$$(x-1)^{a+b}=(x-1)^a\cdot(x-1)^b$$

$$(x-1)^{a+b}=\int_0^{x-1}\int_0^{x-1}\frac{\partial}{\partial t}t^a\cdot\frac{\partial}{\partial u}u^b\,du\,dt$$

$$(x-1)^{a+b}=\int_0^{x-1}\int_0^{x-1}(a t^{a-1})\cdot(b u^{b-1})du\,dt$$

...

$$\{(x-\mathbf{1})^{a+b}\}=\{(x-\mathbf{1})^a\cdot(x-\mathbf{1})^b\}$$

$$\{(x-\mathbf{1})^{a+b}\}=$$

	$\int$	$\sum$
$+$	$\int_0^{x-1}\int_0^{(x-1)-t}\frac{\partial}{\partial t}\{(t-\mathbf{1})^a\}^{+\int}\cdot\frac{\partial}{\partial u}\{(u-\mathbf{1})^b\}^{+\int}du\,dt$	$\sum_{t=1}^{x-1}\sum_{u=1}^{(x-1)-t}\nabla_t\{(t-\mathbf{1})^a\}^{+\Sigma}\cdot\nabla_u\{(u-\mathbf{1})^b\}^{+\Sigma}$
$*$	$\int_1^x\int_1^{\frac{t}{t}}\frac{\partial}{\partial t}\{(t-\mathbf{1})^a\}^{*\int}\cdot\frac{\partial}{\partial u}\{(u-\mathbf{1})^b\}^{*\int}du\,dt$	$\sum_{t=2}^{x}\sum_{u=2}^{\lfloor\frac{x}{t}\rfloor}\nabla_t\{(t-\mathbf{1})^a\}^{*\Sigma}\cdot\nabla_u\{(u-\mathbf{1})^b\}^{*\Sigma}$

	$\int$	$\sum$
$+$	$\frac{(x-1)^{a+b}}{(a+b)!}=\int_0^{x-1}\int_0^{(x-1)-t}\frac{t^{a-1}}{(a-1)!}\cdot\frac{u^{b-1}}{(b-1)!}du\,dt$	$\binom{x-1}{a+b}=\sum_{t=1}^{x-1}\sum_{u=1}^{(x-1)-t}\binom{t-1}{a-1}\cdot\binom{u-1}{b-1}$
$*$	$(-1)^{a+b}\cdot\frac{\gamma(a+b,-\log x)}{\Gamma(a+b)}=\int_1^x\int_1^{\frac{t}{t}}\frac{\log^{a-1}t}{(a-1)!}\cdot\frac{\log^{b-1}u}{(b-1)!}du\,dt$	$D_{a+b}'(x)=\sum_{t=2}^{x}\sum_{u=2}^{\lfloor\frac{x}{t}\rfloor}d_a'(t)\cdot d_b'(u)$

	$\int$	$\sum$
$+$	$\int_0^{x-1} \frac{\partial}{\partial t}\{(t-\mathbf{1})^a\}^{+\int} \cdot \frac{\partial}{\partial t}\{((x-1-t)-\mathbf{1})^b\}^{+\int} dt$	$\sum_{t=1}^{x-1} \nabla_t\{(t-\mathbf{1})^a\}^{+\Sigma}\cdot\nabla_t\{((x-1)-t-\mathbf{1})^b\}^{+\Sigma}$
$*$	$\int_1^x \frac{\partial}{\partial t}\{(t-\mathbf{1})^a\}^{*\int}\cdot\frac{\partial}{\partial t}\{(\frac{x}{t}-\mathbf{1})^b\}^{*\int} dt$	$\sum_{t\mid x,\,1<t<x}\nabla_t\{(t-\mathbf{1})^a\}^{*\Sigma}\cdot\nabla_t\{(\frac{x}{t}-\mathbf{1})^b\}^{*\Sigma}$

	$\int$	$\sum$
$+$	$\frac{(x-1)^{a+b-1}}{(a+b-1)!}=\int_0^{x-1}\frac{t^{a-1}}{(a-1)!}\cdot\frac{(x-1-t)^{b-1}}{(b-1)!}dt$	$\binom{x-2}{a+b-1}=\sum_{t=1}^{(x-1)-1}\binom{t-1}{a-1}\cdot\binom{(x-1-t)-1}{b-1}$
$*$	$\frac{\log^{a+b-1}x}{(a+b-1)!}=\int_1^x\frac{\log^{a-1}t}{(a-1)!}\cdot(\frac{\log^{b-1}(\frac{x}{t})}{(b-1)!}\cdot\frac{1}{t})dt$	$d_{a+b}'(x)=\sum_{t\mid x,\,1<t<x} d_a'(t)\cdot d_b'(\frac{x}{t})$

```
Clear[x,a,b];
x=410;a=3;b=4;
(* *)
{NIntegrate[ t^(a-1)/(a-1)! (x-1-t)^(b-1)/(b-1)!, {t,0,x-1}],N@(x-1)^(a+b-1)/(a+b-1)!}
{NIntegrate[ t^(a-1)/(a-1)! u^(b-1)/(b-1)!, {t,0,x-1},{u,0,x-1-t}],N@(x-1)^(a+b)/(a+b)!}
(* *)
{Sum[Binomial[t-1,a-1]Binomial[(x-1-t)-1,b-1],{t,1,(x-1)-1}],Binomial[x-2,a+b-1]}
{Sum[Binomial[t-1,a-1]Binomial[u-1,b-1],{t,1,x-1},{u,1,x-1-t}],Binomial[x-1,a+b]}
(* *)
{NIntegrate[ ((Log[t]^(a-1))/((a-1)!)) ((Log[x/t]^(b-1))/((b-1)!))(1/t), {t,1,x}],N@Log[x]^(a+b-1)/(a+b-1)!}
{NIntegrate[ (Log[t]^(a-1)/(a-1)!) (Log[u]^(b-1)/(b-1)!), {t,1,x},{u,1,x/t}],N@((-1)^(a+b)Gamma[a+b,0,-Log[x]]/Gamma[a+b])}
(* *)
FI[n_]:=FactorInteger[n];FI[1]:={}
dz[n_,z_]:=Product[(-1)^p[[2]] Binomial[-z,p[[2]]],{p,FI[n]}]
d2[n_,k_]:=Sum[(-1)^(k-j)Binomial[k,j]dz[n,j],{j,0,k}]
{Sum[If[1<t<x,d2[t,a]d2[x/t,b],0],{t,Divisors[x]}],d2[x,a+b]}
{Sum[d2[t,a]d2[u,b],{t,2,x},{u,2,x/t}],Sum[d2[t,a+b],{t,2,x}]}
```