Inversion #2

The coefficient pair $a_k = -1, b_k = -1$ generates another inverse. For some function $f_k(n)$ as found in (L1), if we have

$$g_1(n) = \sum_{k=1} -f_k(n) \quad g_k(n) = \sum_{j|n} g_1(j) g_{k-1}(\frac{n}{j}) \quad g_0(n) = 1 \, if \; n=1, 0 \, otherwise \quad G_k(n) = \sum_{j=1}^{n} g_k(j)$$

then

$$F_1(n) = \sum_{j=1}^{n} g_1(j)(F_1(\frac{n}{j}) - 1) \tag{L30}$$

$$F_1(n) = -G_1(n) + \sum_{j=1}^{n} f_1(j) G_1(\frac{n}{j}) \tag{L31}$$

Additionally,

$$\sum_{j=1} -g_1(j) F_k(\frac{n}{j}) = \sum_{j=1} F_{k+j}(n) \tag{L32}$$

and thus

$$\sum_{j=1} g_1(j)(F_k(\frac{n}{j}) - F_{k-1}(\frac{n}{j})) = F_k(n) \tag{L33}$$

Also,

$$F_k(n) = \sum_{j=0} \binom{k+j-1}{k-1} G_{k+j}(n) \tag{L34}$$

Because this is an inverse, all f's and g's can be swapped in these equations.

With these functions, we have the following inversion. Forward is:

$$\sum_{j=1}^{n} (f_1(j) - f_0(j)) \sum_{m=0}^{k} -1^{k-m} \binom{k}{m} F_m(\frac{n}{j}) = \sum_{m=0}^{k+1} -1^{k-m+1} \binom{k}{m} F_m(n)$$

(the coefficients here mirror those found in the similar $(x-1)\cdot(x-1)^k = (x-1)^{k+1}$)

and going backward,

$$\sum_{j=1}^{n} (g_1(j) - g_0(j)) \sum_{m=0}^{k} -1^{k-m} \binom{k}{m} F_m(\frac{n}{j}) = \sum_{m=0}^{k-1} -1^{k-m-1} \binom{k}{m} F_m(n)$$

(the coefficients here mirror $\frac{(x-1)^k}{(x-1)} = (x-1)^{k-1}$)

with

$$\sum_{j=1}^{n} (g_1(j)-g_0(j))(F_1(\frac{n}{j})-F_0(\frac{n}{j}))=1$$

f's and g's can be swapped here too.

This is very similar to the standard Dirichlet inverse, but with the first term's sign negated.

This leads to the following mobius-like function:

If, for some two functions,

$$G(n)=F(n)-\sum_{j=2}^{n} F(\frac{n}{j})$$

then

$$F(n)=-G(n)-\sum_{j=2}^{n} g_1(j)F(\frac{n}{j})$$

$$p(n)=\frac{1}{a}\text{ if } n=p^a \text{ where } p \text{ is prime}, 0 \text{ otherwise}$$
$$p'(n)=\frac{-1^a}{a}\text{ if } n=p^a \text{ where } p \text{ is prime}, 0 \text{ otherwise}$$

$$\sum_{j=2}^{n}\frac{\Lambda(j)}{\log j} = \sum_{j=2}^{n}1-\frac{1}{2}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}1+\frac{1}{3}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}}1-\frac{1}{4}...$$

$$\sum_{j=2}^{n}1 = \sum_{j=2}^{n}\frac{\Lambda(j)}{\log j}+\frac{1}{2}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\frac{\Lambda(j)}{\log j}\cdot\frac{\Lambda(k)}{\log k}+\frac{1}{6}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}}\frac{\Lambda(j)}{\log j}\cdot\frac{\Lambda(k)}{\log k}\cdot\frac{\Lambda(l)}{\log l}+\frac{1}{24}...$$

$$\sum_{j=2}^{n}\frac{\Lambda(j)}{\log j} = -\sum_{j=2}^{n}\mu(j)+\frac{1}{2}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\mu(j)\cdot\mu(k)-\frac{1}{3}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}}\mu(j)\cdot\mu(k)\cdot\mu(l)+\frac{1}{4}...$$

$$\sum_{j=2}^{n}\mu(j) = -\sum_{j=2}^{n}\frac{\Lambda(j)}{\log j}+\frac{1}{2}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\frac{\Lambda(j)}{\log j}\cdot\frac{\Lambda(k)}{\log k}-\frac{1}{6}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}}\frac{\Lambda(j)}{\log j}\cdot\frac{\Lambda(k)}{\log k}\cdot\frac{\Lambda(l)}{\log l}+\frac{1}{24}...$$

$$\sum_{j=2}^{n}\mu(j) = -\sum_{j=2}^{n}1+\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}1-\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}}1+...$$

$$\sum_{j=2}^{n} 1 \quad = \quad -\sum_{j=2}^{n} \mu(j) + \sum_{j=2}^{n} \sum_{k=2}^{\frac{n}{j}} \mu(j) \cdot \mu(k) - \sum_{j=2}^{n} \sum_{k=2}^{\frac{n}{j}} \sum_{l=2}^{\frac{n}{j \cdot k}} \mu(j) \cdot \mu(k) \cdot \mu(l) + \ldots$$

OR

$$\sum_{j=2}^{n} 1 = \sum_{j=2}^{n} \frac{\Lambda(j)}{\log j} (1 + \sum_{k=2}^{\frac{n}{j}} \frac{\Lambda(k)}{\log k} (\frac{1}{2} + \sum_{m=2}^{\frac{n}{jk}} \frac{\Lambda(m)}{\log m} (\frac{1}{6} + \ldots)))$$

$$\sum_{j=2}^{n} \frac{\Lambda(j)}{\log j} = -\sum_{x=2}^{n} \mu(x) (1 - \sum_{y=2}^{\frac{n}{x}} \mu(y) (\frac{1}{2} - \sum_{z=2}^{\frac{n}{xy}} \mu(z) (\frac{1}{3} - \ldots)))$$

$$\sum_{j=2}^{n} \mu(j) = -\sum_{x=2}^{n} \frac{\Lambda(x)}{\log x} (1 - \sum_{y=2}^{\frac{n}{x}} \frac{\Lambda(y)}{\log y} (\frac{1}{2} - \sum_{z=2}^{\frac{n}{xy}} \frac{\Lambda(z)}{\log z} (\frac{1}{6} - \ldots)))$$

$$\sum_{j=2}^{n} \frac{\Lambda(j)}{\log j} = \sum_{j=2}^{n} 1 - \sum_{k=2}^{\frac{n}{j}} \frac{1}{2} - \sum_{m=2}^{\frac{n}{jk}} \frac{1}{3} - \ldots$$

$$\sum_{j=2}^{n} \mu(j) = -\sum_{x=2}^{n} 1 - \sum_{y=2}^{\frac{n}{x}} 1 - \sum_{z=2}^{\frac{n}{xy}} 1 - \ldots$$

$$\sum_{j=2}^{n} 1 = -\sum_{x=2}^{n} \mu(x) (1 - \sum_{y=2}^{\frac{n}{x}} \mu(y) (1 - \sum_{z=2}^{\frac{n}{xy}} \mu(z) (1 - \ldots)))$$

$$\sum_{j=2}^{n} \Lambda'(j) = \sum_{j=2}^{n} \frac{B_0}{0!} - (\sum_{k=2}^{\frac{n}{j}} \Lambda'(k) (\frac{B_1}{1!} - \sum_{m=2}^{\frac{n}{jk}} \Lambda'(m) (\frac{B_2}{2!} - \ldots)))$$

$$\Lambda'(n) = \frac{B_0}{0!} + \sum_{j|n} \Lambda'(j) (\frac{B_1}{1!} + \sum_{k|\frac{n}{j}} \Lambda'(k) (\frac{B_2}{2!} + \ldots))$$

$$p'(n) = \frac{-1^a}{a} \text{ if } n = p^a \text{ where } p \text{ is prime}, 0 \text{ otherwise}$$

$$\sum_{j=2}^{n} p'(j) \quad = \quad \sum_{j=2}^{n} \lambda(j) - \frac{1}{2} \sum_{j=2}^{n} \sum_{k=2}^{\frac{n}{j}} \lambda(j) \cdot \lambda(k) + \frac{1}{3} \sum_{j=2}^{n} \sum_{k=2}^{\frac{n}{j}} \sum_{l=2}^{\frac{n}{j \cdot k}} \lambda(j) \cdot \lambda(k) \cdot \lambda(l) - \frac{1}{4} \ldots$$

$$\sum_{j=2}^{n} \lambda(j) \quad = \quad \sum_{j=2}^{n} p'(j) + \frac{1}{2} \sum_{j=2}^{n} \sum_{k=2}^{\frac{n}{j}} p'(j) \cdot p'(k) + \frac{1}{6} \sum_{j=2}^{n} \sum_{k=2}^{\frac{n}{j}} \sum_{l=2}^{\frac{n}{j \cdot k}} p'(j) \cdot p'(k) \cdot p'(l) + \frac{1}{24} \ldots$$

$$\sum_{j=2}^{n} p'(j) \quad = \quad -\sum_{j=2}^{n} \mu(j)^2 + \frac{1}{2} \sum_{j=2}^{n} \sum_{k=2}^{\frac{n}{j}} \mu(j)^2 \cdot \mu(k)^2 - \frac{1}{3} \sum_{j=2}^{n} \sum_{k=2}^{\frac{n}{j}} \sum_{l=2}^{\frac{n}{j \cdot k}} \mu(j)^2 \cdot \mu(k)^2 \cdot \mu(l)^2 + \frac{1}{4} \ldots$$

$$\sum_{j=2}^{n}\mu(j)^2 = -\sum_{j=2}^{n} p'(j)+\frac{1}{2}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}} p'(j)\cdot p'(k)-\frac{1}{6}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}} p'(j)\cdot p'(k)\cdot p'(l)+\frac{1}{24}\ldots$$

$$\sum_{j=2}^{n}\mu(j)^2 = -\sum_{j=2}^{n}\lambda(j)+\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\lambda(j)\cdot\lambda(k)-\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}}\lambda(j)\cdot\lambda(k)\cdot\lambda(l)+\ldots$$

$$\sum_{j=2}^{n}\lambda(j) = -\sum_{j=2}^{n}\mu(j)^2+\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\mu(j)^2\cdot\mu(k)^2-\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}}\mu(j)^2\cdot\mu(k)^2\cdot\mu(l)^2+\ldots$$

OR

$$\sum_{j=2}^{n} p'(j)=\sum_{j=2}^{n}\lambda(j)(1-\sum_{k=2}^{\frac{n}{j}}\lambda(k)(\frac{1}{2}-\sum_{m=2}^{\frac{n}{jk}}\lambda(m)(\frac{1}{3}-\ldots)))$$

$$\sum_{j=2}^{n}\lambda(j)=\sum_{j=2}^{n} p'(j)(1+\sum_{k=2}^{\frac{n}{j}} p'(k)(\frac{1}{2}+\sum_{m=2}^{\frac{n}{jk}} p'(m)(\frac{1}{6}+\ldots)))$$

$$\sum_{j=2}^{n}\lambda(j)=-\sum_{j=2}^{n}\mu(j)^2(1-\sum_{k=2}^{\frac{n}{j}}\mu(k)^2(1-\sum_{m=2}^{\frac{n}{jk}}\mu(m)^2(1-\ldots)))$$

$$\sum_{j=2}^{n}\mu(j)^2=-\sum_{j=2}^{n}\lambda(j)(1-\sum_{k=2}^{\frac{n}{j}}\lambda(k)(1-\sum_{m=2}^{\frac{n}{jk}}\lambda(m)(1-\ldots)))$$

$$\sum_{j=2}^{n} p'(j)=-\sum_{j=2}^{n}\mu(j)^2(1-\sum_{k=2}^{\frac{n}{j}}\mu(k)^2(\frac{1}{2}-\sum_{m=2}^{\frac{n}{jk}}\mu(m)^2(\frac{1}{3}-\ldots)))$$

$$\sum_{j=2}^{n}\mu(j)^2=-\sum_{j=2}^{n} p'(j)(1-\sum_{k=2}^{\frac{n}{j}} p'(k)(\frac{1}{2}-\sum_{m=2}^{\frac{n}{jk}} p'(m)(\frac{1}{6}-\ldots)))$$

$$\sum_{j=2}^{n}\frac{\Lambda(j)}{\log j} j^a = \sum_{j=2}^{n} j^a-\frac{1}{2}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}} j^a\cdot k^a+\frac{1}{3}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}} j^a\cdot k^a\cdot l^a-\frac{1}{4}\ldots$$

$$\sum_{j=2}^{n} j^a = \sum_{j=2}^{n}\frac{\Lambda(j)}{\log j} j^a+\frac{1}{2}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\frac{\Lambda(j)}{\log j} j^a\cdot\frac{\Lambda(k)}{\log k} k^a+\frac{1}{6}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}}\frac{\Lambda(j)}{\log j} j^a\cdot\frac{\Lambda(k)}{\log k} k^a\cdot\frac{\Lambda(l)}{\log l} l^a+\frac{1}{24}\ldots$$

$$\sum_{j=2}^{n} p'(j)\cdot j^a = \sum_{j=2}^{n}\lambda(j)\cdot j^a-\frac{1}{2}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\lambda(j)\cdot j^a\cdot\lambda(k)\cdot k^a+\frac{1}{3}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}}\lambda(j)\cdot j^a\cdot\lambda(k)\cdot k^a\cdot\lambda(l)\cdot l^a-\frac{1}{4}\ldots$$

$$\sum_{j=2}^{n}\lambda(j)\cdot j^a = \sum_{j=2}^{n} p'(j)\cdot j^a+\frac{1}{2}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}} p'(j)\cdot j^a\cdot p'(k)\cdot k^a+\frac{1}{6}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}} p'(j)\cdot j^a\cdot p'(k)\cdot k^a\cdot p'(l)\cdot l^a+\frac{1}{24}\ldots$$

OR

$$\sum_{j=2}^{n}\frac{\Lambda(j)}{\log j}j^{a}=\sum_{j=2}^{n}j^{a}(1-\sum_{k=2}^{\frac{n}{j}}k^{a}(\frac{1}{2}-\sum_{m=2}^{\frac{n}{jk}}m^{a}(\frac{1}{3}-...)))$$

$$\sum_{j=2}^{n}\varphi(j) \quad = \quad \sum_{j=2}^{n}\frac{\Lambda(j)}{\log j}(j-1)+\frac{1}{2}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\frac{\Lambda(j)}{\log j}(j-1)\cdot\frac{\Lambda(k)}{\log k}(k-1)+\frac{1}{6}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}}\frac{\Lambda(j)}{\log j}(j-1)\cdot\frac{\Lambda(k)}{\log k}(k-1)\cdot\frac{\Lambda(l)}{\log l}(l-1)+\frac{1}{24}...$$

$$v_{k}(n)=\sum_{j=2}^{n}\frac{\Lambda(j)}{\log j}(j-1)(\frac{1}{k}+v_{k+1}(\frac{n}{j})) \qquad v_{1}(n)=\sum_{j=2}^{n}\varphi(j)$$

$$\sum_{j=2}^{n}\frac{\Lambda(j)}{\log j}(j-1) \quad = \quad \sum_{j=2}^{n}\varphi(j)-\frac{1}{2}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\varphi(j)\cdot\varphi(k)+\frac{1}{3}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}}\varphi(j)\cdot\varphi(k)\cdot\varphi(l)-\frac{1}{4}...$$

$$v_{k}(n)=\sum_{j=2}^{n}\varphi(j)(\frac{1}{k}-v_{k+1}(\frac{n}{j})) \qquad v_{1}(n)=\sum_{j=2}^{n}\frac{\Lambda(j)}{\log j}(j-1)$$

$$v(n)=1+\sum_{j=2}^{n}j-v(\frac{n}{j}) \qquad v(n)=\sum_{j=1}^{n}\varphi(j)$$

$$\sum_{j=2}^{n}\sigma_{1}(j) \quad = \quad \sum_{j=2}^{n}\frac{\Lambda(j)}{\log j}(j+1)+\frac{1}{2}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\frac{\Lambda(j)}{\log j}(j+1)\cdot\frac{\Lambda(k)}{\log k}(k+1)+\frac{1}{6}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}}\frac{\Lambda(j)}{\log j}(j+1)\cdot\frac{\Lambda(k)}{\log k}(k+1)\cdot\frac{\Lambda(l)}{\log l}(l+1)+\frac{1}{24}...$$

$$\sum_{j=2}^{n}\frac{\Lambda(j)}{\log j}(j+1) \quad = \quad \sum_{j=2}^{n}\sigma_{1}(j)-\frac{1}{2}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sigma_{1}(j)\cdot\sigma_{1}(k)+\frac{1}{3}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}}\sigma_{1}(j)\cdot\sigma_{1}(k)\cdot\sigma_{1}(l)-\frac{1}{4}...$$

$$v_{k}(n)=\sum_{j=2}^{n}\sigma_{1}(j)(\frac{1}{k}-v_{k+1}(\frac{n}{j})) \qquad v_{1}(n)=\sum_{j=2}^{n}\frac{\Lambda(j)}{\log j}(j+1)$$

$$2\,\Pi(n) \quad = \quad \sum_{j=2}^{n}\sigma_{1}(j)-\varphi(j)-\frac{1}{2}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sigma_{1}(j)\cdot\sigma_{1}(k)-\varphi(j)\cdot\varphi(k)+\frac{1}{3}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}}\sigma_{1}(j)\cdot\sigma_{1}(k)\cdot\sigma_{1}(l)-\varphi(j)\cdot\varphi(k)\cdot\varphi(l)-\frac{1}{4}...$$

$$\sigma_{a}(n)=\sum_{j|n}j^{a}$$

$$\sum_{j=2}^{n}\sigma_{a}(j) \quad = \quad \sum_{j=2}^{n}\frac{\Lambda(j)}{\log j}(j^{a}+1)+\frac{1}{2}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\frac{\Lambda(j)}{\log j}(j^{a}+1)\cdot\frac{\Lambda(k)}{\log k}(k^{a}+1)+\frac{1}{6}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}}\frac{\Lambda(j)}{\log j}(j^{a}+1)\cdot\frac{\Lambda(k)}{\log k}(k^{a}+1)\cdot\frac{\Lambda(l)}{\log l}(l^{a}+1)+\frac{1}{24}...$$

$$\sum_{j=2}^{n}\frac{\Lambda(j)}{\log j}(j^{a}+1) \quad = \quad \sum_{j=2}^{n}\sigma_{a}(j)-\frac{1}{2}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sigma_{a}(j)\cdot\sigma_{a}(k)+\frac{1}{3}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}}\sigma_{a}(j)\cdot\sigma_{a}(k)\cdot\sigma_{a}(l)-\frac{1}{4}...$$

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// This is Jordan's Totient Function. J_k(n)
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$$J_a(n)=\sum_{j|n} j^a \mu(\frac{n}{j})$$

$$\sum_{j=2}^{n} J_a(j) \quad = \quad \sum_{j=2}^{n} \frac{\Lambda(j)}{\log j}(j^a-1)+\frac{1}{2}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}} \frac{\Lambda(j)}{\log j}(j^a-1)\cdot\frac{\Lambda(k)}{\log k}(k^a-1)+\frac{1}{6}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}} \frac{\Lambda(j)}{\log j}(j^a-1)\cdot\frac{\Lambda(k)}{\log k}(k^a-1)\cdot\frac{\Lambda(l)}{\log l}(l^a-1)+\frac{1}{24}...$$

$$\sum_{j=2}^{n} \frac{\Lambda(j)}{\log j}(j^a-1) \quad = \quad \sum_{j=2}^{n} J_a(j)-\frac{1}{2}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}} J_a(j)\cdot J_a(k)+\frac{1}{3}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}} J_a(j)\cdot J_a(k)\cdot J_a(l)-\frac{1}{4}...$$

Generally speaking, a sum of the form $\sum_{k=1} a_k(\zeta(s))^k$ won't converge if $|\zeta(s)|>1$.

$$F(s)-f(1)=\sum_{k=1} a_k(\zeta(s)-1)^k$$

$$\zeta(s)-1=\sum_{k=1} b_k(F(s)-f(1))^k$$

$$1=\sum_{j=1}^{n} \mu(j)\frac{n}{j}$$

$$-n+1=\sum_{j=2}^{n} \mu(j)\frac{n}{j}$$

$$n=\sum_{j=1}^{n} (\mu(j)^2)\lfloor(\frac{n}{j})^{\frac{1}{2}}\rfloor$$

$$n=\sum_{j=1}^{n^{\frac{1}{2}}} 1+MM(\frac{n}{j^2})$$

$$n=\sum_{j=1}^{n^{\frac{1}{2}}}\sum_{k=0}\frac{-1^k PP_k(\frac{n}{j^2})}{k!}$$

$$n=\sum_{j=1}^{n}(\sum_{k=0}\frac{-1^k p_k{}'(j)}{k!})\lfloor(\frac{n}{j})^{\frac{1}{2}}\rfloor$$

$$n=\sum_{j=1}^{n}(\sum_{k=0} -1^k l_k{}'(j))\lfloor(\frac{n}{j})^{\frac{1}{2}}\rfloor$$

$$n=\sum_{j=1}^{n^{\frac{1}{2}}}(\sum_{k=0} -1^k L_k(\frac{n}{j^2}))$$

$$n=\lfloor n^{\frac{1}{2}}\rfloor-\sum_{j=2}^{n} \lambda(j)\lfloor\frac{n}{j}\rfloor$$

$$n=\lfloor n^{\frac{1}{2}}\rfloor-\sum_{j=1}^{n} L(\frac{n}{j})$$

$$L(n)=\sum_{j=2}^{n}\mu(j)^{2}(-1-L(\frac{n}{j}))$$

$$L(n)=\sum_{j=1}^{n}\mu(j)\lfloor(\frac{n}{j})^{\frac{1}{2}}\rfloor$$

$$L(n)=-1+\sum_{j=1}^{n^{\frac{1}{2}}}M_{1}'(\frac{n}{j^{2}})$$

$$L(n)=\lfloor n^{\frac{1}{2}}\rfloor-\sum_{j=2}^{n}L(\frac{n}{j})$$

$$L(n)=\lfloor n^{\frac{1}{2}}\rfloor-\sum_{j=2}^{n}\lfloor(\frac{n}{j})^{\frac{1}{2}}\rfloor-\sum_{k=2}^{\frac{n}{j}}\lfloor(\frac{n}{jk})^{\frac{1}{2}}\rfloor-\sum_{s=2}^{\frac{n}{jk}}\lfloor(\frac{n}{jks})^{\frac{1}{2}}\rfloor-\ldots$$

$$L(n)=\sum_{j=1}^{n}(\sum_{k=0}-1^{k}d_{k}(j))\lfloor(\frac{n}{j})^{\frac{1}{2}}\rfloor$$

$$L(n)=\sum_{j=1}^{n^{\frac{1}{2}}}(\sum_{k=0}-1^{k}D_{k}'(\frac{n}{j^{2}}))$$

$$L(n)=\sum_{j=1}^{n}(\sum_{k=0}\frac{-1^{k}p_{k}(j)}{k!})\lfloor(\frac{n}{j})^{\frac{1}{2}}\rfloor$$

$$L(n)=-1+\sum_{j=1}^{n^{\frac{1}{2}}}\sum_{k=1}\frac{-1^{k}P_{k}'(\frac{n}{j^{2}})}{k!}$$

$$L(n)=\sum_{j=1}^{n}(\sum_{k=0}\frac{p_{k}'(j)}{k!})$$

$$\sum_{j=1}^{n}L(\frac{n}{j})=\lfloor n^{\frac{1}{2}}\rfloor$$

$$\sum_{j=1}^{n}\sum_{k=0}\frac{PP_{k}(\frac{n}{j})}{k!}=\lfloor n^{\frac{1}{2}}\rfloor$$

$$\sum_{j=1}^{n}\mu(j)\lfloor(\frac{n}{j})^{\frac{1}{2}}\rfloor=L(n)$$

$$\sum_{j=1}^{n}\mu(j)MM(\frac{n}{j})=M(n^{\frac{1}{2}})$$

$$\sum_{j=1}^{n}\mu(j)MM(\frac{n}{j})=M(n^{\frac{1}{2}})$$

$$\sum_{j=1}^{n}M((\frac{n}{j})^{\frac{1}{2}})=MM(n)$$

$$n=\sum_{j=1}^{n^{\frac{1}{2}}}1+MM(\frac{n}{j^{2}})$$

$$n=\sum_{j=1}^{n^{\frac{1}{2}}}1+\sum_{k=1}\frac{-1^{k}}{k!}PP_{k}(\frac{n}{j^{2}})$$

$$PP'(n)=L_1(n)-\frac{1}{2}L_2(n)+\frac{1}{3}L_3(n)-\frac{1}{4}L_4(n)+\frac{1}{5}L_5(n)-...$$

$$\Pi(n)=\pi(n)+\frac{1}{2}\pi(n^{\frac{1}{2}})+\frac{1}{3}\pi(n^{\frac{1}{3}})+\frac{1}{4}\pi(n^{\frac{1}{4}})+\frac{1}{5}\pi(n^{\frac{1}{5}})+...$$

$$PP'(n)=-\pi(n)+\frac{1}{2}\pi(n^{\frac{1}{2}})-\frac{1}{3}\pi(n^{\frac{1}{3}})+\frac{1}{4}\pi(n^{\frac{1}{4}})-\frac{1}{5}\pi(n^{\frac{1}{5}})+...$$

$$PP'(n)=-\Pi(n)+\pi(n^{\frac{1}{2}})+\frac{1}{2}\pi(n^{\frac{1}{4}})+\frac{1}{3}\pi(n^{\frac{1}{6}})-...$$

$$\psi'(n)=\sum_{j=2}^{n}p'(j)\log j$$

$$\psi'(n)=\sum_{j=2}^{n}-\lambda(j)(MM_1(\frac{n}{j})+1)\log j$$

$$\psi'(n)=\sum_{j=2}^{n}-\mu(j)^2(L(\frac{n}{j})+1)\log j$$

$$\psi'(n)=\sum_{j=2}^{n}\lambda(j)(\log j-\psi'(\frac{n}{j}))$$

$$0=\sum_{j=1}^{n}\lambda(j)(\log j-\psi'(\frac{n}{j}))$$

$$\psi'(n)=\sum_{j=2}^{n}\mu(j)^2(-\log j-\psi'(\frac{n}{j}))$$

$$\sum_{j=1}^{n}\lambda(j)\psi'(\frac{n}{j})=\sum_{j=2}^{n}\lambda(j)\log j$$

$$\sum_{j=1}^{n}\mu(j)^2\psi'(\frac{n}{j})=\sum_{j=2}^{n}\mu(j)^2\log j$$

$$\psi'(n)=\sum_{j=2}^{n}l_1(j)(\log j-\psi'(\frac{n}{j}))$$

$$\psi'(n)=\sum_{j=2}^{n}mm_1(j)(-\log j-\psi'(\frac{n}{j}))$$

$$\psi(n)=\sum_{j=2}^{n}\mu(j)(-\log j-\psi'(\frac{n}{j}))$$

$$P'_1(n)=-\sum_{j=2}^{n}\mu(j)^2+\frac{1}{2}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\mu(j)^2\cdot\mu(k)^2-\frac{1}{3}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{2=1}^{\frac{n}{j\cdot k}}\mu(j)^2\cdot\mu(k)^2\cdot\mu(l)^2+\frac{1}{4}...$$

$$PP(n)=v_1(n);\, v_k(n)=\sum_{j=2}^{n}\mu(j)^2(-\frac{1}{k}-v_{k+1}(\frac{n}{j}))$$

$$PP(n)=v_1(n);\, v_k(n)=\sum_{j=2}^{n}\lambda(j)(\frac{1}{k}-v_{k+1}(\frac{n}{j}))$$

$$n=\sum_{j=1}^{n^{\frac{1}{2}}}\sum_{k=0}\frac{-1^k PP_k(\frac{n}{j^2})}{k!}$$

//FIX PP should have sign inverse. So

$$n=\sum_{j=1}^{n^{\frac{1}{2}}}\sum_{k=0}\frac{PP_k(\frac{n}{j^2})}{k!}$$

$$L(n)=\lfloor n^{\frac{1}{2}}\rfloor-\sum_{j=2}^{n}L(\frac{n}{j})$$

$$L(n)=\lfloor n^{\frac{1}{2}}\rfloor-\sum_{j=2}^{n}L(\frac{n}{j})$$

$$L(n)=\sum_{j=2}^{n}s(j)-L(\frac{n}{j})$$

$$L(n)=\sum_{j=2}^{n}s(j)-L(\frac{n}{j})$$

$$L(n)=-1+\sum_{j=1}^{n^{\frac{1}{2}}}M_1'(\frac{n}{j^2})$$

$$L(n)=-1+\sum_{j=1}^{n^{\frac{1}{2}}}(1-\sum_{x=2}^{\frac{n}{j^2}}1-\sum_{y=2}^{\frac{n}{j^2x}}1-\sum_{z=2}^{\frac{n}{j^2xy}}1-...)$$

$$L(n)=-1+\sum_{x=1}^{n^{\frac{1}{2}}}(1-\sum_{y=2}^{\frac{n}{x^2}}1-\sum_{y=2}^{\frac{n}{x^2}}\sum_{z=2}^{\frac{n}{x^2y}}1-\sum_{y=2}^{\frac{n}{x^2}}\sum_{z=2}^{\frac{n}{x^2y}}\sum_{w=2}^{\frac{n}{x^2yz}}1-...)$$

$$L(n)\approx-1+\int_1^{n^{\frac{1}{2}}}(1-\int_1^{\frac{n}{x^2}}dy-\int_1^{\frac{n}{x^2}}\int_1^{\frac{n}{x^2y}}dz-\int_1^{\frac{n}{x^2}}\int_1^{\frac{n}{x^2y}}\int_1^{\frac{n}{x^2yz}}dw-...)\,dx$$

$$L(n)\approx-1+\int_1^{n^{\frac{1}{2}}}(1-\log\frac{n}{j^2})\,dj$$

$$L(n)\approx-1+\int_1^{n^{\frac{1}{2}}}(1-\log n+2\log j)\,dj$$

$$L(n)\approx-1+(1-n^{\frac{1}{2}}+\log n)$$

$$L(n)\approx-n^{\frac{1}{2}}+\log n$$

$$-n^{\frac{1}{2}}+\log n=-1+\int_1^{n^{\frac{1}{2}}}(1-\int_1^{\frac{n}{x^2}}dy-\int_1^{\frac{n}{x^2}}\int_1^{\frac{n}{x^2y}}dz-\int_1^{\frac{n}{x^2}}\int_1^{\frac{n}{x^2y}}\int_1^{\frac{n}{x^2yz}}dw-...)\,dx$$

$$-n^{\frac{1}{2}}+\log n-L(n)=$$

$$-\left(\int_1^{n^{1/2}}\int_1^{\frac{n}{x^2}} dy\,dx-\sum_{x=1}^{n^{1/2}}\sum_{y=2}^{\frac{n}{x^2}} 1\right)$$

$$+\left(\int_1^{n^{1/2}}\int_1^{\frac{n}{x^2}}\int_1^{\frac{n}{x^2 y}} dz\,dy\,dx-\sum_{x=1}^{n^{1/2}}\sum_{y=2}^{\frac{n}{x^2}}\sum_{z=2}^{\frac{n}{x^2 y}} 1\right)$$

$$-\left(\int_1^{n^{1/2}}\int_1^{\frac{n}{x}}\int_1^{\frac{n}{x^{2y}}}\int_1^{\frac{n}{x^{2yz}}} dw\,dz\,dy\,dx-\sum_{x=1}^{n^{1/2}}\sum_{y=2}^{\frac{n}{x^2}}\sum_{z=2}^{\frac{n}{x^{2y}}}\sum_{w=2}^{\frac{n}{x^{2yz}}} 1\right)$$

$$+\ \ldots$$

$$n=\sum_{j=1}^{n^{\frac{1}{2}}} 1+MM\left(\frac{n}{j^2}\right)$$

$$0=-n+1+MM(n)+\sum_{j=2}^{n^{\frac{1}{2}}} 1+MM\left(\frac{n}{j^2}\right)$$

$$-MM(n)=-n+1+\sum_{j=2}^{n^{\frac{1}{2}}} 1+MM\left(\frac{n}{j^2}\right)$$

$$MM(n)=n-1-\sum_{j=2}^{n^{\frac{1}{2}}} 1+MM\left(\frac{n}{j^2}\right)$$

$$MM(n)=n-\lfloor n^{\frac{1}{2}}\rfloor-\sum_{j=2}^{n^{\frac{1}{2}}} MM\left(\frac{n}{j^2}\right)$$

$$MM(n)=\sum_{j=1}^{n^{\frac{1}{2}}}\mu(j)\left\lfloor\frac{n}{j^2}\right\rfloor \quad \text{better}$$

$$L(n)=-1+\sum_{j=1}^{n^{\frac{1}{2}}} M_1{}'\left(\frac{n}{j^2}\right)$$

$$n=\lfloor n^{\frac{1}{2}}\rfloor-\sum_{j=1}^{n} L\left(\frac{n}{j}\right)$$

$$L(n)=-n+\lfloor n^{\frac{1}{2}}\rfloor-\sum_{j=2}^{n} L\left(\frac{n}{j}\right)$$

$$L(n)=\sum_{j=1}^{n}\mu(j)\left(-\frac{n}{j}+\left\lfloor\left(\frac{n}{j}\right)^{\frac{1}{2}}\right\rfloor\right)$$

$$\psi'(n)=\sum_{j=2}^{n}-\lambda(j)\left(MM_1\left(\frac{n}{j}\right)+1\right)\log j$$

$$\psi'(n)=\sum_{j=2}^{n}-\mu(j)^2\left(L\left(\frac{n}{j}\right)+1\right)\log j$$

$$L(n)=\sum_{j=1}^{n}\left(\sum_{k=0}-1^k d_k(j)\right)\left\lfloor\left(\frac{n}{j}\right)^{\frac{1}{2}}\right\rfloor$$

$$L(n)=(\sum_{k=0}\sum_{j=1}^{n}-1^{k}d_{k}(j))\lfloor(\frac{n}{j})^{\frac{1}{2}}\rfloor$$

$$L(n)=\lfloor n^{\frac{1}{2}}\rfloor-\sum_{j=2}^{n}\lfloor(\frac{n}{j})^{\frac{1}{2}}\rfloor+\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\lfloor(\frac{n}{jk})^{\frac{1}{2}}\rfloor-\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{s=2}^{\frac{n}{jk}}\lfloor(\frac{n}{jks})^{\frac{1}{2}}\rfloor-\ldots$$

$$L(n)=\sum_{j=2}^{n}sq(j)-\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}sq(k)+\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{s=2}^{\frac{n}{jk}}sq(s)-\ldots$$

$$v_{k}(n)=\sum_{j=2}^{n}k\log j-v_{k+1}(n/j) \qquad v_{1}(n)=\sum_{j=2}^{n}-\mu(j)\log j$$

$$\sum_{j=2}^{n}-\mu(j)\log j=\sum_{j=2}^{n}\log j-2\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\log k+3\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}}\log l-4\ldots$$

$$v_{k}(n)=\sum_{j=2}^{n}\mu(j)(-k\log j-v_{k+1}(n/j)) \qquad v_{1}(n)=\sum_{j=2}^{n}\log j$$