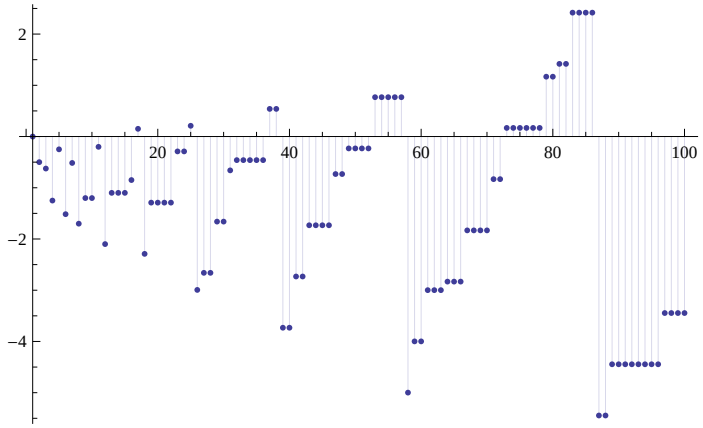
```
t[n_, a_, b_] := b (Floor[n / b] - Floor[(n - 1) / b]) - a (Floor[n / a] - Floor[(n - 1) / a])

Table[ t[n, 51, 50], {n, 1, 100}]
```

```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 50, -51, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 50}
```

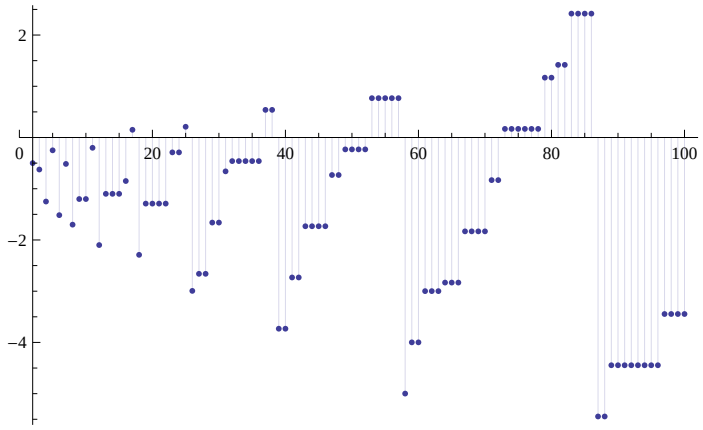
```
Clear[p]
p[n_, k_] := p[n, k] =
  (1 / 2) Sum[ If[t[2 j, 3, 2] == 0, 0, t[2 j, 3, 2] (1 / k - p[n / j, k + 1])], {j, 3 / 2, n, 1 / 2}]

p[100, 1]
```

$$-\frac{8\,149\,753}{2\,365\,440}$$

```
binomial[z_, k_] := binomial[z, k] = Product[z - j, {j, 0, k - 1}] / k!
Ds[n_, 0, s_, a_] := UnitStep[n - 1]
Ds[n_, 1, s_, a_] := Ds[n, 1, s, a] = HarmonicNumber[Floor[n], s] - HarmonicNumber[a, s]
Ds[n_, 2, s_, a_] := Ds[n, 2, s, a] =
  Sum[(m^(-2 s)) + 2 (m^-s) (Ds[Floor[n / m], 1, s, m]), {m, a + 1, Floor[n^(1 / 2)]}]
Ds[n_, k_, s_, a_] := Ds[n, k, s, a] =
  Sum[(m^(-s k)) + k (m^(-s (k - 1))) Ds[Floor[n / (m^(k - 1))], 1, s, m] +
    Sum[binomial[k, j] (m^-s)^j Ds[Floor[n / (m^j)], k - j, s, m], {j, 1, k - 2}],
   {m, a + 1, Floor[n^(1 / k)]}]
Dnsz[n_, s_, z_] := Expand@Sum[binomial[z, k] Ds[n, k, s, 1], {k, 0, Log2@n}]

Dnsyabz[n_, s_, a_, b_, z_] := Expand@Sum[
   (-1)^j binomial[z, j] (a / b)^(j (1 - s)) Dnsz[n / ((a / b)^j), s, z], {j, 0, Log[a / b, n]}]

DiscretePlot[D[Dnsyabz[n, 0, 3, 2, z], z] /. z → 0, {n, 1, 100}]
```



```
DiscretePlot[p[n, 1], {n, 2, 100}]
```



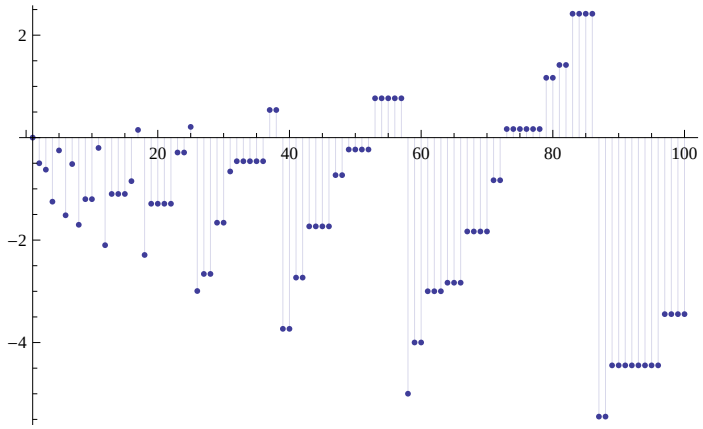

```
(**)
Clear[da, dc]
da[n_, z_, a_, x_, y_] := da[n, z, a, x, y] =
  If[n < y, 1, da[n, z, a, x, y + x] + If[t[x^-1 y, a, x^-1] == 0, 0, Sum[ binomial[z, k]
       x^k t[x^-1 y, a, x^-1]^k da[n / y^k, z - k, a, x, y + x], {k, 1, Log[y, n]}]]]
dc[n_, z_, a_, x_, y_] := dc[n, z, a, x, y] = If[n < x y, 1,
   dc[n, z, a, x, y + 1] + If[t[y, a, x^-1] == 0, 0, Sum[ binomial[z, k] x^k
       t[ y, a, x^-1]^k dc[n / (x y)^k, z - k, a, x, y + 1], {k, 1, Log[x y, n]}]]]

D[da[100, z, 3, 1 / 2, 1 + 1 / 2], z] /. z → 0
```

$$-\frac{8\,149\,753}{2\,365\,440}$$

```
D[dc[100, z, 3, 1 / 2, 3], z] /. z → 0
```

$$-\frac{8\,149\,753}{2\,365\,440}$$

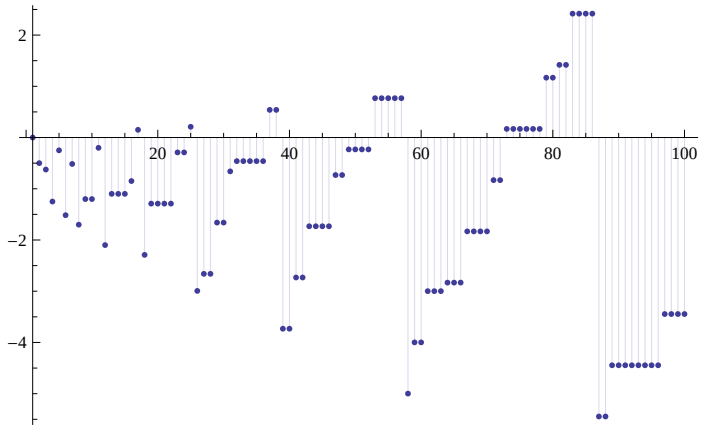
```
DiscretePlot[D[da[n, z, 3, 1 / 2, 1 + 1 / 2], z] /. z → 0, {n, 1, 100}]
```



```
((1 / 2) t[3 / 2 (1 / 2)^-1, 3, (1 / 2)^-1])
```

$$-\frac{3}{2}$$

```
DiscretePlot[D[dc[n, z, 3, 1 / 2, 3], z] /. z → 0, {n, 1, 100}]
```




```
$RecursionLimit = 100 000
```

100 000

```
D[da[100, z, 81, 1 / 80, 2], z] /. z → 0
```

$$-\frac{1\,899\,762\,991\,297}{1\,572\,864\,000\,000}$$