```
1. - 2.28116 \times 10^{-16} i
ztmm[l_{-}] := Product[(1+1/ZetaZero[r]) (1+1/ZetaZero[-r]), \{r, 1, 1\}]
N[Pi/3]
N[ztmm[500]]
1.0472
1.04479 + 3.88578 \times 10^{-16} i
N[3 Zeta[3] / Pi]
zt3m22[1_] := Product[(1+2/ZetaZero[r])(1+2/ZetaZero[-r]), \{r, 1, 1\}]
N[zt3m22[2200]]
1.14788
1.14504 + 1.66533 \times 10^{-16} i
zt4[1_] := Product[(1-1/ZetaZero[r]) (1-1/ZetaZero[-r]), {r, 1, 1}]
N[zt4[800]]
1. - 3.33067 \times 10^{-16} i
zt322b[1_] := Product[(1-2/ZetaZero[r])(1-2/ZetaZero[-r]), \{r, 1, 1\}]
N[zt322b[700]]
1.04528 + 2.22045 \times 10^{-16} i
1.0472
zt33[1_] := Product[(1-3/ZetaZero[r]) (1-3/ZetaZero[-r]), {r, 1, 1}]
N[zt33[2200]]
N[3 Zeta[3] / Pi]
N[2^{(1/2)} Pi^{(3/2)} / 7]
1.14504 + 3.33067 \times 10^{-16} i
1.14788
1.12497
```

```
zt42[1_] := Product[(1-4/ZetaZero[r])(1-4/ZetaZero[-r]), {r, 1, 1}]
N\left[\frac{2\pi^2}{15}\right]
N[zt42[1200]]
1.31595
1.30597 - 7.77156 \times 10^{-16} i
zt35a[1_] := Product[(1-5/ZetaZero[r]) (1-5/ZetaZero[-r]), {r, 1, 1}]
N[zt35a[700]]
N[2^(3/2) Pi^(5/2)/31]
1.57594
1.54728 - 5.55112 \times 10^{-16} i
1.59609
N\left[\frac{4 \pi^3}{63}\right]
zt36b[1_] := Product[(1-6/ZetaZero[r])(1-6/ZetaZero[-r]), \{r, 1, 1\}]
N[zt36b[1100]]
1.96865
1.92926 + 4.44089 \times 10^{-16} \text{ i}
N\left[\frac{315\,\text{Zeta}[7]}{4\,\pi^3}\right]
zt37b[1_] := Product[(1-7/ZetaZero[r])(1-7/ZetaZero[-r]), \{r, 1, 1\}]
N[zt37b[1200]]
N[2^{(5/2)}Pi^{(7/2)}/127]
2.56101
2.4937 + 6.10623 \times 10^{-16} i
2.44791
2.5610139009137747
```

```
N\left[\frac{8 \pi^4}{225}\right]
zt38b[1_] := Product[(1-8/ZetaZero[r]) (1-8/ZetaZero[-r]), {r, 1, 1}]
N[zt38b[1200]]
N\Big[\frac{945\,{\tt Zeta}\,[9]}{2\,\pi^4}\,\Big]
zt399b[1\_] := Product[(1-9 / ZetaZero[r]) (1-9 / ZetaZero[-r]), \{r, 1, 1\}]
N[zt399b[900]]
4.86042
4.5971 - 3.55271 \times 10^{-15} i
N\left[\frac{16 \pi^5}{693}\right]
zt310b[1_{-}] := Product[(1-10/ZetaZero[r])(1-10/ZetaZero[-r]), \{r, 1, 1\}]
N[zt310b[2200]]
7.06539
6.80749 + 8.88178 \times 10^{-16} i
zzz[1_{-}] := Product[(1-(1/2)/ZetaZero[r])(1-(1/2)/ZetaZero[-r]), \{r, 1, 1\}]
N[zzz[3611]]
0.994313 + 4.16334 \times 10^{-17} i
zt4a2[1_] := Product[(-1+1/ZetaZero[r]) (1-1/ZetaZero[-r]), \{r, 1, 1\}]
N[zt4a2[800]]
1. - 3.33067 \times 10^{-16} i
b1[n_{\_}] := (-1) \ ^(n+1) \ BernoulliB[2\,n] \ (2\,Pi) \ ^(2\,n) \ / \ (2\ ((2\,n)\ !))
b1[2]
\pi^4
bla[n_{-}] := (-1) \cdot (Floor[n/2] + 1) BernoulliB[n] (2 Pi) \cdot (n) / (2 ((n)!))
b1a[3]
blb[n_] := 2 ((-1) ^ (Floor[n / 2] + 1) BernoulliB[n] (2 Pi) ^ (n) / (2 ((n)!)))
   (1/2 Pi^{(-n/2)} n (n-1) Gamma[n/2])
```

```
b1b[6]
 4~\pi^3
blc[n_{-}] := 2((-1)^{(Floor[n/2]+1)} BernB[n](2Pi)^{(n)/(2((n)!))}
     (1 / 2 Pi^ (-n / 2) n (n - 1) Gamma[n / 2])
b1c[6]
8 - π<sup>3</sup> BernB[6]
 2 ((-1) ^ (Floor[n / 2] + 1) BernB[n] (2 Pi) ^ (n) / (2 ((n)!)))
  (1/2Pi^{(-n/2)}n(n-1)Gamma[n/2])
\text{FullSimplify} \Big[ \text{Expand} \Big[ \frac{ \left( -1 \right)^{1+\text{Floor} \left[ \frac{n}{2} \right]} \, 2^{-1+n} \, \left( -1+n \right) \, n \, \pi^{n/2} \, \text{BernB} [n] \, \, \text{Gamma} \left[ \frac{n}{2} \right]}{n \, !} \, \Big] \Big]
ea[n_{-}] := -\frac{2 (-1)^{Floor\left[\frac{n}{2}\right]} \pi^{\frac{1+n}{2}} BernB[n]}{Gamma\left[\frac{1}{2} (-1+n)\right]}
ea[4]
-4\pi^2 BernB[4]
\operatorname{Gamma}\left[\frac{1}{2} \left(-1+4\right)\right]
Gamma[(8-1)/2]
 15 \sqrt{\pi}
coes[s_] := Pi^(s/2) / (2(s-1) Gamma[1+s/2])
coes[9]
 2 \pi^4
 945
```

```
N[Pi/3]
N[ztmm[500]]
1.04479 + 3.88578 \times 10^{-16} i
ztmm[1_] :=
  Product[(1+1/(1/2+Im[ZetaZero[r]]I))(1+1/(1/2-Im[ZetaZero[r]]I)), \{r,1,1\}] \\
N[Pi/3]
N[ztmm[100]]
(1+1/ZetaZero[r]) (1+1/ZetaZero[-r])
1.0472
1.0407 - 4.50358 \times 10^{-18} i
\label{eq:fullSimplify} FullSimplify [Expand[(1+1/(1/2+Im[ZetaZero[r]]I)) (1+1/(1/2-Im[ZetaZero[r]]I))]]
\mathtt{Expand}\Big[1+\frac{8}{1+4\,\mathtt{Im}[\mathtt{ZetaZero}[r]\,]^2}\Big]
9 + 4 Im[ZetaZero[r]]<sup>2</sup>
1 + 4 Im[ZetaZero[r]]<sup>2</sup>
N[Pi/3]
N[ztmm[300]]
1.0472
1.04385
```

```
ztmmc[1_] := Product[(1+1/ZetaZero[r])^2 (1+1/ZetaZero[-r])^2, \{r, 1, 1\}]
N[Pi^2/9]
N[ztmmc[800]]
1.09662
1.09295 - 4.996 \times 10^{-16} i
FullSimplify[Expand[(1+1/ZetaZero[r])^2(1+1/ZetaZero[-r])^2]]
 (1 + ZetaZero[-r])^2 (1 + ZetaZero[r])^2
        ZetaZero[-r]<sup>2</sup> ZetaZero[r]<sup>2</sup>
N[Pi^2/9]
N[ztmmc[200]]
1.09662
1.08762 - 3.33067 \times 10^{-16} i
FullSimplify[Expand[(1+1/(a+bI))^2(1+1/(a-bI))^2]]
\frac{\left( (1+a)^2 + b^2 \right)^2}{\left( a^2 + b^2 \right)^2}
FullSimplify[Expand[(1+1/(a+bI))(1+1/(a-bI))]]
1 + \frac{1 + 2 a}{a^2 + b^2}
\frac{\left((1+a)^2+b^2\right)^2}{\left(a^2+b^2\right)^2} /. a \to (1/2)
FullSimplify \left[ \text{Expand} \left[ \frac{\left( \frac{9}{4} + b^2 \right)^2}{\left( \frac{1}{4} + b^2 \right)^2} \right] \right]
ztmmd[l_] := Product \left[\frac{\left(9+4 \text{ Im}\left[\text{ZetaZero}[r]\right]^2\right)^2}{\left(1+4 \text{ Im}\left[\text{ZetaZero}[r]\right]^2\right)^2}, \{r, 1, 1\}\right]
N[Pi^2/9]
N[ztmmd[120]]
```

```
1.08441
N[(9 + 4 Im[ZetaZero[r]])^2 / (1 + 4 Im[ZetaZero[r]])^2 / r \rightarrow 5]
1.12417
N\left[\frac{\left(1+ZetaZero[-r]\right)^{2}\left(1+ZetaZero[r]\right)^{2}}{ZetaZero[-r]^{2}ZetaZero[r]^{2}} /.r \rightarrow 1\right]
1.0201 + 0.i
N\left[\frac{\left(1+ZetaZero[-r]\right)^{2}\left(1+ZetaZero[r]\right)^{2}}{\left(1/2-Im[ZetaZero[r]]I\right)^{2}ZetaZero[r]^{2}}/.r\rightarrow 1\right]
1.0201 + 0. i
N[ZetaZero[-1]2]
 -199.54 - 14.1347 i
N[Im[ZetaZero[-1]]]
-14.1347
N[(1/2 - Im[ZetaZero[1]]I)^2]
 -199.54 - 14.1347 i
 \frac{(1 + (1/2 - Im[ZetaZero[r]]I))^{2} (1 + (1/2 + Im[ZetaZero[r]]I))^{2}}{(1/2 - Im[ZetaZero[r]]I)^{2} (1/2 + Im[ZetaZero[r]]I)^{2}}
Full Simplify \Big[ Expand \Big[ \frac{\left(\frac{3}{2} - i Im[ZetaZero[r]]\right)^2 \left(\frac{3}{2} + i Im[ZetaZero[r]]\right)^2}{\left(\frac{1}{2} - i Im[ZetaZero[r]]\right)^2 \left(\frac{1}{2} + i Im[ZetaZero[r]]\right)^2} \Big] \Big]
ztmmde[1_] := Product \left[\frac{\left(9 + 4 \text{ Im}[ZetaZero[r]]^2\right)^{3/2}}{\left(1 + 4 \text{ Im}[ZetaZero[r]]^2\right)^{3/2}}, \{r, 1, 1\}\right]
N[Pi^(3/2)/(3^(3/2))]
N[ztmmde[240]]
1.07163
```

1.09662

1.06572

```
ztmmd[1_] := Product \left[\frac{(9+4 \text{ Im}[ZetaZero[r]]^2)^2}{(1+4 \text{ Im}[ZetaZero[r]]^2)^2}, \{r, 1, 1\}\right]
N[Pi^2/9]
N[ztmmd[120]]
1.09662
1.08441
\label{eq:fullSimplify} FullSimplify [Expand[(1-4/(1/2+Im[ZetaZero[r]]I)) (1-4/(1/2-Im[ZetaZero[r]]I))]]
 49 + 4 Im[ZetaZero[r]]<sup>2</sup>
 1 + 4 Im[ZetaZero[r]]<sup>2</sup>
zt42d[1_] := Product[(1-4/ZetaZero[r])(1-4/ZetaZero[-r]), \{r, 1, 1\}]
N\left[\frac{2 \pi^2}{15}\right]
N[zt42d[200]]
zt42d[1_] := Product \left[\frac{49 + 4 \text{ Im}[ZetaZero[r]]^2}{1 + 4 \text{ Im}[ZetaZero[r]]^2}, \{r, 1, 1\}\right]
N\left[\frac{2 \pi^2}{15}\right]
N[zt42d[400]]
1.31595
1.29512
N\left[\frac{4\pi^3}{63}\right]
zt36b[1_{-}] := Product \left[ \frac{121 + 4 Im[ZetaZero[r]]^{2}}{1 + 4 Im[ZetaZero[r]]^{2}}, \{r, 1, 1\} \right]
N[zt36b[300]]
1.96865
1.87642
Full Simplify [Expand[(1-6/(1/2+Im[ZetaZero[r]]I)) (1-6/(1/2-Im[ZetaZero[r]]I))]] \\
```

$$\begin{split} & N \Big[\frac{8 \, \pi^4}{225} \Big] \\ & zt38b[1_{-}] := Product \Big[\frac{225 + 4 \, Im[ZetaZero[r]]^2}{1 + 4 \, Im[ZetaZero[r]]^2} \,, \, \{r, 1, 1\} \Big] \\ & N[zt38b[900]] \\ & 3.46343 \\ & 3.3166 \\ & FullSimplify[Expand[(1 - 8 / (1 / 2 + Im[ZetaZero[r]] \, I)) \, (1 - 8 / (1 / 2 - Im[ZetaZero[r]] \, I))]] \\ & \frac{225 + 4 \, Im[ZetaZero[r]]^2}{1 + 4 \, Im[ZetaZero[r]]^2} \end{split}$$

zt38b[l_] := Product
$$\left[\frac{4 \text{ Im}[ZetaZero[r]]^2}{1+4 \text{ Im}[ZetaZero[r]]^2}, \{r, 1, 1\}\right]$$
 N[zt38b[1200]]

0.994399209202834

 $\label{eq:fullSimplify} FullSimplify [Expand[(1-3/(1/2+Im[ZetaZero[r]]I)) (1-3/(1/2-Im[ZetaZero[r]]I))]]$

$$1 + \frac{24}{1 + 4 \operatorname{Im}[\operatorname{ZetaZero}[r]]^2}$$

$$\begin{split} & \mathbf{Expand} \Big[\frac{\left(9 + 4 \, \mathbf{Im} [\mathbf{ZetaZero}[\mathbf{r}]]^2\right)^2}{\left(1 + 4 \, \mathbf{Im} [\mathbf{ZetaZero}[\mathbf{r}]]^2\right)^2} \Big] \\ & \frac{81}{\left(1 + 4 \, \mathbf{Im} [\mathbf{ZetaZero}[\mathbf{r}]]^2\right)^2} + \frac{72 \, \mathbf{Im} [\mathbf{ZetaZero}[\mathbf{r}]]^2}{\left(1 + 4 \, \mathbf{Im} [\mathbf{ZetaZero}[\mathbf{r}]]^2\right)^2} + \frac{16 \, \mathbf{Im} [\mathbf{ZetaZero}[\mathbf{r}]]^4}{\left(1 + 4 \, \mathbf{Im} [\mathbf{ZetaZero}[\mathbf{r}]]^2\right)^2} \end{split}$$

```
Expand [(1 + 4 Im[ZetaZero[r]]^2)^2]
1 + 8 Im[ZetaZero[r]]<sup>2</sup> + 16 Im[ZetaZero[r]]<sup>4</sup>
  (9 + 4 \text{ Im}[ZetaZero[r]]^2)^2
 (1 + 4 Im[ZetaZero[r]]^2)^2
 \left(\frac{49 + 4 \operatorname{Im}[\operatorname{ZetaZero}[r]]^{2}}{1 + 4 \operatorname{Im}[\operatorname{ZetaZero}[r]]^{2}}\right) / \left(\frac{9 + 4 \operatorname{Im}[\operatorname{ZetaZero}[r]]^{2}}{1 + 4 \operatorname{Im}[\operatorname{ZetaZero}[r]]^{2}}\right)
  49 + 4 Im[ZetaZero[r]]<sup>2</sup>
   9 + 4 Im[ZetaZero[r]]<sup>2</sup>
\frac{4 \pi^3}{63} \bigg/ \frac{\pi^2}{9}
\frac{121+4\;\text{Im}[\text{ZetaZero}[\text{r}]]^2}{1+4\;\text{Im}[\text{ZetaZero}[\text{r}]]^2}\bigg/\,\frac{\left(9+4\;\text{Im}[\text{ZetaZero}[\text{r}]]^2\right)^2}{\left(1+4\;\text{Im}[\text{ZetaZero}[\text{r}]]^2\right)^2}
FullSimplify \Big[ \underbrace{ \underbrace{ \Big( 1 + 4 \, \text{Im} [\text{ZetaZero}[\textbf{r}]]^2 \Big) \, \Big( 121 + 4 \, \text{Im} [\text{ZetaZero}[\textbf{r}]]^2 \Big) }_{\Big( 9 + 4 \, \text{Im} [\text{ZetaZero}[\textbf{r}]]^2 \Big)^2} \Big] \Big] \\
 \frac{\left(1+4 \operatorname{Im}[\operatorname{ZetaZero}[r]]^{2}\right) \left(121+4 \operatorname{Im}[\operatorname{ZetaZero}[r]]^{2}\right)}{\left(9+4 \operatorname{Im}[\operatorname{ZetaZero}[r]]^{2}\right)^{2}}
\frac{8 \pi^4}{225} / \frac{4 \pi^3}{63}
 \frac{225 + 4 \operatorname{Im}[\operatorname{ZetaZero}[r]]^{2}}{1 + 4 \operatorname{Im}[\operatorname{ZetaZero}[r]]^{2}} / \frac{121 + 4 \operatorname{Im}[\operatorname{ZetaZero}[r]]^{2}}{1 + 4 \operatorname{Im}[\operatorname{ZetaZero}[r]]^{2}}
 225 + 4 Im[ZetaZero[r]]^2
 121 + 4 Im[ZetaZero[r]]<sup>2</sup>
\frac{49 + 4 \, \text{Im}[\text{ZetaZero}[r]]^2}{9 + 4 \, \text{Im}[\text{ZetaZero}[r]]^2} / \frac{225 + 4 \, \text{Im}[\text{ZetaZero}[r]]^2}{121 + 4 \, \text{Im}[\text{ZetaZero}[r]]^2}
```

 $(49 + 4 Im[ZetaZero[r]]^2)^2$ $(9 + 4 \text{Im}[ZetaZero[r]]^2) (121 + 4 \text{Im}[ZetaZero[r]]^2)$