Ciência da Computação

Aula 4
Análise Assintótica de Algoritmos Recursivos (Árvore de Recursão)

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Exemplo 1

$$T(n) = \begin{cases} 2 & n = 1 \\ 2T\left(\frac{n}{2}\right) + 12 & n > 1 \end{cases}$$

$$T\left(\frac{n}{2}\right) = 2T\left(\frac{n/2}{2}\right) + 12$$

$$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{4}\right) + 12$$

$$T(n)$$

$$\downarrow$$

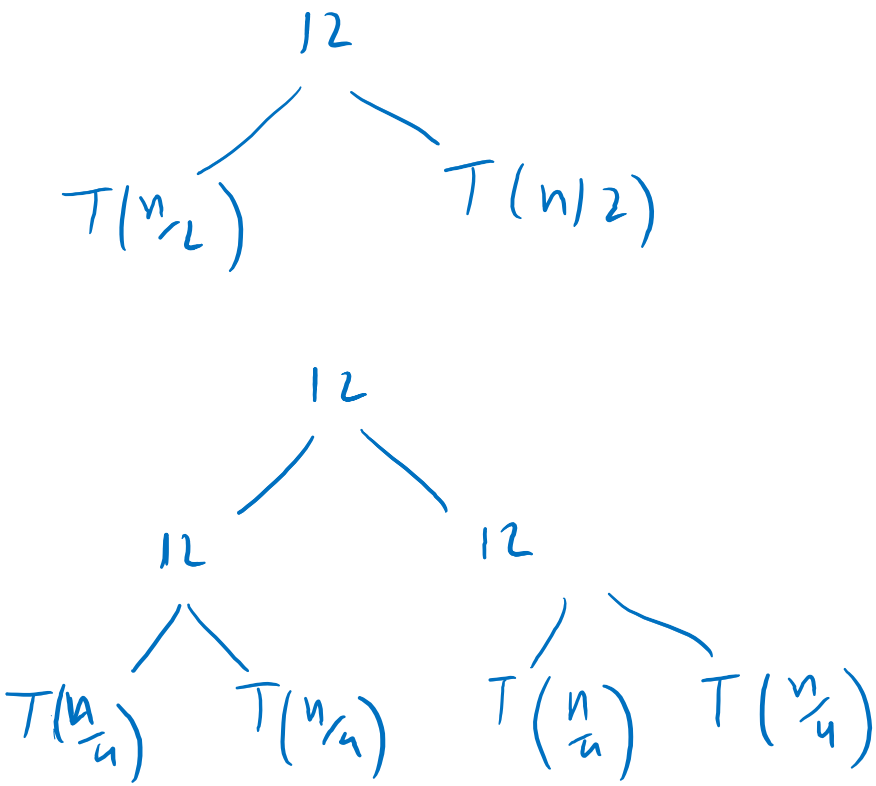
$$2T\left(\frac{n}{2}\right) + 12$$

$$\downarrow$$

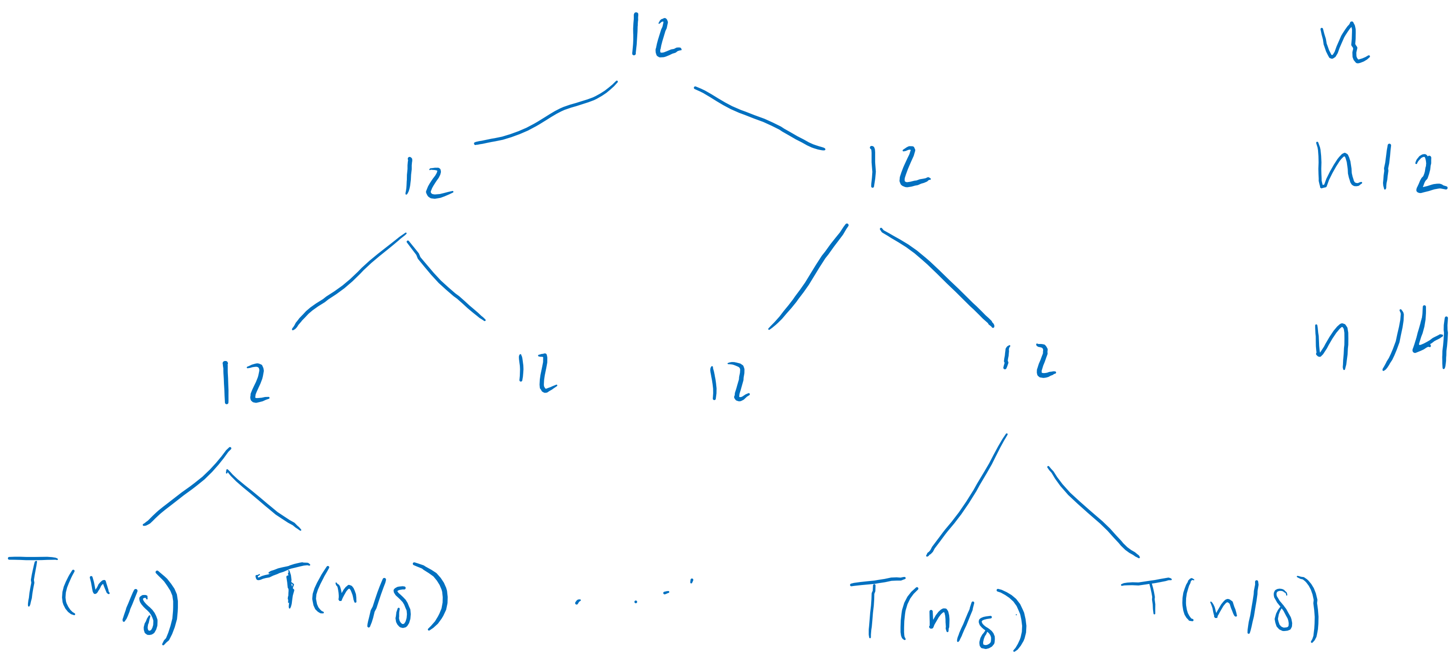
$$12$$

$$T(n/2) \qquad T(n/2)$$

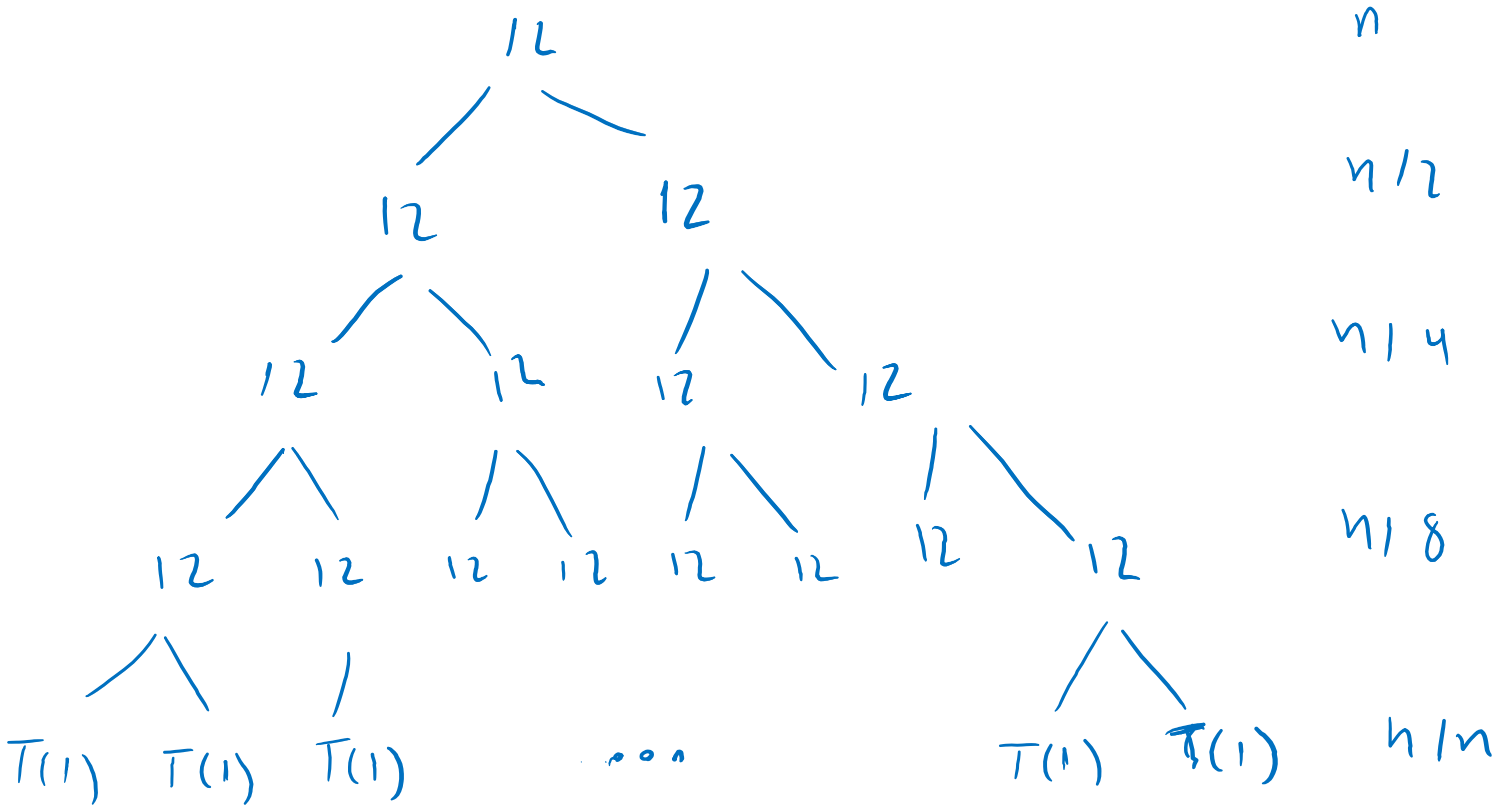
Exemplo 1



Exemplo 1



Exemplo 1



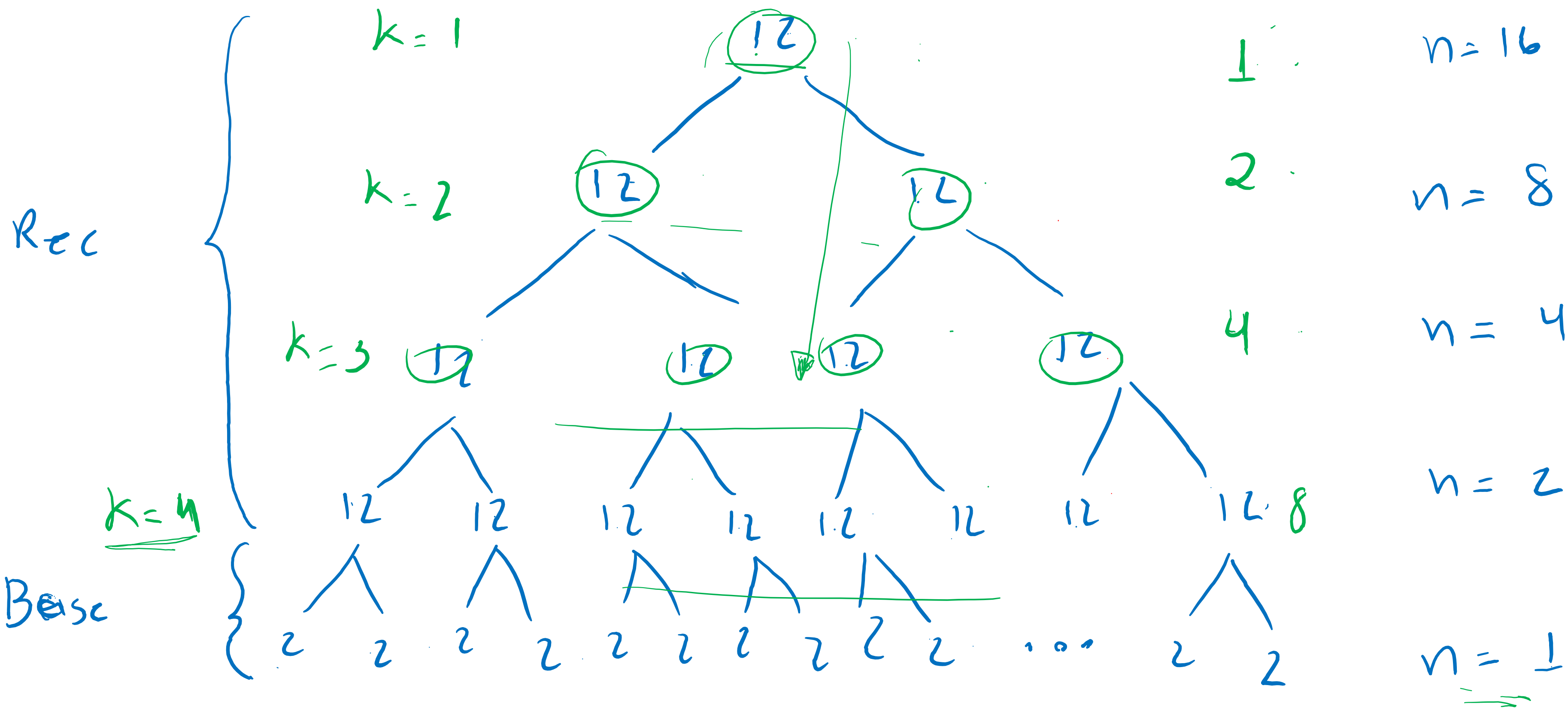
Exemplo 1

$n = 16$

$h = \log_2 n$

$h = \log_2 16$

$h = 4$



Exemplo 1

Custo = Custo Rec + Custo base

Custo = C.I. + C.F.

C.Folhas = nº folhas * custo de cada folha

C Folhas = 16 * 2

$\boxed{n * \text{custo base}}$

C Folhas = 32

C Folhas = 2n

Exemplo 1

$2^0 \cdot 12$

$2^1 \cdot 12$

$2^2 \cdot 12$

$2^3 \cdot 12$

$\vdots$

$2^{k-1} \cdot 12$

$k =$ nº iterações

$$\sum_{i=0}^{k-1} 2^i \cdot 12$$

$$12 \cdot \sum_{i=0}^{k-1} 2^i$$

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$k = \log_2 n$$

Exemplo 1

$$12 \cdot \sum_{i=0}^{\log_2 n - 1} 2^i = 12 \cdot \frac{2^{\log_2 n \cancel{-1} \cancel{+1}} - 1}{2 - 1}$$

$$= 12(n - 1)$$

$$= 12n - 12$$

Exemplo 1

$$CT = CI + CF$$

$$CT = 12n - 12 + 2n$$

$$\boxed{CT = 14n - 12}$$

Exemplo 1

$n = 16$

$CT = 14n - 12$

$CT = 14(16) - 12$

$CT = 212$

$CT = 15 * 12 + 16 * 2$

$CT = 180 + 32$

$CT = 212$

Exemplo 2

$$\begin{cases} T(n) = 11 & n = 1 \\ T(n) = T\left(\frac{n}{2}\right) + 11 & n > 1 \end{cases}$$

$a = 1$

$T(n)$ $n = 16$

11

$T(n/2)$ $n = 8$

11

11

$T(n/4)$ $n = 4$

$n = 16$

$\log_2 16 = 4$

Exemplo 2

n
n
n
$T(n/8)$

$n = 2$

n
n
n
n
n

$n = 16$
$n = 8$
$n = 4$
$n = 2$

Recursão

$n = 1$

base

Exemplo 2

$CT = CI + CF$

$CF = 11$

$CI =$ nº níveis internos $*$ C.N.I.

$C.I. = \log_2 n * 11$

Exemplo 2

$CT = CI + CF$

$CT = 11 \log_2 n + 11$

$O(\log_2 n)$

Exemplo 3

$$\begin{cases} T(n) = 5 & n = 1 \\ T(n) = 2T\left(\frac{n}{2}\right) + 3n & n > 1 \end{cases}$$

$$T\left(\frac{n}{2}\right) = 2T\left(\frac{n/2}{2}\right) + 3\left(\frac{n}{2}\right)$$

$$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{4}\right) + 3\frac{n}{2}$$

$k = 1$ $\quad T(n) = 2T\left(\frac{n}{2}\right) + 3n$

$$T(n) = 2\left(2T\left(\frac{n}{4}\right) + 3\frac{n}{2}\right) + 3n$$

$$T\left(\frac{n}{4}\right) = 2T\left(\frac{n/4}{2}\right) + 3\left(\frac{n}{4}\right)$$

$$T\left(\frac{n}{4}\right) = 2T\left(\frac{n}{8}\right) + 3\frac{n}{4}$$

$k = 2$ $\quad T(n) = 4T\left(\frac{n}{4}\right) + 6n$

Exemplo 3

$$T(n) = 4\left(2T\left(\frac{n}{8}\right) + \frac{3n}{4}\right) + 6n \qquad k = \log_2 n$$

$k = 3$

$$T(n) = 8T\left(n/8\right) + 9n$$

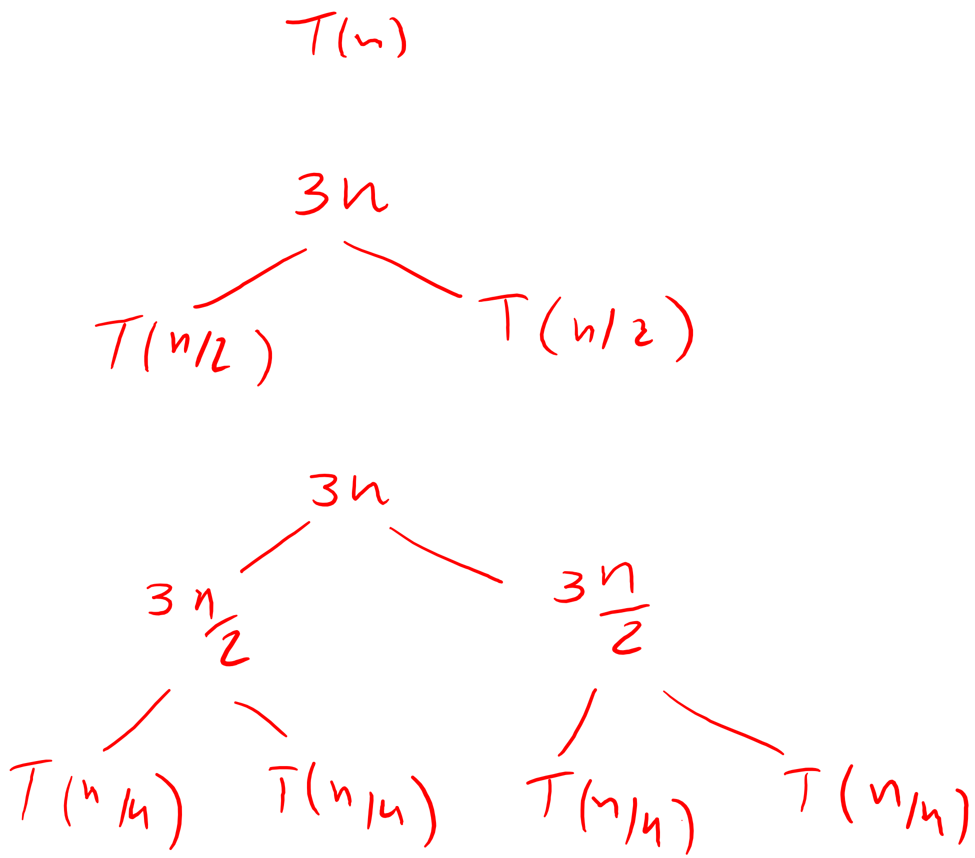
$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + 3nk$$

$$T(n) = 2^{\log_2 n} T\left(\frac{n}{2^{\log_2 n}}\right) + 3n\log_2 n \qquad O(n \log_2 n)$$

$$T(n) = n \cdot 5 + 3n\log_2 n$$

$$T(n) = 3n\log_2 n + 5n$$

Exemplo 3



Exemplo 3

$\frac{16}{2^0}$ $\frac{16}{2^1}$ $\frac{16}{2^2}$ $\frac{16}{2^3}$ $\frac{16}{2^4}$

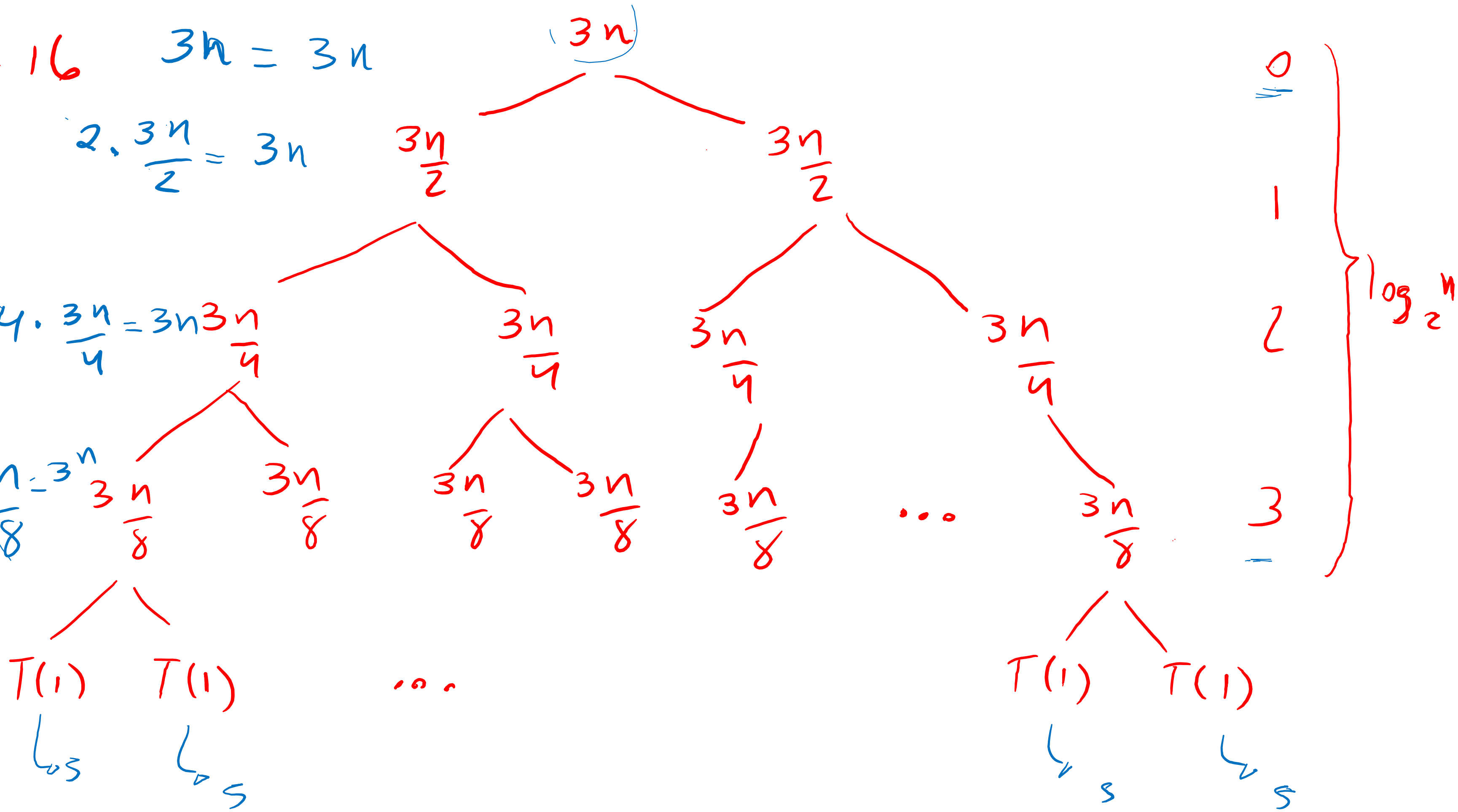
$n = 16$

$3n = 3n$

$2 \cdot \frac{3n}{2} = 3n$

$4 \cdot \frac{3n}{4} = 3n$

$8 \cdot \frac{3n}{8} = 3n$



Exemplo 3

$CT = CI + CE$

$CI =$ Soma Niveis + C.N.

$CI = \log_2 n \cdot 3n$

$CI = 3n \log_2 n$

$$\sum_{i=0}^{\log_2 n - 1} 3n$$

$CE =$ Nº folhas $*$ C Base

$CE = n * 5$

$CE = 5n$

Exemplo 3

$$CT = 3n\log_2 n + 5n$$