```
ClearAll["Global`*"]

K[n_, k_] :=
 K[n, k] = Sum[ FullSimplify[MangoldtLambda[j] / Log[j]] K[Floor[n / j], k - 1], {j, 2, n}];
K[n_, 0] := 1

E2a[n_, k_, a_] :=
 E2a[n, k, a] = Sum[ E2a[n / j, k - 1, a], {j, 2, n}] - a Sum[ E2a[n / (a j), k - 1, a], {j, 1, n / a}];
E2a[n_, 0, a_] := 1

EM2[n_, a_, b_] :=
 EM2[n, a, b] = Sum[(-1)^k Binomial[k - 1, k - a] E2a[n, k, b], {k, 1, Log[If[b < 2, b, 2], n]}];
EM2[n_, 0, b_] := 1
E2d[n_, a_, b_] :=
 E2d[n, a, b] = Sum[(-1)^s Binomial[s - 1, s - a] EM2[n, s, b], {s, 1, Log[If[b < 2, b, 2], n]}];
E2d[n_, 0, b_] := 1
E1e[n_, k_, b_] := Sum[FactorialPower[k, a] / a! E2d[n, a, b], {a, 0, Log[If[b > 2, 2, b], n]}]
D1g[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j E1e[n / b^j, k, b], {j, 0, Log[b, n]}]
D1h[n_, k_, b_] :=
 Sum[ Binomial[k + j - 1, k - 1] b^j (Sum[FactorialPower[k, a] / a! E2d[n / b^j, a, b],
     {a, 0, Log[If[b > 2, 2, b], n]}]), {j, 0, Log[b, n]}]

D1i[n_, k_, b_] :=
 Sum[ Binomial[k + j - 1, k - 1] b^j (1 + Sum[FactorialPower[k, a] / a! (Sum[(-1)^s
          Binomial[s - 1, s - a] EM2[n / b^j, s, b], {s, 1, Log[If[b < 2, b, 2], n / b^j]}]),
      {a, 1, Log[If[b > 2, 2, b], n]}]), {j, 0, Log[b, n]}]
D1j[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j
   (1 + Sum[FactorialPower[k, a] / a! (Sum[(-1)^s Binomial[s - 1, s - a] EM2[n / b^j, s, b],
         {s, 1, Log[If[b < 2, b, 2], n / b^j]}]),
      {a, 1, Log[If[b > 2, 2, b], n]}]), {j, 0, Log[b, n]}]

EP2[n_, a_, b_] :=
 EP2[n, a, b] = Sum[SeriesCoefficient[Series[(Log[x + 1])^a, {x, 0, 230}], k] E2a[n, k, b],
   {k, 1, Log[If[b > 2, 2, b], n]}]; EP2[n_, 0, b_] := 1
E1p[n_, a_, b_] := E1p[n, a, b] = 1 + Sum[a^k / k! EP2[n, k, b], {k, 1, Log[If[b > 2, 2, b], n]}]
D1f[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j E1p[n / b^j, k, b], {j, 0, Log[b, n]}]


D1f2[n_, a_, b_] := Sum[ Binomial[a + j - 1, a - 1] b^j
   (1 + Sum[a^k / k! EP2[n / b^j, k, b], {k, 1, Log[If[b > 2, 2, b], n / b^j]}]), {j,
   0, Log[b, n]}]
D1f3[n_, a_, b_] := Sum[ Binomial[a + j - 1, a - 1] b^j a^k / k! EP2[n / b^j, k, b],
  {j, 0, Log[b, n]}, {k, 0, Log[If[b > 2, 2, b], n / b^j]}]
D1f3a[n_, a_, b_] := Grid[Table[ Binomial[a + j - 1, a - 1] b^j a^k / k! EP[n / b^j, k, b],
   {j, 0, Log[b, n]}, {k, 0, Log[If[b > 2, 2, b], n / b^j]}]]
D1f3a2[n_, a_, b_] := Grid[Table[ Binomial[a + j - 1, a - 1] b^j a^k / k! EP[n / b^j, k, b] / a,
   {j, 0, Log[b, n]}, {k, 0, Log[If[b > 2, 2, b], n / b^j]}]]
D1f3b[n_, a_, b_] := Grid[Table[ Binomial[a + j - 1, a - 1] b^j a^k / k! EP2[n / b^j, k, b],
   {j, 0, Log[b, n]}, {k, 0, Log[If[b > 2, 2, b], n / b^j]}]]
```

D1j[100, 2, 2]

482

D1f3[100, -1, 1.2]

1.

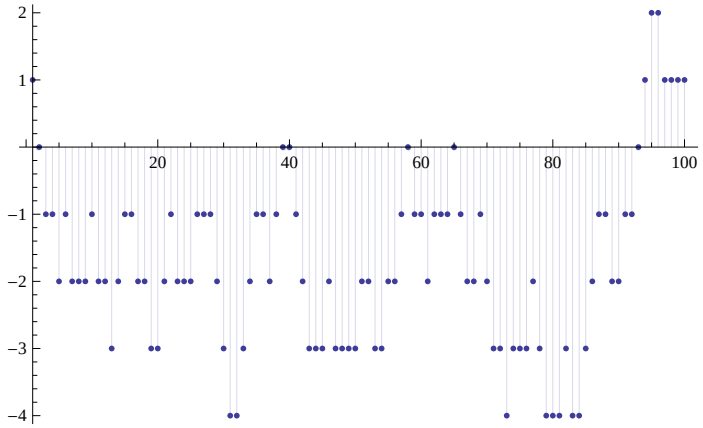
D1f3a[100, -1, 2]

EP[100, 0, 2]	-EP[100, 1, 2]	$\frac{1}{2}$ EP[100, 2, 2]	$-\frac{1}{6}$ EP[100, 3, 2]	$\frac{1}{24}$ EP[100, 4, 2]	$-\frac{1}{120}$ EP[100, 5, 2]	$\frac{1}{720}$ EP[100, 6, 2]
-2 EP[50, 0, 2]	2 EP[50, 1, 2]	-EP[50, 2, 2]	$\frac{1}{3}$ EP[50, 3, 2]	$-\frac{1}{12}$ EP[50, 4, 2]	$\frac{1}{60}$ EP[50, 5, 2]	
0	0	0	0	0		
0	0	0	0			
0	0	0				
0	0					
0						

D1f3b[100, -1, 1.2]

1.	4.1506	6.08731	5.07149	2.85546	0.411142	0.219317	0.161956	0.037866	0.00671787	0.00252624	0.000656331	0.000105289	0.0000119036	1.24035×10^{-6}	1.3385×10^{-7}	1.28887×10^{-8}	9
-1.2	-2.8017	-5.7052	-5.80447	-2.27484	-0.608456	-0.414726	-0.158911	-0.0247354	-0.00839147	-0.00298941	-0.000643128	-0.0000879832	-0.0000100652	-1.182×10^{-6}	-1.2773×10^{-7}	-1.0288×10^{-8}	-
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.			
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.					
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.						
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.							
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.								
0.	0.	0.	0.	0.	0.	0.	0.	0.									
0.	0.	0.	0.	0.	0.	0.	0.										
0.	0.	0.	0.	0.	0.	0.											
0.	0.	0.	0.	0.	0.												
0.	0.	0.	0.	0.													
0.	0.	0.	0.														
0.	0.	0.															
0.	0.																
0.																	

```
DiscretePlot[ D1f3[ n, -1, 2], {n, 1, 100}]
```




```
D1i[n_, k_, b_] :=
 Sum[ Binomial[k + j - 1, k - 1] b^j (1 + Sum[FactorialPower[k, a] / a! (Sum[(-1)^s
           Binomial[s - 1, s - a] EM2[n / b^j, s, b], {s, 1, Log[If[b < 2, b, 2], n / b^j]}]),
      {a, 1, Log[If[b > 2, 2, b], n]}]), {j, 0, Log[b, n]}]

D1j[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j
   (1 + Sum[Sum[FactorialPower[k, a] / a! (-1)^s Binomial[s - 1, s - a] EM2[n / b^j, s, b], {s, 1,
        Log[If[b < 2, b, 2], n / b^j]}], {a, 1, Log[If[b > 2, 2, b], n]}]), {j, 0, Log[b, n]}]

D1k[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j (1 + Sum[
      FactorialPower[k, a] / a! (-1)^s Binomial[s - 1, s - a] EM2[n / b^j, s, b],
      {a, 1, Log[If[b > 2, 2, b], n]}, {s, 1, Log[If[b < 2, b, 2], n / b^j]}
     ]),
  {j,
   0,
   Log[
    b,
    n]}]
D1m[n_, k_, b_] := Sum[ (Binomial[k + j - 1, k - 1] b^j + Sum[
     Binomial[k + j - 1, k - 1] b^j FactorialPower[k, a] / a! (-1)^s Binomial[s - 1, s - a]
      EM2[n / b^j, s, b], {a, 1, Log[If[b > 2, 2, b], n]}, {s, 1, Log[If[b < 2, b, 2], n / b^j]}
    ]), {j, 0, Log[b, n]}]

D1n[n_, k_, b_] := Sum[Binomial[k + j - 1, k - 1] b^j, {j, 0, Log[b, n]}] + Sum[ (Sum[
     Binomial[k + j - 1, k - 1] b^j FactorialPower[k, a] / a! (-1)^s Binomial[s - 1, s - a]
      EM2[n / b^j, s, b], {a, 1, Log[If[b > 2, 2, b], n]}, {s, 1, Log[If[b < 2, b, 2], n / b^j]}
    ]), {j, 0, Log[b, n]}]

D1o[n_, k_, b_] :=
 Sum[Binomial[k + j - 1, k - 1] b^j, {j, 0, Log[b, n]}] + Sum[ Binomial[k + j - 1, k - 1]
    b^j FactorialPower[k, a] / a! (-1)^s Binomial[s - 1, s - a] EM2[n / b^j, s, b],
   {j, 0, Log[b, n]}, {a, 1, Log[If[b > 2, 2, b], n]}, {s, 1, Log[If[b < 2, b, 2], n / b^j]}]
```

```
D1o[100, 2, 2]
```

482

```
D1p[n_, k_, b_] :=
 Sum[Binomial[k + j - 1, k - 1] b^j, {j, 0, Log[b, n]}] + Sum[ Binomial[k + j - 1, k - 1]
    b^j FactorialPower[k, a] / a! (-1)^s Binomial[s - 1, s - a] em2[n / b^j, s, b],
   {a, 1, Log[If[b > 2, 2, b], n]}, {j, 0, Log[b, n]}, {s, 1, Log[If[b < 2, b, 2], n / b^j]}]
```

```
D1p[100, 1, 7]
```

$$57 - 49\,\text{em2}\left[\frac{100}{49}, 1, 7\right] - 7\,\text{em2}\left[\frac{100}{7}, 1, 7\right] + 7\,\text{em2}\left[\frac{100}{7}, 2, 7\right] - 7\,\text{em2}\left[\frac{100}{7}, 3, 7\right] - \text{em2}[100, 1, 7] + \text{em2}[100, 2, 7] - \text{em2}[100, 3, 7] + \text{em2}[100, 4, 7] - \text{em2}[100, 5, 7] + \text{em2}[100, 6, 7]$$

```
ff[n_, b_, k_] := Sum[ FactorialPower[k, a] / a!, {a, 0, Floor[Log[If[b < 2, b, 2], n]]}]
```

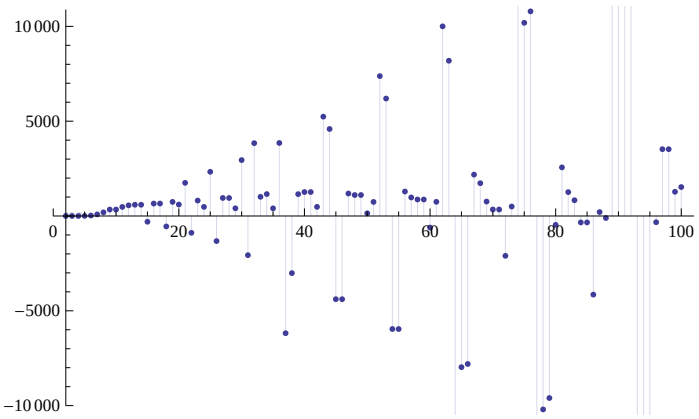
```
ff[100, 1.1, -3]
```

625

```
(2^2) - 1
```

3

```
DiscretePlot[EP2[n, 8, 1.2], {n, 2, 100}]
```



D1f3a[100, 1, 2]

EP[100, 0, 2]	EP[100, 1, 2]	$\frac{1}{2}$ EP[100, 2, 2]	$\frac{1}{6}$ EP[100, 3, 2]	$\frac{1}{24}$ EP[100, 4, 2]	$\frac{1}{120}$ EP[100, 5, 2]	$\frac{1}{720}$ EP[100, 6, 2]
2 EP[50, 0, 2]	2 EP[50, 1, 2]	EP[50, 2, 2]	$\frac{1}{3}$ EP[50, 3, 2]	$\frac{1}{12}$ EP[50, 4, 2]	$\frac{1}{60}$ EP[50, 5, 2]	
4 EP[25, 0, 2]	4 EP[25, 1, 2]	2 EP[25, 2, 2]	$\frac{2}{3}$ EP[25, 3, 2]	$\frac{1}{6}$ EP[25, 4, 2]		
8 EP$\left[\frac{25}{2}, 0, 2\right]$	8 EP$\left[\frac{25}{2}, 1, 2\right]$	4 EP$\left[\frac{25}{2}, 2, 2\right]$	$\frac{4}{3}$ EP$\left[\frac{25}{2}, 3, 2\right]$			
16 EP$\left[\frac{25}{4}, 0, 2\right]$	16 EP$\left[\frac{25}{4}, 1, 2\right]$	8 EP$\left[\frac{25}{4}, 2, 2\right]$				
32 EP$\left[\frac{25}{8}, 0, 2\right]$	32 EP$\left[\frac{25}{8}, 1, 2\right]$					
64 EP$\left[\frac{25}{16}, 0, 2\right]$						

D1f3a[100, -1, 2]

EP[100, 0, 2]	-EP[100, 1, 2]	$\frac{1}{2}$ EP[100, 2, 2]	$-\frac{1}{6}$ EP[100, 3, 2]	$\frac{1}{24}$ EP[100, 4, 2]	$-\frac{1}{120}$ EP[100, 5, 2]	$\frac{1}{720}$ EP[100, 6, 2]
-2 EP[50, 0, 2]	2 EP[50, 1, 2]	-EP[50, 2, 2]	$\frac{1}{3}$ EP[50, 3, 2]	$-\frac{1}{12}$ EP[50, 4, 2]	$\frac{1}{60}$ EP[50, 5, 2]	
0	0	0	0	0		
0	0	0	0			
0	0	0				
0	0					
0						

```
D1f3a2[100, .0000001, 2]
```

$$1.\times 10^{7}\,\text{EP}[100,0,2] \quad 1.\,\text{EP}[100,1,2] \quad 5.\times 10^{-8}\,\text{EP}[100,2,2] \quad 1.66667\times 10^{-15}\,\text{EP}[100,3,2] \quad 4.16667\times 10^{-23}\,\text{EP}[100,4,2] \quad 8.33333\times 10^{-31}\,\text{EP}[100,5,2] \quad 1.38889\times 10^{-38}\,\text{EP}[100,6,2]$$

$$2.\,\text{EP}[50,0,2] \quad 2.\times 10^{-7}\,\text{EP}[50,1,2] \quad 1.\times 10^{-14}\,\text{EP}[50,2,2] \quad 3.33333\times 10^{-22}\,\text{EP}[50,3,2] \quad 8.33333\times 10^{-30}\,\text{EP}[50,4,2] \quad 1.66667\times 10^{-37}\,\text{EP}[50,5,2]$$

$$2.\,\text{EP}[25,0,2] \quad 2.\times 10^{-7}\,\text{EP}[25,1,2] \quad 1.\times 10^{-14}\,\text{EP}[25,2,2] \quad 3.33333\times 10^{-22}\,\text{EP}[25,3,2] \quad 8.33333\times 10^{-30}\,\text{EP}[25,4,2]$$

$$2.66667\,\text{EP}\left[\tfrac{25}{2},0,2\right] \quad 2.66667\times 10^{-7}\,\text{EP}\left[\tfrac{25}{2},1,2\right] \quad 1.33333\times 10^{-14}\,\text{EP}\left[\tfrac{25}{2},2,2\right] \quad 4.44445\times 10^{-22}\,\text{EP}\left[\tfrac{25}{2},3,2\right]$$

$$4.\,\text{EP}\left[\tfrac{25}{4},0,2\right] \quad 4.\times 10^{-7}\,\text{EP}\left[\tfrac{25}{4},1,2\right] \quad 2.\times 10^{-14}\,\text{EP}\left[\tfrac{25}{4},2,2\right]$$

$$6.4\,\text{EP}\left[\tfrac{25}{8},0,2\right] \quad 6.4\times 10^{-7}\,\text{EP}\left[\tfrac{25}{8},1,2\right]$$

$$10.6667\,\text{EP}\left[\tfrac{25}{16},0,2\right]$$

```
(D1f3[100, aa = .00001, 2] - 1) / aa
```

```
28.5338
```

```
EP2[100, 1, 1.02]
```

```
$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

General::stop : Further output of $RecursionLimit::reclim will be suppressed during this calculation. ≫

$Aborted
```

```
D1ha[n_, k_, b_] :=
 Sum[ Binomial[k + j - 1, k - 1] b^j (Sum[FactorialPower[k, a] / a! E2d[n / b^j, a, b],
     {a, 0, Log[If[b > 2, 2, b], n]}]), {j, 0, Log[b, n]}]
```

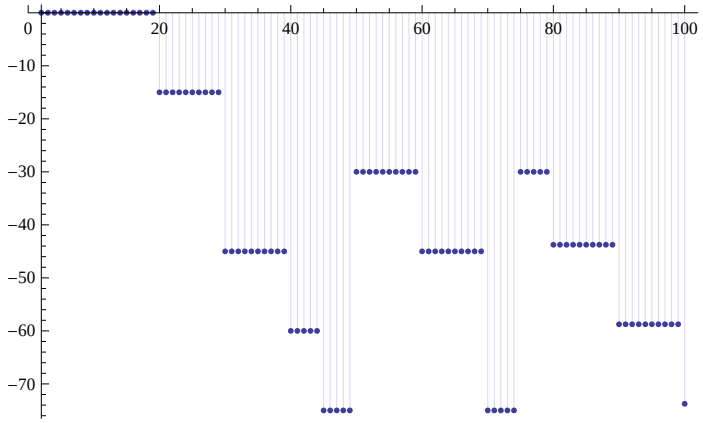
```
D1ha[100, 1, 2]
```

```
100
```

```
D1hb[n_, k_, b_] :=
 Sum[ (Sum[Binomial[k + j - 1, k - 1] b^j FactorialPower[k, a] / a! E2[n / b^j, a, b],
    {a, 0, Log[If[b > 2, 2, b], n]}]), {j, 0, Log[b, n]}]
```

```
D1hb[100, 1, 2]
```

```
DiscretePlot[EP2[n, 3, 5] - K[n, 3], {n, 2, 100}]
```




```
DiscretePlot[K[n, 2], {n, 2, 100}]
```