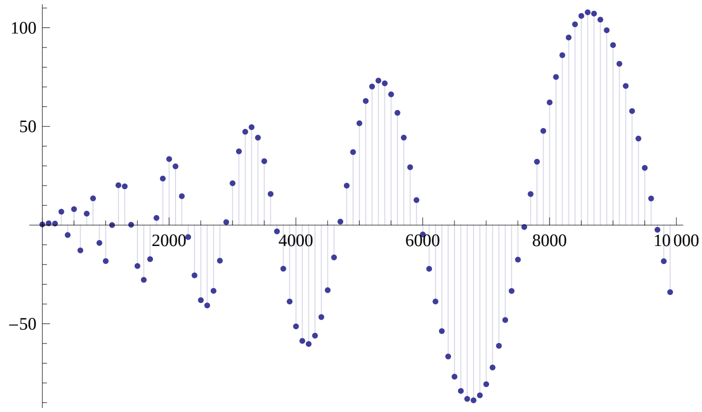
```
DiscretePlot[ Re@Zeta[.2 + 13 I, n], {n, 1, 10 000, 100}]
```




```
ps[n_, s_] :=
 (s - 1) (Zeta[s] - HarmonicNumber[n, s]) + s n^(1 - 2 s) ( Zeta[1 - s] - HarmonicNumber[n, 1 - s])
ps2[n_, s_] := (Zeta[s] - HarmonicNumber[n, s]) +
  (s / (s - 1)) n^(1 - 2 s) ( Zeta[1 - s] - HarmonicNumber[n, 1 - s])
ps3[n_, s_] := (2^s Pi^(s - 1) Sin[ Pi s / 2] Gamma[1 - s] Zeta[1 - s] - HarmonicNumber[n, s]) +
  (s / (s - 1)) n^(1 - 2 s) ( Zeta[1 - s] - HarmonicNumber[n, 1 - s])
ps4[n_, s_] := (2^s Pi^(s - 1) Sin[ Pi s / 2] Gamma[1 - s]) Zeta[1 - s] - HarmonicNumber[n, s] +
  (s / (s - 1)) n^(1 - 2 s) Zeta[1 - s] - (s / (s - 1)) n^(1 - 2 s) HarmonicNumber[n, 1 - s]
ps5[n_, s_] := ((2^s Pi^(s - 1) Sin[ Pi s / 2] Gamma[1 - s]) + (s / (s - 1)) n^(1 - 2 s)) Zeta[1 - s] -
  (HarmonicNumber[n, s] + (s / (s - 1)) n^(1 - 2 s) HarmonicNumber[n, 1 - s])
ps6[n_, s_] := (HarmonicNumber[n, s] + (s / (s - 1)) n^(1 - 2 s) HarmonicNumber[n, 1 - s]) /
  ((2^s Pi^(s - 1) Sin[ Pi s / 2] Gamma[1 - s]) + (s / (s - 1)) n^(1 - 2 s))
```

$$\text{ps7}[n\_,\ s\_] := \frac{\text{HarmonicNumber}[n,\ 1-s] - \frac{n^{1-2\,(1-s)}\ (1-s)\ \text{HarmonicNumber}[n,s]}{s}}{-\frac{n^{1-2\,(1-s)}\ (1-s)}{s} + 2^{1-s}\,\pi^{-s}\ \text{Gamma}[s]\ \text{Sin}\left[\frac{1}{2}\,\pi\ (1-s)\right]}$$

$$\text{ps8}[n\_,\ s\_] := \left((2\,\pi)^{s}\ \left(n\ s\ \text{HarmonicNumber}[n,\ 1-s] + n^{2\,s}\ (-1+s)\ \text{HarmonicNumber}[n,\ s]\right)\right) \Big/ \left(n^{2\,s}\ (2\,\pi)^{s}\ (-1+s) + 2\ n\ s\ \text{Cos}\left[\frac{\pi\,s}{2}\right]\ \text{Gamma}[s]\right)$$

$$\text{ps8a}[n\_,\ s\_] := (2\,\pi)^{s}\ \left(n\ s\ \text{HarmonicNumber}[n,\ 1-s] + n^{2\,s}\ (-1+s)\ \text{HarmonicNumber}[n,\ s]\right) \left(n^{2\,s}\ (2\,\pi)^{s}\ (-1+s) + 2\ n\ s\ \text{Cos}\left[\frac{\pi\,s}{2}\right]\ \text{Gamma}[s]\right) \hat{}\, {-1}$$

$$\text{ps8b}[n\_,\ s\_] := (2\,\pi)^{s}\ \left(n\ s\ \text{HarmonicNumber}[n,\ 1-s] + n^{2\,s}\ (-1+s)\ \text{HarmonicNumber}[n,\ s]\right) \left(n^{2\,s}\ (2\,\pi)^{s}\ (-1+s) + 2\ n\ s\ \text{Cos}\left[\frac{\pi\,s}{2}\right]\ \text{Gamma}[s]\right) \hat{}\, {-1}$$

$$\text{ps8c}[n\_,\ s\_] := \left\{\frac{n\ (2\,\pi)^{s}\ s\ \text{HarmonicNumber}[n,\ 1-s]}{n^{2\,s}\ (2\,\pi)^{s}\ (-1+s) + 2\ n\ s\ \text{Cos}\left[\frac{\pi\,s}{2}\right]\ \text{Gamma}[s]},\right.$$
$$\left. -\frac{n^{2\,s}\ (2\,\pi)^{s}\ \text{HarmonicNumber}[n,\ s]}{n^{2\,s}\ (2\,\pi)^{s}\ (-1+s) + 2\ n\ s\ \text{Cos}\left[\frac{\pi\,s}{2}\right]\ \text{Gamma}[s]},\ +\frac{n^{2\,s}\ (2\,\pi)^{s}\ s\ \text{HarmonicNumber}[n,\ s]}{n^{2\,s}\ (2\,\pi)^{s}\ (-1+s) + 2\ n\ s\ \text{Cos}\left[\frac{\pi\,s}{2}\right]\ \text{Gamma}[s]}\right\}$$

$$\text{ps9}[n\_,\ s\_] := (2\,\pi)^{s}\ \left(n\ s\ \text{HarmonicNumber}[n,\ 1-s] + n^{2\,s}\ (-1+s)\ \text{HarmonicNumber}[n,\ s]\right) \left(n^{2\,s}\ (2\,\pi)^{s}\ (-1+s) + 2\ n\ s\ \text{Cos}\left[\frac{\pi\,s}{2}\right]\ \text{Gamma}[s]\right) \hat{}\, {-1}$$

$$\text{ps10}[n\_,\ s\_] := \left(n\ s\ \text{HarmonicNumber}[n,\ 1-s] - n^{2\,s}\ (1-s)\ \text{HarmonicNumber}[n,\ s]\right) \left(n^{2\,s}\ (-1+s) + 2\ n\ s\ (2\,\pi)^{-s}\ \text{Cos}\left[\frac{\pi\,s}{2}\right]\ \text{Gamma}[s]\right) \hat{}\, {-1}$$

```
ps11[n_, s_] := (n^(1 - s) s HarmonicNumber[n, 1 - s] - n^s (1 - s) HarmonicNumber[n, s])
  (2 n^(1 - s) s (2 π)^-s Cos[π s/2] Gamma[s] - n^s (1 - s))^-1
ps11a[n_, s_] := (n^(1 - s) s HarmonicNumber[n, 1 - s] - n^s (1 - s) HarmonicNumber[n, s]) /
  (2^(1 - s) n^(1 - s) s π^-s Cos[π s/2] Gamma[s] - n^s (1 - s))
ps11b[n_, s_] := (-n^(1/2 - i s) (1/2 + i s) HarmonicNumber[n, 1/2 - i s] +
    n^(1/2 + i s) (1/2 - i s) HarmonicNumber[n, 1/2 + i s]) /
  (-n^(1/2 - i s) (1/2 + i s) + 2^(1/2 + i s) n^(1/2 + i s) π^(-1/2 + i s) (1/2 - i s) Cos[1/2 π (1/2 - i s)] Gamma[1/2 - i s])
ps11c[n_, s_] := ps11b[n, s I - I / 2]
ps12[n_, s_] :=
 (n^(1 - s) s HarmonicNumber[n, 1 - s]) (2 n^(1 - s) s (2 π)^-s Cos[π s/2] Gamma[s] - n^s (1 - s))^-1 -
  (n^s (1 - s) HarmonicNumber[n, s]) (2 n^(1 - s) s (2 π)^-s Cos[π s/2] Gamma[s] - n^s (1 - s))^-1
ps13[n_, s_] := HarmonicNumber[n, 1 - s] / (2 (2 π)^-s Cos[π s/2] Gamma[s] - n^(2 s-1) (1 - s) / s) -
  (n^s (1 - s) HarmonicNumber[n, s]) / (2 n^(1 - s) s (2 π)^-s Cos[π s/2] Gamma[s] - n^s (1 - s))
ps14[n_, s_] := HarmonicNumber[n, s]
   (1 - 2 n^(1 - 2 s) (s / (1 - s)) (2 π)^-s Cos[π s/2] Gamma[s])^-1 -
  HarmonicNumber[n, 1 - s] (n^(2 s-1) (1 - s) / s - 2 (2 π)^-s Cos[π s/2] Gamma[s])^-1

ps11a[100 000 000 000, (.65 + I)]
```

0.243869 - 0.814324 i

```
N@Zeta[.65 + I]
```

0.243869 - 0.814324 i

```
FullSimplify@ps6[n, 1 - s]
```

$$\left((2\pi)^{s}\left(n\, s\, \text{HarmonicNumber}[n, 1-s] + n^{2s}(-1+s)\, \text{HarmonicNumber}[n, s]\right)\right) \Big/ \left(n^{2s}(2\pi)^{s}(-1+s) + 2\, n\, s\, \text{Cos}\left[\frac{\pi s}{2}\right] \text{Gamma}[s]\right)$$

```
N@ps8c[100 000 000 000 000 000, N@ZetaZero@1]
```

$\{-2.07228\times10^{7} - 3.29905\times10^{6}\, i,\ -181312. + 1.47251\times10^{6}\, i,\ 2.09042\times10^{7} + 1.82654\times10^{6}\, i\}$

```
Expand[((2 π)^s (n s HarmonicNumber[n, 1 - s] + n^(2 s) (-1 + s) HarmonicNumber[n, s])) /
   (n^(2 s) (2 π)^s (-1 + s) + 2 n s Cos[π s / 2] Gamma[s])]
```

$$\frac{n\,(2\pi)^s\,s\,\text{HarmonicNumber}[n,1-s]}{n^{2s}\,(2\pi)^s\,(-1+s)+2\,n\,s\,\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]}-\frac{n^{2s}\,(2\pi)^s\,\text{HarmonicNumber}[n,s]}{n^{2s}\,(2\pi)^s\,(-1+s)+2\,n\,s\,\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]}+\frac{n^{2s}\,(2\pi)^s\,s\,\text{HarmonicNumber}[n,s]}{n^{2s}\,(2\pi)^s\,(-1+s)+2\,n\,s\,\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]}$$

```
2^s Pi^(s - 1) Sin[ Pi s / 2] Gamma[1 - s] Zeta[1 - s] /. s → 1 / 2 + s
```

$$2^{\frac{1}{2}+s}\,\pi^{-\frac{1}{2}+s}\,\text{Gamma}\left[\frac{1}{2}-s\right]\,\text{Sin}\left[\frac{1}{2}\,\pi\,\left(\frac{1}{2}+s\right)\right]\,\text{Zeta}\left[\frac{1}{2}-s\right]$$

```
n^x / (n^(-x))
```

n^{2x}

```
FullSimplify[(-1 / 2 + x) / (-1 / 2 - x)] /. x → s
```

$$-1+\frac{2}{1+2s}$$

```
ps11b[100 000, (.3 + 2 I) I - I / 2]
```

0.385062 − 0.282542 i

```
Zeta[.3 + 2 I]
```

0.38531 − 0.282528 i

```
Expand[(1 / 2 - s) / I]
```

$$-\frac{i}{2}+i\,s$$

```
ts[n_, s_] := (-1 / 2 - s) n^(-s) (Zeta[1 / 2 - s] - HarmonicNumber[n, 1 / 2 - s]) -
  (-1 / 2 + s) n^(s) (Zeta[1 / 2 + s] - HarmonicNumber[n, 1 / 2 + s])
ts2[n_, s_] := n^(-s) (Zeta[1 / 2 - s] - HarmonicNumber[n, 1 / 2 - s]) -
  (-1 / 2 + s) n^(s) / (-1 / 2 - s) (Zeta[1 / 2 + s] - HarmonicNumber[n, 1 / 2 + s])
ts3[n_, s_] := n^(-s) (Zeta[1 / 2 - s] - HarmonicNumber[n, 1 / 2 - s]) -
  (-1 / 2 + s) n^(s) / (-1 / 2 - s)
   (2^(1/2+s) π^(-1/2+s) Gamma[1/2 - s] Sin[1/2 π (1/2 + s)] Zeta[1/2 - s] - HarmonicNumber[n, 1 / 2 + s])
ts4[n_, s_] := n^-s Zeta[1 / 2 - s] - n^-s HarmonicNumber[n, 1 / 2 - s] -
  (-1 / 2 + s) n^(s) / (-1 / 2 - s)
   (2^(1/2+s) π^(-1/2+s) Gamma[1/2 - s] Sin[1/2 π (1/2 + s)] Zeta[1/2 - s] - HarmonicNumber[n, 1 / 2 + s])
ts5[n_, s_] := n^-s Zeta[1 / 2 - s] - n^-s HarmonicNumber[n, 1 / 2 - s] -
  (-1 / 2 + s) n^(s) / (-1 / 2 - s) 2^(1/2+s) π^(-1/2+s) Gamma[1/2 - s] Sin[1/2 π (1/2 + s)] Zeta[1/2 - s] +
  (-1 / 2 + s) n^(s) / (-1 / 2 - s) HarmonicNumber[n, 1 / 2 + s]
ts6[n_, s_] := n^-s Zeta[1 / 2 - s] - (-1 / 2 + s) n^(s) / (-1 / 2 - s)
   2^(1/2+s) π^(-1/2+s) Gamma[1/2 - s] Sin[1/2 π (1/2 + s)] Zeta[1/2 - s] +
  (-1 / 2 + s) n^(s) / (-1 / 2 - s) HarmonicNumber[n, 1 / 2 + s] - n^-s HarmonicNumber[n, 1 / 2 - s]
ts7[n_, s_] := (n^-s - (-1 / 2 + s) n^(s) / (-1 / 2 - s) 2^(1/2+s) π^(-1/2+s) Gamma[1/2 - s] Sin[1/2 π (1/2 + s)])
   Zeta[1/2 - s] + (-1 / 2 + s) n^s / (-1 / 2 - s)
   HarmonicNumber[n, 1 / 2 + s] - n^-s HarmonicNumber[n, 1 / 2 - s]
ts8[n_, s_] := (n^-s - (-1 / 2 + s) n^(s) / (-1 / 2 - s) 2^(1/2+s) π^(-1/2+s) Gamma[1/2 - s] Sin[1/2 π (1/2 + s)])
   Zeta[1/2 - s] + (-1 + 2/(1 + 2 s)) n^s HarmonicNumber[n, 1 / 2 + s] -
  n^-s HarmonicNumber[n, 1 / 2 - s]
(* So this is Zeta[1/2-s] *)
ts9[n_, s_] :=
 (-(-1 + 2/(1 + 2 s)) n^s HarmonicNumber[n, 1 / 2 + s] + n^-s HarmonicNumber[n, 1 / 2 - s]) /
  (n^-s - (-1 / 2 + s) n^(s) / (-1 / 2 - s) 2^(1/2+s) π^(-1/2+s) Gamma[1/2 - s] Sin[1/2 π (1/2 + s)])
zt[n_, s_] := ts9[n, 1 / 2 - s]
ts10[n_, s_] :=
 (Sqrt[π] ((1 + 2 s) HarmonicNumber[n, 1/2 - s] + n^(2 s) (-1 + 2 s) HarmonicNumber[n, 1/2 + s])) /
  (Sqrt[π] (1 + 2 s) - n^(2 s) ((2 π)^s - 2^(1+s) π^s s) Gamma[1/2 - s] (Cos[π s/2] + Sin[π s/2]))
zt10[n_, s_] := ts10[n, 1 / 2 - s]
N@ts9[10 000 000 000 000, 3 I + .1]
0.513629 + 0.0713081 ⅈ
```

```
N@Zeta[-1.2]
```

-0.0547884

```
zt[10 000 000 000, -1.2]
```

-0.0547884

```
FullSimplify@
 Expand[ (-(-1 + 2/(1 + 2 s)) n^s HarmonicNumber[n, 1 / 2 + s] + n^-s HarmonicNumber[n, 1 / 2 - s]) /
   (n^-s - (-1 / 2 + s) n^(s) / (-1 / 2 - s) 2^(1/2+s) π^(-1/2+s) Gamma[1/2 - s] Sin[1/2 π (1/2 + s)])]
```

$$\left(\sqrt{\pi}\left((1+2\,s)\,\text{HarmonicNumber}\left[n,\frac{1}{2}-s\right]+n^{2\,s}\,(-1+2\,s)\,\text{HarmonicNumber}\left[n,\frac{1}{2}+s\right]\right)\right)\Big/\left(\sqrt{\pi}\,(1+2\,s)-n^{2\,s}\left((2\,\pi)^{s}-2^{1+s}\,\pi^{s}\,s\right)\text{Gamma}\left[\frac{1}{2}-s\right]\left(\text{Cos}\left[\frac{\pi\,s}{2}\right]+\text{Sin}\left[\frac{\pi\,s}{2}\right]\right)\right)$$

```
zt10[10 000 000 000, N@ZetaZero@1]
```

$-8.96391\times 10^{-7}+5.63064\times 10^{-6}\,i$

```
qs[n_, s_] := (-1 / 2 + s) n^(s) (Zeta[1 / 2 + s] - HarmonicNumber[n, 1 / 2 + s]) -
  (-1 / 2 - s) n^(-s) (Zeta[1 / 2 - s] - HarmonicNumber[n, 1 / 2 - s])
qs2[n_, s_] := (-1 / 2 + s) n^(s)
   ((2^(1/2+s) π^(-1/2+s) Gamma[1/2 - s] Sin[1/2 π (1/2 + s)] Zeta[1/2 - s]) - HarmonicNumber[n, 1 / 2 + s]) -
  (-1 / 2 - s) n^(-s) (Zeta[1 / 2 - s] - HarmonicNumber[n, 1 / 2 - s])
qs3[n_, s_] := n^s (2^(1/2+s) π^(-1/2+s) Gamma[1/2 - s] Sin[1/2 π (1/2 + s)] Zeta[1/2 - s]) -
  n^s HarmonicNumber[n, 1 / 2 + s] - ((1 + 2 s)/(1 - 2 s)) n^-s Zeta[1 / 2 - s] +
  ((1 + 2 s)/(1 - 2 s)) n^-s HarmonicNumber[n, 1 / 2 - s]
qs4[n_, s_] := (n^s (2^(1/2+s) π^(-1/2+s) Gamma[1/2 - s] Sin[1/2 π (1/2 + s)]) - ((1 + 2 s)/(1 - 2 s)) n^-s) Zeta[1 / 2 - s] +
  ((1 + 2 s)/(1 - 2 s)) n^-s HarmonicNumber[n, 1 / 2 - s] - n^s HarmonicNumber[n, 1 / 2 + s]
qs5[n_, s_] := (n^s HarmonicNumber[n, 1 / 2 + s] - ((1 + 2 s)/(1 - 2 s)) n^-s HarmonicNumber[n, 1 / 2 - s]) /
  ((n^s (2^(1/2+s) π^(-1/2+s) Gamma[1/2 - s] Sin[1/2 π (1/2 + s)]) - ((1 + 2 s)/(1 - 2 s)) n^-s))
qt[n_, s_] := qs5[n, 1 / 2 - s]
qs6[n_, s_] := ((1 + 2 s) HarmonicNumber[n, 1/2 - s] + n^(2 s) (-1 + 2 s) HarmonicNumber[n, 1/2 + s]) /
  (1 + 2 s - 2^s n^(2 s) π^(-1/2+s) (1 - 2 s) Gamma[1/2 - s] (Cos[π s/2] + Sin[π s/2]))
qt6[n_, s_] := qs6[n, 1 / 2 - s]
qs7[n_, s_] := HarmonicNumber[n, 1/2 - s] + n^(2 s) (-1 + 2 s) / (1 + 2 s) HarmonicNumber[n, 1/2 + s] /
  (1 - 2^s n^(2 s) π^(-1/2+s) (1 - 2 s) / (1 + 2 s) Gamma[1/2 - s] (Cos[π s/2] + Sin[π s/2]))
qt7[n_, s_] := qs7[n, 1 / 2 - s]
qs8[n_, s_] :=
 n^-s HarmonicNumber[n, 1/2 - s] + n^s (-1 + 2 s) / (1 + 2 s) HarmonicNumber[n, 1/2 + s] /
  (n^-s - 2^s n^s π^(-1/2+s) (1 - 2 s) / (1 + 2 s) Gamma[1/2 - s] (Cos[π s/2] + Sin[π s/2]))
qt8[n_, s_] := qs8[n, 1 / 2 - s]
N@qt8[10 000, 3 I + .1]
0.457347 - 0.0455962 i
Zeta[3 I + .1]
0.457485 - 0.0457237 i
```

FullSimplify[(n^s HarmonicNumber[n, 1 / 2 + s] - ((1 + 2 s)/(1 - 2 s)) n^-s HarmonicNumber[n, 1 / 2 - s]) / ((n^s (2^(1/2+s) π^(-1/2+s) Gamma[1/2 - s] Sin[1/2 π (1/2 + s)]) - ((1 + 2 s)/(1 - 2 s)) n^-s))]

$$\left(\sqrt{\pi}\left((1+2s)\,\mathrm{HarmonicNumber}\left[n,\frac{1}{2}-s\right]+n^{2s}(-1+2s)\,\mathrm{HarmonicNumber}\left[n,\frac{1}{2}+s\right]\right)\right)\Big/\left(\sqrt{\pi}\,(1+2s)-n^{2s}\left((2\pi)^s-2^{1+s}\pi^s s\right)\mathrm{Gamma}\left[\frac{1}{2}-s\right]\left(\mathrm{Cos}\left[\frac{\pi s}{2}\right]+\mathrm{Sin}\left[\frac{\pi s}{2}\right]\right)\right)$$

FullSimplify[((2 π)^s - 2^(1+s) π^s s) / (Pi^(1 / 2))]

$$2^s \pi^{-\frac{1}{2}+s}(1-2s)$$

((1 + 2 s) HarmonicNumber[n, 1/2 - s] + n^(2 s) (-1 + 2 s) HarmonicNumber[n, 1/2 + s]) / ((1 + 2 s) - n^(2 s) 2^s π^(-1/2+s) (1 - 2 s) Gamma[1/2 - s] (Cos[π s/2] + Sin[π s/2]))

$$\left((1+2s)\,\mathrm{HarmonicNumber}\left[n,\frac{1}{2}-s\right]+n^{2s}(-1+2s)\,\mathrm{HarmonicNumber}\left[n,\frac{1}{2}+s\right]\right)\Big/\left(1+2s-2^s n^{2s}\pi^{-\frac{1}{2}+s}(1-2s)\,\mathrm{Gamma}\left[\frac{1}{2}-s\right]\left(\mathrm{Cos}\left[\frac{\pi s}{2}\right]+\mathrm{Sin}\left[\frac{\pi s}{2}\right]\right)\right)$$

FullSimplify[2^s n^(2 s) π^(-1/2+s) (1 - 2 s) Gamma[1/2 - s] (Cos[π s/2] + Sin[π s/2])]

$$2^{1+s} n^{2s}\pi^{-\frac{1}{2}+s}\,\mathrm{Gamma}\left[\frac{3}{2}-s\right]\left(\mathrm{Cos}\left[\frac{\pi s}{2}\right]+\mathrm{Sin}\left[\frac{\pi s}{2}\right]\right)$$

FullSimplify[(1 - 2 s) / (1 + 2 s)]

$$-1+\frac{2}{1+2s}$$

2^s n^(2 s) π^(-1/2+s) (-1 + 2/(1 + 2 s)) Gamma[1/2 - s] (Cos[π s/2] + Sin[π s/2])

$$2^s n^{2s}\pi^{-\frac{1}{2}+s}\left(-1+\frac{2}{1+2s}\right)\mathrm{Gamma}\left[\frac{1}{2}-s\right]\left(\mathrm{Cos}\left[\frac{\pi s}{2}\right]+\mathrm{Sin}\left[\frac{\pi s}{2}\right]\right)$$

(n^(1 - s) s HarmonicNumber[n, 1 - s] - n^s (1 - s) HarmonicNumber[n, s]) (2 n^(1 - s) s (2 π)^-s Cos[π s/2] Gamma[s] - n^s (1 - s))^-1

$$\frac{n^{1-s}\,s\,\mathrm{HarmonicNumber}[n,1-s]-n^s(1-s)\,\mathrm{HarmonicNumber}[n,s]}{-n^s(1-s)+2^{1-s}n^{1-s}\pi^{-s}s\,\mathrm{Cos}\left[\frac{\pi s}{2}\right]\mathrm{Gamma}[s]}$$

(n^(1 - s) s HarmonicNumber[n, 1 - s]) (2 n^(1 - s) s (2 π)^-s Cos[π s/2] Gamma[s] - n^s (1 - s))^-1

$$\frac{n^{1-s}\,s\,\mathrm{HarmonicNumber}[n,1-s]}{-n^s(1-s)+2^{1-s}n^{1-s}\pi^{-s}s\,\mathrm{Cos}\left[\frac{\pi s}{2}\right]\mathrm{Gamma}[s]}$$

$$\texttt{(n^(1-s) s HarmonicNumber[n, 1-s])}\left(\texttt{2 n^(1-s) s }(2\pi)^{-s}\,\texttt{Cos}\left[\frac{\pi s}{2}\right]\texttt{Gamma[s]} - n^{s}\ (1-s)\right)\texttt{^-1}$$

$$\frac{n^{1-s}\ s\ \text{HarmonicNumber}[n,\ 1-s]}{-n^{s}\ (1-s) + 2^{1-s}\, n^{1-s}\, \pi^{-s}\ s\ \text{Cos}\left[\frac{\pi s}{2}\right]\ \text{Gamma}[s]}$$

```
xi[s_] := 1 / 2 s (s - 1) Pi^(-s / 2) Gamma[ s / 2] Zeta[s]
Limit[xi[s], s → ZetaZero[1]]
```

0

$$\texttt{ps11[n\_, s\_] := (n^(1-s) s HarmonicNumber[n, 1-s]} - n^{s}\,\texttt{(1-s) HarmonicNumber[n, s])}\left(\texttt{2 n^(1-s) s }(2\pi)^{-s}\,\texttt{Cos}\left[\frac{\pi s}{2}\right]\texttt{Gamma[s]} - n^{s}\ (1-s)\right)\texttt{^-1}$$

```
Expand@ps11[n, s]
```

$$\frac{n^{1-s}\ s\ \text{HarmonicNumber}[n,\ 1-s]}{-n^{s}\ (1-s) + 2^{1-s}\, n^{1-s}\, \pi^{-s}\ s\ \text{Cos}\left[\frac{\pi s}{2}\right]\ \text{Gamma}[s]} - \frac{n^{s}\ \text{HarmonicNumber}[n,\ s]}{-n^{s}\ (1-s) + 2^{1-s}\, n^{1-s}\, \pi^{-s}\ s\ \text{Cos}\left[\frac{\pi s}{2}\right]\ \text{Gamma}[s]} + \frac{n^{s}\ s\ \text{HarmonicNumber}[n,\ s]}{-n^{s}\ (1-s) + 2^{1-s}\, n^{1-s}\, \pi^{-s}\ s\ \text{Cos}\left[\frac{\pi s}{2}\right]\ \text{Gamma}[s]}$$

$$\texttt{ps14[n\_, s\_] :=}\left(\texttt{1 - 2^(1-s) n^(1-2 s) (s / (1-s))}\ \pi^{-s}\,\texttt{Cos}\left[\frac{\pi s}{2}\right]\texttt{Gamma[s]}\right)\texttt{^-1 HarmonicNumber[n, s]} - \left(n^{(2s-1)}\ \texttt{(1-s) / s - 2^(1-s)}\ \pi^{-s}\,\texttt{Cos}\left[\frac{\pi s}{2}\right]\texttt{Gamma[s]}\right)\texttt{^-1 HarmonicNumber[n, 1-s]}$$

$$\texttt{ps14a[n\_, s\_] :=}\ \left\{\left(\texttt{1 - 2^(1-s) n^(1-2 s) (s / (1-s))}\ \pi^{-s}\,\texttt{Cos}\left[\frac{\pi s}{2}\right]\texttt{Gamma[s]}\right)\texttt{^-1, HarmonicNumber[n, s],} -\left(n^{(2s-1)}\ \texttt{(1-s) / s - 2^(1-s)}\ \pi^{-s}\,\texttt{Cos}\left[\frac{\pi s}{2}\right]\texttt{Gamma[s]}\right)\texttt{^-1, HarmonicNumber[n, 1-s]}\right\}$$

```
N@ps14a[100 000 000 000, 2.0]
```

$\{1.,\ 1.64493,\ 2.\times 10^{-33},\ 5.\times 10^{21}\}$

```
Cos[Pi .5 / 2]
```

0.707107

$$\texttt{Expand}\left[\left(\texttt{1 - 2 n^(1-2 s) (s / (1-s))}\ (2\pi)^{-s}\,\texttt{Cos}\left[\frac{\pi s}{2}\right]\texttt{Gamma[s]}\right)\texttt{^-1}\right]\ \texttt{/. s → .5}$$

-4.5036×10^{15}

```
ps14[100 000, 2.0]
```

1.64493

```
Zeta[.3]
```

-0.904559

```
(n^(2 s-1) (1 - s) / s - 2 (2 π)^-s Cos[π s/2] Gamma[s])^-1
```

$$\frac{1}{\frac{n^{-1+2 s}\,(1-s)}{s}-2^{1-s}\,\pi^{-s}\,\text{Cos}\left[\frac{\pi s}{2}\right]\,\text{Gamma}[s]}$$

```
ps15[n_, s_] := Sum[ ((1 - 2^(1 - s) n^(1 - 2 s) (s / (1 - s)) π^-s Cos[π s/2] Gamma[s])^-1) j^-s -
  ((n^(2 s-1) (1 - s) / s - 2^(1 - s) π^-s Cos[π s/2] Gamma[s])^-1) j^(s - 1), {j, 1, n}]
```

```
ps15[10 000, .5 + I]
```

0.14191 − 0.711931 i

```
Zeta[.5 + I]
```

0.143936 − 0.7221 i

```
((1 - 2^(1 - s) n^(1 - 2 s) (s / (1 - s)) π^-s Cos[π s/2] Gamma[s])^-1) j^-s -
 ((n^(2 s-1) (1 - s) / s - 2^(1 - s) π^-s Cos[π s/2] Gamma[s])^-1) j^(s - 1)
```

$$-\frac{j^{-1+s}}{\frac{n^{-1+2 s}\,(1-s)}{s}-2^{1-s}\,\pi^{-s}\,\text{Cos}\left[\frac{\pi s}{2}\right]\,\text{Gamma}[s]}+\frac{j^{-s}}{1-\frac{2^{1-s}\,n^{1-2 s}\,\pi^{-s}\,s\,\text{Cos}\left[\frac{\pi s}{2}\right]\,\text{Gamma}[s]}{1-s}}$$

```
ps11[n_, s_] := (n^(1 - s) s HarmonicNumber[n, 1 - s] - n^s (1 - s) HarmonicNumber[n, s])
  (2 n^(1 - s) s (2 π)^-s Cos[π s/2] Gamma[s] - n^s (1 - s))^-1
ps11a[n_, s_] := Sum[(n^(1 - s) s j^(s - 1) - n^s (1 - s) j^-s)
  (2 n^(1 - s) s (2 π)^-s Cos[π s/2] Gamma[s] - n^s (1 - s))^-1, {j, 1, n}]
```

```
ps11a[10 000, .5 + I]
```

0.14191 − 0.711931 i

```
qs9[n_, s_] :=
 ((1 + 2 s) n^-s HarmonicNumber[n, 1/2 - s] + n^s (-1 + 2 s) HarmonicNumber[n, 1/2 + s]) /
  ((1 + 2 s) n^-s - 2^s n^s π^(-1/2+s) (1 - 2 s) Gamma[1/2 - s] (Cos[π s/2] + Sin[π s/2]))
qt9[n_, s_] := qs9[n, 1 / 2 - s]
qs9a[n_, s_] := Sum[((1 + 2 s) n^-s j^(-1/2 + s) + n^s (-1 + 2 s) j^(-1/2 - s)) /
   ((1 + 2 s) n^-s - 2^s n^s π^(-1/2+s) (1 - 2 s) Gamma[1/2 - s] (Cos[π s/2] + Sin[π s/2])), {j, 1, n}]
qt9a[n_, s_] := qs9a[n, 1 / 2 - s]
qs9b[n_, s_] := Sum[1 / j^(1 / 2) ((1 + 2 s) (j / n)^s + (-1 + 2 s) (j / n)^-s) /
    ((1 + 2 s) n^-s - 2^s n^s π^(-1/2+s) (1 - 2 s) Gamma[1/2 - s] (Cos[π s/2] + Sin[π s/2])), {j, 1, n}]
qt9b[n_, s_] := qs9b[n, 1 / 2 - s]
qs9c[n_, s_] := Sum[1 / j^(1 / 2) (2 s ((j / n)^s + (j / n)^-s) + ((j / n)^s - (j / n)^-s)) /
    ((1 + 2 s) n^-s - 2^s n^s π^(-1/2+s) (1 - 2 s) Gamma[1/2 - s] (Cos[π s/2] + Sin[π s/2])), {j, 1, n}]
qt9c[n_, s_] := qs9c[n, 1 / 2 - s]
qs9d[n_, s_] := Sum[1 / j^(1 / 2) (4 s Cosh[s Log[j / n]] + 2 Sinh[s Log[j / n]]) /
    ((1 + 2 s) n^-s - 2^s n^s π^(-1/2+s) (1 - 2 s) Gamma[1/2 - s] (Cos[π s/2] + Sin[π s/2])), {j, 1, n}]
qt9d[n_, s_] := qs9d[n, 1 / 2 - s]

qt9d[10 000, -.5]
-0.207886
```

```
qs9ca[n_, s_] := Sum[1 / j^(1 / 2) (2 s ((j / n)^s + (j / n)^-s) + ((j / n)^s - (j / n)^-s)) /
    ((1 + 2 s) E^(-s Log[n]) - (1 - 2 s) E^(s Log[2]) E^(s Log[n]) E^((-1/2 + s) Log[π]) Gamma[1/2 - s]
      (1 / 2 (E^(π s/2 I) + E^(-π s/2 I)) + 1 / (2 I) (E^(π s/2 I) - E^(-π s/2 I)))), {j, 1, n}]
qt9ca[n_, s_] := qs9ca[n, 1 / 2 - s]
qs9cb[n_, s_] := Sum[1 / j^(1 / 2) (2 s ((j / n)^s + (j / n)^-s) + ((j / n)^s - (j / n)^-s)) /
    ((1 + 2 s) E^(-s Log[n]) - (1 - 2 s) E^(s Log[n]) E^(s Log[2]) E^((-1/2 + s) Log[π]) Gamma[1/2 - s]
      (1 / 2 (E^(π s/2 I) + E^(-π s/2 I)) + 1 / (2 I) (E^(π s/2 I) - E^(-π s/2 I)))), {j, 1, n}]
qt9cb[n_, s_] := qs9cb[n, 1 / 2 - s]
qs9cc[n_, s_] := Sum[1 / j^(1 / 2) (2 s ((j / n)^s + (j / n)^-s) + ((j / n)^s - (j / n)^-s)) /
    ((1 + 2 s) E^(-s Log[n]) - (1 - 2 s) E^(s Log[n]) E^(s Log[2]) E^((-1/2 + s) Log[π]) Gamma[1/2 - s]
      (1 / 2 (E^(π s/2 I) + E^(-π s/2 I)) + 1 / (2 I) (E^(π s/2 I) - E^(-π s/2 I)))), {j, 1, n}]
qt9cc[n_, s_] := qs9cc[n, 1 / 2 - s]
```

```
qt9cc[10 000, -.5]
```

$-0.207886 + 0.\, i$

```
Expand[(1 + 2 s) E^(-s Log[n]) - (1 - 2 s) E^(s Log[n]) E^(s Log[2]) E^((-1/2 + s) Log[π])
    Gamma[1/2 - s] (1 / 2 (E^(π s/2 I) + E^(-π s/2 I)) + 1 / (2 I) (E^(π s/2 I) - E^(-π s/2 I)))]
```

$$n^{-s} + 2\, n^{-s}\, s - (1+i)\, 2^{-1+s}\, e^{-\frac{1}{2} i \pi s}\, n^s\, \pi^{-\frac{1}{2}+s}\, \mathrm{Gamma}\left[\frac{1}{2} - s\right] - (1-i)\, 2^{-1+s}\, e^{\frac{i \pi s}{2}}\, n^s\, \pi^{-\frac{1}{2}+s}\, \mathrm{Gamma}\left[\frac{1}{2} - s\right] +$$
$$(1+i)\, 2^s\, e^{-\frac{1}{2} i \pi s}\, n^s\, \pi^{-\frac{1}{2}+s}\, s\, \mathrm{Gamma}\left[\frac{1}{2} - s\right] + (1-i)\, 2^s\, e^{\frac{i \pi s}{2}}\, n^s\, \pi^{-\frac{1}{2}+s}\, s\, \mathrm{Gamma}\left[\frac{1}{2} - s\right]$$

```
ps11[n_, s_] := (n^(1 - s) s HarmonicNumber[n, 1 - s] - n^s (1 - s) HarmonicNumber[n, s]) /
  (2^(1 - s) n^(1 - s) s π^-s Cos[π s/2] Gamma[s] - n^s (1 - s))
ps14[n_, s_] := HarmonicNumber[n, s]
   (1 - 2 n^(1 - 2 s) (s / (1 - s)) (2 π)^-s Cos[π s/2] Gamma[s])^-1 -
  HarmonicNumber[n, 1 - s] (n^(2 s-1) (1 - s) / s - 2 (2 π)^-s Cos[π s/2] Gamma[s])^-1
ps14a1[n_, s_] := HarmonicNumber[n, s]
  (1 - 2 n^(1 - 2 s) (s / (1 - s)) (2 π)^-s Cos[π s/2] Gamma[s])^-1
ps14a2[n_, s_] := HarmonicNumber[n, 1 - s] (n^(2 s-1) (1 - s) / s - 2 (2 π)^-s Cos[π s/2] Gamma[s])^-1
```

```
ps14[n, 1 - s]
```

$$\frac{\text{HarmonicNumber}[n,1-s]}{1-\frac{2^s n^{1-2(1-s)}\pi^{-1+s}(1-s)\,\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]}{s}}-\frac{\text{HarmonicNumber}[n,s]}{\frac{n^{-1+2(1-s)}s}{1-s}-2^s\pi^{-1+s}\,\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]}$$

```
Expand[1 - 2 (1 - s)]
```

$$-1+2s$$

```
Expand[-1 + 2 (1 - s)]
```

$$1-2s$$

```
FullSimplify[ps14[n, s] + ps14[n, 1 - s]]
```

$$\left(\left(n\,s\,\text{HarmonicNumber}[n,1-s]+n^{2s}(-1+s)\,\text{HarmonicNumber}[n,s]\right)\left(n\pi\left((2\pi)^s s+2\,\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[1+s]\right)-n^{2s}(2\pi)^s\left(\pi-\pi s+(2\pi)^s\,\text{Gamma}[2-s]\,\text{Sin}\left[\frac{\pi s}{2}\right]\right)\right)\right)\Big/\left(\left(n^{2s}(2\pi)^s(-1+s)+2\,n\,s\,\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]\right)\left(n\pi s-n^{2s}(2\pi)^s\,\text{Gamma}[2-s]\,\text{Sin}\left[\frac{\pi s}{2}\right]\right)\right)$$

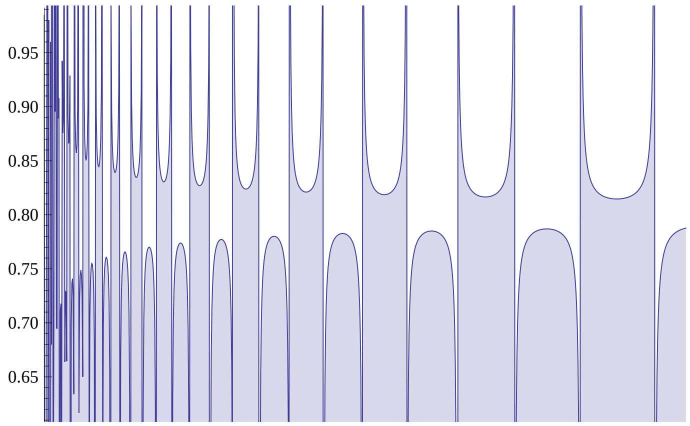
```
ps14[n, s] - ps14[n, 1 - s]
```

$$-\frac{\text{HarmonicNumber}[n,1-s]}{1-\frac{2^s n^{1-2(1-s)}\pi^{-1+s}(1-s)\,\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]}{s}}-\frac{\text{HarmonicNumber}[n,1-s]}{\frac{n^{-1+2s}(1-s)}{s}-2^{1-s}\pi^{-s}\,\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]}+\frac{\text{HarmonicNumber}[n,s]}{\frac{n^{-1+2(1-s)}s}{1-s}-2^s\pi^{-1+s}\,\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]}+\frac{\text{HarmonicNumber}[n,s]}{1-\frac{2^{1-s}n^{1-2s}\pi^{-s}s\,\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]}{1-s}}$$

```
ps14[n, s] ps14[n, 1 - s]
```

$$\left(\frac{\text{HarmonicNumber}[n,1-s]}{1-\frac{2^s n^{1-2(1-s)}\pi^{-1+s}(1-s)\,\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]}{s}}-\frac{\text{HarmonicNumber}[n,s]}{\frac{n^{-1+2(1-s)}s}{1-s}-2^s\pi^{-1+s}\,\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]}\right)$$
$$\left(-\frac{\text{HarmonicNumber}[n,1-s]}{\frac{n^{-1+2s}(1-s)}{s}-2^{1-s}\pi^{-s}\,\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]}+\frac{\text{HarmonicNumber}[n,s]}{1-\frac{2^{1-s}n^{1-2s}\pi^{-s}s\,\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]}{1-s}}\right)$$

```
DiscretePlot[{Re@ps14[n, .5 + 24.14 I]}, {n, 1, 1000}]
```




```
Zeta[.8 + 1.14 I]
```

0.419338 − 0.765218 ⅈ

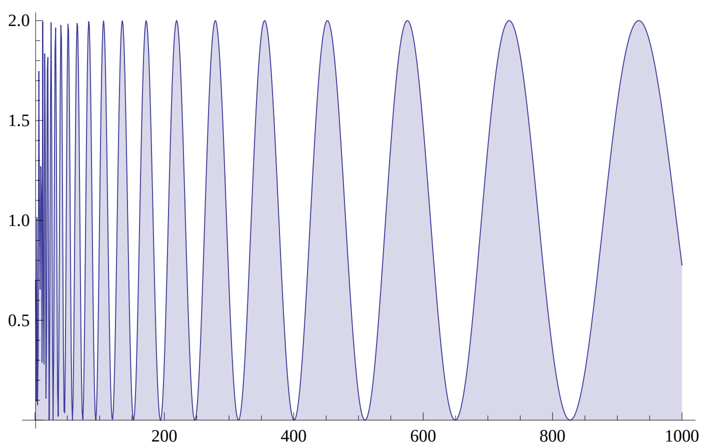
```
D[ps14[n, s], n]
```

$$\frac{n^{-2+2 s} (1-s) (-1+2 s) \text{HarmonicNumber}[n, 1-s]}{s \left(\frac{n^{-1+2 s} (1-s)}{s} - 2^{1-s} \pi^{-s} \text{Cos}\left[\frac{\pi s}{2}\right] \text{Gamma}[s]\right)^2} + \frac{2^{1-s} n^{-2 s} \pi^{-s} (1-2 s) s \text{Cos}\left[\frac{\pi s}{2}\right] \text{Gamma}[s] \text{HarmonicNumber}[n, s]}{(1-s) \left(1 - \frac{2^{1-s} n^{1-2 s} \pi^{-s} s \text{Cos}\left[\frac{\pi s}{2}\right] \text{Gamma}[s]}{1-s}\right)^2} - \frac{(1-s) (-\text{HarmonicNumber}[n, 2-s] + \text{Zeta}[2-s])}{\frac{n^{-1+2 s} (1-s)}{s} - 2^{1-s} \pi^{-s} \text{Cos}\left[\frac{\pi s}{2}\right] \text{Gamma}[s]} + \frac{s (-\text{HarmonicNumber}[n, 1+s] + \text{Zeta}[1+s])}{1 - \frac{2^{1-s} n^{1-2 s} \pi^{-s} s \text{Cos}\left[\frac{\pi s}{2}\right] \text{Gamma}[s]}{1-s}}$$

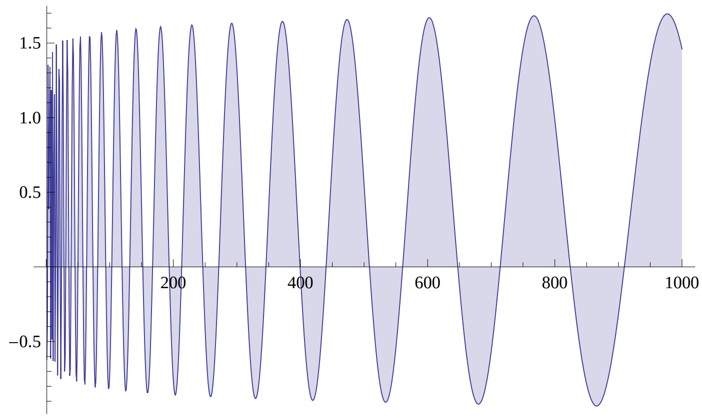
$$\text{px}[n\_, s\_] := 1 - 2 n\verb|^|(1 - 2 s) (s / (1 - s)) (2 \pi)^{-s} \text{Cos}\left[\frac{\pi s}{2}\right] \text{Gamma}[s]$$

$$\text{px2}[n\_, s\_] := n^{(2 s - 1)} (1 - s) / s - 2 (2 \pi)^{-s} \text{Cos}\left[\frac{\pi s}{2}\right] \text{Gamma}[s]$$

```
DiscretePlot[Re@px[n, .5 + 13 I], {n, 1, 1000}]
```



```
DiscretePlot[Re@px2[n, .52 + 13 I], {n, 1, 1000}]
```



$$\text{ps20}[n\_, s\_] := \text{HarmonicNumber}[n, s]\ 1/2\ s\ (s-1)\ \text{Pi}\char`^(-s/2)\ \text{Gamma}[s/2] \Big/ \left(1 - 2\ n\char`^(1-2\ s)\ (s/(1-s))\ (2\ \pi)^{-s}\ \text{Cos}\left[\frac{\pi\ s}{2}\right]\ \text{Gamma}[s]\right) - \text{HarmonicNumber}[n, 1-s]\ 1/2\ s\ (s-1)\ \text{Pi}\char`^(-s/2)\ \text{Gamma}[s/2] \Big/ \left(n^{(2\ s-1)}\ (1-s)/s - 2\ (2\ \pi)^{-s}\ \text{Cos}\left[\frac{\pi\ s}{2}\right]\ \text{Gamma}[s]\right)$$

$$\text{ps21}[n\_, s\_] := \text{HarmonicNumber}[n, s]\ 1/2\ \text{Pi}\char`^(-s/2)\ \text{Gamma}[s/2] \Big/ \left(1/(s\ (s-1)) + 2\ n\char`^(1-2\ s)\ (1/(1-s))\ (2\ \pi)^{-s}\ \text{Cos}\left[\frac{\pi\ s}{2}\right]\ \text{Gamma}[s]\right) - \text{HarmonicNumber}[n, 1-s]\ 1/2\ s\ (s-1)\ \text{Pi}\char`^(-s/2)\ \text{Gamma}[s/2] \Big/ \left(n^{(2\ s-1)}\ (1-s)/s - 2\ (2\ \pi)^{-s}\ \text{Cos}\left[\frac{\pi\ s}{2}\right]\ \text{Gamma}[s]\right)$$

```
xi[s_] := 1 / 2 s (s - 1) Pi^(-s / 2) Gamma[s / 2] Zeta[s]
N@xi[2]
0.523599
ps21[10 000 000, 2.0]
0.523599
```

$$\text{ps30}[n\_, s\_] := \text{HarmonicNumber}[n, s]\ \left(1 - 2\ n\char`^(1-2\ s)\ (s/(1-s))\ (2\ \pi)^{-s}\ \text{Cos}\left[\frac{\pi\ s}{2}\right]\ \text{Gamma}[s]\right)\char`^-1 - \text{HarmonicNumber}[n, 1-s]\ \left(n^{(2\ s-1)}\ (1-s)/s - 2\ (2\ \pi)^{-s}\ \text{Cos}\left[\frac{\pi\ s}{2}\right]\ \text{Gamma}[s]\right)\char`^-1$$

$$\text{ps31}[n\_, s\_] := \frac{\text{HarmonicNumber}[n, 1-s]}{1 - \frac{2^{s}\ n^{1-2\ (1-s)}\ \pi^{-1+s}\ (1-s)\ \text{Cos}\left[\frac{1}{2}\ \pi\ (1-s)\right]\ \text{Gamma}[1-s]}{s}} - \frac{\text{HarmonicNumber}[n, s]}{\frac{n^{-1+2\ (1-s)}\ s}{1-s} - 2^{s}\ \pi^{-1+s}\ \text{Cos}\left[\frac{1}{2}\ \pi\ (1-s)\right]\ \text{Gamma}[1-s]}$$

$$\text{ps32}[n\_, s\_] := -\frac{\text{HarmonicNumber}[n, 1-s]}{\frac{n^{-1+2\ s}\ (1-s)}{s} - 2^{1-s}\ \pi^{-s}\ \text{Cos}\left[\frac{\pi\ s}{2}\right]\ \text{Gamma}[s]} + \frac{\text{HarmonicNumber}[n, s]}{1 - \frac{2^{1-s}\ n^{1-2\ s}\ \pi^{-s}\ s\ \text{Cos}\left[\frac{\pi\ s}{2}\right]\ \text{Gamma}[s]}{1-s}}$$

$$\text{ps33}[n\_, s\_] := \left(\frac{1}{1-\frac{2^{s} n^{1-2(1-s)} \pi^{-1+s} (1-s) \text{Cos}\left[\frac{1}{2}\pi(1-s)\right] \text{Gamma}[1-s]}{s}} - \frac{1}{\frac{n^{-1+2s}(1-s)}{s} - 2^{1-s}\pi^{-s}\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]}\right)$$
$$\text{HarmonicNumber}[n, 1-s] + \left(\frac{1}{1-\frac{2^{1-s} n^{1-2s} \pi^{-s} s \,\text{Cos}\left[\frac{\pi s}{2}\right] \text{Gamma}[s]}{1-s}} - \frac{1}{\frac{n^{-1+2(1-s)} s}{1-s} - 2^{s}\pi^{-1+s}\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]}\right) \text{HarmonicNumber}[n, s]$$

$$\text{ps33a}[n\_, s\_] := \left\{\left(\frac{1}{1-\frac{2^{s} n^{1-2(1-s)} \pi^{-1+s} (1-s) \text{Cos}\left[\frac{1}{2}\pi(1-s)\right] \text{Gamma}[1-s]}{s}} - \frac{1}{\frac{n^{-1+2s}(1-s)}{s} - 2^{1-s}\pi^{-s}\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]}\right)\right.,$$
$$\text{HarmonicNumber}[n, 1-s],$$
$$\left(\frac{1}{1-\frac{2^{1-s} n^{1-2s} \pi^{-s} s \,\text{Cos}\left[\frac{\pi s}{2}\right] \text{Gamma}[s]}{1-s}} - \frac{1}{\frac{n^{-1+2(1-s)} s}{1-s} - 2^{s}\pi^{-1+s}\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]}\right),$$
$$\left.\text{HarmonicNumber}[n, s]\right\}$$

$$\text{ps33b}[n\_, s\_] := \left\{\left(\frac{1}{1-\frac{2^{s} n^{1-2(1-s)} \pi^{-1+s} (1-s) \text{Cos}\left[\frac{1}{2}\pi(1-s)\right] \text{Gamma}[1-s]}{s}} - \frac{1}{\frac{n^{-1+2s}(1-s)}{s} - 2^{1-s}\pi^{-s}\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]}\right)\right.$$
$$\text{HarmonicNumber}[n, 1-s],$$
$$\left(\frac{1}{1-\frac{2^{1-s} n^{1-2s} \pi^{-s} s \,\text{Cos}\left[\frac{\pi s}{2}\right] \text{Gamma}[s]}{1-s}} - \frac{1}{\frac{n^{-1+2(1-s)} s}{1-s} - 2^{s}\pi^{-1+s}\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]}\right)$$
$$\left.\text{HarmonicNumber}[n, s]\right\}$$

$$\text{ps35}[n\_, s\_] := \left(\frac{1}{1-\frac{2^{s} n^{1-2(1-s)} \pi^{-1+s} (1-s) \text{Cos}\left[\frac{1}{2}\pi(1-s)\right] \text{Gamma}[1-s]}{s}} - \frac{1}{\frac{n^{-1+2s}(1-s)}{s} - 2^{1-s}\pi^{-s}\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]}\right) \Big/$$
$$\left(\frac{1}{1-\frac{2^{1-s} n^{1-2s} \pi^{-s} s \,\text{Cos}\left[\frac{\pi s}{2}\right] \text{Gamma}[s]}{1-s}} - \frac{1}{\frac{n^{-1+2(1-s)} s}{1-s} - 2^{s}\pi^{-1+s}\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]}\right)$$

$$\text{ps36}[n\_, s\_] := \left(\frac{1}{1-\frac{2^{1-s} n^{1-2s} \pi^{-s} s \,\text{Cos}\left[\frac{\pi s}{2}\right] \text{Gamma}[s]}{1-s}} - \frac{1}{\frac{n^{-1+2(1-s)} s}{1-s} - 2^{s}\pi^{-1+s}\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]}\right)$$
$$\left(\frac{n^{1-2s}\, s}{-1+s}\text{HarmonicNumber}[n, 1-s] + \text{HarmonicNumber}[n, s]\right)$$

$$\text{ps37}[n\_, s\_] := \left(\frac{1}{1 - \frac{2^{1-s} n^{1-2s} \pi^{-s} s \operatorname{Cos}\left[\frac{\pi s}{2}\right] \operatorname{Gamma}[s]}{1-s}} - \frac{1}{\frac{n^{-1+2(1-s)} s}{1-s} - 2^s \pi^{-1+s} \operatorname{Cos}\left[\frac{1}{2}\pi(1-s)\right] \operatorname{Gamma}[1-s]}\right)$$
$$(1/(s-1)\, n\verb|^|-s)\left(n^{1-s} s \operatorname{HarmonicNumber}[n, 1-s] + (s-1)\, n\verb|^|s \operatorname{HarmonicNumber}[n, s]\right)$$

$$\text{ps38}[n\_, s\_] := \left(-\frac{1}{(1-s) - 2^{1-s} n^{1-2s} \pi^{-s} s \operatorname{Cos}\left[\frac{\pi s}{2}\right] \operatorname{Gamma}[s]} + \frac{1}{n^{-1+2(1-s)} s - (1-s)\, 2^s \pi^{-1+s} \operatorname{Cos}\left[\frac{1}{2}\pi(1-s)\right] \operatorname{Gamma}[1-s]}\right)$$
$$(n\verb|^|-s)\left(n^{1-s} s \operatorname{HarmonicNumber}[n, 1-s] + (s-1)\, n\verb|^|s \operatorname{HarmonicNumber}[n, s]\right)$$

$$\text{ps39}[n\_, s\_] := \left(\frac{1}{s\, n^{(1-s)} - (1-s)\, n\verb|^|s\, 2^s \pi^{-1+s} \operatorname{Cos}\left[\frac{1}{2}\pi(1-s)\right] \operatorname{Gamma}[1-s]} - \frac{1}{(1-s)\, n\verb|^|s - s\, n^{1-s}\, 2^{1-s} \pi^{-s} \operatorname{Cos}\left[\frac{\pi s}{2}\right] \operatorname{Gamma}[s]}\right)$$
$$\left(n^{1-s} s \operatorname{HarmonicNumber}[n, 1-s] - (1-s)\, n\verb|^|s \operatorname{HarmonicNumber}[n, s]\right)$$

$$\text{ps39a}[n\_, s\_] := n^{1-s} s \operatorname{HarmonicNumber}[n, 1-s] - (1-s)\, n\verb|^|s \operatorname{HarmonicNumber}[n, s]$$

$$\text{ps39a2}[n\_, s\_] := \operatorname{Sum}\left[n^{1-s} s / j\verb|^|(1-s) - (1-s)\, n\verb|^|s / (j\verb|^|s), \{j, 1, n\}\right]$$

$$\text{ps39b}[n\_, s\_] := \left(\frac{1}{s\, n^{(1-s)} - (1-s)\, n\verb|^|s\, 2^s \pi^{-1+s} \operatorname{Cos}\left[\frac{1}{2}\pi(1-s)\right] \operatorname{Gamma}[1-s]} - \frac{1}{(1-s)\, n\verb|^|s - s\, n^{1-s}\, 2^{1-s} \pi^{-s} \operatorname{Cos}\left[\frac{\pi s}{2}\right] \operatorname{Gamma}[s]}\right)$$

ps39[100 000 000, .3 + 13 I]

0.880295 - 0.227194 i

Zeta[.3 + 13 I] + Zeta[.7 - 13 I]

0.880292 - 0.227193 i

ps39[100 000 000, .3 + 13 I]

0.880295 - 0.227194 i

Zeta[.3 + 13 I] - Zeta[.7 - 13 I]

-0.129108 - 1.33067 i

ps32[1 000 000, .3 + 13 I] - ps31[1 000 000, .3 + 13 I]

-0.129069 - 1.33057 i

ps44[1 000 000, .3 + 13 I]

-0.129069 - 1.33057 i

Chop@ps39a[1000, N@ZetaZero@5]

0. + 32.9351 i

```
N@Im@ZetaZero@5
```

32.9351

```
Zeta[.3 + 13 I]
```

0.375592 − 0.778932 i

```
FullSimplify[ps43a[n, s]]
```

$$\frac{n^{-1+2s}(-1+s)}{s}$$

$$\text{ps40}[n\_, s\_] := \left(-\frac{\text{HarmonicNumber}[n, 1-s]}{\frac{n^{-1+2s}(1-s)}{s} - 2^{1-s}\pi^{-s}\,\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]} + \frac{\text{HarmonicNumber}[n, s]}{1 - \frac{2^{1-s} n^{1-2s}\pi^{-s} s\,\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]}{1-s}}\right) - \left(\frac{\text{HarmonicNumber}[n, 1-s]}{1 - \frac{2^{s} n^{1-2(1-s)}\pi^{-1+s}(1-s)\,\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]}{s}} - \frac{\text{HarmonicNumber}[n, s]}{\frac{n^{-1+2(1-s)} s}{1-s} - 2^{s}\pi^{-1+s}\,\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]}\right)$$

$$\text{ps41}[n\_, s\_] := -\frac{\text{HarmonicNumber}[n, 1-s]}{\frac{n^{-1+2s}(1-s)}{s} - 2^{1-s}\pi^{-s}\,\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]} + \frac{\text{HarmonicNumber}[n, s]}{1 - \frac{2^{1-s} n^{1-2s}\pi^{-s} s\,\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]}{1-s}} - \frac{\text{HarmonicNumber}[n, 1-s]}{1 - \frac{2^{s} n^{1-2(1-s)}\pi^{-1+s}(1-s)\,\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]}{s}} + \frac{\text{HarmonicNumber}[n, s]}{\frac{n^{-1+2(1-s)} s}{1-s} - 2^{s}\pi^{-1+s}\,\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]}$$

$$\text{ps42}[n\_, s\_] := \left(\frac{1}{1 - \frac{2^{1-s} n^{1-2s}\pi^{-s} s\,\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]}{1-s}} + \frac{1}{\frac{n^{-1+2(1-s)} s}{1-s} - 2^{s}\pi^{-1+s}\,\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]}\right)\text{HarmonicNumber}[n, s] - \left(\frac{1}{\frac{n^{-1+2s}(1-s)}{s} - 2^{1-s}\pi^{-s}\,\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]} + \frac{1}{1 - \frac{2^{s} n^{1-2(1-s)}\pi^{-1+s}(1-s)\,\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]}{s}}\right)\text{HarmonicNumber}[n, 1-s]$$

$$\text{ps43}[n\_, s\_] := \left(\frac{1}{1 - \frac{2^{1-s} n^{1-2s}\pi^{-s} s\,\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]}{1-s}} + \frac{1}{\frac{n^{-1+2(1-s)} s}{1-s} - 2^{s}\pi^{-1+s}\,\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]}\right)\text{HarmonicNumber}[n, s] - \left(\frac{1}{\frac{n^{-1+2s}(1-s)}{s} - 2^{1-s}\pi^{-s}\,\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]} + \frac{1}{1 - \frac{2^{s} n^{1-2(1-s)}\pi^{-1+s}(1-s)\,\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]}{s}}\right)\text{HarmonicNumber}[n, 1-s]$$

$$\text{ps43a}[n\_, s\_] := \left(\frac{1}{1-\frac{2^{1-s}\, n^{1-2\,s}\, \pi^{-s}\, s\, \text{Cos}\left[\frac{\pi s}{2}\right] \text{Gamma}[s]}{1-s}} + \frac{1}{\frac{n^{-1+2\,(1-s)}\, s}{1-s} - 2^{s}\, \pi^{-1+s}\, \text{Cos}\left[\frac{1}{2}\,\pi\,(1-s)\right] \text{Gamma}[1-s]}\right) \Big/$$

$$-\left(\frac{1}{\frac{n^{-1+2\,s}\,(1-s)}{s} - 2^{1-s}\, \pi^{-s}\, \text{Cos}\left[\frac{\pi s}{2}\right] \text{Gamma}[s]} + \frac{1}{1-\frac{2^{s}\, n^{1-2\,(1-s)}\, \pi^{-1+s}\,(1-s)\, \text{Cos}\left[\frac{1}{2}\,\pi\,(1-s)\right] \text{Gamma}[1-s]}{s}}\right)$$

$$\text{ps44}[n\_, s\_] := \left(\frac{1}{\frac{n^{-1+2\,s}\,(1-s)}{s} - 2^{1-s}\, \pi^{-s}\, \text{Cos}\left[\frac{\pi s}{2}\right] \text{Gamma}[s]} + \frac{1}{1-\frac{2^{s}\, n^{1-2\,(1-s)}\, \pi^{-1+s}\,(1-s)\, \text{Cos}\left[\frac{1}{2}\,\pi\,(1-s)\right] \text{Gamma}[1-s]}{s}}\right)$$

$$(1/s)\left(n^{-1+2\,s}\,(1-s)\, \text{HarmonicNumber}[n, s] - s\, \text{HarmonicNumber}[n, 1-s]\right)$$

$$\text{ps45}[n\_, s\_] := \left(\frac{1}{\frac{n^{-1+2\,s}\,(1-s)}{s} - 2^{1-s}\, \pi^{-s}\, \text{Cos}\left[\frac{\pi s}{2}\right] \text{Gamma}[s]} + \frac{1}{1-\frac{2^{s}\, n^{1-2\,(1-s)}\, \pi^{-1+s}\,(1-s)\, \text{Cos}\left[\frac{1}{2}\,\pi\,(1-s)\right] \text{Gamma}[1-s]}{s}}\right)$$

$$(1/s)\, n\hat{\ }s \left(n^{-1+s}\,(1-s)\, \text{HarmonicNumber}[n, s] - s\, n\hat{\ }{-s}\, \text{HarmonicNumber}[n, 1-s]\right)$$

$$\text{ps46}[n\_, s\_] := \left(\frac{1}{(1-s)\, n^{s-1} - s\, n\hat{\ }{-s}\, 2^{1-s}\, \pi^{-s}\, \text{Cos}\left[\frac{\pi s}{2}\right] \text{Gamma}[s]} + \frac{1}{s\, n\hat{\ }{-s} - (1-s)\, n^{s-1}\, 2^{s}\, \pi^{-1+s}\, \text{Cos}\left[\frac{1}{2}\,\pi\,(1-s)\right] \text{Gamma}[1-s]}\right)$$

$$\left((1-s)\, n^{s-1}\, \text{HarmonicNumber}[n, s] - s\, n\hat{\ }{-s}\, \text{HarmonicNumber}[n, 1-s]\right)$$

$$\text{ps46a}[n\_, s\_] := \left((1-s)\, n^{s-1}\, \text{HarmonicNumber}[n, s] - s\, n\hat{\ }{-s}\, \text{HarmonicNumber}[n, 1-s]\right)$$

Zeta[.4 + 17 I] - Zeta[1 - (.4 + 17 I)]

0.157956 + 1.79925 i

ps46[1 000 000, .4 + 17 I]

0.157881 + 1.79905 i

ps46a[1000, N@ZetaZero@5] * -1000

0. + 32.9351 i

N@Im@ZetaZero@5

32.9351

FullSimplify$\left[n^{1-2\,(1-s)}\, n\hat{\ }{-s}\right]$

n^{-1+s}

ps50[n_, s_] :=

$$\left(\frac{\text{HarmonicNumber}[n, 1-s]}{1-\frac{2^{s}\, n^{1-2\,(1-s)}\, \pi^{-1+s}\,(1-s)\, \text{Cos}\left[\frac{1}{2}\,\pi\,(1-s)\right] \text{Gamma}[1-s]}{s}} - \frac{\text{HarmonicNumber}[n, s]}{\frac{n^{-1+2\,(1-s)}\, s}{1-s} - 2^{s}\, \pi^{-1+s}\, \text{Cos}\left[\frac{1}{2}\,\pi\,(1-s)\right] \text{Gamma}[1-s]}\right)$$

$$\left(-\frac{\text{HarmonicNumber}[n,1-s]}{\frac{n^{-1+2s}(1-s)}{s}-2^{1-s}\pi^{-s}\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]}+\frac{\text{HarmonicNumber}[n,s]}{1-\frac{2^{1-s}n^{1-2s}\pi^{-s}s\,\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]}{1-s}}\right)$$

$$\text{ps51}[n\_,s\_]:=-\text{HarmonicNumber}[n,1-s]^2\Big/\left(\left(1-\frac{2^s n^{1-2(1-s)}\pi^{-1+s}(1-s)\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]}{s}\right)\left(\frac{n^{-1+2s}(1-s)}{s}-2^{1-s}\pi^{-s}\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]\right)\right)+$$
$$(2\,\text{HarmonicNumber}[n,1-s]\,\text{HarmonicNumber}[n,s])\Big/\left(\left(\frac{n^{-1+2(1-s)}s}{1-s}-2^s\pi^{-1+s}\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]\right)\left(\frac{n^{-1+2s}(1-s)}{s}-2^{1-s}\pi^{-s}\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]\right)\right)-\text{HarmonicNumber}[n,s]^2\Big/$$
$$\left(\left(\frac{n^{-1+2(1-s)}s}{1-s}-2^s\pi^{-1+s}\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]\right)\left(1-\frac{2^{1-s}n^{1-2s}\pi^{-s}s\,\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]}{1-s}\right)\right)$$

$$\text{ps52}[n\_,s\_]:=(2\,\text{HarmonicNumber}[n,1-s]\,\text{HarmonicNumber}[n,s])\Big/\left(\left(\frac{n^{-1+2(1-s)}s}{1-s}-2^s\pi^{-1+s}\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]\right)\left(\frac{n^{-1+2s}(1-s)}{s}-2^{1-s}\pi^{-s}\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]\right)\right)-$$
$$\left(\text{HarmonicNumber}[n,1-s]^2+\frac{n^{-2+4s}(-1+s)^2}{s^2}\text{HarmonicNumber}[n,s]^2\right)\Big/\left(\left(1-\frac{2^s n^{1-2(1-s)}\pi^{-1+s}(1-s)\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]}{s}\right)\left(\frac{n^{-1+2s}(1-s)}{s}-2^{1-s}\pi^{-s}\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]\right)\right)$$

$$\text{ps53}[n\_,s\_]:=(2\,\text{HarmonicNumber}[n,1-s]\,\text{HarmonicNumber}[n,s])\Big/\left(\left(\frac{n^{-1+2(1-s)}s}{1-s}-2^s\pi^{-1+s}\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]\right)\left(\frac{n^{-1+2s}(1-s)}{s}-2^{1-s}\pi^{-s}\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]\right)\right)-$$
$$\left(s\hat{\ }2\,\text{HarmonicNumber}[n,1-s]^2+n^{-2+4s}(-1+s)^2\,\text{HarmonicNumber}[n,s]^2\right)\Big/\left(\left(s-2^s n^{1-2(1-s)}\pi^{-1+s}(1-s)\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]\right)\left(n^{-1+2s}(1-s)-s\,2^{1-s}\pi^{-s}\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]\right)\right)$$

$$\textbf{ps54}[\text{n\_},\ \text{s\_}] := (2\ \text{HarmonicNumber}[n,\ 1-s]\ \text{HarmonicNumber}[n,\ s]) \Big/ \left(\left(n^{1-2s}\,\frac{s}{1-s} - 2^{s}\,\pi^{-1+s}\,\text{Cos}\left[\frac{1}{2}\,\pi\,(1-s)\right]\,\text{Gamma}[1-s]\right)\left(n^{-1+2s}\,\frac{1-s}{s} - 2^{1-s}\,\pi^{-s}\,\text{Cos}\left[\frac{\pi\,s}{2}\right]\,\text{Gamma}[s]\right)\right) - \left(s\text{^}2\,n\text{^}(-2\,s)\ \text{HarmonicNumber}[n,\ 1-s]^{2} + n^{-2+2s}\,(-1+s)^{2}\ \text{HarmonicNumber}[n,\ s]^{2}\right) \Big/ \left(\left(s\,n\text{^}-s\ - 2^{s}\,n^{-1+s}\,\pi^{-1+s}\,(1-s)\,\text{Cos}\left[\frac{1}{2}\,\pi\,(1-s)\right]\,\text{Gamma}[1-s]\right)\left(n^{-1+s}\,(1-s) - s\,n\text{^}-s\ 2^{1-s}\,\pi^{-s}\,\text{Cos}\left[\frac{\pi\,s}{2}\right]\,\text{Gamma}[s]\right)\right)$$

$$\textbf{ps54a}[\text{n\_},\ \text{s\_}] := \Big\{1 \Big/ \left(\left(n^{1-2s}\,\frac{s}{1-s} - 2^{s}\,\pi^{-1+s}\,\text{Cos}\left[\frac{1}{2}\,\pi\,(1-s)\right]\,\text{Gamma}[1-s]\right)\left(n^{-1+2s}\,\frac{1-s}{s} - 2^{1-s}\,\pi^{-s}\,\text{Cos}\left[\frac{\pi\,s}{2}\right]\,\text{Gamma}[s]\right)\right),\ 2\ \text{HarmonicNumber}[n,\ 1-s]\ \text{HarmonicNumber}[n,\ s],\ -1 \Big/ \left(\left(s\,n\text{^}-s\ - 2^{s}\,n^{-1+s}\,\pi^{-1+s}\,(1-s)\,\text{Cos}\left[\frac{1}{2}\,\pi\,(1-s)\right]\,\text{Gamma}[1-s]\right)\left(n^{-1+s}\,(1-s) - s\,n\text{^}-s\ 2^{1-s}\,\pi^{-s}\,\text{Cos}\left[\frac{\pi\,s}{2}\right]\,\text{Gamma}[s]\right)\right),\ s\text{^}2\,n\text{^}(-2\,s)\ \text{HarmonicNumber}[n,\ 1-s]^{2} + n^{-2+2s}\,(-1+s)^{2}\ \text{HarmonicNumber}[n,\ s]^{2}\Big\}$$

$0.0263648 + 1.12757 \times 10^{-17}\,i$

ps54[100 000, .3 + 10 I]

$2.40802 + 0.0200895\,i$

Zeta[.3 + 10 I] Zeta[1 - (.3 + 10 I)]

$2.40847 + 0.0191575\,i$

```
Expand[(HarmonicNumber[n, 1 - s] / (1 - (2^s n^(1-2 (1-s)) π^(-1+s) (1-s) Cos[1/2 π (1-s)] Gamma[1-s])/s) - HarmonicNumber[n, s] / ((n^(-1+2 (1-s)) s)/(1-s) - 2^s π^(-1+s) Cos[1/2 π (1 - s)] Gamma[1 - s]))
  (-HarmonicNumber[n, 1 - s] / ((n^(-1+2 s) (1-s))/s - 2^(1-s) π^(-s) Cos[π s/2] Gamma[s]) + HarmonicNumber[n, s] / (1 - (2^(1-s) n^(1-2 s) π^(-s) s Cos[π s/2] Gamma[s])/(1-s)))]
```

```
-HarmonicNumber[n, 1 - s]^2 / ((1 - (2^s n^(1-2 (1-s)) π^(-1+s) (1 - s) Cos[1/2 π (1 - s)] Gamma[1 - s])/s)
    ((n^(-1+2 s) (1 - s))/s - 2^(1-s) π^(-s) Cos[π s/2] Gamma[s])) +
  (HarmonicNumber[n, 1 - s] HarmonicNumber[n, s]) /
   (((n^(-1+2 (1-s)) s)/(1 - s) - 2^s π^(-1+s) Cos[1/2 π (1 - s)] Gamma[1 - s])
    ((n^(-1+2 s) (1 - s))/s - 2^(1-s) π^(-s) Cos[π s/2] Gamma[s])) +
  (HarmonicNumber[n, 1 - s] HarmonicNumber[n, s]) /
   ((1 - (2^s n^(1-2 (1-s)) π^(-1+s) (1 - s) Cos[1/2 π (1 - s)] Gamma[1 - s])/s)
    (1 - (2^(1-s) n^(1-2 s) π^(-s) s Cos[π s/2] Gamma[s])/(1 - s))) - HarmonicNumber[n, s]^2 /
   (((n^(-1+2 (1-s)) s)/(1 - s) - 2^s π^(-1+s) Cos[1/2 π (1 - s)] Gamma[1 - s]) (1 - (2^(1-s) n^(1-2 s) π^(-s) s Cos[π s/2] Gamma[s])/(1 - s)))
```

```
FullSimplify[(((n^(-1+2 (1-s)) s)/(1 - s) - 2^s π^(-1+s) Cos[1/2 π (1 - s)] Gamma[1 - s])
    ((n^(-1+2 s) (1 - s))/s - 2^(1-s) π^(-s) Cos[π s/2] Gamma[s])) /
  ((1 - (2^s n^(1-2 (1-s)) π^(-1+s) (1 - s) Cos[1/2 π (1 - s)] Gamma[1 - s])/s)
    (1 - (2^(1-s) n^(1-2 s) π^(-s) s Cos[π s/2] Gamma[s])/(1 - s)))]
```

```
1
```

$$\text{FullSimplify}\left[\left(\left(1-\frac{2^s n^{1-2(1-s)} \pi^{-1+s} (1-s) \text{Cos}\left[\frac{1}{2}\pi(1-s)\right] \text{Gamma}[1-s]}{s}\right)\left(\frac{n^{-1+2s}(1-s)}{s} - 2^{1-s}\pi^{-s}\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]\right)\right)\Big/\left(\left(\frac{n^{-1+2(1-s)} s}{1-s} - 2^s\pi^{-1+s}\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]\right)\left(1-\frac{2^{1-s} n^{1-2s}\pi^{-s} s\,\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]}{1-s}\right)\right)\right]$$

$$\text{FullSimplify}\left[\frac{n^{-2+4s}(-1+s)^2}{s^2}\right]$$

$$\frac{n^{-2+4s}(-1+s)^2}{s^2}$$

$$\text{FullSimplify}\left[n^{1-2(1-s)}\, n\text{^}-s\right]$$

$$n^{-1+s}$$

$$\text{FullSimplify}\left[n^{-1+2(1-s)}\right]$$

$$n^{1-2s}$$

$$\text{FullSimplify}\left[\left(n^{1-2s}\frac{s}{1-s} - 2^s\pi^{-1+s}\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]\right)\left(n^{-1+2s}\frac{1-s}{s} - 2^{1-s}\pi^{-s}\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]\right)\right]$$

$$2-\frac{2^s n^{-1+2s}\pi^{-1+s}\text{Gamma}[2-s]\,\text{Sin}\left[\frac{\pi s}{2}\right]}{s}+\frac{n^{1-2s}(2\pi)^{-s}\text{Csc}\left[\frac{\pi s}{2}\right]\text{Gamma}[1+s]\,\text{Sin}[\pi s]}{-1+s}$$

$$\text{FullSimplify}\left[\text{Expand}\left[\left(s\, n\text{^}-s - 2^s n^{-1+s}\pi^{-1+s}(1-s)\text{Cos}\left[\frac{1}{2}\pi(1-s)\right]\text{Gamma}[1-s]\right)\left(n^{-1+s}(1-s) - s\, n\text{^}-s\; 2^{1-s}\pi^{-s}\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]\right)\right]\right]$$

$$2^{-s} n^{-2(1+s)}\pi^{-1-s}\left(n^{2s}(2\pi)^s(-1+s)+2\,n\,s\,\text{Cos}\left[\frac{\pi s}{2}\right]\text{Gamma}[s]\right)\left(-n\pi s+n^{2s}(2\pi)^s\text{Gamma}[2-s]\,\text{Sin}\left[\frac{\pi s}{2}\right]\right)$$

ps54[100 000 000, N@ZetaZero@20]

$2.56114\times10^{-9} - 1.45646\times10^{-12}\,i$

ps54a[10 000 000 000, .5 + 14 I]

$\{0.365273 + 0.\,i,\ 1.01913\times10^{8} + 0.\,i,\ -1.86126\times10^{7} + 5.7312\times10^{-10}\,i,\ 2.00003 + 0.\,i\}$

HarmonicNumber[100, .5 + I] HarmonicNumber[100, 1 - (.5 + I)]

$73.9495 + 0.\,i$

n^(1 / 2 + s I) n^(1 / 2 - s I)

n

```
(1 - (1 / 2 + s I))
```

$$\frac{1}{2} - i s$$

```
Limit[s^2 n^(-2 s) HarmonicNumber[n, 1 - s]^2 + n^(-2+2 s) (-1 + s)^2 HarmonicNumber[n, s]^2 /.
  s → 1 / 2 + 10 I, n → Infinity]
```

2

```
Limit[s^2 n^(-2 s) HarmonicNumber[n, 1 - s]^2 + n^(-2+2 s) (-1 + s)^2 HarmonicNumber[n, s]^2 /.
  s → 2 / 3 + 10 I, n → Infinity]
```

2

```
ps54x[n_, s_] := (2 HarmonicNumber[n, 1 - s] HarmonicNumber[n, s]) /
   ((n^(1-2 s) s/(1 - s) - 2^s π^(-1+s) Cos[1/2 π (1 - s)] Gamma[1 - s]) (n^(-1+2 s) (1 - s)/s - 2^(1-s) π^-s Cos[π s/2] Gamma[s])) -
  2 / ((s n^-s - 2^s n^(-1+s) π^(-1+s) (1 - s) Cos[1/2 π (1 - s)] Gamma[1 - s])
     (n^(-1+s) (1 - s) - s n^-s 2^(1-s) π^-s Cos[π s/2] Gamma[s]))
```

```
FullSimplify[
 ((n^(1-2 s) s/(1 - s) - 2^s π^(-1+s) Cos[1/2 π (1 - s)] Gamma[1 - s]) (n^(-1+2 s) (1 - s)/s - 2^(1-s) π^-s Cos[π s/2] Gamma[s])) /
  - ((s n^-s - 2^s n^(-1+s) π^(-1+s) (1 - s) Cos[1/2 π (1 - s)] Gamma[1 - s])
     (n^(-1+s) (1 - s) - s n^-s 2^(1-s) π^-s Cos[π s/2] Gamma[s]))]
```

$$\frac{n}{(-1 + s)\, s}$$

```
ps54[n_, s_] := (2 HarmonicNumber[n, 1 - s] HarmonicNumber[n, s]) /
   ((n^(1-2 s) s/(1 - s) - 2^s π^(-1+s) Cos[1/2 π (1 - s)] Gamma[1 - s]) (n^(-1+2 s) (1 - s)/s - 2^(1-s) π^-s Cos[π s/2] Gamma[s])) -
  (s^2 n^(-2 s) HarmonicNumber[n, 1 - s]^2 + n^(-2+2 s) (-1 + s)^2 HarmonicNumber[n, s]^2) /
   ((s n^-s - 2^s n^(-1+s) π^(-1+s) (1 - s) Cos[1/2 π (1 - s)] Gamma[1 - s])
     (n^(-1+s) (1 - s) - s n^-s 2^(1-s) π^-s Cos[π s/2] Gamma[s]))
```

```
ps55[n_, s_] :=
 (2 HarmonicNumber[n, 1 - s] HarmonicNumber[n, s] + (s^2 n^(-2 s) HarmonicNumber[n, 1 - s]^2 +
      n^(-2+2 s) (-1 + s)^2 HarmonicNumber[n, s]^2) (n/((-1 + s) s))) /
  ((n^(1-2 s) s/(1 - s) - 2^s π^(-1+s) Cos[1/2 π (1 - s)] Gamma[1 - s]) (n^(-1+2 s) (1 - s)/s - 2^(1-s) π^(-s) Cos[π s/2] Gamma[s]))
ps56[n_, s_] := (2 HarmonicNumber[n, 1 - s] HarmonicNumber[n, s] +
    n^(1-2 s) s HarmonicNumber[n, 1 - s]^2/(-1 + s) + n^(-1+2 s) (-1 + s) HarmonicNumber[n, s]^2/s) /
  ((n^(1-2 s) s/(1 - s) - 2^s π^(-1+s) Cos[1/2 π (1 - s)] Gamma[1 - s]) (n^(-1+2 s) (1 - s)/s - 2^(1-s) π^(-s) Cos[π s/2] Gamma[s]))
ps56a[n_, s_] := {2 HarmonicNumber[n, 1 - s] HarmonicNumber[n, s],
  n^(1-2 s) s HarmonicNumber[n, 1 - s]^2/(-1 + s), n^(-1+2 s) (-1 + s) HarmonicNumber[n, s]^2/s,
  ((n^(1-2 s) s/(1 - s) - 2^s π^(-1+s) Cos[1/2 π (1 - s)] Gamma[1 - s]) (n^(-1+2 s) (1 - s)/s - 2^(1-s) π^(-s) Cos[π s/2] Gamma[s]))}
ps56x[n_, s_] := 2 HarmonicNumber[n, 1 - s] HarmonicNumber[n, s] +
  n^(1-2 s) s HarmonicNumber[n, 1 - s]^2/(-1 + s) + n^(-1+2 s) (-1 + s) HarmonicNumber[n, s]^2/s
```

```
ps56a[100 000, N@ZetaZero@1]
```

{999.803 + 0. ⅈ, −499.901 − 0.0706595 ⅈ, −499.901 + 0.0706595 ⅈ, 0.718153 + 0. ⅈ}

```
Zeta[.3 + 11 I] Zeta[1 - (.3 + 11 I)]
```

2.26421 − 0.0818805 ⅈ

```
FullSimplify[
 (s^2 n^(-2 s) HarmonicNumber[n, 1 - s]^2 + n^(-2+2 s) (-1 + s)^2 HarmonicNumber[n, s]^2) (n/((-1 + s) s))]
```

$$\frac{n^{1-2s}\, s\, \mathrm{HarmonicNumber}[n, 1-s]^2}{-1+s} + \frac{n^{-1+2s}\,(-1+s)\, \mathrm{HarmonicNumber}[n, s]^2}{s}$$

```
N@ps56x[1 000 000 000, .51 + 2 I]
```

1.14854 + 0.117791 ⅈ

```
Integrate[ j^-s, {j, 0, n}]
```

$$\mathrm{ConditionalExpression}\left[-\frac{n^{1-s}}{-1+s},\ \mathrm{Re}[s] < 1\right]$$

```
fa1[n_, s_] := -n^(1-s)/(-1 + s)
fa2[n_, s_] := 1 / (1 - s) n^(1 - s)
fa3[n_, s_] := 1 / (1 - s) E^((1 - s) Log[n])
fa4[n_, s_, t_] := 1 / (1 - s - t I) E^((1 - s - t I) Log[n])
fa5[n_, s_, t_] := 1 / (1 - s - t I) E^((1 - s) Log[n]) E^((-t I) Log[n])
fa6[n_, s_, t_] := 1 / (1 - s - t I) n^(1 - s) (Cos[((-t) Log[n])] + I Sin[((-t) Log[n])])
fa7[n_, s_, t_] :=
 (1 - s + t I) / (1 - 2 s + s^2 + t^2) n^(1 - s) (Cos[((-t) Log[n])] + I Sin[((-t) Log[n])])
fa8[n_, s_, t_] :=
 (1 - s) / (1 - 2 s + s^2 + t^2) n^(1 - s) (Cos[((-t) Log[n])] + I Sin[((-t) Log[n])]) +
  t I / (1 - 2 s + s^2 + t^2) n^(1 - s) (Cos[((-t) Log[n])] + I Sin[((-t) Log[n])])
fa9[n_, s_, t_] := (1 - s) / (1 - 2 s + s^2 + t^2) n^(1 - s) (Cos[((-t) Log[n])]) +
  (1 - s) / (1 - 2 s + s^2 + t^2) n^(1 - s) (I Sin[((-t) Log[n])]) +
  t I / (1 - 2 s + s^2 + t^2) n^(1 - s) (Cos[((-t) Log[n])]) +
  t I / (1 - 2 s + s^2 + t^2) n^(1 - s) (I Sin[((-t) Log[n])])
fa10[n_, s_, t_] := (1 - s) / (1 - 2 s + s^2 + t^2) n^(1 - s) (Cos[((-t) Log[n])]) +
  I (1 - s) / (1 - 2 s + s^2 + t^2) n^(1 - s) ( Sin[((-t) Log[n])]) +
  I t / (1 - 2 s + s^2 + t^2) n^(1 - s) (Cos[((-t) Log[n])]) -
  t / (1 - 2 s + s^2 + t^2) n^(1 - s) (Sin[((-t) Log[n])])
fa11[n_, s_, t_] := (1 - s) / (1 - 2 s + s^2 + t^2) n^(1 - s) (Cos[((-t) Log[n])]) -
  t / (1 - 2 s + s^2 + t^2) n^(1 - s) (Sin[((-t) Log[n])]) +
  I (t / (1 - 2 s + s^2 + t^2) n^(1 - s) (Cos[((-t) Log[n])]) +
     (1 - s) / (1 - 2 s + s^2 + t^2) n^(1 - s) ( Sin[((-t) Log[n])]))
fa12[n_, s_, t_] := ((1 - s) n^(1 - s) (Cos[t Log[n]]) - t n^(1 - s) (-Sin[t Log[n]]) +
    I (t n^(1 - s) (Cos[t Log[n]]) + (1 - s) n^(1 - s) ( -Sin[t Log[n]]))) / (1 - 2 s + s^2 + t^2)
fa13[n_, s_, t_] := n^(1 - s) / ((1 - s)^2 + t^2) ((1 - s) Cos[t Log[n]] + t Sin[t Log[n]]) +
  I (n^(1 - s) / ((1 - s)^2 + t^2) (t Cos[t Log[n]] - (1 - s) Sin[t Log[n]]))
fa13a[n_, s_, t_] := n^(1 - s) / ((1 - s)^2 + t^2) ((1 - s) Cos[t Log[n]] + t Sin[t Log[n]])
fa13b[n_, s_, t_] := (n^(1 - s) / ((1 - s)^2 + t^2) (t Cos[t Log[n]] - (1 - s) Sin[t Log[n]]))
```

```
fa1[100, .5 + 3 I]
```

3.24779 + 0.512496 i

```
fa13[100, .5, 3]
```

3.24779 + 0.512496 i

```
fa3a[n_, s_] := E^((1 - s) Log[n] - Log[1 - s])
fa4a[n_, s_, t_] := E^((1 - s - t I) Log[n] - Log[1 - s - t I])
```

```
fa4a[100, .5, 3]
```

3.24779 + 0.512496 i

```
1 / (1 - s - t I)
```

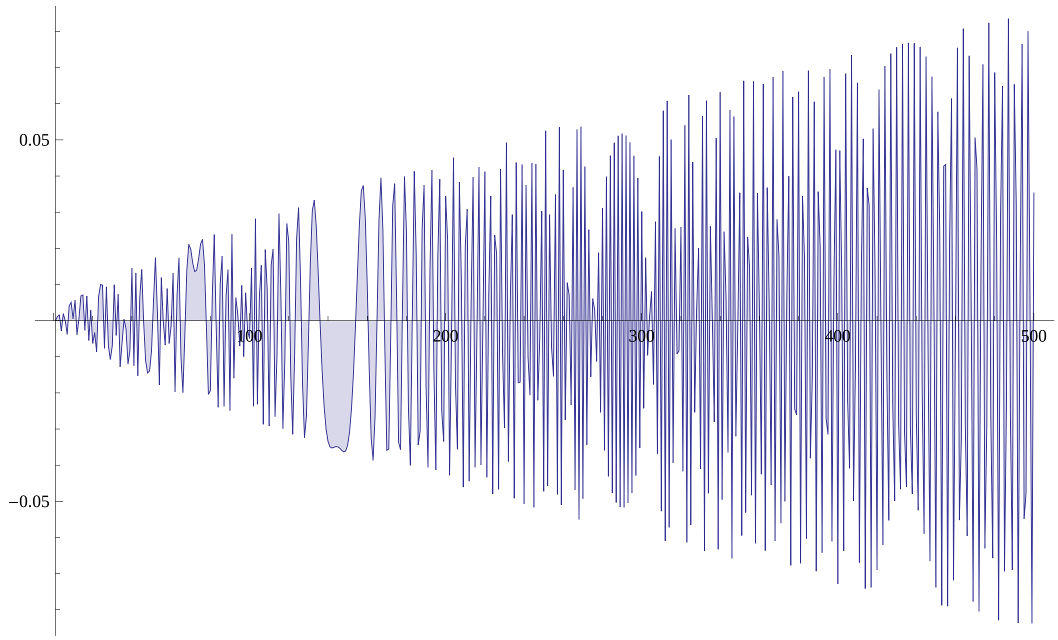
$$\frac{1}{1 - s - i\, t}$$

```
Expand[(1 - s - I t) (1 - s + I t)]
```

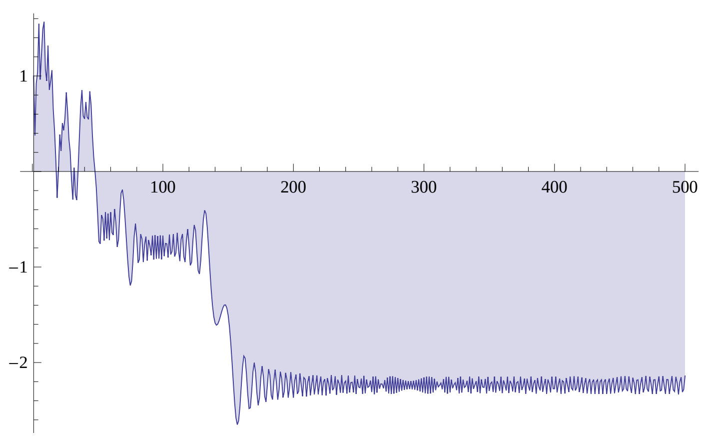
$1 - 2 s + s^2 + t^2$

```
fa13a[n_, s_, t_] := n^(1 - s) / ((1 - s)^2 + t^2) ((1 - s) Cos[t Log[n]] + t Sin[t Log[n]])
fa13b[n_, s_, t_] := (n^(1 - s) / ((1 - s)^2 + t^2) (t Cos[t Log[n]] - (1 - s) Sin[t Log[n]]))
ga13a[n_, s_, t_] := Sum[ Cos[t Log[j]] / j^s, {j, 1, n}]
ga13b[n_, s_, t_] := -Sum[ Sin[t Log[j]] / j^s, {j, 1, n}]
```

```
DiscretePlot[fa13a[n, .3, 910], {n, 1, 500}]
```




```
DiscretePlot[ga13a[n, .3, 910], {n, 1, 500}]
```



$$2\,\text{HarmonicNumber}[n, 1-s]\,\text{HarmonicNumber}[n, s] + \frac{n^{1-2s}\, s\, \text{HarmonicNumber}[n, 1-s]^2}{-1+s} + \frac{n^{-1+2s}\,(-1+s)\,\text{HarmonicNumber}[n, s]^2}{s}$$

$$\frac{n^{1-2s}\, s\, \text{HarmonicNumber}[n, 1-s]^2}{-1+s} + 2\,\text{HarmonicNumber}[n, 1-s]\,\text{HarmonicNumber}[n, s] + \frac{n^{-1+2s}\,(-1+s)\,\text{HarmonicNumber}[n, s]^2}{s}$$

```
Expand[(n^(s - 1 / 2) (s - 1)^(1 / 2) / s^(1 / 2) HarmonicNumber[n, s] +
    n^(1 / 2 - s) s^(1 / 2) / (s - 1)^(1 / 2) HarmonicNumber[n, 1 - s])^2]
```

$$\frac{n^{1-2s}\, s\, \text{HarmonicNumber}[n, 1-s]^2}{-1+s} + 2\, \text{HarmonicNumber}[n, 1-s]\, \text{HarmonicNumber}[n, s] + n^{-1+2s}\, \text{HarmonicNumber}[n, s]^2 - \frac{n^{-1+2s}\, \text{HarmonicNumber}[n, s]^2}{s}$$

FullSimplify$\Bigg[$

$$\left(\left(n^{1-2s} \frac{s}{1-s} - 2^s \pi^{-1+s} \text{Cos}\left[\frac{1}{2} \pi (1-s)\right] \text{Gamma}[1-s]\right) \left(n^{-1+2s} \frac{1-s}{s} - 2^{1-s} \pi^{-s} \text{Cos}\left[\frac{\pi s}{2}\right] \text{Gamma}[s]\right)\right)^{\wedge}$$

$(1/2)\Bigg]$

$$\sqrt{\left(2 - \frac{2^s\, n^{-1+2s}\, \pi^{-1+s}\, \text{Gamma}[2-s]\, \text{Sin}\left[\frac{\pi s}{2}\right]}{s} + \frac{n^{1-2s}\, (2\pi)^{-s}\, \text{Csc}\left[\frac{\pi s}{2}\right] \text{Gamma}[1+s]\, \text{Sin}[\pi s]}{-1+s}\right)}$$

$$\text{ps60}[n\_, s\_] := \frac{1}{\sqrt{2 - \frac{2^s n^{-1+2s} \pi^{-1+s} \text{Gamma}[2-s] \text{Sin}\left[\frac{\pi s}{2}\right]}{s} + \frac{n^{1-2s} (2\pi)^{-s} \text{Csc}\left[\frac{\pi s}{2}\right] \text{Gamma}[1+s] \text{Sin}[\pi s]}{-1+s}}}$$

```
  (n^(s - 1/2) (s - 1)^(1/2) / s^(1/2) HarmonicNumber[n, s] +
    n^(1/2 - s) s^(1/2) / (s - 1)^(1/2) HarmonicNumber[n, 1 - s])
ps60x[n_, s_] := n^(s - 1/2) (s - 1)^(1/2) / s^(1/2) HarmonicNumber[n, s] +
  n^(1/2 - s) s^(1/2) / (s - 1)^(1/2) HarmonicNumber[n, 1 - s]
```

$$\text{ps61}[n\_, s\_] := \frac{1}{\sqrt{2 - \frac{2^s n^{-1+2s} \pi^{-1+s} \text{Gamma}[2-s] \text{Sin}\left[\frac{\pi s}{2}\right]}{s} + \frac{n^{1-2s} (2\pi)^{-s} \text{Csc}\left[\frac{\pi s}{2}\right] \text{Gamma}[1+s] \text{Sin}[\pi s]}{-1+s}}}$$

```
  (1 / (s^(1/2)) / ((s - 1)^(1/2)))
  (n^(s - 1/2) (s - 1) HarmonicNumber[n, s] + n^(1/2 - s) s HarmonicNumber[n, 1 - s])
ps61x[n_, s_] := n^(s - 1/2) (s - 1) HarmonicNumber[n, s] +
  n^(1/2 - s) s HarmonicNumber[n, 1 - s]
```

$$\text{ps62}[n\_, s\_] := 1 \Big/ \left(\sqrt{\left(s (s-1) \left(2 - \frac{2^s n^{-1+2s} \pi^{-1+s} \text{Gamma}[2-s] \text{Sin}\left[\frac{\pi s}{2}\right]}{s} + \frac{n^{1-2s} (2\pi)^{-s} \text{Csc}\left[\frac{\pi s}{2}\right] \text{Gamma}[1+s] \text{Sin}[\pi s]}{-1+s}\right)\right)}\right)$$

```
  (n^(s - 1/2) (1 - s) HarmonicNumber[n, s] - n^(1/2 - s) s HarmonicNumber[n, 1 - s])
```

$$\text{ps63}[n\_, s\_] := 1 \Big/ \left(\sqrt{\left(n\, s (s-1) \left(2 - \frac{2^s n^{-1+2s} \pi^{-1+s} \text{Gamma}[2-s] \text{Sin}\left[\frac{\pi s}{2}\right]}{s} + \frac{n^{1-2s} (2\pi)^{-s} \text{Csc}\left[\frac{\pi s}{2}\right] \text{Gamma}[1+s] \text{Sin}[\pi s]}{-1+s}\right)\right)}\right)$$

```
  (n^s (1 - s) HarmonicNumber[n, s] - n^(1 - s) s HarmonicNumber[n, 1 - s])
```

$$\text{ps64}[n\_, s\_] := 1 \Big/ \left(\sqrt{\left(2 s (s-1) n - 2^s (s-1) n^{2s} \pi^{-1+s} \text{Gamma}[2-s] \text{Sin}\left[\frac{\pi s}{2}\right] + n^{2-2s} (2\pi)^{-s} s\, \text{Csc}\left[\frac{\pi s}{2}\right] \text{Gamma}[1+s] \text{Sin}[\pi s]\right)}\right)$$

```
  (n^s (1 - s) HarmonicNumber[n, s] - n^(1 - s) s HarmonicNumber[n, 1 - s])
```

```
ps64[100 000 000 000 000, N@ZetaZero@2 + 3 I + .2]
```

1.11394 + 0.155879 i

```
(Zeta[N@ZetaZero@2 + 3 I + .2] Zeta[1 - (N@ZetaZero@2 + 3 I + .2)])^(1/2)
```

1.11394 + 0.155879 i

```
n^(s - 1/2) (s - 1) HarmonicNumber[n, s] + n^(1/2 - s) s HarmonicNumber[n, 1 - s]
```

$n^{\frac{1}{2}-s}\, s\, \text{HarmonicNumber}[n, 1-s] + n^{-\frac{1}{2}+s} (-1+s)\, \text{HarmonicNumber}[n, s]$

```
n^(s - 1/2) (s - 1) HarmonicNumber[n, s] + n^(1/2 - s) s HarmonicNumber[n, 1 - s] /. s → 1/2 - x
```

$$n^{-x}\left(-\frac{1}{2}-x\right)\text{HarmonicNumber}\left[n,\frac{1}{2}-x\right]+n^{x}\left(\frac{1}{2}-x\right)\text{HarmonicNumber}\left[n,\frac{1}{2}+x\right]$$

```
n^(s - 1/2) (s - 1) HarmonicNumber[n, s] + n^(1/2 - s) s HarmonicNumber[n, 1 - s] /.
  s → N@ZetaZero@1 /. n → 100 000 000 000
```

0. + 0.0000446902 i

```
(n^(s - a) (1 - s) HarmonicNumber[n, s] - n^(1 - a - s) s HarmonicNumber[n, 1 - s]) /.
   s → N@ZetaZero@1 /. n → 1000 /. a → 0
```

0. − 14.1347 i

```
N@ZetaZero@1
```

0.5 + 14.1347 i

```
(n^(s - a) (1 - b - s) HarmonicNumber[n, s] - n^(1 - a - s) (s + b) HarmonicNumber[n, 1 - s]) /.
    s → N@ZetaZero@1 /. n → 1 000 000 /. a → 1/2 /. b → 1/2
```

−2.49999 − 0.0141347 i

```
(n^(s - a) (1 - b - s) HarmonicNumber[n, s] - n^(1 - a - s) (s + b) HarmonicNumber[n, 1 - s]) /.
    s → N@ZetaZero@1 /. n → 100 000 000 /. a → 1/2 /. b → 0
```

0. − 0.00141347 i

```
ps[n_, s_, a_, b_, c_] :=
 (n^(s - a) (1 - b - s) HarmonicNumber[n, s - c] - n^(1 - a - s) (s + b) HarmonicNumber[n, 1 - c - s])
psa[n_, s_, a_, b_, c_] := -ps[n, s, a, b, c] n^a
```

```
N@ps[10 000, N@ZetaZero@1, 0, 0, 0]
```

0. − 14.1347 i

```
(n^s (1 - s) HarmonicNumber[n, s] - n^(1 - s) s HarmonicNumber[n, 1 - s]) /. s → N@ZetaZero@1 /.
 n → 1000
```

0. − 14.1347 i

$$\textbf{FullSimplify}\left[\sqrt{\left(2\,s\,(s-1)\,n-2^{s}\,(s-1)\,n^{2s}\,\pi^{-1+s}\,\text{Gamma}[2-s]\,\text{Sin}\left[\frac{\pi s}{2}\right]+n^{2-2s}\,(2\pi)^{-s}\,s\,\text{Csc}\left[\frac{\pi s}{2}\right]\text{Gamma}[1+s]\,\text{Sin}[\pi s]\right)}\right]$$

$$\sqrt{\left(-2^{s}\,n^{2s}\,\pi^{-1+s}\,(-1+s)\,\text{Gamma}[2-s]\,\text{Sin}\left[\frac{\pi s}{2}\right]+n\,s\left(-2+2\,s+n^{1-2s}\,(2\pi)^{-s}\,\text{Csc}\left[\frac{\pi s}{2}\right]\text{Gamma}[1+s]\,\text{Sin}[\pi s]\right)\right)}$$

$$-2^{s}\,n^{2s}\,\pi^{-1+s}\,(-1+s)\,\text{Gamma}[2-s]\,\text{Sin}\left[\frac{\pi s}{2}\right]+n^{2-2s}\,(2\pi)^{-s}\,s\Big/\text{Sin}\left[\frac{\pi s}{2}\right]\text{Gamma}[1+s]\,\text{Sin}[\pi s]$$

$$-2^{s}\,n^{2s}\,\pi^{-1+s}\,(-1+s)\,\text{Gamma}[2-s]\,\text{Sin}\left[\frac{\pi s}{2}\right]+n^{2-2s}\,(2\pi)^{-s}\,s\,\text{Csc}\left[\frac{\pi s}{2}\right]\text{Gamma}[1+s]\,\text{Sin}[\pi s]$$

```
pt1[n_, s_] := (n^s (1 - s) HarmonicNumber[n, s] - n^(1 - s) s HarmonicNumber[n, 1 - s])
pt2[n_, s_] := (n^s (1 - s) Sum[j^-s, {j, 1, n}] - n^(1 - s) s Sum[j^(s - 1), {j, 1, n}])
pt3[n_, s_] := Sum[(1 - s) (n / j)^s - s (n / j)^(1 - s), {j, 1, n}]
pt4[n_, s_] :=
 Sum[(1 - (1 / 2 - s)) (n / j)^(1 / 2 - s) - (1 / 2 - s) (n / j)^(1 - (1 / 2 - s)), {j, 1, n}]
pt4a[n_, s_] := pt4[n, 1 / 2 - s]
```

pt5[n_, s_] := $\text{Sum}\left[\frac{1}{2}\left(\left(\frac{n}{j}\right)^{\frac{1}{2}-s}-\left(\frac{n}{j}\right)^{\frac{1}{2}+s}\right)+s\left(\left(\frac{n}{j}\right)^{\frac{1}{2}-s}+\left(\frac{n}{j}\right)^{\frac{1}{2}+s}\right),\ \{j,1,n\}\right]$

```
pt5a[n_, s_] := pt5[n, 1 / 2 - s]
```

pt6[n_, s_] := $\text{Sum}\left[(n / j)^\wedge(1/2)\ \left(\frac{1}{2}\left(\left(\frac{n}{j}\right)^{-s}-\left(\frac{n}{j}\right)^{+s}\right)+s\left(\left(\frac{n}{j}\right)^{-s}+\left(\frac{n}{j}\right)^{+s}\right)\right),\ \{j,1,n\}\right]$

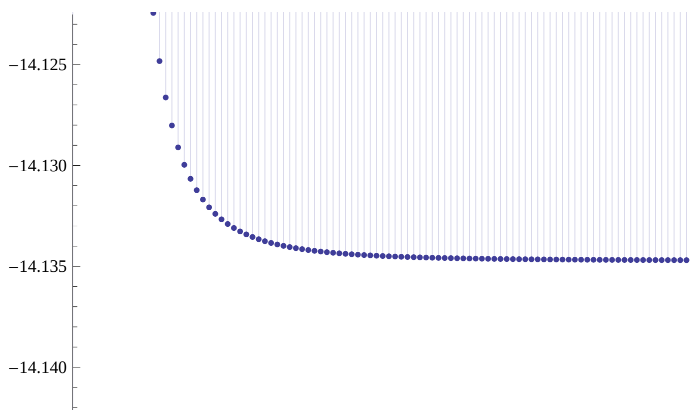
```
pt6a[n_, s_] := pt6[n, 1 / 2 - s]
pt6a[1000, N@ZetaZero@1]
```

0. - 14.1347 i

```
Expand[(1 - (1 / 2 - s)) (n / j)^(1 / 2 - s) - (1 / 2 - s) (n / j)^(1 - (1 / 2 - s))]
```

$$\frac{1}{2}\left(\frac{n}{j}\right)^{\frac{1}{2}-s}-\frac{1}{2}\left(\frac{n}{j}\right)^{\frac{1}{2}+s}+\left(\frac{n}{j}\right)^{\frac{1}{2}-s}s+\left(\frac{n}{j}\right)^{\frac{1}{2}+s}s$$

```
DiscretePlot[Im@ pt5a[n, N@ZetaZero@1], {n, 1, 100}]
```




```
pro3[n_, x_] :=
```

$$n^{\frac{1}{2}-i x}\,(-i/2+x)\ \text{HarmonicNumber}\left[n,\ \frac{1}{2}-i x\right]+n^{\frac{1}{2}+i x}\,(i/2+x)\ \text{HarmonicNumber}\left[n,\ \frac{1}{2}+i x\right]$$

```
Chop@pro3[1000, N@Im@ZetaZero@20]
```

77.1448

```
N@Im@ZetaZero@20
```

77.1448

```
po[n_, s_] := Sum[ (n / j)^(1 / 2) (2 s Cos[ s Log[j / n]] + Sin[s Log[j / n]]), {j, 1, n}]
po2[n_, s_] := Sum[ (n / j)^(1 / 2) (2 s Cos[ s Log[n / j]] - Sin[s Log[n / j]]), {j, 1, n}]

Chop@po2[1000, N@Im@ZetaZero@20]
```

77.1448

```
ps14x[n_, s_] :=
 HarmonicNumber[n, s] / (1 - 2^(1 - s) n^(1 - 2 s) (s / (1 - s)) π^-s Cos[π s / 2] Gamma[s]) -
  HarmonicNumber[n, 1 - s] / (n^(2 s-1) (1 - s) / s - 2^(1 - s) π^-s Cos[π s / 2] Gamma[s])
ps14y[n_, s_] := n^(2 s-1) (1 - s) / s
   HarmonicNumber[n, s] / (n^(2 s-1) (1 - s) / s - 2^(1 - s) π^-s Cos[π s / 2] Gamma[s]) -
  HarmonicNumber[n, 1 - s] / (n^(2 s-1) (1 - s) / s - 2^(1 - s) π^-s Cos[π s / 2] Gamma[s])
ps14y2[n_, s_] := (n^(2 s-1) (1 - s) / s HarmonicNumber[n, s] - HarmonicNumber[n, 1 - s]) /
  (n^(2 s-1) (1 - s) / s - 2^(1 - s) π^-s Cos[π s / 2] Gamma[s])
ps14y3[n_, s_] := ((n^(2 s-1) (1 - s) / s - 1) Re@HarmonicNumber[n, s] +
     I (n^(2 s-1) (1 - s) / s + 1) Im@HarmonicNumber[n, s]) /
  (n^(2 s-1) (1 - s) / s - 2^(1 - s) π^-s Cos[π s / 2] Gamma[s])
ps14y32[n_, s_] := (n^(2 s-1) (1 - s) / s - 1) Re@HarmonicNumber[n, s] +
  I (n^(2 s-1) (1 - s) / s + 1) Im@HarmonicNumber[n, s]
ps14y33[n_, s_] := {(n^(2 s-1) (1 - s) / s - 1), Re@HarmonicNumber[n, s],
  I (n^(2 s-1) (1 - s) / s + 1), Im@HarmonicNumber[n, s]}
```

```
ps14y33[10 000 000 000, N@ZetaZero@1]
```

{-0.228237 + 0.63591 ⅈ, -6654.71, -0.63591 + 1.77176 ⅈ, 2388.47}

```
Zeta[.5 + 17 I]
```

1.94665 + 0.895405 ⅈ

```
FullSimplify[(n^(2 s-1) (1 - s) / s - 1) Re@HarmonicNumber[n, s] +
  I (n^(2 s-1) (1 - s) / s + 1) Im@HarmonicNumber[n, s]]
```

$$-\text{Conjugate}[\text{HarmonicNumber}[n, s]] - \frac{n^{-1+2 s} (-1 + s)\ \text{HarmonicNumber}[n, s]}{s}$$

$$-\text{Conjugate}[\text{HarmonicNumber}[n, s]] - \frac{n^{-1+2 s} (-1 + s)\ \text{HarmonicNumber}[n, s]}{s} \ /.$$

```
  n → 10 000 000 000 000 /. s → N@ZetaZero@1
```

$1.80793 \times 10^{-7} - 2.67493 \times 10^{-7}\ ⅈ$

```
FullSimplify[n^(2 s-1) (1 - s) / s - 1]
```

$$-1+n^{-1+2 s}\left(-1+\frac{1}{s}\right)$$

```
FullSimplify[(n^(2 s-1) (1 - s) / s + 1)]
```

$$1+n^{-1+2 s}\left(-1+\frac{1}{s}\right)$$

```
FullSimplify[(1 - s) n^(1 - s) / (s - 1)]
```

$$-n^{1-s}$$

```
(n^(1 - s - x) / (s + x - 1)) (s - 1 + x) n^x
```

$$n^{1-s}$$

```
FullSimplify[Expand[-s (1 - s) Integrate[ FractionalPart[t] / t^(s + 1), {t, n, Infinity}] +
   - (s - 1 + x) n^x (s + x) Integrate[ FractionalPart[t] / t^(s + 1 + x), {t, n, Infinity}]]]
```

$$(-1+s)\, s \int_n^{\infty} t^{-1-s}\, \text{FractionalPart}[t]\, dt - n^x (-1+s+x)(s+x) \int_n^{\infty} t^{-1-s-x}\, \text{FractionalPart}[t]\, dt$$

$$\text{Limit}\left[(-1+s)\, s \int_n^{\infty} t^{-1-s}\, \text{FractionalPart}[t]\, dt - n^x (-1+s+x)(s+x) \int_n^{\infty} t^{-1-s-x}\, \text{FractionalPart}[t]\, dt,\ n \to \text{Infinity}\right]$$

```
$Aborted
```

```
Integrate[(-s) (1 - s) FractionalPart[t] / t^(s + 1) +
  (-1) (s - 1 + x) n^x (s + x) FractionalPart[t] / t^(s + 1 + x), {t, n, Infinity}]
```

```
FullSimplify[Expand[(-s) (1 - s) FractionalPart[t] / t^(s + 1) +
   (-1) (s - 1 + x) n^x (s + x) FractionalPart[t] / t^(s + 1 + x)]]
```

$$t^{-1-s-x}\left((-1+s)\, s\, t^x - n^x (-1+s+x)(s+x)\right) \text{FractionalPart}[t]$$

```
Integrate[((-s) (1 - s) / t^(s + 1) + (-1) (s - 1 + x) n^x (s + x) / t^(s + 1 + x)) FractionalPart[t],
 {t, n, Infinity}]
```

```
FullSimplify[
 ((-s) (1 - s) / t^(s + 1) + (-1) (s - 1 + x) n^x (s + x) / t^(s + 1 + x)) FractionalPart[t]]
```

$$t^{-1-s}\left((-1+s)\, s - n^x t^{-x} (-1+s+x)(s+x)\right) \text{FractionalPart}[t]$$

$$\text{Integrate}\left[t^{-1-s}\left((-1+s)\, s - n^x t^{-x} (-1+s+x)(s+x)\right) \text{FractionalPart}[t],\ \{t, n, \text{Infinity}\}\right]$$

$$\int_n^{\infty} t^{-1-s}\left((-1+s)\, s - n^x t^{-x} (-1+s+x)(s+x)\right) \text{FractionalPart}[t]\, dt$$

$$\text{pr}[n\_, s\_, x\_] := \int_n^{\infty} t^{-1-s}\left((-1+s)\, s - n^x t^{-x} (-1+s+x)(s+x)\right) \text{FractionalPart}[t]\, dt$$

$$\text{pr2}[n\_, s\_, x\_] := s\,(s-1) \int_n^{\infty} \left(1 - (n/t)^x (1 + x/(s-1))(1 + x/s)\right) \text{FractionalPart}[t] / t^{\wedge}(s+1)\, dt$$

```
N@pr2[10 000 000 000 000 000, .1 + I, .1 + 3 I]
```

```
0. + 0. i
```

```
N[10 000 000 000^(-1 / 2)]
```

```
0.00001
```

$$\text{FullSimplify}\left[\text{Expand}\left[t^{-1-s}\,((-1+s)\,s - n^x\,t^{-x}\,(-1+s+x)\,(s+x))\right]\right]$$

$$t^{-1-s-x}\,((-1+s)\,s\,t^x - n^x\,(-1+s+x)\,(s+x))$$

```
FullSimplify[(s - 1 + x) (s + x) / (s - 1) / s]
```

$$\frac{(-1+s+x)\,(s+x)}{(-1+s)\,s}$$

$$\text{FullSimplify}\left[t^{-1-s}\,s\,(s-1)\,(1-(n/t)^x\,(1+x/(s-1))\,(1+x/s))\right]$$

$$t^{-1-s}\left((-1+s)\,s - \left(\frac{n}{t}\right)^x\,(-1+s+x)\,(s+x)\right)$$

```
N@pr[10 000, .3 + I, .3 + 3 I]
```

0.0464083 + 0.133533 ⅈ

```
N@pr2[10 000, .3 + I, .3 + 3 I]
```

0.0464083 + 0.133533 ⅈ

```
fl[s2_] := Limit[(1 / 2) s (s - 1) Pi^(-s / 2) Gamma[s / 2], s → s2]
```

```
fl[1 - s]
```

$$-\frac{1}{2}\,\pi^{\frac{1}{2}(-1+s)}\,(1-s)\,s\,\text{Gamma}\left[\frac{1-s}{2}\right]$$

```
so[n_, s_] :=
 (1 - s) (Zeta[s] - HarmonicNumber[n, s]) - s n^(1 - 2 s) (Zeta[1 - s] - HarmonicNumber[n, 1 - s])
so2[n_, s_] := ((1 / fl[s]) (1 - s) n^s (fl[s] Zeta[s] - fl[s] HarmonicNumber[n, s]) -
     (1 / fl[1 - s]) s n^(1 - s) (fl[1 - s] Zeta[1 - s] - fl[1 - s] HarmonicNumber[n, 1 - s])) n^(-s)
so2a[n_, s_] := ((1 / fl[s]) (1 - s) n^s (fl[s] Zeta[s] - fl[s] HarmonicNumber[n, s]) -
    (1 / fl[1 - s]) s n^(1 - s) (fl[1 - s] Zeta[1 - s] - fl[1 - s] HarmonicNumber[n, 1 - s]))
so3[n_, s_] := ((1 / fl[s]) (1 - s) n^s (fl[s] Zeta[s] - fl[s] HarmonicNumber[n, s]) -
    (1 / fl[1 - s]) s n^(1 - s) (fl[s] Zeta[s] - fl[1 - s] HarmonicNumber[n, 1 - s]))
so4[n_, s_] := (1 - s) n^s / fl[s] (fl[s] Zeta[s] - fl[s] HarmonicNumber[n, s]) -
   s n^(1 - s) / fl[1 - s] (fl[s] Zeta[s] - fl[1 - s] HarmonicNumber[n, 1 - s])
so5[n_, s_] := (1 - s) n^s / fl[s] fl[s] Zeta[s] - s n^(1 - s) / fl[1 - s] fl[s] Zeta[s] -
   (1 - s) n^s HarmonicNumber[n, s] + s n^(1 - s) HarmonicNumber[n, 1 - s]
so6[n_, s_] := ((1 - s) n^s / fl[s] - s n^(1 - s) / fl[1 - s]) ( fl[s] Zeta[s]) -
   (1 - s) n^s HarmonicNumber[n, s] - s n^(1 - s) HarmonicNumber[n, 1 - s]
so7[n_, s_] := ( fl[s] Zeta[s]) -
   ((1 - s) n^s HarmonicNumber[n, s] - s n^(1 - s) HarmonicNumber[n, 1 - s]) /
    ((1 - s) n^s / fl[s] - s n^(1 - s) / fl[1 - s])
so8[n_, s_] := ((1 - s) n^s HarmonicNumber[n, s] - s n^(1 - s) HarmonicNumber[n, 1 - s]) /
   ((1 - s) n^s / fl[s] - s n^(1 - s) / fl[1 - s])
```

$$\text{so9}[n\_,\,s\_] := \frac{n^s\,(1-s)\,\text{HarmonicNumber}[n,\,s] - n^{1-s}\,s\,\text{HarmonicNumber}[n,\,1-s]}{\frac{2\,n^{1-s}\,\pi^{\frac{1-s}{2}}}{(1-s)\,\text{Gamma}\left[\frac{1-s}{2}\right]} - \frac{2\,n^s\,\pi^{s/2}}{s\,\text{Gamma}\left[\frac{s}{2}\right]}}$$

$$\text{so10}[n\_,\,s\_] := \frac{n^s\,(1-s)\,\text{HarmonicNumber}[n,\,s] - n^{1-s}\,s\,\text{HarmonicNumber}[n,\,1-s]}{\frac{2\,n^{1-s}\,\pi^{\frac{1-s}{2}}}{(1-s)\,\text{Gamma}\left[\frac{1}{2}-\frac{s}{2}\right]} - \frac{2\,n^s\,\pi^{s/2}}{s\,\text{Gamma}\left[\frac{s}{2}\right]}}$$

$$\texttt{so11[n\_, s\_] :=} \left(n^{s}\,(1-s)\ \texttt{HarmonicNumber[n, s]} - n^{1-s}\,s\ \texttt{HarmonicNumber[n, 1-s]}\right) \Big/ \left(2\,n^{1-s}\,\pi^{\frac{1-s}{2}}\ (\texttt{Gamma[-s / 2] / (2\^{}(1 + s) Pi\^{}(1 / 2) Gamma[-s] (1 - s)))} - 2\,n^{s}\,\pi^{s/2}\ (\texttt{Gamma[s / 2 + 1 / 2] / (2\^{}(1 - s) Pi\^{}(1 / 2) Gamma[s] s))}\right)$$

$$\texttt{so12[n\_, s\_] :=} \left(n^{s}\,(1-s)\ \texttt{HarmonicNumber[n, s]} - n^{1-s}\,s\ \texttt{HarmonicNumber[n, 1-s]}\right) \Big/ \left(2\,n^{1-s}\,\pi^{\frac{1-s}{2}}\ \texttt{Gamma[-s / 2] / (2\^{}(1 + s) Pi\^{}(1 / 2) Gamma[-s] (1 - s))} - 2\,n^{s}\,\pi^{s/2}\ \texttt{Gamma[s / 2 + 1 / 2] / (2\^{}(1 - s) Pi\^{}(1 / 2) Gamma[s] s)}\right)$$

$$\texttt{so13[n\_, s\_] :=} \frac{n^{s}\,(1-s)\ \texttt{HarmonicNumber[n, s]} - n^{1-s}\,s\ \texttt{HarmonicNumber[n, 1-s]}}{\frac{n^{1-s}\,\pi^{\frac{1}{2}-\frac{s}{2}}}{\texttt{Gamma}\left[\frac{3}{2}-\frac{s}{2}\right]} - \frac{n^{s}\,\pi^{s/2}}{\texttt{Gamma}\left[1+\frac{s}{2}\right]}}$$

$$\texttt{so14[n\_, s\_] :=} \frac{n^{s}\,(1-s)\ \texttt{HarmonicNumber[n, s]} - n^{1-s}\,s\ \texttt{HarmonicNumber[n, 1-s]}}{\frac{n^{1-s}\,\pi^{\frac{(1-s)}{2}}}{\texttt{Gamma}\left[1+\frac{1-s}{2}\right]} - \frac{n^{s}\,\pi^{s/2}}{\texttt{Gamma}\left[1+\frac{s}{2}\right]}}$$

$$\texttt{zet14[n\_, s\_] := so14[n, s] / fl[s]}$$

$$\texttt{so15[n\_, s\_] :=} \frac{n^{s}\,(1-s)\ \texttt{HarmonicNumber[n, s]} - n^{1-s}\,s\ \texttt{HarmonicNumber[n, 1-s]}}{\frac{n^{1-s}\,\pi^{\frac{(1-s)}{2}}\ \texttt{Gamma[(s-1)/2]}}{\texttt{Gamma}\left[1-\frac{(s-1)}{2}\right]\ \texttt{Gamma[(s-1)/2]}} - \frac{n^{s}\,\pi^{s/2}\ \texttt{Gamma}\left[-\frac{s}{2}\right]}{\texttt{Gamma}\left[1+\frac{s}{2}\right]\ \texttt{Gamma}\left[-\frac{s}{2}\right]}}$$

$$\texttt{so16[n\_, s\_] :=} \frac{n^{s}\,(1-s)\ \texttt{HarmonicNumber[n, s]} - n^{1-s}\,s\ \texttt{HarmonicNumber[n, 1-s]}}{\frac{n^{1-s}\,\pi^{\frac{(1-s)}{2}}\ \texttt{Gamma[(s-1)/2]}}{\texttt{Pi/Sin[Pi (s-1)/2]}} - \frac{n^{s}\,\pi^{s/2}\ \texttt{Gamma}\left[-\frac{s}{2}\right]}{\texttt{Pi/Sin}\left[\texttt{Pi}\left(1+\frac{s}{2}\right)\right]}}$$

$$\texttt{so17[n\_, s\_] :=} \left(n^{s}\,(1-s)\ \texttt{HarmonicNumber[n, s]} - n^{1-s}\,s\ \texttt{HarmonicNumber[n, 1-s]}\right) \Big/ \left(n^{1-s}\,\pi^{-1/2-\frac{s}{2}}\ \texttt{Gamma[(s - 1) / 2] Sin[Pi (s - 1) / 2]} - n^{s}\,\pi^{s/2-1}\ \texttt{Gamma}\left[-\frac{s}{2}\right]\ \texttt{Sin}\left[\texttt{Pi}\left(1+\frac{s}{2}\right)\right]\right)$$

$$\texttt{so18[n\_, s\_] :=} \frac{n^{s}\,(1-s)\ \texttt{HarmonicNumber[n, s]} - n^{1-s}\,s\ \texttt{HarmonicNumber[n, 1-s]}}{n^{s}\,\pi^{s/2-1}\ \texttt{Gamma}\left[-\frac{s}{2}\right]\ \texttt{Sin}\left[\frac{\pi s}{2}\right] - n^{1-s}\,\pi^{-\frac{(s+1)}{2}}\ \texttt{Gamma}\left[\frac{s-1}{2}\right]\ \texttt{Cos}\left[\frac{\pi s}{2}\right]}$$

$$\texttt{zet[n\_, s\_] := so18[n, s] / fl[s]}$$

```
so18[1 000 000 000 000 000, -.3 + 17 I] / fl[-.3 + 17 I]
```

2.83973 + 1.93155 i

```
N@zet14[1 000 000 000 000 000, .53]
```

-1.5857

```
N@Zeta[.53]
```

-1.5857

((1 - s) n^s HarmonicNumber[n, s] - s n^(1 - s) HarmonicNumber[n, 1 - s]) / ((1 - s) n^s / fl[s] - s n^(1 - s) / fl[1 - s])

$$\frac{-n^{1-s}\, s\, \text{HarmonicNumber}[n, 1-s] + n^s (1-s)\, \text{HarmonicNumber}[n, s]}{\frac{2\, n^{1-s}\, \pi^{\frac{1-s}{2}}}{(1-s)\, \text{Gamma}\left[\frac{1-s}{2}\right]} + \frac{2\, n^s\, \pi^{s/2}\, (1-s)}{(-1+s)\, s\, \text{Gamma}\left[\frac{s}{2}\right]}}$$

$$\textbf{FullSimplify}\left[\frac{2\, n^{1-s}\, \pi^{\frac{1-s}{2}}}{(1-s)\, \text{Gamma}\left[\frac{1-s}{2}\right]} - \frac{2\, n^s\, \pi^{s/2}}{s\, \text{Gamma}\left[\frac{s}{2}\right]} \text{ /. } s \to 3\right]$$

$$-\frac{4\, n^3\, \pi}{3}$$

Gamma[s / 2] /. s → 5

$$\frac{3\sqrt{\pi}}{4}$$

Gamma[s / 2] /. s → 5

$$\frac{-n^{1-s}\, s\, \text{HarmonicNumber}[n, 1-s] + n^s (1-s)\, \text{HarmonicNumber}[n, s]}{\frac{2\, n^{1-s}\, \pi^{\frac{1-s}{2}}}{(1-s)\, \text{Gamma}\left[\frac{1-s}{2}\right]} - \frac{2\, n^s\, \pi^{s/2}}{s\, \text{Gamma}\left[\frac{s}{2}\right]}} \text{ /. } s \to 1-s$$

$$\frac{n^{1-s}\, s\, \text{HarmonicNumber}[n, 1-s] - n^s (1-s)\, \text{HarmonicNumber}[n, s]}{-\frac{2\, n^{1-s}\, \pi^{\frac{1-s}{2}}}{(1-s)\, \text{Gamma}\left[\frac{1-s}{2}\right]} + \frac{2\, n^s\, \pi^{s/2}}{s\, \text{Gamma}\left[\frac{s}{2}\right]}}$$

$$\textbf{FullSimplify}\left[2\, n^{1-s}\, \pi^{\frac{1-s}{2}}\, \text{Gamma}[-s/2] / (2\text{^}(1+s)\, \text{Pi^}(1/2)\, \text{Gamma}[-s]\, (1-s))\right]$$

$$\frac{n^{1-s}\, \pi^{\frac{1}{2}-\frac{s}{2}}}{\text{Gamma}\left[\frac{3}{2}-\frac{s}{2}\right]}$$

$$\textbf{FullSimplify}\left[2\, n^s\, \pi^{s/2}\, \text{Gamma}[s/2 + 1/2] / (2\text{^}(1-s)\, \text{Pi^}(1/2)\, \text{Gamma}[s]\, s)\right]$$

$$\frac{n^s\, \pi^{s/2}}{\text{Gamma}\left[1+\frac{s}{2}\right]}$$

$$n^s (1-s)\, \textbf{HarmonicNumber}[n, s] - n^{1-s}\, s\, \textbf{HarmonicNumber}[n, 1-s] \text{ /. } n \to 10\,000\,000 \text{ /. } s \to \textbf{N@ZetaZero@1}$$

0. - 14.1347 i

Gamma[(s - 1) / 2] /. s → .3

-3.95656

2 Gamma[(s - 1) / 2 + 1] / (s - 1) /. s → .3

-3.95656

```
Gamma[2]
```

1

$$\frac{n^{1-s}\,\pi^{\frac{(1-s)}{2}}}{\text{Gamma}\left[1-\frac{(s-1)}{2}\right]}-\frac{n^{s}\,\pi^{s/2}}{\text{Gamma}\left[1+\frac{s}{2}\right]}\ \text{/. } s\to 1/2$$

0

```
N@Limit[(1 / 2) s (s - 1) Pi^(-s / 2) Gamma[s / 2], s → 1 / 2]
```

-0.340411

```
Gamma[1]
```

1

```
fl[s2_] := Limit[(1 / 2) s (s - 1) Pi^(-s / 2) Gamma[s / 2], s → s2]
```

$$n^{s}\,(1-s)\ \text{HarmonicNumber}[n,\ s]-n^{1-s}\,s\ \text{HarmonicNumber}[n,\ 1-s]\ \text{/. } s\to 1/2$$

0

$$\text{so10[n\_, s\_]} := \frac{n^{s}\,(1-s)\,\text{HarmonicNumber[n, s]} - n^{1-s}\,s\,\text{HarmonicNumber[n, 1-s]}}{\frac{2\,n^{1-s}\,\pi^{\frac{1-s}{2}}}{(1-s)\,\text{Gamma}\left[\frac{1}{2}-\frac{s}{2}\right]} - \frac{2\,n^{s}\,\pi^{s/2}}{s\,\text{Gamma}\left[\frac{s}{2}\right]}}$$

zet10[n_, s_] := so10[n, s] / fl[s]

$$\text{so10a[n\_, s\_]} := \frac{\text{HarmonicNumber[n, s]}}{\frac{2\,n^{1-2s}\,\pi^{\frac{1-s}{2}}}{(1-s)\,(1-s)\,\text{Gamma}\left[\frac{(1-s)}{2}\right]} - \frac{2\,\pi^{s/2}}{s\,(1-s)\,\text{Gamma}\left[\frac{s}{2}\right]}} - \frac{\text{HarmonicNumber[n, 1-s]}}{\frac{2\,\pi^{\frac{1-s}{2}}}{(1-s)\,s\,\text{Gamma}\left[\frac{(1-s)}{2}\right]} - \frac{2\,n^{2s-1}\,\pi^{s/2}}{s\,s\,\text{Gamma}\left[\frac{s}{2}\right]}}$$

$$\text{so10b[n\_, s\_]} := \frac{(1-s)\,\text{HarmonicNumber[n, s]}}{\frac{n^{1-2s}\,\pi^{\frac{1-s}{2}}}{\text{Gamma}\left[1+\frac{1-s}{2}\right]} - \frac{\pi^{s/2}}{\text{Gamma}\left[1+\frac{s}{2}\right]}} - \frac{s\,\text{HarmonicNumber[n, 1-s]}}{\frac{\pi^{\frac{1-s}{2}}}{\text{Gamma}\left[1+\frac{1-s}{2}\right]} - \frac{n^{2s-1}\,\pi^{s/2}}{\text{Gamma}\left[1+\frac{s}{2}\right]}}$$

zet10b[n_, s_] := so10b[n, s] / fl[s]

$$\text{sosub[n\_, s\_]} := \frac{(1-s)\,\text{HarmonicNumber[n, s]}}{\frac{n^{1-2s}\,\pi^{\frac{1-s}{2}}}{\text{Gamma}\left[1+\frac{1-s}{2}\right]} - \frac{\pi^{s/2}}{\text{Gamma}\left[1+\frac{s}{2}\right]}}$$

so10c[n_, s_] := sosub[n, s] + sosub[n, 1 - s]
zet10c[n_, s_] := so10c[n, s] / fl[s]

$$\text{sosub2[n\_, s\_]} := \frac{(1-s)\;\text{n\^{}s HarmonicNumber[n, s] Gamma}\left[1+\frac{1-s}{2}\right]\text{Gamma}\left[1+\frac{s}{2}\right]}{n^{1-s}\,\pi^{\frac{1-s}{2}}\,\text{Gamma}\left[1+\frac{s}{2}\right] - \text{n\^{}s}\,\pi^{s/2}\,\text{Gamma}\left[1+\frac{1-s}{2}\right]}$$

so10c2[n_, s_] := sosub2[n, s] + sosub2[n, 1 - s]
zet10c2[n_, s_] := so10c2[n, s] / fl[s]

$$\text{sosub3[n\_, s\_]} := ((1-s)\ \text{HarmonicNumber[n, s]}) \Big/ \left(\pi^{s/2-1}\ \text{Gamma[-s / 2] Sin[Pi s / 2]} - n^{1-2s}\,\pi^{\frac{-s-1}{2}}\ \text{Gamma[(s - 1) / 2] Cos[Pi s / 2]}\right)$$

so10c3[n_, s_] := sosub3[n, s] + sosub3[n, 1 - s]
zet10c3[n_, s_] := so10c3[n, s] / fl[s]

$$\text{sosub4[n\_, s\_]} := \frac{(1-s)\,\text{HarmonicNumber[n, s]}}{\frac{n^{1-2s}\,\pi^{\frac{1-s}{2}}}{\text{Gamma}\left[1/2+1+\frac{-s}{2}\right]} - \frac{\pi^{s/2}}{\text{Gamma}\left[1+\frac{s}{2}\right]}}$$

so10c4[n_, s_] := sosub4[n, s] + sosub4[n, 1 - s]
zet10c4[n_, s_] := so10c4[n, s] / fl[s]

so10c4[10 000, N@ZetaZero@1]

$-8.7111 \times 10^{-6} + 0.\,i$

zet10c4[10 000 000, .6]

-1.95268

Zeta[.6]

-1.95266

```
so10[10 000, 1.15 + 3 I]
```

0.40559 + 0.0371621 i

$$\texttt{FullSimplify}\left[\frac{2\, n^{1-2s}\, \pi^{\frac{1-s}{2}}}{(1-s)\,(1-s)\,\texttt{Gamma}\left[\frac{(1-s)}{2}\right]} - \frac{2\,\pi^{s/2}}{s\,(1-s)\,\texttt{Gamma}\left[\frac{s}{2}\right]}\right]$$

$$\frac{\pi^{-s/2}\left(-\frac{n^{1-2s}\sqrt{\pi}}{\texttt{Gamma}\left[\frac{3}{2}-\frac{s}{2}\right]} + \frac{\pi^{s}}{\texttt{Gamma}\left[1+\frac{s}{2}\right]}\right)}{-1+s}$$

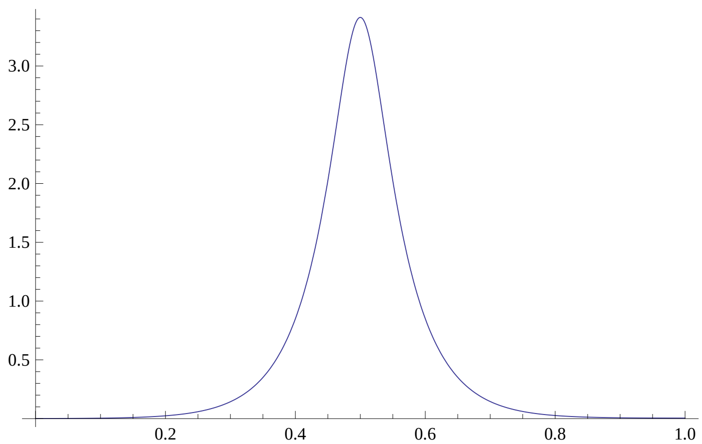
$$\texttt{FullSimplify}\left[\frac{2\,\pi^{\frac{1-s}{2}}}{(1-s)\,s\,\texttt{Gamma}\left[\frac{(1-s)}{2}\right]} - \frac{2\, n^{2s-1}\,\pi^{s/2}}{s\, s\,\texttt{Gamma}\left[\frac{s}{2}\right]}\right]$$

$$\frac{\pi^{-s/2}\left(\frac{\sqrt{\pi}}{\texttt{Gamma}\left[\frac{3}{2}-\frac{s}{2}\right]} - \frac{n^{-1+2s}\,\pi^{s}}{\texttt{Gamma}\left[1+\frac{s}{2}\right]}\right)}{s}$$

$$\frac{(1-s)\,\texttt{HarmonicNumber}[n, s]}{\frac{n^{1-2s}\,\pi^{\frac{1-s}{2}}}{\texttt{Gamma}\left[1+\frac{1-s}{2}\right]} - \frac{\pi^{s/2}}{\texttt{Gamma}\left[1+\frac{s}{2}\right]}} \;/.\; s \to 1-s$$

$$\frac{s\,\texttt{HarmonicNumber}[n, 1-s]}{-\frac{\pi^{\frac{1-s}{2}}}{\texttt{Gamma}\left[1+\frac{1-s}{2}\right]} + \frac{n^{1-2(1-s)}\,\pi^{s/2}}{\texttt{Gamma}\left[1+\frac{s}{2}\right]}}$$

```
Plot[{Im@sosub[100 000 000, x + 12 I]}, {x, 0, 1}]
```




```
fla[s2_] := Limit[(1 / 2) s (s - 1) Pi^(-s / 2) Gamma[s / 2], s → s2]
```

$$\texttt{so10x[n\_, s\_] :=} \frac{n^{s}\,(1-s)\,\texttt{HarmonicNumber}[n, s] - n^{1-s}\, s\,\texttt{HarmonicNumber}[n, 1-s]}{\frac{2\, n^{1-s}\,\pi^{\frac{1-s}{2}}}{(1-s)\,\texttt{Gamma}\left[\frac{1}{2}-\frac{s}{2}\right]} - \frac{2\, n^{s}\,\pi^{s/2}}{s\,\texttt{Gamma}\left[\frac{s}{2}\right]}}$$

```
so10x2[n_, s_] :=
```

$$\left(\left(n^{s}\,(1-s)\,\texttt{HarmonicNumber}[n, s] - n^{1-s}\, s\,\texttt{HarmonicNumber}[n, 1-s]\right) s\,(1-s)\,\texttt{Gamma}\left[\frac{1}{2}-\frac{s}{2}\right]\right.$$

$$\text{Gamma}\left[\frac{s}{2}\right]\Big) \Big/ \left(2\, n^{1-s}\, \pi^{\frac{1-s}{2}}\, s\, \text{Gamma}\left[\frac{s}{2}\right] - 2\, n^s\, \pi^{s/2}\, (1-s)\, \text{Gamma}\left[\frac{1}{2} - \frac{s}{2}\right]\right)$$

$$\text{so10x3}[n\_, s\_] := \Big(\big(n^s\, (1-s)\, \text{HarmonicNumber}[n, s] - n^{1-s}\, s\, \text{HarmonicNumber}[n, 1-s]\big)\, s\, (1-s)\, \text{Gamma}\left[\frac{1}{2} - \frac{s}{2}\right] \text{Gamma}\left[\frac{s}{2}\right]\Big) \Big/ \left(2\, n^{1-s}\, \pi^{\frac{1-s}{2}}\, s\, \text{Gamma}\left[\frac{s}{2}\right] - 2\, n^s\, \pi^{s/2}\, (1-s)\, \text{Gamma}\left[\frac{1}{2} - \frac{s}{2}\right]\right)$$

$$\text{so14x}[n\_, s\_] := \frac{n^s\, (1-s)\, \text{HarmonicNumber}[n, s] - n^{1-s}\, s\, \text{HarmonicNumber}[n, 1-s]}{\frac{n^{1-s}\, \pi^{\frac{(1-s)}{2}}}{\text{Gamma}\left[1+\frac{1-s}{2}\right]} - \frac{n^s\, \pi^{s/2}}{\text{Gamma}\left[1+\frac{s}{2}\right]}}$$

$$\text{so14x2}[n\_, s\_] := \Big(\big(n^s\, (1-s)\, \text{HarmonicNumber}[n, s] - n^{1-s}\, s\, \text{HarmonicNumber}[n, 1-s]\big)\, \text{Gamma}\left[1 + \frac{1-s}{2}\right] \text{Gamma}\left[1 + \frac{s}{2}\right]\Big) \Big/ \left(n^{1-s}\, \pi^{\frac{(1-s)}{2}}\, \text{Gamma}\left[1 + \frac{s}{2}\right] - n^s\, \pi^{s/2}\, \text{Gamma}\left[1 + \frac{1-s}{2}\right]\right)$$

$$\text{zet10x}[n\_, s\_] := \text{so10x3}[n, s] / \text{fla}[s]$$
$$\text{zet14x}[n\_, s\_] := \text{so14x2}[n, s] / \text{fla}[s]$$

$$\text{so14y}[n\_, s\_] := \frac{n^s\, (1-s)\, \text{HarmonicNumber}[n, s]}{\frac{n^{1-s}\, \pi^{\frac{(1-s)}{2}}}{\text{Gamma}\left[1+\frac{1-s}{2}\right]} - \frac{n^s\, \pi^{s/2}}{\text{Gamma}\left[1+\frac{s}{2}\right]}} - \frac{n^{1-s}\, s\, \text{HarmonicNumber}[n, 1-s]}{\frac{n^{1-s}\, \pi^{\frac{(1-s)}{2}}}{\text{Gamma}\left[1+\frac{1-s}{2}\right]} - \frac{n^s\, \pi^{s/2}}{\text{Gamma}\left[1+\frac{s}{2}\right]}}$$

$$\text{so14y2}[n\_, s\_] := \frac{1}{\frac{n^{1-2s}\, \pi^{\frac{(1-s)}{2}}}{\text{Gamma}\left[1+\frac{1-s}{2}\right]\, (1-s)} - \frac{\pi^{s/2}}{\text{Gamma}\left[1+\frac{s}{2}\right]\, (1-s)}}\, \text{HarmonicNumber}[n, s] - \frac{1}{\frac{\pi^{\frac{(1-s)}{2}}}{\text{Gamma}\left[1+\frac{1-s}{2}\right]\, s} - \frac{n^{2s-1}\, \pi^{s/2}}{\text{Gamma}\left[1+\frac{s}{2}\right]\, s}}\, \text{HarmonicNumber}[n, 1-s]$$

$$\text{so14y3}[n\_, s\_] := \frac{1}{\frac{\text{Gamma}\left[1+\frac{s}{2}\right]\, n^{1-2s}\, \pi^{\frac{(1-s)}{2}} - \text{Gamma}\left[1+\frac{1-s}{2}\right]\, \pi^{s/2}}{\text{Gamma}\left[1+\frac{s}{2}\right]\, \text{Gamma}\left[1+\frac{1-s}{2}\right]\, (1-s)}}\, \text{HarmonicNumber}[n, s] - \frac{1}{\frac{\text{Gamma}\left[1+\frac{s}{2}\right]\, \pi^{\frac{(1-s)}{2}} - \text{Gamma}\left[1+\frac{1-s}{2}\right]\, n^{2s-1}\, \pi^{s/2}}{\text{Gamma}\left[1+\frac{s}{2}\right]\, \text{Gamma}\left[1+\frac{1-s}{2}\right]\, s}}\, \text{HarmonicNumber}[n, 1-s]$$

$$\text{so14y4}[n\_, s\_] := \frac{\text{Gamma}\left[1 + \frac{s}{2}\right] \text{Gamma}\left[1 + \frac{1-s}{2}\right] (1-s)}{\text{Gamma}\left[1 + \frac{s}{2}\right] n^{1-2s}\, \pi^{\frac{(1-s)}{2}} - \text{Gamma}\left[1 + \frac{1-s}{2}\right] \pi^{s/2}}\, \text{HarmonicNumber}[n, s] - \frac{\text{Gamma}\left[1 + \frac{s}{2}\right] \text{Gamma}\left[1 + \frac{1-s}{2}\right] s}{\text{Gamma}\left[1 + \frac{s}{2}\right] \pi^{\frac{(1-s)}{2}} - \text{Gamma}\left[1 + \frac{1-s}{2}\right] n^{2s-1}\, \pi^{s/2}}\, \text{HarmonicNumber}[n, 1-s]$$

$$\text{so14y5}[n\_, s\_] := \frac{\text{Gamma}\left[1 + \frac{s}{2}\right] \text{Gamma}\left[1 + \frac{1-s}{2}\right]}{\text{Gamma}\left[1 + \frac{s}{2}\right] n^{1-s}\, \pi^{\frac{(1-s)}{2}} - \text{Gamma}\left[1 + \frac{1-s}{2}\right] \pi^{s/2}\, n\verb|^|s}$$

$$(1-s)\, n\text{^}s\, \text{HarmonicNumber}[n, s] - \frac{\text{Gamma}\left[1+\frac{s}{2}\right] \text{Gamma}\left[1+\frac{1-s}{2}\right]}{\text{Gamma}\left[1+\frac{s}{2}\right] n\text{^}(1-s)\, \pi^{\frac{(1-s)}{2}} - \text{Gamma}\left[1+\frac{1-s}{2}\right] n^s \pi^{s/2}}\, s\, n\text{^}(1-s)\, \text{HarmonicNumber}[n, 1-s]$$

$$\text{so14y6}[n\_, s\_] := \frac{\text{Gamma}\left[1+\frac{s}{2}\right] \text{Gamma}\left[1+\frac{1-s}{2}\right]}{\text{Gamma}\left[1+\frac{s}{2}\right] n^{1-s} \pi^{\frac{(1-s)}{2}} - \text{Gamma}\left[1+\frac{1-s}{2}\right] \pi^{s/2}\, n\text{^}s}$$

((1 - s) n^s HarmonicNumber[n, s] - s n^(1 - s) HarmonicNumber[n, 1 - s])

zet14y[n_, s_] := so14y6[n, s] / fla[s]

zet14y[100 000, .3 + 6 I]

0.818794 + 0.373409 i

Zeta[.3 + 6 I]

0.81858 + 0.373183 i

so14x2[100 000, 0]

$$\frac{100\,000}{199\,999}$$

$$\text{FullSimplify}\left[\text{Gamma}\left[1+\frac{1-s}{2}\right] \text{Gamma}\left[1+\frac{s}{2}\right]\right]$$

$$\text{Gamma}\left[\frac{3}{2}-\frac{s}{2}\right] \text{Gamma}\left[1+\frac{s}{2}\right]$$

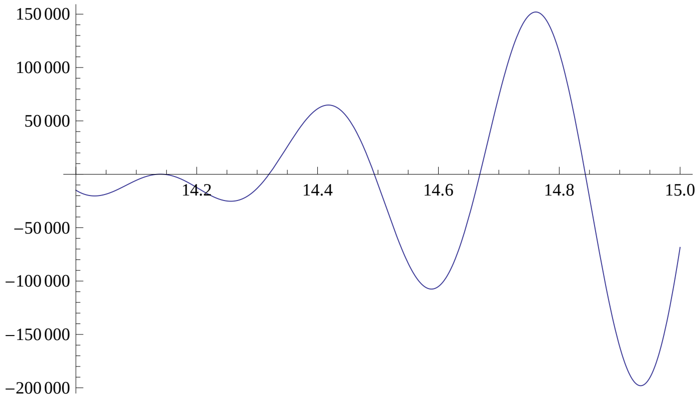
$$\text{sosub2}[n\_, s\_] := \frac{(1-s)\, n\text{^}s\, \text{HarmonicNumber}[n, s]}{\frac{n^{1-s} \pi^{\frac{1-s}{2}}}{\text{Gamma}\left[1+\frac{1-s}{2}\right]} - \frac{n\text{^}s\, \pi^{s/2}}{\text{Gamma}\left[1+\frac{s}{2}\right]}}$$

sosub2a[n_, s_] := (1 - s) n^s HarmonicNumber[n, s]

$$\text{sosub2b}[n\_, s\_] := \frac{n^{1-s} \pi^{\frac{1-s}{2}}}{\text{Gamma}\left[1+\frac{1-s}{2}\right]} - \frac{n\text{^}s\, \pi^{s/2}}{\text{Gamma}\left[1+\frac{s}{2}\right]}$$

$$\text{sosub2x}[n\_, s\_] := \frac{(1-s)\, \text{HarmonicNumber}[n, s]}{\frac{n^{1-2s} \pi^{\frac{1-s}{2}}}{\text{Gamma}\left[1+\frac{1-s}{2}\right]} - \frac{\pi^{s/2}}{\text{Gamma}\left[1+\frac{s}{2}\right]}}$$

```
Plot[Im@(sosub2a[100 000 000, .5 + x I] - sosub2a[100 000 000, 1 - (.5 + x I)]), {x, 14, 15}]
```




```
sosub2x[10 000 000, N@ZetaZero[100] + .2 I] + sosub2x[10 000 000, 1 - (N@ZetaZero[100]) + .2 I]
```

$-7.8426 \times 10^{-78} - 3.21638 \times 10^{-76}\, i$

```
N@ZetaZero[20]
```

$0.5 + 77.1448\, i$

$$\frac{n^{1-2\,s}\,\pi^{\frac{1-s}{2}}}{\text{Gamma}\left[1+\frac{1-s}{2}\right]} \;/.\; n \to 1\,000\,000 \;/.\; s \to \text{N@ZetaZero@100}$$

$-7.01842 \times 10^{78} - 5.52083 \times 10^{77}\, i$

```
Gamma[1 + (1 - s) / 2] (1 - s) /. s → .3
```

0.623806

```
Gamma[(1 - s) / 2] ((1 - s) / 2) (1 - s) /. s → .3
```

0.623806

```
FullSimplify[Gamma[(1 - s) / 2] ((1 - s) / 2) (1 - s)]
```

$$(1-s)\,\text{Gamma}\left[\frac{3}{2}-\frac{s}{2}\right]$$

```
al[s_] := -1 / ((1 / 2) s (1 - s) Pi^(-s / 2) Gamma[s / 2])
al2[s_] := -1 / ((1 / 2) s (1 - s) Pi^(-s / 2) Gamma[s / 2])
al3[s_] := -1 / ((1 / 2) (1 - s) Pi^(-s / 2) Gamma[s / 2])
al4[s_] := -1 / ((1 / 2) Pi^(-s / 2) Gamma[s / 2])
```

$$\text{ssosub[n\_, s\_] :=} \frac{(1-s)\ n\verb|^|s}{\frac{1}{1/2\ (1-s)\ \pi^{-\frac{1-s}{2}}\ \text{Gamma}\left[\frac{1-s}{2}\right]}\ n\verb|^|(1-s) - \frac{1}{1/2\ s\ \pi^{-s/2}\ \text{Gamma}\left[\frac{s}{2}\right]}\ n\verb|^|s}\ \text{HarmonicNumber[n, s]}$$

$$\text{ssosub2[n\_, s\_] :=} \frac{(1-s)\ n\verb|^|s}{(1-s)\ \text{al[s]}\ n\verb|^|s - s\ \text{al[1 - s]}\ n\verb|^|(1-s)}\ \text{HarmonicNumber[n, s]}$$

```
sso10c[n_, s_] := ssosub2[n, s] + ssosub2[n, 1 - s]
szet10c[n_, s_] := al[s] sso10c[n, s]
```

$$\text{ssosub3[n\_, s\_] :=} \frac{s\verb|^|-1\ n\verb|^|s}{\text{al4[s]}\ s\verb|^|-1\ n\verb|^|s - \text{al4[1 - s]}\ (1-s)\verb|^|-1\ n\verb|^|(1-s)}\ \text{HarmonicNumber[n, s]}$$

```
sso10cc[n_, s_] := ssosub3[n, s] + ssosub3[n, 1 - s]
szet10cc[n_, s_] := al4[s] sso10cc[n, s]

szet10cc[10 000 000, .3 + 4 I]
```

0.575751 + 0.10774 i

```
Zeta[.3 + 4 I]
```

0.575756 + 0.10773 i

(*

$$\text{ssosub3[n\_,s\_]:=} \frac{(1-s)\ n\verb|^|s}{(1-s)\ \text{al2[s]}\ n\verb|^|s - s\ \text{al2[1-s]}\ n\verb|^|(1-s)}\ \text{HarmonicNumber[n,s]}$$

```
  sso10cc[n_,s_]:=ssosub3[n,s]+ssosub3[n,1-s]
    szet10cc[n_,s_]:=al2[s] sso10cc[n,s]
```

$$\text{ssosub3[n\_,s\_]:=} \frac{(1-s)\ n\verb|^|s}{\text{al4[s]}/s\ n\verb|^|s - \text{al4[1-s]}/(1-s)\ n\verb|^|(1-s)}\ \text{HarmonicNumber[n,s]}$$

```
     sso10cc[n_,s_]:=ssosub3[n,s]/s/(1-s)+ssosub3[n,1-s]/s/(1-s)
       szet10cc[n_,s_]:=al4[s] sso10cc[n,s]
```

*)

```
FullSimplify[Pi^(-(s / 2)) / Pi^(-(1 - s) / 2)]
```

$\pi^{\frac{1}{2}-s}$

```
FullSimplify[Gamma[s / 2] / Gamma[(1 - s) / 2]] /. s → 2
```

$$-\frac{1}{2\sqrt{\pi}}$$