```
Clear[dz]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, z_] := dz[n, z] = Product[(-1)^p[[2]] bin[-z, p[[2]]], {p, FI[n]}]
Ds[n_, s_, z_] := Sum[ j^-s dz[j, z], {j, 1, n}]
ddz1[n_, m_, s_, z_] :=
 Sum[a^-s b^-s dz[a, z] dz[b, -z], {a, 1, n}, {b, 1, m / a^(N@Log[m] / Log[n])}]
ldz[n_, m_, s_] := D[ ddz1[n, m, s, z], z] /. z → 0
sddz[n_, m_, s_, t_, z_] :=
 Sum[a^-s b^-t dz[a, z] dz[b, -z], {a, 1, n}, {b, 1, (n / a)^(N@Log[m] / Log[n])}]
sldz[n_, m_, s_, t_] := D[ sddz[n, m, s, t, z], z] /. z → 0
pr[n_, s_] := Sum[ FullSimplify[MangoldtLambda[j] / Log[j]] j^-s, {j, 2, n}]
ch[n_] := -Sum[MangoldtLambda[j], {j, 2, n}]


ldz[1200, 320, -3]

13 317 518 631 487
─────────────────
       180

pr[1200, -3] - pr[320, -3]

13 317 518 631 487
─────────────────
       180

sldz[1200, 320, -2, -4]

   159 395 771 066 767
 - ───────────────────
        1260

pr[1200, -2] - pr[320, -4]

   159 395 771 066 767
 - ───────────────────
        1260

fo[n_, s_, k_] :=
 fo[n, s, k] = Sum[ Abs[MoebiusMu[j]] j^-s fo[Floor[n / j], s, k - 1], {j, 2, n}]
fo[n_, s_, 0] := UnitStep[n - 1]
lf[n_, s_] := Sum[ (-1)^(k + 1) / k fo[n, s, k], {k, 1, Log2@n}]

lf[1000, -2]

15 167 671 034
──────────────
     315

pr[1000, -2] - pr[1000^(1 / 2), -4]

15 167 671 034
──────────────
     315


N[D[ldz[1000, 910, s], s] /. s → 0]

-92.6396

N[ch[1000] - ch[910]]

-92.6396
```

Expand[ddz1[100, 50, 1, z]]

$$1 + \frac{5\,709\,300\,759\,326\,076\,208\,729\,z}{38\,875\,468\,949\,180\,319\,102\,336} + \frac{4\,601\,729\,822\,212\,889\,z^2}{190\,035\,075\,607\,977\,600} - \frac{598\,781\,923\,z^3}{33\,418\,344\,960} + \frac{6331\,z^4}{422\,400} + \frac{397\,z^5}{69\,120} - \frac{z^6}{7680}$$

Expand[ddz1[100, 50, 1, z]]

$$1 + \frac{5\,709\,300\,759\,326\,076\,208\,729\,z}{38\,875\,468\,949\,180\,319\,102\,336} + \frac{4\,601\,729\,822\,212\,889\,z^2}{190\,035\,075\,607\,977\,600} - \frac{598\,781\,923\,z^3}{33\,418\,344\,960} + \frac{6331\,z^4}{422\,400} + \frac{397\,z^5}{69\,120} - \frac{z^6}{7680}$$

D[ddz1[100, 50, 1, z], z] /. z → 0

$$\frac{5\,709\,300\,759\,326\,076\,208\,729}{38\,875\,468\,949\,180\,319\,102\,336}$$

pr[100, 1] - pr[50, 1]

$$\frac{5\,709\,300\,759\,326\,076\,208\,729}{38\,875\,468\,949\,180\,319\,102\,336}$$

N[D[D[ddz1[100, 50, s, z], z] /. z → 0, s] /. s → 0]

-44.5599

N[ch[100] - ch[50]]

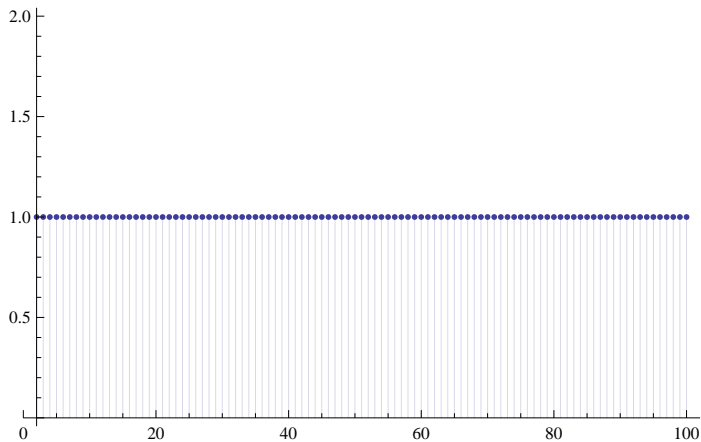-44.5599

Expand@(Ds[100, -1, z] - Ds[50, -1, z])

$$\frac{9155\,z}{12} + \frac{129\,325\,z^2}{72} + \frac{11\,261\,z^3}{12} + \frac{18\,571\,z^4}{72} + \frac{56\,z^5}{3} + \frac{8\,z^6}{9}$$
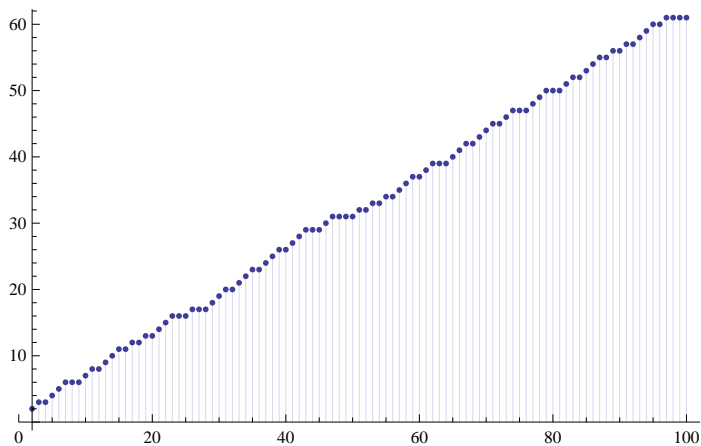
Expand@ddz1[100, 50, -1, z]

$$1 + \frac{9155\,z}{12} + \frac{223\,717\,z^2}{360} - \frac{43\,z^3}{12} + \frac{4223\,z^4}{72} + \frac{92\,z^5}{3} - \frac{4\,z^6}{45}$$

aa[n_, m_] := Sum[MoebiusMu[k], {j, 1, n}, {k, 1, m / (j^(N@Log[m] / Log[n]))}]

DiscretePlot[aa[n, n], {n, 2, 100}]

```
DiscretePlot[ddz1b[n, n^(1/2)], {n, 2, 100}]
```



```
ddz1[n_, m_, s_, z_] :=
 Sum[a^-s b^-s dz[a, z] dz[b, -z], {a, 1, n}, {b, 1, m/a^(N@Log[m]/Log[n])}]
ddz1a[n_, m_] := Sum[dz[a, 1] dz[b, -1], {a, 1, n}, {b, 1, m/a^(N@Log[m]/Log[n])}]
ddz1b[n_, m_] := Sum[dz[a, 1] dz[b, -1], {a, 1, n}, {b, 1, m/a^(N@Log[m]/Log[n])}]
```

```
FullSimplify@D[D[(Zeta[2 s]/Zeta[s])^z, z]/. z → 0, s]
```

$$-\frac{\text{Zeta}'[s]}{\text{Zeta}[s]} + \frac{2\,\text{Zeta}'[2\,s]}{\text{Zeta}[2\,s]}$$

```
ri[] := RandomInteger[{10, 100}]; rr[] := RandomReal[{-4, 4}] + RandomReal[{-4, 4}] I
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, z_] := dz[n, z] = Product[(-1)^p[[2]] bin[-z, p[[2]]], {p, FI[n]}]
Dz[n_, z_] := Sum[dz[j, z], {j, 1, n}]
zeta[n_, s_, z_, k_] := 1 + ((z + 1)/k - 1) Sum[j^-s zeta[n/j, s, z, k + 1], {j, 2, n}]
F[n_, i_, k_, z_] :=
 If[Prime[i] > n, 1, (1 + (z - 1)/k) F[n/Prime[i], i, k + 1, z] + F[n, i + 1, 1, z]]
Grid[Table[Chop[Dz[a = 143, s + t I] - F[a, 1, 1, s + t I]], {s, -1.5, 4, .7}, {t, -1.1, 4, .7}]]
```

```
0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0
```

```
Limit[(1 - (x + 1)^(1 - s)) Sum[j^-s, {j, 1, Floor[x]}], x → Infinity]
```

$$\text{Limit}\left[\left(1 - (1 + x)^{1-s}\right) \text{HarmonicNumber}[\text{Floor}[x], s], x \to \infty\right]$$

```
Grid[
 Table[(FullSimplify[dz[n, z] dz[m, -z]] / z) /. z → 0, {n, 1, 20}, {m, 1, 10 / n^Log[20, 10]}]]
```

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

ComplexInfinity	-1	-1	$-\frac{1}{2}$	-1	0	-1	$-\frac{1}{3}$	$-\frac{1}{2}$	0
1		0	0	0	0				
1		0	0	0					
$\frac{1}{2}$		0	0						
1		0							
0		0							
1		0							
$\frac{1}{3}$		0							
$\frac{1}{2}$									
0									
1									
0									
1									
0									
0									
$\frac{1}{4}$									
1									
0									
1									
0									

```
Grid[Table[
  D[D[Expand[n^-s dz[n, z] m^-s dz[m, -z]], s] /. s → 0, z] /. z → 0, {n, 1, 30}, {m, 1, 10}]]
```

0	Log[2]	Log[3]	Log[2]	Log[5]	0	Log[7]	Log[2]	$\frac{Log[9]}{2}$	0
$-Log[2]$	0	0	0	0	0	0	0	0	0
$-Log[3]$	0	0	0	0	0	0	0	0	0
$-Log[2]$	0	0	0	0	0	0	0	0	0
$-Log[5]$	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
$-Log[7]$	0	0	0	0	0	0	0	0	0
$-Log[2]$	0	0	0	0	0	0	0	0	0
$-\frac{Log[9]}{2}$	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
$-Log[11]$	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
$-Log[13]$	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
$-Log[2]$	0	0	0	0	0	0	0	0	0
$-Log[17]$	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
$-Log[19]$	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
$-Log[23]$	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
$-\frac{Log[25]}{2}$	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
$-Log[3]$	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
$-Log[29]$	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

```
Grid[Table[D[Expand[n^-s dz[n, z] m^-s dz[m, -z]], z] /. z → 0, {n, 1, 30}, {m, 1, 10}]]
```

0	-2^{-s}	-3^{-s}	-2^{-1-2s}	-5^{-s}	0	-7^{-s}	$-\frac{1}{3}2^{-3s}$	$-\frac{9^{-s}}{2}$	0
2^{-s}	0	0	0	0	0	0	0	0	0
3^{-s}	0	0	0	0	0	0	0	0	0
2^{-1-2s}	0	0	0	0	0	0	0	0	0
5^{-s}	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
7^{-s}	0	0	0	0	0	0	0	0	0
$\frac{2^{-3s}}{3}$	0	0	0	0	0	0	0	0	0
$\frac{9^{-s}}{2}$	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
11^{-s}	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
13^{-s}	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
2^{-2-4s}	0	0	0	0	0	0	0	0	0
17^{-s}	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
19^{-s}	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
23^{-s}	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
$\frac{25^{-s}}{2}$	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
3^{-1-3s}	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
29^{-s}	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

```
Grid[Table[Expand[dz[n, z] dz[m, -z]], {n, 1, 30}, {m, 1, 10}]]
```

1	$-z$	$-z$	$-\frac{z}{2}+\frac{z^2}{2}$	$-z$	z^2	$-z$	$-\frac{z}{3}+\frac{z^2}{2}-\frac{z^3}{6}$	$-\frac{z}{2}+\frac{z^2}{2}$	z^2
z	$-z^2$	$-z^2$	$-\frac{z^2}{2}+\frac{z^3}{2}$	$-z^2$	z^3	$-z^2$	$-\frac{z^2}{3}+\frac{z^3}{2}-\frac{z^4}{6}$	$-\frac{z^2}{2}+\frac{z^3}{2}$	z^3
z	$-z^2$	$-z^2$	$-\frac{z^2}{2}+\frac{z^3}{2}$	$-z^2$	z^3	$-z^2$	$-\frac{z^2}{3}+\frac{z^3}{2}-\frac{z^4}{6}$	$-\frac{z^2}{2}+\frac{z^3}{2}$	z^3
$\frac{z}{2}+\frac{z^2}{2}$	$-\frac{z^2}{2}-\frac{z^3}{2}$	$-\frac{z^2}{2}-\frac{z^3}{2}$	$-\frac{z^2}{4}+\frac{z^4}{4}$	$-\frac{z^2}{2}-\frac{z^3}{2}$	$\frac{z^3}{2}+\frac{z^4}{2}$	$-\frac{z^2}{2}-\frac{z^3}{2}$	$-\frac{z^2}{6}+\frac{z^3}{12}+\frac{z^4}{6}-\frac{z^5}{12}$	$-\frac{z^2}{4}+\frac{z^4}{4}$	$\frac{z^3}{2}+\frac{z^4}{2}$
z	$-z^2$	$-z^2$	$-\frac{z^2}{2}+\frac{z^3}{2}$	$-z^2$	z^3	$-z^2$	$-\frac{z^2}{3}+\frac{z^3}{2}-\frac{z^4}{6}$	$-\frac{z^2}{2}+\frac{z^3}{2}$	z^3

z^2	$-z^3$	$-z^3$	$-\frac{z^3}{2}+\frac{z^4}{2}$	$-z^3$	z^4	$-z^3$	$-\frac{z^3}{3}+\frac{z^4}{2}-\frac{z^5}{6}$	$-\frac{z^3}{2}+\frac{z^4}{2}$	z^4
z	$-z^2$	$-z^2$	$-\frac{z^2}{2}+\frac{z^3}{2}$	$-z^2$	z^3	$-z^2$	$-\frac{z^2}{3}+\frac{z^3}{2}-\frac{z^4}{6}$	$-\frac{z^2}{2}+\frac{z^3}{2}$	z^3
$\frac{z}{3}+\frac{z^2}{2}+\frac{z^3}{6}$	$-\frac{z^2}{3}-\frac{z^3}{2}-\frac{z^4}{6}$	$-\frac{z^2}{3}-\frac{z^3}{2}-\frac{z^4}{6}$	$-\frac{z^2}{6}-\frac{z^3}{12}+\frac{z^4}{6}+\frac{z^5}{12}$	$-\frac{z^2}{3}-\frac{z^3}{2}-\frac{z^4}{6}$	$\frac{z^3}{3}+\frac{z^4}{2}+\frac{z^5}{6}$	$-\frac{z^2}{3}-\frac{z^3}{2}-\frac{z^4}{6}$	$-\frac{z^2}{9}+\frac{5z^4}{36}-\frac{z^6}{36}$	$-\frac{z^2}{6}-\frac{z^3}{12}+\frac{z^4}{6}+\frac{z^5}{12}$	$\frac{z^3}{3}+\frac{z^4}{2}+\frac{z^5}{6}$
$\frac{z}{2}+\frac{z^2}{2}$	$-\frac{z^2}{2}-\frac{z^3}{2}$	$-\frac{z^2}{2}-\frac{z^3}{2}$	$-\frac{z^2}{4}+\frac{z^4}{4}$	$-\frac{z^2}{2}-\frac{z^3}{2}$	$\frac{z^3}{2}+\frac{z^4}{2}$	$-\frac{z^2}{2}-\frac{z^3}{2}$	$-\frac{z^2}{6}+\frac{z^3}{12}+\frac{z^4}{6}-\frac{z^5}{12}$	$-\frac{z^2}{4}+\frac{z^4}{4}$	$\frac{z^3}{2}+\frac{z^4}{2}$
z^2	$-z^3$	$-z^3$	$-\frac{z^3}{2}+\frac{z^4}{2}$	$-z^3$	z^4	$-z^3$	$-\frac{z^3}{3}+\frac{z^4}{2}-\frac{z^5}{6}$	$-\frac{z^3}{2}+\frac{z^4}{2}$	z^4
z	$-z^2$	$-z^2$	$-\frac{z^2}{2}+\frac{z^3}{2}$	$-z^2$	z^3	$-z^2$	$-\frac{z^2}{3}+\frac{z^3}{2}-\frac{z^4}{6}$	$-\frac{z^2}{2}+\frac{z^3}{2}$	z^3
$\frac{z^2}{2}+\frac{z^3}{2}$	$-\frac{z^3}{2}-\frac{z^4}{2}$	$-\frac{z^3}{2}-\frac{z^4}{2}$	$-\frac{z^3}{4}+\frac{z^5}{4}$	$-\frac{z^3}{2}-\frac{z^4}{2}$	$\frac{z^4}{2}+\frac{z^5}{2}$	$-\frac{z^3}{2}-\frac{z^4}{2}$	$-\frac{z^3}{6}+\frac{z^4}{12}+\frac{z^5}{6}-\frac{z^6}{12}$	$-\frac{z^3}{4}+\frac{z^5}{4}$	$\frac{z^4}{2}+\frac{z^5}{2}$
z	$-z^2$	$-z^2$	$-\frac{z^2}{2}+\frac{z^3}{2}$	$-z^2$	z^3	$-z^2$	$-\frac{z^2}{3}+\frac{z^3}{2}-\frac{z^4}{6}$	$-\frac{z^2}{2}+\frac{z^3}{2}$	z^3
z^2	$-z^3$	$-z^3$	$-\frac{z^3}{2}+\frac{z^4}{2}$	$-z^3$	z^4	$-z^3$	$-\frac{z^3}{3}+\frac{z^4}{2}-\frac{z^5}{6}$	$-\frac{z^3}{2}+\frac{z^4}{2}$	z^4
z^2	$-z^3$	$-z^3$	$-\frac{z^3}{2}+\frac{z^4}{2}$	$-z^3$	z^4	$-z^3$	$-\frac{z^3}{3}+\frac{z^4}{2}-\frac{z^5}{6}$	$-\frac{z^3}{2}+\frac{z^4}{2}$	z^4
$\frac{z}{4}+\frac{11z^2}{24}+\frac{z^3}{4}+\frac{z^4}{24}$	$-\frac{z^2}{4}-\frac{11z^3}{24}-\frac{z^4}{4}-\frac{z^5}{24}$	$-\frac{z^2}{4}-\frac{11z^3}{24}-\frac{z^4}{4}-\frac{z^5}{24}$	$-\frac{z^2}{8}-\frac{5z^3}{48}+\frac{5z^4}{48}+\frac{5z^5}{48}+\frac{z^6}{48}$	$-\frac{z^2}{4}-\frac{11z^3}{24}-\frac{z^4}{4}-\frac{z^5}{24}$	$\frac{z^3}{4}+\frac{11z^4}{24}+\frac{z^5}{4}+\frac{z^6}{24}$	$-\frac{z^2}{4}-\frac{11z^3}{24}-\frac{z^4}{4}-\frac{z^5}{24}$	$-\frac{z^2}{12}-\frac{z^3}{36}+\frac{5z^4}{48}+\frac{5z^5}{144}-\frac{z^6}{48}-\frac{z^7}{144}$	$-\frac{z^2}{8}-\frac{5z^3}{48}+\frac{5z^4}{48}+\frac{5z^5}{48}+\frac{z^6}{48}$	$\frac{z^3}{4}+\frac{11z^4}{24}+\frac{z^5}{4}+\frac{z^6}{24}$
z	$-z^2$	$-z^2$	$-\frac{z^2}{2}+\frac{z^3}{2}$	$-z^2$	z^3	$-z^2$	$-\frac{z^2}{3}+\frac{z^3}{2}-\frac{z^4}{6}$	$-\frac{z^2}{2}+\frac{z^3}{2}$	z^3
$\frac{z^2}{2}+\frac{z^3}{2}$	$-\frac{z^3}{2}-\frac{z^4}{2}$	$-\frac{z^3}{2}-\frac{z^4}{2}$	$-\frac{z^3}{4}+\frac{z^5}{4}$	$-\frac{z^3}{2}-\frac{z^4}{2}$	$\frac{z^4}{2}+\frac{z^5}{2}$	$-\frac{z^3}{2}-\frac{z^4}{2}$	$-\frac{z^3}{6}+\frac{z^4}{12}+\frac{z^5}{6}-\frac{z^6}{12}$	$-\frac{z^3}{4}+\frac{z^5}{4}$	$\frac{z^4}{2}+\frac{z^5}{2}$
z	$-z^2$	$-z^2$	$-\frac{z^2}{2}+\frac{z^3}{2}$	$-z^2$	z^3	$-z^2$	$-\frac{z^2}{3}+\frac{z^3}{2}-\frac{z^4}{6}$	$-\frac{z^2}{2}+\frac{z^3}{2}$	z^3

$$
\begin{array}{llllllllll}
\frac{z^2}{2}+\frac{z^3}{2} & -\frac{z^3}{2}-\frac{z^4}{2} & -\frac{z^3}{2}-\frac{z^4}{2} & -\frac{z^3}{4}+\frac{z^5}{4} & -\frac{z^3}{2}-\frac{z^4}{2} & \frac{z^4}{2}+\frac{z^5}{2} & -\frac{z^3}{2}-\frac{z^4}{2} & -\frac{z^3}{6}+\frac{z^4}{12}+\frac{z^5}{6}-\frac{z^6}{12} & -\frac{z^3}{4}+\frac{z^5}{4} & \frac{z^4}{2}+\frac{z^5}{2} \\[2mm]
z^2 & -z^3 & -z^3 & -\frac{z^3}{2}+\frac{z^4}{2} & -z^3 & z^4 & -z^3 & -\frac{z^3}{3}+\frac{z^4}{2}-\frac{z^5}{6} & -\frac{z^3}{2}+\frac{z^4}{2} & z^4 \\[2mm]
z^2 & -z^3 & -z^3 & -\frac{z^3}{2}+\frac{z^4}{2} & -z^3 & z^4 & -z^3 & -\frac{z^3}{3}+\frac{z^4}{2}-\frac{z^5}{6} & -\frac{z^3}{2}+\frac{z^4}{2} & z^4 \\[2mm]
z & -z^2 & -z^2 & -\frac{z^2}{2}+\frac{z^3}{2} & -z^2 & z^3 & -z^2 & -\frac{z^2}{3}+\frac{z^3}{2}-\frac{z^4}{6} & -\frac{z^2}{2}+\frac{z^3}{2} & z^3 \\[2mm]
\frac{z^2}{3}+\frac{z^3}{2}+\frac{z^4}{6} & -\frac{z^3}{3}-\frac{z^4}{2}-\frac{z^5}{6} & -\frac{z^3}{3}-\frac{z^4}{2}-\frac{z^5}{6} & -\frac{z^3}{6}+\frac{z^4}{12}+\frac{z^5}{6}+\frac{z^6}{12} & -\frac{z^3}{3}-\frac{z^4}{2}-\frac{z^5}{6} & \frac{z^4}{3}+\frac{z^5}{2}+\frac{z^6}{6} & -\frac{z^3}{3}-\frac{z^4}{2}-\frac{z^5}{6} & -\frac{z^3}{9}+\frac{5z^5}{36}-\frac{z^7}{36} & -\frac{z^3}{6}-\frac{z^4}{12}+\frac{z^5}{6}+\frac{z^6}{12} & \frac{z^4}{3}+\frac{z^5}{2}+\frac{z^6}{6} \\[2mm]
\frac{z}{2}+\frac{z^2}{2} & -\frac{z^2}{2}-\frac{z^3}{2} & -\frac{z^2}{2}-\frac{z^3}{2} & -\frac{z^2}{4}+\frac{z^4}{4} & -\frac{z^2}{2}-\frac{z^3}{2} & \frac{z^3}{2}+\frac{z^4}{2} & -\frac{z^2}{2}-\frac{z^3}{2} & -\frac{z^2}{6}+\frac{z^3}{12}+\frac{z^4}{6}-\frac{z^5}{12} & -\frac{z^2}{4}+\frac{z^4}{4} & \frac{z^3}{2}+\frac{z^4}{2} \\[2mm]
z^2 & -z^3 & -z^3 & -\frac{z^3}{2}+\frac{z^4}{2} & -z^3 & z^4 & -z^3 & -\frac{z^3}{3}+\frac{z^4}{2}-\frac{z^5}{6} & -\frac{z^3}{2}+\frac{z^4}{2} & z^4 \\[2mm]
\frac{z}{3}+\frac{z^2}{2}+\frac{z^3}{6} & -\frac{z^2}{3}-\frac{z^3}{2}-\frac{z^4}{6} & -\frac{z^2}{3}-\frac{z^3}{2}-\frac{z^4}{6} & -\frac{z^2}{6}+\frac{z^3}{12}+\frac{z^4}{6}+\frac{z^5}{12} & -\frac{z^2}{3}-\frac{z^3}{2}-\frac{z^4}{6} & \frac{z^3}{3}+\frac{z^4}{2}+\frac{z^5}{6} & -\frac{z^2}{3}-\frac{z^3}{2}-\frac{z^4}{6} & -\frac{z^2}{9}+\frac{5z^4}{36}-\frac{z^6}{36} & -\frac{z^2}{6}-\frac{z^3}{12}+\frac{z^4}{6}+\frac{z^5}{12} & \frac{z^3}{3}+\frac{z^4}{2}+\frac{z^5}{6} \\[2mm]
\frac{z^2}{2}+\frac{z^3}{2} & -\frac{z^3}{2}-\frac{z^4}{2} & -\frac{z^3}{2}-\frac{z^4}{2} & -\frac{z^3}{4}+\frac{z^5}{4} & -\frac{z^3}{2}-\frac{z^4}{2} & \frac{z^4}{2}+\frac{z^5}{2} & -\frac{z^3}{2}-\frac{z^4}{2} & -\frac{z^3}{6}+\frac{z^4}{12}+\frac{z^5}{6}-\frac{z^6}{12} & -\frac{z^3}{4}+\frac{z^5}{4} & \frac{z^4}{2}+\frac{z^5}{2} \\[2mm]
z & -z^2 & -z^2 & -\frac{z^2}{2}+\frac{z^3}{2} & -z^2 & z^3 & -z^2 & -\frac{z^2}{3}+\frac{z^3}{2}-\frac{z^4}{6} & -\frac{z^2}{2}+\frac{z^3}{2} & z^3 \\[2mm]
z^3 & -z^4 & -z^4 & -\frac{z^4}{2}+\frac{z^5}{2} & -z^4 & z^5 & -z^4 & -\frac{z^4}{3}+\frac{z^5}{2}-\frac{z^6}{6} & -\frac{z^4}{2}+\frac{z^5}{2} & z^5
\end{array}
$$

```
D[Expand[Dz[30, z] Dz[10, -z]], z] /. z → 0
```

$$\frac{85}{12}$$

```
(D[Expand[Dz[30, z]], z] /. z → 0) - (D[Expand[Dz[10, z]], z] /. z → 0)
```

$$\frac{85}{12}$$