```
ClearAll["Global`*"]

E2a[n_, k_, a_] :=
 E2a[n, k, a] = Sum[ E2a[n / j, k - 1, a], {j, 2, n}] - a Sum[ E2a[n / (a j), k - 1, a], {j, 1, n / a}];
E2a[n_, 0, a_] := 1
D2a[n_, k_] := D2a[n, k] = Sum[D2a[Floor[n / j], k - 1], {j, 2, n}]; D2a[n_, 0] := 1
DD[n_, z_] := DD[n, z] = Sum[FactorialPower[z, a] / a! D2a[n, a], {a, 0, Log[2, n]}]
EE[n_, z_, b_] :=
 EE[n, z, b] = Sum[FactorialPower[z, a] / a! E2a[n, a, b], {a, 0, Log[If[b > 2, 2, b], n]}]
D1b[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j E1b[n / b^j, k, b], {j, 0, Log[b, n]}]
E1b[n_, k_, b_] := Sum[FactorialPower[k, a] / a! E2b[n, a, b], {a, 0, Log[If[b > 2, 2, b], n]}]
E2b[n_, k_, a_] :=
 E2b[n, k, a] = Sum[ E2b[n / j, k - 1, a], {j, 2, n}] - a Sum[ E2b[n / (a j), k - 1, a], {j, 1, n / a}];
E2b[n_, 0, a_] := 1
D1c[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j
   Sum[FactorialPower[k, a] / a! E2b[n / b^j, a, b], {a, 0, Log[If[b > 2, 2, b], n / b^j]}],
  {j, 0, Log[b, n]}]
D1d[n_, z_, b_] := Sum[
  Binomial[z + j - 1, z - 1] Binomial[z, k] b^j E2[n / b^j, k, b],
  {j, 0, Log[b, n]}, {k, 0, Log[If[b > 2, 2, b], n / b^j]}]
D1e[n_, k_, b_] := Grid[Table[
   Binomial[k + j - 1, k - 1] Binomial[k, a] b^j E2[n / b^j, a, b],
   {j, 0, Log[b, n]}, {a, 0, Log[If[b > 2, 2, b], n / b^j]}]]
D1e2[n_, k_, b_] := Grid[Table[
   Binomial[k + j - 1, k - 1] FactorialPower[k, a] / a! b^j E2[n / b^j, a, b] / k,
   {j, 0, Log[b, n]}, {a, 0, Log[If[b > 2, 2, b], n / b^j]}]]
D1c2[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j
   Sum[FactorialPower[k, a] / a! E2b[n / b^j, a, b], {a, 0, Log[If[b > 2, 2, b], n / b^j]}],
  {j, 0, Log[b, n]}]
lin[n_, b_] := Sum[(-1)^(k + 1) / k E2b[n, k, b], {k, 1, Log[2, n]}]
M2[n_, a_] := Sum[(-1)^k ( E2b[n, k, a] - a E2b[n / a, k, a]), {k, 0, Log[a, n]}]



M2[30, 1.1]

-3.

D1c[30, -1, 1.1]

-3.

(D1c2[100, .0000001, 2] - 1) / .0000001

28.5333

lin[100, 2]

4
-
5
```

D1e2[100, .0000001, 2]

$1.\times 10^{7}\,E2[100, 0, 2]$	$1.\,E2[100, 1, 2]$	$-0.5\,E2[100, 2, 2]$	$0.333333\,E2[100, 3, 2]$	$-0.25\,E2[100, 4, 2]$	$0.2\,E2[100, 5, 2]$	$-0.166667\,E2[100, 6, 2]$
$2.\,E2[50, 0, 2]$	$2.\times 10^{-7}\,E2[50, 1, 2]$	$-1.\times 10^{-7}\,E2[50, 2, 2]$	$6.66667\times 10^{-8}\,E2[50, 3, 2]$	$-5.\times 10^{-8}\,E2[50, 4, 2]$	$4.\times 10^{-8}\,E2[50, 5, 2]$	
$2.\,E2[25, 0, 2]$	$2.\times 10^{-7}\,E2[25, 1, 2]$	$-1.\times 10^{-7}\,E2[25, 2, 2]$	$6.66667\times 10^{-8}\,E2[25, 3, 2]$	$-5.\times 10^{-8}\,E2[25, 4, 2]$		
$2.66667\,E2\left[\frac{25}{2}, 0, 2\right]$	$2.66667\times 10^{-7}\,E2\left[\frac{25}{2}, 1, 2\right]$	$-1.33333\times 10^{-7}\,E2\left[\frac{25}{2}, 2, 2\right]$	$8.88889\times 10^{-8}\,E2\left[\frac{25}{2}, 3, 2\right]$			
$4.\,E2\left[\frac{25}{4}, 0, 2\right]$	$4.\times 10^{-7}\,E2\left[\frac{25}{4}, 1, 2\right]$	$-2.\times 10^{-7}\,E2\left[\frac{25}{4}, 2, 2\right]$				
$6.4\,E2\left[\frac{25}{8}, 0, 2\right]$	$6.4\times 10^{-7}\,E2\left[\frac{25}{8}, 1, 2\right]$					
$10.6667\,E2\left[\frac{25}{16}, 0, 2\right]$						

D1e[100, -1, 2]

$E2[100, 0, 2]$	$-E2[100, 1, 2]$	$E2[100, 2, 2]$	$-E2[100, 3, 2]$	$E2[100, 4, 2]$	$-E2[100, 5, 2]$	$E2[100, 6, 2]$
$-2\,E2[50, 0, 2]$	$2\,E2[50, 1, 2]$	$-2\,E2[50, 2, 2]$	$2\,E2[50, 3, 2]$	$-2\,E2[50, 4, 2]$	$2\,E2[50, 5, 2]$	
0	0	0	0	0		
0	0	0	0			
0	0	0				
0	0					
0						

D1e[900, 1, 2]

$E2[900, 0, 2]$	$E2[900, 1, 2]$	0 0 0 0 0 0 0 0
$2\,E2[450, 0, 2]$	$2\,E2[450, 1, 2]$	0 0 0 0 0 0 0
$4\,E2[225, 0, 2]$	$4\,E2[225, 1, 2]$	0 0 0 0 0 0
$8\,E2\left[\frac{225}{2}, 0, 2\right]$	$8\,E2\left[\frac{225}{2}, 1, 2\right]$	0 0 0 0 0
$16\,E2\left[\frac{225}{4}, 0, 2\right]$	$16\,E2\left[\frac{225}{4}, 1, 2\right]$	0 0 0 0
$32\,E2\left[\frac{225}{8}, 0, 2\right]$	$32\,E2\left[\frac{225}{8}, 1, 2\right]$	0 0 0
$64\,E2\left[\frac{225}{16}, 0, 2\right]$	$64\,E2\left[\frac{225}{16}, 1, 2\right]$	0 0
$128\,E2\left[\frac{225}{32}, 0, 2\right]$	$128\,E2\left[\frac{225}{32}, 1, 2\right]$	0
$256\,E2\left[\frac{225}{64}, 0, 2\right]$	$256\,E2\left[\frac{225}{64}, 1, 2\right]$	
$512\,E2\left[\frac{225}{128}, 0, 2\right]$		

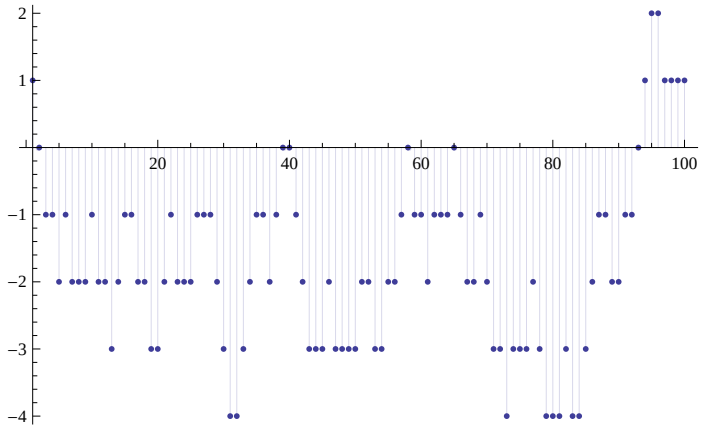
D1e[900, 2, 2]

$E2[900, 0, 2]$	$2\,E2[900, 1, 2]$	$E2[900, 2, 2]$	0 0 0 0 0 0 0
$4\,E2[450, 0, 2]$	$8\,E2[450, 1, 2]$	$4\,E2[450, 2, 2]$	0 0 0 0 0 0
$12\,E2[225, 0, 2]$	$24\,E2[225, 1, 2]$	$12\,E2[225, 2, 2]$	0 0 0 0 0
$32\,E2\left[\frac{225}{2}, 0, 2\right]$	$64\,E2\left[\frac{225}{2}, 1, 2\right]$	$32\,E2\left[\frac{225}{2}, 2, 2\right]$	0 0 0 0
$80\,E2\left[\frac{225}{4}, 0, 2\right]$	$160\,E2\left[\frac{225}{4}, 1, 2\right]$	$80\,E2\left[\frac{225}{4}, 2, 2\right]$	0 0 0
$192\,E2\left[\frac{225}{8}, 0, 2\right]$	$384\,E2\left[\frac{225}{8}, 1, 2\right]$	$192\,E2\left[\frac{225}{8}, 2, 2\right]$	0 0
$448\,E2\left[\frac{225}{16}, 0, 2\right]$	$896\,E2\left[\frac{225}{16}, 1, 2\right]$	$448\,E2\left[\frac{225}{16}, 2, 2\right]$	0
$1024\,E2\left[\frac{225}{32}, 0, 2\right]$	$2048\,E2\left[\frac{225}{32}, 1, 2\right]$	$1024\,E2\left[\frac{225}{32}, 2, 2\right]$	
$2304\,E2\left[\frac{225}{64}, 0, 2\right]$	$4608\,E2\left[\frac{225}{64}, 1, 2\right]$		
$5120\,E2\left[\frac{225}{128}, 0, 2\right]$			

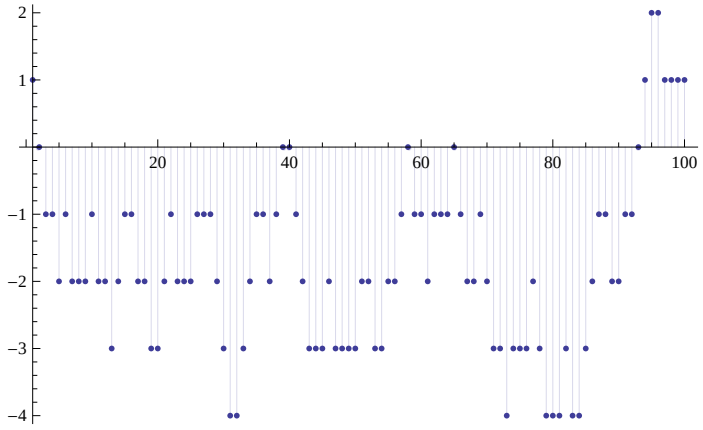
DD[100, -1]

1

DiscretePlot[DD[n, -1], {n, 1, 100}]



```
DiscretePlot[ EE[n, -1, 200], {n, 1, 100}]
```




```
Animate[DiscretePlot[ EE[n, 7, z], {n, 1, 100}], {z, 2, 100}]
```

```
Animate[DiscretePlot[ EE[n, -1, z], {n, 1, 100}], {z, 2, 100}]
```

```
Animate[DiscretePlot[( EE[n, .0001, z] - 1) / .0001, {n, 1, 100}], {z, 2, 100}]
```

D1d[1000, -3 / 2, 2]

$$\frac{143}{128}\,E2\left[\frac{125}{64},0,2\right]+\frac{99}{128}\,E2\left[\frac{125}{32},0,2\right]-\frac{297}{256}\,E2\left[\frac{125}{32},1,2\right]+\frac{9}{16}\,E2\left[\frac{125}{16},0,2\right]-$$
$$\frac{27}{32}\,E2\left[\frac{125}{16},1,2\right]+\frac{135}{128}\,E2\left[\frac{125}{16},2,2\right]+\frac{7}{16}\,E2\left[\frac{125}{8},0,2\right]-\frac{21}{32}\,E2\left[\frac{125}{8},1,2\right]+$$
$$\frac{105}{128}\,E2\left[\frac{125}{8},2,2\right]-\frac{245}{256}\,E2\left[\frac{125}{8},3,2\right]+\frac{3}{8}\,E2\left[\frac{125}{4},0,2\right]-\frac{9}{16}\,E2\left[\frac{125}{4},1,2\right]+$$
$$\frac{45}{64}\,E2\left[\frac{125}{4},2,2\right]-\frac{105}{128}\,E2\left[\frac{125}{4},3,2\right]+\frac{945\,E2\left[\frac{125}{4},4,2\right]}{1024}+\frac{3}{8}\,E2\left[\frac{125}{2},0,2\right]-\frac{9}{16}\,E2\left[\frac{125}{2},1,2\right]+$$
$$\frac{45}{64}\,E2\left[\frac{125}{2},2,2\right]-\frac{105}{128}\,E2\left[\frac{125}{2},3,2\right]+\frac{945\,E2\left[\frac{125}{2},4,2\right]}{1024}-\frac{2079\,E2\left[\frac{125}{2},5,2\right]}{2048}+$$
$$\frac{1}{2}\,E2[125,0,2]-\frac{3}{4}\,E2[125,1,2]+\frac{15}{16}\,E2[125,2,2]-\frac{35}{32}\,E2[125,3,2]+\frac{315}{256}\,E2[125,4,2]-$$
$$\frac{693}{512}\,E2[125,5,2]+\frac{3003\,E2[125,6,2]}{2048}+\frac{3}{2}\,E2[250,0,2]-\frac{9}{4}\,E2[250,1,2]+\frac{45}{16}\,E2[250,2,2]-$$
$$\frac{105}{32}\,E2[250,3,2]+\frac{945}{256}\,E2[250,4,2]-\frac{2079}{512}\,E2[250,5,2]+\frac{9009\,E2[250,6,2]}{2048}-$$
$$\frac{19\,305\,E2[250,7,2]}{4096}-3\,E2[500,0,2]+\frac{9}{2}\,E2[500,1,2]-\frac{45}{8}\,E2[500,2,2]+$$
$$\frac{105}{16}\,E2[500,3,2]-\frac{945}{128}\,E2[500,4,2]+\frac{2079}{256}\,E2[500,5,2]-\frac{9009\,E2[500,6,2]}{1024}+$$
$$\frac{19\,305\,E2[500,7,2]}{2048}-\frac{328\,185\,E2[500,8,2]}{32\,768}+E2[1000,0,2]-\frac{3}{2}\,E2[1000,1,2]+$$
$$\frac{15}{8}\,E2[1000,2,2]-\frac{35}{16}\,E2[1000,3,2]+\frac{315}{128}\,E2[1000,4,2]-\frac{693}{256}\,E2[1000,5,2]+$$
$$\frac{3003\,E2[1000,6,2]}{1024}-\frac{6435\,E2[1000,7,2]}{2048}+\frac{109\,395\,E2[1000,8,2]}{32\,768}-\frac{230\,945\,E2[1000,9,2]}{65\,536}$$

D1e[900, -1, 2]

E2[900, 0, 2]	-E2[900, 1, 2]	E2[900, 2, 2]	-E2[900, 3, 2]	E2[900, 4, 2]	-E2[900, 5, 2]	E2[900, 6, 2]	-E2[900, 7, 2]	E2[900, 8, 2]	-E2[900, 9, 2]
-2 E2[450, 0, 2]	2 E2[450, 1, 2]	-2 E2[450, 2, 2]	2 E2[450, 3, 2]	-2 E2[450, 4, 2]	2 E2[450, 5, 2]	-2 E2[450, 6, 2]	2 E2[450, 7, 2]	-2 E2[450, 8, 2]	
0	0	0	0	0	0	0	0		
0	0	0	0	0	0	0			
0	0	0	0	0	0				
0	0	0	0	0					
0	0	0	0						
0	0	0							
0	0								
0									

```
D1f[n_, k_, b_] := Sum[
  Binomial[k + j - 1, k - 1] Binomial[k, a] b^j E2a[n / b^j, a, b],
  {j, 0, Log[b, n]}, {a, 0, Log[If[b > 2, 2, b], n / b^j]}]

ME2[n_, b_] := Sum[ (-1)^k E2a[n, k, b], {k, 0, Log[If[b < 2, b, 2], n]}]
ME2a[n_, b_] := ME2[n, b] - b ME2[n / b, b]
ME2b[n_, a_] := Sum[ (-1)^k (E2a[n, k, a] - a E2a[n / a, k, a]), {k, 0, Log[If[a < 2, a, 2], n]}]

ME2[100, 2]
```

```
-13
```

```
M2[n_, a_] := Sum[ (-1)^k (k + 1) (E2a[n, k, a] - 2 a E2a[n / a, k, a] + a^2 E2a[n / (a^2), k, a]),
  {k, 0, Log[If[a < 2, a, 2], n]}]

M2[100, 4]
```

```
19
```

```
DD[100, -2]
```

```
19
```

```
D1c[100, -2, 2]
```

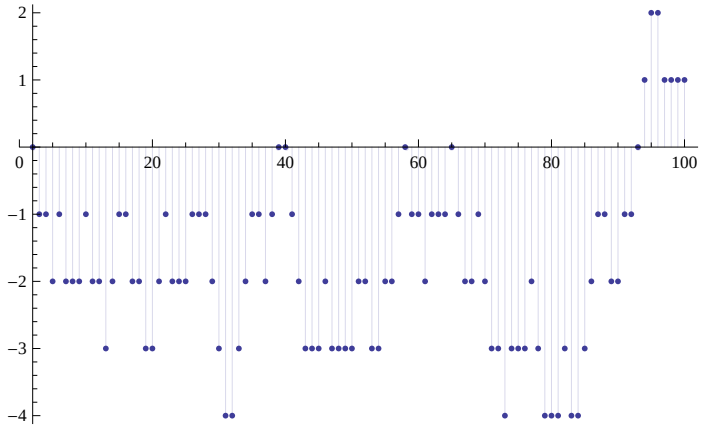
```
19
```

```
D1e[2400, -2, 3]
```

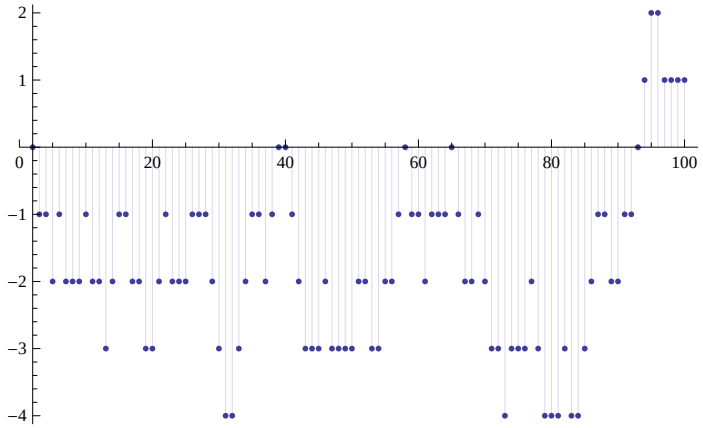
```
E2[2400, 0, 3] - 2 E2[2400, 1, 3] + 3 E2[2400, 2, 3] - 4 E2[2400, 3, 3] + 5 E2[2400, 4, 3] - 6 E2[2400, 5, 3] + 7 E2[2400, 6, 3] - 8 E2[2400, 7, 3] + 9 E2[2400, 8, 3] - 10 E2[2400, 9, 3] + 11 E2[2400, 10, 3] - 12 E2[2400, 11, 3] - 6 E2[800, 0, 3] + 12 E2[800, 1, 3] - 18 E2[800, 2, 3] + 24 E2[800, 3, 3] - 30 E2[800, 4, 3] + 36 E2[800, 5, 3] - 42 E2[800, 6, 3] + 48 E2[800, 7, 3] - 54 E2[800, 8, 3] + 60 E2[800, 9, 3] + 9 E2[800 / 3, 0, 3] - 18 E2[800 / 3, 1, 3] + 27 E2[800 / 3, 2, 3] - 36 E2[800 / 3, 3, 3] + 45 E2[800 / 3, 4, 3] - 54 E2[800 / 3, 5, 3] + 63 E2[800 / 3, 6, 3] - 72 E2[800 / 3, 7, 3] + 81 E2[800 / 3, 8, 3]
```

```
0
```

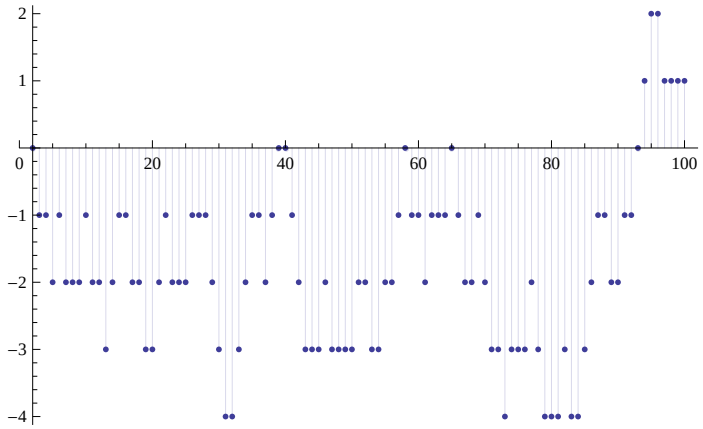
```
DiscretePlot[ME2b[n, 2], {n, 2, 100}]
```



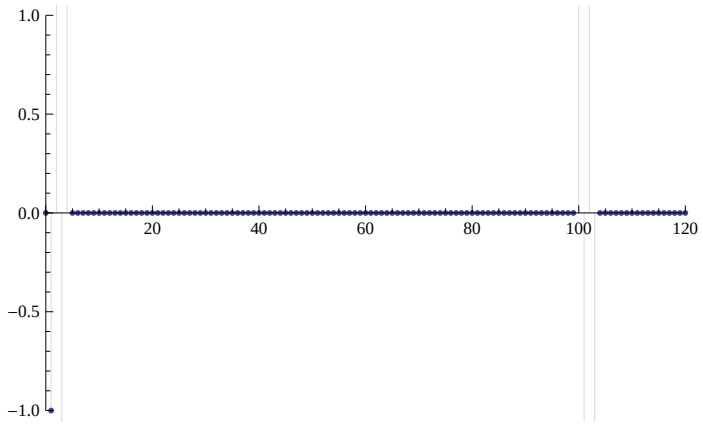

```
DiscretePlot[D1f[n, -1, 2], {n, 2, 100}]
```




```
DiscretePlot[ME2[n, (3 / 2)] - (3 / 2) ME2[n / (3 / 2), (3 / 2)], {n, 2, 100}]
```



```
DiscretePlot[(-1)^k (E2a[100, k, 1.0001] - 1.0001 E2a[100 / 1.0001, k, 1.0001]),
 {k, 0, 120}, PlotRange → {{0, 120}, {-1, 1}}]
```




```
D1e[2400, -2, 7]
```

```
E2[ -2 E2[ 3 E2[ -4 E2[ 5 E2[ -6 E2[ 7 E2[ -8 E2[ 9 E2[ -10 11 E2[ -12
 2400,  2400  2400  2400  2400  2400  2400  2400  2400  E2[  2400  E2[
 0, 7]  ,     ,     ,     ,     ,     ,     ,     ,     2400  ,     2400
        1,    2,    3,    4,    5,    6,    7,    8,    ,     10,   ,
        7]    7]    7]    7]    7]    7]    7]    7]    9,    7]    11,
                                                         7]          7]

-14   28 E2[ -42   56 E2[ -70   84 E2[ -98   112   -126
 E2[    2400   E2[    2400   E2[    2400   E2[   E2[   E2[
  2400  /      2400  /      2400  /      2400  2400  2400
  /     7,     /     7,     /     7,     /     /     /
  7,    1,     7,    3,     7,    5,     7,    7,    7,
  0,    7]     2,    7]     4,    7]     6,    7,    8,
  7]           7]           7]           7]    7]    7]

49 E2[ -98   147   -196  245   -294
  2400  E2[   E2[   E2[   E2[   E2[
  /     2400  2400  2400  2400  2400
  49,   /     /     /     /     /
  0,    49,   49,   49,   49,   49,
  7]    1,    2,    3,    4,    5,
        7]    7]    7]    7]    7]
  0     0     0
```

```
D1e[1200, -3, 2]
```

```
E2[ -3 E2[ 6 E2[ -10 E2[ 15 E2[ -21 E2[ 28 E2[ -36 E2[ 45 E2[ -55 E2[ 66 E2[
 1200,    1200    1200    1200    1200    1200    1200    1200    1200    1200
 0, 2]      ,       ,       ,       ,       ,       ,       ,       ,       ,
          1, 2]   2, 2]   3, 2]   4, 2]   5, 2]   6, 2]   7, 2]   8, 2]   9, 2]
                                                                            10,
                                                                            2]
-6 E2[  18 E2[ -36 E2[ 60 E2[ -90 E2[ 126 E2[ -168    216 E2[ -270    330 E2[
  600,    600,    600,    600,    600,    600,    E2[     600,    E2[     600,
  0, 2]   1, 2]   2, 2]   3, 2]   4, 2]   5, 2]   600,   7, 2]   600,   9, 2]
                                                   6, 2]           8, 2]
12 E2[  -36 E2[ 72 E2[ -120    180 E2[ -252    336 E2[ -432    540 E2[
  300,    300,    300,    E2[     300,    E2[     300,    E2[     300,
  0, 2]   1, 2]   2, 2]   300,   4, 2]   300,   6, 2]   300,   8, 2]
                         3, 2]           5, 2]           7, 2]
-8 E2[  24 E2[ -48 E2[ 80 E2[ -120    168 E2[ -224    288 E2[
  150,    150,    150,    150,    E2[     150,    E2[     150,
  0, 2]   1, 2]   2, 2]   3, 2]   150,   5, 2]   150,   7, 2]
                                  4, 2]           6, 2]
   0       0       0       0       0       0       0
   0       0       0       0       0       0
   0       0       0       0       0
   0       0       0       0
   0       0       0
   0       0
   0
```

```
$RecursionLimit = 10 000
```

```
10 000
```

```
Table[{1.0001^k, (-1)^k (E2a[100, k, 1.0001] - 1.0001 E2a[100 / 1.0001, k, 1.0001])},
  {k, 0, 200}] // TableForm
```

```
1.          -0.0001
1.0001      -1.0001
1.0002      6.9999
1.0003      -12.0001
1.0004      5.9999
1.0005      -0.000104955
1.0006      -0.000105947
1.0007      -0.00010694
1.0008      -0.000107933
1.0009      -0.000108927
1.001       -0.000109921
1.0011      -0.000110916
1.0012      -0.000111911
1.0013      -0.000112907
1.0014      -0.000113903
1.0015      -0.0001149
1.0016      -0.000115897
1.0017      -0.000116894
1.0018      -0.000117892
```

```
1.0019     -0.000118891
1.002      -0.00011989
1.0021     -0.00012089
1.0022     -0.000121889
1.0023     -0.00012289
1.0024     -0.000123891
1.0025     -0.000124892
1.0026     -0.000125894
1.0027     -0.000126896
1.0028     -0.000127899
1.0029     -0.000128903
1.003      -0.000129906
1.0031     -0.000130911
1.0032     -0.000131915
1.00331    -0.000132921
1.00341    -0.000133926
1.00351    -0.000134932
1.00361    -0.000135939
1.00371    -0.000136946
1.00381    -0.000137954
1.00391    -0.000138962
1.00401    -0.00013997
1.00411    -0.000140979
1.00421    -0.000141989
1.00431    -0.000142999
1.00441    -0.000144009
1.00451    -0.00014502
1.00461    -0.000146032
1.00471    -0.000147044
1.00481    -0.000148056
1.00491    -0.000149069
1.00501    -0.000150082
1.00511    -0.000151096
1.00521    -0.00015211
1.00531    -0.000153125
1.00541    -0.00015414
1.00551    -0.000155156
1.00562    -0.000156172
1.00572    -0.000157189
1.00582    -0.000158206
1.00592    -0.000159224
1.00602    -0.000160242
1.00612    -0.000161261
1.00622    -0.00016228
1.00632    -0.000163299
1.00642    -0.00016432
1.00652    -0.00016534
1.00662    -0.000166361
1.00672    -0.000167383
1.00682    -0.000168405
1.00692    -0.000169427
1.00702    -0.00017045
1.00712    -0.000171474
1.00723    -0.000172498
1.00733    -0.000173522
1.00743    -0.000174547
```

```
1.00753    -0.000175572
1.00763    -0.000176598
1.00773    -0.000177625
1.00783    -0.000178651
1.00793    -0.000179679
1.00803    -0.000180707
1.00813    -0.000181735
1.00823    -0.000182764
1.00833    -0.000183793
1.00843    -0.000184823
1.00854    -0.000185853
1.00864    -0.000186884
1.00874    -0.000187915
1.00884    -0.000188946
1.00894    -0.000189979
1.00904    -0.000191011
1.00914    -0.000192044
1.00924    -0.000193078
1.00934    -0.000194112
1.00944    -0.000195147
1.00954    -0.000196182
1.00965    -0.000197217
1.00975    -0.000198254
1.00985    -0.00019929
1.00995    -0.000200327
1.01005    510126.
```

```
N[Table[{k, (-1)^k (E2a[1000, k, ss = 2] - ss E2a[1000 / ss, k, ss])}, {k, 0, 20}] // TableForm]
```

```
0.     -1.
1.     -1.
2.     -4.
3.     13.
4.     98.
5.     -120.
6.     26.
7.     20.
8.     -21.
9.     -8.
10.    0.
11.    0.
12.    0.
13.    0.
14.    0.
15.    0.
16.    0.
17.    0.
18.    0.
19.    0.
20.    0.
```

```
N[Table[{k, (-1)^k (E2a[10 000, k, ss = 2] - ss E2a[1000 / ss, k, ss])}, {k, 0, 20}] // TableForm]
```

```
0.    -1.
1.    -1.
2.    7.
3.    0.
4.    525.
5.    -1069.
6.    -792.
7.    1293.
8.    -154.
9.    1018.
10.   939.
11.   320.
12.   67.
13.   1.
14.   0.
15.   0.
16.   0.
17.   0.
18.   0.
19.   0.
20.   0.
```

```
Binomial[z, 0]
```

```
1
```

```
Limit[ Sum[ Binomial[z + j - 1,  z - 1] a^j, {j, 0, Log[a, n]}] /. z → 1, a → 1]
```

```
DirectedInfinity[-1 + n]
```

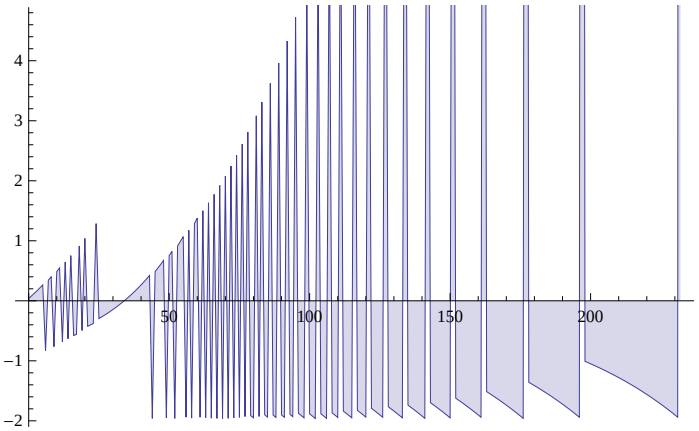
```
D1e[100, -3, 2]
```

$$\begin{array}{llllllll} E2[100, 0, 2] & -3\,E2[100, 1, 2] & 6\,E2[100, 2, 2] & -10\,E2[100, 3, 2] & 15\,E2[100, 4, 2] & -21\,E2[100, 5, 2] & 28\,E2[100, 6, 2] & \\ -6\,E2[50, 0, 2] & 18\,E2[50, 1, 2] & -36\,E2[50, 2, 2] & 60\,E2[50, 3, 2] & -90\,E2[50, 4, 2] & 126\,E2[50, 5, 2] & & \\ 12\,E2[25, 0, 2] & -36\,E2[25, 1, 2] & 72\,E2[25, 2, 2] & -120\,E2[25, 3, 2] & 180\,E2[25, 4, 2] & & & \\ -8\,E2\left[\frac{25}{2}, 0, 2\right] & 24\,E2\left[\frac{25}{2}, 1, 2\right] & -48\,E2\left[\frac{25}{2}, 2, 2\right] & 80\,E2\left[\frac{25}{2}, 3, 2\right] & & & & \\ 0 & 0 & 0 & & & & & \\ 0 & 0 & & & & & & \\ 0 & & & & & & & \end{array}$$

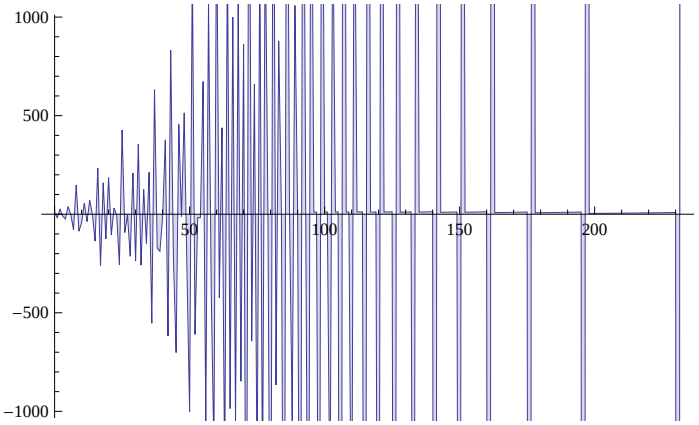
```
$RecursionLimit = 1 000 000
dr[n_, k_, b_] := DiscretePlot[ Binomial[k + j - 1, k - 1] b^j
   Sum[FactorialPower[k, a] / a ! E2b[n / b^j, a, b], {a, 0, Log[If[b > 2, 2, b], n / b^j]}],
  {j, 0, Log[b, n]}]
```

```
1 000 000
```

```
dr[100, 1, 1.02]
```

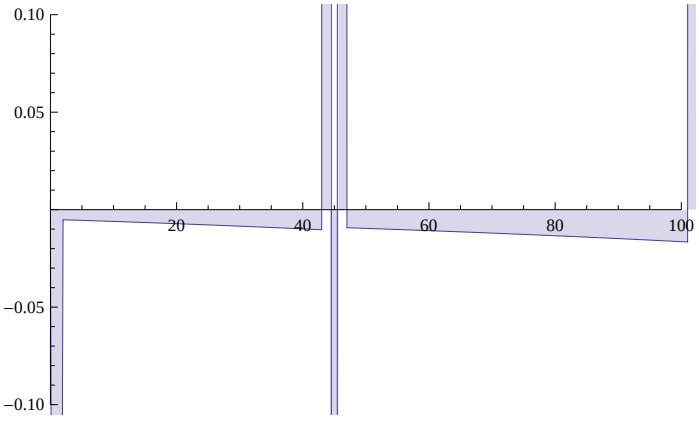



```
dr[100, 2, 1.02]
```




```
Dr2[n_, k_, b_] := DiscretePlot[Sum[ Binomial[k + j - 1, k - 1] b^j
    Sum[FactorialPower[k, a] / a! E2b[n / b^j, a, b], {a, s, s}], {j, 0, Log[b, n]}],
  {s, 0, Log[b, n]}, PlotRange → {{0, 100}, {-.1, .1}}]
```

```
Dr2[5, -1, 1.005]
```



```
Dr3[n_, k_, b_] := Table[{b^s, Sum[ Binomial[k + j - 1, k - 1] b^j
      Sum[FactorialPower[k, a] / a! E2b[n / b^j, a, b], {a, s, s}], {j, 0, Log[b, n]}]},
   {s, 0, Log[b, n]}] // TableForm
```

```
Dr3[12, -1, 1.02]
```

1.	-0.02
1.02	-1.0244
1.0404	3.97097
1.06121	-3.0339
1.08243	4.3773
1.10408	-5.45441
1.12616	-0.0474139
1.14869	-0.052843
1.17166	-0.0585433
1.19509	98.674
1.21899	-229.242
1.24337	131.389
1.26824	-0.0770143
1.29361	-0.083342
1.31948	141.226
1.34587	-299.687
1.37279	158.243
1.40024	-0.100815
1.42825	-0.107714
1.45681	-0.114909
1.48595	2334.
1.51567	-7298.94
1.54598	7596.34
1.5769	-2631.74
1.60844	-0.133239
1.64061	-0.140672
1.67342	-0.148388
1.70689	46.8513
1.74102	-47.9384
1.77584	-0.153128
1.81136	-0.160954
1.84759	-0.169075
1.88454	-0.177499
1.92223	-0.186237
1.96068	-0.195299
1.99989	2498.66
2.03989	-5161.29
2.08069	2663.68
2.1223	-0.180424
2.16474	-0.188061
2.20804	-0.195948
2.2522	-0.204092
2.29724	-0.212502
2.34319	-0.221184
2.39005	107.035
2.43785	-107.749
2.48661	-0.204091
2.53634	-0.212054
2.58707	-0.220274
2.63881	-0.228758

```
2.69159    -0.237514
2.74542    -0.246549
2.80033    -0.255873
2.85633    -0.265493
2.91346    -0.275418
2.97173    4667.71
3.03117    -9409.28
3.09179    4742.68
3.15362    -0.206531
3.2167     -0.213185
3.28103    -0.220022
3.34665    -0.227047
3.41358    -0.234266
3.48186    -0.241682
3.55149    -0.249301
3.62252    -0.257128
3.69497    -0.265169
3.76887    -0.273428
3.84425    -0.281912
3.92114    -0.290625
3.99956    285.269
4.07955    -284.164
4.16114    -0.200714
4.24436    -0.206393
4.32925    -0.212218
4.41584    -0.218194
4.50415    -0.224324
4.59424    -0.230613
4.68612    -0.237063
4.77984    -0.243678
4.87544    -0.250464
4.97295    -0.257423
5.07241    -0.264561
5.17386    -0.271881
5.27733    -0.279388
5.38288    -0.287087
5.49054    -0.294982
5.60035    -0.303078
5.71235    -0.311379
5.8266     -0.319892
5.94313    545.251
6.062      -540.946
6.18324    -0.123665
6.3069     -0.126138
6.43304    -0.128661
6.5617     -0.131234
6.69293    -0.133859
6.82679    -0.136536
6.96333    -0.139267
7.10259    -0.142052
7.24465    -0.144893
7.38954    -0.147791
7.53733    -0.150747
7.68808    -0.153762
7.84184    -0.156837
7.99867    -0.159973
```

```
8.15865    -0.163173
8.32182    -0.166436
8.48826    -0.169765
8.65802    -0.17316
8.83118    -0.176624
9.00781    -0.180156
9.18796    -0.183759
9.37172    -0.187434
9.55916    -0.191183
9.75034    -0.195007
9.94535    -0.198907
10.1443    -0.202885
10.3471    -0.206943
10.5541    -0.211082
10.7652    -0.215303
10.9805    -0.219609
11.2001    -0.224002
11.4241    -0.228482
11.6526    -0.233051
11.8856    11.8856
```

```
DD[5, -1]
```

```
-2
```

```
DD[6, -1]
```

```
-1
```

```
$RecursionLimit = 10 000
```

```
10 000
```

```
Table[{k, (-1)^k ( E2b[5, k, 1.01] - 1.01 E2b[5 / 1.01, k, 1.01]),
    (-1)^(k + 1) / k E2b[5, k, 1.01] }, {k, 1, Log[1.01, 5] + 10}] // TableForm
```

```
1     -1.0104      -0.04
2     -0.010808    -0.5404
3     -0.0112241   -0.374138
4     -0.0116485   -0.291213
5     -0.0120813   -0.241626
6     -0.0125226   -0.20871
7     -0.0129726   -0.185323
8     -0.0134314   -0.167893
9     -0.0138992   -0.154435
10    -0.014376    -0.14376
11    -0.0148621   -0.13511
12    -0.0153576   -0.12798
13    -0.0158626   -0.12202
14    -0.0163773   -0.116981
15    -0.0169018   -0.112679
16    -0.0174364   -0.108977
17    -0.0179811   -0.105771
18    -0.0185361   -0.102978
19    -0.0191016   -0.100535
20    -0.0196778   -0.0983891
21    -0.0202648   -0.0964991
22    318.041      -0.094831
```

23	−661.591	14.9926
24	343.523	−14.3923
25	−0.019173	−0.076692
26	−0.0196212	−0.0754662
27	−0.0200765	−0.0743573
28	−0.0205389	−0.0733531
29	−0.0210085	−0.0724432
30	−0.0214855	−0.0716184
31	−0.0219699	−0.0708708
32	−0.0224619	−0.0701934
33	−0.0229615	−0.0695803
34	−0.0234689	−0.0690261
35	−0.0239841	−0.0685259
36	−0.0245072	−0.0680756
37	−0.0250385	−0.0676715
38	−0.0255779	−0.0673101
39	−0.0261255	−0.0669886
40	−0.0266816	−0.066704
41	−0.0272462	−0.0664542
42	−0.0278194	−0.0662367
43	−0.0284014	−0.0660497
44	−0.0289922	−0.0658913
45	−0.029592	−0.0657599
46	−0.0302008	−0.065654
47	−0.0308189	−0.0655722
48	−0.0314464	−0.0655133
49	−0.0320833	−0.0654761
50	−0.0327298	−0.0654596
51	85.5287	−0.0654628
52	−86.4015	1.6122
53	−0.0258364	−0.0487479
54	−0.0262642	−0.0486374
55	−0.026698	−0.0485418
56	−0.0271378	−0.0484604
57	−0.0275838	−0.0483926
58	−0.028036	−0.0483379
59	−0.0284944	−0.0482956
60	−0.0289592	−0.0482654
61	−0.0294305	−0.0482467
62	−0.0299083	−0.0482392
63	−0.0303927	−0.0482424
64	−0.0308838	−0.0482559
65	−0.0313817	−0.0482795
66	−0.0318864	−0.0483128
67	−0.0323981	−0.0483554
68	−0.0329169	−0.0484072
69	−0.0334428	−0.0484678
70	−0.0339759	−0.048537
71	−0.0345163	−0.0486145
72	−0.0350642	−0.0487002
73	−0.0356195	−0.0487939
74	−0.0361825	−0.0488952
75	−0.0367531	−0.0490042
76	−0.0373316	−0.0491205
77	−0.0379179	−0.049244
78	−0.0385122	−0.0493747

79	−0.0391147	−0.0495122
80	−0.0397253	−0.0496566
81	−0.0403442	−0.0498077
82	−0.0409715	−0.0499653
83	−0.0416074	−0.0501294
84	−0.0422519	−0.0502998
85	−0.042905	−0.0504765
86	−0.0435671	−0.0506594
87	−0.0442381	−0.0508483
88	−0.0449181	−0.0510433
89	−0.0456073	−0.0512442
90	−0.0463058	−0.0514509
91	−0.0470137	−0.0516635
92	232.053	−0.0518817
93	−232.325	2.47072
94	−0.0254806	−0.027107
95	−0.0257354	−0.0270899
96	−0.0259927	−0.0270758
97	−0.0262527	−0.0270646
98	−0.0265152	−0.0270563
99	−0.0267803	−0.0270508
100	−0.0270481	−0.0270481
101	−0.0273186	−0.0270481
102	−0.0275918	−0.0270508
103	−0.0278677	−0.027056
104	−0.0281464	−0.0270638
105	−0.0284279	−0.0270742
106	−0.0287121	−0.0270869
107	−0.0289993	−0.0271021
108	−0.0292893	−0.0271197
109	−0.0295822	−0.0271396
110	−0.029878	−0.0271618
111	−0.0301768	−0.0271863
112	−0.0304785	−0.027213
113	−0.0307833	−0.0272419
114	−0.0310911	−0.0272729
115	−0.031402	−0.0273061
116	−0.0317161	−0.0273414
117	−0.0320332	−0.0273788
118	−0.0323536	−0.0274183
119	−0.0326771	−0.0274597
120	−0.0330039	−0.0275032
121	−0.0333339	−0.0275487
122	−0.0336672	−0.0275961
123	−0.0340039	−0.0276455
124	−0.034344	−0.0276967
125	−0.0346874	−0.0277499
126	−0.0350343	−0.027805
127	−0.0353846	−0.0278619
128	−0.0357385	−0.0279207
129	−0.0360958	−0.0279813
130	−0.0364568	−0.0280437
131	−0.0368214	−0.0281079
132	−0.0371896	−0.0281739
133	−0.0375615	−0.0282417
134	−0.0379371	−0.0283113

135	−0.0383165	−0.0283826
136	−0.0386996	−0.0284556
137	−0.0390866	−0.0285304
138	−0.0394775	−0.0286069
139	−0.0398723	−0.0286851
140	−0.040271	−0.028765
141	−0.0406737	−0.0288466
142	−0.0410804	−0.0289299
143	−0.0414912	−0.0290149
144	−0.0419062	−0.0291015
145	−0.0423252	−0.0291898
146	−0.0427485	−0.0292798
147	−0.043176	−0.0293714
148	−0.0436077	−0.0294647
149	−0.0440438	−0.0295596
150	−0.0444842	−0.0296562
151	−0.0449291	−0.0297544
152	−0.0453784	−0.0298542
153	−0.0458321	−0.0299557
154	−0.0462905	−0.0300587
155	−0.0467534	−0.0301635
156	−0.0472209	−0.0302698
157	−0.0476931	−0.0303778
158	−0.04817	−0.0304874
159	−0.0486517	−0.0305986
160	−0.0491383	−0.0307114
161	4.96296	−0.0308259
162	0.	0.
163	0.	0.
164	0.	0.
165	0.	0.
166	0.	0.
167	0.	0.
168	0.	0.
169	0.	0.
170	0.	0.
171	0.	0.

```
-E2b[5, 161, 1.01] - 1.01 E2b[5 / 1.01, 161, 1.01]
```

4.96296

```
(-1)^159 ( E2b[5, 159, 1.01] - 1.01 E2b[5 / 1.01, 159, 1.01])
```

−0.0486517