```
L2[n_{,}1,b_{,}] := L2[n,1,b] = Sum[Log[j], \{j,2,n\}] - b Sum[Log[jb], \{j,1,n/b\}]
L2[n_{,k_{,b_{,j}}}] := Sum[L2[n/j,k-1,b], {j,2,n}] - bSum[L2[n/(jb),k-1,b], {j,1,n}]
         \texttt{L1}[\texttt{n}\_, \ \texttt{z}\_, \ \texttt{b}\_] \ := \ \texttt{Sum}[\ \texttt{Binomial}[\texttt{z}, \ \texttt{k}] \ \texttt{L2}[\texttt{n}, \ \texttt{k}, \ \texttt{b}], \ \{\texttt{k}, \ \texttt{0}, \ \texttt{Log}[\texttt{If}[\texttt{b} < 2, \ \texttt{b}, \ \texttt{2}], \ \texttt{n}]\}] 
sa[n_, c_] := Sum[c^k Log[c], \{k, 1, Log[c, n]\}]
DiscretePlot[L1[n, -1, 2] - sa[n, 2], {n, 1, 100}]
                                                                                     100
                                                                     80
-20
-40
-60
-80
bin[z_{,k_{]}} := Product[z-j, {j, 0, k-1}]/k!
\mathtt{Sum}[\,(\mathtt{j}+\mathtt{1})\,\,{}^{\wedge}-\mathtt{s}\,\mathtt{D1xD}[\,\mathtt{n}\,/\,\,(\mathtt{j}+\mathtt{1})\,\,,\,\,\mathtt{s},\,\,\mathtt{k}\,-\mathtt{1},\,\,\mathtt{x}]\,\,-\mathtt{x}\,\,(\mathtt{j}\,\mathtt{x})\,\,{}^{\wedge}-\mathtt{s}\,\mathtt{D1xD}[\,\mathtt{n}\,/\,\,(\mathtt{x}\,\mathtt{j})\,\,,\,\,\mathtt{s},\,\,\mathtt{k}\,-\mathtt{1},\,\,\mathtt{x}]\,\,,\,\,\{\mathtt{j},\,\,\mathtt{1},\,\,\mathtt{n}\}]
D1xD[n_, s_, 0, x_] := UnitStep[n-1]
DxD[n_, s_, z_, x_] :=
 Sum[bin[z, k] D1xD[n, s, k, x], \{k, 0, If[x < 2, Log[x, n], Log[2, n]]\}]
N[D[FullSimplify@D1xD[100, s, 2, 2], s] /. s \rightarrow 0]
-12.8961
-2N@L2[100, 2, 2]
-12.8961
\texttt{D[Expand@N[D[DxD[100, s, z, 2], s] /. s \rightarrow 0], z] /. z \rightarrow 0}
-6.70877
N@L1[100, -1, 2]
-6.70877
```