```
F[n_] := Sum[ n / x, {x, 2, n}]
N[F[100]]
```

418.738

```
F2[n_] := n Sum[ 1 / x, {x, 2, n}]
N[F2[100]]
```

418.738

```
F3[x_] := x ( Floor[x] / x - Floor[2] / 2 + Integrate[ Floor[u] / u^2, {u, 2, x}])
N[F3[100]]
```

368.738

```
T1[x_] := Sum[ 1 / n, {n, 1, x}]
T2[x_] := Floor[x] / x + Integrate[ Floor[u] / u^2, {u, 1, x}]
T1[100]
T2[100]
```

$$\frac{14\,466\,636\,279\,520\,351\,160\,221\,518\,043\,104\,131\,447\,711}{2\,788\,815\,009\,188\,499\,086\,581\,352\,357\,412\,492\,142\,272}$$

$$\frac{14\,466\,636\,279\,520\,351\,160\,221\,518\,043\,104\,131\,447\,711}{2\,788\,815\,009\,188\,499\,086\,581\,352\,357\,412\,492\,142\,272}$$

```
T1[x_] := x Sum[ 1 / n, {n, 2, x}]
T2[x_] := x (Floor[x] / x - Floor[2 - 1] / 2 + Integrate[ Floor[u] / u^2, {u, 2, x}])
N[T1[100]]
N[T2[100]]
```

418.738

418.738

```
Fa[x_] := Integrate[ Floor[u] / u^2, {u, 2, x}]
Fb[x_] := Integrate[ u / u^2 - FractionalPart[u] / u^2, {u, 2, x}]

Fa[100]
```

$$\frac{10\,283\,413\,765\,737\,602\,530\,349\,489\,506\,985\,393\,234\,303}{2\,788\,815\,009\,188\,499\,086\,581\,352\,357\,412\,492\,142\,272}$$

```
Fb[100]
```

$$\frac{10\,283\,413\,765\,737\,602\,530\,349\,489\,506\,985\,393\,234\,303}{2\,788\,815\,009\,188\,499\,086\,581\,352\,357\,412\,492\,142\,272}$$

```
Integrate[ u / u^2 , {u, 2, x}]
```

ConditionalExpression[-Log[2] + Log[x], Re[x] ≥ 0 || x ∉ Reals]

```
Integrate[-FractionalPart[u] / u^2, {u, 2, x}]
```

$$\int_2^x -\frac{\text{FractionalPart}[u]}{u^2}\,du$$

$$SS[x\_] := \int_2^x -\frac{\text{FractionalPart}[u]}{u^2}\,du$$

```
N[SS[100]]

-0.224645

N[HarmonicNumber[100]]

5.18738

G1[n_] := Sum[1, {x, 2, n}, {y, 2, n / x}]

G1[1000]

5070

G2[n_] := Sum[ Floor[n / x] - 1, {x, 2, n}]

G2[1000]

5070

G3[n_] := Sum[n / x - 1 - FractionalPart[n / x], {x, 2, n}]

G3[1000]

5070

G4[n_] := -n + 1 + Sum[n / x - FractionalPart[n / x], {x, 2, n}]

G4[100]

283

G5[n_] := -n + 1 - Sum[FractionalPart[n / x], {x, 2, n}] +
  (n (Floor[n] / n - 1 / 2 + Integrate[ Floor[x] / x^2, {x, 2, n}]))

G5[100]

283

G6[n_] := -n + 1 - Sum[FractionalPart[n / x], {x, 2, n}] +
  (n (1 / 2 + Integrate[ Floor[x] / x^2, {x, 2, n}]))


G6[100]

283

G7[n_] :=
 -n + 1 - Sum[FractionalPart[n / x], {x, 2, n}] + (n (1 / 2 + Integrate[ (x) / x^2, {x, 2, n}] -
      Integrate[ (FractionalPart[x]) / x^2, {x, 2, n}]))

N[G7[100]]

283.
```

```
G8[n_] := -n + 1 - Sum[FractionalPart[n / x], {x, 2, n}] +
  (n (1 / 2 + Log[n] - Log[2] - Integrate[ (FractionalPart[x]) / x^2, {x, 2, n}]))
```

```
N[G8[100]]
```

283.

```
Expand[Integrate[ 1, {x, 1, n}, {y, 1, n / x}]]
```

ConditionalExpression[1 - n + n Log[n], Re[n] ≥ 0 || n ∉ Reals]

```
H1[n_] := Integrate[ 1, {x, 1, n}, {y, 1, n / x}] - Sum[ 1, {x, 2, n}, {y, 2, n / x}]
N[H1[100]]
```

78.517

```
H2[n_] := n Log[n] - n + 1 - Sum[ 1, {x, 2, n}, {y, 2, n / x}]
N[H2[100]]
```

78.517

```
H3[n_] := n Log[n] - n + 1 - G8[n]
N[H3[100]]
```

78.517

```
H4[n_] := n Log[n] - n + 1 - (-n + 1 - Sum[FractionalPart[n / x], {x, 2, n}] +
    (n (1 / 2 + Log[n] - Log[2] - Integrate[ (FractionalPart[x]) / x^2, {x, 2, n}]))
  )
N[H4[100]]
```

78.517

```
FullSimplify[ n Log[n] - n + 1 - (-n + 1 - Sum[FractionalPart[n / x], {x, 2, n}] +
    (n (1 / 2 + Log[n] - Log[2] - Integrate[ (FractionalPart[x]) / x^2, {x, 2, n}]))
  ) ]
```

$$n \int_2^n \frac{\text{FractionalPart}[x]}{x^2}\, dx + n\left(-\frac{1}{2} + \text{Log}[2]\right) + \sum_{x=2}^{n} \text{FractionalPart}\left[\frac{n}{x}\right]$$

$$\text{H5}[n\_] := n \int_2^n \frac{\text{FractionalPart}[x]}{x^2}\, dx + n\left(-\frac{1}{2} + \text{Log}[2]\right) + \sum_{x=2}^{n} \text{FractionalPart}\left[\frac{n}{x}\right]$$

```
N[H5[100]]
```

78.517

$$\text{H6}[n\_] := n \int_2^n \frac{\text{FractionalPart}[x]}{x^2}\, dx$$

```
DiscretePlot[H6[n], {n, 2, 100}]
```

```
Table[N[H6[n]], {n, 5, 100, 5}]
```

{0.664787, 1.8047, 2.95011, 4.09691, 5.24426, 6.39189, 7.53968, 8.68757, 9.83552, 10.9835, 12.1316, 13.2796, 14.4277, 15.5758, 16.7239, 17.872, 19.0201, 20.1683, 21.3164, 22.4645}

$$\text{H7[n\_]} := \sum_{x=2}^{n} \text{FractionalPart}\left[\frac{n}{x}\right]$$

```
Table[N[H7[n]], {n, 5, 100, 5}]
```

{1.41667, 2.28968, 4.77343, 5.95479, 8.39895, 8.84961, 14.1373, 13.1417, 15.7727, 17.9603, 21.6487, 19.7922, 25.3529, 26.2986, 29.6017, 29.2383, 32.1877, 32.4314, 40.9529, 36.7378}

```
I1[n_] := (n Log[n] - n + 1) - Sum[1, {x, 2, n}, {y, 2, n / x}]
```

```
N[I1[100]]
```

78.517

```
I2[n_] :=
 (n Log[n] - n + 1) - ( 1 - Floor[n^(1 / 2)]^2 + 2 Sum[ Floor[n / a], {a, 2, Floor[n^(1 / 2)]}] )
```

```
N[I2[100]]
```

78.517

```
I3[n_] := (n Log[n] - n + 1) - ( 1 - (n^(1 / 2))^2 - FractionalPart[n^(1 / 2)]^2 +
    2 Sum[ n / a - FractionalPart[n / a], {a, 2, Floor[n^(1 / 2)]}] )
```

```
N[I3[100]]
```

78.517

```
I4[n_] := (n Log[n] - n + 1) - ( 1 - n - FractionalPart[n^(1 / 2)]^2 +
    2 Sum[ n / a, {a, 2, Floor[n^(1 / 2)]}] -
    2 Sum[ FractionalPart[n / a], {a, 2, Floor[n^(1 / 2)]}]
  )
```

```
N[I4[100]]
```

78.517

```
J1[n_] := Sum[ n / a, {a, 2, Floor[n^(1 / 2)]}]
```

```
J1[100]
```

$\frac{24305}{126}$

```
J2[n_] := n Sum[ 1 / a, {a, 2, Floor[n^(1 / 2)]}]
```

```
J2[100]
```

$\frac{24305}{126}$

```
J3[n_] := n (1 / 2 + Integrate[ Floor[u] / u^2, {u, 2, Floor[n^(1 / 2)]}])
```

```
N[J3[100]]
```

192.897

```
Ja1[n_] := Integrate[ Floor[u] / u^2, {u, 2, Floor[n^(1 / 2)]}]
```

```
N[Ja1[100]]

1.42897

Ja2[n_] := Integrate[u / u^2, {u, 2, Floor[n^(1 / 2)]}] -
  Integrate[FractionalPart[u] / u^2, {u, 2, Floor[n^(1 / 2)]}]

N[Ja2[100]]

1.42897

Integrate[u / u^2, {u, 2, Floor[n^(1 / 2)]}]
```

ConditionalExpression$\left[-\text{Log}[2]+\text{Log}\left[\text{Floor}\left[\sqrt{n}\right]\right],\ \text{Re}\left[\text{Floor}\left[\sqrt{n}\right]\right]\geq 0\ ||\ \text{Floor}\left[\sqrt{n}\right]\notin\text{Reals}\right]$

```
Ja3[n_] :=
 Log[Floor[n^(1 / 2)]] - Log[2] - Integrate[FractionalPart[u] / u^2, {u, 2, Floor[n^(1 / 2)]}]

N[Ja3[100]]

1.42897

Ja4[n_] := Log[n^(1 / 2) - FractionalPart[n^(1 / 2)]] -
  Log[2] - Integrate[FractionalPart[u] / u^2, {u, 2, Floor[n^(1 / 2)]}]

N[Ja4[100]]

1.42897

J4[n_] := n (1 / 2 + Log[n^(1 / 2) - FractionalPart[n^(1 / 2)]] -
    Log[2] - Integrate[FractionalPart[u] / u^2, {u, 2, Floor[n^(1 / 2)]}])

N[J4[100]]

192.897

I3[n_] := (n Log[n] - n + 1) - ( 1 - (n^(1 / 2))^2 - FractionalPart[n^(1 / 2)]^2 +
    2 Sum[ n / a - FractionalPart[n / a], {a, 2, Floor[n^(1 / 2)]}] )

N[I3[100]]

78.517

I4[n_] := (n Log[n] - n + 1) - ( 1 - n - FractionalPart[n^(1 / 2)]^2 +
    2 Sum[ n / a, {a, 2, Floor[n^(1 / 2)]}] -
    2 Sum[ FractionalPart[n / a], {a, 2, Floor[n^(1 / 2)]}]
  )

N[I4[100]]

78.517

I5[n_] := n Log[n] - (-FractionalPart[n^(1 / 2)]^2 +
    n + 2 n Log[Floor[n^(1 / 2)]] - 2 n Log[2] -
    2 n Integrate[FractionalPart[u] / u^2, {u, 2, Floor[n^(1 / 2)]}] -
    2 Sum[ FractionalPart[n / a], {a, 2, Floor[n^(1 / 2)]}])

N[I5[100]]

78.517

I5[n]

$Aborted
```

```
FullSimplify[n Log[n] - (-FractionalPart[n^(1/2)]^2 +
    n + 2 n Log[Floor[n^(1/2)]] - 2 n Log[2] -
    2 n Integrate[FractionalPart[u] / u^2, {u, 2, Floor[n^(1/2)]}] -
    2 Sum[ FractionalPart[n / a], {a, 2, Floor[n^(1/2)]}])]
```

$$\text{Expand}\Big[\text{FractionalPart}\big[\sqrt{n}\big]^2 + n\left(-1 + 2\int_2^{\text{Floor}[\sqrt{n}]}\frac{\text{FractionalPart}[u]}{u^2}\,du + \text{Log}[4] + \text{Log}[n] - 2\,\text{Log}\big[\text{Floor}\big[\sqrt{n}\big]\big]\right) + 2\sum_{a=2}^{\text{Floor}[\sqrt{n}]}\text{FractionalPart}\Big[\frac{n}{a}\Big]\Big]$$

$$-n + \text{FractionalPart}\big[\sqrt{n}\big]^2 + 2\,n\int_2^{\text{Floor}[\sqrt{n}]}\frac{\text{FractionalPart}[u]}{u^2}\,du + n\,\text{Log}[4] + n\,\text{Log}[n] - 2\,n\,\text{Log}\big[\text{Floor}\big[\sqrt{n}\big]\big] + 2\sum_{a=2}^{\text{Floor}[\sqrt{n}]}\text{FractionalPart}\Big[\frac{n}{a}\Big]$$

$$\text{I6}[n\_] := \text{FractionalPart}\big[\sqrt{n}\big]^2 + n\left(-1 + 2\int_2^{\text{Floor}[\sqrt{n}]}\frac{\text{FractionalPart}[u]}{u^2}\,du + \text{Log}[4] + \text{Log}[n] - 2\,\text{Log}\big[\text{Floor}\big[\sqrt{n}\big]\big]\right) + 2\sum_{a=2}^{\text{Floor}[\sqrt{n}]}\text{FractionalPart}\Big[\frac{n}{a}\Big]$$

```
N[I6[100]]
```

78.517

$$\text{FullSimplify}\Big[\int_2^{\text{Floor}[\sqrt{n}]}\frac{\text{FractionalPart}[u]}{u^2}\,du\Big]$$

$$\int_2^{\text{Floor}[\sqrt{n}]}\frac{\text{FractionalPart}[u]}{u^2}\,du$$

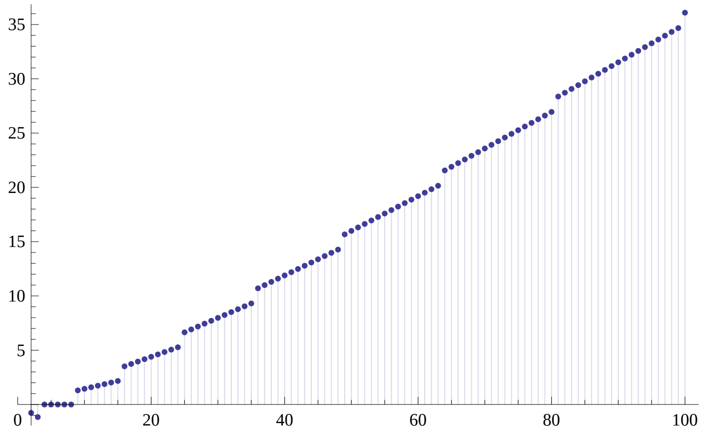
$$\text{P1}[n\_] := -n + \text{FractionalPart}\big[\sqrt{n}\big]^2 + 2\,n\int_2^{\text{Floor}[\sqrt{n}]}\frac{\text{FractionalPart}[u]}{u^2}\,du + n\,\text{Log}[4] + n\,\text{Log}[n] - 2\,n\,\text{Log}\big[\text{Floor}\big[\sqrt{n}\big]\big] + 2\sum_{a=2}^{\text{Floor}[\sqrt{n}]}\text{FractionalPart}\Big[\frac{n}{a}\Big]$$

$$\text{P1a}[n\_] := -n + n\,\text{Log}[4] + n\,\text{Log}[n]$$

$$\text{P2a}[n\_] := \text{FractionalPart}\big[\sqrt{n}\big]^2 + 2\,n\int_2^{\text{Floor}[\sqrt{n}]}\frac{\text{FractionalPart}[u]}{u^2}\,du - 2\,n\,\text{Log}\big[\text{Floor}\big[\sqrt{n}\big]\big] + 2\sum_{a=2}^{\text{Floor}[\sqrt{n}]}\text{FractionalPart}\Big[\frac{n}{a}\Big]$$

$$\text{P2b}[n\_] := 2\,n\int_2^{\text{Floor}[\sqrt{n}]}\frac{\text{FractionalPart}[u]}{u^2}\,du$$

```
DiscretePlot[P2b[n], {n, 2, 100}]
```



$$\mathbf{FullSimplify}\left[-n+\mathrm{FractionalPart}\left[\sqrt{n}\right]^2+2\,n\int_2^{\mathrm{Floor}\left[\sqrt{n}\right]}\frac{\mathrm{FractionalPart}[u]}{u^2}\,du+n\,\mathrm{Log}[4]+n\,\mathrm{Log}[n]-2\,n\,\mathrm{Log}\left[\mathrm{Floor}\left[\sqrt{n}\right]\right]+2\sum_{a=2}^{\mathrm{Floor}\left[\sqrt{n}\right]}\mathrm{FractionalPart}\left[\frac{n}{a}\right]\right]$$

$$\mathrm{FractionalPart}\left[\sqrt{n}\right]^2+n\left(-1+2\int_2^{\mathrm{Floor}\left[\sqrt{n}\right]}\frac{\mathrm{FractionalPart}[u]}{u^2}\,du+\mathrm{Log}[4]+\mathrm{Log}[n]-2\,\mathrm{Log}\left[\mathrm{Floor}\left[\sqrt{n}\right]\right]\right)+2\sum_{a=2}^{\mathrm{Floor}\left[\sqrt{n}\right]}\mathrm{FractionalPart}\left[\frac{n}{a}\right]$$

```
PP[n_] := Integrate[ (FractionalPart[x]) / x^2, {x, 2, n}]
PP[n]
```

$$\int_2^n z\verb|^|-2\,\mathrm{FractionalPart}[z]\,dz$$

$$\int_2^n \frac{\mathrm{FractionalPart}[z]}{z^2}\,dz$$

```
Integrate[ z^(a - 1) FractionalPart[z]]
```

Integrate::argmu : Integrate called with 1 argument; 2 or more arguments are expected. ≫

$$\mathbf{Integrate}\left[z^{-1+a}\,\mathrm{FractionalPart}[z],\ z\right]$$

$$\int z^{-1+a}\,\mathrm{FractionalPart}[z]\,dz$$

```
N[Integrate[z^-2 FractionalPart[z], {z, 2, 10}]]
```

0.18047

```
FF[z_, a_] := (z^a FractionalPart[z]) / a - z^(a + 1) / (a (a + 1))

FF2[z_, z2_, a_] := FF[z2, a] - FF[z, a]

FF2[2, 10, 2]
```

$-\frac{496}{3}$

```
Integrate[(x - 1) / x^2, {x, 1, 2}]
```

$-\frac{1}{2} + \text{Log}[2]$

```
Sum[ Integrate[(x - j) / x^2, {x, j, j + 1}], {j, 2, n - 1}]
```

$$\begin{cases} \frac{1}{3}(-1 - 3\,\text{Log}[2] + 3\,\text{Log}[3]) & n == 3 \\ \frac{1}{2}(3 - 2\,\text{EulerGamma} - 2\,\text{Log}[\text{Pochhammer}[2, -2 + n]] + 2\,\text{Log}[\text{Pochhammer}[3, -2 + n]] - 2\,\text{PolyGamma}[0, 1 + n]) & \text{True} \end{cases}$$

```
N[PP[80]]
```

0.2234

$$\text{PQ}[n\_] := \begin{cases} \frac{1}{3}(-1 - 3\,\text{Log}[2] + 3\,\text{Log}[3]) & n == 3 \\ \frac{1}{2}(3 - 2\,\text{EulerGamma} - 2\,\text{Log}[\text{Pochhammer}[2, -2 + n]] + 2\,\text{Log}[\text{Pochhammer}[3, -2 + n]] - 2\,\text{PolyGamma}[0, 1 + n]) & \text{True} \end{cases}$$

```
N[PQ[4]]
```

0.109814

$$\text{PQQ}[n\_] := \frac{1}{2}(3 - 2\,\text{EulerGamma} - 2\,\text{Log}[\text{Pochhammer}[2, -2 + n]] + 2\,\text{Log}[\text{Pochhammer}[3, -2 + n]] - 2\,\text{PolyGamma}[0, 1 + n])$$

N::argt : N called with 0 arguments; 1 or 2 arguments are expected. >>

```
N[PQQ[4]]
```

0.109814

$$\text{Expand}\left[\frac{1}{2}(3 - 2\,\text{EulerGamma} - 2\,\text{Log}[\text{Pochhammer}[2, -2 + n]] + 2\,\text{Log}[\text{Pochhammer}[3, -2 + n]] - 2\,\text{PolyGamma}[0, 1 + n])\right]$$

$$\text{FullSimplify}\left[\frac{3}{2} - \text{EulerGamma} - \text{Log}[\text{Pochhammer}[2, -2 + n]] + \text{Log}[\text{Pochhammer}[3, -2 + n]] - \text{PolyGamma}[0, 1 + n]\right]$$

$$\frac{3}{2} - \text{HarmonicNumber}[n] - \text{Log}[2\,\text{Gamma}[n]] + \text{Log}[\text{Gamma}[1 + n]]$$

$$\text{PR}[n\_] := \frac{3}{2} - \text{HarmonicNumber}[n] - \text{Log}[2\,\text{Gamma}[n]] + \text{Log}[\text{Gamma}[1 + n]]$$

```
N[PR[4]]
```

0.109814

```
N[Integrate[ (FractionalPart[x]) / x^2, {x, 2, 4}]]
```

0.109814

```
H1[n_] := Integrate[ 1, {x, 1, n}, {y, 1, n / x}] - Sum[ 1, {x, 2, n}, {y, 2, n / x}]
N[H1[100]]
```

78.517

```
Ha[n_] := n Log[n] - n + 1 - (-n + 1 - Sum[FractionalPart[n / x], {x, 2, n}] +
    (n (1 / 2 + Log[n] - Log[2] - Integrate[ (FractionalPart[x]) / x^2, {x, 2, n}])))
N[Ha[100]]
```

78.517

```
Expand[n Log[n] - n + 1 - (-n + 1 - Sum[FractionalPart[n / x], {x, 2, n}] +
    (n (1 / 2 + Log[n] - Log[2] - Integrate[ (FractionalPart[x]) / x^2, {x, 2, n}])))]
```

$$-\frac{n}{2} + n \int_2^n \frac{\text{FractionalPart}[x]}{x^2}\, dx + n\,\text{Log}[2] + \sum_{x=2}^{n} \text{FractionalPart}\left[\frac{n}{x}\right]$$

$$\text{Hb}[n\_] := -\frac{n}{2} + n \int_2^n \frac{\text{FractionalPart}[x]}{x^2}\, dx + n\,\text{Log}[2] + \sum_{x=2}^{n} \text{FractionalPart}\left[\frac{n}{x}\right]$$

$$\text{Hc}[n\_] := -\frac{n}{2} + n\,\text{Log}[2] + \sum_{x=2}^{n} \text{FractionalPart}\left[\frac{n}{x}\right] + n\left(\frac{3}{2} - \text{HarmonicNumber}[n] - \text{Log}[2\,\text{Gamma}[n]] + \text{Log}[\text{Gamma}[1+n]]\right)$$

```
N[Hc[100]]
```

78.517

$$\text{Expand}\left[-\frac{n}{2} + n\,\text{Log}[2] + \sum_{x=2}^{n} \text{FractionalPart}\left[\frac{n}{x}\right] + n\left(\frac{3}{2} - \text{HarmonicNumber}[n] - \text{Log}[2\,\text{Gamma}[n]] + \text{Log}[\text{Gamma}[1+n]]\right)\right]$$

$$\text{FullSimplify}\left[n - n\,\text{HarmonicNumber}[n] + n\,\text{Log}[2] - n\,\text{Log}[2\,\text{Gamma}[n]] + n\,\text{Log}[\text{Gamma}[1+n]] + \sum_{x=2}^{n} \text{FractionalPart}\left[\frac{n}{x}\right]\right]$$

$$\text{Hd}[n\_] := n\,(1 - \text{HarmonicNumber}[n] - \text{Log}[\text{Gamma}[n]] + \text{Log}[\text{Gamma}[1+n]]) + \sum_{x=2}^{n} \text{FractionalPart}\left[\frac{n}{x}\right]$$

```
N[Hd[100]]
```

78.517

```
FullSimplify[ Log[2] - Log[2 Gamma[n]]]
```

```
-Log[Gamma[n]]
```

```
DiscretePlot[1 - HarmonicNumber[n] - Log[Gamma[n]] + Log[Gamma[1 + n]], {n, 2, 100}]
```