```
K[n_] := If[n == 1, 0, FullSimplify[MangoldtLambda[n] / Log[n]]]
P[n_, k_] := P[n, k] = Sum[ K[j] P[Floor[n / j], k - 1], {j, 2, n}]; P[n_, 0] := 1
D2[n_, k_] := D2[n, k] = Sum[D2[Floor[n / j], k - 1], {j, 2, n}]; D2[n_, 0] := 1
DD[n_, z_] := Sum[FactorialPower[z, a] / a! D2[n, a], {a, 0, Log[2, n]}]
P[n_, k_, s_] := P[n, k, s] = Sum[ j^-s K[j] P[Floor[n / j], k - 1, s], {j, 2, n}];
P[n_, 0, s_] := 1
DDa[n_, z_] := Sum[ z^k / k! P[n, k], {k, 0, Log[2, n]}]
DDa[n_, z_, s_] := Sum[ z^k / k! P[n, k, s], {k, 0, Log[2, n]}]
DDa2[n_, z_] := Sum[ z^k / k! P[n, k] / z, {k, 0, Log[2, n]}]
Dhyp[n_, k_, a_, s_] := Dhyp[n, k, a, s] =
   Sum[Binomial[k, j] Dhyp[Floor[n / (m^ (k - j))], j, m + 1], {m, a, n^(1 / k)}, {j, 0, k - 1}]
Dhyp[n_, 1, a_] := Floor[n] - a + 1; Dhyp[n_, 0, a_] := 1
bins[ z_, a_] := Product[ (z - k), {k, 0, a - 1}] / a!
StrictDivisorsHyperbola[A_, k_, n_, s_] :=
 Sum[((m^A) ^ (k - j)) Binomial[k, j] StrictDivisorsHyperbola[A, j, n / (m^ (k - j)), m + 1],
   {m, s, n^(1 / k)}, {j, 0, k - 1}]
StrictDivisorsHyperbola[A_, 1, n_, s_] := Sum[j^A, {j, s, n}]
StrictDivisorsHyperbola[0, 1, n_, s_] := Floor[n] - s + 1
StrictDivisorsHyperbola[1, 1, n_, s_] := Floor[n] (Floor[n] + 1) / 2 - s (s - 1) / 2
StrictDivisorsHyperbola[2, 1, n_, s_] :=
 Floor[n] (Floor[n] + 1) (2 Floor[n] + 1) / 6 - (s - 1) s (2 s - 1) / 6
StrictDivisorsHyperbola[3, 1, n_, s_] := Floor[n]^2 (Floor[n] + 1)^2 / 4 - s^2 (s - 1)^2 / 4
StrictDivisorsHyperbola[A_, 0, n_, s_] := 1
SumPrimesHyperbola[A_, n_] :=
 Sum[(-1) ^ (k + 1) / (j k) MoebiusMu[j] StrictDivisorsHyperbola[j A, k, n^(1 / j), 2],
   {j, 1, Log[2, n]}, {k, 1, Log[2, (n^(1 / j))]}]
DDD[n_, z_, s_] := Expand[Sum[bins[z, a] StrictDivisorsHyperbola[-s, a, n, 2],
    {a, 0, Log[2, n]}]]
lin[n_, s_] := Sum[ (-1) ^ (k + 1) / k StrictDivisorsHyperbola[-s, k, n, 2], {k, 1, Log[2, n]}]
```

(-1 / List @@ NRoots[DDD[1 112 601, x, -2] == 0, x][[All, 2]])

$\{0.0026846, 0.0337774, 0.00339989 - 0.0177528\,\mathbb{i}, 0.00339989 + 0.0177528\,\mathbb{i},$
$0.0129288 - 0.0338084\,\mathbb{i}, 0.0129288 + 0.0338084\,\mathbb{i}, 0.0418972 - 0.0687773\,\mathbb{i},$
$0.0418972 + 0.0687773\,\mathbb{i}, 0.0521515 - 0.0938085\,\mathbb{i}, 0.0521515 + 0.0938085\,\mathbb{i},$
$0.129722 - 0.178989\,\mathbb{i}, 0.129722 + 0.178989\,\mathbb{i}, 0.275047 - 0.19972\,\mathbb{i},$
$0.275047 + 0.19972\,\mathbb{i}, 0.536214 - 0.164698\,\mathbb{i}, 0.536214 + 0.164698\,\mathbb{i}, 1.0008,$
$3.38194 \times 10^{16}, -0.0000911672 - 0.00160732\,\mathbb{i}, -0.0000911672 + 0.00160732\,\mathbb{i}\}$

N[lin[1 112 601, -2]]

3.38194×10^{16}

```
NumberForm[3.381939745558961`*^16, ExponentFunction → (If[-20 < # < 20, Null, #] &)]
NumberForm[3.3819397455589616`*^16, ExponentFunction → (If[-20 < # < 20, Null, #] &)]
```
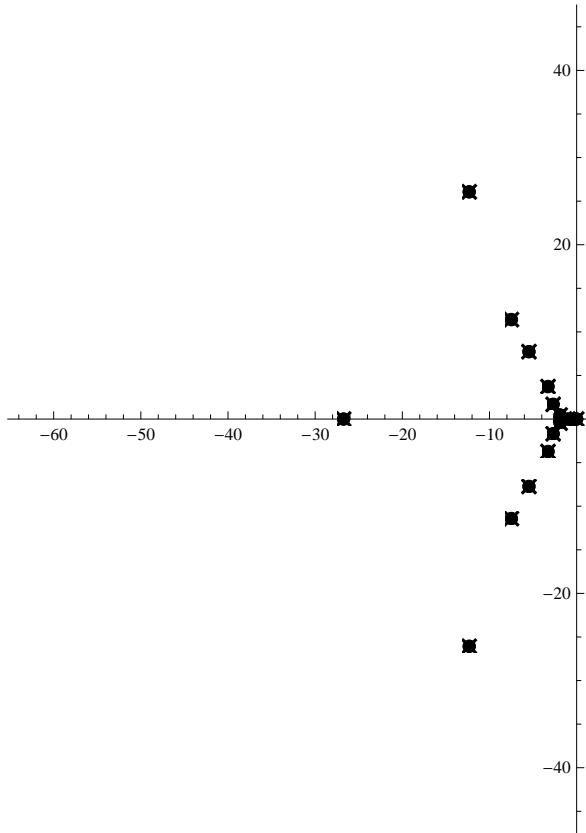
NumberForm::sigz : In addition to the number of digits requested, one or more zeros will appear as placeholders. ≫

33819397455589600.
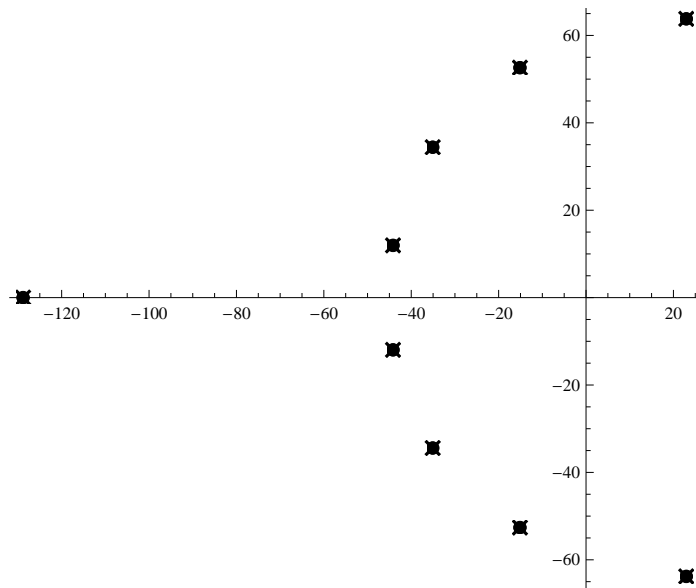
NumberForm::sigz : In addition to the number of digits requested, one or more zeros will appear as placeholders. ≫

33819397455589620.

```
RootLocusPlot[1 / Expand[DDD[1 112 600, x, 0]], {k, 0, 1}, FeedbackType → None]
```

RootLocusPlot[1 / Expand[DDD[1000, x, 4]], {k, 0, 1}, FeedbackType → None]

N[DDD[1000, z, 4]]

$1. + 0.0791099\, z + 0.00312919\, z^2 + 0.0000825164\, z^3 + 1.63192 \times 10^{-6}\, z^4 + 2.58075 \times 10^{-8}\, z^5 +$
$3.3842 \times 10^{-10}\, z^6 + 3.65863 \times 10^{-12}\, z^7 + 3.02665 \times 10^{-14}\, z^8 + 1.11392 \times 10^{-16}\, z^9$

(List @@ NRoots[DDD[1000, x, 4] == 0, x][[All, 2]])

$\{-128.827, -44.1698 - 11.9556\, \dot{\imath}, -44.1698 + 11.9556\, \dot{\imath},$
$-35.0862 - 34.4187\, \dot{\imath}, -35.0862 + 34.4187\, \dot{\imath}, -15.125 - 52.6184\, \dot{\imath},$
$-15.125 + 52.6184\, \dot{\imath}, 22.9387 - 63.7958\, \dot{\imath}, 22.9387 + 63.7958\, \dot{\imath}\}$

(N[D[DDD[100, z, s], s]] /. s → 0)

$-94.0453\, z - 169.15\, z^2 - 81.6195\, z^3 - 17.6846\, z^4 - 1.19616\, z^5 - 0.0438125\, z^6$

(List @@ NRoots[(N[D[DDD[100, z, s], s]] /. s → 0) == 0, z][[All, 2]])

$\{-10.6971 - 12.1994\, \dot{\imath}, -10.6971 + 12.1994\, \dot{\imath},$
$-2.53645 - 1.82621\, \dot{\imath}, -2.53645 + 1.82621\, \dot{\imath}, -0.834706, 0.\}$

vv := {-10.697109591508221` - 12.19935485425952` ḋ,
-10.697109591508221` + 12.19935485425952` ḋ, -2.5364503360079835` - 1.8262103341135916` ḋ,
-2.5364503360079835` + 1.8262103341135916` ḋ, -0.8347063543903029`}