

$$1 / c^2 \text{Sum}[1, \{j, 2, \text{Floor}[c^2 n]\}, \{k, 2, \text{Floor}[c^2 n / j]\}]$$

$$\frac{\sum_{j=2}^{\text{Floor}[c^2 n]} \sum_{k=2}^{\text{Floor}[\frac{c^2 n}{j}]} 1}{c^2}$$

$$1 / c^2 \text{Sum}[1, \{j, 2, c\}, \{k, 2, \text{Floor}[c^2 n / j]\}]$$

$$\frac{\sum_{j=2}^c \sum_{k=2}^{\text{Floor}[\frac{c^2 n}{j}]} 1}{c^2}$$

$$1 / c^2 \text{Sum}[1, \{j, 2, c\}, \{k, 2, c\}]$$

$$\frac{(-1 + c)^2}{c^2}$$

$$f[n_, c_] := 1 / c^2 (\text{Sum}[1, \{j, 2, \text{Floor}[c^2 n]\}, \{k, 2, \text{Floor}[c^2 n / j]\}] - 2 \text{Sum}[1, \{j, 2, c\}, \{k, 2, \text{Floor}[c^2 n / j]\}] + \text{Sum}[1, \{j, 2, c\}, \{k, 2, c\}])$$

$$N[f[100, 100]]$$

$$360.535$$

$$N[\text{Gamma}[2, 0, -\text{Log}[100]]]$$

$$361.517 - 4.41506 \times 10^{-14} i$$

$$\text{Limit}[f[n, c], c \rightarrow \text{Infinity}]$$

$$\$Aborted$$

$$f2[n_, c_] :=$$

$$1 / c^3 (\text{Sum}[1, \{j, 2, \text{Floor}[c^3 n]\}, \{k, 2, \text{Floor}[c^3 n / j]\}, \{l, 2, \text{Floor}[c^3 n / (j k)]\}] - 3 \text{Sum}[1, \{j, 2, c\}, \{k, 2, \text{Floor}[c^3 n / j]\}, \{l, 2, \text{Floor}[c^3 n / (j k)]\}] + 3 \text{Sum}[1, \{j, 2, c\}, \{k, 2, c\}, \{l, 2, \text{Floor}[c^3 n / (j k)]\}] - \text{Sum}[1, \{j, 2, c\}, \{k, 2, c\}, \{l, 2, c\}])$$

$$N[f2[10, 10]]$$

$$10.496$$

$$N[-\text{Gamma}[3, 0, -\text{Log}[100]] / \text{Gamma}[3]]$$

$$698.863 - 1.71417 \times 10^{-13} i$$

$$\text{Dhyp}[n_, k_, a_] := \text{Dhyp}[n, k, a] =$$

$$\text{Sum}[\text{Binomial}[k, j] \text{Dhyp}[n / (m^k (k - j)), j, m + 1], \{m, a, n^k (1 / k)\}, \{j, 0, k - 1\}]$$

$$\text{Dhyp}[n_, 1, a_] := \text{Floor}[n] - a + 1; \text{Dhyp}[n_, 0, a_] := 1$$

$$fa[n_, c_] := 1 / c^2$$

$$(\text{Dhyp}[c^2 n, 2, 2] - 2 \text{Sum}[\text{Dhyp}[c^2 n / j, 1, 2], \{j, 2, c\}] + \text{Sum}[1, \{j, 2, c\}, \{k, 2, c\}])$$

$$N[fa[100, 150]]$$

$$360.86$$

$$f2a[n_, c_] := 1 / c^3 (\text{Dhyp}[n c^3, 3, 2] - 3 \text{Sum}[\text{Dhyp}[\text{Floor}[n c^3 / j], 2, 2], \{j, 2, c\}] + 3 \text{Sum}[\text{Dhyp}[\text{Floor}[n c^3 / (j k)], 1, 2], \{j, 2, c\}, \{k, 2, c\}] - \text{Sum}[1, \{j, 2, c\}, \{k, 2, c\}, \{l, 2, c\}])$$

```

N[f2a[100, 200]]
696.157
N[f2a[4, 800]]
$Aborted
N[f2a[4, 3400]]
g[n_, k_, b_] := b^-k Dhyp[n b^k, k, b+1]
N[g[100, 2, 1000]]
361.418
N[g[100, 3, 200]]
696.157
N[Gamma[3, 0, -Log[100]] / Gamma[3]]
-698.863 + 1.71417 × 10-13 i
f2b[n_, c_] := 1 / c^3 (Dhyp[n c^3, 3, 1] - 3 Sum[Dhyp[Floor[n c^3 / j], 2, 1], {j, 1, c}] +
  3 Sum[Dhyp[Floor[n c^3 / (j k)], 1, 1], {j, 1, c}, {k, 1, c}] -
  Sum[1, {j, 1, c}, {k, 1, c}, {1, 1, c}])
N[f2b[100, 200]]
696.157
f2c[n_, c_] := 1 / c^3 (Dhyp[n c^3, 3, 1] - 3 Sum[Dhyp[Floor[n c^3 / j], 2, 1], {j, 1, c}] +
  3 Sum[Dhyp[Floor[n c^3 / (j k)], 1, 1], {j, 1, c}, {k, 1, c}] -
  Sum[1, {j, 1, c}, {k, 1, c}, {1, 1, c}])
f2d[n_, c_] := 1 / c^3 (Integrate[1, {j, 1, c^3 n}, {k, 1, c^3 n / j}, {1, 1, c^3 n / (j k)}] -
  3 Integrate[1, {j, 1, c}, {k, 1, c^3 n / j}, {1, 1, c^3 n / (j k)}] +
  3 Integrate[1, {j, 1, c}, {k, 1, c}, {1, 1, c^3 n / (j k)}] -
  Integrate[1, {j, 1, c}, {k, 1, c}, {1, 1, c}])
f2d[n, c]
$Aborted
1 / c^3 Integrate[1, {j, 1, c^3 n}, {k, 1, c^3 n / j}, {1, 1, c^3 n / (j k)}]
ConditionalExpression[
$$\frac{-1 + c^3 n + \frac{1}{2} c^3 n (-2 + \text{Log}[c^3 n]) \text{Log}[c^3 n]}{c^3}, \text{Re}[c^3 n] \geq 0 \mid c^3 n \notin \text{Reals}]$$

Expand[
$$\frac{-1 + c^3 n + \frac{1}{2} c^3 n (-2 + \text{Log}[c^3 n]) \text{Log}[c^3 n]}{c^3}$$
]
FullSimplify[
$$-\frac{1}{c^3} + n - n \text{Log}[c^3 n] + \frac{1}{2} n \text{Log}[c^3 n]^2$$
]

$$-\frac{1}{c^3} + n + \frac{1}{2} n (-2 + \text{Log}[c^3 n]) \text{Log}[c^3 n]$$

-3 / c^3 Integrate[1, {j, 1, c}, {k, 1, c^3 n / j}, {1, 1, c^3 n / (j k)}]
3 / c^3 Integrate[1, {j, 1, c}, {k, 1, c}, {1, 1, c^3 n / (j k)}]
1 // c^3 Integrate[1, {j, 1, c}, {k, 1, c}, {1, 1, c}]

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f2[n_, c_] :=
  1 / c^3 (Sum[1, {j, 2, Floor[c^3 n]}, {k, 2, Floor[c^3 n / j]}, {l, 2, Floor[c^3 n / (j k)}}] -
    3 Sum[1, {j, 2, c}, {k, 2, Floor[c^3 n / j]}, {l, 2, Floor[c^3 n / (j k)}}] +
    3 Sum[1, {j, 2, c}, {k, 2, c}, {l, 2, Floor[c^3 n / (j k)}}] -
    Sum[1, {j, 2, c}, {k, 2, c}, {l, 2, c}])
fa[n_, c_] := (Sum[1 / c^2, {j, 2, Floor[c^2 n]}, {k, 2, Floor[c^2 n / j]}] -
  2 Sum[1 / c^2, {j, 2, c}, {k, 2, Floor[c^2 n / j]}] + Sum[1 / c^2, {j, 2, c}, {k, 2, c}])
fa2[n_, c_] := Sum[1, {j, 2, n}, {k, 2, n / j}]
fa2[100, 1]
283
N[fa[100, 40]]
359.061

```

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Dhyp[n_, k_, a_] := Dhyp[n, k, a] = Dhyp[n, k, a] =
  Sum[Binomial[k, j] Dhyp[Floor[n / (m^(k - j))], j, m + 1], {m, a, n^(1 / k)}, {j, 0, k - 1}]
Dhyp[n_, 1, a_] := Floor[n] - a + 1; Dhyp[n_, 0, a_] := 1

g[n_, k_, b_] := b^-k Dhyp[Floor[n b^k], k, b + 1]
g[100, 1, 1]
99

```

```

N[LogIntegral[100] - Log[Log[100]] - EulerGamma]
28.0217

```

```
Table[{k, N[3^-k Dhyp[3^k x 10, k, 2] (-1)^(k+1)/k]}, {k, 1, 30}] // TableForm
```

```
1      9.66667
2     -13.6667
3     20.1728
4    -30.5494
5     46.9654
6    -73.2892
7    115.485
8   -183.429
9   293.204
10  -471.533
11  762.817
12 -1240.41
13 2025.81
14 -3321.38
15 5466.08
16 -9029.4
17 14969.
18 -24895.3
19 41517.7
20 -69399.4
21 116238.
22 -195036.
23 327787.
24 -551755.
25 930191.
26 -1.57066 x 10^6
27 2.65639 x 10^6
28 -4.49981 x 10^6
29 7.63405 x 10^6
30 -1.29692 x 10^7
```

```
gp[n_, s_, t_] := Sum[(-1)^(k+1)/kg[n, k, s], {k, 1, t}]
N[gp[100, 11, 20]]
```

```
28.117
```

```
N[LogIntegral[100] - Log[Log[100]] - EulerGamma]
```

```
28.0217
```

```
Table[{k, N[gp[100, k, 20] - gp[100, k - 1, 24]]}, {k, 2, 16}] // TableForm
```

```
2      0.0711691
3      -0.383071
4      0.238894
5      -0.23735
6      -0.0205683
7      -0.106998
8      0.125742
9      -0.121786
10     0.0905636
11     -0.072895
12     0.0537289
13     -0.0338052
14     -0.01406
15     0.018442
16     0.0237077
```

```
Table[{k, N[ExpIntegralEi[-ZetaZero[k] Log[100]] + ExpIntegralEi[-ZetaZero[-k] Log[100]]]},  
      {k, 1, 16}] // TableForm
```

```
1      0.00246164 + 0. i
2      0.00118841 + 0. i
3      0.00153747 + 0. i
4      0.00136848 + 0. i
5      0.000993297 + 0. i
6      -0.000323772 + 0. i
7      -0.0000799546 + 0. i
8      -0.00100198 + 0. i
9      0.00082397 + 0. i
10     0.000116297 + 0. i
11     -0.000738046 + 0. i
12     0.000562652 + 0. i
13     0.0000200512 + 0. i
14     -0.000359133 + 0. i
15     -0.000656314 + 0. i
16     0.000554059 + 0. i
```