```
ps11b[n_, s_] :=
```

$$\left(-n^{\frac{1}{2}-\mathbb{i}\,s}\left(\frac{1}{2}+\mathbb{i}\,s\right)\text{HarmonicNumber}\left[n,\frac{1}{2}-\mathbb{i}\,s\right]+n^{\frac{1}{2}+\mathbb{i}\,s}\left(\frac{1}{2}-\mathbb{i}\,s\right)\text{HarmonicNumber}\left[n,\frac{1}{2}+\mathbb{i}\,s\right]\right)\Big/$$

$$\left(-n^{\frac{1}{2}-\mathbb{i}\,s}\left(\frac{1}{2}+\mathbb{i}\,s\right)+2^{\frac{1}{2}+\mathbb{i}\,s}\,n^{\frac{1}{2}+\mathbb{i}\,s}\,\pi^{-\frac{1}{2}+\mathbb{i}\,s}\left(\frac{1}{2}-\mathbb{i}\,s\right)\text{Cos}\left[\frac{1}{2}\pi\left(\frac{1}{2}-\mathbb{i}\,s\right)\right]\text{Gamma}\left[\frac{1}{2}-\mathbb{i}\,s\right]\right)$$

```
ps11c[n_, s_] := ps11b[n, s I - I / 2]
ps11d[n_, s_] :=
```

$$\left(n^{\wedge}(1/2)\left(\text{Sum}\left[-n^{-\mathbb{i}\,s}\left(\frac{1}{2}+\mathbb{i}\,s\right)j^{\wedge}\left(-\left(\frac{1}{2}-\mathbb{i}\,s\right)\right)+n^{\mathbb{i}\,s}\left(\frac{1}{2}-\mathbb{i}\,s\right)j^{\wedge}\left(-\left(\frac{1}{2}+\mathbb{i}\,s\right)\right),\{j,1,n\}\right]\right)\right)\Big/$$

$$\left(-n^{\frac{1}{2}-\mathbb{i}\,s}\left(\frac{1}{2}+\mathbb{i}\,s\right)+2^{\frac{1}{2}+\mathbb{i}\,s}\,n^{\frac{1}{2}+\mathbb{i}\,s}\,\pi^{-\frac{1}{2}+\mathbb{i}\,s}\left(\frac{1}{2}-\mathbb{i}\,s\right)\text{Cos}\left[\frac{1}{2}\pi\left(\frac{1}{2}-\mathbb{i}\,s\right)\right]\text{Gamma}\left[\frac{1}{2}-\mathbb{i}\,s\right]\right)$$

```
ps11e[n_, s_] :=
```

$$\left(n^{\wedge}(1/2)\left(\text{Sum}\left[j^{\wedge}(-1/2)\left(-n^{-\mathbb{i}\,s}\left(\frac{1}{2}+\mathbb{i}\,s\right)j^{\wedge}(\mathbb{i}\,s)+n^{\mathbb{i}\,s}\left(\frac{1}{2}-\mathbb{i}\,s\right)j^{\wedge}(-\mathbb{i}\,s)\right),\{j,1,n\}\right]\right)\right)\Big/$$

$$\left(-n^{\frac{1}{2}-\mathbb{i}\,s}\left(\frac{1}{2}+\mathbb{i}\,s\right)+2^{\frac{1}{2}+\mathbb{i}\,s}\,n^{\frac{1}{2}+\mathbb{i}\,s}\,\pi^{-\frac{1}{2}+\mathbb{i}\,s}\left(\frac{1}{2}-\mathbb{i}\,s\right)\text{Cos}\left[\frac{1}{2}\pi\left(\frac{1}{2}-\mathbb{i}\,s\right)\right]\text{Gamma}\left[\frac{1}{2}-\mathbb{i}\,s\right]\right)$$

```
ps11f[n_, s_] :=
```

$$\left(n^{\wedge}(1/2)\left(\text{Sum}\left[j^{\wedge}(-1/2)\left(\left(\frac{1}{2}-\mathbb{i}\,s\right)(n/j)^{\wedge}(\mathbb{i}\,s)-\left(\frac{1}{2}+\mathbb{i}\,s\right)(n/j)^{\wedge}(-\mathbb{i}\,s)\right),\{j,1,n\}\right]\right)\right)\Big/$$

$$\left(-n^{\frac{1}{2}-\mathbb{i}\,s}\left(\frac{1}{2}+\mathbb{i}\,s\right)+2^{\frac{1}{2}+\mathbb{i}\,s}\,n^{\frac{1}{2}+\mathbb{i}\,s}\,\pi^{-\frac{1}{2}+\mathbb{i}\,s}\left(\frac{1}{2}-\mathbb{i}\,s\right)\text{Cos}\left[\frac{1}{2}\pi\left(\frac{1}{2}-\mathbb{i}\,s\right)\right]\text{Gamma}\left[\frac{1}{2}-\mathbb{i}\,s\right]\right)$$

```
ps11g[n_, s_] :=
```

$$\left(\text{Sum}\left[j^{\wedge}(-1/2)\left(\left(\frac{1}{2}-\mathbb{i}\,s\right)(n/j)^{\wedge}(\mathbb{i}\,s)-\left(\frac{1}{2}+\mathbb{i}\,s\right)(n/j)^{\wedge}(-\mathbb{i}\,s)\right),\{j,1,n\}\right]\right)\Big/$$

$$\left(-n^{-\mathbb{i}\,s}\left(\frac{1}{2}+\mathbb{i}\,s\right)+2^{\frac{1}{2}+\mathbb{i}\,s}\,n^{+\mathbb{i}\,s}\,\pi^{-\frac{1}{2}+\mathbb{i}\,s}\left(\frac{1}{2}-\mathbb{i}\,s\right)\text{Cos}\left[\frac{1}{2}\pi\left(\frac{1}{2}-\mathbb{i}\,s\right)\right]\text{Gamma}\left[\frac{1}{2}-\mathbb{i}\,s\right]\right)$$

```
ps11h[n_, s_] := (Sum[j ^ (-1 / 2) ((-I s) ((n / j) ^ (i s) + (n / j) ^ (-i s)) +
        (1 / 2) ((n / j) ^ (i s) - (n / j) ^ (-i s))), {j, 1, n}])
```
$$\Big/$$

$$\left(-n^{-\mathbb{i}\,s}\left(\frac{1}{2}+\mathbb{i}\,s\right)+2^{\frac{1}{2}+\mathbb{i}\,s}\,n^{+\mathbb{i}\,s}\,\pi^{-\frac{1}{2}+\mathbb{i}\,s}\left(\frac{1}{2}-\mathbb{i}\,s\right)\text{Cos}\left[\frac{1}{2}\pi\left(\frac{1}{2}-\mathbb{i}\,s\right)\right]\text{Gamma}\left[\frac{1}{2}-\mathbb{i}\,s\right]\right)$$

```
ps11i[n_, s_] := (I Sum[j ^ (-1 / 2) (2 s Cos[s Log[n / j]] - Sin[s Log[n / j]]), {j, 1, n}])
```
$$\Big/$$

$$\left(n^{-\mathbb{i}\,s}\left(\frac{1}{2}+\mathbb{i}\,s\right)-2^{\frac{1}{2}+\mathbb{i}\,s}\,n^{+\mathbb{i}\,s}\,\pi^{-\frac{1}{2}+\mathbb{i}\,s}\left(\frac{1}{2}-\mathbb{i}\,s\right)\text{Cos}\left[\frac{1}{2}\pi\left(\frac{1}{2}-\mathbb{i}\,s\right)\right]\text{Gamma}\left[\frac{1}{2}-\mathbb{i}\,s\right]\right)$$

```
ps11j[n_, s_] := (I Sum[j ^ (-1 / 2) (2 s Cos[s Log[n / j]] - Sin[s Log[n / j]]), {j, 1, n}])
```
$$\Big/$$

$$\left(\left(\left(\mathbb{i}\,s+\frac{1}{2}\right)n^{-\mathbb{i}\,s}+\left(\mathbb{i}\,s-\frac{1}{2}\right)n^{\mathbb{i}\,s}\,2^{\frac{1}{2}+\mathbb{i}\,s}\,\pi^{-\frac{1}{2}+\mathbb{i}\,s}\,\text{Cos}\left[\frac{\text{Pi}}{4}-\mathbb{i}\,s\,\text{Pi}/2\right]\text{Gamma}\left[\frac{1}{2}-\mathbb{i}\,s\right]\right)\right)$$

```
ps11k[n_, s_] := Sum[j ^ (-1 / 2) (2 s Cos[s Log[n / j]] - Sin[s Log[n / j]]), {j, 1, n}]
```
$$\Big/$$

$$\left(\left(\left(s-\frac{1}{2}\,I\right)n^{-\mathbb{i}\,s}+\left(s+\frac{1}{2}\,I\right)n^{\mathbb{i}\,s}\,2^{\frac{1}{2}+\mathbb{i}\,s}\,\pi^{-\frac{1}{2}+\mathbb{i}\,s}\,\text{Cos}\left[\frac{\text{Pi}}{4}-\mathbb{i}\,s\,\text{Pi}/2\right]\text{Gamma}\left[\frac{1}{2}-\mathbb{i}\,s\right]\right)\right)$$

```
ps11l[n_, s_] := Sum[j ^ (-1 / 2) (2 s Cos[s Log[n / j]] - Sin[s Log[n / j]]), {j, 1, n}]
```
$$\Big/$$

$$\left(s\left(n^{-\mathbb{i}\,s}+n^{\mathbb{i}\,s}\,2^{\frac{1}{2}+\mathbb{i}\,s}\,\pi^{-\frac{1}{2}+\mathbb{i}\,s}\,\text{Cos}\left[\frac{\text{Pi}}{4}-\mathbb{i}\,s\,\text{Pi}/2\right]\text{Gamma}\left[\frac{1}{2}-\mathbb{i}\,s\right]\right)+\right.$$

$$I \,/\, 2 \left(-n^{-i\,s} + n^{i\,s}\, 2^{\frac{1}{2}+i\,s}\, \pi^{-\frac{1}{2}+i\,s}\, \text{Cos}\left[\frac{Pi}{4} - i\,s\,Pi\,/\,2\right] \text{Gamma}\left[\frac{1}{2} - i\,s\right]\right)\Bigg)$$

```
ps11m[n_, s_] := Sum[j^(-1/2) (2 s Cos[s Log[n/j]] - Sin[s Log[n/j]]), {j, 1, n}]/
```
$$\left(s\left(n^{-i\,s} + n^{i\,s}\, 2^{\frac{1}{2}+i\,s}\, \pi^{-\frac{1}{2}+i\,s}\, \text{Cos}\left[\frac{Pi}{4} - i\,s\,Pi\,/\,2\right] \text{Gamma}\left[\frac{1}{2} - i\,s\right]\right) + \right.$$
$$\left. I\,/\,2\left(n^{i\,s}\, 2^{\frac{1}{2}+i\,s}\, \pi^{-\frac{1}{2}+i\,s}\, \text{Cos}\left[\frac{Pi}{4} - i\,s\,Pi\,/\,2\right] \text{Gamma}\left[\frac{1}{2} - i\,s\right] - n^{-i\,s}\right)\right)$$

```
ps11n[n_, s_] := Sum[j^(-1/2) (2 s Cos[s Log[n/j]] - Sin[s Log[n/j]]), {j, 1, n}]/
```
$$\left(s\left(E^{\wedge}(I\,s\,\text{Log}[n])\, 2^{\frac{1}{2}+i\,s}\, \pi^{-\frac{1}{2}+i\,s}\, \text{Cos}\left[\frac{Pi}{4} - i\,s\,Pi\,/\,2\right] \text{Gamma}\left[\frac{1}{2} - i\,s\right] + E^{\wedge}(-I\,s\,\text{Log}[n])\right) + \right.$$
$$\left. I\,/\,2\left(E^{\wedge}(I\,s\,\text{Log}[n])\, 2^{\frac{1}{2}+i\,s}\, \pi^{-\frac{1}{2}+i\,s}\, \text{Cos}\left[\frac{Pi}{4} - i\,s\,Pi\,/\,2\right] \text{Gamma}\left[\frac{1}{2} - i\,s\right] - E^{\wedge}(-I\,s\,\text{Log}[n])\right)\right)$$

```
ps11o[n_, s_] := Sum[j^(-1/2) (2 s Cos[s Log[n/j]] - Sin[s Log[n/j]]), {j, 1, n}]/
```
$$\left(s\left(E^{\wedge}(I\,s\,\text{Log}[n])\, 2^{\frac{1}{2}+i\,s}\, \pi^{-\frac{1}{2}+i\,s}\, \text{Cos}\left[\frac{Pi}{4} - i\,s\,Pi\,/\,2\right] \text{Gamma}\left[\frac{1}{2} - i\,s\right] + E^{\wedge}(-I\,s\,\text{Log}[n])\right) + \right.$$
$$\left. I\,/\,2\left(E^{\wedge}(I\,s\,\text{Log}[n])\, 2^{\frac{1}{2}+i\,s}\, \pi^{-\frac{1}{2}+i\,s}\, \text{Cos}\left[\frac{Pi}{4} - i\,s\,Pi\,/\,2\right] \text{Gamma}\left[\frac{1}{2} - i\,s\right] - E^{\wedge}(-I\,s\,\text{Log}[n])\right)\right)$$

```
N@ps11o[10 000, .3 + 4 I]
```
1.053 + 0.0129554 i

```
N@ps11b[10 000, .3 + 4 I]
```
1.053 + 0.0129554 i

```
N@ps11jx[100 000, .3 + 4 I]
```
0.575524 + 0.107901 i

```
Zeta[.3 + 4 I]
```
0.575756 + 0.10773 i

```
FullSimplify[1 / 2 (E^(1/2 π (1/2 - i s)) + E^(-1/2 π (1/2 - i s)))]
```
$$\text{Cos}\left[\frac{1}{4}\, \pi\,(i + 2\,s)\right]$$

$$E^{\wedge}(I\,s\,\text{Log}[n] + (1\,/\,2 + s\,I)\,\text{Log}[2] + (-1\,/\,2 + s\,I)\,\text{Log}[Pi])\left(1\,/\,2\left(E^{\wedge}\left(\frac{\pi}{4} - \frac{i\,\pi\,s}{2}\right) + E^{\wedge}\left(-\frac{\pi}{4} + \frac{i\,\pi\,s}{2}\right)\right)\right)$$

$$\left(e^{-\frac{1}{2}\,\pi\,\left(\frac{1}{2}-i\,s\right)} + e^{\frac{1}{2}\,\pi\,\left(\frac{1}{2}-i\,s\right)}\right) n^{i\,s}\,(2\,\pi)^{-\frac{1}{2}+i\,s}$$

$$E^{\wedge}(I\,s\,\text{Log}[n] + (1\,/\,2 + s\,I)\,\text{Log}[2] + (-1\,/\,2 + s\,I)\,\text{Log}[Pi])\,/\,2\left(E^{\wedge}\left(\frac{\pi}{4} - \frac{i\,\pi\,s}{2}\right) + E^{\wedge}\left(-\frac{\pi}{4} + \frac{i\,\pi\,s}{2}\right)\right)$$

$$\left(E^{\wedge}\left(I\ s\ Log[n] + (1/2 + s\ I)\ Log[2] + (-1/2 + s\ I)\ Log[Pi] + \frac{\pi}{4} - \frac{i\ \pi\ s}{2} \right) + \right.$$

$$\left. E^{\wedge}\left(I\ s\ Log[n] + (1/2 + s\ I)\ Log[2] + (-1/2 + s\ I)\ Log[Pi] - \frac{\pi}{4} + \frac{i\ \pi\ s}{2} \right) \right) / 2$$

$$\frac{1}{2}\left(e^{\frac{\pi}{4} - \frac{i\pi s}{2} + \left(\frac{1}{2} + i\ s\right) Log[2] + i\ s\ Log[n] + \left(-\frac{1}{2} + i\ s\right) Log[\pi]} + e^{-\frac{\pi}{4} + \frac{i\pi s}{2} + \left(\frac{1}{2} + i\ s\right) Log[2] + i\ s\ Log[n] + \left(-\frac{1}{2} + i\ s\right) Log[\pi]} \right)$$

$$\mathbf{FullSimplify}\left[I\ s\ Log[n] + (1/2 + s\ I)\ Log[2] + (-1/2 + s\ I)\ Log[Pi] + \frac{\pi}{4} - \frac{i\ \pi\ s}{2} \right]$$

$$\frac{1}{4}\left(\pi - 2\ i\ \pi\ s + Log\left[\frac{4}{\pi^2}\right] + 4\ i\ s\ Log[2\ n\ \pi] \right)$$

$$\mathbf{FullSimplify}\left[I\ s\ Log[n] + (1/2 + s\ I)\ Log[2] + (-1/2 + s\ I)\ Log[Pi] - \frac{\pi}{4} + \frac{i\ \pi\ s}{2} \right]$$

$$\frac{1}{4}\left(\pi\ (-1 + 2\ i\ s) + Log\left[\frac{4}{\pi^2}\right] + 4\ i\ s\ Log[2\ n\ \pi] \right)$$

$$\mathbf{Cos}\left[\frac{\mathbf{Pi}}{4} - i\ s\ Pi\ /\ 2 \right]\ /.\ s \to 11$$

$$Cosh\left[\left(\frac{11}{2} + \frac{i}{4} \right) \pi \right]$$

$$\mathbf{N[-Sin[-i\ s\ Pi\ /\ 2]\ /.\ s \to 11]}$$

$$0. + 1.59603 \times 10^7\ i$$

$$\mathbf{FullSimplify}\left[2^{\frac{1}{2} + i\ s}\ \pi^{-\frac{1}{2} + i\ s}\ \mathbf{Cos}\left[\frac{\mathbf{Pi}}{4} - i\ s\ Pi\ /\ 2 \right]\ \mathbf{Gamma}\left[\frac{1}{2} - i\ s \right]\ /.\ s \to 3\ I \right]$$

$$\frac{15}{64\ \pi^3}$$

$$\mathbf{1\ /\ I}$$

$$- i$$

$$\mathbf{fff[n\_, s\_]} := E^{\wedge}(I\ s\ Log[n])\ 2^{\frac{1}{2} + i\ s}\ \pi^{-\frac{1}{2} + i\ s}\ \mathbf{Cos}\left[\frac{\mathbf{Pi}}{4} - i\ s\ Pi\ /\ 2 \right]\ \mathbf{Gamma}\left[\frac{1}{2} - i\ s \right] + E^{\wedge}(-I\ s\ Log[n])$$

$$\mathbf{Plot[Re[fff[100, s]\ ],\ \{s,\ 50,\ 50 + 2\ Pi\}]}$$
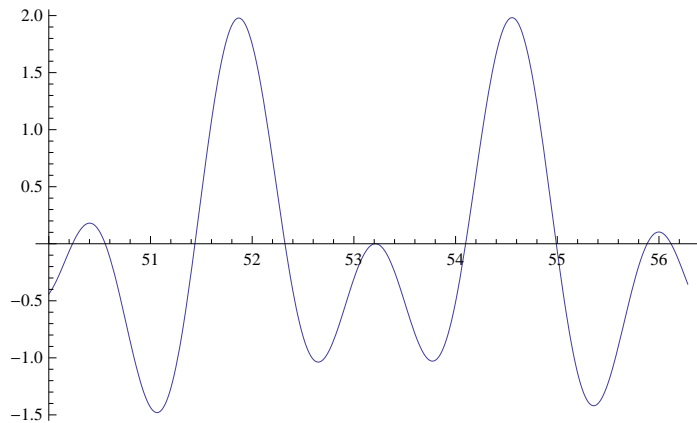
Plot[{Re[fff[100, s]], Re[Cos[s Log[100]]]}, {s, 1130, 1130 + 2 Pi}]