```
-Integrate[x^(-s k) / k, {k, 1, Infinity}]
```

```
ConditionalExpression[-Gamma[0, s Log[x]], Re[s Log[x]] > 0]
```

```
Integrate[x^k, {k, 0, Infinity}]
```

$$\text{ConditionalExpression}\left[-\frac{1}{\text{Log}[x]}, \text{Re}[\text{Log}[x]] < 0\right]$$

```
Integrate[x^k k, {k, 0, Infinity}]
```

$$\text{ConditionalExpression}\left[\frac{1}{\text{Log}[x]^2}, \text{Re}[\text{Log}[x]] < 0\right]$$

```
Integrate[x^k / k^2, {k, 1, Infinity}]
```

```
ConditionalExpression[x + Gamma[0, -Log[x]] Log[x], Re[Log[x]] < 0]
```

```
-Integrate[x^(-s k) / k, {k, 1, Infinity}]
```

```
ConditionalExpression[-Gamma[0, s Log[x]], Re[s Log[x]] > 0]
```

```
FullSimplify@-Sum[x^(-s k) / k, {k, 1, Infinity}]
```

$$\text{Log}[1 - x^{-s}]$$

```
-Integrate[x^(-s k) / k, {k, 1, Infinity}]
```

```
ConditionalExpression[-Gamma[0, s Log[x]], Re[s Log[x]] > 0]
```

```
Integrate[1 / (k!), {k, 0, n}]
```

$$\int_0^n \frac{1}{k!}\, dk$$

```
Integrate[1 / (k!) × 1 / (j!), {k, 0, n}, {j, 0, n - k}]
```

$$\int_0^n \int_0^{-k+n} \frac{1}{j!\,k!}\, dj\, dk$$

```
Integrate[1, {j, 0, n}, {k, 0, n - j}]
```

$$\frac{n^2}{2}$$

```
Integrate[1, {j, 0, n}, {k, 0, (n - j)^(1 / 2)}]
```

$$\frac{2 n^{3/2}}{3}$$

```
Integrate[1, {j, 0, n}, {k, 0, (n - j)^(1 / 7)}]
```

$$\frac{7 n^{8/7}}{8}$$

```
Integrate[1, {j, 0, n}, {k, 0, n - j}, {l, 0, (n - j - k)^(1 / 2)}, {m, 0, (n - j - k - l^2)^(1 / 2)}]
```

$$\frac{n^3 \pi}{24}$$

```
Integrate[1, {j, 0, n}, {k, 0, (n - j) / 2}]
```

$$\frac{n^2}{4}$$

```
Integrate[1, {j, 0, n}, {k, 0, n - j}, {l, 0, (n - j - k) / 2}, {m, 0, (n - j - k - 2 l) / 2}]
```

$$\frac{n^4}{96}$$

```
Integrate[1, {j, 0, n}, {k, 0, n - j}, {l, 0, (n - j - k)}, {m, 0, (n - j - k - l) / 2},
 {o, 0, (n - j - k - l - 2 m) / 2}, {p, 0, (n - j - k - l - 2 m - 2 o) / 2}]
```

$$\frac{n^6}{5760}$$

```
5760 / 6!
```

8

```
aa[n_, z_] := (n + z) ! / (n!) / (z!)
da[n_, z_] := Pochhammer[z, n] / n!
ab[n_, z_] := (n + z - 1) ! / (n!) / ((z - 1) !)
ac[n_, z_] := Sum[da[n - 2 j, z] da[j, -z], {j, 0, n / 2}]
acf[n_, z_] := Sum[da[k, z] da[j, -z], {j, 0, n / 2}, {k, 0, n - 2 j}]
binomial[z_, k_] := binomial[z, k] = Product[z - j, {j, 0, k - 1}] / k!
```

```
FullSimplify@Sum[da[j, z], {j, 0, n}]
```

$$\frac{\text{Gamma}[1+n+z]}{\text{Gamma}[1+n]\ \text{Gamma}[1+z]}$$

```
FullSimplify@Sum[ab[j, z], {j, 0, n}]
```

$$\frac{\text{Gamma}[1+n+z]}{\text{Gamma}[1+n]\ \text{Gamma}[1+z]}$$

```
Expand@ac[10, z]
```

$$-\frac{z}{10}+\frac{7129\,z^2}{25\,200}-\frac{1303\,z^3}{4032}+\frac{4523\,z^4}{22\,680}-\frac{19\,z^5}{256}+\frac{3013\,z^6}{172\,800}-\frac{z^7}{384}+\frac{29\,z^8}{120\,960}-\frac{z^9}{80\,640}+\frac{z^{10}}{3\,628\,800}$$

```
Expand@binomial[z, 10]
```

$$-\frac{z}{10}+\frac{7129\,z^2}{25\,200}-\frac{1303\,z^3}{4032}+\frac{4523\,z^4}{22\,680}-\frac{19\,z^5}{256}+\frac{3013\,z^6}{172\,800}-\frac{z^7}{384}+\frac{29\,z^8}{120\,960}-\frac{z^9}{80\,640}+\frac{z^{10}}{3\,628\,800}$$

```
Expand@acf[10, z]
```

$$1+\frac{1627\,z}{2520}+\frac{19\,013\,z^2}{50\,400}-\frac{2729\,z^3}{25\,920}+\frac{8161\,z^4}{72\,576}-\frac{1339\,z^5}{34\,560}+\frac{1753\,z^6}{172\,800}-\frac{193\,z^7}{120\,960}+\frac{z^8}{6048}-\frac{z^9}{103\,680}+\frac{z^{10}}{3\,628\,800}$$

```
Expand@Sum[binomial[z, k], {k, 0, 10}]
```

$$1+\frac{1627\,z}{2520}+\frac{19\,013\,z^2}{50\,400}-\frac{2729\,z^3}{25\,920}+\frac{8161\,z^4}{72\,576}-\frac{1339\,z^5}{34\,560}+\frac{1753\,z^6}{172\,800}-\frac{193\,z^7}{120\,960}+\frac{z^8}{6048}-\frac{z^9}{103\,680}+\frac{z^{10}}{3\,628\,800}$$

7

```
i2[n_] := If[Log2@n == Floor[Log2@n], Log2@n, 0]
i2a[n_, v_] := If[n == 1, 0, If[Log[v, n] == Floor[Log[v, n]], n / Log[v, n], 0]]
i2b[n_, v_] := If[n / v == Floor[n / v], n - (n / v), 0]
```

```
Table[i2a[n, 2], {n, 2, 32}]
```

$$\left\{2, 0, 2, 0, 0, 0, \frac{8}{3}, 0, 0, 0, 0, 0, 0, 0, 4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{32}{5}\right\}$$

```
Table[2^k / k, {k, 1, 5}]
```

$$\left\{2, 2, \frac{8}{3}, 4, \frac{32}{5}\right\}$$

```
Clear[mo]
mo[n_, 0] := UnitStep[n]
mo[n_, k_] := mo[n, k] = Sum[(1 / j + i2b[j, 2]) mo[n - j, k - 1], {j, 1, n}]
mz[n_, z_] := Sum[z^k / k ! mo[n, k], {k, 0, n}]
ma[n_, 0] := UnitStep[n]
ma[n_, k_] := ma[n, k] = Sum[(1 / j) ma[n - j, k - 1], {j, 1, n}]
maz[n_, z_] := Sum[z^k / k ! ma[n, k], {k, 0, n}]
```

```
mz[10, z] /. z → 1
```

$$\frac{1123}{10}$$

```
Expand[Product[z + k, {k, 0, 10}] / Product[k, {k, 1, 10}]]
```

$$z + \frac{7381 z^2}{2520} + \frac{177133 z^3}{50400} + \frac{84095 z^4}{36288} + \frac{341693 z^5}{362880} + \frac{8591 z^6}{34560} + \frac{7513 z^7}{172800} + \frac{121 z^8}{24192} + \frac{11 z^9}{30240} + \frac{11 z^{10}}{725760} + \frac{z^{11}}{3628800}$$

```
Table[mz[n, 1] - maz[n, 1], {n, 1, 20}]
```

$$\left\{0, 1, 2, \frac{11}{2}, 9, \frac{53}{3}, \frac{79}{3}, \frac{1081}{24}, \frac{255}{4}, \frac{1013}{10}, \frac{2777}{20}, \frac{151789}{720}, \frac{101803}{360}, \frac{524449}{1260}, \frac{92345}{168}, \frac{10628491}{13440}, \frac{6934691}{6720}, \frac{18903079}{12960}, \frac{68409911}{36288}, \frac{9531080581}{3628800}\right\}$$

```
Sum[x^j, {j, 1, n}]
```

$$\frac{x(-1 + x^n)}{-1 + x}$$

$$\text{Limit}\left[\frac{x(-1 + x^n)}{-1 + x}, x \to 1\right]$$

$$n$$

```
Sum[x^(j + k), {j, 1, n}, {k, 1, n - j}]
```

$$\frac{x\left(x - n x^n + (-1 + n) x^{1+n}\right)}{(-1 + x)^2}$$

$$\text{Limit}\left[\frac{x\left(x - n x^n + (-1 + n) x^{1+n}\right)}{(-1 + x)^2}, x \to 1\right]$$

$$\frac{1}{2}(-1 + n) n$$

```
Sum[x^(j + k + l), {j, 1, n}, {k, 1, n - j}, {l, 1, n - j - k}]
```

$$\frac{x\left(-2 x^2 - n x^n + n^2 x^n + 4 n x^{1+n} - 2 n^2 x^{1+n} + 2 x^{2+n} - 3 n x^{2+n} + n^2 x^{2+n}\right)}{2(-1 + x)^3}$$

$$\text{FullSimplify@Limit}\left[\frac{x\left(-2x^2 - n x^n + n^2 x^n + 4 n x^{1+n} - 2 n^2 x^{1+n} + 2 x^{2+n} - 3 n x^{2+n} + n^2 x^{2+n}\right)}{2(-1+x)^3}, x \to 1\right]$$

$$\frac{1}{6}(-2+n)(-1+n)\,n$$

```
Integrate[x^j, {j, 0, x}]
```

$$\frac{-1 + x^x}{\text{Log}[x]}$$

$$\text{Limit}\left[\frac{-1 + x^n}{\text{Log}[x]}, x \to 1\right]$$

$$n$$

```
Integrate[x^(j + k), {j, 0, n}, {k, 0, n - j}]
```

$$\frac{1 - x^n + n x^n \text{Log}[x]}{\text{Log}[x]^2}$$

$$\text{Limit}\left[\frac{1 - x^n + n x^n \text{Log}[x]}{\text{Log}[x]^2}, x \to 1\right]$$

$$\frac{n^2}{2}$$

```
Integrate[x^(j + k + l), {j, 0, n}, {k, 0, n - j}, {l, 0, n - j - k}]
```

$$\frac{-2 + x^n (2 + n \text{Log}[x] (-2 + n \text{Log}[x]))}{2 \text{Log}[x]^3}$$

$$\text{Limit}\left[\frac{-2 + x^n (2 + n \text{Log}[x] (-2 + n \text{Log}[x]))}{2 \text{Log}[x]^3}, x \to 1\right]$$

$$\frac{n^3}{6}$$

```
(x^s)^t /. s → 2 /. t → 3
```

$$x^6$$

```
x^(s t) /. s → 2 /. t → 3
```

$$x^6$$

```
Sum[x^(j), {j, 1, n}]
```

$$\frac{x(-1 + x^n)}{-1 + x}$$

```
Sum[x^(j + k), {k, 1, n - j}]
```

$$-\frac{x\left(x^j - x^n\right)}{-1 + x}$$

```
Sum[x^(j + k + l), {l, 1, n - j - k}]
```

$$-\frac{x\left(x^{j+k} - x^n\right)}{-1 + x}$$

```
Integrate[x^j, {j, 0, x}]
```

$$\frac{-1+x^x}{\mathrm{Log}[x]}$$

```
Integrate[x^(j + k), {k, 0, x - j}]
```

$$\frac{-x^j+x^x}{\mathrm{Log}[x]}$$

```
Integrate[x^(j + k + l), {l, 0, x - j - k}]
```

$$\frac{-x^{j+k}+x^x}{\mathrm{Log}[x]}$$

```
Sum[Binomial[z, k] x^k, {k, 0, Infinity}]
```

$$(1+x)^z$$

```
xx[x_, z_] := Integrate[(z^k) / (k!) x^k / (k!), {k, 0, Infinity}]
xx[x, z]
```

$$\int_0^\infty \frac{x^k z^k}{(k!)^2}\, dk$$

```
Sum[Binomial[z, k] x^k / k!, {k, 0, Infinity}]
```

Hypergeometric1F1[-z, 1, -x]

```
Integrate[Binomial[z, k] x^k / k!, {k, 0, Infinity}]
```

$$\int_0^\infty \frac{x^k\, \mathrm{Binomial}[z, k]}{k!}\, dk$$

```
Sum[Binomial[z, k] Binomial[x, k], {k, 0, Infinity}]
```

$$\frac{\mathrm{Gamma}[1+x+z]}{\mathrm{Gamma}[1+x]\,\mathrm{Gamma}[1+z]}$$

```
N@Table[-(1 / x) LaguerreL[-z - 1, 1, Log[x]] /. z → 1, {x, 1, 10}]
```

{1., 1., 1., 1., 1., 1., 1., 1., 1., 1.}

```
-(1 / x) LaguerreL[-z - 1, 1, Log[x]] /. z → -1
```

$$-\frac{1}{x}$$

```
po[n_] := n^-s
g[n_, f_] := f[n] + Integrate[f[n / k], {k, 1, Infinity}]
f2[n_, f_] := g[n, f] - Integrate[(1 / r) g[n / r, f], {r, 1, Infinity}]
gb[n_, f_] := f[n] - Integrate[(1 / k) f[n / k], {k, 1, Infinity}]
f2b[n_, f_] := gb[n, f] + Integrate[ gb[n / r, f], {r, 1, Infinity}]
{po[n], FullSimplify@g[n, po], FullSimplify@f2[n, po],
 FullSimplify@gb[n, po], FullSimplify@f2b[n, po]}
```

$$\left\{n^{-s},\ \mathrm{ConditionalExpression}\left[\frac{n^{-s}\, s}{1+s},\ \mathrm{Re}[s] < -1\right],\ \mathrm{ConditionalExpression}[n^{-s},\ \mathrm{Re}[s] < -1],\right.$$
$$\left.\mathrm{ConditionalExpression}\left[\frac{n^{-s}\,(1+s)}{s},\ \mathrm{Re}[s] < 0\right],\ \mathrm{ConditionalExpression}[n^{-s},\ \mathrm{Re}[s] < -1]\right\}$$

```
pa[n_] := n^-s
ga[n_, f_] := f[n] + Integrate[f[n - k], {k, 0, n}, Assumptions → n ∈ Reals && n > 0]
f2a[n_, f_] :=
 ga[n, f] - Integrate[E^(-r) ga[n - r, f], {r, 0, n}, Assumptions → n ∈ Reals && n > 0]
gb[n_, f_] := f[n] - Integrate[E^(-k) f[n - k], {k, 0, n}, Assumptions → n ∈ Reals && n > 0]
f2b[n_, f_] := gb[n, f] + Integrate[ gb[n - r, f], {r, 0, n}, Assumptions → n ∈ Reals && n > 0]
{pa[n], FullSimplify@f2a[n, pa], FullSimplify@f2b[n, pa],
 FullSimplify@ga[n, pa], FullSimplify@gb[n, pa]}
```

$\{n^{-s}$, ConditionalExpression$[n^{-s}$, Re[s] $< 1]$,
ConditionalExpression$[n^{-s}$, Re[s] $< 1]$, ConditionalExpression$\left[\frac{n^{-s}(-1-n+s)}{-1+s}\right.$, Re[s] $\left.< 1\right]$,
ConditionalExpression$[n^{-s} - (-1)^s e^{-n}$ (s Gamma$[-s]$ + Gamma$[1-s, -n]$), Re[s] $< 1]\}$

```
N@Table[LaguerreL[z - 1, 1, -x] /. z → 1, {x, 1, 10}]
```

{1., 1., 1., 1., 1., 1., 1., 1., 1., 1.}

```
Table[LaguerreL[z - 1, 1, -x] /. z → -1, {x, 1, 10}]
```

$\left\{-\frac{1}{e}, -\frac{1}{e^2}, -\frac{1}{e^3}, -\frac{1}{e^4}, -\frac{1}{e^5}, -\frac{1}{e^6}, -\frac{1}{e^7}, -\frac{1}{e^8}, -\frac{1}{e^9}, -\frac{1}{e^{10}}\right\}$

```
pv[n_] := 1
gv[n_, f_] := f[n] + Sum[f[n - k], {k, 1, n - 1}]
f2v[n_, f_] := gv[n, f] - gv[n - 1, f]
{pv[n], FullSimplify@gv[n, pv], FullSimplify@f2v[n, pv]}
```

{1, n, 1}

```
Table[LaguerreL[z - 1, 1, -x] /. z → 2, {x, 0, 10}]
```

{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

```
Table[LaguerreL[z - 1, 1, -x] /. z → -2, {x, 0, 10}]
```

$\left\{-2, -\frac{1}{e}, 0, \frac{1}{e^3}, \frac{2}{e^4}, \frac{3}{e^5}, \frac{4}{e^6}, \frac{5}{e^7}, \frac{6}{e^8}, \frac{7}{e^9}, \frac{8}{e^{10}}\right\}$

```
pa[n_] := Sin[n]
ga[n_, f_] := f[n] + Integrate[(k + 2) f[n - k], {k, 0, n}]
f2a[n_, f_] := ga[n, f] + Integrate[(r - 2) E^(-r) ga[n - r, f], {r, 0, n}]
{pa[n], ga[n, pa], FullSimplify@f2a[n, pa]}
```

{Sin[n], 2 + n - 2 Cos[n], Sin[n]}

```
Table[-(1 / x) LaguerreL[-z - 1, 1, Log[x]] /. z → -2, {x, 1, 10}]
```

$\left\{-2, \frac{1}{2}(-2+\text{Log}[2]), \frac{1}{3}(-2+\text{Log}[3]), \frac{1}{4}(-2+\text{Log}[4]), \frac{1}{5}(-2+\text{Log}[5]),\right.$
$\left.\frac{1}{6}(-2+\text{Log}[6]), \frac{1}{7}(-2+\text{Log}[7]), \frac{1}{8}(-2+\text{Log}[8]), \frac{1}{9}(-2+\text{Log}[9]), \frac{1}{10}(-2+\text{Log}[10])\right\}$

```
pa[n_] := n^2
ga[n_, f_] := f[n] + Integrate[(LaguerreL[(3 + I) - 1, 1, -k]) f[n - k], {k, 0, n}]
f2a[n_, f_] := ga[n, f] + Integrate[(LaguerreL[(-3 - I) - 1, 1, -r]) ga[n - r, f], {r, 0, n}]
N@{pa[3], ga[3, pa], FullSimplify@f2a[3, pa]}
```

$\{9., 52.0991 + 32.5644\, i, 9. - 7.81597 \times 10^{-14}\, i\}$

```
pv[n_] := n
gv[n_, f_] := f[n] + Sum[Pochhammer[2, k] / (k!) f[n - k], {k, 1, n - 1}]
f2v[n_, f_] := gv[n, f] + Sum[Pochhammer[-2, r] / (r!) gv[n - r, f], {r, 1, n - 1}]
gvb[n_, f_] := f[n] + Sum[Pochhammer[-2, k] / (k!) f[n - k], {k, 1, n - 1}]
f2vb[n_, f_] := gvb[n, f] + Sum[Pochhammer[2, r] / (r!) gvb[n - r, f], {r, 1, n - 1}]
{pv[n], FullSimplify@gv[n, pv], FullSimplify@f2v[n, pv],
 FullSimplify@gvb[n, pv], FullSimplify@f2v[n, pv]}
```

$$\left\{n, \frac{1}{6} n (1+n) (2+n), n, \frac{(-2+n) n (1+n) \text{Pochhammer}[-2, n]}{4 \text{Gamma}[n]}, n\right\}$$

```
ss[z_, x_] := Sum[Binomial[z + 1, z - k] x^k / k!, {k, 0, Infinity}]
Table[ss[z - 1, j] /. z → 1, {j, 1, 10}]
```

{1, 1, 1, 1, 1, 1, 1, 1, 1, 1}

```
Table[ss[-z - 1, j] /. z → 1, {j, 1, 10}]
```

$$\left\{-\frac{1}{e}, -\frac{1}{e^2}, -\frac{1}{e^3}, -\frac{1}{e^4}, -\frac{1}{e^5}, -\frac{1}{e^6}, -\frac{1}{e^7}, -\frac{1}{e^8}, -\frac{1}{e^9}, -\frac{1}{e^{10}}\right\}$$

```
ms[n_, j_] := 1 - j (Floor[n / j] - Floor[(n - 1) / j])
pn[n_] := n^2
pr[t_, f_] := -Sum[ms[j, 2] Binomial[t, j] f[j], {j, 0, t}]
pr2[t_, f_] := -Sum[ms[j, 2] Binomial[t, j] pr[j, f], {j, 0, t}]
Table[pr2[n, pn], {n, 0, 10}]
```

{0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100}

```
Clear[d2, d1]
binomial[z_, k_] := binomial[z, k] = Product[z - j, {j, 0, k - 1}] / k!
d2[n_, k_] := d2[n, k] = Sum[d2[Floor[n / j], k - 1], {j, 2, n}]
d2[n_, 0] := UnitStep[n - 1]
d1[n_, k_] := d1[n, k] = Sum[d1[Floor[n / j], k - 1], {j, 1, n}]
d1[n_, 0] := UnitStep[n - 1]
d2z[n_, z_] := Sum[(-1)^k binomial[z, k] d1[n, k], {k, 0, Log2@n}]
d1z[n_, z_] := Sum[(-1)^k binomial[z, k] d2z[n, k], {k, 0, Log2@n}]
d2z[100, 1]
```

-99

```
Expand@d1z[100, z]
```

$$1 + \frac{428 z}{15} + \frac{16289 z^2}{360} + \frac{331 z^3}{16} + \frac{611 z^4}{144} + \frac{67 z^5}{240} + \frac{7 z^6}{720}$$

```
Table[ms[n, 3], {n, 1, 10}]
```

{1, 1, -2, 1, 1, -2, 1, 1, -2, 1}

```
pa[n_] := Sin[n]
ga[n_, f_] := f[n] + Integrate[f[n - k], {k, 0, n}]
f2a[n_, f_] := ga[n, f] - Integrate[E^(-r) ga[n - r, f], {r, 0, n}]
gb[n_, f_] := f[n] - Integrate[E^(-k) f[n - k], {k, 0, n}]
f2b[n_, f_] := gb[n, f] + Integrate[ gb[n - r, f], {r, 0, n}]
{pa[n], FullSimplify@f2a[n, pa], FullSimplify@f2b[n, pa],
 FullSimplify@ga[n, pa], FullSimplify@gb[n, pa]}
```

$\{\mathrm{Sin}[n], \mathrm{Sin}[n], \mathrm{Sin}[n], 1 - \mathrm{Cos}[n] + \mathrm{Sin}[n], \frac{1}{2} (\mathrm{Cos}[n] - \mathrm{Cosh}[n] + \mathrm{Sin}[n] + \mathrm{Sinh}[n])\}$

```
Sum[ Binomial[n, k] x^k, {k, 0, n}]
```

$(1 + x)^n$

```
Sum[ Pochhammer[z, j] / j !, {j, 0, n}]
```

$\frac{(1 + n)\,\mathrm{Gamma}[1 + n + z]}{z\,\mathrm{Gamma}[2 + n]\,\mathrm{Gamma}[z]}$

```
Sum[ (z - 1 + j) ! / z ! / (j - 1) !, {j, 0, n}]
```

$\frac{n\,(n + z)\,!}{(1 + z)\,n\,!\,z\,!}$

$\frac{(1 + n)\,\mathrm{Gamma}[1 + n + z]}{z\,\mathrm{Gamma}[2 + n]\,\mathrm{Gamma}[z]}$ /. n → 15 /. z → 7

170 544

```
(n + z) ! / n ! / z ! /. n → 15 /. z → 7
```

170 544

```
Pochhammer[z + 1, j] / (j) !  /. z → 15 /. j → 7
```

170 544

```
Sum[Binomial[z, k] x^k / (k !), {k, 0, Infinity}]
```

Hypergeometric1F1[-z, 1, -x]

```
Sum[Binomial[z, k] x^(k - 1) / ((k - 1) !), {k, 0, Infinity}]
```

z Hypergeometric1F1[1 - z, 2, -x]

```
Sum[Binomial[z, k] x^(k - 1), {k, 0, Infinity}]
```

$\frac{(1 + x)^z}{x}$

```
Sum[Binomial[z, k] x^(k - 1) / ((k - 1) !), {k, 0, Infinity}]
Integrate[ LaguerreL[z - 1, 1, -j], {j, 0, n}]
```

-LaguerreL[z, 0] + LaguerreL[z, -n]

```
Table[N[LaguerreL[z, 0]], {z, 0, 3}]
```

{1., 1., 1., 1.}

```
D[LaguerreL[z, -x], {x, 2}]
```

LaguerreL[-2 + z, 2, -x]

```
Integrate[ LaguerreL[z - 2, 2, -j], {j, 0, n}]
```

$-\text{LaguerreL}[-1+z, 1, 0] + \text{LaguerreL}[-1+z, 1, -n]$

```
Table[-LaguerreL[-1 + z, 1, 0], {z, 0, 4}]
```

{0, -1, -2, -3, -4}

```
Table[-LaguerreL[-2 + z, 2, 0], {z, 0, 8}]
```

{0, 0, -1, -3, -6, -10, -15, -21, -28}

```
Table[-LaguerreL[-3 + z, 3, 0], {z, 0, 8}]
```

{0, 0, 0, -1, -4, -10, -20, -35, -56}

```
FullSimplify@Table[Pochhammer[z, j] / (j!) /. z → 2, {j, 0, 10}]
```

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}

```
pa[n_] := Sin[n]
ga[n_, f_] := f[n] + Integrate[f[n - k], {k, 0, n}]
gb[n_, f_] := f[n] - Integrate[E^(-k) f[n - k], {k, 0, n}]
ga2[n_, f_] := f[n] + Integrate[(k + 2) f[n - k], {k, 0, n}]
gb2[n_, f_] := f[n] + Integrate[(r - 2) E^(-r) ga[n - r, f], {r, 0, n}]
TableForm@{FullSimplify@gb2[n, pa], FullSimplify@gb[n, pa], ga[n, pa], ga2[n, pa]}
```

$\frac{1}{2}(-2+3\,\text{Cos}[n]-\text{Cosh}[n]+\text{Sin}[n]+\text{Sinh}[n])$

$\frac{1}{2}(\text{Cos}[n]-\text{Cosh}[n]+\text{Sin}[n]+\text{Sinh}[n])$

$1-\text{Cos}[n]+\text{Sin}[n]$

$2+n-2\,\text{Cos}[n]$

```
pa[n_] := n
ga[n_, f_] := f[n] + Integrate[f[n - k], {k, 0, n}]
f2a[n_, f_] := ga[n, f] - Integrate[E^(-r) ga[n - r, f], {r, 0, n}]
{pa[n], FullSimplify@ga[n, pa], FullSimplify@f2a[n, pa]}
```

$\left\{n, \frac{1}{2} n (2+n), n\right\}$

```
pa[n_] := n^t
f2a[n_, f_] := f[n] - Integrate[E^(-r) f[n - r], {r, 0, n}, Assumptions → n ∈ Reals && n > 0]
{pa[n], FullSimplify@f2a[n, pa]}
```

$\left\{n^t, \text{ConditionalExpression}\left[n^t + (-1)^{-t} e^{-n} (\text{Gamma}[1+t] - \text{Gamma}[1+t, -n]), \text{Re}[t] > -1\right]\right\}$

$\textbf{FullSimplify}\left[n^t \left(1 + e^{-n} (-n)^{-t} (t\,\text{Gamma}[t] - \text{Gamma}[1+t, -n])\right)\right]$

$n^t \left(1 + e^{-n} (-n)^{-t} (\text{Gamma}[1+t] - \text{Gamma}[1+t, -n])\right)$ /. n → 20. /. t → 2.

$38. + 8.86644 \times 10^{-14}\, i$

$n^t \left(1 + e^{-n} (-1)$^-t $(n)^{-t} (\text{Gamma}[1+t, 0, -n])\right)$ /. n → 20. /. t → 2.

$38. + 8.86644 \times 10^{-14}\, i$

$\left(n^t + e^{-n} (-1)\right.$^-t $\left.(\text{Gamma}[1+t, 0, -n])\right)$ /. n → 20. /. t → 2.

$38. + 1.00968 \times 10^{-24}\, i$

```
po[n_] := n
g[n_, f_] := f[n] + Integrate[f[n / k], {k, 1, n}, Assumptions → n ∈ Reals && n > 1]
f2[n_, f_] := g[n, f] - Integrate[(1 / r) g[n / r, f], {r, 1, n}, Assumptions → n ∈ Reals && n > 1]
gb[n_, f_] := f[n] - Integrate[(1 / k) f[n / k], {k, 1, n}, Assumptions → n ∈ Reals && n > 1]
f2b[n_, f_] := gb[n, f] + Integrate[ gb[n / r, f], {r, 1, n}, Assumptions → n ∈ Reals && n > 1]
{po[n], FullSimplify@g[n, po], FullSimplify@f2[n, po],
 FullSimplify@gb[n, po], FullSimplify@f2b[n, po]}
```

```
{n, n (1 + Log[n]), n, 1, n}
```

```
N@LaguerreL[-1, Log[n]] /. n → 12
```

```
12.
```

```
D[h[n] - Integrate[1 / j h[n / j], {j, 1, n}, Assumptions → n ∈ Reals && n > 1], n]
```

$$-\frac{h[1]}{n}-\frac{-h[1]+h[n]}{n}+h'[n]$$

```
D[h[n] + Integrate[E^-j h[n - j], {j, 0, n}, Assumptions → n ∈ Reals && n > 0], n]
```

$$e^{-n}\, h[0] + \text{Integrate}\left[e^{-j}\, h'[-j+n],\ \{j, 0, n\},\ \text{Assumptions} \to n \in \text{Reals}\ \&\&\ n > 0\right] + h'[n]$$

```
D[h[n] + Integrate[ h[n / j], {j, 1, n}, Assumptions → n ∈ Reals && n > 1], n]
```

$$h[1] + \text{Integrate}\left[\frac{h'\left[\frac{n}{j}\right]}{j},\ \{j, 1, n\},\ \text{Assumptions} \to n \in \text{Reals}\ \&\&\ n > 1\right] + h'[n]$$

```
D[h[n] + Integrate[ h[n - j], {j, 0, n}, Assumptions → n ∈ Reals && n > 0], n]
```

```
h[n] + h′[n]
```

```
Sum[Binomial[z, k] x^(k - 1) / (k - 1) !, {k, 0, Infinity}]
```

```
z Hypergeometric1F1[1 - z, 2, -x]
```

```
Sum[Binomial[z, k] Log[x]^(k - 1) / (k - 1) !, {k, 0, Infinity}]
```

```
z Hypergeometric1F1[1 - z, 2, -Log[x]]
```

```
Integrate[z Hypergeometric1F1[1 - z, 2, -x], {x, 0, n}, Assumptions → n ∈ Reals && n > 0]
```

```
-1 + LaguerreL[z, -n]
```

```
Integrate[z Hypergeometric1F1[1 - z, 2, -Log[x]], {x, 1, n}, Assumptions → n ∈ Reals && n > 1]
```

```
ConditionalExpression[-1 + LaguerreL[-z, Log[n]], n ≤ e]
```

```
Sum[Binomial[z, k] x^(k) / (k) !, {k, 0, Infinity}]
```

```
Hypergeometric1F1[-z, 1, -x]
```

```
Sum[Binomial[-z, k] (-Log[x])^(k) / (k) !, {k, 0, Infinity}]
```

```
Hypergeometric1F1[z, 1, Log[x]]
```

```
Hypergeometric1F1[z, 1, Log[x]] /. x → 10 /. z → 3.
```

```
82.5612
```

```
LaguerreL[-z, Log[x]] /. x → 10. /. z → 3.
```

```
82.5612
```

```
Hypergeometric1F1[-z, 1, -x] /. x → 10. /. z → 3.
```

```
347.667
```

```
LaguerreL[z, -x] /. x → 10. /. z → 3.
```

347.667

```
Table[Binomial[z, k] FullSimplify@Integrate[ x^(k - 1) / (k - 1) !, {x, 0, n}], {k, 0, 10}] //
 TableForm
```

$$0$$
$$n\, z$$
$$\frac{1}{4} n^2 (-1+z)\, z$$
$$\frac{1}{36} n^3 (-2+z)(-1+z)\, z$$
$$\frac{1}{576} n^4 (-3+z)(-2+z)(-1+z)\, z$$
$$\frac{n^5 (-4+z)(-3+z)(-2+z)(-1+z)\, z}{14400}$$
$$\frac{1}{720} n^6\, \text{Binomial}[z, 6]$$
$$\frac{n^7\, \text{Binomial}[z,7]}{5040}$$
$$\frac{n^8\, \text{Binomial}[z,8]}{40320}$$
$$\frac{n^9\, \text{Binomial}[z,9]}{362880}$$
$$\frac{n^{10}\, \text{Binomial}[z,10]}{3628800}$$

```
Table[Binomial[z, k] Expand@Integrate[(Log[x])^(k - 1) / (k - 1) !,
     {x, 1, n}, Assumptions → n ∈ Reals && n > 1], {k, 0, 10}] // TableForm
```

$$0$$
$$(-1+n)\, z$$
$$\frac{1}{2} (-1+z)\, z\, (1-n+n\,\text{Log}[n])$$
$$\frac{1}{6} (-2+z)(-1+z)\, z \left(-1+n-n\,\text{Log}[n]+\frac{1}{2} n\,\text{Log}[n]^2\right)$$
$$\frac{1}{24} (-3+z)(-2+z)(-1+z)\, z \left(1-n+n\,\text{Log}[n]-\frac{1}{2} n\,\text{Log}[n]^2+\frac{1}{6} n\,\text{Log}[n]^3\right)$$
$$\frac{1}{120} (-4+z)(-3+z)(-2+z)(-1+z)\, z \left(-1+n-n\,\text{Log}[n]+\frac{1}{2} n\,\text{Log}[n]^2-\frac{1}{6} n\,\text{Log}[n]^3+\frac{1}{24} n\,\text{Log}[n]^4\right)$$
$$\text{Binomial}[z, 6] \left(1-n+n\,\text{Log}[n]-\frac{1}{2} n\,\text{Log}[n]^2+\frac{1}{6} n\,\text{Log}[n]^3-\frac{1}{24} n\,\text{Log}[n]^4+\frac{1}{120} n\,\text{Log}[n]^5\right)$$
$$\text{Binomial}[z, 7] \left(-1+n-n\,\text{Log}[n]+\frac{1}{2} n\,\text{Log}[n]^2-\frac{1}{6} n\,\text{Log}[n]^3+\frac{1}{24} n\,\text{Log}[n]^4-\frac{1}{120} n\,\text{Log}[n]^5+\frac{1}{720} n\,\text{Log}[n]\right.$$
$$\text{Binomial}[z, 8] \left(1-n+n\,\text{Log}[n]-\frac{1}{2} n\,\text{Log}[n]^2+\frac{1}{6} n\,\text{Log}[n]^3-\frac{1}{24} n\,\text{Log}[n]^4+\frac{1}{120} n\,\text{Log}[n]^5-\frac{1}{720} n\,\text{Log}[n]^6\right.$$
$$\text{Binomial}[z, 9] \left(-1+n-n\,\text{Log}[n]+\frac{1}{2} n\,\text{Log}[n]^2-\frac{1}{6} n\,\text{Log}[n]^3+\frac{1}{24} n\,\text{Log}[n]^4-\frac{1}{120} n\,\text{Log}[n]^5+\frac{1}{720} n\,\text{Log}[n]\right.$$
$$\text{Binomial}[z, 10] \left(1-n+n\,\text{Log}[n]-\frac{1}{2} n\,\text{Log}[n]^2+\frac{1}{6} n\,\text{Log}[n]^3-\frac{1}{24} n\,\text{Log}[n]^4+\frac{1}{120} n\,\text{Log}[n]^5-\frac{1}{720} n\,\text{Log}[n]\right.$$

```
Sum[Pochhammer[z, k] / k ! Binomial[x, k], {k, 0, Infinity}]
```

Hypergeometric2F1[-x, z, 1, -1]

```
z Hypergeometric1F1[1 - z, 2, -x] /. z → -1
```

$$-e^{-x}$$

```
z Hypergeometric1F1[1 - z, 2, -Log[x]] /. z → -1
```

$$-\frac{1}{x}$$

```
Table[binomial[z, k], {k, 0, 5}]
```

$$\left\{1, z, \frac{1}{2}(-1+z)z, \frac{1}{6}(-2+z)(-1+z)z, \frac{1}{24}(-3+z)(-2+z)(-1+z)z, \frac{1}{120}(-4+z)(-3+z)(-2+z)(-1+z)z\right\}$$

```
Table[Pochhammer[z - k + 1, k] / k!, {k, 0, 5}]
```

$$\left\{1, z, \frac{1}{2}(-1+z)z, \frac{1}{6}(-2+z)(-1+z)z, \frac{1}{24}(-3+z)(-2+z)(-1+z)z, \frac{1}{120}(-4+z)(-3+z)(-2+z)(-1+z)z\right\}$$

```
Table[FullSimplify@binomial[z + k - 1, k], {k, 0, 5}]
```

$$\left\{1, z, \frac{1}{2}z(1+z), \frac{1}{6}z(1+z)(2+z), \frac{1}{24}z(1+z)(2+z)(3+z), \frac{1}{120}z(1+z)(2+z)(3+z)(4+z)\right\}$$

```
Table[Pochhammer[z, k] / k!, {k, 0, 5}]
```

$$\left\{1, z, \frac{1}{2}z(1+z), \frac{1}{6}z(1+z)(2+z), \frac{1}{24}z(1+z)(2+z)(3+z), \frac{1}{120}z(1+z)(2+z)(3+z)(4+z)\right\}$$

```
Clear[po]
po[n_, 0] := 0; po[0, k_] := 0; po[1, 1] := 1
po[n_, k_] := po[n, k] = po[n, k - 1] - po[n - 1, k - 1]
Grid@Table[po[n, k], {n, 0, 10}, {k, 0, 10}]
```

0	0	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	1	1	1
0	0	−1	−2	−3	−4	−5	−6	−7	−8	−9
0	0	0	1	3	6	10	15	21	28	36
0	0	0	0	−1	−4	−10	−20	−35	−56	−84
0	0	0	0	0	1	5	15	35	70	126
0	0	0	0	0	0	−1	−6	−21	−56	−126
0	0	0	0	0	0	0	1	7	28	84
0	0	0	0	0	0	0	0	−1	−8	−36
0	0	0	0	0	0	0	0	0	1	9
0	0	0	0	0	0	0	0	0	0	−1

```
Clear[po]
po[n_, 0] := 0; po[0, k_] := 0; po[1, 1] := 1
po[n_, k_] := po[n, k] = Sum[po[n - j, k - 1], {j, 0, n}]
Grid@Table[po[n, k], {n, 1, 10}, {k, 1, 10}]
```

1	1	1	1	1	1	1	1	1	1
0	1	2	3	4	5	6	7	8	9
0	1	3	6	10	15	21	28	36	45
0	1	4	10	20	35	56	84	120	165
0	1	5	15	35	70	126	210	330	495
0	1	6	21	56	126	252	462	792	1287
0	1	7	28	84	210	462	924	1716	3003
0	1	8	36	120	330	792	1716	3432	6435
0	1	9	45	165	495	1287	3003	6435	12 870
0	1	10	55	220	715	2002	5005	11 440	24 310

```
Clear[po]
po[n_, 0] := 0; po[0, k_] := 0; po[1, 1] := 1
po[n_, k_] := po[n, k] = Sum[po[n - j, k - 1], {j, 1, n - 1}]
Grid@Table[po[n, k], {n, 1, 10}, {k, 1, 10}]
```

1	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0
0	1	1	0	0	0	0	0	0	0
0	1	2	1	0	0	0	0	0	0
0	1	3	3	1	0	0	0	0	0
0	1	4	6	4	1	0	0	0	0
0	1	5	10	10	5	1	0	0	0
0	1	6	15	20	15	6	1	0	0
0	1	7	21	35	35	21	7	1	0
0	1	8	28	56	70	56	28	8	1

```
Clear[po]
po[n_, 0] := 0; po[0, k_] := 0; po[1, 1] := 1
po[n_, k_] := po[n, k] = Sum[po[Floor[n / j], k - 1], {j, 1, n}]
Grid@Table[po[n, k], {n, 1, 10}, {k, 1, 10}]
```

1	1	1	1	1	1	1	1	1	1
0	1	2	3	4	5	6	7	8	9
0	2	4	6	8	10	12	14	16	18
0	2	5	9	14	20	27	35	44	54
0	3	7	12	18	25	33	42	52	63
0	3	9	18	30	45	63	84	108	135
0	4	11	21	34	50	69	91	116	144
0	4	12	25	44	70	104	147	200	264
0	5	15	31	54	85	125	175	236	309
0	5	17	37	66	105	155	217	292	381

```
Clear[po]
po[n_, 0] := 0; po[0, k_] := 0; po[1, 1] := 1
po[n_, k_] := po[n, k] = Sum[MoebiusMu[j] po[Floor[n / j], k - 1], {j, 1, n}]
Grid@Table[po[n, k], {n, 1, 10}, {k, 1, 10}]
```

1	1	1	1	1	1	1	1	1	1
0	-1	-2	-3	-4	-5	-6	-7	-8	-9
0	-2	-4	-6	-8	-10	-12	-14	-16	-18
0	-1	-1	0	2	5	9	14	20	27
0	-2	-3	-3	-2	0	3	7	12	18
0	0	3	9	18	30	45	63	84	108
0	-1	1	6	14	25	39	56	76	99
0	-1	0	2	4	5	4	0	-8	-21
0	-1	1	5	10	15	19	21	20	15
0	1	7	17	30	45	61	77	92	105

```
Table[Binomial[z, j] /. z → -1, {j, 0, 10}]
```

{1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1}

```
FullSimplify@Sum[Pochhammer[z, k] / k!, {k, 0, n}]
```

$$\frac{\text{Gamma}[1+n+z]}{\text{Gamma}[1+n]\ \text{Gamma}[1+z]}$$

```
Sum[(-1)^k Binomial[z, k], {k, 0, n}]
```

$(-1)^n$ Binomial[-1 + z, n]

```
vv[n_, k_] := Sum[vv[Floor[n / j], k - 1], {j, 2, n}]
vv[n_, 0] := 1
vx[n_, m_] := Sum[vv[n, k] vv[m, k], {k, 0, Log2@Max[n, m]}]
```

```
Grid@Table[vx[j, k], {j, 1, 10}, {k, 1, 10}]
```

1	1	1	1	1	1	1	1	1	1
1	2	3	4	5	6	7	8	9	10
1	3	5	7	9	11	13	15	17	19
1	4	7	11	14	19	22	27	31	36
1	5	9	14	18	24	28	34	39	45
1	6	11	19	24	35	40	51	59	70
1	7	13	22	28	40	46	58	67	79
1	8	15	27	34	51	58	76	88	105
1	9	17	31	39	59	67	88	102	122
1	10	19	36	45	70	79	105	122	147

```
FullSimplify@Sum[(-1) ^ (k - j) Binomial[k, j] Hypergeometric1F1[j, 1, Log[x]], {j, 0, k}]
```

```
(-1)^k DifferenceRoot[Function[{ỵ, ṇ}, {-ṇ (-1 - ṇ + k) (-ṇ + k) ỵ[ṇ] +
      (1 + ṇ)^2 (2 + ṇ) ỵ[3 + ṇ] + (-1 - ṇ + k) ỵ[1 + ṇ] (-1 - 3 ṇ - 3 ṇ^2 + ṇ k - Log[x] - ṇ Log[x]) +
      (1 + ṇ) ỵ[2 + ṇ] (-3 - 6 ṇ - 3 ṇ^2 + k + 2 ṇ k - Log[x] - ṇ Log[x] + k Log[x]) == 0,
    ỵ[0] == 0, ỵ[1] == 1, ỵ[2] == 1 - k x}]][1 + k]
```

```
FullSimplify@Sum[(-1) ^ (k - j) Binomial[k, j] LaguerreL[-j, Log[x]], {j, 0, k}]
```

```
(-1)^k DifferenceRoot[Function[{ỵ, ṇ}, {-ṇ (-1 - ṇ + k) (-ṇ + k) ỵ[ṇ] +
      (1 + ṇ)^2 (2 + ṇ) ỵ[3 + ṇ] + (-1 - ṇ + k) ỵ[1 + ṇ] (-1 - 3 ṇ - 3 ṇ^2 + ṇ k - Log[x] - ṇ Log[x]) +
      (1 + ṇ) ỵ[2 + ṇ] (-3 - 6 ṇ - 3 ṇ^2 + k + 2 ṇ k - Log[x] - ṇ Log[x] + k Log[x]) == 0,
    ỵ[0] == 0, ỵ[1] == 1, ỵ[2] == 1 - k x}]][1 + k]
```

```
p1[x_, k_] := (-1) ^k Gamma[k, 0, -Log[x]] / Gamma[k]
p2[x_, k_] := (-1) ^k ((-Log[x]) ^k / k Hypergeometric1F1[k, 1 + k, Log[x]]) / Gamma[k]
p3[x_, k_] := Log[x] ^k / k ! Hypergeometric1F1[k, 1 + k, Log[x]]
```

```
p1[12, 3.]
```

$18.2297 - 6.69748 \times 10^{-15}\, i$

```
p3[12, 3.]
```

18.2297

```
Hypergeometric1F1[k, 1 + k, Log[x]]
```

$(\text{Gamma}[1+k] - k\,\text{Gamma}[k, -\text{Log}[x]])\,(-\text{Log}[x])^{-k}$

```
Sum[Pochhammer[k, j] / Pochhammer[k + 1, j] Log[x] ^j / j !, {j, 0, Infinity}]
```

$(\text{Gamma}[1+k] - k\,\text{Gamma}[k, -\text{Log}[x]])\,(-\text{Log}[x])^{-k}$

```
Table[Pochhammer[k, j] / Pochhammer[k + 1, j], {j, 0, 10}]
```

$$\left\{1, \frac{k}{1+k}, \frac{k}{2+k}, \frac{k}{3+k}, \frac{k}{4+k}, \frac{k}{5+k}, \frac{k}{6+k}, \frac{k}{7+k}, \frac{k}{8+k}, \frac{k}{9+k}, \frac{k}{10+k}\right\}$$

```
FullSimplify@Sum[k / (k + j) Log[x] ^j / j !, {j, 0, Infinity}]
```

$(\text{Gamma}[1+k] - k\,\text{Gamma}[k, -\text{Log}[x]])\,(-\text{Log}[x])^{-k}$

```
FullSimplify[D[Log[x]^k / k! Hypergeometric1F1[k, 1 + k, Log[x]], x]]
```

$$\frac{\text{Log}[x]^{-1+k}}{\text{Gamma}[k]}$$

```
D[x^k / (k!), k] /. k → 0
```

EulerGamma + Log[x]

```
D[Binomial[x, k], k] /. k → 0
```

EulerGamma + PolyGamma[0, 1 + x]

```
FullSimplify[D[Hypergeometric1F1[k, k + 1, Log[x]] (Log[x])^k / (k!), k] /. k → 0]
```

$$\frac{1}{2}\left(\text{Log}\left[\frac{1}{\text{Log}[x]}\right] + \text{Log}[\text{Log}[x]]\right) + \text{LogIntegral}[x]$$

```
FullSimplify[Limit[D[x^(k - 1) / (k - 1)!, k], k → 0]]
```

$$\frac{1}{x}$$

```
FullSimplify[Limit[D[Binomial[x - 1, k - 1], k], k → 0]]
```

$$\frac{1}{x}$$

```
FullSimplify[Limit[D[Log[x]^(k - 1) / (k - 1)!, k], k → 0]]
```

$$\frac{1}{\text{Log}[x]}$$

```
Table[FullSimplify[D[x^k / (k!), {k, j}] /. k → 0], {j, 0, 4}]
```

$$\Big\{1,\ \text{EulerGamma} + \text{Log}[x],\ \text{EulerGamma}^2 - \frac{\pi^2}{6} + 2\,\text{EulerGamma}\,\text{Log}[x] + \text{Log}[x]^2,\ \frac{1}{2}(\text{EulerGamma} + \text{Log}[x])\left(2\,\text{EulerGamma}^2 - \pi^2 + 2\,\text{Log}[x]\,(2\,\text{EulerGamma} + \text{Log}[x])\right) + 2\,\text{Zeta}[3],\ \text{EulerGamma}^4 - \text{EulerGamma}^2\,\pi^2 + \frac{\pi^4}{60} + \text{Log}[x]\,(2\,\text{EulerGamma} + \text{Log}[x])\left(2\,\text{EulerGamma}^2 - \pi^2 + 2\,\text{EulerGamma}\,\text{Log}[x] + \text{Log}[x]^2\right) + 8\,(\text{EulerGamma} + \text{Log}[x])\,\text{Zeta}[3]\Big\}$$

```
Table[FullSimplify[D[Binomial[x, k], {k, j}] /. k → 0], {j, 0, 4}]
```

$$\Big\{1,\ \text{HarmonicNumber}[x],\ -\frac{\pi^2}{6} + \text{HarmonicNumber}[x]^2 - \text{PolyGamma}[1, 1 + x],\ \text{HarmonicNumber}[x]^3 - \frac{1}{2}\text{HarmonicNumber}[x]\left(\pi^2 + 6\,\text{PolyGamma}[1, 1 + x]\right) + \text{PolyGamma}[2, 1 + x] + 2\,\text{Zeta}[3],\ \frac{\pi^4}{60} + \text{HarmonicNumber}[x]^4 + \text{PolyGamma}[1, 1 + x]\left(\pi^2 + 3\,\text{PolyGamma}[1, 1 + x]\right) - \text{HarmonicNumber}[x]^2\left(\pi^2 + 6\,\text{PolyGamma}[1, 1 + x]\right) - \text{PolyGamma}[3, 1 + x] + 4\,\text{HarmonicNumber}[x]\,(\text{PolyGamma}[2, 1 + x] + 2\,\text{Zeta}[3])\Big\}$$

```
Table[FullSimplify[
  D[Hypergeometric1F1[k, k + 1, Log[x]] (Log[x])^k / (k!), {k, j}] /. k → 0], {j, 0, 4}]
```

$$\left\{1, \frac{1}{2}\left(\mathrm{Log}\left[\frac{1}{\mathrm{Log}[x]}\right] + \mathrm{Log}[\mathrm{Log}[x]]\right) + \mathrm{LogIntegral}[x],\right.$$
$$(\mathrm{Log}[-\mathrm{Log}[x]] - \mathrm{Log}[\mathrm{Log}[x]])^2 - 2\,\mathrm{ExpIntegralE}[1, -\mathrm{Log}[x]]\,(\mathrm{EulerGamma} + \mathrm{Log}[\mathrm{Log}[x]]) -$$
$$2\,\mathrm{MeijerG}[\{\{\}, \{1, 1\}\}, \{\{0, 0, 0\}, \{\}\}, -\mathrm{Log}[x]],$$
$$-(\mathrm{Log}[-\mathrm{Log}[x]] - \mathrm{Log}[\mathrm{Log}[x]])^3 + \frac{1}{2}\,\mathrm{ExpIntegralE}[1, -\mathrm{Log}[x]]$$
$$\left(-6\,\mathrm{EulerGamma}^2 + \pi^2 - 6\,\mathrm{Log}[\mathrm{Log}[x]]\,(2\,\mathrm{EulerGamma} + \mathrm{Log}[\mathrm{Log}[x]])\right) -$$
$$6\,(\mathrm{EulerGamma} + \mathrm{Log}[\mathrm{Log}[x]])\,\mathrm{MeijerG}[\{\{\}, \{1, 1\}\}, \{\{0, 0, 0\}, \{\}\}, -\mathrm{Log}[x]] -$$
$$6\,\mathrm{MeijerG}[\{\{\}, \{1, 1, 1\}\}, \{\{0, 0, 0, 0\}, \{\}\}, -\mathrm{Log}[x]], (\mathrm{Log}[-\mathrm{Log}[x]] - \mathrm{Log}[\mathrm{Log}[x]])^4 -$$
$$2\left(6\,\mathrm{EulerGamma}^2 - \pi^2 + 6\,\mathrm{Log}[\mathrm{Log}[x]]\,(2\,\mathrm{EulerGamma} + \mathrm{Log}[\mathrm{Log}[x]])\right)$$
$$\mathrm{MeijerG}[\{\{\}, \{1, 1\}\}, \{\{0, 0, 0\}, \{\}\}, -\mathrm{Log}[x]] -$$
$$24\,(\mathrm{EulerGamma} + \mathrm{Log}[\mathrm{Log}[x]])\,\mathrm{MeijerG}[\{\{\}, \{1, 1, 1\}\}, \{\{0, 0, 0, 0\}, \{\}\}, -\mathrm{Log}[x]] -$$
$$24\,\mathrm{MeijerG}[\{\{\}, \{1, 1, 1, 1\}\}, \{\{0, 0, 0, 0, 0\}, \{\}\}, -\mathrm{Log}[x]] +$$
$$2\,\mathrm{ExpIntegralE}[1, -\mathrm{Log}[x]]\,\left(-(\mathrm{EulerGamma} + \mathrm{Log}[\mathrm{Log}[x]])\right.$$
$$\left.\left.\left(2\,\mathrm{EulerGamma}^2 - \pi^2 + 2\,\mathrm{Log}[\mathrm{Log}[x]]\,(2\,\mathrm{EulerGamma} + \mathrm{Log}[\mathrm{Log}[x]])\right) - 4\,\mathrm{Zeta}[3]\right)\right\}$$