```
ClearAll["Global`*"]
Sum[ (-1)^(5 j / 6 + 1) / j, {j, 1, Infinity}]
```

$\text{Log}\left[1-(-1)^{5/6}\right]$

```
cyc := {1, 1, 1, -7, 1, 1, 1, 1}
ee[n_, 0, m_] := 1; ee[n_, k_ , m_] :=
 ee[n, k, m] = Sum[ cyc[[1 + Mod[j - 1, Length[cyc]]]] ee[Floor[n / j], k - 1, m], {j, 2, n}]
e1[n_, z_, m_] := Sum[ Binomial[ z, k] ee[n, k, m], {k, 0, Log[2, n]}]
Table[ {n, Limit[ (e1[n, z, a = 1 / 4] - 1) / z, z → 0] - Limit[ (e1[n - 1, z, a] - 1) / z, z → 0]},
  {n, 2, 30}] // TableForm
```

2	1
3	1
4	$-\frac{15}{2}$
5	1
6	0
7	1
8	$\frac{25}{3}$
9	$\frac{1}{2}$
10	0
11	1
12	0
13	1
14	0
15	0
16	$-\frac{127}{4}$
17	1
18	0
19	1
20	0
21	0
22	0
23	1
24	0
25	$\frac{1}{2}$
26	0
27	$\frac{1}{3}$
28	0
29	1
30	0

```
d1[100, 3]
```

$22-7\,i$

```
cyc[[2]]
```

-1

```
Table[cyc[[1 + Mod[n - 1, Length[cyc]]]], {n, 1, 20}]
```

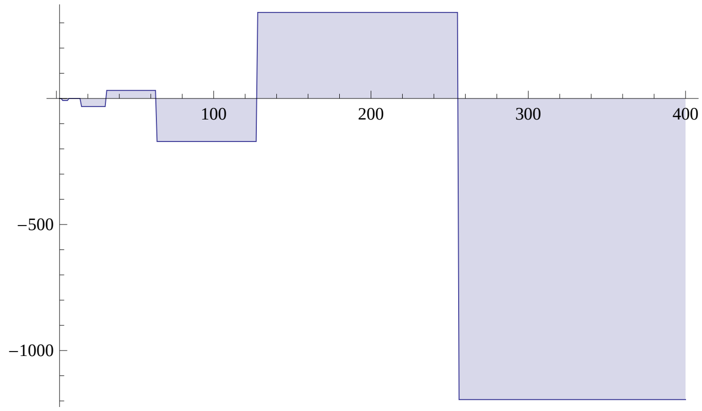
{1, -1, -2, 2, 1, -1, -2, 2, 1, -1, -2, 2, 1, -1, -2, 2, 1, -1, -2, 2}

```
Table[1 + Mod[n - 1, Length[cyc]], {n, 1, 20}]
```

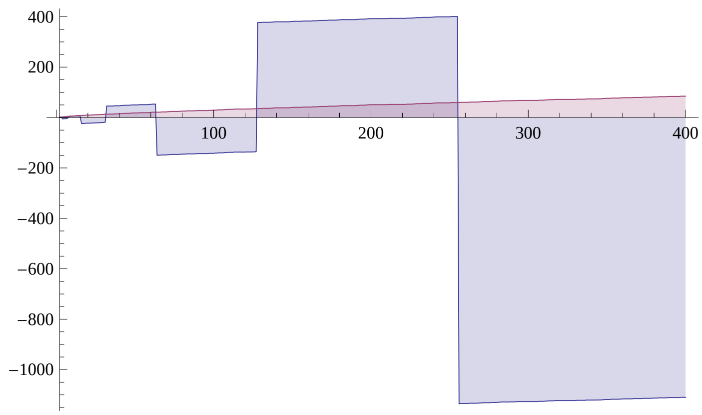
{1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4}

```
p[n_, k_] := p[n, k] = Sum[ 1 / k - p[Floor[n / j], k + 1], {j, 2, n}]
```

```
DiscretePlot[ Limit[ (e1[n, z, a = 1 / 4] - 1) / z, z → 0] - p[n, 1], {n, 2, 400}] // TableForm
```




```
DiscretePlot[ {Limit[ (e1[n, z, a = 1 / 4] - 1) / z, z → 0], p[n, 1]}, {n, 2, 400}] // TableForm
```




```
Table[ {n, Limit[ (e1[n, z, a = 1 / 4] - 1) / z, z → 0] - p[n, 1]}, {n, 2, 100}] // TableForm
```

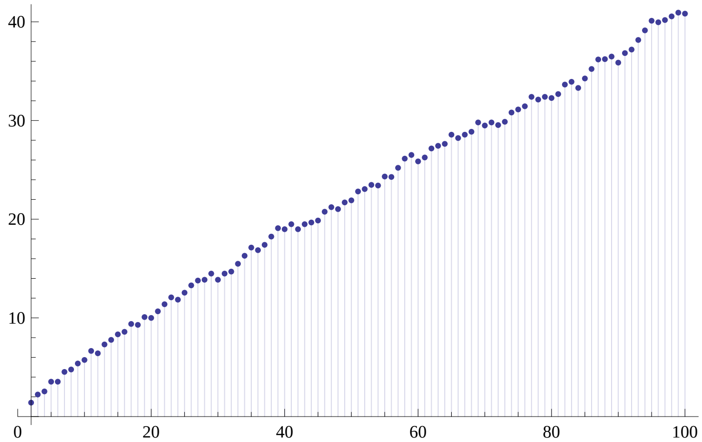
2	0
3	0
4	-8
5	-8
6	-8
7	-8
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0
16	-32
17	-32
18	-32
19	-32

20	-32
21	-32
22	-32
23	-32
24	-32
25	-32
26	-32
27	-32
28	-32
29	-32
30	-32
31	-32
32	32
33	32
34	32
35	32
36	32
37	32
38	32
39	32
40	32
41	32
42	32
43	32
44	32
45	32
46	32
47	32
48	32
49	32
50	32
51	32
52	32
53	32
54	32
55	32
56	32
57	32
58	32
59	32
60	32
61	32
62	32
63	32
64	$-\frac{512}{3}$
65	$-\frac{512}{3}$
66	$-\frac{512}{3}$
67	$-\frac{512}{3}$
68	$-\frac{512}{3}$
69	$-\frac{512}{3}$
70	$-\frac{512}{3}$
71	$-\frac{512}{3}$

72	$-\frac{512}{3}$
73	$-\frac{512}{3}$
74	$-\frac{512}{3}$
75	$-\frac{512}{3}$
76	$-\frac{512}{3}$
77	$-\frac{512}{3}$
78	$-\frac{512}{3}$
79	$-\frac{512}{3}$
80	$-\frac{512}{3}$
81	$-\frac{512}{3}$
82	$-\frac{512}{3}$
83	$-\frac{512}{3}$
84	$-\frac{512}{3}$
85	$-\frac{512}{3}$
86	$-\frac{512}{3}$
87	$-\frac{512}{3}$
88	$-\frac{512}{3}$
89	$-\frac{512}{3}$
90	$-\frac{512}{3}$
91	$-\frac{512}{3}$
92	$-\frac{512}{3}$
93	$-\frac{512}{3}$
94	$-\frac{512}{3}$
95	$-\frac{512}{3}$
96	$-\frac{512}{3}$
97	$-\frac{512}{3}$
98	$-\frac{512}{3}$
99	$-\frac{512}{3}$
100	$-\frac{512}{3}$

```
d[x_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[x]}];
FI[x_] := FactorInteger[x]; FI[1] := {}
Dd[x_, z_] := Sum[d[j, z], {j, 1, x}]
```

```
DiscretePlot[ Abs[Dd[n, -I]], {n, 2, 100}]
```




```
Dd[100, -I]
```

$-\frac{2881}{72} - \frac{65\,i}{8}$

```
Sum[ 1 / 2^j, {j, 1, Infinity}]
```

1

```
Sum[ (-1)^(j+1) / 2^j, {j, 1, Infinity}]
```

$\frac{1}{3}$

```
t [n_, j_] := Mod[n, j] - Mod[n - 1, j]
```

```
Sum[ t[j, 3] / 2^j, {j, 1, Infinity}]
```

$\frac{4}{7}$

```
Sum[ t[j, 4] / 2^j, {j, 1, Infinity}]
```

$\frac{11}{15}$

```
Sum[ t[j, 5] / 2^j, {j, 1, Infinity}]
```

$\frac{26}{31}$

```
Sum[ t[j, 2] / 3^j, {j, 1, Infinity}]
```

$\frac{1}{4}$

```
Sum[ t[j, 3] / 3^j, {j, 1, Infinity}]
```

$\frac{5}{13}$

```
Sum[ t[j, 3] / 4^j, {j, 1, Infinity}]
```

$\frac{2}{7}$

```
Sum[ t[j, 3] / 5^j, {j, 1, Infinity}]
```

$$\frac{7}{31}$$

```
Sum[ Abs[t[j, 3]] / 5^j, {j, 1, Infinity}]
```

$$\frac{8}{31}$$

```
Sum[ t[j, 3] / c^j, {j, 1, Infinity}]
```

$$\frac{2+c}{1+c+c^2}$$

```
Sum[Abs[ t[j, 3]] / c^j, {j, 1, Infinity}]
```

$$\frac{2+c+c^2}{-1+c^3}$$

```
Sum[ t[j, 4] / c^j, {j, 1, Infinity}]
```

$$\frac{3+2c+c^2}{1+c+c^2+c^3}$$

```
Sum[Abs[ t[j, 4]] / c^j, {j, 1, Infinity}]
```

$$\frac{3+c+c^2+c^3}{-1+c^4}$$

```
Sum[ t[j, 5] / c^j, {j, 1, Infinity}]
```

$$\frac{4+3c+2c^2+c^3}{1+c+c^2+c^3+c^4}$$

```
Sum[Abs[ t[j, 5]] / c^j, {j, 1, Infinity}]
```

$$\frac{4+c+c^2+c^3+c^4}{-1+c^5}$$

```
dAlt[x_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[x]}];
FI[x_] := FactorInteger[x]; FI[1] := {}
```

```
D[dAlt[x, z], z] /. z → 0
```

FactorInteger::exact : Argument x in FactorInteger[x] is not an exact number. ≫

Part::partd : Part specification p⟦2⟧ is longer than depth of object. ≫

Part::partd : Part specification p⟦2⟧ is longer than depth of object. ≫

FactorInteger::exact : Argument x in FactorInteger[x] is not an exact number. ≫

Part::partd : Part specification p⟦2⟧ is longer than depth of object. ≫

General::stop : Further output of Part::partd will be suppressed during this calculation. ≫

General::ivar : 0 is not a valid variable. ≫

$$\partial_0 \prod_{p}^{\text{FactorInteger}[x]} (-1)^{p[\![2]\!]}\, \text{Binomial}[0, p[\![2]\!]]$$

```
x^(1/2) Sum[ (-1)^(n-1) (Log[x])^n / (n! 2^(n-1))
   Sum[1/(2k + 1), {k, 0, (n-1)/2}], {n, 1, Infinity}]
```

$$\sqrt{x}\sum_{n=1}^{\infty}\frac{(-1)^{-1+n}\,2^{-n}\,\mathrm{Log}[x]^{n}\left(-\mathrm{PolyGamma}\left[0,\frac{1}{2}\right]+\mathrm{PolyGamma}\left[0,1+\frac{n}{2}\right]\right)}{n!}$$

```
vv[x_] := x^(1/2) Sum[ (-1)^(n-1) (Log[x])^n / ((n!) 2^(n-1))
    Sum[1/(2k + 1), {k, 0, Floor[(n-1)/2]}], {n, 1, Infinity}]
```

```
N[vv[100.]]
```

$Aborted

```
LogIntegral[100.] - Log[Log[100.]] - EulerGamma
```

28.0217

```
LL[x_, 1, a_] := LL[x, 1, a] = Sum[Log[(j + a)/(a - 1)], {j, 0, Floor[x - a]}]
LL[x_, k_, a_] := LL[x, k, a] = Sum[LL[x/(j + a), k - 1, a], {j, 0, Floor[x - a]}]
LC[x_, k_, a_] := a^(-k) LL[x a^k, k, a + 1]
L1[x_, z_, a_] := Sum[Binomial[z, k] LC[x, k, a], {k, 1, Log[2, 2 a x]}]
```

```
N[L1[100, -1, 1]]
```

-94.0453

```
DiscretePlot[L1[n, -1, 1], {n, 1, 100}]
```