```mathematica
FullSimplify@Sum[Pochhammer[z, j] / j!, {j, 0, n}]
```

$$\frac{\text{Gamma}[1 + n + z]}{\text{Gamma}[1 + n]\ \text{Gamma}[1 + z]}$$

```mathematica
FullSimplify@Sum[j / j!, {j, 1, n}]
```

$$\frac{-1 + e\ \text{Gamma}[1 + n, 1]}{\text{Gamma}[1 + n]}$$

```mathematica
FullSimplify@Sum[j^2 / j!, {j, 1, n}]
```

$$-\frac{2 + n - 2\ e\ \text{Gamma}[1 + n, 1]}{\text{Gamma}[1 + n]}$$

```mathematica
FullSimplify@Sum[j^3 / j!, {j, 1, n}]
```

$$\frac{-5 - n\ (3 + n) + 5\ e\ \text{Gamma}[1 + n, 1]}{\text{Gamma}[1 + n]}$$

```mathematica
Sum[z^j / j!, {j, 1, n}]
```

$$-1 + \frac{e^z\ \text{Gamma}[1 + n, z]}{\text{Gamma}[1 + n]}$$

```mathematica
Pochhammer[z, 3]
```

$$z\ (1 + z)\ (2 + z)$$

```mathematica
Expand@z^3
```

$$z^3$$

```mathematica
Sum[Binomial[z, k] FactorialPower[x, k] / k!, {k, 0, Infinity}]
```

$$\frac{\text{Gamma}[1 + x + z]}{\text{Gamma}[1 + x]\ \text{Gamma}[1 + z]}$$

```mathematica
FactorialPower[x - 1, k - 1] / (k - 1)! /. x -> 18 /. k -> 5
```

2380

```mathematica
Binomial[18 - 1, 5 - 1]
```

2380

```mathematica
Binomial[18, 5] - Binomial[17, 5]
```

2380

```mathematica
Clear[d2]
d2[n_, k_] := d2[n, k] = Sum[d2[Floor[n / j], k - 1], {j, 2, n}]
d2[n_, 0] := UnitStep[n - 1]
dd2[n_, k_] := d2[n, k] - d2[n - 1, k]
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, s_, z_] := n^-s Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}]
dz2[n_, k_] := Sum[(-1)^(k - j) Binomial[k, j] dz[n, 0, j], {j, 0, k}]
```

```mathematica
Table[dd2[2^n, 2], {n, 1, 8}]
```

$$\{0, 1, 2, 3, 4, 5, 6, 7\}$$

```
Table[dd2[30^n, 2], {n, 1, 4}]
```

{6, 25, 62, 123}

```
Table[(n + 1)^3 - 2, {n, 1, 4}]
```

{6, 25, 62, 123}

```
Table[dd2[2^n, 3], {n, 1, 8}]
```

{0, 0, 1, 3, 6, 10, 15, 21}

```
Table[Pochhammer[n - 2, 2] / 2!, {n, 1, 8}]
```

{0, 0, 1, 3, 6, 10, 15, 21}

```
Table[dd2[2^n, 4], {n, 1, 8}]
```

{0, 0, 0, 1, 4, 10, 20, 35}

```
Table[Pochhammer[n - 3, 3] / 3!, {n, 1, 8}]
```

{0, 0, 0, 1, 4, 10, 20, 35}

```
Table[Pochhammer[n - 1, 1] / 1!, {n, 1, 8}]
```

{0, 1, 2, 3, 4, 5, 6, 7}

```
d2a[a_, k_] := Pochhammer[a - (k - 1), (k - 1)] / (k - 1)!
```

```
dd2[2^8, 5]
```

35

```
d2a[8, 5]
```

35

```
FullSimplify[Pochhammer[a - (k - 1), (k - 1)] / (k - 1)!]
```

$$\frac{\text{Gamma}[a]}{\text{Gamma}[1 + a - k]\, \text{Gamma}[k]}$$

```
dd2[6, 3]
```

0

```
d2b[1, 1, 2]
```

0

```
Grid@Table[dz2[6^n, k], {k, 1, 9}, {n, 1, 8}]
```

1	1	1	1	1	1	1	1
2	7	14	23	34	47	62	79
0	12	55	153	336	640	1107	1785
0	6	92	471	1584	4210	9596	19607
0	0	70	780	4251	16175	49225	128345
0	0	20	720	7002	39733	164898	555303
0	0	0	350	7238	65226	380731	1685257
0	0	0	70	4592	72660	623576	3716695
0	0	0	0	1638	54390	732618	6077196

```
Grid@Table[dz2[3 × 2^n, k], {k, 1, 9}, {n, 1, 8}]
```

1	1	1	1	1	1	1	1
2	4	6	8	10	12	14	16
0	3	9	18	30	45	63	84
0	0	4	16	40	80	140	224
0	0	0	5	25	75	175	350
0	0	0	0	6	36	126	336
0	0	0	0	0	7	49	196
0	0	0	0	0	0	8	64
0	0	0	0	0	0	0	9

```
Grid@Table[dz2[3 × 2^n, k], {k, 1, 9}, {n, 1, 8}]
```

1	1	1	1	1	1	1	1
2	4	6	8	10	12	14	16
0	3	9	18	30	45	63	84
0	0	4	16	40	80	140	224
0	0	0	5	25	75	175	350
0	0	0	0	6	36	126	336
0	0	0	0	0	7	49	196
0	0	0	0	0	0	8	64
0	0	0	0	0	0	0	9

```
Grid@Table[k Binomial[a, k - 1], {k, 1, 9}, {a, 1, 8}]
```

1	1	1	1	1	1	1	1
2	4	6	8	10	12	14	16
0	3	9	18	30	45	63	84
0	0	4	16	40	80	140	224
0	0	0	5	25	75	175	350
0	0	0	0	6	36	126	336
0	0	0	0	0	7	49	196
0	0	0	0	0	0	8	64
0	0	0	0	0	0	0	9

```
Grid@Table[dz2[3^2 × 2^n, k], {k, 1, 9}, {n, 1, 8}]
```

1	1	1	1	1	1	1	1
4	7	10	13	16	19	22	25
3	12	27	48	75	108	147	192
0	6	28	76	160	290	476	728
0	0	10	55	175	425	875	1610
0	0	0	15	96	351	966	2226
0	0	0	0	21	154	637	1960
0	0	0	0	0	28	232	1072
0	0	0	0	0	0	36	333

```
Grid@Table[ dz2[3 × 5 × 2^n, k], {k, 1, 9}, {n, 1, 8}]
```

1	1	1	1	1	1	1	1
6	10	14	18	22	26	30	34
6	21	45	78	120	171	231	300
0	12	52	136	280	500	812	1232
0	0	20	105	325	775	1575	2870
0	0	0	30	186	666	1806	4116
0	0	0	0	42	301	1225	3724
0	0	0	0	0	56	456	2080
0	0	0	0	0	0	72	657

```
Grid@Table[(k n + 1) k Binomial[n + 1, k - 1] / (n + 1), {k, 1, 9}, {n, 1, 8}]
```

1	1	1	1	1	1	1	1
6	10	14	18	22	26	30	34
6	21	45	78	120	171	231	300
0	12	52	136	280	500	812	1232
0	0	20	105	325	775	1575	2870
0	0	0	30	186	666	1806	4116
0	0	0	0	42	301	1225	3724
0	0	0	0	0	56	456	2080
0	0	0	0	0	0	72	657

```
(k n + 1) k Binomial[n + 1, k - 1] / (n + 1)
```

$$\frac{k\,(1 + k\,n)\,\mathrm{Binomial}[1 + n,\, -1 + k]}{1 + n}$$

```
(k n + 1) k / (n + 1)
```

$$\frac{k\,(1 + k\,n)}{1 + n}$$

```
(k n + 1) k Binomial[n + 1, k - 1] / (n + 1) /. n → a
```

$$\frac{k\,(1 + a\,k)\,\mathrm{Binomial}[1 + a,\, -1 + k]}{1 + a}$$

```
Grid@Table[dz2[3^2 × 2^n, k], {k, 1, 7}, {n, 1, 10}]
```

1	1	1	1	1	1	1	1	1	1
4	7	10	13	16	19	22	25	28	31
3	12	27	48	75	108	147	192	243	300
0	6	28	76	160	290	476	728	1056	1470
0	0	10	55	175	425	875	1610	2730	4350
0	0	0	15	96	351	966	2226	4536	8442
0	0	0	0	21	154	637	1960	4998	11172

```
Grid@Table[(n k (k + 1) / 2 - k (k - 3) / 2) (Binomial[n + 1, k - 1] / (n + 1)), {k, 1, 7}, {n, 1, 10}]
```

1	1	1	1	1	1	1	1	1	1
4	7	10	13	16	19	22	25	28	31
3	12	27	48	75	108	147	192	243	300
0	6	28	76	160	290	476	728	1056	1470
0	0	10	55	175	425	875	1610	2730	4350
0	0	0	15	96	351	966	2226	4536	8442
0	0	0	0	21	154	637	1960	4998	11172

```
FullSimplify@Expand[(n k (k + 1) / 2 - k (k - 3) / 2) (Binomial[n + 1, k - 1] / (n + 1))] /. n → a
```

$$\frac{k\,(3 + a + (-1 + a)\,k)\,\mathrm{Binomial}[1 + a,\, -1 + k]}{2\,(1 + a)}$$

```
FullSimplify[ (k (3 + a + (-1 + a) k) Binomial[1 + a, -1 + k]) / (2 (1 + a)) /. a → 2]
```

```
FullSimplify@Expand[k (1 + a k) Binomial[1 + a, -1 + k] / (1 + a) /. a → 1]
```

$\frac{1}{2}$ k (1 + k) Binomial[2, -1 + k]

```
Table[k (1 + k) / 2 Binomial[2, k - 1], {k, 0, 5}]
```

{0, 1, 6, 6, 0, 0}

```
Binomial[ k + 1, k - 1]
```

$\frac{1}{2}$ k (1 + k)

```
Table[Binomial[k, k - 1], {k, 0, 5}]
```

{0, 1, 2, 3, 4, 5}

```
FullSimplify[Binomial[z, k] (z + 1) / (z - k)]
```

$\frac{(1 + z) \, \text{Binomial}[z, k]}{-k + z}$

```
Table[k Binomial[1, k - 1], {k, 0, 5}]
```

{0, 1, 2, 0, 0, 0}

```
Table[(k) (3 - k) / 2, {k, 1, 6}]
```

{1, 1, 0, -2, -5, -9}

$\frac{k (3 + a + (-1 + a) k) \, \text{Binomial}[1 + a, -1 + k]}{2 (1 + a)}$ /. a → 2

```
Table[1/6 k (5 + k) Binomial[3, -1 + k], {k, 0, 5}]
```

{0, 1, 7, 12, 6, 0}

```
Table[dz2[6^2, j], {j, 0, 5}]
```

{0, 1, 7, 12, 6, 0}

```
Table[1/6 k (5 + k) FactorialPower[3, -1 + k] / (k - 1) !, {k, 0, 5}]
```

{0, 1, 7, 12, 6, 0}

```
FullSimplify[1/6 k (5 + k) FactorialPower[3, -1 + k] / (k - 1) !]
```

$\frac{k (5 + k) \, \text{FactorialPower}[3, -1 + k]}{6 \, \text{Gamma}[k]}$

```
FactorialPower[3, -1 + k]
```

FactorialPower[3, -1 + k]

```
Table[FactorialPower[3, -1 + k], {k, 0, 5}]
```

$\{\frac{1}{4}, 1, 3, 6, 6, 0\}$

```
binx[z_, k_] := Binomial[z, k]
bin[z_, k_] := Gamma[z + 1] / Gamma[z - k + 1] / Gamma[k + 1]
da[a_, k_] := bin[a - 1, k - 1]
da1[a_, k_] := k bin[a, k - 1]
da11[a_, k_] := k (1 + a k) / (1 + a) bin[a + 1, k - 1]
da2[a_, k_] := k ((3 + a + (a - 1) k) / (2 (a + 1))) bin[a + 1, k - 1]
d2f[z_] :=
 da[1, z] + da[1, z] + da[2, z] + da[1, z] + da1[1, z] + da[1, z] + da[3, z] + da[2, z] + da1[1, z]
d2f2[z_] := d2f[z] + da[1, z] + da1[2, z] + da[1, z] + da1[1, z] +
   da1[1, z] + da[4, z] + da[1, z] + da1[2, z] + da[1, z] + da1[2, z]
d2f3[z_] := d2f2[z] + da1[1, z] + da1[1, z] + da[1, z] + da1[3, z] +
   da[2, z] + da1[1, z] + da[3, z] + da1[2, z] + da[1, z] + da11[1, z]
Clear[v]
v[z_, 1] := 0
v[z_, 2] := v[z, 1] + da[1, z]
v[z_, 3] := v[z, 2] + da[1, z]
v[z_, 4] := v[z, 3] + da[2, z]
v[z_, 5] := v[z, 4] + da[1, z]
v[z_, 6] := v[z, 5] + da1[1, z]
v[z_, 7] := v[z, 6] + da[1, z]
v[z_, 8] := v[z, 7] + da[3, z]
v[z_, 9] := v[z, 8] + da[2, z]
v[z_, 10] := v[z, 10] = v[z, 9] + da1[1, z]
v[z_, 11] := v[z, 10] + da[1, z]
v[z_, 12] := v[z, 11] + da1[2, z]
v[z_, 13] := v[z, 12] + da[1, z]
v[z_, 14] := v[z, 13] + da1[1, z]
v[z_, 15] := v[z, 14] + da1[1, z]
v[z_, 16] := v[z, 15] + da[4, z]
v[z_, 17] := v[z, 16] + da[1, z]
v[z_, 18] := v[z, 17] + da1[2, z]
v[z_, 19] := v[z, 18] + da[1, z]
v[z_, 20] := v[z, 20] = v[z, 19] + da1[2, z]
v[z_, 21] := v[z, 20] + da1[1, z]
v[z_, 22] := v[z, 21] + da1[1, z]
v[z_, 23] := v[z, 22] + da[1, z]
v[z_, 24] := v[z, 23] + da1[3, z]
v[z_, 25] := v[z, 24] + da[2, z]
v[z_, 26] := v[z, 25] + da1[1, z]
v[z_, 27] := v[z, 26] + da[3, z]
v[z_, 28] := v[z, 27] + da1[2, z]
v[z_, 29] := v[z, 28] + da[1, z]
v[z_, 30] := v[z, 30] = v[z, 29] + da11[1, z]
v[z_, 31] := v[z, 30] + da[1, z]
v[z_, 32] := v[z, 31] + da[5, z]
v[z_, 33] := v[z, 32] + da1[1, z]
v[z_, 34] := v[z, 33] + da1[1, z]
v[z_, 35] := v[z, 34] + da1[1, z]
v[z_, 36] := v[z, 35] + da2[2, z]
v[z_, 37] := v[z, 36] + da[1, z]
```

```mathematica
v[z_, 38] := v[z, 37] + da1[1, z]
v[z_, 39] := v[z, 38] + da1[1, z]
v[z_, 40] := v[z, 40] = v[z, 39] + da1[3, z]
v[z_, 41] := v[z, 40] + da[1, z]
v[z_, 42] := v[z, 41] + da11[1, z]
v[z_, 43] := v[z, 42] + da[1, z]
v[z_, 44] := v[z, 43] + da1[2, z]
v[z_, 45] := v[z, 44] + da1[2, z]
v[z_, 46] := v[z, 45] + da1[1, z]
v[z_, 47] := v[z, 46] + da[1, z]
v[z_, 48] := v[z, 47] + da1[4, z]
v[z_, 49] := v[z, 48] + da[2, z]
v[z_, 50] := v[z, 50] = v[z, 49] + da1[2, z]
v[z_, 51] := v[z, 50] + da1[1, z]
v[z_, 52] := v[z, 51] + da1[2, z]
v[z_, 53] := v[z, 52] + da[1, z]
v[z_, 54] := v[z, 53] + da1[3, z]
v[z_, 55] := v[z, 54] + da1[1, z]
v[z_, 56] := v[z, 55] + da1[3, z]
v[z_, 57] := v[z, 56] + da1[1, z]
v[z_, 58] := v[z, 57] + da1[1, z]
v[z_, 59] := v[z, 58] + da[1, z]
v[z_, 60] := v[z, 60] = v[z, 59] + da11[2, z]
v[z_, 61] := v[z, 60] + da[1, z]
v[z_, 62] := v[z, 61] + da1[1, z]
v[z_, 63] := v[z, 62] + da1[2, z]
v[z_, 64] := v[z, 63] + da[6, z]
v[z_, 65] := v[z, 64] + da1[1, z]
v[z_, 66] := v[z, 65] + da11[1, z]
v[z_, 67] := v[z, 66] + da[1, z]
v[z_, 68] := v[z, 67] + da1[2, z]
v[z_, 69] := v[z, 68] + da1[1, z]
v[z_, 70] := v[z, 70] = v[z, 69] + da11[1, z]
```

```mathematica
Table[FactorInteger[k], {k, 61, 70}]
```

```
{{{61, 1}}, {{2, 1}, {31, 1}}, {{3, 2}, {7, 1}},
 {{2, 6}}, {{5, 1}, {13, 1}}, {{2, 1}, {3, 1}, {11, 1}}, {{67, 1}},
 {{2, 2}, {17, 1}}, {{3, 1}, {23, 1}}, {{2, 1}, {5, 1}, {7, 1}}}
```

```mathematica
{Limit[D[FullSimplify@v[z, 70], z], z → 0],
 Sum[FullSimplify[MangoldtLambda[j] / Log[j]], {j, 2, 70}]}
```

$$\left\{ \frac{1337}{60}, \frac{1337}{60} \right\}$$

```mathematica
Table[d2[30, k], {k, 1, 6}]
```

```
{29, 52, 32, 5, 0, 0}
```

```mathematica
Table[d2f3[k], {k, 1, 6}]
```
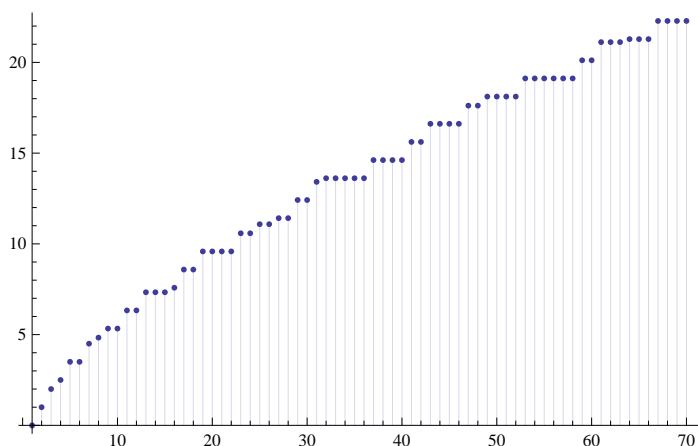
```
{29, 52, 32, 5, 0, 0}
```

Plot[v[x, 63], {x, 0, 7}]
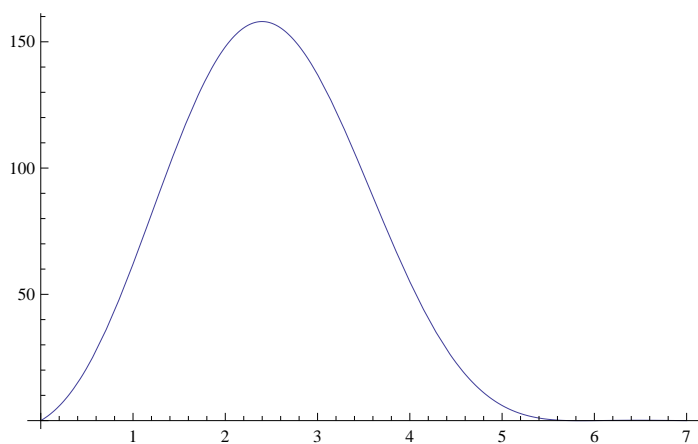
DiscretePlot[Limit[D[v[z, n], z], z → 0], {n, 1, 70}]

Sum[FullSimplify[MangoldtLambda[j] / Log[j]], {j, 2, 35}]

$$\frac{817}{60}$$

FullSimplify@d2f3[z]

$$\frac{(298 + z \, (-159 + (39 - 4 \, z) \, z)) \, \text{Sin}[\pi \, z]}{\pi \, (-4 + z) \, (-3 + z) \, (-2 + z) \, (-1 + z)}$$

Expand@$\dfrac{(298 + z \, (-159 + (39 - 4 \, z) \, z))}{(-4 + z) \, (-3 + z) \, (-2 + z) \, (-1 + z)}$

$$\frac{298}{(-4 + z) \, (-3 + z) \, (-2 + z) \, (-1 + z)} - \frac{159 \, z}{(-4 + z) \, (-3 + z) \, (-2 + z) \, (-1 + z)} +$$

$$\frac{39 \, z^2}{(-4 + z) \, (-3 + z) \, (-2 + z) \, (-1 + z)} - \frac{4 \, z^3}{(-4 + z) \, (-3 + z) \, (-2 + z) \, (-1 + z)}$$

d2[30, 5]

0

```
Limit[D[(298 + z (-159 + (39 - 4 z) z)) / ((-4 + z) (-3 + z) (-2 + z) (-1 + z)), z], z → 0]
```

$$\frac{2771}{144}$$

```
Limit[D[(298 + z (-159 + (39 - 4 z) z)) Sin[π z] / (π (-4 + z) (-3 + z) (-2 + z) (-1 + z)), z], z → 0]
```

$$\frac{149}{12}$$

```
Expand[(298 + z (-159 + (39 - 4 z) z))]
```

$$298 - 159 z + 39 z^2 - 4 z^3$$

```
d2f3[z]
```

```
Table[FullSimplify@Limit[D[d2f3[z], {z, k}], z → 0], {k, 0, 5}]
```

Limit::ztest1 : Unable to decide whether numeric quantity

$$-\frac{35245}{10368} - \frac{145}{12} \text{PolyGamma}[2, 2] + \frac{15}{2} \text{PolyGamma}[2, 3] + \frac{10}{3} \text{PolyGamma}[2, 4] + \frac{5}{4} \text{PolyGamma}[2, 5]$$

is equal to zero. Assuming it is. ≫

$$\left\{0, \frac{149}{12}, \frac{2771}{72}, \frac{1}{288} \left(40\,793 - 3576\,\pi^2\right), \frac{17}{864} \left(31\,867 - 3912\,\pi^2\right), \frac{34\,216\,645 - 4\,895\,160\,\pi^2 + 128\,736\,\pi^4}{10\,368}\right\}$$

```
d2f3[z]
```

```
10 Binomial[0, -1 + z] + 3 Binomial[1, -1 + z] +
  7 z Binomial[1, -1 + z] + 2 Binomial[2, -1 + z] + 4 z Binomial[2, -1 + z] +
  1
  — z (1 + z) Binomial[2, -1 + z] + Binomial[3, -1 + z] + z Binomial[3, -1 + z]
  2
```

```
FullSimplify@d2f[z]
```

```
4 Binomial[0, -1 + z] + 2 (1 + z) Binomial[1, -1 + z] + Binomial[2, -1 + z]
```

```
Table[Expand@FullSimplify@v[z, n], {n, 15, 17}]
```

$$\left\{\frac{44}{\text{Gamma}[4 - z] \text{Gamma}[z]} - \frac{18 z}{\text{Gamma}[4 - z] \text{Gamma}[z]} + \frac{2 z^2}{\text{Gamma}[4 - z] \text{Gamma}[z]}, \right.$$

$$\frac{182}{\text{Gamma}[5 - z] \text{Gamma}[z]} - \frac{116 z}{\text{Gamma}[5 - z] \text{Gamma}[z]} + \frac{26 z^2}{\text{Gamma}[5 - z] \text{Gamma}[z]} - \frac{2 z^3}{\text{Gamma}[5 - z] \text{Gamma}[z]},$$

$$\frac{206 \text{Sin}[\pi z]}{\pi (-4 + z) (-3 + z) (-2 + z) (-1 + z)} - \frac{142 z \text{Sin}[\pi z]}{\pi (-4 + z) (-3 + z) (-2 + z) (-1 + z)} +$$

$$\left. \frac{35 z^2 \text{Sin}[\pi z]}{\pi (-4 + z) (-3 + z) (-2 + z) (-1 + z)} - \frac{3 z^3 \text{Sin}[\pi z]}{\pi (-4 + z) (-3 + z) (-2 + z) (-1 + z)}\right\}$$

```
Table[d2[50, k], {k, 0, 6}]
```

$$\{1, 49, 108, 92, 35, 6, 0\}$$

```
Table[v[k, 50], {k, 0, 6}]
```

$$\{0, 49, 108, 92, 35, 6, 0\}$$

```
Grid@Table[d2[n, k], {n, 1, 10}, {k, 1, 6}]
```

```
0  0  0  0  0  0
1  0  0  0  0  0
2  0  0  0  0  0
3  1  0  0  0  0
4  1  0  0  0  0
5  3  0  0  0  0
6  3  0  0  0  0
7  5  1  0  0  0
8  6  1  0  0  0
9  8  1  0  0  0
```

```
Solve[{ c1 - c2 == v1,
        c1 - 2 c2 == v2},
       {c1, c2}] /. v1 → D1 /. v2 → D2 /. v3 → D3 /. v4 → D4 /. v5 → D5 /. v6 → D6
```

$\{\{c1 \to 2 D1 - D2, c2 \to D1 - D2\}\}$

```
Solve[{ c1 - c2 + c3 == 2 v1,
        c1 - 2 c2 + 4 c3 == v2,
        c1 - 3 c2 + 9 c3 == 2 v3},
       {c1, c2, c3}] /. v1 → D1 /. v2 → D2 /. v3 → D3 /. v4 → D4 /. v5 → D5 /. v6 → D6
```

$\{\{c1 \to 6 D1 - 3 D2 + 2 D3, c2 \to 5 D1 - 4 D2 + 3 D3, c3 \to D1 - D2 + D3\}\}$

```
Expand@Solve[{ c1 - c2 + c3 - c4 == 6 v1,
        c1 - 2 c2 + 4 c3 - 8 c4 == 2 v2,
        c1 - 3 c2 + 9 c3 - 27 c4 == 2 v3,
        c1 - 4 c2 + 16 c3 - 64 c4 == 6 v4},
       {c1, c2, c3, c4}] /. v1 → D1 /. v2 → D2 /. v3 → D3 /. v4 → D4 /. v5 → D5 /. v6 → D6
```

$\{\{c1 \to 24 D1 - 12 D2 + 8 D3 - 6 D4, c2 \to 26 D1 - 19 D2 + 14 D3 - 11 D4,$
$c3 \to 9 D1 - 8 D2 + 7 D3 - 6 D4, c4 \to D1 - D2 + D3 - D4\}\}$

```
Expand@Solve[{ c1 - c2 + c3 - c4 + c5 == 24 v1,
        c1 - 2 c2 + 4 c3 - 8 c4 + 16 c5 == 6 v2,
        c1 - 3 c2 + 9 c3 - 27 c4 + 81 c5 == 4 v3,
        c1 - 4 c2 + 16 c3 - 64 c4 + 256 c5 == 6 v4,
        c1 - 5 c2 + 25 c3 - 125 c4 + 625 c5 == 24 v5},
       {c1, c2, c3, c4, c5}] /. v1 → D1 /. v2 → D2 /. v3 → D3 /. v4 → D4 /. v5 → D5 /. v6 → D6
```

$\{\{c1 \to 120 D1 - 60 D2 + 40 D3 - 30 D4 + 24 D5,$
$c2 \to 154 D1 - 107 D2 + 78 D3 - 61 D4 + 50 D5, c3 \to 71 D1 - 59 D2 + 49 D3 - 41 D4 + 35 D5,$
$c4 \to 14 D1 - 13 D2 + 12 D3 - 11 D4 + 10 D5, c5 \to D1 - D2 + D3 - D4 + D5\}\}$

```
Limit[{1 / (Gamma[5 - z] Gamma[z]), Sin[Pi z] / (Pi FactorialPower[z - 1, 4] )}, z → 3 + I]
```

$\left\{\dfrac{\left(\frac{1}{5} - \frac{i}{10}\right) \operatorname{Sinh}[\pi]}{\pi}, \dfrac{\left(\frac{1}{5} - \frac{i}{10}\right) \operatorname{Sinh}[\pi]}{\pi}\right\}$

```
Table[Gamma[6 - z] Gamma[z], {z, 1, 5}]
```

$\{24, 6, 4, 6, 24\}$

```
Table[Gamma[4 - z] Gamma[z], {z, 1, 3}]
```

{2, 1, 2}

$$\text{Expand@FullSimplify}\Big[D\Big[\frac{182}{\text{Gamma}[5-z]\,\text{Gamma}[z]} -$$

$$\frac{116\,z}{\text{Gamma}[5-z]\,\text{Gamma}[z]} + \frac{26\,z^2}{\text{Gamma}[5-z]\,\text{Gamma}[z]} - \frac{2\,z^3}{\text{Gamma}[5-z]\,\text{Gamma}[z]}, z\Big]\Big]$$

$$-\frac{116}{\text{Gamma}[5-z]\,\text{Gamma}[z]} + \frac{52\,z}{\text{Gamma}[5-z]\,\text{Gamma}[z]} - \frac{6\,z^2}{\text{Gamma}[5-z]\,\text{Gamma}[z]} + \frac{182\,\text{PolyGamma}[0, 5-z]}{\text{Gamma}[5-z]\,\text{Gamma}[z]} -$$

$$\frac{116\,z\,\text{PolyGamma}[0, 5-z]}{\text{Gamma}[5-z]\,\text{Gamma}[z]} + \frac{26\,z^2\,\text{PolyGamma}[0, 5-z]}{\text{Gamma}[5-z]\,\text{Gamma}[z]} - \frac{2\,z^3\,\text{PolyGamma}[0, 5-z]}{\text{Gamma}[5-z]\,\text{Gamma}[z]} -$$

$$\frac{182\,\text{PolyGamma}[0, z]}{\text{Gamma}[5-z]\,\text{Gamma}[z]} + \frac{116\,z\,\text{PolyGamma}[0, z]}{\text{Gamma}[5-z]\,\text{Gamma}[z]} - \frac{26\,z^2\,\text{PolyGamma}[0, z]}{\text{Gamma}[5-z]\,\text{Gamma}[z]} + \frac{2\,z^3\,\text{PolyGamma}[0, z]}{\text{Gamma}[5-z]\,\text{Gamma}[z]}$$

```
{182 / Gamma[5], Sum[FullSimplify[MangoldtLambda[j] / Log[j]], {j, 2, 16}]}
```

$$\Big\{\frac{91}{12}, \frac{91}{12}\Big\}$$

```
Clear[d2]
d2o[n_, k_] := d22[n, k]
d2[n_, k_] := d2[n, k] = Sum[d2[Floor[n / j], k - 1], {j, 2, n}]
d2[n_, 0] := UnitStep[n - 1]
co[n_, 1] := 120 d2[n, 1] - 60 d2[n, 2] + 40 d2[n, 3] - 30 d2[n, 4] + 24 d2[n, 5]
co[n_, 2] := 154 d2[n, 1] - 107 d2[n, 2] + 78 d2[n, 3] - 61 d2[n, 4] + 50 d2[n, 5]
co[n_, 3] := 71 d2[n, 1] - 59 d2[n, 2] + 49 d2[n, 3] - 41 d2[n, 4] + 35 d2[n, 5]
co[n_, 4] := 14 d2[n, 1] - 13 d2[n, 2] + 12 d2[n, 3] - 11 d2[n, 4] + 10 d2[n, 5]
co[n_, 5] := d2[n, 1] - d2[n, 2] + d2[n, 3] - d2[n, 4] + d2[n, 5]
d2z5[n_, z_] :=
  (co[n, 1] - co[n, 2] z + co[n, 3] z^2 - co[n, 4] z^3 + co[n, 5] z^4) / (Gamma[z] Gamma[6 - z])
```

```
d2[100, 1]
```

99

```
FullSimplify@Expand@d2z5[10, z]
```

((-5 + z) (-4 + z) (-3 + z) (-2 + z) d22[10, 1] +
   (-1 + z) (- (-5 + z) (-4 + z) (-3 + z) d22[10, 2] + (-2 + z) ((-5 + z) ((-4 + z) d22[10, 3] -
            (-3 + z) d22[10, 4]) + (-4 + z) (-3 + z) d22[10, 5]))) / (Gamma[6 - z] Gamma[z])

```
Table[d2[10, k], {k, 1, 6}]
```

{9, 8, 1, 0, 0, 0}

(-5 + z) (-4 + z) (-3 + z) (-2 + z) (0) + (-1 + z) (- (-5 + z) (-4 + z) (-3 + z) (0) +
   (-2 + z) ((-5 + z) ((-4 + z) (0) - (-3 + z) (0)) + (-4 + z) (-3 + z) (0)))

0

```
(-5 + z) (-4 + z) (-3 + z) (-2 + z) (A) + (-1 + z) (- (-5 + z) (-4 + z) (-3 + z) (0) +
    (-2 + z) ((-5 + z) ((-4 + z) (0) - (-3 + z) (0)) + (-4 + z) (-3 + z) (0)))
```

A (-5 + z) (-4 + z) (-3 + z) (-2 + z)

```
(-5 + z) (-4 + z) (-3 + z) (-2 + z) (0) + (-1 + z) (- (-5 + z) (-4 + z) (-3 + z) (B) +
    (-2 + z) ((-5 + z) ((-4 + z) (0) - (-3 + z) (0)) + (-4 + z) (-3 + z) (0)))
```

B (5 - z) (-4 + z) (-3 + z) (-1 + z)

```
(-5 + z) (-4 + z) (-3 + z) (-2 + z) (0) + (-1 + z) (- (-5 + z) (-4 + z) (-3 + z) (0) +
    (-2 + z) ((-5 + z) ((-4 + z) (C) - (-3 + z) (0)) + (-4 + z) (-3 + z) (0)))
```

C (-5 + z) (-4 + z) (-2 + z) (-1 + z)

```
(-5 + z) (-4 + z) (-3 + z) (-2 + z) (0) + (-1 + z) (- (-5 + z) (-4 + z) (-3 + z) (0) +
    (-2 + z) ((-5 + z) ((-4 + z) (0) - (-3 + z) (D)) + (-4 + z) (-3 + z) (0)))
```

-D (-5 + z) (-3 + z) (-2 + z) (-1 + z)

```
(-5 + z) (-4 + z) (-3 + z) (-2 + z) (0) + (-1 + z) (- (-5 + z) (-4 + z) (-3 + z) (0) +
    (-2 + z) ((-5 + z) ((-4 + z) (0) - (-3 + z) (0)) + (-4 + z) (-3 + z) (F)))
```

F (-4 + z) (-3 + z) (-2 + z) (-1 + z)

FullSimplify[FactorialPower[z, 5] / Gamma[6 - z] / Gamma[z]]

$$\frac{z \, \text{Sin}[\pi \, z]}{5 \, \pi - \pi \, z}$$

FullSimplify[FactorialPower[z, 6] / Gamma[7 - z] / Gamma[z]]

$$\frac{z \, \text{Sin}[\pi \, z]}{\pi \, (-6 + z)}$$

```
bbo[n_, z_] := Sum[
   (-1) ^ (k + 1) / (z - k) (FactorialPower[z - 1, 5] / Gamma[6 - z] / Gamma[z]) d2[n, k], {k, 1, 5}]
bb2[n_, z_] := Sin[Pi z] / Pi Sum[(-1) ^k d2[n, k] / (z - k), {k, 1, 5}]
bb3[n_, z2_] := Limit[Sin[Pi z] / Pi Sum[(-1) ^k d2[n, k] / (z - k), {k, 1, Log2@n}], z → z2]
```

Table[Limit[FullSimplify[bbo[50, z]], z → k], {k, 1, 7}]

{49, 108, 92, 35, 6, 0, 0}

Table[Limit[FullSimplify[bb2[50, z]], z → k], {k, 1, 7}]

{49, 108, 92, 35, 6, 0, 0}

(FactorialPower[z, 5] / Gamma[6 - z] / Gamma[z])

d2[20, 1]

19

Table[d2[50, k], {k, 1, 7}]

{49, 108, 92, 35, 6, 0, 0}

Limit[FullSimplify@D[bbo[50, z], z], z → 0]

$$\frac{1087}{60}$$

```
FullSimplify[Sum[
  (-1) ^ (k + 1) / (z - k) (FactorialPower[z - 1, 5] / Gamma[6 - z] / Gamma[z]) ff[n, k], {k, 1, 5}]]
```

$$-\frac{\left(\frac{ff[n,1]}{-1+z} - \frac{ff[n,2]}{-2+z} + \frac{ff[n,3]}{-3+z} - \frac{ff[n,4]}{-4+z} + \frac{ff[n,5]}{-5+z}\right) \operatorname{Sin}[\pi z]}{\pi}$$

```
bb3[100, z]
```

$$\frac{\left(\frac{7}{-6+z} - \frac{51}{-5+z} + \frac{184}{-4+z} - \frac{324}{-3+z} + \frac{283}{-2+z} - \frac{99}{-1+z}\right) \operatorname{Sin}[\pi z]}{\pi}$$

```
d2[100, 6]
```

7

```
FullSimplify[Sin[Pi z] / Pi Sum[(-1)^k Binomial[n, k] / (z - k), {k, 0, Infinity}]]
```

$$-\frac{\operatorname{Gamma}[1 + n] \operatorname{Gamma}[-z] \operatorname{Sin}[\pi z]}{\pi \operatorname{Gamma}[1 + n - z]}$$

```
Limit[- Gamma[1 + n] Gamma[-z] Sin[π z] / (π Gamma[1 + n - z]) /. n → 5, z → 3.2]
```

9.22792

```
Binomial[5, 3.2]
```

9.22792

```
FullSimplify[Sin[Pi z] / Pi Sum[(-1)^k x^k / (z - k), {k, 0, Infinity}]]
```

$$-\frac{\operatorname{HurwitzLerchPhi}[-x, 1, -z] \operatorname{Sin}[\pi z]}{\pi}$$

```
vv[x_, z_] := Limit[- HurwitzLerchPhi[-x, 1, -z2] Sin[π z2] / π, z2 → z]
```

```
vv[10., 2]
```

$$\operatorname{Limit}\left[-\frac{\operatorname{HurwitzLerchPhi}[-10., 1, -z2] \operatorname{Sin}[\pi z2]}{\pi}, z2 \to 2\right]$$

```
v[3 / 2, 7]
```

$$\frac{18}{\pi}$$

```
bb3[7, 3 / 2]
```

$$\frac{18}{\pi}$$

```
Plot[v[x, 7], {x, -1, 4}]
```
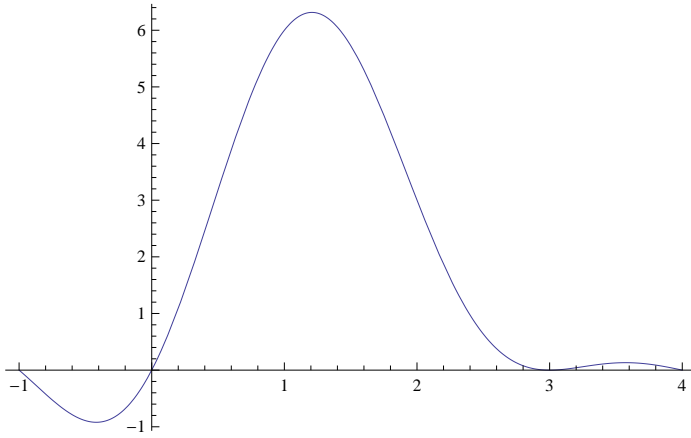
```
Plot[bb3[7, x], {x, -1, 4}]
```
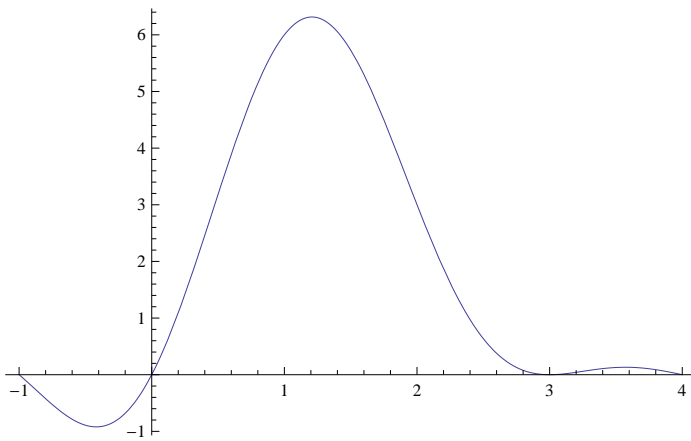


```
FullSimplify[Sin[Pi z] / Pi Sum[(-1)^k (x^(k-1) / (k-1)!) / (z-k), {k, 1, Infinity}]]
```

$$\frac{x^{-1+z} \, (\text{Gamma}[1-z] - \text{Gamma}[1-z, x]) \, \text{Sin}[\pi z]}{\pi}$$

```
Table[Limit[
```
$$\frac{x^{-1+z} \, (\text{Gamma}[1-z] - \text{Gamma}[1-z, x]) \, \text{Sin}[\pi z]}{\pi}$$
```
, z → k], {k, 0, 5}]
```

$$\left\{ 0, \, 1, \, x, \, \frac{x^2}{2}, \, \frac{x^3}{6}, \, \frac{x^4}{24} \right\}$$

```
Table[FullSimplify[(-1)^k Sin[Pi z] / Pi / (z-k)], {k, 0, 5}]
```

$$\left\{ \frac{\text{Sin}[\pi z]}{\pi z}, \, \frac{\text{Sin}[\pi z]}{\pi - \pi z}, \, \frac{\text{Sin}[\pi z]}{\pi (-2+z)}, \, \frac{\text{Sin}[\pi z]}{3\pi - \pi z}, \, \frac{\text{Sin}[\pi z]}{\pi (-4+z)}, \, \frac{\text{Sin}[\pi z]}{5\pi - \pi z} \right\}$$

```
Clear[z]
```

```
bbn[n_, z_] :=
```
$$\frac{\text{Gamma}[1+n] \, \text{Gamma}[1-z] \, \text{Sin}[\pi z]}{\pi z \, \text{Gamma}[1+n-z]}$$

```
bbn[8, 1 / 2]
```

$$\frac{65\,536}{6435\,\pi}$$

```
Binomial[7.8, 3.1]
```

53.3087

FullSimplify@Sin[Pi z] / Pi Sum[(-1)^k Binomial[n, k] / (z - k), {k, 0, Infinity}]

$$\frac{\text{Gamma}[1 + n]\ \text{Gamma}[1 - z]\ \text{Sin}[\pi z]}{\pi\ z\ \text{Gamma}[1 + n - z]}$$

$$\frac{\text{Gamma}[1 + n]\ \text{Gamma}[1 - z]\ \text{Sin}[\pi z]}{\pi\ z\ \text{Gamma}[1 + n - z]}\ /.\ n \rightarrow 7.8\ /.\ z \rightarrow 3.1$$

53.3087

FullSimplify[Sin[Pi z] / Pi Sum[(-1)^k x^(k) / (k)! / (z - k), {k, 0, Infinity}]]

$$\frac{(\text{ExpIntegralE}[1 + z, x] - x^z\ \text{Gamma}[-z])\ \text{Sin}[\pi z]}{\pi}$$

$$\textbf{Table}\Big[\textbf{Limit}\Big[\frac{(\text{ExpIntegralE}[1 + z, x] - x^z\ \text{Gamma}[-z])\ \text{Sin}[\pi z]}{\pi},\ z \rightarrow k\Big],\ \{k,\ 0,\ 6\}\Big]$$

$$\Big\{1,\ x,\ \frac{x^2}{2},\ \frac{x^3}{6},\ \frac{x^4}{24},\ \frac{x^5}{120},\ \frac{x^6}{720}\Big\}$$

FullSimplify[Sin[Pi z] / Pi Sum[(-1)^k / (z - k) 1, {k, 1, Infinity}]]

$$\frac{\text{HurwitzLerchPhi}[-1, 1, 1 - z]\ \text{Sin}[\pi z]}{\pi}$$

$$\textbf{Table}\Big[\textbf{Limit}\Big[\frac{\text{HurwitzLerchPhi}[-1, 1, 1 - z]\ \text{Sin}[\pi z]}{\pi},\ z \rightarrow k\Big],\ \{k,\ 0,\ 6\}\Big]$$

$$\textbf{Plot}\Big[\frac{\text{HurwitzLerchPhi}[-1, 1, 1 - z]\ \text{Sin}[\pi z]}{\pi},\ \{z,\ -10,\ 30\}\Big]$$
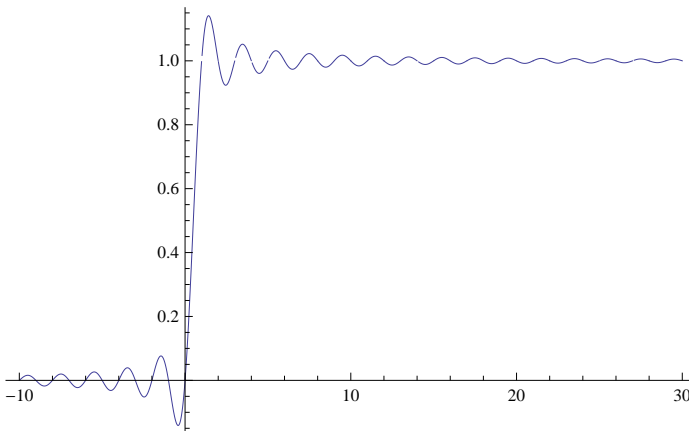
Sum[(-1)^k / (z - k) 1, {k, 1, Infinity}]

HurwitzLerchPhi[-1, 1, 1 - z]

FullSimplify[Sin[Pi z] / Pi Sum[(-1)^k / (z - k) 1 / k, {k, 1, Infinity}]]

$$\frac{(\text{HurwitzLerchPhi}[-1, 1, 1 - z] - \text{Log}[2])\ \text{Sin}[\pi z]}{\pi\ z}$$

pp[x_, z_] := Hypergeometric1F1[z, z + 1, Log[x]] (Log[x]^z) / (z!)

FullSimplify[D[pp[x, z], z] /. z → 0]

$$\frac{1}{2}\Big(\text{Log}\Big[\frac{1}{\text{Log}[x]}\Big] + \text{Log}[\text{Log}[x]]\Big) + \text{LogIntegral}[x]$$

```
Plot[bb3[997, zz] - bb3[996, zz], {zz, -2, 6}]
```

```
FullSimplify@Sin[Pi z] / Pi Sum[((-1)^k) / (z - k) x^k / k!, {k, 0, Infinity}]
```

$$\frac{x^z \ (\text{Gamma}[1-z] + z \ \text{Gamma}[-z, x]) \ \text{Sin}[\pi z]}{\pi z}$$

```
bz[x_, z_] :=
```
$$\frac{(\text{ExpIntegralE}[1 + z, x] - x^z \ \text{Gamma}[-z]) \ \text{Sin}[\pi z]}{\pi}$$

```
FullSimplify@bz[12, -3 / 2]
```

$$-\frac{1}{48 \ \sqrt{3 \ \pi}} + \frac{\text{ExpIntegralE}\left[-\frac{1}{2}, 12\right]}{\pi}$$

```
12 ^ (-3 / 2) / ((-3 / 2) !)
```

$$-\frac{1}{48 \ \sqrt{3 \ \pi}}$$

```
FullSimplify@Sin[Pi z] / Pi Sum[(-1)^k Binomial[n, k] / (z - k), {k, 0, Infinity}]
```

$$\frac{\text{Gamma}[1+n] \ \text{Gamma}[1-z] \ \text{Sin}[\pi z]}{\pi z \ \text{Gamma}[1+n-z]}$$

```
brz[n_, z_] :=
```
$$\frac{\textbf{Gamma}[1+n] \ \textbf{Gamma}[1-z] \ \textbf{Sin}[\pi z]}{\pi z \ \textbf{Gamma}[1+n-z]}$$

```
FullSimplify@brz[12, -3 / 2 + I]
```

$$-\frac{479\,001\,600 \ \text{Cosh}[\pi] \ \text{Gamma}\left[\frac{3}{2} - \mathbb{i}\right]}{\pi \ \text{Gamma}\left[\frac{29}{2} - \mathbb{i}\right]}$$

```
Binomial[12, -3 / 2 + I]
```

$$\text{Binomial}\left[12, -\frac{3}{2} + \mathbb{i}\right]$$

```
FullSimplify@Sin[Pi z] / Pi Sum[(-1)^k (x^(k - 1) / (k - 1) !) / (z - k), {k, 0, Infinity}]
```

$$\frac{x^{-1+z} \ (\text{Gamma}[1-z] - \text{Gamma}[1-z, x]) \ \text{Sin}[\pi z]}{\pi}$$

```
FullSimplify@
  Expand[(1 / Gamma[z] / Gamma[1 - z]) Sum[(-1)^k (x^(k) / (k) !) / (z - k), {k, 0, Infinity}]]
```

$$\frac{x^z}{\text{Gamma}[1+z]} + \frac{\text{ExpIntegralE}[1+z, x] \ \text{Sin}[\pi z]}{\pi}$$

```
FullSimplify@Expand[
  (1 / Gamma[z] / Gamma[1 - z]) Integrate[(-1)^k (x^(k) / (k) !) / (z - k), {k, 0, Infinity}]]
```

$$\frac{\left(\int_0^\infty \frac{(-1)^k \ x^k}{(-k+z) \ k!} \ \mathbb{d}k\right) \ \text{Sin}[\pi z]}{\pi}$$

```
FullSimplify[1 / Gamma[z] / Gamma[1 - z]]
```

$$\frac{\text{Sin}[\pi z]}{\pi}$$

```
Expand@(1 / Gamma[z] / Gamma[1 - z]) Sum[(-1)^k Binomial[n, k] / (z - k), {k, 0, Infinity}]
```

$$\frac{\text{Gamma}[1 + n]}{z\,\text{Gamma}[1 + n - z]\,\text{Gamma}[z]}$$

```
FullSimplify@Expand[(1 / Gamma[z] / Gamma[1 - z])
    Sum[(-1)^k (Log[x]^(k - 1) / (k - 1)!) / (z - k), {k, 1, Infinity}]]
```

$$\frac{(\text{Gamma}[1 - z] - \text{Gamma}[1 - z, \text{Log}[x]])\,\text{Log}[x]^{-1+z}\,\text{Sin}[\pi z]}{\pi}$$

```
FullSimplify@Limit[ (Gamma[1 - z] - Gamma[1 - z, Log[x]]) Log[x]^-1+z Sin[π z] / π , z → -1 / 2]
```

$$-\frac{\sqrt{\pi} - 2\,\text{Gamma}\left[\frac{3}{2},\,\text{Log}[x]\right]}{2\,\pi\,\text{Log}[x]^{3/2}}$$

```
FullSimplify@(Log[x]^(z - 1)) / (z - 1)! /. z → -1 / 2
```

$$-\frac{1}{2\sqrt{\pi}\,\text{Log}[x]^{3/2}}$$

```
Clear[bb3a]
bb3[n_, z2_] := Limit[Sin[Pi z] / Pi Sum[(-1)^k d2[n, k] / (z - k), {k, 1, Log2@n}], z → z2]
bb3a[n_, z_, k_] := bb3a[n, z, k] = Sum[1 / (z - k) - bb3a[Floor[n / j], z, k + 1], {j, 2, n}]
bb3az[n_, z_] := -Sin[Pi z] / Pi bb3a[n, z, 1]
bb3ax[n_, z_, k_, j_] :=
 If[n < j, 0, 1 / (z - k) - bb3ax[n / j, z, k + 1, 2] + bb3ax[n, z, k, j + 1]]
bb3ay[n_, z_] := (-Sin[Pi z] / Pi) Expand@bb3ax[n, z, 1, 2]
FI[n_] := FactorInteger[n]; FI[1] := {}
sp[n_] := Sum[p[[2]], {p, FI[n]}]
dbb3[n_, z_] := dbb3[n, z] = FullSimplify[bb3[n, z] - bb3[n - 1, z]]
dbb3a[n_, z_] :=
 FullSimplify[FullSimplify@dbb3[n, z] / (Sin[Pi z] / Pi) FactorialPower[z - 1, sp[n]]]
Expand@bb3[100, z]
```

$$\frac{7\,\text{Sin}[\pi z]}{\pi\,(-6 + z)} - \frac{51\,\text{Sin}[\pi z]}{\pi\,(-5 + z)} + \frac{184\,\text{Sin}[\pi z]}{\pi\,(-4 + z)} - \frac{324\,\text{Sin}[\pi z]}{\pi\,(-3 + z)} + \frac{283\,\text{Sin}[\pi z]}{\pi\,(-2 + z)} - \frac{99\,\text{Sin}[\pi z]}{\pi\,(-1 + z)}$$

```
Expand@bb3az[100, z]
```

$$\frac{7\,\text{Sin}[\pi z]}{\pi\,(-6 + z)} - \frac{51\,\text{Sin}[\pi z]}{\pi\,(-5 + z)} + \frac{184\,\text{Sin}[\pi z]}{\pi\,(-4 + z)} - \frac{324\,\text{Sin}[\pi z]}{\pi\,(-3 + z)} + \frac{283\,\text{Sin}[\pi z]}{\pi\,(-2 + z)} - \frac{99\,\text{Sin}[\pi z]}{\pi\,(-1 + z)}$$
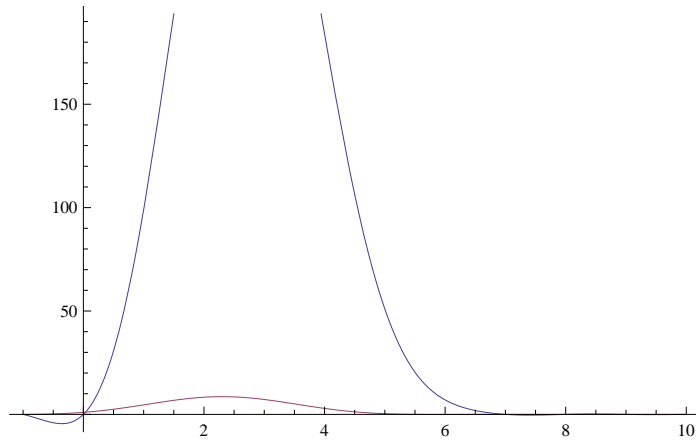
```
bb3ay[100, z]
```

$$-\frac{\left(-\frac{7}{-6+z} + \frac{51}{-5+z} - \frac{184}{-4+z} + \frac{324}{-3+z} - \frac{283}{-2+z} + \frac{99}{-1+z}\right)\,\text{Sin}[\pi z]}{\pi}$$

```
Table[dbb3a[2^n × 3^2 , z], {n, 1, 6}]
```

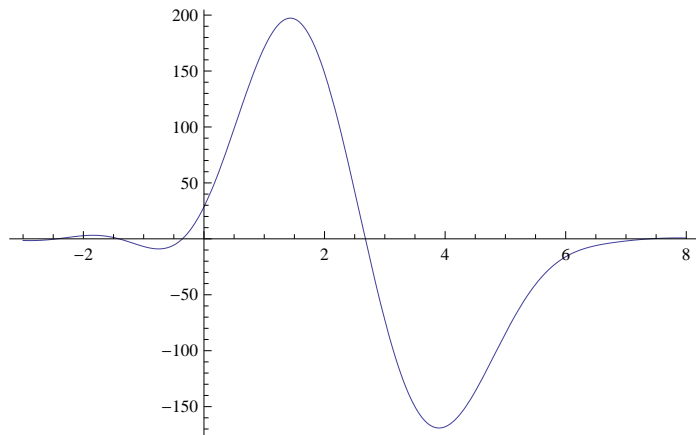$$\{-2\,z,\ z\,(5 + z),\ -6\,z\,(3 + z),\ 12\,z\,(7 + 3\,z),\ -240\,z\,(2 + z),\ 360\,z\,(9 + 5\,z)\}$$

Plot[{bb3az[100, z], Binomial[Log[100], z]}, {z, -1, 10}]

FullSimplify@D[bb3az[100, z], z]

$$\left(\frac{7}{-6+z} - \frac{51}{-5+z} + \frac{184}{-4+z} - \frac{324}{-3+z} + \frac{283}{-2+z} - \frac{99}{-1+z} \right) \text{Cos}[\pi z] +$$

$$\frac{\left(-\frac{7}{(-6+z)^2} + \frac{51}{(-5+z)^2} - \frac{184}{(-4+z)^2} + \frac{324}{(-3+z)^2} - \frac{283}{(-2+z)^2} + \frac{99}{(-1+z)^2} \right) \text{Sin}[\pi z]}{\pi}$$
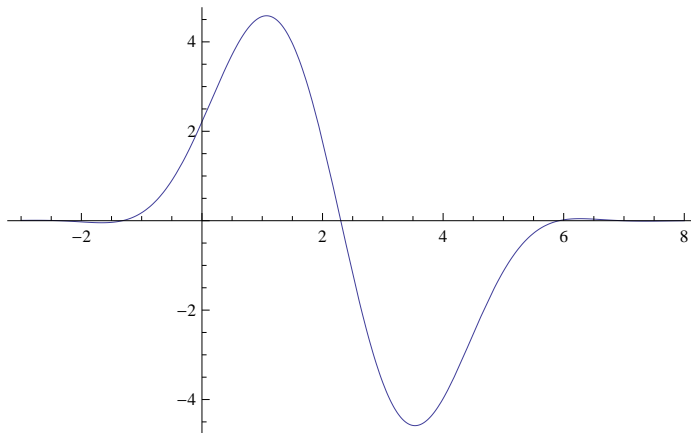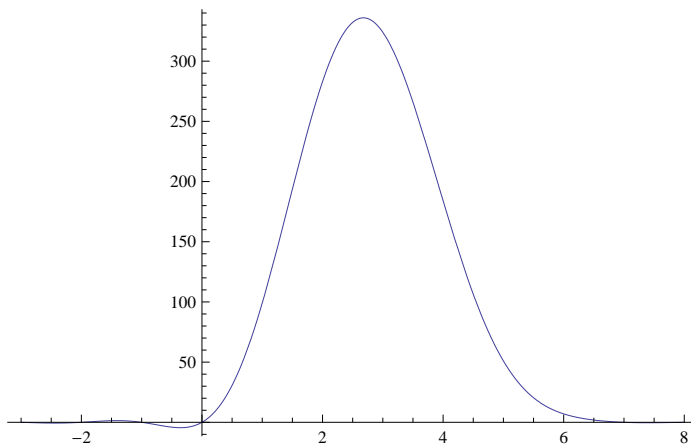
Plot[$\left(\frac{7}{-6+z} - \frac{51}{-5+z} + \frac{184}{-4+z} - \frac{324}{-3+z} + \frac{283}{-2+z} - \frac{99}{-1+z} \right)$ **Cos[π z] +**

$$\frac{\left(-\frac{7}{(-6+z)^2} + \frac{51}{(-5+z)^2} - \frac{184}{(-4+z)^2} + \frac{324}{(-3+z)^2} - \frac{283}{(-2+z)^2} + \frac{99}{(-1+z)^2} \right) \text{Sin}[\pi z]}{\pi}$$ **, {z, -3, 8}]**

FullSimplify@D[Binomial[Log[100], z], z]

Binomial[Log[100], z] (-HarmonicNumber[z] + HarmonicNumber[-z + Log[100]])

```
Plot[Binomial[Log[100], z] (-HarmonicNumber[z] + HarmonicNumber[-z + Log[100]]), {z, -3, 8}]
```
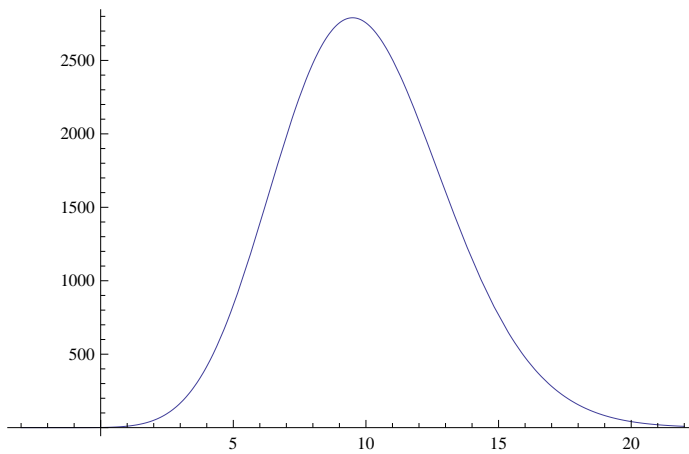


```
Plot[{bb3az[100, z]}, {z, -3, 8}]
```



```
Plot[Hypergeometric1F1[z, z + 1, Log[100]] (Log[100]^z) / z!, {z, -2, 13}]
```
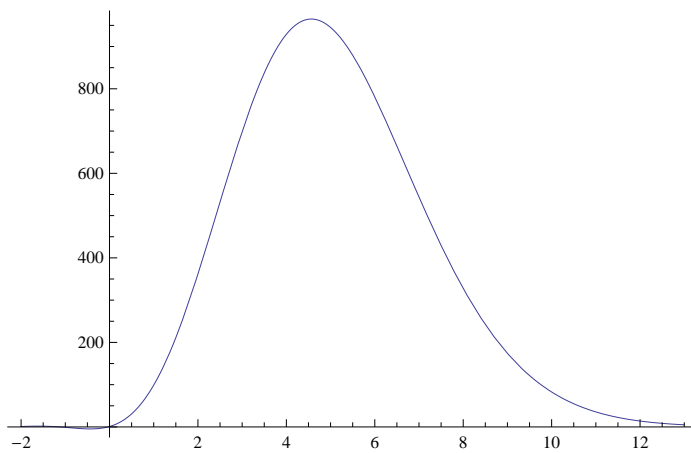
```
Plot[{10^z / z!}, {z, -3, 22}]
```



```
xr[x_, z_] := x^z / z!
```
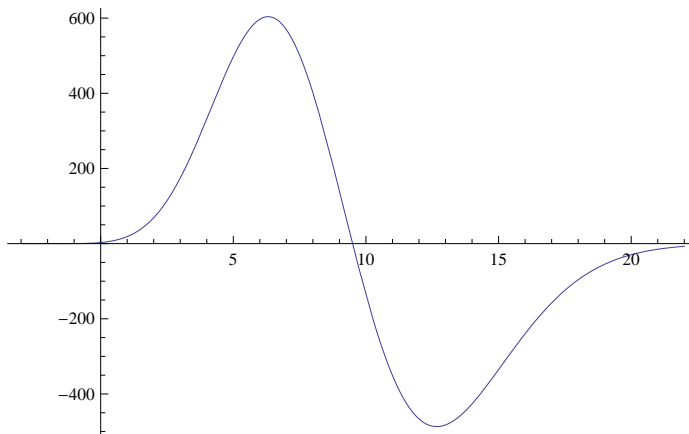
```
xr[10, 10] / xr[10, 11]
```

$$\frac{11}{10}$$

```
FullSimplify[D[10^z / (z!), z]]
```
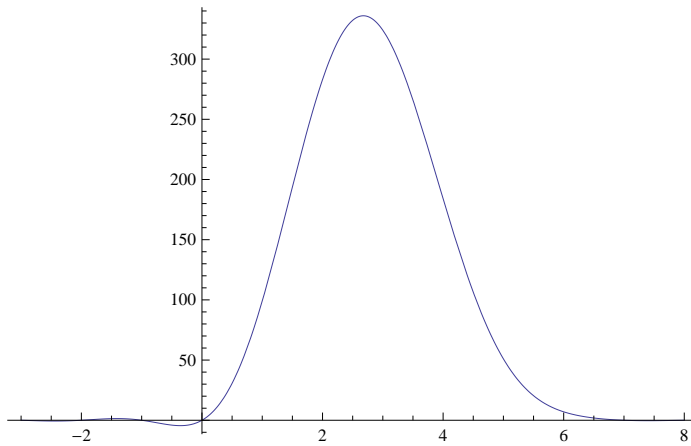
$$\frac{10^z \; (\text{EulerGamma} - \text{HarmonicNumber}[z] + \text{Log}[10])}{\text{Gamma}[1 + z]}$$

```
Plot[{ (10^z (EulerGamma - HarmonicNumber[z] + Log[10])) / (Gamma[1 + z]) }, {z, -3, 22}]
```
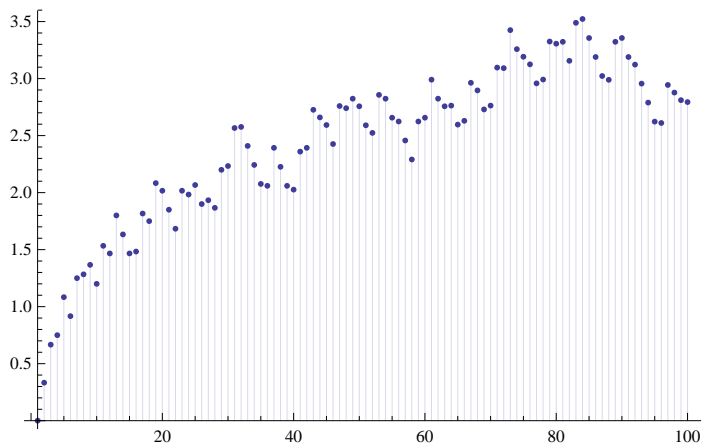
```
Plot[bb3az[100, z], {z, -3, 8}]
```



```
D[bb3az[100, z], z]
```

```
Plot[bb3az[100, z], {z, 6.9, 7.1}]
```

```
DiscretePlot[D[bb3az[n, z], z] /. z → -2, {n, 1, 100}]
```
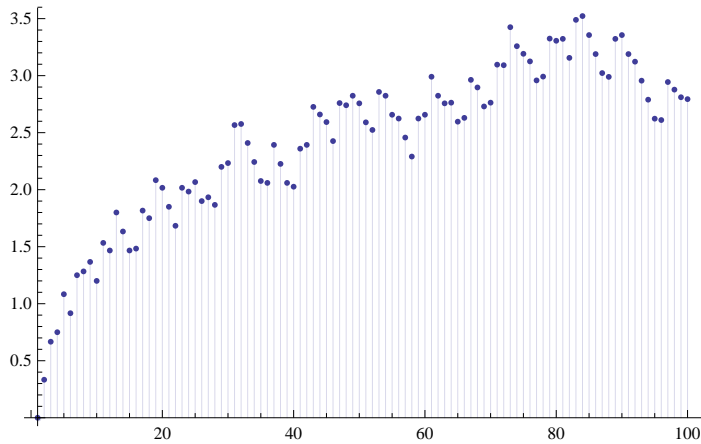


```
Floor[Log2@100]
```

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```
pp[n_, k_] := pp[n, k] = Sum[1 / k - pp[Floor[n / j], k + 1], {j, 2, n}]
```

```
DiscretePlot[pp[n, 3], {n, 1, 100}]
```



```
binomial[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, z_] := Product[(-1)^p[[2]] binomial[-z, p[[2]]], {p, FI[n]}]
Ddz[n_, z_] := Sum[dz[j, z], {j, 1, n}]
```

```
FullSimplify[Sum[dbb3[j, z] dbb3[210 / j, y], {j, Divisors[210]}]]
```

$$\left(2 \left(12 z (1 + z) + y (12 + (7 - 17 z) z) + y^2 (12 + z (-17 + 7 z))\right) \operatorname{Sin}[\pi y] \operatorname{Sin}[\pi z]\right) \Big/ \left(\pi^2 (-3 + y) (-2 + y) (-1 + y) (-3 + z) (-2 + z) (-1 + z)\right)$$

```
dbb3[210, 4]
```

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$$\operatorname{Limit}\Big[\operatorname{Limit}\Big[\left(2 \left(12 z (1 + z) + y (12 + (7 - 17 z) z) + y^2 (12 + z (-17 + 7 z))\right) \operatorname{Sin}[\pi y] \operatorname{Sin}[\pi z]\right) \Big/ \left(\pi^2 (-3 + y) (-2 + y) (-1 + y) (-3 + z) (-2 + z) (-1 + z)\right), y \to -1\Big], z \to 5\Big]$$

0

```
Sum[Binomial[a, z] Binomial[10 - a, y], {a, 0, 10}]
```

Binomial[5, y] Binomial[5, z] + Binomial[4, z] Binomial[6, y] + Binomial[4, y] Binomial[6, z] +
 Binomial[3, z] Binomial[7, y] + Binomial[3, y] Binomial[7, z] + Binomial[2, z] Binomial[8, y] +
 Binomial[2, y] Binomial[8, z] + Binomial[1, z] Binomial[9, y] + Binomial[1, y] Binomial[9, z] +
 Binomial[0, z] Binomial[10, y] + Binomial[0, y] Binomial[10, z]

```
Clear[br3a]
br3a[n_, z_, k_] :=
 br3a[n, z, k] = Sum[(-1)^(j + 1) (1 / (z - k) - br3a[Floor[n / j], z, k + 1]), {j, 2, n}]
br3az[n_, z_] := -Sin[Pi z] / Pi br3a[n, z, 1]
```

DiscretePlot[D[br3az[n, z], z] /. z → 0, {n, 2, 100}]