$$[f_n]^{*0}=1_{[1,\infty)}(|n|)$$

$$[f_n]^{*k}=\sum_{\left|\frac{n}{m_1\cdot m_2\cdot\ldots m_k}\right|\geq 1;\, m_k\geq 1} f(m_1)\cdot f(m_2)\cdot\ldots f(m_k)$$

$$\nabla_1[f_n]^{*z}=[f_n]^{*z}-[f_{n-1}]^{*z}$$

$$[f_n]^{*x}*[g_n]^{*y}=\sum_{j=1}\nabla_1[f_j]^{*x}\cdot[g_{n j^{-1}}]^{*y}=\sum_{j=1}\nabla_1[g_j]^{*y}\cdot[f_{n j^{-1}}]^{*x}$$

$$[f_n]^{*x}/*[g_n]^{*y}=[f_n]^{*x}*[g_n]^{*-y}$$

$$\sum_{k=1}\frac{(-1)^{k+1}}{k}(\zeta(s)-1)^{k+a}=(\zeta(s)-1)^a\cdot\log\zeta(s)$$

Sum[(-1)^(k+1)/k (Zeta[s]-1)^(k+a),{k,1,Infinity}]

$$[\zeta_n(s)-1]^{*k}=$$
$$\sum_{m=0}\frac{1}{m!}\left(\lim_{x\to 0}\frac{\partial^m}{\partial x^m}\frac{x}{\log(1+x)}\right)[\zeta_n(s)-1]^{*k-1+m}*[\log\zeta_n(s)]^{*1}$$

(There are some other ideas from 9-G that are worth keeping)

$$[\zeta_n(s)-1]^{*k+a}=[\zeta_n(s)-1]^{*k}*[\zeta_n(s)-1]^{*a}$$

$$(\zeta(s)-1)^{k+a}=(\zeta(s)-1)^k\cdot(\zeta(s)-1)^a$$

(Zeta[s]-1)^a (Zeta[s]-1)^k

$$[\zeta_n(s)^{-1}-1]^{*k}=$$
$$[\zeta_n(s)-1]^{*1}*(-[\zeta_n(s)^{-1}-1]^{*k-1}-[\zeta_n(s)^{-1}-1]^{*k})$$

$$(\zeta(s)^{-1}-1)^k=$$
$$(\zeta(s)-1)^1\cdot(-(\zeta(s)^{-1}-1)^{k-1}-(\zeta(s)^{-1}-1)^k)$$

FullSimplify[(Zeta[s]-1)(-(Zeta[s]^-1-1)^(k-1)-(Zeta[s]^-1-1)^k)]

$$[\zeta_n(s)^{-1}-1]^{*1}=\sum_{k=1}(-1)^k[\zeta_n(s)-1]^{*k}$$

$$(\zeta(s)^{-1}-1)^1=\sum_{k=1}(-1)^k(\zeta(s)-1)^k$$

((1/(Zeta[s])-1)-FullSimplify[Sum[(-1)^k (Zeta[s]-1)^k,{k,1,Infinity}]])

$$[\zeta_n(s)^{-1}-1]^{*k}=\sum_{j=0}(-1)^{k+j}\binom{k+j-1}{k-1}[\zeta_n(s)-1]^{*k+j}$$

$$(\zeta(s)^{-1}-1)^k=\sum_{j=0}(-1)^{k+j}\binom{k+j-1}{k-1}(\zeta(s)-1)^{k+j}$$

FullSimplify[Sum[(-1)^(k+j) Binomial[k+j-1, k-1] (Zeta[s]-1)^(k+j),{j,0, Infinity}]]

$$[\zeta_n(s)^{-1}-1]^{*1}*[\zeta_n(s)-1]^{*k}=$$
$$\sum_{j=0}(-1)^j[\zeta_n(s)-1]^{*j+k}=$$
$$[\zeta_n(s)-1]^{*k}*[\zeta_n(s)]^{*-1}$$

$$(\zeta(s)^{-1}-1)\cdot(\zeta(s)-1)^k=$$
$$\sum_{j=0}(-1)^j(\zeta(s)-1)^{j+k}=$$
$$(\zeta(s)-1)^k\cdot\zeta(s)^{*-1}$$

Sum[(-1)^j (Zeta[s]-1)^(j+k),{j,0,Infinity}]

$$[\zeta_n(s)-1]^{*k}=\sum_{j=0}(-1)^{k+j}\binom{k+j-1}{k-1}[\zeta_n(s)^{-1}-1]^{*k+j}$$

$$(\zeta(s)-1)^k=\sum_{j=0}(-1)^{k+j}\binom{k+j-1}{k-1}(\zeta(s)^{-1}-1)^{k+j}$$

FullSimplify[Sum[(-1)^(k+j) Binomial[k+j-1, k-1] (Zeta[s]^-1-1)^(k+j),{j,0, Infinity}]]

$$-[\zeta_n(s)^{-1}-1]^{*1}*([\zeta_n(s)-1]^{*k-1}+[\zeta_n(s)-1]^{*k})=[\zeta_n(s)-1]^{*k}$$

$$-(\zeta(s)^{-1}-1)^1\cdot((\zeta(s)-1)^{k-1}+(\zeta(s)-1)^k)=(\zeta(s)-1)^k$$

FullSimplify[-(Zeta[s]^-1 - 1)((Zeta[s]-1)^k + (Zeta[s]-1)^(k-1))]

$$[\log\zeta_n(s)]^{*1}=\sum_{k=1}\frac{(-1)^k}{k}[\zeta_n(s)^{-1}-1]^{*k}$$

$$\log\zeta(s)=\sum_{k=1}\frac{(-1)^k}{k}(\zeta(s)^{-1}-1)^k$$

FullSimplify[Sum[(-1)^(k)/k (Zeta[s]^-1-1)^(k),{k,1,Infinity}]]

$$\sum_{k=1}\frac{(-1)^{k+1}}{k}[\zeta_n(s)-1]^{*k+a}=[\zeta_n(s)-1]^{*a}*[\log\zeta_n(s)]^{*1}$$

(There are some other ideas from 10-G that are worthy keeping)

$$[\zeta_n(s)-1]^{*a}=\sum_{j=0}^{k}(-1)^k\binom{k}{j}[\zeta_n(s)^{-1}-1]^{*k}*[\zeta_n(s)-1]^{*a-j}$$

$$(\zeta(s)-1)^a=\sum_{j=0}^{k}(-1)^k\binom{k}{j}(\zeta(s)^{-1}-1)^k\cdot(\zeta(s)-1)^{a-j}$$

```
Sum[ (-1)^a Binomial[a,b] (Zeta[s]^-1 -1)^a (Zeta[s]-1)^(k-b),{b,0,a}]
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$$[\zeta_n(s)^{-1}-1]^{*a}=\sum_{j=0}^{k}(-1)^k\binom{k}{j}[\zeta_n(s)-1]^{*k}*[\zeta_n(s)^{-1}-1]^{*a-j}$$

$$(\zeta(s)^{-1}-1)^a=\sum_{j=0}^{k}(-1)^k\binom{k}{j}(\zeta(s)-1)^k\cdot(\zeta(s)^{-1}-1)^{a-j}$$

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Sum[ (-1)^a Binomial[a,b] (Zeta[s]^-1 -1)^a (Zeta[s]-1)^(k-b),{b,0,a}]
```

$$[\log\zeta_n(s)]^{*a}=\sum_{k=1}\frac{(-1)^{k+1}}{k}[\zeta_n(s)-1]^{*k}[\log\zeta_n(s)]^{*a-1}$$

$$(\log\zeta(s))^a=\sum_{k=1}\frac{(-1)^{k+1}}{k}(\zeta(s)-1)^k(\log\zeta(s))^{a-1}$$

```
Sum[ (-1)^(k+1)/k (Zeta[s]-1)^k Log[ Zeta[s]]^(a-1),{k,1,Infinity}]
```

$$[\log\zeta_n(s)]^{*1}=\sum_{k=0}\frac{B_k}{k!}[\zeta_n(s)-1]^{*1}*\lim_{z\to0}\frac{\partial^k}{\partial z^k}[\zeta_n(s)]^{*z}$$

$$\log\zeta(s)=\sum_{k=0}\frac{B_k}{k!}(\zeta(s)-1)\cdot\lim_{z\to0}\frac{\partial^k}{\partial z^k}\zeta(s)^z$$

$$\lim_{z\to0}\frac{\partial^a}{\partial z^a}[\zeta_n(s)]^{*z}=\sum_{k=0}\frac{B_k}{k!}[\zeta_n(s)-1]^{*1}*\lim_{z\to0}\frac{\partial^{k+a-1}}{\partial z^{k+a-1}}[\zeta_n(s)]^{*z}$$

$$\lim_{z\to0}\frac{\partial^a}{\partial z^a}\zeta(s)^z=\sum_{k=0}\frac{B_k}{k!}(\zeta(s)-1)^1\cdot\lim_{z\to0}\frac{\partial^{k+a-1}}{\partial z^{k+a-1}}\zeta(s)^z$$

$$[(\log(1+\zeta_n(s)))^{-1}-1]^{*a}=\sum_{k=0}\left(\lim_{x\to0}\frac{\partial^k}{\partial x^k}((\log(1+x)+1)^{-1}-1)^a\right)[\zeta_n(s)-1]^{*k}$$

$$((\log(1+\zeta(s)))^{-1}-1)^a=\sum_{k=0}\left(\lim_{x\to0}\frac{\partial^k}{\partial x^k}((\log(1+x)+1)^{-1}-1)^a\right)(\zeta(s)-1)^k$$