BASIC FUNCTIONS

$$\boxed{n^z = \binom{z}{0}1 + \binom{z}{1}\int_1^n dx + \binom{z}{2}\int_1^n \int_1^n dy\,dx + \binom{z}{3}\int_1^n \int_1^n \int_1^n dz\,dy\,dx + \ldots}$$

$$\boxed{\{n^z\} = L_{-z}(\log n) = \binom{z}{0}1 + \binom{z}{1}\int_1^n dx + \binom{z}{2}\int_1^n \int_1^{\frac{n}{x}} dy\,dx + \binom{z}{3}\int_1^n \int_1^{\frac{n}{x}} \int_1^{\frac{n}{x\,y}} dz\,dy\,dx + \ldots}$$

$$\boxed{[n^z] = D_z(n) = \binom{z}{0}1 + \binom{z}{1}\sum_{j=2}^{n} 1 + \binom{z}{2}\sum_{j=2}^{n} \sum_{k=2}^{\frac{n}{j}} 1 + \binom{z}{3}\sum_{j=2}^{n} \sum_{k=2}^{\frac{n}{j}} \sum_{l=2}^{\frac{n}{j\cdot k}} 1 + \ldots}$$

….

and a note to explore later:

$$\boxed{n^z = \lim_{a \to 0} \binom{z}{0}1 + \binom{z}{1}\int_1^n dx + \binom{z}{2}\int_1^n \int_1^{\frac{n}{x^a}} dy\,dx + \binom{z}{3}\int_1^n \int_1^{\frac{n}{x^a}} \int_1^{\frac{n}{x^a\,y^a}} dz\,dy\,dx + \ldots}$$

VERSIONS WITH EXTRA j^-s

$$\boxed{[\zeta(s)^z]_n = \binom{z}{0}1 + \binom{z}{1}\sum_{j=2}^{n} j^{-s} + \binom{z}{2}\sum_{j=2}^{n} \sum_{k=2}^{n} (j\cdot k)^{-s} + \binom{z}{3}\sum_{j=2}^{n} \sum_{k=2}^{n} \sum_{l=2}^{n} (j\cdot k\cdot l)^{-s} + \ldots}$$

EXPONENTIAL MULTIPLICATION

$$\boxed{n^{a+b}=\int_0^n\int_0^n \frac{\partial(x^a)}{\partial x}\cdot\frac{\partial(y^b)}{\partial y}\,dy\,dx}$$

$$\boxed{n^{a+b}=1+(n^a-1)+(n^b-1)+\int_1^n\int_1^n \frac{\partial(x^a)}{\partial x}\cdot\frac{\partial(y^b)}{\partial y}\,dy\,dx}$$

...

$$\boxed{\{n^{a+b}\}=?\int_0^n\int_0^{\frac{n}{x}} \frac{\partial\{x^a\}}{\partial x}\cdot\frac{\partial\{y^b\}}{\partial y}\,dy\,dx}$$ That division by x is a problem.

$$\boxed{\{n^{a+b}\}=1+\{n^a\text{-}1\}+\{n^b\text{-}1\}+\int_1^n\int_1^{\frac{n}{x}} \frac{\partial\{x^a\}}{\partial x}\cdot\frac{\partial\{y^b\}}{\partial y}\,dy\,dx}$$

$$\boxed{\{n^{a+b}\}=1+\int_1^n \frac{\partial\{x^a\}}{\partial x}\,dx+\int_1^n \frac{\partial\{y^b\}}{\partial y}\,dy+\int_1^n\int_1^{\frac{n}{x}} \frac{\partial\{x^a\}}{\partial x}\cdot\frac{\partial\{y^b\}}{\partial y}\,dy\,dx}$$

...

$$\boxed{[n^{a+b}]=\sum_{j=1}^{n}\sum_{k=1}^{\lfloor\frac{n}{j}\rfloor}[\nabla j^a]\cdot[\nabla k^b]}$$

$$\boxed{[n^{a+b}]=\sum_{j=1}^{1}\sum_{k=1}^{1}[\nabla j^a]\cdot[\nabla k^b]+\sum_{j=2}^{n}\sum_{k=1}^{1}[\nabla j^a]\cdot[\nabla k^b]+\sum_{j=1}^{1}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}[\nabla j^a]\cdot[\nabla k^b]+\sum_{j=2}^{n}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}[\nabla j^a]\cdot[\nabla k^b]}$$

$$\log_x ab = \log_x a + \log_x b$$

$$x^{a+b} = x^a \cdot x^b$$

$$\log_x x^y = y$$

$$L_{-(a+b)}(\log n) = 1 + \int_1^n \frac{\partial L_{-a}(\log x)}{\partial x} dx + \int_1^n \frac{\partial L_{-b}(\log y)}{\partial y} dy + \int_1^n \int_1^{\frac{n}{x}} \frac{\partial L_{-a}(\log x)}{\partial x} \cdot \frac{\partial L_{-b}(\log y)}{\partial y} dy\, dx$$

$$D_{a+b}(n) = \sum_{j=1}^{n} \sum_{k=1}^{\lfloor \frac{n}{j} \rfloor} \nabla[D]_a(j) \cdot \nabla[D]_b(k)$$

$$n \cdot m = \int_0^n \int_0^m \frac{\partial(x)}{\partial x} \cdot \frac{\partial(y)}{\partial y} dy\, dx$$

$$L_{-2}(\log n) = 1 + \int_1^n \frac{\partial L_{-1}(\log x)}{\partial x} dx + \int_1^n \frac{\partial L_{-1}(\log y)}{\partial y} dy + \int_1^n \int_1^{\frac{n}{x}} \frac{\partial L_{-1}(\log x)}{\partial x} \cdot \frac{\partial L_{-1}(\log y)}{\partial y} dy\, dx$$

$$n/((m+1)((x-1)/(n-1))+1)$$

$$D_{a+b}(n) = \sum_{j=1}^{n} \sum_{k=1}^{\lfloor \frac{n}{j} \rfloor} \nabla[D]_a(j) \cdot \nabla[D]_b(k)$$

$$\frac{n}{j \cdot k^{\log_m n}} \geq 1$$

$$\log n - \log\left(j \cdot k^{\log_m n}\right) \geq 0$$

$$\log n - \log j - \left(\frac{\log n}{\log m}\right) \log k \geq 0$$

$$1 \geq \frac{\log j}{\log n} + \frac{\log k}{\log m}$$

$$\boxed{f(n) * g(m) = \sum_{\frac{\log j}{\log n} + \frac{\log k}{\log m} \leq 1} \nabla[f](j) \cdot \nabla[g](k)}$$

$$\boxed{f(n) * g(m) = \sum_{\frac{\log j}{\log n} + \frac{\log k}{\log m} \leq 1} \nabla[f](j) \cdot \nabla[g](k)}$$

$$\boxed{f(n) * g(m) = \sum_{\log_n j + \log_m k \leq 1} \nabla[f](j) \cdot \nabla[g](k)}$$

$$\boxed{f(n) * g(m) = \sum_{j=1}^{n} \sum_{k=1}^{\left\lfloor m^{1 - \frac{\log j}{\log n}} \right\rfloor} \nabla[f](j) \cdot \nabla[g](k)}$$

$$\boxed{f(n) * g(m) = \sum_{j=1}^{n} \sum_{k=1}^{\left\lfloor m^{1 - \log_n j} \right\rfloor} \nabla[f](j) \cdot \nabla[g](k)}$$

$$\boxed{f(m) * g(n) * h(o) = \sum_{j=1}^{m} \sum_{k=1}^{\left\lfloor n^{1 - \log_m j} \right\rfloor} \sum_{l=1}^{\left\lfloor o^{1 - \log_m j - \log_n k} \right\rfloor} \nabla[f](j) \cdot \nabla[g](k) \cdot \nabla[h](l)}$$

$$\log_n j + \log_m k \leq 1$$

$$k \leq m^{1 - \frac{\log j}{\log n}}$$

$$\boxed{n \cdot m = \int_0^n \int_0^m dy\, dx}$$

$$\boxed{n \cdot m = 1 + (m - 1) + (n - 1) + \int_1^n \int_1^m dy\, dx}$$

$$\boxed{\{n \cdot m\} = 1 + \{m - 1\} + \{n - 1\} + \int_1^n \int_1^{\frac{m}{x^{\log_n m}}} dy\, dx}$$

$$[n\cdot m]=\sum_{j=1}^{n}\sum_{k=1}^{\lfloor \frac{m}{j^{\log_n m}} \rfloor} 1$$

$$[n\cdot m]=1+[n-1]+[m-1]+\sum_{j=2}^{n}\sum_{k=2}^{\lfloor \frac{m}{j^{\log_n m}} \rfloor} 1$$

$$[n\cdot m]=1+\sum_{j=2}^{n} 1+\sum_{k=2}^{m} 1+\sum_{j=2}^{n}\sum_{k=2}^{\lfloor \frac{m}{j^{\log_n m}} \rfloor} 1$$

$$\ldots$$

$$\{n\cdot m\}=\frac{m\log m-n\log n}{\log m-\log n}$$

$$\ldots$$

$$[n\cdot m\cdot o]=\sum_{a\cdot b^{\log_m n}\cdot c^{\log_o n}\leq n} 1$$

$$[n\cdot m\cdot o]=\sum_{a=1}^{n}\sum_{b=1}^{\lfloor \frac{m}{a^{\log_n m}} \rfloor}\sum_{c=1}^{\lfloor \frac{o}{a^{\log_n o}\cdot b^{\log_m o}} \rfloor} 1$$

$$\{n\cdot m\cdot o\}=1+\{m\text{-}1\}+\{n\text{-}1\}+\{o\text{-}1\}+\int_1^n\int_1^{\frac{m}{x^{\log_n m}}} dy\,dx+\int_1^n\int_1^{\frac{o}{x^{\log_n o}}} dy\,dx+\int_1^m\int_1^{\frac{o}{x^{\log_m o}}} dy\,dx+\int_1^n\int_1^{\frac{m}{x^{\log_n m}}}\int_1^{\frac{o}{x^{\log_n o}\cdot y^{\log_m o}}} dz\,dy\,dx$$

$$\boxed{[n\cdot m]=\sum_{j=1}^{n}\sum_{k=1}^{\lfloor(\frac{n}{j})^{\log_n m}\rfloor} 1}$$

$$\boxed{[n\cdot m]=\sum_{j\cdot k^{\log_m n}\le n} 1}$$

$$\frac{e}{j^{\frac{1}{\log n}}\cdot k^{\frac{1}{\log m}}}\ge 1 \quad \text{is} \quad 1-\frac{\log j}{\log n}-\frac{\log k}{\log m}\ge 0 \quad \text{so}$$

$$\boxed{[n\cdot m]=\sum_{1-\frac{\log j}{\log n}-\frac{\log k}{\log m}\ge 0} 1}$$

$$\boxed{[n\cdot m\cdot o]=\sum_{1-\frac{\log j}{\log n}-\frac{\log k}{\log m}-\frac{\log l}{\log o}\ge 0} 1}$$

and so on.

$$\boxed{a \log x = \log a^{x}}$$

$$\boxed{a \log x = \lim_{z \to 0} \frac{\partial}{\partial z} x^{a \cdot z}}$$

$$\boxed{a\left(li(n) - \log\log n - \gamma\right) = \lim_{z \to 0} \frac{\partial}{\partial z} L_{-za}(\log n)}$$

$$\boxed{a\,\Pi(n) = \lim_{z \to 0} \frac{\partial}{\partial z} D_{a \cdot z}(n)}$$

$$\boxed{a \log x = \lim_{z \to 0} \frac{x^{a \cdot z} - 1}{z}}$$

$$\boxed{a\left(li(x) - \log\log x - \gamma\right) = \lim_{z \to 0} \frac{L_{-a \cdot z}(\log x) - 1}{z}}$$

$$\boxed{a\,\Pi(x) = \lim_{z \to 0} \frac{D_{a \cdot z}(x) - 1}{z}}$$

$$\boxed{a \log x = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} (x^{a} - 1)^{k}}$$

$$\boxed{a \log n = \int_1^n \frac{\partial x^a}{\partial x} dx - \frac{1}{2} \int_1^n \int_1^n \frac{\partial x^a}{\partial x} \cdot \frac{\partial y^a}{\partial y} dy\, dx + \frac{1}{3} \int_1^n \int_1^n \int_1^n \frac{\partial x^a}{\partial x} \cdot \frac{\partial y^a}{\partial y} \cdot \frac{\partial z^a}{\partial z} dz\, dy\, dx + \ldots}$$

$$\boxed{a(li(n) - \log\log n - \gamma) = \int_1^n \frac{\partial x^a}{\partial x} dx - \frac{1}{2} \int_1^n \int_1^{\frac{n}{x}} \frac{\partial x^a}{\partial x} \cdot \frac{\partial y^a}{\partial y} dy\, dx + \frac{1}{3} \int_1^n \int_1^{\frac{n}{x}} \int_1^{\frac{n}{x \cdot y}} \frac{\partial x^a}{\partial x} \cdot \frac{\partial y^a}{\partial y} \cdot \frac{\partial z^a}{\partial z} dz\, dy\, dx + \ldots}$$ ….XXXXX FIX

$$\boxed{a\,\Pi(n) = \sum_{j=2}^{\lfloor n \rfloor} \nabla[D]_a(j) - \frac{1}{2} \sum_{j=2}^{\lfloor n \rfloor} \sum_{k=2}^{\lfloor \frac{n}{j} \rfloor} \nabla[D]_a(j) \cdot \nabla[D]_a(k) + \frac{1}{3} \sum_{j=2}^{\lfloor n \rfloor} \sum_{k=2}^{\lfloor \frac{n}{j} \rfloor} \sum_{l=2}^{\lfloor \frac{n}{j \cdot k} \rfloor} \nabla[D]_a(j) \cdot \nabla[D]_a(k) \cdot \nabla[D]_a(l) + \ldots}$$

$$\boxed{L_{-1}(\log x)=x}$$

$$\boxed{\log x-\log y=\int_{v}^{x}\frac{\partial \log(z)}{\partial z}dz=\int_{v}^{x}\frac{1}{z}dz}$$

$$\boxed{li(x)-li(y)=\int_{v}^{x}\frac{\partial\, li(z)}{\partial z}dz=\int_{v}^{x}\frac{1}{\log z}dz}$$

$$\boxed{\Pi(x)-\Pi(y)=\sum_{z=v+1}^{x}\nabla[\Pi](z)=\sum_{z=v+1}^{x}\kappa(z)}$$

$$\boxed{\log 1=0}$$

$$\boxed{\log(1-1)=-\infty}$$

$$\boxed{\lim_{x\to 1}(li(x)-\log\log x-\gamma)=0}$$

$$\boxed{li(1)=-\infty}$$

$$\boxed{\Pi(1)=0}$$

$$\boxed{\lim_{x\to 0}\frac{\log(1+x)}{x}=1}$$

$$\boxed{\lim_{x\to 0}\frac{\log(x)}{x}=-\infty}$$

$$\boxed{\lim_{x\to 0}\frac{li(1+x)-\log\log(1+x)-\gamma}{x}=1}$$

$$\boxed{\lim_{x\to 0}\frac{li(1+x)}{x}=-\infty}$$

(can't do with prime counting function as it is not continuous)

$$\boxed{\lim_{x\to 1}\log' x=1}$$

$$\boxed{\lim_{x\to 1}\log'(x-1)=\infty}$$

$$\boxed{\lim_{x\to 1}(li'(x)-\log\log'(x)-\gamma)=1}$$

$$\boxed{\lim_{x\to 1}(li'(x))=\infty}$$

$$\boxed{\log' x=\frac{1}{x}}$$

$$\boxed{\log'(x-1)=\frac{1}{x-1}}$$

$$\boxed{(li'(x)-\log\log'(x)-\gamma)=\frac{1}{\log x}-\frac{1}{x\log x}}$$

$$\boxed{(li'(x))=\frac{1}{\log x}}$$

$$f(x)=\log x,\ f'\left(\frac{1}{x}\right)=\frac{1}{f'(x)}$$

$$f(x)=\log(x-1),\ f'\left(\frac{1}{x}\right)=-x\,f'(x)$$

$$f(x)=li(x)-\log\log x-\gamma,\ f'\left(\frac{1}{x}\right)=x\,f'(x)$$

$$f(x)=li(x),\ f'\left(\frac{1}{x}\right)=-f'(x)$$

$$f(x)=\log x,\ f\left(\frac{1}{x}\right)=-f(x)$$

$$\boxed{\log 3=\frac{1}{1}+\frac{1}{2}-\frac{2}{3}+\frac{1}{4}+\frac{1}{5}-\frac{2}{6}+\frac{1}{7}+\ldots}$$

$$\boxed{\lim_{n\to\infty} H_n-H_{\frac{n}{x}}=\log x}$$

$$\sum_{k=1}^{\infty}-\frac{(1-z)^k}{k}=\log z$$

$$\lim_{z\to 1}\sum_{k=1}^{\lfloor\log_z n\rfloor}\frac{z^k-1}{k}=li(n)-\log\log n-\gamma$$

$$\boxed{\log x=\int_1^x \frac{1}{z}\,dz}$$

$$\boxed{li(x)-\log\log x=\int_1^x \left(\frac{1}{\log z}\right)\left(1-\frac{1}{z}\right)dz}$$

$$\boxed{li(x)-\log\log(x)-\gamma=\int_0^{x-1}\frac{z}{(z+1)\log(z+1)}\,dz}$$

If you go for a taylor series here, you have something like

$$\boxed{\begin{array}{l} a_0=1 \\ a_n=\frac{1}{n!}\cdot\sum_{k=1}^{n}\frac{(-1)^{k+1}}{k}B_k\cdot s(n,k) \end{array}}$$

where s(n,k) are stirling numbers of the first kind

OR!

$$\boxed{\begin{array}{l} G_n=\lim_{x\to 0}\frac{\partial^n}{\partial x^n}\frac{x}{\log(1-x)} \\ a_0=1 \\ a_n=(-1)^{n+1}\cdot\frac{1}{n!}\cdot\sum_{k=1}^{n}G_k \end{array}}$$

then

$$\boxed{li(x)-\log\log(x)-\gamma=\sum_{k=0}^{\infty}a_k x^k}$$

although what, if anything, that converges for is kind of a question.

$$D_1{}'(n)=(\sum_{k=1}^{\infty}\frac{1}{k!}D'_k(n))*(\sum_{k=0}^{\infty}\frac{B_k}{k!}D'_k(n))$$

$$n-1=(\sum_{k=1}^{\infty}\frac{1}{k!}(n-1)^k)\cdot(\sum_{k=0}^{\infty}\frac{B_k}{k!}(n-1)^k)$$

$$D_a{}'(n)=(\sum_{k=1}^{\infty}\frac{1}{k!}D'_k(n))*(\sum_{k=0}^{\infty}\frac{B_k}{k!}D'_{k+a}(n))$$

$$(n-1)^a=(\sum_{k=1}^{\infty}\frac{1}{k!}(n-1)^k)\cdot(\sum_{k=0}^{\infty}\frac{B_k}{k!}(n-1)^{k+a})$$

$$L_{-n}(\log x)=\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\binom{-n}{k}(\log x)^k$$

$$li(n)=\sum_{k=1}^{\infty}\frac{(\log n)^k}{k!k}$$

$$li(n)=\lim_{x\to 1}\sum_{k=1}^{\lfloor\frac{\log n}{\log x}\rfloor}\frac{x^k-1}{k}$$

$$L_{-z}(n)=\sum_{k=0}^{\infty}(-1)^k\cdot\frac{1}{k!}\binom{-z}{k}n^k$$

$$L_{-z}(n)=\sum_{k=0}^{\infty}(-1)^k\cdot\binom{z}{k}\cdot\frac{\gamma(k,-\log n)}{\Gamma(k)}$$

$$\lim_{z\to 0}\frac{\partial}{\partial z}L_{-z}(\log n)=\lim_{z\to 0}\frac{L_{-z}(\log n)-1}{z}=li(n)-\log\log n-\gamma$$

$$\lim_{x\to 1}\sum_{k=1}^{\lfloor\frac{\log n}{\log x}\rfloor}\frac{x^k-1}{k}=li(n)-\log\log n-\gamma$$

$$\sum_{k=1}^{\infty}\frac{(-1)^k\cdot\gamma(k,-\log n)}{\Gamma(k)}=li(n)-\log\log n-\gamma$$

$$\lim_{z\to 0}\frac{\partial}{\partial z}(-1)^z\frac{\gamma(z,-\log n)}{\Gamma(z)}=\lim_{z\to 0}\frac{(-1)^z\frac{\gamma(z,-\log n)}{\Gamma(z)}-1}{z}=li(n)+2\pi i$$

$$\frac{\partial}{\partial x}\log x=\frac{1}{x}$$

$$\frac{\partial}{\partial x}\log(x-1)=\frac{1}{x-1}$$

$$\frac{\partial}{\partial x}li(x)-\log\log x-\gamma=\frac{1}{\log x}-\frac{1}{x\log x}$$

$$\frac{\partial}{\partial x}li(x)=\frac{1}{\log x}$$

$$\lim_{x\to 1}\frac{1}{x}=1$$

$$\lim_{x\to 1}\left(\frac{1}{\log(x-1)}\right)=\infty$$

$$\lim_{x\to 1}\frac{1}{\log x}-\frac{1}{x\log x}=1$$

$$\lim_{x\to 1}\left(\frac{1}{\log x}\right)=\infty$$

$$\log 1=0$$

$$\log(1-1)=-\infty$$

$$\lim_{x\to 1}li(x)-\log\log x-\gamma=0$$

$$li(1)=-\infty$$

$$\lim_{x\to 0}\frac{\log(1+x)}{x}=1$$

$$\lim_{x\to 0}\frac{\log(x)}{x}=-\infty$$

$$\lim_{x\to 0}\frac{li(1+x)-\log\log(1+x)-\gamma}{x}=1$$

$$\lim_{x\to 0}\frac{li(1+x)}{x}=-\infty$$

$$\int \frac{1}{\log x}\,dx = li(x)$$

$$\int \frac{x}{\log x}\,dx = li(x^2)$$

$$\int \frac{x^z}{\log x}\,dx = li(x^{z+1})$$

$$\int \left(1-\frac{1}{x}\right)\left(\frac{x^z}{\log x}\right)dx = li(x^{z+1}) - li(x^z)$$

$$\int \frac{x^{s-1}}{\log x}\,dx = li(x^s)$$

EXCEPT IF s=0, then

$$\int \frac{x^{0-1}}{\log x}\,dx = \log\log x$$

Compare!!!!

$$\int x^{s-1}\,dx = \frac{x^s}{s}$$

EXCEPT IF s=0, then

$$\int x^{0-1}\,dx = \log x$$

$$\int \frac{x^{s-1}}{(\log x)^k}\,dx = -(-s)^{k-1}\,\Gamma(1-k, -s\log x)$$

$$\int \frac{1}{(\log x)^k}\,dx = (-1)^k\,\Gamma(1-k, -\log x)$$

$$\int \left(\log\frac{1}{x}\right)^z dx = \Gamma\left(1+z, \log\frac{1}{x}\right)$$

$$\int_1^n \left(\log\frac{1}{x}\right)^z dx = -\gamma(1+z, -\log x)$$

$$\lim_{x\to 1} \int \left(\log\frac{1}{x}\right)^z dx = z!$$

IN GENERAL

$$\int x^{s-1}(\log x)^{k-1}dx=-(-s)^{-k}\,\Gamma(k,-s\log x)$$

IF k=0, then

$$\int x^{s-1}(\log x)^{0-1}dx=li(x^{s})$$

IF s=0, then

$$\int x^{0-1}(\log x)^{k-1}dx=\frac{\log(x)^{k}}{k}$$

$$\int x^{0-1}(\log x)^{1-1}dx=\log(x)$$ and $$\int x^{1-1}(\log x)^{0-1}dx=li(x)$$

IF s=-1, then

$$\int_{1}^{\infty} x^{-1-1}(\log x)^{k-1}dx=\Gamma(k)$$ which is $$\int_{1}^{\infty}\frac{(\log x)^{k}}{x^{2}}dx=k\,!$$

IF s=0 and k=0, then

$$\int x^{0-1}(\log x)^{0-1}dx=\log\log x$$

$$\int (e^{x})^{s}\cdot x^{k-1}dx=-(-s)^{-k}\,\Gamma(k,-s\,x)$$

IF k=0, then

$$\int (e^{x})^{s}\cdot x^{0-1}dx=Ei(s\,x)$$

IF s=0, then

$$\int (e^{x})^{0}\cdot x^{k-1}dx=\frac{x^{k}}{k}$$

$$\int (e^{x})^{0}\cdot x^{1-1}dx=x$$ and $$\int (e^{x})^{1}\cdot x^{0-1}dx=Ei(x)$$

IF s=-1, then

$$\int_{0}^{\infty}(e^{x})^{-1}\cdot x^{k-1}dx=\Gamma(k)$$

IF s=0 and k=0, then

$$\int (e^{x})^{0}\cdot x^{0-1}dx=\log x$$

$$\int x^{s-1}(\log x)^{k-1}\,dx = -(-s)^{-k}\,\Gamma(k, -s\log x)$$

$$\int (e^{x})^{s}\cdot x^{k-1}\,dx = -(-s)^{-k}\,\Gamma(k, -s\,x)$$

Big question – does this, or how does this, relate to the Laplace transform?

Is there some Laplace transform equivalent using the Lagerre L polynomials where x^n would go? Preliminary investigations look not great.

$$L_{-z}(\log x)=\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\binom{-z}{k}(\log x)^k$$
$$L_{-z}(\log x)=\sum_{k=0}^{\infty}(k!)^{-1}\binom{z+k-1}{k}(\log x)^k$$
$$L_{-z}(n)=\sum_{k=0}^{\infty}(-1)^k\cdot\binom{z}{k}\cdot\frac{\gamma(k,-\log n)}{\Gamma(k)}$$

$$\boxed{\nabla[D]_{a+b}(n)=\sum_{j=1}^{n}\sum_{k=1}^{\lfloor\frac{n}{j}\rfloor}\nabla[D]_a(j)\cdot\nabla[D]_b(k)-\sum_{j=1}^{n-1}\sum_{k=1}^{\lfloor\frac{n-1}{j}\rfloor}\nabla[D]_a(j)\cdot\nabla[D]_b(k)}$$

$$\nabla[D]_{a+1}(n)=\sum_{j=1}^{n}\sum_{k=1}^{\lfloor\frac{n}{j}\rfloor}\nabla[D]_a(j)-\sum_{j=1}^{n-1}\sum_{k=1}^{\lfloor\frac{n-1}{j}\rfloor}\nabla[D]_a(j)$$

$$\nabla[D]_{a+1}(n)=\nabla[D]_a(n)+\sum_{j=1}^{n-1}(\lfloor\frac{n}{j}\rfloor-\lfloor\frac{n-1}{j}\rfloor)\nabla[D]_a(j)$$

$$\nabla[D]_{a+1}(n)=\nabla[D]_a(n)+\sum_{j=1}^{n-1}\nabla[D]_1(\frac{n}{j})\nabla[D]_a(j)$$

$$\nabla[D]_1(n)=0\ if\ n!\in\mathbb{N}$$

How does this connect with the Laguerre L Polynomial derivatives?

$$L_{-z}(n)=\sum_{k=0}^{\infty}(-1)^k\cdot\binom{z}{k}\cdot\frac{\gamma(k,-\log n)}{\Gamma(k)}$$

$$\frac{\partial}{\partial n}L_{-z}(n)=\sum_{k=0}^{\infty}((-1)^k\cdot\binom{z}{k}\cdot\frac{1}{\Gamma(k)})\cdot(-1)^k\cdot(\log n)^{k-1}$$

$$\frac{\partial}{\partial n}L_{-z}(n)=\sum_{k=0}^{\infty}\binom{z}{k}\cdot\frac{(\log n)^{k-1}}{\Gamma(k)}=z\,{}_1F_1(1-z,2,-\log n)$$

$$L_{-(a+b)}(\log n)=1+\int_1^n\frac{\partial L_{-a}(\log x)}{\partial x}dx+\int_1^n\frac{\partial L_{-b}(\log y)}{\partial y}dy+\int_1^n\int_1^{\frac{n}{x}}\frac{\partial L_{-a}(\log x)}{\partial x}\cdot\frac{\partial L_{-b}(\log y)}{\partial y}dy\,dx$$

$$\frac{\partial}{\partial n}L_{-(a+b)}(\log n)=\int_1^n\frac{\partial L_{-a}(\log x)}{\partial x}dx+\int_1^n\frac{\partial L_{-b}(\log y)}{\partial y}dy+\int_1^n\int_1^{\frac{n}{x}}\frac{\partial L_{-a}(\log x)}{\partial x}\cdot\frac{\partial L_{-b}(\log y)}{\partial y}dy\,dx$$

$$\frac{\partial}{\partial n}L_{-(a+1)}(\log n)=\int_1^n\frac{\partial L_{-a}(\log x)}{\partial x}dx+\int_1^n\frac{\partial L_{-1}(\log y)}{\partial y}dy+\int_1^n\int_1^{\frac{n}{x}}\frac{\partial L_{-a}(\log x)}{\partial x}\cdot\frac{\partial L_{-1}(\log y)}{\partial y}dy\,dx$$

$$\frac{\partial}{\partial n}L_{-(a+1)}(\log n)=\int_1^n\frac{\partial L_{-a}(\log x)}{\partial x}dx+1+\int_1^n\int_1^{\frac{n}{x}}\frac{\partial L_{-a}(\log x)}{\partial x}dy\,dx$$

$$\frac{\partial}{\partial n}L_{-(a+1)}(\log n)=\int_1^n\frac{\partial L_{-a}(\log x)}{\partial x}dx+1+\int_1^n\frac{\partial L_{-a}(\log x)}{\partial x}(\frac{n}{x}-1)dx$$

$$\frac{\partial}{\partial n}L_{-(a+1)}(\log n)=1+n\int_1^n\frac{\partial L_{-a}(\log x)}{\partial x}x^{-1}dx$$

$$\frac{\partial}{\partial x}x^a=a\int_0^x\frac{\partial}{\partial y}y^{a-1}dx$$

$$L_{-z}(\log x) = {}_1F_1(z, 1, \log x)$$

$$\frac{\partial}{\partial x} L_{-z}(\log x) = \sum_{k=0}^{\infty} \binom{z}{k} \cdot \frac{(\log x)^{k-1}}{\Gamma(k)} = \frac{z}{x} \cdot {}_1F_1(1+z, 2, \log x)$$

$$L_{-z}(\log n) = 1 + z \cdot \int_1^n x^{-1} \cdot {}_1F_1(1+z, 2, \log x)\, dx$$

$$\boxed{L'_{-z}(\log n) = 1 + z \cdot \int_1^n \frac{1}{x \log x} \cdot (L_{-z-1}(\log x) - L_{-z}(\log x))\, dx}$$

$$\lim_{z \to 0} \frac{\partial}{\partial z} z\, {}_1F_1(1-z, 2, -\log n) = \frac{1}{\log n} - \frac{1}{n \log n}$$

$$\lim_{z \to 0} \frac{\partial^2}{\partial z^2} z\, {}_1F_1(1-z, 2, -\log n) = -2 \sum_{k=0}^{\infty} \frac{(-1)^k H_k (\log n)^k}{(k+1)!}$$

$$L_{-z}(\log n) = \sum_{k=0}^{\infty} \lim_{z \to 0} \frac{\partial^k}{\partial z^k} L_{-z}(\log n) \cdot \frac{z^k}{k!}$$

$$\lim_{z \to 0} \frac{\partial^k}{\partial z^k} L_{-z}(1) = 0$$

$$L_n^m(x) = \frac{(m+n)!}{m!\, n!}\, {}_1F_1(-n, m+1, x)$$

$$L_{-z}(\log x) = {}_1F_1(z, 1, \log x)$$

$$L_{-z}^1(\log x) = (1-z)\, {}_1F_1(z, 2, \log x)$$

$$x L'_n(x) = n L_n(x) - n L_{n-1}(x)$$

$$L'_{-z}(\log n) = \frac{z}{n \log n} \cdot (L_{-z-1}(\log n) - L_{-z}(\log n))$$

$$\boxed{L'_{-z}(\log n) = \frac{z}{n \log n} \cdot (L_{-z-1}(\log n) - L_{-z}(\log n))}$$

$$\frac{\partial}{\partial x} x^{*z} = \frac{z}{x \log x} \cdot (x^{*z+1} - x^{*z})$$

$$\lim_{z \to 0} \frac{\partial}{\partial z} L'_{-z}(\log n) = \frac{L_{-1}(\log n) - L_0(\log n)}{n \log n}$$

$$\boxed{\lim_{z \to 0} \frac{\partial}{\partial z} L'_{-z}(\log n) = \frac{L_{-1}(\log n) - L_0(\log n)}{n \log n}}$$

Viola. That's great.

…?

$$\boxed{1+z\int_{-\log n}^{0} e^{t(s-1)}\,{}_1F_1(1-z;2;t)\,dt}$$

$$\boxed{1+z\int_{1}^{x} t^{-s}\,{}_1F_1(1-z;2;-\log t)\,dt}$$

$$\boxed{L_{-z}(\log x)=1+z\int_{1}^{x} {}_1F_1(1-z;2;-\log t)\,dt}$$

$$L_{-z}(\log x)=1+z\cdot\int_{1}^{x} t^{-1}\cdot{}_1F_1(1+z,2,\log t)\,dt$$

$$\frac{\partial}{\partial x}L_{-z}(\log x)=\frac{z}{x}\cdot{}_1F_1(1+z,2,\log x)$$

$$\frac{\partial}{\partial x}L_{-z}(\log x)=\frac{z}{x\log x}\cdot\left(L_{-z-1}(\log x)-L_{-z}(\log x)\right)$$

$$\boxed{\sum_{k=0}^{\infty}\binom{z}{k}\frac{1}{(s-1)^k}\cdot\frac{\gamma(k,(s-1)\log n)}{\Gamma(k)}}$$

$$\boxed{\begin{aligned}&\Gamma(0,s\log n)-\Gamma(0,(s-1)\log n)+\log(s)-\log(s-1)=\\&\quad\int_1^n x^{-s}\,dx-\frac{1}{2}\int_1^n\int_1^{\frac{n}{x}}(x\cdot y)^{-s}\,dy\,dx+\frac{1}{3}\int_1^n\int_1^{\frac{n}{x}}\int_1^{\frac{n}{x\,y}}(x\cdot y\cdot z)^{-s}\,dz\,dy\,dx-\frac{1}{4}\ldots\end{aligned}}$$

$$L'_{-z}(\log n)=\frac{z}{n\log n}\cdot\left(L_{-z-1}(\log n)-L_{-z}(\log n)\right)$$

$$\lim_{z\to 0}\frac{\partial}{\partial z}L'_{-z}(\log n)=\frac{L_{-1}(\log n)-L_0(\log n)}{n\log n}=\frac{n-1}{n\log n}$$

$$\frac{\partial}{\partial x}li(x^a)-li(x^b)=\frac{x^a-x^b}{x\log x}$$

$$\frac{\partial}{\partial x}li(x^{s+1})-li(x^s)=\frac{(x-1)\,x^{s-1}}{\log x}$$

$$\boxed{\frac{\partial}{\partial x}\Gamma(0,s\log x)-\Gamma(0,(s-1)\log x)+\log(s)-\log(s-1)=\frac{(x-1)\,x^{s-1}}{\log x}}$$

$$L'_{-z}(\log n)=\frac{z}{n\log n}\cdot(L_{-z-1}(\log n)-L_{-z}(\log n))$$

$$\int_1^x \frac{z}{n\log n}\cdot(L_{-z-1}(\log n)-L_{-z}(\log n))\,dn=L_{-z}(\log x)-1$$

$$\int_0^x \frac{z}{n\log n}\cdot(L_{-z-1}(\log n)-L_{-z}(\log n))\,dn=L_{-z}(\log x)$$

$$\int_0^1 \frac{z}{n\log n}\cdot(L_{-z-1}(\log n)-L_{-z}(\log n))\,dn=\int_0^1 L'_{-z}(\log x)\,dx=1$$

$$L'_{-1}(\log x)=1$$

$$L_{-(a+b)}(\log n)=1+\int_1^n \frac{\partial L_{-a}(\log x)}{\partial x}dx+\int_1^n \frac{\partial L_{-b}(\log y)}{\partial y}dy+\int_1^n\int_1^{\frac{n}{x}} \frac{\partial L_{-a}(\log x)}{\partial x}\cdot\frac{\partial L_{-b}(\log y)}{\partial y}dy\,dx$$

$$L_{-(a+b)}(\log n)=1+\int_1^n L'_{-a}(\log x)dx+\int_1^n L'_{-b}(\log y)dy+\int_1^n\int_1^{\frac{n}{x}} L'_{-a}(\log x)\cdot L'_{-b}(\log y)\,dy\,dx$$

$$L_{-(a+b)}(\log n)=1+\int_1^n L'_{-a}(\log x)dx+\int_1^n L'_{-b}(\log y)dy+\int_1^n\int_1^{\frac{n}{x}} L'_{-a}(\log x)\cdot L'_{-b}(\log y)\,dy\,dx$$

$$L_{-(a+b)}(\log n)=1+\int_1^n L'_{-a}(\log x)dx+\int_1^n L'_{-b}(\log y)dy+\int_1^n L'_{-a}(\log x)\cdot(\int_1^{\frac{n}{x}} L'_{-b}(\log y)\,dy)\,dx$$

$$L_{-(a+b)}(\log n)=1+\int_1^n L'_{-a}(\log x)dx+\int_1^n L'_{-b}(\log y)dy+\int_1^n L'_{-a}(\log x)\cdot(L_{-b}(\log\frac{n}{x})-1)\,dx$$

$$L_{-(a+b)}(\log n)=1+\int_1^n L'_{-a}(\log x)dx+\int_1^n L'_{-b}(\log y)dy-\int_1^n L'_{-a}(\log x)dx+\int_1^n L'_{-a}(\log x)\cdot(L_{-b}(\log n-\log x))\,dx$$

$$L_{-(a+b)}(\log n)=1+\int_1^n L'_{-b}(\log y)dy-\int_1^n L'_{-a}(\log x)\cdot(L_{-b}(\log n-\log x))\,dx$$

$$\boxed{L_{-(a+b)}(\log n)=L_{-b}(\log n)-\int_1^n L'_{-a}(\log x)\cdot(L_{-b}(\log\frac{n}{x}))\,dx}$$

$$L_{-(1+b)}(\log n)=L_{-b}(\log n)-\int_1^n L_{-b}(\log n-\log x)dx$$

$$\boxed{x^{*z} = L_{-z}(\log x)}$$

$$\boxed{(x-1)^{*z} = (-1)^z \frac{\gamma(z, -\log x)}{\Gamma(z)}}$$

$$\boxed{\frac{\partial}{\partial x}(x-1)^{*z} = \frac{(\log x)^{z-1}}{z!}} \qquad \text{(versus } \boxed{\frac{\partial}{\partial x}(x-1)^z = z \cdot (x-1)^{z-1}}\text{)}$$

$$\frac{\partial}{\partial x} x^{*z} = \frac{z}{x \log x} \cdot (x^{*z+1} - x^{*z}) \qquad \text{(compare to } \frac{\partial}{\partial x} x^z = \frac{z}{x(x-1)} \cdot (x^{z+1} - x^z)\text{)}$$

$$\lim_{x \to 1} \frac{\partial}{\partial x} x^{*z} = z \qquad \text{(compare to } \lim_{x \to 1} \frac{\partial}{\partial x} x^z = z\text{)}$$

$$x^{*0} = 1 \qquad \text{(compare to } x^0 = 1\text{)}$$
$$x^{*1} = x \qquad \text{(compare to } x^1 = x\text{)}$$
$$\frac{\partial}{\partial x} x^{*1} = 1 \qquad \text{(compare to } \frac{\partial}{\partial x} x^1 = 1\text{)}$$
$$\frac{\partial}{\partial x} x^{*0} = 0 \qquad \text{(compare to } \frac{\partial}{\partial x} x^0 = 0\text{)}$$

$$(x-1)^{*0} = 1 \qquad \text{(compare to } (x-1)^0 = 1\text{)}$$
$$(x-1)^{*1} = x-1 \qquad \text{(compare to } (x-1)^1 = x-1\text{)}$$

$$\boxed{x^{*z} = \sum_{k=0}^{\infty} \binom{z}{k} (x-1)^{*k}}$$

$$\boxed{(x-1)^{*z} = \sum_{k=0}^{\infty} (-1)^k \binom{z}{k} x^{*k}} \text{ if z is positive integer}$$

$$\boxed{(n-1)^{*a+b} = \int_1^n \int_1^{\frac{n}{x}} \frac{\partial (x-1)^{*a}}{\partial x} \cdot \frac{\partial (y-1)^{*b}}{\partial y} \, dy \, dx}$$

$$\int_0^1 \frac{\partial}{\partial x} x^{*z} \, dx = 1 \qquad \text{(compare to } \int_0^1 \frac{\partial}{\partial x} x^z \, dx = 1\text{)}$$

$$\boxed{n^{*a+b} = 1 + \int_1^n \frac{\partial x^{*a}}{\partial x} dx + \int_1^n \frac{\partial y^{*b}}{\partial y} dy + \int_1^n \int_1^{\frac{n}{x}} \frac{\partial a^{*x}}{\partial x} \cdot \frac{\partial y^{*b}}{\partial y} \, dy \, dx}$$

// ugh. The sum bounds are messy. Should start at 1 rather than 0 for logs.

$$f(x)\cdot g(x)=\int_0^x\int_0^x \frac{\partial}{\partial s}f(s)\cdot\frac{\partial}{\partial t}g(t)\,dt\,ds$$

$$f(x)*g(x)=\int_1^x\int_1^{\frac{x}{s}} \frac{\partial}{\partial s}f(s)\cdot\frac{\partial}{\partial t}g(t)\,dt\,ds$$

$$f(x)[*]g(x)=\sum_{s=1}^{x}\sum_{t=1}^{\lfloor\frac{x}{s}\rfloor}\nabla f(s)\cdot\nabla g(t)$$

$$(\log x)^2=\int_1^x\int_1^x \log' s\cdot\log' t\,dt\,ds$$

$$(*\log x)^{*2}=\int_1^x\int_1^{\frac{x}{s}} *\log' s\cdot *\log' t\,dt\,ds$$

$$([\log]x)^{[2]}=\sum_{s=2}^{x}\sum_{t=2}^{\lfloor\frac{x}{s}\rfloor}\nabla[\log]s\cdot\nabla[\log]t$$

$$(\log x)^3=\int_1^x\int_1^x\int_1^x \log' s\cdot\log' t\cdot\log' u\,du\,dt\,ds$$

$$(*\log x)^{*3}=\int_1^x\int_1^{\frac{x}{s}}\int_1^{\frac{x}{s\cdot t}} *\log' s\cdot *\log' t\cdot *\log' u\,du\,dt\,ds$$

$$([\log]x)^{[3]}=\sum_{s=2}^{x}\sum_{t=2}^{\lfloor\frac{x}{s}\rfloor}\sum_{u=2}^{\lfloor\frac{x}{s\cdot t}\rfloor}\nabla[\log]s\cdot\nabla[\log]t\cdot\nabla[\log]u$$

$$(\log x)^{a+b}=\int_1^x\int_1^x \frac{\partial}{\partial s}(\log s)^a\cdot\frac{\partial}{\partial t}(\log t)^b\,dt\,ds$$

$$(*\log x)^{*a+b}=\int_1^x\int_1^{\frac{x}{s}} \frac{\partial}{\partial s}(*\log s)^{*a}\cdot\frac{\partial}{\partial t}(*\log t)^{*b}\,dt\,ds \quad \text{////??????}$$

$$([\log]x)^{[a+b]}=\sum_{s=2}^{x}\sum_{t=2}^{\lfloor\frac{x}{s}\rfloor}\nabla([\log]s)^{[a]}\cdot\nabla([\log]t)^{[b]}$$

$$(\log x)^a=\lim_{z\to 0}\frac{\partial^a}{\partial z^a}x^z$$

$$(*\log x)^{*a}=\lim_{z\to 0}\frac{\partial^a}{\partial z^a}x^{*z}$$

$$([\log]x)^{[a]}=\lim_{z\to 0}\frac{\partial^a}{\partial z^a}x^{[z]}$$

$$\frac{\partial}{\partial x}(\log x)^{a+b} = x^{-1}(a+b)(\log x)^{a+b-1}$$
$$\frac{\partial}{\partial x}(*\log x)^{*a+b} = \sum_{k=1}^{\infty} k^{-1}(k!)^{-1}\frac{\partial^{a+b}}{\partial x^{a+b}}(\log x)^{k}$$
$$\nabla([\log]x)^{[a+b]} = \sum_{s\cdot t=x} \nabla([\log]s)^{[a]} \cdot \nabla([\log]t)^{[b]}$$

$$(\log 1)^{a} = 0$$
$$(*\log 1)^{*a} = 0$$
$$([\log]1)^{[a]} = 0$$

$$x^{z} = \sum_{k=0}^{\infty} \frac{z^{k}\cdot(\log x)^{k}}{k!}$$
$$x^{*z} = \sum_{k=0}^{\infty} \frac{z^{k}\cdot(*\log x)^{*k}}{k!}$$
$$x^{[z]} = \sum_{k=0}^{\infty} \frac{z^{k}\cdot([\log]x)^{[k]}}{k!}$$

$$\frac{\partial}{\partial z}x^{z} = \sum_{k=0}^{\infty} \frac{z^{k}\cdot(\log x)^{k+1}}{k!}$$
$$\frac{\partial}{\partial z}x^{*z} = \sum_{k=0}^{\infty} \frac{z^{k}\cdot(*\log x)^{*k+1}}{k!}$$
$$\frac{\partial}{\partial z}x^{[z]} = \sum_{k=0}^{\infty} \frac{z^{k}\cdot([\log]x)^{[k+1]}}{k!}$$

$$\frac{\partial}{\partial z}x^{z} = \log x \cdot x^{z}$$
$$\frac{\partial}{\partial z}x^{*z} = *\log x * x^{*z}$$
$$\frac{\partial}{\partial z}x^{[z]} = [\log]x[*]x^{[z]}$$

$$x^{z} = 1 + \int_{0}^{z} \log(x)\cdot x^{y}\,dy$$
$$x^{z} = 1 + \int_{1}^{x}\int_{0}^{x}\int_{0}^{z} \frac{\partial}{\partial s}\log(s)\cdot\frac{\partial}{\partial t}t^{y}\,dy\,dt\,ds$$
???
$$x^{[z]} = 1 + \sum_{s=2}^{x}\sum_{t=1}^{\lfloor\frac{x}{s}\rfloor}\int_{0}^{z} \nabla[\log]s\cdot\nabla t^{[y]}\,dy$$

$$(\log x)^{z} = \sum_{k=0}^{\infty} \frac{B_k}{k!}\cdot(\log x)^{k+z-1}\cdot(x-1)$$
$$(\log x)^{z} = \sum_{k=0}^{\infty}\int_{1}^{x}\int_{1}^{x} \frac{B_k}{k!}\cdot\frac{\partial}{\partial s}(\log s)^{k+z-1}\cdot\frac{\partial}{\partial t}(t-1)\,dt\,ds$$
???
$$([\log]x)^{[z]} = \sum_{k=0}^{\infty}\sum_{s=2}^{x}\sum_{t=2}^{\lfloor\frac{x}{s}\rfloor} \frac{B_k}{k!}\cdot\nabla([\log]s)^{[k+z-1]}\cdot\nabla(t-1)^{[1]}$$

$$[\log]x=\sum_{k=0}^{\infty}\sum_{s=2}^{x}\sum_{t=2}^{\lfloor\frac{x}{s}\rfloor}\frac{B_k}{k!}\cdot\nabla([\log]s)^{[k]}\cdot\nabla(t-1)^{[1]}$$

$$(x-1)^z=\sum_{k=0}^{\infty}\frac{C_k}{k!}\cdot(x-1)^{k+z-1}\cdot(\log x)$$

$$(x-1)^z=\sum_{k=0}^{\infty}\int_1^x\int_1^x\frac{C_k}{k!}\cdot\frac{\partial}{\partial s}(s-1)^{k+z-1}\cdot\frac{\partial}{\partial t}(\log t)\,dt\,ds$$

???

$$(x-1)^{[z]}=\sum_{k=0}^{\infty}\sum_{s=2}^{x}\sum_{t=2}^{\lfloor\frac{x}{s}\rfloor}\frac{C_k}{k!}\cdot\nabla(s-1)^{[k+z-1]}\cdot\nabla[\log]t$$

$$a\log x=\lim_{z\to 0}\frac{\partial}{\partial z}x^{a\cdot z}$$

$$a*\log x=\lim_{z\to 0}\frac{\partial}{\partial z}x^{*az}$$

$$a[\log]x=\lim_{z\to 0}\frac{\partial}{\partial z}x^{[az]}$$

$$*\log x^a=\lim_{z\to 0}\frac{\partial}{\partial z}(x^a)^{*z}$$

$$[\log]x^a=\lim_{z\to 0}\frac{\partial}{\partial z}(x^a)^{[z]}$$

$$a\log x=\lim_{z\to 0}\frac{x^{a\cdot z}-1}{z}$$

$$a*\log x=\lim_{z\to 0}\frac{x^{*az}-1}{z}$$

$$a[\log]x=\lim_{z\to 0}\frac{x^{[az]}-1}{z}$$

$$*\log x^a=\lim_{z\to 0}\frac{(x^a)^{*z}-1}{z}$$

$$[\log]x^a=\lim_{z\to 0}\frac{(x^a)^{[z]}-1}{z}$$

$$a\log x=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\left(x^{a}-1\right)^{k}$$

$$a\log x=\log x^{a}=\int_{1}^{x}\frac{\partial s^{a}}{\partial s}sx-\frac{1}{2}\int_{1}^{x}\int_{1}^{x}\frac{\partial s^{a}}{\partial s}\cdot\frac{\partial t^{a}}{\partial t}dt\,ds+\frac{1}{3}\int_{1}^{x}\int_{1}^{x}\int_{1}^{x}\frac{\partial s^{a}}{\partial s}\cdot\frac{\partial t^{a}}{\partial t}\cdot\frac{\partial u^{a}}{\partial u}du\,dt\,ds+\ldots$$

$$*\log x^{a}=\int_{1}^{x}\frac{\partial s^{a}}{\partial s}ds-\frac{1}{2}\int_{1}^{x}\int_{1}^{\frac{x}{s}}\frac{\partial s^{a}}{\partial s}\cdot\frac{\partial t^{a}}{\partial t}dt\,ds+\frac{1}{3}\int_{1}^{x}\int_{1}^{\frac{x}{s}}\int_{1}^{\frac{x}{st}}\frac{\partial s^{a}}{\partial s}\cdot\frac{\partial t^{a}}{\partial t}\cdot\frac{\partial u^{a}}{\partial u}du\,dt\,ds+\ldots$$

$$*\log x^{a}=\int_{1}^{x^{a}}\frac{\partial s}{\partial s}ds-\frac{1}{2}\int_{1}^{x^{a}}\int_{1}^{\frac{x^{a}}{s}}\frac{\partial s}{\partial s}\cdot\frac{\partial t}{\partial t}dt\,ds+\frac{1}{3}\int_{1}^{x^{a}}\int_{1}^{\frac{x^{a}}{s}}\int_{1}^{\frac{x^{a}}{st}}\frac{\partial s}{\partial s}\cdot\frac{\partial t}{\partial t}\cdot\frac{\partial u}{\partial u}du\,dt\,ds+\ldots$$

$$a(*\log x)=\int_{1}^{x}\frac{\partial s^{*a}}{\partial s}ds-\frac{1}{2}\int_{1}^{x}\int_{1}^{\frac{x}{s}}\frac{\partial s^{*a}}{\partial s}\cdot\frac{\partial t^{*a}}{\partial t}dt\,ds+\frac{1}{3}\int_{1}^{x}\int_{1}^{\frac{x}{s}}\int_{1}^{\frac{x}{st}}\frac{\partial s^{*a}}{\partial s}\cdot\frac{\partial t^{*a}}{\partial t}\cdot\frac{\partial u^{*a}}{\partial u}du\,dt\,ds+\ldots$$

$$a[\log]x=\sum_{s=2}^{x}\nabla s^{[a]}-\frac{1}{2}\sum_{s=2}^{x}\sum_{t=2}^{\lfloor\frac{x}{s}\rfloor}\nabla s^{[a]}\cdot\nabla t^{[a]}+\frac{1}{3}\sum_{s=2}^{x}\sum_{t=2}^{\lfloor\frac{x}{s}\rfloor}\sum_{u=2}^{\lfloor\frac{x}{s\cdot t}\rfloor}\nabla s^{[a]}\cdot\nabla t^{[a]}\cdot\nabla u^{[a]}+\ldots$$

$$[\log]x^{a}=\sum_{s=2}^{x^{a}}\nabla s^{[1]}-\frac{1}{2}\sum_{s=2}^{x^{a}}\sum_{t=2}^{\lfloor\frac{x^{a}}{s}\rfloor}\nabla s^{[1]}\cdot\nabla t^{[1]}+\frac{1}{3}\sum_{s=2}^{x^{a}}\sum_{t=2}^{\lfloor\frac{x^{a}}{s}\rfloor}\sum_{u=2}^{\lfloor\frac{x^{a}}{s\cdot t}\rfloor}\nabla s^{[1]}\cdot\nabla t^{[1]}\cdot\nabla u^{[1]}+\ldots$$

$$(x^{a})^{b}=(x^{b})^{a}=x^{a\cdot b}$$

???

???

$$\log a+\log b=\log a\cdot b$$

???

???

$$\log a+\log b=\log a\cdot b$$

$$[\log]x-*\log x=¿$$

$$\log\frac{1}{a} = -\log a$$

$$\log e^{x} = e^{\log x} = x$$
$$\log_{y} y^{x} = y^{\log_{y} x} = x$$

$$f(x) = \log x + \frac{1}{4}(\log x)^{2} + \frac{1}{18}(\log x)^{3} + \frac{1}{96}(\log x)^{4} + \frac{1}{600}(\log x)^{5} + \ldots$$

… inverse function?

$$\lfloor x-1\rfloor=(x-1)^{[1]}=\sum_{s=2}^{x} 1$$

$$(x-1)^{[2]}=\sum_{s=2}^{x}\sum_{t=2}^{\lfloor\frac{x}{s}\rfloor} 1$$

$$(x-1)^{[3]}=\sum_{s=2}^{x}\sum_{t=2}^{\lfloor\frac{x}{s}\rfloor}\sum_{u=2}^{\lfloor\frac{x}{s\cdot t}\rfloor} 1$$

$$(x^{[a]}-1)^{[1]}=\sum_{s=2}^{x}\nabla s^{[a]}$$

$$(x^{[a]}-1)^{[2]}=\sum_{s=2}^{x}\sum_{t=2}^{\lfloor\frac{x}{s}\rfloor}\nabla s^{[a]}\cdot\nabla t^{[a]}$$

$$(x^{[a]}-1)^{[3]}=\sum_{s=2}^{x}\sum_{t=2}^{\lfloor\frac{x}{s}\rfloor}\sum_{u=2}^{\lfloor\frac{x}{s\cdot t}\rfloor}\nabla s^{[a]}\cdot\nabla t^{[a]}\cdot\nabla u^{[a]}$$

$$[\log]x=\sum_{s=2}^{\lfloor x\rfloor}\nabla[\log]s$$

$$([\log]x)^{[2]}=\sum_{s=2}^{\lfloor\frac{x}{2}\rfloor}\sum_{t=2}^{\lfloor\frac{x}{s}\rfloor}\nabla[\log]s\cdot\nabla[\log]t$$

$$([\log]x)^{[3]}=\sum_{s=2}^{\lfloor\frac{x}{4}\rfloor}\sum_{t=2}^{\lfloor\frac{x}{2s}\rfloor}\sum_{u=2}^{\lfloor\frac{x}{s\cdot t}\rfloor}\nabla[\log]s\cdot\nabla[\log]t\cdot\nabla[\log]u$$

$$x^{[z]}=1+\frac{z^1}{1!}\sum_{s=2}^{x}\nabla[\log]s+\frac{z^2}{2!}\sum_{s=2}^{x}\sum_{t=2}^{\lfloor\frac{x}{s}\rfloor}\nabla[\log]s\cdot\nabla[\log]t+\frac{z^3}{3!}\sum_{s=2}^{x}\sum_{t=2}^{\lfloor\frac{x}{s}\rfloor}\sum_{u=2}^{\lfloor\frac{x}{s\cdot t}\rfloor}\nabla[\log]s\cdot\nabla[\log]t\cdot\nabla[\log]u+\frac{z^4}{4!}\ldots$$

$$[\log]x=\sum_{s=2}^{x}1-\frac{1}{2}\sum_{s=2}^{x}\sum_{t=2}^{\lfloor\frac{x}{s}\rfloor}1+\frac{1}{3}\sum_{s=2}^{x}\sum_{t=2}^{\lfloor\frac{x}{s}\rfloor}\sum_{u=2}^{\lfloor\frac{x}{s\cdot t}\rfloor}1+\ldots$$

$$a[\log]x=\sum_{s=2}^{x}\nabla s^{[a]}-\frac{1}{2}\sum_{s=2}^{x}\sum_{t=2}^{\lfloor\frac{x}{s}\rfloor}\nabla s^{[a]}\cdot\nabla t^{[a]}+\frac{1}{3}\sum_{s=2}^{x}\sum_{t=2}^{\lfloor\frac{x}{s}\rfloor}\sum_{u=2}^{\lfloor\frac{x}{s\cdot t}\rfloor}\nabla s^{[a]}\cdot\nabla t^{[a]}\cdot\nabla u^{[a]}+\ldots$$

$$[\log]x^a=\sum_{s=2}^{x^a}\nabla 1-\frac{1}{2}\sum_{s=2}^{x^a}\sum_{t=2}^{\lfloor\frac{x^a}{s}\rfloor}1+\frac{1}{3}\sum_{s=2}^{x^a}\sum_{t=2}^{\lfloor\frac{x^a}{s}\rfloor}\sum_{u=2}^{\lfloor\frac{x^a}{s\cdot t}\rfloor}1+\ldots$$

$$x^{[z]}=1+\frac{z^1}{1!}[\log]x+\frac{z^2}{2!}[\log]x^{[2]}+\frac{z^3}{3!}[\log]x^{[3]}+\frac{z^4}{4!}\ldots$$

$$[\log]x=(x-1)^{[1]}-\frac{1}{2}(x-1)^{[2]}+\frac{1}{3}(x-1)^{[3]}+\ldots$$

$$a[\log]x^a=(x^{[a]}-1)^{[1]}-\frac{1}{2}(x^{[a]}-1)^{[2]}+\frac{1}{3}(x^{[a]}-1)^{[3]}+\ldots$$

$$[\log]x^a=(x^a-1)^{[1]}-\frac{1}{2}(x^a-1)^{[2]}+\frac{1}{3}(x^a-1)^{[3]}+\ldots$$

$$li(n) - \log\log n - \gamma = \int_1^n dx - \frac{1}{2}\int_1^n \int_1^{\frac{n}{x}} dy\, dx + \frac{1}{3}\int_1^n \int_1^{\frac{n}{x}} \int_1^{\frac{n}{xy}} dz\, dy\, dx - \ldots$$

$$\frac{\partial}{\partial n} li(n) - \log\log n - \gamma = \frac{\partial}{\partial n}\int_1^n dx - \frac{1}{2}\int_1^n \int_1^{\frac{n}{x}} dy\, dx + \frac{1}{3}\int_1^n \int_1^{\frac{n}{x}} \int_1^{\frac{n}{xy}} dz\, dy\, dx - \ldots$$

$$\frac{\partial}{\partial n} li(n) - \log\log n - \gamma = 1 - \frac{1}{2}\log n + \frac{1}{3}\frac{(\log n)^2}{2} - \frac{1}{4}\frac{(\log n)^4}{6} + \frac{1}{5}\frac{(\log n)^4}{24} - \ldots$$

$$\frac{\partial}{\partial n} li(n) - \log\log n - \gamma = \frac{1}{1!} - \frac{(\log n)}{2!} + \frac{(\log n)^2}{3!} - \frac{(\log n)^4}{4!} + \frac{(\log n)^4}{5!} - \ldots$$

$$\frac{\partial}{\partial n} li(n) - \log\log n - \gamma = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(\log n)^{k-1}}{k!} = \frac{1}{\log n} - \frac{1}{n \log n}$$

$$\frac{\partial^2}{\partial n^2} li(n) - \log\log n - \gamma = \frac{\partial^2}{\partial n^2}\int_1^n dx - \frac{1}{2}\int_1^n \int_1^{\frac{n}{x}} dy\, dx + \frac{1}{3}\int_1^n \int_1^{\frac{n}{x}} \int_1^{\frac{n}{xy}} dz\, dy\, dx - \ldots$$

$$\frac{\partial^2}{\partial n^2} li(n) - \log\log n - \gamma = \frac{1}{n^2(\log n)^2} - \frac{1}{n(\log n)^2} + \frac{1}{n^2 \log n}$$

SCRATCH AT THE END

$$\frac{\partial}{\partial x} L_{-z}(\log x)$$

$$[\nabla n^z] = \prod_{p^\alpha \| n} (-1)^\alpha \binom{-z}{\alpha}$$

$$L_{-z}(\log x) = \int_{y=0}^{x} \frac{\partial}{\partial y} L_{-z}(\log y)\, dy$$

...

? ? ? Anything here ?

$$[\zeta_n(s)]^{*z} = \sum_{j=1}^{n} [\zeta_{\Delta j}(s)]^{*z}$$

$$[\zeta_n(s)]^{*z} = \sum_{j=1}^{n} \frac{d_z(j)}{j^s}$$

$$[\zeta_n(s)]^{*z} = \sum_{j=1}^{n} j^{-s} \prod_{p^\alpha | j} (-1)^\alpha \binom{-z}{\alpha}$$

$$\boxed{L_{-z}(\log x) = \sum_{k=0}^{\infty} \binom{z}{k} (-1)^k \frac{\gamma(k, -\log x)}{\Gamma(k)}}$$

$$\boxed{[\zeta_n(s)]^{*z} = \sum_{k=0}^{\infty} \binom{z}{k} [\zeta_n(s) - 1]^{*k}}$$

$$\boxed{x^{*z} = \sum_{k=0}^{\infty} \binom{z}{k} (x-1)^{*k}}$$