```
D[(1 - x^(1 - s)) Zeta[s], x]
```

$-(1-s)\,x^{-s}\,\mathrm{Zeta}[s]$

```
D[x^(1 - s) / j^s, x]
```

$j^{-s}\,(1-s)\,x^{-s}$

```
D[x^(1 - s) / (j + n / x)^s, x]
```

$$n\,s\left(j+\frac{n}{x}\right)^{-1-s}x^{-1-s}+(1-s)\left(j+\frac{n}{x}\right)^{-s}x^{-s}$$

$$\mathbf{Expand}\left[\left(j+\frac{n}{x}\right)^{-1-s}x^{-1-s}\right]$$

$$\left(j+\frac{n}{x}\right)^{-1-s}x^{-1-s}$$

```
FullSimplify@D[x^(1 - s) / (j + n / x)^s, {x, 2}] /. x → 1
```

$j\,(j+n)^{-2-s}\,(-2\,n+j\,(-1+s))\,s$

```
Expand[j (-2 n + j (-1 + s)) s]
```

$-j^2\,s-2\,j\,n\,s+j^2\,s^2$

```
tes2[n_, s_] :=  (s - 1) n^s Zeta[s, n + 1]
```

```
FullSimplify[(-1 - 1) (n^-1 Zeta[-1] - Sum[ (n / j)^-1, {j, 1, n}])]
```

$$1+\frac{1}{6\,n}+n$$

```
FullSimplify[(-2 - 1) (n^-2 Zeta[-2] - Sum[ (n / j)^-2, {j, 1, n}])]
```

$$\frac{3}{2}+\frac{1}{2\,n}+n$$

```
FullSimplify[(-3 - 1) (n^-3 Zeta[-3] - Sum[ (n / j)^-3, {j, 1, n}])]
```

$$2-\frac{1}{30\,n^3}+\frac{1}{n}+n$$

```
FullSimplify[(-4 - 1) (n^-4 Zeta[-4] - Sum[ (n / j)^-4, {j, 1, n}])]
```

$$\frac{5}{2}-\frac{1}{6\,n^3}+\frac{5}{3\,n}+n$$

```
FullSimplify[(-5 - 1) (n^-5 Zeta[-5] - Sum[ (n / j)^-5, {j, 1, n}])]
```

$$3+\frac{1}{42\,n^5}-\frac{1}{2\,n^3}+\frac{5}{2\,n}+n$$

```
Expand[(-6 - 1) (n^-6 Zeta[-6] - Sum[ (n / j)^-6, {j, 1, n}])]
```

$$\frac{7}{2}+\frac{1}{6\,n^5}-\frac{7}{6\,n^3}+\frac{7}{2\,n}+n$$

```
Expand[(-7 - 1) (n^-7 Zeta[-7] - Sum[ (n / j)^-7, {j, 1, n}])]
```

$$4-\frac{1}{30\,n^7}+\frac{2}{3\,n^5}-\frac{7}{3\,n^3}+\frac{14}{3\,n}+n$$

```
Expand[(-8 - 1) (n^-8 Zeta[-8] - Sum[ (n / j) ^-8, {j, 1, n}])]
```

$$\frac{9}{2} - \frac{3}{10\, n^7} + \frac{2}{n^5} - \frac{21}{5\, n^3} + \frac{6}{n} + n$$

```
Expand[(-9 - 1) (n^-9 Zeta[-9] - Sum[ (n / j) ^-9, {j, 1, n}])]
```

$$5 + \frac{5}{66\, n^9} - \frac{3}{2\, n^7} + \frac{5}{n^5} - \frac{7}{n^3} + \frac{15}{2\, n} + n$$

```
Expand[(-9 - 1) (n^-9 Zeta[-9] - Sum[ (n / j) ^-9, {j, 1, n}])]
```

$$5 + \frac{5}{66\, n^9} - \frac{3}{2\, n^7} + \frac{5}{n^5} - \frac{7}{n^3} + \frac{15}{2\, n} + n$$

```
FullSimplify[(2 - 1) (n^2 Zeta[2] - Sum[ (n / j) ^2, {j, 1, n}])]
```

n^2 PolyGamma[1, 1 + n]

```
FullSimplify[(3 - 1) (n^3 Zeta[3] - Sum[ (n / j) ^3, {j, 1, n}])]
```

$-n^3$ PolyGamma[2, 1 + n]

```
FullSimplify[(4 - 1) (n^4 Zeta[4] - Sum[ (n / j) ^4, {j, 1, n}])]
```

$3\, n^4$ HurwitzZeta[4, 1 + n]

```
FullSimplify[(5 - 1) (n^5 Zeta[5] - Sum[ (n / j) ^5, {j, 1, n}])]
```

$-\frac{1}{6}\, n^5$ PolyGamma[4, 1 + n]

```
FullSimplify[(6 - 1) (n^6 Zeta[6] - Sum[ (n / j) ^6, {j, 1, n}])]
```

$5\, n^6$ HurwitzZeta[6, 1 + n]

```
FullSimplify[(7 - 1) (n^7 Zeta[7] - Sum[ (n / j) ^7, {j, 1, n}])]
```

$6\, n^7$ HurwitzZeta[7, 1 + n]

```
Expand[FullSimplify[((-5 - 1) (n^-5 Zeta[-5] - Sum[ (n / j) ^-5, {j, 1, n}]) + (-5 - 1) / 2)] / n]
```

$$1 + \frac{1}{42\, n^6} - \frac{1}{2\, n^4} + \frac{5}{2\, n^2}$$

```
Expand[((-5 - 1) n^-(6) (Zeta[-5] - Sum[ (1 / j) ^-5, {j, 1, n}]))]
```

$$1 + \frac{1}{42\, n^6} - \frac{1}{2\, n^4} + \frac{5}{2\, n^2} + \frac{3}{n}$$

Limit$\left[1 + \frac{1}{42\, n^6} - \frac{1}{2\, n^4} + \frac{5}{2\, n^2} + \frac{3}{n},\ n \to \text{Infinity} \right]$

1

```
ta[n_, s_] := ((s - 1) n^(s - 1) (Zeta[s] - Sum[ (1 / j) ^s, {j, 1, n}]))
ta2[n_, s_] := ((s - 1) n^(s - 1) (- Sum[ (1 / j) ^s, {j, 1, n}]))
ta3[n_, s_] := Sum[j^-s, {j, 1, n}] - n^(-s) / (s - 1)
```

```
N@ta2[10 000, -3 + I]
```

1.0002 − 0.0000500058 i

```
ta3[1 000 000, -.5]
```

6.66668×10^8

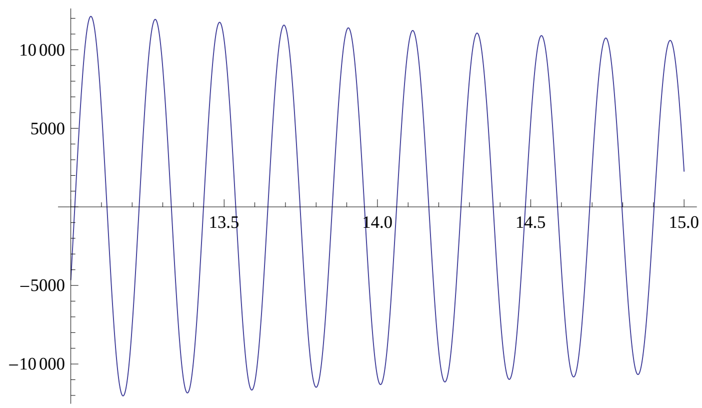
```
Zeta[.5]

-1.46035

tes5[n_, s_] := Zeta[s]
tso[n_, s_] := 1 / (s - 1) / n^(s - 1) + Sum[ (1 / j)^s, {j, 1, n}]
tso2[n_, s_] := Sum[ (1 / j)^s, {j, 1, n}] - n^(1 - s) / (1 - s)
tso2a[n_, s_] := {Sum[ (1 / j)^s, {j, 1, n}], n^(1 - s) / (1 - s)}
tso2b[n_, s_] := Sum[ 1 / j^s - 1 / (n^s (1 - s)), {j, 1, n}]
tso3[n_, s_] := HarmonicNumber[n, s] - n^(1 - s) / (1 - s)
tso4[n_, s_] := n^(1 - s) / (s - 1) + (Zeta[s] - Zeta[s, n])
tso5[n_, s_] := Sum[ 1 / j^s, {j, 1, n}] - 2^(1 - s) Sum[ 1 / j^s, {j, 1, n / 2}]
tso5a[n_, s_] := (HarmonicNumber[n, s] - 2^(1 - s) HarmonicNumber[n / 2, s]) / (1 - 2^(1 - s))

tes5[1 000 000, .5 + I]

0.143936 - 0.7221 i

N@tso2[1 000 000 000 000, .2]

-0.731937

N@tso2b[1 000 000 000 000, .2]

-0.73193


Zeta[.2]

-0.733921

N@tso2a[1 000 000 000 000, N[ZetaZero[1]] - .1]

{967 156. + 565 340. i, 967 156. + 565 340. i}

ag[n_, s_] := -2^(1 - s) Sum[ j^-s, {j, 1, n / 2}]
ag2[n_, s_] := -n^(1 - s) / (1 - s)

ag[100 000 000, .5 + I]

-310.456 - 8937.95 i

ag2[100 000 000, .5 + I]

-310.951 - 8938.87 i

N@Zeta[1 + 1 / 2 + 11 I]

1.18046 - 0.274549 i

N[tso3[100 000 000 000, ZetaZero[1]]]
```

$1.5673 \times 10^{-6} + 2.0663 \times 10^{-7}\, i$

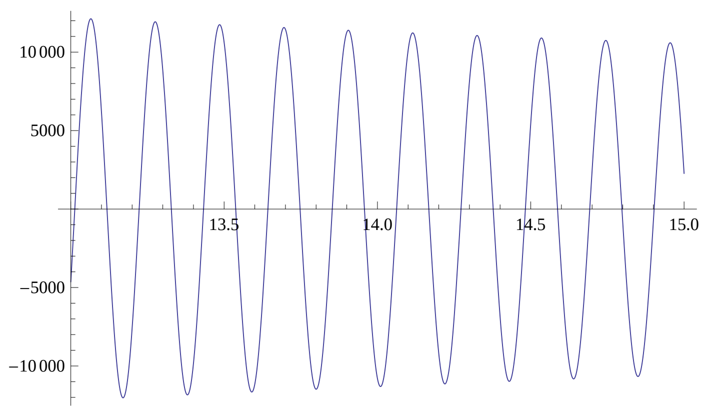
```
N[tso5a[10 000 000 000 000, ZetaZero[1]]]
```

$2.28745 \times 10^{-8} + 6.42544 \times 10^{-8}\, i$

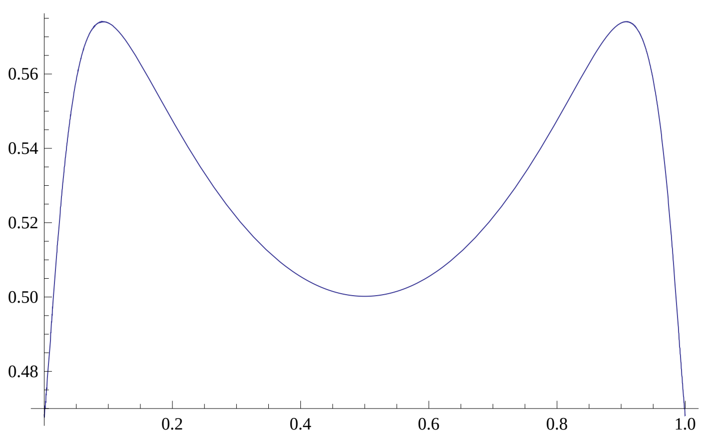
```
p1[n_, s_] := HarmonicNumber[n, s]
p1i[n_, s_] := p1[n, 1 - s]
p2[n_, s_] := n^(1 - s) / (1 - s)
p2i[n_, s_] := p2[n, 1 - s]
p12[n_, s_] := HarmonicNumber[n, s] - n^(1 - s) / (1 - s)
p12i[n_, s_] := p12[n, s] - p12[n, 1 - s]
p12j[n_, s_] := p12[n, s] + p12[n, 1 - s]
p12k[n_, s_] := p12[n, s] p12[n, 1 - s]
Plot[Re[ p2i[10 000 000 000 000, .4 + t I]], {t, 13, 15}]
```




```
Plot[Re[ p1i[10 000 000 000 000, .4 + t I]], {t, 13, 15}]
```




```
Plot[Abs[ p12k[10 000 000 000 000, t + 13.14 I]], {t, 0, 1}]
```



```
tso2b[n_, s_] := Sum[1/j^s - 1/(n^s (1 - s)), {j, 1, n}]

tso2b[10 000 000 000, .5]
```

-1.46035

```
tso2c[10 000 000 000, .5]
```

$2. \times 10^{10}$

```
tc[n_, s_] := (s - 1) (Zeta[s] - Sum[j^-s, {j, 1, n}]) - n^(1 - s)

Chop@tc[10 000 000 000, .75 + I]
```

$1.15745 \times 10^{-8} + 1.14738 \times 10^{-8}\, i$