$$\begin{array}{l} t[n_{-}, a_{-}] := Mod[n, a] - Mod[n-1, a] \\ Sum[k, (k, 1, n)] \\ \frac{1}{2} n (1+n) \\ Sum[(-1)^{n}(k+1) k, (k, 1, n)] \\ \frac{1}{4} (1-(-1)^{n}-2 (-1)^{n} n) \\ Sum[k^{2}, (k, 1, n)] \\ \frac{1}{6} n (1+n) (1+2n) \\ Sum[(-1)^{n}(k+1) k^{2}, (k, 1, n)] \\ -\frac{1}{2} (-1)^{n} n (1+n) \\ Sum[k^{3}, (k, 1, n)] \\ \frac{1}{4} n^{2} (1+n)^{2} \\ Sum[(-1)^{n}(k+1) k^{3}, (k, 1, n)] \\ \frac{1}{8} \left\{ -1 + (-1)^{n} -6 (-1)^{n} n^{2} -4 (-1)^{n} n^{3} \right\} \\ Sum[(2k-1), (k, 1, n)] \\ n^{2} \\ Sum[(2k-1), (k, 1, n)] \\ \frac{1}{2} \left\{ -(n+4)^{n} \right\} \\ Sum[(2k-1)^{n}(k+1) (2k-1), (k, 1, n)] \\ \frac{1}{2} \left\{ -(n+4)^{n} \right\} \\ Sum[(2k-1)^{n}, (k+1) (2k-1)^{n}, (k, 1, n)] \\ \frac{1}{2} \left\{ -(n+4)^{n} -4 (-1)^{n} n^{2} \right\} \\ Sum[(2k-1)^{n}, (k+1) (2k-1)^{n}, (k, 1, n)] \\ -n^{n} + 2n^{4} \\ Sum[(-1)^{n}(k+1) (2k-1)^{3}, (k, 1, n)] \\ -(-1)^{n} n (-3 + 4n^{2}) \\ Sum[tk, 2] k, (k, 1, n)] \\ \begin{bmatrix} 1 \\ -(-1)^{n} n (-3 + 4n^{2}) \\ Sum[tk, 2] k, (k, 1, n)] \\ \end{bmatrix}$$

$$n = 3$$

$$\begin{bmatrix} 1 \\ -(-1)^{n} n (-3 + 4n^{2}) \\ Sum[tk, 2] k, (k, 1, n)] \\ \end{bmatrix}$$
True

Sum[t[k, 3] k, {k, 1, n}]

$$\left\{ \begin{array}{l} 1 & \text{$n=1$} \\ \frac{1}{2} \left(-6 - 18 \, \text{Floor} \left[\frac{1}{3} \, \left(-3 + n \right) \, \right] - 6 \, \text{Floor} \left[\frac{1}{3} \, \left(-3 + n \right) \, \right]^2 + 7 \, \text{Floor} \left[\frac{1}{3} \, \left(-2 + n \right) \, \right] + \, \text{True} \right. \\ \left. 3 \, \text{Floor} \left[\frac{1}{3} \, \left(-2 + n \right) \, \right]^2 + 5 \, \text{Floor} \left[\frac{1}{3} \, \left(-1 + n \right) \, \right] + 3 \, \text{Floor} \left[\frac{1}{3} \, \left(-1 + n \right) \, \right]^2 \right) \end{array} \right.$$

Sum[t[k, 10] k, {k, 1, n}]

$$\begin{array}{l} 1 \\ -45-135\, {\rm Floor} \Big[\frac{1}{10} \, \left(-10+n \right) \, \Big] -45\, {\rm Floor} \Big[\frac{1}{10} \, \left(-10+n \right) \, \Big]^2 +14\, {\rm Floor} \Big[\frac{1}{10} \, \left(-9+n \right) \, \Big] \, + \\ 5\, {\rm Floor} \Big[\frac{1}{10} \, \left(-9+n \right) \, \Big]^2 +13\, {\rm Floor} \Big[\frac{1}{10} \, \left(-8+n \right) \, \Big] +5\, {\rm Floor} \Big[\frac{1}{10} \, \left(-8+n \right) \, \Big]^2 \, + \\ 12\, {\rm Floor} \Big[\frac{1}{10} \, \left(-7+n \right) \, \Big] +5\, {\rm Floor} \Big[\frac{1}{10} \, \left(-7+n \right) \, \Big]^2 +11\, {\rm Floor} \Big[\frac{1}{10} \, \left(-6+n \right) \, \Big] \, + \\ 5\, {\rm Floor} \Big[\frac{1}{10} \, \left(-6+n \right) \, \Big]^2 +10\, {\rm Floor} \Big[\frac{1}{10} \, \left(-5+n \right) \, \Big] +5\, {\rm Floor} \Big[\frac{1}{10} \, \left(-3+n \right) \, \Big]^2 \, + \\ 9\, {\rm Floor} \Big[\frac{1}{10} \, \left(-4+n \right) \, \Big] +5\, {\rm Floor} \Big[\frac{1}{10} \, \left(-2+n \right) \, \Big]^2 +8\, {\rm Floor} \Big[\frac{1}{10} \, \left(-3+n \right) \, \Big] +5\, {\rm Floor} \Big[\frac{1}{10} \, \left(-1+n \right) \, \Big]^2 \, + \\ 7\, {\rm Floor} \Big[\frac{1}{10} \, \left(-2+n \right) \, \Big] +5\, {\rm Floor} \Big[\frac{1}{10} \, \left(-2+n \right) \, \Big]^2 +6\, {\rm Floor} \Big[\frac{1}{10} \, \left(-1+n \right) \, \Big] +5\, {\rm Floor} \Big[\frac{1}{10} \, \left(-1+n \right) \, \Big]^2 \, + \\ \end{array}$$

$Sum[t[k, 3] (2k-1), \{k, 1, n\}]$

$$\left\{ \begin{array}{l} 1 & \text{$n=1$} \\ -8-18\, \text{Floor} \left[\frac{1}{3}\, \left(-3+n\right)\,\right] - 6\, \text{Floor} \left[\frac{1}{3}\, \left(-3+n\right)\,\right]^2 + 6\, \text{Floor} \left[\frac{1}{3}\, \left(-2+n\right)\,\right] + \\ 3\, \text{Floor} \left[\frac{1}{3}\, \left(-2+n\right)\,\right]^2 + 4\, \text{Floor} \left[\frac{1}{3}\, \left(-1+n\right)\,\right] + 3\, \text{Floor} \left[\frac{1}{3}\, \left(-1+n\right)\,\right]^2 + 2\, \text{Floor} \left[\frac{n}{3}\,\right] \end{array} \right. \right.$$

$$pp[n_{-}] := -8 - 18 Floor \left[\frac{1}{3} (-3 + n)\right] - 6 Floor \left[\frac{1}{3} (-3 + n)\right]^{2} + 6 Floor \left[\frac{1}{3} (-2 + n)\right] + 3 Floor \left[\frac{1}{3} (-2 + n)\right]^{2} + 4 Floor \left[\frac{1}{3} (-1 + n)\right] + 3 Floor \left[\frac{1}{3} (-1 + n)\right]^{2} + 2 Floor \left[\frac{n}{3}\right]$$

$$pp[100]$$

1

```
Table[{n, pp[n]}, {n, 1, 50}] // TableForm
1
    1
2
    4
3
    - 6
4
    1
5
    10
6
    -12
7
    1
8
    16
9
    -18
10
11
    22
12
    - 24
13 1
14
    28
15
    - 30
16
17
    34
    - 36
18
19
    1
20 40
21
    -42
22
23
    46
24
    -48
25 1
26 52
27
    - 54
28
    1
29
    58
30 -60
31 1
32 64
33
    -66
34
35
    70
36 - 72
37 1
38 76
39
    - 78
40
    1
41
    82
42
    -84
43
    1
44
    88
45
    - 90
46
47
    94
48
    -96
49
    1
```

 $tri2[n_{,a_{]} := Sum[t[k, a] k, \{k, 1, n\}]$

 $Table[\{n,\, tri2[n,\, 2],\, tri2[n,\, 3],\, tri2[n,\, 4],\, tri2[n,\, 5]\},\, \{n,\, 1,\, 50\}] \,\, //\,\, TableForm$

1	1	1	1	1
2	- 1	3	3	3
3	2	- 3	6	6
4	- 2	1	- б	10
5	3	6	-1	-10
6	- 3	- 6	5	- 4
7	4	1	12	3
8	- 4	9	-12	11
9	5	– 9	- 3	20
10	- 5	1	7	- 20
11	6	12	18	– 9
12	- 6	-12	-18	3
13	7	1	- 5	16
14	- 7	15	9	30
15	8	-15	24	-30
16	- 8	1	-24	-14
17	9	18	- 7	3
18	- 9	-18	11	21
19	10	1	30	40
20	-10	21	- 30	- 40
21	11	-21	- 9	-19
22	-11	1	13	3
23	12	24	36	26
24	-12	- 24	- 36	50
25	13	1	-11	- 50
26	-13	27	15	- 24
27	14	- 27	42	3
28	-14	1	- 42	31
29	15	30	-13	60
30	-15	- 30	17	- 60
31	16	1	48	- 29
32	-16	33	- 48	3
33	17	- 33	-15	36
34	-17	1	19	70
35	18	36	54	- 70
36	-18	- 36	-54	- 34
37	19	1	-17	3
38	-19	39	21	41
39	20	- 39	60	80
40	- 20	1	-60	- 80
41	21	42	-19	- 39
42	-21	-42	23	3
43	22	1	66	46
44	- 22	45	-66	90
45	23	- 45	-21	- 90
46	- 23	1	25	- 44
47	24	48	72	3
48	- 24	-48	-72	51
49	25	1	- 23	100
50	- 25	51	27	-100

```
\texttt{ce} := \texttt{CoefficientList[Series[x/Log[1+x], \{x, 0, 10000\}], x]}
N[ce[[950]]]
0.0000170911
N[Sum[Abs[ce[[k]]], \{k, 2, 9999\}]]
$Aborted
cd[[3]]
 1
 12
Integrate [2/(2-xy), \{x, 0, 2\}, \{y, 0, 2\}]
\frac{1}{2} \pi (\pi - 4 i \text{ Log}[2])
Integrate [\; 1 \; / \; (1 - x \; y \; z \; w) \; , \; \{x, \; 0, \; 1\} \; , \; \{y, \; 0, \; 1\} \; , \; \{z, \; 0, \; 1\} \; , \; \{w, \; 0, \; 1\}]
\pi^4
Integrate[1/(1-xyz), \{x, 0, 1\}, \{y, 0, 1\}, \{z, 0, 1\}]
Zeta[3]
N[Sum[ (-1) ^n / n^Re[ZetaZero[1]] Cos[Im[ZetaZero[1] Log[n]]], \{n, 1, 100 000\}]]
0.00127694
```

```
t[n_{,a_{]} := Mod[n, a] - Mod[n-1, a]
tri2a[n_, a_, p_] := Sum[t[k, a] k^p, \{k, 1, n\}]
Table[{n, tri2a[n, 2, p], tri2a[n, 3, p], tri2a[n, 4, p], tri2a[n, 5, p]}, {n, 1, 50}] /.
  p \rightarrow 2 // TableForm
      1
                1
                          1
1
                                    1
2
      - 3
                5
                          5
                                    5
3
      6
                -13
                          14
                                    14
4
                3
                          -34
                                    30
      -10
5
      15
                28
                          - 9
                                    - 70
6
      -21
                -44
                          27
                                    - 34
7
                5
                          76
                                    15
      28
8
      - 36
                69
                          -116
                                    79
9
      45
                          - 35
                                    160
                - 93
                7
10
      - 55
                          65
                                    -240
11
      66
                128
                          186
                                    -119
                -160
12
      - 78
                          -246
                                    25
13
      91
                          - 77
                                    194
                9
14
      -105
                205
                          119
                                    390
15
                          344
                                    -510
      120
                -245
16
      -136
                11
                          -424
                                    - 254
17
      153
                300
                          -135
                                    35
18
      -171
                -348
                          189
                                    359
      190
                          550
19
                13
                                    720
      -210
                          -650
                                    -880
20
                413
21
      231
                -469
                          -209
                                    -439
22
      - 253
                15
                          275
                                    45
23
      276
                544
                          804
                                    574
24
      -300
                -608
                          -924
                                    1150
25
      325
                          -299
                17
                                    -1350
      -351
                693
                          377
                                    -674
26
27
      378
                -765
                          1106
                                    55
28
      -406
                19
                          -1246
                                    839
29
                          -405
      435
                860
                                    1680
30
      -465
                -940
                          495
                                    -1920
                                    - 959
31
      496
                21
                          1456
32
                1045
                          -1616
                                    65
      -528
33
      561
                -1133
                          -527
                                    1154
                23
                          629
34
      - 595
                                    2310
                                    - 2590
35
      630
                1248
                          1854
36
      -666
                          -2034
                                    -1294
                -1344
37
      703
                25
                          -665
                                    75
                          779
38
      -741
                1469
                                    1519
      780
                -1573
39
                          2300
                                    3040
40
      -820
                27
                          -2500
                                    -3360
41
      861
                1708
                          -819
                                    -1679
                -1820
42
      -903
                          945
                                    85
43
      946
                29
                          2794
                                    1934
44
      -990
                1965
                          -3014
                                    3870
45
      1035
                -2085
                          -989
                                    -4230
46
      -1081
                31
                          1127
                                    -2114
47
      1128
                2240
                          3336
                                    95
                                    2399
48
      -1176
                -2368
                          -3576
49
      1225
                33
                          -1175
                                    4800
50
      -1275
                2533
                          1325
                                    -5200
```

```
t[n_{,a}] := Mod[n, a] - Mod[n-1, a]
tri2a[n_, a_, p_] := Sum[t[k, a] k^p, \{k, 1, n\}]
Table[{n, tri2a[n, 2, p], tri2a[n, 3, p], tri2a[n, 4, p], tri2a[n, 5, p]}, {n, 1, 50}] /.
  p \rightarrow 3 // TableForm
1
      1
                 1
                             1
                                         1
                                         9
2
      - 7
                 9
                             9
3
      20
                 - 45
                             36
                                         36
4
      -44
                 19
                             -156
                                         100
5
      81
                 144
                                         -400
                             - 31
      -135
                 -288
                                         -184
6
                             185
7
      208
                 55
                             528
                                         159
                 567
8
      -304
                             -1008
                                         671
9
      425
                             - 279
                 -891
                                         1400
                             721
10
      - 575
                 109
                                         -2600
11
      756
                 1440
                             2052
                                         -1269
12
      -972
                 -2016
                             -3132
                                         459
13
      1225
                 181
                             -935
                                         2656
14
      -1519
                 2925
                             1809
                                         5400
15
      1856
                 -3825
                             5184
                                         -8100
16
      -2240
                 271
                             -7104
                                         -4004
17
      2673
                 5184
                             -2191
                                         909
18
      -3159
                 -6480
                             3641
                                         6741
19
      3700
                 379
                             10500
                                         13600
20
      -4300
                 8379
                             -13500
                                         -18400
21
      4961
                 -10143
                             -4239
                                         -9139
2.2
      -5687
                 505
                             6409
                                         1509
23
      6480
                 12672
                             18576
                                         13676
24
      -7344
                 -14976
                             -22896
                                         27500
                 649
25
      8281
                             -7271
                                         -35000
                 18 225
                                         -17424
26
      -9295
                             10305
27
      10388
                 -21141
                             29 988
                                         2259
      -11564
                             -35868
                                         24 211
28
                 811
29
      12825
                 25 200
                             -11479
                                         48600
30
      -14175
                 -28800
                             15521
                                         -59400
      15 616
31
                 991
                             45 312
                                         -29609
32
      -17152
                 33 759
                             -52992
                                         3159
33
      18 785
                 -38115
                             -17055
                                         39096
34
      -20519
                 1189
                             22 249
                                         78 400
35
      22356
                 44064
                             65124
                                         -93100
36
      -24300
                 -49248
                             -74844
                                         -46444
37
                             -24191
      26 353
                 1405
                                         4209
38
      -28519
                 56 277
                             30681
                                         59081
39
      30800
                 -62361
                             90000
                                         118 400
40
      -33200
                             -102000
                                         -137600
                 1639
41
      35 721
                 70 560
                             -33079
                                         -68679
42
      -38367
                 -77616
                             41009
                                         5409
      41 140
43
                 1891
                             120516
                                         84916
44
      -44044
                 87 075
                             -135036
                                         170 100
45
      47081
                 -95175
                             -43911
                                         -194400
      -50255
                 2161
                             53 425
                                         -97064
46
47
      53 568
                 105984
                             157 248
                                         6759
48
      -57024
                 -115200
                             -174528
                                         117 351
                                         235 000
49
      60 625
                 2449
                             - 56 879
50
      -64375
                 127 449
                             68121
                                         -265000
```

```
Table[{n, aa = (Abs[tri2a[n, 2, p]] - Abs[tri2a[n-1, 2, p]]), bb = }
      aa - (Abs[tri2a[n - 1, 2, p]] - Abs[tri2a[n - 2, 2, p]])}, \{n, 1, 50\}] /. p \rightarrow 3 // TableForm
1
2
      6
              5
3
      13
              7
4
      24
              11
5
      37
              13
6
      54
              17
7
      73
              19
8
      96
              23
9
      121
              25
10
      150
              29
11
      181
              31
12
      216
              35
13
      253
              37
14
      294
              41
15
      337
              43
16
      384
              47
17
      433
              49
18
      486
              53
19
      541
              55
20
      600
              59
21
      661
              61
22
      726
              65
23
      793
              67
24
      864
              71
25
      937
              73
26
      1014
              77
27
      1093
              79
28
      1176
              83
29
              85
      1261
30
      1350
              89
31
      1441
              91
32
      1536
              95
      1633
              97
33
34
      1734
              101
35
      1837
              103
36
      1944
              107
37
      2053
              109
38
      2166
              113
39
      2281
              115
40
      2400
              119
41
      2521
              121
42
      2646
              125
43
      2773
              127
44
      2904
              131
45
      3037
              133
46
      3174
              137
47
      3313
              139
48
      3456
              143
```