<u>Logic</u>

20 lectures

- Propositional Logic
  - Semantics ('truth tables')
    - <u>Tautologies</u> ~~[illegible]~~
  - Syntax ('proof')
    - <u>Theorems</u>
- 1st order logic
  - Semantics ('1st order structures')
  - Formal deduction
    - Gödel's Completeness Thm.

<u>Set theory</u>

$$\left(\left(\left(p \to q\right) \wedge \left(q \to (\neg p)\right)\right) \to (\neg p)\right)$$

"If $\overbrace{\text{Mr Jones is happy}}^{p}$ then $\overbrace{\text{Mrs Jones is unhappy}}^{q}$
and if Mrs J. is unhappy then Mr J. is unhappy.
So Mr Jones is unhappy."

1. Propositional Logic.

1.1 Propositional Formulas.

'Proposition' = 'Statement'

either True (T)
or False (F).

Combine basic propositions using connectives.

(1.1.1) Connectives + truth table rules.

$p, q, \ldots$ statements.

Connectives

Negation $(\neg p)$

p	$(\neg p)$
T	F
F	T

Conjunction ('and')

$(p \wedge q)$ has value T

$\iff$ p, q have value T.

Disjunction ('or')

$(p \vee q)$ has value T

$\iff$ at least one of p, q has value T

Implication $(p \to q)$

$(p \to q)$ has value F only when p has value T and q has value F.

Biconditional $(p \leftrightarrow q)$

Value T precisely when p, q have the same value.

Summary:

p	q	$(p\wedge q)$	$(p\vee q)$	$(p\to q)$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

(1.1.2) Def. A propositional formula is obtained from propositional variables $p_1, p_2, p_3, \dots$ and connectives in the following way:

(i) Any prop. var. is a formula

(ii) If ϕ, ψ are formulas then $(\neg\phi)$ $(\phi\wedge\psi)$ $(\phi\to\psi)$ $(\phi\vee\psi)$ $(\phi\leftrightarrow\psi)$ are formulas

(iii) Any formula arises in this way.

Eg. Formulas

p_1 $\quad p_2 \quad (\neg p_2)$

$(p_1\to(\neg p_2))$

$((p_1\to(\neg p_2))\to p_2)$

Not formulas $\quad p_1\wedge p_2$ (missing brackets)

$)(\neg p_9$

Remarks: (i) Because of the brackets, every formula is either a prop. variable or is built from 'shorter' formulas in a unique way.

② Any assignment of truth values to the variables in a formula ϕ determines the the truth value of ϕ in a unique way, using (1.1.1)

Eg ϕ: $((p_1 \to (\neg p_2)) \to p_1)$

p_1	p_2	$\neg p_2$	$(p_1 \to (\neg p_2))$	ϕ
T	T	F	F	T
T	F	T	T	T
→ F	T	F	T	F
F	F	T	T	F

(1.1.3) Def. Let $n \in \mathbb{N}$

① A truth function of n variables is a function

$$f : \{T, F\}^n \longrightarrow \{T, F\}$$

(where $\{T, F\}^n = \{(x_1, \dots, x_n) :$ each x_i is T or F $\}$. 5

[Ex: How many?]

② Suppose ϕ is a formula whose variables are amongst $p_1, \dots, p_n$. Obtain a truth function $F_\phi : \{T, F\}^n \longrightarrow \{T, F\}$ whose value at $(x_1, \dots, x_n)$ is the truth value of ϕ when p_i has value x_i, when computed according to (1.1.1).

F_ϕ is the truth function of ϕ

[Eg. In previous $n = 2$ $F_\phi((F, T)) = F$.]