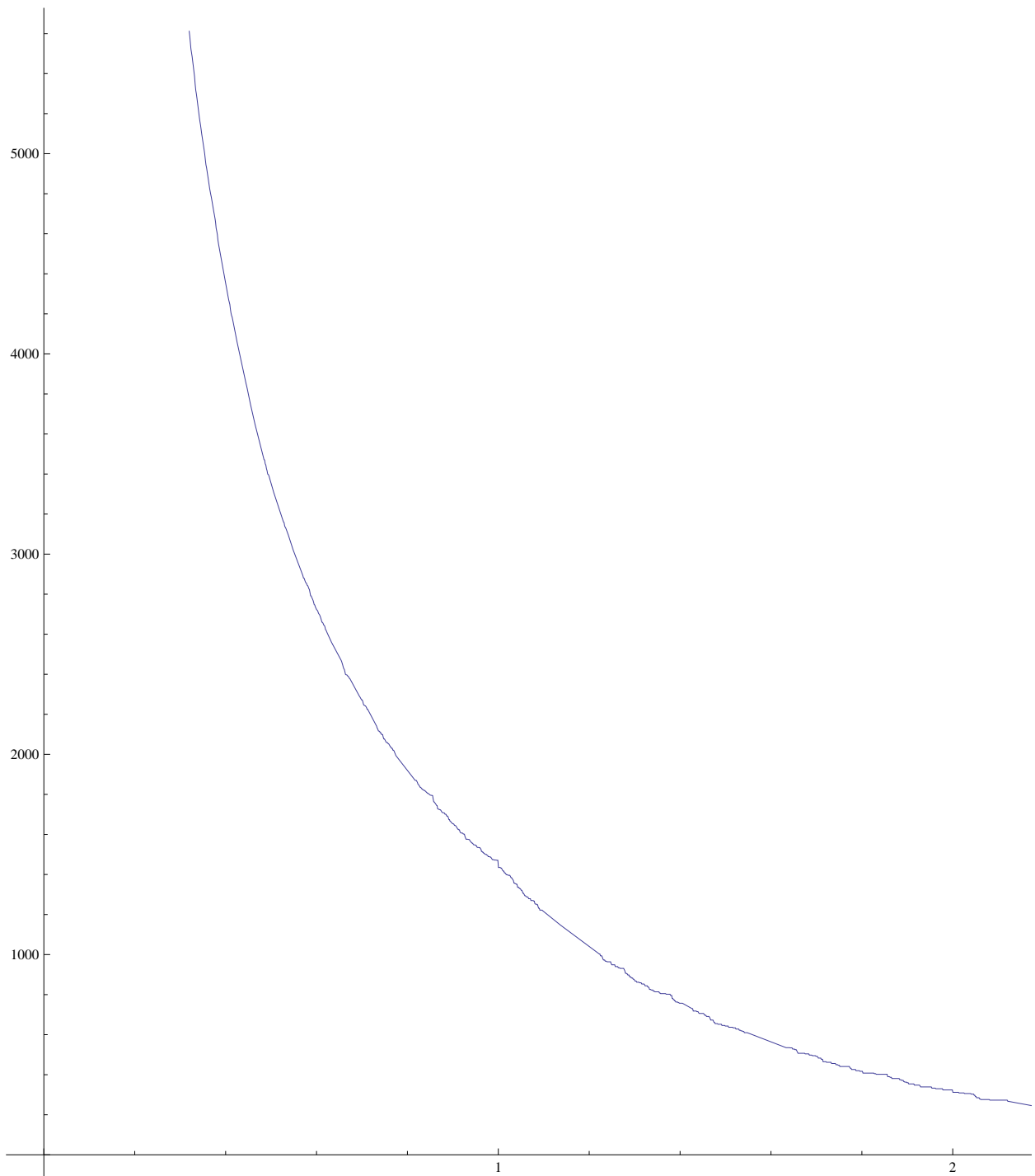


```
ClearAll["Global`*"]
```

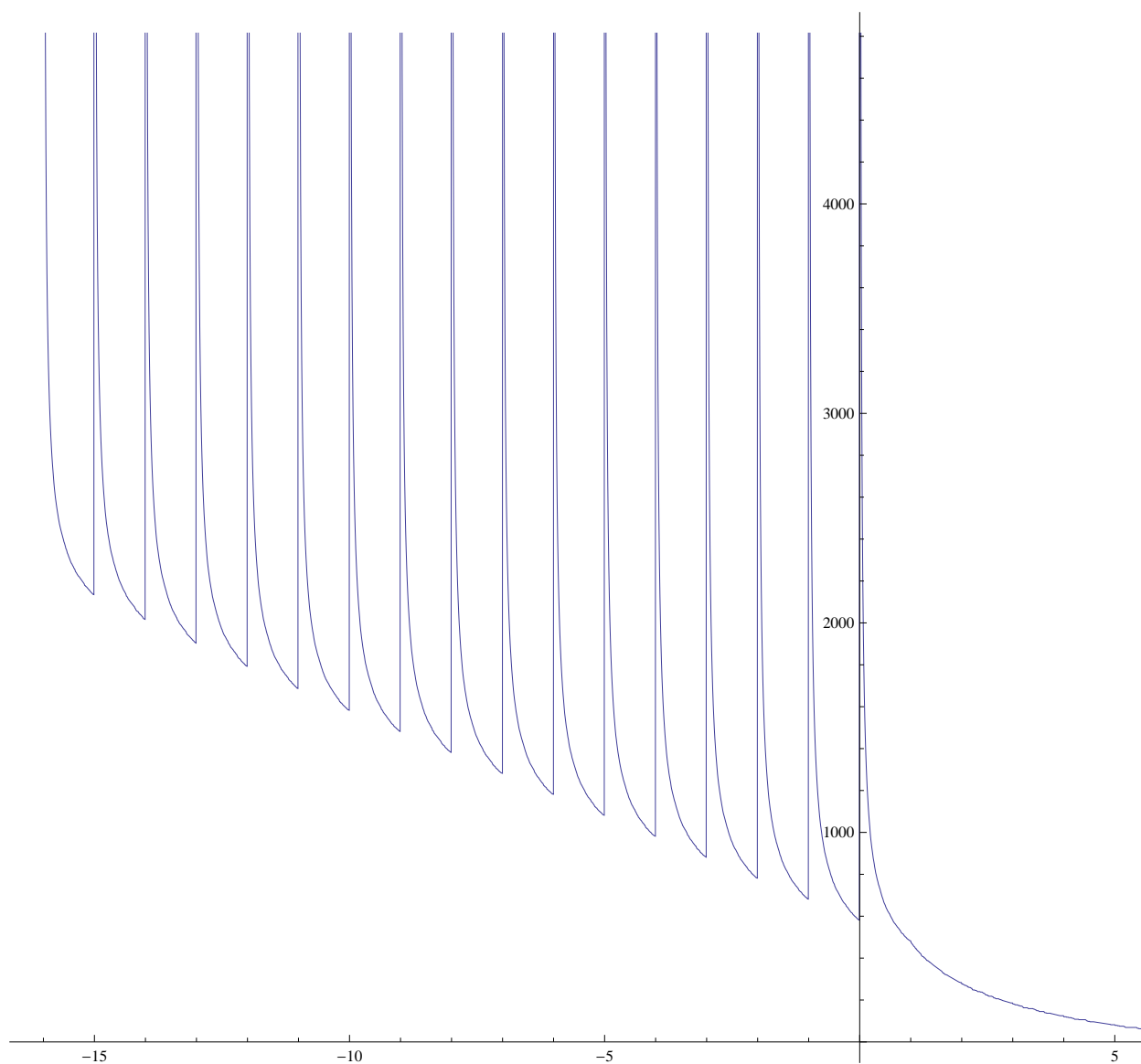
```
f[n_, k_, a_] := f[n, k, a] = Sum[f[n / (a + j), k - 1, a], {j, 0, Floor[n - a]}];
```

```
f[n_, 0, a_] := 1
```

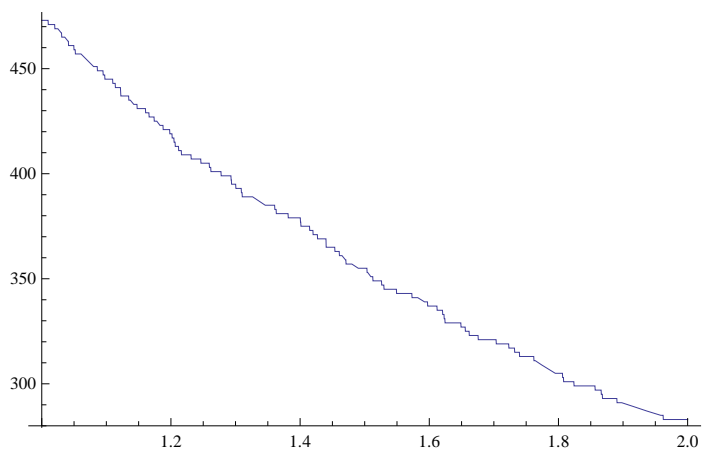
```
Plot[f[100, 3, a], {a, 0, 4}]
```



`Plot[f[100, 2, a], {a, -16, 16}]`



`Plot[f[100, 2, a], {a, 1, 2}]`



```
f[100, 2, 11]
```

```
0
```

```
Dhyp[n_, k_, a_] :=
```

```
Sum[Binomial[k, j] Dhyp[Floor[n / (m^(k - j))], j, m + 1], {m, a, n^(1 / k)}, {j, 0, k - 1}]
```

```
Dhyp[n_, 1, a_] := Floor[n] - a + 1; Dhyp[n_, 0, a_] := 1
```

```
24^-2 f[100 × 24^2, 3, 25]
```

```
4493
```

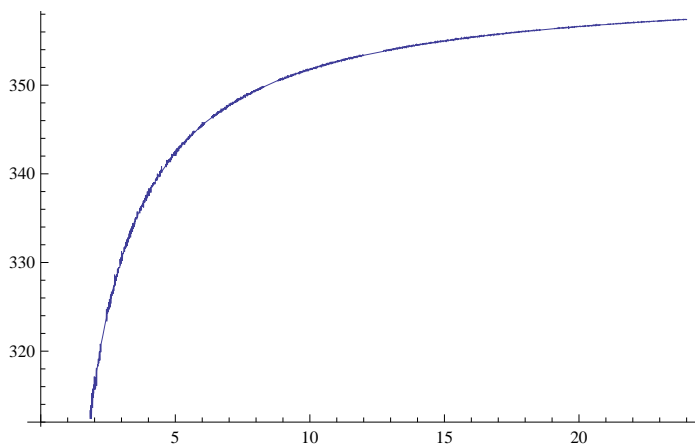
```
144
```

```
24^-2 Dhyp[100 × 24^2, 3, 25]
```

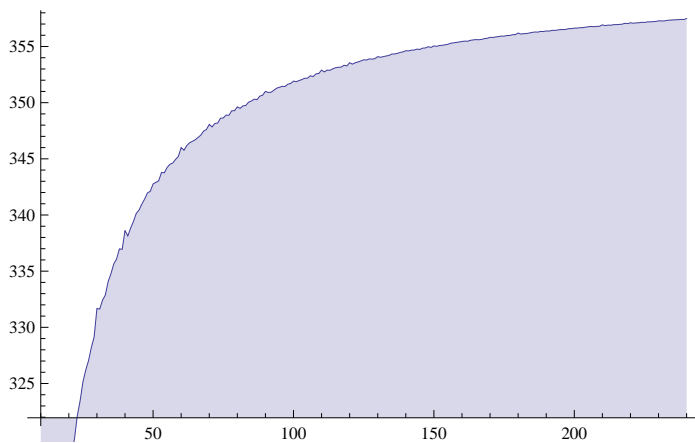
```
4493
```

```
144
```

```
Plot[n^-2 Dhyp[100 n^2, 2, n + 1], {n, 0, 24}]
```



```
DiscretePlot[(n * .1)^-2 f[100 (n * .1)^2, 2, (n * .1) + 1], {n, 10, 240}]
```



```
N[Gamma[2, 0, -Log[100]] / Gamma[2]]
```

```
361.517 - 4.41506 × 10^-14 i
```

f[100, 2, k]

$$\sum_{j=0}^{100+\text{Floor}[-k]} \left(1 + \text{Floor} \left[\frac{100 - j k - k^2}{j + k} \right] \right)$$

f2[k_] := k^-2 f[10 k^2, 2, k + 1]

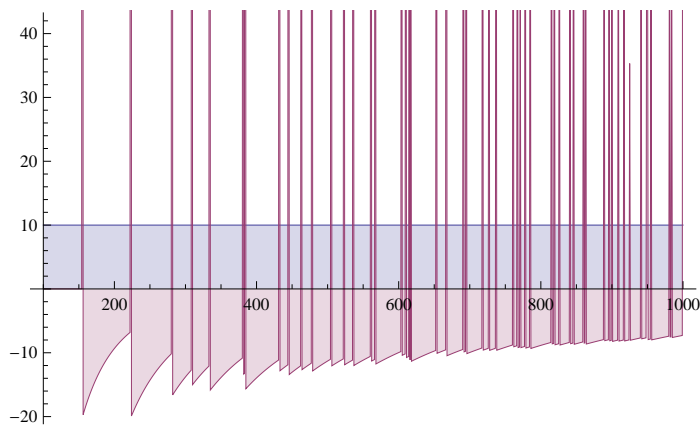
f2'[4]

General::ivar : 4 is not a valid variable. >>

$$-\frac{2351}{32} + \frac{\partial_4 2351}{16}$$

General::ivar : 3 is not a valid variable. >>

DiscretePlot[{10, (f2[n * .003] - f2[(n - 1) * .003]) / .003}, {n, 100, 1000}]



Sum[Binomial[z, k] 1 / (s - 1)^k, {k, 0, Infinity}]

$$\left(\frac{s}{-1 + s} \right)^z$$

Sum[(-1)^(k - 1) / k 1 / (s - 1)^k, {k, 1, Infinity}]

$$\text{Log} \left[\frac{s}{-1 + s} \right]$$

f[n_, z_] := Sum[(-1)^k Binomial[z, k] (1 - Gamma[k, -Log[n]] / Gamma[k]), {k, 0, Infinity}]

N[f[100, 2]]

560.517

f[100, 3, 2]

324

hh[s_, x_] := x^(s - 1) Zeta[s, x + 1]

D[hh[s, x], x]

h2[s_, x_] := (-1 + s) x^-2+s Zeta[s, 1 + x] - s x^-1+s Zeta[1 + s, 1 + x]

h3[s_] := (1 / (s - 1)) - Integrate[h2[s, x], {x, 1, Infinity}]

h3[3]

-1 + Zeta[3]

h3a[s_] :=

(1 / (s - 1)) - Integrate[(-1 + s) x^{-2+s} Zeta[s, 1 + x] - s x^{-1+s} Zeta[1 + s, 1 + x], {x, 1, Infinity}]

h3a[s]

$\frac{1}{-1+s} - \int_1^{\infty} ((-1+s) x^{-2+s} \text{Zeta}[s, 1+x] - s x^{-1+s} \text{Zeta}[1+s, 1+x]) dx$

Integrate[(-1 + s) x^{-2+s} Zeta[s, 1 + x], {x, 1, Infinity}]

$\int_1^{\infty} (-1+s) x^{-2+s} \text{Zeta}[s, 1+x] dx$

Integrate[-s x^{-1+s} Zeta[1 + s, 1 + x], {x, 1, Infinity}]

$\int_1^{\infty} -s x^{-1+s} \text{Zeta}[1+s, 1+x] dx$

hi[s_, z_, x_] := x^z (s - 1) Zeta[s, x + 1]^z

D[hi[s, z, x], x]

$(-1+s) x^{-1+(-1+s)z} z \text{Zeta}[s, 1+x]^z - s x^{(-1+s)z} z \text{Zeta}[s, 1+x]^{-1+z} \text{Zeta}[1+s, 1+x]$

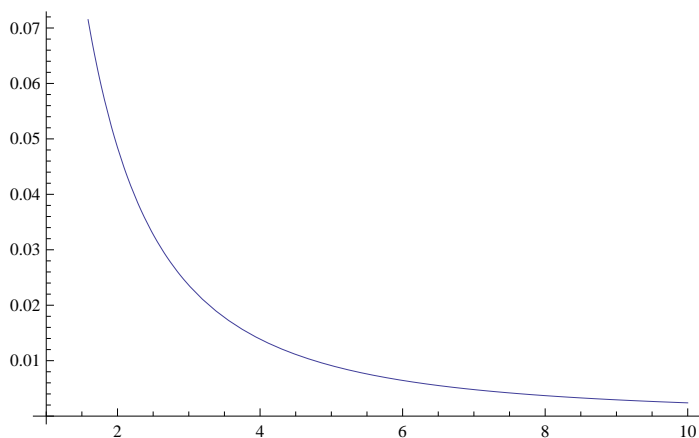
Sum[(-1)^(z-1) / z

((-1 + s) x^{-1+(-1+s)z} z Zeta[s, 1 + x]^z - s x^{(-1+s)z} z Zeta[s, 1 + x]^{-1+z} Zeta[1 + s, 1 + x]), {z, 1, Infinity}]

$-\frac{x^{-1+s} (\text{Zeta}[s, 1+x] - s \text{Zeta}[s, 1+x] + s x \text{Zeta}[1+s, 1+x])}{x + x^s \text{Zeta}[s, 1+x]}$

dlog[s_, x_] := - $\frac{x^{-1+s} (\text{Zeta}[s, 1+x] - s \text{Zeta}[s, 1+x] + s x \text{Zeta}[1+s, 1+x])}{x + x^s \text{Zeta}[s, 1+x]}$

Plot[dlog[2, x], {x, 1, 10}]



Residue[Zeta[s]^z / z^2, {z, -2}]

0

```
f[100, 2, 2]
```

```
283
```

```
f[100, 2, 3]
```

```
186
```

```
f[100, 2, 3] + 2 f[100 / 2, 1, 3] + f[100 / 4, 0, 3]
```

```
283
```

```
tt[a_] := f[100, 2, a + 1] + 2 f[100 / a, 1, a + 1] + f[100 / a^2, 0, a + 1]
```

```
f[n_, k_, a_] := f[n, k, a] = Sum[ f[n / (a + j), k - 1, a], {j, 0, Floor[n - a]}];
```

```
f[n_, 0, a_] := 1
```

```
tt[n_, k_, a_] := Sum[ Binomial[k, j] f[n / a^j, k - j, a + 1], {j, 0, k}]
```

```
ttb[n_, k_, a_] := Sum[ (-1)^j Binomial[k, j] f[n / (a - 1)^j, k - j, a - 1], {j, 0, k}]
```

```
tt[231, 4, 2.5]
```

```
296
```

```
ttb[231, 4, 2.5]
```

```
296
```

```
f[231, 4, 2.5]
```

```
296
```

```
Limit[f[231, 4, z], z → 0]
```

```
$Aborted
```