

```

Approx[x_, t_] := -MangoldtLambda[n] / 2 / Log[n] +
  N[2 Re[ Sum[ ExpIntegralEi[ ZetaZero[k] Log[x] ], {k, 1, t} ]]] -
  NIntegrate[1 / ((y ^ 3 - y) Log[y]), {y, x, Infinity}] + Log[2]
D2Approx[n_, j_] := (-1) ^ j (1 - n Sum[(-Log[n]) ^ k / (k!), {k, 0, j - 1}])
Dhyp[n_, k_, a_] := Sum[ Binomial[k, j] Dhyp[n / (m ^ (k - j)), j, m + 1],
  {m, a, n ^ (1 / k)}, {j, 0, k - 1}]
Dhyp[n_, 1, a_] := If[n < a, 0, Floor[n] - a + 1]; Dhyp[n_, 0, a_] := 1
dhyp[n_, k_, a_] := dhyp[n, k, a] = Dhyp[n, k, a] - Dhyp[n - 1, k, a]
LinnikApprox[ n_ ] := Log[ Log[n]] + EulerGamma +
  Sum[ (-1) ^ (k + 1) / k (D2Approx[n, k] - Dhyp[n, k, 2]), {k, 2, 100}]

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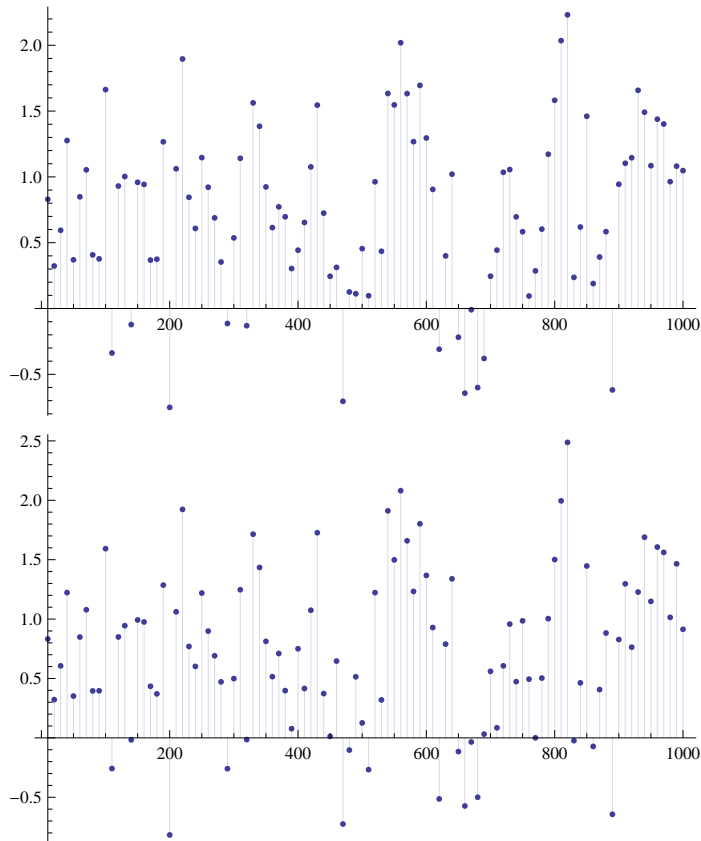
```
Table[{n, N[Approx[n, 200]], N[LinnikApprox[n]]}, {n, 2, 100}] // TableForm
```

2	0.0404408	0.0451638
3	0.168095	0.163589
4	0.470302	0.467585
5	0.137339	0.134588
6	0.716942	0.722222
7	0.252888	0.257052
8	0.420725	0.420385
9	0.387728	0.387904
10	0.829857	0.832266
11	0.267622	0.257676
12	0.641954	0.667214
13	0.050246	0.0632146
14	0.44657	0.447492
15	0.810561	0.821492
16	0.927181	0.936383
17	0.28454	0.293131
18	0.645275	0.642541
19	-0.0116518	-0.0147075
20	0.323054	0.321967
21	0.650298	0.653064
22	0.974469	0.979019
23	0.306377	0.300214
24	0.612812	0.616982
25	0.433014	0.429623
26	0.723349	0.738401
27	0.731966	0.71022
28	1.00717	1.01196
29	0.29671	0.310485
30	0.594435	0.605966
31	-0.102194	-0.101436
32	0.016244	-0.0115745
33	0.294331	0.275685
34	0.567287	0.560464
35	0.842675	0.842878
36	1.12052	1.12303
37	0.417993	0.40102
38	0.659374	0.676936
39	0.927158	0.950862
40	1.27621	1.22288
41	0.466328	0.493056

42	0.769206	0.761465
43	0.0394584	0.0281692
44	0.253858	0.293229
45	0.573268	0.556702
46	0.827592	0.818641
47	0.0877267	0.0790972
48	0.329784	0.338117
49	0.0726235	0.0957474
50	0.370153	0.35203
51	0.620274	0.607005
52	0.832117	0.860712
53	0.109475	0.113187
54	0.397738	0.364464
55	0.622202	0.614578
56	0.860014	0.86356
57	1.09222	1.11144
58	1.30476	1.35824
59	0.584248	0.604005
60	0.848747	0.848746
61	0.113974	0.092492
62	0.394565	0.335268
63	0.604427	0.577097
64	0.675077	0.651335
65	0.912491	0.891336
66	1.12267	1.13045
67	0.40463	0.368709
68	0.680656	0.606119
69	0.872629	0.842704
70	1.05371	1.07848
71	0.297926	0.313464
72	0.524036	0.547673
73	-0.244028	-0.218877
74	0.019978	0.0138289
75	0.238055	0.245805
76	0.486408	0.477066
77	0.728116	0.707625
78	0.926788	0.937496
79	0.168923	0.166691
80	0.40779	0.395224
81	0.367812	0.373105
82	0.591654	0.600347
83	-0.159223	-0.173039
84	0.056158	0.0529582
85	0.227667	0.278349
86	0.491898	0.503143
87	0.777028	0.727352
88	0.971057	0.950984
89	0.164343	0.174049
90	0.3773	0.396556
91	0.589805	0.618515
92	0.852074	0.839934
93	1.07631	1.06082
94	1.2574	1.28118
95	1.50417	1.50103
96	1.77634	1.72037
97	0.950746	0.939214

98	1.07983	1.15756
99	1.38151	1.37542
100	1.66304	1.59281

```
DiscretePlot[Approx[n, 200], {n, 10, 1000, 10}] // TableForm
DiscretePlot[LinnikApprox[n], {n, 10, 1000, 10}] // TableForm
```



```
Table[{n, N[Approx[n, 200]], N[LinnikApprox[n]]}, {n, 2, 100}] // TableForm
```

2	0.0404408	0.0451638
3	0.168095	0.163589
4	0.470302	0.467585
5	0.137339	0.134588
6	0.716942	0.722222
7	0.252888	0.257052
8	0.420725	0.420385
9	0.387728	0.387904
10	0.829857	0.832266
11	0.267622	0.257676
12	0.641954	0.667214
13	0.050246	0.0632146
14	0.44657	0.447492
15	0.810561	0.821492
16	0.927181	0.936383

17	0.28454	0.293131
18	0.645275	0.642541
19	-0.0116518	-0.0147075
20	0.323054	0.321967
21	0.650298	0.653064
22	0.974469	0.979019
23	0.306377	0.300214
24	0.612812	0.616982
25	0.433014	0.429623
26	0.723349	0.738401
27	0.731966	0.71022
28	1.00717	1.01196
29	0.29671	0.310485
30	0.594435	0.605966
31	-0.102194	-0.101436
32	0.016244	-0.0115745
33	0.294331	0.275685
34	0.567287	0.560464
35	0.842675	0.842878
36	1.12052	1.12303
37	0.417993	0.40102
38	0.659374	0.676936
39	0.927158	0.950862
40	1.27621	1.22288
41	0.466328	0.493056
42	0.769206	0.761465
43	0.0394584	0.0281692
44	0.253858	0.293229
45	0.573268	0.556702
46	0.827592	0.818641
47	0.0877267	0.0790972
48	0.329784	0.338117
49	0.0726235	0.0957474
50	0.370153	0.35203
51	0.620274	0.607005
52	0.832117	0.860712
53	0.109475	0.113187
54	0.397738	0.364464
55	0.622202	0.614578
56	0.860014	0.86356
57	1.09222	1.11144
58	1.30476	1.35824
59	0.584248	0.604005
60	0.848747	0.848746
61	0.113974	0.092492
62	0.394565	0.335268
63	0.604427	0.577097
64	0.675077	0.651335
65	0.912491	0.891336
66	1.12267	1.13045
67	0.40463	0.368709
68	0.680656	0.606119
69	0.872629	0.842704
70	1.05371	1.07848
71	0.297926	0.313464
72	0.524036	0.547673

73	-0.244028	-0.218877
74	0.019978	0.0138289
75	0.238055	0.245805
76	0.486408	0.477066
77	0.728116	0.707625
78	0.926788	0.937496
79	0.168923	0.166691
80	0.40779	0.395224
81	0.367812	0.373105
82	0.591654	0.600347
83	-0.159223	-0.173039
84	0.056158	0.0529582
85	0.227667	0.278349
86	0.491898	0.503143
87	0.777028	0.727352
88	0.971057	0.950984
89	0.164343	0.174049
90	0.3773	0.396556
91	0.589805	0.618515
92	0.852074	0.839934
93	1.07631	1.06082
94	1.2574	1.28118
95	1.50417	1.50103
96	1.77634	1.72037
97	0.950746	0.939214
98	1.07983	1.15756
99	1.38151	1.37542
100	1.66304	1.59281