

```

f[n_, t_] := Sum[ (-1) ^ (k + 1) / k (-1) ^ k (1 - Gamma[k, -Log[100]] / Gamma[k]), {k, 1, t}]
N[f[100, 10 000]] + Log[Log[100]] + EulerGamma
30.1261 - 2.09386 × 10-14 i
N[ExpIntegralEi[Log[100]]]
30.1261

f2[n_, t_, s_] :=
Sum[ (-1) ^ (k + 1) / k (-1) ^ k (1 - Gamma[k, - (s) Log[100]] / Gamma[k]), {k, 1, t}]
f2a[n_, t_, s_, s2_] := Sum[ (-1) ^ (k + 1) / k (-1) ^ k
((1 - Gamma[k, -s Log[100]] / Gamma[k]) + (1 - Gamma[k, -s2 Log[100]] / Gamma[k])), {k, 1, t}]

N[f2[100, 1000, 2]] + Log[2 Log[100]] + EulerGamma
1246.14 - 1.13248 × 10-9 i
N[ExpIntegralEi[2 Log[100]]]
1246.14

N[f2[100, 1000, 3]] + Log[3 Log[100]] + EulerGamma
78 627.5 + 9.32697 × 10-6 i
N[ExpIntegralEi[3 Log[100]]]
78 627.5

N[f2a[100, 1600, ZetaZero[1], ZetaZero[-1]]] +
Log[(ZetaZero[1] + ZetaZero[-1]) Log[100]] + EulerGamma
1.88143 × 1012 + 0. i
N[ExpIntegralEi[(ZetaZero[1]) Log[100]] + ExpIntegralEi[(ZetaZero[-1]) Log[100]]]
0.232873 + 0. i
N[ZetaZero[1]]
0.5 + 14.1347 i

Table[ {k, N[(-1) ^ (k + 1) / k (-1) ^ k (1 - Gamma[k, - (s) Log[100]] / Gamma[k])]}, {k, 1, 200}] /.
s → ZetaZero[1] // TableForm
1      -7.36665 + 7.71141 i
2      254.625 + 202.189 i
3      4274.85 - 5622.11 i
4      -92 840.2 - 67 705.8 i
5      -856 579. + 1.22546 × 106 i
6      1.34688 × 107 + 9.01724 × 106 i
7      8.12342 × 107 - 1.26785 × 108 i
8      -1.04346 × 109 - 6.39257 × 108 i
9      -4.46347 × 109 + 7.62791 × 109 i
10     5.01481 × 1010 + 2.79944 × 1010 i
11     1.59284 × 1011 - 2.99498 × 1011 i
12     -1.63845 × 1012 - 8.28913 × 1011 i
13     -3.97218 × 1012 + 8.26807 × 1012 i
14     3.8716 × 1013 + 1.76284 × 1013 i

```

15 $7.28079 \times 10^{13} - 1.69089 \times 10^{14} i$
 16 $-6.91863 \times 10^{14} - 2.81016 \times 10^{14} i$
 17 $-1.01724 \times 10^{15} + 2.6626 \times 10^{15} i$
 18 $9.67121 \times 10^{15} + 3.46407 \times 10^{15} i$
 19 $1.11263 \times 10^{16} - 3.32577 \times 10^{16} i$
 20 $-1.08579 \times 10^{17} - 3.37805 \times 10^{16} i$
 21 $-9.71189 \times 10^{16} + 3.37388 \times 10^{17} i$
 22 $1.00008 \times 10^{18} + 2.64764 \times 10^{17} i$
 23 $6.85063 \times 10^{17} - 2.83372 \times 10^{18} i$
 24 $-7.68993 \times 10^{18} - 1.68303 \times 10^{18} i$
 25 $-3.92494 \times 10^{18} + 2.00209 \times 10^{19} i$
 26 $5.00887 \times 10^{19} + 8.67891 \times 10^{18} i$
 27 $1.81527 \times 10^{19} - 1.20595 \times 10^{20} i$
 28 $-2.79804 \times 10^{20} - 3.57537 \times 10^{19} i$
 29 $-6.57831 \times 10^{19} + 6.26418 \times 10^{20} i$
 30 $1.3548 \times 10^{21} + 1.11368 \times 10^{20} i$
 31 $1.68105 \times 10^{20} - 2.83381 \times 10^{21} i$
 32 $-5.73854 \times 10^{21} - 2.08571 \times 10^{20} i$
 33 $-1.49235 \times 10^{20} + 1.12613 \times 10^{22} i$
 34 $2.14353 \times 10^{22} - 2.14032 \times 10^{20} i$
 35 $-1.32262 \times 10^{21} - 3.96094 \times 10^{22} i$
 36 $-7.11126 \times 10^{22} + 4.05299 \times 10^{21} i$
 37 $1.00329 \times 10^{22} + 1.24139 \times 10^{23} i$
 38 $2.10859 \times 10^{23} - 2.2119 \times 10^{22} i$
 39 $-4.50784 \times 10^{22} - 3.48738 \times 10^{23} i$
 40 $-5.61966 \times 10^{23} + 8.65083 \times 10^{22} i$
 41 $1.58 \times 10^{23} + 8.8286 \times 10^{23} i$
 42 $1.353 \times 10^{24} - 2.76517 \times 10^{23} i$
 43 $-4.65888 \times 10^{23} - 2.0238 \times 10^{24} i$
 44 $-2.95617 \times 10^{24} + 7.58253 \times 10^{23} i$
 45 $1.19521 \times 10^{24} + 4.21893 \times 10^{24} i$
 46 $5.88562 \times 10^{24} - 1.82836 \times 10^{24} i$
 47 $-2.71885 \times 10^{24} - 8.02967 \times 10^{24} i$
 48 $-1.07178 \times 10^{25} + 3.93567 \times 10^{24} i$
 49 $5.55229 \times 10^{24} + 1.40022 \times 10^{25} i$
 50 $1.79118 \times 10^{25} - 7.6417 \times 10^{24} i$
 51 $-1.02698 \times 10^{25} - 2.24439 \times 10^{25} i$
 52 $-2.75566 \times 10^{25} + 1.34875 \times 10^{25} i$
 53 $1.73225 \times 10^{25} + 3.31644 \times 10^{25} i$
 54 $3.91359 \times 10^{25} - 2.17712 \times 10^{25} i$
 55 $-2.6792 \times 10^{25} - 4.52972 \times 10^{25} i$
 56 $-5.1438 \times 10^{25} + 3.23009 \times 10^{25} i$
 57 $3.8171 \times 10^{25} + 5.73239 \times 10^{25} i$
 58 $6.27103 \times 10^{25} - 4.42349 \times 10^{25} i$
 59 $-5.02922 \times 10^{25} - 6.73597 \times 10^{25} i$
 60 $-7.10591 \times 10^{25} + 5.61202 \times 10^{25} i$
 61 $6.14882 \times 10^{25} + 7.36363 \times 10^{25} i$
 62 $7.49729 \times 10^{25} - 6.61723 \times 10^{25} i$

63 $-6.99721 \times 10^{25} - 7.50128 \times 10^{25} i$
 64 $-7.37666 \times 10^{25} + 7.27247 \times 10^{25} i$
 65 $7.43161 \times 10^{25} + 7.13086 \times 10^{25} i$
 66 $6.77703 \times 10^{25} - 7.46895 \times 10^{25} i$
 67 $-7.38474 \times 10^{25} - 6.33288 \times 10^{25} i$
 68 $-5.81924 \times 10^{25} + 7.18505 \times 10^{25} i$
 69 $6.88109 \times 10^{25} + 5.25852 \times 10^{25} i$
 70 $4.67315 \times 10^{25} - 6.48823 \times 10^{25} i$
 71 $-6.02481 \times 10^{25} - 4.08422 \times 10^{25} i$
 72 $-3.51033 \times 10^{25} + 5.51071 \times 10^{25} i$
 73 $4.96613 \times 10^{25} + 2.96687 \times 10^{25} i$
 74 $2.46552 \times 10^{25} - 4.41029 \times 10^{25} i$
 75 $-3.86052 \times 10^{25} - 2.01418 \times 10^{25} i$
 76 $-1.61719 \times 10^{25} + 3.3315 \times 10^{25} i$
 77 $2.83488 \times 10^{25} + 1.27569 \times 10^{25} i$
 78 $9.88211 \times 10^{24} - 2.37908 \times 10^{25} i$
 79 $-1.96944 \times 10^{25} - 7.51287 \times 10^{24} i$
 80 $-5.60088 \times 10^{24} + 1.60847 \times 10^{25} i$
 81 $1.29625 \times 10^{25} + 4.09 \times 10^{24} i$
 82 $2.92118 \times 10^{24} - 1.03096 \times 10^{25} i$
 83 $-8.09364 \times 10^{24} - 2.03639 \times 10^{24} i$
 84 $-1.38152 \times 10^{24} + 6.27274 \times 10^{24} i$
 85 $4.80006 \times 10^{24} + 9.08175 \times 10^{23} i$
 86 $5.74631 \times 10^{23} - 3.62723 \times 10^{24} i$
 87 $-2.70707 \times 10^{24} - 3.46099 \times 10^{23} i$
 88 $-1.94435 \times 10^{23} + 1.99564 \times 10^{24} i$
 89 $1.45336 \times 10^{24} + 9.75338 \times 10^{22} i$
 90 $3.85186 \times 10^{22} - 1.04576 \times 10^{24} i$
 91 $-7.43539 \times 10^{23} - 4.8693 \times 10^{21} i$
 92 $1.24333 \times 10^{22} + 5.2245 \times 10^{23} i$
 93 $3.62829 \times 10^{23} - 1.96878 \times 10^{22} i$
 94 $-2.11456 \times 10^{22} - 2.49069 \times 10^{23} i$
 95 $-1.69023 \times 10^{23} + 1.9567 \times 10^{22} i$
 96 $1.66613 \times 10^{22} + 1.13402 \times 10^{23} i$
 97 $7.5229 \times 10^{22} - 1.34232 \times 10^{22} i$
 98 $-1.03809 \times 10^{22} - 4.93493 \times 10^{22} i$
 99 $-3.20144 \times 10^{22} + 7.77198 \times 10^{21} i$
 100 $5.66371 \times 10^{21} + 2.05406 \times 10^{22} i$
 101 $1.30352 \times 10^{22} - 4.03225 \times 10^{21} i$
 102 $-2.81201 \times 10^{21} - 8.18259 \times 10^{21} i$
 103 $-5.08117 \times 10^{21} + 1.92469 \times 10^{21} i$
 104 $1.29488 \times 10^{21} + 3.1215 \times 10^{21} i$
 105 $1.89721 \times 10^{21} - 8.57305 \times 10^{20} i$
 106 $-5.59102 \times 10^{20} - 1.14089 \times 10^{21} i$
 107 $-6.78839 \times 10^{20} + 3.59446 \times 10^{20} i$
 108 $2.27952 \times 10^{20} + 3.99674 \times 10^{20} i$
 109 $2.32849 \times 10^{20} - 1.42679 \times 10^{20} i$
 110 $-8.81831 \times 10^{19} - 1.34241 \times 10^{20} i$

```

111     $-7.65855 \times 10^{19} + 5.38391 \times 10^{19} i$ 
112     $3.24827 \times 10^{19} + 4.3238 \times 10^{19} i$ 
113     $2.41569 \times 10^{19} - 1.93725 \times 10^{19} i$ 
114     $-1.14241 \times 10^{19} - 1.33559 \times 10^{19} i$ 
115     $-7.30718 \times 10^{18} + 6.66296 \times 10^{18} i$ 
116     $3.84437 \times 10^{18} + 3.95603 \times 10^{18} i$ 
117     $2.11924 \times 10^{18} - 2.19475 \times 10^{18} i$ 
118     $-1.24002 \times 10^{18} - 1.12327 \times 10^{18} i$ 
119     $-5.89027 \times 10^{17} + 6.93483 \times 10^{17} i$ 
120     $3.8395 \times 10^{17} + 3.05552 \times 10^{17} i$ 
121     $1.56774 \times 10^{17} - 2.10481 \times 10^{17} i$ 
122     $-1.14264 \times 10^{17} - 7.95473 \times 10^{16} i$ 
123     $-3.99072 \times 10^{16} + 6.14364 \times 10^{16} i$ 
124     $3.272 \times 10^{16} + 1.97895 \times 10^{16} i$ 
125     $9.69697 \times 10^{15} - 1.72632 \times 10^{16} i$ 
126     $-9.02405 \times 10^{15} - 4.69332 \times 10^{15} i$ 
127     $-2.24258 \times 10^{15} + 4.67408 \times 10^{15} i$ 
128     $2.39911 \times 10^{15} + 1.05722 \times 10^{15} i$ 
129     $4.91346 \times 10^{14} - 1.2204 \times 10^{15} i$ 
130     $-6.15306 \times 10^{14} - 2.24884 \times 10^{14} i$ 
131     $-1.01226 \times 10^{14} + 3.07508 \times 10^{14} i$ 
132     $1.52345 \times 10^{14} + 4.47302 \times 10^{13} i$ 
133     $1.9355 \times 10^{13} - 7.48248 \times 10^{13} i$ 
134     $-3.64364 \times 10^{13} - 8.17183 \times 10^{12} i$ 
135     $-3.34866 \times 10^{12} + 1.75926 \times 10^{13} i$ 
136     $8.42284 \times 10^{12} + 1.32067 \times 10^{12} i$ 
137     $4.94089 \times 10^{11} - 3.99895 \times 10^{12} i$ 
138     $-1.88287 \times 10^{12} - 1.70489 \times 10^{11} i$ 
139     $-5.07341 \times 10^{10} + 8.79236 \times 10^{11} i$ 
140     $4.07216 \times 10^{11} + 1.01599 \times 10^{10} i$ 
141     $-1.45554 \times 10^9 - 1.87068 \times 10^{11} i$ 
142     $-8.52417 \times 10^{10} + 3.45727 \times 10^9 i$ 
143     $2.83017 \times 10^9 + 3.85301 \times 10^{10} i$ 
144     $1.72768 \times 10^{10} - 1.84062 \times 10^9 i$ 
145     $-1.07503 \times 10^9 - 7.68524 \times 10^9 i$ 
146     $-3.39155 \times 10^9 + 5.88657 \times 10^8 i$ 
147     $3.08363 \times 10^8 + 1.48491 \times 10^9 i$ 
148     $6.45023 \times 10^8 - 1.56265 \times 10^8 i$ 
149     $-7.71284 \times 10^7 - 2.77994 \times 10^8 i$ 
150     $-1.18875 \times 10^8 + 3.72445 \times 10^7 i$ 
151     $1.76504 \times 10^7 + 5.04368 \times 10^7 i$ 
152     $2.12331 \times 10^7 - 8.22736 \times 10^6 i$ 
153     $-3.77844 \times 10^6 - 8.86937 \times 10^6 i$ 
154     $-3.6761 \times 10^6 + 1.71186 \times 10^6 i$ 
155     $765900. + 1.51181 \times 10^6 i$ 
156     $616911. - 338669. i$ 
157     $-148105. - 249779. i$ 
158     $-100344. + 64090.4 i$ 

```

```

159 27456.8 + 39995.7 i
160 15816.6 - 11649.6 i
161 -4896.92 - 6205.39 i
162 -2415.24 + 2039.93 i
163 842.358 + 932.51 i
164 357.117 - 344.887 i
165 -140.039 - 135.647 i
166 -51.1014 + 56.3956 i
167 22.5254 + 19.0841 i
168 7.06058 - 8.93145 i
169 -3.51915 - 2.5936 i
170 -0.949199 + 1.3715 i
171 0.525572 + 0.339897 i
172 0.115475 - 0.204398 i
173 -0.0838266 - 0.0428453 i
174 -0.0207221 + 0.0295872 i
175 0.00542271 + 0.00517527 i
176 -0.00391473 - 0.00416271 i
177 -0.00719482 - 0.000595526 i
178 -0.0058158 + 0.000569565 i
179 -0.00537807 + 0.0000646616 i
180 -0.00553481 - 0.0000758275 i
181 -0.00555225 - 6.51196 × 10-6 i
182 -0.0054965 + 9.82688 × 10-6 i
183 -0.00546098 + 5.87169 × 10-7 i
184 -0.00543462 - 1.24016 × 10-6 i
185 -0.00540584 - 4.32001 × 10-8 i
186 -0.00537635 + 1.52458 × 10-7 i
187 -0.00534754 + 1.65482 × 10-9 i
188 -0.00531915 - 1.82626 × 10-8 i
189 -0.00529101 + 2.2713 × 10-10 i
190 -0.00526316 + 2.13211 × 10-9 i
191 -0.0052356 - 7.51998 × 10-11 i
192 -0.00520833 - 2.42648 × 10-10 i
193 -0.00518135 + 1.40416 × 10-11 i
194 -0.00515464 + 2.69229 × 10-11 i
195 -0.00512821 - 2.16606 × 10-12 i
196 -0.00510204 - 2.91263 × 10-12 i
197 -0.00507614 + 3.00757 × 10-13 i
198 -0.00505051 + 3.07244 × 10-13 i
199 -0.00502513 - 3.88763 × 10-14 i
200 -0.005 - 3.16011 × 10-14 i

```

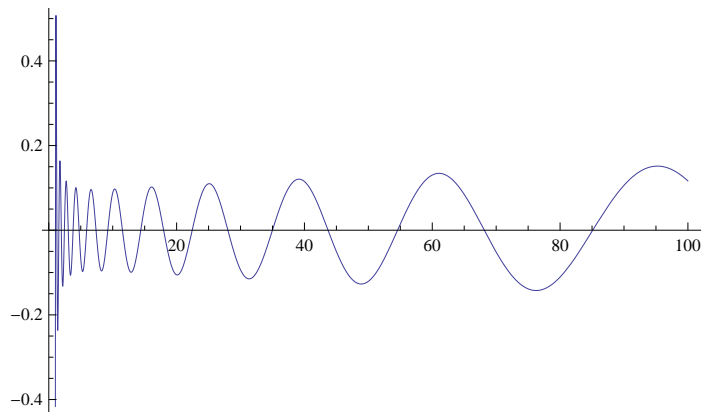
```

FullSimplify[f2a[100, 200, ZetaZero[1], ZetaZero[-1]] +
  Log[(ZetaZero[1] + ZetaZero[-1]) Log[100]] + EulerGamma]
$Aborted

Limit[(-(-1)^z Gamma[z, -(ZetaZero[1]) Log[n]] / Gamma[z]) / z, z -> 0]
-Gamma[0, -Log[n] ZetaZero[1]]

```

```
Plot[Re[-Gamma[0, -Log[n] ZetaZero[1]]], {n, 1, 100}]
```



```
Limit[(-(-1)^z Gamma[z, -Log[n]] / Gamma[z]) / z, z -> 1]
```

n

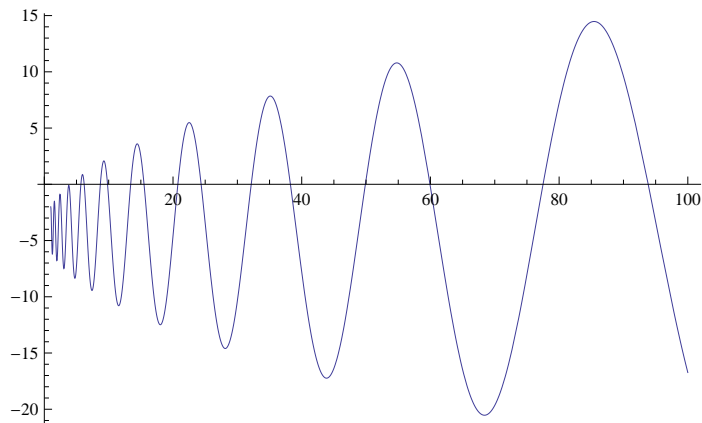
```
Limit[( (-1)^z - (-1)^z Gamma[z, -(ZetaZero[1] Log[n]) / Gamma[z] - 1) / z, z -> 1]
```

- 2 + n^{ZetaZero[1]}

```
Limit[(-(-1)^z Gamma[z, -(ZetaZero[-1]) Log[n]] / Gamma[z]) / z, z -> 1]
```

n^{ZetaZero[-1]}

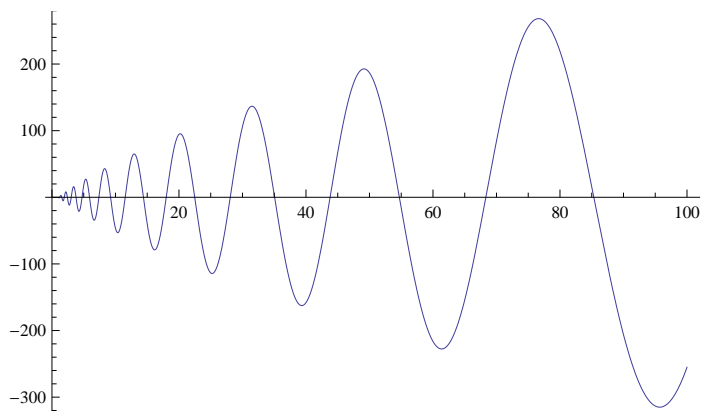
```
Plot[nZetaZero[1] + nZetaZero[-1] - 4, {n, 1, 100}]
```



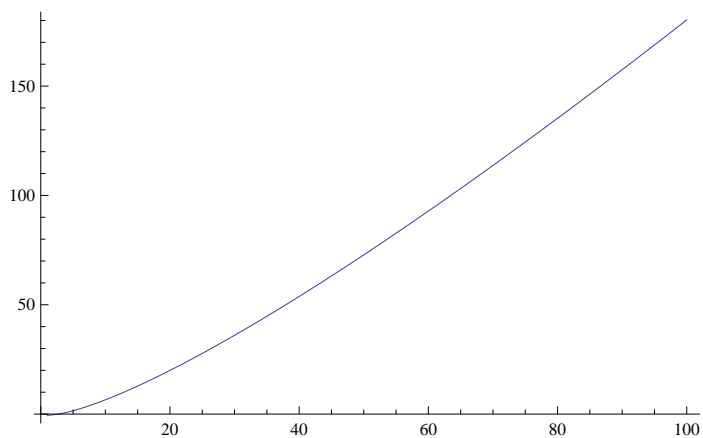
```
Limit[( (-1)^z - (-1)^z Gamma[z, -(ZetaZero[1] Log[n]) / Gamma[z] - 1) / z, z -> 2]
```

$-\frac{1}{2} \Gamma[2, -\text{Log}[n] \text{ZetaZero}[1]]$

```
Plot[Re[-1/2 Gamma[2, -Log[n] ZetaZero[1]]], {n, 1, 100}]
```



```
Plot[Re[-1/2 Gamma[2, -Log[n] ]], {n, 1, 100}]
```



```
ff[n_, t_] := n - Sum[ n^ZetaZero[k] + n^ZetaZero[-k] + 2, {k, 1, 200}]
```

```
N[ff[1000, 60]]
```

```
218.909 - 5.68434 × 10-14 i
```

```
DiscretePlot[Re[ff[n, 50]], {n, 1, 100}]
```

