$$D_3'(n)= 2\sum_{a=2}\sum_{b=a+1}\sum_{c=b+1}^{\lfloor\frac{n}{ab}\rfloor} 1 +\sum_{a=2}\sum_{b=a+1}^{\lfloor(\frac{n}{a})^{\frac{1}{2}}\rfloor} 1 +\sum_{a=2}\sum_{b=a+1}^{\lfloor\frac{n}{a^2}\rfloor} 1 +\frac{1}{3}(\lfloor n^{(1/3)}\rfloor-1)$$

$$D_3'(n)= 2\sum_{a=2}\sum_{b=a+1}(\lfloor\frac{n}{ab}\rfloor-b) +\sum_{a=2}(\lfloor(\frac{n}{a})^{\frac{1}{2}}\rfloor-a) +\sum_{a=2}(\lfloor\frac{n}{a^2}\rfloor-a) +\frac{1}{3}(\lfloor n^{(1/3)}\rfloor-1)$$

$$\begin{gathered} D_3'(n)= \\ 2\sum_{a=2}\sum_{b=a+1}(\lfloor\frac{n}{ab}\rfloor)+2\sum_{a=2}\sum_{b=a+1}(-b) \\ +\sum_{a=2}(\lfloor(\frac{n}{a})^{\frac{1}{2}}\rfloor)+\sum_{a=2}(-a) \\ +\sum_{a=2}(\lfloor\frac{n}{a^2}\rfloor)+\sum_{a=2}(-a) \\ +\frac{1}{3}(\lfloor n^{(1/3)}\rfloor)-\frac{1}{3} \end{gathered}$$

$$5$$
$$5-(2\cdot5,3\cdot5,4\cdot5)$$
$$5-(2\cdot5,3\cdot5,4\cdot5)+2\cdot(2\cdot3\cdot5,2\cdot4\cdot5,3\cdot4\cdot5)$$
$$5-(2\cdot5,3\cdot5,4\cdot5)+2\cdot(2\cdot3\cdot5,2\cdot4\cdot5,3\cdot4\cdot5)-6\cdot(2\cdot3\cdot4\cdot5)$$

$$1-3+2\cdot3-6\cdot1$$

$$-2 \text{ (at 120)}$$

$$4$$
$$4-(2\cdot4,3\cdot4)$$
$$4-(2\cdot4,3\cdot4)+2\cdot(2\cdot3\cdot4)$$

$$1-2+2\cdot1$$

$$1 \text{ (at 24)}$$

Let's suppose we know the values up to 20000. Then for $a\cdot b\cdot c\cdot d\le n$,

$$\sum_{a=2}\sum_{b=a+1}\sum_{c=a+1}\sum_{d=20001}^{\lfloor\frac{n}{abc}\rfloor}1$$

$$\sum_{a=2}\sum_{b=a+1}\sum_{c=b+1}\sum_{d=c+1}^{\lfloor\frac{n}{abc}\rfloor}1$$

$$\sum_{a=2}\sum_{b=a+1}\sum_{c=b+1}^{\lfloor(\frac{n}{ab})^{\frac{1}{2}}\rfloor}\lfloor\frac{n}{abc}\rfloor-c$$

$$\sum_{a=2}\sum_{b=a+1}\sum_{c=b+1}^{\lfloor(\frac{n}{ab})^{\frac{1}{2}}\rfloor}\lfloor\frac{n}{abc}\rfloor-\sum_{a=2}\sum_{b=a+1}\sum_{c=b+1}^{\lfloor(\frac{n}{ab})^{\frac{1}{2}}\rfloor}c$$

$$\sum_{a=2}\sum_{b=a+1}\sum_{c=b+1}^{\lfloor(\frac{n}{ab})^{\frac{1}{2}}\rfloor}\lfloor\frac{n}{abc}\rfloor-\sum_{a=2}\sum_{b=a+1}^{\lfloor(\frac{n}{a})^{(\frac{1}{3})}\rfloor}..$$

$$D_{k,a}(n)=\sum_{j=1}^{k}\sum_{m=a}^{\lfloor n^{1/k}\rfloor}\binom{k}{j}D_{k-j,m+1}\left(\lfloor\frac{n}{m^j}\rfloor\right)$$

$$D_{1,a}(n)=n-a+1$$

$$D_{0,a}(n)=1$$

$$D_{3,2}(n)=\sum_{m=2}^{\lfloor n^{1/3}\rfloor}\binom{3}{3}D_{3-3,m+1}\left(\lfloor\frac{n}{m^3}\rfloor\right)+\binom{3}{2}D_{3-2,m+1}\left(\lfloor\frac{n}{m^2}\rfloor\right)+\binom{3}{1}D_{3-1,m+1}\left(\lfloor\frac{n}{m^1}\rfloor\right)$$

$$D_{3,2}(n)=\sum_{m=2}^{\lfloor n^{1/3}\rfloor}1\,D_{0,m+1}\left(\lfloor\frac{n}{m^3}\rfloor\right)+3\,D_{1,m+1}\left(\lfloor\frac{n}{m^2}\rfloor\right)+3\,D_{2,m+1}\left(\lfloor\frac{n}{m}\rfloor\right)$$

$$D_{3,2}(n)=\sum_{m=2}^{\lfloor n^{1/3}\rfloor}(1)+3\left(\left(\lfloor\frac{n}{m^2}\rfloor\right)-(m+1)+1\right)+3\left(D_{2,m+1}\left(\lfloor\frac{n}{m}\rfloor\right)\right)$$

$$D_{3,2}(n)=\sum_{m=2}^{\lfloor n^{1/3}\rfloor}1+3\lfloor\frac{n}{m^2}\rfloor-3\,m+3\,D_{2,m+1}\left(\lfloor\frac{n}{m}\rfloor\right)$$

$$D_{3,2}(n)=\frac{1}{2}\left(4-\lfloor n^{\frac{1}{3}}\rfloor-3\lfloor n^{\frac{1}{3}}\rfloor^2\right)+3\sum_{m=2}^{\lfloor n^{1/3}\rfloor}\lfloor\frac{n}{m^2}\rfloor+3\sum_{m=2}^{\lfloor n^{1/3}\rfloor}D_{2,m+1}\left(\lfloor\frac{n}{m}\rfloor\right)$$

$$D_{2,a}(n)=\sum_{m=a}^{\lfloor n^{1/2}\rfloor}\binom{2}{2}D_{2-2,m+1}\left(\lfloor\frac{n}{m^2}\rfloor\right)+\binom{2}{1}D_{2-1,m+1}\left(\lfloor\frac{n}{m}\rfloor\right)$$

$$D_{2,a}(n)=\sum_{m=a}^{\lfloor n^{1/2}\rfloor}1+2\,D_{1,m+1}\left(\lfloor\frac{n}{m}\rfloor\right)$$

$$D_{2,a}(n)=\sum_{m=a}^{\lfloor n^{1/2}\rfloor}1+2\left(\lfloor\frac{n}{m}\rfloor-(m+1)+1\right)$$

$$D_{2,a}(n)=\sum_{m=a}^{\lfloor n^{1/2}\rfloor}1+2\left(\lfloor\frac{n}{m}\rfloor-m\right)$$

$$D_{2,a}(n)=\sum_{m=a}^{\lfloor n^{1/2}\rfloor}1+2\lfloor\frac{n}{m}\rfloor-2\,m$$

$$D_{2,a}(n)=1-2\,a+a^2-\lfloor n^{\frac{1}{2}}\rfloor^2+2\sum_{m=a}^{\lfloor n^{1/2}\rfloor}\lfloor\frac{n}{m}\rfloor$$

$$D_{2,2}(n)=1-\lfloor n^{\frac{1}{2}}\rfloor^2+2\sum_{m=2}^{\lfloor n^{1/2}\rfloor}\lfloor\frac{n}{m}\rfloor$$

$$D_{2,a}(n)=1-2\,a+a^2-\lfloor n^{\frac{1}{2}}\rfloor^2+2\sum_{m=a}^{\lfloor n^{1/2}\rfloor}\lfloor\frac{n}{m}\rfloor$$

$$D_{3,2}(n)=\frac{1}{2}(4-\lfloor n^{\frac{1}{3}}\rfloor-3\lfloor n^{\frac{1}{3}}\rfloor^2)+3\sum_{m=2}^{\lfloor n^{1/3}\rfloor}\lfloor\frac{n}{m^2}\rfloor+3\sum_{m=2}^{\lfloor n^{1/3}\rfloor}(1-2(m+1)+(m+1)^2-\lfloor\lfloor\frac{n}{m}\rfloor^{\frac{1}{2}}\rfloor^2+2\sum_{j=m}^{\lfloor\lfloor\frac{n}{m}\rfloor^{1/2}\rfloor}\lfloor\frac{\lfloor\frac{n}{m}\rfloor}{j}\rfloor)$$

$$D_{3,2}(n)=\frac{1}{2}(4-\lfloor n^{\frac{1}{3}}\rfloor-3\lfloor n^{\frac{1}{3}}\rfloor^2)+3\sum_{m=2}^{\lfloor n^{1/3}\rfloor}\lfloor\frac{n}{m^2}\rfloor+3\sum_{m=2}^{\lfloor n^{1/3}\rfloor}(m^2-\lfloor\lfloor\frac{n}{m}\rfloor^{\frac{1}{2}}\rfloor^2+2\sum_{j=m}^{\lfloor\lfloor\frac{n}{m}\rfloor^{1/2}\rfloor}\lfloor\frac{n}{mj}\rfloor)$$

$$D_{3,2}(n)=\frac{1}{2}(4-\lfloor n^{\frac{1}{3}}\rfloor-3\lfloor n^{\frac{1}{3}}\rfloor^2)+3\sum_{m=2}^{\lfloor n^{1/3}\rfloor}\lfloor\frac{n}{m^2}\rfloor+3\sum_{m=2}^{\lfloor n^{1/3}\rfloor}m^2-3\sum_{m=2}^{\lfloor n^{1/3}\rfloor}\lfloor\lfloor\frac{n}{m}\rfloor^{\frac{1}{2}}\rfloor^2+3\sum_{m=2}^{\lfloor n^{1/3}\rfloor}2\sum_{j=m}^{\lfloor\lfloor\frac{n}{m}\rfloor^{1/2}\rfloor}\lfloor\frac{n}{mj}\rfloor$$

$$D_{3,2}(n)=\frac{1}{2}(4-\lfloor n^{\frac{1}{3}}\rfloor-3\lfloor n^{\frac{1}{3}}\rfloor^2)+\frac{1}{2}(-6+\lfloor n^{\frac{1}{3}}\rfloor+3\lfloor n^{\frac{1}{3}}\rfloor^2+\lfloor n^{\frac{1}{3}}\rfloor^3)+$$
$$+3\sum_{m=2}^{\lfloor n^{1/3}\rfloor}\lfloor\frac{n}{m^2}\rfloor-3\sum_{m=2}^{\lfloor n^{1/3}\rfloor}\lfloor\lfloor\frac{n}{m}\rfloor^{\frac{1}{2}}\rfloor^2+6\sum_{m=2}^{\lfloor n^{1/3}\rfloor}\sum_{j=m}^{\lfloor\lfloor\frac{n}{m}\rfloor^{1/2}\rfloor}\lfloor\frac{n}{mj}\rfloor$$

$$D_{3,2}(n)=\frac{1}{2}(4-\lfloor n^{\frac{1}{3}}\rfloor-3\lfloor n^{\frac{1}{3}}\rfloor^2)+\frac{1}{2}(-6+\lfloor n^{\frac{1}{3}}\rfloor+3\lfloor n^{\frac{1}{3}}\rfloor^2+\lfloor n^{\frac{1}{3}}\rfloor^3)+3\sum_{m=2}^{\lfloor n^{1/3}\rfloor}\lfloor\frac{n}{m^2}\rfloor-\lfloor\lfloor\frac{n}{m}\rfloor^{\frac{1}{2}}\rfloor^2+2\sum_{j=m}^{\lfloor\lfloor\frac{n}{m}\rfloor^{1/2}\rfloor}\lfloor\frac{n}{mj}\rfloor$$

$$D_{2,2}(n)=1-\lfloor n^{\frac{1}{2}}\rfloor^2+2\sum_{m=2}^{\lfloor n^{1/2}\rfloor}\lfloor\frac{n}{m}\rfloor$$

$$D_{3,2}(n)=-1+\frac{\lfloor n^{\frac{1}{3}}\rfloor^3}{2}+3\sum_{m=2}^{\lfloor n^{1/3}\rfloor}\lfloor\frac{n}{m^2}\rfloor-\lfloor\lfloor\frac{n}{m}\rfloor^{\frac{1}{2}}\rfloor^2+2\sum_{j=m}^{\lfloor\lfloor\frac{n}{m}\rfloor^{1/2}\rfloor}\lfloor\frac{n}{mj}\rfloor$$

$$D_{2,a}(n)=\sum_{m=a}^{\lfloor n^{1/2}\rfloor} 1+2\lfloor\frac{n}{m}\rfloor-2m$$

$$D_{3,a}(n)=\sum_{m=a}^{\lfloor n^{1/3}\rfloor}\binom{3}{3}D_{3-3,m+1}(\lfloor\frac{n}{m^3}\rfloor)+\binom{3}{2}D_{3-2,m+1}(\lfloor\frac{n}{m^2}\rfloor)+\binom{3}{1}D_{3-1,m+1}(\lfloor\frac{n}{m^1}\rfloor)$$

$$D_{3,a}(n)=\sum_{m=a}^{\lfloor n^{1/3}\rfloor} 1+3(\lfloor\frac{n}{m^2}\rfloor-m)+3D_{2,m+1}(\lfloor\frac{n}{m}\rfloor)$$

$$D_{3,a}(n)=\sum_{m=a}^{\lfloor n^{1/3}\rfloor} 1+3(\lfloor\frac{n}{m^2}\rfloor-m)+3(\sum_{j=m+1}^{\lfloor\lfloor\frac{n}{m}\rfloor^{1/2}\rfloor} 1+2\lfloor\frac{\lfloor\frac{n}{m}\rfloor}{j}\rfloor-2j)$$