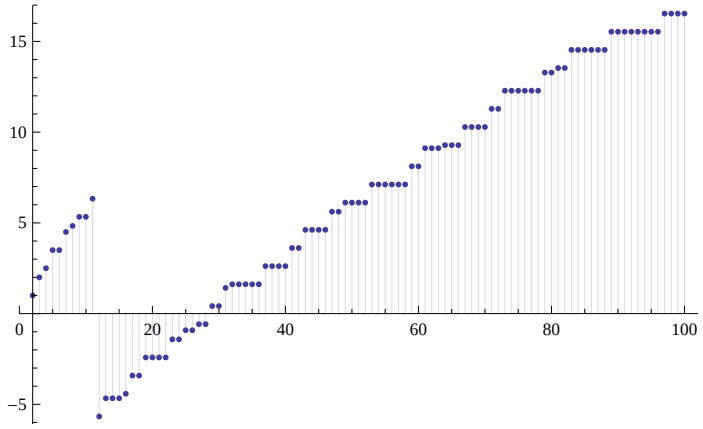
```
ClearAll["Global`*"]
```

```
vv := 2
K[n_] := If[n == 1, 0, FullSimplify[MangoldtLambda[n] / Log[n]]]
K6[n_] := K[n] - If[Floor[Log[vv, n]] == Log[vv, n], n / Log[vv, n], 0]
P[n_, 0] = 1;
P[n_, k_] := P[n, k] = Sum[K6[j] P[Floor[n / j], k - 1], {j, 2, n}]
PO[n_, 0] = 1;
PO[n_, k_] := PO[n, k] = Sum[K[j] PO[Floor[n / j], k - 1], {j, 2, n}]
p[n_, k_] := P[n, k] - P[n - 1, k]
po[n_, k_] := PO[n, k] - PO[n - 1, k]
En[n_] := En[n] = Sum[1 / (k!) P[n, k], {k, 0, Log[2, n]}]
En[n_, z_] := En[n] = Sum[(z^k) / (k!) P[n, k], {k, 0, Log[2, n]}]
en[n_] := En[n] - En[n - 1]
LAdd[n_] := Sum[vv^k / k, {k, 1, Log[vv, n]}]
LAdd2[n_] := Sum[(-1)^k vv^k, {k, 1, Log[vv, n]}]
PP[n_, k_] := PP[n, k] = Sum[1 / k - PP[Floor[n / j], k + 1], {j, 2, n}]
```

```
P[100, 1]
```

$$\frac{248}{15}$$

```
DiscretePlot[{P[n, 1]}, {n, 2, 100}]
```

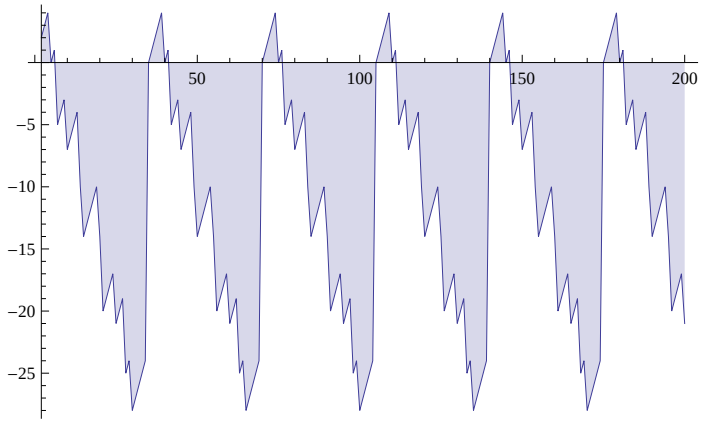


```
Table[{n, p[n, 2], po[n, 2], (p[n, 2] - po[n, 2]),
    ((p[n, 2] - po[n, 2]) - If[Mod[n, vv] == 0, -2 (vv) po[n / vv, 1], 0]) / (vv * vv)

  }, {n, 9, 250, 9}] // TableForm
```

9	4	1	3	1
18	−8	1	−9	−1
27	16	1	15	2
36	−4	$\frac{1}{2}$	$-\frac{9}{2}$	$-\frac{1}{2}$
45	−8	1	−9	−1
54	$-\frac{52}{3}$	$\frac{2}{3}$	−18	−2
63	−8	1	−9	−1
72	$-\frac{8}{3}$	$\frac{1}{3}$	−3	$-\frac{1}{3}$
81	$\frac{152}{3}$	$\frac{11}{12}$	$\frac{199}{4}$	$\frac{23}{4}$
90	0	0	0	0
99	−8	1	−9	−1
108	$-\frac{26}{3}$	$\frac{1}{3}$	−9	−1
117	−8	1	−9	−1
126	0	0	0	0
135	$-\frac{52}{3}$	$\frac{2}{3}$	−18	−2
144	−2	$\frac{1}{4}$	$-\frac{9}{4}$	$-\frac{1}{4}$
153	−8	1	−9	−1
162	−40	$\frac{1}{2}$	$-\frac{81}{2}$	$-\frac{9}{2}$
171	−8	1	−9	−1
180	0	0	0	0
189	$-\frac{52}{3}$	$\frac{2}{3}$	−18	−2
198	0	0	0	0
207	−8	1	−9	−1
216	$-\frac{52}{9}$	$\frac{2}{9}$	−6	$-\frac{2}{3}$
225	−4	$\frac{1}{2}$	$-\frac{9}{2}$	$-\frac{1}{2}$
234	0	0	0	0
243	$\frac{448}{3}$	$\frac{5}{6}$	$\frac{297}{2}$	$\frac{50}{3}$

```
DiscretePlot[En[n], {n, 2, 200}]
```

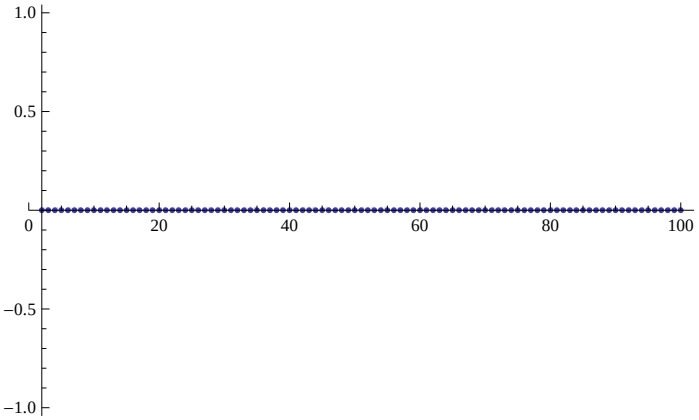


```
Table[{n, En[n], Mod[n, vv]}, {n, 1, 100}] // TableForm
```

1	1	1
2	2	2
3	3	3
4	4	4
5	0	5
6	1	6
7	−5	0
8	−4	1
9	−3	2
10	−7	3
11	−6	4
12	−5	5
13	−4	6
14	−10	0
15	−14	1
16	−13	2
17	−12	3
18	−11	4
19	−10	5
20	−14	6
21	−20	0
22	−19	1
23	−18	2
24	−17	3
25	−21	4
26	−20	5
27	−19	6
28	−25	0
29	−24	1
30	−28	2
31	−27	3
32	−26	4
33	−25	5
34	−24	6
35	0	0
36	1	1
37	2	2
38	3	3
39	4	4
40	0	5
41	1	6
42	−5	0
43	−4	1
44	−3	2
45	−7	3
46	−6	4
47	−5	5
48	−4	6
49	−10	0
50	−14	1
51	−13	2
52	−12	3
53	−11	4

54	−10	5
55	−14	6
56	−20	0
57	−19	1
58	−18	2
59	−17	3
60	−21	4
61	−20	5
62	−19	6
63	−25	0
64	−24	1
65	−28	2
66	−27	3
67	−26	4
68	−25	5
69	−24	6
70	0	0
71	1	1
72	2	2
73	3	3
74	4	4
75	0	5
76	1	6
77	−5	0
78	−4	1
79	−3	2
80	−7	3
81	−6	4
82	−5	5
83	−4	6
84	−10	0
85	−14	1
86	−13	2
87	−12	3
88	−11	4
89	−10	5
90	−14	6
91	−20	0
92	−19	1
93	−18	2
94	−17	3
95	−21	4
96	−20	5
97	−19	6
98	−25	0
99	−24	1
100	−28	2

```
DiscretePlot[P[n, 1] - PP[n, 1] + LAdd[n] + LAdda[n], {n, 2, 100}]
```



```
Table[{n, P[n, 2], PO[n, 2], (P[n, 2] - PO[n, 2]),
    P[n, 2] - PO[n, 2] + 2 Sum[vv^j / j PO[n / vv^j, 1], {j, 1, Log[vv, n]}] -
     Sum[ vv^j vv^k / (j k), {j, 1, Log[vv, n]}, {k, 1, Log[vv, Floor[n / (vv^j)]]}]
  }, {n, 9, 250, 9}] // TableForm
```

9	3	5	−2	0
18	$\frac{47}{12}$	$\frac{167}{12}$	−10	0
27	$-\frac{15}{4}$	$\frac{283}{12}$	$-\frac{82}{3}$	0
36	$\frac{33}{4}$	$\frac{383}{12}$	$-\frac{71}{3}$	0
45	$\frac{19}{12}$	$\frac{463}{12}$	−37	0
54	$-\frac{115}{12}$	$\frac{187}{4}$	$-\frac{169}{3}$	0
63	$-\frac{53}{4}$	$\frac{677}{12}$	$-\frac{209}{3}$	0
72	$\frac{653}{45}$	$\frac{2813}{45}$	−48	0
81	$\frac{1067}{180}$	$\frac{12\,587}{180}$	−64	0
90	$\frac{227}{180}$	$\frac{14\,147}{180}$	$-\frac{232}{3}$	0
99	$-\frac{365}{36}$	$\frac{16\,199}{180}$	$-\frac{1502}{15}$	0
108	$-\frac{695}{36}$	$\frac{16\,829}{180}$	$-\frac{564}{5}$	0
117	$-\frac{893}{36}$	$\frac{17\,999}{180}$	$-\frac{624}{5}$	0
126	$-\frac{929}{36}$	$\frac{19\,979}{180}$	$-\frac{684}{5}$	0
135	$\frac{6113}{180}$	$\frac{4261}{36}$	$-\frac{422}{5}$	0
144	$\frac{2479}{90}$	$\frac{2255}{18}$	$-\frac{1466}{15}$	0
153	$\frac{1969}{90}$	$\frac{2393}{18}$	$-\frac{1666}{15}$	0
162	$\frac{584}{45}$	$\frac{6383}{45}$	$-\frac{1933}{15}$	0
171	$\frac{449}{45}$	$\frac{6608}{45}$	$-\frac{2053}{15}$	0
180	$\frac{43}{90}$	$\frac{13\,801}{90}$	$-\frac{2293}{15}$	0
189	$-\frac{47}{90}$	$\frac{14\,551}{90}$	$-\frac{811}{5}$	0
198	$-\frac{1126}{45}$	$\frac{7403}{45}$	$-\frac{2843}{15}$	0
207	$-\frac{1096}{45}$	$\frac{7913}{45}$	$-\frac{1001}{5}$	0
216	$-\frac{2917}{90}$	$\frac{5567}{30}$	$-\frac{9809}{45}$	0
225	$-\frac{1814}{45}$	$\frac{2917}{15}$	$-\frac{2113}{9}$	0
234	$-\frac{2114}{45}$	$\frac{2957}{15}$	$-\frac{2197}{9}$	0
243	$-\frac{4153}{90}$	$\frac{6119}{30}$	$-\frac{2251}{9}$	0

```
Table[{n, P[n, 3],
   PO[n, 3]
    - 3 Sum[vv^j / j PO[n / vv^j, 2], {j, 1, Log[vv, n]}]
    + 3 Sum[ vv^j vv^k / (j k) PO[n / (vv^j vv^k), 1],
      {j, 1, Log[vv, n]}, {k, 1, Log[vv, Floor[n / (vv^j)]]}]
    - Sum[ vv^j vv^k vv^m / (j k m) PO[n / (vv^j vv^k vv^m), 0], {j, 1, Log[vv, n]},
     {k, 1, Log[vv, Floor[n / (vv^j)]]}, {m, 1, Log[vv, Floor[n / (vv^j vv^k)]]}]
  }, {n, 9, 250, 9}] // TableForm
```

9	-1	-1
18	$-\frac{11}{2}$	$-\frac{11}{2}$
27	$\frac{15}{2}$	$\frac{15}{2}$
36	$-\frac{49}{4}$	$-\frac{49}{4}$
45	$-\frac{13}{4}$	$-\frac{13}{4}$
54	$\frac{29}{2}$	$\frac{29}{2}$
63	$\frac{35}{2}$	$\frac{35}{2}$
72	$-\frac{237}{8}$	$-\frac{237}{8}$
81	$-\frac{59}{8}$	$-\frac{59}{8}$
90	$-\frac{83}{8}$	$-\frac{83}{8}$
99	$\frac{289}{8}$	$\frac{289}{8}$
108	$\frac{309}{8}$	$\frac{309}{8}$
117	$\frac{427}{8}$	$\frac{427}{8}$
126	$\frac{323}{8}$	$\frac{323}{8}$
135	$-\frac{6727}{120}$	$-\frac{6727}{120}$
144	$-\frac{944}{15}$	$-\frac{944}{15}$
153	$-\frac{719}{15}$	$-\frac{719}{15}$
162	$-\frac{1331}{60}$	$-\frac{1331}{60}$
171	$-\frac{1811}{60}$	$-\frac{1811}{60}$
180	$-\frac{209}{15}$	$-\frac{209}{15}$
189	$-\frac{164}{15}$	$-\frac{164}{15}$
198	$\frac{1407}{20}$	$\frac{1407}{20}$
207	$\frac{1237}{20}$	$\frac{1237}{20}$
216	$\frac{408}{5}$	$\frac{408}{5}$
225	$\frac{1131}{10}$	$\frac{1131}{10}$
234	$\frac{1101}{10}$	$\frac{1101}{10}$
243	$\frac{1667}{20}$	$\frac{1667}{20}$

```
Table[{n, P[n, 3],
   PO[n, 3]
    - 3 Sum[vv^j / j PO[n / vv^j, 2], {j, 1, Log[vv, n]}]
    + 3 Sum[ vv^(j + k) / (j k) PO[n / (vv^(j + k)), 1],
      {j, 1, Log[vv, n]}, {k, 1, Log[vv, Floor[n / (vv^j)]]}]
    - Sum[ vv^(j + k + m) / (j k m) PO[n / (vv^(j + k + m)), 0], {j, 1, Log[vv, n]},
     {k, 1, Log[vv, Floor[n / (vv^j)]]}, {m, 1, Log[vv, Floor[n / (vv^(j + k))]]}]
  }, {n, 2, 50, 1}] // TableForm
```

2	0	0
3	0	0
4	0	0
5	0	0
6	0	0
7	0	0
8	−1	−1
9	−1	−1
10	−1	−1
11	−1	−1
12	2	2
13	2	2
14	2	2
15	2	2
16	$-\frac{5}{2}$	$-\frac{5}{2}$
17	$-\frac{5}{2}$	$-\frac{5}{2}$
18	$-\frac{11}{2}$	$-\frac{11}{2}$
19	$-\frac{11}{2}$	$-\frac{11}{2}$
20	$-\frac{5}{2}$	$-\frac{5}{2}$
21	$-\frac{5}{2}$	$-\frac{5}{2}$
22	$-\frac{5}{2}$	$-\frac{5}{2}$
23	$-\frac{5}{2}$	$-\frac{5}{2}$
24	$\frac{13}{2}$	$\frac{13}{2}$
25	$\frac{13}{2}$	$\frac{13}{2}$
26	$\frac{13}{2}$	$\frac{13}{2}$
27	$\frac{15}{2}$	$\frac{15}{2}$
28	$\frac{21}{2}$	$\frac{21}{2}$
29	$\frac{21}{2}$	$\frac{21}{2}$
30	$\frac{9}{2}$	$\frac{9}{2}$
31	$\frac{9}{2}$	$\frac{9}{2}$
32	$-\frac{37}{4}$	$-\frac{37}{4}$
33	$-\frac{37}{4}$	$-\frac{37}{4}$
34	$-\frac{37}{4}$	$-\frac{37}{4}$
35	$-\frac{37}{4}$	$-\frac{37}{4}$
36	$-\frac{49}{4}$	$-\frac{49}{4}$

37	$-\frac{49}{4}$	$-\frac{49}{4}$
38	$-\frac{49}{4}$	$-\frac{49}{4}$
39	$-\frac{49}{4}$	$-\frac{49}{4}$
40	$-\frac{13}{4}$	$-\frac{13}{4}$
41	$-\frac{13}{4}$	$-\frac{13}{4}$
42	$-\frac{37}{4}$	$-\frac{37}{4}$
43	$-\frac{37}{4}$	$-\frac{37}{4}$
44	$-\frac{25}{4}$	$-\frac{25}{4}$
45	$-\frac{13}{4}$	$-\frac{13}{4}$
46	$-\frac{13}{4}$	$-\frac{13}{4}$
47	$-\frac{13}{4}$	$-\frac{13}{4}$
48	$\frac{35}{2}$	$\frac{35}{2}$
49	$\frac{35}{2}$	$\frac{35}{2}$
50	$\frac{29}{2}$	$\frac{29}{2}$

```
fd[n_] := Sum[ vv^(j + k) / (j k ), {j, 1, Log[vv, n]}, {k, 1, Log[vv, Floor[n / (vv^j)]]}]
fe[n_] := Sum[ vv^(j + k + m) / (j k m) , {j, 1, Log[vv, n]},
  {k, 1, Log[vv, Floor[n / (vv^j)]]}, {m, 1, Log[vv, Floor[n / (vv^(j + k))]]}]
```

```
Table[{n, (fe[n] - fe[n - 1]) / n}, {n, 8, 800, 8}] // TableForm
```

8	1
16	$\frac{3}{2}$
24	0
32	$\frac{7}{4}$
40	0
48	0
56	0
64	$\frac{15}{8}$
72	0
80	0
88	0
96	0
104	0
112	0
120	0
128	$\frac{29}{15}$
136	0
144	0
152	0
160	0
168	0
176	0
184	0
192	0
200	0
208	0
216	0
224	0
232	0

240	0
248	0
256	$\frac{469}{240}$
264	0
272	0
280	0
288	0
296	0
304	0
312	0
320	0
328	0
336	0
344	0
352	0
360	0
368	0
376	0
384	0
392	0
400	0
408	0
416	0
424	0
432	0
440	0
448	0
456	0
464	0
472	0
480	0
488	0
496	0
504	0
512	$\frac{29\,531}{15\,120}$
520	0
528	0
536	0
544	0
552	0
560	0
568	0
576	0
584	0
592	0
600	0
608	0
616	0
624	0
632	0
640	0
648	0
656	0
664	0
672	0

680	0
688	0
696	0
704	0
712	0
720	0
728	0
736	0
744	0
752	0
760	0
768	0
776	0
784	0
792	0
800	0

```
Table[{n, (fd[n] - fd[n - 1]) / n}, {n, 4, 400, 4}] // TableForm
```

4	1
8	1
12	0
16	$\frac{11}{12}$
20	0
24	0
28	0
32	$\frac{5}{6}$
36	0
40	0
44	0
48	0
52	0
56	0
60	0
64	$\frac{137}{180}$
68	0
72	0
76	0
80	0
84	0
88	0
92	0
96	0
100	0
104	0
108	0
112	0
116	0
120	0
124	0
128	$\frac{7}{10}$
132	0
136	0
140	0
144	0

148	0
152	0
156	0
160	0
164	0
168	0
172	0
176	0
180	0
184	0
188	0
192	0
196	0
200	0
204	0
208	0
212	0
216	0
220	0
224	0
228	0
232	0
236	0
240	0
244	0
248	0
252	0
256	$\frac{363}{560}$
260	0
264	0
268	0
272	0
276	0
280	0
284	0
288	0
292	0
296	0
300	0
304	0
308	0
312	0
316	0
320	0
324	0
328	0
332	0
336	0
340	0
344	0
348	0
352	0
356	0
360	0
364	0
368	0

```
372   0
376   0
380   0
384   0
388   0
392   0
396   0
400   0
```

```
$Aborted
```

```
$Aborted
```

```
$Aborted
```

```
PO[n, 3]
-3 Sum[vv^j / j PO[n / vv^j, 2], {j, 1, Log[vv, n]}]
+3 Sum[ vv^j vv^k / (j k) PO[n / (vv^j vv^k), 1],
  {j, 1, Log[vv, n]}, {k, 1, Log[vv, Floor[n / (vv^j)]]}]
-Sum[ vv^j vv^k vv^m / (j k m) PO[n / (vv^j vv^k vv^m), 0], {j, 1, Log[vv, n]},
  {k, 1, Log[vv, Floor[n / (vv^j)]]}, {m, 1, Log[vv, Floor[n / (vv^j vv^k)]]}]
```

```
Sum::write : Tag Plus in 2 + Sum`FiniteSumDump`I is Protected. ≫
```

```
Sum::write : Tag Plus in 2 + Sum`FiniteSumDump`I is Protected. ≫
```

```
Sum::write : Tag Plus in 2 + Sum`FiniteSumDump`I is Protected. ≫
```

```
General::stop : Further output of Sum::write will be suppressed during this calculation. ≫
```

```
$Aborted
```

```
$Aborted
```

```
ss[n_] := Ps[n, 3] - 3 Sum[vv^j / j Ps[n / vv^j, 2], {j, 1, Log[vv, n]}] +
  3 Sum[ vv^j vv^k / (j k) Ps[n / (vv^j vv^k), 1], {j, 1, Log[vv, n]},
    {k, 1, Log[vv, Floor[n / (vv^j)]]}] - Sum[ vv^j vv^k vv^m / (j k m) , {j, 1, Log[vv, n]},
   {k, 1, Log[vv, Floor[n / (vv^j)]]}, {m, 1, Log[vv, Floor[n / (vv^j vv^k)]]}]
```

```
ss[10 000]
```

$$-\frac{7889}{2}+3\left(\frac{26\,411}{12}\,\text{Ps}\left[\frac{10\,000}{2401},1\right]+343\,\text{Ps}\left[\frac{10\,000}{343},1\right]+49\,\text{Ps}\left[\frac{10\,000}{49},1\right]\right)-$$
$$3\left(\frac{2401}{4}\,\text{Ps}\left[\frac{10\,000}{2401},2\right]+\frac{343}{3}\,\text{Ps}\left[\frac{10\,000}{343},2\right]+\frac{49}{2}\,\text{Ps}\left[\frac{10\,000}{49},2\right]+7\,\text{Ps}\left[\frac{10\,000}{7},2\right]\right)+\text{Ps}[10\,000,3]$$

```
P[10 000, 3]
```

$$-\frac{1\,798\,920\,407}{475\,200}$$

```
sso[n_, va_] := PO[n, 3] - 3 Sum[va^j / j PO[n / va^j, 2], {j, 1, Log[va, n]}] +
   3 Sum[ va^j va^k / (j k) PO[n / (va^j va^k), 1], {j, 1, Log[va, n]},
     {k, 1, Log[va, Floor[n / (va^j)]]}] - Sum[ va^j va^k va^m / (j k m) , {j, 1, Log[va, n]},
    {k, 1, Log[va, Floor[n / (va^j)]]}, {m, 1, Log[va, Floor[n / (va^j va^k)]]}]
N[sso[100]]
N[P[100, 3]]
```

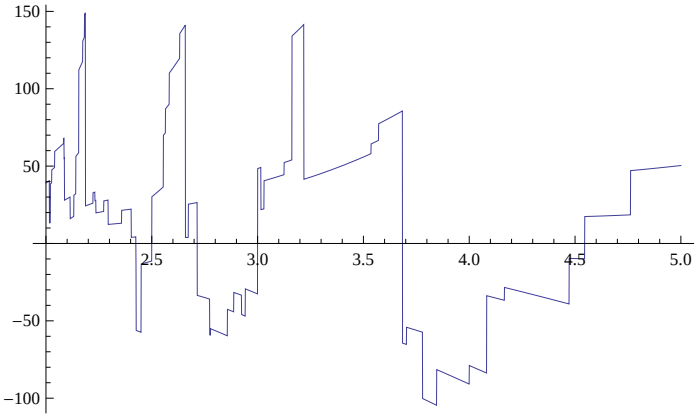
```
33.125
```

```
33.125
```

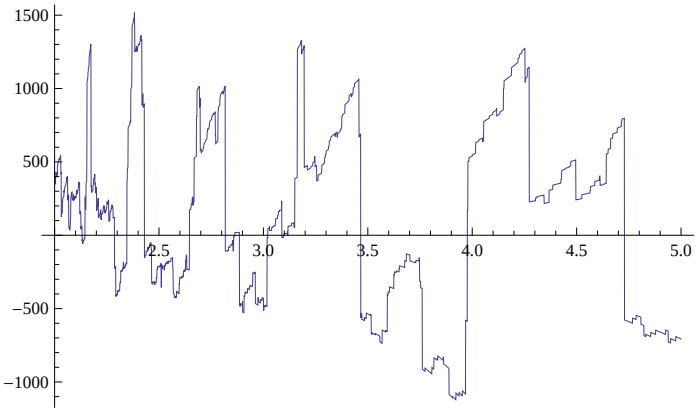
```
N[sso[100, 2]]
```

```
33.125
```

```
Plot[sso[100, k], {k, 2, 5}]
```




```
Plot[sso[1000, k], {k, 2, 5}]
```



```
Plot[sso[411, k], {k, 2, 5}]
```