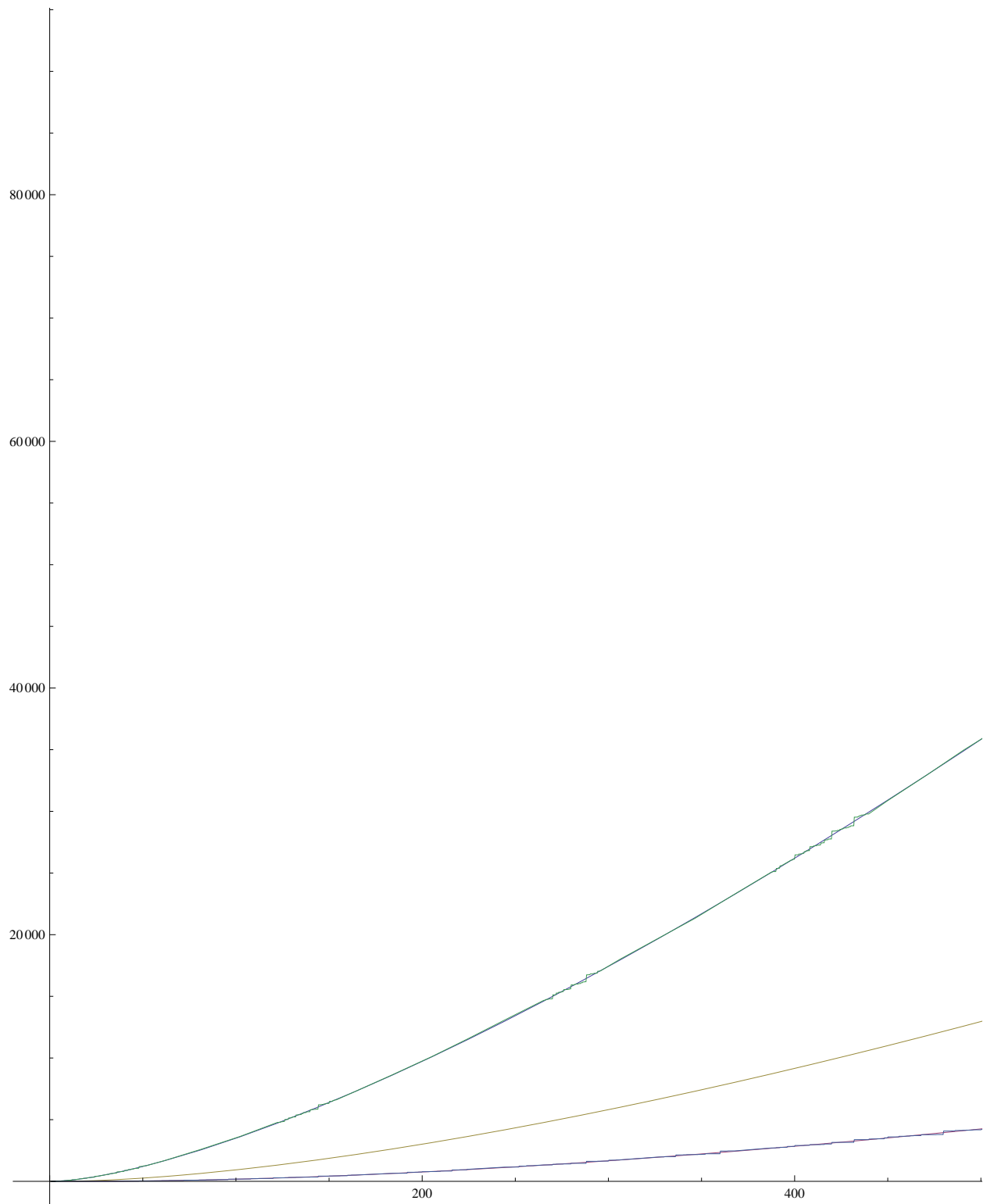


```

D2[n_, k_] := D2[n, k] = Sum[D2[Floor[n / j], k - 1], {j, 2, n}]; D2[n_, 0] := D2[n, 0] = 1
DD[n_, k_] := DD[n, k] = Sum[DD[Floor[n / j], k - 1], {j, 1, n}]; DD[n_, 0] := DD[n, 0] = 1
T1 := T1 = Table[Residue[(Zeta[s])^k x^s s^(-1), {s, 1}], {k, 1, 15}]
T2 := T2 = Table[Residue[(Zeta[s] - 1)^k x^s s^(-1), {s, 1}], {k, 1, 15}]
T2cal := T2cal = Table[T2[[k]] /. x -> 1, {k, 1, 15}]
Ap[n_, k_] := (-1)^(k) (1 - Gamma[k, -Log[n]] / Gamma[k])

```

```
Plot[ {(T1[[tt = 4]] /. x -> n), (T2[[tt]] /. x -> n) - T2cal[[tt]],
      Ap[n, tt], DD[Floor[n], tt], D2[Floor[n], tt] }, {n, 1, 1000}]
```



**T1[[2]]**

$-x + 2 \text{ EulerGamma } x + x \text{ Log}[x]$

**T2[[2]]**

$-3x + 2 \text{EulerGamma} x + x \text{Log}[x]$

**T2a[n\_] := Sum[ (-1)^(k+1) / k ( (T2[[k]] /. x -> n) - T2cal[[k]] ), {k, 1, 15} ]**

**N[T2a[100]]**

30.6209

**N[T2a[100]] - N[LogIntegral[100]]**

0.494743

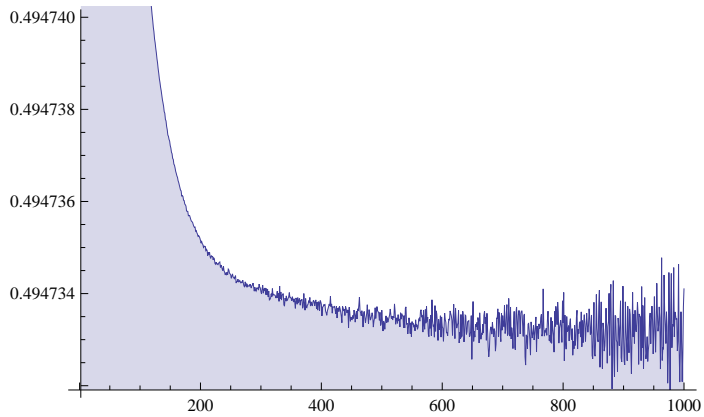
**N[Log[Log[100]]]**

1.52718

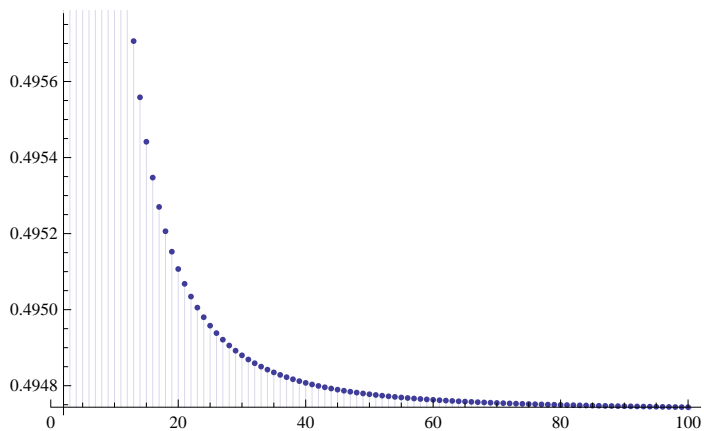
**N[EulerGamma]**

0.577216

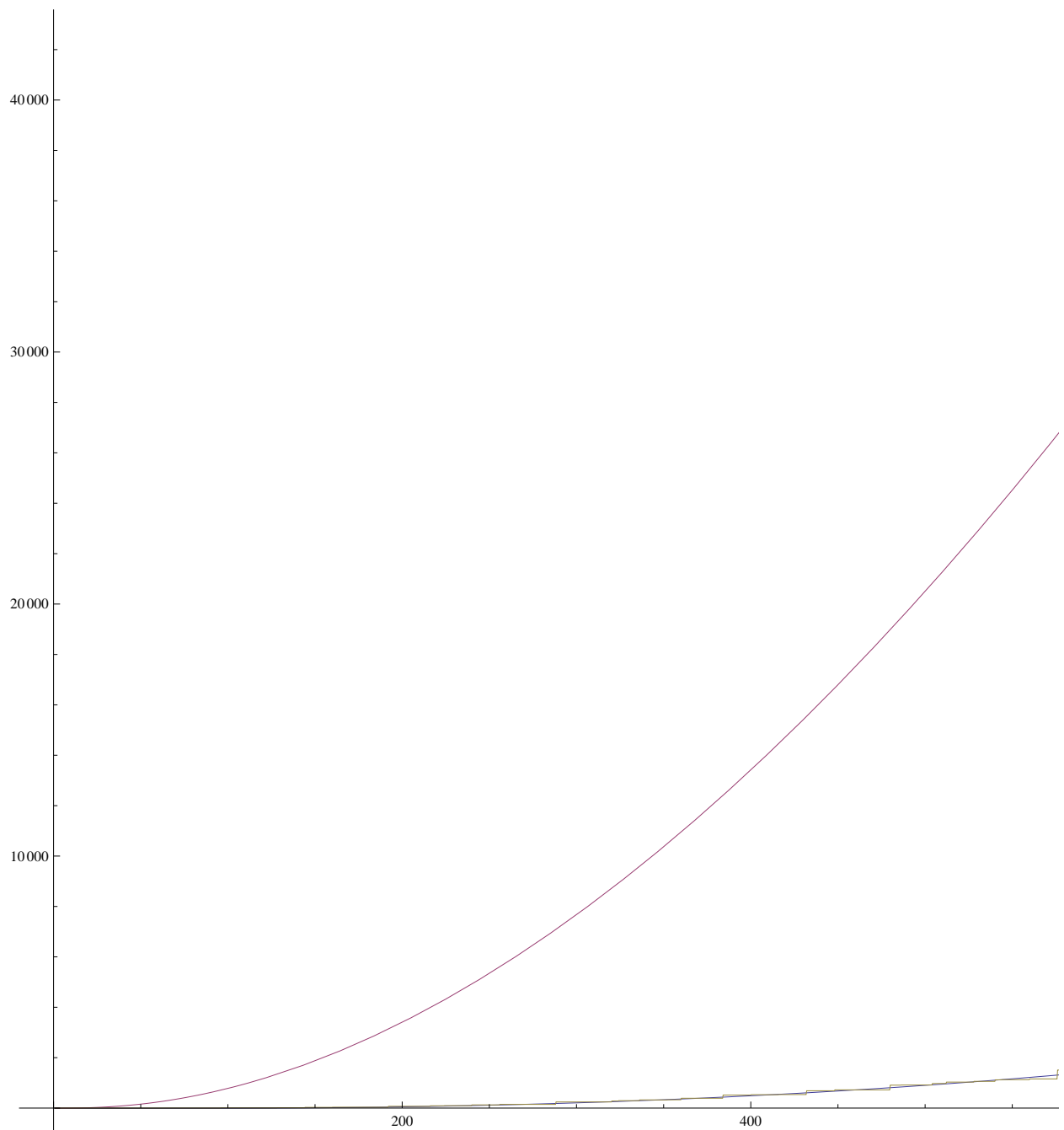
**DiscretePlot[ T2a[n] - LogIntegral[n] , {n, 2, 1000} ]**



**DiscretePlot[ T2a[n] - LogIntegral[n] , {n, 2, 100} ]**



```
Plot[ {(T2[[tt = 6]] /. x -> n), Ap[n, tt], D2[Floor[n], tt] }, {n, 1, 1000}]
```



**T2[[2]]**

$-3x + 2 \text{EulerGamma} x + x \text{Log}[x]$

**T1[[2]]**

$-x + 2 \text{EulerGamma} x + x \text{Log}[x]$

```
Test[j_] := Sum[ T1[[k]] Binomial[j, j - k] (-1)^(j - k), {k, 1, j}]
```

```
Expand[Test[4]]
```

```
-15 x + 28 EulerGamma x - 18 EulerGamma^2 x + 4 EulerGamma^3 x +  
11 x Log[x] - 16 EulerGamma x Log[x] + 6 EulerGamma^2 x Log[x] -  $\frac{5}{2}$  x Log[x]^2 +  
2 EulerGamma x Log[x]^2 +  $\frac{1}{6}$  x Log[x]^3 + 16 x StieltjesGamma[1] -  
12 EulerGamma x StieltjesGamma[1] - 4 x Log[x] StieltjesGamma[1] + 2 x StieltjesGamma[2]
```

```
T2[[6]] - Test[6]
```

```
T2[[4]] /. x -> 1
```

```
 $\frac{1}{6} \left( -90 + 168 \text{EulerGamma} - 108 \text{EulerGamma}^2 + 24 \text{EulerGamma}^3 + \right.$   
96 StieltjesGamma[1] - 72 EulerGamma StieltjesGamma[1] + 12 StieltjesGamma[2] )
```

```
T2cal[[3]]
```

```
 $\frac{1}{2} \left( 14 - 18 \text{EulerGamma} + 6 \text{EulerGamma}^2 - 6 \text{StieltjesGamma}[1] \right)$ 
```