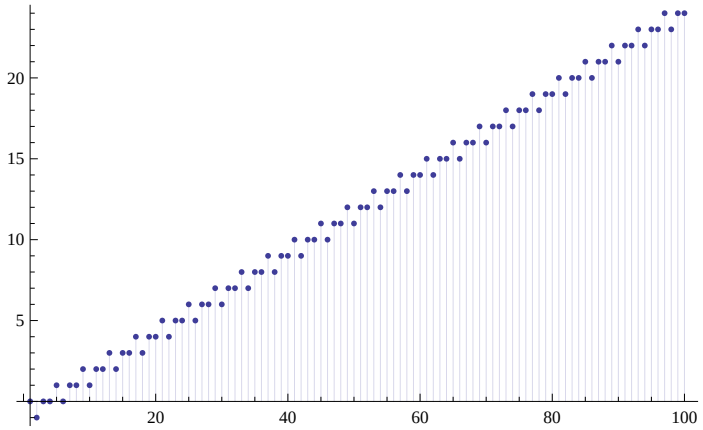
```
ClearAll["Global`*"]
```

```
K[n_] := If[n == 1, 0, FullSimplify[ MangoldtLambda[n] / Log[n]]]
pref9[n_, k_, a_, c_] := pref9[n, k, a, c] =
  Sum[K[Floor[a j]] pref9[Floor[n / j], k - 1, a, c], {j, 2, n}] -
   c Sum[K[Floor[a j c]] pref9[Floor[n / (j c)], k - 1, a, c], {j, 1, n / c}];
pref9[n_, 0, a_, c_] := 1
pref8[n_, k_, a_, c_] := pref8[n, k, a, c] =
  Sum[K[Floor[a j]] pref8[Floor[n / j], k - 1, a, c], {j, 1, n}] -
   c Sum[K[Floor[a j c]] pref8[Floor[n / (j c)], k - 1, a, c], {j, 1, n / c}];
pref8[n_, 0, a_, c_] := 1
pref8a[n_, s_] := Sum[ s^k pref8[n / (s^k), 1, s^k, s], {k, 0, Log[s, n]}]
pref8b[n_, k_, b_] :=
 Sum[Binomial[k + j - 1, k - 1] b^j pref8[n / b^j, k, b^j, b], {j, 0, Log[b, n]}]
pref[n_, k_] := pref[n, k] = Sum[ (-1)^(j + 1) K[j] pref[Floor[n / j], k - 1], {j, 2, n}];
pref[n_, 0] := 1
prefa[n_, k_] := Sum[ (-1)^(j + 1) K[j] prefa[Floor[n / j], k - 1], {j, 1, n}]; prefa[n_, 0] := 1
tt[n_, k_] := Mod[n, k] - Mod[n - 1, k]

pref2[n_, k_, a_] := pref2[n, k, a] = Sum[ tt[j, a] K[j] pref2[Floor[n / j], k - 1, a], {j, 2, n}];
pref2[n_, 0, a_] := 1
pref2a[n_, k_, a_] := Sum[ tt[j, a] K[j] pref2a[Floor[n / j], k - 1, a], {j, 1, n}];
pref2a[n_, 0, a_] := 1

ptest[n_, z_] := Sum[ z^k / (k!) pref[n, k], {k, 1, Log[2, n]}]
ptest2[n_, z_, a_] := Sum[ z^k / (k!) pref2[n, k, a], {k, 1, Log[2, n]}]
ptest3[n_, z_, a_] := Sum[ z^k / (k!) pref9[n, k, 1, a], {k, 1, Log[a, n]}]
```

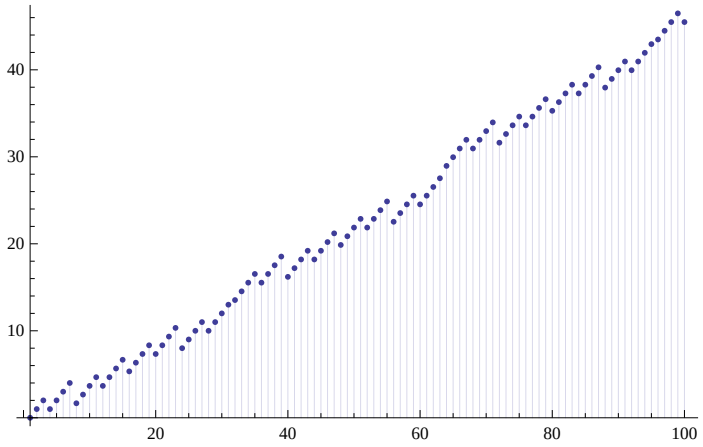
```
pref8a[100, 2]
```

$$\frac{428}{15}$$

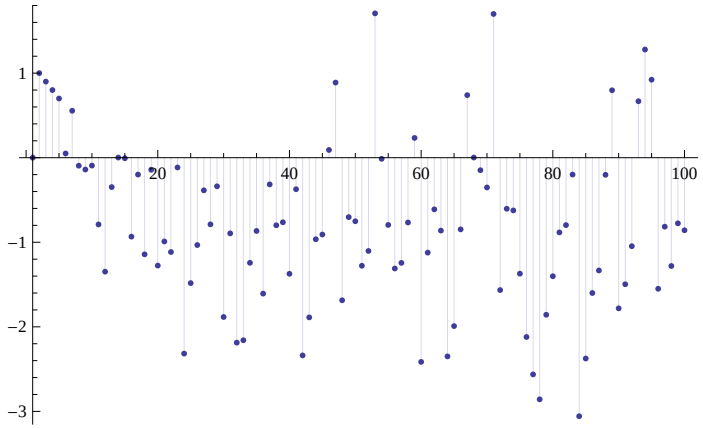
```
DiscretePlot[ptest[n, 1], {n, 1, 100}]
```



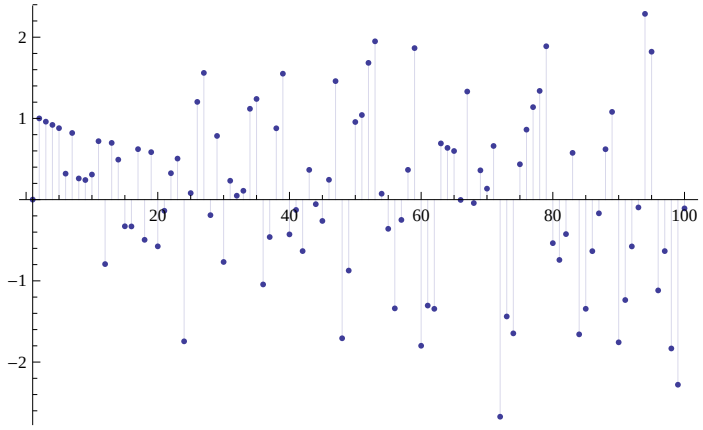
```
DiscretePlot[ptest2[n, 1, 4], {n, 1, 100}]
```



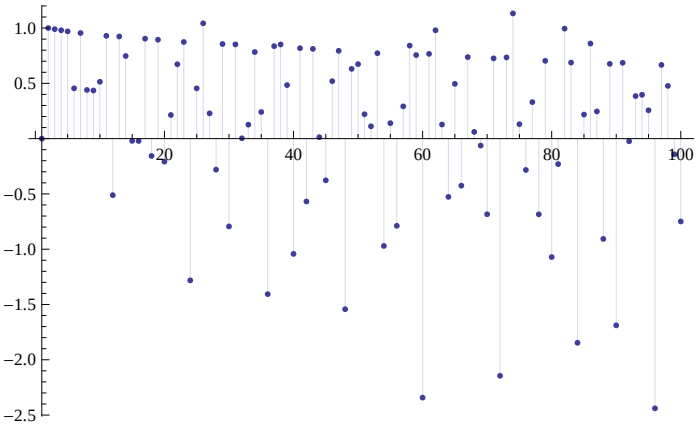

```
DiscretePlot[ptest3[n, 1, 1.1], {n, 1, 100}]
```



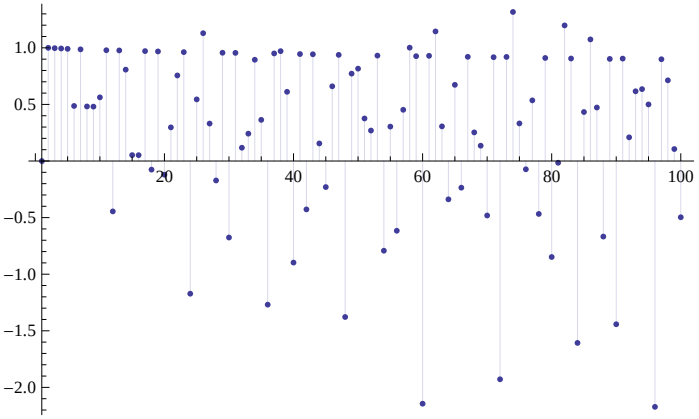

```
DiscretePlot[ptest3[n, 1, 1.04], {n, 1, 100}]
```



```
DiscretePlot[ptest3[n, 1, 1.01], {n, 1, 100}]
```




```
DiscretePlot[ptest3[n, 1, 1.003], {n, 1, 100}]
```




```
DiscretePlot[ptest3[n, -1, 1.003], {n, 1, 100}]
```