```
d2[n_, k_] := d2[n, k] = Sum[d2[j, k - 1] d2[n / j, 1], {j, Divisors[n]}];
d2[n_, 1] := 1; d2[1, 1] := 0; d2[n_, 0] := 0; d2[1, 0] := 1
D2[n_, k_] := D2[n, k] = Sum[d2[j, k], {j, 2, n}]; D2[n_, 0] := 1
d[n_, z_] := d[n, z] = Sum[FactorialPower[z, a] / a! d2[n, a], {a, 0, Log[2, n]}]
FI[n_] := FactorInteger[n]; FI[1] := {}
DD[n_, k_] := DD[n, k] = Sum[d[j, k], {j, 1, n}]; DD[n_, 0] := 1


DT[n_, k_, s_] := Sum[ (Sum[d[j, a] / (a!), {a, 1, s}])
    (Sum[ BernoulliB[b] / (b!) DD[Floor[n / j], b + k - 1], {b, 0, s}]), {j, 1, n}]

N[DT[100, 1, 12]]

100.

DTT[n_, k_, s_] := Sum[ (Sum[DD[Floor[n / j], a] / (a!), {a, 1, s}])
    (Sum[ BernoulliB[b] / (b!) d[j, b + k - 1], {b, 0, s}]), {j, 1, n}]

DTT2[n_, k_, s_] := Sum[ (Sum[DD[Floor[n / j], a + k - 1] / (a!), {a, 1, s}])
    (Sum[ BernoulliB[b] / (b!) d[j, b], {b, 0, s}]), {j, 1, n}]


N[DTT[100, 1, 12]]

100.

N[DTT2[100, 1, 12]]

100.


DT2[n_, k_] := Sum[ (Sum[d2[j, a] / (a!), {a, 1, Log[2, n]}])
    (Sum[ BernoulliB[b] / (b!) D2[Floor[n / j], b + k - 1], {b, 0, Log[2, n]}]), {j, 2, n}]

dt2[n_, k_] := Sum[ (Sum[d2[j, a] / (a!), {a, 1, Log[2, n]}])
    (Sum[ BernoulliB[b] / (b!) d2[n / j, b + k - 1], {b, 0, Log[2, n]}]), {j, Divisors[n]}]


N[dt2[10, 1]]

1.

N[DT2[120, 1]]

119.

ff[j_, s_] := (Sum[d[j, a] / (a!), {a, 1, s}])
```

```
N[Table[{n, ff[n, 20], Log[ff[n, 20]]}, {n, 1, 20}] // TableForm]
```

```
1.     1.71828    0.541325
2.     2.71828    1.
3.     2.71828    1.
4.     4.07742    1.40547
5.     2.71828    1.
6.     5.43656    1.69315
7.     2.71828    1.
8.     5.88961    1.77319
9.     4.07742    1.40547
10.    5.43656    1.69315
11.    2.71828    1.
12.    9.51399    2.25276
13.    2.71828    1.
14.    5.43656    1.69315
15.    5.43656    1.69315
16.    8.26811    2.11241
17.    2.71828    1.
18.    9.51399    2.25276
19.    2.71828    1.
20.    9.51399    2.25276
```

```
d[2, 1]
```

1

```
N[E^2]
```

7.38906

```
N[2^(1 / 2)]
```

1.41421

```
ff2[j_, s_] := (Sum[BernoulliB[a] d[j, a] / (a!), {a, 0, s}])
```

```
N[Table[{n, ff2[n, 30]}, {n, 1, 20}] // TableForm]
```

1.	0.581977
2.	−0.338697
3.	−0.338697
4.	−0.263223
5.	−0.338697
6.	−0.187749
7.	−0.338697
8.	−0.192687
9.	−0.263223
10.	−0.187749
11.	−0.338697
12.	−0.051616
13.	−0.338697
14.	−0.187749
15.	−0.187749
16.	−0.128036
17.	−0.338697
18.	−0.051616
19.	−0.338697
20.	−0.051616

```
N[Log[Zeta[2]]^2]
```

0.247706

```
N[(Zeta[2] - 1) (Sum[BernoulliB[k] / k! Log[Zeta[2]]^(k + 1), {k, 0, 16}])]
```

0.247706

```
dt2[n_, k_] := Sum[ (Sum[d2[j, a] / (a!), {a, 1, Log[2, n]}])
    (Sum[ BernoulliB[b] / (b!) d2[n / j, b + k - 1], {b, 0, Log[2, n]}]), {j, Divisors[n]}]
Table[{n, dt2[n, 1]}, {n, 1, 30}] // TableForm
```

```
1     0
2     1
3     1
4     1
5     1
6     1
7     1
8     1
9     1
10    1
11    1
12    1
13    1
14    1
15    1
16    1
17    1
18    1
19    1
20    1
21    1
22    1
23    1
24    1
25    1
26    1
27    1
28    1
29    1
30    1
```

```
K[n_, 0] := If[n == 1, 1, 0]
K[n_, 1] := If[n == 1, 0, FullSimplify[MangoldtLambda[n] / Log[n]]]
K[n_, k_] := Sum[K[j, k - 1] K[n / j, 1], {j, Divisors[n]}]
```

```
dk2[n_, k_] := Sum[ (Sum[K[j, a] / (a!), {a, 1, Log[2, n]}])
    (Sum[ BernoulliB[b] / (b!) K[n / j, b + k - 1], {b, 0, Log[2, n]}]), {j, Divisors[n]}]
Table[{n, dk2[n, 1]}, {n, 1, 30}] // TableForm
```

1	0
2	1
3	1
4	$\frac{1}{2}$
5	1
6	0
7	1
8	$\frac{1}{3}$
9	$\frac{1}{2}$
10	0
11	1
12	0
13	1
14	0
15	0
16	$\frac{1}{4}$
17	1
18	0
19	1
20	0
21	0
22	0
23	1
24	0
25	$\frac{1}{2}$
26	0
27	$\frac{1}{3}$
28	0
29	1
30	0

```
dk3[n_, k_] := Sum[ (Sum[K[j, a] / (a!), {a, 1, Log[2, n]}])
    (Sum[ BernoulliB[b] / (b!) K[n / j, b + k - 1], {b, 0, Log[2, n]}]), {j, Divisors[n]}]
Table[{n, dk3[n, 1]}, {n, 1, 30}] // TableForm
```

1	0
2	1
3	1
4	$\frac{1}{2}$
5	1
6	0
7	1
8	$\frac{1}{3}$
9	$\frac{1}{2}$
10	0
11	1
12	0
13	1
14	0
15	0
16	$\frac{1}{4}$
17	1
18	0
19	1
20	0
21	0
22	0
23	1
24	0
25	$\frac{1}{2}$
26	0
27	$\frac{1}{3}$
28	0
29	1
30	0

```
dkk[j_] := (Sum[K[j, a] / (a!), {a, 1, Log[2, 30]}])
Table[{n, dkk[n]}, {n, 1, 30}] // TableForm
```

1	0
2	1
3	1
4	1
5	1
6	1
7	1
8	1
9	1
10	1
11	1
12	1
13	1
14	1
15	1
16	1
17	1
18	1
19	1
20	1
21	1
22	1
23	1
24	1
25	1
26	1
27	1
28	1
29	1
30	1

```
dk4[n_, k_] := Sum[ If[j == 1, 0, 1]
    (Sum[ BernoulliB[b] / (b!) K[n / j, b + k - 1], {b, 0, Log[2, n]}]), {j, Divisors[n]}]
Table[{n, dk4[n, 1]}, {n, 1, 30}] // TableForm
```

1	0
2	1
3	1
4	$\frac{1}{2}$
5	1
6	0
7	1
8	$\frac{1}{3}$
9	$\frac{1}{2}$
10	0
11	1
12	0
13	1
14	0
15	0
16	$\frac{1}{4}$
17	1
18	0
19	1
20	0
21	0
22	0
23	1
24	0
25	$\frac{1}{2}$
26	0
27	$\frac{1}{3}$
28	0
29	1
30	0

```
dk5[n_, k_] := Sum[
  d2[j, 1] (Sum[ BernoulliB[b] / (b!) K[n / j, b + k - 1], {b, 0, Log[2, n]}]), {j, Divisors[n]}]
Table[{n, dk5[n, 1]}, {n, 1, 30}] // TableForm
```

1	0
2	1
3	1
4	$\frac{1}{2}$
5	1
6	0
7	1
8	$\frac{1}{3}$
9	$\frac{1}{2}$
10	0
11	1
12	0
13	1
14	0
15	0
16	$\frac{1}{4}$
17	1
18	0
19	1
20	0
21	0
22	0
23	1
24	0
25	$\frac{1}{2}$
26	0
27	$\frac{1}{3}$
28	0
29	1
30	0

```
dk5[n_, k_] := -Sum[ BernoulliB[b] / (b!) K[n, b + k - 1], {b, 0, Log[2, n]}] +
  Sum[ (Sum[ BernoulliB[b] / (b!) K[j, b + k - 1], {b, 0, Log[2, n]}]), {j, Divisors[n]}]
Table[{n, dk5[n, 1]}, {n, 1, 30}] // TableForm
```

1	0
2	1
3	1
4	$\frac{1}{2}$
5	1
6	0
7	1
8	$\frac{1}{3}$
9	$\frac{1}{2}$
10	0
11	1
12	0
13	1
14	0
15	0
16	$\frac{1}{4}$
17	1
18	0
19	1
20	0
21	0
22	0
23	1
24	0
25	$\frac{1}{2}$
26	0
27	$\frac{1}{3}$
28	0
29	1
30	0

Expand[(Zeta[s] - 1) (Sum[BernoulliB[k] / k ! Log[Zeta[s]]^k, {k, 0, 32}])]

$$-1 + \frac{1}{2} \text{Log}[\text{Zeta}[s]] - \frac{1}{12} \text{Log}[\text{Zeta}[s]]^2 + \frac{1}{720} \text{Log}[\text{Zeta}[s]]^4 - \frac{\text{Log}[\text{Zeta}[s]]^6}{30\,240} +$$

$$\frac{\text{Log}[\text{Zeta}[s]]^8}{1\,209\,600} - \frac{\text{Log}[\text{Zeta}[s]]^{10}}{47\,900\,160} + \frac{691\,\text{Log}[\text{Zeta}[s]]^{12}}{1\,307\,674\,368\,000} - \frac{\text{Log}[\text{Zeta}[s]]^{14}}{74\,724\,249\,600} +$$

$$\frac{3617\,\text{Log}[\text{Zeta}[s]]^{16}}{10\,670\,622\,842\,880\,000} - \frac{43\,867\,\text{Log}[\text{Zeta}[s]]^{18}}{5\,109\,094\,217\,170\,944\,000} + \frac{174\,611\,\text{Log}[\text{Zeta}[s]]^{20}}{802\,857\,662\,698\,291\,200\,000} -$$

$$\frac{77\,683\,\text{Log}[\text{Zeta}[s]]^{22}}{14\,101\,100\,039\,391\,805\,440\,000} + \frac{236\,364\,091\,\text{Log}[\text{Zeta}[s]]^{24}}{1\,693\,824\,136\,731\,743\,669\,452\,800\,000} -$$

$$\frac{657\,931\,\text{Log}[\text{Zeta}[s]]^{26}}{186\,134\,520\,519\,971\,831\,808\,000\,000} + \frac{3\,392\,780\,147\,\text{Log}[\text{Zeta}[s]]^{28}}{37\,893\,265\,687\,455\,865\,519\,472\,640\,000\,000} -$$

$$\frac{1\,723\,168\,255\,201\,\text{Log}[\text{Zeta}[s]]^{30}}{759\,790\,291\,646\,040\,068\,357\,842\,010\,112\,000\,000} + \frac{7\,709\,321\,041\,217\,\text{Log}[\text{Zeta}[s]]^{32}}{134\,196\,726\,836\,183\,700\,385\,281\,186\,201\,600\,000\,000} +$$

$$\text{Zeta}[s] - \frac{1}{2} \text{Log}[\text{Zeta}[s]]\,\text{Zeta}[s] + \frac{1}{12} \text{Log}[\text{Zeta}[s]]^2\,\text{Zeta}[s] - \frac{1}{720} \text{Log}[\text{Zeta}[s]]^4\,\text{Zeta}[s] +$$

$$\frac{\text{Log}[\text{Zeta}[s]]^6\,\text{Zeta}[s]}{30\,240} - \frac{\text{Log}[\text{Zeta}[s]]^8\,\text{Zeta}[s]}{1\,209\,600} + \frac{\text{Log}[\text{Zeta}[s]]^{10}\,\text{Zeta}[s]}{47\,900\,160} -$$

$$\frac{691\,\text{Log}[\text{Zeta}[s]]^{12}\,\text{Zeta}[s]}{1\,307\,674\,368\,000} + \frac{\text{Log}[\text{Zeta}[s]]^{14}\,\text{Zeta}[s]}{74\,724\,249\,600} - \frac{3617\,\text{Log}[\text{Zeta}[s]]^{16}\,\text{Zeta}[s]}{10\,670\,622\,842\,880\,000} +$$

$$\frac{43\,867\,\text{Log}[\text{Zeta}[s]]^{18}\,\text{Zeta}[s]}{5\,109\,094\,217\,170\,944\,000} - \frac{174\,611\,\text{Log}[\text{Zeta}[s]]^{20}\,\text{Zeta}[s]}{802\,857\,662\,698\,291\,200\,000} +$$

$$\frac{77\,683\,\text{Log}[\text{Zeta}[s]]^{22}\,\text{Zeta}[s]}{14\,101\,100\,039\,391\,805\,440\,000} - \frac{236\,364\,091\,\text{Log}[\text{Zeta}[s]]^{24}\,\text{Zeta}[s]}{1\,693\,824\,136\,731\,743\,669\,452\,800\,000} +$$

$$\frac{657\,931\,\text{Log}[\text{Zeta}[s]]^{26}\,\text{Zeta}[s]}{186\,134\,520\,519\,971\,831\,808\,000\,000} - \frac{3\,392\,780\,147\,\text{Log}[\text{Zeta}[s]]^{28}\,\text{Zeta}[s]}{37\,893\,265\,687\,455\,865\,519\,472\,640\,000\,000} +$$

$$\frac{1\,723\,168\,255\,201\,\text{Log}[\text{Zeta}[s]]^{30}\,\text{Zeta}[s]}{759\,790\,291\,646\,040\,068\,357\,842\,010\,112\,000\,000} - \frac{7\,709\,321\,041\,217\,\text{Log}[\text{Zeta}[s]]^{32}\,\text{Zeta}[s]}{134\,196\,726\,836\,183\,700\,385\,281\,186\,201\,600\,000\,000}$$

Expand[2 (Zeta[s] - 1) (Sum[BernB[k] / fac[k] Log[Zeta[s]]^k, {k, 0, 32}])]

$$-\frac{2\,\text{BernB}[0]}{\text{fac}[0]} - \frac{2\,\text{BernB}[1]\,\text{Log}[\text{Zeta}[s]]}{\text{fac}[1]} - \frac{2\,\text{BernB}[2]\,\text{Log}[\text{Zeta}[s]]^2}{\text{fac}[2]} -$$

$$\frac{2\,\text{BernB}[3]\,\text{Log}[\text{Zeta}[s]]^3}{\text{fac}[3]} - \frac{2\,\text{BernB}[4]\,\text{Log}[\text{Zeta}[s]]^4}{\text{fac}[4]} - \frac{2\,\text{BernB}[5]\,\text{Log}[\text{Zeta}[s]]^5}{\text{fac}[5]} -$$

$$\frac{2\,\text{BernB}[6]\,\text{Log}[\text{Zeta}[s]]^6}{\text{fac}[6]} - \frac{2\,\text{BernB}[7]\,\text{Log}[\text{Zeta}[s]]^7}{\text{fac}[7]} - \frac{2\,\text{BernB}[8]\,\text{Log}[\text{Zeta}[s]]^8}{\text{fac}[8]} -$$

$$\frac{2\,\text{BernB}[9]\,\text{Log}[\text{Zeta}[s]]^9}{\text{fac}[9]} - \frac{2\,\text{BernB}[10]\,\text{Log}[\text{Zeta}[s]]^{10}}{\text{fac}[10]} - \frac{2\,\text{BernB}[11]\,\text{Log}[\text{Zeta}[s]]^{11}}{\text{fac}[11]} -$$

$$\frac{2\,\text{BernB}[12]\,\text{Log}[\text{Zeta}[s]]^{12}}{\text{fac}[12]} - \frac{2\,\text{BernB}[13]\,\text{Log}[\text{Zeta}[s]]^{13}}{\text{fac}[13]} - \frac{2\,\text{BernB}[14]\,\text{Log}[\text{Zeta}[s]]^{14}}{\text{fac}[14]} -$$

$$\frac{2\,\text{BernB}[15]\,\text{Log}[\text{Zeta}[s]]^{15}}{\text{fac}[15]} - \frac{2\,\text{BernB}[16]\,\text{Log}[\text{Zeta}[s]]^{16}}{\text{fac}[16]} - \frac{2\,\text{BernB}[17]\,\text{Log}[\text{Zeta}[s]]^{17}}{\text{fac}[17]} -$$

$$\frac{2\,\text{BernB}[18]\,\text{Log}[\text{Zeta}[s]]^{18}}{\text{fac}[18]} - \frac{2\,\text{BernB}[19]\,\text{Log}[\text{Zeta}[s]]^{19}}{\text{fac}[19]} - \frac{2\,\text{BernB}[20]\,\text{Log}[\text{Zeta}[s]]^{20}}{\text{fac}[20]} -$$

$$\frac{2 \, \text{BernB}[21] \, \text{Log}[\text{Zeta}[s]]^{21}}{\text{fac}[21]} - \frac{2 \, \text{BernB}[22] \, \text{Log}[\text{Zeta}[s]]^{22}}{\text{fac}[22]} - \frac{2 \, \text{BernB}[23] \, \text{Log}[\text{Zeta}[s]]^{23}}{\text{fac}[23]} -$$

$$\frac{2 \, \text{BernB}[24] \, \text{Log}[\text{Zeta}[s]]^{24}}{\text{fac}[24]} - \frac{2 \, \text{BernB}[25] \, \text{Log}[\text{Zeta}[s]]^{25}}{\text{fac}[25]} - \frac{2 \, \text{BernB}[26] \, \text{Log}[\text{Zeta}[s]]^{26}}{\text{fac}[26]} -$$

$$\frac{2 \, \text{BernB}[27] \, \text{Log}[\text{Zeta}[s]]^{27}}{\text{fac}[27]} - \frac{2 \, \text{BernB}[28] \, \text{Log}[\text{Zeta}[s]]^{28}}{\text{fac}[28]} - \frac{2 \, \text{BernB}[29] \, \text{Log}[\text{Zeta}[s]]^{29}}{\text{fac}[29]} -$$

$$\frac{2 \, \text{BernB}[30] \, \text{Log}[\text{Zeta}[s]]^{30}}{\text{fac}[30]} - \frac{2 \, \text{BernB}[31] \, \text{Log}[\text{Zeta}[s]]^{31}}{\text{fac}[31]} - \frac{2 \, \text{BernB}[32] \, \text{Log}[\text{Zeta}[s]]^{32}}{\text{fac}[32]} +$$

$$\frac{2 \, \text{BernB}[0] \, \text{Zeta}[s]}{\text{fac}[0]} + \frac{2 \, \text{BernB}[1] \, \text{Log}[\text{Zeta}[s]] \, \text{Zeta}[s]}{\text{fac}[1]} + \frac{2 \, \text{BernB}[2] \, \text{Log}[\text{Zeta}[s]]^2 \, \text{Zeta}[s]}{\text{fac}[2]} +$$

$$\frac{2 \, \text{BernB}[3] \, \text{Log}[\text{Zeta}[s]]^3 \, \text{Zeta}[s]}{\text{fac}[3]} + \frac{2 \, \text{BernB}[4] \, \text{Log}[\text{Zeta}[s]]^4 \, \text{Zeta}[s]}{\text{fac}[4]} +$$

$$\frac{2 \, \text{BernB}[5] \, \text{Log}[\text{Zeta}[s]]^5 \, \text{Zeta}[s]}{\text{fac}[5]} + \frac{2 \, \text{BernB}[6] \, \text{Log}[\text{Zeta}[s]]^6 \, \text{Zeta}[s]}{\text{fac}[6]} +$$

$$\frac{2 \, \text{BernB}[7] \, \text{Log}[\text{Zeta}[s]]^7 \, \text{Zeta}[s]}{\text{fac}[7]} + \frac{2 \, \text{BernB}[8] \, \text{Log}[\text{Zeta}[s]]^8 \, \text{Zeta}[s]}{\text{fac}[8]} +$$

$$\frac{2 \, \text{BernB}[9] \, \text{Log}[\text{Zeta}[s]]^9 \, \text{Zeta}[s]}{\text{fac}[9]} + \frac{2 \, \text{BernB}[10] \, \text{Log}[\text{Zeta}[s]]^{10} \, \text{Zeta}[s]}{\text{fac}[10]} +$$

$$\frac{2 \, \text{BernB}[11] \, \text{Log}[\text{Zeta}[s]]^{11} \, \text{Zeta}[s]}{\text{fac}[11]} + \frac{2 \, \text{BernB}[12] \, \text{Log}[\text{Zeta}[s]]^{12} \, \text{Zeta}[s]}{\text{fac}[12]} +$$

$$\frac{2 \, \text{BernB}[13] \, \text{Log}[\text{Zeta}[s]]^{13} \, \text{Zeta}[s]}{\text{fac}[13]} + \frac{2 \, \text{BernB}[14] \, \text{Log}[\text{Zeta}[s]]^{14} \, \text{Zeta}[s]}{\text{fac}[14]} +$$

$$\frac{2 \, \text{BernB}[15] \, \text{Log}[\text{Zeta}[s]]^{15} \, \text{Zeta}[s]}{\text{fac}[15]} + \frac{2 \, \text{BernB}[16] \, \text{Log}[\text{Zeta}[s]]^{16} \, \text{Zeta}[s]}{\text{fac}[16]} +$$

$$\frac{2 \, \text{BernB}[17] \, \text{Log}[\text{Zeta}[s]]^{17} \, \text{Zeta}[s]}{\text{fac}[17]} + \frac{2 \, \text{BernB}[18] \, \text{Log}[\text{Zeta}[s]]^{18} \, \text{Zeta}[s]}{\text{fac}[18]} +$$

$$\frac{2 \, \text{BernB}[19] \, \text{Log}[\text{Zeta}[s]]^{19} \, \text{Zeta}[s]}{\text{fac}[19]} + \frac{2 \, \text{BernB}[20] \, \text{Log}[\text{Zeta}[s]]^{20} \, \text{Zeta}[s]}{\text{fac}[20]} +$$

$$\frac{2 \, \text{BernB}[21] \, \text{Log}[\text{Zeta}[s]]^{21} \, \text{Zeta}[s]}{\text{fac}[21]} + \frac{2 \, \text{BernB}[22] \, \text{Log}[\text{Zeta}[s]]^{22} \, \text{Zeta}[s]}{\text{fac}[22]} +$$

$$\frac{2 \, \text{BernB}[23] \, \text{Log}[\text{Zeta}[s]]^{23} \, \text{Zeta}[s]}{\text{fac}[23]} + \frac{2 \, \text{BernB}[24] \, \text{Log}[\text{Zeta}[s]]^{24} \, \text{Zeta}[s]}{\text{fac}[24]} +$$

$$\frac{2 \, \text{BernB}[25] \, \text{Log}[\text{Zeta}[s]]^{25} \, \text{Zeta}[s]}{\text{fac}[25]} + \frac{2 \, \text{BernB}[26] \, \text{Log}[\text{Zeta}[s]]^{26} \, \text{Zeta}[s]}{\text{fac}[26]} +$$

$$\frac{2 \, \text{BernB}[27] \, \text{Log}[\text{Zeta}[s]]^{27} \, \text{Zeta}[s]}{\text{fac}[27]} + \frac{2 \, \text{BernB}[28] \, \text{Log}[\text{Zeta}[s]]^{28} \, \text{Zeta}[s]}{\text{fac}[28]} +$$

$$\frac{2 \, \text{BernB}[29] \, \text{Log}[\text{Zeta}[s]]^{29} \, \text{Zeta}[s]}{\text{fac}[29]} + \frac{2 \, \text{BernB}[30] \, \text{Log}[\text{Zeta}[s]]^{30} \, \text{Zeta}[s]}{\text{fac}[30]} +$$

$$\frac{2 \, \text{BernB}[31] \, \text{Log}[\text{Zeta}[s]]^{31} \, \text{Zeta}[s]}{\text{fac}[31]} + \frac{2 \, \text{BernB}[32] \, \text{Log}[\text{Zeta}[s]]^{32} \, \text{Zeta}[s]}{\text{fac}[32]}$$

Sum::argmu : Sum called with 1 argument; 2 or more arguments are expected. ≫

Expand[2 (Zeta[s] – 1) (Sum[BernoulliB[k] / k ! Log[Zeta[s]]^k, {k, 0, 32}])]

$$-2 + \text{Log[Zeta[s]]} - \frac{1}{6} \text{Log[Zeta[s]]}^2 + \frac{1}{360} \text{Log[Zeta[s]]}^4 - \frac{\text{Log[Zeta[s]]}^6}{15\,120} +$$

$$\frac{\text{Log[Zeta[s]]}^8}{604\,800} - \frac{\text{Log[Zeta[s]]}^{10}}{23\,950\,080} + \frac{691\,\text{Log[Zeta[s]]}^{12}}{653\,837\,184\,000} - \frac{\text{Log[Zeta[s]]}^{14}}{37\,362\,124\,800} +$$

$$\frac{3617\,\text{Log[Zeta[s]]}^{16}}{5\,335\,311\,421\,440\,000} - \frac{43\,867\,\text{Log[Zeta[s]]}^{18}}{2\,554\,547\,108\,585\,472\,000} + \frac{174\,611\,\text{Log[Zeta[s]]}^{20}}{401\,428\,831\,349\,145\,600\,000} -$$

$$\frac{77\,683\,\text{Log[Zeta[s]]}^{22}}{7\,050\,550\,019\,695\,902\,720\,000} + \frac{236\,364\,091\,\text{Log[Zeta[s]]}^{24}}{846\,912\,068\,365\,871\,834\,726\,400\,000} -$$

$$\frac{657\,931\,\text{Log[Zeta[s]]}^{26}}{93\,067\,260\,259\,985\,915\,904\,000\,000} + \frac{3\,392\,780\,147\,\text{Log[Zeta[s]]}^{28}}{18\,946\,632\,843\,727\,932\,759\,736\,320\,000\,000} -$$

$$\frac{1\,723\,168\,255\,201\,\text{Log[Zeta[s]]}^{30}}{379\,895\,145\,823\,020\,034\,178\,921\,005\,056\,000\,000} + \frac{7\,709\,321\,041\,217\,\text{Log[Zeta[s]]}^{32}}{67\,098\,363\,418\,091\,850\,192\,640\,593\,100\,800\,000\,000} +$$

$$2\,\text{Zeta[s]} - \text{Log[Zeta[s]]}\,\text{Zeta[s]} + \frac{1}{6}\,\text{Log[Zeta[s]]}^2\,\text{Zeta[s]} - \frac{1}{360}\,\text{Log[Zeta[s]]}^4\,\text{Zeta[s]} +$$

$$\frac{\text{Log[Zeta[s]]}^6\,\text{Zeta[s]}}{15\,120} - \frac{\text{Log[Zeta[s]]}^8\,\text{Zeta[s]}}{604\,800} + \frac{\text{Log[Zeta[s]]}^{10}\,\text{Zeta[s]}}{23\,950\,080} -$$

$$\frac{691\,\text{Log[Zeta[s]]}^{12}\,\text{Zeta[s]}}{653\,837\,184\,000} + \frac{\text{Log[Zeta[s]]}^{14}\,\text{Zeta[s]}}{37\,362\,124\,800} - \frac{3617\,\text{Log[Zeta[s]]}^{16}\,\text{Zeta[s]}}{5\,335\,311\,421\,440\,000} +$$

$$\frac{43\,867\,\text{Log[Zeta[s]]}^{18}\,\text{Zeta[s]}}{2\,554\,547\,108\,585\,472\,000} - \frac{174\,611\,\text{Log[Zeta[s]]}^{20}\,\text{Zeta[s]}}{401\,428\,831\,349\,145\,600\,000} +$$

$$\frac{77\,683\,\text{Log[Zeta[s]]}^{22}\,\text{Zeta[s]}}{7\,050\,550\,019\,695\,902\,720\,000} - \frac{236\,364\,091\,\text{Log[Zeta[s]]}^{24}\,\text{Zeta[s]}}{846\,912\,068\,365\,871\,834\,726\,400\,000} +$$

$$\frac{657\,931\,\text{Log[Zeta[s]]}^{26}\,\text{Zeta[s]}}{93\,067\,260\,259\,985\,915\,904\,000\,000} - \frac{3\,392\,780\,147\,\text{Log[Zeta[s]]}^{28}\,\text{Zeta[s]}}{18\,946\,632\,843\,727\,932\,759\,736\,320\,000\,000} +$$

$$\frac{1\,723\,168\,255\,201\,\text{Log[Zeta[s]]}^{30}\,\text{Zeta[s]}}{379\,895\,145\,823\,020\,034\,178\,921\,005\,056\,000\,000} - \frac{7\,709\,321\,041\,217\,\text{Log[Zeta[s]]}^{32}\,\text{Zeta[s]}}{67\,098\,363\,418\,091\,850\,192\,640\,593\,100\,800\,000\,000}$$

$$\text{ll}[s\_] := -2 - \frac{1}{6} \text{Log}[\text{Zeta}[s]]^2 + \frac{1}{360} \text{Log}[\text{Zeta}[s]]^4 - \frac{\text{Log}[\text{Zeta}[s]]^6}{15\,120} + \frac{\text{Log}[\text{Zeta}[s]]^8}{604\,800} -$$

$$\frac{\text{Log}[\text{Zeta}[s]]^{10}}{23\,950\,080} + \frac{691\,\text{Log}[\text{Zeta}[s]]^{12}}{653\,837\,184\,000} - \frac{\text{Log}[\text{Zeta}[s]]^{14}}{37\,362\,124\,800} + \frac{3617\,\text{Log}[\text{Zeta}[s]]^{16}}{5\,335\,311\,421\,440\,000} -$$

$$\frac{43\,867\,\text{Log}[\text{Zeta}[s]]^{18}}{2\,554\,547\,108\,585\,472\,000} + \frac{174\,611\,\text{Log}[\text{Zeta}[s]]^{20}}{401\,428\,831\,349\,145\,600\,000} - \frac{77\,683\,\text{Log}[\text{Zeta}[s]]^{22}}{7\,050\,550\,019\,695\,902\,720\,000} +$$

$$\frac{236\,364\,091\,\text{Log}[\text{Zeta}[s]]^{24}}{846\,912\,068\,365\,871\,834\,726\,400\,000} - \frac{657\,931\,\text{Log}[\text{Zeta}[s]]^{26}}{93\,067\,260\,259\,985\,915\,904\,000\,000} +$$

$$\frac{3\,392\,780\,147\,\text{Log}[\text{Zeta}[s]]^{28}}{18\,946\,632\,843\,727\,932\,759\,736\,320\,000\,000} - \frac{1\,723\,168\,255\,201\,\text{Log}[\text{Zeta}[s]]^{30}}{379\,895\,145\,823\,020\,034\,178\,921\,005\,056\,000\,000} +$$

$$\frac{7\,709\,321\,041\,217\,\text{Log}[\text{Zeta}[s]]^{32}}{67\,098\,363\,418\,091\,850\,192\,640\,593\,100\,800\,000\,000} + 2\,\text{Zeta}[s] -$$

$$\text{Log}[\text{Zeta}[s]]\,\text{Zeta}[s] + \frac{1}{6}\,\text{Log}[\text{Zeta}[s]]^2\,\text{Zeta}[s] - \frac{1}{360}\,\text{Log}[\text{Zeta}[s]]^4\,\text{Zeta}[s] +$$

$$\frac{\text{Log}[\text{Zeta}[s]]^6\,\text{Zeta}[s]}{15\,120} - \frac{\text{Log}[\text{Zeta}[s]]^8\,\text{Zeta}[s]}{604\,800} + \frac{\text{Log}[\text{Zeta}[s]]^{10}\,\text{Zeta}[s]}{23\,950\,080} -$$

$$\frac{691\,\text{Log}[\text{Zeta}[s]]^{12}\,\text{Zeta}[s]}{653\,837\,184\,000} + \frac{\text{Log}[\text{Zeta}[s]]^{14}\,\text{Zeta}[s]}{37\,362\,124\,800} - \frac{3617\,\text{Log}[\text{Zeta}[s]]^{16}\,\text{Zeta}[s]}{5\,335\,311\,421\,440\,000} +$$

$$\frac{43\,867\,\text{Log}[\text{Zeta}[s]]^{18}\,\text{Zeta}[s]}{2\,554\,547\,108\,585\,472\,000} - \frac{174\,611\,\text{Log}[\text{Zeta}[s]]^{20}\,\text{Zeta}[s]}{401\,428\,831\,349\,145\,600\,000} +$$

$$\frac{77\,683\,\text{Log}[\text{Zeta}[s]]^{22}\,\text{Zeta}[s]}{7\,050\,550\,019\,695\,902\,720\,000} - \frac{236\,364\,091\,\text{Log}[\text{Zeta}[s]]^{24}\,\text{Zeta}[s]}{846\,912\,068\,365\,871\,834\,726\,400\,000} +$$

$$\frac{657\,931\,\text{Log}[\text{Zeta}[s]]^{26}\,\text{Zeta}[s]}{93\,067\,260\,259\,985\,915\,904\,000\,000} - \frac{3\,392\,780\,147\,\text{Log}[\text{Zeta}[s]]^{28}\,\text{Zeta}[s]}{18\,946\,632\,843\,727\,932\,759\,736\,320\,000\,000} +$$

$$\frac{1\,723\,168\,255\,201\,\text{Log}[\text{Zeta}[s]]^{30}\,\text{Zeta}[s]}{379\,895\,145\,823\,020\,034\,178\,921\,005\,056\,000\,000} - \frac{7\,709\,321\,041\,217\,\text{Log}[\text{Zeta}[s]]^{32}\,\text{Zeta}[s]}{67\,098\,363\,418\,091\,850\,192\,640\,593\,100\,800\,000\,000}$$

N[ll[3]]

0.184034

N[Log[Zeta[3]]]

0.184034

$$
\begin{aligned}
\text{lo[s\_] := } & -\frac{1}{2}\, \text{Log[Zeta[s]]} - \frac{1}{12}\, \text{Log[Zeta[s]]}^2 + \frac{1}{720}\, \text{Log[Zeta[s]]}^4 - \frac{\text{Log[Zeta[s]]}^6}{30\,240} + \\
& \frac{\text{Log[Zeta[s]]}^8}{1\,209\,600} - \frac{\text{Log[Zeta[s]]}^{10}}{47\,900\,160} + \frac{691\,\text{Log[Zeta[s]]}^{12}}{1\,307\,674\,368\,000} - \frac{\text{Log[Zeta[s]]}^{14}}{74\,724\,249\,600} + \\
& \frac{3617\,\text{Log[Zeta[s]]}^{16}}{10\,670\,622\,842\,880\,000} - \frac{43\,867\,\text{Log[Zeta[s]]}^{18}}{5\,109\,094\,217\,170\,944\,000} + \frac{174\,611\,\text{Log[Zeta[s]]}^{20}}{802\,857\,662\,698\,291\,200\,000} - \\
& \frac{77\,683\,\text{Log[Zeta[s]]}^{22}}{14\,101\,100\,039\,391\,805\,440\,000} + \frac{236\,364\,091\,\text{Log[Zeta[s]]}^{24}}{1\,693\,824\,136\,731\,743\,669\,452\,800\,000} - \\
& \frac{657\,931\,\text{Log[Zeta[s]]}^{26}}{186\,134\,520\,519\,971\,831\,808\,000\,000} + \frac{3\,392\,780\,147\,\text{Log[Zeta[s]]}^{28}}{37\,893\,265\,687\,455\,865\,519\,472\,640\,000\,000} - \\
& \frac{1\,723\,168\,255\,201\,\text{Log[Zeta[s]]}^{30}}{759\,790\,291\,646\,040\,068\,357\,842\,010\,112\,000\,000} + \frac{7\,709\,321\,041\,217\,\text{Log[Zeta[s]]}^{32}}{134\,196\,726\,836\,183\,700\,385\,281\,186\,201\,600\,000\,000} + \\
& \text{Zeta[s]} - \frac{1}{2}\, \text{Log[Zeta[s]]}\, \text{Zeta[s]} + \frac{1}{12}\, \text{Log[Zeta[s]]}^2\, \text{Zeta[s]} - \frac{1}{720}\, \text{Log[Zeta[s]]}^4\, \text{Zeta[s]} + \\
& \frac{\text{Log[Zeta[s]]}^6\, \text{Zeta[s]}}{30\,240} - \frac{\text{Log[Zeta[s]]}^8\, \text{Zeta[s]}}{1\,209\,600} + \frac{\text{Log[Zeta[s]]}^{10}\, \text{Zeta[s]}}{47\,900\,160} - \\
& \frac{691\,\text{Log[Zeta[s]]}^{12}\, \text{Zeta[s]}}{1\,307\,674\,368\,000} + \frac{\text{Log[Zeta[s]]}^{14}\, \text{Zeta[s]}}{74\,724\,249\,600} - \frac{3617\,\text{Log[Zeta[s]]}^{16}\, \text{Zeta[s]}}{10\,670\,622\,842\,880\,000} + \\
& \frac{43\,867\,\text{Log[Zeta[s]]}^{18}\, \text{Zeta[s]}}{5\,109\,094\,217\,170\,944\,000} - \frac{174\,611\,\text{Log[Zeta[s]]}^{20}\, \text{Zeta[s]}}{802\,857\,662\,698\,291\,200\,000} + \\
& \frac{77\,683\,\text{Log[Zeta[s]]}^{22}\, \text{Zeta[s]}}{14\,101\,100\,039\,391\,805\,440\,000} - \frac{236\,364\,091\,\text{Log[Zeta[s]]}^{24}\, \text{Zeta[s]}}{1\,693\,824\,136\,731\,743\,669\,452\,800\,000} + \\
& \frac{657\,931\,\text{Log[Zeta[s]]}^{26}\, \text{Zeta[s]}}{186\,134\,520\,519\,971\,831\,808\,000\,000} - \frac{3\,392\,780\,147\,\text{Log[Zeta[s]]}^{28}\, \text{Zeta[s]}}{37\,893\,265\,687\,455\,865\,519\,472\,640\,000\,000} + \\
& \frac{1\,723\,168\,255\,201\,\text{Log[Zeta[s]]}^{30}\, \text{Zeta[s]}}{759\,790\,291\,646\,040\,068\,357\,842\,010\,112\,000\,000} - \frac{7\,709\,321\,041\,217\,\text{Log[Zeta[s]]}^{32}\, \text{Zeta[s]}}{134\,196\,726\,836\,183\,700\,385\,281\,186\,201\,600\,000\,000}
\end{aligned}
$$

lo[2]

$$\frac{\pi^2}{6} - \frac{1}{2} \text{Log}\left[\frac{\pi^2}{6}\right] - \frac{1}{12} \pi^2 \text{Log}\left[\frac{\pi^2}{6}\right] - \frac{1}{12} \text{Log}\left[\frac{\pi^2}{6}\right]^2 + \frac{1}{72} \pi^2 \text{Log}\left[\frac{\pi^2}{6}\right]^2 + \frac{1}{720} \text{Log}\left[\frac{\pi^2}{6}\right]^4 - \frac{\pi^2 \text{Log}\left[\frac{\pi^2}{6}\right]^4}{4320} -$$

$$\frac{\text{Log}\left[\frac{\pi^2}{6}\right]^6}{30\,240} + \frac{\pi^2 \text{Log}\left[\frac{\pi^2}{6}\right]^6}{181\,440} + \frac{\text{Log}\left[\frac{\pi^2}{6}\right]^8}{1\,209\,600} - \frac{\pi^2 \text{Log}\left[\frac{\pi^2}{6}\right]^8}{7\,257\,600} - \frac{\text{Log}\left[\frac{\pi^2}{6}\right]^{10}}{47\,900\,160} + \frac{\pi^2 \text{Log}\left[\frac{\pi^2}{6}\right]^{10}}{287\,400\,960} + \frac{691 \text{Log}\left[\frac{\pi^2}{6}\right]^{12}}{1\,307\,674\,368\,000} -$$

$$\frac{691 \pi^2 \text{Log}\left[\frac{\pi^2}{6}\right]^{12}}{7\,846\,046\,208\,000} - \frac{\text{Log}\left[\frac{\pi^2}{6}\right]^{14}}{74\,724\,249\,600} + \frac{\pi^2 \text{Log}\left[\frac{\pi^2}{6}\right]^{14}}{448\,345\,497\,600} + \frac{3617 \text{Log}\left[\frac{\pi^2}{6}\right]^{16}}{10\,670\,622\,842\,880\,000} - \frac{3617 \pi^2 \text{Log}\left[\frac{\pi^2}{6}\right]^{16}}{64\,023\,737\,057\,280\,000} -$$

$$\frac{43\,867 \text{Log}\left[\frac{\pi^2}{6}\right]^{18}}{5\,109\,094\,217\,170\,944\,000} + \frac{43\,867 \pi^2 \text{Log}\left[\frac{\pi^2}{6}\right]^{18}}{30\,654\,565\,303\,025\,664\,000} + \frac{174\,611 \text{Log}\left[\frac{\pi^2}{6}\right]^{20}}{802\,857\,662\,698\,291\,200\,000} -$$

$$\frac{174\,611 \pi^2 \text{Log}\left[\frac{\pi^2}{6}\right]^{20}}{4\,817\,145\,976\,189\,747\,200\,000} - \frac{77\,683 \text{Log}\left[\frac{\pi^2}{6}\right]^{22}}{14\,101\,100\,039\,391\,805\,440\,000} + \frac{77\,683 \pi^2 \text{Log}\left[\frac{\pi^2}{6}\right]^{22}}{84\,606\,600\,236\,350\,832\,640\,000} +$$

$$\frac{236\,364\,091 \text{Log}\left[\frac{\pi^2}{6}\right]^{24}}{1\,693\,824\,136\,731\,743\,669\,452\,800\,000} - \frac{236\,364\,091 \pi^2 \text{Log}\left[\frac{\pi^2}{6}\right]^{24}}{10\,162\,944\,820\,390\,462\,016\,716\,800\,000} -$$

$$\frac{657\,931 \text{Log}\left[\frac{\pi^2}{6}\right]^{26}}{186\,134\,520\,519\,971\,831\,808\,000\,000} + \frac{657\,931 \pi^2 \text{Log}\left[\frac{\pi^2}{6}\right]^{26}}{1\,116\,807\,123\,119\,830\,990\,848\,000\,000} +$$

$$\frac{3\,392\,780\,147 \text{Log}\left[\frac{\pi^2}{6}\right]^{28}}{37\,893\,265\,687\,455\,865\,519\,472\,640\,000\,000} - \frac{3\,392\,780\,147 \pi^2 \text{Log}\left[\frac{\pi^2}{6}\right]^{28}}{227\,359\,594\,124\,735\,193\,116\,835\,840\,000\,000} -$$

$$\frac{1\,723\,168\,255\,201 \text{Log}\left[\frac{\pi^2}{6}\right]^{30}}{759\,790\,291\,646\,040\,068\,357\,842\,010\,112\,000\,000} + \frac{1\,723\,168\,255\,201 \pi^2 \text{Log}\left[\frac{\pi^2}{6}\right]^{30}}{4\,558\,741\,749\,876\,240\,410\,147\,052\,060\,672\,000\,000} +$$

$$\frac{7\,709\,321\,041\,217 \text{Log}\left[\frac{\pi^2}{6}\right]^{32}}{134\,196\,726\,836\,183\,700\,385\,281\,186\,201\,600\,000\,000} - \frac{7\,709\,321\,041\,217 \pi^2 \text{Log}\left[\frac{\pi^2}{6}\right]^{32}}{805\,180\,361\,017\,102\,202\,311\,687\,117\,209\,600\,000\,000}$$

lo[s_]

$$-\frac{1}{2}\,\text{Log}[\text{Zeta}[s\_]] - \frac{1}{12}\,\text{Log}[\text{Zeta}[s\_]]^2 + \frac{1}{720}\,\text{Log}[\text{Zeta}[s\_]]^4 - \frac{\text{Log}[\text{Zeta}[s\_]]^6}{30\,240} + \frac{\text{Log}[\text{Zeta}[s\_]]^8}{1\,209\,600} -$$

$$\frac{\text{Log}[\text{Zeta}[s\_]]^{10}}{47\,900\,160} + \frac{691\,\text{Log}[\text{Zeta}[s\_]]^{12}}{1\,307\,674\,368\,000} - \frac{\text{Log}[\text{Zeta}[s\_]]^{14}}{74\,724\,249\,600} + \frac{3617\,\text{Log}[\text{Zeta}[s\_]]^{16}}{10\,670\,622\,842\,880\,000} -$$

$$\frac{43\,867\,\text{Log}[\text{Zeta}[s\_]]^{18}}{5\,109\,094\,217\,170\,944\,000} + \frac{174\,611\,\text{Log}[\text{Zeta}[s\_]]^{20}}{802\,857\,662\,698\,291\,200\,000} - \frac{77\,683\,\text{Log}[\text{Zeta}[s\_]]^{22}}{14\,101\,100\,039\,391\,805\,440\,000} +$$

$$\frac{236\,364\,091\,\text{Log}[\text{Zeta}[s\_]]^{24}}{1\,693\,824\,136\,731\,743\,669\,452\,800\,000} - \frac{657\,931\,\text{Log}[\text{Zeta}[s\_]]^{26}}{186\,134\,520\,519\,971\,831\,808\,000\,000} +$$

$$\frac{3\,392\,780\,147\,\text{Log}[\text{Zeta}[s\_]]^{28}}{37\,893\,265\,687\,455\,865\,519\,472\,640\,000\,000} - \frac{1\,723\,168\,255\,201\,\text{Log}[\text{Zeta}[s\_]]^{30}}{759\,790\,291\,646\,040\,068\,357\,842\,010\,112\,000\,000} +$$

$$\frac{7\,709\,321\,041\,217\,\text{Log}[\text{Zeta}[s\_]]^{32}}{134\,196\,726\,836\,183\,700\,385\,281\,186\,201\,600\,000\,000} + \text{Zeta}[s\_] - \frac{1}{2}\,\text{Log}[\text{Zeta}[s\_]]\,\text{Zeta}[s\_] +$$

$$\frac{1}{12}\,\text{Log}[\text{Zeta}[s\_]]^2\,\text{Zeta}[s\_] - \frac{1}{720}\,\text{Log}[\text{Zeta}[s\_]]^4\,\text{Zeta}[s\_] + \frac{\text{Log}[\text{Zeta}[s\_]]^6\,\text{Zeta}[s\_]}{30\,240} -$$

$$\frac{\text{Log}[\text{Zeta}[s\_]]^8\,\text{Zeta}[s\_]}{1\,209\,600} + \frac{\text{Log}[\text{Zeta}[s\_]]^{10}\,\text{Zeta}[s\_]}{47\,900\,160} - \frac{691\,\text{Log}[\text{Zeta}[s\_]]^{12}\,\text{Zeta}[s\_]}{1\,307\,674\,368\,000} +$$

$$\frac{\text{Log}[\text{Zeta}[s\_]]^{14}\,\text{Zeta}[s\_]}{74\,724\,249\,600} - \frac{3617\,\text{Log}[\text{Zeta}[s\_]]^{16}\,\text{Zeta}[s\_]}{10\,670\,622\,842\,880\,000} +$$

$$\frac{43\,867\,\text{Log}[\text{Zeta}[s\_]]^{18}\,\text{Zeta}[s\_]}{5\,109\,094\,217\,170\,944\,000} - \frac{174\,611\,\text{Log}[\text{Zeta}[s\_]]^{20}\,\text{Zeta}[s\_]}{802\,857\,662\,698\,291\,200\,000} +$$

$$\frac{77\,683\,\text{Log}[\text{Zeta}[s\_]]^{22}\,\text{Zeta}[s\_]}{14\,101\,100\,039\,391\,805\,440\,000} - \frac{236\,364\,091\,\text{Log}[\text{Zeta}[s\_]]^{24}\,\text{Zeta}[s\_]}{1\,693\,824\,136\,731\,743\,669\,452\,800\,000} +$$

$$\frac{657\,931\,\text{Log}[\text{Zeta}[s\_]]^{26}\,\text{Zeta}[s\_]}{186\,134\,520\,519\,971\,831\,808\,000\,000} - \frac{3\,392\,780\,147\,\text{Log}[\text{Zeta}[s\_]]^{28}\,\text{Zeta}[s\_]}{37\,893\,265\,687\,455\,865\,519\,472\,640\,000\,000} +$$

$$\frac{1\,723\,168\,255\,201\,\text{Log}[\text{Zeta}[s\_]]^{30}\,\text{Zeta}[s\_]}{759\,790\,291\,646\,040\,068\,357\,842\,010\,112\,000\,000} - \frac{7\,709\,321\,041\,217\,\text{Log}[\text{Zeta}[s\_]]^{32}\,\text{Zeta}[s\_]}{134\,196\,726\,836\,183\,700\,385\,281\,186\,201\,600\,000\,000}$$

Zeta[s] - 1 + (Zeta[s] - 1) (Sum[BernoulliB[k] / k! Log[Zeta[s]]^k, {k, 1, 32}])

$$-1 + \left(-\frac{1}{2}\,\text{Log}[\text{Zeta}[s]] + \frac{1}{12}\,\text{Log}[\text{Zeta}[s]]^2 - \frac{1}{720}\,\text{Log}[\text{Zeta}[s]]^4 + \frac{\text{Log}[\text{Zeta}[s]]^6}{30\,240} - \frac{\text{Log}[\text{Zeta}[s]]^8}{1\,209\,600} + \right.$$

$$\frac{\text{Log}[\text{Zeta}[s]]^{10}}{47\,900\,160} - \frac{691\,\text{Log}[\text{Zeta}[s]]^{12}}{1\,307\,674\,368\,000} + \frac{\text{Log}[\text{Zeta}[s]]^{14}}{74\,724\,249\,600} - \frac{3617\,\text{Log}[\text{Zeta}[s]]^{16}}{10\,670\,622\,842\,880\,000} +$$

$$\frac{43\,867\,\text{Log}[\text{Zeta}[s]]^{18}}{5\,109\,094\,217\,170\,944\,000} - \frac{174\,611\,\text{Log}[\text{Zeta}[s]]^{20}}{802\,857\,662\,698\,291\,200\,000} + \frac{77\,683\,\text{Log}[\text{Zeta}[s]]^{22}}{14\,101\,100\,039\,391\,805\,440\,000} -$$

$$\frac{236\,364\,091\,\text{Log}[\text{Zeta}[s]]^{24}}{1\,693\,824\,136\,731\,743\,669\,452\,800\,000} + \frac{657\,931\,\text{Log}[\text{Zeta}[s]]^{26}}{186\,134\,520\,519\,971\,831\,808\,000\,000} -$$

$$\frac{3\,392\,780\,147\,\text{Log}[\text{Zeta}[s]]^{28}}{37\,893\,265\,687\,455\,865\,519\,472\,640\,000\,000} + \frac{1\,723\,168\,255\,201\,\text{Log}[\text{Zeta}[s]]^{30}}{759\,790\,291\,646\,040\,068\,357\,842\,010\,112\,000\,000} -$$

$$\left.\frac{7\,709\,321\,041\,217\,\text{Log}[\text{Zeta}[s]]^{32}}{134\,196\,726\,836\,183\,700\,385\,281\,186\,201\,600\,000\,000}\right) (-1 + \text{Zeta}[s]) + \text{Zeta}[s]$$

```
Sum[-BernoulliB[b] / (b!) f[n]^b, {b, 0, Infinity}]
```

$$-\frac{f[n]}{-1 + e^{f[n]}}$$

```
Log[Zeta[s]] + 1 + (1 - Zeta[s]) (Sum[ BernoulliB[k] / k! Log[Zeta[s]]^k, {k, 1, 32}])
```

$1 + \text{Log}[\text{Zeta}[s]] +$

$\left(-\frac{1}{2} \text{Log}[\text{Zeta}[s]] + \frac{1}{12} \text{Log}[\text{Zeta}[s]]^2 - \frac{1}{720} \text{Log}[\text{Zeta}[s]]^4 + \frac{\text{Log}[\text{Zeta}[s]]^6}{30\,240} - \frac{\text{Log}[\text{Zeta}[s]]^8}{1\,209\,600} + \right.$

$\frac{\text{Log}[\text{Zeta}[s]]^{10}}{47\,900\,160} - \frac{691 \, \text{Log}[\text{Zeta}[s]]^{12}}{1\,307\,674\,368\,000} + \frac{\text{Log}[\text{Zeta}[s]]^{14}}{74\,724\,249\,600} - \frac{3617 \, \text{Log}[\text{Zeta}[s]]^{16}}{10\,670\,622\,842\,880\,000} +$

$\frac{43\,867 \, \text{Log}[\text{Zeta}[s]]^{18}}{5\,109\,094\,217\,170\,944\,000} - \frac{174\,611 \, \text{Log}[\text{Zeta}[s]]^{20}}{802\,857\,662\,698\,291\,200\,000} + \frac{77\,683 \, \text{Log}[\text{Zeta}[s]]^{22}}{14\,101\,100\,039\,391\,805\,440\,000} -$

$\frac{236\,364\,091 \, \text{Log}[\text{Zeta}[s]]^{24}}{1\,693\,824\,136\,731\,743\,669\,452\,800\,000} + \frac{657\,931 \, \text{Log}[\text{Zeta}[s]]^{26}}{186\,134\,520\,519\,971\,831\,808\,000\,000} -$

$\frac{3\,392\,780\,147 \, \text{Log}[\text{Zeta}[s]]^{28}}{37\,893\,265\,687\,455\,865\,519\,472\,640\,000\,000} + \frac{1\,723\,168\,255\,201 \, \text{Log}[\text{Zeta}[s]]^{30}}{759\,790\,291\,646\,040\,068\,357\,842\,010\,112\,000\,000} -$

$\left. \frac{7\,709\,321\,041\,217 \, \text{Log}[\text{Zeta}[s]]^{32}}{134\,196\,726\,836\,183\,700\,385\,281\,186\,201\,600\,000\,000} \right) (1 - \text{Zeta}[s])$

```
N[Zeta[2]]
```

1.64493

```
lz + 1 + (1 - z) (Sum[ BernoulliB[k] / k! lz^k, {k, 1, 30}])
```

$1 + lz + \left(-\frac{lz}{2} + \frac{lz^2}{12} - \frac{lz^4}{720} + \frac{lz^6}{30\,240} - \frac{lz^8}{1\,209\,600} + \right.$

$\frac{lz^{10}}{47\,900\,160} - \frac{691 \, lz^{12}}{1\,307\,674\,368\,000} + \frac{lz^{14}}{74\,724\,249\,600} - \frac{3617 \, lz^{16}}{10\,670\,622\,842\,880\,000} +$

$\frac{43\,867 \, lz^{18}}{5\,109\,094\,217\,170\,944\,000} - \frac{174\,611 \, lz^{20}}{802\,857\,662\,698\,291\,200\,000} + \frac{77\,683 \, lz^{22}}{14\,101\,100\,039\,391\,805\,440\,000} -$

$\frac{236\,364\,091 \, lz^{24}}{1\,693\,824\,136\,731\,743\,669\,452\,800\,000} + \frac{657\,931 \, lz^{26}}{186\,134\,520\,519\,971\,831\,808\,000\,000} -$

$\frac{3\,392\,780\,147 \, lz^{28}}{37\,893\,265\,687\,455\,865\,519\,472\,640\,000\,000} + \frac{1\,723\,168\,255\,201 \, lz^{30}}{759\,790\,291\,646\,040\,068\,357\,842\,010\,112\,000\,000} \right) (1 - z)$

$1 + lz + \left(-\frac{lz}{2} + \frac{lz^2}{12} - \frac{lz^4}{720} + \frac{lz^6}{30\,240} - \frac{lz^8}{1\,209\,600} + \frac{lz^{10}}{47\,900\,160} - \frac{691 \, lz^{12}}{1\,307\,674\,368\,000} + \frac{lz^{14}}{74\,724\,249\,600} - \right.$

$\left. \frac{3617 \, lz^{16}}{10\,670\,622\,842\,880\,000} + \frac{43\,867 \, lz^{18}}{5\,109\,094\,217\,170\,944\,000} - \frac{174\,611 \, lz^{20}}{802\,857\,662\,698\,291\,200\,000} \right) \left(1 - \right.$

$\left(1 + lz + \left(-\frac{lz}{2} + \frac{lz^2}{12} - \frac{lz^4}{720} + \frac{lz^6}{30\,240} - \frac{lz^8}{1\,209\,600} + \frac{lz^{10}}{47\,900\,160} - \frac{691 \, lz^{12}}{1\,307\,674\,368\,000} + \frac{lz^{14}}{74\,724\,249\,600} - \right. \right.$

$\left. \left. \left. \frac{3617 \, lz^{16}}{10\,670\,622\,842\,880\,000} + \frac{43\,867 \, lz^{18}}{5\,109\,094\,217\,170\,944\,000} - \frac{174\,611 \, lz^{20}}{802\,857\,662\,698\,291\,200\,000} \right) (1 - z) \right) \right)$

$1 + lz + \frac{lz^2}{2} - \frac{lz^3}{3} + \frac{5 \, lz^4}{24} - \frac{47 \, lz^5}{360} + \frac{59 \, lz^6}{720} - \frac{311 \, lz^7}{6048} + \frac{1301 \, lz^8}{40\,320} - \frac{36\,737 \, lz^9}{1\,814\,400}$ /. lz → Log[Zeta[s]]

```
fa[s_] := 1 + Log[Zeta[s]] + — Log[Zeta[s]]² - — Log[Zeta[s]]³ +
                             2                  3

     5                  47                59
    —— Log[Zeta[s]]⁴ - ——— Log[Zeta[s]]⁵ + ——— Log[Zeta[s]]⁶ -
    24                 360                720

    311 Log[Zeta[s]]⁷   1301 Log[Zeta[s]]⁸   36 737 Log[Zeta[s]]⁹
    ————————————————— + ————————————————— - ————————————————————
         6048                40 320               1 814 400
```

```
N[fa[2]]
```

1.59019

```
N[Zeta[2]]
```

1.64493

```
N[(Zeta[2]) Log[Zeta[2]]]
```

0.818684

```
N[Sum[ Log[Zeta[2]]^(1+k) / (k!), {k, 0, 10}]]
```

0.818684