```
Clear[pae, pap]
pe[n_, k_] := pe[n, k] = Sum[1 / j pe[n - j, k - 1], {j, 1, n - 1}]
pe[n_, 1] := 1 / n
pe[n_, 0] := If[n == 0, 1, 0]
pa[n_, z_] := Sum[ z^k / k ! pe[n, k], {k, 0, n}]
spa[n_, z_] := Sum[pa[j, z], {j, 0, n}]

pae[n_, k_] := pae[n, k] = Sum[If[j == 1, 1, 0] pae[n - j, k - 1], {j, 1, n - 1}]
pae[n_, 1] := If[n == 1, 1, 0]
pae[n_, 0] := If[n == 0, 1, 0]
paa[n_, z_] := Sum[ z^k / k ! pae[n, k], {k, 0, n}]
spaa[n_, z_] := Sum[paa[j, z], {j, 0, n}]

pal[n_] := D[Sum[ paa[j, z], {j, 0, n}], z] /. z -> 0
pel[n_] := D[Sum[ pa[j, z], {j, 0, n}], z] /. z -> 0
pel2[n_]  :=  Sum[pal[j] / j, {j, 1, n}]

pap[n_, k_] := pap[n, k] = Sum[ (-1) ^ (j + 1) pap[n - j, k - 1], {j, 1, n - 1}]
pap[n_, 1] := (-1) ^ (n + 1)
pass[n_, z_] := Sum[ bin[z, k] pap[n, k], {k, 0, n}]
```

```
Table[pa[n, 1], {n, 1, 10}]
```

$\{1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$

```
Table[ D[ Pochhammer[z, n] / n !, z] /. z -> 0, {n, 1, 10}]
```

$\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}\right\}$

```
Table[ D[ z^n / n !, z] /. z -> 0, {n, 1, 10}]
```

$\{1, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$

```
Table[paa[n, 1], {n, 1, 10}]
```

$\left\{1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \frac{1}{720}, \frac{1}{5040}, \frac{1}{40\,320}, \frac{1}{362\,880}, \frac{1}{3\,628\,800}\right\}$

```
Sum[ 1 / k ! x^k, {k, 0, Infinity}]
```

e^x

```
Table[pel[n], {n, 1, 10}]
```

$\left\{1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}, \frac{137}{60}, \frac{49}{20}, \frac{363}{140}, \frac{761}{280}, \frac{7129}{2520}, \frac{7381}{2520}\right\}$

```
Table[pel[n], {n, 1, 10}]
```

$\left\{1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}, \frac{137}{60}, \frac{49}{20}, \frac{363}{140}, \frac{761}{280}, \frac{7129}{2520}, \frac{7381}{2520}\right\}$

```
Table[pel2[n], {n, 1, 10}]
```

$\left\{1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}, \frac{137}{60}, \frac{49}{20}, \frac{363}{140}, \frac{761}{280}, \frac{7129}{2520}, \frac{7381}{2520}\right\}$

```
HarmonicNumber[3] - HarmonicNumber[2] / 2 - HarmonicNumber[1] / 12
```

1

```
1 / 4 - 1 / 6 - 1 / 24
```

$\frac{1}{24}$

```
HarmonicNumber[4] - HarmonicNumber[3] / 2 - HarmonicNumber[2] / 12 - HarmonicNumber[1] / 24
```

1

```
HarmonicNumber[5] - HarmonicNumber[4] / 2 -
 HarmonicNumber[3] / 12 - HarmonicNumber[2] / 24 - 19 HarmonicNumber[1] / 720
```

1

```
Limit[D[z / Log[1 - z], {z, 24}], z → 0] / 24!
```

$\frac{101\,543\,126\,947\,618\,093\,900\,697\,699}{50\,814\,724\,101\,952\,310\,083\,584\,000\,000}$

```
fa[n_] := 1
Sum[ fa[10 / j] / j, {j, 1, 10}]
```

$\frac{7381}{2520}$

```
Sum[MoebiusMu[j] HarmonicNumber[Floor[80 / j]] / j, {j, 1, 80}]
```

1

```
(1 / 9 - 1 / 16) / 4
```

$\frac{7}{576}$

```
1 / 16 - 1 / 36 - 7 / 576
```

$\frac{13}{576}$

```
1 / 9 - 1 / 16
```

$\frac{7}{144}$

```
Table[FullSimplify@pass[n, z], {n, 1, 7}] // TableForm
```

z

$\frac{1}{2}(-3+z)\,z$

$\frac{1}{6}(-7+z)(-2+z)\,z$

$\frac{1}{24}(-5+z)\,z\,(18+(-13+z)\,z)$

$\frac{1}{120}(-4+z)\,z\,(-186+z\,(171+(-26+z)\,z))$

$\frac{1}{720}(-7+z)\,z\,(1080+(-11+z)\,z\,(122+(-27+z)\,z))$

$\frac{(-6+z)(-3+z)\,z\,(5080+(-27+z)\,z\,(202+(-27+z)\,z))}{5040}$

```
FullSimplify@CoefficientList[Series[((1 - x) / (1 - x^2))^z, {x, 0, 10}], x] // TableForm
```

$$1$$
$$-z$$
$$\frac{1}{2} z (1+z)$$
$$-\frac{1}{6} z (1+z) (2+z)$$
$$\frac{1}{24} z (1+z) (2+z) (3+z)$$
$$-\frac{1}{120} z (1+z) (2+z) (3+z) (4+z)$$
$$\frac{1}{720} z (1+z) (2+z) (3+z) (4+z) (5+z)$$
$$-\frac{z (1+z) (2+z) (3+z) (4+z) (5+z) (6+z)}{5040}$$
$$\frac{z (1+z) (2+z) (3+z) (4+z) (5+z) (6+z) (7+z)}{40320}$$
$$-\frac{z (1+z) (2+z) (3+z) (4+z) (5+z) (6+z) (7+z) (8+z)}{362880}$$
$$\frac{z (1+z) (2+z) (3+z) (4+z) (5+z) (6+z) (7+z) (8+z) (9+z)}{3628800}$$

```
FullSimplify[((1 - x) / (1 - x^2))^z]
```

$$\left(\frac{1}{1+x}\right)^z$$

```
FullSimplify@Sum[ Pochhammer[z, 6 - 2 k] / (6 - 2 k) ! Pochhammer[-z, k] / k !, {k, 0, 6 / 2}]
```

$$\frac{1}{720} (-5+z) (-4+z) (-3+z) (-2+z) (-1+z) z$$

```
Table[pa[n, 1], {n, 1, 10}]
```

{1, 1, 1, 1, 1, 1, 1, 1, 1, 1}

```
tn[n_, z_] :=
 Sum[ paa[a, z] paa[b, z / 2] paa[c, z / 3] paa[d, z / 4] paa[e, z / 5], {a, 0, n}, {b, 0, (n - a) / 2},
  {c, 0, (n - a - 2 b) / 3}, {d, 0, (n - a - 2 b - 3 c) / 4}, {e, 0, (n - a - 2 b - 3 c - 4 d) / 5}]
tn2[n_, z_] := Sum[ z^a / a ! (z / 2) ^b / b ! (z / 3) ^c / c ! (z / 4) ^d / d ! (z / 5) ^e,
  {a, 0, n}, {b, 0, (n - a) / 2}, {c, 0, (n - a - 2 b) / 3},
  {d, 0, (n - a - 2 b - 3 c) / 4}, {e, 0, (n - a - 2 b - 3 c - 4 d) / 5}]
tni[n_, z_] := Sum[ pa[a, z] pa[b, -z / 2] pa[c, -z / 3] pa[e, -z / 5], {a, 0, n},
  {b, 0, (n - a) / 2}, {c, 0, (n - a - 2 b) / 3}, {e, 0, (n - a - 2 b - 3 c) / 5}]
tni2[n_, z_] := Sum[ Pochhammer[z, a] / a ! Pochhammer[-z / 2, b] / b !
   Pochhammer[-z / 3, c] / c ! Pochhammer[-z / 5, e] / e !, {a, 0, n},
  {b, 0, (n - a) / 2}, {c, 0, (n - a - 2 b) / 3}, {e, 0, (n - a - 2 b - 3 c) / 5}]
```

```
tn[5, I]
```

$$-\frac{3}{4} + \frac{19 i}{12}$$

```
tn2[5, I]
```

$$-\frac{3}{4} + \frac{19 i}{12}$$

```
spa[5, I]
```

$$-\frac{3}{4} + \frac{19 i}{12}$$

```
tni[4, 1]
```

$\frac{65}{24}$

```
spaa[5, I]
```

$\frac{13}{24} + \frac{101\, i}{120}$

```
FullSimplify@pa[5, z]
```

$\frac{1}{120}\, z\, (1+z)\, (2+z)\, (3+z)\, (4+z)$

```
r1[n_, z_, k_] := If[k > n, 1, Sum[ (z / k) ^j / j ! r1[n - k j, z, k + 1], {j, 0, n / k}]]
r2[n_, z_, k_] :=
 If[k > n, 1, Sum[ Pochhammer[z MoebiusMu[k] / k, j] / j ! r2[n - k j, z, k + 1], {j, 0, n / k}]]
r2[24, 1, 1]
```

$\frac{337\,310\,723\,185\,584\,470\,837\,549}{124\,089\,680\,346\,647\,887\,872\,000}$

```
spaa[24, 1]
```

$\frac{337\,310\,723\,185\,584\,470\,837\,549}{124\,089\,680\,346\,647\,887\,872\,000}$

```
FullSimplify@paa[5, z]
```

$\frac{z^5}{120}$

```
D[Expand[r1[10, z, 1]], z] /. z → 0
```

$\frac{7381}{2520}$

```
D[Expand[r2[30, 10 z, 1]], z] /. z → 0
```

10

```
r1[24, 2.5, 1]
```

1011.6

```
spa[24, 2.5]
```

1011.6

```
Sum[ (z / 2) ^b / b !, {b, 0, Infinity}]
```

$e^{z/2}$

```
E^z E^(z / 2) E^(z / 3) E^(z / 4)
```

$e^{25\, z/12}$

```
HarmonicNumber[4]
```

$\frac{25}{12}$

```
Sum[ x^n Pochhammer[z / 3, n] / n!, {n, 0, Infinity}]
```

$(1-x)^{-z/3}$

```
spaa[10, z]
```

$$1+z+\frac{z^2}{2}+\frac{z^3}{6}+\frac{z^4}{24}+\frac{z^5}{120}+\frac{z^6}{720}+\frac{z^7}{5040}+\frac{z^8}{40320}+\frac{z^9}{362880}+\frac{z^{10}}{3628800}$$

```
Sum[ z^k / k!, {k, 0, n}]
```

$$\frac{e^z\,\text{Gamma}[1+n, z]}{n!}$$

```
Product[ E^(-(2 + I) / k), {k, 1, Infinity}]
```

0

```
Product[ (1 / (1 - x)) ^ (z MoebiusMu[k] / k), {k, 1, Infinity}]
```

1

```
Product[ (1 / (1 - x)) ^ ( MoebiusMu[k] / k), {k, 1, Infinity}]
```

1

```
fa[n_] := 1
Sum[ (-1) ^ (j + 1) fa[10 / j] / j, {j, 1, 10}]
```

$$\frac{1627}{2520}$$

```
Sum[MoebiusMu[j] HarmonicNumber[Floor[80 / j]] / j, {j, 1, 80}]
```

1

```
Sum[ (-1) ^ (j + 1) / j, {j, 1, 10}]
```

$$\frac{1627}{2520}$$

```
Sum[ t[j, 3, 1] fa[10 / j] / j, {j, 1, 10}]
```

$$\frac{2761}{2520}$$

```
Sum[ t[j, 3, 1] / j, {j, 1, 10}]
```

$$\frac{2761}{2520}$$

```
alta[n_, a_] := Sum[t[j, a, 1] / j, {j, 1, n}]
Sum[MoebiusMu[j] alta[Floor[80 / j], 101] / j, {j, 1, 80}]
```

1

```
ff[n_] := Sum[ (-1) ^ (j + 1) fg[n / j] / j, {j, 1, n}]
```

```
Expand[ ff[20] + ff[20 / 2] / 2 - ff[20 / 3] / 3 + ff[20 / 4] / 2 -
  ff[20 / 5] / 5 - ff[20 / 6] / 6 - ff[20 / 7] / 7 + ff[20 / 8] / 2 - ff[20 / 10] / 10]
```

$$\frac{fg[1]}{10} + \frac{1}{19}\,fg\left[\frac{20}{19}\right] + \frac{1}{17}\,fg\left[\frac{20}{17}\right] - \frac{1}{2}\,fg\left[\frac{5}{4}\right] - \frac{1}{15}\,fg\left[\frac{4}{3}\right] + \frac{1}{14}\,fg\left[\frac{10}{7}\right] + \frac{1}{13}\,fg\left[\frac{20}{13}\right] + \frac{1}{6}\,fg\left[\frac{5}{3}\right] + \frac{1}{11}\,fg\left[\frac{20}{11}\right] + fg[20]$$

1, 1 / 2, -1 / 3, 1 / 2, -1 / 5, -1 / 6, -1 / 7, +1 / 2,

```
Product[ (1 + x^(2^k))^z, {k, 0, Infinity}]
```

$$\prod_{k=0}^{\infty}\left(1 + x^{2^k}\right)^z$$

```
FullSimplify[(1 + x^(2^k))^(MoebiusMu[j] / j)]
```

$$\left(1 + x^{2^k}\right)^{\frac{MoebiusMu[j]}{j}}$$

Power::infy : Infinite expression $\frac{1}{0^3}$ encountered. ≫

```
Product[ n^( z MoebiusMu[k] / k), {k, 1, Infinity}]
```

1

```
Product[ 0^(z MoebiusMu[k] / k), {k, 1, Infinity}]
```

$$\prod_{k=1}^{\infty} 0^{\frac{z\,MoebiusMu[k]}{k}}$$

```
Expand[(r2[15, z, 1] + r2[15, -z, 1]) / 2]
```

$$1 + \frac{z^2}{2} + \frac{z^4}{24} + \frac{z^6}{720} + \frac{z^8}{40\,320} + \frac{z^{10}}{3\,628\,800} + \frac{z^{12}}{479\,001\,600} + \frac{z^{14}}{87\,178\,291\,200}$$

```
Limit[((1 - x^E) / (1 - x))^z, x → 1]
```

$$e^z$$

```
Log[(1 / (1 - x))^(z MoebiusMu[k] / k)]
```

$$\mathrm{Log}\left[\left(\frac{1}{1-x}\right)^{\frac{z\,MoebiusMu[k]}{k}}\right]$$

```
(1 / (1 - x))^(z MoebiusMu[k] / k)
```

$$\left(\frac{1}{1-x}\right)^{\frac{z\,MoebiusMu[k]}{k}}$$