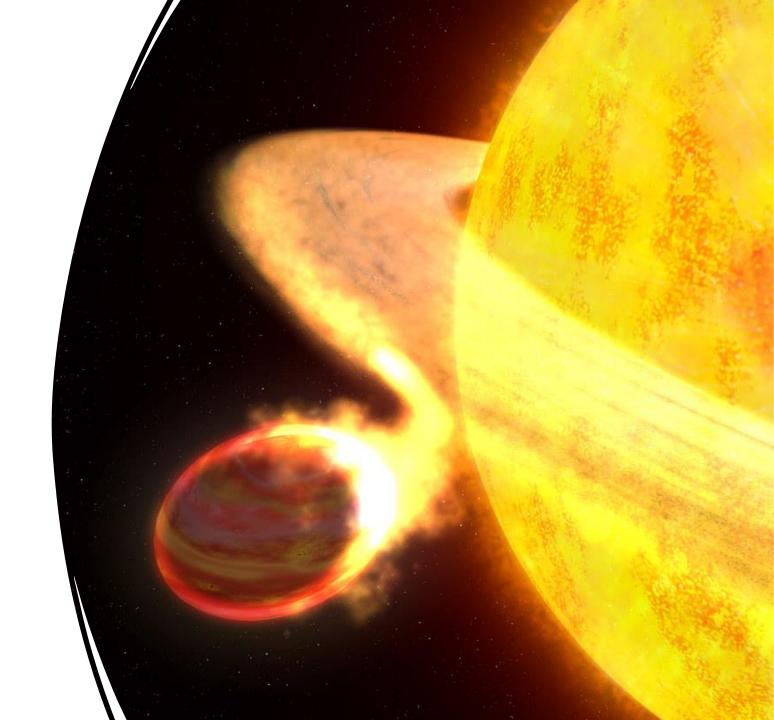
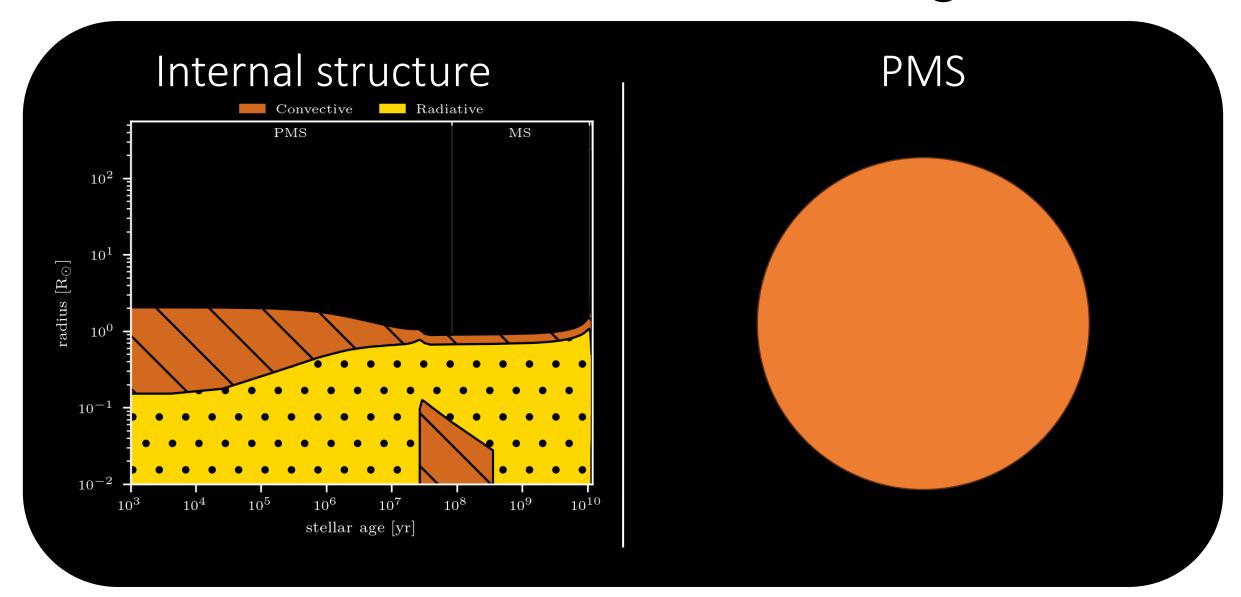
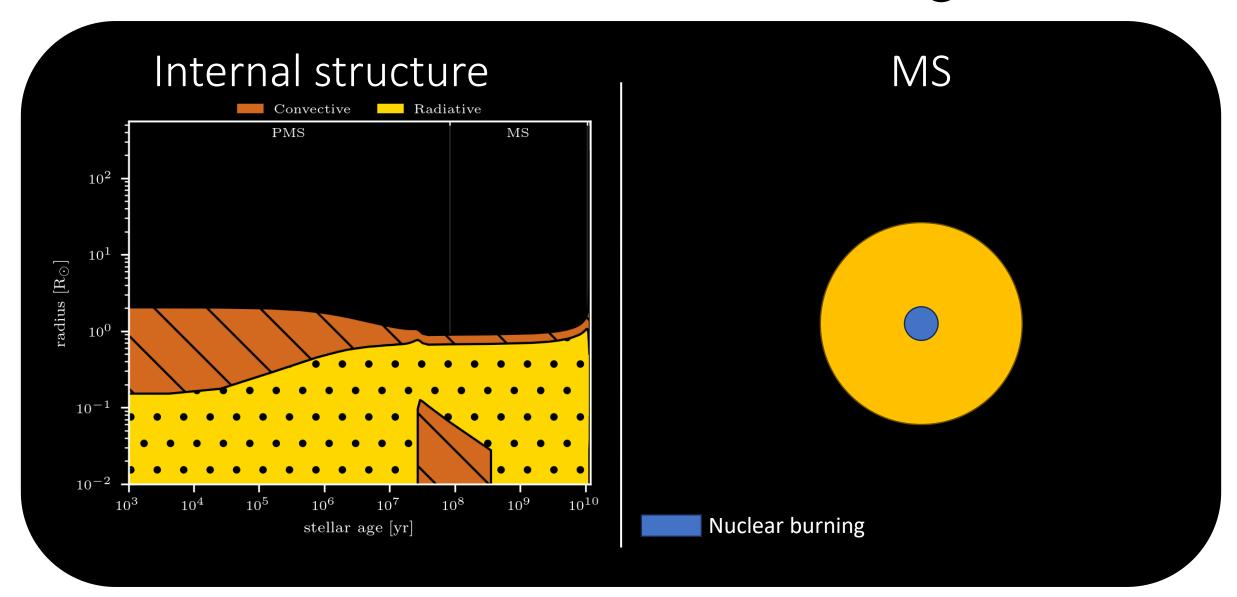
# Tidal Dissipation in Cool Evolved Stars

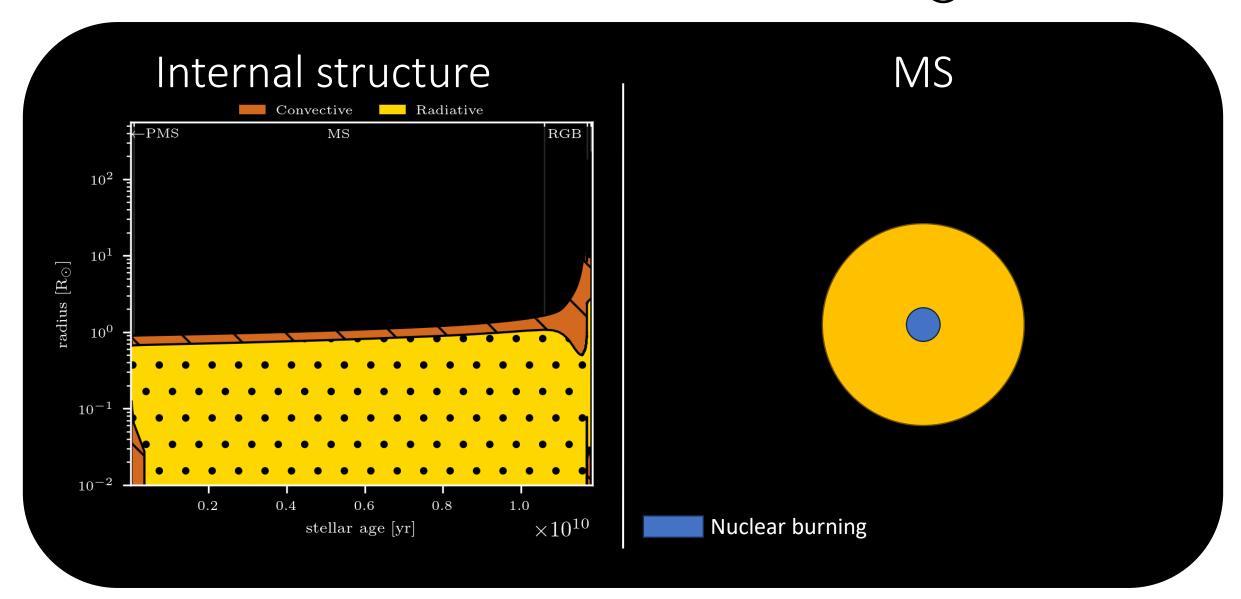
Mats Esseldeurs Stéphane Mathis Leen Decin

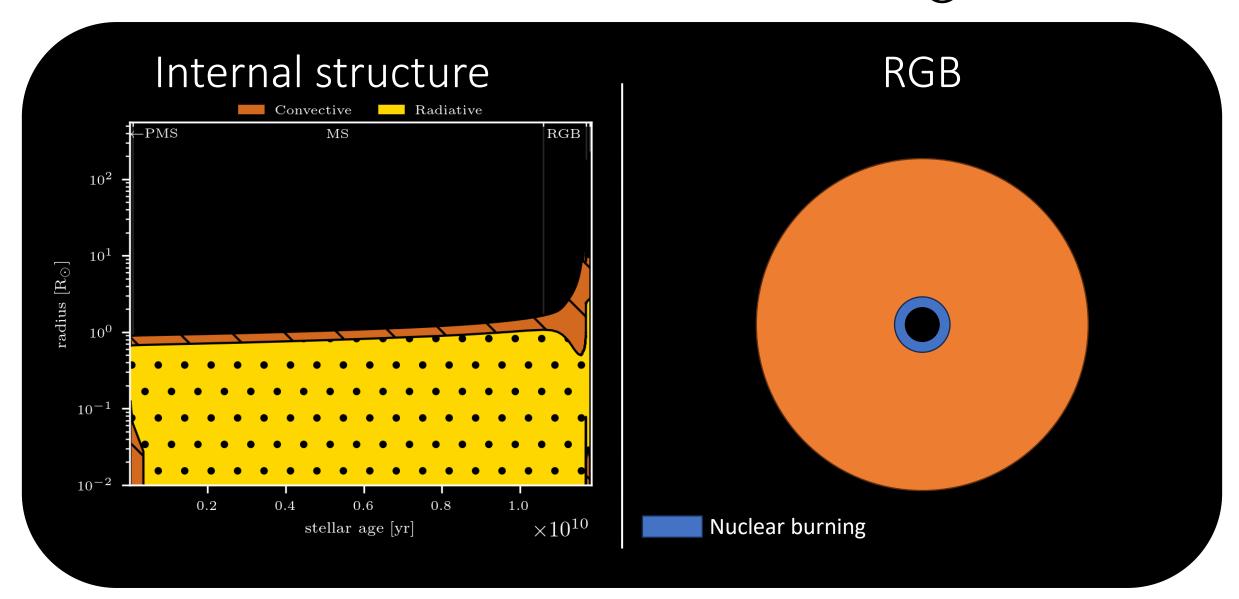


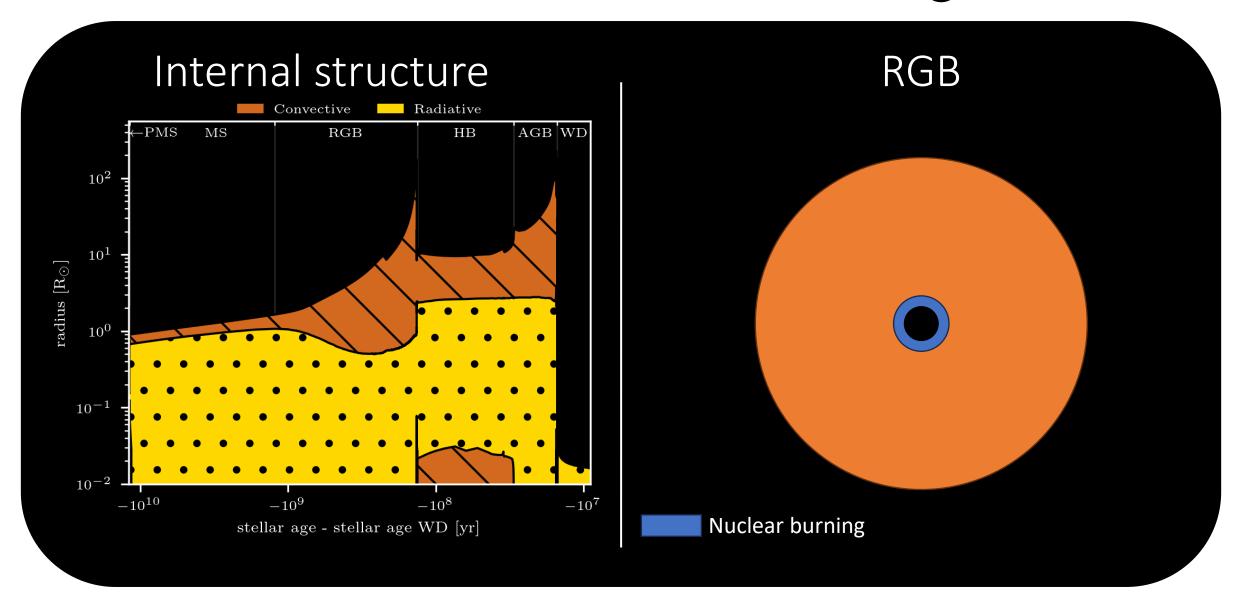


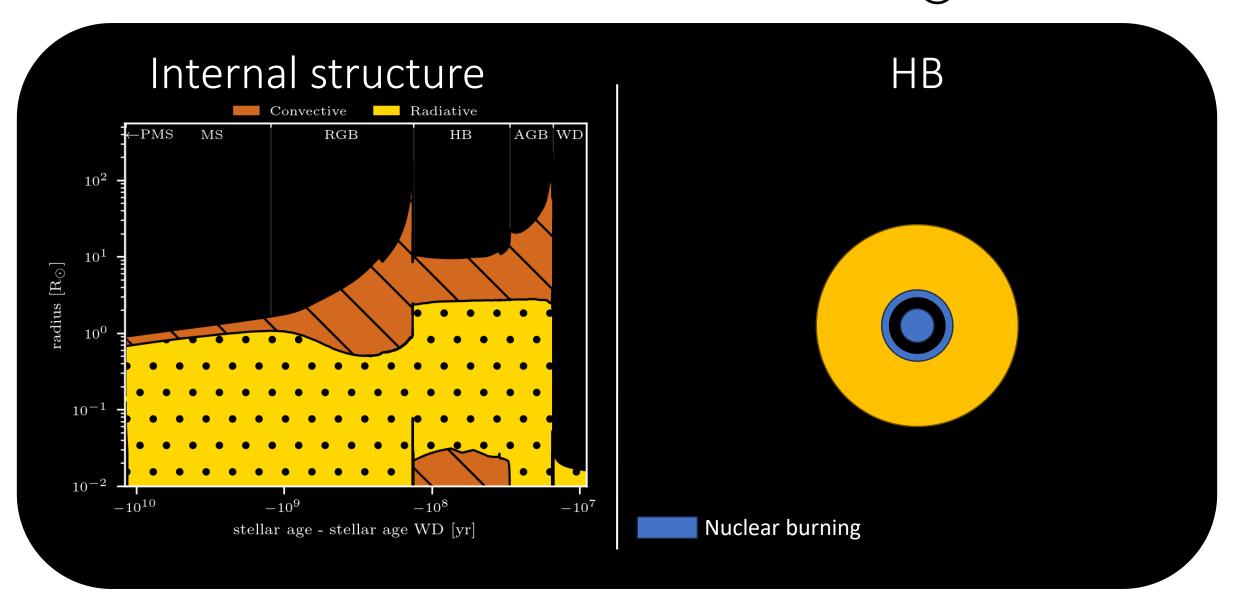


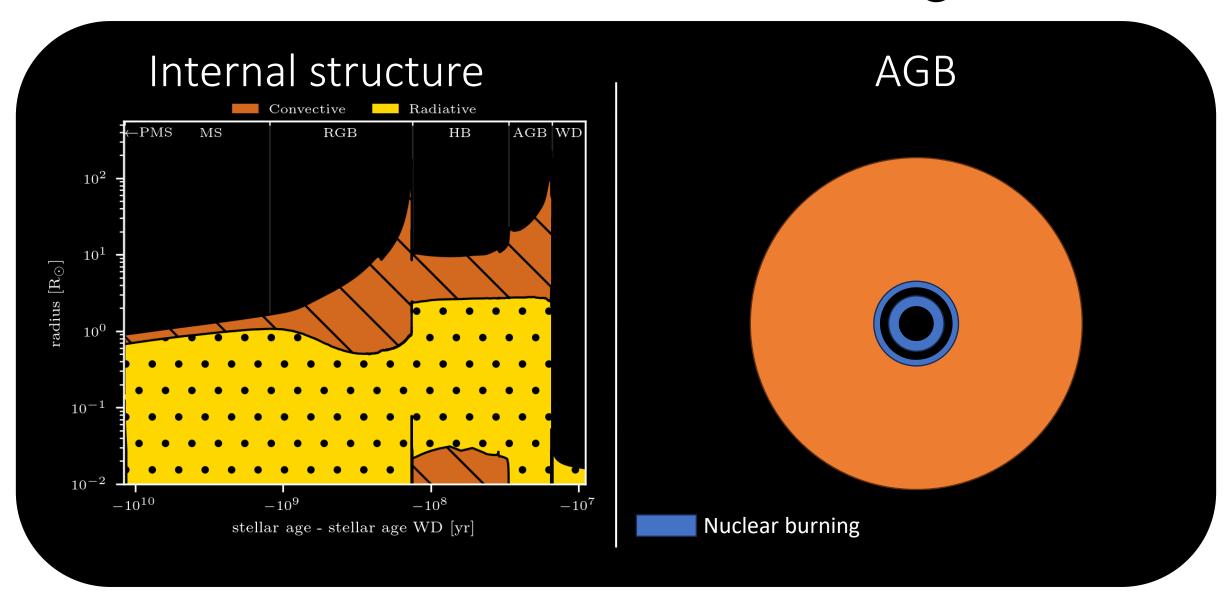


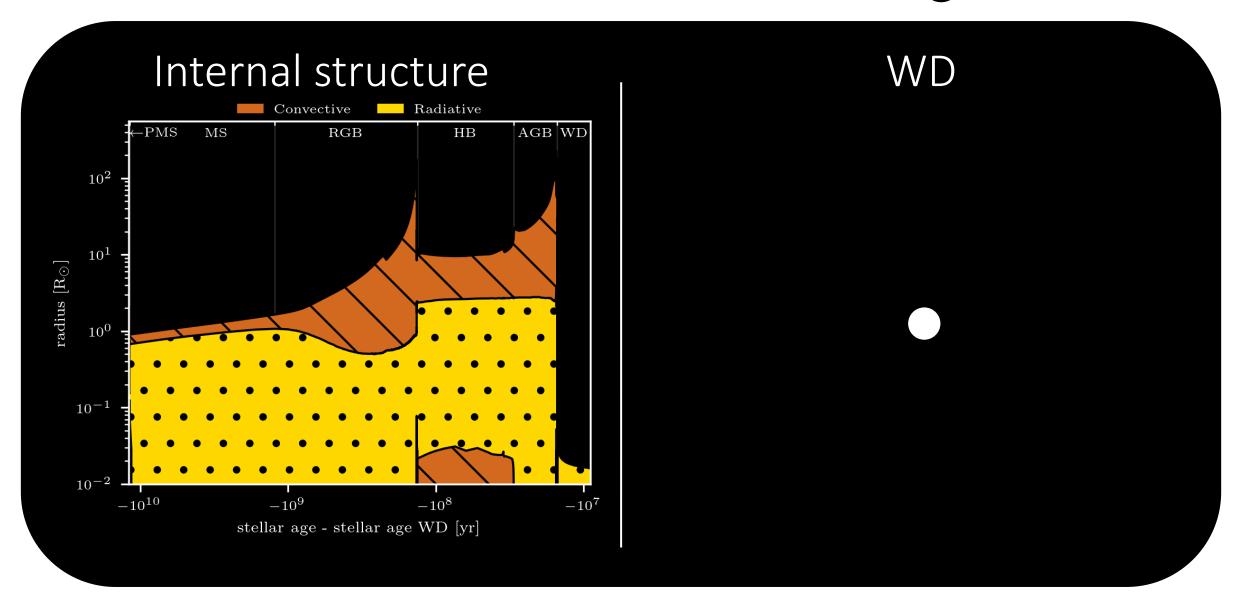


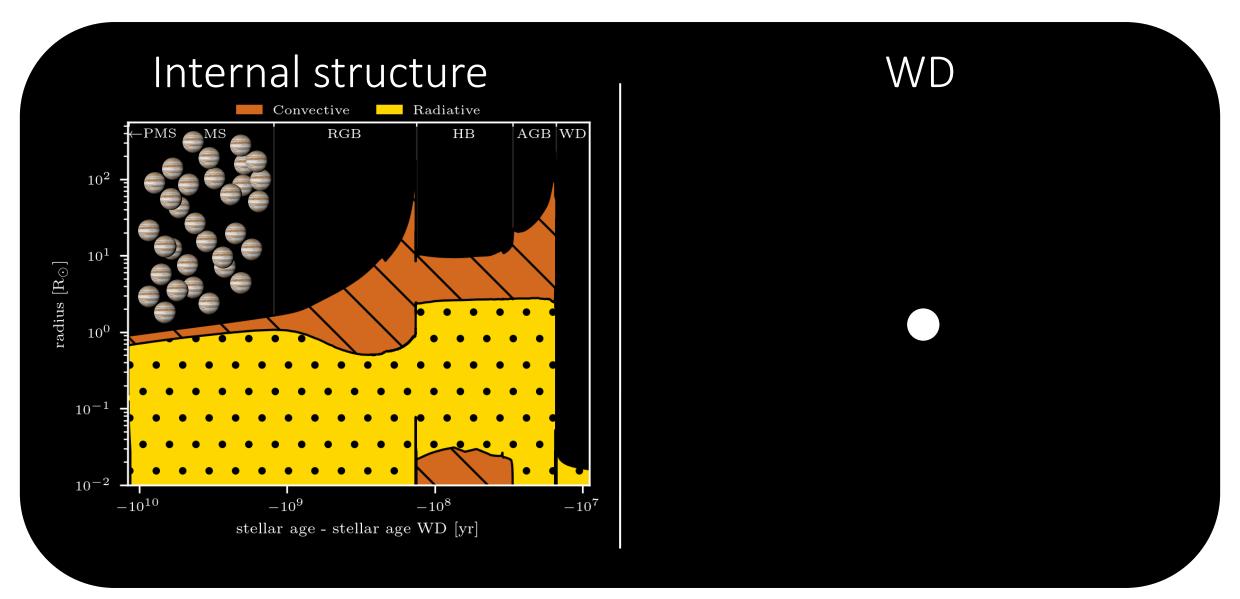


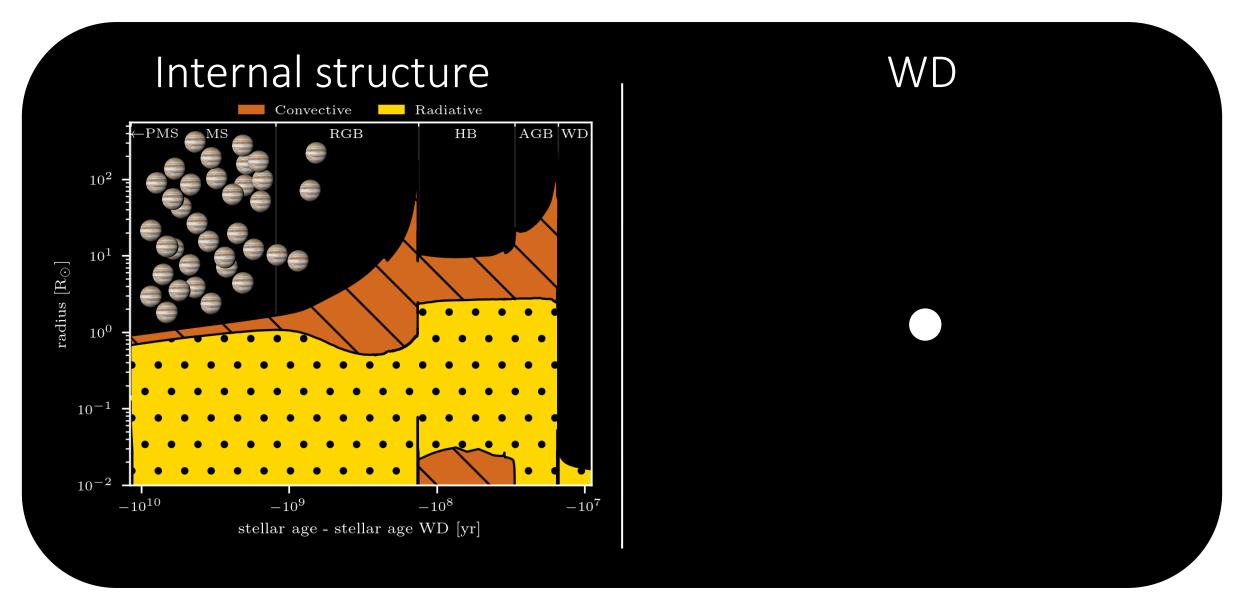


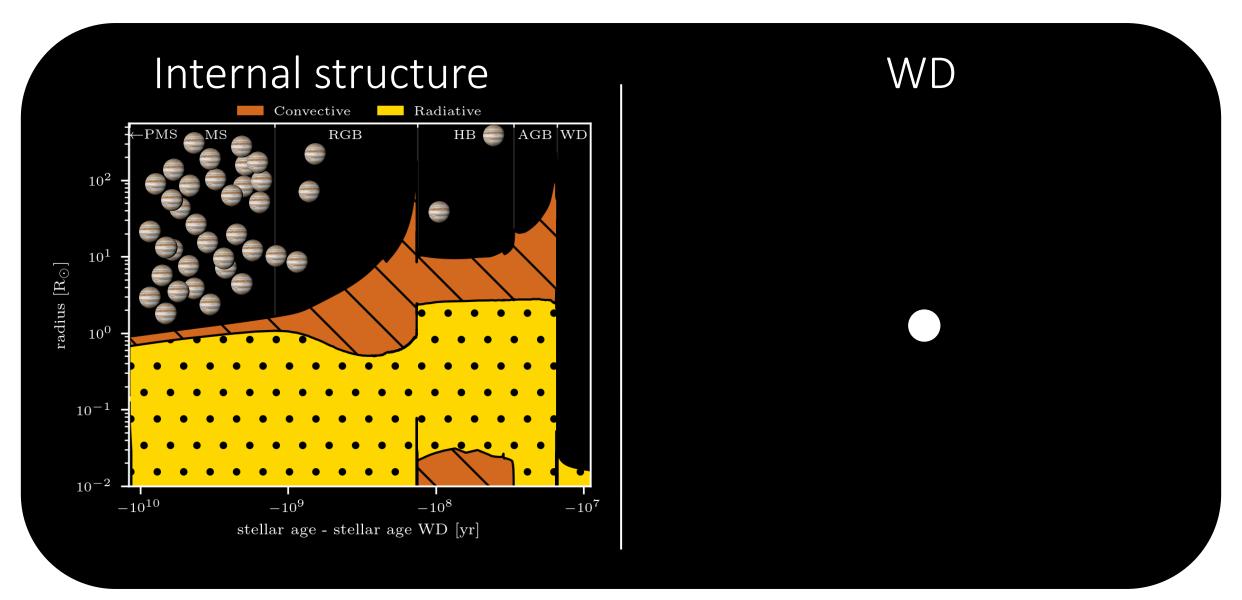


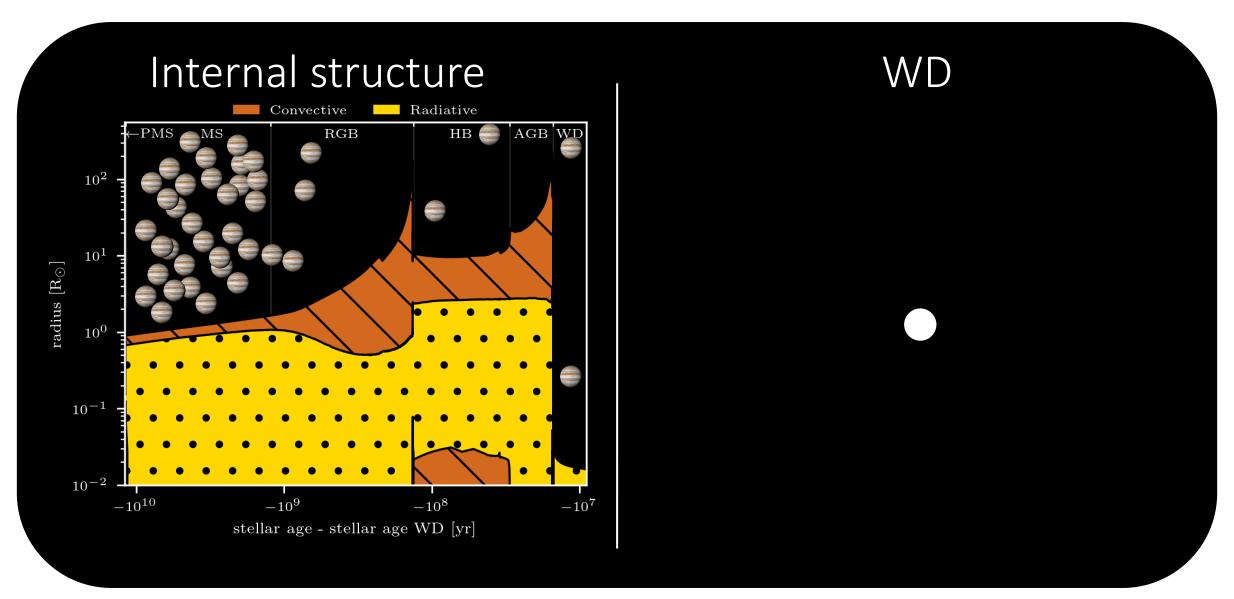


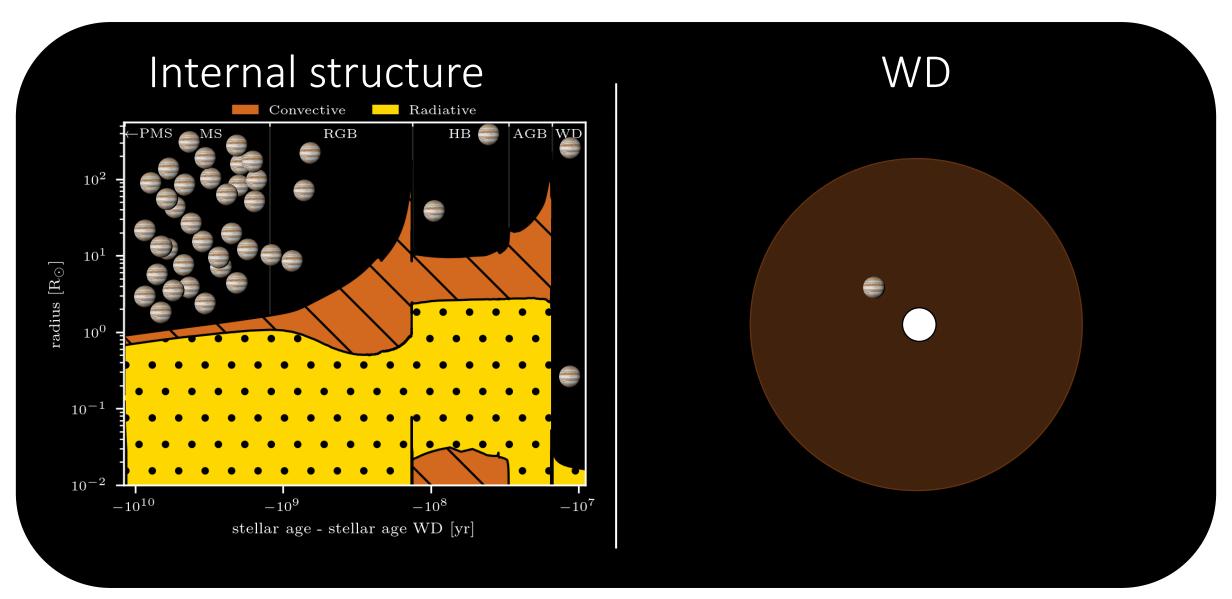




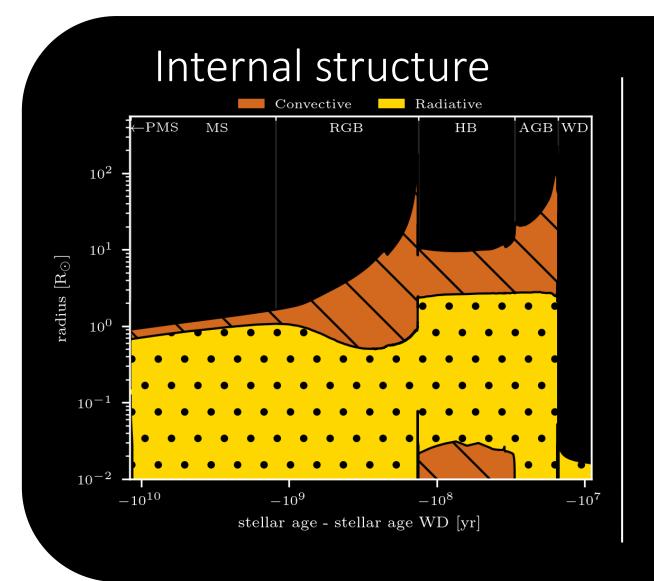








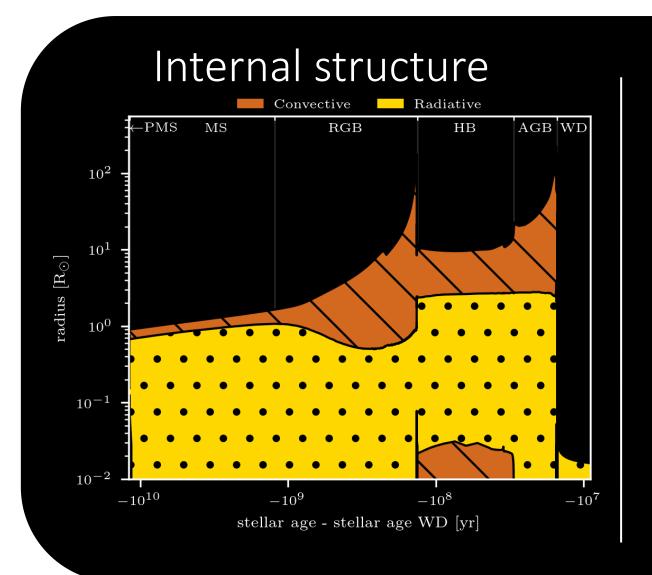
## Tidal dissipation in stars



#### Tidal dissipation

• Equilibrium tide

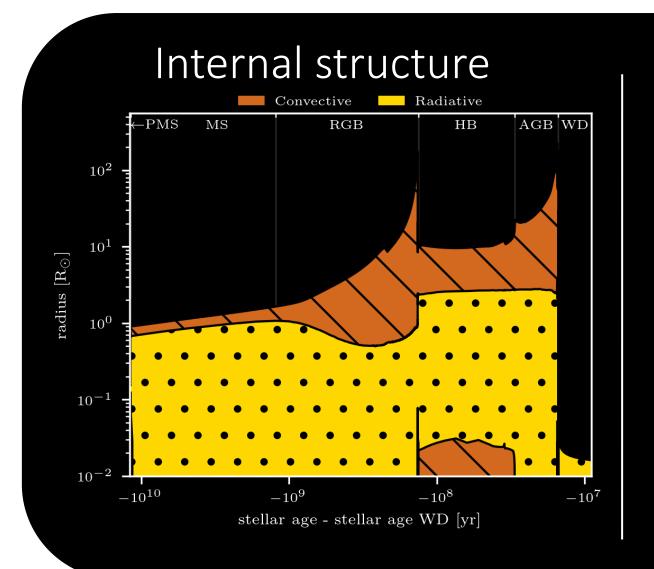
## Tidal dissipation in stars



#### Tidal dissipation

- Equilibrium tide
- Dynamical tide
  - Inertial waves
  - Pressure waves
  - Gravity waves

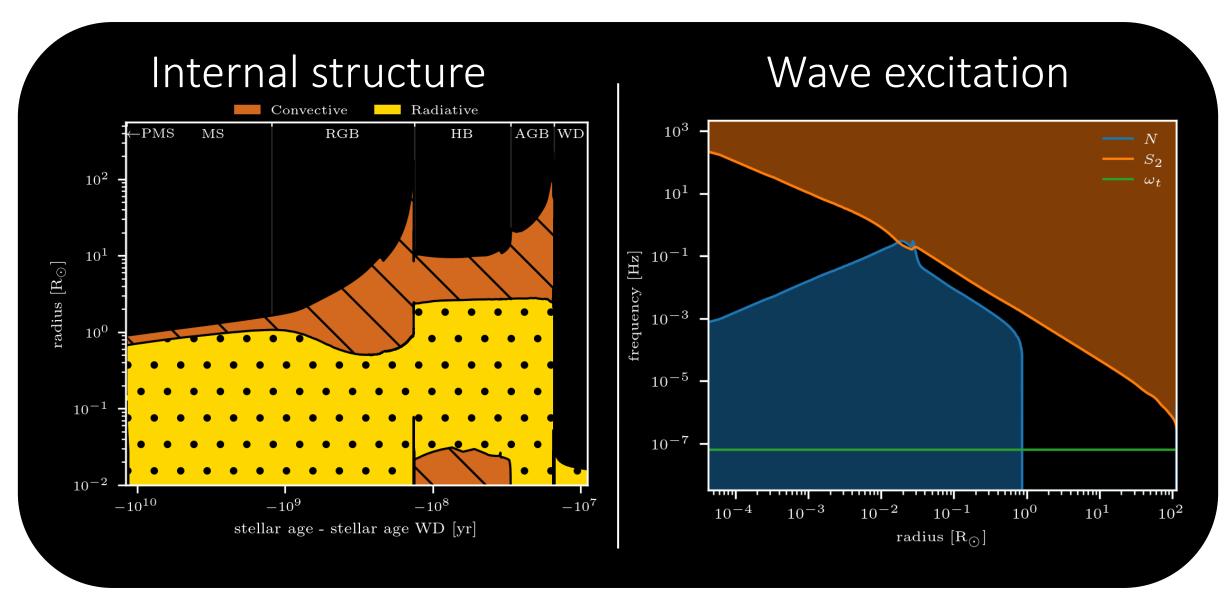
## Tidal dissipation in stars



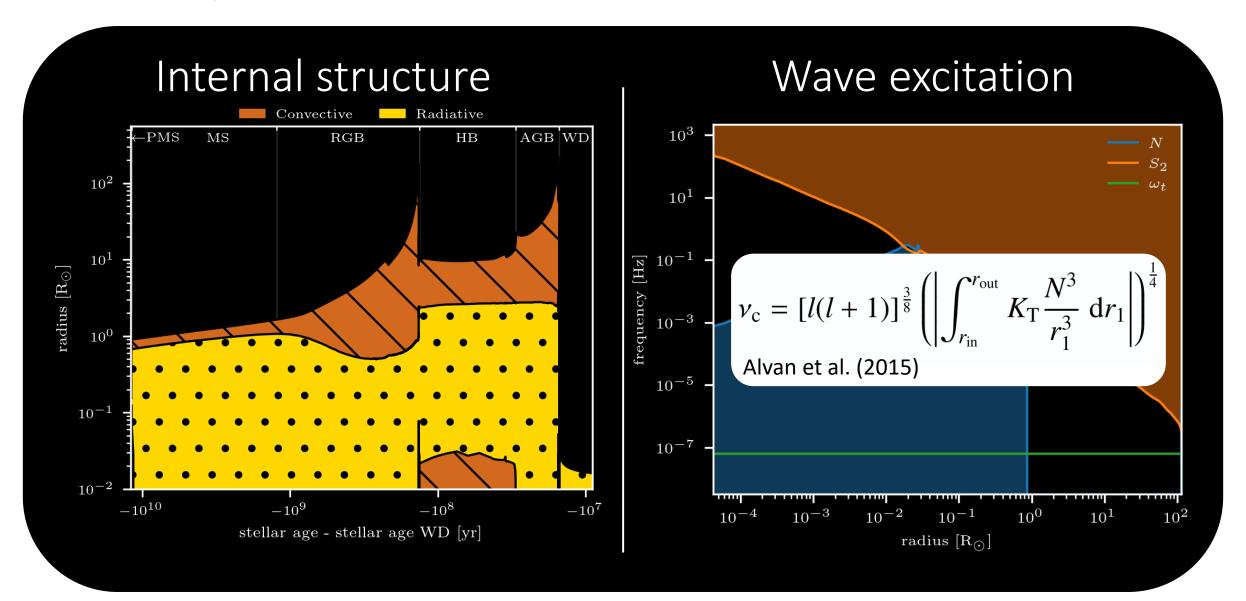
#### Tidal dissipation

- Equilibrium tide
- Dynamical tide
  - Inertial waves
  - Pressure waves
  - Gravity waves

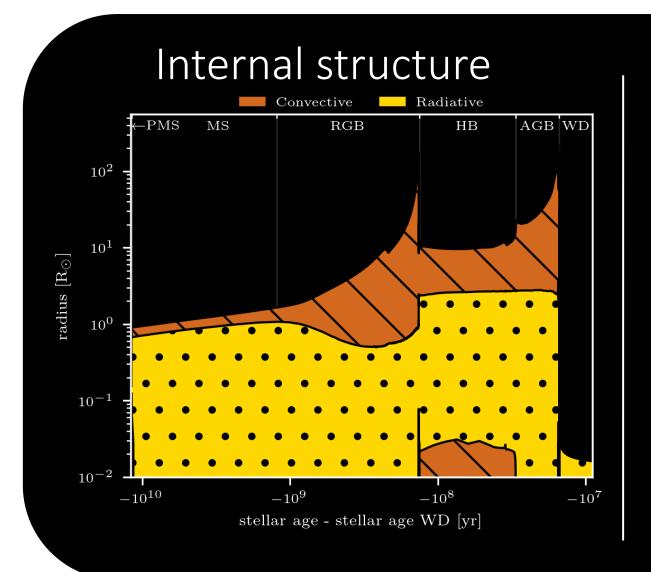
## Tidally Excited Waves



## Tidally Excited Waves

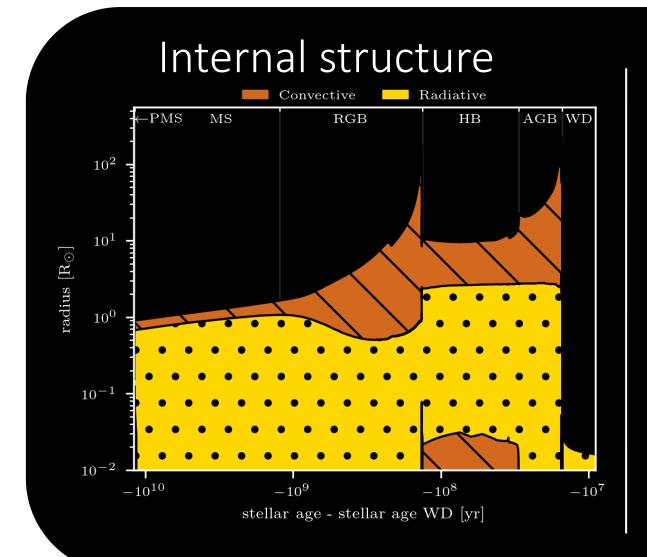


## Tidally Excited Waves



#### Tidal dissipation

- Equilibrium tide
- Dynamical tide
  - Inertial waves
  - Pressure waves
  - Gravity waves
  - → Progressive internal gravity waves



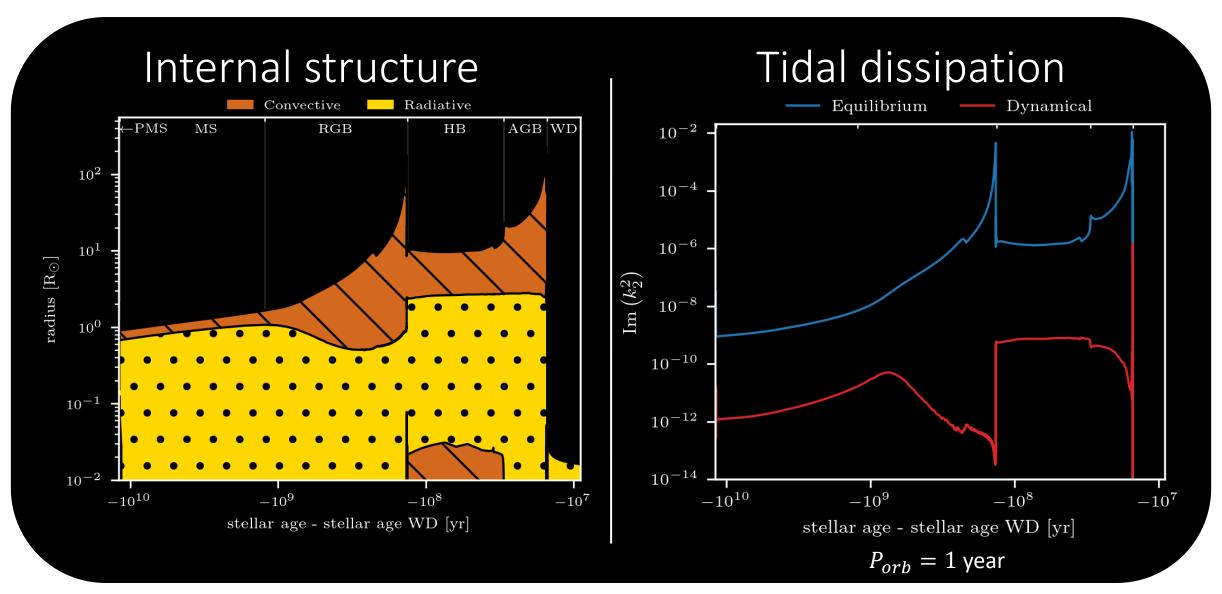
#### Tidal dissipation

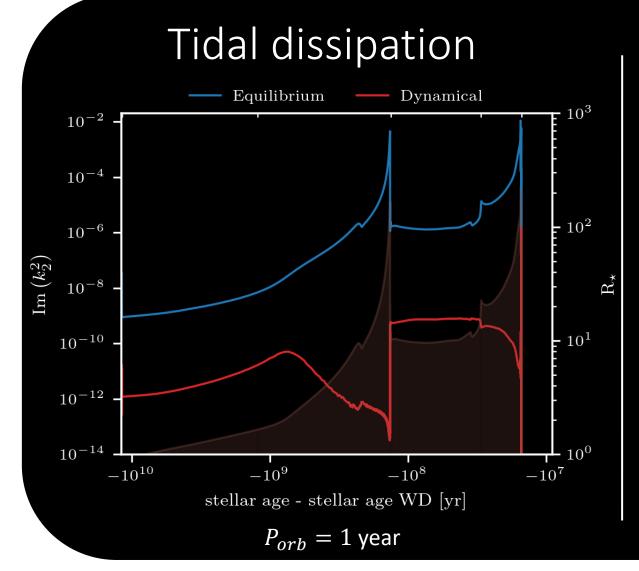
$$\operatorname{Im}\left(k_2^2\right)_{\operatorname{eq}} = \frac{16\pi G\omega_t}{4(2l+1)R_{\star}|\varphi_T(R_{\star})|^2} \int_0^{R_{\star}} r^2 \rho \nu_t D_l(r) dr$$

Barker (2020); Dhouib et al. (2024)

$$\operatorname{Im}(k_{2}^{2})_{\operatorname{IGW}} = \frac{3^{-\frac{1}{3}}\Gamma^{2}(\frac{1}{3})}{2\pi} [l(l+1)]^{-\frac{4}{3}} \omega_{t}^{\frac{8}{3}} \frac{a^{6}}{GM_{2}^{2}R_{\star}^{5}} \times \left(\rho_{0}(r_{\operatorname{in}}) r_{\operatorname{in}} \left| \frac{dN^{2}}{d \ln r} \right|_{r_{\operatorname{in}}}^{-\frac{1}{3}} \mathcal{F}_{\operatorname{in}}^{2} + \rho_{0}(r_{\operatorname{out}}) r_{\operatorname{out}} \left| \frac{dN^{2}}{d \ln r} \right|_{r_{\operatorname{out}}}^{-\frac{1}{3}} \mathcal{F}_{\operatorname{out}}^{2}\right)$$

Ahuir et al. (2021); Esseldeurs et al. (2024)

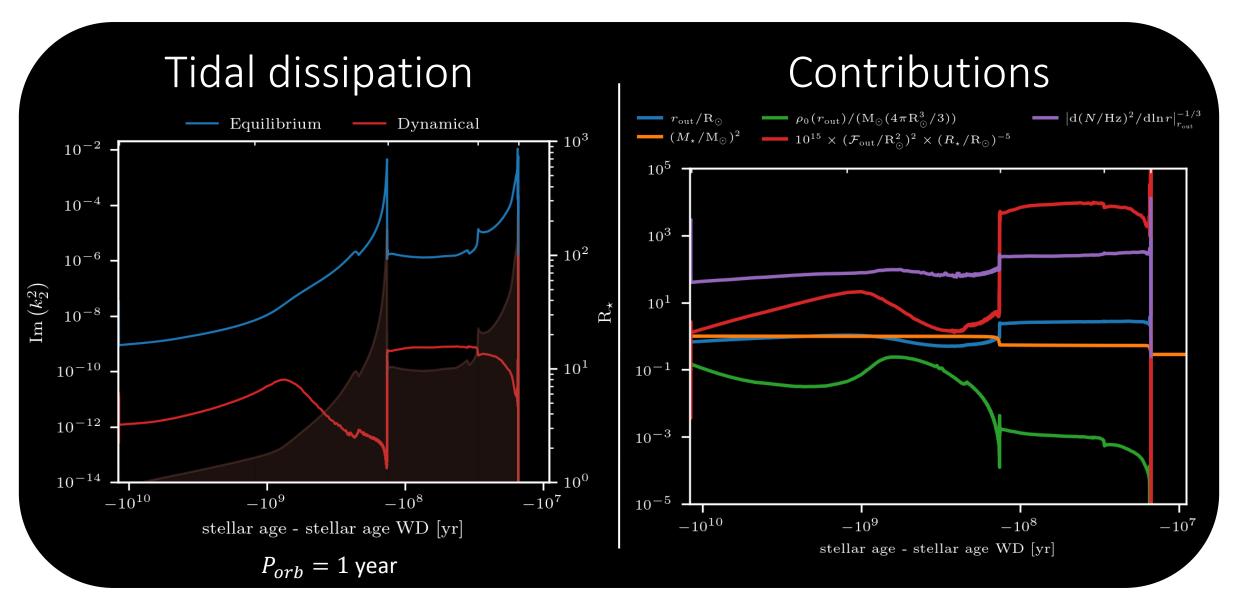


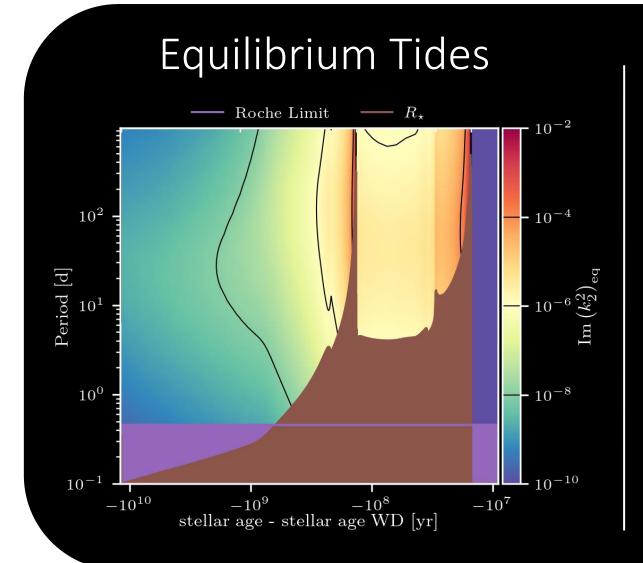


#### Contributions

$${\rm Im} \left(k_2^2\right)_{\rm eq} = \frac{16\pi G \omega_t}{4(2l+1)R_{\star}|\varphi_T(R_{\star})|^2} \int_0^{R_{\star}} r^2 \rho \nu_t D_l(r) dr$$
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Ahuir et al. (2021); Esseldeurs et al. (2024)





#### Contributions

$$\operatorname{Im}\left(k_2^2\right)_{\text{eq}} = \frac{16\pi G\omega_t}{4(2l+1)R_{\star}|\varphi_T(R_{\star})|^2} \int_0^{R_{\star}} r^2 \rho \nu_t D_l(r) dr$$

Barker (2020); Dhouib et al. (2024)

$$v_{t} = V_{c} l_{c} F(\omega_{t}), \ F(\omega_{t}) = \begin{cases} 5 & |\omega_{t}| t_{c} < 10^{-2} \\ \frac{1}{2} (|\omega_{t}| t_{c})^{-\frac{1}{2}} & |\omega_{t}| t_{c} \in \left[10^{-2}, 5\right] \\ \frac{25}{\sqrt{20}} (|\omega_{t}| t_{c})^{-2} & |\omega_{t}| t_{c} > 5 \end{cases},$$

Duguid et al. (2020)

#### Equilibrium Tides Roche Limit $10^{-4}$ $10^{2}$ Period [d] $^{10_1}$ $10^{-8}$ $10^{0}$ $-10^{10}$ $-10^{7}$ stellar age - stellar age WD [yr]

#### Contributions

$$\operatorname{Im}\left(k_2^2\right)_{\text{eq}} = \frac{16\pi G\omega_t}{4(2l+1)R_{\star}|\varphi_T(R_{\star})|^2} \int_0^{R_{\star}} r^2 \rho \nu_t D_l(r) dr$$

Barker (2020); Dhouib et al. (2024)

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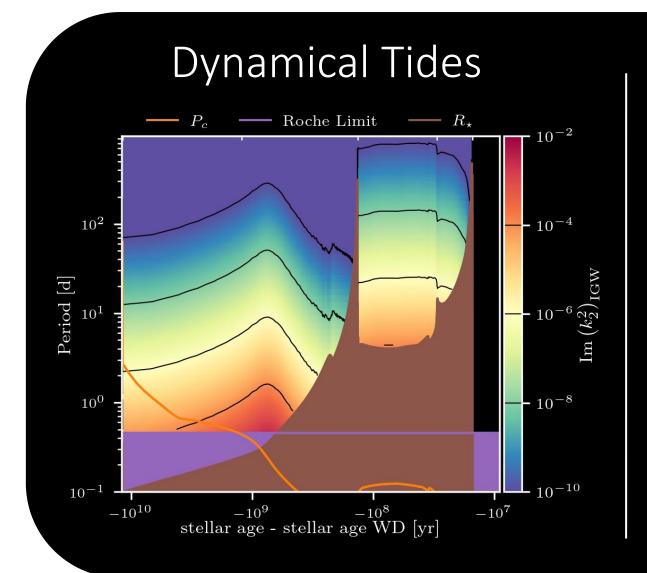
#### Contributions

$$\operatorname{Im}\left(k_2^2\right)_{\text{eq}} = \frac{16\pi G\omega_t}{4(2l+1)R_{\star}|\varphi_T(R_{\star})|^2} \int_0^{R_{\star}} r^2 \rho \nu_t D_l(r) dr$$

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$$v_{t} = V_{c} l_{c} F(\omega_{t}), \ F(\omega_{t}) = \begin{cases} 5 & |\omega_{t}| t_{c} < 10^{-2} \\ \frac{1}{2} (|\omega_{t}| t_{c})^{-\frac{1}{2}} & |\omega_{t}| t_{c} \in \left[10^{-2}, 5\right] \\ \frac{25}{\sqrt{20}} (|\omega_{t}| t_{c})^{-2} & |\omega_{t}| t_{c} > 5 \end{cases},$$

Duguid et al. (2020)



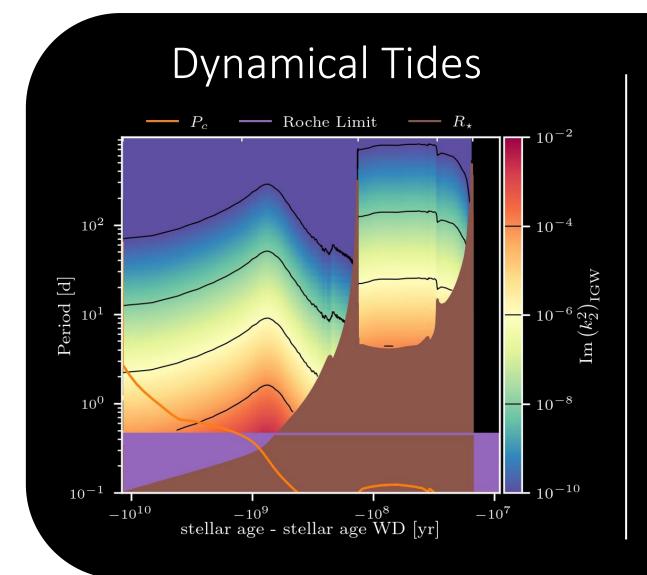
#### Contributions

$$\operatorname{Im}(k_{2}^{2})_{\operatorname{IGW}} = \frac{3^{-\frac{1}{3}}\Gamma^{2}(\frac{1}{3})}{2\pi} [l(l+1)]^{-\frac{4}{3}} \omega_{t}^{\frac{8}{3}} \frac{a^{6}}{GM_{2}^{2}R_{\star}^{5}}$$

$$\times \left(\rho_{0}(r_{\operatorname{in}}) r_{\operatorname{in}} \left| \frac{dN^{2}}{d \ln r} \right|_{r_{\operatorname{in}}}^{-\frac{1}{3}} \mathcal{F}_{\operatorname{in}}^{2} \right)$$

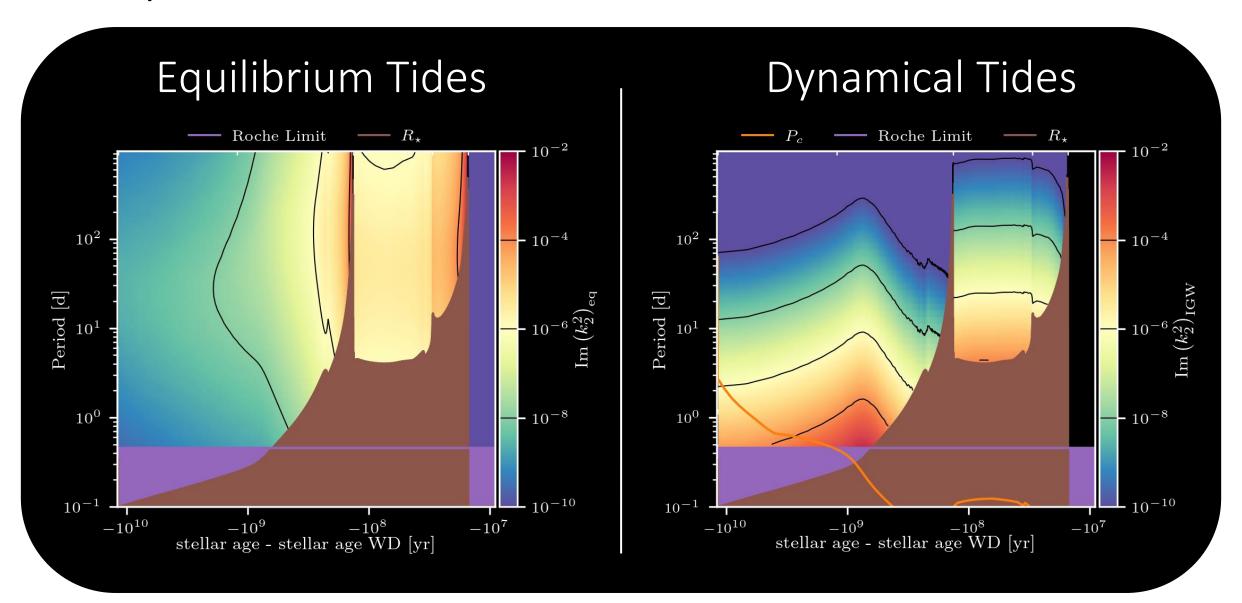
$$+ \rho_{0}(r_{\operatorname{out}}) r_{\operatorname{out}} \left| \frac{dN^{2}}{d \ln r} \right|_{r_{\operatorname{out}}}^{-\frac{1}{3}} \mathcal{F}_{\operatorname{out}}^{2}$$

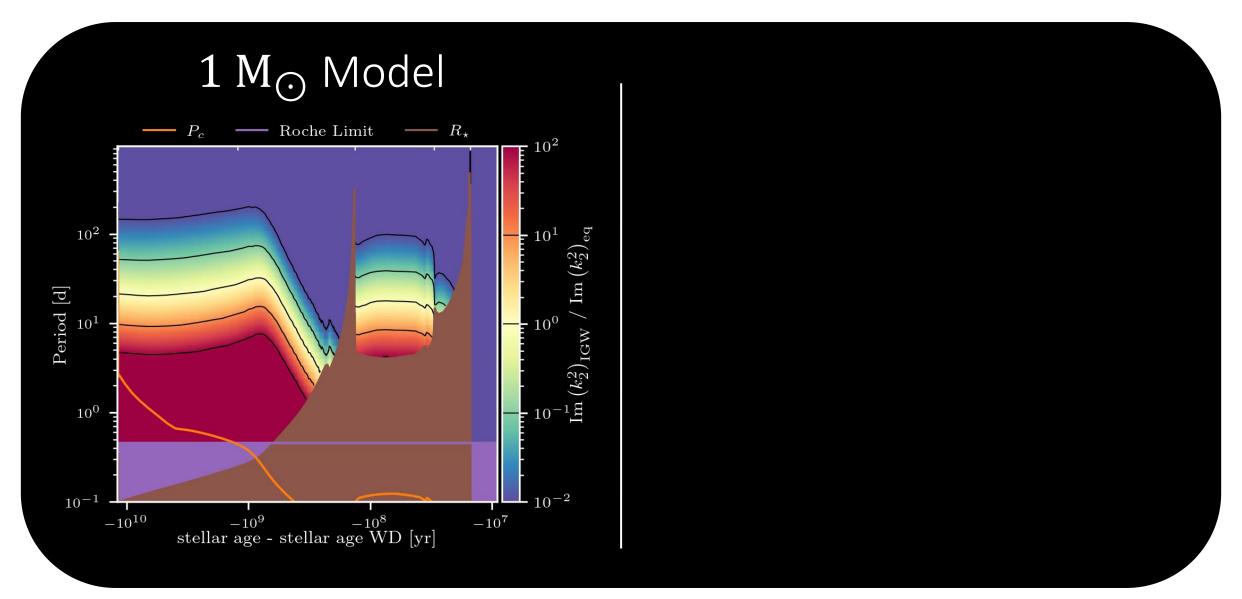
Ahuir et al. (2021); Esseldeurs et al. (2024)

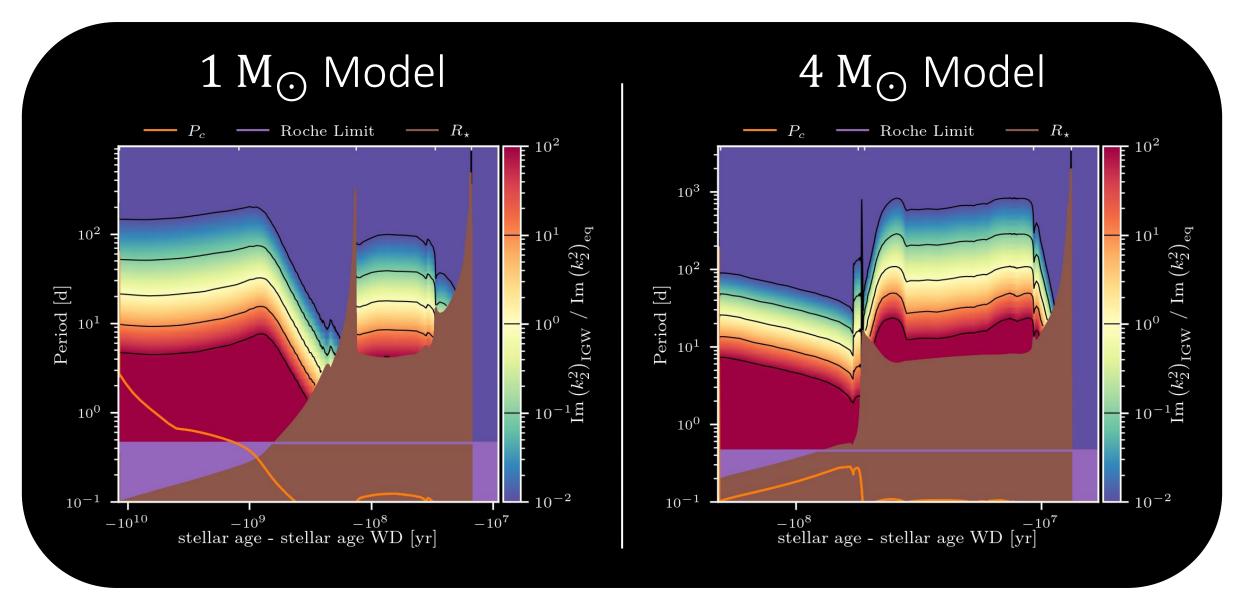


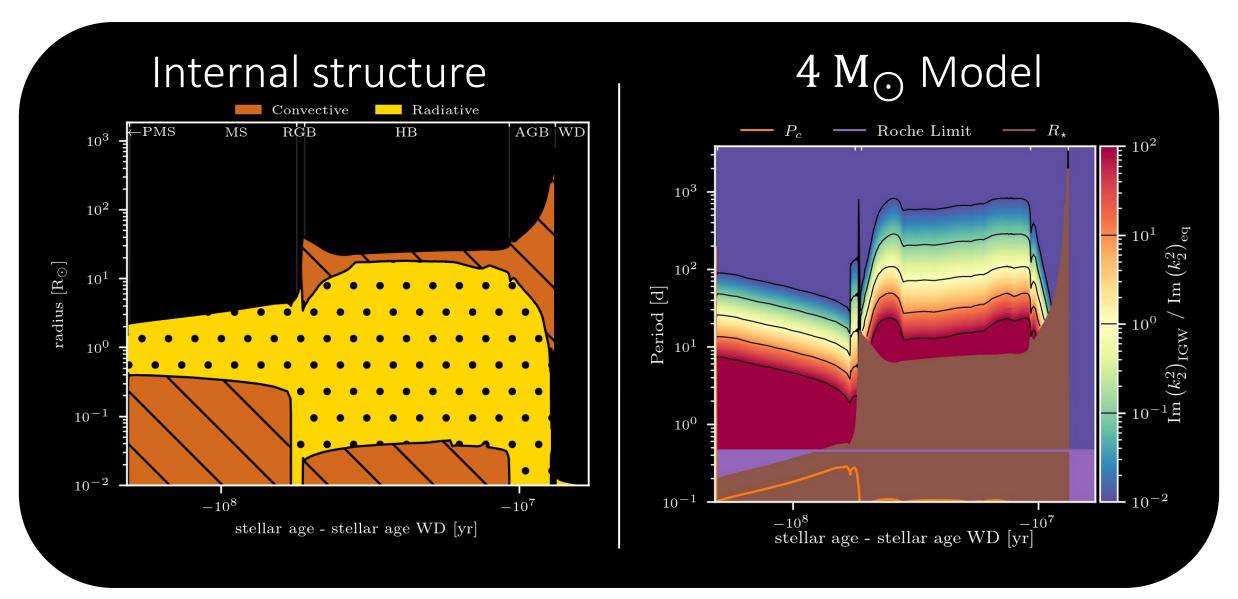
#### Contributions

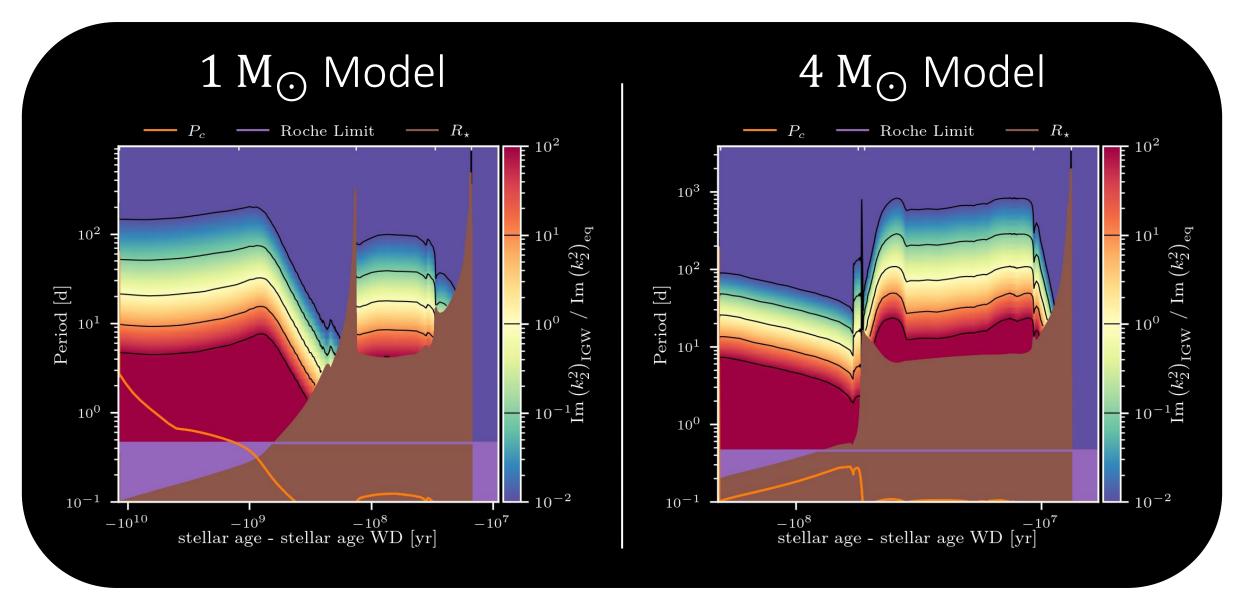
$$\operatorname{Im}(k_{2}^{2})_{\operatorname{IGW}} = \frac{3^{-\frac{1}{3}}\Gamma^{2}\left(\frac{1}{3}\right)}{2\pi} [l(l+1)]^{-\frac{4}{3}} \omega_{t}^{\frac{8}{3}} \frac{a^{6}}{GM_{2}^{2}R_{\star}^{5}} \\ \times \left(\rho_{0}\left(r_{\operatorname{in}}\right)r_{\operatorname{in}}\left|\frac{dN^{2}}{\operatorname{d}\ln r}\right|_{r_{\operatorname{in}}}^{-\frac{1}{3}} \mathcal{F}_{\operatorname{in}}^{2} \right. \\ \left. + \rho_{0}\left(r_{\operatorname{out}}\right)r_{\operatorname{out}}\left|\frac{dN^{2}}{\operatorname{d}\ln r}\right|_{r_{\operatorname{out}}}^{-\frac{1}{3}} \mathcal{F}_{\operatorname{out}}^{2}\right)$$
Ahuir et al. (2021); Esseldeurs et al. (2024)

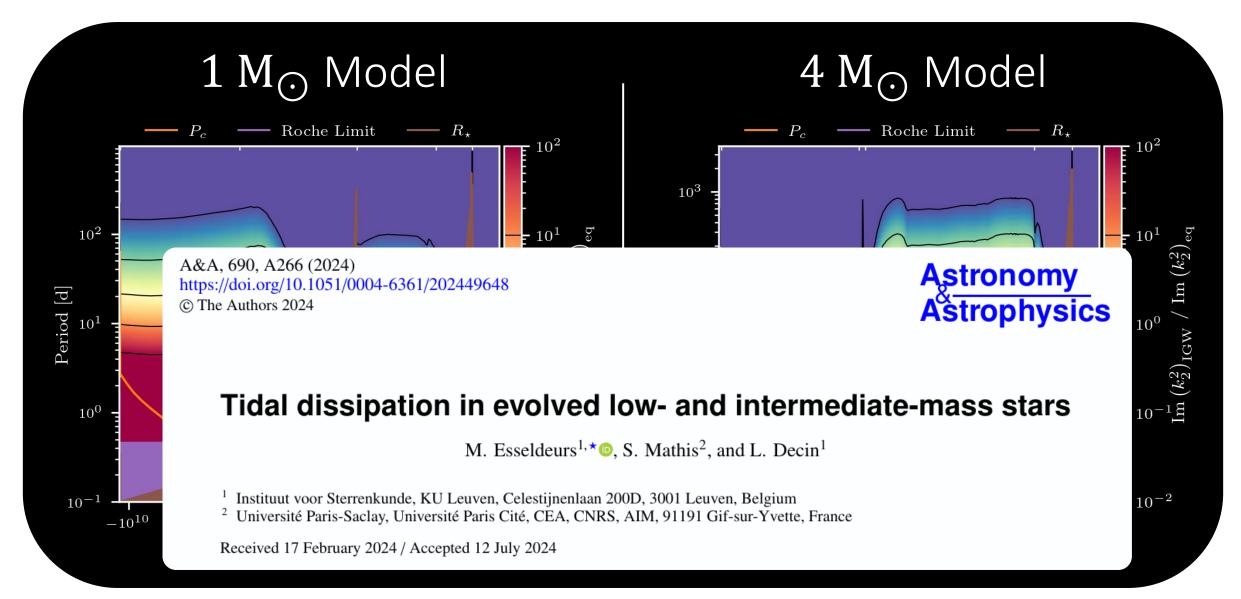












#### Conclusions

• During the MS, the dynamical tide dominates for orbital periods shorter than 10 days, while the equilibrium tide dominates for orbital periods longer than 50 days.

• When the star increases its size, so does the importance of the equilibrium tide, while the effect of the dynamical tide decreases.

• During the giant phases (RGB and AGB) the equilibrium tide dominates, and the dynamical tide is negligible.

#### Future prospects

 These tidal dissipation computations can now be used in orbital evolution codes to compute the orbital evolution for planets starting from the PMS, all the way to the WD phase.

 Improve the equilibrium tide dissipation prescription taking into account non-local effects of the viscosity.

 Potential effect of magnetism in the cores of giant stars, and their effect on tidal dissipation