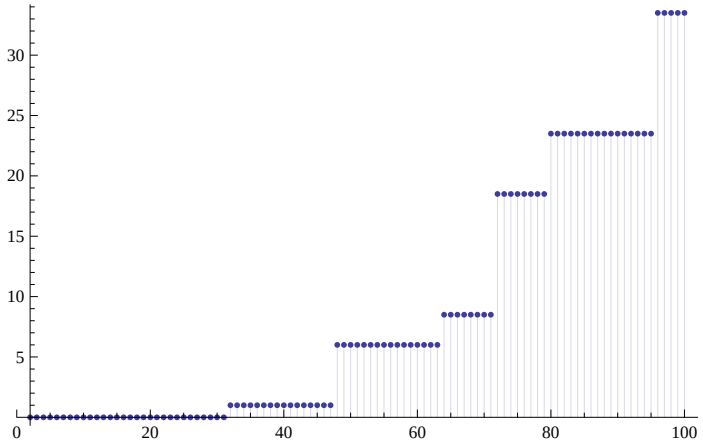
```
ClearAll["Global`*"]
$RecursionLimit = 1000000
d2[n_, k_] := d2[n, k] = Sum[d2[j, k - 1] d2[n / j, 1], {j, Divisors[n]}];
d2[n_, 1] := 1; d2[1, 1] := 0; d2[n_, 0] := 0; d2[1, 0] := 1
D2[n_, k_] := D2[n, k] = D2[n - 1, k] + d2[n, k]; D2[1, k_] := 0; D2[n_, 0] := 1
K[n_] := K[n] = FullSimplify[MangoldtLambda[n] / Log[n]]
k2[n_, k_] := k2[n, k] = Sum[k2[j, k - 1] k2[n / j, 1], {j, Divisors[n]}];
k2[n_, 1] := K[n]; k2[1, 1] := 0; k2[n_, 0] := 0; k2[1, 0] := 1
K2[n_, k_] := K2[n, k] = K2[n - 1, k] + k2[n, k]; K2[1, k_] := 0; K2[n_, 0] := 1
e2[n_, 1] := e2[n, 1] = Sum[ 1 / (k!) d2[n, k], {k, 0, Log[2, n]}]; e2[1, 1] := 0;
e2[n_, k_] := Sum[e2[j, k - 1] e2[n / j, 1], {j, Divisors[n]}]; e2[n_, 0] := 0; e2[1, 0] := 1
E2[n_, k_] := E2[n, k] = E2[n - 1, k] + e2[n, k]; E2[1, k_] := 0; E2[n_, 0] := 1
E1[n_, k_] := Sum[ Binomial[k, j] E2[n, j], {j, 0, Log[2, n]}]
K1[n_, k_] := Sum[ Binomial[k, j] K2[n, j], {j, 0, Log[2, n]}]
D1[n_, k_] := Sum[ Binomial[k, j] D2[n, j], {j, 0, Log[2, n]}]
```

```
1000000
```

```
E1[100, 1]
```

$$\frac{218593}{720}$$

```
DiscretePlot[ K2[n, 5], {n, 2, 100}]
```




```
KD2[n_, k_] := Sum[ k^j / j! K2[n, j], {j, 0, Log[2, n]}]
KD1[n_, k_] := Sum[ k^j / j! K1[n, j], {j, 0, Log[2, n] * 3}]
DE2[n_, k_] := Sum[ k^j / j! D2[n, j], {j, 0, Log[2, n]}]
DE1[n_, k_] := Sum[ k^j / j! D1[n, j], {j, 0, Log[2, n] * 3}]
```

```
N[KD2[1000, 3 I]]
```

```
4101.65 - 2109.86 i
```

```
D1[1000, 2 I]
```

$$\frac{14149}{84} - \frac{736109}{567}\,i$$

```
N[KD1[1000, ss = 3 I] / (E^ss)]
```

```
4101.65 - 2109.86 i
```

```
N[KD1[2, 1]]
```

```
5.16667
```

```
K1[1, 0]
```

```
0
```

```
N[DE2[100, 2]]
```

```
1333.89
```

```
N[E1[100, 2]]
```

```
1333.89
```

```
N[DE1[100, ss = 2] / E^ss]
```

```
1333.89
```

```
Dhyp[n_, k_, a_] := Sum[ d2[m, 2]^(k - j) Binomial[k, j] Dhyp[n / (m^(k - j)), j, m + 1],
  {m, a, n^(1 / k)}, {j, 0, k - 1}]
Dhyp[n_, 1, a_] := Floor[n] - a + 1; Dhyp[n_, 0, a_] := 1
```

```
Dhyp[100, 2, 2]
```

```
116
```

```
D2[100, 8]
```

```
0
```

```
CoefficientList[Series[Cos[x], {x, 0, 30}], x]
```

```
cc := cc = {1, 0, -1/2, 0, 1/24, 0, -1/720, 0, 1/40320, 0,
   -1/3628800, 0, 1/479001600, 0, -1/87178291200, 0, 1/20922789888000, 0,
   -1/6402373705728000, 0, 1/2432902008176640000, 0, -1/1124000727777607680000,
   0, 1/620448401733239439360000, 0, -1/403291461126605635584000000, 0,
   1/304888344611713860501504000000, 0, -1/265252859812191058636308480000000}
```

```
cc[[3]]
```

```
-1/2
```

```
CS2[n_, k_] := Sum[ cc[[j + 1]] D2[n, j], {j, 0, Log[2, n]}]
CS1[n_, k_] := Sum[ cc[[j + 1]] D1[n, j], {j, 0, Log[2, n] * 3}]
```

```
N[CS2[20, 1]]
```

```
-12.4583
```

```
N[CS1[20, 1]]
```

```
-20.896
```

```
DiscretePlot[{CS2[n, 1], CS1[n, 1]}, {n, 2, 500}]
```