Convolution Cheat Sheet

$$[\zeta(s)]_n=\sum_{j=1}^{n} j^{-s}$$

$$[f^0]_n=1_{[1,\infty)}(|n|)$$

$$[\zeta(s)^k]_n=\sum_{j=1} j^{-s}\cdot[\zeta(s)^{k-1}]_{n\cdot j^{-1}}=[(1+\zeta(s,2))^k]_n$$

$$[(\zeta(s)-1)^k]_n=\sum_{j=1} j^{-s}\cdot[(\zeta(s)-1)^{k-1}]_{n\cdot j^{-1}}-[(\zeta(s)-1)^{k-1}]_n$$

$$[\zeta(s,a+1)^k]_n=\sum_{j=1}(j+a)^{-s}\cdot[\zeta(s,a+1)^{k-1}]_{n\cdot(j+a)^{-1}}$$

$$[(1+\zeta(s,a+1))^k]_n=[(1+\zeta(s,a+1))^{k-1}]_n+\sum_{j=1}(j+a)^{-s}\cdot[(1+\zeta(s,a+1))^{k-1}]_{n\cdot(j+a)^{-1}}$$

$$[(t\cdot\zeta(s))^k]_n=t\cdot\sum_{j=1} j^{-s}\cdot[(t\cdot\zeta(s))^{k-1}]_{n\cdot j^{-1}}$$

$$[(t\cdot\zeta(s)-1)^k]_n=(t\cdot\sum_{j=1} j^{-s}\cdot[(t\cdot\zeta(s))^{k-1}]_{n\cdot j^{-1}})-[(t\cdot\zeta(s)-1)^{k-1}]_n$$

$$[(x^{1-s}\zeta(s))^k]_n=x\sum_{j=1}(j\,x)^{-s}\cdot[(x^{1-s}\cdot\zeta(s))^{k-1}]_{n\cdot(j\cdot x)^{-1}}$$

$$[(1+x^{1-s}\zeta(s))^k]_n=[(1+x^{1-s}\cdot\zeta(s))^{k-1}]_n+x\sum_{j=1}(j\,x)^{-s}\cdot[(x^{1-s}\cdot\zeta(s))^{k-1}]_{n(j\,x)^{-1}}$$

$$[(x^{1-s}\cdot\zeta(s,a+1))^k]_n=x\sum_{j=1}(j\,x+a)^{-s}\cdot[(x^{1-s}\cdot\zeta(s,a+1))^{k-1}]_{n(j\,x+a)^{-1}}$$

$$[(1+x^{1-s}\cdot\zeta(s,a+1))^k]_n=[(1+x^{1-s}\cdot\zeta(s,a+1))^{k-1}]_n+x\sum_{j=1}(j\,x+a)^{-s}\cdot[(1+x^{1-s}\cdot\zeta(s,a+1))^{k-1}]_{n(j\,x+a)^{-1}}$$

$$[((1-x^{1-s})\zeta(s)-1)^k]_n=(\sum_{j=1} j^{-s}[((1-x^{1-s})\zeta(s)-1)^{k-1}]_{n\cdot j^{-1}}-x\cdot(j\,x)^{-s}[((1-x^{1-s})\zeta(s))^{k-1}]_{n(j\,x)^{-1}})-[((1-x^{1-s})\zeta(s)-1)^{k-1}]_n$$

$$[((1-x^{1-s})\zeta(s))^k]_n=\sum_{j=1} j^{-s}\cdot[((1-x^{1-s})\zeta(s))^{k-1}]_{n\cdot j^{-1}}-x\cdot(j\,x)^{-s}[((1-x^{1-s})\zeta(s))^{k-1}]_{n\cdot(j\,x)^{-1}}$$

$$[(\zeta(s)^{-1}-1)^k]_n=(\sum_{j=1}[\nabla\zeta(s)^{-1}]_n\cdot[(\zeta(s)^{-1}-1)^{k-1}]_{n\cdot j^{-1}})-[(\zeta(s)^{-1}-1)^{k-1}]_n$$

$$[(\zeta(s)^z-1)^k]_n=(\sum_{j=1}[\nabla\zeta(s)^z]_n\cdot[(\zeta(s)^z-1)^k]_{n\cdot j^{-1}})-[(\zeta(s)^z-1)^{k-1}]_n$$

$$[(\zeta(s)^z)^k]_n=(\sum_{j=1}[\nabla\zeta(s)^z]_n\cdot[(\zeta(s)^z)^{k-1}]_{n\cdot j^{-1}})$$

$$[\zeta(s,y+1)]_n=[\zeta(s)]_n-[\zeta(s)]_y$$

$$[([\zeta(s)]_n - [\zeta(s)]_y)^k] = \sum_{j=1} (j+y)^{-s} \cdot [([\zeta(s)]_{n(j+y)^{-1}} - [\zeta(s)]_y)^{k-1}]$$

$$d_z(n) = [\nabla \zeta(0)^z]_n$$

$$n^{-s} \cdot d_z(n) = [\nabla \zeta(s)^z]_n$$

Properties

$$[\zeta(s,y)^k]_n=\sum_{j=0}^{k}\binom{k}{j}\cdot[\zeta(s,y+1)^j]_{n\cdot y^{j-k}}$$

$$[\zeta(s,y+1)^k]_n=\sum_{j=0}^{k}(-1)^{k-j}\cdot\binom{k}{j}\cdot[\zeta(s,y)^j]_{n\,y^{j-k}}$$

$$[(x^{1-s}\cdot\zeta(s))^k]_n=x^{k(1-s)}\cdot[\zeta(s)^k]_{n\cdot x^{-k}}$$

$$\ldots$$

$$[(1+\zeta(0,y))^z]_n=\sum_{k=0}\binom{z}{k}\cdot[1+\zeta(0,y+1)^{z-k}]_{n/y^k}$$

$$[(1+\zeta(0,y))^z]_n=\sum_{k=0}(-1)^k\binom{z}{k}\cdot[1+\zeta(0,y-1)^{z-k}]_{n/(y-1)^k}$$

$$\ldots?$$

$$[(\zeta(s,2)\cdot x^{1-s})^k]_n=x^{k(1-s)}[\zeta(s,x+1)^k]_{n\cdot x^{-k}}$$

$$[(\zeta(s)^z)^k]_n=[\zeta(s)^{z\cdot k}]_n \quad (!!!)$$

$$[\zeta(s)^z\cdot\zeta(s)^k]_n=[\zeta(s)^{z+k}]_n$$

$$[(1+\zeta(s,2))^z]_n=\sum_{k=0}^{\infty}\binom{z}{k}\cdot[\zeta(s,2)^k]_n$$

$$[(1+x^{1-s}\cdot\zeta(s,2))^z]_n=\sum_{k=0}^{\infty}\binom{z}{k}\cdot[(x^{1-s}\cdot\zeta(s,2))^k]_n$$

$$[(1+\zeta_n(s,2)-x^{1-s}\zeta(s))^z]_n=\sum_{k=0}\binom{z}{k}[(\zeta_n(s,2)-x^{1-s}\zeta(s))^k]_n$$

$$[\zeta(s,2)^k]_n=\sum_{j=0}(-1)^{k-j}\binom{k}{j}[(1+\zeta(s,2))^j]_n$$

$$[((1-x^{1-s})\zeta(s))^z]_n=\sum_{j=0}(-1)^j\binom{z}{j}x^{j(1-s)}\cdot[\zeta(s)^z]_{n\cdot x^{-j}}$$

$$[\zeta(s)^z]_n=\sum_{j=0}(-1)^j\binom{-z}{j}x^{j(1-s)}[((1-x^{1-s})\zeta(s))^z]_{n\cdot x^{-j}}$$

$$[\zeta(s)^z]_n=\sum_{j=0}\sum_{k=0}(-1)^j\binom{-z}{j}\binom{z}{k}x^{j(1-s)}\cdot[((1-x^{1-s})\zeta(s)-1)^k]_{n\cdot x^{-j}}$$

$$[(\log \zeta(s))^k]_n = \sum_{j=2} \frac{\Lambda(j)}{\log j} \cdot j^{-s} \cdot [(\log \zeta(s))^{k-1}]_{n \cdot j^{-1}}$$

$$[\zeta(s)^z]_n = \sum_{k=0}^{\infty} \frac{z^k}{k!} [(\log \zeta(s))^k]_n$$

$$[\zeta(s)^z]_n = \sum_{k=0}^{\infty} \frac{z^k}{k!} \lim_{y \to 0} \frac{\partial^k}{\partial y^k} \cdot [\zeta(s)^y]_n$$

As Sets

$$[\zeta(s)^k]_n = \sum_{\left|\frac{n}{m_1\cdot m_2\cdot..m_k}\right|\geq 1\,;\,m_k\geq 1} m_1^{-s}\cdot m_2^{-s}\cdot\ldots m_k^{-s}$$

$$[\zeta(s,y+1)^k]_n = \sum_{\left|\frac{n}{(n_1+y)\cdot(n_2+y)\cdot..(n_k+y)}\right|\geq 1\,;\,n_k\geq 1} (n_1+y)^{-s}\cdot(n_2+y)^{-s}\cdot\ldots(n_k+y)^{-s}$$

$$[(x^{1-s}\zeta(s))^k]_n = \sum_{\left|\frac{n}{(n_1 x)\cdot(n_2 x)\cdot\ldots(n_k x)}\right|\geq 1\,;\,n_k\geq 1} (n_1 x)^{-s}\cdot(n_2 x)^{-s}\cdot\ldots(n_k x)^{-s}$$

$$[(x^{1-s}\cdot\zeta(s,y+1))^k]_n = \sum_{\left|\frac{n}{(n_1 x+y)\cdot(n_2 x+y)x\cdot..(n_k x+y)}\right|\geq 1\,;\,n_k\geq 1} (n_1 x+y)^{-s}\cdot(n_2 x+y)^{-s}\cdot\ldots(n_k x+y)^{-s}$$

$$[(1+x^{1-s}\cdot\zeta(s,y+1))^k]_n = \sum_{\left|\frac{n}{(1\cup(n_1 x+y))\cdot(1\cup(n_2 x+y))\cdot..(1\cup(n_k x+y))}\right|\geq 1\,;\,n_k\geq 1} (n_1 x+y)^{-s}\cdot(n_2 x+y)^{-s}\cdot\ldots(n_k x+y)^{-s}$$

$$(\zeta_n(s)-x^{1-s}\zeta_n(s))^{*k} =$$

$$[(\zeta(s)-1)-x^{1-s}\zeta(s)]_n^{*k} =$$

$$[(\log\zeta(s))^k]_n = \sum_{\left|\frac{n}{m_1\cdot m_2\cdot..m_k}\right|\geq 1\,;\,m_k\geq 1} \kappa(m_1)\cdot m_1^{-s}\cdot\kappa(m_2)\cdot m_2^{-s}\cdot\ldots\kappa(m_k)\cdot m_k^{-s}$$

$$[(x^{1-s}\cdot\zeta(s,y+1))^k]_n = \sum_{\left|\frac{n}{n_1\cdot n_2\cdot..n_k}\right|\geq 1\,;\,n_k\in\{(jx+y),\,j\in\mathbb{N}\}} n_1^{-s}\cdot n_2^{-s}\cdot\ldots n_k^{-s}$$

$$[(1+x^{1-s}\cdot\zeta(s,y+1))^k]_n = \sum_{\left|\frac{n}{n_1\cdot n_2\cdot..n_k}\right|\geq 1\,;\,n_k\in\{1\cup(jx+y),\,j\in\mathbb{N}\}} n_1^{-s}\cdot n_2^{-s}\cdot\ldots n_k^{-s}$$

Convergence

$[(\zeta(s)-1)^k]_n=[\zeta(s,2)^k]_n$ equals 0 if $n<2^k$

$[\zeta(s)^z]_n=[(1+\zeta(s,2))^z]_n$ does not converge

$[\zeta(s,y)^k]_n$ equals 0 if $n<y^k$

$[(1+\zeta(s,y))^z]_n$ does not converge

$[(x^{1-s}\zeta(s))^k]_n$ equals 0 if $n<x^k$

$[(1+x^{1-s}\zeta(s))^z]_n$ does not converge

$[(x^{1-s}\cdot\zeta(s,y))^k]_n$ equals 0 if $n<(x+y)^k$

$[(1+x^{1-s}\cdot\zeta(s,y))^z]_n$ does not converge

$[((1-x^{1-s})\zeta(s)-1)^k]_n=[(\zeta_n(s,2)-x^{1-s}\zeta(s))^k]_n$ equals 0 if $n<2^k$ and $n<x^k$

$[(1+\zeta_n(s,2)-x^{1-s}\zeta(s))^z]_n$ does not converge

IF $f_1(x)^{*1}\neq 0$ then $f_n(x)^{*k}$ doesn't converge for any k. Otherwise it does.

As Sets

$$[\zeta(s)^k]_n = \sum_{\left|\frac{n}{n_1\cdot n_2\cdot..n_k}\right|\geq 1;\, n_k\in\mathbb{N}} (n_1\cdot n_2\cdot\ldots n_k)^{-s}$$

$$[\zeta(s,y+1)^k]_n = \sum_{\left|\frac{n}{n_1\cdot n_2\cdot..n_k}\right|\geq 1;\, n_k\in\{(j+y),\, j\in\mathbb{N}\}} (n_1\cdot n_2\cdot\ldots n_k)^{-s}$$

$$[(x^{1-s}\zeta(s))^k]_n = \sum_{\left|\frac{n}{n_1\cdot n_2\cdot\ldots n_k}\right|\geq 1;\, n_k\in\{(jx),\, j\in\mathbb{N}\}} (n_1\cdot n_2\cdot\ldots n_k)^{-s}$$

$$[(1+x^{1-s}\zeta(s))^k]_n = \sum_{\left|\frac{n}{n_1\cdot n_2\cdot\ldots n_k}\right|\geq 1;\, n_k\in\{1\cup(jx),\, j\in\mathbb{N}\}} (n_1\cdot n_2\cdot\ldots n_k)^{-s}$$

$$[(x^{1-s}\cdot\zeta(s,y+1))^k]_n = \sum_{\left|\frac{n}{n_1\cdot n_2\cdot..n_k}\right|\geq 1;\, n_k\in\{(jx+y),\, j\in\mathbb{N}\}} (n_1\cdot n_2\cdot\ldots n_k)^{-s}$$

$$[(1+x^{1-s}\cdot\zeta(s,y+1))^k]_n = \sum_{\left|\frac{n}{n_1\cdot n_2\cdot..n_k}\right|\geq 1;\, n_k\in\{1\cup(jx+y),\, j\in\mathbb{N}\}} (n_1\cdot n_2\cdot\ldots n_k)^{-s}$$

$$[(\zeta_n(s)-x^{1-s}\zeta(s))^k]_n = \sum_{\left|\frac{n}{n_1\cdot n_2\cdot..n_k}\right|\geq 1;\, n_k\in\{1\cup(j+1),\, j\in\mathbb{N}\cup -jx,\, j\in\mathbb{N}\}} (n_1\cdot n_2\cdot\ldots n_k)^{-s}$$

$$[(\zeta_n(s)-1-x^{1-s}\zeta(s))^k]_n = \sum_{\left|\frac{n}{n_1\cdot n_2\cdot..n_k}\right|\geq 1;\, n_k\in\{(j+1),\, j\in\mathbb{N}\cup -jx,\, j\in\mathbb{N}\}} (n_1\cdot n_2\cdot\ldots n_k)^{-s}$$

$$[(\log\zeta(s))^k]_n = \sum_{\left|\frac{n}{m_1\cdot m_2\cdot..m_k}\right|\geq 1;\, m_k\geq 1} \kappa(m_1)\cdot m_1^{-s}\cdot\kappa(m_2)\cdot m_2^{-s}\cdot\ldots\kappa(m_k)\cdot m_k^{-s}$$

As Sets

$$f(n,M)=\sum_{\sum_{j=1}^{k}\log a_j\le\log n;\,a_j\in M}\prod_{j=1}^{k}a_j^{-s}$$

$$f(n,g,M)=\sum_{\sum_{j=1}^{k}\log a_j\le\log n;\,a_j\in M}\prod_{j=1}^{k}g(a_j)\cdot a_j^{-s}$$

$$[\zeta(s)^k]_n=f(n,\mathbb{N})$$

$$[\zeta(s,y+1)^k]_n=f(n,\{(j+y),\,j\in\mathbb{N}\})$$

$$[(x^{1-s}\zeta(s))^k]_n=f(n,\{(j\,x),\,j\in\mathbb{N}\})$$

$$[(1+x^{1-s}\zeta(s))^k]_n=f(n,\{1\cup(j\,x),\,j\in\mathbb{N}\})$$

$$[(x^{1-s}\cdot\zeta(s,y+1))^k]_n=f(n,\{(j\,x+y),\,j\in\mathbb{N}\})$$

$$[(1+x^{1-s}\cdot\zeta(s,y+1))^k]_n=f(n,\{1\cup(j\,x+y),\,j\in\mathbb{N}\})$$

$$[(\zeta_n(s)-x^{1-s}\zeta(s))^k]_n=f(n,\{1\cup(j+1),\,j\in\mathbb{N}\cup-j\,x,\,j\in\mathbb{N}\})$$

$$[(\zeta_n(s)-1-x^{1-s}\zeta(s))^k]_n=f(n,\{(j+1),\,j\in\mathbb{N}\cup-j\,x,\,j\in\mathbb{N}\})$$

$$[(\log\zeta(s))^k]_n=f(n,\kappa,\,j\in\mathbb{N})$$

Inversions

$[f]_n=[\prod_{k=1} \zeta_{\frac{1}{k}}(0)^{\frac{1}{k}\cdot[\nabla\zeta(0)^{-1}]_k}]_n$	$[\zeta(0)]_n=[\prod_{k=1} f_{\frac{1}{k}}(0)^{\frac{1}{k}}]_n$
$[f]_n=[\prod_{k=1} \zeta_{\frac{1}{k}}(0)]_n$	$[\zeta(0)]_n=[\prod_{k=1} f_{\frac{1}{k}}(0)^{\mu(k)}]_n$
$[f]_n=[\frac{\zeta(0)}{\zeta_{\frac{1}{2}}(0)}]_n$	$[\zeta(0)]_n=[\prod_{k=0} f_{\frac{1}{2^k}}(0)]_n$
$[f]_n=[\frac{\zeta(0)^2}{\zeta_{\frac{1}{2}}(0)}]_n$	$[\zeta(0)]_n=[\prod_{k=0} f_{\frac{1}{2^k}}(0)^{\frac{1}{2^k}}]_n$
$[f]_n=[\frac{\zeta(0)^3}{\zeta_{\frac{1}{2}}(0)}]_n$	$[\zeta(0)]_n=[\prod_{k=0} f_{\frac{1}{2^k}}(0)^{\frac{1}{3^k}}]_n$
$[f]_n=[\zeta(0)^2]_n$	$[\zeta(0)]_n=[f^{\frac{1}{2}}]_n$
$[f]_n=[\zeta(0)^k]_n$	$[\zeta(0)]_n=[f^{\frac{1}{k}}]_n$
$[f]_n=[\zeta(0)\cdot\zeta_{\frac{1}{2}}(0)]_n$	$[\zeta(0)]_n=[\prod_{k=0} f_{\frac{1}{2^k}}(0)^{(-1)^k}]_n$
$[f]_n=[\zeta(0)\cdot\zeta_{\frac{1}{3}}(0)]_n$	$[\zeta(0)]_n=[\prod_{k=0} f_{\frac{1}{3^k}}(0)^{(-1)^k}]_n$
$[f]_n=[\zeta(0)\cdot\zeta_{\frac{1}{t}}(0)]_n$	$[\zeta(0)]_n=[\prod_{k=0} f_{\frac{1}{t^k}}(0)^{(-1)^k}]_n$
$[f]_n=[\frac{\zeta(0)}{\zeta_{\frac{1}{2}}(0)}]_n$	$[\zeta(0)]_n=[\prod_{k=0} f_{\frac{1}{2^k}}(0)]_n$

$[f]_n=[\frac{\zeta(0)}{\zeta_{\frac{1}{3}}(0)}]_n$	$[\zeta(0)]_n=[\prod_{k=0} f_{\frac{1}{3^k}}(0)]_n$
$[f]_n=[\frac{\zeta(0)}{\zeta_{\frac{1}{t}}(0)}]_n$	$[\zeta(0)]_n=[\prod_{k=0} f_{\frac{1}{t^k}}(0)]_n$

As Products

$$[\nabla \zeta(s)^z]_n = \prod_{p^a|n} \frac{z^{(a)}}{a!} p^{-as}$$

$$[\nabla (\frac{\zeta(s)}{\zeta_{\frac{1}{2}}(2s)})^z]_n = \prod_{p^a|n} (-1)^a \frac{(-z)^{(a)}}{a!} p^{-as} = \prod_{p^a|n} \frac{(z-a+1)^{(a)}}{a!} p^{-as}$$

$$[\nabla (\frac{\zeta(0)}{\zeta_{\frac{1}{3}}(0)})^z]_n = ???$$

$$[(1+\log f_n)^{-1}-1]^{*k}$$

$$[(1+\log f_n)^{-1}]^{*z}=[1+\log f_n]^{*-z}$$

$$[1+\log f_n]^{*z}$$

$$[\log f_n]^{*k}$$

$$[(1+f_n)^{-1}-1]^{*1}=$$
$$-[f_n]^{*1}*(1+[(1+f_n)^{-1}-1]^{*1})$$

$$[(1+\log\zeta_n(s))^{-1}-1]^{*1}=$$
$$-[\log\zeta_n(s)]^{*1}*(1+[(1+\log\zeta_n(s))^{-1}-1]^{*1})$$

$$[(1+\log\zeta_n(s))^{-1}]^{*z}*[1+\log\zeta_n(s)]^{*z}=1$$

$$[(1+\log f_n)^{-1}-1]^{*1}=$$
$$-[\log f_n]^{*1}*(1+[(1+\log f_n)^{-1}-1]^{*1})$$

$$[(1+\log f_n)^{-1}-1]^{*k}=$$
$$\sum_{j=1}(\lim_{x\to 0}\frac{\partial^j}{\partial x^j}((1+\log x)^{-1}-1)^k)[f_n-1]^{*j}$$

$$[f_n]^{*0}=1_{[1,\infty)}(|n|)$$

$$[(f_n)^{-1}]^{*z}=[f_n]^{*-z}$$

$$[f_n]^{*j}*[f_n]^{*k}=[f_n]^{*j+k}$$

$$n=1-\sum_{j=2}^{n}\mu(j)+\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\mu(j)\mu(k)-\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}}\mu(j)\mu(k)\mu(l)+\ldots$$

$$D_2(n)=1-2\sum_{j=2}^{n}\mu(j)+3\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\mu(j)\mu(k)-4\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}}\mu(j)\mu(k)\mu(l)+5\ldots$$

Series for (x-1)^(1/2)

$$n=1+\frac{1}{2}\sum_{j=2}^{n}d_2(j)-\frac{1}{8}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}d_2(j)\cdot d_2(k)+\frac{1}{16}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}}d_2(j)\cdot d_2(k)\cdot d_2(l)-\frac{5}{128}\ldots$$

Series for (x-1)^(3/2)

$$D_3(n)=1+\frac{3}{2}\sum_{j=2}^{n}d_2(j)+\frac{3}{8}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}d_2(j)\cdot d_2(k)-\frac{1}{16}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}}d_2(j)\cdot d_2(k)\cdot d_2(l)+\frac{3}{128}\ldots$$

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FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}]
dx[n_, z_, y_, k_] := SeriesCoefficient[(x + 1)^(z), {x, 0, k}] + Sum[ dz[j, y] dx[n/j, z, y, k + 1], {j, 2, n}]
dx[n, 1/a, a, 0] == n
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$$[\zeta(s)]_n = 1 - [\zeta(s)^{-1} - 1]_n + [(\zeta(s)^{-1} - 1)^2]_n - [(\zeta(s)^{-1} - 1)^3]_n + \ldots$$

$$[\zeta(s)^2]_n = 1 - 2[\zeta(s)^{-1} - 1]_n + 3[(\zeta(s)^{-1} - 1)^2]_n - 4[(\zeta(s)^{-1} - 1)^3]_n + 5\ldots$$

Series for (x-1)^(1/2)

$$[\zeta(s)]_n = 1 + \frac{1}{2}[\zeta(s)^2 - 1]_n - \frac{1}{8}[(\zeta(s)^2 - 1)^2]_n + \frac{1}{16}[(\zeta(s)^2 - 1)^3]_n - \frac{5}{128}\ldots$$

Series for (x-1)^(3/2)

$$[\zeta(s)^3]_n = 1 + \frac{3}{2}[\zeta(s)^2 - 1]_n + \frac{3}{8}[(\zeta(s)^2 - 1)^2]_n - \frac{1}{16}[(\zeta(s)^2 - 1)^3]_n + \frac{3}{128}\ldots$$

$$[\log\zeta(s)]_n=\sum_{k=1}\frac{(-1)^{k+1}}{k}[(\zeta(s)-1)^k]_n$$

$$[(1+\log\zeta(s))^z]_n=\sum_{k=0}\binom{z}{k}[(\log\zeta(s))^k]_n$$

For now, $p_z(x)=[\nabla(1+\log\zeta(0))^z]_x$

$$1+\Pi(n)=1+\frac{1}{2}\sum_{j=2}^{n}p_2(j)-\frac{1}{8}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}p_2(j)\cdot p_2(k)+\frac{1}{16}\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}}p_2(j)\cdot p_2(k)\cdot p_2(l)-\frac{5}{128}\ldots$$

$$1+\Pi(n)=1-\sum_{j=2}^{n}p_{-1}(j)+\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}p_{-1}(j)\,p_{-1}(k)-\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{j\cdot k}}p_{-1}(j)\,p_{-1}(k)\,p_{-1}(l)+\ldots$$

$$*\log x+1=1-((*\log x+1)^{*-1}-1)^{*1}+((*\log x+1)^{*-1}-1)^{*2}-((*\log x+1)^{*-1}-1)^{*3}+\ldots$$

$$(*\log x+1)^{*2}=1-2((*\log x+1)^{*-1}-1)^{*1}+3((*\log x+1)^{*-1}-1)^{*2}-4((*\log x+1)^{*-1}-1)^{*3}+5\ldots$$

Series for (x-1)^(1/2)

$$*\log x+1=1+\frac{1}{2}((*\log x+1)^{*2}-1)^{*1}-\frac{1}{8}((*\log x+1)^{*2}-1)^{*2}+\frac{1}{16}((*\log x+1)^{*2}-1)^{*3}-\frac{5}{128}\ldots$$

Series for (x-1)^(3/2)

$$(*\log x+1)^{*3}=1+\frac{3}{2}((*\log x+1)^{*2}-1)^{*1}+\frac{3}{8}((*\log x+1)^{*2}-1)^{*2}-\frac{1}{16}((*\log x+1)^{*2}-1)^{*3}+\frac{3}{128}\ldots$$

$$\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{jk}}1=\sum_{j=2}^{2}\sum_{k=2}^{2}\sum_{l=2}^{2}1+\sum_{j=2}^{2}\sum_{k=2}^{2}\sum_{l=3}^{\frac{n}{jk}}1+\sum_{j=2}^{2}\sum_{k=3}^{\frac{n}{j}}\sum_{l=2}^{2}1+\sum_{j=2}^{2}\sum_{k=3}^{\frac{n}{j}}\sum_{l=3}^{\frac{n}{jk}}1+\sum_{j=3}^{n}\sum_{k=2}^{2}\sum_{l=2}^{2}1+\sum_{j=3}^{n}\sum_{k=2}^{2}\sum_{l=3}^{\frac{n}{jk}}1+\sum_{j=3}^{n}\sum_{k=3}^{\frac{n}{j}}\sum_{l=2}^{2}1+\sum_{j=3}^{n}\sum_{k=3}^{\frac{n}{j}}\sum_{l=3}^{\frac{n}{jk}}1$$

$$\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{jk}}1=1+3\sum_{j=2}^{2}\sum_{k=2}^{2}\sum_{l=3}^{\frac{n}{jk}}1+3\sum_{j=2}^{2}\sum_{k=3}^{\frac{n}{j}}\sum_{l=3}^{\frac{n}{jk}}1+\sum_{j=3}^{n}\sum_{k=3}^{\frac{n}{j}}\sum_{l=3}^{\frac{n}{jk}}1$$

$$\sum_{j=2}^{n}\sum_{k=2}^{\frac{n}{j}}\sum_{l=2}^{\frac{n}{jk}}1=1+3\sum_{l=3}^{\frac{n}{4}}1+3\sum_{k=3}^{\frac{n}{2}}\sum_{l=3}^{\frac{n}{2k}}1+\sum_{j=3}^{n}\sum_{k=3}^{\frac{n}{j}}\sum_{l=3}^{\frac{n}{jk}}1$$

$$\zeta_n(0,2)^{*3}=\zeta_{n\cdot 2^{-3}}(0,3)^{*0}+3\zeta_{n\cdot 2^{-2}}(0,3)^{*1}+3\zeta_{n\cdot 2^{-1}}(0,3)^{*2}+\zeta_{n\cdot 2^{0}}(0,3)^{*3}$$

$$\zeta_n(0,2)^{*z}=\sum_{k=0}\binom{z}{k}\cdot\zeta_{n\cdot 2^{k-z}}(0,3)^{*k}\ \ldots\ldots\ldots\,?$$

$$\zeta_n(0,a)^{*z}=\sum_{k=0}\binom{z}{k}\cdot\zeta_{n\cdot a^{k-z}}(0,a+1)^{*k}$$

$\zeta_n(s,a)^{*k}$ equals 0 if $n<a^k$

What is the upper limit on this sum? When does it converge?

$$\frac{n}{a^{z-k}}/(a+1)^k\geq 1$$

$$\frac{n}{a^{z-k}}\geq(a+1)^k$$

$$n\geq(a+1)^k\cdot a^{z-k}$$

$$\log n\geq k\log(a+1)+(z-k)\log a$$

$$\log n\geq k\log(a+1)+z\log a-k\log a$$

$$\log n-z\log a\geq k(\log(a+1)-\log a)$$

$$\frac{\log n-z\log a}{(\log(a+1)-\log a)}\geq k$$

$$\log_{\frac{a+1}{a}}\frac{n}{a^z}\geq k$$