```
Gamma[0, s Log[n]] - Gamma[0, (s - 1) Log[n]] + Log[s / (s - 1)]
```

$$-\text{Gamma}[0, (-1+s)\,\text{Log}[n]] + \text{Gamma}[0, s\,\text{Log}[n]] + \text{Log}\left[\frac{s}{-1+s}\right]$$

$$\text{Limit}\left[-\text{Gamma}[0, 2\,\text{Log}[n]] + \text{Gamma}[0, 3\,\text{Log}[n]] + \text{Log}\left[\frac{3}{2}\right], n \to \text{Infinity}\right]$$

$$\text{Log}\left[\frac{3}{2}\right]$$

```
D[Gamma[0, s Log[n]] - Gamma[0, (s - 1) Log[n]] + Log[s / (s - 1)], s]
```

$$\frac{n^{1-s}}{-1+s} - \frac{n^{-s}}{s} + \frac{(-1+s)\left(\frac{1}{-1+s} - \frac{s}{(-1+s)^2}\right)}{s}$$

$$\text{Limit}\left[\frac{n^{1-s}}{-1+s} - \frac{n^{-s}}{s} + \frac{(-1+s)\left(\frac{1}{-1+s} - \frac{s}{(-1+s)^2}\right)}{s} \text{ /. } s \to -1,\ n \to \text{Infinity}\right]$$

$-\infty$

```
Limit[Log[(1 - 2^(1 - s)) Zeta[s]], s → 1]
```

Log[Log[2]]

```
Limit[Log[(1 - 2^(1 - s))] + Log[Zeta[s]], s → 1]
```

Log[Log[2]]

```
binomial[z_, k_] := binomial[z, k] = Product[z - j, {j, 0, k - 1}] / k!

Clear[kappa2, pk]
kappa2[n_] :=
 kappa2[n] = If[ MangoldtLambda[n] / Log[n] == 0, 0, FullSimplify[MangoldtLambda[n] / Log[n]]
    (-1)^(1 / (FullSimplify[MangoldtLambda[n] / Log[n]])))]
pk[n_, 0] := 1
pk[n_, k_] := pk[n, k] = Sum[kappa2[j] pk[Floor[n / j], k - 1], {j, 2, n}]
Dnz12[n_, z_] := Sum[z^k / k! pk[n, k], {k, 0, Log[2, n]}]

FI[n_] := FactorInteger[n]
FI[1] := {}
Dnz13[n_, s_, z_] := Sum[j^-s Product[ binomial[-z, p[[2]]], {p, FI[j]}], {j, 1, n}]

N[Table[FullSimplify[D[-Dnz13[n, s, 1], s] /. s → 0], {n, 90, 100}]]
```

{-11.1004, -6.58952, -11.1113, -6.57871, -2.03542,
 2.51846, 7.08281, 2.5081, -2.07687, -6.67199, -2.06682}

```
Sum[ N[ LiouvilleLambda[j] Log[j]], {j, 2, 100}]
```

-2.06682

```
N[D[D[Dnz13[10 000, s, z], z] /. z → 0, s] /. s → 2]
```

0.442522

```
N[Log[(Zeta[4] / Zeta[2])]]
```

-0.41859

```
Expand[D[Zeta[2 s] / Zeta[s], s] / (Zeta[2 s] / Zeta[s])]
```

$$-\frac{\text{Zeta}'[s]}{\text{Zeta}[s]}+\frac{2\,\text{Zeta}'[2\,s]}{\text{Zeta}[2\,s]}$$

$$\text{N}\left[-\frac{\text{Zeta}'[s]}{\text{Zeta}[s]}+\frac{2\,\text{Zeta}'[2\,s]}{\text{Zeta}[2\,s]}\ /.\ s\to 2\right]$$

0.442621

```
FI[n_] := FactorInteger[n]
FI[1] := {}
Dnz13[n_, s_, z_] := Sum[j^-s Product[ binomial[-z, p[[2]]], {p, FI[j]}], {j, 1, n}]
Dnz13o[n_, s_, z_] :=
 Sum[j^-s Product[(-1)^p[[2]] binomial[-z, p[[2]]], {p, FI[j]}], {j, 1, n}]
pp[n_, s_, z_] := Sum[ (Dnz13o[j, s, -z] - Dnz13o[j - 1, s, -z])
   Dnz13o[Floor[(n / j)^(1 / 2)], 2 s, z], {j, 1, n}]
pp2[n_, s_, z_] := Sum[ (Dnz13o[j, 2 s, z] - Dnz13o[j - 1, 2 s, z]) Dnz13o[n / j^2, s, -z],
  {j, 1, n^(1 / 2)}]
oo[n_, s_, z_] := Sum[ (Dnz13[j, s, -z] - Dnz13[j - 1, s, -z])
   Dnz13[Floor[(n / j)^(1 / 2)], 2 s, z], {j, 1, n}]
oo2[n_, s_, z_] := Sum[ (Dnz13[j, s, -z] - Dnz13[j - 1, s, -z]) Dnz13[n / j^2, 2 s, z],
  {j, 1, n^(1 / 2)}]
oo3[n_] := Sum[MoebiusMu[j] Dnz13[n / j^2, 0, 1], {j, 1, n^(1 / 2)}]
oo3a[n_, s_, z_] :=
 Sum[ (Dnz13o[j, s, -z] - Dnz13o[j - 1, s, -z]) Dnz13[n / j^2, 2 s, z], {j, 1, n^(1 / 2)}]
(* irk *)
oo4a[n_, s_, z_] := Sum[ (Dnz13[k^2, s, -z] - Dnz13[k^2 - 1, s, -z]) Dnz13[n / j^2, 2 s, -z],
  {j, 1, n^(1 / 2)}, {k, 1, n^(1 / 2)}]
pp2a[n_] := Sum[D[(Dnz13o[j, 0, z] - Dnz13o[j - 1, 0, z]) Dnz13o[n / j^2, 0, -z], z] /. z → 0,
  {j, 1, n^(1 / 2)}]
pp2a[n_, s_, z_] := Sum[( Dnz13o[Floor[j^(1 / 2)], s, -z] -
     Dnz13o[Floor[(j - 1)^(1 / 2)], s, -z]) Dnz13o[n / j, 2 s, z], {j, 1, n}]
oo2a[n_, s_, z_] := Sum[( Dnz13[Floor[j^(1 / 2)], s, -z] -
     Dnz13[Floor[(j - 1)^(1 / 2)], s, -z]) Dnz13[n / j, 2 s, z], {j, 1, n}]
pp2b[n_, s_, z_] := Sum[( Dnz13o[Floor[j^(1 / 2)], s, -z] -
     Dnz13o[Floor[(j - 1)^(1 / 2)], s, -z]) Dnz13o[n / j, 2 s, z], {j, 1, n}]
```

```
Expand[oo2a[100, 0, z]]
```

$$1-\frac{298\,z}{15}+\frac{5549\,z^2}{360}-\frac{85\,z^3}{48}+\frac{299\,z^4}{144}+\frac{11\,z^5}{80}+\frac{7\,z^6}{720}$$

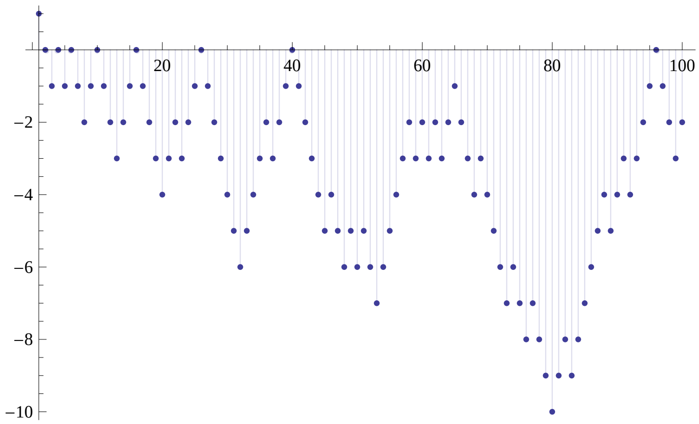
```
Expand[pp2a[100]]
```

$-\frac{116}{5}$

```
Expand[oo2a[100, 0, z]]
```

$1-\frac{298\,z}{15}+\frac{5549\,z^2}{360}-\frac{85\,z^3}{48}+\frac{299\,z^4}{144}+\frac{11\,z^5}{80}+\frac{7\,z^6}{720}$

```
DiscretePlot[ pp[n, 0, 1], {n, 1, 100}]
```




```
pp2b[n_, s_, z_] :=
 Sum[( Dnz13o[Floor[j^(1/4)], s, -z] - Dnz13o[Floor[(j-1)^(1/4)], s, -z])
   Dnz13o[n/j, 4 s, z], {j, 1, n}]
Table[{n, D[(pp2b[2^n, 0, z] - pp2b[2^n - 1, 0, z]), z] /. z → 0}, {n, 1, 10}] // TableForm
```

| | |
|---|---|
| 1 | $1$ |
| 2 | $\frac{1}{2}$ |
| 3 | $\frac{1}{3}$ |
| 4 | $-\frac{3}{4}$ |
| 5 | $\frac{1}{5}$ |
| 6 | $\frac{1}{6}$ |
| 7 | $\frac{1}{7}$ |
| 8 | $-\frac{3}{8}$ |
| 9 | $\frac{1}{9}$ |
| 10 | $\frac{1}{10}$ |

```
K[n_] := K[n] = If[ n ≠ Floor[n], 0, FullSimplify[ MangoldtLambda[ n] / Log[n]]]
kl[n_] := K[n] - K[n^(1/2)]
Table[kl[n], {n, 2, 33}]
```

$\{1, 1, -\frac{1}{2}, 1, 0, 1, \frac{1}{3}, -\frac{1}{2}, 0, 1, 0, 1, 0, 0, -\frac{1}{4}, 1, 0, 1, 0, 0, 0, 1, 0, -\frac{1}{2}, 0, \frac{1}{3}, 0, 1, 0, 1, \frac{1}{5}, 0\}$

```
K[16]
```

$\frac{1}{4}$

```
Clear[dd, jord]
FI[n_] := FactorInteger[n]
FI[0] := {}
FI[1] := {}
dd[n_, s_, z_] :=
 dd[n, s, z] = Sum[j^-s Product[(-1)^p[[2]] binomial[-z, p[[2]]], {p, FI[j]}], {j, 1, n}]
dd[0, s_, z_] := 0
jord[n_, k_] := jord[n, k] = n^k Product[1 - 1 / p[[1]]^k, {p, FI[n]}]
js[n_, s_, k_] := Sum[ j^-s jord[j, k], {j, 1, n}]
sig[n_, s_, k_] := Sum[ j^-s DivisorSigma[k, j], {j, 1, n}]
tot[n_, s_, k_] := Sum[ j^-s jord[j, k], {j, 1, n}]
tot2[n_, s_, k_] := Sum[ (dd[j, s - k, 1] - dd[j - 1, s - k, 1]) dd[n / j, s, -1], {j, 1, n}]
dsigma[n_, s_, k_] := Sum[ (dd[j, s - k, 1] - dd[j - 1, s - k, 1]) dd[n / j, s, 1], {j, 1, n}]
ds2[n_, s_, a_, k_] := Sum[ j^-s DivisorSigma[a, j] ds2[n / j, s, a, k - 1], {j, 2, n}]
ds2[n_, s_, a_, 0] := 1
dsz[n_, s_, a_, z_] := Sum[ Binomial[ z, k] ds2[n, s, a, k], {k, 0, Log[2, n]}]
dsigmaz[n_, s_, a_, z_] := Sum[ (dd[j, s - a, z] - dd[j - 1, s - a, z]) dd[n / j, s, z], {j, 1, n}]
```

```
tot2[100, -2, 2]
```

1 696 967 413

```
js[100, -2, 2]
```

1 696 967 413

```
dsigma[100, -1, 2]
```

30 766 703

```
sig[100, -1, 2]
```

30 766 703

```
dsz[100, -1, -2, -3]
```

$$\frac{2\,682\,015\,571\,623\,969\,862\,865\,333\,635\,975\,968\,635\,064\,127}{363\,126\,954\,321\,419\,151\,898\,613\,588\,204\,751\,581\,025}$$

```
dsigmaz[100, -1, -2, -3]
```

$$\frac{2\,682\,015\,571\,623\,969\,862\,865\,333\,635\,975\,968\,635\,064\,127}{363\,126\,954\,321\,419\,151\,898\,613\,588\,204\,751\,581\,025}$$

```
rr[n_] := Sum[ DivisorSigma[0, j^2], {j, 1, n}]
```

```
rr[100]
```

1194

```
Sum[ (dd[j^2, 0, 3] - dd[j^2 - 1, 0, 3]) dd[Floor[100 / j^2], 0, -1], {j, 1, 10}]
```

35

```
px[n_, s_, z_] :=
 Sum[( dd[Floor[j^(1 / 2)], s, 3 z] - dd[Floor[(j - 1)^(1 / 2)], s, 3 z]) dd[n / j, 2 s, -z],
  {j, 1, n}]
```

```
px[100, 0, 1]
```

5

```
Clear[K]
K[n_] := If[n < 2, 0, FullSimplify[ MangoldtLambda[ Floor[n]] / Log[Floor[n]]]]
k2[n_] := K[n] - (K[n^(1 / 2)] - K[(n - 1)^(1 / 2)])
```

```
Table[ k2[n], {n, 1, 20}]
```

$$\left\{0, 1, 1, -\frac{1}{2}, 1, 0, 1, \frac{1}{3}, \frac{1}{2}, 0, 1, 0, 1, 0, 0, \frac{3}{4}, 1, 0, 1, 0\right\}$$

```
D[Log[Zeta[s + a]] - Log[Zeta[s]], s]
```

$$-\frac{\text{Zeta}'[s]}{\text{Zeta}[s]} + \frac{\text{Zeta}'[a + s]}{\text{Zeta}[a + s]}$$