```
Sum[ (-1)^k (1 / Zeta^a - 1)^k, {k, 0, Infinity}]
```

Zeta^a

```
PP[a_, p_, k_] := PP[a, p, k] = Residue[(1 / Zeta[s]^a - 1)^p, {s, ZetaZero[k]}]
PS[a_, p_, k_] := PS[a, p, k] = Residue[(1 / Zeta[s]^a)^p, {s, ZetaZero[k]}]
PQ[a_, p_, k_] := PQ[a, p, k] = Residue[- (1 / Zeta[s]^a + 1)^p, {s, ZetaZero[k]}]
```

```
Grid[Table[ Re[N[PP[1, k, j]]], {j, 1, 50}, {k, 1, 7}]]
```

1.2451	−1.18437	1.14948	−1.12534	1.10705	−1.09241	1.08027
0.85832	−0.836837	0.835954	−0.833516	0.831555	−0.829826	0.828284
0.68866	−0.715818	0.715803	−0.716315	0.716216	−0.715958	0.715633
0.658799	−0.640736	0.644504	−0.640412	0.641118	−0.639997	0.639913
0.607547	−0.604573	0.594451	−0.595639	0.593272	−0.593304	0.592608
0.494249	−0.562053	0.569326	−0.571862	0.573123	−0.573753	0.574066
0.656872	−0.498974	0.520473	−0.514878	0.514482	−0.514569	0.514457
0.435512	−0.54382	0.520302	−0.525507	0.525684	−0.52552	0.52562
0.420515	−0.586359	0.450583	−0.548093	0.480455	−0.520809	0.498798
0.626261	−0.393902	0.520082	−0.419804	0.486014	−0.444834	0.466347
0.398243	−0.465621	0.475069	−0.477601	0.478824	−0.479485	0.47985
0.395356	−0.465207	0.466864	−0.466851	0.467194	−0.467335	0.467357
0.650527	−0.266017	0.564193	−0.307281	0.48346	−0.411037	0.375647
0.312819	−0.614886	0.311113	−0.566284	0.388806	−0.460498	0.495472
0.315553	−0.466771	0.444108	−0.442947	0.446703	−0.44602	0.446037
0.559569	−0.372811	0.40268	−0.403817	0.400543	−0.401531	0.401741
0.441849	−0.427997	0.412917	−0.411893	0.411201	−0.410838	0.410617
0.316827	−0.395738	0.413059	−0.418306	0.420771	−0.422191	0.423071
0.308119	−0.5786	0.291654	−0.510294	0.373904	−0.4066	0.462665
0.643994	−0.201839	0.488105	−0.275023	0.407539	−0.373193	0.316909
0.282731	−0.425998	0.418094	−0.408831	0.410744	−0.411212	0.410659
0.291342	−0.410311	0.402443	−0.400508	0.402042	−0.402059	0.402008
0.461173	−0.362639	0.371317	−0.374282	0.373177	−0.373408	0.373608
0.530547	−0.267019	0.404471	−0.354124	0.348653	−0.378128	0.349744
0.242811	−0.489577	0.351484	−0.400652	0.405549	−0.375712	0.403865
0.241387	−0.38807	0.400786	−0.391838	0.391562	−0.393091	0.393353
0.581929	−0.196025	0.533822	−0.0771733	0.661739	0.0198168	0.710159
0.343283	−0.534494	0.172406	−0.634206	0.0482369	−0.727353	−0.00343889
0.284255	−0.346339	0.362884	−0.368756	0.371476	−0.372982	0.373907
0.305966	−0.361561	0.366395	−0.366659	0.366885	−0.367068	0.36718
0.408326	−0.377282	0.346971	−0.355916	0.350151	−0.351611	0.350374
0.518743	−0.301055	0.318515	−0.331113	0.333371	−0.327477	0.329838
0.196751	−0.382796	0.389275	−0.367261	0.370631	−0.375021	0.37392
0.0913935	−0.995193	−0.806707	−2.0078	−0.0349164	6.41113	28.5449
0.644192	0.302064	1.5016	1.31545	0.726314	−7.10191	−27.8545
0.390405	−0.345354	0.338092	−0.340028	0.33987	−0.339777	0.339748
0.284809	−0.354536	0.35319	−0.350664	0.350102	−0.349806	0.349572
0.254855	−0.321002	0.339832	−0.346458	0.349454	−0.351093	0.352098
0.303835	−0.443305	0.314305	−0.35519	0.347958	−0.335771	0.346491
0.641374	−0.0599385	0.364489	−0.372882	0.185947	−0.353159	0.345503
0.159171	−0.52993	0.333759	−0.27673	0.46713	−0.309919	0.305448
0.216354	−0.311612	0.338133	−0.344345	0.34653	−0.347919	0.348899
0.369974	−0.365279	0.317264	−0.325345	0.320593	−0.32186	0.321056
0.407479	−0.301761	0.331253	−0.31931	0.323327	−0.321574	0.322007
0.428312	−0.265833	0.314191	−0.322136	0.305204	−0.315082	0.314577
0.186538	−0.387748	0.339242	−0.332576	0.350069	−0.340537	0.341271
0.167988	−0.369576	0.376936	−0.338234	0.340992	−0.350434	0.347596
0.515964	−0.241299	0.297992	−0.278951	0.323411	−0.276024	0.308132
0.406559	−0.359709	0.27472	−0.333937	0.287461	−0.324835	0.295104
0.212329	−0.320516	0.338947	−0.336614	0.335471	−0.335813	0.336124

Grid[Table[Re[N[PQ[1, k, j]]], {j, 1, 4}, {k, 1, 10}]]

-1.2451	-3.796` 03	-8.984` 47	-19.48` 89	-40.69` 15	-83.39` 96	-169.3` 02	-341.9` 05	-688.4` 49	-1383.` 81
-0.858` 32	-2.596` 44	-6.114` 77	-13.23` 63	-27.65` 32	-56.84` 72	-115.9` 84	-235.8` 23	-478.7` 68	-971.4` 97
-0.688` 66	-2.038` 82	-4.684` 81	-9.8676	-20.01` 25	-39.85` 46	-78.62` 58	-154.3	-301.8` 16	-588.9` 59
-0.658` 799	-1.994` 46	-4.705` 67	-10.21` 95	-21.46` 22	-44.462	-91.70` 77	-189.24	-391.7` 61	-815.1` 45

Residue[(1 / Zeta[s] - 1), {s, ZetaZero[1]}]

$$\frac{1}{Zeta'[ZetaZero[1]]}$$

Sum[(-1)^k (1 / Zeta[s] - 1)^k, {k, 0, 8}]

$$2 + \left(-1 + \frac{1}{Zeta[s]}\right)^2 - \left(-1 + \frac{1}{Zeta[s]}\right)^3 + \left(-1 + \frac{1}{Zeta[s]}\right)^4 -$$

$$\left(-1 + \frac{1}{Zeta[s]}\right)^5 + \left(-1 + \frac{1}{Zeta[s]}\right)^6 - \left(-1 + \frac{1}{Zeta[s]}\right)^7 + \left(-1 + \frac{1}{Zeta[s]}\right)^8 - \frac{1}{Zeta[s]}$$

-1 + (1 / Zeta[s])

$$-1 + \frac{1}{Zeta[s]}$$

Sum[- (1 / Zeta^a + 1)^k, {k, 0, Infinity}]

$Zeta^a$

Sum[- (1 / Zeta + 1)^k, {k, 0, Infinity}]

$Zeta$

PQ[1, 1, 1]
PP[1, 1, 1]

$$-\frac{1}{Zeta'[ZetaZero[1]]}$$

$$\frac{1}{Zeta'[ZetaZero[1]]}$$

Sum[(-1) ^ (k + 1) (1 / Zeta^a - 1) ^k / k, {k, 1, Infinity}]

$Log[Zeta^{-a}]$

Sum[(-1) ^ (k + 1) (Zeta^a - 1) ^k / k, {k, 1, Infinity}]

$Log[Zeta^a]$

Sum[- (-Zeta^a + 1) ^k / k, {k, 1, Infinity}]

$Log[Zeta^a]$

Sum[- (1 - x) ^k / k, {k, 1, Infinity}]

$Log[x]$

```
Grid[Table[ Re[N[PS[1, k, j]]], {j, 1, 50}, {k, 1, 7}]]
```

```
N[Log[Zeta[2.5]]]
```

0.293779

```
K[n_] := N[MangoldtLambda[n] / Log[n]]
```

```
SS[s_, t_] := Sum[ K[n] / n^s, {n, 2, t}]
```

```
SS[2.5, 20 000]
```

0.2937788695393159`


```
K[n_, 0] := K[n, 0] = If[n == 1, 1, 0]
K[n_, 1] := K[n, 1] = If[n == 1, 0, FullSimplify[MangoldtLambda[n] / Log[n]]]
K[n_, k_] := K[n, k] = Sum[K[j, k - 1] K[n / j, 1], {j, Divisors[n]}]
```

```
K[4, 1]
```

$\frac{1}{2}$

```
K[6, 2]
```

2

```
N[Log[Zeta[2.5]]^2]
```

0.086306

```
SS2[s_, t_] := Sum[ K[n, 2] / n^s, {n, 2, t}]
```

```
SS2[2.5, 2000]
```

0.0863013

```
Series[Log[x], {x, 0, 4}]
```

$\text{Log}[x] + O[x]^5$

```
Expand[Sum[(-1) ^ (k + 1) / k (x) ^k, {k, 1, 13}] ^ 2]
```

$$x^2 - x^3 + \frac{11\,x^4}{12} - \frac{5\,x^5}{6} + \frac{137\,x^6}{180} - \frac{7\,x^7}{10} + \frac{363\,x^8}{560} - \frac{761\,x^9}{1260} + \frac{7129\,x^{10}}{12\,600} - \frac{671\,x^{11}}{1260} + \frac{83\,711\,x^{12}}{166\,320} -$$
$$\frac{6617\,x^{13}}{13\,860} + \frac{1\,145\,993\,x^{14}}{2\,522\,520} - \frac{785\,633\,x^{15}}{2\,702\,700} + \frac{605\,453\,x^{16}}{2\,882\,880} - \frac{28\,549\,x^{17}}{180\,180} + \frac{395\,243\,x^{18}}{3\,243\,240} -$$
$$\frac{17\,009\,x^{19}}{180\,180} + \frac{263\,111\,x^{20}}{3\,603\,600} - \frac{4319\,x^{21}}{77\,220} + \frac{11\,899\,x^{22}}{283\,140} - \frac{131\,x^{23}}{4290} + \frac{431\,x^{24}}{20\,592} - \frac{x^{25}}{78} + \frac{x^{26}}{169}$$

```
Sum[(-1) ^ (k + 1) / k (x) ^k, {k, 1, 10}]
```

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \frac{x^7}{7} - \frac{x^8}{8} + \frac{x^9}{9} - \frac{x^{10}}{10}$$

```
Expand[Sum[(-1) ^ (k + 1) / k (x) ^k, {k, 1, 13}] ^3]
```

$$x^3 - \frac{3 x^4}{2} + \frac{7 x^5}{4} - \frac{15 x^6}{8} + \frac{29 x^7}{15} - \frac{469 x^8}{240} + \frac{29\,531 x^9}{15\,120} - \frac{1303 x^{10}}{672} + \frac{16\,103 x^{11}}{8400} -$$

$$\frac{190\,553 x^{12}}{100\,800} + \frac{128\,977 x^{13}}{69\,300} - \frac{9061 x^{14}}{4950} + \frac{30\,946\,717 x^{15}}{17\,199\,000} - \frac{34\,841\,661 x^{16}}{22\,422\,400} + \frac{44\,499\,407 x^{17}}{33\,633\,600} -$$

$$\frac{226\,661\,653 x^{18}}{201\,801\,600} + \frac{26\,190\,319 x^{19}}{27\,518\,400} - \frac{4\,868\,331\,551 x^{20}}{6\,054\,048\,000} + \frac{4\,298\,960\,483 x^{21}}{6\,356\,750\,400} - \frac{31\,083\,503 x^{22}}{55\,036\,800} +$$

$$\frac{1\,554\,730\,091 x^{23}}{3\,329\,726\,400} - \frac{6\,082\,401\,761 x^{24}}{15\,982\,686\,720} + \frac{5\,060\,290\,517 x^{25}}{16\,648\,632\,000} - \frac{4\,828\,837 x^{26}}{20\,490\,624} + \frac{40\,799\,043\,101 x^{27}}{233\,746\,793\,280} -$$

$$\frac{84\,547\,811 x^{28}}{706\,717\,440} + \frac{528\,332\,027 x^{29}}{6\,183\,777\,600} - \frac{1\,428\,192\,341 x^{30}}{23\,189\,166\,000} + \frac{34\,218\,281 x^{31}}{772\,972\,200} - \frac{129\,694\,661 x^{32}}{4\,122\,518\,400} +$$

$$\frac{23\,302\,589 x^{33}}{1\,062\,836\,775} - \frac{436\,919 x^{34}}{29\,446\,560} + \frac{15\,737 x^{35}}{1\,635\,920} - \frac{93\,967 x^{36}}{16\,061\,760} + \frac{287 x^{37}}{89\,232} - \frac{x^{38}}{676} + \frac{x^{39}}{2197}$$

```
PrimeKappa[n_, 0] := If[n == 1, 1, 0]
PrimeKappa[n_, 1] := If[n == 1, 0, FullSimplify[MangoldtLambda[n] / Log[n]]]
PrimeKappa[n_, k_] := Sum[PrimeKappa[j, k - 1] PrimeKappa[n / j, 1], {j, Divisors[n]}]
PrimeK[n_] := N[MangoldtLambda[n] / Log[n]]
d[n_, z_] := Product[Pochhammer[z, a = p[[2]]] / a !, {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
dlimit[n_, z_] := Round[N[ (1 / z d[n, z]) ], .0000001]
dlimit[n_, z_] := Round[
  N[ Sum[If[j == 1 || j == n, 0, (d[j, z] / z) (d[n / j, z] / z)], {j, Divisors[n]}]], .0000001]
Table[{n, dlimit[n, 10^-120], N[PrimeKappa[n, 2]]}, {n, 2, 100}] // TableForm
```

2	0.	0.
3	0.	0.
4	1.	1.
5	0.	0.
6	2.	2.
7	0.	0.
8	1.	1.
9	1.	1.
10	2.	2.
11	0.	0.
12	1.	1.
13	0.	0.
14	2.	2.
15	2.	2.
16	0.916667	0.916667
17	0.	0.
18	1.	1.
19	0.	0.
20	1.	1.
21	2.	2.
22	2.	2.
23	0.	0.
24	0.666667	0.666667
25	1.	1.
26	2.	2.
27	1.	1.
28	1.	1.

29	0.	0.
30	0.	0.
31	0.	0.
32	0.833333	0.833333
33	2.	2.
34	2.	2.
35	2.	2.
36	0.5	0.5
37	0.	0.
38	2.	2.
39	2.	2.
40	0.666667	0.666667
41	0.	0.
42	0.	0.
43	0.	0.
44	1.	1.
45	1.	1.
46	2.	2.
47	0.	0.
48	0.5	0.5
49	1.	1.
50	1.	1.
51	2.	2.
52	1.	1.
53	0.	0.
54	0.666667	0.666667
55	2.	2.
56	0.666667	0.666667
57	2.	2.
58	2.	2.
59	0.	0.
60	0.	0.
61	0.	0.
62	2.	2.
63	1.	1.
64	0.761111	0.761111
65	2.	2.
66	0.	0.
67	0.	0.
68	1.	1.
69	2.	2.
70	0.	0.
71	0.	0.
72	0.333333	0.333333
73	0.	0.
74	2.	2.
75	1.	1.
76	1.	1.
77	2.	2.
78	0.	0.
79	0.	0.
80	0.5	0.5
81	0.916667	0.916667
82	2.	2.
83	0.	0.
84	0.	0.

```
85    2.        2.
86    2.        2.
87    2.        2.
88    0.666667  0.666667
89    0.        0.
90    0.        0.
91    2.        2.
92    1.        1.
93    2.        2.
94    2.        2.
95    2.        2.
96    0.4       0.4
97    0.        0.
98    1.        1.
99    1.        1.
100   0.5       0.5
```

((3 ^ (a = .00000001) – 1) / a) ^ 2

1.20695

N[Log[3]] ^ 2

1.20695

PrimeKappa[4, 2]

1

ZZ[s_, t_] := Sum[1 / k^s, {k, 1, t}]

ZZ'[s, 2]

$-2^{-s} \text{Log}[2] - 3^{-s} \text{Log}[3] - 4^{-s} \text{Log}[4] - 5^{-s} \text{Log}[5] - 6^{-s} \text{Log}[6] - 7^{-s} \text{Log}[7] - 8^{-s} \text{Log}[8] - 9^{-s} \text{Log}[9] - 10^{-s} \text{Log}[10]$

ZZ'[s, 10] / ZZ[s, 10]

$(-2^{-s} \text{Log}[2] - 3^{-s} \text{Log}[3] - 4^{-s} \text{Log}[4] - 5^{-s} \text{Log}[5] - 6^{-s} \text{Log}[6] - 7^{-s} \text{Log}[7] - 8^{-s} \text{Log}[8] - 9^{-s} \text{Log}[9] - 10^{-s} \text{Log}[10]) / (1 + 2^{-s} + 3^{-s} + 4^{-s} + 5^{-s} + 6^{-s} + 7^{-s} + 8^{-s} + 9^{-s} + 10^{-s})$

YY[s_] := 1 / 2^s

YY'[s]

$-2^{-s} \text{Log}[2]$

YY[s_] := 1 / s^3

YY'[s]

$-\dfrac{3}{s^4}$

N[TS[100]]

94.0453

```
TS[n_] := TS[n] = Sum[ Log[j] - TS[Floor[n / j]], {j, 2, n}]
TMM[n_] := TMM[n] = 1 - Sum[ TMM[Floor[n / j]], {j, 2, n}]
Sum[ N[Log[ j ] TMM[Floor[100 / j]]], {j, 2, 100}]
```

94.0453

TM[100]

0

DD[n_, z_] := (-1)^z (1 - Gamma[z, -Log[n]] / Gamma[z])

Table[N[DD[100^ZetaZero[1], k]], {k, 1, 24}] // TableForm

$-7.36665 + 7.71141\, \dot{\imath}$
$-24.7283 - 4.34998\, \dot{\imath}$
$-16.0224 - 28.0628\, \dot{\imath}$
$9.17307 - 27.5269\, \dot{\imath}$
$18.3056 - 8.34461\, \dot{\imath}$
$10.5696 + 4.25085\, \dot{\imath}$
$2.05433 + 5.05907\, \dot{\imath}$
$-0.908868 + 2.0808\, \dot{\imath}$
$-0.779311 + 0.297956\, \dot{\imath}$
$-0.250182 - 0.11347\, \dot{\imath}$
$-0.028579 - 0.0768149\, \dot{\imath}$
$0.00933872 - 0.0203138\, \dot{\imath}$
$0.00526977 - 0.0019486\, \dot{\imath}$
$0.00118959 + 0.000546151\, \dot{\imath}$
$0.0000992716 + 0.000266357\, \dot{\imath}$
$-0.0000238935 + 0.0000526388\, \dot{\imath}$
$-0.0000103362 + 3.92011 \times 10^{-6}\, \dot{\imath}$
$-1.82232 \times 10^{-6} - 8.11879 \times 10^{-7}\, \dot{\imath}$
$-1.23432 \times 10^{-7} - 3.17727 \times 10^{-7}\, \dot{\imath}$
$2.20499 \times 10^{-8} - 5.07092 \times 10^{-8}\, \dot{\imath}$
$7.92897 \times 10^{-9} - 3.1696 \times 10^{-9}\, \dot{\imath}$
$1.15898 \times 10^{-9} + 4.89629 \times 10^{-10}\, \dot{\imath}$
$6.76157 \times 10^{-11} + 1.63855 \times 10^{-10}\, \dot{\imath}$
$-9.0572 \times 10^{-12} + 2.21449 \times 10^{-11}\, \dot{\imath}$

Gamma[-2]

ComplexInfinity

N[-Gamma[0, ZetaZero[1] (-Log[1000])]]

$-0.0879017 + 0.311575\, \dot{\imath}$

CC[n_] := Sum[(-1)^(k + 1) / k N[

**(-1)^k (1 - Gamma[k, (-Log[n^ZetaZero[1]])] / Gamma[k])
], {k, 1, 24}]**

CC[1000]

$-5.2116 - 9.77658\, \dot{\imath}$

```
N[ExpIntegralEi[ ZetaZero[1] Log[1000]]]
```

$-0.0879017 + 3.45317\,i$

```
RiePrimeCnt[n_] := Sum[PrimePi[n^(1/j)]/j, {j, 1, Log[2, n]}]
RieExplicitForumla[x_, t_] := (MangoldtLambda[x]/Log[x]/2) +
   LogIntegral[x] - N[2 Re[Sum[ExpIntegralEi[ZetaZero[k] Log[x]], {k, 1, t}]]] +
   NIntegrate[1/((y^3 - y) Log[y]), {y, x, Infinity}] - Log[2]
Table[{n, N[RiePrimeCnt[n]], RieExplicitForumla[n, 200]}, {n, 2, 100}] // TableForm
```

2	1.	1.00472
3	2.	1.99549
4	2.5	2.49728
5	3.5	3.49725
6	3.5	3.50528
7	4.5	4.50416
8	4.83333	4.83299
9	5.33333	5.33351
10	5.33333	5.33574
11	6.33333	6.32339
12	6.33333	6.35859
13	7.33333	7.3463
14	7.33333	7.33426
15	7.33333	7.34426
16	7.58333	7.59254
17	8.58333	8.59192
18	8.58333	8.5806
19	9.58333	9.58028
20	9.58333	9.58225
21	9.58333	9.5861
22	9.58333	9.58788
23	10.5833	10.5772
24	10.5833	10.5875
25	11.0833	11.0799
26	11.0833	11.0984
27	11.4167	11.3949
28	11.4167	11.4215
29	12.4167	12.4304
30	12.4167	12.4282
31	13.4167	13.4174
32	13.6167	13.5888
33	13.6167	13.598
34	13.6167	13.6098
35	13.6167	13.6169
36	13.6167	13.6192
37	14.6167	14.5997
38	14.6167	14.6342
39	14.6167	14.6404
40	14.6167	14.5633
41	15.6167	15.6434
42	15.6167	15.6089
43	16.6167	16.6054
44	16.6167	16.656

45	16.6167	16.6001
46	16.6167	16.6077
47	17.6167	17.608
48	17.6167	17.625
49	18.1167	18.1398
50	18.1167	18.0985
51	18.1167	18.1034
52	18.1167	18.1453
53	19.1167	19.1204
54	19.1167	19.0834
55	19.1167	19.109
56	19.1167	19.1202
57	19.1167	19.1359
58	19.1167	19.1702
59	20.1167	20.1364
60	20.1167	20.1167
61	21.1167	21.0952
62	21.1167	21.0574
63	21.1167	21.0893
64	21.2833	21.2596
65	21.2833	21.2622
66	21.2833	21.2911
67	22.2833	22.2474
68	22.2833	22.2088
69	22.2833	22.2534
70	22.2833	22.3081
71	23.2833	23.2989
72	23.2833	23.307
73	24.2833	24.3085
74	24.2833	24.2772
75	24.2833	24.2911
76	24.2833	24.274
77	24.2833	24.2628
78	24.2833	24.294
79	25.2833	25.2811
80	25.2833	25.2708
81	25.5333	25.5386
82	25.5333	25.542
83	26.5333	26.5195
84	26.5333	26.5301
85	26.5333	26.584
86	26.5333	26.5446
87	26.5333	26.4837
88	26.5333	26.5133
89	27.5333	27.543
90	27.5333	27.5526
91	27.5333	27.562
92	27.5333	27.5212
93	27.5333	27.5178
94	27.5333	27.5571
95	27.5333	27.5302
96	27.5333	27.4774
97	28.5333	28.5218
98	28.5333	28.6111
99	28.5333	28.5273
100	28.5333	28.4631

```
RiePrimeCnt[n_] := Sum[PrimePi[n^(1/j)]/j, {j, 1, Log[2, n]}]
RieExplicitForumla[x_, t_] := (MangoldtLambda[x]/Log[x]/2) +
   LogIntegral[x] - N[2 Re[Sum[ExpIntegralEi[ZetaZero[k] Log[x]], {k, 1, t}]]] +
   NIntegrate[1/((y^3 - y) Log[y]), {y, x, Infinity}] - Log[2]
Table[{n, N[RiePrimeCnt[n]], RieExplicitForumla[n, 200]}, {n, 2, 100}] // TableForm
```

```
Residue[(1/Zeta[s])^1, {s, ZetaZero[1]}] /. ZetaZero[1] → z
```

$$\frac{1}{Zeta'[z]}$$

```
Residue[(1/Zeta[s])^2, {s, ZetaZero[1]}] /. ZetaZero[1] → z
```

$$-\frac{Zeta''[z]}{Zeta'[z]^3}$$

```
Residue[(1/Zeta[s])^3, {s, ZetaZero[1]}] /. ZetaZero[1] → z
```

$$\frac{3\,Zeta''[z]^2 - Zeta'[z]\,Zeta^{(3)}[z]}{2\,Zeta'[z]^5}$$

```
Residue[(1/Zeta[s])^4, {s, ZetaZero[1]}] /. ZetaZero[1] → z
```

$$\frac{-15\,Zeta''[z]^3 + 10\,Zeta'[z]\,Zeta''[z]\,Zeta^{(3)}[z] - Zeta'[z]^2\,Zeta^{(4)}[z]}{6\,Zeta'[z]^7}$$

```
N[Zeta'[ZetaZero[3]]]
```

$1.2958 + 0.450037\,\mathbb{i}$

```
f1[s_] := Sum[-Log[k]/k^s, {k, 1, Infinity}]
```

```
f1[s]
```

$Zeta'[s]$

```
f2[s_] := -Log[2]/2^s - Log[3]/3^s - Log[4]/4^s
```

```
f2'[s]
```

$2^{-s}\,Log[2]^2 + 3^{-s}\,Log[3]^2 + 4^{-s}\,Log[4]^2$

```
f2a[s_] := Sum[Log[k]^2/k^s, {k, 1, Infinity}]
```

```
f2a[s]
```

$Zeta''[s]$

```
f3[s_, t_] := Sum[(-1)^(t) Log[k]^t/k^s, {k, 1, Infinity}]
```

```
f3[s, t]
```

$$\sum_{k=1}^{\infty} (-1)^t\,k^{-s}\,Log[k]^t$$

```
f3[2, 3/2]
```

$$\sum_{k=1}^{\infty} -\frac{\mathbb{i}\,Log[k]^{3/2}}{k^2}$$

```
fa[s_] := 1 / (s - 1)
```

```
fa'[s]
```

$$-\frac{1}{(-1+s)^2}$$

```
fa''[s]
```

$$\frac{2}{(-1+s)^3}$$

```
sa[s_, n_] := (-1)^n / n! StieltjesGamma[n] (s - 1)^n
sb[s_] := sa[s, 1]
sc[s_] := sa[s, 2]
sr[s_] := sa[s, 10]
```

```
sb'[s]
```

$-$StieltjesGamma$[1]$

```
sc'[s]
```

$(-1+s)$ StieltjesGamma$[2]$

```
sc''[s]
```

StieltjesGamma$[2]$

```
sr'[s]
```

$$\frac{(-1+s)^9 \text{StieltjesGamma}[10]}{362\,880}$$

```
sr''[s]
```

$$\frac{(-1+s)^8 \text{StieltjesGamma}[10]}{40\,320}$$

```
sr'''[s]
```

$$\frac{(-1+s)^7 \text{StieltjesGamma}[10]}{5040}$$

```
sr''''[s]
```

$\frac{1}{720} (-1+s)^6$ StieltjesGamma$[10]$

```
sr'''''[s]
```

$\frac{1}{120} (-1+s)^5$ StieltjesGamma$[10]$

```
Gamma[6]
```

120

```
sr[s]
```

$$\frac{(-1+s)^{10} \text{StieltjesGamma}[10]}{3\,628\,800}$$

```
ssl[n_, k_] := (s - 1)^(n - k) StieltjesGamma[n] / Gamma[n + 1 - k]
```

ssl[4, 0]

$\frac{1}{24}(-1+s)^4$ StieltjesGamma[4]

fa[s_] := 1 / (s - 1)

fa''[s]

$\frac{2}{(-1+s)^3}$

fa'''''[s]

$-\frac{120}{(-1+s)^6}$

fa''''[s]

$\frac{24}{(-1+s)^5}$

fa'''''''[s]

$-\frac{5040}{(-1+s)^8}$

Gamma[6]

120

Expand[- (1 - s) ^ 6]

$-1 + 6 s - 15 s^2 + 20 s^3 - 15 s^4 + 6 s^5 - s^6$

Expand[- (s - 1) ^ 6]

$-1 + 6 s - 15 s^2 + 20 s^3 - 15 s^4 + 6 s^5 - s^6$

Expand[(s - 1) ^ 3]

$-1 + 3 s - 3 s^2 + s^3$

fs[n_] := Gamma[n] / (- (1 - s) ^ n)

fs[3]

$-\frac{2}{(1-s)^3}$

fs[n_, s_] := Gamma[n + 1] / (- (1 - s) ^ (n + 1))

fs[7 / 3, 4]

$-\frac{(-1)^{2/3}\,\text{Gamma}\left[\frac{10}{3}\right]}{27 \times 3^{1/3}}$

fa'''[4]

$-\frac{2}{27}$

```
ts[t_, s_] := Gamma[t + 1] / (- (1 - s) ^ (t + 1)) +
   Sum[ (s - 1) ^ (n - t) StieltjesGamma[n] / Gamma[n + 1 - t], {n, 1, Infinity}]
```

```
N[ts[1, 2]]
```

StieltjesGamma::intnm : Non−negative machine−sized integer expected at position 1 in StieltjesGamma[16.]. ≫

StieltjesGamma::intnm : Non−negative machine−sized integer expected at position 1 in StieltjesGamma[17.]. ≫

StieltjesGamma::intnm : Non−negative machine−sized integer expected at position 1 in StieltjesGamma[18.]. ≫

General::stop : Further output of StieltjesGamma::intnm will be suppressed during this calculation. ≫

−1.08106

```
N[Zeta'[2]]
```

−0.937548