```
Sum[ (-1)^j / ((2 j) !) x^(-2 j s), {j, 0, Infinity}]
```

$\text{Cos}[x^{-s}]$

```
FullSimplify[Sum[ (-1)^j / ((2 j) !) x^(-2 j s), {j, 0, n}]]
```

$$\text{Cos}[x^{-s}] + \frac{(-1)^n x^{-2(1+n)s}\,\text{HypergeometricPFQ}\left[\{1\}, \left\{\frac{3}{2}+n, 2+n\right\}, -\frac{1}{4}x^{-2s}\right]}{\text{Gamma}[3+2n]}$$

```
N@Sum[ ((-1)^j / ((2 j) !) x^(-2 j s)) ((-1)^k / ((2 k) !) x^(-2 k s)), {j, 0, 10}, {k, 0, 10}] /.
  s → 0
```

0.291927

```
N[Cos[1]^2]
```

0.291927

```
Plot[Cos[2.^-s], {s, -15, 15}]
```

```
Sum[ (-1)^k Binomial[z, k] x^(-s k), {k, 0, Infinity}]
```

$(1 - x^{-s})^z$

```
Sum[ (-1)^k z^k / (k!) x^(-s k), {k, 0, Infinity}]
```

$e^{-x^{-s} z}$

```
FI[n_] := FactorInteger[n]; FI[1] := {}
dzo[n_, z_] := Product[Pochhammer[z, p[[2]]] / (p[[2]] !), {p, FI[n]}]
dz[n_, z_] := dz[n, z] = Product[z^p[[2]] / (p[[2]] !), {p, FI[n]}]
```

```
Table[D[dz[n, z], z] /. z → 0, {n, 1, 10}] // TableForm
```

0
-1
-1
$\frac{1}{2}$
-1
0
-1
$-\frac{1}{3}$
$\frac{1}{2}$
0

```
FullSimplify[ ((1 - x^(-4 s)) / (1 - x^(-s)))^z]
```

$\left(x^{-3s} (1 + x^s) \left(1 + x^{2s}\right)\right)^z$

Expand$\left[x^{-3s} (1 + x^s) \left(1 + x^{2s}\right)\right]$

$1 + x^{-3s} + x^{-2s} + x^{-s}$

```
Sum[ Pochhammer[z, k] / k! x^(-s k), {k, 0, Infinity}]
```

$(1 - x^{-s})^{-z}$

```
Sum[ Pochhammer[-z, k] / k! x^(-2 s k), {k, 0, Infinity}]
```

$\left(1 - x^{-2s}\right)^z$

```
Limit[FullSimplify[ ((1 - x^(-a s)) / (1 - x^(-s)))^z], x → 1]
```

a^z

```
Limit[FullSimplify[ ((1 - x^(-a s)) / (1 - x^(-s)))^z], s → 0]
```

a^z

```
D[((1 - x^(-a s)) / (1 - x^(-s)))^z, z] /. z → 0
```

$\text{Log}\left[\frac{1 - x^{-a\,s}}{1 - x^{-s}}\right]$

$\text{Limit}\left[\text{Log}\left[\frac{1 - x^{-a\,s}}{1 - x^{-s}}\right], s \to 0\right]$

```
Log[a]
```

```
Clear[p2, pm, do2]
bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k!
do2[n_, s_, k_] := do2[n, s, k] = Sum[j^-s do2[Floor[n / j], s, k - 1], {j, 2, n}]
do2[n_, s_, 0] := UnitStep[n - 1]
doz[n_, s_, z_] := Sum[bin[z, k] do2[n, s, k], {k, 0, Log2@n}]
p2[n_, s_, k_] := p2[n, s, k] = Sum[(-1)^(j + 1) j^-s p2[Floor[n / j], s, k - 1], {j, 2, n}]
p2[n_, s_, 0] := UnitStep[n - 1]
pz[n_, s_, z_] := Sum[bin[z, k] p2[n, s, k], {k, 0, Log2@n}]
pm[n_, s_] :=
 pm[n, s] = Sum[ MoebiusMu[ k] / k D[pz[n^(1 / k), s, z], z] /. z → 0, {k, 1, Log2@n}]
pmi[n_, s_] := Sum[ 1 / k pm[n^(1 / k), s], {k, 1, Log2@n}]
pd[1, s_] := 0
pd[n_, s_] := pm[n, s] - pm[n - 1, s]
pdk[n_, s_, k_] := pdk[n, s, k] = Sum[pd[j, s] pdk[Floor[n / j], k - 1], {j, 2, n}]
pdk[n_, s_, 0] := UnitStep[n - 1]
pdz[n_, s_, z_] := Sum[z^k / k! pdk[n, s, k], {k, 0, Log2@n}]
pmx[n_, s_] :=
 Sum[ MoebiusMu[ k] / k D[pz[n^(1 / k), s, z] - doz[n^(1 / k), s, z], z] /. z → 0, {k, 1, Log2@n}]
```

```
Table[ pm[n, 0] - pm[n - 1, 0], {n, 2, 100}]
```

```
{-1, 1, -1, 1, 0, 1, -2, 0, 0, 1, 0, 1, 0, 0, -3, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, -6, 0,
 0, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, -9, 0, 0,
 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}
```

```
(* https://oeis.org/A001037 *)
(* https://oeis.org/A059966 *)
Table[ (pm[2^k, 0] - pm[2^k - 1, 0]), {k, 1, 16}]
```

```
{-1, -1, -2, -3, -6, -9, -18, -30, -56, -99, -186, -335, -630, -1161, -2182, -4080}
```

```
Table[ (pm2[n, 0] - pm2[n - 1, 0]), {n, 2, 100}]
```

```
{-2, 0, -1, 0, 0, 0, -2, 0, 0, 0, 0, 0, 0, 0, -3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -6, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -9, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

```
Table[D[(pz[n, 0, z] - doz[n, 0, z]) - (pz[n - 1, 0, z] - doz[n - 1, 0, z]), z] /. z → 0, {n, 2, 100}]
```

$\{-2, 0, -2, 0, 0, 0, -\frac{8}{3}, 0, 0, 0, 0, 0, 0, 0, -4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{32}{5}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{32}{3}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$

```
Table[pd[n, 0], {n, 1, 100}]
```

{0, -1, 1, -1, 1, 0, 1, -2, 0, 0, 1, 0, 1, 0, 0, -3, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, -6,
0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, -9, 0, 0,
1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}

```
FullSimplify[Table[ Sum[ MoebiusMu[n / d] ((2^d) - 1) / n, {d, Divisors[n]}], {n, 1, 16}]]
```

{1, 1, 2, 3, 6, 9, 18, 30, 56, 99, 186, 335, 630, 1161, 2182, 4080}

```
Table[ N@(2^n - 1) / n, {n, 1, 16}]
```

{1., 1.5, 2.33333, 3.75, 6.2, 10.5, 18.1429, 31.875,
56.7778, 102.3, 186.091, 341.25, 630.077, 1170.21, 2184.47, 4095.94}

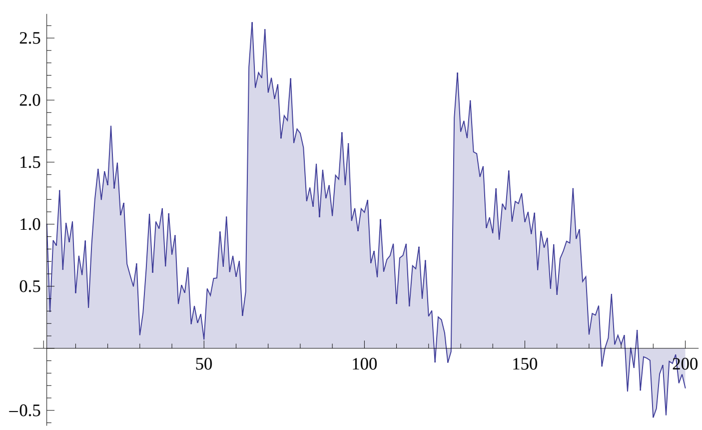
```
Table[ (pm[2^k, -1] - pm[2^k - 1, -1]), {k, 1, 16}]
```

{-2, -5, -18, -57, -198, -661, -2322, -8130, -29 064, -104 655,
-381 114, -1 397 405, -5 161 590, -19 171 629, -71 580 534, -268 427 280}

```
FullSimplify[Table[ Sum[ MoebiusMu[n / d] ((2^d) - 1) / n, {d, Divisors[n]}], {n, 1, 16}]]
```

{1, 1, 2, 3, 6, 9, 18, 30, 56, 99, 186, 335, 630, 1161, 2182, 4080}

```
DiscretePlot[Re[pdz[n, .5, 1]], {n, 1, 200}]
```




```
f[n_] := Block[{d = Divisors@n}, Plus @@ (MoebiusMu[n / d] * 2^d / n)]; Array[f, 32]
```

{2, 1, 2, 3, 6, 9, 18, 30, 56, 99, 186, 335, 630, 1161, 2182, 4080,
7710, 14 532, 27 594, 52 377, 99 858, 190 557, 364 722, 698 870, 1 342 176,
2 580 795, 4 971 008, 9 586 395, 18 512 790, 35 790 267, 69 273 666, 134 215 680}

```
Table[pdz[n, .5, 1], {n, 1, 100}]
```

```
{1, 0.292893, 0.870243, 0.82735, 1.27456, 0.632335, 1.0103, 0.85497, 1.02164, 0.444476,
 0.745988, 0.591016, 0.868367, 0.325831, 0.838113, 1.20326, 1.44579, 1.1965, 1.42591,
 1.31432, 1.79198, 1.28767, 1.49618, 1.07206, 1.17206, 0.679832, 0.586752, 0.498243,
 0.683939, 0.106715, 0.28632, 0.644111, 1.08354, 0.608721, 1.02131, 0.964012, 1.12841,
 0.66015, 1.0875, 0.755559, 0.911733, 0.357592, 0.510091, 0.447066, 0.653047, 0.195237,
 0.341102, 0.204804, 0.276232, 0.0711132, 0.481056, 0.426085, 0.563446, 0.566634,
 0.940996, 0.658106, 1.06149, 0.615088, 0.745277, 0.576454, 0.704491, 0.261135, 0.455575,
 2.26582, 2.62811, 2.09945, 2.22162, 2.17825, 2.57119, 2.06042, 2.1791, 2.0101, 2.12714,
 1.69139, 1.87488, 1.83589, 2.17562, 1.65502, 1.76753, 1.73317, 1.61719, 1.18555,
 1.29531, 1.14092, 1.48579, 1.05599, 1.43751, 1.20878, 1.31477, 1.06682, 1.39447,
 1.36245, 1.74093, 1.31444, 1.65275, 1.02722, 1.12876, 0.943694, 1.12539, 1.09574}
```

```
Table[ (pm[2^k, 0] - pm[2^k - 1, 0]), {k, 1, 12}]
```

```
{-1, -1, -2, -3, -6, -9, -18, -30, -56, -99, -186, -335}
```

```
Table[pmi[n, 0], {n, 2, 40}]
```

$$\left\{-1, 0, -\frac{3}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{11}{6}, -\frac{4}{3}, -\frac{4}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, -\frac{37}{12}, -\frac{25}{12}, -\frac{25}{12}, -\frac{13}{12}, -\frac{13}{12}, -\frac{13}{12}, -\frac{13}{12}, -\frac{1}{12}, -\frac{1}{12}, \frac{5}{12}, \frac{5}{12}, \frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{7}{4}, \frac{11}{4}, -\frac{69}{20}, -\frac{69}{20}, -\frac{69}{20}, -\frac{69}{20}, -\frac{69}{20}, -\frac{49}{20}, -\frac{49}{20}, -\frac{49}{20}, -\frac{49}{20}\right\}$$

```
Table[D[pz[n, 0, z], z] /. z → 0, {n, 2, 40}]
```

$$\left\{-1, 0, -\frac{3}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{11}{6}, -\frac{4}{3}, -\frac{4}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, -\frac{37}{12}, -\frac{25}{12}, -\frac{25}{12}, -\frac{13}{12}, -\frac{13}{12}, -\frac{13}{12}, -\frac{13}{12}, -\frac{1}{12}, -\frac{1}{12}, \frac{5}{12}, \frac{5}{12}, \frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{7}{4}, \frac{11}{4}, -\frac{69}{20}, -\frac{69}{20}, -\frac{69}{20}, -\frac{69}{20}, -\frac{69}{20}, -\frac{49}{20}, -\frac{49}{20}, -\frac{49}{20}, -\frac{49}{20}\right\}$$

```
Table[-z Pochhammer[1 - k, n] Pochhammer[1 - z, n] / Pochhammer[2, n] (-1)^n / n!, {n, 0, 6}] /.
  k → 5 // TableForm
```

$-z$

$-2 (1-z) z$

$-(1-z) (2-z) z$

$-\frac{1}{6} (1-z) (2-z) (3-z) z$

$-\frac{1}{120} (1-z) (2-z) (3-z) (4-z) z$

0

0

```
Table[-z Pochhammer[1 - z, n] Pochhammer[1 - k, n] (-1)^n / n! / (n + 1)!, {n, 0, 6}] /. k → 5 //
 TableForm
```

$-z$

$-2 (1-z) z$

$-(1-z) (2-z) z$

$-\frac{1}{6} (1-z) (2-z) (3-z) z$

$-\frac{1}{120} (1-z) (2-z) (3-z) (4-z) z$

0

0

```
Table[{(-1)^n Pochhammer[1 - k, n] / n!, Binomial[k - 1, n]}, {n, 0, 6}] /. k → 5 // TableForm
```

1	1
4	4
6	6
4	4
1	1
0	0
0	0

```
Table[-z Pochhammer[1 - z, n] / (n + 1)! Binomial[k - 1, n], {n, 0, 6}] /. k → 5 // TableForm
```

$-z$
$-2(1-z)z$
$-(1-z)(2-z)z$
$-\frac{1}{6}(1-z)(2-z)(3-z)z$
$-\frac{1}{120}(1-z)(2-z)(3-z)(4-z)z$
0
0

```
Table[{Expand[(-z Pochhammer[1 - z, n] / (n + 1)!) - (-1)^(n + 1) (bin[z, n + 1])]}, {n, 0, 5}] //
 TableForm
```

0
0
0
0
0
0

```
Table[{Expand[(-z Pochhammer[1 - z, n] / (n + 1)!) - (-1)^(n + 1) (bin[z, n + 1])]}, {n, 0, 5}] //
 TableForm
```

```
Table[(-1)^(n + 1) (Binomial[z, n + 1]) Binomial[k - 1, n], {n, 0, 6}] /. k → 5 // TableForm
```

$-z$
$2(-1+z)z$
$-(-2+z)(-1+z)z$
$\frac{1}{6}(-3+z)(-2+z)(-1+z)z$
$-\frac{1}{120}(-4+z)(-3+z)(-2+z)(-1+z)z$
0
0

```
FullSimplify[(-1)^(n + 1) (Binomial[z, n + 1]) Binomial[k - 1, n]]
```

$-(-1)^n$ Binomial[-1 + k, n] Binomial[z, 1 + n]

```
Table[ (pm[2^k, 1] - pm[2^k - 1, 1]), {k, 1, 12}]
```

$$\left\{-\frac{1}{2}, -\frac{1}{8}, -\frac{1}{8}, -\frac{3}{64}, -\frac{3}{32}, \frac{3}{128}, -\frac{9}{128}, -\frac{15}{2048}, -\frac{7}{512}, \frac{45}{2048}, -\frac{93}{2048}, \frac{235}{16384}\right\}$$

```
Table[ (pmx[2^k, 2] - pmx[2^k - 1, 2]), {k, 1, 12}]
```

$$\left\{-\frac{1}{2}, \frac{1}{8}, \frac{1}{8}, \frac{3}{64}, \frac{3}{32}, -\frac{3}{128}, \frac{9}{128}, \frac{15}{2048}, \frac{7}{512}, -\frac{45}{2048}, \frac{93}{2048}, -\frac{235}{16384}\right\}$$

```
Table[ (pm[2^k, 2] - pm[2^k - 1, 2]), {k, 1, 12}]
```

$$\left\{-\frac{1}{4}, \frac{1}{32}, \frac{3}{64}, \frac{33}{1024}, \frac{45}{1024}, \frac{43}{8192}, \frac{567}{16384}, \frac{3585}{524288}, \frac{3129}{262144}, -\frac{6963}{2097152}, \frac{95139}{4194304}, -\frac{636245}{67108864}\right\}$$

```
eta[s2_] := Limit[ (1 - 2^(1 - s)) Zeta[s], s → s2]
einv[s_] := Product[ Limit[eta[k s]^(MoebiusMu[k] / k), k → k2], {k2, 1, Infinity}]
einv[s_, t_] := Product[ eta[k s]^(MoebiusMu[k] / k), {k, 1, t}]
einv2[s_, t_] := Product[Limit[ eta[k s]^(MoebiusMu[k] / k), k → k2], {k2, 1, t}]
```

```
eta[1]
```

Log[2]

```
einv[1]
```

$$\prod_{k2=1}^{\infty} \text{Limit}\left[\left(2^{-k}\left(-2+2^{k}\right) \text{Zeta}[k]\right)^{\frac{\text{MoebiusMu}[k]}{k}}, k \to k2\right]$$

```
N@einv[1, 80]
```

0.794536

```
N@einv[2, 80]
```

0.849401

```
N@einv[.5, 120]
```

0.811298

```
N@einv[N[ZetaZero[1]], 120]
```

0. + 0. i

```
N@einv[N[ZetaZero[1] / 6], 120]
```

0. + 0. i

```
N[ZetaZero[1] / 6]
```

0.0833333 + 2.35579 i

```
N@einv[0.5` + 14.134725141734695` i, 120]
```

0. + 0. i

```
N@ZetaZero[1]
```

0.5 + 14.1347 i

```
FullSimplify[(1 - 2^(1 - s)) (1 / (1 - 2^-s))]
```

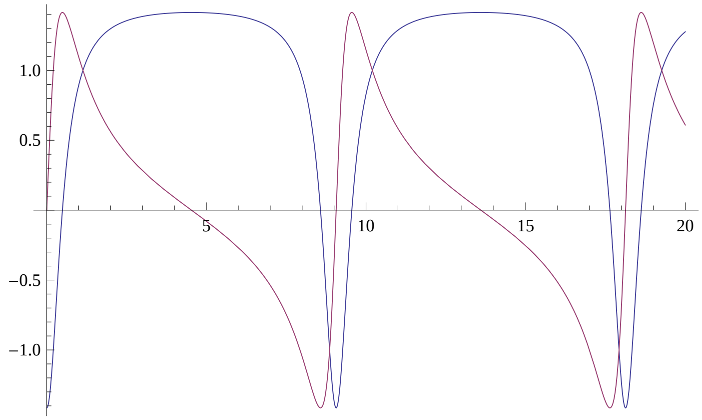
$$1 + \frac{1}{1 - 2^{s}}$$

$$\text{fs}[s\_] := \left(1 + \frac{1}{1 - 2^{s}}\right)$$

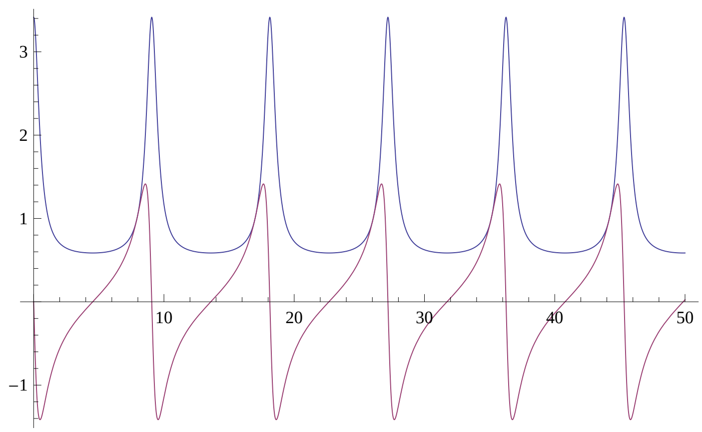
$$\text{fs2}[s\_] := 1 + \frac{1}{-1 + 2^{s}}$$

```
eta[s_] := (1 - 2^(1 - s)) Zeta[s]
```

```
Plot[{Re[fs[.5 + t I]], Im[fs[.5 + t I]]}, {t, 0, 20}]
```




```
Plot[{Re[fs2[.5 + t I]], Im[fs2[.5 + t I]]}, {t, 0, 50}]
```




```
FullSimplify@Sum[2^(-s k) (-1) Hypergeometric2F1[1 - k, 0, 2, -1], {k, 0, Infinity}]
```

$$\frac{1}{-1+2^{-s}}$$

```
FullSimplify@Sum[2^(-s k) Pochhammer[1, k] / k!, {k, 0, Infinity}]
```

$$1+\frac{1}{-1+2^{s}}$$

```
FullSimplify@Sum[2^(-s k), {k, 0, Infinity}]
```

$$1+\frac{1}{-1+2^{s}}$$

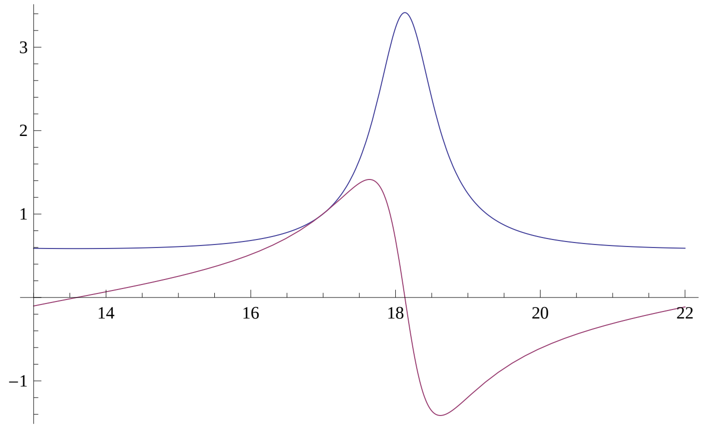
```
(-1) Hypergeometric2F1[1 - k, 0, 2, -1]
```

-1

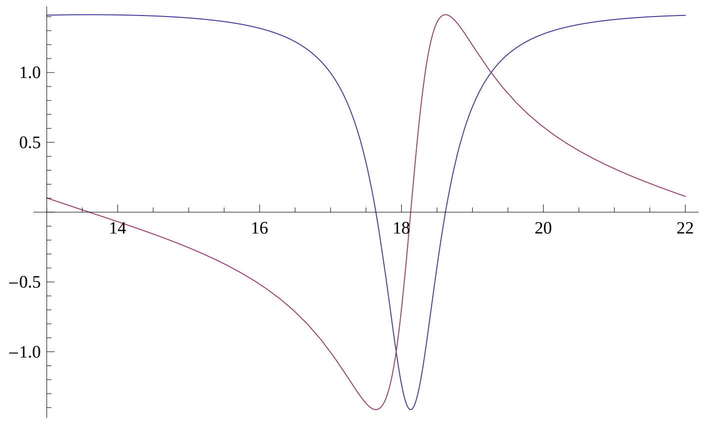
FullSimplify$\left[\left(\frac{1}{-1+2^{-s}}\right)\Big/\left(1+\frac{1}{-1+2^{s}}\right)\right]$

-1

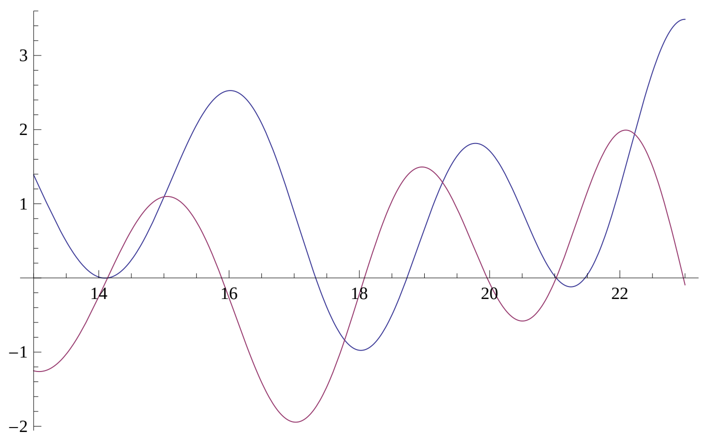
```
Plot[{Re[fs2[.5 + t I]], Im[fs2[.5 + t I]]}, {t, 13, 22}]
```



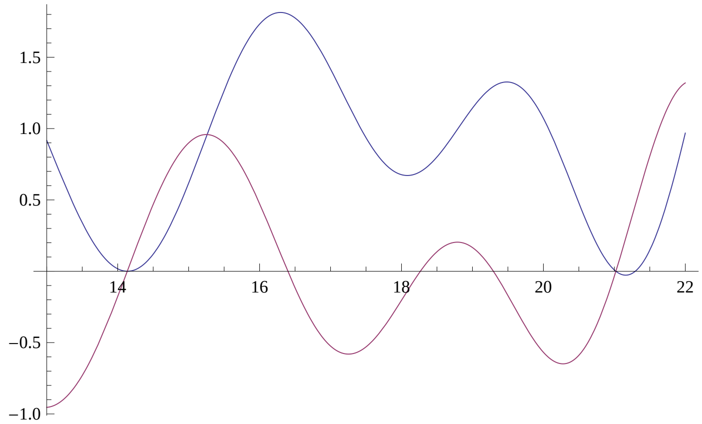

```
Plot[{Re[fs[.5 + t I]], Im[fs[.5 + t I]]}, {t, 13, 22}]
```




```
Plot[{Re[eta[.5 + t I]], Im[eta[.5 + t I]]}, {t, 13, 23}]
```



```
Plot[{Re[ eta[.5 + t I] / fs[.5 + t I]], Im[ eta[.5 + t I] / fs[.5 + t I]]}, {t, 13, 22}]
```




```
ps1[s_] := Product[ 1 / (1 - Prime[j] ^ -s), {j, 1, 6000}]
ps2[s_] := Product[ 1 / (1 - Prime[j] ^ -s), {j, 2, 6000}]
```

ps3[s_] := $\left(1 + \frac{1}{1 - 2^s}\right)$ ps2[s]

```
N@{ps1[.5], ps2[.5], ps3[.5]}
```

$\{4.48127 \times 10^{25}, 1.31253 \times 10^{25}, -1.8562 \times 10^{25}\}$

```
N@Pi^2 / 12
```

0.822467

```
FullSimplify[1 - Sum[2^(-s k), {k, 1, Infinity}]]
```

$1 + \frac{1}{1 - 2^s}$

```
Table[ If[a == 0, 1, -1] Abs[N@(2^a) ^ -s], {a, 0, 18}] /. s → .5 // TableForm
```

1.
-0.707107
-0.5
-0.353553
-0.25
-0.176777
-0.125
-0.0883883
-0.0625
-0.0441942
-0.03125
-0.0220971
-0.015625
-0.0110485
-0.0078125
-0.00552427
-0.00390625
-0.00276214
-0.00195313

```
Sum[ If[a == 0, 1, -1] N@(2^a)^-s, {a, 0, 100}] /. s → N@ZetaZero[4]
```

1.39481 + 0.233473 i

```
N[-2^(1/2)]
```

-1.41421

```
1 - Sum[(2^a)^-s, {a, 1, Infinity}] /. s → 1/2
```

$$1 - \frac{1}{-1 + \sqrt{2}}$$

```
1 - Sum[(2^a)^-s, {a, 1, Infinity}] /. s → 1/2
```

```
et[n_, s_] := Sum[ (-1)^(j + 1) j^-s, {j, 1, n}]
pom[n_, s_] := Sum[ j^-s, {j, 1, Floor[n]}]
po[n_, s_] := Sum[ j^-s, {j, 1, n, 2}]
poi[n_, o_, s_] := Sum[ (j + o)^-s, {j, 1, n, 2}]
pox[n_, s_] := Sum[ (2 j + 1)^-s, {j, 0, (n - 1) / 2}]
poxy[n_, s_] := Sum[ (2 j - 1)^-s, {j, 1, (n + 1) / 2}]
s2[n_, s_] := po[n, s] - Sum[ (2^a)^-s po[n / (2^a), s], {a, 1, Log2@n}]
s2a[n_, s_] := Sum[ If[a == 0, 1, -1] (2^a)^-s po[n / (2^a), s], {a, 0, Log2@n}]
s2aa[n_, s_] := Sum[ If[a == 0, 1, -1] Abs[(2^a)^-s po[n / (2^a), s]], {a, 0, Log2@n}]
s2t[n_, s_] := Table[ If[a == 0, 1, -1] (2^a)^-s po[n / (2^a), s], {a, 0, Log2@n}]
s2ta[n_, s_] := Table[ Abs[If[a == 0, 1, -1] (2^a)^-s po[n / (2^a), s]], {a, 0, Log2@n}]
s2tx[n_, s_] := Table[ If[a == 0, 1, -1] po[n, s] / ((2^a)^-s po[n / (2^a), s]), {a, 0, Log2@n}]
```

```
po[1000000, 1.]
```

7.54294

```
s2a[1000000., -.2 + N[ZetaZero[1]]]
```

-0.450893 + 0.028744 i

```
s2a[1000000000., N[ZetaZero[11]]]
```

0.0000335657 - 0.000121956 i

```
s2t[1 000 000 000 000 000., N[ZetaZero[1]]] // TableForm
```

$-1.07245 \times 10^6 - 315\,585.\, i$
$536\,226. + 157\,793.\, i$
$268\,113. + 78\,896.4\, i$
$134\,056. + 39\,448.2\, i$
$67\,028.2 + 19\,724.1\, i$
$33\,514.1 + 9862.05\, i$
$16\,757.1 + 4931.02\, i$
$8378.53 + 2465.51\, i$
$4189.27 + 1232.76\, i$
$2094.63 + 616.378\, i$
$1047.32 + 308.189\, i$
$523.658 + 154.094\, i$
$261.829 + 77.0472\, i$
$130.915 + 38.5236\, i$
$65.4573 + 19.2618\, i$
$32.7286 + 9.6309\, i$
$16.3643 + 4.81545\, i$
$8.18216 + 2.40773\, i$
$4.09108 + 1.20386\, i$
$2.04554 + 0.601932\, i$
$1.02277 + 0.300966\, i$
$0.511385 + 0.150483\, i$
$0.255692 + 0.0752414\, i$
$0.127846 + 0.0376207\, i$
$0.0639231 + 0.0188103\, i$
$0.0319616 + 0.00940517\, i$
$0.0159808 + 0.00470258\, i$
$0.00799039 + 0.00235128\, i$
$0.0039952 + 0.00117563\, i$
$0.0019976 + 0.000587811\, i$
$0.000998796 + 0.000293921\, i$
$0.000499403 + 0.000146945\, i$
$0.000249696 + 0.0000734876\, i$
$0.000124853 + 0.0000367288\, i$
$0.0000624266 + 0.0000183644\, i$
$0.0000312133 + 9.1822 \times 10^{-6}\, i$
$0.0000156067 + 4.5911 \times 10^{-6}\, i$
$7.80333 \times 10^{-6} + 2.29555 \times 10^{-6}\, i$
$3.90167 \times 10^{-6} + 1.14778 \times 10^{-6}\, i$
$1.9458 \times 10^{-6} + 5.88878 \times 10^{-7}\, i$
$9.77923 \times 10^{-7} + 2.79446 \times 10^{-7}\, i$
$4.83907 \times 10^{-7} + 1.54708 \times 10^{-7}\, i$
$2.4695 \times 10^{-7} + 6.23483 \times 10^{-8}\, i$
$1.23713 \times 10^{-7} + 3.12341 \times 10^{-8}\, i$
$5.60222 \times 10^{-8} + 3.04206 \times 10^{-8}\, i$
$2.89334 \times 10^{-8} + 1.56981 \times 10^{-8}\, i$
$1.65695 \times 10^{-8} + 8.85779 \times 10^{-9}\, i$
$1.55246 \times 10^{-9} - 1.46302 \times 10^{-8}\, i$
$-1.97674 \times 10^{-8} - 1.75985 \times 10^{-8}\, i$
$3.50478 \times 10^{-8} + 2.34095 \times 10^{-8}\, i$

```
po[1 000 000 000, N@ZetaZero[3]]
```

22.843 − 631.642 i

```
2^N[1 - ZetaZero[3]] po[1 000 000 000 / 2, N@ZetaZero[3]]
```

22.843 − 631.642 i

```
4^N[1 - ZetaZero[3]] po[1 000 000 000 / 4, N@ZetaZero[3]]
```

22.8429 − 631.642 i

```
Table[ po[n, .5] - poxy[n, .5], {n, 1, 10}]
```

{0., 0., 0., 0., 0., 0., 0., 0., 0., 0.}

```
(1^N[1 - ZetaZero[1]] pom[10 000 000 000 000 / 1, N@ZetaZero[1]]) -
 (2^N[1 - ZetaZero[1]] pom[10 000 000 000 000 / 2, N@ZetaZero[1]])
```

$8.60891\times 10^{-8}+1.37108\times 10^{-7}\,i$

```
((1 / 2)^N[1 - ZetaZero[1]] pom[10 000 000 000 000 / (1 / 2), N@ZetaZero[1]]) -
 (2^N[1 - ZetaZero[1]] pom[10 000 000 000 000 / 2, N@ZetaZero[1]])
```

$1.28988\times 10^{-7}+2.06011\times 10^{-7}\,i$

```
(9^N[1 - ZetaZero[1]] pom[10 000 000 000 000 / 9, N@ZetaZero[1]]) -
 (2^N[1 - ZetaZero[1]] pom[10 000 000 000 000 / 2, N@ZetaZero[1]])
```

$-4.17174\times 10^{-7}-6.67205\times 10^{-7}\,i$

0.444444

```
FullSimplify[2^(1 - s) (2 j - 1)^-s] /. j → 3 /. s → 4
```

$\frac{1}{5000}$

```
2 (4 j - 2)^-s
```

$2\,(-2+4\,j)^{-s}$

```
Plot[(n - 1)^-s + (n + 1)^-s - 2 n^-(s) /. n → 6.5, {s, -2, 2}]
```

```
N@ZetaZero[1]
```

0.5 + 14.1347 i

```
fr2a[2, s, 4]
```

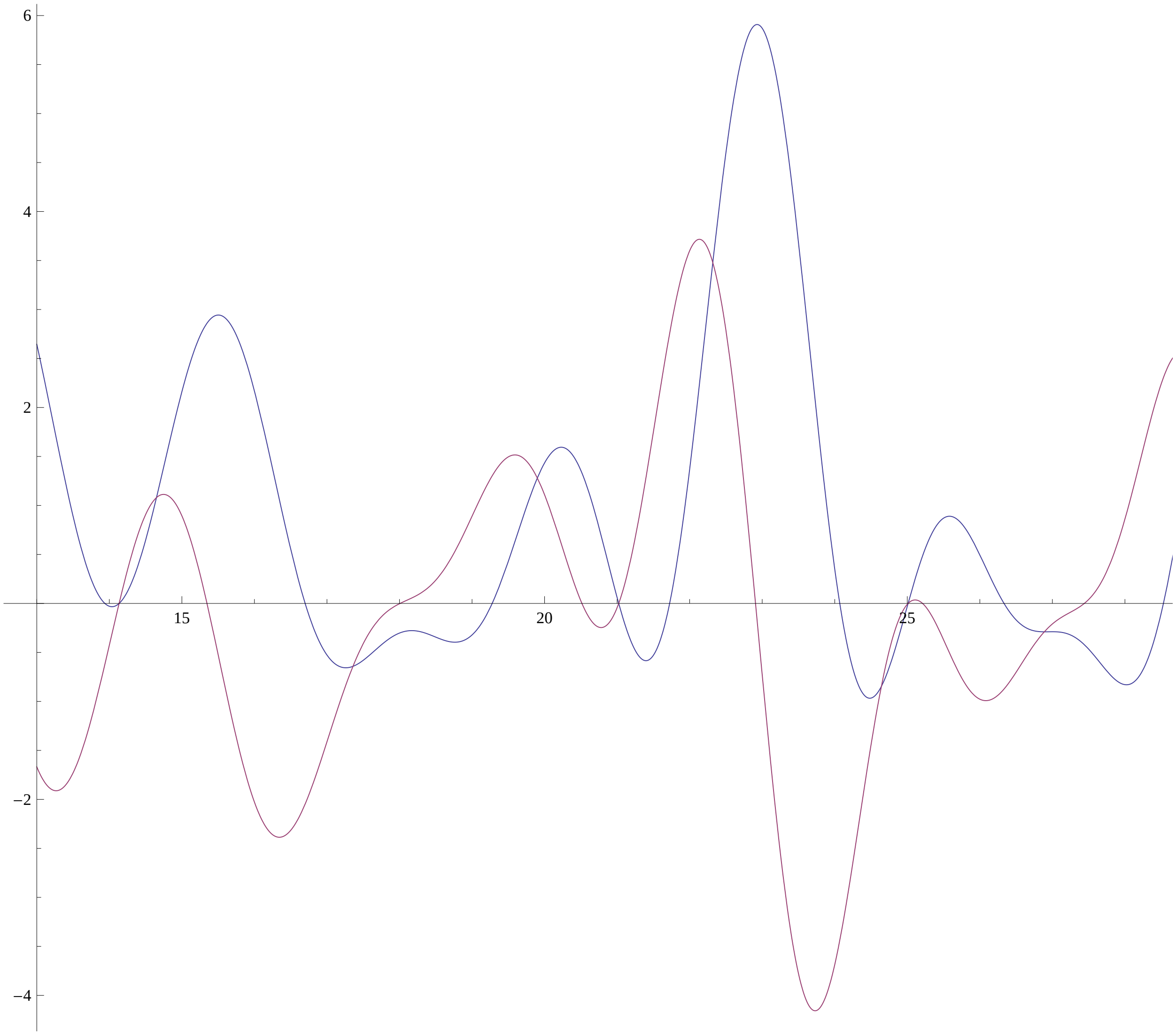
$1-2^{1-2\,s}-2^{1-s}+2\times 3^{-s}-2^{1-s}\,3^{-s}+2\times 5^{-s}+7^{-s}$

```
tt[n_, s_] := po[8 n, s] - 2 × 2^-s po[4 n, s]
tto[n_, o_, s_] := poi[8 n, o, s] - 2 × 2^-s poi[4 n, o, s]
```

```
tt[3, s]
```

$1+3^{-s}+5^{-s}+7^{-s}+9^{-s}+11^{-s}+13^{-s}+15^{-s}+17^{-s}+19^{-s}+21^{-s}+23^{-s}-2^{1-s}\,(1+3^{-s}+5^{-s}+7^{-s}+9^{-s}+11^{-s})$

```
Plot[{Re[tt[1000, .5 + I s]], Im[tt[1000, .5 + I s]]}, {s, 13, 35}]
```




```
N@tt[1000, ZetaZero[1]]
```

$-4.87234 \times 10^{-6} - 8.24266 \times 10^{-7}\, i$

```
po[8 000 000 000, N@ZetaZero[3]]
```

$-1771.17 + 242.715\, i$

```
2 2^N[-ZetaZero[3]] po[8 000 000 000 / 2, N@ZetaZero[3]]
```

$-1771.17 + 242.715\, i$

```
po[1 000 000 000, N@ZetaZero[1]]
 2 × 2^N[ZetaZero[1]] po[1 000 000 000 / 2, N@ZetaZero[1]]
```

```
tt[2, s]
```

$1 + 3^{-s} + 5^{-s} + 7^{-s} + 9^{-s} + 11^{-s} + 13^{-s} + 15^{-s} - 2^{1-s} \left(1 + 3^{-s} + 5^{-s} + 7^{-s}\right)$

```
1 + 3^-s + 5^-s + 7^-s - 2^-s - 2^-s - 6^-s - 6^-s /. s → .5 + I
0.197926 + 0.387535 ⅈ
tt[1, .5 + I]
0.197926 + 0.387535 ⅈ
Sum[ ff[n, s] 2^-a, {a, Log[2, n], Infinity}]
2 ff[n, s]
----------
    n

eet[n_, s_] := Sum[ (-1)^(j + 1) j^-s, {j, 1, n}]
epo[n_, s_] := Sum[ j^-s, {j, 1, n, 2}]
es2[n_, s_] := po[n, s] - Sum[ (2^a)^-s po[n / (2^a), s], {a, 1, Log2@n}]
es2a[n_, s_] :=
 Sum[ po[n, s] / 2^a, {a, 1, Infinity}] - Sum[ (2^a)^-s po[n / (2^a), s], {a, 1, Log2@n}]
es2b[n_, s_] := Sum[ po[n, s] / 2^a, {a, Floor[Log2@n] + 1, Infinity}] +
  Sum[ po[n, s] / 2^a, {a, 1, Floor[Log2@n]}] -
  Sum[ (2^a)^-s po[n / (2^a), s], {a, 1, Log2@n}]
es2c[n_, s_] := Sum[ po[n, s] / 2^a, {a, Floor[Log2@n] + 1, Infinity}] +
  Sum[ po[n, s] / 2^a - (2^a)^-s po[n / (2^a), s], {a, 1, Log2@n}]
es2d[n_, s_] := Sum[ po[n, s] / 2^a, {a, Floor[Log2@n] + 1, Infinity}] +
  Sum[ 2^-a po[n, s] - (2^-a)^s po[n / (2^a), s], {a, 1, Log2@n}]
es2e[n_, s_] := Sum[ po[n, s] / 2^a, {a, Floor[Log2@n] + 1, Infinity}] +
  Sum[ 2^-a po[n, s] - (2^(-a s)) po[n / (2^a), s], {a, 1, Log2@n}]
es2f[n_, s_] := Sum[ po[n, s] / 2^a, {a, Floor[Log2@n] + 1, Infinity}] + Sum[
   2^-a Sum[ j^-s, {j, 1, n, 2}] - (2^(-a s)) Sum[ j^-s, {j, 1, n / (2^a), 2}], {a, 1, Log2@n}]
es2g[n_, s_] := Sum[ po[n, s] / 2^a, {a, Floor[Log2@n] + 1, Infinity}] +
  Sum[ Sum[2^-a j^-s, {j, 1, n, 2}] -
    Sum[ (2^(-a s)) j^-s, {j, 1, n / (2^a), 2}], {a, 1, Log2@n}]
es2h[n_, s_] := Sum[ po[n, s] / 2^a, {a, Floor[Log2@n] + 1, Infinity}] +
  Sum[ Sum[2^-a j^-s, {j, 1, n, 2}] -
    Sum[ (2^(-a s)) j^-s, {j, 1, n / (2^a), 2}], {a, 1, Log2@n}]

N@eet[100, -1 + 13 I]
51.9917 - 23.4411 ⅈ
N@es2h[100, -1 + 13 I]
51.9917 - 23.4411 ⅈ
FullSimplify[(2^(-a))^s]
```

$(2^{-a})^{s}$

```
(2^(3 + I))^6
```

$2^{18+6\,i}$

```
4^6
4096
```

```
2^12
```

4096

```
2^(-a s)
```

$2^{-a s}$

```
2^s × 2^(-a)
```

2^{-a+s}

```
2^3 fn - 2^(3 a) gfn
```

$8\,\text{fn} - 2^{3 a}\,\text{gfn}$

```
tr[n_, s_, a_] := Sum[2^-a j^-s, {j, 1, n, 2}] - Sum[ (2^(-a s)) j^-s, {j, 1, n / (2^a), 2}]
tr1[n_, s_] := 1 / 2 Sum[ j^-s, {j, 1, n, 2}] - (1 / 2) Sum[2 j^-s (2^(-s)), {j, 1, n / 2, 2}]
tr1a[n_, s_] := 1 / 2 ( Sum[ j^-s, {j, 1, n, 2}] -
    Sum[ j^-s (2^(-s)), {j, 1, n / 2, 2}] - Sum[ j^-s (2^(-s)), {j, 1, n / 2, 2}])
tr1b[n_, s_] := 1 / 2 ( Sum[ j^-s, {j, 1, n, 2}] - Sum[ j^-s (2^(-s)), {j, 1, n / 2, 2}] -
    Sum[ j^-s (2^(-s)), {j, 1, n / 2, 2}])
```

```
tr1b[1 000 000 000, N@ZetaZero[1]]
```

$5.78694 \times 10^{-6} - 5.38623 \times 10^{-6}\, i$

```
tr1b[8, s]
```

$\frac{1}{2}\left(1 - 2^{1-s} + 3^{-s} - 2^{1-s}\, 3^{-s} + 5^{-s} + 7^{-s}\right)$

```
N[- (2^(-ZetaZero[1]))]
```

$0.658571 - 0.257458\, i$

```
zetar[n_, s_, a_] := Sum[2^-a j^-s, {j, 1, n, 2}] + Sum[ (2^(-a s)) j^-s, {j, 1, n / (2^a), 2}]
```

```
N@zetar[100 000, 2, 1]
```

0.92527

```
N@tr[100 000., 3, 1]
```

0.394425

```
zt1[n_, s_] := Sum[ N[j^-s], {j, 1, Floor[n]}]
zt2[n_, s_] := Sum[ N[j^-s], {j, 1, Floor[n], 2}]
zt3[n_, s_] := Sum[ N[2^-k s], {k, 0, Log2@n}]
zd1[n_, s_, r_] := zt1[n, s] - r^(1 - s) zt1[n / r, s]
zd2[n_, s_, r_] := zt2[n, s] - r^(1 - s) zt2[n / r, s]
zd3[n_, s_, r_] := zt3[n, s] - r^(1 - s) zt2[n / r, s]
```

```
N@zd1[10 000 000.0, N@ZetaZero[1], 1.2]
```

$-5.63043 \times 10^{-6} - 0.0000947011\, i$

```
Log[2.3]
```

0.832909

```
zt1[100 000 000.0, N@ZetaZero[1]]
```

$239.497 - 665.237\, i$

```
N@zd2[100 000 000.0, 1, 3]
```

0.549306

```
N@zd3[1 000 000 000.0, N[ZetaZero[1]], 1.2]
```

791.103 + 819.146 i

```
pom[n_, s_] := Sum[ j^-s, {j, 1, Floor[n]}]
dif[n_, s_, a_, b_] := (a^(1 - s) pom[n / a, s]) - (b^(1 - s) pom[n / b, s])
div[n_, s_, a_, b_] := (a^(1 - s) pom[n / a, s]) / (b^(1 - s) pom[n / b, s])
(* If Re(s) > 1, the following is true *)
div2[n_, s_, a_, b_] := (b / a)^(s - 1)
```

```
N@dif[10^20, 2, 1, 2]
```

0.822467

```
N@dif[10^20, .5, 1, 2]
```

0.604897

```
N@div[10^200, 2 + I, 1, 2]
```

1.53848 + 1.27792 i

```
N@div[10^24, 1.1 + I, 3.2, 5.4]
```

0.911603 + 0.52468 i

```
N@div2[10^24, 1.1 + I, 3.2, 5.4]
```

0.912731 + 0.526539 i

```
dif[10^20, 1 / 2 + I, 1, 2]
```

$-2^{\frac{1}{2}-i}$ HarmonicNumber$\left[50\,000\,000\,000\,000\,000\,000, \frac{1}{2}+i\right]$ + HarmonicNumber$\left[100\,000\,000\,000\,000\,000\,000, \frac{1}{2}+i\right]$

N$\left[-2^{\frac{1}{2}-i}\right]$

-1.08787 + 0.903628 i

```
dif[10^20, ZetaZero[1], 1, 2]
```

$-2^{1-\text{ZetaZero}[1]}$ HarmonicNumber[50 000 000 000 000 000 000, ZetaZero[1]] + HarmonicNumber[100 000 000 000 000 000 000, ZetaZero[1]]

N$\left[-2^{1-\text{ZetaZero}[1]}\right]$

1.31714 - 0.514916 i

```
5 000 000 003^(.5 + 3 I)
```

-36 726. - 60 425.1 i

N$\left[1-2^{1-\text{ZetaZero}[1]}\right]$

2.31714 - 0.514916 i

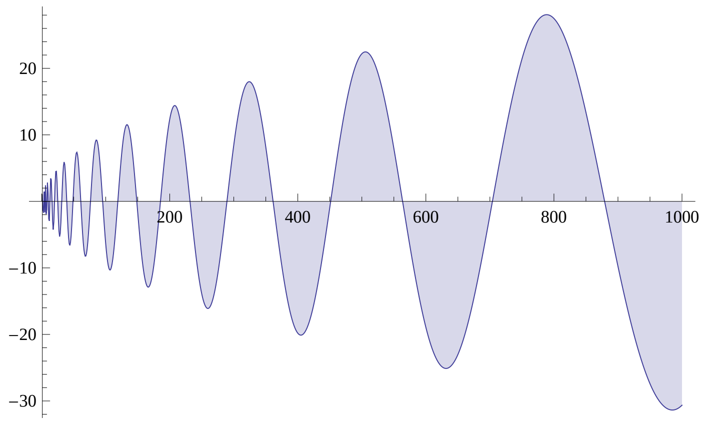
```
N[1 - 2^(1-ZetaZero[1])] (5 000 000 000 - 1) ^ (N[ZetaZero[1]])
```

46 723.1 + 161 209. i

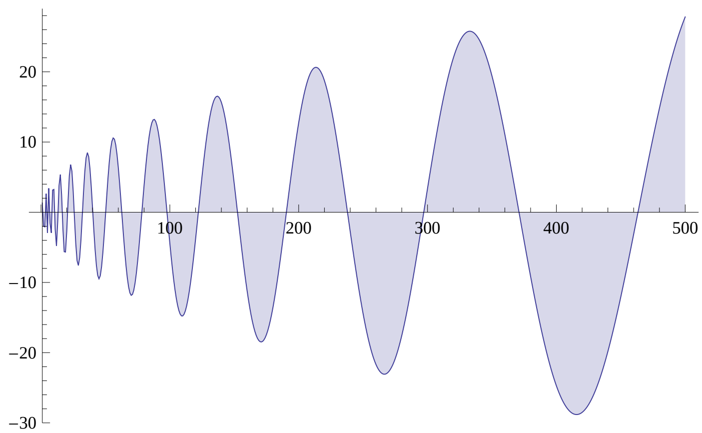
```
(5 000 000 000) ^ (N[ZetaZero[1]])
```

4482.35 + 70 568.5 i

```
DiscretePlot[ { Re[x^N[ZetaZero[1]]]}, {x, 1, 1000}]
```




```
DiscretePlot[ Re[-2^(1-ZetaZero[1]) x^N[ZetaZero[1]]], {x, 1, 500}]
```




```
Sum[MoebiusMu[j] / j HarmonicNumber[Floor[10 / j]], {j, 1, 10}]
```

1

```
md[n_] := Sum[MoebiusMu[j] / j (2), {j, 1, n}] -
  Sum[MoebiusMu[j] / j (HarmonicNumber[Floor[n / j]]), {j, 1, n}]
md2[n_] := Sum[1 / j (2 - HarmonicNumber[Floor[n / j]]), {j, 1, n}]
Table[md[n] - md[n - 1] , {n, 1, 12}]
```

$\left\{1, -1, -\frac{2}{3}, 0, -\frac{2}{5}, \frac{1}{3}, -\frac{2}{7}, 0, 0, \frac{1}{5}, -\frac{2}{11}, 0\right\}$

```
Table[If[n == 1, -1, 0] + 2 MoebiusMu[n] / n , {n, 1, 12}]
```

$\left\{1, -1, -\frac{2}{3}, 0, -\frac{2}{5}, \frac{1}{3}, -\frac{2}{7}, 0, 0, \frac{1}{5}, -\frac{2}{11}, 0\right\}$

```
Clear[pe]
pe[n_, k_] :=
 pe[n, k] = Sum[ (If[j == 1, -1, 0] + 2 MoebiusMu[j] / j) pe[n - j, k - 1], {j, 1, n - 1}]
pe[n_, 1] := (If[n == 1, -1, 0] + 2 MoebiusMu[n] / n)
pe[n_, 0] := If[n == 0, 1, 0]
pa[n_, z_] := Sum[ z^k / k ! pe[n, k], {k, 0, n}]
```

```
Table[pa[n, 1], {n, 1, 12}]
```

$$\left\{1, -\frac{1}{2}, -\frac{3}{2}, -\frac{5}{8}, \frac{11}{40}, \frac{181}{240}, \frac{589}{1680}, -\frac{2869}{13440}, -\frac{2843}{8064}, \frac{103751}{403200}, \frac{677791}{4435200}, -\frac{3547517}{17740800}\right\}$$

```
Table[md[n] , {n, 1, 12}]
```

$$\left\{1, 0, -\frac{2}{3}, -\frac{2}{3}, -\frac{16}{15}, -\frac{11}{15}, -\frac{107}{105}, -\frac{107}{105}, -\frac{107}{105}, -\frac{86}{105}, -\frac{1156}{1155}, -\frac{1156}{1155}\right\}$$

```
Sum[ md[Floor[12 / j]] / j, {j, 1, 12}]
```

$$-\frac{30581}{27720}$$

```
2 - HarmonicNumber[12]
```

$$-\frac{30581}{27720}$$

```
Sum[MoebiusMu[j] md2[Floor[12 / j]] / j, {j, 1, 12}]
```

$$-\frac{30581}{27720}$$

```
Clear[pe, pp]
pe[n_, k_] := pe[n, k] = Sum[ - (2^j - 1) / j pe[n - j, k - 1], {j, 1, n - 1}]
pe[n_, 1] := - (2^n - 1) / n
pe[n_, 0] := If[n == 0, 1, 0]
pa[n_, z_] := Sum[ z^k / k ! pe[n, k], {k, 0, n}]
pp[n_, k_] := pp[n, k] = Sum[ - pp[n - j, k - 1], {j, 1, n - 1}]
pp[n_, 0] := If[n == 0, 1, 0]
pp[n_, 1] := -1
pss[n_, z_] := Sum[ bin[z, k] pp[n, k], {k, 0, n}]
```

```
Table[D[pss[n, z], z] /. z -> 0, {n, 0, 12}]
```

$$\left\{0, -1, -\frac{3}{2}, -\frac{7}{3}, -\frac{15}{4}, -\frac{31}{5}, -\frac{21}{2}, -\frac{127}{7}, -\frac{255}{8}, -\frac{511}{9}, -\frac{1023}{10}, -\frac{2047}{11}, -\frac{1365}{4}\right\}$$

```
FullSimplify[2 - Sum[x^k, {k, 0, Infinity}]]
```

$$2 + \frac{1}{-1 + x}$$

```
Series[2 - 1/(1 - x), {x, 0, 10}]
```

$$1 - x - x^2 - x^3 - x^4 - x^5 - x^6 - x^7 - x^8 - x^9 - x^{10} + O[x]^{11}$$

```
Integrate[1 - x - x^2 - x^3, x]
```

$x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4}$

```
Table[-(2^n - 1) / n, {n, 0, 12}]
```

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

Infinity::indet : Indeterminate expression 0 ComplexInfinity encountered. ≫

$\{\text{Indeterminate}, -1, -\frac{3}{2}, -\frac{7}{3}, -\frac{15}{4}, -\frac{31}{5}, -\frac{21}{2}, -\frac{127}{7}, -\frac{255}{8}, -\frac{511}{9}, -\frac{1023}{10}, -\frac{2047}{11}, -\frac{1365}{4}\}$

```
Table[pa[n, 1], {n, 0, 12}]
```

{1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1}

```
Ak[n_] := Sum[ - (2^j - 1) / j , {j, 1, n}]
invAk[n_] := Sum[ MoebiusMu[j] / j Ak[Floor[n / j]], {j, 1, n}]
invAk2[n_] := Sum[ invAk[Floor[n / j]] / j, {j, 1, n}]

(* https://oeis.org/A097344 *)

Table[Ak[n], {n, 1, 10}]
```

$\{-1, -\frac{5}{2}, -\frac{29}{6}, -\frac{103}{12}, -\frac{887}{60}, -\frac{1517}{60}, -\frac{18239}{420}, -\frac{63253}{840}, -\frac{332839}{2520}, -\frac{118127}{504}\}$

```
Table[HypergeometricPFQ[{1, 1, -n}, {2, 2}, -1] // Numerator, {n, 0, 12}]
```

{1, 5, 29, 103, 887, 1517, 18 239, 63 253, 332 839, 118 127, 2 331 085, 4 222 975, 100 309 579}

```
Table[invAk[n] - invAk[n - 1], {n, 1, 20}]
```

{-1, -1, -2, -3, -6, -9, -18, -30, -56, -99, -186,
-335, -630, -1161, -2182, -4080, -7710, -14 532, -27 594, -52 377}

```
Table[invAk2[n] - invAk2[n - 1], {n, 1, 10}]
```

$\{-1, -\frac{3}{2}, -\frac{7}{3}, -\frac{15}{4}, -\frac{31}{5}, -\frac{21}{2}, -\frac{127}{7}, -\frac{255}{8}, -\frac{511}{9}, -\frac{1023}{10}\}$

```
(* https://oeis.org/A059966 *)

Table[1 / n Apply[Plus, Map[(MoebiusMu[n / #] (2^# - 1)) &, Divisors[n]]], {n, 1, 20}]
```

{1, 1, 2, 3, 6, 9, 18, 30, 56, 99, 186, 335, 630, 1161, 2182, 4080, 7710, 14 532, 27 594, 52 377}

```
Clear[pe]
pe[n_, k_] := pe[n, k] = Sum[ (invAk[j] - invAk[j - 1]) pe[n - j, k - 1], {j, 1, n - 1}]
pe[n_, 1] := invAk[n] - invAk[n - 1]
pe[n_, 0] := If[n == 0, 1, 0]
pa[n_, z_] := Sum[ z^k / k ! pe[n, k], {k, 0, n}]
```

```
Table[pa[n, -1], {n, 1, 20}]
```

$$\left\{1, \frac{3}{2}, \frac{19}{6}, \frac{145}{24}, \frac{507}{40}, \frac{17\,491}{720}, \frac{251\,203}{5040}, \frac{1\,325\,483}{13\,440}, \frac{14\,365\,325}{72\,576}, \frac{1\,431\,651\,331}{3\,628\,800}, \frac{10\,531\,651\,057}{13\,305\,600}, \frac{756\,534\,213\,073}{479\,001\,600}, \frac{19\,695\,041\,269\,489}{6\,227\,020\,800}, \frac{4\,082\,229\,886\,613}{645\,765\,120}, \frac{16\,536\,564\,083\,960\,131}{1\,307\,674\,368\,000}, \frac{529\,078\,125\,772\,064\,641}{20\,922\,789\,888\,000}, \frac{5\,997\,196\,382\,277\,344\,171}{118\,562\,476\,032\,000}, \frac{49\,816\,223\,747\,401\,636\,303}{492\,490\,285\,056\,000}, \frac{4\,922\,208\,068\,556\,923\,587\,127}{24\,329\,020\,081\,766\,400}, \frac{328\,132\,653\,266\,986\,162\,905\,307}{810\,967\,336\,058\,880\,000}\right\}$$

```
Series[(1 - 2 x)/(1 - x), {x, 0, 10}]
```

$$1 - x - x^2 - x^3 - x^4 - x^5 - x^6 - x^7 - x^8 - x^9 - x^{10} + O[x]^{11}$$

```
Product[((1 - 2 x)/(1 - x))^(MoebiusMu[k] / k), {k, 1, 10}]
```

$$\frac{1 - 2x}{\left(\frac{1-2x}{1-x}\right)^{191/210} (1 - x)}$$

```
Clear[pq, n, Pq]
arb[n_] := Sum[ MoebiusMu[n / d] ((2^d) - 1) / n, {d, Divisors[n]}]
pq[1] := -1
pq[2] := -1
pq[n_] := pq[n] =
  Floor[MangoldtLambda[n] / Log[n]] - If[Log[2, n] == Floor[Log[2, n]], arb[Log[2, n]], 0]
Pq[n_, k_] := Pq[n, k] = Sum[ pq[j] Pq[Floor[n / j], k - 1], {j, 2, n}]
Pq[n_, 0] := UnitStep[n - 1]
pqz[n_, z_] := Sum[ z^k / k ! Pq[n, k], {k, 0, Log2@n}]
```

```
Table[pq[n] - (pm[n, 0] - pm[n - 1, 0]), {n, 2, 100}]
```

{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

```
Table[pqz[n, 1], {n, 1, 100}]
```

$$\left\{1, 0, 1, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}, \frac{3}{2}, \frac{1}{3}, \frac{5}{6}, -\frac{1}{6}, \frac{5}{6}, \frac{1}{3}, \frac{4}{3}, \frac{1}{3}, \frac{4}{3}, \frac{3}{8}, \frac{11}{8}, \frac{7}{8}, \frac{15}{8}, \frac{11}{8}, \frac{19}{8}, \frac{11}{8}, \frac{19}{8}, \frac{29}{24}, \frac{41}{24}, \frac{17}{24}, \frac{7}{8}, \frac{3}{8}, \frac{11}{8}, \frac{3}{8}, \frac{11}{8}, -\frac{29}{30}, \frac{1}{30}, -\frac{29}{30}, \frac{1}{30}, -\frac{13}{60}, \frac{47}{60}, -\frac{13}{60}, \frac{47}{60}, -\frac{23}{60}, \frac{37}{60}, -\frac{23}{60}, \frac{37}{60}, \frac{7}{60}, \frac{37}{60}, -\frac{23}{60}, \frac{37}{60}, -\frac{41}{120}, \frac{19}{120}, -\frac{41}{120}, \frac{79}{120}, \frac{19}{120}, \frac{139}{120}, \frac{119}{120}, \frac{239}{120}, \frac{33}{40}, \frac{73}{40}, \frac{33}{40}, \frac{73}{40}, \frac{53}{40}, \frac{93}{40}, \frac{53}{40}, \frac{73}{40}, \frac{101}{144}, \frac{245}{144}, \frac{101}{144}, \frac{245}{144}, \frac{173}{144}, \frac{317}{144}, \frac{173}{144}, \frac{317}{144}, \frac{233}{144}, \frac{377}{144}, \frac{233}{144}, \frac{305}{144}, \frac{233}{144}, \frac{377}{144}, \frac{233}{144}, \frac{377}{144}, \frac{239}{144}, \frac{245}{144}, \frac{101}{144}, \frac{245}{144}, \frac{173}{144}, \frac{317}{144}, \frac{173}{144}, \frac{317}{144}, \frac{149}{144}, \frac{293}{144}, \frac{221}{144}, \frac{365}{144}, \frac{293}{144}, \frac{437}{144}, \frac{293}{144}, \frac{437}{144}, \frac{499}{720}, \frac{1219}{720}, \frac{859}{720}, \frac{1219}{720}, \frac{1039}{720}\right\}$$