```
ClearAll["Global`*"]
```

```
E2a[n_, k_, a_] :=
 E2a[n, k, a] = Sum[ E2a[n / j, k - 1, a], {j, 2, n}] - a Sum[ E2a[n / (a j), k - 1, a], {j, 1, n / a}];
E2a[n_, 0, a_] := 1
EM2[n_, a_, b_] :=
 EM2[n, a, b] = Sum[(-1)^k Binomial[k - 1, k - a] E2a[n, k, b], {k, 1, Log[If[b < 2, b, 2], n]}];
EM2[n_, 0, b_] := 1
E1e2[n_, k_, b_] :=
 Sum[FactorialPower[-k, a] / a! EM2[n, a, b], {a, 0, Log[If[b > 2, 2, b], n]}]
D1g[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j E1e2[n / b^j, k, b], {j, 0, Log[b, n]}]
D1h[n_, k_, b_] :=
 Sum[ Binomial[k + j - 1, k - 1] b^j (Sum[FactorialPower[-k, a] / a! EM2[n / b^j, a, b],
     {a, 0, Log[If[b > 2, 2, b], n / b^j]}]), {j, 0, Log[b, n]}]
D1i[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j FactorialPower[-k, a] / a!
   EM2[n / b^j, a, b], {j, 0, Log[b, n]}, {a, 0, Log[If[b > 2, 2, b], n / b^j]}]
D1ia[n_, k_, b_] := Grid[Table[ Binomial[k + j - 1, k - 1] b^j FactorialPower[-k, a] / a!
    EM[n / b^j, a, b], {j, 0, Log[b, n]}, {a, 0, Log[If[b > 2, 2, b], n / b^j]}]]
D1ib[n_, k_, b_] := Grid[Table[ Binomial[k + j - 1, k - 1] b^j FactorialPower[-k, a] / a!
    EM[n / b^j, a, b] / k, {j, 0, Log[b, n]}, {a, 0, Log[If[b > 2, 2, b], n / b^j]}]]
D1ic[n_, k_, b_] := Grid[Table[ Binomial[k + j - 1, k - 1] FactorialPower[-k, a] / a!,
   {j, 0, 7}, {a, 0, 7}]]
D1id[n_, k2_, b_] := Grid[Table[ Limit[Binomial[k + j - 1, k - 1]
     FactorialPower[-k, a] / a! / k, k → k2], {j, 0, 7}, {a, 0, 7}]]
```

```
D1ia[100, 1, 2]
```

$$\begin{array}{lllllll}
EM[100, 0, 2] & -EM[100, 1, 2] & EM[100, 2, 2] & -EM[100, 3, 2] & EM[100, 4, 2] & -EM[100, 5, 2] & EM[100, 6, 2] \\
2\,EM[50, 0, 2] & -2\,EM[50, 1, 2] & 2\,EM[50, 2, 2] & -2\,EM[50, 3, 2] & 2\,EM[50, 4, 2] & -2\,EM[50, 5, 2] & \\
4\,EM[25, 0, 2] & -4\,EM[25, 1, 2] & 4\,EM[25, 2, 2] & -4\,EM[25, 3, 2] & 4\,EM[25, 4, 2] & & \\
8\,EM\left[\frac{25}{2}, 0, 2\right] & -8\,EM\left[\frac{25}{2}, 1, 2\right] & 8\,EM\left[\frac{25}{2}, 2, 2\right] & -8\,EM\left[\frac{25}{2}, 3, 2\right] & & & \\
16\,EM\left[\frac{25}{4}, 0, 2\right] & -16\,EM\left[\frac{25}{4}, 1, 2\right] & 16\,EM\left[\frac{25}{4}, 2, 2\right] & & & & \\
32\,EM\left[\frac{25}{8}, 0, 2\right] & -32\,EM\left[\frac{25}{8}, 1, 2\right] & & & & & \\
64\,EM\left[\frac{25}{16}, 0, 2\right] & & & & & &
\end{array}$$

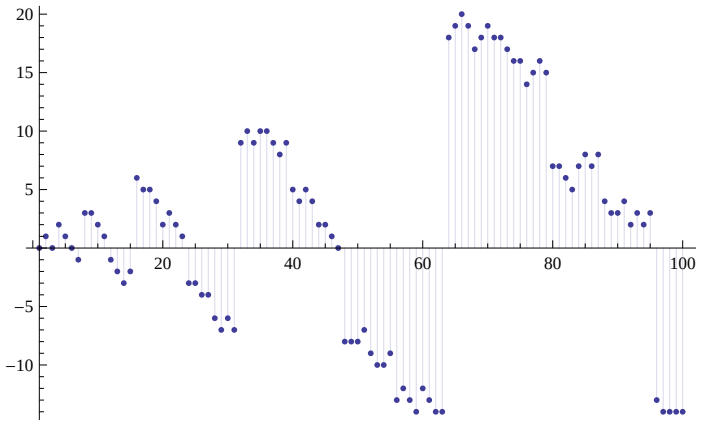
```
D1ia[100, -1, 20]
```

$$\begin{array}{lllllll}
EM[100, 0, 20] & EM[100, 1, 20] & 0 & 0 & 0 & 0 & 0 \\
-20\,EM[5, 0, 20] & -20\,EM[5, 1, 20] & 0 & & & &
\end{array}$$

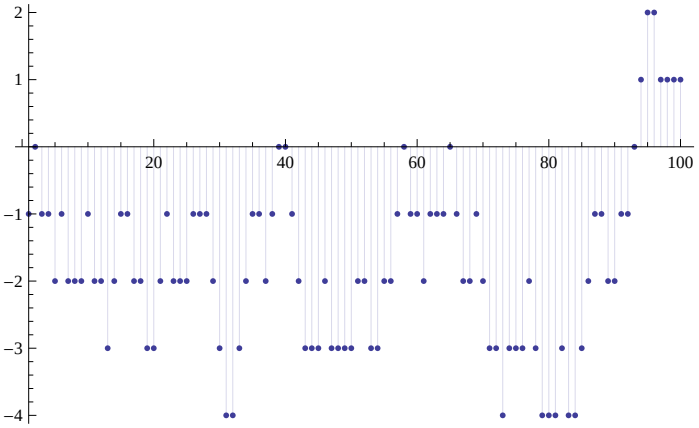
**D1ib[100, .000001, 2]**

| | | | | | | |
|---|---|---|---|---|---|---|
| $1.\times 10^{6}\,EM[100, 0, 2]$ | $-1.\,EM[100, 1, 2]$ | $0.500001\,EM[100, 2, 2]$ | $-0.333334\,EM[100, 3, 2]$ | $0.25\,EM[100, 4, 2]$ | $-0.2\,EM[100, 5, 2]$ | $0.166667\,EM[100, 6, 2]$ |
| $2.\,EM[50, 0, 2]$ | $-2.\times 10^{-6}\,EM[50, 1, 2]$ | $1.\times 10^{-6}\,EM[50, 2, 2]$ | $-6.66668\times 10^{-7}\,EM[50, 3, 2]$ | $5.00001\times 10^{-7}\,EM[50, 4, 2]$ | $-4.00001\times 10^{-7}\,EM[50, 5, 2]$ | |
| $2.\,EM[25, 0, 2]$ | $-2.\times 10^{-6}\,EM[25, 1, 2]$ | $1.\times 10^{-6}\,EM[25, 2, 2]$ | $-6.66668\times 10^{-7}\,EM[25, 3, 2]$ | $5.00001\times 10^{-7}\,EM[25, 4, 2]$ | | |
| $2.66667\,EM\left[\frac{25}{2}, 0, 2\right]$ | $-2.66667\times 10^{-6}\,EM\left[\frac{25}{2}, 1, 2\right]$ | $1.33334\times 10^{-6}\,EM\left[\frac{25}{2}, 2, 2\right]$ | $-8.88892\times 10^{-7}\,EM\left[\frac{25}{2}, 3, 2\right]$ | | | |
| $4.00001\,EM\left[\frac{25}{4}, 0, 2\right]$ | $-4.00001\times 10^{-6}\,EM\left[\frac{25}{4}, 1, 2\right]$ | $2.00001\times 10^{-6}\,EM\left[\frac{25}{4}, 2, 2\right]$ | | | | |
| $6.40001\,EM\left[\frac{25}{8}, 0, 2\right]$ | $-6.40001\times 10^{-6}\,EM\left[\frac{25}{8}, 1, 2\right]$ | | | | | |
| $10.6667\,EM\left[\frac{25}{16}, 0, 2\right]$ | | | | | | |

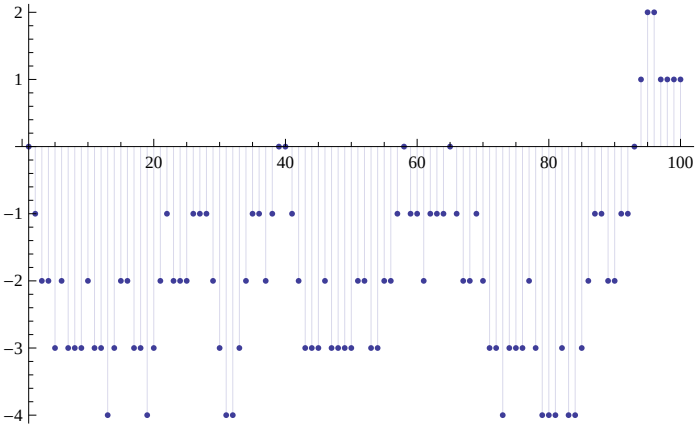
**DiscretePlot[ EM2[n, 1, 2], {n, 1, 100}]**



```
DiscretePlot[ -1 + EM2[n, 1, 2] - 2 EM2[Floor[n / 2], 1, 2], {n, 1, 100}]
```




```
DiscretePlot[If[n > 19, -19, 0] + EM2[n, 1, 20] - 20 EM2[Floor[n / 20], 1, 20], {n, 1, 100}]
```




```
D1ic[100, -2, 2]
```

```
 1   2   1  0  0  0  0  0
-2  -4  -2  0  0  0  0  0
 1   2   1  0  0  0  0  0
 0   0   0  0  0  0  0  0
 0   0   0  0  0  0  0  0
 0   0   0  0  0  0  0  0
 0   0   0  0  0  0  0  0
 0   0   0  0  0  0  0  0
```

```
D1id[100, 0, 2]
```

| ∞ | −1 | $\frac{1}{2}$ | $-\frac{1}{3}$ | $\frac{1}{4}$ | $-\frac{1}{5}$ | $\frac{1}{6}$ | $-\frac{1}{7}$ |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\frac{1}{2}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\frac{1}{3}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\frac{1}{4}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\frac{1}{5}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\frac{1}{6}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\frac{1}{7}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

```
D1i[100, 1, 1.1]
```

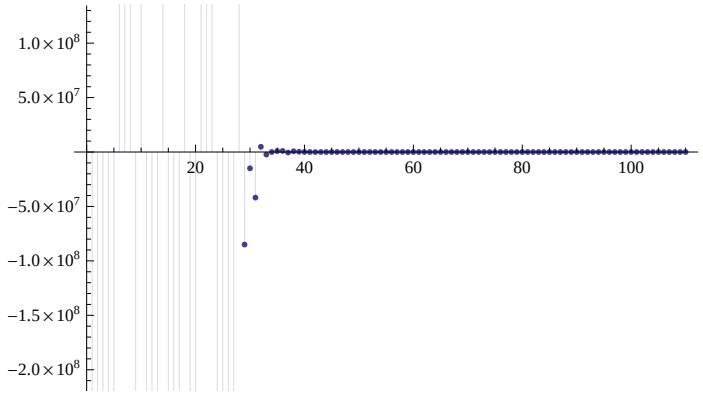
```
100.357
```

```
dra[n_, k_, b_] :=
 DiscretePlot[Sum[ Binomial[k + j - 1, k - 1] b^j FactorialPower[-k, a] / a! EM2[n / b^j, a, b],
    {a, 0, Log[If[b > 2, 2, b], n / b^j]}], {j, 0, Log[b, n]}]
dra2[n_, k_, b_] := DiscretePlot[
  {(1 / k) Sum[ Binomial[k + j - 1, k - 1] b^j FactorialPower[-k, a] / a! EM2[n / b^j, a, b],
     {a, 0, Log[If[b > 2, 2, b], n / b^j]}]}, {j, 0, Log[b, n]}]
```

```
$RecursionLimit = 10 000
```

```
10 000
```

```
dra[3, 1, 1.01]
```




```
Limit[(D1i[100, z, 2] - 1) / z, z → 0]
```

$$\frac{428}{15}$$