```
Clear[rr, paa, pab, psum]
rr[n_, s_] := rr[n, s] = If[ n == 1, 0, FullSimplify[ MangoldtLambda[n] / Log[n]] n^-s]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!

paa[n_, s_, 0, a_] := UnitStep[n - 1]
paa[n_, s_, 1, a_] := paa[n, s, 1, a] = Sum[ rr[j, s], {j, a + 1, n}]
paa[n_, s_, k_, a_] :=
 paa[n, s, k, a] = Sum[If[rr[m, 0] == 0, 0, Sum[Binomial[k, j] (rr[m, s])^j
      paa[Floor[n / (m^j)], s, k - j, m], {j, 1, k}]], {m, a + 1, Floor[n^(1 / k)]}]
dz1[n_, s_, z_] := Sum[ z^k / (k!) paa[n, s, k, 1], {k, 0, Log[2, n]}]
rs[ n_, s_] := rs[n, s] = Sum[ rr[j, s], {j, 1, n}]
pabx[n_, s_, 1, a_] := rs[n, s] - rs[a, s]
pab[n_, s_, 0, a_] := UnitStep[n - 1]
psum[ n_, s_] :=
 psum[n, s] = Sum[ t^-1 (Prime[j]^t)^-s, {t, 1, Log[2, n]}, {j, 1, PrimePi[ n^(1 / t)]}]
pab[n_, s_, 1, a_] := psum[n, s] - psum[a, s]
pab[n_, s_, k_, a_] := pab[n, s, k, a] = Sum[Binomial[k, j] ((t^-1) (Prime[m]^t)^-s)^j
    pab[Floor[n / ((Prime[m]^t)^j)], s, k - j, Prime[m]^t], {t, 1, Log[2, Floor[n^(1 / k)]]},
   {j, 1, k}, {m, PrimePi[a^(1 / t)] + 1, PrimePi[Floor[n^(1 / k)]^(1 / t)]}]
dz2[n_, s_, z_] := Sum[ z^k / (k!) pab[n, s, k, 1], {k, 0, Log[2, n]}]
ez2[n_, s_, z_] :=
 Expand[Sum[ (-1)^j bin[z, j] (2)^(j (1 - s)) dz2[ n / (2^j), s, z], {j, 0, Log[2, n]}]]
dzeros[n_, s_] := List @@ NRoots[dz2[n, s, z] == 0, z][[All, 2]]
ezeros[n_, s_] := List @@ NRoots[ez2[n, s, z] == 0, z][[All, 2]]



Timing[paa[1 000 000, -1, 3, 1]]

{13.962, 23 730 945 231 458 376 567 503 / 25 729 704 000}

Timing[pab[1 000 000, -1, 3, 0]]

{1.794, 23 730 945 231 458 376 567 503 / 25 729 704 000}

Sum[ ra[j] ra[k] ra[l], {j, 2, 100}, {k, 2, 100 / j}, {l, 2, 100 / (j k)}]

77

PrimePi[0]

0

Timing[psum2[1 000 000., .2 + .2 I, 1, 5]]

{0.218, -4849.43 - 3717.59 i}

Timing[pab[1 000 000., .2 + .2 I, 1, 5]]

$Aborted


Timing[dz1[10 000 000, N[ZetaZero[1]], z]]

$Aborted
```

```
Timing[dz2[10 000 000, N[ZetaZero[1]], z]]
```

$$\{42.37,\ 1 + (10.241 - 1.07236\,i)\,z + (51.463 - 4.84059\,i)\,z^2 + (60.1566 - 5.19101\,i)\,z^3 + (56.9535 - 5.99884\,i)\,z^4 + (27.6503 - 2.3643\,i)\,z^5 + (11.4284 - 1.33599\,i)\,z^6 + (2.92756 - 0.251875\,i)\,z^7 + (0.650212 - 0.0862847\,i)\,z^8 + (0.0970048 - 0.00831852\,i)\,z^9 + (0.0125315 - 0.00196848\,i)\,z^{10} + (0.00114164 - 0.0000916494\,i)\,z^{11} + (0.0000872086 - 0.0000171298\,i)\,z^{12} + (4.98819 \times 10^{-6} - 4.52685 \times 10^{-7}\,i)\,z^{13} + (2.14592 \times 10^{-7} - 5.67684 \times 10^{-8}\,i)\,z^{14} + (8.13245 \times 10^{-9} - 5.33729 \times 10^{-10}\,i)\,z^{15} + (1.67979 \times 10^{-10} - 5.71371 \times 10^{-11}\,i)\,z^{16} + (4.49993 \times 10^{-12} - 1.16398 \times 10^{-12}\,i)\,z^{17} + (2.8879 \times 10^{-14} - 1.83615 \times 10^{-14}\,i)\,z^{18} + (5.94298 \times 10^{-16} - 5.5431 \times 10^{-16}\,i)\,z^{19} + (3.24876 \times 10^{-18} + 2.6174 \times 10^{-18}\,i)\,z^{20} - (2.8419 \times 10^{-21} + 7.65419 \times 10^{-21}\,i)\,z^{21} + (4.97408 \times 10^{-23} - 5.40176 \times 10^{-23}\,i)\,z^{22} + (8.77986 \times 10^{-27} + 1.0064 \times 10^{-26}\,i)\,z^{23}\}$$

```
dzeros[n_, s_] := List @@ NRoots[dz2[n, s, z] == 0, z][[All, 2]]
```

```
dzeros[10 000 000, N[ZetaZero[1]]]
```

$$\{-218.628 + 147.425\,i,\ -34.8815 + 45.2531\,i,\ -21.5808 + 27.9845\,i,\ -8.80516 + 12.7127\,i,\ -2.80399 + 6.39738\,i,\ -2.77014 - 12.5732\,i,\ -2.34127 - 35.1931\,i,\ -1.65364 + 10.1088\,i,\ -1.56303 + 4.34153\,i,\ -1.418 - 7.23512\,i,\ -1.21757 - 4.5893\,i,\ -1.01983 + 2.49085\,i,\ -0.830131 - 2.60002\,i,\ -0.641384 - 16.7226\,i,\ -0.576475 + 1.14302\,i,\ -0.508126 - 1.1706\,i,\ -0.129589 + 0.113453\,i,\ -0.101499 - 0.107278\,i,\ 1.35073 + 25.8457\,i,\ 21.297 - 54.5177\,i,\ 48.1709 + 150.908\,i,\ 167.676 - 200.594\,i,\ 662.381 + 5365.95\,i\}$$

```
ez2[100 000 000, N[ZetaZero[1]], z]
```

$$(1. + 0.\,i) + (178.811 - 111.793\,i)\,z - (648.401 - 338.823\,i)\,z^2 + (1006. - 434.963\,i)\,z^3 - (899.348 - 326.894\,i)\,z^4 + (522.784 - 169.271\,i)\,z^5 - (211.134 - 67.6491\,i)\,z^6 + (61.8198 - 22.3314\,i)\,z^7 - (13.5292 - 6.1622\,i)\,z^8 + (2.26506 - 1.38782\,i)\,z^9 - (0.294909 - 0.248205\,i)\,z^{10} + (0.0300519 - 0.03466\,i)\,z^{11} - (0.00236985 - 0.00375092\,i)\,z^{12} + (0.000138919 - 0.000313588\,i)\,z^{13} - (5.54652 \times 10^{-6} - 0.0000201624\,i)\,z^{14} + (1.14854 \times 10^{-7} - 9.88521 \times 10^{-7}\,i)\,z^{15} + (1.63804 \times 10^{-9} + 3.65482 \times 10^{-8}\,i)\,z^{16} - (2.23903 \times 10^{-10} + 1.00481 \times 10^{-9}\,i)\,z^{17} + (8.61981 \times 10^{-12} + 2.01412 \times 10^{-11}\,i)\,z^{18} - (1.88538 \times 10^{-13} + 2.86703 \times 10^{-13}\,i)\,z^{19} + (2.58061 \times 10^{-15} + 2.83791 \times 10^{-15}\,i)\,z^{20} - (2.39143 \times 10^{-17} + 2.01128 \times 10^{-17}\,i)\,z^{21} + (1.59503 \times 10^{-19} + 1.13567 \times 10^{-19}\,i)\,z^{22} - (7.70254 \times 10^{-22} + 5.18787 \times 10^{-22}\,i)\,z^{23} + (1.96369 \times 10^{-24} + 1.83033 \times 10^{-24}\,i)\,z^{24} - (1.58645 \times 10^{-28} + 2.07157 \times 10^{-27}\,i)\,z^{25} - (2.92084 \times 10^{-31} - 7.92366 \times 10^{-32}\,i)\,z^{26}$$

```
ezeros[10 000 000, N[ZetaZero[1]]] // TableForm
```

```
-2.94428 + 18.6403 i
0.010228 - 0.0117525 i
0.915848 - 4665.19 i
0.999973 - 0.0000256921 i
1.0654 + 6.32766 i
1.53388 - 5.96189 i
2.00673 + 0.0150723 i
2.32923 - 1.61325 i
2.39015 - 0.214322 i
2.61838 + 1.01077 i
3.49043 - 2.63485 i
3.68162 + 2.5122 i
6.74177 - 0.0210039 i
10.637 - 15.9654 i
11.4814 + 28.2677 i
16.7079 - 0.11017 i
20.1259 + 7.43797 i
26.7139 - 9.96327 i
47.7687 + 50.9779 i
55.9511 - 112.028 i
57.0493 - 34.2386 i
117.826 + 302.104 i
220.895 - 101.699 i
```

```
ezeros[10 000 000, N[ZetaZero[1]]] // TableForm
```

```
-2.94428 + 18.6403 i
0.010228 - 0.0117525 i
0.915848 - 4665.19 i
0.999973 - 0.0000256921 i
1.0654 + 6.32766 i
1.53388 - 5.96189 i
2.00673 + 0.0150723 i
2.32923 - 1.61325 i
2.39015 - 0.214322 i
2.61838 + 1.01077 i
3.49043 - 2.63485 i
3.68162 + 2.5122 i
6.74177 - 0.0210039 i
10.637 - 15.9654 i
11.4814 + 28.2677 i
16.7079 - 0.11017 i
20.1259 + 7.43797 i
26.7139 - 9.96327 i
47.7687 + 50.9779 i
55.9511 - 112.028 i
57.0493 - 34.2386 i
117.826 + 302.104 i
220.895 - 101.699 i
```

```
ezeros[100 000 000, N[ZetaZero[1]]]
```

```
{-23.4432 + 115.492 i, -3.00749 + 7.24046 i, -0.00399172 - 0.0024432 i,
 1. + 3.60842 × 10^-6 i, 1.59804 + 38.0367 i, 1.61973 - 1.50287 i, 2.00164 - 0.0052567 i,
 2.70904 + 0.134875 i, 3.0931 - 8.44949 i, 3.1986 - 1.21641 i, 3.56358 + 1.39473 i,
 4.17529 - 3.86039 i, 4.4097 + 16.3853 i, 4.60627 + 3.92814 i, 5.18591 + 0.00864528 i,
 9.63811 - 0.0407828 i, 12.277 - 154.547 i, 12.4618 - 21.1794 i,
 21.8501 + 3.95519 i, 24.4498 - 2.96747 i, 32.8254 - 0.284303 i, 66.7752 - 26.1226 i,
 68.0093 + 67.8245 i, 74.2416 - 5866. i, 133.036 - 68.5184 i, 819.951 - 843.178 i}
```

```
N[ZetaZero[1]]
```

```
0.5 + 14.1347 i
```

```
N[Log[2, 100 000 000]]
```

```
26.5754
```

```
Clear[FAlt, f1a, pab, psum, d2]
bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k!
FAlt[n_, 0, s_, a_] := UnitStep[n - 1]
FAlt[n_, 1, s_, a_] := FAlt[n, 1, s, a] = HarmonicNumber[Floor[n], s] - HarmonicNumber[a, s]
FAlt[n_, 2, s_, a_] := FAlt[n, 2, s, a] =
  Sum[ (m^(-2 s)) + 2 (m^-s) (FAlt[Floor[n / m], 1, s, m]), {m, a + 1, Floor[n^(1 / 2)]}]
FAlt[n_, k_, s_, a_] := FAlt[n, k, s, a] =
  Sum[(m^(-s k)) + k (m^(-s (k - 1))) FAlt[Floor[n / (m^(k - 1))], 1, s, m] +
    Sum[bin[k, j] (m^-s)^j FAlt[Floor[n / (m^j)], k - j, s, m], {j, 1, k - 2}],
   {m, a + 1, Floor[n^(1 / k)]}]
f1a[n_, s_, z_] := Expand[Sum[bin[z, k] FAlt[n, k, s, 1], {k, 0, Log[2, n]}]]
ez1a[n_, s_, z_] :=
 Expand[Sum[ (-1)^j bin[z, j] (2)^(j (1 - s)) f1a[ n / (2^j), s, z], {j, 0, Log[2, n]}]]
pab[n_, s_, 0, a_] := UnitStep[n - 1]
psum[ n_, s_] :=
 psum[n, s] = Sum[ t^-1 (Prime[j]^t)^-s, {t, 1, Log[2, n]}, {j, 1, PrimePi[ n^(1 / t)]}]
pab[n_, s_, 1, a_] := psum[n, s] - psum[a, s]
pab[n_, s_, k_, a_] := pab[n, s, k, a] = Sum[Binomial[k, j] ((t^-1) (Prime[m]^t)^-s)^j
    pab[Floor[n / ((Prime[m]^t)^j)], s, k - j, Prime[m]^t], {t, 1, Log[2, Floor[n^(1 / k)]]},
   {j, 1, k}, {m, PrimePi[a^(1 / t)] + 1, PrimePi[Floor[n^(1 / k)]^(1 / t)]}]
dz2[n_, s_, z_] := Sum[ z^k / (k!) pab[n, s, k, 1], {k, 0, Log[2, n]}]
ez2[n_, s_, z_] :=
 Expand[Sum[ (-1)^j bin[z, j] (2)^(j (1 - s)) dz2[ n / (2^j), s, z], {j, 0, Log[2, n]}]]
d2[n_, s_, 0] := UnitStep[n - 1]
d2[n_, s_, k_] := d2[n, s, k] = Sum[ j^-s d2[Floor[n / j], s, k - 1], {j, 2, n}]
dzz[n_, s_, z_] := Sum[ bin[ z, k] d2[n, s, k], {k, 0, Log[2, n]}]
```

```
AbsoluteTiming[ez1a[10 000 000, N[ZetaZero[1]], z]]
```

$$\{39.9082826,\ (1.+0.\,i)-(45.4558+48.4997\,i)\,z+(140.588+164.395\,i)\,z^2-(191.246+238.877\,i)\,z^3+(152.097+199.937\,i)\,z^4-(80.3319+108.801\,i)\,z^5+(30.3631+41.1341\,i)\,z^6-(8.62493+11.2783\,i)\,z^7+(1.90012+2.31031\,i)\,z^8-(0.329937+0.359479\,i)\,z^9+(0.0452778+0.0424413\,i)\,z^{10}-(0.00487763+0.00370671\,i)\,z^{11}+(0.00040802+0.000228966\,i)\,z^{12}-(0.0000261843+9.41988\times10^{-6}\,i)\,z^{13}+(1.26949\times10^{-6}+2.18011\times10^{-7}\,i)\,z^{14}-(4.554\times10^{-8}-3.03041\times10^{-12}\,i)\,z^{15}+(1.18086\times10^{-9}-1.83197\times10^{-10}\,i)\,z^{16}-(2.16758\times10^{-11}-6.08217\times10^{-12}\,i)\,z^{17}+(2.75011\times10^{-13}-9.65399\times10^{-14}\,i)\,z^{18}-(2.34225\times10^{-15}-8.61328\times10^{-16}\,i)\,z^{19}+(1.25981\times10^{-17}-4.23332\times10^{-18}\,i)\,z^{20}-(3.5056\times10^{-20}-1.60456\times10^{-20}\,i)\,z^{21}+(5.09703\times10^{-23}-3.3653\times10^{-23}\,i)\,z^{22}-(8.78027\times10^{-27}+1.00637\times10^{-26}\,i)\,z^{23}\}$$

```
AbsoluteTiming[ez2[10 000 000, N[ZetaZero[1]], z]]
```

$$\{63.4636299,\ (1.+0.\,i)-(45.4558+48.4997\,i)\,z+(140.588+164.395\,i)\,z^2-(191.246+238.877\,i)\,z^3+(152.097+199.937\,i)\,z^4-(80.3319+108.801\,i)\,z^5+(30.3631+41.1341\,i)\,z^6-(8.62493+11.2783\,i)\,z^7+(1.90012+2.31031\,i)\,z^8-(0.329937+0.359479\,i)\,z^9+(0.0452778+0.0424413\,i)\,z^{10}-(0.00487763+0.00370671\,i)\,z^{11}+(0.00040802+0.000228966\,i)\,z^{12}-(0.0000261843+9.41988\times10^{-6}\,i)\,z^{13}+(1.26949\times10^{-6}+2.18011\times10^{-7}\,i)\,z^{14}-(4.554\times10^{-8}-3.03041\times10^{-12}\,i)\,z^{15}+(1.18086\times10^{-9}-1.83197\times10^{-10}\,i)\,z^{16}-(2.16758\times10^{-11}-6.08217\times10^{-12}\,i)\,z^{17}+(2.75011\times10^{-13}-9.65399\times10^{-14}\,i)\,z^{18}-(2.34225\times10^{-15}-8.61328\times10^{-16}\,i)\,z^{19}+(1.25981\times10^{-17}-4.23332\times10^{-18}\,i)\,z^{20}-(3.5056\times10^{-20}-1.60456\times10^{-20}\,i)\,z^{21}+(5.09701\times10^{-23}-3.3653\times10^{-23}\,i)\,z^{22}-(8.77956\times10^{-27}+1.00642\times10^{-26}\,i)\,z^{23}\}$$

```
Table[{Expand[f1a[10 000 / 2^n, 0, z] - Expand[dzz[10 000 / 2^n, 0, z]]],
    Expand[dz2[10 000 / 2^n, 0, z] - Expand[dzz[10 000 / 2^n, 0, z]]]} , {n, 0, 18}] // TableForm
```

0	0
0	0
0	0
0	0
0	0
$\frac{z}{2}$	0
$\frac{z}{4}$	0
$\frac{z}{8}$	0
$\frac{z}{16}$	0
$\frac{17\,z}{32}$	0
$\frac{49\,z}{64}$	0
$\frac{113\,z}{128}$	0
$\frac{113\,z}{256}$	0
0	0
0	0
0	0
0	0
0	0
0	0

$$\left\{5.179999999999836`,\ 1+\frac{546\,736\,922\,027\,278\,097\,z}{14\,549\,535}+\frac{906\,064\,090\,085\,165\,349\,697\,z^2}{7\,718\,911\,200}+\right.$$
$$\frac{23\,730\,945\,231\,458\,376\,567\,503\,z^3}{154\,378\,224\,000}+\frac{242\,646\,876\,607\,500\,345\,427\,z^4}{2\,122\,848\,000}+\frac{2\,540\,504\,723\,192\,757\,736\,147\,z^5}{46\,702\,656\,000}+$$
$$\frac{1\,604\,900\,827\,527\,715\,863\,643\,z^6}{90\,531\,302\,400}+\frac{12\,080\,176\,357\,605\,237\,815\,393\,z^7}{2\,942\,267\,328\,000}+\frac{104\,549\,368\,685\,985\,987\,497\,z^8}{150\,885\,504\,000}+$$
$$\frac{6\,508\,456\,376\,756\,282\,153\,z^9}{75\,442\,752\,000}+\frac{624\,689\,833\,375\,091\,z^{10}}{78\,382\,080}+\frac{237\,240\,552\,758\,791\,z^{11}}{431\,101\,440}+$$
$$\frac{12\,525\,410\,607\,772\,829\,z^{12}}{452\,656\,512\,000}+\frac{373\,620\,760\,522\,951\,z^{13}}{367\,783\,416\,000}+\frac{23\,466\,773\,167\,z^{14}}{891\,596\,160}+$$
$$\left.\frac{15\,520\,900\,817\,z^{15}}{30\,648\,618\,000}+\frac{6\,705\,071\,z^{16}}{1\,094\,593\,500}+\frac{464\,713\,z^{17}}{9\,304\,044\,750}+\frac{299\,z^{18}}{1\,502\,961\,075}+\frac{236\,z^{19}}{1\,856\,156\,927\,625}\right\}$$

$$\{5.18,\ 1+\frac{546\,736\,922\,027\,278\,097\,z}{14\,549\,535}+\frac{906\,064\,090\,085\,165\,349\,697\,z^2}{7\,718\,911\,200}+$$
$$\frac{23\,730\,945\,231\,458\,376\,567\,503\,z^3}{154\,378\,224\,000}+\frac{242\,646\,876\,607\,500\,345\,427\,z^4}{2\,122\,848\,000}+\frac{2\,540\,504\,723\,192\,757\,736\,147\,z^5}{46\,702\,656\,000}+$$
$$\frac{1\,604\,900\,827\,527\,715\,863\,643\,z^6}{90\,531\,302\,400}+\frac{12\,080\,176\,357\,605\,237\,815\,393\,z^7}{2\,942\,267\,328\,000}+\frac{104\,549\,368\,685\,985\,987\,497\,z^8}{150\,885\,504\,000}+$$
$$\frac{6\,508\,456\,376\,756\,282\,153\,z^9}{75\,442\,752\,000}+\frac{624\,689\,833\,375\,091\,z^{10}}{78\,382\,080}+\frac{237\,240\,552\,758\,791\,z^{11}}{431\,101\,440}+$$
$$\frac{12\,525\,410\,607\,772\,829\,z^{12}}{452\,656\,512\,000}+\frac{373\,620\,760\,522\,951\,z^{13}}{367\,783\,416\,000}+\frac{23\,466\,773\,167\,z^{14}}{891\,596\,160}+$$
$$\frac{15\,520\,900\,817\,z^{15}}{30\,648\,618\,000}+\frac{6\,705\,071\,z^{16}}{1\,094\,593\,500}+\frac{464\,713\,z^{17}}{9\,304\,044\,750}+\frac{299\,z^{18}}{1\,502\,961\,075}+\frac{236\,z^{19}}{1\,856\,156\,927\,625}\}$$

```
Clear[f]
f[n_, 0, s_, a_] := 1
f[n_, 1, s_, a_] := HarmonicNumber[n, s] - HarmonicNumber[a, s]
f[n_, k_, s_, a_] :=
 f[n, k, s, a] = N[Sum[Binomial[k, j] (N[m]^-s)^j f[Floor[n / (N[m]^j)], k - j, s, m],
    {j, 1, k}, {m, a + 1., Floor[N[n]^(1. / k)]}]]
fz2[n_, s_, z_] := Sum[ z^k / (k!) f[n, k, s, 1], {k, 0, Log[2, n]}]
```

```
Timing[fz2[1 000 000, N[ZetaZero[1]], z]]
```

$$\{20.015,\ 1+(35.0516+60.8217\,i)\,z+(236.434+393.057\,i)\,z^2+$$
$$(481.519+791.603\,i)\,z^3+(458.6+741.935\,i)\,z^4+(243.349+386.342\,i)\,z^5+$$
$$(79.6143+123.483\,i)\,z^6+(17.085+25.7987\,i)\,z^7+(2.50327+3.67526\,i)\,z^8+$$
$$(0.257254+0.367286\,i)\,z^9+(0.0189479+0.0261787\,i)\,z^{10}+(0.00102107+0.00133649\,i)\,z^{11}+$$
$$(0.0000410039+0.0000484445\,i)\,z^{12}+\left(1.23251\times10^{-6}+1.22181\times10^{-6}\,i\right)z^{13}+$$
$$\left(2.54783\times10^{-8}+2.166\times10^{-8}\,i\right)z^{14}+\left(4.2397\times10^{-10}+2.31233\times10^{-10}\,i\right)z^{15}+$$
$$\left(4.73005\times10^{-12}+1.51223\times10^{-12}\,i\right)z^{16}+\left(3.87252\times10^{-14}+1.51353\times10^{-14}\,i\right)z^{17}+$$
$$\left(6.72051\times10^{-17}+1.51522\times10^{-16}\,i\right)z^{18}-\left(1.7878\times10^{-19}-5.09203\times10^{-20}\,i\right)z^{19}\}$$

```
Binomial[2, 1]
```

2

```
(m^-s)^2
```

$m^{-2\,s}$

```
Sum[(m^(-s k)) , {m, a + 1, Floor[n^(1 / k)]}]
```

$$\text{HurwitzZeta}[k\, s,\, 1 + a] - \text{HurwitzZeta}\left[k\, s,\, 1 + \text{Floor}\left[n^{\frac{1}{k}}\right]\right]$$

```
Binomial[k, k - 1]
```

$$k$$

```
Sum[(m^(-s)) , {m, a + 1, Floor[n]}] /. {n → 1000.5, s → 0, a → 0}
```

$$1000$$

```
Sum[(m^(-s)) , {m, a + 1, Floor[n]}]
```

$$\text{HurwitzZeta}[s,\, 1 + a] - \text{HurwitzZeta}[s,\, 1 + \text{Floor}[n]]$$

```
HurwitzZeta[s, 1 + a] - HurwitzZeta[s, 1 + Floor[n]] /. {n → 1000.5, s → 0, a → 0}
```

$$1000$$

```
HarmonicNumber[n, s] - HarmonicNumber[a, s] /. {n → 1000.5, s → 0, a → 0}
```

$$1000.5$$