Problem class

[1]
1. F
2. F
3. T
4. T

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[2] $\{\neg, \leftrightarrow\}$ is not adequate.

Look at truth fns. with 2 variables

Claim: Always an even number of T appearing.

Pf by ind. on number of connectives

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Another way : Write 1 for T, 0 for F) do arithmetic mod 2, work in the field $\mathbb{F}_2 = \{0,1\}$.

Truth fn. of n variables $f : \{0,1\}^n \to \{0,1\}$

$$\mathbb{F}_2^n \longrightarrow \mathbb{F}_2$$

These form a vector space over $\mathbb{F}_2$.

If ϕ is a fmla. of n variables

$$F_{(\neg\phi)}(\bar{x}) = 1 + F_\phi(\bar{x}) \qquad F_{(\neg\phi)} = \underset{\text{constant fn.}}{\overline{1}} + F_\phi$$

$\bar{x} = (x_1, \dots, x_n) \in \mathbb{F}_2^n$

$F_{(\phi \leftrightarrow \psi)}$?

$$F_\phi(\bar{x}) = F_\psi(\bar{x}) \Leftrightarrow F_\phi(\bar{x}) + F_\psi(\bar{x}) = 0$$

So $F_{(\phi \leftrightarrow \psi)} = \overline{1} + F_\phi + F_\psi$.

If θ is built from $p_1, \dots, p_n$ & $\neg, \leftrightarrow$ then F_θ is a lin. combination of $F_{p_1}, \dots, F_{p_n}$ & $\overline{1}$.

$$\leq 2^{n+1} < 2^{2^n} \quad \text{if} \quad n \geq 2.$$