$$\boxed{E_z(n)-1=\binom{z}{1}\sum_{j=2}^{\lfloor n\rfloor}\Gamma(j)^{-1}+\binom{z}{2}\sum_{j=2}^{\lfloor\frac{n}{2}\rfloor}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}\Gamma(j)^{-1}\cdot\Gamma(k)^{-1}+\binom{z}{3}\sum_{j=2}^{\lfloor\frac{n}{4}\rfloor}\sum_{k=2}^{\lfloor\frac{n}{2j}\rfloor}\sum_{l=2}^{\lfloor\frac{n}{j\cdot k}\rfloor}\Gamma(j)^{-1}\cdot\Gamma(k)^{-1}\cdot\Gamma(l)^{-1}+\dots}$$

$$\boxed{P_z(n)-1=\binom{z}{1}\sum_{j=2}^{\lfloor n\rfloor}\frac{(-1)^{j+1}}{2j-1}+\binom{z}{2}\sum_{j=2}^{\lfloor\frac{n}{2}\rfloor}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}\frac{(-1)^{j+1}}{2j-1}\cdot\frac{(-1)^{k+1}}{2k-1}+\binom{z}{3}\sum_{j=2}^{\lfloor\frac{n}{4}\rfloor}\sum_{k=2}^{\lfloor\frac{n}{2j}\rfloor}\sum_{l=2}^{\lfloor\frac{n}{j\cdot k}\rfloor}\frac{(-1)^{j+1}}{2j-1}\cdot\frac{(-1)^{k+1}}{2k-1}\cdot\frac{(-1)^{l+1}}{2l-1}+\dots}$$

$$\boxed{(-1)^{a+b}\frac{\gamma(a+b,-\log n)}{\Gamma(a+b)}=\int_1^n \frac{\partial}{\partial x}((-1)^a\frac{\gamma(a,-\log x)}{\Gamma(a)})\cdot((-1)^b\frac{\gamma(b,-\log\frac{n}{x})}{\Gamma(b)})dx}$$

$$\boxed{\frac{\gamma(a+b,-\log n)}{\Gamma(a+b)}=\int_1^n \frac{\partial}{\partial x}(\frac{\gamma(a,-\log x)}{\Gamma(a)})\cdot(\frac{\gamma(b,-\log\frac{n}{x})}{\Gamma(b)})dx}$$

$$\boxed{(-1)^{a+b}\frac{\gamma(a+b,(s-1)\log n)}{\Gamma(a+b)}=\int_1^n \frac{\partial}{\partial x}((-1)^a\frac{\gamma(a,(s-1)\log x)}{\Gamma(a)})\cdot((-1)^b\frac{\gamma(b,(s-1)\log\frac{n}{x})}{\Gamma(b)})dx}$$

$$\boxed{\frac{\partial}{\partial x}\gamma(z,-\log x)=-(\log x)^{z-1}}$$

$$\boxed{\frac{\partial}{\partial x}\gamma(z,-\log x)=-(\log x)^{z-1}}$$

$$\boxed{\frac{\partial}{\partial x}\Gamma(0,-\log x)=-\frac{1}{\log x}}$$

$$\boxed{\frac{\partial^2}{\partial x^2}\Gamma(0,-\log x)=\frac{1}{x(\log x)^2}}$$

$$\boxed{\frac{\partial^3}{\partial x^3}\Gamma(0,-\log x)=-\frac{2}{x^2(\log x)^3}-\frac{1}{x^2(\log x)^2}}$$

$$\boxed{\frac{\partial^4}{\partial x^4}\Gamma(0,-\log x)=\frac{6}{x^3(\log x)^4}+\frac{6}{x^3(\log x)^3}+\frac{2}{x^3(\log x)^2}}$$

$$\boxed{\Gamma(0,-\log x)=-li(x)-\pi i}$$

$\boxed{\frac{\partial}{\partial x}\gamma(0,-\log x)=\frac{1}{\log x}}$ …? Some convergence problem.

$$\boxed{G(x,z)=(-1)^z\frac{\gamma(z,-\log x)}{\Gamma(z)}}$$

$$\boxed{\frac{\partial}{\partial x}G(x,z)=\frac{(\log x)^{z-1}}{\Gamma(z)}}$$

$$\boxed{G(n,a+b)=\int_1^n \frac{\partial}{\partial x}G(x,a)\cdot G(\frac{n}{x},b)\,dx}$$

$$\boxed{G(n,a+b)=\int_1^n\int_1^{\frac{n}{x}}\frac{\partial G(x,a)}{\partial x}\cdot\frac{\partial G(y,b)}{\partial y}\,dy\,dx}$$

$$L_{-z}(\log n)=\sum_{k=0}^{\infty}\binom{z}{k}G(n,k)$$

$$\frac{\partial}{\partial x}L_{-z}(\log x)=\sum_{k=0}^{\infty}\binom{z}{k}(-1)^{k+1}\frac{(-\log x)^{k-1}}{\Gamma(k)}=z\cdot{}_1F_1(1-z,2,-\log x)$$

$$L_{-(a+b)}(\log n)=-1+L_{-a}(\log n)+L_{-b}(\log n)+\int_1^n\int_1^{\frac{n}{x}}\frac{\partial L_{-a}(\log x)}{\partial x}\cdot\frac{\partial L_{-b}(\log y)}{\partial y}\,dy\,dx$$

$$G(n,k)=\sum_{k=0}^{\infty}(-1)^k\binom{z}{k}L_{-k}(\log n)$$ if k is positive integer

$$\lim_{z\to 0}\frac{\partial}{\partial z}L_{-z}(\log n)=li(n)-\log\log n-\gamma$$

$$\gamma(a+b,-\log n)=\frac{\Gamma(a+b)}{\Gamma(a)\cdot\Gamma(b)}\cdot\int_1^n\frac{\partial}{\partial x}(\gamma(a,-\log x))\cdot(\gamma(b,-\log\frac{n}{x}))\,dx$$

$$\gamma(a+b,-\log n)=\mathrm{B}(a,b)^{-1}\cdot\int_1^n\frac{\partial}{\partial x}(\gamma(a,-\log x))\cdot(\gamma(b,-\log\frac{n}{x}))\,dx$$

$$\gamma(a+b,-\log n)=\mathrm{B}(a,b)^{-1}\cdot\int_1^n\int_1^{\frac{n}{x}}\frac{\partial\gamma(a,-\log x)}{\partial x}\cdot\frac{\partial\gamma(b,-\log y)}{\partial x}\,dy\,dx$$

$$\gamma(b+1,-\log n)=-b\cdot\int_1^n\gamma(b,-\log\frac{n}{x})\,dx$$

$$\gamma(s,x)=-(s-1)\gamma(s-1,x)-x^{s-1}e^{-x}$$

$$\gamma(b+1,x)=-b\gamma(b,x)-x^{b}e^{-x}$$

$$\gamma(b+1,-\log n)=-b\gamma(b,-\log n)-(-\log n)^{b}e^{-(-\log n)}$$

$$\gamma(b+1,-\log n)=-b\gamma(b,-\log n)-(-\log n)^{b}n$$

$$-b\cdot\int_1^n\gamma(b,-\log\frac{n}{x})\,dx=-b\gamma(b,-\log n)-(-\log n)^{b}n$$

$$\boxed{\int_1^n \gamma\left(b,-\log\frac{n}{x}\right)dx=\gamma(b,-\log n)+\frac{(-\log n)^b n}{b}}$$

$$\boxed{\gamma(b,-\log n)=\int_1^n \gamma\left(b,-\log\frac{n}{x}\right)dx-\frac{(-\log n)^b n}{b}}$$

$$\boxed{\Gamma(a+b,-\log n)=\Gamma(a+b)-\mathrm{B}(a,b)^{-1}\cdot\int_1^n\int_1^{\frac{n}{x}}\frac{\partial}{\partial x}(\Gamma(a,-\log x))\cdot\frac{\partial}{\partial y}(\Gamma(b,-\log y))\,dy\,dx}$$

$$G(x,z)=(-1)^z\frac{\gamma(z,-\log x)}{\Gamma(z)}$$

$$\frac{\partial G(x,z)}{\partial x}=(-1)^{z+1}\frac{(-\log x)^{z-1}}{\Gamma(z)}$$

$$G(n,a+b)=\int_1^n\int_1^{\frac{n}{x}}\frac{\partial G(x,a)}{\partial x}\cdot\frac{\partial G(y,b)}{\partial y}\,dy\,dx$$

Compare to

$$n^{a+b}=\int_0^n\int_0^n\frac{\partial x^a}{\partial x}\cdot\frac{\partial y^b}{\partial y}\,dy\,dx$$

Also works as

$$G(x,z)=\frac{\gamma(z,-\log x)}{\Gamma(z)}$$

$$\frac{\partial}{\partial x}G(x,z)=-\frac{(-\log x)^{z-1}}{\Gamma(z)}$$

$$G(x,z)=\frac{\gamma(z,(s-1)\log x)}{\Gamma(z)}$$

$$\frac{\partial}{\partial x}G(x,z)=\frac{x^{-s}((s-1)\log x)^z}{\log x\cdot\Gamma(z)}$$

$$G(x,z)=(-1)^z\frac{\gamma(z,(s-1)\log x)}{\Gamma(z)}$$

$$\frac{\partial}{\partial x}G(x,z)=\frac{x^{-s}((s-1)\log x)^z}{\log x\cdot\Gamma(z)}$$

Here is what does not work:

$$G(x,z)=\gamma(z,-\log x)$$

$$G(x,z)=\frac{\gamma(z,x)}{\Gamma(z)}$$

$$G(x,z)=\frac{\Gamma(z,-\log x)}{\Gamma(z)}$$

$$\boxed{n^{*z}=(-1)^{z}\frac{\gamma(z,-\log n)}{\Gamma(z)}}$$

$$\boxed{n^{*a+b}=\int_{1}^{n}\int_{1}^{\frac{n}{x}}\frac{\partial x^{*a}}{\partial x}\cdot\frac{\partial y^{*b}}{\partial y}\,dy\,dx}$$

$$\boxed{\left(\frac{x^{1-s}-s}{1-s}\right)^{z}=\binom{z}{0}1+\binom{z}{1}\int_{1}^{n}x^{-s}\,dx+\binom{z}{2}\int_{1}^{n}\int_{1}^{n}(x\,y)^{-s}\,dy\,dx+\binom{z}{3}\int_{1}^{n}\int_{1}^{n}\int_{1}^{n}(x\,y\,z)^{-s}\,dz\,dy\,dx+\ldots}$$