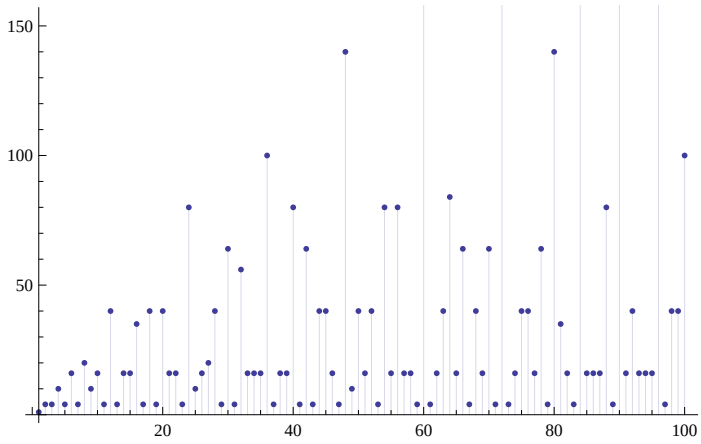
```
ClearAll["Global`*"]


tk[n_, k_, a_] :=
 tk[n, k, a] = Sum[ tk[n / j, k - 1, a], {j, 2, n}] - a Sum[ tk[n / (a j), k - 1, a], {j, 1, n / a}];
tk[n_, 0, a_] := 1

tk1b[n_, k_, a_] := Sum[t[j, a] tk1b[n / j, k - 1, a], {j, 1, n}]; tk1b[n_, 0, a_] := 1

tk1[n_, k_, a_] :=
 tk1[n, k, a] = Sum[ tk1[n / j, k - 1, a], {j, 1, n}] - a Sum[ tk1[n / (a j), k - 1, a], {j, 1, n / a}];
tk1[n_, 0, a_] := 1


tki[n_, k_, k2_, a_] := tki[n, k, k2, a] = Sum[ tki[n / j, k - 1, k2, a], {j, 2, n}]
tki[n_, 0, k2_, a_] := tkp[n, k2, a]
tkp[n_, k_, a_] := tkp[n, k, a] = Sum[ tkp[n / (a j), k - 1, a], {j, 1, n / a}]
tkp[n_, 0, a_] := 1
tk2[n_, k_, a_] :=
 tk2[n, k, a] = Sum[ (-1)^j a^j Binomial[k, j] tki[n, k - j, j, a], {j, 0, k}]
tk2s[n_, k_, j_, a_] := (-1)^j a^j Binomial[k, j] tki[n, k - j, j, a]
tk2s2[n_, k_, j_, a_] := tki[n, k - j, j, a]

tk2a[n_, k_, a_] := tk2a[n, k, a] = Sum[(-1)^j Binomial[k, j] tk1[n, k - j, a], {j, 0, k}]

tk1z[n_, z_, b_] := Sum[FactorialPower[z, j] / j! tk2[n, j, b], {j, 0, Log[If[b ≤ 2, b, 2], n]}]

D1[n_, k_] := Sum[D1[Floor[n / j], k - 1], {j, 1, n}]; D1[n_, 0] := 1
Lina[n_, a_] := Sum[ ((-1)^(k + 1) tk2[n, k, a] + 1) / k + (a^k - 1) / k, {k, 1, Log[a, n]}]
Linb[n_, a_] :=
 Sum[ (-1)^(k + 1) / k tk[n, k, a], {k, 1, Log[2, n]}] + Sum[ a^k / k, {k, 1, Log[a, n]}]
Lin2[n_, a_] := If[a >= 2, Linb[n, a], Lina[n, a]]

d[n_, z_] := Product[Pochhammer[z, a = p[[2]]] / a!, {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}

d2[n_, k_] := Sum[d2[j, k - 1] d2[n / j, 1], {j, Divisors[n]}];
d2[n_, 1] := 1; d2[1, 1] := 0; d2[n_, 0] := 0; d2[1, 0] := 1

DD[n_, z_] := Sum[d[j, z], {j, 1, n}]

st[n_, a_, k_] := If[tk2[a, n, k] ≠ 0, (-1)^(n + 1) / n tk2[a, n, k] + 1 / n, 0]
```

```
DiscretePlot[{tk1[n, 4, 1.0000001]}, {n, 1, 100}]
```




```
Table[{n, d[n, 4], tk1[n, 4, 1.0000001]}, {n, 1, 100}] // TableForm
```

1	1	1.
2	4	4.
3	4	4.
4	10	10.
5	4	4.
6	16	16.
7	4	4.
8	20	20.
9	10	10.
10	16	16.
11	4	4.
12	40	40.
13	4	4.
14	16	16.
15	16	16.
16	35	35.
17	4	4.
18	40	40.
19	4	4.
20	40	40.
21	16	16.
22	16	16.
23	4	4.
24	80	80.
25	10	10.
26	16	16.
27	20	20.
28	40	40.
29	4	4.
30	64	64.
31	4	4.
32	56	56.
33	16	16.
34	16	16.
35	16	16.
36	100	100.
37	4	4.
38	16	16.

```
39    16    16.
40    80    80.
41    4     4.
42    64    64.
43    4     4.
44    40    40.
45    40    40.
46    16    16.
47    4     4.
48    140   140.
49    10    10.
50    40    40.
51    16    16.
52    40    40.
53    4     4.
54    80    80.
55    16    16.
56    80    80.
57    16    16.
58    16    16.
59    4     4.
60    160   160.
61    4     4.
62    16    16.
63    40    40.
64    84    84.
65    16    16.
66    64    64.
67    4     4.
68    40    40.
69    16    16.
70    64    64.
71    4     4.
72    200   200.
73    4     4.
74    16    16.
75    40    40.
76    40    40.
77    16    16.
78    64    64.
79    4     4.
80    140   140.
81    35    35.
82    16    16.
83    4     4.
84    160   160.
85    16    16.
86    16    16.
87    16    16.
88    80    80.
89    4     4.
90    160   160.
91    16    16.
92    40    40.
93    16    16.
94    16    16.
```

```
95    16    16.
96    224   224.
97    4     4.
98    40    40.
99    40    40.
100   100   100.
```

```
Table[{n, d2[n, 4] + 1, tk[n, 4, 1.0000001]}, {n, 1, 100}] // TableForm
```

```
1     1     0.
2     1     1.
3     1     1.
4     1     1.
5     1     1.
6     1     1.
7     1     1.
8     1     1.
9     1     1.
10    1     1.
11    1     1.
12    1     1.
13    1     1.
14    1     1.00001
15    1     1.00001
16    2     2.00001
17    1     1.00001
18    1     1.00001
19    1     1.00001
20    1     1.00001
21    1     1.00001
22    1     1.00001
23    1     1.00001
24    5     5.00001
25    1     1.00001
26    1     1.00001
27    1     1.00001
28    1     1.00001
29    1     1.00001
30    1     1.00001
31    1     1.00001
32    5     5.00001
33    1     1.00001
34    1     1.00001
35    1     1.00001
36    7     7.00001
37    1     1.00001
38    1     1.00001
39    1     1.00002
40    5     5.00002
41    1     1.00002
42    1     1.00002
43    1     1.00002
44    1     1.00002
45    1     1.00002
46    1     1.00002
47    1     1.00002
48    17    17.
```

```
49    1    1.00002
50    1    1.00002
51    1    1.00002
52    1    1.00002
53    1    1.00002
54    5    5.00002
55    1    1.00002
56    5    5.00002
57    1    1.00002
58    1    1.00002
59    1    1.00002
60    13   13.
61    1    1.00002
62    1    1.00002
63    1    1.00002
64    11   11.
65    1    1.00003
66    1    1.00003
67    1    1.00003
68    1    1.00003
69    1    1.00003
70    1    1.00003
71    1    1.00003
72    29   29.
73    1    1.00003
74    1    1.00003
75    1    1.00003
76    1    1.00003
77    1    1.00003
78    1    1.00003
79    1    1.00003
80    17   17.
81    2    2.00003
82    1    1.00003
83    1    1.00003
84    13   13.
85    1    1.00003
86    1    1.00003
87    1    1.00003
88    5    5.00003
89    1    1.00004
90    13   13.
91    1    1.00004
92    1    1.00004
93    1    1.00004
94    1    1.00004
95    1    1.00004
96    41   41.
97    1    1.00004
98    1    1.00004
99    1    1.00004
100   7    7.00004
```

```
Table[{n, d2[n, 3] - 1, tk[n, 3, 1.0000001]}, {n, 1, 100}] // TableForm
```

```
1     -1    0.
2     -1    -1.
3     -1    -1.
```

4	−1	−1.
5	−1	−1.
6	−1	−1.
7	−1	−1.
8	0	-2.1×10^{-6}
9	−1	−1.
10	−1	−1.
11	−1	−1.
12	2	2.
13	−1	−1.
14	−1	−1.
15	−1	−1.
16	2	2.
17	−1	−1.
18	2	1.99999
19	−1	−1.00001
20	2	1.99999
21	−1	−1.00001
22	−1	−1.00001
23	−1	−1.00001
24	8	7.99999
25	−1	−1.00001
26	−1	−1.00001
27	0	-7.8×10^{-6}
28	2	1.99999
29	−1	−1.00001
30	5	4.99999
31	−1	−1.00001
32	5	4.99999
33	−1	−1.00001
34	−1	−1.00001
35	−1	−1.00001
36	11	11.
37	−1	−1.00001
38	−1	−1.00001
39	−1	−1.00001
40	8	7.99999
41	−1	−1.00001
42	5	4.99999
43	−1	−1.00001
44	2	1.99999
45	2	1.99999
46	−1	−1.00001
47	−1	−1.00001
48	17	17.
49	−1	−1.00001
50	2	1.99999
51	−1	−1.00002
52	2	1.99998
53	−1	−1.00002
54	8	7.99998
55	−1	−1.00002
56	8	7.99998
57	−1	−1.00002
58	−1	−1.00002
59	−1	−1.00002

```
60    20    20.
61    -1    -1.00002
62    -1    -1.00002
63    2     1.99998
64    9     8.99998
65    -1    -1.00002
66    5     4.99998
67    -1    -1.00002
68    2     1.99998
69    -1    -1.00002
70    5     4.99998
71    -1    -1.00002
72    26    26.
73    -1    -1.00002
74    -1    -1.00002
75    2     1.99998
76    2     1.99998
77    -1    -1.00002
78    5     4.99998
79    -1    -1.00002
80    17    17.
81    2     1.99998
82    -1    -1.00002
83    -1    -1.00002
84    20    20.
85    -1    -1.00003
86    -1    -1.00003
87    -1    -1.00003
88    8     7.99997
89    -1    -1.00003
90    20    20.
91    -1    -1.00003
92    2     1.99997
93    -1    -1.00003
94    -1    -1.00003
95    -1    -1.00003
96    29    29.
97    -1    -1.00003
98    2     1.99997
99    2     1.99997
100   11    11.
```

```
Table[{n, d2[n, 8] + 1, tk[n, 8, 1.0000001]}, {n, 1, 100}] // TableForm
```

```
1     1    0.
2     1    1.
3     1    1.
4     1    1.
5     1    1.
6     1    1.
7     1    1.
8     1    1.00001
9     1    1.00001
10    1    1.00001
11    1    1.00001
12    1    1.00001
13    1    1.00001
```

```
14    1    1.00001
15    1    1.00001
16    1    1.00001
17    1    1.00001
18    1    1.00001
19    1    1.00001
20    1    1.00002
21    1    1.00002
22    1    1.00002
23    1    1.00002
24    1    1.00002
25    1    1.00002
26    1    1.00002
27    1    1.00002
28    1    1.00002
29    1    1.00002
30    1    1.00002
31    1    1.00002
32    1    1.00002
33    1    1.00003
34    1    1.00003
35    1    1.00003
36    1    1.00003
37    1    1.00003
38    1    1.00003
39    1    1.00003
40    1    1.00003
41    1    1.00003
42    1    1.00003
43    1    1.00003
44    1    1.00003
45    1    1.00004
46    1    1.00004
47    1    1.00004
48    1    1.00004
49    1    1.00004
50    1    1.00004
51    1    1.00004
52    1    1.00004
53    1    1.00004
54    1    1.00004
55    1    1.00004
56    1    1.00004
57    1    1.00004
58    1    1.00005
59    1    1.00005
60    1    1.00005
61    1    1.00005
62    1    1.00005
63    1    1.00005
64    1    1.00005
65    1    1.00005
66    1    1.00005
67    1    1.00005
68    1    1.00005
69    1    1.00005
```

70	1	1.00006
71	1	1.00006
72	1	1.00006
73	1	1.00006
74	1	1.00006
75	1	1.00006
76	1	1.00006
77	1	1.00006
78	1	1.00006
79	1	1.00006
80	1	1.00006
81	1	1.00006
82	1	1.00006
83	1	1.00007
84	1	1.00007
85	1	1.00007
86	1	1.00007
87	1	1.00007
88	1	1.00007
89	1	1.00007
90	1	1.00007
91	1	1.00007
92	1	1.00007
93	1	1.00007
94	1	1.00007
95	1	1.00008
96	1	1.00008
97	1	1.00008
98	1	1.00008
99	1	1.00008
100	1	1.00008

```
Table[{n, aa = (tk1[n, 1, 1.1] - tk1[n - 1, 1, 1.1]),
   bb = (Mod[n + .0001, 1.1] - Mod[n + .0001 - 1, 1.1]), aa - bb}, {n, 2, 120}] // TableForm
```

2	−0.1	−0.1	0.
3	−0.1	−0.1	0.
4	−0.1	−0.1	0.
5	−0.1	−0.1	0.
6	−0.1	−0.1	4.44089×10^{-16}
7	−0.1	−0.1	-4.44089×10^{-16}
8	−0.1	−0.1	-4.44089×10^{-16}
9	−0.1	−0.1	4.44089×10^{-16}
10	−0.1	−0.1	4.44089×10^{-16}
11	−0.1	−0.1	4.44089×10^{-16}
12	1.	1.	0.
13	−0.1	−0.1	-1.33227×10^{-15}
14	−0.1	−0.1	4.44089×10^{-16}
15	−0.1	−0.1	4.44089×10^{-16}
16	−0.1	−0.1	-1.33227×10^{-15}
17	−0.1	−0.1	2.22045×10^{-15}
18	−0.1	−0.1	-1.33227×10^{-15}
19	−0.1	−0.1	-1.33227×10^{-15}
20	−0.1	−0.1	2.22045×10^{-15}
21	−0.1	−0.1	-1.33227×10^{-15}

22	−0.1	−0.1	2.22045×10^{-15}
23	1.	1.	0.
24	−0.1	−0.1	-1.33227×10^{-15}
25	−0.1	−0.1	-1.33227×10^{-15}
26	−0.1	−0.1	2.22045×10^{-15}
27	−0.1	−0.1	-1.33227×10^{-15}
28	−0.1	−0.1	-1.33227×10^{-15}
29	−0.1	−0.1	2.22045×10^{-15}
30	−0.1	−0.1	-1.33227×10^{-15}
31	−0.1	−0.1	-1.33227×10^{-15}
32	−0.1	−0.1	2.22045×10^{-15}
33	−0.1	−0.1	2.22045×10^{-15}
34	1.	1.	0.
35	−0.1	−0.1	-1.33227×10^{-15}
36	−0.1	−0.1	-1.33227×10^{-15}
37	−0.1	−0.1	-1.33227×10^{-15}
38	−0.1	−0.1	-1.33227×10^{-15}
39	−0.1	−0.1	5.77316×10^{-15}
40	−0.1	−0.1	-1.33227×10^{-15}
41	−0.1	−0.1	-1.33227×10^{-15}
42	−0.1	−0.1	-1.33227×10^{-15}
43	−0.1	−0.1	-1.33227×10^{-15}
44	−0.1	−0.1	5.77316×10^{-15}
45	1.	1.	0.
46	−0.1	−0.1	-1.33227×10^{-15}
47	−0.1	−0.1	-1.33227×10^{-15}
48	−0.1	−0.1	-1.33227×10^{-15}
49	−0.1	−0.1	-1.33227×10^{-15}
50	−0.1	−0.1	-1.33227×10^{-15}
51	−0.1	−0.1	5.77316×10^{-15}
52	−0.1	−0.1	-1.33227×10^{-15}
53	−0.1	−0.1	-1.33227×10^{-15}
54	−0.1	−0.1	-1.33227×10^{-15}
55	−0.1	−0.1	-1.33227×10^{-15}
56	1.	1.	0.
57	−0.1	−0.1	5.77316×10^{-15}
58	−0.1	−0.1	-1.33227×10^{-15}
59	−0.1	−0.1	-1.33227×10^{-15}
60	−0.1	−0.1	-1.33227×10^{-15}
61	−0.1	−0.1	-1.33227×10^{-15}
62	−0.1	−0.1	-1.33227×10^{-15}
63	−0.1	−0.1	5.77316×10^{-15}
64	−0.1	−0.1	-1.33227×10^{-15}
65	−0.1	−0.1	-1.33227×10^{-15}
66	−0.1	−0.1	5.77316×10^{-15}
67	1.	1.	0.
68	−0.1	−0.1	-8.43769×10^{-15}
69	−0.1	−0.1	5.77316×10^{-15}

70	−0.1	−0.1	-8.43769×10^{-15}
71	−0.1	−0.1	5.77316×10^{-15}
72	−0.1	−0.1	5.77316×10^{-15}
73	−0.1	−0.1	-8.43769×10^{-15}
74	−0.1	−0.1	5.77316×10^{-15}
75	−0.1	−0.1	-8.43769×10^{-15}
76	−0.1	−0.1	5.77316×10^{-15}
77	−0.1	−0.1	5.77316×10^{-15}
78	1.	1.	0.
79	−0.1	−0.1	-8.43769×10^{-15}
80	−0.1	−0.1	5.77316×10^{-15}
81	−0.1	−0.1	-8.43769×10^{-15}
82	−0.1	−0.1	5.77316×10^{-15}
83	−0.1	−0.1	5.77316×10^{-15}
84	−0.1	−0.1	-8.43769×10^{-15}
85	−0.1	−0.1	5.77316×10^{-15}
86	−0.1	−0.1	-8.43769×10^{-15}
87	−0.1	−0.1	5.77316×10^{-15}
88	−0.1	−0.1	5.77316×10^{-15}
89	1.	1.	0.
90	−0.1	−0.1	-8.43769×10^{-15}
91	−0.1	−0.1	5.77316×10^{-15}
92	−0.1	−0.1	-8.43769×10^{-15}
93	−0.1	−0.1	5.77316×10^{-15}
94	−0.1	−0.1	-8.43769×10^{-15}
95	−0.1	−0.1	5.77316×10^{-15}
96	−0.1	−0.1	5.77316×10^{-15}
97	−0.1	−0.1	-8.43769×10^{-15}
98	−0.1	−0.1	5.77316×10^{-15}
99	−0.1	−0.1	-8.43769×10^{-15}
100	1.	1.	0.
101	−0.1	−0.1	5.77316×10^{-15}
102	−0.1	−0.1	5.77316×10^{-15}
103	−0.1	−0.1	-8.43769×10^{-15}
104	−0.1	−0.1	5.77316×10^{-15}
105	−0.1	−0.1	-8.43769×10^{-15}
106	−0.1	−0.1	5.77316×10^{-15}
107	−0.1	−0.1	5.77316×10^{-15}
108	−0.1	−0.1	-8.43769×10^{-15}
109	−0.1	−0.1	5.77316×10^{-15}
110	−0.1	−0.1	-8.43769×10^{-15}
111	1.	1.	0.
112	−0.1	−0.1	5.77316×10^{-15}
113	−0.1	−0.1	5.77316×10^{-15}
114	−0.1	−0.1	-8.43769×10^{-15}
115	−0.1	−0.1	5.77316×10^{-15}
116	−0.1	−0.1	-8.43769×10^{-15}
117	−0.1	−0.1	5.77316×10^{-15}

118	−0.1	−0.1	5.77316×10^{-15}
119	−0.1	−0.1	-8.43769×10^{-15}
120	−0.1	−0.1	5.77316×10^{-15}

```
Table[{n, tk[n, 2, 1.71], tk2[n, 2, 1.71], tk2a[n, 2, 1.71]}, {n, 1, 100}] // TableForm
```

1	0.	0.	0.
2	0.	0.	0.
3	2.9241	2.9241	2.9241
4	0.5041	0.5041	0.5041
5	0.5041	0.5041	0.5041
6	4.9323	4.9323	4.9323
7	−1.9077	−1.9077	−1.9077
8	0.0923	0.0923	0.0923
9	3.5205	3.5205	3.5205
10	5.5205	5.5205	5.5205
11	−4.7395	−4.7395	−4.7395
12	4.6128	4.6128	4.6128
13	4.6128	4.6128	4.6128
14	−3.6472	−3.6472	−3.6472
15	4.201	4.201	4.201
16	0.361	0.361	0.361
17	0.361	0.361	0.361
18	5.7974	5.7974	5.7974
19	2.3774	2.3774	2.3774
20	6.3774	6.3774	6.3774
21	−2.8744	−2.8744	−2.8744
22	−0.8744	−0.8744	−0.8744
23	−4.2944	−4.2944	−4.2944
24	3.142	3.142	3.142
25	4.142	4.142	4.142
26	−4.118	−4.118	−4.118
27	6.6543	6.6543	6.6543
28	−3.0257	−3.0257	−3.0257
29	−3.0257	−3.0257	−3.0257
30	11.2507	11.2507	11.2507
31	−5.8493	−5.8493	−5.8493
32	−1.8493	−1.8493	−1.8493
33	2.5789	2.5789	2.5789
34	4.5789	4.5789	4.5789
35	−10.5211	−10.5211	−10.5211
36	3.7635	3.7635	3.7635
37	3.7635	3.7635	3.7635
38	−4.4965	−4.4965	−4.4965
39	3.3517	3.3517	3.3517
40	5.9317	5.9317	5.9317
41	17.6281	17.6281	17.6281
42	−0.3119	−0.3119	−0.3119
43	−7.1519	−7.1519	−7.1519
44	8.5445	8.5445	8.5445
45	2.2845	2.2845	2.2845
46	4.2845	4.2845	4.2845
47	8.645	8.645	8.645
48	−0.455	−0.455	−0.455

49	0.545	0.545	0.545
50	6.9732	6.9732	6.9732
51	8.9732	8.9732	8.9732
52	−10.9668	−10.9668	−10.9668
53	6.5778	6.5778	6.5778
54	9.1578	9.1578	9.1578
55	−5.9422	−5.9422	−5.9422
56	5.906	5.906	5.906
57	−2.354	−2.354	−2.354
58	−0.354	−0.354	−0.354
59	6.9306	6.9306	6.9306
60	6.6706	6.6706	6.6706
61	6.6706	6.6706	6.6706
62	−6.993	−6.993	−6.993
63	−2.993	−2.993	−2.993
64	−1.413	−1.413	−1.413
65	2.0234	2.0234	2.0234
66	8.0234	8.0234	8.0234
67	−2.2366	−2.2366	−2.2366
68	7.6116	7.6116	7.6116
69	−14.3284	−14.3284	−14.3284
70	−8.3284	−8.3284	−8.3284
71	11.6444	11.6444	11.6444
72	−2.2956	−2.2956	−2.2956
73	−2.2956	−2.2956	−2.2956
74	5.0567	5.0567	5.0567
75	9.0567	9.0567	9.0567
76	−4.0433	−4.0433	−4.0433
77	−7.4469	−7.4469	−7.4469
78	−1.4469	−1.4469	−1.4469
79	−0.0105	−0.0105	−0.0105
80	7.9895	7.9895	7.9895
81	7.5695	7.5695	7.5695
82	27.1141	27.1141	27.1141
83	−3.6659	−3.6659	−3.6659
84	−0.5059	−0.5059	−0.5059
85	7.3423	7.3423	7.3423
86	−7.7577	−7.7577	−7.7577
87	−5.7577	−5.7577	−5.7577
88	13.3751	13.3751	13.3751
89	−3.7249	−3.7249	−3.7249
90	6.2751	6.2751	6.2751
91	10.7033	10.7033	10.7033
92	14.7033	14.7033	14.7033
93	−7.2367	−7.2367	−7.2367
94	12.3079	12.3079	12.3079
95	4.0479	4.0479	4.0479
96	−9.8921	−9.8921	−9.8921
97	1.8043	1.8043	1.8043
98	−4.4557	−4.4557	−4.4557
99	−0.4557	−0.4557	−0.4557
100	7.9807	7.9807	7.9807

```
Mod[110.0000000001, 1.1]
```

9.99929×10^{-11}

```
11 / 1.1
```

10.

```
tk1[110, 1, 1.1] - tk1[109, 1, 1.1]
```

-0.1

```
Mod[110, 1.1] - Mod[109, 1.1]
```

1.

```
t[n_, a_] := (Mod[n + .0001, a] - Mod[n + .0001 - 1, a])
```

```
Sum[ t[n, 2] / n, {n, 1, 50 000}]
```

0.693137

```
N[Log[2]]
```

0.693147

```
Sum[ t[n, 3] / n, {n, 1, 50 000}]
```

1.09863

```
N[Log[3]]
```

1.09861

```
Sum[ t[n, 1.1] / n, {n, 1, 50 000}]
```

0.846177

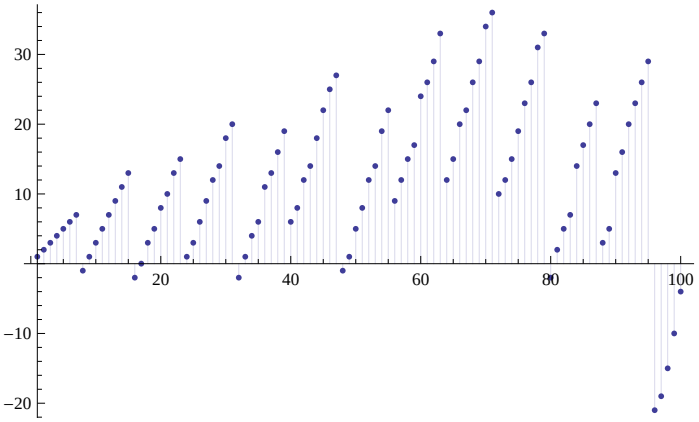
```
Table[ {n, aa = (tk1b[n, cc = 2, dc = 8.71] - tk1b[n - 1, cc, dc]),
    bb = (tk1[n, cc, dc] - tk1[n - 1, cc, dc]), aa - bb}, {n, 1, 100}] // TableForm
```

1	1.	1.	0.
2	1.	2.	-1.
3	1.	2.	-1.
4	1.	3.	-2.
5	1.	2.	-1.
6	1.	4.	-3.
7	1.	2.	-1.
8	1.	4.	-3.
9	-8.42	-14.42	6.
10	2.	4.	-2.
11	2.	2.	0.
12	2.	6.	-4.
13	2.	2.	0.
14	2.	4.	-2.
15	2.	4.	-2.
16	2.	5.	-3.
17	2.	2.	0.
18	-16.13	-28.84	12.71
19	2.	2.	0.
20	3.	6.	-3.
21	2.	4.	-2.
22	3.	4.	-1.
23	2.	2.	0.
24	3.	8.	-5.

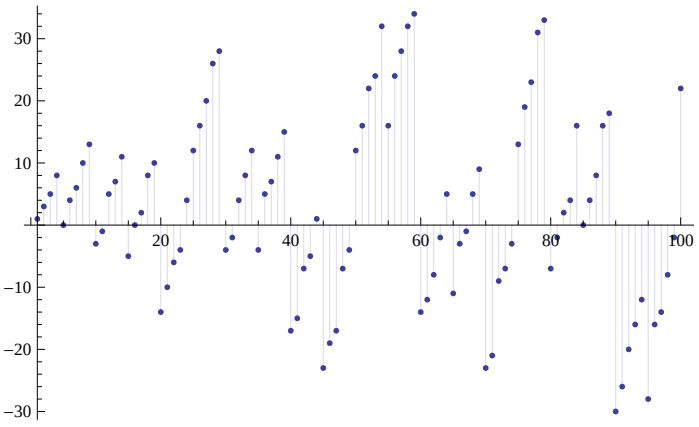
25	2.	3.	−1.
26	3.	4.	−1.
27	−16.13	−30.84	14.71
28	3.	6.	−3.
29	2.	2.	0.
30	4.	8.	−4.
31	2.	2.	0.
32	3.	6.	−3.
33	3.	4.	−1.
34	3.	4.	−1.
35	−25.84	−48.26	22.42
36	5.	9.	−4.
37	2.	2.	0.
38	3.	4.	−1.
39	3.	4.	−1.
40	4.	8.	−4.
41	2.	2.	0.
42	4.	8.	−4.
43	2.	2.	0.
44	−15.13	−28.84	13.71
45	4.	6.	−2.
46	3.	4.	−1.
47	2.	2.	0.
48	5.	10.	−5.
49	2.	3.	−1.
50	4.	6.	−2.
51	3.	4.	−1.
52	4.	6.	−2.
53	−34.55	−67.68	33.13
54	5.	8.	−3.
55	3.	4.	−1.
56	4.	8.	−4.
57	3.	4.	−1.
58	3.	4.	−1.
59	2.	2.	0.
60	7.	12.	−5.
61	−17.13	−32.84	15.71
62	3.	4.	−1.
63	4.	6.	−2.
64	4.	7.	−3.
65	3.	4.	−1.
66	5.	8.	−3.
67	2.	2.	0.
68	4.	6.	−2.
69	3.	4.	−1.
70	−31.55	−61.68	30.13
71	2.	2.	0.
72	7.	12.	−5.
73	2.	2.	0.
74	3.	4.	−1.
75	4.	6.	−2.
76	4.	81.8641	−77.8641
77	3.	4.	−1.
78	5.	8.	−3.
79	−10.9459	−50.26	39.3141
80	6.	10.	−4.

81	−4.71	5.	−9.71
82	3.	4.	−1.
83	2.	2.	0.
84	7.	12.	−5.
85	3.	4.	−1.
86	3.	4.	−1.
87	3.	4.	−1.
88	−31.55	−61.68	30.13
89	2.	2.	0.
90	−0.71	12.	−12.71
91	3.	4.	−1.
92	4.	6.	−2.
93	3.	4.	−1.
94	3.	4.	−1.
95	3.	4.	−1.
96	−12.13	−22.84	10.71
97	2.	2.	0.
98	4.	6.	−2.
99	−3.71	6.	−9.71
100	6.	9.	−3.

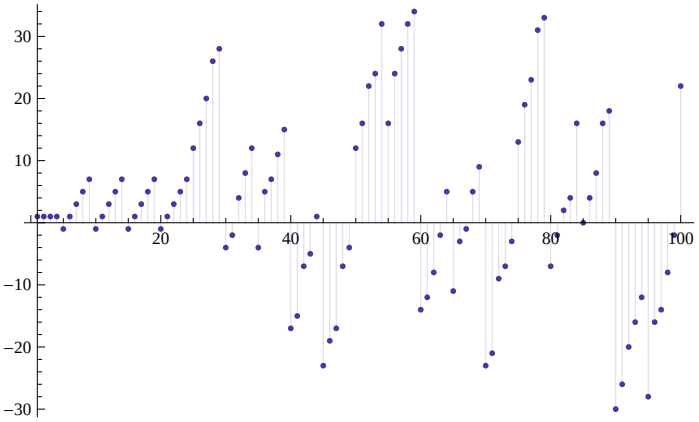
```
DiscretePlot[tk1b[n, 2, 8], {n, 1, 100}]
```



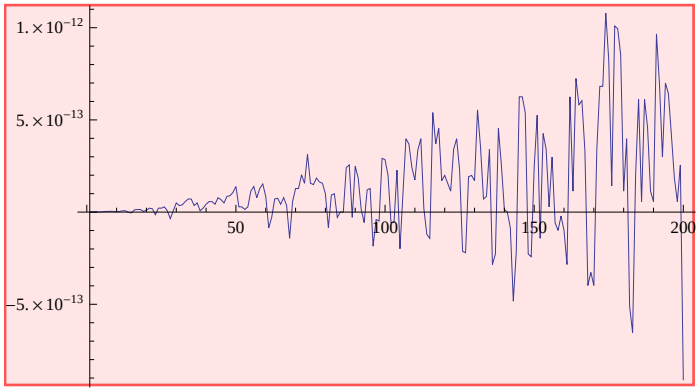

```
DiscretePlot[tk1[n, 2, 5], {n, 1, 100}]
```



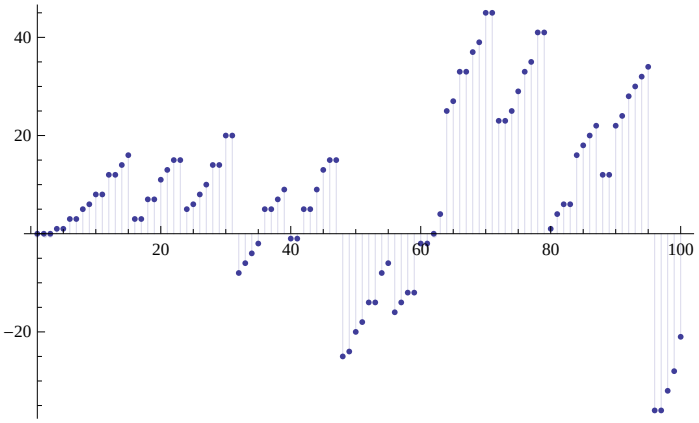
```
DiscretePlot[tk1z[n, 2, 5], {n, 1, 100}]
```



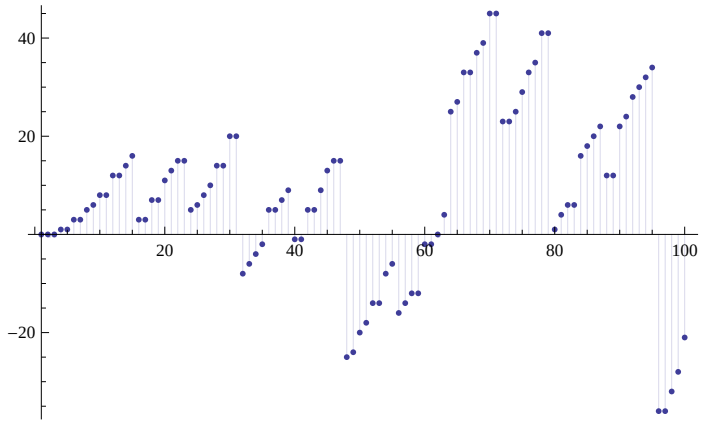

```
DiscretePlot[tk1[n, aa = 3, bb = 1.7] - tk1z[n, aa, bb], {n, 1, 200}]
```



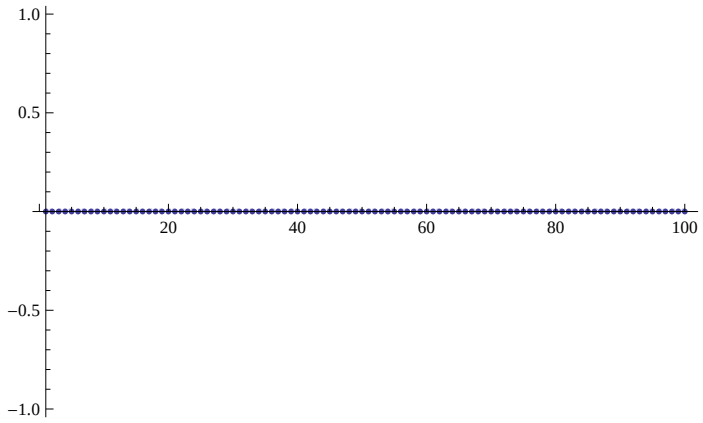

```
DiscretePlot[tk[n, 2, 8], {n, 1, 100}]
```



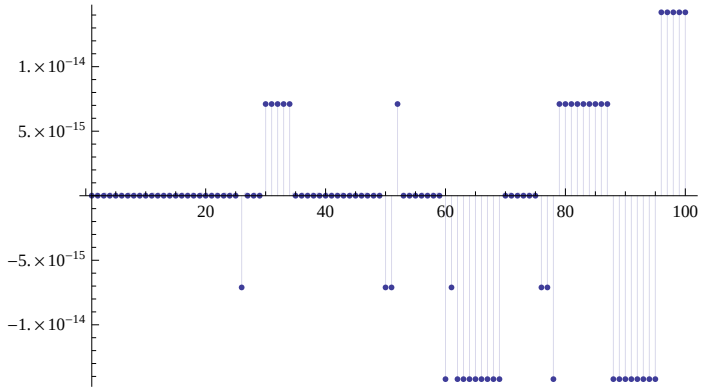
```
DiscretePlot[tk2a[n, 2, 8], {n, 1, 100}]
```




```
DiscretePlot[tk[n, 2, 8] - tk2a[n, 2, 8], {n, 1, 100}]
```




```
DiscretePlot[tk[n, 2, 8.71] - tk2a[n, 2, 8.71], {n, 1, 100}]
```




```
Table[{n, ax = tk1[n, aa = 3, bb = 1.2], bx = tk1z[n, aa, bb], ax - bx}, {n, 1, 100}] // TableForm
```

1	1.	1.	0.
2	2.992	2.992	-8.88178×10^{-16}
3	8.152	8.152	-1.77636×10^{-15}
4	−1.832	−1.832	-2.22045×10^{-15}
5	−7.472	−7.472	0.
6	11.464	11.464	0.
7	4.096	4.096	0.
8	−5.344	−5.344	2.66454×10^{-15}

9	23.552	23.552	-1.42109×10^{-14}
10	−3.448	−3.448	-9.76996×10^{-15}
11	−24.64	−24.64	1.42109×10^{-14}
12	4.16	4.16	9.76996×10^{-15}
13	27.896	27.896	-7.10543×10^{-15}
14	8.816	8.816	-1.06581×10^{-14}
15	−8.104	−8.104	3.55271×10^{-15}
16	−1.312	−1.312	-2.04281×10^{-14}
17	−30.712	−30.712	-3.55271×10^{-15}
18	17.096	17.096	7.10543×10^{-15}
19	33.056	33.056	7.10543×10^{-15}
20	−8.128	−8.128	-1.06581×10^{-14}
21	−2.152	−2.152	-1.77636×10^{-15}
22	−19.072	−19.072	1.77636×10^{-14}
23	−32.056	−32.056	1.42109×10^{-14}
24	−2.056	−2.056	-4.44089×10^{-16}
25	1.352	1.352	1.77636×10^{-15}
26	40.16	40.16	-6.39488×10^{-14}
27	17.76	17.76	-3.55271×10^{-14}
28	12.	12.	-3.01981×10^{-14}
29	−15.24	−15.24	3.19744×10^{-14}
30	−15.024	−15.024	1.06581×10^{-14}
31	26.856	26.856	-3.55271×10^{-14}
32	23.232	23.232	1.06581×10^{-14}
33	−8.952	−8.952	-7.10543×10^{-15}
34	−51.792	−51.792	7.10543×10^{-15}
35	44.904	44.904	3.55271×10^{-14}
36	9.624	9.624	2.13163×10^{-14}
37	−2.928	−2.928	-1.06581×10^{-14}
38	34.152	34.152	-9.23706×10^{-14}
39	−4.8	−4.8	5.32907×10^{-15}
40	−12.384	−12.384	9.76996×10^{-14}
41	35.976	35.976	7.10543×10^{-15}
42	−8.304	−8.304	-7.81597×10^{-14}
43	−5.304	−5.304	-7.4607×10^{-14}
44	−10.632	−10.632	-1.77636×10^{-14}
45	−6.024	−6.024	0.
46	−29.424	−29.424	-4.26326×10^{-14}
47	14.616	14.616	1.24345×10^{-14}
48	−9.504	−9.504	-9.41469×10^{-14}
49	4.272	4.272	-8.26006×10^{-14}
50	11.472	11.472	-1.47438×10^{-13}
51	−43.032	−43.032	6.39488×10^{-14}
52	73.032	73.032	-1.42109×10^{-14}
53	11.232	11.232	-6.21725×10^{-14}
54	−15.792	−15.792	3.90799×10^{-14}
55	32.088	32.088	4.26326×10^{-14}
56	−6.6	−6.6	6.75016×10^{-14}

57	30.48	30.48	-6.75016×10^{-14}
58	−8.472	−8.472	-7.28306×10^{-14}
59	−42.624	−42.624	4.26326×10^{-14}
60	−40.464	−40.464	1.27898×10^{-13}
61	63.624	63.624	1.7053×10^{-13}
62	53.184	53.184	1.20792×10^{-13}
63	−55.824	−55.824	2.20268×10^{-13}
64	33.952	33.952	-1.27898×10^{-13}
65	12.712	12.712	4.61853×10^{-14}
66	−8.24	−8.24	9.05942×10^{-14}
67	33.64	33.64	1.63425×10^{-13}
68	−58.952	−58.952	1.13687×10^{-13}
69	−82.352	−82.352	-4.26326×10^{-14}
70	54.808	54.808	-3.05533×10^{-13}
71	67.744	67.744	9.9476×10^{-14}
72	11.104	11.104	-1.06581×10^{-13}
73	−32.552	−32.552	-1.42109×10^{-13}
74	4.528	4.528	-1.34115×10^{-13}
75	62.704	62.704	1.13687×10^{-13}
76	15.904	15.904	1.81188×10^{-13}
77	−94.04	−94.04	-1.27898×10^{-13}
78	−0.944	−0.944	1.66089×10^{-13}
79	2.056	2.056	1.66089×10^{-13}
80	−26.816	−26.816	2.13163×10^{-14}
81	106.984	106.984	1.42109×10^{-14}
82	46.	46.	2.84217×10^{-14}
83	−22.28	−22.28	1.10134×10^{-13}
84	−26.6	−26.6	-3.55271×10^{-14}
85	−15.008	−15.008	7.10543×10^{-15}
86	−16.808	−16.808	-2.84217×10^{-14}
87	−21.632	−21.632	3.33955×10^{-13}
88	10.528	10.528	1.04805×10^{-13}
89	−34.424	−34.424	-2.13163×10^{-13}
90	−37.448	−37.448	-1.91847×10^{-13}
91	49.312	49.312	-4.83169×10^{-13}
92	−2.672	−2.672	-3.2685×10^{-13}
93	94.888	94.888	4.26326×10^{-14}
94	−6.272	−6.272	4.9738×10^{-14}
95	−8.072	−8.072	3.17968×10^{-13}
96	−5.984	−5.984	2.27374×10^{-13}
97	−41.864	−41.864	-2.84217×10^{-14}
98	−0.104	−0.104	-3.64153×10^{-13}
99	−30.056	−30.056	-2.70006×10^{-13}
100	34.024	34.024	-1.84741×10^{-13}

```
Table[{n, tk1z[n, .5, 1.1]}, {n, 1, 100}] // TableForm
```

1	1.
2	0.546603
3	0.525095

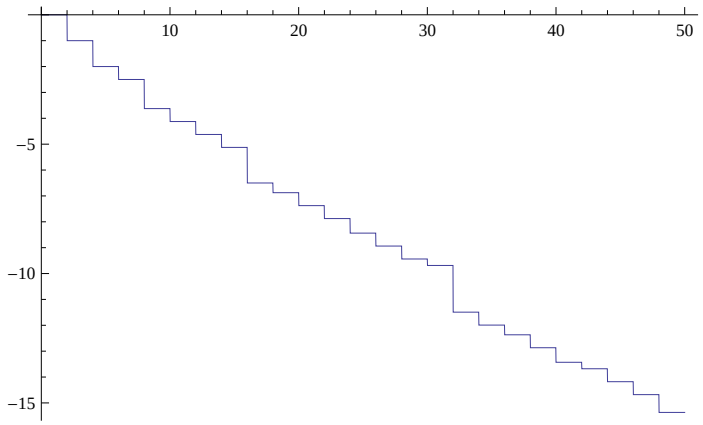
4	0.387578
5	0.505547
6	0.328804
7	0.478961
8	0.346368
9	0.341108
10	0.214264
11	0.38739
12	0.42779
13	0.534728
14	0.49209
15	0.298238
16	0.241158
17	0.483764
18	0.358518
19	0.470429
20	0.36407
21	0.185242
22	0.170834
23	0.500926
24	0.387078
25	0.517538
26	0.293447
27	0.403165
28	0.193134
29	0.454014
30	0.313443
31	0.47445
32	0.33536
33	0.408498
34	0.484454
35	0.344241
36	0.176818
37	0.467123
38	0.338717
39	0.298609
40	0.283349
41	0.393976
42	0.208859
43	0.42788
44	0.437741
45	0.460983
46	0.248976
47	0.587357
48	0.335895
49	0.523482
50	0.338514
51	0.370105
52	0.164122
53	0.470252
54	0.32399
55	0.226865
56	0.280232
57	0.237386
58	0.276794
59	0.418023

```
60     0.266621
61     0.463564
62     0.508225
63     0.383374
64     0.419404
65     0.265102
66     0.190146
67     0.505506
68     0.322644
69     0.294885
70     0.231693
71     0.479097
72     0.316032
73     0.559502
74     0.396346
75     0.385228
76     0.287133
77     0.380846
78     0.378825
79     0.475099
80     0.419391
81     0.285246
82     0.229364
83     0.511004
84     0.357515
85     0.394483
86     0.419498
87     0.301859
88     0.260457
89     0.552852
90     0.351436
91     0.350822
92     0.133348
93     0.266709
94     0.274559
95     0.285037
96     0.0882377
97     0.411169
98     0.148082
99     0.135323
100    0.151354
```

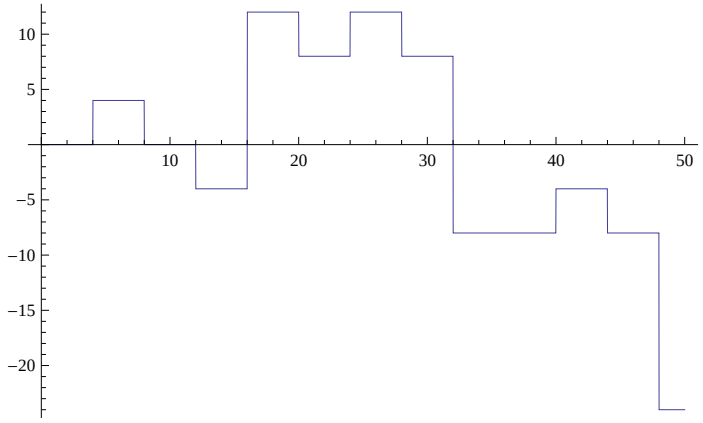
$RecursionLimit = 1 000 000

1 000 000

```
Plot[ tk1z[n, .5, 2] - DD[n, .5], {n, 0, 50}]
```




```
-Plot[ tk1z[n, -1, 4] - DD[n, -1], {n, 0, 50}]
```




```
Table[ {n, ((tk1z[n, aa = -1, 4] - DD[n, aa]) - ((tk1z[n - 1, aa, 4] - DD[n - 1, aa]))),
    If[Mod[n, 4] == 0, 4 MoebiusMu[Floor[n / 4]], 0] +
     If[Mod[n, 16] == 0, 16 MoebiusMu[Floor[n / 16]], 0] +
     If[Mod[n, 64] == 0, 64 MoebiusMu[Floor[n / 64]], 0]}, {n, 2, 100}] // TableForm
```

2	0	0
3	0	0
4	4	4
5	0	0
6	0	0
7	0	0
8	−4	−4
9	0	0
10	0	0
11	0	0
12	−4	−4
13	0	0
14	0	0
15	0	0
16	16	16
17	0	0
18	0	0
19	0	0
20	−4	−4
21	0	0

22	0	0
23	0	0
24	4	4
25	0	0
26	0	0
27	0	0
28	−4	−4
29	0	0
30	0	0
31	0	0
32	−16	−16
33	0	0
34	0	0
35	0	0
36	0	0
37	0	0
38	0	0
39	0	0
40	4	4
41	0	0
42	0	0
43	0	0
44	−4	−4
45	0	0
46	0	0
47	0	0
48	−16	−16
49	0	0
50	0	0
51	0	0
52	−4	−4
53	0	0
54	0	0
55	0	0
56	4	4
57	0	0
58	0	0
59	0	0
60	4	4
61	0	0
62	0	0
63	0	0
64	64	64
65	0	0
66	0	0
67	0	0
68	−4	−4
69	0	0
70	0	0
71	0	0
72	0	0
73	0	0
74	0	0
75	0	0
76	−4	−4
77	0	0

78	0	0
79	0	0
80	−16	−16
81	0	0
82	0	0
83	0	0
84	4	4
85	0	0
86	0	0
87	0	0
88	4	4
89	0	0
90	0	0
91	0	0
92	−4	−4
93	0	0
94	0	0
95	0	0
96	16	16
97	0	0
98	0	0
99	0	0
100	0	0

```
Plot[n - 4.5 Floor[n / 4.5], {n, 0, 100}]
```