$Sum[n/(ab)-b, \{a, 2, Floor[n^{(1/3)}]\}, \{b, a+1, Floor[(n/a)^{1/2}]\}]$ 

\$Aborted

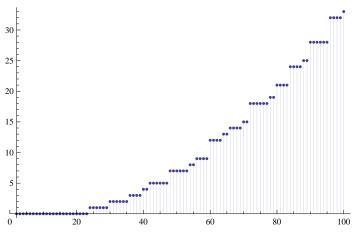
 $FF[n_] := Sum[Floor[n/(ab)] - b, \{a, 2, Floor[n^(1/3)]\}, \{b, a+1, Floor[(n/a)^(1/2)]\}]$  FF[80]

21

-213

-213

## DiscretePlot[FF[n], {n, 2, 100}]

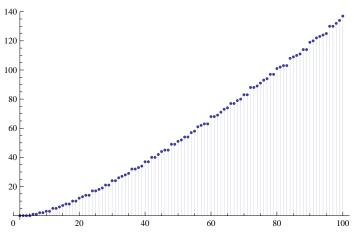


 $\label{eq:GG_n_sigma} \mbox{GG[$n_{$}$} \mbox{ := } \mbox{Sum[Floor[$n / m$] - m, {m, 2, Floor[$n^{(1/2)}$]}} \mbox{ }$ 

GG[103]

140

## DiscretePlot[GG[n], {n, 2, 100}]



FF[n]

$$\sum_{a=2}^{\texttt{Floor}\left[n^{1/3}\right]} \sum_{b=1+a}^{\texttt{Floor}\left[\sqrt{\frac{n}{a}}\right]} \left(-\,b + \texttt{Floor}\left[\frac{n}{a\,b}\right]\right)$$

$$\sum_{a=2}^{\text{Floor}\left[n^{1/3}\right]} \sum_{b=1+a}^{\text{Floor}\left[\sqrt{\frac{n}{a}}\right]} (-b) + \sum_{a=2}^{\text{Floor}\left[n^{1/3}\right]} \sum_{b=1+a}^{\text{Floor}\left[\sqrt{\frac{n}{a}}\right]} \left(\text{Floor}\left[\frac{n}{a \ b}\right]\right)$$

\$Aborted

$$\sum_{a=2}^{\text{Floor}\left[n^{1/3}\right]} \sum_{b=1+a}^{\text{Floor}\left[\sqrt{\frac{n}{a}}\right]} (-b)$$

\$Aborted

$$\operatorname{Floor}\left[\sqrt{\frac{n}{a}}\right] \\
\sum_{b=1+a} (-b)$$

$$\mathtt{Expand}\Big[\frac{1}{2}\left(\mathtt{a-Floor}\Big[\sqrt{\frac{\mathtt{n}}{\mathtt{a}}}\hspace{0.1cm}\Big]\right)\left(\mathtt{1+a+Floor}\Big[\sqrt{\frac{\mathtt{n}}{\mathtt{a}}}\hspace{0.1cm}\Big]\right)\Big]$$

$$\frac{\mathsf{a}}{2} + \frac{\mathsf{a}^2}{2} - \frac{1}{2} \, \mathtt{Floor} \Big[ \sqrt{\frac{\mathsf{n}}{\mathsf{a}}} \, \Big] - \frac{1}{2} \, \mathtt{Floor} \Big[ \sqrt{\frac{\mathsf{n}}{\mathsf{a}}} \, \Big]^2$$

$$\sum_{a=2}^{\text{Floor}\left[n^{1/3}\right]} \frac{a}{2} + \sum_{a=2}^{\text{Floor}\left[n^{1/3}\right]} \frac{a^2}{2} - \sum_{a=2}^{\text{Floor}\left[n^{1/3}\right]} \frac{1}{2} \, \text{Floor}\!\left[\sqrt{\frac{n}{a}}\right] - \sum_{a=2}^{\text{Floor}\left[n^{1/3}\right]} \frac{1}{2} \, \text{Floor}\!\left[\sqrt{\frac{n}{a}}\right]^2$$

$$\frac{1}{4} \left(-2 + \texttt{Floor}\left[n^{1/3}\right] + \texttt{Floor}\left[n^{1/3}\right]^2\right) + \frac{1}{12} \left(-6 + \texttt{Floor}\left[n^{1/3}\right] + 3 \, \texttt{Floor}\left[n^{1/3}\right]^2 + 2 \, \texttt{Floor}\left[n^{1/3}\right]^3\right) - \left(-6 + \frac{1}{12} + \frac{1}{12}$$

$$\sum_{a=2}^{\text{Floor}\left[n^{1/3}\right]} \frac{1}{2} \, \, \text{Floor}\!\left[\sqrt{\frac{n}{a}}\,\,\right] \, - \, \sum_{a=2}^{\text{Floor}\left[n^{1/3}\right]} \frac{1}{2} \, \, \text{Floor}\!\left[\sqrt{\frac{n}{a}}\,\,\right]^2$$

FullSimplify

$$\frac{1}{4} \left(-2 + {\tt Floor}{\left[n^{1/3}\right]} + {\tt Floor}{\left[n^{1/3}\right]}^2\right) + \frac{1}{12} \left(-6 + {\tt Floor}{\left[n^{1/3}\right]} + 3 \, {\tt Floor}{\left[n^{1/3}\right]}^2 + 2 \, {\tt Floor}{\left[n^{1/3}\right]}^3\right) - \frac{1}{4} \left(-2 + {\tt Floor}{\left[n^{1/3}\right]}^2 + 2 \, {\tt Floor}{\left[n^{1/3}\right]}^3\right) + \frac{1}{12} \left(-6 + {\tt Floor}{\left[n^{1/3}\right]}^3$$

$$\sum_{a=2}^{\text{Floor}\left[n^{1/3}\right]} \frac{1}{2} \, \, \text{Floor}\!\left[\sqrt{\frac{n}{a}}\,\,\right] \, - \, \sum_{a=2}^{\text{Floor}\left[n^{1/3}\right]} \frac{1}{2} \, \, \text{Floor}\!\left[\sqrt{\frac{n}{a}}\,\,\right]^2 \, \right]$$

$$6\left(1+\sum_{a=2}^{\lceil floor\left[n^{1/3}\right]}\frac{1}{2}\operatorname{Floor}\left[\sqrt{\frac{n}{a}}\right]+\sum_{a=2}^{\lceil floor\left[n^{1/3}\right]}\frac{1}{2}\operatorname{Floor}\left[\sqrt{\frac{n}{a}}\right]^{2}\right)\right)\right]$$

$$-1 + \frac{1}{3} \operatorname{Floor} \left[ n^{1/3} \right] + \frac{1}{2} \operatorname{Floor} \left[ n^{1/3} \right]^2 + \frac{1}{6} \operatorname{Floor} \left[ n^{1/3} \right]^3 - \frac{1}{6} \operatorname{F$$

$$\sum_{a=2}^{\text{Floor}\left[n^{1/3}\right]} \frac{1}{2} \, \text{Floor}\!\left[\sqrt{\frac{n}{a}}\right] - \sum_{a=2}^{\text{Floor}\left[n^{1/3}\right]} \frac{1}{2} \, \text{Floor}\!\left[\sqrt{\frac{n}{a}}\right]^2$$

$$F1[n_{]} := \sum_{a=2}^{Floor \left[n^{1/3}\right]} \sum_{b=1+a}^{Floor \left[\sqrt{\frac{n}{a}}\right]} (-b)$$

$$\begin{split} & \text{F2}[\text{n}_{\_}] := -1 + \frac{1}{3} \, \text{Floor} \big[ \text{n}^{1/3} \big] + \frac{1}{2} \, \text{Floor} \big[ \text{n}^{1/3} \big]^2 + \\ & \frac{1}{6} \, \text{Floor} \big[ \text{n}^{1/3} \big]^3 - \sum_{\substack{n=2 \ a=2}}^{\text{Floor} \big[ \text{n}^{1/3} \big]} \frac{1}{2} \, \text{Floor} \big[ \sqrt{\frac{\text{n}}{\text{a}}} \, \big] - \sum_{\substack{n=2 \ a=2}}^{\text{Floor} \big[ \text{n}^{1/3} \big]} \frac{1}{2} \, \text{Floor} \big[ \sqrt{\frac{\text{n}}{\text{a}}} \, \big]^2 \end{split}$$

F1[1000]

-750

F2[1000]

-750

$$\texttt{F3} \left[ n_{\_} \right] := -1 + \frac{1}{3} \, \texttt{Floor} \left[ n^{1/3} \right] + \frac{1}{2} \, \texttt{Floor} \left[ n^{1/3} \right]^2 + \frac{1}{6} \, \texttt{Floor} \left[ n^{1/3} \right]^3 - \frac{1}{6} \, \texttt{Floor} \left[ n^{1/3} \right]^3 + \frac{1}{6} \, \texttt{Floor} \left[$$

$$\sum_{a=2}^{\text{Floor}\left[n^{1/3}\right]} \frac{1}{2} \, \text{Floor}\left[\sqrt{\frac{n}{a}}\right] - \sum_{a=2}^{\text{Floor}\left[n^{1/3}\right]} \frac{1}{2} \, \text{Floor}\left[\sqrt{\frac{n}{a}}\right]^2 + \sum_{a=2}^{\text{Floor}\left[n^{1/3}\right]} \sum_{b=1+a}^{\text{Floor}\left[\sqrt{\frac{n}{a}}\right]} \left(\text{Floor}\left[\frac{n}{a\,b}\right]\right)$$

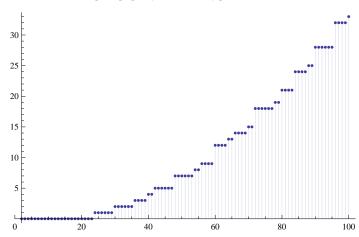
F3[100]

33

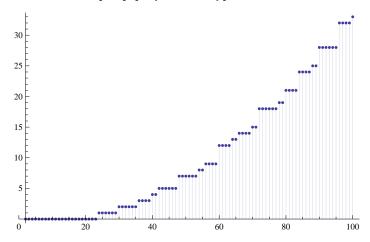
FF[100]

33

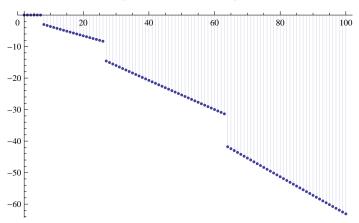
DiscretePlot[FF[n], {n, 2, 100}]



## DiscretePlot[F3[n], {n, 2, 100}]



$$\mbox{DiscretePlot} \left[ - \sum_{a=2}^{Floor \left[n^{1/3}\right]} \frac{1}{2} \, \sqrt{\frac{n}{a}} \, - \sum_{a=2}^{Floor \left[n^{1/3}\right]} \frac{1}{2} \, \sqrt{\frac{n}{a}}^{2} \, , \, \{n, \, 2, \, 100\} \right]$$



 $Full Simplify \left[ -1 + \frac{1}{3} \ Floor \left[ n^{1/3} \right] + \frac{1}{2} \ Floor \left[ n^{1/3} \right]^2 + \frac{1}{6} \ Floor \left[ n^{1/3} \right]^3 - \frac{1}{6} \left[ n^{1/3} \right]^3 + \frac{1}{6} \left[ n^{1/3} \right]^3$ 

$$\sum_{a=2}^{\text{Floor}\left[n^{1/3}\right]} \frac{1}{2} \, \text{Floor}\left[\sqrt{\frac{n}{a}}\right] - \sum_{a=2}^{\text{Floor}\left[n^{1/3}\right]} \frac{1}{2} \, \text{Floor}\left[\sqrt{\frac{n}{a}}\right]^2 + \sum_{a=2}^{\text{Floor}\left[n^{1/3}\right]} \sum_{b=1+a}^{\text{Floor}\left[\frac{n}{a}\right]} \left(\text{Floor}\left[\frac{n}{a}\right]\right) \, \right]$$

$$\frac{1}{6} \left[ \operatorname{Floor} \left[ n^{1/3} \right] \left( 1 + \operatorname{Floor} \left[ n^{1/3} \right] \right) \left( 2 + \operatorname{Floor} \left[ n^{1/3} \right] \right) - \right]$$

$$6\left[1+\sum_{a=2}^{\operatorname{Floor}\left[n^{1/3}\right]}\frac{1}{2}\operatorname{Floor}\left[\sqrt{\frac{n}{a}}\right]+\sum_{a=2}^{\operatorname{Floor}\left[n^{1/3}\right]}\frac{1}{2}\operatorname{Floor}\left[\sqrt{\frac{n}{a}}\right]^2-\sum_{a=2}^{\operatorname{Floor}\left[n^{1/3}\right]}\sum_{b=1+a}^{\operatorname{Floor}\left[\frac{n}{a}\right]}\operatorname{Floor}\left[\frac{n}{a}\right]\right]\right]$$