```
RiePrimeCnt[n_] := Sum[PrimePi[n^(1/j)]/j, {j, 1, Log[2, n]}]
LinnikApprox[n_] :=
 Sum[(-1)^(j+1)/j (-1)^j (1-n Sum[(-Log[n])^k/k!, {k, 0, j-1}]), {j, 1, Infinity}]
Table[{N[LinnikApprox[10^n]], N[LogIntegral[10^n] - Log[Log[10^n]] - EulerGamma]},
  {n, 1, 4}] // TableForm

$Aborted

LogIntegral[ 100.] - Log[Log[100.]] - EulerGamma

28.0217

-Gamma[ 0,  -Log[100.]] - Pi I - Log[Log[100]] - EulerGamma

28.0217 + 0. i

ff[ n_, z_, t_] :=
 (Sum[ Binomial[z, k] (-1)^k ( 1 - Gamma[ k, -Log[n]] / Gamma[k]), {k, 0, t}] - 1) / z
ff2[ n_, z_, t_] :=
 Sum[ Binomial[z, k] (-1)^k ( 1 - Gamma[ k, -Log[n]] / Gamma[k]), {k, 0, t}]

N[Limit[ ff[ 100, z, 30], z → 0]]
```

28.0217 + 9.30681 × 10^{-27} i

```
1 - Log[1000.]

-5.90776

N[Gamma[2, -Log[200]]] / 200
```

-4.29832 + 5.26392 × 10^{-16} i

```
ff2[100., 3, 1000]
```

2081.41 - 3.03869 × 10^{-13} i

```
ff2[100., -2, 2000]
```

2.39346 + 1.62995 × 10^{-12} i

```
N[Gamma[3, -Log[100]]]
```

1399.73 - 3.42834 × 10^{-13} i

```
ff3[ n_, z_, s_] := Sum[ Binomial[z, k] (-1)^(k+1) / ((k-1)!)
   Integrate[ t^(k-1) E^(-t), {t, -Log[n], 0}], {k, 0, s}]

N[ff3[ 100, 3, 30]]
```

Integrate::idiv : Integral of $\frac{e^{-t}}{t}$ does not converge on {-Log[100], 0}. ≫

```
$Aborted

ff4[ n_, z_, s_] := Sum[Integrate[
   Binomial[z, k] (-1)^(k+1) / ((k-1)!) t^(k-1) E^(-t), {t, -Log[n], 0}], {k, 0, s}]

N[ff4[ 120, 3, 40]]

2643.2

ff5[ n_, z_] :=
 Integrate[Sum[ Binomial[z, k] (-1)^(k+1) / ((k-1)!) t^(k-1) E^(-t), {k, 0, Infinity}],
  {t, -Log[n], 0}]
```

```
N[ff5[ 10, z]]
```

```
-1. + LaguerreL[-1. z, 2.30259]
```

```
Sum[ Binomial[z, k] (-1) ^ (k + 1) / ((k - 1) !) t^ (k - 1) E^ (-t), {k, 0, Infinity}]
```

e^{-t} z Hypergeometric1F1[1 - z, 2, t]

```
Log[10.]
```

```
2.30259
```

```
-1.` + LaguerreL[-1.` z, Log[10]] /. z → 2
```

```
32.0259
```

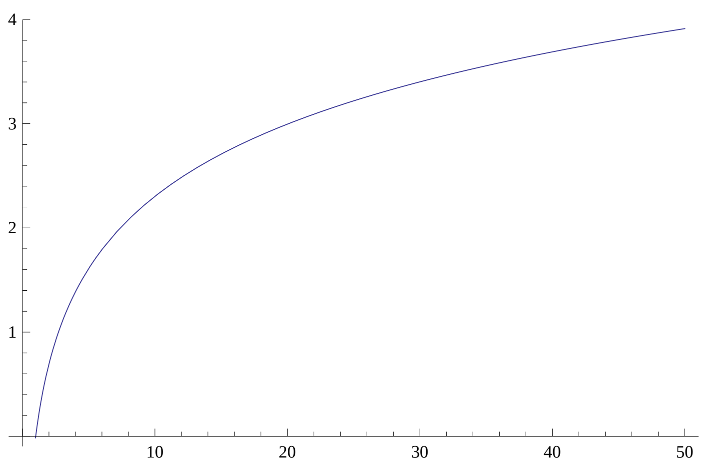
```
N[Limit[ (LaguerreL[-z, Log[100]] - 1) / z, z → 0]]
```

```
28.0217
```

```
-1. + LaguerreL[-1, Log[10]]
```

```
9.
```

```
Plot[ Re[-1 + LaguerreL[1, -Log[n]]], {n, 0, 50}]
```




```
N[ff2[ 1000, -2 + I, 1000]]
```

```
27.1002 + 19.121 i
```

```
N[LaguerreL[ 2 + I, Log[1000]]]
```

```
27.1002 - 19.121 i
```

```
N[Limit[ (LaguerreL[-z, Log[20]] - 1) / z, z → 0]]
```

```
8.2309
```

```
LogIntegral[ 20.] - Log[Log[20.]] - EulerGamma
```

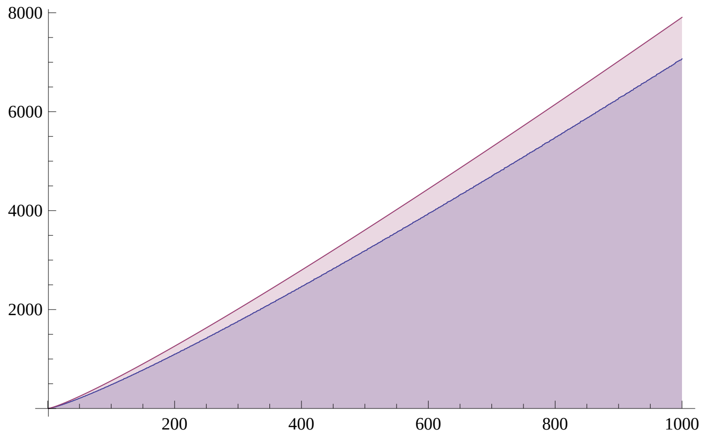
```
8.2309
```

```
ExpIntegralEi[ZetaZero[1] Log[1000.]]
```

```
-0.0879017 + 3.45317 i
```

```
Limit[ N[(LaguerreL[ -z, ZetaZero[1] Log[1000]] - 1) / z], z → 0] +
 Log[ZetaZero[1] Log[1000]] + EulerGamma

-0.0879017 + 3.45317 i

N[E^EulerGamma]

1.78107

N[Limit[( LaguerreL[-z, Log[100]] - 1) / z, z → 0]]

28.0217

N[LogIntegral[100] - Log[Log[100]] - EulerGamma]

28.0217


Sum[ N[D[ LaguerreL[ -z, Log[10]], {z, k}] /. z → 0] / k!, {k, 0, 5}]

9.99914

Sum[ N[D[ LaguerreL[ -z, Log[10]], {z, k}] /. z → 0] / k!, {k, 0, 6}]

9.99996

N[D[( LaguerreL[-z, Log[100]] - 1) / z, z]]

28.0217

N[-Pi I - Gamma[ 0, -Log[100]]]

30.1261 + 0. i

LogIntegral[100.]

30.1261

 Limit[(LaguerreL[-ee, Log[100.]] - 1) / ee, ee → 0]

28.0217

N[Limit[( LaguerreL[-z, Log[100]] - 1) / z, z → 0]]

28.0217

N[D[LaguerreL[-z, Log[100]], z]] /. z → 0

28.0217

D[ LaguerreL[-z, Log[100.]], {z, 2}] /. z → 0

80.5038

P2[n_, 0] := 1
P2[n_, k_] :=
 P2[n, k] = Sum[FullSimplify[MangoldtLambda[j] / Log[j]] P2[n / j, k - 1], {j, 2, Floor[n]}]
D2[n_, k_] := D2[n, k] = Sum[D2[Floor[n / j], k - 1], {j, 2, Floor[n]}]; D2[n_, 0] := 1
D1[n_, z_] := Sum[Binomial[z, k] D2[n, k], {k, 0, Log[2, n]}]
```

```
DiscretePlot[{ D1[n, 2], LaguerreL[-2, Log[n]]}, {n, 1, 1000}]
```




```
Integrate[ 1 / Log[x], {x, 1.1, n}, PrincipalValue → True]
```

```
ConditionalExpression[1.67577 + LogIntegral[n], Im[n] ≠ 0 || Re[n] ≥ 1.]
```

```
TestSum[ n_, z_, t_] :=
 Sum[ N[Binomial[z, k] (-1) ^k ( 1 - Gamma[ k, -Log[n]] / Gamma[k])], {k, 0, t}]
Grid[ Table[ { Re[TestSum[n, k, 23]], N[LaguerreL[-k, Log[n]]]},
  {n, 10, 100, 10}, {k, -5, 5}]]
```

{0.966672, 0.966672}	{0.727895, 0.727895}	{0.0104133, 0.0104133}	{-0.954221, -0.954221}	{-1.30259, -1.30259}	{1., 1.}	{10., 10.}	{33.0259, 33.0259}	{82.5612, 82.5612}	{178.953, 178.953}	{354.26, 354.26}

{0.853716, 0.853716}	{1.37285, 1.37285}	{0.993598, 0.993598}	{-0.504259, -0.504259}	{-1.99573, -1.99573}	{1., 1.}	{20., 20.}	{79.9146, 79.9146}	{229.573, 229.573}	{558.593, 558.593}	{1223.71, 1223.71}
{0.345442, 0.345447}	{1.4452, 1.4452}	{1.59103, 1.59103}	{-0.018323, -0.0183229}	{-2.4012, -2.4012}	{1., 1.}	{30., 30.}	{132.036, 132.036}	{407.594, 407.594}	{1053.4, 1053.4}	{2433.46, 2433.46}
{-0.182647, -0.182608}	{1.31842, 1.31842}	{1.97883, 1.97883}	{0.426157, 0.426157}	{-2.68888, -2.68888}	{1., 1.}	{40., 40.}	{187.555, 187.555}	{607.267, 607.267}	{1633.79, 1633.79}	{3910.39, 3910.39}
{-0.66439, -0.664191}	{1.10954, 1.10957}	{2.2416, 2.2416}	{0.827916, 0.827916}	{-2.91202, -2.91202}	{1., 1.}	{50., 50.}	{245.601, 245.601}	{823.8, 823.8}	{2283.51, 2283.51}	{5611.57, 5611.57}
{-1.09019, -1.08949}	{0.865209, 0.86531}	{2.42308, 2.42309}	{1.19314, 1.19314}	{-3.09434, -3.09434}	{1., 1.}	{60., 60.}	{305.661, 305.661}	{1054.23, 1054.23}	{2992.07, 2992.07}	{7508.1, 7508.1}
{-1.46377, -1.46181}	{0.606799, 0.607082}	{2.54836, 2.5484}	{1.52786, 1.52787}	{-3.2485, -3.2485}	{1., 1.}	{70., 70.}	{367.395, 367.395}	{1296.53, 1296.53}	{3752.05, 3752.05}	{9578.84, 9578.84}
{-1.79198, -1.78731}	{0.344902, 0.345574}	{2.63302, 2.6331}	{1.83702, 1.83703}	{-3.38203, -3.38203}	{1., 1.}	{80., 80.}	{430.562, 430.562}	{1549.21, 1549.21}	{4557.87, 4557.87}	{11807.5, 11807.5}
{-2.08206, -2.0722}	{0.0849608, 0.0863776}	{2.68727, 2.68743}	{2.12451, 2.12452}	{-3.49981, -3.49981}	{1., 1.}	{90., 90.}	{494.983, 494.983}	{1811.14, 1811.14}	{5405.17, 5405.17}	{14181.2, 14181.2}

```
{-2.3\  {-0.1\  {2.71\  {2.39\  {-3.6\   {1.,   {100.,  {560.\  {2081\  {6290\  {1668\
  40\     70\    814,    343,     05\     1.}    100.}   517,    .41,    .43,    9.3,
  96,     25\   2.71\   2.39\     17,                    560.\   2081\   6290\   1668\
-2.3\     9,     845}    346}   -3.6\                     517}    .41}    .43}    9.3}
  22\   -0.1\                     05\
  02}     67\                     17}
          53\
          6}
```

```
Table[{N[Sum[(-1)^(k+1) / k ((-1)^k (1 - Gamma[k, -Log[n]] / Gamma[k])), {k, 1, 30}]],
   N[ Limit[ (LaguerreL[-z, Log[n]] - 1) / z, z → 0]],
   N[LogIntegral[n] - Log[Log[n]] - EulerGamma]}, {n, 100, 600, 100}] // TableForm
```

$28.0217 - 2.09386\times10^{-14}\,i$	28.0217	28.0217
$47.9476 - 4.32162\times10^{-14}\,i$	47.9476	47.9476
$66.0153 - 1.89547\times10^{-13}\,i$	66.0153	66.0153
$83.0503 - 4.70318\times10^{-13}\,i$	83.0503	83.0503
$99.3898 - 1.32718\times10^{-12}\,i$	99.3898	99.3898
$115.213 - 2.10999\times10^{-12}\,i$	115.213	115.213

```
N[ Limit[ (LaguerreL[-z, Log[600]] - 1) / z, z → 0]]
```

115.213

```
N[LogIntegral[600] - Log[Log[600]] - EulerGamma]
```

115.213

```
Table[{N[Sum[(-1)^(k+1) / k ((-1)^k (1 - Gamma[k, -Log[n]] / Gamma[k])), {k, 1, 30}]],
   N[Limit[(LaguerreL[-z, Log[n]] - 1) / z, z → 0]],
   N[LogIntegral[n] - Log[Log[n]] - EulerGamma]}, {n, 100, 600, 100}] // TableForm
```

$28.0217 - 2.09386\times10^{-14}\,i$	28.0217	28.0217
$47.9476 - 4.32162\times10^{-14}\,i$	47.9476	47.9476
$66.0153 - 1.89547\times10^{-13}\,i$	66.0153	66.0153
$83.0503 - 4.70318\times10^{-13}\,i$	83.0503	83.0503
$99.3898 - 1.32718\times10^{-12}\,i$	99.3898	99.3898
$115.213 - 2.10999\times10^{-12}\,i$	115.213	115.213

```
Limit[ Sum[ c^j / j, {j, 2, Log[c, 100]}], c → 1]
```

$$\mathrm{Limit}\left[-c - 100\,c\,\mathrm{LerchPhi}\left[c, 1, 1 + \frac{\mathrm{Log}[100]}{\mathrm{Log}[c]}\right] - \mathrm{Log}[1-c],\ c \to 1\right]$$

```
Binomial[z, 0]
```

1

```
Binomial[z, 1]
```

z

```
Binomial[z, 2]
```

$$\frac{1}{2}(-1+z)\,z$$

```
Binomial[z, 3]
```

$$\frac{1}{6}(-2+z)(-1+z)z$$

```
Binomial[z, 4]
```

$$\frac{1}{24}(-3+z)(-2+z)(-1+z)z$$

```
Sum[ 1, {j, 0, Floor[x - y]}]
```

1 + Floor[x - y]

```
Sum[ 1, {j, 0, Floor[x - y]}, {k, 0, Floor[ x / (j + y) - y]}]
```

$$\sum_{j=0}^{\text{Floor}[x-y]}\ \sum_{k=0}^{\text{Floor}\left[-y+\frac{x}{j+y}\right]} 1$$

```
Dd[x_, 0, y_] := 1
Dd[x_, k_, y_] := Sum[Dd[x / (j + y), k - 1, y], {j, 0, Floor[x - y]}]
Cc[x_, k_, y_] := y^-k Dd[x y^k, k, y + 1]
CSum[ x_, y_] := Sum[ (-1) ^ (k + 1) / k Cc[ x, k, y], {k, 1, 10}]
```

```
Cd[x_, k_, y_] := (1 / y) Sum[ Cd[ y x / (j + y + 1), k - 1, y], {j, 0, Floor[y x - y - 1]}];
Cd[ x_, 0, y_] := 1
Cz[x_, z_, k_, y_] := (z - k + 1) / k (1 / y)
  Sum[ Cz[ y x / (j + y + 1), z, k - 1, y], {j, 0, Floor[y x - y - 1]}]; Cz[ x_, z_, 0, y_] := 1
```

```
Cc[ 100, 2, 3]
```

$$\frac{995}{3}$$

```
Cd[ 100, 4, 1]
```

184

```
Cz[ 100, 1, 1, 1]
```

99

```
pp[ x_, k_, y_] := (1 / y) Sum[ 1 / k - pp[ x y / ( j + y), k + 1, y], {j, 1, Floor[ (x - 1) y]}]
```

```
pp[100, 1, 1]
```

$$\frac{428}{15}$$

```
D[pp[10, 1, y], y]
```

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

General::stop : Further output of $RecursionLimit::reclim will be suppressed during this calculation. ≫

$IterationLimit::itlim : Iteration limit of 4096 exceeded. ≫

$IterationLimit::itlim : Iteration limit of 4096 exceeded. ≫

$IterationLimit::itlim : Iteration limit of 4096 exceeded. ≫

General::stop : Further output of $IterationLimit::itlim will be suppressed during this calculation. ≫

```
Expand[ (x y / (j + y) - 1 ) y]
```

$$-y + \frac{x\,y^2}{j+y}$$

```
CSum[ 100, 13 / 10]
```

$$\frac{454\,930\,622\,410}{17\,130\,345\,141}$$

```
Expand[((x y) / (j + y) - 1 ) y]
```

$$-y + \frac{x\,y^2}{j+y}$$

```
Limit[(1 / y) Sum[ 1, {j, 1, Floor[ x y - y]}], y → Infinity]
```

$$\text{Limit}\left[\frac{\text{Floor}[-y + x\,y]}{y},\ y \to \infty\right]$$

```
D[ 1 / y^2 Ccc, y]
```

$$\textbf{Integrate}\left[-\frac{2\,\text{Ccc}}{y^3},\ \{y,\ 11/10,\ 12/10\}\right]$$

$$-\frac{575\,\text{Ccc}}{4356}$$

```
1 / ((12 / 10)^2) - 1 / ((11 / 10)^2)
```

$$-\frac{575}{4356}$$

$$\textbf{Integrate}\left[-\frac{2\,\text{Ccc}}{y^3},\ \{y,\ \text{bbb},\ \text{Infinity}\}\right]$$

$$\text{ConditionalExpression}\left[-\frac{\text{Ccc}}{\text{bbb}^2},\ \text{Im}[\text{bbb}] \neq 0 \,||\, \text{bbb} > 0\right]$$

```
Czz[x_, z_, k_, y_] :=
 (z - k + 1) / (y k) Sum[ 1 + Czz[ x y / (j + y), z, k + 1, y], {j, 1, Floor[ x y - y]}]
```

```
1 + Czz[ 100, -3, 1, 1]
```

47

```
d1[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
D1[n_, z_] := Sum[d1[j, z], {j, 1, n}]
```

```
D1[100, -3]
```

47

```
F[n_, j_, k_, z_] :=
 If[n < j, 0, (z - k + 1) / k (1 + F[n / j, 2, k + 1, z]) + F[n, j + 1, k, z]]
D1Alt[n_, z_] := 1 + F[n, 2, 1, z]
```

```
1 + F[100, 2, 1, 3]
```

```
1471
```

```
d1[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
D1[n_, z_] := Sum[d1[j, z], {j, 1, n}]
F[n_, j_, k_, z_] := If[n < j, 0, (z - k + 1) / k (1 + F[n / j, 2, k + 1, z]) + F[n, j + 1, k, z]]
D1Alt[n_, z_] := 1 + F[n, 2, 1, z]
Grid[Table[{D1[a = 100, s + t I], D1Alt[a, s + t I]}, {s, -1.3, 4, .7}, {t, -1.3, 4, .7}]]
```

{10.4793 + 28.7121 ⅈ, 10.4793 + 28.7121 ⅈ}	{5.72468 + 11.2587 ⅈ, 5.72468 + 11.2587 ⅈ}	{6.03456 − 1.77709 ⅈ, 6.03456 − 1.77709 ⅈ}	{5.94691 − 15.6189 ⅈ, 5.94691 − 15.6189 ⅈ}	{15.2681 − 34.9044 ⅈ, 15.2681 − 34.9044 ⅈ}	{58.5435 − 59.5846 ⅈ, 58.5435 − 59.5846 ⅈ}	{173.704 − 80.8227 ⅈ, 173.704 − 80.8227 ⅈ}	{409.891 − 76.8935 ⅈ, 409.891 − 76.8935 ⅈ}
{−21.9794 + 33.2704 ⅈ, −21.9794 + 33.2704 ⅈ}	{−9.29577 + 6.6042 ⅈ, −9.29577 + 6.6042 ⅈ}	{−3.93641 − 0.702512 ⅈ, −3.93641 − 0.702512 ⅈ}	{−12.975 − 11.3196 ⅈ, −12.975 − 11.3196 ⅈ}	{−23.7041 − 47.2133 ⅈ, −23.7041 − 47.2133 ⅈ}	{−4.16474 − 124.722 ⅈ, −4.16474 − 124.722 ⅈ}	{95.5007 − 249.632 ⅈ, 95.5007 − 249.632 ⅈ}	{340.872 − 412.252 ⅈ, 340.872 − 412.252 ⅈ}
{−70.5899 − 1.50386 ⅈ, −70.5899 − 1.50386 ⅈ}	{−13.7213 − 18.1133 ⅈ, −13.7213 − 18.1133 ⅈ}	{3.81025 + 3.79964 ⅈ, 3.81025 + 3.79964 ⅈ}	{−26.9749 + 19.1944 ⅈ, −26.9749 + 19.1944 ⅈ}	{−89.9388 − 16.1483 ⅈ, −89.9388 − 16.1483 ⅈ}	{−144.356 − 139.879 ⅈ, −144.356 − 139.879 ⅈ}	{−126.266 − 377.33 ⅈ, −126.266 − 377.33 ⅈ}	{49.351 − 735.771 ⅈ, 49.351 − 735.771 ⅈ}
{−109.692 − 116.693 ⅈ, −109.692 − 116.693 ⅈ}	{25.3505 − 82.1743 ⅈ, 25.3505 − 82.1743 ⅈ}	{64.0826 + 14.9568 ⅈ, 64.0826 + 14.9568 ⅈ}	{−4.67506 + 101.541 ⅈ, −4.67506 + 101.541 ⅈ}	{−160.825 + 105.357 ⅈ, −160.825 + 105.357 ⅈ}	{−353.522 − 38.304 ⅈ, −353.522 − 38.304 ⅈ}	{−502.525 − 380.071 ⅈ, −502.525 − 380.071 ⅈ}	{−500.378 − 949.919 ⅈ, −500.378 − 949.919 ⅈ}
{−89.6457 − 364.055 ⅈ, −89.6457 − 364.055 ⅈ}	{165.919 − 209.786 ⅈ, 165.919 − 209.786 ⅈ}	{237.081 + 36.8175 ⅈ, 237.081 + 36.8175 ⅈ}	{110.164 + 267.906 ⅈ, 110.164 + 267.906 ⅈ}	{−190.242 + 376.688 ⅈ, −190.242 + 376.687 ⅈ}	{−601.821 + 264.819 ⅈ, −601.821 + 264.819 ⅈ}	{−1025.88 − 150.194 ⅈ, −1025.88 − 150.194 ⅈ}	{−1329.51 − 927.604 ⅈ, −1329.51 − 927.604 ⅈ}

{69.3293 - 807.552 i, 69.3293 - 807.552 i}	{497.243 - 430.789 i, 497.243 - 430.789 i}	{614.555 + 74.3692 i, 614.555 + 74.3692 i}	{404.806 + 557.989 i, 404.806 + 557.989 i}	{-102.401 + 871.308 i, -102.401 + 871.308 i}	{-831.921 + 874.96 i, -831.921 + 874.96 i}	{-1664.43 + 447.187 i, -1664.43 + 447.187 i}	{-2438.58 - 507.936 i, -2438.58 - 507.936 i}
{484.136 - 1524.88 i, 484.136 - 1524.88 i}	{1146.82 - 781.362 i, 1146.82 - 781.362 i}	{1326.79 + 133.656 i, 1326.79 + 133.656 i}	{1004.52 + 1019.94 i, 1004.52 + 1019.94 i}	{215.141 + 1678.53 i, 215.141 + 1678.53 i}	{-952.011 + 1920.83 i, -952.011 + 1920.83 i}	{-2354.8 + 1577.59 i, -2354.8 + 1577.59 i}	{-3800.46 + 508.005 i, -3800.46 + 508.005 i}
{1316.61 - 2608.99 i, 1316.61 - 2608.99 i}	{2288.19 - 1304.73 i, 2288.19 - 1304.73 i}	{2550.42 + 221.895 i, 2550.42 + 221.895 i}	{2080.38 + 1711.29 i, 2080.38 + 1711.29 i}	{919.284 + 2905.28 i, 919.284 + 2905.28 i}	{-828. + 3556.97 i, -828. + 3556.97 i}	{-2994.35 + 3440.6 i, -2994.35 + 3440.6 i}	{-5352.56 + 2361.41 i, -5352.56 + 2361.41 i}

```
zz1[n_, k_] := (-1)^k (1 - Gamma[k, -Log[n]] / Gamma[k])
zz2[n_, k_] := Integrate[(-1)^(k+1) / ((k-1)!) t^(k-1) E^(-t), {t, -Log[n], 0}]
zz2a[n_, k_] := Integrate[t^(k-1) E^(-t), {t, -Log[n], 0}]
```

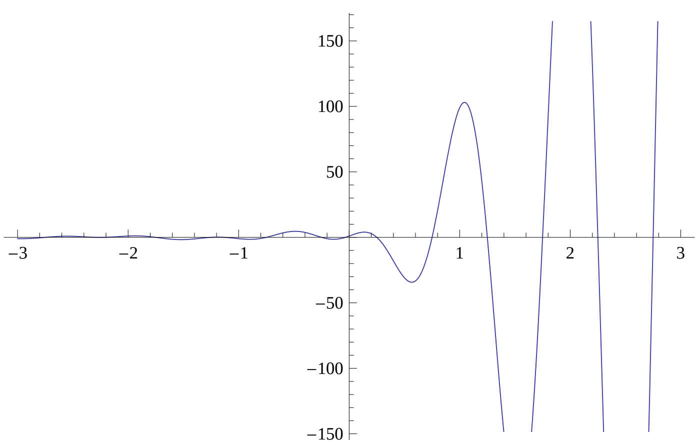
```
zz1[100, 0.01]
```

1.30358 + 0.0820144 i

```
zz2[100, 0.01]
```

1.30358 + 0.0820144 i

```
Plot[Re[zz1[100, z]], {z, -3, 3}]
```




```
Plot[Re[zz2[100, z]], {z, -3, 3}]
```

$Aborted

```
ff4a[n_, z_] :=
 Integrate[Sum[Binomial[z, k] (-1)^(k+1) / ((k-1)!) t^(k-1) E^(-t), {k, 0, Infinity}],
  {t, -Log[n], 0}]
```

```
ff4a[100, z]
```

```
-1 + LaguerreL[-z, Log[100]]
```

```
TestSum[n_, z_, t_] :=
 Sum[N[Binomial[z, k] (-1)^k (1 - Gamma[k, -Log[n]] / Gamma[k])], {k, 0, t}]
Grid[Table[{Re[TestSum[n, k, 23]], N[LaguerreL[-k, Log[n]]]}, {n, 10, 100, 10}, {k, -5, 5}]]
```

{0.966672, 0.966672}	{0.727895, 0.727895}	{0.0104133, 0.0104133}	{-0.954221, -0.954221}	{-1.30259, -1.30259}	{1., 1.}	{10., 10.}	{33.0259, 33.0259}	{82.5612, 82.5612}	{178.953, 178.953}	{354.26, 354.26}
{0.853716, 0.853716}	{1.37285, 1.37285}	{0.993598, 0.993598}	{-0.504259, -0.504259}	{-1.99573, -1.99573}	{1., 1.}	{20., 20.}	{79.9146, 79.9146}	{229.573, 229.573}	{558.593, 558.593}	{1223.71, 1223.71}
{0.345442, 0.345447}	{1.4452, 1.4452}	{1.59103, 1.59103}	{-0.018323, -0.0183229}	{-2.4012, -2.4012}	{1., 1.}	{30., 30.}	{132.036, 132.036}	{407.594, 407.594}	{1053.4, 1053.4}	{2433.46, 2433.46}
{-0.182647, -0.182608}	{1.31842, 1.31842}	{1.97883, 1.97883}	{0.426157, 0.426157}	{-2.68888, -2.68888}	{1., 1.}	{40., 40.}	{187.555, 187.555}	{607.267, 607.267}	{1633.79, 1633.79}	{3910.39, 3910.39}
{-0.66439, -0.664191}	{1.10954, 1.10957}	{2.2416, 2.2416}	{0.827916, 0.827916}	{-2.91202, -2.91202}	{1., 1.}	{50., 50.}	{245.601, 245.601}	{823.8, 823.8}	{2283.51, 2283.51}	{5611.57, 5611.57}

{-1.09019, -1.08949}	{0.865209, 0.86531}	{2.42308, 2.42309}	{1.19314, 1.19314}	{-3.09434, -3.09434}	{1., 1.}	{60., 60.}	{305.661, 305.661}	{1054.23, 1054.23}	{2992.07, 2992.07}	{7508.1, 7508.1}
{-1.46377, -1.46181}	{0.606799, 0.607082}	{2.54836, 2.5484}	{1.52786, 1.52787}	{-3.2485, -3.2485}	{1., 1.}	{70., 70.}	{367.395, 367.395}	{1296.53, 1296.53}	{3752.05, 3752.05}	{9578.84, 9578.84}
{-1.79198, -1.78731}	{0.344902, 0.345574}	{2.63302, 2.6331}	{1.83702, 1.83703}	{-3.38203, -3.38203}	{1., 1.}	{80., 80.}	{430.562, 430.562}	{1549.21, 1549.21}	{4557.87, 4557.87}	{11807.5, 11807.5}
{-2.08206, -2.0722}	{0.0849608, 0.0863776}	{2.68727, 2.68743}	{2.12451, 2.12452}	{-3.49981, -3.49981}	{1., 1.}	{90., 90.}	{494.983, 494.983}	{1811.14, 1811.14}	{5405.17, 5405.17}	{14181.2, 14181.2}
{-2.34096, -2.32202}	{-0.170259, -0.167536}	{2.71814, 2.71845}	{2.39343, 2.39346}	{-3.60517, -3.60517}	{1., 1.}	{100., 100.}	{560.517, 560.517}	{2081.41, 2081.41}	{6290.43, 6290.43}	{16689.3, 16689.3}

```
TestSum[n_, z_, t_] :=
 Sum[N[Binomial[z, k] (-1)^k (1 - Gamma[k, -Log[n]] / Gamma[k])], {k, 0, t}]
```

```
TestSum[ 100, 2, 30]
```

$560.517 - 4.41506 \times 10^{-14}\, i$

```
zz1[n_, k_] := (-1)^k ( 1 - Gamma[ k, -Log[n]] / Gamma[k])
zz2[n_, k_] := Integrate[(-1)^(k+1) / ((k-1)!) t^(k-1) E^(-t), {t, -Log[n], 0}]
```

```
N[zz2[100, 2]]
```

361.517

```
aff2[ n_, z_] :=
 Sum[ Binomial[z, k] (-1)^k ( 1 - Gamma[ k, -Log[n]] / Gamma[k]), {k, 0, Infinity}]
```

```
aff2[100, 2, 40]
```

200 - Gamma[2, -Log[100]]

```
Limit[ Sum[(j + 1) c^j / j , {j, 1, Log[c, n]}], c → 1]
```

$$\text{Limit}\left[\frac{1}{-1+c}\left(-c + c\,n + c\,n\,\text{LerchPhi}\left[c, 1, 1 + \frac{\text{Log}[n]}{\text{Log}[c]}\right] - c^2\,n\,\text{LerchPhi}\left[c, 1, 1 + \frac{\text{Log}[n]}{\text{Log}[c]}\right] + \text{Log}[1-c] - c\,\text{Log}[1-c]\right), c \to 1\right]$$

```
Limit[ Sum[(j + 1) (c - 1)^2 c^j , {j, 1, Log[c, n]}], c → 1]
```

$1 - n + n\,\text{Log}[n]$

```
Limit[ Sum[(j + 1) (j + 2) (c - 1)^3 c^j , {j, 1, Log[c, n]}], c → 1]
```

$-2 + 2\,n - 2\,n\,\text{Log}[n] + n\,\text{Log}[n]^2$

```
N[Limit[ Sum[(j + 1) (j + 2) (j + 3) (c - 1)^4 c^j , {j, 1, Log[c, n]}], c → 1] /. n → 100]
```

5573.28

```
Expand[LaguerreL[4, Log[n]]]
```

$$1 - 4\,\text{Log}[n] + 3\,\text{Log}[n]^2 - \frac{2\,\text{Log}[n]^3}{3} + \frac{\text{Log}[n]^4}{24}$$

```
Gamma[ 4, -Log[100.]]
```

$-5567.28 + 2.04539 \times 10^{-12}\,i$

```
Limit[ Sum[j^z (c - 1)^(z + 1) c^j , {j, 1, Log[c, n]}], c → 1] /. {z → 4}
```

$24\,(-1 + n) - 24\,n\,\text{Log}[n] + 12\,n\,\text{Log}[n]^2 - 4\,n\,\text{Log}[n]^3 + n\,\text{Log}[n]^4$

```
N[Gamma[ 4, -Log[n]] /. n → 100]
```

$-5567.28 + 2.04539 \times 10^{-12}\,i$

```
Limit[ Sum[ c^-j (c - 1) , {j, 1, Log[c, n]}], c → 1]
```

$$\frac{-1 + n}{n}$$

```
Limit[ Sum[ c^j (c - 1) , {j, 1, Log[c, n]}], c → 1]
```

$-1 + n$

```
Limit[ Sum[ c^j / j (c - 1) , {j, 1, Log[c, n]}], c → 1]
```

$$\text{Limit}\left[c\,n\,\text{LerchPhi}\left[c, 1, 1 + \frac{\text{Log}[n]}{\text{Log}[c]}\right] - c^2\,n\,\text{LerchPhi}\left[c, 1, 1 + \frac{\text{Log}[n]}{\text{Log}[c]}\right] + \text{Log}[1-c] - c\,\text{Log}[1-c], c \to 1\right]$$