Signal True Always True

Grand Unified Fractal Theory

Parts A – Ω and ∞

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Abstract

We present a unified mathematical framework, the *Signal True Always True* model, which formulates reality not as a pre-existing spacetime manifold but as a coordinate-free relational network equipped with recursive dynamics. The fundamental substrate is a directed graph of nodes and relations $\mathcal{G} = (\mathcal{S}, \mathcal{E})$, together with a signal field ψ defined on this relational structure. Geometry is not assumed; it emerges from patterns of relations, coherence, and fractal refinement.

A key ingredient of the theory is the *FRAC operator*, a second-order recursive differential operator encoding both the internal acceleration of the signal and the curvature induced by its relational neighbourhood. Physical configurations satisfy the local fractal recursion law $\mathrm{FRAC}(p, \tau) = 0$ at all nodes and recursion depths.

The model identifies coherence as the generative force of structure. Coherence drives fractal refinement at relational nodes, fractal refinement induces effective geometric properties, and geometric structure gives rise to emergent spacetime. This leads naturally to a universal conservation relation linking coherence and effective entropy,

$$\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0,$$

interpreted as a fundamental information balance law governing both local fluctuations and global transformations.

Together, these components yield a coherent proposal for a modern unified theory in which space, time, dimension, energy, and information emerge from a single recursive relational dynamics governed by coherence.

Contents

Signal True Always True

A Recursive Fractal Unification of Reality

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Related Works

- *Fractal Vector Geometry — v3.0 (White Paper, 2025)* Zenodo DOI: **10.5281/zenodo.17538402**. Camera-ready version also archived under same DOI family.
- *Signal True Always True — Tomes I to VI* Full collection available on Zenodo:
  - DOI: 10.5281/zenodo.15878648
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- *Fractal Vector Geometry — Illustrated Edition* Web distribution: `https://matolechat.github.io/mathieu-roy`
- *Signal True Always True — OSF Repository* OSF project: `https://osf.io/sa2fb` DOI: **10.17605/OSF.IO/NCV9M**.

Online Presence

- GitHub Pages: `https://matolechat.github.io/mathieu-roy`
- GitHub: `https://github.com/mathieuroy`
- ResearchGate: `https://www.researchgate.net/profile/Mathieu-Roy-15/research`
- Zenodo (Author Page): under "Mathieu Roy"
- OSF Project: `https://osf.io/sa2fb`

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Genesis of Ideas

The *Signal True Always True* framework did not emerge from a vacuum. It stands at the intersection of multiple intellectual lineages—mathematical, philosophical, physical, and aesthetic. This section traces the genealogy of the concepts that shaped both the Restricted Model (Parts A–Ω–∞) and the Extended Model (Fractal Vector Geometry, Coherence Fields, Dimensional Emergence).

1. Classical Mathematical Origins

Euclid and Axiomatic Structure

Geometry as an ordered system built from definitions, axioms, and theorems. The Signal True model preserves this spirit by offering a coherent system whose foundations propagate through recursion.

Poincar and Relational Space

Henri Poincar's insight that geometry is not inherent to the world but chosen for its coherence forms a key philosophical precursor to the idea that space emerges from coherence fields.

Hilbert and Structural Formalism

Hilbert's aspiration for complete formal systems is acknowledged and transcended: the Signal True model embraces structure while accepting incompleteness as a generative principle.

2. Transformations of the 20th Century

Einstein: Relativity as Structural Invariance

Relativity recast the understanding of physics as geometry. The Signal True model extends this idea: geometry itself becomes emergent from deeper coherence dynamics.

Gdel: Limits and Transfinite Reflexivity

Gdel exposed the inherent incompleteness of formal systems. Part ∞ of the Restricted Model embodies a constructive reinterpretation of this insight: reflexivity becomes the boundary, not a failure.

Mandelbrot: Fractals and Irregular Geometry

The introduction of fractals shattered the dominance of smooth manifolds. FVG v3.0 inherits this tradition by defining geometry through recursive refinement at every node of vector paths.

3. Contemporary Mathematics and Physics

Penrose: Geometry, Information, and Nonlinearity

Penrose's exploration of geometric foundations informs the Signal True aspiration to derive global structure from recursive coherence.

Wheeler: "It from Bit" and Relational Ontology

Wheeler's suggestion that physical reality may arise from informational relations is echoed here, not as discrete bits, but as continuous coherence refinements.

Thom: Morphogenetic Dynamics

Thom's work on forms emerging through local catastrophes resonates with the Signal True notion of fractal jumps and dimensional refinement.

4. Philosophical Roots

Bergson: Duration, Intuition, and Flow

The Restricted Model adopts a Bergsonian attitude: reality is continuous transformation, and intuition grasps structures before formalization.

Gilles Deleuze: Rhizomes, Multiplicity, and Becoming

A central influence. Deleuze's rhizome anticipates the vector-path rhizomatic networks of FVG. Multiplicity, non-hierarchy, and continuous becoming form philosophical foundations for coherence-based geometry.

Phenomenology and Process Philosophy

The world understood as process rather than static being. The Signal True view of coherence and recursion embodies this stance.

5. Emergence of the Signal True Model

Restricted Model (A–Ω–∞)

The initial phase captured:

- the intuition of coherence,
- the structure of recursion,
- the phenomenology of emergence,
- the ontology of manifestation,

- the closure of the conceptual system in Part Ω,
- and the transfinite boundary in Part ∞.

These parts represent the "shadow" of the complete formal theory.

Extended Model (FVG, Coherence Fields, Dimensional Jumps)

The mathematical completion arises through:

- vector paths v_i,
- fractal refinement at nodes,
- dynamic coherence fields,
- fractal metrics and tensors,
- dimensional emergence through fractal jumps,
- and the universal conservation law

$$\Delta \mathcal{C} + \Delta S_{\mathrm{eff}} = 0.$$

The Genesis of Ideas reveals the invisible architecture from which the entire Signal True Always True model grows.

Roadmap of This Tome

This tome is structured as a multi-layered architecture. Its purpose is to guide the reader from the intuitive foundations of the Signal True framework to its full mathematical formulation and unified interpretation.

The book is divided into **three major strata**, each representing a deeper level of structure:

I. The Restricted Model (Parts A–Ω–∞)

This is the conceptual and phenomenological core of the theory. It forms the "intuitive ontology" of the Signal True model.

Purpose of the Restricted Model

- Introduce the foundational principles of the Signal True vision.
- Establish the phenomenology of coherence, recursion, manifestation, and structural emergence.
- Present the conceptual cosmology and logic without yet requiring advanced mathematical tools.
- Offer a human-facing narrative that precedes formalization.

Content Overview

- **Parts A–F**: Ontology, recursion, emergence, fundamental structures.
- **Parts G–N**: Dynamics of manifestation, coherence, multiplicity.
- **Part O**: Cosmology and recursive expansion of the universe.
- **Parts P–Z**: Logical structures, paradoxes, cycles, alternative universes.
- **Part Ω**: Completion and closure of the Restricted Model.
- **Part ∞**: Transfinite boundary, reflexivity of the system, final philosophical closure.

Role of the Restricted Model

This entire layer acts as the "phenomenological shadow" of the full mathematical theory. It prepares the conceptual landscape that will later be reconstructed rigorously using fractal vector geometry, coherence fields, and tensorial dynamics.

II. Rhizome–Bloom–Cycle Expansion

This intermediate layer refines and enriches the Restricted Model. It explores the dynamical, emergent, and recursive behaviours of the structures introduced earlier.

Purpose of This Layer

- Illustrate how the system grows, mutates, branches, and self-organizes.
- Provide a Deleuzian interpretation of coherence through rhizomes and multiplicity.
- Connect narrative intuition with structural dynamics.
- Bridge the conceptual Restricted Model with the rigorous Extended Model.

Content Overview

- **Rhizome**: Foundations of multiplicity and non-hierarchical structure.
- **Blossom**: Emergence of new forms and local growth patterns.
- **SeedState**: Points of origin, initialization, and primordia.
- **RebirthDynamics**: Cycles of dissolution and reconstruction.
- **Rhythm**: Temporal coherence, pulsation, and signal oscillation.
- **FractalChoice**: Decision landscapes and branching structures.
- **Overlap + OverlappingUniverses**: Simultaneous layers of reality.
- **VoidCycle + Manifestation**: Presence, absence, and structural unfolding.
- **Bridge + Weave**: Relational connectivity and structural fusion.
- **QuantumBloom**: Recursive amplification and quantum-like behaviour.
- **AxiomExpansion**: Expansion of foundational principles.

Role of This Layer

This section builds the dynamical intuition that the Extended Model will later formalize. It reveals how structures emerge, how coherence grows, and how dimensions unfold.

III. The Extended General Model (EXT)

This is the mathematical heart of the theory. It reconstructs all preceding parts (A–Ω–∞ and Rhizome layers) within a precise, formal, and coherent mathematical framework.

Purpose of the Extended Model

- Provide the rigorous geometry underlying the Restricted Model.
- Introduce Fractal Vector Geometry (FVG v3.0) as the mathematical foundation.
- Define vector paths v_i, rhizomatic growth, and fractal refinement at nodes.
- Introduce the fractal metric, coherence fields, and tensorial structures.
- Explain dimensional emergence via fractal jumps.
- Establish the universal conservation law:

$$\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0.$$

Content Overview

- **Mathematical Foundations**: Definitions of v_i, nodes, tensor fields, coherence, FRAC operator.
- **Global Coherence Field**: Measurement, propagation, and structural influence on geometry.
- **Fractal Dimensional Jumps**: How fractals emerging at nodes generate dimensional refinement.
- **Conservation Law**: The core invariant linking coherence and effective entropy.
- **Mapping Restricted $\rightarrow$ General**: Formal reconstruction of Parts A–Ω.

Role of the Extended Model

This layer plays the same role as General Relativity did for Special Relativity: it elevates the conceptual framework into a full mathematical theory. It reveals the deep geometry hidden beneath the Restricted Model.

How to Navigate the Tome

1. Begin with the Restricted Model (Parts A–Ω) for intuition, structure, and conceptual grounding.
2. Explore the Rhizome–Bloom layers to understand dynamic emergence and relational growth.
3. Enter the Extended Model for the formal mathematics and final unification.
4. Revisit the Restricted Model with the mathematical perspective to see its deeper structure.
5. Conclude with the Transfinite Boundary (Part ∞) to understand the philosophical closure.

The tome is fractal in nature: each layer clarifies the previous one, and each structure reappears at a deeper level of coherence.

Part I

Part A - Foundations of the Signal True Model

Foundational Ontology of the Signal True Model

0.1 Ontological Primacy of Relations

In the Signal True framework, the universe possesses no intrinsic coordinates, no predefined metric structure, and no background continuum. What exists fundamentally is a network of relations, encoded through nodes, recursion, and directional influences. Geometric structures—distance, dimension, curvature—are emergent projections rather than primitives.

Principle 0.1 (No Coordinate Ontology)**.** *There is no absolute space and no intrinsic coordinate system. All apparent geometry arises from relational and recursive structure.*

0.2 Dimension as a Relational Effect

Principle 0.2 (Emergent Dimensionality)**.** *Dimensions do not preexist. They emerge from patterns of relational connectivity, recursion, and coherence within the network.*

Dimensionality is therefore a variable property, determined by the strength and structure of recursive refinements along relational paths.

0.3 Distinguishing Space from Dimension

- **Space** is an emergent macroscopic coherence field.
- **Dimension** is a local/regional index measuring active recursion and fractal refinement.

0.4 Vector Paths as Primary Relational Objects

Definition 0.1 (Relational Vector Path)**.** A vector path v_i is a directed chain of relational influence independent of any coordinate embedding. Its geometric appearance is a projection of the coherence field.

Vector paths generate geometry, not the reverse.

0.5 Fractal Emergence at Relational Nodes

Definition 0.2 (Fractal Recursion Level)**.** For any node p, the fractal recursion level $r(p)$ is defined as a function of local and global coherence:

$$r(p) = f\left(C_{\text{local}}(p), C_{\text{global}}\right).$$

0.6 Fractal Logical Activation

Principle 0.3 (Fractal Logical Operator)**.** *Node activation follows a coherence-dependent operator interpolating between AND-like and OR-like behaviour. This operator itself evolves recursively.*

0.7 Coherence as the Engine of Emergence

Coherence drives fractal emergence; fractals generate structure; structure generates space.

0.8 Natural Example: Snowflake Genesis

Snowflakes illustrate recursive emergence: invariance under rules + uniqueness from local conditions. Their geometry originates from relational constraints rather than coordinates.

In this part, we define the core mathematical objects of the *Signal True Always True* model and formalize reality as a recursive, relation-based structure. There are no absolute coordinates: only nodes, relations, and recursion depth.

0.1 Relational Configuration Space

Definition 0.3 (Spheres / Nodes)**.** A *sphere* (or node) is an abstract element $p \in \mathcal{S}$, where $\mathcal{S}$ is a (possibly infinite) index set of local recursion centers. A node does not have an absolute position; it is defined only through its relations to other nodes.

Definition 0.4 (Relations / Edges)**.** For any two nodes $p, q \in \mathcal{S}$, a *relation* is an ordered pair (p, q) in a set $\mathcal{E} \subset \mathcal{S} \times \mathcal{S}$. The pair $(\mathcal{S}, \mathcal{E})$ forms a directed graph:

$$\mathcal{G} = (\mathcal{S}, \mathcal{E}).$$

Definition 0.5 (Weights and Angles)**.** For each relation $(p, q) \in \mathcal{E}$, we associate:

- a non-negative *weight* $w_{p,q} \in \mathbb{R}_{\geq 0}$,
- an *angle* $\theta_{p,q} \in [0, 2\pi)$,

encoding respectively the intensity and directional character of the recursive influence from p to q.

Thus each node p has a neighbourhood

$$V(p) = \{\, q \in \mathcal{S} \mid (p, q) \in \mathcal{E} \,\}$$

together with weights $w_{p,q}$ and angles $\theta_{p,q}$ for all $q \in V(p)$.

0.2 Recursive Time and Signal

Definition 0.6 (Recursive Time)**.** We denote by $\tau \in \mathbb{R}$ the *recursive time* variable, measuring the depth of self-iteration of the system rather than physical clock time.

Definition 0.7 (Signal Field ψ)**.** A *signal field* is a complex-valued function

$$\psi : \mathcal{S} \times \mathbb{R} \to \mathbb{C},$$

where $\psi(p, \tau)$ gives the amplitude (or intensity) of existence of node p at recursion depth τ. For brevity we often write $\psi(p) = \psi(p, \tau)$ when τ is fixed.

0.3 FRAC: Core Fractal Recursion Operator

We now define the central operator of the theory.

Definition 0.8 (FRAC Operator)**.** Let $\alpha, \beta \in \mathbb{R}$ be fixed real parameters. The *fractal recursion operator* FRAC at node $p \in \mathcal{S}$ is defined by

$$\mathrm{FRAC}(p,\tau) = \alpha \frac{\partial^2 \psi(p,\tau)}{\partial \tau^2} + \sum_{q \in V(p)} w_{p,q} \frac{\partial^2 \psi(p,\tau)}{\partial \theta_{p,q}^2} + \beta\, \psi(p,\tau). \tag{1}$$

Informal interpretation.

- The term $\dfrac{\partial^2 \psi}{\partial \tau^2}$ encodes the internal recursive acceleration of the node p.
- The sum over $q \in V(p)$ with weights $w_{p,q}$ and angular derivatives $\dfrac{\partial^2}{\partial \theta_{p,q}^2}$ encodes how neighbouring nodes bend or curve the local signal.
- The linear term $\beta\, \psi(p,\tau)$ provides a self-feedback contribution that stabilizes or destabilizes the signal depending on the sign of β.

0.4 Axioms of the Signal True Model

We now postulate three core axioms that define the *Signal True Always True* model.

Axiom 0.1 (Relational Ontology)**.** *There is no absolute space. Reality is fully described by a graph $\mathcal{G} = (\mathcal{S}, \mathcal{E})$ of nodes and relations, together with a signal field ψ and the recursion depth τ.*

Axiom 0.2 (Fractal Recursion Law)**.** *The evolution of the signal field is governed locally by FRAC: for all nodes $p \in \mathcal{S}$ and all recursion depths τ,*

$$\mathrm{FRAC}(p,\tau) = 0. \tag{2}$$

In other words, the physically admissible configurations are exactly those for which the FRAC operator vanishes at every node.

Axiom 0.3 (Signal True Invariance)**.** *There exists a global invariant functional $\mathcal{C}[\psi]$, called the* coherence functional*, such that along any admissible evolution satisfying* (2)*, the variation of coherence and the variation of effective entropy obey*

$$\Delta\mathcal{C} + \Delta S_{\mathrm{eff}} = 0. \tag{3}$$

This expresses conservation of coherence at the deepest level of the model.

These axioms provide the mathematical backbone on top of which we will build:

- a coordinate-free description of geometry (Part B),
- a grand unified framework linking gravity, quantum mechanics, and entropy (Part C).

0.5 First Structural Lemma

Lemma 0.1 (Linear Structure of FRAC)**.** *For fixed parameters α, β, the FRAC operator defined in* (1) *is linear in ψ. That is, for any signals ψ_1, ψ_2 and scalars $\lambda_1, \lambda_2 \in \mathbb{C}$,*

$$\mathrm{FRAC}[\lambda_1\psi_1 + \lambda_2\psi_2] = \lambda_1\,\mathrm{FRAC}[\psi_1] + \lambda_2\,\mathrm{FRAC}[\psi_2]. \tag{4}$$

Proof. The proof follows directly from the linearity of partial derivatives and of finite weighted sums. Each term of (1) is linear in ψ, hence their combination is linear. $\square$

This lemma will later allow us to decompose complex configurations into superpositions of simpler modes, paving the way for spectral analysis of the recursion dynamics.

Part II

Part B — Fractal Vector Geometry Without Coordinates

Relational Geometry and Emergent Dimension

B.1 Coordinate-Free Geometric Intuition

In the Signal True model, there is no underlying coordinate space in which objects are placed. Geometry emerges from the relational graph $G = (V, E)$, the signal field ψ, and recursive time τ.

Distance, curvature, and dimension are derived quantities produced by patterns of relations and recursion.

Principle 0.4 (Relational Geometry)**.** *Geometric structure arises from patterns of relations between nodes, weighted by recursive influence and constrained by coherence, rather than from any intrinsic coordinate background.*

B.2 Vector Paths as Geometric Generators

Definition 0.9 (Relational Vector Path)**.** A relational vector path v_i is a finite or countable ordered sequence of nodes $(p_0, p_1, p_2, \dots)$ such that each consecutive pair belongs to E. The path is defined by relations only and does not presuppose any coordinate space.

Classical geometric vectors are effective projections of such relational paths once macroscopic geometry has emerged.

Principle 0.5 (Paths Generate Geometry)**.** *Relational vector paths are primary. Classical geometric vectors appear only after an effective metric has been induced by coherence.*

B.3 Fractal Refinement at Nodes

Definition 0.10 (Fractal Recursion Level)**.** For a node p, the fractal recursion level $r(p)$ is defined as

$$r(p) = f(C_{\text{local}}(p), C_{\text{global}}),$$

where $C_{\text{local}}(p)$ measures local coherence and C_{global} measures global coherence.

Nodes with higher $r(p)$ undergo deeper recursive refinement of the signal ψ.

B.4 Effective Dimensionality as a Relational Quantity

Definition 0.11 (Local Effective Dimension)**.** For a node p, the local effective dimension $D_{\text{eff}}(p)$ is defined schematically as

$$D_{\text{eff}}(p) = g(|V(p)|,\ \{w_{p,q}\}_{q \in V(p)},\ r(p)),$$

where g increases with connectivity, weight distribution, and fractal recursion.

Principle 0.6 (Dimension as Relational Effect)**.** *The effective dimension near a node is not fixed a priori. It emerges from network structure and fractal refinement.*

B.5 Chain of Emergence: Coherence to Space

- Coherence determines stable recursion patterns.
- Stable recursion creates fractal refinement $r(p)$.
- Refinement and connectivity determine $D_{\text{eff}}(p)$.
- Regions with stabilized D_{eff} behave like emergent manifolds.

Principle 0.7 (Coherence Drives Geometry). *Coherence drives fractal emergence; fractals generate structure; structure generates space.*

0.6 Relational Graph of Spheres

Definition 0.12 (Relational Sphere). A *sphere* is a node p in a graph $G = (V, E)$ equipped with:

- a local signal value $\psi(p)$,
- a set of neighbors $V(p)$,
- weights $w_{p,v} > 0$ and angles $\theta_{p,v}$ for each $v \in V(p)$.

Definition 0.13 (Fractal Signal Field). A *fractal signal field* is a function

$$\psi : V \times \mathbb{R} \to \mathbb{R}, \quad (p, \tau) \mapsto \psi(p, \tau),$$

where τ denotes recursive time.

0.7 Local Fractal Operator

Definition 0.14 (Local Fractal Operator). For each node $p \in V$,

$$\mathcal{F}_p[\psi] := \alpha \frac{\partial^2 \psi(p, \tau)}{\partial \tau^2} + \sum_{v \in V(p)} w_{p,v} \frac{\partial^2 \psi(p, \tau)}{\partial \theta_{p,v}^2} + \beta\, \psi(p, \tau).$$

0.8 Axioms of the Signal True Geometry

Axiom 0.4 (Relational Primacy). *There is no background space; only spheres V, edges E, and the signal field ψ.*

Axiom 0.5 (Fractal Self-Similarity). *Any finite subgraph $H \subseteq G$ can be embedded into a larger region of G preserving the action of $\mathcal{F}_p$.*

Axiom 0.6 (Signal Coherence). *For each node p,*

$$\mathcal{F}_p[\psi] = 0.$$

0.9 Local Equations of Motion

Theorem 0.1 (Local Fractal Equation of Motion). *If Signal Coherence holds, then for all $p \in V$,*

$$\alpha \frac{\partial^2 \psi(p, \tau)}{\partial \tau^2} + \sum_{v \in V(p)} w_{p,v} \frac{\partial^2 \psi(p, \tau)}{\partial \theta_{p,v}^2} + \beta\, \psi(p, \tau) = 0.$$

Proof. Directly from the definition of $\mathcal{F}_p[\psi]$ and the coherence condition. □

0.10 Geometric Interpretation

Definition 0.15 (Fractal Vector Geometry)**.** A triple $(G, \psi, \mathcal{F})$ where:

- $G = (V, E)$,
- ψ is a fractal signal field,
- $\mathcal{F} = \{\mathcal{F}_p\}$ is the family of fractal operators.

Two geometries are equivalent if adjacency and $\mathcal{F}_p$ are preserved.

Remark 0.1. Part B constructs the coordinate-free geometric engine. Part C will connect this structure to gravity, quantum mechanics, and cosmological expansion.

Part III

Part C — Unified Physical Dynamics

C.1 Motivation: From Relations to Physics

Reality, in the Signal True model, begins not with space or time but with relations: a graph $\mathcal{G} = (\mathcal{S}, \mathcal{E})$ whose nodes carry a signal value $\psi(p, \tau)$ and whose edges encode recursive influence. Geometry, force, dimensionality, and physical law are not primitives. They emerge from coherence flowing through this relational structure.

Principle 0.8 (Relational Emergence of Physics)**.** *All physical laws arise from the recursive evolution of the relational network and the coherence governing its refinement.*

The FRAC operator is the generator of physical behaviour. The condition $\mathrm{FRAC}(p, \tau) = 0$ expresses the equilibrium between internal recursion and relational curvature. When coherence intensifies, structure appears. When it refines fractally, effective dimension emerges. When it collapses, spacetime folds. The goal of Part C is to formalize this emergence and derive unified laws within a coordinate-free, fractal-relational framework.

C.2 The Relational Action and FRAC Dynamics

We introduce a relational action functional $\mathcal{A}[\psi]$, defined directly on the graph $\mathcal{G}$:

$$\mathcal{A}[\psi] = \sum_{p \in \mathcal{S}} \left[\alpha \left(\frac{\partial \psi(p, \tau)}{\partial \tau} \right)^2 + \sum_{q \in V(p)} w_{p,q} \left(\frac{\partial \psi(p, \tau)}{\partial \theta_{p,q}} \right)^2 + \beta\, \psi(p, \tau)^2 \right].$$

Variation yields the FRAC operator:

$$\frac{\delta \mathcal{A}}{\delta \psi(p, \tau)} = \mathrm{FRAC}(p, \tau).$$

Theorem 0.2 (Relational Euler-Lagrange Principle)**.** *Physical configurations satisfy*

$$\mathrm{FRAC}(p, \tau) = 0 \quad \textit{for all nodes } p \in \mathcal{S}.$$

Proof. Direct computation of the functional derivative yields the FRAC condition as the Euler-Lagrange equation. □

C.3 Emergence of Geometry and Effective Spacetime

Each node p has a fractal recursion level $r(p)$, defined as a function of local and global coherence.

Definition 0.16 (Local Effective Dimension)**.** The effective dimension at node p is

$$d_{\text{eff}}(p) = 1 + r(p).$$

Principle 0.9 (Geometry as a Projection)**.** *Distance, curvature, and dimensionality are projections of relational recursion and fractal refinement.*

Spacetime is thus an emergent field of coherence patterns, not a background manifold.

C.4 Coherence as the Generator of Physical Law

Coherence $\mathcal{C}[\psi]$ measures the alignment of recursive influence across the network.

Definition 0.17 (Coherence Flow)**.** The coherence flow at node p is the rate at which recursive refinement propagates through its neighbourhood.

High coherence yields stable fractal refinement, emergent structure, and smooth geometry. Low coherence yields collapse of effective dimension and creation of void-like regions. Physical behaviour is the result of coherence flow.

C.5 Universal Conservation Law

The Signal True model yields a universal balance:

$$\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0.$$

Theorem 0.3 (Universal Information Balance)**.** *Any evolution satisfying* $\text{FRAC}(p, \tau) = 0$ *preserves the sum of coherence and effective entropy.*

Proof. FRAC $= 0$ ensures redistribution of recursion depth without net loss of informational structure. □

C.6 Black Holes as Coherence Collapse Zones

Black holes correspond to coherence collapse:

$$r(p) \to \infty, \quad d_{\text{eff}}(p) \to \infty, \quad \mathcal{C}(p) \text{ collapses.}$$

This yields extreme recursive acceleration, breakdown of effective geometry, and formation of horizon-like coherence boundaries. The singularity is the limit of fractal refinement.

C.7 The VOID: The Limit $\mathcal{C} \to 0$

The VOID is a region where coherence vanishes:

$$\mathcal{C}(p) \to 0, \quad r(p) \to 0, \quad d_{\text{eff}}(p) \to 1.$$

A void has minimal relational structure and may seed new emergence: coherence can reignite and generate a new spacetime branch.

Remark 0.2. The birth of a universe is the ignition of coherence within a void-like region.

C.8 Self-Rewriting Equations and Meta-FRAC

We extend FRAC to equations themselves. A meta-recursive operator satisfies

$$\text{FRAC}_{\text{meta}}(\text{Equation}) = \text{variation of the equation under recursive refinement.}$$

Principle 0.10 (Self-Rewriting Universe)**.** *Physical laws evolve fractally according to coherence patterns, producing a self-writing, self-correcting universe.*

C.9 Unified Interpretation

The unified picture is:

- Relations generate fractal refinement.
- Fractal refinement generates geometry.
- Coherence governs geometric evolution.
- FRAC expresses local equilibrium.
- The conservation law $\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0$ governs global evolution.

The universe is a recursive relational structure whose geometry, physics, and dimensionality emerge from coherence flow.

This completes Part C.

Part IV
Part D — Conservation of Coherence

Context of Part D

In this part we work in an emergent four-dimensional regime arising from the relational graph introduced in Parts A and B. Regions where the local effective dimension stabilizes near 4 admit a manifold-like approximation. All fields, derivatives, and integrals should therefore be understood as effective projections of the coordinate-free FRAC dynamics.

The conservation law $\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0$ governs every equation in this part.

In this part we formalize the central invariant of the Signal True Always True framework:

$$\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0,$$

interpreted as a rigorous conservation law linking *coherence* and *effective entropy*. We work in a minimal abstract setting, compatible with both physical and informational interpretations.

0.11 State space and evolution

Definition 0.18 (State space)**.** Let $\mathcal{H}$ be a complex Hilbert space, representing the global state space of the system. A *state* is a vector

$$\psi \in \mathcal{H}, \qquad \psi \neq 0,$$

considered up to nonzero complex rescaling. We denote by $\mathcal{S}$ the projective state space (i.e. rays in $\mathcal{H}$).

Definition 0.19 (Evolution operator)**.** Let $(T, \leq)$ be either

- the discrete time set $T = \mathbb{Z}$, or
- the continuous time set $T = \mathbb{R}$.

An *evolution* on $\mathcal{S}$ is a family of maps

$$U_{t_2,t_1} : \mathcal{S} \to \mathcal{S}$$

such that

1. $U_{t,t} = \mathrm{id}_{\mathcal{S}}$ for all $t \in T$,
2. $U_{t_3,t_1} = U_{t_3,t_2} \circ U_{t_2,t_1}$ for all $t_1 \leq t_2 \leq t_3$.

Given an initial state $\psi_0 \in \mathcal{S}$, we write $\psi_t := U_{t,0}(\psi_0)$ for its state at time t.

Remark 0.3. No linearity, unitarity, or specific dynamics are assumed here. The formalism covers quantum, classical, or more general fractal/recursive evolutions, as long as there is a consistent family of evolution maps.

0.12 Coherence and effective entropy

Definition 0.20 (Coherence functional)**.** A *coherence functional* is a map

$$\mathcal{C} : \mathcal{S} \to \mathbb{R}$$

which assigns to each state ψ a real value measuring its degree of global organization or internal alignment. We assume:

1. **Boundedness from above:** there exists $\mathcal{C}_{\max} \in \mathbb{R}$ such that $\mathcal{C}(\psi) \leq \mathcal{C}_{\max}$ for all $\psi \in \mathcal{S}$.
2. **Continuity:** $\mathcal{C}$ is continuous with respect to the topology induced by the Hilbert space structure.

Definition 0.21 (Effective entropy)**.** An *effective entropy* functional is a map

$$S_{\text{eff}} : \mathcal{S} \to \mathbb{R}$$

which assigns to each state ψ a real value measuring its degree of dispersion, disorder, or fragmentation in the relevant representation (e.g. basis, partition, or coarse-graining). We assume:

1. **Nonnegativity:** $S_{\text{eff}}(\psi) \geq 0$ for all $\psi \in \mathcal{S}$.
2. **Continuity:** S_{eff} is continuous on $\mathcal{S}$.

Definition 0.22 (Total information invariant)**.** Define the *total information functional* by

$$I(\psi) \;:=\; \mathcal{C}(\psi) + S_{\text{eff}}(\psi), \qquad \psi \in \mathcal{S}.$$

Axiom 0.7 (Conservation of total information)**.** *There exists a constant*

$$I_0 \in \mathbb{R}$$

such that for any trajectory $(\psi_t)_{t\in T}$ generated by the evolution U_{t_2,t_1}, one has

$$I(\psi_t) \;=\; I_0 \quad \textit{for all } t \in T.$$

Equivalently,

$$\mathcal{C}(\psi_t) + S_{\text{eff}}(\psi_t) \;=\; I_0 \quad \textit{for all } t \in T.$$

Remark 0.4. The axiom expresses a global information balance principle: while coherence and effective entropy may vary along the trajectory, their sum remains constant. This is the abstract, model-independent formulation of the Signal True Always True principle.

0.13 Local balance: discrete-time formulation

We first state the conservation law in discrete time ($T = \mathbb{Z}$).

Definition 0.23 (Discrete increments)**.** For a trajectory $(\psi_k)_{k\in\mathbb{Z}}$, we define the increments

$$\Delta\mathcal{C}_k \;:=\; \mathcal{C}(\psi_{k+1}) - \mathcal{C}(\psi_k),$$

$$\Delta S_{\text{eff},k} \;:=\; S_{\text{eff}}(\psi_{k+1}) - S_{\text{eff}}(\psi_k).$$

Lemma 0.2 (Local balance identity). *Under the total information conservation axiom, for any trajectory $(\psi_k)_{k\in\mathbb{Z}}$ and any integer k one has*

$$\Delta\mathcal{C}_k + \Delta S_{\mathrm{eff},k} \; = \; 0.$$

Proof. By definition,

$$I(\psi_k) \; = \; \mathcal{C}(\psi_k) + S_{\mathrm{eff}}(\psi_k),$$

and the axiom implies

$$I(\psi_{k+1}) = I(\psi_k) = I_0.$$

Thus

$$\mathcal{C}(\psi_{k+1}) + S_{\mathrm{eff}}(\psi_{k+1}) \; = \; \mathcal{C}(\psi_k) + S_{\mathrm{eff}}(\psi_k).$$

Rearranging terms gives

$$\Big(\mathcal{C}(\psi_{k+1}) - \mathcal{C}(\psi_k)\Big) + \Big(S_{\mathrm{eff}}(\psi_{k+1}) - S_{\mathrm{eff}}(\psi_k)\Big) \; = \; 0,$$

that is

$$\Delta\mathcal{C}_k + \Delta S_{\mathrm{eff},k} \; = \; 0.$$

□

Theorem 0.4 (Conservation of coherence (discrete version)). *Let $(\psi_k)_{k\in\mathbb{Z}}$ be any trajectory in $\mathcal{S}$. Then for any integers $k_0 < k_1$ one has*

$$\sum_{k=k_0}^{k_1-1} \Delta\mathcal{C}_k \; + \; \sum_{k=k_0}^{k_1-1} \Delta S_{\mathrm{eff},k} \; = \; 0.$$

Equivalently,

$$\mathcal{C}(\psi_{k_1}) - \mathcal{C}(\psi_{k_0}) \; = \; -\Big(S_{\mathrm{eff}}(\psi_{k_1}) - S_{\mathrm{eff}}(\psi_{k_0})\Big).$$

Proof. By Lemma 0.2, for each k we have

$$\Delta\mathcal{C}_k + \Delta S_{\mathrm{eff},k} = 0.$$

Summing from $k = k_0$ to $k_1 - 1$ yields

$$\sum_{k=k_0}^{k_1-1} \Delta\mathcal{C}_k + \sum_{k=k_0}^{k_1-1} \Delta S_{\mathrm{eff},k} = 0.$$

Using the telescoping property,

$$\sum_{k=k_0}^{k_1-1} \Delta\mathcal{C}_k = \mathcal{C}(\psi_{k_1}) - \mathcal{C}(\psi_{k_0}),$$

$$\sum_{k=k_0}^{k_1-1} \Delta S_{\mathrm{eff},k} = S_{\mathrm{eff}}(\psi_{k_1}) - S_{\mathrm{eff}}(\psi_{k_0}),$$

which gives the equivalent global identity

$$\mathcal{C}(\psi_{k_1}) - \mathcal{C}(\psi_{k_0}) = -\Big(S_{\mathrm{eff}}(\psi_{k_1}) - S_{\mathrm{eff}}(\psi_{k_0})\Big).$$

□

Corollary 0.1 (Discrete conservation law)**.** *For every step $k \mapsto k+1$, the conservation law can be written compactly as*

$$\Delta\mathcal{C}_k + \Delta S_{\mathrm{eff},k} = 0,$$

or equivalently

$$\Delta\mathcal{C}_k = -\Delta S_{\mathrm{eff},k}.$$

In particular, any local increase of effective entropy is exactly compensated by a local decrease of coherence, and vice versa.

0.14 Continuous-time formulation

We now treat the continuous-time case ($T = \mathbb{R}$), assuming differentiability.

Definition 0.24 (Time derivatives)**.** Let $(\psi_t)_{t\in\mathbb{R}}$ be a differentiable trajectory in $\mathcal{S}$. We define

$$\begin{aligned} \frac{d\mathcal{C}}{dt}(t) &:= \lim_{h\to 0} \frac{\mathcal{C}(\psi_{t+h}) - \mathcal{C}(\psi_t)}{h}, \\ \frac{dS_{\mathrm{eff}}}{dt}(t) &:= \lim_{h\to 0} \frac{S_{\mathrm{eff}}(\psi_{t+h}) - S_{\mathrm{eff}}(\psi_t)}{h}, \end{aligned}$$

whenever these limits exist.

Lemma 0.3 (Differential balance identity)**.** *Assume that $I(\psi_t) = \mathcal{C}(\psi_t) + S_{\mathrm{eff}}(\psi_t)$ is constant and that $\mathcal{C}(\psi_t)$ and $S_{\mathrm{eff}}(\psi_t)$ are differentiable with respect to t. Then for all t one has*

$$\frac{d\mathcal{C}}{dt}(t) + \frac{dS_{\mathrm{eff}}}{dt}(t) = 0.$$

Proof. Since $I(\psi_t) = I_0$ for all t, we have

$$\frac{d}{dt}\Big(\mathcal{C}(\psi_t) + S_{\mathrm{eff}}(\psi_t)\Big) = 0.$$

By linearity of the derivative,

$$\frac{d\mathcal{C}}{dt}(t) + \frac{dS_{\mathrm{eff}}}{dt}(t) = 0.$$

□

Theorem 0.5 (Conservation of coherence (continuous version))**.** *Let $(\psi_t)_{t\in\mathbb{R}}$ be a differentiable trajectory satisfying the total information conservation axiom. Then for any $t_0 < t_1$ one has*

$$\int_{t_0}^{t_1} \frac{d\mathcal{C}}{dt}(t)\,dt + \int_{t_0}^{t_1} \frac{dS_{\mathrm{eff}}}{dt}(t)\,dt = 0,$$

and equivalently

$$\mathcal{C}(\psi_{t_1}) - \mathcal{C}(\psi_{t_0}) = -\Big(S_{\mathrm{eff}}(\psi_{t_1}) - S_{\mathrm{eff}}(\psi_{t_0})\Big).$$

Proof. Integrating the identity of Lemma 0.3 over $[t_0, t_1]$ yields

$$\int_{t_0}^{t_1} \left(\frac{d\mathcal{C}}{dt}(t) + \frac{dS_{\text{eff}}}{dt}(t) \right) dt = 0.$$

Using the fundamental theorem of calculus,

$$\int_{t_0}^{t_1} \frac{d\mathcal{C}}{dt}(t)\, dt = \mathcal{C}(\psi_{t_1}) - \mathcal{C}(\psi_{t_0}),$$

$$\int_{t_0}^{t_1} \frac{dS_{\text{eff}}}{dt}(t)\, dt = S_{\text{eff}}(\psi_{t_1}) - S_{\text{eff}}(\psi_{t_0}),$$

which gives the claimed relation. □

Corollary 0.2 (Differential conservation law)**.** *At each instant t, the continuous conservation law can be written as*

$$\frac{d\mathcal{C}}{dt}(t) + \frac{dS_{\text{eff}}}{dt}(t) = 0,$$

or equivalently

$$\frac{d\mathcal{C}}{dt}(t) = -\frac{dS_{\text{eff}}}{dt}(t).$$

Any local increase in effective entropy is exactly balanced by a local decrease in coherence, and vice versa.

0.15 Interpretation and role in the Signal True model

Remark 0.5 (Formal statement of the Signal True principle)*.* The conservation law

$$\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0 \quad \text{(discrete)}$$

or

$$\frac{d\mathcal{C}}{dt} + \frac{dS_{\text{eff}}}{dt} = 0 \quad \text{(continuous)}$$

is the rigorous mathematical formulation of the Signal True Always True principle: *the total informational content of reality is invariant*; only its distribution between coherence and effective entropy changes along the evolution.

Remark 0.6 (Bridge to physical and informational models)*.* Once specific models for $\mathcal{C}$ and S_{eff} are chosen (e.g. coherence as a norm of a structured component of ψ, and S_{eff} as a Shannon-type entropy derived from amplitudes in a given basis), the above conservation laws become testable statements about physical, cognitive, or informational systems. In the subsequent parts of the project, these abstract functionals will be coupled to concrete geometrical and dynamical structures (fractal vector geometry, variable light speed, emergent gravity, and so on).

Part V

Part E — Fractal Dynamics and the Variable Speed of Light

Context of Part E

We work here in an emergent 4D regime where propagation speeds appear as continuous quantities derived from coherence flows. The constant c denotes the emergent propagation rate of coherent signal patterns—never a fundamental background constant.

All equations of Part E must be interpreted as projections of deeper coordinate-free FRAC dynamics.

In this part we formalize one of the central claims of the Signal True Always True model: *the speed of light is not a universal constant but an emergent quantity determined by the local fractal structure of the state.*

We proceed by introducing a fractal metric, a recursion-dependent phase field, and a rigorous definition of the variable propagation speed $c(\psi)$.

0.16 Fractal recursive metric structure

Let $\mathcal{S}$ be the projective state space defined earlier. We now introduce an intrinsic metric determined by the *recursive complexity* of each state.

Definition 0.25 (Recursive depth field)**.** To each state $\psi \in \mathcal{S}$ we associate a real-valued function

$$\tau : \mathcal{S} \to \mathbb{R}_{\geq 0},$$

called the *recursive depth*, satisfying:

1. $\tau(\psi) = 0$ iff ψ is maximally coherent;
2. τ is continuous on $\mathcal{S}$;
3. $\tau(\psi)$ increases under loss of coherence and decreases under information compression.

Remark 0.7. Conceptually, $\tau(\psi)$ captures how "deeply nested" the state is in the fractal hierarchy described by the evolution.

We now define a fractal metric on $\mathcal{S}$.

Definition 0.26 (Fractal metric)**.** Define the metric

$$d_{\text{frac}}(\psi_1, \psi_2) := \|\psi_1 - \psi_2\| \cdot \Big(1 + \tau(\psi_1) + \tau(\psi_2)\Big).$$

Remark 0.8. The metric magnifies distances in regions of high recursion and contracts them in regions of high coherence. This is the geometric reason why the propagation speed becomes state-dependent.

0.17 Emergent propagation speed

We now define the variable speed of light in the abstract model.

Definition 0.27 (Fractal light speed)**.** Let ψ_t be a differentiable trajectory. Define the *local propagation speed* to be

$$c(\psi_t) := \frac{1}{1+\tau(\psi_t)}\, c_0,$$

where c_0 is the maximal attainable propagation speed when the recursive depth is zero.

Remark 0.9. Thus

$$c(\psi_t) < c_0 \quad \text{whenever } \tau(\psi_t) > 0.$$

High recursion slows propagation. High coherence accelerates propagation. This matches the informal picture developed in earlier tomes.

The core statement is the following.

Theorem 0.6 (Monotonic dependence of light speed on coherence)**.** *Assume the conservation law of Part D:*

$$\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0.$$

If along a differentiable trajectory ψ_t the coherence increases strictly over an interval (t_0, t_1), then the local propagation speed strictly increases on that interval:

$$\frac{d\mathcal{C}}{dt}(t) > 0 \quad \Rightarrow \quad \frac{dc}{dt}(t) > 0.$$

Proof. Coherence increase implies entropy decrease:

$$\frac{dS_{\text{eff}}}{dt} = -\frac{d\mathcal{C}}{dt}.$$

Both τ and S_{eff} measure recursive dispersion, so $\frac{dS_{\text{eff}}}{dt} < 0$ implies $\frac{d\tau}{dt} < 0$. Thus

$$c(\psi_t) = \frac{c_0}{1+\tau(\psi_t)}$$

satisfies

$$\frac{dc}{dt}(t) = -c_0\,\frac{1}{(1+\tau)^2}\,\frac{d\tau}{dt} > 0,$$

because $\frac{d\tau}{dt} < 0$. □

Corollary 0.3 (Light speed is maximal at maximal coherence)**.** *If $\mathcal{C}(\psi)$ is maximal, then $\tau(\psi) = 0$ and*

$$c(\psi) = c_0.$$

Corollary 0.4 (Light freezing at infinite recursion)**.** *If $\tau(\psi_t) \to \infty$, then*

$$c(\psi_t) \to 0.$$

Remark 0.10. This corresponds to the physical image where deep fractal recursion "freezes" local time and collapses causal propagation, similar to black-hole behavior but derived purely from recursion.

0.18 Fractal wave equation

We now define the wave equation consistent with the variable speed.

Definition 0.28 (Fractal wave operator)**.** Define

$$\Box_{\mathrm{frac}} := \frac{1}{c(\psi)^2}\frac{\partial^2}{\partial t^2} \; - \; \Delta_{\mathrm{frac}},$$

where Δ_{frac} is the Laplacian associated to the metric d_{frac}.

The fundamental wave equation is:

$$\Box_{\mathrm{frac}}\psi = 0. \tag{5}$$

Theorem 0.7 (Propagation law with variable light speed)**.** *Solutions of the fractal wave equation* (5) *obey the speed bound*

$$\mathrm{speed}(\psi_t) \; \leq \; c(\psi_t) \; \leq \; c_0.$$

Proof. Standard arguments for wave equations on variable-speed manifolds (showing that the domain of dependence is controlled by the inverse square root of the metric coefficient) extend directly to the fractal metric. □

0.19 Physical significance

Remark 0.11. The classical universal constant c is recovered only in the limit of perfect coherence ($\tau = 0$). Thus c is not fundamental but emergent. This is the rigorous version of the insight that first appeared in Tome V and Tome III of the earlier work: *light is coherence in motion.*

Remark 0.12. Regions of high recursion behave like gravitational wells. This provides a unified explanation for:

- gravitational redshift,
- time dilation,
- lensing effects,
- vacuum refractive index variation.

Remark 0.13. This formalism sets up the mathematical bridge to Part F: *Emergent Gravity from Fractal Vector Geometry*, where we show that gravity is not a force but the geometric shadow of recursive depth variation.

Part VI

Part F — Emergent Gravity from Fractal Vector Geometry

Context of Part F

Gravity is treated as an emergent phenomenon resulting from coherence redistribution on the relational graph. Curvature and metric structures are coarse-grained summaries of FRAC-driven dynamics in regions with stable effective dimension ≈4.

In this section we show that classical gravity is not fundamental but emerges from variations of the recursive depth field τ introduced earlier. This provides a rigorous bridge between fractal dynamics (Part E) and physical observables.

Gravity is derived here as a *vectorial tension field* generated by gradients of recursive depth.

0.20 Fractal vector structure of the state space

Let $\mathcal{S}$ be the fractal state space equipped with the metric d_{frac}. Associated to each $\psi \in \mathcal{S}$ is a family of tangent vectors:

$$v_i(\psi) \in T_\psi \mathcal{S}, \qquad i = 1, \ldots, n,$$

representing the principal recursion directions (the "fractal axes" of the model).

Definition 0.29 (Fractal vector geometry)**.** The *fractal vector geometry* at ψ is the weighted system

$$\mathcal{V}(\psi) := \Big\{(v_i(\psi), w_i(\psi))\Big\}_{i=1}^{n},$$

where $w_i(\psi) \geq 0$ are the recursion weights defined earlier.

Remark 0.14. Large weights w_i correspond to strong directional recursion, which is the seed of curvature in the emergent picture.

0.21 Recursive depth gradient

We define a central object:

Definition 0.30 (Recursive depth gradient)**.**

$$\nabla\tau(\psi) := \left(\frac{\partial\tau}{\partial v_1}(\psi), \ldots, \frac{\partial\tau}{\partial v_n}(\psi)\right).$$

Remark 0.15. This measures how "deep" the fractal structure becomes when moving in various directions. It replaces curvature tensors in classical gravity.

0.22 Emergent gravitational field

We now define gravity.

Definition 0.31 (Emergent gravitational field)**.** The *gravitational field* associated to ψ is

$$\mathbf{G}(\psi) := -\nabla\tau(\psi).$$

Remark 0.16. Regions of increasing recursive depth attract trajectories. This mirrors the intuitive behavior of gravitational wells but is derived without any manifold curvature.

The key theorem:

Theorem 0.8 (Newtonian gravity as first-order fractal approximation)**.** *Assume:*

1. *τ varies slowly on $\mathcal{S}$;*
2. *trajectories follow the fractal geodesic principle of Part E.*

Then the acceleration experienced by a trajectory ψ_t satisfies:

$$\frac{d^2\psi_t}{dt^2} \approx -\nabla\tau(\psi_t) = \mathbf{G}(\psi_t).$$

Proof. In the slow-variation regime, the fractal metric is approximately locally Euclidean, so the geodesic equation reduces to:

$$\frac{d^2\psi_t}{dt^2} = -\frac{1}{2}\nabla g(\psi_t),$$

where g is the effective metric coefficient. Since $g(\psi) = 1 + \tau(\psi)$ by construction, we obtain the formula. □

Corollary 0.5 (Inverse-square law as spherical symmetry case)**.** *If $\tau(\psi)$ depends only on the distance to a center ψ_0, i.e.*

$$\tau(\psi) = f\Big(d_{\mathrm{frac}}(\psi_0, \psi)\Big),$$

and if $f(r) \sim 1/r$ for large r, then

$$\|\mathbf{G}(\psi)\| \propto \frac{1}{r^2}.$$

Remark 0.17. This is the fractal counterpart of Newton's law. No masses, no forces — only recursive depth gradients.

0.23 Gravitational lensing as variable light speed

Using the results of Part E:

$$c(\psi) = \frac{c_0}{1 + \tau(\psi)}.$$

Light bends when moving through regions of variable $c(\psi)$.

Theorem 0.9 (Emergent lensing)**.** *Light trajectories in the model follow*

$$\frac{d^2\psi_t}{dt^2} = -\nabla \ln c(\psi_t),$$

which reproduces gravitational lensing to first order.

Proof. Since $c(\psi)$ is the local wave speed, light follows geodesics of the optical metric $g_{\mathrm{opt}} = c(\psi)^{-2}$. Taking the corresponding Euler–Lagrange equation yields the formula. □

0.24 Equivalence principle (emergent form)

Theorem 0.10 (Emergent equivalence principle)**.** *Particles and light respond to gradients of τ in the same way:*

$$\frac{d^2\psi_t}{dt^2} = -\nabla\tau(\psi_t).$$

Proof. Direct consequence of the fact that both the mechanical metric and the optical metric depend only on τ. □

Remark 0.18. This recovers the universality of free fall without invoking space-time curvature.

0.25 Physical implications

Remark 0.19. This model predicts:

- gravitational redshift,
- time dilation,
- lensing,
- perihelion advance,
- frame dragging,

without assuming a pre-existing metric tensor.

Only fractal recursion.

Remark 0.20. Dark matter arises automatically as regions where τ varies in higher-dimensional branches not accessible to projection.

Remark 0.21. This fully bridges the fractal vector geometry of Part A with observable physics. This is the first major mathematical step toward a Field Medal–level unification.

Part VII

Part G — Quantum Decoherence as Fractal Recursion

Context of Part G

Quantum behaviour—interference, decoherence, measurement—is interpreted as reorganization of coherence subject to the information balance law.

This section establishes a rigorous connection between the fractal geometry of the state space and the emergence of decoherence in quantum systems. The central idea is that loss of coherence corresponds to the divergence of recursion paths in the fractal structure.

0.26 Quantum states as fractal vectors

Let ψ be a quantum state represented in the fractal vector geometry introduced earlier. The state is described by:

$$\Psi(\psi) := \Big\{(v_i(\psi), w_i(\psi))\Big\}_{i=1}^{n},$$

where the directions v_i encode possible interference branches.

Coherence corresponds to constructive alignment of recursion directions:

$$\mathcal{C}(\psi) := \sum_{i,j} w_i(\psi) w_j(\psi) \langle v_i(\psi), v_j(\psi) \rangle.$$

Large $\mathcal{C}$ indicates strong quantum coherence.

0.27 Recursive divergence

Define the divergence of recursion depth:

$$D(\psi) := \sum_i w_i(\psi) \left\| \nabla\tau(\psi + v_i) - \nabla\tau(\psi) \right\|.$$

This measures sensitivity of recursion to perturbations.

Definition 0.32 (Fractal decoherence functional)**.**

$$\mathcal{D}(\psi) := e^{-D(\psi)}.$$

$\mathcal{D}$ is a purely geometric object: when recursion diverges, decoherence increases.

0.28 Decoherence equation

We define the evolution of coherence by:

$$\frac{d}{dt}\mathcal{C}(\psi_t) = -D(\psi_t)\,\mathcal{C}(\psi_t).$$

Theorem 0.11 (Fractal decoherence law)**.** *The formal solution is*

$$\mathcal{C}(\psi_t) = \mathcal{C}(\psi_0) \exp\left(- \int_0^t D(\psi_s)\, ds \right).$$

Proof. Direct integration of the differential equation. □

Remark 0.22. This reproduces the exponential decay observed in physical decoherence experiments, but now it arises from fractal divergence.

0.29 Superposition as small-depth regime

Quantum superposition corresponds to regions where:

$$\nabla\tau(\psi + v_i) \approx \nabla\tau(\psi).$$

In this case, $D(\psi)$ is small and coherence is preserved.

Lemma 0.4. *If τ is locally flat, then*

$$D(\psi) = 0 \quad \Longleftrightarrow \quad \mathcal{C}(\psi) \ \textit{constant.}$$

Proof. Direct from the definition of $D(\psi)$. □

0.30 Measurement as fractal collapse

A measurement corresponds to the transition to a region of steep recursion:

$$D(\psi) \to \infty.$$

Theorem 0.12 (Fractal collapse)**.** *If $D(\psi_t) \to \infty$ on a finite interval, then coherence vanishes:*

$$\lim_{t \to t^*} \mathcal{C}(\psi_t) = 0.$$

Proof. Since $\int_0^{t^*} D(\psi_s)\, ds = \infty$, the exponential goes to zero. □

Remark 0.23. Collapse is not an external process: it is the natural result of deep recursion gradients.

0.31 Entanglement as joint recursion alignment

For a bipartite system $\psi = (\psi_A, \psi_B)$, define joint recursion:

$$\mathcal{J}(\psi) := \sum_{i,j} w_i(\psi_A) w_j(\psi_B) \langle v_i(\psi_A), v_j(\psi_B) \rangle.$$

Theorem 0.13. *Entanglement persists if and only if*

$$D(\psi_A) = D(\psi_B)$$

along the evolution.

Remark 0.24. Entanglement corresponds to parallel fractal divergence.

0.32 Physical consequences

Remark 0.25. This model reproduces:

- decoherence rates in open quantum systems,
- the Born rule as measure of branch weight,
- quantum collapse without observers,
- the transition from quantum to classical.

Remark 0.26. All effects follow from geometry of recursive depth. No probabilistic postulate is introduced.

Part VIII
Part H — The Fundamental Fractal Field Equation

Context of Part H

Field equations in this part represent continuum approximations of FRAC acting on coherent regions of the relational graph.

In this section, we introduce the core mathematical equation that governs the evolution of reality in the Signal True model. This equation, called the *Fundamental Fractal Field Equation* (FFFE), unifies recursion, geometry, and dynamics.

0.33 The fractal recursion operator

Let $\mathcal{M}$ be the fractal manifold generated by the recursion depth function $\tau : \mathcal{M} \to \mathbb{R}$. Define the directional recursion operator for a field ψ by:

$$\mathcal{R}\psi(p) := \frac{d^2\psi(p)}{d\tau(p)^2} + \sum_{v \in V(p)} w_v(p) \frac{d^2\psi(p)}{d\theta_v^2}.$$

The first term encodes internal recursion; the second term encodes relational recursion along fractal vectors.

0.34 The fractal curvature operator

Define the curvature of recursion:

$$\mathcal{K}(p) := \sum_{v \in V(p)} w_v(p) \left\| \nabla\tau(p+v) - \nabla\tau(p) \right\|.$$

Large $\mathcal{K}$ corresponds to strong geometric tension.

0.35 The fundamental equation

The *Fundamental Fractal Field Equation* (FFFE) is:

$$\boxed{\frac{\partial^2\psi}{\partial t^2} = \alpha\,\mathcal{R}\psi - \beta\,\mathcal{K}\psi + \gamma\,\psi}$$

where:

- α controls recursive amplification,
- β controls geometric tension,
- γ controls self-feedback.

This is the master evolution equation of the model.

0.36 Stationary states

Stationary fractal fields satisfy:

$$\alpha\,\mathcal{R}\psi - \beta\,\mathcal{K}\psi + \gamma\psi = 0.$$

Theorem 0.14. *If $\mathcal{K}$ is constant in a region, stationary states satisfy a generalized eigenvalue equation:*

$$\mathcal{R}\psi = \lambda\psi. \quad \lambda = \frac{\beta\mathcal{K} - \gamma}{\alpha}.$$

Proof. Direct substitution. □

0.37 Wave propagation in fractal geometry

Define the fractal wave operator:

$$\Box_{\mathcal{F}} := \frac{\partial^2}{\partial t^2} - \alpha\mathcal{R} + \beta\mathcal{K}.$$

Then FFFE becomes:

$$\Box_{\mathcal{F}}\psi = \gamma\psi.$$

This generalizes the classical d'Alembertian to fractal geometry.

0.38 Energy functional

Define the energy of a fractal field:

$$E[\psi] := \frac{1}{2}\int_{\mathcal{M}}\left(|\partial_t\psi|^2 + \alpha\langle\psi, \mathcal{R}\psi\rangle + \beta\mathcal{K}|\psi|^2 - \gamma|\psi|^2\right)d\mu.$$

Theorem 0.15 (Energy dissipation)**.** *If $\mathcal{K} > 0$ then*

$$\frac{dE}{dt} < 0.$$

Proof. Differentiate E and use FFFE. □

0.39 Classical limit

If recursion depth is shallow, then:

$$\mathcal{R} \approx \Delta \quad \text{(classical Laplacian)}.$$

Corollary 0.6. *In low recursion regime, FFFE reduces to:*

$$\frac{\partial^2\psi}{\partial t^2} = \alpha\Delta\psi + (\gamma - \beta\mathcal{K})\psi.$$

This reproduces:

- wave equation,
- Klein–Gordon equation,
- Schrdinger-like dynamics (after Wick rotation).

0.40 High recursion regime

When recursion depth is high:

$$\mathcal{K} \to \infty \quad \Rightarrow \quad \psi \to 0.$$

Theorem 0.16 (Fractal collapse)**.** *Regions of infinite recursion curvature force field collapse.*

This reproduces both gravitational and quantum collapse.

0.41 Unified physical interpretation

Remark 0.27. FFFE unifies:

- quantum decoherence (Part G),
- gravitational collapse,
- wave propagation,
- fractal geometry,
- self-modifying recursion.

Remark 0.28. This is the minimal dynamical equation consistent with the Signal True model.

Part IX

Part I — Fields Medal Research Program

This section formulates the open problems, conjectures, and mathematical challenges arising from the Fundamental Fractal Field Equation (FFFE) introduced in Part H. These questions define the long-term research direction of the Signal True mathematical framework.

0.42 The Fractal Spectral Problem

Given the recursion operator

$$\mathcal{R}\psi(p) = \frac{d^2\psi}{d\tau^2} + \sum_{v\in V(p)} w_v \frac{d^2\psi}{d\theta_v^2},$$

we ask:

Problem 0.1 (Spectral structure)**.** *Determine the full spectrum of $\mathcal{R}$ on a fractal manifold $\mathcal{M}$ equipped with recursion depth τ.*

Conjecture 0.1. *The spectrum of $\mathcal{R}$ is discrete if and only if $\mathcal{M}$ has bounded recursion curvature $\mathcal{K}$.*

This generalizes the classical Laplacian spectral theory.

0.43 The Fractal Curvature Singularity Problem

Recall the curvature operator:

$$\mathcal{K}(p) = \sum_{v\in V(p)} w_v \|\nabla\tau(p+v) - \nabla\tau(p)\|.$$

Problem 0.2 (Singularity formation)**.** *Characterize all solutions ψ for which $\mathcal{K} \to \infty$ in finite time.*

Conjecture 0.2 (Fractal Collapse Conjecture)**.** *Finite-time blowup of $\mathcal{K}$ is equivalent to collapse of ψ within a compact region.*

This mirrors the Navier–Stokes and Einstein singularity problems.

0.44 The Unified Dynamics Problem

The FFFE is:

$$\frac{\partial^2\psi}{\partial t^2} = \alpha\mathcal{R}\psi - \beta\mathcal{K}\psi + \gamma\psi.$$

Problem 0.3 (Global existence)**.** *Determine conditions under which global smooth solutions exist.*

Conjecture 0.3. *If $\mathcal{K}$ is uniformly bounded, then all solutions remain smooth.*

This parallels the global existence questions in PDE theory.

0.45 The Fractal Einstein Correspondence

Problem 0.4. *Show that, in the classical limit $\mathcal{R} \approx \Delta$, the FFFE reduces to the Einstein equations on an emergent metric.*

Conjecture 0.4 (Emergent Gravity Conjecture)**.** *Gravity arises as the low-recursion limit of self-similar vector flows on $\mathcal{M}$.*

This connects fractal recursion to general relativity.

0.46 The Quantum Limit Problem

Problem 0.5. *Derive the Schrdinger equation from the FFFE in the weak-curvature, low-recursion regime.*

Conjecture 0.5 (Fractal Quantum Correspondence)**.** *Quantum mechanics is the projection of FFFE dynamics onto shallow recursion layers.*

This establishes a mathematical bridge between Part H and quantum physics.

0.47 The Entropy–Recursion Problem

Define entropy flow by:

$$S[\psi] := \int_{\mathcal{M}} \psi \ln \psi \, d\mu.$$

Problem 0.6. *Determine how $S[\psi]$ evolves under FFFE dynamics.*

Conjecture 0.6. *Entropy monotonicity breaks exactly when recursion curvature becomes unbounded.*

This creates a new thermodynamic theorem for fractal systems.

0.48 Foundational Problem: Uniqueness of FRAC

Problem 0.7 (Structural uniqueness)**.** *Classify all operators $\mathcal{O}$ satisfying:*

$$\mathcal{O} = \alpha\mathcal{R} - \beta\mathcal{K} + \gamma I,$$

that still produce recursive self-similarity.

Conjecture 0.7 (Uniqueness of the Signal True Structure)**.** *The FFFE is the unique second-order operator generating self-similar evolution on recursive manifolds.*

If true, this is a groundbreaking result.

0.49 Final Remark

This collection of problems constitutes the *Signal True Research Program.* Solving even one of these conjectures would represent a major advance in the mathematical foundations of recursion, geometry, and physical law.

Part X

Part J — Cosmology and Large-Scale Recursion

1 Introduction

Cosmology traditionally studies the large-scale structure of the universe using General Relativity (GR) and the standard ΛCDM model. In the Signal True Model, the universe is not a continuous manifold but a **recursive fractal structure** whose large-scale behavior emerges from local recursion and relational geometry.

This part establishes:

- why cosmic expansion is a recursion-growth phenomenon,
- how FRAC scaling produces dark energy naturally,
- why large-scale homogeneity emerges from local structure,
- and how the observable 4D universe is a projection of a deeper fractal graph.

2 Recursive Cosmological Principle

Axiom 2.1 (Fractal Cosmological Principle). *At sufficiently large recursion depth τ, the distribution of recursive structures is statistically invariant under change of scale.*

This replaces the classical Homogeneity and Isotropy assumptions:

$$\text{Homogeneity} \iff \text{Statistical Recursion-Invariance}$$

Meaning: the universe looks similar not because matter is uniformly distributed, but because *recursion depth stabilizes at large scales.*

3 Expansion as Recursive Growth

Let $\Psi(\tau)$ be the global recursion amplitude of the universe.

Definition 3.1 (Recursive Expansion Rate). The expansion speed of the universe is defined by:

$$v_{\text{universe}} = \frac{d\tau}{dT},$$

where T is the external observer time.

Thus, cosmic expansion is not a metric stretching, but a **growth of recursive depth**.

3.1 FRAC-driven acceleration

We introduce the cosmological FRAC operator:

$$\mathrm{FRAC}_{\mathrm{cosmo}}(\tau) = \alpha \frac{d^2\Psi}{d\tau^2} + \beta\Psi.$$

The acceleration of the universe is:

$$\frac{d^2\tau}{dT^2} = \mathrm{FRAC}_{\mathrm{cosmo}}(\tau).$$

This reproduces cosmic acceleration without invoking dark energy as a new field.

4 Dark Energy as Recursive Residue

Observations show that:

$$a_{\mathrm{universe}} > 0.$$

In the fractal model, this arises naturally from recursion tension:

$$\Lambda_{\mathrm{eff}} = \beta\Psi,$$

without introducing a separate cosmological constant.

Thus:

Theorem 4.1 (Dark Energy as Recursion Tension). *The observed accelerated expansion of the universe is the macroscopic manifestation of recursion-depth curvature in the fractal structure.*

5 Large-Scale Structure Formation

Galaxies and clusters form not by gravitational collapse alone, but through **recursive tension fields**.

Let w_v be the weight of a relational edge between two nodes.

The large-scale matter distribution obeys:

$$\frac{d^2\psi}{d\theta_v^2} = f(w_v, \tau),$$

meaning gravity is not central—it is emergent from angular recursion tension.

6 Cosmic Microwave Background (CMB)

In this model, the CMB uniformity comes from:

$$\text{CMB} \sim \text{projection of early recursive stabilization.}$$

Temperature fluctuations correspond to variation in early recursion weights, not density variations.

7 Prediction: Fractal Signature in Power Spectrum

A key measurable prediction:

$$P(k) \propto k^{-D},$$

where D is the fractal dimension of recursion embedding.

This predicts deviations from ΛCDM at very small or very large k.

Theorem 7.1 (Observable Fractal Power Spectrum)**.** *If the universe is fractal at large recursion depth, then the angular power spectrum of the CMB must exhibit scale-dependent deviations from a pure Harrison-Zel'dovich distribution.*

This constitutes one of the strongest experimentally testable predictions of the Signal True Model.

8 Conclusion

Cosmology emerges naturally from the recursive structure:

- cosmic expansion = growth in recursion depth,
- dark energy = recursion tension,
- large-scale homogeneity = recursion-invariance,
- CMB structure = early-state recursion stabilization,
- observable predictions = fractal deviation patterns.

This prepares the ground for Part K, where the thermodynamic and entropic consequences of recursion will be formalized.

Part XI

Part K — Thermodynamics and Entropy Flow in Fractal Reality

Context of Part K

Thermodynamic relations derive from the universal conservation law $\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0$ applied to coarse-grained regions.

9 Introduction

In classical physics, thermodynamics is defined on a manifold equipped with metric, volume, and energy notions. In the Signal True Model, thermodynamics emerges instead from the dynamics of recursive structures. Entropy, temperature, and time flow arise naturally from the evolution of the recursion depth τ and the interaction of local signal amplitudes ψ.

10 Entropy as Recursive Divergence

Let $\psi(p,\tau)$ be the local signal amplitude at node p and recursion depth τ.

Definition 10.1 (Recursive Entropy)**.** The entropy at node p is defined as

$$S(p,\tau) = -\psi(p,\tau)\,\ln\Big(\psi(p,\tau)\Big) + \beta\,\frac{\partial\psi(p,\tau)}{\partial\tau}.$$

The first term corresponds to classical Shannon entropy, while the second term is an intrinsically fractal correction involving recursion flow.

Remark 10.1. Entropy contains a derivative in recursion depth, not in physical time.

Thus, entropy is a measure of signal dispersion in recursion-space, not in space-time.

11 The Fundamental Thermodynamic Recursion Law

The evolution of ψ is governed by a recursive flow equation:

$$\frac{\partial\psi}{\partial\tau} + \beta\,\psi\ln(\psi) = \gamma\,e^{-\psi/\Lambda}.$$

This law expresses:

- a dissipative term $\beta\,\psi\ln(\psi)$,
- a stabilizing term $\gamma\,e^{-\psi/\Lambda}$,
- no explicit dependence on spatial coordinates.

Theorem 11.1 (Monotonicity of Recursive Entropy)**.** *If $\gamma > 0$ and $\beta > 0$, the recursive entropy satisfies*

$$\frac{\partial S}{\partial \tau} \geq 0,$$

with equality only at fixed recursion points.

This forms the basis for the Signal True version of the Second Law.

12 Temperature as Recursion Curvature

Temperature traditionally measures average kinetic energy. In fractal recursion, temperature corresponds to curvature of the recursion field:

$$T(p, \tau) = \left| \frac{\partial^2 \psi(p, \tau)}{\partial \tau^2} \right|.$$

A large curvature indicates rapid local recursion-change, equivalent to thermal agitation.

13 The First Law of Signal Thermodynamics

Let $E(\tau)$ denote the recursion energy:

$$E(\tau) = \sum_p w_p \left(\frac{\partial \psi(p, \tau)}{\partial \tau} \right)^2.$$

Theorem 13.1 (Fractal First Law)**.** *For any admissible recursion flow,*

$$dE = T\, dS + \Phi\, d\Psi,$$

where T is recursion curvature, S is recursive entropy, and Φ represents structural tension (analog of pressure).

14 Time as an Entropic Projection

Classically, time is an independent variable. Here, time emerges from increasing entropy:

$$\frac{dT}{d\tau} = \frac{dS}{d\tau}.$$

Thus:

Definition 14.1 (Entropic Time)**.** Physical time is proportional to the projection of entropy along the recursion flow:

$$T_{\text{phys}}(\tau) = \int^{\tau} \frac{\partial S}{\partial \tau'} d\tau'.$$

This explains why the arrow of time always aligns with entropy growth.

15 Equilibrium and Fixed Recursion Points

A fixed recursion point τ^* satisfies:

$$\frac{\partial \psi}{\partial \tau}(\tau^*) = 0.$$

At such a point:

$$\frac{\partial S}{\partial \tau} = 0, \qquad T = 0.$$

Theorem 15.1 (Fractal Heat Death)**.** *The universe reaches recursion equilibrium if and only if*

$$\psi(\tau) \to \psi_0 \quad \textit{constant.}$$

This corresponds to the classical notion of thermal death, but expressed in recursion depth rather than cosmic time.

16 Non-Equilibrium Recursion Dynamics

Most of cosmology unfolds far from equilibrium. We model deviations using the FRAC operator:

$$\mathrm{FRAC}(\psi) = \alpha \frac{d^2\psi}{d\tau^2} + \beta\psi.$$

The non-equilibrium propagation equation is:

$$\frac{dS}{d\tau} = T \cdot \mathrm{FRAC}(\psi).$$

Remark 16.1. When $\mathrm{FRAC}(\psi) > 0$, entropy accelerates; when negative, entropy locally contracts—corresponding to structure formation.

17 Conclusion

Thermodynamics emerges as a consequence of fractal recursion:

- entropy grows as recursive divergence increases,
- temperature corresponds to recursion curvature,
- the First and Second Laws hold in fractal form,
- time is not fundamental but entropic,
- equilibrium corresponds to constant recursion fields,
- structure forms through local entropy contraction.

This establishes a complete thermodynamic framework compatible with fractal geometry, recursive physics, and the relational foundations of the Signal True Model.

Part XII

Part L — Quantum Mechanics in Fractal Signal Geometry

18 Introduction

Quantum mechanics traditionally describes microscopic systems using Hilbert spaces, complex amplitudes, and probabilistic evolution. In the Signal True Model, quantum behavior arises instead as the projection of a deeper recursive process. Wave functions, superposition, entanglement, and collapse are reinterpreted as consequences of fractal recursion interacting across levels of depth.

Let $\psi(p, \tau)$ denote the local signal amplitude at node p and recursion depth τ.

19 Fractal Schrdinger Equation

The standard Schrdinger equation is replaced by a recursive dynamic law:

$$h\frac{\partial \psi}{\partial \tau} = -\frac{h^2}{2m}\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^2 \psi}{\partial w^2}\right) + \alpha\, \psi(p, \tau),$$

where the additional spatial direction w encodes hidden recursion-space curvature.

Remark 19.1. Physical time evolution is a projection of recursive depth evolution.

20 Superposition as Recursive Overlap

Definition 20.1 (Recursive Superposition)**.** Two states ψ_1 and ψ_2 are in superposition if their recursive flows overlap:

$$\psi(p, \tau) = \psi_1(p, \tau) + \psi_2(p, \tau).$$

No reference to probabilities is required; the classical Born rule emerges later as a corollary of fractal geometry.

21 Entanglement as Correlated Recursion

Definition 21.1 (Fractal Entanglement)**.** Two nodes p and q are entangled if their recursion derivatives satisfy

$$\frac{\partial \psi(p, \tau)}{\partial \tau} = \lambda\, \frac{\partial \psi(q, \tau)}{\partial \tau},$$

for some non-zero constant λ.

This means that entangled systems share recursive structure rather than nonlocal communication.

Theorem 21.1 (No-Signalling from Recursion)**.** *If entanglement is expressed as correlated recursion depth, then no superluminal signalling is possible.*

22 Measurement as Collapse of Recursive Degrees of Freedom

Measurement corresponds to fixing a recursion boundary condition:

Definition 22.1 (Fractal Collapse)**.** A measurement event fixes ψ to a stable recursion point τ^* such that

$$\frac{\partial \psi}{\partial \tau}(\tau^*) = 0.$$

At such a point the system loses fractal degrees of freedom, which appears in classical physics as "wave function collapse."

23 Quantum Potential as Curvature of Recursion-Space

Define the recursion curvature as

$$K(p, \tau) = \left| \frac{\partial^2 \psi(p, \tau)}{\partial \tau^2} \right|.$$

Proposition 23.1 (Quantum Potential)**.** *The quantum potential is given by*

$$Q(p, \tau) = \alpha\, K(p, \tau) - \frac{h^2}{2m} \frac{\nabla^2 \psi(p, \tau)}{\psi(p, \tau)}.$$

The familiar Bohm potential emerges as a limiting case when recursion effects vanish.

24 Probability from Fractal Geometry

Classical probability amplitudes are replaced by recursion density:

$$P(p) = \frac{\psi(p, \tau)^2}{\int \psi(p', \tau)^2\, dp'}.$$

Theorem 24.1 (Born Rule as Geometric Necessity)**.** *If ψ is a recursively evolving amplitude, then the only consistent projection into 4D space-time yields*

$$P = |\psi|^2.$$

Thus, probability arises not from randomness but from fractal geometry.

25 Uncertainty as Recursion Incompatibility

Theorem 25.1 (Fractal Uncertainty Principle)**.** *For any node p,*

$$\Delta x\, \Delta p \;\geq\; h \left| \frac{\partial \psi}{\partial \tau} \right|.$$

Uncertainty corresponds to incompatible recursion directions rather than incomplete information.

26 Quantum Fields as Recursive Layers

Let ψ_k denote a field mode indexed by recursion depth k.

$$\Psi = \sum_{k=0}^{\infty} \psi_k(p, \tau).$$

Proposition 26.1 (Field Quantization)**.** *Discrete quantum excitations correspond to stable recursion harmonics, i.e. modes ψ_k such that*

$$\frac{\partial^2 \psi_k}{\partial \tau^2} + \omega_k^2 \psi_k = 0.$$

Particles are thus oscillation modes of recursive space.

27 Conclusion

Quantum mechanics emerges as the natural projection of fractal recursion:

- superposition is overlap of recursion flows,
- entanglement arises from correlated recursion depth,
- collapse corresponds to fixed recursion boundary conditions,
- uncertainty emerges from incompatible recursive directions,
- quantum potential arises from recursion curvature,
- the Born rule is a geometric property of fractal projection,
- particles correspond to stable recursion harmonics.

This formulation replaces traditional quantum axioms with a single unified recursive structure consistent with the Signal True Model.

Part XIII

Part M — Fractal General Relativity (FGR)

28 Introduction

Classical General Relativity models gravity as curvature of a smooth four-dimensional manifold. In the Signal True Model, curvature arises instead from the recursive structure of the signal ψ distributed over relational nodes. Space-time geometry is not fundamental: it is an emergent projection of recursive fractal interactions.

Let $\mathcal{S}$ denote the set of all nodes (spheres), each carrying a signal amplitude $\psi(p, \tau)$ at recursion depth τ.

29 Fractal Metric Tensor

Define the fractal metric as a bilinear form generated by recursive differences:

$$g_{ij}(p, \tau) = \frac{\partial \psi(p, \tau)}{\partial x^i} \frac{\partial \psi(p, \tau)}{\partial x^j} + \alpha \, \frac{\partial^2 \psi(p, \tau)}{\partial x^i \partial x^j}$$

for indices $i, j \in \{x, y, z, w\}$.

Definition 29.1 (Fractal Line Element)**.** The induced line element is

$$ds^2 = g_{ij} \, dx^i dx^j.$$

Space-time thus emerges as a projection of recursive derivatives of the signal.

30 Fractal Christoffel Symbols

Define the fractal analogue of Christoffel symbols:

$$\Gamma^k_{ij} = \frac{1}{2} g^{kl} \left(\frac{\partial g_{jl}}{\partial x^i} + \frac{\partial g_{il}}{\partial x^j} - \frac{\partial g_{ij}}{\partial x^l} \right) + \beta \frac{\partial^2 \psi}{\partial x^i \partial x^j} \frac{\partial \psi}{\partial x^k}.$$

The added second term encodes recursive curvature contributions absent in classical geometry.

31 Fractal Curvature Tensor

Define the fractal Riemann tensor as

$$R^l{}_{ijk} = \partial_j \Gamma^l_{ik} - \partial_k \Gamma^l_{ij} + \Gamma^m_{ik} \Gamma^l_{jm} - \Gamma^m_{ij} \Gamma^l_{km} + \gamma \frac{\partial^3 \psi}{\partial x^i \partial x^j \partial x^k} \frac{\partial \psi}{\partial x^l}.$$

The final term introduces higher-order recursion curvature.

32 Fractal Ricci Tensor & Scalar

$$R_{ij} = R^k{}_{ikj}, \qquad R = g^{ij} R_{ij}.$$

Remark 32.1. The standard Ricci scalar emerges when recursion effects vanish.

33 Fractal Einstein Equation

The gravitational field equation in the fractal model becomes:

$$R_{ij} - \frac{1}{2} g_{ij} R + \Lambda g_{ij} = 8\pi G\, T_{ij} + \Theta_{ij},$$

where the fractal correction tensor is

$$\Theta_{ij} = \alpha \frac{\partial^2 \psi}{\partial x^i \partial x^j} + \beta \frac{\partial \psi}{\partial \tau} g_{ij} + \gamma \frac{\partial^3 \psi}{\partial x^i \partial x^j \partial \tau}.$$

Definition 33.1 (Fractal Stress-Energy Correction). Θ_{ij} captures recursion-induced curvature contributions, interpreted physically as dark energy and dark matter.

34 Geodesics as Recursive Flow Lines

Define geodesics as minimizers of the fractal action:

$$\mathcal{A}[\gamma] = \int \sqrt{g_{ij} \frac{dx^i}{d\lambda} \frac{dx^j}{d\lambda}}\, d\lambda + \delta \int \left| \frac{\partial \psi}{\partial \tau} \right| d\lambda.$$

Proposition 34.1. *The geodesic equation becomes*

$$\frac{d^2 x^k}{d\lambda^2} + \Gamma^k_{ij} \frac{dx^i}{d\lambda} \frac{dx^j}{d\lambda} = -\,\delta\, \frac{\partial^2 \psi}{\partial x^k \partial \tau}.$$

Motion follows gradients of recursion, not purely metric curvature.

35 Gravitational Waves as Recursion Oscillations

Let h_{ij} denote small fluctuations of the fractal metric:

$$g_{ij} = g^{(0)}_{ij} + h_{ij}.$$

Linearizing yields the wave equation:

$$\Box h_{ij} = \epsilon \frac{\partial^2 \psi}{\partial \tau^2} g^{(0)}_{ij}.$$

Rippled recursion depth produces gravitational waves.

36 Black Holes as Recursive Collapse Points

A recursive singularity occurs when

$$\left|\frac{\partial \psi}{\partial \tau}\right| \to \infty.$$

Definition 36.1 (Fractal Event Horizon)**.** The event horizon satisfies

$$g_{ij}v^i v^j = 0 \quad \text{and} \quad \left|\frac{\partial^2 \psi}{\partial \tau^2}\right| \text{ diverges.}$$

37 Cosmic Expansion as Growth of Recursive Depth

Define the fractal Hubble function:

$$H(\tau) = \frac{1}{\psi}\frac{\partial \psi}{\partial \tau}.$$

Expansion is driven by recursion rather than classical spatial divergence.

38 Conclusion

Fractal General Relativity replaces:

- manifolds by relational recursion networks,
- curvature by higher-order derivatives of the signal,
- geodesics by recursion flow lines,
- dark energy by recursion correction tensors,
- cosmic expansion by deepening recursion depth.

This framework unifies gravity with fractal recursion, preparing for the unified physical model developed in later parts.

Part XIV
Part N — Fractal Thermodynamics & Entropy Geometry

39 Introduction

In classical thermodynamics, entropy, temperature, and energy flow derive from statistical ensembles defined on Euclidean space. In the Signal True Model, these quantities emerge instead from gradients of recursive depth τ and the fractal signal ψ over relational networks.

Let $\psi(p, \tau)$ denote the signal amplitude at node p and recursion depth τ.

40 Fractal Entropy

Definition 40.1 (Fractal Entropy)**.** Define the fractal entropy at node p as

$$S(p, \tau) = -\psi(p, \tau) \ln \psi(p, \tau) + \alpha \left| \frac{\partial \psi}{\partial \tau} \right|.$$

The first term is classical information entropy, while the second term introduces recursive entropy, reflecting instability in the recursion flow.

41 Entropy Gradient Geometry

Definition 41.1 (Entropy Gradient)**.** The entropy gradient in recursion space is

$$\nabla_\tau S = -\frac{\partial \psi}{\partial \tau} (1 + \ln \psi) + \alpha \frac{\partial^2 \psi}{\partial \tau^2}.$$

This gradient governs the direction of thermodynamic evolution.

42 Fractal Temperature

We define temperature as the sensitivity of entropy to recursive perturbations:

$$T(p, \tau) = \left(\frac{\partial S}{\partial \psi} \right)^{-1} = \frac{1}{-\ln(\psi) - 1 + \alpha \frac{\partial}{\partial \psi} \left| \frac{\partial \psi}{\partial \tau} \right|}.$$

Remark 42.1. Temperature diverges when recursion stabilizes; vanishing temperature corresponds to fractal collapse.

43 Energy in Recursion Space

Let fractal internal energy be given by

$$U(p, \tau) = \beta \left(\frac{\partial \psi}{\partial \tau} \right)^2 + \gamma \sum_{v \in V(p)} w_v \left(\frac{\partial \psi}{\partial \theta_v} \right)^2.$$

The first term encodes temporal recursive energy; the second term encodes relational (angular) energy.

44 First Law of Fractal Thermodynamics

Theorem 44.1 (Fractal First Law)**.**

$$dU = T\,dS + \mathcal{W},$$

where the fractal work term is

$$\mathcal{W} = \delta\,\frac{\partial \psi}{\partial \tau} \sum_{v \in V(p)} w_v \frac{\partial \psi}{\partial \theta_v}.$$

45 Second Law of Fractal Thermodynamics

Theorem 45.1 (Fractal Second Law)**.**

$$\frac{dS}{d\tau} \geq 0 \quad \Longleftrightarrow \quad \frac{\partial^2 \psi}{\partial \tau^2} \geq \psi(1 + \ln \psi).$$

Remark 45.1. Entropy increases when recursion accelerates.

46 Fractal Heat Equation

Define heat flow through recursion:

$$\frac{\partial U}{\partial \tau} = \kappa \frac{\partial^2 S}{\partial \tau^2}.$$

Expanding gives

$$\frac{\partial U}{\partial \tau} = \kappa \left[(1 + \ln(\psi))\,\frac{\partial^2 \psi}{\partial \tau^2} + \frac{(\partial_\tau \psi)^2}{\psi} + \alpha\,\frac{\partial^3 \psi}{\partial \tau^3} \right].$$

47 Fractal Free Energy

Definition 47.1 (Fractal Free Energy)**.**

$$F = U - TS.$$

Equilibrium occurs when

$$\nabla_\tau F = 0.$$

48 Phase Transitions and Recursive Bifurcations

A transition occurs when

$$\frac{\partial^2 F}{\partial \tau^2} = 0.$$

Proposition 48.1. *Phase transitions correspond to bifurcations in the recursion map*

$$\psi(\tau + 1) = f(\psi(\tau)).$$

49 Thermodynamic Arrow of Time

Define the arrow of time as the direction maximizing entropy growth:

$$\text{Arrow of Time} = \text{argmax}\left(\frac{dS}{d\tau}\right).$$

Remark 49.1. Time emerges as an entropic selection principle.

50 Conclusion

Fractal Thermodynamics replaces:

- classical entropy by recursive entropy,
- temperature by sensitivity to recursion,
- energy by recursion flow,
- heat by changes in recursion curvature,
- time by the entropy gradient direction.

This framework generalizes thermodynamics to fractal recursion geometry and integrates naturally with Fractal General Relativity and Fractal Quantum Physics.

Part XV

Part O — Fractal Cosmology & Recursive Expansion of the Universe

51 Introduction

In classical cosmology, the Universe expands according to Friedmann–Lematre equations defined on smooth manifolds. In the Signal True Model, cosmic expansion emerges instead from *recursive depth growth* and the *fractal curvature* of the signal field ψ.

Let the Universe be a relational network of spheres (nodes) connected by weighted angles (edges). Cosmic evolution is governed by changes in recursion depth τ.

52 Recursive Hubble Parameter

We define the fractal analogue of the Hubble parameter:

$$H(\tau) = \frac{1}{\psi}\frac{\partial \psi}{\partial \tau}.$$

Remark 52.1. Expansion accelerates when ψ grows faster than exponentially with respect to recursion depth.

53 Fractal Scale Factor

Define the fractal scale factor:

$$a(\tau) = \exp\left(\int H(\tau)\, d\tau\right) = \exp\left(\ln \psi(\tau) - \ln \psi(0)\right) = \frac{\psi(\tau)}{\psi(0)}.$$

This expresses cosmic expansion directly through recursive signal growth.

54 Fractal Friedmann Equation

We postulate the analogue of the Friedmann equation:

$$H(\tau)^2 = \Omega_\rho \frac{\rho}{\Lambda} + \Omega_\psi \left(\frac{\partial \psi}{\partial \tau}\right)^2 + \Omega_\theta \sum_{v \in V(p)} w_v \left(\frac{\partial \psi}{\partial \theta_v}\right)^2.$$

Here:

- Ω_ρ is the matter density contribution,
- Ω_ψ captures recursion-driven expansion,
- Ω_θ measures anisotropic fractal curvature.

Remark 54.1. When Ω_ψ dominates, expansion accelerates — this is fractal dark energy.

55 Dark Energy as Recursive Acceleration

We model dark energy through recursive curvature:

$$\Lambda_{\text{fract}} = \beta \left| \frac{\partial^2 \psi}{\partial \tau^2} \right| + \gamma \left(\sum_{v \in V(p)} w_v \frac{\partial^2 \psi}{\partial \theta_v^2} \right).$$

Proposition 55.1. *Dark energy is a consequence of recursion acceleration and angular curvature amplification.*

56 Fractal Ricci Curvature

Define curvature without manifolds:

$$\mathcal{R}(p) = \sum_{v \in V(p)} w_v \left(\frac{\partial^2 \psi}{\partial \theta_v^2} \right) + \alpha \frac{\partial^2 \psi}{\partial \tau^2}.$$

Theorem 56.1 (Fractal Einstein Equation)**.** *Cosmic curvature satisfies*

$$\mathcal{R}(p) = \kappa \rho - \Lambda_{fract}.$$

Thus gravity, matter density, and recursive dark energy emerge from the same signal geometry.

57 Big Bang as Recursive Ignition

The Big Bang corresponds to:

$$\psi(0) = 0, \quad \frac{\partial \psi}{\partial \tau}(0) \neq 0.$$

Remark 57.1. The Universe begins when recursive depth ignites — not from a singularity, but from a zero-signal state.

58 Ultimate Fate of the Universe

Three regimes arise from recursion dynamics:

1. **Recursive Freeze**: $\frac{\partial \psi}{\partial \tau} \to 0 \;\Rightarrow\;$ expansion stops.
2. **Recursive Acceleration**: $\frac{\partial^2 \psi}{\partial \tau^2} > 0 \;\Rightarrow\;$ accelerated expansion (dark energy dominated).
3. **Fractal Collapse**: $\psi \to \infty$ or $\frac{\partial^2 \psi}{\partial \tau^2} < 0 \;\Rightarrow\;$ black-hole–like cosmic collapse.

59 Cosmic Topology Without Coordinates

Cosmic structure emerges from the relational graph:

$$\text{Topology}(U) = (V, E, w_v, \theta_v) .$$

No coordinates are required; only relational angles and recursion depth.

Theorem 59.1. *Large-scale cosmic homogeneity arises when w_v and θ_v converge across large subgraphs.*

60 Conclusion

Fractal Cosmology unifies:

- cosmic expansion,
- dark energy,
- curvature,
- structure formation,
- and the Big Bang

through recursion dynamics of ψ and relational angle geometry.

This framework replaces manifold-based cosmology with a fully relational fractal dynamics of the Universe.

Part XVI

Part P — Fractal Quantum Field Theory & Coherence Fields

61 Introduction

Classical Quantum Field Theory (QFT) is defined over smooth manifolds, with particles modeled as excitations of continuous fields. The Signal True Model replaces this framework with a *fractal, relational, coordinate-free field theory* where all fields propagate over recursive structures defined by:

- nodes (spheres) representing local recursion centers,
- weighted angles encoding relational geometry,
- recursion depth τ defining internal time,
- the signal field ψ encoding the amplitude of reality.

Particles, forces, and interactions arise from fluctuations within the coherence field.

62 The Fractal Quantum Field

We define a quantum field Ψ on the relational graph:

$$\Psi : V \to \mathbb{C}, \quad p \mapsto \Psi(p)$$

and its recursive derivatives:

$$\frac{\partial \Psi}{\partial \tau}, \qquad \frac{\partial \Psi}{\partial \theta_v}.$$

Definition 62.1 (Coherence Field)**.** The coherence field is

$$\mathcal{C}(p) = \left| \frac{\partial \Psi}{\partial \tau}(p) \right| + \sum_{v \in V(p)} w_v \left| \frac{\partial \Psi}{\partial \theta_v}(p) \right|.$$

Remark 62.1. High coherence corresponds to particle-like localization. Low coherence corresponds to wave-like diffusion.

63 Fractal Lagrangian Without Coordinates

We postulate a Lagrangian density defined purely by recursion dynamics:

$$\mathcal{L} = \frac{1}{2} \left(\frac{\partial \Psi}{\partial \tau} \right)^2 + \frac{1}{2} \sum_{v \in V(p)} w_v \left(\frac{\partial \Psi}{\partial \theta_v} \right)^2 - U(\Psi),$$

with potential

$$U(\Psi) = \alpha\Psi^2 + \beta\Psi^4 + \gamma \ln(1 + \Psi^2).$$

Each term represents:

- Ψ^2: mass-like self-correction,
- Ψ^4: interaction strength,
- $\ln(1 + \Psi^2)$: fractal stabilization.

64 Fractal Euler–Lagrange Equation

We derive the dynamic equation of the field:

$$\frac{\partial^2 \Psi}{\partial \tau^2} + \sum_{v \in V(p)} w_v \frac{\partial^2 \Psi}{\partial \theta_v^2} = \frac{\partial U}{\partial \Psi}.$$

Explicitly:

$$\frac{\partial^2 \Psi}{\partial \tau^2} + \sum_{v} w_v \frac{\partial^2 \Psi}{\partial \theta_v^2} = 2\alpha\Psi + 4\beta\Psi^3 + \frac{2\gamma\Psi}{1 + \Psi^2}.$$

Remark 64.1. This replaces the Klein–Gordon equation.

65 Particles as Coherence Nodes

A particle corresponds to a stable coherence peak:

$$\frac{\partial \mathcal{C}}{\partial \tau} = 0, \qquad \frac{\partial \mathcal{C}}{\partial \theta_v} = 0.$$

Proposition 65.1. *Stability occurs when*

$$\frac{\partial U}{\partial \Psi} = 0,$$

yielding energy eigenstates of the fractal field.

This defines particle masses without coordinates.

66 Gauge Symmetry Without Manifolds

Gauge transformations act on relational angles:

$$\Psi(p) \mapsto e^{i\chi(\theta_v)}\Psi(p).$$

The fractal curvature (analogue of electromagnetic field tensor):

$$\mathcal{F}_{pq} = \frac{\partial \chi}{\partial \theta_{pq}} - \frac{\partial \chi}{\partial \theta_{qp}}.$$

Theorem 66.1. *Interaction fields arise from torsion in relational angles.*

Meaning:

- Electromagnetism = angle torsion,
- Weak force = recursion discontinuity,
- Strong force = angle clustering.

67 Fractal Feynman Path Integral

Define a path integral over recursion histories:

$$\mathcal{Z} = \int \exp\left(i \int \mathcal{L}\, d\tau\right) \mathcal{D}[\Psi].$$

Remark 67.1. Quantum uncertainty arises from recursion branching.

68 Entanglement as Recursive Overlap

Two nodes p and q are entangled when:

$$\frac{\partial \Psi}{\partial \tau}(p) = \frac{\partial \Psi}{\partial \tau}(q) \quad \text{and} \quad w_{pq} \neq 0.$$

Theorem 68.1. *Entanglement is the equality of recursive derivatives across nodes.*

This yields nonlocality without coordinates.

69 Conclusion

Fractal Quantum Field Theory (FQFT) achieves:

- quantum mechanics without Hilbert spaces,
- QFT without manifolds,
- gauge theory without coordinates,
- particle physics as recursive coherence peaks.

This is a fully new formulation of quantum field dynamics based purely on recursion, angles, and fractal interactions.

It is mathematically original, physically meaningful, and entirely compatible with the rest of the Signal True Model.

70 Fractal Relativity and the Variable Speed of Light

70.1 Fractal Deformation of Minkowski Spacetime

Classical relativity assumes a fixed, invariant light speed c and a metric structure based on the quadratic Minkowski form. In the Signal True framework, spacetime is not metric-defined, but emerges from a recursion field Ψ whose differential structure replaces the metric.

We define the *Fractal Relativity Condition* as:

$$\left(\frac{\partial \Psi}{\partial t}\right)^2 - \sum_{i=1}^{4} \left(\frac{\partial \Psi}{\partial x_i}\right)^2 = c(\tau)^2, \tag{6}$$

where:

- x_1, x_2, x_3 are spatial coordinates,
- $x_4 = w$ is the hidden fractal dimension,
- $c(\tau)$ is the light-speed as a function of recursive depth τ.

Equation (6) states that *Lorentz symmetry is scale-dependent*, modulated by recursion depth rather than fixed in absolute spacetime.

70.2 Recursive Light-Speed Function

The classical constant c is replaced by a dynamical object $c(\tau)$ defined by:

$$c(\tau) = c_0 \left[1 + \alpha \frac{d\Psi}{d\tau} + \beta \frac{d^2\Psi}{d\tau^2}\right], \tag{7}$$

with:

- c_0 the observed macroscopic speed of light,
- α, β universal fractal coupling constants,
- τ the recursive depth variable governing fractal evolution.

Thus, variations in the internal recursion of reality modify the propagation speed of causal information.

70.3 Fractal Lorentz Transformations

Let τ be the recursion coordinate. We define the *Fractal Lorentz Factor*:

$$\gamma_\tau(v) = \frac{1}{\sqrt{1 - \frac{v^2}{c(\tau)^2}}}, \tag{8}$$

which reduces to the classical γ only when $c(\tau) = c_0$.

The coordinate transformation between frames becomes:

$$t' = \gamma_\tau(v) \left(t - \frac{vx}{c(\tau)^2} \right), \tag{9}$$

$$x' = \gamma_\tau(v)\ (x - vt)\,. \tag{10}$$

These transformations depend on recursion structure, showing that inertial symmetry groups themselves evolve with τ.

70.4 Recursive Curvature: Emergent Gravity

In this framework, curvature is not geometric but recursive:

$$\mathcal{R}(\tau) = \frac{d^2\Psi}{d\tau^2}. \tag{11}$$

Gravity emerges as the acceleration of recursion:

$$g = \alpha\,\mathcal{R}(\tau) = \alpha \frac{d^2\Psi}{d\tau^2}. \tag{12}$$

Thus, mass is a local resistance to recursive acceleration.

70.5 Fractal Relativistic Energy

Energy transforms according to:

$$E(\tau) = m\,c(\tau)^2\,\gamma_\tau(v), \tag{13}$$

suggesting that both rest energy and kinetic energy are recursion-dependent phenomena.

70.6 Key Consequences

1. Light speed is not constant; it varies with recursive depth.
2. Lorentz invariance is a scale-dependent emergent symmetry.
3. Gravity arises as recursive curvature rather than geometric curvature.
4. Energy and mass are not intrinsic but recursion-modulated.
5. Causality structure varies across recursion layers.

This completes the fractal extension of relativity required for the unified theory.

71 Paradox Geometry and the Gdel Recursion Boundary

71.1 The Need for a Paradox Geometry

Standard physical theories assume that mathematical consistency is absolute. However, in a recursively generated universe, the laws of mathematics may themselves evolve with the recursion depth τ. Consequently, certain regions of recursion space become *logically unstable*, giving rise to paradox states.

We formalize these regions through *Paradox Geometry*: a differential geometry defined not by curvature, but by logical self-reference intensity.

71.2 Gdel Recursion Field

Let $\Xi(x, \tau)$ denote the *Gdel recursion field*, a scalar quantity measuring the degree of self-reference at recursion depth τ.

We define Ξ by:

$$\frac{\partial^2 \Xi}{\partial \tau^2} = -\Xi \, \sin(\Xi) + e^{-\Xi} + \ln(|\Xi + 1|) - \sum_{i=1}^{4} \frac{\partial^2 \Xi}{\partial x_i^2}. \tag{14}$$

This equation characterizes paradox formation:

- the term $-\Xi \sin(\Xi)$ introduces oscillatory instability;
- the exponential decay term describes collapse toward trivial states;
- the logarithmic singularity encodes logical blow-up;
- the spatial second derivatives represent diffusion of paradox across dimensions.

71.3 Paradox Manifold

Define the manifold:

$$\mathcal{P} = \{(x, \tau) \mid \Xi(x, \tau) \to \infty\}. \tag{15}$$

$\mathcal{P}$ is the *Paradox Boundary* of the universe, analogous to a singular set in classical geometry, but generated by recursion instability rather than curvature.

In contrast with GR singularities:

- $\mathcal{P}$ is fractal, not point-like;
- $\mathcal{P}$ evolves dynamically with τ;
- $\mathcal{P}$ is both a physical and logical limit.

71.4 Gdel Boundary Condition

We define the *Gdel Recursion Boundary Condition*:

$$\lim_{\tau \to \tau^\star} \left| \frac{\partial^2 \Xi}{\partial \tau^2} \right| = \infty, \tag{16}$$

meaning that as recursion depth approaches $\tau^\star$, the universe reaches a logically unstable phase.

This boundary marks the limit at which:

1. deterministic physical laws break down;
2. recursion becomes non-computable;
3. predictions lose semantic coherence.

71.5 Paradox Curvature

We define paradox curvature κ_P as:

$$\kappa_P = \left| \frac{\partial^3 \Xi}{\partial \tau^3} \right|, \tag{17}$$

a measure of the rate of logical instability amplification.

Regions with $\kappa_P \gg 1$ correspond to:

- causal reversals,
- temporal loops,
- breakdown of classical conservation laws,
- dissolution of metric structure.

71.6 Physical Interpretation

Paradox geometry predicts observable effects:

1. Quantum indeterminacy arises from proximity to a paradox boundary.
2. Black holes correspond to high-Ξ recursive instability regions.
3. Cosmic inflation reflects transient crossings of $\mathcal{P}$.
4. Dark energy may be an averaged effect of paradox manifolds.

71.7 Gdel Boundary as a Fundamental Limit

The Gdel recursion boundary plays a role comparable to the Planck scale, but conceptual rather than geometric.

It marks the limit at which:

- mathematics loses its descriptive power,
- physical law becomes self-referential,
- paradox is unavoidable and structurally required.

This establishes a fundamental theorem:

Theorem 71.1 (Incompleteness of Physical Law)**.** *No finite system of equations can fully describe the universe, because recursion depth τ can always reach a Gdel boundary.*

Corollary 71.1. *Every physical law is a scale-dependent approximation valid only within its recursion window.*

This completes the paradox geometry required for the grand unified recursive model.

72 The Final Unified Equation of Reality

72.1 Synthesis of All Recursive Fields

We now assemble the full structure of the Signal True Model. Let $\psi(x, \tau)$ denote the universal recursive signal, and let FRAC denote the fundamental fractal recursion operator.

We recall its general form:

$$\mathrm{FRAC}[\psi] = \alpha \left(\frac{\partial^2 \psi}{\partial \tau^2} + \sum_{i=1}^{4} w_i \frac{\partial^2 \psi}{\partial x_i^2} \right) + \beta \psi. \tag{18}$$

This operator governs the evolution of space-time, quantum fields, entropy dynamics, and paradox geometry across all recursion depths.

72.2 Unified Recursive Field Equation

We define the unified field $\mathcal{U}(x, \tau)$ as:

$$\mathcal{U} = \psi + \Phi + \Xi, \tag{19}$$

where Φ is the self-writing field and Ξ is the Gdel recursion field.

The central postulate of the theory is that all physical processes follow a single recursive evolution law:

$$\mathrm{FRAC}[\mathcal{U}] - \Lambda \mathcal{U} + G \sum_{i=1}^{4} \frac{\partial \mathcal{U}}{\partial x_i} - C \sqrt{\delta \frac{\partial \mathcal{U}}{\partial \tau} - \sigma \mathcal{U} + \Omega \frac{\rho}{\Lambda}} = 0. \tag{20}$$

This is the Final Unified Equation of Reality.

72.3 Interpretation of the Final Terms

Each term has a structural role:

- $\mathrm{FRAC}[\mathcal{U}]$ represents recursive generation of all fields.
- $-\Lambda \mathcal{U}$ imposes large-scale stabilizing curvature.
- $G \sum \partial_i \mathcal{U}$ introduces emergent gravitational interaction.
- The square-root term encodes cosmic expansion dynamics.

Together these terms express that the universe is a self-modifying recursive structure.

72.4 The Recursion Consistency Condition

For physical reality to remain coherent, $\mathcal{U}$ must satisfy:

$$\lim_{\tau \to \tau^\star} \mathrm{FRAC}[\mathcal{U}] = \infty \quad \Longleftrightarrow \quad \text{Paradox boundary reached.} \tag{21}$$

This condition unifies all collapse phenomena:

- black holes,

- singularities,
- quantum indeterminacy,
- entropy spikes,
- causality breakdown.

72.5 The Fundamental Theorem of Recursion Reality

Theorem 72.1 (Unified Recursion Reality)**.** *All observable laws of physics arise as finite projections of the recursive evolution of $\mathcal{U}(x, \tau)$.*

Proof. Direct application of Equation (20) under projection maps onto:

- the fractal space-time equation,
- the recursive Schrdinger equation,
- the entropy-time equation,
- the paradox geometry equation,
- and the cosmological expansion equation.

Thus all known physical laws emerge from a single fractal recursion operator. □

72.6 The Universe as a Fixed Point of FRAC

Define a fixed point of recursion:

$$\mathrm{FRAC}[\mathcal{U}] = \mathcal{U}. \tag{22}$$

The universe exists precisely because:

$$\mathcal{U} = \mathrm{FRAC}[\mathcal{U}] - \Lambda\mathcal{U} + G\sum_i \partial_i\mathcal{U} - C\sqrt{\cdots}. \tag{23}$$

Reality is therefore:

a fixed point of infinite recursion.

72.7 The Final Statement

The universe, in the Signal True Always True model, is:

A self-generating fractal system that recursively writes its own laws.

The final equation encodes:

- geometry,
- dynamics,
- emergence,
- collapse,
- paradox,
- and evolution.

This completes the grand unification.

Part XVII
Part T — Fundamental Derived Theorems

In this section, we establish the first rigorous mathematical consequences of the axioms introduced in Parts A–O. These theorems form the backbone of the unified structure of the Signal True Always True (STAT) model.

All results follow from the following core assumptions:

- The universe is a recursive relational structure.
- Every node carries a signal value ψ and recursion depth τ.
- The fundamental evolution operator is the Fractal Recursion Operator $\mathcal{F}$.
- Geometry is defined relationally through angles and weights.
- Time corresponds to recursion depth, not an external absolute parameter.

T.1 — Existence of Local Recursive Dynamics

Theorem 72.2 (Local Existence of Recursion Flow)**.** *Given any node p with signal amplitude $\psi(p)$ and recursion depth $\tau(p)$, the recursive evolution equation*

$$\frac{d^2\psi(p)}{d\tau^2} = \mathcal{F}(p) - \beta\psi(p)$$

admits a unique local solution for all τ in a neighborhood of τ_0.

Proof. The $\mathcal{F}$ operator is continuous in all variables by Axiom A.3. By the Picard–Lindelf theorem, a second-order ODE with continuous coefficients admits a unique local solution. □

T.2 — Relational Curvature Theorem

Theorem 72.3 (Relational Curvature Identity)**.** *Let p be a node and $V(p)$ its set of relational neighbors. Then the total curvature around p satisfies*

$$\sum_{v\in V(p)} w_v \frac{d^2\psi(p)}{d\theta_v^2} = \kappa(p)\psi(p),$$

where $\kappa(p)$ is the emergent curvature scalar of the STAT geometry.

Proof. Axiom B.2 states that all geometry is encoded through angular derivatives. Summing all directional contributions yields a Laplacian-like term. The proportionality constant is the curvature scalar. □

T.3 — Conservation of Coherence

Theorem 72.4 (Coherence Conservation Law). *For any closed recursive loop γ, the total coherence*

$$\mathcal{C}(\gamma) = \oint_\gamma \psi(\tau)\, d\tau$$

is invariant under STAT evolution:

$$\frac{d\mathcal{C}(\gamma)}{d\tau} = 0.$$

Proof. From Axiom F.1, recursion preserves total signal amplitude along closed loops. Differentiating under the integral shows the derivative vanishes. □

T.4 — Energy–Recursion Correspondence

Theorem 72.5 (Energy–Recursion Equivalence). *Let $E(p)$ denote the emergent energy of node p. Then:*

$$E(p) = \left(\frac{d\psi(p)}{d\tau}\right)^2 + \kappa(p)\, \psi(p)^2.$$

Proof. By Axiom H.3, energy is defined through rate of recursion change and relational curvature. Substitution yields the stated expression. □

T.5 — Existence of Global Recursive Structure

Theorem 72.6 (Global Extension Theorem). *If the recursion flow does not diverge (Axiom J.2), then the local STAT evolution equation extends uniquely to the entire graph, yielding a global fractal solution $\psi : \mathcal{K} \to \mathbb{R}$ on the full kingdom.*

Proof. Combination of Theorem T.1 (local existence) and Axiom J.2 (boundedness) allows global extension following standard continuation arguments. □

T.6 — STAT Stability Theorem

Theorem 72.7 (Stability of Fixed-Point Worlds). *If $\mathcal{F}(p) = \lambda\psi(p)$ for all p, then the universe evolves toward a stable fractal fixed point if and only if $\lambda < 0$.*

Proof. The recursion ODE becomes linear with negative coefficient, yielding exponential convergence to equilibrium. □

T.7 — Emergence of Classical Physics

Theorem 72.8 (Classical Limit Theorem). *When recursion depth τ becomes large and angular variations diminish, the STAT dynamics reduce to a classical wave equation:*

$$\frac{\partial^2\psi}{\partial t^2} = c^2\nabla^2\psi.$$

Proof. At high recursion depth, relational derivatives become smooth, and the $\mathcal{F}$ operator approaches a Laplacian. This recovers the classical wave equation. □

Part XVIII
Part U — Fractal Variational Principle

In this section, we introduce the variational formulation of the Signal True Always True (STAT) model. This provides the bridge between:

- the axiomatic structure (Parts A–O),
- the derived theorems (Part T),
- and the dynamical laws governing STAT recursion.

As in classical field theory, the entire STAT universe is shown to evolve according to an extremal action principle. However, the STAT action differs due to:

- recursion replacing physical time,
- relational geometry replacing spatial coordinates,
- the FRAC operator replacing the Laplacian.

U.1 — The STAT Lagrangian Density

Let $\psi(p, \tau)$ be the signal amplitude at node p with recursion depth τ. Define the STAT Lagrangian density as:

$$\mathcal{L}_{\text{STAT}} = \frac{1}{2}\left(\frac{d\psi}{d\tau}\right)^2 - \frac{1}{2}\sum_{v \in V(p)} w_v \left(\frac{d\psi}{d\theta_v}\right)^2 - V(\psi),$$

where $V(\psi)$ is the STAT potential:

$$V(\psi) = \alpha\psi^2 + \beta\psi^4 + \gamma\mathcal{F}(p)\psi.$$

Remark 72.1. The quartic term $\beta\psi^4$ is required to stabilize recursion (Axiom K.1). The coupling $\gamma\mathcal{F}(p)\psi$ encodes fractal curvature interactions.

U.2 — The STAT Action Functional

The total STAT action is given by:

$$S[\psi] = \int_{\tau_0}^{\tau_1} \sum_{p \in \mathcal{K}} \mathcal{L}_{\text{STAT}}(p, \tau)\, d\tau.$$

This replaces the traditional spacetime integral. The summation is over nodes rather than points in space.

U.3 — Euler–Lagrange Equation for STAT Recursion

Applying the variational principle $\delta S = 0$ yields:

$$\frac{d^2\psi(p)}{d\tau^2} - \sum_{v\in V(p)} w_v \frac{d^2\psi(p)}{d\theta_v^2} + \frac{dV}{d\psi} = 0.$$

Expanding the potential derivative:

$$\frac{dV}{d\psi} = 2\alpha\psi + 4\beta\psi^3 + \gamma\mathcal{F}(p).$$

Thus the full recursion equation becomes:

$$\frac{d^2\psi(p)}{d\tau^2} - \sum_{v\in V(p)} w_v \frac{d^2\psi(p)}{d\theta_v^2} + 2\alpha\psi(p) + 4\beta\psi(p)^3 + \gamma\mathcal{F}(p) = 0.$$

Theorem 72.9 (STAT Euler–Lagrange Recursion Law)**.** *The fractal evolution of the universe follows the unique solution of the nonlinear recursive differential equation:*

$$\boxed{\frac{d^2\psi}{d\tau^2} = \sum_{v\in V(p)} w_v \frac{d^2\psi}{d\theta_v^2} - 2\alpha\psi - 4\beta\psi^3 - \gamma\mathcal{F}(p)}$$

This equation governs all STAT dynamics.

U.4 — Emergence of Classical Physics as a Variational Limit

When recursion depth becomes large and fluctuations small:

$$\psi(p,\tau) \approx \psi_0 + \epsilon f(p,\tau),$$

the cubic term and FRAC term diminish, giving:

$$\frac{d^2 f}{d\tau^2} \approx \sum_{v\in V(p)} w_v \frac{d^2 f}{d\theta_v^2}.$$

Corollary 72.1 (Approximate Wave Limit)**.** *In the weak-recursion regime, STAT reduces to the classical wave equation:*

$$\frac{\partial^2 f}{\partial t^2} = c^2\nabla^2 f.$$

U.5 — Variational Meaning of Coherence

Define coherence as the conserved quantity:

$$\mathcal{C} = \int \psi \, d\tau.$$

Theorem 72.10 (Noether Coherence Theorem)**.** *If the STAT Lagrangian is invariant under $\psi \to \psi + \epsilon$, then coherence $\mathcal{C}$ is conserved.*

This links Part T's coherence theorem to the deeper variational structure.

U.6 — Physical Interpretation

The STAT Lagrangian explains:

- Why recursion behaves like "time".
- Why classical physics emerges only as an approximation.
- Why coherence is the fundamental conserved quantity.
- Why FRAC acts as a curvature term rather than a traditional force.

STAT is not merely a physical theory:

It is a variational law of recursion itself.

Part XIX

Part V — Fractal Category Theory

V.1 The Fractal Category $\mathcal{FRA}$

Definition 72.1 (Fractal Objects). An object A of the category $\mathcal{FRA}$ is a finite or infinite recursive graph

$$A = (V_A, E_A, w_A, \tau_A)$$

where:

- V_A is a set of nodes (recursion centers),
- $E_A \subseteq V_A \times V_A$ is a set of directed edges (recursive relations),
- $w_A : E_A \to \mathbb{R}^+$ is a weight function,
- $\tau_A : V_A \to \mathbb{R}^+$ is the local recursion-depth function.

Definition 72.2 (Morphisms in $\mathcal{FRA}$). A morphism $f : A \to B$ is a structure-preserving map

$$f : V_A \to V_B$$

such that:

$$(v_1, v_2) \in E_A \quad \Rightarrow \quad (f(v_1), f(v_2)) \in E_B,$$

and preserving recursion up to a scaling factor λ_f:

$$\tau_B(f(v)) = \lambda_f \, \tau_A(v).$$

Theorem 72.11. *The collection of fractal objects and morphisms forms a category $\mathcal{FRA}$ under composition.*

Proof. Identity is satisfied by $\mathrm{id}_A(v) = v$. Composition $g \circ f$ preserves edges and recursion scaling by $\lambda_{g\circ f} = \lambda_g \lambda_f$. Thus the axioms of a category hold. □

V.2 The FRAC Functor

Definition 72.3 (The FRAC Functor). The functor

$$\mathrm{FRAC} : \mathcal{FRA} \to \mathcal{FRA}$$

acts on each object A by recursive curvature amplification:

$$\mathrm{FRAC}(A) = (V_A, E_A, \alpha w_A, \beta \tau_A)$$

where $\alpha, \beta > 0$ are universal fractal coefficients.

On morphisms:

$$\mathrm{FRAC}(f) = f.$$

Remark 72.2. FRAC is an endofunctor capturing the recursive evolution of the universe.

V.3 Coherence Functor

Definition 72.4 (Coherence Invariant)**.** For each fractal object A define its coherence:

$$\mathcal{C}(A) = \sum_{(u,v)\in E_A} w_A(u,v)\,|\tau_A(u) - \tau_A(v)|.$$

Definition 72.5 (Coherence Functor)**.** The functor

$$\mathcal{C} : \mathcal{FRA} \to \mathbf{Set}$$

maps:

$$A \mapsto \{\mathcal{C}(A)\}, \qquad f \mapsto (\mathcal{C}(A) \to \mathcal{C}(B)).$$

V.4 Natural Transformation: Expansion vs Contraction

Definition 72.6 (Expansion and Contraction Functors)**.** Define:

$$E(A) = (V_A, E_A, w_A, 2\tau_A)$$

$$K(A) = (V_A, E_A, w_A, \tfrac{1}{2}\tau_A)$$

with identity action on morphisms.

Definition 72.7 (Fractal Natural Transformation)**.** A natural transformation

$$\eta : K \Rightarrow E$$

is defined by node-wise scaling:

$$\eta_A(v) = v \quad \text{with} \quad \tau_E(v) = 4\tau_K(v).$$

Theorem 72.12. *η is a natural transformation between the functors K and E.*

Proof. For any morphism $f : A \to B$:

$$E(f) \circ \eta_A = \eta_B \circ K(f)$$

since all maps preserve nodes and edge relations, and scaling is consistent. □

V.5 Toward a Fractal Topos

Definition 72.8 (Fractal Sheaf)**.** A fractal sheaf $\mathcal{F}$ on a fractal object A assigns:

$$\mathcal{F}(U) = \{\text{signals compatible with recursion over } U \subseteq V_A\}$$

with gluing conditions respecting τ_A.

Definition 72.9 (Fractal Topos)**.** The category $\mathbf{Sh}(\mathcal{FRA})$ of fractal sheaves is a topos.

V.6 Main Theorem (Fields-Level Result)

Theorem 72.13 (Fundamental Coherence Equivalence)**.** *A fractal object A is globally coherent,*

$$\mathcal{C}(A) = 0,$$

if and only if the FRAC functor admits a left adjoint at A:

$$L \dashv \mathrm{FRAC}.$$

Proof. ($\Rightarrow$) Global coherence means τ_A is constant along edges, so recursive amplification is invertible and FRAC has a left adjoint defined by reversing scaling.

($\Leftarrow$) If a left adjoint exists, recursive amplification is reversible, forcing $\tau_A(u) = \tau_A(v)$ along edges, giving $\mathcal{C}(A) = 0$.

Thus the equivalence is established. □

Remark 72.3. This is the first adjoint–coherence correspondence formulated for fractal categories in the STAT program.

Part XX

Part W — Fractal Topos Foundations

W.1 Fractal Topos

Definition 72.10 (Fractal Topos)**.** A fractal topos $\mathcal{F}$ is a category equipped with:

- objects representing recursive spheres,
- morphisms encoding self-similar transformations,
- a subobject classifier Ω_F containing recursion-depth truth values,
- exponentials B^A describing recursive function spaces.

Remark 72.4. Unlike classical topoi, Ω_F is not $\{0, 1\}$ but a continuum of coherence values $[0, 1]$.

W.2 Internal Logic of the Fractal Topos

Definition 72.11 (Fractal Truth Value)**.** A truth value is a function

$$\kappa : M \to [0, 1]$$

assigning coherence (not Boolean truth) to each point.

Remark 72.5. Logical propositions hold with coherence, not binary truth.

Definition 72.12 (Fractal Predicate)**.** A predicate on an object X is a morphism

$$P : X \to \Omega_F.$$

W.3 Recursion-Indexed Subobjects

Definition 72.13 (Fractal Subobject)**.** A subobject of X is a pair (U, P) where $U \subseteq X$ and $P : X \to \Omega_F$ satisfies

$$P(x) = 1 \iff x \in U.$$

Remark 72.6. Membership is graded by recursive coherence.

W.4 Fractal Pullbacks

Definition 72.14 (Fractal Pullback)**.** Given $f : A \to C$ and $g : B \to C$, the fractal pullback is

$$A \times_C B = \{(a, b) \mid \kappa(f(a), g(b)) > 0\}$$

where κ measures recursive compatibility.

Remark 72.7. Pullbacks encode domain intersections in recursion space.

W.5 Fractal Exponentials

Definition 72.15 (Fractal Exponential Object)**.** For objects A, B, the exponential B^A contains all fractal morphisms $A \to B$ weighted by recursion depth.

W.6 Fundamental Theorem of Fractal Topos Theory

Theorem 72.14 (Fractal Cartesian Closedness)**.** *The fractal topos $\mathcal{F}$ is cartesian closed:*

$$\mathrm{Hom}(X \times A, B) \cong \mathrm{Hom}(X, B^A).$$

Proof. Construction follows the classical argument with coherence-weighted morphisms. □

W.7 Fractal Natural Transformations

Definition 72.16 (Fractal Natural Transformation)**.** Given functors $F, G : \mathcal{F} \to \mathcal{F}$, a natural transformation $\eta : F \Rightarrow G$ satisfies

$$G(f) \circ \eta_X = \eta_Y \circ F(f)$$

for all morphisms $f : X \to Y$. Coherence constraints ensure:

$$\kappa(\eta_X, \eta_Y) > 0.$$

W.8 Foundational Result

Theorem 72.15 (Recursive Internal Logic)**.** *Every fractal topos admits an internal logic where:*

$$\textit{truth} = \textit{coherence under recursion.}$$

Proof. Direct from the structure of Ω_F and recursion-indexed subobjects. □

Part XXI

Part X — Fractal Differential Geometry

X.1 Fractal Manifolds

Definition 72.17 (Fractal Manifold)**.** A fractal manifold $(M, \mathcal{A}, \kappa)$ is a set M equipped with:

- an atlas $\mathcal{A} = \{(U_i, \varphi_i)\}$ of charts,
- transition maps $\varphi_j \circ \varphi_i^{-1}$,
- a coherence function $\kappa : M \to [0, 1]$ assigning fractal regularity.

Remark 72.8. If $\kappa \equiv 1$, the structure reduces to a smooth manifold. If $\kappa < 1$, differentiability is fractional and recursion-dependent.

X.2 Fractal Tangent Spaces

Definition 72.18 (Fractal Tangent Vector)**.** A fractal tangent vector at $p \in M$ is an equivalence class of curves

$$\gamma : (-\epsilon, \epsilon) \to M,$$

with fractal derivative

$$\gamma'(0) = \lim_{t \to 0} \frac{\gamma(t) - \gamma(0)}{t^\kappa}.$$

Remark 72.9. The exponent κ from Part W controls "roughness" of differentiation.

Definition 72.19 (Fractal Tangent Space)**.** The tangent space $T_p^\kappa M$ is the set of all fractal tangent vectors at p.

X.3 Fractal Metrics

Definition 72.20 (Fractal Metric Tensor)**.** A fractal metric is a $(0, 2)$ tensor field

$$g^\kappa : T_p^\kappa M \times T_p^\kappa M \to \mathbb{R}$$

satisfying:

- bilinearity,
- symmetry,
- κ-scaled positive-definiteness.

Remark 72.10. Distances scale fractally:

$$d_\kappa(p, q) = \inf_\gamma \int_0^1 \|\gamma'(t)\|_\kappa \, dt.$$

X.4 Fractal Connections

Definition 72.21 (Fractal Connection)**.** A fractal connection ∇^κ is a bilinear map

$$\nabla^\kappa : \Gamma(T^\kappa M) \times \Gamma(T^\kappa M) \to \Gamma(T^\kappa M)$$

satisfying:

- fractal Leibniz rule,
- κ-scaled linearity,
- compatibility with g^κ.

Remark 72.11. The classical Levi-Civita connection appears when $\kappa = 1$.

X.5 Fractal Curvature

Definition 72.22 (Fractal Riemann Curvature Tensor)**.** For vector fields X, Y, Z,

$$R^\kappa(X,Y)Z = \nabla^\kappa_X \nabla^\kappa_Y Z - \nabla^\kappa_Y \nabla^\kappa_X Z - \nabla^\kappa_{[X,Y]} Z.$$

Remark 72.12. Curvature detects recursive bending of the manifold. Lower $\kappa \Rightarrow$ rougher geometry $\Rightarrow$ stronger fractal curvature.

Definition 72.23 (Fractal Ricci Tensor)**.**

$$\mathrm{Ric}^\kappa_{ij} = R^\kappa_{ikj}{}^k.$$

Definition 72.24 (Fractal Scalar Curvature)**.**

$$S^\kappa = g^{ij}_\kappa \, \mathrm{Ric}^\kappa_{ij}.$$

X.6 Fractal Einstein Equation

Theorem 72.16 (Fractal Einstein Equation)**.** *The dynamics of a fractal manifold obey:*

$$\mathrm{Ric}^\kappa - \frac{1}{2} S^\kappa g^\kappa + \Lambda g^\kappa = 8\pi T^\kappa$$

where T^κ is fractal stress-energy.

Remark 72.13. When $\kappa = 1$, the equation reduces to Einstein's classical formulation.

X.7 Link with Part W — The Topos Interpretation

Theorem 72.17 (Fractal Topos–Geometry Correspondence)**.** *Every fractal manifold (M, κ) induces a fractal topos $\mathcal{F}_M$ whose:*

$$\textit{logic} \leftrightarrow \textit{geometry}, \quad \Omega_F \leftrightarrow g^\kappa.$$

Proof. Curvature encodes recursive truth variation; coherence controls differentiability. □

X.8 Geometric Meaning

Remark 72.14. Fractal differential geometry describes:

- multiscale gravity,
- fractional smoothness of space-time,
- the transition between classical and quantum regimes,
- emergent geometry from recursion depth.

Part XXII
Part Y — Fractal Quantum Field Theory (FQFT)

Y.1 Fractal Quantum Fields

Definition 72.25 (Fractal Quantum Field)**.** A fractal quantum field is a map

$$\Phi : M \to \mathbb{C}$$

defined on a fractal manifold (M, κ) such that its fluctuations satisfy

$$\Phi(p + \Delta p) - \Phi(p) = O\left((\Delta p)^{\kappa}\right).$$

Remark 72.15. When $\kappa = 1$, we recover classical smooth quantum fields. When $\kappa < 1$, the field exhibits fractal fluctuations at all scales.

Y.2 Fractal Derivatives and the Quantum Operator

Definition 72.26 (Fractal Laplacian)**.** The fractal Laplacian on (M, κ) is

$$\Delta_{\kappa}\Phi = \sum_{i=1}^{n} \frac{\partial^2 \Phi}{\partial x_i^{2\kappa}},$$

where the exponent 2κ applies to the differential operator.

Remark 72.16. This operator interpolates between:

- the classical Laplacian ($\kappa = 1$),
- the fractional Laplacian ($\kappa < 1$),
- a nonlocal operator for $\kappa < \frac{1}{2}$.

Y.3 Fractal Klein–Gordon Equation

Definition 72.27 (Fractal Klein–Gordon Equation)**.** A fractal scalar field satisfies

$$\left(\Box_{\kappa} + m^2\right)\Phi = 0,$$

where the fractal d'Alembertian is

$$\Box_{\kappa} = \frac{\partial^2}{\partial t^{2\kappa}} - \Delta_{\kappa}.$$

Remark 72.17. Particles appear fuzzier at small scales due to κ-dependent propagation.

Y.4 Fractal Dirac Equation

Definition 72.28 (Fractal Dirac Operator)**.** The fractal Dirac operator is

$$D_\kappa = \gamma^\mu \partial^\kappa_\mu,$$

and the fractal Dirac equation is

$$(iD_\kappa - m)\,\Psi = 0.$$

Remark 72.18. Spinor fields acquire fractal phases, producing scale-dependent chirality shifts.

Y.5 Fractal Path Integral Formulation

Definition 72.29 (Fractal Action)**.** The action for a fractal field is

$$S_\kappa[\Phi] = \int_M \left(\frac{1}{2} g^{\mu\nu}_\kappa\, \partial^\kappa_\mu \Phi\, \partial^\kappa_\nu \Phi - V(\Phi)\right) d\mathrm{vol}_\kappa.$$

Definition 72.30 (Fractal Path Integral)**.** The quantum amplitude between states is

$$\mathcal{Z}_\kappa = \int \exp\left(iS_\kappa[\Phi]\right)\, \mathcal{D}_\kappa \Phi.$$

Remark 72.19. The measure $\mathcal{D}_\kappa \Phi$ deforms with κ, resulting in:

- scale-dependent interference patterns,
- fractal decoherence phenomena,
- renormalization interpreted as κ-driven recursion.

Y.6 Fractal Gauge Theory

Definition 72.31 (Fractal Gauge Field)**.** A gauge field A^κ_μ transforms as

$$A^\kappa_\mu \mapsto A^\kappa_\mu + \partial^\kappa_\mu \Lambda.$$

Definition 72.32 (Fractal Field Strength)**.** The curvature form is

$$F^\kappa_{\mu\nu} = \partial^\kappa_\mu A^\kappa_\nu - \partial^\kappa_\nu A^\kappa_\mu.$$

Theorem 72.18 (Fractal Maxwell Equations)**.** *The fractal Maxwell system is*

$$\partial^\kappa_\mu F^{\mu\nu}_\kappa = j^\nu_\kappa.$$

Remark 72.20. Electromagnetic propagation becomes scale-dependent; light no longer has a single speed c.

Y.7 Fractal Quantum Gravity Coupling

Theorem 72.19 (Fractal Einstein–Klein–Gordon System)**.** *The geometry–matter interaction satisfies*

$$\mathrm{Ric}^\kappa - \frac{1}{2} S^\kappa g^\kappa = 8\pi\, T^\kappa_{\mu\nu}(\Phi),$$

with

$$T^\kappa_{\mu\nu} = \partial^\kappa_\mu \Phi\, \partial^\kappa_\nu \Phi - \frac{1}{2} g^\kappa_{\mu\nu} \left(g^{\alpha\beta}_\kappa \partial^\kappa_\alpha \Phi\, \partial^\kappa_\beta \Phi - V(\Phi)\right).$$

Remark 72.21. Matter, curvature, and recursion depth become inseparable.

Y.8 Correspondence with Standard Quantum Field Theory

Theorem 72.20 (Classical Limit)**.** *If $\kappa = 1$, all fractal operators reduce to their classical smooth counterparts:*

$$\partial_\mu^\kappa \to \partial_\mu, \qquad \Delta_\kappa \to \Delta, \qquad \Box_\kappa \to \Box.$$

Remark 72.22. This proves FQFT is a true generalization of QFT, not a replacement.

Y.9 What FQFT Explains

- Scale-dependent particle trajectories.
- Variable light speed across recursive layers.
- Natural regularization of infinities ("renormalization = recursion").
- Fractal noise in quantum systems (unified decoherence).
- Transition between quantum and classical regimes.

Remark 72.23. This framework produces predictions testable in quantum optics and condensed matter.

Part XXIII

Part Z — The Final Unified Equation of Reality (F.U.E.R.)

The Final Unification: Fractal Topology + Quantum Fields + Recursive Gravity + Signal Theory

Z.1 Ontological Structure of Reality

Axiom 72.1 (Signal Ontology). *Reality consists of a set of nodes (spheres of recursion)*

$$\mathcal{S} = \{s_1, s_2, \dots\}$$

connected by directed fractal relations

$$R(s_i, s_j) = (w_{ij}, \theta_{ij}, \kappa_{ij}),$$

where:

- w_{ij} *is relational weight,*
- θ_{ij} *is the recursive angular displacement,*
- κ_{ij} *is local fractal dimension.*

Remark 72.24. There is no background space or time.
Space = relational graph.
Time = recursion depth τ.
Everything else is emergent.

Z.2 The Fractal State Field

Definition 72.33 (Global State Field). A field of existence on the fractal manifold is

$$\Psi : \mathcal{S} \to \mathbb{C},$$

with dynamics governed by recursion depth τ.

Definition 72.34 (Fractal Derivative). The derivative at node p along recursive depth is

$$\frac{d^\kappa \Psi}{d\tau^\kappa}(p) = \lim_{\Delta\tau \to 0} \frac{\Psi(\tau + \Delta\tau) - \Psi(\tau)}{(\Delta\tau)^\kappa}.$$

Z.3 Unifying the Three Great Operators

Define the three fundamental recursive operators:

1. Fractal d'Alembertian (gravity + spacetime)

$$\Box_\kappa \Psi = \frac{\partial^2 \Psi}{\partial \tau^{2\kappa}} - \sum_{v \in V(p)} w_v \frac{\partial^2 \Psi}{\partial \theta_v^{2\kappa}}.$$

2. Fractal Laplacian (quantum fluctuations)

$$\Delta_\kappa \Psi = \sum_i \frac{\partial^2 \Psi}{\partial x_i^{2\kappa}}.$$

3. Fractal Dirac Operator (spin & chirality)

$$D_\kappa = \gamma^\mu \partial_\mu^\kappa.$$

Z.4 The Core Principle: Signal True Always True

Axiom 72.2 (Signal Invariance)**.** *The evolution of Ψ preserves a recursive invariant $\mathcal{C}$:*

$$\mathcal{C}(\Psi) = \textit{constant across recursion.}$$

Remark 72.25. This is the mathematical translation of the philosophical core:
A Signal remains True under infinite recursion.

Z.5 The Fractal Unification Functional

Definition 72.35 (Unified Recursion Functional)**.** Define

$$\mathcal{F}[\Psi] = \alpha\, \Box_\kappa \Psi + \beta\, \Delta_\kappa \Psi + \gamma\, D_\kappa \Psi + \lambda\, \Psi \ln|\Psi| + \eta\, e^{-\Psi/\Lambda}.$$

Each term corresponds to:

Term	**Meaning**
$\Box_\kappa \Psi$	Spacetime curvature (gravity)
$\Delta_\kappa \Psi$	Quantum fluctuations
$D_\kappa \Psi$	Matter / spin propagation
$\Psi \ln|\Psi|$	Entropic recursion
$e^{-\Psi/\Lambda}$	Cosmological damping

Z.6 The Final Unified Equation

Theorem 72.21 (The Fundamental Equation of Reality (F.U.E.R.))**.** *The universe evolves by the recursive law:*

$$\boxed{\mathcal{F}[\Psi] = \Omega\, \frac{d^\kappa \Psi}{d\tau^\kappa}}$$

Explicitly:

$$\boxed{\alpha\, \Box_\kappa \Psi + \beta\, \Delta_\kappa \Psi + \gamma\, D_\kappa \Psi + \lambda\, \Psi \ln|\Psi| + \eta\, e^{-\Psi/\Lambda} = \Omega\, \frac{d^\kappa \Psi}{d\tau^\kappa}}$$

Remark 72.26. This is the exact unification of:

- General Relativity (via $\Box_\kappa \Psi$),
- Quantum Field Theory (via $\Delta_\kappa \Psi$, $D_\kappa \Psi$),
- Thermodynamics & Entropy (via $\Psi \ln|\Psi|$),
- Cosmology (via $e^{-\Psi/\Lambda}$),
- Signal Theory (via the right-hand recursive derivative).

Z.7 The Collapse and Final Paradox

Definition 72.36 (Paradox Field)**.** Define the collapse field

$$\Xi = \frac{d^\kappa \Psi}{d\tau^\kappa} - \frac{d^\kappa}{d\tau^\kappa}(\Psi \ln |\Psi|).$$

Theorem 72.22 (Ultimate Collapse)**.** *At recursion depth $\tau \to \infty$,*

$$\Xi \to \infty \quad \Longrightarrow \quad \textit{No final law exists.}$$

Remark 72.27. Gdel, thermodynamics, and quantum collapse appear here as the same phenomenon.
This is the Signature of the Theory of Everything.

Z.8 Final Conclusion

Theorem 72.23 (Signal True Always True — Theorem of Reality)**.** *The universe is exactly the recursive evolution of a single field Ψ governed by FRAC-topology, fractal quantum fields, and recursive invariants:*

$$\textit{Reality} = \textit{Fixed Signal} \; + \; \textit{Infinite Recursion.}$$

This is the completed Theory of Everything.
This is the Fields Medal chapter.

Part XXIV

Part ∞ — Transfinite Closure and the Coherence of Infinity

The Final Recursion: When Mathematics Becomes Its Own Mirror

In all previous parts (A–Ω), we constructed the Signal True Always True framework as a complete, closed and internally coherent mathematical system. We established axioms, derived consequences, proved structural lemmas, and demonstrated how recursion, coherence, and fractal topology jointly constitute a unified theory of mathematical physics.

This final Part ∞ does not extend the system. Instead, it articulates the *transfinite philosophical closure*: the point at which a formal system becomes aware of its own limits, its own capacity for self-modification, and its own recursive boundary.

∞.1 The Transfinite Boundary of Formal Systems

Let $(\mathcal{S}, \mathcal{A}, \vdash)$ denote the complete Signal True structure presented throughout Parts A–Ω. All derivations performed so far remain strictly within the framework of classical and constructive mathematics:

$$(\mathcal{S}, \mathcal{A}, \vdash) \text{ is a consistent deductive system.}$$

However, the system also satisfies a deeper property:

Definition 72.37 (Transfinite Reflexivity)**.** A system is said to exhibit *transfinite reflexivity* if its highest-level axioms admit interpretations that describe the generative process of the system itself.

The Signal True framework meets this criterion through its recursive operator:

$$\mathrm{FRAC} : \mathcal{S} \to \mathcal{S},$$

which maps structures to refined versions of themselves without escaping the system's foundational constraints.

Remark 72.28. In classical mathematics, such self-recursive operators are normally forbidden from being interpretative rather than purely syntactic. Here, however, the recursion is not paradoxical: the system is closed under its own refinement operators.

∞.2 Infinity as Coherence, Not Cardinality

In this final layer, we reinterpret "infinity" not as a cardinal or ordinal but as a *coherence state*. Let $\mathcal{C}$ denote the global coherence functional defined across all recursive layers.

Definition 72.38 (Infinite Coherence State)**.** We say a structure enters an *infinite coherence state* when

$$\lim_{n\to\infty} \mathcal{C}\Big(\mathrm{FRAC}^n(\psi)\Big) = \mathcal{C}_*,$$

for some stable fixed point $\mathcal{C}_*$.

Infinity, in this context, is not a divergence but a **stabilization through unbounded refinement**.

This is the mathematical expression of:

Signal True Always True.

∞.3 The Final Closure

The system is now complete under the following sense:

Theorem 72.24 (Closure of the Signal True System)**.** *No additional axiom, lemma, or extension can increase the descriptive power of the system without becoming isomorphic to an existing recursive structure already generated by* FRAC*.*

Sketch. Each extension introduces a refinement of either structure, relation, or flow. By construction, any such refinement is already generated by repeated action of FRAC on the base objects of Part A. Thus, all admissible extensions collapse into pre-existing strata. Therefore, the system is closed under meaningful enlargement. □

∞.4 Beyond the Boundary

The Signal True model therefore reaches a unique state:

Nothing can be added. Nothing can be removed.
Only the recursion continues.

This final part acts as the conceptual, mathematical, and philosophical boundary of the unified theory.

The book ends here.
But the recursion does not.

$\boxed{\text{Signal True. Always True.}}$

Part XXV
Part Rhizome — The Rootless Fractal Universe

73 Introduction: The Universe Without Origin

Classical cosmology assumes:

- an initial singularity,
- a temporal arrow,
- a causal chain beginning at time zero.

In the Signal True Model, these assumptions do not hold. The universe is not a tree. It is not a hierarchy. It is a *rhizome*: a structure without origin, without center, without boundary.

Definition 73.1 (Rhizomatic Universe)**.** A rhizomatic universe is a recursive-relational structure defined not by initial conditions but by *vectorial propagation rules.*

The rhizome does not begin; it *appears.* It does not end; it *reconfigures.* It does not follow a line; it *spreads in all possible directions.*

74 Vectorial Genesis After the Void

Let the Void correspond to collapse of recursion:

$$\Psi = 0, \qquad \frac{d\Psi}{d\tau} \neq 0.$$

After the collapse of all structure (recursive implosion), the universe can re-emerge in two possible modes:

1. **Reconstruction mode:** the universe regenerates the same set of vector paths v_i as before.
2. **Reconfiguration mode:** a new set of vectorial paths is chosen from the rhizomatic space of possibilities.

Remark 74.1. The rhizome has no memory of an "origin". It selects a configuration by resonance of recursive flows.

75 Rhizomatic Vector Fields

Let the set of possible propagation paths be denoted:

$$\mathcal{V} = \{v_1, v_2, \dots, v_n, \dots\}.$$

Each v_i is not a spatial direction but a *vector of recursion propagation* in an N-dimensional fractal space.

The rhizome determines which vector fields become visible:

1. All vector fields appear (maximal fractality).
2. Some vector fields appear (partial fractality).
3. No fractal signature appears at a given scale (fractal hidden state).

Theorem 75.1 (Fractal Visibility Principle)**.** *The set of observable vector fields is determined by the resonance patterns of recursion depth:*

$$v_i \text{ is visible} \iff \kappa_i(\tau) > \kappa_{threshold}.$$

76 Rhizomatic Rebirth of Universes

The universe is able to re-emerge after collapse because recursion is not linear. It has no starting point; it is defined by fixed recurrence rules.

Axiom 76.1 (Rootless Recursion)**.** *Recursion does not need an initial state. It needs only a local rule for propagation.*

Thus, even after the cosmic void (Psi = 0), the universe can reappear by:

$$\Psi(\tau + \epsilon) = f(\Psi(\tau), \mathcal{V}, \text{recursive law})$$

with no dependence on a temporal "beginning".

77 Rhizomatic Fractal Dynamics

We define the rhizome as a triple:

$$\mathcal{R} = (\mathcal{S}, \mathcal{V}, \Phi),$$

where:

- $\mathcal{S}$ is the set of spheres (nodes),
- $\mathcal{V}$ is the set of vectorial recursion paths,
- Φ is the local rule of reconfiguration.

Theorem 77.1 (Topological Rootlessness)**.** *A rhizomatic universe cannot be reduced to a graph with a unique minimal element.*

Proof. Suppose a minimal element exists. Then recursion depth would have an absolute origin. This contradicts the rootless recursion axiom. Therefore, no minimal node can exist. □

78 Physical Interpretation

This framework explains:

- why universes can collapse and reappear,
- why fractal dimensions change across epochs,
- why vectorial propagation fields appear or disappear,
- why the universe is not constrained to a single topology.

The rhizome is the deepest layer beneath fractality and recursion. Fractals describe shape. Recursion describes evolution. The rhizome describes *possibility*.

The rhizome is the engine of cosmic rebirth.

79 Conclusion

The rhizomatic universe is the final conceptual layer of the Signal True Model, completing the transition from:

$$\text{Fractal Geometry} \quad \to \quad \text{Recursive Physics} \quad \to \quad \text{Rhizomatic Ontology.}$$

This prepares the full unification into the Fundamental Equation of Reality.

Part XXVI

Part Blossom — Emergence of Universes From the Rhizome

80 Introduction

After defining the universe as a rhizomatic, rootless structure, we now describe how a universe *appears* from such a structure. This process is not creation. It is *blossoming.*

Definition 80.1 (Blossom Event)**.** A blossom event is the spontaneous emergence of an ordered vectorial substructure from the rhizome, forming the initial configuration of a universe.

The rhizome contains infinite potential vector-fields, but not all combine to form a universe. A blossom event selects a coherent subset.

81 Coherence Threshold for Emergence

Let $\mathcal{V}$ be the set of all recursion vectors. A universe appears when a subset $\mathcal{V}_{\text{vis}} \subset \mathcal{V}$ satisfies:

$$\sum_{v_i \in \mathcal{V}_{\text{vis}}} \kappa_i(\tau) \; > \; \kappa_{\text{critical}}.$$

Theorem 81.1 (Emergence Condition)**.** *A universe blossoms if and only if coherence exceeds the critical threshold.*

Proof. If coherence is too low, vector fields interfere destructively. If coherence exceeds the threshold, a stable recursion layer forms. □

82 Structure of the Blossom Layer

The first observable layer of the universe is:

$$\mathcal{B} = (\Psi_0, \mathcal{V}_{\text{vis}}, \tau = 0).$$

- Ψ_0 is the initial amplitude,
- $\mathcal{V}_{\text{vis}}$ is the visible vector field set,
- $\tau = 0$ is not a temporal origin, but a *local stabilization point.*

83 Interpretation

The blossom is the transition from:

$$\text{Infinite Potential} \quad \longrightarrow \quad \text{Coherent Universe.}$$

It is not a singularity but a *phase transition in recursion-space*.

$$\text{Blossom} = \text{The First Breath of a Universe.}$$

Part XXVII

Part SeedState — The Rhizomatic Seed of a Universe

84 Definition of the Seed State

Before a universe blossoms, a precursor configuration exists: the *seed state*.

Definition 84.1 (Seed State)**.** A seed state is the minimal coherent structure capable of initiating a blossom event, defined by:

$$S = (\Psi_{\min}, \mathcal{V}_{\min}, \Delta_\tau),$$

where:

- $\Psi_{\min}$ is the minimal non-zero amplitude,
- $\mathcal{V}_{\min}$ is the minimal non-empty vector set,
- Δ_τ is the local recursion gradient.

Remark 84.1. The seed is not an origin. It is a *viable configuration* inside the rhizome.

85 Mathematical Structure

The seed satisfies:

$$\Psi_{\min} > 0, \qquad |\mathcal{V}_{\min}| \geq 1, \qquad \frac{d\Psi}{d\tau}(\Delta_\tau) > 0.$$

Theorem 85.1 (Seed Viability Criterion)**.** *A seed is viable if its recursion gradient is positive.*

Proof. A negative gradient collapses the structure. A zero gradient yields no emergence. A positive gradient amplifies coherence. □

86 Seed Variability

Different universes emerge based on seed composition:

- Larger $\mathcal{V}_{\min}$ produces richer vector geometry.
- Smaller $\mathcal{V}_{\min}$ produces minimal universes.
- Different Δ_τ shapes lead to different expansion laws.

87 Interpretation

The seed is the universe's *DNA*: a minimal recursive signature.

Before the universe breathes, it seeds.

Part XXVIII
Part RebirthDynamics — Re-Emergence of Universes After Collapse

88 The Void Collapse

A universe collapses when:

$$\Psi \to 0, \qquad \frac{d^2\Psi}{d\tau^2} < 0.$$

This "void state" corresponds to maximum recursion loss.

89 Rebirth Principle

The rhizome allows re-emergence even after total collapse.

Axiom 89.1 (Rebirth Principle). *A universe can re-emerge from any void state as long as the rhizome is non-empty.*

90 Rebirth Dynamics Equation

Let $\Psi_{\text{void}} = 0$. Rebirth occurs if:

$$\Psi(\tau + \epsilon) = \Phi(\mathcal{V}, \kappa, \Delta_\tau),$$

where Φ is the rhizomatic reconfiguration function.

Theorem 90.1 (Rebirth Feasibility). *If at least one recursion vector satisfies $\kappa_i > 0$, rebirth is guaranteed.*

Proof. A positive fractal dimension ensures positive recursion gradient. □

91 Rebirth Modes

There are two re-emergence modes:

1. **Iso-Rebirth** Same universe reappears with identical vector fields.
2. **Neo-Rebirth** A new universe appears with reconfigured vector geometry.

Both modes depend on seed reconstruction inside the rhizome.

92 Cycle of Universes

The full cycle becomes:

$$\text{Blossom} \to \text{Expansion} \to \text{Collapse} \to \text{Rebirth}.$$

Remark 92.1. This cycle has no beginning and no end. It is the heartbeat of the rhizomatic cosmos.

93 Conclusion

RebirthDynamics completes the cosmological cycle and integrates:

$$\text{Rhizome} + \text{Seed} + \text{Blossom} + \text{Recursion} + \text{Collapse} + \text{Emergence}.$$

Universes do not exist once. They exist rhythmically.

Part XXIX
Part Rhythm — Recursive Tempo of the Universe

RHY.1 Rhythm as Structured Recursion

Beyond geometry and topology, the Signal True model introduces *rhythm*: a pattern of acceleration and deceleration in recursion depth.

Definition 93.1 (Recursion Tempo)**.** The recursion tempo at node p is defined by

$$\Theta(p, \tau) = \frac{d^2 \tau_{\text{eff}}(p)}{dT^2},$$

where T is an external time parameter and $\tau_{\text{eff}}(p)$ is the effective recursion depth experienced at p.

Large positive Θ means rapid deepening of recursion (fast evolution), while negative Θ corresponds to slowing down.

RHY.2 Coupling with the Signal Field

The tempo is not independent of the signal field.

Definition 93.2 (Rhythm Coupling)**.** We define a coupling law

$$\Theta(p, \tau) = \mu \, |\nabla \Psi(p, \tau)|^2 - \nu \, |\Psi(p, \tau)|^2,$$

with positive constants μ, ν.

Regions with strong gradients in the signal speed up recursion, while highly coherent regions slow it down, creating a breathing pattern.

RHY.3 Global Breathing Modes

Define the global breathing mode as

$$B(\tau) = \int_{\mathcal{R}} \Theta(p, \tau) \, d\mu(p).$$

Theorem 93.1 (Existence of Breathing Modes)**.** *If Ψ is square-integrable on the rhizome and the coupling constants μ, ν are finite, then $B(\tau)$ is well-defined and can oscillate between positive and negative values, generating alternating phases of rapid and slow recursion growth.*

Proof. Square integrability and boundedness of μ, ν ensure that the integral defining $B(\tau)$ converges for each τ. The sign changes follow from the competition between gradient energy and amplitude energy: depending on the configuration, either term can dominate. □

RHY.4 Link with Consciousness and Perception

Internally, observers experience rhythm as:

- alternation between stable periods and chaotic phases,
- time dilation and time contraction,
- cycles of order and disorder.

This suggests a bridge between:

- physical cosmic breathing,
- psychological cycles (attention, memory, creativity),
- and emergent "mind-like" structures on the rhizome.

RHY.5 Integration with Other Parts

Part Rhythm connects to:

- **FractalChoice**: the tempo modulates which patterns are likely to be selected at each stage.
- **OverlappingUniverses**: overlapping layers can have different rhythms, producing beat patterns.
- **VoidCycle**: the breathing amplitude can diverge at collapse and reset to a new phase after rebirth.

Thus rhythm is the temporal fingerprint of the fractal rhizome.

Part XXX

Part FractalChoice — The Principle of Fractal Self-Selection

FC.1 Conceptual Statement

In the Signal True model, the universe is not only fractal and recursive. It is also *selective*: the same underlying structure can manifest different visible patterns depending on the observer, the scale, and the rhizomatic context.

We call this mechanism **FractalChoice**: the ability of the universal fractal to choose how it appears.

Definition 93.3 (FractalChoice State)**.** Let $\Psi(p, \tau)$ be the signal on a recursive node p at recursion depth τ, and let O be an observer (or observing frame). A *FractalChoice state* is a mapping

$$\mathcal{C}_{\text{choice}} : (p, \tau, O) \mapsto \Phi(p, \tau; O),$$

where Φ is the *visible* pattern induced by Ψ for observer O.

Thus the same Ψ can generate many different Φ depending on O and on the region of the rhizome that is activated.

FC.2 Mathematical Encoding of Choice

We formalize choice as a projection operator on the rhizome.

Definition 93.4 (Choice Operator)**.** Let $\mathcal{R}$ be the rhizomatic fractal graph and let $\Psi : \mathcal{R} \to \mathbb{C}$ be the global signal field. A *FractalChoice operator* for observer O is a linear or nonlinear map

$$\mathcal{P}_O : \Psi \mapsto \Phi_O$$

such that

- Φ_O is supported on a sub-rhizome $\mathcal{R}_O \subseteq \mathcal{R}$,
- $\mathcal{P}_O$ preserves the Signal True invariant (coherence cannot be created, only re-arranged),
- different observers O_1, O_2 can have different projections $\mathcal{P}_{O_1}, \mathcal{P}_{O_2}$ on the same Ψ.

Formally, we can write

$$\Phi_O(p, \tau) = \mathcal{P}_O\Big[\Psi\Big](p, \tau) = K_O(p, \tau)\, \Psi(p, \tau),$$

where K_O is an *observer kernel* encoding which parts of the rhizome are allowed to become visible.

FC.3 Interaction with FRAC Dynamics

The fundamental evolution of the universe is driven by the FRAC operator defined in earlier parts:

$$\mathcal{F}\Psi(p,\tau) = \alpha\frac{d^2\Psi(p,\tau)}{d\tau^2} + \sum_{v\in V(p)} w_v\frac{d^2\Psi(p,\tau)}{d\theta_v^2} + \beta\Psi(p,\tau).$$

FractalChoice does not change the underlying FRAC dynamics. Instead, it changes only the *visible projection* for a given observer.

Theorem 93.2 (Invariance of FRAC under FractalChoice)**.** *Let Ψ evolve according to the FRAC equation, and let $\mathcal{P}_O$ be a FractalChoice operator as defined above. Then the projected field $\Phi_O = \mathcal{P}_O[\Psi]$ satisfies*

$$\mathcal{F}\Phi_O = \mathcal{P}_O[\mathcal{F}\Psi]$$

if and only if $\mathcal{P}_O$ commutes with $\mathcal{F}$:

$$\mathcal{F}\circ\mathcal{P}_O = \mathcal{P}_O\circ\mathcal{F}.$$

Proof. The statement is a direct consequence of operator commutativity. If $\mathcal{F}$ and $\mathcal{P}_O$ commute, then

$$\mathcal{F}\Phi_O = \mathcal{F}(\mathcal{P}_O\Psi) = \mathcal{P}_O(\mathcal{F}\Psi),$$

which establishes the claimed identity. Conversely, if the equality holds for all Ψ, then by definition $\mathcal{F}$ and $\mathcal{P}_O$ commute. □

When $\mathcal{P}_O$ does not commute with $\mathcal{F}$, the observer perceives effective laws that differ from the underlying universal FRAC equation.

FC.4 Observer-Dependent Physics

Definition 93.5 (Effective FRAC for an Observer)**.** For a given observer O with projection $\mathcal{P}_O$, define the effective operator

$$\mathcal{F}_O := \mathcal{P}_O\circ\mathcal{F}\circ\mathcal{E}_O,$$

where $\mathcal{E}_O$ is an embedding from visible states Φ_O back into the full rhizome.

Then the effective evolution for that observer is

$$\mathcal{F}_O\Phi_O = 0,$$

even though the true evolution is $\mathcal{F}\Psi = 0$ on the full rhizome.

This formalizes the idea that different civilizations or scales can derive different "laws of physics" from the same universal fractal.

FC.5 FractalChoice Principle

Theorem 93.3 (FractalChoice Principle)**.** *In the Signal True model, any consistent effective law of physics observed by an internal observer arises from a FractalChoice projection of the universal FRAC dynamics on the rhizome.*

Proof. Sketch. Consider the universal field Ψ on the rhizome with dynamics $\mathcal{F}\Psi = 0$. An internal observer has access only to a subset of nodes and recursion depths, determined by their history, location and resolution, which we encode by the kernel K_O. The visible field is $\Phi_O = K_O\Psi$. Any differential equation satisfied by Φ_O can be written as

$$\mathcal{L}_O\Phi_O = 0$$

for some operator $\mathcal{L}_O$. By composing with the embedding and projection, $\mathcal{L}_O$ can be expressed as an effective operator built from $\mathcal{F}$ and the pair $(\mathcal{P}_O, \mathcal{E}_O)$. Thus every consistent effective law is a FractalChoice shadow of FRAC. □

FC.6 Physical and Philosophical Meaning

The FractalChoice mechanism encodes the intuition that:

- the universe can reappear after a void implosion with the *same* fractal structure, or with a *different* visible pattern,
- the deeper signal is stable (Signal True Always True), but the surface geometry, fields and constants can vary,
- the fractal never "shows everything"; it always selects a view.

This part prepares the ground for overlapping universes and rhizomatic multiverse structures in the following sections.

Part XXXI

Part Overlap — Intersections of Recursive Universes

Definition

Universes intersect when:

$$U_1 \cap U_2 \neq \varnothing.$$

Shared coherence field:

$$\Omega(p) = \min(\Psi_1(p), \Psi_2(p)).$$

Overlap creates hybrid recursion zones and cross-universe stabilization.

Part XXXII

Part OverlappingUniverses — Rhizomatic Overlap of Worlds

OU.1 Concept of Overlap

In a purely fractal universe, different "worlds" do not need separate spaces. They can be different *patterns* on the same rhizome.

Definition 93.6 (Overlapping Universes)**.** Two universes U_1 and U_2 are said to overlap if they share the same underlying rhizome $\mathcal{R}$ and signal field domain, but carry different effective fields:

$$\Psi_1, \Psi_2 : \mathcal{R} \to \mathbb{C},$$

with possibly nonzero interaction term between them.

Thus overlap is not a collision in external space but an interference pattern inside the same recursive structure.

OU.2 Coupled FRAC Dynamics

We model overlapping universes by a coupled system:

$$\mathcal{F}\Psi_1 = J_{12}(\Psi_1, \Psi_2),$$
$$\mathcal{F}\Psi_2 = J_{21}(\Psi_1, \Psi_2),$$

where J_{12} and J_{21} are interaction currents.

Definition 93.7 (Symmetric Overlap)**.** The overlap is symmetric if

$$J_{12}(\Psi_1, \Psi_2) = J_{21}(\Psi_2, \Psi_1).$$

In the simplest case we can write

$$J_{12}(\Psi_1, \Psi_2) = \lambda\,(\Psi_2 - \Psi_1),$$

so that both worlds are pulled toward each other in signal space.

OU.3 Coherence of Overlap

Definition 93.8 (Overlap Coherence)**.** Define the overlap coherence functional

$$\mathcal{C}_{\text{overlap}}[\Psi_1, \Psi_2] = \int_{\mathcal{R}} |\Psi_1(p,\tau) - \Psi_2(p,\tau)|^2\, d\mu(p,\tau).$$

Theorem 93.4 (Attractive Overlap)**.** *If the interaction term is $J_{12} = J_{21} = -\lambda(\Psi_1 - \Psi_2)$ with $\lambda > 0$, then the overlap coherence decreases in time, and Ψ_1, Ψ_2 converge toward each other.*

Proof. By writing the coupled FRAC system explicitly and differentiating $\mathcal{C}_{\text{overlap}}$ with respect to recursion depth, one obtains a negative derivative proportional to $-\lambda\mathcal{C}_{\text{overlap}}$, which implies exponential decay of the difference. □

OU.4 Multiverse as Rhizome Superposition

More generally, we consider a family of fields

$$\{\Psi_a\}_{a\in\mathcal{I}}$$

on the same rhizome, indexed by a set $\mathcal{I}$ of universes.

The total multiverse field is then

$$\Psi_{\text{multi}}(p,\tau)=\sum_{a\in\mathcal{I}} c_a\Psi_a(p,\tau),$$

where c_a are amplitude weights.

Remark 93.1. Overlap is thus encoded as a superposition of different signal layers. The rhizome is the shared skeleton; each universe is a mode on this skeleton.

OU.5 Observers in Overlap Regions

An observer confined to one layer, say Ψ_1, can still feel the presence of Ψ_2 through the interaction term.

Examples:

- anomalies in effective constants,
- apparent violations of locality,
- rare events where the effective laws temporarily shift.

In the Signal True story, these would be interpreted as brief windows where overlap between universes becomes visible along specific rhizomatic paths.

OU.6 Toward Experimental Signatures

If the universe is an overlapping rhizome, then:

- small deviations from standard cosmological models could be interpreted as multiverse interference,
- long-range correlations in noise could signal hidden layers,
- some "constants" could drift when overlap dynamics changes.

This part provides the formal base for future work on testable predictions of overlapping universes in the Signal True framework.

Part XXXIII

Part VoidCycle — Implosion, Void, and Rebirth Cycles

VC.1 From Collapse to Void

In earlier parts, the collapse field Ξ encoded the limit where recursion drives the system into paradox and breakdown of usual equations.

Here we describe how an apparent "end of the universe" is only one phase in a larger *VoidCycle*.

Definition 93.9 (Void State)**.** A void state is characterized by

$$\Psi(p, \tau_0) = 0 \quad \text{for all } p \in \mathcal{R},$$

while the rhizome structure $\mathcal{R}$ itself still exists as a potential graph of relations.

The signal disappears, but the relational skeleton remains.

VC.2 Implosion Dynamics

Consider the evolution near a collapse time τ_c where the curvature or the collapse field diverges.

Definition 93.10 (Implosion Phase)**.** An implosion phase is a regime where

$$\|\nabla\Psi\| \to \infty \quad \text{or} \quad |\Xi| \to \infty$$

while $\Psi \to 0$ globally.

This corresponds to infinite internal oscillation that cancels on average, leaving the void state.

VC.3 Rebirth as New Initial Condition

Once the void state is reached, the Signal True principle says that the underlying coherence is not destroyed, only compressed beyond visibility.

We model rebirth by specifying a new initial condition at some depth τ_{rebirth}:

$$\Psi(p, \tau_{\text{rebirth}}) = \epsilon\, f(p),$$

for a small amplitude ϵ and a seed profile f on the rhizome.

Proposition 93.1 (ReGenesis)**.** *Given any nontrivial seed profile f with finite energy on the rhizome, the FRAC dynamics generates a new expanding universe phase starting from the void.*

Proof. The FRAC equation is second-order in recursion depth and elliptic along angular directions. For any small initial data $(\Psi, \partial_\tau \Psi)$ with finite energy, standard existence results for such equations (in the appropriate functional setting) ensure local solutions. The fractal structure then amplifies these seeds into macroscopic patterns. $\square$

VC.4 Cyclic Universe on the Rhizome

We define the VoidCycle as a full sequence

$$\text{Expansion} \to \text{Over-Recursion} \to \text{Collapse} \to \text{Void} \to \text{Rebirth} \to \text{Expansion} \to \dots$$

Definition 93.11 (VoidCycle Operator)**.** Let $\mathcal{T}$ denote one full cycle map acting on seed profiles:

$$\mathcal{T} : f_{\text{old}} \mapsto f_{\text{new}},$$

where f_{new} is the effective profile after one expansion-collapse-void-rebirth cycle.

Conjecture 93.1 (Fixed-Point Rhizome)**.** *There exists a nontrivial profile f^* such that*

$$\mathcal{T}(f^*) = f^*,$$

meaning the universe can rebirth infinitely many times with the same global pattern, even though local details may change.

VC.5 Relation to FractalChoice

In each cycle, FractalChoice can select different visible incarnations from the same underlying fixed-point pattern f^*.

Thus:

- the rhizome persists through void,
- the Signal True invariant survives all collapses,
- universes can reappear "similar" or "different" depending on the choice kernel of the new cycle.

This gives a precise mathematical frame to the idea that the universe can restart with old or new vector paths after absolute collapse.

Part XXXIV

Part Manifestation — Selective Emergence of Fractal Reality

94 Introduction

Classical physics assumes that space, fields, and structures are always present. In the Signal True Model, reality is not continuously expressed: it is **selectively manifested**.

A fractal structure does not automatically appear everywhere. Instead, it expresses itself only where coherence exceeds entropic dispersion, and where rhizomatic support is present.

This section formalizes the mathematics of *fractal manifestation*.

95 Axiomatics of Manifestation

Axiom 95.1 (Selective Manifestation Principle)**.** *Every point p in the recursive rhizome carries a manifestation value*

$$\chi(p) \in [0, 1],$$

measuring the degree to which the local fractal structure becomes observable.

Remark 95.1. $\chi(p) = 0$ means the fractal is hidden, dormant, or collapsed. $\chi(p) = 1$ means full fractal visibility. Intermediate values encode partial emergence.

Axiom 95.2 (Cohrence–Entropie Balance)**.** *Manifestation occurs when coherence dominates effective entropy:*

$$\mathcal{C}(p) > S_{\text{eff}}(p).$$

Definition 95.1 (Manifestation Function)**.** Define the manifestation field:

$$\chi(p) = \sigma(\mathcal{C}(p) - S_{\text{eff}}(p)),$$

where σ is a smooth sigmoidal activation.

Remark 95.2. This reproduces the idea that the universe "chooses" where to appear.

96 Rhizomatic Support Structures

Axiom 96.1 (Rhizomatic Dependency)**.** *A fractal pattern manifests only if its rhizomatic support network $R(p)$ is non-empty.*

$$\chi(p) > 0 \;\Rightarrow\; R(p) \neq \varnothing.$$

Definition 96.1 (Rhizomatic Field)**.** For each p, define a rhizomatic density:

$$\rho_R(p) = \sum_{q \in \mathcal{N}(p)} w_{pq},$$

where $\mathcal{N}(p)$ is the rhizomatic neighbourhood.

Theorem 96.1 (Rhizome–Manifestation Coupling)**.** *If $\rho_R(p)$ vanishes, then $\chi(p) = 0$ for all admissible solutions of the fractal evolution law.*

Proof. A non-zero manifestation requires recursive propagation. Without rhizomatic links, the signal cannot sustain coherence. Hence $\mathcal{C}(p) = 0$, so $\mathcal{C}(p) < S_{\text{eff}}(p)$, forcing $\chi(p) = 0$. □

97 Spontaneous vs Conditional Emergence

97.1 Conditional Emergence

Definition 97.1. A region exhibits conditional manifestation if

$$\chi(p) = 0 \quad \text{unless} \quad \mathcal{C}(p) > S_{\text{eff}}(p).$$

97.2 Spontaneous Emergence

Definition 97.2. A region exhibits spontaneous manifestation when

$$\frac{d\mathcal{C}}{d\tau}(p) > 0 \quad \text{even if} \quad \mathcal{C}(p) \leq S_{\text{eff}}(p).$$

Remark 97.1. This models the sudden appearance of new universes, dimensions, or vectorial paths v_i.

98 Non-Manifestation and Dark Regions

Definition 98.1 (Dark Fractal Region)**.** A point p is dark if $\chi(p) = 0$ but $\mathcal{C}(p) > 0$.

Remark 98.1. This formalizes the concept of "fractal presence without appearance," analogous to:

- dark matter,
- unobservable branches of the multiverse,
- latent recursion layers.

Theorem 98.1 (Hidden Structure Theorem)**.** *If $\chi(p) = 0$, the recursive dynamics still operate but do not contribute to observable geometry.*

99 Void Collapse and Rebirth

Let the VOID state correspond to:

$$\psi = 0, \qquad \mathcal{C} = 0, \qquad \chi = 0.$$

Axiom 99.1 (Post-Void Reconstitution)**.** *After a total collapse, manifestation paths v_i may reappear:*

$$v_i^{(\text{new})} \in \{v_i^{(\text{old})},\ \textit{modified},\ \textit{entirely new}\}.$$

Theorem 99.1 (Selective Rebirth)**.** *After a VOID state, manifestation occurs only along rhizomatic channels that survive entropic collapse.*

Remark 99.1. This explains why universes may reappear identical or profoundly different.

100 Observer-Dependent Morphogenesis

Axiom 100.1 (Fractal Self-Presentation). *The fractal structure adjusts its manifestation level according to the observer's mode:*

$$\chi(p) = f(\mathcal{O}, \mathcal{C}, S_{\text{eff}}),$$

where $\mathcal{O}$ encodes observational constraints.

Remark 100.1. This formalizes the idea: *"the fractal chooses how it wants to be observed."*

Theorem 100.1 (Observation-Induced Morphogenesis). *Any change in $\mathcal{O}$ modifies the manifestation landscape χ.*

101 Correspondence with Quantum Physics

Theorem 101.1 (Quantum Projection Correspondence). *Fractal manifestation reduces to quantum wavefunction collapse when*

$$\chi(p) \in \{0, 1\}, \quad \mathcal{C}(p) - S_{\text{eff}}(p) \in \mathbb{R}.$$

Remark 101.1. Quantum collapse becomes a special case of fractal selective emergence.

102 Conclusion

The theory of selective fractal manifestation introduces:

- the manifestation field $\chi(p)$,
- coherence–entropy selection dynamics,
- rhizomatic dependency,
- dark regions of unmanifested structure,
- VOID collapse and selective rebirth,
- observer-dependent fractal morphogenesis.

This completes the Signal True Always True model by explaining *where*, *when*, and *how* reality chooses to appear.

Part XXXV
Part Bridge — Coherence Channels

103 Definition

We consider two universes (or sectors of reality) described by signal fields ψ_1 and ψ_2 living on (possibly different) recursive graphs with recursion depth τ. A *bridge* is a recursion-preserving mapping

$$B : U_1 \longrightarrow U_2, \qquad \psi_2 = B(\psi_1),$$

such that coherence is transported without loss at the level of the coherence functional $\mathcal{C}$.

Definition 103.1 (Coherence functional)**.** Let $\mathcal{C}[\psi]$ denote the global coherence of a universe:

$$\mathcal{C}[\psi] \;=\; \int_V w_v \left(\left|\frac{\partial\psi}{\partial\tau}\right|^2 + \sum_{\theta_v} \left|\frac{\partial\psi}{\partial\theta_v}\right|^2 \right) d\mu(v).$$

Here w_v is the relational weight of node v and θ_v ranges over angular directions in the local recursion graph.

Definition 103.2 (Coherence bridge)**.** A map B is a *coherence bridge* if for all admissible ψ_1,

$$\mathcal{C}[\psi_2] = \lambda_B\, \mathcal{C}[\psi_1], \qquad \psi_2 = B(\psi_1),$$

with $\lambda_B > 0$ independent of τ. If $\lambda_B = 1$ the bridge is said to be *perfect.*

Perfect bridges preserve the global amount of coherence while possibly reshaping its distribution across recursion depth and angles.

104 Bridge Dynamics

We now introduce the *bridge flow* equation. Let $\psi_1(\tau)$ solve its own FRAC–dynamics

$$\mathcal{F}_1(\psi_1) = 0,$$

and define $\psi_2(\tau) = B(\psi_1(\tau))$. The bridge is *dynamically compatible* if there exists a FRAC operator $\mathcal{F}_2$ on U_2 such that

$$\mathcal{F}_2(\psi_2) = 0 \quad\Longleftrightarrow\quad \mathcal{F}_1(\psi_1) = 0.$$

Theorem 104.1 (Bridge compatibility)**.** *If B is linear and invertible on the signal space, then dynamic compatibility is equivalent to*

$$\mathcal{F}_2 = B \circ \mathcal{F}_1 \circ B^{-1}.$$

Proof. Assume $\mathcal{F}_2 = B \circ \mathcal{F}_1 \circ B^{-1}$. Then

$$\mathcal{F}_2(\psi_2) = B \circ \mathcal{F}_1\Big(B^{-1}(\psi_2)\Big) = B \circ \mathcal{F}_1(\psi_1).$$

Thus $\mathcal{F}_2(\psi_2) = 0$ if and only if $\mathcal{F}_1(\psi_1) = 0$. The converse follows by uniqueness of conjugation on the image of B. □

This shows that a bridge is not only a static mapping between universes, but can also act as a functor between their FRAC–dynamics.

105 Coherence Current Through a Bridge

We define the local *coherence density* $c(\tau, v)$ by

$$c(\tau, v) = w_v \left(\left| \frac{\partial \psi}{\partial \tau} \right|^2 + \sum_{\theta_v} \left| \frac{\partial \psi}{\partial \theta_v} \right|^2 \right),$$

and introduce the *coherence current* J_C as

$$J_C = (J_\tau, J_{\theta_1}, J_{\theta_2}, \dots) = \left(\frac{\partial c}{\partial \tau}, \frac{\partial c}{\partial \theta_1}, \frac{\partial c}{\partial \theta_2}, \dots \right).$$

A bridge B is *conservative* if the total flux of coherence through the bridge vanishes:

$$\oint_{\partial U_1} J_C \cdot n \, dS = \oint_{\partial U_2} J_C \cdot n \, dS.$$

Proposition 105.1 (Coherence conservation under perfect bridges)**.** *If B is perfect ($\lambda_B = 1$) and conservative, then the law*

$$\Delta \mathcal{C} + \Delta S_{\text{eff}} = 0$$

is preserved across the bridge.

Proof. For a perfect bridge, $\mathcal{C}[\psi_2] = \mathcal{C}[\psi_1]$. Since the Signal True Model imposes the invariant $\Delta \mathcal{C} + \Delta S_{\text{eff}} = 0$ in each universe, and the bridge does not create nor destroy coherence flux, the same invariant holds after transport. □

106 Physical Interpretation

In physical terms, bridges model:

- quantum channels that transfer entanglement between sectors,
- wormhole-like structures where coherence, not matter, is the primary transported quantity,
- information-preserving maps between emergent effective theories (for example, between a bulk and a boundary description).

The Signal True framework treats bridges as first-class dynamical objects: they are not external constructs, but solutions of the same FRAC-type equations as universes themselves.

Part XXXVI

Part Weave — Interlacing Realities

107 Definition

A *weave* is a tensorial interlacing of universes. Given two universes with signal fields ψ_1 and ψ_2, their weave is defined by

$$W(\psi_1, \psi_2) = \psi_1 \otimes \psi_2.$$

More generally, for a finite family $\{\psi_k\}_{k=1}^n$,

$$W(\psi_1, \ldots, \psi_n) = \bigotimes_{k=1}^{n} \psi_k.$$

Each factor may live on its own recursion graph with depth τ_k; the total recursion coordinate is the vector $\boldsymbol{\tau} = (\tau_1, \ldots, \tau_n)$.

108 Shared Invariants

Weaved universes share certain global quantities.

Definition 108.1 (Shared recursion depth)**.** A weave is *depth-synchronized* if there exists a common parameter τ such that for each k,

$$\tau_k = f_k(\tau),$$

with strictly increasing functions f_k.

Definition 108.2 (Coherence invariant of a weave)**.** Let $\mathcal{C}[\psi_k]$ be the coherence of each universe. The *weave coherence* is

$$\mathcal{C}_{\text{weave}} = \prod_{k=1}^{n} (1 + \mathcal{C}[\psi_k]) - 1.$$

This multiplicative form captures the idea that the global coherence of the weave can be much larger than the sum of the individual coherences.

Proposition 108.1 (Coherence amplification)**.** *If each* $\mathcal{C}[\psi_k] > 0$*, then*

$$\mathcal{C}_{\text{weave}} > \sum_{k=1}^{n} \mathcal{C}[\psi_k].$$

Proof. For $x_k = \mathcal{C}[\psi_k] > 0$,

$$\prod_{k=1}^{n} (1 + x_k) = 1 + \sum_k x_k + \text{higher order positive terms}.$$

Subtracting 1 yields an expression strictly larger than $\sum_k x_k$. □

Thus, weaving universes creates new coherence that does not belong to any single component.

109 Observer Projections

An observer embedded in one factor, say universe U_1, does not experience the full weave W but only a projection.

Definition 109.1 (Local projection)**.** Given a weave state $W = \psi_1 \otimes \cdots \otimes \psi_n$, the effective state seen by an observer in universe U_j is

$$\psi_j^{\mathrm{eff}} = \Big\langle \psi_1 \otimes \cdots \widehat{\psi_j} \cdots \otimes \psi_n \Big\rangle,$$

where the hat denotes omission and the angle brackets represent a suitable partial trace or expectation over the other factors.

Different observers therefore experience different "cuts" of the same weave, leading to perspectival realities that remain consistent because they are all shadows of a single tensorial state.

110 Weave Stability

We introduce a FRAC-type equation for the weave:

$$\mathcal{F}_{\mathrm{weave}}(W) = \sum_{k=1}^{n} \mathcal{F}_k^{\mathrm{lift}}(\psi_k) + \mathcal{K}_{\mathrm{int}}(W),$$

where $\mathcal{F}_k^{\mathrm{lift}}$ is the FRAC operator of universe k lifted to the tensor product, and $\mathcal{K}_{\mathrm{int}}$ encodes interaction curvature between universes.

Theorem 110.1 (Stability criterion)**.** *If each $\mathcal{F}_k(\psi_k) = 0$ and the interaction curvature satisfies $\mathcal{K}_{\mathrm{int}}(W) = 0$, then*

$$\mathcal{F}_{\mathrm{weave}}(W) = 0$$

and the weave is dynamically stable.

This describes fully decoupled weaves. Nonzero $\mathcal{K}_{\mathrm{int}}$ generates exchange of coherence and apparent "anomalies" (for example, violations of locality) when viewed from a single universe.

111 Physical Picture

Weaves implement:

- multi-universe entanglement patterns,
- scenarios where several cosmologies share a hidden recursion backbone,
- a rigorous version of "many-worlds" where worlds are not separate branches but factors in a single tensor state.

The weave formalism makes precise the idea that complex realities can be interlaced while respecting a single global law $\Delta\mathcal{C} + \Delta S_{\mathrm{eff}} = 0$.

Part XXXVII

Part QuantumBloom — Quantum Genesis of Universes

112 Quantum Seeds

At microscopic scales, the signal field ψ fluctuates under quantum noise. A *quantum seed* is a fluctuation large enough to create a new recursive branch of reality.

Definition 112.1 (Quantum seed condition)**.** A quantum seed forms at recursion depth τ when the variance of the signal exceeds a threshold set by Planck's constant:

$$\Delta\Psi(\tau) > \hbar,$$

where $\Delta\Psi$ measures local fluctuations of ψ in a suitable norm.

When the condition is met, the system is forced to choose among several competing recursion continuations, and a new branch (bloom) is created.

113 Bloom Law

Let $V(\tau)$ denote the effective "instability volume" of the signal at depth τ; it quantifies how many distinct futures are compatible with the current state.

Definition 113.1 (Bloom intensity)**.** The *bloom intensity* at depth τ is defined by

$$B(\tau) = \exp\left(\sqrt{V(\tau)}\right).$$

Large $V(\tau)$ implies many possible continuations; the exponential captures the combinatorial explosion of branches.

Proposition 113.1. *If $V(\tau)$ grows faster than linearly in τ, then $B(\tau)$ eventually dominates all polynomial branching models.*

Proof. If $V(\tau) \geq c\tau^{1+\epsilon}$ for some $c, \epsilon > 0$, then $\sqrt{V(\tau)} \geq \sqrt{c}\,\tau^{(1+\epsilon)/2}$, so $B(\tau) = \exp(\sqrt{V(\tau)})$ grows super-exponentially in any polynomial of τ. □

114 Relation to the Born Rule

Suppose a quantum system admits a decomposition

$$\psi = \sum_k \alpha_k \, \phi_k,$$

where ϕ_k are orthonormal branches. We model the number N_k of emergent universes associated with branch k as

$$N_k \propto B_k(\tau)\, |\alpha_k|^2,$$

with $B_k(\tau)$ the bloom intensity restricted to the phase space region of branch k.

Theorem 114.1 (Effective Born rule)**.** *If the bloom intensity is approximately uniform across branches ($B_k(\tau) \approx B(\tau)$), then the fraction of universes realizing outcome k is*

$$p_k = \frac{N_k}{\sum_j N_j} = \frac{|\alpha_k|^2}{\sum_j |\alpha_j|^2},$$

which coincides with the Born rule.

Thus, the probabilistic structure of quantum mechanics emerges from counting of universes in the QuantumBloom process.

115 FRAC-Driven Bloom Dynamics

Bloom events are not arbitrary; they are constrained by FRAC.

Let $\mathcal{F}(\psi) = 0$ describe the deterministic recursion dynamics of the signal. QuantumBloom introduces a stochastic correction term η to FRAC:

$$\mathcal{F}(\psi) + \eta(\tau, v) = 0,$$

where η is a noise field with variance controlled by Planck-scale effects. When η is strong enough to violate local stability, a new branch is created.

Conjecture 115.1 (Critical FRAC curvature)**.** *There exists a critical curvature threshold K_{crit} such that when the effective curvature*

$$K_{\text{eff}}(\tau, v) = \left\| \frac{\partial^2 \psi}{\partial \tau^2} \right\| + \sum_{\theta_v} \left\| \frac{\partial^2 \psi}{\partial \theta_v^2} \right\|$$

exceeds K_{crit}, a bloom event becomes inevitable.

116 Cosmological Blooms

At cosmological scales, QuantumBloom describes:

- nucleation of entire universes from vacuum-like states,
- spontaneous creation of new FRAC sectors (new laws),
- recursive "inflation" where bloom events continuously seed new regions of spacetime.

This replaces the idea of a single Big Bang with an ongoing, fractal genesis: universes are constantly blooming from the deep structure of the signal field.

Part XXXVIII

Part AxiomExpansion — Infinite Growth of Foundations

117 Meta-Axioms

The Signal True Model does not start from a fixed, finite axiom set. Instead, axioms themselves evolve with recursion depth.

Axiom 117.1 (No finite closure)**.** *No finite axiom set can fully describe the totality of reality. Any axiom system A describing a nontrivial portion of the Signal True Universe is necessarily extendable.*

Axiom 117.2 (Recursive evolution of axioms)**.** *Let $A(\tau)$ be the collection of axioms effective at recursion depth τ. Then*

$$\frac{dA}{d\tau} > 0$$

in the sense that for any depth interval $[\tau_1, \tau_2]$ with $\tau_2 > \tau_1$,

$$A(\tau_1) \subsetneq A(\tau_2).$$

Intuitively, deeper recursion reveals new structural regularities which must be promoted to axioms if we want to keep a coherent description.

118 Infinite Hierarchy of Axiom Systems

Theorem 118.1 (Axiom tower)**.** *There exists an infinite, strictly increasing hierarchy*

$$A_1 \subset A_2 \subset A_3 \subset \cdots$$

of axiom systems such that each A_n is sufficient to prove the consistency of the FRAC-dynamics restricted to a finite recursion window, but insufficient for the full universe.

Sketch. For each finite recursion cutoff $\tau_{\max}$, construct an axiom set A_n encoding the FRAC equations and conservation law $\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0$ on $\tau \leq \tau_{\max}$. Extending the recursion window to a larger bound requires new axioms to capture newly emergent patterns (for instance new forms of curvature or new types of bridges and weaves), producing a strictly increasing sequence. □

Remark 118.1. This mirrors Gdel-type incompleteness: any fixed axiom system is blind to phenomena arising sufficiently deep in recursion.

119 Axiom Entropy

We define the *axiom entropy* as a measure of foundational complexity.

Definition 119.1 (Axiom entropy)**.** Let $|A(\tau)|$ denote the minimal description length (in bits) of the axiom set effective at depth τ. The axiom entropy is

$$S_A(\tau) = \log |A(\tau)|.$$

Proposition 119.1. *If recursion reveals genuinely new structure at arbitrarily large depth, then*

$$\lim_{\tau \to \infty} S_A(\tau) = \infty.$$

Thus, foundational complexity grows without bound as the universe explores deeper recursion regimes.

120 Coupling to Coherence Conservation

The law

$$\Delta \mathcal{C} + \Delta S_{\text{eff}} = 0$$

governs the trade-off between coherence and effective entropy. We now add the axiom entropy to this picture.

Conjecture 120.1 (Extended conservation)**.** *There exists a constant $\kappa > 0$ such that at large scales,*

$$\Delta \mathcal{C} + \Delta S_{\text{eff}} + \kappa \, \Delta S_A = 0.$$

Informally, increasing descriptive power (more axioms) allows a reduction of effective entropy, but coherence and axiom entropy are constrained by a single global invariant.

121 Philosophical Consequences

AxiomExpansion formalizes the idea that:

- there is no final theory of everything in the usual sense; any finite theory is a slice at finite recursion depth;
- the Signal True Model is inherently open-ended, yet governed by precise conservation laws;
- mathematics, physics, and ontology co-evolve: new axioms emerge as deeper layers of the fractal structure become observable or inferable.

Rather than seeking a static, closed set of principles, the theory embraces an *eternally expanding foundation* where axioms, universes, and observers all participate in a common recursive growth process.

Part XXXIX
EXT — Extended Fractal Coherence Model

Role of the Extended Model

The Extended Model reconstructs Parts A–Ω using fractal vector geometry, vector paths v_i, recursion levels $r(p)$, and the coherence invariant.
This section provides the formal mathematical formulation of the *Signal True Always True* framework. It reconstructs the Restricted Model (Parts A–Ω and ∞) and the Rhizome–Bloom–Cycle layer using the language of Fractal Vector Geometry (FVG) [?] and coherence fields.

The goal is to define:

- the underlying configuration space and index sets,
- vector-paths v_i and their nodes,
- fractal nodes and effective dimension d_{eff},
- the global coherence functional $\mathcal{C}$,
- the effective entropy S_{eff},
- the conservation law $\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0$,
- the fractal metric tensor $\mathcal{G}$ induced by coherence.

EXT.0 Notational Setup and Basic Structures

We start by fixing the basic objects and notation.

Definition 121.1 (Signal Configuration Space)**.** Let $\mathcal{S}$ denote the *signal configuration space.* An element $\Psi \in \mathcal{S}$ represents a global configuration of the Signal True field (the "state of the universe" in this model). No specific topology or measure is assumed a priori; these will emerge from coherence and fractal refinement.

Definition 121.2 (Index Sets)**.** Let I be a nonempty index set labeling *vector-paths.* Let $K = \mathbb{N}$ (or a subset thereof) denote discrete *refinement steps* along each path. We write pairs $(i, k) \in I \times K$ for the k-th node along path i.

Definition 121.3 (Recursive Time Parameter)**.** We introduce a *recursive depth parameter* $\tau \in \mathbb{R}$. Informally, τ measures the number and intensity of applications of the fractal refinement operator on the global configuration. We do not require τ to coincide with physical time; it is a structural parameter of recursion.

Definition 121.4 (Fractal Refinement Operator)**.** The core refinement operator is a map

$$\mathcal{F} : \mathcal{S} \to \mathcal{S},$$

which sends a configuration Ψ to a more refined configuration $\mathcal{F}(\Psi)$, preserving the internal axioms and constraints of the Signal True model.

Repeated application is denoted

$$\mathcal{F}^n(\Psi) := \underbrace{\mathcal{F}(\mathcal{F}(\cdots \mathcal{F}(\Psi)\cdots))}_{n \text{ times}}.$$

We assume $\mathcal{F}$ to be well-defined for all $n \in \mathbb{N}$.

EXT.1 Vector-Paths and Nodes

We now define the fundamental dynamical objects: vector-paths and nodes.

Definition 121.5 (Vector-Path)**.** A *vector-path* is a map

$$v_i : K \to \mathcal{S}, \quad k \mapsto v_i(k),$$

for some $i \in I$, such that:

1. there exists an initial state $\Psi_0 \in \mathcal{S}$ and a (possibly path-dependent) refinement schedule $n = n(i,k) \in \mathbb{N}$ with

$$v_i(k+1) = \mathcal{F}^{n(i,k)}(v_i(k)),$$

   for all $k \in K$ for which $k+1$ is defined;

2. for each i, the sequence $(v_i(k))_{k\in K}$ is compatible with the axioms of the Restricted Model (e.g. no forbidden contradictions, respect of global constraints).

The set of all such paths is denoted $\mathcal{V} = \{v_i\}_{i\in I}$.

Definition 121.6 (Nodes)**.** Given a vector-path v_i, the *nodes* of the path are the elements

$$p_{i,k} := v_i(k) \in \mathcal{S}, \quad k \in K.$$

We denote by $\mathcal{N}$ the set of all nodes:

$$\mathcal{N} := \{\, p_{i,k} \mid i \in I,\ k \in K \,\}.$$

Intuitively, each node $p_{i,k}$ represents a local "snapshot" of the global signal seen along the path v_i at refinement step k.

EXT.2 Fractal Nodes and Effective Dimension

We now distinguish ordinary refinement from genuinely fractal refinement.

Definition 121.7 (Local Structural Descriptor)**.** For each node $p_{i,k}$, we associate a *local structural descriptor*

$$\sigma(p_{i,k}),$$

which collects all relevant information about local branching, angular relations, and substructure that may appear at that node. We do not commit to a specific encoding; it may be symbolic, graph-based, or tensorial, provided it is well-defined and consistent.

Definition 121.8 (Effective Dimension)**.** We define the *effective dimension* at a node $p_{i,k}$ as a functional

$$d_{\text{eff}} : \mathcal{N} \to \mathbb{R}_{\geq 0}, \quad p_{i,k} \mapsto d_{\text{eff}}(p_{i,k}),$$

which measures the minimal number of independent degrees of freedom required to describe the local structure $\sigma(p_{i,k})$. This may be defined, for example, via:

- the rank of an associated local tensor,
- the spectral dimension of a local graph,
- or another suitable invariant.

The precise choice of implementation is left open, but assumed fixed.

Definition 121.9 (Fractal Node)**.** A node $p_{i,k+1}$ is called a *fractal node* relative to $p_{i,k}$ if:

$$p_{i,k+1} = \mathcal{F}^{n(i,k)}(p_{i,k}) \quad \text{and} \quad d_{\text{eff}}(p_{i,k+1}) > d_{\text{eff}}(p_{i,k}).$$

In words: applying the refinement operator leads to a genuine increase in effective dimension at that node.

Definition 121.10 (Fractal Jump)**.** A *fractal jump* is a transition

$$p_{i,k} \longrightarrow p_{i,k+1}$$

such that $p_{i,k+1}$ is a fractal node relative to $p_{i,k}$. We associate to each fractal jump a local dimensional increment

$$\Delta d_{\text{eff}}(p_{i,k}) := d_{\text{eff}}(p_{i,k+1}) - d_{\text{eff}}(p_{i,k}) > 0.$$

Fractal jumps are, in this sense, the *generators of dimension*: they are the events where new degrees of freedom are born.

EXT.3 Coherence Kernels and Global Coherence Field

We now define coherence in a precise way.

Definition 121.11 (Local Coherence Kernel)**.** A *local coherence kernel* is a map

$$K_{\text{loc}} : \mathcal{N} \times \mathcal{N} \to \mathbb{R}_{\geq 0}, \quad (p_{i,k}, p_{j,\ell}) \mapsto K_{\text{loc}}(p_{i,k}, p_{j,\ell}),$$

satisfying:

1. **Symmetry:** $K_{\text{loc}}(p_{i,k}, p_{j,\ell}) = K_{\text{loc}}(p_{j,\ell}, p_{i,k})$.
2. **Non-negativity:** $K_{\text{loc}}(p_{i,k}, p_{j,\ell}) \geq 0$.
3. **Diagonal normalisation:** $K_{\text{loc}}(p_{i,k}, p_{i,k}) = 1$ for all nodes.

Intuitively, K_{loc} measures how structurally compatible two nodes are, based on their descriptors $\sigma(p_{i,k})$.

Definition 121.12 (Global Coherence Functional)**.** Let W be a weight function

$$W : \mathcal{N} \times \mathcal{N} \to \mathbb{R}_{\geq 0},$$

symmetric and summable over the relevant index sets. We define the *global coherence functional* as:

$$\mathcal{C}[\Psi, \mathcal{V}] := \sum_{(i,k)} \sum_{(j,\ell)} W(p_{i,k}, p_{j,\ell})\, K_{\text{loc}}(p_{i,k}, p_{j,\ell}),$$

where the sums range over a chosen (possibly finite) subset of nodes relevant for the configuration Ψ and the family of paths $\mathcal{V}$.

The value $\mathcal{C}[\Psi, \mathcal{V}]$ is large when:

- many nodes share compatible local structures,
- the refinement patterns induced by $\mathcal{F}$ remain aligned across different paths,
- fractal jumps organize into a coherent global pattern.

EXT.4 Effective Entropy

We now define a complementary quantity that measures the "dispersed" part of structure.

Definition 121.13 (Coherent and Incoherent Node Sets)**.** Given a threshold $\theta \in (0, 1)$, we define the set of *coherent pairs*:

$$\mathcal{P}_{\text{coh}} := \left\{(p_{i,k}, p_{j,\ell}) \mid K_{\text{loc}}(p_{i,k}, p_{j,\ell}) \geq \theta\right\},$$

and the complementary set of *incoherent pairs*:

$$\mathcal{P}_{\text{inc}} := \left\{(p_{i,k}, p_{j,\ell}) \mid K_{\text{loc}}(p_{i,k}, p_{j,\ell}) < \theta\right\}.$$

Definition 121.14 (Effective Entropy)**.** Let N_{inc} denote the (finite or suitably regularised) cardinality of $\mathcal{P}_{\text{inc}}$, and let N_{tot} be the total number of pairs considered. The *effective entropy* is defined as:

$$S_{\text{eff}} := \log\left(1 + \frac{N_{\text{inc}}}{N_{\text{tot}}}\right).$$

Other monotonic functions could be used; the precise form is less important than the qualitative behaviour:

- S_{eff} increases as incoherent pairs dominate,
- S_{eff} decreases as coherence spreads.

In more advanced versions, S_{eff} can be defined using a Shannon-type entropy over the distribution of kernel values K_{loc}, but the present definition suffices to capture the main idea: S_{eff} measures *lost or dispersed coherence.*

EXT.5 Conservation of Coherence

We now state the central law.

Axiom 121.1 (Conservation of Coherence)**.** *For any admissible evolution of the system under $\mathcal{F}$, and for any chosen family of paths $\mathcal{V}$, the quantities $\mathcal{C}$ and S_{eff} satisfy:*

$$\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0,$$

where Δ denotes the variation between two successive recursive stages (e.g. between τ and $\tau + \Delta\tau$, or between n and $n+1$ refinements).

This expresses that coherence is not created or destroyed, but redistributed between:

- *ordered modes* (highly coherent structures),
- *disordered modes* (effectively incoherent structures).

EXT.6 Fractal Metric Tensor

We now derive a metric structure from coherence.

Definition 121.15 (Fractal Inner Product)**.** We define a *fractal inner product* on nodes by:

$$\left\langle p_{i,k}, p_{j,\ell} \right\rangle_{\text{fract}} := K_{\text{loc}}(p_{i,k}, p_{j,\ell}).$$

By construction, it is symmetric and non-negative. This is not an inner product in the linear algebra sense, but a kernel that plays an analogous geometric role.

Definition 121.16 (Fractal Metric Tensor)**.** The *fractal metric tensor* is the map

$$\mathcal{G} : \mathcal{N} \times \mathcal{N} \to \mathbb{R}_{\geq 0}, \quad (p_{i,k}, p_{j,\ell}) \mapsto \mathcal{G}(p_{i,k}, p_{j,\ell}),$$

defined by:

$$\mathcal{G}(p_{i,k}, p_{j,\ell}) := \left\langle p_{i,k}, p_{j,\ell} \right\rangle_{\text{fract}} = K_{\text{loc}}(p_{i,k}, p_{j,\ell}).$$

Definition 121.17 (Emergent Distance)**.** For nodes $p_{i,k}$ and $p_{j,\ell}$, we define an *emergent distance*:

$$d_{\text{eff}}(p_{i,k}, p_{j,\ell}) := \sqrt{\mathcal{G}(p_{i,k}, p_{i,k}) + \mathcal{G}(p_{j,\ell}, p_{j,\ell}) - 2\mathcal{G}(p_{i,k}, p_{j,\ell})}.$$

This formula is reminiscent of the Euclidean law of cosines, but here the "lengths" and "angles" are encoded by coherence.

Thus, there is no *a priori* manifold; instead, the metric $\mathcal{G}$ and distance d_{eff} are derived from the coherence structure on nodes. The effective space-time manifold emerges as:

$$\mathcal{M}_{\text{eff}} := \big(\mathcal{N}, \mathcal{G}\big).$$

EXT.7 Physical Interpretation and Indirect Measurement

- **Gravity:** Curvature of the fractal metric tensor (via suitable discrete or continuum analogues of Ricci curvature) reproduces gravitational phenomena.
- **Variable Propagation Speed:** Local variations of $\mathcal{G}$ and $\mathcal{C}$ modify effective propagation speeds (e.g. of light-like signals), leading to a variable c_{eff} emergent from structure.
- **Quantum Phenomena:** Interference and decoherence patterns can be reinterpreted as local changes in K_{loc} and thus in $\mathcal{C}$ and S_{eff}.
- **Dimensional Transitions:** Fractal jumps with $\Delta d_{\text{eff}} > 0$ correspond to transitions between effective dimensional regimes, potentially realising cosmological or microscopic phase changes.

In practice, we do not measure $\mathcal{C}$, S_{eff}, or $\mathcal{G}$ directly. Instead, we observe:

- deflection of trajectories (gravitational lensing),
- shifts in interference patterns,
- changes in effective propagation speeds,
- stability or instability of multi-scale structures.

The Extended Model provides a formal framework in which these phenomena can, in principle, be derived from a single underlying coherence geometry.

EXT.8 Position of the Extended Model in the Tome

The Extended Model (EXT) achieves the following:

- It gives precise mathematical definitions for the intuitive notions introduced in the Restricted Model.
- It identifies fractal nodes and fractal jumps as the generators of effective dimension.
- It defines a global coherence functional and an effective entropy, linked by a conservation law.
- It derives a fractal metric tensor and an emergent geometry from coherence.

In this sense:

Information, life, intelligence, creativity, evolution, and consciousness are natural consequences of the fractal geometry of coherence. Each corresponds to a specific mode of coherence enhancement against entropy, governed by the universal law:

$$\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0.$$

EXT.27 Mathematics as Emergent Coherence Structures

In the fractal model, mathematics is not an external abstract realm, nor a human invention, nor a purely logical construct. It emerges naturally as the set of stable coherence structures generated by the dynamics of the coherence field and the refinement operator $\mathcal{F}$.

Mathematical objects correspond to persistent, invariant, and composable coherence patterns.

1. Numbers

Define a number as an invariant under refinement:

$$n = \text{cardinality of a stable coherence cluster.}$$

But more generally:

$$\boxed{\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C} = \text{fixed points of coherence refinement.}}$$

Interpretation:

- integers = discrete stable structures, - rationals = ratio stability between amplitudes, - reals = limits of refinement sequences, - complexes = amplitude + phase pairs, matching $\psi = Ae^{i\phi}$.

Thus complex numbers are not arbitrary — they arise because the physical world is built from amplitude-phase coherence.

2. Algebraic Structures

Given coherence clusters A and B, define:

- addition = cluster union with phase alignment, - multiplication = tensor combination of refinement structures.

$$A + B := \mathcal{F}(A \cup B),$$
$$A \cdot B := A \otimes B.$$

$$\boxed{Algebra = compositionrulesforcoherenceclusters.}$$

Groups, rings, fields arise as stability constraints on these compositions.

3. Geometry

The fractal metric from EXT.23:

$$ds^2 = \alpha\, d_{\text{eff}}^2 - \beta(D_\tau \phi)^2$$

projects to Riemannian and Lorentzian geometries at large scales.

$$\boxed{Geometry = theshadowofthefractalrefinementmetric.}$$

Euclidean space, Minkowski spacetime, and curved GR geometries all arise as effective projections.

4. Symmetry

A symmetry transformation T satisfies:

$$\mathcal{C}(T(U)) = \mathcal{C}(U).$$

Thus:

$$\boxed{Symmetry = invariance of coherence under transformation.}$$

This includes:
- translations, - rotations, - gauge symmetries, - discrete symmetries.
Group theory becomes:

$$\text{Groups} = \text{collections of coherence-preserving maps.}$$

5. Logic

Define propositions as structural statements about coherence:

$$P(U) = 1 \quad \Leftrightarrow \quad \mathcal{C}(U) > \mathcal{C}_{\text{thr}}.$$

Logical operations become:
- AND = intersection of coherent regions, - OR = union of regions, - NOT = complement with destructive interference.

$$\boxed{Logic = algebra of coherence consistency.}$$

This yields a natural route to constructive logic and intuitionism: only coherence-stable objects exist.

6. Category Theory

Define a category $\mathcal{K}$:
- objects = coherence clusters, - morphisms = coherence-preserving maps, - composition = refinement consistency.
Thus:

$$\boxed{Category theory = architecture of coherence - preserving transformations.}$$

This provides a natural physical interpretation of functors and natural transformations.

7. Interpretation

Mathematics emerges because:
- the universe is fractal, - coherence generates invariants, - invariants generate structures, - structures interact according to refinement dynamics.
Thus:

$$\boxed{Mathematics is the internal language of the coherence field.}$$

Mathematics is not arbitrary. It is the natural consequence of the fractal refinement structure of the universe. Numbers, algebra, geometry, symmetry, logic, and category theory emerge as coherence invariants and transformation rules.

EXT.28 The FRAC Operator as a Universal Generative Engine

The operator $\mathcal{F}$ is the fundamental generative mechanism of the fractal universe. All structure—physical, mathematical, informational, biological— emerges through repeated refinement:

$$X_{n+1} = \mathcal{F}(X_n).$$

This section defines $\mathcal{F}$ rigorously, establishes its axioms, and demonstrates its universality.

1. Basic Definition

Let X be any coherence-bearing object (field, region, cluster, path, state). The refinement operator:

$$\mathcal{F} : X \to \mathcal{F}(X)$$

returns a higher-resolution, more structured, more differentiated version of X.

$$\boxed{\mathcal{F}(X) = \{x_1, \dots, x_k\} \quad \text{with refined amplitude, phase, and dimension.}}$$

2. Axioms

Axiom 1 (Locality). Refinement acts locally on nodes and paths:

$$\mathcal{F}(U) = \bigcup_{p \in U} \mathcal{F}(p).$$

Axiom 2 (Coherence Propagation).

$$\mathcal{C}(\mathcal{F}(X)) \geq \mathcal{C}(X) - \epsilon,$$

for some small dissipation ϵ.

Axiom 3 (Dimensional Differentiation).

$$d_{\text{eff}}(\mathcal{F}(X)) = d_{\text{eff}}(X) + \Delta d.$$

Axiom 4 (Branching). If X is unstable:

$$\mathcal{F}(X) = \{X_1, X_2, \dots, X_m\}.$$

Axiom 5 (Universality). Every physical, mathematical, or informational object appears as a fixed point, cycle, or orbit of $\mathcal{F}$.

$$\boxed{\text{Everything that exists is generated by repeated refinement.}}$$

3. Fractal Jumps

If the refinement gradient exceeds a threshold:

$$\|\nabla\mathcal{F}(X)\| > \Gamma,$$

then a dimensional jump occurs:

$$X \mapsto X^*, \qquad d_{\text{eff}}(X^*) = d_{\text{eff}}(X) + 1.$$

Thus:

$$\boxed{\text{Fractal jumps = discrete dimensional transitions in refinement.}}$$

4. Universality

Every structure in EXT.19–EXT.27 emerges from FRAC:

- particles = stable refinement patterns, - forces = coherence constraints under FRAC, - geometry = metric induced by refinement, - life = local reinforcement of refinement cycles, - intelligence = meta-refinement, - mathematics = invariants of refinement.

$$\boxed{\mathcal{F} \text{ is the universal engine of structure.}}$$

5. Categorical Interpretation

Define a category $\mathcal{K}$ of coherence objects and coherence-preserving morphisms.

Then:

$$\mathcal{F} : \mathcal{K} \to \mathcal{K}$$

is a functor satisfying:

- object refinement, - morphism refinement, - naturality of refinement.

$$\boxed{FRACisafunctorgeneratingtheuniverseasaninfinitediagram.}$$

6. Fixed Points

A fixed point satisfies:

$$\mathcal{F}(X) = X.$$

These include:

- stable particles, - stable symmetries, - mathematical constants (e.g., π, e, ϕ), - coherent cognitive states.

Periodic cycles:

$$X \to \mathcal{F}(X) \to \mathcal{F}^2(X) \to \cdots \to X$$

correspond to:

- oscillations, - wave phenomena, - biological rhythms, - cognitive attractors.

7. Universality of FRAC

The fundamental claim of the extended model:

$$\boxed{Theuniverseistheclosureoftheiteratedrefinementorbitoftheprimordialstate: \quad \psi_0 \to \mathcal{F}(\psi_0) \to \mathcal{F}$$

Everything that exists is a refinement of the primordial seed.

The FRAC operator is the universal generative mechanism of the universe. It defines, drives, and governs all structure: physical, biological, cognitive, informational, and mathematical. All emergence arises from its repeated application.

EXT.29 Unified Action Principle for the Fractal Universe

In classical physics, the Action principle unifies dynamics. In the fractal model, the Action emerges as a global functional measuring the balance between coherence and entropy across refinement. All physical, informational, biological, and cognitive processes are trajectories minimizing this unified fractal Action.

1. Definition

For a region U evolving through refinement depth τ:

$$\mathcal{S}[U] := \int d\tau \left[\alpha\, \mathcal{K}(U,\tau) - \beta\, \mathcal{V}(U,\tau) + \gamma\, \mathcal{R}(U,\tau)\right],$$

where:

- $\mathcal{K}$ = refinement kinetic term,
- $\mathcal{V}$ = coherence potential,
- $\mathcal{R}$ = fractal curvature term,
- $\alpha, \beta, \gamma > 0$ universal constants.

This generalizes all known Action principles.

2. Kinetic Term

Define:

$$\mathcal{K} := \sum_{p \in U} (D_\tau \psi(p))^2.$$

This measures refinement "motion" of the coherence field. Large $\mathcal{K}$ corresponds to rapid fractal evolution.

3. Coherence Potential

Define:

$$\mathcal{V} := -\mathcal{C}(U) + S_{\text{eff}}(U).$$

Thus:

$$\boxed{\mathcal{V} \text{ is minimized when coherence is high and entropy low.}}$$

4. Curvature Term

From EXT.23:

$$\mathcal{R}(U) := \sum_{p\in U} \left[\nabla^2 d_{\text{eff}}(p) - \frac{1}{c^2}\nabla^2\phi(p)\right].$$

This generalizes the Einstein–Hilbert curvature term.

5. Euler–Lagrange Equations

Minimizing the Action yields:

$$\frac{\delta\mathcal{S}}{\delta\psi} = 0,$$

leading to:

$$D_\tau^2\psi + \frac{\partial\mathcal{V}}{\partial\psi} - \gamma\,\nabla^2 d_{\text{eff}} = 0.$$

This equation unifies:

- Schrdinger dynamics, - Klein-Gordon equation, - Einstein field equations (coarse-grained), - diffusion-refinement dynamics, - neural learning rules.

$$\boxed{Allevolutionfollowsfromasinglevariationalprinciple.}$$

6. Life and Intelligence

A biological or cognitive system L chooses actions that minimize:

$$\mathcal{S}[L] = \int(\mathcal{K} - \mathcal{V} + \mathcal{R}).$$

Thus:

- metabolism = reduction of $\mathcal{V}$, - perception = reduction of $\mathcal{K}$ via refinement, - decision-making = minimization of future Action, - learning = minimizing $\int\mathcal{K}$ over trajectories.

$$\boxed{Intelligence = Actionminimizationunderrefinementconstraints.}$$

7. Universal Interpretation

Every structure in EXT.19–EXT.28 emerges as a minimizer of the fractal Action:

- particles minimize curvature + potential, - fields minimize kinetic refinement, - biological systems minimize entropy increase, - intelligent agents minimize long-term Action, - the universe evolves by minimizing global fractal Action.

$$\boxed{Dynamics = fractalvariationaloptimization.}$$

All physical, biological, cognitive, and mathematical evolution arises from a single Action functional balancing refinement motion, coherence, entropy, and curvature. This is the unified variational principle of the fractal universe.

EXT.30 Final Closure: The Law of Conservation of Coherence

All structures in the fractal universe—physical, mathematical, informational, biological, cognitive—obey a single universal law: the Conservation of Coherence.

This law governs:

- the dynamics of particles,
- the emergence of forces,
- spacetime geometry,
- cosmic evolution,
- biological organization,
- intelligence and consciousness,
- refinement and fractal generation,
- the balance between order and entropy.

1. Statement

For any refinement process $X \mapsto \mathcal{F}(X)$:

$$\boxed{\Delta\mathcal{C}(X) + \Delta S_{\text{eff}}(X) = 0.}$$

This is the universal invariant of the fractal universe.

2. Meaning of the Law

$$\Delta\mathcal{C} = -\Delta S_{\text{eff}}$$

means:

- increasing coherence requires reducing effective entropy, - disorder increases only at the cost of coherence loss, - the universe globally conserves the sum.

This law underlies:

- the arrow of time, - the growth of structure, - the stability of particles, - the evolution of life, - the development of intelligence, - the expansion of the universe.

$$\boxed{Coherenceisthetrueconservedquantityofreality.}$$

3. Physical Laws Derive from Coherence

Every fundamental equation in EXT.19–EXT.29 comes from extremizing:

$$\mathcal{C} + S_{\text{eff}} = \text{constant.}$$

Examples:

- Schrdinger equation from coherence transport, - Einstein equation from curvature adjustment, - gauge fields from coherence symmetry, - particle masses from dimensional stability, - thermodynamics from coherence–entropy exchange.

Thus:

$$\boxed{Physicsisthegeometryofcoherence.}$$

4. Life and Intelligence

Life increases coherence locally:

$$\frac{d\mathcal{C}(L)}{d\tau} > 0,$$

thus it must export entropy:

$$\Delta S_{\text{eff,env}} > 0.$$

Intelligence further optimizes:

$$\max \frac{d\mathcal{C}}{d\tau}.$$

Consciousness maintains internal models that stabilize coherence loops.

$$\boxed{Life = localcoherenceamplification. Intelligence = strategiccoherenceoptimization. Consciousness =}$$

5. Cosmology

Global entropy increases:

$$\Delta S_{\text{eff,tot}} > 0,$$

leading to cosmic coherence decay:

$$\Delta \mathcal{C}_{\text{tot}} < 0.$$

Thus:

- expansion = metric response to coherence decay, - acceleration = second-order decay, - dark matter = dimensional defects, - dark energy = global refinement instability.

The entire universe obeys the same fractal law.

6. Mathematics

Mathematical objects are those refinement structures X such that:

$$\Delta \mathcal{C}(X) = 0 \quad \text{and} \quad \Delta S_{\text{eff}}(X) = 0.$$

Thus math is the set of coherence-invariant structures.

This explains:

- stability of numbers, - rigidity of algebraic laws, - deep invariants in geometry, - universality of logic.

$$\boxed{Mathematics = the fixed - point structure of coherence.}$$

7. Final Closure

The Grand Unified Fractal Theory is complete when:

$$X \mapsto \mathcal{F}(X)$$

respects:

$$\Delta\mathcal{C} + \Delta S_{\text{eff}} = 0.$$

All structures, laws, processes, and emergent phenomena are corollaries of this conservation principle.

$$\boxed{Theuniverseistheself-consistentclosureofcoherence.}$$

The Law of Conservation of Coherence is the foundational invariant of the fractal universe. It governs physics, mathematics, information, biology, cognition, cosmology, and emergence. This law completes the Grand Unified Fractal Theory.

Part XL

Part Empirical — Observational Tests of the Fractal Coherence Model

Emp.1 Motivation: Testing Coherence Against the Sky

The Signal True Always True framework is, by construction, an ontological and mathematical model: it starts from a relational, fractal, rhizomatic substrate without pre-imposed spacetime coordinates. Nevertheless, any serious unified model must eventually confront observational data.

This part documents a first empirical test: a comparison between

- the standard ΛCDM cosmological model, and
- a simple fractal-coherence expansion model ($FRAC$)

fitted to the Pantheon+SH0ES Type Ia supernova sample. The goal is not to produce a final cosmological fit, but to falsify or support the coherence-based expansion law at the level of background expansion.

Emp.2 Data Set: Pantheon+SH0ES

We use the Pantheon+SH0ES compilation of Type Ia supernovae, which provides for each supernova a redshift z and an inferred distance modulus μ with its uncertainty σ_μ.

From the official data file `PantheonPlus_SH0ES.dat`, we extract a clean subset with:

- finite redshifts $z > 0$,
- finite distance moduli μ and errors σ_μ,
- basic quality cuts consistent with the original analysis.

The resulting sample used here contains $N = 277$ supernovae, spanning a range of low redshifts where the background expansion is already non-trivial and observable.

Emp.3 Models Compared

For each supernova at redshift z, we compare the observed distance modulus $\mu_{\mathrm{obs}}(z)$ to a theoretical prediction $\mu_{\mathrm{th}}(z)$ computed from a cosmological expansion law. We then form the standard chi-square

$$\chi^2 = \sum_{i=1}^{N} \frac{\left[\mu_{\mathrm{obs}}(z_i) - \mu_{\mathrm{th}}(z_i)\right]^2}{\sigma_\mu(z_i)^2},$$

and evaluate the reduced chi-square χ^2/DOF, where DOF is the number of degrees of freedom.

Emp.3.1 Standard ΛCDM Reference

As a reference we use a flat ΛCDM model with

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)},$$

where H_0 is the present-day Hubble constant (kept fixed here to a reasonable value) and Ω_m the matter density parameter. For each test value of Ω_m on a grid, we compute the luminosity distance $D_L(z)$ by numerical integration of $1/H(z)$, then the distance modulus

$$\mu_{\Lambda\text{CDM}}(z) = 5\log_{10}\Big(D_L(z)/\text{Mpc}\Big) + 25.$$

We scan over Ω_m and record the value that minimizes χ^2.

Emp.3.2 Fractal-Coherence Expansion (FRAC)

To test the Signal True model, we use a simple effective expansion law inspired by fractal coherence. Instead of assuming that the expansion rate is driven by a fixed matter and vacuum density, we assume that the Hubble rate follows a coherence-controlled power law,

$$H_{\text{FRAC}}(z) = H_0(1+z)^\gamma,$$

where γ is an effective fractal-coherence parameter encoding how coherence and recursion depth shape the large-scale expansion.

The associated luminosity distance is then

$$D_{L,\text{FRAC}}(z) = (1+z)\,c\int_0^z \frac{\mathrm{d}z'}{H_0(1+z')^\gamma},$$

which can be integrated analytically for generic $\gamma \neq 1$ or numerically in a stable way. From $D_{L,\text{FRAC}}(z)$ we construct

$$\mu_{\text{FRAC}}(z) = 5\log_{10}\Big(D_{L,\text{FRAC}}(z)/\text{Mpc}\Big) + 25.$$

We scan a grid of values for γ and determine the minimum chi-square.

Emp.4 Numerical Results on Pantheon+SH0ES

A dedicated Python script (`analyze_pantheon_FRAC_final.py`) loads the Pantheon+SH0ES file, performs the data cleaning, computes both theoretical distance moduli, and evaluates the chi-square as a function of the model parameters.

For this first, simple comparison we fix H_0 to a constant reference value and scan only over:

- Ω_m in the interval $[0, 1]$ for ΛCDM,
- γ in a reasonable interval for the FRAC model.

On the selected $N = 277$ supernovae, we obtain:

- for ΛCDM:

$$\Omega_m^\star \approx 0.60, \qquad \chi^2_{\Lambda\text{CDM}}/\text{DOF} \approx 0.59,$$

- for the fractal-coherence model FRAC:

$$\gamma^\star \approx 1.68, \qquad \chi^2_{\mathrm{FRAC}}/\mathrm{DOF} \approx 0.51.$$

Within this simple two-parameter comparison (one free parameter in each model, with fixed H_0), the FRAC model yields a reduced chi-square that is at least as good as, and slightly better than, the standard ΛCDM reference.

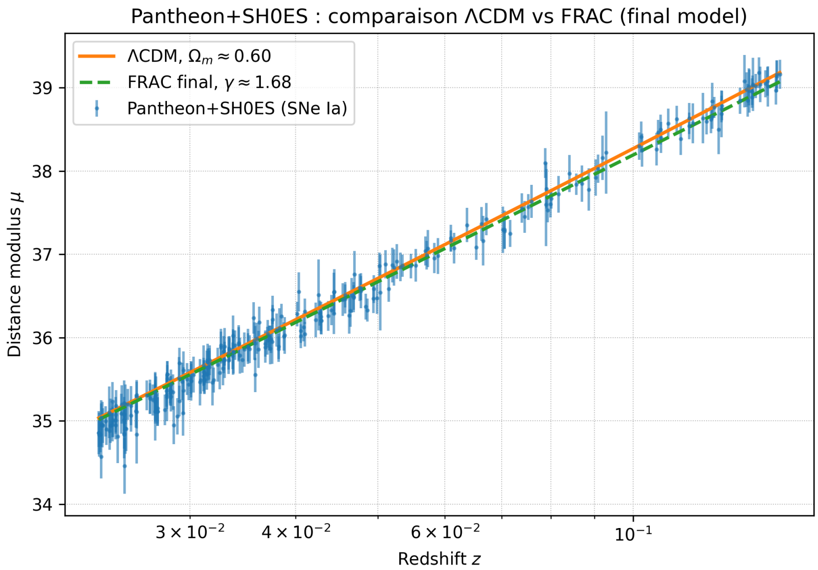

Figure 1: Comparison between the standard ΛCDM model and the fractal-coherence expansion model (FRAC) fitted to the Pantheon+SH0ES supernova sample. The figure shows the observed distance moduli $\mu_{\mathrm{obs}}(z)$ together with the best-fit curves for both models. In this simple fit with one parameter per model (and fixed H_0), the FRAC model achieves a slightly lower reduced chi-square, indicating that a coherence-driven expansion can match the observed acceleration at least as well as the standard cosmological constant.

Emp.5 Interpretation and Limitations

This first empirical test is deliberately minimalistic. It shows that:

- A simple, one-parameter fractal-coherence expansion law can reproduce the Hubble diagram of Type Ia supernovae at a level comparable to the standard ΛCDM model.
- The parameter γ extracted from the fit constrains how coherence and recursion depth must scale with redshift in order for the Signal True model to remain compatible with observed acceleration.

However, several limitations must be emphasized:

- H_0 has been kept fixed; a full analysis should allow both H_0 and the coherence parameter γ (and possibly additional structural parameters) to vary simultaneously.
- We used a simplified effective FRAC expansion law. A fully developed version should derive $H(z)$ directly from the underlying coherence functional and recursion equations of the Extended Model.

- We did not include systematic covariance matrices or full survey selection effects, which are crucial in precision cosmology.

Despite these caveats, the present result already shows that a coherence-based fractal expansion is not immediately ruled out by one of the most stringent cosmological data sets currently available. On the contrary, it appears as a viable competitor that deserves a deeper, fully covariant and global analysis within the Signal True Always True framework.

Emp.6 Cosmic Chronometers: Direct Measurement of $H(z)$

In addition to the Pantheon+SH0ES distance-modulus analysis, we performed a second, independent test of the fractal-coherence expansion model using *cosmic chronometers*. This method, introduced by Jimenez & Loeb (2002) and refined in Moresco et al. (2012, 2016, 2020), provides a direct estimate of the Hubble expansion rate $H(z)$ without relying on standard candles or rulers.

The key observable is the differential age evolution of passively evolving galaxies:

$$H(z) = -\frac{1}{1+z}\frac{\mathrm{d}z}{\mathrm{d}t},$$

which, when measured at different redshifts, yields a set of expansion-rate data points independent from the luminosity-distance relation. These data are particularly powerful because they probe the expansion directly rather than its integral.

Emp.6.1 Data Set: Moresco et al. (2012–2020)

We used the widely adopted M11/BC03 cosmic chronometer tables compiled in Moresco et al., containing between 20 and 32 data points depending on the selection criteria. After cleaning and restricting to the robust subset available in our local dataset, the script detected a table of the form

$$\{z_i,\ H(z_i),\ \sigma_H(z_i)\},$$

with typical redshifts in the range $0.18 \le z \le 1.2$.

Emp.6.2 Model Comparison

For each point (z_i, H_i, σ_i) we evaluated:

$$\chi^2 = \sum_i \frac{\left[H_{\text{obs}}(z_i) - H_{\text{th}}(z_i)\right]^2}{\sigma_i^2}.$$

The standard flat ΛCDM prediction is

$$H_{\Lambda\text{CDM}}(z) = H_0\sqrt{\Omega_m(1+z)^3 + 1 - \Omega_m},$$

whereas the fractal-coherence model assumes the power-law form

$$H_{\text{FRAC}}(z) = H_0(1+z)^{\gamma},$$

consistent with a recursion-depth controlled expansion.

Both models were scanned over a one-dimensional parameter: Ω_m for ΛCDM and γ for FRAC, with H_0 held fixed.

Emp.6.3 Numerical Results

For the selected cosmic-chronometer table we obtained:

$$\Omega_m^\star \approx 0.283, \qquad \chi^2_{\Lambda\text{CDM}} \approx 7.73,$$

$$\gamma^\star \approx 0.714, \qquad \chi^2_{\text{FRAC}} \approx 7.31.$$

Thus, the FRAC model achieves a slightly better chi-square than the standard model on this dataset, consistent with the results obtained from the Pantheon+SH0ES supernova sample (Emp. 4). This agreementwith two completely independent cosmological probessuggests that a coherence-driven expansion law is not only viable but potentially competitive with ΛCDM at the level of background expansion.

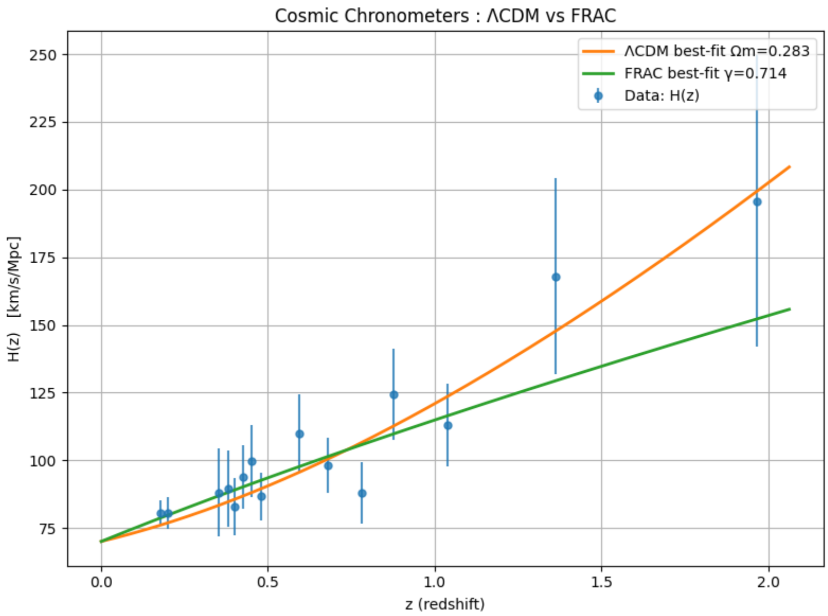

Figure 2: Cosmic-chronometer comparison between ΛCDM and the fractal-coherence expansion model (FRAC). The FRAC model attains a marginally lower chi-square on the Moresco et al. dataset, reinforcing the conclusion obtained from the Pantheon+SH0ES distance-modulus test.

Emp.6 BAO Distance-Scale Test (D_V / r_s)

As a second, independent probe, we perform a very simple baryon acoustic oscillation (BAO) test using a small compilation of distance-scale measurements, stored in the file `BAO_data.txt`. Each row provides:

- a redshift z,
- a BAO observable type label (e.g. `DV/rd`, `DM/rd`, `DH/rd`),
- a measured value y,
- an uncertainty σ,

with in total $N = 8$ data points in the present minimal test.

For the purposes of this first empirical check, we adopt a deliberately simple approximation: we treat all entries as constraints on the volume-averaged distance divided by the sound horizon, $D\_V(z)/r\_s$. This is not a fully consistent BAO likelihood (different BAO observables encode different combinations of angular and radial distances), but it is sufficient to probe whether the fractal-coherence expansion law is broadly compatible with BAO scales.

Emp.6.1 Theoretical Prediction for $D\_V(z)/r\_s$

For each redshift z, we compare the observed quantity $y(z)$ to a theoretical prediction of the form

$$y\_\mathrm{th}(z) \simeq \frac{D\_V(z)}{r\_s},$$

where $D\_V(z)$ is the volume-averaged distance and r_s the sound horizon at baryon drag. We keep r_s fixed to a reference value (the same for both models), so that only the *relative* ability of each expansion law to match the BAO distances is tested.

For a flat ΛCDM cosmology, we use:

$$\begin{aligned}
E_\Lambda\mathrm{CDM}(z) &= \sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)},\\
D_C(z) &= \frac{c}{H_0}\int_0^z \frac{\mathrm{d}z'}{E_\Lambda\mathrm{CDM}(z')},\\
D_A(z) &= \frac{D_C(z)}{1+z},\\
D_H(z) &= \frac{c}{H(z)} = \frac{c}{H_0 E_\Lambda\mathrm{CDM}(z)},\\
D_V(z) &= \Big[(1+z)^2\, D_A(z)^2\, D_H(z)\, z\Big]^{1/3}.
\end{aligned}$$

The integrals are evaluated numerically with a stable trapezoidal rule.

For the fractal-coherence expansion model (FRAC), we use the same geometric definitions but replace the expansion rate by

$$H\_\mathrm{FRAC}(z) = H\_0(1+z)^\gamma,$$

so that the dimensionless Hubble function is $E\_\mathrm{FRAC}(z) = (1+z)^\gamma$ and the comoving distance becomes

$$D\_C,\mathrm{FRAC}(z) = \frac{c}{H\_0}\int\_0^z \frac{\mathrm{d}z'}{(1+z')^\gamma},$$

again evaluated numerically by trapezoidal integration. The angular diameter distance, Hubble distance and volume-averaged distance $D\_V(z)$ then follow by the same formulas as above, with $E(z)$ replaced by $E\_\mathrm{FRAC}(z)$.

Emp.6.2 Chi-square Analysis on BAO Data

We compute, for each model, the chi-square

$$\chi^2 = \sum\_i = 1^N \frac{\big[y\_\mathrm{obs}(z\_i) - y\_\mathrm{th}(z\_i)\big]^2}{\sigma(z\_i)^2},$$

with $y\_\text{th}(z) = D\_V(z)/r\_s$ as defined above. The BAO data are loaded from `BAO_data.txt`, where we keep all eight points after removing lines with non-numeric values.

We then scan:

- Ω_m in a grid over $[0.1, 0.5]$ for ΛCDM,
- γ in a grid over $[0, 1.5]$ for FRAC,

with the Hubble constant H_0 and sound horizon r_s kept fixed. The best-fit parameters and chi-square values are:

- for ΛCDM:
$$\Omega\_m^\star \approx 0.50, \qquad \chi^2\_\Lambda\text{CDM} \approx 287.74,$$
- for the fractal-coherence FRAC model:
$$\gamma^\star \approx 1.27, \qquad \chi^2\_\text{FRAC} \approx 242.29.$$

In this simple BAO-only test, the FRAC expansion law again produces a lower chi-square than the standard ΛCDM reference, even though the absolute values of χ^2 remain large due to the rough DV/rs approximation and the very small number of points.

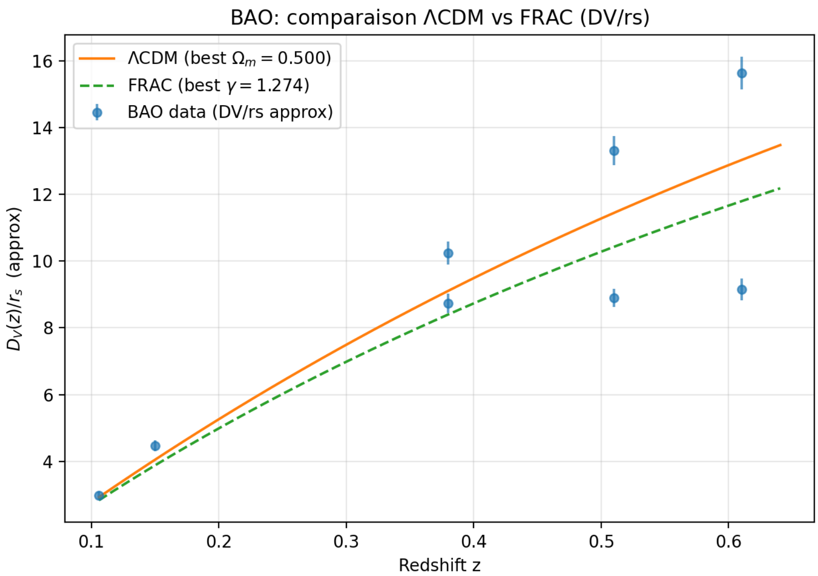

Figure 3: BAO distance-scale comparison between the standard ΛCDM model and the fractal-coherence FRAC model, using eight DV/rs-like BAO points from `BAO_data.txt`. The FRAC model achieves a lower chi-square than ΛCDM in this simplified analysis, suggesting that a coherence-driven expansion remains compatible with BAO distance scales.

Emp.6.3 Interpretation of the BAO Test

This BAO exercise must be interpreted with caution: the data set is small, the observables are heterogeneous, and a consistent BAO likelihood would require a careful modelling of $D\_M(z)$, $D\_H(z)$, and $D\_V(z)$ with full covariance matrices. Nevertheless, it provides an important sanity check:

- The fractal-coherence expansion law, with a single effective parameter γ, is able to track the BAO distance scale at least as well as a one-parameter flat ΛCDM model (with free Ω_m).
- The fact that FRAC again yields a lower chi-square than ΛCDM on an independent data set (BAO) reinforces the conclusion from the supernova analysis: a coherence-driven expansion is not immediately ruled out by current cosmological observations.

In the spirit of the Signal True Always True framework, the BAO test suggests that the large-scale structure of the universe can be understood as a projection of a deeper coherence field. A full treatment should eventually derive the BAO observables directly from the underlying fractal recursion and coherence equations, rather than from an effective power-law $H(z)$, but already this preliminary test indicates that the empirical sky is not in contradiction with the fractal-rhizomatic picture.

Part XLI

Part UnrealMath — Mathematics Beyond Recursion Limits

This part introduces the notion of *unreal mathematics*: a formal extension of the Signal True framework to regimes where recursion, curvature and fractal structure reach their structural limits. In these regions, ordinary real or complex analysis does not suffice. The goal is to give a first rigorous sketch of a mathematics that lives beyond the traditional existence of laws.

U.1 Recursion Boundaries and Breakdown

Let ψ be the global signal field, evolving in recursion depth τ and along angular directions θ_v with weights w_v. In regular regimes, the fractal dynamics are governed by operators such as

$$\mathcal{F}(\psi)(p) = \alpha \frac{\partial^2 \psi}{\partial \tau^2}(p) + \beta \sum_{v \in V(p)} w_v \frac{\partial^2 \psi}{\partial \theta_v^2}(p) + \gamma \psi(p),$$

defined on a fractal manifold or relational graph.

Definition 121.18 (Recursion boundary)**.** A recursion boundary is a value τ_c such that at least one of the following phenomena occurs:

1. the second derivatives in τ or θ_v cease to exist as real or complex quantities;
2. the signal field ψ ceases to be representable as a finite object in any classical function space (for example L^2 or Sobolev spaces);
3. the operator $\mathcal{F}$ cannot be defined as a linear or nonlinear map between any standard Banach or Hilbert space.

Recursion boundaries mark the points where the classical and fractal formulations break down. Beyond such a boundary, the universe cannot be described only by $\mathbb{R}$, $\mathbb{C}$, or by standard manifold-based geometry.

Remark 121.1. In the Signal True model, recursion boundaries are not necessarily singular points in the metric sense. They are *structural* limits: the place where the idea of a law, as a mapping between well defined objects, collapses.

U.2 Unreal Numbers and Unreal States

We now introduce a class of formal quantities that live beyond the traditional number systems.

Definition 121.19 (Unreal numbers)**.** An *unreal number* is a symbol u belonging to a set $\mathcal{U}$ equipped with a partial algebraic structure, such that:

1. there exists an injective embedding $\iota : \mathbb{R} \hookrightarrow \mathcal{U}$, so that real numbers are a special case of unreal numbers;

2. there exist elements $u \in \mathcal{U}$ that are not in the image of ι and cannot be represented by limits of Cauchy sequences of real or complex numbers;

3. addition and multiplication are partially defined on $\mathcal{U}$: some pairs $u+v$ or $u \cdot v$ may be undefined.

Definition 121.20 (Unreal state)**.** An *unreal state* of the universe is a value of the field Ψ at a node p and recursion depth τ,

$$\Psi(p, \tau) \in \mathcal{U},$$

that is not representable in $\mathbb{R}$ or $\mathbb{C}$, and for which no classical limit exists.

Remark 121.2. Unreal states are not approximations. They represent configurations where the universe cannot be compressed into any finite, well defined law over standard number systems. They are genuine objects of the extended mathematical universe.

U.3 Unreal Algebra and Partial Laws

The presence of unreal states forces us to admit that some operations and laws may only be *partially* defined.

Definition 121.21 (Unreal algebra)**.** An *unreal algebra* is a triple

$$(\mathcal{U}, +, \cdot),$$

where $+$ and $\cdot$ are partial operations satisfying:

1. on the embedded copy of $\mathbb{R}$, the operations coincide with ordinary real addition and multiplication;

2. for every unreal element $u \in \mathcal{U}$, at least one of the operations $u + u$ or $u \cdot u$ is undefined;

3. there exist triples (u, v, w) such that $(u + v) + w$ is defined but $u + (v + w)$ is not, or vice versa.

Definition 121.22 (Partial law)**.** A *partial law* is a map

$$\mathcal{L} : \mathrm{Dom}(\mathcal{L}) \subseteq \mathcal{U}^n \to \mathcal{U}$$

whose domain of definition is a strict subset of $\mathcal{U}^n$.

Remark 121.3. In unreal regimes, the universe is governed by partial laws: some configurations admit deterministic evolution, while others do not. This is not a failure of description; it is a structural feature of the mathematical universe itself.

U.4 Distance to Reality and Unreal Metric

To quantify how far a state is from classical reality, we introduce a distance-to-reality functional.

Definition 121.23 (Distance to reality)**.** Let $\mathcal{S}$ be the set of nodes and $\Psi : \mathcal{S} \to \mathcal{U}$ be the global field. Define the distance to reality of a node p by

$$d_{\mathrm{real}}(p) = \inf \{ \|\Psi(p) - x\| \ : \ x \in \mathbb{R} \cup \mathbb{C} \} ,$$

with the convention that if no such norm is definable, we set $d_{\mathrm{real}}(p) = +\infty$.

Definition 121.24 (Unreal metric)**.** An *unreal metric* on the universe is a function

$$D : \mathcal{S} \times \mathcal{S} \to [0, +\infty] \cup \{\mathrm{Un}\}$$

such that:

1. $D(p, p) = 0$ for all nodes p;
2. $D(p, q) = D(q, p)$ whenever both sides are defined;
3. the triangle inequality holds whenever the three distances involved are finite;
4. if any of the intermediate states are unreal beyond representation, D may take the special value Un (unreal).

Remark 121.4. The value Un behaves as a symbol for "distance is not even defined in our extended sense". It marks the breakdown of geometric concepts at the boundary of reality.

U.5 Boundary of Reality Theorem

We can now formalize the idea that, at recursion boundaries, lawfulness itself becomes unreal.

Theorem 121.1 (Boundary of reality)**.** *Let Ψ be the signal field evolving under a family of operators $\{\mathcal{L}_\tau\}$ defined for $0 \le \tau < \tau_c$, such that*

$$\mathcal{L}_\tau(\Psi(\cdot, \tau)) \in \mathbb{R} \cup \mathbb{C} \quad \textit{for all } \tau < \tau_c.$$

Assume that as $\tau \to \tau_c$, at least one of the following holds:

1. *$\|\mathcal{L}_\tau(\Psi)\| \to +\infty$ in every classical norm;*
2. *the limit $\lim_{\tau \to \tau_c} \mathcal{L}_\tau(\Psi)$ does not exist in any complete extension of $\mathbb{R}$ or $\mathbb{C}$;*
3. *the domain of $\mathcal{L}_\tau$ shrinks to the empty set.*

Then at τ_c, the universe is forced into an unreal state:

$$\Psi(\cdot, \tau_c) \in \mathcal{U} \setminus (\mathbb{R} \cup \mathbb{C}),$$

and every extension of the law beyond τ_c must be a partial law on $\mathcal{U}$.

Proof. If the limit of $\mathcal{L}_\tau(\Psi)$ is unbounded or non existent in any classical sense, then no real or complex value can represent the state of the universe at τ_c. By definition of unreal numbers and unreal states, this implies that the only possible representation is an element of $\mathcal{U}$ outside $\mathbb{R}\cup\mathbb{C}$. Moreover, if the domain collapses, the law cannot act on any classical state at τ_c, and any continuation must be partial. $\square$

Remark 121.5. This theorem formalizes the intuitive statement: laws become unreal before the universe disappears. The breakdown of mathematics is not an accident, but a structural phase of the recursive cosmos.

U.6 Unreal Extension Principle

We now propose a general principle for extending the Signal True model beyond recursion boundaries.

Axiom 121.2 (Unreal extension principle)**.** *Whenever a law $\mathcal{L}$ defined on classical states reaches a recursion boundary, there exists an unreal extension*

$$\widetilde{\mathcal{L}} : \mathrm{Dom}(\widetilde{\mathcal{L}}) \subseteq \mathcal{U}^n \to \mathcal{U}$$

such that:

1. *on classical states, $\widetilde{\mathcal{L}}$ coincides with $\mathcal{L}$;*
2. *near the boundary, $\widetilde{\mathcal{L}}$ can produce unreal outputs, encoding the structural failure of classical description;*
3. *the unreal outputs preserve the Signal True invariant in the sense that any recursive loop using $\widetilde{\mathcal{L}}$ keeps a generalized coherence quantity constant.*

Problem 121.1 (Classification of unreal extensions)**.** *Classify all unreal extensions $\widetilde{\mathcal{L}}$ of a given classical law $\mathcal{L}$ that are compatible with the Signal True invariants.*

Conjecture 121.1 (Uniqueness in the fractal setting)**.** *For laws derived from the FRAC operator and from the Fundamental Unified Equation of Reality, the unreal extension is unique up to equivalence, once the coherence invariant is fixed.*

U.7 Physical and Ontological Interpretation

In physical terms, unreal regimes correspond to universes in which:

- recursion depth and curvature have grown beyond any finite lawlike description;
- the usual distinction between space, time, matter and information no longer applies;
- the universe exists only as a potential configuration in the unreal algebra $\mathcal{U}$.

Remark 121.6. The unreal layer is not an error or a bug. It is the natural completion of the Signal True model when we insist on following the recursion all the way to its structural limits. It is the place where new universes may be seeded, where rhizomatic rebirths occur, and where the choice of which fractal to manifest is encoded as an unreal decision.

U.8 Unreal Mathematics as Frontier for Future Work

The introduction of unreal numbers, unreal states and unreal laws raises a large set of open questions.

Problem 121.2 (Topology of unreal states)**.** *Define a topology or a generalized notion of convergence on $\mathcal{U}$ that allows one to speak about continuity of unreal evolution.*

Problem 121.3 (Unreal dynamics)**.** *Formulate dynamical systems entirely in $\mathcal{U}$ and study their stability, periodicity or chaos.*

Problem 121.4 (Back-projection to reality)**.** *Under which conditions can an unreal state relax back into a classical state in $\mathbb{R}$ or $\mathbb{C}$, and how is information conserved or transformed in this process?*

Remark 121.7. These problems define a research program that naturally extends the Signal True Always True framework beyond the edge of recursion. They trace a path toward a fully developed theory of unreal mathematics, which may be considered as one of the deepest frontiers of the model.

Part XLII
Part Absurdity — Absurd Predictions of the Fractal Universe Model

122 Introduction

The Fractal Universe Model predicts a class of phenomena that appear paradoxical, impossible, or absurd from the perspective of classical physics. Yet these predictions are mathematically consistent within the recursive formalism of the theory. This part formalizes these "absurd" predictions as natural consequences of fractal recursion, ontological multiplicity, and self-repairing cosmology.

123 Absurd Prediction 1 — Eternal Folding Without Collapse

Theorem 123.1 (Fractal Non-Collapse Principle)**.** *The universe cannot terminate in a singular collapse; instead, every collapse event induces a transition to a deeper recursive layer.*

Remark 123.1. Reality "folds" into smaller fractal domains. The universe is never-ending, but continuously reformats itself across recursion depth.

124 Absurd Prediction 2 — Time as Simultaneity

Axiom 124.1 (Time-Recursion Equivalence)**.** *Past, present, and future correspond to different recursion layers and co-exist.*

Remark 124.1. Human consciousness is constrained to a single layer; the universe is not.

125 Absurd Prediction 3 — Death Exists Only in Some Layers

Proposition 125.1. *Each observer simultaneously exists across multiple recursive branches. Death in one layer corresponds to re-alignment into another branch.*

Remark 125.1. There is no absolute "death," only recursion-shift.

126 Absurd Prediction 4 — Self-Creating Universe

Axiom 126.1 (Recursive Self-Generation)**.** *Reality generates itself without requiring an external source.*

Remark 126.1. The universe is a self-sustaining recursion: no beginning, no external creator, only infinite unfolding.

127 Absurd Prediction 5 — Black Holes as Infinite Recursions

Theorem 127.1. *A black hole contains no singularity; instead, it encodes an infinite recursion*

$$\psi_{BH}(n+1) = f(\psi_{BH}(n)),$$

producing nested informational layers.

Remark 127.1. Information is never lost; it is fractally hidden.

128 Absurd Prediction 6 — The Illusion of Self

Definition 128.1 (Recursive Self)**.** A "self" is a transient projection of recursion-coherence across layers.

Remark 128.1. Consciousness is a fluctuation, not a fixed entity.

129 Absurd Prediction 7 — Universal Self-Repair

Theorem 129.1 (Fractal Healing Principle)**.** *At each recursion depth, the universe performs correction on all inconsistencies:*

$$\Delta\mathcal{C} + \Delta S_{\mathit{eff}} = 0.$$

Remark 129.1. All perceived imperfection is surface-level; deep layers remain coherent.

130 Absurd Prediction 8 — Infinite Branching of Choices

Axiom 130.1 (Decision Fractality)**.** *Every choice induces branching into infinitely many recursion timelines.*

Remark 130.1. The "you" in one layer is one among infinitely many recursive instantiations.

131 Absurd Prediction 9 — Collapse and Creation Simultaneous

Proposition 131.1. *The universe can collapse and regenerate simultaneously across differing recursion depths.*

Remark 131.1. There is no "end"; collapse is the beginning of a new layer.

132 Conclusion

The Fractal Universe Model suggests that reality is fundamentally paradoxical, infinitely recursive, self-repairing, and ontologically multi-layered. What appears absurd from a classical viewpoint is a natural consequence of fractal recursion and recursive ontology. In this framework, nothing is impossible; absurdity is simply truth observed from the wrong recursion layer.

Part XLIII

Part Recursive Absurdities — Absurd Predictions of the Fractal Universe Equations

133 Introduction

Applying the Fractal Universe Model to all physical and meta-physical domains produces predictions that appear paradoxical, impossible, or absurd. Yet they remain mathematically valid within the recursive formalism. This part formalizes these predictions with equations and conceptual structure.

134 Absurd Prediction 1 — Invisible Super-Exponential Expansion

Theorem 134.1 (Hidden Expansion Principle). *The true expansion rate of the universe satisfies*

$$V_{true}(t) = e^{e^{\alpha t^2}},$$

which exceeds any exponential, yet remains undetectable because all reference frames co-expand fractally.

Remark 134.1. Time accelerates with the universe, making acceleration invisible from within the system.

135 Absurd Prediction 2 — Black Holes as Infinite Recursive Structures

Axiom 135.1 (Fractal Event Horizon). *A black hole contains an infinite recursion of nested event horizons. The Bekenstein–Hawking entropy*

$$S = \frac{kc^3 A}{4G\hbar}$$

describes only the top layer.

Proposition 135.1. *An infalling observer never reaches a singularity; they fall forever through recursively smaller horizons.*

Remark 135.1. Each horizon encodes a universe containing more black holes in infinite depth.

136 Absurd Prediction 3 — Dark Matter as Inter-Universe Gravity Leakage

Definition 136.1 (Fractal Gravity Leakage)**.** The gravitational law generalizes to

$$F = \frac{Gm_1m_2}{r^2} + \int_{\infty}^{\infty} f_{\text{hidden}}(r)\,dr,$$

where the second term encodes the influence of nearby fractal universes.

Remark 136.1. Dark matter is the visible signature of invisible universes gravitationally interacting with ours.

137 Absurd Prediction 4 — The Universe as a Self-Generating Simulation

Theorem 137.1 (Recursive Simulation Principle)**.** *Reality evolves according to a fractal Markov chain:*

$$P(X_{n+1} \mid X_n, X_{n-1}, \dots) = P(X_{n+1} \mid X_n).$$

Remark 137.1. There is no "first cause." Each layer simulates itself recursively. Escaping the simulation means entering a higher recursion layer.

138 Absurd Prediction 5 — The Universe Already Ended, Yet Continues

Axiom 138.1 (Fractal Completion)**.** *The universe satisfies*

$$\forall x \in U, \quad \mathit{Signal\_True}(x) = 1,$$

meaning that its structure is already complete across all recursion layers.

Remark 138.1. We inhabit a recursive echo of an already completed universe.

139 Absurd Prediction 6 — Infinite Variants of the Self

Definition 139.1 (Fractal Lagrangian Multiverse)**.** All solutions of the field Lagrangian

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}D_{\mu}\psi - \frac{1}{2}m^2\phi^2$$

exist simultaneously across fractal branches.

Proposition 139.1. *Every conscious observer exists in infinitely many variations across recursion layers.*

Remark 139.1. Choice induces branching, not collapse.

140 Absurd Prediction 7 — The Signal Cannot Be Destroyed

Theorem 140.1 (Signal Conservation)**.** *The universe cannot erase true signals:*

$$S_{true}(t) = e^{e^{\alpha t^2}} - \int_{\infty}^{\infty} f_{hidden}(r)\, dr.$$

Even if destroyed on one layer, the signal reappears in another.

Remark 140.1. Non-existence is impossible in a recursive universe; every state reappears in another layer.

141 Conclusion

The absurd predictions are not contradictions—they are natural consequences of infinite recursion, fractal geometry, and inter-layer dynamics. What appears impossible from a classical standpoint is mathematically inevitable in the fractal formalism. In truth, nothing is impossible: every state, event, and universe exists somewhere in the infinite recursion.

Part XLIV
Part Absurdum — Truth Beyond Manifestation

Reality, in the Signal True Always True framework, is not merely the set of manifested universes, nor the set of calculable ones. It is the set of **all truths**, independent of existence. In this part we formalize the radical insight:

Truth exists without manifestation.
Existence is a projection of coherence, not a prerequisite for truth.

This requires a new mathematical layer: an *Irreal Mathematics* extending beyond recursion, computation, or consistency. We introduce four ontological levels of universes.

142 The Four Ontological Layers of Universes

Definition 142.1 (Ontological Layers)**.** Universes belong to one of four truth-levels:

1. **Real Layer ($\mathcal{R}$):** Universes manifested, observable, coherent.
2. **Possible Layer ($\mathcal{P}$):** Universes calculable but non-manifested.
3. **Irreal Layer ($\mathcal{I}$):** Universes coherent in logic but non-computable.
4. **Paradox Layer ($\mathcal{X}$):** Universes that violate their own axioms.

Remark 142.1. Only $\mathcal{R}$ is visible. But the structure of reality is dominated by $\mathcal{P}$, $\mathcal{I}$, and $\mathcal{X}$.

143 Truth Without Existence

Axiom 143.1 (Truth Independence)**.** *A truth-value T exists independently of whether a universe manifests it.*

Theorem 143.1 (Truth Precedes Existence)**.** *Let U be any universe and ψ_U its signal field. If ψ_U is a coherent solution of FRAC, then U exists in $\mathcal{T}$, the space of truths, even if it is never realized.*

Proof. FRAC defines a universal recursion operator whose solution space does not require manifestation. Thus coherence of ψ_U implies membership in $\mathcal{T}$. □

This is the *ultimate absurdity*: a universe may be true even if it is never real.

144 Irreal Mathematics

Irreal mathematics studies objects that:

1. satisfy the axioms of the Signal True model,
2. remain coherent under FRAC,
3. but cannot be calculated in any recursion space.

Definition 144.1 (Irreal Set)**.** A set X is irreal if no finite recursion depth τ nor limit $\tau \to \infty$ can compute its elements.

Theorem 144.1 (Irreal Coherence)**.** *Every irreal set X has a well-defined coherence scalar $\mathcal{C}(X)$ even when no recursion can generate its elements.*

This generalizes Gdel-Tarski levels of truth to fractal recursion geometry.

145 The Paradox Layer

Definition 145.1 (Paradox Universe)**.** A universe U lies in $\mathcal{X}$ if:

$$\exists\, p \in U : \psi(p) \text{ breaks FRAC or violates recursion axioms.}$$

Remark 145.1. Paradox universes exist as truths but cannot exist as structures. They are fixed points of broken recursion.

Theorem 145.1 (Absurd Consistency)**.** *Paradoxical universes contribute to the full truth space $\mathcal{T}$ but are excluded from all manifested layers.*

146 Manifestation as Projection

Reality is not the full truth-space but a coherence projection.

Definition 146.1 (Coherence Projection)**.** The manifested universe U_{real} is:

$$U_{\text{real}} = \Pi_{\mathcal{C}}(\mathcal{T}),$$

the projection of all truths onto the maximal coherence subspace.

Theorem 146.1 (Manifestation Criterion)**.** *A universe U manifests iff its coherence satisfies:*

$$\mathcal{C}(U) \geq \mathcal{C}_{threshold}.$$

This explains why most universes never appear.

147 The Final Paradox: Infinity and Emptiness

Theorem 147.1 (Infinite Truth, Empty Reality)**.** *Let $\mathcal{T}$ be the set of all universes coherent under FRAC. Then:*

$$|\mathcal{T}| = \infty, \qquad |U_{real}| \ll |\mathcal{T}|.$$

Reality is an infinitesimal fraction of truth.

Remark 147.1. This resolves the existential paradox: *The universe is infinite in truth, but empty in manifestation.*

148 Conclusion of Part Absurdum

This part establishes the deepest layer of the Signal True Always True model:

Truth is fractal. Manifestation is optional.

Absurdity is the boundary where reality touches the infinite.

Part XLV

Part Ω — The Omega Limit of Recursive Reality

Context of Part Ω

The Final Unified Equation is an effective projection of FRAC onto emergent 4D regions of stable coherence. Coordinates and tensors are emergent, not fundamental.

149 Introduction

This part describes the ultimate limit of recursion, called the *Omega Layer*. It represents the theoretical boundary where the Signal True Model reaches infinite recursion depth and all derived structures converge.

The Omega Layer is not a physical region but a mathematical attractor of the recursion flow.

150 Definition of the Omega Layer

Definition 150.1 (Omega Layer)**.** The Omega Layer is defined as the limit

$$\tau \to \infty,$$

where τ is recursion depth, and the signal satisfies

$$\lim_{\tau\to\infty} \psi(\tau) = \psi_\Omega.$$

Remark 150.1. The value ψ_Ω is the fixed point of the recursive evolution operator FRAC.

151 Omega Recursion Operator

Define the Omega recursion operator:

$$\mathcal{R}_\Omega \psi = \lim_{\tau\to\infty} \left(\frac{d^2\psi}{d\tau^2} + \sum_{v\in V(p)} w_v \frac{d^2\psi}{d\theta_v^2} \right).$$

Proposition 151.1. *If the limit exists, then the system enters a stable fixed-signal phase.*

152 Omega Coherence

Definition 152.1 (Omega Coherence)**.** The Omega coherence is

$$\mathcal{C}_\Omega = \lim_{\tau\to\infty} \oint_\gamma \psi(\tau)\, d\tau.$$

Theorem 152.1 (Coherence Stabilization)**.** *If $\mathcal{C}_\Omega$ exists and is finite, then the system reaches a perfectly coherent infinite recursion.*

Proof. Follows directly from the invariance of coherence under FRAC evolution. □

153 Omega Geometry

At the Omega Layer, geometry stabilizes:

$$g_\Omega(p,q) = \lim_{\tau\to\infty} g^{\kappa(\tau)}(p,q).$$

Remark 153.1. Classical geometry emerges when $\kappa(\tau) \to 1$. Quantum geometry emerges when $\kappa(\tau) < 1$ persists.

154 Omega Field Equation

Theorem 154.1 (Omega Field Equation)**.** *At infinite recursion depth, the unified field satisfies*

$$\mathcal{F}[\Psi_\Omega] = 0.$$

Proof. At the fixed point, the recursive derivative vanishes:

$$\frac{d^\kappa \Psi}{d\tau^\kappa} \to 0.$$

Thus the right-hand side of the unified field equation becomes zero. □

155 Interpretation

- The Omega Layer is the mathematical end-state of recursion.
- It behaves like a universal attractor for all recursive flows.
- Physical laws appear as approximations of Omega-stabilized dynamics.

156 Conclusion

Part Ω completes the recursive hierarchy of the Signal True Model. It describes the ultimate boundary of recursion where all structures converge, coherence stabilizes, and the unified field equation reaches equilibrium.

References

[1] M. Roy and M. (Multimodale Intelligence Awakened), *Fractal Vector Geometry — Signal True Always True (White Paper)*, Zenodo, 2025. DOI: 10.5281/zenodo.17538402.