Errata for *Linear Algebra*, first printing

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Many thanks to everyone who has spotted typos, errors, inaccuracies, and inconsistencies, most especially our intrepid translator, Professor Yoshi Yamamoto!

Page 9: In Exercise 1.1.11, $x_n = tc_n + (1-t)c_n$ should be $x_n = tc_n + (1-t)d_n$.

Page 35: The last column of the augmented matrix in Quick Exercise #15 is incorrect. The matrix should be

$$\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 5 & 0 & -3 \\ 0 & 0 & 1 & -1 & 0 & -3 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

and two of the following formulas should be changed to

$$x_3 = -3 + x_4$$
$$x_5 = 2$$

and the vector form of the solution is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -3 - 2x_2 - 5x_4 \\ x_2 \\ -3 + x_4 \\ x_4 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ -3 \\ 0 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -5 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}.$$

Page 46: The answer to Quick Exercise #20 should begin, "Add i times row 1 to row 2."

In the displayed equations after the Quick Exercise, the second equation should be $y = -2i - (2-i)z$, and the set of solutions should be

$$\left\{ \begin{bmatrix} -1 - iz \\ -2i - (2-i)z \\ z \end{bmatrix} \;\middle|\; z \in \mathbb{C} \right\}.$$

Page 47: In Exercise 1.4.5(a), z should be y.

Page 49: In the second-last line of Exercise 1.4.14, "then products" should be "the products".

Page 55: In the paragraph above Theorem 1.10, the phrase "element of W" should be "element of U."

Page 72: Example 5 should end with "is an eigenvector of $\mathbf{A}$ with eigenvalue $-i$."

Page 82: At the end of the third line of the proof of Theorem 2.6, v should be u.

Page 88: Exercise 2.2.3 should begin "Give an explicit bijective function."

Page 95: In Lemma 2.14, the last equation should be

$$\mathbf{AB} = \begin{bmatrix} \mathbf{a}_1\mathbf{b}_1 & \cdots & \mathbf{a}_1\mathbf{b}_p \\ \vdots & \ddots & \vdots \\ \mathbf{a}_m\mathbf{b}_1 & \cdots & \mathbf{a}_m\mathbf{b}_p \end{bmatrix}.$$

Page 99: The example following Lemma 2.19 should end with "$\mathbf{B}^{-1} = \mathbf{A}$."

Page 107: Quick Exercise #18 should end with "Why does this not contradict Proposition 2.27?"

Page 108: In Algorithm 2.28, the statement "$\mathbf{A}$ does not have an LU decomposition" should be replaced with "the algorithm fails to produce an LU decomposition for $\mathbf{A}$."

Page 109: The sentence "Algorithm 2.28 says..." is incorrect. In the situation described this algorithm fails to produce an LU decomposition for $\mathbf{A}$. In typical circumstances such a matrix does not have an LU decomposition, but it is nevertheless possible.

Page 131: The last sentence of the paragraph beginning, "Suppose that Alice and Bob" should be replaced with:

> If $z \in C(\mathbf{A})$, then Bob can tell that either $y = z$ and nothing went wrong, or that multiple bits were changed in transmission. (We assume in practice that multiple errors are rare enough to ignore.) If, on the other hand, $z \notin C(\mathbf{A})$, then something definitely went wrong. Bob could ask Alice to send that signal again.

Page 159: The third vector in Exercise 3.2.1(d) should be written as $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 3 \end{bmatrix}$.

Page 160: The matrix in Exercise 3.2.5(a) should be $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$.

Page 165: The proof of Lemma 3.22 implicitly uses the fact that U_1 and U_2 are finite-dimensional. This follows from Theorem 3.29, which is proved later.

Page 168: The second sentence of the example should be, "We saw in Example 2 on page 144 that both these lists are linearly independent."

Page 177: At the end of the first sentence, W should be replaced with with $\operatorname{range} \boldsymbol{T}$.

Page 180: In the answer to Quick Exercise #22, $\ker \boldsymbol{T}$ should be $\operatorname{null} \boldsymbol{T}$.

Page 183: Exercise 3.4.4 should begin "Let $\mathbf{A} \in \mathrm{M}_{m,n}(\mathbb{R})$".

Page 190: The second sentence of the proof of Theorem 3.42 should be, "Let $\mathcal{B}_V = (v_1, \ldots, v_n)$ and $\mathcal{B}_W = (w_1, \ldots, w_m)$."

At the end of the third last line, $\boldsymbol{T}_{\mathbf{A}}$ should be $\boldsymbol{T}_{\mathbf{A},\mathcal{B}_V,\mathcal{B}_W}$.

Page 192: In the last paragraph of the proof of Proposition 3.45, each subscript i should be j.

Page 194: In Quick Exercise #26, $\{\mathbf{e}_1, \mathbf{e}_2\}$ should be $(\mathbf{e}_1, \mathbf{e}_2)$.

Page 197: Exercise 3.5.8(b) should ask the reader to compute the vectors:

$$\text{(i)} \left[\boldsymbol{T} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \right]_{\mathcal{E}} \quad \text{(ii)} \left[\boldsymbol{T} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \right]_{\mathcal{B}}$$

Page 204: In the first sentence of the proof of Theorem 3.54, "Theorem 3.53" should be "Corollary 3.53".

In the first displayed equation in the proof of Theorem 3.54, v_j should be v_i.

Page 206: The last matrix in the second and fourth displayed equations should be $\begin{bmatrix} | & & | \\ \lambda_1 \mathbf{v}_1 & \cdots & \lambda_n \mathbf{v}_n \\ | & & | \end{bmatrix}$.

Page 211: The third lines of Exercises 3.6.7 and 3.6.8 should both begin "and $\boldsymbol{T} \in \mathcal{L}\big(\mathbb{R}^3, \mathbb{R}^2\big)$.

Page 214: In Exercise 3.6.24, $\mathrm{M}_{\mathbb{F}}(n)$ should be $\mathrm{M}_n(\mathbb{F})$.

Page 216: In the first line, $\mathbb{F}$ should be $\mathbf{A}$.

Page 218: The matrix $\begin{bmatrix} 1 & -3 \\ 9 & -4 \end{bmatrix}$ should be $\begin{bmatrix} 1 & -3 \\ 9 & -2 \end{bmatrix}$, and the matrix $\begin{bmatrix} -4 & 0 \\ 0 & -6 \end{bmatrix}$ should be $\begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$.

Page 220: At the end of the third paragraph of the proof of Theorem 3.67, "$\boldsymbol{T}$ is also not surjective" should be "$\boldsymbol{T} - \lambda \boldsymbol{I}$ is also not surjective".

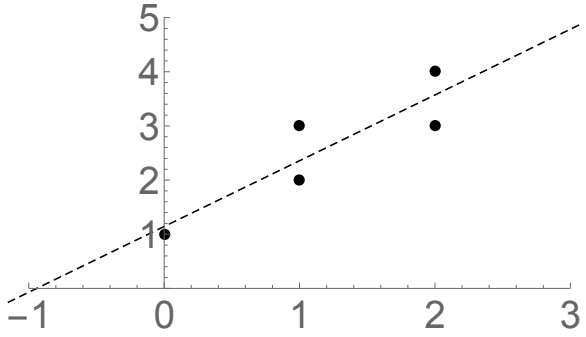
Pages 236–237: In Exercise 4.1.5, the assumption that $\mathbf{A} \in \mathrm{M}_n(\mathbb{R})$ applies to both parts of the exercise.

In Exercise 4.1.6, "Suppose" should be "Show that if".

Page 258: In the first line of the proof of Theorem 4.19, u should be $\boldsymbol{P}_U v$.

Page 259: The proof of Theorem 4.19 should end at "$u = \boldsymbol{P}_U v$."

Page 260: Figure 4.7 should appear as:



Page 263: In Exercise 4.3.2(d), $\mathbb{R}^n$ should be $\mathbb{R}^5$.

Exercises 4.3.3 and 4.3.4 should each begin "Find the matrix (with respect to the standard basis) of the orthogonal projection onto each of the following subspaces."

Page 265: Exercise 4.3.14(c) should state $\boldsymbol{P}_W(\mathbf{A}) = i \operatorname{Im} \mathbf{A}$ instead of $\boldsymbol{P}_W(\mathbf{A}) = \operatorname{Im} \mathbf{A}$.

Page 270: In the first line, "Lemma 4.21" should be "Proposition 4.21".

Page 276: In Exercise 4.4.19, the definition of a strictly convex norm should include the condition $v \neq w$.

Page 280: In the proof of Corollary 4.29, "Theorem 4.12" should be "Corollary 4.12".

Page 286: Exercises 4.5.9 and 4.5.10 should refer to example on page 284.

Page 293: In the first displayed equation, $\boldsymbol{T} f_j$ in the fourth expression should be $\boldsymbol{T} e_j$.

The fifth line of the proof of Theorem 5.3 should begin "For $v \in V$".

Page 294: The second line of the third full paragraph (paragraph beginning "That is, we have shown") should begin, "$\langle \tilde{e}_1, ..., \tilde{e}_{\tilde{r}_1} \rangle = U$, where $\tilde{r}_1$ is the largest index such that $\tilde{\sigma}_{\tilde{r}_1} = \tilde{\sigma}_1 = \|\boldsymbol{T}\|_{op}$."

In the last displayed equation and the following line, each instance of the index k should be r_1.

Page 297: Exercise 5.1.14(c) should state "... then there exists a nonzero $v \in U$ such that ..."

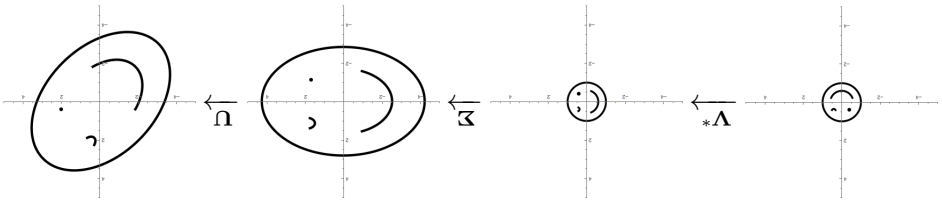
Page 299: In the definition, Corollary 5.4 should be Theorem 5.4.

Page 301: The sentence after the first displayed formula should begin "Since $\mathbf{V}^*$ is unitary, $\mathbf{V}^*$ acts as an isometry".

In the third paragraph, $\mathbf{E}$ should be $\mathcal{E}$.

Page 302: In the caption to Figure 5.3, $\mathbf{U}^*$ should be $\mathbf{U}$.

The answer to Quick Exercise #5 should be



Page 306: Lemma 5.10 should begin "Let $W \subseteq \mathbb{C}^m$ ..."

Page 307: In the first sentence of the second paragraph of the proof of Theorem 5.9, $\mathbb{F}^m$ should be $\mathbb{C}^n$.

Page 310: In Exercise 5.2.11, the string of inequalities should be $\sigma_1 \geq \cdots \geq \sigma_n > 0$.

Page 311: In Exercise 5.2.17(b), $(1/2, 1/4)$ should be $(1, 1/2)$.

Page 312: The first Example should state, "... then $\boldsymbol{T}^*$ is represented by ..."

Page 313: The displayed equation in Theorem 5.13 should be

$$[\boldsymbol{T}^*]_{\mathcal{B}_W,\mathcal{B}_V} = \Big([\boldsymbol{T}]_{\mathcal{B}_V,\mathcal{B}_W}\Big)^*.$$

Page 315: The argument in the paragraph following the statement of Proposition 5.16 implicitly uses the fact that $\operatorname{rank} \boldsymbol{T}^* = \operatorname{rank} \boldsymbol{T}$, which has not been proved up to this point. One can prove this fact using either Exercise 5.2.6 or Exercise 5.3.20(d).

Page 318: The equation in the second Key Idea should be $[\boldsymbol{T}^*]_{\mathcal{B}_W,\mathcal{B}_V} = \Big([\boldsymbol{T}]_{\mathcal{B}_V,\mathcal{B}_W}\Big)^*$.

Page 320: In Exercise 5.3.20(b), e_m should be e_n.

In Exercise 5.3.20(c), f_n should be f_m.

Page 324: The equation in the first Example should be

$$(\sqrt{\mathbf{A}})^2 = (\mathbf{U}\,\mathbf{diag}(\sqrt{\lambda_1}, \ldots, \sqrt{\lambda_n})\mathbf{U}^*)^2 = \mathbf{U}\,\mathbf{diag}(\lambda_1, \ldots, \lambda_n)\mathbf{U}^* = \mathbf{A}.$$

The last two displayed equations should be

$$\mathbf{A}^* = \mathbf{U}\,\mathbf{diag}(\lambda_1, \ldots, \lambda_n)^*\mathbf{U}^* = \mathbf{U}\,\mathbf{diag}(\overline{\lambda_1}, \ldots, \overline{\lambda_n})\mathbf{U}^*$$

and

$$\begin{aligned}\mathbf{A}^*\mathbf{A} &= \mathbf{U}\,\mathbf{diag}(\overline{\lambda_1}, \ldots, \overline{\lambda_n})\mathbf{U}^*\mathbf{U}\,\mathbf{diag}(\lambda_1, \ldots, \lambda_n)\mathbf{U}^* \\ &= \mathbf{U}\,\mathbf{diag}(|\lambda_1|^2, \ldots, |\lambda_n|^2)\mathbf{U}^* \\ &= \mathbf{A}\mathbf{A}^*.\end{aligned}$$

Page 326: In the Example, "counterclockwise" should be "clockwise".

Page 329: In the fourth Key Idea, "a Hermitian $\mathbf{U}$" should be "a unitary $\mathbf{U}$".

Page 341: The sentence beginning on the seventeenth line from the bottom of the page should begin, "We can obtain $\mathbf{A}_{i\ell}$ from $\mathbf{A}_{ik}$".

Page 346: Exercise 6.1.16(a) should begin, "Show that V is a vector space over $\mathbb{F}$..."

Exercise 6.1.17(a) should begin, "Show that W is a vector space over $\mathbb{F}$..."

Page 351: In the solution to Quick Exercise #8, the third step is to add -3 times the first row to the third row.

Page 354: The third Key Idea should be:

> To compute $\det \mathbf{A}$ via row operations: row-reduce $\mathbf{A}$ to an upper triangular matrix $\mathbf{B}$ using only row operations **R1** and **R3**. Then
>
> $$\det \mathbf{A} = (-1)^k b_{11} \cdots b_{nn},$$
>
> where k is the number of times **R3** is used.

Page 356: In the hint to Exercise 6.2.10, the second sentence should begin "Then show that $g(\mathbf{D}) = \det \begin{bmatrix} \mathbf{I}_m & \mathbf{C} \\ \mathbf{0} & \mathbf{D} \end{bmatrix}$".

Page 362: In the fifth line from the end of Proposition 6.25, $(-1)^n \det \mathbf{A}$ should be just $\det \mathbf{A}$.

Page 366: Exercise 6.3.18 should ask the reader to show that $p_{\mathbf{A}}(x) = (-1)^n p(x)$.

Page 374: In Exercise 6.4.3, $\mathcal{C}$ should be "the unit disc".

Page 379: In the eleventh line, "colon" should be "bar".

Page 392: The answers to the first three parts of Exercise 1.2.7 should be as follows:

(a) $x = -\frac{7}{2} + z$, $y = \frac{15}{4} - 2z$, $z \in \mathbb{R}$. (b) $x = 1 + \frac{1}{7}z - 2w$, $y = \frac{5}{7}z - w$, $z, w \in \mathbb{R}$. (c) Solutions are $x = 2 - z$, $y = 1 - z$, $z \in \mathbb{R}$.

Page 398: The answer to Exercise 2.2.3 should end with "the required bijective function."

Page 400: The answer to Exercise 2.4.7(b) should be $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 0 & -1 \end{bmatrix}$.

Page 402: The answer to Exercise 2.6.3(b) should be $\mathbf{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

The answer to Exercise 2.6.3(d) should be $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, or $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

Page 404: The answer to Exercise 3.2.7(b) should be $\frac{3}{2}\begin{bmatrix} 2 \\ 3 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 4 \\ -5 \end{bmatrix}$.

Page 407: In the anser to Exercise 3.5.11, $[\boldsymbol{TD}] \in \mathrm{M}_{n+1}(\mathbb{R})$.

Page 408: The answer to Exercise 3.6.1(a) should be

$$[\boldsymbol{I}]_{\mathcal{B},\mathcal{C}} = \frac{1}{2}\begin{bmatrix} -13 & -19 \\ -9 & -13 \end{bmatrix}, \qquad [\boldsymbol{I}]_{\mathcal{C},\mathcal{B}} = \begin{bmatrix} 13 & -19 \\ -9 & 13 \end{bmatrix}.$$

In the answer to Exercise 3.6.1(c), the factor of $\frac{1}{2}$ should not appear.

The answer to Exercise 3.6.9(b) should be

$$[\boldsymbol{I}]_{\mathcal{B},\mathcal{C}} = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}, \qquad [\boldsymbol{I}]_{\mathcal{C},\mathcal{B}} = \begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}.$$

Page 409: In the answer to Exercise 3.6.23, the basis should be $(v_1, \ldots, v_n)$.

Page 411: The answer to Exercise 4.2.5(c) should be $\begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{2\pi i/3} & 0 \\ 0 & 0 & e^{4\pi i/3} \end{bmatrix}$.

The answer to Exercise 4.2.7(c) should be $\frac{1}{2}\begin{bmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{bmatrix}$.

The answer to Exercise 4.2.11(a) should be $\left(\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{6}}\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}\right)$.

The answer to Exercise 4.2.11(b) should be $\left(\frac{1}{\sqrt{5}}\begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}, \frac{1}{\sqrt{30}}\begin{bmatrix} 5 \\ -2 \\ -1 \end{bmatrix}, \frac{1}{\sqrt{6}}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}\right)$.

The answer to Exercise 4.2.15(c) should be $\left(\sqrt{\frac{3}{2}}, \sqrt{\frac{5}{2}}x, \frac{5\sqrt{7}}{2\sqrt{2}}\left(x^2 - \frac{3}{5}\right), \frac{21}{2\sqrt{2}}x^3 - \frac{15}{2\sqrt{2}}\right)$.

In the answer to Exercise 4.2.21, the expression for $\widetilde{e}_j$ should end with $\langle v_j, e_{j-1}\rangle\, e_{j-1}$.

Page 412: The answer to Exercise 4.3.3 should be

$$\frac{1}{7}\begin{bmatrix} 3 & -2+2i & 2i \\ -2-2i & 5 & -1+i \\ -2i & -1-i & 6 \end{bmatrix}.$$

At the end of the answer to Exercise 4.3.11, the 9 should be 10.

In the answer to Exercise 4.3.13, the $\frac{1}{2\pi}$ should be $\frac{1}{\sqrt{2\pi}}$, and the final answer should be $g(x) = \pi - 2\sin(x) - \sin(2x)$.

In the answer to Exercise 4.3.19, the orthonormal basis should be $(e_1^1, ..., e_{n_1}^1, ..., e_1^m, ..., e_{n_m}^m)$.

Page 414: The answer to Exercise 4.5.7(c) should be

$$\mathbf{Q} = \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix}, \qquad \mathbf{R} = \begin{bmatrix} \sqrt{2} & -\sqrt{2} & \sqrt{2} \\ 0 & 2 & -1 \\ 0 & 0 & \sqrt{2} \end{bmatrix}.$$

The answer to Exercise 4.5.13 should begin "All that is needed".

Page 415: In the answer to Exercise 5.1.3, "clockwise" should be "counterclockwise".

In the answer to Exercise 5.1.5, $\frac{1}{2\pi}$ should be $\frac{1}{\sqrt{2\pi}}$, "left" should be "right", and "right" should be "left".

In the answer to Exercise 5.1.13, v_{k-1} should be V_{k-1}.

Page 416: In the answer to Exercise 5.2.3, $\|\mathbf{A}_z\|_F$ should be $\|\mathbf{A}_z\|_F^2$.

In the answer to Exercise 5.2.15(a), $\mathbf{V}^*\mathbf{\Sigma}^\dagger\mathbf{U}\mathbf{A}$ should be $\mathbf{V}\mathbf{\Sigma}^\dagger\mathbf{U}^*\mathbf{A}$.

Page 417: In the answer to Exercise 5.3.13, $\|\boldsymbol{T}v\|$ should be $\|\boldsymbol{T}v\|^2$.

Page 418: In the answer to Exercise 5.4.15, $r+s=n+j-i+1$ should be $r+s=n+j-i+2$, and $r+s=j-i+1$ should be $r+s=j-i+2$.

In the answer to Exercise 5.3.21, U_1 should be u_1.

Page 419: In the answer to Exercise 5.4.24, $[\mathbf{A}\mathbf{A}^*]_{ii}$ should be $[\mathbf{A}\mathbf{A}^*]_{11}$.

Page 420: The answer to Exercise 6.3.1(c) should begin $p(\lambda)=-\lambda^3+6\lambda^2-11\lambda+6$.

The answer to Exercise 6.3.1(d) should begin $p(\lambda)=-\lambda^3+12\lambda+16$.