```
Limit[Sum[(a^k - 1) / k, {k, 1, Log[a, n]}], {a → 1}]
```

$$\left\{\text{Limit}\left[-\text{HarmonicNumber}\left[\frac{\text{Log}[n]}{\text{Log}[a]}\right]-a\,n\,\text{LerchPhi}\left[a,1,1+\frac{\text{Log}[n]}{\text{Log}[a]}\right]-\text{Log}[1-a],\ a\to 1\right]\right\}$$

```
Limit[Sum[(a^k - 1) / k, {k, 1, Log[a, 2 n]}], {a → 1}]
```

$$\left\{\text{Limit}\left[-\text{HarmonicNumber}\left[\frac{\text{Log}[2\,n]}{\text{Log}[a]}\right]-2\,a\,n\,\text{LerchPhi}\left[a,1,1+\frac{\text{Log}[2\,n]}{\text{Log}[a]}\right]-\text{Log}[1-a],\ a\to 1\right]\right\}$$

```
Limit[Sum[(a^k - 1) / k, {k, 1, Log[a, n^2]}], {a → 1}]
```

$$\left\{\text{Limit}\left[-\text{HarmonicNumber}\left[\frac{\text{Log}\left[n^2\right]}{\text{Log}[a]}\right]-a\,n^2\,\text{LerchPhi}\left[a,1,1+\frac{\text{Log}\left[n^2\right]}{\text{Log}[a]}\right]-\text{Log}[1-a],\ a\to 1\right]\right\}$$

```
Limit[Sum[(a^k - 1) / k, {k, 1, Log[2 a, n]}], {a → 1}]
```

$$\{\infty\}$$

```
Limit[Sum[(a^k - 1) / k, {k, 1, Log[a^2, n]}], {a → 1}]
```

$$\left\{\text{Limit}\left[-\text{HarmonicNumber}\left[\frac{\text{Log}[n]}{\text{Log}\left[a^2\right]}\right]-a^{1+\frac{\text{Log}[n]}{\text{Log}\left[a^2\right]}}\,\text{LerchPhi}\left[a,1,1+\frac{\text{Log}[n]}{\text{Log}\left[a^2\right]}\right]-\text{Log}[1-a],\ a\to 1\right]\right\}$$

```
Limit[Sum[(a^k - 1) / k, {k, 1, Log[a^(1 / 2), n]}], {a → 1}]
```

$$\left\{\text{Limit}\left[-\text{HarmonicNumber}\left[\frac{2\,\text{Log}[n]}{\text{Log}[a]}\right]-a\,n^2\,\text{LerchPhi}\left[a,1,1+\frac{2\,\text{Log}[n]}{\text{Log}[a]}\right]-\text{Log}[1-a],\ a\to 1\right]\right\}$$

```
Limit[Sum[(a^k - 1) / k^2, {k, 1, Log[a, n]}], {a → 1}]
```

$$\left\{\text{Limit}\left[-\text{HarmonicNumber}\left[\frac{\text{Log}[n]}{\text{Log}[a]},2\right]-a\,n\,\text{LerchPhi}\left[a,2,1+\frac{\text{Log}[n]}{\text{Log}[a]}\right]+\text{PolyLog}[2,a],\ a\to 1\right]\right\}$$

```
Limit[Sum[(a^k) / k^2, {k, 1, Log[a, n]}], {a → 1}]
```

$$\left\{\text{Limit}\left[-a\,n\,\text{LerchPhi}\left[a,2,1+\frac{\text{Log}[n]}{\text{Log}[a]}\right]+\text{PolyLog}[2,a],\ a\to 1\right]\right\}$$

```
Limit[Sum[(a^k) / k^2, {k, Log[a, n], Infinity}], {a → 1}]
```

$$\left\{\text{Limit}\left[n\,\text{HurwitzLerchPhi}\left[a,2,\frac{\text{Log}[n]}{\text{Log}[a]}\right],\ a\to 1\right]\right\}$$

$$\boldsymbol{\left\{\text{Limit}\left[-\text{HarmonicNumber}\left[\frac{\text{Log}[n]}{\text{Log}[a]},2\right]+\text{PolyLog}[2,a],\ a\to 1\right]\right\}}$$

$$\left\{\text{Limit}\left[-\text{HarmonicNumber}\left[\frac{\text{Log}[n]}{\text{Log}[a]},2\right]+\text{PolyLog}[2,a],\ a\to 1\right]\right\}$$

$$\boldsymbol{\left\{\text{Limit}\left[-\text{HarmonicNumber}\left[\frac{\text{Log}[100]}{\text{Log}[a]},2\right]+\text{PolyLog}[2,a],\ a\to 1\right]\right\}}$$

$$\{0\}$$

$$\boldsymbol{\left\{\text{Limit}\left[-\text{HarmonicNumber}\left[\frac{\text{Log}[100]}{\text{Log}[a]},3\right]+\text{PolyLog}[3,a],\ a\to 1\right]\right\}}$$

$$\{0\}$$

```
{Limit[-HarmonicNumber[Log[100]/Log[a], 4] + PolyLog[4, a], a → 1]}
```

```
{0}
```

```
{Limit[-HarmonicNumber[Log[n]/Log[a], 1] + PolyLog[1, a], a → 1]}
Limit[-HarmonicNumber[Log[n]/Log[a]] - Log[1 - a], a → 1]
Limit[-HarmonicNumber[Log[100]/Log[a]] - Log[1 - a], a → 1]
```

```
-EulerGamma - i π - Log[Log[100]]
```

```
Limit[Sum[(a^k - 1) / k^3, {k, 1, Log[a, n]}], {a → 1}]
```

```
{Limit[-HarmonicNumber[Log[n]/Log[a], 3] - a n LerchPhi[a, 3, 1 + Log[n]/Log[a]] + PolyLog[3, a], a → 1]}
```

```
Limit[Sum[(a^k - 1) / k^2, {k, 1, Log[a, 100]}], {a → 1}]
```

```
{Limit[
  -HarmonicNumber[Log[100]/Log[a], 2] - 100 a LerchPhi[a, 2, 1 + Log[100]/Log[a]] + PolyLog[2, a], a → 1]}
```

```
Limit[-HarmonicNumber[Log[100]/Log[a], 2] + PolyLog[2, a], a → 1]
```

```
0
```

```
Limit[Sum[(a^k - 1) / k^b, {k, 1, Log[a, n]}], {a → 1}]
```

```
{Limit[-HarmonicNumber[Log[n]/Log[a], b] - a n LerchPhi[a, b, 1 + Log[n]/Log[a]] + PolyLog[b, a], a → 1]}
```

```
Limit[-HarmonicNumber[Log[n]/Log[a], b] + PolyLog[b, a], a → 1]
```

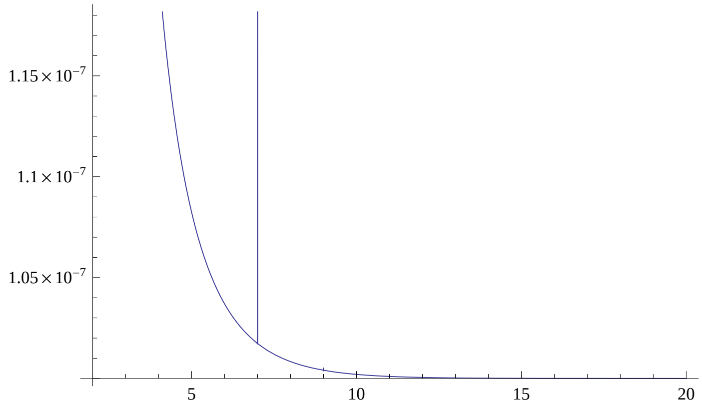
```
Limit[-HarmonicNumber[Log[n]/Log[a], b] + PolyLog[b, a], a → 1]
```

```
Limit[-HarmonicNumber[Log[100]/Log[a], b] + PolyLog[b, a], a → 1]
```

```
Limit[-HarmonicNumber[Log[100]/Log[a], b] + PolyLog[b, a], a → 1]
```

```
fb[a_, b_] := -HarmonicNumber[Log[100]/Log[a], b] + PolyLog[b, a]
```

```
Plot[Re[fb[1.0000001, b]], {b, 2, 20}]
```




```
Limit[Sum[(a^k - 1) (k^b) , {k, 1, Log[a, n]}], {a → 1}]
```

$$\left\{\text{Limit}\left[\text{HurwitzZeta}\left[-b,\ 1+\frac{\text{Log}[n]}{\text{Log}[a]}\right]-\text{a n LerchPhi}\left[a,\ -b,\ 1+\frac{\text{Log}[n]}{\text{Log}[a]}\right]+\text{PolyLog}[-b,\ a]-\text{Zeta}[-b],\ a\to 1\right]\right\}$$

```
Limit[Sum[(a^k - 1) (k^0) , {k, 1, Log[a, n]}], {a → 1}]
```

```
{DirectedInfinity[-1 + n - Log[n]]}
```

$$\text{Limit}\left[\text{HurwitzZeta}\left[-b,\ 1+\frac{\text{Log}[100]}{\text{Log}[a]}\right]+\text{PolyLog}[-b,\ a]-\text{Zeta}[-b]\ \text{/.}\ b\to 1,\ a\to 1\right]$$

$-\infty$

$$\text{Limit}\left[\text{HurwitzZeta}\left[-b,\ 1+\frac{\text{Log}[n]}{\text{Log}[a]}\right]-\text{a n LerchPhi}\left[a,\ -b,\ 1+\frac{\text{Log}[n]}{\text{Log}[a]}\right]+\text{PolyLog}[-b,\ a]-\text{Zeta}[-b]\ \text{/.}\ \{b\to -1/2,\ n\to 100\},\ a\to 1\right]$$

```
$Aborted
```

```
Limit[Sum[(a^k ) (k^b) , {k, 1, Log[a, n]}], {a → 1}]
```

$$\left\{\text{Limit}\left[-\text{a n LerchPhi}\left[a,\ -b,\ 1+\frac{\text{Log}[n]}{\text{Log}[a]}\right]+\text{PolyLog}[-b,\ a],\ a\to 1\right]\right\}$$

```
Limit[Sum[(a^k ) (k^b) , {k, Log[a, n], Infinity}], {a → 1}]
```

$$\left\{\text{Limit}\left[\text{n HurwitzLerchPhi}\left[a,\ -b,\ \frac{\text{Log}[n]}{\text{Log}[a]}\right],\ a\to 1\right]\right\}$$

```
Limit[Sum[(a^k) (k^-1) , {k, Log[a, 100], Infinity}], {a → 1}]
```

$$\left\{\text{Limit}\left[100\ \text{HurwitzLerchPhi}\left[a,\ 1,\ \frac{\text{Log}[100]}{\text{Log}[a]}\right],\ a\to 1\right]\right\}$$

```
Limit[Sum[(a^k) (k^2) , {k, Log[a, 100], Infinity}], {a → 1}]
```

$\{-\infty\}$

```
lerch[z_, s_, a_] :=
 1 / (2 a^s) + (Log[1 / z]^(s - 1) / z^a) Gamma[1 - s, a Log[1 / z]] + 2 / (a^(s - 1)) Integrate[
    Sin[ s ArcTan[t] - t a Log[z]] / ((1 + t^2)^(s / 2) (E^(2 Pi a t) - 1)), {t, 0, Infinity}]

lch[z_, s_, a_] :=
 1 / (2 a^s) + Log[1 / z]^(s - 1) / z^a Gamma[1 - s, a Log[1 / z]] + 2 / (a^(s - 1)) Integrate[
    Sin[s ArcTan[t] - t a Log[z]] / ( (1 + t^2 )^(s / 2) (E^(2 Pi a t) - 1)), {t, 0, Infinity}]

lch[1.00000001, 1, 1]
```

$$(18.3435 - 3.14159\, i) + 2 \int_0^{\infty} -\frac{\mathrm{Sin}\left[1. \times 10^{-8}\, t - \mathrm{ArcTan}[t]\right]}{\left(-1 + e^{2 \pi t}\right) \sqrt{1 + t^2}}\, dt$$

$$N\left[2 \int_0^{\infty} -\frac{\mathrm{Sin}[9.999999889225291\text{`*^-9}\, t - \mathrm{ArcTan}[t]]}{\left(-1 + e^{2 \pi t}\right) \sqrt{1 + t^2}}\, dt\right]$$

0.0772157

```
lch[1.00000001, 1, 2]
```

$$(17.4003 - 3.14159\, i) + 2 \int_0^{\infty} -\frac{\mathrm{Sin}\left[2. \times 10^{-8}\, t - \mathrm{ArcTan}[t]\right]}{\left(-1 + e^{4 \pi t}\right) \sqrt{1 + t^2}}\, dt$$

$$N\left[2 \int_0^{\infty} -\frac{\mathrm{Sin}[1.9999999778450582\text{`*^-8}\, t - \mathrm{ArcTan}[t]]}{\left(-1 + e^{4 \pi t}\right) \sqrt{1 + t^2}}\, dt\right]$$

0.0203628

```
lch[a, 1, Log[n] / Log[a]] /. {n → 100, a → 1.0000001}
```

GCD::exact : Argument 2.8935138979524446`*^8 in GCD$\left[0, 1, 2.89351 \times 10^8, 2.89351 \times 10^8\right]$ is not an exact number. ≫

GCD::exact : Argument 2.8935138979524446`*^8 in GCD$\left[0, 1, 2.89351 \times 10^8, 2.89351 \times 10^8\right]$ is not an exact number. ≫

$$100 \left((-0.30126140506335297\text{`} - 0.031415926535897934\text{`}\, i) + 2 \int_0^{\infty} \frac{\mathrm{Sin}[\mathrm{ArcTan}[t] - t\, \mathrm{Log}[100]]}{\left(-1 + 100^{6.283185617670301\text{`*^7}\, t}\right) \sqrt{1 + t^2}}\, dt \right)$$

$$100 \left((-0.301261 - 0.0314159\, i) + 2 \int_0^{\infty} \frac{\mathrm{Sin}[\mathrm{ArcTan}[t] - t\, \mathrm{Log}[100]]}{\left(-1 + 100^{6.28319 \times 10^7\, t}\right) \sqrt{1 + t^2}}\, dt \right)$$

$$\mathbf{N}\left[100\left(2\int_0^\infty \frac{\mathrm{Sin}[\mathrm{ArcTan}[t]-t\,\mathrm{Log}[100]]}{\left(-1+100^{6.283185617670301`*^7\,t}\right)\sqrt{1+t^2}}\,dt\right)\right]$$

NIntegrate::ncvb :

NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in t near {t} = $\{2.29781\times10^{-7}\}$.

NIntegrate obtained -7.08309×10^{-17} and 1.3674898494317748`*^-19 for the integral and error estimates. ≫

-1.41662×10^{-14}

lch[a, 0, Log[n] / Log[a]] /. {n → 100, a → 1.000001}

$$-1.\times10^{6}+9.21034\times10^{6}\int_0^\infty -\frac{\mathrm{Sin}[t\,\mathrm{Log}[100]]}{-1+100^{6.28319\times10^{6}\,t}}\,dt$$

$$\mathbf{N}\left[(1/(100^{\wedge}2))\left(-1.0000000001583302`*^{\wedge}6+9.210344977903306`*^{\wedge}6\int_0^\infty -\frac{\mathrm{Sin}[t\,\mathrm{Log}[100]]}{-1+100^{6.283188449288612`*^{\wedge}6\,t}}\,dt\right)\right]$$

Power::infy : Infinite expression $\frac{1}{0.}$ encountered. ≫

NIntegrate::ncvb :

NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in t near {t} = $\{4.59562\times10^{-7}\}$.

NIntegrate obtained -9.04779×10^{-15} and 5.0316117479614096`*^-18 for the integral and error estimates. ≫

-100.

Gamma[1, -Log[100]]

100

lch[a, 0, Log[n] / Log[a]] /. {n → 100, a → 1.001}

$$-999.999916708291`+9214.944775018343`\int_0^\infty -\frac{\mathrm{Sin}[t\,\mathrm{Log}[100]]}{-1+100^{6286.326376496726`\,t}}\,dt$$

$$\mathbf{N}\left[-999.999916708291`+9214.944775018343`\int_0^\infty -\frac{\mathrm{Sin}[t\,\mathrm{Log}[100]]}{-1+100^{6286.326376496726`\,t}}\,dt\right]$$

NIntegrate::deorela :

The relative error 4.121882867389303` is larger than expected for the integrand $-\frac{\mathrm{Sin}[t\,\mathrm{Log}[100]]}{-1+100^{6286.33\,t}}$ over {0, ∞} with DoubleExponentialOscillatory method and automatic tuning parameters, TuningParameters -> {10, 5}. The integration will proceed with TuningParameters -> {1, 5}. ≫

Power::infy : Infinite expression $\frac{1}{0.}$ encountered. ≫

-1000.

lch[a, 0, Log[n] / Log[a]] /. {n → 100, a → 1.0001}

$$-10\,000.+92\,108.\int_0^\infty -\frac{\mathrm{Sin}[t\,\mathrm{Log}[100]]}{-1+100^{62\,835.\,t}}\,dt$$

lch[a, 0, Log[n] / Log[a]] /. {n → 100, a → 1.00001}

$$-100\,000. + 921\,039. \int_0^\infty -\frac{\text{Sin}[t\,\text{Log}[100]]}{-1 + 100^{628\,322.\,t}}\,dt$$

N[1 / (100^2)]

0.0001

.01^2

0.0001

lch[a, 0, Log[n] / Log[a]] /. {n → 100, a → 1.0001}

$$-10\,000. + 92\,108. \int_0^\infty -\frac{\text{Sin}[t\,\text{Log}[100]]}{-1 + 100^{62\,835.\,t}}\,dt$$

$$\mathbf{N\left[-9999.999991678147` + 92108.00881320897` \int_0^\infty -\frac{\text{Sin}[t\,\text{Log}[100]]}{-1 + 100^{62834.99461209911`\,t}}\,dt\right]}$$

NIntegrate::deorela :

The relative error 2.685513989782156` is larger than expected for the integrand $-\frac{\text{Sin}[t\,\text{Log}[100]]}{-1 + 100^{62835.\,t}}$ over {0, ∞} with DoubleExponentialOscillatory method and automatic tuning parameters, TuningParameters –> {10, 5}. The integration will proceed with TuningParameters –> {1, 5}. ≫

Power::infy : Infinite expression $\frac{1}{0.}$ encountered. ≫

(-1 / (100^1)) (-10000.000000011063`)

100.

NIntegrate::deorela :

The relative error 2.685513989782156` is larger than expected for the integrand $-\frac{\text{Sin}[t\,\text{Log}[100]]}{-1 + 100^{62835.\,t}}$ over {0, ∞} with DoubleExponentialOscillatory method and automatic tuning parameters, TuningParameters –> {10, 5}. The integration will proceed with TuningParameters –> {1, 5}. ≫

Power::infy : Infinite expression $\frac{1}{0.}$ encountered. ≫

0.01

(-1 / (10^3)) lch[a, 0, Log[100] / Log[a]] /. a → (1 + .1^3)

$Aborted

```
lch[a, -1, Log[n] / Log[a]] /. {n → 100, a → 1.000001}
```

GCD::exact : Argument 2.8935151119608667`*^7 in GCD$[0, 1, 2.89352\times10^{7}, 2.89352\times10^{7}]$ is not an exact number. ≫

GCD::exact : Argument 2.8935151119608667`*^7 in GCD$[0, 1, 2.89352\times10^{7}, 2.89352\times10^{7}]$ is not an exact number. ≫

GCD::exact : Argument 2.8935151119608667`*^7 in GCD$[0, 1, 2.89352\times10^{7}, 2.89352\times10^{7}]$ is not an exact number. ≫

General::stop : Further output of GCD::exact will be suppressed during this calculation. ≫

$$\left(-3.60517\times10^{12} + 0.000441506\, i\right) + 4.24152\times10^{13}\int_0^\infty -\frac{\sqrt{1+t^2}\ \mathrm{Sin}[\mathrm{ArcTan}[t] + t\,\mathrm{Log}[100]]}{-1 + 100^{6.28319\times10^{6}\, t}}\, dt$$

```
N[Gamma[2, -Log[100]]]
```

$-360.517 + 4.41506\times10^{-14}\, i$

```
N[(-3.605171489461148`*^12 + 0.0004415064544800765` i) +
  4.241522730599433`*^13 ∫_0^∞ -(Sqrt[1 + t^2] Sin[ArcTan[t] + t Log[100]])/(-1 + 100^(6.283188449288612`*^6 t)) dt]
```

GCD::exact : Argument 2.8935151119608667`*^7 in GCD$[0, 1, 2.89352\times10^{7}, 2.89352\times10^{7}]$ is not an exact number. ≫

NIntegrate::ncvb :

NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in t near {t} = $\{2.29781\times10^{-7}\}$.

NIntegrate obtained -1.10125×10^{-14} and 1.979528576393144`*^-18 for the integral and error estimates. ≫

$-3.60517\times10^{12} + 0.000441506\, i$

```
(1 / 100^6) (-3.605171489461615`*^12 + 0.0004415064544800765` i)
```

$-3.60517 + 4.41506\times10^{-16}\, i$

```
N[Gamma[2, -Log[100]]]
```

$-360.517 + 4.41506\times10^{-14}\, i$

```
{(-1 / (100^1)) N[lch[a, 0, Log[n] / Log[a]] /. {n → 100, a → 1.0001}], Gamma[1, -Log[100]]}
```

NIntegrate::deorela :

The relative error 2.685513989782156` is larger than expected for the integrand $-\frac{\mathrm{Sin}[t\,\mathrm{Log}[100]]}{-1 + 100^{62835.\, t}}$ over

{0, ∞} with DoubleExponentialOscillatory method and automatic tuning parameters,
TuningParameters -> {10, 5}. The integration will proceed with TuningParameters -> {1, 5}. ≫

Power::infy : Infinite expression $\frac{1}{0.}$ encountered. ≫

```
{100., 100}
```

```
N[1 / (100^4)]
```

$1. \times 10^{-8}$

```
{(-1 / (10^1)) N[lch[a, 0, Log[n] / Log[a]] /. {n → 10, a → 1 + (.1)^2}], Gamma[1, -Log[10]]}
```

Power::infy : Infinite expression $\frac{1}{0.}$ encountered. ≫

```
{10., 10}
```

```
{(-1 / (10^3)) N[lch[a, 0, Log[n] / Log[a]] /. {n → 10, a → 1 + (.1)^4}], Gamma[1, -Log[10]]}
```

NIntegrate::deorela :

The relative error 2.685513989782156` is larger than expected for the integrand $-\frac{\text{Sin}[t\,\text{Log}[10]]}{-1+10^{62835.\,t}}$ over {0, ∞} with DoubleExponentialOscillatory method and automatic tuning parameters, TuningParameters –> {10, 5}. The integration will proceed with TuningParameters –> {1, 5}. ≫

Power::infy : Infinite expression $\frac{1}{0.}$ encountered. ≫

```
{10., 10}
```

```
N[1 / 7]
```

```
0.142857
```

```
{(-1 / (7^3)) N[lch[a, 0, Log[n] / Log[a]] /. {n → 7, a → 1 + (0.14285714285714285`)^4}],
 Gamma[1, -Log[7]]}
```

NIntegrate::deorela :

The relative error 4.0474465243134885` is larger than expected for the integrand $-\frac{\text{Sin}[t\,\text{Log}[7]]}{-1+7^{15089.1\,t}}$ over {0, ∞} with DoubleExponentialOscillatory method and automatic tuning parameters, TuningParameters –> {10, 5}. The integration will proceed with TuningParameters –> {1, 5}. ≫

Power::infy : Infinite expression $\frac{1}{0.}$ encountered. ≫

```
{7., 7}
```

```
{(-1 / (7^5)) N[lch[a, 0, Log[n] / Log[a]] /. {n → 7, a → 1 + (0.14285714285714285`)^6}],
 Gamma[1, -Log[7]]}
```

Power::infy : Infinite expression $\frac{1}{0.}$ encountered. ≫

NIntegrate::ncvb :

NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in t near {t} = {0.0000179875}. NIntegrate obtained -1.54698×10^{-12} and 9.438044133734867`*^-18 for the integral and error estimates. ≫

```
{7., 7}
```

```
{(-1 / (7^5)) N[lch[a, 0, Log[n] / Log[a]] /. {n → 2, a → 1 + (0.14285714285714285`)^6}],
 Gamma[1, -Log[13]]}
```

Power::infy : Infinite expression $\frac{1}{0.}$ encountered. ≫

NIntegrate::ncvb :
NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in t near {t} = {0.0000179875}.
NIntegrate obtained -4.34293×10^{-12} and 1.1835952518080663`*^-17 for the integral and error estimates. ≫

```
{7., 13}

lch[a, -1, Log[n] / Log[a]] /. {n → 10, a → 1.001}

$Aborted

lch2[z_, s_, a_] := 1 / (2 a^s) + Log[1 / z]^(s - 1) / z^a Gamma[1 - s, a Log[1 / z]]


(1 / 10)^5 lch2[a, -1, Log[n] / Log[a]] /. {n → 10, a → 1 + (1 / 10.)^3}
```

$-13.0274+1.5968\times10^{-15}\,i$

```
N[Gamma[2, -Log[17]]]
```

$-31.1646+3.81657\times10^{-15}\,i$

```
{(-1)^(r + 1) (1 / m)^((1 - r) b - 1) lch2[a, r, Log[n] / Log[a]] /. {n → m, a → 1 + (1 / m)^b},
  N[Gamma[1 - r, -Log[m]]]} /. {b → 5, m → 27., r → -2}
```

$\{169.313-4.14698\times10^{-14}\,i,\ 169.313-4.14698\times10^{-14}\,i\}$

```
{(-1)^r (1 / m)^(r b - 1) lch2[a, 1 - r, Log[n] / Log[a]] /. {n → m, a → 1 + (1 / m)^b},
  N[Gamma[r, -Log[m]]]} /. {b → 5, m → 27., r → 3}
```

$\{169.313-4.14698\times10^{-14}\,i,\ 169.313-4.14698\times10^{-14}\,i\}$

```
{(-1)^s m^(1 - s b) lch2[a, 1 - s, Log[n] / Log[a]] /. {n → m, a → 1 + (1 / m)^b},
  N[Gamma[s, -Log[m]]]} /. {b → 5, m → 27., s → 2}
```

$\{-61.9876+7.59129\times10^{-15}\,i,\ -61.9876+7.59129\times10^{-15}\,i\}$

```
{(-1)^s n^(1 - s b) lch2[a, 1 - s, Log[n] / Log[a]] /. {a → 1 + (1 / n)^b},
  N[Gamma[s, -Log[n]]]} /. {b → 5, n → 17., s → 3}
```

$\{74.1314-2.65007\times10^{-14}\,i,\ 74.1314-2.65007\times10^{-14}\,i\}$

```
{(-1)^s n^(1 - s a) lch2[1. + (1. / n)^a, 1 - s, Log[n] / Log[1. + (1. / n)^a]],
  N[Gamma[s, -Log[n]]]} /. {a → 5, n → 117., s → 3}
```

$\{1773.02-4.34264\times10^{-13}\,i,\ 1773.01-4.34263\times10^{-13}\,i\}$

```
lch3[z_, s_, a_] := Log[1 / z]^(s - 1) / z^a Gamma[1 - s, a Log[1 / z]]

{(-1)^s n^(1 - s a) lch3[1. + (1. / n)^a, 1 - s, Log[n] / Log[1. + (1. / n)^a]],
  N[Gamma[s, -Log[n]]]} /. {a → 5, n → 100., s → 1}

{100., 100.}
```

```
Limit[Sum[(a^k - 1) (k^b), {k, 1, Log[a, n]}], {a → 1}]
```

$$\left\{\text{Limit}\left[\text{HurwitzZeta}\left[-b, 1+\frac{\text{Log}[n]}{\text{Log}[a]}\right] - a\, n\, \text{LerchPhi}\left[a, -b, 1+\frac{\text{Log}[n]}{\text{Log}[a]}\right] + \text{PolyLog}[-b, a] - \text{Zeta}[-b], a \to 1\right]\right\}$$

```
lch3[z_, s_, a_] := Log[1 / z] ^ (s - 1) / z ^ a Gamma[1 - s, a Log[1 / z]]
lch4[z_, s_, a_] := (-Log[z]) ^ (s - 1) / z ^ a Gamma[1 - s, -a Log[z]]
Limit[Sum[(a^k) (k^s), {k, Log[a, n], Infinity}], {a → 1}]
```

$$\left\{\text{Limit}\left[n\, \text{HurwitzLerchPhi}\left[a, -s, \frac{\text{Log}[n]}{\text{Log}[a]}\right], a \to 1\right]\right\}$$

```
Limit[(-1)^s n^(1 - s a) Sum[(a^k - 1) (k^s), {k, 1, Log[a, n]}] /. s → 1, {a → 1}]
pp[n_, s_, a_] := (-1)^s n^(1 - s a) lch3[1. + (1. / n)^a, 1 - s, Log[n] / Log[1. + (1. / n)^a]]
pp2[n_, s_, a_] := (-1)^s n^(1 - s a) lch4[1. + (1. / n)^a, 1 - s, Log[n] / Log[1. + (1. / n)^a]]
pp3[n_, s_, a_] := (-1)^s n^(1 - s a) lch4[1. + n^-a, 1 - s, Log[n] / Log[1. + n^-a]]
pp4[n_, s_, a_] := (-1)^s n^(1 - s a) lch4[1. + n^-a, 1 - s, Log[n] / Log[1. + n^-a]]
pp5[n_, s_, a_] := (-1)^s n^(1 - s a) lch4[1. + n^-a, 1 - s, Log[n] / Log[1. + n^-a]]
pp[100, 3, 4]
```

$1399.73 - 3.42834 \times 10^{-13}\, i$

```
pp2[100, 3, 4]
```

$1399.73 - 3.42834 \times 10^{-13}\, i$

```
pp3[100, 3, 4]
```

$1399.73 - 3.42834 \times 10^{-13}\, i$

```
pp5[100, 3, 4]
```

$1399.73 - 3.42834 \times 10^{-13}\, i$

```
N[Gamma[3, -Log[100]]]
```

$1399.73 - 3.42834 \times 10^{-13}\, i$

```
Limit[(-1)^s n^(1 - s a) Sum[(a^k) (k^s), {k, Log[a, n], Infinity}], {a → 1}]
```

```
n^(1 - s a) /. {n → 3, s → 2, a → 2}
```

$\frac{1}{27}$

```
n^(a (1 / a - s)) /. {n → 3, s → 2, a → 2}
```

$\frac{1}{27}$

```
n^(1 - s a) / n^-a
```

$n^{1+a-a s}$

```
n^-a n^(1+a-a s) /. {n → 3, s → 2, a → 2}
```

$\frac{1}{27}$

```
FullSimplify[1 + a - a s]
```

1 + a - a s

```
Expand[Log[n^(1+a-a s)] / Log[n^a]]
```

$\texttt{v1[n\_, a\_, s\_] :=} \frac{\texttt{Log}[n^{1+a-a s}]}{\texttt{Log}[n^{-a}]}$

```
N[v1[100, 2, 3]]
```

1.5

$\texttt{v2[n\_, a\_, s\_] :=} \frac{\texttt{Log}[n^{1+a-a s}]}{-a\,\texttt{Log}[n]}$

```
N[v2[100, 2, 3]]
```

1.5

$\texttt{v3[n\_, a\_, s\_] :=} \frac{(1 + a - a s)\,\texttt{Log}[n]}{-a\,\texttt{Log}[n]}$

```
N[v3[100, 2, 3]]
```

1.5

$\texttt{v4[n\_, a\_, s\_] :=} \frac{(1 + a - a s)}{-a}$

```
N[v4[100, 2, 3]]
```

1.5

```
lch4[z_, s_, a_] := (-Log[z])^(s - 1) / z^a Gamma[1 - s, -a Log[z]]
pp6[n_, s_, a_] := (-1)^s n^(1 - s a) lch4[1. + t, 1 - s, Log[n] / Log[1. + t]] /. t → n^-a
pp7[n_, s_, a_] := (-1)^s t n^(1+a-a s) lch4[1. + t, 1 - s, Log[n] / Log[1. + t]] /. t → n^-a
pp8[n_, s_, a_] := (-1)^s t^(s - a^-1) lch4[1. + t, 1 - s, Log[n] / Log[1. + t]] /. t → n^-a
pp9[n_, s_, a_] := (-1)^s n a^s lch4[1. + a, 1 - s, Log[1 + a, n]]
pp10[n_, s_, a_] := (-1)^s n a^s lch[1. + a, 1 - s, Log[1 + a, n]]
pp11[n_, s_, a_] := (-1)^s n (a - 1)^s lch4[a, 1 - s, Log[a, n]]
```

pp6[10, 2, 4]

$-13.0272 + 1.59537 \times 10^{-15}\, i$

pp9[164, 2, .00001]

$-672.385 + 8.23434 \times 10^{-14}\, i$

pp10[10, 1, .00001]

$10. + 0.\, i$

pp10[10, 2, .00001]

$-13.0259 + 1.59522 \times 10^{-15}\, i$

N[Gamma[2, -Log[10]]]

$-13.0259 + 1.59521 \times 10^{-15}\, i$

pp11[164, 2, 1.00001]

$-672.385 + 8.23434 \times 10^{-14}\, i$

pp5[100, 2, 4]

$-360.517 + 4.41506 \times 10^{-14}\, i$

pp6[100, 2, 4]

$-360.517 + 4.41506 \times 10^{-14}\, i$

pp7[100, 2, 4]

$-360.517 + 4.41506 \times 10^{-14}\, i$

pp8[100, 2, 4]

$-360.517 + 4.41506 \times 10^{-14}\, i$

n^(1 - s a) / n^-a

FullSimplify$\left[n^{1+a-a s}\right]$

$n^{1+a-a s}$

Log[n^(1 - s a)] / Log[n^-a]

$$\frac{\mathrm{Log}\left[n^{1-a s}\right]}{\mathrm{Log}\left[n^{-a}\right]}$$

$$\mathrm{E}\,\hat{}\left(\frac{(1-a s)}{(-a)}\right)$$

$e^{-\frac{1-a s}{a}}$

n^(1 - s a) / n^-a

$n^{1+a-a s}$

n^(1 - s a) /. {n → 100, s → 2, a → 2}

$$\frac{1}{1\,000\,000}$$

```
n^(1 - s a) /. {n → 100, s → 2, a → 2, t → n^-a}
```

$$\frac{1}{1000000}$$

```
n^(1 - s a) / n^-a
```

$$n^{1+a-a\,s}$$

```
t = n^-a
```

$$n^{-a}$$

```
(n^(1 + a - a s) t) /. {n → 100, s → 2, a → 2}
```

$$\frac{1}{1000000}$$

```
E^Log[n^(1 + a - a s) t] /. {n → 100, s → 2, a → 2}
```

$$\frac{1}{1000000}$$

```
E^(Log[n^(1 + a - a s)] + Log[t]) /. {n → 100, s → 2, a → 2}
```

$$\frac{1}{1000000}$$

```
E^((1 + a - a s) Log[n] + Log[t]) /. {n → 100, s → 2, a → 2}
```

$$\frac{1}{1000000}$$

```
E^((1 + a - a s) / -a * -a Log[n] + Log[t]) /. {n → 100, s → 2, a → 2}
```

$$\frac{1}{1000000}$$

```
E^((1 + a - a s) / -a * Log[t] + Log[t]) /. {n → 100, s → 2, a → 2}
```

$$\frac{1}{1000000}$$

```
E^(((1 + a - a s) / -a + 1) Log[t]) /. {n → 100, s → 2, a → 2}
```

$$\frac{1}{1000000}$$

```
Expand[((1 + a - a s) / -a + 1)]
```

$$-\frac{1}{a}+s$$

$$\texttt{E\^{}}\left(\left(-\frac{1}{a}+s\right)\texttt{Log[t]}\right)\ \texttt{/.}\ \{n\to 100,\ s\to 2,\ a\to 2\}$$

$$\frac{1}{1000000}$$

$$\texttt{t\^{}}\left(\left(-\frac{1}{a}+s\right)\right)\ \texttt{/.}\ \{n\to 100,\ s\to 2,\ a\to 2\}$$

$$\frac{1}{1000000}$$

```
t^(s - a^-1) /. {n → 100, s → 2, a → 2}
```

$\frac{1}{1\,000\,000}$

```
FullSimplify[(n^-a)^(s - a^-1)]
```

$(n^{-a})^{-\frac{1}{a}+s}$

```
3^7
```

2187

```
-Log[3, 2187]^-1
```

$-\frac{1}{7}$

```
FullSimplify[(-1)^s n^2 t^(s - Log[n, t]^-1)]
```

$(-1)^s\, n\, t^s$

```
lch[z_, s_, a_] :=
 1 / (2 a^s) + Log[1 / z]^(s - 1) / z^a Gamma[1 - s, a Log[1 / z]] + 2 / (a^(s - 1)) Integrate[
    Sin[s ArcTan[t] - t a Log[z]] / ((1 + t^2)^(s / 2) (E^(2 Pi a t) - 1)), {t, 0, Infinity}]

Limit[Sum[(a^k) (k^(s - 1)), {k, Log[a, n], Infinity}], {a → 1}]
```

$\left\{\text{Limit}\left[n\,\text{HurwitzLerchPhi}\left[a, 1-s, \frac{\text{Log}[n]}{\text{Log}[a]}\right], a \to 1\right]\right\}$

```
Limit[(-1)^s Sum[(a - 1)^s (a^k) (k^(s - 1)), {k, Log[a, n], Infinity}], {a → 1}]
```

$\left\{\text{Limit}\left[(-1)^s\,(-1+a)^s\, n\,\text{HurwitzLerchPhi}\left[a, 1-s, \frac{\text{Log}[n]}{\text{Log}[a]}\right], a \to 1\right]\right\}$

$\text{so}[n\_, s\_, a\_] := (-1)^s\,(-1+a)^s\, n\,\text{HurwitzLerchPhi}\left[a, 1-s, \frac{\text{Log}[n]}{\text{Log}[a]}\right]$

```
so[100, 2, 1.00001]
```

$-360.518 + 1.22466\times 10^{-16}\, i$

```
N[Gamma[3, -Log[100]]]
```

$1399.73 - 3.42834\times 10^{-13}\, i$

```
so2[n_, s_, a_] := (-1)^s (a - 1)^s Sum[(a^k - 1) (k^(s - 1)), {k, 1, Log[a, n]}]

so2[100, 0, 1.000001]
```

$28.0218 - 2.46016\times 10^{-10}\, i$

```
so2[100, 1, 1.000001]
```

-94.3949

```
so3[n_, s_, a_] := (-1)^s Sum[(a - 1)^s (a^k - 1) (k^(s - 1)), {k, 1, Log[a, n]}]
so3a[n_, s_, a_] := (-1)^s (a - 1)^s Sum[(a^k - 1) (k^(s - 1)), {k, 1, Log[a, n]}]

so3[100, 0, 1.000001]
```

$28.0218 - 2.46016\times 10^{-10}\, i$

so3a[100, 0, 1.000001]

$28.0218 - 2.46016\times 10^{-10}\, i$

so3[100, 1, 1.000001] - (1 + Log[100])

$-100.$

so3a[100, 1, 1.000001] - (1 + Log[100])

$-100.$

$$\texttt{so3[100, 2, 1.00001]} - \left(1 - \frac{\texttt{Log[n]}^2}{2}\ \texttt{/. n} \to 100\right)$$

360.521

Limit[(-1)^s Sum[(a - 1)^s (a^k - 1) (k^(s - 1)), {k, 1, Log[a, n]}], {a → 1}]

$$\left\{\texttt{Limit}\left[-(-1)^s(-1+a)^s\left(\texttt{HarmonicNumber}\left[\frac{\texttt{Log[n]}}{\texttt{Log[a]}}, 1-s\right] + a\, n\, \texttt{LerchPhi}\left[a, 1-s, 1+\frac{\texttt{Log[n]}}{\texttt{Log[a]}}\right] - \texttt{PolyLog}[1-s, a]\right), a\to 1\right]\right\}$$

$$\texttt{Limit}\left[-(-1)^s(-1+a)^s\left(\texttt{HarmonicNumber}\left[\frac{\texttt{Log[n]}}{\texttt{Log[a]}}, 1-s\right] - \texttt{PolyLog}[1-s, a]\right), a\to 1\right]$$

$$\texttt{Limit}\left[-(-1)^s(-1+a)^s\left(\texttt{HarmonicNumber}\left[\frac{\texttt{Log[n]}}{\texttt{Log[a]}}, 1-s\right] - \texttt{PolyLog}[1-s, a]\right)\ \texttt{/. s}\to 0,\ a\to 1\right]$$

$$\texttt{Limit}\left[-\texttt{HarmonicNumber}\left[\frac{\texttt{Log[n]}}{\texttt{Log[a]}}\right] - \texttt{Log}[1-a],\ a\to 1\right]$$

$$\texttt{Limit}\left[-\texttt{HarmonicNumber}\left[\frac{\texttt{Log[100]}}{\texttt{Log[a]}}\right] - \texttt{Log}[1-a],\ a\to 1\right]$$

$-\texttt{EulerGamma} - i\pi - \texttt{Log[Log[100]]}$

$$\texttt{Limit}\left[-(-1)^s(-1+a)^s\left(\texttt{HarmonicNumber}\left[\frac{\texttt{Log[n]}}{\texttt{Log[a]}}, 1-s\right] - \texttt{PolyLog}[1-s, a]\right)\ \texttt{/. s}\to 1,\ a\to 1\right]$$

$1 + \texttt{Log[n]}$

$$\texttt{Limit}\left[-(-1)^s(-1+a)^s\left(\texttt{HarmonicNumber}\left[\frac{\texttt{Log[n]}}{\texttt{Log[a]}}, 1-s\right] - \texttt{PolyLog}[1-s, a]\right)\ \texttt{/. s}\to 2,\ a\to 1\right]$$

$$1 - \frac{\texttt{Log[n]}^2}{2}$$

$$\texttt{Expand}\left[\texttt{Limit}\left[-(-1)^s(-1+a)^s\left(\texttt{HarmonicNumber}\left[\frac{\texttt{Log[n]}}{\texttt{Log[a]}}, 1-s\right] - \texttt{PolyLog}[1-s, a]\right)\ \texttt{/. s}\to 3,\ a\to 1\right]\right]$$

$$2 + \frac{\texttt{Log[n]}^3}{3}$$

```
N[1 + Log[n] /. n → 100]
```

5.60517

```
Limit[-(-1)^s (-1 + a)^s (HarmonicNumber[Log[n]/Log[a], 1 - s] - PolyLog[1 - s, a]) /. s → 4, a → 1]
```

$$6 - \frac{\text{Log}[n]^4}{4}$$

```
Limit[-(-1)^s (-1 + a)^s (HarmonicNumber[Log[n]/Log[a], 1 - s] - PolyLog[1 - s, a]) /. s → 5, a → 1]
```

$$24 + \frac{\text{Log}[n]^5}{5}$$

```
Limit[-(-1)^s (-1 + a)^s (HarmonicNumber[Log[n]/Log[a], 1 - s] - PolyLog[1 - s, a]) /. s → 6, a → 1]
```

$$120 - \frac{\text{Log}[n]^6}{6}$$

```
so4[n_, s_, a_] := (-1)^(s + 1) Sum[(a - 1)^s (a^k - 1) (k^(s - 1)), {k, 1, Log[a, n]}] +
  Gamma[s] - (-1)^(s) Log[n]^s / s
so4a[n_, s_, a_] := (-1)^(s + 1) (a - 1)^s Sum[(a^k - 1) (k^(s - 1)), {k, 1, Log[a, n]}] +
  Gamma[s] - (-1)^(s) Log[n]^s / s
```

```
so4a[100, 4, 1.00001]
```

-5567.41

```
N[Gamma[4, -Log[100]]]
```

$-5567.28 + 2.04539 \times 10^{-12}\, i$

```
FullSimplify[(-1)^(s + 1) Sum[(a - 1)^s (a^k - 1) (k^(s - 1)), {k, 1, Log[a, n]}] +
  Gamma[s] + (-1)^(s + 1) Log[n]^s / s]
```

$$\text{Gamma}[s] - \frac{(-1)^s\,\text{Log}[n]^s}{s} + (-1)^s\,(-1 + a)^s \left(\text{HarmonicNumber}\left[\frac{\text{Log}[n]}{\text{Log}[a]}, 1 - s\right] + a\,n\,\text{LerchPhi}\left[a, 1 - s, 1 + \frac{\text{Log}[n]}{\text{Log}[a]}\right] - \text{PolyLog}[1 - s, a]\right)$$

```
so5[n_, s_, a_] := Gamma[s] - ((-1)^s Log[n]^s)/s + (-1)^s (-1 + a)^s
  (HarmonicNumber[Log[n]/Log[a], 1 - s] + a n LerchPhi[a, 1 - s, 1 + Log[n]/Log[a]] - PolyLog[1 - s, a])
so5[200, 3, 1.00001]
```

$3895.19 + 4.89866 \times 10^{-16}\, i$

```
N[Gamma[3, -Log[200]]]
```

$3895.11 - 9.54026 \times 10^{-13}\, i$

```
(-1)^(s+1) (a-1)^s Sum[(a^k-1) (k^(s-1)), {k, 1, Log[a, n]}] +
 Gamma[s] - (-1)^(s) Log[n]^s / s
```

$$\text{Gamma}[s] - \frac{(-1)^s\,\text{Log}[n]^s}{s} + (-1)^{1+s}\,(-1+a)^s \left(-\text{HarmonicNumber}\left[\frac{\text{Log}[n]}{\text{Log}[a]}, 1-s\right] - a\,n\,\text{LerchPhi}\left[a, 1-s, 1+\frac{\text{Log}[n]}{\text{Log}[a]}\right] + \text{PolyLog}[1-s, a]\right)$$

```
FullSimplify[(-1)^(s+1) (a-1)^s Fn + Gamma[s] - (-1)^(s) Log[n]^s / s]
```

$$\text{Gamma}[s] - \frac{(-1)^s\,\left((-1+a)^s\,\text{Fn}\,s + \text{Log}[n]^s\right)}{s}$$

```
FullSimplify[Gamma[s] s]
```

Gamma[1 + s]

$$\text{so6}[n\_, s\_, a\_] := \frac{1}{s}\left(s! - (-1)^s\left((-1+a)^s\,\text{Sum}[(a\verb|^|k-1)\,(k\verb|^|(s-1)), \{k, 1, \text{Log}[a, n]\}]\,s + \text{Log}[n]^s\right)\right)$$

```
so6[100, 3, 1.00001]
```

1399.75

```
N[Gamma[3, -Log[100]]]
```

$1399.73 - 3.42834 \times 10^{-13}\,i$

```
so7[n_, s_, a_] := (-1)^(s+1) Sum[(a-1)^s (a^k-1) (k^(s-1)), {k, 1, Log[a, n]}] +
  Gamma[s] + (-1)^(s+1) Log[n]^s / s
so7[100, 2, 1.0001]
so8[n_, s_, a_] :=
 (-1)^s Sum[(a-1)^s (a^k-1) (k^(s-1)), {k, 1, Log[a, n]}] + (-1)^(s) Log[n]^s / s
so8[130, 4, 1.0001]
```

8776.14

```
N[Gamma[4, 0, -Log[130]]]
```

$8774.82 - 3.22161 \times 10^{-12}\,i$

```
so9[n_, s_, a_] :=
 (-1)^s ( (a-1)^s Sum[(a^k-1) (k^(s-1)), {k, 1, Log[a, n]}] + Log[n]^s / s)
so9[130, 4, 1.0001]
```

8776.14

```
Limit[(-1)^b Sum[(a^(1-b) - 1)^b ((a)^k - 1) (k^(b-1)), {k, 1, Log[a, n]}], {a → 1}]
```

$$\left\{\text{Limit}\left[-(-1)^b\left(-1+a^{1-b}\right)^b\left(\text{HarmonicNumber}\left[\frac{\text{Log}[n]}{\text{Log}[a]},1-b\right]+a\,n\,\text{LerchPhi}\left[a,1-b,1+\frac{\text{Log}[n]}{\text{Log}[a]}\right]-\text{PolyLog}[1-b,a]\right),a\to 1\right]\right\}$$

```
Limit[(-1)^s Sum[(a^(1-t) - 1)^s ((a^(1-t))^k - 1) (k^(s-1)), {k, 1, Log[a, n]}], {a → 1}]
```

$$\left\{\text{Limit}\left[-(-1)^s a^{-t}\left(-1+a^{1-t}\right)^s\left(a^t\,\text{HarmonicNumber}\left[\frac{\text{Log}[n]}{\text{Log}[a]},1-s\right]+a\left(a^{1-t}\right)^{\frac{\text{Log}[n]}{\text{Log}[a]}}\text{LerchPhi}\left[a^{1-t},1-s,1+\frac{\text{Log}[n]}{\text{Log}[a]}\right]-a^t\,\text{PolyLog}\left[1-s,a^{1-t}\right]\right),a\to 1\right]\right\}$$

$$\textbf{Limit}\left[-(-1)^s a^{-t}\left(-1+a^{1-t}\right)^s\left(a^t\,\textbf{HarmonicNumber}\left[\frac{\textbf{Log}[n]}{\textbf{Log}[a]},1-s\right]-a^t\,\textbf{PolyLog}\left[1-s,a^{1-t}\right]\right),a\to 1\right]$$

$$\text{Limit}\left[-(-1)^s a^{-t}\left(-1+a^{1-t}\right)^s\left(a^t\,\text{HarmonicNumber}\left[\frac{\text{Log}[n]}{\text{Log}[a]},1-s\right]-a^t\,\text{PolyLog}\left[1-s,a^{1-t}\right]\right),a\to 1\right]$$

$$\textbf{Limit}\left[-(-1)^s a^{-t}\left(-1+a^{1-t}\right)^s\left(a^t\,\textbf{HarmonicNumber}\left[\frac{\textbf{Log}[n]}{\textbf{Log}[a]},1-s\right]-a^t\,\textbf{PolyLog}\left[1-s,a^{1-t}\right]\right)\text{ /. }s\to 0,a\to 1\right]$$

$$\text{Limit}\left[-a^{-t}\left(a^t\,\text{HarmonicNumber}\left[\frac{\text{Log}[n]}{\text{Log}[a]}\right]+a^t\,\text{Log}\left[1-a^{1-t}\right]\right),a\to 1\right]$$

$$\textbf{Limit}\left[-(-1)^s a^{-t}\left(-1+a^{1-t}\right)^s\left(a^t\,\textbf{HarmonicNumber}\left[\frac{\textbf{Log}[n]}{\textbf{Log}[a]},1-s\right]-a^t\,\textbf{PolyLog}\left[1-s,a^{1-t}\right]\right)\text{ /. }s\to 1,a\to 1\right]$$

$$1+\text{Log}[n]-t\,\text{Log}[n]$$

$$\textbf{Limit}\left[-(-1)^s a^{-t}\left(-1+a^{1-t}\right)^s\left(a^t\,\textbf{HarmonicNumber}\left[\frac{\textbf{Log}[n]}{\textbf{Log}[a]},1-s\right]-a^t\,\textbf{PolyLog}\left[1-s,a^{1-t}\right]\right)\text{ /. }s\to 2,a\to 1\right]$$

$$\textbf{FullSimplify}\left[1-\frac{1}{2}(-1+t)^2\,\textbf{Log}[n]^2\right]$$

$$1-\frac{1}{2}(-1+t)^2\,\text{Log}[n]^2$$

$$\textbf{Limit}\left[-(-1)^s a^{-t}\left(-1+a^{1-t}\right)^s\left(a^t\,\textbf{HarmonicNumber}\left[\frac{\textbf{Log}[n]}{\textbf{Log}[a]},1-s\right]-a^t\,\textbf{PolyLog}\left[1-s,a^{1-t}\right]\right)\text{ /. }s\to 3,a\to 1\right]$$

$$\textbf{FullSimplify}\left[\frac{1}{3}\left(6-(-1+t)^3\,\textbf{Log}[n]^3\right)\right]$$

$$2-\frac{1}{3}(-1+t)^3\,\text{Log}[n]^3$$

FullSimplify[
Limit[$-(-1)^s a^{-t} \left(-1+a^{1-t}\right)^s \left(a^t \,\text{HarmonicNumber}\left[\frac{\text{Log}[n]}{\text{Log}[a]}, 1-s\right] - a^t \,\text{PolyLog}\left[1-s, a^{1-t}\right]\right)$ /. s → 4,
a → 1]]

$6 - \frac{1}{4} (-1+t)^4 \text{Log}[n]^4$

FullSimplify[
Limit[$-(-1)^s a^{-t} \left(-1+a^{1-t}\right)^s \left(a^t \,\text{HarmonicNumber}\left[\frac{\text{Log}[n]}{\text{Log}[a]}, 1-s\right] - a^t \,\text{PolyLog}\left[1-s, a^{1-t}\right]\right)$ /. s → 5,
a → 1]]

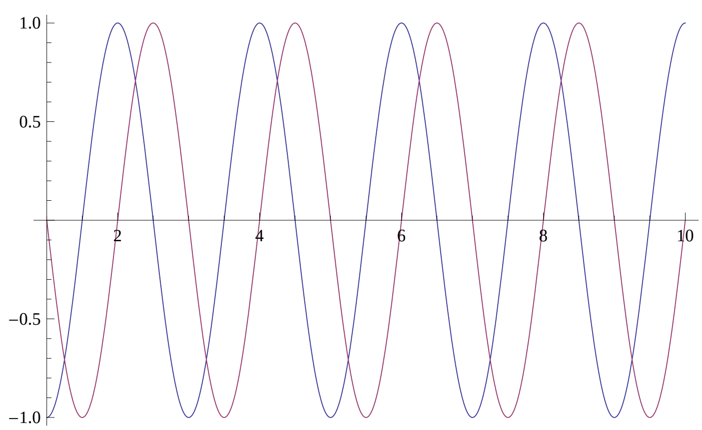
$24 - \frac{1}{5} (-1+t)^5 \text{Log}[n]^5$

```
N[Gamma[2.5, 0, -Log[130]]]
so9[n_, s_, a_] :=
 (-1)^s ( (a - 1)^s Sum[(a^k - 1) (k^(s - 1)) , {k, 1, Log[a, n]}] + Log[n]^s / s)
so9[130, 2.5, 1.0001]
```

$3.11437 \times 10^{-13} + 1017.23\, i$

$3.11464 \times 10^{-13} + 1017.32\, i$

```
Plot[{Re[(-1)^(j) ], Im[(-1)^(j) ]}, {j, 1, 10}]
```



```
Plot[{Cos[j Pi], Sin[j Pi]}, {j, 1, 10}]
```