笔记上课已检查

习题1.1 P6

T4 甲,乙,丙三人各向目标射击一发子弹,以A,B,C表示甲,乙,丙命中目标,用A,B,C的运算关系表示下列事件

(1) 至少有一人命中目标: $A\cup B\cup C$

(2) 恰有一人命中目标: $(A\cap\bar{B}\cap\bar{C})\cup(\bar{A}\cap B\cap\bar{C})\cup(\bar{A}\cap\bar{B}\cap C)$

(3) 恰有两人命中目标: $(A\cap B\cap\bar{C})\cup(\bar{A}\cap B\cap C)\cup(A\cap\bar{B}\cap C)$

(4) 最多有一人命中目标: $(A\bar{B}\cap\bar{C})\cup(\bar{A}B\bar{C})\cup(\bar{A}\bar{B}C)\cup(\bar{A}\bar{B}\bar{C})$

(5) 三人均命中目标: $\cancel{\bar{A}\cap\bar{B}\cap\bar{C}}$ $A\cap B\cap C$

(6) 三人均未命中目标: $\bar{A}\cap\bar{B}\cap\bar{C}$

T5 $A\cup B$: $\{1,2,3,4,8\}$

AB : $\{2,4\}$

$A-B$: $\{1,3\}$

$B-A$: $\{6,8\}$

$B\cup C$: $\{1,2,3,4,5,6,7,8\}$

$(A\cup B)C$: $\{1,3\}$

习题1.2 P_{12}

1. $A \subset B$, $P(A)=0.4$, $P(B)=0.6$

(1) $P(\bar{A})$, $P(\bar{B})$ $\quad$ $P(\bar{A})=0.6$, $P(\bar{B})=0.4$

(2) $P(AB)$ $\quad$ $P(AB)=P(A)=0.4$

(3) $P(A\cup B)$ $\quad$ $P(A\cup B)=P(A)+P(B)-P(AB)=P(B)=0.6$

(4) $P(\bar{A}B)$ $\quad$ $P(\bar{A}B)=P(B)-P(A)=0.2$

(5) $P(\bar{A}\bar{B})$, $P(\bar{B}|A)$ $\quad$ $P(\bar{A}\bar{B})=1-P(B)=0.4$, $P(\bar{B}|A)=\varnothing$

~~(6)~~

$P(\bar{A}\bar{B})=P(\overline{A\cup B})=1-P(A\cup B)$ $\quad$ $P(\bar{B}A)=0$

2. $P(A)=\frac{1}{2}$ (1)若A、B互不相容，求$P(A\bar{B})$；(2) $P(AB)=\frac{1}{8}$ 求$P(A\bar{B})$

(1) $P(A\bar{B})=P(A)-P(AB)=P(A)=\frac{1}{2}$

(2) $P(A\bar{B})=P(A)-P(AB)=\frac{3}{8}$

习题1.4 P24

T5 设 B={任取一个球为玻璃球}

A={任取一个球为蓝球}

$$P(A|B)=\frac{P(AB)}{P(B)}=\frac{\frac{C_4^1}{C_{16}^1}}{\frac{C_6^1}{C_{16}^1}}=\frac{2}{3}$$

T10 设 B={任意选一位考生答对}

A={被选的学生为 A_1, A_2, A_3 对应完全掌握,部分掌握,完全不掌握}

(1)

$$P(B)=P(A_1)P(B|A_1)+P(A_2)P(B|A_2)+P(A_3)P(B|A_3)$$

$$=0.6\times 1+0.2\times 0.5+0.2\times 0.25$$

$$=0.75$$

(2) 设 C={任意一位考生完全掌握知识}

$$P(C|B)=\frac{P(BC)}{P(B)}=\frac{P(A_1)P(B|A_1)}{0.75}=\frac{0.6}{0.75}=\frac{4}{5}=0.8$$

习题1.5

T3 设 A={三人都不能翻译出密码}

B={能翻译出密码}

$P(B)=1-P(A)=1-\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}=\frac{3}{5}$

$A_1\ A_2\ A_3$

T7 设 A_i={甲,乙,丙中靶}

$\cancel{P(A_1\cup A_2\cup A_3)}$

(1) $P(A_1\cap\bar{A}_2\cap\bar{A}_3)+P(\bar{A}_1\cap A_2\cap\bar{A}_3)+P(\bar{A}_1\cap\bar{A}_2\cap A_3)=0.5\times0.4\times0.2+0.6\times0.5\times0.2$

$+0.8\times0.5\times0$

$=0.26$

(2) $P(\bar{A}_1\cap\bar{A}_2\cap\bar{A}_3)=0.5\times0.4\times0.2=0.04$

(3) $P(A_1\cap\bar{A}_2\cap\bar{A}_3)=0.5\times0.4\times0.2=0.04$

习题2.1 P39

T4

~~$P\{X=0\}=0.5$~~ $P\{X=0\}=0.5$

~~$X\{$ ——~~ $P\{X=1\}=0.3$

$P\{X=2\}=0.2$

$$F(x)=P\{X\le x\}=\begin{cases} 0, & x<0 \\ 0.5, & 0\le x<1 \\ 0.8, & 1\le x<2 \\ 1, & x\ge 3 \end{cases}$$

习题2.2 P44

T3

$p(\frac{1}{2}<X<\frac{5}{2})=p(X=1)+p(X=2)$

$=\frac{c}{2}+\frac{c}{6}$

$=\frac{2}{3}c$

$P(X=1)+P(X=2)+P(X=3)=1$ 得 $\frac{9}{12}c=1$

$P(\frac{1}{2}<X<\frac{5}{2})=\frac{2}{3}c=\frac{8}{9}$ $\therefore c=\frac{4}{3}$

T5 (1) $X=0,1,2,3$

$P(X=0)=\frac{C_{37}^{3}}{C_{40}^{3}}$

$P(X=1)=\frac{C_{37}^{2}C_{3}^{1}}{C_{40}^{3}}$

$P(X=2)=\frac{C_{37}^{1}C_{3}^{2}}{C_{40}^{3}}$

$P(X=3)=\frac{C_{37}^{0}C_{3}^{3}}{C_{40}^{3}}$

x	0	1	2	3
P	$\frac{7770}{9880}$	$\frac{1998}{9880}$	$\frac{111}{9880}$	$\frac{1}{9880}$

$p(X\geqslant 1)=1-P(X<1)=1-p(X=0)$

$=1-\frac{777}{988}=\frac{211}{988}$

$$F(x)=\begin{cases}\frac{7770}{9880}, & X<0\\ \frac{1998}{9880}, & 0\leq x<1\\ \frac{9768}{9880}, & 1\leq X<2\\ \frac{9879}{9880}, & 2\leq X<3\\ 1, & X\geq 3\end{cases}$$

习题2.2 P44

T_4 $B \sim (5000, 0.001)$

$\lambda = np = 5000 \times 0.001 = 5$

$P\{X \geq 2\} = 1 - P\{X=0\} - P\{X=1\}$

$\approx 1 - 5^0 \cdot e^{-5} - 5^1 e^{-5}$

$\approx 1 - 0.0067 - 0.0336$

≈ 0.9597

T_{10} $B \sim (365, 0.01)$

$\lambda = np = 3.65 \approx 3 + 0.6$

$P\{X \geq 1\} = 1 - P\{X = 0\}$

$\approx 1 - e^{-0.6} \cdot e^{-3}$

≈ 0.9726

习题2.3 P5.

T_2

$$F(x) = \begin{cases} 0 & , x < 0 \\ Ax^2 & , 0 \leq x \leq 1 \\ 1 & , x > 1 \end{cases}$$

(1) $F(1) = 1$ 得 $\lim\limits_{x \to 1} F(x) = 1$

$A = 1$ $\lim\limits_{x \to 1} Ax^2 = 1$

(2) $P(\frac{1}{3} < X < 2)$

$= F(2) - F(\frac{1}{3})$

$= 1 - \frac{1}{9}$

$= \frac{8}{9}$

$P(-1 < X < \frac{1}{2})$

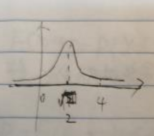
$= F(\frac{1}{2}) - F(-1) = \frac{1}{4}$

(3)

$$p(x) = \begin{cases} 2x & , 0 \leq x < 1 \\ 0 & , 其他 \end{cases}$$

习题2.5 P50

T4 设 $X \sim N(2,\sigma^2)$ 且 $p(2<X<4)=0.3$，求 $p(X<0)$



$$p(X<0)=\frac{1}{2}-p(0<X<2)$$
$$=\frac{1}{2}-p(2<X<4)$$
$$=0.2$$

T5 设k在(0,5)上服从均匀分布，求方程 $4x^2+4kx+k+2=0$ 有实根的概率

$k \sim U(0,5)$

$$\Delta=(4k)^2-4\times4(k+2)$$
$$=16k^2-16k-32$$
$$=16(k-2)(k+1)\geq 0$$

当 ~~$-1\leq k$~~ $k\leq -1$ 或 $k\geq 2$ 时 $\Delta\geq 0$

$$p(k\geq 2)=\frac{5-2}{5-0}=\frac{3}{5}$$

P64 习题 3.1

T_3 设随机变量 (X,Y) 的概率密度为

$$f(x,y)=\begin{cases} k(6-x-y), & 0<x<2,\ 2<y<4 \\ 0, & 其他 \end{cases}$$

(1) 确定常数 k (2) $P\{X<1, Y<3\}$ (3) $P\{X+Y<4\}$

(1) 则 $\int_2^4\int_0^2 k(6-x-y)\,dx\,dy=1$

则 左式 $= \int_2^4 \left(kx(6-y)-\frac{1}{2}kx^2\right)\Big|_0^2 dy$

$= \int_2^4 2k(6-y)-2k\,dy$

$= \int_2^4 10k-2ky\,dy$

$= 10ky-ky^2\Big|_2^4$

$= (40k-16k)-(20k-4k)$

$= 8k=1$

得 $k=\frac{1}{8}$

(2) $P\{X<1, Y<3\}$

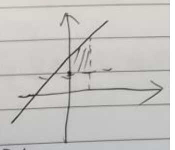
$= \int_2^3\int_0^1 \frac{1}{8}(6-x-y)\,dx\,dy$

$= \int_2^3 \left(\frac{1}{8}x(6-y)-\frac{1}{2}\cdot\frac{1}{8}x^2\right)\Big|_0^2 dy$

$= \frac{1}{8}\int_2^3 6-\frac{1}{2}-y\,dy$

$= \frac{1}{8}\left(\frac{11}{2}y-\frac{1}{2}y^2\right)\Big|_2^3 = \frac{3}{8}$

(3) $P\{x+y<4\}$



$= \frac{1}{8}\int_0^2\int_2^{-x+4}(6-x-y)\,dy\,dx$

$= \frac{1}{8}\int_0^2\left[(6-x)y - \frac{1}{2}y^2\Big|_2^{-x+4}\right]dx$

$= \frac{1}{8}\int_0^2\left[(6-x)(4-x) - \frac{1}{2}(4-x^2) - 2(6-x) + 2\right]dx$

$= \frac{1}{8}\int_0^2(24-10x+x^2-\frac{1}{2}x^2+4x-8-12+2x+2)\,dx$

$= \frac{1}{8}\int_0^2 \frac{1}{2}x^2-4x+6\,dx$

$= \frac{1}{8}\left(\frac{1}{6}x^3-2x^2+6x\Big|_0^2\right)$

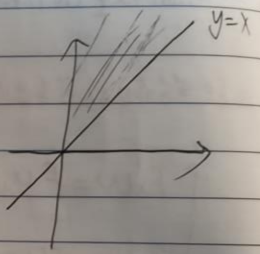
$= \frac{1}{8}\times\frac{16}{3}$

$= \frac{2}{3}$

习题3.2 P67

T3 设二维随机变量 (X, Y) 的概率密度为

$$f(x,y) = \begin{cases} e^{-y}, & 0 < x < y \\ 0, & \text{其他} \end{cases}$$



求边缘概率密度

$\cancel{F_X(x) = \lim_{x \to +\infty} f(x,y)}$

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \begin{cases} \int_x^{+\infty} e^{-y}\,dy = e^{-x}, & x > 0 \\ 0, & x \le 0 \end{cases}$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \begin{cases} \int_0^y e^{-y}\,dy = y e^{-y}, & y > 0 \\ 0, & y \le 0 \end{cases}$$

3.5 两个随机变量的函数分布

1. 离散

P	p_1	p_2	p_3
(X,Y)	(x_1,y_1)	(x_2,y_2)	(x_3,y_3)
$Z_1=X+Y$	x_1+y_1	x_2+y_2	x_3+y_3
$Z_2=X-Y$	x_1-y_1	x_2-y_2	x_3-y_3
$\cdots$			

合并 $z_1, z_2 \dots$ 相同的值，得出它们的分布律

2. 连续

对于 $Z=X+Y$ 的分布

有以下两个公式 $\begin{cases} f_Z(z)=\int_{-\infty}^{+\infty} f(x,z-x)\,dx \\ f_Z(z)=\int_{-\infty}^{+\infty} f(z-y,y)\,dy \end{cases}$

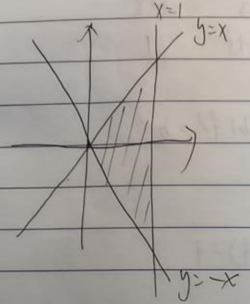
$\boxed{\text{延伸～}}$ 对于 $X\sim N(\mu_1,\sigma_1^2)$，$Y\sim N(\mu_2,\sigma_2^2)$ $\Rightarrow Z\sim N(\mu_1+\mu_2,\sigma_1^2+\sigma_2^2)$

设 X和Y是两个相互独立的随机变量，它们服从 $N(0,1)$，$Z=X+Y$，求Z的密度函数

习题3.4 P71

13 设二维连续型随机变量 (X, Y) 的概率密度函数为

$$f(x,y)=\begin{cases}1, & |y|<x,\ 0<x<1, \\ 0, & \text{其他},\end{cases}$$



$$f_X(x)=\int_{-\infty}^{+\infty} f(x,y)\,dy=\begin{cases}\int_{-x}^{x} 1\,dy=2x, & 0<x<1 \\ 0, & \text{其他}\end{cases}$$

$$f_Y(y)=\int_{-\infty}^{+\infty} f(x,y)\,dx=\begin{cases}\int_{y}^{1} 1\,dx=1-y, & 0<y<1 \\ \int_{-y}^{1} 1\,dx=1+y, & -1<y\leq 0 \\ 0, & \text{其他}.\end{cases}$$

$f_X(x)\cdot f_Y(y)\neq f(x,y)$ 故 X 与 Y 不独立.

T4 设二维随机变量 (X,Y) 分布律如表，并且 X 与 Y 相互独立.

$X \backslash Y$	1	2	3	$X=i$
1	a	$\frac{1}{9}$	c	$a+c+\frac{1}{9}$
2	$\frac{1}{9}$	b	$\frac{1}{3}$	$b+\frac{4}{9}$
$y=j$	$a+\frac{1}{9}$	$b+\frac{1}{9}$	$c+\frac{1}{3}$	1

$p(x=2,y=2)=p(x=2)\cdot p(y=2)=(b+\frac{1}{9})(b+\frac{4}{9})=b$

得 $b=\frac{2}{9}$

$p(x=2,y=1)=p(x=2)\cdot p(y=1)=\frac{6}{9}(a+\frac{1}{9})=\frac{1}{9}$

得 $a=\frac{1}{18}$

又∵ $a+c+\frac{1}{9}+b+\frac{4}{9}=1$ 得 $c=\frac{3}{18}$

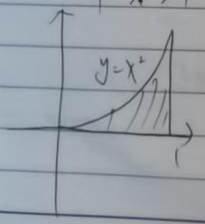
T5 设 X 和 Y 是2个相互独立的随机变量，$X\sim U(0,1)$，$Y\sim E(2)$，求 a 有实根的概率

$a^2+2Xa+Y=0$

$\Delta=\cancel{(2a)^2-4a^2}\ (2X)^2-4Y=4X^2-4Y\geq 0$

即 $X^2\geq Y$

$f_X(x)=\begin{cases}1, & 0<x<1\\ 0, & else\end{cases}$



$f_Y(y)=\begin{cases}\frac{1}{2}e^{-\frac{y}{2}}, & 0<y\\ 0, & else\end{cases}$

$P(X^2\geq Y)=\int_0^1 dx\int_0^{x^2}\frac{1}{2}e^{-\frac{y}{2}}dy$

$=\int_0^1(1-e^{-\frac{x^2}{2}})dx$

$=1-\int_0^1 e^{-\frac{x^2}{2}}dx$

$=1-0.8555=$

3.3 条件分布

引言:

对于考虑 在事件 $\{Y=y_j\}$ 已发生的条件下事件 $\{X=x_i\}$ 发生的概率

即事件 $\{X=x_i | Y=y_j\}$, $i=1,2\cdots$

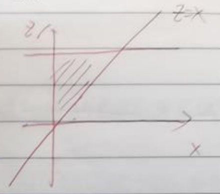
由条件概率公式可得

$$P\{X=x_i | Y=y_j\} = \frac{P\{X=x_i, Y=y_j\}}{P\{Y=y_j\}} = \frac{p_{ij}}{p_{\cdot j}}$$

P_{84} 习题3.5

T4 设随机变量(X,Y)的概率密度为 ，求 $Z=X+Y$ 的概率密度

$$f(x,y)=\begin{cases}\frac{1}{2}(x+y)e^{-(x+y)}, & x>0,\ y>0\\ 0, & \text{其他}\end{cases}$$

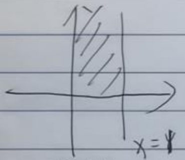


$\begin{cases}z-x>0\\ x>0\end{cases}$ 得 $0<x<z$

$$f_Z(z)=\begin{cases}\int_0^{z} f(x,z-x)\,dx=\int_0^{z}\frac{1}{2}(x+z-x)e^{-(x+z-x)}\,dx=\frac{1}{2}z^2e^{-z}\\ 0\end{cases}$$

T5 设随机变量(X,Y)的概率密度为

$$f(x,y)=\begin{cases}be^{-(x+y)}, & 0<x<1,\ y>0\\ 0, & \text{其他}\end{cases}$$



(1) $\int_0^1 dx\int_0^{+\infty} be^{-x}\cdot e^{-y}\,dy$

$=\int_0^{+\infty}\int_0^1 -be^{-x}\,d(-x)\cdot e^{-y}\,dy$

$=\int_0^{+\infty} -b(\frac{1}{e}-1)\cdot e^{-y}\,dy$

$=b(\frac{1}{e}-1)e^{-y}\Big|_0^{+\infty}$

$=b(\frac{1}{e}-1)(0-1)=1$

得 $b=\frac{e}{e-1}$

$$f(x,y)=\begin{cases}\frac{e}{e-1}e^{-(x+y)}, & 0<x<1,\ y>0\\ 0, & \text{其他}\end{cases}$$

(2) $U=\max(X,Y)$

$F_U(u)=P\{X<u,\ Y<u\}$

对于 $f_X(x)=\begin{cases}\int_0^{+\infty}\frac{e}{e-1}e^{-x}\cdot e^{-y}\,dy=\frac{e}{e-1}e^{-x}, & 0<x<1\\ 0, & \text{其他}\end{cases}$

$f_Y(y)=\begin{cases}\int_0^1\frac{e}{e-1}\cdot e^{-y}\cdot e^{-x}\,dx=e^{-y}, & y>0\\ 0, & \text{其他}\end{cases}$

有 $f(x,y)=f_X(x)\cdot f_Y(y)$ (相互独立)

$\therefore F_X(x)=\int_0^x f(t)\,dt=\frac{(1-e^{-x})e}{e-1}, \quad 0\le x<1$

$F_Y(y)=\int_0^y f(t)\,dt=1-e^{-y},\quad y>0$

$$\therefore F(u)=\begin{cases}0, & u<0\\ \frac{(1-e^{-u})^2 e}{e-1}, & 0\le u<1\\ 1-e^{-u}, & u\ge1\end{cases}$$

P85 T7 设(X,Y)的概率分布

X \ Y	1	2
0	$\frac{1}{5}$	$\frac{1}{5}$
1	$\frac{1}{5}$	$\frac{2}{5}$

求 $U=XY$ 的期望 $E(U)$

解 (x,y): $(0,1)$ $(0,2)$ $(1,1)$ $(1,2)$

xy	0	1	1	2
p	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$

①

U	0	1	2
P	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

$E(U)=0\times\frac{1}{5}+1\times\frac{2}{5}+2\times\frac{2}{5}$
$=\frac{6}{5}$

② $E(U)=\sum_{i=1,j=1}^{i=2,j=2} P_{ij}=0\times\frac{1}{5}+1\times\frac{1}{5}+1\times\frac{1}{5}+2\times\frac{2}{5}=\frac{6}{5}$

P_{41} T_5 T_7 T_{10}

T_5 设随机变量X的分布律如表4-5所示.

x	-2	0	2
P	0.4	0.3	0.3

4-5表

求 $E(X)$, $E(X^2)$, $E(-3X+5)$

① $E(X) = (-2)\times0.4 + 0\times0.3 + 2\times0.3 = -0.8+0.6 = -0.2$

② $E(X^2) = (-2)^2\times0.4 + 0^2\times0.3 + 2^2\times0.3 = 2.8$

③ $E(-3X+5) = -3E(X)+5 = 0.6+5 = 5.6$

T_7 设随机变量的概率密度

$$f(x)=\begin{cases} x, & 0\le x\le 1 \\ 2-x, & 1\le x\le 2 \\ 0, & 其他 \end{cases}$$

① $E(X) = \int_{-\infty}^{+\infty} x f(x)\,dx$

$$E(X) = \begin{cases} \int_0^1 x^2\,dx \\ \int_1^2 2x-x^2\,dx \\ 0 \end{cases} = \begin{cases} \frac{1}{3}, & 0\le x\le 1 \\ \frac{2}{3}, & 1\le x\le 2 \\ 0, & 其他 \end{cases} \quad E(X)=\frac{1}{3}+\frac{2}{3}=1$$

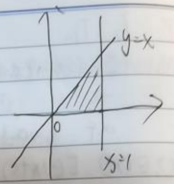
② $E(X^2) = \int_{-\infty}^{+\infty} x^2 f(x)\,dx$

$$E(X) = \begin{cases} \int_0^1 x^3\,dx \\ \int_1^2 2x^2-x^3\,dx \\ 0 \end{cases} = \begin{cases} \frac{1}{4}, & 0\le x\le 1 \\ \frac{11}{12}, & 1\le x\le 2 \\ 0, & 其他 \end{cases} \quad E(X)=\frac{1}{4}+\frac{11}{12}=\frac{7}{6}$$

③ $E(-3X+5) = -3E(X)+5 = 2$

T6 设随机变量(X,Y)的概率密度为

$$f(x,y)=\begin{cases}12y^2, & 0\le y\le x\le 1\\ 0, & 其他\end{cases}$$



① $E(X)=\int_0^1 x\,dx\int_0^x 12y^2\,dy$

$=\int_0^1 4x^3 x\,dx$

$=\frac{4}{5}x^5\Big|_0^1$

$=\frac{4}{5}$

$f_X(x,y)=\begin{cases}\int_0^x 12y^2\,dy=4x^3\\ 0\end{cases}$

② $E(Y)=\int_0^1 dx\int_0^x 12y^2\cdot y\,dy$

$=\int_0^1 3x^4\,dx$

$=\frac{3}{5}x^5\Big|_0^1$

$=\frac{3}{5}$

$f_Y(x,y)=\begin{cases}\int_y^1 12y^2\,dx=12y^2(1-y)\\ 0\end{cases}$

③ $E(X+Y)=E(X)+E(Y)=\frac{7}{5}$

④ $E(XY)=\int_0^1 x\,dx\int_0^x 12y^3\,dy$

$=\int_0^1 x\cdot 3x^4\,dx$

$=\int_0^1 3x^5\,dx$

$=\frac{x^6}{2}\Big|_0^1$

$=\frac{1}{2}$

⑤ $E\{\min(X,Y)\}=E(Y)=\frac{3}{5}$

$y\le x \quad \therefore \min(x,y)=y$

P111

T1 /前三问

1. 设 (X,Y) 的联合分布律为表

求：(1) $Cov(X,Y)$

(2) ρ_{XY}

(3) 相关性和独立性

$x \backslash y$	-1	0	1
-1	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
0	$\frac{1}{8}$	0	$\frac{1}{8}$
1	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

(1) $E(X) = -1\times\frac{3}{8} + 0\times\frac{2}{8} + 1\times\frac{3}{8} = 0$

$E(Y) = -1\times\frac{3}{8} + 0\times\frac{2}{8} + 1\times\frac{3}{8} = 0$

$E(XY) = 1\times\frac{1}{8} + 0\times\frac{1}{8} + (-1)\times\frac{1}{8} + 0\times\frac{1}{8} + 0\times\frac{1}{8} + 0\times 0 + (-1)\times\frac{1}{8} + 0\times\frac{1}{8} + 1\times\frac{1}{8} = 0$

$Cov(X,Y) = E(XY) - E(X)E(Y) = 0$

(2) $\rho_{XY} = \frac{Cov(X,Y)}{\sqrt{D(X)}\sqrt{D(Y)}} = 0$

(3) 故 X与Y不相关 $(\rho_{XY}=0)$

故 X与Y相互独立 $(E(XY)=E(X)E(Y))$

T2 /前三问

2. 设随机变量 (X,Y) 的联合密度函数为

$$f(x,y) = \begin{cases} 15xy^2, & 0 \le y \le x \le 1 \\ 0, & \text{其他} \end{cases}$$

求 (1) $Cov(X,Y)$

(2) ρ_{XY}

(3) $D(2X-3Y+7)$

~~(2) $\rho_{XY} = \frac{Cov(X,Y)}{\sqrt{D(X)}\sqrt{D(Y)}}$~~

(1) $E(X) = \int_0^1 x\,dx \int_0^x 15xy^2\,dy =$

$E(Y) = \int_0^1 dx \int_0^x 15xy^2\cdot y\,dy =$

$E(XY) = \int_0^1 x\,dx \int_0^x 15xy^2\cdot y\,dy =$

$Cov(X,Y) = E(XY) - E(X)E(Y) =$

$E(X) = \frac{5}{6}$, $E(Y) = \frac{15}{24} = \frac{5}{8}$, $E(X,Y) = \frac{15}{28}$, $Cov(X,Y) = \frac{5}{336}$

(2) $E(X^2) = \int_0^1 x^2 dx \int_0^x 15xy^2 dy = \int_0^1 x^2 \cdot 5x^4 dx = \frac{5}{7}$

$E(Y^2) = \int_0^1 dx \int_0^x 15xy^4 dy = \int_0^1 15x \left(\frac{y^5}{5}\Big|_0^x\right) dx = \int_0^1 3x^6 dx = \frac{3}{7}$

$D(X) = E(X^2) - [E(X)]^2 = \frac{5}{7} - \left(\frac{5}{6}\right)^2 = \frac{5}{252}$

$D(Y) = E(Y^2) - [E(Y)]^2 = \frac{3}{7} - \left(\frac{5}{8}\right)^2 = \frac{17}{448}$

$\rho_{XY} = \frac{\frac{5}{336}}{\sqrt{\frac{5}{252} \times \frac{17}{448}}} = \frac{\sqrt{112896}}{336\sqrt{17}} = \frac{\sqrt{17}}{17}$

(3) $D(2X-3Y+7)$

$= D(2X-3Y)$

$= D(2X) + D(-3Y) + 2(E(-6XY) - E(2X) \cdot E(-3Y))$

$= 4D(X) + 9D(Y) - 12E(XY) + 12E(X)E(Y)$

$= 4 \times \frac{5}{252} + 9 \times \frac{17}{448} - 12\left(\frac{15}{28} - \frac{5}{6} \times \frac{15}{24}\right)$

$= \frac{5}{63} + \frac{153}{448} - \frac{5}{28}$

$= \frac{5}{63} + \frac{73}{448}$

$= \frac{477}{4032}$

p111

1. 后两问

(4)

三阶矩 $E(X^3) = (-1)^3 \times \frac{3}{8} + 0 \times \frac{2}{8} + 1^3 \times \frac{3}{8} = 0$

~~E5?~~

~~三阶矩 $E[X-E(X)]^3$~~

$E(X) = -1 \times \frac{3}{8} + 0 \times \frac{2}{8} + 1 \times \frac{3}{8} = 0$

$E(X^2) = 1 \times \frac{3}{8} + 0 \times \frac{2}{8} + 1 \times \frac{3}{8} = \frac{3}{4}$

三阶中心矩.

$E\{[X-E(X)]^3\}$

$= E(X^3) - 3E(X)E(X^2) + 3E(X)^3 - E(X)^3$

$= E(X^3) - 3E(X)E(X^2) + 2E(X)^3$

$= 0 - 3 \times 0 \times \frac{3}{4} + 0 = 0$

(5) 协方差矩阵

$$\begin{pmatrix} D(X) & Cov(Y,X) \\ Cov(X,Y) & D(Y) \end{pmatrix} = \begin{pmatrix} \frac{3}{4} & 0 \\ 0 & \frac{3}{4} \end{pmatrix}$$

T2/后两问

(4) X的三阶矩

$$E(X^3)=\int_0^1\int_0^x x^3 \times 15xy^2\,dxdy$$

$$=\int_0^1 15x^4\,dx\int_0^x y^2\,dy$$

$$=\int_0^1 15x^4\cdot\frac{y^3}{3}\Big|_0^x dx$$

$$=\int_0^1 5x^7\,dx$$

$$=\frac{5}{8}x^8\Big|_0^1$$

$$=\frac{5}{8}$$

∴X的三阶中心矩

$$E\{[X-E(X)]^3\}=E(X^3)-3E(X)E(X^2)-E(X)^3$$

$$=\frac{5}{8}-3\times\frac{5}{6}\times\frac{5}{7}-\left(\frac{5}{6}\right)^3$$

$$=-\frac{5}{1512}$$

(5) 协方差矩阵

$$\begin{pmatrix} D(X) & Cov(Y,X) \\ Cov(X,Y) & D(Y) \end{pmatrix}=\begin{pmatrix} \frac{2}{252} & \frac{5}{336} \\ \frac{5}{336} & \frac{17}{448} \end{pmatrix}$$