

(*

```
$E_{k,a}=\displaystyle\sum_{j=2}^{\lfloor n\rfloor}E_{k-1,a}(\frac{n}{j})-
a\sum_{j=1}^{\lfloor\frac{n}{a}\rfloor}E_{k-1,a}(\frac{n}{ja})$
```

```
$E_{0,k}(n)=1$
```

```
$\Pi(n)=\text{li}(n)-\log\log n-\gamma+\displaystyle\lim_{a\rightarrow 1^+}\sum_{k=1}^{\lfloor\log_a n\rfloor}\frac{(-1)^{k-1}E_{k,a}(n)+1}{k}$
$M(n)=\displaystyle\lim_{a\rightarrow 1^+}\sum_{k=1}^{\lfloor\log_a n\rfloor}(-1)^k(E_{k,a}(n)-aE_{k,a}(\frac{n}{a}))$
```

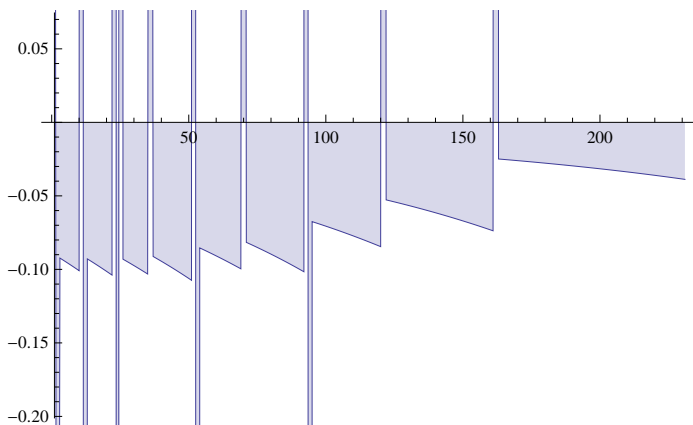
*)

```
ClearAll["Global`*"]
```

```
EE[n_, k_, a_] :=
  EE[n, k, a] = Sum[EE[n / j, k - 1, a], {j, 2, n}] - a Sum[EE[n / (a j), k - 1, a], {j, 1, n / a}];
EE[n_, 0, a_] := 1
lin[n_, b_] := Sum[(-1)^(k + 1) / k E2a[n, k, b], {k, 1, Log[b, n]}]
E2ax[n_, k_, a_, c_] := E2ax[n, k, a, c] = Sum[E2ax[n / j, k - 1, a, c], {j, 2, n}] -
  a^c Sum[E2ax[n / (a j), k - 1, a, c], {j, 1, n / a}]; E2ax[n_, 0, a_, c_] := 1
linx[n_, b_, c_] := Sum[(-1)^(k + 1) / k E2a[n, k, b], {k, 1, Log[2, n]}]
E2ay[n_, k_, a_, c_] := E2ay[n, k, a, c] = Sum[E2ay[n / j, k - 1, a, c], {j, 2, n}] +
  a^c Sum[E2ay[n / (a j), k - 1, a, c], {j, 1, n / a}]; E2ay[n_, 0, a_, c_] := 1
liny[n_, b_, c_] := Sum[(-1)^(k + 1) / k E2a[n, k, b], {k, 1, Log[2, n]}]
$RecursionLimit = 10 000
```

```
10 000
```

```
DiscretePlot[((-1)^(k + 1) EE[10, k, 1.01] + 1) / k, {k, 1, Log[1.01, 10]}]
```

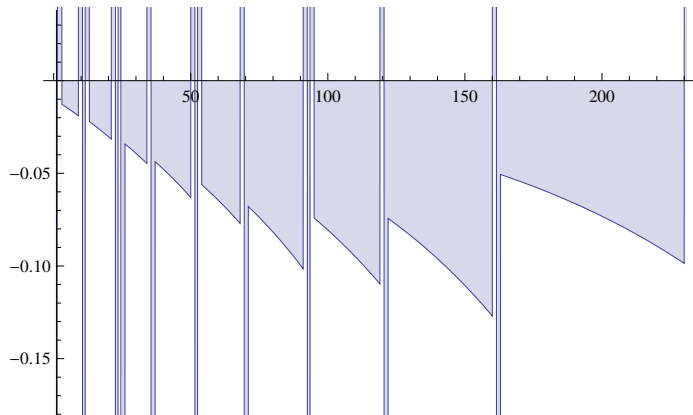


```
P2[x_, a_] := LogIntegral[x] - Log[Log[x]] -
  EulerGamma + Sum[((-1)^(k + 1) EE[x, k, a] + 1) / k, {k, 1, Log[a, x]}]
```

```
DiscretePlot[P2[n, 1.03], {n, 1, 100}]
```

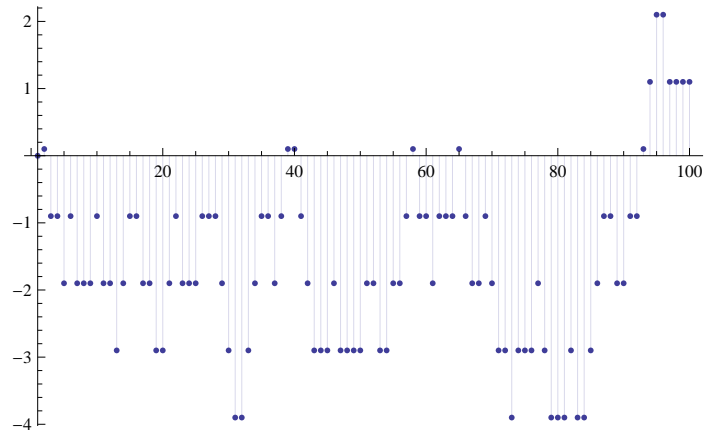
Infinity::indet: Indeterminate expression -EulerGamma + -∞ + ∞ encountered. >>

```
DiscretePlot[(-1)^k (EE[10, k, 1.01] - 1.01 EE[10 / 1.01, k, 1.01]), {k, 1, Log[1.01, 10]}]
```

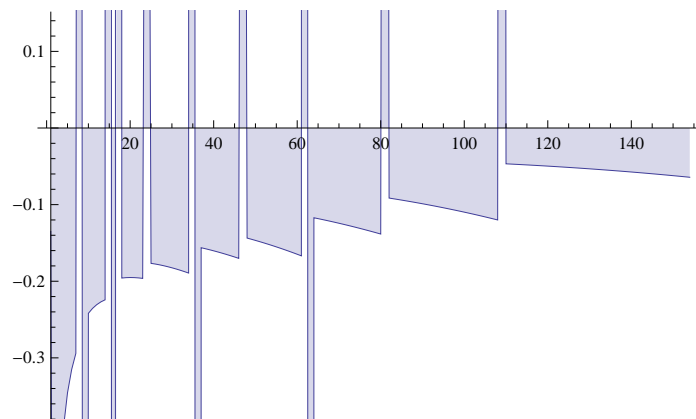


```
M2[x_, a_] := Sum[(-1)^k (EE[x, k, a] - a EE[x / a, k, a]), {k, 1, Log[a, x]}]
```

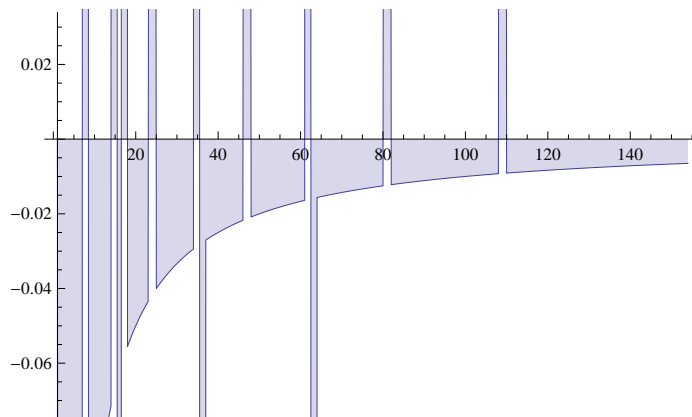
```
DiscretePlot[M2[n, 1.1], {n, 1, 100}]
```



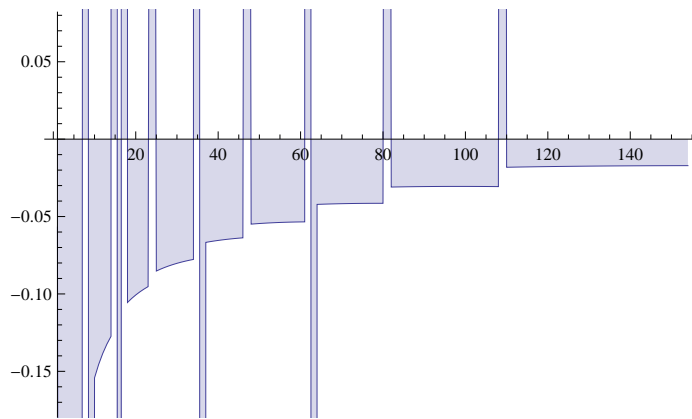
```
DiscretePlot[(-1)^(k+1) / k E2ax[10, k, 1.015, 1], {k, 1, Log[1.015, 10]}]
```



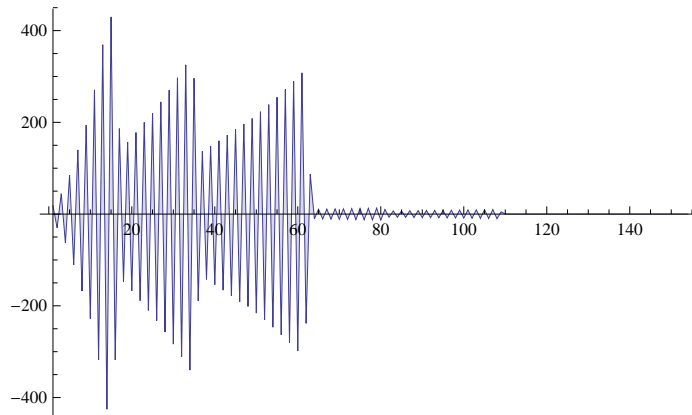
```
DiscretePlot[(-1)^(k+1)/k E2ax[10, k, 1.015, 0], {k, 1, Log[1.015, 10]}]
```



```
DiscretePlot[(-1)^(k+1)/k E2ax[10, k, 1.015, 1 - EulerGamma], {k, 1, Log[1.015, 10]}]
```



```
DiscretePlot[(-1)^(k+1)/k E2ay[10, k, 1.015, 1], {k, 1, Log[1.015, 10]}]
```



```
pp[k_, a_] := (-1)^(k+1)/k E2ax[10, k, 1.015, a]
```

```
pp2[k_, a_] := (-1)^(k+1)/k E2ay[10, k, 1.015, a]
```

```
Table[{1.015^k, pp[k, 1], pp[k, 0], pp[k, 1/2]}, {k, 1, Log[1.015, 10]}] // TableForm
```

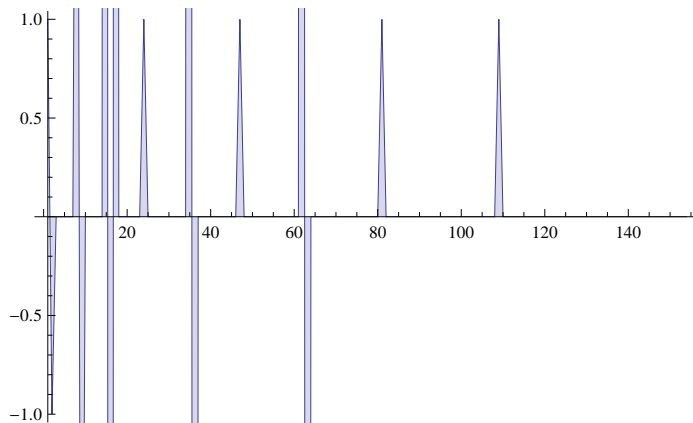
1.015	-0.135	0.	-0.0672488
1.03022	-1.63759	-1.5	-1.56789
1.04568	-0.473558	-0.333333	-0.401872

1.06136	-0.392912	-0.25	-0.319193
1.07728	-0.345651	-0.2	-0.269854
1.09344	-0.315108	-0.166667	-0.237188
1.10984	-0.294143	-0.142857	-0.214051
1.12649	4.79003	4.375	4.57926
1.14339	-4.68973	-4.11111	-4.38947
1.16054	-0.241858	-0.1	-0.164996
1.17795	-0.235432	-0.0909091	-0.156513
1.19562	-0.230571	-0.0833333	-0.14955
1.21355	-0.226927	-0.0769231	-0.143759
1.23176	-0.224252	-0.0714286	-0.13889
1.25023	56.4548	45.2667	50.5542
1.26899	-105.534	-85.0625	-94.7468
1.28802	49.0735	39.9412	44.275
1.30734	-0.195833	-0.0555556	-0.116006
1.32695	-0.195325	-0.0526316	-0.1136
1.34686	-0.195152	-0.05	-0.11149
1.36706	-0.195272	-0.047619	-0.109636
1.38756	-0.195652	-0.0454545	-0.108003
1.40838	-0.196265	-0.0434783	-0.106562
1.4295	1.23242	0.958333	1.09033
1.45095	-0.176658	-0.04	-0.0952362
1.47271	-0.177525	-0.0384615	-0.0941809
1.4948	-0.178548	-0.037037	-0.0932439
1.51722	-0.179717	-0.0357143	-0.092413
1.53998	-0.18102	-0.0344828	-0.0916776
1.56308	-0.182451	-0.0333333	-0.0910287
1.58653	-0.184002	-0.0322581	-0.0904584
1.61032	-0.185666	-0.03125	-0.0899598
1.63448	-0.187438	-0.030303	-0.0895269
1.659	-0.189315	-0.0294118	-0.0891541
1.68388	58.7446	34.9714	45.3287
1.70914	-58.2199	-35.0278	-45.1571
1.73478	-0.156254	-0.027027	-0.0759759
1.7608	-0.157538	-0.0263158	-0.0756363
1.78721	-0.15889	-0.025641	-0.0753362
1.81402	-0.160309	-0.025	-0.0750728
1.84123	-0.161792	-0.0243902	-0.0748436
1.86885	-0.163337	-0.0238095	-0.0746466
1.89688	-0.164944	-0.0232558	-0.0744796
1.92533	-0.166611	-0.0227273	-0.0743409
1.95421	-0.168336	-0.0222222	-0.0742288
1.98353	-0.17012	-0.0217391	-0.0741418
2.01328	1.84132	0.978723	1.34482
2.04348	-0.143658	-0.0208333	-0.0634356
2.07413	-0.145157	-0.0204082	-0.0633369
2.10524	-0.146705	-0.02	-0.0632578
2.13682	-0.148301	-0.0196078	-0.0631975
2.16887	-0.149945	-0.0192308	-0.0631549
2.20141	-0.151636	-0.0188679	-0.0631292
2.23443	-0.153373	-0.0185185	-0.0631197
2.26794	-0.155158	-0.0181818	-0.0631257
2.30196	-0.156989	-0.0178571	-0.0631465
2.33649	-0.158867	-0.0175439	-0.0631815
2.37154	-0.160792	-0.0172414	-0.0632302
2.40711	-0.162763	-0.0169492	-0.0632919

2.44322	-0.164782	-0.0166667	-0.0633663
2.47987	-0.166846	-0.0163934	-0.0634528
2.51707	79.1186	31.4839	49.912
2.55482	-76.992	-31.0159	-48.8666
2.59314	-0.117163	-0.015625	-0.0490478
2.63204	-0.118287	-0.0153846	-0.0490243
2.67152	-0.119439	-0.0151515	-0.0490096
2.7116	-0.120617	-0.0149254	-0.0490034
2.75227	-0.121822	-0.0147059	-0.0490054
2.79355	-0.123054	-0.0144928	-0.0490154
2.83546	-0.124313	-0.0142857	-0.049033
2.87799	-0.125599	-0.0140845	-0.0490581
2.92116	-0.126911	-0.0138889	-0.0490903
2.96498	-0.128251	-0.0136986	-0.0491295
3.00945	-0.129618	-0.0135135	-0.0491754
3.05459	-0.131011	-0.0133333	-0.049228
3.10041	-0.132433	-0.0131579	-0.0492869
3.14692	-0.133881	-0.012987	-0.049352
3.19412	-0.135358	-0.0128205	-0.0494232
3.24203	-0.136862	-0.0126582	-0.0495003
3.29066	-0.138394	-0.0125	-0.0495832
3.34002	3.20007	0.987654	1.7779
3.39012	-0.0914433	-0.0121951	-0.0361098
3.44097	-0.0923094	-0.0120482	-0.0361071
3.49259	-0.0931931	-0.0119048	-0.0361088
3.54498	-0.0940945	-0.0117647	-0.0361149
3.59815	-0.0950137	-0.0116279	-0.0361253
3.65213	-0.0959507	-0.0114943	-0.0361398
3.70691	-0.0969058	-0.0113636	-0.0361583
3.76251	-0.097879	-0.011236	-0.0361808
3.81895	-0.0988704	-0.0111111	-0.0362072
3.87623	-0.0998802	-0.010989	-0.0362374
3.93438	-0.100908	-0.0108696	-0.0362712
3.99339	-0.101955	-0.0107527	-0.0363087
4.05329	-0.103021	-0.0106383	-0.0363497
4.11409	-0.104106	-0.0105263	-0.0363942
4.1758	-0.105209	-0.0104167	-0.036442
4.23844	-0.106332	-0.0103093	-0.0364932
4.30202	-0.107475	-0.0102041	-0.0365477
4.36655	-0.108637	-0.010101	-0.0366054
4.43205	-0.109819	-0.01	-0.0366663
4.49853	-0.111021	-0.00990099	-0.0367303
4.566	-0.112243	-0.00980392	-0.0367973
4.63449	-0.113485	-0.00970874	-0.0368674
4.70401	-0.114748	-0.00961538	-0.0369404
4.77457	-0.116032	-0.00952381	-0.0370163
4.84619	-0.117337	-0.00943396	-0.0370951
4.91888	-0.118664	-0.00934579	-0.0371767
4.99267	-0.120012	-0.00925926	-0.0372611
5.06756	4.94618	0.990826	2.21378
5.14357	-0.0467597	-0.00909091	-0.0206177
5.22072	-0.0470335	-0.00900901	-0.0205846
5.29903	-0.0473128	-0.00892857	-0.0205532
5.37852	-0.0475975	-0.00884956	-0.0205236
5.4592	-0.0478877	-0.00877193	-0.0204955
5.54109	-0.0481834	-0.00869565	-0.0204691

5.6242	-0.0484845	-0.00862069	-0.0204443
5.70856	-0.0487912	-0.00854701	-0.020421
5.79419	-0.0491033	-0.00847458	-0.0203993
5.88111	-0.0494211	-0.00840336	-0.020379
5.96932	-0.0497444	-0.00833333	-0.0203602
6.05886	-0.0500732	-0.00826446	-0.0203428
6.14975	-0.0504078	-0.00819672	-0.0203268
6.24199	-0.0507479	-0.00813008	-0.0203122
6.33562	-0.0510937	-0.00806452	-0.0202989
6.43066	-0.0514452	-0.008	-0.020287
6.52712	-0.0518025	-0.00793651	-0.0202764
6.62502	-0.0521655	-0.00787402	-0.020267
6.7244	-0.0525344	-0.0078125	-0.0202589
6.82526	-0.052909	-0.00775194	-0.0202521
6.92764	-0.0532896	-0.00769231	-0.0202465
7.03156	-0.053676	-0.00763359	-0.020242
7.13703	-0.0540684	-0.00757576	-0.0202388
7.24409	-0.0544668	-0.0075188	-0.0202367
7.35275	-0.0548713	-0.00746269	-0.0202358
7.46304	-0.0552818	-0.00740741	-0.020236
7.57498	-0.0556984	-0.00735294	-0.0202373
7.68861	-0.0561212	-0.00729927	-0.0202397
7.80394	-0.0565503	-0.00724638	-0.0202431
7.921	-0.0569856	-0.00719424	-0.0202477
8.03981	-0.0574272	-0.00714286	-0.0202533
8.16041	-0.0578752	-0.0070922	-0.0202599
8.28282	-0.0583297	-0.00704225	-0.0202675
8.40706	-0.0587906	-0.00699301	-0.0202762
8.53316	-0.0592581	-0.00694444	-0.0202858
8.66116	-0.0597321	-0.00689655	-0.0202964
8.79108	-0.0602129	-0.00684932	-0.0203081
8.92294	-0.0607003	-0.00680272	-0.0203206
9.05679	-0.0611945	-0.00675676	-0.0203341
9.19264	-0.0616956	-0.00671141	-0.0203486
9.33053	-0.0622035	-0.00666667	-0.0203639
9.47049	-0.0627185	-0.00662252	-0.0203802
9.61255	-0.0632404	-0.00657895	-0.0203974
9.75673	-0.0637695	-0.00653595	-0.0204155
9.90308	-0.0643057	-0.00649351	-0.0204345

`DiscretePlot[((-1)^(k+1) E2ax[10, k, 1.015, 0] + 1) / k, {k, 1, Log[1.015, 10]}]`



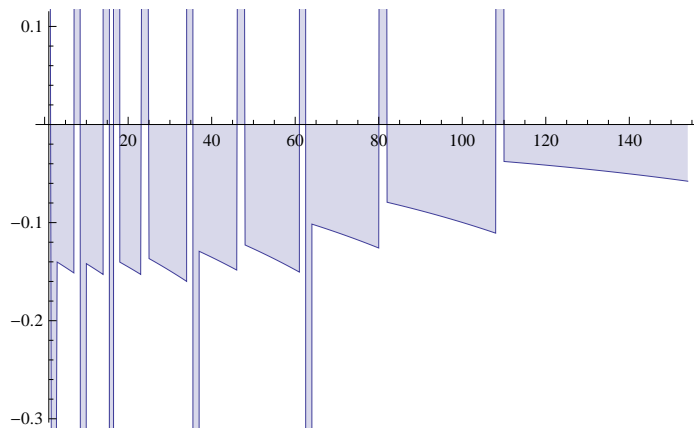
```
Table[{10 / 1.015^k, ((-1)^(k + 1) E2ax[10, k, 1.015, 0] + 1) / k},
      {k, 1, Log[1.015, 10]}] // TableForm
```

9.85222	1.
9.70662	-1.
9.56317	0.
9.42184	0.
9.2826	0.
9.14542	0.
9.01027	0.
8.87711	4.5
8.74592	-4.
8.61667	0.
8.48933	0.
8.36387	0.
8.24027	0.
8.11849	0.
7.99852	45.3333
7.88031	-85.
7.76385	40.
7.64912	0.
7.53607	0.
7.4247	0.
7.31498	0.
7.20688	0.
7.10037	0.
6.99544	1.
6.89206	0.
6.79021	0.
6.68986	0.
6.59099	0.
6.49359	0.
6.39762	0.
6.30308	0.
6.20993	0.
6.11816	0.
6.02774	0.
5.93866	35.
5.8509	-35.
5.76443	0.
5.67924	0.
5.59531	0.
5.51262	0.
5.43116	0.
5.35089	0.
5.27182	0.
5.19391	0.
5.11715	0.
5.04153	0.
4.96702	1.
4.89362	0.
4.8213	0.
4.75005	0.
4.67985	0.
4.61069	0.
4.54255	0.
4.47542	0.

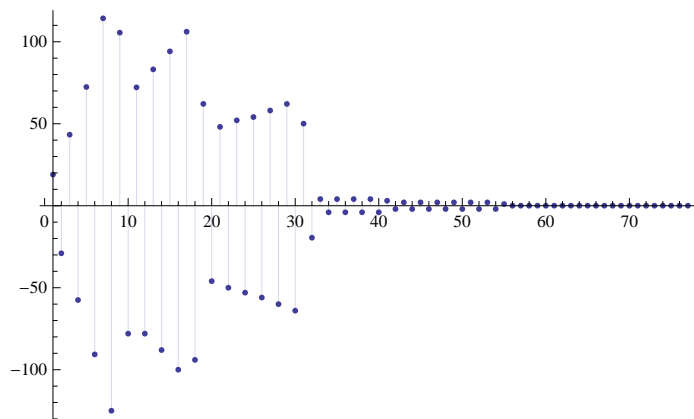
4.40928	0.
4.34412	0.
4.27992	0.
4.21667	0.
4.15435	0.
4.09296	0.
4.03247	0.
3.97288	31.5
3.91417	-31.
3.85632	0.
3.79933	0.
3.74318	0.
3.68787	0.
3.63337	0.
3.57967	0.
3.52677	0.
3.47465	0.
3.4233	0.
3.37271	0.
3.32287	0.
3.27376	0.
3.22538	0.
3.17771	0.
3.13075	0.
3.08448	0.
3.0389	0.
2.99399	1.
2.94975	0.
2.90615	0.
2.86321	0.
2.82089	0.
2.7792	0.
2.73813	0.
2.69767	0.
2.6578	0.
2.61852	0.
2.57982	0.
2.5417	0.
2.50414	0.
2.46713	0.
2.43067	0.
2.39475	0.
2.35936	0.
2.32449	0.
2.29014	0.
2.25629	0.
2.22295	0.
2.1901	0.
2.15773	0.
2.12585	0.
2.09443	0.
2.06348	0.
2.03298	0.
2.00294	0.
1.97334	1.
1.94417	0.

1.91544	0.
1.88714	0.
1.85925	0.
1.83177	0.
1.8047	0.
1.77803	0.
1.75175	0.
1.72587	0.
1.70036	0.
1.67523	0.
1.65047	0.
1.62608	0.
1.60205	0.
1.57838	0.
1.55505	0.
1.53207	0.
1.50943	0.
1.48712	0.
1.46514	0.
1.44349	0.
1.42216	0.
1.40114	0.
1.38044	0.
1.36004	0.
1.33994	0.
1.32013	0.
1.30063	0.
1.2814	0.
1.26247	0.
1.24381	0.
1.22543	0.
1.20732	0.
1.18948	0.
1.1719	0.
1.15458	0.
1.13752	0.
1.12071	0.
1.10414	0.
1.08783	0.
1.07175	0.
1.05591	0.
1.04031	0.
1.02493	0.
1.00979	0.

```
DiscretePlot[((-1)^(k+1) E2ax[10, k, 1.015, 1] + 1) / k, {k, 1, Log[1.015, 10]}]
```



```
DiscretePlot[((-1)^(k+1) E2ay[10, k, 1.03, 0] + 1) / k, {k, 1, Log[1.03, 10]}]
```



```
Table[{10 / 1.03^k, ((-1)^(k+1) E2ay[10, k, 1.03, 0] + 1) / k},  
      {k, 1, Log[1.03, 10]}] // TableForm
```

9.70874	19.
9.42596	-29.
9.15142	43.3333
8.88487	-57.5
8.62609	72.4
8.37484	-90.6667
8.13092	114.286
7.89409	-125.
7.66417	105.556
7.44094	-78.
7.22421	72.1818
7.0138	-78.
6.80951	83.1538
6.61118	-88.
6.41862	94.1333
6.23167	-100.
6.05016	106.118
5.87395	-94.
5.70286	62.1053
5.53676	-46.
5.37549	48.0952

5.21893	-50.
5.06692	52.087
4.91934	-53.
4.77606	54.08
4.63695	-56.
4.50189	58.0741
4.37077	-60.
4.24346	62.069
4.11987	-64.
3.99987	50.0645
3.88337	-19.5
3.77026	4.06061
3.66045	-4.
3.55383	4.05714
3.45032	-4.
3.34983	4.05405
3.25226	-4.
3.15754	4.05128
3.06557	-4.
2.97628	3.04878
2.88959	-2.
2.80543	2.04651
2.72372	-2.
2.64439	2.04444
2.56737	-2.
2.49259	2.04255
2.41999	-2.
2.3495	2.04082
2.28107	-2.
2.21463	2.03922
2.15013	-2.
2.0875	2.03774
2.0267	-2.
1.96767	1.03636
1.91036	0.
1.85472	0.0350877
1.8007	0.
1.74825	0.0338983
1.69733	0.
1.64789	0.0327869
1.5999	0.
1.5533	0.031746
1.50806	0.
1.46413	0.0307692
1.42149	0.
1.38009	0.0298507
1.33989	0.
1.30086	0.0289855
1.26297	0.
1.22619	0.028169
1.19047	0.
1.1558	0.0273973
1.12214	0.
1.08945	0.0266667
1.05772	0.
1.02691	0.025974