

```

Clear[dda, dz]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, z_] := dz[n, z] = Product[(-1)^p[[2]] bin[-z, p[[2]]], {p, FI[n]}]
dda[n_, m_, s_, z_] := Sum[a^(-s b^(-s dz[a, z] dz[b, -z]), {a, 1, n}, {b, 1, (n/a)^m}]
ddz1[n_, m_, s_, z_] :=
  Sum[a^(-s b^(-s dz[a, z] dz[b, -z]), {a, 1, n}, {b, 1, (n/a)^(N@Log[m] / Log[n])}]
d2[n_, m_, s_, k_] := Sum[(-1)^(k - j) bin[k, j] dda[n, m, s, j], {j, 0, k}]
ddz[n_, m_, s_, z_] := Sum[bin[z, k] d2[n, N@Log[m] / Log[n], s, k], {k, 0, Log2@n}]
ld[n_, m_, s_] := Sum[(-1)^(k + 1) / k d2[n, N@Log[m] / Log[n], s, k], {k, 1, Log2@n}]
ldz[n_, m_, s_] := D[ddz1[n, m, s, z], z] /. z -> 0
sdda[n_, m_, s_, t_, z_] := Sum[a^(-s b^(-t dz[a, z] dz[b, -z]), {a, 1, n}, {b, 1, (n/a)^m}]
sddz[n_, m_, s_, t_, z_] :=
  Sum[a^(-s b^(-t dz[a, z] dz[b, -z]), {a, 1, n}, {b, 1, (n/a)^(N@Log[m] / Log[n])}]
sd2[n_, m_, s_, t_, k_] := Sum[(-1)^(k - j) bin[k, j] sdda[n, m, s, t, j], {j, 0, k}]
sld[n_, m_, s_, t_] := Sum[(-1)^(k + 1) / k sd2[n, N@Log[m] / Log[n], s, t, k], {k, 1, Log2@n}]
sldz[n_, m_, s_, t_] := D[sddz[n, m, s, t, z], z] /. z -> 0
pr[n_, s_] := Sum[FullSimplify[MangoldtLambda[j] / Log[j]] j^(-s), {j, 2, n}]
ch[n_] := -Sum[MangoldtLambda[j], {j, 2, n}]

ldz[1200, 32, -3]
23 461 752 552 682
-----
315
pr[1200, -3] - pr[32, -3]
23 461 752 552 682
-----
315
sldz[1200, 320, -2, -4]
159 395 771 066 767
-----
1260
pr[1200, -2] - pr[320, -4]
159 395 771 066 767
-----
1260
fo[n_, s_, k_] :=
  fo[n, s, k] = Sum[Abs[MoebiusMu[j]] j^(-s fo[Floor[n / j], s, k - 1], {j, 2, n}]
fo[n_, s_, 0] := UnitStep[n - 1]
lf[n_, s_] := Sum[(-1)^(k + 1) / k fo[n, s, k], {k, 1, Log2@n}]
lf[1000, -2]
15 167 671 034
-----
315
pr[1000, -2] - pr[1000^(1/2), -4]
15 167 671 034
-----
315

```

**N[D[ld[1000, 910, s], s] /. s → 0]**

-92.6396

**N[ch[1000] - ch[910]]**

-92.6396

**Expand[ddz1[100, 50, 1, z]]**

$$1 + \frac{5\,709\,300\,759\,326\,076\,208\,729\,z}{38\,875\,468\,949\,180\,319\,102\,336} + \frac{4\,601\,729\,822\,212\,889\,z^2}{190\,035\,075\,607\,977\,600} - \frac{598\,781\,923\,z^3}{33\,418\,344\,960} + \frac{6331\,z^4}{422\,400} + \frac{397\,z^5}{69\,120} - \frac{z^6}{7680}$$

**Expand[ddz[100, 50, 1, z]]**

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**D[ddz1[100, 50, 1, z], z] /. z → 0**

$$\frac{5\,709\,300\,759\,326\,076\,208\,729}{38\,875\,468\,949\,180\,319\,102\,336}$$

**pr[100, 1] - pr[50, 1]**

$$\frac{5\,709\,300\,759\,326\,076\,208\,729}{38\,875\,468\,949\,180\,319\,102\,336}$$

**N[D[D[ddz1[100, 50, s, z], z] /. z → 0, s] /. s → 0]**

-44.5599

**N[ch[100] - ch[50]]**

-44.5599