

```

D2Cache[n_, k_, s_] :=
  Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1/k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]

LinnikSumFast[n_] := D2Cache[n, 1, 2] +

  Sum[(-1)^(k + 1) / k (

    Sum[D2Cache[n / j, k - 1, 2], {j, Floor[n^(1/3)] + 1, n^(1/2)}] +

    Sum[(Floor[n / j] - (Floor[n / (j + 1)])) D2Cache[j, k - 1, 2],
      {j, 1, n / Floor[n^(1/2)] - 1}] +

    Sum[d2cache[j, k - 1] (Floor[n / j] - 1), {j, 2, n^(1/3)}] +

    Sum[d2cache[j, m] D2Cache[n / (j s), k - m - 1, 2], {j, 2, n^(1/3)},
      {s, Floor[Floor[n^(1/3)] / j] + 1, Floor[n / j]^(1/2)}, {m, 1, k - 2}] +

    Sum[(Sum[1, {m, Floor[n / (j (s + 1))] + 1, n / (j s)}])
      (Sum[d2cache[j, m] D2Cache[s, k - m - 1, 2], {m, 1, k - 2}]),
      {j, 2, n^(1/3)}, {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1/2)] - 1}]

  ), {k, 2, Log[2, n]}]
RiePrimeCnt[n_] := Sum[PrimePi[n^(1/j)] / j, {j, 1, Log[2, n]}]
Table[{n, LinnikSumFast[n], RiePrimeCnt[n]}, {n, 1, 100}] // TableForm

```

1	0	0
2	1	1
3	2	2
4	$\frac{5}{2}$	$\frac{5}{2}$
5	$\frac{7}{2}$	$\frac{7}{2}$
6	$\frac{7}{2}$	$\frac{7}{2}$
7	$\frac{9}{2}$	$\frac{9}{2}$
8	$\frac{29}{6}$	$\frac{29}{6}$
9	$\frac{16}{3}$	$\frac{16}{3}$
10	$\frac{16}{3}$	$\frac{16}{3}$
11	$\frac{19}{3}$	$\frac{19}{3}$
12	$\frac{19}{3}$	$\frac{19}{3}$
13	$\frac{22}{3}$	$\frac{22}{3}$
14	$\frac{22}{3}$	$\frac{22}{3}$
15	$\frac{22}{3}$	$\frac{22}{3}$
16	$\frac{91}{12}$	$\frac{91}{12}$
17	$\frac{103}{12}$	$\frac{103}{12}$
18	$\frac{103}{12}$	$\frac{103}{12}$
19	$\frac{115}{12}$	$\frac{115}{12}$

	--	--
20	<u>115</u>	<u>115</u>
	12	12
21	<u>115</u>	<u>115</u>
	12	12
22	<u>115</u>	<u>115</u>
	12	12
23	<u>127</u>	<u>127</u>
	12	12
24	<u>127</u>	<u>127</u>
	12	12
25	<u>133</u>	<u>133</u>
	12	12
26	<u>133</u>	<u>133</u>
	12	12
27	<u>137</u>	<u>137</u>
	12	12
28	<u>137</u>	<u>137</u>
	12	12
29	<u>149</u>	<u>149</u>
	12	12
30	<u>149</u>	<u>149</u>
	12	12
31	<u>161</u>	<u>161</u>
	12	12
32	<u>817</u>	<u>817</u>
	60	60
33	<u>817</u>	<u>817</u>
	60	60
34	<u>817</u>	<u>817</u>
	60	60
35	<u>817</u>	<u>817</u>
	60	60
36	<u>817</u>	<u>817</u>
	60	60
37	<u>877</u>	<u>877</u>
	60	60
38	<u>877</u>	<u>877</u>
	60	60
39	<u>877</u>	<u>877</u>
	60	60
40	<u>877</u>	<u>877</u>
	60	60
41	<u>937</u>	<u>937</u>
	60	60
42	<u>937</u>	<u>937</u>
	60	60
43	<u>997</u>	<u>997</u>
	60	60
44	<u>997</u>	<u>997</u>
	60	60
45	<u>997</u>	<u>997</u>
	60	60
46	<u>997</u>	<u>997</u>
	60	60
47	<u>1057</u>	<u>1057</u>
	60	60
48	<u>1057</u>	<u>1057</u>
	60	60
49	<u>1087</u>	<u>1087</u>
	60	60
50	<u>1087</u>	<u>1087</u>
	60	60
51	<u>1087</u>	<u>1087</u>
	60	60
52	<u>1087</u>	<u>1087</u>
	60	60
53	<u>1147</u>	<u>1147</u>
	60	60
54	<u>1147</u>	<u>1147</u>
	60	60
55	<u>1147</u>	<u>1147</u>
	60	60
56	<u>1147</u>	<u>1147</u>
	60	60
57	<u>1147</u>	<u>1147</u>
	60	60
58	<u>1147</u>	<u>1147</u>
	60	60

	~	~
59	<u>1207</u>	<u>1207</u>
	60	60
60	<u>1207</u>	<u>1207</u>
	60	60
61	<u>1267</u>	<u>1267</u>
	60	60
62	<u>1267</u>	<u>1267</u>
	60	60
63	<u>1267</u>	<u>1267</u>
	60	60
64	<u>1277</u>	<u>1277</u>
	60	60
65	<u>1277</u>	<u>1277</u>
	60	60
66	<u>1277</u>	<u>1277</u>
	60	60
67	<u>1337</u>	<u>1337</u>
	60	60
68	<u>1337</u>	<u>1337</u>
	60	60
69	<u>1337</u>	<u>1337</u>
	60	60
70	<u>1337</u>	<u>1337</u>
	60	60
71	<u>1397</u>	<u>1397</u>
	60	60
72	<u>1397</u>	<u>1397</u>
	60	60
73	<u>1457</u>	<u>1457</u>
	60	60
74	<u>1457</u>	<u>1457</u>
	60	60
75	<u>1457</u>	<u>1457</u>
	60	60
76	<u>1457</u>	<u>1457</u>
	60	60
77	<u>1457</u>	<u>1457</u>
	60	60
78	<u>1457</u>	<u>1457</u>
	60	60
79	<u>1517</u>	<u>1517</u>
	60	60
80	<u>1517</u>	<u>1517</u>
	60	60
81	<u>383</u>	<u>383</u>
	15	15
82	<u>383</u>	<u>383</u>
	15	15
83	<u>398</u>	<u>398</u>
	15	15
84	<u>398</u>	<u>398</u>
	15	15
85	<u>398</u>	<u>398</u>
	15	15
86	<u>398</u>	<u>398</u>
	15	15
87	<u>398</u>	<u>398</u>
	15	15
88	<u>398</u>	<u>398</u>
	15	15
89	<u>413</u>	<u>413</u>
	15	15
90	<u>413</u>	<u>413</u>
	15	15
91	<u>413</u>	<u>413</u>
	15	15
92	<u>413</u>	<u>413</u>
	15	15
93	<u>413</u>	<u>413</u>
	15	15
94	<u>413</u>	<u>413</u>
	15	15
95	<u>413</u>	<u>413</u>
	15	15
96	<u>413</u>	<u>413</u>
	15	15
97	<u>428</u>	<u>428</u>
	15	15

```

98      428      428
      15      15
99      428      428
      15      15
100     428      428
      15      15

D2Cache[n_, k_, s_] :=
  Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1/k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]

LinnikSumFast[n_] := D2Cache[n, 1, 2] +
  Sum[(-1)^(k + 1) / k ( Sum[D2Cache[n / j, k - 1, 2], {j, Floor[n^(1/3)] + 1, n^(1/2)}]),
    {k, 2, Log[2, n]}] +
  Sum[(-1)^(k + 1) / k ( Sum[(Floor[n / j] - (Floor[n / (j + 1)])) D2Cache[j, k - 1, 2],
    {j, 1, n / Floor[n^(1/2)] - 1}], {k, 2, Log[2, n]}] +
  Sum[(-1)^(k + 1) / k ( Sum[d2cache[j, k - 1] (Floor[n / j] - 1), {j, 2, n^(1/3)}]),
    {k, 2, Log[2, n]}] +
  Sum[(-1)^(k + 1) / k ( Sum[d2cache[j, m] D2Cache[n / (j s), k - m - 1, 2],
    {j, 2, n^(1/3)}, {s, Floor[Floor[n^(1/3)] / j] + 1, Floor[n / j]^(1/2)},
    {m, 1, k - 2}], {k, 2, Log[2, n]}] +
  Sum[(-1)^(k + 1) / k ( Sum[(Sum[1, {m, Floor[n / (j (s + 1))] + 1, n / (j s)}])
    (Sum[d2cache[j, m] D2Cache[s, k - m - 1, 2], {m, 1, k - 2}], {j, 2, n^(1/3)},
    {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1/2)] - 1}], {k, 2, Log[2, n]}]
RiePrimeCnt[n_] := Sum[PrimePi[n^(1/j)] / j, {j, 1, Log[2, n]}]
Table[{n, LinnikSumFast[n], RiePrimeCnt[n]}, {n, 1, 100}] // TableForm

```

1	0	0
2	1	1
3	2	2
4	$\frac{5}{2}$	$\frac{5}{2}$
5	$\frac{7}{2}$	$\frac{7}{2}$
6	$\frac{7}{2}$	$\frac{7}{2}$
7	$\frac{9}{2}$	$\frac{9}{2}$
8	$\frac{29}{6}$	$\frac{29}{6}$
9	$\frac{16}{3}$	$\frac{16}{3}$
10	$\frac{16}{3}$	$\frac{16}{3}$
11	$\frac{19}{3}$	$\frac{19}{3}$
12	$\frac{19}{3}$	$\frac{19}{3}$
13	$\frac{22}{3}$	$\frac{22}{3}$
14	$\frac{22}{3}$	$\frac{22}{3}$
15	$\frac{22}{3}$	$\frac{22}{3}$
16	$\frac{91}{12}$	$\frac{91}{12}$
17	$\frac{103}{12}$	$\frac{103}{12}$
18	$\frac{103}{12}$	$\frac{103}{12}$
19	$\frac{115}{12}$	$\frac{115}{12}$
20	$\frac{115}{12}$	$\frac{115}{12}$

	--	--
21	<u>115</u>	<u>115</u>
	12	12
22	<u>115</u>	<u>115</u>
	12	12
23	<u>127</u>	<u>127</u>
	12	12
24	<u>127</u>	<u>127</u>
	12	12
25	<u>133</u>	<u>133</u>
	12	12
26	<u>133</u>	<u>133</u>
	12	12
27	<u>137</u>	<u>137</u>
	12	12
28	<u>137</u>	<u>137</u>
	12	12
29	<u>149</u>	<u>149</u>
	12	12
30	<u>149</u>	<u>149</u>
	12	12
31	<u>161</u>	<u>161</u>
	12	12
32	<u>817</u>	<u>817</u>
	60	60
33	<u>817</u>	<u>817</u>
	60	60
34	<u>817</u>	<u>817</u>
	60	60
35	<u>817</u>	<u>817</u>
	60	60
36	<u>817</u>	<u>817</u>
	60	60
37	<u>877</u>	<u>877</u>
	60	60
38	<u>877</u>	<u>877</u>
	60	60
39	<u>877</u>	<u>877</u>
	60	60
40	<u>877</u>	<u>877</u>
	60	60
41	<u>937</u>	<u>937</u>
	60	60
42	<u>937</u>	<u>937</u>
	60	60
43	<u>997</u>	<u>997</u>
	60	60
44	<u>997</u>	<u>997</u>
	60	60
45	<u>997</u>	<u>997</u>
	60	60
46	<u>997</u>	<u>997</u>
	60	60
47	<u>1057</u>	<u>1057</u>
	60	60
48	<u>1057</u>	<u>1057</u>
	60	60
49	<u>1087</u>	<u>1087</u>
	60	60
50	<u>1087</u>	<u>1087</u>
	60	60
51	<u>1087</u>	<u>1087</u>
	60	60
52	<u>1087</u>	<u>1087</u>
	60	60
53	<u>1147</u>	<u>1147</u>
	60	60
54	<u>1147</u>	<u>1147</u>
	60	60
55	<u>1147</u>	<u>1147</u>
	60	60
56	<u>1147</u>	<u>1147</u>
	60	60
57	<u>1147</u>	<u>1147</u>
	60	60
58	<u>1147</u>	<u>1147</u>
	60	60
59	<u>1207</u>	<u>1207</u>
	60	60

	~	~
60	<u>1207</u>	<u>1207</u>
	60	60
61	<u>1267</u>	<u>1267</u>
	60	60
62	<u>1267</u>	<u>1267</u>
	60	60
63	<u>1267</u>	<u>1267</u>
	60	60
64	<u>1277</u>	<u>1277</u>
	60	60
65	<u>1277</u>	<u>1277</u>
	60	60
66	<u>1277</u>	<u>1277</u>
	60	60
67	<u>1337</u>	<u>1337</u>
	60	60
68	<u>1337</u>	<u>1337</u>
	60	60
69	<u>1337</u>	<u>1337</u>
	60	60
70	<u>1337</u>	<u>1337</u>
	60	60
71	<u>1397</u>	<u>1397</u>
	60	60
72	<u>1397</u>	<u>1397</u>
	60	60
73	<u>1457</u>	<u>1457</u>
	60	60
74	<u>1457</u>	<u>1457</u>
	60	60
75	<u>1457</u>	<u>1457</u>
	60	60
76	<u>1457</u>	<u>1457</u>
	60	60
77	<u>1457</u>	<u>1457</u>
	60	60
78	<u>1457</u>	<u>1457</u>
	60	60
79	<u>1517</u>	<u>1517</u>
	60	60
80	<u>1517</u>	<u>1517</u>
	60	60
81	<u>383</u>	<u>383</u>
	15	15
82	<u>383</u>	<u>383</u>
	15	15
83	<u>398</u>	<u>398</u>
	15	15
84	<u>398</u>	<u>398</u>
	15	15
85	<u>398</u>	<u>398</u>
	15	15
86	<u>398</u>	<u>398</u>
	15	15
87	<u>398</u>	<u>398</u>
	15	15
88	<u>398</u>	<u>398</u>
	15	15
89	<u>413</u>	<u>413</u>
	15	15
90	<u>413</u>	<u>413</u>
	15	15
91	<u>413</u>	<u>413</u>
	15	15
92	<u>413</u>	<u>413</u>
	15	15
93	<u>413</u>	<u>413</u>
	15	15
94	<u>413</u>	<u>413</u>
	15	15
95	<u>413</u>	<u>413</u>
	15	15
96	<u>413</u>	<u>413</u>
	15	15
97	<u>428</u>	<u>428</u>
	15	15
98	<u>428</u>	<u>428</u>
	15	15

	--	--
99	$\frac{428}{15}$	$\frac{428}{15}$
100	$\frac{428}{15}$	$\frac{428}{15}$

```

D2Cache[n_, k_, s_] :=
  Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1/k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]

LinnikSumFast[n_] := D2Cache[n, 1, 2] +
  Sum[(-1)^(k + 1) / k ( Sum[D2Cache[n / j, k - 1, 2], {j, Floor[n^(1/3)] + 1, n^(1/2)}]),
    {k, 2, Log[2, n]}] +
  Sum[(-1)^(k + 1) / k ( Sum[(Floor[n / j] - (Floor[n / (j + 1)])) D2Cache[j, k - 1, 2],
    {j, 1, n / Floor[n^(1/2)] - 1}], {k, 2, Log[2, n]}] +
  Sum[(-1)^(k + 1) / k ( Sum[d2cache[j, k - 1] (Floor[n / j] - 1), {j, 2, n^(1/3)}]),
    {k, 2, Log[2, n]}] +
  Sum[(-1)^(k + 1) / k ( Sum[d2cache[j, m] D2Cache[n / (j s), k - m - 1, 2],
    {j, 2, n^(1/3)}, {s, Floor[Floor[n^(1/3)] / j] + 1, Floor[n / j]^(1/2)},
    {m, 1, k - 2}], {k, 2, Log[2, n]}] +
  Sum[(-1)^(k + 1) / k ( Sum[(Sum[1, {m, Floor[n / (j (s + 1))] + 1, n / (j s)}])
    (Sum[d2cache[j, m] D2Cache[s, k - m - 1, 2], {m, 1, k - 2}], {j, 2, n^(1/3)},
    {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1/2)] - 1}], {k, 2, Log[2, n]}]
RiePrimeCnt[n_] := Sum[PrimePi[n^(1/j)] / j, {j, 1, Log[2, n]}]
Table[{n, LinnikSumFast[n], RiePrimeCnt[n]}, {n, 1, 100}]

```

$$\begin{aligned}
& \{ \{1, 0, 0\}, \{2, 1, 1\}, \{3, 2, 2\}, \{4, \frac{5}{2}, \frac{5}{2}\}, \{5, \frac{7}{2}, \frac{7}{2}\}, \{6, \frac{7}{2}, \frac{7}{2}\}, \{7, \frac{9}{2}, \frac{9}{2}\}, \\
& \{8, \frac{29}{6}, \frac{29}{6}\}, \{9, \frac{16}{3}, \frac{16}{3}\}, \{10, \frac{16}{3}, \frac{16}{3}\}, \{11, \frac{19}{3}, \frac{19}{3}\}, \{12, \frac{19}{3}, \frac{19}{3}\}, \{13, \frac{22}{3}, \frac{22}{3}\}, \\
& \{14, \frac{22}{3}, \frac{22}{3}\}, \{15, \frac{22}{3}, \frac{22}{3}\}, \{16, \frac{91}{12}, \frac{91}{12}\}, \{17, \frac{103}{12}, \frac{103}{12}\}, \{18, \frac{103}{12}, \frac{103}{12}\}, \\
& \{19, \frac{115}{12}, \frac{115}{12}\}, \{20, \frac{115}{12}, \frac{115}{12}\}, \{21, \frac{115}{12}, \frac{115}{12}\}, \{22, \frac{115}{12}, \frac{115}{12}\}, \{23, \frac{127}{12}, \frac{127}{12}\}, \\
& \{24, \frac{127}{12}, \frac{127}{12}\}, \{25, \frac{133}{12}, \frac{133}{12}\}, \{26, \frac{133}{12}, \frac{133}{12}\}, \{27, \frac{137}{12}, \frac{137}{12}\}, \{28, \frac{137}{12}, \frac{137}{12}\}, \\
& \{29, \frac{149}{12}, \frac{149}{12}\}, \{30, \frac{149}{12}, \frac{149}{12}\}, \{31, \frac{161}{12}, \frac{161}{12}\}, \{32, \frac{817}{60}, \frac{817}{60}\}, \{33, \frac{817}{60}, \frac{817}{60}\}, \\
& \{34, \frac{817}{60}, \frac{817}{60}\}, \{35, \frac{817}{60}, \frac{817}{60}\}, \{36, \frac{817}{60}, \frac{817}{60}\}, \{37, \frac{877}{60}, \frac{877}{60}\}, \{38, \frac{877}{60}, \frac{877}{60}\}, \\
& \{39, \frac{877}{60}, \frac{877}{60}\}, \{40, \frac{877}{60}, \frac{877}{60}\}, \{41, \frac{937}{60}, \frac{937}{60}\}, \{42, \frac{937}{60}, \frac{937}{60}\}, \{43, \frac{997}{60}, \frac{997}{60}\}, \\
& \{44, \frac{997}{60}, \frac{997}{60}\}, \{45, \frac{997}{60}, \frac{997}{60}\}, \{46, \frac{997}{60}, \frac{997}{60}\}, \{47, \frac{1057}{60}, \frac{1057}{60}\}, \{48, \frac{1057}{60}, \frac{1057}{60}\}, \\
& \{49, \frac{1087}{60}, \frac{1087}{60}\}, \{50, \frac{1087}{60}, \frac{1087}{60}\}, \{51, \frac{1087}{60}, \frac{1087}{60}\}, \{52, \frac{1087}{60}, \frac{1087}{60}\}, \\
& \{53, \frac{1147}{60}, \frac{1147}{60}\}, \{54, \frac{1147}{60}, \frac{1147}{60}\}, \{55, \frac{1147}{60}, \frac{1147}{60}\}, \{56, \frac{1147}{60}, \frac{1147}{60}\}, \\
& \{57, \frac{1147}{60}, \frac{1147}{60}\}, \{58, \frac{1147}{60}, \frac{1147}{60}\}, \{59, \frac{1207}{60}, \frac{1207}{60}\}, \{60, \frac{1207}{60}, \frac{1207}{60}\}, \\
& \{61, \frac{1267}{60}, \frac{1267}{60}\}, \{62, \frac{1267}{60}, \frac{1267}{60}\}, \{63, \frac{1267}{60}, \frac{1267}{60}\}, \{64, \frac{1277}{60}, \frac{1277}{60}\}, \\
& \{65, \frac{1277}{60}, \frac{1277}{60}\}, \{66, \frac{1277}{60}, \frac{1277}{60}\}, \{67, \frac{1337}{60}, \frac{1337}{60}\}, \{68, \frac{1337}{60}, \frac{1337}{60}\}, \\
& \{69, \frac{1337}{60}, \frac{1337}{60}\}, \{70, \frac{1337}{60}, \frac{1337}{60}\}, \{71, \frac{1397}{60}, \frac{1397}{60}\}, \{72, \frac{1397}{60}, \frac{1397}{60}\}, \\
& \{73, \frac{1457}{60}, \frac{1457}{60}\}, \{74, \frac{1457}{60}, \frac{1457}{60}\}, \{75, \frac{1457}{60}, \frac{1457}{60}\}, \{76, \frac{1457}{60}, \frac{1457}{60}\}, \\
& \{77, \frac{1457}{60}, \frac{1457}{60}\}, \{78, \frac{1457}{60}, \frac{1457}{60}\}, \{79, \frac{1517}{60}, \frac{1517}{60}\}, \{80, \frac{1517}{60}, \frac{1517}{60}\}, \\
& \{81, \frac{383}{15}, \frac{383}{15}\}, \{82, \frac{383}{15}, \frac{383}{15}\}, \{83, \frac{398}{15}, \frac{398}{15}\}, \{84, \frac{398}{15}, \frac{398}{15}\}, \{85, \frac{398}{15}, \frac{398}{15}\}, \\
& \{86, \frac{398}{15}, \frac{398}{15}\}, \{87, \frac{398}{15}, \frac{398}{15}\}, \{88, \frac{398}{15}, \frac{398}{15}\}, \{89, \frac{413}{15}, \frac{413}{15}\}, \{90, \frac{413}{15}, \frac{413}{15}\}, \\
& \{91, \frac{413}{15}, \frac{413}{15}\}, \{92, \frac{413}{15}, \frac{413}{15}\}, \{93, \frac{413}{15}, \frac{413}{15}\}, \{94, \frac{413}{15}, \frac{413}{15}\}, \{95, \frac{413}{15}, \frac{413}{15}\}, \\
& \{96, \frac{413}{15}, \frac{413}{15}\}, \{97, \frac{428}{15}, \frac{428}{15}\}, \{98, \frac{428}{15}, \frac{428}{15}\}, \{99, \frac{428}{15}, \frac{428}{15}\}, \{100, \frac{428}{15}, \frac{428}{15}\} \}
\end{aligned}$$


```

D2Cache[n_, k_, s_] :=
  Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1/k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]

LinnikSumFast[n_] := D2Cache[n, 1, 2] +
  Sum[(Sum[(-1)^(k + 1) / k D2Cache[n / j, k - 1, 2], {j, Floor[n^(1/3)] + 1, n^(1/2)}]),
    {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k (Floor[n / j] - (Floor[n / (j + 1)])) D2Cache[j, k - 1, 2],
    {j, 1, n / Floor[n^(1/2)] - 1}], {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k d2cache[j, k - 1] (Floor[n / j] - 1), {j, 2, n^(1/3)}]),
    {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k d2cache[j, m] D2Cache[n / (j s), k - m - 1, 2],
    {j, 2, n^(1/3)}, {s, Floor[Floor[n^(1/3)] / j] + 1, Floor[n / j]^(1/2)},
    {m, 1, k - 2}], {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k (Sum[1, {m, Floor[n / (j (s + 1))] + 1, n / (j s)}])
    (Sum[d2cache[j, m] D2Cache[s, k - m - 1, 2], {m, 1, k - 2}], {j, 2, n^(1/3)},
    {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1/2)] - 1}), {k, 2, Log[2, n]}]
RiePrimeCnt[n_] := Sum[PrimePi[n^(1/j)] / j, {j, 1, Log[2, n]}]
Table[{n, LinnikSumFast[n], RiePrimeCnt[n]}, {n, 1, 100}]

```

$$\begin{aligned}
& \{ \{1, 0, 0\}, \{2, 1, 1\}, \{3, 2, 2\}, \{4, \frac{5}{2}, \frac{5}{2}\}, \{5, \frac{7}{2}, \frac{7}{2}\}, \{6, \frac{7}{2}, \frac{7}{2}\}, \{7, \frac{9}{2}, \frac{9}{2}\}, \\
& \{8, \frac{29}{6}, \frac{29}{6}\}, \{9, \frac{16}{3}, \frac{16}{3}\}, \{10, \frac{16}{3}, \frac{16}{3}\}, \{11, \frac{19}{3}, \frac{19}{3}\}, \{12, \frac{19}{3}, \frac{19}{3}\}, \{13, \frac{22}{3}, \frac{22}{3}\}, \\
& \{14, \frac{22}{3}, \frac{22}{3}\}, \{15, \frac{22}{3}, \frac{22}{3}\}, \{16, \frac{91}{12}, \frac{91}{12}\}, \{17, \frac{103}{12}, \frac{103}{12}\}, \{18, \frac{103}{12}, \frac{103}{12}\}, \\
& \{19, \frac{115}{12}, \frac{115}{12}\}, \{20, \frac{115}{12}, \frac{115}{12}\}, \{21, \frac{115}{12}, \frac{115}{12}\}, \{22, \frac{115}{12}, \frac{115}{12}\}, \{23, \frac{127}{12}, \frac{127}{12}\}, \\
& \{24, \frac{127}{12}, \frac{127}{12}\}, \{25, \frac{133}{12}, \frac{133}{12}\}, \{26, \frac{133}{12}, \frac{133}{12}\}, \{27, \frac{137}{12}, \frac{137}{12}\}, \{28, \frac{137}{12}, \frac{137}{12}\}, \\
& \{29, \frac{149}{12}, \frac{149}{12}\}, \{30, \frac{149}{12}, \frac{149}{12}\}, \{31, \frac{161}{12}, \frac{161}{12}\}, \{32, \frac{817}{60}, \frac{817}{60}\}, \{33, \frac{817}{60}, \frac{817}{60}\}, \\
& \{34, \frac{817}{60}, \frac{817}{60}\}, \{35, \frac{817}{60}, \frac{817}{60}\}, \{36, \frac{817}{60}, \frac{817}{60}\}, \{37, \frac{877}{60}, \frac{877}{60}\}, \{38, \frac{877}{60}, \frac{877}{60}\}, \\
& \{39, \frac{877}{60}, \frac{877}{60}\}, \{40, \frac{877}{60}, \frac{877}{60}\}, \{41, \frac{937}{60}, \frac{937}{60}\}, \{42, \frac{937}{60}, \frac{937}{60}\}, \{43, \frac{997}{60}, \frac{997}{60}\}, \\
& \{44, \frac{997}{60}, \frac{997}{60}\}, \{45, \frac{997}{60}, \frac{997}{60}\}, \{46, \frac{997}{60}, \frac{997}{60}\}, \{47, \frac{1057}{60}, \frac{1057}{60}\}, \{48, \frac{1057}{60}, \frac{1057}{60}\}, \\
& \{49, \frac{1087}{60}, \frac{1087}{60}\}, \{50, \frac{1087}{60}, \frac{1087}{60}\}, \{51, \frac{1087}{60}, \frac{1087}{60}\}, \{52, \frac{1087}{60}, \frac{1087}{60}\}, \\
& \{53, \frac{1147}{60}, \frac{1147}{60}\}, \{54, \frac{1147}{60}, \frac{1147}{60}\}, \{55, \frac{1147}{60}, \frac{1147}{60}\}, \{56, \frac{1147}{60}, \frac{1147}{60}\}, \\
& \{57, \frac{1147}{60}, \frac{1147}{60}\}, \{58, \frac{1147}{60}, \frac{1147}{60}\}, \{59, \frac{1207}{60}, \frac{1207}{60}\}, \{60, \frac{1207}{60}, \frac{1207}{60}\}, \\
& \{61, \frac{1267}{60}, \frac{1267}{60}\}, \{62, \frac{1267}{60}, \frac{1267}{60}\}, \{63, \frac{1267}{60}, \frac{1267}{60}\}, \{64, \frac{1277}{60}, \frac{1277}{60}\}, \\
& \{65, \frac{1277}{60}, \frac{1277}{60}\}, \{66, \frac{1277}{60}, \frac{1277}{60}\}, \{67, \frac{1337}{60}, \frac{1337}{60}\}, \{68, \frac{1337}{60}, \frac{1337}{60}\}, \\
& \{69, \frac{1337}{60}, \frac{1337}{60}\}, \{70, \frac{1337}{60}, \frac{1337}{60}\}, \{71, \frac{1397}{60}, \frac{1397}{60}\}, \{72, \frac{1397}{60}, \frac{1397}{60}\}, \\
& \{73, \frac{1457}{60}, \frac{1457}{60}\}, \{74, \frac{1457}{60}, \frac{1457}{60}\}, \{75, \frac{1457}{60}, \frac{1457}{60}\}, \{76, \frac{1457}{60}, \frac{1457}{60}\}, \\
& \{77, \frac{1457}{60}, \frac{1457}{60}\}, \{78, \frac{1457}{60}, \frac{1457}{60}\}, \{79, \frac{1517}{60}, \frac{1517}{60}\}, \{80, \frac{1517}{60}, \frac{1517}{60}\}, \\
& \{81, \frac{383}{15}, \frac{383}{15}\}, \{82, \frac{383}{15}, \frac{383}{15}\}, \{83, \frac{398}{15}, \frac{398}{15}\}, \{84, \frac{398}{15}, \frac{398}{15}\}, \{85, \frac{398}{15}, \frac{398}{15}\}, \\
& \{86, \frac{398}{15}, \frac{398}{15}\}, \{87, \frac{398}{15}, \frac{398}{15}\}, \{88, \frac{398}{15}, \frac{398}{15}\}, \{89, \frac{413}{15}, \frac{413}{15}\}, \{90, \frac{413}{15}, \frac{413}{15}\}, \\
& \{91, \frac{413}{15}, \frac{413}{15}\}, \{92, \frac{413}{15}, \frac{413}{15}\}, \{93, \frac{413}{15}, \frac{413}{15}\}, \{94, \frac{413}{15}, \frac{413}{15}\}, \{95, \frac{413}{15}, \frac{413}{15}\}, \\
& \{96, \frac{413}{15}, \frac{413}{15}\}, \{97, \frac{428}{15}, \frac{428}{15}\}, \{98, \frac{428}{15}, \frac{428}{15}\}, \{99, \frac{428}{15}, \frac{428}{15}\}, \{100, \frac{428}{15}, \frac{428}{15}\} \}
\end{aligned}$$

```

D2Cache[n_, k_, s_] :=
  Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1/k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]

LinnikSumFast[n_] := D2Cache[n, 1, 2] +
  Sum[(Sum[(-1)^(k + 1) / k D2Cache[n / j, k - 1, 2], {j, Floor[n^(1/3)] + 1, n^(1/2)}]),
    {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k (Floor[n / j] - (Floor[n / (j + 1)])) D2Cache[j, k - 1, 2],
    {j, 1, n / Floor[n^(1/2)] - 1}], {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k d2cache[j, k - 1] (Floor[n / j] - 1), {j, 2, n^(1/3)}]),
    {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k d2cache[j, m] D2Cache[n / (j s), k - m - 1, 2],
    {j, 2, n^(1/3)}, {s, Floor[Floor[n^(1/3)] / j] + 1, Floor[n / j]^(1/2)},
    {m, 1, k - 2}], {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k (Sum[1, {m, Floor[n / (j (s + 1))] + 1, n / (j s)]
    (Sum[d2cache[j, m] D2Cache[s, k - m - 1, 2], {m, 1, k - 2}]), {j, 2, n^(1/3)},
    {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1/2)] - 1}), {k, 2, Log[2, n]}]
RiePrimeCnt[n_] := Sum[PrimePi[n^(1/j)] / j, {j, 1, Log[2, n]}]
Table[{n, LinnikSumFast[n], RiePrimeCnt[n]}, {n, 1, 100}]

```

$$\begin{aligned}
& \{ \{1, 0, 0\}, \{2, 1, 1\}, \{3, 2, 2\}, \{4, \frac{5}{2}, \frac{5}{2}\}, \{5, \frac{7}{2}, \frac{7}{2}\}, \{6, \frac{7}{2}, \frac{7}{2}\}, \{7, \frac{9}{2}, \frac{9}{2}\}, \\
& \{8, \frac{29}{6}, \frac{29}{6}\}, \{9, \frac{16}{3}, \frac{16}{3}\}, \{10, \frac{16}{3}, \frac{16}{3}\}, \{11, \frac{19}{3}, \frac{19}{3}\}, \{12, \frac{19}{3}, \frac{19}{3}\}, \{13, \frac{22}{3}, \frac{22}{3}\}, \\
& \{14, \frac{22}{3}, \frac{22}{3}\}, \{15, \frac{22}{3}, \frac{22}{3}\}, \{16, \frac{91}{12}, \frac{91}{12}\}, \{17, \frac{103}{12}, \frac{103}{12}\}, \{18, \frac{103}{12}, \frac{103}{12}\}, \\
& \{19, \frac{115}{12}, \frac{115}{12}\}, \{20, \frac{115}{12}, \frac{115}{12}\}, \{21, \frac{115}{12}, \frac{115}{12}\}, \{22, \frac{115}{12}, \frac{115}{12}\}, \{23, \frac{127}{12}, \frac{127}{12}\}, \\
& \{24, \frac{127}{12}, \frac{127}{12}\}, \{25, \frac{133}{12}, \frac{133}{12}\}, \{26, \frac{133}{12}, \frac{133}{12}\}, \{27, \frac{137}{12}, \frac{137}{12}\}, \{28, \frac{137}{12}, \frac{137}{12}\}, \\
& \{29, \frac{149}{12}, \frac{149}{12}\}, \{30, \frac{149}{12}, \frac{149}{12}\}, \{31, \frac{161}{12}, \frac{161}{12}\}, \{32, \frac{817}{60}, \frac{817}{60}\}, \{33, \frac{817}{60}, \frac{817}{60}\}, \\
& \{34, \frac{817}{60}, \frac{817}{60}\}, \{35, \frac{817}{60}, \frac{817}{60}\}, \{36, \frac{817}{60}, \frac{817}{60}\}, \{37, \frac{877}{60}, \frac{877}{60}\}, \{38, \frac{877}{60}, \frac{877}{60}\}, \\
& \{39, \frac{877}{60}, \frac{877}{60}\}, \{40, \frac{877}{60}, \frac{877}{60}\}, \{41, \frac{937}{60}, \frac{937}{60}\}, \{42, \frac{937}{60}, \frac{937}{60}\}, \{43, \frac{997}{60}, \frac{997}{60}\}, \\
& \{44, \frac{997}{60}, \frac{997}{60}\}, \{45, \frac{997}{60}, \frac{997}{60}\}, \{46, \frac{997}{60}, \frac{997}{60}\}, \{47, \frac{1057}{60}, \frac{1057}{60}\}, \{48, \frac{1057}{60}, \frac{1057}{60}\}, \\
& \{49, \frac{1087}{60}, \frac{1087}{60}\}, \{50, \frac{1087}{60}, \frac{1087}{60}\}, \{51, \frac{1087}{60}, \frac{1087}{60}\}, \{52, \frac{1087}{60}, \frac{1087}{60}\}, \\
& \{53, \frac{1147}{60}, \frac{1147}{60}\}, \{54, \frac{1147}{60}, \frac{1147}{60}\}, \{55, \frac{1147}{60}, \frac{1147}{60}\}, \{56, \frac{1147}{60}, \frac{1147}{60}\}, \\
& \{57, \frac{1147}{60}, \frac{1147}{60}\}, \{58, \frac{1147}{60}, \frac{1147}{60}\}, \{59, \frac{1207}{60}, \frac{1207}{60}\}, \{60, \frac{1207}{60}, \frac{1207}{60}\}, \\
& \{61, \frac{1267}{60}, \frac{1267}{60}\}, \{62, \frac{1267}{60}, \frac{1267}{60}\}, \{63, \frac{1267}{60}, \frac{1267}{60}\}, \{64, \frac{1277}{60}, \frac{1277}{60}\}, \\
& \{65, \frac{1277}{60}, \frac{1277}{60}\}, \{66, \frac{1277}{60}, \frac{1277}{60}\}, \{67, \frac{1337}{60}, \frac{1337}{60}\}, \{68, \frac{1337}{60}, \frac{1337}{60}\}, \\
& \{69, \frac{1337}{60}, \frac{1337}{60}\}, \{70, \frac{1337}{60}, \frac{1337}{60}\}, \{71, \frac{1397}{60}, \frac{1397}{60}\}, \{72, \frac{1397}{60}, \frac{1397}{60}\}, \\
& \{73, \frac{1457}{60}, \frac{1457}{60}\}, \{74, \frac{1457}{60}, \frac{1457}{60}\}, \{75, \frac{1457}{60}, \frac{1457}{60}\}, \{76, \frac{1457}{60}, \frac{1457}{60}\}, \\
& \{77, \frac{1457}{60}, \frac{1457}{60}\}, \{78, \frac{1457}{60}, \frac{1457}{60}\}, \{79, \frac{1517}{60}, \frac{1517}{60}\}, \{80, \frac{1517}{60}, \frac{1517}{60}\}, \\
& \{81, \frac{383}{15}, \frac{383}{15}\}, \{82, \frac{383}{15}, \frac{383}{15}\}, \{83, \frac{398}{15}, \frac{398}{15}\}, \{84, \frac{398}{15}, \frac{398}{15}\}, \{85, \frac{398}{15}, \frac{398}{15}\}, \\
& \{86, \frac{398}{15}, \frac{398}{15}\}, \{87, \frac{398}{15}, \frac{398}{15}\}, \{88, \frac{398}{15}, \frac{398}{15}\}, \{89, \frac{413}{15}, \frac{413}{15}\}, \{90, \frac{413}{15}, \frac{413}{15}\}, \\
& \{91, \frac{413}{15}, \frac{413}{15}\}, \{92, \frac{413}{15}, \frac{413}{15}\}, \{93, \frac{413}{15}, \frac{413}{15}\}, \{94, \frac{413}{15}, \frac{413}{15}\}, \{95, \frac{413}{15}, \frac{413}{15}\}, \\
& \{96, \frac{413}{15}, \frac{413}{15}\}, \{97, \frac{428}{15}, \frac{428}{15}\}, \{98, \frac{428}{15}, \frac{428}{15}\}, \{99, \frac{428}{15}, \frac{428}{15}\}, \{100, \frac{428}{15}, \frac{428}{15}\} \}
\end{aligned}$$

```

D2Cache[n_, k_, s_] :=
  Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1/k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]

LinnikSumFast[n_] := D2Cache[n, 1, 2] +
  Sum[(Sum[(-1)^(k + 1) / k D2Cache[n / j, k - 1, 2], {j, Floor[n^(1/3)] + 1, n^(1/2)}]),
    {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k (Floor[n / j] - (Floor[n / (j + 1)])) D2Cache[j, k - 1, 2],
    {j, 1, n / Floor[n^(1/2)] - 1}], {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k d2cache[j, k - 1] (Floor[n / j] - 1), {j, 2, n^(1/3)}]),
    {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k d2cache[j, m] D2Cache[n / (j s), k - m - 1, 2],
    {j, 2, n^(1/3)}, {s, Floor[Floor[n^(1/3)] / j] + 1, Floor[n / j]^(1/2)},
    {m, 1, k - 2}], {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k (Floor[n / (j s)] - Floor[n / (j (s + 1)]))
    (Sum[d2cache[j, m] D2Cache[s, k - m - 1, 2], {m, 1, k - 2}], {j, 2, n^(1/3)},
    {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1/2)] - 1}), {k, 2, Log[2, n]}]
RiePrimeCnt[n_] := Sum[PrimePi[n^(1/j)] / j, {j, 1, Log[2, n]}]
Table[{n, LinnikSumFast[n], RiePrimeCnt[n]}, {n, 1, 100}]

```

$$\begin{aligned}
& \{ \{1, 0, 0\}, \{2, 1, 1\}, \{3, 2, 2\}, \{4, \frac{5}{2}, \frac{5}{2}\}, \{5, \frac{7}{2}, \frac{7}{2}\}, \{6, \frac{7}{2}, \frac{7}{2}\}, \{7, \frac{9}{2}, \frac{9}{2}\}, \\
& \{8, \frac{29}{6}, \frac{29}{6}\}, \{9, \frac{16}{3}, \frac{16}{3}\}, \{10, \frac{16}{3}, \frac{16}{3}\}, \{11, \frac{19}{3}, \frac{19}{3}\}, \{12, \frac{19}{3}, \frac{19}{3}\}, \{13, \frac{22}{3}, \frac{22}{3}\}, \\
& \{14, \frac{22}{3}, \frac{22}{3}\}, \{15, \frac{22}{3}, \frac{22}{3}\}, \{16, \frac{91}{12}, \frac{91}{12}\}, \{17, \frac{103}{12}, \frac{103}{12}\}, \{18, \frac{103}{12}, \frac{103}{12}\}, \\
& \{19, \frac{115}{12}, \frac{115}{12}\}, \{20, \frac{115}{12}, \frac{115}{12}\}, \{21, \frac{115}{12}, \frac{115}{12}\}, \{22, \frac{115}{12}, \frac{115}{12}\}, \{23, \frac{127}{12}, \frac{127}{12}\}, \\
& \{24, \frac{127}{12}, \frac{127}{12}\}, \{25, \frac{133}{12}, \frac{133}{12}\}, \{26, \frac{133}{12}, \frac{133}{12}\}, \{27, \frac{137}{12}, \frac{137}{12}\}, \{28, \frac{137}{12}, \frac{137}{12}\}, \\
& \{29, \frac{149}{12}, \frac{149}{12}\}, \{30, \frac{149}{12}, \frac{149}{12}\}, \{31, \frac{161}{12}, \frac{161}{12}\}, \{32, \frac{817}{60}, \frac{817}{60}\}, \{33, \frac{817}{60}, \frac{817}{60}\}, \\
& \{34, \frac{817}{60}, \frac{817}{60}\}, \{35, \frac{817}{60}, \frac{817}{60}\}, \{36, \frac{817}{60}, \frac{817}{60}\}, \{37, \frac{877}{60}, \frac{877}{60}\}, \{38, \frac{877}{60}, \frac{877}{60}\}, \\
& \{39, \frac{877}{60}, \frac{877}{60}\}, \{40, \frac{877}{60}, \frac{877}{60}\}, \{41, \frac{937}{60}, \frac{937}{60}\}, \{42, \frac{937}{60}, \frac{937}{60}\}, \{43, \frac{997}{60}, \frac{997}{60}\}, \\
& \{44, \frac{997}{60}, \frac{997}{60}\}, \{45, \frac{997}{60}, \frac{997}{60}\}, \{46, \frac{997}{60}, \frac{997}{60}\}, \{47, \frac{1057}{60}, \frac{1057}{60}\}, \{48, \frac{1057}{60}, \frac{1057}{60}\}, \\
& \{49, \frac{1087}{60}, \frac{1087}{60}\}, \{50, \frac{1087}{60}, \frac{1087}{60}\}, \{51, \frac{1087}{60}, \frac{1087}{60}\}, \{52, \frac{1087}{60}, \frac{1087}{60}\}, \\
& \{53, \frac{1147}{60}, \frac{1147}{60}\}, \{54, \frac{1147}{60}, \frac{1147}{60}\}, \{55, \frac{1147}{60}, \frac{1147}{60}\}, \{56, \frac{1147}{60}, \frac{1147}{60}\}, \\
& \{57, \frac{1147}{60}, \frac{1147}{60}\}, \{58, \frac{1147}{60}, \frac{1147}{60}\}, \{59, \frac{1207}{60}, \frac{1207}{60}\}, \{60, \frac{1207}{60}, \frac{1207}{60}\}, \\
& \{61, \frac{1267}{60}, \frac{1267}{60}\}, \{62, \frac{1267}{60}, \frac{1267}{60}\}, \{63, \frac{1267}{60}, \frac{1267}{60}\}, \{64, \frac{1277}{60}, \frac{1277}{60}\}, \\
& \{65, \frac{1277}{60}, \frac{1277}{60}\}, \{66, \frac{1277}{60}, \frac{1277}{60}\}, \{67, \frac{1337}{60}, \frac{1337}{60}\}, \{68, \frac{1337}{60}, \frac{1337}{60}\}, \\
& \{69, \frac{1337}{60}, \frac{1337}{60}\}, \{70, \frac{1337}{60}, \frac{1337}{60}\}, \{71, \frac{1397}{60}, \frac{1397}{60}\}, \{72, \frac{1397}{60}, \frac{1397}{60}\}, \\
& \{73, \frac{1457}{60}, \frac{1457}{60}\}, \{74, \frac{1457}{60}, \frac{1457}{60}\}, \{75, \frac{1457}{60}, \frac{1457}{60}\}, \{76, \frac{1457}{60}, \frac{1457}{60}\}, \\
& \{77, \frac{1457}{60}, \frac{1457}{60}\}, \{78, \frac{1457}{60}, \frac{1457}{60}\}, \{79, \frac{1517}{60}, \frac{1517}{60}\}, \{80, \frac{1517}{60}, \frac{1517}{60}\}, \\
& \{81, \frac{383}{15}, \frac{383}{15}\}, \{82, \frac{383}{15}, \frac{383}{15}\}, \{83, \frac{398}{15}, \frac{398}{15}\}, \{84, \frac{398}{15}, \frac{398}{15}\}, \{85, \frac{398}{15}, \frac{398}{15}\}, \\
& \{86, \frac{398}{15}, \frac{398}{15}\}, \{87, \frac{398}{15}, \frac{398}{15}\}, \{88, \frac{398}{15}, \frac{398}{15}\}, \{89, \frac{413}{15}, \frac{413}{15}\}, \{90, \frac{413}{15}, \frac{413}{15}\}, \\
& \{91, \frac{413}{15}, \frac{413}{15}\}, \{92, \frac{413}{15}, \frac{413}{15}\}, \{93, \frac{413}{15}, \frac{413}{15}\}, \{94, \frac{413}{15}, \frac{413}{15}\}, \{95, \frac{413}{15}, \frac{413}{15}\}, \\
& \{96, \frac{413}{15}, \frac{413}{15}\}, \{97, \frac{428}{15}, \frac{428}{15}\}, \{98, \frac{428}{15}, \frac{428}{15}\}, \{99, \frac{428}{15}, \frac{428}{15}\}, \{100, \frac{428}{15}, \frac{428}{15}\} \}
\end{aligned}$$

```

D2Cache[n_, k_, s_] :=
  Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1/k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]

LinnikSumFast[n_] := D2Cache[n, 1, 2] +
  Sum[(Sum[(-1)^(k + 1) / k D2Cache[n / j, k - 1, 2], {j, Floor[n^(1/3)] + 1, n^(1/2)}]),
    {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k (Floor[n / j] - (Floor[n / (j + 1)])) D2Cache[j, k - 1, 2],
    {j, 1, n / Floor[n^(1/2)] - 1}], {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k d2cache[j, k - 1] (Floor[n / j] - 1), {j, 2, n^(1/3)}]),
    {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k d2cache[j, m] D2Cache[n / (j s), k - m - 1, 2],
    {j, 2, n^(1/3)}, {s, Floor[Floor[n^(1/3)] / j] + 1, Floor[n / j]^(1/2)},
    {m, 1, k - 2}], {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k (Floor[n / (j s)] - Floor[n / (j (s + 1)]))
    (Sum[d2cache[j, m] D2Cache[s, k - m - 1, 2], {m, 1, k - 2}], {j, 2, n^(1/3)},
    {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1/2)] - 1}), {k, 2, Log[2, n]}]
RiePrimeCnt[n_] := Sum[PrimePi[n^(1/j)] / j, {j, 1, Log[2, n]}]
Table[{n, LinnikSumFast[n], RiePrimeCnt[n]}, {n, 1, 100}]

```

$$\begin{aligned}
& \{ \{1, 0, 0\}, \{2, 1, 1\}, \{3, 2, 2\}, \{4, \frac{5}{2}, \frac{5}{2}\}, \{5, \frac{7}{2}, \frac{7}{2}\}, \{6, \frac{7}{2}, \frac{7}{2}\}, \{7, \frac{9}{2}, \frac{9}{2}\}, \\
& \{8, \frac{29}{6}, \frac{29}{6}\}, \{9, \frac{16}{3}, \frac{16}{3}\}, \{10, \frac{16}{3}, \frac{16}{3}\}, \{11, \frac{19}{3}, \frac{19}{3}\}, \{12, \frac{19}{3}, \frac{19}{3}\}, \{13, \frac{22}{3}, \frac{22}{3}\}, \\
& \{14, \frac{22}{3}, \frac{22}{3}\}, \{15, \frac{22}{3}, \frac{22}{3}\}, \{16, \frac{91}{12}, \frac{91}{12}\}, \{17, \frac{103}{12}, \frac{103}{12}\}, \{18, \frac{103}{12}, \frac{103}{12}\}, \\
& \{19, \frac{115}{12}, \frac{115}{12}\}, \{20, \frac{115}{12}, \frac{115}{12}\}, \{21, \frac{115}{12}, \frac{115}{12}\}, \{22, \frac{115}{12}, \frac{115}{12}\}, \{23, \frac{127}{12}, \frac{127}{12}\}, \\
& \{24, \frac{127}{12}, \frac{127}{12}\}, \{25, \frac{133}{12}, \frac{133}{12}\}, \{26, \frac{133}{12}, \frac{133}{12}\}, \{27, \frac{137}{12}, \frac{137}{12}\}, \{28, \frac{137}{12}, \frac{137}{12}\}, \\
& \{29, \frac{149}{12}, \frac{149}{12}\}, \{30, \frac{149}{12}, \frac{149}{12}\}, \{31, \frac{161}{12}, \frac{161}{12}\}, \{32, \frac{817}{60}, \frac{817}{60}\}, \{33, \frac{817}{60}, \frac{817}{60}\}, \\
& \{34, \frac{817}{60}, \frac{817}{60}\}, \{35, \frac{817}{60}, \frac{817}{60}\}, \{36, \frac{817}{60}, \frac{817}{60}\}, \{37, \frac{877}{60}, \frac{877}{60}\}, \{38, \frac{877}{60}, \frac{877}{60}\}, \\
& \{39, \frac{877}{60}, \frac{877}{60}\}, \{40, \frac{877}{60}, \frac{877}{60}\}, \{41, \frac{937}{60}, \frac{937}{60}\}, \{42, \frac{937}{60}, \frac{937}{60}\}, \{43, \frac{997}{60}, \frac{997}{60}\}, \\
& \{44, \frac{997}{60}, \frac{997}{60}\}, \{45, \frac{997}{60}, \frac{997}{60}\}, \{46, \frac{997}{60}, \frac{997}{60}\}, \{47, \frac{1057}{60}, \frac{1057}{60}\}, \{48, \frac{1057}{60}, \frac{1057}{60}\}, \\
& \{49, \frac{1087}{60}, \frac{1087}{60}\}, \{50, \frac{1087}{60}, \frac{1087}{60}\}, \{51, \frac{1087}{60}, \frac{1087}{60}\}, \{52, \frac{1087}{60}, \frac{1087}{60}\}, \\
& \{53, \frac{1147}{60}, \frac{1147}{60}\}, \{54, \frac{1147}{60}, \frac{1147}{60}\}, \{55, \frac{1147}{60}, \frac{1147}{60}\}, \{56, \frac{1147}{60}, \frac{1147}{60}\}, \\
& \{57, \frac{1147}{60}, \frac{1147}{60}\}, \{58, \frac{1147}{60}, \frac{1147}{60}\}, \{59, \frac{1207}{60}, \frac{1207}{60}\}, \{60, \frac{1207}{60}, \frac{1207}{60}\}, \\
& \{61, \frac{1267}{60}, \frac{1267}{60}\}, \{62, \frac{1267}{60}, \frac{1267}{60}\}, \{63, \frac{1267}{60}, \frac{1267}{60}\}, \{64, \frac{1277}{60}, \frac{1277}{60}\}, \\
& \{65, \frac{1277}{60}, \frac{1277}{60}\}, \{66, \frac{1277}{60}, \frac{1277}{60}\}, \{67, \frac{1337}{60}, \frac{1337}{60}\}, \{68, \frac{1337}{60}, \frac{1337}{60}\}, \\
& \{69, \frac{1337}{60}, \frac{1337}{60}\}, \{70, \frac{1337}{60}, \frac{1337}{60}\}, \{71, \frac{1397}{60}, \frac{1397}{60}\}, \{72, \frac{1397}{60}, \frac{1397}{60}\}, \\
& \{73, \frac{1457}{60}, \frac{1457}{60}\}, \{74, \frac{1457}{60}, \frac{1457}{60}\}, \{75, \frac{1457}{60}, \frac{1457}{60}\}, \{76, \frac{1457}{60}, \frac{1457}{60}\}, \\
& \{77, \frac{1457}{60}, \frac{1457}{60}\}, \{78, \frac{1457}{60}, \frac{1457}{60}\}, \{79, \frac{1517}{60}, \frac{1517}{60}\}, \{80, \frac{1517}{60}, \frac{1517}{60}\}, \\
& \{81, \frac{383}{15}, \frac{383}{15}\}, \{82, \frac{383}{15}, \frac{383}{15}\}, \{83, \frac{398}{15}, \frac{398}{15}\}, \{84, \frac{398}{15}, \frac{398}{15}\}, \{85, \frac{398}{15}, \frac{398}{15}\}, \\
& \{86, \frac{398}{15}, \frac{398}{15}\}, \{87, \frac{398}{15}, \frac{398}{15}\}, \{88, \frac{398}{15}, \frac{398}{15}\}, \{89, \frac{413}{15}, \frac{413}{15}\}, \{90, \frac{413}{15}, \frac{413}{15}\}, \\
& \{91, \frac{413}{15}, \frac{413}{15}\}, \{92, \frac{413}{15}, \frac{413}{15}\}, \{93, \frac{413}{15}, \frac{413}{15}\}, \{94, \frac{413}{15}, \frac{413}{15}\}, \{95, \frac{413}{15}, \frac{413}{15}\}, \\
& \{96, \frac{413}{15}, \frac{413}{15}\}, \{97, \frac{428}{15}, \frac{428}{15}\}, \{98, \frac{428}{15}, \frac{428}{15}\}, \{99, \frac{428}{15}, \frac{428}{15}\}, \{100, \frac{428}{15}, \frac{428}{15}\} \}
\end{aligned}$$


```

D2Cache[n_, k_, s_] :=
  Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1/k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]

LinnikSumFast[n_] := D2Cache[n, 1, 2] +
  ( Sum[(-1)^(k + 1) / k D2Cache[n / j, k - 1, 2],
    {j, Floor[n^(1/3)] + 1, n^(1/2)}, {k, 2, Log[2, n]}] +
  ( Sum[(-1)^(k + 1) / k (Floor[n / j] - (Floor[n / (j + 1)])) D2Cache[j, k - 1, 2],
    {j, 1, n / Floor[n^(1/2)] - 1}, {k, 2, Log[2, n]}] +
  ( Sum[(-1)^(k + 1) / k d2cache[j, k - 1] (Floor[n / j] - 1),
    {j, 2, n^(1/3)}, {k, 2, Log[2, n]}] +
  Sum[( Sum[(-1)^(k + 1) / k d2cache[j, m] D2Cache[n / (j s), k - m - 1, 2],
    {j, 2, n^(1/3)}, {s, Floor[Floor[n^(1/3)] / j] + 1, Floor[n / j]^(1/2)},
    {m, 1, k - 2}], {k, 2, Log[2, n]}] +
  Sum[( Sum[(-1)^(k + 1) / k (Floor[n / (j s)] - Floor[n / (j (s + 1)]))
    (Sum[d2cache[j, m] D2Cache[s, k - m - 1, 2], {m, 1, k - 2}], {j, 2, n^(1/3)},
    {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1/2)] - 1}), {k, 2, Log[2, n]}]
RiePrimeCnt[n_] := Sum[PrimePi[n^(1/j)] / j, {j, 1, Log[2, n]}]
Table[{n, LinnikSumFast[n], RiePrimeCnt[n]}, {n, 1, 100}]

```

$$\begin{aligned}
& \{ \{1, 0, 0\}, \{2, 1, 1\}, \{3, 2, 2\}, \{4, \frac{5}{2}, \frac{5}{2}\}, \{5, \frac{7}{2}, \frac{7}{2}\}, \{6, \frac{7}{2}, \frac{7}{2}\}, \{7, \frac{9}{2}, \frac{9}{2}\}, \\
& \{8, \frac{29}{6}, \frac{29}{6}\}, \{9, \frac{16}{3}, \frac{16}{3}\}, \{10, \frac{16}{3}, \frac{16}{3}\}, \{11, \frac{19}{3}, \frac{19}{3}\}, \{12, \frac{19}{3}, \frac{19}{3}\}, \{13, \frac{22}{3}, \frac{22}{3}\}, \\
& \{14, \frac{22}{3}, \frac{22}{3}\}, \{15, \frac{22}{3}, \frac{22}{3}\}, \{16, \frac{91}{12}, \frac{91}{12}\}, \{17, \frac{103}{12}, \frac{103}{12}\}, \{18, \frac{103}{12}, \frac{103}{12}\}, \\
& \{19, \frac{115}{12}, \frac{115}{12}\}, \{20, \frac{115}{12}, \frac{115}{12}\}, \{21, \frac{115}{12}, \frac{115}{12}\}, \{22, \frac{115}{12}, \frac{115}{12}\}, \{23, \frac{127}{12}, \frac{127}{12}\}, \\
& \{24, \frac{127}{12}, \frac{127}{12}\}, \{25, \frac{133}{12}, \frac{133}{12}\}, \{26, \frac{133}{12}, \frac{133}{12}\}, \{27, \frac{137}{12}, \frac{137}{12}\}, \{28, \frac{137}{12}, \frac{137}{12}\}, \\
& \{29, \frac{149}{12}, \frac{149}{12}\}, \{30, \frac{149}{12}, \frac{149}{12}\}, \{31, \frac{161}{12}, \frac{161}{12}\}, \{32, \frac{817}{60}, \frac{817}{60}\}, \{33, \frac{817}{60}, \frac{817}{60}\}, \\
& \{34, \frac{817}{60}, \frac{817}{60}\}, \{35, \frac{817}{60}, \frac{817}{60}\}, \{36, \frac{817}{60}, \frac{817}{60}\}, \{37, \frac{877}{60}, \frac{877}{60}\}, \{38, \frac{877}{60}, \frac{877}{60}\}, \\
& \{39, \frac{877}{60}, \frac{877}{60}\}, \{40, \frac{877}{60}, \frac{877}{60}\}, \{41, \frac{937}{60}, \frac{937}{60}\}, \{42, \frac{937}{60}, \frac{937}{60}\}, \{43, \frac{997}{60}, \frac{997}{60}\}, \\
& \{44, \frac{997}{60}, \frac{997}{60}\}, \{45, \frac{997}{60}, \frac{997}{60}\}, \{46, \frac{997}{60}, \frac{997}{60}\}, \{47, \frac{1057}{60}, \frac{1057}{60}\}, \{48, \frac{1057}{60}, \frac{1057}{60}\}, \\
& \{49, \frac{1087}{60}, \frac{1087}{60}\}, \{50, \frac{1087}{60}, \frac{1087}{60}\}, \{51, \frac{1087}{60}, \frac{1087}{60}\}, \{52, \frac{1087}{60}, \frac{1087}{60}\}, \\
& \{53, \frac{1147}{60}, \frac{1147}{60}\}, \{54, \frac{1147}{60}, \frac{1147}{60}\}, \{55, \frac{1147}{60}, \frac{1147}{60}\}, \{56, \frac{1147}{60}, \frac{1147}{60}\}, \\
& \{57, \frac{1147}{60}, \frac{1147}{60}\}, \{58, \frac{1147}{60}, \frac{1147}{60}\}, \{59, \frac{1207}{60}, \frac{1207}{60}\}, \{60, \frac{1207}{60}, \frac{1207}{60}\}, \\
& \{61, \frac{1267}{60}, \frac{1267}{60}\}, \{62, \frac{1267}{60}, \frac{1267}{60}\}, \{63, \frac{1267}{60}, \frac{1267}{60}\}, \{64, \frac{1277}{60}, \frac{1277}{60}\}, \\
& \{65, \frac{1277}{60}, \frac{1277}{60}\}, \{66, \frac{1277}{60}, \frac{1277}{60}\}, \{67, \frac{1337}{60}, \frac{1337}{60}\}, \{68, \frac{1337}{60}, \frac{1337}{60}\}, \\
& \{69, \frac{1337}{60}, \frac{1337}{60}\}, \{70, \frac{1337}{60}, \frac{1337}{60}\}, \{71, \frac{1397}{60}, \frac{1397}{60}\}, \{72, \frac{1397}{60}, \frac{1397}{60}\}, \\
& \{73, \frac{1457}{60}, \frac{1457}{60}\}, \{74, \frac{1457}{60}, \frac{1457}{60}\}, \{75, \frac{1457}{60}, \frac{1457}{60}\}, \{76, \frac{1457}{60}, \frac{1457}{60}\}, \\
& \{77, \frac{1457}{60}, \frac{1457}{60}\}, \{78, \frac{1457}{60}, \frac{1457}{60}\}, \{79, \frac{1517}{60}, \frac{1517}{60}\}, \{80, \frac{1517}{60}, \frac{1517}{60}\}, \\
& \{81, \frac{383}{15}, \frac{383}{15}\}, \{82, \frac{383}{15}, \frac{383}{15}\}, \{83, \frac{398}{15}, \frac{398}{15}\}, \{84, \frac{398}{15}, \frac{398}{15}\}, \{85, \frac{398}{15}, \frac{398}{15}\}, \\
& \{86, \frac{398}{15}, \frac{398}{15}\}, \{87, \frac{398}{15}, \frac{398}{15}\}, \{88, \frac{398}{15}, \frac{398}{15}\}, \{89, \frac{413}{15}, \frac{413}{15}\}, \{90, \frac{413}{15}, \frac{413}{15}\}, \\
& \{91, \frac{413}{15}, \frac{413}{15}\}, \{92, \frac{413}{15}, \frac{413}{15}\}, \{93, \frac{413}{15}, \frac{413}{15}\}, \{94, \frac{413}{15}, \frac{413}{15}\}, \{95, \frac{413}{15}, \frac{413}{15}\}, \\
& \{96, \frac{413}{15}, \frac{413}{15}\}, \{97, \frac{428}{15}, \frac{428}{15}\}, \{98, \frac{428}{15}, \frac{428}{15}\}, \{99, \frac{428}{15}, \frac{428}{15}\}, \{100, \frac{428}{15}, \frac{428}{15}\} \}
\end{aligned}$$

```

D2Cache[n_, k_, s_] :=
  Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1/k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]

LinnikSumFast[n_] := D2Cache[n, 1, 2] +
  ( Sum[(-1)^(k + 1) / k D2Cache[n / j, k - 1, 2],
    {j, Floor[n^(1/3)] + 1, n^(1/2)}, {k, 2, Log[2, n]}] +
  ( Sum[(-1)^(k + 1) / k (Floor[n / j] - (Floor[n / (j + 1)])) D2Cache[j, k - 1, 2],
    {j, 1, n / Floor[n^(1/2)] - 1}, {k, 2, Log[2, n]}] +
  ( Sum[(-1)^(k + 1) / k d2cache[j, k - 1] (Floor[n / j] - 1),
    {j, 2, n^(1/3)}, {k, 2, Log[2, n]}] +
  ( Sum[(-1)^(k + 1) / k d2cache[j, m] D2Cache[n / (j s), k - m - 1, 2], {j, 2, n^(1/3)}, {s,
    Floor[Floor[n^(1/3)] / j] + 1, Floor[n / j]^(1/2)}, {k, 2, Log[2, n]}, {m, 1, k - 2}] +
  ( Sum[(-1)^(k + 1) / k ( Floor[n / (j s)] - Floor[n / (j (s + 1))] )
    (Sum[d2cache[j, m] D2Cache[s, k - m - 1, 2], {m, 1, k - 2}], {j, 2, n^(1/3)},
    {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1/2)] - 1}, {k, 2, Log[2, n]}])
RiePrimeCnt[n_] := Sum[PrimePi[n^(1/j)] / j, {j, 1, Log[2, n]}]
Table[{n, LinnikSumFast[n], RiePrimeCnt[n]}, {n, 1, 100}]

```

$$\begin{aligned}
& \{ \{1, 0, 0\}, \{2, 1, 1\}, \{3, 2, 2\}, \{4, \frac{5}{2}, \frac{5}{2}\}, \{5, \frac{7}{2}, \frac{7}{2}\}, \{6, \frac{7}{2}, \frac{7}{2}\}, \{7, \frac{9}{2}, \frac{9}{2}\}, \\
& \{8, \frac{29}{6}, \frac{29}{6}\}, \{9, \frac{16}{3}, \frac{16}{3}\}, \{10, \frac{16}{3}, \frac{16}{3}\}, \{11, \frac{19}{3}, \frac{19}{3}\}, \{12, \frac{19}{3}, \frac{19}{3}\}, \{13, \frac{22}{3}, \frac{22}{3}\}, \\
& \{14, \frac{22}{3}, \frac{22}{3}\}, \{15, \frac{22}{3}, \frac{22}{3}\}, \{16, \frac{91}{12}, \frac{91}{12}\}, \{17, \frac{103}{12}, \frac{103}{12}\}, \{18, \frac{103}{12}, \frac{103}{12}\}, \\
& \{19, \frac{115}{12}, \frac{115}{12}\}, \{20, \frac{115}{12}, \frac{115}{12}\}, \{21, \frac{115}{12}, \frac{115}{12}\}, \{22, \frac{115}{12}, \frac{115}{12}\}, \{23, \frac{127}{12}, \frac{127}{12}\}, \\
& \{24, \frac{127}{12}, \frac{127}{12}\}, \{25, \frac{133}{12}, \frac{133}{12}\}, \{26, \frac{133}{12}, \frac{133}{12}\}, \{27, \frac{137}{12}, \frac{137}{12}\}, \{28, \frac{137}{12}, \frac{137}{12}\}, \\
& \{29, \frac{149}{12}, \frac{149}{12}\}, \{30, \frac{149}{12}, \frac{149}{12}\}, \{31, \frac{161}{12}, \frac{161}{12}\}, \{32, \frac{817}{60}, \frac{817}{60}\}, \{33, \frac{817}{60}, \frac{817}{60}\}, \\
& \{34, \frac{817}{60}, \frac{817}{60}\}, \{35, \frac{817}{60}, \frac{817}{60}\}, \{36, \frac{817}{60}, \frac{817}{60}\}, \{37, \frac{877}{60}, \frac{877}{60}\}, \{38, \frac{877}{60}, \frac{877}{60}\}, \\
& \{39, \frac{877}{60}, \frac{877}{60}\}, \{40, \frac{877}{60}, \frac{877}{60}\}, \{41, \frac{937}{60}, \frac{937}{60}\}, \{42, \frac{937}{60}, \frac{937}{60}\}, \{43, \frac{997}{60}, \frac{997}{60}\}, \\
& \{44, \frac{997}{60}, \frac{997}{60}\}, \{45, \frac{997}{60}, \frac{997}{60}\}, \{46, \frac{997}{60}, \frac{997}{60}\}, \{47, \frac{1057}{60}, \frac{1057}{60}\}, \{48, \frac{1057}{60}, \frac{1057}{60}\}, \\
& \{49, \frac{1087}{60}, \frac{1087}{60}\}, \{50, \frac{1087}{60}, \frac{1087}{60}\}, \{51, \frac{1087}{60}, \frac{1087}{60}\}, \{52, \frac{1087}{60}, \frac{1087}{60}\}, \\
& \{53, \frac{1147}{60}, \frac{1147}{60}\}, \{54, \frac{1147}{60}, \frac{1147}{60}\}, \{55, \frac{1147}{60}, \frac{1147}{60}\}, \{56, \frac{1147}{60}, \frac{1147}{60}\}, \\
& \{57, \frac{1147}{60}, \frac{1147}{60}\}, \{58, \frac{1147}{60}, \frac{1147}{60}\}, \{59, \frac{1207}{60}, \frac{1207}{60}\}, \{60, \frac{1207}{60}, \frac{1207}{60}\}, \\
& \{61, \frac{1267}{60}, \frac{1267}{60}\}, \{62, \frac{1267}{60}, \frac{1267}{60}\}, \{63, \frac{1267}{60}, \frac{1267}{60}\}, \{64, \frac{1277}{60}, \frac{1277}{60}\}, \\
& \{65, \frac{1277}{60}, \frac{1277}{60}\}, \{66, \frac{1277}{60}, \frac{1277}{60}\}, \{67, \frac{1337}{60}, \frac{1337}{60}\}, \{68, \frac{1337}{60}, \frac{1337}{60}\}, \\
& \{69, \frac{1337}{60}, \frac{1337}{60}\}, \{70, \frac{1337}{60}, \frac{1337}{60}\}, \{71, \frac{1397}{60}, \frac{1397}{60}\}, \{72, \frac{1397}{60}, \frac{1397}{60}\}, \\
& \{73, \frac{1457}{60}, \frac{1457}{60}\}, \{74, \frac{1457}{60}, \frac{1457}{60}\}, \{75, \frac{1457}{60}, \frac{1457}{60}\}, \{76, \frac{1457}{60}, \frac{1457}{60}\}, \\
& \{77, \frac{1457}{60}, \frac{1457}{60}\}, \{78, \frac{1457}{60}, \frac{1457}{60}\}, \{79, \frac{1517}{60}, \frac{1517}{60}\}, \{80, \frac{1517}{60}, \frac{1517}{60}\}, \\
& \{81, \frac{383}{15}, \frac{383}{15}\}, \{82, \frac{383}{15}, \frac{383}{15}\}, \{83, \frac{398}{15}, \frac{398}{15}\}, \{84, \frac{398}{15}, \frac{398}{15}\}, \{85, \frac{398}{15}, \frac{398}{15}\}, \\
& \{86, \frac{398}{15}, \frac{398}{15}\}, \{87, \frac{398}{15}, \frac{398}{15}\}, \{88, \frac{398}{15}, \frac{398}{15}\}, \{89, \frac{413}{15}, \frac{413}{15}\}, \{90, \frac{413}{15}, \frac{413}{15}\}, \\
& \{91, \frac{413}{15}, \frac{413}{15}\}, \{92, \frac{413}{15}, \frac{413}{15}\}, \{93, \frac{413}{15}, \frac{413}{15}\}, \{94, \frac{413}{15}, \frac{413}{15}\}, \{95, \frac{413}{15}, \frac{413}{15}\}, \\
& \{96, \frac{413}{15}, \frac{413}{15}\}, \{97, \frac{428}{15}, \frac{428}{15}\}, \{98, \frac{428}{15}, \frac{428}{15}\}, \{99, \frac{428}{15}, \frac{428}{15}\}, \{100, \frac{428}{15}, \frac{428}{15}\} \}
\end{aligned}$$

```

D2Cache[n_, k_, s_] :=
  Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1/k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]

LinnikSumFast[n_] := D2Cache[n, 1, 2] +
  ( Sum[(-1)^(k + 1) / k D2Cache[n / j, k - 1, 2],
    {j, Floor[n^(1/3)] + 1, n^(1/2)}, {k, 2, Log[2, n]}] ) +
  ( Sum[(-1)^(k + 1) / k (Floor[n / j] - (Floor[n / (j + 1)])) D2Cache[j, k - 1, 2],
    {j, 1, n / Floor[n^(1/2)] - 1}, {k, 2, Log[2, n]}] ) +
  ( Sum[(-1)^(k + 1) / k d2cache[j, k - 1] (Floor[n / j] - 1),
    {j, 2, n^(1/3)}, {k, 2, Log[2, n]}] ) +
  ( Sum[(-1)^(k + 1) / k d2cache[j, m] D2Cache[n / (j s), k - m - 1, 2], {j, 2, n^(1/3)}, {s,
    Floor[Floor[n^(1/3)] / j] + 1, Floor[n / j]^(1/2)}, {k, 2, Log[2, n]}, {m, 1, k - 2}}] ) +
  ( Sum[(-1)^(k + 1) / k ( Floor[n / (j s)] - Floor[n / (j (s + 1))] )
    (Sum[d2cache[j, m] D2Cache[s, k - m - 1, 2], {m, 1, k - 2}], {j, 2, n^(1/3)},
    {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1/2)] - 1}, {k, 2, Log[2, n]}] )

RiePrimeCnt[n_] := Sum[PrimePi[n^(1/j)] / j, {j, 1, Log[2, n]}]
Table[{n, LinnikSumFast[n], RiePrimeCnt[n]}, {n, 1, 100}]

```

$$\begin{aligned}
& \{ \{1, 0, 0\}, \{2, 1, 1\}, \{3, 2, 2\}, \{4, \frac{5}{2}, \frac{5}{2}\}, \{5, \frac{7}{2}, \frac{7}{2}\}, \{6, \frac{7}{2}, \frac{7}{2}\}, \{7, \frac{9}{2}, \frac{9}{2}\}, \\
& \{8, \frac{29}{6}, \frac{29}{6}\}, \{9, \frac{16}{3}, \frac{16}{3}\}, \{10, \frac{16}{3}, \frac{16}{3}\}, \{11, \frac{19}{3}, \frac{19}{3}\}, \{12, \frac{19}{3}, \frac{19}{3}\}, \{13, \frac{22}{3}, \frac{22}{3}\}, \\
& \{14, \frac{22}{3}, \frac{22}{3}\}, \{15, \frac{22}{3}, \frac{22}{3}\}, \{16, \frac{91}{12}, \frac{91}{12}\}, \{17, \frac{103}{12}, \frac{103}{12}\}, \{18, \frac{103}{12}, \frac{103}{12}\}, \\
& \{19, \frac{115}{12}, \frac{115}{12}\}, \{20, \frac{115}{12}, \frac{115}{12}\}, \{21, \frac{115}{12}, \frac{115}{12}\}, \{22, \frac{115}{12}, \frac{115}{12}\}, \{23, \frac{127}{12}, \frac{127}{12}\}, \\
& \{24, \frac{127}{12}, \frac{127}{12}\}, \{25, \frac{133}{12}, \frac{133}{12}\}, \{26, \frac{133}{12}, \frac{133}{12}\}, \{27, \frac{137}{12}, \frac{137}{12}\}, \{28, \frac{137}{12}, \frac{137}{12}\}, \\
& \{29, \frac{149}{12}, \frac{149}{12}\}, \{30, \frac{149}{12}, \frac{149}{12}\}, \{31, \frac{161}{12}, \frac{161}{12}\}, \{32, \frac{817}{60}, \frac{817}{60}\}, \{33, \frac{817}{60}, \frac{817}{60}\}, \\
& \{34, \frac{817}{60}, \frac{817}{60}\}, \{35, \frac{817}{60}, \frac{817}{60}\}, \{36, \frac{817}{60}, \frac{817}{60}\}, \{37, \frac{877}{60}, \frac{877}{60}\}, \{38, \frac{877}{60}, \frac{877}{60}\}, \\
& \{39, \frac{877}{60}, \frac{877}{60}\}, \{40, \frac{877}{60}, \frac{877}{60}\}, \{41, \frac{937}{60}, \frac{937}{60}\}, \{42, \frac{937}{60}, \frac{937}{60}\}, \{43, \frac{997}{60}, \frac{997}{60}\}, \\
& \{44, \frac{997}{60}, \frac{997}{60}\}, \{45, \frac{997}{60}, \frac{997}{60}\}, \{46, \frac{997}{60}, \frac{997}{60}\}, \{47, \frac{1057}{60}, \frac{1057}{60}\}, \{48, \frac{1057}{60}, \frac{1057}{60}\}, \\
& \{49, \frac{1087}{60}, \frac{1087}{60}\}, \{50, \frac{1087}{60}, \frac{1087}{60}\}, \{51, \frac{1087}{60}, \frac{1087}{60}\}, \{52, \frac{1087}{60}, \frac{1087}{60}\}, \\
& \{53, \frac{1147}{60}, \frac{1147}{60}\}, \{54, \frac{1147}{60}, \frac{1147}{60}\}, \{55, \frac{1147}{60}, \frac{1147}{60}\}, \{56, \frac{1147}{60}, \frac{1147}{60}\}, \\
& \{57, \frac{1147}{60}, \frac{1147}{60}\}, \{58, \frac{1147}{60}, \frac{1147}{60}\}, \{59, \frac{1207}{60}, \frac{1207}{60}\}, \{60, \frac{1207}{60}, \frac{1207}{60}\}, \\
& \{61, \frac{1267}{60}, \frac{1267}{60}\}, \{62, \frac{1267}{60}, \frac{1267}{60}\}, \{63, \frac{1267}{60}, \frac{1267}{60}\}, \{64, \frac{1277}{60}, \frac{1277}{60}\}, \\
& \{65, \frac{1277}{60}, \frac{1277}{60}\}, \{66, \frac{1277}{60}, \frac{1277}{60}\}, \{67, \frac{1337}{60}, \frac{1337}{60}\}, \{68, \frac{1337}{60}, \frac{1337}{60}\}, \\
& \{69, \frac{1337}{60}, \frac{1337}{60}\}, \{70, \frac{1337}{60}, \frac{1337}{60}\}, \{71, \frac{1397}{60}, \frac{1397}{60}\}, \{72, \frac{1397}{60}, \frac{1397}{60}\}, \\
& \{73, \frac{1457}{60}, \frac{1457}{60}\}, \{74, \frac{1457}{60}, \frac{1457}{60}\}, \{75, \frac{1457}{60}, \frac{1457}{60}\}, \{76, \frac{1457}{60}, \frac{1457}{60}\}, \\
& \{77, \frac{1457}{60}, \frac{1457}{60}\}, \{78, \frac{1457}{60}, \frac{1457}{60}\}, \{79, \frac{1517}{60}, \frac{1517}{60}\}, \{80, \frac{1517}{60}, \frac{1517}{60}\}, \\
& \{81, \frac{383}{15}, \frac{383}{15}\}, \{82, \frac{383}{15}, \frac{383}{15}\}, \{83, \frac{398}{15}, \frac{398}{15}\}, \{84, \frac{398}{15}, \frac{398}{15}\}, \{85, \frac{398}{15}, \frac{398}{15}\}, \\
& \{86, \frac{398}{15}, \frac{398}{15}\}, \{87, \frac{398}{15}, \frac{398}{15}\}, \{88, \frac{398}{15}, \frac{398}{15}\}, \{89, \frac{413}{15}, \frac{413}{15}\}, \{90, \frac{413}{15}, \frac{413}{15}\}, \\
& \{91, \frac{413}{15}, \frac{413}{15}\}, \{92, \frac{413}{15}, \frac{413}{15}\}, \{93, \frac{413}{15}, \frac{413}{15}\}, \{94, \frac{413}{15}, \frac{413}{15}\}, \{95, \frac{413}{15}, \frac{413}{15}\}, \\
& \{96, \frac{413}{15}, \frac{413}{15}\}, \{97, \frac{428}{15}, \frac{428}{15}\}, \{98, \frac{428}{15}, \frac{428}{15}\}, \{99, \frac{428}{15}, \frac{428}{15}\}, \{100, \frac{428}{15}, \frac{428}{15}\} \}
\end{aligned}$$

```

ClearAll["Global`*"]
d[n_, z_] := Product[Pochhammer[z, p[[2]]] / p[[2]]!, {p, FI[n]}};
FI[n_] := FactorInteger[n]; FI[1] := {}
dCache := dCache = Table[d[n, z], {n, 1, 500}, {z, 0, Log[2, 500]}]
d2[n_, k_] := Sum[(-1)^(k-j) Binomial[k, j] dCache[[n]][[j+1]], {j, 0, k}]
d2Cache := d2Cache = Table[d2[n, z], {n, 1, 500}, {z, 1, Log[2, 500]}]
D2[n_, k_] := D2[n-1, k] + d2Cache[[n]][[k]]; D2[1, k_] := 0
D2Cache := D2Cache = Table[D2[n, z], {n, 1, 500}, {z, 1, Log[2, 500]}]

```

```
D2[100, 2]
```

```
324
```

```
D2Cache[[99]]
```

```
$RecursionLimit::reclim: Recursion depth of 256 exceeded. >>
```

```
$RecursionLimit::reclim: Recursion depth of 256 exceeded. >>
```

```
$RecursionLimit::reclim: Recursion depth of 256 exceeded. >>
```

```
General::stop: Further output of $RecursionLimit::reclim will be suppressed during this calculation. >>
```

```
{98, 276, 312, 178, 51, 7, 0, 0}
```

```
8 - 3
```

```
5
```