```
Limit[dx Sum[ (dx + 1) ^k , {k, 1, Log[dx + 1, n]}], dx → 0]
Integrate[ E^k, {k, 0, Log[n]}]
```

$$-1 + n$$

$$-1 + n$$

```
Limit[dx Sum[ (dx + 1) ^-k , {k, 1, Log[dx + 1, n]}], dx → 0]
Integrate[E^-k, {k, 0, Log[n]}]
```

$$\frac{-1 + n}{n}$$

$$\frac{-1 + n}{n}$$

```
Limit[dx Sum[ (dx + 1) ^(2 k) , {k, 1, Log[dx + 1, n]}], dx → 0]
Integrate[ E^(2 k), {k, 0, Log[n]}]
```

$$\frac{1}{2}\left(-1 + n^2\right)$$

$$\frac{1}{2}\left(-1 + n^2\right)$$

```
Limit[dx Sum[ (dx + 1) ^(3 k) , {k, 1, Log[dx + 1, n]}], dx → 0]
Integrate[ E^(3 k), {k, 0, Log[n]}]
```

$$\frac{1}{3}\left(-1 + n^3\right)$$

$$\frac{1}{3}\left(-1 + n^3\right)$$

```
Limit[dx Sum[ (dx + 1) ^(c k) , {k, 1, Log[dx + 1, n]}], dx → 0]
Integrate[ E^(c k) , {k, 0, Log[n]}]
```

$$\frac{-1 + n^c}{c}$$

$$\frac{-1 + n^c}{c}$$

```
-1 + n^c
--------  /. c → 2
   c
```

$$\frac{1}{2}\left(-1 + n^2\right)$$

```
Limit[ dx Sum[ 1, {k, 1, Log[dx + 1, n]}], dx → 0]
Integrate[ 1, {k, 0, Log[n]}]
```

$$\text{Log}[n]$$

$$\text{Log}[n]$$

```
Limit[ Sum[ dx^2, {k, 1, Log[dx + 1, n]}, {j, 1, Log[dx + 1, n]}], dx → 0]
Integrate[2 k, {k, 0, Log[n]}]
```

$$\text{Log}[n]^2$$

$$\text{Log}[n]^2$$

```
Limit[ dx^2 Sum[k , {k, 1, Log[dx + 1, n]}], dx → 0]
Integrate[ k Log[n]^2, {k, 0, 1}]
Integrate[ k, {k, 0, Log[n]}]
```

$$\frac{\text{Log}[n]^2}{2}$$

$$\frac{\text{Log}[n]^2}{2}$$

$$\frac{\text{Log}[n]^2}{2}$$

```
Limit[ dx^3 Sum[k^2, {k, 1, Log[dx + 1, n]}], dx → 0]
Integrate[ k^2 Log[n]^3, {k, 0, 1}]
Integrate[ k^2, {k, 0, Log[n]}]
```

$$\frac{\text{Log}[n]^3}{3}$$

$$\frac{\text{Log}[n]^3}{3}$$

$$\frac{\text{Log}[n]^3}{3}$$

```
Limit[dx^3 Sum[k^(3 - 1), {k, 1, Log[dx + 1, n]}], dx → 0]
Integrate[ k^(m - 1) Log[n]^m, {k, 0, 1}]
Integrate[ k^(m - 1), {k, 0, Log[n]}]
```

$$\frac{\text{Log}[n]^3}{3}$$

$$\text{ConditionalExpression}\left[\frac{\text{Log}[n]^m}{m}, \text{Re}[m] > 0\right]$$

$$\text{ConditionalExpression}\left[\frac{\text{Log}[n]^m}{m}, \text{Re}[m] > 0\right]$$

```
Limit[ dx Sum[ (dx + 1)^k, {k, 1, Log[dx + 1, n]}], dx → 0]
Integrate[ n^k Log[n], {k, 0, 1}]
Integrate[ E^k, {k, 0, Log[n]}]
```

$-1 + n$

$-1 + n$

$-1 + n$

```
Limit[ dx^2 Sum[k (dx + 1)^k , {k, 1, Log[dx + 1, n]}], dx → 0]
Integrate[ n^k k Log[n]^2, {k, 0, 1}]
Expand[Integrate[ E^k k, {k, 0, Log[n]}]]
```

$1 - n + n \text{Log}[n]$

$1 - n + n \text{Log}[n]$

$1 - n + n \text{Log}[n]$

```
Limit[ dx^3 Sum[k^2 (dx + 1)^k , {k, 1, Log[dx + 1, n]}], dx → 0]
Integrate[ n^k k^2 Log[n]^3, {k, 0, 1}]
Expand[Integrate[ E^k k^2, {k, 0, Log[n]}]]
```

$-2 + 2 n - 2 n \text{Log}[n] + n \text{Log}[n]^2$

$-2 + 2 n - 2 n \text{Log}[n] + n \text{Log}[n]^2$

$-2 + 2 n - 2 n \text{Log}[n] + n \text{Log}[n]^2$

```
Limit[ dx^4 Sum[k^3 (dx + 1)^k , {k, 1, Log[dx + 1, n]}], dx → 0]
Expand[Integrate[ n^k k^3 Log[n]^4, {k, 0, 1}]]
Expand[Integrate[E^k k^3, {k, 0, Log[n]}]]
```

$6 - 6 n + 6 n \text{Log}[n] - 3 n \text{Log}[n]^2 + n \text{Log}[n]^3$

$6 - 6 n + 6 n \text{Log}[n] - 3 n \text{Log}[n]^2 + n \text{Log}[n]^3$

$6 - 6 n + 6 n \text{Log}[n] - 3 n \text{Log}[n]^2 + n \text{Log}[n]^3$

```
Expand[Limit[ dx^5 Sum[k^4 (dx + 1)^k , {k, 1, Log[dx + 1, n]}], dx → 0]]
Expand[Integrate[ n^k k^4 Log[n]^5, {k, 0, 1}]]
Expand[Integrate[ E^k k^4, {k, 0, Log[n]}]]
```

$$-24 + 24\,n - 24\,n\,\text{Log}[n] + 12\,n\,\text{Log}[n]^2 - 4\,n\,\text{Log}[n]^3 + n\,\text{Log}[n]^4$$

$$-24 + 24\,n - 24\,n\,\text{Log}[n] + 12\,n\,\text{Log}[n]^2 - 4\,n\,\text{Log}[n]^3 + n\,\text{Log}[n]^4$$

$$-24 + 24\,n - 24\,n\,\text{Log}[n] + 12\,n\,\text{Log}[n]^2 - 4\,n\,\text{Log}[n]^3 + n\,\text{Log}[n]^4$$

```
Limit[ dx^m Sum[k^(m - 1) (dx + 1)^k , {k, 1, Log[dx + 1, n]}], dx → 0]
Expand[Integrate[ n^k k^(m - 1) Log[n]^m, {k, 0, 1}]]
Expand[Integrate[ E^k k^(m - 1), {k, 0, Log[n]}]]
```

$$\text{Limit}\left[dx^m \left(-n\,\text{LerchPhi}\left[1 + dx,\ 1 - m,\ 1 + \frac{\text{Log}[n]}{\text{Log}[1 + dx]}\right] - dx\,n\,\text{LerchPhi}\left[1 + dx,\ 1 - m,\ 1 + \frac{\text{Log}[n]}{\text{Log}[1 + dx]}\right] + \text{PolyLog}[1 - m,\ 1 + dx]\right),\ dx \to 0\right]$$

$$\text{ConditionalExpression}[\text{Gamma}[m]\,(-\text{Log}[n])^{-m}\,\text{Log}[n]^m - \text{Gamma}[m,\ -\text{Log}[n]]\,(-\text{Log}[n])^{-m}\,\text{Log}[n]^m,\ \text{Re}[\text{Log}[n]] < 0 \&\&\ \text{Re}[m] > 0]$$

$$\text{ConditionalExpression}[\text{Gamma}[m]\,(-\text{Log}[n])^{-m}\,\text{Log}[n]^m - \text{Gamma}[m,\ -\text{Log}[n]]\,(-\text{Log}[n])^{-m}\,\text{Log}[n]^m,\ \text{Re}[m] > 0]$$

```
Limit[ (a - 1)^m Sum[k^(m - 1) a^-k , {k, 1, Log[a, n]}], a → 1]
Integrate[ n^-t t^(s - 1) Log[n]^s, {t, 0, 1}]
```

$$\text{Limit}\left[\frac{1}{a}(-1 + a)^m \left(-\left(\frac{1}{a}\right)^{\frac{\text{Log}[n]}{\text{Log}[a]}}\,\text{LerchPhi}\left[\frac{1}{a},\ 1 - m,\ 1 + \frac{\text{Log}[n]}{\text{Log}[a]}\right] + a\,\text{PolyLog}\left[1 - m,\ \frac{1}{a}\right]\right),\ a \to 1\right]$$

$$\text{ConditionalExpression}[\text{Gamma}[s] - \text{Gamma}[s,\ \text{Log}[n]],\ \text{Re}[\text{Log}[n]] > 0 \&\&\ \text{Re}[s] > 0]$$

```
Limit[ (a - 1)^m Sum[k^(m - 1) a^k , {k, 1, Log[a, n]}] /. m → 4, a → 1]
```

$$6 - 6\,n + 6\,n\,\text{Log}[n] - 3\,n\,\text{Log}[n]^2 + n\,\text{Log}[n]^3$$

```
Expand[Limit[ dx^2 Sum[k (dx + 1)^(2 k) , {k, 1, Log[dx + 1, n]}], dx → 0]]
Expand[Integrate[ n^(2 k) k Log[n]^2, {k, 0, 1}]]
Expand[Integrate[ E^(2 k) k, {k, 0, Log[n]}]]
```

$$\frac{1}{4} - \frac{n^2}{4} + \frac{1}{2}\,n^2\,\text{Log}[n]$$

$$\frac{1}{4} - \frac{n^2}{4} + \frac{1}{2}\,n^2\,\text{Log}[n]$$

$$\frac{1}{4} - \frac{n^2}{4} + \frac{1}{2}\,n^2\,\text{Log}[n]$$

```
Limit[ dx^0 Sum[k^-1 (dx + 1)^k , {k, 1, Log[dx + 1, n]}], dx → 0]
```

$$\text{Limit}\left[-n\,\text{LerchPhi}\left[1+dx,\,1,\,1+\frac{\text{Log}[n]}{\text{Log}[1+dx]}\right]-dx\,n\,\text{LerchPhi}\left[1+dx,\,1,\,1+\frac{\text{Log}[n]}{\text{Log}[1+dx]}\right]-\text{Log}[-dx],\,dx\to 0\right]$$

```
Limit[ dx^4 Sum[k^(4 - 1) (dx + 1)^k , {k, 1, Log[dx + 1, n]}], dx → 0]
Expand[Integrate[ n^k k^(4 - 1) Log[n]^4, {k, 0, 1}]]
Expand[Integrate[ E^k k^(4 - 1), {k, 0, Log[n]}]]
```

$$6-6\,n+6\,n\,\text{Log}[n]-3\,n\,\text{Log}[n]^2+n\,\text{Log}[n]^3$$

$$6-6\,n+6\,n\,\text{Log}[n]-3\,n\,\text{Log}[n]^2+n\,\text{Log}[n]^3$$

$$6-6\,n+6\,n\,\text{Log}[n]-3\,n\,\text{Log}[n]^2+n\,\text{Log}[n]^3$$

```
Limit[Integrate[ s^(a - 1) Log[n]^a, {s, 0, 1}], a → 2]
```

$$\frac{\text{Log}[n]^2}{2}$$

```
Limit[Integrate[ n^s s^(a - 1) Log[n]^a, {s, 1, Infinity}], a → 2]
```

```
ConditionalExpression[Gamma[2, -Log[n]], Re[Log[n]] < 0]
```

```
Limit[Integrate[ n^s s^(a - 1) Log[n]^a, {s, 0, Infinity}], a → c]
```

```
ConditionalExpression[Gamma[c] (-Log[n])^-c Log[n]^c, Re[Log[n]] < 0 && Re[c] ≥ 0]
```

```
Limit[Integrate[ n^s s^(a - 1) Log[n]^a, {s, 0, 1}], a → 4]
```

```
ConditionalExpression[6 - Gamma[4, -Log[n]], Re[Log[n]] < 0]
```

```
Limit[Integrate[ n^s s^(a - 1) Log[n]^a, {s, 1, Infinity}], a → 1]
```

```
ConditionalExpression[-n, Re[Log[n]] < 0]
```

```
Integrate[ n^s s^(a - 1) Log[n]^a, {s, 0, 1}]
```

```
ConditionalExpression[
  (Gamma[a] - Gamma[a, -Log[n]]) (-Log[n])^-a Log[n]^a, Re[Log[n]] < 0 && Re[a] > 0]
```

```
Integrate[ n^s s^(a - 1) Log[n]^a, {s, 0, Infinity}]
```

```
ConditionalExpression[Gamma[a] (-Log[n])^-a Log[n]^a, Re[Log[n]] < 0 && Re[a] > 0]
```

```
Limit[Integrate[ n^s s^(a - 1) Log[n]^a, {s, 0, 1}], a → c]
```

```
ConditionalExpression[
  (Gamma[c] - Gamma[c, -Log[n]]) (-Log[n])^-c Log[n]^c, Re[Log[n]] < 0 && Re[c] ≥ 0]
```

```
Limit[Integrate[ E^s s^(a - 1), {s, 0, Log[n]}], a → c]
```

```
ConditionalExpression[(Gamma[c] - Gamma[c, -Log[n]]) (-Log[n])^-c Log[n]^c, Re[c] ≥ 0]
```

```
N[Integrate[ 100^s s^(7 / 4 - 1) Log[100]^(7 / 4), {s, 0, 1}]]
```

```
259.651 + 1.77636 × 10^-15 i
```

```
Abs[N[Gamma[7 / 4, 0, -Log[100]]]]
```

```
259.651
```

```
Integrate[ Log[1 / t]^(k - 1), {t, 0, 1}]
```

```
ConditionalExpression[Gamma[k], Re[k] > 0]
```

```
Limit[ dx Sum[ (dx + 1)^k, {k, 1, Log[dx + 1, x]}], dx → 0]
Limit[Sum[ (dx + 1)^(k + 1) - (dx + 1)^k, {k, 1, Log[dx + 1, x]}], dx → 0]
Limit[ dx Sum[ E^(Log[dx + 1] k), {k, 1, Log[x] / Log[dx + 1]}], dx → 0]
Integrate[ x^s Log[x], {s, 0, 1}]
```

```
-1 + x
```

```
-1 + x
```

```
-1 + x
```

```
-1 + x
```

```
Expand[Limit[ dx^3 Sum[ (dx + 1)^(k) k^2, {k, Log[dx + 1, x], Infinity}], dx → 0]]
Expand[Limit[ dx^3 Sum[ (dx + 1)^k k^2, {k, 0, Log[dx + 1, x]}], dx → 0]]
Expand[Limit[ dx^3 Sum[ (dx + 1)^(k) k^2, {k, -Infinity, -Log[dx + 1, x]}], dx → 0]]
Expand[Limit[ dx^3 Sum[ (dx + 1)^k k^2, {k, -Log[dx + 1, x], 0}], dx → 0]]
Expand[Limit[ dx^3 Sum[ (dx + 1)^k k^2, {k, -Infinity, 0}], dx → 0]]
```

```
-2 x + 2 x Log[x] - x Log[x]^2
```

```
-2 + 2 x - 2 x Log[x] + x Log[x]^2
```

$$\frac{2}{x} + \frac{2\,\mathrm{Log}[x]}{x} + \frac{\mathrm{Log}[x]^2}{x}$$

$$2 - \frac{2}{x} - \frac{2\,\mathrm{Log}[x]}{x} - \frac{\mathrm{Log}[x]^2}{x}$$

```
2
```

```
Limit[dx Sum[ (dx + 1) ^k , {k, 1, Log[dx + 1, x]}], dx → 0]
Integrate[ x^t Log[x], {t, 0, 1}]
```

```
-1 + x
```

```
-1 + x
```