

```
ClearAll["Global`*"]
```

```
E1a[n_, k_, a_] :=
```

```
  E1a[n, k, a] = Sum[E1a[n / j, k - 1, a], {j, 1, n}] - a Sum[E1a[n / (a j), k - 1, a], {j, 1, n / a}];
```

```
E1a[n_, 0, a_] := 1
```

```
DDa[n_, k_] := DDa[n, k] = Sum[DDa[Floor[n / j], k - 1], {j, 1, n}]; DDa[n_, 0] := 1
```

```
D2a[n_, k_] := D2a[n, k] = Sum[D2a[Floor[n / j], k - 1], {j, 2, n}]; D2a[n_, 0] := 1
```

```
DDd[n_, z_] := DDd[n, z] = Sum[FactorialPower[z, a] / a! D2a[n, a], {a, 0, Log[2, n]}]
```

```
DDc[n_, k_, b_] := Sum[Binomial[k + j - 1, k - 1] b^j E1a[n / b^j, k, b], {j, 0, Log[b, n]}]
```

```
E1c[n_, k_, b_] := Sum[(-1)^j Binomial[k, j] b^j DDa[n / b^j, k], {j, 0, k}]
```

```
E2a[n_, k_, a_] :=
```

```
  E2a[n, k, a] = Sum[E2a[n / j, k - 1, a], {j, 2, n}] - a Sum[E2a[n / (a j), k - 1, a], {j, 1, n / a}];
```

```
E2a[n_, 0, a_] := 1
```

```
E2z[n_, k_, a_] :=
```

```
  E2z[n, k, a] = Sum[E2z[n / j, k - 1, a], {j, 2, n}] + a Sum[E2z[n / (a j), k - 1, a], {j, 1, n / a}];
```

```
E2z[n_, 0, a_] := 1
```

```
E2r[n_, k_, a_] :=
```

```
  E2r[n, k, a] = Sum[E2r[n / j, k - 1, a], {j, 2, n}] - Sum[E2r[n / (a j), k - 1, a], {j, 1, n / a}];
```

```
E2r[n_, 0, a_] := 1
```

```
E2q[n_, k_, a_, b_] := Sum[E2q[n / j, k - 1, a, b], {j, 2, n}] -
```

```
  Sum[E2q[n / (a j), k - 1, a, b], {j, 1, n / a}] - Sum[E2q[n / (b j), k - 1, a, b], {j, 1, n / b}]
```

```
E2q[n_, 0, a_, b_] := 1
```

```
Linr[n_, a_] := Sum[(-1)^(k + 1) / k E2r[n, k, a], {k, 1, Log[a, n]}]
```

```
Linq[n_, a_, b_] := Sum[(-1)^(k + 1) / k E2q[n, k, a, b], {k, 1, Log[a, n]}]
```

```
Linr2[n_, a_] := Sum[(-1)^(k + 1) / k E2a[n, k, a], {k, 1, Log[a, n]}]
```

```
Linz[n_, a_] := Sum[(-1)^(k + 1) / k E2z[n, k, a], {k, 1, Log[a, n]}]
```

```
DDa[100, 1]
```

```
100
```

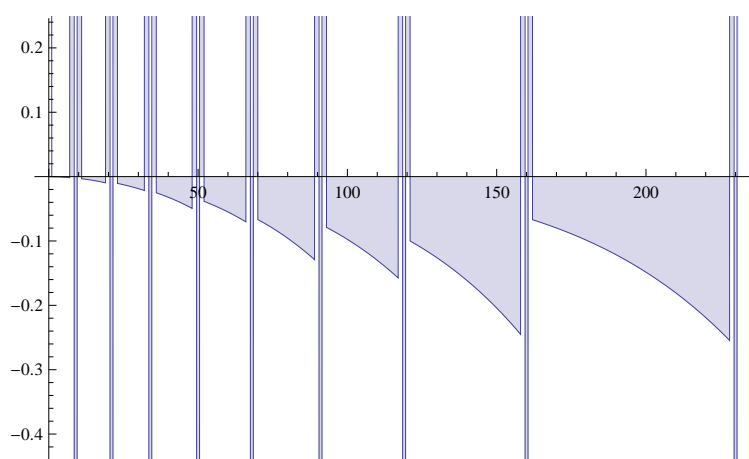
```
DDc[100, 1, 3]
```

```
100
```

```
fo[n_, k_, b_] :=
```

```
  DiscretePlot[{Binomial[k + j - 1, k - 1] b^j E1a[n / b^j, k, b]}, {j, 0, Log[b, n]}]
```

fo[10, 3, 1.01]



DDa[10, 1]

10

DDc[10, 1, .9]

0

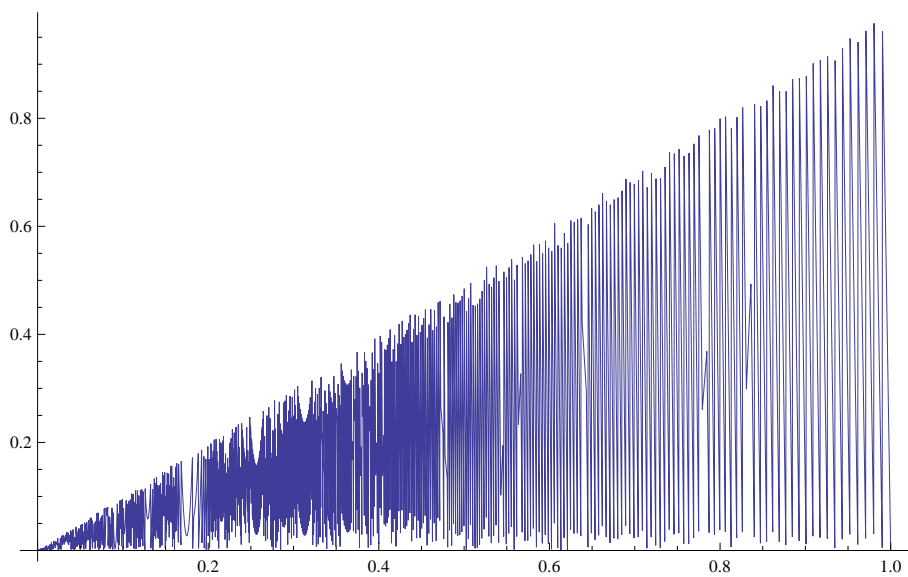
E1a[10, 2, .8]

-0.52

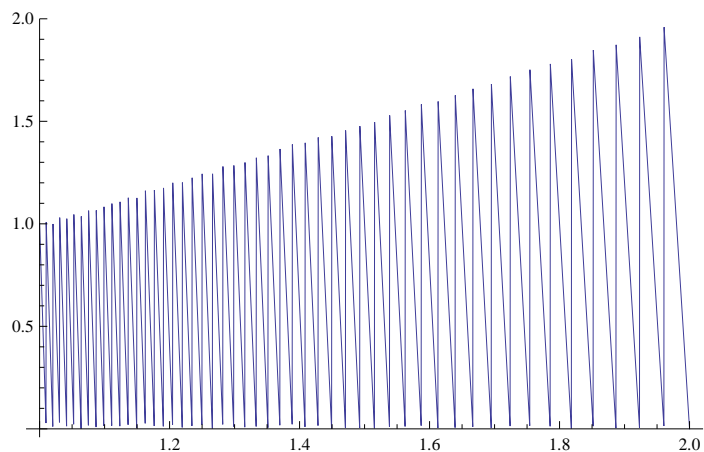
E1c[10, 2, .8]

-0.2

Plot[E1a[100, 1, n], {n, 0, 1}]



```
Plot[E1a[100, 1, n], {n, 1, 2}]
```



```
E1a[10, 1, .8]
```

```
0.4
```

```
-E1a[10, 1, 1 / .8]
```

```
0.
```

```
N[Lin[10, 1 / 2, 13]]
```

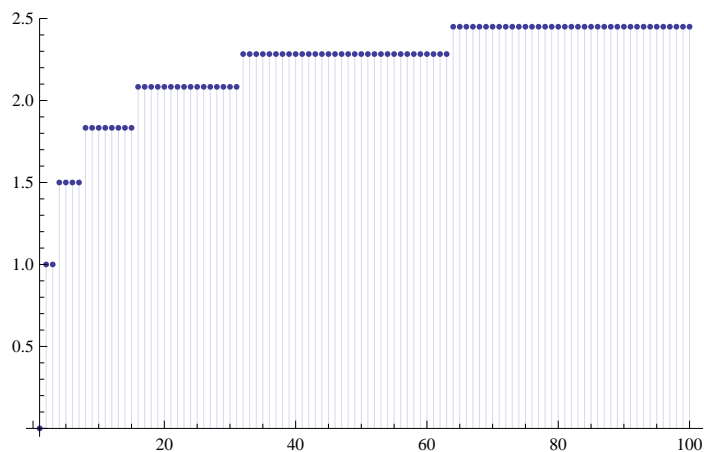
```
-19.7409
```

```
Linr[100, 2] + 1 + 1 / 2 + 1 / 3 + 1 / 4 + 1 / 5 + 1 / 6
```

```
428
```

```
15
```

```
DiscretePlot[ Limit[ (DDd[n, z] - 1) / z, z → 0] - Linr[n, 2], {n, 1, 100}]
```

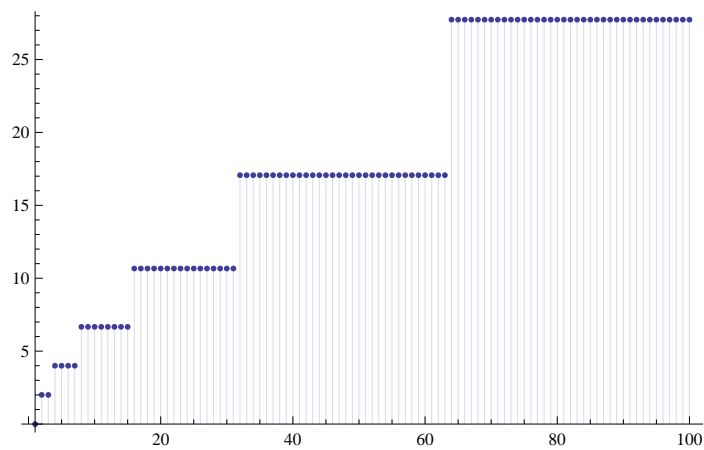


```
Limit[ (DDd[100, z] - 1) / z, z → 0]
```

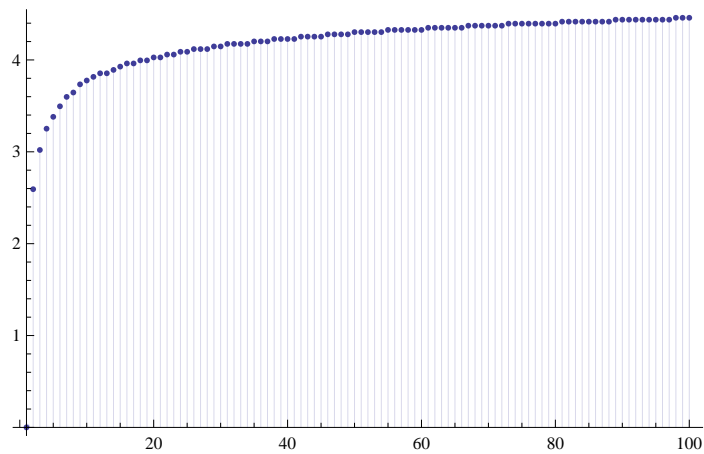
```
428
```

```
15
```

```
DiscretePlot[ Limit[ (DDd[n, z] - 1) / z, z → 0] - Linr2[n, 2], {n, 1, 100}]
```

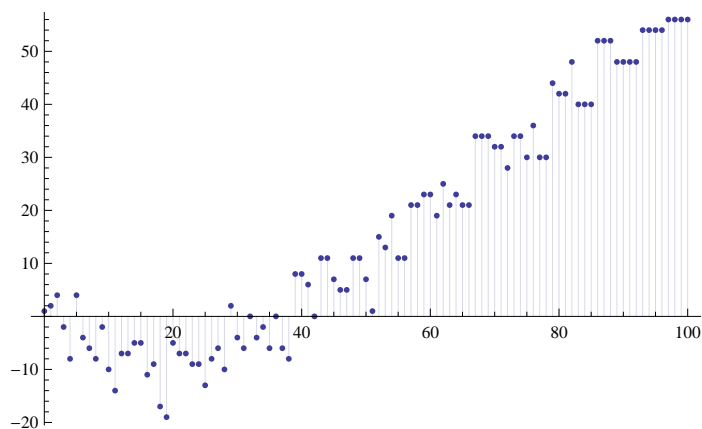


```
DiscretePlot[ Limit[ (DDd[n, z] - 1) / z, z → 0] - Linr[n, 1.1], {n, 1, 100}]
```

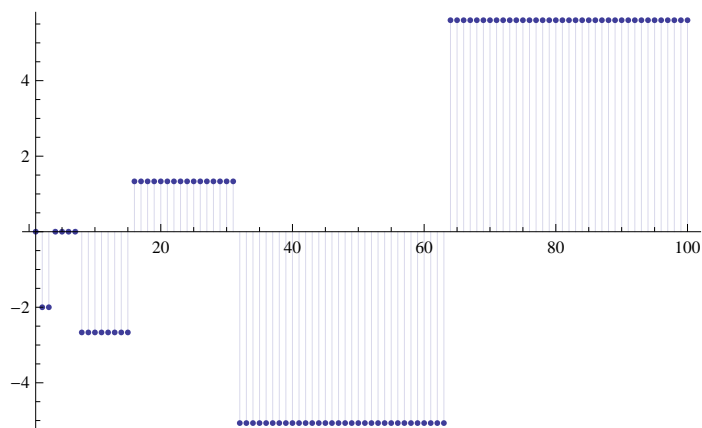


```
ff[n_, a_, b_] :=
  Sum[ 1 / j, {j, 1, Log[a, n]}] + Sum[ 1 / j, {j, 1, Log[b, n]}] - Sum[ 1 / j, {j, 1, Log[a b, n]}]
Linq[100, 3, 5]
251
12
Limit[ (DDd[100, z] - 1) / z, z → 0] - ff[100, 3, 5]
519
20
```

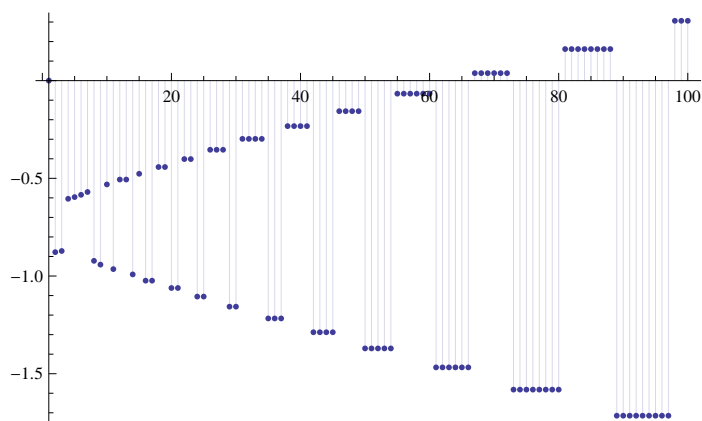
```
DiscretePlot[E2r[100, 2, 1 + n / 100], {n, 0, 100}]
```



```
DiscretePlot[Limit[(DDd[n, z] - 1) / z, z -> 0] - Linz[n, 2], {n, 1, 100}]
```

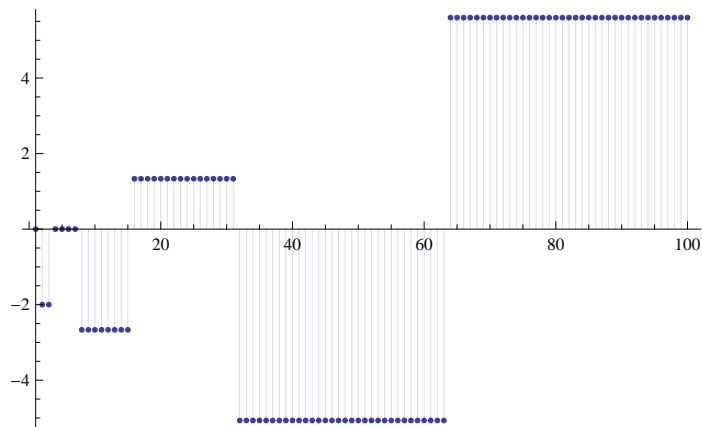


```
DiscretePlot[Limit[(DDd[n, z] - 1) / z, z -> 0] - Linz[n, 1.1], {n, 1, 100}]
```

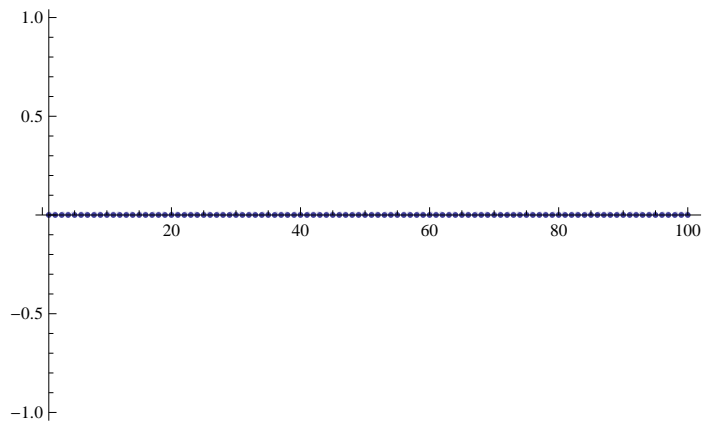


```
fa[n_, a_] := Sum[(-1)^(k) a^k / k, {k, 1, Log[a, n]}]
```

```
DiscretePlot[ fa[n, 2], {n, 1, 100}]
```



```
DiscretePlot[ Limit[ (DDd[n, z] - 1) / z, z → 0] - Linz[n, 2] - fa[n, 2], {n, 1, 100}]
```



```
Limit[Sum[ (-1) ^ (k) a ^ k / k, {k, 1, Log[a, 100]}], a → 1]
```

```
Limit[1001 +  $\frac{i\pi}{\text{Log}[a]}$  a LerchPhi[-a, 1,  $\frac{\text{Log}[100 a]}{\text{Log}[a]}$ ] - Log[1 + a], a → 1]
```

```
fr[a_] := 1001 +  $\frac{i\pi}{\text{Log}[a]}$  a LerchPhi[-a, 1,  $\frac{\text{Log}[100 a]}{\text{Log}[a]}$ ] - Log[1 + a]
```

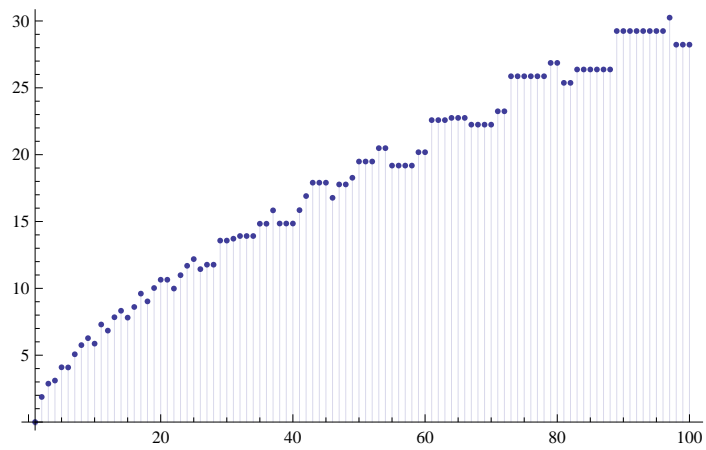
```
fr[1.000001]
```

```
-0.693147 + 0.0000108508 i
```

```
N[-Log[2]]
```

```
-0.693147
```

```
DiscretePlot[ Linz[n, 1.1], {n, 1, 100}]
```



```
$RecursionLimit = 10 000
```

```
10 000
```

```
Linz[100, 1.05]
```

```
28.7193
```