M4P70 Markov Processes, Problems 1

Lecture 2, 7 Oct; Solutions Lecture 5, 14 Oct

Borel-Cantelli Lemmas
    For A_n a sequence of events, its limit superior, or the event A_n *i.o.* (i.o. for 'infinitely often') is

$$\{A_n \ i.o.\} := \bigcap_m \bigcup_{n>m} A_n.$$

[Compare Analysis, where a sequence of reals has $\limsup x_n := \inf_m \sup_{n>m} x_n.$]

    Q1. Prove the first Borel-Cantelli Lemma:
If $\sum \mathbb{P}(A_n) < \infty$, then $\mathbb{P}(A_n \ i.o.) = 0$.

    Q2. Prove the second Borel-Cantelli Lemma:
If the events A_n are *independent* and $\sum \mathbb{P}(A_n) = \infty$, then $\mathbb{P}(A_n \ i.o.) = 1$.   NHB