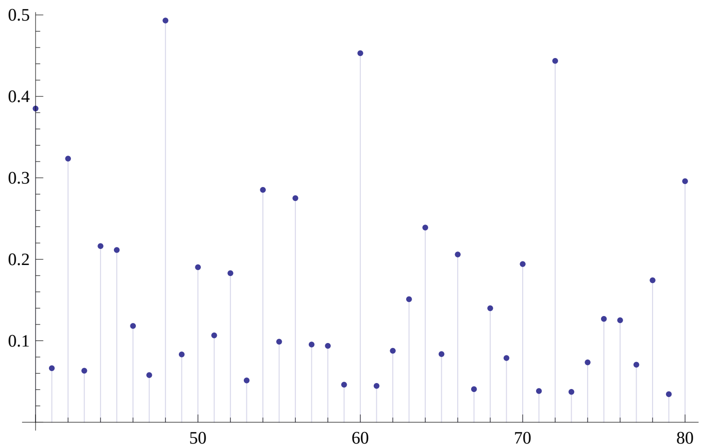
```
Clear[pp]
pp[n_, k_] := pp[n, k] = Sum[ 1 / j pp[Floor[n / j], k - 1], {j, 1, n}]
pp[n_, 0] := UnitStep[n - 1]
ppz[n_, z_, t_] := Sum[ z^k / (k!) pp[n, k], {k, 0, t}]
DiscretePlot[ N@ppz[n, 1, 30] - N@ppz[n - 1, 1, 30], {n, 40, 80}]
```




```
Clear[pp]
bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k!
pp[n_, k_] := pp[n, k] = 1 / j Sum[ pp[n - j, k - 1], {j, 1, n - 1}]
pp[n_, 1] := 1 / n
pp[n_, 0] := 0
pss[n_, z_] := Sum[ bin[z, k] pp[n, k], {k, 0, n}]
psf[n_, z_] := Sum[ pss[j, z], {j, 1, n}]
rootsa[n_] := If[(c = Exponent[f = psf[n, z], z]) == 0, {},
  If[c == 1, List@Roots[f == 0, z][[2]], List @@ Roots[f == 0, z][[All, 2]]]]

Table[D[pss[10, z], {z, k}] /. z -> 0, {k, 0, 12}]
```

$$\left\{0, \frac{3391}{151200}, \frac{33541}{453600}, \frac{949}{6720}, \frac{9563}{30240}, \frac{23}{72}, \frac{49}{60}, \frac{7}{24}, \frac{4}{3}, 0, 1, 0, 0\right\}$$

```
Sum[1^k / k! D[pss[10, z], {z, k}] /. z -> 0, {k, 0, 12}]
```

$$\frac{1}{10}$$

```
HarmonicNumber[10]
```

$$\frac{7381}{2520}$$

```
Expand@psf[10, z]
```

$$\frac{184963\, z}{129600} + \frac{1735487\, z^2}{1814400} + \frac{226\, z^3}{567} + \frac{16999\, z^4}{145152} + \frac{107\, z^5}{4320} + \frac{23\, z^6}{5400} + \frac{59\, z^7}{120960} + \frac{z^8}{17280} + \frac{z^9}{362880} + \frac{z^{10}}{3628800}$$

```
N@rootsa[40]

{0., -8.60779, -8.38004 - 2.92561 i, -8.38004 + 2.92561 i, -7.70477 - 5.81542 i,
 -7.70477 + 5.81542 i, -6.60429 - 8.63163 i, -6.60429 + 8.63163 i, -5.10743 - 11.3408 i,
 -5.10743 + 11.3408 i, -3.50359 - 13.8299 i, -3.50359 + 13.8299 i, -2.69771 - 15.5657 i,
 -2.69771 + 15.5657 i, -2.21913 - 17.7475 i, -2.21913 + 17.7475 i, -2.18353 - 11.3591 i,
 -2.18353 + 11.3591 i, -1.55446 - 20.0452 i, -1.55446 + 20.0452 i, -1.04005 - 7.51469 i,
 -1.04005 + 7.51469 i, -0.815345 - 22.4795 i, -0.815345 + 22.4795 i,
 -0.276448 - 3.72125 i, -0.276448 + 3.72125 i, 0.02227 - 25.0754 i, 0.02227 + 25.0754 i,
 0.971128 - 27.8583 i, 0.971128 + 27.8583 i, 2.05107 - 30.8664 i, 2.05107 + 30.8664 i,
 3.29214 - 34.1579 i, 3.29214 + 34.1579 i, 4.744 - 37.8293 i, 4.744 + 37.8293 i,
 6.50164 - 42.0671 i, 6.50164 + 42.0671 i, 8.80847 - 47.3563 i, 8.80847 + 47.3563 i}

pp[10, 1]
```

$$\frac{1}{10}$$

```
(* THIS IS https://oeis.org/A089064 - Expansion of Log[1-Log[1-x]]*)
Table[D[pss[n, z], {z, 1}] /. z → 0, {n, 1, 10}]
```

$$\left\{1, 0, \frac{1}{6}, \frac{1}{24}, \frac{1}{15}, \frac{13}{360}, \frac{97}{2520}, \frac{571}{20160}, \frac{1217}{45360}, \frac{3391}{151200}\right\}$$

```
120 / 15

8

7!

5040

97 * 2

194

CoefficientList[Series[Log[1 - Log[1 - x]], {x, 0, 10}], x]
```

$$\left\{0, 1, 0, \frac{1}{6}, \frac{1}{24}, \frac{1}{15}, \frac{13}{360}, \frac{97}{2520}, \frac{571}{20160}, \frac{1217}{45360}, \frac{3391}{151200}\right\}$$

```
CoefficientList[Series[-Log[1 - x], {x, 0, 10}], x]
```

$$\left\{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}\right\}$$

```
CoefficientList[Series[1 / (1 - x), {x, 0, 10}], x]

{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}
```

```
CoefficientList[
 Expand[(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9) (1 + x^2 + x^4 + x^6 + x^8)
   (1 + x^3 + x^6 + x^9) (1 + x^4 + x^8) (1 + x^5) (1 + x^6) (1 + x^7) (1 + x^8) (1 + x^9)], x]

{1, 1, 2, 3, 5, 7, 11, 15, 22, 30, 38, 51, 64, 82, 101, 126, 150, 182, 211,
 247, 284, 324, 367, 409, 453, 493, 537, 572, 613, 643, 677, 698, 718, 728,
 735, 735, 728, 718, 698, 677, 643, 613, 572, 537, 493, 453, 409, 367, 324, 284,
 247, 211, 182, 150, 126, 101, 82, 64, 51, 38, 30, 22, 15, 11, 7, 5, 3, 2, 1, 1}

Table[PartitionsP[n], {n, 1, 9}]

{1, 2, 3, 5, 7, 11, 15, 22, 30}


Clear[pp]
bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k!
pp[n_, r_, k_] := pp[n, r, k] = Sum[ If[Mod[j, r] ≠ 0, 0, 1] pp[n - j, r, k - 1], {j, 1, n - 1}]
pp[n_, r_, 1] := If[Mod[n, r] ≠ 0, 0, 1]
pp[n_, r_, 0] := 0
pss[n_, r_, z_] := Sum[ bin[z, k] pp[n, r, k], {k, 0, n}]
psf[n_, r_, z_] := Sum[ pss[j, r, z], {j, 1, n}]
rootsa[n_, r_] := If[(c = Exponent[f = psf[n, r, z], z]) == 0, {},
  If[c == 1, List@Roots[f == 0, z][[2]], List @@ Roots[f == 0, z][[All, 2]]]]


Table[D[pss[n, 1, z] + pss[n, 2, z] + pss[n, 3, z] + pss[n, 4, z] + pss[n, 5, z] + pss[n, 6, z] +
    pss[n, 7, z] + pss[n, 8, z] + pss[n, 9, z] + pss[n, 10, z], z] /. z → 0, {n, 1, 10}]
```

$$\left\{1, \frac{3}{2}, \frac{4}{3}, \frac{7}{4}, \frac{6}{5}, 2, \frac{8}{7}, \frac{15}{8}, \frac{13}{9}, \frac{9}{5}\right\}$$

```
Table[DivisorSigma[1, n] / n, {n, 1, 10}]
```

$$\left\{1, \frac{3}{2}, \frac{4}{3}, \frac{7}{4}, \frac{6}{5}, 2, \frac{8}{7}, \frac{15}{8}, \frac{13}{9}, \frac{9}{5}\right\}$$

```
Table[D[psf[n, 1, z] + psf[n, 2, z] + psf[n, 3, z] + psf[n, 4, z] + psf[n, 5, z] + psf[n, 6, z] +
    psf[n, 7, z] + psf[n, 8, z] + psf[n, 9, z] + psf[n, 10, z], z] /. z → 0, {n, 1, 10}]
```

$$\left\{1, \frac{5}{2}, \frac{23}{6}, \frac{67}{12}, \frac{407}{60}, \frac{527}{60}, \frac{4169}{420}, \frac{9913}{840}, \frac{33379}{2520}, \frac{7583}{504}\right\}$$

```
Sum[ HarmonicNumber[Floor[10 / n]], {n, 1, 10}]
```

$$\frac{7583}{504}$$

```
Sum[ 1, {j, 0, 6}, {k, 0, (6 - j) / 2}, {l, 0, (6 - j - 2 k) / 3}, {m, 0, (6 - j - 2 k - 3 l) / 4},
 {n, 0, (6 - j - 2 k - 3 l - 4 m) / 5}, {o, 0, (6 - j - 2 k - 3 l - 4 m - 5 n) / 6}]

30

Sum[ PartitionsP[j], {j, 1, 6}]

29
```

```
Clear[pp]
bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k!
pp[n_, r_, k_] := pp[n, r, k] = Sum[ If[Mod[j, r] ≠ 0, 0, 1] pp[n - j, r, k - 1], {j, 1, n - 1}]
pp[n_, r_, 1] := If[Mod[n, r] ≠ 0, 0, 1]
pp[n_, r_, 0] := 0
pss[n_, r_, z_] := Sum[ bin[z, k] pp[n, r, k], {k, 0, n}]
psf[n_, r_, z_] := Sum[ pss[j, r, z], {j, 1, n}]
rootsa[n_, r_] := If[(c = Exponent[f = psf[n, r, z], z]) == 0, {},
  If[c == 1, List@Roots[f == 0, z][[2]], List @@ Roots[f == 0, z][[All, 2]]]]

Clear[lp, p2, pz]
bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k!
lp[n_, s_, k_] := lp[n, s, k] = Sum[ (1 - (s + 1) ^j) / j lp[n - j, s, k - 1], {j, 1, n - 1}]
lp[n_, s_, 1] := (1 - (s + 1) ^n) / n
lp[n_, s_, 0] := If[n == 0, 1, 0]
p2[n_, s_, k_] := p2[n, k] = Sum[(-s) p2[n - j, s, k - 1], {j, 1, n - 1}]
p2[n_, s_, 1] := -s
p2[n_, s_, 0] := If[n == 0, 1, 0]
pz[n_, s_, z_] := Sum[ z^k / k! lp[n, s, k], {k, 0, n}]
pzx[n_, s_, z_] := Sum[ bin[z, k] p2[n, s, k], {k, 0, n}]
```

```
Table[lp[n, 1, 1], {n, 1, 10}]
```

$$\left\{-1, -\frac{3}{2}, -\frac{7}{3}, -\frac{15}{4}, -\frac{31}{5}, -\frac{21}{2}, -\frac{127}{7}, -\frac{255}{8}, -\frac{511}{9}, -\frac{1023}{10}\right\}$$

```
Table[D[pzx[n, 1, z], z] /. z → 0, {n, 1, 10}]
```

$$\left\{-1, -\frac{3}{2}, -\frac{7}{3}, -\frac{15}{4}, -\frac{31}{5}, -\frac{21}{2}, -\frac{127}{7}, -\frac{255}{8}, -\frac{511}{9}, -\frac{1023}{10}\right\}$$

```
Table[(1 - 2^n) / n, {n, 1, 10}]
```

$$\left\{-1, -\frac{3}{2}, -\frac{7}{3}, -\frac{15}{4}, -\frac{31}{5}, -\frac{21}{2}, -\frac{127}{7}, -\frac{255}{8}, -\frac{511}{9}, -\frac{1023}{10}\right\}$$

```
Table[D[pzx[n, 2, z], z] /. z → 0, {n, 1, 10}]
```

$$\left\{-2, -4, -\frac{26}{3}, -20, -\frac{242}{5}, -\frac{364}{3}, -\frac{2186}{7}, -820, -\frac{19682}{9}, -\frac{29524}{5}\right\}$$

```
Table[-(3^n - 1) / n, {n, 1, 10}]
```

$$\left\{-2, -4, -\frac{26}{3}, -20, -\frac{242}{5}, -\frac{364}{3}, -\frac{2186}{7}, -820, -\frac{19682}{9}, -\frac{29524}{5}\right\}$$

```
FullSimplify[Sum[(1 - (1 + s) ^t) / t, {t, 1, n}]]
```

$$\text{HarmonicNumber}[n] + (1+s)^{1+n}\,\text{LerchPhi}[1+s, 1, 1+n] + \text{Log}[-s]$$

```
FullSimplify[HarmonicNumber[t] + a^(1+t) LerchPhi[a, 1, 1 + t] + Log[1 - a] /. {a → 2, t → 10}]
```

$$-\frac{118127}{504}$$

```
Table[D[pzx[n, 3, z], z] /. z → 0, {n, 1, 10}]
```

$$\left\{-3, -\frac{15}{2}, -21, -\frac{255}{4}, -\frac{1023}{5}, -\frac{1365}{2}, -\frac{16383}{7}, -\frac{65535}{8}, -29127, -\frac{209715}{2}\right\}$$

```
Table[(1 - 4^n) / n, {n, 1, 10}]
```

$$\left\{-3, -\frac{15}{2}, -21, -\frac{255}{4}, -\frac{1023}{5}, -\frac{1365}{2}, -\frac{16383}{7}, -\frac{65535}{8}, -29127, -\frac{209715}{2}\right\}$$

```
Table[D[pzx[n, -1, z], z] /. z → 0, {n, 1, 10}]
```

$$\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}\right\}$$

```
Table[(1 - 0^n) / n, {n, 1, 10}]
```

$$\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}\right\}$$

```
Table[D[pzx[n, -2, z], z] /. z → 0, {n, 1, 10}]
```

$$\left\{2, 0, \frac{2}{3}, 0, \frac{2}{5}, 0, \frac{2}{7}, 0, \frac{2}{9}, 0\right\}$$

```
Table[(1 - (-1)^n) / n, {n, 1, 10}]
```

$$\left\{2, 0, \frac{2}{3}, 0, \frac{2}{5}, 0, \frac{2}{7}, 0, \frac{2}{9}, 0\right\}$$

```
Table[D[pzx[n, -3, z], z] /. z → 0, {n, 1, 10}]
```

$$\left\{3, -\frac{3}{2}, 3, -\frac{15}{4}, \frac{33}{5}, -\frac{21}{2}, \frac{129}{7}, -\frac{255}{8}, 57, -\frac{1023}{10}\right\}$$

```
Table[(1 - (-2)^n) / n, {n, 1, 10}]
```

$$\left\{3, -\frac{3}{2}, 3, -\frac{15}{4}, \frac{33}{5}, -\frac{21}{2}, \frac{129}{7}, -\frac{255}{8}, 57, -\frac{1023}{10}\right\}$$

```
Table[FullSimplify@pz[n, 1, z], {n, 0, 6}] // TableForm
```

$1$

$-z$

$\frac{1}{2}(-3+z)z$

$-\frac{1}{6}(-7+z)(-2+z)z$

$\frac{1}{24}(-5+z)z(18+(-13+z)z)$

$-\frac{1}{120}(-4+z)z(-186+z(171+(-26+z)z))$

$\frac{1}{720}(-7+z)z(1080+(-11+z)z(122+(-27+z)z))$

```
Sum[pzx[j, -1, 2], {j, 0, 10}]
```

66

```
Pochhammer[3, 10] / (10!)
```

66

```
Table[Expand@Sum[pz[j, -1, z], {j, 0, n}], {n, 0, 6}] // TableForm
```

$1$
$1+z$
$1+\frac{3z}{2}+\frac{z^2}{2}$
$1+\frac{11z}{6}+z^2+\frac{z^3}{6}$
$1+\frac{25z}{12}+\frac{35z^2}{24}+\frac{5z^3}{12}+\frac{z^4}{24}$
$1+\frac{137z}{60}+\frac{15z^2}{8}+\frac{17z^3}{24}+\frac{z^4}{8}+\frac{z^5}{120}$
$1+\frac{49z}{20}+\frac{203z^2}{90}+\frac{49z^3}{48}+\frac{35z^4}{144}+\frac{7z^5}{240}+\frac{z^6}{720}$

```
Table[FullSimplify@pz[n, -2, z + 1], {n, 0, 6}] // TableForm
```

$1$
$2(1+z)$
$2(1+z)^2$
$\frac{2}{3}(1+z)(3+2z(2+z))$
$\frac{2}{3}(1+z)^2\left(2+(1+z)^2\right)$
$\frac{2}{15}(1+z)(15+2z(2+z)(7+z(2+z)))$
$\frac{2}{45}(1+z)^2(45+2z(2+z)(12+z(2+z)))$

```
zp[z_, n_, s_] := Product[((-s) z + (-s) j), {j, 0, n - 1}] / n!
```

```
Table[ FullSimplify[zp[z, n, -2]], {n, 0, 5}] // TableForm
```

$1$
$2z$
$2z(1+z)$
$\frac{4}{3}z(1+z)(2+z)$
$\frac{2}{3}z(1+z)(2+z)(3+z)$
$\frac{4}{15}z(1+z)(2+z)(3+z)(4+z)$

```
Sum[ 1, {j, 0, 5}]
```

$6$

```
FullSimplify@Sum[2^j / j, {j, 1, n}]
```

$-i\pi-2^{1+n}\,\mathrm{LerchPhi}[2,1,1+n]$

```
Sum[(-s) (-s), {j, 1, 10 - 1}]
```

$9s^2$

```
p2[10, s, 2]
```

$9s^2$

```
p2[10, s, 1]
```

$-s$

```
Table[ p2[n, s, k], {k, 1, 6}, {n, 1, 10}] // TableForm
```

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $-s$ | $-s$ | $-s$ | $-s$ | $-s$ | $-s$ | $-s$ | $-s$ | $-s$ | $-s$ |
| 0 | $s^2$ | $2s^2$ | $3s^2$ | $4s^2$ | $5s^2$ | $6s^2$ | $7s^2$ | $8s^2$ | $9s^2$ |
| 0 | 0 | $-s^3$ | $-3s^3$ | $-6s^3$ | $-10s^3$ | $-15s^3$ | $-21s^3$ | $-28s^3$ | $-36s^3$ |
| 0 | 0 | 0 | $s^4$ | $4s^4$ | $10s^4$ | $20s^4$ | $35s^4$ | $56s^4$ | $84s^4$ |
| 0 | 0 | 0 | 0 | $-s^5$ | $-5s^5$ | $-15s^5$ | $-35s^5$ | $-70s^5$ | $-126s^5$ |
| 0 | 0 | 0 | 0 | 0 | $s^6$ | $6s^6$ | $21s^6$ | $56s^6$ | $126s^6$ |

```
Table[ s^2 Pochhammer[k - 1, 1] / 1!, {k, 1, 10}]
```

$\{0, s^2, 2 s^2, 3 s^2, 4 s^2, 5 s^2, 6 s^2, 7 s^2, 8 s^2, 9 s^2\}$

```
Table[ -s^3 Pochhammer[k - 2, 2] / 2!, {k, 1, 10}]
```

$\{0, 0, -s^3, -3 s^3, -6 s^3, -10 s^3, -15 s^3, -21 s^3, -28 s^3, -36 s^3\}$

```
Table[ s^4 Pochhammer[k - 3, 3] / 3!, {k, 1, 10}]
```

$\{0, 0, 0, s^4, 4 s^4, 10 s^4, 20 s^4, 35 s^4, 56 s^4, 84 s^4\}$

```
Table[ (-s) ^k Pochhammer[n - k + 1, k - 1] / (k - 1) !, {k, 1, 6}, {n, 1, 10}] // TableForm
```

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $-s$ | $-s$ | $-s$ | $-s$ | $-s$ | $-s$ | $-s$ | $-s$ | $-s$ | $-s$ |
| 0 | $s^2$ | $2 s^2$ | $3 s^2$ | $4 s^2$ | $5 s^2$ | $6 s^2$ | $7 s^2$ | $8 s^2$ | $9 s^2$ |
| 0 | 0 | $-s^3$ | $-3 s^3$ | $-6 s^3$ | $-10 s^3$ | $-15 s^3$ | $-21 s^3$ | $-28 s^3$ | $-36 s^3$ |
| 0 | 0 | 0 | $s^4$ | $4 s^4$ | $10 s^4$ | $20 s^4$ | $35 s^4$ | $56 s^4$ | $84 s^4$ |
| 0 | 0 | 0 | 0 | $-s^5$ | $-5 s^5$ | $-15 s^5$ | $-35 s^5$ | $-70 s^5$ | $-126 s^5$ |
| 0 | 0 | 0 | 0 | 0 | $s^6$ | $6 s^6$ | $21 s^6$ | $56 s^6$ | $126 s^6$ |

```
FullSimplify[Sum[ (-s) ^k Pochhammer[n - k + 1, k - 1] / (k - 1) !, {n, 1, 10}]]
```

$$-\left((-11+k)\ (966\,240+(-11+k)\ k\ (97\,416+(-11+k)\ k\ (4708+(-11+k)\ k\ (110+(-11+k)\ k))))\ (-s)^k\right) \Big/ (\text{Gamma}[11-k]\ \text{Gamma}[k])$$

```
Expand@Sum[Binomial[z, k] (-s) ^k Pochhammer[n - k + 1, k - 1] / (k - 1) !, {k, 0, Infinity}]
```

$-s\, z\, \text{Hypergeometric2F1}[1-n, 1-z, 2, -s]$

```
pzn[n_, s_, z_] := -s z Hypergeometric2F1[1 - n, 1 - z, 2, -s]
```

```
Sum[-s z Hypergeometric2F1[1 - n, 1 - z, 2, -s], {n, 1, m}] /. s → -1
```

$$z\left(\frac{-1+0^z}{z} + \text{DifferenceRoot}\Big[\text{Function}\Big[\{\dot{y}, \dot{n}\}, \{(\dot{n}+z)\ \dot{y}[\dot{n}] + (-1-2\dot{n}-z)\ \dot{y}[1+\dot{n}] + (1+\dot{n})\ \dot{y}[2+\dot{n}] == 0,\ \dot{y}[0] == 0,\ \dot{y}[1] == -\frac{-1+0^z}{z},\ \dot{y}[2] == 1-\frac{-1+0^z}{z}\}\Big]\Big][1+m]\right)$$

```
FullSimplify[Sum[ (s) ^k Binomial[-n - k + 1, k - 1], {n, 1, m}]]
```

$$\frac{1}{k} s^k\ (-(-1+k)\ \text{Binomial}[-k, -1+k] + (-1+k+m)\ \text{Binomial}[-k-m, -1+k])$$

```
(-s) ^k Pochhammer[n - k + 1, k - 1] / (k - 1) ! /. {n → 10, s → 5, k → 4}
```

52 500

```
p2[10, 5, 4]
```

52 500

```
- (s) ^k Binomial[- (n - k + 1), k - 1] /. {n → 10, s → 5, k → 4}
```

52 500

```
(-1) ^k (-1) ^ (k - 1)
```

$(-1)^{-1+2k}$

```
(-1)^(-1)
```

$-1$

```
Sum[ - (s)^k Binomial[-(n - k + 1), k - 1], {n, 1, m}]
```

$$\frac{(-1+k-m)\, s^k\, \text{Binomial}[-2+k-m, -1+k]}{k} \;/.\; \{m \to 10, s \to 5, k \to 4\}$$

131 250

$$\text{st}[m\_, s\_, k\_] := \frac{(-1+k-m)\, s^k\, \text{Binomial}[-2+k-m, -1+k]}{k}$$

$$\text{s3}[m\_, s\_, k\_] := \frac{(-1+k-m)\, s^k\, (-1)\text{^}(k-1)\, \text{Pochhammer}[-(-2+k-m), -1+k]}{k!}$$

$$\text{s4}[m\_, s\_, k\_] := \frac{(m-k+1)\, (-s)^k\, \text{Pochhammer}[m-k+2, k-1]}{k!}$$

```
s5[n_, s_, k_] := s^k Pochhammer[-n, k] / k!
s2[n_, s_, k_] := Sum[ p2[j, s, k], {j, 1, n}]
```

```
Table[ s5[n, s, 1] - s2[n, s, 1], {n, 2, 8}, {s, -2, 3}]
```

{{0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}}

$$\frac{(m-k+1)\, (-s)^k\, \text{Pochhammer}[m-k+2, k-1]}{k!} \;/.\; m \to n$$

$$\text{Expand@Table}\left[\frac{(1-k+n)\, (-s)^k\, \text{Pochhammer}[2-k+n, -1+k]}{k!}, \{k, 0, 6\}\right] \;//\; \text{TableForm}$$

$1$

$-n\, s$

$-\frac{n s^2}{2} + \frac{n^2 s^2}{2}$

$-\frac{n s^3}{3} + \frac{n^2 s^3}{2} - \frac{n^3 s^3}{6}$

$-\frac{n s^4}{4} + \frac{11 n^2 s^4}{24} - \frac{n^3 s^4}{4} + \frac{n^4 s^4}{24}$

$-\frac{n s^5}{5} + \frac{5 n^2 s^5}{12} - \frac{7 n^3 s^5}{24} + \frac{n^4 s^5}{12} - \frac{n^5 s^5}{120}$

$-\frac{n s^6}{6} + \frac{137 n^2 s^6}{360} - \frac{5 n^3 s^6}{16} + \frac{17 n^4 s^6}{144} - \frac{n^5 s^6}{48} + \frac{n^6 s^6}{720}$

```
Expand@Table[s^k Pochhammer[-n, k] / k!, {k, 0, 6}] // TableForm
```

$1$

$-n\, s$

$-\frac{n s^2}{2} + \frac{n^2 s^2}{2}$

$-\frac{n s^3}{3} + \frac{n^2 s^3}{2} - \frac{n^3 s^3}{6}$

$-\frac{n s^4}{4} + \frac{11 n^2 s^4}{24} - \frac{n^3 s^4}{4} + \frac{n^4 s^4}{24}$

$-\frac{n s^5}{5} + \frac{5 n^2 s^5}{12} - \frac{7 n^3 s^5}{24} + \frac{n^4 s^5}{12} - \frac{n^5 s^5}{120}$

$-\frac{n s^6}{6} + \frac{137 n^2 s^6}{360} - \frac{5 n^3 s^6}{16} + \frac{17 n^4 s^6}{144} - \frac{n^5 s^6}{48} + \frac{n^6 s^6}{720}$

```
s5[n_, s_, k_] := s^k Pochhammer[-n, k] / k!
```

```
Sum[ p2[j, 3, 4], {j, 1, 12}]
```

40 095

```
3^4 Pochhammer[-12, 4] / 4!
```

40 095

```
Sum[ Binomial[z, k] s^k Pochhammer[-n, k] / k!, {k, 0, Infinity}]
```

Hypergeometric2F1[-n, -z, 1, -s]

```
Table[ Hypergeometric2F1[-n, -z, 1, -s] /. {z → 1, s → -1}, {n, 1, 12}]
```

{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13}

```
Sum[pz[j, -3, -3], {j, 1, 15}]
```

-9 248 769

```
Table[Sum[pz[j, -1, 1], {j, 1, n}], {n, 1, 10}]
```

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

```
Table[ FullSimplify[Sum[ (-1)^(k - j) Binomial[k, j] Hypergeometric2F1[-n, -j, 1, -s],
    {j, 0, k}]], {k, 1, 5}] // TableForm
```

$$-n\, s$$
$$\frac{1}{2}(-1+n)\, n\, s^2$$
$$-\frac{1}{6}(-2+n)(-1+n)\, n\, s^3$$
$$\frac{1}{24}(-3+n)(-2+n)(-1+n)\, n\, s^4$$
$$-\frac{1}{120}(-4+n)(-3+n)(-2+n)(-1+n)\, n\, s^5$$

```
N@D[Hypergeometric2F1[-n, -z, 1, -s], z] /. {z → 0, n → 10, s → -1}
```

2.92897

```
HarmonicNumber[10.]
```

2.92897

```
Hypergeometric2F1[-n, -z, 1, -s] /. {z → 5, n → 10, s → -1}
```

3003

```
Pochhammer[5 + 1, 10] / (10!)
```

3003

```
Pochhammer[10 + 1, 5] / (5!)
```

3003

```
Sum[ Binomial[z, k] (-1)^k Pochhammer[-n, k] / k!, {k, 0, Infinity}]
```

$$\frac{\text{Gamma}[1+n+z]}{\text{Gamma}[1+n]\ \text{Gamma}[1+z]}$$

```
Hypergeometric2F1[-n, -z, 1, 1] /. {n → 7, z → 9}
```

11 440

```
Pochhammer[9 + 1, 7] / 7 !
```

11 440

```
Integrate[ 1, {j, 0, n}, {k, 0, n - j}]
```

$$\frac{n^2}{2}$$

```
Integrate[ 1, {j, 0, n}, {k, 0, n - j}, {l, 0, n - j - k}]
```

$$\frac{n^3}{6}$$

```
Integrate[ 1, {j, 0, n}, {k, 0, n - j}, {l, 0, n - j - k}, {m, 0, n - j - k - l}]
```

$$\frac{n^4}{24}$$

```
Sum[ Binomial[z, k] n^k / k !, {k, 0, Infinity}]
```

Hypergeometric1F1[-z, 1, -n]

```
Integrate[ (-s) (-s), {j, 0, n}, {k, 0, n - j}]
```

$$\frac{n^2 s^2}{2}$$

```
Integrate[ (-s) (-s) (-s), {j, 0, n}, {k, 0, n - j}, {l, 0, n - j - k}]
```

$$-\frac{1}{6} n^3 s^3$$

```
Sum[ Binomial[z, k] (-s) ^k n^k / k !, {k, 0, Infinity}]
```

Hypergeometric1F1[-z, 1, n s]

```
Sum[Binomial[z, k] (1 / (1 - x) - 1) ^k, {k, 0, Infinity}]
```

$$\left(-\frac{1}{-1+x}\right)^z$$

```
Limit[(1 - x^m) / (1 - x), x → 1]
```

m

```
Series[(2 / (1 - x)) ^2, {x, 0, 10}]
```

$$4 + 8 x + 12 x^2 + 16 x^3 + 20 x^4 + 24 x^5 + 28 x^6 + 32 x^7 + 36 x^8 + 40 x^9 + 44 x^{10} + O[x]^{11}$$

```
Sum[ Binomial[z, k] (2 / (1 - x) - 1) ^k, {k, 0, Infinity}]
```

$$2^z \left(-\frac{1}{-1+x}\right)^z$$

```
FullSimplify@Sum[Binomial[z, k] (2 / (1 - x) - 1) ^k, {k, 0, n}]
```

$$2^z \left(\frac{1}{1-x}\right)^z + \frac{1}{-1+x} (1+x) \left(\frac{1+x}{1-x}\right)^n \text{Binomial}[z, 1+n]\ \text{Hypergeometric2F1}\left[1, 1+n-z, 2+n, \frac{1+x}{-1+x}\right]$$

$$\text{Limit}\left[2^z \left(\frac{1}{1-x}\right)^z + \frac{1}{-1+x} (1+x) \left(\frac{1+x}{1-x}\right)^n \text{Binomial}[z, 1+n]\ \text{Hypergeometric2F1}\left[1, 1+n-z, 2+n, \frac{1+x}{-1+x}\right], x \to 1\right]$$

$$\text{Limit}\left[2^z \left(\frac{1}{1-x}\right)^z + \frac{1}{-1+x} (1+x) \left(\frac{1+x}{1-x}\right)^n \text{Binomial}[z, 1+n]\ \text{Hypergeometric2F1}\left[1, 1+n-z, 2+n, \frac{1+x}{-1+x}\right], x \to 1\right]$$

```
FullSimplify@Sum[Binomial[z, k] ((-s) / (1 - x) - 1)^k, {k, 0, n}]
```

$$\left(\frac{s}{-1+x}\right)^z - (1+s-x)^{1+n} (-1+x)^{-1-n} \text{Binomial}[z, 1+n]\ \text{Hypergeometric2F1}\left[1, 1+n-z, 2+n, 1-\frac{s}{-1+x}\right]$$

$$\text{Limit}\left[\left(\frac{s}{-1+x}\right)^z - (1+s-x)^{1+n} (-1+x)^{-1-n} \text{Binomial}[z, 1+n]\ \text{Hypergeometric2F1}\left[1, 1+n-z, 2+n, 1-\frac{s}{-1+x}\right], x \to 1\right]$$

$$\text{Limit}\left[\left(\frac{s}{-1+x}\right)^z - (1+s-x)^{1+n} (-1+x)^{-1-n} \text{Binomial}[z, 1+n]\ \text{Hypergeometric2F1}\left[1, 1+n-z, 2+n, 1-\frac{s}{-1+x}\right], x \to 1\right]$$

```
Clear[lpt, p2t, pzt]
t[n_, a_, b_] := b (Floor[n / b] - Floor[(n - 1) / b]) - a (Floor[n / a] - Floor[(n - 1) / a])
bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k!
lpt[n_, s_, a_, b_, k_] :=
 lpt[n, s, a, b, k] = Sum[ (1 - (s + 1)^j) / j lpt[n - j, s, a, b, k - 1], {j, 1, n - 1}] -
   a Sum[ (1 - (s + 1)^j) / j lpt[n - j, s, a, b, k - 1], {j, a, n - 1, a}]
lpt[n_, s_, a_, b_, 1] := t[n, a, b] (1 - (s + 1)^n) / n
lpt[n_, s_, a_, b_, 0] := If[n == 0, 1, 0]
p2t[n_, s_, a_, b_, k_] :=
 p2t[n, s, a, b, k] = Sum[(-s) p2t[n - j, s, a, b, k - 1], {j, 1, n - 1}]
p2t[n_, s_, a_, b_, 1] := -s
p2t[n_, s_, a_, b_, 0] := If[n == 0, 1, 0]
pzt[n_, s_, a_, b_, z_] := Sum[ z^k / k! lpt[n, s, a, b, k], {k, 0, n}]
pzxt[n_, s_, a_, b_, z_] := Sum[ bin[z, k] p2t[n, s, a, b, k], {k, 0, n}]


Table[pzt[k, -1, 2, 1, n], {n, 0, 6}, {k, 0, n}] // TableForm
```

Power::indet : Indeterminate expression $0^0$ encountered. ≫

| | | | | | | |
|---|---|---|---|---|---|---|
| Indeterminate | | | | | | |
| 1 | 1 | | | | | |
| 1 | 2 | 1 | | | | |
| 1 | 3 | 3 | 1 | | | |
| 1 | 4 | 6 | 4 | 1 | | |
| 1 | 5 | 10 | 10 | 5 | 1 | |
| 1 | 6 | 15 | 20 | 15 | 6 | 1 |

```
tbs[t_, n_] := Table[ ff[z + 1, n - m k] ff[-z, k], {m, 1, t}, {k, 0, n / m}] // TableForm
tbs2[t_, n_] := Table[ ff[z + 1, n - m k] ff[-z, k], {k, 1, t}, {m, 1, t / k}] // TableForm
```

**tbs[10, 12]**

```
ff[-z, 0] ff[1 + z, 12]    ff[-z, 1] ff[1 + z, 11]    ff[-z, 2] ff[1 + z, 10]    ff[-z, 3] ff[1 + z,
ff[-z, 0] ff[1 + z, 12]    ff[-z, 1] ff[1 + z, 10]    ff[-z, 2] ff[1 + z, 8]     ff[-z, 3] ff[1 + z,
ff[-z, 0] ff[1 + z, 12]    ff[-z, 1] ff[1 + z, 9]     ff[-z, 2] ff[1 + z, 6]     ff[-z, 3] ff[1 + z,
ff[-z, 0] ff[1 + z, 12]    ff[-z, 1] ff[1 + z, 8]     ff[-z, 2] ff[1 + z, 4]     ff[-z, 3] ff[1 + z,
ff[-z, 0] ff[1 + z, 12]    ff[-z, 1] ff[1 + z, 7]     ff[-z, 2] ff[1 + z, 2]
ff[-z, 0] ff[1 + z, 12]    ff[-z, 1] ff[1 + z, 6]     ff[-z, 2] ff[1 + z, 0]
ff[-z, 0] ff[1 + z, 12]    ff[-z, 1] ff[1 + z, 5]
ff[-z, 0] ff[1 + z, 12]    ff[-z, 1] ff[1 + z, 4]
ff[-z, 0] ff[1 + z, 12]    ff[-z, 1] ff[1 + z, 3]
ff[-z, 0] ff[1 + z, 12]    ff[-z, 1] ff[1 + z, 2]
```

**tbs[14, 9]**

```
ff[-z, 0] ff[1 + z, 9]    ff[-z, 1] ff[1 + z, 8]    ff[-z, 2] ff[1 + z, 7]    ff[-z, 3] ff[1 + z, 6]
ff[-z, 0] ff[1 + z, 9]    ff[-z, 1] ff[1 + z, 7]    ff[-z, 2] ff[1 + z, 5]    ff[-z, 3] ff[1 + z, 3]
ff[-z, 0] ff[1 + z, 9]    ff[-z, 1] ff[1 + z, 6]    ff[-z, 2] ff[1 + z, 3]    ff[-z, 3] ff[1 + z, 0]
ff[-z, 0] ff[1 + z, 9]    ff[-z, 1] ff[1 + z, 5]    ff[-z, 2] ff[1 + z, 1]
ff[-z, 0] ff[1 + z, 9]    ff[-z, 1] ff[1 + z, 4]
ff[-z, 0] ff[1 + z, 9]    ff[-z, 1] ff[1 + z, 3]
ff[-z, 0] ff[1 + z, 9]    ff[-z, 1] ff[1 + z, 2]
ff[-z, 0] ff[1 + z, 9]    ff[-z, 1] ff[1 + z, 1]
ff[-z, 0] ff[1 + z, 9]    ff[-z, 1] ff[1 + z, 0]
ff[-z, 0] ff[1 + z, 9]
ff[-z, 0] ff[1 + z, 9]
ff[-z, 0] ff[1 + z, 9]
ff[-z, 0] ff[1 + z, 9]
ff[-z, 0] ff[1 + z, 9]
```

**Sum[(-1)^k Binomial[-z, k], {k, 1, n, 3}]**

$$3^{-1-z}\left(0^{-z}-(-1)^{1/3}\left(1+(-1)^{1/3}\right)^{z}+(-1)^{2/3}\left(1-(-1)^{2/3}\right)^{z}+(-1)^{n}\,3^{1+z}\,\text{Binomial}[-z,3+n]\right.$$
$$\left.\text{HypergeometricPFQ}\left[\left\{1,1+\frac{n}{3}+\frac{z}{3},\frac{4}{3}+\frac{n}{3}+\frac{z}{3},\frac{5}{3}+\frac{n}{3}+\frac{z}{3}\right\},\left\{\frac{4}{3}+\frac{n}{3},\frac{5}{3}+\frac{n}{3},2+\frac{n}{3}\right\},1\right]\right)$$

**Limit[Sum[Pochhammer[z + 1, k] / k!, {k, 0, n, 3}], z → 0]**

$$\text{Limit}\Big[$$
$$\Big(3^{-1-z}\Big(\Big(-\left(1+(-1)^{1/3}\right)^{z}+(-1)^{2/3}\left(1-(-1)^{2/3}\right)^{z}+0^{-1-z}\,3^{z}\left(-1+(-1)^{2/3}\right)\Big)\,\text{Gamma}[4+n]\,\text{Gamma}[1+z]-$$
$$3^{1+z}\left(-1+(-1)^{2/3}\right)\text{Gamma}[4+n+z]$$
$$\text{HypergeometricPFQ}\left[\left\{1,\frac{4}{3}+\frac{n}{3}+\frac{z}{3},\frac{5}{3}+\frac{n}{3}+\frac{z}{3},2+\frac{n}{3}+\frac{z}{3}\right\},\left\{\frac{4}{3}+\frac{n}{3},\frac{5}{3}+\frac{n}{3},2+\frac{n}{3}\right\},1\right]\Big)\Big)\Big/$$
$$\left(\left(-1+(-1)^{2/3}\right)\text{Gamma}[4+n]\,\text{Gamma}[1+z]\right),\ z\to 0\Big]$$

**Table[ Pochhammer[-3, k] / k!, {k, 0, 5}]**

```
{1, -3, 3, -1, 0, 0}
```

**CoefficientList[Series[(1 / (1 - x)) ^ (-5 / 3), {x, 0, 4}], x]**

$$\left\{1,-\frac{5}{3},\frac{5}{9},\frac{5}{81},\frac{5}{243}\right\}$$

**CoefficientList[Series[(1 + x + x^2 + x^3 + x^4)^(-5 / 3), {x, 0, 4}], x]**

$\left\{1, -\frac{5}{3}, \frac{5}{9}, \frac{5}{81}, \frac{5}{243}\right\}$

**Sum[ Pochhammer[k, 2] / 2 ! x^k, {k, 0, Infinity}]**

$-\frac{x}{(-1+x)^3}$

**Sum[ Pochhammer[k, 3] / 3 ! x^k, {k, 0, Infinity}]**

$\frac{x}{(-1+x)^4}$

**Sum[ Pochhammer[k, 4] / 4 ! x^k, {k, 0, Infinity}]**

$-\frac{x}{(-1+x)^5}$

**Sum[Binomial[k, 3] x^k, {k, 0, Infinity}]**

$\frac{x^3}{(-1+x)^4}$

**Sum[Binomial[k, 5] x^k, {k, 0, Infinity}]**

$\frac{x^5}{(-1+x)^6}$

**Sum[ Pochhammer[k, 4] / 4 ! x^k, {k, 0, Infinity}]**

$-\frac{x}{(-1+x)^5}$

**Sum[x^(4 k), {k, 0, Infinity}]**

$\frac{1}{1-x^4}$

**CoefficientList[Series[ 1 / (1 - x)^2, {x, 0, 10}], x]**

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}

**CoefficientList[Series[ 1 / (1 - x)^3, {x, 0, 10}], x]**

{1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66}

**Sum[Binomial[k, 2] x^k, {k, 0, Infinity}]**

$-\frac{x^2}{(-1+x)^3}$

**CoefficientList** $\left[\mathbf{Series}\left[-\frac{x^2}{(-1+x)^3}, \{x, 0, 10\}\right], x\right]$

{0, 0, 1, 3, 6, 10, 15, 21, 28, 36, 45}

**FullSimplify[Sum[1, {j, 0, n}, {k, 0, n - j}, {l, 0, n - j - k}]]**

$\frac{1}{6}(1+n)(2+n)(3+n)$

```
FullSimplify[Sum[1, {j, 1, n - 2}, {k, 1, n - j - 1}, {l, 1, n - j - k}]]
```

$$\frac{1}{6}(-2+n)(-1+n)n$$

```
FullSimplify[Sum[1, {j, 0, n}, {k, 0, n - j}, {l, 0, n - j - k}, {m, 0, n - j - k - l}]]
```

$$\frac{1}{24}(1+n)(2+n)(3+n)(4+n)$$

```
FullSimplify[Sum[1, {j, 1, n - 3}, {k, 1, n - j - 2}, {l, 1, n - j - k - 1}, {m, 1, n - j - k - l}]]
```

$$\frac{1}{24}(-3+n)(-2+n)(-1+n)n$$

```
Table[D[Binomial[n, k], {k, 2}] /. k → 0, {n, 1, 10}]
```

$$\left\{2-\frac{\pi^2}{3},\ \frac{7}{2}-\frac{\pi^2}{3},\ \frac{85}{18}-\frac{\pi^2}{3},\ \frac{415}{72}-\frac{\pi^2}{3},\ \frac{12\,019}{1800}-\frac{\pi^2}{3},\ \frac{13\,489}{1800}-\frac{\pi^2}{3},\ \frac{726\,301}{88\,200}-\frac{\pi^2}{3},\ \frac{3\,144\,919}{352\,800}-\frac{\pi^2}{3},\ \frac{30\,300\,391}{3\,175\,200}-\frac{\pi^2}{3},\ \frac{32\,160\,403}{3\,175\,200}-\frac{\pi^2}{3}\right\}$$

```
HarmonicNumber[10]
```

$$\frac{7381}{2520}$$

```
Table[D[Pochhammer[z + 1, k], {z, 2}] / k ! /. z → 0, {k, 1, 10}]
```

$$\left\{0,\ 1,\ 2,\ \frac{35}{12},\ \frac{15}{4},\ \frac{203}{45},\ \frac{469}{90},\ \frac{29\,531}{5040},\ \frac{6515}{1008},\ \frac{177\,133}{25\,200}\right\}$$

```
FullSimplify[Sum[1, {j, 2, n - 4}, {k, 2, n - j - 2}, {l, 2, n - j - k}]]
```

$$\frac{1}{6}(-5+n)(-4+n)(-3+n)$$

```
FullSimplify[Sum[1, {j, 2, n - 6}, {k, 2, n - j - 4}, {l, 2, n - j - k - 2}, {m, 2, n - j - k - l}]]
```

$$\frac{1}{24}(-7+n)(-6+n)(-5+n)(-4+n)$$

```
FullSimplify[Sum[1, {j, -1, n + 2}, {k, -1, n - j + 1}, {l, -1, n - j - k}]]
```

$$\frac{1}{6}(4+n)(5+n)(6+n)$$

```
Table[ Pochhammer[n + 1 - k y, k] / k !, {y, -2, 2}] /. k → 3 // TableForm
```

$\frac{1}{6}(7+n)(8+n)(9+n)$

$\frac{1}{6}(4+n)(5+n)(6+n)$

$\frac{1}{6}(1+n)(2+n)(3+n)$

$\frac{1}{6}(-2+n)(-1+n)n$

$\frac{1}{6}(-5+n)(-4+n)(-3+n)$

```
FullSimplify[
 Table[ Pochhammer[n + 1 - k y, k] / k ! - Pochhammer[n - k y, k] / k !, {y, -2, 2}] /. k → 3 //
  TableForm]
```

$\frac{1}{2}(7+n)(8+n)$

$\frac{1}{2}(4+n)(5+n)$

$\frac{1}{2}(1+n)(2+n)$

$\frac{1}{2}(-2+n)(-1+n)$

$\frac{1}{2}(-5+n)(-4+n)$

```
Table[ Pochhammer[n + 1 - k y, k - 1] / (k - 1) !, {y, -2, 2}] /. k → 3 // TableForm
```

$\frac{1}{2}(7+n)(8+n)$

$\frac{1}{2}(4+n)(5+n)$

$\frac{1}{2}(1+n)(2+n)$

$\frac{1}{2}(-2+n)(-1+n)$

$\frac{1}{2}(-5+n)(-4+n)$

```
FullSimplify[
 Table[ Pochhammer[n + 1 - k y, k] / k ! - Pochhammer[n - k y, k] / k !, {y, -2, 2}] /. k → 4 //
  TableForm]
```

$\frac{1}{6}(9+n)(10+n)(11+n)$

$\frac{1}{6}(5+n)(6+n)(7+n)$

$\frac{1}{6}(1+n)(2+n)(3+n)$

$\frac{1}{6}(-3+n)(-2+n)(-1+n)$

$\frac{1}{6}(-7+n)(-6+n)(-5+n)$

```
Table[ Pochhammer[n + 1 - k y, k - 1] / (k - 1) !, {y, -2, 2}] /. k → 4 // TableForm
```

$\frac{1}{6}(9+n)(10+n)(11+n)$

$\frac{1}{6}(5+n)(6+n)(7+n)$

$\frac{1}{6}(1+n)(2+n)(3+n)$

$\frac{1}{6}(-3+n)(-2+n)(-1+n)$

$\frac{1}{6}(-7+n)(-6+n)(-5+n)$

```
Expand[(k / (k - 1)) (k - 1) y]
```

$k\,y$

```
FullSimplify[Sum[1, {j, 0, n / 2}, {k, 0, (n - 2 j) / 2}, {l, 0, (n - 2 j - 2 k) / 2}]]
```

$$\frac{1}{48}(2+n)(4+n)(6+n)$$

```
FullSimplify[Sum[1, {j, 0, n}, {k, 0, (n - j)}, {l, 0, (n - j - k)}]]
```

$$\frac{1}{6}(1+n)(2+n)(3+n)$$

```
FullSimplify[Sum[1, {j, 0, n / 3}, {k, 0, (n - 3 j) / 3}, {l, 0, (n - 3 j - 3 k) / 3}]]
```

$$\frac{1}{162}(3+n)(6+n)(9+n)$$

```
FullSimplify[Sum[1, {j, 0, n / 4}, {k, 0, (n - 4 j) / 4}, {l, 0, (n - 4 j - 4 k) / 4}]]
```

$$\frac{1}{384} (4+n) (8+n) (12+n)$$

```
48 / 6
```

8

```
162 / 6
```

27

```
384 / 6
```

64

```
Sum[1, {j, 0, n}, {k, 0, n - j}]
```

$$\frac{1}{2} (1+n) (2+n)$$

```
Limit[Sum[1 / (a^2), {j, 0, a n}, {k, 0, a n - j}], a → Infinity]
```

$$\frac{n^2}{2}$$

```
Limit[Sum[1 / (a^3), {j, 0, a n}, {k, 0, a n - j}, {l, 0, a n - j - k}], a → Infinity]
```

$$\frac{n^3}{6}$$

```
Limit[Sum[1 / (a^2), {j, 1, a n - 1}, {k, 1, a n - j}], a → Infinity]
```

$$\frac{n^2}{2}$$

```
Limit[Sum[1 / (a^3), {j, 1, a n - 2}, {k, 1, a n - j - 1}, {l, 1, a n - j - k}], a → Infinity]
```

$$\frac{n^3}{6}$$

```
ba[n_, k_] := Sum[ Binomial[k, j] n^j / (j !), {j, 0, k}]
Table[Expand[ba[n, k]], {k, 0, 8}] // TableForm
```

$1$

$1+n$

$1+2n+\frac{n^2}{2}$

$1+3n+\frac{3n^2}{2}+\frac{n^3}{6}$

$1+4n+3n^2+\frac{2n^3}{3}+\frac{n^4}{24}$

$1+5n+5n^2+\frac{5n^3}{3}+\frac{5n^4}{24}+\frac{n^5}{120}$

$1+6n+\frac{15n^2}{2}+\frac{10n^3}{3}+\frac{5n^4}{8}+\frac{n^5}{20}+\frac{n^6}{720}$

$1+7n+\frac{21n^2}{2}+\frac{35n^3}{6}+\frac{35n^4}{24}+\frac{7n^5}{40}+\frac{7n^6}{720}+\frac{n^7}{5040}$

$1+8n+14n^2+\frac{28n^3}{3}+\frac{35n^4}{12}+\frac{7n^5}{15}+\frac{7n^6}{180}+\frac{n^7}{630}+\frac{n^8}{40320}$

```
Expand[Limit[Sum[1 / (a^2), {j, 1, a n}, {k, a, a n - j}], a → Infinity] -
  Limit[Sum[1 / (a), {j, 1, a n}], a → Infinity] + 1]
```

$$1-2n+\frac{n^2}{2}$$

```
Limit[Sum[1 / (a), {j, 1, a n}], a → Infinity]
```

n

```
Sum[ Binomial[z, j] n^j / (j!), {j, 0, Infinity}]
```

Hypergeometric1F1[-z, 1, -n]

```
FullSimplify[Sum[1 / (a^1), {j, 1, a n}]] /. a → x
FullSimplify[Sum[1 / (a^2), {j, 1, a n - 1}, {k, 1, a n - j}]] /. a → x
FullSimplify[Sum[1 / (a^3), {j, 1, a n - 2}, {k, 1, a n - j - 1}, {l, 1, a n - j - k}]] /. a → x
FullSimplify[Sum[1 / (a^4), {j, 1, a n - 3},
   {k, 1, a n - j - 2}, {l, 1, a n - j - k - 1}, {m, 1, a n - j - k - l}]] /. a → x
```

n

$$\frac{n(-1+n x)}{2 x}$$

$$\frac{n(-2+n x)(-1+n x)}{6 x^2}$$

$$\frac{n(-3+n x)(-2+n x)(-1+n x)}{24 x^3}$$

```
pr[n_, k_, x_] := Product[(n x - j), {j, 0, k - 1}] / (k! x^k)
```

```
Table[pr[n, k, x], {k, 1, 4}] // TableForm
```

n

$$\frac{n(-1+n x)}{2 x}$$

$$\frac{n(-2+n x)(-1+n x)}{6 x^2}$$

$$\frac{n(-3+n x)(-2+n x)(-1+n x)}{24 x^3}$$

$$\text{Limit}\left[\frac{n(-3+n x)(-2+n x)(-1+n x)}{24 x^3}, x \to \text{Infinity}\right]$$

$$\frac{n^4}{24}$$

```
Sum[(-1)^(j + 1) / j n^j / (j!), {j, 1, Infinity}]
```

EulerGamma + Gamma[0, n] + Log[n]

$$\text{FullSimplify}\left[D\left[\frac{n(-1+n x)}{2 x}, x\right]\right]$$

$$\frac{n}{2 x^2}$$

$$\text{FullSimplify}\left[D\left[\frac{n(-2+n x)(-1+n x)}{6 x^2}, x\right]\right]$$

$$\frac{n(-4+3 n x)}{6 x^3}$$

$$\text{FullSimplify}\left[D\left[\frac{n(-3+n x)(-2+n x)(-1+n x)}{24 x^3}, x\right]\right]$$

$$\frac{n(9+n x(-11+3 n x))}{12 x^4}$$

```
Sum[ Binomial[z, k] x^k Product[(n / x - j), {j, 0, k - 1}] / ( k! ), {k, 0, Infinity}]
```

$$\text{Hypergeometric2F1}\left[-\frac{n}{x}, -z, 1, x\right]$$

$$\textbf{Limit}\left[\textbf{Hypergeometric2F1}\left[-\frac{n}{x}, -z, 1, x\right], x \to 0\right]$$

$$\text{Limit}\left[\text{Hypergeometric2F1}\left[-\frac{n}{x}, -z, 1, x\right], x \to 0\right]$$

$$\textbf{D}\left[\textbf{Hypergeometric2F1}\left[-\frac{n}{x}, -z, 1, x\right], x\right]$$

$$\frac{n\, z\, \text{Hypergeometric2F1}\left[1-\frac{n}{x}, 1-z, 2, x\right]}{x} + \frac{n\, \text{Hypergeometric2F1}^{(1,0,0,0)}\left[-\frac{n}{x}, -z, 1, x\right]}{x^2}$$

```
Series[(1 - x) / (1 - x^2), {x, 0, 10}]
```

$$1 - x + x^2 - x^3 + x^4 - x^5 + x^6 - x^7 + x^8 - x^9 + x^{10} + O[x]^{11}$$

```
Series[(1 - x) / (1 - x^2), {x, 0, 10}]
```