```
ClearAll["Global`*"]

lo[n_, k_] := Sum[(-1)^(j+1) lo[Floor[n/j], k-1], {j, 1, n}];
lo[n_, 1] := Sum[(-1)^(j+1) Log[j], {j, 1, n}]
t[n_, a_] := Mod[n, a] - Mod[n-1, a]
lp[n_, k_, b_] := Sum[t[j, b] lp[Floor[n/j], k-1, b], {j, 1, n}];
lp[n_, 1, b_] := Sum[t[j, b] Log[j], {j, 1, n}]
fa[n_, k_] := Sum[2^j Binomial[k, j] (-1)^j l1[n/2^j, k], {j, 0, k}] +
  Sum[2^j Binomial[k-1, j-1] (-1)^j Log[2] d1[n/2^j, k], {j, 1, k}]

L1[n_, k_] := Sum[L1[Floor[n/j], k-1], {j, 1, n}];
L1[n_, 1] := Sum[Log[j], {j, 1, n}]; L1[n_, 0] := 1
L2[n_, k_] := Sum[L2[Floor[n/j], k-1], {j, 2, n}]; L2[n_, 1] := Sum[Log[j], {j, 2, n}]
D1[n_, k_] := Sum[D1[Floor[n/j], k-1], {j, 1, n}]; D1[n_, 0] := 1
L2toL1[n_, z_] := Sum[FactorialPower[z-1, a]/a! L2[n, a+1], {a, 0, Log[2, n]}]
L2toL1x[n_, z_] := Sum[Binomial[z-1, a] L2[n, a+1], {a, 0, Log[2, n]}]
L1toL2[n_, k_] := Sum[(-1)^(k-j) Binomial[k-1, j-1] L1[n, j], {j, 1, k}]

EL[n_, k_, b_] :=
 EL[n, k, b] = Sum[EL[n/j, k-1, b], {j, 1, n}] - b Sum[EL[n/(j b), k-1, b], {j, 1, n}];
EL[n_, 1, b_] := EL[n, 1, b] = Sum[Log[j], {j, 1, n}] - b Sum[Log[j b], {j, 1, n/b}]
LtoEL[n_, k_, b_] := Sum[b^j Binomial[k, j] (-1)^j L1[n/b^j, k], {j, 0, k}] +
  Sum[b^j Binomial[k-1, j-1] (-1)^j Log[b] D1[n/b^j, k], {j, 1, k}]

EL1toL1[n_, b_] := Sum[b^j EL[n/b^j, 1, b], {j, 0, Log[b, n]}] +
  Log[b] Sum[b^j D1[n/b^j, 1], {j, 1, Log[b, n]}]

EL2[n_, k_, b_] :=
 EL2[n, k, b] = Sum[EL2[n/j, k-1, b], {j, 2, n}] - b^2 Sum[EL2[n/(j b), k-1, b], {j, 1, n}];
EL2[n_, 1, b_] := EL2[n, 1, b] = Sum[Log[j], {j, 2, n}] - b^2 Sum[Log[j b], {j, 1, n/b}]
EL2toEL1[n_, z_, b_] :=
 Sum[FactorialPower[z-1, a]/a! EL2[n, a+1, b], {a, 0, Log[If[b < 2, b, 2], n]}]
EL1toEL2[n_, k_, b_] := Sum[(-1)^(k-j) Binomial[k-1, j-1] EL[n, j, b], {j, 1, k}]
```

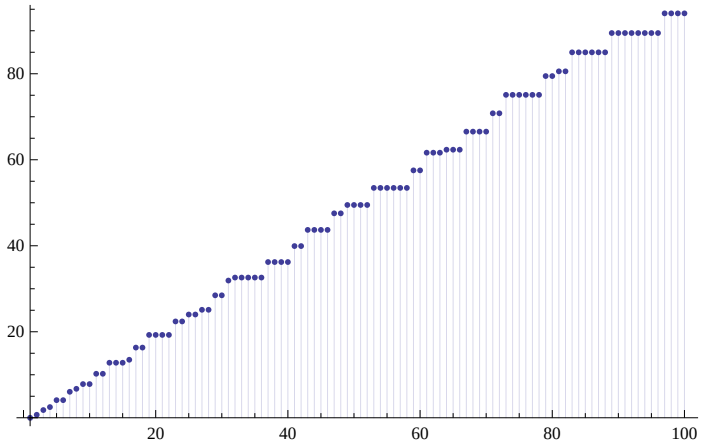
```
N[L2toL1x[100, 0]]
```

```
94.0453
```

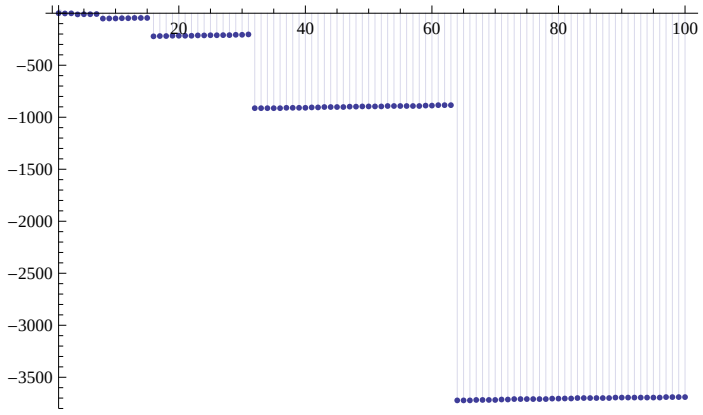
```
N[EL2toEL1[100, 0, 101]]
```

```
94.0453
```

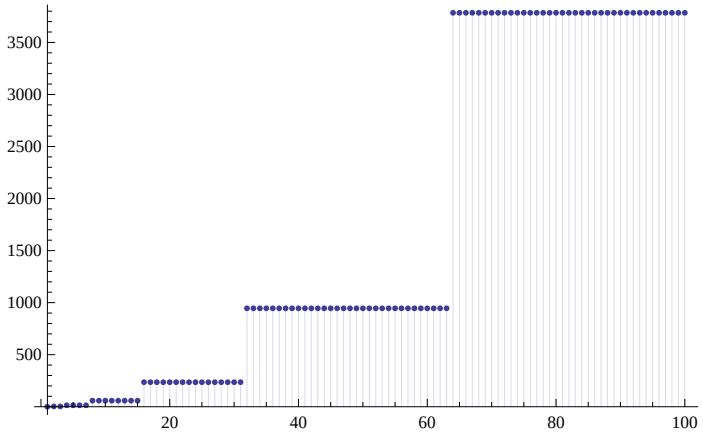
```
DiscretePlot[L2toL1[n, 0], {n, 1, 100}]
```



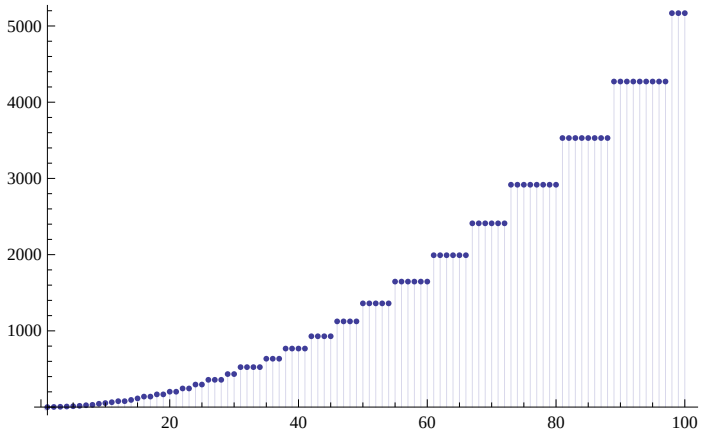

```
DiscretePlot[EL2toEL1[n, 0, 2], {n, 1, 100}]
```




```
DiscretePlot[L2toL1[n, 0] - EL2toEL1[n, 0, 2], {n, 1, 100}]
```



```
DiscretePlot[L2toL1[n, 0] - EL2toEL1[n, 0, 1.1], {n, 1, 100}]
```




```
DiscretePlot[L2toL1[n, 0] - EL2toEL1[n, 0, 1.05], {n, 1, 100}]
```

```
$Aborted
```

```
N[Log[2 Pi]]
```

```
1.83788
```

```
fdif[n_, b_] := Sum[ Log[b^((b)^(2 j))], {j, 1, Log[b, n]}]
fdif2[n_, b_] := Log[b] Sum[ (b^(2 j)), {j, 1, Log[b, n]}]
fdif3[n_, b_] := Log[b] Sum[ E^(2 j Log[b]), {j, 1, Log[b, n]}]
```

```
N[fdif[100, 1.1]]
```

```
5168.41
```

```
L2toL1[100, 0] - EL2toEL1[100, 0, 1.1]
```

```
5168.41
```

```
N[fdif2[100, 1.1]]
```

```
5168.41
```

```
N[fdif3[100, 1.1]]
```

```
5168.41
```

```
fdifx[n_, b_, a_] := Sum[ Log[b^((b)^(a j))], {j, 1, Log[b, n]}]
```

```
N[fdifx[100, 1.00001, 2]]
```

```
4999.52
```

```
(100^2 - 1) / 2.
```

```
4999.5
```

```
N[fdifx[100, 1.00001, 3]]
```

```
333 335.
```

```
(100^3 - 1) / 3
```

```
333 333
```

```
N[fdifx[100, 1.00001, 4]]
```

2.50002×10^{7}

```
(100^4 - 1) / 4.
```

2.5×10^{7}

```
N[fdifx[100, 1.00001, 1.5]]
```

666.002

```
(100^1.5 - 1) / 1.5
```

666.

```
N[fdifx[100, 1.00001, -1]]
```

0.989995

```
(100^-1 - 1) / -1.
```

0.99

```
N[fdifx[100, 1.00001, -2]]
```

0.499945

```
(100^-2 - 1) / -2.
```

0.49995

```
N[fdifx[100, 1.00001, I]]
```

$-0.994263 + 1.10701\,i$

```
N[(100^I - 1) / I]
```

$-0.994258 + 1.10701\,i$