Applied Probability Progress Test 1

29 October 2019

Question 1: Let $\{X_n\}_{n\in\{0,1,2,\dots\}}$ denote a discrete-time homogeneous Markov chain with transition matrix $\mathbf{P} = (p_{ij})_{i,j\in E}$ for a state space $E \subseteq \mathbb{Z}$.

(a) (1 point) Define what it means that state j is *accessible* from state i (for $i, j \in E$).

(b) (1 point) Define what it means that states i and j *communicate* (for $i, j \in E$).

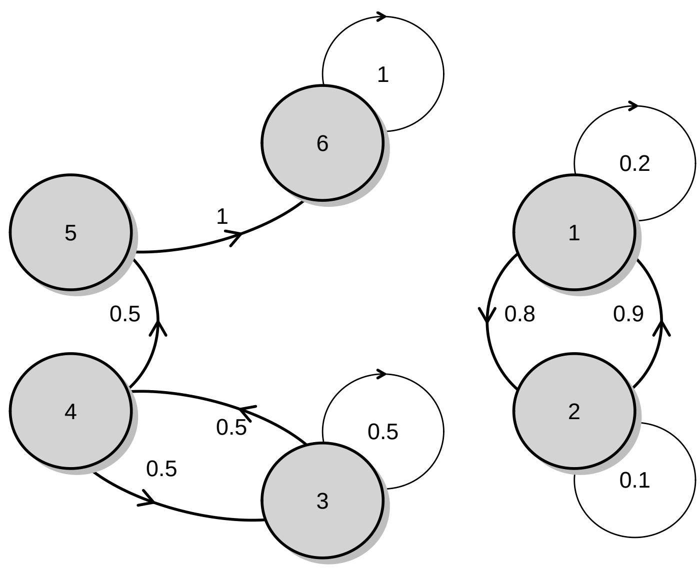
(c) (2 points) Describe (in about two to three sentences) why it can be useful to find the communicating classes of a Markov chain.

Question 2: A die is rolled repeatedly. We denote by Y_n the number of fours in n rolls for $n \in \mathbb{N}$.

(a) (3 points) Show that $Y = (Y_n)_{n\in\mathbb{N}}$ is a Markov chain.

(b) (3 points) State the transition probabilities of the Markov chain Y.

Question 3: Let $X = (X_n)_{n\in\{0,1,2,\dots\}}$ denote a discrete-time, homogeneous Markov chain with state space $E = \{1, 2, 3, 4, 5, 6\}$ and transition diagram given by



(a) (2 points) Find the transition matrix $\mathbf{P} = (p_{ij})_{i,j\in E}$.

(b) (2 points) Determine the communicating classes.

(c) (2 points) For each class, specify whether the class is transient, positive recurrent or null recurrent and justify your answer.

(d) Suppose that the Markov chain starts (at time 0) in state 1 with probability 1/2 and it starts in state 2 with probability 1/2. Find the following probabilities:

i. (1 point) $\mathbb{P}(X_1 = 0.5)$.

ii. (1 point) $\mathbb{P}(X_0 = 1, X_1 = 1)$.

iii. (2 points) $\mathbb{P}(X_0 = 1, X_2 = 1)$.