EXERCISES 1

Do not forget to start your Python programs with

```python
import numpy as np
import matplotlib.pyplot as plt
```

`numpy as np` lets us to use numpy functions using the format `np.function()`. Plotting will be mainly done using `matplotlib.pyplot`, so it is useful to get a short name for it as `plt` which is what the second line is doing.

Exercise 1.1. Implement a sampler for a given discrete distribution $p(s_k)$. For numerical purposes, you can use (or anything you like)

```python
p = np.array([0.2, 0.3, 0.2, 0.1, 0.2])
```

You can use `np.cumsum` function to compute the CDF. You can also use `np.uniform.random` for uniform random variate generation. Plot the PMF, histogram, and CDF.

Exercise 1.2. Implement the sampler for Exponential distribution using uniform random numbers. Choose any valid λ, generate samples, plot the pdf as well as the histogram.

Exercise 1.3. Box-Müller method can be defined directly from uniform random numbers (instead of exponential, as described in the class). Define $A = 2\pi U_1, R = (-2\log U_2)^{\frac{1}{2}}$, where U_1, U_2 are independent Uniform(0,1) quantities. Then

$$X = R\cos A, Y = R\sin A$$

are independent $\mathcal{N}(0,1)$ random quantities. Implement the Box-Müller method as described above, using just uniform random numbers. Sample 10000 X and Y as described above and plot the histograms.

Exercise 1.4. Prove that if $X \sim \mathcal{N}(0,1)$, then $Z = \mu + \sigma X$ has

$$Z \sim \mathcal{N}(\mu, \sigma^2).$$

Simulate $Z \sim \mathcal{N}(\mu, \sigma^2)$ using $\mathcal{N}(0,1)$ as generated in Exercise 1.4, with $n = 10000$ and plot the histogram, mean estimate, and variance estimate.

Exercise 1.5. Show that if $X \sim \text{Exponential}(1)$ then $W = \alpha X^{1/\beta}$ has the Weibull distribution with p.d.f.

$$f_W(w) = \beta\alpha^{-\beta}w^{\beta-1}\exp\left[-\left(\frac{w}{\alpha}\right)^{\beta}\right]$$

Hence, explain how you could generate Weibull random quantities using the inversion method. Implement and test this algorithm.