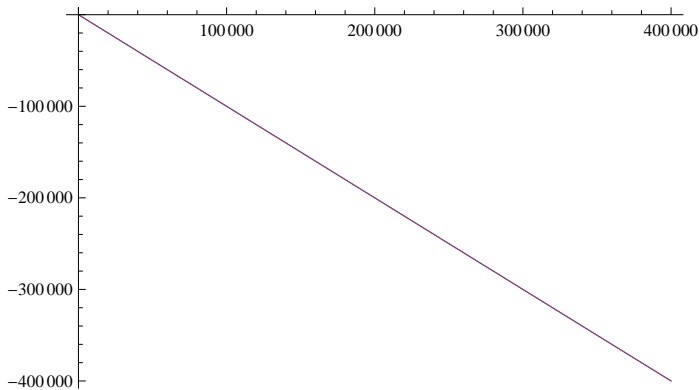


s

```
aa[n_, x_] := (-1 / 2 + x) n^(1 / 2 + x) HarmonicNumber[n, 1 / 2 + x] -  
  (-1 / 2 - x) n^(1 / 2 - x) HarmonicNumber[n, 1 / 2 - x]  
pll[s_, t_] := Plot[{Re@aa[n, s + t I], Re@aa[n, -s + t I]}, {n, 1, 400 000}]  
pllI[s_, t_] := Plot[{Im@aa[n, s + t I], Im@aa[n, -s + t I]}, {n, 1, 400 000}]  
  
aal[n_, x_] := (-1 / 2 - x) n^(1 / 2 - x) HarmonicNumber[n, 1 / 2 - x]  
aa2[n_, x_] := (-1 / 2 + x) n^(1 / 2 + x) HarmonicNumber[n, 1 / 2 + x]  
pall[s_, t_] := Plot[{Re@aal[n, s + t I], Re@aa2[n, s + t I]}, {n, 1, 400 000}]  
pallI[s_, t_] := Plot[{Im@aal[n, s + t I], Im@aa2[n, s + t I]}, {n, 1, 400 000}]  
pallt[s_, t_] :=  
  N@Table[{Re@aal[n, s + t I], Re@aa2[n, s + t I]}, {n, 1, 400 000, 400 000 / 30}]  
pallIt[s_, t_] :=  
  N@Table[{Im@aal[n, s + t I], Im@aa2[n, s + t I]}, {n, 1, 400 000, 400 000 / 30}]  
  
naa[n_, x_] :=  
  (-1 / 2 + x) n^(x) HarmonicNumber[n, 1 / 2 + x] - (-1 / 2 - x) n^(-x) HarmonicNumber[n, 1 / 2 - x]  
nab[n_, x_] := (-1 / 2 + x) n^(x) HarmonicNumber[n, 1 / 2 + x] +  
  (-1 / 2 - x) n^(-x) HarmonicNumber[n, 1 / 2 - x]  
npl[s_, t_] := Plot[{Re@naa[n, s + t I], Im@naa[n, s + t I]}, {n, 1, 400 000}]  
nplb[s_, t_] := Plot[{Re@nab[n, s + t I], Im@nab[n, s + t I]}, {n, 1, 400 000}]  
npll[s_, t_] := Plot[{Re@naa[n, s + t I], Re@naa[n, -s + t I]}, {n, 1, 400 000}]  
npllI[s_, t_] := Plot[{Im@naa[n, s + t I], Im@naa[n, -s + t I]}, {n, 1, 400 000}]  
npllb[s_, t_] := Plot[{Re@nab[n, s + t I], Re@nab[n, -s + t I]}, {n, 1, 400 000}]  
npllIb[s_, t_] := Plot[{Im@nab[n, s + t I], Im@nab[n, -s + t I]}, {n, 1, 400 000}]  
naa1[n_, x_] := (-1 / 2 - x) n^(-x) HarmonicNumber[n, 1 / 2 - x]  
naa2[n_, x_] := (-1 / 2 + x) n^(x) HarmonicNumber[n, 1 / 2 + x]  
npall[s_, t_] := Plot[{Re@naa1[n, s + t I], Re@naa2[n, s + t I]}, {n, 1, 400 000}]  
npallI[s_, t_] := Plot[{Im@naa1[n, s + t I], Im@naa2[n, s + t I]}, {n, 1, 400 000}]  
npallt[s_, t_] :=  
  N@Table[{Re@naa1[n, s + t I], Re@naa2[n, s + t I]}, {n, 1, 400 000, 400 000 / 30}]  
npallIt[s_, t_] :=  
  N@Table[{Im@naa1[n, s + t I], Im@naa2[n, s + t I]}, {n, 1, 400 000, 400 000 / 30}]  
pall[0, N@Im@ZetaZero@1]
```



N@ZetaZero@100 000

0.5 + 74 920.8 i

```
pal1It[0, Im@ZetaZero@2 + 3]
```

```
{{-24.022, 24.022}, {-450.608, 450.608}, {-3095.42, 3095.42},
 {4720.61, -4720.61}, {5994.6, -5994.6}, {4955.02, -4955.02}, {-6616.89, 6616.89},
 {4029.32, -4029.32}, {-3830.66, 3830.66}, {6359.44, -6359.44}, {-9475.36, 9475.36},
 {7950.78, -7950.78}, {1500.87, -1500.87}, {-10760.1, 10760.1}, {4610.99, -4610.99},
 {10320.1, -10320.1}, {-5679.09, 5679.09}, {-11727.7, 11727.7},
 {2193.11, -2193.11}, {13201.3, -13201.3}, {5781.68, -5781.68}, {-9352.27, 9352.27},
 {-13747.7, 13747.7}, {-3249.65, 3249.65}, {10684., -10684.}, {14890.9, -14890.9},
 {6320.96, -6320.96}, {-7416.76, 7416.76}, {-15778.1, 15778.1}, {-13245.2, 13245.2}}
```

```
N@aa2[400000, .3 + Im@ZetaZero@2 I + 3 I]
```

```
-1.08787 × 106 - 47113.5 i
```

```
Integrate[j^(-(1/2 + x)), {j, 0, n}]
```

```
ConditionalExpression[ $\frac{2 n^{\frac{1}{2}-x}}{1-2x}$ , Re[x] <  $\frac{1}{2}$ ]
```

```
FullSimplify[ $\left(\frac{2 n^{\frac{1}{2}-x}}{1-2x}\right)^{-1}$ ]
```

```
 $\frac{1}{2} n^{-\frac{1}{2}+x} (1-2x)$ 
```

```
 $n^{-\frac{1}{2}+x} (1/2 - x)$ 
```

```
(-1 + A + f I) Sum[(n / j) ^ (A + f I), {j, 1, n}]
```

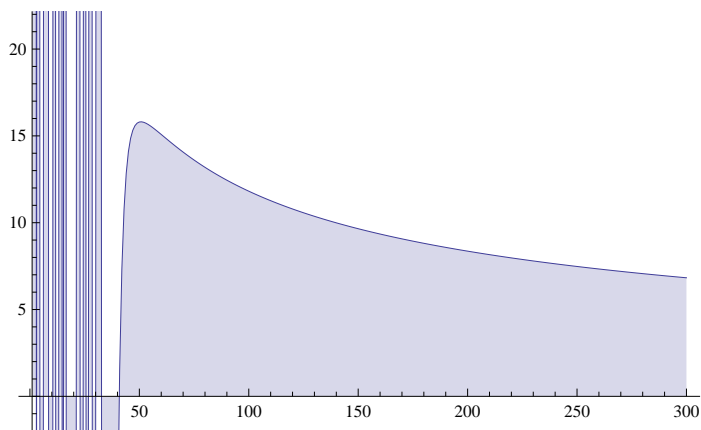
```
(-1 + A + i f) nA+i f (-HurwitzZeta[A + i f, 1 + n] + Zeta[A + i f])
```

```
pr[n_, A_, f_] := n^A Sum[j^(-A (f Cos[f Log[n / j]] + (A - 1) Sin[f Log[n / j]]), {j, 1, n}]
```

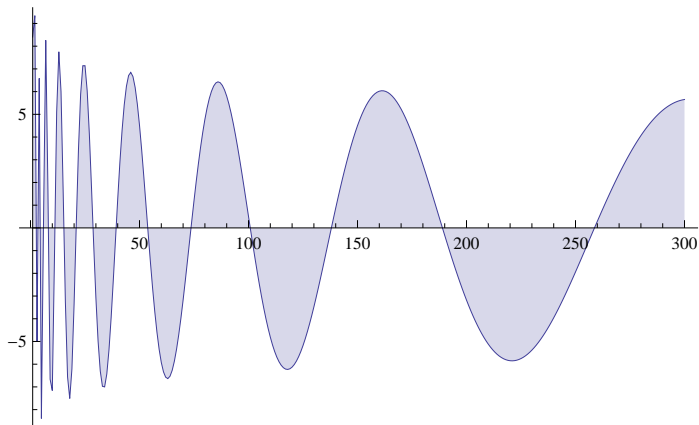
```
pr2[n_, A_, f_] := Sum[j^(-A (f Cos[f Log[n / j]] + (A - 1) Sin[f Log[n / j]]), {j, 1, n}]
```

```
pr2a[n_, A_, f_] := DiscretePlot[j^(-A (f Cos[f Log[n / j]] + (A - 1) Sin[f Log[n / j]]), {j, 1, n}]
```

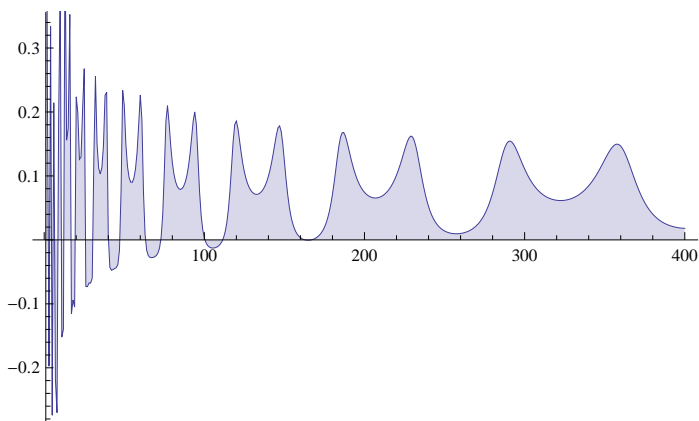
```
DiscretePlot[pr2[n, .5, N@Im@ZetaZero@100], {n, 1, 300}]
```



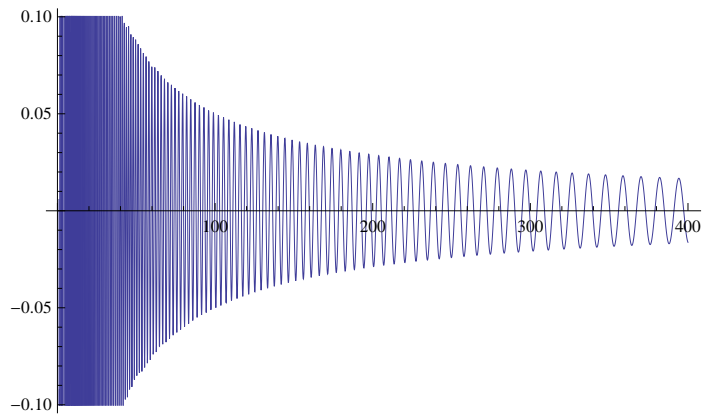
```
pr2a[300, .1, 10]
```



```
rr[n_, t_] :=  
  Sum[j^(-1/2) (Cos[t Log[j]] + Tan[t Log[n] + ArcCot[2 t]] Sin[t Log[j]]), {j, 1, n}]  
rrs[n_, t_] := Tan[t Log[n] + ArcCot[2 t]]  
rr[1000, .4 I + 100]  
1.76709 + 0.0543434 i  
Zeta[.5 - 100 I]  
2.69262 + 0.020386 i  
DiscretePlot[Re@rr[n, N@Im@ZetaZero@1 + .1 I], {n, 1, 400}]
```



```
Plot[Re@rrs[n, 100 + .4 I], {n, 1, 400}]
```



```
FullSimplify[TrigToExp[Tan[t Log[n] + ArcCot[2 t]]], Element[n, Integers]]
```

$$-\frac{i}{n} + \frac{-2i + 4t}{-1 + n^{2i}t(1 - 2it) - 2it}$$