```
riemannPrimeCount[n_] := Sum[k^-1 PrimePi[n^(1 / k)], {k, 1, Log[2, n]}]
Table[{n, riemannPrimeCount[n]}, {n, 1, 100}] // TableForm
```

1	0
2	1
3	2
4	$\frac{5}{2}$
5	$\frac{7}{2}$
6	$\frac{7}{2}$
7	$\frac{9}{2}$
8	$\frac{29}{6}$
9	$\frac{16}{3}$
10	$\frac{16}{3}$
11	$\frac{19}{3}$
12	$\frac{19}{3}$
13	$\frac{22}{3}$
14	$\frac{22}{3}$
15	$\frac{22}{3}$
16	$\frac{91}{12}$
17	$\frac{103}{12}$
18	$\frac{103}{12}$
19	$\frac{115}{12}$
20	$\frac{115}{12}$
21	$\frac{115}{12}$
22	$\frac{115}{12}$
23	$\frac{127}{12}$
24	$\frac{127}{12}$
25	$\frac{133}{12}$
26	$\frac{133}{12}$
27	$\frac{137}{12}$
28	$\frac{137}{12}$
29	$\frac{149}{12}$
30	$\frac{149}{12}$
31	$\frac{161}{12}$
32	$\frac{817}{60}$
33	$\frac{817}{60}$
34	$\frac{817}{60}$
35	$\frac{817}{60}$
36	$\frac{817}{60}$
37	$\frac{877}{60}$

38	$\frac{877}{60}$
39	$\frac{877}{60}$
40	$\frac{877}{60}$
41	$\frac{937}{60}$
42	$\frac{937}{60}$
43	$\frac{997}{60}$
44	$\frac{997}{60}$
45	$\frac{997}{60}$
46	$\frac{997}{60}$
47	$\frac{1057}{60}$
48	$\frac{1057}{60}$
49	$\frac{1087}{60}$
50	$\frac{1087}{60}$
51	$\frac{1087}{60}$
52	$\frac{1087}{60}$
53	$\frac{1147}{60}$
54	$\frac{1147}{60}$
55	$\frac{1147}{60}$
56	$\frac{1147}{60}$
57	$\frac{1147}{60}$
58	$\frac{1147}{60}$
59	$\frac{1207}{60}$
60	$\frac{1207}{60}$
61	$\frac{1267}{60}$
62	$\frac{1267}{60}$
63	$\frac{1267}{60}$
64	$\frac{1277}{60}$
65	$\frac{1277}{60}$
66	$\frac{1277}{60}$
67	$\frac{1337}{60}$
68	$\frac{1337}{60}$
69	$\frac{1337}{60}$
70	$\frac{1337}{60}$
71	$\frac{1397}{60}$
72	$\frac{1397}{60}$
73	$\frac{1457}{60}$
74	$\frac{1457}{60}$
75	$\frac{1457}{60}$
76	$\frac{1457}{60}$

77	$\frac{1457}{60}$
78	$\frac{1457}{60}$
79	$\frac{1517}{60}$
80	$\frac{1517}{60}$
81	$\frac{383}{15}$
82	$\frac{383}{15}$
83	$\frac{398}{15}$
84	$\frac{398}{15}$
85	$\frac{398}{15}$
86	$\frac{398}{15}$
87	$\frac{398}{15}$
88	$\frac{398}{15}$
89	$\frac{413}{15}$
90	$\frac{413}{15}$
91	$\frac{413}{15}$
92	$\frac{413}{15}$
93	$\frac{413}{15}$
94	$\frac{413}{15}$
95	$\frac{413}{15}$
96	$\frac{413}{15}$
97	$\frac{428}{15}$
98	$\frac{428}{15}$
99	$\frac{428}{15}$
100	$\frac{428}{15}$

```
Zk[n_, k_, s_] := Sum[j^-s Zk[n / j, k - 1, s], {j, 1, n}]; Zk[n_, 0, s_] := UnitStep[n - 1]
Table[Zk[n, k, 0], {n, 1, 50}, {k, 1, 7}] // TableForm
```

1	1	1	1	1	1	1
2	3	4	5	6	7	8
3	5	7	9	11	13	15
4	8	13	19	26	34	43
5	10	16	23	31	40	50
6	14	25	39	56	76	99
7	16	28	43	61	82	106
8	20	38	63	96	138	190
9	23	44	73	111	159	218
10	27	53	89	136	195	267
11	29	56	93	141	201	274
12	35	74	133	216	327	470
13	37	77	137	221	333	477
14	41	86	153	246	369	526
15	45	95	169	271	405	575
16	50	110	204	341	531	785
17	52	113	208	346	537	792
18	58	131	248	421	663	988
19	60	134	252	426	669	995
20	66	152	292	501	795	1191
21	70	161	308	526	831	1240
22	74	170	324	551	867	1289
23	76	173	328	556	873	1296
24	84	203	408	731	1209	1884
25	87	209	418	746	1230	1912
26	91	218	434	771	1266	1961
27	95	228	454	806	1322	2045
28	101	246	494	881	1448	2241
29	103	249	498	886	1454	2248
30	111	276	562	1011	1670	2591
31	113	279	566	1016	1676	2598
32	119	300	622	1142	1928	3060
33	123	309	638	1167	1964	3109
34	127	318	654	1192	2000	3158
35	131	327	670	1217	2036	3207
36	140	363	770	1442	2477	3991
37	142	366	774	1447	2483	3998
38	146	375	790	1472	2519	4047
39	150	384	806	1497	2555	4096
40	158	414	886	1672	2891	4684
41	160	417	890	1677	2897	4691
42	168	444	954	1802	3113	5034
43	170	447	958	1807	3119	5041
44	176	465	998	1882	3245	5237
45	182	483	1038	1957	3371	5433
46	186	492	1054	1982	3407	5482
47	188	495	1058	1987	3413	5489
48	198	540	1198	2337	4169	6959
49	201	546	1208	2352	4190	6987
50	207	564	1248	2427	4316	7183

```
Zk1[n_, s_] := Sum[j^-s, {j, 1, n}]
Zk2[n_, s_] := Sum[j^-s k^-s, {j, 1, n}, {k, 1, n / j}]
Zk3[n_, s_] := Sum[j^-s k^-s m^-s, {j, 1, n}, {k, 1, n / j}, {m, 1, n / (j k)}]
Zk[n_, k_, s_] := Sum[j^-s Zk[n / j, k - 1, s], {j, 1, n}]; Zk[n_, 0, s_] := UnitStep[n - 1]
FullSimplify[
 Table[{Zk1[n, s] - Zk[n, 1, s], Zk2[n, s] - Zk[n, 2, s], Zk3[n, s] - Zk[n, 3, s]}, {n, 1, 50}] //
  TableForm]
```

0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
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0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0

```
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k !
ds[n_, k_, s_] := ds[n, k, s] = Sum[ j^-s ds[Floor[n / j], k - 1, s], {j, 2, n}];
ds[n_, 0, s_] := UnitStep[n - 1]
dz[n_, z_, s_] := Expand[Sum[ bin[z, k] ds[n, k, s], {k, 0, Log[2, n]}]]
zeros[n_, s_] := List @@ Roots[dz[n, z, s] == 0, z][[All, 2]]
```

```
FullSimplify[dz[100, z, -1 / 5]]
```

$Aborted

```
((1 / (Zeta[s]) - 1) - FullSimplify[Sum[ (-1) ^k (Zeta[s] - 1) ^k, {k, 1, Infinity}]])
```

0

```
1 / (Zeta[s]) - 1
```

$$-1 + \frac{1}{\text{Zeta}[s]}$$

```
Sum[ (-1) ^j (Zeta[s] - 1) ^ (j + k), {j, 0, Infinity}]
```

$$\frac{(-1 + \text{Zeta}[s])^k}{\text{Zeta}[s]}$$

```
FullSimplify[- (Zeta[s] ^-1 - 1 ) ( (Zeta[s] - 1) ^k + (Zeta[s] - 1) ^ (k - 1) )]
```

$(-1 + \text{Zeta}[s])^k$

```
Sum[ (-1) ^ (k + 1) / k (Zeta[s] - 1) ^ (k + a), {k, 1, Infinity}]
```

$\text{Log}[\text{Zeta}[s]]\ (-1 + \text{Zeta}[s])^a$

```
(Zeta[s] - 1) ^a (Zeta[s] - 1) ^k
```

$(-1 + \text{Zeta}[s])^{a+k}$

```
FullSimplify[(Zeta[s] - 1) ( - (Zeta[s] ^-1 - 1) ^ (k - 1) - (Zeta[s] ^-1 - 1) ^k)]
```

$$\left(-1 + \frac{1}{\text{Zeta}[s]}\right)^k$$

```
FullSimplify[
 Sum[ (-1) ^ (k + j) Binomial[ k + j - 1, k - 1] (Zeta[s] - 1) ^ (k + j), {j, 0, Infinity}]]
```

$$(-1)^k \left(1 - \frac{1}{\text{Zeta}[s]}\right)^k$$

```
FullSimplify[Sum[(-1) ^ (k) / k (Zeta[s] ^-1 - 1) ^ (k), {k, 1, Infinity}]]
```

$$-\text{Log}\left[\frac{1}{\text{Zeta}[s]}\right]$$

```
Sum[ (-1) ^a Binomial[a, b] (Zeta[s] ^-1 - 1) ^a (Zeta[s] - 1) ^ (k - b), {b, 0, a}]
```

$(-1 + \text{Zeta}[s])^k$

```
Sum[ (-1) ^ (k + 1) / k (Zeta[s] - 1) ^k Log[ Zeta[s]] ^ (a - 1), {k, 1, Infinity}]
```

$\text{Log}[\text{Zeta}[s]]^a$

```
N[Log[Zeta[3]] ^4]
```

0.00114708

```
Sum[ Log[ Zeta[s]] ^{a + k}, {k, 1, Infinity}]
```

$$\left\{-\frac{\text{Log}[\text{Zeta}[s]]^{1+a}}{-1 + \text{Log}[\text{Zeta}[s]]}\right\}$$

```
(Zeta[s] - 1) (Log[Zeta[s]])^a
```

$\text{Log}[\text{Zeta}[s]]^a\ (-1+\text{Zeta}[s])$

```
Series[(1 / (Log[x + 1] + 1) - 1)^2, {x, 0, 10}]
```

$$x^2 - 3x^3 + \frac{83x^4}{12} - \frac{43x^5}{3} + \frac{2521x^6}{90} - \frac{791x^7}{15} + \frac{81251x^8}{840} - \frac{187979x^9}{1080} + \frac{61723x^{10}}{200} + O[x]^{11}$$

```
Series[(1 / (Log[x + 1] + 1))^2, {x, 0, 10}]
```

$$1 - 2x + 4x^2 - \frac{23x^3}{3} + \frac{57x^4}{4} - \frac{259x^5}{10} + \frac{2777x^6}{60} - \frac{1714x^7}{21} + \frac{19937x^8}{140} - \frac{372413x^9}{1512} + \frac{7992833x^{10}}{18900} + O[x]^{11}$$

```
FullSimplify[((Log[x + 1] + 1)^-1 - 1)]
```

$$-1 + \frac{1}{1 + \text{Log}[1 + x]}$$

```
Series[(1 / (x + 1) - 1)^1, {x, 0, 10}]
```

$$-x + x^2 - x^3 + x^4 - x^5 + x^6 - x^7 + x^8 - x^9 + x^{10} + O[x]^{11}$$

```
Series[(1 / (Log[x + 1] + 1) - 1), {x, 0, 10}]
```

$$-x + \frac{3x^2}{2} - \frac{7x^3}{3} + \frac{11x^4}{3} - \frac{347x^5}{60} + \frac{3289x^6}{360} - \frac{1011x^7}{70} + \frac{38371x^8}{1680} - \frac{136553x^9}{3780} + \frac{4320019x^{10}}{75600} + O[x]^{11}$$

```
Series[((Log[x + 1] + 1)^-1), {x, 0, 10}]
```

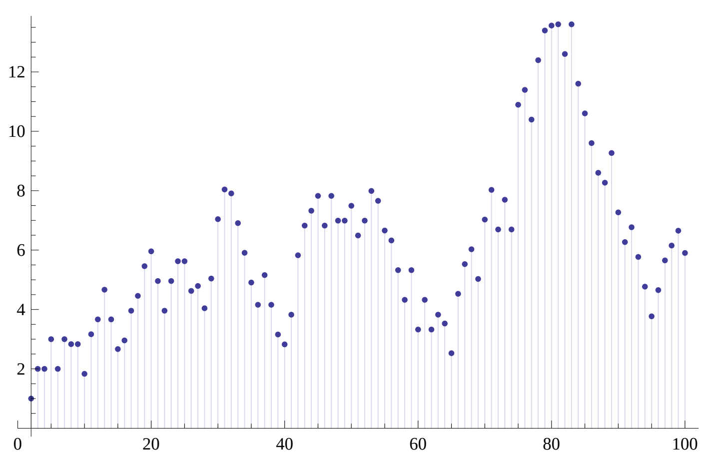
$$1 - x + \frac{3x^2}{2} - \frac{7x^3}{3} + \frac{11x^4}{3} - \frac{347x^5}{60} + \frac{3289x^6}{360} - \frac{1011x^7}{70} + \frac{38371x^8}{1680} - \frac{136553x^9}{3780} + \frac{4320019x^{10}}{75600} + O[x]^{11}$$

```
(((Log[x + 1] + 1)^-1)) (Log[x + 1] + 1)
```

1

```
K[n_] := FullSimplify[MangoldtLambda[n] / Log[n]]
logD[n_, k_] := logD[n, k] = Sum[K[j] logD[Floor[n / j], k - 1], {j, 2, n}];
logD[n_, 0] := UnitStep[n - 1]
logDz[n_, z_] := Sum[ bin[z, k] logD[n, k], {k, 0, Log[2 n]}]
```

```
DiscretePlot[D[Expand[logDz[n, z]], {z, 1}] /. z → 0, {n, 2, 100}]
```



```
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
K[n_] := FullSimplify[MangoldtLambda[n] / Log[n]]
logD[n_, k_, s_] := logD[n, k, s] =
  Sum[If[ K[j] == 0, 0, (-1)^(j + 1) j^-s K[j] logD[Floor[n / j], k - 1, s]], {j, 2, n}];
logD[n_, 0, s_] := UnitStep[n - 1]
logDz[n_, z_, s_] := Sum[ bin[z, k] logD[n, k, s], {k, 0, Log[2 n]}]
Dz[n_, z_, s_] := Sum[ z^k / k! logD[n, k, s], {k, 0, Log[2 n]}]
zeros[n_, s_] := List @@ NRoots[Dz[n, z, s] == 0, z][[All, 2]]


Dz[100, 1, -1]
```

$\frac{45442}{9}$

```
Table[ {N[logD[100, k, 2]], N[Log[Zeta[2]]^k]}, {k, 0, 14}]
```

```
{{1., 1.}, {0.495776, 0.4977}, {0.240154, 0.247706}, {0.105174, 0.123283},
 {0.0369013, 0.0613581}, {0.00755238, 0.0305379}, {0.000895182, 0.0151987},
 {0., 0.00756441}, {0., 0.00376481}, {0., 0.00187375}, {0., 0.000932565},
 {0., 0.000464138}, {0., 0.000231002}, {0., 0.00011497}, {0., 0.0000572204}}
```

```
zeros[ 100, ZetaZero[1]]
```

$\{-2.62683 - 0.291815\,i,\ 0.982159 + 4.91784\,i,\ 1.12502 - 0.282495\,i,\ 1.6139 - 2.81973\,i,\ 1.86201 + 0.66778\,i\}$

```
Log[2, 10000.]
```

13.2877

```
Product[ 1 - (1 / j), {j, zeros[ 10000, 2]}]
```

$0.92525 + 1.38778 \times 10^{-17}\,i$

```
N[ Dz[10000, 1, ZetaZero[1]]]
```

$Aborted

```
N[Pi^2 / 6]
```

1.64493

```
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
filt[ n_, m_] := Sum[ If[ Log[j, n] == Floor[Log[j, n]], 1, 0], {j, 2, m}]
Ka[n_, m_] := If[ filt[n, m] > 0, 0, FullSimplify[MangoldtLambda[n] / Log[n]]]
logDa[n_, k_, s_, m_] := logDa[n, k, s] =
  Sum[If[ Ka[j, m] == 0, 0, j^-s Ka[j, m] logDa[Floor[n / j], k - 1, s, m]], {j, 2, n}];
logDa[n_, 0, s_, m_] := UnitStep[n - 1]
logDaz[n_, z_, s_, m_] := Sum[ bin[z, k] logDa[n, k, s, m], {k, 0, Log[2 n]}]
Daz[n_, z_, s_, m_] := Sum[ z^k / k! logDa[n, k, s, m], {k, 0, Log[2 n]}]
zerosa[n_, s_, m_] := List @@ NRoots[Daz[n, z, s, m] == 0, z][[All, 2]]


logDa[100, 1, 0] + HarmonicNumber[ Floor[Log[2, 100]]] +
 HarmonicNumber[ Floor[Log[3, 100]]] + HarmonicNumber[ Floor[Log[5, 100]]]
```

$\frac{181}{30}$ + logDa[100, 1, 0]

```
HarmonicNumber[Floor[Log[2, 100]]]
```

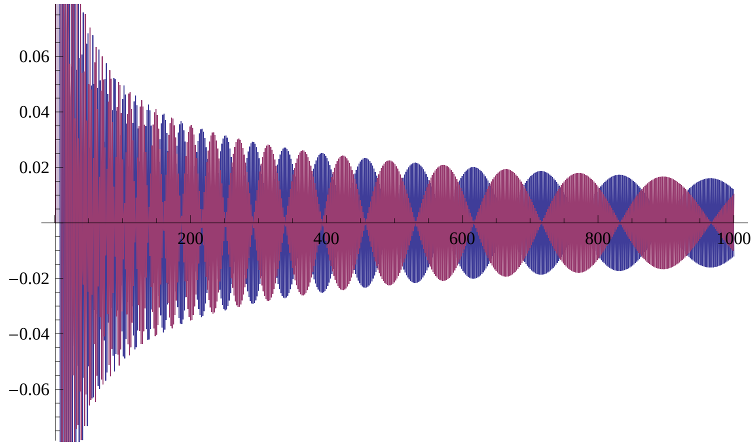
$\frac{49}{20}$

```
zerosa[1000, 0, 1]

{-4.8878, -4.13806 - 5.52305 i, -4.13806 + 5.52305 i,
 -2.05117 - 1.10317 i, -2.05117 + 1.10317 i, -0.961669, -0.00572997}

zerosa[1000, 0, 2]

{-265.031, -10.7617 - 2.65172 i, -10.7617 + 2.65172 i, -3.27504, -1.16496, -0.0057963}

zerosa[1000, 0, 3]

{-69.8934, -8.28722, -1.41355, -0.00586255}

zerosa[1000, 0, 5]

{-32.9481, -1.92101, -0.00592479}

zerosa[1000, 0, 7]

{-2.80914, -0.00598286}

zerosa[1000, 0, 11]

{-4.14397, -0.00603287}

zerosa[1000, 0, 13]

{-6.7082, -0.00608454}

zerosa[1000, 0, 17]

{-10.8605, -0.00613844}

bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
Dz[n_, z_, k_, s_] :=
 Dz[n, z, k, s] = 1 + ((z + 1) / k - 1) Sum[(-1) ^ (j + 1) j ^ - s Dz[n / j, z, k + 1, s], {j, 2, n}]
zeros[n_, s_] := List @@ NRoots[Dz[n, z, 1, s] == 0, z][[All, 2]]


Expand[Dz[100, z, 1, N[ZetaZero[1]]]]
```

$1 - (0.764664 - 0.423453\,i)\,z - (0.694574 + 0.47739\,i)\,z^2 + (0.612768 + 0.0349837\,i)\,z^3 - (0.129055 - 0.0723845\,i)\,z^4 + (0.0100174 - 0.0179348\,i)\,z^5 - (0.000011119 - 0.000708482\,i)\,z^6$

```
zeros[5000, N[ZetaZero[1]]]

{-0.122278 - 0.302615 i, 0.988031 + 0.000854469 i, 1.66008 - 3.88178 i, 1.97964 - 0.321138 i,
 1.9824 + 0.266293 i, 2.05348 + 4.00239 i, 3.50892 + 13.7615 i, 6.35835 - 1.05213 i,
 9.92667 - 1.40671 i, 31.9434 + 42.1149 i, 43.9777 - 15.061 i, 111.277 - 542.104 i}

zeros[1000, .1 + N[ZetaZero[1]]]

{-2.58444 + 1.07135 i, 0.744939 - 0.749314 i, 0.844225 + 0.853938 i,
 2.04714 - 0.675841 i, 2.16373 + 0.8309 i, 4.88379 - 0.62227 i,
 11.5722 - 1.88375 i, 17.1958 + 15.5409 i, 89.5536 - 52.2708 i}
```

```
DiscretePlot[
 {Re[Dz[n, 1, 1, N[ZetaZero[2]]]], Im[Dz[n, 1, 1, N[ZetaZero[2]]]]}, {n, 1, 1000}]
```




```
bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k!
dz[n_, z_] := dz[n, z] = Product[(-1)^p[[2]] bin[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FI[n] = FactorInteger[n]; FI[1] := {}
Dz1[n_, z_, s_] := Dz1[n, z, s] = Sum[FullSimplify[ j^-s dz[j, z]], {j, 1, n}]
DxDAlt[n_, z_, x_, s_] :=
 Sum[(-1)^j bin[z, j] x^(j (1 - s)) Dz1[n / x^j, z, s], {j, 0, Log[x, n]}]
zeros2[n_, s_] := List @@ NRoots[DxDAlt[n, z, 2, s] == 0, z][[All, 2]]
zeros3[n_, s_] := List @@ Roots[DxDAlt[n, z, 2, s] == 0, z][[All, 2]]
```

```
Expand[Dz1[100 000, z, 0]]
```

$$1+\frac{991892879\,z}{102960}+\frac{16611877533197\,z^2}{605404800}+\frac{27613425421567\,z^3}{864864000}+\frac{8883298064606291\,z^4}{435891456000}+\frac{82938597121\,z^5}{10264320}+\frac{12123475378339\,z^6}{5748019200}+\frac{987114594581\,z^7}{2612736000}+\frac{6832898553167\,z^8}{146313216000}+\frac{53237749\,z^9}{13063680}+\frac{1772592397\,z^{10}}{7315660800}+\frac{20466961\,z^{11}}{2052864000}+\frac{30323737\,z^{12}}{114960384000}+\frac{841\,z^{13}}{186810624}+\frac{9773\,z^{14}}{209227898880}+\frac{71\,z^{15}}{373621248000}+\frac{17\,z^{16}}{20922789888000}$$

```
DxDAlt[ 1231, 1.5, 2, -1]
```

-1796.7

```
Dz[1231, 1.5, 1, -1]
```

-1796.7

Expand[DxDAlt[100 000, z, 2, 0]]

$$1+\frac{63100897\,z}{80080}-\frac{180348209849\,z^2}{55036800}+\frac{97254541679083\,z^3}{18162144000}-\frac{397875843476297\,z^4}{87178291200}+\frac{819629656441\,z^5}{359251200}-\frac{90242719681\,z^6}{127733760}+\frac{359231217229\,z^7}{2612736000}-\frac{162434105119\,z^8}{9754214400}+\frac{71104889\,z^9}{57153600}-\frac{15739609\,z^{10}}{270950400}+\frac{23627123\,z^{11}}{14370048000}-\frac{487\,z^{12}}{15482880}+\frac{1063\,z^{13}}{1334361600}-\frac{6373\,z^{14}}{348713164800}+\frac{433\,z^{15}}{2615348736000}-\frac{z^{16}}{1394852659200}$$

zeros2[100 000, N[ZetaZero[1]]]

{-1.5782 + 2.56046 ⅈ, 0.099237 - 0.00129244 ⅈ, 1.00102 + 0.000529329 ⅈ, 1.51313 - 8.06227 ⅈ, 1.97177 - 0.010541 ⅈ, 2.9512 + 1.09857 ⅈ, 3.05508 - 1.61428 ⅈ, 3.39272 + 0.0222148 ⅈ, 5.55241 - 0.00674025 ⅈ, 10.2484 + 11.7983 ⅈ, 13.0121 + 44.0801 ⅈ, 16.1591 + 2.96883 ⅈ, 17.1949 - 9.11333 ⅈ, 43.7242 - 21.1622 ⅈ, 120.418 - 118.71 ⅈ, 121.553 + 200.507 ⅈ}

zeros2[100 000, .1 + N[ZetaZero[1]]]

{-1.58961 + 2.61753 ⅈ, 0.433581 - 0.213656 ⅈ, 0.5472 + 0.271463 ⅈ, 1.47342 - 7.99666 ⅈ, 2.08921 - 0.0510959 ⅈ, 2.94335 + 1.09229 ⅈ, 3.05787 - 1.61562 ⅈ, 3.38832 + 0.0294925 ⅈ, 5.55214 - 0.00687741 ⅈ, 10.2124 + 11.7658 ⅈ, 13.1309 + 44.1903 ⅈ, 16.1775 + 3.07072 ⅈ, 17.0161 - 9.05257 ⅈ, 43.7773 - 20.9222 ⅈ, 120.592 - 118.335 ⅈ, 122.483 + 202.34 ⅈ}

zeros2[100 000, .1 I + N[ZetaZero[1]]]

{-1.64513 + 2.54763 ⅈ, 0.0321829 + 0.0859746 ⅈ, 1.11228 - 0.0735677 ⅈ, 1.44684 - 8.10087 ⅈ, 1.91921 - 0.0420792 ⅈ, 2.96582 + 1.09079 ⅈ, 3.04473 - 1.60261 ⅈ, 3.38918 + 0.0248579 ⅈ, 5.55256 - 0.00702265 ⅈ, 10.2845 + 11.7613 ⅈ, 12.9031 + 44.1993 ⅈ, 16.058 + 2.98967 ⅈ, 17.1283 - 9.28908 ⅈ, 43.4956 - 21.1092 ⅈ, 119.73 + 201.411 ⅈ, 120.056 - 118.551 ⅈ}

zeros2[100 000, N[ZetaZero[2]]]

{-11.5782 + 180.192 ⅈ, -5.18725 - 11.1497 ⅈ, 0.0992719 + 0.0123424 ⅈ, 0.997808 - 0.000207433 ⅈ, 1.93441 + 0.181163 ⅈ, 1.94229 - 0.220858 ⅈ, 2.34694 + 47.1573 ⅈ, 2.80181 - 2.37204 ⅈ, 2.99031 + 2.103 ⅈ, 3.44301 - 0.0212634 ⅈ, 5.5488 - 0.0375321 ⅈ, 9.0019 + 16.1821 ⅈ, 11.0893 + 2.21452 ⅈ, 11.28 - 14.7059 ⅈ, 39.5162 - 32.7796 ⅈ, 106.874 - 160.032 ⅈ}

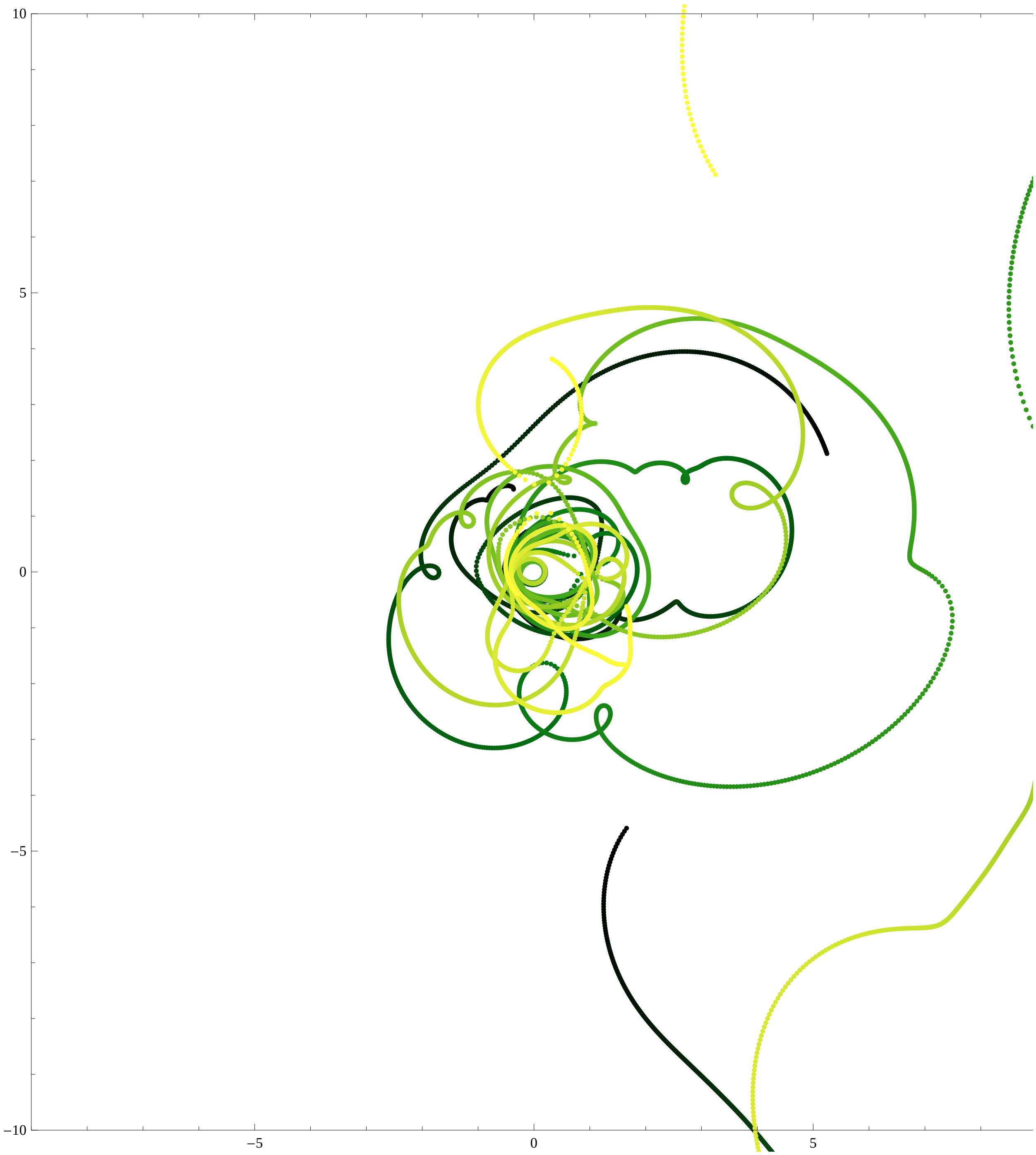
FullSimplify[DxDAlt[100 000, z, 2, 0]]

$$-\frac{1}{20922789888000}(-1+z)(20922789888000+z(16507521312307200+z(-52053654143888640+z(59983577870414976+z(-35506624563896304+z(12228606627227536+z(-2553150856520264+z(323572731049568+z(-24848424430687+z(1181653334433+z(-33759272547+z(641818541+z(-16288909+z(378931+z(-3449+15z)))))))))))))))$$

FullSimplify[DxDAlt[100, z, 2, ZetaZero[1]]]

$Aborted

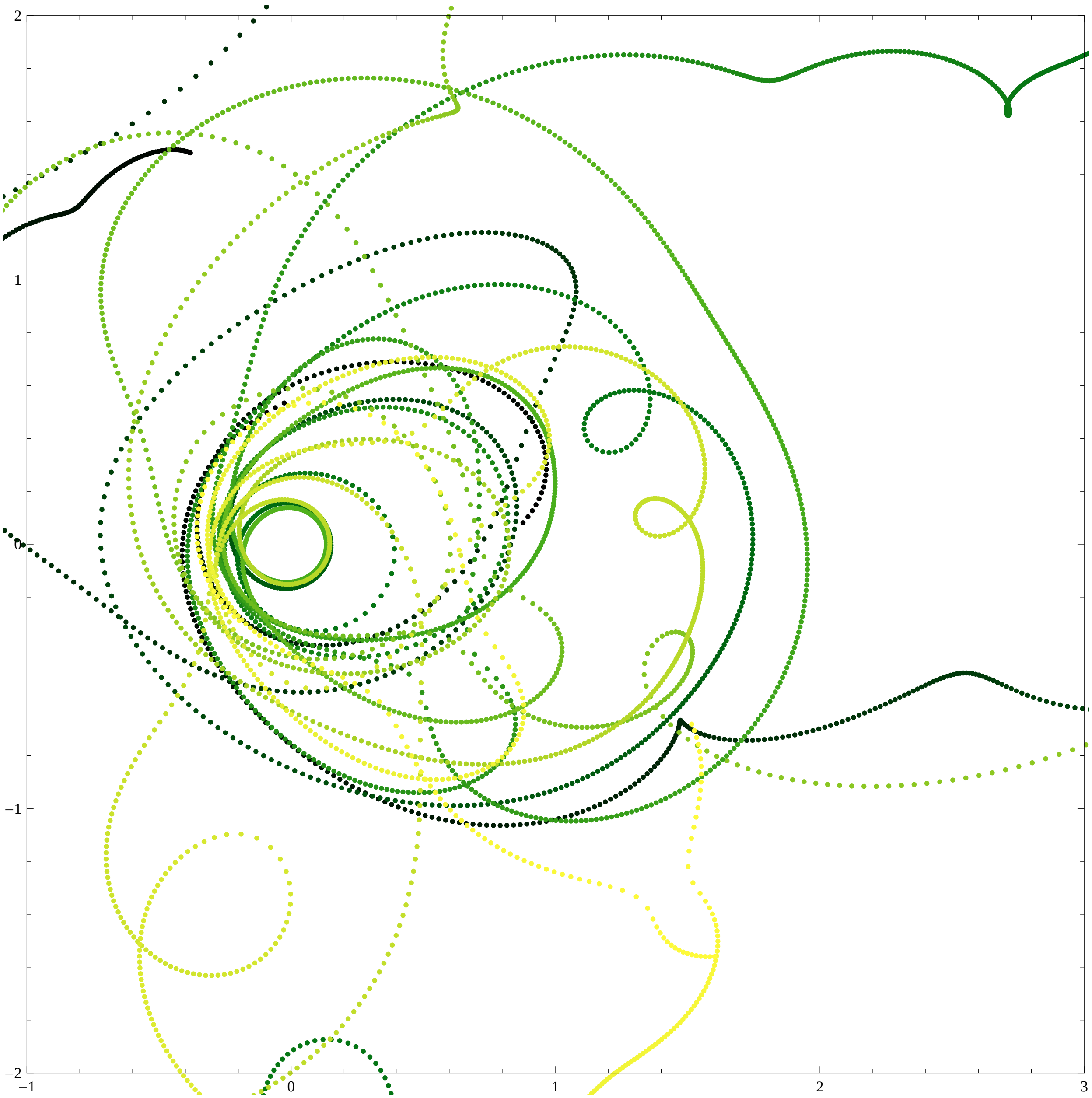
```
colfunc = ColorData["AvocadoColors"]; aa = 0; bb = 1600;
pts1 = Table[{colfunc[(n - aa) / bb], Point[{Re[#], Im[#]}]} & /@
    zeros2[100, n * .02 I + N[ZetaZero[12]]], {n, aa, aa + bb}];
Graphics[pts1, Frame → True, PlotRange → {{-10 + 1, 10 + 1}, {-10, 10}}]
```



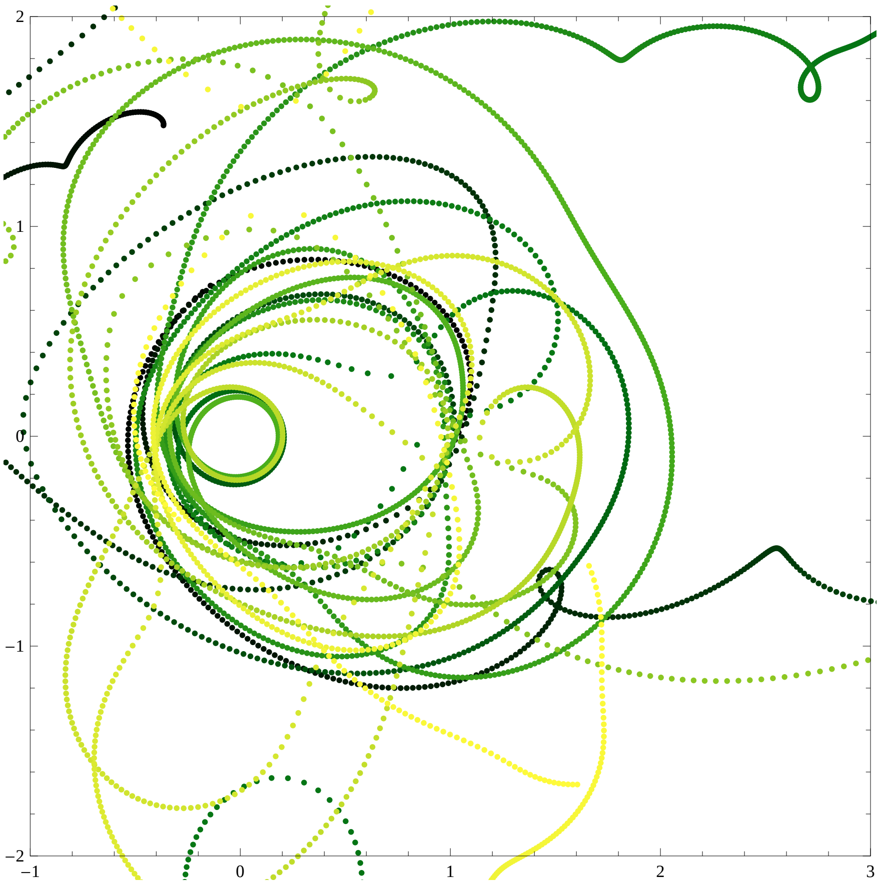
```
N[ZetaZero[45]]
```

0.5 + 133.498 ⅈ

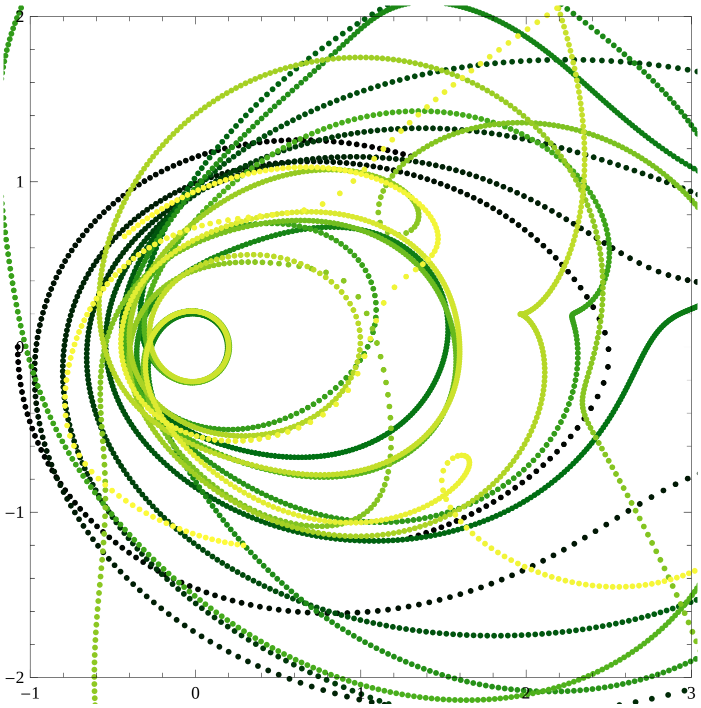
```
colfunc = ColorData["AvocadoColors"]; aa = 0; bb = 1600;
pts1 = Table[{colfunc[(n - aa) / bb], Point[{Re[#], Im[#]}]} & /@
    zeros2[100, -.1 + n * .02 I + N[ZetaZero[12]]], {n, aa, aa + bb}];
Graphics[pts1, Frame → True, PlotRange → {{-2 + 1, 2 + 1}, {-2, 2}}]
```



```
colfunc = ColorData["AvocadoColors"]; aa = 0; bb = 1600;
pts1 = Table[{colfunc[(n - aa) / bb], Point[{Re[#], Im[#]}]} & /@
    zeros2[100, n * .02 I + N[ZetaZero[12]]], {n, aa, aa + bb}];
Graphics[pts1, Frame → True, PlotRange → {{-2 + 1, 2 + 1}, {-2, 2}}]
```



```
colfunc = ColorData["AvocadoColors"]; aa = 0; bb = 1600;
pts1 = Table[{colfunc[(n - aa) / bb], Point[{Re[#], Im[#]}]} & /@
    zeros2[100, .5 + n * .02 I], {n, aa, aa + bb}];
Graphics[pts1, Frame → True, PlotRange → {{-2 + 1, 2 + 1}, {-2, 2}}]
```




```
ss[n_, s_] := Sum[ 2^(x (1 - s)) / x, {x, 1, Log[2, n]}]
```

```
f[s_] := Sum[ k^-s, {k, 1, 5}] - 1
```

```
bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k!
dz[n_, z_] := dz[n, z] = Product[(-1)^p[[2]] bin[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FI[n] = FactorInteger[n]; FI[1] := {}
Dz1[n_, z_, s_] := Dz1[n, z, s] = Sum[FullSimplify[ j^-s dz[j, z]], {j, 1, n}]
zerosd[n_, s_] := List @@ NRoots[Dz1[n, z, s] == 0, z][[All, 2]]
zerosdr[n_, s_] := List @@ Roots[Dz1[n, z, s] == 0, z][[All, 2]]
DxDAlt[n_, z_, x_, s_] :=
 Sum[(-1)^j bin[z, j] x^(j (1 - s)) Dz1[n / x^j, z, s], {j, 0, Log[x, n]}]
zerose[n_, s_] := List @@ NRoots[DxDAlt[n, z, 2, s] == 0, z][[All, 2]]
zeroser[n_, s_] := List @@ Roots[DxDAlt[n, z, 2, s] == 0, z][[All, 2]]
```

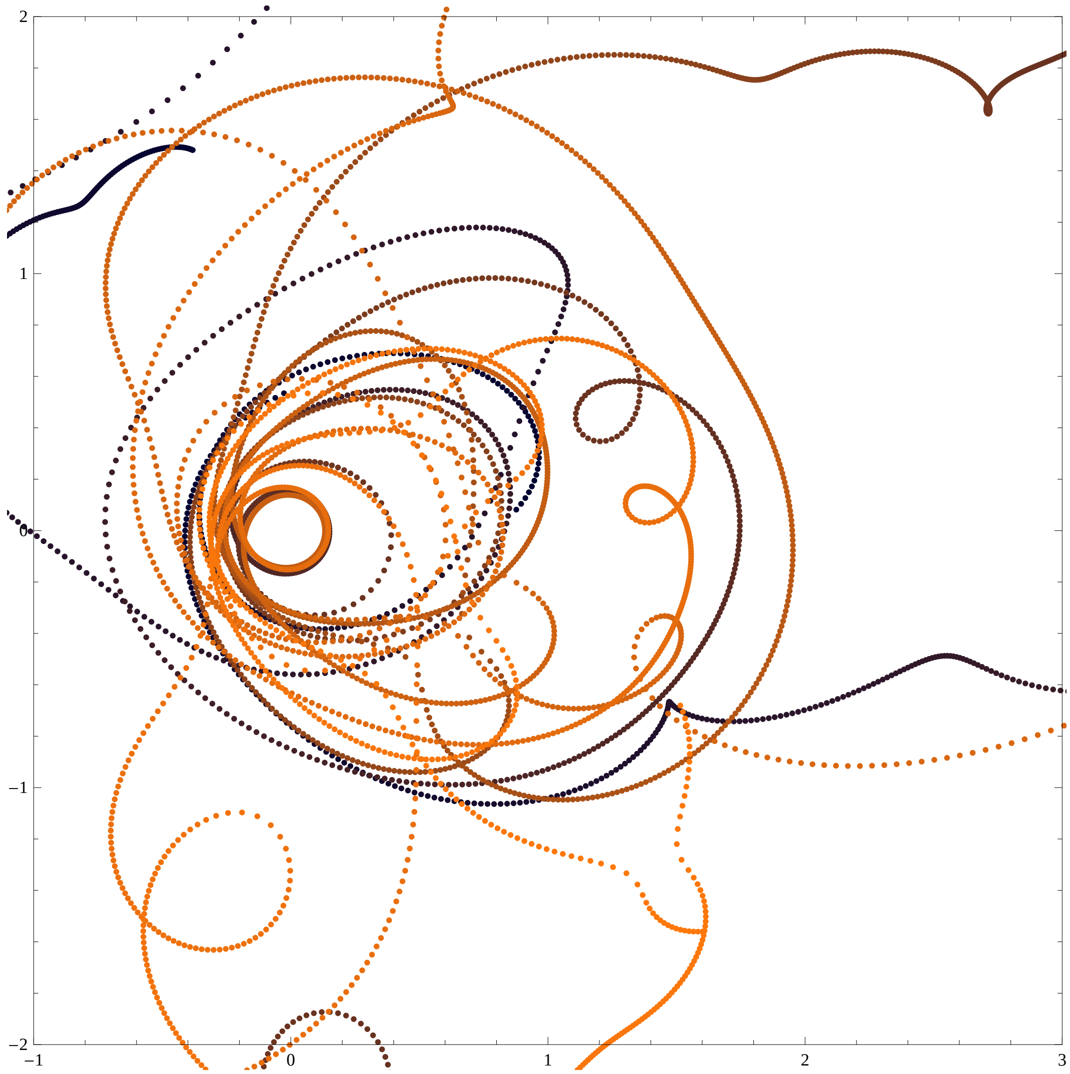
```
1 - 1 / zerosd[30 000, N[ZetaZero[1]]]
```

```
{1.01019 + 0.0246656 i, 1.00576 + 0.0310077 i, 1.01835 + 0.0754149 i,
 1.03902 + 0.144207 i, 1.05578 + 0.294042 i, 1.01884 - 0.257244 i, 1.08487 + 0.684835 i,
 1.00305 - 0.133428 i, 1.03747 - 0.606286 i, 0.137877 - 2.31485 i, -0.421362 + 2.69743 i,
 0.994531 - 0.0594043 i, 0.999821 + 0.00312797 i, 0.992283 - 0.0168883 i}
```

```
1 - 1 / zeros2[30 000, N[ZetaZero[1]]]

{1.16051 + 0.0818891 i, -0.918653 - 3.93515 i, -0.00395532 - 0.00199566 i,
 0.46003 - 0.0130028 i, 0.901701 - 0.18711 i, 0.816009 + 0.220098 i, 0.672987 + 0.104833 i,
 0.693862 - 0.0998235 i, 0.980764 + 0.0388616 i, 0.909317 - 0.0123339 i, 0.934593 - 0.0120227 i,
 0.989332 + 0.0156057 i, 0.982859 - 0.00641856 i, 0.99883 - 0.00343033 i}

Dz1[10 000, 1, 0]

10 000
```

```
colfunc = ColorData["AvocadoColors"]; aa = 0; bb = 16;
pts1 = Table[Point[{Re[#], Im[#]}] & /@ zeros2[100, .5 + n * .02 I], {n, aa, aa + bb}];

{{Point[{-1.07553, 0}], Point[{1.30206, -1.15423}], Point[{1.30206, 1.15423}],
  Point[{3.57559, 0}], Point[{10.0652, -5.05697}], Point[{10.0652, 5.05697}]},
 {Point[{-1.0748, -0.0456319}], Point[{1.2554, 1.16747}], Point[{1.34863, -1.14}],
  Point[{3.57503, 0.0136508}], Point[{10.0488, 5.04717}], Point[{10.0817, -5.0668}]},
 {Point[{-1.07261, -0.0912289}], Point[{1.20867, 1.17972}], Point[{1.39507, -1.12475}],
  Point[{3.57337, 0.0272438}], Point[{10.0323, 5.03739}], Point[{10.0982, -5.07665}]},
 {Point[{-1.06895, -0.136756}], Point[{1.1619, 1.19099}], Point[{1.44135, -1.10846}],
  Point[{3.57059, 0.0407208}], Point[{10.0158, 5.02763}], Point[{10.1147, -5.08653}]},
 {Point[{-1.06384, -0.182179}], Point[{1.11513, 1.20131}], Point[{1.48745, -1.09112}],
  Point[{3.5667, 0.0540229}], Point[{9.99935, 5.0179}], Point[{10.1312, -5.09643}]},
 {Point[{-1.05727, -0.227461}], Point[{1.06838, 1.21067}], Point[{1.53334, -1.0727}],
  Point[{3.56169, 0.0670898}], Point[{9.98289, 5.0082}], Point[{10.1477, -5.10636}]},
 {Point[{-1.04925, -0.27257}], Point[{1.02168, 1.21908}], Point[{1.57899, -1.05317}],
  Point[{3.55557, 0.0798597}], Point[{9.96644, 4.99852}], Point[{10.1642, -5.11631}]},
 {Point[{-1.03977, -0.31747}], Point[{0.97506, 1.22654}], Point[{1.62439, -1.0325}],
  Point[{3.54832, 0.0922685}], Point[{9.95, 4.98886}], Point[{10.1807, -5.1263}]},
 {Point[{-1.02886, -0.362126}], Point[{0.928552, 1.23306}], Point[{1.66952, -1.01066}],
  Point[{3.53994, 0.104249}], Point[{9.93358, 4.97922}], Point[{10.1972, -5.13631}]},
 {Point[{-1.01651, -0.406504}], Point[{0.882182, 1.23865}], Point[{1.71435, -0.987593}],
  Point[{3.53043, 0.115732}], Point[{9.91716, 4.9696}], Point[{10.2137, -5.14634}]},
 {Point[{-1.00272, -0.45057}], Point[{0.835974, 1.24329}], Point[{1.75887, -0.963267}],
  Point[{3.51977, 0.126641}], Point[{9.90076, 4.96001}], Point[{10.2302, -5.15641}]},
 {Point[{-0.987511, -0.494289}], Point[{0.789956, 1.24701}], Point[{1.80307, -0.937627}],
  Point[{3.50795, 0.136896}], Point[{9.88437, 4.95045}], Point[{10.2467, -5.1665}]},
 {Point[{-0.970885, -0.537627}], Point[{0.74415, 1.24978}], Point[{1.84695, -0.910616}],
  Point[{3.49497, 0.14641}], Point[{9.86799, 4.9409}], Point[{10.2632, -5.17662}]},
 {Point[{-0.952851, -0.580551}], Point[{0.698581, 1.25163}], Point[{1.8905, -0.882165}],
  Point[{3.48081, 0.155088}], Point[{9.85162, 4.93138}], Point[{10.2796, -5.18676}]},
 {Point[{-0.93342, -0.623028}], Point[{0.65327, 1.25254}], Point[{1.93372, -0.852198}],
  Point[{3.46545, 0.162822}], Point[{9.83528, 4.92188}], Point[{10.2961, -5.19694}]},
 {Point[{-0.9126, -0.665022}], Point[{0.60824, 1.25252}], Point[{1.97661, -0.820624}],
  Point[{3.44886, 0.169494}], Point[{9.81895, 4.9124}], Point[{10.3126, -5.20714}]},
 {Point[{-0.890404, -0.706502}], Point[{0.563512, 1.25157}], Point[{2.0192, -0.787333}],
  Point[{3.43103, 0.174966}], Point[{9.80263, 4.90295}], Point[{10.3291, -5.21738}]}}
```

```
colfunc = ColorData["AlpineColors"]; aa = 0; bb = 1600;
pts1 = Table[{colfunc[(n - aa) / bb], Point[{Re[#], Im[#]}]} & /@
    zeros2[100, -.1 + n * .02 I + N[ZetaZero[12]]], {n, aa, aa + bb}];
Graphics[pts1, Frame → True, PlotRange → {{-2 + 1, 2 + 1}, {-2, 2}}]
```




```
colfunc = ColorData["AlpineColors"]; aa = 0; bb = 1600;
pts1 = Table[{colfunc[(n - aa) / bb], Point[{Re[#], Im[#]}]} & /@
    zeros2[1000, -.1 + n * .02 I + N[ZetaZero[12]]], {n, aa, aa + bb}];
Graphics[pts1, Frame → True, PlotRange → {{-2 + 1, 2 + 1}, {-2, 2}}]
```

```
$Aborted
```

```
$Aborted[]
```