```
Clear[pp, pe, pa, ppo, ppr, pwe]
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, z_] := dz[n, z] = Product[(-1)^p[[2]] bin[-z, p[[2]]], {p, FI[n]}]
bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k!
pp[n_, k_] := pp[n, k] = Sum[ PartitionsP[j] pp[n - j, k - 1], {j, 1, n - 1}]
pp[n_, 1] := PartitionsP[n]
pe[n_, k_] := pe[n, k] = Sum[ DivisorSigma[1, j] / j pe[n - j, k - 1], {j, 1, n - 1}]
pe[n_, 1] := DivisorSigma[1, n] / n
pe[n_, 0] := 0
pa[z_, 0] := 1
pa[n_, z_] := Sum[ z^k / k! pe[n, k], {k, 0, n}]
pss[n_, z_] := Sum[ bin[z, k] pp[n, k], {k, 0, n}]
ppe[n_, k_] :=
 ppe[n, k] = Sum[ (-1)^(j + 1) DivisorSigma[1, j] / j ppe[n - j, k - 1], {j, 1, n - 1}]
ppe[n_, 1] := (-1)^(n + 1) DivisorSigma[1, n] / n
ppe[n_, 0] := 0
ppa[n_, z_] := Sum[ z^k / k! ppe[n, k], {k, 0, n}]
ppa[n_, 0] := 1
ppa[0, z_] := 1
ppo[n_, k_] := ppo[n, k] = Sum[ (-1)^(j + 1) / j ppo[n - j, k - 1], {j, 1, n - 1}]
ppo[n_, 1] := (-1)^(n + 1) / n
ppo[n_, 0] := 0
ppu[n_, z_] := Sum[ z^k / k! ppo[n, k], {k, 0, n}]
ppu[0, z_] := 1
ppr[n_, k_] := ppr[n, k] = Sum[ 1 / j ppr[n - j, k - 1], {j, 1, n - 1}]
ppr[n_, 1] := 1 / n
ppr[n_, 0] := 0
ppt[n_, z_] := Sum[ z^k / k! ppr[n, k], {k, 0, n}]
ppt[0, z_] := 1

pwe[n_, k_] := pe[n, k] = Sum[ (D[dz[j, z], z] /. z → 0) / j pwe[n - j, k - 1], {j, 1, n - 1}]
pwe[n_, 1] := (D[dz[n, z], z] /. z → 0) / n
pwe[n_, 0] := 0
pwa[z_, 0] := 1
pwa[0, n_] := 1
pwa[n_, z_] := Sum[ z^k / k! pwe[n, k], {k, 0, n}]
```

```
Table[pa[n, 1], {n, 1, 10}]
```

```
{1, 2, 3, 5, 7, 11, 15, 22, 30, 42}
```

```
Table[ppa[j, k], {k, 0, 8}, {j, 0, k}] // TableForm
```

```
1
1   1
1   2   -1
1   3   0    -5
1   4   2    -8    -5
1   5   5    -10   -15   6
1   6   9    -10   -30   0     11
1   7   14   -7    -49   -21   35   -41
1   8   20   0     -70   -64   56   0     -125
```

```
Table[ppu[j, k], {k, 0, 8}, {j, 0, k}] // TableForm
```

```
1
1   1
1   2   1
1   3   3    1
1   4   6    4    1
1   5   10   10   5    1
1   6   15   20   15   6    1
1   7   21   35   35   21   7    1
1   8   28   56   70   56   28   8   1
```

```
ppu[3, 10]
```

```
120
```

```
2492 + 1547 + 490
```

```
4529
```

```
Table[pe[k, j], {k, 0, 8}, {j, 0, k + 1}] // TableForm
```

```
Power::infy : Infinite expression 1/0 encountered. >>

0   ComplexInfinity
0   1                 0
0   3/2               1       0
0   4/3               3       1       0
0   7/4               59/12   9/2     1       0
0   6/5               15/2    43/4    6       1       0
0   2                 1697/180  165/8   113/6   15/2    1       0
0   8/7               184/15  2021/60 89/2    175/6   9       1       0
0   15/8              8147/560  4049/80  21127/240  165/2   167/4   21/2    1   0
```

```
Table[ppe[k, j], {k, 0, 8}, {j, 0, k + 1}] // TableForm
```

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

0	ComplexInfinity							
0	1	0						
0	$-\frac{3}{2}$	1	0					
0	$\frac{4}{3}$	-3	1	0				
0	$-\frac{7}{4}$	$\frac{59}{12}$	$-\frac{9}{2}$	1	0			
0	$\frac{6}{5}$	$-\frac{15}{2}$	$\frac{43}{4}$	-6	1	0		
0	-2	$\frac{1697}{180}$	$-\frac{165}{8}$	$\frac{113}{6}$	$-\frac{15}{2}$	1	0	
0	$\frac{8}{7}$	$-\frac{184}{15}$	$\frac{2021}{60}$	$-\frac{89}{2}$	$\frac{175}{6}$	-9	1	0
0	$-\frac{15}{8}$	$\frac{8147}{560}$	$-\frac{4049}{80}$	$\frac{21\,127}{240}$	$-\frac{165}{2}$	$\frac{167}{4}$	$-\frac{21}{2}$	1	0

```
Table[ppo[k, j], {k, 0, 8}, {j, 0, k + 1}] // TableForm
```

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

0	ComplexInfinity							
0	1	0						
0	$-\frac{1}{2}$	1	0					
0	$\frac{1}{3}$	-1	1	0				
0	$-\frac{1}{4}$	$\frac{11}{12}$	$-\frac{3}{2}$	1	0			
0	$\frac{1}{5}$	$-\frac{5}{6}$	$\frac{7}{4}$	-2	1	0		
0	$-\frac{1}{6}$	$\frac{137}{180}$	$-\frac{15}{8}$	$\frac{17}{6}$	$-\frac{5}{2}$	1	0	
0	$\frac{1}{7}$	$-\frac{7}{10}$	$\frac{29}{15}$	$-\frac{7}{2}$	$\frac{25}{6}$	-3	1	0
0	$-\frac{1}{8}$	$\frac{363}{560}$	$-\frac{469}{240}$	$\frac{967}{240}$	$-\frac{35}{6}$	$\frac{23}{4}$	$-\frac{7}{2}$	1	0

```
Table[ppr[k, j], {k, 0, 8}, {j, 0, k + 1}] // TableForm
```

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

0	ComplexInfinity							
0	1	0						
0	$\frac{1}{2}$	1	0					
0	$\frac{1}{3}$	1	1	0				
0	$\frac{1}{4}$	$\frac{11}{12}$	$\frac{3}{2}$	1	0			
0	$\frac{1}{5}$	$\frac{5}{6}$	$\frac{7}{4}$	2	1	0		
0	$\frac{1}{6}$	$\frac{137}{180}$	$\frac{15}{8}$	$\frac{17}{6}$	$\frac{5}{2}$	1	0	
0	$\frac{1}{7}$	$\frac{7}{10}$	$\frac{29}{15}$	$\frac{7}{2}$	$\frac{25}{6}$	3	1	0
0	$\frac{1}{8}$	$\frac{363}{560}$	$\frac{469}{240}$	$\frac{967}{240}$	$\frac{35}{6}$	$\frac{23}{4}$	$\frac{7}{2}$	1	0

```
11 / 12 - 1 / 3
```

$$\frac{7}{12}$$

```
7 / 4 - 11 / 12
```

$$\frac{5}{6}$$

```
17 / 6 - 7 / 4
```

$$\frac{13}{12}$$

```
25 / 6 - 17 / 6
```

$$\frac{4}{3}$$

```
Table[ ppr[k + 2, k], {k, 0, 10}]
```

$$\left\{0, \frac{1}{3}, \frac{11}{12}, \frac{7}{4}, \frac{17}{6}, \frac{25}{6}, \frac{23}{4}, \frac{91}{12}, \frac{29}{3}, 12, \frac{175}{12}\right\}$$

```
Table[( ppr[k + 2, k] - ppr[k + 1, k - 1]) - (3 k + 1) / 12, {k, 1, 10}]
```

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$$

```
Table[( ppr[k + 3, k] - ppr[k + 2, k - 1]) 24, {k, 1, 10}]
```

$$\{6, 14, 25, 39, 56, 76, 99, 125, 154, 186\}$$

```
Table[( ppr[k + 1, k] - ppr[k, k - 1]) - 1 / 2, {k, 1, 10}]
```

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$$

```
Table[pwa[j, k], {k, 0, 10}, {j, 0, k}] // TableForm
```

1										
1	1									
1	0	1								
1	0	$\frac{3}{2}$	1							
1	0	2	$\frac{4}{3}$	$\frac{5}{2}$						
1	0	$\frac{5}{2}$	$\frac{5}{3}$	$\frac{15}{4}$	$\frac{31}{6}$					
1	0	3	2	$\frac{21}{4}$	$\frac{36}{5}$	$\frac{35}{4}$				
1	0	$\frac{7}{2}$	$\frac{7}{3}$	7	$\frac{287}{30}$	$\frac{931}{72}$	$\frac{667}{30}$			
1	0	4	$\frac{8}{3}$	9	$\frac{184}{15}$	$\frac{164}{9}$	$\frac{1104}{35}$	$\frac{3419}{90}$		
1	0	$\frac{9}{2}$	3	$\frac{45}{4}$	$\frac{153}{10}$	$\frac{99}{4}$	$\frac{6039}{140}$	$\frac{17\,643}{320}$	$\frac{1285}{14}$	
1	0	5	$\frac{10}{3}$	$\frac{55}{4}$	$\frac{56}{3}$	$\frac{1175}{36}$	$\frac{2405}{42}$	$\frac{22\,265}{288}$	$\frac{74\,645}{567}$	$\frac{351\,907}{2016}$

```
Table[ {(-1)^(5 + j) ppr[5, j], ppo[5, j]}, {j, 0, 5}]
```

$$\left\{\{0, 0\}, \left\{\frac{1}{5}, \frac{1}{5}\right\}, \left\{-\frac{5}{6}, -\frac{5}{6}\right\}, \left\{\frac{7}{4}, \frac{7}{4}\right\}, \{-2, -2\}, \{1, 1\}\right\}$$

```
Table[ {(-1)^(4 + j) ppr[4, j], ppo[4, j]}, {j, 0, 5}]
```

$$\left\{\{0, 0\}, \left\{-\frac{1}{4}, -\frac{1}{4}\right\}, \left\{\frac{11}{12}, \frac{11}{12}\right\}, \left\{-\frac{3}{2}, -\frac{3}{2}\right\}, \{1, 1\}, \{0, 0\}\right\}$$

```
ppu[4, z]
```

$$-\frac{z}{4}+\frac{11\,z^2}{24}-\frac{z^3}{4}+\frac{z^4}{24}$$

```
(-1)^4 ppt[4, -z]
```

$$-\frac{z}{4}+\frac{11\,z^2}{24}-\frac{z^3}{4}+\frac{z^4}{24}$$