```
bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k!
ff[n_, s_, k_] :=
 ff[n, s, k] = N[Sum[(-1) ^ (j + 1) j^-s ff[Floor[n / j], s, k - 1], {j, 2, n}]];
ff[n_, s_, 0] := UnitStep[n - 1]
FAlt[n_, 0, s_, a_] := UnitStep[n - 1]
FA1[n_, s_] := FA1[n, s] = Sum[N[m^-s (-1) ^ (m + 1)], {m, 1, n}]
FAlt[n_, 1, s_, a_] := FAlt[n, 1, s, a] = FA1[Floor[n], s] - FA1[a, s]
FAlt[n_, k_, s_, a_] := FAlt[n, k, s, a] =
  N[Sum[Binomial[k, j] (m^-s (-1) ^ (m + 1)) ^ j FAlt[Floor[n / (m^j)], k - j, s, m],
    {j, 1, k}, {m, a + 1, Floor[n ^ (1 / k)]}]]
f1a[n_, s_, z_] := Sum[bin[z, k] FAlt[n, k, s, 1], {k, 0, Log[2, n]}]
zerosy[n_, s_] := List @@ NRoots[f1a[n, s, z] == 0, z][[All, 2]]

ff[1 000 000, N[ZetaZero[4]], 4]

-0.998844 - 1.59766 i

N[FAlt[1 000 000, 4, N[ZetaZero[4]], 1]]

-0.998844 - 1.59766 i

zeros[10 000 000, N[ZetaZero[1]]] // TableForm

-2.94428 + 18.6403 i
0.010228 - 0.0117525 i
0.999973 - 0.0000256921 i
1.0591 - 4665.17 i
1.0654 + 6.32766 i
1.53388 - 5.96189 i
2.00673 + 0.0150723 i
2.32923 - 1.61325 i
2.39015 - 0.214322 i
2.61838 + 1.01077 i
3.49043 - 2.63485 i
3.68162 + 2.5122 i
6.74177 - 0.0210039 i
10.637 - 15.9654 i
11.4814 + 28.2677 i
16.7079 - 0.11017 i
20.1259 + 7.43797 i
26.7139 - 9.96327 i
47.7687 + 50.9779 i
55.9512 - 112.028 i
57.0493 - 34.2386 i
117.826 + 302.103 i
220.895 - 101.701 i

Product[ 1 - 1 / j, {j, zeros[10 000 000, N[ZetaZero[1]]]}]

9.3841 × 10^-6 + 0.000157835 i

N[Pi^2 / 12]

0.822467

f1a[100 000, -2, z]

0.
```

FAlt[1000, 1, 2, 1]

-0.177533

ff[1000, 2, 1]

-0.177533

zeros[10 000 000, 2] // TableForm
Product[1 – 1 / j, {j, zeros[10 000 000, 2]}]

$-25.0413 - 122.576\,\mathtt{i}$
$-25.0413 + 122.576\,\mathtt{i}$
$-16.1421 - 5.78035\,\mathtt{i}$
$-16.1421 + 5.78035\,\mathtt{i}$
$-9.93523 - 14.4648\,\mathtt{i}$
$-9.93523 + 14.4648\,\mathtt{i}$
$-1.84852 - 18.1465\,\mathtt{i}$
$-1.84852 + 18.1465\,\mathtt{i}$
$6.14385 - 17.5536\,\mathtt{i}$
$6.14385 + 17.5536\,\mathtt{i}$
$13.4978 - 13.5529\,\mathtt{i}$
$13.4978 + 13.5529\,\mathtt{i}$
$17.6717 - 24.4519\,\mathtt{i}$
$17.6717 + 24.4519\,\mathtt{i}$
$18.979 - 7.16057\,\mathtt{i}$
$18.979 + 7.16057\,\mathtt{i}$
20.9263
$24.9227 - 8.541\,\mathtt{i}$
$24.9227 + 8.541\,\mathtt{i}$
$66.6905 - 49.8393\,\mathtt{i}$
$66.6905 + 49.8393\,\mathtt{i}$
217.321
3721.25

$0.822467 + 8.65515 \times 10^{-18}\,\mathtt{i}$

```
zeros[10 000 000, 1] // TableForm
Product[ 1 - 1 / j, {j, zeros[10 000 000, 1]}]
```

-11.3964 - 131.995 ⅈ
-11.3964 + 131.995 ⅈ
-1.49532 - 1.34739 ⅈ
-1.49532 + 1.34739 ⅈ
0.112435 - 11.1991 ⅈ
0.112435 + 11.1991 ⅈ
0.378816 - 3.52632 ⅈ
0.378816 + 3.52632 ⅈ
2.78102 - 4.05344 ⅈ
2.78102 + 4.05344 ⅈ
4.6394 - 2.89851 ⅈ
4.6394 + 2.89851 ⅈ
6.2508 - 0.0889317 ⅈ
6.2508 + 0.0889317 ⅈ
17.0579
17.1421 - 23.8002 ⅈ
17.1421 + 23.8002 ⅈ
25.1407 - 8.23549 ⅈ
25.1407 + 8.23549 ⅈ
65.2672 - 49.4014 ⅈ
65.2672 + 49.4014 ⅈ
226.288
4392.01

$0.693147 - 3.74406 \times 10^{-17}$ ⅈ

```
bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k !
DAlt[n_, 0, s_, a_] := UnitStep[n - 1]
DA1[n_, s_] := DA1[n, s] = Sum[N[m^-s], {m, 1, n}]
DAlt[n_, 1, s_, a_] := DAlt[n, 1, s, a] = DA1[Floor[n], s] - DA1[a, s]
DAlt[n_, k_, s_, a_] :=
 DAlt[n, k, s, a] = N[Sum[Binomial[k, j] (m^-s) ^j DAlt[Floor[n / (m^j)], k - j, s, m],
    {j, 1, k}, {m, a + 1, Floor[n^(1 / k)]}]]
d1a[n_, s_, z_] := Sum[bin[z, k] DAlt[n, k, s, 1], {k, 0, Log[2, n]}]
dzeros[n_, s_] := List @@ NRoots[d1a[n, s, z] ⩵ 0, z][[All, 2]]

dzeros[10 000 000, 2] // TableForm
Product[ 1 - 1 / j, {j, dzeros[10 000 000, 2]}]
```

$Aborted

$Aborted

```
zeros[10 000 000, .1 I + N[ZetaZero[2]]] // TableForm
zeros[10 000 000, N[ZetaZero[2]]] // TableForm
zeros[10 000 000, -.1 I + N[ZetaZero[2]]] // TableForm
```

$-4204.28 - 2915.36\,i$
$-11.0475 + 286.348\,i$
$-5.32712 + 1.48361\,i$
$-3.40911 + 33.033\,i$
$-0.0107495 - 0.000741138\,i$
$0.988473 + 0.0232957\,i$
$1.15667 - 6.84743\,i$
$2.02357 + 0.0186354\,i$
$2.59232 + 0.150767\,i$
$2.86791 - 1.04211\,i$
$3.17251 + 1.58288\,i$
$3.80794 + 3.28481\,i$
$3.93298 - 2.70173\,i$
$6.73833 - 0.031841\,i$
$7.66962 + 19.7787\,i$
$7.87704 - 72.2511\,i$
$10.7394 - 17.0016\,i$
$15.227 - 0.834028\,i$
$15.4542 + 7.84042\,i$
$25.353 - 20.2619\,i$
$29.9927 + 52.82\,i$
$59.6086 - 58.059\,i$
$220.118 - 210.04\,i$

$-4163.71 - 2949.26\,i$
$-9.69325 + 288.018\,i$
$-5.42227 + 1.48108\,i$
$-3.22951 + 32.9884\,i$
$-0.000465589 + 0.0106046\,i$
$0.999991 + 0.0000153019\,i$
$1.20061 - 6.83745\,i$
$2.00285 - 0.00504836\,i$
$2.59246 + 0.16043\,i$
$2.86219 - 1.02459\,i$
$3.17864 + 1.58091\,i$
$3.81472 + 3.28919\,i$
$3.93135 - 2.69123\,i$
$6.73839 - 0.0316826\,i$
$7.67628 + 19.7609\,i$
$8.40552 - 72.5916\,i$
$10.8025 - 17.0402\,i$
$15.2951 - 0.811878\,i$
$15.5569 + 7.8492\,i$
$25.3491 - 19.8491\,i$
$30.3968 + 52.894\,i$
$59.2606 - 57.5567\,i$
$219.087 - 207.599\,i$

-4122.19 - 2982.91 i
-8.30471 + 289.647 i
-5.52228 + 1.47999 i
-3.04785 + 32.9464 i
0.0105599 - 0.000197582 i
0.985483 + 0.0211906 i
1.24023 - 6.82881 i
1.99095 - 0.030665 i
2.60645 + 0.167777 i
2.86383 - 1.02044 i
3.18607 + 1.58045 i
3.82191 + 3.29378 i
3.92989 - 2.68576 i
6.73845 - 0.0315245 i
7.68215 + 19.7399 i
8.93876 - 72.9356 i
10.8527 - 17.0816 i
15.3584 - 0.79023 i
15.6566 + 7.85717 i
25.3637 - 19.4611 i
30.7939 + 52.9607 i
58.9415 - 57.0573 i
218.137 - 205.16 i

zeros[10 000 000, .1 + N[ZetaZero[2]]] // TableForm
zeros[10 000 000, N[ZetaZero[2]]] // TableForm
zeros[10 000 000, -.1 + N[ZetaZero[2]]] // TableForm

-4130.41 - 2908.34 i
-11.3599 + 289.41 i
-5.418 + 1.3829 i
-3.1872 + 33.1676 i
-0.00489119 + 0.0519684 i
1.05701 - 0.114192 i
1.19395 - 6.79633 i
1.90741 + 0.0770104 i
2.62862 + 0.150263 i
2.86818 - 1.02622 i
3.18105 + 1.58725 i
3.81005 + 3.29614 i
3.92502 - 2.69142 i
6.73823 - 0.0316248 i
7.69602 + 19.7687 i
8.74534 - 72.0589 i
10.8501 - 16.9829 i
15.2755 - 0.746169 i
15.5501 + 7.95078 i
24.9396 - 19.8295 i
30.3301 + 53.2981 i
58.7444 - 57.8882 i
216.606 - 208.588 i

```
- 4163.71 - 2949.26 i
- 9.69325 + 288.018 i
- 5.42227 + 1.48108 i
- 3.22951 + 32.9884 i
- 0.000465589 + 0.0106046 i
0.999991 + 0.0000153019 i
1.20061 - 6.83745 i
2.00285 - 0.00504836 i
2.59246 + 0.16043 i
2.86219 - 1.02459 i
3.17864 + 1.58091 i
3.81472 + 3.28919 i
3.93135 - 2.69123 i
6.73839 - 0.0316826 i
7.67628 + 19.7609 i
8.40552 - 72.5916 i
10.8025 - 17.0402 i
15.2951 - 0.811878 i
15.5569 + 7.8492 i
25.3491 - 19.8491 i
30.3968 + 52.894 i
59.2606 - 57.5567 i
219.087 - 207.599 i

- 4197.95 - 2990.44 i
- 8.0608 + 286.667 i
- 5.4217 + 1.57781 i
- 3.27389 + 32.8065 i
- 0.000116287 + 0.00210849 i
0.996938 + 0.00494952 i
1.21147 - 6.87775 i
1.99649 - 0.00103789 i
2.59247 + 0.1565 i
2.86586 - 1.02219 i
3.17634 + 1.57418 i
3.81896 + 3.28207 i
3.93754 - 2.68764 i
6.73855 - 0.0317401 i
7.65732 + 19.7562 i
8.06093 - 73.1207 i
10.7681 - 17.095 i
15.3194 - 0.877081 i
15.5668 + 7.74844 i
25.7402 - 19.8435 i
30.4704 + 52.497 i
59.7478 - 57.2222 i
221.489 - 206.608 i
```

```
zeros[10 000 000, .1 I + N[ZetaZero[3]]] // TableForm
zeros[10 000 000, N[ZetaZero[3]]] // TableForm
zeros[10 000 000, -.1 I + N[ZetaZero[3]]] // TableForm
```

```
- 4990.67 - 1262.48 i
- 19.4465 + 198.668 i
- 9.84319 + 36.7571 i
- 9.70263 - 59.5131 i
- 3.61981 + 2.5083 i
0.00340154 - 0.00827635 i
0.822055 - 9.92686 i
0.982429 + 0.00937009 i
2.02427 + 0.00084531 i
2.62821 + 0.139758 i
2.99487 - 0.988604 i
3.18778 + 1.56269 i
3.85055 + 3.21737 i
4.20601 - 2.65992 i
6.73574 - 0.0382688 i
9.26501 + 16.1002 i
10.3973 - 17.0902 i
14.4101 + 2.61136 i
15.4866 + 35.1224 i
18.5597 - 5.43954 i
39.7484 - 26.2006 i
83.6147 - 61.5231 i
308.61 - 271.491 i
```

```
- 4990.47 - 1312.75 i
- 20.6692 + 200.738 i
- 9.63863 + 36.6222 i
- 9.30708 - 59.8327 i
- 3.62445 + 2.44802 i
- 0.00857301 - 0.00315621 i
0.764138 - 9.75655 i
```
$0.999989 + 9.86576 \times 10^{-6}$ i
```
2.00288 - 0.00348785 i
2.62961 + 0.14334 i
2.98768 - 0.993729 i
3.18022 + 1.55855 i
3.85829 + 3.21508 i
4.19012 - 2.66475 i
6.73581 - 0.0381014 i
9.02819 + 16.0786 i
10.444 - 17.138 i
14.3428 + 2.60299 i
15.2922 + 35.7922 i
18.4471 - 5.48761 i
39.4686 - 26.3154 i
83.1344 - 61.7653 i
305.905 - 271.159 i
```

```
- 4989.35 - 1362.52 i
- 21.8199 + 202.868 i
- 9.43726 + 36.4813 i
- 8.91164 - 60.1528 i
- 3.63147 + 2.3923 i
- 0.00296793 + 0.00916663 i
0.715787 - 9.59469 i
0.978474 + 0.00660488 i
1.98284 - 0.0174171 i
2.6347 + 0.134251 i
2.97841 - 0.99883 i
3.18325 + 1.54467 i
3.87622 + 3.21711 i
4.17493 - 2.66903 i
6.73588 - 0.0379338 i
8.76831 + 16.075 i
10.4855 - 17.1846 i
14.2699 + 2.59276 i
15.1319 + 36.5044 i
18.3303 - 5.53572 i
39.1813 - 26.428 i
82.6431 - 62.0021 i
303.198 - 270.765 i
```

```
zeros[10 000 000, N[ZetaZero[31]]] // TableForm
zeros[10 000 000, N[ZetaZero[32]]] // TableForm
zeros[10 000 000, N[ZetaZero[33]]] // TableForm
```

```
- 20.0507 + 265.966 i
- 6.45622 - 21.9698 i
- 1.05536 + 9.2149 i
- 0.241732 + 3.86816 i
- 0.215938 - 2.17054 i
0.0068937 + 0.0116644 i
1.00001 + 0.0000110904 i
1.99736 - 0.00218885 i
2.83533 + 0.393639 i
3.04595 - 0.670442 i
3.90502 + 2.11698 i
4.2368 - 2.87486 i
7.66246 + 2.1129 i
8.25611 + 0.503007 i
8.94342 - 5.68011 i
12.736 - 27.7639 i
25.8096 - 3.84079 i
34.1647 + 6.36382 i
61.4713 + 44.1556 i
79.3177 - 27.1908 i
157.233 - 132.262 i
209.517 + 39.6198 i
4844.16 + 2159.97 i
```

$-44.6222 + 251.445\,\mathrm{i}$

$-6.64528 - 21.7207\,\mathrm{i}$

$-1.0271 + 9.4543\,\mathrm{i}$

$-0.857409 - 2.05595\,\mathrm{i}$

$-0.561243 + 3.97324\,\mathrm{i}$

$-0.00960582 - 0.00779953\,\mathrm{i}$

$1.00001 + 9.50978 \times 10^{-6}\,\mathrm{i}$

$1.99818 - 0.00161087\,\mathrm{i}$

$2.92633 + 0.355582\,\mathrm{i}$

$3.17361 - 0.602857\,\mathrm{i}$

$3.85814 + 2.20642\,\mathrm{i}$

$4.41698 - 3.18613\,\mathrm{i}$

$7.1915 - 0.415166\,\mathrm{i}$

$8.60989 + 3.06306\,\mathrm{i}$

$10.3952 - 5.92528\,\mathrm{i}$

$16.5615 - 31.4747\,\mathrm{i}$

$24.1893 - 4.60009\,\mathrm{i}$

$33.1276 + 7.2716\,\mathrm{i}$

$59.1746 + 45.1887\,\mathrm{i}$

$81.4894 - 33.4167\,\mathrm{i}$

$110.989 - 117.393\,\mathrm{i}$

$212.775 + 27.6752\,\mathrm{i}$

$4889.47 + 1093.08\,\mathrm{i}$

$-60.6868 + 230.026\,\mathrm{i}$

$-6.21395 - 21.1772\,\mathrm{i}$

$-1.66418 - 2.33564\,\mathrm{i}$

$-0.986386 + 9.67848\,\mathrm{i}$

$-0.946073 + 4.42506\,\mathrm{i}$

$0.00718708 + 0.00208407\,\mathrm{i}$

$1.00001 + 5.84232 \times 10^{-6}\,\mathrm{i}$

$1.99894 - 0.000925843\,\mathrm{i}$

$3.02281 + 0.280098\,\mathrm{i}$

$3.29783 - 0.459241\,\mathrm{i}$

$4.05808 + 2.24967\,\mathrm{i}$

$4.84149 - 3.57697\,\mathrm{i}$

$6.90884 - 0.468197\,\mathrm{i}$

$9.09868 + 3.37316\,\mathrm{i}$

$10.9954 - 5.81689\,\mathrm{i}$

$20.596 - 33.8174\,\mathrm{i}$

$23.6714 - 4.74623\,\mathrm{i}$

$32.2917 + 7.84324\,\mathrm{i}$

$57.1252 + 45.8016\,\mathrm{i}$

$72.3799 - 120.645\,\mathrm{i}$

$88.8028 - 32.8224\,\mathrm{i}$

$214.583 + 13.426\,\mathrm{i}$

$4803.38 - 79.3219\,\mathrm{i}$

```
zeros[10 000 000, 2 - .1 I] // TableForm
zeros[10 000 000, 2] // TableForm
zeros[10 000 000, 2 + .1 I] // TableForm
```

```
– 26.0477 + 124.08 i
– 24.0095 – 121.092 i
– 16.7309 – 3.21318 i
– 15.2691 + 8.25589 i
– 11.5401 – 12.6865 i
– 8.168 + 16.0419 i
– 3.81473 – 17.3953 i
0.21863 + 18.6458 i
3.98502 – 17.7962 i
8.37941 + 17.0459 i
11.4723 – 14.7864 i
15.3999 + 12.0628 i
17.4597 – 9.1601 i
17.6016 – 24.5014 i
17.7478 + 24.394 i
20.1121 + 4.90175 i
20.7577 – 1.94088 i
24.9442 + 8.63862 i
24.9496 – 8.55715 i
66.662 – 49.9772 i
66.7206 + 49.7039 i
217.321 + 0.9084 i
3721.08 + 65.3744 i

– 25.0413 – 122.576 i
– 25.0413 + 122.576 i
– 16.1421 – 5.78035 i
– 16.1421 + 5.78035 i
– 9.93523 – 14.4648 i
– 9.93523 + 14.4648 i
– 1.84852 – 18.1465 i
– 1.84852 + 18.1465 i
6.14385 – 17.5536 i
6.14385 + 17.5536 i
13.4978 – 13.5529 i
13.4978 + 13.5529 i
17.6717 – 24.4519 i
17.6717 + 24.4519 i
18.979 – 7.16057 i
18.979 + 7.16057 i
20.9263
24.9227 – 8.541 i
24.9227 + 8.541 i
66.6905 – 49.8393 i
66.6905 + 49.8393 i
217.321
3721.25
```

```
- 26.0477 - 124.08 i
- 24.0095 + 121.092 i
- 16.7309 + 3.21318 i
- 15.2691 - 8.25589 i
- 11.5401 + 12.6865 i
- 8.168 - 16.0419 i
- 3.81473 + 17.3953 i
0.21863 - 18.6458 i
3.98502 + 17.7962 i
8.37941 - 17.0459 i
11.4723 + 14.7864 i
15.3999 - 12.0628 i
17.4597 + 9.1601 i
17.6016 + 24.5014 i
17.7478 - 24.394 i
20.1121 - 4.90175 i
20.7577 + 1.94088 i
24.9442 - 8.63862 i
24.9496 + 8.55715 i
66.662 + 49.9772 i
66.7206 - 49.7039 i
217.321 - 0.9084 i
3721.08 - 65.3744 i
```
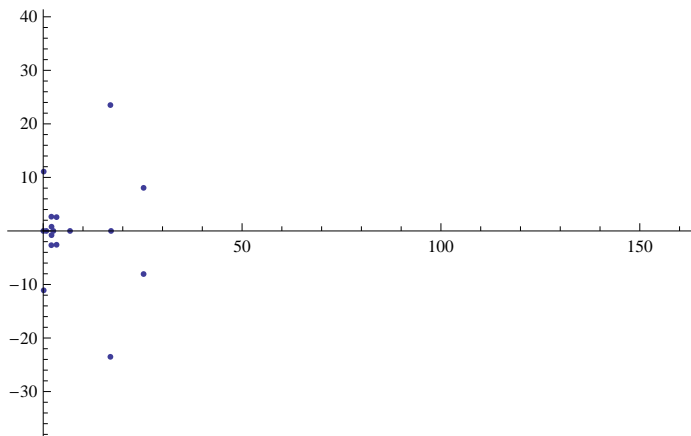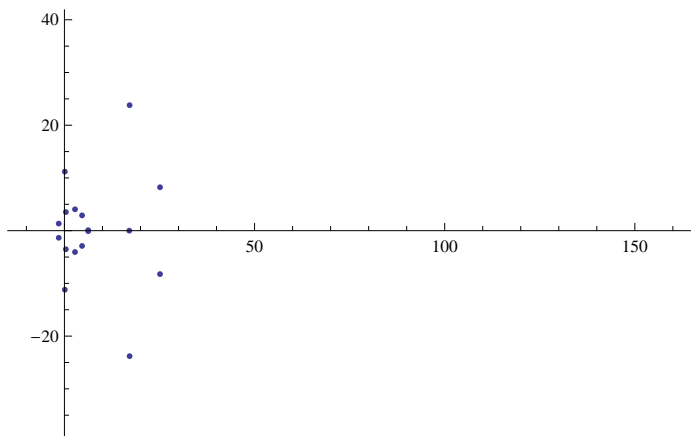
zeros[10 000 000, .5] // TableForm

```
- 5.5649 - 136.275 i
- 5.5649 + 136.275 i
0.0334422
0.123065 - 11.0902 i
0.123065 + 11.0902 i
0.78539
2.04628 - 2.66277 i
2.04628 + 2.66277 i
2.08928 - 0.787744 i
2.08928 + 0.787744 i
2.54582
3.35264 - 2.58016 i
3.35264 + 2.58016 i
6.74435
16.8925 - 23.5118 i
16.8925 + 23.5118 i
17.0552
25.2386 - 8.04952 i
25.2386 + 8.04952 i
64.5474 - 49.0656 i
64.5474 + 49.0656 i
230.619
4741.16
```
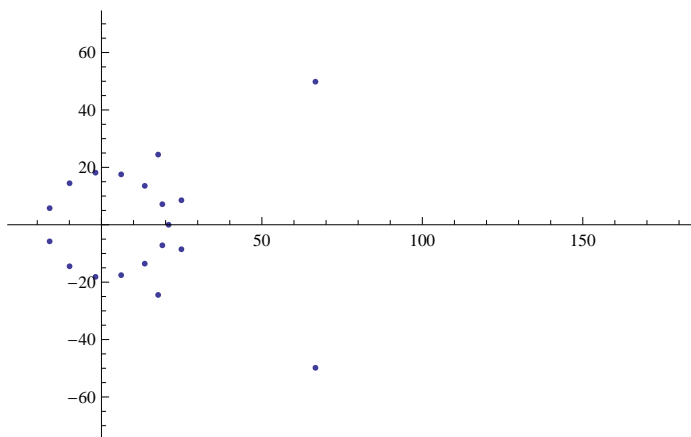
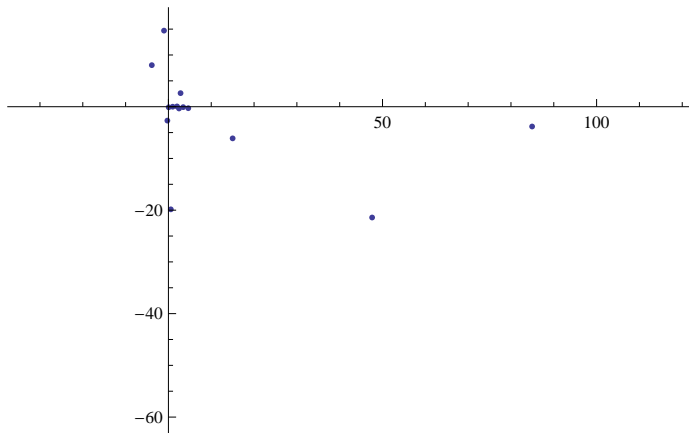**ListPlot[Table[{Re[n], Im[n]}, {n, zeros[10 000 000, .5]}]]**

ListPlot[Table[{Re[n], Im[n]}, {n, zeros[10 000 000, 1]}]]

ListPlot[Table[{Re[n], Im[n]}, {n, zeros[10 000 000, 2]}]]

```
ListPlot[Table[{Re[n], Im[n]}, {n, zeros[100 000, N[ZetaZero[10]]]}]]
```



```
zeros[10 000 000, N[ZetaZero[100]]] // TableForm
```

$-4615.92 - 1588.86\,i$
$-7.84663 - 8.87265\,i$
$-6.73516 + 12.3041\,i$
$-3.19784 + 170.011\,i$
$-2.00683 - 2.12074\,i$
$0.00444354 + 0.000169567\,i$
$0.999997 + 5.19924 \times 10^{-6}\,i$
$2.00133 - 0.000566946\,i$
$2.43427 - 1.15814\,i$
$2.83351 - 0.0603766\,i$
$3.80246 + 0.433095\,i$
$4.55094 - 1.4197\,i$
$6.36689 - 138.999\,i$
$7.29178 + 1.58652\,i$
$8.44806 + 6.44521\,i$
$9.55876 - 15.6586\,i$
$9.84331 - 4.76613\,i$
$17.9574 + 39.9512\,i$
$19.9389 + 18.032\,i$
$33.5793 - 10.7918\,i$
$40.9349 - 31.9254\,i$
$108.099 - 7.60183\,i$
$280.984 + 71.0008\,i$

zeros[100 000 000, N[ZetaZero[100]]] // TableForm

```
-5905.92 - 2033.42 i
-215.271 + 430.476 i
-8.96512 + 11.7435 i
-8.25917 - 18.581 i
-2.92865 - 1.17318 i
-0.000391594 + 0.00173884 i
1. - 3.50065 × 10⁻⁷ i
1.77344 - 1.65967 i
1.99897 - 0.000385591 i
2.96919 + 0.141964 i
3.20758 - 0.593734 i
4.35534 + 0.425791 i
5.46722 - 0.37634 i
6.37063 - 1.97553 i
8.57171 + 8.48716 i
9.15015 + 1.90223 i
11.2755 - 17.7177 i
13.9419 - 7.18019 i
20.1989 + 18.1931 i
23.9654 + 38.2192 i
39.9825 + 104.366 i
45.1272 - 17.731 i
49.0879 - 32.5719 i
49.4894 - 141.701 i
100.886 - 282.796 i
131.729 - 6.05992 i
```

zeros[1 000 000 000, N[ZetaZero[100]]] // TableForm

```
$Aborted
```

zerosy[1000, N[ZetaZero[1]]] // TableForm

```
-2.47962 + 1.04229 i
0.74741 - 0.0262707 i
0.890334 + 0.0467483 i
1.96769 - 0.745176 i
2.15607 + 0.923377 i
4.88614 - 0.650277 i
11.7714 - 1.82294 i
17.0848 + 15.4822 i
90.5792 - 53.3749 i
```