PRELIMINARY EXERCISES

Exercise 0.1. Find and identify the density function of $Y = X^2$, where $X \sim \mathcal{N}(0, 1)$.

Exercise 0.2. If X and Y are independent, identify the distribution of $X + Y$ when:

(i) $X \sim N(\mu_1, \sigma_1^2), Y \sim N(\mu_2, \sigma_2^2)$;

(ii) X, Y are Poisson, means μ, λ, respectively;

(iii) X, Y are Exponential, means λ^{-1}, μ^{-1}, respectively.

What happens in (iii) if $\lambda = \mu$? Propose and verify generalizations of the results you obtain in (i) and (ii) for the sums of n independent random quantities.

Exercise 0.3. Derive the density function of the random quantity $X = -\lambda^{-1} \log U$, where $U \sim \text{Uniform}(0, 1)$.

Exercise 0.4. If $X_1, \ldots, X_n$ are independent Exponential(1) random quantities, find the density of

$$Y = \max(X_1, X_2, \ldots, X_n).$$

Exercise 0.5. If $U_1, \ldots, U_{12}$ are independent Uniform(0,1), what (approximately) will the distribution of

$$X = \left[\sum_{i=1}^{12} U_i - 6\right]$$

be, and why?

Exercise 0.6. If $X \sim \text{Exponential}(\lambda)$ and $Y = \lfloor X \rfloor$ (= integer part of X) show that Y has a geometric distribution. Does Exercise 0.3 suggest a way of generating a sequence of independent geometric quantities?