

```

Clear[Dd]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
Dd[n_, 0, x_] := UnitStep[n - 1]
Dd[n_, k_, x_] :=
  Dd[n, k, x] = Sum[Binomial[k, j] Dd[n / (m^(k - j)), j, m + 1], {m, x, n^(1 / k)}, {j, 0, k - 1}]
yD[n_, k_, x_] := x^k - k Dd[n x^k, k, x + 1]
yDl[n_, z_, x_] := Sum[bin[z, k] yD[n, k, x], {k, 0, 30}]
yDm[n_, k_, x_, m_] := Sum[(-1)^(k - j) bin[k, j] yDl[n, m j, x], {j, 0, k}]
yDm1[n_, z_, x_, m_] := Sum[bin[z, k] yDm[n, k, x, m], {k, 0, 30}]
yDt[n_, k_, x_, t_] := Sum[(-1)^(k - j) bin[k, j] t^j yDl[n, j, x], {j, 0, k}]
yDt1[n_, z_, x_, t_] := Sum[bin[z, k] yDt[n, k, x, t], {k, 0, 40}]

```

```

D[Expand@yDl[100, z, 2], z] /. z -> 0

```

$$\frac{202986703}{7096320}$$

```

Table[yD[100, k, 2], {k, 0, 20}]

```

```

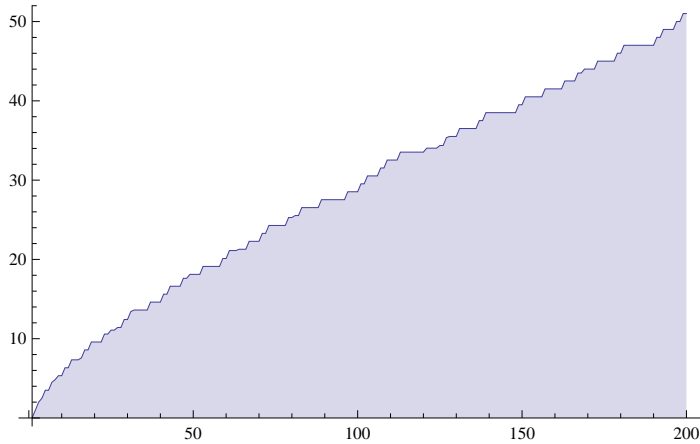
{1, 99, 318,  $\frac{1901}{4}$ ,  $\frac{3379}{8}$ ,  $\frac{1937}{8}$ ,  $\frac{1487}{16}$ ,  $\frac{2983}{128}$ ,  $\frac{1087}{256}$ ,  $\frac{229}{512}$ ,  $\frac{21}{1024}$ ,  $\frac{1}{2048}$ , 0, 0, 0, 0, 0, 0, 0, 0, 0}

```

```

DiscretePlot[D[Expand@yDl[n, z, 1], z] /. z -> 0, {n, 1, 200}]

```



```

yDm[100, 2, 2, 2]

```

$$\frac{28763}{8}$$

```

yD[100, 2, 2]

```

```

318

```

```

D[Expand[yDm1[100, z, 1, 1]], z] /. z -> 0

```

$$\frac{428}{15}$$

```

D[Expand[yDm1[100, z, 1, 2]], z] /. z -> 0

```

$$\frac{856}{15}$$

```
D[Expand[yDm1[100, z, 2, 1]], z] /. z -> 0
```

```
202986703
```

```
7096320
```

```
(D[Expand[yDm1[100, z, 2, -1]], z] /. z -> 0) / -1
```

```
202986703
```

```
7096320
```

```
(D[Expand[yDt1[10, z, 3, .4]], z] /. z -> 0) - D[yD1[10, z, 3], z] /. z -> 0
```

```
-0.916291
```

```
E^-0.9162911597887128`
```

```
0.4
```

```
E^-0.6931471806678493`
```

```
0.5
```

```
Clear[Dly1, Dds, yD1]
```

```
Dly1[n_, s_, k_, x_] :=
```

```
  Dly1[n, s, k, x] = x Sum[(1 + j x)^-s Dly1[n (1 + j x)^-1, s, k - 1, x], {j, 1, (n - 1) / x}]
```

```
Dly1[n_, s_, 0, x_] := UnitStep[n - 1]
```

```
Dn[n_, s_, k_, x_] := Dly1[n, s, k, 1 / x]
```

```
Dds[n_, s_, 0, x_] := UnitStep[n - 1]
```

```
Dds[n_, s_, k_, x_] :=
```

```
  Dds[n, s, k, x] = Sum[Binomial[k, j] m^(-(k - j) s) Dds[Floor[n / (m^(k - j))], s, j, m + 1],  
    {m, x, n^(1 / k)}, {j, 0, k - 1}]
```

```
yDs[n_, s_, k_, x_] := x^(k (s - 1)) Dds[n x^k, s, k, x + 1]
```

```
yDlo[n_, s_, z_, x_] := yD1[n, s, z, x] = Sum[bin[z, k] yDs[n, s, k, x], {k, 0, 30}]
```

```
yD1[n_, s_, z_, x_] :=
```

```
  Sum[bin[z, k] x^(k (s - 1)) Dds[n x^k, s, k, x + 1], {k, 0, Log[n] / (Log[x + 1] - Log[x])}]
```

```
yDm[n_, s_, k_, x_, m_] := Sum[(-1)^(k - j) bin[k, j] yD1[n, s, m j, x], {j, 0, k}]
```

```
yDm1[n_, s_, z_, x_, m_] :=
```

```
  Sum[bin[z, k] yDm[n, s, k, x, m], {k, 0, Log[If[1 + 1 / x < 2, 1 + 1 / x, 2], n]}]
```

```
yDm1a[n_, s_, z_, x_, m_] :=
```

```
  Table[bin[z, k] yDm[n, s, k, x, m], {k, 0, Log[If[1 + 1 / x < 2, 1 + 1 / x, 2], n]}]
```

```
yDt[n_, s_, k_, x_, t_] := Sum[(-1)^(k - j) bin[k, j] t^j yD1[n, s, j, x], {j, 0, k}]
```

```
yDt1[n_, s_, z_, x_, t_] := Sum[bin[z, k] yDt[n, s, k, x, t], {k, 0, 40}]
```

```
l[n_] := Sum[Log[j] - l[n / j], {j, 2, n}]
```

```
Dn[100, 0, 2, 3]
```

```
995
```

```
3
```

```
yD[100, 2, 3]
```

```
995
```

```
3
```

```

D[Expand[yDm1[100, -2, z, 1, 1]], z] /. z -> 0
1 404 271
20
(D[Expand[yDm1[100, -2, z, 1, 2]], z] /. z -> 0) / 2
1 404 271
20
D[Expand[yDm1[100, -2, z, 2, 1]], z] /. z -> 0
251 454 465 703 053 807
3 307 124 817 920
(D[Expand[yDm1[100, -2, z, 2, -3]], z] /. z -> 0) / -3
251 454 465 703 053 807
3 307 124 817 920
N[D[D[yDm1[20, r, z, 3, 1], z] /. z -> 0, r] /. r -> 0]
-16.6612
N[D[D[yDm1[20, r, z, 3, 3], z] /. z -> 0, r] /. r -> 0] / 3
-16.6612
N[D[D[yDt1[20, r, z, 1, 1], z] /. z -> 0, r] /. r -> 0]
-19.3301
N[D[D[yDt1[20, r, z, 1, .5], z] /. z -> 0, r] /. r -> 0]
-19.2657

(D[Expand[yDt1[20, 3, z, 4, .73]], z] /. z -> 0) - D[yD1[20, 3, z, 4], z] /. z -> 0
-0.31471
Log[.73]
-0.314711
Clear[ds]
ds[n_, a_, k_, x_] :=
  ds[n, a, k, x] = Sum[ds[n / (i j^a), a, k - 1], {i, 1 + x, n, x}, {j, 1 + x, (n / i)^(1 / a), x}]
ds[n_, a_, 0, x_] := UnitStep[n - 1]
aa[n_, k_] := Sum[j^k, {j, 1, n^(1 / k)}]
100^(1 / (-2))
1
10
N@Sum[1 / (m!) (-1)^(k + 1) / k (Limit[D[x / Log[1 + x], {x, m}], x -> 0])
  (y^(r - 1) Zeta[r, 1 + y])^(m + k), {k, 1, 20}, {m, 0, 20}] /. {y -> 2, r -> 2}
0.789602
N[y^(r - 1) Zeta[r, 1 + y] /. {y -> 2, r -> 2}]
0.789868

```

```

Clear[Dds]
Dds[n_, s_, 0, x_] := UnitStep[n - 1]
Dds[n_, s_, k_, x_] :=
  Dds[n, s, k, x] = Sum[Binomial[k, j] m^(-(k - j) s) Dds[Floor[n / (m^(k - j))], s, j, m + 1],
    {m, x, n^(1 / k)}, {j, 0, k - 1}]
yDs[n_, s_, k_, x_] := x^(k (s - 1)) Dds[n x^k, s, k, x + 1]

N@Sum[1 / (m!) (-1)^(k + 1) / k (Limit[D[x / Log[1 + x], {x, m}], x → 0])
  yDs[100, 2, m + k + 1, 3], {k, 1, 13}, {m, 0, 13}]

0.672226

N@yDs[100, 2, 2, 3]

0.672226

Table[Chop[1 / ((r + I) - 1) - N[y^((r + I) - 1) Zeta[(r + I), 1 + y] /. y → 100 000]], {r, 1, 5}]
{4.99999 × 10-6, 4.99998 × 10-6, 4.99997 × 10-6, 4.99997 × 10-6, 4.99996 × 10-6}

```