```
DD[n_, k_] := Sum[ DD[Floor[n / j], k - 1], {j, 2, n}]
DD[n_, 0] := 1
PI[n_] := Sum[ (-1)^(k + 1) / k DD[n, k], {k, 1, Log[n] / Log[2]}]
```

```
PI[100]
```

$$\frac{428}{15}$$

```
PI[n_, k_] := Sum[ 1 / k - PI[Floor[n / j], k + 1], {j, 2, n}]
PI[n_] := PI[n, 1]
```

```
PI[100]
```

$$\frac{428}{15}$$

```
PI[n_, j_, k_] := 1 / k - PI[Floor[n / j], Floor[n / j], k + 1] + PI[n, j - 1, k]
PI[n_, 1, k_] := 0
PI[n_] := PI[n, n, 1]
```

```
PI[96]
```

$$\frac{413}{15}$$

```
Kappa[n_] := N[MangoldtLambda[n] / Log[n]]
F[n_, k_ ] := Sum[ Kappa[j] ( 1 / (k!) + F[Floor[n / j], k + 1]), {j, 2, n}]
F[n_] := F[n, 1]
```

```
F[108]
```

```
107.
```

```
Kappa[6]
```

```
0.
```

```
Kappa[n_] := N[MangoldtLambda[n] / Log[n]]
F[n_, j_, k_ ] :=  Kappa[j] ( 1 / (k!) + F[Floor[n / j], k + 1]) + F[n, j - 1, k]
F[n_, 1, k_] := 0
F[n_] := F[n, n, 1]
```

```
F[107]
```

```
106.
```

```
DivisorSum[ 20, f, PrimeQ]
```

```
f[2] + f[5]
```

```
PrimeQ[7]
```

```
True
```

```
DivisorSum[ n, Sin[Pi #1 / 2] &]
```

$$\frac{1}{4}\,\text{SquaresR}[2, n]$$

```
Divisors[300]
```

```
{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 25, 30, 50, 60, 75, 100, 150, 300}
```

```
Sum[ j, {j, Divisors[10]}]
```

```
18
```

```
Divisors[10]
```

```
{1, 2, 5, 10}
```

```
dp[n_, k_ ] := Sum[ dp[j, k - 1] dp[n / j, 1], {j, Divisors[n]}]
dp[n_, 1 ] := If[n < 2, 0, 1]
Kappa[n_] := Sum[ (-1)^(k + 1) / k dp[n, k], {k, 1, N[Log[n] / Log[2]]}]
```

```
Kappa[12]
```

```
0
```

```
DD[8, 2]
```

```
2
```

```
Table[ Kappa[j], {j, 2, 21}]
```

$$\left\{1, 1, \frac{1}{2}, 1, 0, 1, \frac{1}{3}, \frac{1}{2}, 0, 1, 0, 1, 0, 0, \frac{1}{4}, 1, 0, 1, 0, 0\right\}$$

```
pd[n_, k_ ] := Sum[ pd[j, k - 1] pd[n / j, 1], {j, Divisors[n]}]
pd[n_, 1 ] := Kappa[n]
InvKappa[n_] := Sum[ 1 / (k!) pd[n, k], {k, 1, N[Log[n] / Log[2]]}]
```

```
Kappa2[9]
```

```
1
```

```
Table[InvKappa[n], {n, 2, 21}]
```

```
{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}
```

```
InvKappa[1]
```

```
0
```

```
PI[n_] := Sum[ 1 / k PrimePi[Floor[n^(1 / k)]], {k, 1, N[Log[n] / Log[2]]}]
PP[n_] := Sum[ MoebiusMu[k] 1 / k PI[Floor[n^(1 / k)]], {k, 1, N[Log[n] / Log[2]]}]
```

```
FactorInteger[9000]
```

```
{{2, 3}, {3, 2}, {5, 3}}
```

```
Product[ j, {j, FactorInteger[9000]}]
```

```
{30, 18}
```

```
Length[FactorInteger[9000]]
```

```
3
```

```
FactorInteger[9000][[2]][[1]]
```

```
3
```

```
DDD[n_, z_] :=
 1 / z Product[ 1 / (FactorInteger[n][[j]][[2]]!) Pochhammer[z, FactorInteger[n][[j]][[2]]],
   {j, 1, Length[FactorInteger[n]]}]
```

```
PI2[n_] := N[Sum[ DDD[j, 10^(-120)], {j, 2, n}]]

PI2[100]
```

```
28.5333
```

```
Table[ Round[N[DDD[n, 10^-120]], .0000001], {n, 2, 100}]
```

```
{1., 1., 0.5, 1., 0., 1., 0.333333, 0.5, 0., 1., 0., 1., 0., 0., 0.25, 1., 0., 1.,
 0., 0., 0., 1., 0., 0.5, 0., 0.333333, 0., 1., 0., 1., 0.2, 0., 0., 0., 0., 1., 0.,
 0., 0., 1., 0., 1., 0., 0., 0., 1., 0., 0.5, 0., 0., 0., 1., 0., 0., 0., 0., 0., 1.,
 0., 1., 0., 0., 0.166667, 0., 0., 1., 0., 0., 0., 1., 0., 1., 0., 0., 0., 0., 0., 1.,
 0., 0.25, 0., 1., 0., 0., 0., 0., 0., 1., 0., 0., 0., 0., 0., 0., 0., 1., 0., 0., 0.}
```

```
FactInteger[ n_] := If[ n == 1, {}, FactorInteger[n]]
dd[n_, z_] := Product[ 1 / (j[[2]]!) Pochhammer[z, j[[2]]], {j, FactInteger[n]}]

Dd[101, .00001]
```

```
0.00001
```

```
PrimeKappa[n_] := N[MangoldtLambda[n] / Log[n]]
FactInteger[ n_] := If[ n == 1, {}, FactorInteger[n]]
d[n_, z_] := Product[ 1 / (j[[2]]!) Pochhammer[z, j[[2]]], {j, FactInteger[n]}]
Limitd[n_, z_] := Round[N[1 / z d[n, z]], .0000001]
Table[{ n, Limitd[n, 10^(-120)], PrimeKappa[n]}, {n, 2, 100}] // TableForm
```

2	1.	1.
3	1.	1.
4	0.5	0.5
5	1.	1.
6	0.	0.
7	1.	1.
8	0.333333	0.333333
9	0.5	0.5
10	0.	0.
11	1.	1.
12	0.	0.
13	1.	1.
14	0.	0.
15	0.	0.
16	0.25	0.25
17	1.	1.
18	0.	0.
19	1.	1.
20	0.	0.
21	0.	0.
22	0.	0.
23	1.	1.
24	0.	0.
25	0.5	0.5
26	0.	0.
27	0.333333	0.333333
28	0.	0.
29	1.	1.
30	0.	0.
31	1.	1.
32	0.2	0.2
33	0.	0.

34	0.	0.
35	0.	0.
36	0.	0.
37	1.	1.
38	0.	0.
39	0.	0.
40	0.	0.
41	1.	1.
42	0.	0.
43	1.	1.
44	0.	0.
45	0.	0.
46	0.	0.
47	1.	1.
48	0.	0.
49	0.5	0.5
50	0.	0.
51	0.	0.
52	0.	0.
53	1.	1.
54	0.	0.
55	0.	0.
56	0.	0.
57	0.	0.
58	0.	0.
59	1.	1.
60	0.	0.
61	1.	1.
62	0.	0.
63	0.	0.
64	0.166667	0.166667
65	0.	0.
66	0.	0.
67	1.	1.
68	0.	0.
69	0.	0.
70	0.	0.
71	1.	1.
72	0.	0.
73	1.	1.
74	0.	0.
75	0.	0.
76	0.	0.
77	0.	0.
78	0.	0.
79	1.	1.
80	0.	0.
81	0.25	0.25
82	0.	0.
83	1.	1.
84	0.	0.
85	0.	0.
86	0.	0.
87	0.	0.
88	0.	0.
89	1.	1.

```
90    0.          0.
91    0.          0.
92    0.          0.
93    0.          0.
94    0.          0.
95    0.          0.
96    0.          0.
97    1.          1.
98    0.          0.
99    0.          0.
100   0.          0.
```

```
PI[n_] := N[Sum[Dd[j, 10^(-120)], {j, 2, n}]]
```

```
PI2[100]
```

```
28.5333
```

```
Dd[n_, z_] := Product[1 / (a[[2]]!) Pochhammer[z, a[[2]]], {a, FactorInteger[n]}]
```

```
Dd[102, .000000001]
```

$1. \times 10^{-27}$

```
Round[ .000000000001, .000000001]
```

```
0.
```

```
RiemannPrimeCounting[n_] := Sum[ PrimePi[ n^(1 / j)] / j, {j, 1, N[Log[n] / Log[2]]}]
FactInteger[ n_] := If[ n == 1, {}, FactorInteger[n]]
d[n_, z_] := Product[ 1 / (j[[2]]!) Pochhammer[z, j[[2]]], {j, FactInteger[n]}]
DD[n_, z_] := Sum[ d[j, z], {j, 1, n}]
LimitD[n_, z_] := 1 / z (DD[n, z] - 1)
Table[{ n, N[LimitD[n, 10^(-120)]], N[RiemannPrimeCounting[n]]}, {n, 1, 100}] //
 TableForm
```

```
1     0.         0.
2     1.         1.
3     2.         2.
4     2.5        2.5
5     3.5        3.5
6     3.5        3.5
7     4.5        4.5
8     4.83333    4.83333
9     5.33333    5.33333
10    5.33333    5.33333
11    6.33333    6.33333
12    6.33333    6.33333
13    7.33333    7.33333
14    7.33333    7.33333
15    7.33333    7.33333
16    7.58333    7.58333
17    8.58333    8.58333
18    8.58333    8.58333
19    9.58333    9.58333
20    9.58333    9.58333
21    9.58333    9.58333
22    9.58333    9.58333
```

23	10.5833	10.5833
24	10.5833	10.5833
25	11.0833	11.0833
26	11.0833	11.0833
27	11.4167	11.4167
28	11.4167	11.4167
29	12.4167	12.4167
30	12.4167	12.4167
31	13.4167	13.4167
32	13.6167	13.6167
33	13.6167	13.6167
34	13.6167	13.6167
35	13.6167	13.6167
36	13.6167	13.6167
37	14.6167	14.6167
38	14.6167	14.6167
39	14.6167	14.6167
40	14.6167	14.6167
41	15.6167	15.6167
42	15.6167	15.6167
43	16.6167	16.6167
44	16.6167	16.6167
45	16.6167	16.6167
46	16.6167	16.6167
47	17.6167	17.6167
48	17.6167	17.6167
49	18.1167	18.1167
50	18.1167	18.1167
51	18.1167	18.1167
52	18.1167	18.1167
53	19.1167	19.1167
54	19.1167	19.1167
55	19.1167	19.1167
56	19.1167	19.1167
57	19.1167	19.1167
58	19.1167	19.1167
59	20.1167	20.1167
60	20.1167	20.1167
61	21.1167	21.1167
62	21.1167	21.1167
63	21.1167	21.1167
64	21.2833	21.2833
65	21.2833	21.2833
66	21.2833	21.2833
67	22.2833	22.2833
68	22.2833	22.2833
69	22.2833	22.2833
70	22.2833	22.2833
71	23.2833	23.2833
72	23.2833	23.2833
73	24.2833	24.2833
74	24.2833	24.2833
75	24.2833	24.2833
76	24.2833	24.2833
77	24.2833	24.2833
78	24.2833	24.2833

```
79     25.2833    25.2833
80     25.2833    25.2833
81     25.5333    25.5333
82     25.5333    25.5333
83     26.5333    26.5333
84     26.5333    26.5333
85     26.5333    26.5333
86     26.5333    26.5333
87     26.5333    26.5333
88     26.5333    26.5333
89     27.5333    27.5333
90     27.5333    27.5333
91     27.5333    27.5333
92     27.5333    27.5333
93     27.5333    27.5333
94     27.5333    27.5333
95     27.5333    27.5333
96     27.5333    27.5333
97     28.5333    28.5333
98     28.5333    28.5333
99     28.5333    28.5333
100    28.5333    28.5333
```

```
Dd[1, 4]
```

```
1
```

```
FactInteger[1]
```

```
{}
```

```
RiemannPrimeCounting[n_] := Sum[ PrimePi[ n^(1 / j)] / j, {j, 1, N[Log[n] / Log[2]]}]
DD[n_, k_] := Sum[DD[Floor[n / j], k - 1], {j, 2, n}]
DD[n_, 0] := 1
PI[n_] := Sum[(-1)^(k + 1) / k DD[n, k], {k, 1, N[Log[n] / Log[2]]}]
Table[{ n, PI[n], RiemannPrimeCounting[n]}, {n, 1, 100}] // TableForm
```

1	0	0
2	1	1
3	2	2
4	$\frac{5}{2}$	$\frac{5}{2}$
5	$\frac{7}{2}$	$\frac{7}{2}$
6	$\frac{7}{2}$	$\frac{7}{2}$
7	$\frac{9}{2}$	$\frac{9}{2}$
8	$\frac{29}{6}$	$\frac{29}{6}$
9	$\frac{16}{3}$	$\frac{16}{3}$
10	$\frac{16}{3}$	$\frac{16}{3}$
11	$\frac{19}{3}$	$\frac{19}{3}$
12	$\frac{19}{3}$	$\frac{19}{3}$
13	$\frac{22}{3}$	$\frac{22}{3}$
14	$\frac{22}{3}$	$\frac{22}{3}$
15	$\frac{22}{3}$	$\frac{22}{3}$

16	$\frac{91}{12}$	$\frac{91}{12}$
17	$\frac{103}{12}$	$\frac{103}{12}$
18	$\frac{103}{12}$	$\frac{103}{12}$
19	$\frac{115}{12}$	$\frac{115}{12}$
20	$\frac{115}{12}$	$\frac{115}{12}$
21	$\frac{115}{12}$	$\frac{115}{12}$
22	$\frac{115}{12}$	$\frac{115}{12}$
23	$\frac{127}{12}$	$\frac{127}{12}$
24	$\frac{127}{12}$	$\frac{127}{12}$
25	$\frac{133}{12}$	$\frac{133}{12}$
26	$\frac{133}{12}$	$\frac{133}{12}$
27	$\frac{137}{12}$	$\frac{137}{12}$
28	$\frac{137}{12}$	$\frac{137}{12}$
29	$\frac{149}{12}$	$\frac{149}{12}$
30	$\frac{149}{12}$	$\frac{149}{12}$
31	$\frac{161}{12}$	$\frac{161}{12}$
32	$\frac{817}{60}$	$\frac{817}{60}$
33	$\frac{817}{60}$	$\frac{817}{60}$
34	$\frac{817}{60}$	$\frac{817}{60}$
35	$\frac{817}{60}$	$\frac{817}{60}$
36	$\frac{817}{60}$	$\frac{817}{60}$
37	$\frac{877}{60}$	$\frac{877}{60}$
38	$\frac{877}{60}$	$\frac{877}{60}$
39	$\frac{877}{60}$	$\frac{877}{60}$
40	$\frac{877}{60}$	$\frac{877}{60}$
41	$\frac{937}{60}$	$\frac{937}{60}$
42	$\frac{937}{60}$	$\frac{937}{60}$
43	$\frac{997}{60}$	$\frac{997}{60}$
44	$\frac{997}{60}$	$\frac{997}{60}$
45	$\frac{997}{60}$	$\frac{997}{60}$
46	$\frac{997}{60}$	$\frac{997}{60}$
47	$\frac{1057}{60}$	$\frac{1057}{60}$
48	$\frac{1057}{60}$	$\frac{1057}{60}$
49	$\frac{1087}{60}$	$\frac{1087}{60}$
50	$\frac{1087}{60}$	$\frac{1087}{60}$
51	$\frac{1087}{60}$	$\frac{1087}{60}$
52	$\frac{1087}{60}$	$\frac{1087}{60}$
53	$\frac{1147}{60}$	$\frac{1147}{60}$
54	$\frac{1147}{60}$	$\frac{1147}{60}$

55	$\frac{1147}{60}$	$\frac{1147}{60}$
56	$\frac{1147}{60}$	$\frac{1147}{60}$
57	$\frac{1147}{60}$	$\frac{1147}{60}$
58	$\frac{1147}{60}$	$\frac{1147}{60}$
59	$\frac{1207}{60}$	$\frac{1207}{60}$
60	$\frac{1207}{60}$	$\frac{1207}{60}$
61	$\frac{1267}{60}$	$\frac{1267}{60}$
62	$\frac{1267}{60}$	$\frac{1267}{60}$
63	$\frac{1267}{60}$	$\frac{1267}{60}$
64	$\frac{1277}{60}$	$\frac{1277}{60}$
65	$\frac{1277}{60}$	$\frac{1277}{60}$
66	$\frac{1277}{60}$	$\frac{1277}{60}$
67	$\frac{1337}{60}$	$\frac{1337}{60}$
68	$\frac{1337}{60}$	$\frac{1337}{60}$
69	$\frac{1337}{60}$	$\frac{1337}{60}$
70	$\frac{1337}{60}$	$\frac{1337}{60}$
71	$\frac{1397}{60}$	$\frac{1397}{60}$
72	$\frac{1397}{60}$	$\frac{1397}{60}$
73	$\frac{1457}{60}$	$\frac{1457}{60}$
74	$\frac{1457}{60}$	$\frac{1457}{60}$
75	$\frac{1457}{60}$	$\frac{1457}{60}$
76	$\frac{1457}{60}$	$\frac{1457}{60}$
77	$\frac{1457}{60}$	$\frac{1457}{60}$
78	$\frac{1457}{60}$	$\frac{1457}{60}$
79	$\frac{1517}{60}$	$\frac{1517}{60}$
80	$\frac{1517}{60}$	$\frac{1517}{60}$
81	$\frac{383}{15}$	$\frac{383}{15}$
82	$\frac{383}{15}$	$\frac{383}{15}$
83	$\frac{398}{15}$	$\frac{398}{15}$
84	$\frac{398}{15}$	$\frac{398}{15}$
85	$\frac{398}{15}$	$\frac{398}{15}$
86	$\frac{398}{15}$	$\frac{398}{15}$
87	$\frac{398}{15}$	$\frac{398}{15}$
88	$\frac{398}{15}$	$\frac{398}{15}$
89	$\frac{413}{15}$	$\frac{413}{15}$
90	$\frac{413}{15}$	$\frac{413}{15}$
91	$\frac{413}{15}$	$\frac{413}{15}$
92	$\frac{413}{15}$	$\frac{413}{15}$
93	$\frac{413}{15}$	$\frac{413}{15}$

94	$\frac{413}{15}$	$\frac{413}{15}$
95	$\frac{413}{15}$	$\frac{413}{15}$
96	$\frac{413}{15}$	$\frac{413}{15}$
97	$\frac{428}{15}$	$\frac{428}{15}$
98	$\frac{428}{15}$	$\frac{428}{15}$
99	$\frac{428}{15}$	$\frac{428}{15}$
100	$\frac{428}{15}$	$\frac{428}{15}$

```
RiemannPrimeCounting[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, N[Log[n] / Log[2]]}]
PI[n_, k_] := Sum[1 / k - PI[Floor[n / j], k + 1], {j, 2, n}]
Table[{n, PI[n, 1], RiemannPrimeCounting[n]}, {n, 1, 100}] // TableForm
```

1	0	0
2	1	1
3	2	2
4	$\frac{5}{2}$	$\frac{5}{2}$
5	$\frac{7}{2}$	$\frac{7}{2}$
6	$\frac{7}{2}$	$\frac{7}{2}$
7	$\frac{9}{2}$	$\frac{9}{2}$
8	$\frac{29}{6}$	$\frac{29}{6}$
9	$\frac{16}{3}$	$\frac{16}{3}$
10	$\frac{16}{3}$	$\frac{16}{3}$
11	$\frac{19}{3}$	$\frac{19}{3}$
12	$\frac{19}{3}$	$\frac{19}{3}$
13	$\frac{22}{3}$	$\frac{22}{3}$
14	$\frac{22}{3}$	$\frac{22}{3}$
15	$\frac{22}{3}$	$\frac{22}{3}$
16	$\frac{91}{12}$	$\frac{91}{12}$
17	$\frac{103}{12}$	$\frac{103}{12}$
18	$\frac{103}{12}$	$\frac{103}{12}$
19	$\frac{115}{12}$	$\frac{115}{12}$
20	$\frac{115}{12}$	$\frac{115}{12}$
21	$\frac{115}{12}$	$\frac{115}{12}$
22	$\frac{115}{12}$	$\frac{115}{12}$
23	$\frac{127}{12}$	$\frac{127}{12}$
24	$\frac{127}{12}$	$\frac{127}{12}$
25	$\frac{133}{12}$	$\frac{133}{12}$
26	$\frac{133}{12}$	$\frac{133}{12}$
27	$\frac{137}{12}$	$\frac{137}{12}$
28	$\frac{137}{12}$	$\frac{137}{12}$
29	$\frac{149}{12}$	$\frac{149}{12}$

30	$\frac{149}{12}$	$\frac{149}{12}$
31	$\frac{161}{12}$	$\frac{161}{12}$
32	$\frac{817}{60}$	$\frac{817}{60}$
33	$\frac{817}{60}$	$\frac{817}{60}$
34	$\frac{817}{60}$	$\frac{817}{60}$
35	$\frac{817}{60}$	$\frac{817}{60}$
36	$\frac{817}{60}$	$\frac{817}{60}$
37	$\frac{877}{60}$	$\frac{877}{60}$
38	$\frac{877}{60}$	$\frac{877}{60}$
39	$\frac{877}{60}$	$\frac{877}{60}$
40	$\frac{877}{60}$	$\frac{877}{60}$
41	$\frac{937}{60}$	$\frac{937}{60}$
42	$\frac{937}{60}$	$\frac{937}{60}$
43	$\frac{997}{60}$	$\frac{997}{60}$
44	$\frac{997}{60}$	$\frac{997}{60}$
45	$\frac{997}{60}$	$\frac{997}{60}$
46	$\frac{997}{60}$	$\frac{997}{60}$
47	$\frac{1057}{60}$	$\frac{1057}{60}$
48	$\frac{1057}{60}$	$\frac{1057}{60}$
49	$\frac{1087}{60}$	$\frac{1087}{60}$
50	$\frac{1087}{60}$	$\frac{1087}{60}$
51	$\frac{1087}{60}$	$\frac{1087}{60}$
52	$\frac{1087}{60}$	$\frac{1087}{60}$
53	$\frac{1147}{60}$	$\frac{1147}{60}$
54	$\frac{1147}{60}$	$\frac{1147}{60}$
55	$\frac{1147}{60}$	$\frac{1147}{60}$
56	$\frac{1147}{60}$	$\frac{1147}{60}$
57	$\frac{1147}{60}$	$\frac{1147}{60}$
58	$\frac{1147}{60}$	$\frac{1147}{60}$
59	$\frac{1207}{60}$	$\frac{1207}{60}$
60	$\frac{1207}{60}$	$\frac{1207}{60}$
61	$\frac{1267}{60}$	$\frac{1267}{60}$
62	$\frac{1267}{60}$	$\frac{1267}{60}$
63	$\frac{1267}{60}$	$\frac{1267}{60}$
64	$\frac{1277}{60}$	$\frac{1277}{60}$
65	$\frac{1277}{60}$	$\frac{1277}{60}$
66	$\frac{1277}{60}$	$\frac{1277}{60}$
67	$\frac{1337}{60}$	$\frac{1337}{60}$
68	$\frac{1337}{60}$	$\frac{1337}{60}$

69	1337/60	1337/60
70	1337/60	1337/60
71	1397/60	1397/60
72	1397/60	1397/60
73	1457/60	1457/60
74	1457/60	1457/60
75	1457/60	1457/60
76	1457/60	1457/60
77	1457/60	1457/60
78	1457/60	1457/60
79	1517/60	1517/60
80	1517/60	1517/60
81	383/15	383/15
82	383/15	383/15
83	398/15	398/15
84	398/15	398/15
85	398/15	398/15
86	398/15	398/15
87	398/15	398/15
88	398/15	398/15
89	413/15	413/15
90	413/15	413/15
91	413/15	413/15
92	413/15	413/15
93	413/15	413/15
94	413/15	413/15
95	413/15	413/15
96	413/15	413/15
97	428/15	428/15
98	428/15	428/15
99	428/15	428/15
100	428/15	428/15

```
RiemannPrimeCounting[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, N[Log[n] / Log[2]]}]
PI[n_, j_, k_] := 1 / k - PI[Floor[n / j], Floor[n / j], k + 1] + PI[n, j - 1, k]
PI[n_, 1, k_] := 0
Table[{n, PI[n, n, 1], RiemannPrimeCounting[n]}, {n, 1, 100}] // TableForm
```

```
1      0      0
2      1      1
3      2      2
```

4	$\frac{5}{2}$	$\frac{5}{2}$
5	$\frac{7}{2}$	$\frac{7}{2}$
6	$\frac{7}{2}$	$\frac{7}{2}$
7	$\frac{9}{2}$	$\frac{9}{2}$
8	$\frac{29}{6}$	$\frac{29}{6}$
9	$\frac{16}{3}$	$\frac{16}{3}$
10	$\frac{16}{3}$	$\frac{16}{3}$
11	$\frac{19}{3}$	$\frac{19}{3}$
12	$\frac{19}{3}$	$\frac{19}{3}$
13	$\frac{22}{3}$	$\frac{22}{3}$
14	$\frac{22}{3}$	$\frac{22}{3}$
15	$\frac{22}{3}$	$\frac{22}{3}$
16	$\frac{91}{12}$	$\frac{91}{12}$
17	$\frac{103}{12}$	$\frac{103}{12}$
18	$\frac{103}{12}$	$\frac{103}{12}$
19	$\frac{115}{12}$	$\frac{115}{12}$
20	$\frac{115}{12}$	$\frac{115}{12}$
21	$\frac{115}{12}$	$\frac{115}{12}$
22	$\frac{115}{12}$	$\frac{115}{12}$
23	$\frac{127}{12}$	$\frac{127}{12}$
24	$\frac{127}{12}$	$\frac{127}{12}$
25	$\frac{133}{12}$	$\frac{133}{12}$
26	$\frac{133}{12}$	$\frac{133}{12}$
27	$\frac{137}{12}$	$\frac{137}{12}$
28	$\frac{137}{12}$	$\frac{137}{12}$
29	$\frac{149}{12}$	$\frac{149}{12}$
30	$\frac{149}{12}$	$\frac{149}{12}$
31	$\frac{161}{12}$	$\frac{161}{12}$
32	$\frac{817}{60}$	$\frac{817}{60}$
33	$\frac{817}{60}$	$\frac{817}{60}$
34	$\frac{817}{60}$	$\frac{817}{60}$
35	$\frac{817}{60}$	$\frac{817}{60}$
36	$\frac{817}{60}$	$\frac{817}{60}$
37	$\frac{877}{60}$	$\frac{877}{60}$
38	$\frac{877}{60}$	$\frac{877}{60}$
39	$\frac{877}{60}$	$\frac{877}{60}$
40	$\frac{877}{60}$	$\frac{877}{60}$
41	$\frac{937}{60}$	$\frac{937}{60}$
42	$\frac{937}{60}$	$\frac{937}{60}$

43	$\frac{997}{60}$	$\frac{997}{60}$
44	$\frac{997}{60}$	$\frac{997}{60}$
45	$\frac{997}{60}$	$\frac{997}{60}$
46	$\frac{997}{60}$	$\frac{997}{60}$
47	$\frac{1057}{60}$	$\frac{1057}{60}$
48	$\frac{1057}{60}$	$\frac{1057}{60}$
49	$\frac{1087}{60}$	$\frac{1087}{60}$
50	$\frac{1087}{60}$	$\frac{1087}{60}$
51	$\frac{1087}{60}$	$\frac{1087}{60}$
52	$\frac{1087}{60}$	$\frac{1087}{60}$
53	$\frac{1147}{60}$	$\frac{1147}{60}$
54	$\frac{1147}{60}$	$\frac{1147}{60}$
55	$\frac{1147}{60}$	$\frac{1147}{60}$
56	$\frac{1147}{60}$	$\frac{1147}{60}$
57	$\frac{1147}{60}$	$\frac{1147}{60}$
58	$\frac{1147}{60}$	$\frac{1147}{60}$
59	$\frac{1207}{60}$	$\frac{1207}{60}$
60	$\frac{1207}{60}$	$\frac{1207}{60}$
61	$\frac{1267}{60}$	$\frac{1267}{60}$
62	$\frac{1267}{60}$	$\frac{1267}{60}$
63	$\frac{1267}{60}$	$\frac{1267}{60}$
64	$\frac{1277}{60}$	$\frac{1277}{60}$
65	$\frac{1277}{60}$	$\frac{1277}{60}$
66	$\frac{1277}{60}$	$\frac{1277}{60}$
67	$\frac{1337}{60}$	$\frac{1337}{60}$
68	$\frac{1337}{60}$	$\frac{1337}{60}$
69	$\frac{1337}{60}$	$\frac{1337}{60}$
70	$\frac{1337}{60}$	$\frac{1337}{60}$
71	$\frac{1397}{60}$	$\frac{1397}{60}$
72	$\frac{1397}{60}$	$\frac{1397}{60}$
73	$\frac{1457}{60}$	$\frac{1457}{60}$
74	$\frac{1457}{60}$	$\frac{1457}{60}$
75	$\frac{1457}{60}$	$\frac{1457}{60}$
76	$\frac{1457}{60}$	$\frac{1457}{60}$
77	$\frac{1457}{60}$	$\frac{1457}{60}$
78	$\frac{1457}{60}$	$\frac{1457}{60}$
79	$\frac{1517}{60}$	$\frac{1517}{60}$
80	$\frac{1517}{60}$	$\frac{1517}{60}$
81	$\frac{383}{15}$	$\frac{383}{15}$

82	$\frac{383}{15}$	$\frac{383}{15}$
83	$\frac{398}{15}$	$\frac{398}{15}$
84	$\frac{398}{15}$	$\frac{398}{15}$
85	$\frac{398}{15}$	$\frac{398}{15}$
86	$\frac{398}{15}$	$\frac{398}{15}$
87	$\frac{398}{15}$	$\frac{398}{15}$
88	$\frac{398}{15}$	$\frac{398}{15}$
89	$\frac{413}{15}$	$\frac{413}{15}$
90	$\frac{413}{15}$	$\frac{413}{15}$
91	$\frac{413}{15}$	$\frac{413}{15}$
92	$\frac{413}{15}$	$\frac{413}{15}$
93	$\frac{413}{15}$	$\frac{413}{15}$
94	$\frac{413}{15}$	$\frac{413}{15}$
95	$\frac{413}{15}$	$\frac{413}{15}$
96	$\frac{413}{15}$	$\frac{413}{15}$
97	$\frac{428}{15}$	$\frac{428}{15}$
98	$\frac{428}{15}$	$\frac{428}{15}$
99	$\frac{428}{15}$	$\frac{428}{15}$
100	$\frac{428}{15}$	$\frac{428}{15}$

```
PrimeKappa[n_] := FullSimplify[MangoldtLambda[n] / Log[n]]
RiemannPrimeCounting[n_] := Sum[PrimeKappa[j], {j, 2, n}]
PI[n_, k_] := Sum[1 / k - PI[Floor[n / j], k + 1], {j, 2, n}]
Table[{n, PI[n, 1], RiemannPrimeCounting[n]}, {n, 1, 100}] // TableForm
```

1	0	0
2	1	1
3	2	2
4	$\frac{5}{2}$	$\frac{5}{2}$
5	$\frac{7}{2}$	$\frac{7}{2}$
6	$\frac{7}{2}$	$\frac{7}{2}$
7	$\frac{9}{2}$	$\frac{9}{2}$
8	$\frac{29}{6}$	$\frac{29}{6}$
9	$\frac{16}{3}$	$\frac{16}{3}$
10	$\frac{16}{3}$	$\frac{16}{3}$
11	$\frac{19}{3}$	$\frac{19}{3}$
12	$\frac{19}{3}$	$\frac{19}{3}$
13	$\frac{22}{3}$	$\frac{22}{3}$
14	$\frac{22}{3}$	$\frac{22}{3}$
15	$\frac{22}{3}$	$\frac{22}{3}$
16	$\frac{91}{12}$	$\frac{91}{12}$

17	$\frac{103}{12}$	$\frac{103}{12}$
18	$\frac{103}{12}$	$\frac{103}{12}$
19	$\frac{115}{12}$	$\frac{115}{12}$
20	$\frac{115}{12}$	$\frac{115}{12}$
21	$\frac{115}{12}$	$\frac{115}{12}$
22	$\frac{115}{12}$	$\frac{115}{12}$
23	$\frac{127}{12}$	$\frac{127}{12}$
24	$\frac{127}{12}$	$\frac{127}{12}$
25	$\frac{133}{12}$	$\frac{133}{12}$
26	$\frac{133}{12}$	$\frac{133}{12}$
27	$\frac{137}{12}$	$\frac{137}{12}$
28	$\frac{137}{12}$	$\frac{137}{12}$
29	$\frac{149}{12}$	$\frac{149}{12}$
30	$\frac{149}{12}$	$\frac{149}{12}$
31	$\frac{161}{12}$	$\frac{161}{12}$
32	$\frac{817}{60}$	$\frac{817}{60}$
33	$\frac{817}{60}$	$\frac{817}{60}$
34	$\frac{817}{60}$	$\frac{817}{60}$
35	$\frac{817}{60}$	$\frac{817}{60}$
36	$\frac{817}{60}$	$\frac{817}{60}$
37	$\frac{877}{60}$	$\frac{877}{60}$
38	$\frac{877}{60}$	$\frac{877}{60}$
39	$\frac{877}{60}$	$\frac{877}{60}$
40	$\frac{877}{60}$	$\frac{877}{60}$
41	$\frac{937}{60}$	$\frac{937}{60}$
42	$\frac{937}{60}$	$\frac{937}{60}$
43	$\frac{997}{60}$	$\frac{997}{60}$
44	$\frac{997}{60}$	$\frac{997}{60}$
45	$\frac{997}{60}$	$\frac{997}{60}$
46	$\frac{997}{60}$	$\frac{997}{60}$
47	$\frac{1057}{60}$	$\frac{1057}{60}$
48	$\frac{1057}{60}$	$\frac{1057}{60}$
49	$\frac{1087}{60}$	$\frac{1087}{60}$
50	$\frac{1087}{60}$	$\frac{1087}{60}$
51	$\frac{1087}{60}$	$\frac{1087}{60}$
52	$\frac{1087}{60}$	$\frac{1087}{60}$
53	$\frac{1147}{60}$	$\frac{1147}{60}$
54	$\frac{1147}{60}$	$\frac{1147}{60}$
55	$\frac{1147}{60}$	$\frac{1147}{60}$

56	$\frac{1147}{60}$	$\frac{1147}{60}$
57	$\frac{1147}{60}$	$\frac{1147}{60}$
58	$\frac{1147}{60}$	$\frac{1147}{60}$
59	$\frac{1207}{60}$	$\frac{1207}{60}$
60	$\frac{1207}{60}$	$\frac{1207}{60}$
61	$\frac{1267}{60}$	$\frac{1267}{60}$
62	$\frac{1267}{60}$	$\frac{1267}{60}$
63	$\frac{1267}{60}$	$\frac{1267}{60}$
64	$\frac{1277}{60}$	$\frac{1277}{60}$
65	$\frac{1277}{60}$	$\frac{1277}{60}$
66	$\frac{1277}{60}$	$\frac{1277}{60}$
67	$\frac{1337}{60}$	$\frac{1337}{60}$
68	$\frac{1337}{60}$	$\frac{1337}{60}$
69	$\frac{1337}{60}$	$\frac{1337}{60}$
70	$\frac{1337}{60}$	$\frac{1337}{60}$
71	$\frac{1397}{60}$	$\frac{1397}{60}$
72	$\frac{1397}{60}$	$\frac{1397}{60}$
73	$\frac{1457}{60}$	$\frac{1457}{60}$
74	$\frac{1457}{60}$	$\frac{1457}{60}$
75	$\frac{1457}{60}$	$\frac{1457}{60}$
76	$\frac{1457}{60}$	$\frac{1457}{60}$
77	$\frac{1457}{60}$	$\frac{1457}{60}$
78	$\frac{1457}{60}$	$\frac{1457}{60}$
79	$\frac{1517}{60}$	$\frac{1517}{60}$
80	$\frac{1517}{60}$	$\frac{1517}{60}$
81	$\frac{383}{15}$	$\frac{383}{15}$
82	$\frac{383}{15}$	$\frac{383}{15}$
83	$\frac{398}{15}$	$\frac{398}{15}$
84	$\frac{398}{15}$	$\frac{398}{15}$
85	$\frac{398}{15}$	$\frac{398}{15}$
86	$\frac{398}{15}$	$\frac{398}{15}$
87	$\frac{398}{15}$	$\frac{398}{15}$
88	$\frac{398}{15}$	$\frac{398}{15}$
89	$\frac{413}{15}$	$\frac{413}{15}$
90	$\frac{413}{15}$	$\frac{413}{15}$
91	$\frac{413}{15}$	$\frac{413}{15}$
92	$\frac{413}{15}$	$\frac{413}{15}$
93	$\frac{413}{15}$	$\frac{413}{15}$
94	$\frac{413}{15}$	$\frac{413}{15}$

95	$\frac{413}{15}$	$\frac{413}{15}$
96	$\frac{413}{15}$	$\frac{413}{15}$
97	$\frac{428}{15}$	$\frac{428}{15}$
98	$\frac{428}{15}$	$\frac{428}{15}$
99	$\frac{428}{15}$	$\frac{428}{15}$
100	$\frac{428}{15}$	$\frac{428}{15}$