```
FactInteger[n_] := If[n == 1, {}, FactorInteger[n]]
d[n_, z_] := Product[1 / (p[[2]]!) Pochhammer[z, p[[2]]], {p, FactInteger[n]}]

pk[n_, 0] := pk[n, 0] = If[n == 1, 1, 0]
pk[n_, 1] := pk[n, 1] = If[n == 1, 0, FullSimplify[MangoldtLambda[n] / Log[n]]]
pk[n_, k_] := pk[n, k] = Sum[pk[j, k - 1] pk[n / j, 1], {j, Divisors[n]}]
dv2[n_, k_] := Sum[k^j / (j!) pk[n, j], {j, 0, N[Log[n] / Log[2]]}]
dv3[n_, k_] :=
 dv3[n, k] = pk[n, 0] + k pk[n, 1] + k^2 / 2 pk[n, 2] + k^3 / 6 pk[n, 3] + k^4 / 24 pk[n, 4] +
   k^5 / 120 pk[n, 5] + k^6 / 720 pk[n, 6] + k^7 / (7!) pk[n, 7] + k^8 / (8!) pk[n, 8]
cosp[n_, k_] := cosp[n, k] = pk[n, 0] - k^2 / 2 pk[n, 2] +
   k^4 / 24 pk[n, 4] - k^6 / 720 pk[n, 6] + k^8 / (8!) pk[n, 8]
sinp[n_, k_] := sinp[n, k] = k pk[n, 1] - k^3 / 6 pk[n, 3] +
   k^5 / 120 pk[n, 5] - k^7 / (7!) pk[n, 7]

Table[{n, a = dv3[n, I], b = (cosp[n, 1] + I sinp[n, 1]), a - b}, {n, 1, 100}] // TableForm
```

1	1	1	0
2	i	i	0
3	i	i	0
4	$-\frac{1}{2}+\frac{i}{2}$	$-\frac{1}{2}+\frac{i}{2}$	0
5	i	i	0
6	-1	-1	0
7	i	i	0
8	$-\frac{1}{2}+\frac{i}{6}$	$-\frac{1}{2}+\frac{i}{6}$	0
9	$-\frac{1}{2}+\frac{i}{2}$	$-\frac{1}{2}+\frac{i}{2}$	0
10	-1	-1	0
11	i	i	0
12	$-\frac{1}{2}-\frac{i}{2}$	$-\frac{1}{2}-\frac{i}{2}$	0
13	i	i	0
14	-1	-1	0
15	-1	-1	0
16	$-\frac{5}{12}$	$-\frac{5}{12}$	0
17	i	i	0
18	$-\frac{1}{2}-\frac{i}{2}$	$-\frac{1}{2}-\frac{i}{2}$	0
19	i	i	0
20	$-\frac{1}{2}-\frac{i}{2}$	$-\frac{1}{2}-\frac{i}{2}$	0
21	-1	-1	0
22	-1	-1	0
23	i	i	0
24	$-\frac{1}{6}-\frac{i}{2}$	$-\frac{1}{6}-\frac{i}{2}$	0
25	$-\frac{1}{2}+\frac{i}{2}$	$-\frac{1}{2}+\frac{i}{2}$	0
26	-1	-1	0
27	$-\frac{1}{2}+\frac{i}{6}$	$-\frac{1}{2}+\frac{i}{6}$	0
28	$-\frac{1}{2}-\frac{i}{2}$	$-\frac{1}{2}-\frac{i}{2}$	0
29	i	i	0
30	$-i$	$-i$	0
31	i	i	0
32	$-\frac{1}{3}-\frac{i}{12}$	$-\frac{1}{3}-\frac{i}{12}$	0
33	-1	-1	0
34	-1	-1	0

35	-1	-1	0
36	$-\frac{i}{2}$	$-\frac{i}{2}$	0
37	i	i	0
38	-1	-1	0
39	-1	-1	0
40	$-\frac{1}{6}-\frac{i}{2}$	$-\frac{1}{6}-\frac{i}{2}$	0
41	i	i	0
42	$-i$	$-i$	0
43	i	i	0
44	$-\frac{1}{2}-\frac{i}{2}$	$-\frac{1}{2}-\frac{i}{2}$	0
45	$-\frac{1}{2}-\frac{i}{2}$	$-\frac{1}{2}-\frac{i}{2}$	0
46	-1	-1	0
47	i	i	0
48	$-\frac{5\,i}{12}$	$-\frac{5\,i}{12}$	0
49	$-\frac{1}{2}+\frac{i}{2}$	$-\frac{1}{2}+\frac{i}{2}$	0
50	$-\frac{1}{2}-\frac{i}{2}$	$-\frac{1}{2}-\frac{i}{2}$	0
51	-1	-1	0
52	$-\frac{1}{2}-\frac{i}{2}$	$-\frac{1}{2}-\frac{i}{2}$	0
53	i	i	0
54	$-\frac{1}{6}-\frac{i}{2}$	$-\frac{1}{6}-\frac{i}{2}$	0
55	-1	-1	0
56	$-\frac{1}{6}-\frac{i}{2}$	$-\frac{1}{6}-\frac{i}{2}$	0
57	-1	-1	0
58	-1	-1	0
59	i	i	0
60	$\frac{1}{2}-\frac{i}{2}$	$\frac{1}{2}-\frac{i}{2}$	0
61	i	i	0
62	-1	-1	0
63	$-\frac{1}{2}-\frac{i}{2}$	$-\frac{1}{2}-\frac{i}{2}$	0
64	$-\frac{19}{72}-\frac{i}{8}$	$-\frac{19}{72}-\frac{i}{8}$	0
65	-1	-1	0
66	$-i$	$-i$	0
67	i	i	0
68	$-\frac{1}{2}-\frac{i}{2}$	$-\frac{1}{2}-\frac{i}{2}$	0
69	-1	-1	0
70	$-i$	$-i$	0
71	i	i	0
72	$\frac{1}{6}-\frac{i}{3}$	$\frac{1}{6}-\frac{i}{3}$	0
73	i	i	0
74	-1	-1	0
75	$-\frac{1}{2}-\frac{i}{2}$	$-\frac{1}{2}-\frac{i}{2}$	0
76	$-\frac{1}{2}-\frac{i}{2}$	$-\frac{1}{2}-\frac{i}{2}$	0
77	-1	-1	0
78	$-i$	$-i$	0
79	i	i	0
80	$-\frac{5\,i}{12}$	$-\frac{5\,i}{12}$	0
81	$-\frac{5}{12}$	$-\frac{5}{12}$	0
82	-1	-1	0

83	i	i	0
84	$\frac{1}{2} - \frac{i}{2}$	$\frac{1}{2} - \frac{i}{2}$	0
85	-1	-1	0
86	-1	-1	0
87	-1	-1	0
88	$-\frac{1}{6} - \frac{i}{2}$	$-\frac{1}{6} - \frac{i}{2}$	0
89	i	i	0
90	$\frac{1}{2} - \frac{i}{2}$	$\frac{1}{2} - \frac{i}{2}$	0
91	-1	-1	0
92	$-\frac{1}{2} - \frac{i}{2}$	$-\frac{1}{2} - \frac{i}{2}$	0
93	-1	-1	0
94	-1	-1	0
95	-1	-1	0
96	$\frac{1}{12} - \frac{i}{3}$	$\frac{1}{12} - \frac{i}{3}$	0
97	i	i	0
98	$-\frac{1}{2} - \frac{i}{2}$	$-\frac{1}{2} - \frac{i}{2}$	0
99	$-\frac{1}{2} - \frac{i}{2}$	$-\frac{1}{2} - \frac{i}{2}$	0
100	$-\frac{i}{2}$	$-\frac{i}{2}$	0

```
FF[n_] := Sin[n]^2 + Cos[n]^2
```

```
N[FF[8]]
```

```
1.
```

```
sinp[8, 3]
```

$$-\frac{7}{2}$$

```
sinp[9, 2] + cosp[9, 2]
```

```
-1
```

```
Sinp[n_, k_] := Sum[ sinp[j, k], {j, 1, n}]
```

```
Sinp[100, 2]
```

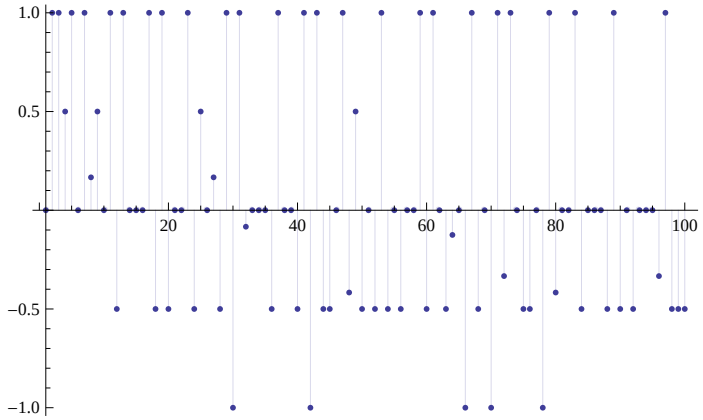
$$-\frac{199}{2}$$

```
Cosp[n_, k_] := Sum[ cosp[j, k], {j, 1, n}]
```

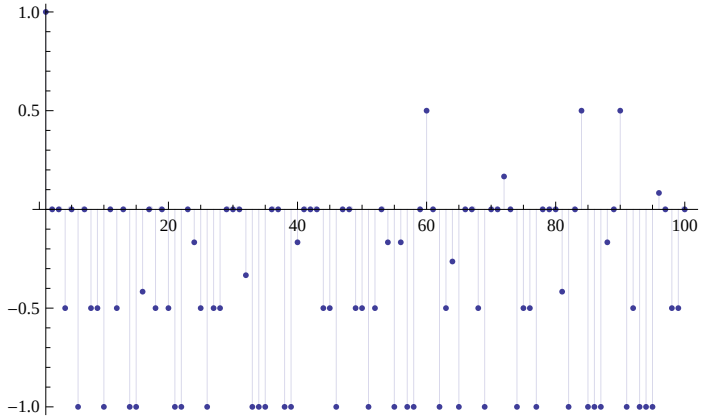
```
Cosp[100, 2]
```

$$-\frac{2029}{18}$$

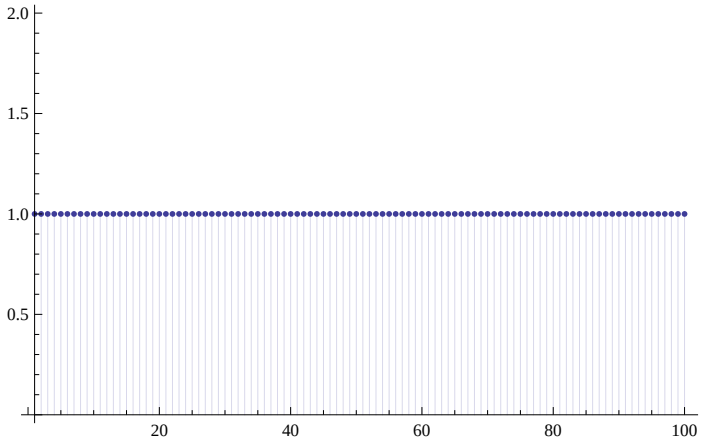
```
DiscretePlot[sinp[n, 1], {n, 1, 100}]
```




```
DiscretePlot[cosp[n, 1], {n, 1, 100}]
```




```
DiscretePlot[dv3[n, 1], {n, 1, 100}]
```



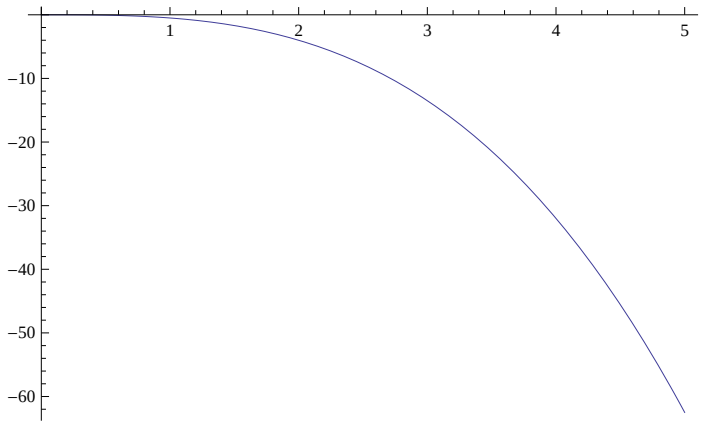

```
Table[{n, a = cosp[n, 1], b = (sinp[n, 1])}, {n, 1, 100}] // TableForm
```

1	1	0
2	0	1
3	0	1
4	$-\frac{1}{2}$	$\frac{1}{2}$
5	0	1

6	-1	0
7	0	1
8	$-\frac{1}{2}$	$\frac{1}{6}$
9	$-\frac{1}{2}$	$\frac{1}{2}$
10	-1	0
11	0	1
12	$-\frac{1}{2}$	$-\frac{1}{2}$
13	0	1
14	-1	0
15	-1	0
16	$-\frac{5}{12}$	0
17	0	1
18	$-\frac{1}{2}$	$-\frac{1}{2}$
19	0	1
20	$-\frac{1}{2}$	$-\frac{1}{2}$
21	-1	0
22	-1	0
23	0	1
24	$-\frac{1}{6}$	$-\frac{1}{2}$
25	$-\frac{1}{2}$	$\frac{1}{2}$
26	-1	0
27	$-\frac{1}{2}$	$\frac{1}{6}$
28	$-\frac{1}{2}$	$-\frac{1}{2}$
29	0	1
30	0	-1
31	0	1
32	$-\frac{1}{3}$	$-\frac{1}{12}$
33	-1	0
34	-1	0
35	-1	0
36	0	$-\frac{1}{2}$
37	0	1
38	-1	0
39	-1	0
40	$-\frac{1}{6}$	$-\frac{1}{2}$
41	0	1
42	0	-1
43	0	1
44	$-\frac{1}{2}$	$-\frac{1}{2}$
45	$-\frac{1}{2}$	$-\frac{1}{2}$
46	-1	0
47	0	1
48	0	$-\frac{5}{12}$
49	$-\frac{1}{2}$	$\frac{1}{2}$
50	$-\frac{1}{2}$	$-\frac{1}{2}$
51	-1	0
52	$-\frac{1}{2}$	$-\frac{1}{2}$
53	0	1

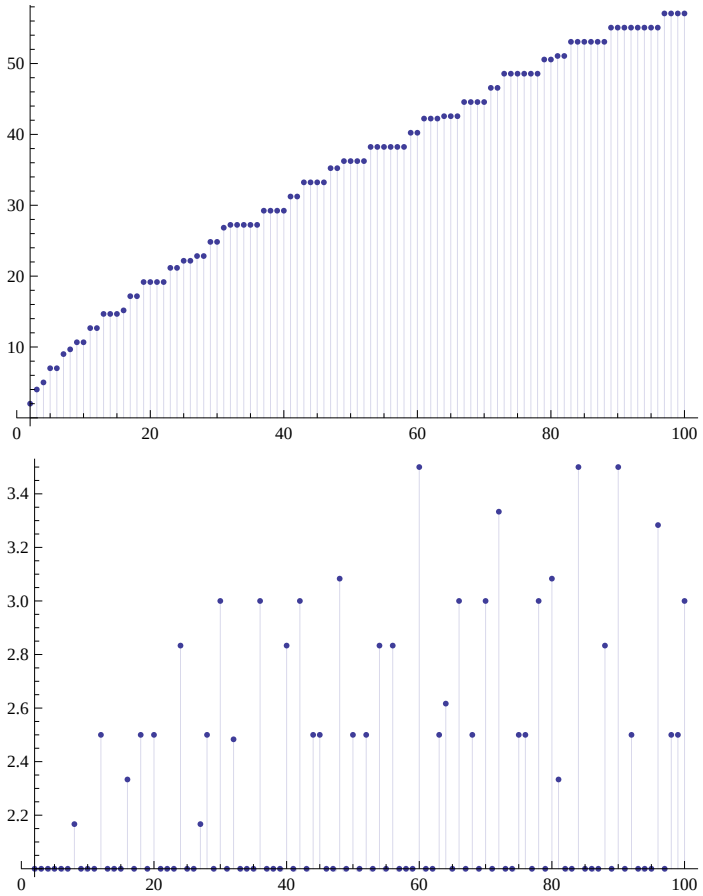
54	$-\frac{1}{6}$	$-\frac{1}{2}$
55	-1	0
56	$-\frac{1}{6}$	$-\frac{1}{2}$
57	-1	0
58	-1	0
59	0	1
60	$\frac{1}{2}$	$-\frac{1}{2}$
61	0	1
62	-1	0
63	$-\frac{1}{2}$	$-\frac{1}{2}$
64	$-\frac{19}{72}$	$-\frac{1}{8}$
65	-1	0
66	0	-1
67	0	1
68	$-\frac{1}{2}$	$-\frac{1}{2}$
69	-1	0
70	0	-1
71	0	1
72	$\frac{1}{6}$	$-\frac{1}{3}$
73	0	1
74	-1	0
75	$-\frac{1}{2}$	$-\frac{1}{2}$
76	$-\frac{1}{2}$	$-\frac{1}{2}$
77	-1	0
78	0	-1
79	0	1
80	0	$-\frac{5}{12}$
81	$-\frac{5}{12}$	0
82	-1	0
83	0	1
84	$\frac{1}{2}$	$-\frac{1}{2}$
85	-1	0
86	-1	0
87	-1	0
88	$-\frac{1}{6}$	$-\frac{1}{2}$
89	0	1
90	$\frac{1}{2}$	$-\frac{1}{2}$
91	-1	0
92	$-\frac{1}{2}$	$-\frac{1}{2}$
93	-1	0
94	-1	0
95	-1	0
96	$\frac{1}{12}$	$-\frac{1}{3}$
97	0	1
98	$-\frac{1}{2}$	$-\frac{1}{2}$
99	$-\frac{1}{2}$	$-\frac{1}{2}$
100	0	$-\frac{1}{2}$

```
Plot[ N[sinp[24, x]], {x, 0, 5}]
```




```
PP[n_, k_, a_] := PP[n, k, a] = Sum[ d[j, a] / k - PP[Floor[n / j], k + 1, a], {j, 2, n}]
PS[n_, k_, a_] := PS[n, k, a] = Sum[ d[j, a] (1 / k - PS[Floor[n / j], k + 1, a]), {j, 2, n}]
```

```
DiscretePlot[ PS[n, 1, 2], {n, 2, 100}]
```




```
PP[n_, k_, a_] := PP[n, k, a] = Sum[ d[j, a] / k - PP[Floor[n / j], k + 1, a], {j, 2, n}]
PS[n_, k_, a_] := PS[n, k, a] = Sum[ d[j, a] (1 / k - PS[Floor[n / j], k + 1, a]), {j, 2, n}]
PS[100, 2, .0000000001] / .0000000001 * 2
```

```
28.5333
```

```
PS[100, 3, .0000000001] / .0000000001 * 3
```

28.5333

```
N[PS[100, 3, 1] * 3]
```

8.38214

```
DD[n_, k_] := Sum[d[j, k], {j, 1, n}]
Sum[ d[j, 3], {j, 2, 100}]
```

1470

```
DD[100, 3] - DD[100, 0]
```

1470

```
(x^3 - 1)
```

$-1 + x^3$

```
Expand[(x^3 - 1) ^2]
```

$1 - 2\,x^3 + x^6$

```
Sum[ d[j, 3 * .2] d[k, 3 * .2], {j, 2, 100}, {k, 2, 100 / j}]
```

67.767

```
DD[100, 6 * .2] - 2 DD[100, 3 * .2] + DD[100, 0 * .2]
```

67.767

```
Sum[ d[j, 2] d[k, 2], {j, 2, 100}, {k, 2, 100 / j}]
```

2612

```
DD[100, 4] - 2 DD[100, 2] + DD[100, 0]
```

2612

```
Expand[(x^2 - 1) ^4]
```

$1 - 4\,x^2 + 6\,x^4 - 4\,x^6 + x^8$

```
Sum[ d[j, 2] d[k, 2] d[m, 2] d[l, 2], {j, 2, 1000},
 {k, 2, 1000 / j}, {l, 2, 1000 / (j k)}, {m, 2, 1000 / (j k l)}]
```

695 709

```
DD[1000, 8] - 4 DD[1000, 6] + 6 DD[1000, 4] - 4 DD[1000, 2] + DD[1000, 0]
```

695 709

```
DDD[ n_, k_, a_ ] := Sum[(-1) ^ (k - j) Binomial[k, j] DD[n, a j], {j, 0, k}]
```

```
DDD[1000, 9, 2]
```

5120

```
FactInteger[n_] := If[n == 1, {}, FactorInteger[n]]
d[n_, z_] := Product[1 / (p[[2]] !) Pochhammer[z, p[[2]]], {p, FactInteger[n]}]
DD[n_, k_] := Sum[d[j, k], {j, 1, n}]
PrimeCount[n_, a_ ] :=
 Sum[ (-1) ^ (k + 1) / k Sum[(-1) ^ (k - j) Binomial[k, j] DD[n, a j], {j, 0, k}],
  {k, 1, N[Log[n] / Log[2]]}]
```

```
N[PPP[100, I] / I]
```

28.5333

```
Sum[ (-1)^(k + 1) / k (x^a - 1)^k, {k, 1, Infinity}]
```

```
Log[x^a] == a Log[x]
```

$\text{Log}[x^a] == a\ \text{Log}[x]$

```
Integrate[ d[2, -t], {t, 1, 2}]
```

$-\frac{3}{2}$

```
d[3, 3]
```

3

```
N[PrimeCount[100, ZetaZero[1]]]
```

$14.2667 + 403.311\ i$

```
Kappa[n_] := FullSimplify[ MangoldtLambda[ n] / Log[n]]
PB[n_, k_] := Sum[ BernoulliB[k] / (k!) + Kappa[ j] PB[Floor[n / j], k + 1], {j, 2, n}]
PB[100, 0]
```

$\frac{428}{15}$

```
Kappa[n_] := FullSimplify[ MangoldtLambda[ n] / Log[n]]
PB[n_, k_, a_] := Sum[ BernoulliB[k] / (k!) + Kappa[j] PB[Floor[n / j], k + 1, a], {j, 2, n}]
PB[100, 0, 1]
```

$\frac{428}{15}$