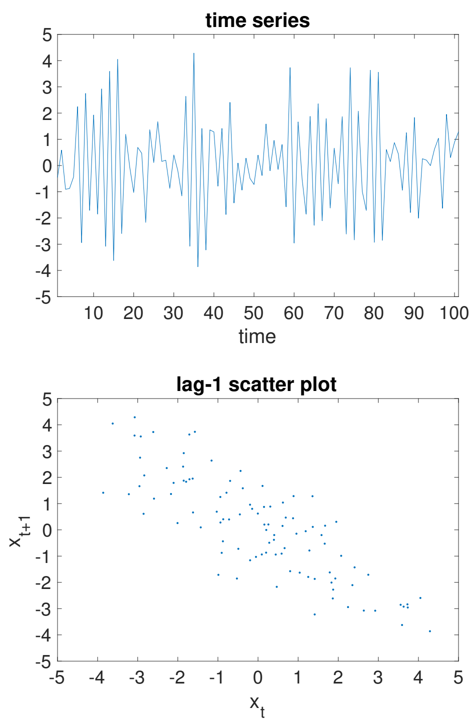
Quiz questions

Chapter 1.2

1. Consider the time series and lag-1 scatter plot shown here. Which of the following is a correct inference from visual inspection?



A Successive values are uncorrelated.

B Successive values are positively correlated.

C Negative values tend to be followed by negative values.

D Successive values are negatively correlated.

Chapter 1.3

2. Which of the following is a valid covariance matrix?

$$A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 4 & -1 \\ 1 & 4 \end{pmatrix} \quad C = \begin{pmatrix} 1 & -3 \\ -3 & 9 \end{pmatrix} \quad D = \begin{pmatrix} 4 & 3 \\ 3 & 1 \end{pmatrix}$$

3. X and Y are random variables such that $Y = 2X + \epsilon$ where X is a random variable with mean 0 and variance 2, and $\epsilon \sim N(0, 1)$ that is independent of X. What is $\mathrm{Corr}(X, Y)$?

   A $4/\sqrt{18}$

   B 0

   C $-2/\sqrt{5}$

   D 1

Chapter 2.1

4. Which of the following statements is TRUE?

   A A random process is completely stationary if it has a constant mean.

   B A random process is completely stationary if it is second order stationary.

   C A random process is completely stationary if and only if it is second order stationary.

   D A random process is second order stationary if it is completely stationary.

   E A random process is second order stationary if it has a constant variance.

5. Let $\{X_t\}$ be a stationary process with mean 1, variance σ^2 and autocovariance sequence $\{s_\tau\}$. Which of the following statements is FALSE?

A $E\{X_t + X_s\} = 2,\ s \neq t.$

B $E\{X_{10}^2\} = \sigma^2.$

C $E\{X_0\} = E\{X_{1283}\}.$

D $\mathrm{Cov}\{X_2, X_{231}\} = \mathrm{Cov}\{X_{-233}, X_{-4}\}.$

E $E\{X_{546}X_{536}\} = E\{X_{-230}X_{-220}\}.$

6. Let $\{X_t\}$ be a (second-order) stationary process with mean 0 and autocovariance sequence $\{s_\tau\}$, and let $a \neq 0$ be a fixed constant. Which of the following is FALSE?

   A $\{aX_t\}$ is a stationary process.

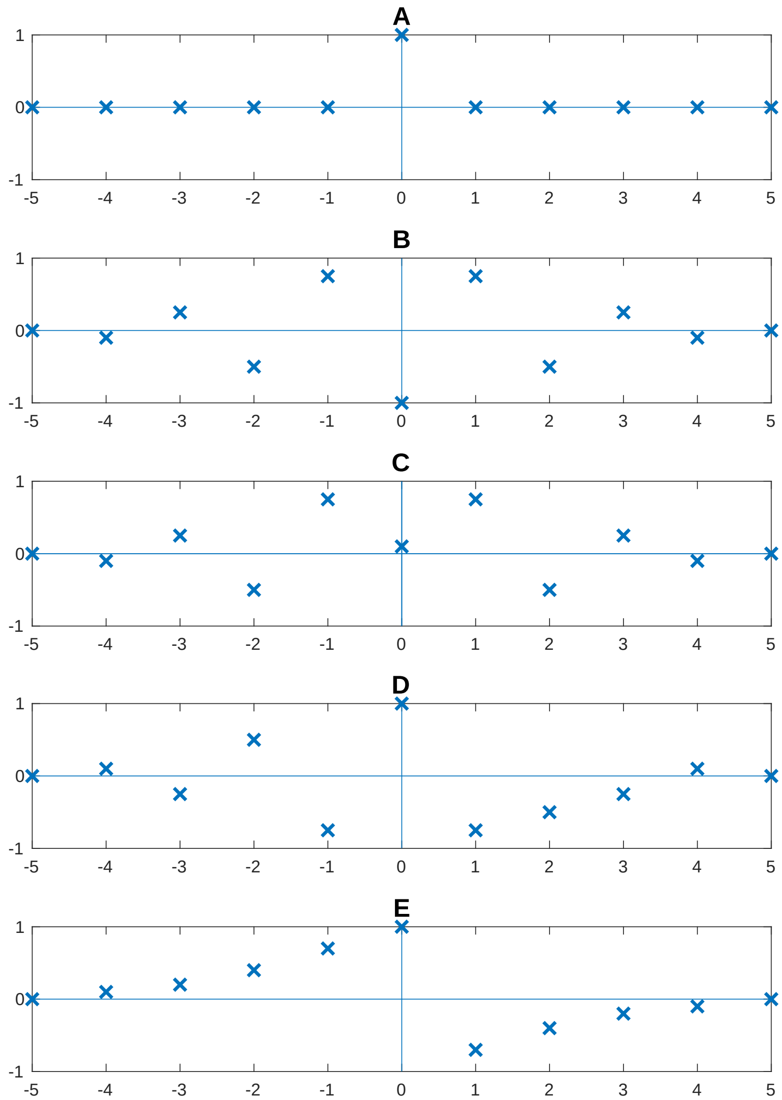
   B $\{-X_t\}$ is a stationary process.

   C The autocovariance sequence of $\{aX_t\}$ is $\{a^2 s_\tau\}$

   D $\{(-1)^t X_t\}$ is a non-stationary process.

   E $\{X_t + at\}$ is a non-stationary process.

7. Which of the following is a valid autocovariance sequence?



8. Let $\{\epsilon_t\}$ be a zero mean white noise process of variance σ_ϵ^2. Which of the following statements is FALSE?

   A. $\{\epsilon_t\}$ is a stationary process.

   B. $E\{\epsilon_t \epsilon_{t+\tau}\} = 0$ for all $\tau \neq 0$.

   C. The random variable $(\epsilon_t + \epsilon_{t+1})$ is correlated with the random variable $(\epsilon_{t+1} + \epsilon_{t+2})$.

   D. $\sum_{\tau=-\infty}^{\infty} s_\tau = \sigma_\epsilon^2$.

   E. $\text{Cov}\{\epsilon_{t+\tau}, \epsilon_{t-\tau}\} = 0$ for all τ.

9. Let $\{X_t\}$ be the MA(2) process

$$X_t = \epsilon_t - \frac{1}{2}\epsilon_{t-1} + \frac{1}{4}\epsilon_{t-2}, \quad \epsilon_t \sim N(0,1).$$

Which of the following statements is FALSE?

A. $s_2 = \frac{1}{4}$

B. $s_{-3} = 0$

C. X_t is Gaussian distributed.

D. $s_0 = \frac{7}{4}$

E. $E\{X_t\} = 0$.

10. Consider the *stationary* AR(2) process

$$X_t = \frac{1}{2}X_{t-1} + \frac{1}{8}X_{t-2} + \epsilon_t + c,$$

where $\{\epsilon_t\}$ is a zero mean white noise process and c is a fixed constant. What is $\mu = E\{X_t\}$?

A. 0

B. $8c/3$

C. $3c/5$

D. $1 + c$

E. c.

11. Consider an AR(1) process $X_t = \phi X_{t-1} + \epsilon_t$, where $0 < \phi < 1$ and $\{\epsilon_t\}$ is a zero mean white noise process with $\sigma_\epsilon^2 = 1$. What is $\sum_{\tau=-\infty}^{\infty} s_\tau$ equal to?

A. 1

B. $(\phi - 1)/(1 - \phi)$

C. $1/(1 - \phi)^2$

D. $\phi/(1 - \phi)$

E. $2/(1 - \phi)$.

12. Which of the following is an ARMA(1,2)?

A. $X_{t-1} + \epsilon_t = \epsilon_{t-2} + X_t$

B. $X_t - X_{t-1} = \epsilon_t$

C. $\epsilon_{t-1} + X_t = X_{t-1} + \epsilon_t$

D. $X_{t-2} + \epsilon_t = \epsilon_{t-2} + X_t$

E. $\epsilon_{t-2} = 2X_{t-2} + X_t - \epsilon_t$.

13. Which of the following is the correct model name for this process?

$$\frac{1}{2}X_{t-3} - \epsilon_t = 3\epsilon_{t-2} - X_t$$

A. MA(2)
B. ARMA(3,2)
C. AR(5)
D. AR(3,2)
E. ARMA(1/2,3)

Chapter 2.2

14. Consider the model $X_t = \mu_t + Y_t$, where $\{Y_t\}$ is a zero mean stationary process. Which one of the following statements is FALSE?

    A. $\{Y_t^{(d)}\}$ is stationary for all $d \geq 1$.

    B. If $\mu_t = \alpha + \beta t$, then $\{X_t^{(1)}\}$ is a stationary process.

    C. If $\mu_t = \alpha + \beta t$, then $\Delta X_t = \Delta Y_t$.

    D. $B^d X_t = \mu_{t-d} + Y_{t-d}$.

    E. $\Delta^3 = 1 - 3B + 3B^2 - B^3$.

15. Consider the model $X_t = \nu_t + Y_t$, where $\{Y_t\}$ is a zero mean stationary process, and $\nu_t = \sin(\pi t/10)$. Which of the following operations would remove seasonality?

   A. $1 - B^{10}$

   B. $1 - B^{5}$

   C. $1 - B^{12}$

   D. Δ^{10}

   E. $1 - B^{20}$

   F. Δ^{5}

   G. B^{10}

   H. Δ^{20}

16. Consider the model

$$X_t = \mu_t + \nu_t + Y_t$$

where $\{Y_t\}$ is a zero mean stationary process, μ_t is a second order polynomial trend and ν_t is a seasonality component with period 52. Which of the following is the resulting process when the operator $(1 - B^{52})\Delta^3$ is applied to $\{X_t\}$?

A. $Y_t^{(3)} - Y_{t-52}^{(3)}$

B. $Y_t^{(3)}$

C. $Y_t - Y_{t-52}$

D. $Y_t^{(3)} + \nu_t^{(3)}$

E. $Y_t^{(3)} + \mu_t - \mu_{t-52}$

Chapter 2.3

17. Consider the ARMA process

$$X_t = \sum_{k=1}^{3} \frac{1}{2^k} X_{t-k} + \sum_{l=0}^{3} \frac{1}{2^l} \epsilon_{t-l}$$

Which of the following is the correct form of the characteristic polynomial $\Theta(B)$?

A. $\Theta(B) = 1 - \frac{1}{2}B - \frac{1}{4}B^2 - \frac{1}{8}B^3$
B. $\Theta(B) = 1 + \frac{1}{2}B + \frac{1}{4}B^2 + \frac{1}{8}B^3$
C. $\Theta(B) = 1 - \frac{1}{2}B + \frac{1}{4}B^2 - \frac{1}{8}B^3$
D. $\Theta(B) = 1 - \frac{1}{2}B$
E. $\Theta(B) = 1 - \frac{1}{8}B^3$

18. Consider the General Linear Process

$$X_t = \sum_{k=0}^{\infty} \frac{1}{2^k} \epsilon_{t-k}$$

where $\{\epsilon_t\}$ is a white noise process with variance σ_ϵ^2. What is the variance of $\{X_t\}$?

A. $2\sigma_\epsilon^2$

B. $4\sigma_\epsilon^2/3$

C. 1

D. σ_ϵ^2

E. $4\sigma_\epsilon^2/5$

19. Consider again the General Linear Process

$$X_t = \sum_{k=0}^{\infty} \frac{1}{2^k} \epsilon_{t-k}$$

where $\{\epsilon_t\}$ is a white noise process with variance σ_ϵ^2. What is $s_1 \equiv \mathrm{Cov}\{X_t, X_{t+1}\}$?

A. $2\sigma_\epsilon^2/3$

B. $4\sigma_\epsilon^2/3$

C. $1/2$

D. $\sigma_\epsilon^2/2$

E. $2\sigma_\epsilon^2/5$

20. Consider a process of the form

$$X_t = \frac{G_1(B)}{G_2(B)}\epsilon_t.$$

Which of the following can be written in General Linear Process form?

A. $G_1(z) = z^2 + 3z - 4$, $G_2(z) = z^2 - z + \frac{1}{4}$.

B. $G_1(z) = z^2 - 1$, $G_2(z) = z^2 + \frac{3}{2}z - 1$.

C. $G_1(z) = z^2 + 3z + 2$, $G_2(z) = z^2 - 4$.

21. Consider the MA(2) process

$$X_t = \epsilon_t + \frac{1}{4}\epsilon_{t-2}.$$

Which of the following statements is TRUE?

A. It is stationary and invertible.

B. It is stationary but not invertible.

C. It is not stationary but is invertible.

D. It is not stationary and it is not invertible.

22. Consider the ARMA(2,1) process

$$X_t = \frac{31}{20}X_{t-1} - \frac{3}{5}X_{t-2} + \epsilon_t - \frac{4}{3}\epsilon_{t-1}.$$

Which of the following statements is TRUE?

A. It is stationary and invertible.

B. It is stationary but not invertible.

C. It is not stationary but is invertible.

D. It is not stationary and it is not invertible.

23. Consider the stationary and invertible (check this if you want) ARMA(1,1) process

$$X_t = \frac{1}{2}X_{t-1} + \epsilon_t + \frac{1}{8}\epsilon_{t-1},$$

where $\{\epsilon_t\}$ is a white noise process with variance σ_ϵ^2. What is the variance of $\{X_t\}$?

A. $\frac{72}{48}\sigma_\epsilon^2$.

B. $\frac{73}{49}\sigma_\epsilon^2$.

C. $\frac{71}{47}\sigma_\epsilon^2$.

D. $\frac{73}{48}\sigma_\epsilon^2$.

E. $\frac{74}{49}\sigma_\epsilon^2$.

Chapter 3.1

24. Let $\{X_t\}$ be a stationary random process with mean zero, autocovariance sequence $\{s_\tau\}$, integrated spectrum $S^{(I)}(f)$ and spectral density function $S(f)$. Which of the following statements is FALSE?

    A. $S(0) = \sum_{\tau=-\infty}^{\infty} s_\tau$.

    B. $\frac{\mathrm{d}}{\mathrm{df}} S^{(I)}(f)$ is an even function.

    C. $S^{(I)}(f)$ is an even function.

    D. $\int_{-1/2}^{0} S(f)\mathrm{df} = \mathrm{s}_0/2$.

25. Let $\{X_t\}$ is a stationary random process with mean zero and autocovariance sequence

$$s_\tau = \begin{cases} 1 & \tau = 0 \\ -1/2 & |\tau| = 1 \\ 0 & |\tau| > 1. \end{cases}$$

Which of the following is the spectral density function for $\{X_t\}$?

A. $S(f) = 1$

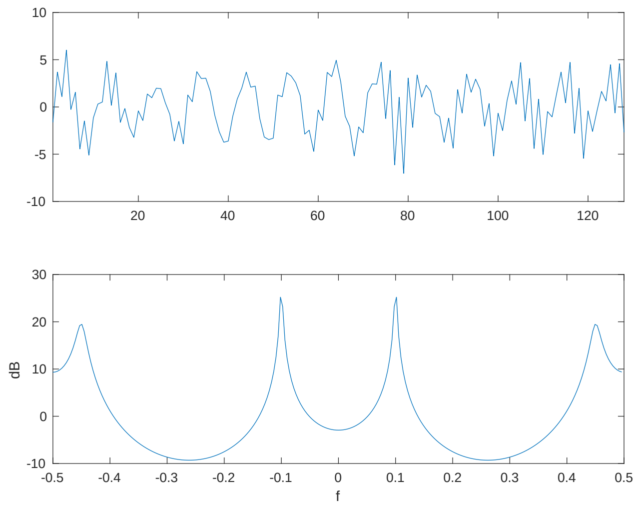
B. $S(f) = 1 - \cos(2\pi f)$

C. $S(f) = 1 + \frac{1}{2}\mathrm{e}^{\mathrm{i}2\pi f}$

D. $S(f) = 1 + \sin(2\pi f)$

E. $S(f) = 1 - \sin(2\pi f)$

26. Here is shown a single realization and the spectral density function for a stationary random process.



Which of the following is a correct statement?

A. This is a white noise process.

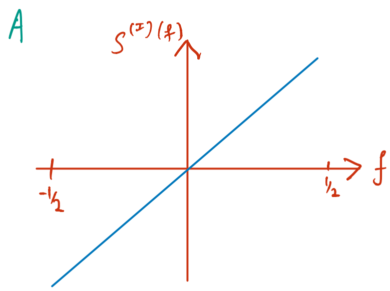
B. This process has a purely discrete spectra.

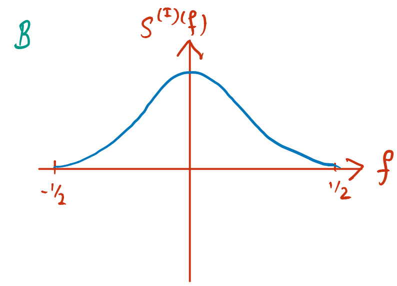
C. This process exhibits strong oscillatory behaviour at frequencies 0.1 and 0.45.

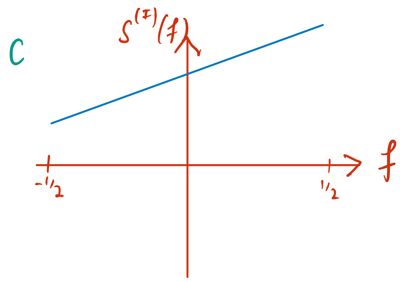
D. This process has a mixed spectra.

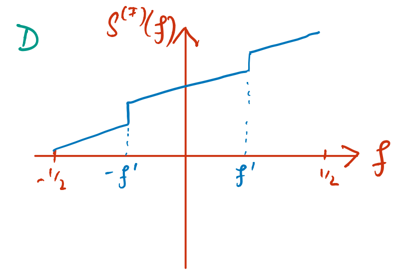
E. Successive values of this process are uncorrelated.

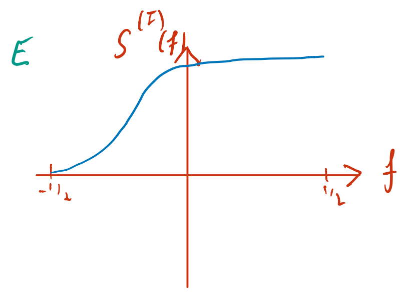
27. Which of the following is a valid integrated spectrum?











28. How would you classify the valid integrated spectrum from Question 27 (using the classification system given in the notes)?

   A. Purely continuous.

   B. Purely discrete.

   C. Mixed.

   D. Discrete.

Chapter 3.2

29. I sample a continuous time random process at 9am GMT every Monday. What is the Nyquist frequency in units hour^{-1}?

    A. 1/14.

    B. 1/336.

    C. 1/48.

    D. 1/168.

    E. 1/7.

30. A continuous-time stationary process $\{X(t)\}$, with t in seconds (s), has spectral density function

$$S_{X(t)}(f) = \begin{cases} 1 - \frac{1}{4}(|f| - 6), & 6 < |f| \le 10, \\ 0, & \text{otherwise}, \end{cases}$$

with f in cycles/s. It is sampled with a sample interval $\Delta t = 0.1$s to produce the discrete-time process $\{X_t\}$.

What is the spectral density function $S_{X_t}(f)$ of $\{X_t\}$ for $|f| < f_{\mathcal{N}}$, where $f_{\mathcal{N}}$ is the Nyquist frequency?

A.
$$S_{X_t}(f) = \begin{cases} \frac{1}{4}|f|, & |f| \le 4, \\ 0, & 4 < |f| \le 5 \end{cases}$$

B.
$$S_{X_t}(f) = \begin{cases} 1 - \frac{1}{4}|f|, & |f| \le 4, \\ 0, & 4 < |f| \le 5 \end{cases}$$

C.
$$S_{X_t}(f) = S_{X(t)}(f), \qquad |f| < 10.$$

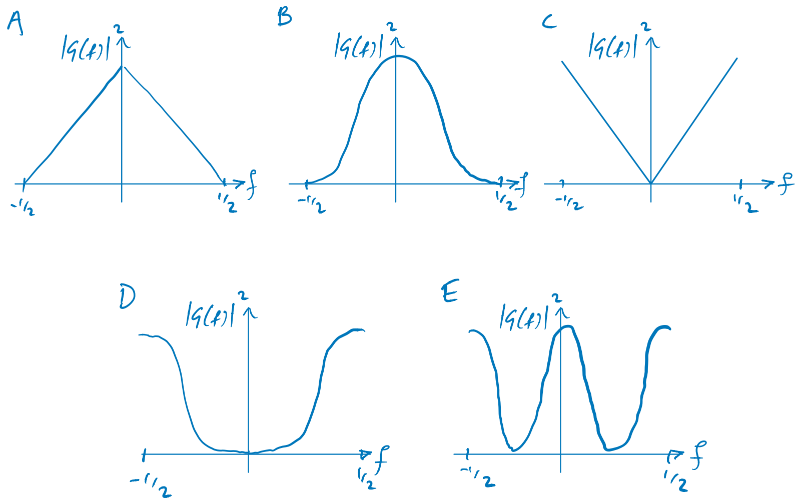
D.
$$S_{X_t}(f) = 1 - |f|, \qquad |f| < 10.$$

E.
$$S_{X_t}(f) = \frac{1}{4}|f|, \qquad |f| < 5.$$

Chapter 3.3

31. Consider the linear filter $L\{X_t\} = X_t - \frac{1}{2}X_{t-1} - \frac{1}{2}X_{t+1}$.

    Which of the following is $|G(f)|^2$?



32. The filter from Question 31 is which of the following types?

   A. Low band-pass filter.

   B. High band-pass filter.

33. Consider the process

$$X_t = \frac{1}{2}X_{t-1} + \epsilon_t + \frac{1}{2}\epsilon_{t-1} + \frac{1}{4}\epsilon_{t-2},$$

where $\{\epsilon_t\}$ is a white noise process with variance σ_ϵ^2. What is its spectral density function?

A.

$$S(f) = \sigma_\epsilon^2 \frac{21 + 8\cos(2\pi f) + 20\cos(4\pi f)}{16 + 20\cos(2\pi f)}$$

B.

$$S(f) = \sigma_\epsilon^2 \frac{20 + 16\cos(2\pi f)}{20 + 18\cos(2\pi f) + 8\cos(4\pi f)}$$

C.

$$S(f) = \sigma_\epsilon^2 \frac{20 + 18\cos(2\pi f) + 8\cos(4\pi f)}{20 + 16\cos(2\pi f)}$$

D.

$$S(f) = \sigma_\epsilon^2 \frac{20 + 16\cos(2\pi f)}{21 + 20\cos(2\pi f) + 8\cos(4\pi f)}$$

E.

$$S(f) = \sigma_\epsilon^2 \frac{21 + 20\cos(2\pi f) + 8\cos(4\pi f)}{20 - 16\cos(2\pi f)}$$

34. Let $\Phi(B)X_t = \epsilon_t$ be an AR(2) process where $\Phi(z)$ has complex conjugate roots and $\{X_t\}$ has has spectral density function

$$S(f) = \frac{\sigma_\epsilon^2}{[1 - \cos(2\pi(0.125 - f)) + 0.25][1 - \cos(2\pi(0.125 + f)) + 0.25]}.$$

Expressing $\{X_t\}$ in the form $X_t = \phi_{1,2}X_{t-1} + \phi_{2,2}X_{t-2} + \epsilon_t$, what are the parameters $\phi_{1,2}$ and $\phi_{2,2}$?

A. $\phi_{1,2} = 1/\sqrt{2}$, $\phi_{2,2} = -1/4$

B. $\phi_{1,2} = 1/4$, $\phi_{2,2} = 1/\sqrt{2}$

C. $\phi_{1,2} = -1/\sqrt{2}$, $\phi_{2,2} = -1/\sqrt{2}$

D. $\phi_{1,2} = -1/4$, $\phi_{2,2} = 1/\sqrt{2}$

E. $\phi_{1,2} = 1/\sqrt{2}$, $\phi_{2,2} = 1/4$

Chapter 4.1

35. An estimator $\hat{\theta}$ of a parameter θ has $\text{MSE}\{\hat{\theta}\} = 3$ and $\text{Var}\{\hat{\theta}\} = 3$. What is $E\{\hat{\theta}\}$?

    A. 0.

    B. 3.

    C. $-\theta$.

    D. $\theta + 3$.

    E. θ.

36. An estimator $\hat{\theta}$ of a parameter θ has $E\{\hat{\theta}\} = \theta + c$, and $\text{Var}\{\hat{\theta}\} = c^2/2$. What is the mean square error of $\hat{\theta}$?

A. $c^2/2$.

B. $c + c^2/2$.

C. $3c^2/2$.

D. $\theta + c + c^2/2$.

E. $(\theta + c)^2 + c^2/2$.

37. Let X_1, X_2 be a portion of an AR(1) process $X_t = \frac{1}{2}X_{t-1} + \epsilon_t$, where $\{\epsilon_t\}$ is a white noise process with zero mean and $\sigma_\epsilon^2 = 1$. What is $\mathrm{Var}\{\bar{X}\}$, where $\bar{X} = \frac{1}{2}(X_1 + X_2)$?

A. $\frac{10}{9}$.

B. 1.

C. $\frac{7}{2}$.

D. $\frac{5}{4}$.

E. $\frac{8}{3}$.

38. Let $X_1, X_2, ..., X_{100}$ be a portion of an MA(2) process $X_t = \epsilon_t - \frac{1}{2}\epsilon_{t-1} + \frac{1}{4}\epsilon_{t-2}$, where $\{\epsilon_t\}$ is zero mean white noise process with $\sigma_\epsilon^2 = 2$. What is $E\{\hat{s}_1^{(p)}\}$ computed from this portion?

A. $\frac{99}{100}$.

B. $-\frac{99}{100}$.

C. $\frac{100}{99}$.

D. $-\frac{99}{80}$.

E. $-\frac{80}{99}$.

Chapter 4.2

39. Which of the following statements is FALSE

    A. The periodogram is an asymptotically ($N \to \infty$) unbiased estimator of $S(f)$.

    B. The effect of side-lobe leakage is greater for processes with large dynamic range.

    C. The periodogram for a white noise process is unbiased but the direct spectral estimator for a white noise process is biased.

    D. Tapering (with a non-rectangular taper such as those given in the notes) reduces side-lobe leakage.

    E. $\hat{s}_0^{(p)} = \int_{-1/2}^{1/2} \hat{S}^{(p)}(f)\mathrm{df}$.

40. For any taper, it can be shown that

$$\int_{-1/2}^{1/2} E\{S^{(d)}(f)\}\mathrm{df} = \mathrm{s}_0.$$

Which of the following is a true statement about the periodogram?

A. $\int_{-1/2}^{1/2} E\{S^{(p)}(f)\}\mathrm{df}$ is less than s_0.

B. $\int_{-1/2}^{1/2} E\{S^{(p)}(f)\}\mathrm{df}$ is greater than s_0.

C. $\int_{-1/2}^{1/2} E\{S^{(p)}(f)\}\mathrm{df}$ is equal to s_0.

Chapter 5

41. Which of the following parametric model fitting methods for AR(p) processes makes assumptions on the probability distribution of the random process as well as the model and order.?

    A. Yule-Walker (untapered).

    B. Yule-Walker (tapered).

    C. Least squares.

    D. Maximum likelihood.

42. Consider the AR(2) process $X_t - 0.5X_{t-2} = \epsilon_t$. Given $\mathrm{Var}\{X_t\} = 2$, and using the Yule-Walker equations, what is the value of σ_ϵ^2?

A. 4/3.

B. 2.

C. 2/3.

D. 1.

E. 3/2

43. Suppose for an AR(2) process we obtain $\hat{s}_0 = 5$, $\hat{s}_1 = 4$, and $\hat{s}_2 = 2$. What is the estimated AR(2) model?

A. $\phi_{1,2} = 4/3,\ \phi_{2,2} = -2/3,\ \sigma_\epsilon^2 = 1$

B. $\phi_{1,2} = -2/3,\ \phi_{2,2} = 4/3,\ \sigma_\epsilon^2 = 3/2.$

C. $\phi_{1,2} = -4/3,\ \phi_{2,2} = -2/3,\ \sigma_\epsilon^2 = 3/2.$

D. $\phi_{1,2} = -4/3,\ \phi_{2,2} = 2/3,\ \sigma_\epsilon^2 = 1/2.$

E. $\phi_{1,2} = 2/3,\ \phi_{2,2} = -4/3,\ \sigma_\epsilon^2 = 1/2.$

44. Suppose you observe a portion $X_1, ..., X_8$ and wish to fit an AR(2) model. Which of the following is F as defined in the notes?

A. $\begin{bmatrix} X_2 & X_1 \\ X_3 & X_2 \\ \vdots & \vdots \\ X_8 & X_7 \end{bmatrix}$ B. $\begin{bmatrix} X_1 & X_2 & \cdots & X_6 \\ X_2 & X_3 & \cdots & X_7 \end{bmatrix}$ C. $\begin{bmatrix} X_2 & X_3 & \cdots & X_8 \\ X_1 & X_2 & \cdots & X_7 \end{bmatrix}$

D. $\begin{bmatrix} X_2 & X_1 \\ X_3 & X_2 \\ \vdots & \vdots \\ X_7 & X_6 \end{bmatrix}$ E. $\begin{bmatrix} X_1 & X_2 \\ X_2 & X_3 \\ \vdots & \vdots \\ X_7 & X_8 \end{bmatrix}$ F. $\begin{bmatrix} X_2 & X_3 & \cdots & X_8 \end{bmatrix}$

45. Which of the following probabilistic statements is FALSE for an AR(4) process?

A. $f(X_{10}|X_9, ..., X_1, \boldsymbol{\phi}, \sigma_\epsilon^2) = f(X_{10}|X_9, ..., X_6, \boldsymbol{\phi}, \sigma_\epsilon^2)$

B. $f(X_{12}|X_{11}, ..., X_1, \boldsymbol{\phi}, \sigma_\epsilon^2) = f(X_{12}|X_{11}, ..., X_6, \boldsymbol{\phi}, \sigma_\epsilon^2)$.

C. $f(X_{12}|X_{11}, ..., X_1, \boldsymbol{\phi}, \sigma_\epsilon^2) = f(X_{12}|X_{11}, X_{10}, \boldsymbol{\phi}, \sigma_\epsilon^2)$.

D. $f(X_6|X_5, ..., X_2, \boldsymbol{\phi}, \sigma_\epsilon^2) = f(X_6|X_5, ..., X_1, \boldsymbol{\phi}, \sigma_\epsilon^2)$.

E. $f(X_{1004}|X_{1003}, ..., X_1, \boldsymbol{\phi}, \sigma_\epsilon^2) = f(X_{1004}|X_{1003}, X_{1002}, X_{1001}, X_{1000}, \boldsymbol{\phi}, \sigma_\epsilon^2)$.

46. You fit AR(p) processes for $p = 1, ..., 6$. The table below reports the AIC value for each model.

p	AIC
1	26.51
2	20.46
3	14.18
4	-21.71
5	-17.99
6	-18.56

Which model do you select?

Chapter 6

47. Forecast $X_{100}(7)$ is which of the following?

    A. A forecast made at time 100 of the value at time 7.

    B. A forecast made at time 7 for the value 107.

    C. A forecast made at time 7 of the value at time 100.

    D. A forecast made at time 100 of the value at time 107.

    E. A forecast made at time 107 of the value at time 100.

48. Consider the AR(1) process $X_t = -\frac{1}{3}X_{t-1} + \epsilon_t$, where $\{\epsilon_t\}$ is a zero mean white noise process with variance σ_ϵ^2. Which of the following is the 2-step ahead prediction variance?

A. $\frac{81}{100}\sigma_\epsilon^2$.

B. $\frac{8}{10}\sigma_\epsilon^2$.

C. $\frac{10}{9}\sigma_\epsilon^2$.

D. σ_ϵ^2.

E. $\frac{91}{81}\sigma_\epsilon^2$.

49. Consider the AR(2) model $X_t = \frac{1}{2}X_{t-1} - \frac{1}{5}X_{t-2} + \epsilon_t$. I observe the following realisation $X_0 = 3, X_1 = 0, X_2 = 1$. What is $X_2(3)$?

A. $-3/40$.

B. $1/20$.

C. $-1/20$.

D. $1/2$.

E. 0.

50. Consider the MA(3) model $X_t = \epsilon_t - \frac{1}{2}\epsilon_{t-1} + \epsilon_{t-2} - \frac{1}{2}\epsilon_{t-3}$, $\{\epsilon_t\}$ is a white noise process with variance 2. What is the 3-step ahead prediction variance?

A. 4/9.

B. 2.

C. 9/4.

D. 9/2.

E. 5

51. Consider the ARMA(1,1) model

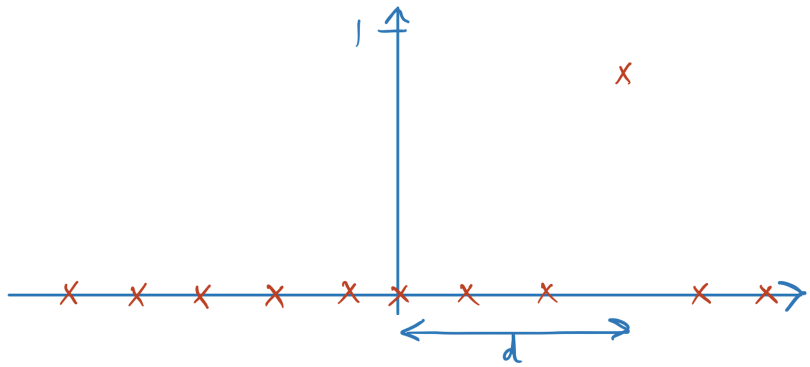
$$X_t = \frac{1}{2}X_{t-1} + \epsilon_t + \frac{1}{8}\epsilon_{t-1}$$

Which of the following is the forecast $X_t(2)$ in the form $X_t(2) = \sum_{k=0}^{\infty} \pi_k X_{t-k}$?
HINT: You may want to refer back to Question 23.

A. $\sum_{k=0}^{\infty} \frac{5}{8} \cdot \left(-\frac{1}{2}\right)^k X_{t-k}$
B. $\sum_{k=0}^{\infty} \frac{5}{16} \cdot \left(-\frac{1}{8}\right)^k X_{t-k}$
C. $\sum_{k=0}^{\infty} \frac{5}{8} \cdot \left(-\frac{1}{4}\right)^k X_{t-k}$.
D. $\sum_{k=0}^{\infty} \frac{1}{16} \cdot \left(-\frac{1}{8}\right)^k X_{t-k}$.
E. $\sum_{k=0}^{\infty} \frac{5}{16} \cdot \left(\frac{1}{4}\right)^k X_{t-k}$

Chapter 7.1

52. Consider the following cross correlation sequence $\rho_{X_1 X_2,\tau}$ for two jointly stationary processes $\{X_{1,t}\}$ and $\{X_{2,t}\}$.



Which of the following is a correct statement?

A. $X_{1,t}$ is strongly correlated with $X_{2,t+d}$.

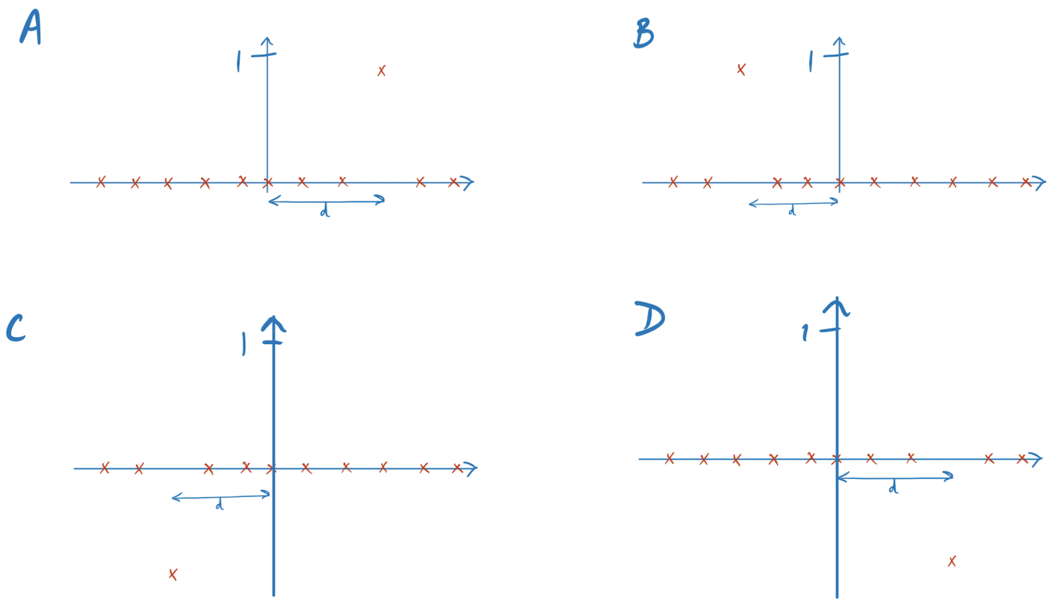
B. $X_{1,t}$ is strongly correlated with $X_{2,t-d}$.

C. $X_{1,t+d}$ is strongly correlated with $X_{2,t}$.

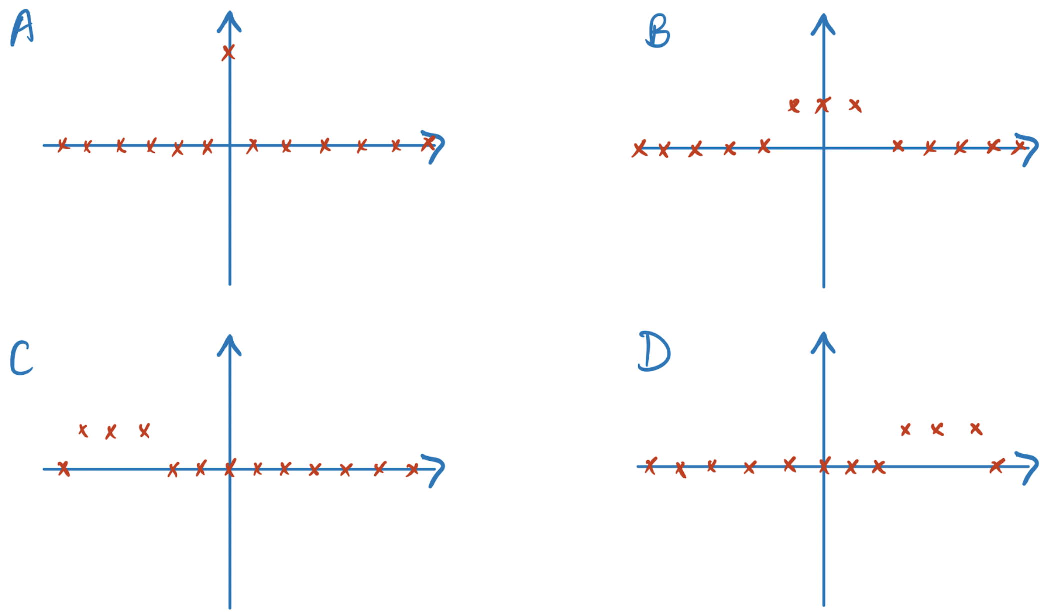
D. $X_{1,t}$ is strongly correlated with $X_{2,t}$.

E. Process $\{X_{1,t}\}$ shows no correlation with process $\{X_{2,t}\}$

53. Consider again the jointly stationary processes from Question 52. Which of the following is $\rho_{X_2,X_1,\tau}$?



54. Let $\{X_{1,t}\}$ be a stationary zero mean white noise process with variance $\sigma^2_{X_1}$, and let $X_{2,t} = \frac{1}{3}X_{1,t-3} + \frac{1}{3}X_{1,t-4} + \frac{1}{3}X_{1,t-5}$. Which of the following represents the cross covariance sequence $s_{X_1,X_2,\tau}$?



Chapter 7.2

55. For the same processes as in Question 54, what is the group delay?

    A. $3/2$
    B. $-1/2$
    C. 3
    D. $3\pi/2$
    E. -3π

56. For the same processes as in Question 54, what is the magnitude squared coherence $\gamma^2_{X_1 X_2}(f)$?

A. $\gamma^2_{X_1 X_2}(f) = \left[1 + \frac{\sigma_\epsilon^2}{4\sin^2(\pi f) S_X(f)}\right]^{-1}$

B. $\gamma^2_{X_1 X_2}(f) = \left[1 + \frac{1}{4\cos(\pi f) S_X(f)}\right]^{-1}$

C. $\gamma^2_{X_1 X_2}(f) = \left[1 + \frac{\sigma_\epsilon^2}{4\cos^2(\pi f) S_X(f)}\right]^{-1}$

D. $\gamma^2_{X_1 X_2}(f) = \left[1 + \frac{1}{4\cos^2(\pi f) S_X(f)}\right]^{-1}$

E. $\gamma^2_{X_1 X_2}(f) = \left[1 + \frac{\sigma_\epsilon^2}{4\sin^2(\pi f) S_X(f)}\right]^{-1}$

Chapter 7.3

57. Let $\{X_t\}$ be a zero mean stationary process with autocovariance sequence $\{s_{X,\tau}\}$ and spectral density function $S_X(f)$. Consider the process

$$Y_t = \sum_{u=-\infty}^{\infty} g_u X_{t-u} + \epsilon_t,$$

where $\{\epsilon_t\}$ is a zero mean white noise process with variance σ_ϵ^2 that is uncorrelated with $\{X_t\}$, and the impulse response sequence is $g_1 = 1$, $g_2 = 1$ and $g_u = 0$ for all $u \neq 1, 2$. Which of the following is the cross spectrum $S_{XY}(f)$? Hint: You may use without proof that $\mathrm{e}^{\mathrm{i}a} + \mathrm{e}^{\mathrm{i}b} = \mathrm{e}^{\mathrm{i}(a+b)/2} \cdot 2\cos((a-b)/2)$

A. $S_{XY}(f) = S_X(f)\cos(\pi f)$

B. $S_{XY}(f) = 2S_X(f)\sin(\pi f)\mathrm{e}^{-\mathrm{i}3\pi f}$

C. $S_{XY}(f) = S_X(f)\cos(\pi f)\mathrm{e}^{-\mathrm{i}6\pi f}$

D. $S_{XY}(f) = S_X(f)\sin(\pi f)$

E. $S_{XY}(f) = 2S_X(f)\cos(\pi f)\mathrm{e}^{-\mathrm{i}3\pi f}$