```
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
referenced[n_, z_] := Product[(-1)^p[[2]] bin[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
referenceD[n_, z_] := Sum[referenced[j, z], {j, 1, n}]
DAlt[n_, z_] := Sum[z^k / k! (D[referenceD[n, y], {y, k}] /. y → 0), {k, 0, Log[2, n]}]
Grid[Table[{referenceD[a = 143, s + t I], DAlt[a, s + t I]}, {s, -1.5, 4, .7}, {t, -1.1, 4, .7}]]
```

{16.0605 + 14.7882 i, 16.0605 + 14.7882 i}	{7.01799 + 5.33418 i, 7.01799 + 5.33418 i}	{6.67848 - 3.98695 i, 6.67848 - 3.98695 i}	{13.6614 - 13.5084 i, 13.6614 - 13.5084 i}	{45.7809 - 19.3357 i, 45.7809 - 19.3357 i}	{139.117 - 3.518 i, 139.117 - 3.518 i}	{346.087 + 75.6627 i, 346.087 + 75.6627 i}	{732.518 + 293.202 i, 732.518 + 293.202 i}
{-7.42241 + 29.7552 i, -7.42241 + 29.7552 i}	{-5.12015 + 5.79344 i, -5.12015 + 5.79344 i}	{-4.56204 - 4.08102 i, -4.56204 - 4.08102 i}	{-7.87033 - 24.6323 i, -7.87033 - 24.6323 i}	{12.4009 - 75.9507 i, 12.4009 - 75.9507 i}	{112.261 - 162.582 i, 112.261 - 162.582 i}	{374.193 - 262.779 i, 374.193 - 262.779 i}	{903.99 - 317.942 i, 903.99 - 317.942 i}
{-56.0223 + 9.25 i, -56.0223 + 9.25 i}	{-10.5689 - 8.8744 i, -10.5689 - 8.8744 i}	{-6.97805 + 7.28175 i, -6.97805 + 7.28175 i}	{-48.191 - 2.39535 i, -48.191 - 2.39535 i}	{-96.1249 - 92.8639 i, -96.1249 - 92.8639 i}	{-72.7546 - 301.934 i, -72.7546 - 301.934 i}	{137.642 - 638.389 i, 137.642 - 638.389 i}	{685.071 - 1070.32 i, 685.071 - 1070.32 i}
{-99.9941 - 101.923 i, -99.9941 - 101.923 i}	{32.7006 - 58.9696 i, 32.7006 - 58.9696 i}	{42.2795 + 45.338 i, 42.2795 + 45.338 i}	{-75.1011 + 103.495 i, -75.1011 + 103.495 i}	{-269.64 + 13.6667 i, -269.64 + 13.6667 i}	{-439.049 - 307.107 i, -439.049 - 307.107 i}	{-430.959 - 909.836 i, -430.959 - 909.836 i}	{-46.5707 - 1800.69 i, -46.5707 - 1800.69 i}
{-69.5115 - 377.855 i, -69.5115 - 377.855 i}	{208.047 - 171.681 i, 208.047 - 171.681 i}	{227.472 + 130.492 i, 227.472 + 130.492 i}	{-16.0705 + 360.517 i, -16.0705 + 360.517 i}	{-459.903 + 356.475 i, -459.903 + 356.475 i}	{-975.07 - 22.8456 i, -975.07 - 22.8456 i}	{-1368.9 - 883.571 i, -1368.9 - 883.571 i}	{-1388.91 - 2282.58 i, -1388.91 - 2282.58 i}
{157.887 - 913.433 i, 157.887 - 913.433 i}	{652.798 - 381.775 i, 652.798 - 381.775 i}	{686.921 + 288.831 i, 686.921 + 288.831 i}	{254.333 + 855.063 i, 254.333 + 855.063 i}	{-568.176 + 1080.41 i, -568.176 + 1080.41 i}	{-1622.42 + 751.038 i, -1622.42 + 751.038 i}	{-2671.53 - 308.491 i, -2671.53 - 308.491 i}	{-3404.11 - 2219.65 i, -3404.11 - 2219.65 i}

{771.1 − 1826.36 i, 771.1 − 1826.36 i}	{1572.15 − 732.333 i, 1572.15 − 732.333 i}	{1626.9 + 552.68 i, 1626.9 + 552.68 i}	{928.264 + 1694.27 i, 928.264 + 1694.27 i}	{−431.562 + 2365.48 i, −431.562 + 2365.48 i}	{−2262.43 + 2264.07 i, −2262.43 + 2264.07 i}	{−4279.16 + 1129.77 i, −4279.16 + 1129.77 i}	{−6105.87 − 1238.79 i, −6105.87 − 1238.79 i}
{2040.59 − 3259.3 i, 2040.59 − 3259.3 i}	{3254.36 − 1275.53 i, 3254.36 − 1275.53 i}	{3336.88 + 961.187 i, 3336.88 + 961.187 i}	{2279.75 + 3008. i, 2279.75 + 3008. i}	{191.98 + 4430.16 i, 191.98 + 4430.16 i}	{−2701.44 + 4819.84 i, −2701.44 + 4819.84 i}	{−6062.62 + 3814.9 i, −6062.62 + 3814.9 i}	{−9445.37 + 1117.38 i, −9445.37 + 1117.38 i}

```
P2[n_, k_] :=
 P2[n, k] = Sum[FullSimplify[MangoldtLambda[j] / Log[j]] P2[n / j, k - 1], {j, 2, Floor[n]}];
P2[n_, 0] := 1
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
referenced[n_, z_] := Product[(-1)^p[[2]] bin[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
referenceD[n_, z_] := Sum[referenced[j, z], {j, 1, n}]
Grid[Table[{FullSimplify[P2[n, k]], "=",
   (D[referenceD[n, z], {z, k}] /. z → 0)}, {n, 1, 50}, {k, 1, 5}]]
```

{0, =, 0}	{0, =, 0}	{0, =, 0}	{0, =, 0}	{0, =, 0}
{1, =, 1}	{0, =, 0}	{0, =, 0}	{0, =, 0}	{0, =, 0}
{2, =, 2}	{0, =, 0}	{0, =, 0}	{0, =, 0}	{0, =, 0}
$\{\frac{5}{2}, =, \frac{5}{2}\}$	{1, =, 1}	{0, =, 0}	{0, =, 0}	{0, =, 0}
$\{\frac{7}{2}, =, \frac{7}{2}\}$	{1, =, 1}	{0, =, 0}	{0, =, 0}	{0, =, 0}
$\{\frac{7}{2}, =, \frac{7}{2}\}$	{3, =, 3}	{0, =, 0}	{0, =, 0}	{0, =, 0}
$\{\frac{9}{2}, =, \frac{9}{2}\}$	{3, =, 3}	{0, =, 0}	{0, =, 0}	{0, =, 0}
$\{\frac{29}{6}, =, \frac{29}{6}\}$	{4, =, 4}	{1, =, 1}	{0, =, 0}	{0, =, 0}
$\{\frac{16}{3}, =, \frac{16}{3}\}$	{5, =, 5}	{1, =, 1}	{0, =, 0}	{0, =, 0}
$\{\frac{16}{3}, =, \frac{16}{3}\}$	{7, =, 7}	{1, =, 1}	{0, =, 0}	{0, =, 0}
$\{\frac{19}{3}, =, \frac{19}{3}\}$	{7, =, 7}	{1, =, 1}	{0, =, 0}	{0, =, 0}
$\{\frac{19}{3}, =, \frac{19}{3}\}$	{8, =, 8}	{4, =, 4}	{0, =, 0}	{0, =, 0}
$\{\frac{22}{3}, =, \frac{22}{3}\}$	{8, =, 8}	{4, =, 4}	{0, =, 0}	{0, =, 0}
$\{\frac{22}{3}, =, \frac{22}{3}\}$	{10, =, 10}	{4, =, 4}	{0, =, 0}	{0, =, 0}
$\{\frac{22}{3}, =, \frac{22}{3}\}$	{12, =, 12}	{4, =, 4}	{0, =, 0}	{0, =, 0}
$\{\frac{91}{12}, =, \frac{91}{12}\}$	$\{\frac{155}{12}, =, \frac{155}{12}\}$	$\{\frac{11}{2}, =, \frac{11}{2}\}$	{1, =, 1}	{0, =, 0}
$\{\frac{103}{12}, =, \frac{103}{12}\}$	$\{\frac{155}{12}, =, \frac{155}{12}\}$	$\{\frac{11}{2}, =, \frac{11}{2}\}$	{1, =, 1}	{0, =, 0}
$\{\frac{103}{12}, =, \frac{103}{12}\}$	$\{\frac{167}{12}, =, \frac{167}{12}\}$	$\{\frac{17}{2}, =, \frac{17}{2}\}$	{1, =, 1}	{0, =, 0}
$\{\frac{115}{12}, =, \frac{115}{12}\}$	$\{\frac{167}{12}, =, \frac{167}{12}\}$	$\{\frac{17}{2}, =, \frac{17}{2}\}$	{1, =, 1}	{0, =, 0}
$\{\frac{115}{12}, =, \frac{115}{12}\}$	$\{\frac{179}{12}, =, \frac{179}{12}\}$	$\{\frac{23}{2}, =, \frac{23}{2}\}$	{1, =, 1}	{0, =, 0}
$\{\frac{115}{12}, =, \frac{115}{12}\}$	$\{\frac{203}{12}, =, \frac{203}{12}\}$	$\{\frac{23}{2}, =, \frac{23}{2}\}$	{1, =, 1}	{0, =, 0}

$\{\frac{115}{12}, =, \frac{115}{12}\}$	$\{\frac{227}{12}, =, \frac{227}{12}\}$	$\{\frac{23}{2}, =, \frac{23}{2}\}$	$\{1, =, 1\}$	$\{0, =, 0\}$
$\{\frac{127}{12}, =, \frac{127}{12}\}$	$\{\frac{227}{12}, =, \frac{227}{12}\}$	$\{\frac{23}{2}, =, \frac{23}{2}\}$	$\{1, =, 1\}$	$\{0, =, 0\}$
$\{\frac{127}{12}, =, \frac{127}{12}\}$	$\{\frac{235}{12}, =, \frac{235}{12}\}$	$\{\frac{29}{2}, =, \frac{29}{2}\}$	$\{5, =, 5\}$	$\{0, =, 0\}$
$\{\frac{133}{12}, =, \frac{133}{12}\}$	$\{\frac{247}{12}, =, \frac{247}{12}\}$	$\{\frac{29}{2}, =, \frac{29}{2}\}$	$\{5, =, 5\}$	$\{0, =, 0\}$
$\{\frac{133}{12}, =, \frac{133}{12}\}$	$\{\frac{271}{12}, =, \frac{271}{12}\}$	$\{\frac{29}{2}, =, \frac{29}{2}\}$	$\{5, =, 5\}$	$\{0, =, 0\}$
$\{\frac{137}{12}, =, \frac{137}{12}\}$	$\{\frac{283}{12}, =, \frac{283}{12}\}$	$\{\frac{31}{2}, =, \frac{31}{2}\}$	$\{5, =, 5\}$	$\{0, =, 0\}$
$\{\frac{137}{12}, =, \frac{137}{12}\}$	$\{\frac{295}{12}, =, \frac{295}{12}\}$	$\{\frac{37}{2}, =, \frac{37}{2}\}$	$\{5, =, 5\}$	$\{0, =, 0\}$
$\{\frac{149}{12}, =, \frac{149}{12}\}$	$\{\frac{295}{12}, =, \frac{295}{12}\}$	$\{\frac{37}{2}, =, \frac{37}{2}\}$	$\{5, =, 5\}$	$\{0, =, 0\}$
$\{\frac{149}{12}, =, \frac{149}{12}\}$	$\{\frac{295}{12}, =, \frac{295}{12}\}$	$\{\frac{49}{2}, =, \frac{49}{2}\}$	$\{5, =, 5\}$	$\{0, =, 0\}$
$\{\frac{161}{12}, =, \frac{161}{12}\}$	$\{\frac{295}{12}, =, \frac{295}{12}\}$	$\{\frac{49}{2}, =, \frac{49}{2}\}$	$\{5, =, 5\}$	$\{0, =, 0\}$
$\{\frac{817}{60}, =, \frac{817}{60}\}$	$\{\frac{305}{12}, =, \frac{305}{12}\}$	$\{\frac{105}{4}, =, \frac{105}{4}\}$	$\{7, =, 7\}$	$\{1, =, 1\}$
$\{\frac{817}{60}, =, \frac{817}{60}\}$	$\{\frac{329}{12}, =, \frac{329}{12}\}$	$\{\frac{105}{4}, =, \frac{105}{4}\}$	$\{7, =, 7\}$	$\{1, =, 1\}$
$\{\frac{817}{60}, =, \frac{817}{60}\}$	$\{\frac{353}{12}, =, \frac{353}{12}\}$	$\{\frac{105}{4}, =, \frac{105}{4}\}$	$\{7, =, 7\}$	$\{1, =, 1\}$
$\{\frac{817}{60}, =, \frac{817}{60}\}$	$\{\frac{377}{12}, =, \frac{377}{12}\}$	$\{\frac{105}{4}, =, \frac{105}{4}\}$	$\{7, =, 7\}$	$\{1, =, 1\}$
$\{\frac{817}{60}, =, \frac{817}{60}\}$	$\{\frac{383}{12}, =, \frac{383}{12}\}$	$\{\frac{117}{4}, =, \frac{117}{4}\}$	$\{13, =, 13\}$	$\{1, =, 1\}$
$\{\frac{877}{60}, =, \frac{877}{60}\}$	$\{\frac{383}{12}, =, \frac{383}{12}\}$	$\{\frac{117}{4}, =, \frac{117}{4}\}$	$\{13, =, 13\}$	$\{1, =, 1\}$
$\{\frac{877}{60}, =, \frac{877}{60}\}$	$\{\frac{407}{12}, =, \frac{407}{12}\}$	$\{\frac{117}{4}, =, \frac{117}{4}\}$	$\{13, =, 13\}$	$\{1, =, 1\}$
$\{\frac{877}{60}, =, \frac{877}{60}\}$	$\{\frac{431}{12}, =, \frac{431}{12}\}$	$\{\frac{117}{4}, =, \frac{117}{4}\}$	$\{13, =, 13\}$	$\{1, =, 1\}$
$\{\frac{877}{60}, =, \frac{877}{60}\}$	$\{\frac{439}{12}, =, \frac{439}{12}\}$	$\{\frac{129}{4}, =, \frac{129}{4}\}$	$\{17, =, 17\}$	$\{1, =, 1\}$
$\{\frac{937}{60}, =, \frac{937}{60}\}$	$\{\frac{439}{12}, =, \frac{439}{12}\}$	$\{\frac{129}{4}, =, \frac{129}{4}\}$	$\{17, =, 17\}$	$\{1, =, 1\}$
$\{\frac{937}{60}, =, \frac{937}{60}\}$	$\{\frac{439}{12}, =, \frac{439}{12}\}$	$\{\frac{153}{4}, =, \frac{153}{4}\}$	$\{17, =, 17\}$	$\{1, =, 1\}$
$\{\frac{997}{60}, =, \frac{997}{60}\}$	$\{\frac{439}{12}, =, \frac{439}{12}\}$	$\{\frac{153}{4}, =, \frac{153}{4}\}$	$\{17, =, 17\}$	$\{1, =, 1\}$
$\{\frac{997}{60}, =, \frac{997}{60}\}$	$\{\frac{451}{12}, =, \frac{451}{12}\}$	$\{\frac{165}{4}, =, \frac{165}{4}\}$	$\{17, =, 17\}$	$\{1, =, 1\}$
$\{\frac{997}{60}, =, \frac{997}{60}\}$	$\{\frac{463}{12}, =, \frac{463}{12}\}$	$\{\frac{177}{4}, =, \frac{177}{4}\}$	$\{17, =, 17\}$	$\{1, =, 1\}$
$\{\frac{997}{60}, =, \frac{997}{60}\}$	$\{\frac{487}{12}, =, \frac{487}{12}\}$	$\{\frac{177}{4}, =, \frac{177}{4}\}$	$\{17, =, 17\}$	$\{1, =, 1\}$
$\{\frac{1057}{60}, =, \frac{1057}{60}\}$	$\{\frac{487}{12}, =, \frac{487}{12}\}$	$\{\frac{177}{4}, =, \frac{177}{4}\}$	$\{17, =, 17\}$	$\{1, =, 1\}$
$\{\frac{1057}{60}, =, \frac{1057}{60}\}$	$\{\frac{493}{12}, =, \frac{493}{12}\}$	$\{47, =, 47\}$	$\{23, =, 23\}$	$\{6, =, 6\}$
$\{\frac{1087}{60}, =, \frac{1087}{60}\}$	$\{\frac{505}{12}, =, \frac{505}{12}\}$	$\{47, =, 47\}$	$\{23, =, 23\}$	$\{6, =, 6\}$
$\{\frac{1087}{60}, =, \frac{1087}{60}\}$	$\{\frac{517}{12}, =, \frac{517}{12}\}$	$\{50, =, 50\}$	$\{23, =, 23\}$	$\{6, =, 6\}$

```
referenceRiemanPrimeCount[n_] := Sum[FullSimplify[MangoldtLambda[j] / Log[j]], {j, 2, n}]
referenced[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
referenceD[n_, z_] := Sum[referenced[j, z], {j, 1, n}]
Table[{referenceRiemanPrimeCount[n], "=", Residue[referenceD[n, z] / z^2, {z, 0}]},
  {n, 1, 100}] // TableForm
```

0	=	0
1	=	1
2	=	2
$\frac{5}{2}$	=	$\frac{5}{2}$
$\frac{7}{2}$	=	$\frac{7}{2}$

$$\frac{7}{2} = \frac{7}{2}$$
$$\frac{9}{2} = \frac{9}{2}$$
$$\frac{29}{6} = \frac{29}{6}$$
$$\frac{16}{3} = \frac{16}{3}$$
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```
bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k!
d[x_, k_] := d[x, k] = Sum[d[j, k - 1] d[x / j, 1], {j, Divisors[x]}];
d[x_, 1] := 1; d[x_, 0] := 0; d[1, 0] := 1
dAlt[x_, z_] := Product[(-1)^p[[2]] bin[-z, p[[2]]], {p, FI[x]}];
FI[x_] := FactorInteger[x]; FI[1] := {}
Grid[Table[{d[x, k], dAlt[x, k]}, {x, 1, 100}, {k, 1, 10}]]
```

{1, 1}	{1, 1}	{1, 1}	{1, 1}	{1, 1}	{1, 1}	{1, 1}	{1, 1}	{1, 1}	{1, 1}
{1, 1}	{2, 2}	{3, 3}	{4, 4}	{5, 5}	{6, 6}	{7, 7}	{8, 8}	{9, 9}	{10, 10}
{1, 1}	{2, 2}	{3, 3}	{4, 4}	{5, 5}	{6, 6}	{7, 7}	{8, 8}	{9, 9}	{10, 10}
{1, 1}	{3, 3}	{6, 6}	{10, 10}	{15, 15}	{21, 21}	{28, 28}	{36, 36}	{45, 45}	{55, 55}
{1, 1}	{2, 2}	{3, 3}	{4, 4}	{5, 5}	{6, 6}	{7, 7}	{8, 8}	{9, 9}	{10, 10}
{1, 1}	{4, 4}	{9, 9}	{16, 16}	{25, 25}	{36, 36}	{49, 49}	{64, 64}	{81, 81}	{100, 100}
{1, 1}	{2, 2}	{3, 3}	{4, 4}	{5, 5}	{6, 6}	{7, 7}	{8, 8}	{9, 9}	{10, 10}
{1, 1}	{4, 4}	{10, 10}	{20, 20}	{35, 35}	{56, 56}	{84, 84}	{120, 120}	{165, 165}	{220, 220}
{1, 1}	{3, 3}	{6, 6}	{10, 10}	{15, 15}	{21, 21}	{28, 28}	{36, 36}	{45, 45}	{55, 55}
{1, 1}	{4, 4}	{9, 9}	{16, 16}	{25, 25}	{36, 36}	{49, 49}	{64, 64}	{81, 81}	{100, 100}
{1, 1}	{2, 2}	{3, 3}	{4, 4}	{5, 5}	{6, 6}	{7, 7}	{8, 8}	{9, 9}	{10, 10}
{1, 1}	{6, 6}	{18, 18}	{40, 40}	{75, 75}	{126, 126}	{196, 196}	{288, 288}	{405, 405}	{550, 550}
{1, 1}	{2, 2}	{3, 3}	{4, 4}	{5, 5}	{6, 6}	{7, 7}	{8, 8}	{9, 9}	{10, 10}
{1, 1}	{4, 4}	{9, 9}	{16, 16}	{25, 25}	{36, 36}	{49, 49}	{64, 64}	{81, 81}	{100, 100}
{1, 1}	{4, 4}	{9, 9}	{16, 16}	{25, 25}	{36, 36}	{49, 49}	{64, 64}	{81, 81}	{100, 100}
{1, 1}	{5, 5}	{15, 15}	{35, 35}	{70, 70}	{126, 126}	{210, 210}	{330, 330}	{495, 495}	{715, 715}

```
{1, 1}  {2, 2}   {3, 3}   {4, 4}   {5, 5}   {6, 6}   {7, 7}   {8, 8}   {9, 9}   {10, 10}
{1, 1}  {6, 6}  {18, 18} {40, 40} {75, 75} {126,    {196,    {288,    {405,    {550,
                                            126}     196}     288}     405}     550}
{1, 1}  {2, 2}   {3, 3}   {4, 4}   {5, 5}   {6, 6}   {7, 7}   {8, 8}   {9, 9}   {10, 10}
{1, 1}  {6, 6}  {18, 18} {40, 40} {75, 75} {126,    {196,    {288,    {405,    {550,
                                            126}     196}     288}     405}     550}
{1, 1}  {4, 4}   {9, 9}   {16, 16} {25, 25} {36, 36} {49, 49} {64, 64} {81, 81} {100,
                                                                                 100}
{1, 1}  {4, 4}   {9, 9}   {16, 16} {25, 25} {36, 36} {49, 49} {64, 64} {81, 81} {100,
                                                                                 100}
{1, 1}  {2, 2}   {3, 3}   {4, 4}   {5, 5}   {6, 6}   {7, 7}   {8, 8}   {9, 9}   {10, 10}
{1, 1}  {8, 8}  {30, 30} {80, 80} {175,    {336,    {588,    {960,    {1485,   {2200,
                                   175}     336}     588}     960}     1485}    2200}
{1, 1}  {3, 3}   {6, 6}   {10, 10} {15, 15} {21, 21} {28, 28} {36, 36} {45, 45} {55, 55}
{1, 1}  {4, 4}   {9, 9}   {16, 16} {25, 25} {36, 36} {49, 49} {64, 64} {81, 81} {100,
                                                                                 100}
{1, 1}  {4, 4}  {10, 10} {20, 20} {35, 35} {56, 56} {84, 84} {120,    {165,    {220,
                                                                    120}     165}     220}
{1, 1}  {6, 6}  {18, 18} {40, 40} {75, 75} {126,    {196,    {288,    {405,    {550,
                                            126}     196}     288}     405}     550}
{1, 1}  {2, 2}   {3, 3}   {4, 4}   {5, 5}   {6, 6}   {7, 7}   {8, 8}   {9, 9}   {10, 10}
{1, 1}  {8, 8}  {27, 27} {64, 64} {125,    {216,    {343,    {512,    {729,    {1000,
                                   125}     216}     343}     512}     729}     1000}
{1, 1}  {2, 2}   {3, 3}   {4, 4}   {5, 5}   {6, 6}   {7, 7}   {8, 8}   {9, 9}   {10, 10}
{1, 1}  {6, 6}  {21, 21} {56, 56} {126,    {252,    {462,    {792,    {1287,   {2002,
                                   126}     252}     462}     792}     1287}    2002}
{1, 1}  {4, 4}   {9, 9}   {16, 16} {25, 25} {36, 36} {49, 49} {64, 64} {81, 81} {100,
                                                                                 100}
{1, 1}  {4, 4}   {9, 9}   {16, 16} {25, 25} {36, 36} {49, 49} {64, 64} {81, 81} {100,
                                                                                 100}
{1, 1}  {4, 4}   {9, 9}   {16, 16} {25, 25} {36, 36} {49, 49} {64, 64} {81, 81} {100,
                                                                                 100}
{1, 1}  {9, 9}  {36, 36} {100,    {225,    {441,    {784,    {1296,   {2025,   {3025,
                         100}     225}     441}     784}     1296}    2025}    3025}
{1, 1}  {2, 2}   {3, 3}   {4, 4}   {5, 5}   {6, 6}   {7, 7}   {8, 8}   {9, 9}   {10, 10}
{1, 1}  {4, 4}   {9, 9}   {16, 16} {25, 25} {36, 36} {49, 49} {64, 64} {81, 81} {100,
                                                                                 100}
{1, 1}  {4, 4}   {9, 9}   {16, 16} {25, 25} {36, 36} {49, 49} {64, 64} {81, 81} {100,
                                                                                 100}
{1, 1}  {8, 8}  {30, 30} {80, 80} {175,    {336,    {588,    {960,    {1485,   {2200,
                                   175}     336}     588}     960}     1485}    2200}
{1, 1}  {2, 2}   {3, 3}   {4, 4}   {5, 5}   {6, 6}   {7, 7}   {8, 8}   {9, 9}   {10, 10}
{1, 1}  {8, 8}  {27, 27} {64, 64} {125,    {216,    {343,    {512,    {729,    {1000,
                                   125}     216}     343}     512}     729}     1000}
{1, 1}  {2, 2}   {3, 3}   {4, 4}   {5, 5}   {6, 6}   {7, 7}   {8, 8}   {9, 9}   {10, 10}
{1, 1}  {6, 6}  {18, 18} {40, 40} {75, 75} {126,    {196,    {288,    {405,    {550,
                                            126}     196}     288}     405}     550}
{1, 1}  {6, 6}  {18, 18} {40, 40} {75, 75} {126,    {196,    {288,    {405,    {550,
                                            126}     196}     288}     405}     550}
{1, 1}  {4, 4}   {9, 9}   {16, 16} {25, 25} {36, 36} {49, 49} {64, 64} {81, 81} {100,
                                                                                 100}
{1, 1}  {2, 2}   {3, 3}   {4, 4}   {5, 5}   {6, 6}   {7, 7}   {8, 8}   {9, 9}   {10, 10}
```

```
{1, 1}  {10, 10}  {45, 45}  {140,   {350,   {756,   {1470,  {2640,  {4455,  {7150,
                            140}    350}    756}    1470}   2640}   4455}   7150}
{1, 1}  {3, 3}    {6, 6}    {10, 10}  {15, 15}  {21, 21}  {28, 28}  {36, 36}  {45, 45}  {55, 55}
{1, 1}  {6, 6}    {18, 18}  {40, 40}  {75, 75}  {126,   {196,   {288,   {405,   {550,
                                                126}    196}    288}    405}    550}
{1, 1}  {4, 4}    {9, 9}    {16, 16}  {25, 25}  {36, 36}  {49, 49}  {64, 64}  {81, 81}  {100,
                                                                                      100}
{1, 1}  {6, 6}    {18, 18}  {40, 40}  {75, 75}  {126,   {196,   {288,   {405,   {550,
                                                126}    196}    288}    405}    550}
{1, 1}  {2, 2}    {3, 3}    {4, 4}    {5, 5}    {6, 6}    {7, 7}    {8, 8}    {9, 9}    {10, 10}
{1, 1}  {8, 8}    {30, 30}  {80, 80}  {175,   {336,   {588,   {960,   {1485,  {2200,
                                        175}    336}    588}    960}    1485}   2200}
{1, 1}  {4, 4}    {9, 9}    {16, 16}  {25, 25}  {36, 36}  {49, 49}  {64, 64}  {81, 81}  {100,
                                                                                      100}
{1, 1}  {8, 8}    {30, 30}  {80, 80}  {175,   {336,   {588,   {960,   {1485,  {2200,
                                        175}    336}    588}    960}    1485}   2200}
{1, 1}  {4, 4}    {9, 9}    {16, 16}  {25, 25}  {36, 36}  {49, 49}  {64, 64}  {81, 81}  {100,
                                                                                      100}
{1, 1}  {4, 4}    {9, 9}    {16, 16}  {25, 25}  {36, 36}  {49, 49}  {64, 64}  {81, 81}  {100,
                                                                                      100}
{1, 1}  {2, 2}    {3, 3}    {4, 4}    {5, 5}    {6, 6}    {7, 7}    {8, 8}    {9, 9}    {10, 10}
{1, 1}  {12, 12}  {54, 54}  {160,   {375,   {756,   {1372,  {2304,  {3645,  {5500,
                            160}    375}    756}    1372}   2304}   3645}   5500}
{1, 1}  {2, 2}    {3, 3}    {4, 4}    {5, 5}    {6, 6}    {7, 7}    {8, 8}    {9, 9}    {10, 10}
{1, 1}  {4, 4}    {9, 9}    {16, 16}  {25, 25}  {36, 36}  {49, 49}  {64, 64}  {81, 81}  {100,
                                                                                      100}
{1, 1}  {6, 6}    {18, 18}  {40, 40}  {75, 75}  {126,   {196,   {288,   {405,   {550,
                                                126}    196}    288}    405}    550}
{1, 1}  {7, 7}    {28, 28}  {84, 84}  {210,   {462,   {924,   {1716,  {3003,  {5005,
                                        210}    462}    924}    1716}   3003}   5005}
{1, 1}  {4, 4}    {9, 9}    {16, 16}  {25, 25}  {36, 36}  {49, 49}  {64, 64}  {81, 81}  {100,
                                                                                      100}
{1, 1}  {8, 8}    {27, 27}  {64, 64}  {125,   {216,   {343,   {512,   {729,   {1000,
                                        125}    216}    343}    512}    729}    1000}
{1, 1}  {2, 2}    {3, 3}    {4, 4}    {5, 5}    {6, 6}    {7, 7}    {8, 8}    {9, 9}    {10, 10}
{1, 1}  {6, 6}    {18, 18}  {40, 40}  {75, 75}  {126,   {196,   {288,   {405,   {550,
                                                126}    196}    288}    405}    550}
{1, 1}  {4, 4}    {9, 9}    {16, 16}  {25, 25}  {36, 36}  {49, 49}  {64, 64}  {81, 81}  {100,
                                                                                      100}
{1, 1}  {8, 8}    {27, 27}  {64, 64}  {125,   {216,   {343,   {512,   {729,   {1000,
                                        125}    216}    343}    512}    729}    1000}
{1, 1}  {2, 2}    {3, 3}    {4, 4}    {5, 5}    {6, 6}    {7, 7}    {8, 8}    {9, 9}    {10, 10}
{1, 1}  {12, 12}  {60, 60}  {200,   {525,   {1176,  {2352,  {4320,  {7425,  {12 100,
                            200}    525}    1176}   2352}   4320}   7425}   12 100}
{1, 1}  {2, 2}    {3, 3}    {4, 4}    {5, 5}    {6, 6}    {7, 7}    {8, 8}    {9, 9}    {10, 10}
{1, 1}  {4, 4}    {9, 9}    {16, 16}  {25, 25}  {36, 36}  {49, 49}  {64, 64}  {81, 81}  {100,
                                                                                      100}
{1, 1}  {6, 6}    {18, 18}  {40, 40}  {75, 75}  {126,   {196,   {288,   {405,   {550,
                                                126}    196}    288}    405}    550}
{1, 1}  {6, 6}    {18, 18}  {40, 40}  {75, 75}  {126,   {196,   {288,   {405,   {550,
                                                126}    196}    288}    405}    550}
{1, 1}  {4, 4}    {9, 9}    {16, 16}  {25, 25}  {36, 36}  {49, 49}  {64, 64}  {81, 81}  {100,
                                                                                      100}
```

{1, 1}	{8, 8}	{27, 27}	{64, 64}	{125, 125}	{216, 216}	{343, 343}	{512, 512}	{729, 729}	{1000, 1000}
{1, 1}	{2, 2}	{3, 3}	{4, 4}	{5, 5}	{6, 6}	{7, 7}	{8, 8}	{9, 9}	{10, 10}
{1, 1}	{10, 10}	{45, 45}	{140, 140}	{350, 350}	{756, 756}	{1470, 1470}	{2640, 2640}	{4455, 4455}	{7150, 7150}
{1, 1}	{5, 5}	{15, 15}	{35, 35}	{70, 70}	{126, 126}	{210, 210}	{330, 330}	{495, 495}	{715, 715}
{1, 1}	{4, 4}	{9, 9}	{16, 16}	{25, 25}	{36, 36}	{49, 49}	{64, 64}	{81, 81}	{100, 100}
{1, 1}	{2, 2}	{3, 3}	{4, 4}	{5, 5}	{6, 6}	{7, 7}	{8, 8}	{9, 9}	{10, 10}
{1, 1}	{12, 12}	{54, 54}	{160, 160}	{375, 375}	{756, 756}	{1372, 1372}	{2304, 2304}	{3645, 3645}	{5500, 5500}
{1, 1}	{4, 4}	{9, 9}	{16, 16}	{25, 25}	{36, 36}	{49, 49}	{64, 64}	{81, 81}	{100, 100}
{1, 1}	{4, 4}	{9, 9}	{16, 16}	{25, 25}	{36, 36}	{49, 49}	{64, 64}	{81, 81}	{100, 100}
{1, 1}	{4, 4}	{9, 9}	{16, 16}	{25, 25}	{36, 36}	{49, 49}	{64, 64}	{81, 81}	{100, 100}
{1, 1}	{8, 8}	{30, 30}	{80, 80}	{175, 175}	{336, 336}	{588, 588}	{960, 960}	{1485, 1485}	{2200, 2200}
{1, 1}	{2, 2}	{3, 3}	{4, 4}	{5, 5}	{6, 6}	{7, 7}	{8, 8}	{9, 9}	{10, 10}
{1, 1}	{12, 12}	{54, 54}	{160, 160}	{375, 375}	{756, 756}	{1372, 1372}	{2304, 2304}	{3645, 3645}	{5500, 5500}
{1, 1}	{4, 4}	{9, 9}	{16, 16}	{25, 25}	{36, 36}	{49, 49}	{64, 64}	{81, 81}	{100, 100}
{1, 1}	{6, 6}	{18, 18}	{40, 40}	{75, 75}	{126, 126}	{196, 196}	{288, 288}	{405, 405}	{550, 550}
{1, 1}	{4, 4}	{9, 9}	{16, 16}	{25, 25}	{36, 36}	{49, 49}	{64, 64}	{81, 81}	{100, 100}
{1, 1}	{4, 4}	{9, 9}	{16, 16}	{25, 25}	{36, 36}	{49, 49}	{64, 64}	{81, 81}	{100, 100}
{1, 1}	{4, 4}	{9, 9}	{16, 16}	{25, 25}	{36, 36}	{49, 49}	{64, 64}	{81, 81}	{100, 100}
{1, 1}	{12, 12}	{63, 63}	{224, 224}	{630, 630}	{1512, 1512}	{3234, 3234}	{6336, 6336}	{11583, 11583}	{20020, 20020}
{1, 1}	{2, 2}	{3, 3}	{4, 4}	{5, 5}	{6, 6}	{7, 7}	{8, 8}	{9, 9}	{10, 10}
{1, 1}	{6, 6}	{18, 18}	{40, 40}	{75, 75}	{126, 126}	{196, 196}	{288, 288}	{405, 405}	{550, 550}
{1, 1}	{6, 6}	{18, 18}	{40, 40}	{75, 75}	{126, 126}	{196, 196}	{288, 288}	{405, 405}	{550, 550}
{1, 1}	{9, 9}	{36, 36}	{100, 100}	{225, 225}	{441, 441}	{784, 784}	{1296, 1296}	{2025, 2025}	{3025, 3025}

dAlt[100, k]

$$\frac{1}{4}(-1-k)^2 k^2$$

P2[100, 4] - P2[99, 4]

6

FI[100]

{{2, 2}, {5, 2}}

```
alt[j_] := Sum[(D[Log[1 + x]^j, {x, k}] /. x → 0) n^k, {k, 0, 10}]
```

```
Series[ Log[x + 1]^2, {x, 0, 10}]
```

$$x^2 - x^3 + \frac{11\,x^4}{12} - \frac{5\,x^5}{6} + \frac{137\,x^6}{180} - \frac{7\,x^7}{10} + \frac{363\,x^8}{560} - \frac{761\,x^9}{1260} + \frac{7129\,x^{10}}{12\,600} + 0[x]^{11}$$

```
Series[ Log[x + 1]^3, {x, 0, 10}]
```

$$x^3 - \frac{3\,x^4}{2} + \frac{7\,x^5}{4} - \frac{15\,x^6}{8} + \frac{29\,x^7}{15} - \frac{469\,x^8}{240} + \frac{29\,531\,x^9}{15\,120} - \frac{1303\,x^{10}}{672} + 0[x]^{11}$$

```
Series[ (E^x - 1)^2, {x, 0, 10}]
```

$$x^2 + x^3 + \frac{7\,x^4}{12} + \frac{x^5}{4} + \frac{31\,x^6}{360} + \frac{x^7}{40} + \frac{127\,x^8}{20\,160} + \frac{17\,x^9}{12\,096} + \frac{73\,x^{10}}{259\,200} + 0[x]^{11}$$

```
Series[ (E^x)^2, {x, 0, 10}]
```

$$1 + 2\,x + 2\,x^2 + \frac{4\,x^3}{3} + \frac{2\,x^4}{3} + \frac{4\,x^5}{15} + \frac{4\,x^6}{45} + \frac{8\,x^7}{315} + \frac{2\,x^8}{315} + \frac{4\,x^9}{2835} + \frac{4\,x^{10}}{14\,175} + 0[x]^{11}$$

```
FI[100]
```

```
{{2, 2}, {5, 2}}
```

```
D[dAlt[ 100, k], {k, 4}] /. k → 0
```

```
6
```

```
dAlt[ 1000, k]
```

$$\frac{1}{36}\,(-2-k)^2\,(-1-k)^2\,k^2$$

```
dAlt[ 10 000, k]
```

$$\frac{1}{576}\,(-3-k)^2\,(-2-k)^2\,(-1-k)^2\,k^2$$

```
FI[30]
```

```
{{2, 1}, {3, 1}, {5, 1}}
```

```
D[dAlt[ 2 * 2 * 3 * 3 * 5 * 5, k], {k, 6}] /. k → 0
```

```
90
```

```
dAlt[30, k]
```

$$k^3$$

```
D[ (k - 1)^2, k]
```

$$2\,(-1+k)$$