

```
ClearAll["Global`*"]
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```

lo[n_, k_] := Sum[ (-1)^(j+1) lo[Floor[n/j], k-1], {j, 1, n}];
lo[n_, 1] := Sum[ (-1)^(j+1) Log[j], {j, 1, n}]
t[n_, a_] := Mod[n, a] - Mod[n-1, a]
lp[n_, k_, b_] := Sum[ t[j, b] lp[Floor[n/j], k-1, b], {j, 1, n}];
lp[n_, 1, b_] := Sum[ t[j, b] Log[j], {j, 1, n}]
fa[n_, k_] := Sum[ 2^j Binomial[k, j] (-1)^j l1[n/2^j, k], {j, 0, k}] +
  Sum[ 2^j Binomial[k-1, j-1] (-1)^j Log[2] d1[n/2^j, k], {j, 1, k}]

L1[n_, k_] := Sum[ L1[Floor[n/j], k-1], {j, 1, n}];
L1[n_, 1] := Sum[ Log[j], {j, 1, n}]; L1[n_, 0] := 1
L2[n_, k_] := Sum[ L2[Floor[n/j], k-1], {j, 2, n}]; L2[n_, 1] := Sum[ Log[j], {j, 2, n}]
D1[n_, k_] := Sum[ D1[Floor[n/j], k-1], {j, 1, n}]; D1[n_, 0] := 1
L2toL1[n_, z_] := Sum[ FactorialPower[z-1, a] / a! L2[n, a+1], {a, 0, Log[2, n]}]
L2toL1x[n_, z_] := Sum[ Binomial[z-1, a] L2[n, a+1], {a, 0, Log[2, n]}]
L1toL2[n_, k_] := Sum[ (-1)^(k-j) Binomial[k-1, j-1] L1[n, j], {j, 1, k}]

EL[n_, k_, b_] :=
  EL[n, k, b] = Sum[ EL[n/j, k-1, b], {j, 1, n}] - b Sum[ EL[n/(j b), k-1, b], {j, 1, n}];
EL[n_, 1, b_] := EL[n, 1, b] = Sum[ Log[j], {j, 1, n}] - b Sum[ Log[j b], {j, 1, n/b}]
LtoEL[n_, k_, b_] := Sum[ b^j Binomial[k, j] (-1)^j L1[n/b^j, k], {j, 0, k}] +
  Sum[ b^j Binomial[k-1, j-1] (-1)^j Log[b] D1[n/b^j, k], {j, 1, k}]

EL1toL1[n_, b_] := Sum[ b^j EL[n/b^j, 1, b], {j, 0, Log[b, n]}] +
  Log[b] Sum[ b^j D1[n/b^j, 1], {j, 1, Log[b, n]}]

EL2[n_, k_, b_] :=
  EL2[n, k, b] = Sum[ EL2[n/j, k-1, b], {j, 2, n}] - Sum[ EL2[n/(j b), k-1, b], {j, 1, n}];
EL2[n_, 1, b_] := EL2[n, 1, b] = Sum[ Log[j], {j, 2, n}] - Sum[ Log[j b], {j, 1, n/b}]
EL2toEL1[n_, z_, b_] :=
  Sum[ FactorialPower[z-1, a] / a! EL2[n, a+1, b], {a, 0, Log[If[b < 2, b, 2], n]}]
EL1toEL2[n_, k_, b_] := Sum[ (-1)^(k-j) Binomial[k-1, j-1] EL[n, j, b], {j, 1, k}]

```

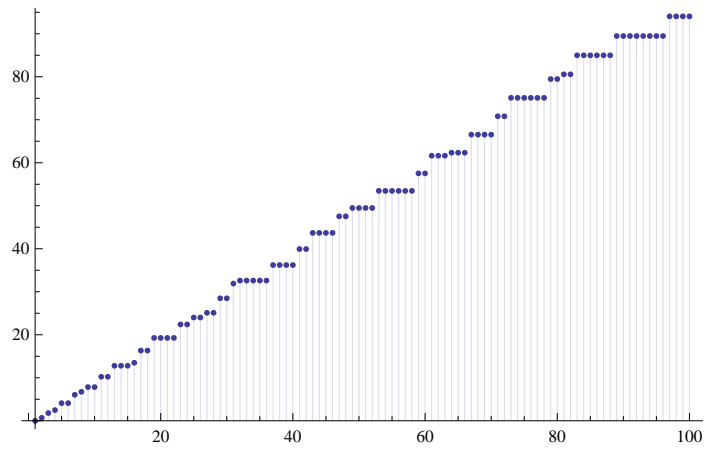
```
N[L2toL1x[100, 0]]
```

```
94.0453
```

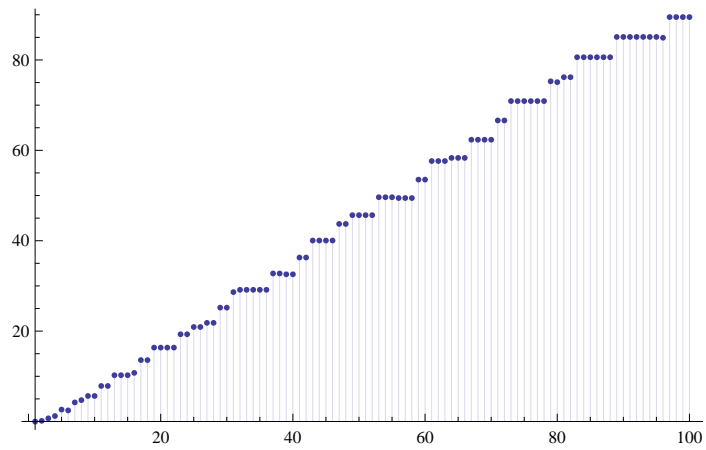
```
N[EL2toEL1[100, 0, 101]]
```

```
94.0453
```

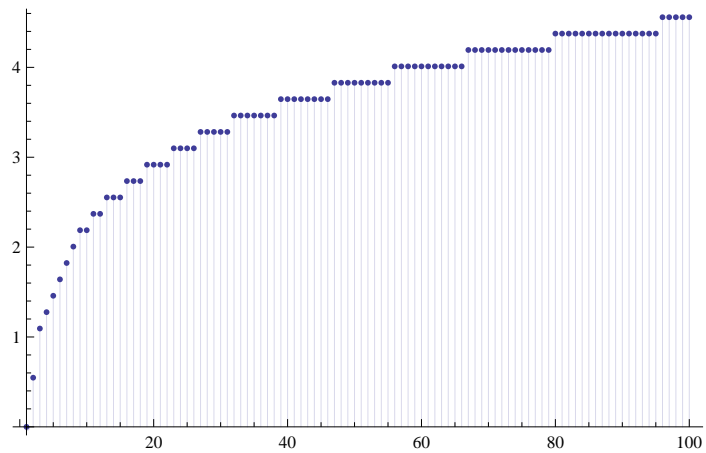
`DiscretePlot[L2toL1[n, 0], {n, 1, 100}]`



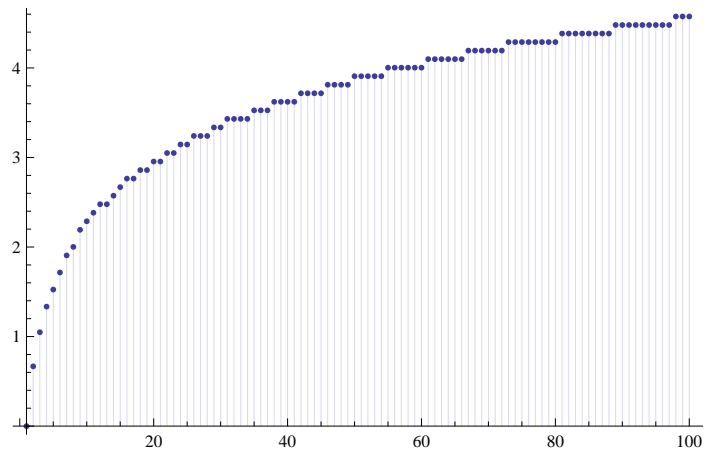
`DiscretePlot[EL2toEL1[n, 0, 1.2], {n, 1, 100}]`



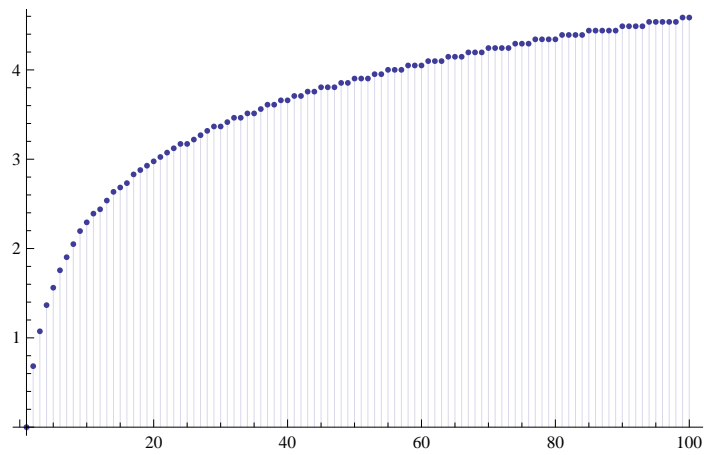
`DiscretePlot[L2toL1[n, 0] - EL2toEL1[n, 0, 1.2], {n, 1, 100}]`



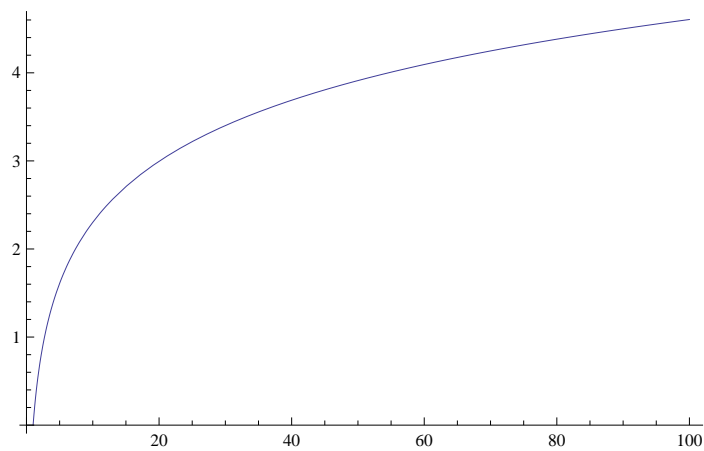
```
DiscretePlot[L2toL1[n, 0] - EL2toEL1[n, 0, 1.1], {n, 1, 100}]
```



```
DiscretePlot[L2toL1[n, 0] - EL2toEL1[n, 0, 1.05], {n, 1, 100}]
```



```
Plot[Log[n], {n, 1, 100}]
```



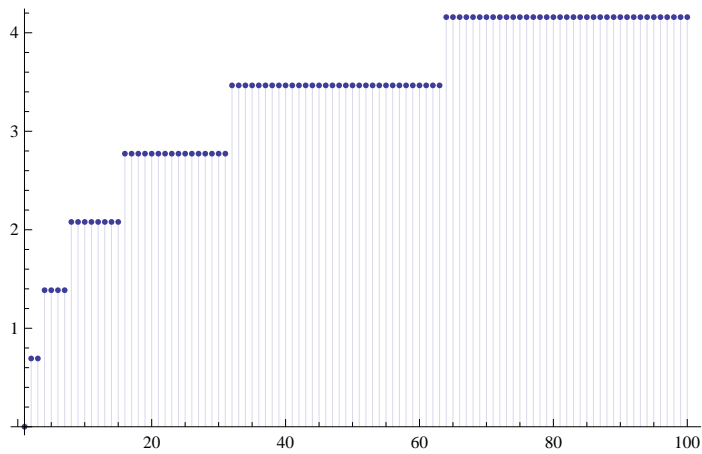
```
N[Log[2 Pi]]
```

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1.83788
```

```

fdif[n_, b_] := Sum[Log[b], {j, 1, Log[b, n]}]
DiscretePlot[L2toL1[n, 0] - EL2toEL1[n, 0, 2], {n, 1, 100}]

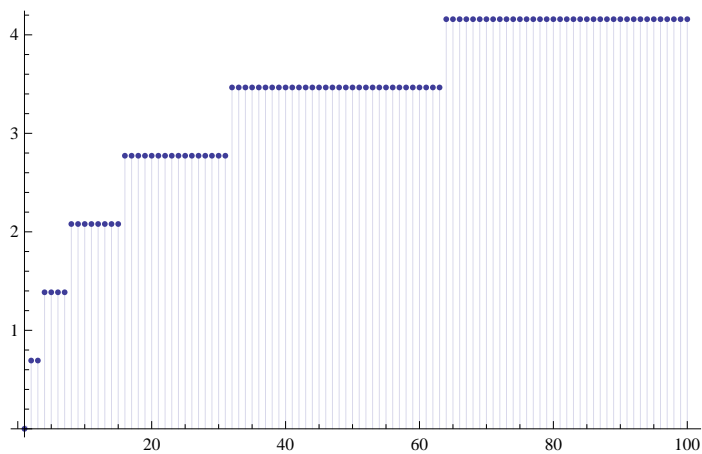
```



```

DiscretePlot[fdif[n, 2], {n, 1, 100}]

```



```

N[Table[{n, L2toL1[n, 0] - EL2toEL1[n, 0, 2]}, {n, 1, 100}]] // TableForm

```

1.	0.
2.	0.693147
3.	0.693147
4.	1.38629
5.	1.38629
6.	1.38629
7.	1.38629
8.	2.07944
9.	2.07944
10.	2.07944
11.	2.07944
12.	2.07944
13.	2.07944
14.	2.07944
15.	2.07944
16.	2.77259
17.	2.77259
18.	2.77259
19.	2.77259

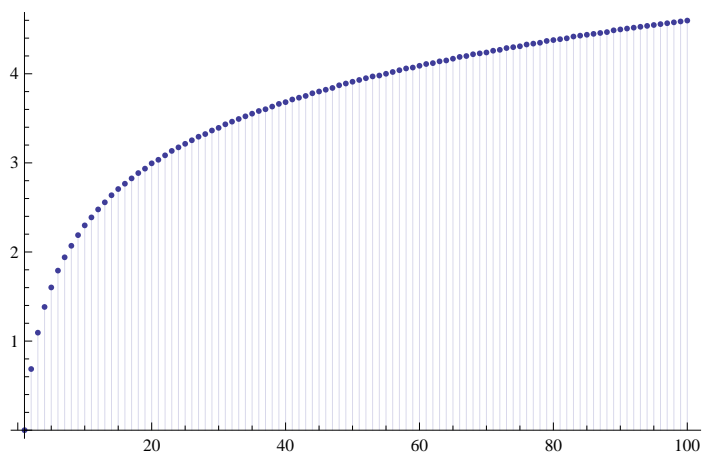
20.	2.77259
21.	2.77259
22.	2.77259
23.	2.77259
24.	2.77259
25.	2.77259
26.	2.77259
27.	2.77259
28.	2.77259
29.	2.77259
30.	2.77259
31.	2.77259
32.	3.46574
33.	3.46574
34.	3.46574
35.	3.46574
36.	3.46574
37.	3.46574
38.	3.46574
39.	3.46574
40.	3.46574
41.	3.46574
42.	3.46574
43.	3.46574
44.	3.46574
45.	3.46574
46.	3.46574
47.	3.46574
48.	3.46574
49.	3.46574
50.	3.46574
51.	3.46574
52.	3.46574
53.	3.46574
54.	3.46574
55.	3.46574
56.	3.46574
57.	3.46574
58.	3.46574
59.	3.46574
60.	3.46574
61.	3.46574
62.	3.46574
63.	3.46574
64.	4.15888
65.	4.15888
66.	4.15888
67.	4.15888
68.	4.15888
69.	4.15888
70.	4.15888
71.	4.15888
72.	4.15888
73.	4.15888
74.	4.15888
75.	4.15888

```

76.      4.15888
77.      4.15888
78.      4.15888
79.      4.15888
80.      4.15888
81.      4.15888
82.      4.15888
83.      4.15888
84.      4.15888
85.      4.15888
86.      4.15888
87.      4.15888
88.      4.15888
89.      4.15888
90.      4.15888
91.      4.15888
92.      4.15888
93.      4.15888
94.      4.15888
95.      4.15888
96.      4.15888
97.      4.15888
98.      4.15888
99.      4.15888
100.     4.15888

```

```
DiscretePlot[fdif[n, 1.01], {n, 1, 100}]
```



```
Limit[Sum[Log[b], {j, 1, Log[b, n]}], b -> 1]
```

```
Log[n]
```