```
ClearAll["Global`*"]
```

```
Cb1c[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j
    Sum[FactorialPower[k, a] / a ! CbE2b[n / b^j, a, b], {a, 1, Log[If[b > 2, 2, b], n / b^j]}],
  {j, 0, Log[b, n]}]
CbE2b[n_, k_, a_] := Sum[ Log[j] CbE2b[n / j, k - 1, a], {j, 2, n}] -
  a Sum[ Log[j a] CbE2b[n / (a j), k - 1, a], {j, 1, n / a}]; CbE2b[n_, 0, a_] := 1
CbE1a[n_, k_, a_] := CbE1a[n, k, a] = Sum[Log[j] CbE1a[n / j, k - 1, a], {j, 1, n}] -
    a Sum[ Log[a j] CbE1a[n / (a j), k - 1, a], {j, 1, n / a}]; CbE1a[n_, 0, a_] := 1
CbDDc[n_, k_, b_] := Sum[Binomial[k + j - 1, k - 1] b^j CbE1a[n / b^j, k, b], {j, 0, Log[b, n]}]
CbE1b[n_, k_, b_] := Sum[ Binomial[k, j] CbE2b[n, k - j, b], {j, 0, k}]
CbDDd[n_, k_, b_] :=
 Sum[Binomial[k + j - 1, k - 1] b^j CbE1b[n / b^j, k, b], {j, 0, Log[b, n]}]
```

```
N[CbDDc[100, 1, 2]]
```

```
-18.8779
```

```
N[CbE1a[100, 1, 200]]
```

```
363.739
```

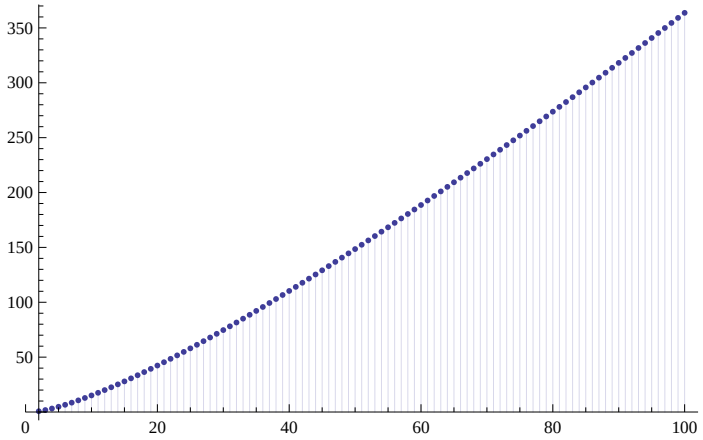
```
N[CbE1b[100, 2, 101]]
```

```
1627.7
```

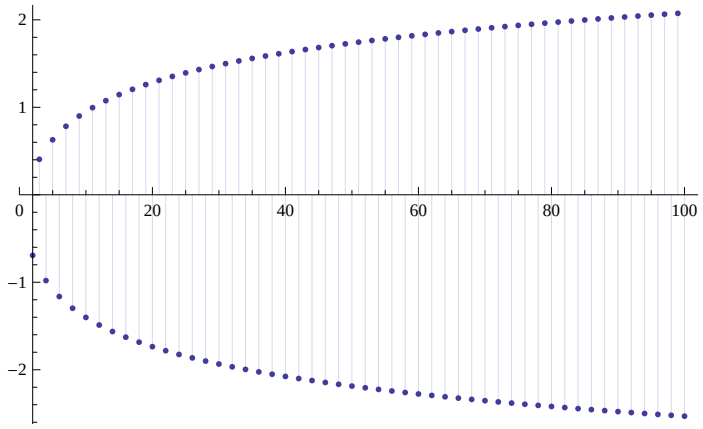
```
N[Sum[ Log[j], {j, 1, 100}]]
```

```
363.739
```

```
DiscretePlot[CbDDc[n, 1, 2], {n, 2, 100}]
```



```
DiscretePlot[ftb3[n, 2^0, 2], {n, 2, 100}]
```




```
t[n_, a_] := Mod[n, a] - Mod[n - 1, a]
f2[n_] := Sum[ (-1)^(j + 1) Log[j], {j, 1, n}]
f2b[n_, a_] := Sum[ (-1)^(j + 1) Log[j a], {j, 1, n}]
ftb[n_, a_, c_] := Sum[ t[j, c] Log[j a], {j, 1, n}]
f2a[n_] := f[n, 1] - 2 f[n / 2, 2]
f[n_, a_] := Sum[ Log[j a], {j, 1, n}]
f1[n_] := f2[n] + 2 f[n / 2, 2]
f1b[n_] := f2b[n, 1] + 2 f2b[n / 2, 2] + 4 f[n / 4, 4]
f1c[n_] := f2b[n, 1] + 2 f2b[n / 2, 2] + 4 (f2b[n / 4, 4] + 2 f[n / 8, 8])
f4[n_] := Sum[ 2^k f2b[n / (2^k), 2^k], {k, 0, Log[2, n]}]
f43[n_] := Sum[ 3^k ftb[n / (3^k), 3^k, 3], {k, 0, Log[3, n]}]
f43[n_, s_] := Sum[ s^k ftb[n / (s^k), s^k, s], {k, 0, Log[s, n]}]
ftb1[n_, a_, c_] := Sum[ t[j, c] (Log[a j]), {j, 1, n}]
f431[n_, s_] := Sum[ s^k ftb1[n / (s^k), s^k, s], {k, 0, Log[s, n]}]
ftb2[n_, a_, c_] := Log[a] Sum[ t[j, c] , {j, 1, n}] + Sum[ t[j, c] (Log[j]), {j, 1, n}]
f432[n_, s_] := Sum[ s^k ftb2[n / (s^k), s^k, s], {k, 0, Log[s, n]}]
ftb3[n_, a_, c_] := Log[a] Mod[n, c] + Sum[ t[j, c] Log[j], {j, 2, n}]
f433[n_, b_] := Sum[ b^k ftb3[Floor[n / (b^k)], b^k, b], {k, 0, Log[b, n]}]
ftb1a[n_, a_, c_] := Sum[ t[j, c] (Log[a j]), {j, 1, n}]

ftb4[n_, a_, c_] := Sum[ (Log[a j]), {j, 1, n}] - c Sum[ Log[a c j], {j, 1, n / c}]
f434[n_, s_] := Sum[ s^k ftb4[n / (s^k), s^k, s], {k, 0, Log[s, n]}]
```

```
FullSimplify[ftb1[100, 3^0, 3]]
```

```
-96 Log[3] + Log[5 279 378 867 581 417 425 990 115 212 984 319 055 708 066 000]
```

```
FullSimplify[ftb1[100, 3^1, 3]]
```

```
-95 Log[3] + Log[5 279 378 867 581 417 425 990 115 212 984 319 055 708 066 000]
```

```
FullSimplify[ftb1[100, 3^2, 3]]
```

```
-94 Log[3] + Log[5 279 378 867 581 417 425 990 115 212 984 319 055 708 066 000]
```

```
FullSimplify[ftb1[100, 2^2, 2]]
```

```
-97 Log[2] + Log[12 611 418 068 195 524 166 851 562 157]
```

```
FullSimplify[ftb4[100, 2^2, 2]]
```

```
-97 Log[2] + Log[12 611 418 068 195 524 166 851 562 157]
```

```
N[f2[100]]
```

```
-2.53088
```

```
N[CbE1a[100, 1, 2]]
```

```
-2.53088
```

```
N[f2a[100]]
```

```
-2.53088
```

```
N[f3[100, 1]]
```

```
363.739
```

```
N[f4[100]]
```

```
363.739
```

```
N[f1b[100]]
```

```
363.739
```

```
N[f1c[100]]
```

```
363.739
```

```
N[f43[100, 2]]
```

```
363.739
```

```
N[f432[100, 2]]
```

```
363.739
```

```
N[f433[100, 2]]
```

```
363.739
```

```
N[f431[100, 2]]
```

```
363.7393755555634`
```

```
N[f434[100, 1.01]]
```

```
363.739
```

```
ss[n_, c_] := Sum[ t[j, c] , {j, 1, n}]
```

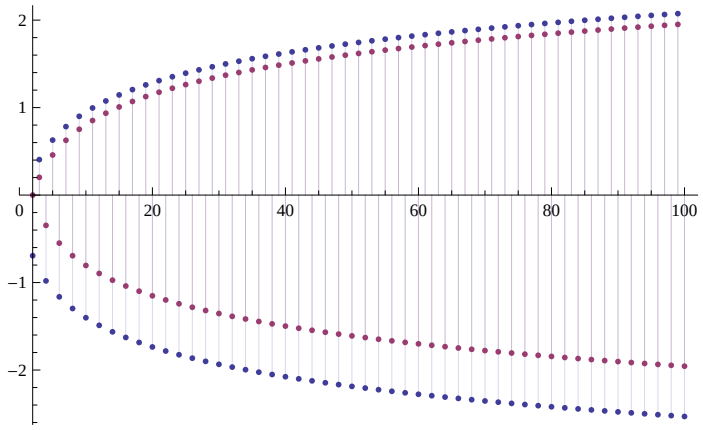
```
Table[{Mod[n, 7] - ss[n, 7]}, {n, 2, 100}]
```

```
{{0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0},
 {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0},
 {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0},
 {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0},
 {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0},
 {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}, {0}}
```

```
Mod[1323, 3]
```

```
0
```

```
DiscretePlot[{ftb3[n, 2^0, 2], (-1)^(n + 1) Log[n / 2] / 2}, {n, 2, 100}]
```




```
ftb4[n_, a_, c_] := Sum[ (Log[a j]), {j, 1, n}] - c Sum[ Log[a c j], {j, 1, n / c}]
f434[n_, s_] := Sum[ s^k ftb4[n / (s^k), s^k, s], {k, 0, Log[s, n]}]
```

```
N[f434[100, 1.01]]
```

```
363.739
```

```
N[Sum[Log[j], {j, 1, 100}]]
```

```
363.739
```

```
ftb5[n_, a_, c_] :=
 Sum[ (Log[c^a] + Log[j]), {j, 1, n}] - c Sum[ Log[c^a] + Log[c] + Log[j], {j, 1, n / c}]
f435[n_, s_] := Sum[ s^k ftb5[n / (s^k), k, s], {k, 0, Log[s, n]}]
```

```
N[f435[100, 1.01]]
```

```
363.739
```

```
ftb6[n_, a_, c_] :=
 Sum[ (a Log[c] + Log[j]), {j, 1, n}] - c Sum[ a Log[c] + Log[c] + Log[j], {j, 1, n / c}]
f436[n_, s_] := Sum[ s^k ftb6[n / (s^k), k, s], {k, 0, Log[s, n]}]
```

```
N[f436[100, 1.01]]
```

```
363.739
```

```
ftb7[n_, a_, c_] := Sum[ (a Log[c]), {j, 1, n}] + Sum[ (Log[j]), {j, 1, n}] -
  c Sum[ (a + 1) Log[c], {j, 1, n / c}] - c Sum[ Log[j], {j, 1, n / c}]
f437[n_, s_] := Sum[ s^k ftb7[n / (s^k), k, s], {k, 0, Log[s, n]}]
```

```
N[f437[100, 1.01]]
```

```
363.739
```

```
ftb8[n_, a_, c_] := (a Log[c]) Floor[n] + Sum[ (Log[j]), {j, 1, n}] -
  (a + 1) Log[c] c Floor[n / c] - c Sum[ Log[j], {j, 1, n / c}]
f438[n_, s_] := Sum[ s^k ftb8[n / (s^k), k, s], {k, 0, Log[s, n]}]
```

```
N[f438[100, 1.01]]
```

```
363.739
```

```
ftb9[n_, a_, c_] := a Log[c] Floor[n] + Sum[ (Log[j]), {j, 1, n}] -
  (a + 1) Log[c] c Floor[n / c] - c Sum[ Log[j], {j, 1, n / c}]
f439[n_, s_] := Sum[ s^k ftb9[n / (s^k), k, s], {k, 0, Log[s, n]}]
```

```
N[f439[100, 1.01]]
```

363.739

```
FullSimplify[Expand[(a Log[c] Floor[n] - (a + 1) Log[c] c Floor[n / c])]]
```

$$\left(a\,\text{Floor}[n]-(1+a)\,c\,\text{Floor}\left[\frac{n}{c}\right]\right)\text{Log}[c]$$

```
ftb10[n_, a_, c_] := (a Floor[n] - (1 + a) c Floor[n / c]) Log[c] +
  Sum[ (Log[j]), {j, 1, n}] - c Sum[ Log[j], {j, 1, n / c}]
f4310[n_, s_] := Sum[ s^k ftb10[n / (s^k), k, s], {k, 0, Log[s, n]}]
```

```
N[f4310[100, 1.01]]
```

363.739

```
ftb11[n_, k_, s_] := (k Floor[n] - (1 + k) s Floor[n / s]) Log[s] +
  Sum[ (Log[j]), {j, 1, n}] - s Sum[ Log[j], {j, 1, n / s}]
f4311[n_, s_] := Sum[ s^k ftb11[n / (s^k), k, s], {k, 0, Log[s, n]}]
```

```
N[f4311[100, 1.01]]
```

363.739

```
ftb12[n_, k_, s_] := (k Floor[n / (s^k)] - (1 + k) s Floor[n / (s^k) / s]) Log[s] +
  Sum[ (Log[j]), {j, 1, n / (s^k)}] - s Sum[ Log[j], {j, 1, n / (s^k) / s}]
f4312[n_, s_] := Sum[ s^k ftb12[n, k, s], {k, 0, Log[s, n]}]
```

```
N[f4312[100, 1.01]]
```

363.739

```
ftb13[n_, k_, s_] := (k Floor[n / (s^k)] - (1 + k) s Floor[n / (s^(k + 1))]) Log[s] +
  Sum[ (Log[j]), {j, 1, n / (s^k)}] - s Sum[ Log[j], {j, 1, n / (s^(k + 1))}]
f4313[n_, s_] := Sum[ s^k ftb13[n, k, s], {k, 0, Log[s, n]}]
```

```
N[f4313[100, 1.01]]
```

363.739

```
ftb14[n_, k_, s_] := (k Floor[n / (s^k)] - (1 + k) s Floor[n / (s^(k + 1))]) Log[s] +
  Sum[ (Log[j]), {j, 1, n / (s^k)}] - s Sum[ Log[j], {j, 1, n / (s^(k + 1))}]
f4314[n_, s_] := Sum[ s^k ((k Floor[n / (s^k)] - (1 + k) s Floor[n / (s^(k + 1))]) Log[s] + Sum[
      (Log[j]), {j, 1, n / (s^k)}] - s Sum[ Log[j], {j, 1, n / (s^(k + 1))}]), {k, 0, Log[s, n]}]
```

```
N[f4314[100, 1.01]]
```

363.739

```
f4315[n_, s_] := Sum[ s^k
    ((k Floor[n / (s^k)] - (1 + k) s Floor[n / (s^(k + 1))]) Log[s])
   + s^k (Sum[ (Log[j]), {j, 1, n / (s^k)}] - s Sum[ Log[j], {j, 1, n / (s^(k + 1))}]),
  {k, 0, Log[s, n]}]
```

```
N[f4315[100, 1.01]]
```

363.739

```
f4316[n_, s_] := Sum[ s^k
     (k Floor[n / s^k] - (1 + k) s Floor[n / s^(k + 1)]) Log[s]
   + s^k (Sum[ (Log[j]), {j, 1, n / (s^k)}] - s Sum[ Log[j], {j, 1, n / (s^(k + 1))}]),

  {k, 0, Log[s, n]}]
N[f4316[100, 1.01]]
```

363.739

```
f4317[n_, s_] := Sum[
  k Floor[n / s^k] Log[s] s^k - Log[s] (1 + k) s^(k + 1) Floor[n / s^(k + 1)]
   + s^k Sum[ (Log[j]), {j, 1, n / (s^k)}] - (s^(k + 1)) Sum[ Log[j], {j, 1, n / (s^(k + 1))}],

  {k, 0, Log[s, n]}]
N[f4317[100, 1.01]]
```

363.739

```
f4318[n_, s_] := Sum[
   k Floor[n / s^k] Log[s] s^k - Log[s] (1 + k) s^(k + 1) Floor[n / s^(k + 1)], {k, 0, Log[s, n]}] +
  Sum[s^k Sum[ Log[j], {j, 1, n / (s^k)}] - s^(k + 1) Sum[ Log[j], {j, 1, n / (s^(k + 1))}],

   {k, 0, Log[s, n]}]
N[f4318[100, 1.01]]
```

363.739

```
f4319[n_, s_] :=
 Sum[ k Floor[n / s^k] Log[s] s^k -
    Log[s] (1 + k) s^(k + 1) Floor[n / s^(k + 1)], {k, 0, Log[s, n]}] +

  Sum[s^k Sum[ Log[j], {j, 1, n / (s^k)}] -
    s^(k + 1) Sum[ Log[j], {j, 1, n / (s^(k + 1))}], {k, 0, Log[s, n]}]
N[f4319[100, 1.01]]
```

363.739

```
Sum[ k Floor[n / s^k] Log[s] s^k - Log[s] (1 + k) s^(k + 1) Floor[n / s^(k + 1)],
 {k, 0, Floor[Log[s, n]]}]
```

$$-s^{1+\text{Floor}\left[\frac{\text{Log}[n]}{\text{Log}[s]}\right]}\ \text{Floor}\left[n\, s^{-1-\text{Floor}\left[\frac{\text{Log}[n]}{\text{Log}[s]}\right]}\right]\ \left(1+\text{Floor}\left[\frac{\text{Log}[n]}{\text{Log}[s]}\right]\right)\ \text{Log}[s]$$

```
f4320[n_, s_] :=
 - s^(1 + Floor[Log[n] / Log[s]]) Floor[n s^(-1 - Floor[Log[n] / Log[s]])] (1 + Floor[Log[n] / Log[s]]) Log[s] +
  Sum[s^k Sum[ Log[j], {j, 1, n / (s^k)}] -
    s^(k + 1) Sum[ Log[j], {j, 1, n / (s^(k + 1))}], {k, 0, Log[s, n]}]


N[f4320[100, 2]]
```

363.739

```
fa[n_, s_] := -s^(1 + Floor[Log[n] / Log[s]]) Floor[n s^(-1 - Floor[Log[n] / Log[s]])] (1 + Floor[Log[n] / Log[s]]) Log[s]
```

```
f4321[n_, s_] :=
 Sum[s^k Sum[ Log[j], {j, 1, n / (s^k)}] -
   s^(k + 1) Sum[ Log[j], {j, 1, n / (s^(k + 1))}], {k, 0, Log[s, n]}]
N[f4321[100, 1.01]]
```

```
363.739
```

```
N[Sum[ MoebiusMu[j] f4321[Floor[100 / j], 2], {j, 1, 100}]]
```

```
94.0453
```

```
ClearAll["Global`*"]
eb2[n_, s_, a_] :=
 Sum[ (Log[a^s j]), {j, 1, n / (a^s)}] - a Sum[ Log[a^(s + 1) j], {j, 1, n / a^(s + 1)}]
eb2a[n_, a_] := Sum[ a^s eb2[n, s, a], {s, 0, Log[a, n]}]

ebk2[n_, k_, s_, a_] := Sum[ ebk2[Floor[n / j], k - 1, s, a], {j, 1, n / (a^s)}] -
  a Sum[ ebk2[Floor[n / j], k - 1, s, a], {j, 1, n / a^(s + 1)}]
ebk2[n_, 1, s_, a_] := Sum[ (Log[a^s j]), {j, 1, n / (a^s)}] -
  a Sum[ Log[a^(s + 1) j], {j, 1, n / a^(s + 1)}]

cb2[n_, k_] := cb2[n, k] = Sum[ cb2[Floor[n / j], k - 1], {j, 2, n}];
cb2[n_, 1] := cb2[n, 1] = Sum[Log[j], {j, 1, n}]
cbm[n_] := Sum[ (-1)^(k + 1) cb2[n, k], {k, 1, Log[2, n]}]
cb1[n_, z_] := cb1[n, z] = Sum[FactorialPower[z, a] / a! cb2[n, a], {a, 0, Log[2, n]}];
cb1[0, z_] := 0
N[cb2[100, 2]]
```

```
557.102
```

```
N[cb1[100, -1]]
```

```
-94.0453
```

```
N[eb2[100, 0, 2]]
```

```
-2.53088
```

```
N[ebk2[100, 3, 0, 2]]
```

```
-87.2678
```

```
N[cbm[100]]
```

```
94.0453
```

```
N[eb2a[100, 2]]
```

```
363.739
```