```
Clear[D2]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
D2[n_, k_] := D2[n, k] = Sum[ D2[Floor[n / j], k - 1], {j, 2, n}];
D2[n_, 0] := UnitStep[n - 1]
d2[n_, k_] := D2[n, k] - D2[n - 1, k]
Dz[n_, z_] := Sum[ bin[ z, k] D2[n, k], {k, 0, Log[2, n]}]
Ez[n_, t_] := Sum[ Dz[n, k] / k!, {k, 0, t}]
ez[n_, t_] := Ez[n, t] - Ez[n - 1, t]
Ez2[n_] := Sum[ D2[n, k] / k!, {k, 0, Log[2, n]}]
ez2[n_] := Ez2[n] - Ez2[n - 1]
Sn[n_, t_] := Sn[n, t] = Sum[(D[Sin[ x ], {x, k}] /. x → 0) / k! Dz[n, k], {k, 0, t}]
Cs[n_, t_] := Cs[n, t] = Sum[(D[Cos[ x ], {x, k}] /. x → 0) / k! Dz[n, k], {k, 0, t}]
sn[n_, t_] := Sn[n, t] - Sn[n - 1, t]
cs[n_, t_] := Cs[n, t] - Cs[n - 1, t]
Sn2[n_] := Sn2[n] = Sum[(D[Sin[ x ], {x, k}] /. x → 0) / k! D2[n, k], {k, 0, Log[2, n]}]
Cs2[n_] := Cs2[n] = Sum[(D[Cos[ x ], {x, k}] /. x → 0) / k! D2[n, k], {k, 0, Log[2, n]}]
sn2[n_] := Sn2[n] - Sn2[n - 1]
cs2[n_] := Cs2[n] - Cs2[n - 1]
l[n_, z_] := LaguerreL[-z, Log[n]]
l2[n_, z_] := (-1)^(z) Gamma[z, 0, -Log[n]] / Gamma[z]
lEz[n_, t_] := Sum[ l[n, k] / k!, {k, 0, t}]
lEz2[n_, t_] := 1 + Sum[l2[n, k] / k!, {k, 1, t}]
lSn[n_, t_] := lSn[n, t] = Sum[(D[Sin[ x ], {x, k}] /. x → 0) / k! l[n, k], {k, 0, t}]
lCs[n_, t_] := lCs[n, t] = Sum[(D[Cos[ x ], {x, k}] /. x → 0) / k! l[n, k], {k, 0, t}]
lSn2[n_, t_] := lSn2[n] = Sum[(D[Sin[ x ], {x, k}] /. x → 0) / k! l2[n, k], {k, 0, t}]
lCs2[n_, t_] := lCs2[n] = Sum[(D[Cos[ x ], {x, k}] /. x → 0) / k! l2[n, k], {k, 0, t}]
```

```
N@Sum[ cs[j, 30] cs[k, 30], {j, 1, 100}, {k, 1, 100 / j}] +
 Sum[ sn[j, 30] sn[k, 30], {j, 1, 100}, {k, 1, 100 / j}]
```

1.

```
N@Sum[ cs2[j] cs2[k], {j, 1, 100}, {k, 1, 100 / j}] +
 Sum[ sn2[j] sn2[k], {j, 1, 100}, {k, 1, 100 / j}]
```

1.

```
2 N@Sum[sn[j, 30] cs[k, 30], {j, 1, 100}, {k, 1, 100 / j}]
```

-311.06

```
N@Sn[2 × 100, 30]
```

-249.411

```
2 N@Sum[sn2[j] cs2[k], {j, 1, 100}, {k, 1, 100 / j}]
```

-220.4

```
N@Sn2[100]
```

45.425

```
N@lEz[10, 15] / N@lEz2[10, 15]
```

$2.71828 + 2.43642 \times 10^{-16}\, i$

```
N@lEz[13, 15] / N@lEz2[13, 15]
```

$2.71828 + 2.83305 \times 10^{-16}\, i$

```
N@lEz[3 I, 40]
```

$-5.63354 + 7.4274\, i$

```
N[lCs[3, 40] + I lSn[3, 40]]
```

$-1.41744 + 1.33588\, i$

```
Clear[ff, ff2, gg2]
(*ff[ n_] :=
 ff[n]=Sum[ (Floor[j^(1/2)]-Floor[(j-1)^(1/2)]) MoebiusMu[k],{j,1,n},{k,1,n/j}]*)
fn[j_] := ((Floor[j^(2 / 3)] - Floor[(j - 1)^(2 / 3)]))
ff[ n_] := ff[n] = Sum[ fn[j] fn[k] , {j, 1, n}, {k, 1, n / j}]
ffd[n_] := ff[n] - ff[n - 1]
ff2[n_, k_] := ff2[n, k] = Sum[ fn[j] ff2[Floor[n / j], k - 1], {j, 2, n}]
ff2[n_, 0] := UnitStep[n - 1]
ffl[n_] := Sum[ (-1)^(k + 1) / k ff2[n, k], {k, 1, Log2@n}]
gg2[n_, k_] := gg2[n, k] = Sum[fn[j] gg2[Floor[n / j], k - 1], {j, 2, n}]
gg2[n_, 0] := UnitStep[n - 1]
ggl[n_] := Sum[ (-1)^(k + 1) / k gg2[n, k], {k, 1, Log2@n}]
pr[n_] := Sum[ PrimePi[ n^(1 / k)] / k, {k, 1, Log2@n}]
```

```
Table[ ffl[n] - ffl[n - 1], {n, 1, 20}]
```

$\left\{0, 0, 1, 0, 0, 1, 0, 1, -\frac{1}{2}, 0, 0, 1, 0, 0, 1, 0, 0, -1, 1, 0\right\}$

```
Table[ LiouvilleLambda[n], {n, 2, 20}]
```

{-1, -1, 1, -1, 1, -1, -1, 1, 1, -1, -1, -1, 1, 1, 1, -1, -1, -1, -1}

```
fg[n_] := Sum[ j * MoebiusMu[k], {j, 1, n}, {k, 1, n / j}]
```

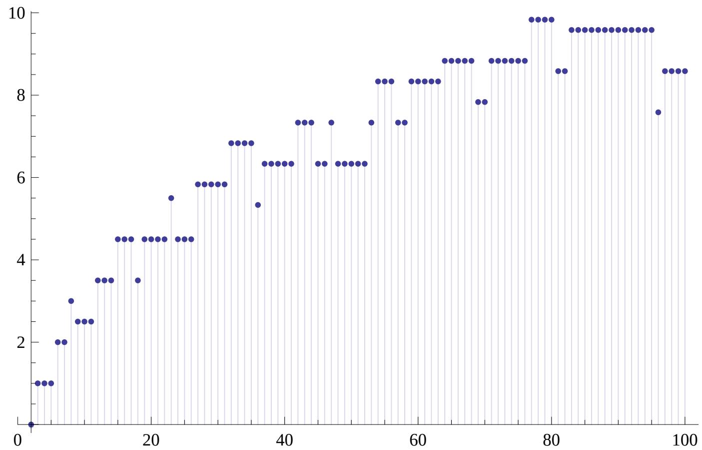
```
fg[100] - fg[99]
```

40

```
EulerPhi[100]
```

40

```
DiscretePlot[ffl[n] , {n, 2, 100}]
```




```
Log[ (1 - x^(1 - s)) Zeta[s]] /. {x → 2, s → 2}
```

$\text{Log}\left[\frac{\pi^2}{12}\right]$

```
N@Log[ (1 - x^(1 - s)) Zeta[s]^2] /. {x → 2, s → 2}
```

0.302253

```
N@2 Log[ (1 - x^(1 - s))^(1 / 2) Zeta[s]] /. {x → 2, s → 2}
```

0.302253

```
Log[a b^c]
```

$\text{Log}[a\, b^c]$

```
N@Log[ (1 - x^(1 - s))] + 2 Log[Zeta[s]] /. {x → 2, s → 2}
```

0.302253

```
FullSimplify@Sum[ -x^(k (1 - s)) / k, {k, 1, Infinity}]
```

$\text{Log}\left[1 - x^{1-s}\right]$

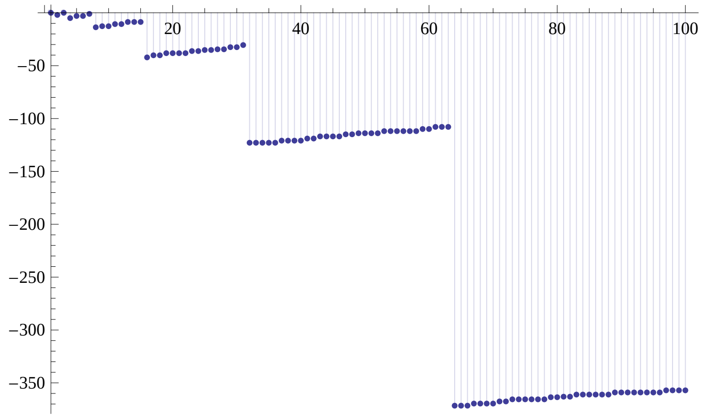
```
FullSimplify@Sum[ - x^(k (1 - s)) / k, {k, 1, Infinity}]
```

$\text{Log}\left[1 - x^{1-s}\right]$

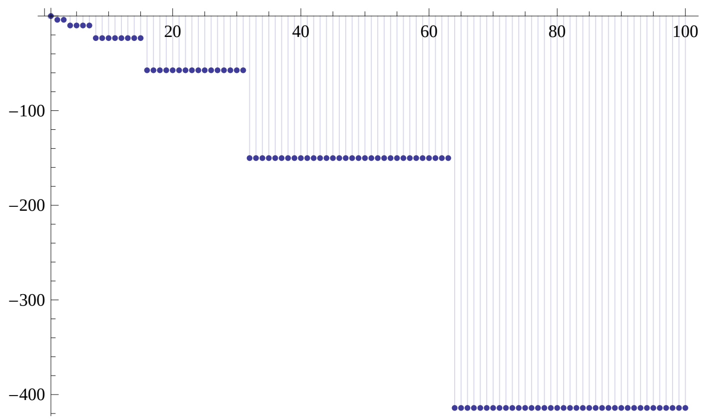
```
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}]

Clear[fk]
fk[n_, z_, k_] :=
 fk[n, z, k] = Sum[ (-1)^(j + 1) dz[j, 2] fk[Floor[n / j], z, k - 1], {j, 2, n}]
fk[n_, z_, 0] := UnitStep[n - 1]
fl[n_, z_] := Sum[ (-1)^(k + 1) / k fk[n, z, k], {k, 1, Log2@n}]
```

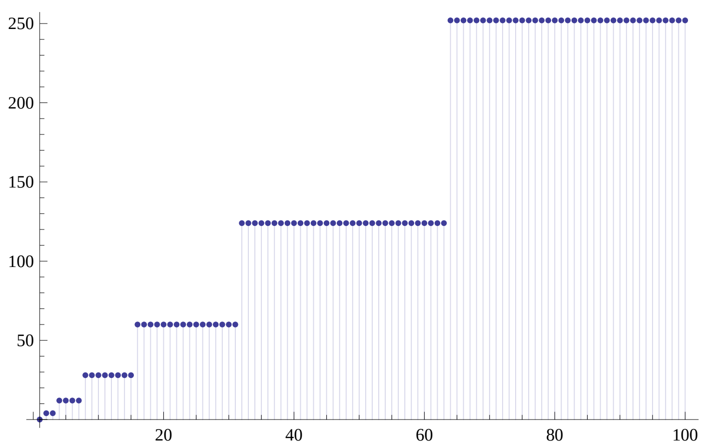
```
DiscretePlot[ fl[n, 1], {n, 1, 100}]
```




```
DiscretePlot[ fl[n, 2] - 2 pr[n], {n, 1, 100}]
```




```
ad[n_] := -Sum[ 2^k × 2 dz[2, -1 k] / k, {k, 1, Log[2, n]}]
DiscretePlot[ad[n], {n, 1, 100}]
```



```
Clear[D1xD, E2, Dz]
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, z_] := Product[(-1)^p[[2]] bin[-z, p[[2]]], {p, FI[n]}]
Dz[n_, z_, z2_, k_] :=
 Dz[n, z, z2, k] = 1 + ((z + 1) / k - 1) Sum[dz[j, z2] Dz[Floor[n / j], z, z2, k + 1], {j, 2, n}]
E2[n_, k_] := E2[n, k] = Sum[(-1)^(j + 1) E2[Floor[n / j], k - 1], {j, 2, n}];
E2[n_, 0] := UnitStep[n - 1]
Etz[n_, z_] := Sum[ bin[z, k] E2[n, k], {k, 0, Log[2, n]}]
etz[n_, z_] := Etz[n, z] - Etz[n - 1, z]
D1xD[n_, k_, z2_] := D1xD[n, k, z2] = Sum[etz[j, z2] D1xD[n / j, k - 1, z2], {j, 2, n}];
D1xD[n_, 0, z2_] := UnitStep[n - 1]
E1[n_, z_] := Sum[ (-1)^(k + 1) / k D1xD[n, k, z], {k, 1, Log2@n}]
D1xDa[n_, k_, z2_] := Sum[ (-1)^(k - j) bin[k, j] Etz[n, j z2], {j, 0, k}]
D1xDbo[n_, k_, z2_] :=
 Sum[ (-1)^(k - j) bin[k, j] Sum[ bin[ j z2, r] E2[n, r], {r, 0, Log[2, n]}], {j, 0, k}]
D1xDb[n_, k_, z2_] :=
 Sum[ (-1)^(k - j) bin[k, j] bin[ j z2, r] E2[n, r], {j, 0, k}, {r, 0, Log[2, n]}]
D1xDb1[n_, k_, z2_] :=
 Sum[ E2[n, r] Sum[(-1)^(k - j) bin[k, j] bin[ j z2, r], {j, 0, k}], {r, 0, Log[2, n]}]
D1xF[n_, k_, z_] := Sum[ (-1)^(k - j) bin[k, j] bin[ j z, r] E2[n, r],
  {j, 0, k}, {r, 0, Log[2, n]}]
DzAlt[n_, z_, z2_] := Sum[(-1)^j bin[-z, j] bin[z, k] 2^j D1xD[n / 2^j, k, z2],
  {j, 0, Log[2, n]}, {k, 0, Log[2, n / 2^j]}]
D2x[n_, k_, z_] := Sum[ (-1)^(k - j) bin[k, j] bin[ j z, r] D2[n, r],
  {j, 0, k}, {r, 0, Log[2, n]}]
```

```
{Etz[121, 6] - 2 Etz[121, 3] + 1, D1xD[121, 2, 3], D1xDa[121, 2, 3], D1xDb1[121, 2, 3]}
```

{301, 301, 301, 301}

```
{Etz[121, 3 × 2.3] - 3 Etz[121, 2 × 2.3] + 3 Etz[121, 2.3] - 1,
 D1xD[121, 3, 2.3], D1xDa[121, 3, 2.3], D1xDb1[121, 3, 2.3]}
```

{153.061, 153.061, 153.061, 153.061}

```
Dz[100, 2, 3, 1]
```

14 393

```
Dz[100, 6, 1, 1]
```

14 393

```
Dz[100, 1, 6, 1]
```

14 393

```
Sum[ dz[j, 2] dz[k, 2], {j, 2, 100}, {k, 2, 100 / j}]
```

2612

```
Sum[1, {j, 1, 100}, {k, 1, 100/j}, {l, 1, 100/(j k)}, {m, 1, 100/(j k l)}] -
 2 Sum[1, {j, 1, 100}, {k, 1, 100/j}] + 1
```

2612

```
Expand[(x^2 - 1)^2]
```

$1 - 2x^2 + x^4$

```
Etz[100, 4] - 2 Etz[100, 2] + 1
```

-12

```
Sum[etz[j, 2] etz[k, 2], {j, 2, 100}, {k, 2, 100/j}]
```

-12

```
Sum[(-1)^(j + k + l + m), {j, 1, 100}, {k, 1, 100/j}, {l, 1, 100/(j k)}, {m, 1, 100/(j k l)}] -
 2 (Sum[(-1)^(j + k), {j, 1, 100}, {k, 1, 100/j}]) + 1
```

-12

```
Sum[(-1)^(j + k + l + m), {j, 1, 100}, {k, 1, 100/j}, {l, 1, 100/(j k)}, {m, 1, 100/(j k l)}] -
 2 (Sum[1, {j, 1, 100}, {k, 1, 100/j}] -
    2×2 Sum[1, {j, 1, 50}, {k, 1, 50/j}] + 4 Sum[1, {j, 1, 25}, {k, 1, 25/j}]) + 1
```

-12

```
Sum[(-1)^(j + k + l + m), {j, 1, 100}, {k, 1, 100/j}, {l, 1, 100/(j k)}, {m, 1, 100/(j k l)}] -
 2 (Dz[100, 2, 1, 1] - 2×2 Dz[50, 2, 1, 1] + 4 Dz[25, 2, 1, 1]) + 1
```

-12

```
Sum[(-1)^(j + k + l + m), {j, 1, 100}, {k, 1, 100/j}, {l, 1, 100/(j k)}, {m, 1, 100/(j k l)}] -
 2 (Dz[100, 2, 1, 1] - 2×2 Dz[50, 2, 1, 1] + 4 Dz[25, 2, 1, 1]) + 1
```

-12

```
(Dz[100, 4, 1, 1] - 2×4 Dz[50, 4, 1, 1] +
   4×6 Dz[25, 4, 1, 1] - 8×4 Dz[12, 4, 1, 1] + 16 Dz[6, 4, 1, 1]) -
 2 (Dz[100, 2, 1, 1] - 2×2 Dz[50, 2, 1, 1] + 4 Dz[25, 2, 1, 1]) + 1
```

-12

```
Sum[etz[j, 3] etz[k, 3], {j, 2, 100}, {k, 2, 100/j}]
```

-116

```
Etz[100, 6] - 2 Etz[100, 3] + 1
```

-116

```
Sum[etz[j, -5/2] etz[k, -5/2], {j, 2, 100}, {k, 2, 100/j}]
```

$-\frac{1\,093\,425}{512}$

```
Etz[100, -5] - 2 Etz[100, -5/2] + 1
```

$-\frac{1\,093\,425}{512}$

```
Sum[etz[j, 3] etz[k, 3] etz[l, 3], {j, 2, 100}, {k, 2, 100/j}, {l, 2, 100/(j k)}]
```

-189

```
Etz[100, 9] - 3 Etz[100, 6] + 3 Etz[100, 3] - 1
```

-189

```
Sum[etz[j, x] etz[k, x] etz[l, x], {j, 2, 100}, {k, 2, 100 / j}, {l, 2, 100 / (j k)}] /. x → (3)
```

-189

```
Sum[etz[j, x] etz[k, x], {j, 2, 100}, {k, 2, 100 / j}] /. x → (3)
```

-116

```
bb[n_, k_, x_] := Sum[ (-1) ^ (k - j) bin[k, j] Etz[n, j x], {j, 0, k}]
bc[n_, k_, x_] :=
 Sum[ (-1) ^ (k - j) bin[k, j] bin[ j x, r] E2[n, r], {j, 0, k}, {r, 0, Log[2, n]}]
bd[n_, z_, x_] :=
 Sum[ bin[ z, k] Sum[ (-1) ^ (k - j) bin[k, j] bin[ j x, r] E2[n, r], {j, 0, k}, {r, 0, Log[2, n]}],
  {k, 0, Log[2, n]}]
```

```
bc[100, 2, 3]
```

-116

```
bd[100, 2, 7]
```

-5361

```
D1xD[100, 2, 7]
```

-5782

```
Etz[100, 14]
```

-6201

```
etz[100, 1]
```

-1

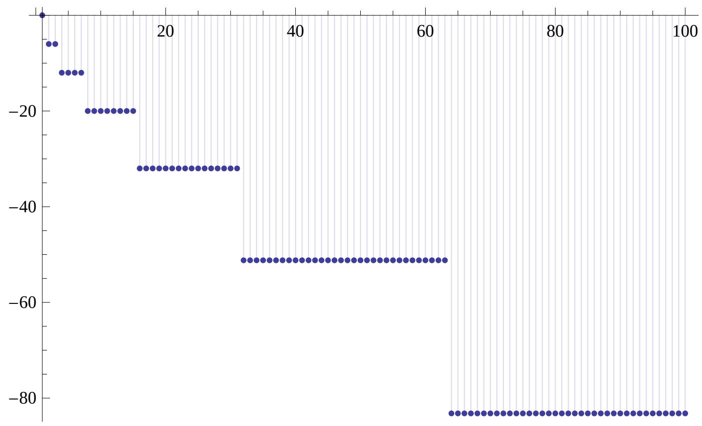
```
D1xDb1a[n_, k_, z2_] :=
 Table[ Sum[(-1) ^ (k - j) bin[k, j] bin[ j z2, r], {j, 0, k}], {r, 0, Log[2, n]}]
D1xDb1a[1208, 1, 5]
```

{0, 5, 10, 10, 5, 1, 0, 0, 0, 0, 0}

```
{Dz[121, 6, 1, 1] - 2 Dz[121, 3, 1, 1] + 1, D2x[121, 2, 3]}
```

{16 213, 16 213}

```
DiscretePlot[E1[n, 3] - 3 pr[n], {n, 1, 100}]
```




```
Table[etz[j, 2], {j, 1, 10}]
```

{1, -2, 2, -1, 2, -4, 2, 0, 3, -4}