```
ClearAll["Global`*"]

WheelEntries := WheelEntries = 5
WheelSize := WheelSize = Product[Prime[j], {j, 1, WheelEntries}]
ExcludedPrimes[n_] := Sum[1 / k, {j, 1, WheelEntries}, {k, 1, Log[Prime[j], n]}]

(*Use[n] is 1 if n isn't rejected by the wheel,and 0 otherwise*)
CoprimeCache := CoprimeCache = Table[CoprimeQ[WheelSize, n], {n, 1, WheelSize}]
Use[n_] := Use[n] = If[CoprimeCache[[Mod[n - 1, WheelSize] + 1]] == True, 1, 0]

(*Coprimes[n] is the count of numbers≤n coprime to our wheel*)
LegendrePhi[x_, a_] := LegendrePhi[x, a - 1] - LegendrePhi[x / Prime[a], a - 1];
LegendrePhi[x_, 0] := Floor[x]
LegPhiCache := LegPhiCache = Table[LegendrePhi[n, WheelEntries], {n, 1, WheelSize}]
FullWheel := FullWheel = LegendrePhi[WheelSize, WheelEntries]
Coprimes[n_] :=
 Coprimes[n] = LegPhiCache[[Mod[n - 1, WheelSize] + 1]] + Floor[(n - 1) / WheelSize] FullWheel

Dhyp[n_, k_, s_] := Sum[If[Use[m] == 0, 0, Binomial[k, j] Dhyp[Floor[n / (m^(k - j))], j, m + 1]],
   {m, s, n^(1 / k)}, {j, 0, k - 1}]
Dhyp[n_, 1, s_] := Dhyp[n, 1, s] = Coprimes[n] - Coprimes[s - 1]
Dhyp[n_, 0, s_] := Dhyp[n, 0, s] = 1

d2cache[n_, k_] := Dhyp[n, k, 2] - Dhyp[n - 1, k, 2]
D2Cache[n_, k_] := Dhyp[n, k, 2]



D2Fast[n_, k_] :=
 Sum[If[Use[j] == 0, 0, d2cache[j, 1] D2Cache[Floor[n / j], k - 1]],
   {j, Floor[n^(1 / 3)] + 1, Floor[n^(1 / 2)]}] +
  Sum[(D2Cache[Floor[n / r], 1] - D2Cache[Floor[n / (r + 1)], 1]) D2Cache[r, k - 1],
   {r, 1, n / Floor[n^(1 / 2)] - 1}] +
  Sum[If[Use[j] == 0, 0, d2cache[j, k - 1] D2Cache[Floor[n / j], 1]], {j, 2, n^(1 / 3)}] +
  Sum[If[Use[j] == 0, 0, d2cache[j, m] D2Cache[Floor[n / (j s)], k - m - 1]],
   {j, 2, Floor[n^(1 / 3)]},
   {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[Floor[n / j]^(1 / 2)]}, {m, 1, k - 2}] +
  Sum[If[Use[j] == 0 || Use[s] == 0, 0,
     (D2Cache[Floor[n / (j s)], 1] - D2Cache[Floor[n / (j (s + 1))], 1])
      (Sum[d2cache[j, m] D2Cache[s, k - m - 1], {m, 1, k - 2}])],
   {j, 2, Floor[n^(1 / 3)]}, {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]

D2Fast[n_, 1] := Coprimes[Floor[n]] - 1

LinnikSumFast[n_] := Sum[(-1)^(k + 1) / k D2Fast[n, k], {k, 1, Log[2, n]}]



RiePrimeCnt[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Table[{n, a = LinnikSumFast[n] + ExcludedPrimes[n], b = RiePrimeCnt[n], a - b},
  {n, 100, 3000, 100}] // TableForm
```

100	$\dfrac{428}{15}$	$\dfrac{428}{15}$	0
200	$\dfrac{5356}{105}$	$\dfrac{5356}{105}$	0
300	$\dfrac{56\,981}{840}$	$\dfrac{56\,981}{840}$	0
400	$\dfrac{23\,707}{280}$	$\dfrac{23\,707}{280}$	0
500	$\dfrac{28\,467}{280}$	$\dfrac{28\,467}{280}$	0
600	$\dfrac{293\,023}{2520}$	$\dfrac{293\,023}{2520}$	0
700	$\dfrac{333\,973}{2520}$	$\dfrac{333\,973}{2520}$	0
800	$\dfrac{369\,673}{2520}$	$\dfrac{369\,673}{2520}$	0
900	$\dfrac{408\,733}{2520}$	$\dfrac{408\,733}{2520}$	0
1000	$\dfrac{445\,273}{2520}$	$\dfrac{445\,273}{2520}$	0
1100	$\dfrac{97\,169}{504}$	$\dfrac{97\,169}{504}$	0
1200	$\dfrac{103\,217}{504}$	$\dfrac{103\,217}{504}$	0
1300	$\dfrac{110\,777}{504}$	$\dfrac{110\,777}{504}$	0
1400	$\dfrac{116\,741}{504}$	$\dfrac{116\,741}{504}$	0
1500	$\dfrac{125\,309}{504}$	$\dfrac{125\,309}{504}$	0
1600	$\dfrac{131\,357}{504}$	$\dfrac{131\,357}{504}$	0
1700	$\dfrac{139\,169}{504}$	$\dfrac{139\,169}{504}$	0
1800	$\dfrac{145\,217}{504}$	$\dfrac{145\,217}{504}$	0
1900	$\dfrac{151\,517}{504}$	$\dfrac{151\,517}{504}$	0
2000	$\dfrac{158\,069}{504}$	$\dfrac{158\,069}{504}$	0
2100	$\dfrac{1\,816\,879}{5544}$	$\dfrac{1\,816\,879}{5544}$	0
2200	$\dfrac{1\,983\,991}{5544}$	$\dfrac{1\,874\,959}{5544}$	$\dfrac{59}{3}$
2300	$\dfrac{2\,077\,315}{5544}$	$\dfrac{1\,960\,891}{5544}$	21
2400	$\dfrac{2\,167\,867}{5544}$	$\dfrac{2\,044\,051}{5544}$	$\dfrac{67}{3}$
2500	$\dfrac{2\,233\,933}{5544}$	$\dfrac{2\,100\,877}{5544}$	24
2600	$\dfrac{2\,298\,613}{5544}$	$\dfrac{2\,161\,861}{5544}$	$\dfrac{74}{3}$
2700	$\dfrac{2\,394\,709}{5544}$	$\dfrac{2\,245\,021}{5544}$	27
2800	$\dfrac{2\,481\,565}{5544}$	$\dfrac{2\,322\,637}{5544}$	$\dfrac{86}{3}$
2900	$\dfrac{2\,558\,257}{5544}$	$\dfrac{2\,391\,937}{5544}$	30
3000	$\dfrac{2\,626\,633}{5544}$	$\dfrac{2\,452\,921}{5544}$	$\dfrac{94}{3}$