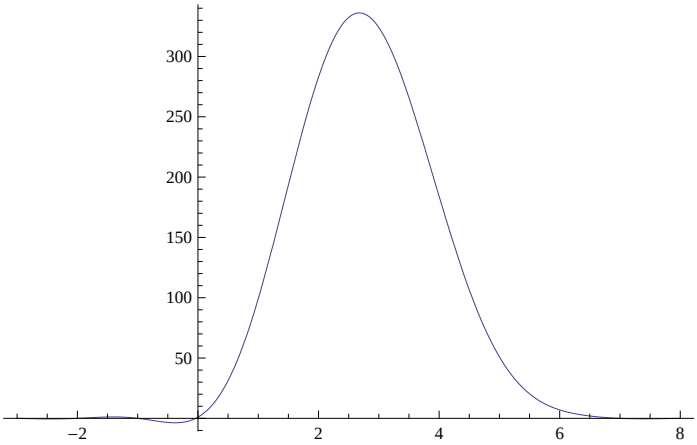
```
Clear[d2]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
d2[n_, 0] := UnitStep[n - 1]
d2[n_, k_] := d2[n, k] = Sum[d2[Floor[n / j], k - 1], {j, 2, n}]
d2z[n_, z_] := Sin[Pi z] / Pi Sum[(-1)^k / (z - k) d2[n, k], {k, 0, Log2@n}]
dz[n_, z_] := Sum[bin[z, k] d2[n, k], {k, 0, Log2@n}]
d2zx[n_, z_] := Sin[Pi z] / Pi Sum[(-1)^k / (z - k) d2x[n, k], {k, 0, Log2@n}]
Plot[d2z[100, z], {z, -3, 8}]
```




```
FullSimplify@Expand@d2z[100, z]
```

$$\frac{(720 + z\,(18\,780 + z\,(-9400 + z\,(1947 + z\,(-165 + (-3 + z)\,z)))))\,\text{Sin}[\pi z]}{\pi\,(-6 + z)\,(-5 + z)\,(-4 + z)\,(-3 + z)\,(-2 + z)\,(-1 + z)\,z}$$

```
FullSimplify@Expand@dz[100, z]
```

$$\frac{1}{720}\,(720 + z\,(1 + z)\,(20\,544 + z\,(12\,034 + z\,(2861 + z\,(194 + 7\,z)))))$$

```
Expand[(720 + z (18 780 + z (-9400 + z (1947 + z (-165 + (-3 + z) z)))))]
```

$$720 + 18\,780\,z - 9400\,z^2 + 1947\,z^3 - 165\,z^4 - 3\,z^5 + z^6$$

```
roots[n_] := If[(c = Exponent[f = bz[z], z]) == 0, {},
  If[c == 1, List@NRoots[f == 0, z][[2]], List @@ NRoots[f == 0, z][[All, 2]]]]


bz[z_] := 720 + 18 780 z - 9400 z^2 + 1947 z^3 - 165 z^4 - 3 z^5 + z^6
Sum[-1 / r, {r, roots[3]}] + Sum[-1 / -r, {r, 1, 6}]
28.5333 + 0. i
d2z[100, 3.5593656126426385` - 4.024461568516174` i]
-2.8716 × 10^-11 + 4.17758 × 10^-10 i
roots[3]
{-16.4989, -0.0376246, 3.55937 - 4.02446 i,
 3.55937 + 4.02446 i, 6.20891 - 1.27718 i, 6.20891 + 1.27718 i}
```

```
Clear[a2]
a2[n_, 0] := UnitStep[n - 1]
a2[n_, k_] := a2[n, k] = Sum[a2[n - j, k - 1], {j, 1, n}]
a2z[n_, z_] := Sin[Pi z] / Pi Sum[(-1)^k / (z - k) a2[n, k], {k, 0, n}]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
az[n_, z_] := Sum[bin[z, k] a2[n, k], {k, 0, n}]
```

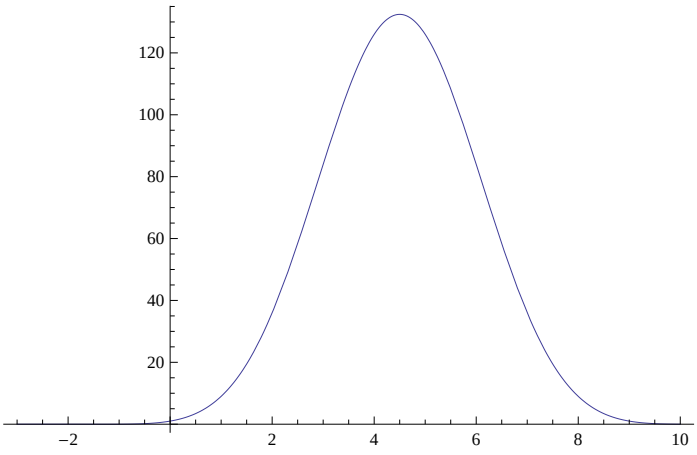
```
FullSimplify@Expand@a2z[10, z]
```

$$-\frac{362\,880\,\sin(\pi z)}{\pi\,(-9+z)\,(-8+z)\,(-7+z)\,(-6+z)\,(-5+z)\,(-4+z)\,(-3+z)\,(-2+z)\,(-1+z)\,z}$$

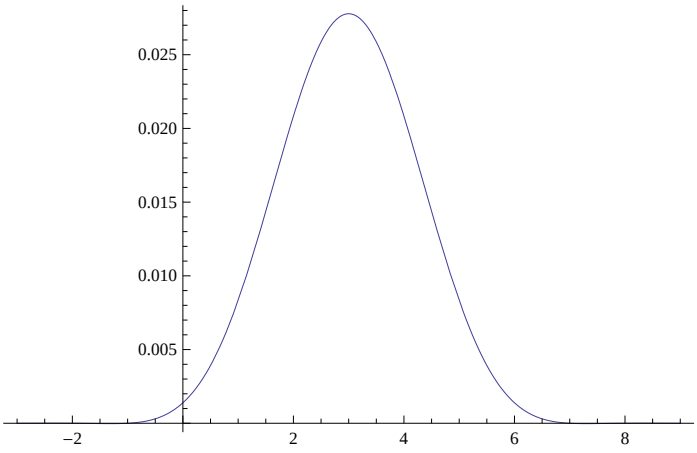
```
FullSimplify@Expand@az[10, z]
```

$$\frac{(1+z)\,(2+z)\,(3+z)\,(4+z)\,(5+z)\,(6+z)\,(7+z)\,(8+z)\,(9+z)}{362\,880}$$

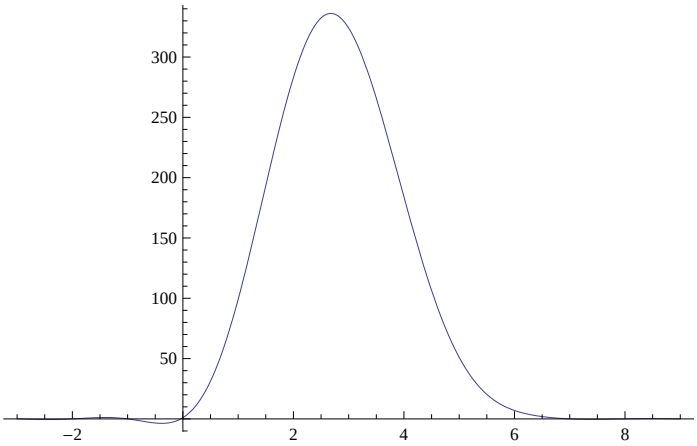
```
Plot[a2z[10, z], {z, -3, 10}]
```



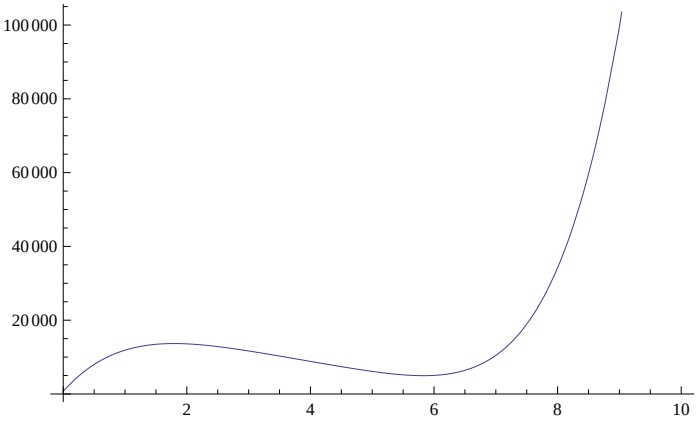

```
Plot[Sin[π z] / (π (-6 + z) (-5 + z) (-4 + z) (-3 + z) (-2 + z) (-1 + z) z), {z, -3, 9}]
```



```
Plot[((720 + z (18 780 + z (-9400 + z (1947 + z (-165 + (-3 + z) z))))) Sin[π z]) /
   (π (-6 + z) (-5 + z) (-4 + z) (-3 + z) (-2 + z) (-1 + z) z), {z, -3, 9}]
```




```
Plot[(720 + z (18 780 + z (-9400 + z (1947 + z (-165 + (-3 + z) z))))), {z, 0, 10}]
```




```
FullSimplify@Expand@d2z[100, z]
```

$$\frac{(720 + z\,(18\,780 + z\,(-9400 + z\,(1947 + z\,(-165 + (-3 + z)\,z))))\,\text{Sin}[\pi\,z]}{\pi\,(-6 + z)\,(-5 + z)\,(-4 + z)\,(-3 + z)\,(-2 + z)\,(-1 + z)\,z}$$