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ClearAll["Global`*"]
$RecursionLimit = 10 000 000
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10 000 000
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K[n_] := K[n] = FullSimplify[MangoldtLambda[n] / Log[n]]; K[1] := 0;
K[n_, k_] := Sum[K[j, k - 1] K[n / j, 1], {j, Divisors[n]}];
K[n_, 1] := K[n]; K[n_, 0] := 0; K[1, 0] := 1
Kl[n_, z_] := Kl[n, z] = Sum[FactorialPower[z, a] / a! K[n, a], {a, 0, Log[2, n]}];
Kl[1, z_] := 1
P[n_, k_] := P[n, k] = P[n - 1, k] + K[n, k]; P[1, k_] := 0; P[n_, 0] := 1
Pl[n_, z_] := Pl[n, z] = Sum[FactorialPower[z, a] / a! P[n, a], {a, 0, Log[2, n]}];
Pl[1, z_] := 1

LL[n_] := LL[n] = Sum[(-1)^(k + 1) / k K[n, k], {k, 1, Log[2, n]}]; LL[1] := 0
LL[n_, k_] := LL[n, k] = Sum[LL[j, k - 1] LL[n / j, 1], {j, Divisors[n]}]; LL[n_, 1] := LL[n]
LL[n_, 0] := 0; LL[1, 0] := 1
PL[n_, k_] := PL[n, k] = PL[n - 1, k] + LL[n, k]; PL[1, k_] := 0; PL[n_, 0] := 1
LLl[n_, z_] := LLl[n, z] = Sum[FactorialPower[z, a] / a! LL[n, a], {a, 0, Log[2, n]}];
LLl[1, z_] := 1
PLl[n_, z_] := PLl[n, z] = Sum[FactorialPower[z, a] / a! PL[n, a], {a, 0, Log[2, n]}];
PLl[1, z_] := 1

LLL[n_] := LLL[n] = Sum[(-1)^(k + 1) / k LL[n, k], {k, 1, Log[2, n]}]; LLL[1] := 0
LLL[n_, k_] := LLL[n, k] = Sum[LLL[j, k - 1] LLL[n / j, 1], {j, Divisors[n]}];
LLL[n_, 1] := LLL[n]
LLL[n_, 0] := 0; LLL[1, 0] := 1
PLL[n_, k_] := PLL[n, k] = PLL[n - 1, k] + LLL[n, k]; PLL[1, k_] := 0; PLL[n_, 0] := 1
LLLl[n_, z_] := LLLl[n, z] = Sum[FactorialPower[z, a] / a! LLL[n, a], {a, 0, Log[2, n]}];
PLLl[n_, z_] := PLLl[n, z] = Sum[FactorialPower[z, a] / a! PLL[n, a], {a, 0, Log[2, n]}];
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PLS[n_, a_] := Sum[a^k / (k!) PL[n, k], {k, 0, Log[2, n]}]
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Table[{n, P[n, 1], Pl[n, 1], Pl[n, 0], Pl[n, -1]}, {n, 1, 100}] // TableForm
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1	0	1	1	1
2	1	2	1	0
3	2	3	1	-1
4	$\frac{5}{2}$	$\frac{7}{2}$	1	$-\frac{1}{2}$
5	$\frac{7}{2}$	$\frac{9}{2}$	1	$-\frac{3}{2}$
6	$\frac{7}{2}$	$\frac{9}{2}$	1	$\frac{1}{2}$
7	$\frac{9}{2}$	$\frac{11}{2}$	1	$-\frac{1}{2}$
8	$\frac{29}{6}$	$\frac{35}{6}$	1	$-\frac{5}{6}$
9	$\frac{16}{3}$	$\frac{19}{3}$	1	$-\frac{1}{3}$
10	$\frac{16}{3}$	$\frac{19}{3}$	1	$\frac{5}{3}$
11	$\frac{19}{3}$	$\frac{22}{3}$	1	$\frac{2}{3}$
12	$\frac{19}{3}$	$\frac{22}{3}$	1	$-\frac{4}{3}$
13	$\frac{22}{3}$	$\frac{25}{3}$	1	$-\frac{7}{3}$

14	$\frac{22}{3}$	$\frac{25}{3}$	1	$-\frac{1}{3}$
15	$\frac{22}{3}$	$\frac{25}{3}$	1	$\frac{5}{3}$
16	$\frac{91}{12}$	$\frac{103}{12}$	1	$\frac{11}{6}$
17	$\frac{103}{12}$	$\frac{115}{12}$	1	$\frac{5}{6}$
18	$\frac{103}{12}$	$\frac{115}{12}$	1	$-\frac{7}{6}$
19	$\frac{115}{12}$	$\frac{127}{12}$	1	$-\frac{13}{6}$
20	$\frac{115}{12}$	$\frac{127}{12}$	1	$-\frac{25}{6}$
21	$\frac{115}{12}$	$\frac{127}{12}$	1	$-\frac{13}{6}$
22	$\frac{115}{12}$	$\frac{127}{12}$	1	$-\frac{1}{6}$
23	$\frac{127}{12}$	$\frac{139}{12}$	1	$-\frac{7}{6}$
24	$\frac{127}{12}$	$\frac{139}{12}$	1	$\frac{1}{2}$
25	$\frac{133}{12}$	$\frac{145}{12}$	1	1
26	$\frac{133}{12}$	$\frac{145}{12}$	1	3
27	$\frac{137}{12}$	$\frac{149}{12}$	1	$\frac{8}{3}$
28	$\frac{137}{12}$	$\frac{149}{12}$	1	$\frac{2}{3}$
29	$\frac{149}{12}$	$\frac{161}{12}$	1	$-\frac{1}{3}$
30	$\frac{149}{12}$	$\frac{161}{12}$	1	$-\frac{19}{3}$
31	$\frac{161}{12}$	$\frac{173}{12}$	1	$-\frac{22}{3}$
32	$\frac{817}{60}$	$\frac{877}{60}$	1	$-\frac{149}{20}$
33	$\frac{817}{60}$	$\frac{877}{60}$	1	$-\frac{109}{20}$
34	$\frac{817}{60}$	$\frac{877}{60}$	1	$-\frac{69}{20}$
35	$\frac{817}{60}$	$\frac{877}{60}$	1	$-\frac{29}{20}$
36	$\frac{817}{60}$	$\frac{877}{60}$	1	$\frac{41}{20}$
37	$\frac{877}{60}$	$\frac{937}{60}$	1	$\frac{21}{20}$
38	$\frac{877}{60}$	$\frac{937}{60}$	1	$\frac{61}{20}$
39	$\frac{877}{60}$	$\frac{937}{60}$	1	$\frac{101}{20}$
40	$\frac{877}{60}$	$\frac{937}{60}$	1	$\frac{403}{60}$
41	$\frac{937}{60}$	$\frac{997}{60}$	1	$\frac{343}{60}$
42	$\frac{937}{60}$	$\frac{997}{60}$	1	$-\frac{17}{60}$
43	$\frac{997}{60}$	$\frac{1057}{60}$	1	$-\frac{77}{60}$
44	$\frac{997}{60}$	$\frac{1057}{60}$	1	$-\frac{197}{60}$
45	$\frac{997}{60}$	$\frac{1057}{60}$	1	$-\frac{317}{60}$
46	$\frac{997}{60}$	$\frac{1057}{60}$	1	$-\frac{197}{60}$
47	$\frac{1057}{60}$	$\frac{1117}{60}$	1	$-\frac{257}{60}$
48	$\frac{1057}{60}$	$\frac{1117}{60}$	1	$-\frac{83}{15}$
49	$\frac{1087}{60}$	$\frac{1147}{60}$	1	$-\frac{151}{30}$
50	$\frac{1087}{60}$	$\frac{1147}{60}$	1	$-\frac{211}{30}$
51	$\frac{1087}{60}$	$\frac{1147}{60}$	1	$-\frac{151}{30}$
52	$\frac{1087}{60}$	$\frac{1147}{60}$	1	$-\frac{211}{30}$

53	$\frac{1147}{60}$	$\frac{1207}{60}$	1	$-\frac{241}{30}$
54	$\frac{1147}{60}$	$\frac{1207}{60}$	1	$-\frac{191}{30}$
55	$\frac{1147}{60}$	$\frac{1207}{60}$	1	$-\frac{131}{30}$
56	$\frac{1147}{60}$	$\frac{1207}{60}$	1	$-\frac{27}{10}$
57	$\frac{1147}{60}$	$\frac{1207}{60}$	1	$-\frac{7}{10}$
58	$\frac{1147}{60}$	$\frac{1207}{60}$	1	$\frac{13}{10}$
59	$\frac{1207}{60}$	$\frac{1267}{60}$	1	$\frac{3}{10}$
60	$\frac{1207}{60}$	$\frac{1267}{60}$	1	$\frac{93}{10}$
61	$\frac{1267}{60}$	$\frac{1327}{60}$	1	$\frac{83}{10}$
62	$\frac{1267}{60}$	$\frac{1327}{60}$	1	$\frac{103}{10}$
63	$\frac{1267}{60}$	$\frac{1327}{60}$	1	$\frac{83}{10}$
64	$\frac{1277}{60}$	$\frac{1337}{60}$	1	$\frac{3007}{360}$
65	$\frac{1277}{60}$	$\frac{1337}{60}$	1	$\frac{3727}{360}$
66	$\frac{1277}{60}$	$\frac{1337}{60}$	1	$\frac{1567}{360}$
67	$\frac{1337}{60}$	$\frac{1397}{60}$	1	$\frac{1207}{360}$
68	$\frac{1337}{60}$	$\frac{1397}{60}$	1	$\frac{487}{360}$
69	$\frac{1337}{60}$	$\frac{1397}{60}$	1	$\frac{1207}{360}$
70	$\frac{1337}{60}$	$\frac{1397}{60}$	1	$-\frac{953}{360}$
71	$\frac{1397}{60}$	$\frac{1457}{60}$	1	$-\frac{1313}{360}$
72	$\frac{1397}{60}$	$\frac{1457}{60}$	1	$-\frac{2813}{360}$
73	$\frac{1457}{60}$	$\frac{1517}{60}$	1	$-\frac{3173}{360}$
74	$\frac{1457}{60}$	$\frac{1517}{60}$	1	$-\frac{2453}{360}$
75	$\frac{1457}{60}$	$\frac{1517}{60}$	1	$-\frac{3173}{360}$
76	$\frac{1457}{60}$	$\frac{1517}{60}$	1	$-\frac{3893}{360}$
77	$\frac{1457}{60}$	$\frac{1517}{60}$	1	$-\frac{3173}{360}$
78	$\frac{1457}{60}$	$\frac{1517}{60}$	1	$-\frac{5333}{360}$
79	$\frac{1517}{60}$	$\frac{1577}{60}$	1	$-\frac{5693}{360}$
80	$\frac{1517}{60}$	$\frac{1577}{60}$	1	$-\frac{6143}{360}$
81	$\frac{383}{15}$	$\frac{398}{15}$	1	$-\frac{6083}{360}$
82	$\frac{383}{15}$	$\frac{398}{15}$	1	$-\frac{5363}{360}$
83	$\frac{398}{15}$	$\frac{413}{15}$	1	$-\frac{5723}{360}$
84	$\frac{398}{15}$	$\frac{413}{15}$	1	$-\frac{2483}{360}$
85	$\frac{398}{15}$	$\frac{413}{15}$	1	$-\frac{1763}{360}$
86	$\frac{398}{15}$	$\frac{413}{15}$	1	$-\frac{1043}{360}$
87	$\frac{398}{15}$	$\frac{413}{15}$	1	$-\frac{323}{360}$
88	$\frac{398}{15}$	$\frac{413}{15}$	1	$\frac{277}{360}$
89	$\frac{413}{15}$	$\frac{428}{15}$	1	$-\frac{83}{360}$
90	$\frac{413}{15}$	$\frac{428}{15}$	1	$\frac{3157}{360}$
91	$\frac{413}{15}$	$\frac{428}{15}$	1	$\frac{3877}{360}$

92	$\frac{413}{15}$	$\frac{428}{15}$	1	$\frac{3157}{360}$
93	$\frac{413}{15}$	$\frac{428}{15}$	1	$\frac{3877}{360}$
94	$\frac{413}{15}$	$\frac{428}{15}$	1	$\frac{4597}{360}$
95	$\frac{413}{15}$	$\frac{428}{15}$	1	$\frac{5317}{360}$
96	$\frac{413}{15}$	$\frac{428}{15}$	1	$\frac{5641}{360}$
97	$\frac{428}{15}$	$\frac{443}{15}$	1	$\frac{5281}{360}$
98	$\frac{428}{15}$	$\frac{443}{15}$	1	$\frac{4561}{360}$
99	$\frac{428}{15}$	$\frac{443}{15}$	1	$\frac{3841}{360}$
100	$\frac{428}{15}$	$\frac{443}{15}$	1	$\frac{5101}{360}$

`Sum[ P1[Floor[50 / j], 2], {j, 1, 50}]`

$$\frac{6723}{20}$$

`P[50, 2] + 2 P[50, 1] + P[50, 0] +`  
`Sum[ P[50, k + 2] / (k!), {k, 1, Log[2, 50]}] +`  
`Sum[ 2 P[50, k + 1] / (k!), {k, 1, Log[2, 50]}] +`  
`Sum[ P[50, k] / (k!), {k, 1, Log[2, 50]}]`

$$\frac{6723}{20}$$

`Sum[ P[50, k + 2] / (k!), {k, 0, Log[2, 50]}] +`  
`Sum[ 2 P[50, k + 1] / (k!), {k, 0, Log[2, 50]}] +`  
`Sum[ P[50, k] / (k!), {k, 0, Log[2, 50]}]`

$$\frac{6723}{20}$$

`Sum[ P1[50, k] / (k!), {k, 0, Log[2, 50]}]`

$$\frac{63\,373}{480}$$

`P[50, 2] + 2 P[50, 1] + P[50, 0]`

$$\frac{4819}{60}$$

`P1[50, 2]`

$$\frac{4819}{60}$$