```
Clear[dlcrp1, dcr, cr, lcrp1]

bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k!

dlcrp1[fn_, n_, s_, k_] :=
 dlcrp1[fn, n, s, k] = Sum[ fn[j] j^-s dlcrp1[fn, n - j, s, k - 1], {j, 1, n - 1}]
dlcrp1[fn_, n_, s_, 1] := fn[n] n^-s
dcrp1[fn_, n_, s_, z_] := Sum[ z^k / k! dlcrp1[fn, n, s, k], {k, 1, n}]
dcrroots[fn_, n_, s_] := If[(c = Exponent[f = dcrp1[fn, n, s, z], z]) == 0,
  {}, If[c == 1, List@NRoots[f == 0, z][[2]], List @@ NRoots[f == 0, z][[All, 2]]]]
dcrrootsa[fn_, n_, s_] := If[(c = Exponent[f = dcrp1[n, n, s, z], z]) == 0, {},
  If[c == 1, List@Roots[f == 0, z][[2]], List @@ Roots[f == 0, z][[All, 2]]]]

dcr[fn_, n_, s_, k_] := dcr[fn, n, s, k] = Sum[ fn[j] j^-s dcr[fn, n - j, s, k - 1], {j, 1, n - 1}]
dcr[fn_, n_, s_, 1] := fn[n] n^-s
dcr[fn_, n_, s_, 0] := 0
dcrp1a[fn_, n_, s_, z_] := Sum[ bin[z, k] dcr[fn, n, s, k], {k, 0, n}]
dcrrootsb[fn_, n_, s_] := If[(c = Exponent[f = dcrp1a[fn, n, s, z], z]) == 0,
  {}, If[c == 1, List@NRoots[f == 0, z][[2]], List @@ NRoots[f == 0, z][[All, 2]]]]

lcrp1[fn_, n_, s_, k_] :=
 lcrp1[fn, n, s, k] = Sum[ fn[j] j^-s lcrp1[fn, n - j, s, k - 1], {j, 1, n}]
lcrp1[fn_, n_, s_, 0] := UnitStep[n]
crp1[fn_, n_, s_, z_] := Sum[ z^k / k! lcrp1[fn, n, s, k], {k, 0, n}]
crroots[fn_, n_, s_] := If[(c = Exponent[f = crp1[fn, n, s, z], z]) == 0, {},
  If[c == 1, List@NRoots[f == 0, z][[2]], List @@ NRoots[f == 0, z][[All, 2]]]]
crrootsa[fn_, n_, s_] := If[(c = Exponent[f = crp1[n, n, s, z], z]) == 0, {},
  If[c == 1, List@Roots[f == 0, z][[2]], List @@ Roots[f == 0, z][[All, 2]]]]

cr[fn_, n_, s_, k_] := cr[fn, n, s, k] = Sum[ fn[j] j^-s cr[fn, n - j, s, k - 1], {j, 1, n}]
cr[fn_, n_, s_, 0] := UnitStep[n]
crp1a[fn_, n_, s_, z_] := Sum[ bin[z, k] cr[fn, n, s, k], {k, 0, n}]
crrootsb[fn_, n_, s_] := If[(c = Exponent[f = crp1a[fn, n, s, z], z]) == 0,
  {}, If[c == 1, List@NRoots[f == 0, z][[2]], List @@ NRoots[f == 0, z][[All, 2]]]]

fn1[n_] := DivisorSigma[1, n] / n
fn1a[n_] := DivisorSigma[1, n]
fn2[n_] := PartitionsP[n] n


crp1[fn1, 30, 0, z]

Expand@crp1a[PartitionsP, 30, 0, z]

Expand@(crp1[fn1, 30, 0, z] - crp1[fn1, 29, 0, z]) /. z → 1

5604

Expand@(dcrp1a[PartitionsP, 30, 0, z]) /. z → 1

5604

PartitionsP[30]

5604
```

```
D[dcrp1a[PartitionsP, 30, 0, z], z] /. z → 0
```

$$\frac{12}{5}$$

```
Table[ dcrp1[fn1, n, 0, 1], {n, 1, 30}]
```

```
{1, 2, 3, 5, 7, 11, 15, 22, 30, 42, 56, 77, 101, 135, 176, 231, 297,
 385, 490, 627, 792, 1002, 1255, 1575, 1958, 2436, 3010, 3718, 4565, 5604}
```

```
Table[D[dcrp1a[PartitionsP, n, 0, z], z] /. z → 0, {n, 1, 30}]
```

$$\left\{1, \frac{3}{2}, \frac{4}{3}, \frac{7}{4}, \frac{6}{5}, 2, \frac{8}{7}, \frac{15}{8}, \frac{13}{9}, \frac{9}{5}, \frac{12}{11}, \frac{7}{3}, \frac{14}{13}, \frac{12}{7}, \frac{8}{5}, \frac{31}{16}, \frac{18}{17}, \frac{13}{6}, \frac{20}{19}, \frac{21}{10}, \frac{32}{21}, \frac{18}{11}, \frac{24}{23}, \frac{5}{2}, \frac{31}{25}, \frac{21}{13}, \frac{40}{27}, 2, \frac{30}{29}, \frac{12}{5}\right\}$$