```
ps[n_, s_] := Sum[(1 - s) j^(-s), {j, 1, n}] - n^(1-s)
pt[n_, s_] :=
  (1 + 4 s^2) Sum[Cos[s Log[j]] / j^(1 / 2), {j, 1, n}] - 2 Sqrt[n] (Cos[s Log[n]] + 2 s Sin[s Log[n]])
pta[n_, s_] := (1 + 4 s^2) Sum[Cos[s Log[j]] / j^(1 / 2), {j, 1, n}] -
   2 Sqrt[n] (Cos[s Log[n]] + 2 s Sin[s Log[n]])

pt[1000000., N@Im@ZetaZero@1]
```

0.351161

```
pta[100., N@Im@ZetaZero@1]
```

-26.1775

```
(1 + 4 s^2)^(1 / 2) /. s -> N@Im@ZetaZero@1
```

28.2871

```
Limit[Zeta[s] (1 - s), s → 1]
```

-1

```
(1 + 4 s^2) /. s → I / 2
```

0

```
Integrate[(1 - s) j^-s, {j, 0, n}]
```

ConditionalExpression$\left[n^{1-s}, \text{Re}[s] < 1\right]$

```
mm[s_] := (Zeta[1 / 2 + s] + Zeta[1 / 2 - s]) / 2

Limit[mm[s], s → .5001]
```

5000.04

```
Integrate[Cos[s Log[j]] / j^(1 / 2), {j, 1, n}] + Integrate[Cos[s Log[j]] / j^(1 / 2), {j, 0, 1}]
```

$$\text{ConditionalExpression}\left[\frac{2}{1+4 s^2}+\frac{-2+2 \sqrt{n}\ (\text{Cos}[s \text{Log}[n]]+2 s \text{Sin}[s \text{Log}[n]])}{1+4 s^2},\ s \in \text{Reals} \&\& (\text{Re}[n] \geq 0 \mid\mid n \notin \text{Reals})\right]$$

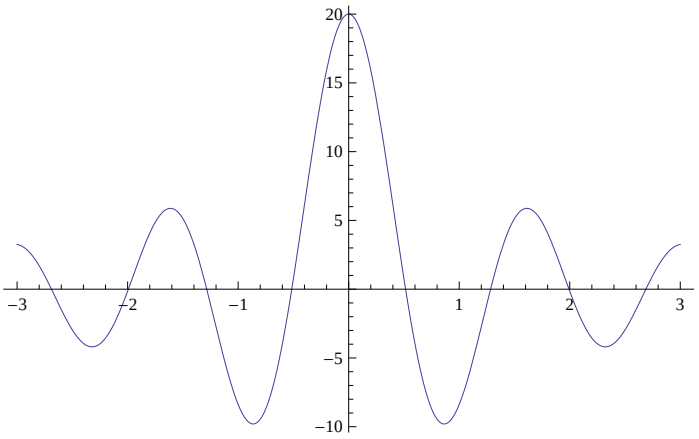
```
FullSimplify[2 / (1 + 4 s^2) + (-2 + 2 Sqrt[n] (Cos[s Log[n]] + 2 s Sin[s Log[n]])) / (1 + 4 s^2)]
```

$$\frac{2 \sqrt{n}\ (\text{Cos}[s \text{Log}[n]]+2 s \text{Sin}[s \text{Log}[n]])}{1+4 s^2}$$

```
(2 Sqrt[n] (Cos[s Log[n]] + 2 s Sin[s Log[n]])) / (1 + 4 s^2) /. s → 1 / 2
```

$$\sqrt{n}\left(\text{Cos}\left[\frac{\text{Log}[n]}{2}\right]+\text{Sin}\left[\frac{\text{Log}[n]}{2}\right]\right)$$

```
Plot[(2 Sqrt[n] (Cos[s Log[n]] + 2 s Sin[s Log[n]]))/(1 + 4 s^2) /. n → 100, {s, -3, 3}]
```




```
(2 Sqrt[n] (Cos[s Log[n]] + 2 s Sin[s Log[n]]))/(1 + 4 s^2) /. s → (s I)
```

$$\frac{2\sqrt{n}\,(\text{Cosh}[s\,\text{Log}[n]] - 2\,s\,\text{Sinh}[s\,\text{Log}[n]])}{1 - 4\,s^2}$$

```
(-1 / 4) ^ (1 / 2)
```

$$\frac{i}{2}$$

```
(1 + 4 s^2) ^ (1 / 2) Integrate[Sin[s Log[j] + ArcTan[2 s]] / j^(1 / 2), {j, 1, n}] +
 Integrate[Sin[s Log[j] + ArcTan[2 s]] / j^(1 / 2), {j, 0, 1}]
```

$$\text{ConditionalExpression}\left[2\sqrt{n}\,\text{Sin}[s\,\text{Log}[n]], -\frac{1}{2} < \text{Im}[s] < \frac{1}{2} \&\& (\text{Re}[n] \geq 0 \,||\, n \notin \text{Reals})\right]$$

```
(1 + 4 s^2) ^ (1 / 2) FullSimplify[Integrate[Cos[s Log[j] + ArcTan[2 s]] / j^(1 / 2), {j, 1, n}] +
   Integrate[Cos[s Log[j] + ArcTan[2 s]] / j^(1 / 2), {j, 0, 1}]]
```

$$\text{ConditionalExpression}\left[2\sqrt{n}\,\text{Cos}[s\,\text{Log}[n]], -\frac{1}{2} < \text{Im}[s] < \frac{1}{2} \&\& (\text{Re}[n] \geq 0 \,||\, n \notin \text{Reals})\right]$$

```
(1 + 4 s^2) FullSimplify[Integrate[Sin[s Log[j]] / j^(1 / 2), {j, 1, n}] +
   Integrate[Sin[s Log[j]] / j^(1 / 2), {j, 0, 1}]]
```

$$\text{ConditionalExpression}\left[2\sqrt{n}\,(-2\,s\,\text{Cos}[s\,\text{Log}[n]] + \text{Sin}[s\,\text{Log}[n]]), -\frac{1}{2} < \text{Im}[s] < \frac{1}{2} \&\& (\text{Re}[n] \geq 0 \,||\, n \notin \text{Reals})\right]$$

```
ba[n_, s_, a_] := 2 Sum[Cos[s Log[j] + a] / j^(1 / 2), {j, 1, n}] -
  2 Integrate[Cos[s Log[j] + a] / j^(1 / 2), {j, 0, n}]
baso[n_, s_, a_] := 2 Sum[Cos[s Log[j] + a] / j^(1 / 2), {j, 1, n}] -
  4 n^(1 / 2) Cos[ArcTan[2 s]] Cos[s Log[n] + a - ArcTan[2 s]] - Cos[s Log[n] + a] / n^(1 / 2)
basf[n_, s_] := 2 Sum[Cos[s Log[j]] / j^(1 / 2), {j, 1, n}] -
  4 n^(1 / 2) Cos[ArcTan[2 s]] Cos[s Log[n] - ArcTan[2 s]] - Cos[s Log[n]] / n^(1 / 2)
basfr[n_, s_] := 2 Sum[Cos[s Log[j]] / j^(1 / 2), {j, 1, n}] -
  4 n^(1 / 2) Cos[ArcTan[2 s]] Cos[s Log[n] - ArcTan[2 s]]
ba2[s_, a_] := Cos[a] (Zeta[1 / 2 - s I] + Zeta[1 / 2 + s I]) +
  I Sin[a] (Zeta[1 / 2 - s I] - Zeta[1 / 2 + s I])


N@basf[100 000., 11.3 + .4 I]
```

```
2.69978 - 0.283166 i
```

```
ba2[11.3 + .4 I, 0]
```

```
2.69978 - 0.283167 i
```

```
FullSimplify[Integrate[Cos[s Log[j] + a] / j^(1 / 2), {j, 1, n}] +
  Integrate[Cos[s Log[j] + a] / j^(1 / 2), {j, 0, 1}]]
```

$$\text{ConditionalExpression}\left[\frac{2\sqrt{n}\,(\text{Cos}[a + s\,\text{Log}[n]] + 2\,s\,\text{Sin}[a + s\,\text{Log}[n]])}{1 + 4\,s^2},\ \text{Re}[n] \geq 0 \,||\, n \notin \text{Reals}\right]$$

```
$Aborted
```

```
bb[n_, s_] := Cos[s Log[n]] / n^(1 / 2)
```

```
Limit[bb[n, -.4 I + 100], n → Infinity]
```

```
0.
```

```
basf[n_, s_] := (1 + 4 s^2) (2 Sum[Cos[s Log[j]] / j^(1 / 2), {j, 1, n}] -
    4 n^(1 / 2) Cos[ArcTan[2 s]] Cos[s Log[n] - ArcTan[2 s]] - Cos[s Log[n]] / n^(1 / 2))
basf1[n_, s_] := 2 Sum[(Cos[(s Log[j])]) / j^(1 / 2), {j, 1, n}] -
  4 n^(1 / 2) Cos[ArcTan[2 s]] (Cos[(s Log[n] - ArcTan[2 s])]) - (Cos[(s Log[n])]) / n^(1 / 2)
basf2[n_, s_] := 2 Sum[(Cos[(s Log[j])]) / j^(1 / 2), {j, 1, n}] -
  4 n^(1 / 2) Cos[ArcTan[2 s]] (Cos[(s Log[n] - ArcTan[2 s])]) -
  (Sum[(-1)^k / (2 k)! (s Log[n])^(2 k), {k, 0, Infinity}]) / n^(1 / 2)
basf3[n_, s_] := (1 + 4 s^2) (2 Sum[(Cos[(s Log[j])]) / j^(1 / 2), {j, 1, n}] - 4 n^(1 / 2)
     Cos[ArcTan[2 s]] (Sum[(-1)^k / (2 k)! (s Log[n] - ArcTan[2 s])^(2 k), {k, 0, Infinity}]) -
    (Sum[(-1)^k / (2 k)! (s Log[n])^(2 k), {k, 0, Infinity}]) / n^(1 / 2))
basf3a[n_, s_, a_] := (2 (1 + 4 s^2) Sum[(Cos[(s Log[j])]) / j^(1 / 2), {j, 1, n}] -
   4 n^(1 / 2) (Sum[(-1)^k / (2 k)! (s Log[n])^(2 k), {k, 0, Infinity}]) -
   4 n^(1 / 2) 2 s (Sum[(-1)^k / (2 k + 1)! (s Log[n])^(2 k + 1), {k, 0, Infinity}]) -
   ((1 + 4 s^2) Sum[(-1)^k / (2 k)! (s Log[n])^(2 k), {k, 0, Infinity}]) / n^(1 / 2))
basf4y3[n_, s_] := 2 (1 + 4 s^2)
   (1 + Sum[(Sum[(-1)^k / (2 k)! (s Log[j])^(2 k), {k, 0, Infinity}]) / j^(1 / 2), {j, 2, n}]) -
  4 n^(1 / 2) Cos[ArcTan[2 s]]
   (Sum[(1 + 4 s^2) (-1)^k / (2 k)! (s Log[n] - ArcTan[2 s])^(2 k), {k, 0, Infinity}]) -
  (Sum[(1 + 4 s^2) (-1)^k / (2 k)! (s Log[n])^(2 k), {k, 0, Infinity}]) / n^(1 / 2)
basf5[n_, s_] :=
 2 ((1 + 4 s^2) + Sum[(Sum[(1 + 4 s^2) (-1)^k / (2 k)! (s Log[j])^(2 k), {k, 0, Infinity}]) /
       j^(1 / 2), {j, 2, n}]) -
```

```
  4 n^(1 / 2) Cos[ArcTan[2 s]] (Sum[(1 + 4 s^2) (-1)^k / (2 k)!
       (s Log[n] - ArcTan[2 s])^(2 k), {k, 0, Infinity}]) -
  (Sum[(1 + 4 s^2) (-1)^k / (2 k)! (s Log[n])^(2 k), {k, 0, Infinity}]) / n^(1 / 2)
basf6[n_, s_] := 2 (1 + 4 s^2) +
  2 Sum[(Sum[(1 + 4 s^2) (-1)^k / (2 k)! (s Log[j])^(2 k), {k, 0, Infinity}]) / j^(1 / 2),
    {j, 2, n}] -
  4 n^(1 / 2) Cos[ArcTan[2 s]] (Sum[(1 + 4 s^2) (-1)^k / (2 k)!
       (s Log[n] - ArcTan[2 s])^(2 k), {k, 0, Infinity}]) -
  Sum[(1 + 4 s^2) (-1)^k / (2 k)! (s Log[n])^(2 k), {k, 0, Infinity}] / n^(1 / 2)
basf6a[n_, s_, a_] := 2 (1 + 4 s^2) +
  2 Sum[(Sum[(1 + 4 s^2) (-1)^k / (2 k)! (s Log[j])^(2 k), {k, 0, Infinity}]) / j^(1 / 2),
    {j, 2, n}] -
  4 n^(1 / 2) (Sum[(-1)^k / (2 k)! (s Log[n])^(2 k), {k, 0, Infinity}]) -
  4 n^(1 / 2) 2 s (Sum[(-1)^k / (2 k + 1)! (s Log[n])^(2 k + 1), {k, 0, Infinity}]) -
  Sum[(1 + 4 s^2) (-1)^k / (2 k)! (s Log[n])^(2 k), {k, 0, Infinity}] / n^(1 / 2)
basf7[n_, s_, l_] := 2 (1 + 4 s^2) +
  Sum[(-1)^k / (2 k)! (2 Sum[((1 + 4 s^2) (s Log[j])^(2 k)) / j^(1 / 2), {j, 2, n}] -
       4 n^(1 / 2) Cos[ArcTan[2 s]] ((1 + 4 s^2) (s Log[n] - ArcTan[2 s])^(2 k)) -
       (1 + 4 s^2) (s Log[n])^(2 k) / n^(1 / 2)), {k, 0, l}]
basf7a[n_, s_, l_] := 2 (1 + 4 s^2) +
  Sum[(-1)^k / (2 k)! (2 Sum[((1 + 4 s^2) (s Log[j])^(2 k)) / j^(1 / 2), {j, 2, n}] -
       4 n^(1 / 2) ((s Log[n])^(2 k) + 2 s (s Log[n])^(2 k + 1) / (2 k + 1)) -
       (1 + 4 s^2) (s Log[n])^(2 k) / n^(1 / 2)), {k, 0, l}]

zn[n_, s_, k_] :=
 (2 Sum[((1 + 4 s^2) (s Log[j])^k) / j^(1 / 2), {j, 2, n}] - 4 n^(1 / 2) Cos[ArcTan[2 s]]
    ((1 + 4 s^2) (s Log[n] - ArcTan[2 s])^k) - (1 + 4 s^2) (s Log[n])^k / n^(1 / 2))
basf8[n_, s_, l_] := 2 (1 + 4 s^2) + Sum[(-1)^k / (2 k)! zn[n, s, 2 k], {k, 0, l}]
zna[n_, s_, k_] := 2 Sum[((1 + 4 s^2) (s Log[j])^k) / j^(1 / 2), {j, 2, n}] -
  4 n^(1 / 2) Cos[ArcTan[2 s]] (1 + 4 s^2) (s Log[n] - ArcTan[2 s])^k -
  (1 + 4 s^2) (s Log[n])^k / n^(1 / 2)
basf9[n_, s_, l_] := 2 (1 + 4 s^2) + Sum[(-1)^k / (2 k)! zna[n, s, 2 k], {k, 0, l}]
znb[n_, s_, k_] := (1 + 4 s^2) (s^k 2 Sum[((Log[j])^k) / j^(1 / 2), {j, 2, n}] -
    4 n^(1 / 2) Cos[ArcTan[2 s]] (s Log[n] - ArcTan[2 s])^k - s^k (Log[n])^k / n^(1 / 2))
basf10[n_, s_, l_] := 2 (1 + 4 s^2) + Sum[(-1)^k / (2 k)! znb[n, s, 2 k], {k, 0, l}]
znc[n_, s_, k_] := (1 + 4 s^2) (s^k 2 Sum[((Log[j])^k) / j^(1 / 2), {j, 2, n}] -
    4 n^(1 / 2) Cos[ArcTan[2 s]] (s Log[n] - ArcTan[2 s])^k - s^k (Log[n])^k / n^(1 / 2))
basf11[n_, s_, l_] := 2 (1 + 4 s^2) + Sum[(-1)^k / (2 k)! znc[n, s, 2 k], {k, 0, l}]


N@basf7a[10., 11.3 + .4 I, 50]

1355.85 - 70.3787 i

N@basf7[10., 11.3 + .4 I, 50]

1355.85 - 70.3775 i

N@basf[10., 11.3 + .4 I]

1355.85 - 70.3782 i

N@basf7[100., 1.3 + .4 I, 100]

3.74215 + 4.60718 i
```

```
N@basf[100., 1.3 + .4 I]
```

$3.74215 + 4.60718\,i$

```
Integrate[ Cos[s Log[x]] / x^(1 / 2), {x, 0, 1}]
```

$$\text{ConditionalExpression}\left[\frac{2}{1+4\,s^2},\ s \in \text{Reals}\right]$$

```
Integrate[ Cos[ Log[x]] / x^(1 / 2), {x, 0, 1}]
```

$$\frac{2}{5}$$

```
Integrate[ Sum[(s Log[x])^(2 k) (-1)^k / (2 k)! , {k, 0, Infinity}] / x^(1 / 2), {x, 0, 1}]
```

$$\text{ConditionalExpression}\left[\frac{2}{1+4\,s^2},\ s \in \text{Reals}\right]$$

```
Sum[Integrate[ s^(2 k) ( Log[x])^(2 k) (-1)^k / (2 k)! / x^(1 / 2), {x, 0, 1}], {k, 0, Infinity}]
```

$$\frac{2}{1+4\,s^2}$$

```
Sum[ s^(2 k) (-1)^k / (2 k)! Integrate[( Log[x])^(2 k)  / x^(1 / 2), {x, 0, 1}],
 {k, 0, Infinity}]
```

$$\frac{2}{1+4\,s^2}$$

```
Sum[ s^(2 k) (-1)^k / (2 k)! Integrate[( Log[x])^(2 k)  / x^(1 / 2), {x, 0, 1}],
 {k, 0, Infinity}]
```

```
FullSimplify[Integrate[( Log[x])^(2 k)  / x^(1 / 2), {x, 0, 1}], Element[k, Integers]]
```

$$\text{ConditionalExpression}\left[2^{1+2k}\,(2k)!,\ k > -\frac{1}{2}\right]$$

```
Sum[ s^(2 k) (-1)^k / (2 k)! (2^(1+2 k) (2 k)!), {k, 0, Infinity}]
```

$$\frac{2}{1+4\,s^2}$$

```
Integrate[ Cos[s Log[x]] / x^(1 / 2), {x, 0, 100.}] /. s → .3 + .4 I
```

$18.7469 - 28.6753\,i$

```
Integrate[ Sum[(s Log[x])^(2 k) (-1)^k / (2 k)! , {k, 0, Infinity}] / x^(1 / 2), {x, 0, 100}] /.
 s → .3 + .4 I
```

$18.7469 - 28.6753\,i$

```
Sum[ s^(2 k) (-1)^k / (2 k)! Integrate[( Log[x])^(2 k)  / x^(1 / 2), {x, 1, 100}],
  {k, 0, Infinity}] /. s → .3 + .4 I
```

$Aborted

```
FullSimplify[Integrate[( Log[x])^(2 k)  / x^(1 / 2), {x, 1, 100}], Element[k, Integers]]
```

$$\text{ConditionalExpression}\left[2^{1+2k}\,(-(2k)! + \text{Gamma}[1+2k, -\text{Log}[10]]),\ k > -\frac{1}{2}\right]$$

```
Sum[ s^(2 k) (-1)^k / (2 k)! (2^(1+2 k) (-(2 k)! + Gamma[1 + 2 k, -Log[10.]])), {k, 0, 40}] /.
 s → .3 + .4 I
```

$17.7469 - 27.342\,i$

```
FullSimplify[
 Integrate[ Cos[Log[x]] / x^(1 / 2), {x, 1, n}] + Integrate[ Cos[Log[x]] / x^(1 / 2), {x, 0, 1}]]
```

$$\text{ConditionalExpression}\left[\frac{2}{5}\sqrt{n}\ (\text{Cos}[\text{Log}[n]]+2\,\text{Sin}[\text{Log}[n]]),\ \text{Re}[n]\geq 0\ ||\ n\notin \text{Reals}\right]$$

```
Cos[ArcTan[2]]
```

$$\frac{1}{\sqrt{5}}$$

```
FullSimplify[Cos@ArcTan[2 s]]
```

$$\frac{1}{\sqrt{1+4\,s^2}}$$

```
FullSimplify[Sin@ArcTan[2 s]]
```

$$\frac{2\,s}{\sqrt{1+4\,s^2}}$$

```
FullSimplify[(1 + 4 s^2) 4 n^(1 / 2) Cos[ArcTan[2 s]] Cos[s Log[n] - ArcTan[2 s]]]
```

$$4\sqrt{n}\ (\text{Cos}[s\,\text{Log}[n]]+2\,s\,\text{Sin}[s\,\text{Log}[n]])$$

```
Sum[(-1)^k / (2 k + 1)! (s Log[n])^(2 k + 1), {k, 0, Infinity}]
```

$$\text{Sin}[s\,\text{Log}[n]]$$

```
FullSimplify[2 Sum[((1 + 4 s^2) (s Log[j])^(2 k)) / j^(1 / 2), {j, 2, n}] -
  4 n^(1 / 2) ((s Log[n])^(2 k) + 2 s (s Log[n])^(2 k + 1) / (2 k + 1)) -
  (1 + 4 s^2) (s Log[n])^(2 k) / n^(1 / 2)]
```

$$-\frac{(s\,\text{Log}[n])^{2k}\left((1+2k)\left(1+4n+4s^2\right)+8\,n\,s^2\,\text{Log}[n]\right)}{(1+2k)\sqrt{n}}+2\sum_{j=2}^{n}\frac{\left(1+4s^2\right)(s\,\text{Log}[j])^{2k}}{\sqrt{j}}$$

```
f1[n_, s_] := ((1 + s) Sum[Exp[s Log[j]] / j^(1 / 2), {j, 1, n}] -
    (1 + s) Integrate[Exp[s Log[j]] / j^(1 / 2), {j, 0, n}] - n^(s I - 1 / 2) / 2)
```

```
f1[100, 3.]
```

$$200\,833.-0.0474357\,i$$