```
s1[n_, 0] := 1; s1[n_, k_] := s1[n, k] = Sum[ s1[Floor[n / j], k - 1], {j, 1, n}]
s2[n_, 0] := 1; s2[n_, k_] := s2[n, k] = Sum[(-1)^(j + 1) s2[Floor[n / j], k - 1], {j, 1, n}]
```

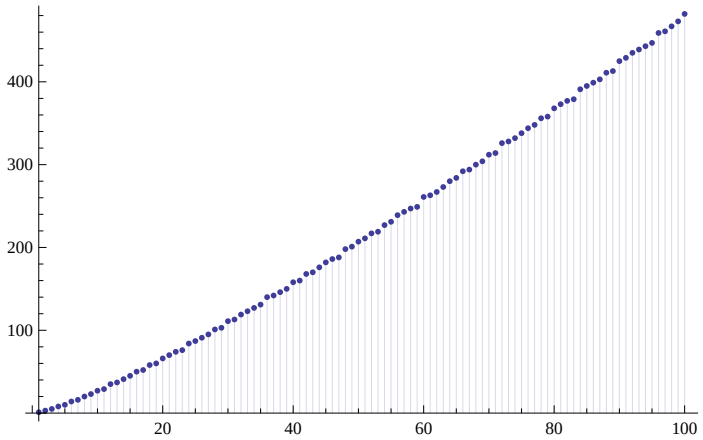
```
s1[100, 1]
```

```
100
```

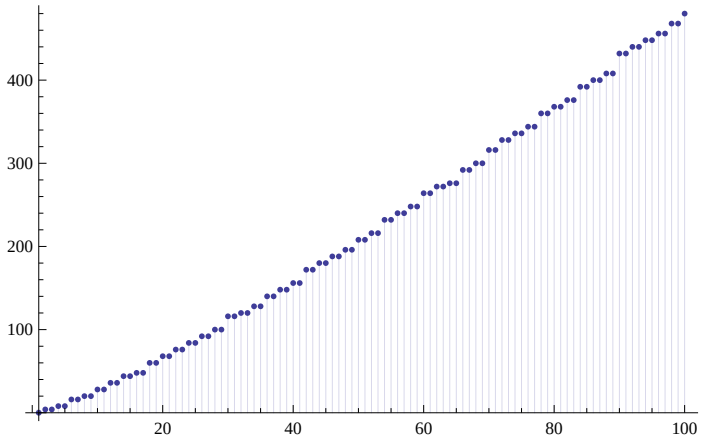
```
s2[100, 1]
```

```
0
```

```
DiscretePlot[s1[n, 2], {n, 1, 100}]
```




```
DiscretePlot[s1[n, 2] - s2[n, 2], {n, 1, 100}]
```




```
f[n_] := (s1[2 n, 2] - s2[2 n, 2]) / 4
```

```
Table[{n, s2[n, 1], s1[n, 1] - 2 s1[Floor[n / 2], 1]}, {n, 1, 100}] // TableForm
```

```
1     1    1
2     0    0
3     1    1
4     0    0
5     1    1
6     0    0
7     1    1
8     0    0
9     1    1
10    0    0
```

```
11    1    1
12    0    0
13    1    1
14    0    0
15    1    1
16    0    0
17    1    1
18    0    0
19    1    1
20    0    0
21    1    1
22    0    0
23    1    1
24    0    0
25    1    1
26    0    0
27    1    1
28    0    0
29    1    1
30    0    0
31    1    1
32    0    0
33    1    1
34    0    0
35    1    1
36    0    0
37    1    1
38    0    0
39    1    1
40    0    0
41    1    1
42    0    0
43    1    1
44    0    0
45    1    1
46    0    0
47    1    1
48    0    0
49    1    1
50    0    0
51    1    1
52    0    0
53    1    1
54    0    0
55    1    1
56    0    0
57    1    1
58    0    0
59    1    1
60    0    0
61    1    1
62    0    0
63    1    1
64    0    0
65    1    1
66    0    0
```

```
67    1    1
68    0    0
69    1    1
70    0    0
71    1    1
72    0    0
73    1    1
74    0    0
75    1    1
76    0    0
77    1    1
78    0    0
79    1    1
80    0    0
81    1    1
82    0    0
83    1    1
84    0    0
85    1    1
86    0    0
87    1    1
88    0    0
89    1    1
90    0    0
91    1    1
92    0    0
93    1    1
94    0    0
95    1    1
96    0    0
97    1    1
98    0    0
99    1    1
100   0    0
```

```
Table[{n, s2[n, 2], s1[n, 2] - 4 s1[Floor[n / 2], 2] + 4 s1[Floor[n / 4], 2]}, {n, 1, 100}] //
 TableForm
```

```
1     1     1
2     -1    -1
3     1     1
4     0     0
5     2     2
6     -2    -2
7     0     0
8     0     0
9     3     3
10    -1    -1
11    1     1
12    -1    -1
13    1     1
14    -3    -3
15    1     1
16    2     2
17    4     4
18    -2    -2
19    0     0
```

20	−2	−2
21	2	2
22	−2	−2
23	0	0
24	0	0
25	3	3
26	−1	−1
27	3	3
28	1	1
29	3	3
30	−5	−5
31	−3	−3
32	−1	−1
33	3	3
34	−1	−1
35	3	3
36	0	0
37	2	2
38	−2	−2
39	2	2
40	2	2
41	4	4
42	−4	−4
43	−2	−2
44	−4	−4
45	2	2
46	−2	−2
47	0	0
48	2	2
49	5	5
50	−1	−1
51	3	3
52	1	1
53	3	3
54	−5	−5
55	−1	−1
56	−1	−1
57	3	3
58	−1	−1
59	1	1
60	−3	−3
61	−1	−1
62	−5	−5
63	1	1
64	4	4
65	8	8
66	0	0
67	2	2
68	0	0
69	4	4
70	−4	−4
71	−2	−2
72	−2	−2
73	0	0
74	−4	−4
75	2	2

```
76    0    0
77    4    4
78    -4   -4
79    -2   -2
80    0    0
81    5    5
82    1    1
83    3    3
84    -1   -1
85    3    3
86    -1   -1
87    3    3
88    3    3
89    5    5
90    -7   -7
91    -3   -3
92    -5   -5
93    -1   -1
94    -5   -5
95    -1   -1
96    3    3
97    5    5
98    -1   -1
99    5    5
100   2    2
```

```
Expand[(x - 2)^3]
```

$-8 + 12\,x - 6\,x^2 + x^3$

```
Table[
  {n, s2[n, 3], s1[n, 3] - 6 s1[Floor[n / 2], 3] + 12 s1[Floor[n / 4], 3] - 8 s1[Floor[n / 8], 3]},
  {n, 1, 100}] // TableForm
```

```
1     1     1
2     -2    -2
3     1     1
4     1     1
5     4     4
6     -5    -5
7     -2    -2
8     0     0
9     6     6
10    -3    -3
11    0     0
12    0     0
13    3     3
14    -6    -6
15    3     3
16    6     6
17    9     9
18    -9    -9
19    -6    -6
20    -6    -6
21    3     3
22    -6    -6
23    -3    -3
```

24	3	3
25	9	9
26	0	0
27	10	10
28	10	10
29	13	13
30	−14	−14
31	−11	−11
32	−8	−8
33	1	1
34	−8	−8
35	1	1
36	1	1
37	4	4
38	−5	−5
39	4	4
40	10	10
41	13	13
42	−14	−14
43	−11	−11
44	−11	−11
45	7	7
46	−2	−2
47	1	1
48	10	10
49	16	16
50	−2	−2
51	7	7
52	7	7
53	10	10
54	−20	−20
55	−11	−11
56	−5	−5
57	4	4
58	−5	−5
59	−2	−2
60	−2	−2
61	1	1
62	−8	−8
63	10	10
64	12	12
65	21	21
66	−6	−6
67	−3	−3
68	−3	−3
69	6	6
70	−21	−21
71	−18	−18
72	−6	−6
73	−3	−3
74	−12	−12
75	6	6
76	6	6
77	15	15
78	−12	−12
79	−9	−9

80	0	0
81	15	15
82	6	6
83	9	9
84	9	9
85	18	18
86	9	9
87	18	18
88	24	24
89	27	27
90	−27	−27
91	−18	−18
92	−18	−18
93	−9	−9
94	−18	−18
95	−9	−9
96	0	0
97	3	3
98	−15	−15
99	3	3
100	3	3

```
Expand[(x - 2)^4]
```

$16 - 32\,x + 24\,x^2 - 8\,x^3 + x^4$

```
Table[{n, s2[n, 4], s1[n, 4] - 8 s1[Floor[n / 2], 4] + 24 s1[Floor[n / 4], 4] -
    32 s1[Floor[n / 8], 4] + 16 s1[Floor[n / 16], 4]}, {n, 1, 100}] // TableForm
```

1	1	1
2	−3	−3
3	1	1
4	3	3
5	7	7
6	−9	−9
7	−5	−5
8	−1	−1
9	9	9
10	−7	−7
11	−3	−3
12	5	5
13	9	9
14	−7	−7
15	9	9
16	12	12
17	16	16
18	−24	−24
19	−20	−20
20	−12	−12
21	4	4
22	−12	−12
23	−8	−8
24	8	8
25	18	18
26	2	2
27	22	22

28	30	30
29	34	34
30	−30	−30
31	−26	−26
32	−26	−26
33	−10	−10
34	−26	−26
35	−10	−10
36	10	10
37	14	14
38	−2	−2
39	14	14
40	30	30
41	34	34
42	−30	−30
43	−26	−26
44	−18	−18
45	22	22
46	6	6
47	10	10
48	22	22
49	32	32
50	−8	−8
51	8	8
52	16	16
53	20	20
54	−60	−60
55	−44	−44
56	−28	−28
57	−12	−12
58	−28	−28
59	−24	−24
60	8	8
61	12	12
62	−4	−4
63	36	36
64	32	32
65	48	48
66	−16	−16
67	−12	−12
68	−4	−4
69	12	12
70	−52	−52
71	−48	−48
72	−8	−8
73	−4	−4
74	−20	−20
75	20	20
76	28	28
77	44	44
78	−20	−20
79	−16	−16
80	−4	−4
81	31	31
82	15	15
83	19	19

```
84     51     51
85     67     67
86     51     51
87     67     67
88     83     83
89     87     87
90     -73    -73
91     -57    -57
92     -49    -49
93     -33    -33
94     -49    -49
95     -33    -33
96     -33    -33
97     -29    -29
98     -69    -69
99     -29    -29
100    -9     -9
```

```
Table[{n, s1[n, 1], Sum[ Binomial[k + 0, 0] 2^k s2[ Floor[n / 2^k], 1], {k, 0, Log[2, n]}]},
  {n, 1, 100}] // TableForm
```

```
1      1      1
2      2      2
3      3      3
4      4      4
5      5      5
6      6      6
7      7      7
8      8      8
9      9      9
10     10     10
11     11     11
12     12     12
13     13     13
14     14     14
15     15     15
16     16     16
17     17     17
18     18     18
19     19     19
20     20     20
21     21     21
22     22     22
23     23     23
24     24     24
25     25     25
26     26     26
27     27     27
28     28     28
29     29     29
30     30     30
31     31     31
32     32     32
33     33     33
34     34     34
35     35     35
36     36     36
```

```
37    37    37
38    38    38
39    39    39
40    40    40
41    41    41
42    42    42
43    43    43
44    44    44
45    45    45
46    46    46
47    47    47
48    48    48
49    49    49
50    50    50
51    51    51
52    52    52
53    53    53
54    54    54
55    55    55
56    56    56
57    57    57
58    58    58
59    59    59
60    60    60
61    61    61
62    62    62
63    63    63
64    64    64
65    65    65
66    66    66
67    67    67
68    68    68
69    69    69
70    70    70
71    71    71
72    72    72
73    73    73
74    74    74
75    75    75
76    76    76
77    77    77
78    78    78
79    79    79
80    80    80
81    81    81
82    82    82
83    83    83
84    84    84
85    85    85
86    86    86
87    87    87
88    88    88
89    89    89
90    90    90
91    91    91
92    92    92
```

```
93     93     93
94     94     94
95     95     95
96     96     96
97     97     97
98     98     98
99     99     99
100    100    100
```

```
Sum[ 2^k s2[ Floor[100 / (2^k)], 1], {k, 0, Log[2, 100]}]
```

```
100
```

```
Expand[
  (x^0 + 2 x^1 + 4 x^2 + 8 x^3 + 16 x^4 + 32 x^5 + 64 x^6 + 128 x^7 + 256 x^8 + 512 x^9 + 1024 x^10) ^2]
```

$$1 + 4\,x + 12\,x^2 + 32\,x^3 + 80\,x^4 + 192\,x^5 + 448\,x^6 + 1024\,x^7 + 2304\,x^8 + 5120\,x^9 + 11\,264\,x^{10} + 20\,480\,x^{11} + 36\,864\,x^{12} + 65\,536\,x^{13} + 114\,688\,x^{14} + 196\,608\,x^{15} + 327\,680\,x^{16} + 524\,288\,x^{17} + 786\,432\,x^{18} + 1\,048\,576\,x^{19} + 1\,048\,576\,x^{20}$$

```
ff[n_] := 2^(n - 1) n
```

```
ff[5]
```

```
80
```

```
Table[{n, s1[n, 2], Sum[ Binomial[k + 1, 1] 2^k s2[ Floor[n / 2^k], 2], {k, 0, Log[2, n]}]},
  {n, 1, 100}] // TableForm
```

```
1      1      1
2      3      3
3      5      5
4      8      8
5      10     10
6      14     14
7      16     16
8      20     20
9      23     23
10     27     27
11     29     29
12     35     35
13     37     37
14     41     41
15     45     45
16     50     50
17     52     52
18     58     58
19     60     60
20     66     66
21     70     70
22     74     74
23     76     76
24     84     84
25     87     87
26     91     91
27     95     95
28     101    101
29     103    103
30     111    111
```

31	113	113
32	119	119
33	123	123
34	127	127
35	131	131
36	140	140
37	142	142
38	146	146
39	150	150
40	158	158
41	160	160
42	168	168
43	170	170
44	176	176
45	182	182
46	186	186
47	188	188
48	198	198
49	201	201
50	207	207
51	211	211
52	217	217
53	219	219
54	227	227
55	231	231
56	239	239
57	243	243
58	247	247
59	249	249
60	261	261
61	263	263
62	267	267
63	273	273
64	280	280
65	284	284
66	292	292
67	294	294
68	300	300
69	304	304
70	312	312
71	314	314
72	326	326
73	328	328
74	332	332
75	338	338
76	344	344
77	348	348
78	356	356
79	358	358
80	368	368
81	373	373
82	377	377
83	379	379
84	391	391
85	395	395
86	399	399

```
87     403    403
88     411    411
89     413    413
90     425    425
91     429    429
92     435    435
93     439    439
94     443    443
95     447    447
96     459    459
97     461    461
98     467    467
99     473    473
100    482    482
```

```
Expand[
  (x^0 + 2 x^1 + 4 x^2 + 8 x^3 + 16 x^4 + 32 x^5 + 64 x^6 + 128 x^7 + 256 x^8 + 512 x^9 + 1024 x^10) ^3]
```

$$1 + 6\,x + 24\,x^2 + 80\,x^3 + 240\,x^4 + 672\,x^5 + 1792\,x^6 + 4608\,x^7 + 11\,520\,x^8 + 28\,160\,x^9 + 67\,584\,x^{10} + 153\,600\,x^{11} + 335\,872\,x^{12} + 712\,704\,x^{13} + 1\,474\,560\,x^{14} + 2\,981\,888\,x^{15} + 5\,898\,240\,x^{16} + 11\,403\,264\,x^{17} + 21\,495\,808\,x^{18} + 39\,321\,600\,x^{19} + 69\,206\,016\,x^{20} + 115\,343\,360\,x^{21} + 188\,743\,680\,x^{22} + 301\,989\,888\,x^{23} + 469\,762\,048\,x^{24} + 704\,643\,072\,x^{25} + 1\,006\,632\,960\,x^{26} + 1\,342\,177\,280\,x^{27} + 1\,610\,612\,736\,x^{28} + 1\,610\,612\,736\,x^{29} + 1\,073\,741\,824\,x^{30}$$

```
Table[{n, s1[n, 3], Sum[ Binomial[k + 2, 2] 2^k s2[ Floor[n / 2^k], 3], {k, 0, Log[2, n]}]},
  {n, 1, 100}] // TableForm
```

```
1      1      1
2      4      4
3      7      7
4      13     13
5      16     16
6      25     25
7      28     28
8      38     38
9      44     44
10     53     53
11     56     56
12     74     74
13     77     77
14     86     86
15     95     95
16     110    110
17     113    113
18     131    131
19     134    134
20     152    152
21     161    161
22     170    170
23     173    173
24     203    203
25     209    209
26     218    218
27     228    228
28     246    246
29     249    249
30     276    276
31     279    279
```

32	300	300
33	309	309
34	318	318
35	327	327
36	363	363
37	366	366
38	375	375
39	384	384
40	414	414
41	417	417
42	444	444
43	447	447
44	465	465
45	483	483
46	492	492
47	495	495
48	540	540
49	546	546
50	564	564
51	573	573
52	591	591
53	594	594
54	624	624
55	633	633
56	663	663
57	672	672
58	681	681
59	684	684
60	738	738
61	741	741
62	750	750
63	768	768
64	796	796
65	805	805
66	832	832
67	835	835
68	853	853
69	862	862
70	889	889
71	892	892
72	952	952
73	955	955
74	964	964
75	982	982
76	1000	1000
77	1009	1009
78	1036	1036
79	1039	1039
80	1084	1084
81	1099	1099
82	1108	1108
83	1111	1111
84	1165	1165
85	1174	1174
86	1183	1183
87	1192	1192

```
88     1222    1222
89     1225    1225
90     1279    1279
91     1288    1288
92     1306    1306
93     1315    1315
94     1324    1324
95     1333    1333
96     1396    1396
97     1399    1399
98     1417    1417
99     1435    1435
100    1471    1471
```

```
Expand[(x^0 + 2 x^1 + 4 x^2 + 8 x^3 + 16 x^4 +
     32 x^5 + 64 x^6 + 128 x^7 + 256 x^8 + 512 x^9 + 1024 x^10) ^4]
```

$$1 + 8\,x + 40\,x^2 + 160\,x^3 + 560\,x^4 + 1792\,x^5 + 5376\,x^6 + 15\,360\,x^7 + 42\,240\,x^8 + 112\,640\,x^9 + 292\,864\,x^{10} + 737\,280\,x^{11} + 1\,798\,144\,x^{12} + 4\,259\,840\,x^{13} + 9\,830\,400\,x^{14} + 22\,151\,168\,x^{15} + 48\,824\,320\,x^{16} + 105\,381\,888\,x^{17} + 222\,822\,400\,x^{18} + 461\,373\,440\,x^{19} + 934\,281\,216\,x^{20} + 1\,845\,493\,760\,x^{21} + 3\,565\,158\,400\,x^{22} + 6\,744\,440\,832\,x^{23} + 12\,499\,025\,920\,x^{24} + 22\,682\,796\,032\,x^{25} + 40\,265\,318\,400\,x^{26} + 69\,793\,218\,560\,x^{27} + 117\,843\,165\,184\,x^{28} + 193\,273\,528\,320\,x^{29} + 307\,090\,161\,664\,x^{30} + 472\,446\,402\,560\,x^{31} + 708\,669\,603\,840\,x^{32} + 1\,030\,792\,151\,040\,x^{33} + 1\,443\,109\,011\,456\,x^{34} + 1\,924\,145\,348\,608\,x^{35} + 2\,405\,181\,685\,760\,x^{36} + 2\,748\,779\,069\,440\,x^{37} + 2\,748\,779\,069\,440\,x^{38} + 2\,199\,023\,255\,552\,x^{39} + 1\,099\,511\,627\,776\,x^{40}$$

```
Table[{n, s1[n, 4], Sum[Binomial[k + 3, 3] 2^k s2[Floor[n / 2^k], 4], {k, 0, Log[2, n]}]},
  {n, 1, 100}] // TableForm
```

```
1      1      1
2      5      5
3      9      9
4      19     19
5      23     23
6      39     39
7      43     43
8      63     63
9      73     73
10     89     89
11     93     93
12     133    133
13     137    137
14     153    153
15     169    169
16     204    204
17     208    208
18     248    248
19     252    252
20     292    292
21     308    308
22     324    324
23     328    328
24     408    408
25     418    418
26     434    434
27     454    454
28     494    494
29     498    498
```

30	562	562
31	566	566
32	622	622
33	638	638
34	654	654
35	670	670
36	770	770
37	774	774
38	790	790
39	806	806
40	886	886
41	890	890
42	954	954
43	958	958
44	998	998
45	1038	1038
46	1054	1054
47	1058	1058
48	1198	1198
49	1208	1208
50	1248	1248
51	1264	1264
52	1304	1304
53	1308	1308
54	1388	1388
55	1404	1404
56	1484	1484
57	1500	1500
58	1516	1516
59	1520	1520
60	1680	1680
61	1684	1684
62	1700	1700
63	1740	1740
64	1824	1824
65	1840	1840
66	1904	1904
67	1908	1908
68	1948	1948
69	1964	1964
70	2028	2028
71	2032	2032
72	2232	2232
73	2236	2236
74	2252	2252
75	2292	2292
76	2332	2332
77	2348	2348
78	2412	2412
79	2416	2416
80	2556	2556
81	2591	2591
82	2607	2607
83	2611	2611
84	2771	2771
85	2787	2787

86	2803	2803
87	2819	2819
88	2899	2899
89	2903	2903
90	3063	3063
91	3079	3079
92	3119	3119
93	3135	3135
94	3151	3151
95	3167	3167
96	3391	3391
97	3395	3395
98	3435	3435
99	3475	3475
100	3575	3575

```
Expand[ (x - 2) ^ 4]
```

$$16 - 32\,x + 24\,x^2 - 8\,x^3 + x^4$$

```
Sum[(-1) ^ j Binomial[4, j] 2^j st[Floor[n / 2^j], 4], {j, 0, 4}]
```

$$16\,\text{st}\left[\text{Floor}\left[\frac{n}{16}\right], 4\right] - 32\,\text{st}\left[\text{Floor}\left[\frac{n}{8}\right], 4\right] + 24\,\text{st}\left[\text{Floor}\left[\frac{n}{4}\right], 4\right] - 8\,\text{st}\left[\text{Floor}\left[\frac{n}{2}\right], 4\right] + \text{st}[\text{Floor}[n], 4]$$

```
Table[{n, s2[n, 4], s1[n, 4] - 8 s1[Floor[n / 2], 4] + 24 s1[Floor[n / 4], 4] -
    32 s1[Floor[n / 8], 4] + 16 s1[Floor[n / 16], 4]}, {n, 1, 100}] // TableForm
```

```
Table[{n, s2[n, 4], Sum[(-1) ^ j Binomial[4, j] 2^j s1[Floor[n / 2^j], 4], {j, 0, 4}]},
  {n, 1, 100}] // TableForm
```

1	1	1
2	-3	-3
3	1	1
4	3	3
5	7	7
6	-9	-9
7	-5	-5
8	-1	-1
9	9	9
10	-7	-7
11	-3	-3
12	5	5
13	9	9
14	-7	-7
15	9	9
16	12	12
17	16	16
18	-24	-24
19	-20	-20
20	-12	-12
21	4	4
22	-12	-12
23	-8	-8
24	8	8

25	18	18
26	2	2
27	22	22
28	30	30
29	34	34
30	−30	−30
31	−26	−26
32	−26	−26
33	−10	−10
34	−26	−26
35	−10	−10
36	10	10
37	14	14
38	−2	−2
39	14	14
40	30	30
41	34	34
42	−30	−30
43	−26	−26
44	−18	−18
45	22	22
46	6	6
47	10	10
48	22	22
49	32	32
50	−8	−8
51	8	8
52	16	16
53	20	20
54	−60	−60
55	−44	−44
56	−28	−28
57	−12	−12
58	−28	−28
59	−24	−24
60	8	8
61	12	12
62	−4	−4
63	36	36
64	32	32
65	48	48
66	−16	−16
67	−12	−12
68	−4	−4
69	12	12
70	−52	−52
71	−48	−48
72	−8	−8
73	−4	−4
74	−20	−20
75	20	20
76	28	28
77	44	44
78	−20	−20
79	−16	−16
80	−4	−4

```
81     31     31
82     15     15
83     19     19
84     51     51
85     67     67
86     51     51
87     67     67
88     83     83
89     87     87
90     -73    -73
91     -57    -57
92     -49    -49
93     -33    -33
94     -49    -49
95     -33    -33
96     -33    -33
97     -29    -29
98     -69    -69
99     -29    -29
100    -9     -9
```