

$$\text{Fa3}[n_, a_, s_] := (-1)^a \frac{(\text{Gamma}[a, 0, -(1-s) \text{Log}[n]]) (1-s)^{-a}}{\text{Gamma}[a]}$$

$$\text{Fa3}[n, 1, 0]$$

$$-1 + n$$

$$\text{Fa3}[n, 1, 2]$$

$$1 - \frac{1}{n}$$

$$\text{Fa3}[n, 1, -1]$$

$$\frac{1}{2} (-1 + n^2)$$

$$\text{Fa3}[n, 1, -2]$$

$$\frac{1}{3} (-1 + n^3)$$

$$\text{Fa3}[n, 1, -3]$$

$$\frac{1}{4} (-1 + n^4)$$

$$\text{Fa3}[n, 1, -1/2]$$

$$-\frac{2}{3} (1 - n^{3/2})$$

$$\text{Fa3}[n, 1, 1/2]$$

$$-2 (1 - \sqrt{n})$$

$$\text{Fa3}[n, 2, 0]$$

$$\text{Gamma}[2, 0, -\text{Log}[n]]$$

$$\text{Fa3}[n, 2, -1]$$

$$\frac{1}{4} \text{Gamma}[2, 0, -2 \text{Log}[n]]$$

$$\text{Grid}[\text{Table}[\text{Fa3}[n, aa, ss], \{ss, -5, 7\}, \{aa, 1, 6\}]]$$

$$\text{Power::infy: Infinite expression } \frac{1}{0} \text{ encountered. } \gg$$

$$\text{Infinity::indet: Indeterminate expression } 0 \text{ ComplexInfinity encountered. } \gg$$

$$\text{Power::infy: Infinite expression } \frac{1}{0^2} \text{ encountered. } \gg$$

$$\text{Infinity::indet: Indeterminate expression } 0 \text{ ComplexInfinity encountered. } \gg$$

$$\text{Power::infy: Infinite expression } \frac{1}{0^3} \text{ encountered. } \gg$$

$$\text{General::stop: Further output of Power::infy will be suppressed during this calculation. } \gg$$

$$\text{Infinity::indet: Indeterminate expression } 0 \text{ ComplexInfinity encountered. } \gg$$

$$\text{General::stop: Further output of Infinity::indet will be suppressed during this calculation. } \gg$$

| | | | | | |
|-----------------------------------|---|---|--|---|--|
| $\frac{1}{6} (-1 + n^6)$ | $\frac{1}{36} \text{Gamma}[2, 0, -6 \text{Log}[n]]$ | $-\frac{1}{432} \text{Gamma}[3, 0, -6 \text{Log}[n]]$ | $\text{Gamma}[4, 0, -6 \text{Log}[n]] / 7776$ | $-\text{Gamma}[5, 0, -6 \text{Log}[n]] / 186624$ | $\text{Gamma}[6, 0, -6 \text{Log}[n]] / 5598720$ |
| $\frac{1}{5} (-1 + n^5)$ | $\frac{1}{25} \text{Gamma}[2, 0, -5 \text{Log}[n]]$ | $-\frac{1}{250} \text{Gamma}[3, 0, -5 \text{Log}[n]]$ | $\text{Gamma}[4, 0, -5 \text{Log}[n]] / 3750$ | $-\text{Gamma}[5, 0, -5 \text{Log}[n]] / 75000$ | $\text{Gamma}[6, 0, -5 \text{Log}[n]] / 1875000$ |
| $\frac{1}{4} (-1 + n^4)$ | $\frac{1}{16} \text{Gamma}[2, 0, -4 \text{Log}[n]]$ | $-\frac{1}{128} \text{Gamma}[3, 0, -4 \text{Log}[n]]$ | $\text{Gamma}[4, 0, -4 \text{Log}[n]] / 1536$ | $-\text{Gamma}[5, 0, -4 \text{Log}[n]] / 24576$ | $\text{Gamma}[6, 0, -4 \text{Log}[n]] / 491520$ |
| $\frac{1}{3} (-1 + n^3)$ | $\frac{1}{9} \text{Gamma}[2, 0, -3 \text{Log}[n]]$ | $-\frac{1}{54} \text{Gamma}[3, 0, -3 \text{Log}[n]]$ | $\frac{1}{486} \text{Gamma}[4, 0, -3 \text{Log}[n]]$ | $-\text{Gamma}[5, 0, -3 \text{Log}[n]] / 5832$ | $\text{Gamma}[6, 0, -3 \text{Log}[n]] / 87480$ |
| $\frac{1}{2} (-1 + n^2)$ | $\frac{1}{4} \text{Gamma}[2, 0, -2 \text{Log}[n]]$ | $-\frac{1}{16} \text{Gamma}[3, 0, -2 \text{Log}[n]]$ | $\frac{1}{96} \text{Gamma}[4, 0, -2 \text{Log}[n]]$ | $-\frac{1}{768} \text{Gamma}[5, 0, -2 \text{Log}[n]]$ | $\text{Gamma}[6, 0, -2 \text{Log}[n]] / 7680$ |
| $-1 + n$ | $\text{Gamma}[2, 0, -\text{Log}[n]]$ | $-\frac{1}{2} \text{Gamma}[3, 0, -\text{Log}[n]]$ | $\frac{1}{6} \text{Gamma}[4, 0, -\text{Log}[n]]$ | $-\frac{1}{24} \text{Gamma}[5, 0, -\text{Log}[n]]$ | $\frac{1}{120} \text{Gamma}[6, 0, -\text{Log}[n]]$ |
| Indeterminate | Indeterminate | Indeterminate | Indeterminate | Indeterminate | Indeterminate |
| $1 - \frac{1}{n}$ | $\text{Gamma}[2, 0, \text{Log}[n]]$ | $\frac{1}{2} \text{Gamma}[3, 0, \text{Log}[n]]$ | $\frac{1}{6} \text{Gamma}[4, 0, \text{Log}[n]]$ | $\frac{1}{24} \text{Gamma}[5, 0, \text{Log}[n]]$ | $\frac{1}{120} \text{Gamma}[6, 0, \text{Log}[n]]$ |
| $\frac{1}{2} (1 - \frac{1}{n^2})$ | $\frac{1}{4} \text{Gamma}[2, 0, 2 \text{Log}[n]]$ | $\frac{1}{16} \text{Gamma}[3, 0, 2 \text{Log}[n]]$ | $\frac{1}{96} \text{Gamma}[4, 0, 2 \text{Log}[n]]$ | $\frac{1}{768} \text{Gamma}[5, 0, 2 \text{Log}[n]]$ | $\text{Gamma}[6, 0, 2 \text{Log}[n]] / 7680$ |
| $\frac{1}{3} (1 - \frac{1}{n^3})$ | $\frac{1}{9} \text{Gamma}[2, 0, 3 \text{Log}[n]]$ | $\frac{1}{54} \text{Gamma}[3, 0, 3 \text{Log}[n]]$ | $\frac{1}{486} \text{Gamma}[4, 0, 3 \text{Log}[n]]$ | $\text{Gamma}[5, 0, 3 \text{Log}[n]] / 5832$ | $\text{Gamma}[6, 0, 3 \text{Log}[n]] / 87480$ |
| $\frac{1}{4} (1 - \frac{1}{n^4})$ | $\frac{1}{16} \text{Gamma}[2, 0, 4 \text{Log}[n]]$ | $\frac{1}{128} \text{Gamma}[3, 0, 4 \text{Log}[n]]$ | $\text{Gamma}[4, 0, 4 \text{Log}[n]] / 1536$ | $\text{Gamma}[5, 0, 4 \text{Log}[n]] / 24576$ | $\text{Gamma}[6, 0, 4 \text{Log}[n]] / 491520$ |
| $\frac{1}{5} (1 - \frac{1}{n^5})$ | $\frac{1}{25} \text{Gamma}[2, 0, 5 \text{Log}[n]]$ | $\frac{1}{250} \text{Gamma}[3, 0, 5 \text{Log}[n]]$ | $\text{Gamma}[4, 0, 5 \text{Log}[n]] / 3750$ | $\text{Gamma}[5, 0, 5 \text{Log}[n]] / 75000$ | $\text{Gamma}[6, 0, 5 \text{Log}[n]] / 1875000$ |
| $\frac{1}{6} (1 - \frac{1}{n^6})$ | $\frac{1}{36} \text{Gamma}[2, 0, 6 \text{Log}[n]]$ | $\frac{1}{432} \text{Gamma}[3, 0, 6 \text{Log}[n]]$ | $\text{Gamma}[4, 0, 6 \text{Log}[n]] / 7776$ | $\text{Gamma}[5, 0, 6 \text{Log}[n]] / 186624$ | $\text{Gamma}[6, 0, 6 \text{Log}[n]] / 5598720$ |

Expand[Sum[j^5, {j, 2, n}]]

$$-1 - \frac{n^2}{12} + \frac{5n^4}{12} + \frac{n^5}{2} + \frac{n^6}{6}$$

Expand[Sum[j^(-2), {j, 2, n}]]

$$-1 + \text{HarmonicNumber}[n, 2]$$

```
Grid[Table[Limit[(Fa3[n, a2, ss] - 1) / a2, a2 → aa], {ss, -2, 4}, {aa, -3, 1}]]
```

Infinity::indet: Indeterminate expression 0(-∞) encountered. >>

Infinity::indet: Indeterminate expression 0(-∞) encountered. >>

$$\begin{array}{ccccccc} \frac{28}{3} & -4 & 4 & i \pi - \Gamma[0, -3 \log[n]] - \log[3] & \frac{1}{3} (-4 + n^3) \\ 3 & -\frac{3}{2} & 3 & i \pi - \Gamma[0, -2 \log[n]] - \log[2] & \frac{1}{2} (-3 + n^2) \\ \frac{2}{3} & 0 & 2 & i \pi - \Gamma[0, -\log[n]] & -2 + n \\ \frac{1}{3} & \frac{1}{2} & 1 & -\infty & 0 \\ 0 & 0 & 0 & -\Gamma[0, \log[n]] & -\frac{1}{n} \\ -\frac{7}{3} & -\frac{3}{2} & -1 & -\Gamma[0, 2 \log[n]] - \log[2] & -\frac{1+n^2}{2n^2} \\ -\frac{26}{3} & -4 & -2 & -\Gamma[0, 3 \log[n]] - \log[3] & -\frac{2}{3} - \frac{1}{3n^3} \end{array}$$

```
Grid[Table[Limit[(Fa3[n, a2, ss] - 1) / a2, a2 → aa], {ss, -3, 5}, {aa, -6, 3}]]
```

$$\begin{array}{cccccccc} -\frac{1365}{2} & 205 & -\frac{255}{4} & \frac{65}{3} & -\frac{15}{2} & 5 & i \pi - \Gamma[0, -4 \log[n]] - \log[4] & \frac{1}{4} (-5 + n^4) & \frac{1}{32} & \frac{1}{384} (-130 + \Gamma[3, -4 \log[n]]) \\ -\frac{364}{3} & \frac{244}{5} & -20 & \frac{28}{3} & -4 & 4 & i \pi - \Gamma[0, -3 \log[n]] - \log[3] & \frac{1}{3} (-4 + n^3) & \frac{1}{18} (-8 - \Gamma[2, -3 \log[n]]) & \frac{1}{162} (-56 + \Gamma[3, -3 \log[n]]) \\ -\frac{21}{2} & \frac{33}{5} & -\frac{15}{4} & 3 & -\frac{3}{2} & 3 & i \pi - \Gamma[0, -2 \log[n]] - \log[2] & \frac{1}{2} (-3 + n^2) & \frac{1}{8} (-3 - \Gamma[2, -2 \log[n]]) & \frac{1}{48} (-18 + \Gamma[3, -2 \log[n]]) \\ 0 & \frac{2}{5} & 0 & \frac{2}{3} & 0 & 2 & i \pi - \Gamma[0, -\log[n]] & -2 + n & -\frac{1}{2} \Gamma[2, -\log[n]] & \frac{1}{6} (-4 + \Gamma[3, -\log[n]]) \\ \frac{1}{6} & \frac{1}{5} & \frac{1}{4} & \frac{1}{3} & \frac{1}{2} & 1 & -\infty & 0 & \text{ComplexInfinity} & \text{ComplexInfinity} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\Gamma[0, \log[n]] & -\frac{1}{n} & -\frac{1}{2} \Gamma[2, \log[n]] & -\frac{1}{6} \Gamma[3, \log[n]] \\ -\frac{21}{2} & -\frac{31}{5} & -\frac{15}{4} & -\frac{7}{3} & -\frac{3}{2} & -1 & -\Gamma[0, 2 \log[n]] - \log[2] & -\frac{1+n^2}{2n^2} & \frac{1}{8} (-3 - \Gamma[2, 2 \log[n]]) & \frac{1}{48} (-14 - \Gamma[3, 2 \log[n]]) \\ -\frac{364}{3} & -\frac{242}{5} & -20 & -\frac{26}{3} & -4 & -2 & -\Gamma[0, 3 \log[n]] - \log[3] & -\frac{2}{3} - \frac{1}{3n^3} & \frac{1}{18} (-8 - \Gamma[2, 3 \log[n]]) & \frac{1}{162} (-52 - \Gamma[3, 3 \log[n]]) \\ -\frac{1365}{2} & -\frac{1023}{5} & -\frac{255}{4} & -21 & -\frac{15}{2} & -3 & -\Gamma[0, 4 \log[n]] - \log[4] & -\frac{3}{4} - \frac{1}{4n^4} & \frac{1}{32} (-15 - \Gamma[2, 4 \log[n]]) & \frac{1}{384} (-126 - \Gamma[3, 4 \log[n]]) \end{array}$$

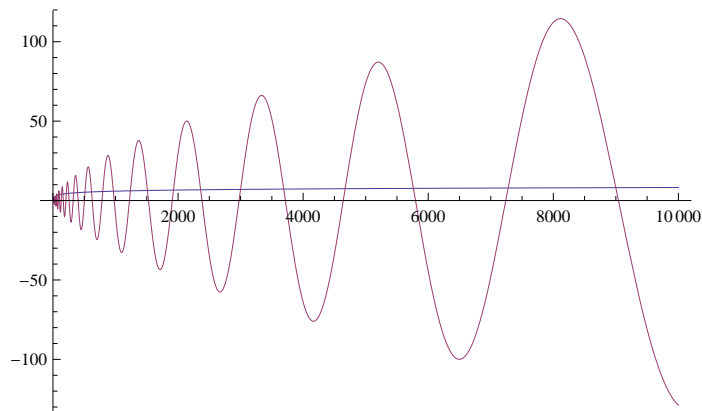
$-\Gamma[1, 0, -\log[n]]$

$-1 + n$

$-\Gamma[1, 0, -(1 - N[\text{ZetaZero}[1]]) \log[n]] - \Gamma[1, 0, -(1 - N[\text{ZetaZero}[-1]]) \log[n]]$

$\text{aa}[n_]:= -2. \cdot n^{0.5 - 14.134725141734695} i + n^{0.5 + 14.134725141734695} i$

```
Plot[{Fa3[j, aa = 2, bb = 0] / j,
      Re[Fa3[j, aa, ZetaZero[cc = 1]] + Fa3[j, aa, ZetaZero[-cc]]]}, {j, 1, 10 000}]
```



```
N[-Gamma[0, -Log[100]]]
```

```
30.1261 + 3.14159 i
```

```
N[LogIntegral[100]]
```

```
30.1261
```

```
N[-Gamma[0, -(1 - ZetaZero[1]) Log[100]]] - Pi I
```

```
0.116437 - 3.24171 i
```

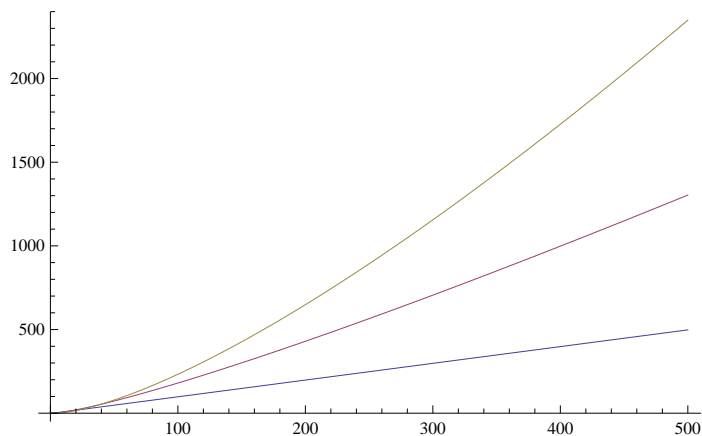
```
N[LogIntegral[100^(1 - ZetaZero[1])]]
```

```
1.35421 - 6.31436 i
```

```
N[ExpIntegralEi[(1 - ZetaZero[1]) Log[100]]]
```

```
0.116437 - 3.24171 i
```

```
Plot[{(Fa3[n, 1, 0] - 1) / 1, (Fa3[n, 2, 0] - 1) / 2, (Fa3[n, 3, 0] - 1) / 3}, {n, 1, 500}]
```



```
Limit[
$$\frac{(-1)^a (\text{Gamma}[a, 0, -(1-s) \text{Log}[n]]) (1-s)^{-a}}{\text{Gamma}[a+1]} - 1/a, \{a \rightarrow 0\}]$$

```

```
{i π - Gamma[0, (-1 + s) Log[n]] - Log[1 - s]}
```

```

N[Gamma[3, 0, -Log[100]]]
-1397.73 + 3.42834 × 10-13 i
N[(3 - 1) Gamma[3 - 1, 0, -Log[100]] - (-Log[100])^(3 - 1) E^(-(-Log[100]))]
-1397.73 - 8.83012 × 10-14 i
fz[n_, s_] := (s - 1) Gamma[s - 1, 0, n] - (n)^(s - 1) E^(-n)
fz1[n_, s_] := - (n)^(s - 1) E^(-n)
N[fz[-Log[100], 3]]
-1397.73 - 8.83012 × 10-14 i
Limit[fz[-Log[n], a], {a → 1}]
{1 - n}

N[Sum[(1 - k Log[k + 1] + k Log[k]) / k, {k, 1, 20 000}]]
0.577191
Limit[1 / n Sum[Ceiling[n / j] - n / j, {j, 1, n}], n → Infinity]
$Aborted

Fa3a[n_, a_, s_] := If[a == 0, Limit[Fa3[n, b, s], {b → 0}], Fa3[n, a, s]]
D2E2a[n_, k_, b_, s_] := Sum[(-1)^j b^(j (1 - s)) Binomial[k, j]
  Sum[Binomial[j, m] If[n / b^j < 1, 0, Fa3a[n / b^j, k - m, s]], {m, 0, j}], {j, 0, k}]

N[D2E2a[50, 3, 3, 0]]
{-1. - 9.81513 × 10-16 i}
DiscretePlot[Re[D2E2a[j, 3, 2, 0]], {j, 3, 100}]
Map::level: Level specification {System`DiscretePlotDump`n} is not of the form n, {n}, or {m, n}. >>
Map::level: Level specification {System`DiscretePlotDump`n} is not of the form n, {n}, or {m, n}. >>
Transpose::tperm: Permutation {2, 1} is longer than the dimensions {2} of the array. >>
Transpose::tperm: Permutation {2, 1} is longer than the dimensions {2} of the array. >>
Map::level: Level specification
  Transpose[{{{3}, {4}, {5}, {6}, {7}, {8}, {9}, {10}, {11}, {12}, {13}, {14}, {15}, {16}, {17}, {18}, {19}, {20}, {21}, {22}, {23}, {24}, {
    25}, {26}, {27}, {28}, {29}, {30}, {31}, {32}, {33}, {34}, {35}, {36}, {37}, {38}, {39}, {40}, {41}, {42}, {43}, {44}, {45}, {
    46}, {47}, {48}, {49}, {50}, {51}, {52}, <<48>>}, {System`DiscretePlotDump`n}}, {2, 1}]
  is not of the form n, {n}, or {m, n}. >>
General::stop: Further output of Map::level will be suppressed during this calculation. >>
Transpose::tperm: Permutation {2, 1} is longer than the dimensions {2} of the array. >>

```


Pick::incomp : Expressions

```
{Map[If[Head[Slot[<<1>>]] == List && Length[Slot[<<1>>]] != System`DiscretePlotDump`max, Table[Indeterminate, {
  System`DiscretePlotDump`max}], N[#, System`DiscretePlotDump`modelData$150676[
    WorkingPrecision]]] &, {-0.225222, -1.58427, -0.147778, 1.7961, 4.19297, {7.}, {-1.}, {-1.}, <<35>>, {-1.},
  {-1.}, {-1.}, {-1.}, {-1.}, {-1.}, {-1.}, <<48>>, {System`DiscretePlotDump`n}]] and
{Map[Charting`realNumericQ[If[Head[<<1>>]] == List && Length[<<1>>]] != System`DiscretePlotDump`max, Table[
  Indeterminate, {System`DiscretePlotDump`max}], N[#, System`DiscretePlotDump`modelData$150676
    [
      WorkingPrecision]]]]] &, {True, True, True, True, True, False, False, False, False, False, False, False, False,
  False, False, False, False, False, False, False, False, False, False, False, False, False, False, False, False,
  False, False, False, False, False, False, False, False, False, False, False, False, False, False, False, False,
  False, <<48>>, {False}}] have incompatible shapes. >
```

Pick::incomp : Expressions

```
{Map[If[Head[Slot[<1>]] == List && Length[Slot[<1>]] != System`DiscretePlotDump`max, Table[Indeterminate, {
  System`DiscretePlotDump`max}], N[#, System`DiscretePlotDump`modelData$150676[
  WorkingPrecision]]] &, {-0.225222, -1.58427, -0.147778, 1.7961, 4.19297, {7.}, {-1.}, {-1.}, <<35>, {-1.
}, {-1.}, {-1.}, {-1.}, {-1.}, {-1.}, <<48>}, {System`DiscretePlotDump`n}]] and
{Map[Charting`realNumericQ[If[Head[<1>]] == List && Length[<1>]] != System`DiscretePlotDump`max, Table[
  Indeterminate, {System`DiscretePlotDump`max}], N[#, System`DiscretePlotDump`modelData$150676
  [
    WorkingPrecision]]]]] &, {True, True, True, True, True, False, False, False, False, False, False, False, False,
  False, False, False, False, False, False, False, False, False, False, False, False, False, False, False, False,
  False, False, False, False, False, False, False, False, False, False, False, False, False, False, False, False,
  False, <<48>}, {False}]] have incompatible shapes. >
```

General::stop : Further output of Pick::incomp will be suppressed during this calculation. >>

Set::shape : Lists {System`DiscretePlotDump`low, System`DiscretePlotDump`high} and <<1>> are not the same shape. >>

Transpose::list : List expected at position 2 in

```
Transpose[If[Head[#1] === List && Length[#1] ≠ System`DiscretePlotDump`max, Table[Indeterminate, {
    System`DiscretePlotDump`max}], N[#1, System`DiscretePlotDump`modelData$150676[
    WorkingPrecision]] &, 1]. >>
```

General::stop : Further output of Transpose::list will be suppressed during this calculation. >>

Transpose::nmtx : The first two levels of the one-dimensional list {} cannot be transposed. >>

Transpose::nmtx : The first two levels of the one-dimensional list {} cannot be transposed. >>

Transpose::nmtx : The first two levels of the one-dimensional list {Transpose[{}], Transpose[{}]} cannot be transposed. >>

General::stop : Further output of Transpose::nmtx will be suppressed during this calculation. >>

MapThread::mptc : Incompatible dimensions of objects at positions {2, 1} and {2, 2} of

[illegible]

MapThread::mptc : Incompatible dimensions of objects at positions {2, 1} and {2, 2} of

[illegible]

Split::normal : Nonatomic expression expected at position 1 in Split[And]. >>

Split::normal : Nonatomic expression expected at position 1 in Split[0]. >>

First::normal : Nonatomic expression expected at position 1 in First[And]. >>

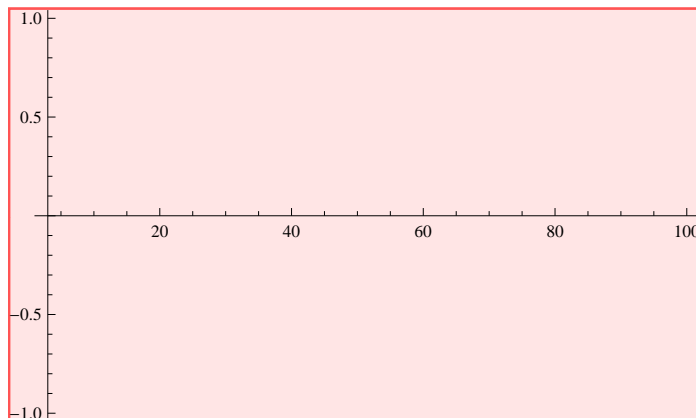
First::normal : Nonatomic expression expected at position 1 in First[And]. >>

Split::normal : Nonatomic expression expected at position 1 in Split[2]. >>

General::stop : Further output of Split::normal will be suppressed during this calculation. >>

First::normal : Nonatomic expression expected at position 1 in First[2]. >>

General::stop : Further output of First::normal will be suppressed during this calculation. >>



Fa3[100, 0, -1]

Infinity::indet: Indeterminate expression 0 ComplexInfinity encountered. >>

Indeterminate

$$(-1)^a \frac{\Gamma[a, 0, -(1-s) \log[n]] (1-s)^{-a}}{\Gamma[a]} /. \{a \rightarrow 0, s \rightarrow -1\}$$

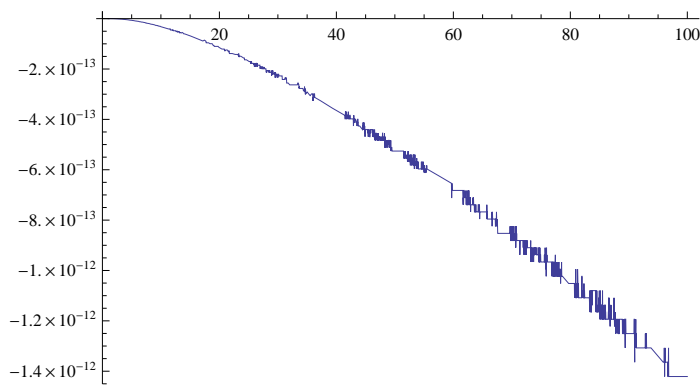
Infinity::indet: Indeterminate expression 0 ComplexInfinity encountered. >>

Indeterminate

Gamma[0]

ComplexInfinity

Plot[Re[Fa3[n, 2.75, 0]], {n, 1, 100}]



$$\text{Fa3}[n_, a_, s_] := (-1)^a \frac{(\text{Gamma}[a, 0, -(1-s) \text{Log}[n]]) (1-s)^{-a}}{\text{Gamma}[a]}$$

$$\text{Fa3b}[n_, a_, s_] := \text{Abs} \left[\frac{(\text{Gamma}[a, 0, -(1-s) \text{Log}[n]]) (1-s)^{-a}}{\text{Gamma}[a]} \right]$$

$$\text{Fa3c}[n_, a_, s_] := \frac{(\text{Gamma}[a, 0, -(1-s) \text{Log}[n]]) (1-s)^{-a}}{\text{Gamma}[a]}$$

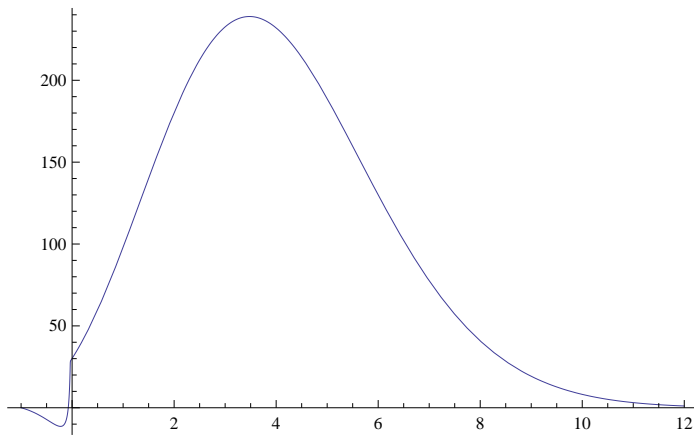
N[Fa3[100, 2.25, 0]]

$1.7053 \times 10^{-13} + 445.721 i$

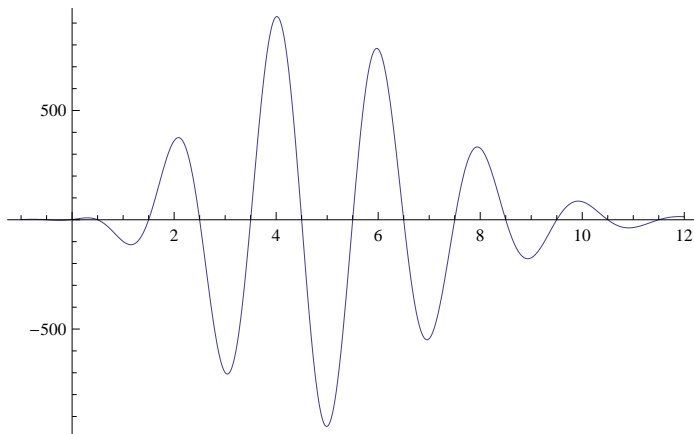
N[Fa3b[100, 3, 0]]

698.863

Plot[{(Fa3b[100, n, aa] - 1) / n}, {n, -1, 12}]



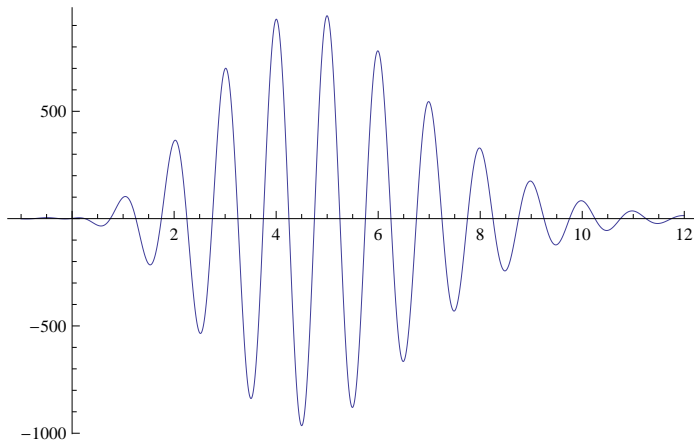
Plot[{(Re[Fa3c[100, n, aa = 0]])}, {n, -1, 12}]



N[Fa3c[100, 2, 0]]

$361.517 - 4.41506 \times 10^{-14} i$

```
Plot[{(Re[Fa3[100, n, aa = 0]])}, {n, -1, 12}]
```



```
Plot[{Fa3b[n, 2, aa], Fa3b[n, 2.5, aa], Fa3b[n, 3, aa]}, {n, -1, 120}]
```

