```
ClearAll["Global`*"]

Residue[ (Zeta[s]^2) x^s / s, {s, 1}]

-x + 2 EulerGamma x + x Log[x]

Residue[ ((1 / Zeta[s])) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z
```

$$\frac{x^z}{z\,\text{Zeta}'[z]}$$

```
Residue[ Zeta[s], {s, 1}]

1

bins[ z_, a_] := Product[ (z - k), {k, 0, a - 1}] / a!
D2[n_, k_] := D2[n, k] = Sum[D2[Floor[n / j], k - 1], {j, 2, n}]; D2[n_, 0] := 1
DD[n_, z_] := (Sum[bins[z, a] D2[n, a], {a, 0, Log[2, n]}])
gg[n_, z_] := Expand[FullSimplify[(DD[n, z + 1] - 1) / (z + 1)]]
Dhyp[n_, k_, a_] := Dhyp[n, k, a] =
  Sum[Binomial[k, j] Dhyp[Floor[n / (m^(k - j))], j, m + 1], {m, a, n^(1 / k)}, {j, 0, k - 1}]
Dhyp[n_, 1, a_] := Floor[n] - a + 1; Dhyp[n_, 0, a_] := 1
bins[ z_, a_] := Product[ (z - k), {k, 0, a - 1}] / a!
DDD[n_, z_] := Expand[Sum[bins[z, a] Dhyp[n, a, 2], {a, 0, Log[2, n]}]]
DDD2[n_, z_] := Sum[bins[z, a] Dhyp[n, a, 2], {a, 0, Log[2, n]}]
ggr[n_, z_] := Expand[FullSimplify[(DDD[n, z + 1] - 1) / (z + 1)]]
ggr2[n_, z_] := FullSimplify[(DDD[n, z + 1] - 1) / (z + 1)]
ggr3[n_, z_] := FullSimplify[(DDD[n, z] - 1) / (z)]
ggr3a[n_, z_] := FullSimplify[(DDD[n, z]) / (z)]
ggr3b[n_, z_] := FullSimplify[(DDD[n, z] - 1)]
ggr3e[n_, z_] := FullSimplify[(DDD[n, z])]
ggr3f[n_, z_] := FullSimplify[(DDD[n, z + 1])]
ggr4[n_, z_] := FullSimplify[(DDD[n, z - 1] - 1) / (z - 1)]
D2z[n_, k_, s_] := D2z[n, k] = Sum[ j^-s D2z[Floor[n / j], k - 1, s], {j, 2, n}];
D2z[n_, 0, s_] := 1
DDz[n_, z_, s_] := (Sum[bins[z, a] D2z[n, a, s], {a, 0, Log[2, n]}])
ggz[n_, z_, s_] := Expand[FullSimplify[(DDz[n, z + 1, s] - 1) / (z + 1)]]
DDa[n_, z_] := (Sum[bins[z, a] D2a[n, a], {a, 0, Log[2, n]}])
K[n_] := If[n == 1, 0, FullSimplify[MangoldtLambda[n] / Log[n]]]
P[n_, k_] := P[n, k] = Sum[ K[j] P[Floor[n / j], k - 1], {j, 2, n}]; P[n_, 0] := 1
DDa[n_, z_] := Sum[ z^k / k! P[n, k], {k, 0, Log[2, n]}]


DD[100, 0]
```

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

Infinity::indet : Indeterminate expression 0 ComplexInfinity encountered. ≫

```
Indeterminate
```

```
Residue[gg[100, s] / s^3 , {s, 0}]
```

$\frac{3167}{90}$

```
Limit[gg[100, z], z → -1]
```

$\frac{428}{15}$

```
Limit[ggz[100, z, 0], z → -1]
```

$\frac{428}{15}$

```
Table[{n, FullSimplify[Residue[DD[n, s] / s^2 , {s, 0}]]}, {n, 1, 100}] // TableForm
```

1	0
2	1
3	2
4	$\frac{5}{2}$
5	$\frac{7}{2}$
6	$\frac{7}{2}$
7	$\frac{9}{2}$
8	$\frac{29}{6}$
9	$\frac{16}{3}$
10	$\frac{16}{3}$
11	$\frac{19}{3}$
12	$\frac{19}{3}$
13	$\frac{22}{3}$
14	$\frac{22}{3}$
15	$\frac{22}{3}$
16	$\frac{91}{12}$
17	$\frac{103}{12}$
18	$\frac{103}{12}$
19	$\frac{115}{12}$
20	$\frac{115}{12}$
21	$\frac{115}{12}$
22	$\frac{115}{12}$
23	$\frac{127}{12}$
24	$\frac{127}{12}$
25	$\frac{133}{12}$
26	$\frac{133}{12}$
27	$\frac{137}{12}$
28	$\frac{137}{12}$
29	$\frac{149}{12}$

30	$\frac{149}{12}$
31	$\frac{161}{12}$
32	$\frac{817}{60}$
33	$\frac{817}{60}$
34	$\frac{817}{60}$
35	$\frac{817}{60}$
36	$\frac{817}{60}$
37	$\frac{877}{60}$
38	$\frac{877}{60}$
39	$\frac{877}{60}$
40	$\frac{877}{60}$
41	$\frac{937}{60}$
42	$\frac{937}{60}$
43	$\frac{997}{60}$
44	$\frac{997}{60}$
45	$\frac{997}{60}$
46	$\frac{997}{60}$
47	$\frac{1057}{60}$
48	$\frac{1057}{60}$
49	$\frac{1087}{60}$
50	$\frac{1087}{60}$
51	$\frac{1087}{60}$
52	$\frac{1087}{60}$
53	$\frac{1147}{60}$
54	$\frac{1147}{60}$
55	$\frac{1147}{60}$
56	$\frac{1147}{60}$
57	$\frac{1147}{60}$
58	$\frac{1147}{60}$
59	$\frac{1207}{60}$
60	$\frac{1207}{60}$
61	$\frac{1267}{60}$
62	$\frac{1267}{60}$
63	$\frac{1267}{60}$
64	$\frac{1277}{60}$
65	$\frac{1277}{60}$
66	$\frac{1277}{60}$
67	$\frac{1337}{60}$
68	$\frac{1337}{60}$

69	$\frac{1337}{60}$
70	$\frac{1337}{60}$
71	$\frac{1397}{60}$
72	$\frac{1397}{60}$
73	$\frac{1457}{60}$
74	$\frac{1457}{60}$
75	$\frac{1457}{60}$
76	$\frac{1457}{60}$
77	$\frac{1457}{60}$
78	$\frac{1457}{60}$
79	$\frac{1517}{60}$
80	$\frac{1517}{60}$
81	$\frac{383}{15}$
82	$\frac{383}{15}$
83	$\frac{398}{15}$
84	$\frac{398}{15}$
85	$\frac{398}{15}$
86	$\frac{398}{15}$
87	$\frac{398}{15}$
88	$\frac{398}{15}$
89	$\frac{413}{15}$
90	$\frac{413}{15}$
91	$\frac{413}{15}$
92	$\frac{413}{15}$
93	$\frac{413}{15}$
94	$\frac{413}{15}$
95	$\frac{413}{15}$
96	$\frac{413}{15}$
97	$\frac{428}{15}$
98	$\frac{428}{15}$
99	$\frac{428}{15}$
100	$\frac{428}{15}$

```
Residue[n^s / (s^3), {s, 0}]
```

$$\frac{\text{Log}[n]^2}{2}$$

```
Residue[n^s / (s^4), {s, 0}]
```

$$\frac{\text{Log}[n]^3}{6}$$

```
K[n_] := If[n == 1, 0, FullSimplify[MangoldtLambda[n] / Log[n]]]
P[n_, k_] := P[n, k] = Sum[ K[j] P[Floor[n / j], k - 1], {j, 2, n}]; P[n_, 0] := 1

Residue[DD[100, s] / s^2 , {s, 0}]
```

$\frac{428}{15}$

```
P[100, 1]
```

$\frac{428}{15}$

```
Residue[DD[100, s] / s^3, {s, 0}]
```

$\frac{16\,289}{360}$

```
P[100, 2] / 2
```

$\frac{16\,289}{360}$

```
Residue[DD[100, s] / s^4, {s, 0}]
```

$\frac{331}{16}$

```
P[100, 3] / 6
```

$\frac{331}{16}$

```
f[x_] := x^4

Residue[ f[n] / s^3, {s, 0}]
```

0

```
da[ n_, z_] := Sum[ z^k Residue[DD[100, m] / m^(k + 1), {m, 0}], {k, 0, Log[2, n]}]

da[100, 2]
```

482

```
Limit[ DD[100, z] / z , z → 0]
```

∞

```
Residue[ Zeta[s] s^2, {s, 1}]
```

1

Series[Zeta[s], {s, 0, 20}]

$$-\frac{1}{2}-\frac{1}{2}\,\mathrm{Log}[2\,\pi]\,s+$$
$$\frac{1}{48}\left(12\,\mathrm{EulerGamma}^2-\pi^2-12\,\mathrm{Log}[2]^2-24\,\mathrm{Log}[2]\,\mathrm{Log}[\pi]-12\,\mathrm{Log}[\pi]^2+24\,\mathrm{StieltjesGamma}[1]\right)s^2+$$
$$\frac{1}{48}\Big(8\,\mathrm{EulerGamma}^3+12\,\mathrm{EulerGamma}^2\,\mathrm{Log}[2\,\pi]-\pi^2\,\mathrm{Log}[2\,\pi]-4\,\mathrm{Log}[2\,\pi]^3+24\,\mathrm{EulerGamma}\,\mathrm{StieltjesGamma}[1]+24\,\mathrm{Log}[2\,\pi]\,\mathrm{StieltjesGamma}[1]+12\,\mathrm{StieltjesGamma}[2]-8\,\mathrm{Zeta}[3]\Big)s^3+$$
$$\frac{1}{24}\,\mathrm{Zeta}^{(4)}[0]\,s^4+\frac{1}{120}\,\mathrm{Zeta}^{(5)}[0]\,s^5+\frac{1}{720}\,\mathrm{Zeta}^{(6)}[0]\,s^6+\frac{\mathrm{Zeta}^{(7)}[0]\,s^7}{5040}+$$
$$\frac{\mathrm{Zeta}^{(8)}[0]\,s^8}{40\,320}+\frac{\mathrm{Zeta}^{(9)}[0]\,s^9}{362\,880}+\frac{\mathrm{Zeta}^{(10)}[0]\,s^{10}}{3\,628\,800}+\frac{\mathrm{Zeta}^{(11)}[0]\,s^{11}}{39\,916\,800}+$$
$$\frac{\mathrm{Zeta}^{(12)}[0]\,s^{12}}{479\,001\,600}+\frac{\mathrm{Zeta}^{(13)}[0]\,s^{13}}{6\,227\,020\,800}+\frac{\mathrm{Zeta}^{(14)}[0]\,s^{14}}{87\,178\,291\,200}+\frac{\mathrm{Zeta}^{(15)}[0]\,s^{15}}{1\,307\,674\,368\,000}+$$
$$\frac{\mathrm{Zeta}^{(16)}[0]\,s^{16}}{20\,922\,789\,888\,000}+\frac{\mathrm{Zeta}^{(17)}[0]\,s^{17}}{355\,687\,428\,096\,000}+\frac{\mathrm{Zeta}^{(18)}[0]\,s^{18}}{6\,402\,373\,705\,728\,000}+$$
$$\frac{\mathrm{Zeta}^{(19)}[0]\,s^{19}}{121\,645\,100\,408\,832\,000}+\frac{\mathrm{Zeta}^{(20)}[0]\,s^{20}}{2\,432\,902\,008\,176\,640\,000}+O[s]^{21}$$

Zeta''[0]

$$\frac{\mathrm{EulerGamma}^2}{2}-\frac{\pi^2}{24}-\frac{1}{2}\,(\mathrm{Log}[2]+\mathrm{Log}[\pi])^2+\mathrm{StieltjesGamma}[1]$$

Integrate[DD[100, z], z]

$$z+\frac{214\,z^2}{15}+\frac{16\,289\,z^3}{1080}+\frac{331\,z^4}{64}+\frac{611\,z^5}{720}+\frac{67\,z^6}{1440}+\frac{z^7}{720}$$

Expand[DD[100, z]]

$$1+\frac{428\,z}{15}+\frac{16\,289\,z^2}{360}+\frac{331\,z^3}{16}+\frac{611\,z^4}{144}+\frac{67\,z^5}{240}+\frac{7\,z^6}{720}$$

gg[1000, z]

$$999+\frac{103\,337\,z}{72}+\frac{1\,647\,349\,z^2}{2016}+\frac{5\,439\,209\,z^3}{22\,680}+\frac{14\,163\,823\,z^4}{362\,880}+\frac{1\,301\,627\,z^5}{362\,880}+\frac{64\,843\,z^6}{362\,880}+\frac{1853\,z^7}{362\,880}+\frac{z^8}{36\,288}$$

(List @@ NRoots[gg[100, x] == 0, x][[All, 2]])

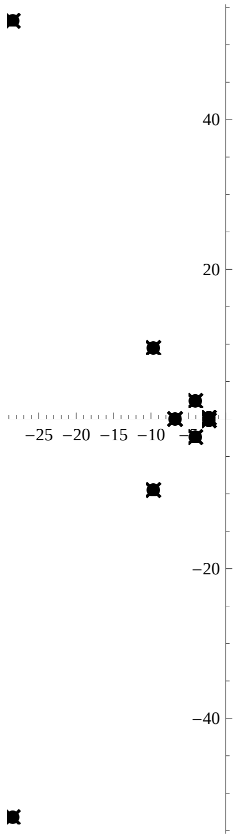
{-12.1997 - 12.3983 ⅈ, -12.1997 + 12.3983 ⅈ, -3.65742 - 1.85785 ⅈ, -3.65742 + 1.85785 ⅈ, -2.}

**vv := {-12.199718313735843` - 12.398284753324177` ⅈ,
-12.199718313735843` + 12.398284753324177` ⅈ, -3.6574245434070125` - 1.8578482376600203` ⅈ,
-3.6574245434070125` + 1.8578482376600203` ⅈ, -2.`}**

99 Product[1 + 1 / j, {j, vv}]

28.5333 + 0. ⅈ

```
RootLocusPlot[1 / Expand[gg[1600, x]], {k, 0, 1}, FeedbackType → None]
```




```
(List @@ NRoots[ DD[100, x] == 0, x][[All, 2]])
```

{−11.1997 − 12.3982 ⅈ, −11.1997 + 12.3982 ⅈ,
 −2.67195 − 1.86184 ⅈ, −2.67195 + 1.86184 ⅈ, −0.933809, −0.0372047}

```
(1 + 3 / (List @@ NRoots[ gg[200, x] == 0, x][[All, 2]]))
```

{0.92523 + 0.0922223 ⅈ, 0.92523 − 0.0922223 ⅈ, 0.593448 + 0.280651 ⅈ,
 0.593448 − 0.280651 ⅈ, −0.247394 + 0.366558 ⅈ, −0.247394 − 0.366558 ⅈ}

```
( Product[ 1 + 2 / j, {j, vv}])
```

0. + 0. ⅈ

```
(99 Product[ 1 − 2 / j, {j, vv}]) 3 + 1
```

$1471. + 3.29736 \times 10^{-14}$ ⅈ

```
(99 Product[ 1 + 2 / j, {j, vv}]) (−1) + 1
```

1. + 0. ⅈ

```
(Sum[ −1 / j, {j, vv}])
```

1.01532 + 0. ⅈ

```
Table[{n, Expand[Roots[ Expand[gg[n, x]] == 0, x]]}, {n, 2, 32}] // TableForm
```

```
Table[{n, NRoots[gg[n, x] == 0, x]}, {n, 2, 210}] // TableForm
```

NRoots::nnumeq : False is expected to be a polynomial equation in the variable x with numeric coefficients. ≫

NRoots::nnumeq : False is expected to be a polynomial equation in the variable x with numeric coefficients. ≫

```
2     NRoots[False, x]
3     NRoots[False, x]
4     x == −6.
5     x == −8.
6     x == −3.33333
7     x == −4.
8     x == −9.64575 || x == −4.35425
9     x == −13.4244 || x == −3.57557
10    x == −20.3459 || x == −2.6541
11    x == −20. || x == −3.
12    x == −4. − 0.707107 ⅈ || x == −4. + 0.707107 ⅈ
13    x == −4. − 1.41421 ⅈ || x == −4. + 1.41421 ⅈ
14    x == −6.5 || x == −3.
15    x == −8.54138 || x == −2.45862
16    x == −11.2727 − 4.42556 ⅈ || x == −11.2727 + 4.42556 ⅈ || x == −2.45468
17    x == −11.1434 − 4.1661 ⅈ || x == −11.1434 + 4.1661 ⅈ || x == −2.71316
18    x == −29.3693 || x == −4.63068 || x == −3.
19    x == −29.406 || x == −3.797 − 0.523141 ⅈ || x == −3.797 + 0.523141 ⅈ
20    x == −42.8546 || x == −3.07271 − 1.09503 ⅈ || x == −3.07271 + 1.09503 ⅈ
21    x == −42.2094 || x == −3.79063 || x == −3.
22    x == −41.5406 || x == −5.06306 || x == −2.39632
23    x == −41.5574 || x == −4.79027 || x == −2.65232
24    x == −5.96489 − 2.40103 ⅈ || x == −5.96489 + 2.40103 ⅈ || x == −2.67023
25    x == −6.00599 − 2.90531 ⅈ || x == −6.00599 + 2.90531 ⅈ || x == −2.58802
26    x == −6.14747 − 3.77731 ⅈ || x == −6.14747 + 3.77731 ⅈ || x == −2.30506
27    x == −6.55036 − 3.37202 ⅈ || x == −6.55036 + 3.37202 ⅈ || x == −2.29929
28    x == −9.80776 || x == −5.65592 || x == −2.33632
29    x == −9.95434 || x == −5.29649 || x == −2.54917
30    x == −17.1815 || x == −2.70926 − 0.872723 ⅈ || x == −2.70926 + 0.872723 ⅈ
31    x == −17.2042 || x == −2.69788 − 1.04473 ⅈ || x == −2.69788 + 1.04473 ⅈ
32    x == −16.8095 − 12.8145 ⅈ || x == −16.8095 + 12.8145 ⅈ || x == −2.69051 − 1.04288 ⅈ || x == −2.690
33    x == −16.6177 − 12.6273 ⅈ || x == −16.6177 + 12.6273 ⅈ || x == −2.88226 − 0.71277 ⅈ || x == −2.882
34    x == −16.4136 − 12.4346 ⅈ || x == −16.4136 + 12.4346 ⅈ || x == −3.51862 || x == −2.65418
35    x == −16.1951 − 12.2362 ⅈ || x == −16.1951 + 12.2362 ⅈ || x == −4.31448 || x == −2.29524
36    x == −55.9842 || x == −5.36265 − 1.99868 ⅈ || x == −5.36265 + 1.99868 ⅈ || x == −2.29053
37    x == −55.9833 || x == −5.2723 − 1.84864 ⅈ || x == −5.2723 + 1.84864 ⅈ || x == −2.47211
38    x == −56.0311 || x == −5.35957 − 2.54895 ⅈ || x == −5.35957 + 2.54895 ⅈ || x == −2.24977
39    x == −56.0787 || x == −5.40778 − 3.06127 ⅈ || x == −5.40778 + 3.06127 ⅈ || x == −2.10574
40    x == −77.9886 || x == −4.45185 − 2.94149 ⅈ || x == −4.45185 + 2.94149 ⅈ || x == −2.10769
41    x == −77.9883 || x == −4.39383 − 2.89281 ⅈ || x == −4.39383 + 2.89281 ⅈ || x == −2.22402
42    x == −76.2124 || x == −6.05457 || x == −4.1856 || x == −2.54741
43    x == −76.2121 || x == −6.27489 || x == −3.51301 || x == −3.
44    x == −75.3 || x == −7.93271 || x == −2.88365 − 0.56828 ⅈ || x == −2.88365 + 0.56828 ⅈ
45    x == −74.3616 || x == −9.26209 || x == −2.68818 − 0.663199 ⅈ || x == −2.68818 + 0.663199 ⅈ
46    x == −74.3879 || x == −8.89038 || x == −3. || x == −2.72176
47    x == −74.3875 || x == −8.94002 || x == −2.83624 − 0.506135 ⅈ || x == −2.83624 + 0.506135 ⅈ
48    x == −8.73692 − 5.99905 ⅈ || x == −8.73692 + 5.99905 ⅈ || x == −2.84642 − 0.516416 ⅈ || x == −2.84
49    x == −8.65424 − 5.92882 ⅈ || x == −8.65424 + 5.92882 ⅈ || x == −2.9291 − 0.379417 ⅈ || x == −2.929
50    x == −8.84235 − 6.87153 ⅈ || x == −8.84235 + 6.87153 ⅈ || x == −2.74098 − 0.549257 ⅈ || x == −2.74
51    x == −8.68808 − 6.76931 ⅈ || x == −8.68808 + 6.76931 ⅈ || x == −3.26792 || x == −2.52258
52    x == −8.85076 − 7.58847 ⅈ || x == −8.85076 + 7.58847 ⅈ || x == −2.73258 − 0.193384 ⅈ || x == −2.73
```

```
53   x == −8.86431 − 7.58559 ⅈ || x == −8.86431 + 7.58559 ⅈ || x == −2.71903 − 0.497377 ⅈ || x == −2.71
54   x == −10.5282 − 5.1973 ⅈ || x == −10.5282 + 5.1973 ⅈ || x == −2.72179 − 0.530173 ⅈ || x == −2.7217
55   x == −10.3716 − 5.00381 ⅈ || x == −10.3716 + 5.00381 ⅈ || x == −3.25357 || x == −2.50318
56   x == −15.4864 || x == −8.5194 || x == −3.30432 || x == −2.52319
57   x == −15.7351 || x == −7.77056 || x == −4.0857 || x == −2.24197
58   x == −15.9594 || x == −6.68146 || x == −5.09353 || x == −2.09893
59   x == −15.945 || x == −6.93096 || x == −4.74544 || x == −2.21188
60   x == −30.4075 || x == −3.61536 − 2.14655 ⅈ || x == −3.61536 + 2.14655 ⅈ || x == −2.1951
61   x == −30.4065 || x == −3.55952 − 2.10496 ⅈ || x == −3.55952 + 2.10496 ⅈ || x == −2.30777
62   x == −30.4353 || x == −3.61081 − 2.31952 ⅈ || x == −3.61081 + 2.31952 ⅈ || x == −2.17639
63   x == −30.0198 || x == −3.81327 − 2.08473 ⅈ || x == −3.81327 + 2.08473 ⅈ || x == −2.187
64   x == −23.1014 − 23.8156 ⅈ || x == −23.1014 + 23.8156 ⅈ || x == −3.80529 − 2.08901 ⅈ || x == −3.805
65   x == −23.1175 − 23.8118 ⅈ || x == −23.1175 + 23.8118 ⅈ || x == −3.84181 − 2.31104 ⅈ || x == −3.841
66   x == −22.6567 − 23.5126 ⅈ || x == −22.6567 + 23.5126 ⅈ || x == −4.23649 − 1.37208 ⅈ || x == −4.236
67   x == −22.6565 − 23.5131 ⅈ || x == −22.6565 + 23.5131 ⅈ || x == −4.16816 − 1.25972 ⅈ || x == −4.168
68   x == −22.4219 − 23.3619 ⅈ || x == −22.4219 + 23.3619 ⅈ || x == −4.3811 − 0.15363 ⅈ || x == −4.3811
69   x == −22.4393 − 23.3567 ⅈ || x == −22.4393 + 23.3567 ⅈ || x == −4.45888 − 1.13933 ⅈ || x == −4.458
70   x == −21.9254 − 23.062 ⅈ || x == −21.9254 + 23.062 ⅈ || x == −6.76899 || x == −2.69014 − 0.106337
71   x == −21.9251 − 23.0626 ⅈ || x == −21.9251 + 23.0626 ⅈ || x == −6.82439 || x == −2.66269 − 0.45111
72   x == −96.1136 || x == −7.27409 − 4.44394 ⅈ || x == −7.27409 + 4.44394 ⅈ || x == −2.66913 − 0.4422
73   x == −96.1136 || x == −7.2961 − 4.44609 ⅈ || x == −7.2961 + 4.44609 ⅈ || x == −2.64712 − 0.61742 ⅈ
74   x == −96.1126 || x == −7.16026 − 4.33243 ⅈ || x == −7.16026 + 4.33243 ⅈ || x == −2.78345 − 0.2455
75   x == −96.1592 || x == −7.26186 − 4.8925 ⅈ || x == −7.26186 + 4.8925 ⅈ || x == −2.65856 − 0.398483
76   x == −96.2057 || x == −7.33313 − 5.3758 ⅈ || x == −7.33313 + 5.3758 ⅈ || x == −2.56403 − 0.463693
77   x == −96.2047 || x == −7.23305 − 5.30245 ⅈ || x == −7.23305 + 5.30245 ⅈ || x == −2.83367 || x == −2
78   x == −96.2984 || x == −7.42509 − 6.19962 ⅈ || x == −7.42509 + 6.19962 ⅈ || x == −2.42572 − 0.5184
79   x == −96.2984 || x == −7.43527 − 6.19822 ⅈ || x == −7.43527 + 6.19822 ⅈ || x == −2.41554 − 0.6237
80   x == −128.657 || x == −6.25813 − 5.6474 ⅈ || x == −6.25813 + 5.6474 ⅈ || x == −2.41341 − 0.630298
81   x == −128.408 || x == −6.38319 − 5.60717 ⅈ || x == −6.38319 + 5.60717 ⅈ || x == −2.41298 − 0.6258
82   x == −128.407 || x == −6.30608 − 5.56445 ⅈ || x == −6.30608 + 5.56445 ⅈ || x == −2.49028 − 0.4688
83   x == −128.407 || x == −6.31715 − 5.56085 ⅈ || x == −6.31715 + 5.56085 ⅈ || x == −2.47921 − 0.5873
84   x == −125.2 || x == −7.7844 − 2.36701 ⅈ || x == −7.7844 + 2.36701 ⅈ || x == −2.61555 − 0.607586 ⅈ |
85   x == −125.2 || x == −7.63324 − 2.08383 ⅈ || x == −7.63324 + 2.08383 ⅈ || x == −2.76692 − 0.244581
86   x == −125.199 || x == −7.43383 − 1.69746 ⅈ || x == −7.43383 + 1.69746 ⅈ || x == −3.59362 || x == −2
87   x == −125.199 || x == −7.10589 − 0.982123 ⅈ || x == −7.10589 + 0.982123 ⅈ || x == −4.41001 || x ==
88   x == −124.105 || x == −10.5034 || x == −4.60437 − 0.904495 ⅈ || x == −4.60437 + 0.904495 ⅈ || x ==
89   x == −124.105 || x == −10.4818 || x == −4.55637 − 0.644862 ⅈ || x == −4.55637 + 0.644862 ⅈ || x ==
90   x == −120.625 || x == −16.2611 || x == −3.42106 − 1.63576 ⅈ || x == −3.42106 + 1.63576 ⅈ || x == −2
91   x == −120.624 || x == −16.3057 || x == −3.4572 − 1.82538 ⅈ || x == −3.4572 + 1.82538 ⅈ || x == −2.1
92   x == −120.654 || x == −15.9803 || x == −3.60076 − 1.65521 ⅈ || x == −3.60076 + 1.65521 ⅈ || x == −2
93   x == −120.654 || x == −16.0278 || x == −3.62335 − 1.84559 ⅈ || x == −3.62335 + 1.84559 ⅈ || x == −2
94   x == −120.653 || x == −16.0745 || x == −3.63601 − 2.01049 ⅈ || x == −3.63601 + 2.01049 ⅈ || x == −2
95   x == −120.653 || x == −16.1207 || x == −3.64243 − 2.15752 ⅈ || x == −3.64243 + 2.15752 ⅈ || x == −1
96   x == −12.2338 − 11.283 ⅈ || x == −12.2338 + 11.283 ⅈ || x == −3.65266 − 2.19783 ⅈ || x == −3.65266
97   x == −12.2331 − 11.2843 ⅈ || x == −12.2331 + 11.2843 ⅈ || x == −3.62405 − 2.16584 ⅈ || x == −3.62
98   x == −12.074 − 11.1913 ⅈ || x == −12.074 + 11.1913 ⅈ || x == −3.78317 − 2.02334 ⅈ || x == −3.78317
99   x == −11.9064 − 11.0997 ⅈ || x == −11.9064 + 11.0997 ⅈ || x == −3.95075 − 1.84742 ⅈ || x == −3.950
100  x == −12.1997 − 12.3983 ⅈ || x == −12.1997 + 12.3983 ⅈ || x == −3.65742 − 1.85785 ⅈ || x == −3.657
101  x == −12.1993 − 12.3994 ⅈ || x == −12.1993 + 12.3994 ⅈ || x == −3.62427 − 1.81947 ⅈ || x == −3.624
102  x == −11.8988 − 12.2682 ⅈ || x == −11.8988 + 12.2682 ⅈ || x == −3.87396 − 1.17951 ⅈ || x == −3.873
103  x == −11.8984 − 12.2694 ⅈ || x == −11.8984 + 12.2694 ⅈ || x == −3.82066 − 1.08726 ⅈ || x == −3.820
104  x == −12.0439 − 13.0382 ⅈ || x == −12.0439 + 13.0382 ⅈ || x == −3.67259 − 1.11924 ⅈ || x == −3.672
105  x == −11.7617 − 12.9332 ⅈ || x == −11.7617 + 12.9332 ⅈ || x == −4.87028 || x == −2.66029 − 0.3315
106  x == −11.7773 − 12.9237 ⅈ || x == −11.7773 + 12.9237 ⅈ || x == −4.34542 || x == −3.47444 || x == −2
107  x == −11.7772 − 12.9248 ⅈ || x == −11.7772 + 12.9248 ⅈ || x == −4.54426 || x == −3. || x == −2.6157
108  x == −15.7224 − 4.6639 ⅈ || x == −15.7224 + 4.6639 ⅈ || x == −5.23562 || x == −3. || x == −2.60534
```

```
109  x == -15.7178 - 4.66164 ⅈ || x == -15.7178 + 4.66164 ⅈ || x == -5.35828 || x == -2.74592 - 0.41612
110  x == -14.9535 - 3.19075 ⅈ || x == -14.9535 + 3.19075 ⅈ || x == -7.49518 || x == -2.44179 - 0.66018
111  x == -15.0553 - 3.31693 ⅈ || x == -15.0553 + 3.31693 ⅈ || x == -7.10072 || x == -2.53718 - 0.51684
112  x == -25.6693 || x == -7.91464 - 1.91422 ⅈ || x == -7.91464 + 1.91422 ⅈ || x == -2.53645 - 0.52387
113  x == -25.6699 || x == -7.93896 - 1.95197 ⅈ || x == -7.93896 + 1.95197 ⅈ || x == -2.51183 - 0.63653
114  x == -26.0218 || x == -7.9481 - 3.46931 ⅈ || x == -7.9481 + 3.46931 ⅈ || x == -2.3267 - 0.724836 ⅈ
115  x == -26.0083 || x == -7.88404 - 3.33549 ⅈ || x == -7.88404 + 3.33549 ⅈ || x == -2.39754 - 0.63549
116  x == -26.1684 || x == -7.84803 - 3.81216 ⅈ || x == -7.84803 + 3.81216 ⅈ || x == -2.35348 - 0.6317
117  x == -26.3232 || x == -7.80893 - 4.22033 ⅈ || x == -7.80893 + 4.22033 ⅈ || x == -2.31516 - 0.62675
118  x == -26.3107 || x == -7.75348 - 4.12558 ⅈ || x == -7.75348 + 4.12558 ⅈ || x == -2.37689 - 0.52916
119  x == -26.2981 || x == -7.69346 - 4.02616 ⅈ || x == -7.69346 + 4.02616 ⅈ || x == -2.4432 - 0.38961
120  x == -49.1479 || x == -4.82762 - 4.1295 ⅈ || x == -4.82762 + 4.1295 ⅈ || x == -2.4556 - 0.375248 ⅈ
121  x == -49.1473 || x == -4.80076 - 4.11365 ⅈ || x == -4.80076 + 4.11365 ⅈ || x == -2.48274 - 0.34541
122  x == -49.1461 || x == -4.73355 - 4.08876 ⅈ || x == -4.73355 + 4.08876 ⅈ || x == -2.73083 || x == -2
123  x == -49.145 || x == -4.66345 - 4.06658 ⅈ || x == -4.66345 + 4.06658 ⅈ || x == -3.06978 || x == -2.
124  x == -49.172 || x == -4.71161 - 4.23507 ⅈ || x == -4.71161 + 4.23507 ⅈ || x == -2.93454 || x == -2.
125  x == -49.1806 || x == -4.70808 - 4.28063 ⅈ || x == -4.70808 + 4.28063 ⅈ || x == -2.93594 || x == -2
126  x == -47.8377 || x == -5.22182 - 3.27114 ⅈ || x == -5.22182 + 3.27114 ⅈ || x == -3.2646 || x == -2.
127  x == -47.8377 || x == -5.24292 - 3.26705 ⅈ || x == -5.24292 + 3.26705 ⅈ || x == -3.1026 || x == -2.
128  x == -30.0652 - 38.1714 ⅈ || x == -30.0652 + 38.1714 ⅈ || x == -5.23968 - 3.27773 ⅈ || x == -5.239
129  x == -30.0651 - 38.1721 ⅈ || x == -30.0651 + 38.1721 ⅈ || x == -5.13151 - 3.2272 ⅈ || x == -5.1315
130  x == -30.0948 - 38.1571 ⅈ || x == -30.0948 + 38.1571 ⅈ || x == -5.36023 - 3.73814 ⅈ || x == -5.360
131  x == -30.0948 - 38.1571 ⅈ || x == -30.0948 + 38.1571 ⅈ || x == -5.37487 - 3.73542 ⅈ || x == -5.374
132  x == -29.3695 - 37.8081 ⅈ || x == -29.3695 + 37.8081 ⅈ || x == -5.94649 - 2.16528 ⅈ || x == -5.946
133  x == -29.3694 - 37.8089 ⅈ || x == -29.3694 + 37.8089 ⅈ || x == -5.79002 - 1.99311 ⅈ || x == -5.790
134  x == -29.3693 - 37.8096 ⅈ || x == -29.3693 + 37.8096 ⅈ || x == -5.55763 - 1.78329 ⅈ || x == -5.557
135  x == -29.1278 - 37.6917 ⅈ || x == -29.1278 + 37.6917 ⅈ || x == -5.89714 || x == -5. || x == -4.6990
136  x == -28.8813 - 37.5758 ⅈ || x == -28.8813 + 37.5758 ⅈ || x == -7.68743 || x == -4.19983 - 0.82926
137  x == -28.8813 - 37.5758 ⅈ || x == -28.8813 + 37.5758 ⅈ || x == -7.64831 || x == -4.17088 - 0.59905
138  x == -28.9125 - 37.5567 ⅈ || x == -28.9125 + 37.5567 ⅈ || x == -6.50394 - 1.82814 ⅈ || x == -6.503
139  x == -28.9124 - 37.5567 ⅈ || x == -28.9124 + 37.5567 ⅈ || x == -6.53155 - 1.85367 ⅈ || x == -6.531
140  x == -28.1109 - 37.242 ⅈ || x == -28.1109 + 37.242 ⅈ || x == -10.2096 || x == -4. || x == -2.78432 -
141  x == -28.1109 - 37.2428 ⅈ || x == -28.1109 + 37.2428 ⅈ || x == -10.287 || x == -3.54618 - 0.746427
142  x == -28.1109 - 37.2437 ⅈ || x == -28.1109 + 37.2437 ⅈ || x == -10.3611 || x == -3.59156 - 1.09581
143  x == -28.1108 - 37.2445 ⅈ || x == -28.1108 + 37.2445 ⅈ || x == -10.4322 || x == -3.60659 - 1.32818
144  x == -152.703 || x == -9.44113 - 7.65529 ⅈ || x == -9.44113 + 7.65529 ⅈ || x == -3.64071 - 1.31265
145  x == -152.703 || x == -9.46705 - 7.64819 ⅈ || x == -9.46705 + 7.64819 ⅈ || x == -3.65185 - 1.49978
146  x == -152.703 || x == -9.49299 - 7.64148 ⅈ || x == -9.49299 + 7.64148 ⅈ || x == -3.65531 - 1.65872
147  x == -152.703 || x == -9.36711 - 7.56743 ⅈ || x == -9.36711 + 7.56743 ⅈ || x == -3.7816 - 1.5215 ⅈ
148  x == -152.702 || x == -9.2332 - 7.49522 ⅈ || x == -9.2332 + 7.49522 ⅈ || x == -3.91594 - 1.34837 ⅈ
149  x == -152.702 || x == -9.23191 - 7.49756 ⅈ || x == -9.23191 + 7.49756 ⅈ || x == -3.88772 - 1.28789
150  x == -152.828 || x == -9.66151 - 8.91424 ⅈ || x == -9.66151 + 8.91424 ⅈ || x == -3.39576 - 1.5139
151  x == -152.828 || x == -9.6609 - 8.91553 ⅈ || x == -9.6609 + 8.91553 ⅈ || x == -3.36509 - 1.47662 ⅈ
152  x == -152.87 || x == -9.70027 - 9.28907 ⅈ || x == -9.70027 + 9.28907 ⅈ || x == -3.30427 - 1.44726
153  x == -152.869 || x == -9.6161 - 9.24135 ⅈ || x == -9.6161 + 9.24135 ⅈ || x == -3.3865 - 1.3364 ⅈ ||
154  x == -152.867 || x == -9.42357 - 9.15576 ⅈ || x == -9.42357 + 9.15576 ⅈ || x == -3.51962 - 0.78837
155  x == -152.868 || x == -9.43928 - 9.14853 ⅈ || x == -9.43928 + 9.14853 ⅈ || x == -3.55867 - 1.04322
156  x == -152.994 || x == -9.71747 - 10.2701 ⅈ || x == -9.71747 + 10.2701 ⅈ || x == -3.22066 - 1.27222
157  x == -152.994 || x == -9.71722 - 10.2709 ⅈ || x == -9.71722 + 10.2709 ⅈ || x == -3.18185 - 1.23154
158  x == -152.994 || x == -9.72838 - 10.2659 ⅈ || x == -9.72838 + 10.2659 ⅈ || x == -3.21414 - 1.36303
159  x == -152.994 || x == -9.73957 - 10.2609 ⅈ || x == -9.73957 + 10.2609 ⅈ || x == -3.23641 - 1.47788
160  x == -197.807 || x == -8.33599 - 9.28383 ⅈ || x == -8.33599 + 9.28383 ⅈ || x == -3.23311 - 1.48759
161  x == -197.807 || x == -8.34745 - 9.27648 ⅈ || x == -8.34745 + 9.27648 ⅈ || x == -3.24894 - 1.59158
162  x == -196.654 || x == -8.94511 - 8.87931 ⅈ || x == -8.94511 + 8.87931 ⅈ || x == -3.22768 - 1.6029
163  x == -196.654 || x == -8.94475 - 8.88047 ⅈ || x == -8.94475 + 8.88047 ⅈ || x == -3.20211 - 1.57572
164  x == -196.654 || x == -8.87163 - 8.84307 ⅈ || x == -8.87163 + 8.84307 ⅈ || x == -3.27467 - 1.49753
```

```
165    x == -196.653 || x == -8.70546 - 8.77824 i || x == -8.70546 + 8.77824 i || x == -3.4037 - 1.15555
166    x == -196.653 || x == -8.71911 - 8.77079 i || x == -8.71911 + 8.77079 i || x == -3.42588 - 1.30617
167    x == -196.653 || x == -8.71881 - 8.77209 i || x == -8.71881 + 8.77209 i || x == -3.39467 - 1.26366
168    x == -191.809 || x == -10.9977 - 4.64931 i || x == -10.9977 + 4.64931 i || x == -3.53847 - 1.41758
169    x == -191.809 || x == -11.0146 - 4.65857 i || x == -11.0146 + 4.65857 i || x == -3.52345 - 1.47704
170    x == -191.809 || x == -10.6846 - 4.26045 i || x == -10.6846 + 4.26045 i || x == -3.80283 - 0.82435
171    x == -191.808 || x == -10.5036 - 4.03785 i || x == -10.5036 + 4.03785 i || x == -4. || x == -3.9548
172    x == -191.808 || x == -10.2838 - 3.783 i || x == -10.2838 + 3.783 i || x == -5.1323 || x == -3.2464
173    x == -191.808 || x == -10.2757 - 3.78281 i || x == -10.2757 + 3.78281 i || x == -5.24578 || x == -3
174    x == -191.807 || x == -9.37041 - 3.0915 i || x == -9.37041 + 3.0915 i || x == -7.54603 || x == -2.4
175    x == -191.806 || x == -9.40118 || x == -8.48657 - 3.03156 i || x == -8.48657 + 3.03156 i || x == -2
176    x == -190.571 || x == -13.4252 || x == -7.09365 - 2.69998 i || x == -7.09365 + 2.69998 i || x == -2
177    x == -190.571 || x == -13.3624 || x == -7.05617 - 2.52469 i || x == -7.05617 + 2.52469 i || x == -2
178    x == -190.571 || x == -13.2965 || x == -7.01038 - 2.32211 i || x == -7.01038 + 2.32211 i || x == -2
179    x == -190.571 || x == -13.302 || x == -7.02966 - 2.35064 i || x == -7.02966 + 2.35064 i || x == -2.
180    x == -182.561 || x == -26.7964 || x == -4.32342 - 3.27378 i || x == -4.32342 + 3.27378 i || x == -2
181    x == -182.561 || x == -26.7965 || x == -4.33601 - 3.26876 i || x == -4.33601 + 3.26876 i || x == -2
182    x == -182.56 || x == -26.8667 || x == -4.42768 - 3.52818 i || x == -4.42768 + 3.52818 i || x == -2.
183    x == -182.56 || x == -26.864 || x == -4.38022 - 3.50472 i || x == -4.38022 + 3.50472 i || x == -2.4
184    x == -182.591 || x == -26.5937 || x == -4.50479 - 3.42913 i || x == -4.50479 + 3.42913 i || x == -2
185    x == -182.591 || x == -26.5909 || x == -4.45382 - 3.40358 i || x == -4.45382 + 3.40358 i || x == -2
186    x == -182.59 || x == -26.6629 || x == -4.53262 - 3.65405 i || x == -4.53262 + 3.65405 i || x == -2.
187    x == -182.59 || x == -26.6601 || x == -4.48923 - 3.63101 i || x == -4.48923 + 3.63101 i || x == -2.
188    x == -182.589 || x == -26.6944 || x == -4.50189 - 3.73721 i || x == -4.50189 + 3.73721 i || x == -2
189    x == -182.62 || x == -26.4213 || x == -4.6274 - 3.66812 i || x == -4.6274 + 3.66812 i || x == -2.35
190    x == -182.619 || x == -26.4945 || x == -4.68038 - 3.89882 i || x == -4.68038 + 3.89882 i || x == -2
191    x == -182.619 || x == -26.4946 || x == -4.68795 - 3.89689 i || x == -4.68795 + 3.89689 i || x == -2
192    x == -16.165 - 18.2884 i || x == -16.165 + 18.2884 i || x == -4.70521 - 3.97397 i || x == -4.70521
193    x == -16.165 - 18.2884 i || x == -16.165 + 18.2884 i || x == -4.71292 - 3.97198 i || x == -4.71292
194    x == -16.1647 - 18.29 i || x == -16.1647 + 18.29 i || x == -4.6759 - 3.9488 i || x == -4.6759 + 3.94
195    x == -16.1999 - 18.2708 i || x == -16.1999 + 18.2708 i || x == -4.71583 - 4.17293 i || x == -4.715
196    x == -15.9925 - 18.1742 i || x == -15.9925 + 18.1742 i || x == -4.92235 - 4.03062 i || x == -4.922
197    x == -15.9924 - 18.1742 i || x == -15.9924 + 18.1742 i || x == -4.9297 - 4.02937 i || x == -4.9297
198    x == -15.5355 - 18.0034 i || x == -15.5355 + 18.0034 i || x == -5.36773 - 3.54685 i || x == -5.367
199    x == -15.5354 - 18.0034 i || x == -15.5354 + 18.0034 i || x == -5.37708 - 3.54753 i || x == -5.377
200    x == -15.9136 - 19.6281 i || x == -15.9136 + 19.6281 i || x == -4.99758 - 3.44993 i || x == -4.997
201    x == -15.9135 - 19.6295 i || x == -15.9135 + 19.6295 i || x == -4.95794 - 3.41911 i || x == -4.957
202    x == -15.9135 - 19.6309 i || x == -15.9135 + 19.6309 i || x == -4.91629 - 3.3883 i || x == -4.9162
203    x == -15.9134 - 19.6323 i || x == -15.9134 + 19.6323 i || x == -4.87246 - 3.35763 i || x == -4.872
204    x == -15.5156 - 19.5042 i || x == -15.5156 + 19.5042 i || x == -5.22712 - 2.74955 i || x == -5.227
205    x == -15.5157 - 19.5057 i || x == -15.5157 + 19.5057 i || x == -5.16812 - 2.69709 i || x == -5.168
206    x == -15.5158 - 19.5072 i || x == -15.5158 + 19.5072 i || x == -5.10307 - 2.64341 i || x == -5.103
207    x == -15.5297 - 19.4966 i || x == -15.5297 + 19.4966 i || x == -5.13386 - 2.81026 i || x == -5.133
208    x == -15.6571 - 20.2321 i || x == -15.6571 + 20.2321 i || x == -5.00719 - 2.75883 i || x == -5.007
209    x == -15.6572 - 20.2334 i || x == -15.6572 + 20.2334 i || x == -4.94329 - 2.71347 i || x == -4.943
210    x == -14.8507 - 20.0715 i || x == -14.8507 + 20.0715 i || x == -7.71104 || x == -3.3637 - 1.29063
```

```
(List @@ NRoots[ gg[1000, x] == 0, x][[All, 2]])
```

```
{-146.722, -9.80186 - 14.3448 i, -9.80186 + 14.3448 i, -5.45475 - 3.16891 i,
 -5.45475 + 3.16891 i, -3.04215 - 1.06292 i, -3.04215 + 1.06292 i, -1.98069}
```

```
v2 := {-146.72179794904417`, -9.801858582665915` - 14.344794580825395` i,
  -9.801858582665915` + 14.344794580825395` i, -5.454749067855133` - 3.1689148842190615` i,
  -5.454749067855133` + 3.1689148842190615` i, -3.0421483609826154` - 1.062918530300729` i,
  -3.0421483609826154` + 1.062918530300729` i, -1.9806900279484432`}
```

```
999 Product[1 + 1 / j, {j, v2}]
```

$176.696 + 3.46598 \times 10^{-15}\, i$

```
N[Limit[ (DD[1000, z] - 1) / z, {z → 0}]]
```

{176.696}

```
ggo[n_, z_] := Expand[FullSimplify[(DDa[n, z + 1] - 1) / (z + 1)]]
ggo[100, z]
```

$$99 + \frac{6031\, z}{60} + \frac{3167\, z^2}{90} + \frac{3929\, z^3}{720} + \frac{59\, z^4}{180} + \frac{7\, z^5}{720}$$

```
(1 - 1 / List @@ NRoots[ gg[100, x] == 0, x][[All, 2]])
```

{1.04032 − 0.0409792 i, 1.04032 + 0.0409792 i, 1.21734 − 0.1104 i, 1.21734 + 0.1104 i, 1.5}

```
ff[z_] := z (n - 1) (1 - (z - 1) / a) (1 - (z - 1) / b) (1 - (z - 1) / c) (1 - (z - 1) / d) (1 - (z - 1) / e)
ffp[z_] := (n - 1) (1 + 1 / a) (1 + 1 / b) (1 + 1 / c) (1 + 1 / d) (1 + 1 / e)
FullSimplify[Expand[ff[p]] / Expand[ff[q]]]
```

$$\frac{(1 + a - p)\,(1 + b - p)\,(1 + c - p)\,(1 + d - p)\,(1 + e - p)\,p}{(1 + a - q)\,q\,(-1 - b + q)\,(-1 - c + q)\,(-1 - d + q)\,(-1 - e + q)}$$

```
FullSimplify[ff[p] / ff[q]]
```

$$\frac{(1 + a - p)\,p\,(-1 - b + p)\,(-1 - c + p)\,(-1 - d + p)\,(-1 - e + p)}{(1 + a - q)\,q\,(-1 - b + q)\,(-1 - c + q)\,(-1 - d + q)\,(-1 - e + q)}$$

```
FullSimplify[ff[2] / ffp[q]]
```

$$\frac{2\,(-1 + a)\,(-1 + b)\,(-1 + c)\,(-1 + d)\,(-1 + e)}{(1 + a)\,(1 + b)\,(1 + c)\,(1 + d)\,(1 + e)}$$

```
FullSimplify[ff[-1] / ffp[q]]
```

$$-\frac{(2 + a)\,(2 + b)\,(2 + c)\,(2 + d)\,(2 + e)}{(1 + a)\,(1 + b)\,(1 + c)\,(1 + d)\,(1 + e)}$$

```
FullSimplify[ff[5] / ff[2]]
```

$$\frac{5\,(-4 + a)\,(-4 + b)\,(-4 + c)\,(-4 + d)\,(-4 + e)}{2\,(-1 + a)\,(-1 + b)\,(-1 + c)\,(-1 + d)\,(-1 + e)}$$

```
FullSimplify[ff[2] / ffp[q]]
```

$$\frac{2\,(-1 + a)\,(-1 + b)\,(-1 + c)\,(-1 + d)\,(-1 + e)}{(1 + a)\,(1 + b)\,(1 + c)\,(1 + d)\,(1 + e)}$$

```
FullSimplify[ff[1] / ffp[q]]
```

$$\frac{a\,b\,c\,d\,e}{(1 + a)\,(1 + b)\,(1 + c)\,(1 + d)\,(1 + e)}$$

```
FullSimplify[ff[2] / ff[1]]
```

$$\frac{2\,(-1+a)\,(-1+b)\,(-1+c)\,(-1+d)\,(-1+e)}{a\,b\,c\,d\,e}$$

```
Table[{n, Expand[Roots[ Expand[gg[n, x]] == 0, x]]}, {n, 2, 32}] // TableForm
```

2	False
3	False
4	$x == -6$
5	$x == -8$
6	$x == -\frac{10}{3}$
7	$x == -4$
8	$x == -7-\sqrt{7}\ \vert\vert\ x == -7+\sqrt{7}$
9	$x == -\frac{17}{2}-\frac{\sqrt{97}}{2}\ \vert\vert\ x == -\frac{17}{2}+\frac{\sqrt{97}}{2}$
10	$x == -\frac{23}{2}-\frac{\sqrt{313}}{2}\ \vert\vert\ x == -\frac{23}{2}+\frac{\sqrt{313}}{2}$
11	$x == -20\ \vert\vert\ x == -3$
12	$x == -4-\frac{i}{\sqrt{2}}\ \vert\vert\ x == -4+\frac{i}{\sqrt{2}}$
13	$x == -4-i\sqrt{2}\ \vert\vert\ x == -4+i\sqrt{2}$
14	$x == -\frac{13}{2}\ \vert\vert\ x == -3$
15	$x == -\frac{11}{2}-\frac{\sqrt{37}}{2}\ \vert\vert\ x == -\frac{11}{2}+\frac{\sqrt{37}}{2}$
16	$x == -\frac{25}{3}+\frac{1}{3}\left(2240-9\sqrt{61\,861}\right)^{1/3}+\frac{1}{3}\left(2240+9\sqrt{61\,861}\right)^{1/3}\ \vert\vert\ x == -\frac{25}{3}-\frac{1}{6}\left(2240-9\sqrt{61\,861}\right)^{1/3} -$
17	$x == -\frac{25}{3}+\frac{1}{3}\left(1916-9\sqrt{45\,237}\right)^{1/3}+\frac{1}{3}\left(1916+9\sqrt{45\,237}\right)^{1/3}\ \vert\vert\ x == -\frac{25}{3}-\frac{1}{6}\left(1916-9\sqrt{45\,237}\right)^{1/3} -$
18	$x == -17-3\sqrt{17}\ \vert\vert\ x == -17+3\sqrt{17}\ \vert\vert\ x == -3$
19	$x == -\frac{37}{3}-\frac{655}{3\left(16\,858-9\sqrt{39\,269}\right)^{1/3}}-\frac{1}{3}\left(16\,858-9\sqrt{39\,269}\right)^{1/3}\ \vert\vert\ x == -\frac{37}{3}+\frac{655}{6\left(16\,858-9\sqrt{39\,269}\right)^{1/3}}+\frac{}{2\sqrt{3}\ (16\,85}$
20	$x == -\frac{49}{3}-\frac{1579}{3\left(63\,388-9\sqrt{1\,002\,605}\right)^{1/3}}-\frac{1}{3}\left(63\,388-9\sqrt{1\,002\,605}\right)^{1/3}\ \vert\vert\ x == -\frac{49}{3}+\frac{1579}{6\left(63\,388-9\sqrt{1\,002\,605}\right)^{1/3}}+\frac{}{2\sqrt{}}$
21	$x == -23-3\sqrt{41}\ \vert\vert\ x == -23+3\sqrt{41}\ \vert\vert\ x == -3$
22	$x == -\frac{49}{3}+\frac{205\times 7^{2/3}}{3\left(-7636+9\,i\sqrt{24\,659}\right)^{1/3}}+\frac{1}{3}\left(7\left(-7636+9\,i\sqrt{24\,659}\right)\right)^{1/3}\ \vert\vert\ x == -\frac{49}{3}-\frac{205\times 7^{2/3}}{6\left(-7636+9\,i\sqrt{24\,659}\right)^{1/3}}-\frac{}{2}$
23	$x == -\frac{49}{3}+\frac{1435}{3\left(-53\,776+9\,i\sqrt{779\,379}\right)^{1/3}}+\frac{1}{3}\left(-53\,776+9\,i\sqrt{779\,379}\right)^{1/3}\ \vert\vert\ x == -\frac{49}{3}-\frac{1435}{6\left(-53\,776+9\,i\sqrt{779\,379}\right)^{1/3}} -$
24	$x == -\frac{73}{15}-\frac{161}{15\left(25\,838+45\sqrt{331\,741}\right)^{1/3}}+\frac{1}{15}\left(25\,838+45\sqrt{331\,741}\right)^{1/3}\ \vert\vert\ x == -\frac{73}{15}+\frac{161}{30\left(25\,838+45\sqrt{331\,741}\right)^{1/3}}+\frac{}{1}$
25	$x == -\frac{73}{15}-\frac{11\times 31^{2/3}}{15\left(1208+45\sqrt{741}\right)^{1/3}}+\frac{1}{15}\left(31\left(1208+45\sqrt{741}\right)\right)^{1/3}\ \vert\vert\ x == -\frac{73}{15}+\frac{11\times 31^{2/3}}{30\left(1208+45\sqrt{741}\right)^{1/3}}+\frac{11}{10\sqrt{3}\ (120}$
26	$x == -\frac{73}{15}-\frac{701}{15\left(68\,768+45\sqrt{2\,505\,437}\right)^{1/3}}+\frac{1}{15}\left(68\,768+45\sqrt{2\,505\,437}\right)^{1/3}\ \vert\vert\ x == -\frac{73}{15}+\frac{701}{30\left(68\,768+45\sqrt{2\,505\,437}\right)^{1/3}}$

27 $x == -\frac{77}{15} - \frac{401}{15\left(63982+45\sqrt{2053421}\right)^{1/3}} + \frac{1}{15}\left(63982+45\sqrt{2053421}\right)^{1/3} \;||\; x == -\frac{77}{15} + \frac{401}{30\left(63982+45\sqrt{2053421}\right)^{1/3}}$

28 $x == -\frac{89}{15} + \frac{1051}{15\left(-6524+45\,i\sqrt{552283}\right)^{1/3}} + \frac{1}{15}\left(-6524+45\,i\sqrt{552283}\right)^{1/3} \;||\; x == -\frac{89}{15} - \frac{1051}{30\left(-6524+45\,i\sqrt{552283}\right)^{1/3}}$

29 $x == -\frac{89}{15} + \frac{1051}{15\left(-14624+45\,i\sqrt{467691}\right)^{1/3}} + \frac{1}{15}\left(-14624+45\,i\sqrt{467691}\right)^{1/3} \;||\; x == -\frac{89}{15} - \frac{1051}{30\left(-14624+45\,i\sqrt{467691}\right.}$

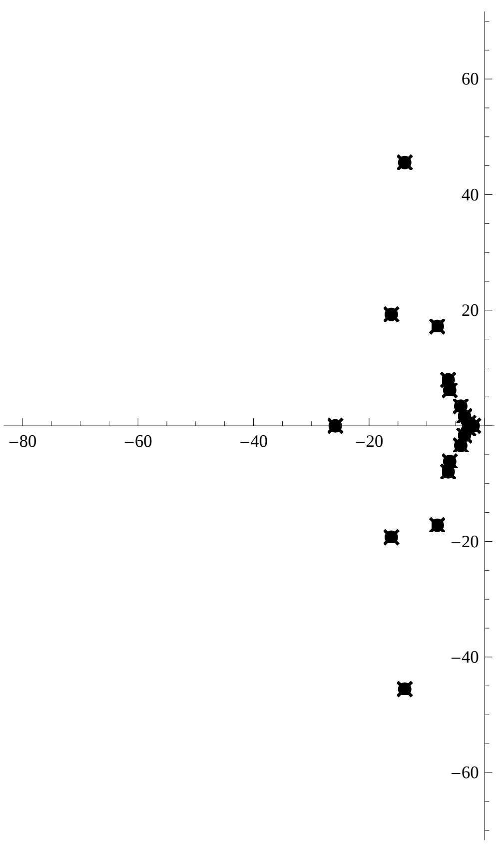
30 $x == -\frac{113}{15} - \frac{5179}{15\left(391292-45\sqrt{7011397}\right)^{1/3}} - \frac{1}{15}\left(391292-45\sqrt{7011397}\right)^{1/3} \;||\; x == -\frac{113}{15} + \frac{5179}{30\left(391292-45\sqrt{70113\right.}}$

31 $x == -\frac{113}{15} - \frac{5179}{15\left(399392-45\sqrt{10174133}\right)^{1/3}} - \frac{1}{15}\left(399392-45\sqrt{10174133}\right)^{1/3} \;||\; x == -\frac{113}{15} + \frac{5179}{30\left(399392-45\sqrt{101\right.}}$

32 $x == -\frac{39}{4} - \frac{1}{2}\sqrt{-\frac{175}{4} + \frac{7506}{\left(\frac{1}{2}\left(427225+i\sqrt{28922954483}\right)\right)^{1/3}} + 2^{2/3}\left(427225+i\sqrt{28922954483}\right)^{1/3}} - \frac{1}{2}\sqrt{-\frac{175}{2}}$

Roots[gg[100, x] == 0, x]

RootLocusPlot[1 / Expand[ggr2[25 524 000, x]], {k, 0, 1}, FeedbackType → None]



```
(1 - 1 / List @@ NRoots[ggr2[25524000, x] == 0, x][[All, 2]])
```

{1.00514, 1.00027 − 0.00185502 i, 1.00027 + 0.00185502 i, 1.00221 − 0.00766598 i,
1.00221 + 0.00766598 i, 1.03868, 1.02554 − 0.0304718 i, 1.02554 + 0.0304718 i,
1.0061 − 0.0200984 i, 1.0061 + 0.0200984 i, 1.02258 − 0.0473686 i, 1.02258 + 0.0473686 i,
1.06122 − 0.0771102 i, 1.06122 + 0.0771102 i, 1.08099 − 0.0826102 i,
1.08099 + 0.0826102 i, 1.14421 − 0.118623 i, 1.14421 + 0.118623 i, 1.23313 − 0.111055 i,
1.23313 + 0.111055 i, 1.3438 − 0.0612693 i, 1.3438 + 0.0612693 i, 1.50299}

```
N[1 / (3 + I)]
```

0.3 − 0.1 i

```
N[1 / (3 - 30 I)] * N[1 / (3 + 30 I)]
```

0.00110011 + 0. i

```
gg[100, z]
```

$$99+\frac{6031\,z}{60}+\frac{3167\,z^2}{90}+\frac{3929\,z^3}{720}+\frac{59\,z^4}{180}+\frac{7\,z^5}{720}$$

```
gg2[n_, z_] := Expand[(gg[n, z] - (n - 1)) / z]
gg3[n_, z_] := Expand[(gg[n, z] / (n - 1) - 1) / z]
```

```
gg2[100, z]
```

$$\frac{6031}{60}+\frac{3167\,z}{90}+\frac{3929\,z^2}{720}+\frac{59\,z^3}{180}+\frac{7\,z^4}{720}$$

```
gg3[100, z]
```

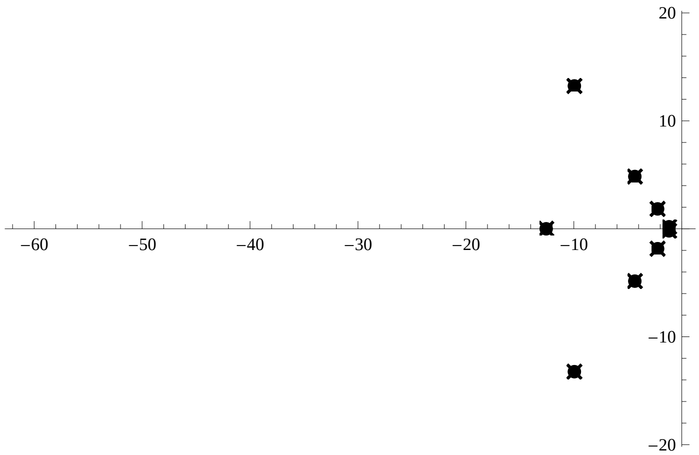
$$\frac{6031}{5940}+\frac{3167\,z}{8910}+\frac{3929\,z^2}{71280}+\frac{59\,z^3}{17820}+\frac{7\,z^4}{71280}$$

```
Table[{n, gg2[n, z] - gg2[n - 1, z]}, {n, 2, 60}] // TableForm
```

2	0
3	0
4	$\frac{1}{2}$
5	0
6	1
7	0
8	$\frac{5}{6}+\frac{z}{6}$
9	$\frac{1}{2}$
10	1
11	0
12	$\frac{3}{2}+\frac{z}{2}$
13	0
14	1
15	1
16	$\frac{13}{12}+\frac{3z}{8}+\frac{z^2}{24}$
17	0
18	$\frac{3}{2}+\frac{z}{2}$
19	0
20	$\frac{3}{2}+\frac{z}{2}$
21	1
22	1

23	0
24	$\frac{11}{6} + z + \frac{z^2}{6}$
25	$\frac{1}{2}$
26	1
27	$\frac{5}{6} + \frac{z}{6}$
28	$\frac{3}{2} + \frac{z}{2}$
29	0
30	$2 + z$
31	0
32	$\frac{77}{60} + \frac{71 z}{120} + \frac{7 z^2}{60} + \frac{z^3}{120}$
33	1
34	1
35	1
36	$2 + \frac{5 z}{4} + \frac{z^2}{4}$
37	0
38	1
39	1
40	$\frac{11}{6} + z + \frac{z^2}{6}$
41	0
42	$2 + z$
43	0
44	$\frac{3}{2} + \frac{z}{2}$
45	$\frac{3}{2} + \frac{z}{2}$
46	1
47	0
48	$\frac{25}{12} + \frac{35 z}{24} + \frac{5 z^2}{12} + \frac{z^3}{24}$
49	$\frac{1}{2}$
50	$\frac{3}{2} + \frac{z}{2}$
51	1
52	$\frac{3}{2} + \frac{z}{2}$
53	0
54	$\frac{11}{6} + z + \frac{z^2}{6}$
55	1
56	$\frac{11}{6} + z + \frac{z^2}{6}$
57	1
58	1
59	0
60	$\frac{5}{2} + 2 z + \frac{z^2}{2}$

```
RootLocusPlot[1 / Expand[ggr3[10 000, x]], {k, 0, 1}, FeedbackType → None]
```




```
(List @@ NRoots[ggr2[100, x] == 0, x][[All, 2]])
```

{-12.1997 - 12.3983 i, -12.1997 + 12.3983 i, -3.65742 - 1.85785 i, -3.65742 + 1.85785 i, -2.}

```
(List @@ NRoots[ggr3[100, x] == 0, x][[All, 2]])
```

{-11.1997 - 12.3983 i, -11.1997 + 12.3983 i, -2.65742 - 1.85785 i, -2.65742 + 1.85785 i, -1.}

```
(List @@ NRoots[ggr4[100, x] == 0, x][[All, 2]])
```

{-10.1997 - 12.3983 i, -10.1997 + 12.3983 i, -1.65742 - 1.85785 i, -1.65742 + 1.85785 i, 0.}

```
vv := {-11.199718313735842` - 12.398284753324173` i,
  -11.199718313735842` + 12.398284753324173` i, -2.6574245434070156` - 1.8578482376600212` i,
  -2.6574245434070156` + 1.8578482376600212` i, -1.`}
```

99. + 1.58392 × 10^{-15} i

```
vv2 := {-12.199718313735842` - 12.398284753324173` i,
  -12.199718313735842` + 12.398284753324173` i, -3.6574245434070143` - 1.8578482376600207` i,
  -3.6574245434070143` + 1.8578482376600207` i, -2.`}
Product[1 + 1 / j, {j, vv2}] 99
```

28.5333 + 0. i

```
vv3 := {-10.199718313735842` - 12.39828475332417` i,
  -10.199718313735842` + 12.39828475332417` i, -1.6574245434070143` - 1.857848237660022` i,
  -1.6574245434070143` + 1.857848237660022` i, 0.`}
Product[1 - 1 / j, {j, vv3}] (DD[100, -1] - 1)
```

Power::infy : Infinite expression $\frac{1}{0.}$ encountered. ≫

Infinity::indet : Indeterminate expression 0 ComplexInfinity encountered. ≫

Indeterminate

```
(List @@ NRoots[ggr3[100, x] == 0, x][[All, 2]])
```

```
{-11.199718313735842` - 12.398284753324173` i,
 -11.199718313735842` + 12.398284753324173` i, -2.6574245434070156` - 1.8578482376600212` i,
 -2.6574245434070156` + 1.8578482376600212` i, -1.`}
vv := {-11.199718313735842` - 12.398284753324173` i,
  -11.199718313735842` + 12.398284753324173` i, -2.6574245434070156` - 1.8578482376600212` i,
  -2.6574245434070156` + 1.8578482376600212` i, -1.`}
{-11.1997 - 12.3983 i, -11.1997 + 12.3983 i, -2.65742 - 1.85785 i, -2.65742 + 1.85785 i, -1.}
Product[ 1 - 1 / j, {j, vv}] 428 / 15
99. + 1.58392 × 10^-15 i
Product[ (j - 1) / j, {j, vv}] 428 / 15
99. + 0. i
Product[ 1 + 1 / (j - 1), {j, vv}] 99
28.5333 + 0. i
Product[ (j) / (j - 1), {j, vv}] 99
28.5333 + 0. i
```

Power::infy : Infinite expression $\frac{1}{0.}$ encountered. ≫

```
Product[ (j + 1) / j, {j, vv}] 428 / 15
0. + 0. i
(List @@ NRoots[ggr3[100, x] == 0, x][[All, 2]])
{-11.1997 - 12.3983 i, -11.1997 + 12.3983 i, -2.65742 - 1.85785 i, -2.65742 + 1.85785 i, -1.}
(List @@ NRoots[ggr3a[100, x] == 0, x][[All, 2]])
```

NRoots::nnumeq : $\frac{720 + x (1 + x) (20544 + x (12034 + \text{Times}[\ll 2 \gg]))}{720 x} == 0$
is expected to be a polynomial equation in the variable x with numeric coefficients. ≫

Part::partd : Part specification NRoots$\left[\frac{720 + x (1 + x) (20544 + \text{Times}[\ll 2 \gg])}{720 x} == 0, x\right]$[[All, 2]] is longer than depth of object. ≫

$\left\{\text{NRoots}\left[\frac{720 + x (1 + x) (20544 + x (12034 + x (2861 + x (194 + 7 x))))}{720 x} == 0, x\right], \text{All}, 2\right\}$

```
(List @@ NRoots[ggr3b[100, x] == 0, x][[All, 2]])
{-11.1997 - 12.3983 i, -11.1997 + 12.3983 i, -2.65742 - 1.85785 i, -2.65742 + 1.85785 i, -1., 0.}
(List @@ NRoots[ggr3e[100, x] == 0, x][[All, 2]])
{-11.1997 - 12.3982 i, -11.1997 + 12.3982 i,
 -2.67195 - 1.86184 i, -2.67195 + 1.86184 i, -0.933809, -0.0372047}
vx := {-11.199718313735842` - 12.398284753324173` i,
  -11.199718313735842` + 12.398284753324173` i, -2.6574245434070156` - 1.8578482376600212` i,
  -2.6574245434070156` + 1.8578482376600212` i, -1.`, 0.`}
```

```
Sum[If[j == 0, 0, -1/j], {j, vx}]

1.58577 + 0. i

vy := {-11.199685576035794` - 12.398224487807218` i,
   -11.199685576035794` + 12.398224487807218` i,
   -2.6719503346754907` - 1.8618449055430242` i, -2.6719503346754907` + 1.8618449055430242` i,
   -0.9338092178222003`, -0.03720467504094745`}

Sum[-1/j, {j, vy}]

28.5333 + 0. i

(List @@ NRoots[ggr3[200, x] == 0, x][[All, 2]])

{-14.9136 - 19.6281 i, -14.9136 + 19.6281 i, -3.99758 - 3.44993 i,
 -3.99758 + 3.44993 i, -1.21384 - 0.650558 i, -1.21384 + 0.650558 i}

vt := {-11.199718313735842` - 12.398284753324173` i,
   -11.199718313735842` + 12.398284753324173` i, -2.6574245434070156` - 1.8578482376600212` i,
   -2.6574245434070156` + 1.8578482376600212` i, -1.`}

tt[s_] := 1 + s (428/15) Product[1 - s/j, {j, vt}]

tt[0]

1. + 0. i

tv[z_] := 1 + z (99) Product[1 - (z - 1) / (j - 1), {j, vt}]
tv2[s_] := (99) Product[1 + 1 / (j - 1), {j, vt}]

tv2[4]

28.5333 + 0. i

(List @@ NRoots[ggr3f[100, x] == 0, x][[All, 2]])

{-12.1997 - 12.3982 i, -12.1997 + 12.3982 i,
 -3.67195 - 1.86184 i, -3.67195 + 1.86184 i, -1.93381, -1.0372}

(List @@ NRoots[ggr3e[100, x] == 0, x][[All, 2]])

{-11.1997 - 12.3982 i, -11.1997 + 12.3982 i,
 -2.67195 - 1.86184 i, -2.67195 + 1.86184 i, -0.933809, -0.0372047}

vx := {-12.19968557603579` - 12.398224487807214` i,
   -12.19968557603579` + 12.398224487807214` i, -3.6719503346754956` - 1.8618449055430237` i,
   -3.6719503346754956` + 1.8618449055430237` i, -1.933809217822195`, -1.0372046750409485`}

Product[1 - 2/j, {j, vx}] 100

1471. - 8.88178 × 10^-14 i

Product[1 + 1/j, {j, vx}] 100

1. + 2.1684 × 10^-17 i

Product[1 + 3/j, {j, vx}] 100

19. + 1.38778 × 10^-15 i

Sum[-1/(j), {j, vx}]

1.99517 + 0. i
```