```
Expand[Sum[ 4 / (4 k + 1) - 4 / (4 k + 3), {k, 0, Infinity}]]
```

π

```
Expand[Sum[ 2 / (4 k + 2) - 2 / (4 k + 4), {k, 0, Infinity}]]
```

$\text{Log}[2]$

```
FullSimplify[Sum[ 4 / (6 k + 1) + 4 / (6 k + 3) - 8 / (6 k + 5), {k, 0, Infinity}]]
```

$\sqrt{3}\ \pi - \text{Log}[3]$

```
FullSimplify[Sum[ -8 / (6 k + 1) + 4 / (6 k + 3) + 4 / (6 k + 5), {k, 0, Infinity}]]
```

$-\sqrt{3}\ \pi - \text{Log}[3]$

```
FullSimplify[Sum[ 2 / (6 k + 1) - 4 / (6 k + 3) + 2 / (6 k + 5), {k, 0, Infinity}]]
```

$\text{Log}[3]$

```
FullSimplify[Sum[ 4 / (6 k + 1) - 8 / (6 k + 3) + 4 / (6 k + 5), {k, 0, Infinity}]]
```

$\text{Log}[9]$

```
FullSimplify[Sum[2 (3^(1 / 2)) / (6 k + 1) - 2 (3^(1 / 2)) / (6 k + 5), {k, 0, Infinity}]]
```

π

```
FullSimplify[Sum[ 1 / (3 k + 1) + 1 / (3 k + 2) - 2 / (3 k + 3), {k, 0, Infinity}]]
```

$\text{Log}[3]$

```
FullSimplify[Sum[ -12 / (3 k + 1) + 6 / (3 k + 2) + 6 / (3 k + 3), {k, 0, Infinity}]]
```

$-\sqrt{3}\ \pi - 3\,\text{Log}[3]$

```
FullSimplify[Sum[ 6 / (3 k + 1) - 12 / (3 k + 2) + 6 / (3 k + 3), {k, 0, Infinity}]]
```

$\sqrt{3}\ \pi - 3\,\text{Log}[3]$

```
FullSimplify[Sum[ 3 (3^(1 / 2)) / (3 k + 1) - 3 (3^(1 / 2)) / (3 k + 2), {k, 0, Infinity}]]
```

π

```
FullSimplify[Sum[12 (3^(1 / 2)) / (6 k + 2) - 12 (3^(1 / 2)) / (6 k + 6), {k, 0, Infinity}]]
```

$\pi + \sqrt{3}\ \text{Log}[27]$

```
FullSimplify[Sum[6 (3^(1 / 2)) / (6 k + 2) - 6 (3^(1 / 2)) / (6 k + 4), {k, 0, Infinity}]]
```

π

```
FullSimplify[Sum[12 (3^(1 / 2)) / (6 k + 4) - 12 (3^(1 / 2)) / (6 k + 6), {k, 0, Infinity}]]
```

$-\pi + \sqrt{3}\ \text{Log}[27]$

```
FullSimplify[Sum[1 / (6 k + 1) + 1 / (6 k + 2) -
    0 / (6 k + 3) - 2 / (6 k + 4) - 1 / (6 k + 5) + 1 / (6 k + 6), {k, 0, Infinity}]]
```

$$\frac{1}{4}\left(\sqrt{3}\ \pi - \text{Log}[3]\right)$$

```
FullSimplify[Sum[(2 × 3^(1 / 2)) / (6 k + 1) + (2 × 3^(1 / 2)) / (6 k + 2) - (2 × 3^(1 / 2)) / (6 k + 3) -
    2 (2 × 3^(1 / 2)) / (6 k + 4) - 0 / (6 k + 5) + (2 × 3^(1 / 2)) / (6 k + 6), {k, 0, Infinity}]]
```

$$\pi$$

```
FullSimplify[Sum[0 / (6 k + 1) + 12 / (6 k + 2) -
    18 / (6 k + 3) - 6 / (6 k + 4) + 6 / (6 k + 5) + 6 / (6 k + 6), {k, 0, Infinity}]]
```

$$\text{Log}\left[\frac{27}{16}\right]$$

```
FullSimplify[Sum[(3 / 4) / (6 k + 1) - (3 / 4) / (6 k + 2) + 2 (3 / 4) / (6 k + 3) -
    (3 / 4) / (6 k + 4) + (3 / 4) / (6 k + 5) - 2 (3 / 4) / (6 k + 6), {k, 0, Infinity}]]
```

$$\text{Log}[2]$$

```
FullSimplify[Sum[1 / (6 k + 1) - 1 / (6 k + 2) +
    1 / (6 k + 3) - 1 / (6 k + 4) + 1 / (6 k + 5) - 1 / (6 k + 6), {k, 0, Infinity}]]
```

$$\text{Log}[2]$$

```
FullSimplify[Sum[(3 / 4) / (6 k + 1) - (3 / 4) / (6 k + 2) + 2 (3 / 4) / (6 k + 3) -
     (3 / 4) / (6 k + 4) + (3 / 4) / (6 k + 5) - 2 (3 / 4) / (6 k + 6), {k, 0, Infinity}]] -
 FullSimplify[Sum[1 / (6 k + 1) - 1 / (6 k + 2) + 1 / (6 k + 3) - 1 / (6 k + 4) +
     1 / (6 k + 5) - 1 / (6 k + 6), {k, 0, Infinity}]]
```

$$0$$

```
((3 / 4) / (6 k + 1) - (3 / 4) / (6 k + 2) + 2 (3 / 4) / (6 k + 3) -
    (3 / 4) / (6 k + 4) + (3 / 4) / (6 k + 5) - 2 (3 / 4) / (6 k + 6)) -
 (1 / (6 k + 1) - 1 / (6 k + 2) + 1 / (6 k + 3) - 1 / (6 k + 4) + 1 / (6 k + 5) - 1 / (6 k + 6))
```

$$-\frac{1}{4\,(1+6\,k)}+\frac{1}{4\,(2+6\,k)}+\frac{1}{2\,(3+6\,k)}+\frac{1}{4\,(4+6\,k)}-\frac{1}{4\,(5+6\,k)}-\frac{1}{2\,(6+6\,k)}$$

```
FullSimplify[
 Sum[-1 / (4 (1 + 6 k)) + 1 / (4 (2 + 6 k)) + 1 / (2 (3 + 6 k)) + 1 / (4 (4 + 6 k)) - 1 / (4 (5 + 6 k)) - 1 / (2 (6 + 6 k)), {k, 0, Infinity}]]
```

$$0$$

$$-\frac{4}{4\,(1+6\,k)}+\frac{4}{4\,(2+6\,k)}+\frac{4}{2\,(3+6\,k)}+\frac{4}{4\,(4+6\,k)}-\frac{4}{4\,(5+6\,k)}-\frac{4}{2\,(6+6\,k)}$$

$$-\frac{1}{1+6\,k}+\frac{1}{2+6\,k}+\frac{2}{3+6\,k}+\frac{1}{4+6\,k}-\frac{1}{5+6\,k}-\frac{2}{6+6\,k}$$

```
FullSimplify[Sum[-1/(1+6k) + 1/(2+6k) + 2/(3+6k) + 1/(4+6k) - 1/(5+6k) - 2/(6+6k), {k, 0, Infinity}]]
```

```
0
```

```
FullSimplify[Sum[-1/(4 (1+6k)^3) + 1/(4 (2+6k)^3) +
    1/(2 (3+6k)^3) + 1/(4 (4+6k)^3) - 1/(4 (5+6k)^3) - 1/(2 (6+6k)^3), {k, 0, Infinity}]]
```

$$-\frac{\text{Zeta}[3]}{6}$$

```
FullSimplify[Sum[-1/(1+6k) + 1/(2+6k) + 2/(3+6k) + 1/(4+6k) - 1/(5+6k) - 2/(6+6k), {k, 0, Infinity}]]
```

```
0
```

```
FullSimplify[Sum[-1/(1+6k) + 1/(2+6k) + 1/(3+6k) + 1/(4+6k) - 1/(5+6k) - 1/(6+6k), {k, 0, Infinity}]]
```

$$-\frac{\text{Log}[2]}{3}$$

```
FullSimplify[Sum[-0/(1+6k) + 0/(2+6k) + 1/(3+6k) + 0/(4+6k) - 0/(5+6k) - 1/(6+6k), {k, 0, Infinity}]]
```

$$\frac{\text{Log}[2]}{3}$$

```
FullSimplify[Sum[-1/(1+6k) + 1/(2+6k) + 0/(3+6k) + 1/(4+6k) - 1/(5+6k) - 0/(6+6k), {k, 0, Infinity}]]
```

$$-\frac{2\,\text{Log}[2]}{3}$$

```
FullSimplify[Sum[-1/(1+6k) + 1/(2+6k) + 0/(3+6k) + 1/(4+6k) - 1/(5+6k) - 0/(6+6k), {k, 0, Infinity}]]
```

$$-\frac{2\,\text{Log}[2]}{3}$$

```
FullSimplify[
 Sum[12 / (2 + 3^(1 / 2)) / (12 k + 1) - 12 / (2 + 3^(1 / 2)) / (12 k + 11), {k, 0, Infinity}]]
```

$$\pi$$

```
FullSimplify[Sum[ 1 / (3 k + 1) ^3 - 1 / (3 k + 2) ^3, {k, 0, Infinity}]]
FullSimplify[Sum[ 1 / (4 k + 1) ^3 - 1 / (4 k + 3) ^3, {k, 0, Infinity}]]
FullSimplify[Sum[ 1 / (5 k + 1) ^3 - 1 / (5 k + 4) ^3, {k, 0, Infinity}]]
FullSimplify[Sum[ 1 / (5 k + 2) ^3 - 1 / (5 k + 3) ^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (6 k + 1) ^3 - 1 / (6 k + 5) ^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (6 k + 2) ^3 - 1 / (6 k + 4) ^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (8 k + 1) ^3 - 1 / (8 k + 7) ^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (8 k + 2) ^3 - 1 / (8 k + 6) ^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (8 k + 3) ^3 - 1 / (8 k + 5) ^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (12 k + 1) ^3 - 1 / (12 k + 11) ^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (12 k + 2) ^3 - 1 / (12 k + 10) ^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (12 k + 3) ^3 - 1 / (12 k + 9) ^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (12 k + 4) ^3 - 1 / (12 k + 8) ^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (12 k + 5) ^3 - 1 / (12 k + 7) ^3, {k, 0, Infinity}]]
FullSimplify[Sum[ 1 / (16 k + 1) ^3 - 1 / (16 k + 15) ^3, {k, 0, Infinity}]]
FullSimplify[Sum[ 1 / (16 k + 2) ^3 - 1 / (16 k + 14) ^3, {k, 0, Infinity}]]
FullSimplify[Sum[ 1 / (16 k + 3) ^3 - 1 / (16 k + 13) ^3, {k, 0, Infinity}]]
FullSimplify[Sum[ 1 / (16 k + 4) ^3 - 1 / (16 k + 12) ^3, {k, 0, Infinity}]]
FullSimplify[Sum[ 1 / (16 k + 5) ^3 - 1 / (16 k + 11) ^3, {k, 0, Infinity}]]
FullSimplify[Sum[ 1 / (16 k + 6) ^3 - 1 / (16 k + 10) ^3, {k, 0, Infinity}]]
FullSimplify[Sum[ 1 / (16 k + 7) ^3 - 1 / (16 k + 9) ^3, {k, 0, Infinity}]]
```

$$\frac{4\,\pi^3}{81\sqrt{3}}$$

$$\frac{\pi^3}{32}$$

$$\frac{2}{625}\sqrt{50+22\sqrt{5}}\;\pi^3$$

$$\frac{2}{625}\sqrt{50-22\sqrt{5}}\;\pi^3$$

$$\frac{\pi^3}{18\sqrt{3}}$$

$$\frac{\pi^3}{162\sqrt{3}}$$

$$\frac{1}{256}\left(4+3\sqrt{2}\right)\pi^3$$

$$\frac{\pi^3}{256}$$

$$\frac{1}{256}\left(-4+3\sqrt{2}\right)\pi^3$$

$$\frac{1}{432}\left(7+4\sqrt{3}\right)\pi^3$$

$$\frac{\pi^3}{144\sqrt{3}}$$

$$\frac{\pi^3}{864}$$

$$\frac{\pi^3}{1296\sqrt{3}}$$

$$\frac{1}{432}\left(7-4\sqrt{3}\right)\pi^3$$

$$\frac{\left(16+12\sqrt{2}+\sqrt{548+386\sqrt{2}}\right)\pi^3}{2048}$$

$$\frac{\left(4+3\sqrt{2}\right)\pi^3}{2048}$$

$$\frac{\left(-8+6\sqrt{2}+\sqrt{137-\frac{193}{\sqrt{2}}}\right)\pi^3}{1024}$$

$$\frac{\pi^3}{2048}$$

$$\frac{\left(8-6\sqrt{2}+\sqrt{137-\frac{193}{\sqrt{2}}}\right)\pi^3}{1024}$$

$$\frac{\left(-4+3\sqrt{2}\right)\pi^3}{2048}$$

$$\frac{\left(-4\left(4+3\sqrt{2}\right)+\sqrt{548+386\sqrt{2}}\right)\pi^3}{2048}$$

```
FullSimplify[
 Sum[1 / (6 k + 1) + 1 / (6 k + 2) - 1 / (6 k + 3) - 2 / (6 k + 4) + 1 / (6 k + 6), {k, 0, Infinity}]]
```

$$\frac{\pi}{2\sqrt{3}}$$

```
FullSimplify[Sum[1 / (6 k + 1)^3 + 1 / (6 k + 2)^3 -
    1 / (6 k + 3)^3 - 2 / (6 k + 4)^3 + 1 / (6 k + 6)^3, {k, 0, Infinity}]]
```

$$\frac{1}{81}\left(\sqrt{3}\,\pi^3+27\,\text{Zeta}[3]\right)$$

```
FullSimplify[Sum[1 / (8 k + 1)^3 - 1 / (8 k + 7)^3, {k, 0, Infinity}]]
```

$$\frac{1}{256}\left(4+3\sqrt{2}\right)\pi^3$$

```
FullSimplify[Sum[1 / (8 k + 2)^3 - 1 / (8 k + 6)^3, {k, 0, Infinity}]]
```

$$\frac{\pi^3}{256}$$

```
FullSimplify[Sum[1 / (8 k + 3)^3 - 1 / (8 k + 5)^3, {k, 0, Infinity}]]
```

$$\frac{1}{256}\left(-4+3\sqrt{2}\right)\pi^3$$

```
FullSimplify[Sum[1 / (8 k + 4)^3 - 1 / (8 k + 8)^3, {k, 0, Infinity}]]
```

$$\frac{3\,\text{Zeta}[3]}{256}$$

```
FullSimplify[Sum[1 / (12 k + 2)^3 - 1 / (12 k + 10)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (12 k + 3)^3 - 1 / (12 k + 9)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (12 k + 4)^3 - 1 / (12 k + 8)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (12 k + 5)^3 - 1 / (12 k + 7)^3, {k, 0, Infinity}]]
```

$$\frac{\pi^3}{144\sqrt{3}}$$

$$\frac{\pi^3}{864}$$

$$\frac{\pi^3}{1296\sqrt{3}}$$

$$\frac{1}{432}\left(7-4\sqrt{3}\right)\pi^3$$

```
FullSimplify[Sum[1 / (12 k + 6)^3, {k, 0, Infinity}]]
```

$$\frac{7\,\text{Zeta}[3]}{1728}$$

```
FullSimplify[Sum[1 / (12 k + 11)^3, {k, 0, Infinity}]]
```

$$-\frac{\text{PolyGamma}\left[2,\frac{11}{12}\right]}{3456}$$

```
FullSimplify[Sum[1 / (5 k + 1)^3 - 1 / (5 k + 4)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (5 k + 2)^3 - 1 / (5 k + 3)^3, {k, 0, Infinity}]]
```

$$\frac{2}{625}\sqrt{50+22\sqrt{5}}\ \pi^3$$

$$\frac{2}{625}\sqrt{50-22\sqrt{5}}\ \pi^3$$

```
FullSimplify[Sum[1 / (5 k + 5)^3, {k, 0, Infinity}]]
```

$$\frac{\text{Zeta}[3]}{125}$$

```
FullSimplify[Sum[1 / (16 k + 1 / 2)^3 - 1 / (16 k + 31 / 2)^3, {k, 0, Infinity}]]
```

$$\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\ \pi^3}{1024\left(2-\sqrt{2+\sqrt{2+\sqrt{2}}}\right)^{3/2}}$$

```
FullSimplify[Sum[1 / (16 k + 1)^3 - 1 / (16 k + 15)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (16 k + 2)^3 - 1 / (16 k + 14)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (16 k + 3)^3 - 1 / (16 k + 13)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (16 k + 4)^3 - 1 / (16 k + 12)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (16 k + 5)^3 - 1 / (16 k + 11)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (16 k + 6)^3 - 1 / (16 k + 10)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (16 k + 7)^3 - 1 / (16 k + 9)^3, {k, 0, Infinity}]]
```

$$\frac{\left(16+12\sqrt{2}+\sqrt{548+386\sqrt{2}}\right)\pi^3}{2048}$$

$$\frac{\left(4+3\sqrt{2}\right)\pi^3}{2048}$$

$$\frac{\left(-8+6\sqrt{2}+\sqrt{137-\frac{193}{\sqrt{2}}}\right)\pi^3}{1024}$$

$$\frac{\pi^3}{2048}$$

$$\frac{\left(8-6\sqrt{2}+\sqrt{137-\frac{193}{\sqrt{2}}}\right)\pi^3}{1024}$$

$$\frac{\left(-4+3\sqrt{2}\right)\pi^3}{2048}$$

$$\frac{\left(-4\left(4+3\sqrt{2}\right)+\sqrt{548+386\sqrt{2}}\right)\pi^3}{2048}$$

```
FullSimplify[Sum[ 1 / (16 k + 16)^3, {k, 0, Infinity}]]
```

$$\frac{\text{Zeta}[3]}{4096}$$

```
FullSimplify[Sum[ 1 / (20 k + 1)^3 - 1 / (20 k + 19)^3, {k, 0, Infinity}]]
FullSimplify[Sum[ 1 / (20 k + 2)^3 - 1 / (20 k + 18)^3, {k, 0, Infinity}]]
FullSimplify[Sum[ 1 / (20 k + 3)^3 - 1 / (20 k + 17)^3, {k, 0, Infinity}]]
FullSimplify[Sum[ 1 / (20 k + 4)^3 - 1 / (20 k + 16)^3, {k, 0, Infinity}]]
FullSimplify[Sum[ 1 / (20 k + 5)^3 - 1 / (20 k + 15)^3, {k, 0, Infinity}]]
FullSimplify[Sum[ 1 / (20 k + 6)^3 - 1 / (20 k + 14)^3, {k, 0, Infinity}]]
FullSimplify[Sum[ 1 / (20 k + 7)^3 - 1 / (20 k + 13)^3, {k, 0, Infinity}]]
FullSimplify[Sum[ 1 / (20 k + 8)^3 - 1 / (20 k + 12)^3, {k, 0, Infinity}]]
FullSimplify[Sum[ 1 / (20 k + 9)^3 - 1 / (20 k + 11)^3, {k, 0, Infinity}]]
```

$$\frac{\left(31 + 15\sqrt{5} + 4\sqrt{130 + 58\sqrt{5}}\right)\pi^3}{4000}$$

$$\frac{\pi^3\,\text{Root}\left[5 - 65\,\#1^2 + \#1^4\,\&,\,4\right]}{2000}$$

$$\frac{\left(-31 + 15\sqrt{5} + 4\sqrt{130 - 58\sqrt{5}}\right)\pi^3}{4000}$$

$$\frac{\pi^3\,\text{Root}\left[5 - 25\,\#1^2 + \#1^4\,\&,\,4\right]}{10\,000}$$

$$\frac{\pi^3}{4000}$$

$$\frac{\pi^3}{200\sqrt{650 + 290\sqrt{5}}}$$

$$\frac{\left(-31 + 15\sqrt{5} - 4\sqrt{130 - 58\sqrt{5}}\right)\pi^3}{4000}$$

$$\frac{\pi^3\,\text{Root}\left[5 - 25\,\#1^2 + \#1^4\,\&,\,3\right]}{10\,000}$$

$$\frac{\left(31 + 15\sqrt{5} - 4\sqrt{130 + 58\sqrt{5}}\right)\pi^3}{4000}$$

```
FullSimplify[Sum[1 / (32 k + 1)^3 - 1 / (32 k + 31)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (32 k + 2)^3 - 1 / (32 k + 30)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (32 k + 3)^3 - 1 / (32 k + 29)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (32 k + 4)^3 - 1 / (32 k + 28)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (32 k + 5)^3 - 1 / (32 k + 27)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (32 k + 6)^3 - 1 / (32 k + 26)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (32 k + 7)^3 - 1 / (32 k + 25)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (32 k + 8)^3 - 1 / (32 k + 24)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (32 k + 9)^3 - 1 / (32 k + 23)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (32 k + 10)^3 - 1 / (32 k + 22)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (32 k + 11)^3 - 1 / (32 k + 21)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (32 k + 12)^3 - 1 / (32 k + 20)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (32 k + 13)^3 - 1 / (32 k + 19)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (32 k + 14)^3 - 1 / (32 k + 18)^3, {k, 0, Infinity}]]
FullSimplify[Sum[1 / (32 k + 15)^3 - 1 / (32 k + 17)^3, {k, 0, Infinity}]]
```

$$\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\ \pi^3}{8192\left(2-\sqrt{2+\sqrt{2+\sqrt{2}}}\right)^{3/2}}$$

$$\frac{\left(16+12\sqrt{2}+\sqrt{548+386\sqrt{2}}\right)\pi^3}{16\,384}$$

$$\frac{\sqrt{2+\sqrt{2+\sqrt{2-\sqrt{2}}}}\ \pi^3}{8192\left(2-\sqrt{2+\sqrt{2-\sqrt{2}}}\right)^{3/2}}$$

$$\frac{\left(4+3\sqrt{2}\right)\pi^3}{16\,384}$$

$$\frac{\sqrt{2+\sqrt{2-\sqrt{2-\sqrt{2}}}}\ \pi^3}{8192\left(2-\sqrt{2-\sqrt{2-\sqrt{2}}}\right)^{3/2}}$$

$$\frac{\left(-8+6\sqrt{2}+\sqrt{137-\frac{193}{\sqrt{2}}}\right)\pi^3}{8192}$$

$$\frac{\sqrt{2+\sqrt{2-\sqrt{2+\sqrt{2}}}}\ \pi^3}{8192\left(2-\sqrt{2-\sqrt{2+\sqrt{2}}}\right)^{3/2}}$$

$$\frac{\pi^3}{16\,384}$$

$$\frac{\sqrt{2-\sqrt{2-\sqrt{2+\sqrt{2}}}}\ \pi^3}{8192\left(2+\sqrt{2-\sqrt{2+\sqrt{2}}}\right)^{3/2}}$$

$$\frac{\left(8-6\sqrt{2}+\sqrt{137-\frac{193}{\sqrt{2}}}\right)\pi^3}{8192}$$

$$\frac{\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2}}}}\ \pi^3}{8192\left(2+\sqrt{2-\sqrt{2-\sqrt{2}}}\right)^{3/2}}$$

$$\frac{\left(-4+3\sqrt{2}\right)\pi^3}{16\,384}$$

$$\frac{\sqrt{2-\sqrt{2+\sqrt{2-\sqrt{2}}}}\ \pi^3}{8192\left(2+\sqrt{2+\sqrt{2-\sqrt{2}}}\right)^{3/2}}$$

$$\frac{\left(-4\left(4+3\sqrt{2}\right)+\sqrt{548+386\sqrt{2}}\right)\pi^3}{16\,384}$$

$$\frac{\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2}}}}\ \pi^3}{8192\left(2+\sqrt{2+\sqrt{2+\sqrt{2}}}\right)^{3/2}}$$

```
FullSimplify[Sum[ 1 / (64 k + 1) ^3 - 1 / (64 k + 63) ^3, {k, 0, Infinity}]]
```

$$\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}\ \pi^3}{65\,536\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{3/2}}$$

```
FullSimplify[Sum[ 1 / (16 k + 1 - 1 / 2) ^3 - 1 / (16 k + 15 + 1 / 2) ^3, {k, 0, Infinity}]]
```

```
FullSimplify[Sum[ 1 / (16 k + 1 - 3 / 4) ^3 - 1 / (16 k + 15 + 3 / 4) ^3, {k, 0, Infinity}]]
```

$$\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\ \pi^3}{1024\left(2-\sqrt{2+\sqrt{2+\sqrt{2}}}\right)^{3/2}}$$

$$\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}\ \pi^3}{1024\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{3/2}}$$

```
FullSimplify[Sum[ 1 / (16 k + 1 - 7 / 8) ^3 - 1 / (16 k + 15 + 7 / 8) ^3, {k, 0, Infinity}]]
```

```
$Aborted
```

$$\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}{1024\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{3/2}}$$

```
Expand[Sqrt[2 + Sqrt[2 + Sqrt[2 + Sqrt[2 + Sqrt[2]]]]] / (1024 (2 - Sqrt[2 + Sqrt[2 + Sqrt[2 + Sqrt[2]]]])^(3/2))]
```

$$\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}{1024\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{3/2}}$$

```
N[Sqrt[2 + Sqrt[2 + Sqrt[2 + Sqrt[2 + Sqrt[2]]]]] / (1024 (2 - Sqrt[2 + Sqrt[2 + Sqrt[2 + Sqrt[2]]]])^(3/2))]
```

$$\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}{1024\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{3/2}}$$

```
2.0641
```

```
FullSimplify[Sum[1 / (16 k + 1 - 1 / 2)^3 - 1 / (16 k + 15 + 1 / 2)^3, {k, 0, Infinity}]]
```

$$\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\ \pi^3}{1024\left(2-\sqrt{2+\sqrt{2+\sqrt{2}}}\right)^{3/2}}$$

```
FullSimplify[Sum[1 / (16 k + 16)^3, {k, 0, Infinity}]]
```

$$\frac{\text{Zeta}[3]}{4096}$$

```
FullSimplify[Sum[1 / (4 k + 1 - 1 / 2)^3 - 1 / (4 k + 3 + 1 / 2)^3, {k, 0, Infinity}]]
```

$$\frac{1}{32}\left(4+3\sqrt{2}\right)\pi^3$$

```
FullSimplify[Sum[1 / (4 k + 1 - 3 / 4)^3 - 1 / (4 k + 3 + 3 / 4)^3, {k, 0, Infinity}]]
```

$$\frac{1}{32}\left(16+12\sqrt{2}+\sqrt{548+386\sqrt{2}}\right)\pi^3$$

```
FullSimplify[Sum[1 / (4 k + 1 / (2^4))^3 - 1 / (4 k + 4 - 1 / (2^4))^3, {k, 0, Infinity}]]
```

$$N\left[\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}\ \pi^3}{16\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{3/2}}\right]$$

4096.

```
FullSimplify[Sum[ 1 / (16 k + 1 / 8)^3 - 1 / (16 k + 16 - 1 / 8)^3, {k, 0, Infinity}]]
```

$Aborted

$$N\left[\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}\ \pi^3}{1024\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{3/2}}\right]$$

64.

$$N\left[\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}\ \pi^3}{16\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{3/2}}\right]$$

4096.

$$N\left[\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}{1024\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{3/2}}\right]$$

2.0641

```
N[64 / Pi^3]
```

2.0641

```
N[Sqrt[2 + Sqrt[2 + Sqrt[2 + Sqrt[2 + Sqrt[2]]]]]]
```

```
1.99759
```

```
N[1024 (2 - Sqrt[2 + Sqrt[2 + Sqrt[2 + Sqrt[2]]]])^(3/2)]
```

```
0.967779
```

```
FullSimplify[Sum[1 / (64 k + 1)^3 - 1 / (64 k + 63)^3, {k, 0, Infinity}]]
```

$$\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}\ \pi^3}{65\,536\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{3/2}}$$

```
FullSimplify[Sum[1 / (4 k + 1 / (2^4))^3 - 1 / (4 k + 4 - 1 / (2^4))^3, {k, 0, Infinity}]]
```

$$\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}\ \pi^3}{16\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{3/2}}$$

```
N[Sqrt[2 + Sqrt[2 + Sqrt[2 + Sqrt[2 + Sqrt[2]]]]] π^3 / (65536 (2 - Sqrt[2 + Sqrt[2 + Sqrt[2 + Sqrt[2]]]])^(3/2))]
```

```
1.
```

```
N[Sqrt[2 + Sqrt[2 + Sqrt[2 + Sqrt[2 + Sqrt[2]]]]] π^3 / (16 (2 - Sqrt[2 + Sqrt[2 + Sqrt[2 + Sqrt[2]]]])^(3/2))]
```

```
4096.
```

$$\text{Log}\left[\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}{65\,536\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{3/2}}\right]$$

$$\text{FullSimplify}\left[\text{Log}\left[\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}{65\,536\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{3/2}}\right]\right]$$

$$\text{Log}\left[\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}{65\,536\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{3/2}}\right]$$

```
N[1 / Pi^3]
```

0.0322515

```
sq[k_] := If[k == 0, 2^(1 / 2), (2 + sq[k - 1])^(1 / 2)]
sq2[k_] := 2 - sq[k - 1]
```

```
sq[4]
```

$$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}$$

```
sq2[4]
```

$$2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}$$

```
ff[k_] := Pi^3 sq[k] / (65 536 sq2[k]^(3 / 2))
```

```
N[ff[8]]
```

4096.

```
N[1 / Pi^3]
```

0.0322515

```
FullSimplify[Sum[1 / (4 k + 1 / (2^1))^3 - 1 / (4 k + 4 - 1 / (2^1))^3, {k, 0, Infinity}]]
```

```
N[1/32 (4 + 3 Sqrt[2]) π^3]
```

```
7.98667
```

```
FullSimplify[Sum[1/(4 k + 1/(2^2))^3 - 1/(4 k + 4 - 1/(2^2))^3, {k, 0, Infinity}]]
```

```
N[1/32 (16 + 12 Sqrt[2] + Sqrt[548 + 386 Sqrt[2]]) π^3]
```

```
63.9936
```

```
FullSimplify[Sum[1/(4 k + 1/(2^3))^3 - 1/(4 k + 4 - 1/(2^3))^3, {k, 0, Infinity}]]
```

$$N\left[\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\ \pi^3}{16\left(2-\sqrt{2+\sqrt{2+\sqrt{2}}}\right)^{3/2}}\right]$$

```
511.997
```

```
FullSimplify[Sum[1/(4 k + 1/(2^4))^3 - 1/(4 k + 4 - 1/(2^4))^3, {k, 0, Infinity}]]
```

$$N\left[\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}\ \pi^3}{16\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{3/2}}\right]$$

```
4096.
```

```
FullSimplify[Sum[1/(4 k + 1/(2^5))^3 - 1/(4 k + 4 - 1/(2^5))^3, {k, 0, Infinity}]]
```

```
$Aborted
```

```
ff[k_] := Pi^3 sq[k] / (65536 sq2[k]^(3/2))
```

```
N[ff[9]]
```

```
32768.
```

```
N[FullSimplify[Sum[1 / (32 k + 1)^3 - 1 / (32 k + 31)^3, {k, 0, Infinity}]]]
N[FullSimplify[Sum[1 / (32 k + 2)^3 - 1 / (32 k + 30)^3, {k, 0, Infinity}]]]
N[FullSimplify[Sum[1 / (32 k + 3)^3 - 1 / (32 k + 29)^3, {k, 0, Infinity}]]]
N[FullSimplify[Sum[1 / (32 k + 4)^3 - 1 / (32 k + 28)^3, {k, 0, Infinity}]]]
N[FullSimplify[Sum[1 / (32 k + 5)^3 - 1 / (32 k + 27)^3, {k, 0, Infinity}]]]
N[FullSimplify[Sum[1 / (32 k + 6)^3 - 1 / (32 k + 26)^3, {k, 0, Infinity}]]]
N[FullSimplify[Sum[1 / (32 k + 7)^3 - 1 / (32 k + 25)^3, {k, 0, Infinity}]]]
N[FullSimplify[Sum[1 / (32 k + 8)^3 - 1 / (32 k + 24)^3, {k, 0, Infinity}]]]
N[FullSimplify[Sum[1 / (32 k + 9)^3 - 1 / (32 k + 23)^3, {k, 0, Infinity}]]]
N[FullSimplify[Sum[1 / (32 k + 10)^3 - 1 / (32 k + 22)^3, {k, 0, Infinity}]]]
N[FullSimplify[Sum[1 / (32 k + 11)^3 - 1 / (32 k + 21)^3, {k, 0, Infinity}]]]
N[FullSimplify[Sum[1 / (32 k + 12)^3 - 1 / (32 k + 20)^3, {k, 0, Infinity}]]]
N[FullSimplify[Sum[1 / (32 k + 13)^3 - 1 / (32 k + 19)^3, {k, 0, Infinity}]]]
N[FullSimplify[Sum[1 / (32 k + 14)^3 - 1 / (32 k + 18)^3, {k, 0, Infinity}]]]
N[FullSimplify[Sum[1 / (32 k + 15)^3 - 1 / (32 k + 17)^3, {k, 0, Infinity}]]]
```

```
0.999994

0.124987

0.0370179

0.015599

0.00796654

0.00458806

0.0028649

0.00189247

0.00129958

0.000914533

0.000650275

0.000459191

0.000313451

0.000195665

0.0000941002
```

```
FullSimplify[Sum[1 / (32 k + 1)^1 - 1 / (32 k + 31)^1, {k, 0, Infinity}]]
```

$$\frac{1}{32}\sqrt{\frac{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}{2-\sqrt{2+\sqrt{2+\sqrt{2}}}}}\,\pi$$

```
FullSimplify[Sum[1 / (64 k + 1)^1 - 1 / (64 k + 63)^1, {k, 0, Infinity}]]
```

$$\frac{1}{64}\sqrt{\frac{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}\,\pi$$

0.999197

```
N[1/64 Sqrt[(2 + Sqrt[2 + Sqrt[2 + Sqrt[2 + Sqrt[2]]]])/(2 - Sqrt[2 + Sqrt[2 + Sqrt[2 + Sqrt[2]]]])]]
```

0.318054

```
N[1/Pi]
```

0.31831

```
FullSimplify[Sum[1/(32 k + 1)^5 - 1/(32 k + 31)^5, {k, 0, Infinity}]]
```

$$N\left[\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\left(10+\sqrt{2+\sqrt{2+\sqrt{2}}}\right)\pi^5}{25\,165\,824\left(2-\sqrt{2+\sqrt{2+\sqrt{2}}}\right)^{5/2}}\right]$$

1.

Sum::div : Sum does not converge. ≫

$$N\left[\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\left(10+\sqrt{2+\sqrt{2+\sqrt{2}}}\right)}{25\,165\,824\left(2-\sqrt{2+\sqrt{2+\sqrt{2}}}\right)^{5/2}}\right]$$

0.00326776

```
N[1/Pi^5]
```

0.00326776

```
FullSimplify[Sum[ 1 / (4 k + 1)^1 - 1 / (4 k + 3)^1, {k, 0, Infinity}]]
```

$$\frac{\pi}{4}$$

```
FullSimplify[Sum[ 1 / (8 k + 1)^1 - 1 / (8 k + 7)^1, {k, 0, Infinity}]]
```

$$\frac{1}{8}\left(1+\sqrt{2}\right)\pi$$

```
FullSimplify[Sum[ 1 / (16 k + 1)^1 - 1 / (16 k + 15)^1, {k, 0, Infinity}]]
```

$$\frac{1}{16}\left(1+\sqrt{2}+\sqrt{2\left(2+\sqrt{2}\right)}\right)\pi$$

```
FullSimplify[Sum[ 1 / (32 k + 1)^1 - 1 / (32 k + 31)^1, {k, 0, Infinity}]]
```

$$\frac{1}{32}\sqrt{\frac{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}{2-\sqrt{2+\sqrt{2+\sqrt{2}}}}}\,\pi$$

```
FullSimplify[Sum[ 1 / (64 k + 1)^1 - 1 / (64 k + 31)^1, {k, 0, Infinity}]]
```

```
$Aborted
```

```
FullSimplify[Sum[ 1 / (4 k + 1)^1 - 1 / (4 k + 3 )^1, {k, 0, Infinity}]]
```

```
N[π/4]
```

```
0.785398
```

```
FullSimplify[Sum[ 1 / (4 k + 1 / 2)^1 - 1 / (4 k + 3 + 1 / 2)^1, {k, 0, Infinity}]]
```

```
N[1/4 (1 + Sqrt[2]) π]
```

```
1.89612
```

```
FullSimplify[Sum[ 1 / (4 k + 1 / 4)^1 - 1 / (4 k + 3 + 3 / 4)^1, {k, 0, Infinity}]]
```

```
N[1/4 (1 + Sqrt[2] + Sqrt[2 (2 + Sqrt[2])]) π]
```

```
3.94846
```

```
FullSimplify[Sum[ 1 / (4 k + 1 / 8)^1 - 1 / (4 k + 3 + 7 / 8)^1, {k, 0, Infinity}]]
```

```
N[1/4 Sqrt[(2 + Sqrt[2 + Sqrt[2 + Sqrt[2]]])/(2 - Sqrt[2 + Sqrt[2 + Sqrt[2]]])] π]
```

```
7.97428
```

```
FullSimplify[Sum[ 1 / (4 k + 1 / 16)^1 - 1 / (4 k + 3 + 15 / 16)^1, {k, 0, Infinity}]]
```

$$N\left[\frac{1}{4}\sqrt{\frac{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}\,\pi\right]$$

15.9871

```
FullSimplify[Sum[1 / (4 k + 1 / 32)^1 - 1 / (4 k + 3 + 31 / 32)^1, {k, 0, Infinity}]]
```

$$\frac{1}{4}\sqrt{\frac{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}\,\pi$$

$$N\left[\frac{1}{4}\sqrt{\frac{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}\,\pi\right]$$

31.9936

```
FullSimplify[Sum[1 / (2 k + 1 / 2)^1 - 1 / (2 k + 1 + 1 / 2)^1, {k, 0, Infinity}]]
```

$$\frac{\pi}{2}$$

```
FullSimplify[Sum[1 / (2 k + 1 / 2)^3 - 1 / (2 k + 1 + 1 / 2)^3, {k, 0, Infinity}]]
```

$$\frac{\pi^3}{4}$$

```
FullSimplify[Sum[1 / (2 k + 1 / 2)^5 - 1 / (2 k + 1 + 1 / 2)^5, {k, 0, Infinity}]]
```

$$\frac{5\,\pi^5}{48}$$

```
FullSimplify[Sum[1 / (2 k + 1 / 2)^1 - 1 / (2 k + 1 + 1 / 2)^1, {k, 0, Infinity}]]
```

$$\frac{\pi}{2}$$

```
FullSimplify[Sum[1 / (2 k + 1 / 4) ^1 - 1 / (2 k + 1 + 3 / 4 ) ^1, {k, 0, Infinity}]]
```

$$\frac{1}{2}\left(1+\sqrt{2}\right)\pi$$

```
FullSimplify[Sum[1 / (2 k + 1 / 8) ^1 - 1 / (2 k + 1 + 7 / 8 ) ^1, {k, 0, Infinity}]]
```

$$\frac{1}{2}\left(1+\sqrt{2}+\sqrt{2\left(2+\sqrt{2}\right)}\right)\pi$$

```
FullSimplify[Sum[1 / (2 k + 1 / 16) ^1 - 1 / (2 k + 1 + 15 / 16 ) ^1, {k, 0, Infinity}]]
```

$$\frac{1}{2}\sqrt{\frac{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}{2-\sqrt{2+\sqrt{2+\sqrt{2}}}}}\,\pi$$

```
FullSimplify[Sum[1 / (k + 1 / 4) ^1 - 1 / (k + 3 / 4 ) ^1, {k, 0, Infinity}]]
```

$$\pi$$

```
FullSimplify[Sum[1 / (k + 1 / 8) ^1 - 1 / (k + 7 / 8 ) ^1, {k, 0, Infinity}]]
```

$$\left(1+\sqrt{2}\right)\pi$$

```
FullSimplify[Sum[1 / (k + 1 / 16) ^1 - 1 / (k + 15 / 16 ) ^1, {k, 0, Infinity}]]
```

$$\left(1+\sqrt{2}+\sqrt{2\left(2+\sqrt{2}\right)}\right)\pi$$

```
FullSimplify[Sum[1 / (k + 1 / 32) ^1 - 1 / (k + 31 / 32 ) ^1, {k, 0, Infinity}]]
```

$$N\left[\sqrt{\frac{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}{2-\sqrt{2+\sqrt{2+\sqrt{2}}}}}\,\pi\right]$$

```
31.8971
```

```
FullSimplify[Sum[1 / (k + 1 / 64) ^1 - 1 / (k + 63 / 64 ) ^1, {k, 0, Infinity}]]
```

$$N\left[\sqrt{\frac{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}\,\pi\right]$$

```
63.9486
```

```
FullSimplify[Sum[1 / (k + 1 / 128) - 1 / (k + 127 / 128), {k, 0, Infinity}]]
```

$$\sqrt{\frac{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}}\,\pi$$

```
N[Sqrt[(2 + Sqrt[2 + Sqrt[2 + Sqrt[2 + Sqrt[2 + Sqrt[2]]]]]) / (2 - Sqrt[2 + Sqrt[2 + Sqrt[2 + Sqrt[2 + Sqrt[2]]]]])] π]
```

```
127.974
```

```
FullSimplify[Sum[1 / (k + 1 / 128) - 1 / (k + 127 / 128), {k, 0, Infinity}]]
```

$$\sqrt{\frac{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}}\,\pi$$

```
FullSimplify[Sum[1 / (k + 1 / 4)^3 - 1 / (k + 3 / 4)^3, {k, 0, Infinity}]]
```

$$2\,\pi^3$$

```
FullSimplify[Sum[1 / (k + 1 / 8)^3 - 1 / (k + 7 / 8)^3, {k, 0, Infinity}]]
```

$$\left(8+6\sqrt{2}\right)\pi^3$$

```
FullSimplify[Sum[1 / (k + 1 / 16)^3 - 1 / (k + 15 / 16)^3, {k, 0, Infinity}]]
```

$$2\left(16+12\sqrt{2}+\sqrt{548+386\sqrt{2}}\right)\pi^3$$

```
FullSimplify[Sum[1 / (k + 1 / 32)^3 - 1 / (k + 31 / 32)^3, {k, 0, Infinity}]]
```

$$\frac{4\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\,\pi^3}{\left(2-\sqrt{2+\sqrt{2+\sqrt{2}}}\right)^{3/2}}$$

```
FullSimplify[Sum[1 / (k + 1 / 64)^3 - 1 / (k + 63 / 64)^3, {k, 0, Infinity}]]
```

$$\frac{4\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}\ \pi^3}{\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{3/2}}$$

```
$Aborted
```

```
FullSimplify[Sum[ (1 / 2)^6 / (k + 1 / 64)^3 - (1 / 2)^6 / (k + 63 / 64 )^3, {k, 0, Infinity}]]
```

$$N\left[\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}\ \pi^3}{16\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{3/2}}\right]$$

```
4096.
```

$$N\left[\frac{4\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}\ \pi^3}{\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{3/2}}\right]$$

```
262144.
```

```
FullSimplify[Sum[ 1 / (k + 1 / 64)^3, {k, 0, Infinity}]]
```

$$-\frac{1}{2}\,\text{PolyGamma}\left[2,\ \frac{1}{64}\right]$$

```
FullSimplify[Sum[ (1 / 2)^18 / (k + 1 / 64)^3 - (1 / 2)^18 / (k + 63 / 64 )^3, {k, 0, Infinity}]]
```

$$N\left[\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}\ \pi^3}{65536\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{3/2}}\right]$$

```
1.
```

$$N\left[\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}\ \pi^3}{1024\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{3/2}}\right]$$

64.

```
FullSimplify[Sum[ (1 / 2)^15 / (k + 1 / 32)^3 - (1 / 2)^15 / (k + 31 / 32 )^3, {k, 0, Infinity}]]
```

$$N\left[\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\ \pi^3}{8192\left(2-\sqrt{2+\sqrt{2+\sqrt{2}}}\right)^{3/2}}\right]$$

0.999994

$$N\left[\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\ \pi^3}{256\left(2-\sqrt{2+\sqrt{2+\sqrt{2}}}\right)^{3/2}}\right]$$

31.9998

```
FullSimplify[Sum[ (1 / 2)^21 / (k + 1 / 128)^3 - (1 / 2)^21 / (k + 127 / 128 )^3, {k, 0, Infinity}]]
```

$$\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}\ \pi^3}{524\,288\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}\right)^{3/2}}$$

$$\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}\ \pi^3}{524\,288\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}\right)^{3/2}}$$

```
N[2^19]
```

524 288.

$$N\left[\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}\ \pi^3}{65\,536\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{3/2}}\right]$$

1.

$$N\left[\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}\ \pi\text{\^{}}3}{65\,536\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{3/2}}\right] 1$$

1.

```
0.031237439560626443`
```

32.0129

$$N\left[1/128\ \sqrt{\frac{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}\ \text{Pi}\right]$$

0.999799

$$N\left[\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}\ \pi\text{\^{}}1}{\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{1/2}}\right]$$

63.9486

$$N\left[\frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}\ \pi^2}{\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{2/2}}\right]$$

2047.18

```
sq[k_] := If[k == 0, 2^(1 / 2), (2 + sq[k - 1])^(1 / 2)]
sq2[k_] := (2 - sq[k - 1])^(1 / 2)
sq2a[k_, a_] := (2 - sq[k - 1])^(a / 2)
Pi1[k_] := 2^(k + 2) sq2[k] / sq[k]
Pi2[k_] := 2^(2 k + 3) sq2a[k, 2] / sq[k]
Pia[k_, a_] := 2^(a k + a + 1) sq2a[k, a] / sq[k]
```

```
Pi2[4]
```

$$\frac{2048\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)}{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}$$

```
N[Pi2[4]]
```

9.87357

```
N[Pia[7, 1 / 2]]
```

1.77248

```
N[Pi^(1 / 2)]
```

1.77245

```
N[Pi^4]
```

97.4091

```
N[Pi1[4]]
```

3.14412

```
N[Pi1[5]]
```

3.14222

```
N[Pi1[6]]
```

3.14175

```
N[Pi1[7]]
```

3.14163

```
2^9
```

512

```
2^8
```

256

```
sq3[k_] := If[k == 0, 2^(1 / 2), (2 + sq3[k - 1])^(1 / 2)]
sq3a[k_, a_] := (2 - sq3[k - 1])^(a / 2)
Pia3[k_, a_] := 2^(a k + a + 1) sq2a[k, a] / sq3[k]
```

```
Pia3[11, 3]
```

$$\left(137\,438\,953\,472\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}}}}}\right)^{3/2}\right)\Big/$$

$$\left(\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}}}}}\right)$$

```
N[sq3[10]]
```

2.

```
sq4[k_, b_] := If[k == 0, b^(1 / 2), (b + sq4[k - 1, b])^(1 / 2)]
sq4a[k_, a_, b_] := (b - sq4[k - 1, b])^(a / 2)
Pia4[k_, a_, b_] := b^(a k + a + 1) sq4a[k, a, b] / sq4[k, b]
```

```
N[Pia4[11, 1, 2]]
```

3.14159

```
N[Pia4[11, 1, 2]]
```

3.14159

Pia3[11, 2]

$$\left(33\,554\,432\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}}}}}}\right)\right)\Big/$$

$$\left(\sqrt{\left(2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}}}}}\right)}\right)$$

Pia[11, 2] / Pia3[11, 1]

$$4096\sqrt{\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}}}}}\right)}$$

Pia[10, 3] / Pia3[10, 2]

$$2048\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}}}}}$$

$$N\left[4096\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}}}}}}}\right]$$

3.14159

```
Pia[3, 2] / Pia3[3, 1]
```

$$16\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2}}}}$$

```
(Pia[11, 2] / Pia3[11, 1])^2
```

$$N\left[16\,777\,216\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}}}}}\right)\right]$$

9.8696

```
N[Pi^2]
```

9.8696

```
N[sq[8]]
```

1.99999

```
sqq[k_] := If[k == 0, 4^(1 / 2), (4 + sqq[k - 1])^(1 / 2)]
```

```
N[sqq[30]]
```

2.56155

```
N[Log[3]]
```

```
1.0986122886681098`
```

```
sqq8[k_] := If[k == 0, 8^(1 / 8), (8 + sqq8[k - 1])^(1 / 8)]
```

```
N[sqq8[30]]
```

1.32187