$$\sum_{k=0}^{\infty}\frac{(-1)^k}{z-k}\cdot\frac{(-1)^{-k}\gamma(k,-\log x)}{\Gamma(k)}=-\Gamma(1-z)\,\gamma(z,-\log x)(-1)^{-z}+\int_1^x E_z(\log t)\,dt$$

$$-\frac{1}{\Gamma(1-z)\Gamma(z)}(-\int_1^x E_z(\log t)\,dt+\sum_{k=0}^{\infty}\frac{(-1)^k}{z-k}\cdot\frac{(-1)^{-k}\gamma(k,-\log x)}{\Gamma(k)})=(-1)^{-z}\frac{\gamma(z,-\log x)}{\Gamma(z)}$$

$$-\frac{\sin(\pi z)}{\pi}(-\int_1^x E_z(\log t)\,dt+\sum_{k=0}^{\infty}\frac{(-1)^k}{z-k}\cdot\frac{(-1)^{-k}\gamma(k,-\log x)}{\Gamma(k)})=(-1)^{-z}\frac{\gamma(z,-\log x)}{\Gamma(z)}$$

$$\ldots$$

$$(-1)^{-z}\frac{\gamma(z,-\log x)}{\Gamma(z)}=\frac{\sin(\pi z)}{\pi}(\int_1^x E_z(\log t)\,dt+\sum_{k=0}^{\infty}\frac{(-1)^k}{z-k}\cdot\frac{(-1)^{-k}\gamma(k,-\log x)}{\Gamma(k)})$$

which is

$$f(x,z)=\frac{\sin(\pi z)}{\pi}(\int_1^x E_z(\log t)\,dt+\sum_{k=0}^{\infty}\frac{(-1)^k}{z-k}\cdot f(x,k))$$

and

$$\frac{x^z}{z!}=\frac{\sin(\pi z)}{\pi}\cdot(\frac{1}{P(x,z)}\cdot\sum_{k=0}^{\infty}\frac{(-1)^k}{z-k}\cdot\frac{x^k}{k!})$$

and

$$\binom{x}{z}=\frac{\sin(\pi z)}{\pi}\cdot\sum_{k=0}^{\infty}\frac{(-1)^k}{z-k}\cdot\binom{x}{k}$$

and

$$D_z'(x)=\frac{\sin(\pi z)}{\pi}\cdot\sum_{k=0}^{\infty}\frac{(-1)^k}{z-k}\cdot D_k'(x)$$

Oh ho! Here it is:

$$E_{1+z}(x)=x^z\cdot\Gamma(-z)+\sum_{k=0}^{\infty}\frac{(-1)^k}{z-k}\cdot\frac{x^k}{k!}$$

Now! Note!

$$-E_1(-\log x)=li(x)$$

$$E_{1+z}(x)=x^{z}\cdot\Gamma(-z)+\sum_{k=0}^{\infty}\frac{(-1)^{k}}{z-k}\cdot\frac{x^{k}}{k\,!}$$

$$E_{z}(\log t)=\log(t)^{z-1}\cdot\Gamma(1-z)+\sum_{k=0}^{\infty}\frac{(-1)^{k}}{(z-1)-k}\cdot\frac{\log^{k}t}{k\,!}$$

$$\lim_{z\to 0}\frac{\partial}{\partial z}\frac{\sin(\pi z)}{\pi}(\int_{1}^{x}E_{z}(\log t)\,dt)=\log\log x+\gamma$$

$$f(x,z)=(-1)^{-z}\frac{\gamma(z,-\log x)}{\Gamma(z)}=(-1)^{-z}\cdot P(z,-\log x)$$

$$f(x,z)=\frac{\sin(\pi z)}{\pi}(\int_{1}^{x}E_{z}(\log t)\,dt+\sum_{k=0}^{\infty}\frac{(-1)^{k}}{z-k}\cdot f(x,k))$$

$$f(x,z)=\frac{\sin(\pi z)}{\pi}(\int_{1}^{x}\log(t)^{z-1}\cdot\Gamma(1-z)+\sum_{k=0}^{\infty}\frac{(-1)^{k}}{(z-1)-k}\cdot\frac{\log^{k}t}{k\,!}\,dt+\sum_{k=0}^{\infty}\frac{(-1)^{k}}{z-k}\cdot f(x,k))$$

$$f(x,z)=\frac{\sin(\pi z)}{\pi}(\int_{1}^{x}\log(t)^{z-1}\cdot\Gamma(1-z)dt+\int_{1}^{x}\sum_{k=0}^{\infty}\frac{(-1)^{k}}{(z-1)-k}\cdot\frac{\log^{k}t}{k\,!}\,dt+\sum_{k=0}^{\infty}\frac{(-1)^{k}}{z-k}\cdot f(x,k))$$

$$f(x,z)=\frac{\sin(\pi z)}{\pi}(\int_{1}^{x}\log(t)^{z-1}\cdot\Gamma(1-z)dt+\sum_{k=0}^{\infty}\frac{(-1)^{k}}{(z-1)-k}\cdot\int_{1}^{x}\frac{\log^{k}t}{k\,!}\,dt+\sum_{k=0}^{\infty}\frac{(-1)^{k}}{z-k}\cdot f(x,k))$$

$$f(x,z)=\frac{\sin(\pi z)}{\pi}(\int_{1}^{x}\log(t)^{z-1}\cdot\Gamma(1-z)dt+\sum_{k=0}^{\infty}\frac{(-1)^{k}}{(z-1)-k}\cdot f(x,k+1)+\sum_{k=0}^{\infty}\frac{(-1)^{k}}{z-k}\cdot f(x,k))$$

$$f(x,z)=\frac{\sin(\pi z)}{\pi}(\int_{1}^{x}\log(t)^{z-1}\cdot\Gamma(1-z)dt+\sum_{k=0}^{\infty}\frac{(-1)^{k}}{(z-1)-k}\cdot f(x,k+1)+\sum_{k=0}^{\infty}\frac{(-1)^{k}}{z-k}\cdot f(x,k))$$

$$\int_{1}^{x}\log(t)^{z-1}\cdot\Gamma(1-z)\,dt=(-1)^{-z}\cdot P(z,-\log x)\cdot\frac{\pi}{\sin(\pi z)}$$

$$f(x,z)=f(x,z)+\frac{\sin(\pi z)}{\pi}(\sum_{k=0}^{\infty}\frac{(-1)^{k}}{(z-1)-k}\cdot f(x,k+1)+\sum_{k=0}^{\infty}\frac{(-1)^{k}}{z-k}\cdot f(x,k))$$

$$E_{1+z}(x) = x^z \cdot \Gamma(-z) + \sum_{k=0}^{\infty} \frac{(-1)^k}{z-k} \cdot \frac{x^k}{k!}$$

Now! Note!

$$-E_1(-\log x) = li(x)$$

$$E_{1+z}(x) = x^z \cdot \Gamma(-z) + \sum_{k=0}^{\infty} \frac{(-1)^k}{z-k} \cdot \frac{x^k}{k!}$$

$$f(x,z) = \frac{\sin(\pi z)}{\pi} \left(\int_1^x E_z(\log t)\, dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{z-k} \cdot f(x,k) \right)$$

$$D_z'(x) = \frac{\sin(\pi z)}{\pi} \cdot \sum_{k=0}^{\infty} \frac{(-1)^k}{z-k} \cdot D_k'(x)$$

$$f(x,z) - D_z'(x) = \frac{\sin(\pi z)}{\pi} \left(\int_1^x E_z(\log t)\, dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{z-k} \cdot (f(x,k) - D_k'(x)) \right)$$

$$\lim_{z \to 0} \frac{\partial}{\partial z}(f(x,z) - D_z'(x)) = \lim_{z \to 0} \frac{\partial}{\partial z} \left(\frac{\sin(\pi z)}{\pi} \left(\int_1^x E_z(\log t)\, dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{z-k} \cdot ((-1)^{-k} P(k, -\log x) - D_k'(x)) \right) \right)$$

$$li(x) - \Pi(x) = \lim_{z \to 0} \frac{\partial}{\partial z} \left(\frac{\sin(\pi z)}{\pi} \left(\int_1^x E_z(\log t)\, dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{z-k} \cdot ((-1)^{-k} P(k, -\log x) - D_k'(x)) \right) \right)$$

$$g_k(x,t)=d\left(\frac{1}{z-k}-g_{k+1}\left(\frac{x}{t},1+d\right)\right)+g_x(n,t+d)\, if\ j<n,\ 0\, otherwise$$

$$f(x,z,s)=\lim_{d\to s}\left(\frac{1}{z}-g_1(x,1+d)\right)$$

$$li(x)-\Pi(x)=\lim_{z\to 0}\frac{\partial}{\partial z}\left(\frac{\sin(\pi z)}{\pi}\left(\int_1^x E_z(\log t)\,dt+\left(f(x,z,0)-f(x,z,1)\right)\right)\right)$$

...

$$\theta_k(x,t)=d\left(\frac{1}{(z-k)^2}-\theta_{k+1}\left(\frac{x}{t},1+d\right)\right)+\theta_x(n,t+d)\, if\ j<n,\ 0\, otherwise$$

$$h(x,z,s)=\lim_{d\to s}\left(-\frac{1}{z^2}+\theta_1(x,1+d)\right)$$

$$li(x)-\Pi(x)=\lim_{z\to 0}\frac{\partial}{\partial z}\left(\frac{\sin(\pi z)}{\pi}\left(\int_1^x E_z(\log t)\,dt\right)\right)+\cos(\pi z)\left(f(x,z,0)-f(x,z,1)\right)+\frac{\sin(\pi z)}{\pi}\cdot\left(h(x,z,0)-h(x,z,1)\right)$$

$$\lim_{z\to 0}(-1)^k\left(\cos(\pi z)\cdot\frac{f(x)}{z-k}-\frac{\sin(\pi z)}{\pi}\cdot\frac{f(x)}{(z-k)^2}\right)=(-1)^{k+1}\frac{f(x)}{k}\text{ except if k = 0, where it is 0}$$

$$li(x)-\log\log x-\gamma-\Pi(x)=\lim_{z\to 0}\cos(\pi z)\left(f(x,z,0)-f(x,z,1)\right)+\frac{\sin(\pi z)}{\pi}\cdot\left(h(x,z,0)-h(x,z,1)\right)$$