```
r[n_, t_, 0, a_] := UnitStep[n - 1]
r[n_, t_, k_, a_] := Sum[j^-t r[n / j, t, k - 1, a], {j, a, Floor[n]}]


r[100, 1, 1, 8]
```

$$\frac{36\,178\,186\,814\,193\,714\,071\,499\,343\,581\,922\,991\,251\,243}{13\,944\,075\,045\,942\,495\,432\,906\,761\,787\,062\,460\,711\,360}$$

```
r[100, 1, 1, 1] - HarmonicNumber[7] r[100, 1, 0, 8]
```

$$\frac{36\,178\,186\,814\,193\,714\,071\,499\,343\,581\,922\,991\,251\,243}{13\,944\,075\,045\,942\,495\,432\,906\,761\,787\,062\,460\,711\,360}$$

```
Clear[s]
b[y_, s_, 0] := 1
b[1, s_, k_] := Zeta[s]^k
b[y_, s_, 1] := Zeta[s] - Zn[y, s]
bo[y_, s_, k_] := Sum[ (-1)^j Binomial[k, j] y^(-j s) Zeta[ s, y - 1]^(k - j), {j, 0, k}]
b[y_, s_, k_] := Sum[ (-1)^j Binomial[k, j] y^(-j s) b[y - 1, s, k - j], {j, 0, k}]

b[10, s, 2]
```

$$2^{-2s} + 3^{-2s} + 4^{-2s} + 5^{-2s} + 6^{-2s} + 7^{-2s} + 8^{-2s} + 9^{-2s} + 10^{-2s} - 2^{1-s}\,\text{Zeta}[s] + \text{Zeta}[s]^2 - 2\times 3^{-s}\,(-1 - 2^{-s} + \text{Zeta}[s]) - 2^{1-2s}\,(-1 - 2^{-s} - 3^{-s} + \text{Zeta}[s]) - 2\times 5^{-s}\,(-1 - 2^{-s} - 3^{-s} - 4^{-s} + \text{Zeta}[s]) - 2^{1-s}\,3^{-s}\,(-1 - 2^{-s} - 3^{-s} - 4^{-s} - 5^{-s} + \text{Zeta}[s]) - 2\times 7^{-s}\,(-1 - 2^{-s} - 3^{-s} - 4^{-s} - 5^{-s} - 6^{-s} + \text{Zeta}[s]) - 2^{1-3s}\,(-1 - 2^{-s} - 3^{-s} - 4^{-s} - 5^{-s} - 6^{-s} - 7^{-s} + \text{Zeta}[s]) - 2\times 9^{-s}\,(-1 - 2^{-s} - 3^{-s} - 4^{-s} - 5^{-s} - 6^{-s} - 7^{-s} - 8^{-s} + \text{Zeta}[s]) - 2^{1-s}\,5^{-s}\,(-1 - 2^{-s} - 3^{-s} - 4^{-s} - 5^{-s} - 6^{-s} - 7^{-s} - 8^{-s} - 9^{-s} + \text{Zeta}[s])$$

```
Zn[s_, y_] := Sum[ j^-s, {j, 1, y}]

N[Zeta[4, 3 + 1]^2]
```

0.0000559138

```
N[Zeta[4]^2 - 2 (Zeta[4] Zn[4, 3]) + (Zn[4, 3])^2]
```

0.0000559138

```
N[Zeta[2, 3 + 1]^3]
```

0.0228635

```
N[Zeta[2]^3 - 3 (Zeta[2]^2 Zn[2, 3]) + 3 (Zeta[2] Zn[2, 3]^2) - (Zn[2, 3])^3]
```

0.0228635

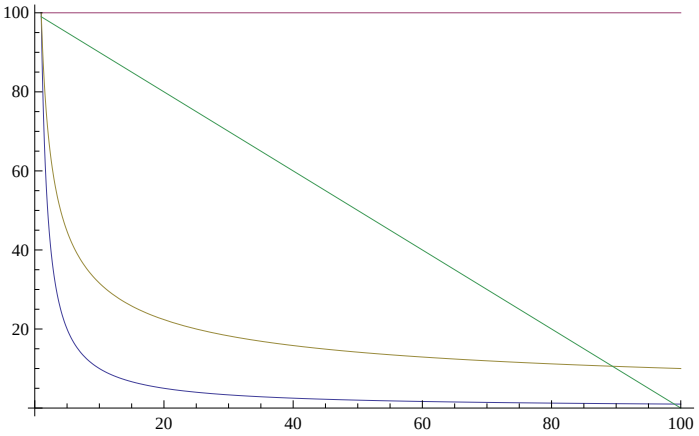
```
h[s_, y_, k_] := Sum[ (-1)^(k - j) Binomial[k, j] Zeta[s]^j Zn[s, y]^(k - j), {j, 0, k}]

N[h[-1, 4, 4]]
```

10 337.5

```
N[Zeta[-1, 4 + 1]^4]
```

10 337.5

```
Plot[{100 / x^1, 100 / x^0, 100 / x^(1 / 2), 100 - x}, {x, 1, 100}]
```




```
Clear[D2]
D2[n_, 0] := UnitStep[n - 1]
D2[n_, k_] := D2[n, k] = Sum[ D2[Floor[n / j], k - 1], {j, 2, n}]
eD2[n_, z_] := Sum[ z^k / k ! D2[n, k], {k, 0, Log2@n}]
```

```
N@eD2[100, 1]
```

```
303.601
```

```
D[gg[x, z], x] = z / (x Log[x]) (gg[x, z + 1] - gg[x, z])
```

```
Set::write : Tag D in ∂x gg[x, z] is Protected. ≫
```

$$\frac{z\,(-gg[x,\,z]+gg[x,\,1+z])}{x\,Log[x]}$$

```
Sum[(-1)^(k + 1) / (k^a) (x - 1)^k, {k, 1, Infinity}]
```

```
-PolyLog[a, 1 - x]
```

```
Sum[(-1)^(k + 1) / (k^4) (x - 1)^k, {k, 1, Infinity}]
```

```
-PolyLog[1, 1 - x]
```

```
Log[x]
```

```
N[-PolyLog[a, 1 - 8] /. a → 1]
```

```
2.07944
```

```
FullSimplify[Expand[z / (x (x - 1)) (x^(z + 1) - x^z)]]
```

$$x^{-1+z}\,z$$

```
D[x^z, x]
```

$$x^{-1+z}\,z$$

```
Log[8.]
```

```
2.07944
```

```
D[(x - 1)^z, x]
```

$$(-1+x)^{-1+z}\,z$$

```
Expand@Integrate[ (Log[x])^4 / 4!, {x, 1, n}]
```

$$\text{ConditionalExpression}\left[-1+n-n\,\text{Log}[n]+\frac{1}{2}n\,\text{Log}[n]^2-\frac{1}{6}n\,\text{Log}[n]^3+\frac{1}{24}n\,\text{Log}[n]^4,\ \text{Re}[n]\geq 0\ ||\ n\notin\text{Reals}\right]$$

```
N@Expand[(-z Gamma[z] + Gamma[1 + z, -Log[n]]) (-Log[n])^-z Log[n]^z] /. {z → 5, n → 100}
```

$$93\,707.7-6.88553\times 10^{-11}\,i$$

```
Gamma[5, 0, -Log[100.]]
```

$$-22\,683.1+1.38894\times 10^{-11}\,i$$

```
Integrate[ (x - 1)^4, {x, 1, n}]
```

$$\frac{1}{5}(-1+n)^5$$

```
Expand@Integrate[ (PolyLog[1, 1 - x])^4 / 4!, {x, 1, n}]
```

$$\text{ConditionalExpression}\left[-1+n-n\,\text{Log}[n]+\frac{1}{2}n\,\text{Log}[n]^2-\frac{1}{6}n\,\text{Log}[n]^3+\frac{1}{24}n\,\text{Log}[n]^4,\ \text{Re}[n]\geq 0\ ||\ n\notin\text{Reals}\right]$$

```
Expand@Integrate[ (PolyLog[2, 1 - x])^4 / 4!, {x, 1, n}]
```

$$\int_1^n \frac{1}{24}\,\text{PolyLog}[2,\,1-x]^4\,dx$$

```
Integrate[ Log[t] / t, {t, 1, z}]
```

$$\text{ConditionalExpression}\left[\frac{\text{Log}[z]^2}{2},\ \text{Re}[z]\geq 0\ ||\ z\notin\text{Reals}\right]$$

```
Integrate[ Log[t]^2 / t, {t, 1, z}]
```

$$\text{ConditionalExpression}\left[\frac{\text{Log}[z]^3}{3},\ \text{Re}[z]\geq 0\ ||\ z\notin\text{Reals}\right]$$

```
Integrate[ Log[t]^3 / t, {t, 1, z}]
```

$$\text{ConditionalExpression}\left[\frac{\text{Log}[z]^4}{4},\ \text{Re}[z]\geq 0\ ||\ z\notin\text{Reals}\right]$$

```
Integrate[ PolyLog[s, t] / t, {t, 0, z}]
```

$$\text{PolyLog}[1+s,\,z]$$

```
Table[D[(-1)^z Gamma[z, 0, -Log[x]] / Gamma[z], x], {z, 1, 8}]
```

$$\left\{1,\ \text{Log}[x],\ \frac{\text{Log}[x]^2}{2},\ \frac{\text{Log}[x]^3}{6},\ \frac{\text{Log}[x]^4}{24},\ \frac{\text{Log}[x]^5}{120},\ \frac{\text{Log}[x]^6}{720},\ \frac{\text{Log}[x]^7}{5040}\right\}$$

```
Table[D[ (-1)^z Gamma[z,  0, -Log[x]], x], {z, 1, 8}]
```

$$\left\{1,\ \text{Log}[x],\ \text{Log}[x]^2,\ \text{Log}[x]^3,\ \text{Log}[x]^4,\ \text{Log}[x]^5,\ \text{Log}[x]^6,\ \text{Log}[x]^7\right\}$$

```
Expand@Integrate[ 1,  {x, 1, n}, {y, 1, n / x}, {z, 1, n / (x y)}]
```

$$\text{ConditionalExpression}\left[-1+n-n\,\text{Log}[n]+\frac{1}{2}n\,\text{Log}[n]^2,\ \text{Re}[n]\geq 0\ ||\ n\notin\text{Reals}\right]$$

```
Expand[(-1)^3 Gamma[3, 0, -Log[n]] / Gamma[3]]
```

$$-\frac{1}{2}\,\text{Gamma}[3, 0, -\text{Log}[n]]$$

```
Sum[Binomial[z, k] k / ((k!)^(1 / a)) Integrate[
   Sum[(-1)^(j + 1) / j^a (x - 1)^j, {j, 1, Infinity}]^(k - 1), {x, 1, n}], {k, 1, Infinity}]
```

$$\sum_{k=1}^{\infty} k\,\text{Binomial}[z, k]\,(k!)^{-1/a} \int_1^n (-\text{PolyLog}[a, 1-x])^{-1+k}\,dx$$

```
Integrate[Sum[(-1)^(j + 1) / j^a (x - 1)^j, {j, 1, Infinity}]^3, {x, 1, n}]
```

$$\int_1^n -\text{PolyLog}[a, 1-x]^3\,dx$$

```
N@LaguerreL[5, Log[20]]
```

0.853716

```
N@20 LaguerreL[-5 - 1, -Log[20]]
```

0.853716

```
N@Sum[j^-2 k^-2, {j, 2, n}, {k, 2, n}] /. n → 100
```

0.403205

```
Integrate[j^-s k^-s, {j, 1, n}, {k, 1, n}]
```

$$\text{ConditionalExpression}\left[\frac{n^{-2s}\,(-n+n^s)^2}{(-1+s)^2},\ \text{Re}[n] \geq 0 \,||\, n \notin \text{Reals}\right]$$

```
N@Sum[j^-2 k^-2 l^-2, {j, 2, n}, {k, 2, n}, {l, 2, n}] /. n → 100
```

0.256028

```
N@Sum[j^-s k^-s l^-s, {j, 2, n}, {k, 2, n}, {l, 2, n}]
```

$$(-1. + \text{HarmonicNumber}[n, s])^3$$

```
Sum[(-1)^(k + 1) / k (Sum[j^2, {j, 2, n}])^k, {k, 1, Infinity}]
```

$$\text{Log}\left[\frac{1}{6}\,n\,\left(1+3n+2n^2\right)\right]$$

```
Expand[Sum[j, {j, 2, n}]]
```

$$-1+\frac{n}{2}+\frac{n^2}{2}$$

```
-Expand[Sum[j k, {j, 2, n}, {k, 2, n}] / 2]
```

$$-\frac{1}{2}+\frac{n}{2}+\frac{3n^2}{8}-\frac{n^3}{4}-\frac{n^4}{8}$$

```
Expand[Sum[j k l, {j, 2, n}, {k, 2, n}, {l, 2, n}] / 2]
```

$$-\frac{1}{2}+\frac{3n}{4}+\frac{3n^2}{8}-\frac{11n^3}{16}-\frac{3n^4}{16}+\frac{3n^5}{16}+\frac{n^6}{16}$$