```
FullSimplify[(E^(I x) + 9 E^(-I x)) / 2]

5 Cos[x] - 4 i Sin[x]

FullSimplify[Gamma[s / 2] Gamma[s / 2]]

Gamma[s/2]^2

pa11[n_, s_] := (n^(1 - s) s HarmonicNumber[n, 1 - s] - n^s (1 - s) HarmonicNumber[n, s])
   (2 n^(1 - s) s (2 π)^-s Cos[π s/2] Gamma[s] - n^s (1 - s))^-1

pa11[100 000, .23 + 12 I]

1.05615 - 0.90073 i

Zeta[.23 + 12 I]

1.05611 - 0.900568 i

pa1[n_, s_] := (1 / 2) Pi^(-s / 2) s (s - 1) Gamma[s / 2]
   (n^(1 - s) s HarmonicNumber[n, 1 - s] - n^s (1 - s) HarmonicNumber[n, s])
   (2 n^(1 - s) s (2 π)^-s Cos[π s/2] Gamma[s] - n^s (1 - s))^-1
pa2[n_, s_] := (1 / 2) Pi^(-s / 2) Gamma[s / 2]
   (n^(1 - s) s HarmonicNumber[n, 1 - s] - n^s (1 - s) HarmonicNumber[n, s]) /
    (2^(1 - s) n^(1 - s) π^-s Cos[π s/2] Gamma[s] / (s - 1) - n^s / s)
pa3[n_, s_] := (1 / 2) Gamma[s / 2]
   (n^(1 - s) s HarmonicNumber[n, 1 - s] - n^s (1 - s) HarmonicNumber[n, s]) /
    (2^(1 - s) n^(1 - s) π^(-s/2) Cos[π s/2] Gamma[s] / (s - 1) - n^s / s Pi^(s / 2))
pa4[n_, s_] := Gamma[s / 2]
   (n^(1 - s) s HarmonicNumber[n, 1 - s] - n^s (1 - s) HarmonicNumber[n, s]) /
    (2^(2 - s) n^(1 - s) π^(-s/2) Cos[π s/2] Gamma[s] / (s - 1) - 2 n^s / s Pi^(s / 2))
pa5[n_, s_] := Gamma[s / 2] (n^(1 - s) s HarmonicNumber[n, 1 - s] -
       n^s (1 - s) HarmonicNumber[n, s]) / (2^(2 - s) n^(1 - s) π^(-s/2) Cos[π s/2]
        (Gamma[s / 2] Gamma[(s + 1) / 2] / (2^(1 - s) Pi^(1 / 2))) / (s - 1) - 2 n^s / s Pi^(s / 2))
pa6[n_, s_] := (n^(1 - s) s HarmonicNumber[n, 1 - s] - n^s (1 - s) HarmonicNumber[n, s]) /
   (2 / Gamma[s / 2] n^(1 - s) π^(-(s+1)/2) Cos[π s/2] (Gamma[s / 2] Gamma[(s + 1) / 2]) / (s - 1) -
     2 n^s / s Pi^(s / 2) / Gamma[s / 2])
pa7[n_, s_] := (n^(1 - s) s HarmonicNumber[n, 1 - s] - n^s (1 - s) HarmonicNumber[n, s]) /
   (2 n^(1 - s) π^(-(s+1)/2) Cos[π s/2] Gamma[(s + 1) / 2] / (s - 1) - 2 n^s / s Pi^(s / 2) / Gamma[s / 2])
pa7[100 000, .23 + 12 I]

0.00864027 - 0.00222696 i
```

```
(1 / 2) Pi^(-s / 2) s (s - 1) Gamma[s / 2] Zeta[s] /. s -> .23 + 12 I
```

0.00863991 - 0.00218371 i

```
Expand[π^(-s/2) / Pi^(1 / 2)]
```

$\pi^{-\frac{1}{2}-\frac{s}{2}}$

```
ps5[n_, s_] := ((2^s Pi^(s - 1) Sin[ Pi s / 2] Gamma[1 - s]) + (s / (s - 1)) n^(1 - 2 s)) Zeta[1 - s] -
   (HarmonicNumber[n, s] + (s / (s - 1)) n^(1 - 2 s) HarmonicNumber[n, 1 - s])
ps5a[n_, s_] := ((2^s Pi^(s - 1) Sin[ Pi s / 2] Gamma[1 - s] n^s (s - 1)) + s n^(1 - s)) Zeta[1 - s] -
   ((s - 1) n^s HarmonicNumber[n, s] + s n^(1 - s) HarmonicNumber[n, 1 - s])
ps5b[n_, s_] := ((s - 1) n^s HarmonicNumber[n, s] + s n^(1 - s) HarmonicNumber[n, 1 - s]) /
   ((2^s Pi^(s - 1) Sin[ Pi s / 2] Gamma[1 - s] n^s (s - 1)) + s n^(1 - s))
ps5x[n_, s_] := ((2^(1 - s) Pi^((1 - s) - 1) Sin[ Pi (1 - s) / 2] Gamma[1 - (1 - s)]) +
      ((1 - s) / ((1 - s) - 1)) n^(1 - 2 (1 - s))) Zeta[1 - (1 - s)] - (HarmonicNumber[n, (1 - s)] +
     ((1 - s) / ((1 - s) - 1)) n^(1 - 2 (1 - s)) HarmonicNumber[n, 1 - (1 - s)])
```

```
ps5x[1 000 000 000 000, .3 + 2 I]
```

$-2.0091\times 10^{-9} + 3.41047\times 10^{-9}$ i

```
Zeta[1 - (.3 + 2 I)]
```

0.501262 + 0.334245 i

```
n^(1 - s) s HarmonicNumber[n, 1 - s] - n^s (1 - s) HarmonicNumber[n, s]
```

n^{1-s} s HarmonicNumber[n, 1 - s] - n^s (1 - s) HarmonicNumber[n, s]

```
n^(1 - s) s HarmonicNumber[n, 1 - s] - n^s (1 - s) HarmonicNumber[n, s] /. n → 1 000 000 /.
 s → N@ZetaZero@1
```

0. + 14.1347 i

```
Integrate[ j^-s, {j, 0, n}]
```

ConditionalExpression$\left[-\frac{n^{1-s}}{-1+s}\right.$, Re[s] < 1$\left.\right]$

```
n^(1 - s) s Sum[j^(s - 1), {j, 1, n}] - n^s (1 - s) Sum[j^-s, {j, 1, n}] /. n → 10 000 000 /.
 s -> N@ZetaZero@1
```

0. + 14.1347 i

```
FullSimplify[n^(1 - s) s Integrate[j^(s - 1), {j, 0, n}] - n^s (1 - s) Integrate[j^-s, {j, 0, n}]]
```

ConditionalExpression[0, 0 < Re[s] < 1]

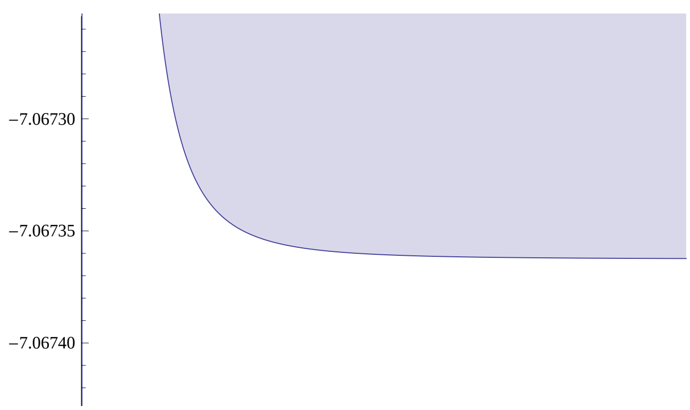
```
FullSimplify[
 n^(1 / 2 - s) s Integrate[j^(s - 1), {j, 0, n}] - n^(s-1/2) (1 - s) Integrate[j^-s, {j, 0, n}]]
```

ConditionalExpression[0, 0 < Re[s] < 1]

```
FullSimplify[ Integrate[s (n / j)^(1 - s), {j, 0, n}] - (1 - s) Integrate[(n / j)^s, {j, 0, n}]]
```

ConditionalExpression[0, 0 < Re[s] < 1 && n > 0]

```
Integrate[s (n / j) ^ (1 - s), {j, 0, n}]

ConditionalExpression[n, Re[s] > 0 && n > 0]

FullSimplify[(1 - s) Integrate[(n / j) ^s, {j, 0, n}]]

ConditionalExpression[n, Re[s] < 1 && n > 0]

bb[n_, s_] := (1 - s) Sum[(n / j) ^s, {j, 1, n}]
bba[n_, s_] := (1 - s) Sum[E^ (s Log[n / j]), {j, 1, n}]
bba2[n_, s_, t_] := (1 - s - t I) Sum[E^ ((s + t I) Log[n / j]), {j, 1, n}]
bba3[n_, s_, t_] := (1 - s - t I) Sum[(n / j) ^s E^ (I t Log[n / j]), {j, 1, n}]
bba4[n_, s_, t_] :=
 (1 - s - t I) Sum[(n / j) ^s (Cos[t Log[n / j]] + I Sin[t Log[n / j]]), {j, 1, n}]
bba5[n_, s_, t_] := (1 - s) Sum[(n / j) ^s (Cos[t Log[n / j]] + I Sin[t Log[n / j]]), {j, 1, n}] -
  t I Sum[(n / j) ^s (Cos[t Log[n / j]] + I Sin[t Log[n / j]]), {j, 1, n}]
bba6[n_, s_, t_] := (1 - s) Sum[(n / j) ^s (Cos[t Log[n / j]]), {j, 1, n}] -
  t I Sum[(n / j) ^s (Cos[t Log[n / j]]), {j, 1, n}] +
  (1 - s) Sum[(n / j) ^s (I Sin[t Log[n / j]]), {j, 1, n}] -
  t I Sum[(n / j) ^s (I Sin[t Log[n / j]]), {j, 1, n}]
bba7[n_, s_, t_] := (1 - s) Sum[(n / j) ^s Cos[t Log[n / j]], {j, 1, n}] -
  t I Sum[(n / j) ^s Cos[t Log[n / j]], {j, 1, n}] +
  (1 - s) Sum[(n / j) ^s I Sin[t Log[n / j]], {j, 1, n}] -
  t I Sum[(n / j) ^s I Sin[t Log[n / j]], {j, 1, n}]
bba8[n_, s_, t_] := Sum[(n / j) ^s (1 - s) Cos[t Log[n / j]], {j, 1, n}] +
  Sum[(n / j) ^s t Sin[t Log[n / j]], {j, 1, n}] - I Sum[(n / j) ^s t Cos[t Log[n / j]], {j, 1, n}] +
  I Sum[(n / j) ^s (1 - s) Sin[t Log[n / j]], {j, 1, n}]
bba9[n_, s_, t_] := Sum[(n / j) ^s ((1 - s) Cos[t Log[n / j]] + t Sin[t Log[n / j]]), {j, 1, n}] -
  I Sum[(n / j) ^s (t Cos[t Log[n / j]] - (1 - s) Sin[t Log[n / j]]), {j, 1, n}]
bba9a[n_, s_, t_] := Sum[(n / j) ^s ((1 - s) Cos[t Log[n / j]] + t Sin[t Log[n / j]]), {j, 1, n}]
bba9b[n_, s_, t_] := 2 Sum[(n / j) ^s (t Cos[t Log[n / j]] - (1 - s) Sin[t Log[n / j]]), {j, 1, n}]
bb2[n_, s_] := s Sum[(n / j) ^ (1 - s), {j, 1, n}]

DiscretePlot[Im@ (bba9[n, .5, N@Im@ZetaZero@1]), {n, 1, 400}]
```



```
DiscretePlot[bba9a[n, .5, N@Im@ZetaZero@1], {n, 1, 400}]
```