```
Grid[Table[{n^z, N[Sum[z^k / k! (Limit[ D[n^y, {y, k}], y → 0]), {k, 0, 50}]]},
  {n, -1.5, 8}, {z, 1, 5}]]
```

$\{-1.5, -1.5 - 1.71613 \times 10^{-16}\, i\}$	$\{2.25, 2.25 + 5.64243 \times 10^{-15}\, i\}$	$\{-3.375, -3.375 - 4.08332 \times 10^{-14}\, i\}$	$\{5.0625, 5.0625 + 1.13725 \times 10^{-10}\, i\}$	$\{-7.59375, -7.59375 + 9.8075 \times 10^{-6}\, i\}$
$\{-0.5, -0.5 - 4.25927 \times 10^{-16}\, i\}$	$\{0.25, 0.25 - 2.04993 \times 10^{-14}\, i\}$	$\{-0.125, -0.125 - 2.20264 \times 10^{-13}\, i\}$	$\{0.0625, 0.0625 + 7.13404 \times 10^{-11}\, i\}$	$\{-0.03125, -0.0312316 + 6.86142 \times 10^{-6}\, i\}$
{0.5, 0.5}	{0.25, 0.25}	{0.125, 0.125}	{0.0625, 0.0625}	{0.03125, 0.03125}
{1.5, 1.5}	{2.25, 2.25}	{3.375, 3.375}	{5.0625, 5.0625}	{7.59375, 7.59375}
{2.5, 2.5}	{6.25, 6.25}	{15.625, 15.625}	{39.0625, 39.0625}	{97.6563, 97.6563}
{3.5, 3.5}	{12.25, 12.25}	{42.875, 42.875}	{150.063, 150.062}	{525.219, 525.219}
{4.5, 4.5}	{20.25, 20.25}	{91.125, 91.125}	{410.063, 410.063}	{1845.28, 1845.28}
{5.5, 5.5}	{30.25, 30.25}	{166.375, 166.375}	{915.063, 915.063}	{5032.84, 5032.84}
{6.5, 6.5}	{42.25, 42.25}	{274.625, 274.625}	{1785.06, 1785.06}	{11 602.9, 11 602.9}
{7.5, 7.5}	{56.25, 56.25}	{421.875, 421.875}	{3164.06, 3164.06}	{23 730.5, 23 730.5}

```
Grid[Table[Chop[n^z - N[Sum[z^k / k! (Limit[D[n^y, {y, k}], y → 0]), {k, 0, 50}]]],
  {n, -1.5, 8}, {z, 1, 5}]]
```

0	0	0	$0. - 1.13725 \times 10^{-10}\, i$	$4.14602 \times 10^{-7} - 9.8075 \times 10^{-6}\, i$
0	0	0	-2.13024×10^{-10}	$-0.0000184176 - 6.86142 \times 10^{-6}\, i$
0	0	0	0	0
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```
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
referenced1[n_, z_] := Product[(-1)^p[[2]] bin[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
referenceD1[n_, z_] := Sum[referenced1[j, z], {j, 1, n}]
D1Alt[n_, z_] :=
 Sum[z^k / k! (Limit[D[referenceD1[n, y], {y, k}], y → 0]), {k, 0, Log[2, n]}]
Grid[Table[Chop[referenceD1[a = 143, s + t I] - D1Alt[a, s + t I]],
  {s, -1.5, 4, .7}, {t, -1.1, 4, .7}]]
```

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```

```
{Log[n], Limit[D[n^z, z], z → 0]}
```

```
{Log[n], Log[n]}
```

```
referenceRiemanPrimeCount[n_] := Sum[FullSimplify[MangoldtLambda[j] / Log[j]], {j, 2, n}]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
referenced1[n_, z_] := Product[(-1)^p[[2]] bin[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
referenceD1[n_, z_] := Sum[referenced1[j, z], {j, 1, n}]
Table[referenceRiemanPrimeCount[n] - (Limit[Expand[D[referenceD1[n, z], z]], z → 0]),
  {n, 1, 100}] // TableForm
```

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{Log[n]^j, Limit[D[n^z, z]^j, z → 0]}
```

$\{\text{Log}[n]^j, \text{Log}[n]^j\}$

```
P2[n_, k_] := Sum[MangoldtLambda[j] / Log[j] P2[n / j, k - 1], {j, 2, Floor[n]}];
P2[n_, 0] := UnitStep[n - 1]
D2[n_, k_] := Sum[D2[n / j, k - 1], {j, 2, Floor[n]}]; D2[n_, 0] := UnitStep[n - 1]
P2Alt[n_, j_] := Sum[1 / k! (Limit[D[Log[1 + y]^j, {y, k}], y → 0]) D2[n, k], {k, 0, Log[2, n]}]
Table[FullSimplify[P2[n, k] - P2Alt[n, k]], {n, 1, 50}, {k, 1, 5}] // TableForm
```

0	0	0	0	0
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```
P2[n_, k_] :=
 P2[n, k] = Sum[FullSimplify[MangoldtLambda[j] / Log[j]] P2[n / j, k - 1], {j, 2, Floor[n]}];
P2[n_, 0] := UnitStep[n - 1]
Dd[n_, k_, a_] :=
 Sum[Binomial[k, j] Dd[n / (m^(k - j)), j, m], {m, a + 1, n^(1 / k)}, {j, 0, k - 1}];
Dd[n_, 0, a_] := UnitStep[n - 1]; Dd[n_, 1, a_] := Floor[n] - a
P2Alt[n_, j_] :=
 Sum[1 / k! (Limit[D[Log[1 + y]^j, {y, k}], y → 0]) Dd[n, k, 1], {k, 0, Log[2, n]}]
Grid[Table[P2[n, k] - P2Alt[n, k], {n, 10, 500, 10}, {k, 1, 5}]]
```

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```
num[c_] := Numerator[c]; den[c_] := Denominator[c]
alpha[n_, c_] := den[c] (Floor[n / den[c]] - Floor[(n - 1) / den[c]]) -
  num[c] (Floor[n / num[c]] - Floor[(n - 1) / num[c]])
E1[n_, k_, c_] := E1[n, k, c] = (1 / den[c])
   Sum[If[alpha[j, c] == 0, 0, alpha[j, c] E1[(den[c] n) / j, k - 1, c]], {j, 1, den[c] n}];
E1[n_, 0, c_] := UnitStep[n - 1]
E2[n_, k_, c_] := E2[n, k, c] = (1 / den[c]) Sum[
    If[alpha[j, c] == 0, 0, alpha[j, c] E2[(den[c] n) / j, k - 1, c]], {j, den[c] + 1, den[c] n}]
E2[n_, 0, c_] := UnitStep[n - 1]
E1Alt[n_, z_, c_] := Sum[bin[z, k] E2[n, k, c], {k, 0, Floor[Log[n] / Log[c]]}]
logE[ n_, k_, c_] := Limit[ D[E1Alt[n, y, c], {y, k}], y → 0]
E1B[ n_, z_, c_] :=
 Sum[ z^k / (k!) Limit[ D[E1Alt[n, y, c], {y, k}], y → 0], {k, 0, Log[If[c < 2, c, 2], n]}]
```

```
Limit[ D[ E1Alt[100, z, 2], {z, 7}], z → 0]
```

0

```
E1B[100, -3, 4 / 3]
```

$$\frac{86\,569\,035\,001}{14\,348\,907}$$

```
E1Alt[100, -3, 4 / 3]
```

$$\frac{86\,569\,035\,001}{14\,348\,907}$$

```
logE[ 100, 1, 3 / 2]
```

$$-\frac{8\,149\,753}{2\,365\,440}$$

```
P2[n_, k_] := Sum[MangoldtLambda[j] / Log[j] P2[n / j, k - 1], {j, 2, Floor[n]}];
P2[n_, 0] := UnitStep[n - 1]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
referenced1[n_, z_] := Product[(-1)^p[[2]] bin[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
referenceD1[n_, z_] := Sum[referenced1[j, z], {j, 1, n}]
Grid[Table[FullSimplify[P2[n, k]] - (Limit[D[referenceD1[n, z], {z, k}], z → 0]),
  {n, 1, 50}, {k, 1, 5}]]
```

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```
coef[j_, k_] := coef[j, k] = Limit[ D[ Log[1 + y]^j, {y, k}], y → 0]
fn[n_, j_] := N[Sum[ (k!)^-1 coef[j, k] (n - 1)^k, {k, 0, 150}]]
Grid[ Table[ Chop[ Log[n]^k - fn[n, k]], {n, .12, 1.8, .1}, {k, 1, 5}]]
```

-2.18324×10^{-10}	2.46071×10^{-9}	-1.97289×10^{-8}	1.34234×10^{-7}	-8.21432×10^{-7}
0	0	0	0	0
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```
P2[n_, k_] := Sum[MangoldtLambda[j] / Log[j] P2[n / j, k - 1], {j, 2, Floor[n]}];
P2[n_, 0] := UnitStep[n - 1]
D2[n_, k_] := Sum[D2[n / j, k - 1], {j, 2, Floor[n]}]; D2[n_, 0] := UnitStep[n - 1]
P2Alt[n_, j_] := Sum[1 / k! (Limit[D[Log[1 + y]^j, {y, k}], y → 0]) D2[n, k], {k, 0, Log[2, n]}]
Table[FullSimplify[P2[n, k] - P2Alt[n, k]], {n, 1, 50}, {k, 1, 5}] // TableForm
```

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```
F2[f_, n_, k_] := F2[f, n, k] = Sum[f[j] F2[f, n / j, k - 1], {j, 2, Floor[n]}];
F2[f_, n_, 0] := UnitStep[n - 1]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
F1[f_, n_, z_] := Sum[bin[z, k] F2[f, n, k], {k, 0, Log[2, n]}]
LF[f_, n_, k_] := Limit[D[F1[f, n, z], {z, k}], z → 0]

logD[n_, 0] := UnitStep[n - 1]
logD[n_, k_] := Sum[MangoldtLambda[j] / Log[j] logD[n / j, k - 1], {j, 2, Floor[n]}]
Table[FullSimplify[logD[n, k]], {n, 1, 50}, {k, 1, 5}] // TableForm
```

0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
$\frac{5}{2}$	1	0	0	0
$\frac{7}{2}$	1	0	0	0
$\frac{7}{2}$	3	0	0	0
$\frac{9}{2}$	3	0	0	0
$\frac{29}{6}$	4	1	0	0
$\frac{16}{3}$	5	1	0	0
$\frac{16}{3}$	7	1	0	0
$\frac{19}{3}$	7	1	0	0
$\frac{19}{3}$	8	4	0	0
$\frac{22}{3}$	8	4	0	0
$\frac{22}{3}$	10	4	0	0
$\frac{22}{3}$	12	4	0	0
$\frac{91}{12}$	$\frac{155}{12}$	$\frac{11}{2}$	1	0
$\frac{103}{12}$	$\frac{155}{12}$	$\frac{11}{2}$	1	0
$\frac{103}{12}$	$\frac{167}{12}$	$\frac{17}{2}$	1	0
$\frac{115}{12}$	$\frac{167}{12}$	$\frac{17}{2}$	1	0
$\frac{115}{12}$	$\frac{179}{12}$	$\frac{23}{2}$	1	0
$\frac{115}{12}$	$\frac{203}{12}$	$\frac{23}{2}$	1	0
$\frac{115}{12}$	$\frac{227}{12}$	$\frac{23}{2}$	1	0
$\frac{127}{12}$	$\frac{227}{12}$	$\frac{23}{2}$	1	0
$\frac{127}{12}$	$\frac{235}{12}$	$\frac{29}{2}$	5	0
$\frac{133}{12}$	$\frac{247}{12}$	$\frac{29}{2}$	5	0
$\frac{133}{12}$	$\frac{271}{12}$	$\frac{29}{2}$	5	0
$\frac{137}{12}$	$\frac{283}{12}$	$\frac{31}{2}$	5	0
$\frac{137}{12}$	$\frac{295}{12}$	$\frac{37}{2}$	5	0
$\frac{149}{12}$	$\frac{295}{12}$	$\frac{37}{2}$	5	0
$\frac{149}{12}$	$\frac{295}{12}$	$\frac{49}{2}$	5	0
$\frac{161}{12}$	$\frac{295}{12}$	$\frac{49}{2}$	5	0
$\frac{817}{60}$	$\frac{305}{12}$	$\frac{105}{4}$	7	1

$\frac{817}{60}$	$\frac{329}{12}$	$\frac{105}{4}$	7	1
$\frac{817}{60}$	$\frac{353}{12}$	$\frac{105}{4}$	7	1
$\frac{817}{60}$	$\frac{377}{12}$	$\frac{105}{4}$	7	1
$\frac{817}{60}$	$\frac{383}{12}$	$\frac{117}{4}$	13	1
$\frac{877}{60}$	$\frac{383}{12}$	$\frac{117}{4}$	13	1
$\frac{877}{60}$	$\frac{407}{12}$	$\frac{117}{4}$	13	1
$\frac{877}{60}$	$\frac{431}{12}$	$\frac{117}{4}$	13	1
$\frac{877}{60}$	$\frac{439}{12}$	$\frac{129}{4}$	17	1
$\frac{937}{60}$	$\frac{439}{12}$	$\frac{129}{4}$	17	1
$\frac{937}{60}$	$\frac{439}{12}$	$\frac{153}{4}$	17	1
$\frac{997}{60}$	$\frac{439}{12}$	$\frac{153}{4}$	17	1
$\frac{997}{60}$	$\frac{451}{12}$	$\frac{165}{4}$	17	1
$\frac{997}{60}$	$\frac{463}{12}$	$\frac{177}{4}$	17	1
$\frac{997}{60}$	$\frac{487}{12}$	$\frac{177}{4}$	17	1
$\frac{1057}{60}$	$\frac{487}{12}$	$\frac{177}{4}$	17	1
$\frac{1057}{60}$	$\frac{493}{12}$	47	23	6
$\frac{1087}{60}$	$\frac{505}{12}$	47	23	6
$\frac{1087}{60}$	$\frac{517}{12}$	50	23	6

```
Dm11[n_] := Sum[1, {j, 2, n}]; Dm12[n_] := Sum[1, {j, 2, n}, {k, 2, n / j}];
Dm13[n_] := Sum[1, {j, 2, n}, {k, 2, n / j}, {m, 2, n / (j k)}]
Dm1[n_, k_] := Sum[Dm1[n / j, k - 1], {j, 2, Floor[n]}]; Dm1[n_, 0] := UnitStep[n - 1]
Table[{Dm11[n] - Dm1[n, 1], Dm12[n] - Dm1[n, 2], Dm13[n] - Dm1[n, 3]}, {n, 1, 50}] // TableForm
```

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```
K[n_] := FullSimplify[MangoldtLambda[n] / Log[n]]
logD1[n_] := Sum[K[j], {j, 2, n}]; logD2[n_] := Sum[K[j] K[k], {j, 2, n}, {k, 2, n / j}]
logD3[n_] := Sum[K[j] K[k] K[m], {j, 2, n}, {k, 2, n / j}, {m, 2, n / (j k)}]
logD[n_, k_] := Sum[K[j] logD[n / j, k - 1], {j, 2, Floor[n]}]; logD[n_, 0] := UnitStep[n - 1]
Table[{logD1[n] - logD[n, 1], logD2[n] - logD[n, 2], logD3[n] - logD[n, 3]}, {n, 1, 50}] //
 TableForm
```

0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
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0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0

```
Dm1[n_, 0] := UnitStep[n - 1]
Dm1[n_, k_] := Sum[D2[n / j, k - 1], {j, 2, Floor[n]}]
Table[Dm1[n, k], {n, 1, 50}, {k, 1, 7}] // TableForm
```

0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	1	0	0	0	0	0
4	1	0	0	0	0	0
5	3	0	0	0	0	0
6	3	0	0	0	0	0
7	5	1	0	0	0	0
8	6	1	0	0	0	0
9	8	1	0	0	0	0
10	8	1	0	0	0	0
11	12	4	0	0	0	0
12	12	4	0	0	0	0
13	14	4	0	0	0	0
14	16	4	0	0	0	0
15	19	7	1	0	0	0
16	19	7	1	0	0	0
17	23	10	1	0	0	0
18	23	10	1	0	0	0
19	27	13	1	0	0	0
20	29	13	1	0	0	0
21	31	13	1	0	0	0
22	31	13	1	0	0	0
23	37	22	5	0	0	0
24	38	22	5	0	0	0
25	40	22	5	0	0	0
26	42	23	5	0	0	0
27	46	26	5	0	0	0
28	46	26	5	0	0	0
29	52	32	5	0	0	0
30	52	32	5	0	0	0
31	56	38	9	1	0	0
32	58	38	9	1	0	0
33	60	38	9	1	0	0
34	62	38	9	1	0	0
35	69	50	15	1	0	0
36	69	50	15	1	0	0
37	71	50	15	1	0	0
38	73	50	15	1	0	0
39	79	59	19	1	0	0
40	79	59	19	1	0	0
41	85	65	19	1	0	0
42	85	65	19	1	0	0
43	89	68	19	1	0	0
44	93	71	19	1	0	0
45	95	71	19	1	0	0
46	95	71	19	1	0	0
47	103	89	35	6	0	0
48	104	89	35	6	0	0
49	108	92	35	6	0	0

```
dk[n_, k_] := Sum[dk[j, k - 1] dk[x / j, 1], {j, Divisors[n]}];
dk[n_, 1] := 1; dk[n_, 0] := 0; dk[1, 0] := 1
Dk[n_, k_] := Sum[Dk[n / j, k - 1], {j, 1, Floor[n]}]; Dk[n_, 0] := UnitStep[n - 1]
Grid[Table[dk[n, k] - (Dk[n, k] - Dk[n - 1, k]), {n, 1, 50}, {k, 1, 7}]]
```

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
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0	0	0	0	0	0	0
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0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

```
Dk1[n_] := Sum[1, {j, 1, n}]; Dk2[n_] := Sum[1, {j, 1, n}, {k, 1, n / j}];
Dk3[n_] := Sum[1, {j, 1, n}, {k, 1, n / j}, {m, 1, n / (j k)}]
Dk[n_, k_] := Sum[Dk[n / j, k - 1], {j, 1, Floor[n]}]; Dk[n_, 0] := UnitStep[n - 1]
Table[{Dk1[n] - Dk[n, 1], Dk2[n] - Dk[n, 2], Dk3[n] - Dk[n, 3]}, {n, 1, 50}] // TableForm
```

0	0	0
0	0	0
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0	0	0
0	0	0

```
Dk[n_, k_] := Sum[Dk[n / j, k - 1], {j, 1, Floor[n]}]; Dk[n_, 0] := UnitStep[n - 1]
Table[Dk[n, k], {n, 1, 50}, {k, 1, 7}] // TableForm
```

1	1	1	1	1	1	1
2	3	4	5	6	7	8
3	5	7	9	11	13	15
4	8	13	19	26	34	43
5	10	16	23	31	40	50
6	14	25	39	56	76	99
7	16	28	43	61	82	106
8	20	38	63	96	138	190
9	23	44	73	111	159	218
10	27	53	89	136	195	267
11	29	56	93	141	201	274
12	35	74	133	216	327	470
13	37	77	137	221	333	477
14	41	86	153	246	369	526
15	45	95	169	271	405	575
16	50	110	204	341	531	785
17	52	113	208	346	537	792
18	58	131	248	421	663	988
19	60	134	252	426	669	995
20	66	152	292	501	795	1191
21	70	161	308	526	831	1240
22	74	170	324	551	867	1289
23	76	173	328	556	873	1296
24	84	203	408	731	1209	1884
25	87	209	418	746	1230	1912
26	91	218	434	771	1266	1961
27	95	228	454	806	1322	2045
28	101	246	494	881	1448	2241
29	103	249	498	886	1454	2248
30	111	276	562	1011	1670	2591
31	113	279	566	1016	1676	2598
32	119	300	622	1142	1928	3060
33	123	309	638	1167	1964	3109
34	127	318	654	1192	2000	3158
35	131	327	670	1217	2036	3207
36	140	363	770	1442	2477	3991
37	142	366	774	1447	2483	3998
38	146	375	790	1472	2519	4047
39	150	384	806	1497	2555	4096
40	158	414	886	1672	2891	4684
41	160	417	890	1677	2897	4691
42	168	444	954	1802	3113	5034
43	170	447	958	1807	3119	5041
44	176	465	998	1882	3245	5237
45	182	483	1038	1957	3371	5433
46	186	492	1054	1982	3407	5482
47	188	495	1058	1987	3413	5489
48	198	540	1198	2337	4169	6959
49	201	546	1208	2352	4190	6987
50	207	564	1248	2427	4316	7183

```
d1[n_, k_] := Sum[d1[j, k - 1] d1[n / j, 1], {j, Divisors[n]}];
d1[n_, 1] := 1; d1[n_, 0] := 0; d1[1, 0] := 1
```

```
d1[0, 1]
```

1

```
dk[n_, k_] := Sum[dk[j, k - 1] dk[n / j, 1], {j, Divisors[n]}];
dk[n_, 1] := 1; dk[n_, 0] := 0; dk[1, 0] := 1
```

```
dk[0, 2]
```

Divisors[0]

```
Divisors[0]
```

Divisors[0]

```
Dk[n_, k_] := Sum[Dk[n / j, k - 1], {j, 1, Floor[n]}]; Dk[n_, 0] := UnitStep[n - 1]
Dm1[n_, k_] := Sum[Dm1[n / j, k - 1], {j, 2, Floor[n]}]; Dm1[n_, 0] := UnitStep[n - 1]
logD[n_, k_] := Sum[FullSimplify[MangoldtLambda[j] / Log[j]] logD[n / j, k - 1],
  {j, 2, Floor[n]}]; logD[n_, 0] := UnitStep[n - 1]
Table[{n, Dm1[n, 4], Dm1[n, 5], Dm1[n, 6], logD[n, 4], logD[n, 5],
   logD[n, 6], Dk[n, 4], Dk[n, 5], Dk[n, 6]}, {n, 1, 64}] // TableForm
```

1	0	0	0	0	0	0	1	1	1
2	0	0	0	0	0	0	5	6	7
3	0	0	0	0	0	0	9	11	13
4	0	0	0	0	0	0	19	26	34
5	0	0	0	0	0	0	23	31	40
6	0	0	0	0	0	0	39	56	76
7	0	0	0	0	0	0	43	61	82
8	0	0	0	0	0	0	63	96	138
9	0	0	0	0	0	0	73	111	159
10	0	0	0	0	0	0	89	136	195
11	0	0	0	0	0	0	93	141	201
12	0	0	0	0	0	0	133	216	327
13	0	0	0	0	0	0	137	221	333
14	0	0	0	0	0	0	153	246	369
15	0	0	0	0	0	0	169	271	405
16	1	0	0	1	0	0	204	341	531
17	1	0	0	1	0	0	208	346	537
18	1	0	0	1	0	0	248	421	663
19	1	0	0	1	0	0	252	426	669
20	1	0	0	1	0	0	292	501	795
21	1	0	0	1	0	0	308	526	831
22	1	0	0	1	0	0	324	551	867
23	1	0	0	1	0	0	328	556	873
24	5	0	0	5	0	0	408	731	1209
25	5	0	0	5	0	0	418	746	1230
26	5	0	0	5	0	0	434	771	1266
27	5	0	0	5	0	0	454	806	1322
28	5	0	0	5	0	0	494	881	1448

29	5	0	0	5	0	0	498	886	1454
30	5	0	0	5	0	0	562	1011	1670
31	5	0	0	5	0	0	566	1016	1676
32	9	1	0	7	1	0	622	1142	1928
33	9	1	0	7	1	0	638	1167	1964
34	9	1	0	7	1	0	654	1192	2000
35	9	1	0	7	1	0	670	1217	2036
36	15	1	0	13	1	0	770	1442	2477
37	15	1	0	13	1	0	774	1447	2483
38	15	1	0	13	1	0	790	1472	2519
39	15	1	0	13	1	0	806	1497	2555
40	19	1	0	17	1	0	886	1672	2891
41	19	1	0	17	1	0	890	1677	2897
42	19	1	0	17	1	0	954	1802	3113
43	19	1	0	17	1	0	958	1807	3119
44	19	1	0	17	1	0	998	1882	3245
45	19	1	0	17	1	0	1038	1957	3371
46	19	1	0	17	1	0	1054	1982	3407
47	19	1	0	17	1	0	1058	1987	3413
48	35	6	0	23	6	0	1198	2337	4169
49	35	6	0	23	6	0	1208	2352	4190
50	35	6	0	23	6	0	1248	2427	4316
51	35	6	0	23	6	0	1264	2452	4352
52	35	6	0	23	6	0	1304	2527	4478
53	35	6	0	23	6	0	1308	2532	4484
54	39	6	0	27	6	0	1388	2707	4820
55	39	6	0	27	6	0	1404	2732	4856
56	43	6	0	31	6	0	1484	2907	5192
57	43	6	0	31	6	0	1500	2932	5228
58	43	6	0	31	6	0	1516	2957	5264
59	43	6	0	31	6	0	1520	2962	5270
60	55	6	0	43	6	0	1680	3337	6026
61	55	6	0	43	6	0	1684	3342	6032
62	55	6	0	43	6	0	1700	3367	6068
63	55	6	0	43	6	0	1740	3442	6194
64	65	11	1	$\frac{275}{6}$	$\frac{17}{2}$	1	1824	3652	6656

```
dk[n_, k_] := Sum[dk[j, k - 1] dk[n / j, 1], {j, Divisors[n]}];
dk[n_, 1] := 1; dk[n_, 0] := 0; dk[1, 0] := 1
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
Grid[Table[dk[n, k] - dz[n, k], {n, 1, 100}, {k, 1, 10}]]
```

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

```
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
Dz[n_, z_] := Sum[dz[j, z], {j, 1, n}]
Grid[Table[Dz[100, s + t I], {s, -1.3, 4, .7}, {t, -1.3, 4, .7}]]
```

10.4793 + 28.7121 i	5.72468 + 11.2587 i	6.03456 − 1.77709 i	5.94691 − 15.6189 i	15.2681 − 34.9044 i	58.5435 − 59.5846 i	173.704 − 80.8227 i	409.891 − 76.8935 i
−21.9794 + 33.2704 i	−9.29577 + 6.6042 i	−3.93641 − 0.702512 i	−12.975 − 11.3196 i	−23.7041 − 47.2133 i	−4.16474 − 124.722 i	95.5007 − 249.632 i	340.872 − 412.252 i
−70.5899 − 1.50386 i	−13.7213 − 18.1133 i	3.81025 + 3.79964 i	−26.9749 + 19.1944 i	−89.9388 − 16.1483 i	−144.356 − 139.879 i	−126.266 − 377.33 i	49.351 − 735.771 i
−109.692 − 116.693 i	25.3505 − 82.1743 i	64.0826 + 14.9568 i	−4.67506 + 101.541 i	−160.825 + 105.357 i	−353.522 − 38.304 i	−502.525 − 380.071 i	−500.378 − 949.919 i
−89.6457 − 364.055 i	165.919 − 209.786 i	237.081 + 36.8175 i	110.164 + 267.906 i	−190.242 + 376.688 i	−601.821 + 264.819 i	−1025.88 − 150.194 i	−1329.51 − 927.604 i
69.3293 − 807.552 i	497.243 − 430.789 i	614.555 + 74.3692 i	404.806 + 557.989 i	−102.401 + 871.308 i	−831.921 + 874.96 i	−1664.43 + 447.187 i	−2438.58 − 507.936 i
484.136 − 1524.88 i	1146.82 − 781.362 i	1326.79 + 133.656 i	1004.52 + 1019.94 i	215.141 + 1678.53 i	−952.011 + 1920.83 i	−2354.8 + 1577.59 i	−3800.46 + 508.005 i
1316.61 − 2608.99 i	2288.19 − 1304.73 i	2550.42 + 221.895 i	2080.38 + 1711.29 i	919.284 + 2905.28 i	−828. + 3556.97 i	−2994.35 + 3440.6 i	−5352.56 + 2361.41 i

```
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
Dz[n_, z_] := Sum[dz[j, z], {j, 1, n}]
Dm1[n_, k_] := Sum[Dm1[n / j, k - 1], {j, 2, Floor[n]}]; Dm1[n_, 0] := UnitStep[n - 1]
DzAlt[n_, z_] := Sum[Binomial[z, k] Dm1[n, k], {k, 0, Log[2, n]}]
Grid[Table[Chop[Dz[a = 100, aa = RandomComplex[]] - DzAlt[a, aa]],
  {s, -1.3, 4, .7}, {t, -1.3, 4, .7}]]
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```
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
Dz[n_, z_] := Sum[dz[j, z], {j, 1, n}]
M2[n_, k_] := Sum[MoebiusMu[j] M2[n / j, k - 1], {j, 2, Floor[n]}];
M2[n_, 0] := UnitStep[n - 1]
DzAlt[n_, z_] := Sum[Binomial[-z, k] M2[n, k], {k, 0, Log[2, n]}]
Grid[Table[Chop[Dz[a = 111, s + t I] - D1Alt[a, s + t I]], {s, -1.3, 4, .7}, {t, -1.3, 4, .7}]]
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dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
Dz[n_, z_] := Sum[dz[j, z], {j, 1, n}]
Aa[n_, a_, k_] := Sum[referenced1[j, a] Aa[n / j, a, k - 1], {j, 2, Floor[n]}];
Aa[n_, a_, 0] := UnitStep[n - 1]
DzAlt[n_, z_, a_] := Sum[Binomial[z / a, k] Aa[n, a, k], {k, 0, Log[2, n]}]
Grid[
 Table[Chop[Dz[b = 111, s + 2.3 I] - DzAlt[b, s + 2.3 I, t]], {s, -1.3, 4, .7}, {t, -1.3, 4, .7}]]
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dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
Dz[n_, z_] := Sum[dz[j, z], {j, 1, n}]
F[n_, j_, k_, z_] := If[n < j, 0, ((z + 1) / k - 1) (1 + F[n / j, 2, k + 1, z]) + F[n, j + 1, k, z]]
DzAlt[n_, z_] := 1 + F[n, 2, 1, z]
Grid[Table[Chop[Dz[a = 100, s + t I] - DzAlt[a, s + t I]], {s, -1.3, 4, .7}, {t, -1.3, 4, .7}]]
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```
Dz[n_, z_] := Sum[dz[j, z], {j, 1, n}];
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
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Dz[100, 2]
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```
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
logD[n_, k_] := Sum[MangoldtLambda[j] / Log[j] logD[n / j, k - 1], {j, 2, Floor[n]}];
logD[n_, 0] := UnitStep[n - 1]
DzAlt[n_, z_] := Sum[z^k / k! logD[n, k], {k, 0, Log[2, n]}]
Grid[
 Table[Chop[Dz[a = 123, s + t I, 1] - DzAlt[a, s + t I]], {s, -1.5, 4, .7}, {t, -1.1, 4, .7}]]
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Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
logD[n_, k_] :=
 Sum[FullSimplify[MangoldtLambda[j] / Log[j]] logD[n / j, k - 1], {j, 2, Floor[n]}];
logD[n_, 0] := UnitStep[n - 1]
DzAlt[n_, z_] := Sum[z^k / k! logD[n, k + 1], {k, 0, Log[2, n] - 1}]
Table[Expand[D[Dz[n, z, 1], z]] - DzAlt[n, z], {n, 1, 100}] // TableForm
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Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
logD[n_, k_] :=
 Sum[FullSimplify[MangoldtLambda[j] / Log[j]] logD[n / j, k - 1], {j, 2, Floor[n]}];
logD[n_, 0] := UnitStep[n - 1]
DzAlt[n_, z_, a_] := Sum[z^k / k ! logD[n, k + a], {k, 0, Log[2, n] - a}]
Grid[Table[Expand[D[Dz[n, z, 1], {z, a}]] - DzAlt[n, z, a], {n, 1, 50}, {a, 0, 5}]]
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```
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
DzAlt[n_, z_] := Sum[z^k / k! (Limit[D[Dz[n, y, 1], {y, k}], y → 0]), {k, 0, Log[2, n]}]
Grid[Table[Chop[Dz[a = 143, s + t I, 1] - DzAlt[a, s + t I]], {s, -1.5, 4, .7}, {t, -1.1, 4, .7}]]
```

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```
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
M2[n_, k_] := Sum[MoebiusMu[j] M2[n / j, k - 1], {j, 2, Floor[n]}];
M2[n_, 0] := UnitStep[n - 1]
DzAlt[n_, z_] := Sum[Binomial[-z, k] M2[n, k], {k, 0, Log[2, n]}]
Grid[
 Table[Chop[Dz[a = 111, s + t I, 1] - DzAlt[a, s + t I]], {s, -1.3, 4, .7}, {t, -1.3, 4, .7}]]
```

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```

```
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
Aa[n_, a_, k_] := Sum[dz[j, a] Aa[n / j, a, k - 1], {j, 2, Floor[n]}];
Aa[n_, a_, 0] := UnitStep[n - 1]
DzAlt[n_, z_, a_] := Sum[Binomial[z / a, k] Aa[n, a, k], {k, 0, Log[2, n]}]
Grid[Table[Chop[Dz[b = 111, s + 2.3 I, 1] - DzAlt[b, s + 2.3 I, t]],
  {s, -1.3, 4, .7}, {t, -1.3, 4, .7}]]
```

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```
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
Mm1[n_, k_] := Sum[MoebiusMu[j] Mm1[n / j, k - 1], {j, 2, Floor[n]}];
Mm1[n_, 0] := UnitStep[n - 1]
DzAlt[n_, z_] := Sum[Binomial[-z, k] Mm1[n, k], {k, 0, Log[2, n]}]
Grid[
 Table[Chop[Dz[a = 111, s + t I, 1] - DzAlt[a, s + t I]], {s, -1.3, 4, .7}, {t, -1.3, 4, .7}]]
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```
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
Am1[n_, a_, k_] := Sum[dz[j, a] Am1[n / j, a, k - 1], {j, 2, Floor[n]}];
Am1[n_, a_, 0] := UnitStep[n - 1]
DzAlt[n_, z_, a_] := Sum[Binomial[z / a, k] Am1[n, a, k], {k, 0, Log[2, n]}]
Grid[Table[Chop[Dz[b = 111, s + 2.3 I, 1] - DzAlt[b, s + 2.3 I, t]],
  {s, -1.3, 4, .7}, {t, -1.3, 4, .7}]]
```

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```
Dz[n_, z_] := Sum[dz[j, z], {j, 1, n}];
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {};
Grid[Table[
  {Dz[a = 155, s + t I], N[Sum[(s + t I)^k Residue[Dz[a, m] / (m^(k + 1)), {m, 0}], {k, 0, 50}]]},
  {s, -1.5, 8}, {t, 1, 5}]]
```

```
$Aborted
```

```
RiemannPrimeCount[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
Table[RiemannPrimeCount[n] - Limit[(Dz[n, z, 1] - 1) / z, z → 0], {n, 1, 100}] // TableForm
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RiemannPrimeCount[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Dm1[n_, k_] := Sum[Dm1[n / j, k - 1], {j, 2, Floor[n]}]; Dm1[n_, 0] := UnitStep[n - 1]
AltPrimeCount[n_] := Sum[(-1)^(k + 1) / k Dm1[n, k], {k, 1, Log[2, n]}]
Table[RiemannPrimeCount[n] - AltPrimeCount[n], {n, 1, 100}] // TableForm
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RiemannPrimeCount[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
Table[RiemannPrimeCount[n] - (Limit[Expand[D[Dz[n, z, 1], z]], z → 0]), {n, 1, 100}] //
 TableForm
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RiemannPrimeCount[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
Table[RiemannPrimeCount[n] - Residue[Dz[n, z, 1] / z^2, {z, 0}], {n, 1, 100}] // TableForm

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RiemannPrimeCount[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
(*PAlt is trunctated and stops working after n=2^6-1*)
logDAlt[n_] := Sum[1, {j, 2, n}] - 1 / 2 Sum[1, {j, 2, n}, {k, 2, n / j}] +
  1 / 3 Sum[1, {j, 2, n}, {k, 2, n / j}, {l, 2, n / (j k)}] -
  1 / 4 Sum[1, {j, 2, n}, {k, 2, n / j}, {l, 2, n / (j k)}, {m, 2, n / (j k l)}] +
  1 / 5 Sum[1, {j, 2, n}, {k, 2, n / j}, {l, 2, n / (j k)}, {m, 2, n / (j k l)}, {o, 2, n / (j k l m)}]
Table[RiemannPrimeCount[n] - logDAlt[n], {n, 1, 63}] // TableForm
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```
RiemannPrimeCount[n_] := Sum[PrimePi[n^(1/j)]/j, {j, 1, Log[2, n]}]
Ff[n_, k_] := Sum[1/k - Ff[Floor[n/j], k + 1], {j, 2, n}]
Table[RiemannPrimeCount[n] - Ff[n, 1], {n, 1, 100}] // TableForm
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```
Ff[n_, j_, k_] := If[n < j, 0, 1 / k-Ff[n / j, 2, k + 1] + Ff[n, j + 1, k]]
```

```
RiemannPrimeCount[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Ff[n_, j_, k_] := If[n < j, 0, 1 / k - Ff[n / j, 2, k + 1] + Ff[n, j + 1, k]]
Table[RiemannPrimeCount[n] - Ff[n, 2, 1], {n, 1, 100}] // TableForm
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```
logD[n_, k_] := Sum[FullSimplify[MangoldtLambda[j] / Log[j]] logD[n / j, k - 1],
  {j, 2, Floor[n]}]; logD[n_, 0] := UnitStep[n - 1]
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
Grid[Table[
  FullSimplify[logD[n, k]] - (Limit[D[Dz[n, z, 1], {z, k}], z → 0]), {n, 1, 50}, {k, 1, 5}]]
```

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```
logD[n_, k_] := Limit[D[Dz[n, z, 1], {z, k}], z → 0];
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
```

```
logD[100, 1]
```

$$\frac{428}{15}$$

```
logD[n_, k_] := Limit[D[Dz[n, z, 1], {z, k}], z → 0];
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
D2[n_, k_] := Sum[D2[n / j, k - 1], {j, 2, Floor[n]}]; D2[n_, 0] := UnitStep[n - 1]
logDAlt[n_, j_] :=
 Sum[1 / k ! (Limit[D[Log[1 + y] ^ j, {y, k}], y → 0]) D2[n, k], {k, 0, Log[2, n]}]
Table[logD[n, k] - logDAlt[n, k], {n, 1, 50}, {k, 1, 5}] // TableForm
```

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```
logD[n_, k_] := Limit[D[Dz[n, z, 1], {z, k}], z → 0];
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
Dm1[n_, k_] := Sum[Dm1[n / j, k - 1], {j, 2, Floor[n]}]; Dm1[n_, 0] := UnitStep[n - 1]
logDAlt[n_, j_] :=
 Sum[1 / k ! (Limit[D[Log[1 + y] ^ j, {y, k}], y → 0]) Dm1[n, k], {k, 0, Log[2, n]}]
Table[logD[n, k] - logDAlt[n, k], {n, 1, 50}, {k, 1, 5}] // TableForm
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```
fn[n_, j_] := N[Sum[(k!)^-1 coef[j, k] (n - 1)^k, {k, 0, 150}]];
coef[j_, k_] := coef[j, k] = Limit[D[Log[1 + y]^j, {y, k}], y → 0]
Grid[Table[Chop[Log[n]^k - fn[n, k]], {n, .12, 1.8, .1}, {k, 1, 5}]]
```

-2.18324×10^{-10}	2.46071×10^{-9}	-1.97289×10^{-8}	1.34234×10^{-7}	-8.21432×10^{-7}
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```
logD[n_, k_] := Limit[D[Dz[n, z, 1], {z, k}], z → 0];
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
Grid[Table[logD[n, k] - k ! Residue[Dz[n, z, 1] / z^(k + 1), {z, 0}], {n, 1, 50}, {k, 1, 5}]]
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```
logD[n_, k_] := Sum[FullSimplify[MangoldtLambda[j] / Log[j]] logD[n / j, k - 1],
  {j, 2, Floor[n]}]; logD[n_, 0] := UnitStep[n - 1]
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
Grid[Table[logD[n, k] - (Limit[D[Dz[n, z, 1], {z, k}], z → 0]), {n, 1, 50}, {k, 1, 5}]]
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```
logD[n_, k_] := Limit[D[Dz[n, z, 1], {z, k}], z → 0];
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
Dm1[n_, k_] := Sum[Dm1[n / j, k - 1], {j, 2, n}]; Dm1[n_, 0] := UnitStep[n - 1]
logDAlt[n_, j_] :=
 Sum[1 / k ! (Limit[D[Log[1 + y]^j, {y, k}], y → 0]) Dm1[n, k], {k, 0, Log[2, n]}]
Table[logD[n, k] - logDAlt[n, k], {n, 1, 50}, {k, 1, 5}] // TableForm
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```
Dz[n_, z_] := Sum[dz[j, z], {j, 1, n}];
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
Dm1[n_, k_] := Sum[Dm1[n / j, k - 1], {j, 2, n}]; Dm1[n_, 0] := UnitStep[n - 1]
DzAlt[n_, z_] := Sum[Binomial[z, k] Dm1[n, k], {k, 0, Log[2, n]}]
Grid[Table[Chop[Dz[a = 100, s + t I] - DzAlt[a, s + t I]], {s, -1.3, 4, .7}, {t, -1.3, 4, .7}]]
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```
Dz[n_, z_] := Sum[dz[j, z], {j, 1, n}];
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
D1[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[D1[n / j, z, k + 1], {j, 2, n}]
Grid[Table[{Dz[a = 100, s + t I], D1[a, s + t I, 1]}, {s, -1.3, 4, .7}, {t, -1.3, 4, .7}]]
```

{10.4793 + 28.7121 i, 10.4793 + 28.7121 i}	{5.72468 + 11.2587 i, 5.72468 + 11.2587 i}	{6.03456 - 1.77709 i, 6.03456 - 1.77709 i}	{5.94691 - 15.6189 i, 5.94691 - 15.6189 i}	{15.2681 - 34.9044 i, 15.2681 - 34.9044 i}	{58.5435 - 59.5846 i, 58.5435 - 59.5846 i}	{173.704 - 80.8227 i, 173.704 - 80.8227 i}	{409.891 - 76.8935 i, 409.891 - 76.8935 i}
{-21.9794 + 33.2704 i, -21.9794 + 33.2704 i}	{-9.29577 + 6.6042 i, -9.29577 + 6.6042 i}	{-3.93641 - 0.7025\12 i, -3.93641 - 0.7025\12 i}	{-12.975 - 11.3196 i, -12.975 - 11.3196 i}	{-23.7041 - 47.2133 i, -23.7041 - 47.2133 i}	{-4.16474 - 124.722 i, -4.16474 - 124.722 i}	{95.5007 - 249.632 i, 95.5007 - 249.632 i}	{340.872 - 412.252 i, 340.872 - 412.252 i}
{-70.5899 - 1.50386 i, -70.5899 - 1.50386 i}	{-13.7213 - 18.1133 i, -13.7213 - 18.1133 i}	{3.81025 + 3.79964 i, 3.81025 + 3.79964 i}	{-26.9749 + 19.1944 i, -26.9749 + 19.1944 i}	{-89.9388 - 16.1483 i, -89.9388 - 16.1483 i}	{-144.356 - 139.879 i, -144.356 - 139.879 i}	{-126.266 - 377.33 i, -126.266 - 377.33 i}	{49.351 - 735.771 i, 49.351 - 735.771 i}
{-109.692 - 116.693 i, -109.692 - 116.693 i}	{25.3505 - 82.1743 i, 25.3505 - 82.1743 i}	{64.0826 + 14.9568 i, 64.0826 + 14.9568 i}	{-4.67506 + 101.541 i, -4.67506 + 101.541 i}	{-160.825 + 105.357 i, -160.825 + 105.357 i}	{-353.522 - 38.304 i, -353.522 - 38.304 i}	{-502.525 - 380.071 i, -502.525 - 380.071 i}	{-500.378 - 949.919 i, -500.378 - 949.919 i}

{-89.6457 - 364.055 ⅈ, -89.6457 - 364.055 ⅈ}	{165.919 - 209.786 ⅈ, 165.919 - 209.786 ⅈ}	{237.081 + 36.8175 ⅈ, 237.081 + 36.8175 ⅈ}	{110.164 + 267.906 ⅈ, 110.164 + 267.906 ⅈ}	{-190.242 + 376.688 ⅈ, -190.242 + 376.688 ⅈ}	{-601.821 + 264.819 ⅈ, -601.821 + 264.819 ⅈ}	{-1025.88 - 150.194 ⅈ, -1025.88 - 150.194 ⅈ}	{-1329.51 - 927.604 ⅈ, -1329.51 - 927.604 ⅈ}
{69.3293 - 807.552 ⅈ, 69.3293 - 807.552 ⅈ}	{497.243 - 430.789 ⅈ, 497.243 - 430.789 ⅈ}	{614.555 + 74.3692 ⅈ, 614.555 + 74.3692 ⅈ}	{404.806 + 557.989 ⅈ, 404.806 + 557.989 ⅈ}	{-102.401 + 871.308 ⅈ, -102.401 + 871.308 ⅈ}	{-831.921 + 874.96 ⅈ, -831.921 + 874.96 ⅈ}	{-1664.43 + 447.187 ⅈ, -1664.43 + 447.187 ⅈ}	{-2438.58 - 507.936 ⅈ, -2438.58 - 507.936 ⅈ}
{484.136 - 1524.88 ⅈ, 484.136 - 1524.88 ⅈ}	{1146.82 - 781.362 ⅈ, 1146.82 - 781.362 ⅈ}	{1326.79 + 133.656 ⅈ, 1326.79 + 133.656 ⅈ}	{1004.52 + 1019.94 ⅈ, 1004.52 + 1019.94 ⅈ}	{215.141 + 1678.53 ⅈ, 215.141 + 1678.53 ⅈ}	{-952.011 + 1920.83 ⅈ, -952.011 + 1920.83 ⅈ}	{-2354.8 + 1577.59 ⅈ, -2354.8 + 1577.59 ⅈ}	{-3800.46 + 508.005 ⅈ, -3800.46 + 508.005 ⅈ}
{1316.61 - 2608.99 ⅈ, 1316.61 - 2608.99 ⅈ}	{2288.19 - 1304.73 ⅈ, 2288.19 - 1304.73 ⅈ}	{2550.42 + 221.895 ⅈ, 2550.42 + 221.895 ⅈ}	{2080.38 + 1711.29 ⅈ, 2080.38 + 1711.29 ⅈ}	{919.284 + 2905.28 ⅈ, 919.284 + 2905.28 ⅈ}	{-828. + 3556.97 ⅈ, -828. + 3556.97 ⅈ}	{-2994.35 + 3440.6 ⅈ, -2994.35 + 3440.6 ⅈ}	{-5352.56 + 2361.41 ⅈ, -5352.56 + 2361.41 ⅈ}

```
Dz[n_, z_] := Sum[dz[j, z], {j, 1, n}];
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
D1[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[D1[n / j, z, k + 1], {j, 2, n}]
Grid[Table[Chop[Dz[a = 100, s + t I] - D1[a, s + t I, 1]], {s, -1.3, 4, .7}, {t, -1.3, 4, .7}]]
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0 0 0 0 0 0 0 0
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```
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
Mm1[n_, k_] := Sum[MoebiusMu[j] Mm1[n / j, k - 1], {j, 2, n}]; Mm1[n_, 0] := UnitStep[n - 1]
DzAlt[n_, z_] := Sum[Binomial[-z, k] Mm1[n, k], {k, 0, Log[2, n]}]
Grid[
 Table[Chop[Dz[a = 111, s + t I, 1] - DzAlt[a, s + t I]], {s, -1.3, 4, .7}, {t, -1.3, 4, .7}]]
```

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

```
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
Am1[n_, a_, k_] := Sum[dz[j, a] Am1[n / j, a, k - 1], {j, 2, n}];
Am1[n_, a_, 0] := UnitStep[n - 1]
DzAlt[n_, z_, a_] := Sum[Binomial[z / a, k] Am1[n, a, k], {k, 0, Log[2, n]}]
Grid[Table[Chop[Dz[b = 111, s + 2.3 I, 1] - DzAlt[b, s + 2.3 I, t]],
  {s, -1.3, 4, .7}, {t, -1.3, 4, .7}]]
```

0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

```
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
logD[n_, k_] := Sum[MangoldtLambda[j] / Log[j] logD[n / j, k - 1], {j, 2, n}];
logD[n_, 0] := UnitStep[n - 1]
DzAlt[n_, z_] := Sum[z^k / k ! logD[n, k], {k, 0, Log[2, n]}]
Grid[
 Table[Chop[Dz[a = 123, s + t I, 1] - DzAlt[a, s + t I]], {s, -1.5, 4, .7}, {t, -1.1, 4, .7}]]
```

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0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

```
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
logD[n_, k_] := Sum[FullSimplify[MangoldtLambda[j] / Log[j]] logD[n / j, k - 1], {j, 2, n}];
logD[n_, 0] := UnitStep[n - 1]
DzAlt[n_, z_] := Sum[z^k / k ! logD[n, k + 1], {k, 0, Log[2, n] - 1}]
Table[Expand[D[Dz[n, z, 1], z]] - DzAlt[n, z], {n, 1, 100}] // TableForm
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```
Dk[n_, k_] := Sum[Dk[n / j, k - 1], {j, 1, n}]; Dk[n_, 0] := UnitStep[n - 1]
Table[Dk[n, k], {n, 1, 50}, {k, 1, 7}] // TableForm
```

1	1	1	1	1	1	1
2	3	4	5	6	7	8
3	5	7	9	11	13	15
4	8	13	19	26	34	43
5	10	16	23	31	40	50
6	14	25	39	56	76	99
7	16	28	43	61	82	106
8	20	38	63	96	138	190
9	23	44	73	111	159	218
10	27	53	89	136	195	267
11	29	56	93	141	201	274
12	35	74	133	216	327	470
13	37	77	137	221	333	477
14	41	86	153	246	369	526
15	45	95	169	271	405	575
16	50	110	204	341	531	785
17	52	113	208	346	537	792
18	58	131	248	421	663	988
19	60	134	252	426	669	995
20	66	152	292	501	795	1191
21	70	161	308	526	831	1240
22	74	170	324	551	867	1289
23	76	173	328	556	873	1296
24	84	203	408	731	1209	1884
25	87	209	418	746	1230	1912
26	91	218	434	771	1266	1961
27	95	228	454	806	1322	2045
28	101	246	494	881	1448	2241
29	103	249	498	886	1454	2248
30	111	276	562	1011	1670	2591
31	113	279	566	1016	1676	2598
32	119	300	622	1142	1928	3060
33	123	309	638	1167	1964	3109
34	127	318	654	1192	2000	3158
35	131	327	670	1217	2036	3207
36	140	363	770	1442	2477	3991
37	142	366	774	1447	2483	3998
38	146	375	790	1472	2519	4047
39	150	384	806	1497	2555	4096
40	158	414	886	1672	2891	4684
41	160	417	890	1677	2897	4691
42	168	444	954	1802	3113	5034
43	170	447	958	1807	3119	5041
44	176	465	998	1882	3245	5237
45	182	483	1038	1957	3371	5433
46	186	492	1054	1982	3407	5482
47	188	495	1058	1987	3413	5489
48	198	540	1198	2337	4169	6959
49	201	546	1208	2352	4190	6987
50	207	564	1248	2427	4316	7183

```
Dk1[n_] := Sum[1, {j, 1, n}]; Dk2[n_] := Sum[1, {j, 1, n}, {k, 1, n / j}];
Dk3[n_] := Sum[1, {j, 1, n}, {k, 1, n / j}, {m, 1, n / (j k)}]
Dk[n_, k_] := Sum[Dk[n / j, k - 1], {j, 1, n}]; Dk[n_, 0] := UnitStep[n - 1]
Table[{Dk1[n] - Dk[n, 1], Dk2[n] - Dk[n, 2], Dk3[n] - Dk[n, 3]}, {n, 1, 50}] // TableForm
```

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```
Dm1[n_, k_] := Sum[Dm1[n / j, k - 1], {j, 2, n}]; Dm1[n_, 0] := UnitStep[n - 1]
Table[Dm1[n, k], {n, 1, 50}, {k, 1, 7}] // TableForm
```

0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	1	0	0	0	0	0
4	1	0	0	0	0	0
5	3	0	0	0	0	0
6	3	0	0	0	0	0
7	5	1	0	0	0	0
8	6	1	0	0	0	0
9	8	1	0	0	0	0
10	8	1	0	0	0	0
11	12	4	0	0	0	0
12	12	4	0	0	0	0
13	14	4	0	0	0	0
14	16	4	0	0	0	0
15	19	7	1	0	0	0
16	19	7	1	0	0	0
17	23	10	1	0	0	0
18	23	10	1	0	0	0
19	27	13	1	0	0	0
20	29	13	1	0	0	0
21	31	13	1	0	0	0
22	31	13	1	0	0	0
23	37	22	5	0	0	0
24	38	22	5	0	0	0
25	40	22	5	0	0	0
26	42	23	5	0	0	0
27	46	26	5	0	0	0
28	46	26	5	0	0	0
29	52	32	5	0	0	0
30	52	32	5	0	0	0
31	56	38	9	1	0	0
32	58	38	9	1	0	0
33	60	38	9	1	0	0
34	62	38	9	1	0	0
35	69	50	15	1	0	0
36	69	50	15	1	0	0
37	71	50	15	1	0	0
38	73	50	15	1	0	0
39	79	59	19	1	0	0
40	79	59	19	1	0	0
41	85	65	19	1	0	0
42	85	65	19	1	0	0
43	89	68	19	1	0	0
44	93	71	19	1	0	0
45	95	71	19	1	0	0
46	95	71	19	1	0	0
47	103	89	35	6	0	0
48	104	89	35	6	0	0
49	108	92	35	6	0	0

```
logD[n_, 0] := UnitStep[n - 1]
logD[n_, k_] := Sum[MangoldtLambda[j] / Log[j] logD[n / j, k - 1], {j, 2, n}]
Table[FullSimplify[logD[n, k]], {n, 1, 50}, {k, 1, 5}] // TableForm
```

0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
$\frac{5}{2}$	1	0	0	0
$\frac{7}{2}$	1	0	0	0
$\frac{7}{2}$	3	0	0	0
$\frac{9}{2}$	3	0	0	0
$\frac{29}{6}$	4	1	0	0
$\frac{16}{3}$	5	1	0	0
$\frac{16}{3}$	7	1	0	0
$\frac{19}{3}$	7	1	0	0
$\frac{19}{3}$	8	4	0	0
$\frac{22}{3}$	8	4	0	0
$\frac{22}{3}$	10	4	0	0
$\frac{22}{3}$	12	4	0	0
$\frac{91}{12}$	$\frac{155}{12}$	$\frac{11}{2}$	1	0
$\frac{103}{12}$	$\frac{155}{12}$	$\frac{11}{2}$	1	0
$\frac{103}{12}$	$\frac{167}{12}$	$\frac{17}{2}$	1	0
$\frac{115}{12}$	$\frac{167}{12}$	$\frac{17}{2}$	1	0
$\frac{115}{12}$	$\frac{179}{12}$	$\frac{23}{2}$	1	0
$\frac{115}{12}$	$\frac{203}{12}$	$\frac{23}{2}$	1	0
$\frac{115}{12}$	$\frac{227}{12}$	$\frac{23}{2}$	1	0
$\frac{127}{12}$	$\frac{227}{12}$	$\frac{23}{2}$	1	0
$\frac{127}{12}$	$\frac{235}{12}$	$\frac{29}{2}$	5	0
$\frac{133}{12}$	$\frac{247}{12}$	$\frac{29}{2}$	5	0
$\frac{133}{12}$	$\frac{271}{12}$	$\frac{29}{2}$	5	0
$\frac{137}{12}$	$\frac{283}{12}$	$\frac{31}{2}$	5	0
$\frac{137}{12}$	$\frac{295}{12}$	$\frac{37}{2}$	5	0
$\frac{149}{12}$	$\frac{295}{12}$	$\frac{37}{2}$	5	0
$\frac{149}{12}$	$\frac{295}{12}$	$\frac{49}{2}$	5	0
$\frac{161}{12}$	$\frac{295}{12}$	$\frac{49}{2}$	5	0
$\frac{817}{60}$	$\frac{305}{12}$	$\frac{105}{4}$	7	1
$\frac{817}{60}$	$\frac{329}{12}$	$\frac{105}{4}$	7	1
$\frac{817}{60}$	$\frac{353}{12}$	$\frac{105}{4}$	7	1
$\frac{817}{60}$	$\frac{377}{12}$	$\frac{105}{4}$	7	1
$\frac{817}{60}$	$\frac{383}{12}$	$\frac{117}{4}$	13	1

$\frac{877}{60}$	$\frac{383}{12}$	$\frac{117}{4}$	13	1
$\frac{877}{60}$	$\frac{407}{12}$	$\frac{117}{4}$	13	1
$\frac{877}{60}$	$\frac{431}{12}$	$\frac{117}{4}$	13	1
$\frac{877}{60}$	$\frac{439}{12}$	$\frac{129}{4}$	17	1
$\frac{937}{60}$	$\frac{439}{12}$	$\frac{129}{4}$	17	1
$\frac{937}{60}$	$\frac{439}{12}$	$\frac{153}{4}$	17	1
$\frac{997}{60}$	$\frac{439}{12}$	$\frac{153}{4}$	17	1
$\frac{997}{60}$	$\frac{451}{12}$	$\frac{165}{4}$	17	1
$\frac{997}{60}$	$\frac{463}{12}$	$\frac{177}{4}$	17	1
$\frac{997}{60}$	$\frac{487}{12}$	$\frac{177}{4}$	17	1
$\frac{1057}{60}$	$\frac{487}{12}$	$\frac{177}{4}$	17	1
$\frac{1057}{60}$	$\frac{493}{12}$	47	23	6
$\frac{1087}{60}$	$\frac{505}{12}$	47	23	6
$\frac{1087}{60}$	$\frac{517}{12}$	50	23	6

```
K[n_] := FullSimplify[MangoldtLambda[n] / Log[n]]
logD1[n_] := Sum[K[j], {j, 2, n}]; logD2[n_] := Sum[K[j] K[k], {j, 2, n}, {k, 2, n / j}]
logD3[n_] := Sum[K[j] K[k] K[m], {j, 2, n}, {k, 2, n / j}, {m, 2, n / (j k)}]
logD[n_, k_] := Sum[K[j] logD[n / j, k - 1], {j, 2, n}]; logD[n_, 0] := UnitStep[n - 1]
Table[{logD1[n] - logD[n, 1], logD2[n] - logD[n, 2], logD3[n] - logD[n, 3]}, {n, 1, 50}] //
 TableForm
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```
Dk[n_, k_] := Sum[Dk[n / j, k - 1], {j, 1, n}]; Dk[n_, 0] := UnitStep[n - 1]
Dm1[n_, k_] := Sum[Dm1[n / j, k - 1], {j, 2, n}]; Dm1[n_, 0] := UnitStep[n - 1]
logD[n_, k_] := Sum[FullSimplify[MangoldtLambda[j] / Log[j]] logD[n / j, k - 1], {j, 2, n}];
logD[n_, 0] := UnitStep[n - 1]
Table[{n, Dm1[n, 4], Dm1[n, 5], Dm1[n, 6], logD[n, 4], logD[n, 5],
   logD[n, 6], Dk[n, 4], Dk[n, 5], Dk[n, 6]}, {n, 1, 64}] // TableForm
```

1	0	0	0	0	0	0	1	1	1
2	0	0	0	0	0	0	5	6	7
3	0	0	0	0	0	0	9	11	13
4	0	0	0	0	0	0	19	26	34
5	0	0	0	0	0	0	23	31	40
6	0	0	0	0	0	0	39	56	76
7	0	0	0	0	0	0	43	61	82
8	0	0	0	0	0	0	63	96	138
9	0	0	0	0	0	0	73	111	159
10	0	0	0	0	0	0	89	136	195
11	0	0	0	0	0	0	93	141	201
12	0	0	0	0	0	0	133	216	327
13	0	0	0	0	0	0	137	221	333
14	0	0	0	0	0	0	153	246	369
15	0	0	0	0	0	0	169	271	405
16	1	0	0	1	0	0	204	341	531
17	1	0	0	1	0	0	208	346	537
18	1	0	0	1	0	0	248	421	663
19	1	0	0	1	0	0	252	426	669
20	1	0	0	1	0	0	292	501	795
21	1	0	0	1	0	0	308	526	831
22	1	0	0	1	0	0	324	551	867
23	1	0	0	1	0	0	328	556	873
24	5	0	0	5	0	0	408	731	1209
25	5	0	0	5	0	0	418	746	1230
26	5	0	0	5	0	0	434	771	1266
27	5	0	0	5	0	0	454	806	1322
28	5	0	0	5	0	0	494	881	1448
29	5	0	0	5	0	0	498	886	1454
30	5	0	0	5	0	0	562	1011	1670
31	5	0	0	5	0	0	566	1016	1676
32	9	1	0	7	1	0	622	1142	1928
33	9	1	0	7	1	0	638	1167	1964
34	9	1	0	7	1	0	654	1192	2000
35	9	1	0	7	1	0	670	1217	2036
36	15	1	0	13	1	0	770	1442	2477
37	15	1	0	13	1	0	774	1447	2483
38	15	1	0	13	1	0	790	1472	2519
39	15	1	0	13	1	0	806	1497	2555
40	19	1	0	17	1	0	886	1672	2891
41	19	1	0	17	1	0	890	1677	2897
42	19	1	0	17	1	0	954	1802	3113
43	19	1	0	17	1	0	958	1807	3119
44	19	1	0	17	1	0	998	1882	3245
45	19	1	0	17	1	0	1038	1957	3371
46	19	1	0	17	1	0	1054	1982	3407
47	19	1	0	17	1	0	1058	1987	3413
48	35	6	0	23	6	0	1198	2337	4169

49	35	6	0	23	6	0	1208	2352	4190
50	35	6	0	23	6	0	1248	2427	4316
51	35	6	0	23	6	0	1264	2452	4352
52	35	6	0	23	6	0	1304	2527	4478
53	35	6	0	23	6	0	1308	2532	4484
54	39	6	0	27	6	0	1388	2707	4820
55	39	6	0	27	6	0	1404	2732	4856
56	43	6	0	31	6	0	1484	2907	5192
57	43	6	0	31	6	0	1500	2932	5228
58	43	6	0	31	6	0	1516	2957	5264
59	43	6	0	31	6	0	1520	2962	5270
60	55	6	0	43	6	0	1680	3337	6026
61	55	6	0	43	6	0	1684	3342	6032
62	55	6	0	43	6	0	1700	3367	6068
63	55	6	0	43	6	0	1740	3442	6194
64	65	11	1	$\frac{275}{6}$	$\frac{17}{2}$	1	1824	3652	6656

```
RiemannPrimeCount[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Dm1[n_, k_] := Sum[Dm1[n / j, k - 1], {j, 2, n}]; Dm1[n_, 0] := UnitStep[n - 1]
AltPrimeCount[n_] := Sum[(-1)^(k + 1) / k Dm1[n, k], {k, 1, Log[2, n]}]
Table[RiemannPrimeCount[n] - AltPrimeCount[n], {n, 1, 100}] // TableForm
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```
RiemannPrimeCount[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Ff[n_, k_] := Sum[1 / k - Ff[n / j, k + 1], {j, 2, n}]
Table[RiemannPrimeCount[n] - Ff[n, 1], {n, 1, 100}] // TableForm
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logD[n_, k_] := Sum[FullSimplify[MangoldtLambda[j] / Log[j]] logD[n / j, k - 1], {j, 2, n}];
logD[n_, 0] := UnitStep[n - 1]
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
Grid[Table[logD[n, k] - (Limit[D[Dz[n, z, 1], {z, k}], z → 0]), {n, 1, 50}, {k, 1, 5}]]
```

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```

```
logD[n_, k_] := Sum[FullSimplify[MangoldtLambda[j] / Log[j]] logD[n / j, k - 1], {j, 2, n}];
logD[n_, 0] := UnitStep[n - 1]
Dz[n_, z_] := Sum[z^k / k ! logD[n, k], {k, 0, Log[2, n]}]
Dz[100, z]
```

$$1+\frac{428\,z}{15}+\frac{16\,289\,z^2}{360}+\frac{331\,z^3}{16}+\frac{611\,z^4}{144}+\frac{67\,z^5}{240}+\frac{7\,z^6}{720}$$

```
logD[n_, k_] := Sum[FullSimplify[MangoldtLambda[j] / Log[j]] logD[n / j, k - 1], {j, 2, n}];
logD[n_, 0] := UnitStep[n - 1]
Dz[n_, z_] := Sum[z^k / k ! logD[n, k], {k, 0, Log[2, n]}]
Table[{n, Roots[Dz[n, z] == 0, z]}, {n, 2, 31}] // TableForm
```

2	$z == -1$
3	$z == -\frac{1}{2}$
4	$z == \frac{1}{2}\left(-5-\sqrt{17}\right) \,\|\|\, z == \frac{1}{2}\left(-5+\sqrt{17}\right)$
5	$z == \frac{1}{2}\left(-7-\sqrt{41}\right) \,\|\|\, z == \frac{1}{2}\left(-7+\sqrt{41}\right)$
6	$z == -\frac{1}{3} \,\|\|\, z == -2$
7	$z == \frac{1}{6}\left(-9-\sqrt{57}\right) \,\|\|\, z == \frac{1}{6}\left(-9+\sqrt{57}\right)$
8	$z == \frac{1}{2}\left(-9-\sqrt{73}\right) \,\|\|\, z == \frac{1}{2}\left(-9+\sqrt{73}\right) \,\|\|\, z == -3$
9	$z == -5+\frac{\left(-432+i\sqrt{51\,897}\right)^{1/3}}{3^{2/3}}+\frac{43}{\left(3\left(-432+i\sqrt{51\,897}\right)\right)^{1/3}} \,\|\|\, z == -5-\frac{\left(1+i\sqrt{3}\right)\left(-432+i\sqrt{51\,897}\right)^{1/3}}{2\times 3^{2/3}}-\frac{43\left(1-i\sqrt{3}\right)}{2\left(3\left(-432+i\sqrt{5189}\right.\right.}$
10	$z == -7+\frac{\left(-2106+i\sqrt{127\,389}\right)^{1/3}}{3^{2/3}}+\frac{115}{\left(3\left(-2106+i\sqrt{127\,389}\right)\right)^{1/3}} \,\|\|\, z == -7-\frac{\left(1+i\sqrt{3}\right)\left(-2106+i\sqrt{127\,389}\right)^{1/3}}{2\times 3^{2/3}}-\frac{115\left(\right.}{2\left(3\left(-2106+i\right.\right.}$
11	$z == -7+\frac{\left(-1917+i\sqrt{210\,198}\right)^{1/3}}{3^{2/3}}+\frac{109}{\left(3\left(-1917+i\sqrt{210\,198}\right)\right)^{1/3}} \,\|\|\, z == -7-\frac{\left(1+i\sqrt{3}\right)\left(-1917+i\sqrt{210\,198}\right)^{1/3}}{2\times 3^{2/3}}-\frac{109\left(\right.}{2\left(3\left(-1917+i\right.\right.}$
12	$z == \frac{1}{2}\left(-3-\sqrt{7}\right) \,\|\|\, z == \frac{1}{2}\left(-3+\sqrt{7}\right) \,\|\|\, z == -3$
13	$z == -2+\frac{1}{6}\left(486-6\sqrt{6513}\right)^{1/3}+\frac{\left(81+\sqrt{6513}\right)^{1/3}}{6^{2/3}} \,\|\|\, z == -2-\frac{1}{12}\left(1+i\sqrt{3}\right)\left(486-6\sqrt{6513}\right)^{1/3}-\frac{\left(1-i\sqrt{3}\right.}{}$
14	$z == \frac{1}{2}\left(-5+\frac{\left(-189+2\,i\sqrt{13\,413}\right)^{1/3}}{3^{2/3}}+\frac{31}{\left(3\left(-189+2\,i\sqrt{13\,413}\right)\right)^{1/3}}\right) \,\|\|\, z == -\frac{5}{2}-\frac{\left(1+i\sqrt{3}\right)\left(-189+2\,i\sqrt{13\,413}\right)^{1/3}}{4\times 3^{2/3}}-\frac{3}{4\left(3\left(-18\right.\right.}$
15	$z == -3+\frac{\left(-405+i\sqrt{32\,583}\right)^{1/3}}{6^{2/3}}+\frac{16\times 2^{2/3}}{\left(3\left(-405+i\sqrt{32\,583}\right)\right)^{1/3}} \,\|\|\, z == -3-\frac{\left(1+i\sqrt{3}\right)\left(-405+i\sqrt{32\,583}\right)^{1/3}}{2\times 6^{2/3}}-\frac{8\times 2^{2/3}\left(1-i\sqrt{3}\right.}{\left(3\left(-405+i\sqrt{32\,583}\right.\right.}$
16	$z == -\frac{11}{2}-\frac{1}{2\sqrt{\frac{3}{53+\left(1\,401\,155-18\sqrt{314\,550\,701}\right)^{1/3}+\left(1\,401\,155+18\sqrt{314\,550\,701}\right)^{1/3}}}}-\frac{1}{2}\sqrt{\frac{106}{3}-\frac{1}{3}\left(1\,401\,155-18\sqrt{314\,550\,701}\right)^{1/3}-}$
17	$z == -\frac{11}{2}-\frac{1}{2\sqrt{\frac{3}{53+\left(1\,158\,587-18\sqrt{343\,927\,669}\right)^{1/3}+\left(1\,158\,587+18\sqrt{343\,927\,669}\right)^{1/3}}}}-\frac{1}{2}\sqrt{\frac{106}{3}-\frac{1}{3}\left(1\,158\,587-18\sqrt{343\,927\,669}\right)^{1/3}-}$
18	$z == -\frac{17}{2}+\frac{1}{2}\sqrt{\frac{1}{3}\left(533+\frac{7165}{\left(197\,099+18\,i\sqrt{1\,015\,380\,251}\right)^{1/3}}+\left(197\,099+18\,i\sqrt{1\,015\,380\,251}\right)^{1/3}\right)}-\frac{1}{2}\sqrt{\frac{1066}{3}-}$

19 $$z == \frac{1}{3}\left(-31+\frac{739}{\left(-19576+9\,i\sqrt{251403}\right)^{1/3}}+\left(-19576+9\,i\sqrt{251403}\right)^{1/3}\right) \;||\; z == -\frac{31}{3}-\frac{739\left(1+i\sqrt{3}\right)}{6\left(-19576+9\,i\sqrt{251403}\right)^{1/3}}$$

20 $$z == -\frac{23}{2}+\frac{1}{2\sqrt{\frac{3}{1229-\frac{589}{\left(1541773-18\sqrt{7335986565}\right)^{1/3}}-\left(1541773-18\sqrt{7335986565}\right)^{1/3}}}}-\frac{1}{2}\sqrt{\frac{2458}{3}+\frac{589}{3\left(1541773-18\sqrt{7335986565}\right)^{1/3}}+\frac{1}{3}\,(1\,54}$$

21 $$z == \frac{1}{3}\left(-43+\frac{1627}{\left(-65296+9\,i\sqrt{534707}\right)^{1/3}}+\left(-65296+9\,i\sqrt{534707}\right)^{1/3}\right) \;||\; z == -\frac{43}{3}-\frac{1627\left(1+i\sqrt{3}\right)}{6\left(-65296+9\,i\sqrt{534707}\right)^{1/3}}$$

22 $$z == -\frac{23}{2}+\frac{1}{2}\sqrt{\frac{1}{3}\left(1133+\frac{20077}{\left(2093219+18\,i\sqrt{11454291403}\right)^{1/3}}+\left(2093219+18\,i\sqrt{11454291403}\right)^{1/3}\right)}-\frac{1}{2}$$

23 $$z == -\frac{23}{2}+\frac{1}{2}\sqrt{\frac{1}{3}\left(1133+\frac{16765}{\left(1122299+18\,i\sqrt{10655874851}\right)^{1/3}}+\left(1122299+18\,i\sqrt{10655874851}\right)^{1/3}\right)}-\frac{1}{2}$$

24 $$z == -\frac{29}{10}-\frac{1}{10\sqrt{\frac{3}{173+5\left(1828351-18\sqrt{4334066733}\right)^{1/3}+5\left(1828351+18\sqrt{4334066733}\right)^{1/3}}}}-\frac{1}{2}\sqrt{\frac{346}{75}-\frac{1}{15}\,(1828351-18\sqrt{4334066733}}$$

25 $$z == -\frac{29}{10}-\frac{1}{10\sqrt{\frac{3}{53+5\left(2719927-18\sqrt{9796208621}\right)^{1/3}+5\left(2719927+18\sqrt{9796208621}\right)^{1/3}}}}-\frac{1}{2}\sqrt{\frac{106}{75}-\frac{1}{15}\,(2719927-18\sqrt{9796208621}}$$

26 $$z == -\frac{29}{10}-\frac{1}{10\sqrt{\frac{3}{-187+5\left(5783311-18\sqrt{31050589477}\right)^{1/3}+5\left(5783311+18\sqrt{31050589477}\right)^{1/3}}}}-\frac{1}{2}\sqrt{-\frac{374}{75}-\frac{1}{15}\,(5783311-18\sqrt{3105058}}$$

27 $$z == -\frac{31}{10}-\frac{1}{10\sqrt{\frac{3}{53+5\left(6121495-90\sqrt{1101023509}\right)^{1/3}+5\left(5\left(1224299+18\sqrt{1101023509}\right)\right)^{1/3}}}}-\frac{1}{2}\sqrt{\frac{106}{75}-\frac{1}{15}\,(6121495-90\sqrt{110102350}}$$

28 $$z == -\frac{37}{10}+\frac{1}{10}\sqrt{\frac{1}{3}\left(1157+\frac{138185}{\left(4323439+18\,i\sqrt{7460246843}\right)^{1/3}}+5\left(4323439+18\,i\sqrt{7460246843}\right)^{1/3}\right)}-\frac{1}{2}$$

29 $$z == -\frac{37}{10}+\frac{1}{10}\sqrt{\frac{1}{3}\left(1157+\frac{111545}{\left(2892439+18\,i\sqrt{8446900867}\right)^{1/3}}+5\left(2892439+18\,i\sqrt{8446900867}\right)^{1/3}\right)}-\frac{1}{2}$$

30 $$z == -\frac{49}{10}+\frac{1}{10\sqrt{\frac{3}{4253-\frac{4265}{\left(5264369-18\sqrt{85533828141}\right)^{1/3}}-5\left(5264369-18\sqrt{85533828141}\right)^{1/3}}}}-\frac{1}{2}\sqrt{\frac{8506}{75}+\frac{853}{15\left(5264369-18\sqrt{85533828141}\right)^{1/3}}+\frac{1}{15}}$$

31 $$z == -\frac{49}{10}+\frac{1}{10\sqrt{\frac{3}{4253-\frac{31015}{\left(-7382249+18\sqrt{168939118597}\right)^{1/3}}+5\left(-7382249+18\sqrt{168939118597}\right)^{1/3}}}}-\frac{1}{2}\sqrt{\frac{8506}{75}+\frac{6203}{15\left(-7382249+18\sqrt{168939118597}\right)^{1/3}}-}$$

```
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
zeros[n_] := List @@ NRoots[Dz[n, z, 1] == 0, z][[All, 2]]
DzAlt[n_, z_] := Product[1 - z / r, {r, zeros[n]}]
Grid[
 Table[Chop[Dz[a = 111, s + t I, 1] - DzAlt[a, s + t I]], {s, -1.3, 4, .7}, {t, -1.3, 4, .7}]]
```

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

```
logD[n_, k_] := Sum[FullSimplify[MangoldtLambda[j] / Log[j]] logD[n / j, k - 1], {j, 2, n}];
logD[n_, 0] := UnitStep[n - 1]
Dz[n_, z_] := Sum[z^k / k ! logD[n, k], {k, 0, Log[2, n] 10}]
Dz[100, z]
```

$$1+\frac{428\,z}{15}+\frac{16\,289\,z^2}{360}+\frac{331\,z^3}{16}+\frac{611\,z^4}{144}+\frac{67\,z^5}{240}+\frac{7\,z^6}{720}$$

```
logD[n_, k_] := Sum[FullSimplify[MangoldtLambda[j] / Log[j]] logD[n / j, k - 1], {j, 2, n}];
logD[n_, 0] := UnitStep[n - 1]
Dz[n_, z_] := Sum[z^k / k ! logD[n, k], {k, 0, Log[2, n]}]
Table[{n, Roots[Dz[n, z] == 0, z]}, {n, 2, 31}] // TableForm
```

2	$z == -1$
3	$z == -\frac{1}{2}$
4	$z == \frac{1}{2}\left(-5-\sqrt{17}\right) \;\|\|\; z == \frac{1}{2}\left(-5+\sqrt{17}\right)$
5	$z == \frac{1}{2}\left(-7-\sqrt{41}\right) \;\|\|\; z == \frac{1}{2}\left(-7+\sqrt{41}\right)$
6	$z == -\frac{1}{3} \;\|\|\; z == -2$
7	$z == \frac{1}{6}\left(-9-\sqrt{57}\right) \;\|\|\; z == \frac{1}{6}\left(-9+\sqrt{57}\right)$
8	$z == \frac{1}{2}\left(-9-\sqrt{73}\right) \;\|\|\; z == \frac{1}{2}\left(-9+\sqrt{73}\right) \;\|\|\; z == -3$
9	$z == -5+\frac{\left(-432+i\sqrt{51\,897}\right)^{1/3}}{3^{2/3}}+\frac{43}{\left(3\left(-432+i\sqrt{51\,897}\right)\right)^{1/3}} \;\|\|\; z == -5-\frac{\left(1+i\sqrt{3}\right)\left(-432+i\sqrt{51\,897}\right)^{1/3}}{2\times 3^{2/3}}-\frac{43\left(1-i\sqrt{3}\right)}{2\left(3\left(-432+i\sqrt{5189\ldots\right.\right.}}$
10	$z == -7+\frac{\left(-2106+i\sqrt{127\,389}\right)^{1/3}}{3^{2/3}}+\frac{115}{\left(3\left(-2106+i\sqrt{127\,389}\right)\right)^{1/3}} \;\|\|\; z == -7-\frac{\left(1+i\sqrt{3}\right)\left(-2106+i\sqrt{127\,389}\right)^{1/3}}{2\times 3^{2/3}}-\frac{115\,(\ldots}{2\left(3\left(-2106+i\ldots\right.\right.}$
11	$z == -7+\frac{\left(-1917+i\sqrt{210\,198}\right)^{1/3}}{3^{2/3}}+\frac{109}{\left(3\left(-1917+i\sqrt{210\,198}\right)\right)^{1/3}} \;\|\|\; z == -7-\frac{\left(1+i\sqrt{3}\right)\left(-1917+i\sqrt{210\,198}\right)^{1/3}}{2\times 3^{2/3}}-\frac{109\,(\ldots}{2\left(3\left(-1917+i\ldots\right.\right.}$
12	$z == \frac{1}{2}\left(-3-\sqrt{7}\right) \;\|\|\; z == \frac{1}{2}\left(-3+\sqrt{7}\right) \;\|\|\; z == -3$
13	$z == -2+\frac{1}{6}\left(486-6\sqrt{6513}\right)^{1/3}+\frac{\left(81+\sqrt{6513}\right)^{1/3}}{6^{2/3}} \;\|\|\; z == -2-\frac{1}{12}\left(1+i\sqrt{3}\right)\left(486-6\sqrt{6513}\right)^{1/3}-\frac{\left(1-i\sqrt{3}\ldots\right.}{}$
14	$z == \frac{1}{2}\left(-5+\frac{\left(-189+2\,i\sqrt{13\,413}\right)^{1/3}}{3^{2/3}}+\frac{31}{\left(3\left(-189+2\,i\sqrt{13\,413}\right)\right)^{1/3}}\right) \;\|\|\; z == -\frac{5}{2}-\frac{\left(1+i\sqrt{3}\right)\left(-189+2\,i\sqrt{13\,413}\right)^{1/3}}{4\times 3^{2/3}}-\frac{3\ldots}{4\left(3\left(-18\ldots\right.\right.}$

15 $z == -3+\frac{(-405+i\sqrt{32583})^{1/3}}{6^{2/3}}+\frac{16\times 2^{2/3}}{(3(-405+i\sqrt{32583}))^{1/3}} \mid\mid z == -3-\frac{(1+i\sqrt{3})(-405+i\sqrt{32583})^{1/3}}{2\times 6^{2/3}}-\frac{8\times 2^{2/3}(1-i\sqrt{3}\cdots}{(3(-405+i\sqrt{32583}\cdots}$

16 $z == -\frac{11}{2}-\frac{1}{2\sqrt{\frac{3}{53+(1401155-18\sqrt{314550701})^{1/3}+(1401155+18\sqrt{314550701})^{1/3}}}}-\frac{1}{2}\sqrt{\frac{106}{3}-\frac{1}{3}(1401155-18\sqrt{314550701})^{1/3}-\cdots}$

17 $z == -\frac{11}{2}-\frac{1}{2\sqrt{\frac{3}{53+(1158587-18\sqrt{343927669})^{1/3}+(1158587+18\sqrt{343927669})^{1/3}}}}-\frac{1}{2}\sqrt{\frac{106}{3}-\frac{1}{3}(1158587-18\sqrt{343927669})^{1/3}-\cdots}$

18 $z == -\frac{17}{2}+\frac{1}{2}\sqrt{\frac{1}{3}\left(533+\frac{7165}{(197099+18i\sqrt{1015380251})^{1/3}}+(197099+18i\sqrt{1015380251})^{1/3}\right)}-\frac{1}{2}\sqrt{\frac{1066}{3}-\cdots}$

19 $z == \frac{1}{3}\left(-31+\frac{739}{(-19576+9i\sqrt{251403})^{1/3}}+(-19576+9i\sqrt{251403})^{1/3}\right) \mid\mid z == -\frac{31}{3}-\frac{739(1+i\sqrt{3})}{6(-19576+9i\sqrt{251403})^{1/3}}\cdots$

20 $z == -\frac{23}{2}+\frac{1}{2\sqrt{\frac{3}{1229-\frac{589}{(1541773-18\sqrt{7335986565})^{1/3}}-(1541773-18\sqrt{7335986565})^{1/3}}}}-\frac{1}{2}\sqrt{\frac{2458}{3}+\frac{589}{3(1541773-18\sqrt{7335986565})^{1/3}}+\frac{1}{3}(154\cdots}$

21 $z == \frac{1}{3}\left(-43+\frac{1627}{(-65296+9i\sqrt{534707})^{1/3}}+(-65296+9i\sqrt{534707})^{1/3}\right) \mid\mid z == -\frac{43}{3}-\frac{1627(1+i\sqrt{3})}{6(-65296+9i\sqrt{534707})^{1/3}}\cdots$

22 $z == -\frac{23}{2}+\frac{1}{2}\sqrt{\frac{1}{3}\left(1133+\frac{20077}{(2093219+18i\sqrt{11454291403})^{1/3}}+(2093219+18i\sqrt{11454291403})^{1/3}\right)}-\frac{1}{2}\sqrt{\cdots}$

23 $z == -\frac{23}{2}+\frac{1}{2}\sqrt{\frac{1}{3}\left(1133+\frac{16765}{(1122299+18i\sqrt{10655874851})^{1/3}}+(1122299+18i\sqrt{10655874851})^{1/3}\right)}-\frac{1}{2}\sqrt{\cdots}$

24 $z == -\frac{29}{10}-\frac{1}{10\sqrt{\frac{3}{173+5(1828351-18\sqrt{4334066733})^{1/3}+5(1828351+18\sqrt{4334066733})^{1/3}}}}-\frac{1}{2}\sqrt{\frac{346}{75}-\frac{1}{15}(1828351-18\sqrt{4334066733\cdots}}$

25 $z == -\frac{29}{10}-\frac{1}{10\sqrt{\frac{3}{53+5(2719927-18\sqrt{9796208621})^{1/3}+5(2719927+18\sqrt{9796208621})^{1/3}}}}-\frac{1}{2}\sqrt{\frac{106}{75}-\frac{1}{15}(2719927-18\sqrt{9796208621\cdots}}$

26 $z == -\frac{29}{10}-\frac{1}{10\sqrt{\frac{3}{-187+5(5783311-18\sqrt{31050589477})^{1/3}+5(5783311+18\sqrt{31050589477})^{1/3}}}}-\frac{1}{2}\sqrt{-\frac{374}{75}-\frac{1}{15}(5783311-18\sqrt{3105058\cdots}}$

27 $z == -\frac{31}{10}-\frac{1}{10\sqrt{\frac{3}{53+5(6121495-90\sqrt{1101023509})^{1/3}+5(5(1224299+18\sqrt{1101023509}))^{1/3}}}}-\frac{1}{2}\sqrt{\frac{106}{75}-\frac{1}{15}(6121495-90\sqrt{110102350\cdots}}$

28 $z == -\frac{37}{10} + \frac{1}{10}\sqrt{\frac{1}{3}\left(1157 + \frac{138185}{\left(4323439+18\,i\sqrt{7460246843}\right)^{1/3}} + 5\left(4323439+18\,i\sqrt{7460246843}\right)^{1/3}\right)} - \frac{1}{2}\sqrt{\cdots}$

29 $z == -\frac{37}{10} + \frac{1}{10}\sqrt{\frac{1}{3}\left(1157 + \frac{111545}{\left(2892439+18\,i\sqrt{8446900867}\right)^{1/3}} + 5\left(2892439+18\,i\sqrt{8446900867}\right)^{1/3}\right)} - \frac{1}{2}\sqrt{\cdots}$

30 $z == -\frac{49}{10} + \frac{1}{10\sqrt{\frac{3}{4253-\frac{4265}{\left(5264369-18\sqrt{85533828141}\right)^{1/3}}-5\left(5264369-18\sqrt{85533828141}\right)^{1/3}}}} - \frac{1}{2}\sqrt{\frac{8506}{75} + \frac{853}{15\left(5264369-18\sqrt{85533828141}\right)^{1/3}} + \frac{1}{15}\cdots}$

31 $z == -\frac{49}{10} + \frac{1}{10\sqrt{\frac{3}{4253-\frac{31015}{\left(-7382249+18\sqrt{168939118597}\right)^{1/3}}+5\left(-7382249+18\sqrt{168939118597}\right)^{1/3}}}} - \frac{1}{2}\sqrt{\frac{8506}{75} + \frac{6203}{15\left(-7382249+18\sqrt{168939118597}\right)^{1/3}} - \cdots}$

```
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
zeros[n_] := List @@ Roots[Dz[n, z, 1] == 0, z][[All, 2]]
DzAlt[n_, z_] := Product[1 - z / r, {r, zeros[n]}]
Grid[Table[Chop[Dz[a = 111, s + t I] - DzAlt[a, s + t I]], {s, -1.3, 4, .7}, {t, -1.3, 4, .7}]]
```

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

```
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
zeros[n_] := List @@ Roots[Dz[n, z, 1] == 0, z][[All, 2]]
DzAlt[n_, z_] := Product[1 - z / r, {r, zeros[n]}]
DzAlt2[n_, z_] := n Product[1 - (z - 1) / (r - 1), {r, zeros[n]}]
Grid[Table[Chop[Dz[a = 111, s + t I] - DzAlt[a, s + t I]], {s, -1.3, 4, .7}, {t, -1.3, 4, .7}]]
Grid[Table[Chop[Dz[a = 131, s + t I] - DzAlt2[a, s + t I]], {s, -1.1, 4, .7}, {t, -1.4, 4, .7}]]
```

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0

```
RiemannPrimeCount[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
zeros[n_] := List @@ Roots[Dz[n, z, 1] == 0, z][[All, 2]]
logD[n_] := -Sum[1 / r, {r, zeros[n]}]
Table[RiemannPrimeCount[n] - logD[n], {n, 4, 100}] // TableForm
```

A very large output was generated. Here is a sample of it:

≪1≫

Show Less | Show More | Show Full Output | Set Size Limit...

```
logD[10]
```

$$-\frac{1}{-7+\frac{\left(-2106+i\sqrt{127389}\right)^{1/3}}{3^{2/3}}+\frac{115}{\left(3\left(-2106+i\sqrt{127389}\right)\right)^{1/3}}}-\frac{1}{-7-\frac{\left(1+i\sqrt{3}\right)\left(-2106+i\sqrt{127389}\right)^{1/3}}{2\times 3^{2/3}}-\frac{115\left(1-i\sqrt{3}\right)}{2\left(3\left(-2106+i\sqrt{127389}\right)\right)^{1/3}}}-\frac{1}{-7-\frac{\left(1-i\sqrt{3}\right)\left(-2106+i\sqrt{127389}\right)^{1/3}}{2\times 3^{2/3}}-\frac{115\left(1+i\sqrt{3}\right)}{2\left(3\left(-2106+i\sqrt{127389}\right)\right)^{1/3}}}$$

```
zeros[10]
```

$$\left\{-7+\frac{\left(-2106+i\sqrt{127389}\right)^{1/3}}{3^{2/3}}+\frac{115}{\left(3\left(-2106+i\sqrt{127389}\right)\right)^{1/3}},\right.$$
$$-7-\frac{\left(1+i\sqrt{3}\right)\left(-2106+i\sqrt{127389}\right)^{1/3}}{2\times 3^{2/3}}-\frac{115\left(1-i\sqrt{3}\right)}{2\left(3\left(-2106+i\sqrt{127389}\right)\right)^{1/3}},$$
$$\left.-7-\frac{\left(1-i\sqrt{3}\right)\left(-2106+i\sqrt{127389}\right)^{1/3}}{2\times 3^{2/3}}-\frac{115\left(1+i\sqrt{3}\right)}{2\left(3\left(-2106+i\sqrt{127389}\right)\right)^{1/3}}\right\}$$

```
zeros2[10]
```

$$\left\{-7+\frac{\left(-2106+i\sqrt{127389}\right)^{1/3}}{3^{2/3}}+\frac{115}{\left(3\left(-2106+i\sqrt{127389}\right)\right)^{1/3}},\right.$$
$$-7-\frac{\left(1+i\sqrt{3}\right)\left(-2106+i\sqrt{127389}\right)^{1/3}}{2\times 3^{2/3}}-\frac{115\left(1-i\sqrt{3}\right)}{2\left(3\left(-2106+i\sqrt{127389}\right)\right)^{1/3}},$$
$$\left.-7-\frac{\left(1-i\sqrt{3}\right)\left(-2106+i\sqrt{127389}\right)^{1/3}}{2\times 3^{2/3}}-\frac{115\left(1+i\sqrt{3}\right)}{2\left(3\left(-2106+i\sqrt{127389}\right)\right)^{1/3}}\right\}$$

```
ReferenceRiemannPrimeCnt[n_] := Sum[MangoldtLambda[j] / Log[j], {j, 2, n}]
P2[n_, k_] := Sum[FullSimplify[MangoldtLambda[j] / Log[j]] P2[n / j, k - 1],
  {j, 2, Floor[n]}]; P2[n_, 0] := UnitStep[n - 1]
D1[n_, z_] := Sum[z^k / k ! P2[n, k], {k, 0, Log[2, n]}]
zeros[n_] := List @@ NRoots[D1[n, z] == 0, z][[All, 2]]
zeros2[n_] := List @@ Roots[D1[n, z] == 0, z][[All, 2]]
P21Alt[n_] := -Sum[1 / r, {r, zeros[n]}]
Table[{N[ReferenceRiemannPrimeCnt[n]], P21Alt[n]}, {n, 4, 100}] // TableForm
```

2.5	2.5
3.5	3.5
3.5	3.5
4.5	4.5
4.83333	4.83333
5.33333	5.33333
5.33333	5.33333
6.33333	6.33333
6.33333	6.33333
7.33333	7.33333 + 0. i
7.33333	7.33333
7.33333	7.33333
7.58333	7.58333 + 0. i
8.58333	8.58333 + 0. i
8.58333	8.58333
9.58333	9.58333
9.58333	9.58333 + 0. i

9.58333	9.58333
9.58333	9.58333
10.5833	10.5833
10.5833	10.5833 + 0. i
11.0833	11.0833 + 0. i
11.0833	11.0833 + 0. i
11.4167	11.4167 + 0. i
11.4167	11.4167
12.4167	12.4167
12.4167	12.4167 + 0. i
13.4167	13.4167 + 0. i
13.6167	13.6167 + 0. i
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16.6167	16.6167
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17.6167	17.6167 + 0. i
17.6167	17.6167 + 0. i
18.1167	18.1167 + 0. i
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18.1167	18.1167 + 0. i
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20.1167	20.1167 + 0. i
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21.1167	21.1167 + 0. i
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21.2833	21.2833 + 0. i
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22.2833	22.2833 + 0. i
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24.2833	24.2833 + 0. i
24.2833	24.2833 + 0. i
24.2833	24.2833 + 0. i
24.2833	24.2833 + 0. i

```
24.2833    24.2833 + 0. i
24.2833    24.2833 + 0. i
25.2833    25.2833 + 0. i
25.2833    25.2833 + 0. i
25.5333    25.5333 + 0. i
25.5333    25.5333 + 0. i
26.5333    26.5333 + 0. i
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26.5333    26.5333 + 0. i
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28.5333    28.5333 + 0. i
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```

```
RiemannPrimeCount[n_] := Sum[PrimePi[n^(1/j)] / j, {j, 1, Log[2, n]}]
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
zeros[n_] := List @@ Roots[Dz[n, z, 1] == 0, z][[All, 2]]
logD[n_] := FullSimplify[-Sum[1 / r, {r, zeros[n]}]]
Table[RiemannPrimeCount[n] - logD[n], {n, 4, 25}] // TableForm
```

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```
RiemannPrimeCount[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
zeros[n_] := List @@ Roots[Dz[n, z, 1] == 0, z][[All, 2]]
logD[n_] := FullSimplify[-Sum[1 / r, {r, zeros[n]}]]
Table[RiemannPrimeCount[n] - logD[n], {n, 4, 25}] // TableForm
```

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```
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
zeros[n_] := List @@ NRoots[Dz[n, z, 1] == 0, z][[All, 2]]
prod[n_, z_] := Product[1 - z / r, {r, zeros[n]}]
Table[
  {Chop[Sum[MoebiusMu[j], {j, 1, n}] - prod[n, -1]], Chop[1 - prod[n, 0]], Chop[n - prod[n, 1]],
   Chop[Sum[1, {j, 1, n}, {k, 1, Floor[n / j]}] - prod[n, 2]]}, {n, 4, 100}] // TableForm
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```
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
zeros[n_] := List @@ NRoots[Dz[n, z, 1] == 0, z][[All, 2]]
prod[n_, z_] := n Product[1 - (z - 1) / (r - 1), {r, zeros[n]}]
Table[
  {Chop[Sum[MoebiusMu[j], {j, 1, n}] - prod[n, -1]], Chop[1 - prod[n, 0]], Chop[n - prod[n, 1]],
   Chop[Sum[1, {j, 1, n}, {k, 1, Floor[n / j]}] - prod[n, 2]]}, {n, 4, 100}] // TableForm
```

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```
RiemannPrimeCount[n_] := Sum[PrimePi[n^(1/j)]/j, {j, 1, Log[2, n]}]
Dz[n_, z_, k_] := 1 + ((z + 1)/k - 1) Sum[Dz[n/j, z, k + 1], {j, 2, n}]
zeros[n_] := List @@ NRoots[(Dz[n, z, 1] - 1)/z == 0, z][[All, 2]]
logD[n_] := (n - 1) Product[1 + 1/(r - 1), {r, zeros[n]}]
Table[Chop[RiemannPrimeCount[n] - logD[n]], {n, 8, 60}]
```

```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

```
Dz[n_, z_, k_] := 1 + ((z + 1)/k - 1) Sum[Dz[n/j, z, k + 1], {j, 2, n}]
zeros[n_] := List @@ NRoots[Dz[n, z, 1] == 0, z][[All, 2]]
prod[n_, z_] := Product[1 - z/r, {r, zeros[n]}]
Table[
  {Chop[Sum[MoebiusMu[j], {j, 1, n}] - prod[n, -1]], Chop[1 - prod[n, 0]], Chop[n - prod[n, 1]],
   Chop[Sum[1, {j, 1, n}, {k, 1, Floor[n/j]}] - prod[n, 2]]}, {n, 4, 100}] // TableForm
prod2[n_, z_] := n Product[1 - (z - 1)/(r - 1), {r, zeros[n]}]
Table[{Chop[Sum[MoebiusMu[j], {j, 1, n}] - prod2[n, -1]],
   Chop[1 - prod2[n, 0]], Chop[n - prod2[n, 1]],
   Chop[Sum[1, {j, 1, n}, {k, 1, Floor[n/j]}] - prod2[n, 2]]}, {n, 4, 100}] // TableForm
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```
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
zeros[n_] := List @@ NRoots[(Dz[n, z, 1] - 1) / z == 0, z][[All, 2]]
prod[n_, z_] := 1 + z (n - 1) Product[1 - (z - 1) / (r - 1), {r, zeros[n]}]
Table[{Chop[Sum[MoebiusMu[j], {j, 1, n}] - prod[n, -1]], Chop[n - prod[n, 1]],
   Chop[Sum[1, {j, 1, n}, {k, 1, Floor[n / j]}] - prod[n, 2]]}, {n, 8, 100}] // TableForm
prod2[n_, z_] := 1 + z Limit[D[ Dz[n, y, 1], y], y → 0] Product[1 - z / r, {r, zeros[n]}]
Table[{Chop[Sum[MoebiusMu[j], {j, 1, n}] - prod2[n, -1]], Chop[n - prod2[n, 1]],
   Chop[Sum[1, {j, 1, n}, {k, 1, Floor[n / j]}] - prod2[n, 2]]}, {n, 8, 100}] // TableForm
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```
Series[(Log[1 + z] + 1)^4, {z, 0, 20}]
```

$$1 + 4 z + 4 z^2 - \frac{2 z^3}{3} - \frac{z^4}{2} + \frac{4 z^5}{5} - \frac{23 z^6}{30} + \frac{127 z^7}{210} - \frac{223 z^8}{560} + \frac{173 z^9}{945} + \frac{947 z^{10}}{37\,800} - \frac{91\,361 z^{11}}{415\,800} + \frac{397\,961 z^{12}}{997\,920} - \frac{6\,076\,799 z^{13}}{10\,810\,800} + \frac{21\,504\,823 z^{14}}{30\,270\,240} - \frac{959\,074\,397 z^{15}}{1\,135\,134\,000} + \frac{119\,446\,409 z^{16}}{123\,552\,000} - \frac{1\,259\,904\,797 z^{17}}{1\,169\,532\,000} + \frac{24\,789\,542\,959 z^{18}}{21\,051\,576\,000} - \frac{19\,381\,860\,529 z^{19}}{15\,277\,011\,750} + \frac{471\,969\,850\,249 z^{20}}{349\,188\,840\,000} + O[z]^{21}$$

```
logD[n_, k_] := Limit[D[Dz[n, z, 1], {z, k}], z → 0];
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k !
logDp1[n_, z_] := Sum[bin[z, k] logD[n, k], {k, 0, Log[2, n]}]
Series[ (Log[1 + z]) ^1, {z, 0, 20}]
```

$$z - \frac{z^2}{2} + \frac{z^3}{3} - \frac{z^4}{4} + \frac{z^5}{5} - \frac{z^6}{6} + \frac{z^7}{7} - \frac{z^8}{8} + \frac{z^9}{9} - \frac{z^{10}}{10} + \frac{z^{11}}{11} - \frac{z^{12}}{12} + \frac{z^{13}}{13} - \frac{z^{14}}{14} + \frac{z^{15}}{15} - \frac{z^{16}}{16} + \frac{z^{17}}{17} - \frac{z^{18}}{18} + \frac{z^{19}}{19} - \frac{z^{20}}{20} + O[z]^{21}$$

```
logDp1[ 100, 2]
```

$$\frac{26741}{180}$$

```
logDalt[100, 2]
```

$$\frac{428}{15}$$

```
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k !
logD[n_, k_] := Limit[D[Dz[n, z, 1], {z, k}], z → 0];
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
logDplus1[x_, z_] := Sum[bin[z, k] logD[x, k], {k, 0, Log[2, x]}]
Expand[logDplus1[12, z]]
```

$$1 + \frac{11 z}{3} + 2 z^2 + \frac{2 z^3}{3}$$

```
RiemannPrimeCount[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
logD[n_, k_] := Limit[D[Dz[n, z, 1], {z, k}], z → 0];
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
logDplus1[x_, z_] := Sum[Binomial[z, k] logD[x, k], {k, 0, Log[2, x]}]
zeros[n_] := List @@ NRoots[logDplus1[n, z] == 0, z][[All, 2]]
Table[Chop[N[RiemannPrimeCount[n]] - (-1 + Product[1 - 1 / r, {r, zeros[n]}])],
  {n, 4, 100}] // TableForm
```

NRoots::nnumeq :

$$1 + \frac{1277 z}{60} + \frac{2573}{90} (-1 + z) z + \frac{535}{48} (-2 + z) (-1 + z) z + \frac{275}{144} (-3 + z) (-2 + z) (-1 + z) z + \frac{17}{240} (-4 + z) (-3 + z) (-2 + z) (-1 + z) z + \text{Binomial}[z, 6] == 0$$ is

expected to be a polynomial equation in the variable z with numeric coefficients. ≫

Part::partd : Part specification

$$\text{NRoots}\left[1 + \frac{1277 z}{60} + \frac{2573}{90} (-1 + z) z + \frac{535}{48} (-2 + z) (-1 + z) z + \frac{275}{144} (-3 + z) (-2 + z) (-1 + z) z + \frac{17}{240} (-4 + z) (-3 + z) (-2 + z) (-1 + z) z + \text{Binomial}[z, 6] == 0, z\right][[\text{All}, 2]]$$

is longer than depth of object. ≫

NRoots::nnumeq :

$$1 + \frac{1277 z}{60} + \frac{2663}{90} (-1 + z) z + \frac{535}{48} (-2 + z) (-1 + z) z + \frac{275}{144} (-3 + z) (-2 + z) (-1 + z) z + \frac{17}{240} (-4 + z) (-3 + z) (-2 + z) (-1 + z) z + \text{Binomial}[z, 6] == 0$$ is

expected to be a polynomial equation in the variable z with numeric coefficients. ≫

Part::partd : Part specification

$$\text{NRoots}\left[1 + \frac{1277\,z}{60} + \frac{2663}{90}(-1+z)\,z + \frac{535}{48}(-2+z)(-1+z)\,z + \frac{275}{144}(-3+z)(-2+z)(-1+z)\,z + \frac{17}{240}(-4+z)(-3+z)(-2+z)(-1+z)\,z + \text{Binomial}[z,6] == 0,\ z\right][\![\text{All}, 2]\!]$$

is longer than depth of object. ≫

NRoots::nnumeq :

$$1 + \frac{1277\,z}{60} + \frac{2663}{90}(-1+z)\,z + \frac{583}{48}(-2+z)(-1+z)\,z + \frac{275}{144}(-3+z)(-2+z)(-1+z)\,z + \frac{17}{240}(-4+z)(-3+z)(-2+z)(-1+z)\,z + \text{Binomial}[z,6] == 0$$ is

expected to be a polynomial equation in the variable z with numeric coefficients. ≫

General::stop : Further output of NRoots::nnumeq will be suppressed during this calculation. ≫

Part::partd : Part specification

$$\text{NRoots}\left[1 + \frac{1277\,z}{60} + \frac{2663}{90}(-1+z)\,z + \frac{583}{48}(-2+z)(-1+z)\,z + \frac{275}{144}(-3+z)(-2+z)(-1+z)\,z + \frac{17}{240}(-4+z)(-3+z)(-2+z)(-1+z)\,z + \text{Binomial}[z,6] == 0,\ z\right][\![\text{All}, 2]\!]$$

is longer than depth of object. ≫

General::stop : Further output of Part::partd will be suppressed during this calculation. ≫

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$22.2833-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{1277\,z}{60}+\frac{2573}{90}(-1+z)\,z+\frac{535}{48}(-2+z)(-1+z)\,z+\frac{275}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{17}{240}(-4+z)(-3+z)(-2+z)(-\right.}\right.$

$22.2833-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{1277\,z}{60}+\frac{2663}{90}(-1+z)\,z+\frac{535}{48}(-2+z)(-1+z)\,z+\frac{275}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{17}{240}(-4+z)(-3+z)(-2+z)(-\right.}\right.$

$22.2833-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{1277\,z}{60}+\frac{2663}{90}(-1+z)\,z+\frac{583}{48}(-2+z)(-1+z)\,z+\frac{275}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{17}{240}(-4+z)(-3+z)(-2+z)(-\right.}\right.$

$23.2833-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{1337\,z}{60}+\frac{2663}{90}(-1+z)\,z+\frac{583}{48}(-2+z)(-1+z)\,z+\frac{275}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{17}{240}(-4+z)(-3+z)(-2+z)(-\right.}\right.$

$23.2833-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{1337\,z}{60}+\frac{1354}{45}(-1+z)\,z+\frac{607}{48}(-2+z)(-1+z)\,z+\frac{275}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{17}{240}(-4+z)(-3+z)(-2+z)(-\right.}\right.$

$23.2833-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{1337\,z}{60}+\frac{1399}{45}(-1+z)\,z+\frac{607}{48}(-2+z)(-1+z)\,z+\frac{275}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{17}{240}(-4+z)(-3+z)(-2+z)(-\right.}\right.$

$23.2833-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{1337\,z}{60}+\frac{1399}{45}(-1+z)\,z+\frac{655}{48}(-2+z)(-1+z)\,z+\frac{275}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{17}{240}(-4+z)(-3+z)(-2+z)(-\right.}\right.$

$24.2833-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{1397\,z}{60}+\frac{1399}{45}(-1+z)\,z+\frac{655}{48}(-2+z)(-1+z)\,z+\frac{275}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{17}{240}(-4+z)(-3+z)(-2+z)(-\right.}\right.$

$24.2833-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{1397\,z}{60}+\frac{2813}{90}(-1+z)\,z+\frac{225}{16}(-2+z)(-1+z)\,z+\frac{323}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{37}{240}(-4+z)(-3+z)(-2+z)(-\right.}\right.$

$25.2833-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{1457\,z}{60}+\frac{2813}{90}(-1+z)\,z+\frac{225}{16}(-2+z)(-1+z)\,z+\frac{323}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{37}{240}(-4+z)(-3+z)(-2+z)(-\right.}\right.$

$25.2833-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{1457\,z}{60}+\frac{2903}{90}(-1+z)\,z+\frac{225}{16}(-2+z)(-1+z)\,z+\frac{323}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{37}{240}(-4+z)(-3+z)(-2+z)(-\right.}\right.$

$25.2833-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{1457\,z}{60}+\frac{1474}{45}(-1+z)\,z+\frac{233}{16}(-2+z)(-1+z)\,z+\frac{323}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{37}{240}(-4+z)(-3+z)(-2+z)(-\right.}\right.$

$25.2833-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{1457\,z}{60}+\frac{2993}{90}(-1+z)\,z+\frac{241}{16}(-2+z)(-1+z)\,z+\frac{323}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{37}{240}(-4+z)(-3+z)(-2+z)(-\right.}\right.$

$$25.2833-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{1457\,z}{60}+\frac{3083}{90}(-1+z)\,z+\frac{241}{16}(-2+z)(-1+z)\,z+\frac{323}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{37}{240}(-4+z)(-3+z)(-2+z)\ldots\right.}\right.$$

$$25.2833-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{1457\,z}{60}+\frac{3083}{90}(-1+z)\,z+\frac{257}{16}(-2+z)(-1+z)\,z+\frac{323}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{37}{240}(-4+z)(-3+z)(-2+z)\ldots\right.}\right.$$

$$26.2833-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{1517\,z}{60}+\frac{3083}{90}(-1+z)\,z+\frac{257}{16}(-2+z)(-1+z)\,z+\frac{323}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{37}{240}(-4+z)(-3+z)(-2+z)\ldots\right.}\right.$$

$$26.2833-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{1517\,z}{60}+\frac{6211}{180}(-1+z)\,z+\frac{793}{48}(-2+z)(-1+z)\,z+\frac{359}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{47}{240}(-4+z)(-3+z)(-2+z)\ldots\right.}\right.$$

$$26.5333-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{383\,z}{15}+\frac{12587}{360}(-1+z)\,z+\frac{805}{48}(-2+z)(-1+z)\,z+\frac{365}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{47}{240}(-4+z)(-3+z)(-2+z)\ldots\right.}\right.$$

$$26.5333-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{383\,z}{15}+\frac{12947}{360}(-1+z)\,z+\frac{805}{48}(-2+z)(-1+z)\,z+\frac{365}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{47}{240}(-4+z)(-3+z)(-2+z)\ldots\right.}\right.$$

$$27.5333-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{398\,z}{15}+\frac{12947}{360}(-1+z)\,z+\frac{805}{48}(-2+z)(-1+z)\,z+\frac{365}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{47}{240}(-4+z)(-3+z)(-2+z)\ldots\right.}\right.$$

$$27.5333-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{398\,z}{15}+\frac{12947}{360}(-1+z)\,z+\frac{829}{48}(-2+z)(-1+z)\,z+\frac{437}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{47}{240}(-4+z)(-3+z)(-2+z)\ldots\right.}\right.$$

$$27.5333-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{398\,z}{15}+\frac{13307}{360}(-1+z)\,z+\frac{829}{48}(-2+z)(-1+z)\,z+\frac{437}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{47}{240}(-4+z)(-3+z)(-2+z)\ldots\right.}\right.$$

$$27.5333-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{398\,z}{15}+\frac{13667}{360}(-1+z)\,z+\frac{829}{48}(-2+z)(-1+z)\,z+\frac{437}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{47}{240}(-4+z)(-3+z)(-2+z)\ldots\right.}\right.$$

$$27.5333-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{398\,z}{15}+\frac{14027}{360}(-1+z)\,z+\frac{829}{48}(-2+z)(-1+z)\,z+\frac{437}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{47}{240}(-4+z)(-3+z)(-2+z)\ldots\right.}\right.$$

$$27.5333-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{398\,z}{15}+\frac{14147}{360}(-1+z)\,z+\frac{853}{48}(-2+z)(-1+z)\,z+\frac{461}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{47}{240}(-4+z)(-3+z)(-2+z)\ldots\right.}\right.$$

$$28.5333-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{413\,z}{15}+\frac{14147}{360}(-1+z)\,z+\frac{853}{48}(-2+z)(-1+z)\,z+\frac{461}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{47}{240}(-4+z)(-3+z)(-2+z)\ldots\right.}\right.$$

$$28.5333-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{413\,z}{15}+\frac{14147}{360}(-1+z)\,z+\frac{877}{48}(-2+z)(-1+z)\,z+\frac{533}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{47}{240}(-4+z)(-3+z)(-2+z)\ldots\right.}\right.$$

$$28.5333-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{413\,z}{15}+\frac{14507}{360}(-1+z)\,z+\frac{877}{48}(-2+z)(-1+z)\,z+\frac{533}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{47}{240}(-4+z)(-3+z)(-2+z)\ldots\right.}\right.$$

$$28.5333-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{413\,z}{15}+\frac{14687}{360}(-1+z)\,z+\frac{901}{48}(-2+z)(-1+z)\,z+\frac{533}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{47}{240}(-4+z)(-3+z)(-2+z)\ldots\right.}\right.$$

$$28.5333-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{413\,z}{15}+\frac{15047}{360}(-1+z)\,z+\frac{901}{48}(-2+z)(-1+z)\,z+\frac{533}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{47}{240}(-4+z)(-3+z)(-2+z)\ldots\right.}\right.$$

$$28.5333-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{413\,z}{15}+\frac{15407}{360}(-1+z)\,z+\frac{901}{48}(-2+z)(-1+z)\,z+\frac{533}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{47}{240}(-4+z)(-3+z)(-2+z)\ldots\right.}\right.$$

$$28.5333-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{413\,z}{15}+\frac{15767}{360}(-1+z)\,z+\frac{901}{48}(-2+z)(-1+z)\,z+\frac{533}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{47}{240}(-4+z)(-3+z)(-2+z)\ldots\right.}\right.$$

$$28.5333-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{413\,z}{15}+\frac{15839}{360}(-1+z)\,z+\frac{307}{16}(-2+z)(-1+z)\,z+\frac{575}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{67}{240}(-4+z)(-3+z)(-2+z)\ldots\right.}\right.$$

$$29.5333-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{428\,z}{15}+\frac{15839}{360}(-1+z)\,z+\frac{307}{16}(-2+z)(-1+z)\,z+\frac{575}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{67}{240}(-4+z)(-3+z)(-2+z)\ldots\right.}\right.$$

$$29.5333-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{428\,z}{15}+\frac{16019}{360}(-1+z)\,z+\frac{315}{16}(-2+z)(-1+z)\,z+\frac{575}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{67}{240}(-4+z)(-3+z)(-2+z)\ldots\right.}\right.$$

$$29.5333-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{428\,z}{15}+\frac{16199}{360}(-1+z)\,z+\frac{323}{16}(-2+z)(-1+z)\,z+\frac{575}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{67}{240}(-4+z)(-3+z)(-2+z)\ldots\right.}\right.$$

$$29.5333-\frac{1}{2}\left(1-\frac{1}{\text{All}}\right)\left(1-\frac{1}{\text{NRoots}\left[1+\frac{428\,z}{15}+\frac{16289}{360}(-1+z)\,z+\frac{331}{16}(-2+z)(-1+z)\,z+\frac{611}{144}(-3+z)(-2+z)(-1+z)\,z+\frac{67}{240}(-4+z)(-3+z)(-2+z)(-\right.}\right.$$

```
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
Lm1[n_, k_] := Sum[Lm1[n / j, k - 1], {j, 2, n}];
Lm1[n_, 1] := Sum[Log[j], {j, 2, n}]; Lm1[n_, 0] := UnitStep[n - 1]
Lz[n_, z_] := Sum[bin[z, k] Lm1[n, k], {k, 0, Log[2, n]}]
zeros[n_] := List @@ NRoots[Lz[n, z] == 0, z][[All, 2]]
Table[{Chop[-1 + Product[1 - 1 / r, {r, zeros[n]}] - N[Sum[Log[j], {j, 2, n}]]],
   Chop[1 - Product[1 + 1 / r, {r, zeros[n]}] - N[Sum[MangoldtLambda[j], {j, 2, n}]]]}, {n,
   4, 100}] // TableForm
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F[f_, n_, 0, a_] := UnitStep[n - 1]
F[f_, n_, k_, a_] := Sum[f[j] F[f, n / j, k - 1, a], {j, a + 1, Floor[n]}]
FAlt[f_, n_, k_, a_] :=
 If[n < (a + 1) ^k, 0, Sum[Binomial[k, j] f[a + 1] ^j F[f, n / (a + 1) ^j, k - j, a + 1], {j, 0, k}]]
Grid[Table[{F[MoebiusMu, n, k, 2] - FAlt[MoebiusMu, n, k, 2]}, {n, 10, 500, 10}, {k, 1, 5}]]
Grid[Table[{F[LiouvilleLambda, n, k, 4] - FAlt[LiouvilleLambda, n, k, 4]},
  {n, 10, 500, 10}, {k, 1, 5}]]
Grid[Table[{F[MoebiusMu, n, k, 2] - FAlt[MoebiusMu, n, k, 2]}, {n, 10, 500, 10}, {k, 1, 5}]]
Grid[Table[{F[LiouvilleLambda, n, k, 4] - FAlt[LiouvilleLambda, n, k, 4]},
  {n, 10, 500, 10}, {k, 1, 5}]]
```

```
F[f_, n_, 0, a_] := UnitStep[n - 1]
F[f_, n_, k_, a_] := Sum[f[j] F[f, n / j, k - 1, a], {j, a + 1, Floor[n]}]
F1[f_, n_, a_] := Sum[f[b], {b, a + 1, Floor[n]}]
F2[f_, n_, a_] := Sum[f[b] ^2, {b, a + 1, Floor[n^ (1 / 2)]}] +
  2 Sum[f[b] f[c], {b, a + 1, Floor[n^ (1 / 2)]}, {c, b + 1, Floor[n / b]}]
F3[f_, n_, a_] := Sum[f[b] ^3, {b, a + 1, Floor[n^ (1 / 3)]}] +
  3 Sum[f[b] ^2 f[c], {b, a + 1, Floor[n^ (1 / 3)]}, {c, b + 1, Floor[n / b^2]}] +
  3 Sum[f[b] f[c] ^2, {b, a + 1, Floor[n^ (1 / 3)]}, {c, b + 1, Floor[(n / b) ^ (1 / 2)]}] +
  6 Sum[f[b] f[c] f[d], {b, a + 1, Floor[n^ (1 / 3)]},
    {c, b + 1, Floor[(n / b) ^ (1 / 2)]}, {d, c + 1, Floor[n / (b c)]}]
l[n_] := LiouvilleLambda[n]; m[n_] := MoebiusMu
Table[{F[l, n, 1, 2] - F1[l, n, 2],
  F[l, n, 2, 2] - F2[l, n, 2], F[l, n, 3, 2] - F3[l, n, 2]}, {n, 10, 500, 10}]
Table[{F[m, n, 1, 3] - F1[m, n, 3], F[m, n, 2, 3] - F2[m, n, 3], F[m, n, 3, 3] - F3[m, n, 3]},
 {n, 10, 500, 10}]
```

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{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0},
 {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0},
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 {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}}
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{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0},
 {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0},
 {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0},
 {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0},
 {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0},
 {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}}
```

```
F[f_, n_, 0, a_] := UnitStep[n - 1]
F[f_, n_, k_, a_] := Sum[f[j] F[f, n / j, k - 1, a], {j, a + 1, n}]
F1[f_, n_, a_] := Sum[f[b], {b, a + 1, n}]
F2[f_, n_, a_] := Sum[f[b] ^2, {b, a + 1, Floor[n^ (1 / 2)]}] +
  2 Sum[f[b] f[c], {b, a + 1, n^ (1 / 2)}, {c, b + 1, n / b}]
F3[f_, n_, a_] := Sum[f[b] ^3, {b, a + 1, Floor[n^ (1 / 3)]}] +
  3 Sum[f[b] ^2 f[c], {b, a + 1, n^ (1 / 3)}, {c, b + 1, n / b^2}] +
  3 Sum[f[b] f[c] ^2, {b, a + 1, n^ (1 / 3)}, {c, b + 1, (n / b) ^ (1 / 2)}] +
  6 Sum[f[b] f[c] f[d], {b, a + 1, n^ (1 / 3)}, {c, b + 1, (n / b) ^ (1 / 2)}, {d, c + 1, n / (b c)}]
l[n_] := LiouvilleLambda[n]; m[n_] := MoebiusMu
Table[{F[l, n, 1, 2] - F1[l, n, 2],
  F[l, n, 2, 2] - F2[l, n, 2], F[l, n, 3, 2] - F3[l, n, 2]}, {n, 10, 500, 10}]
Table[{F[m, n, 1, 3] - F1[m, n, 3], F[m, n, 2, 3] - F2[m, n, 3], F[m, n, 3, 3] - F3[m, n, 3]},
 {n, 10, 500, 10}]
```

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{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0},
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 {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0},
 {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0},
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 {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}}
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{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0},
 {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0},
 {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0},
 {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0},
 {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0},
 {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}}
```

```
Da[n_, 0, a_] := UnitStep[n - 1]; Da[n_, 1, a_] := Floor[n] - a
Da[n_, k_, a_] :=
 Sum[Binomial[k, j] Da[n / (m^(k - j)), j, m], {m, a + 1, n^(1 / k)}, {j, 0, k - 1}]
refD1[n_, k_] := Sum[refD1[n / j, k - 1], {j, 1, n}]; refD1[n_, 0] := UnitStep[n - 1]
refD2[n_, k_] := Sum[refD2[n / j, k - 1], {j, 2, n}]; refD2[n_, 0] := UnitStep[n - 1]
Grid[Table[Da[n, k, 0] - refD1[n, k], {n, 7, 100, 5}, {k, 1, 7}]]
Grid[Table[Da[n, k, 1] - refD2[n, k], {n, 7, 100, 5}, {k, 1, 7}]]
```

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```
refD2[n_, k_] := Sum[refD2[n / j, k - 1], {j, 2, n}]; refD2[n_, 0] := UnitStep[n - 1]
D1[n_, a_] := -a + Floor[n]
D2[n_, a_] := a^2 + -Floor[n^(1 / 2)]^2 + 2 Sum[Floor[(n / b)], {b, a + 1, n^(1 / 2)}]
D3[n_, a_] := -a^3 + Floor[n^(1 / 3)]^3 + 3 Sum[Floor[n / (b^2)], {b, a + 1, n^(1 / 3)}] +
  -3 Sum[Floor[(n / b)^(1 / 2)]^2, {b, a + 1, n^(1 / 3)}] +
  6 Sum[Floor[n / (b c)], {b, a + 1, n^(1 / 3)}, {c, b + 1, (n / b)^(1 / 2)}]
Table[refD2[n, 1] - D1[n, 1], {n, 10, 500, 10}]
Table[refD2[n, 2] - D2[n, 1], {n, 10, 500, 10}]
Table[refD2[n, 3] - D3[n, 1], {n, 10, 500, 10}]
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{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

```
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
Da[n_, k_, a_] :=
 Sum[Binomial[k, j] Da[n / (m^(k - j)), j, m], {m, a + 1, n^(1 / k)}, {j, 0, k - 1}];
Da[n_, 0, a_] := UnitStep[n - 1]; Da[n_, 1, a_] := Floor[n] - a
DzAlt[n_, z_] := Sum[Binomial[z, k] Da[n, k, 1], {k, 0, Log[2, n]}]
Grid[Table[Chop[Dz[721, s + t I, 1] - DzAlt[721, s + t I]], {s, -1.3, 4, .7}, {t, -1.3, 4, .7}]]
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0	0	0	0	0	0	0	-1.01863×10^{-10}
0	0	0	0	0	0	0	0
0	0	0	0	0	0	1.12777×10^{-10}	$2.74667 \times 10^{-10} + 3.6016 \times 10^{-10} i$
$0. + 1.49157 \times 10^{-10} i$	-1.52795×10^{-10}	-1.27329×10^{-10}	$0. - 1.10958 \times 10^{-10} i$	0	-1.38243×10^{-10}	2.6921×10^{-10}	$2.32831 \times 10^{-10} + 2.61934 \times 10^{-10} i$
$2.27374 \times 10^{-10} + 2.54659 \times 10^{-10} i$	$-2.14641 \times 10^{-10} - 1.34605 \times 10^{-10} i$	-1.96451×10^{-10}	$-1.63709 \times 10^{-10} - 1.49157 \times 10^{-10} i$	$1.38016 \times 10^{-10} - 1.96451 \times 10^{-10} i$	$-1.16415 \times 10^{-10} + 3.7835 \times 10^{-10} i$	-3.63798×10^{-10}	$-4.22006 \times 10^{-10} - 8.94943 \times 10^{-10} i$

```
Dd[n_, k_, a_] :=
 Sum[Binomial[k, j] Dd[n / (m^(k - j)), j, m], {m, a + 1, n^(1 / k)}, {j, 0, k - 1}];
Dd[n_, 0, a_] := UnitStep[n - 1]; Dd[n_, 1, a_] := Floor[n] - a
logD[n_] := Sum[(-1)^(k - 1) / k Dd[n, k, 1], {k, 1, Log[2, n]}]
RiemannPrimeCount[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Table[RiemannPrimeCount[n] - logD[n], {n, 2, 100}]
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{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
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```
RiemannPrimeCount[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
Table[RiemannPrimeCount[n] - (Limit[D[Dz[n, z, 1], z], z → 0]), {n, 1, 100}] // TableForm
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RiemannPrimeCount[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
Table[RiemannPrimeCount[n] - Residue[Dz[n, z, 1] / z^2, {z, 0}], {n, 1, 100}] // TableForm
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```
Dd[n_, k_, a_] :=
 Sum[Binomial[k, j] Dd[n / (m^(k - j)), j, m], {m, a + 1, n^(1 / k)}, {j, 0, k - 1}];
Dd[n_, 0, a_] := UnitStep[n - 1]; Dd[n_, 1, a_] := Floor[n] - a
Mertens[n_] := Sum[(-1)^k Dd[n, k, 1], {k, 0, Log[2, n]}]
Table[Mertens[n] - Sum[MoebiusMu[j], {j, 1, n}], {n, 2, 100}]

{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
logD[n_, k_] := Limit[D[Dz[n, z, 1], {z, k}], z → 0];
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
Da[n_, k_, a_] :=
 Sum[Binomial[k, j] Da[n / (m^(k - j)), j, m], {m, a + 1, n^(1 / k)}, {j, 0, k - 1}];
Da[n_, 0, a_] := UnitStep[n - 1]; Da[n_, 1, a_] := Floor[n] - a
logDAlt[n_, j_] :=
 Sum[1 / k ! (Limit[D[Log[1 + y]^j, {y, k}], y → 0]) Da[n, k, 1], {k, 0, Log[2, n]}]
Grid[Table[logD[n, k] - logDAlt[n, k], {n, 10, 100, 10}, {k, 1, 5}]]

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F[fn_, n_, 0] := UnitStep[n - 1];
F[fn_, n_, k_] := F[fn, n, k] = Sum[fn[j] F[fn, n / j, k - 1], {j, 1, Floor[n]}]
f[fn_, n_, k_] := F[fn, n, k] - F[fn, n - 1, k]
FAlt[fn_, n_, k_, t_] := F[fn, t, k] + Sum[fn[j] F[fn, n / j, k - 1], {j, t + 1, Floor[n]}] +
  Sum[fn[s] f[fn, j, m] F[fn, n / (j s), k - m - 1],
   {j, 1, t}, {s, Floor[t / j] + 1, Floor[n / j]}, {m, 1, k - 1}]
id[n_] := 1
Grid[Table[F[id, n, k] - FAlt[id, n, k, Floor[n^(1 / 2)]], {n, 10, 500, 10}, {k, 1, 7}]]
Grid[Table[F[MoebiusMu, n, k] - FAlt[MoebiusMu, n, k, Floor[n^(1 / 2)]],
  {n, 10, 500, 10}, {k, 1, 7}]]
```

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F[fn_, n_, k_, s_] :=
 F[fn, n, k, s] = Sum[(fn[m]^(k - j)) Binomial[k, j] F[fn, n / (m^(k - j)), j, m + 1],
   {m, s, n^(1 / k)}, {j, 0, k - 1}]
F[fn_, n_, 0, s_] := UnitStep[n - 1]
F[fn_, n_, k_] := F[fn, n, k, 1]
f[fn_, n_, k_] := F[fn, n, k] - F[fn, n - 1, k]
FAlt[fn_, n_, k_, t_] := F[fn, t, k] + Sum[fn[j] F[fn, n / j, k - 1], {j, t + 1, n^(1 / 2)}] +
  Sum[Sum[fn[m], {m, Floor[n / (j + 1)] + 1, n / j}] F[fn, j, k - 1],
   {j, 1, n / Floor[n^(1 / 2)] - 1}] + Sum[fn[s] f[fn, j, m] F[fn, n / (j s), k - m - 1],
   {j, 1, t}, {s, Floor[t / j] + 1, Floor[n / j]^(1 / 2)}, {m, 1, k - 1}] +
  Sum[(Sum[fn[m], {m, Floor[n / (j (s + 1))] + 1, n / (j s)}])
    (Sum[f[fn, j, m] F[fn, s, k - m - 1], {m, 1, k - 1}]),
   {j, 1, t}, {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]
FAlt[fn_, n_, 1, t_] := Sum[fn[j], {j, 1, n}]
Grid[Table[F[MoebiusMu, n, k, 1] - FAlt[MoebiusMu, n, k, Floor[n^(1 / 3)]],
  {n, 10, 500, 10}, {k, 1, 7}]]
Grid[Table[F[LiouvilleLambda, n, k, 1] - FAlt[LiouvilleLambda, n, k, Floor[n^(1 / 3)]],
  {n, 10, 500, 10}, {k, 1, 7}]]
```

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0	0	0	0	0	0	0
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0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
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0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

```
dm1[n_, k_] := Sum[dm1[j, k - 1] dm1[n / j, 1], {j, Divisors[n]}];
dm1[n_, 1] := If[n > 1, 1, 0]; dm1[n_, 0] := 0; dm1[1, 0] := 1
Grid[Table[dm1[n, k], {n, 1, 50}, {k, 1, 7}]]
```

0	0	0	0	0	0	0
1	0	0	0	0	0	0
1	0	0	0	0	0	0
1	1	0	0	0	0	0
1	0	0	0	0	0	0
1	2	0	0	0	0	0
1	0	0	0	0	0	0
1	2	1	0	0	0	0
1	1	0	0	0	0	0
1	2	0	0	0	0	0
1	0	0	0	0	0	0
1	4	3	0	0	0	0
1	0	0	0	0	0	0
1	2	0	0	0	0	0
1	2	0	0	0	0	0
1	3	3	1	0	0	0
1	0	0	0	0	0	0
1	4	3	0	0	0	0
1	0	0	0	0	0	0
1	4	3	0	0	0	0
1	2	0	0	0	0	0
1	2	0	0	0	0	0
1	0	0	0	0	0	0
1	6	9	4	0	0	0
1	1	0	0	0	0	0
1	2	0	0	0	0	0
1	2	1	0	0	0	0
1	4	3	0	0	0	0
1	0	0	0	0	0	0
1	6	6	0	0	0	0
1	0	0	0	0	0	0
1	4	6	4	1	0	0
1	2	0	0	0	0	0
1	2	0	0	0	0	0
1	2	0	0	0	0	0
1	7	12	6	0	0	0
1	0	0	0	0	0	0
1	2	0	0	0	0	0
1	2	0	0	0	0	0
1	6	9	4	0	0	0
1	0	0	0	0	0	0
1	6	6	0	0	0	0
1	0	0	0	0	0	0
1	4	3	0	0	0	0
1	4	3	0	0	0	0
1	2	0	0	0	0	0
1	0	0	0	0	0	0
1	8	18	16	5	0	0
1	1	0	0	0	0	0
1	4	3	0	0	0	0

```
Dm1[n_, k_] := Dm1[n, k] = Sum[Dm1[n / j, k - 1], {j, 2, Floor[n]}]; Dm1[n_, 0] := UnitStep[n - 1]
dm1[n_, k_] := Dm1[n, k] - Dm1[n - 1, k]
Dm1Alt[n_, k_] :=
 Dm1[n^(1 / 3), k] + Sum[Dm1[n / j, k - 1], {j, Floor[n^(1 / 3)] + 1, n^(1 / 2)}] +
  Sum[(Floor[n / j] - Floor[n / (j + 1)]) Dm1[j, k - 1], {j, 1, n / Floor[n^(1 / 2)] - 1}] +
  Sum[dm1[j, m] Dm1[n / (j s), k - m - 1], {j, 2, n^(1 / 3)},
   {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[n / j]^(1 / 2)}, {m, 1, k - 1}] +
  Sum[(Floor[n / (j s)] - Floor[n / (j (s + 1))]) (Sum[dm1[j, m] D2[s, k - m - 1], {m, 1, k - 1}]),
   {j, 2, n^(1 / 3)}, {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]
Dm1Alt[n_, 1] := Floor[n] - 1
Grid[Table[Dm1[n, k] - Dm1Alt[n, k], {n, 10, 500, 10}, {k, 1, 7}]]
```

0	0	0	0	0	0	0
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```
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
Grid[Table[dz[n, k], {n, 1, 50}, {k, 1, 7}]]
```

1	1	1	1	1	1	1
1	2	3	4	5	6	7
1	2	3	4	5	6	7
1	3	6	10	15	21	28
1	2	3	4	5	6	7
1	4	9	16	25	36	49
1	2	3	4	5	6	7
1	4	10	20	35	56	84
1	3	6	10	15	21	28
1	4	9	16	25	36	49
1	2	3	4	5	6	7
1	6	18	40	75	126	196
1	2	3	4	5	6	7
1	4	9	16	25	36	49
1	4	9	16	25	36	49
1	5	15	35	70	126	210
1	2	3	4	5	6	7
1	6	18	40	75	126	196
1	2	3	4	5	6	7
1	6	18	40	75	126	196
1	4	9	16	25	36	49
1	4	9	16	25	36	49
1	2	3	4	5	6	7
1	8	30	80	175	336	588
1	3	6	10	15	21	28
1	4	9	16	25	36	49
1	4	10	20	35	56	84
1	6	18	40	75	126	196
1	2	3	4	5	6	7
1	8	27	64	125	216	343
1	2	3	4	5	6	7
1	6	21	56	126	252	462
1	4	9	16	25	36	49
1	4	9	16	25	36	49
1	4	9	16	25	36	49
1	9	36	100	225	441	784
1	2	3	4	5	6	7
1	4	9	16	25	36	49
1	4	9	16	25	36	49
1	8	30	80	175	336	588
1	2	3	4	5	6	7
1	8	27	64	125	216	343
1	2	3	4	5	6	7
1	6	18	40	75	126	196
1	6	18	40	75	126	196
1	4	9	16	25	36	49
1	2	3	4	5	6	7
1	10	45	140	350	756	1470
1	3	6	10	15	21	28
1	6	18	40	75	126	196

```
dm1[n_, k_] := Sum[dm1[j, k - 1] dm1[n / j, 1], {j, Divisors[n]}];
dm1[n_, 1] := If[n > 1, 1, 0]; dm1[n_, 0] := 0; dm1[1, 0] := UnitStep[n - 1]
dz[n_, z_] := Product[(-1) ^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
dm1Alt[n_, k_] := Sum[(-1) ^j Binomial[k, j] dz[n, k - j], {j, 0, k}]
Grid[Table[dm1[n, k] - dm1Alt[n, k], {n, 1, 50}, {k, 1, 7}]]
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0	0	0	0	0	0	0

```
Dm1[n_, k_] := Sum[Dm1[n / j, k - 1], {j, 2, Floor[n]}]; Dm1[n_, 0] := UnitStep[n - 1]
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
dm1[n_, k_] := Sum[(-1)^j Binomial[k, j] dz[n, k - j], {j, 0, k}]
Dm1Alt[n_, k_] := Dm1[n - 1, k] + dm1[n, k]
Grid[Table[Dm1[n, k] - Dm1Alt[n, k], {n, 1, 50}, {k, 1, 7}]]
```

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0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

```
Dm2[n_, k_] := Sum[Dm2[n / j, k - 1], {j, 3, Floor[n]}]; Dm2[n_, 0] := 1
{(1 / 2) Dm2[2 n, 1], (1 / 2) Sum[1, {j, 3, Floor[2 n]}]}
```

$$\left\{\frac{1}{2}(-2+\text{Floor}[2 n]), \frac{1}{2}(-2+\text{Floor}[2 n])\right\}$$

```
Dd[x_, 1, y_] := Sum[(j + y)^0, {j, 0, Floor[x - y - 1]}]
Grid[Table[y^-1 Sum[1, {j, y + 1, Floor[y x]}] - y^-1 (Dd[x y, 1, y]), {x, 1, 40}, {y, 1, 5}]]
```

```
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
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0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
```

```
Cm1[x_, y_, k_] := y Sum[Cm1[x (j y + 1)^-1, y, k - 1], {j, 1, x y - y}];
Cm1[x_, y_, 0] := UnitStep[x - 1]
```

```
Cm1[x_, y_, k_] := y^-1 Sum[Cm1[x y / (j + y), y, k - 1], {j, 1, Floor[x y - y]}];
Cm1[x_, y_, 0] := UnitStep[x - 1]
Cm1[ 100, 10, 2]
```

$$\frac{8798}{25}$$

```
Cm1[x_, y_, k_] := y Sum[Cm1[x (j y + 1) ^-1, y, k - 1], {j, 1, (x - 1) / y}];
Cm1[x_, y_, 0] := UnitStep[x - 1]
```

```
Cm1[ 100, 1 / 10, 2]
```

$$\frac{8798}{25}$$

```
Sum[ Binomial[z, k] (x - 1) ^k, {k, 0, Infinity}]
```

x^z

```
Sum[ Binomial[z, k] (y (x - 1)) ^k, {k, 0, Infinity}] /. z → 1
```

$1 + (-1 + x)\, y$

```
Sum[ Binomial[z, k] (x y - 1) ^k, {k, 0, Infinity}]
```

$(x\, y)^z$

```
Sum[ (-1) ^ (k + 1) / k (y (D - 1)) ^k, {k, 1, Infinity}]
```

Log[1 - y + D y]

```
Log[1 + (D - 1) y]
```

```
Cc[x_, 1, a_, y_] := y^-1 (Floor[y (x - 1) - a + 1]); Cc[x_, 0, a_, y_] := UnitStep[x - 1]
Cc[x_, k_, a_, y_] := Sum[y^-j Binomial[k, j] Cc[x (m y^-1 + 1) ^-j, k - j, m + 1, y],
  {m, a, Floor[y (x^ (1 / k) - 1)]}, {j, 1, k}]
```

```
Cc[ 100, 2, 1, 1]
```

283

```
Cc2[x_, 1, a_, y_] := y^-1 (Floor[y (x - 1) - a]); Cc2[x_, 0, a_, y_] := UnitStep[x - 1]
Cc2[x_, k_, a_, y_] := Sum[y^-j Binomial[k, j] Cc2[x (m y^-1 + 1) ^-j, k - j, m, y],
  {m, a + 1, Floor[y (x^ (1 / k) - 1)]}, {j, 1, k}]
```

```
Cc2[ 100, 2, 0, 1]
```

283

```
Da[x_, 1, a_, y_] := y (Floor[ (x - 1) / y - a]); Da[x_, 0, a_, y_] := UnitStep[x - 1]
Da[x_, k_, a_, y_] := Sum[y^j Binomial[k, j] Da[x (m y + 1) ^-j, k - j, m, y],
  {m, a + 1, Floor[ (x^ (1 / k) - 1) / y]}, {j, 1, k}]
```

```
Da[ 100, 2, 0, .001]
```

361.418

```
N[1 - Gamma[2, -Log[100]] / Gamma[2]]
```

361.517 − 4.41506 × 10^{-14} i

```
Dy[x_, y_, k_] := y Sum[Dy[x (j y + 1) ^-1, y, k - 1], {j, 1, (x - 1) / y}];
Dy[x_, y_, 0] := UnitStep[x - 1]
```

```
Dy[100, 1, 2]
```

283

```
Dm1[n_, k_] := Sum[Dm1[n (1 + j)^-1, k - 1], {j, 1, n - 1}]; Dm1[n_, 0] := UnitStep[n - 1]
Table[Dm1[n, k], {n, 1, 50}, {k, 1, 7}] // TableForm
```

0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	1	0	0	0	0	0
4	1	0	0	0	0	0
5	3	0	0	0	0	0
6	3	0	0	0	0	0
7	5	1	0	0	0	0
8	6	1	0	0	0	0
9	8	1	0	0	0	0
10	8	1	0	0	0	0
11	12	4	0	0	0	0
12	12	4	0	0	0	0
13	14	4	0	0	0	0
14	16	4	0	0	0	0
15	19	7	1	0	0	0
16	19	7	1	0	0	0
17	23	10	1	0	0	0
18	23	10	1	0	0	0
19	27	13	1	0	0	0
20	29	13	1	0	0	0
21	31	13	1	0	0	0
22	31	13	1	0	0	0
23	37	22	5	0	0	0
24	38	22	5	0	0	0
25	40	22	5	0	0	0
26	42	23	5	0	0	0
27	46	26	5	0	0	0
28	46	26	5	0	0	0
29	52	32	5	0	0	0
30	52	32	5	0	0	0
31	56	38	9	1	0	0
32	58	38	9	1	0	0
33	60	38	9	1	0	0
34	62	38	9	1	0	0
35	69	50	15	1	0	0
36	69	50	15	1	0	0
37	71	50	15	1	0	0
38	73	50	15	1	0	0
39	79	59	19	1	0	0
40	79	59	19	1	0	0
41	85	65	19	1	0	0
42	85	65	19	1	0	0
43	89	68	19	1	0	0
44	93	71	19	1	0	0
45	95	71	19	1	0	0
46	95	71	19	1	0	0
47	103	89	35	6	0	0
48	104	89	35	6	0	0
49	108	92	35	6	0	0

```
Dm1[100, 2]
```

283

```
Sum[ Binomial[ z, k] (d - 1 - x d) ^k, {k, 0, Infinity}]
```

$(d - d x)^z$

```
d^z - Sum[(-1) ^j Binomial[ -z, j] c^j (d - c d) ^z , {j, 0, Infinity}] /. {z → 2, c → 2 }
```

0

```
Grid[Table[((n - y n) ^z) - (Sum[(-1) ^j Binomial[z, j] y^j n^z, {j, 0, Infinity}]),
  {z, 0, 6}, {y, 2, 5, 1 / 3}]]
```

```
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
```

```
Grid[Table[(n^z) - (Sum[(-1) ^j Binomial[-z, j] y^j (n - y n) ^z, {j, 0, Infinity}]),
  {z, 2, 3}, {y, 2, 3}]]
```

Sum::div : Sum does not converge. ≫

Sum::div : Sum does not converge. ≫

Sum::div : Sum does not converge. ≫

General::stop : Further output of Sum::div will be suppressed during this calculation. ≫

$n^2 - \sum_{j=0}^{\infty} (-2)^j n^2$ Binomial[-2, j]	$n^2 - \sum_{j=0}^{\infty} 4 (-3)^j n^2$ Binomial[-2, j]
$n^3 - \sum_{j=0}^{\infty} -(-2)^j n^3$ Binomial[-3, j]	$n^3 - \sum_{j=0}^{\infty} -8 (-3)^j n^3$ Binomial[-3, j]

```
Table[ n^z - Sum[(-1) ^j Binomial[ -z, j] y^j (n - y n) ^z , {j, 0, Infinity}], {z, -3, 6}]
```

{0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

```
Table[ {n^z, n^z (1 - y) ^z Sum[(-1) ^j Binomial[ -z, j] y^j  , {j, 0, Infinity}]}, {z, -3, 6}]
```

$\left\{\left\{\frac{1}{n^3}, \frac{1}{n^3}\right\}, \left\{\frac{1}{n^2}, \frac{1}{n^2}\right\}, \left\{\frac{1}{n}, \frac{1}{n}\right\}, \{1, 1\}, \{n, n\}, \{n^2, n^2\}, \{n^3, n^3\}, \{n^4, n^4\}, \{n^5, n^5\}, \{n^6, n^6\}\right\}$

```
Sum[(-1) ^j Binomial[ -z, j] y^j  , {j, 0, Infinity}]
```

$(1 - y)^{-z}$

```
(n - x n ) ^z - Sum[ Binomial[ z, k] (n - 1 - x n) ^k, {k, 0, Infinity}]
```

0

```
FullSimplify[Table[ n^z - Sum[ (-1) ^j Binomial[ -z, j] Binomial[ z, k] y^j ( n - 1 - y n ) ^k,
    {j, 0, Infinity}, {k, 0, Infinity}], {z, -4, 6}]]
```

{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

```
FullSimplify[Table[ Sum[ (-1) ^j Binomial[ -z, j] Binomial[ z, k] y^j ( n (1 - y) - 1 ) ^k,
   {j, 0, Infinity}, {k, 0, Infinity}], {z, -4, 6}]]
```

$\left\{\frac{1}{n^4}, \frac{1}{n^3}, \frac{1}{n^2}, \frac{1}{n}, 1, n, n^2, n^3, n^4, n^5, n^6\right\}$

```
Sum[ (1 / k) ( x^k  + (-1) ^{k + 1} (n - 1 - x n) ^k) ^k,  {k, 1, 40}] /. {x → .5,  n → 1.5}
```

{0.268017}

```
Log[1.5]
```

0.405465

```
FullSimplify[
 Table[Limit[(Sum[(-1) ^j Binomial[-z, j] Binomial[z, k] y^j (n - 1 - y n) ^k, {j, 0, Infinity},
       {k, 0, Infinity}] - 1) / z, z → 0],  {y, -5, -1}]]
```

{Log[n], Log[n], Log[n], Log[n], Log[n]}

```
FullSimplify[
 Table[Limit[(Sum[(-1) ^j Binomial[-z, j] Binomial[z, k] y^j (n - 1 - y n) ^k, {j, 0, Infinity},
     {k, 0, Infinity}]), z → -1],  {y, -5, -1}]]
```

$\left\{\frac{1}{n}, \frac{1}{n}, \frac{1}{n}, \frac{1}{n}, \frac{1}{n}\right\}$

```
FullSimplify[ Limit[(Sum[(-1) ^j Binomial[-z, j] Binomial[z, k] y^j (n - 1 - y n) ^k,
    {j, 0, Infinity}, {k, 0, Infinity}]), z → -1]]
```

$\frac{1}{n}$

```
{n^2,  Sum[(-1) ^j Binomial[-2, j] Binomial[2, k] y^j (n - 1 - y n) ^k,
  {j, 0, Infinity}, {k, 0, Infinity}]}
```

$\{n^2, n^2\}$

```
{1 / n, FullSimplify[Sum[(-1) ^j Binomial[1, j] Binomial[-1, k] y^j (n - 1 - y n) ^k,
   {j, 0, Infinity}, {k, 0, Infinity}]]}
```

$\left\{\frac{1}{n}, \frac{1}{n}\right\}$

```
Table[n^z - Sum[(-1) ^j Binomial[-z, j] x^j (n - x n) ^z, {j, 0, Infinity}], {z, -3, 6}]
```

{0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

```
FullSimplify[Table[n^z - Sum[(-1) ^j Binomial[-z, j] Binomial[z, k] x^j (n - 1 - x n) ^k,
    {j, 0, Infinity}, {k, 0, Infinity}], {z, -4, 6}]]
```

{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

```
Sum[ Binomial[ k, j] (n - (a + 1)) ^ (k - j), {j, 0, k}]
```

$(-a + n)^k$

```
Sum[ (-1) ^j Binomial[ k, j] (n - (a - 1)) ^ (k - j), {j, 0, k}]
```

$(-a + n)^k$

```
ff[n_, z_] := Sum[ z^k / k ! Log[n] ^k, {k, 0, 20}]
```

```
ff[10]
```

```
(n - a) ^k - Sum[Binomial[k, j] (n - (a + 1)) ^ (k - j), {j, 0, k}]
```

```
0
```

```
Dz[n_, z_, k_] := Dz[n, z, k] = 1 + ((z + 1) / k - 1) Sum[Dz[Floor[n / j], z, k + 1], {j, 2, n}]
Dm1[ n_, k_] := Sum[ (-1) ^ {k - j} Binomial[k, j] Dz[ n, j, 1], {j, 0, Log[2, n]}]
logD[ n_, k_] := Limit[D[Dz[n, z, 1], {z, k}], z → 0]
```

```
logDAlt[n_, j_] :=
 Sum[1 / k ! (Limit[D[Log[1 + y] ^j, {y, k}], y → 0]) Dm1[n, k], {k, 0, Log[2, n]}]
Dm1Alt[n_, j_] := Sum[(-1) ^ (j - k) Binomial[j, k] Dz[n, k, 1], {k, 0, j}]
Dm1Alt2[n_, j_] :=
 Sum[(Limit[D[(E^y - 1) ^j, {y, k}], y → 0]) / k ! logD[n, k], {k, 0, Log[2, n]}]
```

```
Dm1Alt[100, 3]
```

```
324
```

```
{(n - 1) ^j, Sum[ (-1) ^ (j - k) Binomial[j, k] n^k, {k, 0, Infinity}]}
```

$\left\{(-1+n)^j,\ (-1)^j\,(1-n)^j\right\}$

```
Sum[ N[((Limit[D[(E^y - 1) ^j, {y, k}], y → 0]) / k !) Log[n] ^k], {k, 0, 6}] /. {n → 10, j → 1}
```

```
$Aborted
```

```
((Limit[D[(E^y - 1) ^j, {y, k}], y → 0]) / k !) /. {j → 1, k → 4}
```

$\frac{1}{24}$

```
Dm1[n_, k_] := Sum[Dm1[n (1 + j) ^ -1, k - 1], {j, 1, n - 1}]; Dm1[n_, 0] := UnitStep[n - 1]
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
Dm1Alt[n_, j_] := Sum[(-1) ^ (j - k) Binomial[j, k] Dz[n, k, 1], {k, 0, j}]
Grid[Table[Dm1[n, k] - Dm1Alt[n, k], {n, 1, 50}, {k, 1, 7}]]
```

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

```
Dm1[n_, k_] := Sum[Dm1[n (1 + j) ^-1, k - 1], {j, 1, n - 1}]; Dm1[n_, 0] := UnitStep[n - 1]
logD[n_, k_] := Sum[FullSimplify[MangoldtLambda[j] / Log[j]] logD[n / j, k - 1], {j, 2, n}];
logD[n_, 0] := UnitStep[n - 1]
Dm1Alt[n_, k_] :=
 Sum[(Limit[D[(E^y - 1) ^k, {y, j}], y → 0]) / j ! logD[n, j], {j, 0, Log[2, n]}]
Grid[Table[Dm1[n, k] - Dm1Alt[n, k], {n, 1, 50}, {k, 1, 7}]]
```

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

```
Grid[Table[Chop[(n - 1)^k -
    N[Sum[Limit[D[x / Log[1 + x], {x, j}], x → 0] / (j!) (n - 1)^(k - 1 + j) Log[n], {j, 0, 50}]]],
  {n, 0.35, 1.75, .2}, {k, -3, 3}]]
```

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
3.1367×10^{-10}	2.35251×10^{-10}	1.76439×10^{-10}	1.32329×10^{-10}	0	0	0

```
Grid[Table[
  Chop[(n - 1)^k - N[Sum[Limit[D[(E^x - 1)^k, {x, j}], x → 0] / (j!) Log[n]^j, {j, 0, 50}]]],
  {n, 0.35, 1.75, .2}, {k, 1, 5}]]
```

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

```
Series[ x / Log[1 + x], {x, 0, 20}]
```

$$1+\frac{x}{2}-\frac{x^2}{12}+\frac{x^3}{24}-\frac{19\,x^4}{720}+\frac{3\,x^5}{160}-\frac{863\,x^6}{60\,480}+\frac{275\,x^7}{24\,192}-\frac{33\,953\,x^8}{3\,628\,800}+\frac{8183\,x^9}{1\,036\,800}-\frac{3\,250\,433\,x^{10}}{479\,001\,600}+\frac{4671\,x^{11}}{788\,480}-\frac{13\,695\,779\,093\,x^{12}}{2\,615\,348\,736\,000}+\frac{2\,224\,234\,463\,x^{13}}{475\,517\,952\,000}-\frac{132\,282\,840\,127\,x^{14}}{31\,384\,184\,832\,000}+\frac{2\,639\,651\,053\,x^{15}}{689\,762\,304\,000}-\frac{111\,956\,703\,448\,001\,x^{16}}{32\,011\,868\,528\,640\,000}+\frac{50\,188\,465\,x^{17}}{15\,613\,165\,568}-\frac{2\,334\,028\,946\,344\,463\,x^{18}}{786\,014\,494\,949\,376\,000}+\frac{301\,124\,035\,185\,049\,x^{19}}{109\,285\,437\,800\,448\,000}-\frac{12\,365\,722\,323\,469\,980\,029\,x^{20}}{4\,817\,145\,976\,189\,747\,200\,000}+O[x]^{21}$$

```
pi[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
(*sum is tructated and stops working after n=2^6-1*)
sum[n_] :=
 pi[n] + (1 / 2) Sum[pi[n / j], {j, 2, n}] - 1 / 12 Sum[pi[n / (j k)], {j, 2, n}, {k, 2, n / j}] +
  1 / 24 Sum[pi[n / (j k l)], {j, 2, n}, {k, 2, n / j}, {l, 2, n / (j k)}] -
  19 / 720 Sum[pi[n / (j k l m)], {j, 2, n}, {k, 2, n / j}, {l, 2, n / (j k)}, {m, 2, n / (j k l)}] +
  3 / 160 Sum[pi[n / (j k l m o)], {j, 2, n}, {k, 2, n / j},
    {l, 2, n / (j k)}, {m, 2, n / (j k l)}, {o, 2, n / (j k l m)}]
Table[{n - 1, sum[n]}, {n, 1, 63}] // TableForm
```

0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7

8	8
9	9
10	10
11	11
12	12
13	13
14	14
15	15
16	16
17	17
18	18
19	19
20	20
21	21
22	22
23	23
24	24
25	25
26	26
27	27
28	28
29	29
30	30
31	31
32	32
33	33
34	34
35	35
36	36
37	37
38	38
39	39
40	40
41	41
42	42
43	43
44	44
45	45
46	46
47	47
48	48
49	49
50	50
51	51
52	52
53	53
54	54
55	55
56	56
57	57
58	58
59	59
60	60
61	61
62	62

```
Grid[Table[Chop[(n - 1) ^k -
    N[Sum[Limit[D[x / Log[1 + x], {x, m}], x → 0] / (m !) (n - 1) ^ (k - 1 + m) Log[n], {m, 0, 50}]]],
  {n, 0.35, 1.75, .2}, {k, -3, 3}]]
```

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
3.1367×10^{-10}	2.35251×10^{-10}	1.76439×10^{-10}	1.32329×10^{-10}	0	0	0

```
RiemannPrimeCount[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Dm1[n_, k_] := Dm1[n, k] = Sum[Dm1[Floor[n (1 + j) ^-1], k - 1], {j, 1, n - 1}];
Dm1[n_, 0] := UnitStep[n - 1]
dm1[n_, k_] := Dm1[n, k] - Dm1[n - 1, k]; dm1[n_, 0] := If[ n == 1, 1, 0]
nm1[ n_ ] := Sum[ Limit[D[x / Log[1 + x], {x, m}], x → 0] / (m !)
   Sum[ dm1[ j, m] RiemannPrimeCount[ n / j], {j, 1, n}], {m, 0, Log[2, n]}]
Table[ {n - 1, nm1[n]}, {n, 1, 100}]
```

```
{{0, 0}, {1, 1}, {2, 2}, {3, 3}, {4, 4}, {5, 5}, {6, 6}, {7, 7}, {8, 8}, {9, 9}, {10, 10},
 {11, 11}, {12, 12}, {13, 13}, {14, 14}, {15, 15}, {16, 16}, {17, 17}, {18, 18}, {19, 19},
 {20, 20}, {21, 21}, {22, 22}, {23, 23}, {24, 24}, {25, 25}, {26, 26}, {27, 27}, {28, 28},
 {29, 29}, {30, 30}, {31, 31}, {32, 32}, {33, 33}, {34, 34}, {35, 35}, {36, 36}, {37, 37},
 {38, 38}, {39, 39}, {40, 40}, {41, 41}, {42, 42}, {43, 43}, {44, 44}, {45, 45}, {46, 46},
 {47, 47}, {48, 48}, {49, 49}, {50, 50}, {51, 51}, {52, 52}, {53, 53}, {54, 54}, {55, 55},
 {56, 56}, {57, 57}, {58, 58}, {59, 59}, {60, 60}, {61, 61}, {62, 62}, {63, 63}, {64, 64},
 {65, 65}, {66, 66}, {67, 67}, {68, 68}, {69, 69}, {70, 70}, {71, 71}, {72, 72}, {73, 73},
 {74, 74}, {75, 75}, {76, 76}, {77, 77}, {78, 78}, {79, 79}, {80, 80}, {81, 81}, {82, 82},
 {83, 83}, {84, 84}, {85, 85}, {86, 86}, {87, 87}, {88, 88}, {89, 89}, {90, 90}, {91, 91},
 {92, 92}, {93, 93}, {94, 94}, {95, 95}, {96, 96}, {97, 97}, {98, 98}, {99, 99}}
```

```
nm1[1000]
```

```
999
```

```
dm1[1, 0]
```

```
UnitStep[-1 + n]
```

```
RiemannPrimeCount[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Dm1[n_, k_] := Dm1[n, k] = Sum[Dm1[Floor[n (1 + j) ^-1], k - 1], {j, 1, n - 1}];
Dm1[n_, 0] := UnitStep[n - 1]
dm1[n_, k_] := Dm1[n, k] - Dm1[n - 1, k]; dm1[n_, 0] := If[ n == 1, 1, 0]
nm1[ n_ ] := Sum[ Limit[D[x / Log[1 + x], {x, m}], x → 0] / (m !)
   Sum[ dm1[ j, m] RiemannPrimeCount[ n / j], {j, 1, n}], {m, 0, Log[2, n]}]
Table[ {n - 1, nm1[n]}, {n, 1, 100}]
```

```
Table[Chop[
  (n - 1) - N[Sum[Limit[D[x / Log[1 + x], {x, m}], x → 0] / (m !) (n - 1) ^m Log[n], {m, 0, 50}]]],
 {n, 0.35, 1.75, .2}]
```

```
{0, 0, 0, 0, 0, 0, 0, 0}
```

```
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
RiemannPrimeCount[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Dm1[n_, k_] := Dm1[n, k] = Sum[Dm1[Floor[n (1 + j)^-1], k - 1], {j, 1, n - 1}];
Dm1[n_, 0] := UnitStep[n - 1]
dm1[n_, k_] := Dm1[n, k] - Dm1[n - 1, k]; dm1[n_, 0] := If[n == 1, 1, 0]
Dm1Alt[ n_, k_ ] := Sum[ Limit[D[x / Log[1 + x], {x, m}], x → 0] / (m!)
   Sum[ dm1[ j, k - 1 + m] RiemannPrimeCount[ n / j], {j, 1, n}], {m, 0, Log[2, n]}]
DzAlt[ n_, z_ ] := 1 + Sum[ Binomial[z, k] Sum[ Limit[D[x / Log[1 + x], {x, m}], x → 0] / (m!)
      Sum[ dm1[ j, k - 1 + m] RiemannPrimeCount[ n / j], {j, 1, n}],
     {m, 0, Log[2, n]}], {k, 1, Log[2, n]}]
Grid[Table[Chop[Dz[a = 55, s + t I, 1] - DzAlt[a, s + t I]], {s, -1.5, 4, .7}, {t, -1.1, 4, .7}]]
```

```
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
```

```
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
Dm1[n_, k_] := Dm1[n, k] = Sum[Dm1[Floor[n (1 + j)^-1], k - 1], {j, 1, n - 1}];
Dm1[n_, 0] := UnitStep[n - 1]
dm1[n_, k_] := Dm1[n, k] - Dm1[n - 1, k]; dm1[n_, 0] := If[n == 1, 1, 0]
DzAlt[ n_, z_ ] := 1 + Sum[ bin[z, k] Sum[ Limit[D[x / Log[1 + x], {x, m}], x → 0] / (m!)
      Sum[ dm1[ j, k - 1 + m] primes[ n / j], {j, 1, n}], {m, 0, Log[2, n]}], {k, 1, Log[2, n]}]
```

```
Sum[ Binomial[2, k] Sum[Limit[D[x / Log[1 + x], {x, m}], x → 0] / (m!) (n - 1)^(k - 1 + m) Log[n],
    {m, 0, 50}], {k, 0, Infinity}] /. n → 1.4
```

```
2.70101
```

```
1.4^2
```

```
1.96
```

```
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
dzMul[ n_, z_, y_] := Sum[ dz[j, z] dz[ n / j, y], {j, Divisors[n]}]
Dz[n_, z_, k_] := Dz[n, z, k] = 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}];
Dz[0, z_, k_] := 0
DzMul[ n_, z_, y_] := Sum[ (Dz[j, z, 1] - Dz[j - 1, z, 1]) Dz[ n / j, y, 1], {j, 1, n}]
```

```
{n^(x + y), n^x n^y}
```

$\{n^{x+y}, n^{x+y}\}$

```
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]; Dz[0, z_, k_] := 0
F[n_, i_, z_] := If[ Prime[i] > n, 1,
  Sum[ (-1) ^a Binomial[ -z, a] F[ n / Prime[i] ^a, i + 1, z], {a, 0, Log[Prime[i], n]}]]
Grid[Table[Chop[Dz[a = 143, s + t I, 1] - F[a, 1, s + t I]], {s, -1.5, 4, .7}, {t, -1.1, 4, .7}]]
```

```
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
```

```
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]; Dz[0, z_, k_] := 0
F[n_, i_, k_, z_] :=
 If[ Prime[i] > n || n <= 1, 1, (1 + (z - 1) / k) F[ n / Prime[i], i, k + 1, z] + F[n, i + 1, 1, z]]
Grid[Table[Chop[Dz[a = 143, s + t I, 1] - F[a, 1, 1, s + t I]],
  {s, -1.5, 4, .7}, {t, -1.1, 4, .7}]]
```

```
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
```

```
Limit[D[ x / Log[1 + x], {x, 3}], x → 0]
```

$$\frac{1}{4}$$

```
Sum[BernoulliB[b] / b ! Log[x] ^ (b), {b, 0, Infinity}]
```

$$\frac{\text{Log}[x]}{-1+x}$$

```
Series[ x / (E^x - 1), {x, 0, 20}]
```

$$1-\frac{x}{2}+\frac{x^2}{12}-\frac{x^4}{720}+\frac{x^6}{30240}-\frac{x^8}{1209600}+\frac{x^{10}}{47900160}-\frac{691\,x^{12}}{1307674368000}+\frac{x^{14}}{74724249600}-\frac{3617\,x^{16}}{10670622842880000}+\frac{43867\,x^{18}}{5109094217170944000}-\frac{174611\,x^{20}}{802857662698291200000}+O[x]^{21}$$

```
Table[BernoulliB[k] / k!, {k, 0, 20}]
```

$$\left\{1, -\frac{1}{2}, \frac{1}{12}, 0, -\frac{1}{720}, 0, \frac{1}{30240}, 0, -\frac{1}{1209600}, 0, \frac{1}{47900160}, 0, -\frac{691}{1307674368000}, 0, \frac{1}{74724249600}, 0, -\frac{3617}{10670622842880000}, 0, \frac{43867}{5109094217170944000}, 0, -\frac{174611}{802857662698291200000}\right\}$$

```
Dz[n_, z_, k_] := Dz[n, z, k] = 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}];
Dz[0, z_, k_] := 0
```

```
logD[n_] := Limit[D[Dz[n, z, 1], z], z → 0]
```

```
logD[100]
```

$$\frac{428}{15}$$

```
logD[n_, k_] := logD[n, k] =
  Sum[FullSimplify[MangoldtLambda[j] / Log[j]] logD[Floor[n / j], k - 1], {j, 2, n}];
logD[n_, 0] := UnitStep[n - 1]
eD[n_, z_] := Sum[z^k / k! logD[n, k], {k, 0, Log[2, n]}]
lapD[n_, s_] := Integrate[eD[n, -s t], {t, 0, Infinity}]
lapD2[s_] := Integrate[E^(-s t), {t, 0, Infinity}]
```

```
Expand[eD[100, z]]
```

$$1 + \frac{428 z}{15} + \frac{16289 z^2}{360} + \frac{331 z^3}{16} + \frac{611 z^4}{144} + \frac{67 z^5}{240} + \frac{7 z^6}{720}$$

```
lapD[100, 2]
```

Integrate::idiv : Integral of $1 - \frac{856 t}{15} + \frac{16289 t^2}{90} - \frac{331 t^3}{2} + \frac{611 t^4}{9} - \frac{134 t^5}{15} + \frac{28 t^6}{45}$ does not converge on {0, ∞}. ≫

$$\int_0^\infty \left(1 - \frac{856 t}{15} + \frac{16289 t^2}{90} - \frac{331 t^3}{2} + \frac{611 t^4}{9} - \frac{134 t^5}{15} + \frac{28 t^6}{45}\right) dt$$

```
lapD2[1 / 5]
```

5

```
n^x
```

n^x

```
E^(Log[n] x)
```

n^x

```
Limit[D[n^x, {x, 3}], x → 0]
```

$\text{Log}[n]^3$

```
Limit[(1 + n / x)^x, {x → Infinity}]
```

$\{e^n\}$

```
Limit[ (n^x - n^0) / x, x → 0]
```

Log[n]

```
Log[E]
```

1

```
Limit[D[z / (E^z - 1), {z, 3}], z → 0]
```

0

```
Sum[Limit[D[z / (E^z - 1), {z, m}], z → 0] / m! (x - 1) Log[x] ^ (k - 1 + m), {m, 0, Infinity}]
```

$$\sum_{m=0}^{\infty} \frac{(-1+x)\ \text{Limit}\left[\partial_{\{z,m\}} \frac{z}{-1+e^z},\ z \to 0\right]\ \text{Log}[x]^{-1+k+m}}{m!}$$

```
Table[{ Limit[D[z / (E^z - 1), {z, m}], z → 0], BernoulliB[m]}, {m, 0, 20}]
```

$$\left\{\{1, 1\}, \left\{-\frac{1}{2}, -\frac{1}{2}\right\}, \left\{\frac{1}{6}, \frac{1}{6}\right\}, \{0, 0\}, \left\{-\frac{1}{30}, -\frac{1}{30}\right\}, \{0, 0\}, \left\{\frac{1}{42}, \frac{1}{42}\right\}, \{0, 0\}, \left\{-\frac{1}{30}, -\frac{1}{30}\right\}, \{0, 0\}, \left\{\frac{5}{66}, \frac{5}{66}\right\}, \{0, 0\}, \left\{-\frac{691}{2730}, -\frac{691}{2730}\right\}, \{0, 0\}, \left\{\frac{7}{6}, \frac{7}{6}\right\}, \{0, 0\}, \left\{-\frac{3617}{510}, -\frac{3617}{510}\right\}, \{0, 0\}, \left\{\frac{43867}{798}, \frac{43867}{798}\right\}, \{0, 0\}, \left\{-\frac{174611}{330}, -\frac{174611}{330}\right\}\right\}$$

```
Fm1[f_, n_, k_] := Fm1[f, n, k] = Sum[f[j] Fm1[f, n / j, k - 1], {j, 2, n}];
Fm1[f_, n_, 0] := UnitStep[n - 1]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
Fz[f_, n_, z_] := Sum[bin[z, k] Fm1[f, n, k], {k, 0, Log[2, n]}]

Fz[LiouvilleLambda, 100, 2.3]
```

49.7959

```
F[ f_, n_, j_, k_, z_] :=
 If[ n < j, 0, ((z - k + 1) / k) f[j] (1 + F[f, n / j, 2, k + 1, z]) + F[f, n, j + 1, k, z]]
```

```
1 + F[LiouvilleLambda, 100, 2, 1, 2.3]
```

49.7959

```
F[f_, n_, z_, k_] := 1 + ((z - k + 1) / k) Sum[ f[j] F[f, n / j, z, k + 1], {j, 2, n}]
```

```
F[LiouvilleLambda, 100, 2.3, 1]
```

49.7959

```
F[f_, n_, z_, k_] := 1 + ((z - k + 1) / k) Sum[ f[j] F[f, n / j, z, k + 1], {j, 2, n}];
F[f_, 0, z_, k_] := 0
FMul[ f_, n_, z_, y_] := Sum[ (F[f, j, z, 1] - F[f, j - 1, z, 1]) F[f, n / j, y, 1], {j, 1, n}]
fz[ f_, n_, z_] := F[f, n, z, 1] - F[f, n - 1, z, 1]
fzMul[f_, n_, z_, y_] := Sum[ fz[f, j, z] fz[ f, n / j, y], {j, Divisors[n]}]

F[LiouvilleLambda, 100, 2.3, 1]
```

49.7959

```
FMul[ LiouvilleLambda, 100, 1, 1.3]
```

49.7959

```
fz[ LiouvilleLambda, 28, 3.5]

-27.5625

fzMul[LiouvilleLambda, 28, 1.5, 2]

-27.5625

fz[MangoldtLambda, 2^4 × 3^3, 3.3]

24.7983

fz[MangoldtLambda, 2^4, 3.3] fz[MangoldtLambda, 3^3, 3.3]

141.118

F[f_, n_, z_, k_] := 1 + ((z - k + 1) / k) Sum[ f[j] F[f, n / j, z, k + 1], {j, 2, n}]

F[LiouvilleLambda, 100, 2.3, 1]

49.7959

Fa[f_, n_, i_, z_] := If[Prime[i] > n, 1,
  Sum[fz[f, Prime[i]^a, z] Fa[f, n / Prime[i]^a, i + 1, z], {a, 0, Log[Prime[i], n]}]]

Fa[LiouvilleLambda, 120, 1, 3.3]

-218.507

Fb[f_, n_, i_, k_, z_] := If[Prime[i] > n || n ≤ 1, 1,
  (1 + (z - 1) / k) f[Prime[i]] Fb[f, n / Prime[i], i, k + 1, z] + Fb[f, n, i + 1, 1, z]]

Fb[MangoldtLambda, 120, 1, 1, 3.3]

3751.62

F[f_, n_, z_, k_] := 1 + ((z - k + 1) / k) Sum[ f[j] F[f, n / j, z, k + 1], {j, 2, n}];
F[f_, 0, z_, k_] := 0

F[MangoldtLambda, 120, 3.3, 1]

2154.49

RiemannPrimeCount[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Dm12D[n_, k_] := Sum[(-1)^(j + 1) Dm12D[n / j, k - 1], {j, 2, n}];
Dm12D[n_, 0] := UnitStep[n - 1]
CountAlt[n_] := Sum[1 / k (2^k Dm12D[n / 2^k, 0] + (-1)^(k + 1) Dm12D[n, k]), {k, 1, Log[2, n]}]
Table[RiemannPrimeCount[n] - CountAlt[n], {n, 1, 100}] // TableForm

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E1[n_, k_, x_] := Sum[E1[n / j, k - 1, x], {j, 1, n}] - x Sum[E1[n / (x j), k - 1, x], {j, 1, n / x}];
E1[n_, 0, x_] := 1

E2[n_, k_, x_] := Sum[E2[n / j, k - 1, x], {j, 2, n}] - x Sum[E2[n / (x j), k - 1, x], {j, 1, n / x}];
E2[n_, 0, x_] := 1

DxD[n_, k_, x_] := Sum[DxD[n / j, k - 1, x] - x DxD[n / (x j), k - 1, x], {j, 1, n}];
DxD[n_, 0, x_] := UnitStep[n - 1]
D1xD[n_, k_, x_] :=
 -x D1xD[n / x, k - 1, x] + Sum[D1xD[n / j, k - 1, x] - x D1xD[n / (x j), k - 1, x], {j, 2, n}];
D1xD[n_, 0, x_] := UnitStep[n - 1]

E1[100, 3, 1.5]

25.375

DmxD[ 100, 3, 1.5]

25.375

E2[100, 3, 1.5]

21.375
```

```
Dm1xD[100, 3, 1.5]
```

```
21.375
```

```
D1xD[n_, k_, x_] := D1xD[n, k, x] =
  -x D1xD[n / x, k - 1, x] + Sum[D1xD[n / j, k - 1, x] - x D1xD[n / (x j), k - 1, x], {j, 2, n}];
D1xD[n_, 0, x_] := UnitStep[n - 1]
DAlt[n_, x_] := Sum[(j + 1) x^j
   (D1xD[n / x^j, 0, x] + 2 D1xD[n / x^j, 1, x] + D1xD[n / x^j, 2, x]), {j, 0, Log[x, n]}]
MertensAlt[n_, x_] := Sum[(-1)^k (D1xD[n, k, x] - x D1xD[n / x, k, x]), {k, 0, Log[x, n]}]
Grid[Table[Sum[1, {j, 1, n}, {k, 1, n / j}] - DAlt[n, (b + 1) / b], {n, 10, 100, 10}, {b, 1, 7}]]
Grid[
 Table[Sum[MoebiusMu[j], {j, 1, n}] - MertensAlt[n, (b + 1) / b], {n, 10, 100, 10}, {b, 1, 5}]]
```

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

```
Dz[n_, z_] := Sum[dz[j, z], {j, 1, n}];
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
D1xD[n_, k_, x_] := D1xD[n, k, x] =
  -x D1xD[n / x, k - 1, x] + Sum[D1xD[n / j, k - 1, x] - x D1xD[n / (x j), k - 1, x], {j, 2, n}];
D1xD[n_, 0, x_] := UnitStep[n - 1]
DzAlt[n_, z_, x_] := Sum[(-1)^j Binomial[-z, j] Binomial[z, k] x^j D1xD[n / x^j, k, x],
  {j, 0, Log[x, n]}, {k, 0, Log[x, n / x^j]}]
Grid[Table[Dz[123, j + 1 / 3] - DzAlt[123, j + 1 / 3, (b + 1) / b], {j, 1, 5}, {b, 1, 5}]]
```

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

```
D1xDa[n_, k_, x_] := Sum[D1xD[n / (j + 1), k - 1, x] - x D1xDa[n / (x j), k - 1, x], {j, 1, n}];
D1xDa[n_, 0, x_] := UnitStep[n - 1]
D1xD[n_, k_, x_] := D1xD[n, k, x] =
  -x D1xD[n / x, k - 1, x] + Sum[D1xD[n / j, k - 1, x] - x D1xD[n / (x j), k - 1, x], {j, 2, n}];
D1xD[n_, 0, x_] := UnitStep[n - 1]
```

```
D1xD[100, 3, 2.2]
```

21.808

```
D1xDa[100, 3, 2.2]
```

21.808

```
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
D1xD[n_, k_, x_] :=
 D1xD[n, k, x] = Sum[D1xD[n / (j + 1), k - 1, x] - x D1xD[n / (x j), k - 1, x], {j, 1, n}];
D1xD[n_, 0, x_] := UnitStep[n - 1]
DzAlt[n_, z_, x_] := Sum[(-1)^j Binomial[-z, j] Binomial[z, k] x^j D1xD[n / x^j, k, x],
  {j, 0, Log[x, n]}, {k, 0, Log[x, n / x^j]}]
Grid[Table[Dz[123, j + 1 / 3, 1] - DzAlt[123, j + 1 / 3, (b + 1) / b], {j, 1, 5}, {b, 1, 5}]]
```

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

```
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
D1xD[n_, k_, x_] :=
 D1xD[n, k, x] = Sum[D1xD[n / (j + 1), k - 1, x] - x D1xD[n / (x j), k - 1, x], {j, 1, n}];
D1xD[n_, 0, x_] := UnitStep[n - 1]
DxD[n_, z_, x_] := Sum[Binomial[z, k] D1xD[n, k, x], {k, 0, Log[x, n]}]
DzAlt[n_, z_, x_] := Sum[(-1)^j Binomial[-z, j] x^j DxD[n / x^j, z, x], {j, 0, Log[x, n]}]
Grid[
 Table[Chop[Dz[a = 111, s + t I, 1] - DzAlt[a, s + t I, 5 / 4]], {s, -1.3, 4, .7}, {t, -1.3, 4, .7}]]
```

0	0	0	0	0	0	$0. - 1.00826\times10^{-10}\,i$	$-1.16722\times10^{-9} + 9.57698\times10^{-10}\,i$
0	0	0	0	0	0	$0. + 1.83718\times10^{-10}\,i$	$6.57053\times10^{-10} + 9.6702\times10^{-10}\,i$
0	0	0	0	0	0	0	$2.37307\times10^{-10} + 1.4245\times10^{-10}\,i$
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-2.31012×10^{-10}
0	0	0	0	0	-1.52568×10^{-10}	$0. - 6.99856\times10^{-10}\,i$	$-1.41608\times10^{-9} + 1.14937\times10^{-9}\,i$
$-1.18234\times10^{-10} + 2.54659\times10^{-10}\,i$	0	-2.89219×10^{-10}	0	0	$1.59594\times10^{-9} + 2.13277\times10^{-10}\,i$	$-2.71075\times10^{-9} + 2.83262\times10^{-9}\,i$	$-2.31194\times10^{-9} + 5.38603\times10^{-9}\,i$

```
Dz[n_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[Dz[n / j, z, k + 1], {j, 2, n}]
D1xD[n_, k_, x_] :=
 D1xD[n, k, x] = Sum[D1xD[n / (j + 1), k - 1, x] - x D1xD[n / (x j), k - 1, x], {j, 1, n}];
D1xD[n_, 0, x_] := UnitStep[n - 1]
DxD[n_, z_, x_] := Sum[Binomial[z, k] D1xD[n, k, x], {k, 0, Log[x, n]}]
DxDAlt[n_, z_, x_] := Sum[(-1)^j Binomial[z, j] x^j Dz[n / x^j, z, 1], {j, 0, Log[x, n]}]
Grid[Table[Chop[DxD[a = 111, s + t I, 4 / 3] - DxDAlt[a, s + t I, 4 / 3]],
  {s, -1.3, 4, .7}, {t, -1.3, 4, .7}]]
```

0	0	0	0	0	0	0	$0. + 1.05501 \times 10^{-10}\,i$
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

```
DxD[n_, k_, x_] := Sum[DxD[n / j, k - 1, x] - x DxD[n / (x j), k - 1, x], {j, 1, n}];
DxD[n_, 0, x_] := UnitStep[n - 1]
D1xD[n_, k_, x_] :=
 D1xD[n, k, x] = Sum[D1xD[n / (j + 1), k - 1, x] - x D1xD[n / (x j), k - 1, x], {j, 1, n}];
D1xD[n_, 0, x_] := UnitStep[n - 1]
DxDAlt[n_, z_, x_] := Sum[Binomial[z, k] D1xD[n, k, x], {k, 0, Log[x, n]}]
Grid[Table[Chop[DxD[n, 3, (b + 1) / b] - DxDAlt[n, 3, (b + 1) / b]], {n, 10, 80, 10}, {b, 1, 6}]]
```

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

```
RiemanPrimeCount[n_] := Sum[PrimePi[n^(1 / k)] / k, {k, 1, Log[2, n]}]
D1xD[n_, k_, x_] :=
 D1xD[n, k, x] = Sum[D1xD[n / (j + 1), k - 1, x] - x D1xD[n / (x j), k - 1, x], {j, 1, n}];
D1xD[n_, 0, x_] := UnitStep[n - 1]
logD[n_, x_] := Sum[x^j / j, {j, 1, Log[x, n]}] +
  Sum[(-1)^(k + 1) / k D1xD[n, k, x], {k, 1, Log[If[x < 2, x, 2], n]}]
Table[{n, RiemanPrimeCount[n], logD[n, 5 / 2], logD[n, 3 / 2], logD[n, 4 / 3]}, {n, 1, 100}] //
 TableForm
```

1	0	0	0	0
2	1	1	1	1
3	2	2	2	2
4	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$
5	$\frac{7}{2}$	$\frac{7}{2}$	$\frac{7}{2}$	$\frac{7}{2}$
6	$\frac{7}{2}$	$\frac{7}{2}$	$\frac{7}{2}$	$\frac{7}{2}$
7	$\frac{9}{2}$	$\frac{9}{2}$	$\frac{9}{2}$	$\frac{9}{2}$
8	$\frac{29}{6}$	$\frac{29}{6}$	$\frac{29}{6}$	$\frac{29}{6}$
9	$\frac{16}{3}$	$\frac{16}{3}$	$\frac{16}{3}$	$\frac{16}{3}$

10	$\frac{16}{3}$	$\frac{16}{3}$	$\frac{16}{3}$	$\frac{16}{3}$
11	$\frac{19}{3}$	$\frac{19}{3}$	$\frac{19}{3}$	$\frac{19}{3}$
12	$\frac{19}{3}$	$\frac{19}{3}$	$\frac{19}{3}$	$\frac{19}{3}$
13	$\frac{22}{3}$	$\frac{22}{3}$	$\frac{22}{3}$	$\frac{22}{3}$
14	$\frac{22}{3}$	$\frac{22}{3}$	$\frac{22}{3}$	$\frac{22}{3}$
15	$\frac{22}{3}$	$\frac{22}{3}$	$\frac{22}{3}$	$\frac{22}{3}$
16	$\frac{91}{12}$	$\frac{91}{12}$	$\frac{91}{12}$	$\frac{91}{12}$
17	$\frac{103}{12}$	$\frac{103}{12}$	$\frac{103}{12}$	$\frac{103}{12}$
18	$\frac{103}{12}$	$\frac{103}{12}$	$\frac{103}{12}$	$\frac{103}{12}$
19	$\frac{115}{12}$	$\frac{115}{12}$	$\frac{115}{12}$	$\frac{115}{12}$
20	$\frac{115}{12}$	$\frac{115}{12}$	$\frac{115}{12}$	$\frac{115}{12}$
21	$\frac{115}{12}$	$\frac{115}{12}$	$\frac{115}{12}$	$\frac{115}{12}$
22	$\frac{115}{12}$	$\frac{115}{12}$	$\frac{115}{12}$	$\frac{115}{12}$
23	$\frac{127}{12}$	$\frac{127}{12}$	$\frac{127}{12}$	$\frac{127}{12}$
24	$\frac{127}{12}$	$\frac{127}{12}$	$\frac{127}{12}$	$\frac{127}{12}$
25	$\frac{133}{12}$	$\frac{133}{12}$	$\frac{133}{12}$	$\frac{133}{12}$
26	$\frac{133}{12}$	$\frac{133}{12}$	$\frac{133}{12}$	$\frac{133}{12}$
27	$\frac{137}{12}$	$\frac{137}{12}$	$\frac{137}{12}$	$\frac{137}{12}$
28	$\frac{137}{12}$	$\frac{137}{12}$	$\frac{137}{12}$	$\frac{137}{12}$
29	$\frac{149}{12}$	$\frac{149}{12}$	$\frac{149}{12}$	$\frac{149}{12}$
30	$\frac{149}{12}$	$\frac{149}{12}$	$\frac{149}{12}$	$\frac{149}{12}$
31	$\frac{161}{12}$	$\frac{161}{12}$	$\frac{161}{12}$	$\frac{161}{12}$
32	$\frac{817}{60}$	$\frac{817}{60}$	$\frac{817}{60}$	$\frac{817}{60}$
33	$\frac{817}{60}$	$\frac{817}{60}$	$\frac{817}{60}$	$\frac{817}{60}$
34	$\frac{817}{60}$	$\frac{817}{60}$	$\frac{817}{60}$	$\frac{817}{60}$
35	$\frac{817}{60}$	$\frac{817}{60}$	$\frac{817}{60}$	$\frac{817}{60}$
36	$\frac{817}{60}$	$\frac{817}{60}$	$\frac{817}{60}$	$\frac{817}{60}$
37	$\frac{877}{60}$	$\frac{877}{60}$	$\frac{877}{60}$	$\frac{877}{60}$
38	$\frac{877}{60}$	$\frac{877}{60}$	$\frac{877}{60}$	$\frac{877}{60}$
39	$\frac{877}{60}$	$\frac{877}{60}$	$\frac{877}{60}$	$\frac{877}{60}$
40	$\frac{877}{60}$	$\frac{877}{60}$	$\frac{877}{60}$	$\frac{877}{60}$
41	$\frac{937}{60}$	$\frac{937}{60}$	$\frac{937}{60}$	$\frac{937}{60}$
42	$\frac{937}{60}$	$\frac{937}{60}$	$\frac{937}{60}$	$\frac{937}{60}$
43	$\frac{997}{60}$	$\frac{997}{60}$	$\frac{997}{60}$	$\frac{997}{60}$
44	$\frac{997}{60}$	$\frac{997}{60}$	$\frac{997}{60}$	$\frac{997}{60}$
45	$\frac{997}{60}$	$\frac{997}{60}$	$\frac{997}{60}$	$\frac{997}{60}$
46	$\frac{997}{60}$	$\frac{997}{60}$	$\frac{997}{60}$	$\frac{997}{60}$
47	$\frac{1057}{60}$	$\frac{1057}{60}$	$\frac{1057}{60}$	$\frac{1057}{60}$
48	$\frac{1057}{60}$	$\frac{1057}{60}$	$\frac{1057}{60}$	$\frac{1057}{60}$

49	$\frac{1087}{60}$	$\frac{1087}{60}$	$\frac{1087}{60}$	$\frac{1087}{60}$
50	$\frac{1087}{60}$	$\frac{1087}{60}$	$\frac{1087}{60}$	$\frac{1087}{60}$
51	$\frac{1087}{60}$	$\frac{1087}{60}$	$\frac{1087}{60}$	$\frac{1087}{60}$
52	$\frac{1087}{60}$	$\frac{1087}{60}$	$\frac{1087}{60}$	$\frac{1087}{60}$
53	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$
54	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$
55	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$
56	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$
57	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$
58	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$
59	$\frac{1207}{60}$	$\frac{1207}{60}$	$\frac{1207}{60}$	$\frac{1207}{60}$
60	$\frac{1207}{60}$	$\frac{1207}{60}$	$\frac{1207}{60}$	$\frac{1207}{60}$
61	$\frac{1267}{60}$	$\frac{1267}{60}$	$\frac{1267}{60}$	$\frac{1267}{60}$
62	$\frac{1267}{60}$	$\frac{1267}{60}$	$\frac{1267}{60}$	$\frac{1267}{60}$
63	$\frac{1267}{60}$	$\frac{1267}{60}$	$\frac{1267}{60}$	$\frac{1267}{60}$
64	$\frac{1277}{60}$	$\frac{1277}{60}$	$\frac{1277}{60}$	$\frac{1277}{60}$
65	$\frac{1277}{60}$	$\frac{1277}{60}$	$\frac{1277}{60}$	$\frac{1277}{60}$
66	$\frac{1277}{60}$	$\frac{1277}{60}$	$\frac{1277}{60}$	$\frac{1277}{60}$
67	$\frac{1337}{60}$	$\frac{1337}{60}$	$\frac{1337}{60}$	$\frac{1337}{60}$
68	$\frac{1337}{60}$	$\frac{1337}{60}$	$\frac{1337}{60}$	$\frac{1337}{60}$
69	$\frac{1337}{60}$	$\frac{1337}{60}$	$\frac{1337}{60}$	$\frac{1337}{60}$
70	$\frac{1337}{60}$	$\frac{1337}{60}$	$\frac{1337}{60}$	$\frac{1337}{60}$
71	$\frac{1397}{60}$	$\frac{1397}{60}$	$\frac{1397}{60}$	$\frac{1397}{60}$
72	$\frac{1397}{60}$	$\frac{1397}{60}$	$\frac{1397}{60}$	$\frac{1397}{60}$
73	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$
74	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$
75	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$
76	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$
77	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$
78	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$
79	$\frac{1517}{60}$	$\frac{1517}{60}$	$\frac{1517}{60}$	$\frac{1517}{60}$
80	$\frac{1517}{60}$	$\frac{1517}{60}$	$\frac{1517}{60}$	$\frac{1517}{60}$
81	$\frac{383}{15}$	$\frac{383}{15}$	$\frac{383}{15}$	$\frac{383}{15}$
82	$\frac{383}{15}$	$\frac{383}{15}$	$\frac{383}{15}$	$\frac{383}{15}$
83	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$
84	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$
85	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$
86	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$
87	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$

88	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$
89	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$
90	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$
91	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$
92	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$
93	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$
94	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$
95	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$
96	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$
97	$\frac{428}{15}$	$\frac{428}{15}$	$\frac{428}{15}$	$\frac{428}{15}$
98	$\frac{428}{15}$	$\frac{428}{15}$	$\frac{428}{15}$	$\frac{428}{15}$
99	$\frac{428}{15}$	$\frac{428}{15}$	$\frac{428}{15}$	$\frac{428}{15}$
100	$\frac{428}{15}$	$\frac{428}{15}$	$\frac{428}{15}$	$\frac{428}{15}$

```
(n - x n) ^z - Sum[Binomial[z, k] (n - 1 - x n) ^k, {k, 0, Infinity}]
```

0

```
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
Dm1[ n_, k_, s_] := Sum[ j^-s Dm1[ n / j, k - 1, s], {j, 2, n}];
Dm1[n_, 0, s_] := UnitStep[n - 1]
Dz[ n_, z_, s_] := Sum[ bin[z, k] Dm1[n, k, s], {k, 0, Log[2, n]}]
```

```
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
Lm1[n_, k_] := Sum[Lm1[n / j, k - 1], {j, 2, n}];
Lm1[n_, 1] := Sum[Log[j], {j, 2, n}]; Lm1[n_, 0] := UnitStep[n - 1]
Lz[n_, z_] := Sum[bin[z, k] Lm1[n, k], {k, 0, Log[2, n]}]
```

```
N[Expand[Lz[100, -1]]]
```

-93.0453

```
D1xD[n_, k_, x_] :=
 D1xD[n, k, x] = Sum[D1xD[n / (j + 1), k - 1, x] - x D1xD[n / (x j), k - 1, x], {j, 1, n}];
D1xD[n_, 0, x_] := UnitStep[n - 1]
DxD[n_, z_, x_] := Sum[bin[z, k] D1xD[n, k, x], {k, 0, Log[x, n]}]; DxD[0, z_, x_] := 0
logDxD[ n_, k_, x_] := Limit[ D[DxD[ n, z, x], {z, k}], z → 0]
logDxDAlt[n_, x_] := Sum[ (-1) ^ (k + 1) / k D1xD[ n, k, x], {k, 1, Log[If[x < 2, x, 2], n]}]
logDxDAlt2[n_, j_, x_] := Sum[
  1 / k ! (Limit[D[Log[1 + y] ^ j, {y, k}], y → 0]) D1xD[n, k, x], {k, 0, Log[If[x < 2, x, 2], n]}]
DxDAlt[ n_, z_, x_] := Sum[ z ^ k / k ! logDxD[n, k, x], {k, 0, Log[If[x < 2, x, 2], n]}]
D1xDAlt[n_, k_, x_] := Sum[(-1) ^ (k - j) Binomial[k, j] DxD[n, j, x], {j, 0, k}]
D1xDAlt2[n_, k_, x_] := Sum[
  (Limit[D[(E^y - 1) ^k, {y, j}], y → 0]) / j ! logDxD[n, j, x], {j, 0, Log[If[x < 2, x, 2], n]}]
DxD[100, -3, 2.2]

-5375.9

DxDAlt[100, -3, 2.2]

-5375.9

D1xD[ 100, 3, 1.5]

21.375

D1xDAlt[ 100, 3, 1.5]

21.375

D1xDAlt2[ 100, 3, 1.5]

21.375

logDxD[ 100, 1, 1.5]

-3.44534

logDxDAlt[100, 1.5]

-3.44534

logDxDAlt2[100, 1, 1.5]

-3.44534

Ss[ n_, z_, x_] := Sum[ Binomial[z, k] (x^k - 1), {k, 0, Log[x, n]}]

Ss[20, -1, 1.001]

-9.50295

D[LaguerreL[-z, Log[100.]], {z, 5}] /. z → 0

154.116

N[LaguerreL[-3, -Log[10]]]

-0.0954221

Sum[Binomial[k, j] (Zeta[s] - 1^-s - 2^-s - 3^-s - 4^-s) ^ (k - j), {j, 0, k}]
```

$\left(-2^{-2 s} 3^{-s} \left(2^{2 s}+3^{s}+6^{s}\right)+\text{Zeta}[s]\right)^{k}$

```
Sum[(-1)^j Binomial[k, j] (n - (a - 1))^(k - j), {j, 0, k}]
```

$(-a+n)^k$

```
FullSimplify[Sum[Binomial[k, j] (Zeta[s] / 2^(-s) - 1^-s - 2^-s)^(k - j), {j, 0, k}] /. k → 2]
```

$4^{-s}\,(-1+4^s\,\text{Zeta}[s])^2$

```
Expand[Sum[(-1)^j Binomial[k, j] (Zeta[s] / (2^(-s (j))) - 1^-s)^(k - j), {j, 0, k}] /. k → 3]
```

$-8+3\,\text{Zeta}[s]+3\times 2^{2s}\,\text{Zeta}[s]+3\times 2^{1+s}\,\text{Zeta}[s]-3\,\text{Zeta}[s]^2-3\times 2^{2s}\,\text{Zeta}[s]^2+\text{Zeta}[s]^3$

```
Expand[Sum[(-1)^j Binomial[k, j] (Zeta[s] / (1^(-s (j))))^(k - j), {j, 0, k}]]
```

$(-1+\text{Zeta}[s])^k$

```
Expand[( Zeta[s] - 1^-s - 2^-s)^2]
```

$1+2^{1-s}+2^{-2s}-2\,\text{Zeta}[s]-2^{1-s}\,\text{Zeta}[s]+\text{Zeta}[s]^2$

```
( Zeta[s] - 1^-s - 2^-s)^k
```

$(-1-2^{-s}+\text{Zeta}[s])^k$

```
Expand[( Zeta[s] - 1^-s - 2^-s)^k -
   Sum[(-1)^j Binomial[k, j] ((Zeta[s] / 2^-s - 1^-s) 2^-s)^(k - j), {j, 0, k}] /. k → 7]
```

0

```
Sum[(-1)^j Binomial[k, j] (Zeta[s] - 1^-s)^(k - j), {j, 0, k}]
```

$(-2+\text{Zeta}[s])^k$

```
Expand[( Zeta[s] - 1^-s - 2^-s)^k -
   Sum[(-1)^j Binomial[k, j] ((Zeta[s] / 2^-s - 1^-s) 2^-s)^(k - j), {j, 0, k}] /. k → 2]
```

0

```
Expand[( Zeta[s] - 1^-s - 2^-s)^2]
```

$1+2^{1-s}+2^{-2s}-2\,\text{Zeta}[s]-2^{1-s}\,\text{Zeta}[s]+\text{Zeta}[s]^2$

```
Expand[Sum[(-1)^j Binomial[k, j] ((Zeta[s] / 2^-s - 1^-s) 2^-s)^(k - j), {j, 0, k}] /. k → 2]
```

$1+2^{1-s}+2^{-2s}-2\,\text{Zeta}[s]-2^{1-s}\,\text{Zeta}[s]+\text{Zeta}[s]^2$

```
Expand[( Zeta[s] - 1^-s - 2^-s - 3^-s)^2]
```

$1+2^{1-s}+2^{-2s}+3^{-2s}+2\times 3^{-s}+2^{1-s}\,3^{-s}-2\,\text{Zeta}[s]-2^{1-s}\,\text{Zeta}[s]-2\times 3^{-s}\,\text{Zeta}[s]+\text{Zeta}[s]^2$

```
Expand[
 Sum[(-1)^j Binomial[k, j] ((Zeta[s] / 3^-s - 1^-s - 2^-s) 3^-s)^(k - j), {j, 0, k}] /. k → 2]
```

$$\frac{625}{576}-\frac{25\,\text{Zeta}[3]}{12}+\text{Zeta}[3]^2$$

```
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
bin[z, a] bin[-z, a] /. {z → 2, a → 1}
```

-4

```
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
```

```
dz[2, -1]
```

-1

```
Fm1[f_, n_, k_] := Fm1[f, n, k] = Sum[f[j] Fm1[f, n / j, k - 1], {j, 2, n}];
Fm1[f_, n_, 0] := UnitStep[n - 1]
Fz[f_, n_, z_] := Sum[bin[z, k] Fm1[f, n, k], {k, 0, Log[2, n]}];
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
logF[f_, n_, k_] := Limit[D[Fz[f, n, z], {z, k}], z → 0]
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
Dz[n_, z_] := Sum[dz[j, z], {j, 1, n}]
```

```
Fz[ EulerPhi, 100, 2.5]
```

19 968.8

```
FullSimplify[(((1 - x^(1 - s)) Zeta[s])^z) -
  (Sum[Binomial[z, k] ((1 - x^(1 - s)) Zeta[s] - 1)^k, {k, 0, Infinity}])]
```

0

```
Grid[Table[((1 - x^(1 - s)) Zeta[s]^z) -
    (Sum[(-1)^j Binomial[z, j] x^(j (1 - s)) Zeta[s]^z, {j, 0, Infinity}]),
   {z, 0, 6}, {x, 2, 5, 1 / 3}]] /. s → 2
```

$-\frac{1}{2}$	$-\frac{3}{7}$	$-\frac{3}{8}$	$-\frac{1}{3}$	$-\frac{3}{10}$	$-\frac{3}{11}$	$-\frac{1}{4}$	$-\frac{3}{13}$	$-\frac{3}{14}$	$-\frac{1}{5}$
0	0	0	0	0	0	0	0	0	0
$\frac{\pi^4}{144}$	$\frac{\pi^4}{147}$	$\frac{5\pi^4}{768}$	$\frac{\pi^4}{162}$	$\frac{7\pi^4}{1200}$	$\frac{2\pi^4}{363}$	$\frac{\pi^4}{192}$	$\frac{5\pi^4}{1014}$	$\frac{11\pi^4}{2352}$	$\frac{\pi^4}{225}$
$\frac{\pi^6}{576}$	$\frac{11\pi^6}{6174}$	$\frac{65\pi^6}{36864}$	$\frac{5\pi^6}{2916}$	$\frac{119\pi^6}{72000}$	$\frac{19\pi^6}{11979}$	$\frac{7\pi^6}{4608}$	$\frac{115\pi^6}{79092}$	$\frac{275\pi^6}{197568}$	$\frac{\pi^6}{750}$
$\frac{7\pi^8}{20736}$	$\frac{31\pi^8}{86436}$	$\frac{215\pi^8}{589824}$	$\frac{19\pi^8}{52488}$	$(511\,\pi^8)/1440000$	$\frac{91\pi^8}{263538}$	$\frac{37\pi^8}{110592}$	$(665\,\pi^8)/2056392$	$(1727\,\pi^8)/5531904$	$\frac{61\pi^8}{202500}$
$\frac{5\pi^{10}}{82944}$	$(715\,\pi^{10})/10890936$	$(5785\,\pi^{10})/84934656$	$\frac{65\pi^{10}}{944784}$	$(17731\,\pi^{10})/259200000$	$(3515\,\pi^{10})/52180524$	$(175\,\pi^{10})/2654208$	$(30935\,\pi^{10})/481195728$	$(87175\,\pi^{10})/1394039808$	$\frac{41\pi^{10}}{675000}$
$(31\,\pi^{12})/2985984$	$(5261\,\pi^{12})/457419312$	$(49405\,\pi^{12})/4076863488$	$(211\,\pi^{12})/17006112$	$(194117\,\pi^{12})/15552000000$	$(42761\,\pi^{12})/3443914584$	$(781\,\pi^{12})/63700992$	$(452155\,\pi^{12})/37533266784$	$(1381501\,\pi^{12})/117099343872$	$(2101\,\pi^{12})/1822500000$

```
{Log[Zeta[s]], Limit[(Zeta[s]^z - 1) / z, z → 0]}
```

```
{Log[Zeta[0]], Limit[(Zeta[0]^z - 1) / z, z → 0]}
```

{ⅈ π - Log[2], ⅈ π - Log[2]}

{Log[Zeta[0]], Limit[D[Zeta[0]^z, z], z → 0]}

{ⅈ π - Log[2], ⅈ π - Log[2]}

{Log[Zeta[0]], Residue[Zeta[0]^z / z^2, {z, 0}]}

{ⅈ π - Log[2], ⅈ (π + ⅈ Log[2])}

Expand[Integrate[1, {x, 1, n}, {y, 1, n / x}]]

ConditionalExpression[1 - n + n Log[n], Re[n] ≥ 0 || n ∉ Reals]

Expand[Integrate[x y, {x, 1, n}, {y, 1, n / x}]]

ConditionalExpression$\left[\frac{1}{4} - \frac{n^2}{4} + \frac{1}{2} n^2 \text{Log}[n], \text{Re}[n] \geq 0 \mid\mid n \notin \text{Reals}\right]$

Expand[Integrate[x^5 y^5, {x, 1, n}, {y, 1, n / x}]]

ConditionalExpression$\left[\frac{1}{36} - \frac{n^6}{36} + \frac{1}{6} n^6 \text{Log}[n], \text{Re}[n] \geq 0 \mid\mid n \notin \text{Reals}\right]$

Expand[Integrate[x^-s y^-s, {x, 1, n}, {y, 1, n / x}]]

ConditionalExpression$\left[\frac{1}{(-1+s)^2} - \frac{n^{1-s}}{(-1+s)^2} + \frac{n^{1-s} \text{Log}[n]}{(-1+s)^2} - \frac{n^{1-s} s \text{Log}[n]}{(-1+s)^2}, \text{Re}[n] \geq 0 \mid\mid n \notin \text{Reals}\right]$

Expand[Integrate[x^-3 y^-3, {x, 1, n}, {y, 1, n / x}]]

ConditionalExpression$\left[\frac{1}{4} - \frac{1}{4 n^2} - \frac{\text{Log}[n]}{2 n^2}, \text{Re}[n] \geq 0 \mid\mid n \notin \text{Reals}\right]$

Fa3[n_, a_, s_] := (-1) ^a $\mathbf{\frac{(\text{Gamma}[a, 0, -(1-s) \text{Log}[n]]) (1-s)^{-a}}{\text{Gamma}[a]}}$

Fa4[n_, a_, s_] := (-1) ^a $\mathbf{\left(1 - \frac{(\text{Gamma}[a, -(1-s) \text{Log}[n]]) (1-s)^{-a}}{\text{Gamma}[a]}\right)}$

$\mathbf{N\left[\frac{1}{4} - \frac{1}{4 n^2} - \frac{\text{Log}[n]}{2 n^2} \text{ /. } \{n \to 100,\ s \to 3\}\right]}$

0.249745

Fa3[n, 5, s]

$-\frac{\text{Gamma}[5, 0, (-1+s) \text{Log}[n]]}{24 (1-s)^5}$

$\mathbf{\text{FullSimplify}\left[\frac{1}{(-1+s)^2} - \frac{n^{1-s}}{(-1+s)^2} + \frac{n^{1-s} \text{Log}[n]}{(-1+s)^2} - \frac{n^{1-s} s \text{Log}[n]}{(-1+s)^2}\right]}$

$\frac{n^{-s} (-n + n^s + (n - n s) \text{Log}[n])}{(-1+s)^2}$

Integrate[(1 - 1 / y) / Log[y], {y, 1, n}]

ConditionalExpression[-EulerGamma - Gamma[0, -Log[n]] - Log[-Log[n]], Im[n] ≠ 0 || Re[n] ≥ 0]

```
Integrate[1 / Log[y], {y, 0, n}]
```

ConditionalExpression[LogIntegral[n], Re[n] ≤ 1 || n ∉ Reals]

```
TestSum[n_, z_, t_, s_] := 1 + Sum[
   N[Binomial[z, k] (-1)^k ((1 - Gamma[k, (s - 1) Log[n]]) / (Gamma[k] (1 - s)^k))], {k, 1, t}]
TestSum[100, 2, 10, 2]
```

3.92395

```
N[LaguerreL[-2, -1 Log[100]]]
```

-0.0360517

```
Integrate[ j^-s k^-s m^-s, {j, 1, x}, {k, 1, x / j}, {m, 1, x / (j k)}]
```

ConditionalExpression$\left[\frac{x^{-s} (2 x^{s} + x (-2 + (-1 + s) \mathrm{Log}[x] (-2 + \mathrm{Log}[x] - s \mathrm{Log}[x])))}{2 (-1 + s)^{3}}, \mathrm{Re}[x] \geq 0 \,||\, x \notin \mathrm{Reals}\right]$

$\mathrm{N}\left[\frac{x^{-s} (2 x^{s} + x (-2 + (-1 + s) \mathrm{Log}[x] (-2 + \mathrm{Log}[x] - s \mathrm{Log}[x])))}{2 (-1 + s)^{3}} \text{ /. } \{x \to 100, s \to 2\}\right]$

0.83791

```
N[(-1) (((Gamma[ 3, 0, (s - 1) Log[ 100]]) / (Gamma[3] (1 - s)^3)))] /. s → 2
```

0.83791

$\mathrm{FullSimplify}\left[(-1)\text{^}a \frac{(\mathrm{Gamma}[a, 0, -(1 - s) \mathrm{Log}[n]]) (1 - s)^{-a}}{\mathrm{Gamma}[a]}\right]$

$\frac{(-1)^{a} (1 - s)^{-a} \mathrm{Gamma}[a, 0, (-1 + s) \mathrm{Log}[n]]}{\mathrm{Gamma}[a]}$

```
TestSum[n_, z_, t_, s_] := 1 +
  Sum[N[Binomial[z, k] (-1)^k (Gamma[k, 0, (s - 1) Log[n]] / (Gamma[k] (1 - s)^k))], {k, 1, t}]

TestSum[ 100, 2, 30, -1]
```

$30526.1 - 2.51369 \times 10^{-12}\, i$

```
N[LaguerreL[-2, 2 Log[100]]]
```

102103.

```
{((1 - x^(1 - s)) Zeta[s])^z,
 FullSimplify[Sum[ (-1)^j Binomial[ z, j ] x^(j (1 - s)) Zeta[s]^z, {j, 0, Infinity}]]}
```

$\left\{\left(\left(1 - x^{1-s}\right) \mathrm{Zeta}[s]\right)^{z}, \left(1 - x^{1-s}\right)^{z} \mathrm{Zeta}[s]^{z}\right\}$

```
{Zeta[s]^z, FullSimplify[Expand[
   Sum[ (-1)^j Binomial[ -z, j ] x^(j (1 - s)) ((1 - x^(1 - s)) Zeta[s])^z, {j, 0, Infinity}]]]}
```

$\left\{\mathrm{Zeta}[s]^{z}, \left(1 - x^{1-s}\right)^{-z} \left(\left(1 - x^{1-s}\right) \mathrm{Zeta}[s]\right)^{z}\right\}$

```
FullSimplify[Table[Zeta[s]^z -
   Sum[(-1)^j Binomial[-z, j] Binomial[z, k] x^(j (1 - s)) ((1 - x^(1 - s)) Zeta[s] - 1)^k,
    {j, 0, Infinity}, {k, 0, Infinity}], {z, -3, 3}]]

{0, 0, 0, 0, 0, 0, 0}

FullSimplify[
  Sum[ (-1)^k (((1 - x^(1 - s)) Zeta[s] - 1)^k - x^(1 - s) ((1 - x^(1 - s)) Zeta[s] - 1)^k),
   {k, 0, Infinity}]] /. s → 0

-2

1 / Zeta[0]

-2

{Zeta[0]^2,
 Sum[ (j + 1) x^(j (1 - s)) (((1 - x^(1 - s)) Zeta[s] - 1)^0 + 2 ((1 - x^(1 - s)) Zeta[s] - 1)^1 +
      ((1 - x^(1 - s)) Zeta[s] - 1)^2), {j, 0, Infinity}] /. s → 0}
```

$\left\{\frac{1}{4}, \frac{1}{4}\right\}$

```
Zeta[0]^2
```

$\frac{1}{4}$

```
{1 / Zeta[0], FullSimplify[
   Sum[(-1)^k (((1 - x^(1 - s)) Zeta[s] - 1)^k - x^(1 - s) ((1 - x^(1 - s)) Zeta[s] - 1)^k),
    {k, 0, Infinity}]] /. s → 0}

{-2, -2}

FullSimplify[
 {Log[Zeta[s]], Sum[(x^(1 - s))^j ((1 - x^(1 - s)) Zeta[s] - 1)^0 / j, {j, 1, Infinity}] +
   Sum[(-1)^(k - 1) / k ((1 - x^(1 - s)) Zeta[s] - 1)^k, {k, 1, Infinity}]}]
```

$\left\{\text{Log}[\text{Zeta}[s]], -\text{Log}\left[1 - x^{1-s}\right] + \text{Log}\left[\left(1 - x^{1-s}\right) \text{Zeta}[s]\right]\right\}$

$\text{Limit}\left[-\text{Log}\left[1 - x^{1-s}\right] + \text{Log}\left[\left(1 - x^{1-s}\right) \text{Zeta}[s]\right], x \to 1\right]$

```
-Log[-1 + s] + Log[(-1 + s) Zeta[s]]
```