```
d2[n_, k_] := Sum[d2[j, k - 1] d2[n / j, 1], {j, Divisors[n]}];
d2[n_, 1] := 1; d2[1, 1] := 0; d2[n_, 0] := 0; d2[1, 0] := 1

d[n_, k_] := Sum[d[j, k - 1] d[n / j, 1], {j, Divisors[n]}];
d[n_, 1] := 1; d[n_, 0] := 0; d[1, 0] := 1

K[n_, 0] := If[n == 1, 1, 0]
K[n_, 1] := If[n == 1, 0, FullSimplify[MangoldtLambda[n] / Log[n]]]
K[n_, k_] := Sum[K[j, k - 1] K[n / j, 1], {j, Divisors[n]}]
K1[n_, k_] := K1[n, k] = Sum[Binomial[k, j] K[n, k - j], {j, 0, k}]

sc[f_, k_, t_] := SeriesCoefficient[Series[f[x], {x, 0, Floor[t]}], k]

q2[b_, f_, n_, 0] := q2[b, f, n, 0] = 1
q2[b_, f_, n_, 1] :=
 q2[b, f, n, 1] = Sum[ b[n, k] sc[f, k, N[Floor[Log[2, n]]], {k, 0, N[Log[2, n]]}]
q2[b_, f_, n_, k_] := q2[b, f, n, k] =
  Sum[q2[b, f, n / j, k - 1] q2[b, f, j, 1], {j, Divisors[n]}]

q1[b_, f_, n_, 0] := q1[b, f, n, 0] = 1
q1[b_, f_, n_, 1] := q1[b, f, n, 1] = Sum[ b[n, k] sc[f, k, 20], {k, 0, 20}]
q1[b_, f_, n_, k_] :=
 q1[b, f, n, k] = Sum[q1[b, f, n / j, k - 1] q1[b, f, j, 1], {j, Divisors[n]}]


XLX[x_] := x / Log[-x + 1]
XLX2[x_] := x / Log[x + 1]
Mcos[x_] := -Cos[x]
Msin[x_] := -Sin[x]
Expd[x_] := E^x
Lg1[x_] := Log[x + 1]
Lg2[x_] := Log[1 - x]
lg1[n_, k_] := q2[d2, Lg1, n, k]
lg2[n_, k_] := q2[d2, Lg2, n, k]
lg2d[n_, k_] := q1[d, Lg2, n, k]
xlx[n_, k_] := q2[d2, XLX, n, k]
xlx2[n_, k_] := q2[K, XLX2, n, k]
expd[n_, k_] := q1[d, Expd, n, k]
expd2[n_, k_] := q2[d2, Expd, n, k]
expk[n_, k_] := q2[K, Expd, n, k]
sind[n_, k_] := q1[K1, Sin, n, k]
cosd[n_, k_] := q1[K1, Cos, n, k]
mcosd[n_, k_] := q1[K1, Mcos, n, k]
msind[n_, k_] := q1[K1, Msin, n, k]
tand[n_, k_] := q2[d2, Tan, n, k]
asinsind[n_, k_] := q2[sind, ArcSin, n, k]
atantand[n_, k_] := q2[tand, ArcTan, n, k]
```

```
Table[{n, xlx2[n, 1]}, {n, 1, 100}] // TableForm
```

1	1
2	$\frac{1}{2}$

3	$\frac{1}{2}$
4	$\frac{1}{6}$
5	$\frac{1}{2}$
6	$-\frac{1}{6}$
7	$\frac{1}{2}$
8	$\frac{1}{8}$
9	$\frac{1}{6}$
10	$-\frac{1}{6}$
11	$\frac{1}{2}$
12	$\frac{1}{24}$
13	$\frac{1}{2}$
14	$-\frac{1}{6}$
15	$-\frac{1}{6}$
16	$\frac{61}{720}$
17	$\frac{1}{2}$
18	$\frac{1}{24}$
19	$\frac{1}{2}$
20	$\frac{1}{24}$
21	$-\frac{1}{6}$
22	$-\frac{1}{6}$
23	$\frac{1}{2}$
24	$-\frac{13}{360}$
25	$\frac{1}{6}$
26	$-\frac{1}{6}$
27	$\frac{1}{8}$
28	$\frac{1}{24}$
29	$\frac{1}{2}$
30	$\frac{1}{4}$
31	$\frac{1}{2}$
32	$\frac{5}{72}$
33	$-\frac{1}{6}$
34	$-\frac{1}{6}$
35	$-\frac{1}{6}$
36	$-\frac{3}{40}$
37	$\frac{1}{2}$
38	$-\frac{1}{6}$
39	$-\frac{1}{6}$
40	$-\frac{13}{360}$
41	$\frac{1}{2}$

42 $-\frac{1}{4}$

43 $\frac{1}{2}$

44 $\frac{1}{24}$

45 $\frac{1}{24}$

46 $-\frac{1}{6}$

47 $\frac{1}{2}$

48 $\frac{1}{120}$

49 $\frac{1}{6}$

50 $\frac{1}{24}$

51 $-\frac{1}{6}$

52 $\frac{1}{24}$

53 $\frac{1}{2}$

54 $-\frac{13}{360}$

55 $-\frac{1}{6}$

56 $-\frac{13}{360}$

57 $-\frac{1}{6}$

58 $-\frac{1}{6}$

59 $\frac{1}{2}$

60 $-\frac{23}{120}$

61 $\frac{1}{2}$

62 $-\frac{1}{6}$

63 $\frac{1}{24}$

64 $\frac{3379}{60\,480}$

65 $-\frac{1}{6}$

66 $\frac{1}{4}$

67 $\frac{1}{2}$

68 $\frac{1}{24}$

69 $-\frac{1}{6}$

70 $\frac{1}{4}$

71 $\frac{1}{2}$

72 $\frac{19}{360}$

73 $\frac{1}{2}$

74 $-\frac{1}{6}$

75 $\frac{1}{24}$

76 $\frac{1}{24}$

77 $-\frac{1}{6}$

78 $\frac{1}{4}$

79 $\frac{1}{2}$

80 $\frac{1}{120}$

81	$\frac{61}{720}$
82	$-\frac{1}{6}$
83	$\frac{1}{2}$
84	$-\frac{23}{120}$
85	$-\frac{1}{6}$
86	$-\frac{1}{6}$
87	$-\frac{1}{6}$
88	$-\frac{13}{360}$
89	$\frac{1}{2}$
90	$-\frac{23}{120}$
91	$-\frac{1}{6}$
92	$\frac{1}{24}$
93	$-\frac{1}{6}$
94	$-\frac{1}{6}$
95	$-\frac{1}{6}$
96	$-\frac{121}{10\,080}$
97	$\frac{1}{2}$
98	$\frac{1}{24}$
99	$\frac{1}{24}$
100	$-\frac{3}{40}$

Table[{n, mcosd[n, 1], cosd[n, 1], msind[n, 1], sind[n, 1]}, {n, 1, 10}] // TableForm

1	-1	$\frac{1\,314\,502\,564\,969\,066\,301}{2\,432\,902\,008\,176\,640\,000}$	0	$\frac{102\,360\,822\,438\,075\,317}{121\,645\,100\,408\,832\,000}$
2	0	$-\frac{102\,360\,822\,438\,075\,317}{121\,645\,100\,408\,832\,000}$	-1	$\frac{691\,843\,455\,246\,877}{1\,280\,474\,741\,145\,600}$
3	0	$-\frac{102\,360\,822\,438\,075\,317}{121\,645\,100\,408\,832\,000}$	-1	$\frac{691\,843\,455\,246\,877}{1\,280\,474\,741\,145\,600}$
4	$\frac{1}{2}$	$-\frac{21\,010\,743\,835\,816\,079}{30\,411\,275\,102\,208\,000}$	$-\frac{1}{2}$	$-\frac{275\,456\,347\,290\,391}{1\,829\,249\,630\,208\,000}$
5	0	$-\frac{102\,360\,822\,438\,075\,317}{121\,645\,100\,408\,832\,000}$	-1	$\frac{691\,843\,455\,246\,877}{1\,280\,474\,741\,145\,600}$
6	1	$-\frac{691\,843\,455\,246\,877}{1\,280\,474\,741\,145\,600}$	0	$-\frac{23\,023\,126\,954\,133}{27\,360\,571\,392\,000}$
7	0	$-\frac{102\,360\,822\,438\,075\,317}{121\,645\,100\,408\,832\,000}$	-1	$\frac{691\,843\,455\,246\,877}{1\,280\,474\,741\,145\,600}$
8	$\frac{1}{2}$	$-\frac{23\,041\,246\,706\,418\,097}{56\,143\,892\,496\,384\,000}$	$-\frac{1}{6}$	$-\frac{6\,351\,508\,922\,783\,491}{19\,207\,121\,117\,184\,000}$
9	$\frac{1}{2}$	$-\frac{21\,010\,743\,835\,816\,079}{30\,411\,275\,102\,208\,000}$	$-\frac{1}{2}$	$-\frac{275\,456\,347\,290\,391}{1\,829\,249\,630\,208\,000}$
10	1	$-\frac{691\,843\,455\,246\,877}{1\,280\,474\,741\,145\,600}$	0	$-\frac{23\,023\,126\,954\,133}{27\,360\,571\,392\,000}$

Table[{n, lg1[n, 1], lg2[n, 1]}, {n, 1, 100}] // TableForm

1	0	0
2	1	-1
3	1	-1
4	$\frac{1}{2}$	$-\frac{3}{2}$
5	1	-1
6	0	-2
7	1	-1
8	$\frac{1}{3}$	$-\frac{7}{3}$

9	$\frac{1}{2}$	$-\frac{3}{2}$
10	0	−2
11	1	−1
12	0	−4
13	1	−1
14	0	−2
15	0	−2
16	$\frac{1}{4}$	$-\frac{15}{4}$
17	1	−1
18	0	−4
19	1	−1
20	0	−4
21	0	−2
22	0	−2
23	1	−1
24	0	−8
25	$\frac{1}{2}$	$-\frac{3}{2}$
26	0	−2
27	$\frac{1}{3}$	$-\frac{7}{3}$
28	0	−4
29	1	−1
30	0	−6
31	1	−1
32	$\frac{1}{5}$	$-\frac{31}{5}$
33	0	−2
34	0	−2
35	0	−2
36	0	−10
37	1	−1
38	0	−2
39	0	−2
40	0	−8
41	1	−1
42	0	−6
43	1	−1
44	0	−4
45	0	−4
46	0	−2
47	1	−1
48	0	−16
49	$\frac{1}{2}$	$-\frac{3}{2}$
50	0	−4
51	0	−2
52	0	−4
53	1	−1
54	0	−8
55	0	−2
56	0	−8
57	0	−2
58	0	−2
59	1	−1
60	0	−16
61	1	−1

62	0	-2
63	0	-4
64	$\frac{1}{6}$	$-\frac{21}{2}$
65	0	-2
66	0	-6
67	1	-1
68	0	-4
69	0	-2
70	0	-6
71	1	-1
72	0	-24
73	1	-1
74	0	-2
75	0	-4
76	0	-4
77	0	-2
78	0	-6
79	1	-1
80	0	-16
81	$\frac{1}{4}$	$-\frac{15}{4}$
82	0	-2
83	1	-1
84	0	-16
85	0	-2
86	0	-2
87	0	-2
88	0	-8
89	1	-1
90	0	-16
91	0	-2
92	0	-4
93	0	-2
94	0	-2
95	0	-2
96	0	-32
97	1	-1
98	0	-4
99	0	-4
100	0	-10