$$\nabla[(\frac{1}{1-x_1})^z]_n = \frac{z^{(n)}}{n!}$$

$$[(\frac{1}{1-x_1})^z]_n = \frac{(z+1)^{(n)}}{n!}$$

$$\sum_{k=0}^{n} \frac{z^{(k)}}{k!} = \frac{(z+1)^{(n)}}{n!}$$

$$\sum_{k=0}^{n} [(\frac{1}{1-x})^z]_k = [(\frac{1}{1-x})^{z+1}]_n$$

$$\sum_{k=0}^{n} \nabla[(\frac{1}{1-x})^z]_k = \nabla[(\frac{1}{1-x})^{z+1}]_n = [(\frac{1}{1-x})^z]_n$$

$$\nabla[(\frac{1}{1-x})^z]_n = [(\frac{1}{1-x})^{z-1}]_n$$

$$\nabla[\log(\frac{1}{1-x})]_k = \frac{1}{k}$$

$$[\log(\frac{1}{1-x})]_n = H_n$$

$$\nabla[(\frac{1}{1-x})^{a+b}]_n = \sum_{j+k=n} \nabla[(\frac{1}{1-x})^a]_j \cdot \nabla[(\frac{1}{1-x})^b]_k$$

$$\nabla[(\frac{1}{1-x})^{a+b+1}]_n = \sum_{j+k\le n} \nabla[(\frac{1}{1-x})^a]_j \cdot \nabla[(\frac{1}{1-x})^b]_k$$

$$\ldots$$

$$[(\frac{1}{1-x})^{a+b}]_n = \sum_{j+k\le n} \nabla[(\frac{1}{1-x})^a]_j \cdot \nabla[(\frac{1}{1-x})^b]_k$$

$$[(\frac{1}{1-x})^{a+b+1}]_n = \sum_{j+k=n} [(\frac{1}{1-x})^a]_j \cdot [(\frac{1}{1-x})^b]_k$$

HUH. Really need to think more about this.

$$\nabla[(1+x_1)^z]_n = (-1)^n \cdot \frac{(-z)^{(n)}}{n!} = (-1)^n \cdot \nabla[(\frac{1}{1-x_1})^{-z}]_n = \binom{z}{n}$$

$$[(1+x_1)^z]_k = \sum_{k=0}^{n} \nabla[(1+x_1)^z]_k = \sum_{k=0}^{n} (-1)^k \nabla[(\frac{1}{1-x_1})^{-z}]_k = \nabla[(\frac{1}{1-x_1})^{-z+1}]_n - 2\sum_{k=0}^{\frac{n}{2}} \nabla[(\frac{1}{1-x_1})^{-z}]_{2k} \text{ …. MEH.}$$

$$\nabla[\log(1+x_1)]_k = \frac{(-1)^{k+1}}{k}$$

$$t_{\frac{a}{b}}(n)=b(\lfloor\frac{n}{b}\rfloor-\lfloor\frac{n-1}{b}\rfloor)-a(\lfloor\frac{n}{a}\rfloor-\lfloor\frac{n-1}{a}\rfloor)$$

$$t_{\frac{a}{b}}(n)=-t_{\frac{b}{a}}(n)$$

$$t_{\frac{a}{a}}(n)=t_1(n)=0$$

$$\lim_{x\to\infty} t_x(n)=1$$

$$\lim_{x\to\infty} t_{x^{-1}}(n)=-1$$

$$t_{-a}(n)=???$$

$$\nabla[\log\frac{1-x_1^m}{1-x_1}]_k=\frac{t_m(k)}{k}$$

$$\nabla[(\frac{1-x_1^m}{1-x_1})^z]_k=\nabla[(\frac{1-x_1}{1-x_1^m})^{-z}]_k$$

$$\nabla[\boldsymbol{m}^{\boldsymbol{a}+\boldsymbol{b}}]_n=\sum_{j+k=n}\nabla[\boldsymbol{m}^{\boldsymbol{a}}]_j\cdot\nabla[\boldsymbol{m}^{\boldsymbol{b}}]_k$$

$$[\boldsymbol{m}^{\boldsymbol{a}+\boldsymbol{b}}]_n=\sum_{j+k\le n}\nabla[\boldsymbol{m}^{\boldsymbol{a}}]_j\cdot\nabla[\boldsymbol{m}^{\boldsymbol{b}}]_k$$

...

$$[(\log\boldsymbol{m})^k]_n=\sum_{j\le n}\nabla[\log\boldsymbol{m}]_j\cdot[(\log\boldsymbol{m})^{k-1}]_{n-j}$$

$$[(\log\boldsymbol{m})^a]_n=\sum_{j+k\le n}\nabla[\log\boldsymbol{m}]_j\cdot\nabla[(\log\boldsymbol{m})^{a-1}]_k$$

$$[(\log\boldsymbol{m})^{a+b}]_n=\sum_{j+k\le n}\nabla[(\log\boldsymbol{m})^a]_j\cdot\nabla[(\log\boldsymbol{m})^b]_k$$

$$[(\log\boldsymbol{m})^k]_n=\sum_{j\le n}t_m(j)\cdot[(\log\boldsymbol{m})^{k-1}]_{n-j}$$

$$\nabla[\boldsymbol{m}^z]_n = \sum_{j=0}^{m-1} \nabla[\boldsymbol{m}^{z-1}]_{n-j}$$

$$\nabla[\boldsymbol{m}^z]_n = \nabla[\boldsymbol{m}^z]_{(m-1)z-n}$$

$$[\boldsymbol{m}^z]_n = \sum_{j=0}^{m-1} [\boldsymbol{m}^{z-1}]_{n-j}$$

$$[\boldsymbol{m}^z]_n = [\boldsymbol{m}^z]_{(m-1)z} - [\boldsymbol{m}^z]_{(m-1)z-n-1}$$

$$\sum_{j=0}^{(m-1)k} \nabla[\boldsymbol{m}^k]_j = m^k$$

$$\sum_{j=0} \nabla[\boldsymbol{m}^z]_j = m^z \ for \ \Re(z) > 0$$

$$\sum_{j=0}^{(m-1)k} t_m(j) \cdot [\boldsymbol{m}^k]_j = 0$$

$$\sum_{j=0} t_m(j) \cdot [\boldsymbol{m}^z]_j = 0$$

$$f(n)=(g(n)/g(n/2))$$

$$f(n)=\frac{g(n)}{g(n)/2}$$

$$lf(n)=lg(n)-lg(\frac{n}{2})$$

$$lg=\sum_{k=0} lf(\frac{n}{2^k})$$

NOW EXPRESS $[\infty^z]_n$ IN TERMS OF $[2^z]_n$ WITH THIS!!!

$$[\mathbf{2}^z]_n=[(\frac{\boldsymbol{\infty}}{\boldsymbol{\infty}_{\mathbf{2}}})^z]_n$$

$$[3^z]_n=[(\frac{\boldsymbol{\infty}}{\boldsymbol{\infty}_{\mathbf{3}}})^z]_n$$

$$[\boldsymbol{\infty}^z]_n=[\prod_{k=0}\mathbf{2}_{\mathbf{2}^k}{}^z]_n$$

$$\nabla[\boldsymbol{\infty}^z]_n=\sum_{a+2b+4c+8d+...=n}\nabla[\mathbf{2}^z]_a\cdot\nabla[\mathbf{2}^z]_b\cdot\nabla[\mathbf{2}^z]_c\cdots$$

$$\nabla[\boldsymbol{\infty}^z]_n=\sum_{a+3b+9c+27d+...=n}\nabla[3^z]_a\cdot\nabla[3^z]_b\cdot\nabla[3^z]_c\cdots$$

$$\nabla[\mathbf{2}^z]_n=\sum_{a+2b=n}\nabla[\boldsymbol{\infty}^z]_a\cdot\nabla[\boldsymbol{\infty}^{-z}]_b$$

$$\nabla[3^z]_n=\sum_{a+3b=n}\nabla[\boldsymbol{\infty}^z]_a\cdot\nabla[\boldsymbol{\infty}^{-z}]_b$$

$$\nabla[3^z]_n=\sum_{a+3b=n}\nabla[\boldsymbol{\infty}^z]_a\cdot\nabla[\boldsymbol{\infty}^{-z}]_b$$

$$[\log\boldsymbol{\infty}]_n=\sum_{k=0}[\log\mathbf{2}_{\mathbf{2}^k}]_n$$

$$[\log\boldsymbol{\infty}]_n=\sum_{k=0}[\log 3]_{\frac{n}{3^k}}$$

$$[\log 1]_n=0=[\log\boldsymbol{\infty}]_n-[\log\boldsymbol{\infty}]_n$$

$$[\log\mathbf{2}]_n=[\log\boldsymbol{\infty}]_n-[\log\boldsymbol{\infty}_{\mathbf{2}}]_n$$

$$[\log 3]_n=[\log\boldsymbol{\infty}]_n-[\log\boldsymbol{\infty}_{\mathbf{3}}]_n$$

$$[\log 3]_n=\sum_{k=0}[\log\mathbf{2}]_{\frac{n}{2^k}}-\sum_{k=0}[\log\mathbf{2}]_{\frac{n}{3\cdot 2^k}}$$

$$[\log b]_n=\sum_{k=0}[\log a-\log a_{1/b}]_{\frac{n}{a^k}}$$

$$\log b = \lim_{n\to\infty} \sum_{k=0} [\log a]_{\frac{n}{a^k}} - \sum_{k=0} [\log a]_{\frac{n}{b\cdot a^k}} \qquad \text{OR} \qquad \log b = \lim_{n\to\infty} \sum_{k=0} \sum_{j=\lfloor \frac{n}{b\cdot a^k}\rfloor+1}^{\lfloor \frac{n}{a^k}\rfloor} \nabla[\log a]_j$$

$$\log b = \lim_{n\to\infty} \sum_{k=0} [\log a]_{\frac{n}{a^k}} - [\log a]_{\frac{n}{b\cdot a^k}} \qquad \log b = \lim_{n\to\infty} \sum_{k=0} [\log a - \log a_{1/b}]_{\frac{n}{a^k}}$$

$$[\log \mathbf{4}]_n = [\log \mathbf{2}]_n + [\log \mathbf{2}_{\mathbf{2}}]_n$$

$$[\log \mathbf{8}]_n = [\log \mathbf{2}]_n + [\log \mathbf{2}]_{\frac{n}{\mathbf{2}}} + [\log \mathbf{2}]_{\frac{n}{4}}$$

$$[\log \mathbf{9}]_n = [\log \mathbf{3}]_n + [\log \mathbf{3}]_{\frac{n}{3}}$$

$$\nabla[\mathbf{4}^z]_n = \sum_{a+2b=n} \nabla[\mathbf{2}^z]_a \cdot \nabla[\mathbf{2}^z]_b$$

$$\nabla[\mathbf{8}^z]_n = \sum_{a+2b+4c=n} \nabla[\mathbf{2}^z]_a \cdot \nabla[\mathbf{2}^z]_b \dot{\nabla}[\mathbf{2}^z]_c$$

$$\nabla[\mathbf{9}^z]_n = \sum_{a+3b=n} \nabla[\mathbf{3}^z]_a \cdot \nabla[\mathbf{3}^z]_b$$

$$[\log \mathbf{2}]_n = \sum_{k=0} (-1)^k [\log \mathbf{4}]_{\frac{n}{2^k}}$$

$$[\log \mathbf{2}]_n = [\prod_{k=0} \mathbf{4}^{(-\mathbf{1})^k\cdot z} / (2^k)]_n$$

$$[\mathbf{2}^z]_n = \sum_{a+2b+4c+8d+\ldots\le n} \nabla[\mathbf{4}^z]_a \cdot \nabla[\mathbf{4}^{-z}]_b \cdot \nabla[\mathbf{4}^z]_c \cdot \nabla[\mathbf{4}^{-z}]_d \cdot \ldots$$

$$\nabla[(\boldsymbol{m}^{\mathbf{2}})^z]_n = \sum_{a+m\cdot b=n} \nabla[\boldsymbol{m}^z]_a \cdot \nabla[\boldsymbol{m}^z]_b$$

$$[(\boldsymbol{m}^{\mathbf{2}})^z]_n = \sum_{a+m\cdot b\le n} \nabla[\boldsymbol{m}^z]_a \cdot \nabla[\boldsymbol{m}^z]_b$$

(is this fine for m as a non-integer?)

...

$$\nabla[\boldsymbol{m}^z]_n = \sum_{a+m\cdot b=n} \nabla[\boldsymbol{\infty}^z]_a \cdot \nabla[\boldsymbol{\infty}^{-z}]_b$$

$$[\boldsymbol{m}^z]_n = \sum_{k=0}^{\lfloor \frac{n}{m} \rfloor} \nabla[\boldsymbol{\infty}^{z+1}]_{n-m\cdot k} \cdot \nabla[\boldsymbol{\infty}^{-z}]_k$$

$$\sum_{m=1}^{t} [\boldsymbol{m}^z]_n = \sum_{m=1}^{t} \sum_{k=0}^{\lfloor \frac{n}{m} \rfloor} \nabla[\boldsymbol{\infty}^{z+1}]_{n-m\cdot k} \cdot \nabla[\boldsymbol{\infty}^{-z}]_k$$

$$a\nabla[\log\boldsymbol{\infty}]_n \rightarrow ???$$

$$\cdots$$

$$[m^z]_n = \sum_{k=0}^{\frac{n}{m}} [\boldsymbol{\infty}^z]_{n-m\cdot k} \cdot [\boldsymbol{\infty}^{-z-\mathbf{1}}]_k$$

$$\cdots$$

$$[\mathbf{2}^z]_n = \sum_{k=0}^{n} \nabla[\mathbf{2}^z]_k$$

$$[\mathbf{2}^{z-\mathbf{1}}]_n = \sum_{k=0}^{n} \nabla[\mathbf{2}^{z-\mathbf{1}}]_k$$

$$[\mathbf{2}^{z+\mathbf{1}}]_n = \sum_{k=0}^{\frac{n}{2}} [\boldsymbol{\infty}^{z+\mathbf{1}}]_{n-2\cdot k} \cdot [\boldsymbol{\infty}^{-z-\mathbf{2}}]_k$$

$$[\mathbf{2}^z]_n = \sum_{k=0}^{\frac{n}{2}} [\boldsymbol{\infty}^z]_{n-2\cdot k} \cdot [\boldsymbol{\infty}^{-z-\mathbf{1}}]_k$$

$$[\mathbf{2}^{z-\mathbf{1}}]_n = \sum_{k=0}^{\frac{n}{2}} [\boldsymbol{\infty}^{z-\mathbf{1}}]_{n-2\cdot k} \cdot [\boldsymbol{\infty}^{-z}]_k$$

$$\nabla[\mathbf{2}^{z-\mathbf{1}}]_n = \sum_{k=0}^{\frac{n}{2}} [\boldsymbol{\infty}^{z-\mathbf{2}}]_{n-2\cdot k} \cdot [\boldsymbol{\infty}^{-z}]_k$$

$$\nabla[\mathbf{2}^z]_n = \sum_{k=0}^{\frac{n}{2}} [\boldsymbol{\infty}^{z-\mathbf{1}}]_{n-2\cdot k} \cdot [\boldsymbol{\infty}^{-z-\mathbf{1}}]_k$$

$$\nabla[\mathbf{2}^{z+\mathbf{1}}]_n = \sum_{k=0}^{\frac{n}{2}} [\boldsymbol{\infty}^z]_{n-2\cdot k} \cdot [\boldsymbol{\infty}^{-z-\mathbf{2}}]_k$$

$$[\log \Sigma\, \boldsymbol{p(n)}]_n = \sum_{k=1}^{n} [\log \boldsymbol{\infty}_{\boldsymbol{1/k}}]_n$$

$$[\log \boldsymbol{\infty}]_n = \sum_{k=1}^{n} \mu(k) [\log \Sigma\, \boldsymbol{p(n)}]_{\frac{n}{k}}$$

$$[\Sigma\, \boldsymbol{p(n)}^z]_n = [\prod_{k=1} \boldsymbol{\infty}_{\boldsymbol{1/k}}^z]_n$$

$$p(n) = \nabla [\prod_{k=1} \boldsymbol{\infty}_{\boldsymbol{1/k}}]_n$$

$$a(n) = b(n) + b(\frac{n}{2}) + b(\frac{n}{3}) + b(\frac{n}{4}) \ldots$$

$$a(n) - a(\frac{n}{2}) = b(n) + b(\frac{n}{3}) + b(\frac{n}{5}) + b(\frac{n}{7}) \ldots$$

$$a(n) - a(\frac{n}{2}) - a(\frac{n}{3}) = b(n) + b(\frac{n}{5}) - b(\frac{n}{6}) + b(\frac{n}{7}) \ldots$$

Investigate relationship between additive and multiplicative identities. For example,

$$[\log \Sigma \boldsymbol{p}(\boldsymbol{n})]_n = \sum_{k=1}^{n} [\log \boldsymbol{\infty}_{\mathbf{1/k}}]_n$$

vs

$$[\log \Sigma \boldsymbol{a}(\boldsymbol{n})]_n = \sum_{k=1}^{n} [\log \boldsymbol{\zeta}_{\mathbf{1/k}}(\mathbf{0})]_n$$

ALSO. Is there an additive equivalence to the s parameter in $\zeta(s)$? Probably a multiplication by s instead, with s=-1 disappearing the way s=0 does in the multiplicative case? And with s=0 being a weird nullity the way s=1 is in the multiplicative case?

YEP

...

$$\nabla[\log \boldsymbol{\infty}(\boldsymbol{s})]_n = \frac{1-(1+s)^n}{n}$$

$$\nabla[\boldsymbol{\infty}(\mathbf{-1})]_n = 1$$

$$[\boldsymbol{\infty}(\mathbf{-1})^z]_n = \frac{z^{(n)}}{n!}$$

$$\sum_{j=0}^{n} \nabla[\boldsymbol{\infty}(\mathbf{-1})^z]_j = [\boldsymbol{\infty}(\mathbf{-1})^{z+1}]$$

...

$$\nabla[\boldsymbol{\infty}(\boldsymbol{s})]_n = -s$$

$$[\boldsymbol{\infty}(\boldsymbol{s})]_n = -s \cdot n$$

$$\nabla[\log \boldsymbol{\infty}(\boldsymbol{s})]_n = \frac{1-(1+s)^n}{n}$$

$$[\log \boldsymbol{\infty}(\boldsymbol{s})]_n = H_n + (1+s)^{n+1} \cdot \Phi(1+s, 1, 1+n) + \log(-s)$$

Lerch transcendental here.

$$[\boldsymbol{\infty}(\boldsymbol{s})^z]_n = \sum_{k=0}^{n} \binom{z}{k} [(\boldsymbol{\infty}(\boldsymbol{s})-1)^k]_n$$

...

$$[(\boldsymbol{\infty}(\boldsymbol{s})-1)]_n = \sum_{j=1}^{n} -s$$

$$[(\boldsymbol{\infty}(\boldsymbol{s})-1)^2]_n = \sum_{j+k=n;\, j,k>0} (-s)^2$$

$$[(\boldsymbol{\infty}(\boldsymbol{s})-1)^2]_n = \sum_{j+k=n;\, j,k>0} (-s)^2$$

...

$$\nabla[\boldsymbol{\infty}(\boldsymbol{s})-1]_n = -s$$

$$\nabla[(\boldsymbol{\infty}(\boldsymbol{s})-1)^k]_n = (-s)^k \cdot \frac{(n-k+1)^{(k-1)}}{(k-1)!}$$

$$\nabla[\boldsymbol{\infty}(\boldsymbol{s})^z]_n = -s z \,{}_2F_1(1-n, 1-z, 2, -s)$$

Good.

$$[(\boldsymbol{\infty}(\boldsymbol{s})-1)^k]_n = s^k \cdot \frac{(-n)^{(k)}}{k!}$$

$$\frac{(z+1)^{(n)}}{n!} = \sum_{k=0} \binom{z}{k} (-1)^k \cdot \frac{(-n)^{(k)}}{k!}$$

$$[\boldsymbol{\infty}(\boldsymbol{s})^z]_n = \sum_{k=0} \binom{z}{k} \cdot s^k \cdot \frac{(-n)^{(k)}}{k!}$$

$$[\boldsymbol{\infty}(\boldsymbol{s})^z]_n = {}_2F_1(-n, -z, 1, -s)$$

(Remember, $[\boldsymbol{\infty}^z]_n = \frac{(z+1)^{(n)}}{n!}$)

$$[\boldsymbol{\infty}(\boldsymbol{s})^z]_n = [\boldsymbol{\infty}(\boldsymbol{s})^{\boldsymbol{n}}]_z$$

...

What happens for $[\mathbf{2}(\boldsymbol{s})^z]_n$? For $[\boldsymbol{m}(\boldsymbol{s})^z]_n$?