```
Clear[f]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
a[n_] := FiniteAbelianGroupCount[n]
f[n_, k_] := f[n, k] = Sum[ a[j] f[Floor[n / j], k - 1], {j, 2, n}]
f[n_, 0] := UnitStep[n - 1]
lf[n_] := Sum[ (-1)^(k + 1) / k f[n, k], {k, 1, Log2@n}]
fz[n_, z_] := Sum[bin[z, k] f[n, k], {k, 0, Log2@n}]
ffz[n_, z_] := fz[n, z] - fz[n - 1, z]
am[n_] := ffz[n, -1]
dfz[n_, z_] := Expand[fz[n, z] - fz[n - 1, z]]
pr[n_] := Sum[ PrimePi[ n^(1 / k)] / k, {k, 1, Log2@n}]
pr2[n_] := Sum[ pr[ n^(1 / k)], {k, 1, Log2@n}]
```

```
lf[100]
```

$$\frac{428}{15}$$

```
pr2[100]
```

$$\frac{1211}{30}$$

```
Table[FullSimplify@dfz[2^k, z], {k, 0, 7}] // TableForm
```

$$1$$
$$z$$
$$\frac{1}{2} z (3 + z)$$
$$\frac{1}{6} z (1 + z) (8 + z)$$
$$\frac{1}{24} z (1 + z) (3 + z) (14 + z)$$
$$\frac{1}{120} z (3 + z) (6 + z) (8 + z (21 + z))$$
$$\frac{1}{720} z (1 + z) (10 + z) (144 + z (181 + z (34 + z)))$$
$$\frac{z (2+z) (3+z) (8+z) (120+z (529+z (50+z)))}{5040}$$

$$1 + \frac{1211\, z}{30} + \frac{31\,949\, z^2}{360} + \frac{2215\, z^3}{48} + \frac{1187\, z^4}{144} + \frac{39\, z^5}{80} + \frac{7\, z^6}{720}$$

$$1 + \frac{1211\, z}{30} + \frac{31\,949\, z^2}{360} + \frac{2215\, z^3}{48} + \frac{1187\, z^4}{144} + \frac{39\, z^5}{80} + \frac{7\, z^6}{720}$$

```
Sum[1, {j, 1, 100}, {k, 1, (100 / j)^(1 / 2)}]
```

153

```
tl[n_, k_, t_] := If[t < 1, 1, Sum[ tl[n / (j^k), k + 1, t - 1], {j, 1, n^(1 / k)}]]
```

```
tl[100, 1, 8]
```

185

```
Sum[ a[j], {j, 1, 100}]
```

185

```
{ffz[12, 1], a[12]}
```

```
{2, 2}
```

```
Sum[ ffz[aa, 1] ffz[b, -1] ffz[c, -1] ffz[d, -1] ffz[e, 1] ffz[ff, -1], {aa, 1, 100},
 {b, 1, Floor[(100 / aa)^(1 / 2)]}, {c, 1, Floor[(100 / (aa b^2))^(1 / 3)]},
 {d, 1, Floor[(100 / (aa b^2 c^3))^(1 / 5)]}, {e, 1, Floor[(100 / (aa b^2 c^3 d^5))^(1 / 6)]},
 {ff, 1, Floor[(100 / (aa b^2 c^3 d^5 e^6))^(1 / 7)]}]
```

```
100
```

```
Sum[ ffz[aa, 1] ffz[b, -1] ffz[c, -1] ffz[d, -1], {aa, 1, 100},
 {b, 1, Floor[(100 / aa)^(1 / 2)]}, {c, 1, Floor[(100 / (aa b^2))^(1 / 3)]},
 {d, 1, Floor[(100 / (aa b^2 c^3))^(1 / 4)]}]
```

```
95
```

```
Sum[a[aa] am[b] am[c] am[d] a[e] am[ff], {aa, 1, 100},
 {b, 1, Floor[(100 / aa)^(1 / 2)]}, {c, 1, Floor[(100 / (aa b^2))^(1 / 3)]},
 {d, 1, Floor[(100 / (aa b^2 c^3))^(1 / 5)]}, {e, 1, Floor[(100 / (aa b^2 c^3 d^5))^(1 / 6)]},
 {ff, 1, Floor[(100 / (aa b^2 c^3 d^5 e^6))^(1 / 7)]}]
```

```
100
```

```
(*https://oeis.org/A129667*)
Table[{n, ffz[n, -1]}, {n, 1, 40}] // TableForm
```

1	1
2	−1
3	−1
4	−1
5	−1
6	1
7	−1
8	0
9	−1
10	1
11	−1
12	1
13	−1
14	1
15	1
16	0
17	−1
18	1
19	−1
20	1
21	1
22	1
23	−1
24	0
25	−1
26	1
27	0
28	1
29	−1
30	−1
31	−1
32	1
33	1
34	1
35	1
36	1
37	−1
38	1
39	1
40	0