```
Integrate[ t^(a-1)/(a-1)! (1/u - E^-u/u), {t, 0, x}, {u, 0, x-t}]
```

$$\int_0^x \frac{t^{-1+a}\,(\text{EulerGamma} + \text{Gamma}[0, -t+x] + 2\,\text{Log}[-t+x])}{(-1+a)!}\,dt$$

```
Sum[(-1)^(k+1)/k Binomial[x, k+a], {k, 1, Infinity}] /. x → 13 /. a → 5
```

$$\frac{323171}{280}$$

```
tt[x_, a_] := Sum[Binomial[t-1, a-1] (1/u), {t, 1, x}, {u, 1, x-t}]
tt[13, 5]
```

$$\frac{323171}{280}$$

```
Sum[Binomial[t-1, k+j-2] (1/u), {t, 1, x}, {u, 1, x-t}]
```

$$\sum_{t=1}^{x}\sum_{u=1}^{-t+x}\frac{\text{Binomial}[-1+t, -2+j+k]}{u}$$

```
Sum[(-1)^(k+1)/k x^k Log[1+x]^(a-1), {k, 1, Infinity}]
```

$\text{Log}[1+x]^a$

```
Sum[BernoulliB[k]/k! x Log[1+x]^k, {k, 0, Infinity}]
```

$\text{Log}[1+x]$

```
Sum[BernoulliB[k]/k! x Log[1+x]^(k+a-1), {k, 0, Infinity}]
```

$\text{Log}[1+x]^a$

```
Series[Log[1+x] (1+x), {x, 0, 10}]
```

$$x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{12} - \frac{x^5}{20} + \frac{x^6}{30} - \frac{x^7}{42} + \frac{x^8}{56} - \frac{x^9}{72} + \frac{x^{10}}{90} + O[x]^{11}$$

```
FullSimplify[x/(x+1) + Sum[(-1)^k/(k (k-1)) x^k/(x+1), {k, 2, Infinity}]]
```

$\text{Log}[1+x]$

```
Series[Log[1+x]/(1+x), {x, 0, 10}]
```

$$x - \frac{3x^2}{2} + \frac{11x^3}{6} - \frac{25x^4}{12} + \frac{137x^5}{60} - \frac{49x^6}{20} + \frac{363x^7}{140} - \frac{761x^8}{280} + \frac{7129x^9}{2520} - \frac{7381x^{10}}{2520} + O[x]^{11}$$

```
Table[HarmonicNumber[k], {k, 1, 10}]
```

$$\left\{1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}, \frac{137}{60}, \frac{49}{20}, \frac{363}{140}, \frac{761}{280}, \frac{7129}{2520}, \frac{7381}{2520}\right\}$$

```
Sum[(-1)^(k+1) HarmonicNumber[k] x^k (1+x), {k, 1, Infinity}]
```

$\text{Log}[1+x]$

```
Series[ Log[1+x]/(1+x), {x, 0, 10}]
```

$$x - \frac{3x^2}{2} + \frac{11x^3}{6} - \frac{25x^4}{12} + \frac{137x^5}{60} - \frac{49x^6}{20} + \frac{363x^7}{140} - \frac{761x^8}{280} + \frac{7129x^9}{2520} - \frac{7381x^{10}}{2520} + O[x]^{11}$$

```
Pochhammer[u, 1]/(1!)
```

u

```
bo[t_, u_] := (t+u)!/t!/u!
```

```
FullSimplify[bo[t, u] - bo[t, u - 1]] /. u → 1 /. t → 9
```

9

```
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}]
```

```
dz[10, 1]
```

1

```
FullSimplify@Expand[Sum[u, {u, 1, (x - t)}]] /. x → 15 /. t → 5
```

55

```
Binomial[t - x, 2] /. x → 15 /. t → 5
```

55

```
Sum[Binomial[t - 1, k - 1] Binomial[t - x, 2], {t, 0, x}]
```

```
pl[x_] := Sum[(-1)^(k + 1) HarmonicNumber[k] (Binomial[x, k] + Binomial[x, k + 1]), {k, 0, x}]
```

```
pl[10]
```

$$\frac{7381}{2520}$$

```
HarmonicNumber[10]
```

$$\frac{7381}{2520}$$

```
Series[ Log[1 + x], {x, 0, 10}]
```

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \frac{x^7}{7} - \frac{x^8}{8} + \frac{x^9}{9} - \frac{x^{10}}{10} + O[x]^{11}$$

```
Sum[Binomial[z, k] x^k, {k, 0, Infinity}]
```

$(1 + x)^z$

```
Sum[(-1)^k Binomial[z, k] x^k, {k, 0, Infinity}]
```

$(1 - x)^z$

```
Sum[1 / k x^k, {k, 1, Infinity}]
```

-Log[1 - x]

```
p1[n_, k_] := Sum[1 / k - p1[n / j, k + 1], {j, 2, n}]
p2[n_, k_] := Sum[1 / k + p2[n / j, k + 1], {j, 2, n}]
```

```
D[(1 - x)^z, z] /. z → 0
```

Log[1 - x]

```
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
Clear[D2, p2a]
D2[n_, 0] := UnitStep[n - 1]
D2[n_, k_] := D2[n, k] = Sum[D2[Floor[n / j], k - 1], {j, 2, n}]
Dz[n_, z_] := Sum[bin[z, k] D2[n, k], {k, 0, Log2@n}]
Dzm[n_, z_] := Sum[(-1)^k bin[z, k] D2[n, k], {k, 0, Log2@n}]
dzm[n_, z_] := Dzm[n, z] - Dzm[n - 1, z]
p2a[n_, k_, z_] := p2a[n, k, z] = Sum[dzm[j, z] (1 / k - p2a[Floor[n / j], k + 1, z]), {j, 2, n}]
```

```
p2a[100, 1, 1]
```

$-\frac{6088}{15}$

```
Table[dzm[n, 1], {n, 1, 10}]
```

{1, -1, -1, -1, -1, -1, -1, -1, -1, -1}

```
Expand[- (1 + x) ^3 + 6 (1 + x) ^2 - 12 (1 + x) ^1 + 8 (1 + x) ^0]
```

$1 - 3x + 3x^2 - x^3$

```
Expand[(1 - x) ^4 - (1 + x) ^4 + 8 (1 + x) ^3 - 24 (1 + x) ^2 + 32 (1 + x) ^1 - 16 (1 + x) ^0]
```

0

```
Expand[(1 + x) ^4 - 8 (1 + x) ^3 + 24 (1 + x) ^2 - 32 (1 + x) ^1 + 16 (1 + x) ^0]
```

$1 - 4x + 6x^2 - 4x^3 + x^4$

```
Sum[(-1) ^k 2^ (z - k) Binomial[z, k] (1 + x) ^k, {k, 0, Infinity}]
```

$(1 - x)^z$

```
Expand[(1 + x) ^4 - (1 - x) ^4 + 8 (1 - x) ^3 - 24 (1 - x) ^2 + 32 (1 - x) ^1 - 16 (1 - x) ^0]
```

0

```
Sum[(-1) ^k 2^ (z - k) Binomial[z, k] (1 - x) ^k, {k, 0, Infinity}]
```

$(1 + x)^z$

```
Dza[n_, z_, t_] := Sum[(-1) ^k 2^ (z - k) bin[z, k] Dzm[n, k], {k, 0, t}]
LDza[n_, t_] := Log[2] - Sum[1 / (2^k k) Dzm[n, k], {k, 1, t}]
```

$$\text{LDza2}[n\_, t\_] := \text{Log}[2] + \text{Sum}\left[-\frac{(-1)^{2k}\, 2^{-k}\, \text{Pochhammer}[1, -1 + k]}{k!}\ \text{Dzm}[n, k], \{k, 1, t\}\right]$$

```
Dzma[n_, z_, t_] := Sum[(-1) ^k 2^ (z - k) bin[z, k] Dz[n, k], {k, 0, t}]
```

```
Dza[100, 2.5, 12]
```

873.753

```
Dz[100, 2.5]
```

873.751

```
N@D[Dzm[100, z], z] /. z → 0
```

-405.867

```
Dzma[100, 3.5 + 6 I, 60]
```

9397.36 - 3024.69 i

```
Dzm[100, 3.5 + 6 I]
```

9397.36 - 3024.69 i

```
D[Expand@N@Dzma[100, z, 40], z] /. z → 0
```

-405.867

```
D[Expand@N@Dza[100, z, 40], z] /. z → 0
```

28.5333

```
D[(-1)^k 2^(z-k) bin[z, k], z] /. z → 0
```

$$-\frac{(-1)^{2k}\, 2^{-k}\, \text{Pochhammer}[1, -1+k]}{k!}$$

```
Table[-(2^-k Pochhammer[1, -1 + k])/k!, {k, 1, 5}]
```

$$\left\{-\frac{1}{2}, -\frac{1}{8}, -\frac{1}{24}, -\frac{1}{64}, -\frac{1}{160}\right\}$$

```
N@LDza2[100, 40]
```

28.5333

```
tDza[n_, z_, t_] := Table[(-1)^k 2^(z-k) bin[z, k] Dzm[n, k], {k, 0, t}]
tLDza[n_, t_] := -Table[1/(2^k k) Dzm[n, k], {k, 1, t}]
```

```
D[tDza[100., z, 6], z] /. z → 0
```

$$\left\{\text{Log}[2], 49, -\frac{43}{4}, -\frac{229}{24}, -\frac{191}{64}, \frac{7}{32}, \frac{367}{384}\right\}$$

```
tLDza[100., 6]
```

$$\left\{49, -\frac{43}{4}, -\frac{229}{24}, -\frac{191}{64}, \frac{7}{32}, \frac{367}{384}\right\}$$

```
Log[2] - Sum[1/(2^k k) (1-x)^k, {k, 1, Infinity}]
```

Log[1 + x]

```
Table[-1/(2^k k), {k, 1, 6}]
```

$$\left\{-\frac{1}{2}, -\frac{1}{8}, -\frac{1}{24}, -\frac{1}{64}, -\frac{1}{160}, -\frac{1}{384}\right\}$$

```
Expand[(1/2) Sum[(-1/k x^k), {k, 1, 6}] + Sum[(-1/k x^(k+1)), {k, 1, 6}]]
```

$$-\frac{x}{2} - \frac{5x^2}{4} - \frac{2x^3}{3} - \frac{11x^4}{24} - \frac{7x^5}{20} - \frac{17x^6}{60} - \frac{x^7}{6}$$

```
D[(1-x)^z, {z, 3}] /. z → 0
```

$\text{Log}[1-x]^3$

```
Sum[Binomial[z, k] 2^(z-k) × 3^k x^k, {k, 0, Infinity}]
```

$$2^z \left(1 + \frac{3x}{2}\right)^z$$

```
Sum[ Binomial[z, k] 7^(z-k) (3 + x)^k, {k, 0, Infinity}]
```

$(10 + x)^z$

```
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
Clear[D2, p2a]
D2[n_, 0] := UnitStep[n - 1]
D2[n_, k_] := D2[n, k] = Sum[D2[Floor[n / j], k - 1], {j, 2, n}]
Dz[n_, z_] := Sum[bin[z, k] D2[n, k], {k, 0, Log2@n}]
Daz[n_, a_, z_] := Sum[bin[z, k] a^(z - k) D2[n, k], {k, 0, Log2@n}]
Dabz[n_, a_, b_, z_] := Sum[bin[z, k] a^(z - k) b^k D2[n, k], {k, 0, Log2@n}]
Dabz2[n_, a_, b_, z_] := a^z Sum[bin[z, k] (b / a)^k D2[n, k], {k, 0, Log2@n}]

D[Expand@FullSimplify@Dabz[100, 1, 1, z], z] /. z → 0
```

$$\frac{428}{15}$$

```
Sum[ Binomial[z, k] 3^(z - k) (4 + x)^k, {k, 0, Infinity}]
```

$(7 + x)^z$

```
FullSimplify[((1 - x^3) / (1 - x))^z]
```

$\left(1 + x + x^2\right)^z$

```
Expand@Dabz[100, a, 1, z] /. z → 1.5 /. a → -.01
```

$2.90681 \times 10^{-8} - 5.42807 \times 10^{7}\, i$

```
Dz[100, 1.5]
```

239.138

```
Sum[x^k, {k, 0, 6}]
```

$1 + x + x^2 + x^3 + x^4 + x^5 + x^6$

```
Sum[x^(2 k), {k, 0, 3}]
```

$1 + x^2 + x^4 + x^6$

```
Sum[(1 + x)^(2 k), {k, 0, Infinity}] / Sum[(1 + x)^k, {k, 0, Infinity}]
```

$$\frac{1}{2 + x}$$

```
m[n_, z_] := Pochhammer[z, n] / (n!)
bo[n_, z_] := Sum[m[a, z] m[b, -z], {a, 0, n}, {b, 0, (n - a) / 2}]
Table[bo[k, j], {k, 0, 5}, {j, 0, k}] // Grid
```

```
1
1  2
1  2  4
1  2  4  8
1  2  4  8  16
1  2  4  8  16  32
```