```mathematica
Limit[ (1 / Log[z]) (1 - 1 / z), z → 1]
```

1

```mathematica
Integrate[ (1 / Log[z]) (1 - 1 / z), z]
```

$-\text{Log}[\text{Log}[z]] + \text{LogIntegral}[z]$


```mathematica
Limit[ (x^(a z) - 1) / (z), z → 0]
```

$a\,\text{Log}[x]$

```mathematica
Integrate[ 1, {x, 1, n^a}]
```

$-1 + n^a$

```mathematica
Expand@Integrate[ 1, {x, 1, n^a}, {y, 1, n^a / x}]
```

$\text{ConditionalExpression}[1 - n^a + n^a\,\text{Log}[n^a],\ \text{Re}[n^a] \geq 0\ ||\ n^a \notin \text{Reals}]$

```mathematica
Expand@Integrate[ 1, {x, 1, n^a}, {y, 1, n^a / x}, {z, 1, n^a / (x y)}]
```

$\text{ConditionalExpression}\Big[$
$\quad -1 + n^a - \frac{1}{2} n^a\,\text{Log}[n^{-a}]^2 - n^a\,\text{Log}[n^a] - n^a\,\text{Log}[n^{-a}]\,\text{Log}[n^a],\ \text{Re}[n^a] \geq 0\ ||\ n^a \notin \text{Reals}\Big]$

```mathematica
N[ -1 + n^a - 1/2 n^a Log[n^-a]^2 - n^a Log[n^a] - n^a Log[n^-a] Log[n^a] /. {n → 10, a → 2}]
```

698.863

```mathematica
Chop[(-1) N@Gamma[3, 0, -Log[100]] / Gamma[3]]
```
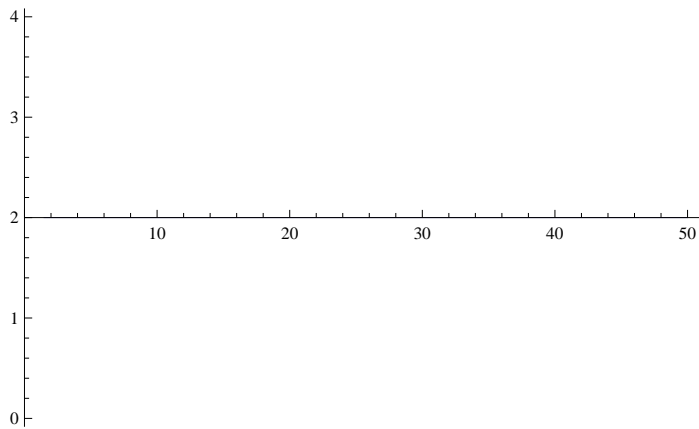
698.863

```mathematica
N[LogIntegral[ 10^2] - Log[Log[10^2]] - EulerGamma]
```

28.0217

```mathematica
Plot[ {LogIntegral[LaguerreL[-2, Log[n]]] / LogIntegral[ n ], Log[n]}, {n, 1, 1000}]
```
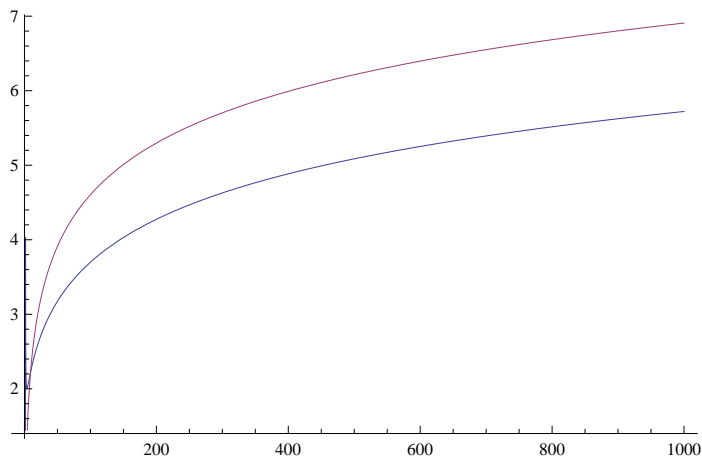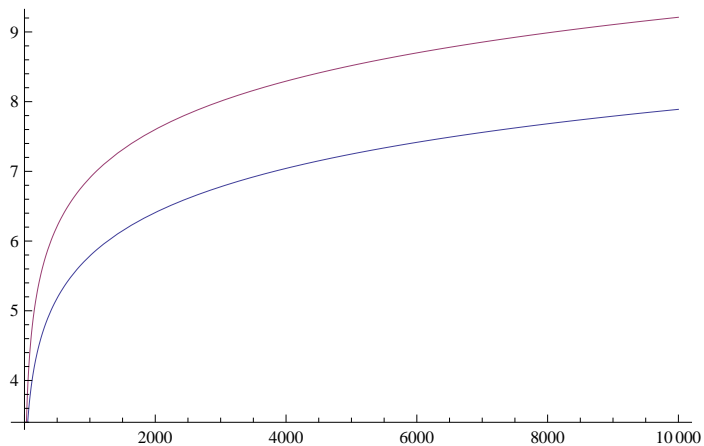
```
Plot[ {Log[n^2] / Log[ n ]}, {n, 1, 50}]
```


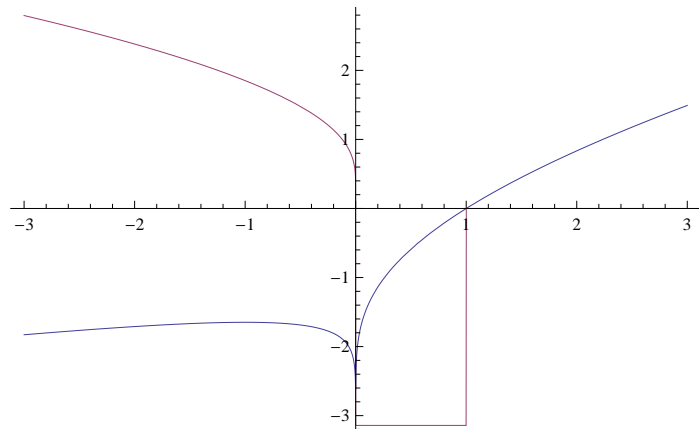
```
aa[n_] := LogIntegral[n] - Log[Log[n]] - EulerGamma
Plot[ {aa[LaguerreL[-2, Log[n]]] / aa[ n ], Log[n]}, {n, 0, 10 000}]
```



```
Plot[{ Re[aa[n]], Im[aa[n]]}, {n, -3, 3}]
```



```
Table[ {N[aa[n^3] / aa[ n ]]} / n^2, {n, 1, 100, 1}] // TableForm
```

Infinity::indet : Indeterminate expression −EulerGamma + −∞ + ∞ encountered. ≫

Infinity::indet : Indeterminate expression −EulerGamma + −∞ + ∞ encountered. ≫

```
Indeterminate
1.18172
0.771132
0.603645
0.515533
0.462063
0.426513
0.401333
0.382652
0.368294
0.356948
0.34778
0.340235
0.333928
0.328588
0.324016
0.320064
0.316618
0.313591
0.310915
0.308535
0.306407
0.304496
0.302771
0.301209
0.299789
0.298494
0.29731
0.296224
0.295224
0.294303
0.293453
0.292665
0.291935
0.291256
0.290624
0.290035
0.289486
0.288972
0.288491
0.288041
0.287618
0.287221
0.286848
0.286497
0.286167
0.285855
0.285561
0.285284
0.285022
0.284774
0.28454
0.284319
0.284109
0.28391
0.283722
```

```
0.283544
0.283375
0.283214
0.283062
0.282917
0.28278
0.28265
0.282526
0.282409
0.282297
0.282191
0.28209
0.281994
0.281903
0.281817
0.281734
0.281656
0.281582
0.281512
0.281445
0.281381
0.281321
0.281264
0.28121
0.281158
0.281109
0.281063
0.281019
0.280978
0.280939
0.280902
0.280867
0.280834
0.280803
0.280774
0.280747
0.280721
0.280697
0.280675
0.280654
0.280634
0.280616
0.280599
0.280583
```

N@LaguerreL[-2, Log[10]]

33.0259

Plot[{n LogIntegral[n], LogIntegral[n^2]}, {n, 0, 100}]
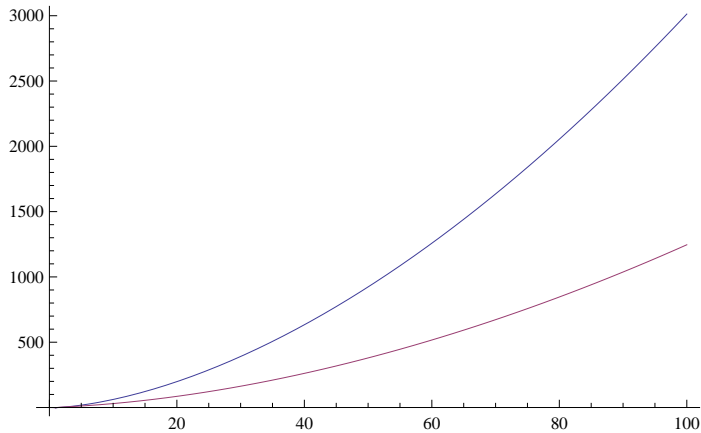
```
Expand[D[LogIntegral[n^2] / LogIntegral[ n ], n]]
```

$$\frac{2\,n}{\text{Log}\left[n^2\right]\,\text{LogIntegral}[n]} - \frac{\text{LogIntegral}\left[n^2\right]}{\text{Log}[n]\,\text{LogIntegral}[n]^2}$$

```
Expand[Integrate[ D[ x^a, x] D[y^a, y], {x, 1, n}, {y, 1, n / x}]]
```

ConditionalExpression$[1 - n^a + a\,n^a\,\text{Log}[n]\,,\,\text{Re}[n] \geq 0\,||\,n \notin \text{Reals}]$

```
Expand[Integrate[ 1, {x, 1, n^a}, {y, 1, n^a / x}]]
```

ConditionalExpression$[1 - n^a + n^a\,\text{Log}[n^a]\,,\,\text{Re}[n^a] \geq 0\,||\,n^a \notin \text{Reals}]$

```
Chop@N[Gamma[2, 0, -Log[n^a]] / Gamma[2] /. {n → 20, a → 3}]
```

63 898.6

```
N[1 - n^a + a n^a Log[n] /. {n → 20, a → 3}]
```

63 898.6

```
N[1 - n^a + n^a Log[n^a] /. {n → 20, a → 3}]
```

63 898.6

```
Chop@N@Sum[ (-1) ^ (k + 1) / k (-1) ^k Gamma[k, 0, -2 Log[30]] / Gamma[k], {k, 1, 80}]
```

160.529

```
N@(LogIntegral[30^2] - Log[Log[30^2]] - EulerGamma)
```

160.529

```
N[Limit[ (LaguerreL[- (z), Log[30^2]] - 1) / z, z → 0]]
```

160.529

```
N[LogIntegral[30^2] - Log[Log[30^2]] - EulerGamma]
```

160.529

```
Expand@Integrate[ 1, {x, 1, n^2}, {y, 1, n^2 / x}]
```

ConditionalExpression$\left[1 - n^2 + n^2\,\text{Log}\left[n^2\right]\,,\,\text{Re}\left[n^2\right] \geq 0\,||\,n^2 \notin \text{Reals}\right]$

```
FullSimplify[( a x^a Log[ x] - x^a + 1) - (x Log[x] - x + 1)]
```

$x - x^a + (-x + a\,x^a)\,\text{Log}[x]$

Expand[(x^a - x) (a Log[x] - 1)]

$x - x^a - a\,x\,\text{Log}[x] + a\,x^a\,\text{Log}[x]$

Expand@Integrate[1, {x, 1, n^a}, {y, 1, n^a / x}] / Integrate[1, {x, 1, n}, {y, 1, n / x}]

$\text{ConditionalExpression}\left[\dfrac{1 - n^a + n^a\,\text{Log}[n^a]}{1 + n\,(-1 + \text{Log}[n])},\ (\text{Re}[n] \geq 0\ ||\ n \notin \text{Reals})\ \&\&\ (\text{Re}[n^a] \geq 0\ ||\ n^a \notin \text{Reals})\right]$

-n^a - n (-1 + Log[n]) + n^a Log[n^a]

$-n^a - n\,(-1 + \text{Log}[n]) + n^a\,\text{Log}[n^a]$

FullSimplify[-n^a - n (-1 + Log[n]) + a n^a Log[n]]

$n - n^a + (-n + a\,n^a)\,\text{Log}[n]$

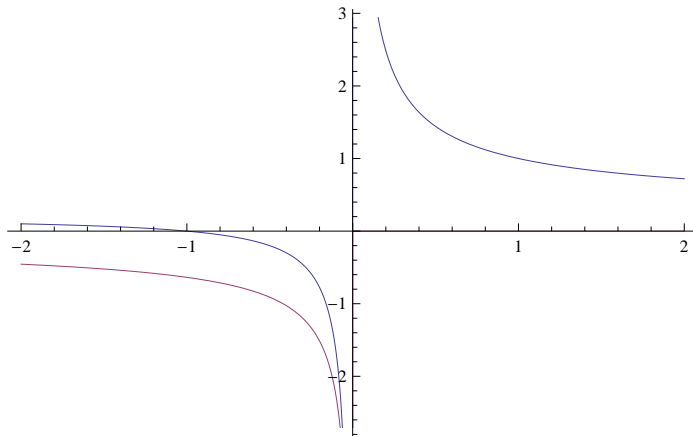**Expand$\left[\dfrac{1 - n^a + n^a\,a\,\text{Log}[n]}{1 + n\,(-1 + \text{Log}[n])}\right]$**

FullSimplify$\left[\dfrac{1}{1 + n\,(-1 + \text{Log}[n])} - \dfrac{n^a}{1 + n\,(-1 + \text{Log}[n])} + \dfrac{a\,n^a\,\text{Log}[n]}{1 + n\,(-1 + \text{Log}[n])}\right]$

$\dfrac{1 + n^a\,(-1 + a\,\text{Log}[n])}{1 - n + n\,\text{Log}[n]}$

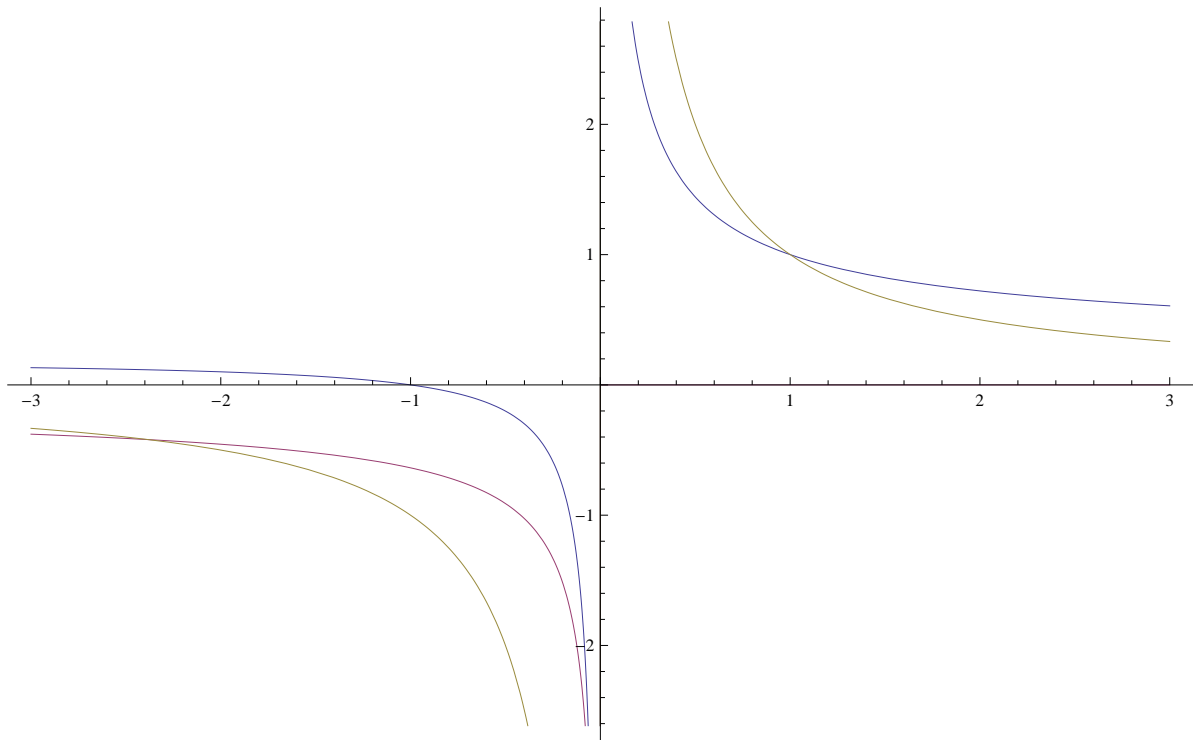**Integrate[1, {x, 1, (n)^a}] / Integrate[1, {x, 1, (n)}]**

$\dfrac{-1 + n^a}{-1 + n}$

Plot[{Re[(1 / Log[n]) (1 - 1 / n)], Im[(1 / Log[n]) (1 - 1 / n)]}, {n, -2, 2}]

Plot[{Re[(1 / Log[n]) (1 - 1 / n)], Im[(1 / Log[n]) (1 - 1 / n)], 1 / n}, {n, -3, 3}]

Integrate[(1 / Log[n]) (1 - 1 / n), n]

-Log[Log[n]] + LogIntegral[n]

Limit[(1 / Log[n]) (1 - 1 / n), n → -Infinity]

0

D[-Log[Log[n]] + LogIntegral[n] - EulerGamma, n]

$$\frac{1}{\text{Log}[n]} - \frac{1}{n\,\text{Log}[n]}$$

Expand[Limit[(1 / Log[n]) (1 - 1 / n) - EulerGamma, n → -1]]

$-\text{EulerGamma} - \dfrac{2\,\dot{\mathbb{i}}}{\pi}$

$\mathbf{N\left[-\dfrac{2\,\dot{\mathbb{i}}}{\pi}\right]}$

0. - 0.63662 $\dot{\mathbb{i}}$

Sum[(-1) ^k / (k!) Binomial[z, k] x^k, {k, 0, Infinity}]

Hypergeometric1F1[-z, 1, x]

Sum[(-1) ^k / (k!) Binomial[z, k] Log[n] ^k, {k, 0, Infinity}]

Hypergeometric1F1[-z, 1, Log[n]]

Expand@LaguerreL[3, Log[12]]

$1 - 3\,\text{Log}[12] + \dfrac{3\,\text{Log}[12]^2}{2} - \dfrac{\text{Log}[12]^3}{6}$

Expand@Hypergeometric1F1[- 3, 1, Log[12]]

$$1 - 3 \operatorname{Log}[12] + \frac{3 \operatorname{Log}[12]^2}{2} - \frac{\operatorname{Log}[12]^3}{6}$$

14 * 30 000

420 000