

```

f[n_, s_] := n1-s s HarmonicNumber[n, 1 - s] - ns (1 - s) HarmonicNumber[n, s] - s + 1 / 2
f2[n_, s_] := (n1-s s HarmonicNumber[n, 1 - s] + ns (1 - s) HarmonicNumber[n, s]) - 2 n - 1 / 2
f3[n_, s_] := (ns (1 - s) HarmonicNumber[n, s] - n + s / 2 - 1 / 2) / nRe[s]
f3y[n_, s_] := (1 - s) HarmonicNumber[n, s] + (-n + s / 2 - 1 / 2) / ns
f3x[n_, s_] := ns (1 - s) HarmonicNumber[n, s]

```

```

N[f3y[31 000 000 000, s]] - (1 - s) Zeta[s] /. s -> -.5 + 12 I

```

```

-54.5543 - 53.3869 i

```

```

(1 - s) Zeta[s] /. s -> .5 + 1 I

```

```

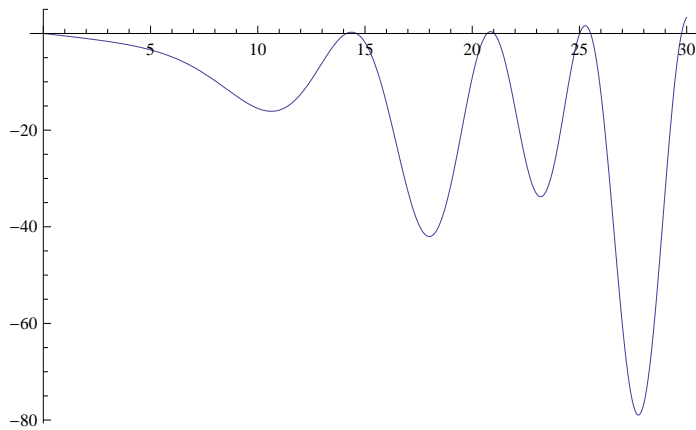
-0.650132 - 0.504986 i

```

```

Plot[Im@(f3y[1 000 000, .5 + x I]), {x, 0, 30}]

```



```

0.5 + 14.1347 i

```

```

FullSimplify[D[f3[n, s], s]]

```

$$\frac{1}{2\sqrt{n}} \left( 1 - 2n^s \left( \text{HarmonicNumber}[n, s] (1 + (-1 + s) \text{Log}[n]) + (-1 + s) \text{HarmonicNumber}^{(0,1)}[n, s] \right) \right)$$

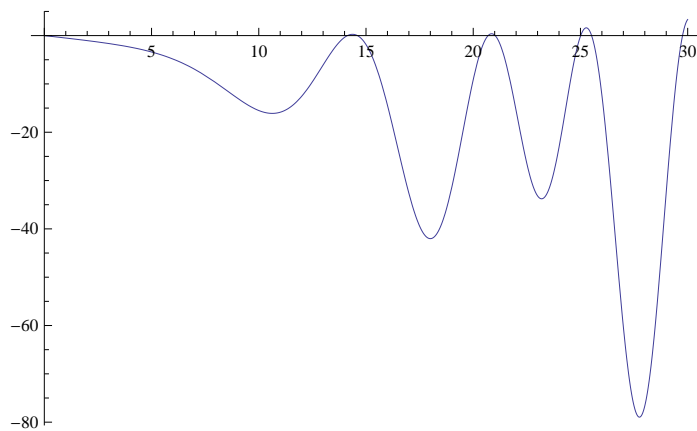
```

FullSimplify[D[Abs[n^s (1 - s) j^-s], s]]

```

$$e^{-\text{Re}[s \text{Log}[j]]} (-n^s (1 + (-1 + s) \text{Log}[n]) \text{Abs}'[-n^s (-1 + s)] - \text{Abs}[n^s (-1 + s)] \text{Log}[j] \text{Re}'[s \text{Log}[j]])$$

```
Plot[Im@((1 - (.5 + x I)) Zeta[.5 + x I]), {x, 0, 30}]
```



```
FullSimplify[(n^s (1 - s) HarmonicNumber[n, s] - n + s / 2 - 1 / 2) / n^(Re[s]) /. s -> 5 + x I]
```

$$\frac{1}{2} n^{-5+\text{Im}[x]} \left( 4 - 2n + i x - 2 i n^{5+i x} (-4 i + x) \text{HarmonicNumber}[n, 5 + i x] \right)$$

```
Expand[(n^s (1 - s) HarmonicNumber[n, s] - n + s / 2 - 1 / 2) / n^s]
```

$$-n^{1-s} - \frac{n^{-s}}{2} + \frac{n^{-s} s}{2} + \text{HarmonicNumber}[n, s] - s \text{HarmonicNumber}[n, s]$$