

$$[x]^0=1\,;\,[x]^k=\sum_{j=1}^{\lfloor x\rfloor}[\frac{x}{j}]^{k-1}$$

$$(\lfloor x\rfloor-1)^0=1\,;\,(\lfloor x\rfloor-1)^k=\sum_{j=2}^{\lfloor x\rfloor}(\lfloor\frac{x}{j}\rfloor-1)^{k-1}$$

$$\Delta[n]^k=\sum_{a_1\cdot a_2\cdot...\cdot a_k=n}1\quad \Delta[n]^k=\sum_{j\mid x}\Delta[j]^{k-1}\quad \Delta[n]^1=1\quad \Delta[n]^0=1\,if\,n=1,0\,otherwise\quad \Delta[n]^k=[n]^k-[n-1]^k$$

$$\log [x]^0=1\,;\,\log [x]^k=\sum_{j=2}^{\lfloor x\rfloor}\frac{\Lambda(j)}{\log j}\log [\frac{x}{j}]^{k-1}$$

$$[x,f]^0=1\,;\,[x,f]^k=\sum_{j=1}^{\lfloor x\rfloor}f(j)[\frac{x}{j},f]^{k-1}$$

$$\log [x]=\Pi(x)=\sum_{j=2}^{\lfloor x\rfloor}\frac{\Lambda(j)}{\log j}\qquad \log [x]^2=\sum_{j=2}^{\lfloor x\rfloor}\sum_{k=2}^{\lfloor \frac{x}{j}\rfloor}\frac{\Lambda(j)}{\log j}\frac{\Lambda(k)}{\log k}\qquad \log [x]^3=\sum_{j=2}^{\lfloor x\rfloor}\sum_{k=2}^{\lfloor \frac{x}{j}\rfloor}\sum_{m=2}^{\lfloor \frac{x}{jk}\rfloor}\frac{\Lambda(j)}{\log j}\frac{\Lambda(k)}{\log k}\frac{\Lambda(m)}{\log m}$$

$$(\lfloor x\rfloor-a)^0=1\,;\,(\lfloor x\rfloor-a)^k=\sum_{j=0}^{\lfloor x-a\rfloor}(\lfloor\frac{x}{j+a}\rfloor-a)^{k-1}$$

$$(\lfloor x,f\rfloor-a)^0=1\,;\,(\lfloor x,f\rfloor-a)^k=\sum_{j=0}^{\lfloor x-a\rfloor}f(j+a)(\lfloor\frac{x}{j+a},f\rfloor-a)^{k-1}$$

$$[x]^{-1}=\sum_{k=0}^{\lfloor \log_2 x\rfloor}(-1)^k(\lfloor x\rfloor-1)^k$$

$$(\lfloor x\rfloor-a)^k=\sum_{j=0}^k\binom{k}{j}(\lfloor\frac{x}{a^j}\rfloor-(a+1))^{k-j}$$

$$(\lfloor x\rfloor-a)^k=\sum_{j=0}^k(-1)^j\binom{k}{j}(\lfloor\frac{x}{(a-1)^j}\rfloor-(a-1))^{k-j}$$

$$(\lfloor x\rfloor-a)^k=\sum_{j=1}^k\binom{k}{j}\sum_{m=a}^{\lfloor x^{\frac{1}{j}}\rfloor}(\lfloor\frac{x}{m^j}\rfloor-(m+1))^{k-j}\\ (\lfloor x\rfloor-a)^1=\lfloor x\rfloor-a+1\\ (\lfloor x\rfloor-a)^0=1$$

$$[x]^z = \sum_{k=0}^{\lfloor \log_2 x \rfloor} \binom{z}{k} ([x]-1)^k$$

$$[x]^z = \sum_{k=0}^{\lfloor \log_2 x \rfloor} \frac{z^k}{k!} \log [x]^k$$

$$\frac{\partial}{\partial z} [x]^z = \sum_{k=0}^{\lfloor \log_2 x \rfloor - 1} \frac{z^k}{k!} \log [x]^{k+1}$$

$$\log [x]=\lim_{z\rightarrow 0}\frac{[x]^z-1}{z}$$

$$\log [x]=\sum_{k=1}^{[\log _ex]} \frac{(-1)^{k+1}}{k}([x]-1)^k$$

$$\log [x]=\frac{\partial}{\partial z}[x]^z\;at\;z=0$$

$$\log [x]=\operatorname{Res}_{z=0}\frac{[x]^z}{z^2}$$

$$(\log [x])^k=\frac{\partial^k}{\partial z^k}[x]^z\;at\;z=0$$

$$\log [x]^k=k!\operatorname{Res}_{z=0}\frac{[x]^z}{z^{k+1}}$$

$$\log [x]^0\!=\!1\,;\,\log [x]^k\!=\!\sum_{j=2}^{\lfloor x\rfloor}\frac{\Lambda(j)}{\log j}\log [\frac{x}{j}]^{k-1}$$

$$[x]^z=\sum_{k=0}^{\lfloor \log_2 x\rfloor}\frac{z^k}{k!}\log [x]^k$$

$$[100]^z=\sum_{k=0}^{\lfloor \log_2 100\rfloor}\frac{z^k}{k!}\log [100]^k\!=\!1\!+\!\frac{428}{15}\,z\!+\!\frac{16289}{360}\,z^2\!+\!\frac{331}{16}\,z^3\!+\!\frac{611}{144}\,z^4\!+\!\frac{67}{240}\,z^5\!+\!\frac{7}{720}\,z^6$$

$$[x]^z\!=\!\prod_{\mathfrak{p}}\!\left(1\!-\!\frac{z}{\mathfrak{p}}\right)$$

$$[x]^z\!=\![x]\!\cdot\!\prod_{\mathfrak{p}}\!\left(1\!-\!\frac{z\!-\!1}{\mathfrak{p}\!-\!1}\right)$$

$$\log [x]\!=\!-\sum_{\mathfrak{p}}\frac{1}{\mathfrak{p}}$$

$$[x]^{-1}\!=\!\prod_{\mathfrak{p}}\!\left(1\!+\!\frac{1}{\mathfrak{p}}\right)$$

$$[x]^0\!=\!1\!=\!\prod_{\mathfrak{p}}\!\left(1\!-\!\frac{0}{\mathfrak{p}}\right)$$

$$[x]^1\!=\![x]\!=\!\prod_{\mathfrak{p}}\!\left(1\!-\!\frac{1}{\mathfrak{p}}\right)$$

$$[x]^2\!=\!D(x)\!=\!\prod_{\mathfrak{p}}\!\left(1\!-\!\frac{2}{\mathfrak{p}}\right)$$

$$([x]-a)^0=1\,;\, ([x]-a)^k=\sum_{j=0}^{\lfloor x-a\rfloor} ([\frac{x}{j+a}]-a)^{k-1}$$

$$C_k(x,a)=a^{-k}([x\,a^k]-(a+1))^k$$

$$\begin{aligned}
&([n]-a)^0=1\\
&([n]-a)^k=\sum_{j=0}^k \binom{k}{j} ([\frac{n}{a^j}]- (a+1))^{k-j}
\end{aligned}$$

$$\log [x]=\sum_{j=2}^{\lfloor x\rfloor}\sum_{k=0}^{\lfloor\log_2x\rfloor}\frac{B_k}{k!}\log\big[\frac{x}{j}\big]^k$$

$$\log x=\sum_{k=0}^{\lfloor\log_2x\rfloor}(x-1)\frac{B_k}{k!}(\log x)^k$$

$$\log [x]^a=\sum_{j=2}^{\lfloor x\rfloor}\sum_{k=0}^{\lfloor\log_2x\rfloor}\frac{B_k}{k!}\log\big[\frac{x}{j}\big]^{(k+a)}$$

$$(\log x)^a=\sum_{k=0}^{\lfloor\log_2x\rfloor}(x-1)\frac{B_k}{k!}(\log x)^{k+a}$$