```
t[n_, a_] := Mod[n, a] - Mod[n - 1, a]
Sum[ (-1)^(k) 1 / (2 k - 1), {k, 0, Infinity}]
```

$$\frac{1}{4}\ (-4-\pi)$$

```
Sum[ t[k, 2] 1 / (2 k - 1), {k, 0, Infinity}]
```

$$\sum_{k=0}^{\infty} \frac{-\text{Mod}[-1+k,\ 2]+\text{Mod}[k,\ 2]}{-1+2\,k}$$

```
Series[Tan[x], {x, 0, 20}]
```

$$x+\frac{x^3}{3}+\frac{2\,x^5}{15}+\frac{17\,x^7}{315}+\frac{62\,x^9}{2835}+\frac{1382\,x^{11}}{155\,925}+\frac{21\,844\,x^{13}}{6\,081\,075}+\frac{929\,569\,x^{15}}{638\,512\,875}+\frac{6\,404\,582\,x^{17}}{10\,854\,718\,875}+\frac{443\,861\,162\,x^{19}}{1\,856\,156\,927\,625}+O[x]^{21}$$

```
Series[ArcTan[x], {x, 0, 20}]
```

$$x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\frac{x^9}{9}-\frac{x^{11}}{11}+\frac{x^{13}}{13}-\frac{x^{15}}{15}+\frac{x^{17}}{17}-\frac{x^{19}}{19}+O[x]^{21}$$

```
Tan[Pi / 10]
```

$$\sqrt{1-\frac{2}{\sqrt{5}}}$$

```
Tan[Pi / 6]
```

$$\frac{1}{\sqrt{3}}$$

```
Tan[Pi / 12]
```

$$2-\sqrt{3}$$

```
Tan[Pi / 3]
```

$$\sqrt{3}$$

```
Tan[Pi / 5]
```

$$\sqrt{5-2\sqrt{5}}$$

```
N[Pi / 4]
```

0.785398

```
N[Sum[ t[k, 2] 1 / (2 k - 1), {k, 1, Infinity}]]
```

0.785398

```
N[Sum[ t[k, 3] 1 / (2 k - 1), {k, 1, Infinity}]]
```

$Aborted

```
N[Sum[ t[k, 4] 1 / (2 k - 1), {k, 1, Infinity}]]
```

```
Sum[ (-1)^(k) 1 / (k!), {k, 0, Infinity}]
```

$$\frac{1}{e}$$

```
Sum[ t[k, 2] 1 / (k!), {k, 0, Infinity}]
```

```
FullSimplify[ (-1 + e^2 - e Sqrt[2 π] BesselI[-1/2, 1]) / (2 e) ]
```

$$-\frac{1}{e}$$

```
Sum[ t[k, 3] 1 / (k!), {k, 0, Infinity}]
```

```
FullSimplify[ -(2 Cos[Sqrt[3]/2]) / Sqrt[e] ]
```

$$-\frac{2 \cos\left[\frac{\sqrt{3}}{2}\right]}{\sqrt{e}}$$

```
Sum[ t[k, 4] 1 / (k!), {k, 0, Infinity}]
```

```
FullSimplify[ 1/2 ( -Sqrt[2 π] BesselI[-1/2, 1] + Sqrt[2 π] BesselI[1/2, 1] - 2 Sqrt[2 π] BesselJ[-1/2, 1] ) ]
```

$$-\frac{1}{e} - 2\cos[1]$$

```
Sum[ t[k, 5] 1 / (k!), {k, 0, Infinity}]
```

```
FullSimplify[ 1/24 ( -96 HypergeometricPFQ[{}, {1/5, 2/5, 3/5, 4/5}, 1/3125] +
    24 HypergeometricPFQ[{}, {2/5, 3/5, 4/5, 6/5}, 1/3125] +
    12 HypergeometricPFQ[{}, {3/5, 4/5, 6/5, 7/5}, 1/3125] +
    4 HypergeometricPFQ[{}, {4/5, 6/5, 7/5, 8/5}, 1/3125] +
    HypergeometricPFQ[{}, {6/5, 7/5, 8/5, 9/5}, 1/3125] ) ]
```

```
Expand[ -e^-(-1)^(1/5) - e^(-1)^(2/5) - e^-(-1)^(3/5) - e^(-1)^(4/5) ]
```

$$-e^{-(-1)^{1/5}} - e^{(-1)^{2/5}} - e^{-(-1)^{3/5}} - e^{(-1)^{4/5}}$$

```
FullSimplify[Sum[ t[k, 6] 1 / (k!), {k, 0, Infinity}]]
```

$$-\frac{1}{e} - 4\cos\left[\frac{\sqrt{3}}{2}\right]\cosh\left[\frac{1}{2}\right]$$

```
FullSimplify[Sum[ t[k, 7] 1 / (k!), {k, 0, Infinity}]]
```

$$-e^{-(-1)^{1/7}} - e^{(-1)^{2/7}} - e^{-(-1)^{3/7}} - e^{(-1)^{4/7}} - e^{-(-1)^{5/7}} - e^{(-1)^{6/7}}$$

```
FullSimplify[Sum[ t[k, 8] 1 / (k!), {k, 0, Infinity}]]
```

$$-\frac{1}{e}-2\left(\text{Cos}[1]+2\,\text{Cos}\left[\frac{1}{\sqrt{2}}\right]\text{Cosh}\left[\frac{1}{\sqrt{2}}\right]\right)$$

```
FullSimplify[Sum[ t[k, 9] 1 / (k!), {k, 0, Infinity}]]
```

```
$Aborted
```

```
FullSimplify[Sum[ t[k, 10] 1 / (k!), {k, 0, Infinity}]]
```

```
1 + 1 / 3 - 2 / 5 + 1 / 7 + 1 / 9 - 2 / 11 + 1 / 13 + 1 / 15 - 2 / 17 +
1 - 1 / 3 + 1 / 5 - 1 / 7 + 1 / 9 - 1 / 11 + 1 / 13 - 1 / 15 + 1 / 17 -
2 / 3 - 3 / 5 + 2 / 7 - 3 / 11 + 2 / 15 - 3 / 17
2 - 1 / 5 + 2 / 9 - 3 / 11 + 2 / 13 - 1 / 17
Expand[Sum[ 1 / (4 k + 1) - 1 / (4 k + 3), {k, 0, Infinity}]]
```

$$\frac{\pi}{4}$$

```
FullSimplify[Sum[ 1 / (6 k + 1) + 1 / (6 k + 3) - 2 / (6 k + 5), {k, 0, Infinity}]]
```

$$\frac{1}{4}\left(\sqrt{3}\ \pi-\text{Log}[3]\right)$$

```
Expand[Sum[ 1 / (8 k + 1) + 1 / (8 k + 3) + 1 / (8 k + 5) - 3 / (8 k + 7), {k, 0, Infinity}]]
```

$$\text{FullSimplify}\left[\frac{\pi}{4}+\frac{\pi}{2\sqrt{2}}-\frac{\text{Log}[2]}{2\sqrt{2}}+\frac{\text{Log}\left[2-\sqrt{2}\right]}{\sqrt{2}}\right]$$

$$\frac{1}{4}\left(\pi+\sqrt{2}\ \pi-\sqrt{2}\ \text{Log}\left[3+2\sqrt{2}\right]\right)$$

```
Expand[Sum[1 / (8 k + 1) + 1 / (8 k + 3) - 3 / (8 k + 5) + 1 / (8 k + 7), {k, 0, Infinity}]]
```

$$-\frac{\pi}{4}+\frac{\pi}{2\sqrt{2}}-\frac{\text{Log}[2]}{2\sqrt{2}}+\frac{\text{Log}\left[2+\sqrt{2}\right]}{\sqrt{2}}$$

```
Expand[
 Sum[ 1 / (10 k + 1) + 1 / (10 k + 3) + 1 / (10 k + 5) + 1 / (10 k + 7) - 4 / (10 k + 9), {k, 0, Infinity}]]
```

```
FullSimplify[ Sqrt[1/2 (5 + Sqrt[5])] π / (2 (-1 + Sqrt[5])) + Log[4] / (2 (-1 + Sqrt[5])) - Sqrt[5] Log[4] / (2 (-1 + Sqrt[5])) + 3 Log[1/8 (5 - Sqrt[5])] / (4 (-1 + Sqrt[5])) -
  Sqrt[5] Log[1/8 (5 - Sqrt[5])] / (4 (-1 + Sqrt[5])) + Log[-1 + Sqrt[5]] / (-1 + Sqrt[5]) - 3 Log[1 + Sqrt[5]] / (2 (-1 + Sqrt[5])) + Sqrt[5] Log[1 + Sqrt[5]] / (2 (-1 + Sqrt[5])) - Log[1/8 (5 + Sqrt[5])] / (2 (-1 + Sqrt[5]))]
```

$$\frac{1}{16}\left(1+\sqrt{5}\right)\left(\sqrt{2\left(5+\sqrt{5}\right)}\,\pi+\sqrt{5}\,\text{Log}\left[1+\frac{2}{\sqrt{5}}\right]+\text{Log}\left[1525-682\sqrt{5}\right]\right)$$

```
FullSimplify[Sum[ 1 / (12 k + 1) + 1 / (12 k + 3) +
    1 / (12 k + 5) + 1 / (12 k + 7) + 1 / (12 k + 9) - 5 / (12 k + 11), {k, 0, Infinity}]]
```

$$\frac{1}{4}\left(\left(2+\sqrt{3}\right)\pi-4\sqrt{3}\,\text{ArcCoth}\left[\sqrt{3}\right]-\text{Log}[3]\right)$$

```
FullSimplify[Sum[ 1 / (14 k + 1) + 1 / (14 k + 3) + 1 / (14 k + 5) +
    1 / (14 k + 7) + 1 / (14 k + 9) + 1 / (14 k + 11) - 6 / (14 k + 13), {k, 0, Infinity}]]
```

$$\frac{1}{4}\pi\,\text{Cot}\left[\frac{\pi}{14}\right]+\text{Cos}\left[\frac{\pi}{7}\right]\text{Log}\left[\text{Tan}\left[\frac{\pi}{14}\right]\right]+\text{Log}\left[\text{Tan}\left[\frac{3\pi}{14}\right]\right]\text{Sin}\left[\frac{\pi}{14}\right]+\text{Log}\left[\text{Tan}\left[\frac{\pi}{7}\right]\right]\text{Sin}\left[\frac{3\pi}{14}\right]$$

```
FullSimplify[Sum[ 1 / (12 k + 1) + 1 / (12 k + 3) + 1 / (12 k + 5) -
    1 / (12 k + 7) - 1 / (12 k + 9) - 1 / (12 k + 11), {k, 0, Infinity}]]
```

$$\frac{5\pi}{12}$$

```
FullSimplify[Sum[ 1 / (16 k + 1) + 1 / (16 k + 3) + 1 / (16 k + 5) + 1 / (16 k + 7) -
    1 / (16 k + 9) - 1 / (16 k + 11) - 1 / (16 k + 13) - 1 / (16 k + 15), {k, 0, Infinity}]]
```

$$\frac{1}{4}\sqrt{2+\sqrt{2}}\,\pi$$

```
FullSimplify[Sum[ 1 / (12 k + 1)^3 + 1 / (12 k + 3)^3 + 1 / (12 k + 5)^3 -
    1 / (12 k + 7)^3 - 1 / (12 k + 9)^3 - 1 / (12 k + 11)^3, {k, 0, Infinity}]]
```

$$\frac{29\,\pi^3}{864}$$

```
FullSimplify[Sum[ 1 / (16 k + 1)^3 + 1 / (16 k + 3)^3 + 1 / (16 k + 5)^3 + 1 / (16 k + 7)^3 -
    1 / (16 k + 9)^3 - 1 / (16 k + 11)^3 - 1 / (16 k + 13)^3 - 1 / (16 k + 15)^3, {k, 0, Infinity}]]
```

$$\frac{1}{512}\sqrt{274+17\sqrt{2}}\,\pi^3$$

```
FullSimplify[Sum[ 2 / (12 k + 1)^3 + 1 / (12 k + 3)^3 + 1 / (12 k + 5)^3 -
    1 / (12 k + 7)^3 - 1 / (12 k + 9)^3 - 2 / (12 k + 11)^3, {k, 0, Infinity}]]
```

$$\frac{1}{864}\left(43+8\sqrt{3}\right)\pi^3$$

```
FullSimplify[Sum[ 4 / (12 k + 1) + 2 / (12 k + 3) + 1 / (12 k + 5) -
    1 / (12 k + 7) - 2 / (12 k + 9) - 4 / (12 k + 11), {k, 0, Infinity}]]
```

$$\frac{1}{4}\left(4+\sqrt{3}\right)\pi$$

```
FullSimplify[Sum[ 6 / (12 k + 1) + 3 / (12 k + 3) + 1 / (12 k + 5) -
    1 / (12 k + 7) - 3 / (12 k + 9) - 6 / (12 k + 11), {k, 0, Infinity}]]
```

$$\frac{1}{12}\left(17+5\sqrt{3}\right)\pi$$

```
FullSimplify[Sum[ 6 / (12 k + 1) - 3 / (12 k + 3) + 1 / (12 k + 5) -
    1 / (12 k + 7) + 3 / (12 k + 9) - 6 / (12 k + 11), {k, 0, Infinity}]]
```

$$\frac{1}{12}\left(11+5\sqrt{3}\right)\pi$$

```
FullSimplify[Sum[ 2 / (12 k + 2) + 1 / (12 k + 4) + 1 / (12 k + 6) -
    1 / (12 k + 8) - 1 / (12 k + 10) - 2 / (12 k + 12), {k, 0, Infinity}]]
```

$$\frac{1}{72}\left(11\sqrt{3}\,\pi+9\,\text{Log}[3]+12\,\text{Log}[4]\right)$$

```
Sum[ (-1)^(k+1) 1 / (2 k-1)^3, {k, 1, Infinity}]
```

$$\frac{\pi^3}{32}$$

```
Sum[ 1 / (4 k-3)^3 - 1 / (4 k-1)^3, {k, 1, Infinity}]
```

$$\frac{\pi^3}{32}$$

```
Sum[ 1 / (6 k-5)^3 + 1 / (4 k-1)^3, {k, 1, Infinity}]
```

```
Sum[ 1 / (6 k+1)^3 + 1 / (6 k+3)^3 - 2 / (6 k+5)^3, {k, 0, Infinity}]
```

$$\frac{1}{36}\left(\sqrt{3}\ \pi^3 - 14\,\text{Zeta}[3]\right)$$

```
Sum[ 1 / (6 k+1) + 1 / (6 k+3) - 2 / (6 k+5), {k, 0, Infinity}]
```

$$\frac{1}{4}\left(\sqrt{3}\ \pi - \text{Log}[3]\right)$$

```
Sum[ 1 / (3 k+1) + 1 / (3 k+2) - 2 / (3 k+3), {k, 0, Infinity}]
```

```
Log[3]
```

```
Sum[ 1 / (3 k+1)^2 + 1 / (3 k+2)^2 - 2 / (3 k+3)^2, {k, 0, Infinity}]
```

$$\frac{\pi^2}{9}$$

```
Sum[ 1 / (3 k+1)^3 + 1 / (3 k+2)^3 - 2 / (3 k+3)^3, {k, 0, Infinity}]
```

$$\frac{8\,\text{Zeta}[3]}{9}$$

```
Sum[1 / (3 k + 1) ^4 + 1 / (3 k + 2) ^4 - 2 / (3 k + 3) ^4, {k, 0, Infinity}]
```

$$\frac{13\,\pi^4}{1215}$$

```
Sum[1 / ((3 k + 1)) + 1 / ((3 k + 2)) - 2 / ((3 k + 3)), {k, 0, Infinity}]
```

$$\text{Log}[3]$$

```
Sum[1 / ((3 k + 1)) - 2 / ((3 k + 2)) + 1 / ((3 k + 3)), {k, 0, Infinity}]
```

$$\frac{1}{6}\left(\sqrt{3}\ \pi - \text{Log}[27]\right)$$

```
Sum[-2 / ((3 k + 1)) + 1 / ((3 k + 2)) + 1 / ((3 k + 3)), {k, 0, Infinity}]
```

$$\frac{1}{6}\left(-\sqrt{3}\ \pi - 3\,\text{Log}[3]\right)$$

```
Sum[-3 / ((3 k + 1)) + 2 / ((3 k + 2)) + 1 / ((3 k + 3)), {k, 0, Infinity}]
```

$$\frac{1}{18}\left(-5\sqrt{3}\ \pi - 9\,\text{Log}[3]\right)$$

```
Sum[-4 / ((3 k + 1)) + 3 / ((3 k + 2)) + 1 / ((3 k + 3)), {k, 0, Infinity}]
```

$$\frac{1}{18}\left(-7\sqrt{3}\ \pi - 9\,\text{Log}[3]\right)$$

```
Sum[-1 / ((3 k + 1)) + 1 / ((3 k + 2)), {k, 0, Infinity}]
```

$$-\frac{\pi}{3\sqrt{3}}$$

```
Sum[-1 / ((3 k + 1)) + 1 / ((3 k + 3)), {k, 0, Infinity}]
```

$$\frac{1}{18}\left(-\sqrt{3}\ \pi - 9\,\text{Log}[3]\right)$$

```
Sum[-1 / ((3 k + 1)) ^3 + 1 / ((3 k + 2)) ^3, {k, 0, Infinity}]
```

$$-\frac{4\,\pi^3}{81\sqrt{3}}$$

```
Sum[-1 / ((4 k + 3)) ^3 + 1 / ((4 k + 1)) ^3, {k, 0, Infinity}]
```

$$\frac{\pi^3}{32}$$

```
Sum[-1 / ((5 k + 4)) ^3 + 1 / ((5 k + 1)) ^3, {k, 0, Infinity}]
```

$$-\frac{8\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}\ \pi^3}{125\left(-5+\sqrt{5}\right)}$$

```
Sum[-1 / ((5 k + 4)) + 1 / ((5 k + 1)), {k, 0, Infinity}]
```

$$\frac{1}{5}\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}\ \pi$$

```
Expand[Sum[ 1 / (8 k + 1) + 1 / (8 k + 3) - 1 / (8 k + 5) - 1 / (8 k + 7), {k, 0, Infinity}]]
```

$$\frac{\pi}{2\sqrt{2}}$$

```
Expand[Sum[ 1 / (8 k + 1)^3 + 1 / (8 k + 3)^3 - 1 / (8 k + 5)^3 - 1 / (8 k + 7)^3, {k, 0, Infinity}]]
```

$$\frac{3\pi^3}{64\sqrt{2}}$$

```
Expand[Sum[ 1 / (8 k + 1) - 0 / (8 k + 5) - 1 / (8 k + 7), {k, 0, Infinity}]]
```

$$\frac{\pi}{8} + \frac{\pi}{4\sqrt{2}}$$

```
Expand[Sum[ 1 / (8 k + 1)^3 - 1 / (8 k + 7)^3, {k, 0, Infinity}]]
```

$$\frac{\sqrt{2+\sqrt{2}}\ \pi^3}{128\left(2-\sqrt{2}\right)^{3/2}}$$

```
Expand[Sum[ 1 / (10 k + 1) - 1 / (10 k + 9), {k, 0, Infinity}]]
```

$$\frac{1}{10}\sqrt{5+2\sqrt{5}}\ \pi$$

```
Expand[Sum[ 1 / (10 k + 1)^3 - 1 / (10 k + 9)^3, {k, 0, Infinity}]]
```

$$\frac{1}{100}\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}\ \pi^3 + \frac{3}{500}\sqrt{5+2\sqrt{5}}\ \pi^3$$

```
Expand[Sum[ 1 / (12 k + 1) - 1 / (12 k + 11), {k, 0, Infinity}]]
```

$$\frac{\pi}{6} + \frac{\pi}{4\sqrt{3}}$$

```
Expand[Sum[ 1 / (20 k + 1) - 1 / (20 k + 19), {k, 0, Infinity}]]
```

$$\frac{\pi}{10\sqrt{2}\left(\sqrt{2}+\sqrt{10}-2\sqrt{5-\sqrt{5}}\right)} + \frac{\pi}{2\sqrt{10}\left(\sqrt{2}+\sqrt{10}-2\sqrt{5-\sqrt{5}}\right)} + \frac{\sqrt{5-\sqrt{5}}\ \pi}{10\left(\sqrt{2}+\sqrt{10}-2\sqrt{5-\sqrt{5}}\right)}$$

```
Expand[Sum[ 1 / (20 k + 1)^3 - 1 / (20 k + 19)^3, {k, 0, Infinity}]]
```

$$\frac{\sqrt{\frac{2}{5}}\ \pi^3}{25\left(\sqrt{2}+\sqrt{10}-2\sqrt{5-\sqrt{5}}\right)^3} + \frac{\sqrt{2}\ \pi^3}{125\left(\sqrt{2}+\sqrt{10}-2\sqrt{5-\sqrt{5}}\right)^3} + \frac{2\sqrt{5-\sqrt{5}}\ \pi^3}{125\left(\sqrt{2}+\sqrt{10}-2\sqrt{5-\sqrt{5}}\right)^3}$$

```
Expand[Sum[ 1 / (22 k + 1) - 1 / (22 k + 21), {k, 0, Infinity}]]
```

$$\frac{1}{22}\pi\,\mathrm{Cot}\left[\frac{\pi}{22}\right]$$

```
Expand[Sum[ 1 / (20 k + 1)^5 - 1 / (20 k + 19)^5, {k, 0, Infinity}]]
```

$$\frac{4\sqrt{\frac{2}{5}}\,\pi^5}{625\left(\sqrt{2}+\sqrt{10}-2\sqrt{5-\sqrt{5}}\right)^5}+\frac{2\sqrt{2}\,\pi^5}{1875\left(\sqrt{2}+\sqrt{10}-2\sqrt{5-\sqrt{5}}\right)^5}+$$

$$\frac{\sqrt{\frac{1}{5}\left(5-\sqrt{5}\right)}\,\pi^5}{1875\left(\sqrt{2}+\sqrt{10}-2\sqrt{5-\sqrt{5}}\right)^5}+\frac{23\sqrt{5-\sqrt{5}}\,\pi^5}{9375\left(\sqrt{2}+\sqrt{10}-2\sqrt{5-\sqrt{5}}\right)^5}$$

```
Sum[1 / 3 / ((3 k + 1)) - 2 / 3 / ((3 k + 2)) + 1 / 3 / ((3 k + 3)), {k, 0, Infinity}]
```

$$\frac{1}{18}\left(\sqrt{3}\,\pi-3\,\text{Log}[3]\right)$$

```
Sum[ 1 / ((3 k + 1)) -  3 / ((3 k + 2)) + 2 / ((3 k + 3)), {k, 0, Infinity}]
```

$$\frac{1}{9}\left(2\sqrt{3}\,\pi-9\,\text{Log}[3]\right)$$

```
Sum[ 1 / ((3 k + 1)) + 1 / ((3 k + 2)) - 2 / ((3 k + 3)), {k, 0, Infinity}]
```

$$\text{Log}[3]$$

```
Sum[ 1 / ((3 k + 1)) + 1 / ((3 k + 2)) - 2 / ((3 k + 3)), {k, 0, Infinity}]
```

$$\text{Log}[3]$$

```
Sum[ 3 / ((3 k + 1)) + 4 / ((3 k + 2)) - 7 / ((3 k + 3)), {k, 0, Infinity}]
```

$$\frac{1}{18}\left(-\sqrt{3}\,\pi+21\,\text{Log}[27]\right)$$

```
Sum[ 2 / ((3 k + 1)) + 2 / ((3 k + 2)) - 4 / ((3 k + 3)), {k, 0, Infinity}]
```

$$\text{Log}[9]$$

```
Sum[ 0 / ((3 k + 1)) + 1 / ((3 k + 2)) - 1 / ((3 k + 3)), {k, 0, Infinity}]
```

$$\frac{1}{18}\left(-\sqrt{3}\,\pi+9\,\text{Log}[3]\right)$$

```
Sum[ -2 / ((3 k + 1)) + 2 / ((3 k + 2)) - 0 / ((3 k + 3)), {k, 0, Infinity}]
```

$$-\frac{2\,\pi}{3\sqrt{3}}$$

```
FullSimplify[Sum[ 1 / (16 k + 2) ^3 + 1 / (16 k + 4) ^3 + 1 / (16 k + 6) ^3 + 1 / (16 k + 8) ^3 -
   1 / (16 k + 10) ^3 - 1 / (16 k + 12) ^3 - 1 / (16 k + 14) ^3 - 1 / (16 k + 16) ^3, {k, 0, Infinity}]]
```

$$\frac{\left(1+6\sqrt{2}\right)\pi^3+3\,\text{Zeta}[3]}{2048}$$

```
FullSimplify[Sum[ 1 / (16 k + 2) + 1 / (16 k + 4) + 1 / (16 k + 6) + 1 / (16 k + 8) +
   1 / (16 k + 10) + -7 / (16 k + 12) + 1 / (16 k + 14) + 1 / (16 k + 16), {k, 0, Infinity}]]
```

$$\frac{\pi}{4}$$

```
FullSimplify[Sum[ 1 / (16 k + 2) ^3 + 1 / (16 k + 4) ^3 + 1 / (16 k + 6) ^3 + 1 / (16 k + 8) ^3 +
   1 / (16 k + 10) ^3 + -7 / (16 k + 12) ^3 + 1 / (16 k + 14) ^3 + 1 / (16 k + 16) ^3, {k, 0, Infinity}]]
```

Power::infy : Infinite expression $\frac{1}{0^3}$ encountered. ≫

```
FullSimplify[Sum[ 1 / (16 k + 2) + 1 / (16 k + 4) + 1 / (16 k + 6) - 7 / (16 k + 8) +
   1 / (16 k + 10) + +1 / (16 k + 12) + 1 / (16 k + 14) + 1 / (16 k + 16), {k, 0, Infinity}]]
```

$$\frac{\text{Log}[2]}{2}$$

```
FullSimplify[Sum[ 1 / (16 k + 2) - 7 / (16 k + 4) + 1 / (16 k + 6) + 1 / (16 k + 8) +
   1 / (16 k + 10) + +1 / (16 k + 12) + 1 / (16 k + 14) + 1 / (16 k + 16), {k, 0, Infinity}]]
```

$$-\frac{\pi}{4}$$

```
FullSimplify[Sum[ 1 / (16 k + 2) + 1 / (16 k + 4) + 1 / (16 k + 6) + 1 / (16 k + 8) +
   1 / (16 k + 10) + +1 / (16 k + 12) + 1 / (16 k + 14) - 7 / (16 k + 16), {k, 0, Infinity}]]
```

$$\frac{\text{Log}[8]}{2}$$

```
FullSimplify[Sum[ 1 / (8 k + 1) - 7 / (8 k + 2) + 1 / (8 k + 3) + 1 / (8 k + 4) +
   1 / (8 k + 5) + +1 / (8 k + 6) + 1 / (8 k + 7) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$$-\frac{\pi}{2}$$

```
FullSimplify[Sum[ 1 / (8 k + 1) + 1 / (8 k + 2) + 1 / (8 k + 3) + 1 / (8 k + 4) +
   1 / (8 k + 5) - 7 / (8 k + 6) + 1 / (8 k + 7) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$$\frac{\pi}{2}$$

```
FullSimplify[Sum[ 1 / (4 k + 1) - 3 / (4 k + 2) + 1 / (4 k + 3) + 1 / (4 k + 4), {k, 0, Infinity}]]
```

$$0$$

```
FullSimplify[
 Sum[ 1 / (4 k + 1) ^3 - 3 / (4 k + 2) ^3 + 1 / (4 k + 3) ^3 + 1 / (4 k + 4) ^3, {k, 0, Infinity}]]
```

Power::infy : Infinite expression $\frac{1}{0^3}$ encountered. ≫

```
N[Sum[ 1 / (4 k + 1) ^3 - 3 / (4 k + 2) ^3 + 1 / (4 k + 3) ^3 + 1 / (4 k + 4) ^3, {k, 0, 10 000}]]
```

```
0.6761570080272795
```

```
N[Sum[ 1 / (4 k + 1) - 3 / (4 k + 2) + 1 / (4 k + 3) + 1 / (4 k + 4), {k, 0, 100 000}]]
```

1.24998×10^{-6}

```
FullSimplify[Sum[ 1 / (8 k + 1) ^3 + 1 / (8 k + 2) ^3 + 1 / (8 k + 3) ^3 + 1 / (8 k + 4) ^3 +
    1 / (8 k + 5) ^3 - 7 / (8 k + 6) ^3 + 1 / (8 k + 7) ^3 + 1 / (8 k + 8) ^3, {k, 0, Infinity}]]
```

```
N[Sum[ 1 / (8 k + 1) ^3 + 1 / (8 k + 2) ^3 + 1 / (8 k + 3) ^3 + 1 / (8 k + 4) ^3 +
    1 / (8 k + 5) ^3 - 7 / (8 k + 6) ^3 + 1 / (8 k + 7) ^3 + 1 / (8 k + 8) ^3, {k, 0, 30 000}]]
```

```
1.16063
```

Power::infy : Infinite expression $\frac{1}{0^3}$ encountered. ≫

```
1.1606300811540349`
```

```
1.1606300811569534`
```

```
N[Sum[ 1 / (8 k + 1) ^3 - 7 / (8 k + 2) ^3 + 1 / (8 k + 3) ^3 + 1 / (8 k + 4) ^3 +
    1 / (8 k + 5) ^3 +  +1 / (8 k + 6) ^3 + 1 / (8 k + 7) ^3 + 1 / (8 k + 8) ^3, {k, 0, 30 000}]]
```

```
0.191684
```

```
31.006276680299816` / 0.19168393489758725`
```

```
161.757
```

```
1 / N[Pi^3] ^ (1 / 2)
```

```
0.179587
```

```
1 / 31.006276680299816`
```

```
0.0322515
```

```
FullSimplify[Sum[ -7 / (8 k + 1) ^2 + 1 / (8 k + 2) ^2 + 1 / (8 k + 3) ^2 + 1 / (8 k + 4) ^2 +
    1 / (8 k + 5) ^2 + 1 / (8 k + 6) ^2 + 1 / (8 k + 7) ^2 + 1 / (8 k + 8) ^2, {k, 0, Infinity}]]
```

$\frac{\pi^2}{6} - \frac{1}{8}\,\text{PolyGamma}\left[1, \frac{1}{8}\right]$

```
Sum[ 1 / (8 k + 1) ^3 + 1 / (8 k + 2) ^3 + 1 / (8 k + 3) ^3 + 1 / (8 k + 4) ^3 +
  1 / (8 k + 5) ^3 + 1 / (8 k + 6) ^3 + 1 / (8 k + 7) ^3 - 7 / (8 k + 8) ^3, {k, 0, Infinity}]
```

$\frac{63\,\text{Zeta}[3]}{64}$

```
Sum[ -7 / (8 k + 1) ^3 + 1 / (8 k + 2) ^3 + 1 / (8 k + 3) ^3 + 1 / (8 k + 4) ^3 +
  1 / (8 k + 5) ^3 + 1 / (8 k + 6) ^3 + 1 / (8 k + 7) ^3 + 1 / (8 k + 8) ^3, {k, 0, Infinity}]
```

$$\frac{1}{128}\left(\text{PolyGamma}\left[2, \frac{1}{8}\right] + 128\,\text{Zeta}[3]\right)$$

```
N[Sum[ 1 / (8 k + 1) + 1 / (8 k + 2) + 1 / (8 k + 3) + 1 / (8 k + 4) +
   1 / (8 k + 5) - 7 / (8 k + 6) + 1 / (8 k + 7) + 1 / (8 k + 8), {k, 0, 1000}]]
```

```
1.57061
```

```
N[Pi / 2]
```

```
1.5708
```

```
FullSimplify[Sum[ 1 / (8 k + 2) + 2 / (8 k + 3) +
   1 / (8 k + 4) - 7 / (8 k + 6) + 2 / (8 k + 7) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$$\frac{\pi}{4}$$

```
FullSimplify[Sum[ 2 / (8 k + 1) + 1 / (8 k + 2) +
   1 / (8 k + 4) + 2 / (8 k + 5) - 7 / (8 k + 6) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$$\frac{3\pi}{4}$$

```
FullSimplify[Sum[ -7 / (8 k + 2) + 2 / (8 k + 3) +
   1 / (8 k + 4) + 1 / (8 k + 6) + 2 / (8 k + 7) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$$-\frac{3\pi}{4}$$

```
FullSimplify[Sum[ 2 / (8 k + 1) - 7 / (8 k + 2) +
   1 / (8 k + 4) + 2 / (8 k + 5) + +1 / (8 k + 6) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$$-\frac{\pi}{4}$$

```
FullSimplify[Sum[ -7 / (8 k + 2) ^3 + 2 / (8 k + 3) ^3 + 1 / (8 k + 4) ^3 +
   1 / (8 k + 6) ^3 + 2 / (8 k + 7) ^3 + 1 / (8 k + 8) ^3, {k, 0, Infinity}]]
```

$$-\frac{3}{64}\left(\pi^3 - 12\,\text{Zeta}[3]\right)$$

```
FullSimplify[Sum[ 2 / (8 k + 1) ^2 + 1 / (8 k + 2) ^2 + 1 / (8 k + 4) ^2 +
   2 / (8 k + 5) ^2 - 7 / (8 k + 6) ^2 + 1 / (8 k + 8) ^2, {k, 0, Infinity}]]
```

Power::infy : Infinite expression $\frac{1}{0^2}$ encountered. ≫

```
FullSimplify[Sum[ -8 / (8 k + 4) ^2, {k, 0, Infinity}]]
```

$$-\frac{\pi^2}{16}$$

```
FullSimplify[Sum[ 1 / (8 k + 4) ^2, {k, 0, Infinity}]]
```

$$\frac{\pi^2}{128}$$

```
FullSimplify[
 Sum[ 1 / (4 k + 1) ^2 - 3 / (4 k + 2) ^2 + 1 / (4 k + 3) ^2 + 1 / (4 k + 4) ^2, {k, 0, Infinity}]]
```

Power::infy : Infinite expression $\frac{1}{0^2}$ encountered. ≫

```
FullSimplify[
  Sum[ 1 / (4 k + 1) ^2 + 1 / (4 k + 2) ^2 + 1 / (4 k + 3) ^2 + 1 / (4 k + 4) ^2, {k, 0, Infinity}]] +
 FullSimplify[Sum[ -4 / (4 k + 2) ^2, {k, 0, Infinity}]]
```

$$\frac{\pi^2}{24}$$

```
FullSimplify[
  Sum[ 1 / (4 k + 1) ^3 + 1 / (4 k + 2) ^3 + 1 / (4 k + 3) ^3 + 1 / (4 k + 4) ^3, {k, 0, Infinity}]] +
 FullSimplify[Sum[ -4 / (4 k + 2) ^3, {k, 0, Infinity}]]
```

$$\frac{9\,\text{Zeta}[3]}{16}$$

```
N[
 FullSimplify[Sum[ 1 / (4 k + 1) ^3 - 3 / (4 k + 2) ^3 + 1 / (4 k + 3) ^3 + 1 / (4 k + 4) ^3, {k, 0, 1000}]]]
```

```
0.676157
```

```
N[ 9 Zeta[3] / 16]
```

```
0.676157
```

```
FullSimplify[Sum[ 1 / (4 k + 2) ^2, {k, 0, Infinity}]]
```

$$\frac{\pi^2}{32}$$

```
FullSimplify[Sum[ 1 / (4 k + 4) ^2, {k, 0, Infinity}]]
```

$$\frac{\pi^2}{96}$$

```
FullSimplify[Sum[ 1 / (6 k + 3) ^2, {k, 0, Infinity}]]
```

$$\frac{\pi^2}{72}$$

```
FullSimplify[Sum[ 1 / (6 k + 6)^2, {k, 0, Infinity}]]
```

$$\frac{\pi^2}{216}$$

```
FullSimplify[Sum[ 2 / (8 k + 1) + 1 / (8 k + 2) + 2 / (8 k + 3) + 1 / (8 k + 4) +
   2 / (8 k + 5) - 11 / (8 k + 6) + 2 / (8 k + 7) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$$\frac{1}{4} (3\pi + \text{Log}[4])$$

```
FullSimplify[Sum[ 1 / (8 k + 1) + 2 / (8 k + 2) + 1 / (8 k + 3) + 2 / (8 k + 4) +
   1 / (8 k + 5) - 10 / (8 k + 6) + 1 / (8 k + 7) + 2 / (8 k + 8), {k, 0, Infinity}]]
```

$$\frac{1}{4} (3\pi - \text{Log}[4])$$

```
FullSimplify[Sum[ 1 / (8 k + 1) + 3 / (8 k + 2) + 1 / (8 k + 3) + 1 / (8 k + 4) +
   1 / (8 k + 5) - 9 / (8 k + 6) + 1 / (8 k + 7) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$$\frac{3\pi}{4}$$

```
FullSimplify[Sum[ 1 / (8 k + 1) + 5 / (8 k + 2) + 1 / (8 k + 3) + 1 / (8 k + 4) +
   1 / (8 k + 5) - 11 / (8 k + 6) + 1 / (8 k + 7) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$$\pi$$

```
FullSimplify[Sum[ 1 / (8 k + 1) - 1 / (8 k + 2) + 1 / (8 k + 3) + 1 / (8 k + 4) +
   1 / (8 k + 5) - 5 / (8 k + 6) + 1 / (8 k + 7) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$$\frac{\pi}{4}$$

```
FullSimplify[Sum[ 1 / (8 k + 1) - 3 / (8 k + 2) + 1 / (8 k + 3) + 1 / (8 k + 4) +
   1 / (8 k + 5) - 3 / (8 k + 6) + 1 / (8 k + 7) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$$0$$

```
FullSimplify[Sum[ 1 / (8 k + 1) - 11 / (8 k + 2) + 1 / (8 k + 3) + 1 / (8 k + 4) +
   1 / (8 k + 5) + 5 / (8 k + 6) + 1 / (8 k + 7) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$$-\pi$$

```
FullSimplify[Sum[ 1 / (8 k + 1)^3 + 5 / (8 k + 2)^3 + 1 / (8 k + 3)^3 + 1 / (8 k + 4)^3 +
   1 / (8 k + 5)^3 - 11 / (8 k + 6)^3 + 1 / (8 k + 7)^3 + 1 / (8 k + 8)^3, {k, 0, Infinity}]]
```

Power::infy : Infinite expression $\frac{1}{0^3}$ encountered. >>

```
FullSimplify[Sum[6 / (8 k + 2) ^ 3 - 12 / (8 k + 6) ^ 3, {k, 0, Infinity}]] +
 FullSimplify[Sum[ 1 / (8 k + 1) ^ 3 + 1 / (8 k + 2) ^ 3 + 1 / (8 k + 3) ^ 3 + 1 / (8 k + 4) ^ 3 +
    1 / (8 k + 5) ^ 3 + 1 / (8 k + 6) ^ 3 + 1 / (8 k + 7) ^ 3 + 1 / (8 k + 8) ^ 3, {k, 0, Infinity}]]
```

$$\frac{3}{256}\left(3\,\pi^3 - 28\,\text{Zeta}[3]\right) + \text{Zeta}[3]$$

```
Expand[Sum[ 1 / (4 k + 1) - 1 / (4 k + 3), {k, 0, Infinity}]]
```

$$\frac{\pi}{4}$$

```
FullSimplify[Sum[ 1 / (8 k + 1) + 1 / (8 k + 2) + 1 / (8 k + 3) + 1 / (8 k + 4) +
    1 / (8 k + 5) - 7 / (8 k + 6) + 1 / (8 k + 7) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$$\frac{\pi}{2}$$

```
Expand[Sum[ 1 / (8 k + 2) - 1 / (8 k + 6), {k, 0, Infinity}]]
```

$$\frac{\pi}{8}$$

```
FullSimplify[Sum[ 1 / (8 k + 1) ^ 2 + 1 / (8 k + 2) ^ 2 + 1 / (8 k + 3) ^ 2 + 1 / (8 k + 4) ^ 2 +
    1 / (8 k + 5) ^ 2 + 1 / (8 k + 6) ^ 2 + 1 / (8 k + 7) ^ 2 + 1 / (8 k + 8) ^ 2, {k, 0, Infinity}]] -
 FullSimplify[Sum[-4 / (8 k + 2) ^ 2 - 4 / (8 k + 6) ^ 2, {k, 0, Infinity}]]
```

$$\frac{7\,\pi^2}{24}$$

```
FullSimplify[
 Sum[ 1 / (16 k + 1) + 1 / (16 k + 2) + 1 / (16 k + 3) + 1 / (16 k + 4) + 1 / (16 k + 5) + -7 / (16 k + 6) +
    1 / (16 k + 7) + 1 / (16 k + 8) + 1 / (16 k + 9) + 1 / (16 k + 10) + 1 / (16 k + 11) + 1 / (16 k + 12) +
    1 / (16 k + 13) - 7 / (16 k + 14) + 1 / (16 k + 15) + 1 / (16 k + 16), {k, 0, Infinity}]]
```

$$\frac{\pi}{2}$$

```
FullSimplify[
 Sum[ 1 / (16 k + 1) - 7 / (16 k + 2) + 1 / (16 k + 3) + 1 / (16 k + 4) + 1 / (16 k + 5) + 1 / (16 k + 6) +
    1 / (16 k + 7) + 1 / (16 k + 8) + 1 / (16 k + 9) - 7 / (16 k + 10) + 1 / (16 k + 11) + 1 / (16 k + 12) +
    1 / (16 k + 13) + 1 / (16 k + 14) + 1 / (16 k + 15) + 1 / (16 k + 16), {k, 0, Infinity}]]
```

$$-\frac{\pi}{2}$$

```
FullSimplify[Sum[ 1 / (16 k + 1) ^ 3 + 1 / (16 k + 2) ^ 3 + 1 / (16 k + 3) ^ 3 + 1 / (16 k + 4) ^ 3 +
    1 / (16 k + 5) ^ 3 + -7 / (16 k + 6) ^ 3 + 1 / (16 k + 7) ^ 3 + 1 / (16 k + 8) ^ 3 + 1 / (16 k + 9) ^ 3 +
    1 / (16 k + 10) ^ 3 + 1 / (16 k + 11) ^ 3 + 1 / (16 k + 12) ^ 3 + 1 / (16 k + 13) ^ 3 -
    7 / (16 k + 14) ^ 3 + 1 / (16 k + 15) ^ 3 + 1 / (16 k + 16) ^ 3, {k, 0, Infinity}]]
```

Power::infy : Infinite expression $\frac{1}{0^3}$ encountered. ≫