```
ClearAll["Global`*"]

D1[n_, 0] := 1
D1[n_, 1] := D1[n, 1] = Sum[N[Log[j]], {j, 2, n}]
D1[n_, k_] := D1[n, k] = Sum[ D1[Floor[n / j], k - 1], {j, 2, n}]
M1[n_] := Sum[ (-1)^(k + 1) D1[n, k], {k, 1, Log[2, n]}]
MM[n_, k_] := MM[n, k] = Sum[ MoebiusMu[j] MM[Floor[n / j], k - 1], {j, 2, n}]; MM[n_, 0] := 1
mm[n_, k_] := MM[n, k] - MM[n - 1, k]
Cb[n_, k_] := Sum[ MangoldtLambda[j] MM[Floor[n / j], k - 1], {j, 2, n}]; Cb[n_, 0] := 1
K[n_] := If[ n == 1, 0, FullSimplify[MangoldtLambda[n] / Log[n]]]
man[n_, k_] := Log[n] - Sum[ If[ j == 1, 0, man[n / j, k + 1]], {j, Divisors[n]}]
manp[n_, k_] := Log[n] / k - Sum[ If[ j == 1, 0, manp[n / j, k + 1]], {j, Divisors[n]}]
man22[n_, v_] := If[ Floor[Log[v, n]] == Log[v, n], n Log[n] / Log[v, n], 0]
manp2[n_] := FullSimplify[manp[n, 1]] - man22[n, 5]
PO[n_, 0] = 1;
PO[n_, k_] := PO[n, k] = Sum[ K[j] PO[Floor[n / j], k - 1], {j, 2, n}]
Pb[n_, k_] := Pb[n, k] = Sum[ manp2[j] PO[Floor[n / j], k - 1], {j, 2, n}]; Pb[n_, 0] := 1
Dp[n_, z_] := Dp[n, z] = Sum[ (z^k) / (k!) Pb[n, k], {k, 1, Log[2, n]}]



N[Dp[100, 1]]
D1[100, 1]
N[Dp[100, -1]]
M1[100]
```

98.7298

363.739

-115.046

94.0453

```
Sum[ (-1)^(k) MM[n = 120, k], {k, 0, Log[2, 120]}]
```

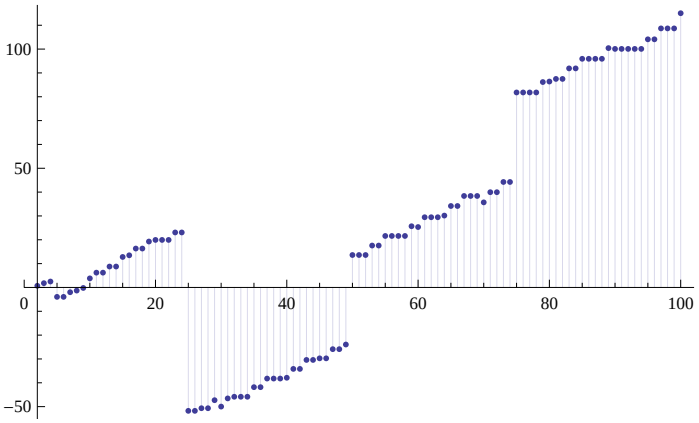
120

```
D1[100, 1]
N[Sum[ (-1)^(k + 1) Cb[100, k], {k, 1, Log[2, 100]}]]
```

363.739

363.739

```
DiscretePlot[Dp[n, ss = -1] / ss, {n, 2, 100}]
```




```
Table[{n, FullSimplify[(Dp[n, ss = -1] / ss) - (Dp[n - 1, ss = -1] / ss) - MangoldtLambda[n]]},
  {n, 2, 100}] // TableForm
```

2	0
3	0
4	0
5	$-5 \text{Log}[5]$
6	0
7	0
8	0
9	0
10	$\text{Log}\left[25 \sqrt{5}\right]$
11	0
12	0
13	0
14	0
15	$\text{Log}\left[25 \sqrt{5}\right]$
16	0
17	0
18	0
19	0
20	$\frac{5 \text{Log}[5]}{12}$
21	0
22	0
23	0
24	0
25	$-\frac{95 \text{Log}[5]}{2}$
26	0
27	0
28	0
29	0
30	$-\frac{5 \text{Log}[5]}{3}$
31	0
32	0
33	0
34	0

35 $\text{Log}\left[25\sqrt{5}\right]$

36 0

37 0

38 0

39 0

40 $\frac{5\,\text{Log}[5]}{24}$

41 0

42 0

43 0

44 0

45 $\frac{5\,\text{Log}[5]}{12}$

46 0

47 0

48 0

49 0

50 $\text{Log}\left[11\,920\,928\,955\,078\,125\times 5^{1/3}\right]$

51 0

52 0

53 0

54 0

55 $\text{Log}\left[25\sqrt{5}\right]$

56 0

57 0

58 0

59 0

60 $-\frac{5\,\text{Log}[5]}{24}$

61 0

62 0

63 0

64 0

65 $\text{Log}\left[25\sqrt{5}\right]$

66 0

67 0

68 0

69 0

70 $-\frac{5\,\text{Log}[5]}{3}$

71 0

72 0

73 0

74 0

75 $\text{Log}\left[11\,920\,928\,955\,078\,125\times 5^{1/3}\right]$

76 0

77 0

78 0

79 0

80 $\frac{19\,\text{Log}[5]}{144}$

81 0

82 0

83 0

84 0

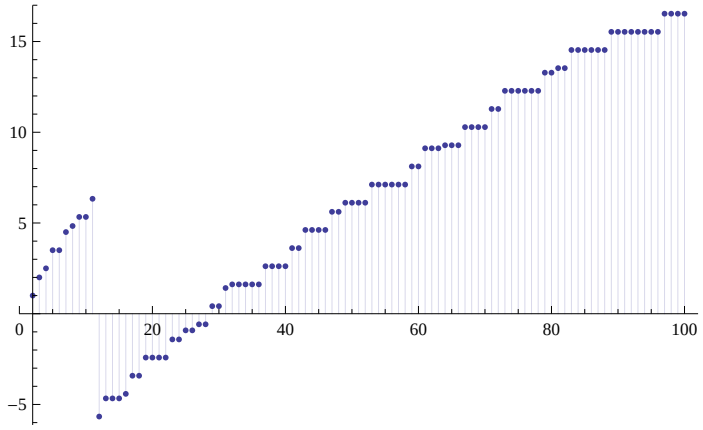
85 $\text{Log}\left[25\sqrt{5}\right]$
86 0
87 0
88 0
89 0
90 $-\frac{5\,\text{Log}[5]}{24}$
91 0
92 0
93 0
94 0
95 $\text{Log}\left[25\sqrt{5}\right]$
96 0
97 0
98 0
99 0
100 $\text{Log}\left[125\times 5^{23/24}\right]$

```
vv := 2
K[n_] := If[ n == 1, 0, FullSimplify[MangoldtLambda[n] / Log[n]]]
K6[n_] := K[n] - If[ Floor[Log[vv, n]] == Log[vv, n], n / Log[vv, n], 0]
P[n_, 0] = 1;
P[n_, k_] := P[n, k] = Sum[ K6[j] P[Floor[n / j], k - 1], {j, 2, n}]
PO[n_, 0] = 1;
PO[n_, k_] := PO[n, k] = Sum[ K[j] PO[Floor[n / j], k - 1], {j, 2, n}]
p[n_, k_] := P[n, k] - P[n - 1, k]
po[n_, k_] := PO[n, k] - PO[n - 1, k]
En[n_] := En[n] = Sum[ 1 / (k!) P[n, k], {k, 0, Log[2, n]}]
En[n_, z_] := En[n] = Sum[ (z^k) / (k!) P[n, k], {k, 0, Log[2, n]}]
en[n_] := En[n] - En[n - 1]
LAdd[n_] := Sum[ vv^k / k, {k, 1, Log[vv, n]}]
LAdd2[n_] := Sum[(-1)^k vv^k, {k, 1, Log[vv, n]}]
PP[n_, k_] := PP[n, k] = Sum[ 1 / k - PP[Floor[n / j], k + 1], {j, 2, n}]
```

```
P[100, 1]
```

$\frac{248}{15}$

```
DiscretePlot[{P[n, 1]}, {n, 2, 100}]
```

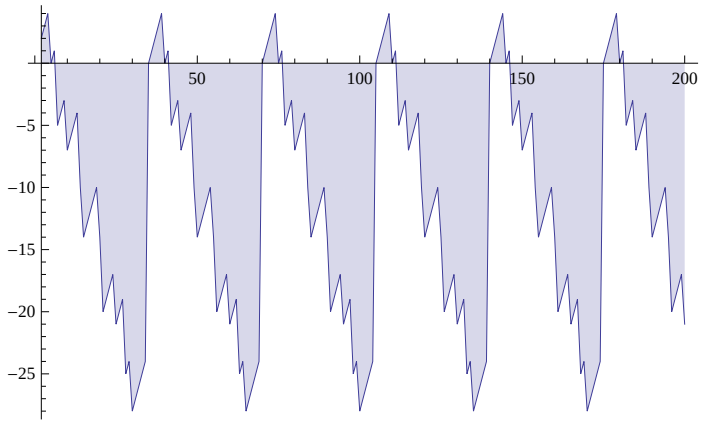



```
Table[{n, p[n, 2], po[n, 2], (p[n, 2] - po[n, 2]),
    ((p[n, 2] - po[n, 2]) - If[Mod[n, vv] == 0, -2 (vv) po[n / vv, 1], 0]) / (vv * vv)

  }, {n, 9, 250, 9}] // TableForm
```

9	4	1	3	1
18	−8	1	−9	−1
27	16	1	15	2
36	−4	$\frac{1}{2}$	$-\frac{9}{2}$	$-\frac{1}{2}$
45	−8	1	−9	−1
54	$-\frac{52}{3}$	$\frac{2}{3}$	−18	−2
63	−8	1	−9	−1
72	$-\frac{8}{3}$	$\frac{1}{3}$	−3	$-\frac{1}{3}$
81	$\frac{152}{3}$	$\frac{11}{12}$	$\frac{199}{4}$	$\frac{23}{4}$
90	0	0	0	0
99	−8	1	−9	−1
108	$-\frac{26}{3}$	$\frac{1}{3}$	−9	−1
117	−8	1	−9	−1
126	0	0	0	0
135	$-\frac{52}{3}$	$\frac{2}{3}$	−18	−2
144	−2	$\frac{1}{4}$	$-\frac{9}{4}$	$-\frac{1}{4}$
153	−8	1	−9	−1
162	−40	$\frac{1}{2}$	$-\frac{81}{2}$	$-\frac{9}{2}$
171	−8	1	−9	−1
180	0	0	0	0
189	$-\frac{52}{3}$	$\frac{2}{3}$	−18	−2
198	0	0	0	0
207	−8	1	−9	−1
216	$-\frac{52}{9}$	$\frac{2}{9}$	−6	$-\frac{2}{3}$
225	−4	$\frac{1}{2}$	$-\frac{9}{2}$	$-\frac{1}{2}$
234	0	0	0	0
243	$\frac{448}{3}$	$\frac{5}{6}$	$\frac{297}{2}$	$\frac{50}{3}$

```
DiscretePlot[En[n], {n, 2, 200}]
```



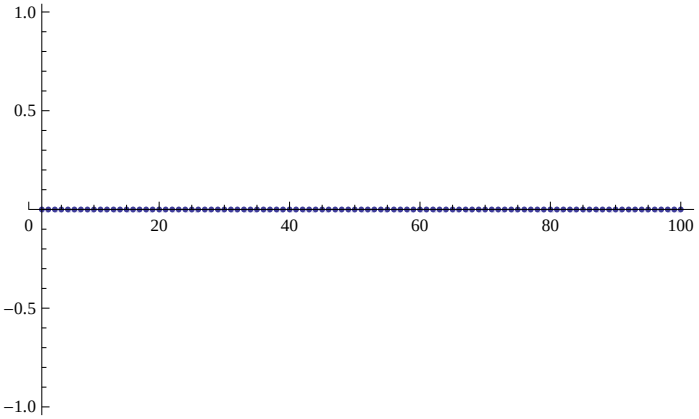

```
Table[ {n, En[n], Mod[n, vv]}, {n, 1, 100}] // TableForm
```

1	1	1
2	2	2
3	3	3
4	4	4
5	0	5
6	1	6
7	−5	0
8	−4	1
9	−3	2
10	−7	3
11	−6	4
12	−5	5
13	−4	6
14	−10	0
15	−14	1
16	−13	2
17	−12	3
18	−11	4
19	−10	5
20	−14	6
21	−20	0
22	−19	1
23	−18	2
24	−17	3
25	−21	4
26	−20	5
27	−19	6
28	−25	0
29	−24	1
30	−28	2
31	−27	3
32	−26	4
33	−25	5
34	−24	6
35	0	0
36	1	1

```
37    2     2
38    3     3
39    4     4
40    0     5
41    1     6
42    -5    0
43    -4    1
44    -3    2
45    -7    3
46    -6    4
47    -5    5
48    -4    6
49    -10   0
50    -14   1
51    -13   2
52    -12   3
53    -11   4
54    -10   5
55    -14   6
56    -20   0
57    -19   1
58    -18   2
59    -17   3
60    -21   4
61    -20   5
62    -19   6
63    -25   0
64    -24   1
65    -28   2
66    -27   3
67    -26   4
68    -25   5
69    -24   6
70    0     0
71    1     1
72    2     2
73    3     3
74    4     4
75    0     5
76    1     6
77    -5    0
78    -4    1
79    -3    2
80    -7    3
81    -6    4
82    -5    5
83    -4    6
84    -10   0
85    -14   1
86    -13   2
87    -12   3
88    -11   4
89    -10   5
90    -14   6
91    -20   0
92    -19   1
```

```
93     -18    2
94     -17    3
95     -21    4
96     -20    5
97     -19    6
98     -25    0
99     -24    1
100    -28    2
```

```
DiscretePlot[P[n, 1] - PP[n, 1] + LAdd[n] + LAdda[n], {n, 2, 100}]
```



```
Table[{n, P[n, 2], PO[n, 2], (P[n, 2] - PO[n, 2]),
    P[n, 2] - PO[n, 2] + 2 Sum[vv^j / j PO[n / vv^j, 1], {j, 1, Log[vv, n]}] -
     Sum[ vv^j vv^k / (j k), {j, 1, Log[vv, n]}, {k, 1, Log[vv, Floor[n / (vv^j)]]}]
  }, {n, 9, 250, 9}] // TableForm
```

9	3	5	−2	0
18	$\frac{47}{12}$	$\frac{167}{12}$	−10	0
27	$-\frac{15}{4}$	$\frac{283}{12}$	$-\frac{82}{3}$	0
36	$\frac{33}{4}$	$\frac{383}{12}$	$-\frac{71}{3}$	0
45	$\frac{19}{12}$	$\frac{463}{12}$	−37	0
54	$-\frac{115}{12}$	$\frac{187}{4}$	$-\frac{169}{3}$	0
63	$-\frac{53}{4}$	$\frac{677}{12}$	$-\frac{209}{3}$	0
72	$\frac{653}{45}$	$\frac{2813}{45}$	−48	0
81	$\frac{1067}{180}$	$\frac{12\,587}{180}$	−64	0
90	$\frac{227}{180}$	$\frac{14\,147}{180}$	$-\frac{232}{3}$	0
99	$-\frac{365}{36}$	$\frac{16\,199}{180}$	$-\frac{1502}{15}$	0
108	$-\frac{695}{36}$	$\frac{16\,829}{180}$	$-\frac{564}{5}$	0
117	$-\frac{893}{36}$	$\frac{17\,999}{180}$	$-\frac{624}{5}$	0
126	$-\frac{929}{36}$	$\frac{19\,979}{180}$	$-\frac{684}{5}$	0
135	$\frac{6113}{180}$	$\frac{4261}{36}$	$-\frac{422}{5}$	0
144	$\frac{2479}{90}$	$\frac{2255}{18}$	$-\frac{1466}{15}$	0
153	$\frac{1969}{90}$	$\frac{2393}{18}$	$-\frac{1666}{15}$	0
162	$\frac{584}{45}$	$\frac{6383}{45}$	$-\frac{1933}{15}$	0
171	$\frac{449}{45}$	$\frac{6608}{45}$	$-\frac{2053}{15}$	0
180	$\frac{43}{90}$	$\frac{13\,801}{90}$	$-\frac{2293}{15}$	0
189	$-\frac{47}{90}$	$\frac{14\,551}{90}$	$-\frac{811}{5}$	0
198	$-\frac{1126}{45}$	$\frac{7403}{45}$	$-\frac{2843}{15}$	0
207	$-\frac{1096}{45}$	$\frac{7913}{45}$	$-\frac{1001}{5}$	0
216	$-\frac{2917}{90}$	$\frac{5567}{30}$	$-\frac{9809}{45}$	0
225	$-\frac{1814}{45}$	$\frac{2917}{15}$	$-\frac{2113}{9}$	0
234	$-\frac{2114}{45}$	$\frac{2957}{15}$	$-\frac{2197}{9}$	0
243	$-\frac{4153}{90}$	$\frac{6119}{30}$	$-\frac{2251}{9}$	0

```
Table[{n, P[n, 3],
   PO[n, 3]
    - 3 Sum[vv^j / j PO[n / vv^j, 2], {j, 1, Log[vv, n]}]
    + 3 Sum[ vv^j vv^k / (j k) PO[n / (vv^j vv^k), 1],
      {j, 1, Log[vv, n]}, {k, 1, Log[vv, Floor[n / (vv^j)]]}]
    - Sum[ vv^j vv^k vv^m / (j k m) PO[n / (vv^j vv^k vv^m), 0], {j, 1, Log[vv, n]},
     {k, 1, Log[vv, Floor[n / (vv^j)]]}, {m, 1, Log[vv, Floor[n / (vv^j vv^k)]]}]
  }, {n, 9, 250, 9}] // TableForm
```

9	-1	-1
18	$-\frac{11}{2}$	$-\frac{11}{2}$
27	$\frac{15}{2}$	$\frac{15}{2}$
36	$-\frac{49}{4}$	$-\frac{49}{4}$
45	$-\frac{13}{4}$	$-\frac{13}{4}$
54	$\frac{29}{2}$	$\frac{29}{2}$
63	$\frac{35}{2}$	$\frac{35}{2}$
72	$-\frac{237}{8}$	$-\frac{237}{8}$
81	$-\frac{59}{8}$	$-\frac{59}{8}$
90	$-\frac{83}{8}$	$-\frac{83}{8}$
99	$\frac{289}{8}$	$\frac{289}{8}$
108	$\frac{309}{8}$	$\frac{309}{8}$
117	$\frac{427}{8}$	$\frac{427}{8}$
126	$\frac{323}{8}$	$\frac{323}{8}$
135	$-\frac{6727}{120}$	$-\frac{6727}{120}$
144	$-\frac{944}{15}$	$-\frac{944}{15}$
153	$-\frac{719}{15}$	$-\frac{719}{15}$
162	$-\frac{1331}{60}$	$-\frac{1331}{60}$
171	$-\frac{1811}{60}$	$-\frac{1811}{60}$
180	$-\frac{209}{15}$	$-\frac{209}{15}$
189	$-\frac{164}{15}$	$-\frac{164}{15}$
198	$\frac{1407}{20}$	$\frac{1407}{20}$
207	$\frac{1237}{20}$	$\frac{1237}{20}$
216	$\frac{408}{5}$	$\frac{408}{5}$
225	$\frac{1131}{10}$	$\frac{1131}{10}$
234	$\frac{1101}{10}$	$\frac{1101}{10}$
243	$\frac{1667}{20}$	$\frac{1667}{20}$

```
Table[{n, P[n, 3],
   PO[n, 3]
    - 3 Sum[vv^j / j PO[n / vv^j, 2], {j, 1, Log[vv, n]}]
    + 3 Sum[ vv^(j + k) / (j k) PO[n / (vv^(j + k)), 1],
      {j, 1, Log[vv, n]}, {k, 1, Log[vv, Floor[n / (vv^j)]]}]
    - Sum[ vv^(j + k + m) / (j k m) PO[n / (vv^(j + k + m)), 0], {j, 1, Log[vv, n]},
     {k, 1, Log[vv, Floor[n / (vv^j)]]}, {m, 1, Log[vv, Floor[n / (vv^(j + k))]]}]
  }, {n, 2, 50, 1}] // TableForm
```

2	0	0
3	0	0
4	0	0
5	0	0
6	0	0
7	0	0
8	-1	-1
9	-1	-1
10	-1	-1
11	-1	-1
12	2	2
13	2	2
14	2	2
15	2	2
16	$-\frac{5}{2}$	$-\frac{5}{2}$
17	$-\frac{5}{2}$	$-\frac{5}{2}$
18	$-\frac{11}{2}$	$-\frac{11}{2}$
19	$-\frac{11}{2}$	$-\frac{11}{2}$
20	$-\frac{5}{2}$	$-\frac{5}{2}$
21	$-\frac{5}{2}$	$-\frac{5}{2}$
22	$-\frac{5}{2}$	$-\frac{5}{2}$
23	$-\frac{5}{2}$	$-\frac{5}{2}$
24	$\frac{13}{2}$	$\frac{13}{2}$
25	$\frac{13}{2}$	$\frac{13}{2}$
26	$\frac{13}{2}$	$\frac{13}{2}$
27	$\frac{15}{2}$	$\frac{15}{2}$
28	$\frac{21}{2}$	$\frac{21}{2}$
29	$\frac{21}{2}$	$\frac{21}{2}$
30	$\frac{9}{2}$	$\frac{9}{2}$
31	$\frac{9}{2}$	$\frac{9}{2}$
32	$-\frac{37}{4}$	$-\frac{37}{4}$
33	$-\frac{37}{4}$	$-\frac{37}{4}$
34	$-\frac{37}{4}$	$-\frac{37}{4}$
35	$-\frac{37}{4}$	$-\frac{37}{4}$
36	$-\frac{49}{4}$	$-\frac{49}{4}$

37	$-\frac{49}{4}$	$-\frac{49}{4}$
38	$-\frac{49}{4}$	$-\frac{49}{4}$
39	$-\frac{49}{4}$	$-\frac{49}{4}$
40	$-\frac{13}{4}$	$-\frac{13}{4}$
41	$-\frac{13}{4}$	$-\frac{13}{4}$
42	$-\frac{37}{4}$	$-\frac{37}{4}$
43	$-\frac{37}{4}$	$-\frac{37}{4}$
44	$-\frac{25}{4}$	$-\frac{25}{4}$
45	$-\frac{13}{4}$	$-\frac{13}{4}$
46	$-\frac{13}{4}$	$-\frac{13}{4}$
47	$-\frac{13}{4}$	$-\frac{13}{4}$
48	$\frac{35}{2}$	$\frac{35}{2}$
49	$\frac{35}{2}$	$\frac{35}{2}$
50	$\frac{29}{2}$	$\frac{29}{2}$

```
fd[n_] := Sum[ vv^(j + k) / (j k ), {j, 1, Log[vv, n]}, {k, 1, Log[vv, Floor[n / (vv^j)]]}]
fe[n_] := Sum[ vv^(j + k + m) / (j k m) , {j, 1, Log[vv, n]},
  {k, 1, Log[vv, Floor[n / (vv^j)]]}, {m, 1, Log[vv, Floor[n / (vv^(j + k))]]}]
```

```
Table[{n, (fe[n] - fe[n - 1]) / n}, {n, 8, 800, 8}] // TableForm
```

8	1
16	$\frac{3}{2}$
24	0
32	$\frac{7}{4}$
40	0
48	0
56	0
64	$\frac{15}{8}$
72	0
80	0
88	0
96	0
104	0
112	0
120	0
128	$\frac{29}{15}$
136	0
144	0
152	0
160	0
168	0
176	0
184	0
192	0
200	0
208	0
216	0
224	0
232	0

240 0
248 0
256 $\frac{469}{240}$
264 0
272 0
280 0
288 0
296 0
304 0
312 0
320 0
328 0
336 0
344 0
352 0
360 0
368 0
376 0
384 0
392 0
400 0
408 0
416 0
424 0
432 0
440 0
448 0
456 0
464 0
472 0
480 0
488 0
496 0
504 0
512 $\frac{29\,531}{15\,120}$
520 0
528 0
536 0
544 0
552 0
560 0
568 0
576 0
584 0
592 0
600 0
608 0
616 0
624 0
632 0
640 0
648 0
656 0
664 0
672 0

```
680   0
688   0
696   0
704   0
712   0
720   0
728   0
736   0
744   0
752   0
760   0
768   0
776   0
784   0
792   0
800   0
```

```
ff[s_] := f[5] / 5^s
```

```
ff'[s]
```

$$-5^{-s}\, f[5]\, \text{Log}[5]$$