$$[\log\zeta_n(s)]^{*1}=$$
$$\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}\left(\frac{1}{(s-1)^k}\frac{\gamma(k,(s-1)\log n)}{\Gamma(k)}-\int_1^{\infty}\frac{\partial}{\partial y}[y^{s-1}\cdot\zeta_n(s,1+y)]^{*k}dy\right)$$

$$\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}\left(\frac{1}{(s-1)^k}\frac{\gamma(k,(s-1)\log n)}{\Gamma(k)}\right)$$

$$\gamma(k,(s-1)\log n)=\int_0^{(s-1)\log n}t^{k-1}e^{-t}dt$$

$$\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}\left(\frac{1}{(s-1)^k}\frac{\gamma(k,(s-1)\log n)}{\Gamma(k)}\right)=-\Gamma(0,(s-1)\log n)+\Gamma(0,s\log n)-\log((s-1)\log n)+\log(s\log n)$$

$$\sum_{k=1}^{\infty}\binom{1}{k}\left(\frac{1}{(s-1)^k}\frac{\gamma(k,(s-1)\log n)}{\Gamma(k)}\right)=\frac{1-n^{1-s}}{s-1}$$

$$\sum_{k=1}^{\infty}\binom{-1}{k}\left(\frac{1}{(s-1)^k}\frac{\gamma(k,(s-1)\log n)}{\Gamma(k)}\right)=\frac{n^{-s}-1}{s}$$

$$\sum_{k=1}^{\infty}\binom{z}{k}\left(\frac{1}{(s-1)^k}\frac{\gamma(k,(s-1)\log n)}{\Gamma(k)}\right)=\frac{z}{s-1}\cdot\int_0^{(s-1)\log n}e^{-r}{}_1F_1\left(1-z;2;-\frac{r}{s-1}\right)dr$$

$$[\log\zeta_n(s)]^{*1}=$$
$$-\Gamma(0,(s-1)\log n)+\Gamma(0,s\log n)-\log((s-1)\log n)+\log(s\log n)-\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}\left(\int_1^{\infty}\frac{\partial}{\partial y}[y^{s-1}\cdot\zeta_n(s,1+y)]^{*k}dy\right)$$
$$\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}\left(\frac{1}{(0-1)^k}\frac{\gamma(k,(0-1)\log n)}{\Gamma(k)}\right)=-\Gamma(0,(0-1)\log n)+\Gamma(0,0\log n)-\log((0-1)\log n)+\log(0\log n)$$
$$\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}\left(\frac{1}{(0-1)^k}\frac{\gamma(k,-\log n)}{\Gamma(k)}\right)=-\Gamma(0,-\log n)+\Gamma(0,0)-\log(-\log n)+\log(0)$$

$$\Pi(n)=\lim_{z\to 0}\frac{\partial}{\partial z}\sum_{k=1}^{\infty}\binom{z}{k}\lim_{s\to 0}[\zeta_n(s)-1]^{*k}$$

$$\sum_{k=1}^{\lfloor\log_2 n\rfloor}\binom{\rho}{k}\lim_{s\to 0}[\zeta_n(s)-1]^{*k}=0$$

$$\lim_{s\to 0}[\zeta_n(s)]^{*\rho}=0$$

$$\Pi(n)=-\sum_{\rho}\rho^{-1}$$

$$\lim_{s\to 0}[\zeta_n(s)]^{*z}=\prod_{\rho}(1-\frac{z}{\rho})$$

$$\psi(n)=-\lim_{z\to 0}\frac{\partial}{\partial z}\sum_{k=1}^{\infty}\binom{z}{k}\lim_{s\to 0}\frac{\partial}{\partial s}[\zeta_n(s)-1]^{*k}$$

$$-1+\sum_{k=1}^{\lfloor\log_2 n\rfloor}\binom{\rho}{k}\lim_{s\to 0}\frac{\partial}{\partial s}[\zeta_n(s)-1]^{*k}=0$$

$$-1+\lim_{s\to 0}\frac{\partial}{\partial s}[\zeta_n(s)]^{*\rho}=0$$

$$\psi(n)=-\sum_{\rho}\rho^{-1}$$

$$\lim_{s\to 0}\frac{\partial}{\partial s}[\zeta_n(s)]^{*z}=1-\prod_{\rho}(1-\frac{z}{\rho})$$

$$\lim_{s\to 0}\frac{\partial}{\partial s}[\zeta_n(s)-1]^{*0}=0$$

$$\lim_{s\to 0}\frac{\partial}{\partial s}[\zeta_n(s)-1]^{*1}=\sum_{j=2}-\log j$$

$$\lim_{s\to 0}\frac{\partial}{\partial s}[\zeta_n(s)-1]^{*k}=\frac{k}{k-1}\sum_{j=2}\lim_{s\to 0}\frac{\partial}{\partial s}[\zeta_{nj^{-1}}(s)-1]^{*k-1}$$

$$[\zeta_n(s)]^{*z}=$$

$$\binom{z}{0}1+\binom{z}{1}\sum_{j=2}^{\lfloor n\rfloor} j^{-s}+\binom{z}{2}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}(j\cdot k)^{-s}\binom{z}{3}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}\sum_{l=2}^{\lfloor\frac{n}{j\cdot k}\rfloor}(j\cdot k\cdot l)^{-s}+\binom{z}{4}\ldots$$

$$\sum_{k=0}^{\infty}\binom{z}{k}\cdot\frac{1}{(s-1)^k}\cdot\frac{\gamma(k,(s-1)\log n)}{\Gamma(k)}=$$

$$\binom{z}{0}1+\binom{z}{1}\int_1^n x^{-s}dx+\binom{z}{2}\int_1^n\int_1^{\frac{n}{x}}(x\cdot y)^{-s}dy\,dx+\binom{z}{3}\int_1^n\int_1^{\frac{n}{x}}\int_1^{\frac{n}{x\,y}}(x\cdot y\cdot z)^{-s}dz\,dy\,dx+\binom{z}{4}\ldots$$

$$[\log\zeta_n(s)]^{*z}=(-\Gamma(0,(s-1)\log n)+\Gamma(0,s\log n)-\log((s-1)\log n)+\log(s\log n))-\int_1^{\infty}\frac{\partial}{\partial y}[\log(1+y^{s-1}\cdot\zeta_n(s,1+y))]^{*1}dy$$

$$[1+y^{s-1}\cdot\zeta_n(s,1+y)]^{*z}=\sum_{k=0}^{\infty}\binom{z}{k}[y^{s-1}\cdot\zeta_n(s,1+y)]^{*k}$$

$$[\log(1+y^{s-1}\cdot\zeta_n(s,1+y))]^{*1}=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}[y^{s-1}\cdot\zeta_n(s,1+y)]^{*k}$$

$$\begin{aligned}&[1+y^{s-1}\cdot\zeta_n(s,1+y)]^{*k}=\\&\quad[1+y^{s-1}\cdot\zeta_n(s,1+y)]^{*k-1}+\\&\quad y^{s-1}\cdot\sum_{j=1}(j+y)^{-s}[1+y^{s-1}\cdot\zeta_{n\cdot y(j+y)^{-1}}(s,1+y)]^{*k-1}\end{aligned}$$

$$\begin{aligned}&[y^{s-1}\cdot\zeta_n(s,1+y)]^{*k}=\\&\quad y^{-1}\cdot\sum_{j=1}(1+j\,y^{-1})^{-s}[y^{s-1}\cdot\zeta_{n(1+j\,y^{-1})^{-1}}(s,1+y)]^{*k-1}\end{aligned}$$

$$[y^{s-1}\cdot\zeta_n(s,1+y)]^{*1}=y^{-1}\sum_{j=1}\left(1+\frac{j}{y}\right)^{-s}$$

$$[y^{s-1}\cdot\zeta_n(s,1+y)]^{*2}=y^{-1}\sum_{j=1}\sum_{k=1}\left(\left(1+\frac{j}{y}\right)\cdot\left(1+\frac{k}{y}\right)\right)^{-s}$$

$$[y^{s-1}\cdot\zeta_n(s,1+y)]^{*3}=y^{-3}\sum_{j=1}\sum_{k=1}\sum_{l=1}\left(\left(1+\frac{j}{y}\right)\cdot\left(1+\frac{k}{y}\right)\cdot\left(1+\frac{l}{y}\right)\right)^{-s}$$

$$[y^{s-1}\cdot\zeta_n(s,1+y)]^{*4}=y^{-4}\sum_{j=1}\sum_{k=1}\sum_{l=1}\sum_{m=1}\left(\left(1+\frac{j}{y}\right)\cdot\left(1+\frac{k}{y}\right)\cdot\left(1+\frac{l}{y}\right)\cdot\left(1+\frac{m}{y}\right)\right)^{-s}$$

$$\boxed{[(1-x^{1-s})\zeta_n(s)]^{*z}=\sum_{k=0}\binom{z}{k}[(1-x^{1-s})\zeta_n(s)-1]^{*k}}$$

$$[(1-x^{1-s})\zeta_n(s)-1]^{*k}=\sum_{j=1}(j+1)^{-s}[(1-x^{1-s})\zeta_{n(j+1)^{-1}}(s)-1]^{*k-1}-x\cdot(j\,x)^{-s}[(1-x^{1-s})\zeta_{n(j\,x)^{-1}}(s)]^{*k-1}$$

$$[(1-x^{1-s})\zeta_n(s)]^{*k}=\sum_{j=1}j^{-s}[(1-x^{1-s})\zeta_{n\,j^{-1}}(s)]^{*k-1}-x\cdot(j\,x)^{-s}[(1-x^{1-s})\zeta_{n(j\,x)^{-1}}(s)]^{*k-1}$$

$$\boxed{\begin{aligned}\Pi(n)=&\,li(n)-\log\log n-\gamma+\\&\lim_{x\to 1+}\lim_{s\to 0}[\log((1-x^{1-s})\zeta_n(s))]^{*1}+H_{\lfloor\frac{\log n}{\log x}\rfloor}\end{aligned}}$$

$$\boxed{\psi(n)=n-1-\lim_{x\to 1+}\lim_{s\to 0}\frac{\partial}{\partial s}[\log((1-x^{1-s})\zeta_n(s))]^{*1}}$$

$$\lim_{s\to 0}\frac{\partial}{\partial s}(-\Gamma(0,(s-1)\log n)+\Gamma(0,s\log n)-\log((s-1)\log n)+\log(s\log n))=n-\log n-1$$

$$\lim_{s\to 0}\frac{\partial}{\partial s}li(n^{1-s})-\log\log n^{1-s}-\gamma=1-n$$

$$\lim_{s \to 0} \frac{\partial}{\partial s} [\zeta_n(s) - 1]^{*0} = 0$$

$$\lim_{s \to 0} \frac{\partial}{\partial s} [\zeta_n(s) - 1]^{*1} = \sum_{j=2}^{n} -\log j = -\log \prod_{j=2}^{n} j$$

$$\lim_{s \to 0} \frac{\partial}{\partial s} [\zeta_n(s) - 1]^{*k} = \frac{k}{k-1} \sum_{j=2}^{n} \lim_{s \to 0} \frac{\partial}{\partial s} [\zeta_{n j^{-1}}(s) - 1]^{*k-1}$$

$$\boxed{[(1-x^{1-s})\zeta_n(s)]^{*z}=\sum_{j=0}(-1)^j\binom{z}{j}x^{j(1-s)}[\zeta_{n\cdot x^{-j}}(s)]^{*z}}$$

$$\boxed{[\zeta_n(s)]^{*z}=\sum_{j=0}(-1)^j\binom{-z}{j}x^{j(1-s)}[(1-x^{1-s})\zeta_{n\cdot x^{-j}}(s)]^{*z}}$$