```
K[n_] := If[n == 1, 0, FullSimplify[MangoldtLambda[n] / Log[n]]]
P[n_, k_] := P[n, k] = Sum[ K[j] P[Floor[n / j], k - 1], {j, 2, n}]; P[n_, 0] := 1
D2[n_, k_] := D2[n, k] = Sum[D2[Floor[n / j], k - 1], {j, 2, n}]; D2[n_, 0] := 1
DD[n_, z_] := Sum[FactorialPower[z, a] / a! D2[n, a], {a, 0, Log[2, n]}]
P[n_, k_, s_] := P[n, k, s] = Sum[ j^-s K[j] P[Floor[n / j], k - 1, s], {j, 2, n}];
P[n_, 0, s_] := 1
DDa[n_, z_] := Sum[ z^k / k! P[n, k] / z, {k, 1, Log[2, n]}]
DDa[n_, z_, s_] := Sum[ z^k / k! P[n, k, s], {k, 0, Log[2, n]}]
DDa2[n_, z_] := Sum[ z^k / k! P[n, k] / z, {k, 0, Log[2, n]}]
```

Limit[DDa[100, s], s → 0]

$$\frac{428}{15}$$

DDa[100, z]

$$\frac{428}{15} + \frac{16\,289\,z}{360} + \frac{331\,z^2}{16} + \frac{611\,z^3}{144} + \frac{67\,z^4}{240} + \frac{7\,z^5}{720}$$

N[Roots[Expand[DDa[100, x]] == 0, x]]

$x == -11.1997 + 12.3983\,i\ ||\ x == -11.1997 - 12.3983\,i\ ||$
$x == -2.65742 - 1.85785\,i\ ||\ x == -2.65742 + 1.85785\,i\ ||\ x == -1.$

```
f1[z_] := (z - (-11.199718313735843` + 12.398284753324173` i))
   (z - (-11.199718313735843` - 12.398284753324173` i))
   (z - (-2.65742454340702` - 1.8578482376600183` i))
   (z - (-2.657424543407010103` + 1.85784823766017` i)) (z - (-1.`))
```

f1[0] (7 / 720)

$28.5333 - 3.75798 \times 10^{-14}\,i$

DDa[100, 1]

99

Table[{n, N[Roots[Expand[DDa[n, x]] == 0, x]]}, {n, 2, 100}] // TableForm

```
2      False
3      False
4      x == -5.
5      x == -7.
6      x == -2.33333
7      x == -3.
8      x == -8.64575 || x == -3.35425
9      x == -12.4244 || x == -2.57557
10     x == -19.3459 || x == -1.6541
11     x == -19. || x == -2.
12     x == -3. - 0.707107 i || x == -3. + 0.707107 i
13     x == -3. - 1.41421 i || x == -3. + 1.41421 i
14     x == -5.5 || x == -2.
15     x == -7.54138 || x == -1.45862
16     x == -1.45468 || x == -10.2727 + 4.42556 i || x == -10.2727 - 4.42556 i
17     x == -1.71316 || x == -10.1434 + 4.1661 i || x == -10.1434 - 4.1661 i
18     x == -28.3693 || x == -3.63068 || x == -2.
19     x == -28.406 || x == -2.797 + 0.523141 i || x == -2.797 - 0.523141 i
20     x == -41.8546 || x == -2.07271 + 1.09503 i || x == -2.07271 - 1.09503 i
21     x == -41.2094 || x == -2.79063 || x == -2.
```

22	$x = -1.39632 - 3.55271 \times 10^{-15}$ i || $x = -40.5406 - 4.44089 \times 10^{-16}$ i || $x = -4.06306 + 1.77636 \times$
23	$x = -1.65232 - 5.92119 \times 10^{-15}$ i || $x = -40.5574 + 0.$ i || $x = -3.79027 + 7.10543 \times 10^{-15}$ i
24	$x = -1.67023$ || $x = -4.96489 + 2.40103$ i || $x = -4.96489 - 2.40103$ i
25	$x = -1.58802$ || $x = -5.00599 + 2.90531$ i || $x = -5.00599 - 2.90531$ i
26	$x = -1.30506$ || $x = -5.14747 + 3.77731$ i || $x = -5.14747 - 3.77731$ i
27	$x = -1.29929$ || $x = -5.55036 + 3.37202$ i || $x = -5.55036 - 3.37202$ i
28	$x = -1.33632 + 2.36848 \times 10^{-16}$ i || $x = -8.80776 + 2.22045 \times 10^{-16}$ i || $x = -4.65592 - 4.44089 \times$
29	$x = -1.54917 + 4.73695 \times 10^{-16}$ i || $x = -8.95434 + 1.11022 \times 10^{-16}$ i || $x = -4.29649 - 8.88178 \times$
30	$x = -16.1815$ || $x = -1.70926 + 0.872723$ i || $x = -1.70926 - 0.872723$ i
31	$x = -16.2042$ || $x = -1.69788 + 1.04473$ i || $x = -1.69788 - 1.04473$ i
32	$x = -15.8095 - 12.8145$ i || $x = -15.8095 + 12.8145$ i || $x = -1.69051 - 1.04288$ i || $x = -1.690$
33	$x = -15.6177 - 12.6273$ i || $x = -15.6177 + 12.6273$ i || $x = -1.88226 - 0.71277$ i || $x = -1.882$
34	$x = -15.4136 - 12.4346$ i || $x = -15.4136 + 12.4346$ i || $x = -2.51862$ || $x = -1.65418$
35	$x = -15.1951 - 12.2362$ i || $x = -15.1951 + 12.2362$ i || $x = -3.31448$ || $x = -1.29524$
36	$x = -4.36265 - 1.99868$ i || $x = -4.36265 + 1.99868$ i || $x = -54.9842$ || $x = -1.29053$
37	$x = -4.2723 - 1.84864$ i || $x = -4.2723 + 1.84864$ i || $x = -54.9833$ || $x = -1.47211$
38	$x = -4.35957 - 2.54895$ i || $x = -4.35957 + 2.54895$ i || $x = -55.0311$ || $x = -1.24977$
39	$x = -4.40778 - 3.06127$ i || $x = -4.40778 + 3.06127$ i || $x = -55.0787$ || $x = -1.10574$
40	$x = -3.45185 - 2.94149$ i || $x = -3.45185 + 2.94149$ i || $x = -76.9886$ || $x = -1.10769$
41	$x = -3.39383 - 2.89281$ i || $x = -3.39383 + 2.89281$ i || $x = -76.9883$ || $x = -1.22402$
42	$x = -3.1856 - 4.70346 \times 10^{-17}$ i || $x = -1.54741 - 4.70346 \times 10^{-17}$ i || $x = -75.2124 - 3.60421 \times 10$
43	$x = -2.51301 + 4.73695 \times 10^{-15}$ i || $x = -75.2121 + 3.55271 \times 10^{-15}$ i || $x = -5.27489 - 7.10543 \times$
44	$x = -1.88365 - 0.56828$ i || $x = -1.88365 + 0.56828$ i || $x = -74.3$ || $x = -6.93271$
45	$x = -1.68818 - 0.663199$ i || $x = -1.68818 + 0.663199$ i || $x = -73.3616$ || $x = -8.26209$
46	$x = -73.3879 + 0.$ i || $x = -7.89038 + 1.42109 \times 10^{-14}$ i || $x = -1.72176 - 9.4739 \times 10^{-15}$ i || $x =$
47	$x = -1.83624 - 0.506135$ i || $x = -1.83624 + 0.506135$ i || $x = -73.3875$ || $x = -7.94002$
48	$x = -7.73692 - 5.99905$ i || $x = -7.73692 + 5.99905$ i || $x = -1.84642 - 0.516416$ i || $x = -1.84$
49	$x = -7.65424 - 5.92882$ i || $x = -7.65424 + 5.92882$ i || $x = -1.9291 - 0.379417$ i || $x = -1.929$
50	$x = -7.84235 + 6.87153$ i || $x = -7.84235 - 6.87153$ i || $x = -1.74098 - 0.549257$ i || $x = -1.74$
51	$x = -7.68808 - 6.76931$ i || $x = -7.68808 + 6.76931$ i || $x = -2.26792$ || $x = -1.52258$
52	$x = -7.85076 - 7.58847$ i || $x = -7.85076 + 7.58847$ i || $x = -1.73258 + 0.193384$ i || $x = -1.73$
53	$x = -7.86431 - 7.58559$ i || $x = -7.86431 + 7.58559$ i || $x = -1.71903 - 0.497377$ i || $x = -1.71$
54	$x = -9.52821 - 5.1973$ i || $x = -9.52821 + 5.1973$ i || $x = -1.72179 - 0.530173$ i || $x = -1.7217$
55	$x = -9.37163 - 5.00381$ i || $x = -9.37163 + 5.00381$ i || $x = -2.25357$ || $x = -1.50318$
56	$x = -2.30432 + 1.08576 \times 10^{-17}$ i || $x = -1.52319 + 1.08576 \times 10^{-17}$ i || $x = -14.4864 - 1.08576 \times$
57	$x = -3.0857 - 7.40039 \times 10^{-16}$ i || $x = -1.24197 + 4.64282 \times 10^{-16}$ i || $x = -14.7351 - 5.72756 \times 1$
58	$x = -4.09353 - 1.99402 \times 10^{-16}$ i || $x = -1.09893 + 9.71918 \times 10^{-17}$ i || $x = -14.9594 + 3.24007 \times$
59	$x = -3.74544 - 3.37317 \times 10^{-16}$ i || $x = -1.21188 + 1.88531 \times 10^{-16}$ i || $x = -14.945 - 4.87724 \times 1$
60	$x = -2.61536 - 2.14655$ i || $x = -2.61536 + 2.14655$ i || $x = -29.4075$ || $x = -1.1951$
61	$x = -2.55952 - 2.10496$ i || $x = -2.55952 + 2.10496$ i || $x = -29.4065$ || $x = -1.30777$
62	$x = -2.61081 - 2.31952$ i || $x = -2.61081 + 2.31952$ i || $x = -29.4353$ || $x = -1.17639$
63	$x = -2.81327 - 2.08473$ i || $x = -2.81327 + 2.08473$ i || $x = -29.0198$ || $x = -1.187$
64	$x = -1.18658$ || $x = -22.1014 - 23.8156$ i || $x = -22.1014 + 23.8156$ i || $x = -2.80529 - 2.0890]$
65	$x = -1.08139$ || $x = -22.1175 - 23.8118$ i || $x = -22.1175 + 23.8118$ i || $x = -2.84181 - 2.31104$
66	$x = -1.21353$ || $x = -21.6567 - 23.5126$ i || $x = -21.6567 + 23.5126$ i || $x = -3.23649 - 1.37208$
67	$x = -1.35069$ || $x = -21.6565 - 23.5131$ i || $x = -21.6565 + 23.5131$ i || $x = -3.16816 - 1.25972$
68	$x = -1.39405$ || $x = -21.4219 - 23.3619$ i || $x = -21.4219 + 23.3619$ i || $x = -3.3811 - 0.15363$
69	$x = -1.20354$ || $x = -21.4393 - 23.3567$ i || $x = -21.4393 + 23.3567$ i || $x = -3.45888 - 1.1393$
70	$x = -5.76899$ || $x = -20.9254 - 23.062$ i || $x = -20.9254 + 23.062$ i || $x = -1.69014 - 0.106337$
71	$x = -5.82439$ || $x = -20.9251 - 23.0626$ i || $x = -20.9251 + 23.0626$ i || $x = -1.66269 - 0.4511]$
72	$x = -95.1136$ || $x = -6.27409 - 4.44394$ i || $x = -6.27409 + 4.44394$ i || $x = -1.66913 - 0.4422$
73	$x = -95.1136$ || $x = -6.2961 - 4.44609$ i || $x = -6.2961 + 4.44609$ i || $x = -1.64712 - 0.61742$ i
74	$x = -95.1126$ || $x = -6.16026 - 4.33243$ i || $x = -6.16026 + 4.33243$ i || $x = -1.78345 - 0.2455($
75	$x = -95.1592$ || $x = -6.26186 - 4.8925$ i || $x = -6.26186 + 4.8925$ i || $x = -1.65856 - 0.398483$

76   x == -95.2057 || x == -6.33313 - 5.3758 i || x == -6.33313 + 5.3758 i || x == -1.56403 - 0.463693

77   x == -95.2047 || x == -1.83367 || x == -1.49555 || x == -6.23305 - 5.30245 i || x == -6.23305 + 5.:

78   x == -95.2984 || x == -6.42509 - 6.19962 i || x == -6.42509 + 6.19962 i || x == -1.42572 - 0.51843

79   x == -95.2984 || x == -6.43527 - 6.19822 i || x == -6.43527 + 6.19822 i || x == -1.41554 - 0.62376

80   x == -127.657 || x == -5.25813 - 5.6474 i || x == -5.25813 + 5.6474 i || x == -1.41341 - 0.630298

81   x == -127.408 || x == -5.38319 - 5.60717 i || x == -5.38319 + 5.60717 i || x == -1.41298 - 0.62585

82   x == -127.407 || x == -5.30608 - 5.56445 i || x == -5.30608 + 5.56445 i || x == -1.49028 - 0.46887

83   x == -127.407 || x == -5.31715 - 5.56085 i || x == -5.31715 + 5.56085 i || x == -1.47921 - 0.58736

84   x == -124.2 || x == -6.7844 - 2.36701 i || x == -6.7844 + 2.36701 i || x == -1.61555 - 0.607586 i |

85   x == -124.2 || x == -6.63324 - 2.08383 i || x == -6.63324 + 2.08383 i || x == -1.76692 - 0.244581

86   x == -124.199 || x == -2.59362 || x == -1.33948 || x == -6.43383 - 1.69746 i || x == -6.43383 + 1.(

87   x == -124.199 || x == -3.41001 || x == -1.1794 || x == -6.10589 - 0.982123 i || x == -6.10589 + 0.!

88   x == -123.105 || x == -9.50339 || x == -1.18246 || x == -3.60437 - 0.904495 i || x == -3.60437 + 0

89   x == -123.105 || x == -9.4818 || x == -1.30005 || x == -3.55637 - 0.644862 i || x == -3.55637 + 0.(

90   x == -119.625 || x == -15.2611 || x == -1.27194 || x == -2.42106 - 1.63576 i || x == -2.42106 + 1.(

91   x == -119.624 || x == -15.3057 || x == -1.15554 || x == -2.4572 - 1.82538 i || x == -2.4572 + 1.82!

92   x == -119.654 || x == -14.9803 || x == -1.16372 || x == -2.60076 - 1.65521 i || x == -2.60076 + 1.(

93   x == -119.654 || x == -15.0278 || x == -1.0716 || x == -2.62335 - 1.84559 i || x == -2.62335 + 1.8

94   x == -2.63601 - 2.01049 i || x == -2.63601 + 2.01049 i || x == -119.653 || x == -15.0745 || x == -1

95   x == -119.653 || x == -15.1207 || x == -0.941552 || x == -2.64243 - 2.15752 i || x == -2.64243 + 2

96   x == -0.941379 || x == -11.2338 - 11.283 i || x == -11.2338 + 11.283 i || x == -2.65266 - 2.19783

97   x == -11.2331 + 11.2843 i || x == -11.2331 - 11.2843 i || x == -2.62405 - 2.16584 i || x == -2.624

98   x == -11.074 + 11.1913 i || x == -11.074 - 11.1913 i || x == -2.78317 - 2.02334 i || x == -2.78317

99   x == -10.9064 + 11.0997 i || x == -10.9064 - 11.0997 i || x == -2.95075 - 1.84742 i || x == -2.950

100   x == -11.1997 + 12.3983 i || x == -11.1997 - 12.3983 i || x == -2.65742 - 1.85785 i || x == -2.657