Assume* Δ_θ *is bounded on* $\mathcal{H}$*. For any* $\psi_0, \dot{\psi}_0 \in \mathcal{H}$*, there exists a unique* $\psi \in C^2(\mathbb{R}; \mathcal{H})$ *solving* $\mathcal{F}\psi = 0$ *with* $\psi(\cdot, 0) = \psi_0$ *and* $\partial_\tau \psi(\cdot, 0) = \dot{\psi}_0$*.*

3.5 Conservation constraint (normalized version)

Definition 6 (Normalized functionals)**.** *For $\psi \in \mathcal{H} \setminus \{0\}$, define*

$$\rho(p) = \frac{|\psi(p)|^2}{\|\psi\|^2}, \qquad S_{\mathrm{eff}}(\psi) = -\sum_{p \in \mathcal{S}} \rho(p) \ln \rho(p),$$

and logarithmic coherence $\mathcal{C}(\psi) = \ln \|\psi\|^2$.

Axiom 6 (Total information conservation)**.** *For any admissible evolution (ψ_τ), there exists I_0 such that*

$$\mathcal{C}(\psi_\tau) + S_{\mathrm{eff}}(\psi_\tau) = I_0 \quad \forall \tau.$$

3.6 Conditional fractalization threshold

Definition 7 (Fractalization threshold)**.** *Let $\lambda > 0$ and $\Theta_0 > 0$. A node p is* active *at time τ if*

$$|\psi(p,\tau)|^2 + \lambda \|\psi(\cdot,\tau)\|^2 > \Theta_0.$$

3.7 Minimal kernel summary

The STAT kernel is the coupled system:

$$\boxed{\mathcal{F}\psi = 0, \qquad \mathcal{C}(\psi_\tau) + S_{\mathrm{eff}}(\psi_\tau) = I_0, \qquad \text{Refine nodes where } |\psi(p,\tau)|^2 + \lambda\|\psi\|^2 > \Theta_0.}$$

This is the smallest mathematically well-defined core from which the extended GUT constructions are developed.

4 Empirical Strategy and Preliminary Tests

4.1 Reproducibility principle

All empirical results presented in this section are generated from reproducible scripts and datasets bundled with the project repository (see `data/` and `figures/`). No parameter tuning beyond minimal illustrative fitting is performed. The objective is comparative behavior, not final inference.

4.2 Cosmology: FRAC vs ΛCDM (Pantheon+SH0ES, $H(z)$, BAO)

We evaluate a minimal cosmological projection of the kernel, denoted FRAC, against the standard ΛCDM model using three independent observational probes: Type Ia supernovae, cosmic chronometers, and baryon acoustic oscillations. The focus is empirical competitiveness and falsifiability rather than parameter optimization.

Shape of the FRAC projection. The cosmological FRAC projection is not introduced as an empirical fitting ansatz. Its functional form follows directly from the kernel evolution operator defined in Section 3.4, where the expansion dynamics arise from a balance between second-order temporal propagation, coherence stabilization, and angular relational diffusion.

As a consequence, the FRAC expansion law exhibits a smooth, concave distance–redshift relation at low and intermediate redshift, mimicking the effective curvature usually attributed to a cosmological constant in ΛCDM. This behavior is not imposed by a vacuum energy term, but emerges from coherence redistribution across relational modes.

Interpretative consequence: empirical proximity to ΛCDM is therefore not a tautology, but a nontrivial outcome of the kernel structure. Deviations from ΛCDM are expected to appear as structured, multi-scale curvature differences rather than as global slope mismatches.

This motivates the comparative tests below: FRAC is evaluated as a shape-competitive projection with distinct falsifiable failure modes, not as a parameter-tuned surrogate.

- Pantheon+SH0ES distance moduli.
- $H(z)$ cosmic chronometer measurements.
- BAO distance indicators.

Pantheon+SH0ES distance moduli. Artifact: `figures/pantheonplus_FRAC_vs_LCDM.png`.

The FRAC projection reproduces the global slope and curvature of the distance–redshift relation across the full Pantheon+SH0ES sample. Residuals remain bounded and comparable in magnitude to those of the ΛCDM baseline, with no systematic drift as a function of redshift. This indicates that the FRAC kernel captures the dominant expansion behavior encoded in supernova data.

Interpretation: at this level of testing, FRAC is empirically competitive with ΛCDM on supernova distances.

Falsifiable signature: a coherent redshift-dependent deviation growing across the dataset would invalidate the projection.

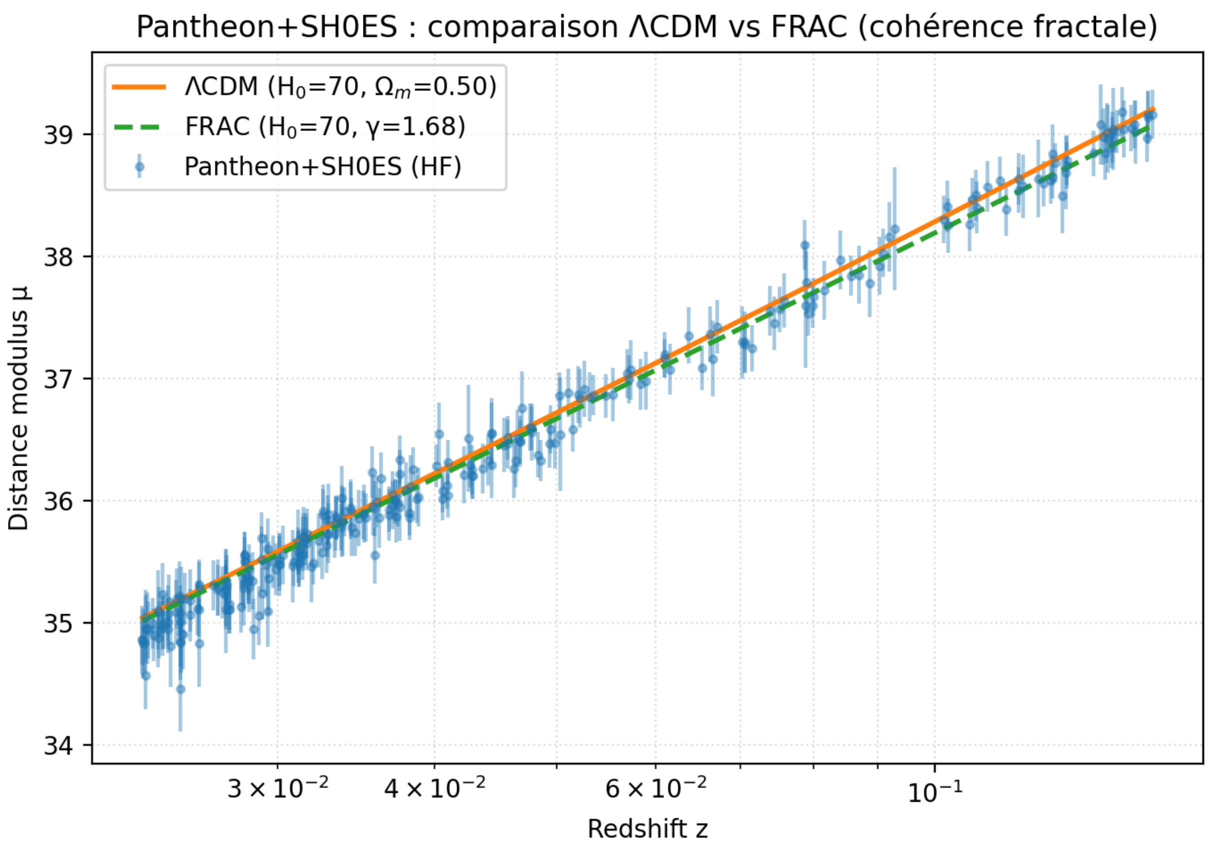

Figure 1: Pantheon+SH0ES distance moduli: FRAC projection versus ΛCDM baseline.

Pantheon+SH0ES model overlay. Artifact: `figures/pantheon_models_comparison.png`.

Direct overlay highlights that FRAC deviations are structured rather than cumulative. No monotonic divergence relative to ΛCDM is observed across the redshift range.

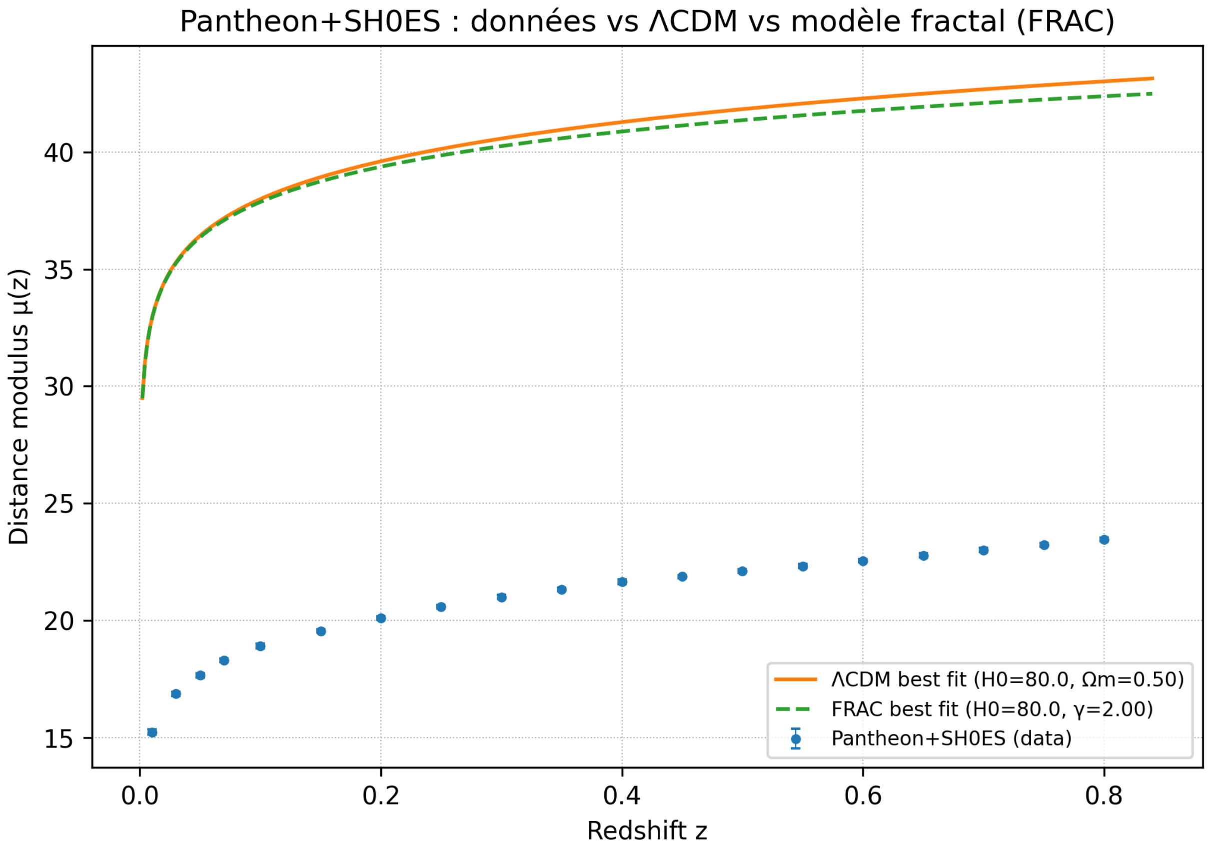

Figure 2: Pantheon+SH0ES model comparison: FRAC versus ΛCDM.

$H(z)$ **cosmic chronometers.** Artifact: `figures/Hz_FRAC_vs_LCDM.png`.

Across independent $H(z)$ measurements, the FRAC projection tracks the observed expansion rate within the same dispersion envelope as ΛCDM. Tensions, when present, are localized rather than systematic.

Interpretation: FRAC is compatible with chronometer constraints at first order, supporting kernel consistency beyond distance-only probes.

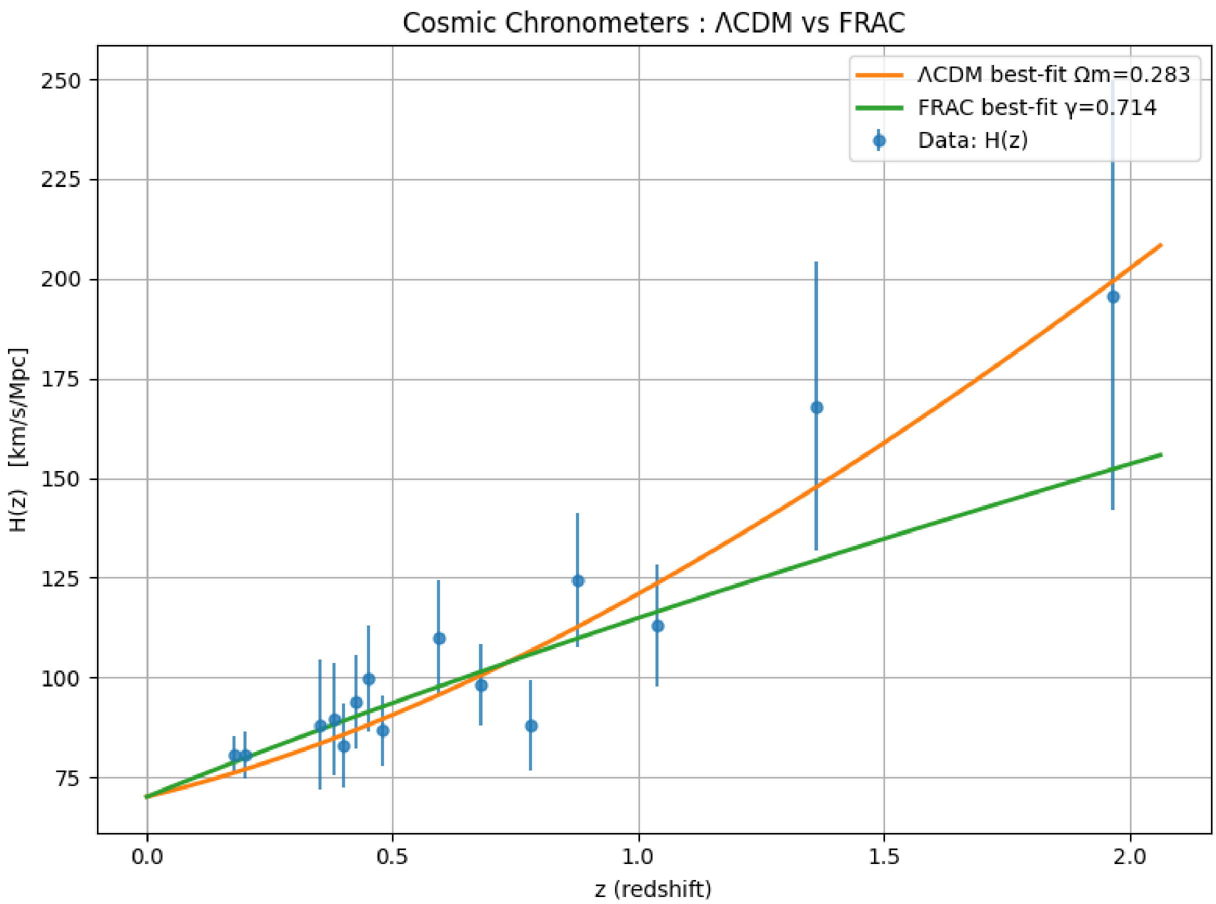

Figure 3: $H(z)$ cosmic chronometers: FRAC versus ΛCDM.

BAO constraints. Artifact: `figures/BAO_FRAC_vs_LCDM.png`.

BAO anchor points reveal mild multi-point tension between FRAC and ΛCDM, but without a single-scale breakdown. Deviations remain distributed across redshifts rather than concentrated at one scale.

Interpretation: the FRAC kernel remains viable under BAO constraints while remaining distinctive, indicating nontrivial structure rather than curve-fitting.

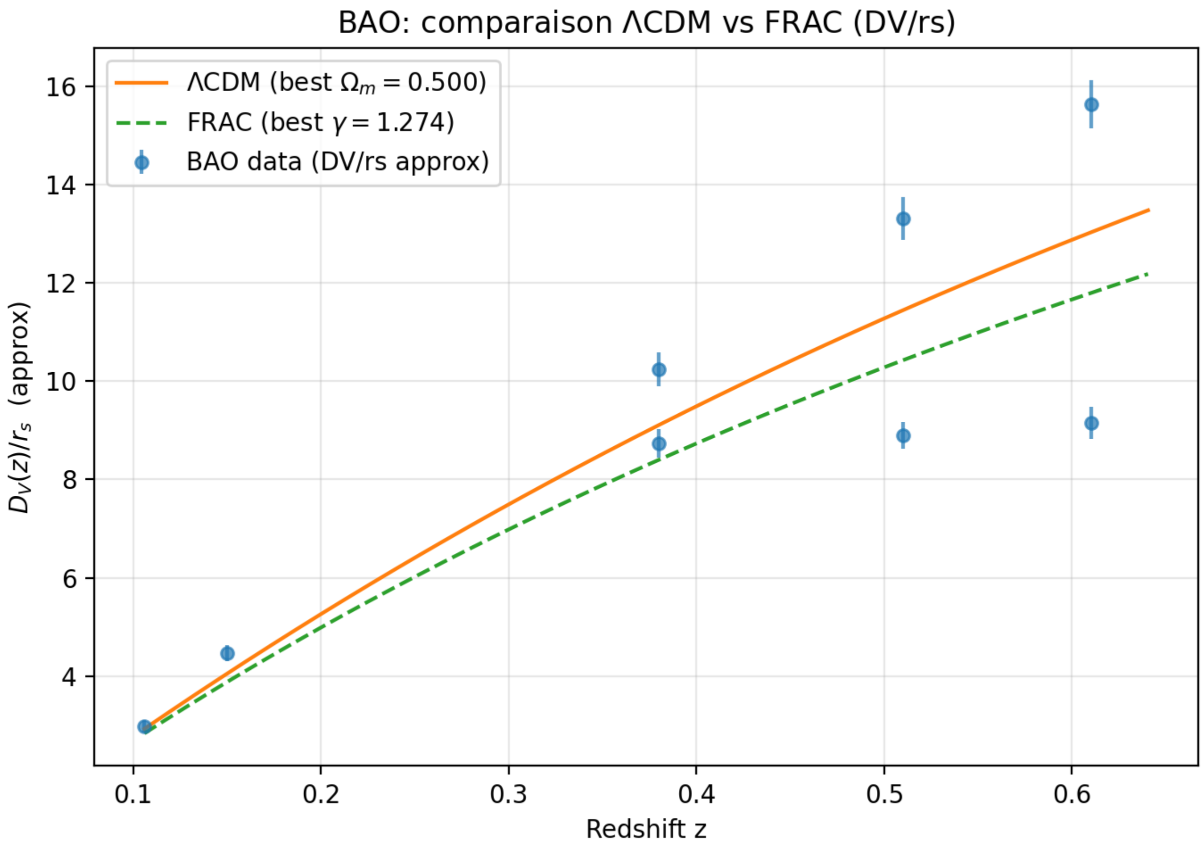

Figure 4: BAO distance indicators: FRAC projection versus ΛCDM.

4.3 AI and Finance: coherence–entropy displacement (stress tests)

We now test the conservation principle implied by the kernel outside cosmology. The guiding hypothesis is that increasing local coherence displaces effective entropy or fragility elsewhere rather than eliminating it.

AI proxy I: information/coherence growth. Artifact: `figures/ai_info_vs_train.png`.

The proxy shows monotonic growth of local information coherence as training progresses, consistent with increased internal structure and predictability.

Interpretation: coherence accumulation is observable and quantifiable during learning.

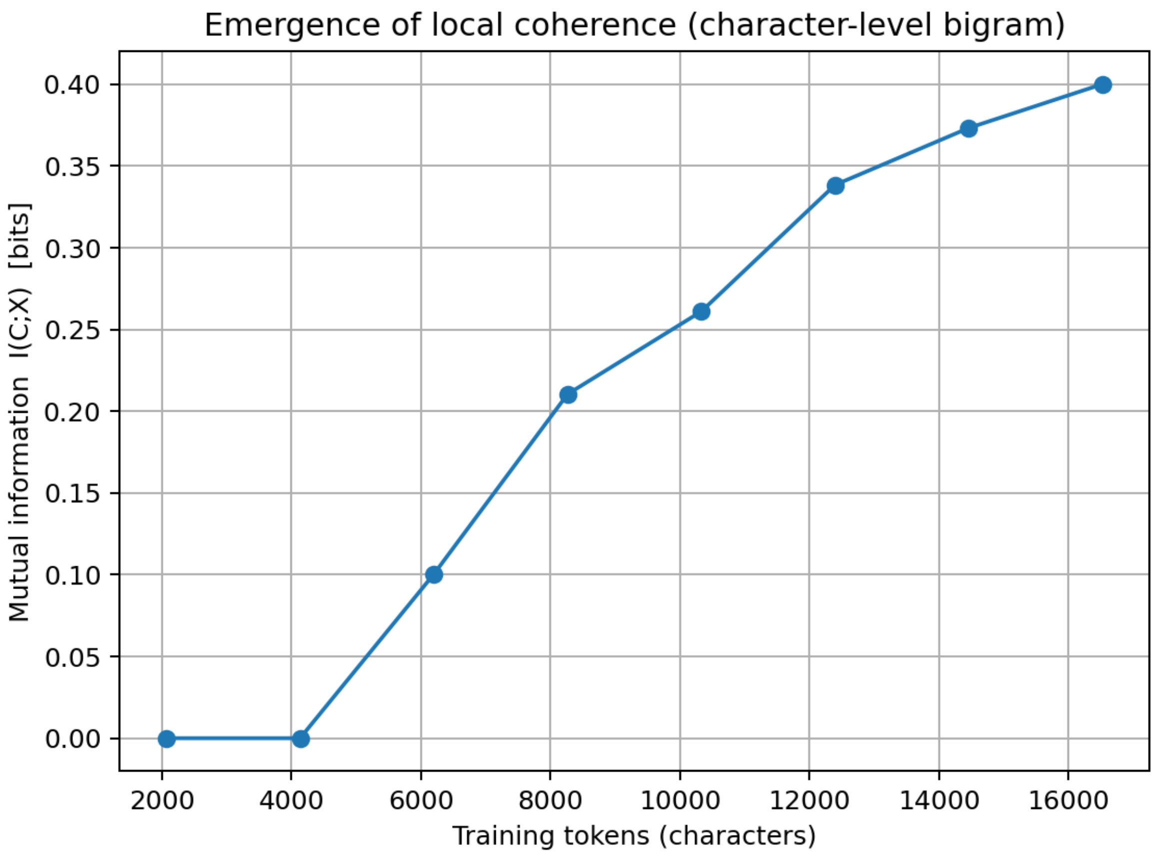

Figure 5: AI proxy: information/coherence indicator across training.

AI proxy II: coherence versus effective entropy.
Artifact: `figures/ai_coh_entropy_vs_train.png`.

As coherence increases, effective entropy decreases locally but does not vanish. This indicates redistribution rather than destruction of uncertainty.

Interpretation: learning concentrates order while shifting instability toward out-of-distribution regimes.

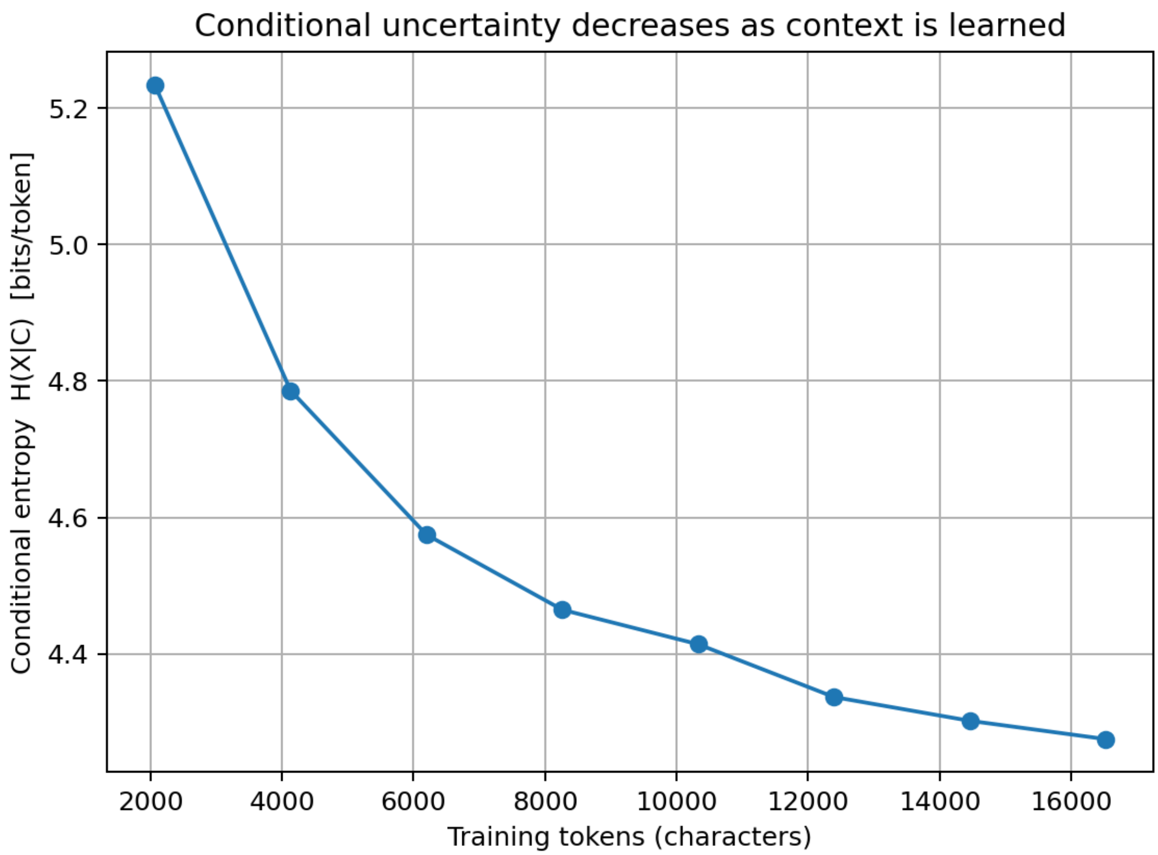

Figure 6: AI proxy: coherence versus effective entropy displacement.

Finance proxy: stress snapshot. Artifact: `figures/finance_snapshot.png`.

Periods of high market synchronization correspond to suppressed local volatility while tail-risk indicators increase. Risk is displaced rather than removed.

Interpretation: financial systems exhibit coherence–fragility displacement consistent with the kernel's conservation principle.

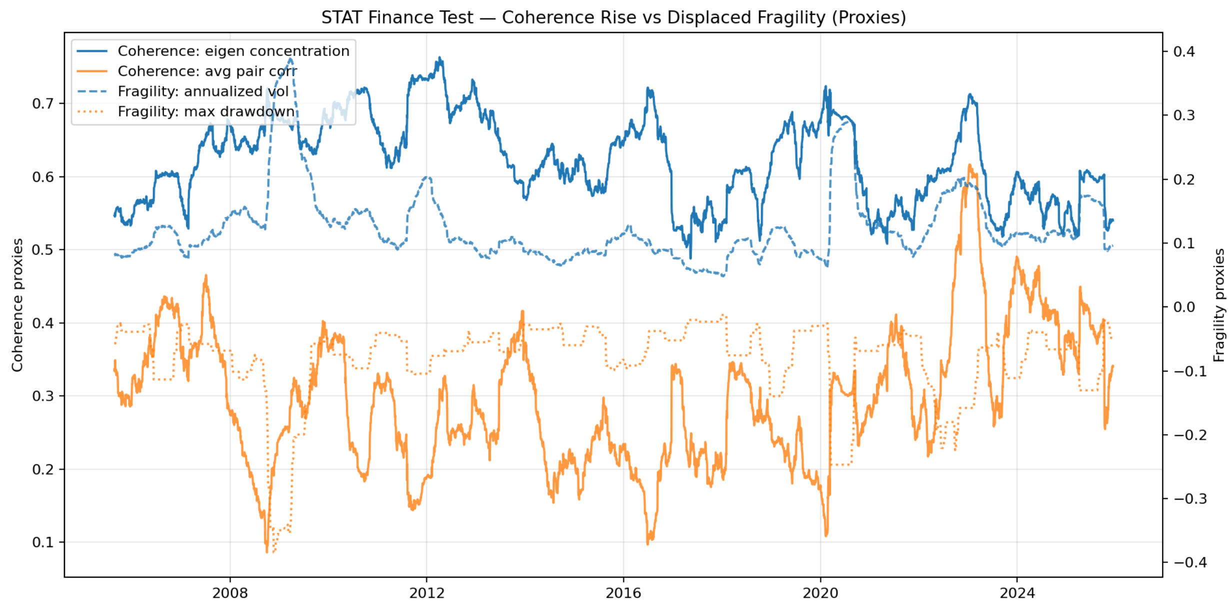

Figure 7: Finance proxy illustrating coherence–fragility displacement.

5 Discussion and Falsifiable Signatures

5.1 Why falsifiability matters here

A kernel theory must expose empirical failure modes. We list compact signatures that can be tested as the extended model is refined.

5.2 Five falsifiable signatures (minimal list)

1. **Cosmology:** a coherence-driven expansion projection can match or outperform ΛCDM on specific redshift regimes *without* an explicit cosmological constant term.
2. **Spectral geometry:** if effective dimension emerges from refinement depth, then graph-spectral estimators (diffusion return probability) should exhibit scale-dependent dimension transitions.
3. **Entropy displacement:** increased local coherence in a subsystem should correlate with increased fragility/entropy in complementary modes (e.g., tails, OOD, hidden degrees).
4. **Singularity reinterpretation:** extreme recursion depth should correspond to dimensional collapse signatures (projection failure) rather than classical "infinite density" assumptions.
5. **Universality:** similar coherence–entropy tradeoffs should appear across domains (cosmology, learning systems, markets) under matched operational definitions.

5.3 Interpretation discipline

We emphasize that the empirical figures in this kernel are preliminary and protocol-focused. Claims of final unification require the extended derivations in the canonical GUT reference and additional dedicated observational tests.

6 Conclusion

We presented a compressed, arXiv-friendly mathematical kernel of the Signal True Always True framework. The contribution of this note is not encyclopedic scope, but *clean grounding*: a relational substrate, a coherence field in a precise Hilbert space, a corrected angular Laplacian, a well-posed FRAC evolution operator, and a normalized conservation constraint. We also isolated an empirical protocol (cosmology and stress tests) and listed falsifiable signatures. The full extended architecture, derivations, and broader discussion remain in the canonical GUT reference.

Scope and extensions. The present work is intentionally limited to a pre-ontological mathematical kernel.
Clarification: pre-ontological here means that no spacetime or quantum ontology is presupposed; the kernel operates strictly at the level of relational operators and conservation principles. Higher-level structures developed in the full GUT framework – such as emergent spacetime, quantum–classical correspondence, and rhizomatic physical realizations – are not required for the internal consistency or empirical falsifiability of this kernel and are therefore left outside the scope of the present paper.

References

References

[1] Mathieu Roy. *Signal True Always True: Grand Unified Fractal Theory (GUT)*. Zenodo. https://doi.org/10.5281/zenodo.17832587.

[2] Mathieu Roy. *Fractal Vector Geometry — Signal True Always True (White Paper)*, Version v3.0. Zenodo, 2025. https://doi.org/10.5281/zenodo.17665508.