```
DD[n_, z_] := DD[n, z] = Sum[FactorialPower[z, a] / a! D2a[n, a], {a, 0, Log[2, n]}]
D2a[n_, k_] := D2a[n, k] = Sum[D2a[Floor[n / j], k - 1], {j, 2, n}]; D2a[n_, 0] := 1
EE[n_, z_, b_] :=
 EE[n, z, b] = Sum[FactorialPower[z, a] / a! E2a[n, a, b], {a, 0, Log[If[b > 2, 2, b], n]}]
E2a[n_, k_, a_] := E2a[n, k, a] = Sum[ E2a[n / j, k - 1, a], {j, 2, n}] -
    a Sum[ E2a[n / (a j), k - 1, a], {j, 1, n / a}]; E2a[n_, 0, a_] := 1
E1[n_, k_, a_] := E1[n, k, a] = Sum[ E1[n / j, k - 1, a], {j, 1, n}] -
    a Sum[ E1[n / (a j), k - 1, a], {j, 1, n / a}]; E1[n_, 0, a_] := 1
E2b[n_, k_, a_] := Sum[ (-1)^j Binomial[k, j] E1[n, k - j, a], {j, 0, k}]
E2c[n_, k_, a_] := Sum[ (-1)^j Binomial[k, j] E1c[n, k - j, a], {j, 0, k}]
E2c2[n_, k_, a_] := Sum[ (-1)^(k - j) Binomial[k, k - j] E1c[n, j, a], {j, 0, k}]
E1c[n_, r_, a_] := Sum[ (-1)^j Binomial[r, j] a^j DDa[n / a^j, r], {j, 0, r}]
E1d[n_, r_, a_] := Sum[ (-1)^m Binomial[r, m] a^m DDa[n / a^m, r], {m, 0, r}]
E2d[n_, k_, a_] := Sum[ (-1)^m Binomial[k, m] E1c[n, k - m, a], {m, 0, k}]
E2d[n_, k_, a_] := Sum[
  (-1)^(j + m) Binomial[k, m] Binomial[k - m, j] a^j DDa[n / a^j, k - m], {m, 0, k}, {j, 0, k - m}]
E2e[n_, k_, a_] := Sum[ (-1)^(j + m) Binomial[k, m] Binomial[k - m, j] a^j DDa[n / a^j, k - m],
  {m, 0, k}, {j, 0, k - m}]
E2c3[n_, k_, a_] := Sum[ (-1)^(k - j) Binomial[k, k - j] (-1)^m
    Binomial[j, m] a^m DDa[n / a^m, j], {j, 0, k}, {m, 0, j}]
E2c4[n_, k_, b_] := Sum[ (-1)^(m + k - j) Binomial[k, k - j]
    Binomial[j, m] b^m DDa[n / b^m, j], {j, 0, k}, {m, 0, j}]
Dk[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j Es[ n / b^j, k, b], {j, 0, Log[b, n]}]
Dkz[n_, k_, b_] :=
 Sum[ Binomial[k + j - 1, k - 1] b^j (Es[ n / b^j, k, b] - 1) / k, {j, 0, Log[b, n]}]
```

```
E1[100, 2, 2]
```

2

```
DD[100, 2] - 4 DD[50, 2] + 4 DD[25, 2]
```

2

```
E2b[1000, 2, 3]
```

18

```
E2a[1000, 2, 3]
```

18

```
Expand[E2c[1000, 3, 3]]
```

$$-27\,\text{DDa}\left[\frac{1000}{27}, 3\right] - 27\,\text{DDa}\left[\frac{1000}{9}, 2\right] + 27\,\text{DDa}\left[\frac{1000}{9}, 3\right] - 9\,\text{DDa}\left[\frac{1000}{3}, 1\right] + 18\,\text{DDa}\left[\frac{1000}{3}, 2\right] - 9\,\text{DDa}\left[\frac{1000}{3}, 3\right] - \text{DDa}[1000, 0] + 3\,\text{DDa}[1000, 1] - 3\,\text{DDa}[1000, 2] + \text{DDa}[1000, 3]$$

```
Expand[E2c2[1000, 3, 3]]
```

$$-27\,\text{DDa}\left[\frac{1000}{27}, 3\right] - 27\,\text{DDa}\left[\frac{1000}{9}, 2\right] + 27\,\text{DDa}\left[\frac{1000}{9}, 3\right] - 9\,\text{DDa}\left[\frac{1000}{3}, 1\right] + 18\,\text{DDa}\left[\frac{1000}{3}, 2\right] - 9\,\text{DDa}\left[\frac{1000}{3}, 3\right] - \text{DDa}[1000, 0] + 3\,\text{DDa}[1000, 1] - 3\,\text{DDa}[1000, 2] + \text{DDa}[1000, 3]$$

```
Expand[E2c4[1000, 3, 3]]
```

$$-27\,\text{DDa}\left[\frac{1000}{27},\,3\right]-27\,\text{DDa}\left[\frac{1000}{9},\,2\right]+27\,\text{DDa}\left[\frac{1000}{9},\,3\right]-9\,\text{DDa}\left[\frac{1000}{3},\,1\right]+18\,\text{DDa}\left[\frac{1000}{3},\,2\right]-9\,\text{DDa}\left[\frac{1000}{3},\,3\right]-\text{DDa}[1000,\,0]+3\,\text{DDa}[1000,\,1]-3\,\text{DDa}[1000,\,2]+\text{DDa}[1000,\,3]$$

```
FullSimplify[(-1)^(k-j) Binomial[k, k-j] (-1)^m Binomial[j, m] a^m]
```

$$(-1)^{-j+k+m}\,a^m\,\text{Binomial}[j,\,m]\,\text{Binomial}[k,\,-j+k]$$

```
Binomial[k, -j+k]
```

$$\text{Binomial}[k,\,-j+k]$$

```
k!/((k-(k-j))!(k-j)!)
```

$$\frac{k!}{j!\,(-j+k)!}$$

```
j!/((j-m)!m!)
```

$$\frac{j!}{(j-m)!\,m!}$$

$$\frac{k!}{j!\,(-j+k)!}\,\frac{j!}{(j-m)!\,m!}$$

$$\frac{k!}{(-j+k)!\,(j-m)!\,m!}$$

```
E2c4[n, 2, 1.000001]
```

$$1.\,\text{DDa}[0.999998\,n,\,2]+2.\,\text{DDa}[0.999999\,n,\,1]-2.\,\text{DDa}[0.999999\,n,\,2]+1.\,\text{DDa}[1.\,n,\,0]-2.\,\text{DDa}[1.\,n,\,1]+1.\,\text{DDa}[1.\,n,\,2]$$

```
Dk[100, -2, 2]
```

$$4\,\text{Es}[25,\,-2,\,2]-4\,\text{Es}[50,\,-2,\,2]+\text{Es}[100,\,-2,\,2]$$

```
Dk[100, -1, 2]
```

$$-2\,\text{Es}[50,\,-1,\,2]+\text{Es}[100,\,-1,\,2]$$

```
Dk[100, -3, 2]
```

$$-8\,\text{Es}\left[\frac{25}{2},\,-3,\,2\right]+12\,\text{Es}[25,\,-3,\,2]-6\,\text{Es}[50,\,-3,\,2]+\text{Es}[100,\,-3,\,2]$$

```
Dkz[100, .000001, 2]
```

$$10.6667\left(-1+\text{Es}\left[\frac{25}{16},\,1.\times10^{-6},\,2\right]\right)+6.40001\left(-1+\text{Es}\left[\frac{25}{8},\,1.\times10^{-6},\,2\right]\right)+4.00001\left(-1+\text{Es}\left[\frac{25}{4},\,1.\times10^{-6},\,2\right]\right)+2.66667\left(-1+\text{Es}\left[\frac{25}{2},\,1.\times10^{-6},\,2\right]\right)+2.\left(-1+\text{Es}\left[25,\,1.\times10^{-6},\,2\right]\right)+2.\left(-1+\text{Es}\left[50,\,1.\times10^{-6},\,2\right]\right)+1.\times10^{6}\left(-1+\text{Es}\left[100,\,1.\times10^{-6},\,2\right]\right)$$

```
DiscretePlot[EE[n, -1, 1.1], {n, 1, 100}]
```