

```

StrictDivisorSummatory[n_, k_, a_] :=
  Sum[ Binomial[ k, j] StrictDivisorSummatory[Floor[n / (m^j)], k - j, m + 1],
    {j, 1, k}, {m, a, n^(1/k)}]
StrictDivisorSummatory[n_, 1, a_] := n - a + 1
StrictDivisorSummatory[n_, 0, a_] := 1
CountOfPrimes[ n_ ] :=
  Sum[ (-1)^(k+1) StrictDivisorSummatory[Floor[n^(1/j)], k, 2] / k / j MoebiusMu[ j],
    {j, 1, Log[ 2, n ]}, {k, 1, Log[ 2, n^(1/j)]}]
CountOfPrimes[ 10 000 ]

1229

DD[200, 2, 2]

699

D2[n_, a_] := 1 - 2 a + a^2 - Floor[n^(1/2)] + 2 Sum[ Floor[n/m], {m, a, Floor[n^(1/2)]]]
D2[200, 2]

881

D2a[n_, a_] := Sum[ 1 + 2 ( Floor[n/m] - (m+1) + 1), {m, a, Floor[n^(1/2)]]]
D2a[200, 2]

699

D2b[n_, a_] := Sum[ 1 + 2 Floor[n/m] - 2 m, {m, a, Floor[n^(1/2)]]]
D2b[200, 2]

699

D2c[n_, a_] := Sum[ 1 - 2 m, {m, a, Floor[n^(1/2)]]]
D2c[n, a]


$$\left(-1 + a - \text{Floor}\left[\sqrt{n}\right]\right) \left(-1 + a + \text{Floor}\left[\sqrt{n}\right]\right)$$

D2d[n_, a_] :=
  
$$\left(-1 + a - \text{Floor}\left[\sqrt{n}\right]\right) \left(-1 + a + \text{Floor}\left[\sqrt{n}\right]\right) + 2 \text{Sum}\left[\text{Floor}\left[n/m\right], \{m, a, \text{Floor}\left[n^{1/2}\right]\}\right]$$

D2d[200, 2]

699

Expand[  $\left(-1 + a - \text{Floor}\left[\sqrt{n}\right]\right) \left(-1 + a + \text{Floor}\left[\sqrt{n}\right]\right)$  ]


$$1 - 2 a + a^2 - \text{Floor}\left[\sqrt{n}\right]^2$$

D2e[n_, a_] := 1 - 2 a + a^2 - Floor[ $\sqrt{n}$ ]^2 + 2 Sum[ Floor[n/m], {m, a, Floor[n^(1/2)]]]
D2e[200, 2]

699

D2f[n_] := 1 - Floor[ $\sqrt{n}$ ]^2 + 2 Sum[ Floor[n/m], {m, 2, Floor[n^(1/2)]]]
D2f[200]

699

```

```

D3a[n_] :=
  Sum[1 + 3 Floor[n / (m^2)] - 3 m + 3 DD[Floor[n / m], 2, m + 1], {m, 2, Floor[n^(1/3)]}]
D3a[200]
1027

DD[200, 3, 2]
1027

D3b[n_] := (4 - Floor[n^(1/3)] - 3 Floor[n^(1/3)]^2) / 2 +
  Sum[3 Floor[n / (m^2)] + 3 DD[Floor[n / m], 2, m + 1], {m, 2, Floor[n^(1/3)]}]
D3b[200]
1027

D3c[n_] := (4 - Floor[n^(1/3)] - 3 Floor[n^(1/3)]^2) / 2 +
  3 Sum[Floor[n / (m^2)] + DD[Floor[n / m], 2, m + 1], {m, 2, Floor[n^(1/3)]}]
D3c[200]
1027

D3d[n_] := (4 - Floor[n^(1/3)] - 3 Floor[n^(1/3)]^2) / 2 +
  3 Sum[Floor[n / (m^2)], {m, 2, Floor[n^(1/3)]}] + 3 Sum[
    DD[Floor[n / m], 2, m + 1], {m, 2, Floor[n^(1/3)]}]
D3d[200]
1027

D3e[n_] := (4 - Floor[n^(1/3)] - 3 Floor[n^(1/3)]^2) / 2 +
  3 Sum[Floor[n / (m^2)], {m, 2, Floor[n^(1/3)]}] + 3 Sum[
    DD[Floor[n / m], 2, m + 1], {m, 2, Floor[n^(1/3)]}]

D2g[n_, a_] := 1 - 2 a + a^2 - Floor[ $\sqrt{n}$ ]^2 + 2 Sum[Floor[n / m], {m, a, Floor[n^(1/2)]}]
D2g[200, 2]
699

D2h[n_, m_] := 1 - 2 m + m^2 - Floor[ $\sqrt{n}$ ]^2 + 2 Sum[Floor[n / j], {j, m, Floor[n^(1/2)]}]
D2h[200, 2]
699

D3f[n_] := (4 - Floor[n^(1/3)] - 3 Floor[n^(1/3)]^2) / 2 +
  3 Sum[Floor[n / (m^2)], {m, 2, Floor[n^(1/3)]}] + 3 Sum[

$$m^2 - \text{Floor}\left[\sqrt{\text{Floor}\left[\frac{n}{m}\right]}\right]^2 + 2 \sum_{j=1+m}^{\text{Floor}\left[\sqrt{\text{Floor}\left[\frac{n}{m}\right]}\right]} \text{Floor}\left[\frac{\text{Floor}\left[\frac{n}{m}\right]}{j}\right], \{m, 2, \text{Floor}[n^{(1/3)}]\}]$$

D3f[200]
1027

D2h[Floor[n / m], m + 1]

```

$$\text{FullSimplify}\left[1 - 2(1+m) + (1+m)^2 - \text{Floor}\left[\sqrt{\text{Floor}\left[\frac{n}{m}\right]}\right]^2 + 2 \sum_{j=1+m}^{\text{Floor}\left[\sqrt{\text{Floor}\left[\frac{n}{m}\right]}\right]} \text{Floor}\left[\frac{\text{Floor}\left[\frac{n}{m}\right]}{j}\right]\right]$$

$$m^2 - \text{Floor}\left[\sqrt{\text{Floor}\left[\frac{n}{m}\right]}\right]^2 + 2 \sum_{j=1+m}^{\text{Floor}\left[\sqrt{\text{Floor}\left[\frac{n}{m}\right]}\right]} \text{Floor}\left[\frac{\text{Floor}\left[\frac{n}{m}\right]}{j}\right]$$

D3f[200]

1027

DD[200, 3, 2]

1027

D2h[Floor[n / m], m + 1]

$$1 - 2(1+m) + (1+m)^2 - \text{Floor}\left[\sqrt{\text{Floor}\left[\frac{n}{m}\right]}\right]^2 + 2 \sum_{j=1+m}^{\text{Floor}\left[\sqrt{\text{Floor}\left[\frac{n}{m}\right]}\right]} \text{Floor}\left[\frac{\text{Floor}\left[\frac{n}{m}\right]}{j}\right]$$

Expand[FullSimplify[3 Sum[m^2, {m, 2, Floor[n^(1/3)]]}]]

$$-3 + \frac{1}{2} \text{Floor}[n^{1/3}] + \frac{3}{2} \text{Floor}[n^{1/3}]^2 + \text{Floor}[n^{1/3}]^3$$

$$\begin{aligned} &\text{FullSimplify}\left[\left(4 - \text{Floor}[n^{1/3}] - 3 \text{Floor}[n^{1/3}]^2\right) / 2 + \left(-6 + \text{Floor}[n^{1/3}] + 3 \text{Floor}[n^{1/3}]^2 + \text{Floor}[n^{1/3}]^3\right) / 2\right] \\ &\frac{1}{2} \left(-2 + \text{Floor}[n^{1/3}]^3\right) \end{aligned}$$

D3g[n_] := (4 - Floor[n^(1/3)] - 3 Floor[n^(1/3)]^2) / 2 +

$$\left(-3 + \frac{1}{2} \text{Floor}[n^{1/3}] + \frac{3}{2} \text{Floor}[n^{1/3}]^2 + \text{Floor}[n^{1/3}]^3\right) +$$

3 Sum[Floor[n / (m^2)], {m, 2, Floor[n^(1/3)]]] + 3 Sum[

$$-\text{Floor}\left[\sqrt{\text{Floor}\left[\frac{n}{m}\right]}\right]^2 + 2 \sum_{j=1+m}^{\text{Floor}\left[\sqrt{\text{Floor}\left[\frac{n}{m}\right]}\right]} \text{Floor}\left[\frac{\text{Floor}\left[\frac{n}{m}\right]}{j}\right], \{m, 2, \text{Floor}[n^{1/3}]\}]$$

D3g[200]

1027

FullSimplify[(4 - Floor[n^(1/3)] - 3 Floor[n^(1/3)]^2) / 2 +

$$\left(-3 + \frac{1}{2} \text{Floor}[n^{1/3}] + \frac{3}{2} \text{Floor}[n^{1/3}]^2 + \text{Floor}[n^{1/3}]^3\right)]$$

$$-1 + \text{Floor}[n^{1/3}]^3$$

D3h[n_] := -1 + Floor[n^(1/3)]^3 + 3 Sum[Floor[n / (m^2)]

$$-\text{Floor}[\text{Floor}[n / m]^{1/2}]^2 + 2 \sum_{j=1+m}^{\text{Floor}\left[\sqrt{\text{Floor}\left[\frac{n}{m}\right]}\right]} \text{Floor}\left[\frac{\text{Floor}\left[\frac{n}{m}\right]}{j}\right], \{m, 2, \text{Floor}[n^{1/3}]\}]$$

D3h[200]

1027

```
D3Unrolled[n_] := -1 + Floor[n^(1/3)]^3 +
  3 Sum[Floor[n/(m^2)] - Floor[Floor[n/m]^(1/2)]^2 + 2 Sum[Floor[Floor[n/m]/j],
    {j, m+1, Floor[Floor[n/m]^(1/2)]}], {m, 2, Floor[n^(1/3)]}
```

D3Unrolled[200]

1027

```
D3i[n_, a_] := Sum[1 + 3 (Floor[n/(m^2)] - m) +
  3 Sum[1 + 2 Floor[n/(m j)] - 2 j, {j, m+1, Floor[Floor[n/m]^(1/2)]}], {m,
  a, Floor[n^(1/2)]}
```

D3i[200, 2]

826

```
D3j[n_, a_] :=
  Sum[1 + 3 (Floor[n/(m^2)] - m) + 3 DD[Floor[n/m], 2, m+1], {m, a, Floor[n^(1/2)]}]
```

D3j[100, 2]

228

StrictDivisorSummatory[n, 2, 2]

\$RecursionLimit::reclim: Recursion depth of 256 exceeded. >>

\$RecursionLimit::reclim: Recursion depth of 256 exceeded. >>

\$RecursionLimit::reclim: Recursion depth of 256 exceeded. >>

General::stop: Further output of \$RecursionLimit::reclim will be suppressed during this calculation. >>

\$IterationLimit::itlim: Iteration limit of 4096 exceeded. >>

\$IterationLimit::itlim: Iteration limit of 4096 exceeded. >>

StrictDivisorSummatory[n, 1, 2]

StrictDivisorSummatory[n, 1, 2]

EE[n_, 0, a_] := 1

EE[n_, 1, a_] := n - a + 1

EE[n_, k_, a_] :=

```
Sum[Binomial[k, j] EE[Floor[n/(m^j)], k - j, m+1], {j, 1, k}, {m, a, n^(1/k)}]
```

EE[100, 3, 2]

324

```
D32Unrolled[n_] := -1 + Floor[n^(1/3)]^3 +
```

```
  3 Sum[Floor[n/(m^2)] - Floor[Floor[n/m]^(1/2)]^2 + 2 Sum[Floor[Floor[n/m]/j],
    {j, m+1, Floor[Floor[n/m]^(1/2)]}], {m, 2, Floor[n^(1/3)]}
```

```

D2f[n_] := 1 - Floor[ $\sqrt{n}$ ]^2 + 2 Sum[ Floor[n / m], {m, 2, Floor[n^(1 / 2)]}]
D22Unrolled[n_] := 1 - Floor[n^(1 / 2)]^2 + 2 Sum[ Floor[n / m], {m, 2, Floor[n^(1 / 2)]}]
D22Unrolled[10 000]
73 669
EE[10 000, 2, 2]
73 669

```