```
ClearAll["Global`*"]

bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k!
M1[n_, k_, s_] := Sum[ (-1)^(j + 1) j^(-s) M1[n / j, k - 1, s], {j, 1, n}];
M1[n_, 0, s_] := UnitStep[n - 1]
M2[n_, k_, s_] := Sum[ (-1)^(j + 1) j^(-s) M2[n / j, k - 1, s], {j, 2, n}];
M2[n_, 0, s_] := UnitStep[n - 1]

E2a[n_, k_, a_, s_] := E2a[n, k, a, s] = Sum[ j^(-s) E2a[n / j, k - 1, a, s], {j, 2, n}] -
   a Sum[ (j a)^(-s) E2a[n / (a j), k - 1, a, s], {j, 1, n / a}];
E2a[n_, 0, a_, s_] := UnitStep[n - 1]
E1a[n_, k_, a_, s_] := E1a[n, k, a, s] = Sum[j^(-s) E1a[n / j, k - 1, a, s], {j, 1, n}] -
   a Sum[(j a)^(-s) E1a[n / (a j), k - 1, a, s], {j, 1, n / a}];
E1a[n_, 0, a_, s_] := UnitStep[n - 1]

DDa[n_, k_, s_] := DDa[n, k, s] = Sum[j^(-s) DDa[Floor[n / j], k - 1, s], {j, 1, n}];
DDa[n_, 0, s_] := UnitStep[n - 1]
D2a[n_, k_, s_] := D2a[n, k, s] = Sum[j^(-s) D2a[Floor[n / j], k - 1, s], {j, 2, n}];
D2a[n_, 0, s_] := UnitStep[n - 1]

D2b[n_, k_, s_] := Sum[(-1)^j Binomial[k, j] DDa[n, k - j, s], {j, 0, k}]
DDb[n_, k_, s_] := Sum[Binomial[k, j] D2a[n, k - j, s], {j, 0, k}]
E2b[n_, k_, b_, s_] := Sum[(-1)^j Binomial[k, j] E1a[n, k - j, b, s], {j, 0, k}]
E1b[n_, z_, b_, s_] := Sum[ bin[z, k] E2a[n, k, b, s], {k, 0, If[ b < 2, Log[b, n], Log[2, n]]}]
DDc[n_, k_, b_, s_] :=
 Sum[Binomial[k + j - 1, k - 1] b^(j (1 - s)) E1a[n / (b^j), k, b, s], {j, 0, Log[b, n]}]

E1c[n_, k_, b_, s_] := Sum[(-1)^j Binomial[k, j] b^(j (1 - s)) DDa[n / b^j, k, s], {j, 0, k}]
E2c[n_, k_, b_, s_] := Sum[(-1)^j b^(j (1 - s))
   Binomial[k, j] Binomial[j, m] D2a[n / b^j, k - m, s], {j, 0, k}, {m, 0, j}]
D2E2[n_, k_, b_, s_] := Sum[ (-1)^j b^(j (1 - s)) Binomial[k, j]
   Sum[ Binomial[j, m] If[n / b^j < 1, 0, D2a[n / b^j, k - m, s]], {m, 0, j}], {j, 0, k}]
E2D2[n_, k_, b_, s_] := (-1)^k + Sum[b^(a (1 - s)) / ((k - 1)!) Binomial[k, j]
    Pochhammer[a - k + j + 1, k - 1] E2a[b^-a n, j, b, s], {a, 0, Log[b, n]}, {j, 0, k}]

Lin[n_, s_] := Sum[ (-1)^(k + 1) / k D2a[n, k, s], {k, Log[2, n]}]
LinE[n_, b_, s_] := Sum[ (-1)^(k + 1) / k E2a[n, k, b, s], {k, Log[2, n]}]


{E1a[100, 4, 3, -1], E1c[100, 4, 3, -1]}

{3761, 3761}

{E2a[100, 3, 3, 2], E2c[100, 3, 3, 2], D2E2[100, 3, 3, 2]}

{1 559 137 560 828 081 735 079 / 1 517 098 224 317 626 798 848 000,
 1 559 137 560 828 081 735 079 / 1 517 098 224 317 626 798 848 000,
 1 559 137 560 828 081 735 079 / 1 517 098 224 317 626 798 848 000}

{E2D2[100, 4, 4, 2], D2a[100, 4, 2]}

{383 724 634 331 / 7 113 173 760 000, 383 724 634 331 / 7 113 173 760 000}
```

```
{Lin[100, -1], LinE[100, 2, -1], Lin[100, -1] - LinE[100, 2, -1]}
```

$$\left\{\frac{69\,389}{60}, \frac{10\,301}{60}, \frac{4924}{5}\right\}$$

```
{Lin[100, 1], LinE[100, 2, 1], Lin[100, 1] - LinE[100, 2, 1]}
```

$$\left\{\frac{292\,149\,953\,504\,274\,361\,788\,974\,787\,095\,433\,526\,022\,627}{139\,440\,750\,459\,424\,954\,329\,067\,617\,870\,624\,607\,113\,600},\right.$$
$$\left.-\frac{49\,479\,885\,121\,316\,776\,317\,240\,876\,687\,596\,761\,405\,693}{139\,440\,750\,459\,424\,954\,329\,067\,617\,870\,624\,607\,113\,600}, \frac{49}{20}\right\}$$

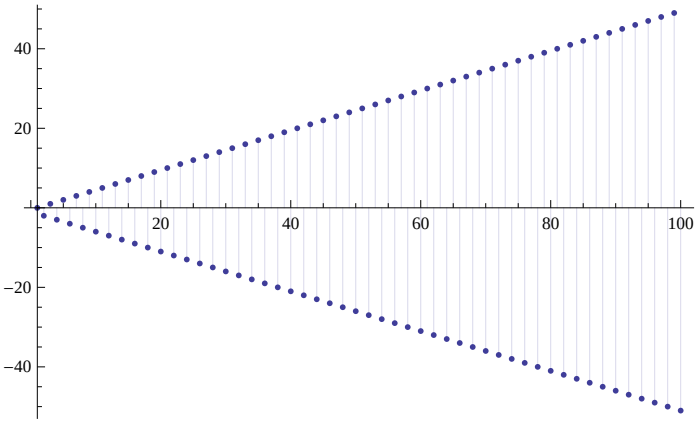
```
ll[n_, b_, s_] := Lin[n, s] - LinE[n, b, s] - (Lin[n - 1, s] - LinE[n - 1, b, s])
ss[n_, b_, s_] := If[ Log[b, n] == Floor[Log[b, n]], (n^(1 - s)) / Log[b, n], 0]
Table[{n, ll[n, cc = 3, -2], ll[n, cc, 0], ll[n, cc, 2],
   ss[n, cc, -2], ss[n, cc, 0], ss[n, cc, 2]}, {n, 2, 100}] // TableForm
```

2	0	0	0	0	0	0
3	27	3	$\frac{1}{3}$	27	3	$\frac{1}{3}$
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0
7	0	0	0	0	0	0
8	0	0	0	0	0	0
9	$\frac{729}{2}$	$\frac{9}{2}$	$\frac{1}{18}$	$\frac{729}{2}$	$\frac{9}{2}$	$\frac{1}{18}$
10	0	0	0	0	0	0
11	0	0	0	0	0	0
12	0	0	0	0	0	0
13	0	0	0	0	0	0
14	0	0	0	0	0	0
15	0	0	0	0	0	0
16	0	0	0	0	0	0
17	0	0	0	0	0	0
18	0	0	0	0	0	0
19	0	0	0	0	0	0
20	0	0	0	0	0	0
21	0	0	0	0	0	0
22	0	0	0	0	0	0
23	0	0	0	0	0	0
24	0	0	0	0	0	0
25	0	0	0	0	0	0
26	0	0	0	0	0	0
27	6561	9	$\frac{1}{81}$	6561	9	$\frac{1}{81}$
28	0	0	0	0	0	0
29	0	0	0	0	0	0
30	0	0	0	0	0	0
31	0	0	0	0	0	0
32	0	0	0	0	0	0
33	0	0	0	0	0	0
34	0	0	0	0	0	0
35	0	0	0	0	0	0
36	0	0	0	0	0	0

37	0	0	0	0	0	0
38	0	0	0	0	0	0
39	0	0	0	0	0	0
40	0	0	0	0	0	0
41	0	0	0	0	0	0
42	0	0	0	0	0	0
43	0	0	0	0	0	0
44	0	0	0	0	0	0
45	0	0	0	0	0	0
46	0	0	0	0	0	0
47	0	0	0	0	0	0
48	0	0	0	0	0	0
49	0	0	0	0	0	0
50	0	0	0	0	0	0
51	0	0	0	0	0	0
52	0	0	0	0	0	0
53	0	0	0	0	0	0
54	0	0	0	0	0	0
55	0	0	0	0	0	0
56	0	0	0	0	0	0
57	0	0	0	0	0	0
58	0	0	0	0	0	0
59	0	0	0	0	0	0
60	0	0	0	0	0	0
61	0	0	0	0	0	0
62	0	0	0	0	0	0
63	0	0	0	0	0	0
64	0	0	0	0	0	0
65	0	0	0	0	0	0
66	0	0	0	0	0	0
67	0	0	0	0	0	0
68	0	0	0	0	0	0
69	0	0	0	0	0	0
70	0	0	0	0	0	0
71	0	0	0	0	0	0
72	0	0	0	0	0	0
73	0	0	0	0	0	0
74	0	0	0	0	0	0
75	0	0	0	0	0	0
76	0	0	0	0	0	0
77	0	0	0	0	0	0
78	0	0	0	0	0	0
79	0	0	0	0	0	0
80	0	0	0	0	0	0
81	$\frac{531\,441}{4}$	$\frac{81}{4}$	$\frac{1}{324}$	$\frac{531\,441}{4}$	$\frac{81}{4}$	$\frac{1}{324}$
82	0	0	0	0	0	0
83	0	0	0	0	0	0
84	0	0	0	0	0	0
85	0	0	0	0	0	0
86	0	0	0	0	0	0
87	0	0	0	0	0	0
88	0	0	0	0	0	0
89	0	0	0	0	0	0
90	0	0	0	0	0	0
91	0	0	0	0	0	0
92	0	0	0	0	0	0

93	0	0	0	0	0	0
94	0	0	0	0	0	0
95	0	0	0	0	0	0
96	0	0	0	0	0	0
97	0	0	0	0	0	0
98	0	0	0	0	0	0
99	0	0	0	0	0	0
100	0	0	0	0	0	0

```
DiscretePlot[E2a[n, 1, 2, -1], {n, 1, 100}]
```




```
M1[100, 2, -1]
```

175

```
E1a[100, 2, 2, -1]
```

175

```
ll[n_, b_, s_] := Lin[n, s] - LinE[n, b, s]
ss[n_, b_, s_] := Sum[(b^(1 - s))^k / k, {k, 1, Log[b, n]}]
Table[{n, ll[n, cc = 3, -2], ll[n, cc, 0], ll[n, cc, 2],
    ss[n, cc, -2], ss[n, cc, 0], ss[n, cc, 2]}, {n, 2, 100}] // TableForm
```

2	0	0	0	0	0	0
3	27	3	$\frac{1}{3}$	27	3	$\frac{1}{3}$
4	27	3	$\frac{1}{3}$	27	3	$\frac{1}{3}$
5	27	3	$\frac{1}{3}$	27	3	$\frac{1}{3}$
6	27	3	$\frac{1}{3}$	27	3	$\frac{1}{3}$
7	27	3	$\frac{1}{3}$	27	3	$\frac{1}{3}$
8	27	3	$\frac{1}{3}$	27	3	$\frac{1}{3}$
9	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$
10	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$
11	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$
12	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$
13	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$
14	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$
15	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$

16	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$
17	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$
18	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$
19	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$
20	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$
21	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$
22	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$
23	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$
24	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$
25	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$
26	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$	$\frac{783}{2}$	$\frac{15}{2}$	$\frac{7}{18}$
27	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
28	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
29	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
30	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
31	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
32	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
33	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
34	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
35	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
36	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
37	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
38	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
39	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
40	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
41	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
42	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
43	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
44	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
45	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
46	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
47	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
48	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
49	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
50	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
51	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
52	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
53	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
54	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$

55	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
56	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
57	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
58	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
59	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
60	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
61	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
62	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
63	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
64	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
65	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
66	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
67	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
68	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
69	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
70	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
71	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
72	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
73	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
74	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
75	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
76	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
77	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
78	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
79	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
80	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$	$\frac{13\,905}{2}$	$\frac{33}{2}$	$\frac{65}{162}$
81	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$
82	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$
83	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$
84	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$
85	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$
86	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$
87	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$
88	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$
89	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$
90	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$
91	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$
92	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$
93	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$

94	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$
95	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$
96	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$
97	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$
98	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$
99	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$
100	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$	$\frac{559\,251}{4}$	$\frac{147}{4}$	$\frac{131}{324}$

```
fg[n_, b_, s_] := Sum[((b^(1 - s))^k - 1) / k, {k, 1, Log[b, n]}]
```

```
N[fg[100, 1.001, 0]]
```

```
28.0218
```

```
N[LogIntegral[100] - Log[Log[100]] - EulerGamma]
```

```
28.0217
```

```
N[fg[100, 1.001, 1]]
```

```
0.
```

```
N[fg[100, 1.0001, -1]]
```

```
1243.45
```

```
N[fg[100, 1.001, -2]]
```

```
78 630.3
```

```
N[fg[100, 1.001, 2]]
```

```
-2.10573
```

```
N[fg[100, 1.001, 1 / 2 + 3 I]]
```

```
-2.51851 - 1.57252 i
```

```
Table[{n, N[fg[100, 1.01, 1 / 2 + n I]]}, {n, -50, 50, 1}] // TableForm
```

```
-50    -6.03848 + 1.36328 i
-49    -6.0166 + 1.30252 i
-48    -5.92975 + 1.31723 i
-47    -5.92623 + 1.38897 i
-46    -5.97071 + 1.3693 i
-45    -5.91648 + 1.31071 i
-44    -5.83322 + 1.35627 i
-43    -5.85847 + 1.41913 i
-42    -5.8885 + 1.36921 i
-41    -5.80261 + 1.32656 i
-40    -5.73647 + 1.40051 i
-39    -5.78616 + 1.44064 i
-38    -5.78758 + 1.3659 i
-37    -5.6765 + 1.35266 i
-36    -5.64002 + 1.4466 i
-35    -5.70371 + 1.45156 i
-34    -5.66423 + 1.36338 i
-33    -5.54012 + 1.39045 i
-32    -5.5425 + 1.49028 i
-31    -5.60405 + 1.45134 i
-30    -5.51538 + 1.36643 i
```

−29	−5.3952 + 1.43995 i
−28	−5.44011 + 1.52681 i
−27	−5.47859 + 1.44106 i
−26	−5.33833 + 1.38023 i
−25	−5.24236 + 1.49969 i
−24	−5.3259 + 1.55143 i
−23	−5.31692 + 1.42343 i
−22	−5.12988 + 1.41012 i
−21	−5.07992 + 1.56691 i
−20	−5.18886 + 1.55953 i
−19	−5.10543 + 1.40279 i
−18	−4.88449 + 1.46195 i
−17	−4.90228 + 1.638 i
−16	−5.01179 + 1.54631 i
−15	−4.8234 + 1.38503 i
−14	−4.59023 + 1.5439 i
−13	−4.69743 + 1.70935 i
−12	−4.76563 + 1.50447 i
−11	−4.43103 + 1.37851 i
−10	−4.21884 + 1.67405 i
−9	−4.44241 + 1.77907 i
−8	−4.39042 + 1.4141 i
−7	−3.82432 + 1.40038 i
−6	−3.69308 + 1.91871 i
−5	−4.10098 + 1.85114 i
−4	−3.69578 + 1.17243 i
−3	−2.52221 + 1.55255 i
−2	−2.74353 + 2.86457 i
−1	−4.20165 + 1.71319 i
0	4.74572
1	−4.20165 − 1.71319 i
2	−2.74353 − 2.86457 i
3	−2.52221 − 1.55255 i
4	−3.69578 − 1.17243 i
5	−4.10098 − 1.85114 i
6	−3.69308 − 1.91871 i
7	−3.82432 − 1.40038 i
8	−4.39042 − 1.4141 i
9	−4.44241 − 1.77907 i
10	−4.21884 − 1.67405 i
11	−4.43103 − 1.37851 i
12	−4.76563 − 1.50447 i
13	−4.69743 − 1.70935 i
14	−4.59023 − 1.5439 i
15	−4.8234 − 1.38503 i
16	−5.01179 − 1.54631 i
17	−4.90228 − 1.638 i
18	−4.88449 − 1.46195 i
19	−5.10543 − 1.40279 i
20	−5.18886 − 1.55953 i
21	−5.07992 − 1.56691 i
22	−5.12988 − 1.41012 i
23	−5.31692 − 1.42343 i
24	−5.3259 − 1.55143 i
25	−5.24236 − 1.49969 i
26	−5.33833 − 1.38023 i

```
27      -5.47859 - 1.44106 i
28      -5.44011 - 1.52681 i
29      -5.3952 - 1.43995 i
30      -5.51538 - 1.36643 i
31      -5.60405 - 1.45134 i
32      -5.5425 - 1.49028 i
33      -5.54012 - 1.39045 i
34      -5.66423 - 1.36338 i
35      -5.70371 - 1.45156 i
36      -5.64002 - 1.4466 i
37      -5.6765 - 1.35266 i
38      -5.78758 - 1.3659 i
39      -5.78616 - 1.44064 i
40      -5.73647 - 1.40051 i
41      -5.80261 - 1.32656 i
42      -5.8885 - 1.36921 i
43      -5.85847 - 1.41913 i
44      -5.83322 - 1.35627 i
45      -5.91648 - 1.31071 i
46      -5.97071 - 1.3693 i
47      -5.92623 - 1.38897 i
48      -5.92975 - 1.31723 i
49      -6.0166 - 1.30252 i
50      -6.03848 - 1.36328 i
```

```
fga[n_, b_, s_] := Sum[((b^(1 - s))^k - 1) / k, {k, 1, Log[b, n]}]
```

```
Expand[fga[100, b, s]]
```

$$-\text{HarmonicNumber}\left[\frac{\text{Log}[100]}{\text{Log}[b]}\right] - \left(b^{1-s}\right)^{1+\frac{\text{Log}[100]}{\text{Log}[b]}}\ \text{LerchPhi}\left[b^{1-s},\ 1,\ 1+\frac{\text{Log}[100]}{\text{Log}[b]}\right] - \text{Log}\left[b^{-s}\ (-b+b^{s})\right]$$

$$\text{Limit}\left[-\text{HarmonicNumber}\left[\frac{\text{Log}[100]}{\text{Log}[b]}\right] - \text{Log}\left[b^{-s}\ (-b+b^{s})\right],\ \{b\to 1\}\right]$$

```
{-EulerGamma - Log[(-1 + s) Log[100]]} /. s → -3
```

$$\{-\text{EulerGamma} - i\,\pi - \text{Log}[4\,\text{Log}[100]]\}$$

$$\text{Limit}\left[\left(b^{1-s}\right)^{1+\frac{\text{Log}[100]}{\text{Log}[b]}}\ \text{LerchPhi}\left[b^{1-s},\ 1,\ 1+\frac{\text{Log}[100]}{\text{Log}[b]}\right],\ b\to 1\right]$$

$$\text{Limit}\left[\left(b^{1-s}\right)^{1+\frac{\text{Log}[100]}{\text{Log}[b]}}\ \text{LerchPhi}\left[b^{1-s},\ 1,\ 1+\frac{\text{Log}[100]}{\text{Log}[b]}\right],\ b\to 1\right]$$

```
fga[100, 1.01, 1]
```

```
0.
```

$$\text{Limit}\left[\left(b^{1-s}\right)^{1+\frac{\text{Log}[100]}{\text{Log}[b]}}\ \text{LerchPhi}\left[b^{1-s},\ 1,\ 1+\frac{\text{Log}[100]}{\text{Log}[b]}\right] /. s\to \text{ZetaZero}[1],\ b\to 1\right]$$

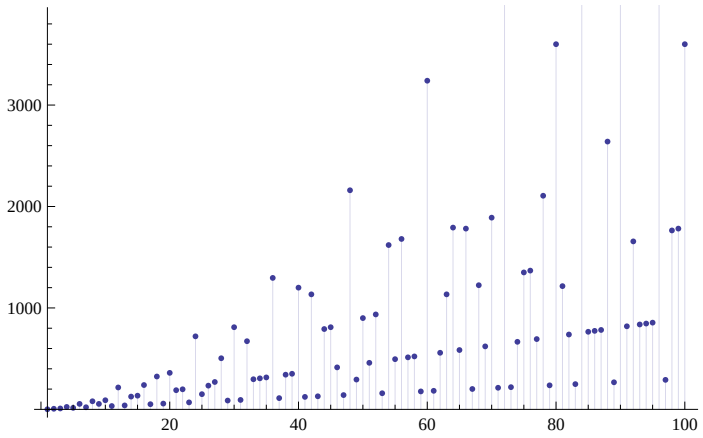
$$\text{Limit}\left[\left(b^{1-\text{ZetaZero}[1]}\right)^{1+\frac{\text{Log}[100]}{\text{Log}[b]}}\ \text{LerchPhi}\left[b^{1-\text{ZetaZero}[1]},\ 1,\ 1+\frac{\text{Log}[100]}{\text{Log}[b]}\right],\ b\to 1\right]$$

$$\text{bb}[b\_] := \left(b^{1-\text{ZetaZero}[1]}\right)^{1+\frac{\text{Log}[100]}{\text{Log}[b]}}\ \text{LerchPhi}\left[b^{1-\text{ZetaZero}[1]},\ 1,\ 1+\frac{\text{Log}[100]}{\text{Log}[b]}\right]$$

```
N[bb[1.00001]]
```

```
-0.11643 + 0.100124 i
```

```
DiscretePlot[E1a[n, 3, 1.00000001, -1], {n, 1, 100}]
```



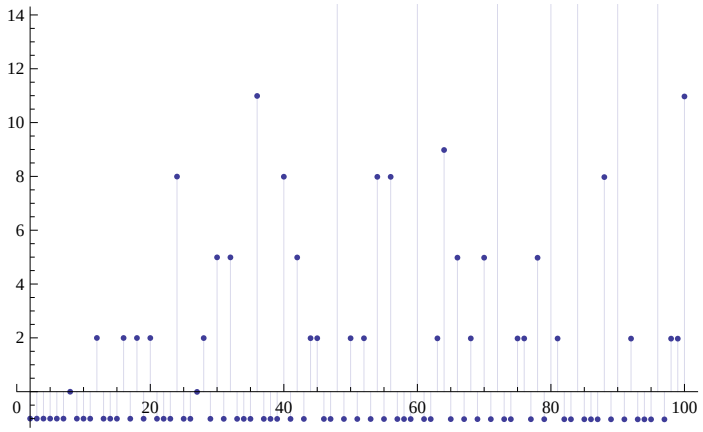

```
E2D2[100, 3, 1.1, 0]
```

324.

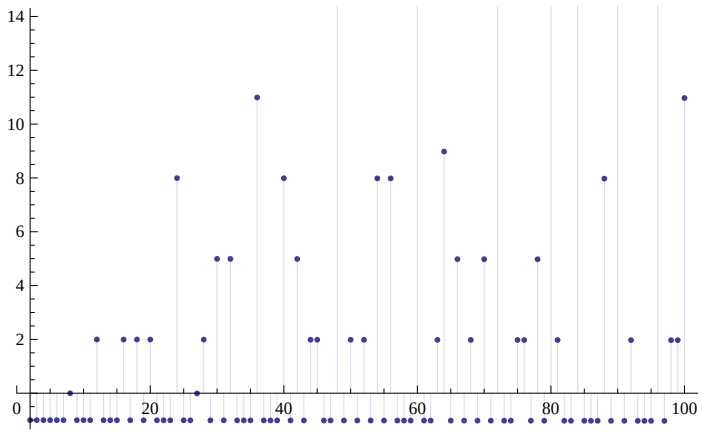
```
D2a[100, 3, 0]
```

324

```
DiscretePlot[E2a[n, 3, 1.0001, 0], {n, 2, 100}]
```




```
DiscretePlot[D2E2[n, 3, 1.0001, 0], {n, 2, 100}]
```



```
LinE[100, 2, 0]
```

$$\frac{4}{5}$$

```
E2a[100, 1, 101, 0]
```

99

```
D[Expand[E1b[100, z, 1.5, 0]], z] /. z → 0
```

−3.44534