Tutorial: Random Walk

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Tutorial: Simulating a random walk in R

Here we consider the simple random walk on the state space $E = \mathbb{Z}$. Note that the simple random walk can be written as the sum

$$X_n = \sum_{i=0}^{n} Y_i,$$

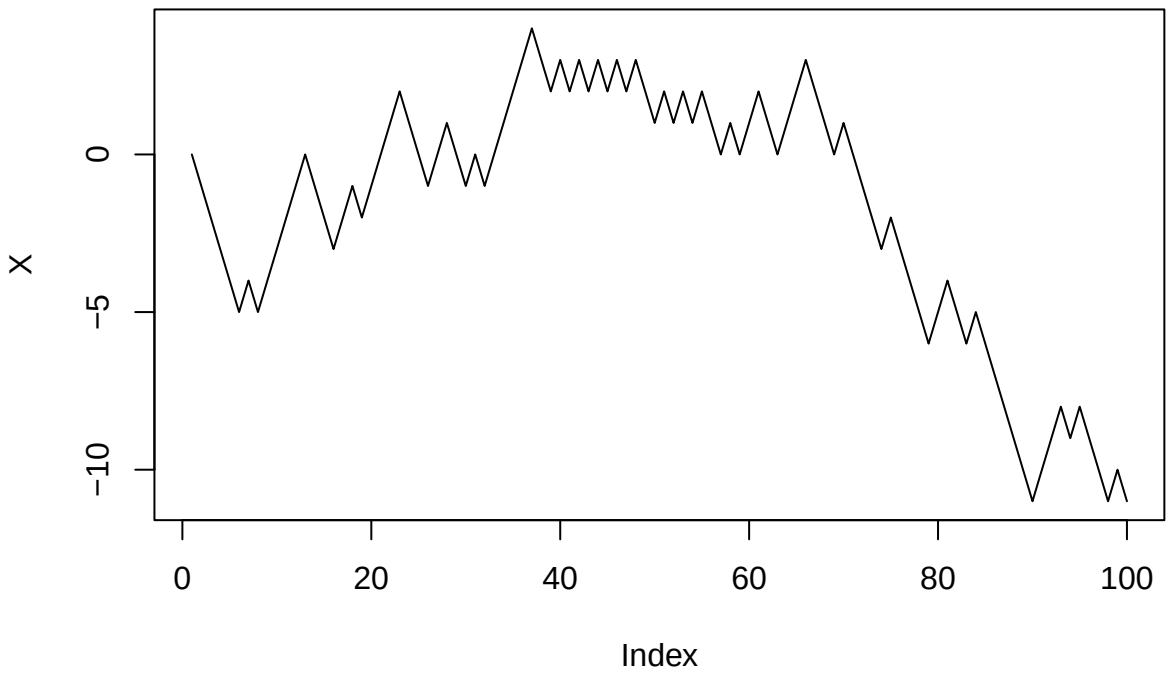
where $Y_1, Y_2, \ldots$ are independent random variables taking the values -1, 1 with probabilities $(1-p)$ and p, respectively, for $p \in (0,1)$. Also $X_0 = Y_0$ denotes the initial value. The transition probabilities of the random walk are given by

$$p_{ij} = \begin{cases} p & \text{if } j = i+1 \\ 1-p & \text{if } j = i-1 \\ 0 & o/w. \end{cases}$$

```
#Simulation of a simple random walk on Z
simRW <- function(initialvalue=0, length=100, p=0.5){

  n<-length
  X<-numeric(n)
  #Assign the initial value
  X[1] <-initialvalue
  #Simulate the Y_i's
  Y<-rbinom(n,1,p) #Simulate n rvs from the Bern(p) distribution
  #Replace the 0s by -1
  Y[Y == 0] <- -1
  for(i in 2:n){
    X[i]<- X[i-1]+Y[i]
  }
  X
}
```

Let us simulate one path of a simple symmetric random walk.

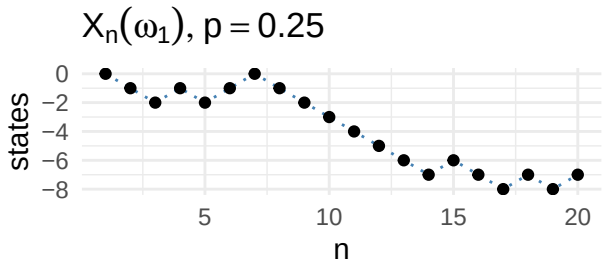
```
X <- simRW()
plot(X, type="l")
```

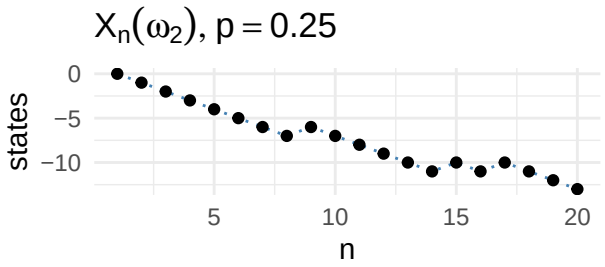


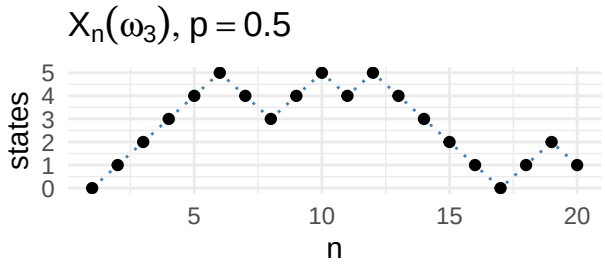
Creating the plots from the lecture notes

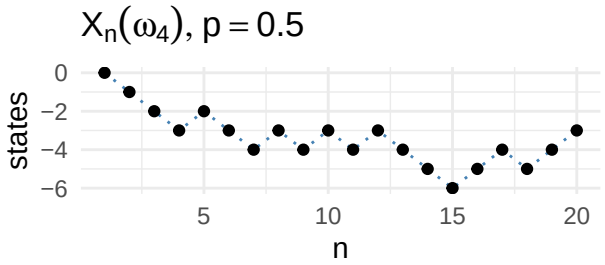
```
library(ggplot2)  #For very pretty plots
library(latex2exp) #For LaTex annotations in the graphs
library(gridExtra) #For combining several plots in one picture

set.seed(1)
len <-20
X1 <- simRW(0, len, 0.25)
X2 <- simRW(0, len, 0.25)
X3 <- simRW(0, len, 0.5)

X4 <- simRW(0, len, 0.5)
X5 <- simRW(0, len, 0.75)
X6 <- simRW(0, len, 0.75)

df_RW1 <- data.frame(n=(1:len), states=X1)
p_RW1 <-ggplot(data=df_RW1, aes(x=n, y=states,group=1)) +
  ggtitle(TeX("$X_n(\\omega_1), p=0.25$"))+
  geom_line( color="steelblue", linetype="dotted") +
  geom_point()+
  theme_minimal()

df_RW2 <- data.frame(n=(1:len), states=X2)
p_RW2 <-ggplot(data=df_RW2, aes(x=n, y=states,group=1)) +
```

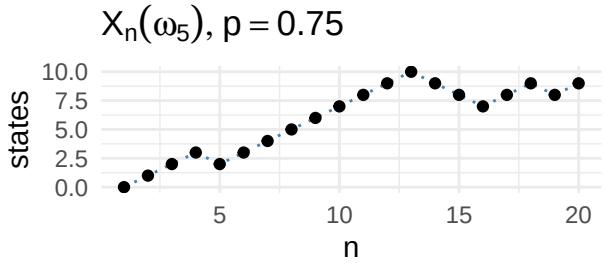
```
  ggtitle(TeX("$X_n(\\omega_2), p=0.25$"))+
  geom_line( color="steelblue", linetype="dotted") +
  geom_point()+
  theme_minimal()

df_RW3 <- data.frame(n=(1:len), states=X3)
p_RW3 <-ggplot(data=df_RW3, aes(x=n, y=states,group=1)) +
  ggtitle(TeX("$X_n(\\omega_3), p=0.5$"))+
  geom_line( color="steelblue", linetype="dotted") +
  geom_point()+
  theme_minimal()


df_RW4 <- data.frame(n=(1:len), states=X4)
p_RW4 <-ggplot(data=df_RW4, aes(x=n, y=states,group=1)) +
  ggtitle(TeX("$X_n(\\omega_4), p=0.5$"))+
  geom_line( color="steelblue", linetype="dotted") +
  geom_point()+
  theme_minimal()

df_RW5 <- data.frame(n=(1:len), states=X5)
p_RW5 <-ggplot(data=df_RW5, aes(x=n, y=states,group=1)) +
  ggtitle(TeX("$X_n(\\omega_5), p=0.75$"))+
  geom_line( color="steelblue", linetype="dotted") +
  geom_point()+
  theme_minimal()

df_RW6 <- data.frame(n=(1:len), states=X6)
p_RW6 <-ggplot(data=df_RW6, aes(x=n, y=states,group=1)) +
  ggtitle(TeX("$X_n(\\omega_6), p=0.75$"))+
  geom_line( color="steelblue", linetype="dotted") +
  geom_point()+
  theme_minimal()

grid.arrange(p_RW1, p_RW2, p_RW3,  p_RW4, p_RW5, p_RW6, ncol=2)
```

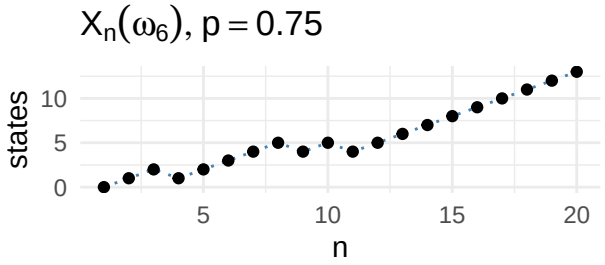

$X_n(\omega_1)$, p = 0.25
states
0
−2
−4
−6
−8
5
10
15
20
n


$X_n(\omega_2)$, p = 0.25
states
0
−5
−10
5
10
15
20
n


$X_n(\omega_3)$, p = 0.5
states
5
4
3
2
1
0
5
10
15
20
n


$X_n(\omega_4)$, p = 0.5
states
0
−2
−4
−6
5
10
15
20
n


$X_n(\omega_5)$, p = 0.75
states
10.0
7.5
5.0
2.5
0.0
5
10
15
20
n


$X_n(\omega_6)$, p = 0.75
states
10
5
0
5
10
15
20
n