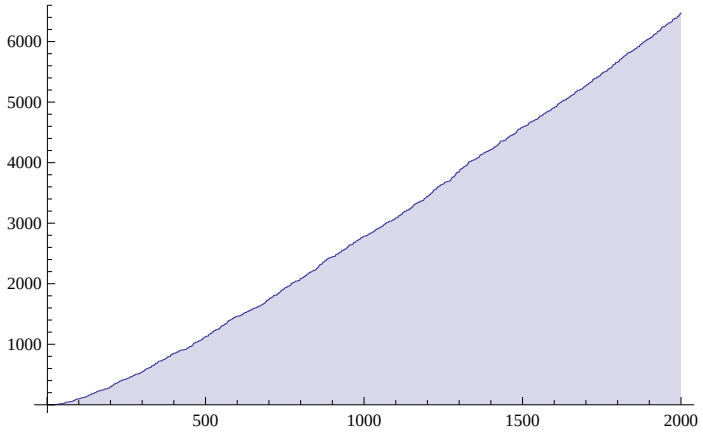
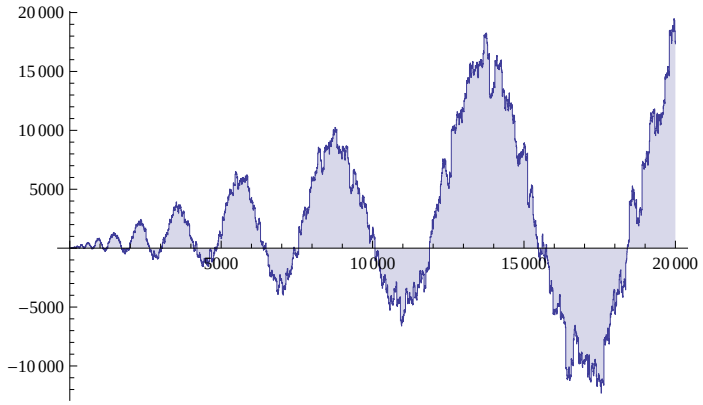
```
ClearAll["Global`*"]
```

```
Dd[n_, 0, a_] := 1; Dd[n_, 1, a_] := Floor[n] - a + 1
Dd[n_, k_, a_] :=
 Dd[n, k, a] = Sum[Binomial[k, j] Dd[n / (m^(k - j)), j, m + 1], {m, a, n^(1 / k)}, {j, 0, k - 1}]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
P2[n_, k_] :=
 P2[n, k] = Sum[FullSimplify[MangoldtLambda[j] / Log[j]] P2[n / j, k - 1], {j, 2, Floor[n]}];
P2[n_, 0] := 1
P1[x_, z_] := Sum[bin[z, k] P2[x, k], {k, 0, Log[2, x]}]
PP2[n_, k_] := D[P1[n, z], {z, k}] /. z → 0
P2Alt[n_, j_] := Sum[1 / k! (D[Log[1 + x]^j, {x, k}] /. x → 0) Dd[n, k, 2], {k, 0, Log[2, n]}]
PP2Alt[n_, j_] :=
 Sum[1 / k! (D[Log[1 + Log[1 + x]]^j, {x, k}] /. x → 0) Dd[n, k, 2], {k, 0, Log[2, n]}]
PP3Alt[n_, j_] :=
 Sum[1 / k! (D[Log[1 + Log[1 + Log[1 + x]]]^j, {x, k}] /. x → 0) Dd[n, k, 2], {k, 0, Log[2, n]}]
PP4Alt[n_, j_] := Sum[1 / k! (D[Log[1 + Log[1 + Log[1 + Log[1 + x]]]]^j, {x, k}] /. x → 0)
   Dd[n, k, 2], {k, 0, Log[2, n]}]
PP5Alt[n_, j_] := Sum[1 / k! (D[Log[1 + Log[1 + Log[1 + Log[1 + Log[1 + x]]]]]^j, {x, k}] /. x → 0)
   Dd[n, k, 2], {k, 0, Log[2, n]}]
```

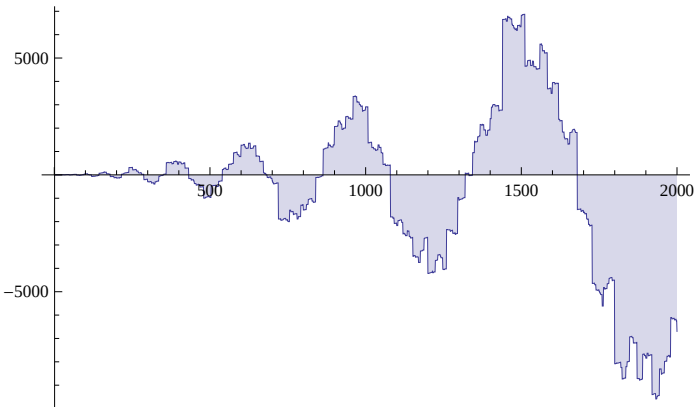
```
DiscretePlot[ P2Alt[n, 4], {n, 1, 2000}]
```



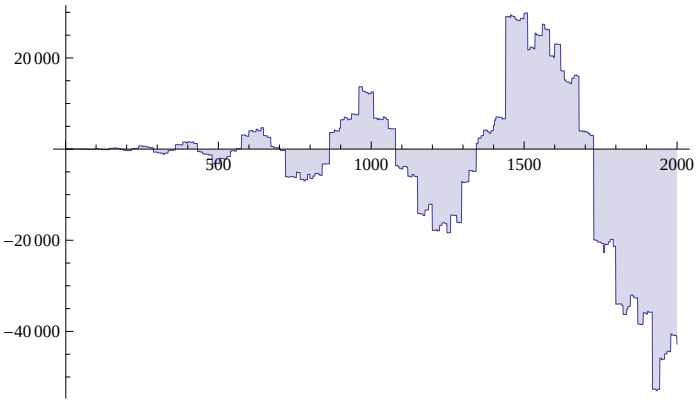

```
DiscretePlot[ PP2Alt[n, 4], {n, 1, 20 000}]
```



```
DiscretePlot[PP3Alt[n, 4], {n, 1, 2000}]
```




```
DiscretePlot[PP4Alt[n, 4], {n, 1, 2000}]
```




```
DiscretePlot[PP5Alt[n, 4], {n, 1, 2000}]
```

$Aborted

```
Limit[D[x / Log[1 + x], {x, 4}], x → 0]
```

$$-\frac{19}{30}$$