

Just a handful of ways to express the count of primes function. Not sure why I was collecting these at this point.

## Count of Primes

$$\pi(n) = \sum_{k=1}^{\lfloor \log_2 n \rfloor} \frac{\mu(k)}{k} \lim_{z \rightarrow 0} \frac{1}{z} \left( -1 + \sum_{j=1}^{\lfloor \frac{1}{n^k} \rfloor} \prod_{p^\alpha | j} \frac{z(z+1) \dots (z+\alpha-1)}{\alpha!} \right)$$

$$\pi(n) = \sum_{k=1}^{\lfloor \log_2 n \rfloor} \frac{\mu(k)}{k} \sum_{j=2}^{\lfloor \frac{1}{n^k} \rfloor} 1 - \frac{1}{2} \sum_{j=2}^{\lfloor \frac{1}{n^k} \rfloor} \sum_{k=2}^{\lfloor \frac{1}{n^k} \rfloor} 1 + \frac{1}{3} \sum_{j=2}^{\lfloor \frac{1}{n^k} \rfloor} \sum_{k=2}^{\lfloor \frac{1}{n^k} \rfloor} \sum_{l=2}^{\lfloor \frac{1}{n^k} \rfloor} 1 - \frac{1}{4} \sum_{j=2}^{\lfloor \frac{1}{n^k} \rfloor} \sum_{k=2}^{\lfloor \frac{1}{n^k} \rfloor} \sum_{l=2}^{\lfloor \frac{1}{n^k} \rfloor} \sum_{m=2}^{\lfloor \frac{1}{n^k} \rfloor} 1 + \frac{1}{5} \dots$$

$$\pi(n) = \sum_{k=1}^{\lfloor \log_2 n \rfloor} \frac{\mu(k)}{k} f_1\left(\left\lfloor \frac{1}{n^k} \right\rfloor\right) \text{ where } f_k(n) = \sum_{j=2}^n \frac{1}{k} - f_{k+1}\left(\left\lfloor \frac{n}{j} \right\rfloor\right)$$

$$\pi(n) = \sum_{k=1}^{\lfloor \log_2 n \rfloor} \frac{\mu(k)}{k} f\left(\left\lfloor \frac{1}{n^k} \right\rfloor, \left\lfloor \frac{1}{n^k} \right\rfloor, 1\right) \text{ where } f(n, j, k) = \frac{1}{k} - f\left(\left\lfloor \frac{n}{j} \right\rfloor, \left\lfloor \frac{n}{j} \right\rfloor, k+1\right) + f(n, j-1, k) \text{ and } f(n, 1, k) = 0$$

$$\pi(n) = \sum_{k=1}^{\lfloor \log_2 n \rfloor} \sum_{j=1}^{\lfloor \log_2 n^{\frac{1}{k}} \rfloor} \frac{(-1)^{j-1} \mu(k)}{j k} f_{j,2}\left(n^{\frac{1}{k}}\right) \text{ where } f_{k,a}(n) = \sum_{j=1}^k \binom{k}{j} \sum_{m=a}^{\lfloor \frac{1}{n^k} \rfloor} f_{k-j,m+1}\left(\frac{n}{m^j}\right) \text{ and } f_{0,a}(n) = 1$$