

```
ClearAll["Global`*"]
```

```
E2a[n_, k_, a_] :=
```

```
  E2a[n, k, a] = Sum[ E2a[n / j, k - 1, a], {j, 2, n}] - a Sum[ E2a[n / (a j), k - 1, a], {j, 1, n / a}];
```

```
E2a[n_, 0, a_] := 1
```

```
E2D2[n_, k_, b_] :=
```

```
  (-1)^k + Sum[b^a / ((k - 1)!) Binomial[k, j] Pochhammer[a - k + j + 1, k - 1] E2a[b^-a n, j, b],  
    {a, 0, Log[b, n]}, {j, 0, k}]
```

```
D2a[n_, k_] := D2a[n, k] = Sum[D2a[Floor[n / j], k - 1], {j, 2, n}]; D2a[n_, 0] := 1
```

```
d2[n_, k_] := Sum[d2[j, k - 1] d2[n / j, 1], {j, Divisors[n]}];
```

```
d2[n_, 1] := 1; d2[1, 1] := 0; d2[n_, 0] := 0; d2[1, 0] := 1
```

```
bin[n_, k_] := (n)! / ((k!) ((n - k)!))
```

```
D2a[100, 2]
```

```
283
```

```
E2D2[100, 2, 11 / 10]
```

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283
```

```
E2D2a[n_, k_, a_] :=
```

```
  (-1)^k + Sum[a^j / ((k - 1)!) Binomial[k, m] Pochhammer[j - k + m + 1, k - 1] E2a[a^-j n, m, a],  
    {j, 0, Log[a, n]}, {m, 0, k}]
```

```
E2D2a[100, 3, 3 / 2]
```

```
324
```

```
E2D2b[n_, k_, a_] :=
```

```
  (-1)^k + Sum[a^j Binomial[k, m] Pochhammer[j - k + m + 1, k - 1] E2a[a^-j n, m, a],  
    {j, 0, Log[a, n]}, {m, 0, k}] / Gamma[k]
```

```
E2D2b[100, 3, 3 / 2]
```

```
324
```

```
E2D2c[n_, k_, a_] := (-1)^k +
```

```
  Sum[a^j Sum[ Binomial[k, m] Pochhammer[j - k + m + 1, k - 1] E2a[a^-j n, m, a], {m, 0, k}],  
    {j, 0, Log[a, n]}] / Gamma[k]
```

```
E2D2c[100, 3, 3 / 2]
```

```
324
```

```
  (-1)^k + Sum[a^j Sum[ Binomial[k, m] Pochhammer[j - k + m + 1, k - 1] E2[a^-j n, m, a], {m, 0, k}],  
    {j, 0, Log[a, n]}] / Gamma[k] /. k -> 1
```

$$-1 + \sum_{j=0}^{\frac{\log[n]}{\log[a]}} a^j \left(E2[a^{-j} n, 0, a] + E2[a^{-j} n, 1, a] \right)$$

```
  (-1)^k + Sum[a^j Sum[ Binomial[k, m] Pochhammer[j - k + m + 1, k - 1] E2[a^-j n, m, a], {m, 0, k}],  
    {j, 0, Log[a, n]}] / Gamma[k] /. k -> 2
```

$$1 + \sum_{j=0}^{\frac{\log[n]}{\log[a]}} a^j \left((-1 + j) E2[a^{-j} n, 0, a] + 2 j E2[a^{-j} n, 1, a] + (1 + j) E2[a^{-j} n, 2, a] \right)$$

$$(-1)^k + \text{Sum}[a^j \text{Sum}[\text{Binomial}[k, m] \text{Pochhammer}[j - k + m + 1, k - 1] \text{E2}[a^{-j} n, m, a], \{m, 0, k\}], \{j, 0, \text{Log}[a, n]\}] / \text{Gamma}[k] /. k \rightarrow 3$$

- 1 +

$$\frac{1}{2} \sum_{j=0}^{\frac{\text{Log}[n]}{\text{Log}[a]}} a^j \left((-2+j) (-1+j) \text{E2}[a^{-j} n, 0, a] + 3 (-1+j) j \text{E2}[a^{-j} n, 1, a] + 3 j (1+j) \text{E2}[a^{-j} n, 2, a] + (1+j) (2+j) \text{E2}[a^{-j} n, 3, a] \right)$$

$$(-1)^k + \text{Sum}[a^j \text{Sum}[\text{Binomial}[k, m] \text{Pochhammer}[j - k + m + 1, k - 1] \text{E2}[a^{-j} n, m, a], \{m, 0, k\}], \{j, 0, \text{Log}[a, n]\}] / \text{Gamma}[k] /. k \rightarrow 4$$

$$f[n_, a_] := 1 + \sum_{j=0}^{\frac{\text{Log}[n]}{\text{Log}[a]}} a^j j (a - 1)^2$$

$$f[100, 1.01]$$

$$364.674$$

$$N[\text{Gamma}[2, 0, -\text{Log}[100]]]$$

$$361.517 - 4.41506 \times 10^{-14} i$$

$$g[n_, a_] := \text{Sum}[a^j (a - 1)^2 (-1)^2 j, \{j, 1, \text{Log}[a, n]\}]$$

$$D2a[100, 2] - g[100, 1.0001]$$

$$-78.576$$

$$E2D2d[n_, k_, a_] :=$$

$$(-1)^k + \text{Sum}[a^j (-j^k (k - 1) (a - 1)^k + \text{Sum}[\text{Binomial}[k, m] \text{Pochhammer}[j - k + m + 1, k - 1] \text{E2a}[a^{-j} n, m, a], \{m, 0, k\}]), \{j, 0, \text{Log}[a, n]\}] / \text{Gamma}[k]$$

$$E2D2d[100, 2, 1.001]$$

$$-78.8913$$

$$-78.517 + 4.41506 \times 10^{-14} i$$

$$E2D2e[n_, k_, a_] :=$$

$$(-1)^k + \text{Sum}[a^j (-j^k (k - 1) (a - 1)^k + \text{Sum}[\text{Binomial}[k, m] \text{Pochhammer}[j - k + m + 1, k - 1] \text{E2}[a^{-j} n, m, a], \{m, 0, k\}]), \{j, 0, \text{Log}[a, n]\}] / \text{Gamma}[k]$$

$$\text{Expand}[(- ((a - 1)^k))]$$

$$- (-1 + a)^k$$

$$E2D2e[n, 2, a]$$

$$1 + \sum_{j=0}^{\frac{\text{Log}[n]}{\text{Log}[a]}} a^j \left(- (-1 + a)^2 j + (-1 + j) \text{E2}[a^{-j} n, 0, a] + 2 j \text{E2}[a^{-j} n, 1, a] + (1 + j) \text{E2}[a^{-j} n, 2, a] \right)$$

$$E2D2e[n, 3, a]$$

$$-1 + \frac{1}{2} \sum_{j=0}^{\frac{\text{Log}[n]}{\text{Log}[a]}} a^j \left(- (-1 + a)^3 j^2 + (-2 + j) (-1 + j) \text{E2}[a^{-j} n, 0, a] + 3 (-1 + j) j \text{E2}[a^{-j} n, 1, a] + 3 j (1 + j) \text{E2}[a^{-j} n, 2, a] + (1 + j) (2 + j) \text{E2}[a^{-j} n, 3, a] \right)$$

E2D2f[n_, k_, a_] := $(-1)^k +$
 $\text{Sum}[a^j j^k (k-1) (- (a-1)^k + \text{Sum}[\text{Binomial}[k, m] (\text{Pochhammer}[j-k+m+1, k-1] / j^k (k-1))$
 $\text{E2}[a^{-j} n, m, a], \{m, 0, k\}], \{j, 0, \text{Log}[a, n]\}] / \text{Gamma}[k]$

E2D2f[n, 2, a]

$$1 + \sum_{j=0}^{\frac{\text{Log}[n]}{\text{Log}[a]}} a^j j \left(-(-1+a)^2 + \frac{(-1+j) \text{E2}[a^{-j} n, 0, a]}{j} + 2 \text{E2}[a^{-j} n, 1, a] + \frac{(1+j) \text{E2}[a^{-j} n, 2, a]}{j} \right)$$

E2D2f[n, 3, a]

$$-1 + \frac{1}{2} \sum_{j=0}^{\frac{\text{Log}[n]}{\text{Log}[a]}} a^j j^2 \left(-(-1+a)^3 + \frac{(-2+j) (-1+j) \text{E2}[a^{-j} n, 0, a]}{j^2} + \right.$$

$$\left. \frac{3 (-1+j) \text{E2}[a^{-j} n, 1, a]}{j} + \frac{3 (1+j) \text{E2}[a^{-j} n, 2, a]}{j} + \frac{(1+j) (2+j) \text{E2}[a^{-j} n, 3, a]}{j^2} \right)$$

Expand[(a-1)^3]

$$-1 + 3a - 3a^2 + a^3$$

E2D2g[n_, k_, a_] :=

$(-1)^k + \text{Sum}[a^j j^k (k-1) (\text{Sum}[\text{Binomial}[k, m] (\text{Pochhammer}[j-k+m+1, k-1] / j^k (k-1))$
 $(\text{E2a}[a^{-j} n, m, a] - (\text{Binomial}[k, m] a^m)), \{m, 0, k\}], \{j, 1, \text{Log}[a, n]\}] / \text{Gamma}[k]$

E2D2g[n, 3, a]

$$-1 + \frac{1}{2 (-1+a)^3 \text{Log}[a]^2}$$

$$(38 a^3 \text{Log}[a]^2 + 6 a^4 \text{Log}[a]^2 - 6 a^5 \text{Log}[a]^2 + 2 a^6 \text{Log}[a]^2 - 38 a^3 n \text{Log}[a]^2 - 6 a^4 n \text{Log}[a]^2 +$$

$$6 a^5 n \text{Log}[a]^2 - 2 a^6 n \text{Log}[a]^2 + a n \text{Log}[a] \text{Log}[n] - 13 a^2 n \text{Log}[a] \text{Log}[n] - 24 a^3 n \text{Log}[a] \text{Log}[n] +$$

$$40 a^4 n \text{Log}[a] \text{Log}[n] - a^5 n \text{Log}[a] \text{Log}[n] - 3 a^6 n \text{Log}[a] \text{Log}[n] - a n \text{Log}[n]^2 -$$

$$7 a^2 n \text{Log}[n]^2 + 8 a^3 n \text{Log}[n]^2 + 8 a^4 n \text{Log}[n]^2 - 7 a^5 n \text{Log}[n]^2 - a^6 n \text{Log}[n]^2)$$

E2D2g[100, 3, 2]

$$-122001$$

E2D2h[n_, k_, a_] :=

$(-1)^k + \text{Sum}[$
 $a^j ($
 $-j^k (k-1) (a-1)^k$
 $+$
 $\text{Sum}[$
 $(\text{Binomial}[k, m] \text{Pochhammer}[j-k+m+1, k-1] \text{E2a}[a^{-j} n, m, a])$
 $, \{m, 0, k\}], \{j, 0, \text{Log}[a, n]\}] / \text{Gamma}[k]$

E2D2h[100, 2, 1.001]

$$-78.8913$$

```

E2D2i[n_, k_, a_] :=
  (-1)^k + Sum[

    a^j (

      Sum[
        Binomial[k, m]
        (Pochhammer[j - k + m + 1, k - 1] E2a[a^-j n, m, a]
          - j^k (k - 1) (a^m (-1)^(k - m)))
      ], {m, 0, k}], {j, 0, Log[a, n]] / Gamma[k]

E2D2i[50, 2, 1.001]
-38.6541

D2a[50, 2] - g[50, 1.0001]
-38.622

fe1[j_, k_, a_] := (a - 1)^k
Expand[fe1[3, 4, a]]
1 - 4 a + 6 a^2 - 4 a^3 + a^4

fe2[j_, k_, a_] := Sum[Binomial[k, s] a^s (-1)^(k - s), {s, 0, k}]
fe2[3, 4, a]
1 - 4 a + 6 a^2 - 4 a^3 + a^4

E2D2j[n_, k_, a_] :=
  (-1)^k + Sum[a^j (Sum[ Binomial[k, m] (Pochhammer[j - k + m + 1, k - 1] E2a[a^-j n, m, a] -
    j^k (k - 1) (a^m (-1)^(k - m))), {m, 0, k}], {j, 0, Log[a, n]] / Gamma[k]

E2D2j[50, 2, 1.001]
-38.6541

N[D2a[50, 2] - Gamma[2, 0, -Log[50]] / Gamma[2]]

-38.6012 + 1.7831 x 10^-14 i

a^j (Sum[ Binomial[k, m] (Pochhammer[j - k + m + 1, k - 1] E2[a^-j n, m, a] -
  j^k (k - 1) (a^m (-1)^(k - m))), {m, 0, k}]) /. j -> 0

$Aborted

FullSimplify[Pochhammer[j - k + m + 1, k - 1] E2[a^-j n, m, a] - j^k (k - 1) (a^m (-1)^(k - m))]
- (-1)^(k - m) a^m j^(-1 + k) + E2[a^-j n, m, a] Pochhammer[1 + j - k + m, -1 + k]

Gamma[(1 + j - k + m) + 1] / Gamma[(1 + j - k + m) - (-1 + k) + 1]

Gamma[2 + j - k + m]
-----
Gamma[3 + j - 2 k + m]

```

`FullSimplify` $\left[\frac{\text{Gamma}[j+m]}{\text{Gamma}[-1+k]} \text{E2}[a^{-j}n, m, a] - j^{(k-1)} (a^m (-1)^{(k-m)})\right]$

$- (-1)^{k-m} a^m j^{-1+k} + \frac{\text{E2}[a^{-j}n, m, a] \text{Gamma}[j+m]}{\text{Gamma}[-1+k]}$

`E2D2k[n_, k_, a_] :=`

$(-1)^k + \text{Sum}\left[a^j j \left(\text{Sum}\left[\text{Binomial}[k, m] \left(\frac{\text{Gamma}[j+m]}{\text{Gamma}[1+j-k+m]} \text{E2a}[a^{-j}n, m, a] - j^{(k-1)} (a^m (-1)^{(k-m)}) \right), \{m, 0, k\}\right], \{j, 0, \text{Log}[a, n]\}\right] \right] / \text{Gamma}[k]$

`E2D2k[50, 2, 1.001]`

Infinity::indet: Indeterminate expression 0 ComplexInfinity encountered. >>

Indeterminate

`bb[j_, k_, m_] := Pochhammer[j - k + m + 1, k - 1]`

`bb[3, 4, 5]`

210

`bb2[j_, k_, m_] := $\frac{\text{Gamma}[(j - k + m + 1) + (k - 1)]}{\text{Gamma}[(j - k + m + 1)]}$`

`bb2[3, 4, 5]`

210

`bb3[j_, k_, m_] := Binomial[j - k + m + 1 + (k - 1) - 1, k - 1] ((k - 1) !)`

`bb3[3, 4, 5]`

210

`$\frac{\text{Gamma}[(j - k + m + 1) + (k - 1)]}{\text{Gamma}[(j - k + m + 1)]}$`
 `$\frac{\text{Gamma}[j + m]}{\text{Gamma}[1 + j - k + m]}$`

`bb4[j_, k_, m_] := Binomial[j - k + m + 1 + (k - 1) - 1, k - 1] ((k - 1) !)`

`bb4[3, 4, 5]`

210

`bin[n_, k_] := (n) ! / ((k!) ((n - k) !))`

`bb5[j_, k_, m_] := bin[j - k + m + 1 + (k - 1) - 1, k - 1] ((k - 1) !)`

`bb5[3, 4, 5]`

210

`bin[j - k + m + 1 + (k - 1) - 1, k - 1] ((k - 1) !)`

$$\frac{(-1+j+m)!}{(j-k+m)!}$$

$$\frac{(-1+j+m)!}{(j-k+m)!}$$

```
E2D2j[n_, k_, a_] := (-1)^k +
  (a^0 (Sum[ Binomial[k, m] (Pochhammer[0 - k + m + 1, k - 1] E2a[a^-0 n, m, a] -
    0^(k - 1) (a^m (-1)^(k - m))), {m, 0, k}]) +
    Sum[a^j (Sum[ Binomial[k, m] (Pochhammer[j - k + m + 1, k - 1] E2a[a^-j n, m, a] -
      j^(k - 1) (a^m (-1)^(k - m))), {m, 0, k}]), {j, 1, Log[a, n]}]) / Gamma[k]
```

```
E2D2j[100, 2, 1.001]
```

```
-78.8913
```

```
E2D2k[n_, k_, a_] := (-1)^k +
  ((Sum[ Binomial[k, m]
    (Pochhammer[-k + m + 1, k - 1] E2a[a^-0 n, m, a] - (a^m (-1)^(k - m))), {m, 0, k}]) +
    Sum[a^j (Sum[ Binomial[k, m] (Pochhammer[j - k + m + 1, k - 1] E2a[a^-j n, m, a] -
      j^(k - 1) (a^m (-1)^(k - m))), {m, 0, k}]), {j, 1, Log[a, n]}]) / Gamma[k]
```

```
E2D2k[100, 2, 1.001]
```

```
-78.8913
```

```
E2D2m[n_, k_, a_] := (-1)^k +
  ((Sum[ Binomial[k, m]
    (Pochhammer[-k + m + 1, k - 1] E2a[n, m, a] - (a^m (-1)^(k - m))), {m, 0, k}]) +
    Sum[a^j (Sum[ Binomial[k, m] (Pochhammer[j - k + m + 1, k - 1] E2a[a^-j n, m, a] -
      j^(k - 1) (a^m (-1)^(k - m))), {m, 0, k}]), {j, 1, Log[a, n]}]) / Gamma[k]
```

```
E2D2m[100, 2, 1.001]
```

```
-78.8913
```

```
Pochhammer[-k + m + 1, k - 1] /. {m -> 1, k -> 5}
```

```
0
```

```
E2D2n[n_, k_, a_] := (-1)^k +
  ((Sum[ Binomial[k, m] (Pochhammer[-k + m + 1, k - 1] E2a[n, m, a]), {m, 0, k}]) +
    (Sum[ Binomial[k, m] (- (a^m (-1)^(k - m))), {m, 0, k}]) +
    Sum[a^j (Sum[ Binomial[k, m] (Pochhammer[j - k + m + 1, k - 1] E2a[a^-j n, m, a] -
      j^(k - 1) (a^m (-1)^(k - m))), {m, 0, k}]), {j, 1, Log[a, n]}]) / Gamma[k]
```

```
E2D2n[100, 2, 1.001]
```

```
-78.8913
```

```
E2D2o[n_, k_, a_] := (-1)^k +
  ((Sum[ (Pochhammer[-k + m + 1, k - 1] E2a[n, m, a]), {m, 0, 0}]) +
    Sum[ (Pochhammer[-k + m + 1, k - 1] E2a[n, m, a]), {m, k, k}]) +
    (Sum[ Binomial[k, m] (- (a^m (-1)^(k - m))), {m, 0, k}]) +
    Sum[a^j (Sum[ Binomial[k, m] (Pochhammer[j - k + m + 1, k - 1] E2a[a^-j n, m, a] -
      j^(k - 1) (a^m (-1)^(k - m))), {m, 0, k}]), {j, 1, Log[a, n]}]) / Gamma[k]
```

E2D2o[100, 2, 1.001]

-78.8913

```
E2D2p[n_, k_, a_] := (-1)^k +
(
  Pochhammer[-k+1, k-1] E2a[n, 0, a] +
  Pochhammer[-k+k+1, k-1] E2a[n, k, a] +
  (Sum[ Binomial[k, m] (- (a^m (-1)^(k-m))), {m, 0, k}]) +
  Sum[a^j (Sum[ Binomial[k, m] (Pochhammer[j-k+m+1, k-1] E2a[a^-j n, m, a] -
    j^(k-1) (a^m (-1)^(k-m))), {m, 0, k}]), {j, 1, Log[a, n]})] / Gamma[k]
```

E2D2p[100, 2, 1.001]

-78.8913

```
E2D2q[n_, k_, a_] := (-1)^k +
((-1)^(k+1) Gamma[k] E2a[n, 0, a] +
  Gamma[k] E2a[n, k, a] +
  (Sum[ Binomial[k, m] (- (a^m (-1)^(k-m))), {m, 0, k}]) +
  Sum[a^j (Sum[ Binomial[k, m] (Pochhammer[j-k+m+1, k-1] E2a[a^-j n, m, a] -
    j^(k-1) (a^m (-1)^(k-m))), {m, 0, k}]), {j, 1, Log[a, n]})] / Gamma[k]
```

E2D2q[100, 2, 1.001]

-78.8913

Pochhammer[1-5, 5-1]

24

$(-1)^{(k+1)} \Gamma[k] /. k \rightarrow 5$

24

Pochhammer[1, 4-1]

6

```
E2D2r[n_, k_, a_] := (-1)^k + (-1)^(k+1) E2a[n, 0, a] + E2a[n, k, a] + (
  (Sum[ Binomial[k, m] (- (a^m (-1)^(k-m))), {m, 0, k}]) +
  Sum[a^j (Sum[ Binomial[k, m] (Pochhammer[j-k+m+1, k-1] E2a[a^-j n, m, a] -
    j^(k-1) (a^m (-1)^(k-m))), {m, 0, k}]), {j, 1, Log[a, n]})] / Gamma[k]
```

E2D2r[100, 2, 1.001]

-78.8913

```
E2D2s[n_, k_, a_] := E2a[n, k, a] + (
  (Sum[ Binomial[k, m] (- (a^m (-1)^(k-m))), {m, 0, k}]) +
  Sum[a^j (Sum[ Binomial[k, m] (Pochhammer[j-k+m+1, k-1] E2a[a^-j n, m, a] -
    j^(k-1) (a^m (-1)^(k-m))), {m, 0, k}]), {j, 1, Log[a, n]})] / Gamma[k]
```

E2D2s[100, 2, 1.001]

-78.8913

pp[k_, a_] := Sum[Binomial[k, m] (- (a^m (-1)^(k-m))), {m, 0, k}]

pp[2, 1.001]

$-1. \times 10^{-6}$

```

E2D2t[n_, k_, a_] := E2a[n, k, a] + (
  Sum[a^j (Sum[ Binomial[k, m] (Pochhammer[j - k + m + 1, k - 1] E2a[a^-j n, m, a] -
    j^(k - 1) (a^m (-1)^(k - m))), {m, 0, k}], {j, 1, Log[a, n]})] / Gamma[k]
E2D2t[100, 2, 1.001]
-78.8913
{E2a[8, 2, 1.0001], d2[8, 2] + 1}
{3.0014, 3}
{E2a[12, 3, 1.0001], d2[12, 3] - 1}
{1.9967, 2}
{E2a[64, 5, 1.0001], d2[64, 5] - 1}
{3.96847, 4}

```

```

E2D2u[n_, k_, a_] := d2[n, k] + (-1)^(k) + (
  Sum[a^j (Sum[ Binomial[k, m] (Pochhammer[j - k + m + 1, k - 1] E2a[a^-j n, m, a] -
    j^(k - 1) (a^m (-1)^(k - m))), {m, 0, k}], {j, 1, Log[a, n]})] / Gamma[k]
E2D2u[100, 2, 1.001]
-79.0897
E2D2v[n_, k_, a_] :=
  d2[n, k] + (-1)^(k) + (Sum[a^j (Sum[ Binomial[k, m] (
    ((-1 + j + m)!) / ((j - k + m)!) E2a[a^-j n, m, a] -
    j^(k - 1) (a^m (-1)^(k - m))], {m, 0, k}], {j, 1, Log[a, n]})] / Gamma[k]
E2D2v[100, 2, 1.001]
-79.0897

```

```

E2D2w[n_, k_, a_] := d2[n, k] + (-1)^(k) +
  (Sum[a^j j^(k - 1) (Sum[ Binomial[k, m] (
    ((-1 + j + m)!) / ((j - k + m)!) (j^(k - 1)) E2a[a^-j n, m, a] -
    (a^m (-1)^(k - m))], {m, 0, k}], {j, 1, Log[a, n]})] / Gamma[k]
E2D2w[100, 2, 1.001]
-79.0897

```

```

E2D2x[n_, k_, a_] := d2[n, k] + (-1)^(k) +
  (Sum[a^j j^(k - 1) (Sum[ Binomial[k, m] ((j^(1 - k) Binomial[-1 + j + m, -1 + k]) E2a[a^-j n, m, a] -
    (a^m (-1)^(k - m)) / Gamma[k]), {m, 0, k}], {j, 1, Log[a, n]})]
E2D2x[100, 2, 1.001]
-79.0897

```


`FullSimplify` $\left[\left(\frac{(-1+j+m)!}{(j-k+m)!}\right) / (j^k (k-1)!)\right]$

$\frac{j^{1-k} \text{Gamma}[j+m]}{(j-k+m)!}$

`Binomial` $[j-k+m+1+(k-1)-1, k-1] (k-1)!$

`Binomial` $[-1+j+m, -1+k] (-1+k)! / (j^k (k-1)!)$

$j^{1-k} \text{Binomial}[-1+j+m, -1+k] (-1+k)!$

$j^{1-k} \text{Binomial}[-1+j+m, -1+k]$

`kk := 5; Table` $[\text{Expand}[j^{1-kk} \text{Binomial}[-1+j+m, -1+kk]], \{m, 0, kk\}] // \text{TableForm}$

$\frac{1}{24} + \frac{1}{j^4} - \frac{25}{12 j^3} + \frac{35}{24 j^2} - \frac{5}{12 j}$

$\frac{1}{24} - \frac{1}{4 j^3} + \frac{11}{24 j^2} - \frac{1}{4 j}$

$\frac{1}{24} + \frac{1}{12 j^3} - \frac{1}{24 j^2} - \frac{1}{12 j}$

$\frac{1}{24} - \frac{1}{12 j^3} - \frac{1}{24 j^2} + \frac{1}{12 j}$

$\frac{1}{24} + \frac{1}{4 j^3} + \frac{11}{24 j^2} + \frac{1}{4 j}$

$\frac{1}{24} + \frac{1}{j^4} + \frac{25}{12 j^3} + \frac{35}{24 j^2} + \frac{5}{12 j}$

`E2D2y` $[n_, k_, a_] := d2[n, k] + (-1)^k + (\text{Sum}[a^j j^k (k-1) (\text{Sum}[\text{Binomial}[k, m] ($
 $(j^{1-k} \text{Binomial}[-1+j+m, -1+k])$

`E2a` $[a^{-j n}, m, a]$

+

$a^m (-1)^{k-m+1} / \text{Gamma}[k]$

$), \{m, 0, k\}], \{j, 1, \text{Log}[a, n]\}])$

`E2D2y` $[100, 2, 1.001]$

-79.0897

$$E2D2z[n_ , k_ , a_] := d2[n, k] + (-1)^{(k)} + \left(\text{Sum}[a^{^j} / \text{Gamma}[k] \left(\text{Sum}[\text{Binomial}[k, m] \left(\right. \right. \right.$$

$$\left(\begin{array}{l} \\ E2a[a^j - j n, m, a] \end{array} \right)$$

+

$$\left. \right), \{m, 0, k\} \bigr] \bigr), \{j, 1, \text{Log}[a, n]\} \bigr] \bigr)$$

E2D2z[100, 2, 1.001]

Power::infy : Infinite expression $\frac{1}{0}$ encountered. >>

Infinity::indet: Indeterminate expression 0 ComplexInfinity encountered. >>

Indeterminate

```
bin[-1 + j + m, -1 + k]
```

$$\frac{(-1+j+m)!}{(-1+k)!(j-k+m)!} \frac{(-1+j+m)!}{(j-k+m)!}$$

$$\text{FullSimplify}\left[j^{(k-1)} a^m (-1)^{(k-m+1)} \bigg/ \left(\frac{(-1+j+m)!}{(j-k+m)!} \right)\right]$$

$$- \frac{(-1)^{k-m} a^m j^{-1+k} (j-k+m)!}{\text{Gamma}[j+m]}$$

$$-\frac{(-1)^{k-m} a^m j^{-1+k} (j-k+m)!}{\text{Gamma}[j+m]} /. \{k \rightarrow 4, m \rightarrow 3\}$$

$$\frac{a^3 j^3 (-1 + j)!}{\text{Gamma}[3 + j]}$$

$$\mathbf{E2D2x2}[n_, k_, a_] := d2[n, k] + (-1)^k +$$

$$a \left(\text{Sum} \left[\text{Binomial}[k, m] \left(\text{Binomial}[m, -1 + k] \right) \text{E2a}[a^{-1} n, m, a] - \frac{a^m (-1)^{(k-m)}}{\text{Gamma}[k]} \right], \{m, 0, k\} \right] +$$

$$\text{Sum}\left[a^j j^{k-1} \left(\text{Sum}\left[\text{Binomial}[k, m] \left((j^{1-k} \text{Binomial}[-1+j+m, -1+k]) \text{E2a}[a^{-j n}, m, a] - (a^m (-1)^{k-m}) / \text{Gamma}[k] \right), \{m, 0, k\} \right] \right), \{j, 2, \text{Log}[a, n]\} \right]$$

E2D2x2[100, 2, 1.001]

-79.0897

E2D2x3[n_, k_, a_] := d2[n, k] + (-1)^(k) +

Sum[Binomial[k, m] Binomial[m, -1 + k] E2a[a^(-1 n, m, a), {m, 0, k}] +

Sum[a^j j^(k-1) (Sum[Binomial[k, m] ((j^(1-k) Binomial[-1 + j + m, -1 + k]) E2a[a^(-j n, m, a) -
(a^m (-1)^(k-m)) / Gamma[k]), {m, 0, k}]), {j, 2, Log[a, n]}]

E2D2x3[100, 2, 1.0005]

-78.7641

D2a[100, 2] - N[Gamma[2, 0, -Log[100]]]

-78.517 + 4.41506 × 10⁻¹⁴ i

bin[k, m] bin[m, -1 + k] /. {k → 5, m → 4}

5

E2D2x4[n_, k_, a_] := d2[n, k] + (-1)^(k) +

Sum[a^j j^(k-1) (Sum[Binomial[k, m] ((j^(1-k) Binomial[-1 + j + m, -1 + k]) E2a[a^(-j n, m, a) -
(a^m (-1)^(k-m)) / Gamma[k]), {m, 0, k}]), {j, 2, Log[a, n]}]

E2D2x4[100, 2, 1.0005]

-78.8633

E2a[1.0001^(-1 × 100, 3, 1.00001]

-1.00297

p2[n_, k_, a_] := k E2a[a^(-1 n, k-1, a) + k E2a[a^(-1 n, k, a]

5 p2[130, 5, 1.00005]

-0.161412

E2a[n_, k_, a_] :=

E2a[n, k, a] = Sum[E2a[n / j, k-1, a], {j, 2, n}] - a Sum[E2a[n / (a j), k-1, a], {j, 1, n / a}];

E2a[n_, 0, a_] := 1

E2D2x5[n_, k_, a_] :=

d2[n, k] + (-1)^(k) + (Sum[a^j / Gamma[k] (Sum[Binomial[k, m] (((-1 + j + m) !)
(j - k + m) !)

E2a[a^(-j n, m, a) + j^(k-1) a^m (-1)^(k-m+1) / (((-1 + j + m) !)
(j - k + m) !)

)

, {m, 0, k}]), {j, 2, Log[a, n]}]

E2D2x5[100, 2, 1.0005]

-78.8633

$$\text{bin}[k, m] \left(\frac{(-1 + j + m)!}{(j - k + m)!} \right)$$

$$\text{p3}[j_, k_, m_] := \frac{k! (-1 + j + m)!}{(k - m)! m! (j - k + m)!}$$

p3[j, 4, 0]

$$\frac{(-1 + j)!}{(-4 + j)!}$$

Pochhammer[j, j + 2]

Pochhammer[j, 2 + j]

$$\text{E2}[a^{-j} n, m, a] + j^k (k - 1) a^m (-1)^{(k - m + 1)} \left/ \left(\frac{(-1 + j + m)!}{(j - k + m)!} \right) \right.$$

$$\text{E2}[a^{-j} n, m, a] + \frac{(-1)^{1+k-m} a^m j^{-1+k} (j - k + m)!}{(-1 + j + m)!}$$

E2b[n_, m_, a_] := Sum[E2b[n/s, m - 1, a], {s, 2, n}] -
a Sum[E2b[n/(a s), m - 1, a], {s, 1, n/a}]; E2b[n_, 0, a_] := 1

$$\text{Table} \left[\left\{ m, \text{E2}[a^{-j} n, m, a] + \frac{(-1)^{1+k-m} a^m j^{-1+k} (j - k + m)!}{(-1 + j + m)!} \right\}, \{m, 0, k = 1\} \right] // \text{TableForm}$$

$$\begin{array}{l} 0 \quad 1 + \text{E2}[a^{-j} n, 0, a] \\ 1 \quad -a + \text{E2}[a^{-j} n, 1, a] \end{array}$$

$$\text{Table} \left[\left\{ m, \text{E2}[a^{-j} n, m, a] + \frac{(-1)^{1+k-m} a^m j^{-1+k} (j - k + m)!}{(-1 + j + m)!} \right\}, \{m, 0, k = 2\} \right] // \text{TableForm}$$

$$\begin{array}{l} 0 \quad \text{E2}[a^{-j} n, 0, a] - \frac{j(-2+j)!}{(-1+j)!} \\ 1 \quad \text{E2}[a^{-j} n, 1, a] + \frac{a j(-1+j)!}{j!} \\ 2 \quad \text{E2}[a^{-j} n, 2, a] - \frac{a^2 j j!}{(1+j)!} \end{array}$$

$$\text{Table} \left[\left\{ m, \text{E2}[a^{-j} n, m, a] + \frac{(-1)^{1+k-m} a^m j^{-1+k} (j - k + m)!}{(-1 + j + m)!} \right\}, \{m, 0, k = 3\} \right] // \text{TableForm}$$

$$\begin{array}{l} 0 \quad \text{E2}[a^{-j} n, 0, a] + \frac{j^2(-3+j)!}{(-1+j)!} \\ 1 \quad \text{E2}[a^{-j} n, 1, a] - \frac{a j^2(-2+j)!}{j!} \\ 2 \quad \text{E2}[a^{-j} n, 2, a] + \frac{a^2 j^2(-1+j)!}{(1+j)!} \\ 3 \quad \text{E2}[a^{-j} n, 3, a] - \frac{a^3 j^2 j!}{(2+j)!} \end{array}$$

Table $\left[\left\{ m, E2[a^{-j} n, m, a] + \frac{(-1)^{1+k-m} a^m j^{-1+k} (j-k+m)!}{(-1+j+m)!} \right\}, \{m, 0, k=4\} \right] // \text{TableForm}$

$$\begin{aligned} 0 \quad & E2[a^{-j} n, 0, a] - \frac{j^3 (-4+j)!}{(-1+j)!} \\ 1 \quad & E2[a^{-j} n, 1, a] + \frac{a j^3 (-3+j)!}{j!} \\ 2 \quad & E2[a^{-j} n, 2, a] - \frac{a^2 j^3 (-2+j)!}{(1+j)!} \\ 3 \quad & E2[a^{-j} n, 3, a] + \frac{a^3 j^3 (-1+j)!}{(2+j)!} \\ 4 \quad & E2[a^{-j} n, 4, a] - \frac{a^4 j^3 j!}{(3+j)!} \end{aligned}$$

Table $\left[\left\{ m, E2[a^{-j} n, m, a] + \frac{(-1)^{1+k-m} a^m j^{-1+k} (j-k+m)!}{(-1+j+m)!} \right\}, \{m, 0, k=5\} \right] // \text{TableForm}$

$$\begin{aligned} 0 \quad & E2[a^{-j} n, 0, a] + \frac{j^4 (-5+j)!}{(-1+j)!} \\ 1 \quad & E2[a^{-j} n, 1, a] - \frac{a j^4 (-4+j)!}{j!} \\ 2 \quad & E2[a^{-j} n, 2, a] + \frac{a^2 j^4 (-3+j)!}{(1+j)!} \\ 3 \quad & E2[a^{-j} n, 3, a] - \frac{a^3 j^4 (-2+j)!}{(2+j)!} \\ 4 \quad & E2[a^{-j} n, 4, a] + \frac{a^4 j^4 (-1+j)!}{(3+j)!} \\ 5 \quad & E2[a^{-j} n, 5, a] - \frac{a^5 j^4 j!}{(4+j)!} \end{aligned}$$

E2a[100, 1, 1.2]

-0.6

Sum[1, {j, 2, 100}] - 1.2 Sum[1, {j, 1, 100 / 1.2}]

-0.6

E2a[100, 2, 1.2]

10.36

Sum[1, {j, 2, 100}, {k, 2, 100 / j}] - 2 * 1.2 Sum[1, {j, 2, 100}, {k, 1, 100 / (1.2 j)}] + 1.2^2 Sum[1, {j, 1, 100 / 1.2}, {k, 1, 100 / (1.2^2 j)}]

10.36

s1[n_, a_] := Sum[1, {j, 2, n}] - a Sum[1, {j, 1, n / a}]

s2[n_, a_] := Sum[s1[n / j, a], {j, 2, n}] - a Sum[s1[n / (j a), a], {j, 1, n / a}]

s3[n_, a_] := Sum[Sum[1, {k, 2, (n / j)}] - a Sum[1, {k, 1, n / (j a)}], {j, 2, n}] - a Sum[Sum[1, {k, 2, n / (j a)}] - a Sum[1, {k, 1, n / (j a a)}], {j, 1, n / a}]

```

s4[n_, a_] :=
  Sum[ Sum[ 1, {k, 2, (n / j)}], {j, 2, n}] - Sum[a Sum[ 1, {k, 1, n / (j a)}], {j, 2, n}] -
  a Sum[ Sum[ 1, {k, 2, n / (j a)}], {j, 1, n / a}] +
  Sum[ a^2 Sum[ 1, {k, 1, n / (j a a)}], {j, 1, n / a}]

s5[n_, a_] :=
  Sum[ 1, {j, 2, n}, {k, 2, n / j}] -
  2 a Sum[ 1, {j, 2, n}, {k, 1, n / (j a)}] + a^2 Sum[ 1, {j, 1, n / a}, {k, 1, n / (j a a)}]

s5[100, 1.2]

10.36

n a^-s / a

a^-1-s n

E2D2x6[n_, k_, a_] := d2[n, k] + (-1)^(k) +
  Sum[ a^j j^(k-1) (Sum[ a^m / Gamma[k] (-1)^(k-m) Binomial[k, m]
    ((Gamma[k] a^-m j^(1-k) / (-1)^(k-m) Binomial[-1+j+m, -1+k]) E2a[a^-j n, m, a] - 1),
    {m, 0, k}]], {j, 2, Log[a, n]}]

E2D2x6[100, 2, 1.0005]

-5.78664 × 10^9

FullSimplify[Gamma[kk] a^-m j^(1-kk) / (-1)^(kk-m)]

(-1)^-kk+m a^-m j^(1-kk) Gamma[kk]

(-1)^-k+m a^-m j^(1-k) Gamma[k]

E2D2x7[n_, k_, a_] := d2[n, k] + (-1)^(k) +
  Sum[ a^j j^(k-1) (Sum[
    a^m (-1)^(k-m) Binomial[k, m] (
      ((-1)^-k+m a^-m j^(1-k) Gamma[k] Binomial[-1+j+m, -1+k]) E2a[a^-j n, m, a] - 1
    ), {m, 0, k}]], {j, 2, Log[a, n]}] / Gamma[k]

E2D2x7[100, 2, 1.0005]

-78.8633

E2D2x8[n_, k_, a_] := d2[n, k] + (-1)^(k) +
  Sum[ (Sum[
    (a^j j^(k-1) a^m (-1)^(k-m) Binomial[k, m] / Gamma[k]) (
      ((-1)^-k+m a^-m j^(1-k) Gamma[k] Binomial[-1+j+m, -1+k]) E2a[a^-j n, m, a] - 1
    ), {m, 0, k}]], {j, 2, Log[a, n]}]

E2D2x8[100, 2, 1.0005]

-78.8633

```

Expand[a^j j^ (k-1) a^m (-1) ^ (k-m) Binomial[k, m] / Gamma[k]]

$$\frac{(-1)^{k-m} a^{j+m} j^{-1+k} \text{Binomial}[k, m]}{\text{Gamma}[k]}$$

bin[k, m] / (k+1) !

$$\frac{k!}{(1+k)! (k-m)! m!}$$

$$\frac{1}{(1+k) (k-m)! m!}$$

$$\frac{1}{(1+k) (k-m)! m!}$$

FullSimplify[k! / ((k+1) !)]

$$\frac{1}{1+k}$$

FullSimplify[a^j j^ (k-1) a^m (-1) ^ (k-m) / ((1+k) (k-m) ! m!)]

$$\frac{(-1)^{k-m} a^{j+m} j^{-1+k}}{(1+k) (k-m)! m!}$$

E2D2x5a2[n_, k_, a_] :=

$$d2[n, k] + (-1)^{(k)} + \left(\text{Sum}\left[a^j / \text{Gamma}[k] \left(\text{Sum}\left[\text{Binomial}[k, m] \left(\frac{(-1+j+m)!}{(j-k+m)!} \right) \right] \right) \right. \right. \\ \left. \left. \text{E2a}[a^{j-n}, m, a] + j^{(k-1)} a^m (-1)^{(k-m+1)} \right] / \left(\frac{(-1+j+m)!}{(j-k+m)!} \right) \right. \\ \left. \right), \{m, 0, k\} \right], \{j, 2, \text{Log}[a, n]\} \right]$$

E2D2x5a2[100, 2, 1.0005]

-78.8633

E2D2x5a2[n_, k_, a_] :=

$$d2[n, k] + (-1)^{(k)} + \left(\text{Sum}\left[a^j \left(\text{Sum}\left[\text{Binomial}[k, m] \left(\frac{(-1+j+m)!}{(j-k+m)!} \right) \right] / \text{Gamma}[k] \right) \right. \right. \\ \left. \left. \text{E2a}[a^{j-n}, m, a] + j^{(k-1)} a^m (-1)^{(k-m+1)} \right] / \left(\frac{(-1+j+m)!}{(j-k+m)!} \right) \right. \\ \left. \right), \{m, 0, k\} \right], \{j, 2, \text{Log}[a, n]\} \right]$$

E2D2x5a2[100, 2, 1.001]

-79.2889

$$\text{bin}[k, m] \left(\frac{(-1 + j + m)!}{(j - k + m)!} \right) / (k - 1)!$$

$$\frac{k! (-1 + j + m)!}{(-1 + k)! (k - m)! m! (j - k + m)!}$$

$$\text{b}[j_, k_, m_] := \text{Binomial}[k, m] \left(\frac{(-1 + j + m)!}{(j - k + m)!} \right) / \text{Gamma}[k]$$

b[5, 4, 3]

140

$$\text{b2}[j_, k_, m_] := \frac{k! (-1 + j + m)!}{(-1 + k)! (k - m)! m! (j - k + m)!}$$

b2[5, 4, 3]

140

$$\frac{k! (-1 + j + m)!}{(-1 + k)! (k - m)! m! (-k + j + m)!}$$

$$((j + m - 1)!) / ((j + m - k)!)$$

$$\frac{(-1 + j + m)!}{(j - k + m)!} = (n!) / (n - k + 1)! \text{ where } n = m + j - 1$$

$$\text{b3}[j_, k_, m_] := \frac{k (-1 + j + m)!}{(k - m)! m! (j - k + m)!}$$

b3[5, 4, 3]

140

$$\text{b4}[j_, k_, m_] := \frac{(-1 + j + m)!}{(j - k + m)!}$$

b4[5, 4, 3]

210

$$\text{b5}[j_, k_, m_] := \frac{\text{Gamma}[j + m]}{(j - k + m) \text{Gamma}[j - k + m]}$$

b5[5, 4, 3]

210

$$\text{b6}[j_, k_, m_] := \frac{\text{FactorialPower}[j + m - 1, k]}{(j - k + m)}$$

b6[5, 4, 3]

210

$$\text{b7}[j_, k_, m_] := \frac{k}{(k - m)! m!} \frac{\text{FactorialPower}[j + m - 1, k]}{(j - k + m)}$$

b7[5, 4, 3]

140

$$\frac{k}{(k-m)! m!} \frac{\text{FactorialPower}[j+m-1, k]}{(j-k+m)}$$

$$\frac{k \text{FactorialPower}[-1+j+m, k]}{(j-k+m) (k-m)! m!}$$

$$\text{Table}\left[\text{Binomial}[k, m] \left(\frac{(-1+j+m)!}{(j-k+m)!} \right) /. \{k \rightarrow 5\}, \{m, 0, 5\}\right]$$

$$\left\{ \frac{(-1+j)!}{(-5+j)!}, \frac{5j!}{(-4+j)!}, \frac{10(1+j)!}{(-3+j)!}, \frac{10(2+j)!}{(-2+j)!}, \frac{5(3+j)!}{(-1+j)!}, \frac{(4+j)!}{j!} \right\}$$

$$\text{Table}[\text{Binomial}[k, m] \text{Pochhammer}[j+m-k+1, k-1]^{-1} /. \{k \rightarrow 5\}, \{m, 0, 5\}]$$

$$\left\{ \frac{1}{(-4+j)(-3+j)(-2+j)(-1+j)}, \frac{5}{(-3+j)(-2+j)(-1+j)j}, \frac{10}{(-2+j)(-1+j)j(1+j)}, \right.$$

$$\left. \frac{10}{(-1+j)j(1+j)(2+j)}, \frac{5}{j(1+j)(2+j)(3+j)}, \frac{1}{(1+j)(2+j)(3+j)(4+j)} \right\}$$

E2D2x5a3[n_, k_, a_] :=

$$\text{d2}[n, k] + (-1)^{(k)} + \text{Sum}[a^s \text{Sum}[\text{Binomial}[k, m] \text{Pochhammer}[s+m-k+1, k-1] ($$

$$\text{E2a}[a^{-s} n, m, a] + a^m (-1)^{(k-m+1)} s^{(k-1)} / \text{Pochhammer}[s+m-k+1, k-1]$$

$$), \{m, 0, k\}], \{s, 2, \text{Log}[a, n]\}] / \text{Gamma}[k]$$

E2D2x5a3[100, 2, 1.001]

-79.2889

E2D2x5a4[n_, k_, a_] := d2[n, k] + (-1)^k +

$$\text{Sum}[a^s \text{Binomial}[k, m] \text{Pochhammer}[s+m-k+1, k-1] / \text{Gamma}[k] ($$

$$\text{E2a}[a^{-s} n, m, a] + a^m (-1)^{(k-m+1)} s^{(k-1)} / \text{Pochhammer}[s+m-k+1, k-1]$$

$$), \{s, 2, \text{Log}[a, n]\}, \{m, 0, k\}]$$

E2D2x5a4[100, 2, 1.001]

-79.2889

N[D2a[100, 2] - (Gamma[2, 0, -Log[100]] / Gamma[2])]

-78.517 + 4.41506 × 10⁻¹⁴ i

E2D2x5a4a1[n_, k_, a_, m_] :=

$$\text{Sum}[a^s \text{Binomial}[k, m] \text{Pochhammer}[s+m-k+1, k-1] / \text{Gamma}[k]$$

$$(\text{E2}[a^{-s} n, m, a] + a^m (-1)^{(k-m+1)} s^{(k-1)} / \text{Pochhammer}[s+m-k+1, k-1]), \{s,$$

$$2, \text{Log}[a, n]\}]$$

E2D2x5a4a1[n, 1, a, 0]

E2D2x5a4a1[n, 1, a, 1]

$$\sum_{s=2}^{\frac{\text{Log}[n]}{\text{Log}[a]}} a^s (1 + \text{E2}[a^{-s} n, 0, a])$$

$$\sum_{s=2}^{\frac{\text{Log}[n]}{\text{Log}[a]}} a^s (-a + \text{E2}[a^{-s} n, 1, a])$$

E2D2x5a4a1[n, 2, a, 0]

E2D2x5a4a1[n, 2, a, 1]

E2D2x5a4a1[n, 2, a, 2]

$$\sum_{s=2}^{\frac{\text{Log}[n]}{\text{Log}[a]}} a^s (-1 + s) \left(-\frac{s}{-1 + s} + \text{E2}[a^{-s} n, 0, a] \right)$$

$$\sum_{s=2}^{\frac{\text{Log}[n]}{\text{Log}[a]}} 2 a^s s (a + \text{E2}[a^{-s} n, 1, a])$$

$$\sum_{s=2}^{\frac{\text{Log}[n]}{\text{Log}[a]}} a^s (1 + s) \left(-\frac{a^2 s}{1 + s} + \text{E2}[a^{-s} n, 2, a] \right)$$

E2D2x5a4a1[n, 3, a, 0]

E2D2x5a4a1[n, 3, a, 1]

E2D2x5a4a1[n, 3, a, 2]

E2D2x5a4a1[n, 3, a, 3]

$$\sum_{s=2}^{\frac{\text{Log}[n]}{\text{Log}[a]}} \frac{1}{2} a^s (-2 + s) (-1 + s) \left(\frac{s^2}{(-2 + s) (-1 + s)} + \text{E2}[a^{-s} n, 0, a] \right)$$

$$\sum_{s=2}^{\frac{\text{Log}[n]}{\text{Log}[a]}} \frac{3}{2} a^s (-1 + s) s \left(-\frac{a s}{-1 + s} + \text{E2}[a^{-s} n, 1, a] \right)$$

$$\sum_{s=2}^{\frac{\text{Log}[n]}{\text{Log}[a]}} \frac{3}{2} a^s s (1 + s) \left(\frac{a^2 s}{1 + s} + \text{E2}[a^{-s} n, 2, a] \right)$$

$$\sum_{s=2}^{\frac{\text{Log}[n]}{\text{Log}[a]}} \frac{1}{2} a^s (1 + s) (2 + s) \left(-\frac{a^3 s^2}{(1 + s) (2 + s)} + \text{E2}[a^{-s} n, 3, a] \right)$$

```

E2D2x5a4a1[n, 4, a, 0]
E2D2x5a4a1[n, 4, a, 1]
E2D2x5a4a1[n, 4, a, 2]
E2D2x5a4a1[n, 4, a, 3]
E2D2x5a4a1[n, 4, a, 4]

```

$$\sum_{s=2}^{\frac{\text{Log}[n]}{\text{Log}[a]}} \frac{1}{6} a^s (-3+s) (-2+s) (-1+s) \left(-\frac{s^3}{(-3+s) (-2+s) (-1+s)} + \text{E2}[a^{-s} n, 0, a] \right)$$

$$\sum_{s=2}^{\frac{\text{Log}[n]}{\text{Log}[a]}} \frac{2}{3} a^s (-2+s) (-1+s) s \left(\frac{a s^2}{(-2+s) (-1+s)} + \text{E2}[a^{-s} n, 1, a] \right)$$

$$\sum_{s=2}^{\frac{\text{Log}[n]}{\text{Log}[a]}} a^s (-1+s) s (1+s) \left(-\frac{a^2 s^2}{(-1+s) (1+s)} + \text{E2}[a^{-s} n, 2, a] \right)$$

$$\sum_{s=2}^{\frac{\text{Log}[n]}{\text{Log}[a]}} \frac{2}{3} a^s s (1+s) (2+s) \left(\frac{a^3 s^2}{(1+s) (2+s)} + \text{E2}[a^{-s} n, 3, a] \right)$$

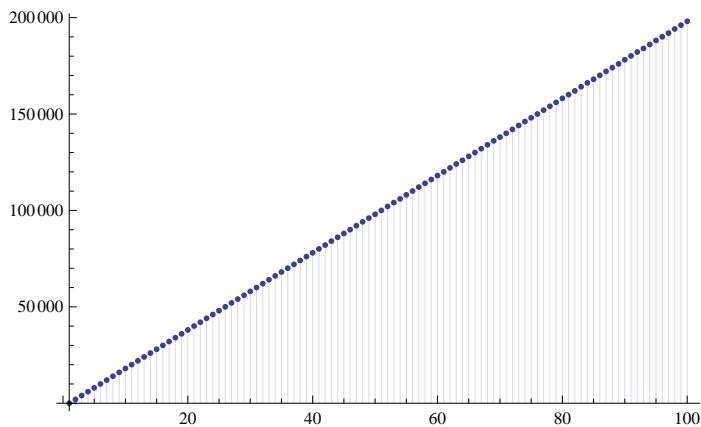
$$\sum_{s=2}^{\frac{\text{Log}[n]}{\text{Log}[a]}} \frac{1}{6} a^s (1+s) (2+s) (3+s) \left(-\frac{a^4 s^3}{(1+s) (2+s) (3+s)} + \text{E2}[a^{-s} n, 4, a] \right)$$

```

E2pt[n_, k_, a_, m_] :=
  Sum[a^s Binomial[k, m] Pochhammer[s+m-k+1, k-1] / Gamma[k]
    (E2a[a^-s n, m, a] + a^m (-1)^(k-m+1) s^(k-1) / Pochhammer[s+m-k+1, k-1]), {s,
    2, Log[a, n]]]

```

```
DiscretePlot[E2pt[n, 1, 1.001, 0], {n, 1, 100}]
```



```
E2pt[100, 1, 1.0001, 1]
```

-1.9802×10^6

```
E2pa[n_, k_, a_, s_, m_] := a^s ( Binomial[k, m] Pochhammer[s + m - k + 1, k - 1] / Gamma[k]
    ((E2[a^-s n, m, a] + a^m (-1)^(k - m + 1) s^(k - 1) / Pochhammer[s + m - k + 1, k - 1])) )
```

```
E2pa[n, 1, a, s, 0]
```

```
E2pa[n, 1, a, s, 1]
```

```
a^s (1 + E2[a^-s n, 0, a])
```

```
a^s (-a + E2[a^-s n, 1, a])
```

```
E2pa[n, 2, a, s, 0]
```

```
E2pa[n, 2, a, s, 1]
```

```
E2pa[n, 2, a, s, 2]
```

```
a^s (-1 + s) ( - s / (-1 + s) + E2[a^-s n, 0, a] )
```

```
2 a^s s (a + E2[a^-s n, 1, a])
```

```
a^s (1 + s) ( - a^2 s / (1 + s) + E2[a^-s n, 2, a] )
```

```
E2pa[n, 3, a, s, 0]
```

```
E2pa[n, 3, a, s, 1]
```

```
E2pa[n, 3, a, s, 2]
```

```
E2pa[n, 3, a, s, 3]
```

```
1 / 2 a^s (-2 + s) (-1 + s) ( s^2 / ((-2 + s) (-1 + s)) + E2[a^-s n, 0, a] )
```

```
3 / 2 a^s (-1 + s) s ( - a s / (-1 + s) + E2[a^-s n, 1, a] )
```

```
3 / 2 a^s s (1 + s) ( a^2 s / (1 + s) + E2[a^-s n, 2, a] )
```

```
1 / 2 a^s (1 + s) (2 + s) ( - a^3 s^2 / ((1 + s) (2 + s)) + E2[a^-s n, 3, a] )
```

```

E2pa[n, 4, a, s, 0]
E2pa[n, 4, a, s, 1]
E2pa[n, 4, a, s, 2]
E2pa[n, 4, a, s, 3]
E2pa[n, 4, a, s, 4]

```

$$\begin{aligned}
& \frac{1}{6} a^s (-3+s) (-2+s) (-1+s) \left(-\frac{s^3}{(-3+s) (-2+s) (-1+s)} + E2[a^{-s} n, 0, a] \right) \\
& \frac{2}{3} a^s (-2+s) (-1+s) s \left(\frac{a s^2}{(-2+s) (-1+s)} + E2[a^{-s} n, 1, a] \right) \\
& a^s (-1+s) s (1+s) \left(-\frac{a^2 s^2}{(-1+s) (1+s)} + E2[a^{-s} n, 2, a] \right) \\
& \frac{2}{3} a^s s (1+s) (2+s) \left(\frac{a^3 s^2}{(1+s) (2+s)} + E2[a^{-s} n, 3, a] \right) \\
& \frac{1}{6} a^s (1+s) (2+s) (3+s) \left(-\frac{a^4 s^3}{(1+s) (2+s) (3+s)} + E2[a^{-s} n, 4, a] \right)
\end{aligned}$$

$$a^s (-1+s) (-s / (-1+s) + 1)$$

$$\text{FullSimplify}\left[a^s (-1+s) \left(1 - \frac{s}{-1+s}\right)\right]$$

$$-a^s$$

$$(-1+s) (-s / (-1+s) + 1)$$

$$(-1+s) \left(1 - \frac{s}{-1+s}\right)$$

$$a^s \left((-1+s) - \frac{s(-1+s)}{-1+s}\right)$$

$$-a^s$$

```
FullSimplify[
```

```
  2 a^s s (a + Sum[1, {j, 2, Floor[n / (a^s)]]] - a Sum[1, {j, 1, Floor[n / (a^(s+1))]]])]
```

$$2 a^s s (-1 + a - a \text{Floor}[a^{-1-s} n] + \text{Floor}[a^{-s} n])$$

$$\text{FullSimplify}\left[2 a^s s (-1 + a - a (a^{-1-s} n - 1) + (a^{-s} n - 1))\right]$$

$$4 (-1 + a) a^s s$$