EXERCISES 2

Exercise 2.1. Sample from a truncated Normal with

$$\bar{p}(x) = \mathcal{N}(x; 0, 1)\mathbf{1}_{x\in[-0.8,0.8]}(x)$$

using $q(x) = \mathcal{N}(x; 0, 1)$. Plot the histogram and the unnormalised density. Can you plot the pdf? You can use `np.random.normal` for sampling from $q(x)$. Try to choose M and the rejection procedure clearly!

Exercise 2.2. Simulate data from the nonlinear model

$$p(x) = \text{Unif}(x; -10, 10)$$
$$p(y|x) = \mathcal{N}(y; a\cos(x) + b, \sigma^2),$$

with $a = 0.5$, $b = 0.5$, and $\sigma = 0.15$. You can use `np.random.normal` for this. Scatter plot (x, y) samples and discuss the behaviour. Plot $p(y)$ using samples/histogram, do you think $p(y)$ is computable any other way?

Exercise 2.3. Sample the following mixture of Gaussians using only uniforms:

$$p(x) = \sum_{k=1}^{5} w_k q_k(x),$$

where

$$q_k(x) = \mathcal{N}(x; \mu_k, \sigma_k^2).$$

Use the Box-Müller transform and a transformation to sample Gaussians with a certain mean and covariance – and inversion to sample from the discrete distribution. The weights are defined as

$$w_1 = 0.1,\ w_2 = 0.2,\ w_3 = 0.3,\ w_4 = 0.2,\ w_5 = 0.2,$$

and

$$\mu_1 = -2,\ \mu_2 = -1,\ \mu_3 = 0,\ \mu_4 = 1,\ \mu_5 = 2$$

and

$$\sigma_1 = 0.5,\ \sigma_2 = 0.1,\ \sigma_3 = 0.5,\ \sigma_4 = 0.2,\ \sigma_5 = 0.5.$$

Note that these are standard deviations, not variances. What would you do if each Gaussian was truncated around their mean $[\mu_k - 0.1, \mu_k + 0.1]$? Think about combining your code with a rejection sampler, if mixture components themselves are hard to sample from.