```
s1[s_, a_] := (1 / (1 - (1 + a)^(1 - s))) ( 1 / (1^s) - (1 + a) / ((1 + a)^s) )
s2[s_, a_, c_] := (( 1 / (c^s) - (1 + a) / ((c (1 + a))^s) )) / a
```

```
Expand[s2[s, tt = .0001, 3] / tt]
```

$$10\,000. \times 3^{-s} - 10\,001. \times 3.0003^{-s}$$

```
FullSimplify[(1 / (1 - (1 + a)^(1 - s))) ( 1 / (1^s) - (1 + a) / ((1 + a)^s) )]
```

$$1$$

```
s1[s, a]
```

$$1$$

```
FullSimplify[s2[s, a]]
```

$$2^{-s} \left(1 - (1 + a)^{1-s}\right)$$

```
Limit[s2[s, a, 2], {a → 0}]
```

$$\{2^{-s} (-1 + s)\}$$

```
Expand[(1 / (1 - (1 + a)^(1 - s)))]
```

$$\frac{1}{1 - (1 + a)^{1-s}}$$

```
s3[s_, a_] := (1 - (1 + a)^(1 - s)) Zeta[s] / a
```

```
s3[2, .00001]
```

$$1.64492$$

```
Limit[s3[s, a], {a → 0}]
```

$$\{(-1 + s)\ \text{Zeta}[s]\}$$

```
s4[s_, a_] := (1 - (1 + a)^(1 - s)) / (s - 1) / a Zeta[s]
s5[s_, a_] := ((s - 1) a) / (1 - (1 + a)^(1 - s)) Zeta[s]
```

```
Limit[s4[s, a], {a → 0}]
```

$$\{\text{Zeta}[s]\}$$

```
Limit[s5[s, a], {a → 0}]
```

$$\{\text{Zeta}[s]\}$$

```
FullSimplify[(1 - (1 + a)^(1 - s)) / (s - 1) / a]
```

$$\text{Limit}\left[\frac{1 - (1 + a)^{1-s}}{a\,(-1 + s)}, a \to 0\right]$$

$$1$$

```
f[a_, s_] := (1 - (1 + a)^(1 - s)) / (a (-1 + s))
```

```
Plot[f[a, .999], {a, -15, 5}]
```