```
K[n_] := FullSimplify[MangoldtLambda[n] / Log[n]]
P21[n_] := Sum[K[j], {j, 2, n}]
P22[n_] := Sum[K[j] K[k], {j, 2, n}, {k, 2, n / j}]
P23[n_] := Sum[K[j] K[k] K[m], {j, 2, n}, {k, 2, n / j}, {m, 2, n / (j k)}]
P2[n_, 0] := UnitStep[n - 1]
P2[n_, k_] := Sum[K[j] P2[n / j, k - 1], {j, 2, Floor[n]}]
Table[{P21[n] - P2[n, 1], P22[n] - P2[n, 2], P23[n] - P2[n, 3]}, {n, 1, 50}] // TableForm
```

```
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
0    0    0
```

```
Floor[20 / (3 / 2)^8]
```

```
0
```

```
N[Log[20 / 2] / Log[3 / 2]]
```

5.67887

```
N[Log[20 / 3] / Log[3 / 2]]
```

4.67887

```
N[Log[20 / 4] / Log[3 / 2]]
```

3.96936

```
N[Log[20 / 13] / Log[3 / 2]]
```

1.06244

```
N[Log[20 / 8] / Log[3 / 2]]
```

2.25985

```
N[Log[20 / 2] / Log[3 / 2]]
N[(3 / 2) ^5]
```

7.59375

```
Floor[N[Log[20 / 9] / Log[3 / 2]]]
```

1

```
d1[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
ReferenceD1[n_, z_] := Sum[d1[j, z], {j, 1, n}]
num[c_] := Numerator[c]; den[c_] := Denominator[c]
alpha[n_, c_] := den[c] (Floor[n / den[c]] - Floor[(n - 1) / den[c]]) -
  num[c] (Floor[n / num[c]] - Floor[(n - 1) / num[c]])
E2[n_, k_, c_] := E2[n, k, c] = (1 / den[c]) Sum[
    If[alpha[j, c] == 0, 0, alpha[j, c] E2[(den[c] n) / j, k - 1, c]], {j, den[c] + 1, den[c] n}]
E2[n_, 0, c_] := UnitStep[n - 1]
E1[n_, z_, c_] := Sum[Binomial[z, k] E2[n, k, c], {k, 0, Floor[Log[n] / Log[c]]}]
E1Alt[n_, z_, c_] :=
 Sum[(-1)^j Binomial[z, j] c^j ReferenceD1[n / c^j, z], {j, 0, Floor[Log[n] / Log[c]]}]
E1a[ n_, z_, c_] :=
 Sum[ d1[k, z] Sum[ (-1)^j Binomial[ z, j] c^j, {j, 0, Floor[ (Log[c, n / k])]}], {k, 1, n}]
```

E1aa[n_, z_, c_] := $\text{Sum}\left[\text{d1}[k, z]\left((1-c)^z + (-c)^{\text{Floor}\left[\frac{\text{Log}\left[\frac{n}{k}\right]}{\text{Log}[c]}\right]} c\,\text{Binomial}\left[z, 1+\text{Floor}\left[\frac{\text{Log}\left[\frac{n}{k}\right]}{\text{Log}[c]}\right]\right]\right.\right.$

$\left.\left.\text{Hypergeometric2F1}\left[1, 1-z+\text{Floor}\left[\frac{\text{Log}\left[\frac{n}{k}\right]}{\text{Log}[c]}\right], 2+\text{Floor}\left[\frac{\text{Log}\left[\frac{n}{k}\right]}{\text{Log}[c]}\right], c\right]\right), \{k, 1, n\}\right]$

```
E1a[ 20, 3, 1.0001]
```

18.

Divide::infy : Infinite expression $\frac{1}{0.}$ encountered. ≫

Indeterminate

$$\frac{168\,613\,837\,054\,282\,593\,734\,036\,748\,436\,658\,202\,640\,366\,746\,381\,549}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

```
E1aa[100, -2, 11 / 10]
```

$$\frac{168\,613\,837\,054\,282\,593\,734\,036\,748\,436\,658\,202\,640\,366\,746\,381\,549}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

```
E1Alt[ 100, -2, 11 / 10]
```

$$\frac{168\,613\,837\,054\,282\,593\,734\,036\,748\,436\,658\,202\,640\,366\,746\,381\,549}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

```
E1[ 1200, 2, 101 / 100]
```

$Aborted

```
Sum[ (-1)^j Binomial[ z, j] c^j, {j, 0, Floor[ (Log[c, n / k])]}]
```

$$(1-c)^z + (-c)^{\text{Floor}\left[\frac{\text{Log}\left[\frac{n}{k}\right]}{\text{Log}[c]}\right]} c\,\text{Binomial}\left[z, 1+\text{Floor}\left[\frac{\text{Log}\left[\frac{n}{k}\right]}{\text{Log}[c]}\right]\right]$$

$$\text{Hypergeometric2F1}\left[1, 1-z+\text{Floor}\left[\frac{\text{Log}\left[\frac{n}{k}\right]}{\text{Log}[c]}\right], 2+\text{Floor}\left[\frac{\text{Log}\left[\frac{n}{k}\right]}{\text{Log}[c]}\right], c\right]$$

$$\text{Limit}\left[(1-c)^z + (-c)^{\text{Floor}\left[\frac{\text{Log}\left[\frac{n}{k}\right]}{\text{Log}[c]}\right]} c\,\text{Binomial}\left[z, 1+\text{Floor}\left[\frac{\text{Log}\left[\frac{n}{k}\right]}{\text{Log}[c]}\right]\right]\right.$$
$$\left.\text{Hypergeometric2F1}\left[1, 1-z+\text{Floor}\left[\frac{\text{Log}\left[\frac{n}{k}\right]}{\text{Log}[c]}\right], 2+\text{Floor}\left[\frac{\text{Log}\left[\frac{n}{k}\right]}{\text{Log}[c]}\right], c\right], c\to 1\right]$$

$$\text{Limit}\left[(1-c)^z + (-c)^{\text{Floor}\left[\frac{\text{Log}\left[\frac{n}{k}\right]}{\text{Log}[c]}\right]} c\,\text{Binomial}\left[z, 1+\text{Floor}\left[\frac{\text{Log}\left[\frac{n}{k}\right]}{\text{Log}[c]}\right]\right]\right.$$
$$\left.\text{Hypergeometric2F1}\left[1, 1-z+\text{Floor}\left[\frac{\text{Log}\left[\frac{n}{k}\right]}{\text{Log}[c]}\right], 2+\text{Floor}\left[\frac{\text{Log}\left[\frac{n}{k}\right]}{\text{Log}[c]}\right], c\right], c\to 1\right]$$

$$\text{Limit}\left[(-c)^{\text{Floor}\left[\frac{\text{Log}\left[\frac{n}{k}\right]}{\text{Log}[c]}\right]} c\,\text{Binomial}\left[z, 1+\text{Floor}\left[\frac{\text{Log}\left[\frac{n}{k}\right]}{\text{Log}[c]}\right]\right]\right.$$
$$\left.\text{Hypergeometric2F1}\left[1, 1-z+\text{Floor}\left[\frac{\text{Log}\left[\frac{n}{k}\right]}{\text{Log}[c]}\right], 2+\text{Floor}\left[\frac{\text{Log}\left[\frac{n}{k}\right]}{\text{Log}[c]}\right], c\right], c\to 1\right]$$

```
$Aborted
```

```
ff[n_ , z_, c_] := Sum[ (-1)^j bin[ z, j] c^j, {j, 0, Floor[ (Log[c, n])]}]
ffp[n_ , z_, c_] :=
 DiscretePlot[ Re[(-1)^j bin[ z, j] c^j], {j, 0, .0001 Floor[ (Log[c, n])]}]
f[n_, k_, z_, c_] := If[ n < 1, 0, 1 - ((z + 1) / k - 1) c f[n / c, k + 1, z, c]]
```

```
d1[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
ReferenceD1[n_, z_] := Sum[d1[j, z], {j, 1, n}]
num[c_] := Numerator[c]; den[c_] := Denominator[c]
alpha[n_, c_] := den[c] (Floor[n / den[c]] - Floor[(n - 1) / den[c]]) -
  num[c] (Floor[n / num[c]] - Floor[(n - 1) / num[c]])
E2[n_, k_, c_] := E2[n, k, c] = (1 / den[c]) Sum[
    If[alpha[j, c] == 0, 0, alpha[j, c] E2[(den[c] n) / j, k - 1, c]], {j, den[c] + 1, den[c] n}]
E2[n_, 0, c_] := UnitStep[n - 1]
E1[n_, z_, c_] := Sum[Binomial[z, k] E2[n, k, c], {k, 0, Floor[Log[n] / Log[c]]}]
D1Alt[n_, z_, c_] :=
 Sum[(-1)^j Binomial[-z, j] c^j E1[n / c^j, z, c], {j, 0, Floor[Log[n] / Log[c]]}]

ee[ n_, 0, x_] := UnitStep[n - 1];
ee[ n_, k_, x_] := Sum[ ee[ n / j, k - 1, x] - x ee[n / (j x), k - 1, x], {j, 1}]
```

```
ff[ n_, z_, c_] := Sum[ (-1)^j Binomial[ z, j] c^j, {j, 0, Floor[ (Log[c, n])]}]
```

```
ff[1, z, 2]

1

E1a[ n_, z_, c_] :=
 Sum[ d1[k, z] Sum[ (-1)^j Binomial[ z, j] c^j, {j, 0, Floor[ (Log[c, n / k])]}], {k, 1, n}]
E1a1[ n_, z_, c_] := d1[n, z] Sum[ (-1)^j Binomial[ z, j] c^j, {j, 0, 0}]

E1a[ 19.1, .5, 1.0001]

0.0325601

d1[ 19, .5]

0.5

E1a1[ 19, .5, 1.0001]

0.5

Binomial[z, 0]

1

(Log[n] - Log[n - 1]) / Log[c] /. {n → 8, c → 1.01}

13.4198

N[10 / 9]

1.11111

Limit[ (Log[10] - Log[9]) / Log[c], c → 1]

∞

c^( (Log[10] - Log[9]) / Log[c] )
```

$$\frac{10}{9}$$

```
d11[n_, z_] := Product[Pochhammer[z, a = p[[2]]] / a!, {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
d111[n_, z_] := Product[Binomial[z + p[[2]] - 1, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
d1[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}

d111[100, 3 / 2]
```

$$\frac{225}{64}$$

```
d11[100, 3 / 2]
```

$$\frac{225}{64}$$

```
d1[100, 3 / 2]
```

$$\frac{225}{64}$$

```
Binomial[ z + Floor[(Log[10] - Log[9]) / Log[c]] - 1, Floor[(Log[10] - Log[9]) / Log[c]]]
```

$$\text{bin}\left[-1+3+\text{Floor}\left[\frac{-\text{Log}[9]+\text{Log}[10]}{\text{Log}[c]}\right],\ \text{Floor}\left[\frac{-\text{Log}[9]+\text{Log}[10]}{\text{Log}[c]}\right]\right]\ /.\ c\to 1.00001$$

```
55 519 453

Limit[Binomial[-1 + z + c, c], c → Infinity]

Limit[Binomial[-1 + c + z, c], c → ∞]


d1[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
E1[n_, k_, x_] :=
 E1[n, k, x] = Sum[E1[n / j, k - 1, x], {j, 1, n}] - x Sum[E1[n / (x j), k - 1, x], {j, 1, n / x}];
E1[n_, 0, x_] := UnitStep[n - 1]
E1a[ n_, z_, c_] :=
 Sum[ d1[k, z] Sum[ (-1)^j Binomial[ z, j] c^j, {j, 0, Floor[ (Log[c, n / k])]}], {k, 1, n}]
E2[n_, k_, x_] := E2[n, k, x] = Sum[E2[n / j, k - 1, x], {j, 2, n}] -
   x Sum[E2[n / (x j), k - 1, x], {j, 1, n / x}]; E2[n_, 0, x_] := UnitStep[n - 1]
E1b[n_, z_, x_] := Sum[Binomial[z, k] E2[n, k, x], {k, 0, Log[x, n]}]
d1[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
ReferenceD1[n_, z_] := Sum[d1[j, z], {j, 1, n}]
E1c[n_, z_, c_] :=
 Sum[(-1)^j Binomial[z, j] c^j ReferenceD1[n / c^j, z], {j, 0, Floor[Log[n] / Log[c]]}]


E1a[4, -2, 1.001]

1.67164 × 10^6

E1c[4, -2, 1.001]

1.67164 × 10^6


eta[s_, c_] := (1 - c^(1 - s)) Zeta[s]



ff[n_ , z_, c_] := Sum[ (-1)^j Binomial[ z, j] c^j, {j, 0, Floor[ (Log[c, n])]}]

(ff[10, c, 1.00001] - 1) / c /. c → .00001

-17.6771
```