```
Sum[ Binomial[k, j] (x - 1)^j, {j, 0, Infinity}]
```

x^k

```
Sum[ Binomial[k, j] (x - 1)^j, {j, 0, Infinity}]
eta[s_, a_] := (1 - a^(1 - s)) Zeta[s]


N[eta[5, 1.001] - 1]
```

-0.995863

```
f2[ s_, k_, a_] := FullSimplify[(1 - a^(1 - s))^-1 (eta[s, a] - 1) (-1)^(k - 1) / k]
FullSimplify[Sum[f2[s, k, a], {k, 1, Infinity}] /. a → 2]
```

$\mathrm{Log}[2] \left(\frac{1}{-1+2^{1-s}}+\mathrm{Zeta}[s]\right)$

```
FullSimplify[Log[eta[s, 2]]]
```

$\mathrm{Log}[2^{-s}(-2+2^{s})\,\mathrm{Zeta}[s]]$

```
zeta[s_, a_] := (1 - a^(1 - s))^-1 eta[s, a]
zeta[s, a]
```

$\mathrm{Zeta}[s]$

```
zetak[s_, k_, a_] :=
 (1 - a^(1 - s))^-k Sum[Binomial[k, j] (eta[s, a] - 1)^(j), {j, 0, Infinity}]
zetak2[s_, k_, a_] :=
 ((1 - a^(1 - s))^-k Sum[Binomial[k, j] (eta[s, a] - 1)^(j), {j, 0, Infinity}] - 1) / k
FullSimplify[Limit[ (zetak[4, z, 3 / 2] - 1) / z, z → 0]]
```

$\mathrm{Log}\left[\frac{\pi^4}{90}\right]$

```
FullSimplify[Limit[(zetak2[4, z, 2]), z → 0]]
```

$\mathrm{Log}\left[\frac{\pi^4}{90}\right]$

```
Zeta[2]^2
```

$\frac{\pi^4}{36}$

```
zetal[s_, r_, a_] := Sum[(-1)^(j - 1) / j (eta[s, a] - 1)^(j), {j, 1, Infinity}]
FullSimplify[zetal[s, k, s]]
```

$\left(1-s^{1-s}\right)\mathrm{Log}\left[\left(1-s^{1-s}\right)\mathrm{Zeta}[s]\right]$

```
Limit[
 ((1 - a^(1 - s))^-k Sum[Binomial[k, j] (eta[s, a] - 1)^(j), {j, 0, Infinity}] - 1) / k, k → 0]
```

$-\mathrm{Log}\left[1-a^{1-s}\right]+\mathrm{Log}\left[\left(1-a^{1-s}\right)\mathrm{Zeta}[s]\right]$

```
Limit[ -Log[1 - a^(1-s)] + Log[(1 - a^(1-s)) Zeta[s]], a → 1]
FullSimplify[-Log[-1 + s] + Log[(-1 + s) Zeta[s]]]
```

$-\mathrm{Log}[-1+s]+\mathrm{Log}[(-1+s)\,\mathrm{Zeta}[s]]$

```
-Log[-1 + s] + Log[(-1 + s) Zeta[s]] /. s → 3
```

$-\text{Log}[2] + \text{Log}[2\,\text{Zeta}[3]]$

```
Log[Zeta[4]]
```

$\text{Log}\left[\frac{\pi^4}{90}\right]$

```
a1[s_] := (1 / 3) Sum[ 1 / (k / 3) ^s, {k, 4, 10}]
a2[s_] := 3^(s - 1) Sum[ 1 / k^s, {k, 4, 10}]
a1[3]
```

$\frac{568\,025\,947}{1\,778\,112\,000}$

```
FullSimplify[a1[s] / a2[s]]
```

1

```
Integrate[1 / x^s, {x, 1, Infinity}]
```

$\text{ConditionalExpression}\left[\frac{1}{-1+s}, \text{Re}[s] > 1\right]$

```
Integrate[(1 / x^s) (1 / y^s), {x, 1, Infinity}, {y, 1, Infinity}]
```

$\text{ConditionalExpression}\left[\frac{1}{(-1+s)^2}, \text{Re}[s] > 1\right]$

```
Integrate[(x y ) ^-s, {x, 1, Infinity}, {y, 1, Infinity}]
```

$\text{ConditionalExpression}\left[\frac{1}{(-1+s)^2}, \text{Re}[s] > 1\right]$

```
Sum[ (-1) ^ (k - 1) / k (1 / (s - 1)), {k, 1, Infinity}]
```

$\frac{\text{Log}[2]}{-1+s}$

```
Log[1 / (s - 1)]
```

$\text{Log}\left[\frac{1}{-1+s}\right]$

```
Limit[ ( (1 / (s - 1)) ^z - 1) / z, z → 0]
```

$\text{Log}\left[\frac{1}{-1+s}\right]$

```
Expand[(F - 2^-s) ^5]
```

$-2^{-5\,s} + 5 \times 2^{-4\,s}\,F - 5 \times 2^{1-3\,s}\,F^2 + 5 \times 2^{1-2\,s}\,F^3 - 5 \times 2^{-s}\,F^4 + F^5$

```
f2[b_] := Expand[Sum[2^(-j s) Binomial[b, j] (-1) ^j F^(b - j), {j, 0, b}]]
f2[5]
```

$-2^{-5\,s} + 5 \times 2^{-4\,s}\,F - 5 \times 2^{1-3\,s}\,F^2 + 5 \times 2^{1-2\,s}\,F^3 - 5 \times 2^{-s}\,F^4 + F^5$

```
zetak2[s_, k2_, a_] :=
 Limit[ ((1 - a^(1 - s)) ^-k Sum[Binomial[k, j] (et2[s, a]) ^j, {j, 0, Infinity}] - 1) / k, k → k2]
FullSimplify[zetak2[s, 0, a]]
```

$-\text{Log}\left[1 - a^{1-s}\right] + \text{Log}[1 + \text{et2}[s, a]]$

```
fl[s_, a_] := -Log[1 - a^(1-s)] + Log[eta[s, a]]

fl[s, a]

-Log[1 - a^(1-s)] + Log[(1 - a^(1-s)) Zeta[s]]

Sum[(2^s)^k, {k, 0, Infinity}]

-1/(-1 + 2^s)

v := a^0 + a^1 + a^2 + a^3 + a^4 + a^5 + a^6 + a^7 + a^8

Expand[v^2]
```

$1 + 2 a + 3 a^2 + 4 a^3 + 5 a^4 + 6 a^5 + 7 a^6 + 8 a^7 + 9 a^8 + 8 a^9 + 7 a^{10} + 6 a^{11} + 5 a^{12} + 4 a^{13} + 3 a^{14} + 2 a^{15} + a^{16}$

```
Sum[ (x^(1 - s))^j / j, {j, 1, Infinity}] /. x → 1

∞

Sum[ (-1)^(k + 1) (et[s, x] - 1)^k / k, {k, 1, Infinity}]

Log[et[s, x]]

Limit[ FullSimplify[Sum[ (x^(1 - s))^j / j, {j, 1, Infinity}] +
   Sum[ (-1)^(k + 1) (eta[s, x] - 1)^k / k, {k, 1, Infinity}]], x → 1]

-Log[-1 + s] + Log[(-1 + s) Zeta[s]]


bins[ z_, a_] := Product[ (z - k), {k, 0, a - 1}] / a!

Expand[bins[ -z, k]] /. {z → 8, k → 3}

-120

Binomial[ z + k - 1, z - 1] /. {z → 8, k → 3}

120


Binomial[8 + 3 - 1, 8 - 1]

120

Binomial[-8, 3]

-120

Table[ {(-1)^(12) Binomial[ -z, 12], Binomial[z + 12 - 1, z - 1]}, {z, 0, 8}] // TableForm
```

0	0
1	1
13	13
91	91
455	455
1820	1820
6188	6188
18 564	18 564
50 388	50 388

```
Limit[ (1 - x^(1 - s)) Zeta[s] - 1, x → 1]

-1
```

```
Limit[ FullSimplify[Sum[ (1 - t) ^j / j, {j, 1, Infinity}]], x → 1]
```

$-\text{Log}[t]$

```
Limit[ Sum[ ((x^ (1 - s)) ^j) / j, {j, 1, Infinity}] +
  Sum[ (-1) ^ (k + 1) (eta[s, x] - 1) ^k / k , {k, 1, Infinity}], x → 1]
```

$-\text{Log}[-1+s]+\text{Log}[(-1+s)\ \text{Zeta}[s]]$

```
Limit[ Sum[ ((x^ (1 - s)) ^j) / j, {j, 1, Infinity}] +
   Sum[ (-1) ^ (k + 1) (eta[s, x] - 1) ^k / k , {k, 1, Infinity}], x → a] /. a → x
```

$-\text{Log}\left[1-x^{1-s}\right]+\text{Log}\left[\left(1-x^{1-s}\right)\ \text{Zeta}[s]\right]$

```
Expand[Sum[ ((x^ (1 - s)) ^j) / j, {j, 1, Infinity}] /. x → 2]
```

$-\text{Log}[2^{-s}\ (-2+2^{s})]$

```
Sum[ (-1) ^ (k + 1) (eta[s, x] - 1) ^k / k , {k, 1, Infinity}]
```

$\text{Log}[x^{-s}\ (-x+x^{s})\ \text{Zeta}[s]]$

```
Limit[ Sum[ ((x^ (1 - s)) ^k) / k + (-1) ^ (k + 1) (eta[s, x] - 1) ^k / k , {k, 1, Infinity}], x → 1]
```

$-\text{Log}[-1+s]+\text{Log}[(-1+s)\ \text{Zeta}[s]]$

```
FullSimplify[((x^ (1 - s)) ^k) / k + (-1) ^ (k + 1) (eta[s, x] - 1) ^k / k]
```

$$\frac{\left(x^{1-s}\right)^{k}-(-1)^{k}\ \left(-1+\text{Zeta}[s]-x^{1-s}\ \text{Zeta}[s]\right)^{k}}{k}$$

$$\text{Limit}\left[\ \text{Sum}\left[\frac{\left(x^{1-s}\right)^{k}-(-1)^{k}\ \left(-1+\text{Zeta}[s]-x^{1-s}\ \text{Zeta}[s]\right)^{k}}{k},\ \{k,\ 1,\ \text{Infinity}\}\right],\ x\to 1\right]$$

$-\text{Log}[-1+s]+\text{Log}[(-1+s)\ \text{Zeta}[s]]$

$$\text{Limit}\left[\ \text{Sum}\left[\frac{\left(x^{1-s}\right)^{k}-(-1)^{k}\ \left(-1+\text{Zeta}[s]-x^{1-s}\ \text{Zeta}[s]\right)^{k}}{k},\ \{k,\ 1,\ \text{Infinity}\}\right],\ x\to 1\right]$$

∞

$\textbf{Expand}[2^{-s}\ (-2+2^{s})]$

$\textbf{Log}\left[1-2^{1-s}\right]$

$\text{Log}\left[1-2^{1-s}\right]$

```
Limit[ eta[s, x] - 1, x → 1]
```

-1

```
zet2[ s_,  z_, x_]  :=
 Sum[ (-1) ^j Binomial[-z, j] Binomial[z, k] (x^ (1 - s)) ^j (eta[s, x] - 1) ^k,
  {j, 0, Infinity}, {k, 0, Infinity}]
FullSimplify[zet2[2, k, 2]]
```

$$-\frac{6}{-18+\pi^{2}}-\frac{6}{-6+\pi^{2}}$$

```
Expand[(-1) ^j Binomial[-z, j] Binomial[z, k] (x^ (1 - s)) ^j (eta[s, x] - 1) ^k]
```

$(-1)^{j}\ \left(x^{1-s}\right)^{j}\ \text{Binomial}[-z,\ j]\ \text{Binomial}[z,\ k]\ \left(-1+\left(1-x^{1-s}\right)\ \text{Zeta}[s]\right)^{k}$

```
D[eta[s, a], a]
```

$-a^{-s}(1-s)\,\mathrm{Zeta}[s]$

```
N[eta[2, -.1]]
```

18.0943