```mathematica
E2[n_, k_, b_] :=
 E2[n, k, b] = Sum[E2[n / j, k - 1, b], {j, 2, n}] - b Sum[E2[n / (b j), k - 1, b], {j, 1, n / b}];
E2[n_, 0, a_] := 1
D1[n_, k_, b_] := Sum[Binomial[k + j - 1, k - 1] b^j
    Sum[FactorialPower[k, a] / a ! E2[n / b^j, a, b], {a, 0, Log[If[b > 2, 2, b], n / b^j]}],
   {j, 0, Log[b, n]}]
D11[n_, b_] := Sum[b^j (E2[n / b^j, 1, b] + E2[n / b^j, 0, b]), {j, 0, Log[b, n]}]
E3[n_, b_] := Sum[1, {j, 2, n}] - b Sum[1, {j, 1, n / b}]
D12[n_, b_] := Sum[b^j (E3[n / b^j, b] + 1), {j, 0, Log[b, n]}]
D13[n_, b_] :=
 Sum[b^j (Sum[1, {j, 2, n / b^j}] - b Sum[1, {j, 1, n / b^(j + 1)}] + 1), {j, 0, Log[b, n]}]
D14[n_, b_] :=
 Sum[b^j (Sum[1, {j, 2, Floor[n / b^j]}] - b Sum[1, {j, 1, Floor[n / b^(j + 1)]}] + 1),
   {j, 0, Log[b, n]}]
D15[n_, b_] := Sum[b^j (-b Floor[b^(-1-j) n] + Floor[b^(-j) n]), {j, 0, Log[b, n]}]
```
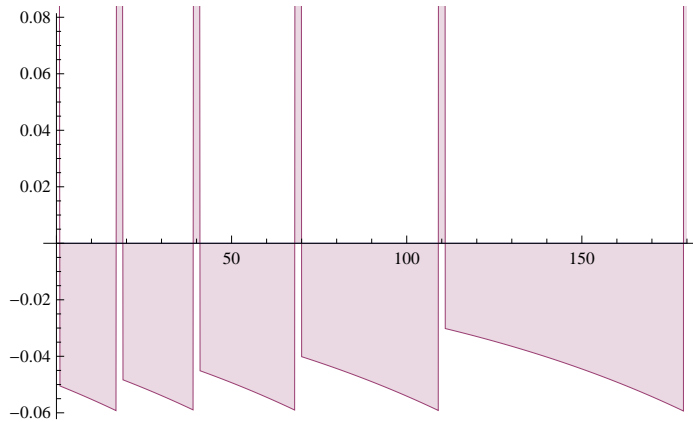
```mathematica
D15[6, 1.01]
```

6.

```mathematica
Log[1.01, 6 / 1]
```

180.07

```mathematica
DiscretePlot[ {0, b^j (-b Floor[b^(-1-j) n] + Floor[b^(-j) n])}, {j, 0, Log[b, n]}] /. {b -> 1.01, n -> 6}
```

DiscretePlot::iterb : Iterator {j, 0, Log[b, n]} does not have appropriate bounds. ≫


```mathematica
FullSimplify[
 Expand[b^j (Sum[1, {j, 2, Floor[n / b^j]}] - b Sum[1, {j, 1, Floor[n / b^(j + 1)]}] + 1)]]
```

$b^j \left(-b \, \text{Floor}\left[b^{-1-j} \, n\right] + \text{Floor}\left[b^{-j} \, n\right]\right)$

```mathematica
Limit[Sum[ b^j (-b Floor[b^(-1-j) 6] + Floor[b^(-j) 6]), {j, 0, Log[b, 6]}], b -> 1]
```

$Aborted

```mathematica
Limit[Sum[ b^j (-b Floor[b^(-1-j) 6] + Floor[b^(-j) 6]), {j, 1, Log[b, 6 / 5]}], b -> 1]
```

$Aborted

$\text{Limit}\left[\text{Sum}\left[b^j (-b\, 5 + 5), \{j, \text{Log}[b, 6 / 6], \text{Log}[b, 6 / 5]\}\right], b \to 1\right]$

-1

$\text{Limit}\left[\text{Sum}\left[b^j (-b\, 4 + 4), \{j, \text{Log}[b, 6 / 5], \text{Log}[b, 6 / 4]\}\right], b \to 1\right]$

$-\dfrac{6}{5}$

$\text{Limit}\left[\text{Sum}\left[b^j (-b\, 3 + 3), \{j, \text{Log}[b, 6 / 4], \text{Log}[b, 6 / 3]\}\right], b \to 1\right]$

$-\dfrac{3}{2}$

$\text{Limit}\left[\text{Sum}\left[b^j (-b\, 2 + 2), \{j, \text{Log}[b, 6 / 3], \text{Log}[b, 6 / 2]\}\right], b \to 1\right]$

-2

$\text{Limit}\left[\text{Sum}\left[b^j (-b\, 1 + 1), \{j, \text{Log}[b, 6 / 2], \text{Log}[b, 6 / 1]\}\right], b \to 1\right]$

-3

-1 - 6 / 5 - 3 / 2 - 2 - 3

$-\dfrac{87}{10}$

(6 / 6 + 6 / 5 + 6 / 4 + 6 / 3 + 6 / 2 + 6 / 1) + (-1 - 6 / 5 - 3 / 2 - 2 - 3)

6

```
D21[n_, b_] := Sum[ (j + 1) b^j
    Sum[FactorialPower[2, a] / a! E2[n / b^j, a, b], {a, 0, Log[If[b > 2, 2, b], n / b^j]}],
   {j, 0, Log[b, n]}]
D22[n_, b_] := Sum[ (j + 1) b^j
    (E2[n / b^j, 2, b] + 2 E2[n / b^j, 1, b] + E2[n / b^j, 0, b]), {j, 0, Log[b, n]}]
D23[n_, b_] := Sum[ (j + 1) b^j
    (E2[n / b^j, 2, b] +
      2 (Sum[1, {r, 2, Floor[n / b^j]}] - b Sum[1, {r, 1, Floor[n / b^ (j + 1)]}]) +
      1), {j, 0, Log[b, n]}]
D25[n_, b_] := Sum[ (j + 1) b^j
    ( (Sum[1, {r, 2, n / b^j}, {k, 2, Floor[n / r / b^j]}] -
        2 b Sum[1, {r, 1, Floor[n / b / b^j]}, {k, 2, Floor[n / (r b) / b^j]}] +
        b^2 Sum[1, {r, 1, Floor[n / b / b^j]}, {k, 1, Floor[n / (b^2 r) / b^j]}]) +
      2 (Sum[1, {r, 2, Floor[n / b^j]}] - b Sum[1, {r, 1, Floor[n / b^ (j + 1)]}]) +
      1), {j, 0, Log[b, n]}]
D26[n_, b_] := Sum[ (j + 1) b^j
    ( (Sum[1, {r, 2, n / b^j}, {k, 2, Floor[n / r / b^j]}] -
        2 b Sum[1, {r, 1, Floor[n / b / b^j]}, {k, 2, Floor[n / (r b) / b^j]}] +
        b^2 Sum[1, {r, 1, Floor[n / b / b^j]}, {k, 1, Floor[n / (b^2 r) / b^j]}]) +
      2 (Sum[1, {r, 2, Floor[n / b^j]}] - b Sum[1, {r, 1, Floor[n / b^ (j + 1)]}]) +
      1), {j, 0, Log[b, n]}]
D27[n_, b_] := Sum[ (j + 1) b^j
    ( (Sum[1, {r, 2, n / b^j}, {k, 2, Floor[n / r / b^j]}] -
        2 b Sum[1, {r, 1, Floor[n / b / b^j]}, {k, 2, Floor[n / (r b) / b^j]}] +
        b^2 Sum[1, {r, 1, Floor[n / b / b^j]}, {k, 1, Floor[n / (b^2 r) / b^j]}]) +
      2 (Floor[n / b^j] - 1 - b Floor[n / b^ (j + 1)]) + 1), {j, 0, Log[b, n]}]


D27[100, 2]

482

Binomial[1 + j, 1]

1 + j

E3a[n_, b_] := Sum[1, {r, 2, n}, {k, 2, Floor[n / r]}] -
  2 b Sum[ 1, {r, 1, Floor[n / b]}, {k, 2, Floor[n / (r b)]}] +
  b^2 Sum[1, {r, 1, Floor[n / b]}, {k, 1, Floor[n / (b^2 r)]}]

E3a[1200, 3]

-20

E2[1200, 2, 3]

-20

FullSimplify[(j + 1) b^j
   ( (Sum[1, {r, 2, n / b^j}, {k, 2, Floor[n / r / b^j]}] -
        2 b Sum[1, {r, 1, Floor[n / b / b^j]}, {k, 2, Floor[n / (r b) / b^j]}] +
        b^2 Sum[1, {r, 1, Floor[n / b / b^j]}, {k, 1, Floor[n / (b^2 r) / b^j]}]) +
      2 (Sum[1, {r, 2, Floor[n / b^j]}] - b Sum[1, {r, 1, Floor[n / b^ (j + 1)]}]) + 1)]
```

$$\text{Expand}\left[b^j (1+j) \left(-1 + 2 \text{ Floor}\left[b^{-j} n\right] + \right.\right.$$

$$b\left(-2 \text{ Floor}\left[b^{-1-j} n\right] + b \sum_{r=1}^{\text{Floor}\left[b^{-1-j} n\right]} \sum_{k=1}^{\text{Floor}\left[\frac{b^{-2-j} n}{r}\right]} 1 - 2 \sum_{r=1}^{\text{Floor}\left[b^{-1-j} n\right]} \sum_{k=2}^{\text{Floor}\left[\frac{b^{-1-j} n}{r}\right]} 1\right) + \sum_{r=2}^{b^{-j} n} \sum_{k=2}^{\text{Floor}\left[\frac{b^{-j} n}{r}\right]} 1\left.\left.\right)\right]$$

$$-b^j - b^j j - 2 b^{1+j} \text{ Floor}\left[b^{-1-j} n\right] - 2 b^{1+j} j \text{ Floor}\left[b^{-1-j} n\right] + 2 b^j \text{ Floor}\left[b^{-j} n\right] +$$

$$2 b^j j \text{ Floor}\left[b^{-j} n\right] + b^{2+j} \sum_{r=1}^{\text{Floor}\left[b^{-1-j} n\right]} \sum_{k=1}^{\text{Floor}\left[\frac{b^{-2-j} n}{r}\right]} 1 + b^{2+j} j \sum_{r=1}^{\text{Floor}\left[b^{-1-j} n\right]} \sum_{k=1}^{\text{Floor}\left[\frac{b^{-2-j} n}{r}\right]} 1 -$$

$$2 b^{1+j} \sum_{r=1}^{\text{Floor}\left[b^{-1-j} n\right]} \sum_{k=2}^{\text{Floor}\left[\frac{b^{-1-j} n}{r}\right]} 1 - 2 b^{1+j} j \sum_{r=1}^{\text{Floor}\left[b^{-1-j} n\right]} \sum_{k=2}^{\text{Floor}\left[\frac{b^{-1-j} n}{r}\right]} 1 + b^j \sum_{r=2}^{b^{-j} n} \sum_{k=2}^{\text{Floor}\left[\frac{b^{-j} n}{r}\right]} 1 + b^j j \sum_{r=2}^{b^{-j} n} \sum_{k=2}^{\text{Floor}\left[\frac{b^{-j} n}{r}\right]} 1$$