```
Timing[PrimePi[10^14]]
```

```
{60.138, 3 204 941 750 802}
```

```
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
(* f is the partial sum of the hurwitzzeta
 function dirichlet convolved to the kth power*)
f[n_, s_, q_, 0] := UnitStep[n - 1]
f[n_, s_, q_, k_] :=
 f[n, s, q, k] = Sum[ (j + q)^(-s) f[n / (j + q), s, q, k - 1], {j, 0, n - q}]
(* g is the partial sum of (the hurwitzzeta function+1)
 convolved to the zth complex power*)
g[n_, s_, q_, z_] := Sum[ bin[z, k] f[n, s, q, k], {k, 0, Log[q, n] + 1}]
Grid[Table[
  FullSimplify[g[n, 0, 2 + j (1 / 3), z] - g[n - 1, 0, 2 + j (1 / 3), z]], {n, 2, 40}, {j, 0, 10}]]
```

z	0	0	0	0	0	0	0	0	0	0
z	z	z	z	0	0	0	0	0	0	0
$\frac{1}{2} z (1+z)$	z	z	z	z	z	z	0	0	0	0
z	z	z	z	z	z	z	z	z	z	0
z^2	$\frac{1}{2} z (1+z)$	z	z	z	z	z	z	z	z	z
z	z	z	z	z	z	z	z	z	z	z
$\frac{1}{6} z (1+z)(2+z)$	z^2	$\frac{1}{2} z (1+z)$	z	z	z	z	z	z	z	z
$\frac{1}{2} z (1+z)$	z	z	$\frac{1}{2} z (1+z)$	z	z	z	z	z	z	z
z^2	z	z^2	z	z	z	z	z	z	z	z
z	z^2	z	z	z	z	z	z	z	z	z
$\frac{1}{2} z^2 (1+z)$	$\frac{1}{2} z (1+z)$	z	z^2	$\frac{1}{2} z (1+z)$	z	z	z	z	z	z
z	$\frac{1}{6} z (1+z)(2+z)$	z^2	z	z	z	z	z	z	z	z
z^2	z	$\frac{1}{2} z (1+z)$	z	z	$\frac{1}{2} z (1+z)$	z	z	z	z	z
z^2	$z (-1+2z)$	z	z^2	z^2	z	z	z	z	z	z
$\frac{1}{24} z (1+z)(2+z)(3+z)$	z	z^2	$\frac{1}{2} z (1+z)$	z	z	$\frac{1}{2} z (1+z)$	z	z	z	z
z	z	z	z	z	z	z	z	z	z	z
$\frac{1}{2} z^2 (1+z)$	$z (-1+2z)$	$z (-1+2z)$	z^2	z^2	z^2	z	z	z	z	z

z	$\frac{1}{2} z (3 + (-2 + z) z)$	$\frac{1}{6} z (8 + (-3 + z) z)$	z	$\frac{1}{2} z (1 + z)$	z	z	$\frac{1}{2} z (1 + z)$	z	z	z
$\frac{1}{2} z^2 (1 + z)$	z^2	z	z^2	z	z	z^2	z	z	z	z
z^2	z	$z (-1 + 2 z)$	z^2	z	z^2	z	z	z	z	z
z^2	$z (-1 + 2 z)$	$\frac{1}{2} z (1 + z)$	z	z^2	$\frac{1}{2} z (1 + z)$	z	z	$\frac{1}{2} z (1 + z)$	z	z
z	z	z	z	z	z	z	z	z	z	z
$\frac{1}{6} z^2 (1 + z) (2 + z)$	$\frac{1}{2} z (2 + (-1 + z) z)$	z^2	$z (-1 + 2 z)$	z^2	z	z^2	z^2	z	z	z
$\frac{1}{2} z (1 + z)$	$z (-1 + 2 z)$	z^2	$\frac{1}{2} z (1 + z)$	z^2	z^2	$\frac{1}{2} z (1 + z)$	z	z	$\frac{1}{2} z (1 + z)$	z
z^2	$\frac{1}{2} z (4 + (-3 + z) z)$	z^2	z	z	z	z	z	z	z	z
$\frac{1}{6} z (1 + z) (2 + z)$	z^2	$\frac{1}{2} z (2 + (-1 + z) z)$	$\frac{1}{6} z (1 + z) (2 + z)$	z	z^2	z	z	z^2	z	z
$\frac{1}{2} z^2 (1 + z)$	$z (-1 + 2 z)$	z	z^2	$z (-1 + 2 z)$	z	z^2	z^2	z	z	z
z	$\frac{1}{2} z (-1 + 3 z)$	$z (-1 + 2 z)$	z	$\frac{1}{2} z (1 + z)$	z^2	z	$\frac{1}{2} z (1 + z)$	z	z	$\frac{1}{2} z (1 + z)$
z^3	$\frac{1}{24} z (42 + z (-25 + z (6 + z)))$	z	$z (-1 + 2 z)$	z	z	z^2	z	z	z^2	z
z	z	z	z	z	z	z	z	z	z	z

$\frac{1}{120} z (1+z)(2+z)(3+z)(4+z)$	$z(-2+3z)$	$z(-2+3z)$	z^2	$z(-1+2z)$	$z(-1+2z)$	z^2	z^2	z^2	z	z
z^2	z	$\frac{1}{2} z (1+z)$	z^2	z	$\frac{1}{2} z (1+z)$	z	z	$\frac{1}{2} z (1+z)$	z	z
z^2	$z(1+(-1+z)z)$	$\frac{1}{2} z (2+(-1+z)z)$	z	z^2	z	z	z^2	z	z	z^2
z^2	$\frac{1}{2} z (2+(-1+z)z)$	z	z^2	z^2	z	z^2	z	z	z^2	z
$\frac{1}{4} z^2 (1+z)^2$	z^2	$\frac{1}{2} z^2 (1+z)$	$\frac{1}{2} z (-1+z(2+z))$	z	$z(-1+2z)$	$\frac{1}{2} z (-1+3z)$	z	z^2	$\frac{1}{2} z (1+z)$	z
z	z^2	z^2	z	z^2	z	z	z^2	z	z	z
z^2	$\frac{1}{6} z (1+z)(2+z)$	z^2	z	$\frac{1}{6} z (1+z)(2+z)$	z^2	z	z	z^2	z	z
z^2	z^2	z	z^2	z	z	z	z	z	z	z
$\frac{1}{6} z^2 (1+z)(2+z)$	$\frac{1}{2} z (2+(-1+z)z)$	$z(-1+2z)$	$z(-1+2z)$	z^2	z^2	$z(-1+2z)$	z^2	z	z^2	z^2

```
N[g[8000, 3, 4.5, -3]] - N[(HurwitzZeta[3, 4.5] + 1)^-3]
```

-7.85653×10^{-8}

```
Grid[Table[N[g[8000, 2 + j * .3, 2 + n * .15, -3]] -
   N[(HurwitzZeta[2 + j * .3, 4.5] + 1)^-3], {n, 1, 10}, {j, 0, 5}]]
```

$Aborted

```
f[n_, q_, 0] := If[n == 1, 1, 0]
f[n_, q_, 1] := If[ n ≥ q, 1, 0]
f[n_, q_, k_] := Sum[ f[j, q, 1] f[n / j, q, k - 1], {j, Divisors[n]}]
falt[n_, q_, k_] :=
 Sum[ (-1)^j Binomial[ k, j] (q - 1)^(k - j) f[ Floor[n / q^(k - j)], q - 1, j], {j, 0, k}]
falt2[n_, q_, k_] := Sum[ Binomial[k, j] If[n / q^(k - j) == Floor[n / q^(k - j)], q^(k - j), 1]
   f[ Floor[n / If[n / (q + 1)^(k - j) == Floor[n / (q + 1)^(k - j)], (q + 1)^(k - j), 1]],
    q + 1, j], {j, 0, k}]
```

```
f[100 × 2^5, 2, 3]
```

147

```
falt2[100 × 2^5, 2, 3]
```

225