```
Clear[Oz]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
FI[n_] := FactorInteger[n]; FI[1] := {}
oz[n_, z_] := Product[z^p[[2]], {p, FI[n]}]
ooz[n_, z_] := Product[z^p[[2]] / p[[2]]!, {p, FI[n]}]
Oz[n_, z_] := Oz[n, z] = Sum[oz[j, z], {j, 1, n}]
Oz2[n_, k_] := Sum[(-1)^(k - j) bin[k, j] Oz[n, j], {j, 0, 20}]
dOz2[n_, k_] := Oz2[n, k] - Oz2[n - 1, k]
Ozz[n_, z_] := Sum[bin[z, k] Oz2[n, k], {k, 0, Log2@n}]
LOz[n_, k_] := D[Oz[n, z], {z, k}] /. z → 0
dLOz[n_, k_] := LOz[n, k] - LOz[n - 1, k]
Ozzz[n_, z_] := Sum[z^k / k! LOz[n, k], {k, 0, Log2@n}]
roots[n_] := If[(c = Exponent[f = Oz[n, z], z]) == 0, {},
  If[c == 1, List@NRoots[f == 0, z][[2]], List @@ NRoots[f == 0, z][[All, 2]]]]
rroots[n_] := If[(c = Exponent[f = Oz[n, z], z]) == 0, {},
  If[c == 1, List@Roots[f == 0, z][[2]], List @@ Roots[f == 0, z][[All, 2]]]]


Table[Oz2[100, n], {n, 1, 10}] // TableForm
```

```
99
612
2244
4368
4080
1440
0
0
0
0
```

```
Expand@Oz[100, z]
```

$$1 + 25\,z + 34\,z^2 + 22\,z^3 + 12\,z^4 + 4\,z^5 + 2\,z^6$$

```
Ozz[100, 1]
```

```
100
```

```
Table[dLoz[n, k], {n, 1, 20}, {k, 0, 10}] // Grid
```

```
1  0  0  0  0  0  0  0  0  0  0
0  1  0  0  0  0  0  0  0  0  0
0  1  0  0  0  0  0  0  0  0  0
0  0  2  0  0  0  0  0  0  0  0
0  1  0  0  0  0  0  0  0  0  0
0  0  2  0  0  0  0  0  0  0  0
0  1  0  0  0  0  0  0  0  0  0
0  0  0  6  0  0  0  0  0  0  0
0  0  2  0  0  0  0  0  0  0  0
0  0  2  0  0  0  0  0  0  0  0
0  1  0  0  0  0  0  0  0  0  0
0  0  0  6  0  0  0  0  0  0  0
0  1  0  0  0  0  0  0  0  0  0
0  0  2  0  0  0  0  0  0  0  0
0  0  2  0  0  0  0  0  0  0  0
0  0  0  0  24 0  0  0  0  0  0
0  1  0  0  0  0  0  0  0  0  0
0  0  0  6  0  0  0  0  0  0  0
0  1  0  0  0  0  0  0  0  0  0
0  0  0  6  0  0  0  0  0  0  0
```

```
roots[30]
```

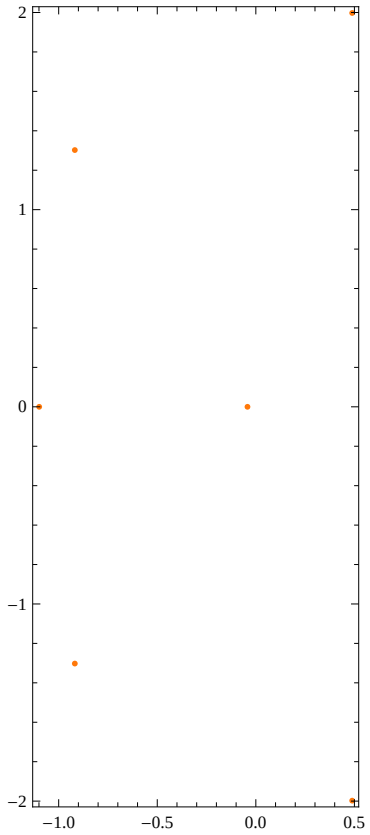
```
{-2.21592, -0.586297 - 1.29619 i, -0.586297 + 1.29619 i, -0.111491}
```

```
rr[n_] := Chop@FullSimplify@Sum[-rho^-1, {rho, roots[n]}]
```

```
rr[100]
```

```
25.
```

```
Graphics[Table[{ColorData["RustTones"][n / 100], Point[{Re[#], Im[#]}]} & /@ roots[n],
  {n, 100, 100}], Frame → True]
```




```
Expand@Sum[Binomial[z, k] (x)^(z - k), {k, 0, Infinity}]
```

$$\left(1+\frac{1}{x}\right)^{z} x^{z}$$