```
Dma[n_, k_, a_, s_] := Sum[ (j + a)^-s Dma[ n (j + a)^-1, k - 1, a, s], {j, 1, n}];
Dma[n_, 0, a_, s_] := UnitStep[n - 1]
Dma2[n_, k_, a_, s_] := Sum[ Binomial[k, j] Dma[n / (a + 1)^(k - j), j, a + 1, s], {j, 0, k}]
Dma3[n_, k_, a_, s_] :=
 Sum[ (-1)^(k - j) Binomial[k, j] Dma[n / a^(k - j), j, a - 1, s], {j, 0, k}]
```

```
Dma[100, 2, 2.31, 0]
```

```
159.
```

```
Dma2[100, 2, 2.31, 0]
```

```
159.
```

```
Dma3[100, 2, 2.31, 0]
```

```
159.
```

```
Dp1[n_, k_, s_] := Dp1[n, k - 1, s] + Sum[ j^-s Dp1[n j^-1, k - 1, s], {j, 1, n}];
Dp1[n_, 0, s_] := UnitStep[n - 1]
Dk[n_, k_, s_] := Sum[ j^-s Dk[n j^-1, k - 1, s], {j, 1, n}];
Dk[n_, 0, s_] := UnitStep[n - 1]
Dp1a[n_, k_, s_] := Sum[ Binomial[ k, j] Dk[ n, j, s], {j, 0, k}]
```

```
E1ab1[n_, k_, x_, s_] :=
 Sum[j^(-s) E1ab1[n / j, k - 1, x, s] - x (j x)^(-s) E1ab1[n / (x j), k - 1, x, s], {j, 1, n}];
E1ab1[n_, 0, a_, s_] := UnitStep[n - 1]
E1ab2[n_, k_, x_, s_] :=
 Sum[j^(-s) E1ab2[n / j, k - 1, x, s] - x (j x)^(-s) E1ab2[n / (x j), k - 1, x, s], {j, 1, n}];
E1ab2[n_, 0, a_, s_] := UnitStep[n - 1]
```

```
(1 / 4) Sum[ 1, {x, 1, 40}, {y, 1, 40 / x}]
```

$\frac{79}{2}$

```
f1[n_, a_, s_] :=
 a^(-2 (s - 1)) Sum[ x^-s y^-s, {x, 1, Floor[n a^-2]}, {y, 1, Floor[a^-2 n / x]}]
f2[n_, a_, s_] := Sum[ a^2 x^-s y^-s, {x, a, n / a, a}, {y, a, n / x, a}]
```

```
f1[20, 1 / 3, -2]
```

$\frac{11\,960\,423}{729}$

```
f2[20, 1 / 3, -2]
```

$\frac{11\,960\,423}{729}$

```
(1 / 2)^(2 (1 - (-1)))
```

$\frac{1}{16}$

```
ds[n_, k_, y_, s_] := y^(1 - s) Sum[j^-s ds[n (j y)^-1, k - 1, y, s], {j, 1, n y}];
ds[n_, 0, y_, s_] := UnitStep[n - 1]
```

```
ds[100, 2, 1/10, 0]
```

$$\frac{291}{100}$$

```
f[n_, j_] := 1 - j (Floor[n/j] - Floor[(n - 1)/j])
```

```
N[Sum[f[k, 2400]/k, {k, 1, 2400}]]
```

7.36065

```
f[3, 3]
```

-2

```
N[Log[1200]]
```

7.09008

```
N[Sum[1/j, {j, 1, 2400}]]
```

8.36065

```
N[HarmonicNumber[s] - (HarmonicNumber[90 000] - HarmonicNumber[Floor[90 000/s]])] /. s → 400
```

0.58068

```
N[Log[3]]
```

1.09861

```
N[HarmonicNumber[900 000] - HarmonicNumber[900 000/5000]]
```

8.51442

```
Log[5000.]
```

8.51719

```
{Limit[(y^(s - 1) HurwitzZeta[s, y + 1])^k, y → Infinity], 1/(s - 1)^k}
```

$$\left\{\left(\frac{1}{-1+s}\right)^k, (-1+s)^{-k}\right\}$$

```
{Limit[(y^(1 - s) HurwitzZeta[s, 1/y + 1])^k, y → 0], 1/(s - 1)^k}
```

$$\left\{\left(\frac{1}{-1+s}\right)^k, (-1+s)^{-k}\right\}$$

```
Grid[Table[Chop[N[1/((s - 1)^k) + Integrate[D[(Zeta[s, 1/y + 1] y^(1 - s))^k, y], {y, 0, 1}]] -
    N[(Zeta[s] - 1)^k]], {s, 2, 4}, {k, 1, 4}]]
```

```
0 0 0 0
0 0 0 0
0 0 0 0
```

```
Grid[Table[
  Chop[N[1 / ((s - 1) ^k) - Integrate[D[(Zeta[s, y + 1] y^ (s - 1)) ^k, y], {y, 1, Infinity}]] -
    N[(Zeta[s] - 1) ^k]], {s, 2, 4}, {k, 1, 4}]]
```

```
0 0 0 0
0 0 0 0
0 0 0 0
```

```
Grid[
 Table[N[Integrate[D[(Zeta[s, 1 / y + 1] y^ (1 - s)) ^k, y], {y, 0, 1}]], {s, 2, 4}, {k, 1, 4}]]
```

```
-0.355066  -0.58406   -0.731746   -0.826994
-0.297943  -0.209173  -0.116751   -0.0608332
 -0.25101  -0.104334  -0.0364791  -0.0122997
```

```
Grid[Table[
  N[Integrate[D[(Zeta[s, y + 1] y^ (s - 1)) ^k, y], {y, 1, Infinity}]], {s, 2, 4}, {k, 1, 4}]]
```

```
0.355066  0.58406   0.731746   0.826994
0.297943  0.209173  0.116751   0.0608332
 0.25101  0.104334  0.0364791  0.0122997
```

```
{Limit[(y^ (1 - s) HurwitzZeta[s, y^-1 + 1]) ^k, y → 0], 1 / (s - 1) ^k}
```

$$\left\{\left(\frac{1}{-1+s}\right)^{k},\ (-1+s)^{-k}\right\}$$

```
Grid[
 Table[Chop[N[1 / ((s - 1) ^k) + Integrate[D[(Zeta[s, y^-1 + 1] y^ (1 - s)) ^k, y], {y, 0, 1}]] -
    N[(Zeta[s] - 1) ^k]], {s, 2, 4}, {k, 1, 4}]]
```

```
0 0 0 0
0 0 0 0
0 0 0 0
```

```
Table[{Zeta[s] - 1,
  1 / (s - 1) - Integrate[D[y^ (s - 1) Zeta[s, y + 1], y], {y, 1, Infinity}]}, {s, 2, 6}]
```

$$\left\{\left\{-1+\frac{\pi^2}{6},\ -1+\frac{\pi^2}{6}\right\},\ \{-1+\text{Zeta}[3],\ -1+\text{Zeta}[3]\},\ \left\{-1+\frac{\pi^4}{90},\ -1+\frac{\pi^4}{90}\right\},\ \{-1+\text{Zeta}[5],\ -1+\text{Zeta}[5]\},\ \left\{-1+\frac{\pi^6}{945},\ -1+\frac{\pi^6}{945}\right\}\right\}$$

```
Table[-Integrate[D[y^ (s - 1) Zeta[s, y + 1], y], {y, 1, Infinity}], {s, 2, 6}]
```

$$\left\{-2+\frac{\pi^2}{6},\ -\frac{3}{2}+\text{Zeta}[3],\ -\frac{4}{3}+\frac{\pi^4}{90},\ -\frac{5}{4}+\text{Zeta}[5],\ -\frac{6}{5}+\frac{\pi^6}{945}\right\}$$

```
Table[Integrate[D[y^(1-s) Zeta[s, y^-1+1], y], {y, 0, 1}], {s, 2, 6}]
```

$$\left\{\int_0^1\left(-\frac{\text{Zeta}\left[2,1+\frac{1}{y}\right]}{y^2}+\frac{2\,\text{Zeta}\left[3,1+\frac{1}{y}\right]}{y^3}\right)dy,\ \int_0^1\left(-\frac{2\,\text{Zeta}\left[3,1+\frac{1}{y}\right]}{y^3}+\frac{3\,\text{Zeta}\left[4,1+\frac{1}{y}\right]}{y^4}\right)dy,\ \int_0^1\left(-\frac{3\,\text{Zeta}\left[4,1+\frac{1}{y}\right]}{y^4}+\frac{4\,\text{Zeta}\left[5,1+\frac{1}{y}\right]}{y^5}\right)dy,\ \int_0^1\left(-\frac{4\,\text{Zeta}\left[5,1+\frac{1}{y}\right]}{y^5}+\frac{5\,\text{Zeta}\left[6,1+\frac{1}{y}\right]}{y^6}\right)dy,\ \int_0^1\left(-\frac{5\,\text{Zeta}\left[6,1+\frac{1}{y}\right]}{y^6}+\frac{6\,\text{Zeta}\left[7,1+\frac{1}{y}\right]}{y^7}\right)dy\right\}$$

```
Grid[
 Table[Chop[N[1/((s-1)^k) + Integrate[D[(Zeta[s, y^-1+1] y^(1-s))^k, y], {y, 0, 1}]] -
    N[(Zeta[s]-1)^k]], {s, 2, 4}, {k, 1, 4}]]
```

0.355066	0.58406	0.731746	0.826994
0.375	0.215111	0.117208	0.0608684
0.270833	0.104727	0.0364869	0.0122999

```
{Limit[y^(1-s) HurwitzZeta[s, y^-1+1], y → 0], 1/(s-1)}
```

$$\left\{\frac{1}{-1+s},\ \frac{1}{-1+s}\right\}$$

```
{Limit[y^(s-1) HurwitzZeta[s, y+1], y → Infinity], 1/(s-1)}
```

$$\left\{\frac{1}{-1+s},\ \frac{1}{-1+s}\right\}$$

```
Integrate[ y^s, {y, 0, 1}]
```

$$\text{ConditionalExpression}\left[\frac{1}{1+s},\ \text{Re}[s]>-1\right]$$

```
N[Gamma[3, 0, (s-1) Log[x]] / Gamma[3] /. {x → 100000, s → 11}]
```

1.

```
(Zeta[s, y^-1+1] y^(1-s) + 1)^z -
 FullSimplify[Sum[Binomial[z, k] (Zeta[s, y^-1+1] y^(1-s))^k, {k, 0, Infinity}]]
```

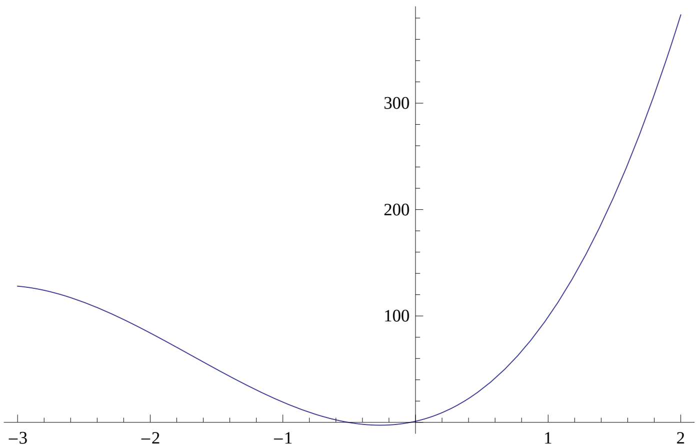
0

```
D1y1[x_, s_, k_, y_] := D1y1[x, s, k, y] =
  D1y1[x, s, k-1, y] + y Sum[(1+j y)^-s D1y1[x (1+j y)^-1, s, k-1, y], {j, 1, (x-1)/y}];
D1y1[x_, s_, 0, y_] := UnitStep[x-1]


bin[z_, k_] := Product[z-j, {j, 0, k-1}] / k!
Da[ n_, k_, a_, s_] :=
 Da[n, k, a, s] = Sum[ (j+a)^-s Da[ n/(j+a), k-1, a, s], {j, 1, n-a}];
Da[n_, 0, a_, s_] := UnitStep[n-1]
Daz[n_, z_, a_, s_] := Sum[ bin[ z, k] Da[n, k, a-1, s], {k, 0, Log[a, n]}]
D[N[Limit[(Daz[100, z, 2, s]-1)/z, z → 0]], s] /. s → 0
```

-94.0453

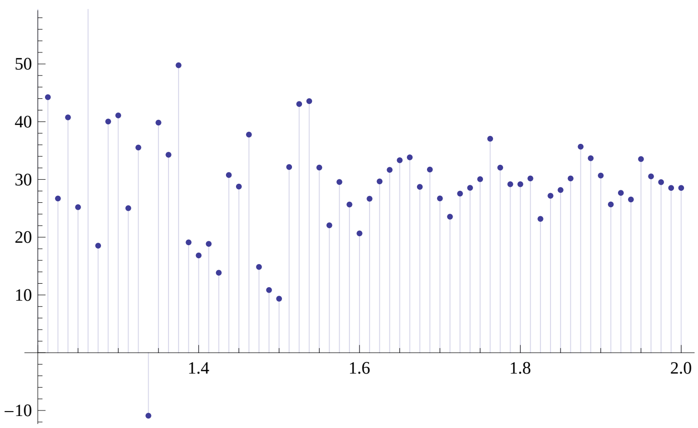
```
Plot[Re[Daz[100, z, 3, 0]], {z, -3, 2}]
```




```
DiscretePlot[-D[Limit[(Daz[n, z, 2, s] - 1) / z, z → 0], s] /. s → 0, {n, 2, 100}]
```

$Aborted

```
DiscretePlot[Limit[(Daz[100, z, a, 0] - 1) / z, z → 0], {a, 1.2, 2, .0125}]
```




```
Expand[Daz[100, z, 3, 0]]
```

$$1 + \frac{341 z}{12} + \frac{1391 z^2}{24} + \frac{139 z^3}{12} + \frac{z^4}{24}$$

```
FullSimplify[D[1 / (s - 1) ^k (Gamma[k, 0, (s - 1) Log[n]] / Gamma[k]), s] /. s → 0]
```

$$\frac{(-1)^{-k} \left(k\,\mathrm{Gamma}[k, 0, -\mathrm{Log}[n]] - n\,(-\mathrm{Log}[n])^k\right)}{\mathrm{Gamma}[k]}$$

```
1 / (s - 1) ^k (Gamma[k, 0, (s - 1) Log[n]] / Gamma[k])
```

$$\frac{(-1+s)^{-k}\,\mathrm{Gamma}[k, 0, (-1+s)\,\mathrm{Log}[n]]}{\mathrm{Gamma}[k]}$$

```
FullSimplify[
 Table[ Zeta[s, a]^k - Sum[(a^-s)^(k - j) Binomial[k, j] Zeta[s, a + 1]^j, {j, 0, k}],
  {k, 1, 5}, {a, 2, 5}, {s, 2, 4}]]
```

```
{{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}},
 {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}}, {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}},
 {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}}, {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}}}
```

```
FullSimplify[Zeta[s, a] - Zeta[s, a + 1]] /. {s → 3, a → 3}
```

```
N[Zeta[3, 3] - Zeta[3, 4]]
```

```
0.037037
```

```
N[3^-3]
```

```
0.037037
```

```
(a^-s)
```

```
FullSimplify[
 Table[ Zeta[s, a]^k - Sum[a^(-s (k - j)) Binomial[k, j] Zeta[s, a + 1]^j, {j, 0, k}],
  {k, 1, 5}, {a, 2, 5}, {s, 2, 4}]]
```

```
{{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}},
 {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}}, {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}},
 {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}}, {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}}}
```

```
N[Table[ Zeta[s, a]^k - Sum[a^(-s (j)) Binomial[k, j] Zeta[s, a + 1]^(k - j), {j, 0, k}],
  {k, 2, 3}, {a, 2, 3}, {s, 2, 3}]]
```

$\{\{\{0., -2.08167\times10^{-17}\}, \{2.77556\times10^{-17}, 8.67362\times10^{-19}\}\},$
$\{\{-2.77556\times10^{-17}, -6.50521\times10^{-18}\}, \{2.77556\times10^{-17}, 5.42101\times10^{-20}\}\}\}$

```
N[Table[ Zeta[s, a]^k - Sum[(-1)^(k - j) (a - 1)^(-s j) Binomial[k, j] Zeta[s, a - 1]^(k - j),
    {j, 0, k}], {k, 2, 3}, {a, 4, 6}, {s, 2, 3}]]
```

$\{\{\{-2.77556\times10^{-17}, -8.67362\times10^{-19}\}, \{0., -2.1684\times10^{-19}\}, \{-1.38778\times10^{-17}, 1.0842\times10^{-19}\}\},$
$\{\{0.045727, 0.000128191\}, \{0.0216825, 0.0000290352\}, \{0.0119231, 8.81361\times10^{-6}\}\}\}$

```
FullSimplify[
 Table[ Zeta[s, a]^k - Sum[(-1)^(k - j) (a - 1)^(-s (k - j)) Binomial[k, j] Zeta[s, a - 1]^j,
    {j, 0, k}], {k, 1, 5}, {a, 2, 5}, {s, 2, 4}]]
```

```
{{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}},
 {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}}, {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}},
 {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}}, {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}}}
```

```
FullSimplify[
 Table[ Chop[Zeta[s, a]^z - Sum[(-1)^j (a - 1)^(-s j) Binomial[z, j] Zeta[s, a - 1]^(z - j),
     {j, 0, Infinity}]], {z, 2.5, 5, .7}, {a, 2, 5}, {s, 2, 4}]]
```

```
{{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}}, {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}},
 {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}}, {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}}}
```

```
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
Dz[n_, z_] := Sum[dz[j, z], {j, 1, n}]
D2z[n_, z_, t_] := Sum[(-1)^j Binomial[z, j] Dz[n, z - j], {j, 0, t}]
```

```
Table[ D2z[100, 2.5, j], {j, 2, 50}]

{331.104, 332.443, 332.065, 331.713, 331.34, 330.829, 330.118, 329.183,
 328.018, 326.623, 324.994, 323.124, 320.995, 318.583, 315.852, 312.761,
 309.255, 305.275, 300.751, 295.607, 289.758, 283.111, 275.57, 267.027, 257.372,
 246.487, 234.247, 220.524, 205.183, 188.083, 169.079, 148.02, 124.751, 99.1129,
 70.9396, 40.0625, 6.30812, -30.5014, -70.5482, -114.018, -161.102, -211.992,
 -266.888, -325.991, -389.506, -457.643, -530.613, -608.634, -691.923}

FullSimplify[
 Table[Zeta[s, a]^k - Sum[(-1)^j (m - 1)^(-s j) Binomial[k, j] Zeta[s, m - 1]^(k - j),
    {j, 1, k}, {m, a + 1, Infinity}], {k, 1, 5}, {a, 2, 5}, {s, 2, 4}]]

$Aborted

Zeta[s, a]^k -
 Sum[m^(-s j) Binomial[k, j] Zeta[s, m + 1]^(k - j), {j, 1, k}, {m, a + 1, Infinity}]

$Aborted

 /. {k → 1, a → 2, s → 2}

Binomial[2, 1]

2

Sum[ m^-s l^-s, {m, 2, Infinity}, {l, m + 1, Infinity}]
```

$$\sum_{m=2}^{\infty} \sum_{l=1+m}^{\infty} l^{-s} m^{-s}$$

```
Sum[ m^-s l^-s, {m, 4, Infinity}, {l, 4, Infinity}]
```

$$(-1 - 2^{-s} - 3^{-s} + \mathrm{Zeta}[s])^2$$

```
Dy[n_, x_, k_] := x Sum[x Dy[n (j x + 1)^-1, x, k - 1], {j, 1, (n - 1) / x}];
Dy[x_, y_, 0] := UnitStep[x - 1]
Ds[ n_, k_] := Sum[ Ds[Floor[n / j], k - 1], {j, 1, n}]; Ds[n_, 0] := UnitStep[n - 1]
Dss[n_, k_, x_] := x^(k (1 - 0)) Ds[n x^-k, k]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
Da[ n_, k_, a_, s_] :=
 Da[n, k, a, s] = Sum[ (j + a)^-s Da[ n / (j + a), k - 1, a, s], {j, 1, n - a}];
Da[n_, 0, a_, s_] := UnitStep[n - 1]
Daz[n_, z_, a_, s_] := Sum[ bin[ z, k] Da[n, k, a - 1, s], {k, 0, Log[a, n]}]
```

```
Dy[100, 2, 2]

896

2^-2 Da[100 × 2^2, 2, 2, 0]

318

Dd[x_, 0, y_] := UnitStep[x - 1]; Dd[x_, 1, y_] := Floor[x] - y + 1
Dd[x_, k_, y_] :=
 Sum[Binomial[k, j] Dd[x / (m^(k - j)), j, m + 1], {m, y, x^(1 / k)}, {j, 0, k - 1}]
Cc[x_, k_, y_] := y^-k Dd[x y^k, k, y + 1]

Cc[100, 2, 2]

318

E1a[n_, k_, x_, s_] := E1a[n, k, x, s] = Sum[j^(-s) E1a[n / j, k - 1, x, s], {j, 1, n}] -
   x Sum[(j x)^(-s) E1a[n / (x j), k - 1, x, s], {j, 1, n / x}];
E1a[n_, 0, a_, s_] := UnitStep[n - 1]
E1ab[n_, k_, x_, s_] := E1ab[n, k, x, s] =
  Sum[j^(-s) E1ab[n / j, k - 1, x, s] - x (j x)^(-s) E1ab[n / (x j), k - 1, x, s], {j, 1, n}];
E1ab[n_, 0, a_, s_] := UnitStep[n - 1]
E1c[n_, k_, x_, s_] :=
 Sum[j^(-s) E1c[n / j, k - 1, x, s] - x (j x)^(-s) E1c[n / (x j), k - 1, x, s], {j, 1, n}];
E1c[n_, 0, a_, s_] := UnitStep[n - 1]

E1a[100, 3, 1.1, 0]

-34.702


Dk[n_, k_, s_] := Dk[n, k, s] = Sum[j^(-s) Dk[Floor[n / j], k - 1, s], {j, 1, n}];
Dk[n_, 0, s_] := UnitStep[n - 1]
E1cc[n_, k_, x_, s_] := Sum[(-1)^j Binomial[k, j] x^(j (1 - s)) Dk[n / x^j, k, s], {j, 0, k}]

E1cc[101, 3, .5, 0]

-3.875

N[(1 / 2)^3 Dk[ 101 (1 / 2)^-3, 3, 0]]

2805.38

Limit[(n x^-1 + n^0)^x, x → Infinity]
```

e^n

```
Limit[ (n^x - n^0) / x, x → 0]

Log[n]

Limit[ (Dz[n, x] - Dz[n, 0]) / x, x → 0]

dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
Dz[n_, z_] := Sum[dz[j, z], {j, 1, n}]
```

```
Dz[100, 0]

1

Limit[ (Dz[100, x] - Dz[100, 0]) / x, x → 0]

428
───
15

ff[ n_, z_, t_] := Sum[ z^k / k! Dz[n, k], {k, 0, t}]

N[ff[100, -4, 22]]

-0.350427

N[HarmonicNumber[2 000 000] - HarmonicNumber[2 000 000 / 12]]

2.4849

N[Log[12]]

2.48491

HarmonicNumber[2 000 000 / 20] + N[Log[20]]

15.0859

N[HarmonicNumber[2 000 000]]

15.0859

Limit[ HarmonicNumber[71 n] - HarmonicNumber[n], n → Infinity]

Log[71]

Limit[ HarmonicNumber[n] - Log[n], n → Infinity]

EulerGamma


Limit[ (1 - 3^(1 - s)) Zeta[s], s → 1]

Log[3]

f1[n_] := Sum[ (-1)^(j + 1) / j, {j, 1, 2 n}]

f2[n_] := Sum[ 1 / j, {j, 1, 2 n}] - 2 Sum[ 1 / (2 j), {j, 1, n}]

f3[n_] := Sum[ 1 / j, {j, 1, 2 n}] - Sum[ 1 / (j), {j, 1, n}]

f4[n_] := (Sum[ 1 / j, {j, 1, 2 n}] - Log[2 n]) - (Sum[ 1 / (j), {j, 1, n}] - Log[n] - Log[2])

Limit[ f4[n], n → Infinity]

Log[2]

N[Sum[ (-1)^(k + 1) / k 1 / (s - 1)^k Gamma[ k, 0, (s - 1) Log[n]] / Gamma[k], {k, 1, 20}] /.
  {s → 0, n → 100}]

28.0217 - 2.09386 × 10^-14 i

-N[Gamma[0, -Log[100]]] - N[Log[Log[100]]] - EulerGamma

28.0217 + 3.14159 i

N[Sum[ ((-1)^(k + 1) / k) (1 / ((s - 1)^k)) Gamma[ k, 0, (s - 1) Log[n]] / Gamma[k], {k, 1, 30}] /.
  {s → -1, n → 100}]

1215.32 - 5.98153 × 10^-13 i
```

```
-N[Gamma[0, (s - 1) Log[100]]] - N[Log[(s - 1) Log[100]]] - EulerGamma /. s → -1
```

1243.34 + 0. ⅈ

```
Sum[ ((-1) ^ (k + 1) / k) (1 / ((s - 1) ^k)) Gamma[ k, 0, (s - 1) Log[n]] / Gamma[k], {k, 1, Infinity}]
```

$Aborted

```
Sum[ ((-1) ^ (k + 1) / k) (1 / ((s - 1) ^k))
   Integrate[ t^ (k - 1) E^-t, {t, 0, (s - 1) Log[n]}] /Gamma[k], {k, 1, Infinity}] /. s → 0
```

$Aborted

```
N[Zeta[ 2, 8]]
```

0.133137

```
N[1 / (8) ^2 Sum[ Zeta[ 2, 1 + k / 8], {k, 0, 7}]]
```

0.133137

```
f1[ s_, m_, z_] := Zeta[ s, m z]
f2[ s_, m_, z_] := 1 / (m^s) Sum[ Zeta[ s, z + k / m], {k, 0, m - 1}]
N[f1[2, 4, 1]]
```

0.283823

```
N[f2[2, 4, 1]]
```

0.283823

```
N[Zeta[2, 5]]
```

0.221323

```
N[ 1 / 5^2 + Zeta[2, 6]]
```

0.221323

```
FullSimplify[(1 / (a - 1) ^s + 1 / a^s + Zeta[ s, a + 1]) ^2]
```

$((-1 + a)^{-s} + a^{-s} + \text{Zeta}[s, 1 + a])^2$

```
5040 - 720
4320 - 600
```

3720

```
s Integrate[ Floor[x] / (x^ (s + 1)), {x, 1, Infinity}]
```

$s \int_1^{\infty} x^{-1-s}\, \text{Floor}[x]\, dx$

```
dz[n_, z_] := dz[n, z] = Product[(-1) ^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
dsz[n_, z_, s_] := n^-s dz[n, z]
Dsz[n_, z_, s_] := Sum[dsz[j, z, s], {j, 1, n}]

N[Dsz[10 000, -1, 3]]
```

0.831907

```
N[Zeta[3]]^-1
```

0.831907

```
N[D[Dsz[18000, 1, s], s]] /. s → ZetaZero[1]
```

-27.2736 - 88.4625 i

```
E1a[n_, k_, x_, s_] := E1a[n, k, x, s] = Sum[j^(-s) E1a[n/j, k-1, x, s], {j, 1, n}] -
   x Sum[(j x)^(-s) E1a[n/(x j), k-1, x, s], {j, 1, n/x}];
E1a[n_, 0, a_, s_] := UnitStep[n-1]
E11[n_, s_] := Sum[j^(-s) UnitStep[n/j-1], {j, 1, n}] -
  2 Sum[(2 j)^(-s) UnitStep[n/(2 j)-1], {j, 1, n/2}]
DDc[n_, s_] := Sum[2^(j (1-s)) E11[n/(2^j), s], {j, 0, Log[2, n]}]
```

```
E11[1000000, 1, 2, N[ZetaZero[1]]]
```

-0.000438861 + 0.000239584 i

```
DDc[100000, N[ZetaZero[1]]]
```

-12.5386 + 18.5117 i

```
Sum[1/j^3, {j, 1, 10}] - 3 Integrate[Floor[x]/x^(3+1), {x, 1, 10}]
```

$\frac{1}{100}$

```
Sum[1/j^3, {j, 1, 10}]
```

$\frac{19164113947}{16003008000}$

```
Sum[1/j^s, {j, 1, n}] - n^(-(s-1)) -
  s Integrate[Floor[x]/x^(s+1), {x, 1, n}] /. {s → 3, n → 20}
```

0

```
Sum[1/j^s, {j, 2, n}] + 2^(-s) - n^(-(s-1)) -
  s Integrate[Floor[x]/x^(s+1), {x, 2, n}] /. {s → 3, n → 15}
```

0

```
Dz[n_, z_, k_] := Dz[n, z, k] = 1 + ((z+1)/k-1) Sum[Dz[n/j, z, k+1], {j, 2, n}]
(Sum[1/j^2, {j, 1, 10}])^2 - 10^-1 - 2 Integrate[Dz[Floor[x], 2, 1]/x^(2+1), {x, 1, 10}]
```

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

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General::stop : Further output of $RecursionLimit::reclim will be suppressed during this calculation. ≫

$IterationLimit::itlim : Iteration limit of 4096 exceeded. ≫

$IterationLimit::itlim : Iteration limit of 4096 exceeded. ≫

0

```
f[fn_, n_, k_, s_] := f[fn, n, k, s] = Sum[ j^-s fn[j] f[fn, n / j, k - 1, s], {j, 1, n}];
f[fn_, n_, 0, s_] := UnitStep[n - 1]
fm1[fn_, n_, k_, s_] :=
 fm1[fn, n, k, s] = Sum[ j^-s fn[j] fm1[fn, n / j, k - 1, s], {j, 2, n}];
fm1[fn_, n_, 0, s_] := UnitStep[n - 1]
fz[fn_, n_, z_, s_] := Sum[ Binomial[z, k] fm1[fn, n, k, s], {k, 0, Log[2, n]}]
dz[n_, z_] := dz[n, z] = Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
dsz[n_, z_, s_] := n^-s dz[n, z]
Dsz[n_, z_, s_] := Sum[dsz[j, z, s], {j, 1, n}]
```

```
Limit[ D[f[id, 10, 1, s], s], s → 0]
```

```
-id[2] Log[2] - id[3] Log[3] - id[4] Log[4] - id[5] Log[5] -
 id[6] Log[6] - id[7] Log[7] - id[8] Log[8] - id[9] Log[9] - id[10] Log[10]
```

```
Limit[ D[fz[id, 10, 1, s], s], s → 0]
```

```
-id[2] Log[2] - id[3] Log[3] - id[4] Log[4] - id[5] Log[5] -
 id[6] Log[6] - id[7] Log[7] - id[8] Log[8] - id[9] Log[9] - id[10] Log[10]
```

```
FullSimplify[Limit[ D[Limit[D[fz[id, 10, z, s], z], z → 0], s], s → 0]]
```

$\text{id}[2]^2 \,\text{Log}[2] - \text{id}[2]^3 \,\text{Log}[2] + (-1 + \text{id}[3])\,\text{id}[3]\,\text{Log}[3] - \text{id}[4]\,\text{Log}[4] -$
$\text{id}[5]\,\text{Log}[5] - \text{id}[6]\,\text{Log}[6] - \text{id}[7]\,\text{Log}[7] - \text{id}[8]\,\text{Log}[8] - \text{id}[9]\,\text{Log}[9] -$
$\text{id}[10]\,\text{Log}[10] + \text{id}[2]\,(-\text{Log}[2] + \text{id}[3]\,\text{Log}[6] + \text{id}[4]\,\text{Log}[8] + \text{id}[5]\,\text{Log}[10])$

```
N[Limit[ D[fz[LiouvilleLambda, 10, 3, s], s], s → 0]]
```

```
-21.5145
```

```
N[-Sum[LiouvilleLambda[j] Log[j] dz[j, 3], {j, 1, 10}]]
```

```
-21.5145
```