```
L1[n_, k_] := Sum[ L1[Floor[n / j], k - 1], {j, 1, n}]; L1[n_, 1] := Sum[Log[j], {j, 2, n}]
L2[n_, k_] := Sum[ (-1)^(j + 1) L2[(n / j), k - 1], {j, 1, n}];
L2[n_, 1] := Sum[(-1)^(j + 1) Log[j], {j, 2, n}]
```

```
N[L2[100, 2]]
```

```
3.9172
```

```
fa[n_] := Sum[ (-1)^(j + 1) If[Mod[k, 2] == 1, Log[k], 0], {j, 1, n}, {k, 1, n / j}] -
  Sum[ (-1)^(j + 1) If[Mod[k, 2] == 0, Log[k], 0], {j, 1, n}, {k, 1, n / j}]
```

```
fb[n_] := Sum[ (-1)^(j + 1) (-1)^(k + 1) Log[k], {j, 1, n}, {k, 1, n / j}]
```

```
N[fa[100]]
```

```
3.9172
```

```
N[fb[100]]
```

```
3.9172
```

```
fc[n_] := Sum[ If[Mod[j, 2] == 1, If[Mod[k, 2] == 1, Log[k], 0], 0], {j, 1, n}, {k, 1, n / j}] -
  Sum[ If[Mod[j, 2] == 0, If[Mod[k, 2] == 1, Log[k], 0], 0], {j, 1, n}, {k, 1, n / j}] -
  Sum[ If[Mod[j, 2] == 1, If[Mod[k, 2] == 0, Log[k], 0], 0], {j, 1, n}, {k, 1, n / j}] +
  Sum[ If[Mod[j, 2] == 0, If[Mod[k, 2] == 0, Log[k], 0], 0], {j, 1, n}, {k, 1, n / j}]
```

```
N[fc[100]]
```

```
3.9172
```

```
fd[n_] := Sum[ Log[k], {j, 1, n}, {k, 1, n / j}] -
  2 Sum[ If[Mod[j, 2] == 0, If[Mod[k, 2] == 1, Log[j], 0], 0], {j, 1, n}, {k, 1, n / j}] -
  2 Sum[ If[Mod[j, 2] == 0, If[Mod[k, 2] == 1, Log[k], 0], 0], {j, 1, n}, {k, 1, n / j}]
```

```
N[fd[100]]
```

```
3.9172
```

```
fe[n_] := Sum[ Log[k], {j, 1, n}, {k, 1, n / j}] -
  2 Sum[ If[Mod[j, 2] == 0, If[Mod[k, 2] == 1, Log[j] + Log[k], 0], 0], {j, 1, n}, {k, 1, n / j}]
```

```
N[fe[100]]
```

```
3.9172
```

```
fac[n_] := -Sum[ Log[k], {j, 1, n}, {k, 1, n / j}] +
  2 Sum[ If[Mod[j, 2] == 1, If[Mod[k, 2] == 1, Log[k], 0], 0], {j, 1, n}, {k, 1, n / j}] +
  2 Sum[ If[Mod[j, 2] == 0, If[Mod[k, 2] == 0, Log[k], 0], 0], {j, 1, n}, {k, 1, n / j}]
```

```
N[fac[100]]
```

```
3.9172
```

```
fad[n_] := -Sum[ Log[j], {j, 1, n}, {k, 1, n / j}] +
  2 Sum[ If[Mod[j, 2] == 1, If[Mod[k, 2] == 1, Log[k], 0], 0], {j, 1, n}, {k, 1, n / j}] +
  2 Sum[ Log[j], {j, 2, n, 2}, {k, 2, n / j, 2}]
```

```
N[fad[100]]
```

```
3.9172
```

```
fae[n_] := -Sum[ Log[j], {j, 1, n}, {k, 1, n / j}] +
  2 Sum[ Log[j], {j, 1, n, 2}, {k, 1, (n) / j, 2}] + 2 Sum[ Log[j], {j, 2, n, 2}, {k, 2, n / j, 2}]
```

```
N[fae[100]]
```

```
3.9171973447936352`
```

```
faf[n_] := -Sum[ Log[j], {j, 1, n}, {k, 1, n / j}] +
  2 Sum[ Log[j], {j, 1, n, 2}, {k, 1, (n) / j, 2}] + 2 Sum[ Log[2 j], {j, 1, n / 2}, {k, 1, n / (4 j)}]
```

```
N[faf[100]]
```

```
3.9172
```