```
D2[n_, k_] := D2[n, k] = Sum[D2[Floor[n / j], k - 1], {j, 2, n}]; D2[n_, 0] := 1
DD[n_, z_] := Sum[FactorialPower[z, a] / a ! D2[n, a], {a, 0, Log[2, n]}]
DDc[n_, z_] := Sum[FactorialPower[z, a] / a ! D2c[n, a], {a, 0, Log[2, n]}]
DDo[n_, z_, a_] := FactorialPower[z, a] / a ! D2[n, a]
DDa[n_, z_] := Sum[FactorialPower[z, a] / a ! D2[n, a], {a, 0, 12}]

(DD[100, 0.001] - DD[100, -0.001]) / (2 * .001)
```

28.5334

```
f[t_] := FullSimplify[(DD[100, t] - 1) / t]
f2[s_] := Integrate[ FullSimplify[E^(-s t) f[t]], {t, 0, Infinity}]

Expand[f2[s]]
```

ConditionalExpression$\left[\frac{7}{6\,s^6}+\frac{67}{10\,s^5}+\frac{611}{24\,s^4}+\frac{331}{8\,s^3}+\frac{16289}{360\,s^2}+\frac{428}{15\,s},\ \mathrm{Re}[s]>0\right]$

f3[s_] := $\frac{7}{6\,s^6}+\frac{67}{10\,s^5}+\frac{611}{24\,s^4}+\frac{331}{8\,s^3}+\frac{16289}{360\,s^2}+\frac{428}{15\,s}$

```
f3[s]
```

$\frac{7}{6\,s^6}+\frac{67}{10\,s^5}+\frac{611}{24\,s^4}+\frac{331}{8\,s^3}+\frac{16289}{360\,s^2}+\frac{428}{15\,s}$

```
Limit[f3[s], s → 0]
```

∞

```
f4[ t_, g_ ] := 1 / (2 Pi I ) Limit[ Integrate[E^(s t) f3[s], {s, g - I T, g + I T}], T → Infinity]

f4[1, 1]
```

99

```
f4[2, 1]
```

$\frac{481}{2}$

```
(DD[100, 1 / 2] - 1) / (1 / 2)
```

$\frac{29121}{512}$

```
f4[1 / 2, 1]
```

$\frac{29121}{512}$

```
(DD[100, -2] - 1) / (-2)
```

-9

```
f4[-2, 1]
```

9

(DD[100, -3] - 1) / (-3)

$-\frac{46}{3}$

f4[-3, 1]

$\frac{46}{3}$

f4[0, 1]

$\frac{214}{15}$

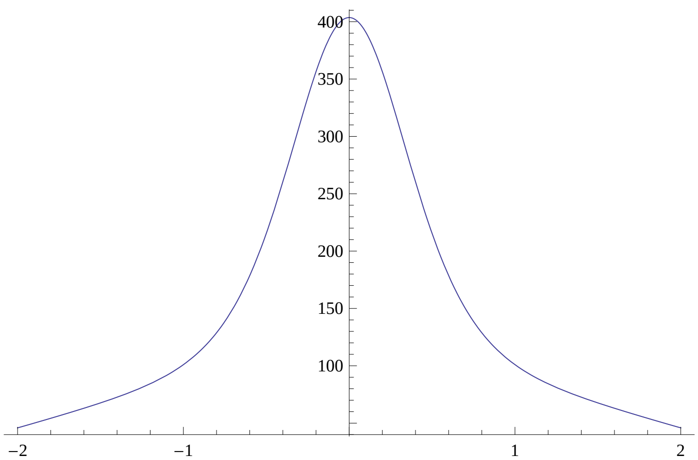
$\left(\frac{214}{15} * 2\right)$

$\frac{428}{15}$

Limit[(DD[100, s] - 1) / (s), s → 0]

$\frac{428}{15}$

Plot[Re[E^((1 + s I) (1)) f3[1 + s I]], {s, -2, 2 }]



$\text{Solve}\left[\frac{7}{6\,s^6} + \frac{67}{10\,s^5} + \frac{611}{24\,s^4} + \frac{331}{8\,s^3} + \frac{16\,289}{360\,s^2} + \frac{428}{15\,s} == 0,\ s\right]$

$\{\{s \to \text{Root}[420 + 2412\,\#1 + 9165\,\#1^2 + 14\,895\,\#1^3 + 16\,289\,\#1^4 + 10\,272\,\#1^5\ \&,\ 1]\},$
$\{s \to \text{Root}[420 + 2412\,\#1 + 9165\,\#1^2 + 14\,895\,\#1^3 + 16\,289\,\#1^4 + 10\,272\,\#1^5\ \&,\ 2]\},$
$\{s \to \text{Root}[420 + 2412\,\#1 + 9165\,\#1^2 + 14\,895\,\#1^3 + 16\,289\,\#1^4 + 10\,272\,\#1^5\ \&,\ 3]\},$
$\{s \to \text{Root}[420 + 2412\,\#1 + 9165\,\#1^2 + 14\,895\,\#1^3 + 16\,289\,\#1^4 + 10\,272\,\#1^5\ \&,\ 4]\},$
$\{s \to \text{Root}[420 + 2412\,\#1 + 9165\,\#1^2 + 14\,895\,\#1^3 + 16\,289\,\#1^4 + 10\,272\,\#1^5\ \&,\ 5]\}\}$

$\text{Residue}\left[\frac{7}{6\,s^6} + \frac{67}{10\,s^5} + \frac{611}{24\,s^4} + \frac{331}{8\,s^3} + \frac{16\,289}{360\,s^2} + \frac{428}{15\,s},\ \{s, 0\}\right]$

$\frac{428}{15}$

```
K[n_] := If[n == 1, 0, FullSimplify[MangoldtLambda[n] / Log[n]]]
P[n_, k_] := Sum[ K[j] P[Floor[n / j], k - 1], {j, 2, n}]; P[n_, 0] := 1
```

```
Sum[ P[100, k] / k / s^(k), {k, 1, Log[2, 100]}]
```

$$\frac{7}{6 s^6}+\frac{67}{10 s^5}+\frac{611}{24 s^4}+\frac{331}{8 s^3}+\frac{16289}{360 s^2}+\frac{428}{15 s}$$

$$\mathrm{N}\left[\mathrm{Solve}\left[\frac{7}{6 s^6}+\frac{67}{10 s^5}+\frac{611}{24 s^4}+\frac{331}{8 s^3}+\frac{16289}{360 s^2}+\frac{428}{15 s}=0, s\right]\right]$$

$\{\{s\to -0.80859\}, \{s\to -0.228604-0.733022\, i\}, \{s\to -0.228604+0.733022\, i\}, \{s\to -0.159985-0.245301\, i\}, \{s\to -0.159985+0.245301\, i\}\}$

$$\mathrm{fa}[s\_] := \frac{7}{6 s^6}+\frac{67}{10 s^5}+\frac{611}{24 s^4}+\frac{331}{8 s^3}+\frac{16289}{360 s^2}+\frac{428}{15 s}$$

```
Expand[Integrate[ FullSimplify[E^(-s t) ((DD[bb = 200, t] - 1) / t)], {t, 0, Infinity}]]
Sum[ P[bb, k] / k / s^(k), {k, 1, Log[2, bb]}]
```

$$\mathrm{ConditionalExpression}\left[\frac{8}{7 s^7}+\frac{23}{3 s^6}+\frac{553}{15 s^5}+\frac{901}{12 s^4}+\frac{18523}{180 s^3}+\frac{3709}{45 s^2}+\frac{5356}{105 s}, \mathrm{Re}[s]>0\right]$$

$$\frac{8}{7 s^7}+\frac{23}{3 s^6}+\frac{553}{15 s^5}+\frac{901}{12 s^4}+\frac{18523}{180 s^3}+\frac{3709}{45 s^2}+\frac{5356}{105 s}$$

```
Expand[Integrate[ FullSimplify[E^(-s t) ((DD[bb = 100, t] - 1))], {t, 0, Infinity}]]
Sum[ P[bb, k] / s^(k + 1), {k, 1, Log[2, bb]}]
```

$$\mathrm{ConditionalExpression}\left[\frac{7}{s^7}+\frac{67}{2 s^6}+\frac{611}{6 s^5}+\frac{993}{8 s^4}+\frac{16289}{180 s^3}+\frac{428}{15 s^2}, \mathrm{Re}[s]>0\right]$$

$$\frac{7}{s^7}+\frac{67}{2 s^6}+\frac{611}{6 s^5}+\frac{993}{8 s^4}+\frac{16289}{180 s^3}+\frac{428}{15 s^2}$$

$$\mathrm{f5}[s\_] := \frac{7}{s^7}+\frac{67}{2 s^6}+\frac{611}{6 s^5}+\frac{993}{8 s^4}+\frac{16289}{180 s^3}+\frac{428}{15 s^2}$$

```
f6[ t_, g_ ] :=
 1 / (2 Pi I ) Limit[ Integrate[E^(s t) f5[s], {s, g - I T, g + I T}], T -> Infinity]
```

```
f6[1, 1]
```

99

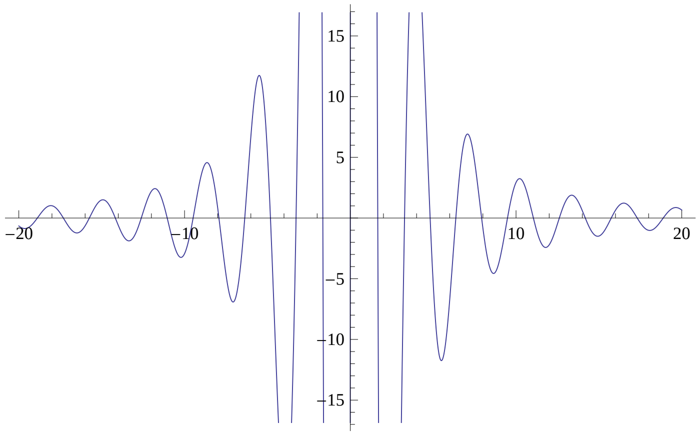
```
f6[2, 1]
```

481

```
f6[3, 1]
```

1470

```
Plot[Re[E^((1 + s I) 2) f5[1 + s I] / 2 Pi I], {s, -20, 20}]
```



$$\text{Solve}\left[\frac{7}{s^7} + \frac{67}{2 s^6} + \frac{611}{6 s^5} + \frac{993}{8 s^4} + \frac{16289}{180 s^3} + \frac{428}{15 s^2} == 0, s\right]$$

```
{{s → Root[2520 + 12060 #1 + 36660 #1^2 + 44685 #1^3 + 32578 #1^4 + 10272 #1^5 &, 1]},
 {s → Root[2520 + 12060 #1 + 36660 #1^2 + 44685 #1^3 + 32578 #1^4 + 10272 #1^5 &, 2]},
 {s → Root[2520 + 12060 #1 + 36660 #1^2 + 44685 #1^3 + 32578 #1^4 + 10272 #1^5 &, 3]},
 {s → Root[2520 + 12060 #1 + 36660 #1^2 + 44685 #1^3 + 32578 #1^4 + 10272 #1^5 &, 4]},
 {s → Root[2520 + 12060 #1 + 36660 #1^2 + 44685 #1^3 + 32578 #1^4 + 10272 #1^5 &, 5]}}
```

```
Sum[P[bb = 10, k] / s^(k + 1), {k, 1, Log[2, bb]}]
```

$$\frac{1}{s^4} + \frac{7}{s^3} + \frac{16}{3 s^2}$$

$$\text{Solve}\left[\frac{1}{s^4} + \frac{7}{s^3} + \frac{16}{3 s^2} == 0, s\right]$$

$$\left\{\left\{s \to \frac{1}{32}\left(-21 - \sqrt{249}\right)\right\}, \left\{s \to \frac{1}{32}\left(-21 + \sqrt{249}\right)\right\}\right\}$$

$$\text{Expand}\left[\left(s - \frac{1}{32}\left(-21 - \sqrt{249}\right)\right)\left(s - \frac{1}{32}\left(-21 + \sqrt{249}\right)\right)\right]$$

$$\frac{3}{16} + \frac{21 s}{16} + s^2$$

$$\frac{3}{16} + \frac{21 s}{16} + s\text{^}2$$

$$\frac{3}{16} + \frac{21 s}{16} + s^2$$

```
Sum[x^k / k! P[bb = 100, k], {k, 0, Log[2, bb]}]
```

$$1 + \frac{428 x}{15} + \frac{16289 x^2}{360} + \frac{331 x^3}{16} + \frac{611 x^4}{144} + \frac{67 x^5}{240} + \frac{7 x^6}{720}$$

$$\text{fo}[x\_] := 1 + \frac{428 x}{15} + \frac{16289 x^2}{360} + \frac{331 x^3}{16} + \frac{611 x^4}{144} + \frac{67 x^5}{240} + \frac{7 x^6}{720}$$

```
Expand[
 Integrate[FullSimplify[E^(-s t) ((DD[bb = 100, (t + 1)] - 1) / (t + 1))], {t, 0, Infinity}]]
```

$$\text{ConditionalExpression}\left[\frac{7}{6\,s^6}+\frac{118}{15\,s^5}+\frac{3929}{120\,s^4}+\frac{3167}{45\,s^3}+\frac{6031}{60\,s^2}+\frac{99}{s},\ \text{Re}[s]>0\right]$$

$$\textbf{h1}[s\_] := \frac{7}{6\,s^6}+\frac{118}{15\,s^5}+\frac{3929}{120\,s^4}+\frac{3167}{45\,s^3}+\frac{6031}{60\,s^2}+\frac{99}{s}$$

```
h2[t_, g_] := 1 / (2 Pi I) Limit[Integrate[E^(s t) h1[s], {s, g - I T, g + I T}], T → Infinity]
```

```
h2[1, 1]
```

$$\frac{481}{2}$$

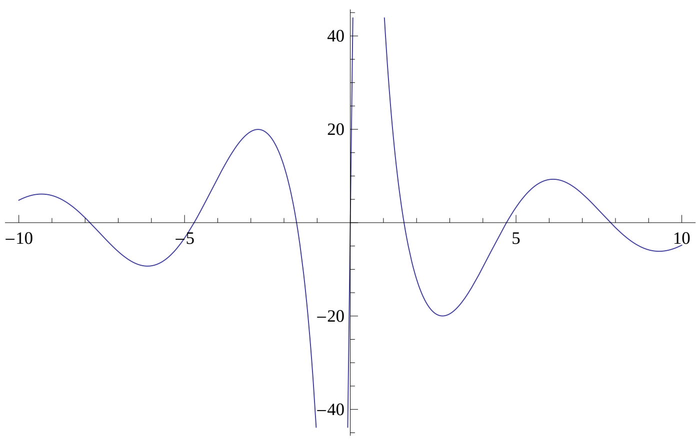
```
h2[-1, 1]
```

$$-\frac{428}{15}$$

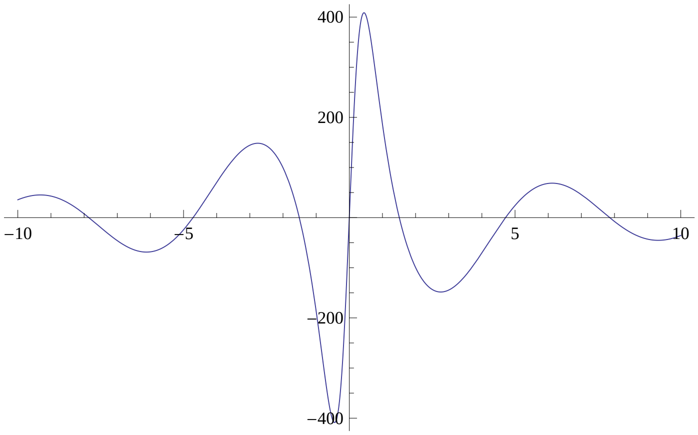
```
h2[-1, 100]
```

$$-\frac{428}{15}$$

```
Plot[Re[E^((1 + s I) (-1)) h1[1 + s I] / 2 Pi I], {s, -10, 10}]
```



```
Plot[ Re[E^((1 + s I) (1)) h1[1 + s I] / 2 Pi I], {s, -10, 10 }]
```




```
Expand[Integrate[ FullSimplify[E^(-s t) ((DD[100, (t + 1)] - 1) / (t + 1))], {t, 0, Infinity}]]
```

$$\text{ConditionalExpression}\left[\frac{7}{6 s^6} + \frac{118}{15 s^5} + \frac{3929}{120 s^4} + \frac{3167}{45 s^3} + \frac{6031}{60 s^2} + \frac{99}{s},\ \text{Re}[s] > 0\right]$$

```
hx[ t_, g_ ] := 1 / (2 Pi I )
  Limit[ Integrate[E^(s t) Integrate[FullSimplify[E^(-s t) ((DD[100, (t + 1)] - 1) / (t + 1))],
      {t, 0, Infinity}], {s, g - I T, g + I T}], T → Infinity]
Integrate[FullSimplify[E^(-s t) ((DDa[n, (t + 1)] - 1) / (t + 1))], {t, 0, Infinity}]
```

$$\text{ConditionalExpression}\left[\frac{840 + 2 s (2832 + s (11787 + 2 s (12668 + 3 s (6031 + 5940 s))))}{720 s^6},\ \text{Re}[s] > 0\right]$$

Integrate::ilim : Invalid integration variable or limit(s) in {1, 0, ∞}. ≫

$$\text{h1}[s\_] := \frac{7}{6 s^6} + \frac{118}{15 s^5} + \frac{3929}{120 s^4} + \frac{3167}{45 s^3} + \frac{6031}{60 s^2} + \frac{99}{s}$$

```
Plot3D[Re[ h1[ x + I y]], {x, -.5, .5}, {y, -.5, .5}]
```

```
(D2[100, 2] - 1) / 2
```

16 980

```
Expand[
 Integrate[ FullSimplify[E^(-s t) ((DD[bb = 20, (t + 1)] - 1) / (t + 1))], {t, 0, Infinity}]]
```

$$\text{ConditionalExpression}\left[\frac{1}{4 s^4} + \frac{49}{12 s^3} + \frac{137}{12 s^2} + \frac{19}{s},\ \text{Re}[s] > 0\right]$$

```
Expand[FullSimplify[E^(-s t) ((DD[bb = 20, (t + 1)] - 1) / (t + 1))]]
```

$$19\, e^{-s t} + \frac{137}{12} e^{-s t} t + \frac{49}{24} e^{-s t} t^2 + \frac{1}{24} e^{-s t} t^3$$

```
Expand[FullSimplify[E^(-s t) ((DDo[bb = 20, (t + 1), 1] - 1) / (t + 1))]]
```

$$\frac{18\, e^{-s t}}{1 + t} + \frac{19\, e^{-s t}\, t}{1 + t}$$

```
Expand[FullSimplify[E^(-s t) ((DDo[bb = 20, (t + 1), 2] - 1) / (t + 1))]]
```

$$-\frac{e^{-s t}}{1+t}+\frac{27 e^{-s t} t}{2 (1+t)}+\frac{27 e^{-s t} t^2}{2 (1+t)}$$

```
Expand[FullSimplify[E^(-s t) ((DDo[bb = 20, (t + 1), 3] - 1) / (t + 1))]]
```

$$-\frac{e^{-s t}}{1+t}+\frac{13 e^{-s t} \text{FactorialPower}[1+t, 3]}{6 (1+t)}$$

```
Expand[FullSimplify[E^(-s t) ((DDo[bb = 20, (t + 1), 4] - 1) / (t + 1))]]
```

$$-\frac{e^{-s t}}{1+t}+\frac{e^{-s t} \text{FactorialPower}[1+t, 4]}{24 (1+t)}$$

```
Sum[Expand[FullSimplify[E^(-s t) ((DDo[bb = 20, (t + 1), k] - 1) / (t + 1))]], {k, 0, 11}]
```

$$\frac{8 e^{-s t}}{1+t}+\frac{65 e^{-s t} t}{2 (1+t)}+\frac{27 e^{-s t} t^2}{2 (1+t)}+\frac{13 e^{-s t} \text{FactorialPower}[1+t, 3]}{6 (1+t)}+\frac{e^{-s t} \text{FactorialPower}[1+t, 4]}{24 (1+t)}$$

```
19 / 1 + 27 / 2
```

$$\frac{65}{2}$$

```
E^(-s t) ((DD[bb = 20, (t + 1)] - 1) / (t + 1))
```

$$\frac{1}{1+t} e^{-s t} \left(19 (1+t)+\frac{27}{2} \text{FactorialPower}[1+t, 2]+\frac{13}{6} \text{FactorialPower}[1+t, 3]+\frac{1}{24} \text{FactorialPower}[1+t, 4]\right)$$

```
FactorialPower[6, 2]
```

30

```
Expand[FullSimplify[E^(-s t) ((DD[bb = 20, (t + 1)] - 1) / (t + 1))]]
```

$$19 e^{-s t}+\frac{137}{12} e^{-s t} t+\frac{49}{24} e^{-s t} t^2+\frac{1}{24} e^{-s t} t^3$$

```
E^(-s t) ((DDc[bb = 20, (t + 1)] - 1) / (t + 1))
```

$$\frac{1}{1+t} e^{-s t} \left(-1+\text{D2c}[20, 0]+(1+t) \text{D2c}[20, 1]+\frac{1}{2} \text{D2c}[20, 2] \text{FactorialPower}[1+t, 2]+\frac{1}{6} \text{D2c}[20, 3] \text{FactorialPower}[1+t, 3]+\frac{1}{24} \text{D2c}[20, 4] \text{FactorialPower}[1+t, 4]\right)$$

$$\frac{1}{1+t} e^{-s t} \left(-1+\text{D2c}[20, 0]+(1+t) \text{D2c}[20, 1]+\frac{1}{2} \text{D2c}[20, 2] (t+1) (t)+\frac{1}{6} \text{D2c}[20, 3] (t+1) (t) (t-1)+\frac{1}{24} \text{D2c}[20, 4] (t+1) (t) (t-1) (t-2)\right)$$

$$\texttt{FullSimplify}\left[\frac{1}{1+t} e^{-s t} \left(-1 + \texttt{D2c}[20, 0] + (1+t)\,\texttt{D2c}[20, 1] + \frac{1}{2} t (1+t)\,\texttt{D2c}[20, 2] + \frac{1}{6}(-1+t)\,t\,(1+t)\,\texttt{D2c}[20, 3] + \frac{1}{24}(-2+t)(-1+t)\,t\,(1+t)\,\texttt{D2c}[20, 4]\right)\right]$$

$$\frac{1}{24(1+t)} e^{-s t} \left(4\left(-6 + 6\,\texttt{D2c}[20, 0] + 3(1+t)(2\,\texttt{D2c}[20, 1] + t\,\texttt{D2c}[20, 2]) + t\left(-1+t^2\right)\texttt{D2c}[20, 3]\right) + (-2+t)(-1+t)\,t\,(1+t)\,\texttt{D2c}[20, 4]\right)$$

$$\frac{1}{1+t} e^{-s t} \left(-1 + \texttt{D2}[20, 0] + (1+t)\,\texttt{D2}[20, 1] + \frac{1}{2}\texttt{D2}[20, 2]\,(t+1)\,(t) + \frac{1}{6}\texttt{D2}[20, 3]\,(t+1)\,(t)\,(t-1) + \frac{1}{24}\texttt{D2}[20, 4]\,(t+1)\,(t)\,(t-1)\,(t-2)\right)$$

$$\frac{1}{1+t} e^{-s t} \left(19(1+t) + \frac{27}{2} t (1+t) + \frac{13}{6}(-1+t)\,t\,(1+t) + \frac{1}{24}(-2+t)(-1+t)\,t\,(1+t)\right)$$

$$\texttt{FullSimplify}\left[\frac{1}{1+t} e^{-s t} \left(19(1+t) + \frac{27}{2} t (1+t) + \frac{13}{6}(-1+t)\,t\,(1+t) + \frac{1}{24}(-2+t)(-1+t)\,t\,(1+t)\right)\right]$$

$$\texttt{Expand}\left[\frac{1}{24} e^{-s t} (456 + t(274 + t(49 + t)))\right]$$

$$19 e^{-s t} + \frac{137}{12} e^{-s t} t + \frac{49}{24} e^{-s t} t^2 + \frac{1}{24} e^{-s t} t^3$$

```
Expand[FullSimplify[E^(-s t) ((DD[bb = 20, (t + 1)] - 1) / (t + 1))]]
```

$$19 e^{-s t} + \frac{137}{12} e^{-s t} t + \frac{49}{24} e^{-s t} t^2 + \frac{1}{24} e^{-s t} t^3$$

```
Expand[t (t - 1) (t - 2) (t - 3)]
```

$$-6 t + 11 t^2 - 6 t^3 + t^4$$