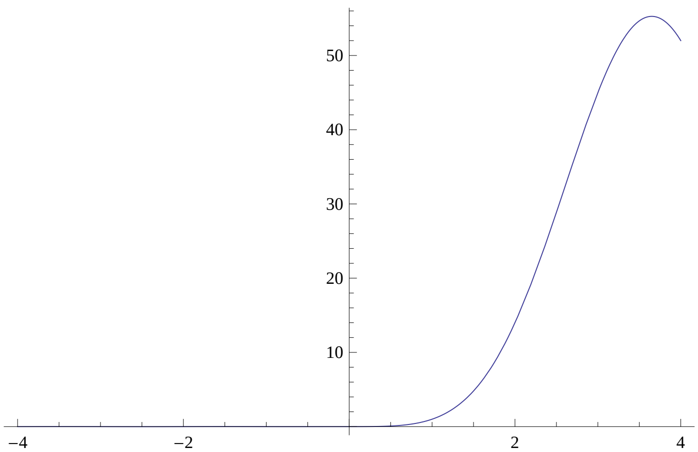
```
Clear[rb]
bin2[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
rb[n_, k_, f_] := rb[n, k, f] = Sum[f[j] rb[Floor[n / j], k - 1, f], {j, 2, n}]
rb[n_, 0, f_] := UnitStep[n - 1]
lrb[n_, f_] := Sum[(-1)^(k + 1) / k rb[n, k, f], {k, 1, Log2@n}]
rbz[n_, z_, f_] := Sum[bin2[z, k] rb[n, k, f], {k, 0, Log2@n}]
lrz[n_, z_, f_] := Sin[Pi z] / Pi Sum[ (-1)^k / (z - k) rb[n, k, f], {k, 0, Log2@n}]
lr[n_, z_] := Limit[lrz[n, z1, id], z1 → z]
id[n_] := 1
df[n_, z_] := df[n, z] = FullSimplify[lrz[n, z, id] - lrz[n - 1, z, id]]
dfa[n_, z_] := df[n, z] = Limit[df[n, z2], z2 → z]
```

```
Plot[df[120, z], {z, -4, 4}]
```




```
FullSimplify[Sum[dfa[j, z] dfa[120 / j, y], {j, Divisors[120]}]]
```

$$\left(\left(720 z^2 (1+3 z)-y^5 (-1+z) z (20+z (-25+9 z))+4 y z^2 (2589+z (-1358+5 (-24+z) z))-y^4 (-1+z) z (-480+z (586+z (-215+9 z)))+y^2 (720+z (10356+z (-13300+z (2355+(1066-45 z) z))))+y^3 (2160+z (-5432+z (2355+z (1252+z (-801+34 z)))))\right) \mathrm{Sin}[\pi y] \mathrm{Sin}[\pi z]\right) / \left(\pi^2 (-5+y) (-4+y) (-3+y) (-2+y) (-1+y) y (-5+z) (-4+z) (-3+z) (-2+z) (-1+z) z\right)$$

```
Limit[
 Limit[((720 z^2 (1 + 3 z) - y^5 (-1 + z) z (20 + z (-25 + 9 z)) + 4 y z^2 (2589 + z (-1358 + 5 (-24 + z) z)) -
       y^4 (-1 + z) z (-480 + z (586 + z (-215 + 9 z))) +
       y^2 (720 + z (10 356 + z (-13 300 + z (2355 + (1066 - 45 z) z)))) +
       y^3 (2160 + z (-5432 + z (2355 + z (1252 + z (-801 + 34 z)))))) Sin[π y] Sin[π z]) /
   (π^2 (-5 + y) (-4 + y) (-3 + y) (-2 + y) (-1 + y) y (-5 + z) (-4 + z) (-3 + z) (-2 + z) (-1 + z) z), z →
   2], y → 2]
```

52

```
dfa[120, 2 + 2]
```

52

```
pl[n_, y_, z_] :=
 {Sum[Binomial[j - 1, z - 1] Binomial[(n - j) - 1, y - 1], {j, 1, n - 1}], Binomial[n - 1, y + z - 1]}
```

```
pl[10, 5, 2]

{84, 84}

pr[x_, a_, b_] :=
 {x^(a + b - 1) / (a + b - 1)!, Integrate[ t^(a - 1) / (a - 1)! (x - t)^(b - 1) / (b - 1)!, {t, 0, x}]}

pr[12, 1.51 I, .5]

{-0.124352 - 1.22808 i, -0.124352 - 1.22808 i}

Sum[Log[x]^(k - 1) / (k - 1)!, {k, 0, Infinity}]

x

lr[100, 2]

283

FI[n_] := FactorInteger[n]; FI[1] := {}
dzeta[j_, s_, z_] := j^-s Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[j]}]
zeta[n_, s_, z_] := Sum[dzeta[j, s, z], {j, 1, n}]
px[n_, k_, a_] := Sum[Binomial[k, j] lr[n, j + a], {j, 0, k}]

Binomial[3, -1]

0

Limit[D[lr[100, z] / lr[50, z], z], z → 0]
```

$$\frac{125}{12}$$

```
N[1 + Integrate[a Hypergeometric1F1[1 - a, 2, -x], {x, 0, n}] +
    Integrate[b Hypergeometric1F1[1 - b, 2, -x], {x, 0, n}] +
    a b Integrate[Hypergeometric1F1[1 - a, 2, -x] Hypergeometric1F1[1 - b, 2, -y],
      {x, 0, n}, {y, 0, n - x}] /. a → 2.2 /. b → -3.5 I /. n → 10.]

157.141 + 268.802 i

LaguerreL[z, -n] /. z → (2.2 + (-3.5 I)) /. n → 10.

157.141 + 268.802 i

Sum[(t + a - 1)! / t! / (a - 1)! (x - t + b - 1)! / (x - t)! / (b - 1)!, {t, 0, x}]
```

$$\frac{(-1+a+b+x)!}{(-1+a+b)!\,x!}$$

```
Sum[Pochhammer[a, t] / t! Pochhammer[b, x - t] / (x - t)!, {t, 0, x}]
```

$$\frac{(-1+a+b+x)!}{(-1+a+b)!\,x!}$$

```
Pochhammer[a + b, x] / x!
```

$$\frac{\text{Pochhammer}[a+b,\ x]}{x!}$$

```
D[LaguerreL[z, -x], x]

LaguerreL[-1 + z, 1, -x]

D[LaguerreL[-z, Log@x], x] /. z → b /. x → u
```

$$-\frac{\text{LaguerreL}[-1-b,\ 1,\ \text{Log}[u]]}{u}$$

```
meh[x_, a_, b_] := 1 + Integrate[-LaguerreL[-1 - a, 1, Log[t]]/t, {t, 1, x}] +
  Integrate[-LaguerreL[-1 - b, 1, Log[t]]/t, {t, 1, x}] + Integrate[
   (-LaguerreL[-1 - a, 1, Log[t]]/t) (-LaguerreL[-1 - b, 1, Log[u]]/u), {t, 1, x}, {u, 1, x / t}]

N[meh[10.3, 2.3, 3.]]

453.812

LaguerreL[-z, Log@x] /. x → 10.3 /. z → 2.3 + 3

453.812

D[Integrate[-LaguerreL[-1 - b, 1, Log[t]]/t, {t, 1, x}], x] /. b → a
```

$$\text{ConditionalExpression}\left[\frac{a\,\text{Hypergeometric1F1}[1+a,\,2,\,\text{Log}[x]]}{x},\ 0 \le \text{Re}[x] \le e\ ||\ x \notin \text{Reals}\right]$$

```
D[Integrate[
  (-LaguerreL[-1 - a, 1, Log[t]]/t) (-LaguerreL[-1 - b, 1, Log[u]]/u), {t, 1, x}, {u, 1, x / t}], x]
```

$$\int_1^x \frac{1}{t\,x}\,a\,b\,\text{Hypergeometric1F1}[1+a,\,2,\,\text{Log}[t]]\,\text{Hypergeometric1F1}\left[1+b,\,2,\,\text{Log}\left[\frac{x}{t}\right]\right]dt$$

```
mee[x_, a_, b_] :=
 a Hypergeometric1F1[1 + a, 2, Log[x]]/x + b Hypergeometric1F1[1 + b, 2, Log[x]]/x +
  ∫_1^x 1/(t x) a b Hypergeometric1F1[1 + a, 2, Log[t]] Hypergeometric1F1[1 + b, 2, Log[x/t]] dt

D[LaguerreL[-(a + b), Log@x], x]
```

$$-\frac{\text{LaguerreL}[-1-a-b,\,1,\,\text{Log}[x]]}{x}$$

```
-LaguerreL[-1 - a - b, 1, Log[x]]/x /. x → 11. /. a → 2 /. b → -3.3

-0.0579115

N@mee[11., 2, -3.3]

-0.0579115

D[(1 + x) ^ (a + b), x]
```

$$(a+b)\,(1+x)^{-1+a+b}$$

```
D[Integrate[a t^(a - 1) b u^(b - 1), {t, 0, x}, {u, 0, x}], x]
```

$$\text{ConditionalExpression}\left[(a+b)\,x^{-1+a+b},\ \text{Re}[a] > 0\right]$$

```
FullSimplify[1 + Integrate[a (1 + t) ^ (a - 1), {t, 0, x}] + Integrate[b (1 + u) ^ (b - 1), {u, 0, x}] +
  Integrate[ a (1 + t) ^ (a - 1) b (1 + u) ^ (b - 1), {t, 0, x}, {u, 0, x}]]
```

$$\text{ConditionalExpression}\left[(1+x)^{a+b},\ \text{Re}[x] \ge -1\ ||\ x \notin \text{Reals}\right]$$

```
D[(1 + t)^a, t]
```

$a\,(1+t)^{-1+a}$

```
1 + Integrate[a (1 + t)^(-1+a), {t, 0, x}]
```

$\text{ConditionalExpression}[(1+x)^{a},\ \text{Re}[x] \geq -1 \,||\, x \notin \text{Reals}]$

```
1 + 3 x + 3 x^2 + x^3 /. x → 3
```

64

```
1 + Integrate[a (1 + t)^(a - 1), {t, 0, x}]
```

$\text{ConditionalExpression}[(1+x)^{a},\ \text{Re}[x] \geq -1 \,||\, x \notin \text{Reals}]$

```
Integrate[ a (1 + t)^(a - 1) b (1 + u)^(b - 1), {t, 0, x}, {u, 0, x}]
```

$\text{ConditionalExpression}\left[(-1+(1+x)^{a})\left(-1+(1+x)^{b}\right),\ \text{Re}[x] \geq -1 \,||\, x \notin \text{Reals}\right]$

```
D[(1 + x)^(a + b), x]
```

$(a+b)\,(1+x)^{-1+a+b}$

```
D[Integrate[ a (1 + t)^(a - 1), {t, 0, x}], x]
```

$\text{ConditionalExpression}\left[a\,(1+x)^{-1+a},\ \text{Re}[x] \geq -1 \,||\, x \notin \text{Reals}\right]$

```
FullSimplify@D[Integrate[ a (1 + t)^(a - 1) b (1 + u)^(b - 1), {t, 0, x}, {u, 0, x}], x]
```

$\text{ConditionalExpression}\left[b\,(1+x)^{-1+b}\,(-1+(1+x)^{a}) + a\,(1+x)^{-1+a}\left(-1+(1+x)^{b}\right),\ \text{Re}[x] \geq -1 \,||\, x \notin \text{Reals}\right]$

```
D[a (1 + t)^(a - 1) b (1 + u)^(b - 1), x]
```

0

```
b (1 + x)^(-1+b) (-1 + (1 + x)^a) + a (1 + x)^(-1+a) (-1 + (1 + x)^b) /. x → 3.3 /. a → 2.2 /. b → 1.1
```

80.5758

```
Integrate[a (1 + t)^(a - 1) b (1 + u)^(b - 1) + a (1 + u)^(a - 1) b (1 + t)^(b - 1),
    {t, 0, x}, {u, 0, t}] /. x → 3.3 /. a → 2.2 /. b → 1.1
```

94.4248

```
(a + b) (1 + x)^(-1+a+b) - a (1 + x)^(a - 1) - b (1 + x)^(b - 1) /. x → 3.3 /. a → 2.2 /. b → 1.1
```

80.5758

```
Expand[b (1 + x)^(-1+b) (-1 + (1 + x)^a) + a (1 + x)^(-1+a) (-1 + (1 + x)^b)]
```

$-a\,(1+x)^{-1+a} - b\,(1+x)^{-1+b} + a\,(1+x)^{-1+a+b} + b\,(1+x)^{-1+a+b}$

```
Integrate[a (1 + t)^(a - 1) b (1 + u)^(b - 1) + a (1 + u)^(a - 1) b (1 + t)^(b - 1), {u, 0, t}]
```

$\text{ConditionalExpression}\left[-a\,(1+t)^{-1+a} - b\,(1+t)^{-1+b} + (a+b)\,(1+t)^{-1+a+b},\ \text{Re}[t] \geq -1 \,||\, t \notin \text{Reals}\right]$

```
Integrate[-a (1 + t)^(a - 1) b (1 + t)^(b - 1), {t, 0, x}]
```

$$\text{ConditionalExpression}\left[-a\,b\left(-\frac{1}{-1+a+b} + \frac{(1+x)^{-1+a+b}}{-1+a+b}\right),\ \text{Re}[x] \geq -1 \,||\, x \notin \text{Reals}\right]$$

```
Integrate[ a (1 + t)^(a - 1) b (1 + u)^(b - 1), {t, 0, x}, {u, 0, x}] /. x → 3.3 /. a → 2.2 /. b → 1.1
```

94.4248

```
Integrate[a (1 + t) ^ (a - 1) b (1 + u) ^ (b - 1) + a (1 + u) ^ (a - 1) b (1 + t) ^ (b - 1), {u, 0, t}] /. t → x
```

ConditionalExpression$\left[-a\,(1+x)^{-1+a} - b\,(1+x)^{-1+b} + (a+b)\,(1+x)^{-1+a+b},\ \mathrm{Re}[x] \geq -1\ ||\ x \notin \mathrm{Reals}\right]$

$-a\,(1+x)^{-1+a} - b\,(1+x)^{-1+b} + (a+b)\,(1+x)^{-1+a+b}$ /. x → 3.3 /. a → 2.2 /. b → 1.1

80.5758

```
(a + b) (1 + x) ^ (a + b - 1) /. x → 3.3 /. a → 2.2 /. b → 1.1
```

94.513

a (1 + x) ^ (a - 1) + b (1 + x) ^ (b - 1) + $\left(b\,(1+x)^{-1+b}\,(-1 + (1+x)^{a}) + a\,(1+x)^{-1+a}\,\left(-1 + (1+x)^{b}\right)\right)$ /.
x → 3.3 /. a → 2.2 /. b → 1.1

94.513

a (1 + x) ^ (a - 1) + b (1 + x) ^ (b - 1) + $\left(-a\,(1+x)^{-1+a} - b\,(1+x)^{-1+b} + (a+b)\,(1+x)^{-1+a+b}\right)$ /. x → 3.3 /.
a → 2.2 /. b → 1.1

94.513

```
a (1 + x) ^ (a - 1) + b (1 + x) ^ (b - 1) +
 Integrate[a (1 + x) ^ (a - 1) b (1 + u) ^ (b - 1) + a (1 + u) ^ (a - 1) b (1 + x) ^ (b - 1), {u, 0, x}]
```

ConditionalExpression$\left[(a+b)\,(1+x)^{-1+a+b},\ \mathrm{Re}[x] \geq -1\ ||\ x \notin \mathrm{Reals}\right]$

```
D[(1 + x) ^a, x] + D[(1 + x) ^b, x] +
 Integrate[D[(1 + x) ^a, x] D[(1 + u) ^b, u] + D[(1 + u) ^a, u] D[(1 + x) ^b, x], {u, 0, x}]
```

ConditionalExpression$\left[(a+b)\,(1+x)^{-1+a+b},\ \mathrm{Re}[x] \geq -1\ ||\ x \notin \mathrm{Reals}\right]$

1 + Integrate$\left[(a+b)\,(1+x)^{-1+a+b}, \{x, 0, n\}\right]$

ConditionalExpression$\left[(1+n)^{a+b},\ \mathrm{Re}[n] \geq -1\ ||\ n \notin \mathrm{Reals}\right]$

```
bla[x_] := Table[Binomial[t - 1, a - 1] Binomial[(x - t) - 1, b - 1], {t, 1, x - 1}]
bla2[x_, a_, b_] := Table[Binomial[t - 1, a - 1] Binomial[(x - t) - 1, b - 1], {t, 1, x - 1}]
bla3[x_, a_, b_] := Sum[Binomial[t - 1, a - 1] Binomial[(x - t) - 1, b - 1], {t, 1, x - 1}]
```

```
Sum[Binomial[t - 1, a - 1] Binomial[u - 1, b - 1], {t, 1, x}, {u, 1, x - t}]
```

$$\sum_{t=1}^{x} \sum_{u=1}^{-t+x} \mathrm{Binomial}[-1+t, -1+a]\ \mathrm{Binomial}[-1+u, -1+b]$$

```
Binomial[5, 3 + 2]
```

1

```
bla2[6, 2, 2] // TableForm
```

0
3
4
3
0

```
Binomial[11 - 1, 3 + 2 - 1]
```

210

```
bla3[11, 3, 2]
```

210

```
FullSimplify[
 (1 / Gamma[z] / Gamma[1 - z]) Sum[(-1)^k / (z - k) Binomial[x, k], {k, 0, Infinity}]]
```

$$\frac{\text{Gamma}[1+x]}{\text{Gamma}[1+x-z]\ \text{Gamma}[1+z]}$$

```
D[Integrate[1 / t LaguerreL[-a - 1, 1, Log[t]] 1 / u LaguerreL[-b - 1, 1, Log[u]],
  {t, 1, x}, {u, 1, x / t}], x]
```

$$\int_1^x \frac{1}{t\,x}\,a\,b\ \text{Hypergeometric1F1}[1+a, 2, \text{Log}[t]]\ \text{Hypergeometric1F1}\left[1+b, 2, \text{Log}\left[\frac{x}{t}\right]\right] \, dt$$

```
Integrate[1 / t LaguerreL[-a - 1, 1, Log[t]] 1 / u LaguerreL[-b - 1, 1, Log[u]],
 {t, 1, x}, {u, 1, x / t}]
```

$$\int_1^x \frac{1}{t}\,a\ \text{Hypergeometric1F1}[1+a, 2, \text{Log}[t]]\ \left(-1+\text{Hypergeometric1F1}\left[b, 1, \text{Log}\left[\frac{x}{t}\right]\right]\right) dt$$