

```

Clear[d2]
Pr[n_] := Product[Prime[j], {j, 1, n}]
d2[n_, k_] := d2[n, k] = Sum[If[j == 1 || j == n, 0, d2[n/j, k-1]], {j, Divisors[n]}]
d2[n_, 1] := If[n == 1, 0, 1]
FI[n_] := FactorInteger[n]; FI[1] := {}
dzz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}]
dz[n_, z_] := Product[Pochhammer[z, p[[2]]] / p[[2]]!, {p, FI[n]}]
d2a[n_, k_] := Sum[(-1)^(k-j) Binomial[k, j] dz[n, j], {j, 0, k}]
binx[z_, k_] := Binomial[z, k]
bin[z_, k_] := Gamma[z+1] / Gamma[z-k+1] / Gamma[k+1]
da[a_, k_] := bin[a-1, k-1]
dal[a_, k_] := k bin[a, k-1]
dal1[a_, k_] := k (1+a k) / (1+a) bin[a+1, k-1]
da2[a_, k_] := k ((3+a+(a-1)k) / (2(a+1))) bin[a+1, k-1]
d2f[z_] :=
  da[1, z] + da[1, z] + da[2, z] + da[1, z] + dal[1, z] + da[1, z] + da[3, z] + da[2, z] + dal[1, z]
d2f2[z_] := d2f[z] + da[1, z] + dal[2, z] + da[1, z] + dal[1, z] +
  dal[1, z] + da[4, z] + da[1, z] + dal[2, z] + da[1, z] + dal[2, z]
d2f3[z_] := d2f2[z] + dal[1, z] + dal[1, z] + da[1, z] + dal[3, z] +
  da[2, z] + dal[1, z] + da[3, z] + dal[2, z] + da[1, z] + dal1[1, z]
Clear[v]
v[z_, 1] := 0
v[z_, 2] := v[z, 1] + da[1, z]
v[z_, 3] := v[z, 2] + da[1, z]
v[z_, 4] := v[z, 3] + da[2, z]
v[z_, 5] := v[z, 4] + da[1, z]
v[z_, 6] := v[z, 5] + dal[1, z]
v[z_, 7] := v[z, 6] + da[1, z]
v[z_, 8] := v[z, 7] + da[3, z]
v[z_, 9] := v[z, 8] + da[2, z]
v[z_, 10] := v[z, 10] = v[z, 9] + dal[1, z]
v[z_, 11] := v[z, 10] + da[1, z]
v[z_, 12] := v[z, 11] + dal[2, z]
v[z_, 13] := v[z, 12] + da[1, z]
v[z_, 14] := v[z, 13] + dal[1, z]
v[z_, 15] := v[z, 14] + dal[1, z]
v[z_, 16] := v[z, 15] + da[4, z]
v[z_, 17] := v[z, 16] + da[1, z]
v[z_, 18] := v[z, 17] + dal[2, z]
v[z_, 19] := v[z, 18] + da[1, z]
v[z_, 20] := v[z, 20] = v[z, 19] + dal[2, z]
v[z_, 21] := v[z, 20] + dal[1, z]
v[z_, 22] := v[z, 21] + dal[1, z]
v[z_, 23] := v[z, 22] + da[1, z]
v[z_, 24] := v[z, 23] + dal[3, z]
v[z_, 25] := v[z, 24] + da[2, z]
v[z_, 26] := v[z, 25] + dal[1, z]
v[z_, 27] := v[z, 26] + da[3, z]
v[z_, 28] := v[z, 27] + dal[2, z]
v[z_, 29] := v[z, 28] + da[1, z]

```

```

v[z_, 30] := v[z, 30] = v[z, 29] + da11[1, z]
v[z_, 31] := v[z, 30] + da[1, z]
v[z_, 32] := v[z, 31] + da[5, z]
v[z_, 33] := v[z, 32] + da1[1, z]
v[z_, 34] := v[z, 33] + da1[1, z]
v[z_, 35] := v[z, 34] + da1[1, z]
v[z_, 36] := v[z, 35] + da2[2, z]
v[z_, 37] := v[z, 36] + da[1, z]
v[z_, 38] := v[z, 37] + da1[1, z]
v[z_, 39] := v[z, 38] + da1[1, z]
v[z_, 40] := v[z, 40] = v[z, 39] + da1[3, z]
v[z_, 41] := v[z, 40] + da[1, z]
v[z_, 42] := v[z, 41] + da11[1, z]
v[z_, 43] := v[z, 42] + da[1, z]
v[z_, 44] := v[z, 43] + da1[2, z]
v[z_, 45] := v[z, 44] + da1[2, z]
v[z_, 46] := v[z, 45] + da1[1, z]
v[z_, 47] := v[z, 46] + da[1, z]
v[z_, 48] := v[z, 47] + da1[4, z]
v[z_, 49] := v[z, 48] + da[2, z]
v[z_, 50] := v[z, 50] = v[z, 49] + da1[2, z]
v[z_, 51] := v[z, 50] + da1[1, z]
v[z_, 52] := v[z, 51] + da1[2, z]
v[z_, 53] := v[z, 52] + da[1, z]
v[z_, 54] := v[z, 53] + da1[3, z]
v[z_, 55] := v[z, 54] + da1[1, z]
v[z_, 56] := v[z, 55] + da1[3, z]
v[z_, 57] := v[z, 56] + da1[1, z]
v[z_, 58] := v[z, 57] + da1[1, z]
v[z_, 59] := v[z, 58] + da[1, z]
v[z_, 60] := v[z, 60] = v[z, 59] + da11[2, z]
v[z_, 61] := v[z, 60] + da[1, z]
v[z_, 62] := v[z, 61] + da1[1, z]
v[z_, 63] := v[z, 62] + da1[2, z]
v[z_, 64] := v[z, 63] + da[6, z]
v[z_, 65] := v[z, 64] + da1[1, z]
v[z_, 66] := v[z, 65] + da11[1, z]
v[z_, 67] := v[z, 66] + da[1, z]
v[z_, 68] := v[z, 67] + da1[2, z]
v[z_, 69] := v[z, 68] + da1[1, z]
v[z_, 70] := v[z, 70] = v[z, 69] + da11[1, z]
Clear[rb]
bin2[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
rb[n_, k_, f_] := rb[n, k, f] = Sum[f[j] rb[Floor[n / j], k - 1, f], {j, 2, n}]
rb[n_, 0, f_] := UnitStep[n - 1]
lrb[n_, f_] := Sum[(-1)^(k + 1) / k rb[n, k, f], {k, 1, Log2@n}]
rbz[n_, z_, f_] := Sum[bin2[z, k] rb[n, k, f], {k, 0, Log2@n}]
lrz[n_, z_, f_] := Sin[Pi z] / Pi Sum[(-1)^k / (z - k) rb[n, k, f], {k, 0, Log2@n}]
id[n_] := 1
ll[n_, z_] := lrz[n, z, id]

```

```
lo[n_, z_] := v[z, n]
```

```
FullSimplify@v[z, 33]
```

$$\frac{1634 + z (-1187 + z (378 + z (-61 + 4 z)))}{\Gamma[6 - z] \Gamma[z]}$$

```
FullSimplify@Expand[(298 + z (-159 + (39 - 4 z) z)) / FactorialPower[z - 1, 4]] Sin[Pi z] / Pi
```

$$\frac{(298 + z (-159 + (39 - 4 z) z)) \sin[\pi z]}{\pi \text{FactorialPower}[-1 + z, 4]}$$

```
ll[33, z]
```

$$\frac{\left(-\frac{1}{-5+z} + \frac{9}{-4+z} - \frac{38}{-3+z} + \frac{58}{-2+z} - \frac{32}{-1+z} + \frac{1}{z}\right) \sin[\pi z]}{\pi}$$

```
binx[z_, k_] := Gamma[z + 1] / Gamma[z - k + 1] / Gamma[k + 1]
```

```
binx2[z_, k_] := Gamma[z + 1] / Gamma[1 - k + z] / Gamma[k] / k
```

```
Expand[FullSimplify@v[z, 36] Gamma[z] Gamma[6 - z]]
```

$$1634 - 1042 z + 284 z^2 - 38 z^3 + 2 z^4$$

```
Table[rb[36, k, id], {k, 0, 5}]
```

```
{1, 35, 69, 50, 15, 1}
```

```
Gamma[6 - z] /. z -> 2.3
```

```
4.17065
```

```
1 / Gamma[z] / Gamma[6 - z] /. z -> 2.3
```

```
0.20551
```

```
-Sin[z Pi] / Pi / FactorialPower[z - 1, 5] /. z -> 2.3
```

```
0.20551
```

```
Limit[(1634 - 1042 z + 284 z^2 - 38 z^3 + 2 z^4) (-Sin[z Pi] / Pi / FactorialPower[z - 1, 5]), z -> 2]
```

```
69
```

$$(1634 - 1042 z + 284 z^2 - 38 z^3 + 2 z^4) / ((1 - z) (2 - z) (3 - z) (4 - z) (5 - z))$$

$$\frac{1634 - 1042 z + 284 z^2 - 38 z^3 + 2 z^4}{(1 - z) (2 - z) (3 - z) (4 - z) (5 - z)}$$

```
FullSimplify[ $\frac{1634 - 1042 z + 284 z^2 - 38 z^3 + 2 z^4}{(1 - z) (2 - z) (3 - z) (4 - z) (5 - z)}$ ]
```

$$\frac{1}{5 - z} + \frac{15}{-4 + z} - \frac{50}{-3 + z} + \frac{69}{-2 + z} - \frac{35}{-1 + z}$$

```
Expand@FullSimplify[
```

$$(\text{Expand}[(1 - z) (2 - z) (3 - z) (4 - z) (5 - z)]) \left(\frac{1}{5 - z} + \frac{15}{-4 + z} - \frac{50}{-3 + z} + \frac{69}{-2 + z} - \frac{35}{-1 + z} \right)$$

$$1634 - 1042 z + 284 z^2 - 38 z^3 + 2 z^4$$

$$1 \times 24 - 15 (1 \times 2 \times 3 \times 5) + 50 (1 \times 2 \times 4 \times 5) - 69 (1 \times 3 \times 4 \times 5) + 35 (2 \times 3 \times 4 \times 5)$$

$$1634$$

```
Table[{n, FullSimplify@Expand[FullSimplify[(v[z, n] - v[z, n - 1]) Gamma[z] Gamma[6 - z]]],
  {n, 33, 60}] // TableForm
```

```
33      - (-5 + z) (-4 + z) (-3 + z) z
34      - (-5 + z) (-4 + z) (-3 + z) z
35      - (-5 + z) (-4 + z) (-3 + z) z
36      - z (-25 + z^2)
37      (-5 + z) (-4 + z) (-3 + z) (-2 + z)
38      - (-5 + z) (-4 + z) (-3 + z) z
39      - (-5 + z) (-4 + z) (-3 + z) z
40      - 6 (-5 + z) z
41      (-5 + z) (-4 + z) (-3 + z) (-2 + z)
42      (-5 + z) (-4 + z) z (1 + z)
43      (-5 + z) (-4 + z) (-3 + z) (-2 + z)
44      2 (-5 + z) (-4 + z) z
45      2 (-5 + z) (-4 + z) z
46      - (-5 + z) (-4 + z) (-3 + z) z
47      (-5 + z) (-4 + z) (-3 + z) (-2 + z)
48      24 z
49      - (-5 + z) (-4 + z) (-3 + z)
50      2 (-5 + z) (-4 + z) z
51      - (-5 + z) (-4 + z) (-3 + z) z
52      2 (-5 + z) (-4 + z) z
53      (-5 + z) (-4 + z) (-3 + z) (-2 + z)
54      - 6 (-5 + z) z
55      - (-5 + z) (-4 + z) (-3 + z) z
56      - 6 (-5 + z) z
57      - (-5 + z) (-4 + z) (-3 + z) z
58      - (-5 + z) (-4 + z) (-3 + z) z
59      (-5 + z) (-4 + z) (-3 + z) (-2 + z)
60      2 z (5 + (9 - 2 z) z)
```

```
{ {33, (5 - z) (-4 + z) (-3 + z) z}, {34, (5 - z) (-4 + z) (-3 + z) z},
  {35, (5 - z) (-4 + z) (-3 + z) z}, {36, -z (-25 + z^2)}, {37, (-5 + z) (-4 + z) (-3 + z) (-2 + z)},
  {38, (5 - z) (-4 + z) (-3 + z) z}, {39, (5 - z) (-4 + z) (-3 + z) z}, {40, -6 (-5 + z) z},
  {41, (-5 + z) (-4 + z) (-3 + z) (-2 + z)}, {42, (-5 + z) (-4 + z) z (1 + z)},
  {43, (-5 + z) (-4 + z) (-3 + z) (-2 + z)}, {44, 2 (-5 + z) (-4 + z) z}, {45, 2 (-5 + z) (-4 + z) z},
  {46, (5 - z) (-4 + z) (-3 + z) z}, {47, (-5 + z) (-4 + z) (-3 + z) (-2 + z)}, {48, 24 z},
  {49, (5 - z) (-4 + z) (-3 + z)}, {50, 2 (-5 + z) (-4 + z) z}, {51, (5 - z) (-4 + z) (-3 + z) z},
  {52, 2 (-5 + z) (-4 + z) z}, {53, (-5 + z) (-4 + z) (-3 + z) (-2 + z)}, {54, -6 (-5 + z) z},
  {55, (5 - z) (-4 + z) (-3 + z) z}, {56, -6 (-5 + z) z}, {57, (5 - z) (-4 + z) (-3 + z) z},
  {58, (5 - z) (-4 + z) (-3 + z) z}, {59, (-5 + z) (-4 + z) (-3 + z) (-2 + z)}, {60, 2 z (5 + (9 - 2 z) z)} }
```

```
Expand[(2 - z) (3 - z) (4 - z) (5 - z)]
```

```
120 - 154 z + 71 z^2 - 14 z^3 + z^4
```

```
Expand[(3 - z) (4 - z) (5 - z)]
```

```
60 - 47 z + 12 z^2 - z^3
```

```
Expand[(4 - z) (5 - z)]
```

```
20 - 9 z + z^2
```

```
FullSimplify[(5 + (9 - 2 z) z) / (z - 5)]
```

```
-1 - 2 z
```

```
Table[{n, FullSimplify@
  Expand[FullSimplify[(v[z, n] - v[z, n - 1]) Pi / Sin[Pi z] FactorialPower[5 - z, 5]]],
  {n, 33, 60}] // TableForm
```

```
33      - (-5 + z) (-4 + z) (-3 + z) z
34      - (-5 + z) (-4 + z) (-3 + z) z
35      - (-5 + z) (-4 + z) (-3 + z) z
36      - z (-25 + z^2)
37      (-5 + z) (-4 + z) (-3 + z) (-2 + z)
38      - (-5 + z) (-4 + z) (-3 + z) z
39      - (-5 + z) (-4 + z) (-3 + z) z
40      - 6 (-5 + z) z
41      (-5 + z) (-4 + z) (-3 + z) (-2 + z)
42      (-5 + z) (-4 + z) z (1 + z)
43      (-5 + z) (-4 + z) (-3 + z) (-2 + z)
44      2 (-5 + z) (-4 + z) z
45      2 (-5 + z) (-4 + z) z
46      - (-5 + z) (-4 + z) (-3 + z) z
47      (-5 + z) (-4 + z) (-3 + z) (-2 + z)
48      24 z
49      - (-5 + z) (-4 + z) (-3 + z)
50      2 (-5 + z) (-4 + z) z
51      - (-5 + z) (-4 + z) (-3 + z) z
52      2 (-5 + z) (-4 + z) z
53      (-5 + z) (-4 + z) (-3 + z) (-2 + z)
54      - 6 (-5 + z) z
55      - (-5 + z) (-4 + z) (-3 + z) z
56      - 6 (-5 + z) z
57      - (-5 + z) (-4 + z) (-3 + z) z
58      - (-5 + z) (-4 + z) (-3 + z) z
59      (-5 + z) (-4 + z) (-3 + z) (-2 + z)
60      2 z (5 + (9 - 2 z) z)
```

```
FactorialPower[5 - z, 5] /. z -> 4.3
```

```
2.07207
```

```
(5 - z) (4 - z) (3 - z) (2 - z) (1 - z) /. z -> 4.3
```

```
2.07207
```

```
Table[{n, FactorInteger[n], FullSimplify@
  Expand[FullSimplify[(v[z, n] - v[z, n - 1]) Pi / Sin[Pi z] FactorialPower[5 - z, 5]]] /
  (-Product[z - k, {k, 1, 5}])}, {n, 2, 63}] // TableForm
```

```
2      2 1      - 1 / (-1 + z)
3      3 1      - 1 / (-1 + z)
4      2 2      1 / ((-2 + z) (-1 + z))
5      5 1      - 1 / (-1 + z)
6      2 1      z / ((-2 + z) (-1 + z))
7      7 1      - 1 / (-1 + z)
8      2 3      2 / ((-3 + z) (-2 + z) (-1 + z))
```

9	3 2	$\frac{1}{(-2+z)(-1+z)}$
10	2 1 5 1	$\frac{z}{(-2+z)(-1+z)}$
11	11 1	$-\frac{1}{-1+z}$
12	2 2 3 1	$-\frac{2z}{(-3+z)(-2+z)(-1+z)}$
13	13 1	$-\frac{1}{-1+z}$
14	2 1 7 1	$\frac{z}{(-2+z)(-1+z)}$
15	3 1 5 1	$\frac{z}{(-2+z)(-1+z)}$
16	2 4	$\frac{6}{(-4+z)(-3+z)(-2+z)(-1+z)}$
17	17 1	$-\frac{1}{-1+z}$
18	2 1 3 2	$-\frac{2z}{(-3+z)(-2+z)(-1+z)}$
19	19 1	$-\frac{1}{-1+z}$
20	2 2 5 1	$-\frac{2z}{(-3+z)(-2+z)(-1+z)}$
21	3 1 7 1	$\frac{z}{(-2+z)(-1+z)}$
22	2 1 11 1	$\frac{z}{(-2+z)(-1+z)}$
23	23 1	$-\frac{1}{-1+z}$
24	2 3 3 1	$\frac{6z}{(-4+z)(-3+z)(-2+z)(-1+z)}$
25	5 2	$\frac{1}{(-2+z)(-1+z)}$
26	2 1 13 1	$\frac{z}{(-2+z)(-1+z)}$
27	3 3	$-\frac{2}{(-3+z)(-2+z)(-1+z)}$
28	2 2 7 1	$-\frac{2z}{(-3+z)(-2+z)(-1+z)}$
29	29 1	$-\frac{1}{-1+z}$
30	2 1 3 1 5 1	$-\frac{z(1+z)}{(-3+z)(-2+z)(-1+z)}$
31	31 1	$-\frac{1}{-1+z}$
32	2 5	$-\frac{24}{(-5+z)(-4+z)(-3+z)(-2+z)(-1+z)}$
33	3 1 11 1	$\frac{z}{(-2+z)(-1+z)}$
34	2 1 17 1	$\frac{z}{(-2+z)(-1+z)}$
35	5 1 7 1	$\frac{z}{(-2+z)(-1+z)}$
36	2 2 3 2	$\frac{z(-25+z^2)}{(-5+z)(-4+z)(-3+z)(-2+z)(-1+z)}$
37	37 1	$-\frac{1}{-1+z}$
38	2 1 19 1	$\frac{z}{(-2+z)(-1+z)}$
39	3 1 13 1	$\frac{z}{(-2+z)(-1+z)}$
40	2 3 5 1	$\frac{6z}{(-4+z)(-3+z)(-2+z)(-1+z)}$
41	41 1	$-\frac{1}{-1+z}$

$$\begin{array}{ll}
42 & \frac{2}{3} \frac{1}{1} - \frac{z(1+z)}{(-3+z)(-2+z)(-1+z)} \\
43 & \frac{43}{-1+z} \\
44 & \frac{2}{11} \frac{2}{1} - \frac{2z}{(-3+z)(-2+z)(-1+z)} \\
45 & \frac{3}{5} \frac{2}{1} - \frac{2z}{(-3+z)(-2+z)(-1+z)} \\
46 & \frac{2}{23} \frac{1}{1} - \frac{z}{(-2+z)(-1+z)} \\
47 & \frac{47}{-1+z} \\
48 & \frac{2}{3} \frac{4}{1} - \frac{24z}{(-5+z)(-4+z)(-3+z)(-2+z)(-1+z)} \\
49 & \frac{7}{-2+z} \frac{2}{(-1+z)} \\
50 & \frac{2}{5} \frac{1}{2} - \frac{2z}{(-3+z)(-2+z)(-1+z)} \\
51 & \frac{3}{17} \frac{1}{1} - \frac{z}{(-2+z)(-1+z)} \\
52 & \frac{2}{13} \frac{2}{1} - \frac{2z}{(-3+z)(-2+z)(-1+z)} \\
53 & \frac{53}{-1+z} \\
54 & \frac{2}{3} \frac{1}{3} - \frac{6z}{(-4+z)(-3+z)(-2+z)(-1+z)} \\
55 & \frac{5}{11} \frac{1}{1} - \frac{z}{(-2+z)(-1+z)} \\
56 & \frac{2}{7} \frac{3}{1} - \frac{6z}{(-4+z)(-3+z)(-2+z)(-1+z)} \\
57 & \frac{3}{19} \frac{1}{1} - \frac{z}{(-2+z)(-1+z)} \\
58 & \frac{2}{29} \frac{1}{1} - \frac{z}{(-2+z)(-1+z)} \\
59 & \frac{59}{-1+z} \\
60 & \frac{2}{3} \frac{2}{1} - \frac{2z(5+(9-2z)z)}{(-5+z)(-4+z)(-3+z)(-2+z)(-1+z)} \\
61 & \frac{61}{-1+z} \\
62 & \frac{2}{31} \frac{1}{1} - \frac{z}{(-2+z)(-1+z)} \\
63 & \frac{3}{7} \frac{2}{1} - \frac{2z}{(-3+z)(-2+z)(-1+z)}
\end{array}$$

(* Note!

60 here is kind of wrong. It should be
 $2z(1+2z)/((4-z)(3-z)(2-z)(1-z))$

36 is also wrong. It should be

$z(5+z)/((4-z)(3-z)(2-z)(1-z))$
 *)

FullSimplify[FactorialPower[5 - z, 5] / Product[z - k, {k, 1, 5}]]

-1

```
Table[Limit[(k - 1)! / FactorialPower[k - z, k] Sin[Pi z] / Pi /. k -> 5, z -> s], {s, 0, 6}]
{0, 1, 4, 6, 4, 1, 0}
```

```
Table[Limit[- $\frac{24}{(-5+z)(-4+z)(-3+z)(-2+z)(-1+z)}$  Sin[Pi z] / Pi, z -> s], {s, 0, 6}]
{0, 1, 4, 6, 4, 1, 0}
```

```
Table[Binomial[5 - 1, z - 1], {z, 0, 6}]
{0, 1, 4, 6, 4, 1, 0}
```

```
Table[Limit[FullSimplify[Expand[l1[210, z] - l1[209, z]]], z -> s], {s, 0, 5}]
{0, 1, 14, 36, 24, 0}
```

```
Table[Limit[6 z (1 + z) / ((4 - z) (3 - z) (2 - z) (1 - z)) Sin[Pi z] / Pi, z -> s], {s, 0, 5}]
{0, 2, 18, 36, 20, 0}
```

```
Expand[(2 z (5 + (9 - 2 z) z))]
```

```
10 z + 18 z^2 - 4 z^3
```

```
Expand[(2 - z) (5 - z)]
```

```
10 - 7 z + z^2
```

```
List@@Roots[(2 z (5 + (9 - 2 z) z)) == 0, z][[All, 2]]
```

```
{0, - $\frac{1}{2}$ , 5}
```

```
Expand[2 z (1 + 2 z) (5 - z)]
```

```
10 z + 18 z^2 - 4 z^3
```

```

$$\frac{z(-25 + z^2)}{(-5 + z)(-4 + z)(-3 + z)(-2 + z)(-1 + z)}$$


$$\frac{z(-25 + z^2)}{(-5 + z)(-4 + z)(-3 + z)(-2 + z)(-1 + z)}$$

```

```
FullSimplify@Expand[z (-25 + z^2) / (5 - z)]
```

```
-z (5 + z)
```

```
Table[Limit[FullSimplify[Expand[l1[36, z] - l1[35, z]]], z -> s], {s, 0, 5}]
{0, 1, 7, 12, 6, 0}
```

```
Table[Limit[z (5 + z) / ((4 - z) (3 - z) (2 - z) (1 - z)) Sin[Pi z] / Pi, z -> s], {s, 0, 5}]
{0, 1, 7, 12, 6, 0}
```

```
FullSimplify@Expand[
```

```
FullSimplify[Expand[l1[210, z] - l1[209, z]]] / (Sin[Pi z] / Pi) (4 - z) (3 - z) (2 - z) (1 - z)]
z^2 (5 + z)
```



```
Table[Limit[FullSimplify[Expand[l1[210, z] - l1[209, z]]], z -> s], {s, 0, 5}]
```

```
{0, 1, 14, 36, 24, 0}
```

```
Table[Limit[z^2 (5 + z) / ((4 - z) (3 - z) (2 - z) (1 - z)) Sin[Pi z] / Pi, z -> s], {s, 0, 5}]
```

```
{0, 1, 14, 36, 24, 0}
```

```
FullSimplify@Expand[
```

```
FullSimplify[Expand[l1[2^2 x 3 x 5 x 7 x 11, z] - l1[2^2 x 3 x 5 x 7 x 11 - 1, z]]] / (Sin[Pi z] / Pi)
```

```
(6 - z) (5 - z) (4 - z) (3 - z) (2 - z) (1 - z)]
```

```
2 z (-6 + z (7 + z (51 + 8 z)))
```

```
FullSimplify@List@@Roots[-42 + 119 z + 42 z^2 + z^3 == 0, z][[All, 2]]
```

$$\left\{ -14 + \frac{67 \times 7^{2/3}}{\left(3 \left(-2430 + i \sqrt{411123}\right)\right)^{1/3}} + \frac{\left(7 \left(-2430 + i \sqrt{411123}\right)\right)^{1/3}}{3^{2/3}}, \right.$$

$$\left. -14 - \frac{67 \times 7^{2/3} \left(1 + i \sqrt{3}\right)}{2 \left(3 \left(-2430 + i \sqrt{411123}\right)\right)^{1/3}} - \frac{\left(1 - i \sqrt{3}\right) \left(7 \left(-2430 + i \sqrt{411123}\right)\right)^{1/3}}{2 \times 3^{2/3}}, \right.$$

$$\left. -14 - \frac{67 \times 7^{2/3} \left(1 - i \sqrt{3}\right)}{2 \left(3 \left(-2430 + i \sqrt{411123}\right)\right)^{1/3}} - \frac{\left(1 + i \sqrt{3}\right) \left(7 \left(-2430 + i \sqrt{411123}\right)\right)^{1/3}}{2 \times 3^{2/3}} \right\}$$

```
Expand[(-42 + z (119 + z (42 + z)))]
```

```
-42 + 119 z + 42 z^2 + z^3
```

```
Table[
```

```
Limit[FullSimplify[Expand[l1[2^2 x 3 x 5 x 7 x 11, z] - l1[2^2 x 3 x 5 x 7 x 11 - 1, z]]], z -> s],
```

```
{s, 0, 7}]
```

```
{0, 1, 46, 345, 900, 960, 360, 0}
```

```
Table[
```

```
Limit[(2 z (-6 + z (7 + z (51 + 8 z)))) / ((6 - z) (5 - z) (4 - z) (3 - z) (2 - z) (1 - z)) Sin[Pi z] / Pi,
```

```
z -> s], {s, 0, 7}]
```

```
{0, 1, 46, 345, 900, 960, 360, 0}
```

```
Table[d2[2^2 x 3 x 5 x 7 x 11, k], {k, 1, 7}]
```

```
{1, 46, 345, 900, 960, 360, 0}
```

```
Table[Limit[l1[2^2 x 3 x 5 x 7 x 11, z] - l1[2^2 x 3 x 5 x 7 x 11 - 1, z], z -> k], {k, 1, 7}]
```

```
{1, 46, 345, 900, 960, 360, 0}
```

```
(5 - z) (4 - z) (3 - z) (2 - z) (1 - z) /. z -> 3
```

```
0
```

```
FactorialPower[5 - z, 5] /. z -> 5
```

```
0
```

```
(-1)^(5) Pochhammer[z - 5, 5] /. z -> 2.3
```

```
1.25307
```

$$\text{Sum}[(-1)^k / (z - k) \text{Product}[k - z, \{k, 1, 6\}] \text{d2}[2 \times 3 \times 5 \times 7 \times 11 \times 13, k], \{k, 1, 6\}]$$

$$\frac{720 (1 - z) (2 - z) (3 - z) (4 - z) (5 - z) (6 - z)}{-6 + z} - \frac{1800 (1 - z) (2 - z) (3 - z) (4 - z) (5 - z) (6 - z)}{-5 + z} +$$

$$\frac{1560 (1 - z) (2 - z) (3 - z) (4 - z) (5 - z) (6 - z)}{-4 + z} - \frac{540 (1 - z) (2 - z) (3 - z) (4 - z) (5 - z) (6 - z)}{-3 + z} +$$

$$\frac{62 (1 - z) (2 - z) (3 - z) (4 - z) (5 - z) (6 - z)}{-2 + z} - \frac{(1 - z) (2 - z) (3 - z) (4 - z) (5 - z) (6 - z)}{-1 + z}$$

Table[StirlingsS2[6, k] k!, {k, 1, 6}]

{1, 62, 540, 1560, 1800, 720}

Sum[(-1)^k / (z - k) Product[k - z, {k, 1, 6}] StirlingsS2[6, k] k!, {k, 1, 6}]

$$\frac{720 (1 - z) (2 - z) (3 - z) (4 - z) (5 - z) (6 - z)}{-6 + z} - \frac{1800 (1 - z) (2 - z) (3 - z) (4 - z) (5 - z) (6 - z)}{-5 + z} +$$

$$\frac{1560 (1 - z) (2 - z) (3 - z) (4 - z) (5 - z) (6 - z)}{-4 + z} - \frac{540 (1 - z) (2 - z) (3 - z) (4 - z) (5 - z) (6 - z)}{-3 + z} +$$

$$\frac{62 (1 - z) (2 - z) (3 - z) (4 - z) (5 - z) (6 - z)}{-2 + z} - \frac{(1 - z) (2 - z) (3 - z) (4 - z) (5 - z) (6 - z)}{-1 + z}$$

Table[

Limit[Sin[Pi z] / Pi Sum[(-1)^k / (z - k) StirlingsS2[6, k] k!, {k, 1, 6}], z -> k], {k, 1, 7}]

{1, 62, 540, 1560, 1800, 720, 0}

Table[d2[2 × 3 × 5 × 7 × 11 × 13, k], {k, 1, 7}]

{1, 62, 540, 1560, 1800, 720, 0}

Table[d2[2 × 3 × 5 × 7 × 11 × 13 × 17 × 19, k], {k, 1, 10}]

{1, 254, 5796, 40 824, 126 000, 191 520, 141 120, 40 320, 0, 0}

Table[StirlingsS2[8, k] k!, {k, 1, 10}]

{1, 254, 5796, 40 824, 126 000, 191 520, 141 120, 40 320, 0, 0}

StirlingsS2[8, 4] 4!

40 824

Table[d2[2^6, k], {k, 0, 10}]

{0, 1, 5, 10, 10, 5, 1, 0, 0, 0, 0}

Table[Binomial[6 - 1, k - 1], {k, 0, 10}]

{0, 1, 5, 10, 10, 5, 1, 0, 0, 0, 0}

{d2[2^8, 4], Binomial[8 - 1, 4 - 1]}

{35, 35}

{d2[2 × 3 × 5 × 7 × 11 × 13 × 17 × 19, 4], StirlingsS2[8, 4] 4!}

{40 824, 40 824}

FullSimplify@

Expand[Sum[(-1)^k / (z - k) Product[k - z, {k, 1, 6}] StirlingS2[6, k] k!, {k, 1, 6}]]

$z^2 (-42 + z (119 + z (42 + z)))$

Expand[z^2 (-42 + z (119 + z (42 + z)))]

$-42 z^2 + 119 z^3 + 42 z^4 + z^5$

Table[

Limit[D[1 / Gamma[z] / Gamma[1 - z] Sum[(-1)^k / (z - k) StirlingS2[a, k] k!, {k, 0, a}], z],
z → 0], {a, 0, 10}]

{0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}

Limit[D[l1[30, z] - l1[29, z], z], z → 0]

0

Limit[D[lr[100, z, id], z], z → 0]

428

15

Sum[(-1)^k / (z - k) StirlingS2[a, k] k!, {k, 0, a}]

$$\sum_{k=0}^a \frac{(-1)^k k! \text{StirlingS2}[a, k]}{-k + z}$$

tt[a_] := Table[(-1)^j / (z - k) j^a Binomial[k, j], {k, 0, a}, {j, 0, k}] // Grid

$(-1)^k (-1)^{(k-j)}$

$(-1)^{-j+2k}$

tt[5]

0

0 $-\frac{1}{-1+z}$

0 $-\frac{2}{-2+z}$ $-\frac{32}{-2+z}$

0 $-\frac{3}{-3+z}$ $-\frac{96}{-3+z}$ $-\frac{243}{-3+z}$

0 $-\frac{4}{-4+z}$ $-\frac{192}{-4+z}$ $-\frac{972}{-4+z}$ $-\frac{1024}{-4+z}$

0 $-\frac{5}{-5+z}$ $-\frac{320}{-5+z}$ $-\frac{2430}{-5+z}$ $-\frac{5120}{-5+z}$ $-\frac{3125}{-5+z}$

```
Sum[(-1)^k / (z - k) Product[k - z, {k, 1, 7}] d2[2^2 × 3 × 5 × 7 × 11 × 13, k], {k, 1, 7}]
```

$$\begin{aligned}
& - \frac{2520 (1-z) (2-z) (3-z) (4-z) (5-z) (6-z) (7-z)}{-7+z} + \\
& \frac{7920 (1-z) (2-z) (3-z) (4-z) (5-z) (6-z) (7-z)}{-6+z} - \\
& \frac{9300 (1-z) (2-z) (3-z) (4-z) (5-z) (6-z) (7-z)}{-5+z} + \\
& \frac{4980 (1-z) (2-z) (3-z) (4-z) (5-z) (6-z) (7-z)}{-4+z} - \\
& \frac{1173 (1-z) (2-z) (3-z) (4-z) (5-z) (6-z) (7-z)}{-3+z} + \\
& \frac{94 (1-z) (2-z) (3-z) (4-z) (5-z) (6-z) (7-z)}{-2+z} - \\
& \frac{(1-z) (2-z) (3-z) (4-z) (5-z) (6-z) (7-z)}{-1+z}
\end{aligned}$$

```
f[r_, n_, m_] := Sum[Binomial[m, l] Binomial[l + r - 1, r] (-1)^(m - l) l^(n - r), {l, 1, m}];
For[n = 2, n ≤ 10, n++, Print[Table[f[2, n, m], {m, 1, n}]]]
```

```
{1, 1}
```

```
{1, 4, 3}
```

```
{1, 10, 21, 12}
```

```
{1, 22, 93, 132, 60}
```

```
{1, 46, 345, 900, 960, 360}
```

```
{1, 94, 1173, 4980, 9300, 7920, 2520}
```

```
{1, 190, 3801, 24612, 71400, 103320, 73080, 20160}
```

```
{1, 382, 11973, 113652, 480060, 1048320, 1234800, 745920, 181440}
```

```
{1, 766, 37065, 502500, 2968560, 9170280, 15981840, 15845760, 8346240, 1814400}
```

```
Sum[(-1)^k / (z - k) Product[k - z, {k, 1, 6}] d2[2^3 × 3 × 5 × 7, k], {k, 1, 6}]
```

$$\begin{aligned}
& \frac{120 (1-z) (2-z) (3-z) (4-z) (5-z) (6-z)}{-6+z} - \frac{360 (1-z) (2-z) (3-z) (4-z) (5-z) (6-z)}{-5+z} + \\
& \frac{388 (1-z) (2-z) (3-z) (4-z) (5-z) (6-z)}{-4+z} - \frac{177 (1-z) (2-z) (3-z) (4-z) (5-z) (6-z)}{-3+z} + \\
& \frac{30 (1-z) (2-z) (3-z) (4-z) (5-z) (6-z)}{-2+z} - \frac{(1-z) (2-z) (3-z) (4-z) (5-z) (6-z)}{-1+z}
\end{aligned}$$

```
f[r_, n_, m_] := Sum[Binomial[m, l] Binomial[l + r - 1, r] (-1)^(m - l) l^(n - r), {l, 1, m}];
For[n = 3, n ≤ 10, n++, Print[Table[f[3, n, m], {m, 1, n}]]]
```

```

{1, 2, 1}
{1, 6, 9, 4}
{1, 14, 45, 52, 20}
{1, 30, 177, 388, 360, 120}
{1, 62, 621, 2260, 3740, 2880, 840}
{1, 126, 2049, 11524, 30000, 39720, 26040, 6720}
{1, 254, 6525, 54292, 207620, 418320, 460320, 262080, 60480}
{1, 510, 20337, 243268, 1309560, 3755640, 6150480, 5779200, 2903040, 604800}

```

```
Table[d2[2^k, j], {k, 1, 7}, {j, 0, k}] // Grid
```

```

0 1
0 1 1
0 1 2 1
0 1 3 3 1
0 1 4 6 4 1
0 1 5 10 10 5 1
0 1 6 15 20 15 6 1

```

```
Table[Binomial[k-1, j-1], {k, 1, 7}, {j, 0, k}] // Grid
```

```

0 1
0 1 1
0 1 2 1
0 1 3 3 1
0 1 4 6 4 1
0 1 5 10 10 5 1
0 1 6 15 20 15 6 1

```

```
Pr[n_] := Product[Prime[j], {j, 1, n}]
```

```
Table[d2a[Pr[k], j], {k, 1, 7}, {j, 0, k}] // Grid
```

```

0 1
0 1 2
0 1 6 6
0 1 14 36 24
0 1 30 150 240 120
0 1 62 540 1560 1800 720
0 1 126 1806 8400 16800 15120 5040

```

```
Table[j! StirlingS2[k, j], {k, 1, 7}, {j, 0, k}] // Grid
```

```

0 1
0 1 2
0 1 6 6
0 1 14 36 24
0 1 30 150 240 120
0 1 62 540 1560 1800 720
0 1 126 1806 8400 16800 15120 5040

```

```
Table[d2[2^k × 3, j], {k, 1, 7}, {j, 0, k + 2}] // Grid
```

```
0 1 2 0
0 1 4 3 0
0 1 6 9 4 0
0 1 8 18 16 5 0
0 1 10 30 40 25 6 0
0 1 12 45 80 75 36 7 0
0 1 14 63 140 175 126 49 8 0
```

```
t[n_, k_] := (k + 1) * Binomial[n, k]; Table[t[n, k - 1], {n, 1, 7}, {k, 0, n + 2}] // Grid
```

```
0 1 2 0
0 1 4 3 0
0 1 6 9 4 0
0 1 8 18 16 5 0
0 1 10 30 40 25 6 0
0 1 12 45 80 75 36 7 0
0 1 14 63 140 175 126 49 8 0
```

```
Table[d2a[2 Pr[k], j], {k, 1, 7}, {j, 0, k + 2}] // Grid
```

```
0 1 1 0
0 1 4 3 0
0 1 10 21 12 0
0 1 22 93 132 60 0
0 1 46 345 900 960 360 0
0 1 94 1173 4980 9300 7920 2520 0
0 1 190 3801 24612 71400 103320 73080 20160 0
```

```
f[r_, n_, k_] := Sum[Binomial[k, j] Binomial[j + r - 1, r] (-1)^(k - j) j^(n - r), {j, 1, k}]
```

```
For[n = 2, n ≤ 8, n++, Print[Table[f[2, n, m], {m, 1, n + 1}]]]
```

```
{1, 1, 0}
```

```
{1, 4, 3, 0}
```

```
{1, 10, 21, 12, 0}
```

```
{1, 22, 93, 132, 60, 0}
```

```
{1, 46, 345, 900, 960, 360, 0}
```

```
{1, 94, 1173, 4980, 9300, 7920, 2520, 0}
```

```
{1, 190, 3801, 24612, 71400, 103320, 73080, 20160, 0}
```

```
Table[d2a[2^2 Pr[k], j], {k, 1, 7}, {j, 0, k + 2}] // Grid
```

```
0 1 2 1
0 1 6 9 4
0 1 14 45 52 20
0 1 30 177 388 360 120
0 1 62 621 2260 3740 2880 840
0 1 126 2049 11524 30000 39720 26040 6720
0 1 254 6525 54292 207620 418320 460320 262080 60480
```

```
f[r_, n_, k_] := Sum[Binomial[k, j] Binomial[j + r - 1, r] (-1)^(k - j) j^(n - r), {j, 1, k}]
```

```
For[n = 3, n ≤ 9, n++, Print[Table[f[3, n, m], {m, 1, n}]]]
```

```

{1, 2, 1}

{1, 6, 9, 4}

{1, 14, 45, 52, 20}

{1, 30, 177, 388, 360, 120}

{1, 62, 621, 2260, 3740, 2880, 840}

{1, 126, 2049, 11524, 30000, 39720, 26040, 6720}

{1, 254, 6525, 54292, 207620, 418320, 460320, 262080, 60480}

Table[d2[2^k × 3 × 5, j], {k, 1, 7}, {j, 0, k + 3}] // Grid

0 1 6 6 0
0 1 10 21 12 0
0 1 14 45 52 20 0
0 1 18 78 136 105 30 0
0 1 22 120 280 325 186 42 0
0 1 26 171 500 775 666 301 56 0
0 1 30 231 812 1575 1806 1225 456 72 0

f[r_, n_, k_] := Sum[(-1)^(k - j) Binomial[k, j] Binomial[j + r - 1, r] j^(n - r), {j, 1, k}]
Table[f[j, 2 + j, m], {j, 1, 7}, {m, 1, 3 + j}] // Grid

1 6 6 0
1 10 21 12 0
1 14 45 52 20 0
1 18 78 136 105 30 0
1 22 120 280 325 186 42 0
1 26 171 500 775 666 301 56 0
1 30 231 812 1575 1806 1225 456 72 0

Table[d2a[2^k × 3^2 × 5, j], {k, 1, 7}, {j, 0, k + 3}] // Grid

0 1 10 21 12
0 1 16 57 72 30
0 1 22 111 220 190 60
0 1 28 183 496 655 420 105
0 1 34 273 940 1675 1626 819 168
0 1 40 381 1592 3575 4656 3535 1456 252
0 1 46 507 2492 6755 11046 11221 6952 2412 360

FI[n_] := FactorInteger[n]; FI[1] := {}
d2b[n_, k_] := Sum[
  (-1)^(k - j) Binomial[k, j] Product[Pochhammer[j, p[[2]]] / p[[2]]!, {p, FI[n]}], {j, 0, k}]

Pochhammer[j, a] / a!

Pochhammer[j, a]
-----
a!

Sum[(-1)^(k - j) Binomial[k, j] Pochhammer[j, a] / a! Pochhammer[j, b] / b!, {j, 0, k}]

- (-1)^k k HypergeometricPFQ[{1 + a, 1 + b, 1 - k}, {1, 2}, 1] +
-----
(-1)^k Pochhammer[0, a] Pochhammer[0, b]
a! b!

```

```

Sum[(-1)^(k-j) Binomial[k, j] Pochhammer[j, a] / a!
  Pochhammer[j, b] / b! Pochhammer[j, c] / c!, {j, 0, k}]
- (-1)^k k HypergeometricPFQ[{1+a, 1+b, 1+c, 1-k}, {1, 1, 2}, 1] +
  (-1)^k Pochhammer[0, a] Pochhammer[0, b] Pochhammer[0, c]
  a! b! c!
Sum[(-1)^(k-j) Binomial[k, j] Pochhammer[j, a] / a!
  Pochhammer[j, b] / b! Pochhammer[j, c] / c! Pochhammer[j, d] / d!, {j, 0, k}]
- (-1)^k k HypergeometricPFQ[{1+a, 1+b, 1+c, 1+d, 1-k}, {1, 1, 1, 2}, 1] +
  1
  a! b! c! d! (-1)^k Pochhammer[0, a] Pochhammer[0, b] Pochhammer[0, c] Pochhammer[0, d]
- (-1)^k k HypergeometricPFQ[{1+a, 1+b, 1+c, 1+d, 1-k}, {1, 1, 1, 2}, 1] + 1
  a! b! c! d!
  (-1)^k Pochhammer[0, a] Pochhammer[0, b] Pochhammer[0, c] Pochhammer[0, d] /.
  a -> 3 /. b -> 3 /. c -> 2 /. d -> 2 /. k -> 5
110190
d2a[2^3 x 3^3 x 5^2, 2]
46
- (-1)^k k HypergeometricPFQ[{1+a, 1+b, 1+c, 1+d, 1-k}, {1, 1, 1, 2}, 1] /. a -> 3 /. b -> 3 /.
  c -> 2 /. d -> 2 /. k -> 5
110190
- (-1)^k k HypergeometricPFQ[{1+a, 1+b, 1+c, 1+d, 1-k}, {1, 1, 1, 2}, 1] /. a -> 3 /. b -> 3 /.
  c -> 2 /. d -> 0 /. k -> 2.3
ComplexInfinity
- (-1)^k k HypergeometricPFQ[{1+a, 1+b, 1+c, 1+d, 1-k}, {1, 1, 1, 2}, 1] /. b -> 0 /. c -> 0 /.
  d -> 0
- (-1)^k k HypergeometricPFQ[{1+a, 1-k}, {2}, 1]
Pochhammer[1, 6]
720
Pochhammer[1+a, k] / k!
Pochhammer[1+a, k] / k! /. a -> 3 /. k -> 3
20
(3+3)! / 3! / 3!
20
(-1)^(k+1) k
Sum[(a+j)! / a! / j! (b+j)! / b! / j! Pochhammer[1-k, j] / (j+1)!, {j, 0, Infinity}]
(-1)^(1+k) k HypergeometricPFQ[{1+a, 1+b, 1-k}, {1, 2}, 1]
(-1)^(k+1) k HypergeometricPFQ[{1+a, 1+b, 1+c, 1+d, 1-k}, {1, 1, 1, 2}, 1] /. a -> 7 /. b -> 3 /.
  c -> 0 /. d -> 0 /. k -> 4
1148

```



```

- (-1)^k k HypergeometricPFQ[{1+a, 1+b, 1+c, 1+d, 1-k}, {1, 1, 1, 2}, 1] /. c -> 0 /. d -> 0
- (-1)^k k HypergeometricPFQ[{1+a, 1+b, 1-k}, {1, 2}, 1]
- (-1)^k k HypergeometricPFQ[{1+a, 1+b, 1+c, 1+d, 1-k}, {1, 1, 1, 2}, 1] /. a -> 7 /. b -> 0 /.
  c -> 0 /. d -> 0 /. k -> 3

```

15

```

Table[FullSimplify[k Pochhammer[1-k, j] / (j+1)!] /. k -> 7, {j, 0, 7}]
{7, -21, 35, -35, 21, -7, 1, 0}
Table[(-1)^j Binomial[7, j+1], {j, 0, 7}]
{7, -21, 35, -35, 21, -7, 1, 0}
Sum[(a+j)! / a! / j! (b+j)! / b! / j! (-1)^(k+1+j) Binomial[k, j+1], {j, 0, Infinity}] /.
  a -> 7 /. b -> 3 /. c -> 0 /. d -> 0 /. k -> 4

```

1148

```

Sum[(-1)^(k+1+j) Binomial[k, j+1] (a+j)! / a! / j! (b+j)! / b! / j!, {j, 0, Infinity}]
- (-1)^k k HypergeometricPFQ[{1+a, 1+b, 1-k}, {1, 2}, 1]

```

```

Sum[(-1)^(k+1+j) Binomial[k, j+1] / j! (a+j)! / a! / j! (b+j)! / b!, {j, 0, Infinity}]
- (-1)^k k HypergeometricPFQ[{1+a, 1+b, 1-k}, {1, 2}, 1]

```

```

FullSimplify[- (-1)^k k HypergeometricPFQ[{1+a, 1-k}, {2}, 1] /. k -> 5]

```

$$\frac{1}{24} (-4+a) (-3+a) (-2+a) (-1+a)$$

```

Binomial[a-1, 5-1]

```

$$\frac{1}{24} (-4+a) (-3+a) (-2+a) (-1+a)$$

```

Sum[(-1)^(k+1+j) Binomial[k, j+1] / j! (a+j)! / a! / j! (b+j)! / b! (c+j)! / c! / j!,
  {j, 0, Infinity}] /. a -> 5 /. b -> 1 /. c -> 3 /. k -> 4

```

2244

```

d2a[2^5 x 3^3 x 5^1, 4]

```

2244

```

(-1)^(k+1) k Hypergeometric2F1[1+x, 1-k, 2, 1] /. k -> 3 /. x -> 15

```

91

```

Binomial[15-1, 3-1]

```

91

```

(-1)^(k+1) k Hypergeometric2F1[1+x, 1-k, 1, 1] /. k -> 3 /. x -> 15

```

315

```

FullSimplify@Sum[Pochhammer[j, a] / a!, {a, 0, x}]

```

$$\frac{\Gamma[1+j+x]}{\Gamma[1+j] \Gamma[1+x]}$$

FullSimplify[Sum[(-1)^(k-j) Binomial[k, j] Pochhammer[j, a] / a!, {j, 0, k}]]

$$(-1)^k \left(-\frac{\Gamma[-a+k]}{\Gamma[1-a] \Gamma[k]} + \frac{\text{Pochhammer}[0, a]}{\Gamma[1+a]} \right)$$

$$(-1)^k \left(-\frac{\Gamma[-a+k]}{\Gamma[1-a] \Gamma[k]} \right)$$

$$-\frac{(-1)^k \Gamma[-a+k]}{\Gamma[1-a] \Gamma[k]}$$

FullSimplify@Sum[(-1)^k \left(-\frac{\Gamma[-a+k]}{\Gamma[1-a] \Gamma[k]} + \frac{\text{Pochhammer}[0, a]}{\Gamma[1+a]} \right), {a, 0, x}]

$$\left((-1)^k (\Gamma[k-x] \Gamma[2+x] + (1-k+x) \Gamma[k] \Gamma[-x] \text{Pochhammer}[0, 1+x]) \right) / (k \Gamma[k] \Gamma[-x] \Gamma[2+x])$$

D[(-1)^z GammaRegularized[z, 0, -Log[100.]], z] /. z -> -0.00000001

30.1261 + 6.28318 i

FullSimplify[D[Hypergeometric1F1[z, z+1, Log[x]] Log[x]^z / z!, z] /. z -> 0]

$$\frac{1}{2} \left(\text{Log}\left[\frac{1}{\text{Log}[x]}\right] + \text{Log}[\text{Log}[x]] \right) + \text{LogIntegral}[x] /. x \rightarrow 100.$$

30.1261

Hypergeometric1F1[z, z+1, Log[x]] Log[x]^z / z!

$$\frac{(\Gamma[1+z] - z \Gamma[z, -\text{Log}[x]]) (-\text{Log}[x])^{-z} \text{Log}[x]^z}{z!}$$

D[\frac{(\Gamma[1+z] - z \Gamma[z, -\text{Log}[x]]) (-\text{Log}[x])^{-z} \text{Log}[x]^z}{z!}, z] /. z -> 0

$$-\Gamma[0, -\text{Log}[x]] - \text{Log}[-\text{Log}[x]] + \text{Log}[\text{Log}[x]]$$

D[GammaRegularized[z, 0, -Log[x]] (-Log[x])^{-z} Log[x]^z, z] /. z -> 0

Infinity::indet: Indeterminate expression 0 ComplexInfinity encountered. >>

Indeterminate

D[\frac{(\Gamma[z+1] - z \Gamma[z, -\text{Log}[x]]) (-\text{Log}[x])^{-z} \text{Log}[x]^z}{z!}, z] /. z -> 0

$$-\Gamma[0, -\text{Log}[x]] - \text{Log}[-\text{Log}[x]] + \text{Log}[\text{Log}[x]]$$

D[(1+x)^z, x]

$$(1+x)^{-1+z} z$$

D[Log[1+x], x]

$$\frac{1}{1+x}$$

Expand[\frac{1}{1+x}]

$$\frac{1}{1+x}$$

```
FullSimplify[(1 + x) (1 - x)]
```

```
1 - x2
```

```
p11[x_, k_] := Sum[k! StirlingS2[j, k] / j! Log[1 + x]^j, {j, 0, Infinity}]
```

```
p11[.5, 4]
```

```
0.0625
```

```
.5^4
```

```
0.0625
```

```
Table[4! StirlingS2[j, 4] / j!, {j, 0, 12}]
```

```
{0, 0, 0, 0, 1, 2,  $\frac{13}{6}$ ,  $\frac{5}{3}$ ,  $\frac{81}{80}$ ,  $\frac{37}{72}$ ,  $\frac{6821}{30240}$ ,  $\frac{265}{3024}$ ,  $\frac{55591}{1814400}$ }
```

```
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
```

```
FI[n_] := FactorInteger[n]; FI[1] := {}
```

```
dz[n_, s_, z_] := n^-s Product[Pochhammer[z, p[[2]]] / p[[2]]!, {p, FI[n]}]
```

```
Dzz[n_, z_] := Sum[dz[j, 0, z], {j, 1, n}]
```

```
D[Dzz[100, z], {z, 7}] /. z -> 0
```

```
0
```

```
StirlingS2[3, 2]
```

```
3
```

```
Table[d2a[2^k, j], {k, 0, 10}, {j, 0, k}] // Grid
```

```
1
0 1
0 1 1
0 1 2 1
0 1 3 3 1
0 1 4 6 4 1
0 1 5 10 10 5 1
0 1 6 15 20 15 6 1
0 1 7 21 35 35 21 7 1
0 1 8 28 56 70 56 28 8 1
0 1 9 36 84 126 126 84 36 9 1
```

```
Table[d2a[Pr[k], j], {k, 0, 10}, {j, 0, k}] // Grid
```

```
1
0 1
0 1 2
0 1 6 6
0 1 14 36 24
0 1 30 150 240 120
0 1 62 540 1560 1800 720
0 1 126 1806 8400 16800 15120 5040
0 1 254 5796 40824 126000 191520 141120 40320
0 1 510 18150 186480 834120 1905120 2328480 1451520 362880
0 1 1022 55980 818520 5103000 16435440 29635200 30240000 16329600 3628800
```

```
Table[j! Stirlings2[k, j], {k, 0, 10}, {j, 0, k}] // Grid
```

```
1
0 1
0 1 2
0 1 6 6
0 1 14 36 24
0 1 30 150 240 120
0 1 62 540 1560 1800 720
0 1 126 1806 8400 16800 15120 5040
0 1 254 5796 40824 126000 191520 141120 40320
0 1 510 18150 186480 834120 1905120 2328480 1451520 362880
0 1 1022 55980 818520 5103000 16435440 29635200 30240000 16329600 3628800
```

```
{Stirlings2[k, j], j Stirlings2[k - 1, j] + Stirlings2[k - 1, j - 1]} /. k -> 8 /. j -> 3
```

```
{966, 966}
```

```
{Binomial[k, j], Binomial[k - 1, j] + Binomial[k - 1, j - 1]} /. k -> 8 /. j -> 3
```

```
{56, 56}
```

```
Grid@Table[
```

```
(-1)^k / (z - k) (-1)^(k - j) / Gamma[z] / Gamma[1 - z] Binomial[k, j] f[j], {k, 0, 7}, {j, 0, k}]
```

```
f[0] /
```

```
(z Gamma[
  1 - z]
Gamma[
  z])
```

```
f[0] / -f[1] /
((-1 + z) ((-1 +
Gamma[ z)
1 - z] Gamma
Gamma[ [
z]) 1 - z]
Gamma
[
z])
```

```
f[0] / - (2 f[1]) / f[2] /
((-2 + z) ((-2 + ((-2 + z)
Gamma[ z) Gamma[
1 - z] Gamma 1 - z]
Gamma[ [
z]) 1 - z] Gamma[
Gamma
[
z])
```

```
f[0] / - (3 f[1]) / (3 f[2]) / -f[3] /
((-3 + z) ((-3 + ((-3 + z) ((-3 +
Gamma[ z) Gamma[ z)
1 - z] Gamma 1 - z] Gamma
Gamma[ [
z]) 1 - z] Gamma[ [
Gamma Gamma
[
z]) Gamma
[
z])
```

$$\begin{aligned}
& \frac{f[0]}{((-4+z)\Gamma[1-z]\Gamma[z])} - \frac{(4f[1])}{((-4+z)\Gamma[1-z]\Gamma[z])} + \frac{(6f[2])}{((-4+z)\Gamma[1-z]\Gamma[z])} - \frac{(4f[3])}{((-4+z)\Gamma[1-z]\Gamma[z])} + \frac{f[4]}{((-4+z)\Gamma[1-z]\Gamma[z])} \\
& \frac{f[0]}{((-5+z)\Gamma[1-z]\Gamma[z])} - \frac{(5f[1])}{((-5+z)\Gamma[1-z]\Gamma[z])} + \frac{(10f[2])}{((-5+z)\Gamma[1-z]\Gamma[z])} - \frac{(10f[3])}{((-5+z)\Gamma[1-z]\Gamma[z])} + \frac{(5f[4])}{((-5+z)\Gamma[1-z]\Gamma[z])} - \frac{f[5]}{((-5+z)\Gamma[1-z]\Gamma[z])} \\
& \frac{f[0]}{((-6+z)\Gamma[1-z]\Gamma[z])} - \frac{(6f[1])}{((-6+z)\Gamma[1-z]\Gamma[z])} + \frac{(15f[2])}{((-6+z)\Gamma[1-z]\Gamma[z])} - \frac{(20f[3])}{((-6+z)\Gamma[1-z]\Gamma[z])} + \frac{(15f[4])}{((-6+z)\Gamma[1-z]\Gamma[z])} - \frac{(6f[5])}{((-6+z)\Gamma[1-z]\Gamma[z])} + \frac{f[6]}{((-6+z)\Gamma[1-z]\Gamma[z])} \\
& \frac{f[0]}{((-7+z)\Gamma[1-z]\Gamma[z])} - \frac{(7f[1])}{((-7+z)\Gamma[1-z]\Gamma[z])} + \frac{(21f[2])}{((-7+z)\Gamma[1-z]\Gamma[z])} - \frac{(35f[3])}{((-7+z)\Gamma[1-z]\Gamma[z])} + \frac{(35f[4])}{((-7+z)\Gamma[1-z]\Gamma[z])} - \frac{(21f[5])}{((-7+z)\Gamma[1-z]\Gamma[z])} + \frac{(7f[6])}{((-7+z)\Gamma[1-z]\Gamma[z])} - \frac{f[7]}{((-7+z)\Gamma[1-z]\Gamma[z])}
\end{aligned}$$

Table $[(-1)^{(j+1)} / (z-j-k+1)$

Pochhammer $[j, k] / k! / \Gamma[z] / \Gamma[1-z] f[j-1], \{k, 0, 5\}] /. j \rightarrow 3$

$$\left\{ \frac{f[2]}{(-2+z)\Gamma[1-z]\Gamma[z]}, \frac{3f[2]}{(-3+z)\Gamma[1-z]\Gamma[z]}, \frac{6f[2]}{(-4+z)\Gamma[1-z]\Gamma[z]}, \frac{10f[2]}{(-5+z)\Gamma[1-z]\Gamma[z]}, \frac{15f[2]}{(-6+z)\Gamma[1-z]\Gamma[z]}, \frac{21f[2]}{(-7+z)\Gamma[1-z]\Gamma[z]} \right\}$$

```

FullSimplify@Sum[(-1)^(j+1)/(z-j-k+1)
  Pochhammer[j, k]/k!/Gamma[z]/Gamma[1-z] f[j-1], {k, 0, Infinity}]

$$\frac{(-1)^j f[-1+j] \Gamma[1-j] \Gamma[-1+j-z] \sin[\pi z]}{\pi \Gamma[-z]}$$

Sum[
$$\frac{(-1)^j f[-1+j] \Gamma[1-j] \Gamma[-1+j-z] \sin[\pi z]}{\pi \Gamma[-z]}$$
 f[j], {j, 0, Infinity}]/. z -> 2
0
(-1)^k/(z-k) (-1)^(k-j) Binomial[k, j]/Gamma[z]/Gamma[1-z]

$$\frac{(-1)^{-j+2k} \text{Binomial}[k, j]}{(-k+z) \Gamma[1-z] \Gamma[z]}$$

dz2[n_, z_] := Sin[Pi z]/Pi Sum[(-1)^k/(z-k) Sum[(-1)^(k-j) Binomial[k, j]
  Product[Pochhammer[j, p[[2]]]/p[[2]]!, {p, FI[n]}], {j, 0, k}], {k, 0, Log2@n}]
Limit[dz2[210, a], a -> 3]
36
d2[210, 3]
36
dz2[n_, z_] := Sin[Pi z]/Pi Sum[(-1)^k/(z-k) Sum[(-1)^(k-j) Binomial[k, j]
  Product[Pochhammer[j, p[[2]]]/p[[2]]!, {p, FI[n]}], {j, 0, k}], {k, 0, Log2@n}]
dz2a[n_, z_] := Sum[Sin[Pi z]/Pi (-1)^k/(z-k) (-1)^(k-j) Binomial[k, j]
  Product[Pochhammer[j, p[[2]]]/p[[2]]!, {p, FI[n]}], {k, 0, Log2@n}, {j, 0, k}]
dz2b[n_, z_] := Sum[Sin[Pi z]/Pi (-1)^(-j)/(z-k) Binomial[k, j]
  Product[Pochhammer[j, p[[2]]]/p[[2]]!, {p, FI[n]}], {k, 0, Log2@n}, {j, 0, k}]
dz2bx[n_, z_] := Table[Sin[Pi z]/Pi (-1)^(-j)/(z-k) Binomial[k, j] f[j],
  {k, 0, Log2@n}, {j, 0, k}]
Limit[dz2b[210, a], a -> 3]
36
Sin[Pi z]/Pi (-1)^k/(z-k) (-1)^(k-j) Binomial[k, j]

$$\frac{(-1)^{-j+2k} \text{Binomial}[k, j] \sin[\pi z]}{\pi (-k+z)}$$


```

dz2bx[100, z] // Grid

$$\begin{aligned}
 & (f[0] \sin[\pi z]) / (\pi z) \\
 & (f[0] \sin[\pi z]) / (\pi z) - (f[1] \sin[\pi z]) / (\pi (-1 + z)) \\
 & (f[0] \sin[\pi z]) / (\pi (-2 + z)) - (2 f[1] \sin[\pi z]) / (\pi (-2 + z)) + (f[2] \sin[\pi z]) / (\pi (-2 + z)) \\
 & (f[0] \sin[\pi z]) / (\pi (-3 + z)) - (3 f[1] \sin[\pi z]) / (\pi (-3 + z)) + (3 f[2] \sin[\pi z]) / (\pi (-3 + z)) - (f[3] \sin[\pi z]) / (\pi (-3 + z)) \\
 & (f[0] \sin[\pi z]) / (\pi (-4 + z)) - (4 f[1] \sin[\pi z]) / (\pi (-4 + z)) + (6 f[2] \sin[\pi z]) / (\pi (-4 + z)) - (4 f[3] \sin[\pi z]) / (\pi (-4 + z)) + (f[4] \sin[\pi z]) / (\pi (-4 + z)) \\
 & (f[0] \sin[\pi z]) / (\pi (-5 + z)) - (5 f[1] \sin[\pi z]) / (\pi (-5 + z)) + (10 f[2] \sin[\pi z]) / (\pi (-5 + z)) - (10 f[3] \sin[\pi z]) / (\pi (-5 + z)) + (5 f[4] \sin[\pi z]) / (\pi (-5 + z)) - (f[5] \sin[\pi z]) / (\pi (-5 + z)) \\
 & (f[0] \sin[\pi z]) / (\pi (-6 + z)) - (6 f[1] \sin[\pi z]) / (\pi (-6 + z)) + (15 f[2] \sin[\pi z]) / (\pi (-6 + z)) - (20 f[3] \sin[\pi z]) / (\pi (-6 + z)) + (15 f[4] \sin[\pi z]) / (\pi (-6 + z)) - (6 f[5] \sin[\pi z]) / (\pi (-6 + z)) + (f[6] \sin[\pi z]) / (\pi (-6 + z))
 \end{aligned}$$