$$\boxed{\lim_{n\to\infty}(1-s)(\zeta(s)-\sum_{j=1}^{n}\frac{1}{j^{s}})+(s-1+x)(n^{x})(\zeta(s+x)-\sum_{j=1}^{n}\frac{1}{j^{s+x}})=0}$$

$$\zeta(s)=\sum_{j=1}^{n} j^{-s}+\frac{n^{1-s}}{s-1}-s\int_{n^{+}}^{\infty}\frac{\{t\}}{t^{s+1}}dt$$

$$\zeta(s)-\sum_{j=1}^{n} j^{-s}=\frac{n^{1-s}}{s-1}-s\int_{n^{+}}^{\infty}\frac{\{t\}}{t^{s+1}}dt$$

$$\lim_{n\to\infty}(1-s)(\frac{n^{1-s}}{s-1}-s\int_{n^{+}}^{\infty}\frac{\{t\}}{t^{s+1}}dt)+(s-1+x)(n^{x})(\frac{n^{1-s-x}}{s+x-1}-(s+x)\int_{n^{+}}^{\infty}\frac{\{t\}}{t^{s+x+1}}dt)=0$$

$$\lim_{n\to\infty}-n^{1-s}-s(1-s)\int_{n^{+}}^{\infty}\frac{\{t\}}{t^{s+1}}dt+n^{1-s}-(s-1+x)n^{x}(s+x)\int_{n^{+}}^{\infty}\frac{\{t\}}{t^{s+x+1}}dt=0$$

$$\lim_{n\to\infty}-s(1-s)\int_{n^{+}}^{\infty}\frac{\{t\}}{t^{s+1}}dt+-(s-1+x)n^{x}(s+x)\int_{n^{+}}^{\infty}\frac{\{t\}}{t^{s+x+1}}dt=0$$

$$\boxed{\lim_{n\to\infty}s(1-s)\int_{n^{+}}^{\infty}\frac{(1-(1+\frac{x}{s})(1+\frac{x}{s-1})(\frac{n}{t})^{x})\{t\}}{t^{s+1}}dt=0}$$

$$\boxed{\lim_{n\to\infty} n^{c+y}(1-s-y)(\zeta(s+y)-\sum_{j=1}^{n}\frac{1}{j^{s+y}})-n^{c+x}(1-s-x)(\zeta(s+x)-\sum_{j=1}^{n}\frac{1}{j^{s+x}})=0}$$

$$\zeta(s)-\sum_{j=1}^{n} j^{-s}=\frac{n^{1-s}}{s-1}-s\int_{n^+}^{\infty}\frac{\{t\}}{t^{s+1}}\,dt$$

$$\lim_{n\to\infty}(1-s)(\frac{n^{1-s}}{s-1}-s\int_{n^+}^{\infty}\frac{\{t\}}{t^{s+1}}\,dt)+(s-1+x)(n^x)(\frac{n^{1-s-x}}{s+x-1}-(s+x)\int_{n^+}^{\infty}\frac{\{t\}}{t^{s+x+1}}\,dt)=0$$

$$\lim_{n\to\infty}(n^{1-s+c}-n^{c+y}(1-s-y)(s+y)\int_{n^+}^{\infty}\frac{\{t\}}{t^{s+y+1}}\,dt)-(n^{1-s+c}-n^{c+x}(1-s-x)(s+x)\int_{n^+}^{\infty}\frac{\{t\}}{t^{s+x+1}}\,dt)=0$$

$$\lim_{n\to\infty} n^{c+y}(1-s-y)(s+y)\int_{n^+}^{\infty}\frac{\{t\}}{t^{s+y+1}}\,dt-n^{c+x}(1-s-x)(s+x)\int_{n^+}^{\infty}\frac{\{t\}}{t^{s+x+1}}\,dt=0$$

$$\lim_{n\to\infty}\int_{n^+}^{\infty}(\frac{n^{c+y}(1-s-y)(s+y)}{t^{s+y+1}}-\frac{n^{c+x}(1-s-x)(s+x)}{t^{s+x+1}})\cdot\{t\}\,dt=0$$

$$\lim_{n\to\infty}\int_{n^+}^{\infty}(\frac{n^{c+y}(1-s-y)(s+y)}{t^{s+y+1}}-\frac{n^{c+x}(1-s-x)(s+x)}{t^{s+x+1}})\cdot\{t\}\,dt=0$$

$$\lim_{n\to\infty}\int_{n^+}^{\infty}\frac{n^c}{t^{s+1}}\cdot((\frac{n}{t})^y(1-s-y)(s+y)-(\frac{n}{t})^x(1-s-x)(s+x))\cdot\{t\}\,dt=0$$

...

Because $\lim_{n\to\infty}\int_{n^+}^{\infty}\frac{n^c}{t^{s+1}}\cdot((\frac{n}{t})^y(1-s-y)(s+y)-(\frac{n}{t})^x(1-s-x)(s+x))\,dt=\lim_{n\to\infty} n^{c-s}\cdot(x-y)=0$
when Re(s+x) > 0 and Re(s+y) > 0 and c-s < 0, it might be reasonable to assume similar bounds for this sum.

Now start with our previous identity.

$$\lim_{n\to\infty} n^{c+y}(1-s-y)(\zeta(s+y)-\sum_{j=1}^{n}\frac{1}{j^{s+y}})-n^{c+x}(1-s-x)(\zeta(s+x)-\sum_{j=1}^{n}\frac{1}{j^{s+x}})=0$$

Set y = 0, and x = 1-2s.

$$\lim_{n\to\infty} n^{c}(1-s)(\zeta(s)-\sum_{j=1}^{n}\frac{1}{j^{s}})-n^{c-1+2s}(-s)(\zeta(1-s)-\sum_{j=1}^{n}\frac{1}{j^{1-s}})=0$$

Now,

$$\alpha(s)=-\frac{1}{2}\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\cdot s(1-s)$$

and

$$\zeta(1-s)=\frac{\alpha(s)}{\alpha(1-s)}\zeta(s)$$

so

$$\lim_{n\to\infty} n^{c}(1-s)(\zeta(s)-\sum_{j=1}^{n}\frac{1}{j^{s}})-n^{c-1+2s}(-s)(\frac{\alpha(s)}{\alpha(1-s)}\zeta(s)-\sum_{j=1}^{n}\frac{1}{j^{1-s}})=0$$

Multiply out

$$\lim_{n\to\infty} n^{c}(1-s)\zeta(s)-n^{c}(1-s)\sum_{j=1}^{n}\frac{1}{j^{s}}+n^{c-1+2s}\cdot s\cdot\frac{\alpha(s)}{\alpha(1-s)}\zeta(s)-n^{c-1+2s}\cdot s\cdot\sum_{j=1}^{n}\frac{1}{j^{1-s}}=0$$

Rearrange terms to isolate zetas

$$\lim_{n\to\infty} n^{c}(1-s)\zeta(s)+n^{c-1+2s}\cdot s\cdot\frac{\alpha(s)}{\alpha(1-s)}\zeta(s)-n^{c}(1-s)\sum_{j=1}^{n}\frac{1}{j^{s}}-n^{c-1+2s}\cdot s\cdot\sum_{j=1}^{n}\frac{1}{j^{1-s}}=0$$

Now factor out zeta

$$\lim_{n\to\infty} (n^{c}(1-s)+n^{c-1+2s}\cdot s\cdot\frac{\alpha(s)}{\alpha(1-s)})\zeta(s)-n^{c}(1-s)\sum_{j=1}^{n}\frac{1}{j^{s}}-n^{c-1+2s}\cdot s\cdot\sum_{j=1}^{n}\frac{1}{j^{1-s}}=0$$

Add non-zeta terms to both side of the equation

$$\lim_{n\to\infty} (n^{c}(1-s)+n^{c-1+2s}\cdot s\cdot\frac{\alpha(s)}{\alpha(1-s)})\zeta(s)=n^{c}(1-s)\sum_{j=1}^{n}\frac{1}{j^{s}}+n^{c-1+2s}\cdot s\cdot\sum_{j=1}^{n}\frac{1}{j^{1-s}}$$

Divide non-zeta terms from left side of equation and move limit

$$\zeta(s)=\lim_{n\to\infty}\frac{n^{c}(1-s)\sum_{j=1}^{n}\frac{1}{j^{s}}+n^{c-1+2s}\cdot s\cdot\sum_{j=1}^{n}\frac{1}{j^{1-s}}}{n^{c}(1-s)+n^{c-1+2s}\cdot s\cdot\frac{\alpha(s)}{\alpha(1-s)}}$$