

```

d[n_, 0, a_] := 1
d[n_, 1, a_] := n - a + 1
d[n_, k_, a_] :=
  Sum[Binomial[k, j] d[Floor[n / (m^j)], k - j, m + 1], {j, 1, k}, {m, a, n^(1/k)}]

D32Unrolled[n_] := -1 + Floor[n^(1/3)]^3 +
  3 Sum[Floor[n / (m^2)] - Floor[Floor[n/m]^(1/2)]^2 + 2 Sum[Floor[Floor[n/m] / j],
    {j, m + 1, Floor[Floor[n/m]^(1/2)]}], {m, 2, Floor[n^(1/3)]}]

D22Unrolled[n_] := 1 - Floor[n^(1/2)]^2 + 2 Sum[Floor[n/m], {m, 2, Floor[n^(1/2)]}]
d0[n_, a_Integer] := 1
d1[n_, a_Integer] := n - a + 1
d2a[n_, a_Integer] :=
  Sum[Binomial[2, 2] d0[Floor[n / (m^2)], m + 1], {m, a, Floor[n^(1/2)]}] +
  Sum[Binomial[2, 1] d1[Floor[n / (m^1)], m + 1], {m, a, Floor[n^(1/2)]}]
d2[n_, a_Integer] :=
  1 - 2 a + a^2 - Floor[n^(1/2)]^2 + 2 Sum[Floor[n/m], {m, a, Floor[n^(1/2)]}]
d2[0, a_Integer] := 0
d3a[n_, a_Integer] :=
  Sum[Binomial[3, 3] d0[Floor[n / (m^3)], m + 1], {m, a, Floor[n^(1/3)]}] +
  Sum[Binomial[3, 2] d1[Floor[n / (m^2)], m + 1], {m, a, Floor[n^(1/3)]}] +
  Sum[Binomial[3, 1] d2a[Floor[n / (m^1)], m + 1], {m, a, Floor[n^(1/3)]}]

d2e[n_, a_Integer] :=
  (1 - a)^2 - Floor[n^(1/2)]^2 + 2 Sum[Floor[n/m], {m, a, Floor[n^(1/2)]}]

dd3e[n_, a_] := (1 - a)^3 + Floor[n^(1/3)]^3 + Sum[3 Floor[n/s^2] - 3 Floor[sqrt(Floor[n/s])]^2 +
  6 Sum[Floor[n/m/s], {m, s + 1, Floor[Floor[n/s]^(1/2)]}], {s, a, Floor[n^(1/3)]}]

d4e[x_, b_] :=
  (-1 + b)^4 +
  -Floor[x^(1/4)]^4 +
  4 Sum[Floor[x / (u^3)], {u, b, x^(1/4)}] +
  -6 Sum[Floor[Floor[x / (u^2)]^(1/2)]^2, {u, b, x^(1/4)}] +
  12 Sum[Floor[x / (u^2 m)],
    {u, b, x^(1/4)}, {m, (u + 1), Floor[Floor[x / (u^2)]^(1/2)]}] +
  4 Sum[Floor[Floor[x / u]^(1/3)]^3, {u, b, x^(1/4)}] +
  12 Sum[Floor[x / (u s^2)], {u, b, x^(1/4)}, {s, (u + 1), Floor[Floor[x / u]^(1/3)]}] +
  -12 Sum[Floor[Floor[x / (u s)]^(1/2)]^2, {u, b, x^(1/4)}, {s, (u + 1), (x / u)^(1/3)}] +
  24 Sum[Floor[x / (u m s)], {u, b, x^(1/4)},
    {s, u + 1, (x / u)^(1/3)}, {m, s + 1, (x / (u s))^(1/2)}]

d4e[100000, 2]
11796070

d[100000, 4, 2]
11796070

FF[x_, b_] := Sum[Floor[x / (u m s)], {u, b, x^(1/4)},
  {s, u + 1, (x / u)^(1/3)}, {m, s + 1, (x / (u s))^(1/2)}]

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FF[10 000, 2]

21 090

GG[x_, b_] :=

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Sum[ Floor[x / (u m s)], {u, 2, x^(1/4)}, {s, 2, (x/u)^(1/3)}, {m, 2, (x/(u s))^(1/2)}}] -
Sum[ Floor[x / (u m s)], {u, 2, x^(1/4)}, {s, 2, (x/u)^(1/3)}, {m, 2, s}] -
Sum[ Floor[x / (u m s)], {u, 2, x^(1/4)}, {s, 2, u}, {m, 2, (x/(u s))^(1/2)}}] +
Sum[ Floor[x / (u m s)], {u, 2, x^(1/4)}, {s, 2, u}, {m, 2, s}] -
Sum[ Floor[x / (u m s)], {u, 2, b-1}, {s, 2, (x/u)^(1/3)}, {m, 2, (x/(u s))^(1/2)}}] +
Sum[ Floor[x / (u m s)], {u, 2, b-1}, {s, 2, (x/u)^(1/3)}, {m, 2, s}] +
Sum[ Floor[x / (u m s)], {u, 2, b-1}, {s, 2, u}, {m, 2, (x/(u s))^(1/2)}}] -
Sum[ Floor[x / (u m s)], {u, 2, b-1}, {s, 2, u}, {m, 2, s}]
```

GG[10 000, 2]

21 090

HH[x_, b_] :=

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Sum[ Floor[x / (u m s)], {u, 2, x^(1/4)}, {s, 2, (x/u)^(1/3)}, {m, 2, (x/(u s))^(1/2)}}] -
Sum[ Floor[x / (u m s)], {u, 2, x^(1/4)}, {s, 2, (x/u)^(1/3)}, {m, 2, s}] -
Sum[ Floor[x / (u m s)], {u, 2, x^(1/4)}, {s, 2, u}, {m, 2, (x/(u s))^(1/2)}}] +
Sum[ Floor[x / (u m s)], {u, 2, x^(1/4)}, {s, 2, u}, {m, 2, s}] -
Sum[ Floor[x / (u m s)], {u, 2, b-1}, {s, 2, (x/u)^(1/3)}, {m, 2, (x/(u s))^(1/2)}}] +
Sum[ Floor[x / (u m s)], {u, 2, b-1}, {s, 2, (x/u)^(1/3)}, {m, 2, s}] +
Sum[ Floor[x / (u m s)], {u, 2, b-1}, {s, 2, u}, {m, 2, (x/(u s))^(1/2)}}] -
Sum[ Floor[x / (u m s)], {u, 2, b-1}, {s, 2, u}, {m, 2, s}]
```

HH[10 000, 2]

21 090

```
dd3f[n_, a_] := (1 - a)^3 + Floor[n^(1/3)]^3 +
+ 3 Sum[Floor[n / (s^2)], {s, a, n^(1/3)}}] +
- 3 Sum[Floor[Floor[n / s]^(1/2)]^2, {s, a, n^(1/3)}}] +
6 Sum[Floor[n / m / s], {s, a, n^(1/3)}, {m, s + 1, Floor[n / s]^(1/2)}}]
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dd3e[100, 2]

324

dd3f[100, 2]

324

II[x_] :=

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Sum[ Floor[x / (u m s)], {u, 2, x^(1/4)}, {s, 2, (x/u)^(1/3)}, {m, 2, (x/(u s))^(1/2)}}] -
Sum[ Floor[x / (u m s)], {u, 2, x^(1/4)}, {s, 2, (x/u)^(1/3)}, {m, 2, s}] -
Sum[ Floor[x / (u m s)], {u, 2, x^(1/4)}, {s, 2, u}, {m, 2, (x/(u s))^(1/2)}}] +
Sum[ Floor[x / (u m s)], {u, 2, x^(1/4)}, {s, 2, u}, {m, 2, s}] -
Sum[ Floor[x / (u m s)], {u, 2, 1}, {s, 2, (x/u)^(1/3)}, {m, 2, (x/(u s))^(1/2)}}] +
Sum[ Floor[x / (u m s)], {u, 2, 1}, {s, 2, (x/u)^(1/3)}, {m, 2, s}] +
Sum[ Floor[x / (u m s)], {u, 2, 1}, {s, 2, u}, {m, 2, (x/(u s))^(1/2)}}] -
Sum[ Floor[x / (u m s)], {u, 2, 1}, {s, 2, u}, {m, 2, s}]
```

II[10 000]

21 090

HH[x, 2]

\$Aborted

JJ[x_] :=

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Sum[ Floor[x / (u m s)], {u, 2, x^(1 / 4)}, {s, 2, (x / u)^(1 / 3)}, {m, 2, (x / (u s))^(1 / 2)}] -
Sum[ Floor[x / (u m s)], {u, 2, x^(1 / 4)}, {s, 2, (x / u)^(1 / 3)}, {m, 2, s}] -
Sum[ Floor[x / (u m s)], {u, 2, x^(1 / 4)}, {s, 2, u}, {m, 2, (x / (u s))^(1 / 2)}] +
Sum[ Floor[x / (u m s)], {u, 2, x^(1 / 4)}, {s, 2, u}, {m, 2, s}]
```

JJ[10 000]

21 090

KK[x_] := Sum[Floor[x / (u m s)], {u, 2, x^(1 / 4)}, {s, 2, u}, {m, 2, s}]

KK[10 000]

17 856

KK[x]

\$Aborted

ddd[n_, a_] := Sum[Floor[n / m / s], {s, a, n^(1 / 3)}, {m, s + 1, Floor[n / s]^(1 / 2)}]

ddd[10 000, 2]

49 245

```
eee[n_] := Sum[Floor[n / (m s)], {s, 2, n^(1 / 3)}, {m, 2, Floor[n / s]^(1 / 2)}] -
Sum[Floor[n / (m s)], {s, 2, n^(1 / 3)}, {m, 2, s}]
```

eee[10 000]

49 245