```
ClearAll["Global`*"]



mm[n_, a_, b_] := If[ Mod[n, a] == 0, a, 0] - If[Mod[n, b] == 0, b, 0]
E1o[n_, k_, x_] := Sum[E1o[n / j, k - 1, x], {j, 1, n}] -
  x Sum[E1o[n / (x j), k - 1, x], {j, 1, n / x}]; E1o[n_, 0, x_] := 1
E1[n_, k_, a_, b_] := E1[n, k, a, b] = Sum[mm[ j, b, a] E1[Floor[n / j], k - 1, a, b], {j, 1, n}];
E1[n_, 0, a_, b_] := 1
E1f[n_, k_, a_, b_] := b^-k E1[ n b^k, k, a, b]
E1h[n_, 0, a_, a1_, b1_] := 1
E1h[n_, k_, a_, a1_, b1_] := Sum[If[ mm[m, b1, a1] == 0, 0, Binomial[k, j] mm[m, b1, a1]^j
    E1h[Floor[n / (m^j)], k - j, m + 1, a1, b1]], {j, 1, k}, {m, a, Floor[n^(1 / k)]}]
E1hf[n_, k_, a_, b_] := b^-k E1h[ n b^k, k, 1, a, b]


{E1f[nn = 150, kk = 4, aa = 4, bb = 3], E1o[nn, kk, aa / bb], E1hf[ nn, kk, aa, bb]}
```

$$\left\{ \frac{23\,951}{81}, \frac{23\,951}{81}, \frac{23\,951}{81} \right\}$$

```
E2o[n_, k_, x_] := Sum[E2o[n / j, k - 1, x], {j, 2, n}] -
  x Sum[E2o[n / (x j), k - 1, x], {j, 1, n / x}]; E2o[n_, 0, x_] := 1
mm[n_, a_, b_] := If[ Mod[n, a] == 0, a, 0] - If[Mod[n, b] == 0, b, 0]
E2[n_, k_, a_, b_] := Sum[mm[ j, b, a] E2[Floor[n / j], k - 1, a, b], {j, b + 1, n}];
E2[n_, 0, a_, b_] := 1
E2f[n_, k_, a_, b_] := b^-k E2[ n b^k, k, a, b]
E1ftoE2f[ n_, k_, a_, b_] := Sum[ (-1)^(k - j) Binomial[ k, j] E1f[ n, j, a, b], {j, 0, k}]


{E2f[nn = 201, aa = 3, bb = 7, cc = 6], E2o[nn, aa, bb / cc], E1ftoE2f[nn, aa, bb, cc]}
```

$$\left\{ -\frac{1903}{108}, -\frac{1903}{108}, -\frac{1903}{108} \right\}$$

```
mm[n_, a_, b_] := If[ Mod[n, a] == 0, a, 0] - If[Mod[n, b] == 0, b, 0]
E2o[n_, k_, x_] := Sum[E2o[n / j, k - 1, x], {j, 2, n}] -
  x Sum[E2o[n / (x j), k - 1, x], {j, 1, n / x}]; E2o[n_, 0, x_] := 1
E2[n_, k_, a_, b_] := E2[n, k, a, b] =
  Sum[mm[ j, b, a] E2[Floor[n / j], k - 1, a, b], {j, b + 1, n}]; E2[n_, 0, a_, b_] := 1
E2a[n_, k_, a_, b_] := E2a[n, k, a, b] = Sum[If[ mm[ j, b, a] == 0, 0,
    mm[ j, b, a] E2a[Floor[n / j], k - 1, a, b]], {j, b + 1, n}]; E2a[n_, 0, a_, b_] := 1
E2b[n_, k_, a_, b_] := E2b[n, k, a, b] = (1 / b) Sum[If[ mm[ j, b, a] == 0, 0,
     mm[ j, b, a] E2b[(b n) / j, k - 1, a, b]], {j, b + 1, b n}]; E2b[n_, 0, a_, b_] := 1
E2f[n_, k_, a_, b_] := b^-k E2[ n b^k, k, a, b]
E2fa[n_, k_, a_, b_] := b^-k E2a[ n b^k, k, a, b]
E2h[n_, 0, a_, a1_, b1_] := 1
E2h[n_, k_, a_, a1_, b1_] :=
 E2h[n, k, a, a1, b1] = Sum[If[ mm[m, b1, a1] == 0, 0, Binomial[k, j] mm[m, b1, a1]^j
    E2h[Floor[n / (m^j)], k - j, m + 1, a1, b1]], {j, 1, k}, {m, a, Floor[n^(1 / k)]}]
E2hf[n_, k_, a_, b_] := b^-k E2h[ n b^k, k, b + 1, a, b]
```

```
{E2f[nn = 120, aa = 4, bb = 4, cc = 3], E2o[nn, aa, bb / cc],
 E2fa[ nn, aa, bb, cc], E2hf[ nn, aa, bb, cc], E2b[ nn, aa, bb, cc]}
```

$$\left\{-\frac{2452}{9}, -\frac{2452}{9}, -\frac{2452}{9}, -\frac{2452}{9}, -\frac{2452}{9}\right\}$$

```
referenceRiemanPrimeCount[n_] := Sum[PrimePi[n^(1 / k)] / k, {k, 1, Floor[Log[2, n]]}]
mm[n_, a_, b_] := If[ Mod[n, a] == 0, a, 0] - If[Mod[n, b] == 0, b, 0]
E2h[n_, 0, a_, a1_, b1_] := 1
E2h[n_, 1, a_, a1_, b1_] := If[ n < a, 0,
  (b1 Floor[n / b1] - a1 Floor[n / a1]) - (b1 Floor[(a - 1) / b1] - a1 Floor[(a - 1) / a1])]
E2h[n_, k_, a_, a1_, b1_] := E2h[n, k, a, a1, b1] =
  Sum[If[ mm[m, b1, a1] == 0, 0, Binomial[k, j] mm[m, b1, a1]^j
    E2h[Floor[n / (m^j)], k - j, m + 1, a1, b1]], {j, 1, k}, {m, a, Floor[n^(1 / k)]}]
E2hf[n_, k_, a_, b_] := b^-k E2h[ n b^k, k, b + 1, a, b]
P[n_, a1_, b1_] := Sum[(a1 / b1)^j / j, {j, 1, Log[(a1 / b1), n]}] + Sum[
   (-1)^(k + 1) / k E2hf[n, k, a1, b1], {k, 1, If[(a1 / b1) < 2, Log[(a1 / b1), n], Log[2, n]]}]
Table[{n, referenceRiemanPrimeCount[n], P[n, 5, 2], P[n, 3, 2], P[n, 4, 3]}, {n, 1, 100}] //
 TableForm
```

1	0	0	0	0
2	1	1	1	1
3	2	2	2	2
4	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$
5	$\frac{7}{2}$	$\frac{7}{2}$	$\frac{7}{2}$	$\frac{7}{2}$
6	$\frac{7}{2}$	$\frac{7}{2}$	$\frac{7}{2}$	$\frac{7}{2}$
7	$\frac{9}{2}$	$\frac{9}{2}$	$\frac{9}{2}$	$\frac{9}{2}$
8	$\frac{29}{6}$	$\frac{29}{6}$	$\frac{29}{6}$	$\frac{29}{6}$
9	$\frac{16}{3}$	$\frac{16}{3}$	$\frac{16}{3}$	$\frac{16}{3}$
10	$\frac{16}{3}$	$\frac{16}{3}$	$\frac{16}{3}$	$\frac{16}{3}$
11	$\frac{19}{3}$	$\frac{19}{3}$	$\frac{19}{3}$	$\frac{19}{3}$

12	$-\frac{19}{3}$	$-\frac{19}{3}$	$-\frac{19}{3}$	$-\frac{19}{3}$
13	$\frac{22}{3}$	$\frac{22}{3}$	$\frac{22}{3}$	$\frac{22}{3}$
14	$\frac{22}{3}$	$\frac{22}{3}$	$\frac{22}{3}$	$\frac{22}{3}$
15	$\frac{22}{3}$	$\frac{22}{3}$	$\frac{22}{3}$	$\frac{22}{3}$
16	$\frac{91}{12}$	$\frac{91}{12}$	$\frac{91}{12}$	$\frac{91}{12}$
17	$\frac{103}{12}$	$\frac{103}{12}$	$\frac{103}{12}$	$\frac{103}{12}$
18	$\frac{103}{12}$	$\frac{103}{12}$	$\frac{103}{12}$	$\frac{103}{12}$
19	$\frac{115}{12}$	$\frac{115}{12}$	$\frac{115}{12}$	$\frac{115}{12}$
20	$\frac{115}{12}$	$\frac{115}{12}$	$\frac{115}{12}$	$\frac{115}{12}$
21	$\frac{115}{12}$	$\frac{115}{12}$	$\frac{115}{12}$	$\frac{115}{12}$
22	$\frac{115}{12}$	$\frac{115}{12}$	$\frac{115}{12}$	$\frac{115}{12}$
23	$\frac{127}{12}$	$\frac{127}{12}$	$\frac{127}{12}$	$\frac{127}{12}$
24	$\frac{127}{12}$	$\frac{127}{12}$	$\frac{127}{12}$	$\frac{127}{12}$
25	$\frac{133}{12}$	$\frac{133}{12}$	$\frac{133}{12}$	$\frac{133}{12}$
26	$\frac{133}{12}$	$\frac{133}{12}$	$\frac{133}{12}$	$\frac{133}{12}$
27	$\frac{137}{12}$	$\frac{137}{12}$	$\frac{137}{12}$	$\frac{137}{12}$
28	$\frac{137}{12}$	$\frac{137}{12}$	$\frac{137}{12}$	$\frac{137}{12}$
29	$\frac{149}{12}$	$\frac{149}{12}$	$\frac{149}{12}$	$\frac{149}{12}$
30	$\frac{149}{12}$	$\frac{149}{12}$	$\frac{149}{12}$	$\frac{149}{12}$
31	$\frac{161}{12}$	$\frac{161}{12}$	$\frac{161}{12}$	$\frac{161}{12}$
32	$\frac{817}{60}$	$\frac{817}{60}$	$\frac{817}{60}$	$\frac{817}{60}$
33	$\frac{817}{60}$	$\frac{817}{60}$	$\frac{817}{60}$	$\frac{817}{60}$
34	$\frac{817}{60}$	$\frac{817}{60}$	$\frac{817}{60}$	$\frac{817}{60}$
35	$\frac{817}{60}$	$\frac{817}{60}$	$\frac{817}{60}$	$\frac{817}{60}$
36	$\frac{817}{60}$	$\frac{817}{60}$	$\frac{817}{60}$	$\frac{817}{60}$
37	$\frac{877}{60}$	$\frac{877}{60}$	$\frac{877}{60}$	$\frac{877}{60}$
38	$\frac{877}{60}$	$\frac{877}{60}$	$\frac{877}{60}$	$\frac{877}{60}$
39	$\frac{877}{60}$	$\frac{877}{60}$	$\frac{877}{60}$	$\frac{877}{60}$
40	$\frac{877}{60}$	$\frac{877}{60}$	$\frac{877}{60}$	$\frac{877}{60}$
41	$\frac{937}{60}$	$\frac{937}{60}$	$\frac{937}{60}$	$\frac{937}{60}$
42	$\frac{937}{60}$	$\frac{937}{60}$	$\frac{937}{60}$	$\frac{937}{60}$
43	$\frac{997}{60}$	$\frac{997}{60}$	$\frac{997}{60}$	$\frac{997}{60}$
44	$\frac{997}{60}$	$\frac{997}{60}$	$\frac{997}{60}$	$\frac{997}{60}$
45	$\frac{997}{60}$	$\frac{997}{60}$	$\frac{997}{60}$	$\frac{997}{60}$
46	$\frac{997}{60}$	$\frac{997}{60}$	$\frac{997}{60}$	$\frac{997}{60}$
47	$\frac{1057}{60}$	$\frac{1057}{60}$	$\frac{1057}{60}$	$\frac{1057}{60}$
48	$\frac{1057}{60}$	$\frac{1057}{60}$	$\frac{1057}{60}$	$\frac{1057}{60}$
49	$\frac{1087}{60}$	$\frac{1087}{60}$	$\frac{1087}{60}$	$\frac{1087}{60}$
50	$\frac{1087}{60}$	$\frac{1087}{60}$	$\frac{1087}{60}$	$\frac{1087}{60}$

51	$\frac{1087}{60}$	$\frac{1087}{60}$	$\frac{1087}{60}$	$\frac{1087}{60}$
52	$\frac{1087}{60}$	$\frac{1087}{60}$	$\frac{1087}{60}$	$\frac{1087}{60}$
53	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$
54	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$
55	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$
56	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$
57	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$
58	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$	$\frac{1147}{60}$
59	$\frac{1207}{60}$	$\frac{1207}{60}$	$\frac{1207}{60}$	$\frac{1207}{60}$
60	$\frac{1207}{60}$	$\frac{1207}{60}$	$\frac{1207}{60}$	$\frac{1207}{60}$
61	$\frac{1267}{60}$	$\frac{1267}{60}$	$\frac{1267}{60}$	$\frac{1267}{60}$
62	$\frac{1267}{60}$	$\frac{1267}{60}$	$\frac{1267}{60}$	$\frac{1267}{60}$
63	$\frac{1267}{60}$	$\frac{1267}{60}$	$\frac{1267}{60}$	$\frac{1267}{60}$
64	$\frac{1277}{60}$	$\frac{1277}{60}$	$\frac{1277}{60}$	$\frac{1277}{60}$
65	$\frac{1277}{60}$	$\frac{1277}{60}$	$\frac{1277}{60}$	$\frac{1277}{60}$
66	$\frac{1277}{60}$	$\frac{1277}{60}$	$\frac{1277}{60}$	$\frac{1277}{60}$
67	$\frac{1337}{60}$	$\frac{1337}{60}$	$\frac{1337}{60}$	$\frac{1337}{60}$
68	$\frac{1337}{60}$	$\frac{1337}{60}$	$\frac{1337}{60}$	$\frac{1337}{60}$
69	$\frac{1337}{60}$	$\frac{1337}{60}$	$\frac{1337}{60}$	$\frac{1337}{60}$
70	$\frac{1337}{60}$	$\frac{1337}{60}$	$\frac{1337}{60}$	$\frac{1337}{60}$
71	$\frac{1397}{60}$	$\frac{1397}{60}$	$\frac{1397}{60}$	$\frac{1397}{60}$
72	$\frac{1397}{60}$	$\frac{1397}{60}$	$\frac{1397}{60}$	$\frac{1397}{60}$
73	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$
74	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$
75	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$
76	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$
77	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$
78	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$	$\frac{1457}{60}$
79	$\frac{1517}{60}$	$\frac{1517}{60}$	$\frac{1517}{60}$	$\frac{1517}{60}$
80	$\frac{1517}{60}$	$\frac{1517}{60}$	$\frac{1517}{60}$	$\frac{1517}{60}$
81	$\frac{383}{15}$	$\frac{383}{15}$	$\frac{383}{15}$	$\frac{383}{15}$
82	$\frac{383}{15}$	$\frac{383}{15}$	$\frac{383}{15}$	$\frac{383}{15}$
83	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$
84	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$
85	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$
86	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$
87	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$
88	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$	$\frac{398}{15}$
89	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$

90	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$
91	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$
92	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$
93	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$
94	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$
95	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$
96	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$	$\frac{413}{15}$
97	$\frac{428}{15}$	$\frac{428}{15}$	$\frac{428}{15}$	$\frac{428}{15}$
98	$\frac{428}{15}$	$\frac{428}{15}$	$\frac{428}{15}$	$\frac{428}{15}$
99	$\frac{428}{15}$	$\frac{428}{15}$	$\frac{428}{15}$	$\frac{428}{15}$
100	$\frac{428}{15}$	$\frac{428}{15}$	$\frac{428}{15}$	$\frac{428}{15}$

Table[{ E1[n, 1, aa = 7, bb = 2], bb Floor[n / bb] – aa Floor[n / aa]}, {n, 1, 30}] // TableForm

0	0
2	2
2	2
4	4
4	4
6	6
–1	–1
1	1
1	1
3	3
3	3
5	5
5	5
0	0
0	0
2	2
2	2
4	4
4	4
6	6
–1	–1
1	1
1	1
3	3
3	3
5	5
5	5
0	0
0	0
2	2

```
Table[ { E2[n, 1, aa = 7, bb = 2],  (bb Floor[(n) / bb] - aa Floor[(n) / aa]) -
     (bb Floor[(bb) / bb] - aa Floor[(bb) / aa]) }, {n, 1, 30}] // TableForm
```

0	− 2
0	0
0	0
2	2
2	2
4	4
− 3	− 3
− 1	− 1
− 1	− 1
1	1
1	1
3	3
3	3
− 2	− 2
− 2	− 2
0	0
0	0
2	2
2	2
4	4
− 3	− 3
− 1	− 1
− 1	− 1
1	1
1	1
3	3
3	3
− 2	− 2
− 2	− 2
0	0

```
Table[ { E2[n, 1, aa = 7, bb = 2],  (bb Floor[(n) / bb] - aa Floor[(n) / aa]) -
     (bb Floor[(bb) / bb] - aa Floor[(bb) / aa]) }, {n, 1, 30}] // TableForm
```

```
Table[ { E2[n, 1, aa = 7, bb = 2],  (bb Floor[(n) / bb] - aa Floor[(n) / aa]) -
    (bb Floor[(bb) / bb] - aa Floor[(bb) / aa])}, {n, 1, 30}] // TableForm
```

0	− 2
0	0
0	0
2	2
2	2
4	4
− 3	− 3
− 1	− 1
− 1	− 1
1	1
1	1
3	3
3	3
− 2	− 2
− 2	− 2
0	0
0	0
2	2
2	2
4	4
− 3	− 3
− 1	− 1
− 1	− 1
1	1
1	1
3	3
3	3
− 2	− 2
− 2	− 2
0	0

```
hh[ n_, 1, a_, a1_, b1_] := If[ n < a, 0,
   (b1 Floor[n / b1] - a1 Floor[n / a1]) - (b1 Floor[(a - 1) / b1] - a1 Floor[(a - 1) / a1])]
Grid[Table[{ hh[n, kk = 1, aa, bb = 11, cc = 5] - E2h[n, kk, aa, bb, cc]},
   {n, 2, 20}, {aa, 2, 16}]]
```

{0} {0} {0} {0} {0} {0} {0} {0} {0} {0} {0} {0} {0} {0} {0}
{0} {0} {0} {0} {0} {0} {0} {0} {0} {0} {0} {0} {0} {0} {0}
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{0} {0} {0} {0} {0} {0} {0} {0} {0} {0} {0} {0} {0} {0} {0}
{0} {0} {0} {0} {0} {0} {0} {0} {0} {0} {0} {0} {0} {0} {0}

```
E2o[n_, k_, x_] :=
 E2o[n, k, x] = Sum[E2o[n / j, k - 1, x], {j, 2, n}] - x Sum[E2o[n / (x j), k - 1, x], {j, 1, n / x}];
E2o[n_, 0, x_] := 1
P[n_, a1_, b1_] := Sum[(a1 / b1) ^ j / j, {j, 1, Log[(a1 / b1), n]}] +
   Sum[(-1) ^ (k + 1) / k E2o[n, k, a1 / b1], {k, 1, If[(a1 / b1) < 2, Log[(a1 / b1), n], Log[2, n]]}]
P[100, 15, 14]
```

$$\frac{428}{15}$$

```
E1o[n_, k_, x_] := Sum[E1o[n / j, k - 1, x], {j, 1, n}] -
   x Sum[E1o[n / (x j), k - 1, x], {j, 1, n / x}]; E1o[n_, 0, x_] := 1
E2o[n_, k_, x_] := Sum[E2o[n / j, k - 1, x], {j, 2, n}] -
   x Sum[E2o[n / (x j), k - 1, x], {j, 1, n / x}]; E2o[n_, 0, x_] := 1
alpha[n_, a_, b_] := b ( Floor[ n / b] - Floor[(n - 1) / b]) - a ( Floor[ n / a] - Floor[(n - 1) / a])
E2[n_, k_, a_, b_] :=
 (1 / b) Sum[If[ alpha[ j, a, b] == 0, 0, alpha[ j, a, b] E2[(b n) / j, k - 1, a, b]],
   {j, b + 1, b n}]; E2[n_, 0, a_, b_] := 1
E1[n_, k_, a_, b_] := (1 / b) Sum[If[ alpha[ j, a, b] == 0, 0,
    alpha[ j, a, b] E1[(b n) / j, k - 1, a, b]], {j, 1, b n}]; E1[n_, 0, a_, b_] := 1
E1o[ 123, 3, 4 / 3]
```

$$-\frac{548}{9}$$

E1[123, 3, 4, 3]

$$-\frac{548}{9}$$