```
ClearAll["Global`*"]
StrictDivisorsHyperbola[k_, n_, s_] :=
 Sum[ Binomial[k, j] StrictDivisorsHyperbola[j, n / (m^(k - j)), m + 1],
  {m, s, n^(1 / k)}, {j, 0, k - 1}]
StrictDivisorsHyperbola[1, n_, s_] := Floor[n] - s + 1
StrictDivisorsHyperbola[0, n_, s_] := 1
Smalld[A_, k_, n_] :=
 StrictDivisorsHyperbola[k, n, 2] - StrictDivisorsHyperbola[k, n - 1, 2]
StrictDivisorsFullReduced[A_, k_, n_] :=
 Sum[j^A StrictDivisorsHyperbola[k - 1, n / j, 2], {j, Floor[n^(1 / 3)] + 1, n^(1 / 2)}] +
  Sum[Sum[m^A, {m, Floor[n / (j + 1)] + 1, n / j}] StrictDivisorsHyperbola[k - 1, j, 2],
   {j, 1, n / Floor[n^(1 / 2)] - 1}] +
  Sum[Smalld[A, k - 1, j] Sum[m^A, {m, 2, n / j}], {j, 2, n^(1 / 3)}] +
  Sum[s^A Smalld[A, m, j] StrictDivisorsHyperbola[k - m - 1, n / (j s), 2], {j, 2, n^(1 / 3)},
   {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[n / j]^(1 / 2)}, {m, 1, k - 2}] +
  Sum[(Sum[m^A, {m, Floor[n / (j (s + 1))] + 1, n / (j s)}])
     (Sum[Smalld[A, m, j] StrictDivisorsHyperbola[k - m - 1, s, 2], {m, 1, k - 2}]),
   {j, 2, n^(1 / 3)}, {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]
StrictDivisorsFullReduced[A_, 1, n_] := Sum[j^A, {j, 2, n}]
SumPrimesFullReduced[A_, n_] :=
 Sum[(-1)^(k + 1) / (j k) MoebiusMu[j] StrictDivisorsFullReduced[j A, k, n^(1 / j)],
  {j, 1, Log[2, n]}, {k, 1, Log[2, (n^(1 / j))]}]
SumPrimesFullReduced[0, 100]
SumPrimesFullReduced[0, 1000]
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```
ClearAll["Global`*"]
HyperbolaD[k_, n_, s_] :=
 Sum[ Binomial[k, j] HyperbolaD[j, n / (m^(k - j)), m + 1], {m, s, n^(1 / k)}, {j, 0, k - 1}]
HyperbolaD[1, n_, s_] := Floor[n] - s + 1
HyperbolaD[0, n_, s_] := 1
Smalld[A_, k_, n_] := HyperbolaD[k, n, 2] - HyperbolaD[k, n - 1, 2]
StrictDivisorsFullReduced[A_, k_, n_] :=
 Sum[j^A HyperbolaD[k - 1, n / j, 2], {j, Floor[n^(1 / 3)] + 1, n^(1 / 2)}] +
  Sum[Sum[m^A, {m, Floor[n / (j + 1)] + 1, n / j}] HyperbolaD[k - 1, j, 2],
   {j, 1, n / Floor[n^(1 / 2)] - 1}] +
  Sum[Smalld[A, k - 1, j] Sum[m^A, {m, 2, n / j}], {j, 2, n^(1 / 3)}] +
  Sum[s^A Smalld[A, m, j] HyperbolaD[k - m - 1, n / (j s), 2], {j, 2, n^(1 / 3)},
   {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[n / j]^(1 / 2)}, {m, 1, k - 2}] +
  Sum[(Sum[m^A, {m, Floor[n / (j (s + 1))] + 1, n / (j s)}])
     (Sum[Smalld[A, m, j] HyperbolaD[k - m - 1, s, 2], {m, 1, k - 2}]),
    {j, 2, n^(1 / 3)}, {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]
StrictDivisorsFullReduced[A_, 1, n_] := Sum[j^A, {j, 2, n}]
SumPrimesFullReduced[A_, n_] :=
 Sum[(-1)^(k + 1) / (j k) MoebiusMu[j] StrictDivisorsFullReduced[j A, k, n^(1 / j)],
  {j, 1, Log[2, n]}, {k, 1, Log[2, (n^(1 / j))]}]
SumPrimesFullReduced[0, 100]
SumPrimesFullReduced[0, 1000]
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ClearAll["Global`*"]
HyperbolaD[k_, n_, s_] :=
 Sum[ Binomial[k, j] HyperbolaD[j, n / (m^(k - j)), m + 1], {m, s, n^(1 / k)}, {j, 0, k - 1}]
HyperbolaD[1, n_, s_] := Floor[n] - s + 1
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StrictDivisorsFullReduced[A_, k_, n_] :=
 Sum[j^A HyperbolaD[k - 1, n / j, 2], {j, Floor[n^(1 / 3)] + 1, n^(1 / 2)}] +
  Sum[Sum[m^A, {m, Floor[n / (j + 1)] + 1, n / j}] HyperbolaD[k - 1, j, 2],
   {j, 1, n / Floor[n^(1 / 2)] - 1}] +
  Sum[dstrict[A, k - 1, j] Sum[m^A, {m, 2, n / j}], {j, 2, n^(1 / 3)}] +
  Sum[s^A dstrict[A, m, j] HyperbolaD[k - m - 1, n / (j s), 2], {j, 2, n^(1 / 3)},
   {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[n / j]^(1 / 2)}, {m, 1, k - 2}] +
  Sum[(Sum[m^A, {m, Floor[n / (j (s + 1))] + 1, n / (j s)}])
     (Sum[dstrict[A, m, j] HyperbolaD[k - m - 1, s, 2], {m, 1, k - 2}]),
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 Sum[(-1)^(k + 1) / (j k) MoebiusMu[j] StrictDivisorsFullReduced[j A, k, n^(1 / j)],
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HyperbolaD[k_, n_, s_] :=
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HyperbolaD[1, n_, s_] := Floor[n] - s + 1
HyperbolaD[0, n_, s_] := 1
dstrict[k_, n_] := HyperbolaD[k, n, 2] - HyperbolaD[k, n - 1, 2]
DStrictFast[A_, k_, n_] :=
 Sum[j^A HyperbolaD[k - 1, n / j, 2], {j, Floor[n^(1 / 3)] + 1, n^(1 / 2)}] +
  Sum[Sum[m^A, {m, Floor[n / (j + 1)] + 1, n / j}] HyperbolaD[k - 1, j, 2],
   {j, 1, n / Floor[n^(1 / 2)] - 1}] +
  Sum[dstrict[k - 1, j] Sum[m^A, {m, 2, n / j}], {j, 2, n^(1 / 3)}] +
  Sum[s^A dstrict[m, j] HyperbolaD[k - m - 1, n / (j s), 2], {j, 2, n^(1 / 3)},
   {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[n / j]^(1 / 2)}, {m, 1, k - 2}] +
  Sum[(Sum[m^A, {m, Floor[n / (j (s + 1))] + 1, n / (j s)}])
     (Sum[dstrict[m, j] HyperbolaD[k - m - 1, s, 2], {m, 1, k - 2}]),
   {j, 2, n^(1 / 3)}, {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]
DStrictFast[A_, 1, n_] := Sum[j^A, {j, 2, n}]
SumPrimesFullReduced[A_, n_] :=
 Sum[(-1)^(k + 1) / (j k) MoebiusMu[j] DStrictFast[j A, k, n^(1 / j)],
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HyperbolaD[1, n_, s_] := Floor[n] - s + 1
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dstrict[n_, k_] := HyperbolaD[k, n, 2] - HyperbolaD[k, n - 1, 2]
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 Sum[j^A HyperbolaD[k - 1, n / j, 2], {j, Floor[n^(1 / 3)] + 1, n^(1 / 2)}] +
  Sum[Sum[m^A, {m, Floor[n / (j + 1)] + 1, n / j}] HyperbolaD[k - 1, j, 2],
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  Sum[dstrict[j, k - 1] Sum[m^A, {m, 2, n / j}], {j, 2, n^(1 / 3)}] +
  Sum[s^A dstrict[j, m] HyperbolaD[k - m - 1, n / (j s), 2], {j, 2, n^(1 / 3)},
   {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[n / j]^(1 / 2)}, {m, 1, k - 2}] +
  Sum[(Sum[m^A, {m, Floor[n / (j (s + 1))] + 1, n / (j s)}])
     (Sum[dstrict[j, m] HyperbolaD[k - m - 1, s, 2], {m, 1, k - 2}]),
   {j, 2, n^(1 / 3)}, {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]
DStrictFast[A_, 1, n_] := Sum[j^A, {j, 2, n}]
SumPrimesFullReduced[A_, n_] :=
 Sum[(-1)^(k + 1) / (j k) MoebiusMu[j] DStrictFast[j A, k, n^(1 / j)],
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SumPrimesFullReduced[0, 100]
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```
ClearAll["Global`*"]
HyperbolaD[n_, k_, s_] :=
 Sum[ Binomial[k, j] HyperbolaD[n / (m^(k - j)), j, m + 1], {m, s, n^(1 / k)}, {j, 0, k - 1}]
HyperbolaD[n_, 1, s_] := Floor[n] - s + 1
HyperbolaD[n_, 0, s_] := 1
dstrict[n_, k_] := HyperbolaD[n, k, 2] - HyperbolaD[n - 1, k, 2]
DStrictFast[A_, k_, n_] :=
 Sum[j^A HyperbolaD[n / j, k - 1, 2], {j, Floor[n^(1 / 3)] + 1, n^(1 / 2)}] +
  Sum[Sum[m^A, {m, Floor[n / (j + 1)] + 1, n / j}] HyperbolaD[j, k - 1, 2],
   {j, 1, n / Floor[n^(1 / 2)] - 1}] +
  Sum[dstrict[j, k - 1] Sum[m^A, {m, 2, n / j}], {j, 2, n^(1 / 3)}] +
  Sum[s^A dstrict[j, m] HyperbolaD[n / (j s), k - m - 1, 2], {j, 2, n^(1 / 3)},
   {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[n / j]^(1 / 2)}, {m, 1, k - 2}] +
  Sum[(Sum[m^A, {m, Floor[n / (j (s + 1))] + 1, n / (j s)}])
     (Sum[dstrict[j, m] HyperbolaD[s, k - m - 1, 2], {m, 1, k - 2}]),
   {j, 2, n^(1 / 3)}, {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]
DStrictFast[A_, 1, n_] := Sum[j^A, {j, 2, n}]
SumPrimesFullReduced[A_, n_] :=
 Sum[(-1)^(k + 1) / (j k) MoebiusMu[j] DStrictFast[j A, k, n^(1 / j)],
  {j, 1, Log[2, n]}, {k, 1, Log[2, (n^(1 / j))]}]
SumPrimesFullReduced[0, 100]
SumPrimesFullReduced[0, 1000]
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```
ClearAll["Global`*"]
HyperbolaD[n_, k_, s_] :=
 Sum[ Binomial[k, j] HyperbolaD[n / (m^(k - j)), j, m + 1], {m, s, n^(1 / k)}, {j, 0, k - 1}]
HyperbolaD[n_, 1, s_] := Floor[n] - s + 1
HyperbolaD[n_, 0, s_] := 1
dstrict[n_, k_] := HyperbolaD[n, k, 2] - HyperbolaD[n - 1, k, 2]
DStrictFast[A_, k_, n_] :=
 Sum[ HyperbolaD[n / j, k - 1, 2], {j, Floor[n^(1 / 3)] + 1, n^(1 / 2)}] +
  Sum[Sum[1, {m, Floor[n / (j + 1)] + 1, n / j}] HyperbolaD[j, k - 1, 2],
   {j, 1, n / Floor[n^(1 / 2)] - 1}] + Sum[dstrict[j, k - 1] Sum[1, {m, 2, n / j}],
   {j, 2, n^(1 / 3)}] + Sum[dstrict[j, m] HyperbolaD[n / (j s), k - m - 1, 2], {j, 2, n^(1 / 3)},
   {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[n / j]^(1 / 2)}, {m, 1, k - 2}] +
  Sum[(Sum[1, {m, Floor[n / (j (s + 1))] + 1, n / (j s)}])
     (Sum[dstrict[j, m] HyperbolaD[s, k - m - 1, 2], {m, 1, k - 2}]),
   {j, 2, n^(1 / 3)}, {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]
DStrictFast[A_, 1, n_] := Sum[1, {j, 2, n}]
SumPrimesFullReduced[A_, n_] :=
 Sum[(-1)^(k + 1) / (j k) MoebiusMu[j] DStrictFast[1, k, n^(1 / j)],
  {j, 1, Log[2, n]}, {k, 1, Log[2, (n^(1 / j))]}]
SumPrimesFullReduced[0, 100]
SumPrimesFullReduced[0, 1000]
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168
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```
ClearAll["Global`*"]
HyperbolaD[n_, k_, s_] :=
 Sum[ Binomial[k, j] HyperbolaD[n / (m^(k - j)), j, m + 1], {m, s, n^(1 / k)}, {j, 0, k - 1}]
HyperbolaD[n_, 1, s_] := Floor[n] - s + 1
HyperbolaD[n_, 0, s_] := 1
dstrict[n_, k_] := HyperbolaD[n, k, 2] - HyperbolaD[n - 1, k, 2]
DStrictFast[k_, n_] := Sum[ HyperbolaD[n / j, k - 1, 2], {j, Floor[n^(1 / 3)] + 1, n^(1 / 2)}] +
  Sum[Sum[1, {m, Floor[n / (j + 1)] + 1, n / j}] HyperbolaD[j, k - 1, 2],
    {j, 1, n / Floor[n^(1 / 2)] - 1}] + Sum[dstrict[j, k - 1] Sum[1, {m, 2, n / j}],
    {j, 2, n^(1 / 3)}] + Sum[dstrict[j, m] HyperbolaD[n / (j s), k - m - 1, 2], {j, 2, n^(1 / 3)},
    {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[n / j]^(1 / 2)}, {m, 1, k - 2}] +
  Sum[(Sum[1, {m, Floor[n / (j (s + 1))] + 1, n / (j s)}])
     (Sum[dstrict[j, m] HyperbolaD[s, k - m - 1, 2], {m, 1, k - 2}]),
    {j, 2, n^(1 / 3)}, {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]
DStrictFast[1, n_] := Sum[1, {j, 2, n}]
SumPrimesFullReduced[n_] := Sum[(-1)^(k + 1) / (j k) MoebiusMu[j] DStrictFast[k, n^(1 / j)],
  {j, 1, Log[2, n]}, {k, 1, Log[2, (n^(1 / j))]}]
SumPrimesFullReduced[100]
SumPrimesFullReduced[1000]
```

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25
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168
```

```
ClearAll["Global`*"]
HyperbolaD[n_, k_, s_] :=
 Sum[ Binomial[k, j] HyperbolaD[n / (m^(k - j)), j, m + 1], {m, s, n^(1 / k)}, {j, 0, k - 1}]
HyperbolaD[n_, 1, s_] := Floor[n] - s + 1
HyperbolaD[n_, 0, s_] := 1
dstrict[n_, k_] := HyperbolaD[n, k, 2] - HyperbolaD[n - 1, k, 2]
DStrictFast[n_, k_] := Sum[ HyperbolaD[n / j, k - 1, 2], {j, Floor[n^(1 / 3)] + 1, n^(1 / 2)}] +
  Sum[Sum[1, {m, Floor[n / (j + 1)] + 1, n / j}] HyperbolaD[j, k - 1, 2],
    {j, 1, n / Floor[n^(1 / 2)] - 1}] + Sum[dstrict[j, k - 1] Sum[1, {m, 2, n / j}],
    {j, 2, n^(1 / 3)}] + Sum[dstrict[j, m] HyperbolaD[n / (j s), k - m - 1, 2], {j, 2, n^(1 / 3)},
    {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[n / j]^(1 / 2)}, {m, 1, k - 2}] +
  Sum[(Sum[1, {m, Floor[n / (j (s + 1))] + 1, n / (j s)}])
     (Sum[dstrict[j, m] HyperbolaD[s, k - m - 1, 2], {m, 1, k - 2}]),
    {j, 2, n^(1 / 3)}, {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]
DStrictFast[n_, 1] := Sum[1, {j, 2, n}]
RiemannPrimeCountingFast[n_] :=
 Sum[(-1)^(k + 1) / k DStrictFast[n, k], {k, 1, N[Log[n] / Log[2]]}]
N[RiemannPrimeCountingFast[100]]
N[RiemannPrimeCountingFast[1000]]
```

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28.5333
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176.696
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```
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HyperbolaD[n_, k_, s_] :=
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 Sum[ HyperbolaD[n / j, k - 1, 2], {j, Floor[n^(1 / 3)] + 1, n^(1 / 2)}] + Sum[
    (Floor[n / j] - (Floor[n / (j + 1)])) HyperbolaD[j, k - 1, 2], {j, 1, n / Floor[n^(1 / 2)] - 1}] +
  Sum[dstrict[j, k - 1] Sum[1, {m, 2, Floor[n / j]}], {j, 2, n^(1 / 3)}] +
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N[RiemannPrimeCountingFast[100]]
N[RiemannPrimeCountingFast[1000]]
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  Sum[(Floor[n / j] - (Floor[n / (j + 1)])) HyperbolaD[j, k - 1, 2],
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  Sum[dstrict[j, m] HyperbolaD[n / (j s), k - m - 1, 2], {j, 2, n^(1 / 3)},
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