$$\begin{split} & \mathtt{zt4}\{1_-] := \mathtt{Product}[(1-1/(\mathtt{ZetaZero}\{r]+2)) \; (1-1/(\mathtt{ZetaZero}\{-r]+2)), \; \{r,1,1\}\} \\ & \mathtt{N}[\mathtt{zt4}[1,800]] \\ & 1, -3,33067 \times 10^{-16} \; \mathbf{i} \\ & \mathtt{F1}[\mathbf{b}_-, \mathbf{c}_-] := \left(1 + \frac{1-2 \; (1/2-\mathbf{c})}{(1/2-\mathbf{c})^2 + \mathbf{b}^2}\right) \\ & \mathtt{F2}[\mathbf{b}_-, \mathbf{c}_-] := \left(1 + \frac{1-2 \; (1/2+\mathbf{c})}{(1/2+\mathbf{c})^2 + \mathbf{b}^2}\right) \\ & \mathtt{F1}[1,1/7] \\ & \frac{277}{221} \\ & \mathtt{F2}[1,1/7] \\ & 221 \\ & 277 \\ & \mathtt{FullSimplify}[(1-1/(\mathbf{a}+\mathbf{b}\,\mathbf{I})) \; (1-1/(\mathbf{a}-\mathbf{b}\,\mathbf{I}))] \\ & 1 + \frac{1-2 \; \mathbf{a}}{\mathbf{a}^2 + \mathbf{b}^2} \\ & \mathtt{FullSimplify}[(1-2/(\mathbf{a}+\mathbf{b}\,\mathbf{I})) \; (1-2/(\mathbf{a}-\mathbf{b}\,\mathbf{I}))] \\ & 1 + \frac{4-4 \; \mathbf{a}}{\mathbf{a}^2 + \mathbf{b}^2} \\ & \left(1 + \frac{4-4 \; (1/2-\mathbf{c})}{(1/2-\mathbf{c})^2 + \mathbf{b}^2}\right) \left(1 + \frac{4-4 \; (1/2+\mathbf{c})}{(1/2+\mathbf{c})^2 + \mathbf{b}^2}\right) \\ & \mathtt{Expand}\left[\left(1 + \frac{4-4 \; (\frac{1}{2}-\mathbf{c})}{\mathbf{b}^2 + (\frac{1}{2}-\mathbf{c})^2}\right) \left(1 + \frac{4-4 \; (\frac{1}{2}+\mathbf{c})}{\mathbf{b}^2 + (\frac{1}{2}+\mathbf{c})^2}\right)\right] \\ & \mathtt{FullSimplify}[1 + \frac{2}{\mathbf{b}^2 + (\frac{1}{2}-\mathbf{c})^2} + \frac{4\mathbf{c}}{\mathbf{b}^2 + (\frac{1}{2}-\mathbf{c})^2} + \frac{2}{\mathbf{b}^2 + (\frac{1}{2}+\mathbf{c})^2} \\ & \frac{4}{\left(\mathbf{b}^2 + (\frac{1}{2}-\mathbf{c})^2\right) \left(\mathbf{b}^2 + (\frac{1}{2}+\mathbf{c})^2\right)} + \frac{4\mathbf{c}}{\mathbf{b}^2 + (\frac{1}{2}+\mathbf{c})^2} - \frac{16 \; \mathbf{b}^2}{\left(\mathbf{b}^2 + (\frac{1}{2}+\mathbf{c})^2\right)^2} \left(\mathbf{b}^2 + (\frac{1}{2}+\mathbf{c})^2\right)^2} \right] \\ & \mathtt{ff}[\mathbf{b}_-, \mathbf{c}_-] := \frac{16 \; \mathbf{b}^4 + (9-4 \; \mathbf{c}^2)^2 + 8 \; \mathbf{b}^2 \; (9+4 \; \mathbf{c}^2)}{(1+4 \; \mathbf{b}^2)^2 + 8 \; (-1+4 \; \mathbf{b}^2) \; \mathbf{c}^2 + 16 \; \mathbf{c}^4} \\ & \mathtt{ff}[1/4, \mathbf{c}_-] \\ & \frac{1}{16} \; (9-4 \; \mathbf{c}^2)^2 + \frac{1}{2} \; (9+4 \; \mathbf{c}^2)}{(1+4 \; \mathbf{b}^2)^2 + 8 \; (-1+4 \; \mathbf{b}^2) \; \mathbf{c}^2 + 16 \; \mathbf{c}^4} \\ & \frac{1}{16} \; (9-4 \; \mathbf{c}^2)^2 + \frac{1}{2} \; (9+4 \; \mathbf{c}^2)}{(1+4 \; \mathbf{b}^2)^2 + 8 \; (-1+4 \; \mathbf{b}^2) \; \mathbf{c}^2 + 16 \; \mathbf{c}^4} \end{aligned}$$

```
ff[1/2,c]
```

$$\frac{1 + (9 - 4 c^{2})^{2} + 2 (9 + 4 c^{2})}{4 + 16 c^{4}}$$

 $N[Table[{n, ff[s, 15]}, {s, 0, 1/2, 1/20}] // TableForm]$

```
0.982282
n
```

0.982282 n

0.982284 n

0.982287 n

0.982291

n 0.982296 n

0.982303 n

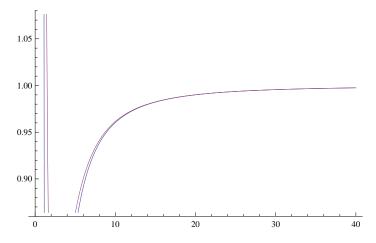
0.982311 n

0.98232 n

0.98233 n

0.982341 n

Plot[{ff[0, c], ff[1, c]}, {c, 0, 40}]



$$zt4[1_] := Product[(1-1/ZetaZero[r]) (1-1/ZetaZero[-r]), {r, 1, 1}] N[zt4[1, 800]]$$

0.99999999999956 - 3.3306690738754696 * 4-16 i

 $zt4a[1_] := Sum[Log[1-1/ZetaZero[r]] + Log[1-1/ZetaZero[-r]], {r, 1, 1}]$ N[zt4a[100]]

 $-8.81897 \times 10^{-17} + 0.$ i

N[Log[1-1/ZetaZero[1]] + Log[1-1/ZetaZero[-1]]]

 $-4.42354 \times 10^{-17} + 0.i$

N[Log[1-1/(.5+3I)] + Log[1-1/(.5-3I)]]

0. + 0. i

N[Log[1-1/(.5+3I)]]

0. + 0.330297 i

$$\begin{split} & \text{N}[\text{Log}[1-1/\left(.3+3\text{ I}\right)] + \text{Log}[1-1/\left(.3-3\text{ I}\right)] \\ & 0.0430637 + 0. \text{ i} \\ & \text{N}[\text{Log}[1-1/\left(.7+3\text{ I}\right)] + \text{Log}[1-1/\left(.7-3\text{ I}\right)] \\ & -0.0430637 + 0. \text{ i} \\ & \text{FullSimplify}[(1+1/\left(a+b\text{ I}\right)) \left(1+1/\left(a-b\text{ I}\right)\right)] \\ & \left(1+\frac{1+2\left(1/2-c\right)}{\left(1/2-c\right)^2+b^2\right)} \left(1+\frac{1+2\left(1/2+c\right)}{\left(1/2+c\right)^2+b^2\right) \\ & \text{FullSimplify}[\text{Expand}\left[\left(1+\frac{1+2\left(\frac{1}{2}-c\right)}{b^2+\left(\frac{1}{2}-c\right)^2\right)}\right) \left(1+\frac{1+2\left(\frac{1}{2}+c\right)}{b^2+\left(\frac{1}{2}+c\right)^2}\right) \right] \right] \\ & \frac{16b^4+\left(9-4c^2\right)^2+8b^2\left(9+4c^2\right)}{\left(1+4b^2\right)^2+8\left(-1+4b^2\right)c^2+16c^4} \\ & \text{FullSimplify}[\left(1-\left(1/2\right)/\left(a+b\text{ I}\right)\right) \left(1-\left(1/2\right)/\left(a-b\text{ I}\right)\right) \right] \\ & 1+\frac{1-4a}{4\left(a^2+b^2\right)} \\ & \left(1+\frac{1-4\left(1/2+c\right)}{4\left(\left(1/2-c\right)^2+b^2\right)}\right) \left(1+\frac{1-4\left(\frac{1}{2}-c\right)}{4\left(b^2+\left(\frac{1}{2}-c\right)^2\right)}\right) \left(1+\frac{1-4\left(\frac{1}{2}+c\right)}{4\left(b^2+\left(\frac{1}{2}+c\right)^2\right)}\right) \right] \\ & \text{ff}[b_-,c_-] := \frac{16\left(b^2+c^2\right)^2}{\left(1+4b^2\right)^2+8\left(-1+4b^2\right)c^2+16c^4} \\ & \text{N}[\text{ff}[0,25]] \\ & 1.0008 \\ & \text{N}[\text{ff}[1/4,25]] \\ & 1.0008002399439064 \\ & \text{zz}[n_-,k_-] := (1-k/\text{ZetaZero}[n]) \left(1-k/\text{ZetaZero}[-n]\right) \\ & \text{N}[\text{zz}[1,1/2]] \\ & 0.99875+0.i \\ & 1.0099979776674461 \\ & \text{FullSimplify}[\text{Expand}[\left(1-1/\left(a-c\text{ I}\right)\right)\left(-1+1/\left(a+c\text{ I}\right)\right)]] \\ & -1+\frac{-1+2a}{a^2+c^2} \\ \end{aligned}$$

$$-1 + \frac{-1 + 2 (1 / 2 - a)}{(1 / 2 - a)^{2} + c^{2}}$$

FullSimplify
$$\left[\left(-1 + \frac{-1+2(1/2-a)}{(1/2-a)^2 + c^2} \right) \left(-1 + \frac{-1+2(1/2+a)}{(1/2+a)^2 + c^2} \right) \right]$$