```
ClearAll["Global`*"]
```

```
Dd[n_, 0, x_] := 1
Dd[n_, k_, x_] := Dd[n, k, x] = Sum[Dd[Floor[n / (j + x)], k - 1, x], {j, 0, n - x}]
Cc[x_, k_, a_] := a^-k Dd[x a^k, k, a + 1]
Table[{Cc[x, 2, 250.], N[x Log[x] - x + 1], 1 - Gamma[2., -Log[x]] / Gamma[2]}, {x, 2, 10}] //
 TableForm
```

0.38248	0.386294	$0.386294 - 9.46148 \times 10^{-17}\,i$
1.28802	1.29584	$1.29584 - 3.17388 \times 10^{-16}\,i$
2.5336	2.54518	$2.54518 - 6.23389 \times 10^{-16}\,i$
4.03157	4.04719	$4.04719 - 9.91276 \times 10^{-16}\,i$
5.73075	5.75056	$5.75056 - 1.40848 \times 10^{-15}\,i$
7.59739	7.62137	$7.62137 - 1.8667 \times 10^{-15}\,i$
9.60808	9.63553	$9.63553 - 1.05755 \times 10^{-15}\,i$
11.7437	11.775	$11.775 - 1.31956 \times 10^{-15}\,i$
13.9904	14.0259	$14.0259 - 1.59521 \times 10^{-15}\,i$

```
Dd[n_, 0, x_] := 1
Dd[n_, k_, x_] := Dd[n, k, x] = Sum[Dd[Floor[n / (j + x)], k - 1, x], {j, 0, n - x}]
Cc[x_, k_, a_] := a^-k Dd[x a^k, k, a + 1]
Table[{Cc[x, 3, 40.], N[x / 2 Log[x]^2 - x Log[x] + x - 1],
    -(1 - Gamma[3., -Log[x]] / Gamma[3])}, {x, 2, 10}] // TableForm
```

0.0803125	0.0941587	$0.0941587 - 3.45933 \times 10^{-17}\,i$
0.467516	0.514587	$0.514587 - 1.89056 \times 10^{-16}\,i$
1.20523	1.29845	$1.29845 - 4.77042 \times 10^{-16}\,i$
2.28005	2.42854	$2.42854 - 8.9223 \times 10^{-16}\,i$
3.66944	3.88065	$3.88065 - 1.42573 \times 10^{-15}\,i$
5.35158	5.63161	$5.63161 - 2.06902 \times 10^{-15}\,i$
7.306	7.66078	$7.66078 - 2.81452 \times 10^{-15}\,i$
9.51539	9.95006	$9.95006 - 3.65559 \times 10^{-15}\,i$
11.9655	12.4836	$12.4836 - 4.58641 \times 10^{-15}\,i$

```
Dd[n_, 0, x_] := 1
Dd[n_, k_, x_] := Sum[Dd[n / (j + x), k - 1, x], {j, 0, n - x}]
DdAlt[n_, 0, a_] := 1; DdAlt[n_, 1, a_] := Floor[n] - a + 1
DdAlt[n_, k_, a_] :=
 Sum[Binomial[k, j] DdAlt[n / (m^(k - j)), j, m + 1], {m, a, n^(1 / k)}, {j, 0, k - 1}]
Grid[Table[{Dd[n, k, 1], DdAlt[n, k, 1]}, {n, 7, 100, 5}, {k, 1, 7}]]
Grid[Table[{Dd[n, k, 3], DdAlt[n, k, 3]}, {n, 7, 200, 5}, {k, 1, 7}]]
```

{7, 7}	{16, 16}	{28, 28}	{43, 43}	{61, 61}	{82, 82}	{106, 106}
{12, 12}	{35, 35}	{74, 74}	{133, 133}	{216, 216}	{327, 327}	{470, 470}
{17, 17}	{52, 52}	{113, 113}	{208, 208}	{346, 346}	{537, 537}	{792, 792}
{22, 22}	{74, 74}	{170, 170}	{324, 324}	{551, 551}	{867, 867}	{1289, 1289}
{27, 27}	{95, 95}	{228, 228}	{454, 454}	{806, 806}	{1322, 1322}	{2045, 2045}
{32, 32}	{119, 119}	{300, 300}	{622, 622}	{1142, 1142}	{1928, 1928}	{3060, 3060}
{37, 37}	{142, 142}	{366, 366}	{774, 774}	{1447, 1447}	{2483, 2483}	{3998, 3998}
{42, 42}	{168, 168}	{444, 444}	{954, 954}	{1802, 1802}	{3113, 3113}	{5034, 5034}
{47, 47}	{188, 188}	{495, 495}	{1058, 1058}	{1987, 1987}	{3413, 3413}	{5489, 5489}
{52, 52}	{217, 217}	{591, 591}	{1304, 1304}	{2527, 2527}	{4478, 4478}	{7428, 7428}
{57, 57}	{243, 243}	{672, 672}	{1500, 1500}	{2932, 2932}	{5228, 5228}	{8709, 8709}
{62, 62}	{267, 267}	{750, 750}	{1700, 1700}	{3367, 3367}	{6068, 6068}	{10 193, 10 193}
{67, 67}	{294, 294}	{835, 835}	{1908, 1908}	{3807, 3807}	{6914, 6914}	{11 712, 11 712}
{72, 72}	{326, 326}	{952, 952}	{2232, 2232}	{4562, 4562}	{8474, 8474}	{14 659, 14 659}
{77, 77}	{348, 348}	{1009, 1009}	{2348, 2348}	{4767, 4767}	{8804, 8804}	{15 156, 15 156}
{82, 82}	{377, 377}	{1108, 1108}	{2607, 2607}	{5342, 5342}	{9944, 9944}	{17 235, 17 235}
{87, 87}	{403, 403}	{1192, 1192}	{2819, 2819}	{5797, 5797}	{10 814, 10 814}	{18 761, 18 761}
{92, 92}	{435, 435}	{1306, 1306}	{3119, 3119}	{6452, 6452}	{12 074, 12 074}	{20 973, 20 973}
{97, 97}	{461, 461}	{1399, 1399}	{3395, 3395}	{7162, 7162}	{13 700, 13 700}	{24 361, 24 361}

{5, 5}	{0, 0}	{0, 0}	{0, 0}	{0, 0}	{0, 0}	{0, 0}
{10, 10}	{3, 3}	{0, 0}	{0, 0}	{0, 0}	{0, 0}	{0, 0}
{15, 15}	{6, 6}	{0, 0}	{0, 0}	{0, 0}	{0, 0}	{0, 0}
{20, 20}	{12, 12}	{0, 0}	{0, 0}	{0, 0}	{0, 0}	{0, 0}
{25, 25}	{19, 19}	{1, 1}	{0, 0}	{0, 0}	{0, 0}	{0, 0}
{30, 30}	{27, 27}	{1, 1}	{0, 0}	{0, 0}	{0, 0}	{0, 0}
{35, 35}	{36, 36}	{4, 4}	{0, 0}	{0, 0}	{0, 0}	{0, 0}
{40, 40}	{46, 46}	{4, 4}	{0, 0}	{0, 0}	{0, 0}	{0, 0}
{45, 45}	{52, 52}	{7, 7}	{0, 0}	{0, 0}	{0, 0}	{0, 0}
{50, 50}	{65, 65}	{10, 10}	{0, 0}	{0, 0}	{0, 0}	{0, 0}
{55, 55}	{77, 77}	{13, 13}	{0, 0}	{0, 0}	{0, 0}	{0, 0}
{60, 60}	{85, 85}	{19, 19}	{0, 0}	{0, 0}	{0, 0}	{0, 0}
{65, 65}	{98, 98}	{23, 23}	{0, 0}	{0, 0}	{0, 0}	{0, 0}
{70, 70}	{114, 114}	{32, 32}	{0, 0}	{0, 0}	{0, 0}	{0, 0}
{75, 75}	{122, 122}	{35, 35}	{0, 0}	{0, 0}	{0, 0}	{0, 0}
{80, 80}	{135, 135}	{41, 41}	{1, 1}	{0, 0}	{0, 0}	{0, 0}
{85, 85}	{147, 147}	{47, 47}	{1, 1}	{0, 0}	{0, 0}	{0, 0}
{90, 90}	{163, 163}	{56, 56}	{1, 1}	{0, 0}	{0, 0}	{0, 0}
{95, 95}	{175, 175}	{65, 65}	{1, 1}	{0, 0}	{0, 0}	{0, 0}
{100, 100}	{190, 190}	{71, 71}	{1, 1}	{0, 0}	{0, 0}	{0, 0}
{105, 105}	{200, 200}	{77, 77}	{1, 1}	{0, 0}	{0, 0}	{0, 0}
{110, 110}	{220, 220}	{92, 92}	{5, 5}	{0, 0}	{0, 0}	{0, 0}
{115, 115}	{232, 232}	{95, 95}	{5, 5}	{0, 0}	{0, 0}	{0, 0}
{120, 120}	{247, 247}	{113, 113}	{5, 5}	{0, 0}	{0, 0}	{0, 0}
{125, 125}	{261, 261}	{123, 123}	{5, 5}	{0, 0}	{0, 0}	{0, 0}
{130, 130}	{279, 279}	{132, 132}	{5, 5}	{0, 0}	{0, 0}	{0, 0}
{135, 135}	{291, 291}	{141, 141}	{9, 9}	{0, 0}	{0, 0}	{0, 0}
{140, 140}	{305, 305}	{147, 147}	{9, 9}	{0, 0}	{0, 0}	{0, 0}
{145, 145}	{324, 324}	{171, 171}	{15, 15}	{0, 0}	{0, 0}	{0, 0}
{150, 150}	{338, 338}	{180, 180}	{15, 15}	{0, 0}	{0, 0}	{0, 0}
{155, 155}	{356, 356}	{189, 189}	{15, 15}	{0, 0}	{0, 0}	{0, 0}
{160, 160}	{374, 374}	{207, 207}	{19, 19}	{0, 0}	{0, 0}	{0, 0}
{165, 165}	{382, 382}	{213, 213}	{19, 19}	{0, 0}	{0, 0}	{0, 0}
{170, 170}	{405, 405}	{234, 234}	{19, 19}	{0, 0}	{0, 0}	{0, 0}
{175, 175}	{421, 421}	{240, 240}	{19, 19}	{0, 0}	{0, 0}	{0, 0}
{180, 180}	{439, 439}	{270, 270}	{31, 31}	{0, 0}	{0, 0}	{0, 0}
{185, 185}	{453, 453}	{270, 270}	{31, 31}	{0, 0}	{0, 0}	{0, 0}
{190, 190}	{475, 475}	{297, 297}	{39, 39}	{0, 0}	{0, 0}	{0, 0}
{195, 195}	{486, 486}	{306, 306}	{39, 39}	{0, 0}	{0, 0}	{0, 0}

```
Dd[n_, 0, a_] := 1; Dd[n_, 1, a_] := Floor[n] - a + 1
Dd[n_, k_, a_] := Dd[n, k, a] =
  Sum[Binomial[k, j] DdAlt[n / (m^(k - j)), j, m + 1], {m, a, n^(1 / k)}, {j, 0, k - 1}]
Cc[x_, k_, a_] := a^-k Dd[x a^k, k, a + 1]
Table[{Cc[x, 2, 3000.], N[x Log[x] - x + 1], 1 - Gamma[2., -Log[x]] / Gamma[2]}, {x, 2, 40}] //
 TableForm
```

0.385964	0.386294	$0.386294 - 9.46148 \times 10^{-17}\,i$
1.29517	1.29584	$1.29584 - 3.17388 \times 10^{-16}\,i$
2.54418	2.54518	$2.54518 - 6.23389 \times 10^{-16}\,i$
4.04585	4.04719	$4.04719 - 9.91276 \times 10^{-16}\,i$
5.74889	5.75056	$5.75056 - 1.40848 \times 10^{-15}\,i$
7.61937	7.62137	$7.62137 - 1.8667 \times 10^{-15}\,i$
9.63321	9.63553	$9.63553 - 1.05755 \times 10^{-15}\,i$
11.7724	11.775	$11.775 - 1.31956 \times 10^{-15}\,i$
14.0229	14.0259	$14.0259 - 1.59521 \times 10^{-15}\,i$
16.3735	16.3768	$16.3768 - 1.88312 \times 10^{-15}\,i$
18.8152	18.8189	$18.8189 - 2.18218 \times 10^{-15}\,i$
21.3403	21.3443	$21.3443 - 2.49146 \times 10^{-15}\,i$
23.9425	23.9468	$23.9468 - 2.81017 \times 10^{-15}\,i$
26.6161	26.6208	$26.6208 - 3.13764 \times 10^{-15}\,i$
29.3564	29.3614	$29.3614 - 3.47327 \times 10^{-15}\,i$
32.1593	32.1646	$32.1646 - 3.81657 \times 10^{-15}\,i$
35.021	35.0267	$35.0267 - 4.16707 \times 10^{-15}\,i$
37.9384	37.9443	$37.9443 - 4.52438 \times 10^{-15}\,i$
40.9083	40.9146	$40.9146 - 4.88813 \times 10^{-15}\,i$
43.9283	43.935	$43.935 - 5.25802 \times 10^{-15}\,i$
46.9959	47.0029	$47.0029 - 5.63373 \times 10^{-15}\,i$
50.109	50.1164	$50.1164 - 6.01502 \times 10^{-15}\,i$
53.2656	53.2733	$53.2733 - 6.40163 \times 10^{-15}\,i$
56.4639	56.4719	$56.4719 - 6.79335 \times 10^{-15}\,i$
59.7022	59.7105	$59.7105 - 7.18996 \times 10^{-15}\,i$
62.9789	62.9876	$62.9876 - 7.59129 \times 10^{-15}\,i$
66.2927	66.3017	$66.3017 - 7.99716 \times 10^{-15}\,i$
69.6422	69.6516	$69.6516 - 8.40739 \times 10^{-15}\,i$
73.0263	73.0359	$73.0359 - 8.82186 \times 10^{-15}\,i$
76.4436	76.4536	$76.4536 - 9.2404 \times 10^{-15}\,i$
79.8932	79.9035	$79.9035 - 9.6629 \times 10^{-15}\,i$
83.3741	83.3847	$83.3847 - 1.00892 \times 10^{-14}\,i$
86.8853	86.8963	$86.8963 - 1.05193 \times 10^{-14}\,i$
90.4258	90.4372	$90.4372 - 1.09529 \times 10^{-14}\,i$
93.995	94.0067	$94.0067 - 1.139 \times 10^{-14}\,i$
97.592	97.604	$97.604 - 1.18306 \times 10^{-14}\,i$
101.216	101.228	$101.228 - 1.22744 \times 10^{-14}\,i$
104.866	104.879	$104.879 - 1.27215 \times 10^{-14}\,i$
108.542	108.555	$108.555 - 1.31717 \times 10^{-14}\,i$

```
Dd[n_, 0, a_] := 1; Dd[n_, 1, a_] := Floor[n] - a + 1
Dd[n_, k_, a_] := Dd[n, k, a] =
  Sum[Binomial[k, j] DdAlt[n / (m^(k - j)), j, m + 1], {m, a, n^(1 / k)}, {j, 0, k - 1}]
Cc[x_, k_, a_] := a^-k Dd[x a^k, k, a + 1]
Table[{Cc[x, 3, 600.], N[x / 2 Log[x]^2 - x Log[x] + x - 1],
   -(1 - Gamma[3., -Log[x]] / Gamma[3])}, {x, 2, 10}] // TableForm
```

0.0931968	0.0941587	$0.0941587 - 3.45933 \times 10^{-17}\,i$
0.511357	0.514587	$0.514587 - 1.89056 \times 10^{-16}\,i$
1.2921	1.29845	$1.29845 - 4.77042 \times 10^{-16}\,i$
2.41844	2.42854	$2.42854 - 8.9223 \times 10^{-16}\,i$
3.8663	3.88065	$3.88065 - 1.42573 \times 10^{-15}\,i$
5.61258	5.63161	$5.63161 - 2.06902 \times 10^{-15}\,i$
7.63672	7.66078	$7.66078 - 2.81452 \times 10^{-15}\,i$
9.92066	9.95006	$9.95006 - 3.65559 \times 10^{-15}\,i$
12.4486	12.4836	$12.4836 - 4.58641 \times 10^{-15}\,i$

```
Dd[n_, 0, a_] := 1; Dd[n_, 1, a_] := Floor[n] - a + 1
Dd[n_, k_, a_] := Dd[n, k, a] =
  Sum[Binomial[k, j] Dd[Floor[n / (m^(k - j))], j, m + 1], {m, a, n^(1 / k)}, {j, 0, k - 1}]
Cc[x_, k_, a_] := a^-k Dd[x a^k, k, a + 1]
Table[{Cc[x, k, 200.], N[(-1)^k (1 - Gamma[k, -Log[x]] / Gamma[k])]},
  {x, 2, 7}, {k, 1, 4}] // TableForm
```

1.	0.3812	0.0913001	0.0159262
1.	$0.386294 - 9.46148 \times 10^{-17}\,i$	$0.0941587 - 3.45933 \times 10^{-17}\,i$	$0.0168496 - 8.25391 \times 10^{-18}\,i$
2.	1.28595	0.504934	0.143316
2.	$1.29584 - 3.17388 \times 10^{-16}\,i$	$0.514587 - 1.89056 \times 10^{-16}\,i$	$0.148398 - 7.2694 \times 10^{-17}\,i$
3.	2.5303	1.27947	0.464829
3.	$2.54518 - 6.23389 \times 10^{-16}\,i$	$1.29845 - 4.77042 \times 10^{-16}\,i$	$0.477685 - 2.33998 \times 10^{-16}\,i$
4.	4.02733	2.3983	1.02148
4.	$4.04719 - 9.91276 \times 10^{-16}\,i$	$2.42854 - 8.9223 \times 10^{-16}\,i$	$1.04556 - 5.12175 \times 10^{-16}\,i$
5.	5.72573	3.83771	1.83311
5.	$5.75056 - 1.40848 \times 10^{-15}\,i$	$3.88065 - 1.42573 \times 10^{-15}\,i$	$1.87162 - 9.16829 \times 10^{-16}\,i$
6.	7.59168	5.57464	2.90884
6.	$7.62137 - 1.8667 \times 10^{-15}\,i$	$5.63161 - 2.06902 \times 10^{-15}\,i$	$2.96476 - 1.45231 \times 10^{-15}\,i$

```
$RecursionLimit = 1 000 000
Dd[n_, 0, a_] := 1; Dd[n_, 1, a_] := Floor[n] - a + 1
Dd[n_, k_, a_] := Dd[n, k, a] =
  Sum[Binomial[k, j] Dd[Floor[n / (m^(k - j))], j, m + 1], {m, a, n^(1 / k)}, {j, 0, k - 1}]
D2[ x_, k_ ] := (-1)^k (1 - Gamma[k, -Log[x]] / Gamma[k]) -
  Integrate[ D[ a^(-k) Dd[ x a^k, k, a + 1 ], a], {a, 1, Infinity}]
```

```
1 000 000
```

```
D2[3, 2]
```

```
2
```