```
Limit[Sum[(a^k - 1) / k, {k, 1, Log[a, 100]}], {a → 1}]

{Limit[-HarmonicNumber[Log[100]/Log[a]] - 100 a LerchPhi[a, 1, 1 + Log[100]/Log[a]] - Log[1 - a], a → 1]}

{Limit[-HarmonicNumber[Log[100]/Log[a]] - Log[1 - a], a → 1]}

{-EulerGamma - ⅈ π - Log[Log[100]]}

Limit[Sum[(a^k) / k, {k, 1, Log[a, 100]}], {a → 1}]

{Limit[-100 a LerchPhi[a, 1, 1 + Log[100]/Log[a]] - Log[1 - a], a → 1]}

Limit[Sum[(a^k + 1) / k, {k, 1, Log[a, 100]}], {a → 1}]

{Limit[HarmonicNumber[Log[100]/Log[a]] - 100 a LerchPhi[a, 1, 1 + Log[100]/Log[a]] - Log[1 - a], a → 1]}

{Limit[-HarmonicNumber[100/Log[a]] - Log[1 - a], a → 1]}

{-EulerGamma - ⅈ π - Log[100]}

{Limit[HarmonicNumber[Log[100]/Log[a]] + Log[1 - a], a → 1]}

{EulerGamma + ⅈ π + Log[Log[100]]}

Log[0]

-∞

N[Integrate[1 / x, {x, 1, 1 / 2}]]

-0.693147

-Log[1 - 2]

-ⅈ π

-Log[1 - 3 / 4]

Log[4]

fc[a_] := -HarmonicNumber[Log[100]/Log[a]] - Log[1 - a]

N[fc[1 - 1 / 1 000 000]]

6.08307



ConditionalExpression[Gamma[s] PolyLog[s, 1], Re[s] > 1]

ts[n_, s_] := Integrate[x^(s - 1) / (E^x - 1), {x, 0, -Log[n]}] / Gamma[s]

N[ts[100, 0]]
```

```
{Limit[PolyGamma[Log[100]/Log[a]] + Log[1 - a], a -> 1]}
```

$\{i\pi + \mathrm{Log}[\mathrm{Log}[100]]\}$

```
Limit[Sum[(a^k) / k, {k, 1, Log[a, 100]}], {a -> 1}]
```

$\left\{\mathrm{Limit}\left[-100\, a\, \mathrm{LerchPhi}\left[a, 1, 1 + \frac{\mathrm{Log}[100]}{\mathrm{Log}[a]}\right] - \mathrm{Log}[1-a],\ a \to 1\right]\right\}$

```
Limit[Sum[ (a^k) / k, {k, 1, Log[a, 100]}], {a -> 1}]
```

$\left\{\mathrm{Limit}\left[-100\, a\, \mathrm{LerchPhi}\left[a, 1, 1 + \frac{\mathrm{Log}[100]}{\mathrm{Log}[a]}\right] - \mathrm{Log}[1-a],\ a \to 1\right]\right\}$

```
Limit[Sum[ (a^k) / k, {k, Log[a, 100], Infinity}], {a -> 7}]
```

$\left\{100\ \mathrm{HurwitzLerchPhi}\left[7, 1, \frac{\mathrm{Log}[100]}{\mathrm{Log}[7]}\right]\right\}$

```
Limit[Sum[ (a^k) / k, {k, 1, Log[a, 100]}], {a -> 7}]
```

$\left\{-i\pi - 700\ \mathrm{LerchPhi}\left[7, 1, \frac{\mathrm{Log}[700]}{\mathrm{Log}[7]}\right] - \mathrm{Log}[6]\right\}$

```
Limit[Sum[ (a^k) / k, {k, 1, Infinity}], {a -> b}]
```

$\{-\mathrm{Log}[1-b]\}$

```
Log[1 - (101 / 100)]
```

$i\pi - \mathrm{Log}[100]$

```
Log[-99]
```

$i\pi + \mathrm{Log}[99]$

```
E^(Pi I + Log[99])
```

-99

```
Limit[Sum[(a^k) / k, {k, Log[a, 100], Infinity}], {a -> 1}]
```

$\left\{\mathrm{Limit}\left[100\ \mathrm{HurwitzLerchPhi}\left[a, 1, \frac{\mathrm{Log}[100]}{\mathrm{Log}[a]}\right],\ a \to 1\right]\right\}$

```
vv[n_, a_] := Sum[(a^k) / k, {k, Log[a, n], Infinity}]
vv[100, 1.1]
```

Sum::div : Sum does not converge. ≫

$$\sum_{k=48.3177}^{\infty} \frac{1.1^k}{k}$$

Integrate::idiv : Integral of $\text{Log}[t]^{-1+a}$ does not converge on $\{1, \infty\}$. ≫

```
Integrate[Log[t^-1]^(a-1), {t, 0, 1}]
ConditionalExpression[Gamma[a], Re[a] > 0]
Integrate[Log[t^-1]^(z-1), {t, 0, 1/2}]
Gamma[z, Log[2]]
Integrate[Log[t^-1]^(z-1), {t, 0, 1/3}]
Gamma[z, Log[3]]
Integrate[Log[t^-1]^(a-1), {t, 1/(n^(1-s)), 1}]
```

$\text{ConditionalExpression}\left[\text{Gamma}[a] - \text{Gamma}\left[a, -\text{Log}\left[n^{-1+s}\right]\right], \text{Re}[a] > 0\right]$

```
Integrate[Log[t^-1]^(z-1), {t, 1/3, 1}]
ConditionalExpression[Gamma[z] - Gamma[z, Log[3]], Re[z] > 0]
N[Gamma[z] - Gamma[z, Log[3]] /. z → 2]
0.300463
N[Gamma[z, 0, Log[3]] /. z → 2]
0.300463
```

$\text{Gamma}[a] - \text{Gamma}\left[a, -\text{Log}\left[n^{-1+s}\right]\right] \text{ /. } \{a \to 2, n \to 100, s \to 2\}$

```
1 - Gamma[2, -Log[100]]
Integrate[ t^(s-1) E^-t, {t, 0, Infinity}]
ConditionalExpression[Gamma[s], Re[s] > 0]
Integrate[ t^(s-1) E^(-n t), {t, 0, Infinity}]
```

$\text{ConditionalExpression}\left[n^{-s}\,\text{Gamma}[s], \text{Re}[s] > 0 \,\&\&\, \text{Re}[n] > 0\right]$

```
Integrate[ t^(s-1) E^(-n t), {t, 0, x}]
```

$\text{ConditionalExpression}\left[x^{s}\,(n\,x)^{-s}\,(\text{Gamma}[s] - \text{Gamma}[s, n\,x]), \text{Re}[s] > 0\right]$

```
Integrate[ Sum[t^(s-1) E^(-n t), {n, 1, Infinity}], {t, 0, Infinity}]
ConditionalExpression[Gamma[s] PolyLog[s, 1], Re[s] > 1]
Integrate[ Sum[t^(s-1) E^(-n t), {n, 1, Infinity}], {t, 0, x}]
```

$$\int_0^x \frac{t^{-1+s}}{-1+e^t}\,dt$$

```
Integrate[ Sum[t^(s-1) E^(-n t), {n, 1, x}], {t, 0, x}]
```

$$\int_0^x \frac{e^{-t\,x}\left(-1+e^{t\,x}\right) t^{-1+s}}{-1+e^t}\,dt$$

$$\text{Expand}\left[\frac{e^{-t\,x}\left(-1+e^{t\,x}\right) t^{-1+s}}{-1+e^t}\right]$$

FullSimplify$\left[\frac{t^{-1+s}}{-1+e^t}-\frac{e^{-t\,x}\,t^{-1+s}}{-1+e^t}\right]$

Integrate$\left[\frac{t^{-1+s}}{-1+e^t}-\frac{e^{-t\,x}\,t^{-1+s}}{-1+e^t},\ \{t,0,x\}\right]$

$$\int_0^x\left(\frac{t^{-1+s}}{-1+e^t}-\frac{e^{-t\,x}\,t^{-1+s}}{-1+e^t}\right)dt$$

Integrate$\left[\frac{t^{-1+s}}{-1+e^t},\ \{t,0,x\}\right]$

$$\int_0^x\frac{t^{-1+s}}{-1+e^t}\,dt$$

Integrate$\left[-\frac{e^{-t\,x}\,t^{-1+s}}{-1+e^t},\ \{t,0,x\}\right]$

$$\int_0^x-\frac{e^{-t\,x}\,t^{-1+s}}{-1+e^t}\,dt$$

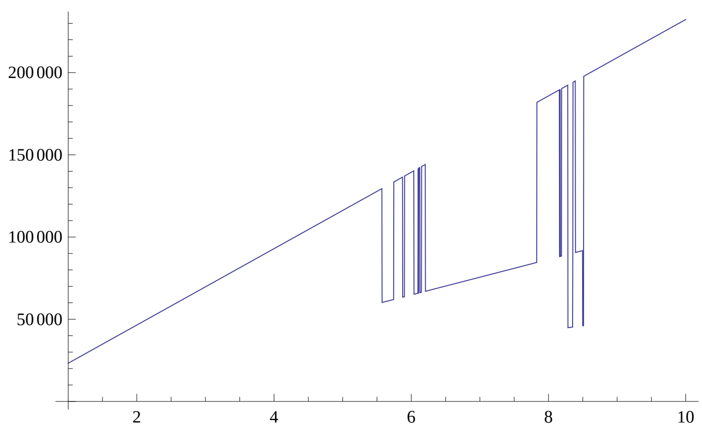
bb[x_, s_] := $\int_0^x\frac{e^{-t\,x}\left(-1+e^{t\,x}\right)t^{-1+s}}{-1+e^t}\,dt$

Plot[**bb**[n, 0], {n, 1, 10}]

Integrate::idiv : Integral of $-\frac{1.}{t-e^t\,t}+\frac{e^{-1.00018\,t}}{t-e^t\,t}$ does not converge on {0, 1.00018}. ≫

NIntegrate::slwcon :
Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or WorkingPrecision too small. ≫

NIntegrate::ncvb :
NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in t near {t} = $\{1.63103\times10^{-29}\}$. NIntegrate obtained 23235.97463501927` and 19755.03376575687` for the integral and error estimates. ≫



Integrate[**Log**[**t**^-**1**]^(**2**), {**t**, **1** / (**n**^(**1** - **p**)), **1**}]

ConditionalExpression$\left[2-n^{-1+p}\left(2+\text{Log}\left[n^{1-p}\right]\left(2+\text{Log}\left[n^{1-p}\right]\right)\right),\right.$
$\left.\left(\frac{n^p}{-n+n^p}\neq0\ \&\&\ \text{Re}\left[\frac{1}{-1+n^{1-p}}\right]\geq0\right)\ ||\ \text{Re}\left[\frac{1}{-1+n^{1-p}}\right]\leq-1\ ||\ \frac{1}{-1+n^{1-p}}\notin\text{Reals}\right]$

Expand$\left[\text{ConditionalExpression}\left[1 - n^{-1+p}\left(1 + \text{Log}\left[n^{1-p}\right]\right),\right.\right.$
$\left.\left.\left(\frac{n^p}{-n + n^p} \neq 0 \,\&\&\, \text{Re}\left[\frac{1}{-1 + n^{1-p}}\right] \geq 0\right) \,||\, \text{Re}\left[\frac{1}{-1 + n^{1-p}}\right] \leq -1 \,||\, \frac{1}{-1 + n^{1-p}} \notin \text{Reals}\right]\right]$

$\text{ConditionalExpression}\left[1 - n^{-1+p} - n^{-1+p}\,\text{Log}\left[n^{1-p}\right],\right.$
$\left.\left(\frac{n^p}{-n + n^p} \neq 0 \,\&\&\, \text{Re}\left[\frac{1}{-1 + n^{1-p}}\right] \geq 0\right) \,||\, \text{Re}\left[\frac{1}{-1 + n^{1-p}}\right] \leq -1 \,||\, \frac{1}{-1 + n^{1-p}} \notin \text{Reals}\right]$

```
Integrate[Log[rr^-1]^(1), {rr, 1/(m^(1-ss)), 1}]
```

```
$Aborted
```