```
ClearAll["Global`*"]

K[n_] := If[n == 1, 0, FullSimplify[MangoldtLambda[n] / Log[n]]]
bins[z_, a_] := Product[ (z - k), {k, 0, a - 1}] / a!
d[f_, n_, k_] := d[f, n, k] = Sum[d[f, j, k - 1] d[f, n / j, 1], {j, Divisors[n]}];
d[f_, n_, 1] := f[n]; d[f_, n_, 0] := 0; d[f_, 1, 0] := 1
d2[f_, n_, k_] := d2[f, n, k] = Sum[(-1) ^ (k - j) Binomial[k, j] d[f, n, j], {j, 0, k}]
dz[f_, n_, z_] := dz[f, n, z] = Sum[bins[z, k] d2[f, n, k], {k, 0, Log[2, n]}]
Dz[f_, n_, z_] := Dz[f, n, z] = Sum[dz[f, j, z], {j, 1, n}]
Dh[f_, n_, k_, a_] := Sum[ f[m] ^ (k - j) Binomial[k, j] Dh[f, Floor[n / m ^ (k - j)], j, m + 1],
    {m, a, n ^ (1 / k)}, {j, 0, k - 1}];
Dh[f_, n_, 0, a_] := 1

zeros[f_, n_] := List @@ NRoots[Dz[f, n, z] == 0, z][[All, 2]]
DzAlt[f_, n_, z_] := Product[1 - z / r, {r, zeros[f, n]}]
Linnik[f_, n_] := Linnik[f, n] = Sum[(-1) ^ (k + 1) / k d2[f, n, k], {k, 1, Log[2, n]}]
RecurseLinnik[f_, n_, k_] :=
 f[n] / k - Sum[If[j == 1 || n == j, 0, f[n / j] RecurseLinnik[f, j, k + 1]], {j, Divisors[n]}]

t[n_, j_] := Mod[n, j] - Mod[n - 1, j]

Ident[n_] := 1
IdentAlt[n_] := (-1) ^ (n + 1)
Ident3[n_] := t[n, 3]
Identn[n_] := n
MuSquared[n_] := MoebiusMu[n] ^ 2
Divisor1[n_] := DivisorSigma[1, n]
PrimeExp[n_, z_] := Product[z ^ p[[2]] / (p[[2]] !), {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
PrimeExp1[n_] := PrimeExp[n, 1]
funcs := { Identn, Ident, IdentAlt, Ident3, MoebiusMu,
   LiouvilleLambda, MuSquared, Divisor1, EulerPhi, PrimeExp1}
```

```
Table[ Limit[ dz[ f, n, z] / z, z → 0],  {n, 2, 40}, {f, funcs}] // TableForm
```

2	1	-1	1	-1	-1	1	3	1	1
3	1	1	-2	-1	-1	1	4	2	1
2	$\frac{1}{2}$	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	0
5	1	1	1	-1	-1	1	6	4	1
0	0	0	0	0	0	0	0	0	0
7	1	1	1	-1	-1	1	8	6	1
$\frac{8}{3}$	$\frac{1}{3}$	$-\frac{7}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	3	$\frac{7}{3}$	0
$\frac{9}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-4	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	5	4	0
0	0	0	0	0	0	0	0	0	0
11	1	1	1	-1	-1	1	12	10	1
0	0	0	0	0	0	0	0	0	0
13	1	1	1	-1	-1	1	14	12	1
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
4	$\frac{1}{4}$	$-\frac{15}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{17}{4}$	$\frac{15}{4}$	0
17	1	1	1	-1	-1	1	18	16	1
0	0	0	0	0	0	0	0	0	0
19	1	1	1	-1	-1	1	20	18	1
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
23	1	1	1	-1	-1	1	24	22	1
0	0	0	0	0	0	0	0	0	0
$\frac{25}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	13	12	0
0	0	0	0	0	0	0	0	0	0
9	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{26}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{28}{3}$	$\frac{26}{3}$	0
0	0	0	0	0	0	0	0	0	0
29	1	1	1	-1	-1	1	30	28	1
0	0	0	0	0	0	0	0	0	0
31	1	1	1	-1	-1	1	32	30	1
$\frac{32}{5}$	$\frac{1}{5}$	$-\frac{31}{5}$	$\frac{1}{5}$	$-\frac{1}{5}$	$-\frac{1}{5}$	$\frac{1}{5}$	$\frac{33}{5}$	$\frac{31}{5}$	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
37	1	1	1	-1	-1	1	38	36	1
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

```
Table[ D[ dz[ f, n, z], z] /. z → 0,  {n, 2, 40}, {f, funcs}] // TableForm
```

2	1	-1	1	-1	-1	1	3	1	1
3	1	1	-2	-1	-1	1	4	2	1
2	$\frac{1}{2}$	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	0
5	1	1	1	-1	-1	1	6	4	1
0	0	0	0	0	0	0	0	0	0
7	1	1	1	-1	-1	1	8	6	1
$\frac{8}{3}$	$\frac{1}{3}$	$-\frac{7}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	3	$\frac{7}{3}$	0
$\frac{9}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-4	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	5	4	0
0	0	0	0	0	0	0	0	0	0
11	1	1	1	-1	-1	1	12	10	1
0	0	0	0	0	0	0	0	0	0
13	1	1	1	-1	-1	1	14	12	1
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
4	$\frac{1}{4}$	$-\frac{15}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{17}{4}$	$\frac{15}{4}$	0
17	1	1	1	-1	-1	1	18	16	1
0	0	0	0	0	0	0	0	0	0
19	1	1	1	-1	-1	1	20	18	1
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
23	1	1	1	-1	-1	1	24	22	1
0	0	0	0	0	0	0	0	0	0
$\frac{25}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	13	12	0
0	0	0	0	0	0	0	0	0	0
9	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{26}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{28}{3}$	$\frac{26}{3}$	0
0	0	0	0	0	0	0	0	0	0
29	1	1	1	-1	-1	1	30	28	1
0	0	0	0	0	0	0	0	0	0
31	1	1	1	-1	-1	1	32	30	1
$\frac{32}{5}$	$\frac{1}{5}$	$-\frac{31}{5}$	$\frac{1}{5}$	$-\frac{1}{5}$	$-\frac{1}{5}$	$\frac{1}{5}$	$\frac{33}{5}$	$\frac{31}{5}$	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
37	1	1	1	-1	-1	1	38	36	1
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

```
Table[ RecurseLinnik[ f, n, 1],  {n, 2, 40}, {f, funcs}] // TableForm
```

2	1	-1	1	-1	-1	1	3	1	1
3	1	1	-2	-1	-1	1	4	2	1
2	$\frac{1}{2}$	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	0
5	1	1	1	-1	-1	1	6	4	1
0	0	0	0	0	0	0	0	0	0
7	1	1	1	-1	-1	1	8	6	1
$\frac{8}{3}$	$\frac{1}{3}$	$-\frac{7}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	3	$\frac{7}{3}$	0
$\frac{9}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-4	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	5	4	0
0	0	0	0	0	0	0	0	0	0
11	1	1	1	-1	-1	1	12	10	1
0	0	0	0	0	0	0	0	0	0
13	1	1	1	-1	-1	1	14	12	1
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
4	$\frac{1}{4}$	$-\frac{15}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{17}{4}$	$\frac{15}{4}$	0
17	1	1	1	-1	-1	1	18	16	1
0	0	0	0	0	0	0	0	0	0
19	1	1	1	-1	-1	1	20	18	1
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
23	1	1	1	-1	-1	1	24	22	1
0	0	0	0	0	0	0	0	0	0
$\frac{25}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	13	12	0
0	0	0	0	0	0	0	0	0	0
9	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{26}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{28}{3}$	$\frac{26}{3}$	0
0	0	0	0	0	0	0	0	0	0
29	1	1	1	-1	-1	1	30	28	1
0	0	0	0	0	0	0	0	0	0
31	1	1	1	-1	-1	1	32	30	1
$\frac{32}{5}$	$\frac{1}{5}$	$-\frac{31}{5}$	$\frac{1}{5}$	$-\frac{1}{5}$	$-\frac{1}{5}$	$\frac{1}{5}$	$\frac{33}{5}$	$\frac{31}{5}$	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
37	1	1	1	-1	-1	1	38	36	1
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0