All of the following follows from the observation that

Zeta Harmonic Number Forms

$$\zeta(s)=\lim_{n\to\infty}\frac{(1-s)\cdot n^{s}\cdot H_n^{(s)}-s\cdot n^{1-s}\,H_n^{(1-s)}}{(1-s)\cdot n^{s}-s\cdot n^{1-s}\cdot 2^{1-s}\cdot\pi^{-s}\cos\left(\frac{\pi s}{2}\right)\cdot\Gamma(s)}$$

$$\zeta(s)=\lim_{n\to\infty}\left(\frac{1}{2}-\frac{1}{2}\tanh\left(\operatorname{Arccoth}(2s-1)+\left(\frac{1}{2}-s\right)\log n\right)\right)H_n^{(s)}+\left(\frac{1}{2}+\frac{1}{2}\tanh\left(\operatorname{Arccoth}(2s-1)+\left(\frac{1}{2}-s\right)\log n\right)\right)H_n^{(1-s)}\ \text{for}$$

re(s)>1/2

Zeta Trig Forms

$$\zeta\left(\frac{1}{2}-t\cdot i\right)=\lim_{n\to\infty}\sum_{j=1}^{n}\frac{1}{\sqrt{j}}\cdot\cos(t\log j)+\tan\left(t\log n+\arctan\left(\frac{1}{2t}\right)\right)\cdot\sum_{j=1}^{n}\frac{1}{\sqrt{j}}\sin(t\log j)\ \text{re(t)} > 0$$

$$\zeta\left(\frac{1}{2}+t\cdot i\right)=\lim_{n\to\infty}\sum_{j=1}^{n}\frac{1}{\sqrt{j}}\cdot\frac{\cos\left(t\log\frac{n}{j}+\cot^{-1}(2t)\right)}{\cos\left(t\log n+\cot^{-1}(2t)\right)}\ \text{re(t)} > 0$$

Trig Identities

$$\Re\left(\lim_{n\to\infty} H_n^{\left(\frac{1}{2}+s\right)}\cdot\left(1-\tanh\left(\operatorname{Arccoth}(2s)-s\log n\right)\right)\right)=0 \text{ at zeta zeros-1/2}$$

$$\lim_{n\to\infty}\sum_{j=1}^{n}\frac{\sin\left(t\log\frac{n}{j}+\tan^{-1}(2t)\right)}{\sqrt{j}}=0 \text{ at t is the imaginary part of zeta zeroes}$$

Identities

$$\lim_{n\to\infty}(1-s)(\zeta(s)-\sum_{j=1}^{n}\frac{1}{j^{s}})+(s-1+x)(n^{x})(\zeta(s+x)-\sum_{j=1}^{n}\frac{1}{j^{s+x}})=0$$

$$\lim_{n\to\infty}(s-1+y)(n^{y})(\zeta(s+y)-\sum_{j=1}^{n}\frac{1}{j^{s+y}})-(s-1+x)(n^{x})(\zeta(s+x)-\sum_{j=1}^{n}\frac{1}{j^{s+x}})=0$$

$$\lim_{n\to\infty}(-\frac{1}{2}-x)(n^{-x})(\zeta(\frac{1}{2}-x)-\sum_{j=1}^{n}\frac{1}{j^{\frac{1}{2}-x}})-(-\frac{1}{2}+x)(n^{x})(\zeta(\frac{1}{2}+x)-\sum_{j=1}^{n}\frac{1}{j^{\frac{1}{2}+x}})=0$$