```
pp1[n_, s_] := Sum[j^(-1/2 + s I) + j^(-1/2 - s I), {j, 1, n}] -
  Integrate[j^(-1/2 + s I) + j^(-1/2 - s I), {j, 0, n}]
pp2[n_, s_] := Sum[j^(-1/2 + s I) + j^(-1/2 - s I), {j, 1, n}] -
```

$$\left(\frac{2\,n^{\frac{1}{2}-i s}}{1+4 s^2}+\frac{2\,n^{\frac{1}{2}+i s}}{1+4 s^2}+\frac{4\,i\,n^{\frac{1}{2}-i s}\,s}{1+4 s^2}-\frac{4\,i\,n^{\frac{1}{2}+i s}\,s}{1+4 s^2}\right)$$

```
pp3[n_, s_] := Sum[j^(-1/2 + s I) + j^(-1/2 - s I), {j, 1, n}] -
```

$$(4/(1+4\,s\verb|^|2))\left((1/2)\,n^{\frac{1}{2}-i s}+(1/2)\,n^{\frac{1}{2}+i s}+i\,n^{\frac{1}{2}-i s}\,s-i\,n^{\frac{1}{2}+i s}\,s\right)$$

```
pp4[n_, s_] := Sum[j^(-1/2) (E^(s Log[j] I) + E^(-s Log[j] I)), {j, 1, n}] -
```

$$(4\,n\verb|^|(1/2)/(1+4\,s\verb|^|2))\left((1/2)\,n^{-i s}+(1/2)\,n^{i s}+i\,n^{-i s}\,s-i\,n^{i s}\,s\right)$$

```
pp5[n_, s_] := 2 Sum[j^(-1/2) Cos[s Log[j]], {j, 1, n}] -
```

$$(4\,n\verb|^|(1/2)/(1+4\,s\verb|^|2))\left((1/2)\,n^{-i s}+(1/2)\,n^{i s}+i\,n^{-i s}\,s-i\,n^{i s}\,s\right)$$

```
pp6[n_, s_] := 2 Sum[j^(-1/2) Cos[s Log[j]], {j, 1, n}] -
```

$$(4\,n\verb|^|(1/2)/(1+4\,s\verb|^|2))\left((1/2)\left(n^{-i s}+n^{i s}\right)+(s\,I)\left(n^{-i s}-n^{i s}\right)\right)$$

```
pp7[n_, s_] := 2 Sum[j^(-1/2) Cos[s Log[j]], {j, 1, n}] -
```

$$(4\,n\verb|^|(1/2)/(1+4\,s\verb|^|2))\left((1/2)\left(E^{-i\,\mathrm{Log}[n]\,s}+E^{i\,\mathrm{Log}[n]\,s}\right)+(s\,I)\left(E^{-i\,\mathrm{Log}[n]\,s}-E^{i\,\mathrm{Log}[n]\,s}\right)\right)$$

```
pp8[n_, s_] := 2 Sum[j^(-1/2) Cos[s Log[j]], {j, 1, n}] -
```

$$(4\,n\verb|^|(1/2)/(1+4\,s\verb|^|2))\left((1/2)\left(E^{-i\,\mathrm{Log}[n]\,s}+E^{i\,\mathrm{Log}[n]\,s}\right)+2\,s\,\mathrm{Sin}[\mathrm{Log}[n]\,s]\right)$$

```
pp9[n_, s_] := Sum[j^(-1/2) Cos[s Log[j]], {j, 1, n}] -
  (2 n^(1/2) / (1 + 4 s^2)) (2 s Sin[Log[n] s] + Cos[Log[n] s])
```

```
pp9[1000, 10. + .2 I]
```

1.58287 + 0.00192283 $i$

```
pp2[1000, 10. + .2 I] / 2
```

1.58287 + 0.00192283 $i$

```
(Zeta[.5 + s] + Zeta[.5 - s]) / 2 /. s -> (10 I + .2)
```

1.54944 − 0.000866905 $i$

```
Integrate[j^(-1/2 + s I) + j^(-1/2 - s I), {j, 0, n}]
```

$$\mathrm{ConditionalExpression}\left[\frac{2\,n^{\frac{1}{2}-i s}\left(1+n^{2 i s}(1-2\,i\,s)+2\,i\,s\right)}{1+4 s^2},\ -\frac{1}{2}<\mathrm{Im}[s]<\frac{1}{2}\right]$$

```
Integrate[j^(-1/2 - s I), {j, 0, n}]
```

$$\mathrm{ConditionalExpression}\left[\frac{2\,i\,n^{\frac{1}{2}-i s}}{i+2 s},\ \mathrm{Im}[s]>-\frac{1}{2}\right]$$

$$\mathrm{Expand}\left[\frac{2\,n^{\frac{1}{2}-i s}\left(1+n^{2 i s}(1-2\,i\,s)+2\,i\,s\right)}{1+4 s^2}\right]$$

$$\frac{2\,n^{\frac{1}{2}-i s}}{1+4 s^2}+\frac{2\,n^{\frac{1}{2}+i s}}{1+4 s^2}+\frac{4\,i\,n^{\frac{1}{2}-i s}\,s}{1+4 s^2}-\frac{4\,i\,n^{\frac{1}{2}+i s}\,s}{1+4 s^2}$$

```
N[E^(s Log[j] I) + E^(-s Log[j] I) /. s → 3 /. j → 2]
```

−0.973989 + 0. $i$

```
N[2 Cos[s Log[j]] /. s → 3 /. j → 2]
```

−0.973989

```
N[(s I) (E^(-I Log[n] s) - E^(I Log[n] s)) /. s → 3 /. n → 2]
```

```
5.24043 + 0. i
```

```
N[2 s Sin[Log[n] s] /. s → 3 /. n → 2]
```

```
5.24043
```

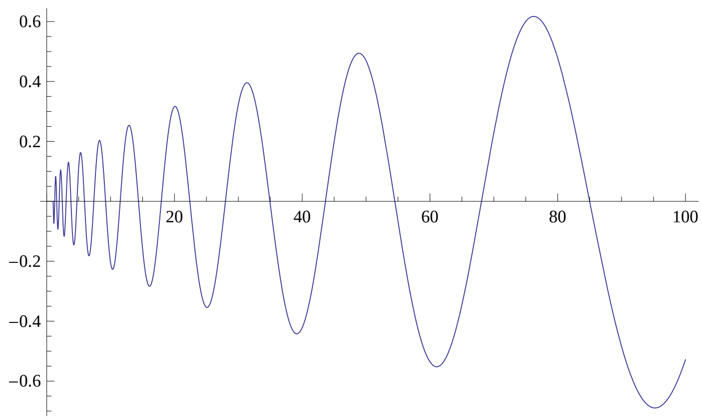
```
FullSimplify[(Zeta[1 / 2 + s] + Zeta[1 / 2 - s]) / 2 /. s -> (-1 / 2 + ZetaZero[1])]
```

```
0
```

```
Sum[j^(-1 / 2) Cos[s Log[j]], {j, 1, n}] -
  (2 n^(1 / 2) / (1 + 4 s^2)) (2 s Sin[Log[n] s] + Cos[Log[n] s])
```

$$-\frac{2\sqrt{n}\,(\mathrm{Cos}[s\,\mathrm{Log}[n]]+2\,s\,\mathrm{Sin}[s\,\mathrm{Log}[n]])}{1+4\,s^2}+\sum_{j=1}^{n}\frac{\mathrm{Cos}[s\,\mathrm{Log}[j]]}{\sqrt{j}}$$

$$\mathbf{Plot}\left[-\frac{2\sqrt{n}\,(\mathrm{Cos}[s\,\mathrm{Log}[n]]+2\,s\,\mathrm{Sin}[s\,\mathrm{Log}[n]])}{1+4\,s^2}\ /.\ s\to \mathrm{Im@ZetaZero@1},\ \{n,1,100\}\right]$$



```
ee1[n_, s_] := Sum[j^(-1/2 + s I) - j^(-1/2 - s I), {j, 1, n}] -
  Integrate[j^(-1/2 + s I) - j^(-1/2 - s I), {j, 0, n}]
ee2[n_, s_] := Sum[j^(-1/2 + s I) - j^(-1/2 - s I), {j, 1, n}] -
```

$$\left(-\frac{2\,n^{\frac{1}{2}-i s}}{1+4 s^2}+\frac{2\,n^{\frac{1}{2}+i s}}{1+4 s^2}-\frac{4\,i\,n^{\frac{1}{2}-i s}\,s}{1+4 s^2}-\frac{4\,i\,n^{\frac{1}{2}+i s}\,s}{1+4 s^2}\right)$$

```
ee3[n_, s_] := Sum[j^(-1/2) (E^(s Log[j] I) - E^(-s Log[j] I)), {j, 1, n}] -
```

$$\left(-\frac{2\,n^{\frac{1}{2}-i s}}{1+4 s^2}+\frac{2\,n^{\frac{1}{2}+i s}}{1+4 s^2}-\frac{4\,i\,n^{\frac{1}{2}-i s}\,s}{1+4 s^2}-\frac{4\,i\,n^{\frac{1}{2}+i s}\,s}{1+4 s^2}\right)$$

```
ee4[n_, s_] := 2 I Sum[j^(-1/2) Sin[s Log[j]], {j, 1, n}] -
```

$$\left(-\frac{2\,n^{\frac{1}{2}-i s}}{1+4 s^2}+\frac{2\,n^{\frac{1}{2}+i s}}{1+4 s^2}-\frac{4\,i\,n^{\frac{1}{2}-i s}\,s}{1+4 s^2}-\frac{4\,i\,n^{\frac{1}{2}+i s}\,s}{1+4 s^2}\right)$$

```
ee5[n_, s_] := 2 I Sum[j^(-1/2) Sin[s Log[j]], {j, 1, n}] -
  (2 n^(1/2) / (1 + 4 s^2)) (- n^(-i s) + n^(+i s) - 2 i n^(-i s) s - 2 i n^(+i s) s)
ee6[n_, s_] := 2 I Sum[j^(-1/2) Sin[s Log[j]], {j, 1, n}] -
  (2 n^(1/2) / (1 + 4 s^2)) ((- n^(-i s) + n^(+i s)) - 2 i s (n^(-i s) + n^(+i s)))
ee7[n_, s_] := 2 I Sum[j^(-1/2) Sin[s Log[j]], {j, 1, n}] -
  (2 n^(1/2) / (1 + 4 s^2)) (2 I Sin[s Log[n]] - 2 i s 2 Cos[s Log[n]])
ee8[n_, s_] := Sum[j^(-1/2) Sin[s Log[j]], {j, 1, n}] +
  (2 n^(1/2) / (1 + 4 s^2)) (2 s Cos[s Log[n]] - Sin[s Log[n]])
```

```
ee8[1000, 10. + .2 I]
```

$0.112631 + 0.101817\, i$

```
ee2[1000, 10. + .2 I] / (2 I)
```

$0.112631 + 0.101817\, i$

```
(Zeta[.5 + s] - Zeta[.5 - s]) / (2 I) /. s -> (10 I + .2)
```

$-0.113829 + 0.0723504\, i$

```
Expand@Integrate[j^(-1/2 + s I) - j^(-1/2 - s I), {j, 0, n}]
```

$$\text{ConditionalExpression}\left[-\frac{2\,n^{\frac{1}{2}-i s}}{1+4 s^2}+\frac{2\,n^{\frac{1}{2}+i s}}{1+4 s^2}-\frac{4\,i\,n^{\frac{1}{2}-i s}\,s}{1+4 s^2}-\frac{4\,i\,n^{\frac{1}{2}+i s}\,s}{1+4 s^2},\ -\frac{1}{2}<\mathrm{Im}[s]<\frac{1}{2}\right]$$

```
N[(E^(s Log[j] I) - E^(-s Log[j] I)) /. s → 3 /. j → 2]
```

$0. + 1.74681\, i$

```
N[2 I Sin[s Log[j]] /. s → 3 /. j → 2]
```

$0. + 1.74681\, i$

```
N[(- n^(-i s) + n^(+i s)) /. s → 3 /. n → 2]
```

$0. + 1.74681\, i$

```
N[ (n^(-i s) + n^(+i s)) /. s → 3 /. n → 2]
```

$-0.973989 + 0.\, i$

```
N[2 Cos[s Log[j]] /. s → 3 /. j → 2]
```

$-0.973989$

```
pp9[n_, s_] := Sum[j^(-1/2) Cos[s Log[j]], {j, 1, n}] -
  (2 n^(1/2) / (1 + 4 s^2)) (Cos[Log[n] s] + 2 s Sin[Log[n] s])
pp9a[s_] := (Zeta[1/2 - s I] + Zeta[1/2 + s I]) / 2
ee9[n_, s_] := Sum[j^(-1/2) Sin[s Log[j]], {j, 1, n}] -
  (2 n^(1/2) / (1 + 4 s^2)) ( Sin[s Log[n]] - 2 s Cos[s Log[n]])
ee9a[s_] := (Zeta[1/2 - s I] - Zeta[1/2 + s I]) / (2 I)
```

```
pp9[100 000, 20 + .3 I]
```

0.376859 - 0.287306 ⅈ

```
pp9a[20 + .3 I]
```

0.391975 - 0.30721 ⅈ

```
ee9[100 000, 10 + .2 I]
```

0.120979 + 0.0688678 ⅈ

```
ee9a[10 + .2 I]
```

0.113829 + 0.0723504 ⅈ

```
pp9[1 000 000, N@Im@ZetaZero@1]
```

0.000438863

```
FullSimplify@pp9a[Im@ZetaZero@1]
```

0

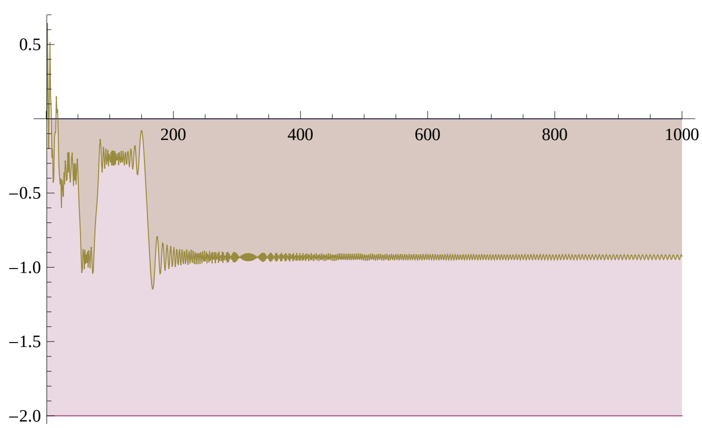
```
ee9[1 000 000, N@Im@ZetaZero@1]
```

-121.644

```
FullSimplify@ee9a[Im@ZetaZero@1]
```

0

```
DiscretePlot[{0, -2, ee9[n, 1000]}, {n, 1, 1000}]
```



```
Plot[{Abs@pp9a[n + .01 I]}, {n, 0, 100}]
```