```
DD[k_, a_, n_] :=
  Sum[Binomial[k, j] DD[k-j, m+1, Floor[n/(m^j)]], {m, a, n^(1/k)}, {j, 1, k}]
DD[1, a_, n_] := Floor[n] - a + 1
DD[0, a_{n}] := 1
DS[n_{k_{1}}, k_{1}] := DD[k, 2, n]
DDD[n_{,k_{]}} := Sum[DDD[n/j, k-1], {j, 2, n}]
DDD[n_{,} 0] := 1
D2[1, a_{n}, n_{p}, r_{1} := n - a + 1]
D2[2, a_{r}, n_{r}, p_{r}] := p/((r+1)(r+2)) + (Floor[n/a] - a)(p/(r+1)) +
     (Floor[n^{(1/2)}] - a) (p/2) + pSum[Floor[n/m] - m, {m, a+1, n^{(1/2)}}]
D2[k_{-}, a_{-}, n_{-}, p_{-}, r_{-}] := D2[k-1, a, n/a, p/(r+1), r+1] +
     Sum[D2[k-1, m, n/m, p, 1], \{m, a+1, n^{(1/k)}]
DD2[n_{,k_{]}} := D2[k, 2, n, k!, 0]
D3[1, a_{n}, n_{r}] := p/(r+1) + p(Floor[n] - a)
D3[k_, a_, n_, p_, r_] :=
  D3[k-1, a, n/a, p/(r+1), r+1] + Sum[D3[k-1, m, n/m, p, 1], {m, a+1, n^(1/k)}]
DD3[n_{k}] := D3[k, 2, n, k!, 0]
D4[1, a_{n}, n_{p}, r] := p(Floor[n] - a + 1) - p/(r + 1)
D4[k_, a_, n_, p_, r_] :=
  Sum[D4[k-1, m, n/m, p, 1], \{m, a, n^{(1/k)}] - D4[k-1, a, n/a, p/(r+1), r+1]
DD4[n_{,k_{-}}] := D4[k, 1, n, k!, 0]
DDD[1000, 4]
13952
DD4[1000, 4]
13952
D5[1, a_{n}, p_{r}, r_{s}] := s!/p/(r+1) + (s!/p) (Floor[n] - a)
D5[k_{-}, a_{-}, n_{-}, p_{-}, r_{-}, s_{-}] := D5[k-1, a, n/a, p(r+1), r+1, s+1] +
     {\tt Sum} [\; {\tt D5} [\; k - 1, \; m, \; n \, / \, m, \; p, \; 1, \; s + 1] \; , \; \{ m, \; a + 1, \; n^{\, \wedge} \; (1 \, / \, k) \; \}]
DD5[n_{,k_{||}} := D5[k, 2, n, 1, 0, 1]
DD5[1000, 3]
11 217
DDD[1000, 3]
11 217
D6[1, a_{n}, n_{p}, r_{s}] := p + (p (r+1)) (Floor[n] - a)
D6[k_{-}, a_{-}, n_{-}, p_{-}, r_{-}, s_{-}] := D6[k-1, a, n/a, p, 1, s+1] +
     Sum[D6[k-1, m, n/m, p(r+1), r+1, s+1], \{m, a+1, n^{(1/k)}\}]
DD6[n_{,k_{||}} := D6[k, 2, n, 1, 0, 1]
DD6[1000, 3]
10655
DA[s_{n}, n_{n}, a_{n}, k_{n}, p_{n}, r_{n}] := DA[s_{n}, n_{n}, a_{n}, k_{n}, k_{n}, r_{n}, r_{n}] + DA[s_{n}, n_{n}, a_{n}, k_{n}, r_{n}, r_{n}] + DA[s_{n}, n_{n}, a_{n}, k_{n}, r_{n}, r_
     Sum[DA[s-1, n/m, m, k+1, kp, 2], \{m, a+1, n^{(1/s)}\}
```

```
DDD[1000, 3]
11 217
DDA[1000, 3]
11 217
DB[1, n_{-}, a_{-}, k_{-}, p_{-}, r_{-}] := p (Floor[n] - a + 1) - p / r
DB[s-1, n/a, a, k+1, kp/r, r+1]
DDB[n_{, s_{, l}} := DB[s, n, 1, 2, 1, 1]
DDD[1000, 3]
11 217
DDB[1000, 3]
11 217
D7[a_, n_, p_, r_, s_] :=
 (s!/p/r + (s!/p) (Floor[n] - a))/s-
  If[n^{(1/2)} \ge a, D7[a, n/a, pr, r+1, s+1] +
      \mathtt{Sum} [ \ \mathtt{D7} \ [\mathtt{m}, \ \mathtt{n} \ / \ \mathtt{m}, \ \mathtt{p}, \ \mathtt{2}, \ \mathtt{s} + \mathtt{1}] \ , \ \{\mathtt{m}, \ \mathtt{a} + \mathtt{1}, \ \mathtt{n} \ \backslash \ (\mathtt{1} \ / \ \mathtt{2}) \, \} \, ] \ , \ \mathtt{0}]
DD7[n_] := D7[2, n, 1, 1, 1]
DD7[100]
428
15
DiscretePlot[ DD7[n], {n, 2, 100}]
25
20
15
10
                                                               100
```

```
D8[s_Number, n_Number, a_Integer, k_Integer, p_Integer, r_Integer] :=
 p(Floor[n/s] *s-a + 1/r) -
  If [n^{(1/2)} \ge a,
  Sum[D8[s, n/m, m, k+1, kp/r, r+1]*s, {m, a, a+.999999, s}], 0] -
  Sum[D8[s, n/m, m, k+1, kp, 2]*s, {m, a+1, n^(1/2), s}]
D8a[n_, a_, k_, p_, r_] :=
 p(Floor[n] - a + 1/r) -
  Sum[D8a[n/m, m, k+1, kp, 2], \{m, a+1, n^{(1/2)}]
DD8a[n_] := D8a[n, 2, 1, 1, 1]
DD8[100, .1]
29.3156
DD8[100, .2]
28.9508
DD8[100, .3]
30.848
DD8[100, .5]
29.2466
N[DD8[100, 1]]
28.5333
DD8[100, .05]
$Aborted
DD8a[100]
428
15
DD9[n_, k_, a_] :=
 Sum[(-1)^{(j-1)} Binomial[k, j] DD9[n/(m^j), k-j, m], {j, 1, k}, {m, a, n^(1/k)}]
DD9[n_, 0, a_] := 1
DD9[1000, 5, 2]
10602
DDD[1000, 5]
10602
DiscretePlot[DD9[n, 4, 2], {n, 2, 1000}]
```