

```
ClearAll["Global`*"]
```

```
E2a[n_, k_, a_] :=
```

```
  E2a[n, k, a] = Sum[E2a[n / j, k - 1, a], {j, 2, n}] - a Sum[E2a[n / (a j), k - 1, a], {j, 1, n / a}];
```

```
E2a[n_, 0, a_] := 1
```

```
lin[n_, b_] := Sum[(-1)^(k + 1) / k E2a[n, k, b], {k, 1, Log[2, n]}]
```

```
E2ax[n_, k_, a_, c_] := E2ax[n, k, a, c] = Sum[E2ax[n / j, k - 1, a, c], {j, 2, n}] -
```

```
  a^c Sum[E2ax[n / (a j), k - 1, a, c], {j, 1, n / a}]; E2ax[n_, 0, a_, c_] := 1
```

```
linx[n_, b_, c_] := Sum[(-1)^(k + 1) / k E2a[n, k, b], {k, 1, Log[2, n]}]
```

```
E2ay[n_, k_, a_, c_] := E2ay[n, k, a, c] = Sum[E2ay[n / j, k - 1, a, c], {j, 2, n}] +
```

```
  a^c Sum[E2ay[n / (a j), k - 1, a, c], {j, 1, n / a}]; E2ay[n_, 0, a_, c_] := 1
```

```
liny[n_, b_, c_] := Sum[(-1)^(k + 1) / k E2a[n, k, b], {k, 1, Log[2, n]}]
```

```
$RecursionLimit = 10 000
```

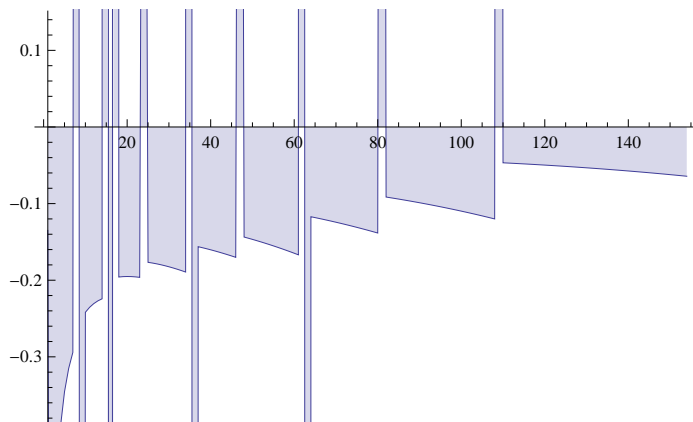
```
10 000
```

```
lin[100, 2]
```

```
4
```

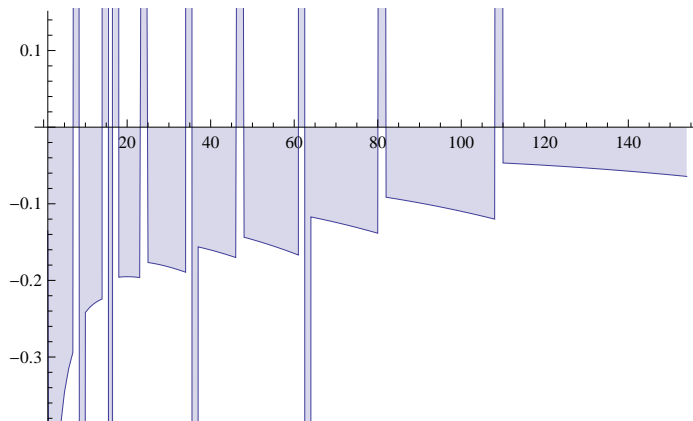
```
5
```

```
DiscretePlot[(-1)^(k + 1) / k E2a[10, k, 1.015], {k, 1, Log[1.015, 10]}]
```

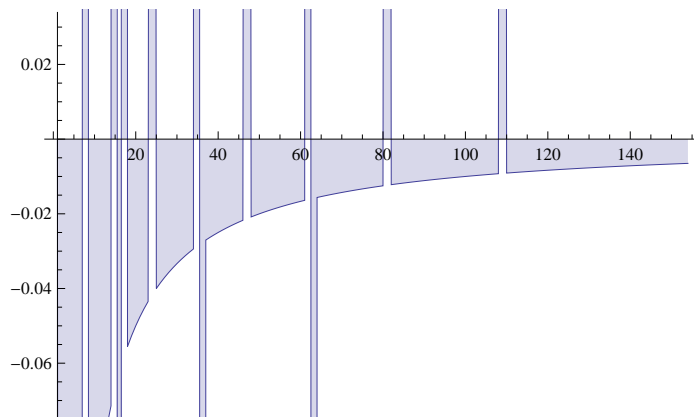


```
10 000
```

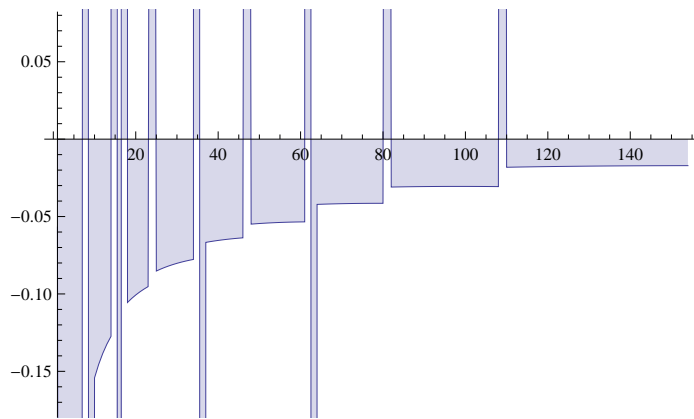
```
DiscretePlot[(-1)^(k + 1) / k E2ax[10, k, 1.015, 1], {k, 1, Log[1.015, 10]}]
```



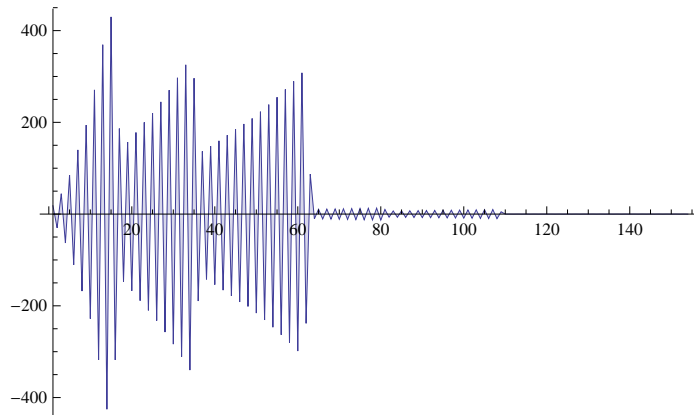
```
DiscretePlot[(-1)^(k+1)/k E2ax[10, k, 1.015, 0], {k, 1, Log[1.015, 10]}]
```



```
DiscretePlot[(-1)^(k+1)/k E2ax[10, k, 1.015, 1 - EulerGamma], {k, 1, Log[1.015, 10]}]
```



```
DiscretePlot[(-1)^(k+1)/k E2ay[10, k, 1.015, 1], {k, 1, Log[1.015, 10]}]
```



```
pp[k_, a_] := (-1)^(k+1)/k E2ax[10, k, 1.015, a]
```

```
pp2[k_, a_] := (-1)^(k+1)/k E2ay[10, k, 1.015, a]
```

```
Table[{1.015^k, pp[k, 1], pp[k, 0], pp[k, 1/2]}, {k, 1, Log[1.015, 10]}] // TableForm
```

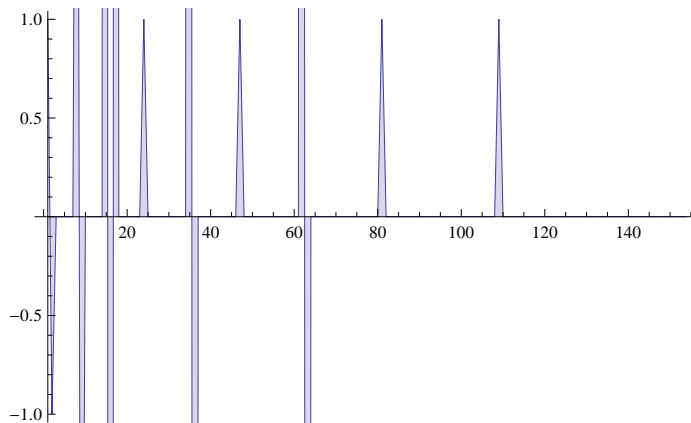
| | | | |
|---------|-----------|-----------|------------|
| 1.015 | -0.135 | 0. | -0.0672488 |
| 1.03022 | -1.63759 | -1.5 | -1.56789 |
| 1.04568 | -0.473558 | -0.333333 | -0.401872 |

| | | | |
|---------|-----------|------------|------------|
| 1.06136 | -0.392912 | -0.25 | -0.319193 |
| 1.07728 | -0.345651 | -0.2 | -0.269854 |
| 1.09344 | -0.315108 | -0.166667 | -0.237188 |
| 1.10984 | -0.294143 | -0.142857 | -0.214051 |
| 1.12649 | 4.79003 | 4.375 | 4.57926 |
| 1.14339 | -4.68973 | -4.11111 | -4.38947 |
| 1.16054 | -0.241858 | -0.1 | -0.164996 |
| 1.17795 | -0.235432 | -0.0909091 | -0.156513 |
| 1.19562 | -0.230571 | -0.0833333 | -0.14955 |
| 1.21355 | -0.226927 | -0.0769231 | -0.143759 |
| 1.23176 | -0.224252 | -0.0714286 | -0.13889 |
| 1.25023 | 56.4548 | 45.2667 | 50.5542 |
| 1.26899 | -105.534 | -85.0625 | -94.7468 |
| 1.28802 | 49.0735 | 39.9412 | 44.275 |
| 1.30734 | -0.195833 | -0.0555556 | -0.116006 |
| 1.32695 | -0.195325 | -0.0526316 | -0.1136 |
| 1.34686 | -0.195152 | -0.05 | -0.11149 |
| 1.36706 | -0.195272 | -0.047619 | -0.109636 |
| 1.38756 | -0.195652 | -0.0454545 | -0.108003 |
| 1.40838 | -0.196265 | -0.0434783 | -0.106562 |
| 1.4295 | 1.23242 | 0.958333 | 1.09033 |
| 1.45095 | -0.176658 | -0.04 | -0.0952362 |
| 1.47271 | -0.177525 | -0.0384615 | -0.0941809 |
| 1.4948 | -0.178548 | -0.037037 | -0.0932439 |
| 1.51722 | -0.179717 | -0.0357143 | -0.092413 |
| 1.53998 | -0.18102 | -0.0344828 | -0.0916776 |
| 1.56308 | -0.182451 | -0.0333333 | -0.0910287 |
| 1.58653 | -0.184002 | -0.0322581 | -0.0904584 |
| 1.61032 | -0.185666 | -0.03125 | -0.0899598 |
| 1.63448 | -0.187438 | -0.030303 | -0.0895269 |
| 1.659 | -0.189315 | -0.0294118 | -0.0891541 |
| 1.68388 | 58.7446 | 34.9714 | 45.3287 |
| 1.70914 | -58.2199 | -35.0278 | -45.1571 |
| 1.73478 | -0.156254 | -0.027027 | -0.0759759 |
| 1.7608 | -0.157538 | -0.0263158 | -0.0756363 |
| 1.78721 | -0.15889 | -0.025641 | -0.0753362 |
| 1.81402 | -0.160309 | -0.025 | -0.0750728 |
| 1.84123 | -0.161792 | -0.0243902 | -0.0748436 |
| 1.86885 | -0.163337 | -0.0238095 | -0.0746466 |
| 1.89688 | -0.164944 | -0.0232558 | -0.0744796 |
| 1.92533 | -0.166611 | -0.0227273 | -0.0743409 |
| 1.95421 | -0.168336 | -0.0222222 | -0.0742288 |
| 1.98353 | -0.17012 | -0.0217391 | -0.0741418 |
| 2.01328 | 1.84132 | 0.978723 | 1.34482 |
| 2.04348 | -0.143658 | -0.0208333 | -0.0634356 |
| 2.07413 | -0.145157 | -0.0204082 | -0.0633369 |
| 2.10524 | -0.146705 | -0.02 | -0.0632578 |
| 2.13682 | -0.148301 | -0.0196078 | -0.0631975 |
| 2.16887 | -0.149945 | -0.0192308 | -0.0631549 |
| 2.20141 | -0.151636 | -0.0188679 | -0.0631292 |
| 2.23443 | -0.153373 | -0.0185185 | -0.0631197 |
| 2.26794 | -0.155158 | -0.0181818 | -0.0631257 |
| 2.30196 | -0.156989 | -0.0178571 | -0.0631465 |
| 2.33649 | -0.158867 | -0.0175439 | -0.0631815 |
| 2.37154 | -0.160792 | -0.0172414 | -0.0632302 |
| 2.40711 | -0.162763 | -0.0169492 | -0.0632919 |

| | | | |
|---------|------------|-------------|------------|
| 2.44322 | -0.164782 | -0.0166667 | -0.0633663 |
| 2.47987 | -0.166846 | -0.0163934 | -0.0634528 |
| 2.51707 | 79.1186 | 31.4839 | 49.912 |
| 2.55482 | -76.992 | -31.0159 | -48.8666 |
| 2.59314 | -0.117163 | -0.015625 | -0.0490478 |
| 2.63204 | -0.118287 | -0.0153846 | -0.0490243 |
| 2.67152 | -0.119439 | -0.0151515 | -0.0490096 |
| 2.7116 | -0.120617 | -0.0149254 | -0.0490034 |
| 2.75227 | -0.121822 | -0.0147059 | -0.0490054 |
| 2.79355 | -0.123054 | -0.0144928 | -0.0490154 |
| 2.83546 | -0.124313 | -0.0142857 | -0.049033 |
| 2.87799 | -0.125599 | -0.0140845 | -0.0490581 |
| 2.92116 | -0.126911 | -0.0138889 | -0.0490903 |
| 2.96498 | -0.128251 | -0.0136986 | -0.0491295 |
| 3.00945 | -0.129618 | -0.0135135 | -0.0491754 |
| 3.05459 | -0.131011 | -0.0133333 | -0.049228 |
| 3.10041 | -0.132433 | -0.0131579 | -0.0492869 |
| 3.14692 | -0.133881 | -0.012987 | -0.049352 |
| 3.19412 | -0.135358 | -0.0128205 | -0.0494232 |
| 3.24203 | -0.136862 | -0.0126582 | -0.0495003 |
| 3.29066 | -0.138394 | -0.0125 | -0.0495832 |
| 3.34002 | 3.20007 | 0.987654 | 1.7779 |
| 3.39012 | -0.0914433 | -0.0121951 | -0.0361098 |
| 3.44097 | -0.0923094 | -0.0120482 | -0.0361071 |
| 3.49259 | -0.0931931 | -0.0119048 | -0.0361088 |
| 3.54498 | -0.0940945 | -0.0117647 | -0.0361149 |
| 3.59815 | -0.0950137 | -0.0116279 | -0.0361253 |
| 3.65213 | -0.0959507 | -0.0114943 | -0.0361398 |
| 3.70691 | -0.0969058 | -0.0113636 | -0.0361583 |
| 3.76251 | -0.097879 | -0.011236 | -0.0361808 |
| 3.81895 | -0.0988704 | -0.0111111 | -0.0362072 |
| 3.87623 | -0.0998802 | -0.010989 | -0.0362374 |
| 3.93438 | -0.100908 | -0.0108696 | -0.0362712 |
| 3.99339 | -0.101955 | -0.0107527 | -0.0363087 |
| 4.05329 | -0.103021 | -0.0106383 | -0.0363497 |
| 4.11409 | -0.104106 | -0.0105263 | -0.0363942 |
| 4.1758 | -0.105209 | -0.0104167 | -0.036442 |
| 4.23844 | -0.106332 | -0.0103093 | -0.0364932 |
| 4.30202 | -0.107475 | -0.0102041 | -0.0365477 |
| 4.36655 | -0.108637 | -0.010101 | -0.0366054 |
| 4.43205 | -0.109819 | -0.01 | -0.0366663 |
| 4.49853 | -0.111021 | -0.00990099 | -0.0367303 |
| 4.566 | -0.112243 | -0.00980392 | -0.0367973 |
| 4.63449 | -0.113485 | -0.00970874 | -0.0368674 |
| 4.70401 | -0.114748 | -0.00961538 | -0.0369404 |
| 4.77457 | -0.116032 | -0.00952381 | -0.0370163 |
| 4.84619 | -0.117337 | -0.00943396 | -0.0370951 |
| 4.91888 | -0.118664 | -0.00934579 | -0.0371767 |
| 4.99267 | -0.120012 | -0.00925926 | -0.0372611 |
| 5.06756 | 4.94618 | 0.990826 | 2.21378 |
| 5.14357 | -0.0467597 | -0.00909091 | -0.0206177 |
| 5.22072 | -0.0470335 | -0.00900901 | -0.0205846 |
| 5.29903 | -0.0473128 | -0.00892857 | -0.0205532 |
| 5.37852 | -0.0475975 | -0.00884956 | -0.0205236 |
| 5.4592 | -0.0478877 | -0.00877193 | -0.0204955 |
| 5.54109 | -0.0481834 | -0.00869565 | -0.0204691 |

| | | | |
|---------|------------|-------------|------------|
| 5.6242 | -0.0484845 | -0.00862069 | -0.0204443 |
| 5.70856 | -0.0487912 | -0.00854701 | -0.020421 |
| 5.79419 | -0.0491033 | -0.00847458 | -0.0203993 |
| 5.88111 | -0.0494211 | -0.00840336 | -0.020379 |
| 5.96932 | -0.0497444 | -0.00833333 | -0.0203602 |
| 6.05886 | -0.0500732 | -0.00826446 | -0.0203428 |
| 6.14975 | -0.0504078 | -0.00819672 | -0.0203268 |
| 6.24199 | -0.0507479 | -0.00813008 | -0.0203122 |
| 6.33562 | -0.0510937 | -0.00806452 | -0.0202989 |
| 6.43066 | -0.0514452 | -0.008 | -0.020287 |
| 6.52712 | -0.0518025 | -0.00793651 | -0.0202764 |
| 6.62502 | -0.0521655 | -0.00787402 | -0.020267 |
| 6.7244 | -0.0525344 | -0.0078125 | -0.0202589 |
| 6.82526 | -0.052909 | -0.00775194 | -0.0202521 |
| 6.92764 | -0.0532896 | -0.00769231 | -0.0202465 |
| 7.03156 | -0.053676 | -0.00763359 | -0.020242 |
| 7.13703 | -0.0540684 | -0.00757576 | -0.0202388 |
| 7.24409 | -0.0544668 | -0.0075188 | -0.0202367 |
| 7.35275 | -0.0548713 | -0.00746269 | -0.0202358 |
| 7.46304 | -0.0552818 | -0.00740741 | -0.020236 |
| 7.57498 | -0.0556984 | -0.00735294 | -0.0202373 |
| 7.68861 | -0.0561212 | -0.00729927 | -0.0202397 |
| 7.80394 | -0.0565503 | -0.00724638 | -0.0202431 |
| 7.921 | -0.0569856 | -0.00719424 | -0.0202477 |
| 8.03981 | -0.0574272 | -0.00714286 | -0.0202533 |
| 8.16041 | -0.0578752 | -0.0070922 | -0.0202599 |
| 8.28282 | -0.0583297 | -0.00704225 | -0.0202675 |
| 8.40706 | -0.0587906 | -0.00699301 | -0.0202762 |
| 8.53316 | -0.0592581 | -0.00694444 | -0.0202858 |
| 8.66116 | -0.0597321 | -0.00689655 | -0.0202964 |
| 8.79108 | -0.0602129 | -0.00684932 | -0.0203081 |
| 8.92294 | -0.0607003 | -0.00680272 | -0.0203206 |
| 9.05679 | -0.0611945 | -0.00675676 | -0.0203341 |
| 9.19264 | -0.0616956 | -0.00671141 | -0.0203486 |
| 9.33053 | -0.0622035 | -0.00666667 | -0.0203639 |
| 9.47049 | -0.0627185 | -0.00662252 | -0.0203802 |
| 9.61255 | -0.0632404 | -0.00657895 | -0.0203974 |
| 9.75673 | -0.0637695 | -0.00653595 | -0.0204155 |
| 9.90308 | -0.0643057 | -0.00649351 | -0.0204345 |

`DiscretePlot[((-1)^(k+1) E2ax[10, k, 1.015, 0] + 1) / k, {k, 1, Log[1.015, 10]}]`



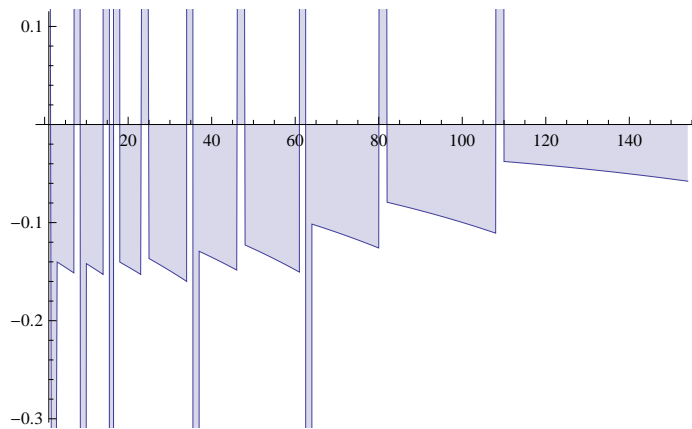
```
Table[{10 / 1.015^k, ((-1)^(k + 1) E2ax[10, k, 1.015, 0] + 1) / k},
      {k, 1, Log[1.015, 10]}] // TableForm
```

| | |
|---------|---------|
| 9.85222 | 1. |
| 9.70662 | -1. |
| 9.56317 | 0. |
| 9.42184 | 0. |
| 9.2826 | 0. |
| 9.14542 | 0. |
| 9.01027 | 0. |
| 8.87711 | 4.5 |
| 8.74592 | -4. |
| 8.61667 | 0. |
| 8.48933 | 0. |
| 8.36387 | 0. |
| 8.24027 | 0. |
| 8.11849 | 0. |
| 7.99852 | 45.3333 |
| 7.88031 | -85. |
| 7.76385 | 40. |
| 7.64912 | 0. |
| 7.53607 | 0. |
| 7.4247 | 0. |
| 7.31498 | 0. |
| 7.20688 | 0. |
| 7.10037 | 0. |
| 6.99544 | 1. |
| 6.89206 | 0. |
| 6.79021 | 0. |
| 6.68986 | 0. |
| 6.59099 | 0. |
| 6.49359 | 0. |
| 6.39762 | 0. |
| 6.30308 | 0. |
| 6.20993 | 0. |
| 6.11816 | 0. |
| 6.02774 | 0. |
| 5.93866 | 35. |
| 5.8509 | -35. |
| 5.76443 | 0. |
| 5.67924 | 0. |
| 5.59531 | 0. |
| 5.51262 | 0. |
| 5.43116 | 0. |
| 5.35089 | 0. |
| 5.27182 | 0. |
| 5.19391 | 0. |
| 5.11715 | 0. |
| 5.04153 | 0. |
| 4.96702 | 1. |
| 4.89362 | 0. |
| 4.8213 | 0. |
| 4.75005 | 0. |
| 4.67985 | 0. |
| 4.61069 | 0. |
| 4.54255 | 0. |
| 4.47542 | 0. |

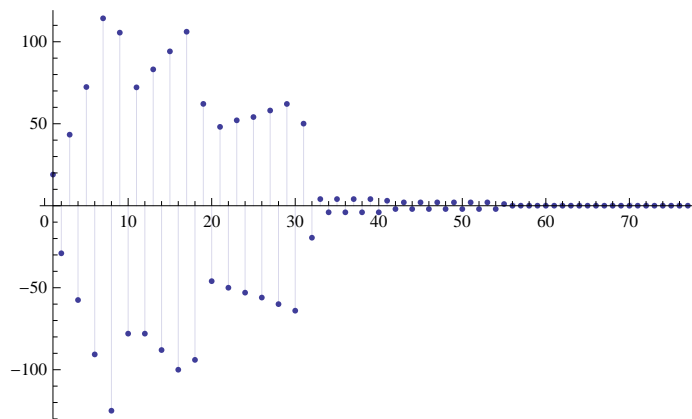
| | |
|---------|------|
| 4.40928 | 0. |
| 4.34412 | 0. |
| 4.27992 | 0. |
| 4.21667 | 0. |
| 4.15435 | 0. |
| 4.09296 | 0. |
| 4.03247 | 0. |
| 3.97288 | 31.5 |
| 3.91417 | -31. |
| 3.85632 | 0. |
| 3.79933 | 0. |
| 3.74318 | 0. |
| 3.68787 | 0. |
| 3.63337 | 0. |
| 3.57967 | 0. |
| 3.52677 | 0. |
| 3.47465 | 0. |
| 3.4233 | 0. |
| 3.37271 | 0. |
| 3.32287 | 0. |
| 3.27376 | 0. |
| 3.22538 | 0. |
| 3.17771 | 0. |
| 3.13075 | 0. |
| 3.08448 | 0. |
| 3.0389 | 0. |
| 2.99399 | 1. |
| 2.94975 | 0. |
| 2.90615 | 0. |
| 2.86321 | 0. |
| 2.82089 | 0. |
| 2.7792 | 0. |
| 2.73813 | 0. |
| 2.69767 | 0. |
| 2.6578 | 0. |
| 2.61852 | 0. |
| 2.57982 | 0. |
| 2.5417 | 0. |
| 2.50414 | 0. |
| 2.46713 | 0. |
| 2.43067 | 0. |
| 2.39475 | 0. |
| 2.35936 | 0. |
| 2.32449 | 0. |
| 2.29014 | 0. |
| 2.25629 | 0. |
| 2.22295 | 0. |
| 2.1901 | 0. |
| 2.15773 | 0. |
| 2.12585 | 0. |
| 2.09443 | 0. |
| 2.06348 | 0. |
| 2.03298 | 0. |
| 2.00294 | 0. |
| 1.97334 | 1. |
| 1.94417 | 0. |

| | |
|---------|----|
| 1.91544 | 0. |
| 1.88714 | 0. |
| 1.85925 | 0. |
| 1.83177 | 0. |
| 1.8047 | 0. |
| 1.77803 | 0. |
| 1.75175 | 0. |
| 1.72587 | 0. |
| 1.70036 | 0. |
| 1.67523 | 0. |
| 1.65047 | 0. |
| 1.62608 | 0. |
| 1.60205 | 0. |
| 1.57838 | 0. |
| 1.55505 | 0. |
| 1.53207 | 0. |
| 1.50943 | 0. |
| 1.48712 | 0. |
| 1.46514 | 0. |
| 1.44349 | 0. |
| 1.42216 | 0. |
| 1.40114 | 0. |
| 1.38044 | 0. |
| 1.36004 | 0. |
| 1.33994 | 0. |
| 1.32013 | 0. |
| 1.30063 | 0. |
| 1.2814 | 0. |
| 1.26247 | 0. |
| 1.24381 | 0. |
| 1.22543 | 0. |
| 1.20732 | 0. |
| 1.18948 | 0. |
| 1.1719 | 0. |
| 1.15458 | 0. |
| 1.13752 | 0. |
| 1.12071 | 0. |
| 1.10414 | 0. |
| 1.08783 | 0. |
| 1.07175 | 0. |
| 1.05591 | 0. |
| 1.04031 | 0. |
| 1.02493 | 0. |
| 1.00979 | 0. |


```
DiscretePlot[((-1)^(k+1) E2ax[10, k, 1.015, 1] + 1) / k, {k, 1, Log[1.015, 10]}]
```



```
DiscretePlot[((-1)^(k+1) E2ay[10, k, 1.03, 0] + 1) / k, {k, 1, Log[1.03, 10]}]
```



```
Table[{10 / 1.03^k, ((-1)^(k+1) E2ay[10, k, 1.03, 0] + 1) / k},  
      {k, 1, Log[1.03, 10]}] // TableForm
```

| | |
|---------|----------|
| 9.70874 | 19. |
| 9.42596 | -29. |
| 9.15142 | 43.3333 |
| 8.88487 | -57.5 |
| 8.62609 | 72.4 |
| 8.37484 | -90.6667 |
| 8.13092 | 114.286 |
| 7.89409 | -125. |
| 7.66417 | 105.556 |
| 7.44094 | -78. |
| 7.22421 | 72.1818 |
| 7.0138 | -78. |
| 6.80951 | 83.1538 |
| 6.61118 | -88. |
| 6.41862 | 94.1333 |
| 6.23167 | -100. |
| 6.05016 | 106.118 |
| 5.87395 | -94. |
| 5.70286 | 62.1053 |
| 5.53676 | -46. |
| 5.37549 | 48.0952 |

| | |
|---------|-----------|
| 5.21893 | - 50. |
| 5.06692 | 52.087 |
| 4.91934 | - 53. |
| 4.77606 | 54.08 |
| 4.63695 | - 56. |
| 4.50189 | 58.0741 |
| 4.37077 | - 60. |
| 4.24346 | 62.069 |
| 4.11987 | - 64. |
| 3.99987 | 50.0645 |
| 3.88337 | - 19.5 |
| 3.77026 | 4.06061 |
| 3.66045 | - 4. |
| 3.55383 | 4.05714 |
| 3.45032 | - 4. |
| 3.34983 | 4.05405 |
| 3.25226 | - 4. |
| 3.15754 | 4.05128 |
| 3.06557 | - 4. |
| 2.97628 | 3.04878 |
| 2.88959 | - 2. |
| 2.80543 | 2.04651 |
| 2.72372 | - 2. |
| 2.64439 | 2.04444 |
| 2.56737 | - 2. |
| 2.49259 | 2.04255 |
| 2.41999 | - 2. |
| 2.3495 | 2.04082 |
| 2.28107 | - 2. |
| 2.21463 | 2.03922 |
| 2.15013 | - 2. |
| 2.0875 | 2.03774 |
| 2.0267 | - 2. |
| 1.96767 | 1.03636 |
| 1.91036 | 0. |
| 1.85472 | 0.0350877 |
| 1.8007 | 0. |
| 1.74825 | 0.0338983 |
| 1.69733 | 0. |
| 1.64789 | 0.0327869 |
| 1.5999 | 0. |
| 1.5533 | 0.031746 |
| 1.50806 | 0. |
| 1.46413 | 0.0307692 |
| 1.42149 | 0. |
| 1.38009 | 0.0298507 |
| 1.33989 | 0. |
| 1.30086 | 0.0289855 |
| 1.26297 | 0. |
| 1.22619 | 0.028169 |
| 1.19047 | 0. |
| 1.1558 | 0.0273973 |
| 1.12214 | 0. |
| 1.08945 | 0.0266667 |
| 1.05772 | 0. |
| 1.02691 | 0.025974 |