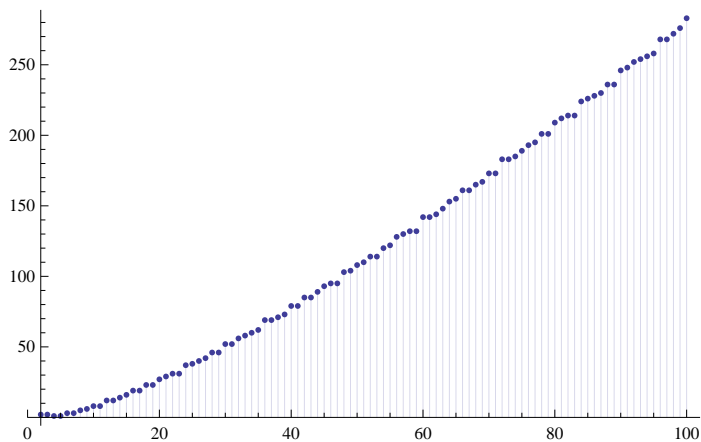


DiscretePlot $\left[1 - \text{Floor}\left[\sqrt{n}\right]^2 + 2 \text{Floor}\left[\frac{n}{2}\right] + 2 \sum_{m=3}^{\text{Floor}\left[\sqrt{n}\right]} \text{Floor}\left[\frac{n}{m}\right], \{n, 2, 100\}\right]$

```

DD[k_, a_, n_] :=
  Sum[Binomial[k, j] DD[k - j, m + 1, Floor[n / (m^j)]]], {m, a, n^(1/k)}, {j, 1, k}
DD[1, a_, n_] := Floor[n] - a + 1
DD[0, a_, n_] := 1
DS[n_, k_] := DD[k, 2, n]
DDD[n_, k_] := Sum[DDD[n / j, k - 1], {j, 2, n}]
DDD[n_, 0] := 1
D2[1, a_, n_, p_, r_] := n - a + 1
D2[2, a_, n_, p_, r_] := p / ((r + 1) (r + 2)) + (Floor[n / a] - a) (p / (r + 1)) +
  (Floor[n^(1/2)] - a) (p / 2) + p Sum[Floor[n / m] - m, {m, a + 1, n^(1/2)}]
D2[k_, a_, n_, p_, r_] := D2[k - 1, a, n / a, p / (r + 1), r + 1] +
  Sum[D2[k - 1, m, n / m, p, 1], {m, a + 1, n^(1/k)}]
DD2[n_, k_] := D2[k, 2, n, k!, 0]
D3[1, a_, n_, p_, r_] := p / (r + 1) + p (Floor[n] - a)
D3[k_, a_, n_, p_, r_] :=
  D3[k - 1, a, n / a, p / (r + 1), r + 1] + Sum[D3[k - 1, m, n / m, p, 1], {m, a + 1, n^(1/k)}]
DD3[n_, k_] := D3[k, 2, n, k!, 0]
D4[1, a_, n_, p_, r_] := p (Floor[n] - a + 1) - p / (r + 1)
D4[k_, a_, n_, p_, r_] :=
  Sum[D4[k - 1, m, n / m, p, 1], {m, a, n^(1/k)}] - D4[k - 1, a, n / a, p / (r + 1), r + 1]
DD4[n_, k_] := D4[k, 1, n, k!, 0]

```



DXX[n_] := $1 - \text{Floor}\left[\sqrt{n}\right]^2 + 2 \text{Floor}\left[\frac{n}{2}\right] + 2 \sum_{m=3}^{\text{Floor}\left[\sqrt{n}\right]} \text{Floor}\left[\frac{n}{m}\right]$

DD2[2000, 2]

11 519

DXX[2000]

11 519

DDD[2000, 2]

11 519

DD2[n, 2]

$$\text{FullSimplify}\left[-1 + \text{Floor}\left[\sqrt{n}\right] + 2\left(-2 + \text{Floor}\left[\frac{n}{2}\right]\right) + 2 \sum_{m=3}^{\sqrt{n}} \left(-m + \text{Floor}\left[\frac{n}{m}\right]\right)\right]$$

$$-5 + \text{Floor}\left[\sqrt{n}\right] + 2 \text{Floor}\left[\frac{n}{2}\right] + 2 \sum_{m=3}^{\sqrt{n}} \left(-m + \text{Floor}\left[\frac{n}{m}\right]\right)$$

$$\text{FullSimplify}\left[2 \sum_{m=3}^{\text{Floor}\left[\sqrt{n}\right]} (-m) + 2 \sum_{m=3}^{\text{Floor}\left[\sqrt{n}\right]} \left(\text{Floor}\left[\frac{n}{m}\right]\right)\right]$$

$$-\text{Floor}\left[\sqrt{n}\right] \left(1 + \text{Floor}\left[\sqrt{n}\right]\right) + 2 \left(3 + \sum_{m=3}^{\text{Floor}\left[\sqrt{n}\right]} \text{Floor}\left[\frac{n}{m}\right]\right)$$

FullSimplify[

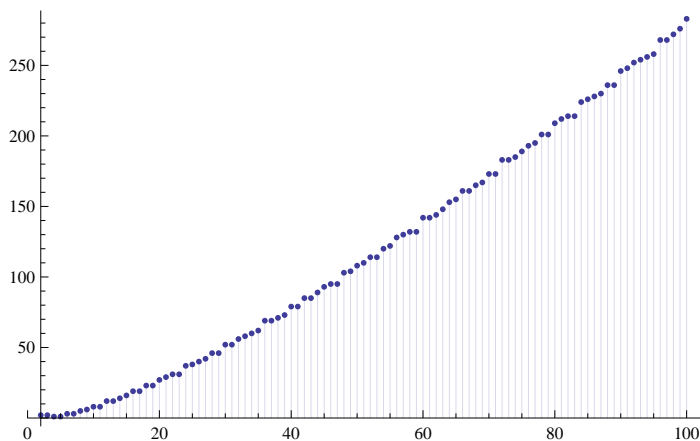
$$-1 + \text{Floor}\left[\sqrt{n}\right] + 2\left(-2 + \text{Floor}\left[\frac{n}{2}\right]\right) - \text{Floor}\left[\sqrt{n}\right] \left(1 + \text{Floor}\left[\sqrt{n}\right]\right) + 2 \left(3 + \sum_{m=3}^{\text{Floor}\left[\sqrt{n}\right]} \text{Floor}\left[\frac{n}{m}\right]\right)]$$

$$1 - \text{Floor}\left[\sqrt{n}\right]^2 + 2 \text{Floor}\left[\frac{n}{2}\right] + 2 \sum_{m=3}^{\text{Floor}\left[\sqrt{n}\right]} \text{Floor}\left[\frac{n}{m}\right]$$

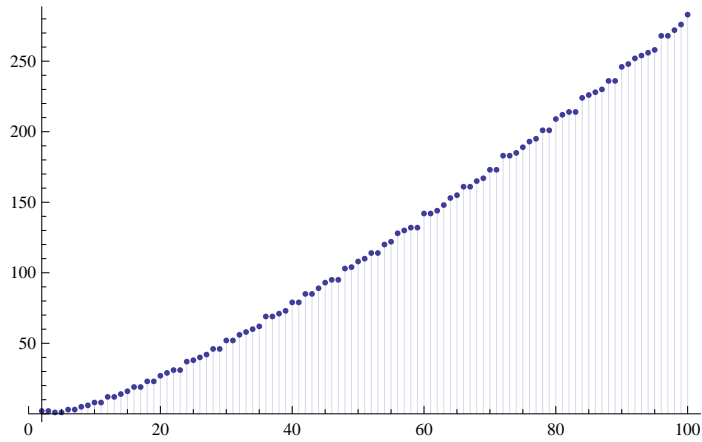
$$\text{FullSimplify}\left[-5 + \text{Floor}\left[\sqrt{n}\right] + 2 \text{Floor}\left[\frac{n}{2}\right] + 6 - \sqrt{n} - n + 2 \sum_{m=3}^{\sqrt{n}} \text{Floor}\left[\frac{n}{m}\right]\right]$$

$$1 - \sqrt{n} - n + \text{Floor}\left[\sqrt{n}\right] + 2 \text{Floor}\left[\frac{n}{2}\right] + 2 \sum_{m=3}^{\sqrt{n}} \text{Floor}\left[\frac{n}{m}\right]$$

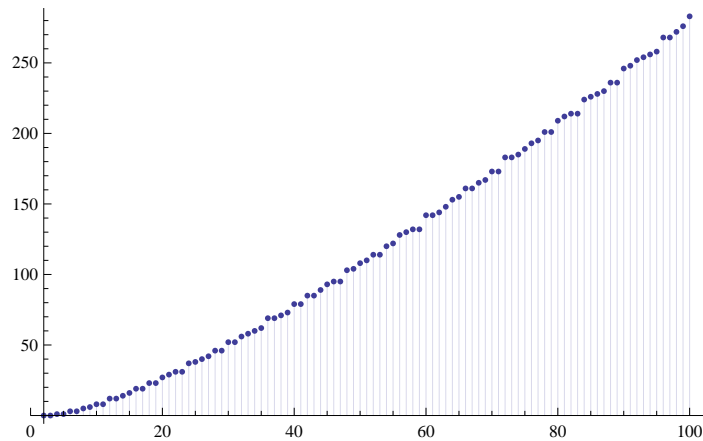
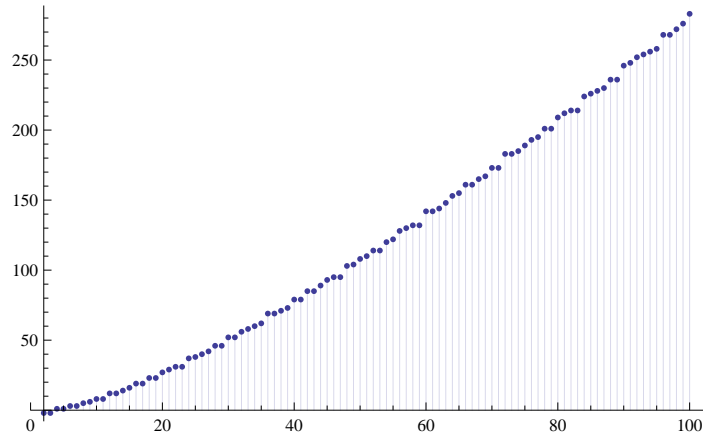
$$\text{DiscretePlot}\left[1 - \text{Floor}\left[\sqrt{n}\right]^2 + 2 \text{Floor}\left[\frac{n}{2}\right] + 2 \sum_{m=3}^{\text{Floor}\left[\sqrt{n}\right]} \text{Floor}\left[\frac{n}{m}\right], \{n, 2, 100\}\right]$$



`DiscretePlot` $\left[-1 + \text{Floor}\left[\sqrt{n}\right] + 2\left(-2 + \text{Floor}\left[\frac{n}{2}\right]\right) + \right.$
 $\left. - \text{Floor}\left[\sqrt{n}\right]\left(1 + \text{Floor}\left[\sqrt{n}\right]\right) + 2\left(3 + \sum_{m=3}^{\text{Floor}\left[\sqrt{n}\right]} \text{Floor}\left[\frac{n}{m}\right]\right), \{n, 2, 100\}\right]$



$$\text{DiscretePlot}\left[-1 + \text{Floor}\left[\sqrt{n}\right] + 2\left(-2 + \text{Floor}\left[\frac{n}{2}\right]\right) + 2 \sum_{m=3}^{\sqrt{n}} \left(-m + \text{Floor}\left[\frac{n}{m}\right]\right), \{n, 2, 100\}\right]$$

$$\text{DiscretePlot}[\text{DDD}[n, 2], \{n, 2, 100\}]$$


```

D5[1, a_, n_, p_, r_, s_] := s! / p / (r + 1) + (s! / p) (Floor[n] - a)
D5[k_, a_, n_, p_, r_, s_] := D5[k - 1, a, n / a, p (r + 1), r + 1, s + 1] +
  Sum[D5[k - 1, m, n / m, p, 1, s + 1], {m, a + 1, n^(1 / k)}]
DD5[n_, k_] := D5[k, 2, n, 1, 0, 1]

DD5[1000, 3]

11 217

DDD[1000, 3]

11 217

D6[1, a_, n_, p_, r_, s_] := p + (p (r + 1)) (Floor[n] - a)
D6[k_, a_, n_, p_, r_, s_] := D6[k - 1, a, n / a, p, 1, s + 1] +
  Sum[D6[k - 1, m, n / m, p (r + 1), r + 1, s + 1], {m, a + 1, n^(1 / k)}]
DD6[n_, k_] := D6[k, 2, n, 1, 0, 1]

DD6[1000, 3]

10 655

```

```

DA[1, n_, a_, k_, p_, r_] := (p/r + p (Floor[n] - a))
DA[s_, n_, a_, k_, p_, r_] := DA[s-1, n/a, a, k+1, kp/r, r+1] +
  Sum[DA[s-1, n/m, m, k+1, kp, 2], {m, a+1, n^(1/s)}]
DDA[n_, s_] := DA[s, n, 2, 2, 1, 1]

DDD[1000, 3]

11 217

DDA[1000, 3]

11 217

DB[1, n_, a_, k_, p_, r_] := p (Floor[n] - a + 1) - p/r
DB[s_, n_, a_, k_, p_, r_] := Sum[DB[s-1, n/m, m, k+1, kp, 2], {m, a, n^(1/s)}] -
  DB[s-1, n/a, a, k+1, kp/r, r+1]
DDB[n_, s_] := DB[s, n, 1, 2, 1, 1]

DDD[1000, 3]

11 217

DDB[1000, 3]

11 217

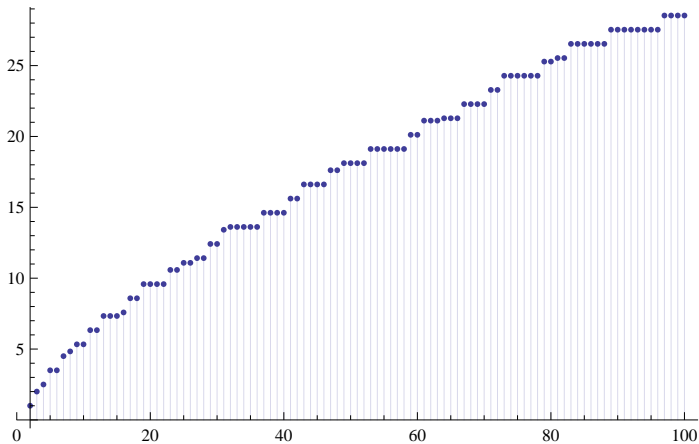
D7[a_, n_, p_, r_, s_] :=
  (s! / p/r + (s! / p) (Floor[n] - a)) / s -
  If[n^(1/2) ≥ a, D7[a, n/a, pr, r+1, s+1] +
    Sum[D7[m, n/m, p, 2, s+1], {m, a+1, n^(1/2)}], 0]
DD7[n_] := D7[2, n, 1, 1, 1]

DD7[100]

428
-----
15

DiscretePlot[DD7[n], {n, 2, 100}]

```



```

D8[s_Number, n_Number, a_Integer, k_Integer, p_Integer, r_Integer] :=
  p (Floor[n/s] * s - a + 1/r) -
  If[n^(1/2) ≥ a,
    Sum[D8[s, n/m, m, k+1, kp/r, r+1] * s, {m, a, a+.999999, s}], 0] -
  Sum[D8[s, n/m, m, k+1, kp, 2] * s, {m, a+1, n^(1/2), s}]
DD8[n_, s_] := D8[s, n, 1+s, 1, 1, 1]

D8a[n_, a_, k_, p_, r_] :=
  p (Floor[n] - a + 1/r) -
  If[n^(1/2) ≥ a, D8a[n/a, a, k+1, kp/r, r+1], 0] -
  Sum[D8a[n/m, m, k+1, kp, 2], {m, a+1, n^(1/2)}]
DD8a[n_] := D8a[n, 2, 1, 1, 1]

DD8[100, .1]
29.3156

DD8[100, .2]
28.9508

DD8[100, .3]
30.848

DD8[100, .5]
29.2466

N[DD8[100, 1]]
28.5333

DD8[100, .05]
$Aborted

DD8a[100]
428
15

DD9[n_, k_, a_] :=
  Sum[(-1)^(j-1) Binomial[k, j] DD9[n/(m^j), k-j, m], {j, 1, k}, {m, a, n^(1/k)}]
DD9[n_, 0, a_] := 1
DD9[1000, 5, 2]
10 602
DDD[1000, 5]
10 602
DiscretePlot[DD9[n, 4, 2], {n, 2, 1000}]

```

