New Generalized Structural Filtering Concept for Active Vibration Control Synthesis

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A new concept of generalized structural filtering and its application to active vibration control synthesis are presented. The concept is a natural extension of the classical notch and phase-lead/-lag filtering, and emphasizes the use of a nonminimum-phase filter, which has zeros in the right-half *s* plane. Application of this concept to single-input/single-output systems with many oscillatory modes results in a robust feedback compensator with much physical insight. The concept also enables the control designer to understand the inherent nature of an "optimal" compensator and to modify the optimal design to be more robust and meaningful. This paper shows that, for certain cases, nonminimum-phase structural filtering provides the proper phase lag to increase the closed-loop damping of the flexible modes while maintaining good performance and robustness to parameter variations.

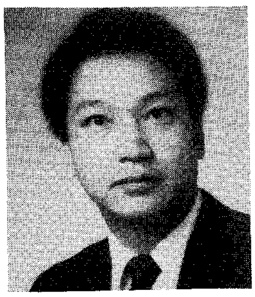
Introduction

ALTHOUGH the last two decades have brought major developments in modern control theory,[1,2] the most usual approach to the design of practical feedback control systems has been repetitive synthesis using the classical single-input/single-output methods in the frequency domain. Application of the standard linear-quadratic-Gaussian (LQG) technique to flexible structure control often results in a feedback compensator that is of high order and sensitive to model uncertainty. As a result, many different ways of improving the robustness of LQG controllers have been developed and applied to flexible structure control problems.[3–9] Some of them are tuned to a specific model of the plane uncertainty. Consequently, the application of LQG/loop-transfer-recovery (LTR) techniques[7,8] often results in cancellations of the plant zeros by the compensator poles. Whenever these cancellations occur near the imaginary axis, the closed-loop system becomes very sensitive to the plant zero uncertainty.

Many different approaches (H^∞ optimization, stable factorization, etc.) have been developed for the robust compensator design in the frequency domain,[10–14] but most of these techniques produce compensators of higher order than the plant. This result may conflict with one's intuition; one might expect the lower-order compensator to be more robust than the higher-order compensator, unless all the compensator poles are placed sufficiently far to the left from the imaginary axis. This is what happens in most of the modern frequency-domain designs, which results in a very high bandwidth controller that becomes sensitive to noises and unmodeled high-frequency dynamics. It seems likely that more ways of improving the modern control techniques will be discovered as control researchers become more familiar with the fundamental nature of controlling many oscillatory modes using a single actuator/sensor pair.

This paper presents an intuitively meaningful design procedure based on a new concept of generalized structural filtering. The concept is based on the classical concepts of gain-phase stabilization, which can be defined briefly as[18]:

1) A gain-stabilized mode is one that is closed-loop stable for the selected loop gain and assumed passive dumping ratio



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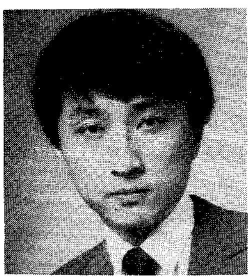

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but can become unstable if the gain is raised or the passive damping ratio reduced. Hence, it has a finite gain margin but is closed-loop stable regardless of the phase uncertainty.

2) A phase-stabilized mode is one that is closed-loop stable for arbitrarily small passive damping ratios. It has a finite-phase margin.

Phase stabilization provides the proper gain and phase characteristics at the desired frequency to obtain a closed-loop damping of the mode greater than the passive damping. Gain stabilization provides attenuation of the control loop gain at the desired frequency to ensure stability regardless of the phase uncertainties. Depending on the specific application, one type may exhibit superior features over the other.

Second-order structural filters with limited pole-zero patterns have been used in practice for proper gain-phase stabilization of flexible modes. For examples, Refs. 15–19 discuss practical implementation of such filters for structural-mode compensation of spacecraft and launch vehicles. Notch filtering (e.g., used for the ARABSAT[15] and Galileo[16] spacecraft) is a conventional way of suppressing an unwanted signal in the control loop, resulting in gain stabilization of a particular mode. The use of notch filtering ensures that the specific mode is not to be destabilized by feedback control; it does not introduce any active damping, which often results in too much "ringing," which may not be acceptable in certain cases. In general, roll-off of the control loop gain at frequencies above the controller bandwidth is also needed to avoid destabilizing unmodeled higher-frequency modes.[17] For the OSO-8 spacecraft,[18] a second-order structural filter was employed to add more phase lead at a particular flexible mode frequency, which results in phase stabilization of the mode. Such a second-order phase-lead filter has often been called a notch filter,[18,20] which may not be a proper use of terminology. The common characteristic of these classical filters used in Refs. 15–19 is that they are minimum-phase filters. While nonminimum-phase filters have rarely been used by practicing control designers, a nonminimum-phase filter with a zero on the positive real axis has been implemented for the active nutation control of the INSAT and ARABSAT spacecraft.[17]

This paper generalizes these classical structural filters for direct synthesis of a robust feedback compensator. Various nonminimum-phase filters are developed for robust compensation of those unstably interacting modes in the "noncolocated" control problem. A successive mode-stabilization approach to the single-input/single-output control synthesis for systems with many oscillatory modes is outlined. The concept and approach presented in this paper do not appear to have been published previously in the open literature. They may also be applicable to multivariable control design where a frequency-shaped estimation technique[21] or a classical "successive-loop-closure" approach[22] is employed.

In the next section, issues in flexible structure control are discussed to emphasize the importance of the classical concepts of gain-phase stabilization and to explain the motivation for developing the generalized structural filtering concept.

Colocated/Noncolocated Control Issues

When the actuator and sensor are colocated on a free-free structure, the rigid-body mode and all the flexible modes are stably interacting with each other. In this case, position feedback with a lead compensator or combined position and rate feedback can be used to stabilize all of the flexible and rigid-body modes. Since all of the modes are phase-stabilized in this case, the phase uncertainty from the control loop time delay and actuator/sensor dynamics must be considered. As frequency increases, the phase lag due to a time delay will eventually exceed the maximum phase lead of 90 deg from the direct rate feedback. In practice, roll-off filtering is often needed to attenuate the control loop gain at frequencies above the controller bandwidth.[17]

The selection of roll-off filter corner frequency depends on many factors. When a colocated actuator/sensor pair is used, the corner frequency is often selected between the primary flexible modes and the secondary modes. An attempt to passive filtering of all flexible modes should be avoided unless the spacecraft or structures are nearly rigid. In practice, the actual phase uncertainty of the control loop must be accounted for the proper tradeoff between phase and gain stabilization, as was done for the ARABSAT spacecraft.[15,17]

When the actuator and sensor are not colocated (e.g., Galileo[16] and OSO-8[18] spacecraft), the rigid-body mode and some of the flexible modes are unstably interacting with each other. Unless passive filtering of all of the flexible modes is possible for a low-bandwidth control, a proper combination of gain-phase stabilization is unavoidable. As demonstrated experimentally in Ref. 23, controlling a flexible structure using a noncolocated actuator/sensor pair is not a trivial control problem. It becomes an extremely difficult problem, especially if a high control bandwidth is required in the presence of many closely spaced, unstably interacting, lightly damped modes with a wide range of parameter variations.

Many of the optimal feedback controllers designed for this type of problem are necessarily tuned closely to the control design model and/or to a specific model of the plant uncertainty. Consequently, such controllers are often of high order and impractical to implement. In this paper, a very simple concept for direct synthesis of a robust feedback compensator is introduced. The concept allows the control designer to properly gain-phase stabilize each mode, one-by-one, resulting in a more meaningful design with physical insight.

Generalized Structural Filtering Concept

This concept is a natural extension of the classical notch and phase-lead/-lag filtering and emphasizes the use of a nonminimum-phase filter. The concept is based on various pole-zero patterns that can be realized from a second-order filter represented as

$$\frac{s^2/\omega_z^2 + 2\zeta_z s/\omega_z + 1}{s^2/\omega_p^2 + 2\zeta_p s/\omega_p + 1}$$

where s is the Laplace transform variable. For different choices of the coefficients of the above second-order filter, several well-known frequency-shaping filters such as notch (band-reject), bandpass, lowpass, highpass, and phase-lead/-lag filters can be realized. In addition to these minimum-phase filters, various nonminimum-phase filters can also be realized from this second-order filter. In this paper, only stable filters with poles in the left-half s plane will be considered. Although it is usually discretized for the microprocessor implementation,[15-17] this paper deals with a continuous-time filter represented in the s domain.

Figure 1 illustrates some of the typical pole-zero patterns and the gain-phase characteristics of various filters that can be realized from this generalized second-order filter. These basic pole-zero patterns of a second-order filter, especially the nonminimum-phase filters, are the essence of the generalized structural filtering concept. Any other compensator pole-zero patterns are basically a combination of the basic filters shown in Fig. 1. Further discussion on these filters can be found in the Appendix.

In the next section, an intuitively meaningful procedure for designing a feedback compensator using the concept of generalized structural filtering is discussed.

Procedure for Robust Compensator Synthesis

The basic idea of this approach is to synthesize the compensator for each mode, one-by-one. This "successive mode-stabilization" way of synthesizing the feedback compensator provides physical insight into how the design can be performed to be more robust and practical. This type of trial-and-error approach is difficult to use, especially for higher-order systems, but the proper use of the basic filter patterns shown in Fig. 1

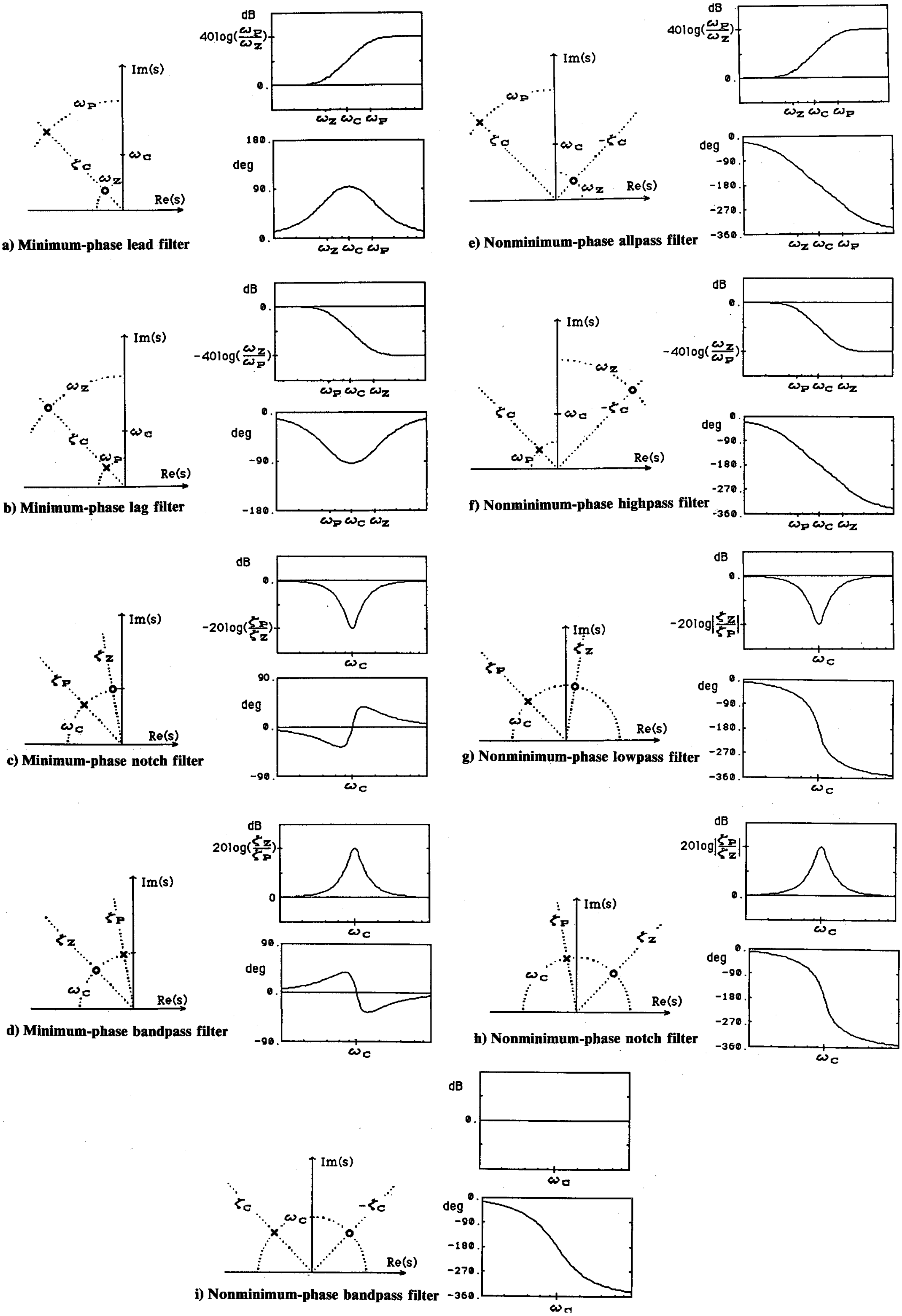

Fig. 1 Pole-zero patterns and gain-phase characteristics of generalized second-order structural filter.

results in a straightforward design with a few iterations. In particular, the use of the nonminimum-phase filtering concept significantly enhances the classical successive mode-stabilization approach. However, the usefulness of this design procedure depends upon the familiarity of the user with the classical control techniques.

A procedure for single-input/single-output compensator synthesis for a system with a rigid body and many oscillatory modes is briefly outlined here as:

Step 1: Control Bandwidth Selection

The control loop bandwidth is one of the key parameters in controller design. Selection of the control bandwidth depends on many factors, including the performance, noise sensitivity, limited control authority, etc. The control bandwidth is closely related to the settling time and determined primarily by the closed-loop poles of the rigid-body mode. In many cases, it is specified a priori.

Step 2: Rigid-Body Mode Compensation

By ignoring the flexible modes, determine the position and rate gains of a rigid-body mode compensator, $C(s) = K(\tau s + 1)$, to achieve the desired control bandwidth. If direct rate feedback is not possible, synthesize a first-order lead filter, $C(s) = K(\tau_1 s + 1)/(\tau_2 s + 1)$, with a lead ratio not much greater than 10. The selection of this ratio depends on the sensor noise, dominant flexible mode frequency, and unmodeled high-frequency modes.

Step 3: Flexible Mode Compensation

Examine the closed-loop stability of each flexible mode after closing the loop with the controller designed in step 2. The types and degrees of the closed-loop behavior of each flexible mode depend on the actuator/sensor location and the relative spectral separation of each mode. Examine whether a simple passive filtering of all the flexible modes is possible or not. If not, then determine the necessary phase-lead or -lag angles for each "destabilized" mode. Synthesize an appropriate structural filter for each "destabilized" mode, one-by-one, using the various second-order filters shown in Fig. 1. In this step, some skill and intuition in the classical direct frequency-shaping approaches are needed, which may be the most significant shortcoming of the successive mode-stabilization approach.

Step 4: Design Iteration

Repeat the design process in order to compromise some interactions between each compensation. A few iterations using the professional software packages (for Bode plot, root locus, simulation, etc.) that have recently become available will result in a quick and straightforward design with physical insight. When a trial design is completed, then perform the closed-loop stability analysis to ensure that the design has adequate robustness to a specified or assumed range of parameter variations. Checking the closed-loop stability for all possible situations is not a trivial problem. The closed-loop stability may be checked by a uniform increase or decrease of each flexible mode frequency. This provides a simple verification of the effect of stiffening or softening the structure on the overall closed-loop stability.[8]

Next, the successive mode-stabilization approach, using the concept of generalized structural filtering, is applied to several examples of flexible structure control problems.

Design Examples

Consider a two-mass spring system shown in Fig. 2, which is a well-known generic example of a flexible spacecraft with noncolocated actuator and sensor.[4,6,20,24] For a direct comparison with the LQG designs presented in Refs. 6 and 20, it can be assumed that the two bodies both have unit mass and are connected by a spring with spring constant k. The spring constant has a nominal value of 1 and is assumed to be uncertain ($0.5 < k < 2$). As a result, the flexible mode frequency can vary between 1 and 2 rad/s, with a nominal value of 1.4 rad/s. The robustness of the control loop to the mass uncertainty is measured by the control loop gain margin.

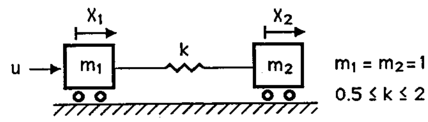

Fig. 2 Two-mass-spring system.

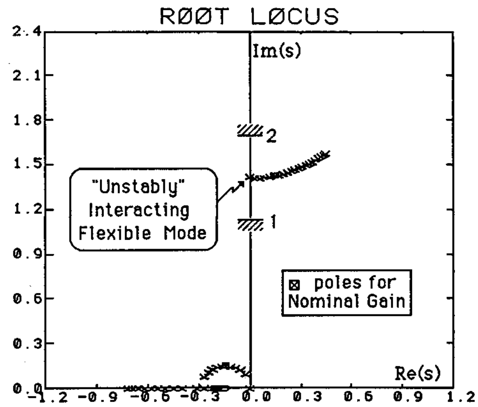

Fig. 3 Root locus vs loop gain using feedback of position and rate of body 2.

A common approach to design when a range of parameter uncertainty is given is to select the "worst case" as a nominal case so that no possible value of the parameters can cause instability[20]; however, in the present case, the nominal design point at $k = 1$ is selected following Ref. 6. A control force acts on body 1 and only the position and rate of body 2 are assumed to be measured. This is a typical noncolocated actuator/sensor problem with uncertainty in model parameters. A transfer-function description of this simple generic model is

$$\begin{bmatrix} x_1(s) \\ x_2(s) \end{bmatrix} = \frac{1}{s^2(s^2+2k)} \begin{bmatrix} s^2+k \\ k \end{bmatrix} u(s)$$

The closed loop poles of the rigid-body mode at $\omega_n = 0.2$ rad/s with a closed-loop damping ratio of 0.7 are chosen. As a first attempt, the rigid-body mode compensator is selected as $C(s) = 0.086(s/0.15 + 1)$. It can be seen from Fig. 3 that the flexible mode becomes unstable due to the "unstable" interaction between the flexible mode and the rigid-body mode control logic.

In order to properly stabilize this "destabilized" mode, approximately $\pm$ 180 deg of phase shift is needed at the flexible mode frequency. The first approach toward solving this problem is to provide 180 deg phase lead at the flexible mode frequency by using the minimum-phase-lead filter shown in Fig. 1a. In this case, the filter zeros with frequency lower than the flexible mode frequency are placed near the imaginary axis. The filter poles associated with the zeros are then usually placed sufficiently far to the left of the imaginary axis. A similar design example based on this approach can be found in Ref. 20 (pp. 457–459), where such phase-lead filtering is misleadingly called notch filtering. This common approach with phase-lead compensation may not be acceptable if the ω_p/ω_z ratio is chosen to be too large, which would amplify any measurement noise intolerably. A typical value for this ratio would be about 2, and some compromise between performance and noise sensitivity should be taken in selecting this ratio.

The second approach is to employ the conventional notch filter (Fig. 1c) or the nonminimum-phase notch filter (Fig. 1h). The conventional notch filtering gain stabilizes the flexible mode without adding any active damping to the system. If more active structural damping is required, or if there is no natural passive damping in the flexible mode, the conventional notch filtering is not an appropriate solution. The passive vibration suppression for a case with no natural damping requires the use of nonminimum-phase notch filtering, which provides the proper gain and phase adjustments at the flexible mode frequency.

The third approach is to employ the nonminimum-phase allpass filter (see Fig. 1e), which maintains the control loop gain and provides the proper phase lag of the flexible mode signals, resulting in an increased closed-loop damping ratio of the flexible mode (active damping!). The filter poles and zeros are selected as $\omega_p = \omega_z = \sqrt{2k}$ ($k = 1$ for nominal case) and $\zeta_p = -\zeta_z = 0.5$. Figure 4 shows the root locus vs overall loop gain. For this design, the gain margin is 5 dB, and the rigid-body and flexible modes have phase margins of 37 and 64 deg., respectively. The flexible mode has a closed-loop damping ratio of 0.1 and the rigid-body mode has a 0.7 damping ratio. Figure 5 shows a locus of closed-loop poles vs spring constant k; the locus indicates that the closed-loop system is stable for $0.5 < k < 2.1$. The Bode plots of the compensator and the loop transfer function are shown in Fig. 6, which illustrates the proper gain and phase margins of the closed-loop system. Response of the nominal system to an impulse disturbance at body 1 is shown in Fig. 7; the closed-loop system has a settling time of about 30 s with almost no "ringing."

The preceding design has a control bandwidth of $\omega_n = 0.2$ rad/s which is relatively low compared to the flexible mode frequency of 1 rad/s. Design of a high-bandwidth controller using a classical approach is also possible, although it was not attempted further in Ref. 20 for this same example. A bandwidth of 0.7 rad/s is selected. Only position measurement of body 2 is assumed in order to illustrate use of a simple first-order lead filter for rigid-body mode compensation. After placing the lead filter zero at $s = -0.2$, the filter pole is placed at $s = -2.0$. The nominal loop gain is chosen as $K = 0.167$, which is twice of the loop gain of the previous low-bandwidth design. Less than 90-deg phase lead at the flexible mode frequency from this filter can be expected. As a result, ω_p and ω_z are selected to be slightly above the flexible mode frequency (Fig. 8). This accounts for a phase-lag requirement less than 180 deg for flexible-mode stabilization. Selection of the damping coefficients of the structural filter is based upon the compromise between performance and robustness. $\zeta_p = -\zeta_z = 0.5$

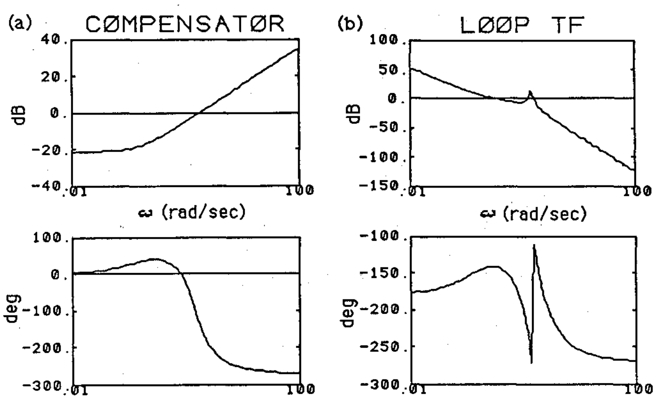

Fig. 6 Bode plots for compensator and loop transfer function.

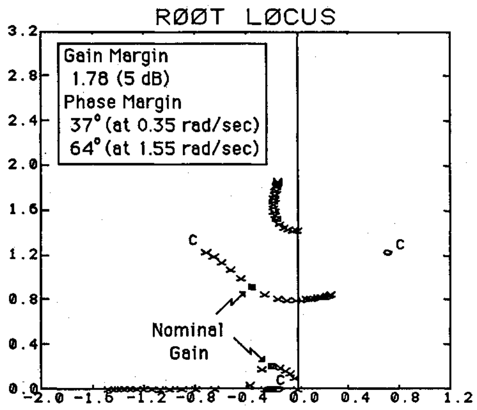

Fig. 4 Root locus vs loop gain using feedback of position and rate of body 2 with nonminimum-phase structural filtering.

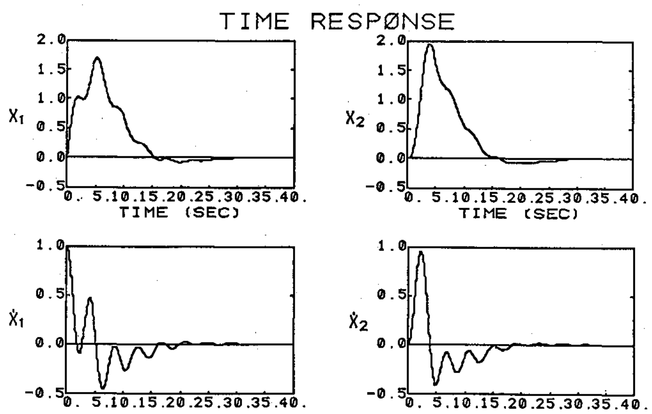

Fig. 7 Closed-loop time response to an impulse disturbance.

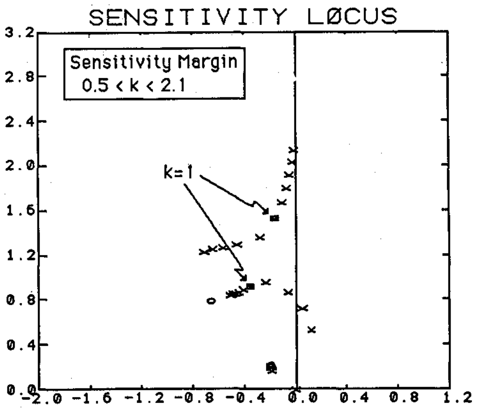

Fig. 5 Sensitivity root locus vs spring constant k.

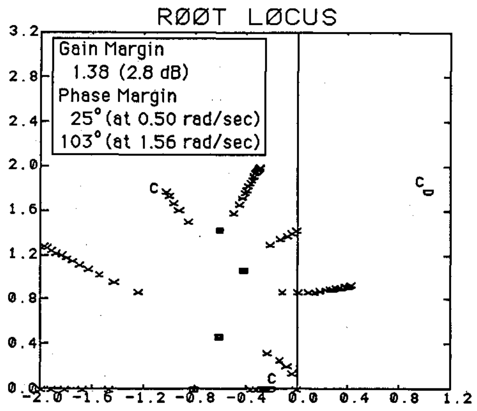

Fig. 8 Root locus vs loop gain for the high-bandwidth control design.

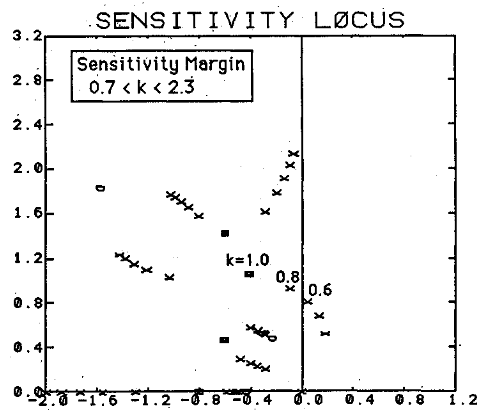

Fig. 9 Sensitivity root locus vs spring constant k (high-bandwidth control design).

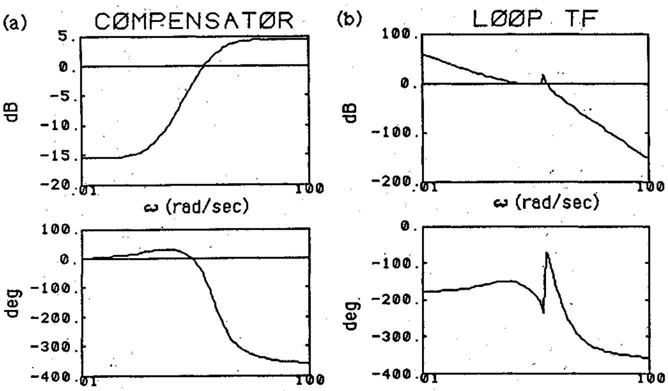

Fig. 10 Bode plots for the high-bandwidth control design.

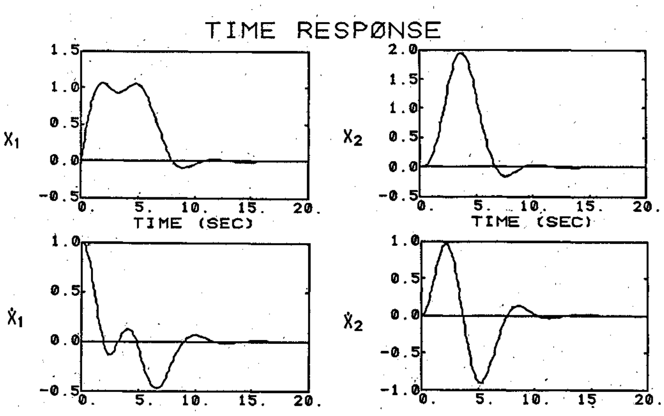

Fig. 11 Closed-loop time response to an impulse disturbance (high-bandwidth control design).

is chosen, and Fig. 8 shows a locus of closed-loop poles vs overall loop gain for this high-bandwidth design. It can be observed that the rigid-body mode has $\omega_n = 0.75$ rad/s and a closed-loop damping ratio of 0.79. The flexible mode has a closed-loop damping ratio of 0.36, while the loop gain margin is now only 2.8 dB (high performance with less robustness!). Figure 9 shows the locus of closed-loop poles vs spring constant k for this high-bandwidth design, while the closed-loop system is stable only for $0.7 < k < 2.3$. As a result, some compromise between performance and robustness is needed, if such a high-bandwidth design is really required. The Bode plots for this high-bandwidth design are shown in Fig. 10, while Fig. 11

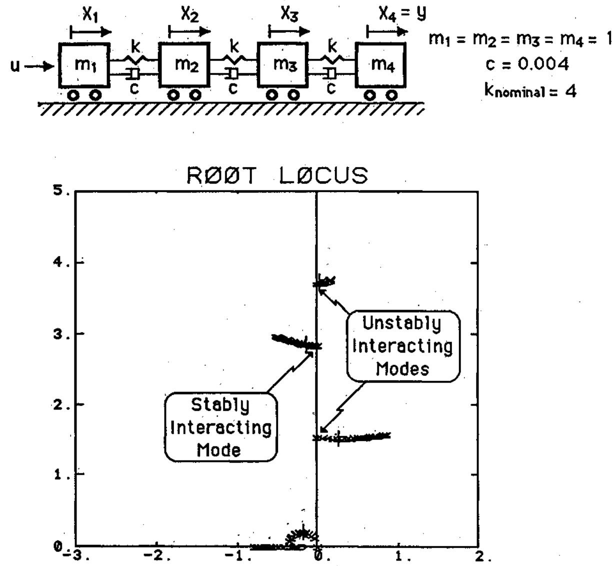

Fig. 12 Root locus vs loop gain for eighth-order example with rigid-body compensation.

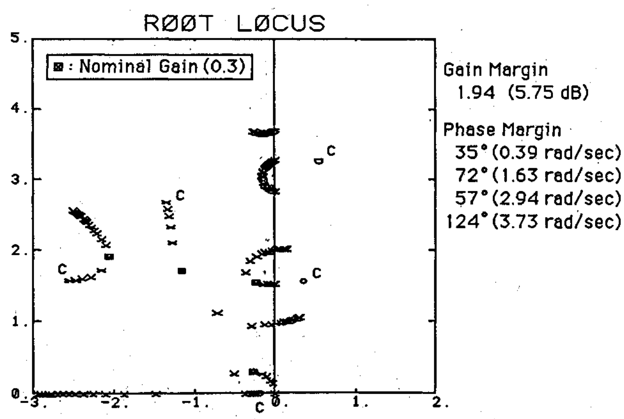

Fig. 13 Root locus vs loop gain for eighth-order example with flexible mode compensation.

shows the response to an impulse disturbance at body 1. A fast transient control with a settling time of about 15 s can be seen; during that time, not much flexible mode "ringing" can be observed.

The preceding high-bandwidth design result is very similar to the modified LQG controller design by Bryson et al.[6] The high-performance robust compensator designed by Bryson et al. for this same problem has a nonminimum-phase zero at $s = 2.4$ and does not have complex zeros near the vibration frequency. Although our compensator has complex nonminimum-phase zeros, the phase plot of our compensator (Fig. 10a) is very similar to that of the robust optimal compensator by Bryson et al.; both designs provide the proper phase "lag" to stabilize the flexible mode. These two different designs ("classical" and "modern") have nearly identical root locus vs overall loop gain and have the same sensitivity and stability margins. It is also interesting to note that, for this same problem, an optimal observer/regulator design by pole placement via the symmetric root locus has resulted in complex phase-lead filtering of the flexible mode,[20] which has accompanied a significant gain increase at higher frequencies. It is emphasized that the proper use of a nonminimum-phase filter, as opposed to minimum-phase filtering, provides the proper compromise between the performance and robustness.

Next, an example studied by Cannon and Rosenthal[23] is used in order to demonstrate the potential usefulness of the

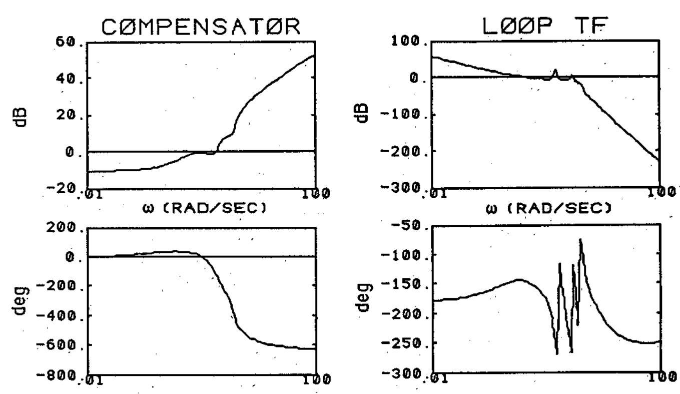

Fig. 14 Bode plots for eighth-order system with fourth-order compensator.

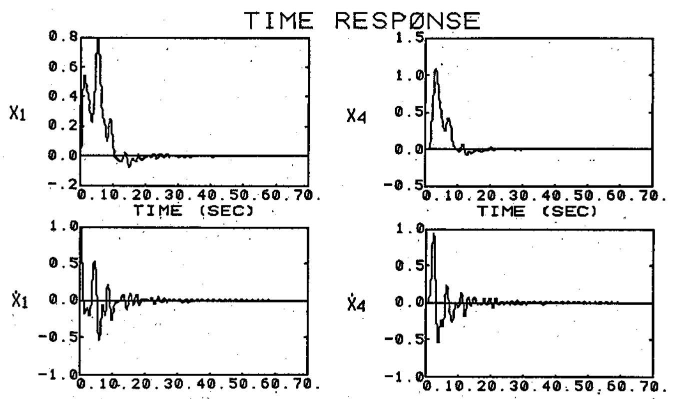

Fig. 15 Closed-loop time response to an impulse disturbance.

generalized structural filtering concept to the feedback compensator synthesis of a higher-order system with many closely spaced modal frequencies. It is assumed that four bodies have unit masses and are connected by springs with spring constant k and dashpots with a damping coefficient of 0.004. The spring constant k has a nominal value of 4 and is assumed to be uncertain. The flexible modes have nominal passive damping ratios of $\zeta_1 = 0.0008$, $\zeta_2 = 0.0014$, and $\zeta_3 = 0.0018$. For this eighth-order flexible structure model, the locus of the closed-loop poles vs overall loop gain, using only rigid-body mode compensation, is shown in Fig. 12. The first and third flexible modes are unstably interacting with the rigid-body mode control, while the second mode is stably interacting with the rigid-body mode control. As a first attempt, two nonminimum-phase second-order filters are used to stabilize the first and third flexible modes for which the root locus is drawn in Fig. 13. The loop gain margin is 5.75 dB, and the closed-loop system has acceptable phase margins for each mode.

The validation of the closed-loop stability with respect to the parameter variations is not a trivial problem, since there are three flexible modes. Instead of considering all possible combinations of the frequency variations, the closed-loop stability is checked by a uniform increase or decrease of each flexible mode frequency. This offers a simple checking of the effect of stiffening or softening the structure. The locus of closed-loop poles vs spring constant k (similar to Figs. 5 and 9) shows that the closed-loop system is stable for -23% and $+26\%$ variation of k from its nominal value of 4. The Bode plots for this fourth-order compensator design are shown in Fig. 14, and the response to an impulse disturbance at the actuator location is shown in Fig. 15. Compared to the optimal LQG design[23] or modified LQG design,[9] the classical design approach provides physical insight into how the design can be performed to be more robust and meaningful. A few more iterations of trial and error could be taken to improve the performance/stability robustness for this example. However, it seems that no further significant improvement can be achieved by using one actuator and a noncolocated sensor for this eighth-order example.

These design examples demonstrate that the classical successive mode-stabiliation approach enhanced by the generalized structural filtering concept provides much physical insight into the control/structure interaction problems. Furthermore, this approach enables the control designer to understand the inherent characteristics of various modern optimal compensators (LQG, LQG/LTR, H^∞-norm, etc.).

Conclusion

We have presented a new concept of generalized structural filtering. The simplicity and potential usefulness of the successive mode-stabilization approach enhanced by the concept have been demonstrated using examples. Throughout the examples, nonminimum-phase filtering has been shown to be an effective way of actively controlling the "unstably" interacting flexible modes. Furthermore, the concept enables the control designer to understand the pole-zero patterns of optimal compensators. The usefulness of the concept and approach, however, depends on the familiarity of the user with the classical control techniques.

Appendix: Generalized Second-Order Filter

This Appendix further discusses various filters that can be realized from the second-order filter represented as

$$\frac{s^2/\omega_z^2 + 2\zeta_z s/\omega_z + 1}{s^2/\omega_p^2 + 2\zeta_p s/\omega_p + 1} \tag{A1}$$

where s is the Laplace transform variable.

Minimum-Phase Lead or Lag Filter

A phase lead or lag filter can be realized from Eq. (A1) with $\zeta_c \triangleq \zeta_p = \zeta_z > 0$ (see Figs. 1a and 1b). The maximum phase lead or lag ϕ_m is obtained at $\omega_c = \sqrt{\omega_z \omega_p}$, i.e.,

$$\phi_m = \cos^{-1}\left[\frac{(2\zeta_c\sqrt{\omega_p/\omega_z})^2 - (\omega_p/\omega_z - 1)^2}{(2\zeta_c\sqrt{\omega_p/\omega_z})^2 + (\omega_p/\omega_z - 1)^2}\right] \tag{A2}$$

The gain increase or decrease at higher frequency can be approximated as

$$K_m = 40\log_{10}\left(\frac{\omega_p}{\omega_z}\right)\text{dB} \tag{A3}$$

For a small ζ_c i.e., a case with the filter poles and zeros near the imaginary axis), the effective phase-lead/-lag region lies between ω_z and ω_p and the maximum phase shift approaches ± 180 deg. For $\zeta_c = 1$, a conventional phase-lead or-lag filter with poles and zeros on the real axis can be realized. At $\omega_c = \sqrt{\omega_z\omega_p}$, one-half of K_m gain is increased (lead filter) or attenuated (lag filter). In practice, ζ_p is often selected to be greater than ζ_z, i.e., the filter poles are placed sufficiently far to the left from the imaginary axis. As an example, the OSO-8 spacecraft[18] employed a phase-lead filter with $\zeta_p = 0.6$, $\zeta_z = 0.3$, and $\omega_p/\omega_z = 2$. The use of a phase-lead filter with a large ω_p/ω_z ratio greater than 2 should be avoided from a practical point of view.

Minimum-Phase Notch or Bandpass Filter

For $\omega_p = \omega_z$, a notch (band-reject) or bandpass filter is obtained as shown in Figs. 1c and 1d. The minimum or maximum gain of the filter is obtained at $\omega_c \triangleq \omega_p = \omega_z$ as

$$K_m = 20\log_{10}\left(\frac{\zeta_z}{\zeta_p}\right)\text{dB} \tag{A4}$$

Both phase lead and lag appear near ω_c. For the notch filter, the maximum-phase lag and lead occur at ω_1 and ω_2, respectively, where

$$\omega_1/\omega_c = \sqrt{2\zeta_z\zeta_p + 1 - \sqrt{(2\zeta_z\zeta_p + 1)^2 - 1}}$$

$$\omega_2/\omega_c = \sqrt{2\zeta_z\zeta_p + 1 + \sqrt{(2\zeta_z\zeta_p + 1)^2 - 1}} \tag{A5}$$

Since ω_1/ω_c and ω_2/ω_c depend only on $\zeta_z\zeta_p$, the filter damping ratios determine the effective notch region. Typical values for the damping ratios of notch filter are $\zeta_p = 1$ and $\zeta_z = 0$ (e.g., the Galileo spacecraft.[16]

Nonminimum-Phase Allpass Filter

For $\omega_c \triangleq \omega_p = \omega_z$ and $\zeta_c \triangleq \zeta_p = |\zeta_z|$ ($\zeta_z < 0$), a large phase lag can be obtained from the second-order filter, with the gain being held constant (Fig. 1e), as

$$\phi = \cos^{-1}\left[\frac{[1-(\omega/\omega_c)^2]^2 - [2\zeta_c(\omega/\omega_c)]^2}{[1-(\omega/\omega_c)^2]^2 + [2\zeta_c(\omega/\omega_c)]^2}\right] \tag{A6}$$

The phase varies from 0 to -360 deg. The slope of the phase change depends on ζ_c (a smaller ζ_c results in a steeper slope). A typical value for ζ_c might be between 0.3 and 0.7, depending on the specific application. This nonminimum-phase filter is useful for stabilizing those unstably interacting, flexible modes that may need 180 deg phase change (lead or lag). This filter, when used for flexible-mode stabilization, maintains the control loop gain at all frequency range and provides the proper phasing of the particular signals to increase the closed-loop damping ratio of the flexible modes.

Nonminimum-Phase Highpass Filter

For $\omega_p > \omega_z$ and $\zeta_c \triangleq \zeta_p = |\zeta_z|$ ($\zeta_z < 0$), a nonminimum-phase filter with a highpass characteristic (a gain increase) is realized (Fig. 1f). The phase lag is continuous from 0 to -360 deg, but the phase curve shows a flat region near -180 deg phase lag, which is accompanied by a gain increase. By adjusting the ω_p/ω_z ratio, a broad regional -180 deg phase lag can be obtained. This filter, as opposed to the previous nonminimum-phase allpass filter, provides more "robust" phase shift to a flexible mode, which needs 180 deg phase change. Special care must be taken about the gain increase at the higher frequency region.

Nonminimum-Phase Lowpass Filter

As shown in Fig. 1g, this filter has a different gain curve compared to the previous nonminimum-phase highpass filter, although the phase curve is very similar to that of the nonminimum-phase highpass filter. If a less active damping is allowed, this filter can be used for stabilizing the unstably interacting, flexible mode since it provides gain attenuation with a proper phase shift.

Nonminimum-Phase Notch Filter

For $\omega_p = \omega_z$ and $\zeta_p > |\zeta_z|$ ($\zeta_z < 0$), a nonminimum-phase notch filter is realized as shown in Fig. 1h. If there is no passive damping (in an ideal case), the conventional notch filter shown in Fig. 2a cannot be used for the stabilization of unstably interacting, flexible modes. On the other hand, this nonminimum-phase notch filter provides the desired phase shift along with sharp gain attenuation at a particular frequency. As the pole-zero pair of this nonminimum-phase notch filter is placed farther from the imaginary axis, the robustness of the closed-loop system is enhanced, while the nominal stability margins become smaller.

Nonminimum-Phase Bandpass Filter

As shown in Fig. 1i, this filter has an opposite gain characteristic to the nonminimum-phase notch filter. This filter can be used for a regional gain increase with a proper phasing to increase the active damping, but the overall stability margin may be decreased if the filter poles are placed too close to the imaginary axis.

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