```
{Integrate[- Zeta'[t] / Zeta[t], {t, s, Infinity}], Log[Zeta[s]]}

{Log[Zeta[s]], Log[Zeta[s]]}

Log[Zeta[s]]

zetaz[n_, s_, z_, k_] :=
 zetaz[n, s, z, k] = 1 + ((z + 1) / k - 1) Sum[j ^ - s zetaz[Floor[n / j], s, z, k + 1], {j, 2, n}]

N[- Expand[Integrate[ D[D[Expand[zetaz[20, s, z, 1]], s], z], {s, 3, Infinity}]]]

0.183719 + 0.0326295 z + 0.00247243 z^2 + 0.0000406901 z^3

N[Expand[D[zetaz[20, s, z, 1], z] /. s → 3]]

0.183719 + 0.0326295 z + 0.00247243 z^2 + 0.0000406901 z^3

{D[Zeta[s], s] / Zeta[s], D[Log[Zeta[s]], s]}
```

$$\left\{\frac{\mathrm{Zeta}'[s]}{\mathrm{Zeta}[s]}, \frac{\mathrm{Zeta}'[s]}{\mathrm{Zeta}[s]}\right\}$$

```
Sum[ (-1) ^j Binomial[k, j] (y - 1) ^ (-s j) Zeta[ s, y - 1] ^ (k - j) , {j, 0, k}]
```

$$\left(-1 - 2^{-s} + \mathrm{Zeta}[s]\right)^{k}$$

```
Chop[
 N[Zeta[s, y + 1] ^k - Sum[ (-1) ^j Binomial[k, j] (y) ^ (-s j) Zeta[ s, y] ^ (k - j) , {j, 0, k}] /.
   {s → 2, k → 2, y → 4}]]

0
```