```
ff[n_, k_] := (-1)^(k + 1) (1 - Gamma[k + 1, -Log[n]] / Gamma[k + 1])

Expand[Integrate[ Log[x], {x, 1, n}]]
```

$\text{ConditionalExpression}[1 - n + n \log[n], \text{Re}[n] \geq 0 \,||\, n \notin \text{Reals}]$

```
Expand[Integrate[ Log[x] (- (1 - Gamma[1, -Log[ n / x]] / Gamma[1])), {x, 1, n}]]
```

$\text{ConditionalExpression}\left[-1 + n - n \log[n] + \frac{1}{2} n \log[n]^2, \text{Re}[n] \geq 0 \,||\, n \notin \text{Reals}\right]$

```
Expand[Integrate[ Log[x] ((1 - Gamma[2, -Log[ n / x]] / Gamma[2])), {x, 1, n}]]
```

$\text{ConditionalExpression}\left[1 - n + n \log[n] - \frac{1}{2} n \log[n]^2 + \frac{1}{6} n \log[n]^3, \text{Re}[n] \geq 0 \,||\, n \notin \text{Reals}\right]$

```
Expand[Integrate[ Log[x] ((- (1 - Gamma[3, -Log[ n / x]] / Gamma[3]))), {x, 1, n}]]
```

$\text{ConditionalExpression}\Big[$
$\quad -1 + n - n \log[n] + \frac{1}{2} n \log[n]^2 - \frac{1}{6} n \log[n]^3 + \frac{1}{24} n \log[n]^4, \text{Re}[n] \geq 0 \,||\, n \notin \text{Reals}\Big]$

```
Expand[Integrate[ Log[x] ((1 - Gamma[4, -Log[ n / x]] / Gamma[4])), {x, 1, n}]]
```

$\text{ConditionalExpression}\Big[$
$\quad 1 - n + n \log[n] - \frac{1}{2} n \log[n]^2 + \frac{1}{6} n \log[n]^3 - \frac{1}{24} n \log[n]^4 + \frac{1}{120} n \log[n]^5, \text{Re}[n] \geq 0 \,||\, n \notin \text{Reals}\Big]$

```
N[-1 + n - n Log[n] + 1/2 n Log[n]^2 - 1/6 n Log[n]^3 + 1/24 n Log[n]^4] /. n → 10
```

3.84941

```
N[- (1 - Gamma[5, -Log[ n]] / Gamma[5]) /. n → 10]
```

$3.84941 - 2.35708 \times 10^{-15} \, \mathbb{i}$

```
N[{Integrate[ Log[x], {x, 1, n}] /. n → 30, ff[30, 1]}]
```

$\left\{73.0359, 73.0359 - 8.82186 \times 10^{-15} \, \mathbb{i}\right\}$

```
{N[Integrate[ Log[x] (1 - Gamma[2, -Log[ n / x]] / Gamma[2]), {x, 1, n}]] /. n → 56, N[ff[56, 3]]}
```

$\left\{325.486, 325.486 - 1.19214 \times 10^{-13} \, \mathbb{i}\right\}$

```
N[1 - n + n Log[n] - 1/2 n Log[n]^2 + 1/6 n Log[n]^3 /. n → 56]
```

325.486

```
N[ (1 - Gamma[4, -Log[ n]] / Gamma[4]) /. n → 56]
```

$325.486 - 1.19214 \times 10^{-13} \, \mathbb{i}$

```
Gamma[0, -Log[n]] / Gamma[0]
```

0

```
N[LaguerreL[-1, Log[100]]]
```

100.

```
Expand[Integrate[ Log[x], {x, 1, n}, {y, 1, n / x}]]
```

$\text{ConditionalExpression}\left[-1 + n - n \log[n] + \frac{1}{2} n \log[n]^2, \text{Re}[n] \geq 0 \,||\, n \notin \text{Reals}\right]$

```
{Expand[Integrate[ Log[x], {x, 1, n}, {y, 1, n / x}, {z, 1, n / (x y)}]],
 Expand[Integrate[1, {x, 1, n}, {y, 1, n / x}, {z, 1, n / (x y)}, {w, 1, n / (x y z)}]]}
```

$$\left\{\text{ConditionalExpression}\left[1 - n + n \, \text{Log}[n] - \frac{1}{2} \, n \, \text{Log}[n]^2 + \frac{1}{6} \, n \, \text{Log}[n]^3, \, \text{Re}[n] \geq 0 \,||\, n \notin \text{Reals}\right],\right.$$

$$\left.\text{ConditionalExpression}\left[1 - n + n \, \text{Log}[n] - \frac{1}{2} \, n \, \text{Log}[n]^2 + \frac{1}{6} \, n \, \text{Log}[n]^3, \, \text{Re}[n] \geq 0 \,||\, n \notin \text{Reals}\right]\right\}$$

```
N[{Expand[Integrate[ Log[x], {x, 1, n}, {y, 1, n / x}, {z, 1, n / (x y)}]], Expand[
     Integrate[1, {x, 1, n}, {y, 1, n / x}, {z, 1, n / (x y)}, {w, 1, n / (x y z)}]]} /. n → 17]
```

{28.3713, 28.3713}

```
ff3[n_, z_] := Sum[ Binomial[z, k] (-1)^(k) / ((k) !)
    Integrate[ t^(k) E^(-t), {t, -Log[n], 0}], {k, 0, Infinity}]
```

```
N[ff3[ 10, 1]]
```

$Aborted

```
ff5[n_, z_] := Integrate[
  Sum[ Binomial[z, k] (-1)^(k) / ((k) !) t^(k) E^(-t), {k, 1, Infinity}], {t, -Log[n], 0}]
```

```
-ff5[100, -1]
```

99 - Log[100]

```
ff5[n, z]
```

$$\int_{-\text{Log}[n]}^{0} e^{-t} \, (-1 + \text{LaguerreL}[z, t]) \, dt$$

```
f6[n_, z_] := (1 - n) + LaguerreL[-1 - z, 3, Log[n]] / (Pochhammer[3, -1 - z] / (-1 - z) !)
```

```
ff6[n_, z_] :=
  (Integrate[Sum[ Binomial[z, k] (-1)^(k) / ((k) !) t^(k) E^(-t), {k, 1, Infinity}],
      {t, -Log[n], 0}] + (n - 1)) * Pochhammer[ 2, -1 - z] / Gamma[ -z]
```

```
N[ff6[100, -2]]
```

-11.9973

```
N[LaguerreL[ 1, -1, Log[100]]]
```

-4.60517

$$\int_{-\text{Log}[n]}^{0} e^{-t} \, (-1 + \text{LaguerreL}[z, t]) \, dt$$

$$\int_{-\text{Log}[n]}^{0} e^{-t} \, (-1 + \text{LaguerreL}[z, t]) \, dt$$

$$\int_{-\text{Log}[n]}^{0} e^{-t} \, (-1) \, dt$$

1 - n

$$\int_{-\text{Log}[n]}^{0} e^{-t} \, (\text{LaguerreL}[z, t]) \, dt$$

$$\int_{-\text{Log}[n]}^{0} e^{-t} \, \text{LaguerreL}[z, t] \, dt \, /. \, z \to 3$$

$$\frac{1}{6}\, n \, \text{Log}[n] \, \left(6 + 6\, \text{Log}[n] + \text{Log}[n]^2\right)$$