

```
ClearAll["Global`*"]
```

```
N[100^ZetaZero[1] / (ZetaZero[1] Zeta'[ZetaZero[1]])]
```

```
0.751611 + 0.479272 i
```

```
Sum[N[Floor[100 / j]^ZetaZero[1] / (ZetaZero[1] Zeta'[ZetaZero[1]])], {j, 2, 100}]
```

```
-1.09186 - 1.9303 i
```

```
N[100^ZetaZero[1] / (ZetaZero[1])]
```

```
0.528969 + 0.469138 i
```

```
Sum[N[Floor[100 / j]^ZetaZero[1] / (ZetaZero[1])], {j, 2, 100}]
```

```
-0.614543 - 1.64816 i
```

```
LL[n_] := Sum[Log[j] - LL[Floor[n / j]], {j, 2, n}]
```

```
MM[n_] := 1 - Sum[MM[Floor[n / j]], {j, 2, n}]
```

```
L2[n_] := Sum[Log[j] MM[Floor[n / j]], {j, 2, n}]
```

```
E2b[n_, k_, a_] :=
```

```
    E2b[n, k, a] = Sum[E2b[n / j, k - 1, a], {j, 2, n}] - a Sum[E2b[n / (a j), k - 1, a], {j, 1, n / a}];
```

```
E2b[n_, 0, a_] := 1
```

```
D1c[n_, k_, b_] := Sum[Binomial[k + j - 1, k - 1] b^j
```

```
    Sum[FactorialPower[k, a] / a! E2b[n / b^j, a, b], {a, 0, Log[If[b > 2, 2, b], n / b^j]}],
```

```
    { j, 0, Log[b, n] }]
```

```
D1a[n_, k_, b_, r1_, r2_] := Sum[Binomial[k + j - 1, k - 1] b^j
```

```
    Sum[FactorialPower[k, a] / a! E2b[n / b^j, a, b], {a, 0, Log[If[b > 2, 2, b], n / b^j]}],
```

```
    { j, r1, r2 }]
```

```
L3[n_, a_] := Sum[Log[j] D1c[Floor[n / j], -1, a], {j, 2, n}]
```

```
M2[n_, a_] := Sum[(-1)^k (E2b[n, k, a] - a E2b[n / a, k, a]), {k, 0, Log[a, n]}]
```

```
L6[n_, a_] := Sum[Log[j] M2[Floor[n / j], a], {j, 2, n / 2}] + Sum[Log[j], {j, n / 2 + 1, n}]
```

```
N[L3[100, 1.5]]
```

```
94.0453
```

```
N[L6[100, 1.5]]
```

```
94.0453
```

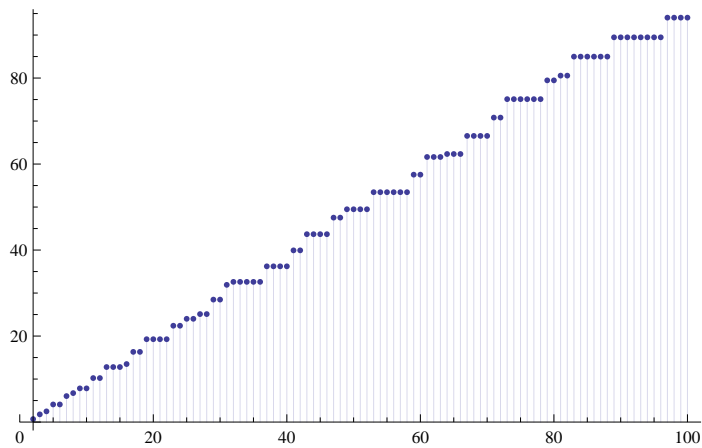
```
N[LL[100]]
```

```
94.0453
```

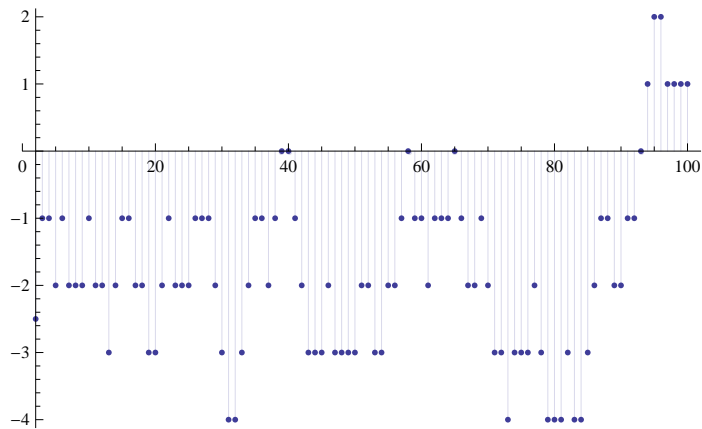
```
(-1)^0 (E2b[10, 0, 3] - 3 E2b[10 / 3, 0, 3])
```

```
- 2
```

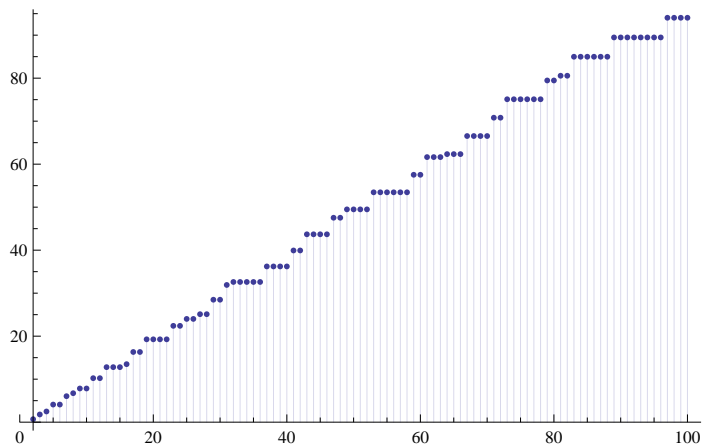
DiscretePlot[LL[n], {n, 2, 100}]



DiscretePlot[M2[n, 2.5], {n, 2, 100}]



DiscretePlot[L2[n], {n, 2, 100}]



Series[(x+1)^-2, {x, 0, 20}]

$$1 - 2x + 3x^2 - 4x^3 + 5x^4 - 6x^5 + 7x^6 - 8x^7 + 9x^8 - 10x^9 + 11x^{10} - 12x^{11} + 13x^{12} - 14x^{13} + 15x^{14} - 16x^{15} + 17x^{16} - 18x^{17} + 19x^{18} - 20x^{19} + 21x^{20} + O[x]^{21}$$

f[n\_, k\_] := 1 - (k+1) Sum[f[Floor[n/j], k+1], {j, 2, n}]

```
f[100, 1]
```

```
14365
```

```
MM[n_, k_] := Sum[MoebiusMu[j] MM[Floor[n/j], k-1], {j, 1, n}]; MM[n_, 0] := 1
```

```
MM[100, 2]
```

```
19
```

```
D2[n_, k_] := Sum[D2[Floor[n/j], k-1], {j, 2, n}]; D2[n_, 0] := 1
```

```
DA[n_, z_] := Sum[FactorialPower[z, a] / a! D2[n, a], {a, 0, Log[2, n]}]
```

```
da[n_, z_] := Sum[FactorialPower[z, a] / a! d2[n, a], {a, 0, Log[2, n]}]
```

```
DA[100, -2]
```

```
19
```

```
da[100, -2]
```

```
d2[100, 0] - 2 d2[100, 1] + 3 d2[100, 2] - 4 d2[100, 3] + 5 d2[100, 4] - 6 d2[100, 5] + 7 d2[100, 6]
```

```
Table[{n, MM[n], M2[n, 1.5], D1c[n, -1, 1.5]}, {n, 0, 100}] // TableForm
```

|    |    |      |     |
|----|----|------|-----|
| 0  | 1  | 0.   | 0.  |
| 1  | 1  | -0.5 | 1.  |
| 2  | 0  | 0.   | 0.  |
| 3  | -1 | -1.  | -1. |
| 4  | -1 | -1.  | -1. |
| 5  | -2 | -2.  | -2. |
| 6  | -1 | -1.  | -1. |
| 7  | -2 | -2.  | -2. |
| 8  | -2 | -2.  | -2. |
| 9  | -2 | -2.  | -2. |
| 10 | -1 | -1.  | -1. |
| 11 | -2 | -2.  | -2. |
| 12 | -2 | -2.  | -2. |
| 13 | -3 | -3.  | -3. |
| 14 | -2 | -2.  | -2. |
| 15 | -1 | -1.  | -1. |
| 16 | -1 | -1.  | -1. |
| 17 | -2 | -2.  | -2. |
| 18 | -2 | -2.  | -2. |
| 19 | -3 | -3.  | -3. |
| 20 | -3 | -3.  | -3. |
| 21 | -2 | -2.  | -2. |
| 22 | -1 | -1.  | -1. |
| 23 | -2 | -2.  | -2. |
| 24 | -2 | -2.  | -2. |
| 25 | -2 | -2.  | -2. |
| 26 | -1 | -1.  | -1. |
| 27 | -1 | -1.  | -1. |
| 28 | -1 | -1.  | -1. |
| 29 | -2 | -2.  | -2. |
| 30 | -3 | -3.  | -3. |
| 31 | -4 | -4.  | -4. |
| 32 | -4 | -4.  | -4. |
| 33 | -3 | -3.  | -3. |

|    |    |     |     |
|----|----|-----|-----|
| 34 | -2 | -2. | -2. |
| 35 | -1 | -1. | -1. |
| 36 | -1 | -1. | -1. |
| 37 | -2 | -2. | -2. |
| 38 | -1 | -1. | -1. |
| 39 | 0  | 0.  | 0.  |
| 40 | 0  | 0.  | 0.  |
| 41 | -1 | -1. | -1. |
| 42 | -2 | -2. | -2. |
| 43 | -3 | -3. | -3. |
| 44 | -3 | -3. | -3. |
| 45 | -3 | -3. | -3. |
| 46 | -2 | -2. | -2. |
| 47 | -3 | -3. | -3. |
| 48 | -3 | -3. | -3. |
| 49 | -3 | -3. | -3. |
| 50 | -3 | -3. | -3. |
| 51 | -2 | -2. | -2. |
| 52 | -2 | -2. | -2. |
| 53 | -3 | -3. | -3. |
| 54 | -3 | -3. | -3. |
| 55 | -2 | -2. | -2. |
| 56 | -2 | -2. | -2. |
| 57 | -1 | -1. | -1. |
| 58 | 0  | 0.  | 0.  |
| 59 | -1 | -1. | -1. |
| 60 | -1 | -1. | -1. |
| 61 | -2 | -2. | -2. |
| 62 | -1 | -1. | -1. |
| 63 | -1 | -1. | -1. |
| 64 | -1 | -1. | -1. |
| 65 | 0  | 0.  | 0.  |
| 66 | -1 | -1. | -1. |
| 67 | -2 | -2. | -2. |
| 68 | -2 | -2. | -2. |
| 69 | -1 | -1. | -1. |
| 70 | -2 | -2. | -2. |
| 71 | -3 | -3. | -3. |
| 72 | -3 | -3. | -3. |
| 73 | -4 | -4. | -4. |
| 74 | -3 | -3. | -3. |
| 75 | -3 | -3. | -3. |
| 76 | -3 | -3. | -3. |
| 77 | -2 | -2. | -2. |
| 78 | -3 | -3. | -3. |
| 79 | -4 | -4. | -4. |
| 80 | -4 | -4. | -4. |
| 81 | -4 | -4. | -4. |
| 82 | -3 | -3. | -3. |
| 83 | -4 | -4. | -4. |
| 84 | -4 | -4. | -4. |
| 85 | -3 | -3. | -3. |
| 86 | -2 | -2. | -2. |
| 87 | -1 | -1. | -1. |
| 88 | -1 | -1. | -1. |
| 89 | -2 | -2. | -2. |

```

90      -2      -2.      -2.
91      -1      -1.      -1.
92      -1      -1.      -1.
93       0       0.       0.
94       1       1.       1.
95       2       2.       2.
96       2       2.       2.
97       1       1.       1.
98       1       1.       1.
99       1       1.       1.
100      1       1.       1.

```

```

Dlcc[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j
  Sum[FactorialPower[k, a] / a! E2[n / b^j, a, b], {a, 0, Log[If[b > 2, 2, b], n / b^j]}],
  {j, 0, Log[b, n]}]

```

```
Dlcc[1, -1, 2]
```

```
E2[1, 0, 2]
```

```
M2d[n_, a_] := Sum[ (-1)^k ( E2[n, k, a] - a E2[n / a, k, a] ), {k, 0, Log[a, n]}]
```

```
M2d[1, a]
```

```
E2[1, 0, a] - a E2[ $\frac{1}{a}$ , 0, a]
```

```
E2b[n_, k_, a_] :=
```

```
  E2b[n, k, a] = Sum[ E2b[n / j, k - 1, a], {j, 2, n}] - a Sum[ E2b[n / (a j), k - 1, a], {j, 1, n / a}];
```

```
E2b[n_, 0, a_] := 1
```

```
E2b[200, 2, 3]
```

```
9
```

```
Et2[n_, a_] := Sum[ 1, {j, 2, n}, {k, 2, n / j}] -
  2 a Sum[ 1, {j, 2, n}, {k, 1, n / (j a)}] + a^2 Sum[1, {j, 1, n / a}, {k, 1, n / (a^2 j)}]
```

```
Et2[200, 3]
```

```
9
```

```
Et3[n_, a_] := Sum[ 1, {j, 2, n}, {k, 2, n / j}, {l, 2, n / (j k)}] -
  3 a Sum[ 1, {j, 2, n}, {k, 2, n / j}, {l, 1, n / (a j k)}] +
  3 a^2 Sum[1, {j, 2, n}, {k, 1, n / (a j)}, {l, 1, n / (j k a^2)}] -
  a^3 Sum[1, {j, 1, n / a}, {k, 1, n / (a^2 j)}, {l, 1, n / (a^3 j k)}]
```

```
Et3[200, 4]
```

```
51
```

```
E2b[200, 3, 4]
```

```
51
```

```
Table[{n, E2b[n, rr = 3, aa = 1.000000000000001] - E2b[n - 1, rr, aa]}, {n, 2, 100}] // TableForm
```

```

2      -1.
3      -3.04201 × 10-14
4      -2.9754 × 10-14
5      -2.93099 × 10-14

```

|    |                            |
|----|----------------------------|
| 6  | $-2.88658 \times 10^{-14}$ |
| 7  | $-3.28626 \times 10^{-14}$ |
| 8  | 1.                         |
| 9  | -1.                        |
| 10 | $-3.19744 \times 10^{-14}$ |
| 11 | $-2.66454 \times 10^{-14}$ |
| 12 | 3.                         |
| 13 | -3.                        |
| 14 | $-2.66454 \times 10^{-14}$ |
| 15 | $-3.37508 \times 10^{-14}$ |
| 16 | 3.                         |
| 17 | -3.                        |
| 18 | 3.                         |
| 19 | -3.                        |
| 20 | 3.                         |
| 21 | -3.                        |
| 22 | $-3.55271 \times 10^{-14}$ |
| 23 | $-3.19744 \times 10^{-14}$ |
| 24 | 9.                         |
| 25 | -9.                        |
| 26 | $-2.4869 \times 10^{-14}$  |
| 27 | 1.                         |
| 28 | 2.                         |
| 29 | -3.                        |
| 30 | 6.                         |
| 31 | -6.                        |
| 32 | 6.                         |
| 33 | -6.                        |
| 34 | $-1.77636 \times 10^{-14}$ |
| 35 | $-3.55271 \times 10^{-14}$ |
| 36 | 12.                        |
| 37 | -12.                       |
| 38 | $-2.13163 \times 10^{-14}$ |
| 39 | $-1.42109 \times 10^{-14}$ |
| 40 | 9.                         |
| 41 | -9.                        |
| 42 | 6.                         |
| 43 | -6.                        |
| 44 | 3.                         |
| 45 | $-4.26326 \times 10^{-14}$ |
| 46 | -3.                        |
| 47 | $-4.9738 \times 10^{-14}$  |
| 48 | 18.                        |
| 49 | -18.                       |
| 50 | 3.                         |
| 51 | -3.                        |
| 52 | 3.                         |
| 53 | -3.                        |
| 54 | 9.                         |
| 55 | -9.                        |
| 56 | 9.                         |
| 57 | -9.                        |
| 58 | $-2.13163 \times 10^{-14}$ |

```

59       $-3.55271 \times 10^{-14}$ 
60      21.
61      -21.
62       $-4.26326 \times 10^{-14}$ 
63      3.
64      7.
65      -10.
66      6.
67      -6.
68      3.
69      -3.
70      6.
71      -6.
72      27.
73      -27.
74      0.
75      3.
76       $-2.84217 \times 10^{-14}$ 
77      -3.
78      6.
79      -6.
80      18.
81      -15.
82      -3.
83       $-5.68434 \times 10^{-14}$ 
84      21.
85      -21.
86       $-5.68434 \times 10^{-14}$ 
87       $1.42109 \times 10^{-14}$ 
88      9.
89      -9.
90      21.
91      -21.
92      3.
93      -3.
94       $1.42109 \times 10^{-14}$ 
95       $-5.68434 \times 10^{-14}$ 
96      30.
97      -30.
98      3.
99       $-5.68434 \times 10^{-14}$ 
100     9.

```

```

d2[n_, k_] := Sum[d2[j, k - 1] d2[n / j, 1], {j, Divisors[n]}];
d2[n_, 1] := 1; d2[1, 1] := 0; d2[n_, 0] := 0; d2[1, 0] := 1

```

```

Table[{n, d2[n, 4], E2b[n, rr = 4, aa = 1.000000001] - aa E2b[n / aa, rr, aa]}, {n, 1, 100}] //
TableForm

```

```

1      0      0.
2      0       $-1. \times 10^{-9}$ 
3      0       $-1. \times 10^{-9}$ 
4      0       $-1. \times 10^{-9}$ 
5      0       $-1. \times 10^{-9}$ 

```

|    |    |                      |
|----|----|----------------------|
| 6  | 0  | $-1. \times 10^{-9}$ |
| 7  | 0  | $-1. \times 10^{-9}$ |
| 8  | 0  | $-1. \times 10^{-9}$ |
| 9  | 0  | $-1. \times 10^{-9}$ |
| 10 | 0  | $-1. \times 10^{-9}$ |
| 11 | 0  | $-1. \times 10^{-9}$ |
| 12 | 0  | $-1. \times 10^{-9}$ |
| 13 | 0  | $-1. \times 10^{-9}$ |
| 14 | 0  | $-1. \times 10^{-9}$ |
| 15 | 0  | $-1. \times 10^{-9}$ |
| 16 | 1  | 1.                   |
| 17 | 0  | $-1. \times 10^{-9}$ |
| 18 | 0  | $-1. \times 10^{-9}$ |
| 19 | 0  | $-1. \times 10^{-9}$ |
| 20 | 0  | $-1. \times 10^{-9}$ |
| 21 | 0  | $-1. \times 10^{-9}$ |
| 22 | 0  | $-1. \times 10^{-9}$ |
| 23 | 0  | $-1. \times 10^{-9}$ |
| 24 | 4  | 4.                   |
| 25 | 0  | $-1. \times 10^{-9}$ |
| 26 | 0  | $-1. \times 10^{-9}$ |
| 27 | 0  | $-1. \times 10^{-9}$ |
| 28 | 0  | $-1. \times 10^{-9}$ |
| 29 | 0  | $-1. \times 10^{-9}$ |
| 30 | 0  | $-1. \times 10^{-9}$ |
| 31 | 0  | $-1. \times 10^{-9}$ |
| 32 | 4  | 4.                   |
| 33 | 0  | $-1. \times 10^{-9}$ |
| 34 | 0  | $-1. \times 10^{-9}$ |
| 35 | 0  | $-1. \times 10^{-9}$ |
| 36 | 6  | 6.                   |
| 37 | 0  | $-1. \times 10^{-9}$ |
| 38 | 0  | $-1. \times 10^{-9}$ |
| 39 | 0  | $-1. \times 10^{-9}$ |
| 40 | 4  | 4.                   |
| 41 | 0  | $-1. \times 10^{-9}$ |
| 42 | 0  | $-1. \times 10^{-9}$ |
| 43 | 0  | $-1. \times 10^{-9}$ |
| 44 | 0  | $-1. \times 10^{-9}$ |
| 45 | 0  | $-1. \times 10^{-9}$ |
| 46 | 0  | $-1. \times 10^{-9}$ |
| 47 | 0  | $-1. \times 10^{-9}$ |
| 48 | 16 | 16.                  |
| 49 | 0  | $-1. \times 10^{-9}$ |
| 50 | 0  | $-1. \times 10^{-9}$ |
| 51 | 0  | $-1. \times 10^{-9}$ |
| 52 | 0  | $-1. \times 10^{-9}$ |
| 53 | 0  | $-1. \times 10^{-9}$ |
| 54 | 4  | 4.                   |



```

55      0      -1. × 10-9
56      4      4.
57      0      -1. × 10-9
58      0      -1. × 10-9
59      0      -1. × 10-9
60     12     12.
61      0      -1. × 10-9
62      0      -1. × 10-9
63      0      -1. × 10-9
64     10     10.
65      0      -1. × 10-9
66      0      -1. × 10-9
67      0      -1. × 10-9
68      0      -1. × 10-9
69      0      -1. × 10-9
70      0      -1. × 10-9
71      0      -1. × 10-9
72     28     28.
73      0      -1. × 10-9
74      0      -1. × 10-9
75      0      -1. × 10-9
76      0      -1. × 10-9
77      0      -1. × 10-9
78      0      -1. × 10-9
79      0      -1. × 10-9
80     16     16.
81      1      1.
82      0      -1. × 10-9
83      0      -1. × 10-9
84     12     12.
85      0      -1. × 10-9
86      0      -1. × 10-9
87      0      -1. × 10-9
88      4      4.
89      0      -1. × 10-9
90     12     12.
91      0      -1. × 10-9
92      0      -1. × 10-9
93      0      -1. × 10-9
94      0      -1. × 10-9
95      0      -1. × 10-9
96     40     40.
97      0      -1. × 10-9
98      0      -1. × 10-9
99      0      -1. × 10-9
100     6      6.

```

```

M2[30, 1.1]

```

```

-3.

```

```

MM[30]
- 3

$RecursionLimit = 10 000

10 000

Table[(-1)^k ( E2b[30, k, aa = 1.01] - aa E2b[30 / aa, k, aa]),
  {k, 0, Log[1.01, 30]}] // TableForm

$Aborted[]

Table[
  {5 / 1.01^j, D1a[5, 1, 1.01, j, j]}, {j, 0, Log[1.01, 5]}] // TableForm

$RecursionLimit::reclim: Recursion depth of 256 exceeded. >>
$RecursionLimit::reclim: Recursion depth of 256 exceeded. >>
$RecursionLimit::reclim: Recursion depth of 256 exceeded. >>
General::stop: Further output of $RecursionLimit::reclim will be suppressed during this calculation. >>
$Aborted

E2b[10, 2, 1.001]
3.01802


$$8 - 2 \times 1.001 \times 14 + 1.001 \times 1.001 \times 23$$

3.01802

```