```
chi[j_] := ((-1)^(j - 1) (2 (j - 1))!) / ((1 - 2 (j - 1)) ((j - 1)!)^2 (4^(j - 1)))
chi2[j_] := Binomial[1 / 2, j - 1]
```

```
Table[{chi[j], chi2[j]}, {j, 1, 10}]
```

$$\left\{\{1, 1\}, \left\{\frac{1}{2}, \frac{1}{2}\right\}, \left\{-\frac{1}{8}, -\frac{1}{8}\right\}, \left\{\frac{1}{16}, \frac{1}{16}\right\}, \left\{-\frac{5}{128}, -\frac{5}{128}\right\}, \left\{\frac{7}{256}, \frac{7}{256}\right\}, \left\{-\frac{21}{1024}, -\frac{21}{1024}\right\}, \left\{\frac{33}{2048}, \frac{33}{2048}\right\}, \left\{-\frac{429}{32\,768}, -\frac{429}{32\,768}\right\}, \left\{\frac{715}{65\,536}, \frac{715}{65\,536}\right\}\right\}$$

```
Sum[Binomial[1 / 2, j - 1], {j, 1, Infinity}]
```

$$\sqrt{2}$$

```
Clear[pp, palt]
pp[n_, j_, k_, z_] := pp[n, j, k] =
  If[n < j, 0, Binomial[1 / 2, j - 1] ((z + 1) / k - 1) (1 + pp[n / j, 2, k + 1, z]) + pp[n, j + 1, k, z]]
S[n_, z_] := 1 + pp[n, 2, 1, z]
palt[n_, z_, y_] := palt[n, z, y] = If[n < y, 1,
   Sum[Binomial[z, k] Binomial[1 / 2, y - 1]^k palt[n / y^k, z - k, y + 1], {k, 0, Log[y, n]}]]
Sv2[n_, z_] := palt[n, z, 2]
SRoots[n_] := If[(c = Exponent[f = S[n, z], z]) == 0, {},
  If[c == 1, List@NRoots[f == 0, z][[2]], List @@ NRoots[f == 0, z][[All, 2]]]]
```

```
Expand@(1 + pp[100, 2, 1, z])
```

$$1 + \frac{130\,856\,389\,339\,316\,386\,294\,377\,686\,430\,089\,439\,999\,946\,485\,663\,219\,916\,659\,871\, z}{376\,626\,104\,123\,200\,845\,830\,147\,365\,392\,459\,984\,966\,141\,326\,667\,842\,070\,773\,760} + \frac{205\,986\,324\,639\,431\,802\,057\,210\,443\,617\, z^2}{3\,565\,267\,313\,141\,895\,191\,709\,477\,765\,120} + \frac{15\,706\,379\,949\,301\, z^3}{1\,688\,849\,860\,263\,936} - \frac{28\,877\, z^4}{75\,497\,472} + \frac{113\, z^5}{491\,520} - \frac{z^6}{92\,160}$$

```
N@(1 + pp[1000, 2, 1, 2])
```

```
2.
```

```
N@Log[2^(1 / 2)]
```

```
0.346574
```

```
$RecursionLimit = 10 000
```

```
10 000
```

```
Table[D[Expand@dp[n, z], z] /. z → 0, {n, 1, 30}]
```

$$\left\{0, \frac{1}{2}, -\frac{1}{8}, -\frac{1}{16}, -\frac{5}{128}, \frac{23}{256}, -\frac{21}{1024}, \frac{163}{6144}, -\frac{685}{32\,768}, \frac{1995}{65\,536}, -\frac{2431}{262\,144}, -\frac{15\,257}{524\,288}, -\frac{29\,393}{4\,194\,304}, \frac{138\,019}{8\,388\,608}, -\frac{349\,565}{33\,554\,432}, -\frac{337\,439}{67\,108\,864}, -\frac{9\,694\,845}{2\,147\,483\,648}, \frac{94\,028\,275}{4\,294\,967\,296}, -\frac{64\,822\,395}{17\,179\,869\,184}, -\frac{319\,681\,525}{34\,359\,738\,368}, -\frac{1\,588\,274\,667}{274\,877\,906\,944}, \frac{4\,190\,118\,361}{549\,755\,813\,888}, -\frac{6\,116\,566\,755}{2\,199\,023\,255\,552}, \frac{59\,590\,124\,679}{4\,398\,046\,511\,104}, -\frac{225\,216\,898\,025}{70\,368\,744\,177\,664}, \frac{815\,608\,746\,719}{140\,737\,488\,355\,328}, -\frac{7\,509\,782\,883\,841}{1\,688\,849\,860\,263\,936}, -\frac{5\,523\,270\,154\,941}{1\,125\,899\,906\,842\,624}, -\frac{17\,383\,387\,729\,001}{9\,007\,199\,254\,740\,992}, \frac{214\,593\,265\,675\,675}{18\,014\,398\,509\,481\,984}\right\}$$

```
proots[16]
```

$$\{-2.0198 - 2.20486\, i, -2.0198 + 2.20486\, i, 2.5198 - 6.0497\, i, 2.5198 + 6.0497\, i\}$$

```
Expand@pp[50, 2, 1, z] + 1
```

$$1+\frac{68\,494\,658\,516\,445\,813\,984\,128\,836\,249\,z}{198\,070\,406\,285\,660\,843\,983\,859\,875\,840}+\frac{8\,951\,771\,050\,407\,z^2}{140\,737\,488\,355\,328}+\frac{44\,819\,z^3}{12\,582\,912}+\frac{7\,z^4}{4096}-\frac{z^5}{15\,360}$$

```
Expand@palt[50, z, 2]
```

$$1+\frac{68\,494\,658\,516\,445\,813\,984\,128\,836\,249\,z}{198\,070\,406\,285\,660\,843\,983\,859\,875\,840}+\frac{8\,951\,771\,050\,407\,z^2}{140\,737\,488\,355\,328}+\frac{44\,819\,z^3}{12\,582\,912}+\frac{7\,z^4}{4096}-\frac{z^5}{15\,360}$$