```
Integrate[x / (E^x + 1), {x, 0, Infinity}]
```

$$\frac{\pi^2}{12}$$

```
Integrate[x / (E^x + 2), {x, 0, Infinity}]
```

$$\frac{1}{12}\left(\pi^2 + 3\,\mathrm{Log}[2]^2 + 6\,\mathrm{PolyLog}\left[2, -\frac{1}{2}\right]\right)$$

```
Sum[ 1 / (2 k + 1) ^2 - 1 / (2 k + 2) ^2, {k, 0, Infinity}]
```

$$\frac{\pi^2}{12}$$

```
Sum[ 1 / (3 k + 1) ^2 + 1 / (3 k + 2) ^2 - 2 / (3 k + 3) ^2, {k, 0, Infinity}]
```

$$\mathrm{N}\left[\frac{\pi^2}{9}\right]$$

1.09662

```
Integrate[x E^ (- (m + 1) x), {x, 0, Infinity}]
```

$$\mathrm{ConditionalExpression}\left[\frac{1}{(1 + m)^2}, \mathrm{Re}[m] > -1\right]$$

```
Integrate[x E^ (- (m + 2) x), {x, 0, Infinity}]
```

$$\mathrm{ConditionalExpression}\left[\frac{1}{(2 + m)^2}, \mathrm{Re}[m] > -2\right]$$

```
Integrate[ 1 / (1 + E^ (-x)), {x, 0, Infinity}]
```

Integrate::idiv : Integral of $\frac{1}{1 + e^{-x}}$ does not converge on $\{0, \infty\}$. ≫

$$\int_0^{\infty} \frac{1}{1 + e^{-x}}\, dx$$

```
Sum[ (-1) ^k E^ (-k x), {k, 0, Infinity}]
```

$$\frac{e^x}{1 + e^x}$$

$$\mathrm{ff}[x\_] := \frac{e^x}{1 + e^x}$$

```
ff[3]
```

$$\frac{e^3}{1 + e^3}$$

```
ff2[x_] := 1 / (1 + E^ (-x))
ff2[3]
```

$$\mathrm{FullSimplify}\left[\mathrm{Expand}\left[\frac{1}{1 + \frac{1}{e^3}}\right]\right]$$

$$\frac{1}{1+\frac{1}{e^3}}$$

```
N[ff[3]]
```

0.952574

```
N[ff2[3]]
```

0.952574

```
Sum[ (-1)^k E^(-k x), {k, 0, Infinity}]
```

$$\frac{e^x}{1+e^x}$$

```
Sum[ E^(-(2 k - 2) x) - E^(-(2 k - 1) x), {k, 1, Infinity}]
```

$$\frac{e^x}{1+e^x}$$

```
Sum[ E^(-(3 k - 3) x) + E^(-(3 k - 2) x) - 2 E^(-(3 k - 1) x), {k, 1, Infinity}]
```

$$\frac{e^x\,(2+e^x)}{1+e^x+e^{2x}}$$

```
Sum[ E^(-(4 k - 4) x) + E^(-(4 k - 3) x) + E^(-(4 k - 2) x) - 3 E^(-(4 k - 1) x), {k, 1, Infinity}]
```

$$\frac{e^x\left(3+2\,e^x+e^{2x}\right)}{1+e^x+e^{2x}+e^{3x}}$$

$$\text{Integrate}\left[x\,E^{\wedge}(-x)\left(\frac{e^x\,(2+e^x)}{1+e^x+e^{2x}}\right),\ \{x,0,\text{Infinity}\}\right]$$

$$\frac{\pi^2}{9}$$

$$\text{Integrate}\left[x\,E^{\wedge}(-x)\left(\frac{e^x\left(3+2\,e^x+e^{2x}\right)}{1+e^x+e^{2x}+e^{3x}}\right),\ \{x,0,\text{Infinity}\}\right]$$

$$\frac{\pi^2}{8}$$

```
Sum[ (-1)^k E^(-k x), {k, 0, 10}]
Sum[ E^(-(2 k - 2) x) - E^(-(2 k - 1) x), {k, 1, 5}]
```

$$1+e^{-10x}-e^{-9x}+e^{-8x}-e^{-7x}+e^{-6x}-e^{-5x}+e^{-4x}-e^{-3x}+e^{-2x}-e^{-x}$$

$$1-e^{-9x}+e^{-8x}-e^{-7x}+e^{-6x}-e^{-5x}+e^{-4x}-e^{-3x}+e^{-2x}-e^{-x}$$

$$\text{FullSimplify}\left[\frac{e^x\left(3+2\,e^x+e^{2x}\right)}{1+e^x+e^{2x}+e^{3x}}\right]$$

$$\frac{1}{1+e^{-x}}+\text{Sech}[x]$$

$$\text{Expand}\left[\frac{e^x\left(3+2\,e^x+e^{2x}\right)}{1+e^x+e^{2x}+e^{3x}}\right]$$

```
Expand[ⅇ^x (3 + 2 ⅇ^x + ⅇ^(2 x))]
```

$3 e^{x} + 2 e^{2x} + e^{3x}$

$$\frac{3 e^{x} + 2 e^{2x} + e^{3x}}{1 + e^{x} + e^{2x} + e^{3x}}$$

```
FullSimplify[ⅇ^x (2 + ⅇ^x) / (1 + ⅇ^x + ⅇ^(2 x))]
```

$$\frac{2 + e^{x}}{1 + 2\,\mathrm{Cosh}[x]}$$

```
Sum[ E^(-(5 k - 5) x) + E^(-(5 k - 4) x) +
  E^(-(5 k - 3) x) + E^(-(5 k - 2) x) - 4 E^(-(5 k - 1) x), {k, 1, Infinity}]
```

$$\frac{4 e^{x} + 3 e^{2x} + 2 e^{3x} + e^{4x}}{1 + e^{x} + e^{2x} + e^{3x} + e^{4x}}$$

```
FullSimplify[ⅇ^x (4 + 3 ⅇ^x + 2 ⅇ^(2 x) + ⅇ^(3 x)) / (1 + ⅇ^x + ⅇ^(2 x) + ⅇ^(3 x) + ⅇ^(4 x))]
```

$$\frac{e^{x} (4 + e^{x} (3 + e^{x} (2 + e^{x})))}{1 + e^{x} (1 + e^{x}) (1 + e^{2x})}$$

```
Expand[ⅇ^x (4 + ⅇ^x (3 + ⅇ^x (2 + ⅇ^x)))]
```

$4 e^{x} + 3 e^{2x} + 2 e^{3x} + e^{4x}$

```
Expand[1 + ⅇ^x + ⅇ^(2 x) + ⅇ^(3 x) + ⅇ^(4 x)]
```

$1 + e^{x} + e^{2x} + e^{3x} + e^{4x}$

```
Integrate[ x^(s - 1) E^(-x) (ⅇ^x / (1 + ⅇ^x)), {x, 0, Infinity}]
```

ConditionalExpression[$2^{-s} (-2 + 2^{s})$ Gamma[s] Zeta[s], Re[s] > 0]

```
fn3[s_] := Integrate[ x^(s - 1) E^(-x) (ⅇ^x (2 + ⅇ^x) / (1 + ⅇ^x + ⅇ^(2 x))), {x, 0, Infinity}]
```

```
fn3[2]
```

$$\frac{\pi^2}{9}$$

```
fn4[s_] := Integrate[ x^(s - 1) E^(-x) ((3 ⅇ^x + 2 ⅇ^(2 x) + ⅇ^(3 x)) / (1 + ⅇ^x + ⅇ^(2 x) + ⅇ^(3 x))), {x, 0, Infinity}]
```

```
fn4[2]
```

$$\frac{\pi^2}{8}$$

```
fn5[s_] := Integrate[ x^(s - 1) E^(-x) ((4 ⅇ^x + 3 ⅇ^(2 x) + 2 ⅇ^(3 x) + ⅇ^(4 x)) / (1 + ⅇ^x + ⅇ^(2 x) + ⅇ^(3 x) + ⅇ^(4 x))), {x, 0, Infinity}]
```

```
fn5[3]
```

$$\frac{48\,\mathrm{Zeta}[3]}{25}$$

```
N[Log[5]]
```

1.60944

$$\text{FullSimplify}\left[\,\text{E}^\wedge(-\text{x})\,\left(\frac{4\,e^{x}+3\,e^{2x}+2\,e^{3x}+e^{4x}}{1+e^{x}+e^{2x}+e^{3x}+e^{4x}}\right)\right]$$

$$\frac{1}{-1+e^{x}+\frac{5}{4+e^{x}\,(3+e^{x}\,(2+e^{x}))}}$$

$$\text{FullSimplify}\left[\text{E}^\wedge(-\text{x})\,\left(\frac{e^{x}}{1+e^{x}}\right)\right]$$

$$\frac{1}{1+e^{x}}$$

$$\text{FullSimplify}\left[\text{E}^\wedge(-\text{x})\,\left(\frac{e^{x}\,(2+e^{x})}{1+e^{x}+e^{2x}}\right)\right]$$

$$\frac{2+e^{x}}{1+e^{x}+e^{2x}}$$

$$\text{FullSimplify}\left[\text{E}^\wedge(-\text{x})\,\left(\frac{3\,e^{x}+2\,e^{2x}+e^{3x}}{1+e^{x}+e^{2x}+e^{3x}}\right)\right]$$

$$1+\frac{1}{1+e^{x}}-\text{Tanh}[x]$$

$$\text{FullSimplify}\left[\,\text{E}^\wedge(-\text{x})\,\left(\frac{4\,e^{x}+3\,e^{2x}+2\,e^{3x}+e^{4x}}{1+e^{x}+e^{2x}+e^{3x}+e^{4x}}\right)\right]$$

$$\frac{1}{-1+e^{x}+\frac{5}{4+e^{x}\,(3+e^{x}\,(2+e^{x}))}}$$

$$\text{fn4}[\text{s\_}] := \text{Integrate}\left[\,\text{x}^\wedge(\text{s}-1)\;\text{E}^\wedge(-\text{x})\,\left(\frac{3\,e^{x}+2\,e^{2x}+e^{3x}}{1+e^{x}+e^{2x}+e^{3x}}\right),\ \{\text{x},\,0,\,\text{Infinity}\}\right]$$

$$\text{N}\left[\text{Integrate}\left[\,\text{E}^\wedge(-\text{x})\,\left(\frac{4\,e^{x}+3\,e^{2x}+2\,e^{3x}+e^{4x}}{1+e^{x}+e^{2x}+e^{3x}+e^{4x}}\right),\ \{\text{x},\,0,\,\text{Infinity}\}\right]-\text{Log}[5]\right]$$

$2.59792\times10^{-14}-1.20015\times10^{-12}\,i$

$$\text{N}\left[\text{Integrate}\left[\,\text{E}^\wedge(-\text{x})\,\left(\frac{3\,e^{x}+2\,e^{2x}+e^{3x}}{1+e^{x}+e^{2x}+e^{3x}}\right),\ \{\text{x},\,0,\,\text{Infinity}\}\right]-\text{Log}[4]\right]$$

0.

$$\text{Integrate}\left[\,\frac{1}{1+e^{x}},\ \{\text{x},\,0,\,\text{Infinity}\}\right]$$

Log[2]

$$\text{Integrate}\left[\,\text{E}^\wedge(-\text{x})\,\left(\frac{3\,e^{x}+2\,e^{2x}+e^{3x}}{1+e^{x}+e^{2x}+e^{3x}}\right),\ \{\text{x},\,0,\,\text{Infinity}\}\right]$$

Log[4]

```
N[Sum[ (-1)^(n+1) / n^ZetaZero[1], {n, 1, 100000}]]
```

$-0.00127694 - 0.000932425\, i$

```
N[Sum[ (-1)^(n+1) / n^ZetaZero[1], {n, 1, 1000000}]]
```

$-0.000438861 + 0.000239584\, i$

$$\mathrm{FullSimplify}\left[\mathrm{Expand}\left[\frac{4 e^{x} + 3 e^{2x} + 2 e^{3x} + e^{4x}}{1 + e^{x} + e^{2x} + e^{3x} + e^{4x}} - \frac{3 e^{x} + 2 e^{2x} + e^{3x}}{1 + e^{x} + e^{2x} + e^{3x}}\right]\right]$$

$$\frac{e^{x} (1 + e^{x} (2 + e^{x} (3 + 4 e^{x})))}{(1 + e^{x}) \left(1 + e^{2x}\right) \left(1 + e^{x} (1 + e^{x}) \left(1 + e^{2x}\right)\right)}$$

$$\mathrm{Expand}\left[\frac{4 e^{x} + 3 e^{2x} + 2 e^{3x} + e^{4x}}{1 + e^{x} + e^{2x} + e^{3x} + e^{4x}} \Big/ \frac{3 e^{x} + 2 e^{2x} + e^{3x}}{1 + e^{x} + e^{2x} + e^{3x}}\right]$$

$$\frac{4 e^{x}}{\left(3 e^{x} + 2 e^{2x} + e^{3x}\right) \left(1 + e^{x} + e^{2x} + e^{3x} + e^{4x}\right)} + \frac{7 e^{2x}}{\left(3 e^{x} + 2 e^{2x} + e^{3x}\right) \left(1 + e^{x} + e^{2x} + e^{3x} + e^{4x}\right)} + \frac{9 e^{3x}}{\left(3 e^{x} + 2 e^{2x} + e^{3x}\right) \left(1 + e^{x} + e^{2x} + e^{3x} + e^{4x}\right)} + \frac{10 e^{4x}}{\left(3 e^{x} + 2 e^{2x} + e^{3x}\right) \left(1 + e^{x} + e^{2x} + e^{3x} + e^{4x}\right)} + \frac{6 e^{5x}}{\left(3 e^{x} + 2 e^{2x} + e^{3x}\right) \left(1 + e^{x} + e^{2x} + e^{3x} + e^{4x}\right)} + \frac{3 e^{6x}}{\left(3 e^{x} + 2 e^{2x} + e^{3x}\right) \left(1 + e^{x} + e^{2x} + e^{3x} + e^{4x}\right)} + \frac{e^{7x}}{\left(3 e^{x} + 2 e^{2x} + e^{3x}\right) \left(1 + e^{x} + e^{2x} + e^{3x} + e^{4x}\right)}$$

$$\mathrm{Expand}\left[\left(3 e^{x} + 2 e^{2x} + e^{3x}\right) \left(1 + e^{x} + e^{2x} + e^{3x} + e^{4x}\right)\right]$$

$$3 e^{x} + 5 e^{2x} + 6 e^{3x} + 6 e^{4x} + 6 e^{5x} + 3 e^{6x} + e^{7x}$$