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E2[n_, k_, a_] := Sum[ E2[n / j, k - 1, a], {j, 2, n}] - a Sum[ E2[n / (j a), k - 1, a], {j, 1, n / a}];
E2[n_, 0, a_] := 1
PP[n_, a_] := Sum[ ((-1)^(k+1) E2[n, k, a] + 1 + a^k - 1) / k, {k, 1, Log[a, n]}]
PP[100, 3 / 2]

428
15
PP2[n_, a_, t_] :=
Sum[ ((-1)^(k+1) E2[n, k, a] + 1 + (a^k - 1) (a - 1)^t (k^(t - 1))), {k, 1, Log[a, n]}]
N[PP2[100, 3 / 2, 1]]

123.94

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f4[n_, s_, a_] := (-1)^s (a - 1)^s Sum[ k^(s - 1) (a^k - 1), {k, 1, Log[a, n]}]
Limit[f4[n, 3 / 2, a], {a -> 1}]

```

$$\left\{ \text{Limit} \left[ -i (-1 + a)^{3/2} \left( \text{HurwitzZeta} \left[ -\frac{1}{2}, 1 + \frac{\text{Log}[n]}{\text{Log}[a]} \right] - a n \text{LerchPhi} \left[ a, -\frac{1}{2}, 1 + \frac{\text{Log}[n]}{\text{Log}[a]} \right] + \text{PolyLog} \left[ -\frac{1}{2}, a \right] - \text{Zeta} \left[ -\frac{1}{2} \right] \right), a \rightarrow 1 \right] \right\}$$

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f4[100, 2, 1.000001]

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350.914

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$$N \left[ 1 - n + n \text{Log}[n] - \frac{\text{Log}[n]^2}{2} /. n \rightarrow 100 \right]$$

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350.913

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f5[n_, s_, a_] := Gamma[s] - (-1)^s Log[n]^s / s - f4[n, s, a]

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f5[100, 1, 1.000001]

```

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100.

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Limit[f5[n, c, a], {a -> 1, c -> 0}]

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$$\left\{ \text{Limit} \left[ \text{Gamma}[c] - \frac{(-1)^c \text{Log}[n]^c}{c} - (-1)^c (-1 + a)^c \left( -\text{HarmonicNumber} \left[ \frac{\text{Log}[n]}{\text{Log}[a]}, 1 - c \right] - a n \text{LerchPhi} \left[ a, 1 - c, 1 + \frac{\text{Log}[n]}{\text{Log}[a]} \right] + \text{PolyLog}[1 - c, a] \right), a \rightarrow 1 \right], -i \pi + a n \text{LerchPhi} \left[ a, 1, 1 + \frac{\text{Log}[n]}{\text{Log}[a]} \right] + \text{Log}[1 - a] - \text{Log}[\text{Log}[n]] + \text{PolyGamma} \left[ 0, 1 + \frac{\text{Log}[n]}{\text{Log}[a]} \right] \right\}$$

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f5[100, 3, 1.000001]
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1399.73
```



```
f6[n_, s_, a_] := (-1)^s (a - 1)^s Sum[k^(s - 1) (a^k - 1), {k, 1, Log[a, n]}]
Limit[f6[n, 2, a], {a -> 1}]
```

$$\left\{1 - n + n \log[n] - \frac{\log[n]^2}{2}\right\}$$

```
f7[n_, s_, a_] := (-1)^s (a - 1)^s Sum[k^(s - 1) (a^k), {k, 1, Log[a, n]}]
Limit[f7[n, 2, a], {a -> 1}]
```

$$\{1 - n + n \log[n]\}$$

```
f8[n_, s_, a_] := (-1)^s (a - 1)^s Sum[k^(s - 1) (-1), {k, 1, Log[a, n]}]
Limit[f8[n, 2, a], {a -> 1}]
```

$$\left\{-\frac{1}{2} \log[n]^2\right\}$$

```
f9[n_, s_, a_] := (-1)^s (a - 1)^s Sum[k^(s - 1) ((k - 1) / k), {k, 1, Log[a, n]}]
Limit[f9[n, 2, a], {a -> 1}]
```

$$\left\{\frac{\log[n]^2}{2}\right\}$$

```
f7[n_, s_, a_] := (-1)^s (a - 1)^s Sum[k^(s - 1) (a^k), {k, 1, Log[a, n]}] / Gamma[s]
Expand[Limit[f7[n, 4, a], {a -> 1}]]
```

$$\left\{1 - n + n \log[n] - \frac{1}{2} n \log[n]^2 + \frac{1}{6} n \log[n]^3\right\}$$

```
Expand[Limit[f7[n, 3 / 2, a], {a -> 1}]]
```

$$\left\{\text{Limit}\left[-\frac{2 \, i \, (-1 + a)^{3/2} \left(-a n \text{LerchPhi}\left[a, -\frac{1}{2}, 1 + \frac{\log[n]}{\log[a]}\right] + \text{PolyLog}\left[-\frac{1}{2}, a\right]\right)}{\sqrt{\pi}}, a \rightarrow 1\right]\right\}$$

```
Expand[Limit[f7[n, c, a], {a -> 1}]]
```

$$\left\{\text{Limit}\left[\frac{1}{\Gamma[c]} (-1)^c (-1 + a)^c \left(-a n \text{LerchPhi}\left[a, 1 - c, 1 + \frac{\log[n]}{\log[a]}\right] + \text{PolyLog}[1 - c, a]\right), a \rightarrow 1\right]\right\}$$

```
f10[n_, s_, a_] := Sum[Gamma[s]^-1 (-1)^s (a - 1)^s k^(s - 1) (a^k), {k, 1, Log[a, n]}]
Expand[Limit[f10[n, 5, a], {a -> 1}]]
```

$$\left\{1 - n + n \log[n] - \frac{1}{2} n \log[n]^2 + \frac{1}{6} n \log[n]^3 - \frac{1}{24} n \log[n]^4\right\}$$

```
Expand[Limit[f10[n, c, a], {a -> 1}]]
```

$$\left\{\text{Limit}\left[-\frac{1}{\Gamma[c]} (-1)^c (-1 + a)^c \left(a n \text{LerchPhi}\left[a, 1 - c, 1 + \frac{\log[n]}{\log[a]}\right] - \text{PolyLog}[1 - c, a]\right), a \rightarrow 1\right]\right\}$$

```
Limit[PolyLog[1 - c, 1] / Gamma[c], c -> 0]
```

```
-1
```

**f11[n\_, s\_, a\_] := Sum[ s (a - 1) ^s k ^ (s - 1) ((k - 1) / k) , {k, 1, Log[a, n]}]**

**Limit[f11[n, 3 / 2, a] , {a → 1}]**

$$\left\{ \text{Limit} \left[ \frac{3}{2} \sqrt{-1+a} \left( -\text{HarmonicNumber} \left[ \frac{\text{Log}[n]}{\text{Log}[a]}, -\frac{1}{2} \right] + a \text{HarmonicNumber} \left[ \frac{\text{Log}[n]}{\text{Log}[a]}, -\frac{1}{2} \right] + \right. \right. \right. \\ \left. \left. \left. \text{HarmonicNumber} \left[ \frac{\text{Log}[n]}{\text{Log}[a]}, \frac{1}{2} \right] - a \text{HarmonicNumber} \left[ \frac{\text{Log}[n]}{\text{Log}[a]}, \frac{1}{2} \right] \right) , a \rightarrow 1 \right] \right\}$$

**Limit[f11[n, 2, a] , {a → 1}]**

$$\{\text{Log}[n]^2\}$$

**Limit[f11[n, 3, a] , {a → 1}]**

$$\{\text{Log}[n]^3\}$$

**Limit[f11[n, 1, a] , {a → 1}]**

$$\left\{ \text{Limit} \left[ \frac{1}{\text{Log}[a]} \left( \text{HarmonicNumber} \left[ \frac{\text{Log}[n]}{\text{Log}[a]} \right] \text{Log}[a] - a \text{HarmonicNumber} \left[ \frac{\text{Log}[n]}{\text{Log}[a]} \right] \text{Log}[a] - \text{Log}[n] + a \text{Log}[n] \right) , \right. \right. \\ \left. \left. a \rightarrow 1 \right] \right\}$$

**Limit[f11[n, c, a] , {a → 1, c → 0}]**

$$\left\{ \text{Limit} \left[ (-1+a)^c c \left( \text{HarmonicNumber} \left[ \frac{\text{Log}[n]}{\text{Log}[a]}, 1-c \right] - \text{HarmonicNumber} \left[ \frac{\text{Log}[n]}{\text{Log}[a]}, 2-c \right] \right) , a \rightarrow 1 \right], 0 \right\}$$

$$\text{Limit} \left[ \frac{1}{\text{Log}[a]} \left( \text{HarmonicNumber} \left[ \frac{\text{Log}[100]}{\text{Log}[a]} \right] \text{Log}[a] - \right. \right. \\ \left. \left. a \text{HarmonicNumber} \left[ \frac{\text{Log}[100]}{\text{Log}[a]} \right] \text{Log}[a] - \text{Log}[100] + a \text{Log}[100] \right) , a \rightarrow 1 \right]$$

$$\text{Log}[100]$$

$$\text{Limit} \left[ (-1+a)^c c \left( \text{HarmonicNumber} \left[ \frac{\text{Log}[100]}{\text{Log}[a]}, 1-c \right] - \text{HarmonicNumber} \left[ \frac{\text{Log}[100]}{\text{Log}[a]}, 2-c \right] \right) , a \rightarrow 1 \right]$$

$$\text{Limit} \left[ \left( (-1+a)^c c \left( \text{HarmonicNumber} \left[ \frac{\text{Log}[100]}{\text{Log}[a]}, 1-c \right] - \text{HarmonicNumber} \left[ \frac{\text{Log}[100]}{\text{Log}[a]}, 2-c \right] \right) \right) /. c \rightarrow 3/2, \right. \\ \left. a \rightarrow 1 \right]$$

$$\text{Limit} \left[ \frac{3}{2} (-1+a)^{3/2} \left( \text{HarmonicNumber} \left[ \frac{\text{Log}[100]}{\text{Log}[a]}, -\frac{1}{2} \right] - \text{HarmonicNumber} \left[ \frac{\text{Log}[100]}{\text{Log}[a]}, \frac{1}{2} \right] \right) , a \rightarrow 1 \right]$$

**Limit[Sum[ Gamma[s] ^-1 (-1) ^s (a - 1) ^s k ^ (s - 1) (a ^k) , {k, 1, Log[a, n]}] , {a → 1}]**

$$\left\{ \text{Limit} \left[ -\frac{1}{\text{Gamma}[s]} (-1)^s (-1+a)^s \left( a \text{LerchPhi} \left[ a, 1-s, 1 + \frac{\text{Log}[n]}{\text{Log}[a]} \right] - \text{PolyLog}[1-s, a] \right) , a \rightarrow 1 \right] \right\}$$