$$\zeta_n(s,2)^1=\sum_{b=2} b^{-s}$$

$$\zeta_n(s,2)^2=\sum_{b=2} b^{-2s}+2\sum_{b=2}\sum_{c=b+1} b^{-s}\cdot c^{-s}$$

$$\begin{aligned}
&\zeta_n(s,2)^3=\\
&\quad\sum_{b=2} b^{-3s}\\
&\quad+3\sum_{b=2}\sum_{c=b+1} b^{-2s}\cdot c^{-s}\\
&\quad+3\sum_{b=2}\sum_{c=b+1} b^{-s}\cdot c^{-2s}\\
&\quad+6\sum_{b=2}\sum_{c=b+1}\sum_{d=c+1} b^{-s}\cdot c^{-s}\cdot d^{-s}
\end{aligned}$$

$$\boxed{[\zeta_n(s,y)]^{*1}=[\zeta_n(s)-\zeta_{y-1}(s)]^{*1}}$$

$$\boxed{[\zeta_n(s,y)]^{*2}=[\zeta_n(s)-\zeta_{y-1}(s)]^{*2}}$$

$$\boxed{[\zeta_n(s,y)]^{*2}=[\zeta_n(s)]^{*2}-2([\zeta_n(s)]^{*1}*[\zeta_{y-1}(s)]^{*1})+[\zeta_{y-1}(s)]^{*2}}$$

$$\boxed{[\zeta_{10}(s,3)]^{*2}=[\zeta_{10}(s)]^{*2}-2([\zeta_{10}(s)]^{*1}*[\zeta_2(s)]^{*1})+[\zeta_2(s)]^{*2}}$$

$$\boxed{\sum_{j=3}^{10}\sum_{k=3}^{\lfloor\frac{10}{j}\rfloor}(j\cdot k)^{-s}=\sum_{j=1}^{10}\sum_{k=1}^{\lfloor\frac{10}{j}\rfloor}(j\cdot k)^{-s}-2\sum_{j=1}^{2}\sum_{k=1}^{\lfloor\frac{10}{j}\rfloor}(j\cdot k)^{-s}+\sum_{j=1}^{2}\sum_{k=1}^{2}(j\cdot k)^{-s}}$$

$$\boxed{[\zeta_n(s,y)]^{*k}=\sum_{j=0}^{k}(-1)^j\binom{k}{j}([\zeta_n(s)]^{*k}*[\zeta_{y-1}(s)]^{*k-j})}$$

$$\boxed{[\zeta_n(s,n)]^{*1}=[\zeta_n(s)-\zeta_{n-1}(s)]^{*1}=n^{-s}}$$

$$[\zeta_n(s,y)]^{*k}=0 \text{ when } n<y^k$$

$$\boxed{[\zeta_{10}(s,10)]^{*2}=[\zeta_{10}(s)]^{*2}-2([\zeta_{10}(s)]^{*1}*[\zeta_9(s)]^{*1})+[\zeta_9(s)]^{*2}}$$

$$[\zeta_n(s,y)]^{*k}=\sum_{j=0}^{k}\binom{k}{j}y^{-sj}[\zeta_{n\cdot(y+1)^{-1}}(s,y+1)]^{*k-j}$$

$$[\zeta_n(s)-\zeta_{y-1}(s)]^{*k}=\sum_{j=0}^{k}\binom{k}{j}y^{-sj}[\zeta_{n\cdot(y+1)^{-1}}(s)-\zeta_{y-1}(s)]^{*k-j}$$

$$[\zeta_n(s,y+1)]^{*k}=\sum_{j=0}^{k}(-1)^j\binom{k}{j}y^{-j\cdot s}[\zeta_{n(y+1)^{-j}}(s,y)]^{*k-j}$$

$$[\zeta_n(s)-\zeta_y(s)]^{*k}=\sum_{j=0}^{k}(-1)^j\binom{k}{j}y^{-j\cdot s}[\zeta_{n(y+1)^{-j}}(s)-\zeta_{y-1}(s)]^{*k-j}$$

$$[f_n]^{*k} = \sum_{j=1} g(j)[f_{n\,g(j)^{-1}}]^{*k-1}$$
$$[f_{\Delta n}]^{*1} = f(g(n)) \cdot \sum_{g(j)=n} 1$$
$$[f_{\Delta n}]^{*k} = \sum_{g(j)\cdot g(r)=n} f(g(j)) \cdot [f_{\Delta n \cdot g(j)^{-1}}]^{*k-1}$$
$$[f_{\Delta n}]^{*1} * [f_{\Delta n}]^{*1} = \sum_{g(j)|n} f(g(j)) \cdot f(n \cdot g(j)^{-1})$$

$$[f_n]^{*a} * [f_n]^{*b} = \sum_{g(j)\cdot g(r)=n} [f_{\Delta g(j)}]^{*a} \cdot [f_{n \cdot g(j)^{-1}}]^{*b}$$

$$[\zeta_n(s)] \rightarrow n$$
$$[\zeta_n(s) - 1] \rightarrow n+1$$
$$[\zeta_n(s, y+1)] \rightarrow n+y$$
$$[\zeta_n(s) - 1] \rightarrow n+1$$
$$[1 + \zeta_n(s, y+1)] \rightarrow n+y$$
$$[x^{1-s} \zeta_n(s)] \rightarrow n \cdot x$$
$$[x^{1-s} \cdot \zeta_n(s, y+1)] \rightarrow n \cdot x + y$$

$$[\zeta_{\Delta n}(s)]^{*k} = \sum_{a_1 \cdot a_2 \cdot \ldots \cdot a_k = n} a_1^{-s} \cdot a_2^{-s} \cdot \ldots \cdot a_k^{-s}$$

$$[\zeta_{\Delta n}(s)]^{*1} = n^{-s}$$
$$[\zeta_{\Delta n}(s)]^{*2} = \sum_{a \cdot b = n} a^{-s} \cdot b^{-s} = n^{-s} \sum_{a \cdot b = n} 1$$

$$\boxed{\sum_{n=1}^{\infty} [\zeta_{\Delta n}(s)]^{*k} = \zeta(s)^k}$$

$$[\zeta_{\Delta n}(s,2)]^{*k} = \sum_{(a_1+1) \cdot (a_2+1) \cdot \ldots \cdot (a_k+1) = n} (a_1+1)^{-s} \cdot (a_2+1)^{-s} \cdot \ldots \cdot (a_k+1)^{-s}$$

$$[\zeta_{\Delta n}(s,2)]^{*1} = n^{-s} \cdot \sum_{(a+1)=n} 1$$
$$[\zeta_{\Delta n}(s,2)]^{*2} = \sum_{(a+1) \cdot (b+1) = n} (a+1)^{-s} \cdot (b+1)^{-s}$$

$$\boxed{\sum_{n=1}^{\infty} [\zeta_{\Delta n}(s,2)]^{*k} = \zeta(s,2)^k}$$

$$[\zeta_{\Delta n}(s,y+1)]^{*k} = \sum_{(a_1+y) \cdot (a_2+y) \cdot \ldots \cdot (a_k+y) = |n|} (a_1+y)^{-s} \cdot (a_2+y)^{-s} \cdot \ldots \cdot (a_k+y)^{-s}$$

$$[\zeta_{\Delta n}(s,y+1)]^{*1} = \sum_{(a+y)=n} (a+y)^{-s}$$
$$[\zeta_{\Delta n}(s,y+1)]^{*2} = \sum_{(a+y) \cdot (b+y) = n} (a+y)^{-s} \cdot (b+y)^{-s}$$

$$\boxed{\lim_{x \to 0} \sum_{n=1}^{\infty} [\zeta_{\Delta (nx)}(s,y)]^{*k} = \zeta(s,y)^k}$$

$$[\log \zeta_{\Delta n}(s)]^{*k} = \sum_{(a_1+1) \cdot (a_2+1) \cdot \ldots \cdot (a_k+1) = n} \kappa(a_1+1) \cdot (a_1+1)^{-s} \cdot \kappa(a_2+1) \cdot (a_2+1)^{-s} \cdot \ldots \cdot \kappa(a_k+1) \cdot (a_k+1)^{-s}$$

$$[\log \zeta_{\Delta n}(s)]^{*1} = \sum_{(a+1)=n} \kappa(a+1) \cdot (a+1)^{-s}$$
$$[\log \zeta_{\Delta n}(s)]^{*2} = \sum_{(a+1) \cdot (b+1) = n} \kappa(a+1) \cdot (a+1)^{-s} \cdot \kappa(b+1) \cdot (b+1)^{-s}$$

$$[1+\zeta_{\Delta n}(s,y+1)]^{*k}=$$
$$\sum_{(a_1+y)=|n|}(a_1+y)^{-s}$$
$$+\sum_{(a_1+y)\cdot(a_2+y)=|n|}(a_1+y)^{-s}\cdot(a_2+y)^{-s}$$
$$+\ldots$$
$$+\sum_{(a_1+y)\cdot(a_2+y)\cdot\ldots\cdot(a_k+y)=|n|}(a_1+y)^{-s}\cdot(a_2+y)^{-s}\cdot\ldots\cdot(a_k+y)^{-s}$$

$$[1+\zeta_{\Delta n}(s,y+1)]^{*1}=\sum_{(a+y)=|n|}(a+y)^{-s}$$
$$[1+\zeta_{\Delta n}(s,y+1)]^{*2}=\sum_{(a+y)=|n|}(a+y)^{-s}+\sum_{(a+y)(b+y)=n}(a+y)^{-s}\cdot(b+y)^{-s}$$

$$[x^{1-s}\zeta_{\Delta n}(s)]^{*k}=\sum_{(a_1 x)\cdot(a_2 x)\cdot\ldots\cdot(a_k x)=|n|}x(a_1 x)^{-s}\cdot x(a_2 x)^{-s}\cdot\ldots\cdot x(a_k x)^{-s}$$

$$[x^{1-s}\zeta_{\Delta n}(s)]^{*1}=\sum_{(a x)=n}x(a x)^{-s}$$
$$[x^{1-s}\zeta_{\Delta n}(s)]^{*2}=\sum_{(a x)\cdot(b x)=n}x(a x)^{-s}\cdot x(b x)^{-s}$$

$$[1+x^{1-s}\zeta_n(s)]^{*k}=[1+x^{1-s}\cdot\zeta_n(s)]^{*k-1}+x\sum_{j=1}(j x)^{-s}[x^{1-s}\cdot\zeta_{n(j x)^{-1}}(s)]^{*k-1}$$

// ???????

$$[x^{1-s}\cdot\zeta_n(s,a+1)]^{*k}=x\sum_{j=1}(j x+a)^{-s}[x^{1-s}\cdot\zeta_{n(j x+a)^{-1}}(s,a+1)]^{*k-1}$$

$$[1+x^{1-s}\cdot\zeta_n(s,a+1)]^{*k}=[1+x^{1-s}\cdot\zeta_n(s,a+1)]^{*k-1}+x\sum_{j=1}(j x+a)^{-s}[1+x^{1-s}\cdot\zeta_{n(j x+a)^{-1}}(s,a+1)]^{*k-1}$$
$$[(1-x^{1-s})\zeta_n(s)-1]^{*k}=\sum_{j=1}(j+1)^{-s}[(1-x^{1-s})\zeta_{n(j+1)^{-1}}(s)-1]^{*k-1}-x\cdot(j x)^{-s}[(1-x^{1-s})\zeta_{n(j x)^{-1}}(s)]^{*k-1}$$
$$[(1-x^{1-s})\zeta_n(s)]^{*k}=\sum_{j=1}j^{-s}[(1-x^{1-s})\zeta_{n j^{-1}}(s)]^{*k-1}-x\cdot(j x)^{-s}[(1-x^{1-s})\zeta_{n(j x)^{-1}}(s)]^{*k-1}$$
$$[\zeta_n(s)^{-1}-1]^{*k}=\sum_{j=1}\mu(j+1)(j+1)^{-s}[\zeta_{n(j+1)^{-1}}(s)^{-1}-1]^{*k-1}$$
$$[\zeta_n(s)^{z}-1]^{*k}=\sum_{j=1}d_z(j+1)(j+1)^{-s}[\zeta_{n(j+1)^{-1}}(s)^{-1}-1]^{*k-1}$$

$$[f_{\Delta n}(y+1)]^{*k} = \sum_{(a+y)\cdot(b+y)=n} f(a+y)\cdot[f_{\Delta(b+y)}(y+1)]^{*k-1}$$

$$((f+y)*(f+y)) = \sum_{(a+y)\cdot(b+y)=n} f(a+y)\cdot f(b+y)$$

$$(x(f+y)*x(f+y)) = \sum_{x(a+y)\cdot x(b+y)=n} f(x(a+y))\cdot f(x(b+y))$$

$$(x(f+y)*x(f+y)*x(f+y)) = \sum_{x(a+y)\cdot x(b+y)\cdot x(c+y)=n} f(x(a+y))\cdot f(x(b+y))\cdot f(x(c+y))$$

Convolutions

$$\lim_{n\to\infty}[\zeta_n(s)]^{*\rho}=0 \quad\rightarrow\quad \zeta(s)=\prod_\rho(1-\frac{1}{\rho})$$

$$\rightarrow\quad \zeta(s)^z=\prod_\rho(1-\frac{z}{\rho})$$

$$\rightarrow\quad \log\zeta(s)=-\sum_\rho\frac{1}{\rho}$$

$$\lim_{s\to0}\frac{\partial}{\partial s}[\zeta_n(s)]^{*\rho}-1=0 \quad\rightarrow\quad \log n!=-1+\prod_\rho(1-\frac{1}{\rho})$$

$$\rightarrow\quad \psi(n)=-\sum_\rho\frac{1}{\rho}$$

$$[\zeta_n(0)]^{*\rho}=0 \quad\rightarrow\quad \Pi(n)=-\sum_\rho\frac{1}{\rho}$$

$$\rightarrow\quad n=\prod_\rho(1-\frac{1}{\rho})$$

$$\rightarrow\quad D(n)=\prod_\rho(1-\frac{2}{\rho})$$

$$\rightarrow\quad D_k(n)=\prod_\rho(1-\frac{k}{\rho})$$

$$\rightarrow\quad M(n)=\prod_\rho(1+\frac{1}{\rho})$$

$$\lim_{y\to\infty}[1+y^{s-1}\cdot\zeta_n(0,1+y)]^{*\rho}=0 \quad\rightarrow\quad L_{-z}(\log n)=\prod_\rho(1-\frac{z}{\rho})$$

$$\rightarrow\quad li(n)-\log\log n-\gamma=-\sum_\rho\frac{1}{\rho}$$

$$[\zeta_n(1)]^{*\rho}=0 \quad\rightarrow\quad H_n=\prod_\rho(1-\frac{1}{\rho})$$

$$\lim_{n\to\infty}[\zeta_n(2)]^{*\rho}=0 \quad\rightarrow\quad \frac{\pi^2}{6}=\prod_\rho(1-\frac{1}{\rho})$$

$$\lim_{n\to\infty}[\eta_n(1)]^{*\rho}=0 \quad\rightarrow\quad \log 2=\prod_\rho(1-\frac{1}{\rho})$$

This can go way more general.

$$[\zeta_n(s)]^{*z} = 1 + \binom{z}{1}\sum_{j=2}^{\lfloor n\rfloor}\frac{1}{j^s} + \binom{z}{2}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor \frac{n}{j}\rfloor}\frac{1}{j^s\cdot k^s} + \binom{z}{3}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor \frac{n}{j}\rfloor}\sum_{l=2}^{\lfloor \frac{n}{j\cdot k}\rfloor}\frac{1}{j^s\cdot k^s\cdot l^s} + \ldots$$

$$[e_n]^{*z} = \binom{z}{0}1 + \binom{z}{1}\sum_{j=2}^{\lfloor n\rfloor}\frac{1}{\Gamma(j)} + \binom{z}{2}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor \frac{n}{j}\rfloor}\frac{1}{\Gamma(j)\cdot\Gamma(k)} + \binom{z}{3}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor \frac{n}{j}\rfloor}\sum_{l=2}^{\lfloor \frac{n}{j\cdot k}\rfloor}\frac{1}{\Gamma(j)\cdot\Gamma(k)\cdot\Gamma(l)} + \ldots$$

$$[e_n]^{*z} = \binom{z}{0}1 + \binom{z}{1}[e_n-1]^{*1} + \binom{z}{2}[e_n-1]^{*2} + \binom{z}{3}[e_n-1]^{*3} + \ldots$$

$$[e_n]^{*z} = \sum_{k=0}^{\infty}\binom{z}{k}[e_n-1]^{*k}$$

$$[e_n]^{*k} = \sum_{j=1}^{n}\frac{1}{j!}[e_{n\cdot j^{-1}}]^{*k-1}$$

$$[e_n-1]^{*k} = \sum_{j=2}^{n}\frac{1}{j!}[e_{n\cdot j^{-1}}-1]^{*k-1}$$

$$[\log e_n]^{*1} = \lim_{z\to 0}\frac{\partial}{\partial z}\sum_{k=0}^{\infty}\binom{z}{k}[e_n-1]^{*k}$$

$$[\log e_n]^{*k} = \lim_{z\to 0}\frac{\partial^k}{\partial z^k}\sum_{k=0}^{\infty}\binom{z}{k}[e_n-1]^{*k}$$

$$[e_n]^{*z} = \sum_{k=0}^{\infty}\frac{z^k}{k!}[\log e_n]^{*k}$$

where it converges,

$$\lim_{n\to\infty}[e_n]^{*z} = e^z$$

$$[\cos_n]^{*z} = \sum_{k=0}^{\infty}\frac{z^k}{k!}\left(\lim_{x\to 0}\frac{\partial^k}{\partial x^k}\cos x\right)[\log e_n]^{*k}$$

$$[\sin_n]^{*z} = \sum_{k=0}^{\infty}\frac{z^k}{k!}\left(\lim_{x\to 0}\frac{\partial^k}{\partial x^k}\sin x\right)[\log e_n]^{*k}$$

$$[e_n]^{*Iz} = [\cos_n]^{*z} + i[\sin_n]^{*z}$$

$$[\log e_n]^{*1} = \sum_{j=2}^{\lfloor n\rfloor}\frac{1}{j!} - \frac{1}{2}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor \frac{n}{j}\rfloor}\frac{1}{j!}\frac{1}{k!} + \frac{1}{3}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor \frac{n}{j}\rfloor}\sum_{l=2}^{\lfloor \frac{n}{j\cdot k}\rfloor}\frac{1}{j!}\frac{1}{k!}\frac{1}{l!} + \ldots$$

$$\lim_{n\to\infty}[\zeta_n(s)]^{*z} = 1 + \binom{z}{1}\sum_{j=2}^{\infty}j^{-s} + \binom{z}{2}\sum_{j=2}^{\infty}\sum_{k=2}^{\infty}(j\cdot k)^{-s} + \binom{z}{3}\sum_{j=2}^{\infty}\sum_{k=2}^{\infty}\sum_{l=2}^{\infty}(j\cdot k\cdot l)^{-s} + \ldots$$

$$\lim_{n\to\infty}[e_n]^{*z}=$$

$$\binom{z}{0}1+\binom{z}{1}\sum_{j=2}^{\infty}\frac{1}{\Gamma(j)}+\binom{z}{2}\sum_{j=2}^{\infty}\sum_{k=2}^{\infty}\frac{1}{\Gamma(j)}\frac{1}{\Gamma(k)}+\binom{z}{3}\sum_{j=2}^{\infty}\sum_{k=2}^{\infty}\sum_{l=2}^{\infty}\frac{1}{\Gamma(j)}\frac{1}{\Gamma(k)}\frac{1}{\Gamma(l)}+\ldots$$

$$[e_{\Delta n}]^{*1}=\sum_{j=n}\frac{1}{\Gamma(j)}$$

$$[e_{\Delta n}]^{*2}=\sum_{j\cdot k=n}\frac{1}{\Gamma(j)}\cdot\frac{1}{\Gamma(k)}$$

$$[e_{\Delta n}]^{*3}=\sum_{j\cdot k\cdot l=n}\frac{1}{\Gamma(j)}\cdot\frac{1}{\Gamma(k)}\cdot\frac{1}{\Gamma(l)}$$

$$[e_{\Delta 15}]^{*2}=2\frac{1}{\Gamma(1)}\cdot\frac{1}{\Gamma(15)}+2\frac{1}{\Gamma(3)}\cdot\frac{1}{\Gamma(5)}$$

$$[e_{\Delta 35}]^{*1}=\frac{1}{\Gamma(35)}$$

$$[e_{\Delta 35}]^{*2}=2\frac{1}{\Gamma(35)}+2\frac{1}{\Gamma(5)}\cdot\frac{1}{\Gamma(7)}$$

$$[e_{\Delta 35}]^{*3}=3\frac{1}{\Gamma(35)}+6\frac{1}{\Gamma(5)}\cdot\frac{1}{\Gamma(7)}$$

$$[e_{\Delta 35}]^{*4}=4\frac{1}{\Gamma(35)}+12\frac{1}{\Gamma(5)}\cdot\frac{1}{\Gamma(7)}$$

$$[e_{\Delta 35}]^{*k}=\binom{k}{1}\frac{1}{\Gamma(35)}+\binom{k}{1}\binom{k-1}{1}\frac{1}{\Gamma(5)}\cdot\frac{1}{\Gamma(7)}$$

$$e^x\cdot e^y=e^{x+y}$$

// For 0 < x < 2

$$[\log x_n]^{*1}=\sum_{j=1}^{\lfloor n\rfloor}-j^{-1}(1-x)^j$$

$$[\log x_n]^{*2}=\sum_{j=1}^{\lfloor n\rfloor}\sum_{k=1}^{\lfloor \frac{n}{j}\rfloor}(j\cdot k)^{-1}(1-x)^{j+k}$$

$$[\log x_n]^{*3}=\sum_{j=1}^{\lfloor n\rfloor}\sum_{k=1}^{\lfloor \frac{n}{j}\rfloor}\sum_{l=1}^{\lfloor \frac{n}{jk}\rfloor}-(j\cdot k\cdot l)^{-1}(1-x)^{j+k+l}$$

$$[\log x_n]^{*k}=\sum_{j=1}^{\infty}\left(-j^{-1}(1-x)^j\right)[\log x_{n\cdot j^{-1}}]^{*k-1}$$

$$[x_n]^{*z}=\sum_{k=0}^{\infty}\frac{z^k}{k!}[\log x_n]^{*k}$$

$$[x_n]^{*-z}=\sum_{k=0}^{\infty}\frac{z^k}{k!}[\log(x^{-1})_n]^{*k}$$

either $\lim_{n\to\infty}[x_n]^{*z}=x^z$ or $\lim_{n\to\infty}[(x^{-1})_n]^{*-z}=x^z$

$$[(?)\zeta_n(s)]^{*z}=1+\binom{z}{1}\sum_{j=2}^{\lfloor n\rfloor}\frac{1}{j^s}x^j+\binom{z}{2}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}\frac{1}{j^s\cdot k^s}x^{j+k}+\binom{z}{3}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}\sum_{l=2}^{\lfloor\frac{n}{j\cdot k}\rfloor}\frac{1}{j^s\cdot k^s\cdot l^s}x^{j+k+l}+\ldots$$

// only converges for -1 to 1

$$\lim_{n\to\infty}[(?)\zeta_n(s)]^{*z}=\left(\frac{1}{1-x}-x\right)^z$$

Not really multiplicatively interesting.

$$[\log\zeta_n(s)]^{*1}=\sum_{j=2}^{\lfloor n\rfloor}\frac{1}{j^s}-\frac{1}{2}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}\frac{1}{j^s\cdot k^s}+\frac{1}{3}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}\sum_{l=2}^{\lfloor\frac{n}{j\cdot k}\rfloor}\frac{1}{j^s\cdot k^s\cdot l^s}-\ldots$$

$$[\log e_n]^{*1}=\sum_{j=2}^{\lfloor n\rfloor}\frac{1}{\Gamma(j)}-\frac{1}{2}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}\frac{1}{\Gamma(j)\cdot\Gamma(k)}+\frac{1}{3}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}\sum_{l=2}^{\lfloor\frac{n}{j\cdot k}\rfloor}\frac{1}{\Gamma(j)\cdot\Gamma(k)\cdot\Gamma(l)}-\ldots$$

what happens in we put a j^-s in there?

$$\boxed{\zeta_n(s)=\sum_{j=1}^{n} j^{-s}}$$

$$\boxed{\zeta_n(s)^2=\sum_{j=1}^{n}\sum_{k=1}^{n}(j\cdot k)^{-s}}$$

$$\boxed{\zeta_n(s)^3=\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n}(j\cdot k\cdot l)^{-s}}$$

$$\boxed{\zeta_n(s)-1=\sum_{j=2}^{n} j^{-s}}$$

$$\boxed{(\zeta_n(s)-1)^2=\sum_{j=2}^{n}\sum_{k=2}^{n}(j\cdot k)^{-s}}$$

$$\boxed{(\zeta_n(s)-1)^3=\sum_{j=2}^{n}\sum_{k=2}^{n}\sum_{l=2}^{n}(j\cdot k\cdot l)^{-s}}$$

$$\boxed{\zeta_n(s)^z=\sum_{k=0}^{\infty}\binom{z}{k}(\zeta_n(s)-1)^k}$$

$$\boxed{\zeta_n(s)^z=1+\binom{z}{1}\sum_{j=2}^{n} j^{-s}+\binom{z}{2}\sum_{j=2}^{n}\sum_{k=2}^{n}(j\cdot k)^{-s}+\binom{z}{3}\sum_{j=2}^{n}\sum_{k=2}^{n}\sum_{l=2}^{n}(j\cdot k\cdot l)^{-s}+\ldots}$$

$$\boxed{[\zeta_n(s)]^{*z}=1+\binom{z}{1}\sum_{j=2}^{\lfloor n\rfloor}\frac{1}{j^s}+\binom{z}{2}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor \frac{n}{j}\rfloor}\frac{1}{j^s\cdot k^s}+\binom{z}{3}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor \frac{n}{j}\rfloor}\sum_{l=2}^{\lfloor \frac{n}{j\cdot k}\rfloor}\frac{1}{j^s\cdot k^s\cdot l^s}+\ldots}$$

$$\boxed{[\zeta_n(s)]^{*2}=\left(\sum_{j=1}^{n} j^{-s}\right)^2-\sum_{j=1}^{n}\sum_{k=\lfloor n j^{-1}+1\rfloor}^{n}(j\cdot k)^{-s}}$$

$$\boxed{[\zeta_n(s)]^{*3}=\left(\sum_{j=1}^{n} j^{-s}\right)^3-\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=\lfloor n(j\cdot k)^{-1}+1\rfloor}^{n}(j\cdot k\cdot l)^{-s}}$$

$$\boxed{[\zeta_n(s)-1]^{*z} = 1 + \binom{z}{1} \sum_{j=3}^{\lfloor \frac{n}{2^{1-z}} \rfloor} \frac{1}{j^s} + \binom{z}{2} \sum_{j=3}^{\lfloor \frac{n}{2^{1-z}} \rfloor} \sum_{k=3}^{\lfloor \frac{n}{2^{2-z}} \cdot j \rfloor} \frac{1}{j^s \cdot k^s} + \binom{z}{3} \sum_{j=3}^{\lfloor \frac{n}{2^{1-z}} \rfloor} \sum_{k=3}^{\lfloor \frac{n}{2^{2-z} j} \rfloor} \sum_{l=3}^{\lfloor \frac{n}{2^{3-z} j \cdot k} \rfloor} \frac{1}{j^s \cdot k^s \cdot l^s} + \ldots}$$

$$\boxed{(a+2)(b+2)\,2^{z-2} \le n}$$

$$\boxed{(a+2)(b+2)(c+2)\,2^{z-3} \le n}$$

$$\boxed{(a+2)(b+2)\,2^{1-2} \le 12}$$

$$\boxed{(a+2)(b+2)\frac{1}{2} \le 12}$$

$\boxed{1 \le 1}$ smallest → 1

$\boxed{(a+2) \le 12}$ smallest → 3

$\boxed{(a+2)(b+2) \le 24}$ smallest → 9

$\boxed{(a+2)(b+2)(c+2) \le 48}$ smallest → 27

$\boxed{(a+2)(b+2)(c+2)(d+2) \le 96}$ smallest → 81

$\boxed{(a+2)(b+2)(c+2)(d+2)(e+2) \le 192}$ smallest → 243

$$\boxed{(a+2)(b+2)\,2^{1.5-2} \le 16.9\ldots}$$

$$\boxed{(a+2)(b+2)(c+2)\,2^{1.5-3} \le 33.9\ldots}$$

$$\boxed{(a+2)(b+2)\,2^{.01-2} \le 47.6\ldots}$$

$$\boxed{(a+2)(b+2)(c+2)\,2^{.01-3} \le 95.3\ldots}$$

$\boxed{\dfrac{n\,a^{k-z}}{(a+1)^k} \ge 0}$ … solve for k.

$$\boxed{n\,a^{k-z} \ge (a+1)^k}$$

$$\boxed{a^{k-z} \ge \frac{(a+1)^k}{n}}$$

$$\boxed{(k-z)\log a \ge k\log(a+1) - \log n}$$

$$\boxed{[\zeta_n(s,y+1)]^{*z}=1+\binom{z}{1}\sum_{j=1}^{\lfloor\frac{n}{(y+1)^{1-z}}\rfloor}(j+y)^{-s}+\binom{z}{2}\sum_{j=1}^{\lfloor\frac{n}{(y+1)^{2-z}}\rfloor}\sum_{k=1}^{\lfloor\frac{n}{(y+1)^{2-z}\cdot j}\rfloor}((j+y)\cdot(k+y))^{-s}+\binom{z}{3}\sum_{j=1}^{\lfloor\frac{n}{(y+1)^{1-z}}\rfloor}\sum_{k=1}^{\lfloor\frac{n}{(y+1)^{1-z}j}\rfloor}\sum_{l=1}^{\lfloor\frac{n}{(y+1)^{3-z}j\cdot k}\rfloor}((j+y)\cdot(k+y)\cdot(l+y))^{-s}+\ldots}$$

$$\boxed{[\log\zeta_n(s)]^{*1}=\frac{\partial}{\partial z}[\zeta_n(s)]^{*z}*[\zeta_n(s)]^{*-z}}$$

$$\boxed{[\log\zeta_n(s)]^{k}=\frac{\partial^k}{\partial z^k}[\zeta_n(s)]^{*z}*[\zeta_n(s)]^{*-z}}$$

$$\boxed{\Pi(n)=\sum_{j=1}^{n}\frac{\partial}{\partial z}d_z(j)[\zeta_{n j^{-1}}(0)]^{*-z}}$$

$$\frac{\partial}{\partial s}[\log\zeta_n(s)]^{*1}=-\frac{\partial}{\partial s}([\zeta_n(s)]^{*1}*[\zeta_n(s)]^{*-1})$$

Generalized harmonic numbers

$$\alpha_b(n)=b\cdot(\lfloor\frac{n}{b}\rfloor-\lfloor\frac{n-1}{b}\rfloor)-(b+1)\cdot(\lfloor\frac{n}{b+1}\rfloor-\lfloor\frac{n-1}{b+1}\rfloor)$$

$$\lim_{b\to\infty}H_{\lfloor\frac{\log n}{\log(b+1)-\log b}\rfloor}+\frac{1}{b}\sum_{j=b+1}^{b\cdot n}\alpha_b(j)-\frac{1}{2}\frac{1}{b^2}\sum_{j=b+1}^{b\cdot n}\sum_{k=b+1}^{\lfloor\frac{b^2\cdot n}{j}\rfloor}\alpha_b(j)\cdot\alpha_b(k)+\frac{1}{3}\frac{1}{b^3}\sum_{j=b+1}^{b\cdot n}\sum_{k=b+1}^{\lfloor\frac{b^2\cdot n}{j}\rfloor}\sum_{l=b+1}^{\lfloor\frac{b^3\cdot n}{j\cdot k}\rfloor}\alpha_b(j)\cdot\alpha_b(k)\cdot\alpha_b(l)-\frac{1}{4}\ldots$$

$$f_k(n)=b^{-1}\sum_{j=b+1}^{\lfloor b\cdot n\rfloor}\alpha_b(j)(k^{-1}-f_{k+1}(n\cdot b\cdot j^{-1}))$$

$$\Pi(n)-li(n)+\log\log n+\gamma=\lim_{b\to\infty}H_{\lfloor\frac{\log n}{\log(b+1)-\log b}\rfloor}+f_1(n)$$

$$\lim_{b\to\infty}\frac{1}{b}\sum_{j=b+1}^{b\cdot n}\alpha_b(j)\log\frac{j}{b}+\frac{1}{b^2}\sum_{j=b+1}^{b\cdot n}\sum_{k=b+1}^{\lfloor\frac{b^2\cdot n}{j}\rfloor}\alpha_b(j)\cdot\alpha_b(k)\log\frac{k}{b}-\frac{1}{b^3}\sum_{j=b+1}^{b\cdot n}\sum_{k=b+1}^{\lfloor\frac{b^2\cdot n}{j}\rfloor}\sum_{l=b+1}^{\lfloor\frac{b^3\cdot n}{j\cdot k}\rfloor}\alpha_b(j)\cdot\alpha_b(k)\cdot\alpha_b(l)\log\frac{l}{b}+\ldots$$

$$f(n)=b^{-1}\sum_{j=b+1}^{\lfloor b\cdot n\rfloor}\alpha_b(j)(\log j\cdot b^{-1}-f(n\cdot b\cdot j^{-1}))$$

$$\psi(n)-n+1=\lim_{b\to\infty}f(n)$$

$$f_z(n,s)=1+\frac{z^1}{1!}\sum_{j=2}^{\lfloor n\rfloor}\kappa(j)\cdot j^{-s}+\frac{z^2}{2!}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}\kappa(j)\kappa(k)\cdot(jk)^{-s}+\frac{z^3}{3!}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}\sum_{l=2}^{\lfloor\frac{n}{j\cdot k}\rfloor}\kappa(j)\kappa(k)\kappa(l)(jkl)^{-s}+\frac{z^4}{4!}\ldots$$

$$\frac{\partial}{\partial z}f_z(n,s)=\sum_{j=2}^{\lfloor n\rfloor}\kappa(j)\cdot j^{-s}+\frac{z}{1!}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}\kappa(j)\kappa(k)\cdot(jk)^{-s}+\frac{z^2}{2!}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}\sum_{l=2}^{\lfloor\frac{n}{j\cdot k}\rfloor}\kappa(j)\kappa(k)\kappa(l)(jkl)^{-s}+\frac{z^3}{3!}\ldots$$

$$\frac{\partial^2}{\partial z^2}f_z(n,s)=\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}\kappa(j)\kappa(k)\cdot(jk)^{-s}+\frac{z}{1!}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}\sum_{l=2}^{\lfloor\frac{n}{j\cdot k}\rfloor}\kappa(j)\kappa(k)\kappa(l)(jkl)^{-s}+\frac{z^2}{2!}\ldots$$

$$\frac{\partial^3}{\partial z^3}f_z(n,s)=\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}\sum_{l=2}^{\lfloor\frac{n}{j\cdot k}\rfloor}\kappa(j)\kappa(k)\kappa(l)(jkl)^{-s}+\frac{z^1}{1!}\ldots$$

$$\frac{\partial}{\partial z}f_z(n,s)=(\lim_{z\to 0}\frac{\partial}{\partial z}f_z(n,s))*f_z(n,s)$$

$$\frac{\partial^2}{\partial z^2}f_z(n,s)=(\lim_{z\to 0}\frac{\partial^2}{\partial z^2}f_z(n,s))*f_z(n,s)$$

$$\frac{n}{(1+\frac{j}{y})}\geq 1 \qquad n\geq(1+\frac{j}{y}) \qquad \lfloor y(n-1)\rfloor\geq j$$

$$\frac{n}{(1+\frac{j}{y})(1+\frac{k}{y})}\geq 1 \qquad \frac{n}{(1+\frac{j}{y})}\geq(1+\frac{k}{y}) \qquad y(\frac{n}{(1+\frac{j}{y})}-1)\geq k$$

$$\frac{ny^3}{(y+j)(y+k)}-y\geq l \qquad y(\frac{n}{(1+\frac{j}{y})(1+\frac{k}{y})}-1)\geq k$$

$$ny-y\geq j \qquad \frac{ny^2}{y+j}-y\geq k \qquad \frac{ny^3}{(y+j)(y+k)}-y\geq l$$