

```

DD2[n_, k_] := DD2[n, k] = Sum[ DD2[Floor[n / j], k - 1], {j, 2, n}]; DD2[n_, 0] := 1
DDD[n_, z_] := Sum[FactorialPower[z, a] / a! DD2[n, a], {a, 0, Log[2, n]}]
K[n_] := If[n == 1, 0, FullSimplify[MangoldtLambda[n] / Log[n]]]
P[n_, k_] := Sum[ K[j] P[Floor[n / j], k - 1], {j, 2, n}]; P[n_, 0] := 1

```

```

da[n_, z_] := Product[Pochhammer[z, a = p[[2]]] / a!, {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
daa[z_] := da[8, z]
Expand[daa'[z]]

```

$$\frac{1}{3} + z + \frac{z^2}{2}$$

```

Table[ P[100, k] / (k - 1)!, {k, 1, 7}]

```

$$\left\{ \frac{428}{15}, \frac{16289}{180}, \frac{993}{16}, \frac{611}{36}, \frac{67}{48}, \frac{7}{120}, 0 \right\}$$

```

faa[z_] := DDDa[100, z]
faa'[z]
Expand[FullSimplify[faa'[z]]]

```

```

DDDa(0,1)[100, z]

```

```

DDDa(0,1)[100, z]

```

```

d2d[n_, z_] := Sum[ 1 / (k!) DD2[n, k] FactorialPower[z, k]
  (-PolyGamma[0, (-k + 1) + z] + PolyGamma[0, 1 + z]), {k, 1, Log[2, n]}]
d2e[n_, z_] := Sum[ 1 / (k!) DD2a[n, k] FactorialPower[z, k]
  (-PolyGamma[0, (-k + 1) + z] + PolyGamma[0, 1 + z]), {k, 1, Log[2, n]}]

```

**d2e[100, 2]**

Infinity::indet: Indeterminate expression 0 DD2a[100, 3] ComplexInfinity encountered. >>

Infinity::indet: Indeterminate expression 0 DD2a[100, 4] ComplexInfinity encountered. >>

Infinity::indet: Indeterminate expression 0 DD2a[100, 5] ComplexInfinity encountered. >>

General::stop: Further output of Infinity::indet will be suppressed during this calculation. >>

Indeterminate

**fa[z\_] := DDD[100, z]**

**Expand[FullSimplify[fa'[z]]]**

$$\frac{428}{15} + \frac{16289z}{180} + \frac{993z^2}{16} + \frac{611z^3}{36} + \frac{67z^4}{48} + \frac{7z^5}{120}$$

**D[DDD[100, z], z]**

$$\begin{aligned} & 99 + \frac{7}{720} \text{FactorialPower}[z, 6] (-\text{PolyGamma}[0, -5 + z] + \text{PolyGamma}[0, 1 + z]) + \\ & \frac{17}{40} \text{FactorialPower}[z, 5] (-\text{PolyGamma}[0, -4 + z] + \text{PolyGamma}[0, 1 + z]) + \\ & \frac{23}{3} \text{FactorialPower}[z, 4] (-\text{PolyGamma}[0, -3 + z] + \text{PolyGamma}[0, 1 + z]) + \\ & 54 \text{FactorialPower}[z, 3] (-\text{PolyGamma}[0, -2 + z] + \text{PolyGamma}[0, 1 + z]) + \\ & \frac{283}{2} \text{FactorialPower}[z, 2] (-\text{PolyGamma}[0, -1 + z] + \text{PolyGamma}[0, 1 + z]) \end{aligned}$$

**FullSimplify[D[DDD[100, z], z]] /. z -> 0**

$$\frac{428}{15}$$

**Expand[FullSimplify[fa[z]]]**

$$\begin{aligned} & 1 + 99z + \frac{283}{2} \text{FactorialPower}[z, 2] + 54 \text{FactorialPower}[z, 3] + \\ & \frac{23}{3} \text{FactorialPower}[z, 4] + \frac{17}{40} \text{FactorialPower}[z, 5] + \frac{7}{720} \text{FactorialPower}[z, 6] \end{aligned}$$

**Expand[FullSimplify[fa[b]]]**

$$\begin{aligned} & 1 + 99b + \frac{283}{2} \text{FactorialPower}[b, 2] + 54 \text{FactorialPower}[b, 3] + \\ & \frac{23}{3} \text{FactorialPower}[b, 4] + \frac{17}{40} \text{FactorialPower}[b, 5] + \frac{7}{720} \text{FactorialPower}[b, 6] \end{aligned}$$

$$\mathbf{fc[z_]} := \frac{428}{15} + \frac{16289z}{180} + \frac{993z^2}{16} + \frac{611z^3}{36} + \frac{67z^4}{48} + \frac{7z^5}{120}$$

**1 + Integrate[fc[z], {z, 0, -3}]**

47

**DDD[100, -3]**

47

`fc[0]`

$$\frac{428}{15}$$

`1 + Integrate[fc[z], {z, 0, 3}]`

1471

`DDD[100, 3]`

1471

`1 + Integrate[fc[z], {z, 0, 1 - 3 I}]`

$$-\frac{5279}{8} + \frac{3335 i}{8}$$

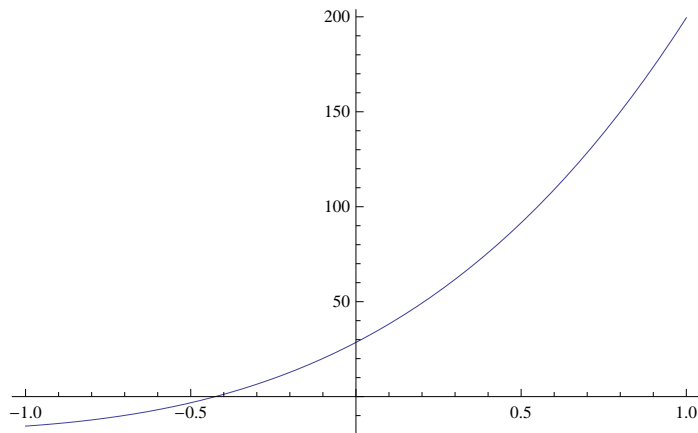
`DDD[100, 1 - 3 I]`

$$-\frac{5279}{8} + \frac{3335 i}{8}$$

`Integrate[fc[z], {z, 1, 2}]`

382

`Plot[{fc[z]}, {z, -1, 1}]`



```

c1 := CoefficientList[Series[Log[1 + x], {x, 0, 20}], x]
c2 := CoefficientList[Series[Log[1 + x]^2, {x, 0, 20}], x]
c3 := CoefficientList[Series[Log[1 + x]^3, {x, 0, 20}], x]
c4 := CoefficientList[Series[Log[1 + x]^4, {x, 0, 20}], x]
c5 := CoefficientList[Series[Log[1 + x]^5, {x, 0, 20}], x]
c6 := CoefficientList[Series[Log[1 + x]^6, {x, 0, 20}], x]
c7 := CoefficientList[Series[Log[1 + x]^7, {x, 0, 20}], x]
ca := {c1, c2, c3, c4, c5, c6, c7}

```

$$\frac{428}{15} + \frac{16289z}{180} + \frac{993z^2}{16} + \frac{611z^3}{36} + \frac{67z^4}{48} + \frac{7z^5}{120}$$

Sum[ca[[1]][[k+1]] DD2[100, k], {k, 1, 7}]

428

15

Sum[Sum[ca[[j]][[k+1]] DD2[100, k] / ((j-1)!) z^(j-1), {k, 1, 7}], {j, 1, 7}]

$$\frac{428}{15} + \frac{16289z}{180} + \frac{993z^2}{16} + \frac{611z^3}{36} + \frac{67z^4}{48} + \frac{7z^5}{120}$$

Sum[DD2a[100, k] Sum[ca[[j]][[k+1]] / ((j-1)!) z^(j-1), {j, 1, 7}], {k, 1, 7}]

$$\begin{aligned} & \text{DD2a}[100, 1] + \left(-\frac{1}{2} + z\right) \text{DD2a}[100, 2] + \left(\frac{1}{3} - z + \frac{z^2}{2}\right) \text{DD2a}[100, 3] + \\ & \left(-\frac{1}{4} + \frac{11z}{12} - \frac{3z^2}{4} + \frac{z^3}{6}\right) \text{DD2a}[100, 4] + \left(\frac{1}{5} - \frac{5z}{6} + \frac{7z^2}{8} - \frac{z^3}{3} + \frac{z^4}{24}\right) \text{DD2a}[100, 5] + \\ & \left(-\frac{1}{6} + \frac{137z}{180} - \frac{15z^2}{16} + \frac{17z^3}{36} - \frac{5z^4}{48} + \frac{z^5}{120}\right) \text{DD2a}[100, 6] + \\ & \left(\frac{1}{7} - \frac{7z}{10} + \frac{29z^2}{30} - \frac{7z^3}{12} + \frac{25z^4}{144} - \frac{z^5}{40} + \frac{z^6}{720}\right) \text{DD2a}[100, 7] \end{aligned}$$

dif[k\_] := (DDD[100, k] - DDD[100, -k]) / (2k)

dif[.0001]

28.5333

D2[n\_, k\_, s\_] := Sum[j^(-s) D2[Floor[n/j], k-1, s], {j, 2, n}]; D2[n\_, 0, s\_] := 1

DD[n\_, z\_, s\_] := Sum[FactorialPower[z, a] / a! D2[n, a, s], {a, 0, Log[2, n]}]

dif[k\_, s\_] := (DD[100, k, s] - DD[100, -k, s]) / (2k)

N[dif[.0001, -1]]

1156.48

(DD[100, .0001, -1] - 1) / .0001

1156.72

fs[s\_] := x^s

fs'[s]

x^s Log[x]

fo[s\_] := (1 - Gamma[s, -Log[100]]) / Gamma[s]

**Limit[fo'[s], s → 0]**

Limit[  

$$-\frac{1}{\Gamma[s]} (\Gamma[s, -\text{Log}[100]] (i \pi + \text{Log}[\text{Log}[100]]) + \text{MeijerG}[\{\{\}, \{1, 1\}\}, \{\{0, 0, s\}, \{\}\}, -\text{Log}[100]]) + \frac{\Gamma[s, -\text{Log}[100]] \text{PolyGamma}[0, s]}{\Gamma[s]}, s \rightarrow 0]$$

```

D2[n_, k_, s_] := D2[n, k, s] = Sum[j^(-s) D2[Floor[n / j], k - 1, s], {j, 2, n}];
D2[n_, 0, s_] := 1
DD[n_, z_, s_] := Sum[FactorialPower[z, a] / a! D2[n, a, s], {a, 0, Log[2, n]}]

```

```

g1[s_] := DD[10, z, s]
g1a[s_] := DD[100, z, s]

```

```
g1[0]
```

$$1 + 99z + \frac{283}{2} \text{FactorialPower}[z, 2] + 54 \text{FactorialPower}[z, 3] + \frac{23}{3} \text{FactorialPower}[z, 4] + \frac{17}{40} \text{FactorialPower}[z, 5] + \frac{7}{720} \text{FactorialPower}[z, 6]$$

```
Integrate[g1a'[s], {s, 0, Infinity}]
```

```
$Aborted
```

```

gg[zz_, s_] := g1'[s] /. z -> zz
gga[zz_, s_] := g1a'[s] /. z -> zz

```

```
g2[s_] := D2[10, 1, s]
```

```
Expand[g2'[s]]
```

$$-2^{-s} \text{Log}[2] - 3^{-s} \text{Log}[3] - 4^{-s} \text{Log}[4] - 5^{-s} \text{Log}[5] - 6^{-s} \text{Log}[6] - 7^{-s} \text{Log}[7] - 8^{-s} \text{Log}[8] - 9^{-s} \text{Log}[9] - 10^{-s} \text{Log}[10]$$

```
Integrate[gg[1, s], {s, 0, -1}]
```

```
45
```

```
DD[10, 2, 0] + Integrate[gg[2, s], {s, 0, -1}]
```

```
170
```

```
DD[10, 2, -1]
```

```
170
```

```
DD[10, 3, 0] + Integrate[gg[3, s], {s, 0, -2}]
```

```
2708
```

```
DD[10, 3, -2]
```

```
2708
```

```
DD[10, 3, 3] + Integrate[gg[3, s], {s, 3, -2}]
```

```
2708
```

```
1 - Integrate[gg[3, s], {s, -2, Infinity}]
```

```
2708
```

```
(DD[10, .00000001, 0] + Integrate[ gg[.00000001, s], {s, 0, -2}] - 1) / .00000001
```

```
156.833
```

```
(DD[10, .00000001, -2] - 1) / .00000001
```

```
156.833
```

```
(-Integrate[ gg[.00000001, s], {s, -2, Infinity}]) / .00000001
```

```
156.833
```

```
(DD[10, .0000001, -2] - Integrate[ gg[.0000001, s], {s, 0, -2}] - 1) / .0000001
```

```
5.33333
```

```
(-Integrate[ gg[.00000001, s], {s, 0, Infinity}]) / .00000001
```

```
5.33333
```

```
(DD[10, .0000001, 0] - 1) / .0000001
```

```
5.33333
```

```
DD[10, .0000001, -2]
```

```
1.00002
```

```
Integrate[ gg[.0000001, s], {s, 0, -2}]
```

```
0.00001515
```

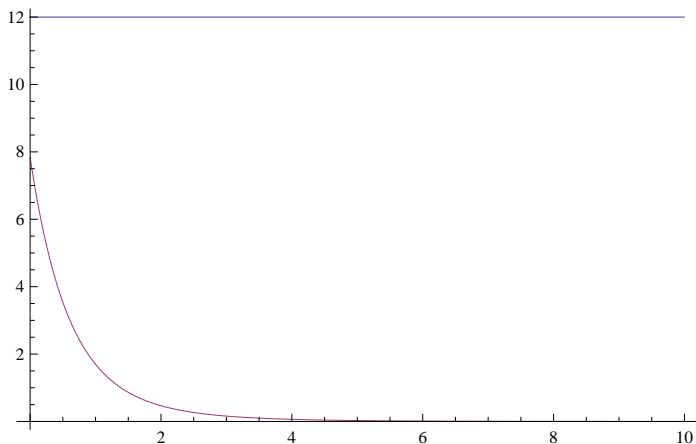
```
(-Integrate[ gga[.00000001, s], {s, 0, Infinity}]) / .00000001
```

```
$Aborted
```

```
(Integrate[- gg[.00000001, s] / .00000001, {s, 0, Infinity}])
```

```
5.33333
```

```
Plot[{12, -gg[.000000001, s] / .000000001}, {s, 0, 10}]
```



```
-gga[.0000000001, 0] / .0000000001
```

```
94.0453
```

```
N[Sum[ MangoldtLambda[ j], {j, 2, 100}]]
```

```
94.0453
```

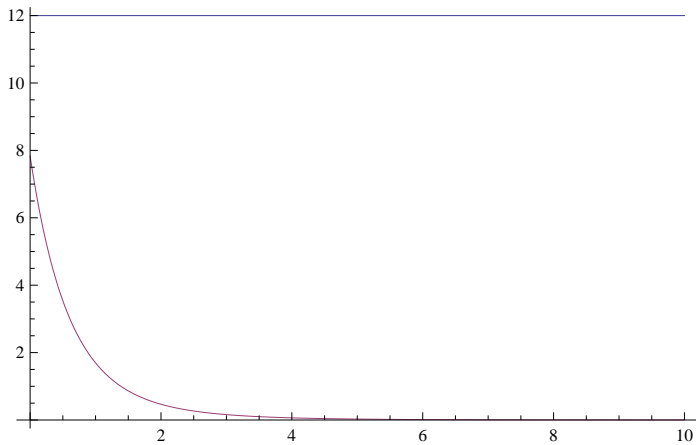
```
N[Limit[-gga[s, 0] / s, {s → 0}]]
```

```
{94.0453}
```

```
-gga[1., 0]
```

```
363.739
```

```
Plot[{12, -gg[.000001, s] / .000001}, {s, 0, 10}]
```



```
Limit[-gg[k, s] / k, {k → 0}]
```

$$\left\{ 4^{-s} (-1 + 2^s) \operatorname{Log}[2] + 9^{-s} (-1 + 3^s) \operatorname{Log}[3] + \right. \\ \left. 2^{-1-3^s} \operatorname{Log}[4] + 4^{-s} \operatorname{Log}[4] + 5^{-s} \operatorname{Log}[5] + 7^{-s} \operatorname{Log}[7] + 9^{-s} \operatorname{Log}[9] \right\}$$

```
ga[s_] := 4-s (-1 + 2s) Log[2] + 9-s (-1 + 3s) Log[3] +  
2-1-3s Log[4] + 4-s Log[4] + 5-s Log[5] + 7-s Log[7] + 9-s Log[9]
```

```
Integrate[ga[s], {s, 0, Infinity}]
```

$$\frac{16}{3}$$

```
Integrate[ga[s], {s, 0, I Infinity}]
```

$$\int_0^{i \infty} \left( 4^{-s} (-1 + 2^s) \operatorname{Log}[2] + 9^{-s} (-1 + 3^s) \operatorname{Log}[3] + \right. \\ \left. 2^{-1-3^s} \operatorname{Log}[4] + 4^{-s} \operatorname{Log}[4] + 5^{-s} \operatorname{Log}[5] + 7^{-s} \operatorname{Log}[7] + 9^{-s} \operatorname{Log}[9] \right) ds$$

```
Sum[Log[2] / 2s, {s, 0, Infinity}]
```

```
2 Log[2]
```

```
Integrate[Log[2] / 2s, {s, 0, Infinity}]
```

```
1
```

```
Integrate[ga[s], {s, 0, 1}]
```

$$\frac{2479}{630}$$

```
(DD[10, .0000001, 1] - 1) / .0000001
```

```
1.39841
```



**D[x^n, x]**

$n x^{-1+n}$

**D[x^n, n]**

$x^n \text{Log}[x]$

**D[DD[10, z, s], s]**

$$-2^{-1-3s} \text{FactorialPower}[z, 3] \text{Log}[2] + \frac{1}{2} \text{FactorialPower}[z, 2] \\ (-2^{-s} (2^{-s} + 3^{-s} + 4^{-s} + 5^{-s}) \text{Log}[2] - 3^{-s} (2^{-s} + 3^{-s}) \text{Log}[3] + 3^{-s} (-2^{-s} \text{Log}[2] - 3^{-s} \text{Log}[3]) + \\ 2^{-s} (-2^{-s} \text{Log}[2] - 3^{-s} \text{Log}[3] - 4^{-s} \text{Log}[4] - 5^{-s} \text{Log}[5]) - 8^{-s} \text{Log}[8] - 10^{-s} \text{Log}[10]) + \\ z (-2^{-s} \text{Log}[2] - 3^{-s} \text{Log}[3] - 4^{-s} \text{Log}[4] - 5^{-s} \text{Log}[5] - 6^{-s} \text{Log}[6] - \\ 7^{-s} \text{Log}[7] - 8^{-s} \text{Log}[8] - 9^{-s} \text{Log}[9] - 10^{-s} \text{Log}[10])$$

**Integrate[-D[ DD[10, .00000001, s], s] / .00000001, {s, 0, Infinity}]**

5.33333

**1 - Integrate[D[ DD[10, 2, s], s], {s, -1, Infinity}]**

170

**DD[10, 2, -1]**

170

**Limit[-z^-1 Integrate[D[ DD[10, z, s], s], {s, 0, Infinity}], {z -> 0}]**

$\left\{\frac{16}{3}\right\}$

**Limit[( DD[10, z, 0] - 1) / z, {z -> 0}]**

$\left\{\frac{16}{3}\right\}$

**FullSimplify[Limit[ Integrate[-D[ DD[10, z, s], s] / z, {s, -1, Infinity}], {z -> 0}]]**

$\left\{\frac{157}{6}\right\}$

**FullSimplify[Limit[ Integrate[-D[ DD[10, z, s], s] / z, {s, 1, Infinity}], {z -> 0}]]**

$\left\{\frac{881}{630}\right\}$

**Limit[( DD[10, z, 1] - 1) / z, {z -> 0}]**

$\left\{\frac{881}{630}\right\}$

1 + 1

**-gga[.0000000001, 0] / .0000000001**

94.0453

```

L2[n_, k_] := Sum[ L2[Floor[n / j], k - 1], {j, 2, n}]; L2[n_, 1] := Sum[ Log[j], {j, 2, n}]
L2toL1x[n_, z_] := Sum[Binomial[z - 1, a] L2[n, a + 1], {a, 0, Log[2, n]}]

```

```

N[Limit[ D[ (-DD[100, z, s] - 1) / z, s] /. s -> 0, z -> 0]]

```

```

94.0453

```

```

Limit[ ( DD[100, z, 0] - 1) / z, z -> 0]

```

```

428

```

```

15

```

```

FullSimplify[D[DD[100, z, 0], z]] /. z -> 0

```

```

428

```

```

15

```

```

N[D[-DD[100, -.5 I, s] / -.5 I, s] /. s -> 0]

```

```

-73.7152 + 82.3658 i

```

```

N[L2toL1x[100, -.5 I]]

```

```

73.7152 - 82.3658 i

```

```

N[D[-DD[100, 1, s] / 1, s] /. s -> 0]

```

```

363.739

```

```

N[L2toL1x[100, 1]]

```

```

363.739

```

```

N[L2toL1x[100, 2]]

```

```

920.841

```

```

N[D[-DD[100, 2, s] / 2, s] /. s -> 0]

```

```

920.841

```

```

FullSimplify[D[DD[100, z, 0], z]] /. z -> 0

```

```

428

```

```

15

```

```

N[Integrate[ D[-DD[100, k, s] / k, s] /. s -> 0, {k, 0, 1}]]

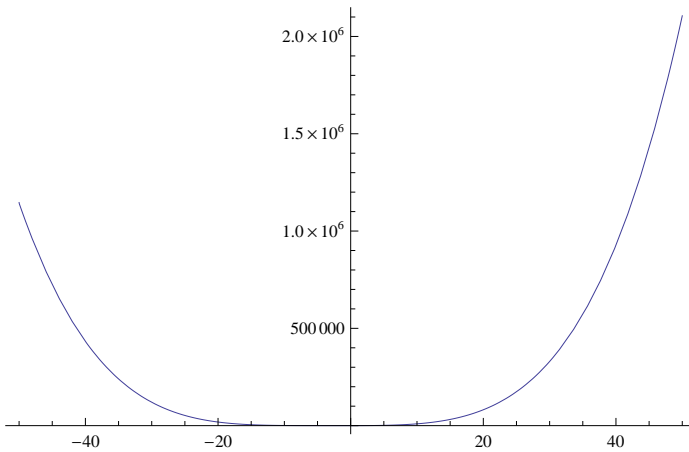
```

```

210.494

```

```
Plot[D[DD[50, z, 0], z] /. z -> z2, {z2, -50, 50}]
```



```
D[DD[50, z, 0], z]
```

```
ss[z_] := 49 +  $\frac{1}{20}$  FactorialPower[z, 5] (-PolyGamma[0, -4 + z] + PolyGamma[0, 1 + z]) +  

 $\frac{35}{24}$  FactorialPower[z, 4] (-PolyGamma[0, -3 + z] + PolyGamma[0, 1 + z]) +  

 $\frac{46}{3}$  FactorialPower[z, 3] (-PolyGamma[0, -2 + z] + PolyGamma[0, 1 + z]) +  

54 FactorialPower[z, 2] (-PolyGamma[0, -1 + z] + PolyGamma[0, 1 + z])
```

```
Expand[FullSimplify[D[DD[100, z, 0], z]]]
```

$$\frac{428}{15} + \frac{16289z}{180} + \frac{993z^2}{16} + \frac{611z^3}{36} + \frac{67z^4}{48} + \frac{7z^5}{120}$$

```
Integrate[-D[DO[n, z, t] / z, z], {t, b, c}]
```

$$\int_b^c \left( \frac{DO[n, z, t]}{z^2} - \frac{DO^{(0,1,0)}[n, z, t]}{z} \right) dt$$

```
D2[n_, k_, s_] := D2[n, k, s] = Sum[j^(-s) D2[Floor[n / j], k - 1, s], {j, 2, n}];
```

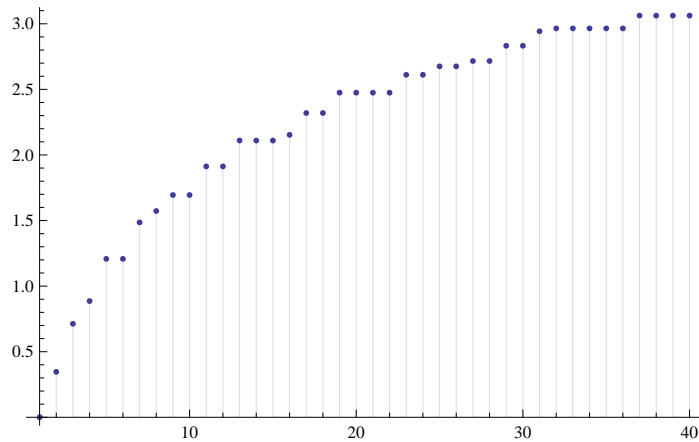
```
D2[n_, 0, s_] := 1
```

```
DD[n_, z_, s_] := Sum[FactorialPower[z, a] / a! D2[n, a, s], {a, 0, Log[2, n]}]
```

```
N[Limit[D[(-DD[100, z, s] - 1) / z, s] /. s -> 0, z -> 0]]
```

```
94.0453
```

```
DiscretePlot[ Limit[ D[ (-DD[n, z, s] - 1) / z, s] /. s -> 1, z -> 0], {n, 1, 40}]
```



```
FullSimplify[
  Limit[ D[ (-DD[30, z, s] - 1) / z, s], z -> 0] - Limit[ D[ (-DD[29, z, s] - 1) / z, s], z -> 0]]
0
```

```
FullSimplify[
  Limit[ D[ (-DD[31, z, s] - 1) / z, s], z -> 0] - Limit[ D[ (-DD[30, z, s] - 1) / z, s], z -> 0]]
31-s Log[31]
31 × 31 × 31
29 791
```

```
FullSimplify[
  Sum[ MangoldtLambda[ j ] / Log[j] Integrate[ Log[j] j-s, {s, 0, Infinity}], {j, 2, 100}]]
428
15
```

```
FullSimplify[
  Limit[ D[ (-DD[31, z, s] - 1) / z, s], z -> 1] - Limit[ D[ (-DD[30, z, s] - 1) / z, s], z -> 1]]
31-s Log[31]
```

```
Table[ FullSimplify[D[DD[100, z, 0], {z, s}]] /. z -> 0, {s, 0, 4}]
```

$$\left\{1, \frac{428}{15}, \frac{16289}{180}, \frac{993}{8}, \frac{611}{6}\right\}$$

```
Table[ P[100, k], {k, 0, 7}]
```

$$\left\{1, \frac{428}{15}, \frac{16289}{180}, \frac{993}{8}, \frac{611}{6}, \frac{67}{2}, 7, 0\right\}$$

```
Table[Limit[FullSimplify[D[(DD[100, z, 0] - 1) / z, {z, s}]], z → 0], {s, 0, 3}]
```

$$\left\{ \frac{428}{15}, \frac{16289}{360}, \frac{331}{8}, \frac{611}{24} \right\}$$

```
Table[P[100, k] / k, {k, 1, 7}]
```

$$\left\{ \frac{428}{15}, \frac{16289}{360}, \frac{331}{8}, \frac{611}{24}, \frac{67}{10}, \frac{7}{6}, 0 \right\}$$

```
N[Limit[D[(DD[100, z, s] - 1) / z, {s, 0}] /. s → 0, z → 0]]
```

```
28.5333
```

```
N[Sum[Log[j]^0 K[j], {j, 2, 100}]]
```

```
28.5333
```

```
N[Limit[D[(DD[100, z, s] - 1) / z, {s, 1}] /. s → 0, z → 0]]
```

```
-94.0453
```

```
N[Sum[(-1)^(1) Log[j] K[j], {j, 2, 100}]]
```

```
-94.0453
```

```
N[Limit[D[(DD[100, z, s] - 1) / z, {s, 2}] /. s → 0, z → 0]]
```

```
342.318
```

```
N[Sum[(-1)^(2) Log[j]^2 K[j], {j, 2, 100}]]
```

```
342.318
```

```
N[Limit[D[(DD[100, z, s] - 1) / z, {s, 3}] /. s → 0, z → 0]]
```

```
-1311.28
```

```
N[Sum[(-1)^(3) Log[j]^3 K[j], {j, 2, 100}]]
```

```
-1311.28
```

```
N[Limit[D[(DD[100, z, s] - 1) / z, {s, 4}] /. s → 0, z → 0]]
```

```
5178.95
```

```
N[Sum[(-1)^4 Log[j]^4 K[j], {j, 2, 100}]]
```

```
5178.95
```

```
N[Limit[D[(DD[100, z, s]), {z, 1}] /. s → 0, z → 0]]
```

```
28.5333
```

```
tt[n_, k_] := Sum[(-1)^(k) Log[j]^k K[j] / k!, {j, 2, 100}]
```

```
lx[n_, x_] := tt[n, 0] + Sum[tt[n, j] x^j, {j, 1, 30}]
```

```
N[lx[100, -1]]
```

```
1156.48
```

```
N[Limit[(DD[100, z, -1] - 1) / z, z → 0]]
```

```
1156.48
```