```
Integrate[1 / j, {j, 1, n}]
```

$$\text{ConditionalExpression}[\text{Log}[n], \text{Re}[n] \geq 0 \,||\, n \notin \text{Reals}]$$

```
Integrate[1 / (j k), {j, 1, n}, {k, 1, n / j}]
```

$$\text{ConditionalExpression}\left[\frac{\text{Log}[n]^2}{2}, \text{Re}[n] \geq 0 \,||\, n \notin \text{Reals}\right]$$

```
Integrate[1 / (j k l), {j, 1, n}, {k, 1, n / j}, {l, 1, n / (j k)}]
```

$$\text{ConditionalExpression}\left[\frac{\text{Log}[n]^3}{6}, \text{Re}[n] \geq 0 \,||\, n \notin \text{Reals}\right]$$

```
Integrate[1 / (j k l m), {j, 1, n}, {k, 1, n / j}, {l, 1, n / (j k)}, {m, 1, n / (j k l)}]
```

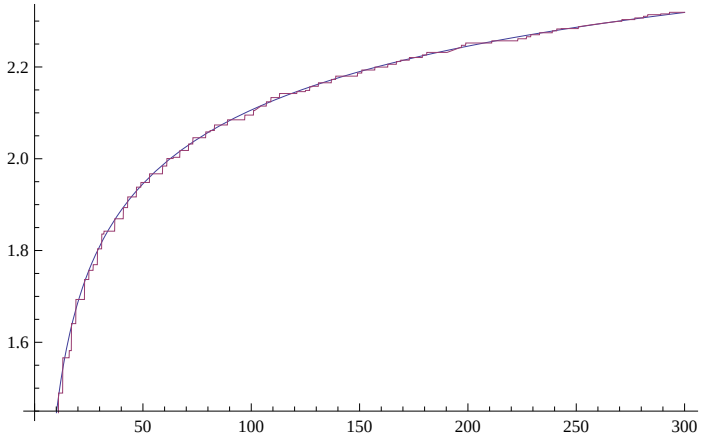
$$\text{ConditionalExpression}\left[\frac{\text{Log}[n]^4}{24}, \text{Re}[n] \geq 0 \,||\, n \notin \text{Reals}\right]$$

```
Sum[(-1)^(k + 1) / k Log[n]^k / k!, {k, 1, Infinity}]
```

```
F[n_] := EulerGamma + Gamma[0, Log[n]] + Log[Log[n]]
F2[n_] := Gamma[0, Log[n]]


p[n_] := FullSimplify[MangoldtLambda[n] / Log[n]]
S[n_] := Sum[If[p[j] ≠ 0, p[j], 0] / j, {j, 2, n}]

S2[n_] := Sum[If[p[j] ≠ 0, p[j], 0] / (j^2), {j, 2, n}]

S[25]
```

$$\frac{188\,126\,532\,397}{107\,084\,577\,600}$$

```
Plot[{F[n], S[n]}, {n, 1, 300}]
```




```
Integrate[1 / j^2, {j, 1, n}]
```

$$\text{ConditionalExpression}\left[1 - \frac{1}{n}, \text{Re}[n] \geq 0 \,||\, n \notin \text{Reals}\right]$$

```
Expand[Integrate[1 / (j^2 k^2), {j, 1, n}, {k, 1, n / j}]]
```

$$\text{ConditionalExpression}\left[1 - \frac{1}{n} - \frac{\text{Log}[n]}{n}, \text{Re}[n] \geq 0 \,||\, n \notin \text{Reals}\right]$$

```
Expand[Integrate[ 1 / ( j^2 k^2 l^2), {j, 1, n}, {k, 1, n / j}, {l, 1, n / (j k)}]]
```

$$\text{ConditionalExpression}\left[1-\frac{1}{n}-\frac{\text{Log}[n]}{n}-\frac{\text{Log}[n]^2}{2\,n},\ \text{Re}[n]\geq 0\ ||\ n\notin \text{Reals}\right]$$

```
Integrate[ 1 / ( j^2 k^2 l^2 m^2), {j, 1, n}, {k, 1, n / j}, {l, 1, n / (j k)}, {m, 1, n / (j k l)}]
```

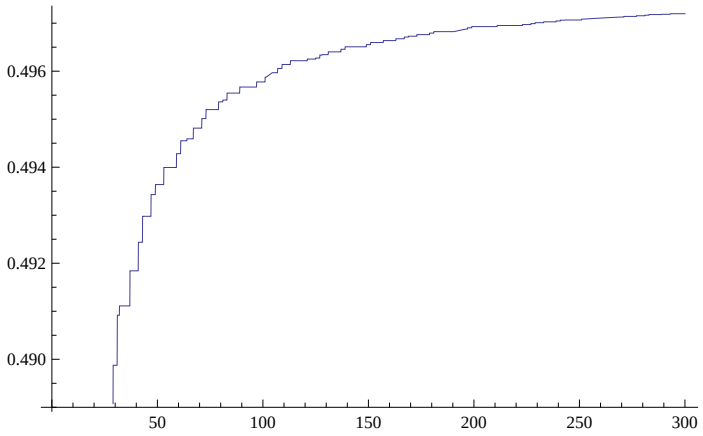
$$\text{Expand}\left[\text{ConditionalExpression}\left[-\frac{6-6\,n+\text{Log}[n]\,(6+\text{Log}[n]\,(3+\text{Log}[n]))}{6\,n},\ \text{Re}[n]\geq 0\ ||\ n\notin \text{Reals}\right]\right]$$

$$\text{ConditionalExpression}\left[1-\frac{1}{n}-\frac{\text{Log}[n]}{n}-\frac{\text{Log}[n]^2}{2\,n}-\frac{\text{Log}[n]^3}{6\,n},\ \text{Re}[n]\geq 0\ ||\ n\notin \text{Reals}\right]$$

```
1 - Sum[ Log[n]^k / (k ! n), {k, 0, Infinity}]
```

```
0
```

```
Plot[{S2[n]}, {n, 1, 300}]
```




```
Integrate[ 1 / ( j^3 k^3 l^3 m^3), {j, 1, n}, {k, 1, n / j}, {l, 1, n / (j k)}, {m, 1, n / (j k l)}]
```

$$\text{Expand}\left[\text{ConditionalExpression}\left[\frac{-3+3\,n^2-6\,\text{Log}[n]-6\,\text{Log}[n]^2-4\,\text{Log}[n]^3}{48\,n^2},\ \text{Re}[n]\geq 0\ ||\ n\notin \text{Reals}\right]\right]$$

$$\text{ConditionalExpression}\left[\frac{1}{16}-\frac{1}{16\,n^2}-\frac{\text{Log}[n]}{8\,n^2}-\frac{\text{Log}[n]^2}{8\,n^2}-\frac{\text{Log}[n]^3}{12\,n^2},\ \text{Re}[n]\geq 0\ ||\ n\notin \text{Reals}\right]$$