```
D2[n_, 0] := UnitStep[n - 1]
D2[n_, k_] := D2[n, k] = Sum[D2[Floor[n / j], k - 1], {j, 2, n}]
Ex[n_] := Sum[D2[n, k] / k!, {k, 0, Log2@n}]
ex[n_] := Ex[n] - Ex[n - 1]
Cz[n_, z_] := Sum[Pochhammer[-z, k] / k! D2[n, k], {k, 0, Log2@n}]
Czx[n_, z_] := FullSimplify@Table[Pochhammer[-z, k] / k! Da2[n, k], {k, 0, Log2@n}]
D2a[n_, z_] := Sum[Pochhammer[-z, k] / k! Cz[n, k], {k, 0, Log2@n}]
```

```
Expand@D2a[100, 1]
```

99

```
Expand@Cz[100, z]
```

$$1 - \frac{6088\,z}{15} + \frac{148\,229\,z^2}{360} - \frac{1873\,z^3}{16} + \frac{1835\,z^4}{144} - \frac{137\,z^5}{240} + \frac{7\,z^6}{720}$$

```
Sum[(-1)^(k - j) Binomial[k, j] (1 + x)^j, {j, 0, k}]
```

x^k

```
Sum[Pochhammer[z, k] / k! x^k, {k, 0, Infinity}]
```

$(1 - x)^{-z}$

```
Sum[ Pochhammer[z, k] / k! (1 - x)^-k, {k, 0, Infinity}]
```

$$\left(\frac{x}{-1 + x}\right)^{-z}$$

```
Sum[(-1)^k Binomial[z, k] x^k, {k, 0, Infinity}]
```

$(1 - x)^z$

```
Sum[(-1)^k Binomial[z, k] (1 - x)^k, {k, 0, Infinity}]
```

x^z

```
Sum[Pochhammer[-z, k] / k! (1 - x)^k, {k, 0, Infinity}]
```

x^z

```
Sum[Pochhammer[-z, k] / k! x^k, {k, 0, Infinity}]
```

$(1 - x)^z$

```
FullSimplify@Sum[Pochhammer[-z, k] / k! Pochhammer[-x, k] / k!, {k, 0, Infinity}]
```

$$\frac{\text{Gamma}[1 + x + z]}{\text{Gamma}[1 + x]\,\text{Gamma}[1 + z]}$$

```
Sum[Binomial[z, k] / k!, {k, 0, Infinity}]
```

Hypergeometric1F1[-z, 1, -1]

```
Sum[(x^k / k!) / k!, {k, 0, Infinity}]
```

$$\text{BesselI}\left[0, 2\sqrt{x}\right]$$

```
(* http://oeis.org/A000262 *)
ex[2 × 3 × 5 × 7 × 11 × 13 × 17]
```

877

1, 2, 5, 15, 52, 203, 877

```
f[n_] := Sum[StirlingS2[n, k], {k, 1, n}]; Table[f[n], {n, 0, 21}]
```

```
{0, 1, 2, 5, 15, 52, 203, 877, 4140, 21 147, 115 975, 678 570,
 4 213 597, 27 644 437, 190 899 322, 1 382 958 545, 10 480 142 147, 82 864 869 804,
 682 076 806 159, 5 832 742 205 057, 51 724 158 235 372, 474 869 816 156 751}
```

```
Table[ex[2^k], {k, 0, 10}]
```

$$\left\{1, 1, \frac{3}{2}, \frac{13}{6}, \frac{73}{24}, \frac{167}{40}, \frac{4051}{720}, \frac{37\,633}{5040}, \frac{43\,817}{4480}, \frac{4\,596\,553}{362\,880}, \frac{58\,941\,091}{3\,628\,800}\right\}$$

```
Range[0, 19]! CoefficientList[Series[E^(x / (1 - x)), {x, 0, 19}], x]
```

```
{1, 1, 3, 13, 73, 501, 4051, 37 633, 394 353, 4 596 553, 58 941 091, 824 073 141,
 12 470 162 233, 202 976 401 213, 3 535 017 524 403, 65 573 803 186 921, 1 290 434 218 669 921,
 26 846 616 451 246 353, 588 633 468 315 403 843, 13 564 373 693 588 558 173}
```

```
167 × 3
```

501

```
8!
```

```
40 320 / 4480
```

9

```
43 817 × 9
```

394 353

```
AEx[n_] := Sum[Binomial[n, k] / k!, {k, 0, n}]
aex[n_] := AEx[n] - AEx[n - 1]
```

```
Table[aex[n], {n, 1, 10}]
```

$$\left\{1, \frac{3}{2}, \frac{13}{6}, \frac{73}{24}, \frac{167}{40}, \frac{4051}{720}, \frac{37\,633}{5040}, \frac{43\,817}{4480}, \frac{4\,596\,553}{362\,880}, \frac{58\,941\,091}{3\,628\,800}\right\}$$