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Clear[zeta, zetaytt]
bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k!
zeta[n_, s_, k_] := zeta[n, k] = Sum[j^(-s) zeta[n - j, s, k - 1], {j, 1, n}]
zeta[n_, s_, 0] := UnitStep[n]
dzeta[n_, s_, k_] := zeta[n, s, k] - zeta[n - 1, s, k]
zetapl[n_, s_, z_] := Sum[bin[z, k] zeta[n, s, k], {k, 0, n}]
dzetapl[n_, s_, z_] := zetapl[n, s, z] - zetapl[n - 1, s, z]
lzetapl[n_, s_] := Sum[(-1)^(k + 1) / k zeta[n, s, k], {k, 1, n}]
lzetapl[k[n_, s_, k_] := D[zetapl[n, s, z], {z, k}] /. z -> 0
dlzetapl[n_, s_] := lzetapl[n, s] - lzetapl[n - 1, s]
zetapl1[n_, s_, z_] := Sum[z^k / k! lzetapl[k[n, s, k], {k, 0, n}]

zetaml[n_, s_, k_] := zetaml[n, k] = Sum[j^(-s) zetaml[n - j, s, k - 1], {j, 2, n}]
zetaml[n_, s_, 0] := UnitStep[n]
dzetaml[n_, s_, k_] := zetaml[n, s, k] - zetaml[n - 1, s, k]
zetatoml[n_, s_, k_] := Sum[(-1)^j bin[k, j] zeta[n - j, s, k - j], {j, 0, k}]
zetamtoz[n_, s_, k_] := Sum[bin[k, j] zetaml[n - j, s, k - j], {j, 0, k}]
zetamm[n_, s_, z_] := Sum[bin[z, k] zetaml[n, s, k], {k, 0, (n) / 2}]

zetayp0[n_, s_, k_, y_] :=
  zetayp0[n, s, k, y] = zetayp0[n, s, k - 1, y] + Sum[j^(-s) zetayp0[n - j, s, k - 1, y], {j, y, n}]
zetayp0[n_, s_, 0, y_] := UnitStep[n]
zetayt[n_, s_, z_, y_] :=
  If[n < y, 1, Sum[bin[z, k] zetayp0[n - yk, s, z - k, y + 1], {k, 0, n / y}]]
zetaytt[n_, s_, z_, y_] := zetaytt[n, s, z, y] = If[n < y, 1, zetaytt[n, s, z, y + 1] +
  Sum[bin[z, k] (y^(-s k)) zetaytt[n - yk, s, z - k, y + 1], {k, 1, n / y}]]

etaml[n_, s_, k_] := etaml[n, k] = Sum[(-1)^(j + 1) j^(-s) etaml[n - j, s, k - 1], {j, 2, n}]
etaml[n_, s_, 0] := UnitStep[n]
etamm[n_, s_, z_] := Sum[bin[z, k] etaml[n, s, k], {k, 0, (n) / 2}]
etaroots[n_, s_] := List@@NRoots[etamm[n, s, z] == 0, z][[All, 2]]

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Expand@zetapl[15, 0, z]

$$\begin{aligned}
& 1 + \frac{1195757z}{360360} + \frac{13215487z^2}{2802800} + \frac{35118025721z^3}{9081072000} + \frac{2065639z^4}{997920} + \\
& \frac{277382447z^5}{359251200} + \frac{2271089z^6}{10886400} + \frac{54576553z^7}{1306368000} + \frac{4783z^8}{762048} + \frac{324509z^9}{457228800} + \frac{109z^{10}}{1814400} + \\
& \frac{26921z^{11}}{7185024000} + \frac{z^{12}}{5987520} + \frac{47z^{13}}{9340531200} + \frac{z^{14}}{10897286400} + \frac{z^{15}}{1307674368000}
\end{aligned}$$

Table[dlzetapl[n, 0], {n, 0, 15}]

$$\left\{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}, \frac{1}{11}, \frac{1}{12}, \frac{1}{13}, \frac{1}{14}, \frac{1}{15}\right\}$$

Table[lzetaPl[n, 0], {n, 0, 15}]

$$\left\{0, 1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}, \frac{137}{60}, \frac{49}{20}, \frac{363}{140}, \frac{761}{280}, \frac{7129}{2520}, \frac{7381}{2520}, \frac{83711}{27720}, \frac{86021}{27720}, \frac{1145993}{360360}, \frac{1171733}{360360}, \frac{1195757}{360360}\right\}$$

Table[HarmonicNumber[n, 1], {n, 0, 15}]

$$\left\{0, 1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}, \frac{137}{60}, \frac{49}{20}, \frac{363}{140}, \frac{761}{280}, \frac{7129}{2520}, \frac{7381}{2520}, \frac{83711}{27720}, \frac{86021}{27720}, \frac{1145993}{360360}, \frac{1171733}{360360}, \frac{1195757}{360360}\right\}$$

Table[dlzetaPl[n, -1], {n, 1, 18}]

$$\left\{1, \frac{3}{2}, \frac{4}{3}, \frac{3}{4}, \frac{1}{5}, 0, \frac{1}{7}, \frac{3}{8}, \frac{4}{9}, \frac{3}{10}, \frac{1}{11}, 0, \frac{1}{13}, \frac{3}{14}, \frac{4}{15}, \frac{3}{16}, \frac{1}{17}, 0\right\}$$

Grid[Table[dzeta[3, p, k], {p, -3, 3}, {k, 1, 3}]]

27	16	1
9	8	1
3	4	1
1	2	1
$\frac{1}{3}$	1	1
$\frac{1}{9}$	$\frac{1}{2}$	1
$\frac{1}{27}$	$\frac{1}{4}$	1

Grid[Table[(3 + 1 - k) ^ -p Binomial[3 - 1, k - 1] , {p, -3, 3}, {k, 1, 3}]]

27	16	1
9	8	1
3	4	1
1	2	1
$\frac{1}{3}$	1	1
$\frac{1}{9}$	$\frac{1}{2}$	1
$\frac{1}{27}$	$\frac{1}{4}$	1

Grid[Table[dzeta[5, p, k] / Binomial[5 - 1, k - 1], {p, -5, 5}, {k, 1, 5}]]

3125	4400	$\frac{1267}{2}$	32	1
625	776	$\frac{337}{2}$	16	1
125	140	$\frac{91}{2}$	8	1
25	26	$\frac{25}{2}$	4	1
5	5	$\frac{7}{2}$	2	1
1	1	1	1	1
$\frac{1}{5}$	$\frac{5}{24}$	$\frac{7}{24}$	$\frac{1}{2}$	1
$\frac{1}{25}$	$\frac{13}{288}$	$\frac{25}{288}$	$\frac{1}{4}$	1
$\frac{1}{125}$	$\frac{35}{3456}$	$\frac{91}{3456}$	$\frac{1}{8}$	1
$\frac{1}{625}$	$\frac{97}{41472}$	$\frac{337}{41472}$	$\frac{1}{16}$	1
$\frac{1}{3125}$	$\frac{275}{497664}$	$\frac{1267}{497664}$	$\frac{1}{32}$	1

FullSimplify@ $\text{bin}[z, k] \text{bin}[n - 1, k], \{k, 0, n\}$

$$\frac{\Gamma[n + z]}{\Gamma[n] \Gamma[1 + z]} - \frac{(\Gamma[1 + n - z] \text{HypergeometricPFQRegularized}[\{1, 2, 1 + n - z\}, \{2 + n, 2 + n\}, 1])}{(\Gamma[1 - n] \Gamma[-z])}$$

Sum $[\text{Binomial}[z, k] \text{Binomial}[n - 1, k], \{k, 0, n\}]$

$$\frac{(-1 + n + z)!}{(-1 + n)! z!} \Big/ . \{z \rightarrow 13, n \rightarrow 4\}$$

560

Pochhammer $[6, 15] / 15!$

15 504

dzeta $\text{p1}[15, 0, 6]$

15 504

FullSimplify@Sum $[\text{Binomial}[t - 1, k], \{t, 1, n\}]$

$$\frac{k \text{Binomial}[0, k] + (-k + n) \text{Binomial}[n, k]}{1 + k}$$

FullSimplify@Sum $[\text{Pochhammer}[z, t] / t!, \{t, 0, n\}]$

$$\frac{\Gamma[1 + n + z]}{\Gamma[1 + n] \Gamma[1 + z]}$$

FullSimplify@Sum $[\text{Binomial}[t + z - 1, t], \{t, 0, n\}]$

$$\frac{(1 + n) \text{Binomial}[n + z, 1 + n]}{z}$$

zetap1 $[13, 0, 8]$

203 490

$$\frac{(1 + n) \text{Binomial}[n + z, 1 + n]}{z} \Big/ . \{n \rightarrow 13, z \rightarrow 8\}$$

203 490

dzeta $[16, 0, 4]$

455

$$\frac{k \text{Binomial}[0, k] + (-k + n) \text{Binomial}[n, k]}{1 + k} \Big/ . \{n \rightarrow 16, k \rightarrow 4\}$$

4368

$$\text{Limit}\left[\text{D}\left[\frac{(1 + n) \text{Binomial}[n + z, 1 + n]}{z}, z\right], z \rightarrow 0\right]$$

EulerGamma + PolyGamma[0, 1 + n] /. n -> 4

$$\frac{25}{12}$$

Table[dzeta[9, 0, k], {k, 1, 10}]

{1, 8, 28, 56, 70, 56, 28, 8, 1, 0}

Table[Binomial[9 - 1, k - 1], {k, 1, 10}]

{1, 8, 28, 56, 70, 56, 28, 8, 1, 0}

Sum[Binomial[t, 5 - 1], {t, 0, 9}]

252

dzeta[10, 0, 5]

126

FullSimplify@Sum[Binomial[t, k - 1], {t, 0, n - 1}]

$$\frac{(1 - k + n) \text{Binomial}[n, -1 + k] - \frac{\sin[k\pi]}{\pi}}{k}$$

$$\frac{(1 - k + n) \text{Binomial}[n, -1 + k] - \frac{\sin[k\pi]}{\pi}}{k} /. \{k \rightarrow 5\}$$

$$\frac{1}{120} (-4 + n) (-3 + n) (-2 + n) (-1 + n) n$$

bin[n, 5]

$$\frac{1}{120} (-4 + n) (-3 + n) (-2 + n) (-1 + n) n$$

bin[9, 4]

126

Table[dlzetapl[n, 0], {n, 1, 12}]

{1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{11}$, $\frac{1}{12}$ }

Table[dlzetapl[n, -1], {n, 1, 12}]

{1, $\frac{3}{2}$, $\frac{4}{3}$, $\frac{3}{4}$, $\frac{1}{5}$, 0, $\frac{1}{7}$, $\frac{3}{8}$, $\frac{4}{9}$, $\frac{3}{10}$, $\frac{1}{11}$, 0}

Table[dlzetapl[n, -2], {n, 1, 15}]

{1, $\frac{7}{2}$, $\frac{16}{3}$, $\frac{11}{4}$, $-\frac{29}{5}$, $-\frac{40}{3}$, $-\frac{27}{7}$, $\frac{243}{8}$, $\frac{520}{9}$, $\frac{47}{10}$, $-\frac{1737}{11}$, $-\frac{784}{3}$, $\frac{729}{13}$, $\frac{1753}{2}$, $\frac{18496}{15}$ }

Table[dlzetapl[n, 1], {n, 1, 15}]

{1, 0, $\frac{1}{6}$, $\frac{1}{24}$, $\frac{1}{15}$, $\frac{13}{360}$, $\frac{97}{2520}$, $\frac{571}{20160}$, $\frac{1217}{45360}$,
 $\frac{3391}{151200}$, $\frac{953}{46200}$, $\frac{13003}{712800}$, $\frac{434737}{25945920}$, $\frac{2767231}{181621440}$, $\frac{383735791}{27243216000}$ }

Table[dlzetapl[n, 2], {n, 1, 15}]

$$\left\{1, -\frac{1}{4}, \frac{7}{36}, -\frac{23}{288}, \frac{11}{150}, -\frac{523}{16200}, \frac{214237}{6350400}, -\frac{1439687}{101606400}, \frac{11825941}{685843200}, -\frac{144510727}{22861440000}, \frac{3278615273}{345779280000}, -\frac{53684706431}{19916886528000}, \frac{93229485781129}{16829769116160000}, -\frac{32337215505289}{33319542896640000}, \frac{2540135864502777503}{742192818022656000000}\right\}$$

Sum[j^-s k^-s 1^-s, {j, 2, Infinity}, {k, 2, Infinity}, {1, 2, Infinity}]

$$(-1 + \text{Zeta}[s])^3$$

Expand@zetapl[10, 0, z]

$$1 + \frac{7381z}{2520} + \frac{177133z^2}{50400} + \frac{84095z^3}{36288} + \frac{341693z^4}{362880} + \frac{8591z^5}{34560} + \frac{7513z^6}{172800} + \frac{121z^7}{24192} + \frac{11z^8}{30240} + \frac{11z^9}{725760} + \frac{z^{10}}{3628800}$$

Expand@zetapl1[10, 0, z]

$$1 + \frac{7381z}{2520} + \frac{177133z^2}{50400} + \frac{84095z^3}{36288} + \frac{341693z^4}{362880} + \frac{8591z^5}{34560} + \frac{7513z^6}{172800} + \frac{121z^7}{24192} + \frac{11z^8}{30240} + \frac{11z^9}{725760} + \frac{z^{10}}{3628800}$$

Table[{zetaml[13, -1, k], zetatoml[13, -1, k]}, {k, 0, 5}]

$$\{\{1, 1\}, \{90, 90\}, \{1210, 1210\}, \{5202, 5202\}, \{8361, 8361\}, \{4712, 4712\}\}$$

zetatoml[10, 0, 3]

$$35$$

zeta[10, 0, 2] - 2 zeta[9, 0, 1] + 1

$$28$$

Table[{zetamtoz[14, 1, k], zeta[14, 1, k]}, {k, 0, 5}]

$$\left\{\{1, 1\}, \left\{\frac{1171733}{360360}, \frac{1171733}{360360}\right\}, \left\{\frac{30946717}{3439800}, \frac{30946717}{3439800}\right\}, \left\{\frac{406841}{19008}, \frac{406841}{19008}\right\}, \left\{\frac{21939781}{498960}, \frac{21939781}{498960}\right\}, \left\{\frac{22463}{288}, \frac{22463}{288}\right\}\right\}$$

Sum[Binomial[z, k] (Zeta[s] - 1)^k, {k, 0, Infinity}]

$$\text{Zeta}[s]^z$$

Expand@etamm[20, N@ZetaZero[1], z]

$$1 - (1.12086 - 0.0339383i)z - (0.357693 - 0.240756i)z^2 + (0.593717 - 0.831922i)z^3 - (0.168903 - 0.630057i)z^4 + (0.0224877 - 0.220685i)z^5 - (0.00230993 - 0.0389419i)z^6 - (0.000104797 + 0.00357847i)z^7 + (0.0000232037 + 0.000155899i)z^8 - (1.41895 \times 10^{-7} + 2.59262 \times 10^{-6}i)z^9 - (7.17928 \times 10^{-9} - 4.7559 \times 10^{-9}i)z^{10}$$

etaroots[20, N@ZetaZero[1]]

$$\{-0.894341 - 0.342431i, 0.073414 + 1.95327i, 0.897335 - 0.100598i, 2.02688 + 0.777048i, 3.65759 - 1.00578i, 3.77978 - 4.07971i, 9.71036 + 10.3563i, 10.517 + 1.54507i, 35.2922 - 6.81704i, 87.4672 - 262.37i\}$$

zetayp0[50, 1, 3, 4]

9 995 969 299 107 722 516 417 376 266 028 561 851

284 735 769 919 272 640 936 826 615 877 120 000

zetam1[10, 0, 2]

28

Expand@zetaytt[50, 1, z, 1]

$$\begin{aligned}
 & 1 + \frac{531\,071\,278\,159\,549\,656\,739\,597\,766\,208\,618\,684\,993\,560\,973\,576\,803\,531\,z}{306\,058\,241\,461\,010\,670\,011\,477\,599\,366\,523\,740\,707\,225\,600\,000\,000\,000} + \\
 & \left(\frac{8\,439\,912\,152\,221\,457\,826\,052\,578\,953\,908\,456\,770\,082\,144\,525\,090\,388\,513\,011\,z^2}{5\,729\,410\,280\,150\,119\,742\,614\,860\,660\,141\,324\,426\,039\,263\,232\,000\,000\,000\,000} + \right. \\
 & \left(\frac{3\,207\,972\,262\,236\,996\,051\,193\,293\,894\,234\,461\,910\,896\,233\,104\,511\,030\,294\,679\,z^3}{3\,928\,738\,477\,817\,224\,966\,364\,475\,881\,239\,765\,320\,712\,637\,644\,800\,000\,000\,000} + \right. \\
 & \left(\frac{1\,206\,605\,019\,393\,266\,194\,512\,335\,582\,433\,359\,510\,458\,179\,267\,895\,510\,802\,633\,z^4}{3\,626\,527\,825\,677\,438\,430\,490\,285\,428\,836\,706\,449\,888\,588\,595\,200\,000\,000\,000} + \right. \\
 & \left(\frac{1\,280\,346\,464\,415\,502\,187\,432\,452\,598\,441\,306\,820\,414\,330\,931\,596\,387\,033\,527\,z^5}{12\,036\,985\,974\,588\,944\,577\,797\,543\,125\,500\,557\,578\,353\,613\,209\,600\,000\,000\,000} + \right. \\
 & \left(\frac{106\,251\,098\,251\,073\,051\,486\,130\,560\,673\,113\,369\,414\,445\,720\,404\,979\,328\,427\,z^6}{3\,821\,265\,388\,758\,395\,104\,062\,712\,103\,333\,510\,342\,334\,480\,384\,000\,000\,000\,000} + \right. \\
 & \frac{555\,460\,507\,392\,110\,474\,090\,325\,378\,092\,165\,137\,109\,845\,640\,728\,690\,357\,z^7}{90\,828\,855\,719\,140\,436\,986\,614\,590\,598\,459\,112\,161\,489\,715\,200\,000\,000\,000} + \\
 & \left(\frac{22\,578\,578\,868\,227\,416\,441\,327\,367\,213\,018\,352\,563\,580\,150\,746\,857\,228\,537\,z^8}{19\,538\,296\,074\,695\,098\,445\,120\,649\,710\,957\,426\,793\,849\,343\,180\,800\,000\,000\,000} + \right. \\
 & \left(\frac{27\,842\,967\,858\,374\,630\,752\,755\,743\,729\,557\,489\,441\,620\,048\,159\,495\,571\,z^9}{146\,050\,385\,940\,411\,865\,453\,559\,673\,919\,116\,977\,694\,887\,116\,800\,000\,000\,000} + \right. \\
 & \left(\frac{24\,989\,702\,114\,589\,831\,647\,611\,060\,058\,512\,965\,098\,074\,301\,278\,166\,291\,z^{10}}{898\,771\,605\,787\,149\,941\,252\,674\,916\,425\,335\,247\,353\,151\,488\,000\,000\,000\,000} + \right. \\
 & \frac{246\,921\,438\,131\,258\,156\,830\,647\,989\,240\,234\,899\,142\,078\,018\,423\,z^{11}}{68\,176\,163\,351\,808\,549\,634\,057\,497\,453\,199\,662\,828\,748\,800\,000\,000\,000} + \\
 & \frac{677\,417\,398\,284\,610\,174\,021\,263\,850\,928\,001\,212\,710\,036\,954\,901\,z^{12}}{1\,594\,273\,358\,380\,753\,776\,057\,959\,940\,444\,053\,653\,841\,510\,400\,000\,000\,000} + \\
 & \frac{137\,432\,942\,445\,730\,462\,869\,300\,564\,374\,392\,578\,937\,523\,124\,901\,z^{13}}{3\,039\,747\,869\,979\,303\,866\,350\,510\,286\,446\,662\,299\,991\,146\,496\,000\,000\,000} + \\
 & \left(\frac{2\,001\,272\,098\,526\,684\,372\,243\,511\,126\,172\,232\,768\,877\,980\,687\,569\,z^{14}}{455\,962\,180\,496\,895\,579\,952\,576\,542\,966\,999\,344\,998\,671\,974\,400\,000\,000\,000} + \right. \\
 & \frac{32\,997\,125\,029\,925\,601\,096\,987\,369\,881\,176\,141\,725\,529\,z^{15}}{84\,452\,454\,602\,465\,545\,717\,864\,017\,115\,450\,032\,783\,360\,000\,000\,000} + \\
 & \frac{2\,597\,110\,783\,568\,126\,940\,101\,673\,201\,933\,423\,214\,680\,779\,z^{16}}{81\,074\,356\,418\,366\,923\,889\,149\,456\,430\,832\,031\,472\,025\,600\,000\,000\,000} + \\
 & \frac{12\,699\,921\,944\,041\,405\,441\,239\,350\,146\,535\,910\,329\,699\,z^{17}}{5\,230\,603\,639\,894\,640\,250\,912\,868\,156\,827\,872\,998\,195\,200\,000\,000\,000} + \\
 & \frac{5\,535\,616\,236\,972\,328\,854\,276\,793\,405\,122\,815\,078\,633\,z^{18}}{32\,429\,742\,567\,346\,769\,555\,659\,782\,572\,332\,812\,588\,810\,240\,000\,000\,000} + \\
 & \frac{1\,256\,982\,024\,530\,935\,057\,804\,259\,816\,859\,907\,447\,z^{19}}{112\,603\,272\,803\,287\,394\,290\,485\,356\,153\,933\,377\,044\,480\,000\,000\,000} +
 \end{aligned}$$

$$\begin{aligned}
& \frac{279\,653\,354\,683\,608\,849\,495\,264\,268\,825\,154\,289\,869}{410\,776\,739\,186\,392\,414\,371\,690\,579\,249\,548\,959\,458\,263\,040\,000\,000\,000} z^{20} + \\
& \frac{1\,138\,288\,936\,175\,287\,150\,163\,294\,742\,631\,667}{29\,333\,373\,423\,258\,017\,936\,052\,094\,149\,861\,153\,177\,600\,000\,000\,000} z^{21} + \\
& \frac{10\,829\,856\,107\,626\,684\,025\,418\,807\,944\,851}{5\,228\,402\,207\,386\,267\,085\,824\,100\,114\,674\,325\,913\,600\,000\,000\,000} z^{22} + \\
& \frac{71\,083\,355\,099\,189\,934\,680\,977\,901\,323\,789}{685\,389\,442\,468\,952\,860\,464\,444\,792\,963\,652\,323\,901\,440\,000\,000\,000} z^{23} + \\
& \frac{11\,831\,151\,839\,125\,428\,360\,762\,174\,338\,147}{2\,425\,224\,181\,043\,987\,044\,720\,343\,113\,563\,692\,838\,420\,480\,000\,000\,000} z^{24} + \\
& \frac{39\,792\,879\,087\,048\,235\,988\,180\,483\,677}{184\,373\,768\,149\,542\,874\,744\,821\,406\,294\,315\,829\,821\,440\,000\,000\,000} z^{25} + \\
& \frac{1\,574\,951\,961\,675\,957\,391\,614\,258\,677\,563}{175\,155\,079\,742\,065\,731\,007\,580\,335\,979\,600\,038\,330\,368\,000\,000\,000\,000} z^{26} + \\
& \frac{55\,205\,332\,746\,252\,566\,395\,439\,417}{156\,388\,464\,055\,415\,831\,256\,768\,157\,124\,642\,891\,366\,400\,000\,000\,000} z^{27} + \\
& \frac{1\,115\,204\,119\,570\,471\,183\,366\,951}{85\,302\,798\,575\,681\,362\,503\,691\,722\,067\,987\,031\,654\,400\,000\,000\,000} z^{28} + \\
& \frac{22\,692\,664\,775\,998\,965\,959\,226\,497}{49\,690\,734\,578\,999\,082\,383\,672\,247\,489\,864\,793\,482\,854\,400\,000\,000\,000} z^{29} + \\
& \frac{1\,248\,146\,798\,539\,881\,371\,308\,447}{82\,817\,890\,964\,998\,470\,639\,453\,745\,816\,441\,322\,471\,424\,000\,000\,000\,000} z^{30} + \\
& \frac{3\,648\,473\,280\,449\,048\,234\,041}{7\,779\,862\,484\,590\,765\,423\,706\,260\,970\,635\,396\,959\,436\,800\,000\,000\,000} z^{31} + \\
& \frac{3\,867\,753\,753\,606\,787\,844\,831}{280\,075\,049\,445\,267\,555\,253\,425\,394\,942\,874\,290\,539\,724\,800\,000\,000\,000} z^{32} + \\
& \frac{3\,247\,613\,499\,597\,151}{8\,491\,239\,675\,153\,636\,770\,962\,448\,306\,538\,754\,867\,200\,000\,000\,000} z^{33} + \\
& \frac{16\,640\,548\,791\,492\,167}{1\,654\,578\,702\,415\,651\,507\,941\,825\,641\,445\,551\,662\,694\,400\,000\,000\,000} z^{34} + \\
& \frac{2\,636\,556\,281\,531}{10\,674\,701\,305\,907\,429\,083\,495\,649\,299\,648\,720\,404\,480\,000\,000\,000} z^{35} + \\
& \frac{1\,882\,644\,951\,661\,687}{324\,297\,425\,673\,467\,695\,556\,597\,825\,723\,328\,125\,888\,102\,400\,000\,000\,000} z^{36} + \\
& \frac{1\,504\,153\,693\,362\,223}{11\,999\,004\,749\,918\,304\,735\,594\,119\,551\,763\,140\,657\,859\,788\,800\,000\,000\,000} z^{37} + \\
& \frac{(1\,204\,136\,467\,557\,823)}{20\,075\,770\,943} z^{38} \Big/ 455\,962\,180\,496\,895\,579\,952\,576\,542\,966\,999\,344\,998\,671\,974\,400\,000\,000\,000 + \\
& \frac{20\,075\,770\,943}{407\,109\,089\,729\,371\,053\,529\,086\,199\,077\,677\,986\,605\,957\,120\,000\,000\,000} z^{39} + \\
& \frac{509\,187\,949}{542\,812\,119\,639\,161\,404\,705\,448\,265\,436\,903\,982\,141\,276\,160\,000\,000\,000} z^{40} + \\
& \frac{297\,131\,929}{20\,232\,088\,095\,641\,470\,539\,021\,253\,529\,920\,966\,607\,083\,929\,600\,000\,000\,000} z^{41} +
\end{aligned}$$

$$\begin{aligned}
 & \frac{536\,623\,z^{42}}{2\,092\,974\,630\,583\,600\,400\,588\,405\,537\,578\,031\,028\,319\,027\,200\,000\,000\,000} + \\
 & \frac{30\,619\,z^{43}}{9\,615\,566\,079\,139\,077\,840\,387\,469\,440\,799\,280\,445\,366\,730\,752\,000\,000\,000} + \\
 & \frac{(845\,381\,z^{44})}{16\,077\,226\,484\,320\,538\,149\,127\,848\,905\,016\,396\,904\,653\,173\,817\,344\,000\,000\,000} + \\
 & \frac{z^{45}}{2\,126\,617\,259\,830\,759\,014\,434\,900\,648\,811\,692\,712\,255\,710\,822\,400\,000\,000} + \\
 & \frac{(53\,z^{46})}{6\,878\,277\,699\,765\,111\,187\,312\,881\,786\,000\,318\,616\,202\,064\,691\,200\,000\,000\,000} + \\
 & \frac{z^{47}}{23\,965\,088\,016\,478\,904\,770\,004\,419\,010\,373\,310\,885\,176\,614\,584\,320\,000\,000\,000} + \\
 & \frac{z^{48}}{1\,379\,323\,954\,726\,230\,296\,762\,476\,560\,819\,263\,893\,169\,054\,039\,408\,640\,000\,000\,000} + \\
 & \frac{z^{49}}{608\,281\,864\,034\,267\,560\,872\,252\,163\,321\,295\,376\,887\,552\,831\,379\,210\,240\,000\,000\,000} + \\
 & \frac{z^{50}}{30\,414\,093\,201\,713\,378\,043\,612\,608\,166\,064\,768\,844\,377\,641\,568\,960\,512\,000\,000\,000\,000}
 \end{aligned}$$