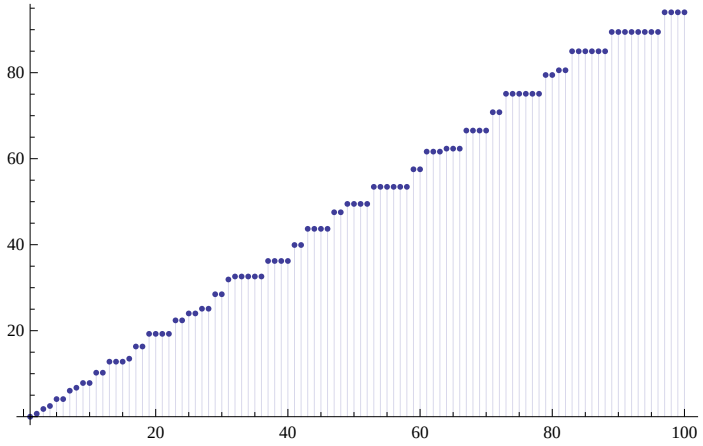
```
ClearAll["Global`*"]

referenceChebyshev[n_] := Sum[ MangoldtLambda[ j], {j, 2, n}]
L2[n_, 1, b_] := L2[n, 1, b] = Sum[ Log[j], {j, 2, n}] - b Sum[ Log[j b], {j, 1, n / b}]
L2[n_, k_, b_ ] := Sum[ L2[ n / j, k - 1, b], {j, 2, n}] - b Sum[ L2[ n / (j b), k - 1, b], {j, 1, n}]
ChebAlt[n_, b_] := Sum[ (-1)^(a - 1) L2[n, a, b] , {a, 1, Log[If[ b < 2, b, 2], n]}] +
  Sum[ b^a Log[b], {a, 1, Log[b, n]}]
Table[ { n, N[referenceChebyshev[ n ]], N[ChebAlt[n, 3 / 2]],
   N[ChebAlt[n, 2]], N[ChebAlt[n, 5]]}, {n, 2, 100}] // TableForm
```

2	0.693147	0.693147	0.693147	0.693147
3	1.79176	1.79176	1.79176	1.79176
4	2.48491	2.48491	2.48491	2.48491
5	4.09434	4.09434	4.09434	4.09434
6	4.09434	4.09434	4.09434	4.09434
7	6.04025	6.04025	6.04025	6.04025
8	6.7334	6.7334	6.7334	6.7334
9	7.83201	7.83201	7.83201	7.83201
10	7.83201	7.83201	7.83201	7.83201
11	10.2299	10.2299	10.2299	10.2299
12	10.2299	10.2299	10.2299	10.2299
13	12.7949	12.7949	12.7949	12.7949
14	12.7949	12.7949	12.7949	12.7949
15	12.7949	12.7949	12.7949	12.7949
16	13.488	13.488	13.488	13.488
17	16.3212	16.3212	16.3212	16.3212
18	16.3212	16.3212	16.3212	16.3212
19	19.2657	19.2657	19.2657	19.2657
20	19.2657	19.2657	19.2657	19.2657
21	19.2657	19.2657	19.2657	19.2657
22	19.2657	19.2657	19.2657	19.2657
23	22.4012	22.4012	22.4012	22.4012
24	22.4012	22.4012	22.4012	22.4012
25	24.0106	24.0106	24.0106	24.0106
26	24.0106	24.0106	24.0106	24.0106
27	25.1092	25.1092	25.1092	25.1092
28	25.1092	25.1092	25.1092	25.1092
29	28.4765	28.4765	28.4765	28.4765
30	28.4765	28.4765	28.4765	28.4765
31	31.9105	31.9105	31.9105	31.9105
32	32.6036	32.6036	32.6036	32.6036
33	32.6036	32.6036	32.6036	32.6036
34	32.6036	32.6036	32.6036	32.6036
35	32.6036	32.6036	32.6036	32.6036
36	32.6036	32.6036	32.6036	32.6036
37	36.2146	36.2146	36.2146	36.2146
38	36.2146	36.2146	36.2146	36.2146
39	36.2146	36.2146	36.2146	36.2146
40	36.2146	36.2146	36.2146	36.2146
41	39.9281	39.9281	39.9281	39.9281
42	39.9281	39.9281	39.9281	39.9281
43	43.6893	43.6893	43.6893	43.6893
44	43.6893	43.6893	43.6893	43.6893
45	43.6893	43.6893	43.6893	43.6893
46	43.6893	43.6893	43.6893	43.6893

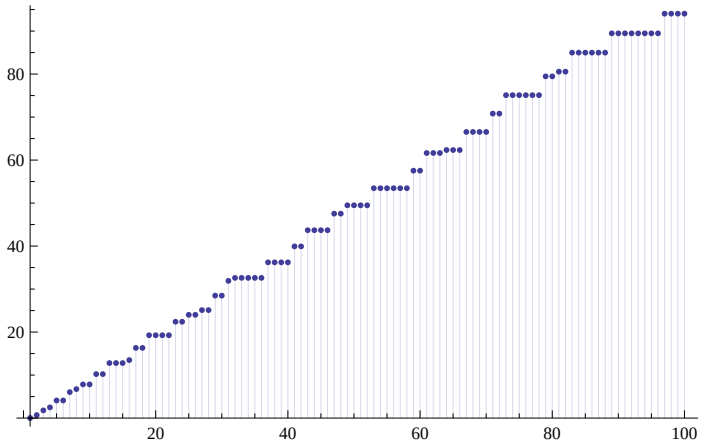
47	47.5395	47.5395	47.5395	47.5395
48	47.5395	47.5395	47.5395	47.5395
49	49.4854	49.4854	49.4854	49.4854
50	49.4854	49.4854	49.4854	49.4854
51	49.4854	49.4854	49.4854	49.4854
52	49.4854	49.4854	49.4854	49.4854
53	53.4557	53.4557	53.4557	53.4557
54	53.4557	53.4557	53.4557	53.4557
55	53.4557	53.4557	53.4557	53.4557
56	53.4557	53.4557	53.4557	53.4557
57	53.4557	53.4557	53.4557	53.4557
58	53.4557	53.4557	53.4557	53.4557
59	57.5332	57.5332	57.5332	57.5332
60	57.5332	57.5332	57.5332	57.5332
61	61.6441	61.6441	61.6441	61.6441
62	61.6441	61.6441	61.6441	61.6441
63	61.6441	61.6441	61.6441	61.6441
64	62.3372	62.3372	62.3372	62.3372
65	62.3372	62.3372	62.3372	62.3372
66	62.3372	62.3372	62.3372	62.3372
67	66.5419	66.5419	66.5419	66.5419
68	66.5419	66.5419	66.5419	66.5419
69	66.5419	66.5419	66.5419	66.5419
70	66.5419	66.5419	66.5419	66.5419
71	70.8046	70.8046	70.8046	70.8046
72	70.8046	70.8046	70.8046	70.8046
73	75.0951	75.0951	75.0951	75.0951
74	75.0951	75.0951	75.0951	75.0951
75	75.0951	75.0951	75.0951	75.0951
76	75.0951	75.0951	75.0951	75.0951
77	75.0951	75.0951	75.0951	75.0951
78	75.0951	75.0951	75.0951	75.0951
79	79.4645	79.4645	79.4645	79.4645
80	79.4645	79.4645	79.4645	79.4645
81	80.5631	80.5631	80.5631	80.5631
82	80.5631	80.5631	80.5631	80.5631
83	84.982	84.982	84.982	84.982
84	84.982	84.982	84.982	84.982
85	84.982	84.982	84.982	84.982
86	84.982	84.982	84.982	84.982
87	84.982	84.982	84.982	84.982
88	84.982	84.982	84.982	84.982
89	89.4706	89.4706	89.4706	89.4706
90	89.4706	89.4706	89.4706	89.4706
91	89.4706	89.4706	89.4706	89.4706
92	89.4706	89.4706	89.4706	89.4706
93	89.4706	89.4706	89.4706	89.4706
94	89.4706	89.4706	89.4706	89.4706
95	89.4706	89.4706	89.4706	89.4706
96	89.4706	89.4706	89.4706	89.4706
97	94.0453	94.0453	94.0453	94.0453
98	94.0453	94.0453	94.0453	94.0453
99	94.0453	94.0453	94.0453	94.0453
100	94.0453	94.0453	94.0453	94.0453

```
DiscretePlot[ChebAlt[ n,  3 / 2], {n, 1, 100}]
```

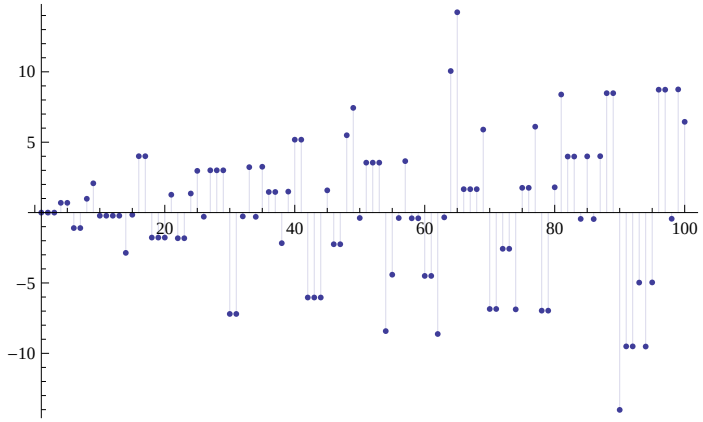



```
referenceChebyshev[n_] := Sum[ MangoldtLambda[ j], {j, 2, n}]
```

```
DiscretePlot[ referenceChebyshev[ n ], {n, 1, 100}]
```




```
DiscretePlot[ L2[ n, 2, 2], {n, 1, 100}]
```




```
Expand[L2[10, 1, 2]]
```

```
-Log[2] + Log[3] - Log[4] + Log[5] - Log[6] + Log[7] - Log[8] + Log[9] - Log[10]
```

```
Sum[ (-1)^(j + 1) / j, {j, 1, Infinity}]
```

```
Log[2]
```