```mathematica
ClearAll["Global`*"]


Dhyp[n_, k_, a_] :=
 Sum[Binomial[k, j] Dhyp[n / (m^ (k - j)), j, m + 1], {m, a, n^(1 / k)}, {j, 0, k - 1}]
Dhyp[n_, 1, a_] := Floor[n] - a + 1; Dhyp[n_, 0, a_] := 1
DA[n_, k_, a_] := Sum[ DA[n / j, k - 1, a], {j, a, n}]; DA[n_, 0, a_] := 1

da[n_, k_, c_] := c Sum[da[n / (j c), k - 1, c], {j, 1 + 1 / c, n / c^k}]; da[n_, 0, c_] := 1
dc[n_, k_, c_] := c^-k Dhyp[n c^k, k, c + 1]
de[n_, k_, c_] := c^-k DA[n c^k, k, c + 1]

df[n_, k_, c_] := c^-k Sum[ DA[n c^k j^-1, k - 1, c + 1], {j, c + 1, n c^k}]
db[n_, k_, c_] := c^-1 Sum[db[n c^1 j^-1, k - 1, c],    {j, c + 1, n c^k}];
db[n_, 0, c_] := 1
dg[n_, k_, c_] := c^-(2 k) Sum[ DA[n c^ (2 k) j^-1, k - 1, (c / 2) + 1], {j, c + 1, n c^k}]


{db[nn = 200, kk = 2, cc = 4], da[nn, kk, 1 / cc],
 dc[nn, kk, cc], de[nn, kk, cc], df[nn, kk, cc], dg[nn, kk, cc]}
```

$$\left\{\frac{6513}{8}, \frac{6513}{8}, \frac{6513}{8}, \frac{6513}{8}, \frac{6513}{8}, \frac{328167}{256}\right\}$$

```mathematica
di[n_, k_, c_] :=
 Sum[ (c^-1 db[n c^1 j^-1, k - 1, c]) - (c^-k DA[n c^k j^-1, k - 1, c + 1]), {j, c + 1, n c^k}]

di[200, 3, 4]
```

0

```mathematica
di2[n_, k_, c_, j_] := (c^-1 db[n c^1 j^-1, k, c]) - (c^-(k + 1) DA[n c^ (k + 1) j^-1, k, c + 1])

di2[200, 2, 2, 4]
```

0

```mathematica
di3[n_, k_, c_] := (c^-1 db[n c^1 , k, c]) - (c^-(k + 1) DA[n c^ (k + 1), k, c + 1])

di3[120, 3, 2]
```

0

```mathematica
di4[n_, k_, c_] := (db[n, k, c]) - (c^-k DA[n c^k, k, c + 1])

di4[160, 4, 2]
```

0

```mathematica
di5[n_, k_, c_] := (c^-2 db[n c^2, k, c]) - (c^-(k + 2) DA[n c^ (k + 2), k, c + 1])

di5[60, 3, 2]
```

0

```mathematica
di6[n_, k_, c_, a_] := (c^-a db[n c^a , k, c]) - (c^-(k + a) DA[n c^ (k + a), k, c + 1])

di6[160, 3, 2, -3]
```

0

```mathematica
di7[n_, k_, c_] := (c^k db[n c^-k, k, c]) - DA[n, k, c + 1]
```

```
di7[160, 3, 2]
```

0

```
di8[n_, k_, c_] := (c^k db[n c^-k, k, c])
```

```
di8[100, 2, 1]
```

283

```
di9[n_, k_, c_] := ((c - 1)^k db[n (c - 1)^-k, k, (c - 1)])
```

```
di9[100, 2, 2]
```

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