```
ps[n_, s_] := n^s / s HarmonicNumber[n, s] - n^(1 - s) / (1 - s) HarmonicNumber[n, 1 - s]
ps2[n_, s_] := n^(s - 1) (1 - s) HarmonicNumber[n, s] - n^(-s) s HarmonicNumber[n, 1 - s]
ps3[n_, s_] := (1 - s) n^(s - 1 / 2) HarmonicNumber[n, s] + s n^(1 / 2 - s) HarmonicNumber[n, 1 - s]
ps4[n_, x_] :=
 (1 / 2 + x) n^(-x) HarmonicNumber[n, 1 / 2 - x] - (1 / 2 - x) n^x HarmonicNumber[n, 1 / 2 + x]
```

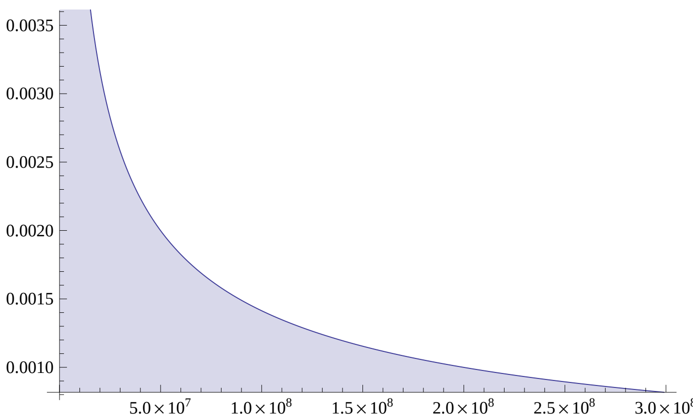
$$\texttt{ps5[n\_, s\_] :=}\; n^{-\frac{1}{2}+s}\,(1-s)\,\texttt{HarmonicNumber[n, s]} - n^{\frac{1}{2}-s}\,s\,\texttt{HarmonicNumber[n, 1 - s]}$$

$$\texttt{ps6[n\_, s\_] :=}\; n^{-\frac{1}{2}+s}\,(1-s)\,\texttt{Zeta[s, n + 1]} - n^{\frac{1}{2}-s}\,s\,\texttt{Zeta[1 - s, n + 1]}$$

```
ps7[n_, s_] :=
 Sum[ j^(-1 / 2) ((1 - 2 s) Cosh[ (1 / 2 - s) Log[j / n]] + Sinh[(1 / 2 - s) Log[j / n]]), {j, 1, n}]
```

$$\texttt{ps8[n\_, s\_, t\_] :=}\; n^{-\frac{1}{2}+s+t}\,(1-s)\,\texttt{HarmonicNumber[n, s]} - n^{\frac{1}{2}-s+t}\,s\,\texttt{HarmonicNumber[n, 1 - s]}$$

```
DiscretePlot[{Abs[ps5[n, N@ZetaZero@1]]}, {n, 1, 300 000 000, 1 000 000}]
```




```
ps4[n, 1 / 2 - x] /. x → s
```

$$-n^{\frac{1}{2}-s}\,s\,\texttt{HarmonicNumber[n, 1 - s]} + n^{-\frac{1}{2}+s}\,(1-s)\,\texttt{HarmonicNumber[n, s]}$$

```
FullSimplify[j^(-1 / 2) ( 2 (1 / 2 - s) Cosh[ (1 / 2 - s) Log[j / n]] + Sinh[(1 / 2 - s) Log[j / n]])]
```

$$\frac{(1-2s)\,\mathrm{Cosh}\left[\left(\frac{1}{2}-s\right)\mathrm{Log}\left[\frac{j}{n}\right]\right] + \mathrm{Sinh}\left[\left(\frac{1}{2}-s\right)\mathrm{Log}\left[\frac{j}{n}\right]\right]}{\sqrt{j}}$$

```
N@ZetaZero@10
```

$0.5 + 49.7738\,i$

```
Sum[FullSimplify[(1 - s) (n / j)^s - s (j / n)^s], {j, 1, n}]
```

```
$Aborted
```

```
Abs[ps8[100 000 000, N@ZetaZero@1, 0]]
```

```
0.00141347
```

```
ps8a[n_, s_, t_] := (n^(-1/2+s+t) (1 - s) HarmonicNumber[n, s] - n^(1/2-s+t) s HarmonicNumber[n, 1 - s]) /
  ((1 - s) n^(s - 1 / 2 + t) - s n^(1 / 2 - s + t) (2^(1 - s) Pi^(-s) Cos[Pi s / 2] Gamma[s]))
ps8b[n_, s_, t_] := (n^(-1/2+s+t) (1 - s) HarmonicNumber[n, s] - n^(1/2-s+t) s HarmonicNumber[n, 1 - s]) /
  ((1 - s) n^(s - 1 / 2 + t) - s n^(1 / 2 - s + t) (Pi^(1 / 2 - s) Gamma[s / 2] / Gamma[(1 - s) / 2]))
ps8c[n_, s_, t_] := {n^(-1/2+s+t) (1 - s) HarmonicNumber[n, s], -n^(1/2-s+t) s HarmonicNumber[n, 1 - s],
  (1 - s) n^(s - 1 / 2 + t), -s n^(1 / 2 - s + t) (Pi^(1 / 2 - s) Gamma[s / 2] / Gamma[(1 - s) / 2])}
ps8c2[n_, s_, t_] := {n^(-1/2+s+t) (1 - s) HarmonicNumber[n, s] - n^(1/2-s+t) s HarmonicNumber[n, 1 - s],
  (1 - s) n^(s - 1 / 2 + t) - s n^(1 / 2 - s + t) (Pi^(1 / 2 - s) Gamma[s / 2] / Gamma[(1 - s) / 2])}
ps8a[10 000 000, N@ZetaZero@10, 0]
-0.0000726868 + 0.000140831 i
Zeta[.3]
-0.904559
ps8c[1 000 000 000, N@ZetaZero@10, 0]
{31 622.8 - 0.000786999 i, -31 622.8 - 0.000786999 i, 43.0282 - 25.0252 i, 45.3245 - 20.576 i}
```

$$\text{fn}[n\_,\ s\_,\ t\_] := n^{\wedge}(1/2+t) \Big/ \left(\frac{n^{-s}}{2} + \frac{n^{1-s}}{1-s} - \frac{1}{12} n^{-1-s} s + \frac{1}{720} n^{-3-s} s\ (1+s)\ (2+s) - \frac{n^{-5-s} s\ (1+s)\ (2+s)\ (3+s)\ (4+s)}{30\,240} + \frac{n^{-7-s} s\ (1+s)\ (2+s)\ (3+s)\ (4+s)\ (5+s)\ (6+s)}{1\,209\,600} - \frac{n^{-9-s} s\ (1+s)\ (2+s)\ (3+s)\ (4+s)\ (5+s)\ (6+s)\ (7+s)\ (8+s)}{47\,900\,160} + \frac{1}{1\,307\,674\,368\,000} 691\, n^{-11-s} s\ (1+s)\ (2+s)\ (3+s)\ (4+s)\ (5+s)\ (6+s)\ (7+s)\ (8+s)\ (9+s)\ (10+s) - \frac{1}{74\,724\,249\,600} n^{-13-s} s\ (1+s)\ (2+s)\ (3+s)\ (4+s)\ (5+s)\ (6+s)\ (7+s)\ (8+s)\ (9+s)\ (10+s)\ (11+s)\ (12+s) + \left(3617\, n^{-15-s} s\ (1+s)\ (2+s)\ (3+s)\ (4+s)\ (5+s)\ (6+s)\ (7+s)\ (8+s)\ (9+s)\ (10+s)\ (11+s)\ (12+s)\ (13+s)\ (14+s)\right) \Big/ 10\,670\,622\,842\,880\,000 - \left(43\,867\, n^{-17-s} s\ (1+s)\ (2+s)\ (3+s)\ (4+s)\ (5+s)\ (6+s)\ (7+s)\ (8+s)\ (9+s)\ (10+s)\ (11+s)\ (12+s)\ (13+s)\ (14+s)\ (15+s)\ (16+s)\right) \Big/ 5\,109\,094\,217\,170\,944\,000 + \left(174\,611\, n^{-19-s} s\ (1+s)\ (2+s)\ (3+s)\ (4+s)\ (5+s)\ (6+s)\ (7+s)\ (8+s)\ (9+s)\ (10+s)\ (11+s)\ (12+s)\ (13+s)\ (14+s)\ (15+s)\ (16+s)\ (17+s)\ (18+s)\right) \Big/ 802\,857\,662\,698\,291\,200\,000\right)$$

$$\text{fna}[n\_,\ s\_,\ t\_] := n^{\wedge}(1/2+t) \Big/ \left(\frac{n^{-s}}{2} + \frac{n^{1-s}}{1-s} - \frac{1}{12} n^{-1-s} s + \frac{1}{720} n^{-3-s} s\ (1+s)\ (2+s) - \frac{n^{-5-s} s\ (1+s)\ (2+s)\ (3+s)\ (4+s)}{30\,240} + \frac{n^{-7-s} s\ (1+s)\ (2+s)\ (3+s)\ (4+s)\ (5+s)\ (6+s)}{1\,209\,600}\right)$$

$$\text{fnb}[n\_,\ s\_,\ t\_] := n^{\wedge}(1/2+t) \Big/ \left(\frac{1}{232\,792\,560\ (-1+s)} n^{-19-s} \left(456\, n^4 \left(-17\, n^2 \left(715\, n^8\, \text{Binomial}[1-s,\ 6] - 1001\, n^6\, \text{Binomial}[1-s,\ 8] + 2275\, n^4\, \text{Binomial}[1-s,\ 10] - 7601\, n^2\, \text{Binomial}[1-s,\ 12] + 35\,035\, \text{Binomial}[1-s,\ 14]\right) + 3\,620\,617\, \text{Binomial}[1-s,\ 16]\right) - 12\,796\,881\,240\, n^2\, \text{Binomial}[1-s,\ 18] + 29\,393 \left(-11\, n^{16} \left(720\, n^4 - 360\, n^3 (-1+s) + 60\, n^2 (-1+s)\, s - (-1+s)\, s\ (1+s)\ (2+s)\right) + 4\,190\,664\, \text{Binomial}[1-s,\ 20]\right)\right)\right)$$

$$\text{fnc}[n\_,\ s\_,\ t\_] := n^{\wedge}(1/2+t) \Big/ \left(\frac{n^{-s}}{2} + \frac{n^{1-s}}{1-s}\right)$$

$$\text{fnd}[n\_,\ s\_,\ t\_] := n^{\wedge}(1/2+t) \Big/ \left(\frac{n^{1-s}}{1-s}\right)$$

```
ps9[n_, s_, t_] := (fn[n, s, t] HarmonicNumber[n, s] - fn[n, 1 - s, t] HarmonicNumber[n, 1 - s]) /
  (fn[n, s, t] - fn[n, 1 - s, t] (Pi^(1 / 2 - s) Gamma[s / 2] / Gamma[(1 - s) / 2]))
ps9a[n_, s_, t_] := (fn[n, s, t] HarmonicNumber[n, s] - fn[n, 1 - s, t] HarmonicNumber[n, 1 - s])
ps9aa[n_, s_, t_] :=
 (fna[n, s, t] HarmonicNumber[n, s] - fna[n, 1 - s, t] HarmonicNumber[n, 1 - s])
ps9ab[n_, s_, t_] :=
 (fnb[n, s, t] HarmonicNumber[n, s] - fnb[n, 1 - s, t] HarmonicNumber[n, 1 - s])
ps9ac[n_, s_, t_] :=
 (fnc[n, s, t] HarmonicNumber[n, s] - fnc[n, 1 - s, t] HarmonicNumber[n, 1 - s])
ps9ad[n_, s_, t_] :=
 (fnd[n, s, t] HarmonicNumber[n, s] - fnd[n, 1 - s, t] HarmonicNumber[n, 1 - s])
ps9b[n_, s_, t_] := (fn[n, s, t] HarmonicNumber[n, s])
```

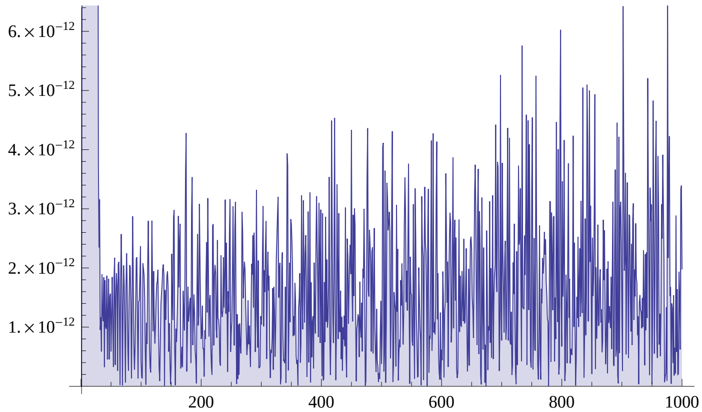
```
N[ps9a[1000, ZetaZero@10, -.5]]
```

0. - 0.279365 i

```
N[ps5[100 000 000, ZetaZero@10]]
```

0. - 0.00497738 i

```
DiscretePlot[ Abs@ps9a[n, N@ZetaZero@10, 0], {n, 1, 1000, 1}]
```




```
Zeta[.5 + 100 000 I]
```

1.07303 + 5.78085 i

```
100 000^.5
```

316.228

ps10[n_, s_, t_] := $n^{-\frac{1}{2}+s+t}$ HarmonicNumber[n, s] - $n^{\frac{1}{2}-s+t}$ HarmonicNumber[n, 1 - s]

ps11[n_, s_, t_, a_, b_] := $n^{-\frac{1}{2}+s+t}$ (1 - s) ^a / s^ (1 - b) HarmonicNumber[n, s] -
  $n^{\frac{1}{2}-s+t}$ s^b / (1 - s) ^ (1 - a) HarmonicNumber[n, 1 - s]

ps12[n_, s_, t_, a_, b_] := $n^{-\frac{1}{2}+s+t}$ (1 - s) ^a s^ (b - 1) HarmonicNumber[n, s] -
  $n^{\frac{1}{2}-s+t}$ s^b (1 - s) ^ (a - 1) HarmonicNumber[n, 1 - s]

```
ps11[1 000 000 000, N@ZetaZero@1 + .1 I, 0, .5, .5]
```

0. - 0.0266785 i

```
(1 - s) ^a / s^b  /.  a → 3 /. b → 2 /. s → .35
```

2.24184

```
((1 - s) / s) s^ (b - a)  /.  a → 3 /. b → 2 /. s → .35
```

5.30612

```
(1 - s) ^a s^-b  /.  a → 3 /. b → 2 /. s → .35
```

2.24184

```
((1 - s) / s) ^a s^ (-b + a)  /.  a → 3 /. b → 2 /. s → .35
```

2.24184

```
(1 / s - 1) ^a s^(a - b)  /. a → 3 /. b → 2 /. s → .35
```

2.24184

```
2 Pi^(s / 2) / Gamma[s / 2] /. s → .3
```

0.381762

```
2 Pi^(s / 2) Gamma[1 - s / 2] Sin[Pi s / 2] / Pi /. s → .3
```

0.381762

```
2 Pi^(s / 2 - 1) Gamma[1 - s / 2] Sin[Pi s / 2] /. s → .3
```

0.381762

```
FullSimplify[2 Pi^(s / 2 - 1) Gamma[1 - s / 2] Sin[Pi s / 2] /. s → 1 - s]
```

$$2\,\pi^{-\frac{1}{2}-\frac{s}{2}}\,\mathrm{Cos}\left[\frac{\pi s}{2}\right]\,\mathrm{Gamma}\left[\frac{1+s}{2}\right]$$

```
2 Pi^(s / 2 - 1) Gamma[1 - s / 2] Sin[Pi s / 2]
```

$$2\,\pi^{-1+\frac{s}{2}}\,\mathrm{Gamma}\left[1-\frac{s}{2}\right]\,\mathrm{Sin}\left[\frac{\pi s}{2}\right]$$

```
N@PolyGamma[2, 10]
```

-0.0110498

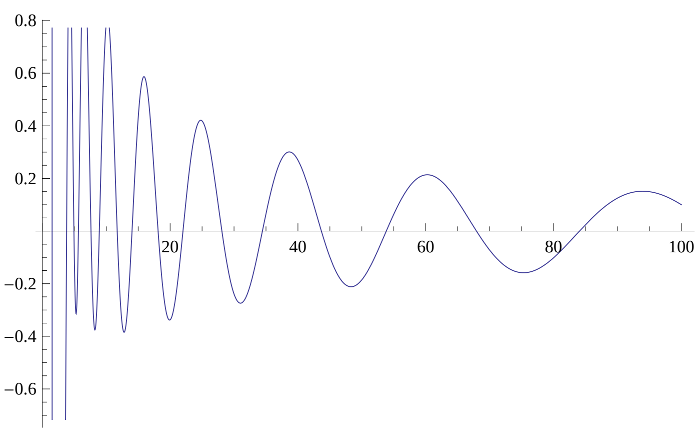
$$\mathrm{D}\left[n^{-\frac{1}{2}+s}\,(1-s)\,\mathrm{HarmonicNumber}[n,s]-n^{\frac{1}{2}-s}\,s\,\mathrm{HarmonicNumber}[n,1-s],\,n\right]$$

$$-n^{-\frac{1}{2}-s}\left(\frac{1}{2}-s\right)s\,\mathrm{HarmonicNumber}[n,1-s]+n^{-\frac{3}{2}+s}\,(1-s)\left(-\frac{1}{2}+s\right)\mathrm{HarmonicNumber}[n,s]-$$
$$n^{\frac{1}{2}-s}\,(1-s)\,s\,(-\mathrm{HarmonicNumber}[n,2-s]+\mathrm{Zeta}[2-s])+$$
$$n^{-\frac{1}{2}+s}\,(1-s)\,s\,(-\mathrm{HarmonicNumber}[n,1+s]+\mathrm{Zeta}[1+s])$$

```
fa[n_, s_] :=
```

$$\frac{1}{2}\,n^{-\frac{3}{2}-s}\left(n\,s\,(-1+2\,s)\,\mathrm{HarmonicNumber}[n,1-s]+(-1+s)\left(2\,n^2\,s\,\mathrm{HurwitzZeta}[2-s,1+n]+n^{2\,s}\,((1-2\,s)\,\mathrm{HarmonicNumber}[n,s]-2\,n\,s\,\mathrm{HurwitzZeta}[1+s,1+n])\right)\right)$$

```
Plot[Im@fa[n, N@ZetaZero@1 + .1], {n, 1, 100}]
```



$$\text{FullSimplify}\Big[-n^{-\frac{1}{2}-s}\left(\frac{1}{2}-s\right) s\,\text{HarmonicNumber}[n,1-s]+n^{-\frac{3}{2}+s}(1-s)\left(-\frac{1}{2}+s\right)\text{HarmonicNumber}[n,s]-n^{\frac{1}{2}-s}(1-s)\,s\,(-\text{HarmonicNumber}[n,2-s]+\text{Zeta}[2-s])+n^{-\frac{1}{2}+s}(1-s)\,s\,(-\text{HarmonicNumber}[n,1+s]+\text{Zeta}[1+s])\Big]$$

$$\frac{1}{2}n^{-\frac{3}{2}-s}\Big(n\,s\,(-1+2s)\,\text{HarmonicNumber}[n,1-s]+(-1+s)\Big(2n^2 s\,\text{HurwitzZeta}[2-s,1+n]+n^{2s}\left((1-2s)\,\text{HarmonicNumber}[n,s]-2n\,s\,\text{HurwitzZeta}[1+s,1+n]\right)\Big)\Big)$$

```
2 Pi^(s / 2) / Gamma[s / 2] /. s → .3
```

0.381762

```
2 Pi^(s / 2) / Gamma[(s - 1) / 2 + 1 / 2] /. s → .3
```

0.381762

$$2^{-1+s}\,\text{Pi}^{\wedge}((s-1)/2)\,\frac{\text{Gamma}[(s-1)/2]}{\text{Gamma}[-1+s]}\ /.\ s\to .3$$

0.381762

$$2^{-1+s}\,\text{Pi}^{\wedge}((s-1)/2)\,\frac{\text{Gamma}[(s-1)/2]}{\text{Gamma}[-1+s]}\ /.\ s\to 1-s$$

$$2^{-s}\,\pi^{-s/2}\,\frac{\text{Gamma}\left[-\frac{s}{2}\right]}{\text{Gamma}[-s]}$$

```
2^(s - 1) Pi^((s - 1) / 2) Gamma[(s - 1) / 2] Gamma[-s] /. s → .3
```

7.05937

```
2^(s - 1) Pi^((s - 1) / 2) Gamma[(s - 1) / 2] Gamma[-s] /. s → .3
```

7.05937

```
(Gamma[s] Gamma[s + 1 / 2] / (2^(1 - 2 s) Pi^(1 / 2) Gamma[2 s])) /. s → -s / 2
```

$$\frac{2^{-1-s}\,\text{Gamma}\left[\frac{1}{2}-\frac{s}{2}\right]\text{Gamma}\left[-\frac{s}{2}\right]}{\sqrt{\pi}\,\text{Gamma}[-s]}$$

$$2^{\wedge}(s-1)\,\text{Pi}^{\wedge}((s-1)/2)\,\text{Gamma}[(s-1)/2]\,\text{Gamma}[-s]\,\frac{2^{-1-s}\,\text{Gamma}\left[\frac{1}{2}-\frac{s}{2}\right]\text{Gamma}\left[-\frac{s}{2}\right]}{\sqrt{\pi}\,\text{Gamma}[-s]}\ /.\ s\to .3\ /.\ s\to .3$$

7.05937

$$2^{\wedge}(s-1)\,\text{Pi}^{\wedge}((s-1)/2)\,\text{Gamma}[(s-1)/2]\,\frac{2^{-1-s}\,\text{Gamma}\left[\frac{1}{2}-\frac{s}{2}\right]\text{Gamma}\left[-\frac{s}{2}\right]}{\sqrt{\pi}}\ /.\ s\to .3\ /.\ s\to .3$$

7.05937

$$2^{\wedge}(-2)\,\text{Pi}^{\wedge}((s-2)/2)\,\text{Gamma}[(s-1)/2]\,\text{Gamma}\left[\frac{1}{2}-\frac{s}{2}\right]\text{Gamma}\left[-\frac{s}{2}\right]\ /.\ s\to .3$$

7.05937

$$2\text{^}(-2)\ \text{Pi}\text{^}((s-2)/2)\ \text{Gamma}[(s-1)/2]\ \text{Gamma}\left[\frac{(1-s)}{2}\right]\ \text{Gamma}\left[-\frac{s}{2}\right]$$

$$\frac{1}{4}\pi^{\frac{1}{2}(-2+s)}\ \text{Gamma}\left[\frac{1-s}{2}\right]\ \text{Gamma}\left[\frac{1}{2}(-1+s)\right]\ \text{Gamma}\left[-\frac{s}{2}\right]$$

$$2\text{^}(-s)\ \text{Pi}\text{^}((-s)/2)\ \text{Gamma}[(-s)/2]\ \text{Gamma}[s]\ /.\ s\to .3$$

-15.1782

$$2\text{^}(-s)\ \text{Pi}\text{^}((-s)/2)\ \text{Gamma}[(-s)/2]\ \text{Gamma}[s]\ \frac{2^{-1+s}\ \text{Gamma}\left[\frac{1}{2}+\frac{s}{2}\right]\ \text{Gamma}\left[\frac{s}{2}\right]}{\sqrt{\pi}\ \text{Gamma}[s]}\ /.\ s\to .3$$

-15.1782

$$2\text{^}(-s)\ \text{Pi}\text{^}((-s)/2)\ \text{Gamma}[(-s)/2]\ \frac{2^{-1+s}\ \text{Gamma}\left[\frac{1}{2}+\frac{s}{2}\right]\ \text{Gamma}\left[\frac{s}{2}\right]}{\sqrt{\pi}}\ /.\ s\to .3$$

-15.1782

$$2\text{^}-1\ \text{Pi}\text{^}((-s-1)/2)\ \text{Gamma}[-s/2]\ \text{Gamma}\left[\frac{1}{2}+\frac{s}{2}\right]\ \text{Gamma}\left[\frac{s}{2}\right]\ /.\ s\to .3$$

-15.1782

```
pb[s_] := 2 Pi^(s / 2) / Gamma[s / 2]
pb2[s_] := pb[s] / pb[1 - s]
```

```
N[pb2[s] pb2[1 - s] /. s → 1 / 2 + 30 I]
```

1.

```
N@Abs[pb2[s]] /. s → 1 / 2 + 10 I
```

1.

$$\begin{aligned}\text{fnx}[n\_,\ s\_] := {} & \frac{n^{-s}}{2}+\frac{n^{1-s}}{1-s}-\frac{1}{12}n^{-1-s}s+\frac{1}{720}n^{-3-s}s(1+s)(2+s)-\\ & \frac{n^{-5-s}s(1+s)(2+s)(3+s)(4+s)}{30\,240}+\frac{n^{-7-s}s(1+s)(2+s)(3+s)(4+s)(5+s)(6+s)}{1\,209\,600}-\\ & \frac{n^{-9-s}s(1+s)(2+s)(3+s)(4+s)(5+s)(6+s)(7+s)(8+s)}{47\,900\,160}+\frac{1}{1\,307\,674\,368\,000}\\ & 691\,n^{-11-s}s(1+s)(2+s)(3+s)(4+s)(5+s)(6+s)(7+s)(8+s)(9+s)(10+s)-\frac{1}{74\,724\,249\,600}\\ & n^{-13-s}s(1+s)(2+s)(3+s)(4+s)(5+s)(6+s)(7+s)(8+s)(9+s)(10+s)(11+s)(12+s)+\\ & \big(3617\,n^{-15-s}s(1+s)(2+s)(3+s)(4+s)(5+s)(6+s)(7+s)(8+s)\\ & \quad(9+s)(10+s)(11+s)(12+s)(13+s)(14+s)\big)\big/10\,670\,622\,842\,880\,000-\\ & \big(43\,867\,n^{-17-s}s(1+s)(2+s)(3+s)(4+s)(5+s)(6+s)(7+s)(8+s)(9+s)(10+s)\\ & \quad(11+s)(12+s)(13+s)(14+s)(15+s)(16+s)\big)\big/5\,109\,094\,217\,170\,944\,000+\\ & \big(174\,611\,n^{-19-s}s(1+s)(2+s)(3+s)(4+s)(5+s)(6+s)(7+s)(8+s)(9+s)(10+s)\\ & \quad(11+s)(12+s)(13+s)(14+s)(15+s)(16+s)(17+s)(18+s)\big)\big/802\,857\,662\,698\,291\,200\,000\end{aligned}$$

```
fn2[n_, s_, t_] := n^(-1/2+s) (1 - s)
eul[n_, s_] := HarmonicNumber[n, s] - fnx[n, s]
eul2[n_, s_] := HarmonicNumber[n, s] - n^(1 - s) / (1 - s) - n^-s / 2
rie[n_, s_] :=
 HarmonicNumber[n, s] + 2^s Pi^(s - 1) Sin[Pi s / 2] Gamma[1 - s] HarmonicNumber[n, 1 - s]
rie2[s_] := rie[Floor[(Im@s / (2 Pi))^(1 / 2)], s]
ps9[n_, s_, t_] :=
 (fn[n, s, t] HarmonicNumber[n, s] - fn[n, 1 - s, t] HarmonicNumber[n, 1 - s]) /
  (fn[n, s, t] - fn[n, 1 - s, t] (Pi^(1 / 2 - s) Gamma[s / 2] / Gamma[(1 - s) / 2]))
ps9x[n_, s_, t_] := (fn2[n, s, t] HarmonicNumber[n, s] -
    fn2[n, 1 - s, t] HarmonicNumber[n, 1 - s]) /
  (fn2[n, s, t] - fn2[n, 1 - s, t] (Pi^(1 / 2 - s) Gamma[s / 2] / Gamma[(1 - s) / 2]))
ps9c[n_, c_, s_, t_] :=
 (fn[n, s, t] HarmonicNumber[c, s] - fn[n, 1 - s, t] HarmonicNumber[c, 1 - s]) /
  (fn[n, s, t] - fn[n, 1 - s, t] (Pi^(1 / 2 - s) Gamma[s / 2] / Gamma[(1 - s) / 2]))
ps92[s_] := ps9[Floor[(Im@s / (2 Pi))^(1 / 2)], s, 0]
ps92x[s_] := ps9x[Floor[Im@s], Floor[(Im@s / (2 Pi))^(1 / 2)], s, 0]
eula[s_] := eul[Floor[(Im@s / (2 Pi))^(1 / 2)], s]
eula2[s_] := eul2[Floor[(Im@s / (2 Pi))^(1 / 2)], s]
dum[s_] := dum2[Floor[(Im@s / (2 Pi))^(1 / 2)], s]
```

```
eul2[10, .7 + 10 000 I]
```

0.314492 − 0.479657 i

```
ps9o[10 000, N@ZetaZero@1 + .1, 0]
```

0.0753346 + 0.0113729 i

```
Zeta[N@ZetaZero@10 000 + .3]
```

0.974392 − 0.319534 i

```
Zeta[N@ZetaZero@121 121 + .1]
```

0.226099 − 0.474933 i

```
rie2[N@ZetaZero@121 121 + .1]
```

0.273798 − 0.467474 i

```
ps92[N@ZetaZero@121 121 + .1]
```

1.06876 + 0.215879 i

```
eula[N@ZetaZero@121 121 + .1]
```

$3.35361 \times 10^{37} + 4.68122 \times 10^{37}$ i

```
eula2[N@ZetaZero@121 121 + .1]
```

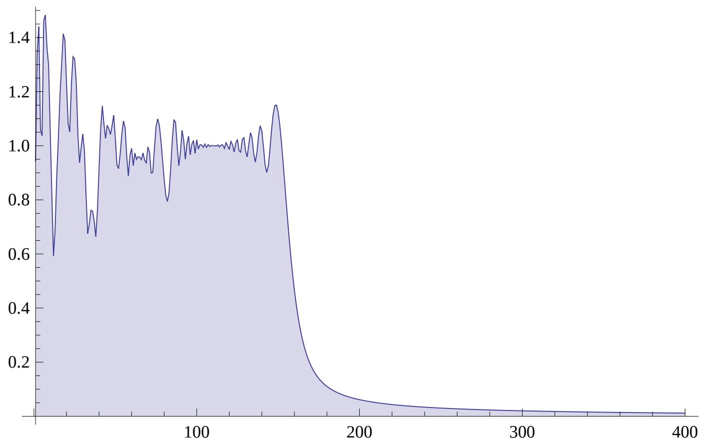
0.569365 − 1.16577 i

```
dum[.6 + 20 000 I]
```

dum2[56, 0.6 + 20 000. i]

```
ps9e[n_, c_, s_, t_] := (fn[n, s, t] c^-s - fn[n, 1 - s, t] c^(s - 1)) /
  (fn[n, s, t] - fn[n, 1 - s, t] (Pi^(1 / 2 - s) Gamma[s / 2] / Gamma[(1 - s) / 2]))
ps9e2[n_, c_, s_, t_] := (fn[n, s, t] c^-s - fn[n, 1 - s, t] c^(s - 1))
```

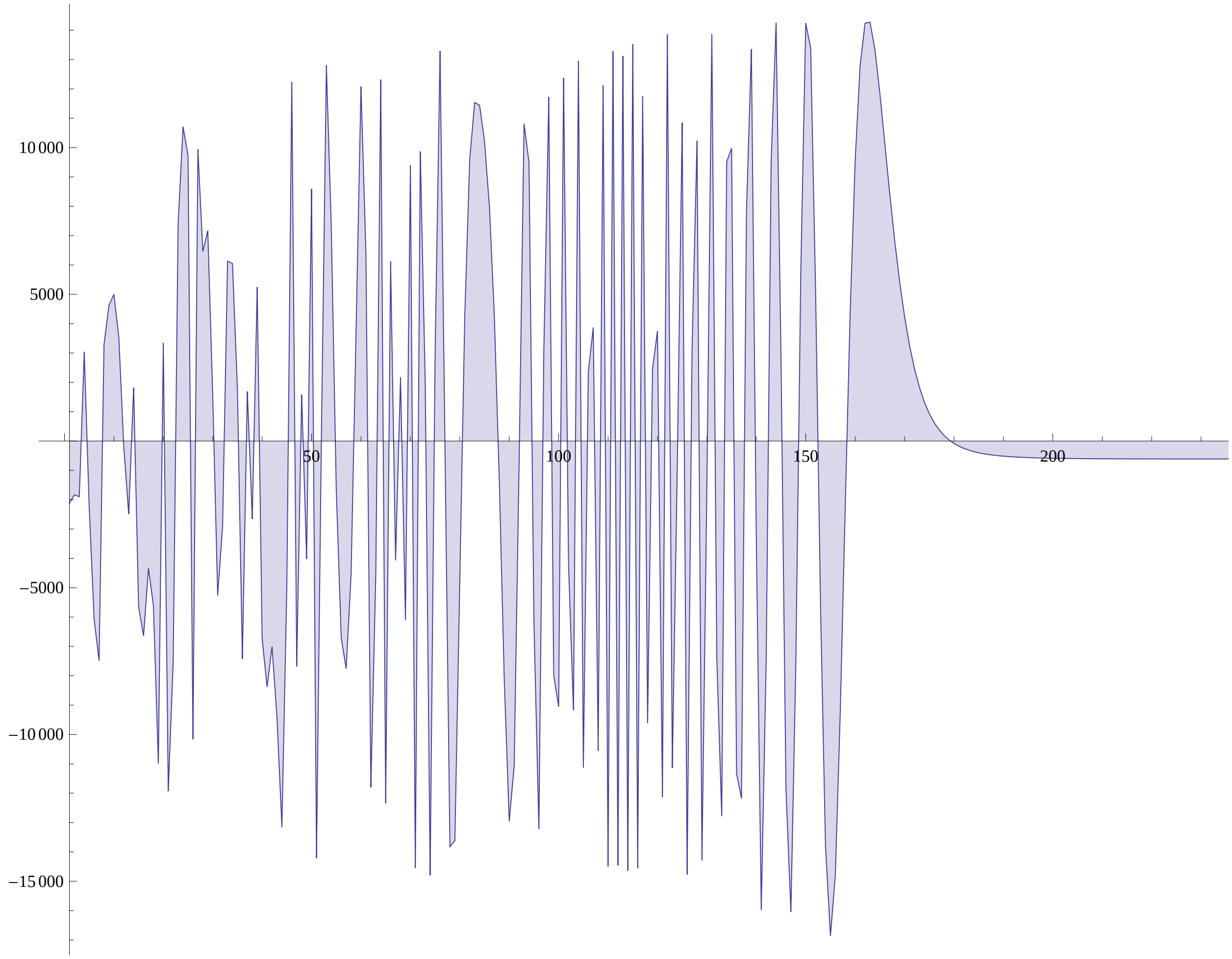
```
DiscretePlot[Abs[eul2[n, .5 + 1000 I] - Zeta[.5 + 1000 I]], {n, 1, 400}]
```



ps5a[n_, j_, s_] := $n^{-\frac{1}{2}+s}$ (1 - s) j^-s - $n^{\frac{1}{2}-s}$ s (j^(s - 1))

```
ps5b[n_, j_, s_] := j^(-1 / 2) ((1 - 2 s) Cosh[(1 / 2 - s) Log[j / n]] + Sinh[(1 / 2 - s) Log[j / n]])
```

```
DiscretePlot[Im[ ps8[n, N@ZetaZero@700, .4]], {n, 1, 300}]
```



$$\texttt{fo[n\_, s\_] :=} \left(\frac{n^{-s}}{2} + \frac{n^{1-s}}{1-s} - \frac{1}{12} n^{-1-s} s + \frac{1}{720} n^{-3-s} s (1+s)(2+s) - \frac{n^{-5-s} s (1+s)(2+s)(3+s)(4+s)}{30\,240} + \frac{n^{-7-s} s (1+s)(2+s)(3+s)(4+s)(5+s)(6+s)}{1\,209\,600} - \frac{n^{-9-s} s (1+s)(2+s)(3+s)(4+s)(5+s)(6+s)(7+s)(8+s)}{47\,900\,160} + \frac{1}{1\,307\,674\,368\,000} 691\, n^{-11-s} s (1+s)(2+s)(3+s)(4+s)(5+s)(6+s)(7+s)(8+s)(9+s)(10+s) - \frac{1}{74\,724\,249\,600} n^{-13-s} s (1+s)(2+s)(3+s)(4+s)(5+s)(6+s)(7+s)(8+s)(9+s)(10+s)(11+s)(12+s) + \left(3617\, n^{-15-s} s (1+s)(2+s)(3+s)(4+s)(5+s)(6+s)(7+s)(8+s)(9+s)(10+s)(11+s)(12+s)(13+s)(14+s)\right) / 10\,670\,622\,842\,880\,000 - \left(43\,867\, n^{-17-s} s (1+s)(2+s)(3+s)(4+s)(5+s)(6+s)(7+s)(8+s)(9+s)(10+s)(11+s)(12+s)(13+s)(14+s)(15+s)(16+s)\right) / 5\,109\,094\,217\,170\,944\,000 + \left(174\,611\, n^{-19-s} s (1+s)(2+s)(3+s)(4+s)(5+s)(6+s)(7+s)(8+s)(9+s)(10+s)(11+s)(12+s)(13+s)(14+s)(15+s)(16+s)(17+s)(18+s)\right) / 802\,857\,662\,698\,291\,200\,000\right)$$

$$\texttt{fp[n\_, s\_] :=} \frac{n^{1-s}}{1-s}$$

$$\texttt{fp2[n\_, s\_] :=} \frac{n^{1-s}}{1-s} + \frac{n^{-s}}{2}$$

$$\texttt{fp3[n\_, s\_] :=} \frac{n^{1-s}}{1-s} + \frac{n^{-s}}{2} - \frac{1}{12} n^{-1-s} s$$

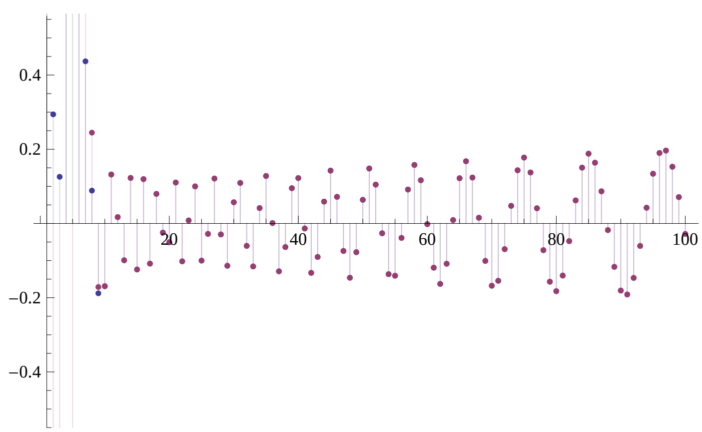
```
fo[100, .5]
```

20.05

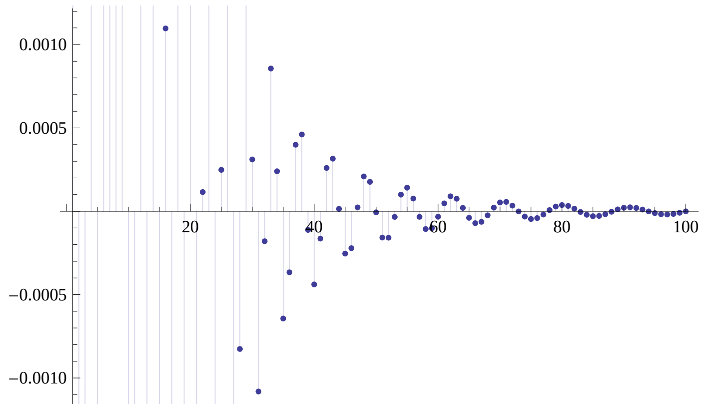
```
(HarmonicNumber[n, s] / fo[n, s] - HarmonicNumber[n, 1 - s] / fo[n, 1 - s]) /. n → 100 /. s → .3 + 2 I
```

-0.164126 + 0.279684 ⅈ

```
DiscretePlot[
 {Re@ HarmonicNumber[ n, N@ZetaZero@10], Re@fo[n, N@ZetaZero@10]}, {n, 1, 100}]
```




```
DiscretePlot[{ Re[fo[n, N@ZetaZero@10]] - Re[fp3[n, N@ZetaZero@10]]}, {n, 1, 100}]
```




```
N[(174611 n^(-19-s) s (1 + s) (2 + s) (3 + s) (4 + s) (5 + s) (6 + s) (7 + s) (8 + s) (9 + s)
    (10 + s) (11 + s) (12 + s) (13 + s) (14 + s) (15 + s) (16 + s) (17 + s) (18 + s)) /
  802857662698291200000 /. s → ZetaZero@10 /. n → 10000000]
```

$-1.87273\times 10^{-120} - 2.98119\times 10^{-122}\, i$

```
FullSimplify[n^(1/2 + t) / (n^-s/2 + n^(1-s)/(1 - s) - 1/12 n^(-1-s) s + 1/720 n^(-3-s) s (1 + s) (2 + s) -
    n^(-5-s) s (1 + s) (2 + s) (3 + s) (4 + s)/30240 + n^(-7-s) s (1 + s) (2 + s) (3 + s) (4 + s) (5 + s) (6 + s)/1209600)]
```

$$\left(1209600\, n^{\frac{1}{2}+s+t}\right) \Big/ \left(604800 - \frac{1209600\, n}{-1+s} - \frac{100800\, s}{n} + \frac{1680\, s\,(1+s)\,(2+s)}{n^3} - \frac{40\, s\,(1+s)\,(2+s)\,(3+s)\,(4+s)}{n^5} + \frac{s\,(1+s)\,(2+s)\,(3+s)\,(4+s)\,(5+s)\,(6+s)}{n^7}\right)$$

```
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
bern[k_] := If[k == 1, 1 / 2, BernoulliB[k]]
bs[n_, s_, t_] := 1 / (1 - s) Sum[FullSimplify[ bin[1 - s, k] bern[k] n^(1 - s - k)], {k, 0, t}]
bs2[n_, s_, t_] :=
 FullSimplify[1 / (1 - s) Sum[FullSimplify[Binomial[1 - s, k] bern[k] n^(1 - s - k)], {k, 0, t}]]
```

```
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
bern[k_] := If[k == 1, 1 / 2, BernoulliB[k]]
bs[n_, s_, t_] := 1 / (1 - s) Sum[FullSimplify[ bin[1 - s, k] bern[k] n^(1 - s - k)], {k, 0, t}]
bbs[n_, s_, a_, b_, c_, d_] :=
 (1 - s)^a / s^b Sum[FullSimplify[ bin[s, k] bern[k] n^(c + s - k)], {k, 0, d}] /
   Sum[FullSimplify[ bin[1 - s, k] bern[k] n^(c + 1 - s - k)], {k, 0, d}]
```

```
FullSimplify[n^(c + y) (1 - s - y) (s + y) / t^(s + y + 1) - n^(c + x) (1 - s - x) (s + x) / t^(s + x + 1)]
```

$n^c\, t^{-1-s}\left(n^x t^{-x}(-1+s+x)(s+x) - n^y t^{-y}(-1+s+y)(s+y)\right)$

```
n^c t^(-1-s) ((n / t)^x (-1 + s + x) (s + x) - (n / t)^y (-1 + s + y) (s + y))
```

$$n^c\, t^{-1-s}\left(\left(\frac{n}{t}\right)^x (-1+s+x)\,(s+x) - \left(\frac{n}{t}\right)^y (-1+s+y)\,(s+y)\right)$$

$$\text{Integrate}\left[n^{c}\,t^{-1-s}\left(\left(\frac{n}{t}\right)^{x}(-1+s+x)\,(s+x)-\left(\frac{n}{t}\right)^{y}(-1+s+y)\,(s+y)\right),\{t,n,\text{Infinity}\}\right]$$

$$\text{ConditionalExpression}\left[n^{c-s}\,(x-y),\ \text{Re}[s+x]>0\ \&\&\ \text{Re}[s+y]>0\ \&\&\ n>0\right]$$

$$\text{Limit}\left[n^{-.01}\,(x-y),\ n\to\text{Infinity}\right]$$

0.

$$\text{Integrate}\left[n^{c}\,t^{-1-s}\left(\left(\frac{n}{t}\right)^{x}(-1+s+x)\,(s+x)-\left(\frac{n}{t}\right)^{y}(-1+s+y)\,(s+y)\right)\text{FractionalPart}[t],\{t,n,\text{Infinity}\}\right]\ /.\ x\to 2\ /.\ y\to 3\ /.\ c\to 2\ /.\ s\to 3$$