

FullSimplify@Integrate[1, {t, a, x}]

-a + x

FullSimplify@Integrate[1, {t, a, x}, {u, a, x - t}]

$\frac{1}{2} (3 a^2 - 4 a x + x^2)$

FullSimplify@Integrate[1, {t, a, x}, {u, a, x - t}, {v, a, x - t - u}]

$-\frac{1}{6} (a - x) (-4 a + x)^2$

FullSimplify@Integrate[1, {t, a, x}, {u, a, x - t}, {v, a, x - t - u}, {w, a, x - t - u - v}]

$\frac{1}{24} (a - x) (5 a - x)^3$

FullSimplify@

Integrate[1, {t, a, x}, {u, a, x - t}, {v, a, x - t - u}, {w, a, x - t - u - v}, {y, a, x - t - u - v - w}]

$-\frac{1}{120} (a - x) (-6 a + x)^4$

bon[x_, k_, a_] := (x - (k + 1) a) ^ (k - 1) (x - a) / k!

bn4[x_, a_] := $\frac{1}{24} (a - x) (5 a - x)^3$

bn5[x_, a_] := $-\frac{1}{120} (a - x) (-6 a + x)^4$

bn4[12, 7]

$-\frac{60835}{24}$

bon[12, 4, 7]

$-\frac{60835}{24}$

FullSimplify@Sum[1, {t, a, x}]

1 - a + x

FullSimplify@Sum[1, {t, a, x}, {u, a, x - t}]

$\frac{1}{2} (-1 + a - x) (-2 + 3 a - x)$

FullSimplify@Sum[1, {t, a, x}, {u, a, x - t}, {v, a, x - t - u}]

$-\frac{1}{6} (-1 + a - x) (-3 + 4 a - x) (-2 + 4 a - x)$

FullSimplify@Sum[1, {t, a, x}, {u, a, x - t}, {v, a, x - t - u}, {w, a, x - t - u - v}]

$\frac{1}{24} (-1 + a - x) (-4 + 5 a - x) (-3 + 5 a - x) (-2 + 5 a - x)$

FullSimplify@

Sum[1, {t, a, x}, {u, a, x - t}, {v, a, x - t - u}, {w, a, x - t - u - v}, {y, a, x - t - u - v - w}]

$-\frac{1}{120} (-1 + a - x) (-5 + 6 a - x) (-4 + 6 a - x) (-3 + 6 a - x) (-2 + 6 a - x)$

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mt[n_, z_] := Pochhammer[n, z] / z!
bina[x_, k_, a_] := (x - a + 1) Product[x - (k + 1) a + j, {j, 2, k}] / k!
bina2[x_, k_, a_] := 
$$\frac{(-1 + a - x) \text{mt}[1 - a (1 + k) + x, k]}{(-1 + a + a k - x)}$$

bina3[x_, k_, a_] := 
$$\left(1 + \frac{a k}{1 - a (1 + k) + x}\right) \text{mt}[1 - a (1 + k) + x, k]$$

bina[x, 5, a]

$$\frac{1}{120} (2 - 6 a + x) (3 - 6 a + x) (4 - 6 a + x) (5 - 6 a + x) (1 - a + x)$$

bin5[x_, a_] := 
$$-\frac{1}{120} (-1 + a - x) (-5 + 6 a - x) (-4 + 6 a - x) (-3 + 6 a - x) (-2 + 6 a - x)$$

bin4[x_, a_] := 
$$\frac{1}{24} (-1 + a - x) (-4 + 5 a - x) (-3 + 5 a - x) (-2 + 5 a - x)$$

bin5[12, 4]
378
bina3[12, 5, 4]
378
FullSimplify@Expand[(x - a + 1) Product[x - (k + 1) a + j, {j, 2, k}] / k!]

$$\frac{(-1 + a - x) \text{Pochhammer}[1 - a (1 + k) + x, k]}{(-1 + a + a k - x) k!}$$

FullSimplify[
$$\frac{(-1 + a - x)}{(-1 + a + a k - x)}$$
]

$$1 + \frac{a k}{1 - a (1 + k) + x}$$

FullSimplify[
$$1 + \frac{a k}{1 - a (1 + k) + x} /. a \rightarrow 1$$
]

$$-\frac{x}{k - x}$$


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