

```

pr[j_] := Product[Prime[a], {a, 1, j}]
FI[n_] := FI[n] = FactorInteger[n]; FI[1] := {}
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}]
d2[n_, k_] := Sum[(-1)^(k-j) Binomial[k, j] dz[n, j], {j, 0, k}]
d2z[n_, z_] := Sin[Pi z] / Pi Sum[(-1)^k / (z-k) d2[n, k], {k, 0, Log2@n}]
d2zl[n_, z2_] := Limit[Sin[Pi z] / Pi Sum[(-1)^k / (z-k) d2[n, k], {k, 0, Log2@n}], z -> z2]

Table[D[(E^t - 1)^k, {t, j}] /. t -> 0, {j, 0, 8}, {k, 1, 6}] // Grid

0 0 0 0 0 0
1 0 0 0 0 0
1 2 0 0 0 0
1 6 6 0 0 0
1 14 36 24 0 0
1 30 150 240 120 0
1 62 540 1560 1800 720
1 126 1806 8400 16800 15120
1 254 5796 40824 126000 191520

Table[k! StirlingsS2[j, k], {j, 0, 8}, {k, 1, 6}] // Grid

0 0 0 0 0 0
1 0 0 0 0 0
1 2 0 0 0 0
1 6 6 0 0 0
1 14 36 24 0 0
1 30 150 240 120 0
1 62 540 1560 1800 720
1 126 1806 8400 16800 15120
1 254 5796 40824 126000 191520

Table[d2zl[pr[j], k], {j, 0, 8}, {k, 1, 6}] // Grid

0 0 0 0 0 0
1 0 0 0 0 0
1 2 0 0 0 0
1 6 6 0 0 0
1 14 36 24 0 0
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1 62 540 1560 1800 720
1 126 1806 8400 16800 15120
1 254 5796 40824 126000 191520

Sum[6 StirlingsS2[j, 3] / (j!) Log[1+x]^j, {j, 0, Infinity}]

x^3

Sum[d2zl[pr[j], 2.5] / (j!) Log[1+.3]^j, {j, 0, 20}]

0.136683

.3^2.5

0.049295

```