```
DD[A_, k_, n_] := Sum[j^A DD[A, k - 1, n / j], {j, 2, n}]
DD[A_, 1, n_] := Sum[j^A, {j, 2, n}]
d[ A_, k_, n_ ] := DDFast[ A, k, n ] - DDFast[ A, k, n - 1]
rng[ 0, start_, end_ ] := Floor[end] - (start - 1)
rng[ 1, start_, end_ ] := Floor[end] (Floor[end] + 1) / 2 - (start - 1) start / 2
rng[ 2, start_, end_ ] :=
 Floor[end] (Floor[end] + 1) (2 Floor[end] + 1) / 6 - (start - 1) start (2 start - 1) / 6
rng[ A_, start_, end_ ] := Sum[m^A, {m, start, end}]
DDFast[A_, 1, n_] := rng[A, 2, n]

DDFast[A_, k_, n_] := Sum[j^A DDFast[A, k - 1, n / j], {j, Floor[n^(1 / 3)] + 1, n^(1 / 2)}] +
  Sum[rng[A, Floor[n / (j + 1)] + 1, n / j] DDFast[A, k - 1, j], {j, 1, n / Floor[n^(1 / 2)] - 1}] +
  Sum[d[A, k - 1, j] rng[A, 2, n / j], {j, 2, n^(1 / 3)}] +
  Sum[s^A d[A, m, j] DDFast[A, k - m - 1, n / (j s)], {j, 2, n^(1 / 3)},
   {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[n / j]^(1 / 2)}, {m, 1, k - 2}] +
  Sum[(rng[A, Floor[n / (j (s + 1))] + 1, n / (j s)])
     (Sum[d[A, m, j] DDFast[A, k - m - 1, s], {m, 1, k - 2}]),
   {j, 2, n^(1 / 3)}, {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]

SumPrimesFast[A_, n_] := Sum[(-1)^(k + 1) / (j k) MoebiusMu[j] DDFast[j A, k, n^(1 / j)],
  {j, 1, Log[2, n]}, {k, 1, Log[2, (n^(1 / j))]}]

SumPrimesFast[ 0, 1000]
```

```
168
```