

```

DD[n_, k_, s_] := Sum[j^s DD[n/j, k-1, s], {j, 1, n}]; DD[n_, 0, s_] := 1
D0to1[n_] := Sum[DD[n-j, 1, 0], {j, 0, n}]
D0to1a[nn_] := Sum[nn-j s, {j s, 0, nn}]
D1to2[n_] := DD[n, 1, 1] + 2 Sum[DD[n-j, 1, 1], {j, 1, n}]
D1to2a[m_] := D0to1a[m] + 2 Sum[D0to1a[m-jj], {jj, 1, m}]
D2to3[n_] := DD[n, 1, 2] + Sum[If[Mod[j, 2] == 0, 2, 4] DD[n-j, 1, 2], {j, 1, n}]
D2to3a[n_] := D1to2a[n] + Sum[If[Mod[j, 2] == 0, 2, 4] D1to2a[n-j], {j, 1, n}]
D2to3b[n_] :=
  ((Sum[n-j s, {j s, 0, n}]) + 2 Sum[(Sum[n-jj-j s, {j s, 0, n-jj}]), {jj, 1, n}]) +
  Sum[If[Mod[j, 2] == 0, 2, 4] ((Sum[n-j-j s, {j s, 0, n-j}]) +
    2 Sum[(Sum[n-j-jj-j s, {j s, 0, n-j-jj}]), {jj, 1, n-j}]), {j, 1, n}]

```

```
DD[100, 1, 1]
```

```
5050
```

```
DD[100, 1, 1]
```

```
5050
```

```
D0to1[100]
```

```
5050
```

```
DD[100, 1, 2]
```

```
338350
```

```
D1to2[100]
```

```
338350
```

```
DD[100, 1, 3]
```

```
25502500
```

```
D2to3[100]
```

```
25502500
```

```
FullSimplify[D2to3b[n]]
```

$$\begin{aligned}
& \frac{1}{6} \left(n (5 + n (-9 + 10 n)) - 4 (-3 + 2 n) (1 + (-3 + n) n) \text{Ceiling}\left[\frac{1}{2} - \frac{n}{2}\right] - \right. \\
& \quad 4 (11 + 6 (-3 + n) n) \text{Ceiling}\left[\frac{1}{2} - \frac{n}{2}\right]^2 + (48 - 32 n) \text{Ceiling}\left[\frac{1}{2} - \frac{n}{2}\right]^3 - 16 \text{Ceiling}\left[\frac{1}{2} - \frac{n}{2}\right]^4 + \\
& \quad \left. 2 \left(-1 + 2 n - 2 \text{Floor}\left[\frac{n}{2}\right] \right) \text{Floor}\left[\frac{n}{2}\right] \left(-1 + (-1 + n) n + \text{Floor}\left[\frac{n}{2}\right] \left(1 - 2 n + 2 \text{Floor}\left[\frac{n}{2}\right] \right) \right) \right) \right)
\end{aligned}$$

$$5/24 - 7/72$$

$$\frac{1}{9}$$

$$f[a_, b_] := 4^b - ((4^a + (2^b - 2^a) 3^a) + 2^a (3^b - (3^a + (2^b - 2^a) 2^a)))$$

$$f[1, 2]$$

$$2$$

$$f[-1, -2]$$

$$-\frac{1}{18}$$

$$f[0, 1]$$

$$1$$

$$\text{FullSimplify[Expand[f[a, b]]]$$

$$2^{2a+b} + 2^{1+a} 3^a - 2^b 3^a - 2^a 3^b - 4^a + 4^b - 8^a$$

$$cc[0, a_, b_] := 1$$

$$cc[k_, a_, b_] := cc[k, a, b] = (k+1)^b - \text{Sum}[cc[k+1-j, a, b] j^a, \{j, 2, k+1\}]$$

$$\text{Table}[\{j, cc[j, 0, -1], -1/(j(j+1))\}, \{j, 1, 20\}] // \text{TableForm}$$

1	$-\frac{1}{2}$	$-\frac{1}{2}$
2	$-\frac{1}{6}$	$-\frac{1}{6}$
3	$-\frac{1}{12}$	$-\frac{1}{12}$
4	$-\frac{1}{20}$	$-\frac{1}{20}$
5	$-\frac{1}{30}$	$-\frac{1}{30}$
6	$-\frac{1}{42}$	$-\frac{1}{42}$
7	$-\frac{1}{56}$	$-\frac{1}{56}$
8	$-\frac{1}{72}$	$-\frac{1}{72}$
9	$-\frac{1}{90}$	$-\frac{1}{90}$
10	$-\frac{1}{110}$	$-\frac{1}{110}$
11	$-\frac{1}{132}$	$-\frac{1}{132}$
12	$-\frac{1}{156}$	$-\frac{1}{156}$
13	$-\frac{1}{182}$	$-\frac{1}{182}$
14	$-\frac{1}{210}$	$-\frac{1}{210}$
15	$-\frac{1}{240}$	$-\frac{1}{240}$
16	$-\frac{1}{272}$	$-\frac{1}{272}$
17	$-\frac{1}{306}$	$-\frac{1}{306}$
18	$-\frac{1}{342}$	$-\frac{1}{342}$
19	$-\frac{1}{380}$	$-\frac{1}{380}$
20	$-\frac{1}{420}$	$-\frac{1}{420}$

```
Table[{j, cc[j, 0, -2], -(1/4) (1 + 2 Binomial[j, 1]) / (Binomial[j + 1, 2]^2)}, {j, 1, 20}] //
TableForm
```

1	$-\frac{3}{4}$	$-\frac{3}{4}$
2	$-\frac{5}{36}$	$-\frac{5}{36}$
3	$-\frac{7}{144}$	$-\frac{7}{144}$
4	$-\frac{9}{400}$	$-\frac{9}{400}$
5	$-\frac{11}{900}$	$-\frac{11}{900}$
6	$-\frac{13}{1764}$	$-\frac{13}{1764}$
7	$-\frac{15}{3136}$	$-\frac{15}{3136}$
8	$-\frac{17}{5184}$	$-\frac{17}{5184}$
9	$-\frac{19}{8100}$	$-\frac{19}{8100}$
10	$-\frac{21}{12100}$	$-\frac{21}{12100}$
11	$-\frac{23}{17424}$	$-\frac{23}{17424}$
12	$-\frac{25}{24336}$	$-\frac{25}{24336}$
13	$-\frac{27}{33124}$	$-\frac{27}{33124}$
14	$-\frac{29}{44100}$	$-\frac{29}{44100}$
15	$-\frac{31}{57600}$	$-\frac{31}{57600}$
16	$-\frac{33}{73984}$	$-\frac{33}{73984}$
17	$-\frac{35}{93636}$	$-\frac{35}{93636}$
18	$-\frac{37}{116964}$	$-\frac{37}{116964}$
19	$-\frac{39}{144400}$	$-\frac{39}{144400}$
20	$-\frac{41}{176400}$	$-\frac{41}{176400}$

```
Table[{j, cc[j, 0, -3], -(1/8) (1 + 6 Binomial[j + 1, 2]) / ((Binomial[j + 1, 2])^3)},
{j, 0, 20}] // TableForm
```

Power::infy : Infinite expression $\frac{1}{0}$ encountered. >>

0	1	ComplexInfinity
---	---	-----------------

1	$-\frac{7}{8}$	$-\frac{7}{8}$
2	$-\frac{19}{216}$	$-\frac{19}{216}$
3	$-\frac{37}{1728}$	$-\frac{37}{1728}$
4	$-\frac{61}{8000}$	$-\frac{61}{8000}$
5	$-\frac{91}{27\,000}$	$-\frac{91}{27\,000}$
6	$-\frac{127}{74\,088}$	$-\frac{127}{74\,088}$
7	$-\frac{169}{175\,616}$	$-\frac{169}{175\,616}$
8	$-\frac{217}{373\,248}$	$-\frac{217}{373\,248}$
9	$-\frac{271}{729\,000}$	$-\frac{271}{729\,000}$
10	$-\frac{331}{1\,331\,000}$	$-\frac{331}{1\,331\,000}$
11	$-\frac{397}{2\,299\,968}$	$-\frac{397}{2\,299\,968}$
12	$-\frac{469}{3\,796\,416}$	$-\frac{469}{3\,796\,416}$
13	$-\frac{547}{6\,028\,568}$	$-\frac{547}{6\,028\,568}$
14	$-\frac{631}{9\,261\,000}$	$-\frac{631}{9\,261\,000}$
15	$-\frac{721}{13\,824\,000}$	$-\frac{721}{13\,824\,000}$
16	$-\frac{817}{20\,123\,648}$	$-\frac{817}{20\,123\,648}$
17	$-\frac{919}{28\,652\,616}$	$-\frac{919}{28\,652\,616}$
18	$-\frac{1027}{40\,001\,688}$	$-\frac{1027}{40\,001\,688}$
19	$-\frac{1141}{54\,872\,000}$	$-\frac{1141}{54\,872\,000}$
20	$-\frac{1261}{74\,088\,000}$	$-\frac{1261}{74\,088\,000}$

```
Table[{j, cc[j, 0, -4], -(1/16) (1 + 24 Binomial[j + 2, 3]) / ((Binomial[j + 1, 2])^4)},
{j, 0, 20}] // TableForm
```

Power::infy : Infinite expression $\frac{1}{0}$ encountered. >>

0	1	ComplexInfinity
1	$-\frac{15}{16}$	$-\frac{25}{16}$
2	$-\frac{65}{1296}$	$-\frac{97}{1296}$
3	$-\frac{175}{20736}$	$-\frac{241}{20736}$
4	$-\frac{369}{160000}$	$-\frac{481}{160000}$
5	$-\frac{671}{810000}$	$-\frac{841}{810000}$
6	$-\frac{1105}{3111696}$	$-\frac{1345}{3111696}$
7	$-\frac{1695}{9834496}$	$-\frac{2017}{9834496}$
8	$-\frac{2465}{26873856}$	$-\frac{2881}{26873856}$
9	$-\frac{3439}{65610000}$	$-\frac{3961}{65610000}$
10	$-\frac{4641}{146410000}$	$-\frac{5281}{146410000}$
11	$-\frac{6095}{303595776}$	$-\frac{6865}{303595776}$
12	$-\frac{7825}{592240896}$	$-\frac{8737}{592240896}$
13	$-\frac{9855}{1097199376}$	$-\frac{10921}{1097199376}$
14	$-\frac{12209}{1944810000}$	$-\frac{13441}{1944810000}$
15	$-\frac{14911}{3317760000}$	$-\frac{16321}{3317760000}$
16	$-\frac{17985}{5473632256}$	$-\frac{19585}{5473632256}$
17	$-\frac{21455}{8767700496}$	$-\frac{23257}{8767700496}$
18	$-\frac{25345}{13680577296}$	$-\frac{27361}{13680577296}$
19	$-\frac{29679}{20851360000}$	$-\frac{31921}{20851360000}$
20	$-\frac{34481}{31116960000}$	$-\frac{36961}{31116960000}$

```
Table[{j, cc[j, 0, 1], cc[j, 0, -1]}, {j, 0, 20}] // TableForm
```

0	1	1
1	1	$-\frac{1}{2}$
2	1	$-\frac{1}{6}$
3	1	$-\frac{1}{12}$
4	1	$-\frac{1}{20}$
5	1	$-\frac{1}{30}$
6	1	$-\frac{1}{42}$
7	1	$-\frac{1}{56}$
8	1	$-\frac{1}{72}$
9	1	$-\frac{1}{90}$
10	1	$-\frac{1}{110}$
11	1	$-\frac{1}{132}$
12	1	$-\frac{1}{156}$
13	1	$-\frac{1}{182}$
14	1	$-\frac{1}{210}$
15	1	$-\frac{1}{240}$
16	1	$-\frac{1}{272}$
17	1	$-\frac{1}{306}$
18	1	$-\frac{1}{342}$
19	1	$-\frac{1}{380}$
20	1	$-\frac{1}{420}$

```
Sum[DD[30 - j, 1, 0] cc[j, 0, 2], {j, 0, 30}]
```

```
9455
```

```
Sum[DD[30 - j, 1, 0] cc[j, 0, -2], {j, 0, 30}]
```

```
8 745 363 341 445 960 333 910 369
```

```
5 424 658 191 543 895 143 840 000
```

```
cc[3, 0, -1]
```

```
 $-\frac{1}{12}$ 
```

```
DD[30, 1, 2]
```

```
9455
```

```
DD[30, 1, -2]
```

```
8 745 363 341 445 960 333 910 369
```

```
5 424 658 191 543 895 143 840 000
```

```
Table[{j, cc[j, 1 / 2, 2]}, {j, 0, 20}] // TableForm  
$Aborted[]
```