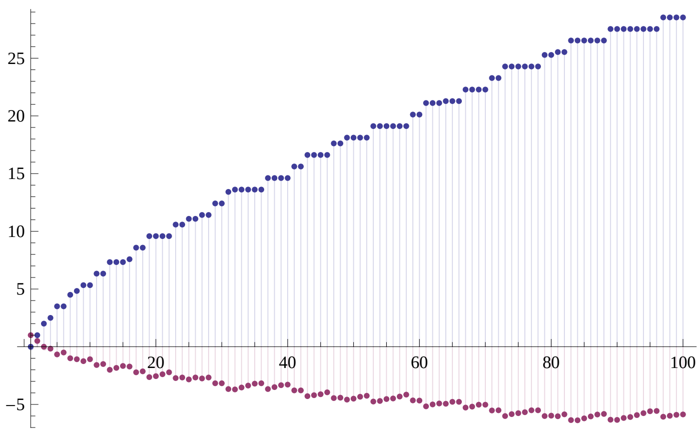
```
fp[n_, k_, y_] :=
 If[y == 1, zt[n, k], Sum[(-1)^j Binomial[k, j] fp[n / y^j, k - j, y - 1], {j, 0, k}]]
fpa[n_, k_, y_] := If[y == 1, zt[m / n, k],
  Sum[(-1)^j Binomial[k, j] fpa[n y^j, k - j, y - 1], {j, 0, k}]]
```

```
fpa[1, 4, 3]
```

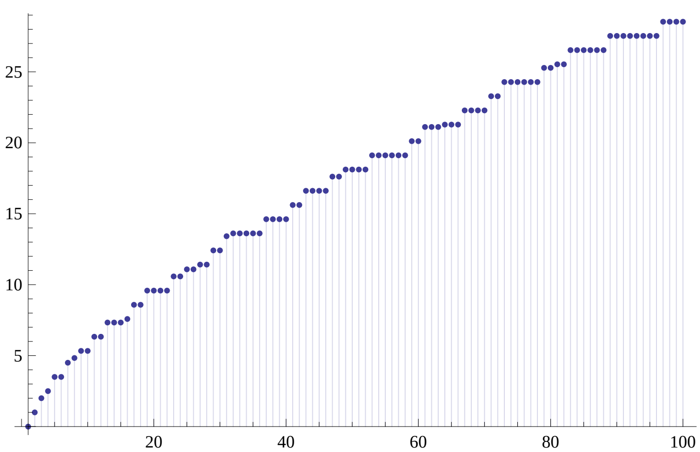
$$\mathrm{zt}\left[\frac{m}{81}, 0\right] - 4\left(-\mathrm{zt}\left[\frac{m}{54}, 0\right] + \mathrm{zt}\left[\frac{m}{27}, 1\right]\right) + \mathrm{zt}\left[\frac{m}{16}, 0\right] + 6\left(\mathrm{zt}\left[\frac{m}{36}, 0\right] - 2\,\mathrm{zt}\left[\frac{m}{18}, 1\right] + \mathrm{zt}\left[\frac{m}{9}, 2\right]\right) - 4\,\mathrm{zt}\left[\frac{m}{8}, 1\right] + 6\,\mathrm{zt}\left[\frac{m}{4}, 2\right] - 4\left(-\mathrm{zt}\left[\frac{m}{24}, 0\right] + 3\,\mathrm{zt}\left[\frac{m}{12}, 1\right] - 3\,\mathrm{zt}\left[\frac{m}{6}, 2\right] + \mathrm{zt}\left[\frac{m}{3}, 3\right]\right) - 4\,\mathrm{zt}\left[\frac{m}{2}, 3\right] + \mathrm{zt}[m, 4]$$

```
Binomial[k, 1]

k

bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, z_] := dz[n, z] = Product[(-1)^p[[2]] bin[-z, p[[2]]], {p, FI[n]}]
dsz[n_, s_, z_] := n^-s dz[n, z]
DZ[n_, s_, z_] := Sum[ j^-s dz[j, z], {j, 1, n}]

m2[n_, t_, s_, z_] :=
 DZ[ t, s, z] - Sum[ dsz[j, s, z] DZ[n / (j k), s, z], {j, 1, t}, {k, Floor[t / j] + 1, n / j}]

m2[200, 10, 0, -1]

-8

DZ[200, 0, -1]

-8
```

```
Clear[Ds]
binomial[z_, k_] := binomial[z, k] = Product[z - j, {j, 0, k - 1}] / k!
Ds[n_, 0, s_, a_] := UnitStep[n - 1]
Ds[n_, 1, s_, a_] := Ds[n, 1, s, a] = HarmonicNumber[Floor[n], s] - HarmonicNumber[a, s]
Ds[n_, 2, s_, a_] := Ds[n, 2, s, a] =
  Sum[(m^(-2 s)) + 2 (m^-s) (Ds[Floor[n / m], 1, s, m]), {m, a + 1, Floor[n^(1 / 2)]}]
Ds[n_, k_, s_, a_] := Ds[n, k, s, a] =
  Sum[(m^(-s k)) + k (m^(-s (k - 1))) Ds[Floor[n / (m^(k - 1))], 1, s, m] +
    Sum[binomial[k, j] (m^-s)^j Ds[Floor[n / (m^j)], k - j, s, m], {j, 1, k - 2}],
   {m, a + 1, Floor[n^(1 / k)]}]

Dnsz[n_, s_, 1] := Expand@Sum[binomial[1, k] Ds[n, k, s, 1], {k, 0, 1}]
Dnsz[n_, s_, z_] := Expand@Sum[binomial[z, k] Ds[n, k, s, 1], {k, 0, Log2@n}]

lgm[n_] := Sum[ BernoulliB[ k] / k! D[Dnsz[n, 0, z], {z, k}] /. z → 0, {k, 0, Log2@n}]

DiscretePlot[ {(D[Dnsz[n, 0, z], z] /. z → 0), lgm[n]}, {n, 1, 100}]
```



```
DiscretePlot[ Sum[ lgm[n / j], {j, 2, n}], {n, 1, 100}]
```



Series[Log[x] / (x - 1), {x, 1, 20}]

$$1-\frac{x-1}{2}+\frac{1}{3}(x-1)^2-\frac{1}{4}(x-1)^3+\frac{1}{5}(x-1)^4-\frac{1}{6}(x-1)^5+\frac{1}{7}(x-1)^6-\frac{1}{8}(x-1)^7+\frac{1}{9}(x-1)^8-\frac{1}{10}(x-1)^9+\frac{1}{11}(x-1)^{10}-\frac{1}{12}(x-1)^{11}+\frac{1}{13}(x-1)^{12}-\frac{1}{14}(x-1)^{13}+\frac{1}{15}(x-1)^{14}-\frac{1}{16}(x-1)^{15}+\frac{1}{17}(x-1)^{16}-\frac{1}{18}(x-1)^{17}+\frac{1}{19}(x-1)^{18}-\frac{1}{20}(x-1)^{19}+\frac{1}{21}(x-1)^{20}+O[x-1]^{21}$$

Series[1 / x, {x, 1, 20}]

$$1-(x-1)+(x-1)^2-(x-1)^3+(x-1)^4-(x-1)^5+(x-1)^6-(x-1)^7+(x-1)^8-(x-1)^9+(x-1)^{10}-(x-1)^{11}+(x-1)^{12}-(x-1)^{13}+(x-1)^{14}-(x-1)^{15}+(x-1)^{16}-(x-1)^{17}+(x-1)^{18}-(x-1)^{19}+(x-1)^{20}+O[x-1]^{21}$$

Series[(x - 1) / Log[x], {x, 1, 20}]

$$1+\frac{x-1}{2}-\frac{1}{12}(x-1)^2+\frac{1}{24}(x-1)^3-\frac{19}{720}(x-1)^4+\frac{3}{160}(x-1)^5-\frac{863\,(x-1)^6}{60\,480}+\frac{275\,(x-1)^7}{24\,192}-\frac{33\,953\,(x-1)^8}{3\,628\,800}+\frac{8183\,(x-1)^9}{1\,036\,800}-\frac{3\,250\,433\,(x-1)^{10}}{479\,001\,600}+\frac{4671\,(x-1)^{11}}{788\,480}-\frac{13\,695\,779\,093\,(x-1)^{12}}{2\,615\,348\,736\,000}+\frac{2\,224\,234\,463\,(x-1)^{13}}{475\,517\,952\,000}-\frac{132\,282\,840\,127\,(x-1)^{14}}{31\,384\,184\,832\,000}+\frac{2\,639\,651\,053\,(x-1)^{15}}{689\,762\,304\,000}-\frac{111\,956\,703\,448\,001\,(x-1)^{16}}{32\,011\,868\,528\,640\,000}+\frac{50\,188\,465\,(x-1)^{17}}{15\,613\,165\,568}-\frac{2\,334\,028\,946\,344\,463\,(x-1)^{18}}{786\,014\,494\,949\,376\,000}+\frac{301\,124\,035\,185\,049\,(x-1)^{19}}{109\,285\,437\,800\,448\,000}-\frac{12\,365\,722\,323\,469\,980\,029\,(x-1)^{20}}{4\,817\,145\,976\,189\,747\,200\,000}+O[x-1]^{21}$$

Series[(x - 1) / Log[x], {x, 1, 20}]

$$1+\frac{x-1}{2}-\frac{1}{12}(x-1)^2+\frac{1}{24}(x-1)^3-\frac{19}{720}(x-1)^4+\frac{3}{160}(x-1)^5-\frac{863\,(x-1)^6}{60\,480}+\frac{275\,(x-1)^7}{24\,192}-\frac{33\,953\,(x-1)^8}{3\,628\,800}+\frac{8183\,(x-1)^9}{1\,036\,800}-\frac{3\,250\,433\,(x-1)^{10}}{479\,001\,600}+\frac{4671\,(x-1)^{11}}{788\,480}-\frac{13\,695\,779\,093\,(x-1)^{12}}{2\,615\,348\,736\,000}+\frac{2\,224\,234\,463\,(x-1)^{13}}{475\,517\,952\,000}-\frac{132\,282\,840\,127\,(x-1)^{14}}{31\,384\,184\,832\,000}+\frac{2\,639\,651\,053\,(x-1)^{15}}{689\,762\,304\,000}-\frac{111\,956\,703\,448\,001\,(x-1)^{16}}{32\,011\,868\,528\,640\,000}+\frac{50\,188\,465\,(x-1)^{17}}{15\,613\,165\,568}-\frac{2\,334\,028\,946\,344\,463\,(x-1)^{18}}{786\,014\,494\,949\,376\,000}+\frac{301\,124\,035\,185\,049\,(x-1)^{19}}{109\,285\,437\,800\,448\,000}-\frac{12\,365\,722\,323\,469\,980\,029\,(x-1)^{20}}{4\,817\,145\,976\,189\,747\,200\,000}+O[x-1]^{21}$$

Series[Log[x] / (x - 1) ^2, {x, 1, 20}]

$$\frac{1}{x-1}-\frac{1}{2}+\frac{x-1}{3}-\frac{1}{4}(x-1)^2+\frac{1}{5}(x-1)^3-\frac{1}{6}(x-1)^4+\frac{1}{7}(x-1)^5-\frac{1}{8}(x-1)^6+\frac{1}{9}(x-1)^7-\frac{1}{10}(x-1)^8+\frac{1}{11}(x-1)^9-\frac{1}{12}(x-1)^{10}+\frac{1}{13}(x-1)^{11}-\frac{1}{14}(x-1)^{12}+\frac{1}{15}(x-1)^{13}-\frac{1}{16}(x-1)^{14}+\frac{1}{17}(x-1)^{15}-\frac{1}{18}(x-1)^{16}+\frac{1}{19}(x-1)^{17}-\frac{1}{20}(x-1)^{18}+\frac{1}{21}(x-1)^{19}-\frac{1}{22}(x-1)^{20}+O[x-1]^{21}$$

```
Series[1 / (x - 1), {x, 2, 20}]
```

$$1-(x-2)+(x-2)^2-(x-2)^3+(x-2)^4-(x-2)^5+(x-2)^6-(x-2)^7+(x-2)^8-(x-2)^9+(x-2)^{10}-(x-2)^{11}+(x-2)^{12}-(x-2)^{13}+(x-2)^{14}-(x-2)^{15}+(x-2)^{16}-(x-2)^{17}+(x-2)^{18}-(x-2)^{19}+(x-2)^{20}+O[x-2]^{21}$$

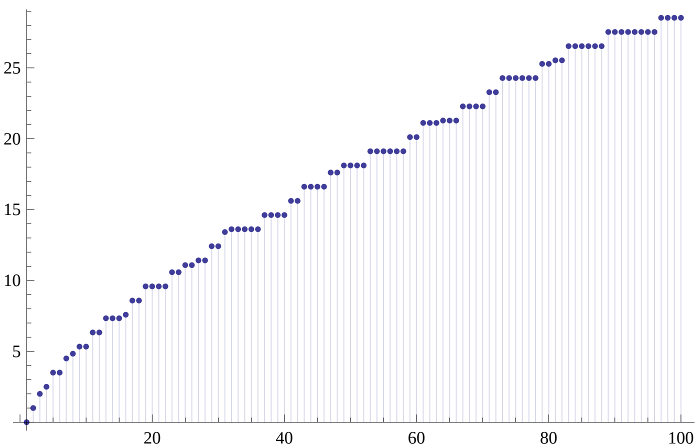
```
Clear[dm]
dm[n_, k_] := dm[n, k] = Sum[ dm[Floor[n / j], k - 1], {j, 2, n}] - dm[n, k - 1]
dm[n_, 0] := UnitStep[n - 1]
```

```
Table[ dm[100, k], {k, 0, 20}]
```

{1, 98, 86, -229, 191, 35, -367, 671, -791, 572, 124, -1409, 3367, -6059, 9535, -13 853, 19 105, -25 450, 33 154, -42 637, 54 527}

```
Clear[d2]
d2[n_, k_] := d2[n, k] = Sum[ d2[Floor[n / j], k - 1], {j, 2, n}]
d2[n_, 0] := UnitStep[n - 1]
pk[n_, j_] := Sum[ (-1) ^ (k + j + 1) / (k + j) d2[n, k], {k, 0, Log2@n}]
pks[n_, j_] := Sum[ (-1) ^ (k + 1) / (k) d2[n, k + j], {k, 1, Log2@n}]
```

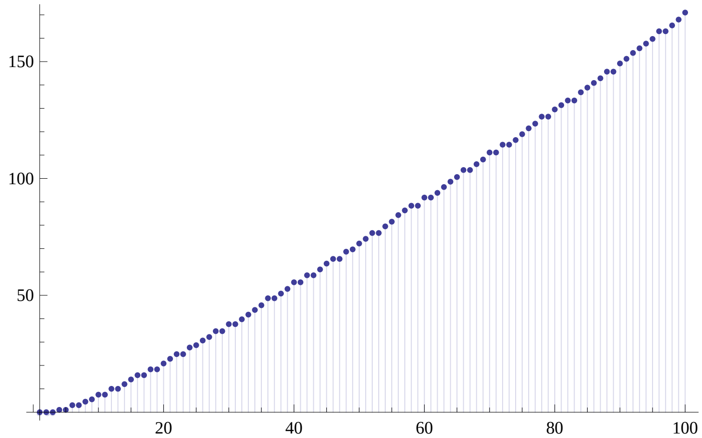
```
DiscretePlot[Sum[ pk[n / j, 1], {j, 2, n}] , {n, 1, 100}]
```




```
Series[Log[x] / x, {x, 1, 20}]
```

$$(x-1)-\frac{3}{2}(x-1)^2+\frac{11}{6}(x-1)^3-\frac{25}{12}(x-1)^4+\frac{137}{60}(x-1)^5-\frac{49}{20}(x-1)^6+\frac{363}{140}(x-1)^7-\frac{761}{280}(x-1)^8+\frac{7129\,(x-1)^9}{2520}-\frac{7381\,(x-1)^{10}}{2520}+\frac{83\,711\,(x-1)^{11}}{27\,720}-\frac{86\,021\,(x-1)^{12}}{27\,720}+\frac{1\,145\,993\,(x-1)^{13}}{360\,360}-\frac{1\,171\,733\,(x-1)^{14}}{360\,360}+\frac{1\,195\,757\,(x-1)^{15}}{360\,360}-\frac{2\,436\,559\,(x-1)^{16}}{720\,720}+\frac{42\,142\,223\,(x-1)^{17}}{12\,252\,240}-\frac{14\,274\,301\,(x-1)^{18}}{4\,084\,080}+\frac{275\,295\,799\,(x-1)^{19}}{77\,597\,520}-\frac{55\,835\,135\,(x-1)^{20}}{15\,519\,504}+O[x-1]^{21}$$

```
DiscretePlot[ pks[n, 1] , {n, 1, 100}]
```




```
Sum[ Sum[(D[Log[x] / x, {x, k}] / k! /. x → 1) d2[100 / j, k], {j, 1, 100}], {k, 1, Log2@100}]
```

$$\frac{428}{15}$$

```
Sum[ (D[Log[x] / x, {x, k}] / k! /. x → 1) Sum[d2[100 / j, k], {j, 1, 100}], {k, 1, Log2@100}]
```

$$\frac{428}{15}$$

```
Sum[ (D[Log[x] / x, {x, k}] / k! /. x → 1) (d2[100, k + 1] + d2[100, k]), {k, 1, Log2@100}]
```

$$\frac{428}{15}$$

```
Sum[ BernoulliB[k] / k! (Log[x]) ^k, {k, 0, Infinity}]
```

$$\frac{\text{Log}[x]}{-1+x}$$

```
Series[Log[x] x, {x, 1, 20}]
```

$$(x-1)+\frac{1}{2}(x-1)^2-\frac{1}{6}(x-1)^3+\frac{1}{12}(x-1)^4-\frac{1}{20}(x-1)^5+\frac{1}{30}(x-1)^6-\frac{1}{42}(x-1)^7+\frac{1}{56}(x-1)^8-\frac{1}{72}(x-1)^9+\frac{1}{90}(x-1)^{10}-\frac{1}{110}(x-1)^{11}+\frac{1}{132}(x-1)^{12}-\frac{1}{156}(x-1)^{13}+\frac{1}{182}(x-1)^{14}-\frac{1}{210}(x-1)^{15}+\frac{1}{240}(x-1)^{16}-\frac{1}{272}(x-1)^{17}+\frac{1}{306}(x-1)^{18}-\frac{1}{342}(x-1)^{19}+\frac{1}{380}(x-1)^{20}+O[x-1]^{21}$$

```
Sum[ (D[Log[x] x, {x, k}] / k! /. x → 1)
  Sum[MoebiusMu[j] d2[100 / j, k], {j, 1, 100}], {k, 1, Log2@100}]
```

$$\frac{428}{15}$$

```
Series[Log[x] / Cos[x], {x, 0, 20}]
```

$$\text{Log}[x]+\frac{1}{2}\text{Log}[x]\,x^2+\frac{5}{24}\text{Log}[x]\,x^4+\frac{61}{720}\text{Log}[x]\,x^6+\frac{277\,\text{Log}[x]\,x^8}{8064}+\frac{50\,521\,\text{Log}[x]\,x^{10}}{3\,628\,800}+\frac{540\,553\,\text{Log}[x]\,x^{12}}{95\,800\,320}+\frac{199\,360\,981\,\text{Log}[x]\,x^{14}}{87\,178\,291\,200}+\frac{3\,878\,302\,429\,\text{Log}[x]\,x^{16}}{4\,184\,557\,977\,600}+\frac{2\,404\,879\,675\,441\,\text{Log}[x]\,x^{18}}{6\,402\,373\,705\,728\,000}+\frac{14\,814\,847\,529\,501\,\text{Log}[x]\,x^{20}}{97\,316\,080\,327\,065\,600}+O[x]^{21}$$

```
Sum[1 / k! (Log[x])^k, {k, 0, Infinity}]
```

x

```
Sum[ (Log[x])^k, {k, 0, Infinity}]
```

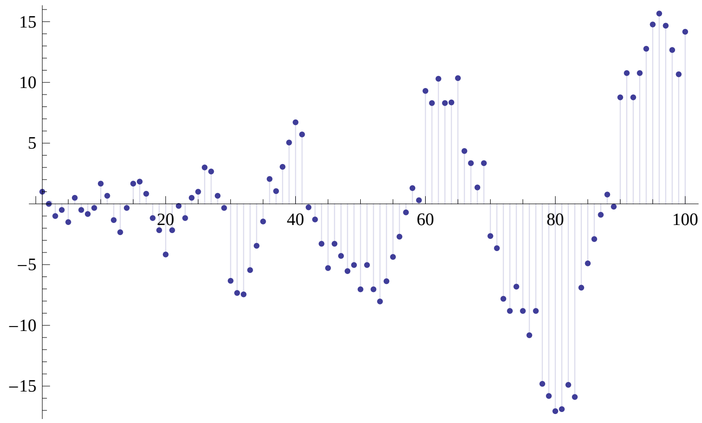
$$\frac{1}{1 - \text{Log}[x]}$$

```
Sum[(-1)^k (Log[x])^k, {k, 0, Infinity}]
```

$$\frac{1}{1 + \text{Log}[x]}$$

```
kap[n_] := kap[n] = FullSimplify[MangoldtLambda[n] / Log[n]]
k2[n_, k_] := k2[n, k] = Sum[ kap[j] k2[Floor[n / j], k - 1], {j, 2, n}]
k2[n_, 0] := UnitStep[n - 1]
```

```
DiscretePlot[ Sum[ (-1)^k k2[n, k], {k, 0, Log2@n}], {n, 1, 100}]
```




```
2 (Sum[ BernoulliB[ k] / k! D[Dnsz[100, 0, z], {z, k}] /. z → 0, {k, 2, Log2@100}] - lgm[100] + 1)
```

$$\frac{428}{15}$$

```
lgmk[n_, t_] := Sum[ BernoulliB[k] / k! (Log[x])^(k + t), {k, 0, Infinity}]
```

```
lgmk[100, -1]
```

$$\frac{1}{-1 + x}$$

```
Table[BernoulliB[k] / k!, {k, 0, 10}]
```

$$\left\{1, -\frac{1}{2}, \frac{1}{12}, 0, -\frac{1}{720}, 0, \frac{1}{30\,240}, 0, -\frac{1}{1\,209\,600}, 0, \frac{1}{47\,900\,160}\right\}$$

```
Series[(x - 1) / Log[x], {x, 1, 20}]
```

$$1+\frac{x-1}{2}-\frac{1}{12}(x-1)^2+\frac{1}{24}(x-1)^3-\frac{19}{720}(x-1)^4+\frac{3}{160}(x-1)^5-\frac{863\,(x-1)^6}{60\,480}+\frac{275\,(x-1)^7}{24\,192}-\frac{33\,953\,(x-1)^8}{3\,628\,800}+\frac{8183\,(x-1)^9}{1\,036\,800}-\frac{3\,250\,433\,(x-1)^{10}}{479\,001\,600}+\frac{4671\,(x-1)^{11}}{788\,480}-\frac{13\,695\,779\,093\,(x-1)^{12}}{2\,615\,348\,736\,000}+\frac{2\,224\,234\,463\,(x-1)^{13}}{475\,517\,952\,000}-\frac{132\,282\,840\,127\,(x-1)^{14}}{31\,384\,184\,832\,000}+\frac{2\,639\,651\,053\,(x-1)^{15}}{689\,762\,304\,000}-\frac{111\,956\,703\,448\,001\,(x-1)^{16}}{32\,011\,868\,528\,640\,000}+\frac{50\,188\,465\,(x-1)^{17}}{15\,613\,165\,568}-\frac{2\,334\,028\,946\,344\,463\,(x-1)^{18}}{786\,014\,494\,949\,376\,000}+\frac{301\,124\,035\,185\,049\,(x-1)^{19}}{109\,285\,437\,800\,448\,000}-\frac{12\,365\,722\,323\,469\,980\,029\,(x-1)^{20}}{4\,817\,145\,976\,189\,747\,200\,000}+O[x-1]^{21}$$

```
Sum[BernoulliB[k] / k! Log[x]^k, {k, 0, Infinity}]
```

$$\frac{\text{Log}[x]}{-1+x}$$

```
Sum[BernoulliB[k] / k! Log[x]^(k + 1), {k, 0, Infinity}]
```

$$\frac{\text{Log}[x]^2}{-1+x}$$

```
Sum[BernoulliB[k] / k! Log[x]^(k - 1), {k, 0, Infinity}]
```

$$\frac{1}{-1+x}$$


```
Series[(x - 1) / Log[x], {x, 1, 20}]
```

$$1+\frac{x-1}{2}-\frac{1}{12}(x-1)^2+\frac{1}{24}(x-1)^3-\frac{19}{720}(x-1)^4+\frac{3}{160}(x-1)^5-\frac{863\,(x-1)^6}{60\,480}+\frac{275\,(x-1)^7}{24\,192}-\frac{33\,953\,(x-1)^8}{3\,628\,800}+\frac{8183\,(x-1)^9}{1\,036\,800}-\frac{3\,250\,433\,(x-1)^{10}}{479\,001\,600}+\frac{4671\,(x-1)^{11}}{788\,480}-\frac{13\,695\,779\,093\,(x-1)^{12}}{2\,615\,348\,736\,000}+\frac{2\,224\,234\,463\,(x-1)^{13}}{475\,517\,952\,000}-\frac{132\,282\,840\,127\,(x-1)^{14}}{31\,384\,184\,832\,000}+\frac{2\,639\,651\,053\,(x-1)^{15}}{689\,762\,304\,000}-\frac{111\,956\,703\,448\,001\,(x-1)^{16}}{32\,011\,868\,528\,640\,000}+\frac{50\,188\,465\,(x-1)^{17}}{15\,613\,165\,568}-\frac{2\,334\,028\,946\,344\,463\,(x-1)^{18}}{786\,014\,494\,949\,376\,000}+\frac{301\,124\,035\,185\,049\,(x-1)^{19}}{109\,285\,437\,800\,448\,000}-\frac{12\,365\,722\,323\,469\,980\,029\,(x-1)^{20}}{4\,817\,145\,976\,189\,747\,200\,000}+O[x-1]^{21}$$


```
Series[(x - 1)^2 / Log[x], {x, 1, 20}]
```

$$(x-1)+\frac{1}{2}(x-1)^2-\frac{1}{12}(x-1)^3+\frac{1}{24}(x-1)^4-\frac{19}{720}(x-1)^5+\frac{3}{160}(x-1)^6-\frac{863\,(x-1)^7}{60\,480}+\frac{275\,(x-1)^8}{24\,192}-\frac{33\,953\,(x-1)^9}{3\,628\,800}+\frac{8183\,(x-1)^{10}}{1\,036\,800}-\frac{3\,250\,433\,(x-1)^{11}}{479\,001\,600}+\frac{4671\,(x-1)^{12}}{788\,480}-\frac{13\,695\,779\,093\,(x-1)^{13}}{2\,615\,348\,736\,000}+\frac{2\,224\,234\,463\,(x-1)^{14}}{475\,517\,952\,000}-\frac{132\,282\,840\,127\,(x-1)^{15}}{31\,384\,184\,832\,000}+\frac{2\,639\,651\,053\,(x-1)^{16}}{689\,762\,304\,000}-\frac{111\,956\,703\,448\,001\,(x-1)^{17}}{32\,011\,868\,528\,640\,000}+\frac{50\,188\,465\,(x-1)^{18}}{15\,613\,165\,568}-\frac{2\,334\,028\,946\,344\,463\,(x-1)^{19}}{786\,014\,494\,949\,376\,000}+\frac{301\,124\,035\,185\,049\,(x-1)^{20}}{109\,285\,437\,800\,448\,000}+O[x-1]^{21}$$

```
Series[(x - 1)^3 / Log[x], {x, 1, 20}]
```

$$(x-1)^2+\frac{1}{2}(x-1)^3-\frac{1}{12}(x-1)^4+\frac{1}{24}(x-1)^5-\frac{19}{720}(x-1)^6+\frac{3}{160}(x-1)^7-\frac{863(x-1)^8}{60480}+\frac{275(x-1)^9}{24192}-\frac{33953(x-1)^{10}}{3628800}+\frac{8183(x-1)^{11}}{1036800}-\frac{3250433(x-1)^{12}}{479001600}+\frac{4671(x-1)^{13}}{788480}-\frac{13695779093(x-1)^{14}}{2615348736000}+\frac{2224234463(x-1)^{15}}{475517952000}-\frac{132282840127(x-1)^{16}}{31384184832000}+\frac{2639651053(x-1)^{17}}{689762304000}-\frac{111956703448001(x-1)^{18}}{32011868528640000}+\frac{50188465(x-1)^{19}}{15613165568}-\frac{2334028946344463(x-1)^{20}}{786014494949376000}+O[x-1]^{21}$$

```
Series[1 / Log[x], {x, 1, 20}]
```

$$\frac{1}{x-1}+\frac{1}{2}-\frac{x-1}{12}+\frac{1}{24}(x-1)^2-\frac{19}{720}(x-1)^3+\frac{3}{160}(x-1)^4-\frac{863(x-1)^5}{60480}+\frac{275(x-1)^6}{24192}-\frac{33953(x-1)^7}{3628800}+\frac{8183(x-1)^8}{1036800}-\frac{3250433(x-1)^9}{479001600}+\frac{4671(x-1)^{10}}{788480}-\frac{13695779093(x-1)^{11}}{2615348736000}+\frac{2224234463(x-1)^{12}}{475517952000}-\frac{132282840127(x-1)^{13}}{31384184832000}+\frac{2639651053(x-1)^{14}}{689762304000}-\frac{111956703448001(x-1)^{15}}{32011868528640000}+\frac{50188465(x-1)^{16}}{15613165568}-\frac{2334028946344463(x-1)^{17}}{786014494949376000}+\frac{301124035185049(x-1)^{18}}{109285437800448000}-\frac{12365722323469980029(x-1)^{19}}{4817145976189747200000}+\frac{8519318716801273673(x-1)^{20}}{3549475982455603200000}+O[x-1]^{21}$$

```
LaplaceTransform[2 t^2 - 7 t + 1, t, s]
```

$$\frac{4}{s^3}-\frac{7}{s^2}+\frac{1}{s}$$

```
Expand@DZ[100, 0, x]
```

$$1+\frac{428x}{15}+\frac{16289x^2}{360}+\frac{331x^3}{16}+\frac{611x^4}{144}+\frac{67x^5}{240}+\frac{7x^6}{720}$$

$$\texttt{LaplaceTransform}\left[1+\frac{428t}{15}+\frac{16289t^2}{360}+\frac{331t^3}{16}+\frac{611t^4}{144}+\frac{67t^5}{240}+\frac{7t^6}{720}, t, s\right]$$

$$\frac{7}{s^7}+\frac{67}{2s^6}+\frac{611}{6s^5}+\frac{993}{8s^4}+\frac{16289}{180s^3}+\frac{428}{15s^2}+\frac{1}{s}$$

$$\texttt{Expand@InverseLaplaceTransform}\left[\frac{7}{s^7}+\frac{67}{2s^6}+\frac{611}{6s^5}+\frac{993}{8s^4}+\frac{16289}{180s^3}+\frac{428}{15s^2}+\frac{1}{s}, s, t\right]$$

$$1+\frac{428t}{15}+\frac{16289t^2}{360}+\frac{331t^3}{16}+\frac{611t^4}{144}+\frac{67t^5}{240}+\frac{7t^6}{720}$$

```
Expand@DZ[100, 0, s]
```

$$1+\frac{428\,s}{15}+\frac{16\,289\,s^2}{360}+\frac{331\,s^3}{16}+\frac{611\,s^4}{144}+\frac{67\,s^5}{240}+\frac{7\,s^6}{720}$$

Expand@InverseLaplaceTransform $\left[1+\frac{428\,s}{15}+\frac{16\,289\,s^2}{360}+\frac{331\,s^3}{16}+\frac{611\,s^4}{144}+\frac{67\,s^5}{240}+\frac{7\,s^6}{720}, s, t\right]$

$$\text{DiracDelta}[t]+\frac{428\,\text{DiracDelta}'[t]}{15}+\frac{16\,289\,\text{DiracDelta}''[t]}{360}+\frac{331}{16}\text{DiracDelta}^{(3)}[t]+\frac{611}{144}\text{DiracDelta}^{(4)}[t]+\frac{67}{240}\text{DiracDelta}^{(5)}[t]+\frac{7}{720}\text{DiracDelta}^{(6)}[t]$$

ff[s_] := $\frac{7}{s^7}+\frac{67}{2\,s^6}+\frac{611}{6\,s^5}+\frac{993}{8\,s^4}+\frac{16\,289}{180\,s^3}+\frac{428}{15\,s^2}+\frac{1}{s}$

D $\left[1+\frac{428\,x}{15}+\frac{16\,289\,x^2}{360}+\frac{331\,x^3}{16}+\frac{611\,x^4}{144}+\frac{67\,x^5}{240}+\frac{7\,x^6}{720}, x\right]$

$$\frac{428}{15}+\frac{16\,289\,x}{180}+\frac{993\,x^2}{16}+\frac{611\,x^3}{36}+\frac{67\,x^4}{48}+\frac{7\,x^5}{120}$$

t100[x_] := $1+\frac{428\,x}{15}+\frac{16\,289\,x^2}{360}+\frac{331\,x^3}{16}+\frac{611\,x^4}{144}+\frac{67\,x^5}{240}+\frac{7\,x^6}{720}$

t100a[x_] := $\left(1+\frac{428\,x}{15}+\frac{16\,289\,x^2}{360}+\frac{331\,x^3}{16}+\frac{611\,x^4}{144}+\frac{67\,x^5}{240}+\frac{7\,x^6}{720}\right)$

```
Chop@N[Sum[DZ[100, 0, E^(2 Pi I / 100 n)], {n, 1, 100}] / 100]
```

1.

```
kap[n_] := kap[n] = FullSimplify[MangoldtLambda[n] / Log[n]]
k2[n_, k_] := k2[n, k] = Sum[ kap[j] k2[Floor[n / j], k - 1], {j, 2, n}]
k2[n_, 0] := UnitStep[n - 1]
```

```
Expand@D[DZ[100, 0, z], z]
```

$$\frac{428}{15}+\frac{16\,289\,z}{180}+\frac{993\,z^2}{16}+\frac{611\,z^3}{36}+\frac{67\,z^4}{48}+\frac{7\,z^5}{120}$$

1 + Integrate $\left[\frac{428}{15}+\frac{16\,289\,z}{180}+\frac{993\,z^2}{16}+\frac{611\,z^3}{36}+\frac{67\,z^4}{48}+\frac{7\,z^5}{120}, \{z, 0, 4\}\right]$

3575

Integrate $\left[\frac{428}{15}, \{z, 0, 3\}\right]$

$$\frac{428}{5}$$

```
m2[n_, k_] := m2[n, k] = Sum[ Abs[MoebiusMu[j]] m2[Floor[n / j], k - 1], {j, 2, n}]
m2[n_, 0] := UnitStep[n - 1]
m1[n_, z_] := Sum[ bin[z, k] m2[n, k], {k, 0, Log2@n}]
lm1[n_, k_] := D[m1[n, z], {z, k}] /. z → 0
lmm[n_] :=
 Sum[ BernoulliB[ k] / k ! Sum[ Abs[MoebiusMu[j]] lm1[n / j, k], {j, 2, n}], {k, 0, Log2@n}]
DiscretePlot[lmm[n], {n, 1, 100}]
```