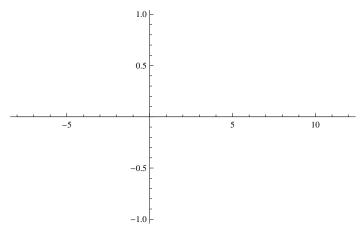
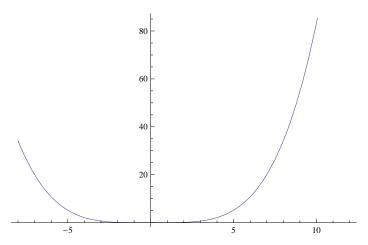
$ff[s_{-}, a_{-}] := (1 - a^{(1-s)}) Zeta[s]$

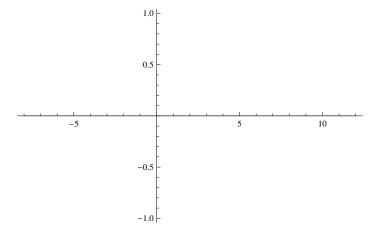
Plot[Abs[ff[-4, k]], $\{k, -8, 12\}$]

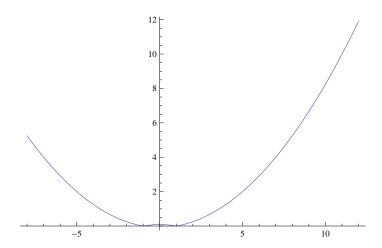


Plot[Abs[ff[-3, k]], {k, -8, 12}]

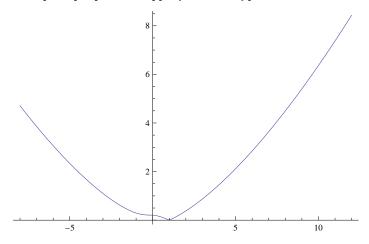


Plot[Abs[ff[-2, k]], {k, -8, 12}]

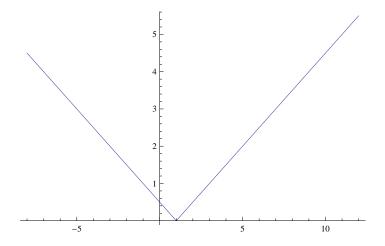




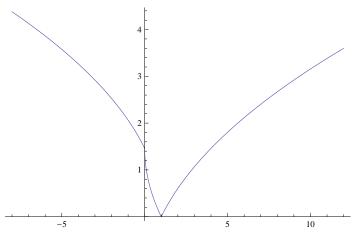
 ${\tt Plot[\,Abs[ff[-1/2,\,k]],\,\{k,\,-8,\,12\}]}$



Plot[Abs[ff[0, k]], {k, -8, 12}]



Plot[Abs[ff[1/2, k]], $\{k, -8, 12\}$]



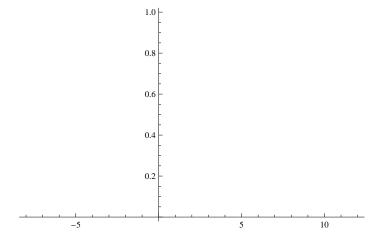
Plot[Abs[ff[1, k]], {k, -8, 12}]

 $Infinity:: indet: \ Indeterminate \ expression \ 0. \ ComplexInfinity \ encountered. \gg$

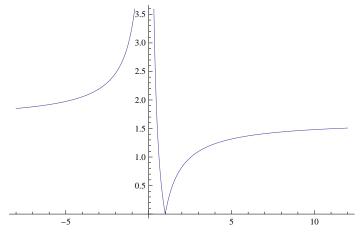
 $Infinity:: indet: \ Indeterminate \ expression \ 0. \ Complex Infinity \ encountered. \gg$

 $Infinity:: indet: Indeterminate\ expression\ 0.\ Complex Infinity\ encountered. \gg$

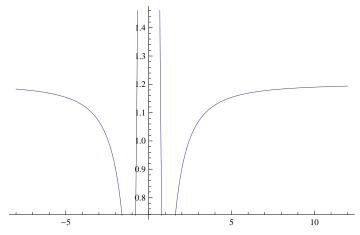
 $\label{eq:General::stop:Further output of Infinity::indet will be suppressed during this calculation. \gg$



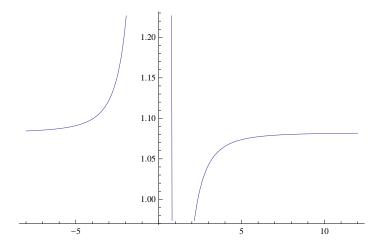
Plot[Abs[ff[2, k]], {k, -8, 12}]

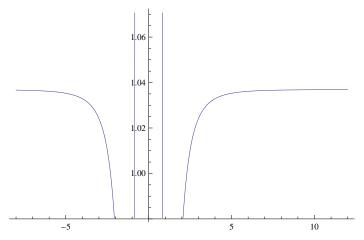


Plot[Abs[ff[3, k]], {k, -8, 12}]

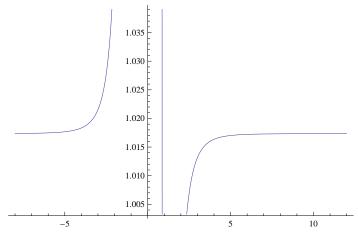


Plot[Abs[ff[4, k]], {k, -8, 12}]

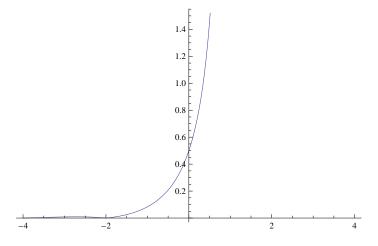




Plot[Abs[ff[6, k]], {k, -8, 12}]



Plot[Abs[ff[k, 0]], {k, -4, 4}]



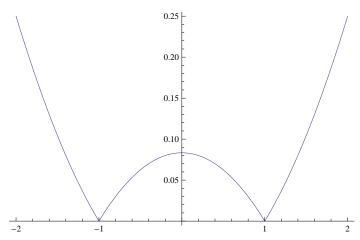
Power::infy : Infinite expression $\frac{1}{0^{3.1}}$ encountered. \gg

Abs[ff[3.12, 0]]

Power::infy : Infinite expression $\frac{1}{0^{2.12}}$ encountered. \gg

α

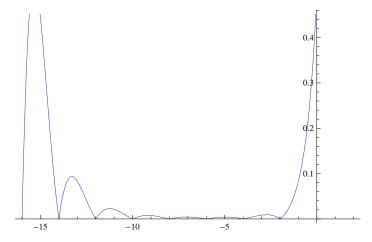
Plot[Abs[ff[-1, k]], $\{k, -2, 2\}$]

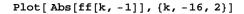


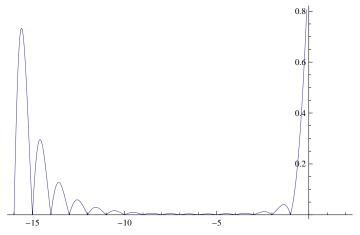
ff[-1/2,0]

$$Zeta\left[-\frac{1}{2}\right]$$

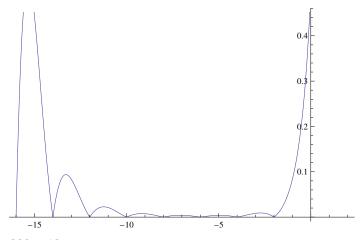
Plot[Abs[ff[k, 0]], {k, -16, 2}]







Plot[Abs[ff[k, 0]], {k, -16, 2}]



ff[1, 2]

Infinity::indet: Indeterminate expression 0 ComplexInfinity encountered. >>>

Indeterminate

 $\label{eq:limit} \mbox{Limit[HarmonicNumber[(-1) x] - HarmonicNumber[x], $\{x \to \mbox{Infinity}\}$]}$

 ${Interval[{-\infty, \infty}]}$

 $Sum[1/(2n+1), \{n, 0, Infinity\}]$

Sum::div : Sum does not converge. \gg

$$\sum_{n=0}^{\infty} \frac{1}{1+2n}$$

Zeta[3.17]

1.17156

 $2^{(3.17)}$ Pi $^{(3.17-1)}$ Sin[Pi $^{(3.17-2)}$ Gamma[1 - $^{(3.17)}$ Zeta[1 - $^{(3.17)}$]

1.17156

Zeta[1-3.17]

0.00429085

```
st[s_] := 2^(s) Pi^(s-1) Sin[Pis/2] Gamma[1-s] Zeta[1-s]
st[2.3]
1.43242
Zeta[1-2.3]
-0.0434641
Zeta[2.3]
1.43242
2^{(s)} Pi^(s-1) Sin[Pis/2] Gamma[1-s] Zeta[1-s] /.s \rightarrow 3.3
1.15194
Zeta[3.3]
1.15194
ff[3.3]
0.918027
zt[s_{-}] := ff[s] / (1 - 2^{(1-s)})
zt[3.3]
1.15194
ff[s_{-}] := (1-t^{(1-s)}) Zeta[s] /. t \rightarrow 2
(2^{(s)} Pi^{(s-1)} Sin[Pis/2] Gamma[1-s] ff[1-s] / (1-t^s)) /. {s \rightarrow 3.3, t \rightarrow 2}
ff[s] / (1-t^{(1-s)}) /. \{s \rightarrow 3.3, t \rightarrow 2\}
1.15194
1.15194
ff[s_] := (1-t^{(1-s)}) Zeta[s] /. t \rightarrow 2
(1-t^{(1-s)})(2^{(s)} Pi^{(s-1)} Sin[Pis/2]Gamma[1-s]ff[1-s]/(1-t^s))/.
 \{s \rightarrow 3.3, t \rightarrow 2\}
ff[s] /. \{s \rightarrow 3.3\}
0.918027
0.918027
ff[s_{-}] := (1-t^{(1-s)}) Zeta[s] /. t \rightarrow 3
(1-t^{(1-s)}) \ (2^{(s)} \ Pi^{(s-1)} \ Sin[Pis/2] \ Gamma[1-s] \ ff[1-s]/(1-t^s)) \ /.
 \{s \rightarrow 4.3, t \rightarrow 3\}
ff[s] /. \{s \rightarrow 4.3\}
1.03593
1.03593
```

```
Full Simplify [\,(1-t^{\, \prime}\,(1-s)\,)\,\,(2^{\, \prime}\,(s)\,\,Pi^{\, \prime}\,(s-1)\,\,Sin[Pi\,s\,/\,\,2]\,\,Gamma\,[1-s]\,\,hh\,[1-s]\,\,/\,\,(1-t^{\, \prime}\,s)\,)\,]
2^{\mathtt{s}} \; \pi^{-1+\mathtt{s}} \; \left( \mathtt{1} - \mathtt{t}^{1-\mathtt{s}} \right) \; \mathsf{Gamma} \left[ \mathtt{1} - \mathtt{s} \right] \; \mathsf{hh} \left[ \mathtt{1} - \mathtt{s} \right] \; \mathsf{Sin} \left[ \frac{\pi \, \mathtt{s}}{2} \; \right]
2^{s} \pi^{-1+s} \left(1-t^{1-s}\right) / \left(1-t^{s}\right)  Gamma [1-s] hh[1-s] sin \left[\frac{\pi s}{2}\right]
\underline{2^{s}\;\pi^{-1+s}\;\left(1-t^{1-s}\right)\;\mathrm{Gamma}\left[1-s\right]}\;\mathrm{hh}\left[1-s\right]\;\mathrm{Sin}\left[\frac{\pi\,s}{2}\right]
\texttt{ff[s\_]} := (1-\texttt{t^(1-s)}) \; \texttt{Zeta[s]} \; /. \; \texttt{t} \rightarrow 3
2^{s} \pi^{-1+s} \left(1-t^{1-s}\right) / (1-t^{s}) Gamma [1-s] ff[1-s] Sin \left[\frac{\pi s}{2}\right] /. {s \rightarrow 3.1, t \rightarrow 3}
ff[s] /. \{s \rightarrow 3.1\}
1.06558
1.06558
ff[s_{-}, t_{-}] := (1-t^{(1-s)}) Zeta[s]
\mathrm{f1[s\_,t\_]} := 2^{\mathrm{s}} \, \pi^{-1+\mathrm{s}} \, \left(1-\mathrm{t}^{1-\mathrm{s}}\right) \, / \, \left(1-\mathrm{t}^{\wedge}\mathrm{s}\right) \, \mathrm{Gamma} \, [1-\mathrm{s}] \, \mathrm{ff} [1-\mathrm{s},\,\mathrm{t}] \, \mathrm{Sin} \left[\frac{\pi \, \mathrm{s}}{2}\right]
N[ff[.5, -2]]
-1.46035 + 2.06525 i
N[f1[.5, -2]]
-1.46035 + 2.06525 i
N[f1[.5, -1]]
-1.46035 + 1.46035 i
N[f1[.5, -.5]]
-1.46035 + 1.03263 i
N[f1[.7, -.5]]
-1.4519 + 1.82575 i
N[f1[.7, -1]]
-1.14529 + 2.24776 i
```

 $gg[s_{-}] := (1-t^{(1-s)}) Zeta[s]$

${\tt Table[\{k,\ N[ff[.5,\,k]]\},\,\{k,\,-2,\,2,\,.1\}]\ //\ TableForm}$

```
-2.
        -1.46035 + 2.06525 i
-1.9
      -1.46035 + 2.01296 i
-1.8
       -1.46035 + 1.95927 i
       -1.46035 + 1.90407 i
-1.7
-1.6
       -1.46035 + 1.84722 i
       -1.46035 + 1.78856 i
-1.5
-1.4
       -1.46035 + 1.72791 i
       -1.46035 + 1.66506 i
-1.3
-1.2
       -1.46035 + 1.59974 i
-1.1
       -1.46035 + 1.53163 i
       -1.46035 + 1.46035 i
-1.
-0.9
       -1.46035 + 1.38541 i
-0.8
      -1.46035 + 1.30618 i
-0.7
       -1.46035 + 1.22182 i
-0.6
       -1.46035 + 1.13119 i
-0.5
        -1.46035 + 1.03263 i
-0.4
       -1.46035 + 0.923609 i
-0.3
       -1.46035 + 0.799869 i
-0.2
       -1.46035 + 0.65309 i
-0.1
       -1.46035 + 0.461805 i
       -1.46035
0.
0.1
       -0.99855
       -0.807264
0.2
0.3
       -0.660485
0.4
       -0.536745
0.5
       -0.427728
0.6
       -0.329169
0.7
       -0.238534
0.8
       -0.154174
0.9
       -0.0749406
1.
        0.
1.1
       0.0712782
1.2
       0.139384
1.3
       0.204706
1.4
       0.26756
1.5
     0.328207
1.6
     0.386864
1.7
       0.443715
1.8
       0.498917
1.9
        0.552605
2.
        0.604899
ff[.5, -1]
-1.46035 + 1.46035 i
ff[x, -1]
```

 $(1 - (-1)^{1-x})$ Zeta[x]

N[Zeta[1/2]]

-1.46035

```
zt2[s_{,} l_{]} := Pi^{(s/2)}
     \label{eq:product}  \  \text{Product} \  \, [ \  \, (1-s \  \, / \  \, \text{ZetaZero}[-r]) \  \, (1-s \  \, / \  \, \text{ZetaZero}[-r]) \  \, (2 \  \, (s-1) \  \, \text{Gamma} \  \, [1+s \  \, / \  \, 2]) 
N[zt2[-.5, 300]]
 -0.207637 + 0.i
N[Zeta[-.5]]
 -0.207886
zt3[s_{-}, 1_{-}] := (1-2^{(1-s)}) Pi^{(s/2)}
      Product[(1-s/ZetaZero[r]) (1-s/ZetaZero[-r]), \{r, 1, 1\}] / (2 (s-1) Gamma[1+s/2]) 
zt3[.5, 300]
0.605141 + 0.i
et[s_] := (1-t^{(1-s)}) Zeta[s] /. t \rightarrow 2
et[.5]
0.604899
zt3a[s_{-}, 1_{-}] := (1 - 2^{(1-s)}) Pi^{(s/2)}
     Product[(1-s / ZetaZero[r]) (1-s / ZetaZero[-r]), \{r, 1, 1\}] / (2 (s-1) Gamma[1+s / 2]) 
FullSimplify[(1-2^{(1-s)}) Pi^{(s/2)}
     Product[(1-s / ZetaZero[r]) (1-s / ZetaZero[-r]), \{r, 1, 1\}] / (2 (s-1) Gamma[1+s / 2])] 
\frac{2^{-s} \left(-2+2^{s}\right) \pi^{s/2} \prod_{r=1}^{l} \left(1-\frac{s}{\text{zetaZero[-r]}}\right) \left(1-\frac{s}{\text{zetaZero[r]}}\right)}{\left(-1+s\right) s \text{Gamma} \left\lceil \frac{s}{2} \right\rceil}
             (-1+s) s Gamma \left[\frac{s}{2}\right]
Gamma[1+1/2]
\operatorname{Limit}\left[\frac{2^{-s} \left(-2+2^{s}\right) \pi^{s/2}}{\left(-1+s\right) s \operatorname{Gamma}\left[\frac{s}{2}\right]}, \left\{s \to 1\right\}\right]
{Log[2]}
\label{eq:limit_limit} \operatorname{Limit}\left[\frac{2^{-s}\;\left(-\,2\,+\,2^{s}\right)\;\pi^{s/2}}{\left(-\,1\,+\,s\right)\;s\;\operatorname{Gamma}\left[\frac{s}{2}\right]}\;\text{, }\left\{s\,\rightarrow\,1\right\}\right]
\label{eq:limit} \text{Limit}[\,(1\text{--}3\,^{\wedge}\,(1\text{--}s)\,)\,\,\text{Pi}\,^{\wedge}\,(s\,/\,2)\,\,/\,(\,2\,\,(s\,-\,1)\,\,\text{Gamma}\,[1+s\,/\,2]\,)\,\,,\,\,s\to1]
Log[3]
```

Limit[(1-2^(1-s)) Pi^(s/2) / (2(s-1) Gamma[1+s/2]), s
$$\rightarrow$$
 2] $\frac{\pi}{4}$

$$\begin{split} & \mathtt{zt4}[\mathbf{s}_-, \mathbf{1}_-] := \mathtt{Product}[(1-s/\mathtt{ZetaZero}[\mathbf{r}]) \; (1-s/\mathtt{ZetaZero}[-\mathbf{r}]), \{\mathbf{r}, \mathbf{1}, \mathbf{1}\}] \\ & \mathtt{N}[\mathtt{zt4}[\mathbf{1}, 300]] \\ & 1. - 2.28116 \times 10^{-16} \; \mathrm{i} \\ & \mathtt{N}[1-1/\mathtt{ZetaZero}[1]] \\ & 0.997501 + 0.0706593 \; \mathrm{i} \\ & \mathtt{N}[1+1/\mathtt{ZetaZero}[1]] \\ & 1.0025 - 0.0706593 \; \mathrm{i} \\ & \mathtt{N}[(1-1/.5) \; (1-1/.5)] \\ & 1. \\ & \mathtt{FullSimplify}[(1-1/(\mathbf{a}+\mathbf{b}\,\mathbf{I})) \; (1-1/(\mathbf{a}-\mathbf{b}\,\mathbf{I}))] \\ & \mathbf{fe}[\mathbf{a}_-, \; \mathbf{b}_-] := 1 + \frac{1-2\,\mathbf{a}}{\mathbf{a}^2 + \mathbf{b}^2} \\ & \mathbf{fe}[.4, \, \mathbf{s}] \\ & 1 + \frac{0.2}{0.16 + \mathbf{s}^2} \\ & \mathtt{FullSimplify}[(1-s/(\mathbf{a}+\mathbf{b}\,\mathbf{I})) \; (1-s/(\mathbf{a}-\mathbf{b}\,\mathbf{I}))] \\ & \frac{\mathbf{b}^2 + (\mathbf{a}-\mathbf{s})^2}{\mathbf{a}^2 + \mathbf{b}^2} \\ & \mathbf{fe}[\mathbf{s}_-, \, \mathbf{a}_-, \, \mathbf{b}_-] := \frac{\mathbf{b}^2 + (\mathbf{a}-\mathbf{s})^2}{\mathbf{a}^2 + \mathbf{b}^2} \\ & \mathbf{fullSimplify}[\mathbf{fe}2[\mathbf{1}, \, \mathbf{a}, \, \mathbf{b}]] \\ & 1 + \frac{1-2\,\mathbf{a}}{\mathbf{a}^2 + \mathbf{b}^2} \\ & [1 + \frac{1-2\,(\mathbf{1}/2-\mathbf{c})}{(\mathbf{1}/2-\mathbf{c})^2 + \mathbf{b}^2}] \left(1 + \frac{1-2\,(\mathbf{1}/2+\mathbf{c})}{(\mathbf{1}/2+\mathbf{c})^2 + \mathbf{b}^2}\right) \\ & \mathbf{Expand} \left[\left(1 + \frac{1-2\,(\frac{1}{2}-\mathbf{c})}{\mathbf{b}^2 + (\frac{1}{2}-\mathbf{c})^2}\right) \left(1 + \frac{1-2\,(\frac{1}{2}+\mathbf{c})}{\mathbf{b}^2 + (\frac{1}{2}+\mathbf{c})^2}\right) \right] \end{aligned}$$

$$\begin{split} & \text{FullSimplify} \Big[1 + \frac{2\,\text{c}}{b^2 + \left(\frac{1}{2} - \text{c}\right)^2} - \frac{2\,\text{c}}{b^2 + \left(\frac{1}{2} + \text{c}\right)^2} - \frac{4\,\text{c}^2}{\left(b^2 + \left(\frac{1}{2} - \text{c}\right)^2\right) \left(b^2 + \left(\frac{1}{2} + \text{c}\right)^2\right)} \Big] \\ & 1 \\ & \text{FullSimplify} \Big[\text{Expand} \Big[\left(1 + \frac{1 - 2\,\left(- \text{c} \right)}{b^2 + \left(\text{a} - \text{c} \right)^2} \right) \left(1 + \frac{1 - 2\,\left(\text{a} + \text{c} \right)}{b^2 + \left(\text{a} + \text{c} \right)^2} \right) \Big] \Big] \\ & \frac{\left(\left(- 1 + \text{a} \right)^2 + \text{b}^2 \right)^2 + 2\,\left(- \left(- 1 + \text{a} \right)^2 + \text{b}^2 \right)\,\text{c}^2 + \text{c}^4}{a^4 + 2\,a^2\,\left(\text{b} - \text{c} \right) \,\left(\text{b} + \text{c} \right) + \left(\text{b}^2 + \text{c}^2 \right)^2} \\ & \text{zt5} \big[\text{s_, 1_} \big] \; := \; \text{Sum} \big[\text{Log} \big[1 - \text{s} \, / \, \text{ZetaZero} \big[\text{r} \big] \big] + \text{Log} \big[1 - \text{s} \, / \, \text{ZetaZero} \big[- \text{r} \big] \big], \; \{ \text{r, 1, 1} \} \big] \\ & \text{N} \big[\text{zt5} \big[- \text{1, 40} \big] \big] \\ & 0.0358945 + 0. \; \text{i} \\ & \text{Log} \big[1 - \text{s} \, / \, \text{ZetaZero} \big[\text{r} \big] \big] \end{split}$$

 $Log \left[1 - \frac{s}{ZetaZero[r]}\right]$

Limit[(1-2^(1-s)) Pi^(s/2) / (2 (s-1) Gamma[1+s/2]), s \rightarrow 2]
$$\frac{\pi}{4}$$

Limit[(1-2^(1-s)) Pi^(s/2) / (2 (s-1) Gamma[1+s/2]), s \rightarrow 3] $\frac{\pi}{4}$

Limit[(1-2^(1-s)) Pi^(s/2) / (2 (s-1) Gamma[1+s/2]), s \rightarrow 4] $\frac{7\pi^2}{96}$

Limit[(1-2^(1-s)) Pi^(s/2) / (2 (s-1) Gamma[1+s/2]), s \rightarrow 5] $\frac{\pi^2}{16}$

ff[s_, a_] := (1-a^(1-s)) Zeta[s] ff[3, 2]

```
0.901543
zt3[s_{-}, 1_{-}] := (1-2^{(1-s)}) Pi^{(s/2)}
   Product[(1-s / ZetaZero[r]) (1-s / ZetaZero[-r]), \{r, 1, 1\}] / (2 (s-1) Gamma[1+s / 2]) 
N[zt3[3, 800]]
0.897019 - 4.36283 \times 10^{-16} i
N[ff[3, 2]]
0.901543
zt33[1_] := Product[(1-3/ZetaZero[r])(1-3/ZetaZero[-r]), \{r, 1, 1\}]
N[zt33[700]]
1.14158 - 6.93889 \times 10^{-17} i
N[3 Zeta[3] / Pi]
1.14788
ff[2, 2]
zt22[1_] := (1-2^{(1-2)}) Pi^{(2/2)}
   Product[(1-2/ZetaZero[r]) (1-2/ZetaZero[-r]), \{r, 1, 1\}] / (2 (2-1) Gamma[1+2/2]) 
N[zt22[500]]
0.82058 + 1.11022 \times 10^{-16} i
(1-2^{(1-2)}) Pi^(2/2)/(2(2-1) Gamma[1+2/2])
ff[2, 2] / \frac{\pi}{4}
N\left[\frac{\pi}{3}\right]
zt222[1_] := Product[(1-2/ZetaZero[r]) (1-2/ZetaZero[-r]), {r, 1, 1}]
N[zt222[400]]
1.04442 + 5.55112 \times 10^{-16} i
ff[-1, 2]
```

```
ztm[1_] := (1-2^{(1-(-1))}) Pi^{((-1)/2)}
  Product[(1-(-1) / ZetaZero[r]) (1-(-1) / ZetaZero[-r]), \{r, 1, 1\}] /
   (2((-1)-1) Gamma[1+(-1)/2])
N[ztm[700]]
0.249542 + 4.16334 \times 10^{-17} i
(1-2^{(1-(-1))}) Pi<sup>((-1)/2)</sup> / (2((-1)-1) Gamma[1+(-1)/2])
4\pi
ztmm[l_] := Product[(1+1/ZetaZero[r])(1+1/ZetaZero[-r]), \{r, 1, 1\}]
N[Pi/3]
1.0472
N[ztmm[500]]
1.04479 + 3.88578 \times 10^{-16} i
zt3[s_{-}, 1_{-}] := (1-2^{(1-s)}) Pi^{(s/2)}
   Product[(1-s / ZetaZero[r]) (1-s / ZetaZero[-r]), \{r, 1, 1\}] / (2 (s-1) Gamma[1+s / 2]) 
ff[-2, 2]
zt3m2[1_] := (1-2^{(1-(-2))}) Pi^{(-2)/2}
  Product[(1 - (-2) / ZetaZero[r]) (1 - (-2) / ZetaZero[-r]), {r, 1, 1}] /
   (2((-2)-1) Gamma[1+(-2)/2])
zt3m2[400]
(1-2^{(1-(-2))}) Pi<sup>((-2)/2)</sup> / (2((-2)-1) Gamma[1+(-2)/2])
zt3m22[1_] := Product[(1+2/ZetaZero[r])(1+2/ZetaZero[-r]), \{r, 1, 1\}]
N[zt3m22[1600]]
1.14432 + 5.55112 \times 10^{-16} \text{ i}
zt3hlf[1_] := (1-2^(1-(1/2))) Pi^((1/2)/2)
  Product[(1-(1/2)/ZetaZero[r])(1-(1/2)/ZetaZero[-r]), \{r, 1, 1\}]/
   (2((1/2)-1) Gamma[1+(1/2)/2])
N[zt3hlf[300]]
0.605141 - 1.11022 \times 10^{-16} i
```

$$\begin{array}{l} (1-2^{\lambda}(1-(1/2))) \; \text{Pi}^{\lambda}(\{1/2\}/2) \; / \; (\; 2\; ((1/2)-1) \; \text{Gamma} [1+(1/2)/2]) \\ -\frac{\left(1-\sqrt{2}\right) \pi^{1/4}}{\text{Gamma} \left[\frac{5}{4}\right]} \\ -\frac{\left(1-\sqrt{2}\right) \pi^{1/4}}{\text{Gamma} \left[\frac{5}{4}\right]} \\ \text{ff}[1/2,2] \\ \text{N}\left[\left(1-\sqrt{2}\right) \; \text{Zeta} \left[\frac{1}{2}\right]\right] \\ 0.604899 \\ \text{ff}[1/2,2] / \left(-\frac{\left(1-\sqrt{2}\right) \pi^{1/4}}{\text{Gamma} \left[\frac{5}{4}\right]}\right) \\ \text{N}\left[-\frac{\text{Gamma} \left[\frac{5}{4}\right] \; \text{Zeta} \left[\frac{1}{2}\right]}{\pi^{1/4}}\right] \\ 0.994242 \\ \text{Zt3hlf2}[1] := \; \text{Product}[\left(1-\left(1/2\right)/\text{ZetaZero}[r]\right) \left(1-\left(1/2\right)/\text{ZetaZero}[-r]\right), \left\{r,1,1\right\}] \\ \text{N[zt3hlf2}[400]] \\ 0.994572 - 5.55112 \times 10^{-17} \; \text{i} \\ \\ \text{Zt3}[s_-,1_-] := \left(1-2^{\lambda}(1-s)\right) \; \text{Pi}^{\lambda}(s/2) \\ \text{Product}[\left(1-s/\text{ZetaZero}[r]\right) \left(1-s/\text{ZetaZero}[-r]\right), \left\{r,1,1\right\}] / \left(2\; (s-1) \; \text{Gamma}[1+s/2]\right) \\ \text{Zt4}[1_-] := \left(1-2^{\lambda}(1-4)\right) \; \text{Pi}^{\lambda}(s/2) \\ \text{Product}[\left(1-4/\text{ZetaZero}[r]\right) \left(1-4/\text{ZetaZero}[-r]\right), \left\{r,1,1\right\}] / \left(2\; (4-1) \; \text{Gamma}[1+4/2]\right) \\ \text{N[zt4}[700]] \\ 0.936661 - 1.11022 \times 10^{-16} \; \text{i} \\ \\ \text{N[fff}[4,2]] \\ 0.947033 \\ \end{array}$$

ff[4, 2]

$$\frac{7 \pi^4}{720}$$
 $(1-2^{(1-4)}) \text{ Pi}^{(4/2)} / (2 (4-1) \text{ Gamma} [1+4/2])$
 $\frac{7 \pi^2}{96}$

```
\frac{7\,\pi^4}{720}\bigg/\,\frac{7\,\pi^2}{96}
N\left[\frac{2 \pi^2}{15}\right]
1.31595
zt42[1_] := Product[(1-4/ZetaZero[r])(1-4/ZetaZero[-r]), \{r, 1, 1\}]
N[zt42[1200]]
1.30597 - 7.77156 \times 10^{-16} i
zt322[s_{-}, 1_{-}] := (1-2^{(1-s)}) Pi^{(s/2)}
   \label{eq:product}  \mbox{Product[(1-s/ZetaZero[-r]), $\{r,1,1\}$]/(2(s-1) $Gamma[1+s/2]$)} 
ff[2, 2]
\frac{\pi^2}{12} \bigg/ \frac{\pi}{4}
N\left[\frac{\pi}{3}\right]
1.0472
zt322a[1_] := (1-2^{(1-2)}) Pi^{(2/2)}
    Product[(1-2/ZetaZero[r]) (1-2/ZetaZero[-r]), \{r, 1, 1\}] / (2 (2-1) Gamma[1+2/2]) 
(1-2^{(1-2)}) Pi<sup>(2/2)</sup> / (2(2-1) Gamma[1+2/2])
zt322b[1_] := Product[(1-2/ZetaZero[r]) (1-2/ZetaZero[-r]), \{r, 1, 1\}]
N[zt322b[700]]
1.04528 + 2.22045 \times 10^{-16} i
zt35[s_{,1}] := (1-2^{(1-s)}) Pi^{(s/2)}
    Product[(1-s / ZetaZero[r]) (1-s / ZetaZero[-r]), \{r, 1, 1\}] / (2 (s-1) Gamma[1+s / 2]) 
zt35a[1_] := (1-2^{(1-5)}) Pi^{(5/2)}
   Product[(1-5/ZetaZero[r]) (1-5/ZetaZero[-r]), \{r, 1, 1\}] / (2 (5-1) Gamma[1+5/2])
(1-2^{(1-5)}) Pi<sup>(5/2)</sup> / (2 (5-1) Gamma[1+5/2])
ff[5, 2]
\frac{15\,\mathtt{Zeta}[5]}{16}\bigg/\,\frac{\pi^2}{16}
N\Big[\frac{15\,\text{Zeta}\,[5]}{\pi^2}\,\Big]
1.57594
zt35a[1_] := Product[(1-5/ZetaZero[r])(1-5/ZetaZero[-r]), \{r, 1, 1\}]
```

```
N[zt35a[700]]
1.54728 - 5.55112 \times 10^{-16} i
zt36[s_{-}, 1_{-}] := (1-2^{(1-s)}) Pi^{(s/2)}
   Product[(1-s / ZetaZero[r]) (1-s / ZetaZero[-r]), \{r, 1, 1\}] / (2 (s-1) Gamma[1+s / 2]) 
zt36a[1_] := (1-2^{(1-6)}) Pi^{(6/2)}
   Product[(1-6 / ZetaZero[r]) (1-6 / ZetaZero[-r]), \{r, 1, 1\}] / (2 (6-1) Gamma[1+6 / 2]) 
(1-2^{(1-6)}) Pi<sup>(6/2)</sup> / (2 (6-1) Gamma[1+6/2])
31~\pi^3
1920
ff[6, 2]
N\Big[\frac{4 \; \pi^3}{63}\,\Big]
1.96865
zt36b[1_] := Product[(1-6/ZetaZero[r]) (1-6/ZetaZero[-r]), {r, 1, 1}]
N[zt36b[1100]]
\texttt{1.92926} + \texttt{4.44089} \times \texttt{10}^{-16} \ \texttt{i}
N[2^{(3/2)} Pi^{(5/2)} / 31]
1.59609
2^(5/2)
4\sqrt{2}
N[3 \text{ Zeta}[3] / Pi] - N[2^{(1/2)} Pi^{(3/2)} / 7]
0.0229076
N[3 Zeta[3] / Pi]
1.14788
-0.020151
zt37[s_{-}, l_{-}] := (1-2^{(1-s)}) Pi^{(s/2)}
  Product[(1-s/ZetaZero[r]) (1-s/ZetaZero[-r]), \{r, 1, 1\}] / (2 (s-1) Gamma[1+s/2])
zt37a[1_] := (1-2^{(1-7)}) Pi^{(7/2)}
  Product[(1-7/ZetaZero[r]) (1-7/ZetaZero[-r]), {r, 1, 1}]/(2(7-1) Gamma[1+7/2])
```

```
(1-2^{(1-7)}) Pi<sup>(7/2)</sup> / (2(7-1) Gamma[1+7/2])
ff[7, 2]
\frac{63\,\mathrm{Zeta}\,[7]}{64}\bigg/\,\frac{\pi^3}{80}
{\tt N}\Big[\frac{{\tt 315\,Zeta}\,{\tt [7]}}{{\tt 4}\,\pi^{\tt 3}}\,\Big]
2.56101
N[2^(5/2) Pi^(7/2) /127]
2.44791
N[zt37b[1200]]
2.4937 + 6.10623 \times 10^{-16} i
zt38[s_1, 1] := (1-2^{(1-s)}) Pi^{(s/2)}
   \label{eq:product} {\tt Product[(1-s/ZetaZero[r])\ (1-s/ZetaZero[-r]), \{r, 1, 1\}]/(2\ (s-1)\ Gamma[1+s/2])}
zt38a[1_] := (1-2^{(1-8)}) Pi^{(8/2)}
    Product[(1-8 / ZetaZero[r]) (1-8 / ZetaZero[-r]), \{r, 1, 1\}] / (2 (8-1) Gamma[1+8/2]) 
N[zt38a[700]]
0.946327 - 3.33067 \times 10^{-16} i
ff[8, 2]
 127~\pi^8
1209600
N\Big[\frac{127\,\pi^8}{1\,209\,600}\Big]
0.996233
(1-2^{(1-8)}) Pi^ (8/2) / (2(8-1) Gamma[1+8/2])
\frac{127 \, \pi^8}{1 \, 209 \, 600} \bigg/ \, \frac{127 \, \pi^4}{43 \, 008}
N\Big[\frac{8 \pi^4}{225}\Big]
3.46343
zt38b[1_] := Product[(1-8/ZetaZero[r])(1-8/ZetaZero[-r]), \{r, 1, 1\}]
```

```
N[zt38b[1200]]
3.34259 - 3.33067 \times 10^{-15} i
zt39[s_{,} l_{]} := Pi^{(s/2)}
    Product[(1-s / ZetaZero[r]) (1-s / ZetaZero[-r]), \{r, 1, 1\}] / (2 (s-1) Gamma[1+s / 2]) 
zt39a[l_] := Pi^(9/2) Product[(1-9/ZetaZero[r]) (1-9/ZetaZero[-r]), {r, 1, 1}] /
    (2(9-1) Gamma[1+9/2])
N[zt39a[1200]]
0.957282 - 2.77556 \times 10^{-16} i
N[Zeta[9]]
1.00201
N\Big[\frac{255\,{\tt Zeta}\,{\tt [9]}}{256}\,\Big]
0.998094
Pi^(9/2)/(2(9-1)Gamma[1+9/2])
2 \pi^4
945
N\Big[\frac{945\,\pi^5}{2}\,\Big]
144594.
Zeta[9] \bigg/ \frac{2 \pi^4}{945}
N\Big[\frac{945\,{\tt Zeta}\,[9]}{2\,\pi^4}\,\Big]
4.86042
zt39b[1_] := Product[(1-9/ZetaZero[r])(1-9/ZetaZero[-r]), \{r, 1, 1\}]
N[zt39b[1200]]
4.64347 + 2.27596 \times 10^{-15} i
zt399[s_{-}, 1_{-}] := (1 - 2^{(1 - s)}) Pi^{(s/2)}
    Product[(1-s / ZetaZero[r]) (1-s / ZetaZero[-r]), \{r, 1, 1\}] / (2 (s-1) Gamma[1+s / 2])
```

```
zt399a[1_] := (1-2^{(1-9)}) Pi^{(9/2)}
    Product[(1-9/ZetaZero[r]) (1-9/ZetaZero[-r]), \{r, 1, 1\}] / (2 (9-1) Gamma[1+9/2]) 
N[zt399a[900]]
0.944022 + 1.11022 \times 10^{-16} i
ff[9, 2]
0.998094
(1-2^{(1-9)}) Pi^(9/2)/(2(9-1)Gamma[1+9/2])
17 \pi^4
8064
\frac{255\,\text{Zeta}[9]}{256}\bigg/\,\frac{17\,\pi^4}{8064}
{\tt N}\Big[\frac{{\tt 945 \, Zeta}\,{\tt [9]}}{2\,\pi^4}\,\Big]
4.86042
zt399b[1_] := Product[(1-9/ZetaZero[r])(1-9/ZetaZero[-r]), \{r, 1, 1\}]
N[zt399b[900]]
4.5971 - 3.55271 \times 10^{-15} i
N[8 Pi ^ 4 / 255]
3.05597
zt310[s_{-}, 1_{-}] := (1-2^{(1-s)}) Pi^{(s/2)}
   Product[(1-s/ZetaZero[r]) (1-s/ZetaZero[-r]), \{r, 1, 1\}] / (2 (s-1) Gamma[1+s/2])
zt310a[1_] := (1-2^(1-10)) Pi^(10/2)
   Product[(1-10 / ZetaZero[r]) (1-10 / ZetaZero[-r]), \{r, 1, 1\}] / (2 (10-1) Gamma[1+10 / 2])
N[zt310a[900]]
0.931849 - 2.77556 \times 10^{-16} i
ff[10, 2]
N\Big[\frac{73\,\pi^{10}}{6\,842\,880}\,\Big]
0.99904
(1-2^{(1-10)}) Pi^(10/2) / (2(10-1) Gamma[1+10/2])
\frac{73\,\pi^{10}}{6\,842\,880}\bigg/\,\frac{511\,\pi^5}{1\,105\,920}
```

```
ff[20, 2]
```

```
91 546 277 357 \pi^{20}
    802 857 662 698 291 200 000
    (1-2^{(1-20)}) Pi ^{(20/2)} / (2(20-1) Gamma[1+20/2])
   \frac{91\,546\,277\,357\,\pi^{20}}{802\,857\,662\,698\,291\,200\,000}\Big/\,\frac{524\,287\,\pi^{10}}{72\,296\,379\,187\,200}
1472.48
 N[zt320a[1700]]
1219.27 + 6.25278 \times 10^{-13} i
 2 ^ 26
   67108864
   ex[s_{-}] := ff[s, 2] / ((1-2^(1-s)) Pi^(s/2) / (2(s-1) Gamma[1+s/2]))
   Table[ex[s], {s, 2, 20, 1 / 2}]
   \left\{\frac{\pi}{2}, \frac{3 \operatorname{Gamma}\left[\frac{9}{4}\right] \operatorname{Zeta}\left[\frac{5}{2}\right]}{\pi^{5/4}}, \frac{3 \operatorname{Zeta}\left[3\right]}{\pi}, \frac{5 \operatorname{Gamma}\left[\frac{11}{4}\right] \operatorname{Zeta}\left[\frac{7}{2}\right]}{\pi^{7/4}}, \frac{2 \pi^2}{15}, \frac{7 \operatorname{Gamma}\left[\frac{13}{4}\right] \operatorname{Zeta}\left[\frac{9}{2}\right]}{\pi^{9/4}}, \frac{7 \operatorname{Gamma}\left[\frac{13}{4}\right] \operatorname{Zeta}\left[\frac{9}{4}\right]}{\pi^{9/4}}, \frac{7 \operatorname{Gamma}\left[\frac{13}{4}\right]}{\pi^{9/4}}, \frac{7 \operatorname{Gamma}\left
             \frac{15 \, \mathrm{Zeta}\,[5]}{\pi^2} \, , \, \frac{9 \, \mathrm{Gamma}\,\left[\frac{15}{4}\,\right] \, \mathrm{Zeta}\left[\frac{11}{2}\,\right]}{\pi^{11/4}} \, , \, \frac{4 \, \pi^3}{63} \, , \, \frac{11 \, \mathrm{Gamma}\left[\frac{17}{4}\,\right] \, \mathrm{Zeta}\left[\frac{13}{2}\,\right]}{\pi^{13/4}} \, , \, \frac{315 \, \mathrm{Zeta}\,[7]}{4 \, \pi^3} \, ,
             \frac{13\,\mathrm{Gamma}\left[\frac{19}{4}\right]\,\mathrm{Zeta}\left[\frac{15}{2}\right]}{\pi^{15/4}}\,,\,\,\frac{8\,\pi^4}{225}\,,\,\,\frac{15\,\mathrm{Gamma}\left[\frac{21}{4}\right]\,\mathrm{Zeta}\left[\frac{17}{2}\right]}{\pi^{17/4}}\,,\,\,\frac{945\,\mathrm{Zeta}\left[9\right]}{2\,\pi^4}\,,\,\,\frac{17\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{2}\right]}{\pi^{19/4}}\,,\,\,\frac{17\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{2}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{2}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{2}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{2}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{2}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{2}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{2}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{2}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{2}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{2}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{2}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{2}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{2}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{2}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{2}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{2}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{4}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{4}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{4}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{4}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{4}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{4}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{4}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,\mathrm{Zeta}\left[\frac{19}{4}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gamma}\left[\frac{23}{4}\right]}{\pi^{19/4}}\,,\,\,\frac{19\,\mathrm{Gam
             \frac{16\,\pi^5}{693}\,,\,\frac{19\,\mathrm{Gamma}\left[\frac{25}{4}\right]\,\mathrm{Zeta}\left[\frac{21}{2}\right]}{\pi^{21/4}}\,,\,\frac{51\,975\,\mathrm{Zeta}\left[11\right]}{16\,\pi^5}\,,\,\frac{21\,\mathrm{Gamma}\left[\frac{27}{4}\right]\,\mathrm{Zeta}\left[\frac{23}{2}\right]}{\pi^{23/4}}\,,\,\frac{22\,112\,\pi^6}{1\,289\,925}
               \frac{23\,\mathrm{Gamma}\!\left[\frac{29}{4}\right]\,\mathrm{Zeta}\!\left[\frac{25}{2}\right]}{\pi^{25/4}}\,\text{,}\;\;\frac{405\,405\,\mathrm{Zeta}\left[13\right]}{16\,\pi^6}\,\text{,}\;\;\frac{25\,\mathrm{Gamma}\!\left[\frac{31}{4}\right]\,\mathrm{Zeta}\!\left[\frac{27}{2}\right]}{\pi^{27/4}}\,\text{,}\;\;\frac{64\,\pi^7}{4455}\,\text{,}
               \frac{27\,\mathrm{Gamma}\left[\frac{33}{4}\right]\,\mathrm{Zeta}\left[\frac{29}{2}\right]}{\pi^{29/4}}\,,\,\,\frac{14\,189\,175\,\mathrm{Zeta}\left[15\right]}{64\,\pi^7}\,,\,\,\frac{29\,\mathrm{Gamma}\left[\frac{35}{4}\right]\,\mathrm{Zeta}\left[\frac{31}{2}\right]}{\pi^{31/4}}\,,\,\,\frac{462\,976\,\pi^8}{34\,459\,425}
            \frac{31\,\mathrm{Gamma}\left[\frac{37}{4}\right]\,\mathrm{Zeta}\left[\frac{33}{2}\right]}{\pi^{33/4}}\,\,,\,\,\frac{34\,459\,425\,\mathrm{Zeta}\left[17\right]}{16\,\pi^8}\,\,,\,\,\frac{33\,\mathrm{Gamma}\left[\frac{39}{4}\right]\,\mathrm{Zeta}\left[\frac{35}{2}\right]}{\pi^{35/4}}\,\,,\,\,\frac{11\,229\,952\,\pi^9}{808\,782\,975}\,\,,
             \frac{35\,\mathrm{Gamma}\left[\frac{41}{4}\right]\,\mathrm{Zeta}\left[\frac{37}{2}\right]}{\pi^{37/4}}\,,\,\,\frac{5\,892\,561\,675\,\mathrm{Zeta}\left[19\right]}{256\,\pi^{9}}\,,\,\,\frac{37\,\mathrm{Gamma}\left[\frac{43}{4}\right]\,\mathrm{Zeta}\left[\frac{39}{2}\right]}{\pi^{39/4}}\,,\,\,\frac{89\,400\,832\,\pi^{10}}{5\,685\,805\,125}\,\Big\}
```

Table[ex[s], {s, 2, 20}]

$$\left\{ \frac{\pi}{3}, \frac{3 \text{ Zeta[3]}}{\pi}, \frac{2 \pi^2}{15}, \frac{15 \text{ Zeta[5]}}{\pi^2}, \frac{4 \pi^3}{63}, \frac{315 \text{ Zeta[7]}}{4 \pi^3}, \frac{8 \pi^4}{225}, \frac{945 \text{ Zeta[9]}}{2 \pi^4}, \frac{16 \pi^5}{693}, \frac{51975 \text{ Zeta[11]}}{16 \pi^5}, \frac{22112 \pi^6}{1289925}, \frac{405 405 \text{ Zeta[13]}}{16 \pi^6}, \frac{64 \pi^7}{4455}, \frac{14189175 \text{ Zeta[15]}}{64 \pi^7}, \frac{462976 \pi^8}{34459425}, \frac{34459425 \text{ Zeta[17]}}{16 \pi^8}, \frac{11229952 \pi^9}{808782975}, \frac{5892561675 \text{ Zeta[19]}}{256 \pi^9}, \frac{89400832 \pi^{10}}{5685805125}, \frac{11229952 \pi^9}{35685805125}, \frac{11229952 \pi^9}{356$$

Table[ex[s], {s, -10, 20}]

Power::infy: Infinite expression $\frac{1}{2}$ encountered. \gg

Infinity::indet: Indeterminate expression 0 ComplexInfinity encountered. >>>

Power::infy : Infinite expression $\frac{1}{2}$ encountered. \gg

Infinity::indet: Indeterminate expression 0 ComplexInfinity encountered. >>>

Power::infy: Infinite expression $\frac{1}{2}$ encountered. \gg

General::stop: Further output of Power::infy will be suppressed during this calculation. ≫

Infinity::indet: Indeterminate expression 0 ComplexInfinity encountered. >>

General::stop: Further output of Infinity::indet will be suppressed during this calculation. \gg

$$\left\{ \text{Indeterminate}, \frac{16\,\pi^5}{693} \right., \, \text{Indeterminate}, \frac{8\,\pi^4}{225} \right., \, \text{Indeterminate}, \frac{4\,\pi^3}{63} \right., \\ \left. \text{Indeterminate}, \frac{2\,\pi^2}{15} \right., \, \left. \text{Indeterminate}, \frac{\pi}{3} \right., \, \left. \text{Indeterminate}, \frac{\pi}{3} \right., \, \frac{3\,\text{Zeta[3]}}{\pi} \right., \, \frac{2\,\pi^2}{15} \right., \\ \left. \frac{15\,\text{Zeta[5]}}{\pi^2} \right., \, \frac{4\,\pi^3}{63} \right., \, \frac{315\,\text{Zeta[7]}}{4\,\pi^3} \right., \, \frac{8\,\pi^4}{225} \left., \frac{945\,\text{Zeta[9]}}{2\,\pi^4} \right., \, \frac{16\,\pi^5}{693} \right., \, \frac{51\,975\,\text{Zeta[11]}}{16\,\pi^5} \right., \\ \left. \frac{22\,112\,\pi^6}{1\,289\,925} \right., \, \frac{405\,405\,\text{Zeta[13]}}{16\,\pi^6} \right., \, \frac{64\,\pi^7}{4455} \left., \frac{14\,189\,175\,\text{Zeta[15]}}{64\,\pi^7} \right., \, \frac{462\,976\,\pi^8}{34\,459\,425\,\text{Zeta[17]}} \right., \\ \left. \frac{34\,459\,425\,\text{Zeta[17]}}{16\,\pi^8} \right., \, \frac{11\,229\,952\,\pi^9}{808\,782\,975} \right., \, \frac{5\,892\,561\,675\,\text{Zeta[19]}}{256\,\pi^9} \right., \, \frac{89\,400\,832\,\pi^{10}}{5\,685\,805\,125} \right\}$$

```
ffa[s_, a_] := (1-a^(1-s)) Zet[s]
ex2[s_] := ffa[s, 2] / ((1-2^(1-s)) Pi^(s/2) / (2(s-1) Gamma[1+s/2]))
Table[ex2[s], {s, -10, 20}]
```

Power::infy: Infinite expression — encountered. ≫

Power::infy : Infinite expression $\stackrel{\text{\tiny 1}}{-}$ encountered. \gg

Power::infy : Infinite expression — encountered. ≫

General::stop: Further output of Power::infy will be suppressed during this calculation. ≫

Infinity::indet: Indeterminate expression 0 ComplexInfinity encountered. >>

$$\left\{ \text{ComplexInfinity, } -\frac{64}{21} \, \pi^5 \, \text{Zet}[-9] \,, \, \text{ComplexInfinity, } \frac{128}{15} \, \pi^4 \, \text{Zet}[-7] \,, \, \text{ComplexInfinity, } \right. \\ \left. -16 \, \pi^3 \, \text{Zet}[-5] \,, \, \text{ComplexInfinity, } 16 \, \pi^2 \, \text{Zet}[-3] \,, \, \text{ComplexInfinity, } -4 \, \pi \, \text{Zet}[-1] \,, \\ \left. -2 \, \text{Zet}[0] \,, \, \text{Indeterminate, } \frac{2 \, \text{Zet}[2]}{\pi} \,, \, \frac{3 \, \text{Zet}[3]}{\pi} \,, \, \frac{12 \, \text{Zet}[4]}{\pi^2} \,, \, \frac{15 \, \text{Zet}[5]}{\pi^2} \,, \, \frac{60 \, \text{Zet}[6]}{\pi^3} \,, \\ \frac{315 \, \text{Zet}[7]}{4 \, \pi^3} \,, \, \frac{336 \, \text{Zet}[8]}{\pi^4} \,, \, \frac{945 \, \text{Zet}[9]}{2 \, \pi^4} \,, \, \frac{2160 \, \text{Zet}[10]}{\pi^5} \,, \, \frac{51 \, 975 \, \text{Zet}[11]}{16 \, \pi^5} \,, \, \frac{15 \, 840 \, \text{Zet}[12]}{\pi^6} \,, \\ \frac{405 \, 405 \, \text{Zet}[13]}{16 \, \pi^6} \,, \, \frac{131 \, 040 \, \text{Zet}[14]}{\pi^7} \,, \, \frac{14 \, 189 \, 175 \, \text{Zet}[15]}{64 \, \pi^7} \,, \, \frac{12 \, 209 \, 600 \, \text{Zet}[16]}{\pi^8} \,, \\ \frac{34 \, 459 \, 425 \, \text{Zet}[17]}{16 \, \pi^8} \,, \, \frac{12 \, 337 \, 920 \, \text{Zet}[18]}{\pi^9} \,, \, \frac{5 \, 892 \, 561 \, 675 \, \text{Zet}[19]}{256 \, \pi^9} \,, \, \frac{137 \, 894 \, 400 \, \text{Zet}[20]}{\pi^{10}} \,\right\}$$

$$\begin{split} & \textbf{ff[s_, a_]} := (1-a^{(1-s)}) \, \textbf{Zeta[s]} \\ & \textbf{ex3[s_]} := \textbf{ff[s, 2]} \, / \, ((1-2^{(1-s)}) \, \textbf{Pi^{(s/2)}} \, / \, (2 \, (s-1) \, \textbf{Gamma[1+s/2]})) \\ & \textbf{Table[ex3[s]} \, / \, (\textbf{Pi^{((s)/2)}}) \, , \, \{s, 2, 20\}] \\ & \{ \frac{1}{3} \, , \, \frac{3 \, \textbf{Zeta[3]}}{\pi^{5/2}} \, , \, \frac{2}{15} \, , \, \frac{15 \, \textbf{Zeta[5]}}{\pi^{9/2}} \, , \, \frac{4}{63} \, , \, \frac{315 \, \textbf{Zeta[7]}}{4 \, \pi^{13/2}} \, , \, \frac{8}{225} \, , \, \frac{945 \, \textbf{Zeta[9]}}{2 \, \pi^{17/2}} \, , \\ & \frac{16}{693} \, , \, \frac{51 \, 975 \, \textbf{Zeta[11]}}{16 \, \pi^{21/2}} \, , \, \frac{22 \, 112}{1289 \, 925} \, , \, \frac{405 \, 405 \, \textbf{Zeta[13]}}{16 \, \pi^{25/2}} \, , \, \frac{64}{4455} \, , \, \frac{14 \, 189 \, 175 \, \textbf{Zeta[15]}}{64 \, \pi^{29/2}} \, , \\ & \frac{462 \, 976}{34 \, 459 \, 425} \, , \, \frac{34 \, 459 \, 425 \, \textbf{Zeta[17]}}{16 \, \pi^{33/2}} \, , \, \frac{11 \, 229 \, 952}{808 \, 782 \, 975} \, , \, \frac{5 \, 892 \, 561 \, 675 \, \textbf{Zeta[19]}}{256 \, \pi^{37/2}} \, , \, \frac{89 \, 400 \, 832}{5 \, 685 \, 805 \, 125} \, \} \end{split}$$

```
N[Zeta[3] / (Pi^(5/2))]
0.0687148
```

 $Grid[Table[N[(a/b) Zeta[3]/(Pi^(5/2)) * Log[5]], {a, 1, 100}, {b, 1, 20}]]$

Grid[Table[N[(a/b) Zeta[2]/(Pi^(4/2))], {a, 2, 100}, {b, 2, 20}]]

$$\begin{split} & \exp 3[\mathbf{s}_{-}] := \pi^{-8/2} \; (-1+\mathbf{s}) \; \mathbf{s} \; \mathbf{Gamma} \left[\frac{\mathbf{s}}{2} \right] \; \mathbf{Zeta}[\mathbf{s}] \\ & \left\{ \frac{1}{3} \; , \; \frac{3 \; \mathbf{Zeta}[3]}{\pi^{5/2}} \; , \; \frac{2}{15} \; , \; \frac{15 \; \mathbf{Zeta}[5]}{\pi^{9/2}} \; , \; \frac{4}{63} \; , \; \frac{315 \; \mathbf{Zeta}[7]}{4 \; \pi^{13/2}} \; , \; \frac{8}{225} \; , \; \frac{945 \; \mathbf{Zeta}[9]}{2 \; \pi^{17/2}} \; , \\ & \frac{16}{693} \; , \; \frac{51 \; 975 \; \mathbf{Zeta}[11]}{16 \; \pi^{21/2}} \; , \; \frac{22 \; 112}{1 \; 289 \; 925} \; , \; \frac{405 \; 405 \; \mathbf{Zeta}[13]}{16 \; \pi^{25/2}} \; , \; \frac{64}{4455} \; , \; \frac{14 \; 189 \; 175 \; \mathbf{Zeta}[15]}{64 \; \pi^{29/2}} \; , \\ & \frac{462 \; 976}{34 \; 459 \; 425} \; , \; \frac{34 \; 459 \; 425 \; \mathbf{Zeta}[17]}{16 \; \pi^{33/2}} \; , \; \frac{11 \; 229 \; 952}{808 \; 782 \; 975} \; , \; \frac{5892 \; 561 \; 675 \; \mathbf{Zeta}[19]}{256 \; \pi^{37/2}} \; , \; \frac{89 \; 400 \; 832}{5 \; 685 \; 805 \; 125} \\ & \mathbf{Expand}[\; ((\mathbf{1} - \mathbf{2}^{\wedge} (\mathbf{1} - \mathbf{s})) \; \mathbf{Zeta}[\mathbf{s}]) \; / \; ((\mathbf{1} - \mathbf{2}^{\wedge} (\mathbf{1} - \mathbf{s})) \; \mathbf{Pi}^{\wedge} (\mathbf{s} \; / \; \mathbf{2}) \; / \; (2 \; (\mathbf{s} - \mathbf{1}) \; \mathbf{Gamma}[\mathbf{1} + \mathbf{s} \; / \; \mathbf{2}]))] \\ & \mathbf{Full Simplify} \left[-2 \; \pi^{-\mathbf{s}/2} \; \mathbf{Gamma} \left[\mathbf{1} + \frac{\mathbf{s}}{2} \right] \; \mathbf{Zeta}[\mathbf{s}] \; \mathbf{Zeta}[\mathbf{s}] \; \right] \; \mathbf{Zeta}[\mathbf{s}] \; \\ & \mathbf{g} \; \mathbf{S} \; \mathbf{Gamma} \left[\frac{\mathbf{s}}{2} \right] \; \mathbf{Zeta}[\mathbf{s}] \; \mathbf{S} \; \mathbf{S} \; \mathbf{Gamma} \left[\mathbf{s} \; \mathbf$$