```
//
Table[{n, Sum[N[(c^j - 1) / j], {j, 1, Floor[Log[n] / Log[c]]}] /. c → 1.00001,
   N[LogIntegral[n] - Log[Log[n]] - EulerGamma]}, {n, 5, 20, 5}] // TableForm
```

5	$2.58148 - 7.59348 \times 10^{-12}\, i$	2.58149
10	$4.75434 + 1.95275 \times 10^{-11}\, i$	4.75435
15	$6.58138 - 1.90723 \times 10^{-11}\, i$	6.58138
20	$8.23088 - 1.74687 \times 10^{-11}\, i$	8.2309

```
Sum[(c^j - 1) / j, {j, 1, Floor[Log[100] / Log[c]]}] /. c → 1.00001
```

$28.0218 - 2.22045 \times 10^{-15}\, i$

```
Integrate[(c^j) / j, {j, 1, Log[c, 100]}]
```

$$\text{ConditionalExpression}\left[\text{LogIntegral}[100] - \text{LogIntegral}[c],\ \left(\text{Re}[\text{Log}[c]] > 0\ \&\&\ \left((\text{Re}[\text{Log}[c]] \le \text{Log}[100]\ \&\&\ \text{Log}[c] \ne \text{Log}[100])\ ||\ \text{Log}[100]\ \text{Re}\left[\frac{1}{\text{Log}[c]}\right] \le 1\right)\right)\ ||\ \text{Log}[c] \notin \text{Reals}\right]$$

```
(N[Sum[(c^j) / j, {j, 1, Log[c, 100]}]] - Integrate[(c^j) / j, {j, 1, Log[c, 100]}]) /.
 c → 1.0000001
```

$0.577217 + 5.71039 \times 10^{-10}\, i$

```
N[Sum[(c^j - 1) / j, {j, 1, Log[c, 100]}] /. c → 1.0000001]
```

$28.0217 + 5.71039 \times 10^{-10}\, i$

```
N[Integrate[(c^j - 1) / j, {j, 1, Log[c, 100]}] /. c → 1.0000001]
```

28.0217

```
N[ EulerGamma]
```

0.577216

```
num[c_] := Numerator[c]; den[c_] := Denominator[c]
alpha[n_, c_] := den[c] (Floor[n / den[c]] - Floor[(n - 1) / den[c]]) -
  num[c] (Floor[n / num[c]] - Floor[(n - 1) / num[c]])
F[n_, 0, s_, c_] := 1
F[n_, 1, s_, c_] := If[n < s, 0, (den[c] Floor[n / den[c]] - num[c] Floor[n / num[c]]) -
   (den[c] Floor[(s - 1) / den[c]] - num[c] Floor[(s - 1) / num[c]])]
F[n_, k_, s_, c_] := F[n, k, s, c] = Sum[If[alpha[m, c] == 0, 0, Binomial[k, j] alpha[m, c]^j
     F[Floor[n / (m^j)], k - j, m + 1, c]], {j, 1, k}, {m, s, Floor[n^(1 / k)]}]
E2Alt[n_, k_, c_] := E2Alt[n, k, c] = den[c]^-k F[n den[c]^k, k, den[c] + 1, c]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
E1Alt[n_, z_, c_] :=
 E1Alt[n, z, c] = Sum[bin[z, k] E2Alt[n, k, c], {k, 0, Floor[Log[n] / Log[c]]}]
L2[n_, k_, c_] := L2[n, k, c] = (1 / den[c])
   Sum[If[alpha[j, c] == 0, 0, alpha[j, c] Log[j / den[c]] E2Alt[den[c] n / j, k - 1, c]],
    {j, den[c] + 1, den[c] n}]; L2[n_, 0, c_] := 1
L1[n_, z_, c_] := L1[n, z, c] = Sum[bin[z, k] L2[n, k, c],
   {k, 0, Floor[Log[n] / Log[If[ c > 2, 2, c]]]}]
```

```
DiscretePlot[ Limit[(E1Alt[n, z, 11 / 10] - 1) / z, z → 0], {n, 2, 100}]
```

```
$Aborted
```

```
Table[ N[Limit[(E1Alt[6, z, (b + 1) / b] - 1) / z, z → 0]], {b, 1, 40}] // TableForm
```

```
-0.5
-1.51563
-2.08171
-2.46503
-2.1324
-2.4491
-2.70366
-2.91508
-3.09506
-2.92935
-3.09283
-3.23761
-3.36724
-3.25313
-3.37347
-3.48343
-3.58452
-3.67797
-3.59272
-3.68134
-3.76414
-3.84178
-3.91483
-3.84678
-3.91688
-3.98327
-4.04629
-4.10624
-4.04961
-4.10758
-4.16299
-4.21602
-4.26686
-4.21837
-4.26779
-4.31532
-4.3611
-4.31823
-4.36283
-4.40589
```

```
Table[ N[L1[6, -1, (b + 1) / b]], {b, 1, 51}] // TableForm
```

```
1.06454
1.84726
2.22047
2.44013
1.45616
1.7077
1.89874
2.04877
2.1697
1.68633
1.81168
1.91895
2.01178
1.67809
1.77117
1.85416
1.92859
1.99574
1.74776
1.81532
1.87724
1.93421
1.98678
1.78944
1.84239
1.89175
1.93788
1.98107
1.81718
1.86069
1.90171
1.94046
1.97712
1.83697
1.87388
1.90897
1.94238
1.81828
1.8518
1.88384
1.9145
1.94385
1.83386
1.86333
1.89163
1.91885
1.94503
1.84626
1.87254
1.89789
1.92236
```

```
zeros[n_, b_] := List @@ NRoots[L1[n, z, (b + 1) / b] == 0, z][[All, 2]]
Table[zeros[100, (b + 1) / b], {b, 1, 10}] // TableForm
```

-41.8797	-2.38343	-0.140031 - 0.362883 i	-0.140031 + 0.3628
-0.216172 - 6.32316 i	-0.216172 + 6.32316 i	-0.103115 - 0.35856 i	-0.103115 + 0.3585
-0.0983532 - 0.440859 i	-0.0983532 + 0.440859 i	0.08166 - 1.87282 i	0.08166 + 1.87282
-16.066	-0.241898	0.203473 - 1.70995 i	0.203473 + 1.70995
-6.75512 - 10.545 i	-6.75512 + 10.545 i	-1.31536	-0.418856
-9.38535	-0.612972 - 1.08019 i	-0.612972 + 1.08019 i	0.222977 - 0.43872
-9.44018	-0.852642 - 1.4015 i	-0.852642 + 1.4015 i	0.162458
-3.38154 - 1.77742 i	-3.38154 + 1.77742 i	0.147524 - 0.268776 i	0.147524 + 0.26877
-2.87395 - 1.92458 i	-2.87395 + 1.92458 i	0.228379 - 0.15916 i	0.228379 + 0.15916
-2.96372	0.256308 - 0.185984 i	0.256308 + 0.185984 i	1.81762 - 1.66988

```
Expand[L1[ 100., z, 1.01]]
```

$1. - 4.33381\,z + 0.696494\,z^2 - 0.0000743469\,z^3 + 8.16064\times10^{-8}\,z^4 - 3.71765\times10^{-11}\,z^5 + 7.30208\times10^{-15}\,z^6$

```
Expand[L1[ 10., z, 1.001]]
```

$1. - 0.0163521\,z + 0.00124768\,z^2 - 1.15525\times10^{-10}\,z^3$

```
N[Sum[ MangoldtLambda[ j ], {j, 1, 10}]]
```

7.83201

```
ff[ c_] := (1 - Sum[ (-1)^k N[L2[ 10., k, c]], {k, 0, Log[c, 10]}] +
   Sum[ c^k Log[c], {k, 1, Log[c, 10.]}])
```

```
Sum[ c^k Log[c], {k, 1, Log[c, 10.]}] /. c → 1.0001
```

9.00045

```
ff[1.5]
```

8.40214

```
num[c_] := Numerator[c]; den[c_] := Denominator[c]
alpha[n_, c_] := den[c] (Floor[n / den[c]] - Floor[(n - 1) / den[c]]) -
  num[c] (Floor[n / num[c]] - Floor[(n - 1) / num[c]])
E2[n_, k_, c_] := E2[n, k, c] = (1 / den[c]) Sum[If[alpha[j, c] == 0, 0,
     alpha[j, c] E2[(den[c] n) / j, k - 1, c]], {j, den[c] + 1, den[c] n}]; E2[n_, 0, c_] := 1
L2[n_, k_, c_] := (1 / den[c]) Sum[If[alpha[j, c] == 0, 0, alpha[j, c] Log[j / den[c]]
     E2[den[c] n / j, k - 1, c]], {j, den[c] + 1, den[c] n}]; L2[n_, 0, c_] := 1
ChebAlt[n_, c_] := Sum[(-1)^(k - 1) L2[n, k, c], {k, 1, Floor[Log[n] / Log[If[c < 2, c, 2]]]}] +
  Sum[c^k Log[c], {k, 1, Floor[Log[n] / Log[c]]}]
```

```
ChebAlt[100, 3 / 2.]
```

99.8686