```
AA[n_, k_] := (-1)^(k+1) / ((k-1)!) Integrate[ t^(k-1) E^(-t), {t, -Log[n], 0}]
```

```
AA[n, 4]
```

$$\frac{1}{6}\left(6-6\,n+6\,n\,\mathrm{Log}[n]-3\,n\,\mathrm{Log}[n]^2+n\,\mathrm{Log}[n]^3\right)$$

$$\mathbf{BB}[n\_,\ k\_]\ :=\ \frac{(-1)^{1+k}\,(-\mathrm{Gamma}[k]+\mathrm{Gamma}[k,\ -\mathrm{Log}[n]])}{(-1+k)\,!}$$

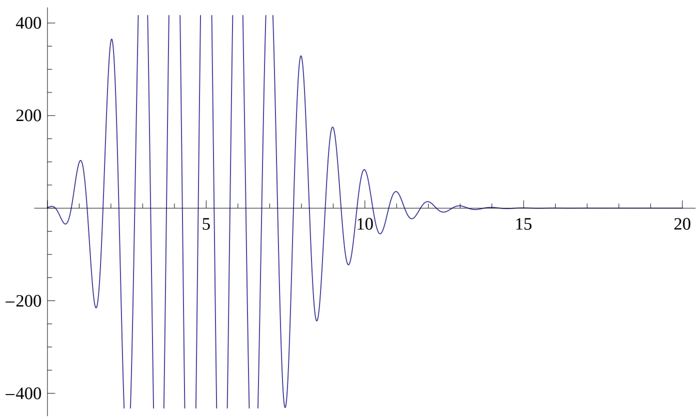
```
N[Re[BB[100, 3]]]
```

698.863

```
N[Re[AA[100, 2.5]]]
```

-532.148

```
Plot[Re[BB[100, k]], {k, 0, 20}]
```




```
DD[ k_, a_, n_ ] :=
 Sum[ Binomial[k, j] DD[ k-j, m+1, Floor[n / (m^j)]], {m, a, n^(1/k)}, {j, 1, k}]
DD[ 1, a_, n_ ] := Floor[n] - a + 1
DD[0, a_, n_] := 1
DS[ n_, k_ ] := DD[k, 2, n]
```

```
DDD[n_, k_ ] := Sum[ DDD[n / j, k - 1], {j, 2, n}]
```

```
DDD[n_, 0] := 1
```

```
DDD[1000, 4]
```

13 952

```
DS[1000, 4]
```

13 952

```
N[Re[BB[1000, 4]]]
```

36 986.5

```
Sum[N[(-1)^k Re[ BB[1000, k]]], {k, 1, 100}]
```

-6.90776

```
Log[100.]
```

4.60517

```
N[Sum[(-1)^k BB[100, k], {k, 1, Infinity}]]
```

-4.60517

```
FullSimplify[Expand[(-1)^4 BB[n, 4]]]
```

$1 - \frac{1}{6}\, \text{Gamma}[4, -\text{Log}[n]]$

```
MM[n_] := N[Log[n] + Sum[(-1)^k BB[n, k], {k, 1, 1000}]]
```

```
MM[25]
```

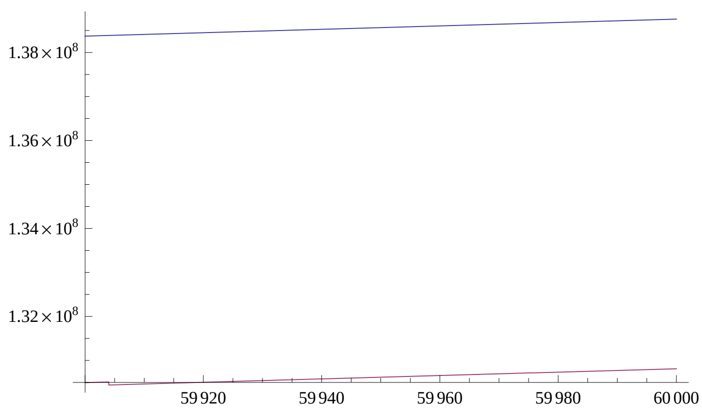
$-2.0373 \times 10^{-11} - 4.21325 \times 10^{-15}\, i$

```
MR[n_] := Table[(-1)^j (N[Re[BB[n, j] - DS[n, j]]]), {j, 1, 40}]
```

```
MR[1000]
```

{0., 838.755, -6732.79, 23 034.5, -47 283.1, 68 136.9, -76 237.1, 70 959.4, -57 374.1, 41 305.1, -26 867., 15 943.6, -8700.17, 4394.66, -2066.48, 908.988, -375.623, 146.364, -53.956, 18.8735, -6.28092, 1.99339, -0.60465, 0.175638, -0.0489469, 0.0131083, -0.0033787, 0.000839374, -0.00020125, 0.0000466253, -0.00001045, 2.26818×10^{-6}, -4.77251×10^{-7}, 9.74388×10^{-8}, -1.93205×10^{-8}, 3.72366×10^{-9}, -6.98111×10^{-10}, 1.2744×10^{-10}, -2.26507×10^{-11}, 3.93932×10^{-12}}

```
Plot[{BB[n, 8], BB[n, 8] - DS[n, 8]}, {n, 59 900, 60 000}]
```




```
DS[n, k]
```

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

General::stop : Further output of $RecursionLimit::reclim will be suppressed during this calculation. ≫

$IterationLimit::itlim : Iteration limit of 4096 exceeded. ≫

$IterationLimit::itlim : Iteration limit of 4096 exceeded. ≫

$IterationLimit::itlim : Iteration limit of 4096 exceeded. ≫

General::stop : Further output of $IterationLimit::itlim will be suppressed during this calculation. ≫

```
D2[1, a_, n_, p_, r_] := n - a + 1
D2[2, a_, n_, p_, r_] := p / ((r + 1) (r + 2)) + (Floor[n / a] - a) (p / (r + 1)) +
  (Floor[n^(1 / 2)] - a) (p / 2) + p Sum[ Floor[n / m] - m, {m, a + 1, n^(1 / 2)}]
D2[k_, a_, n_, p_, r_] :=
 D2[k - 1, a, n / a, p / (r + 1), r + 1] + Sum[ D2[ k - 1, m, n / m, p, 1 ], {m, a + 1, n^(1 / k)}]
DD2[ n_, k_] := D2[ k, 2, n, k !, 0 ]


DDM[ n_] := Sum[ (-1)^k DD2[n, k], {k, 1, Log[n] / Log[2]}]
DDM[100]
0
D3[1, a_, n_, p_, r_] := n - a + 1
D3[2, a_, n_, p_, r_] := p / ((r + 1) (r + 2)) + (n / a - .5 - a) (p / (r + 1)) +
  (n^(1 / 2) - .5 - a) (p / 2) + p Sum[n / m - .5 - m, {m, a + 1, n^(1 / 2)}]
D3[k_, a_, n_, p_, r_] :=
 D3[k - 1, a, n / a, p / (r + 1), r + 1] + Sum[ D3[ k - 1, m, n / m, p, 1 ], {m, a + 1, n^(1 / k)}]
DD3[ n_, k_] := D3[ k, 2, n, k !, 0 ]
DDM3[ n_] := Sum[ (-1)^k DD3[n, k], {k, 1, Log[n] / Log[2]}]
DDM3[1000.]
4.56758
```

```
DiscretePlot[DDM3[n], {n, 2, 100}]
```

Sum::itflrw :

Warning: In evaluating Floor$\left[\frac{\text{Log}[4]}{\text{Log}[2]}\right]$ to find the number of iterations to use for Sum, $MaxExtraPrecision = 50.`

was encountered. An upper estimate will be used for the number of iterations. ≫

Sum::itflrw :

Warning: In evaluating Floor$\left[\frac{\text{Log}[8]}{\text{Log}[2]}\right]$ to find the number of iterations to use for Sum, $MaxExtraPrecision = 50.`

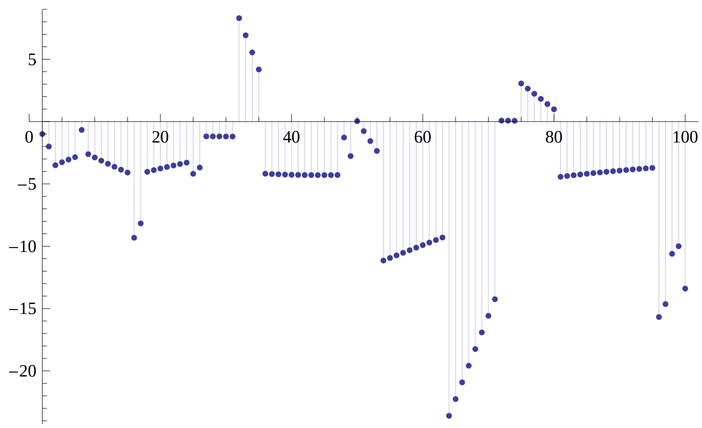
was encountered. An upper estimate will be used for the number of iterations. ≫

Sum::itflrw :

Warning: In evaluating Floor$\left[\frac{\text{Log}[16]}{\text{Log}[2]}\right]$ to find the number of iterations to use for Sum, $MaxExtraPrecision = 50.`

was encountered. An upper estimate will be used for the number of iterations. ≫

General::stop : Further output of Sum::itflrw will be suppressed during this calculation. ≫




```
DD3[ 1001, 4]
```

13 952

```
DS[1000, 4]
```

13 952

```
FullSimplify[DD2[n, 2]]
```

$$-5+\text{Floor}\left[\sqrt{n}\right]+2\,\text{Floor}\left[\frac{n}{2}\right]+2\sum_{m=3}^{\sqrt{n}}\left(-m+\text{Floor}\left[\frac{n}{m}\right]\right)$$

```
DD2[n, 3]
```

$Aborted

```
D2[2, 2, n / 2, (3!) / (0 + 1), 0 + 1] + Sum[ D2[2, m, n / m, 3!, 1], {m, 2 + 1, n^(1 / 3)}]
```

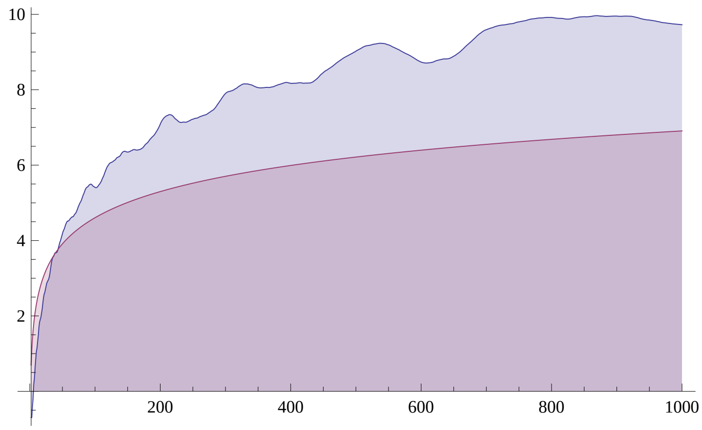
$Aborted

$$\textbf{FullSimplify}\left[2\sum_{m=3}^{\sqrt{n}}(-m)+2\sum_{m=3}^{\sqrt{n}}\left(\text{Floor}\left[\frac{n}{m}\right]\right)\right]$$

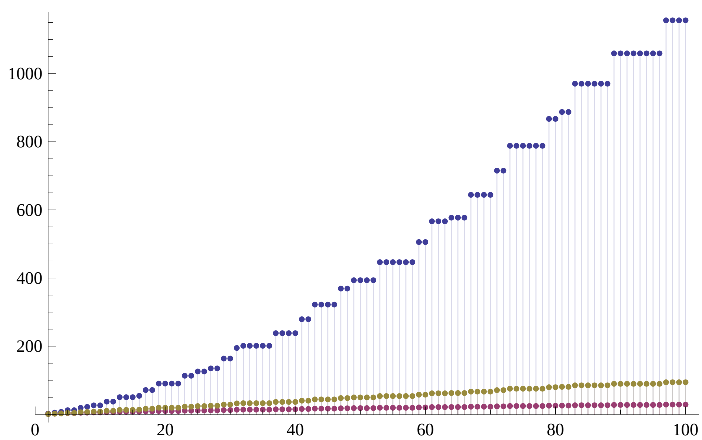
```
FullSimplify[-5 + Floor[√n] + 2 Floor[n/2] + 6 - √n - n + 2 Sum[Floor[n/m], {m, 3, √n}]]
```

$$1 - \sqrt{n} - n + \mathrm{Floor}\left[\sqrt{n}\right] + 2\,\mathrm{Floor}\left[\frac{n}{2}\right] + 2\sum_{m=3}^{\sqrt{n}} \mathrm{Floor}\left[\frac{n}{m}\right]$$

```
EE[n_] := EE[n] = -Log[n] - Sum[ EE[n / j], {j, 2, n}]
DiscretePlot[{EE[n], Log[n]}, {n, 2, 1000}]
```




```
Clear[FF]
FF[n_, k_] := Sum[j (1 / k - FF[n / j, k + 1]), {j, 2, n}]
FG[n_, k_] := Sum[1 / k - FG[n / j, k + 1], {j, 2, n}]
FH[n_] := Sum[Log[j] - FH[n / j], {j, 2, n}]
DiscretePlot[{FF[n, 1], FG[n, 1], FH[n]}, {n, 2, 100}]
```




```
FR[n_, k_, j_] := 1 / k - FR[n / j, k + 1, 2] + FR[n, k, j + 1]
FQ[n_, k_, j_] := If[j < n, 1 / k - FQ[n / j, k + 1, 2] + FQ[n, k, j + 1], 0]

FQ[100, 1, 2]
```

$$\frac{428}{15}$$

```
FS[n_, j_] := If[j < n, Log[j] - FS[n / j, 2] + FS[n, j + 1], 0]
```

```
N[FS[100, 2]]
```

94.0453

```
N[FH[100]]
```

94.0453

```
FS[n, 2]
```

$\text{If}\left[2 < n, \text{Log}[2] - \text{FS}\left[\frac{n}{2}, 2\right] + \text{FS}[n, 2 + 1], 0\right]$

```
FullSimplify[FQ[n, 1, 2]]
```

$\text{If}\left[n > 2, 1 \times \frac{1}{1} - \text{FQ}\left[\frac{n}{2}, 1 + 1, 2\right] + \text{FQ}[n, 1, 2 + 1], 0\right]$

```
PP[n_, j_] := Piecewise[{{Log[j] - PP[n / j, 2] + PP[n, j + 1], n > j}, {0, n ≤ j}}]
```

```
N[PP[100, 2]]
```

94.0453

```
PP[n, 2]
```

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

General::stop : Further output of $RecursionLimit::reclim will be suppressed during this calculation. ≫

$IterationLimit::itlim : Iteration limit of 4096 exceeded. ≫

$IterationLimit::itlim : Iteration limit of 4096 exceeded. ≫

$IterationLimit::itlim : Iteration limit of 4096 exceeded. ≫

General::stop : Further output of $IterationLimit::itlim will be suppressed during this calculation. ≫

$Aborted