```
d[n_, 0, a_] := 1
d[n_, 1, a_] := n - a + 1
d[n_, k_, a_] :=
 Sum[Binomial[k, j] d[Floor[n / (m^j)], k - j, m + 1], {j, 1, k}, {m, a, n^(1 / k)}]

D32Unrolled[ n_ ] := -1 + Floor[ n^(1 / 3)]^3 +
  3 Sum[ Floor[ n / (m^2)] - Floor[ Floor[n / m]^(1 / 2)]^2 + 2 Sum[ Floor[Floor[n / m] / j],
       {j, m + 1, Floor[Floor[n / m]^(1 / 2)]}], {m, 2, Floor[n^(1 / 3)]}]

D22Unrolled[ n_ ] := 1 - Floor[ n^(1 / 2)]^2 + 2 Sum [ Floor[ n / m], {m, 2, Floor[n^(1 / 2)]}]
d0[n_, a_Integer] := 1
d1[n_, a_Integer] := n - a + 1
d2a[n_, a_Integer] :=
 Sum[ Binomial[ 2, 2 ] d0[Floor[n / (m^2)], m + 1] , {m, a, Floor[n^(1 / 2)]}] +
  Sum[Binomial[ 2, 1 ] d1[Floor[n / (m^1)], m + 1], {m, a, Floor[n^(1 / 2)]}]
d2[n_, a_Integer] :=
 1 - 2 a + a^2 - Floor[ n^(1 / 2)]^2 + 2 Sum [ Floor[ n / m], {m, a, Floor[n^(1 / 2)]}]
d2[0, a_Integer] := 0
d3a[n_, a_Integer] :=
 Sum[ Binomial[ 3, 3 ] d0[Floor[n / (m^3)], m + 1] , {m, a, Floor[n^(1 / 3)]}] +
  Sum[Binomial[ 3, 2 ] d1[Floor[n / (m^2)], m + 1], {m, a, Floor[n^(1 / 3)]}] +
  Sum[Binomial[ 3, 1 ] d2a[Floor[n / (m^1)], m + 1], {m, a, Floor[n^(1 / 3)]}]

d2e[n_, a_Integer] :=
 (1 - a)^2 - Floor[ n^(1 / 2)]^2 + 2 Sum [ Floor[ n / m], {m, a, Floor[n^(1 / 2)]}]

dd3e[n_, a_ ] := (1 - a)^3 + Floor[n^(1/3)]^3 + Sum[3 Floor[n / s^2] - 3 Floor[Sqrt[Floor[n / s]]]^2 +
    6 Sum[ Floor[n / m / s], {m, s + 1, Floor[Floor[n / s]^(1 / 2)]}], {s, a, Floor[n^(1 / 3)]}]
d4e[x_, b_ ] :=
 (-1 + b)^4 +
  -Floor[x^(1/4)]^4 +
  4 Sum[ Floor[x / (u^3)], {u, b, x^(1 / 4)}] +
  -6 Sum[Floor[ Floor[x / (u^2)]^(1 / 2)]^2 , {u, b, x^(1 / 4)}] +
  12 Sum [ Floor[x / (u^2 m)],
    {u, b, x^(1 / 4)}, {m, (u + 1), Floor[Floor[x / (u^2)]^(1 / 2)]}] +
  4 Sum[Floor[Floor[x / u]^(1/3)]^3, {u, b, x^(1 / 4)}] +
  12 Sum[ Floor[x / ( u s^2)], {u, b, x^(1 / 4)}, {s, (u + 1), Floor[Floor[x / u]^(1 / 3)]}] +
  -12 Sum[ Floor[Floor[x / (u s)]^(1 / 2)]^2 , {u, b, x^(1 / 4)}, {s, (u + 1), (x / u)^(1 / 3)}] +
  24 Sum[ Floor[x / (u m s)], {u, b, x^(1 / 4)},
    {s, u + 1, (x / u)^(1 / 3)}, {m, s + 1, (x / (u s))^(1 / 2)}]

d4e[100 000, 2]

11 796 070

d[100 000, 4, 2]

11 796 070

FF[x_, b_ ] := Sum[ Floor[x / (u m s)], {u, b, x^(1 / 4)},
  {s, u + 1, (x / u)^(1 / 3)}, {m, s + 1, (x / (u s))^(1 / 2)}]
```

```
FF[10 000, 2]
```

21 090

```
GG[x_, b_] :=
 Sum[ Floor[x / (u m s)], {u, 2, x^(1 / 4)}, {s, 2, (x / u)^(1 / 3)}, {m, 2, (x / (u s))^(1 / 2)}] -
  Sum[ Floor[x / (u m s)], {u, 2, x^(1 / 4)}, {s, 2, (x / u)^(1 / 3)}, {m, 2, s}] -
  Sum[ Floor[x / (u m s)], {u, 2, x^(1 / 4)}, {s, 2, u}, {m, 2, (x / (u s))^(1 / 2)}] +
  Sum[ Floor[x / (u m s)], {u, 2, x^(1 / 4)}, {s, 2, u}, {m, 2, s}] -
  Sum[ Floor[x / (u m s)], {u, 2, b - 1}, {s, 2, (x / u)^(1 / 3)}, {m, 2, (x / (u s))^(1 / 2)}] +
  Sum[ Floor[x / (u m s)], {u, 2, b - 1}, {s, 2, (x / u)^(1 / 3)}, {m, 2, s}] +
  Sum[ Floor[x / (u m s)], {u, 2, b - 1}, {s, 2, u}, {m, 2, (x / (u s))^(1 / 2)}] -
  Sum[ Floor[x / (u m s)], {u, 2, b - 1}, {s, 2, u}, {m, 2, s}]
```

```
GG[10 000, 2]
```

21 090

```
HH[x_, b_] :=
 Sum[ Floor[x / (u m s)], {u, 2, x^(1 / 4)}, {s, 2, (x / u)^(1 / 3)}, {m, 2, (x / (u s))^(1 / 2)}] -
  Sum[ Floor[x / (u m s)], {u, 2, x^(1 / 4)}, {s, 2, (x / u)^(1 / 3)}, {m, 2, s}] -
  Sum[ Floor[x / (u m s)], {u, 2, x^(1 / 4)}, {s, 2, u}, {m, 2, (x / (u s))^(1 / 2)}] +
  Sum[ Floor[x / (u m s)], {u, 2, x^(1 / 4)}, {s, 2, u}, {m, 2, s}] -
  Sum[ Floor[x / (u m s)], {u, 2, b - 1}, {s, 2, (x / u)^(1 / 3)}, {m, 2, (x / (u s))^(1 / 2)}] +
  Sum[ Floor[x / (u m s)], {u, 2, b - 1}, {s, 2, (x / u)^(1 / 3)}, {m, 2, s}] +
  Sum[ Floor[x / (u m s)], {u, 2, b - 1}, {s, 2, u}, {m, 2, (x / (u s))^(1 / 2)}] -
  Sum[ Floor[x / (u m s)], {u, 2, b - 1}, {s, 2, u}, {m, 2, s}]
```

```
HH[10 000, 2]
```

21 090

```
dd3f[n_, a_] := (1 - a)^3 + Floor[n^(1/3)]^3 +
  + 3 Sum[Floor[n / (s^2)], {s, a, n^(1 / 3)}] +
  - 3 Sum[Floor[Floor[n / s]^(1 / 2)]^2, {s, a, n^(1 / 3)}] +
  6 Sum[Floor[n / m / s], {s, a, n^(1 / 3)}, {m, s + 1, Floor[n / s]^(1 / 2)}]
```

```
dd3e[100, 2]
```

324

```
dd3f[100, 2]
```

324

```
II[x_] :=
 Sum[ Floor[x / (u m s)], {u, 2, x^(1 / 4)}, {s, 2, (x / u)^(1 / 3)}, {m, 2, (x / (u s))^(1 / 2)}] -
  Sum[ Floor[x / (u m s)], {u, 2, x^(1 / 4)}, {s, 2, (x / u)^(1 / 3)}, {m, 2, s}] -
  Sum[ Floor[x / (u m s)], {u, 2, x^(1 / 4)}, {s, 2, u}, {m, 2, (x / (u s))^(1 / 2)}] +
  Sum[ Floor[x / (u m s)], {u, 2, x^(1 / 4)}, {s, 2, u}, {m, 2, s}] -
  Sum[ Floor[x / (u m s)], {u, 2, 1}, {s, 2, (x / u)^(1 / 3)}, {m, 2, (x / (u s))^(1 / 2)}] +
  Sum[ Floor[x / (u m s)], {u, 2, 1}, {s, 2, (x / u)^(1 / 3)}, {m, 2, s}] +
  Sum[ Floor[x / (u m s)], {u, 2, 1}, {s, 2, u}, {m, 2, (x / (u s))^(1 / 2)}] -
  Sum[ Floor[x / (u m s)], {u, 2, 1}, {s, 2, u}, {m, 2, s}]
```

```
II[10 000]
```

21 090

```
HH[x, 2]
```

```
$Aborted
```

```
JJ[x_ ] :=
 Sum[ Floor[x / (u m s)], {u, 2, x^(1 / 4)}, {s, 2, (x / u)^(1 / 3)}, {m, 2, (x / (u s))^(1 / 2)}] -
  Sum[ Floor[x / (u m s)], {u, 2, x^(1 / 4)}, {s, 2, (x / u)^(1 / 3)}, {m, 2, s}] -
  Sum[ Floor[x / (u m s)], {u, 2, x^(1 / 4)}, {s, 2, u}, {m, 2, (x / (u s))^(1 / 2)}] +
  Sum[ Floor[x / (u m s)], {u, 2, x^(1 / 4)}, {s, 2, u}, {m, 2, s}]
```

```
JJ[10 000]
```

```
21 090
```

```
KK[x_] := Sum[ Floor[x / (u m s)], {u, 2, x^(1 / 4)}, {s, 2, u}, {m, 2, s}]
```

```
KK[10 000]
```

```
17 856
```

```
KK[x]
```

```
$Aborted
```

```
ddd[n_, a_] := Sum[Floor[n / m / s], {s, a, n^(1 / 3)}, {m, s + 1, Floor[n / s]^(1 / 2)}]
```

```
ddd[10 000, 2]
```

```
49 245
```

```
eee[n_] := Sum[Floor[n / ( m s )], {s, 2, n^(1 / 3)}, {m, 2, Floor[n / s]^(1 / 2)}] -
  Sum[Floor[n / ( m s )], {s, 2, n^(1 / 3)}, {m, 2, s}]
```

```
eee[10 000]
```

```
49 245
```