```
Clear[Dz, Dzo, Dza, Dze, Dzx, dza]
bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k!
FI[n_] := FactorInteger[n]; FI[1] := {}
dza[n_, z_, a_] := dza[n, z, a] = Product[rise[z, p[[2]], a] / p[[2]]!, {p, FI[n]}]
rise[z_, k_] := rise[z, k] = Product[ z + j, {j, 0, k - 1}]
rise[z_, k_, a_] := rise[z, k, a] = Product[ z + j a, {j, 0, k - 1}]
Dz[n_, z_, y_] :=
 Dz[n, z, y] = If[n < y, 1, Sum[bin[z, k] Dz[n / y^k, z - k, y + 1], {k, 0, Log[y, n]}]]
Dzo[n_, z_, y_] := Dzo[n, z, y] =
  If[n < y, 1, Dzo[n, z, y + 1] + Sum[z^k / k! Dzo[n / y^k, z - k, y + 1], {k, 1, Log[y, n]}]]
Dza[n_, z_, y_] := Dza[n, z, y] = If[n < y, 1,
   Sum[Pochhammer[z, k] / k! Dza[n / y^k, z - k, y + 1], {k, 0, Log[y, n]}]]
Dze[n_, z_, y_] := Dze[n, z, y] = If[n < y, 1,
   Sum[(-1)^k Pochhammer[-z, k] / k! Dze[n / y^k, z - k, y + 1], {k, 0, Log[y, n]}]]
Dzx[n_, a_, z_, y_] := Dzx[n, a, z, y] = If[n < y, 1,
   Sum[rise[z, k, a] / k! Dzx[n / y^k, a, z - k, y + 1], {k, 0, Log[y, n]}]]
dzx[n_, a_, z_] := Dzx[n, a, z, 2] - Dzx[n - 1, a, z, 2]
ldzx[n_, a_] := D[dzx[n, a, z], z] /. z → 0
```

```
Expand@Dz[100, z, 2]
```

$$1 + \frac{428\,z}{15} + \frac{16\,289\,z^2}{360} + \frac{331\,z^3}{16} + \frac{611\,z^4}{144} + \frac{67\,z^5}{240} + \frac{7\,z^6}{720}$$

```
Expand@Dzo[100, z, 2]
```

$$1 + \frac{161\,z}{6} + 37\,z^2 + \frac{93\,z^3}{4} + \frac{16\,z^4}{3} + \frac{23\,z^5}{60} + \frac{7\,z^6}{720}$$

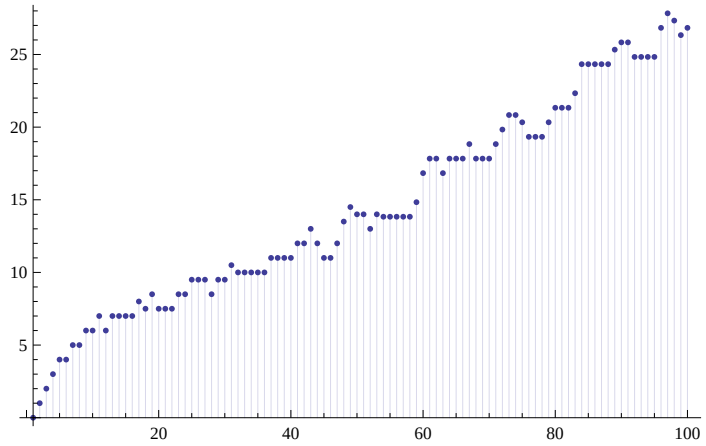
```
Expand@Dza[100, z, 2]
```

$$1 + \frac{193\,z}{15} + \frac{6389\,z^2}{360} + \frac{1375\,z^3}{48} + \frac{1043\,z^4}{144} + \frac{39\,z^5}{80} + \frac{7\,z^6}{720}$$

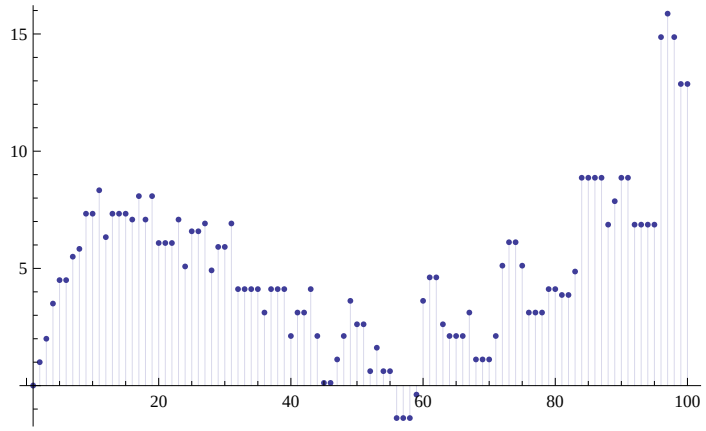
```
Expand@Dze[100, z, 2]
```

$$1 + \frac{428\,z}{15} + \frac{16\,289\,z^2}{360} + \frac{331\,z^3}{16} + \frac{611\,z^4}{144} + \frac{67\,z^5}{240} + \frac{7\,z^6}{720}$$

```
DiscretePlot[D[ Dzo[n, z, 2], z] /. z → 0, {n, 1, 100}]
```



```
DiscretePlot[ D[Dza[n, z, 2], z] /. z → 0, {n, 1, 100}]
```




```
Table[{n, ldzx[n, -3], ldzx[n, -2], ldzx[n, -1],
    ldzx[n, 0], ldzx[n, 1], ldzx[n, 2], ldzx[n, 3]}, {n, 1, 100}] // TableForm
```

1	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1
4	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$
5	1	1	1	1	1	1	1
6	0	0	0	0	0	0	0
7	1	1	1	1	1	1	1
8	3	$\frac{4}{3}$	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{4}{3}$	3
9	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$
10	0	0	0	0	0	0	0
11	1	1	1	1	1	1	1
12	2	1	0	−1	−2	−3	−4
13	1	1	1	1	1	1	1
14	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0
16	$-\frac{21}{4}$	−1	$\frac{1}{4}$	0	$-\frac{1}{4}$	1	$\frac{21}{4}$
17	1	1	1	1	1	1	1
18	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	−1	$-\frac{3}{2}$	−2
19	1	1	1	1	1	1	1
20	2	1	0	−1	−2	−3	−4
21	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0
23	1	1	1	1	1	1	1
24	−6	−2	0	0	−2	−6	−12
25	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$
26	0	0	0	0	0	0	0
27	3	$\frac{4}{3}$	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{4}{3}$	3
28	2	1	0	−1	−2	−3	−4
29	1	1	1	1	1	1	1
30	0	0	0	0	0	0	0
31	1	1	1	1	1	1	1
32	$\frac{56}{5}$	$\frac{17}{10}$	$\frac{1}{5}$	$-\frac{1}{2}$	$-\frac{14}{5}$	$-\frac{43}{10}$	$\frac{11}{5}$
33	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0

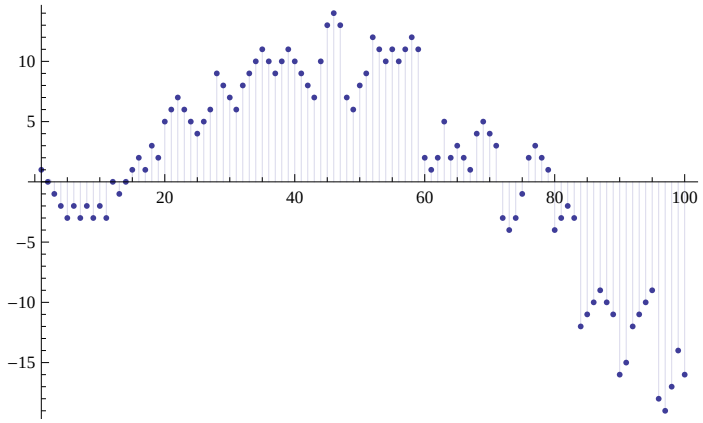
35	0	0	0	0	0	0	0
36	−3	−1	0	0	−1	−3	−6
37	1	1	1	1	1	1	1
38	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0
40	−6	−2	0	0	−2	−6	−12
41	1	1	1	1	1	1	1
42	0	0	0	0	0	0	0
43	1	1	1	1	1	1	1
44	2	1	0	−1	−2	−3	−4
45	2	1	0	−1	−2	−3	−4
46	0	0	0	0	0	0	0
47	1	1	1	1	1	1	1
48	15	$\frac{5}{2}$	0	$\frac{3}{2}$	1	$-\frac{15}{2}$	−30
49	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$
50	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	−1	$-\frac{3}{2}$	−2
51	0	0	0	0	0	0	0
52	2	1	0	−1	−2	−3	−4
53	1	1	1	1	1	1	1
54	$-\frac{5}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{6}$	−1	$-\frac{5}{2}$	$-\frac{14}{3}$
55	0	0	0	0	0	0	0
56	−6	−2	0	0	−2	−6	−12
57	0	0	0	0	0	0	0
58	0	0	0	0	0	0	0
59	1	1	1	1	1	1	1
60	−4	−2	0	2	4	6	8
61	1	1	1	1	1	1	1
62	0	0	0	0	0	0	0
63	2	1	0	−1	−2	−3	−4
64	$-\frac{49}{2}$	−2	$\frac{1}{6}$	1	$-\frac{1}{2}$	$-\frac{16}{3}$	$\frac{11}{2}$
65	0	0	0	0	0	0	0
66	0	0	0	0	0	0	0
67	1	1	1	1	1	1	1
68	2	1	0	−1	−2	−3	−4
69	0	0	0	0	0	0	0
70	0	0	0	0	0	0	0
71	1	1	1	1	1	1	1
72	13	3	0	1	3	3	−2
73	1	1	1	1	1	1	1
74	0	0	0	0	0	0	0
75	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	−1	$-\frac{3}{2}$	−2
76	2	1	0	−1	−2	−3	−4
77	0	0	0	0	0	0	0
78	0	0	0	0	0	0	0
79	1	1	1	1	1	1	1
80	16	3	0	1	0	−9	−32
81	$-\frac{21}{4}$	−1	$\frac{1}{4}$	0	$-\frac{1}{4}$	1	$\frac{21}{4}$
82	0	0	0	0	0	0	0
83	1	1	1	1	1	1	1
84	−4	−2	0	2	4	6	8
85	0	0	0	0	0	0	0
86	0	0	0	0	0	0	0
87	0	0	0	0	0	0	0

88	−6	−2	0	0	−2	−6	−12
89	1	1	1	1	1	1	1
90	−1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
91	0	0	0	0	0	0	0
92	2	1	0	−1	−2	−3	−4
93	0	0	0	0	0	0	0
94	0	0	0	0	0	0	0
95	0	0	0	0	0	0	0
96	−40	−4	0	2	8	0	−64
97	1	1	1	1	1	1	1
98	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	−1	$-\frac{3}{2}$	−2
99	2	1	0	−1	−2	−3	−4
100	−4	$-\frac{3}{2}$	0	$\frac{1}{2}$	0	$-\frac{3}{2}$	−4

```
rise[z, 3, 0]
```

z^3

```
DiscretePlot[Dzx[n, 1, -1, 2], {n, 1, 100}]
```




```
Grid@Table[dzx[n, k, -1], {n, 1, 100}, {k, -3, 3}]
```

0	0	0	0	0	0	0
−1	−1	−1	−1	−1	−1	−1
−1	−1	−1	−1	−1	−1	−1
1	$\frac{1}{2}$	0	$-\frac{1}{2}$	−1	$-\frac{3}{2}$	−2
−1	−1	−1	−1	−1	−1	−1
1	1	1	1	1	1	1
−1	−1	−1	−1	−1	−1	−1
$-\frac{11}{3}$	$-\frac{3}{2}$	0	$\frac{5}{6}$	1	$\frac{1}{2}$	$-\frac{2}{3}$
1	$\frac{1}{2}$	0	$-\frac{1}{2}$	−1	$-\frac{3}{2}$	−2
1	1	1	1	1	1	1
−1	−1	−1	−1	−1	−1	−1
−3	$-\frac{3}{2}$	0	$\frac{3}{2}$	3	$\frac{9}{2}$	6
−1	−1	−1	−1	−1	−1	−1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
$\frac{26}{3}$	$\frac{19}{8}$	0	$\frac{1}{24}$	1	$\frac{11}{8}$	$-\frac{1}{3}$

−1	−1	−1	−1	−1	−1	−1
−2	−1	0	1	2	3	4
−1	−1	−1	−1	−1	−1	−1
−3	$-\frac{3}{2}$	0	$\frac{3}{2}$	3	$\frac{9}{2}$	6
1	1	1	1	1	1	1
1	1	1	1	1	1	1
−1	−1	−1	−1	−1	−1	−1
$\frac{35}{3}$	$\frac{9}{2}$	0	$-\frac{11}{6}$	−1	$\frac{5}{2}$	$\frac{26}{3}$
1	$\frac{1}{2}$	0	$-\frac{1}{2}$	−1	$-\frac{3}{2}$	−2
1	1	1	1	1	1	1
$-\frac{11}{3}$	$-\frac{3}{2}$	0	$\frac{5}{6}$	1	$\frac{1}{2}$	$-\frac{2}{3}$
−3	$-\frac{3}{2}$	0	$\frac{3}{2}$	3	$\frac{9}{2}$	6
−1	−1	−1	−1	−1	−1	−1
−1	−1	−1	−1	−1	−1	−1
−1	−1	−1	−1	−1	−1	−1
$-\frac{59}{3}$	$-\frac{27}{8}$	0	$\frac{19}{120}$	2	$\frac{45}{8}$	$\frac{19}{3}$
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
7	$\frac{11}{4}$	0	$-\frac{5}{4}$	−1	$\frac{3}{4}$	4
−1	−1	−1	−1	−1	−1	−1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
$\frac{35}{3}$	$\frac{9}{2}$	0	$-\frac{11}{6}$	−1	$\frac{5}{2}$	$\frac{26}{3}$
−1	−1	−1	−1	−1	−1	−1
−1	−1	−1	−1	−1	−1	−1
−1	−1	−1	−1	−1	−1	−1
−3	$-\frac{3}{2}$	0	$\frac{3}{2}$	3	$\frac{9}{2}$	6
−3	$-\frac{3}{2}$	0	$\frac{3}{2}$	3	$\frac{9}{2}$	6
1	1	1	1	1	1	1
−1	−1	−1	−1	−1	−1	−1
$-\frac{95}{3}$	$-\frac{59}{8}$	0	$-\frac{49}{24}$	−6	$-\frac{35}{8}$	$\frac{31}{3}$
1	$\frac{1}{2}$	0	$-\frac{1}{2}$	−1	$-\frac{3}{2}$	−2
−2	−1	0	1	2	3	4
1	1	1	1	1	1	1
−3	$-\frac{3}{2}$	0	$\frac{3}{2}$	3	$\frac{9}{2}$	6
−1	−1	−1	−1	−1	−1	−1
$\frac{19}{3}$	$\frac{5}{2}$	0	$-\frac{7}{6}$	−1	$\frac{1}{2}$	$\frac{10}{3}$
1	1	1	1	1	1	1
$\frac{35}{3}$	$\frac{9}{2}$	0	$-\frac{11}{6}$	−1	$\frac{5}{2}$	$\frac{26}{3}$
1	1	1	1	1	1	1
1	1	1	1	1	1	1
−1	−1	−1	−1	−1	−1	−1
9	$\frac{9}{2}$	0	$-\frac{9}{2}$	−9	$-\frac{27}{2}$	−18
−1	−1	−1	−1	−1	−1	−1
1	1	1	1	1	1	1
−3	$-\frac{3}{2}$	0	$\frac{3}{2}$	3	$\frac{9}{2}$	6
$\frac{428}{9}$	$\frac{85}{16}$	0	$-\frac{1049}{720}$	−3	$\frac{9}{16}$	$\frac{32}{9}$

1	1	1	1	1	1	1
−1	−1	−1	−1	−1	−1	−1
−1	−1	−1	−1	−1	−1	−1
−3	$-\frac{3}{2}$	0	$\frac{3}{2}$	3	$\frac{9}{2}$	6
1	1	1	1	1	1	1
−1	−1	−1	−1	−1	−1	−1
−1	−1	−1	−1	−1	−1	−1
$-\frac{86}{3}$	−8	0	$-\frac{2}{3}$	−6	−12	$-\frac{44}{3}$
−1	−1	−1	−1	−1	−1	−1
1	1	1	1	1	1	1
−2	−1	0	1	2	3	4
−3	$-\frac{3}{2}$	0	$\frac{3}{2}$	3	$\frac{9}{2}$	6
1	1	1	1	1	1	1
−1	−1	−1	−1	−1	−1	−1
−1	−1	−1	−1	−1	−1	−1
$-\frac{98}{3}$	$-\frac{63}{8}$	0	$-\frac{37}{24}$	−5	$-\frac{23}{8}$	$\frac{37}{3}$
$\frac{26}{3}$	$\frac{19}{8}$	0	$\frac{1}{24}$	1	$\frac{11}{8}$	$-\frac{1}{3}$
1	1	1	1	1	1	1
−1	−1	−1	−1	−1	−1	−1
9	$\frac{9}{2}$	0	$-\frac{9}{2}$	−9	$-\frac{27}{2}$	−18
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
$\frac{35}{3}$	$\frac{9}{2}$	0	$-\frac{11}{6}$	−1	$\frac{5}{2}$	$\frac{26}{3}$
−1	−1	−1	−1	−1	−1	−1
5	$\frac{5}{2}$	0	$-\frac{5}{2}$	−5	$-\frac{15}{2}$	−10
1	1	1	1	1	1	1
−3	$-\frac{3}{2}$	0	$\frac{3}{2}$	3	$\frac{9}{2}$	6
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
88	$\frac{99}{8}$	0	$\frac{7}{40}$	−9	$-\frac{165}{8}$	1
−1	−1	−1	−1	−1	−1	−1
−2	−1	0	1	2	3	4
−3	$-\frac{3}{2}$	0	$\frac{3}{2}$	3	$\frac{9}{2}$	6
8	$\frac{13}{4}$	0	$-\frac{7}{4}$	−2	$-\frac{3}{4}$	2

```
Grid@Table[dzx[n, k, z], {n, 1, 20}, {k, -2, 2}]
```

0 z z	0 z z	0 z z	0 z z	0 z z
$z+\frac{1}{2}(-2+z)z$	$z+\frac{1}{2}(-1+z)z$	$z+\frac{z^2}{2}$	$z+\frac{1}{2}z(1+z)$	$z+\frac{1}{2}z(2+z)$
z z^2 z	z z^2 z	z z^2 z	z z^2 z	z z^2 z
$z+\frac{1}{6}(-4+z)$ $(-2+z)z-$ $z^2+z(-1+2z)$	$z+\frac{1}{6}(-2+z)$ $(-1+z)z-$ $z^2+z(-1+2z)$	$z-z^2+\frac{z^3}{6}+$ $z(-1+2z)$	$z-z^2+\frac{1}{6}z$ $(1+z)(2+z)+$ $z(-1+2z)$	$z-z^2+\frac{1}{6}z$ $(2+z)(4+z)+$ $z(-1+2z)$
$z+\frac{1}{2}(-2+z)z$	$z+\frac{1}{2}(-1+z)z$	$z+\frac{z^2}{2}$	$z+\frac{1}{2}z(1+z)$	$z+\frac{1}{2}z(2+z)$

$z - z(-1+2z) + z(-2+3z)$	$z - z(-1+2z) + z(-2+3z)$	$z - z(-1+2z) + z(-2+3z)$	$z - z(-1+2z) + z(-2+3z)$	$z - z(-1+2z) + z(-2+3z)$
z	z	z	z	z
$-\frac{1}{2}(-2+z)z + \frac{1}{2}(-2+z)(-1+z)z + z^2 - z(-2+3z) + z(-3+4z)$	$-\frac{1}{2}(-1+z)z + \frac{1}{2}(-1+z)^2 z + z^2 - z(-2+3z) + z(-3+4z)$	$\frac{z^2}{2} + \frac{1}{2}(-1+z)z^2 - z(-2+3z) + z(-3+4z)$	$z^2 - \frac{1}{2}z(1+z) + \frac{1}{2}(-1+z)z(1+z) - z(-2+3z) + z(-3+4z)$	$z^2 - \frac{1}{2}z(2+z) + \frac{1}{2}(-1+z)z(2+z) - z(-2+3z) + z(-3+4z)$
z	z	z	z	z
$z - z(-3+4z) + z(-4+5z)$	$z - z(-3+4z) + z(-4+5z)$	$z - z(-3+4z) + z(-4+5z)$	$z - z(-3+4z) + z(-4+5z)$	$z - z(-3+4z) + z(-4+5z)$
$z - z^2 + z(-1+2z)$	$z - z^2 + z(-1+2z)$	$z - z^2 + z(-1+2z)$	$z - z^2 + z(-1+2z)$	$z - z^2 + z(-1+2z)$
$z + \frac{1}{2}(-2+z)z + \frac{1}{24}(-6+z)(-4+z)(-2+z)z - \frac{1}{2}(-2+z)(-1+z)z + \frac{1}{2}(-2+z)z(-3+2z) - z(-4+5z) + z(-5+6z)$	$z + \frac{1}{2}(-1+z)z + \frac{1}{24}(-3+z)(-2+z)(-1+z)z - \frac{1}{2}(-1+z)^2 z + \frac{1}{2}(-1+z)z(-3+2z) - z(-4+5z) + z(-5+6z)$	$z + \frac{z^2}{2} - \frac{1}{2}(-1+z)z^2 + \frac{z^4}{24} + \frac{1}{2}z^2(-3+2z) - z(-4+5z) + z(-5+6z)$	$z + \frac{1}{2}z(1+z) - \frac{1}{2}(-1+z)z(1+z) + \frac{1}{24}z(1+z)(2+z)(3+z) + \frac{1}{2}z(1+z)(-3+2z) - z(-4+5z) + z(-5+6z)$	$z + \frac{1}{2}z(2+z) - \frac{1}{2}(-1+z)z(2+z) + \frac{1}{24}z(2+z)(4+z)(6+z) + \frac{1}{2}z(2+z)(-3+2z) - z(-4+5z) + z(-5+6z)$
z	z	z	z	z
$z - z(-1+2z) + z(-2+3z) - z(-5+6z) + z\left(-6+\frac{1}{2}(-3+z)(-1+z) + 7z\right)$	$z - z(-1+2z) + z(-2+3z) - z(-5+6z) + z\left(-6+\frac{1}{2}(-2+z)(-1+z) + 7z\right)$	$z - z(-1+2z) + z(-2+3z) - z(-5+6z) + z\left(-6+\frac{1}{2}(-1+z)^2 + 7z\right)$	$z - z(-1+2z) + z(-2+3z) - z(-5+6z) + z\left(-6+7z+\frac{1}{2}(-1+z)z\right)$	$z - z(-1+2z) + z(-2+3z) - z(-5+6z) + z\left(-6+7z+\frac{1}{2}(-1+z)(1+z)\right)$
z	z	z	z	z
$z^2 - \frac{1}{2}(-2+z)z(-3+2z) + \frac{1}{2}(-2+z)z(-5+3z) - z\left(-6+\frac{1}{2}(-3+z)(-1+z) + 7z\right) + z\left(-7+\frac{1}{2}(-3+z)(-1+z) + 8z\right)$	$z^2 - \frac{1}{2}(-1+z)z(-3+2z) + \frac{1}{2}(-1+z)z(-5+3z) - z\left(-6+\frac{1}{2}(-2+z)(-1+z) + 7z\right) + z\left(-7+\frac{1}{2}(-2+z)(-1+z) + 8z\right)$	$z^2 - \frac{1}{2}z^2(-3+2z) + \frac{1}{2}z^2(-5+3z) - z\left(-6+\frac{1}{2}(-1+z)^2 + 7z\right) + z\left(-7+\frac{1}{2}(-1+z)^2 + 8z\right)$	$z^2 - \frac{1}{2}z(1+z)(-3+2z) + \frac{1}{2}z(1+z)(-5+3z) - z\left(-6+7z+\frac{1}{2}(-1+z)z\right) + z\left(-7+8z+\frac{1}{2}(-1+z)z\right)$	$z^2 - \frac{1}{2}z(2+z)(-3+2z) + \frac{1}{2}z(2+z)(-5+3z) - z\left(-6+7z+\frac{1}{2}(-1+z)(1+z)\right) + z\left(-7+8z+\frac{1}{2}(-1+z)(1+z)\right)$

$RecursionLimit = 10000

10000

Expand[dzx[16, 0, z] dzx[27, 0, z]]

$$\frac{z^4}{2} + \frac{7z^5}{12} + \frac{z^6}{8} + \frac{z^7}{144}$$

```
Expand[dzx[16 × 27, 0, z]]
```

$$-4\,z+\frac{11\,z^2}{3}+\frac{25\,z^3}{12}-\frac{37\,z^4}{36}+\frac{z^5}{4}+\frac{z^6}{8}+\frac{z^7}{144}$$

```
(* Only -1 is multiplicative*)
```

```
Grid@Table[dza[n, z, k], {n, 1, 20}, {k, -2, 2}]
```

1 z z $\frac{1}{2}(-2+z)\,z$ z z^2 z $\frac{1}{6}(-4+z)(-2+z)\,z$ $\frac{1}{2}(-2+z)\,z$ z^2 z $\frac{1}{2}(-2+z)\,z^2$ z z^2 z^2 $\frac{1}{24}(-6+z)$ $(-4+z)(-2+z)\,z$ z $\frac{1}{2}(-2+z)\,z^2$ z $\frac{1}{2}(-2+z)\,z^2$	1 z z $\frac{1}{2}(-1+z)\,z$ z z^2 z $\frac{1}{6}(-2+z)(-1+z)\,z$ $\frac{1}{2}(-1+z)\,z$ z^2 z $\frac{1}{2}(-1+z)\,z^2$ z z^2 z^2 $\frac{1}{24}(-3+z)$ $(-2+z)(-1+z)\,z$ z $\frac{1}{2}(-1+z)\,z^2$ z $\frac{1}{2}(-1+z)\,z^2$	1 z z $\frac{z^2}{2}$ z z^2 z $\frac{z^3}{6}$ $\frac{z^2}{2}$ z^2 z $\frac{z^3}{2}$ z z^2 z^2 $\frac{z^4}{24}$ z $\frac{z^3}{2}$ z $\frac{z^3}{2}$	1 z z $\frac{1}{2}z(1+z)$ z z^2 z $\frac{1}{6}z(1+z)(2+z)$ $\frac{1}{2}z(1+z)$ z^2 z $\frac{1}{2}z^2(1+z)$ z z^2 z^2 $\frac{1}{24}z(1+z)$ $(2+z)(3+z)$ z $\frac{1}{2}z^2(1+z)$ z $\frac{1}{2}z^2(1+z)$	1 z z $\frac{1}{2}z(2+z)$ z z^2 z $\frac{1}{6}z(2+z)(4+z)$ $\frac{1}{2}z(2+z)$ z^2 z $\frac{1}{2}z^2(2+z)$ z z^2 z^2 $\frac{1}{24}z(2+z)$ $(4+z)(6+z)$ z $\frac{1}{2}z^2(2+z)$ z $\frac{1}{2}z^2(2+z)$

```
dza[12, z, 1]
```

$$\frac{1}{2}z^2(1+z)$$

```
dzx[12, -1, z]
```

$$-\frac{1}{2}(-1+z)\,z+\frac{1}{2}(-1+z)^2\,z+z^2-z(-2+3\,z)+z(-3+4\,z)$$

```
Grid@Table[D[dza[n, z, k], z] /. z → 0, {n, 1, 100}, {k, -4, 4}]
```

0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1
$\frac{16}{3}$	3	$\frac{4}{3}$	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{4}{3}$	3	$\frac{16}{3}$
-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1

$$\begin{array}{cccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-16 & -\frac{27}{4} & -2 & -\frac{1}{4} & 0 & \frac{1}{4} & 2 & \frac{27}{4} & 16 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2 & -\frac{3}{2} & -1 & -\frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{3}{2} & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{16}{3} & 3 & \frac{4}{3} & \frac{1}{3} & 0 & \frac{1}{3} & \frac{4}{3} & 3 & \frac{16}{3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\frac{256}{5} & \frac{81}{5} & \frac{16}{5} & \frac{1}{5} & 0 & \frac{1}{5} & \frac{16}{5} & \frac{81}{5} & \frac{256}{5} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2 & -\frac{3}{2} & -1 & -\frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{3}{2} & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{512}{3} & -\frac{81}{2} & -\frac{16}{3} & -\frac{1}{6} & 0 & \frac{1}{6} & \frac{16}{3} & \frac{81}{2} & \frac{512}{3}
\end{array}$$

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0
-16	$-\frac{27}{4}$	-2	$-\frac{1}{4}$	0	$\frac{1}{4}$	2	$\frac{27}{4}$	16
0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

```
Sum[ Binomial[z, k] (x - 1)^k, {k, 0, Infinity}]
```

x^z

```
Sum[z^k / k! (x - 1)^k, {k, 0, Infinity}]
```

$e^{(-1+x)\,z}$

```
Sum[Pochhammer[z, k] / k! (x + 1)^k, {k, 0, Infinity}]
```

$(-x)^{-z}$

```
D[Sum[Pochhammer[z, k] / k! (x + 1)^k, {k, 0, Infinity}], z] /. z → 0
```

$-\text{Log}[-x]$

```
Table[Binomial[z, k] Binomial[z - k, j] / z /. z → 0, {k, 0, 5}, {j, 0, 5}] // Grid
```

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

ComplexInfinity	1	$-\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{4}$	$\frac{1}{5}$
1	−1	1	−1	1	−1
$-\frac{1}{2}$	1	$-\frac{3}{2}$	2	$-\frac{5}{2}$	3
$\frac{1}{3}$	−1	2	$-\frac{10}{3}$	5	−7
$-\frac{1}{4}$	1	$-\frac{5}{2}$	5	$-\frac{35}{4}$	14
$\frac{1}{5}$	−1	3	−7	14	$-\frac{126}{5}$

```
Table[Binomial[z, k] Binomial[z - k, j] Binomial[z - k - j, l] / z /. z → 0,
  {k, 0, 5}, {j, 0, 5}, {l, 0, 5}] // Grid
```

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

{ComplexInfinity, 1, $-\frac{1}{2}$, $\frac{1}{3}$, $-\frac{1}{4}$, $\frac{1}{5}$}	{1, −1, 1, −1, 1, −1}	$\{-\frac{1}{2}, 1, -\frac{3}{2}, 2, -\frac{5}{2}, 3\}$	$\{\frac{1}{3}, -1, 2, -\frac{10}{3}, 5, -7\}$	$\{-\frac{1}{4}, 1, -\frac{5}{2}, 5, -\frac{35}{4}, 14\}$	$\{\frac{1}{5}, -1, 3, -7, 14, -\frac{126}{5}\}$
{1, −1, 1, −1, 1, −1}	{−1, 2, −3, 4, −5, 6}	{1, −3, 6, −10, 15, −21}	{−1, 4, −10, 20, −35, 56}	{1, −5, 15, −35, 70, −126}	{−1, 6, −21, 56, −126, 252}
$\{-\frac{1}{2}, 1, -\frac{3}{2}, 2, -\frac{5}{2}, 3\}$	{1, −3, 6, −10, 15, −21}	$\{-\frac{3}{2}, 6, -15, 30, -\frac{105}{2}, 84\}$	{2, −10, 30, −70, 140, −252}	$\{-\frac{5}{2}, 15, -\frac{105}{2}, 140, -315, 630\}$	{3, −21, 84, −252, 630, −1386}
$\{\frac{1}{3}, -1, 2, -\frac{10}{3}, 5, -7\}$	{−1, 4, −10, 20, −35, 56}	{2, −10, 30, −70, 140, −252}	$\{-\frac{10}{3}, 20, -70, \frac{560}{3}, -420, 840\}$	{5, −35, 140, −420, 1050, −2310}	{−7, 56, −252, 840, −2310, 5544}
$\{-\frac{1}{4}, 1, -\frac{5}{2}, 5, -\frac{35}{4}, 14\}$	{1, −5, 15, −35, 70, −126}	$\{-\frac{5}{2}, 15, -\frac{105}{2}, 140, -315, 630\}$	{5, −35, 140, −420, 1050, −2310}	$\{-\frac{35}{4}, 70, -315, 1050, -\frac{5775}{2}, 6930\}$	{14, −126, 630, −2310, 6930, −18 018}
$\{\frac{1}{5}, -1, 3, -7, 14, -\frac{126}{5}\}$	{−1, 6, −21, 56, −126, 252}	{3, −21, 84, −252, 630, −1386}	{−7, 56, −252, 840, −2310, 5544}	{14, −126, 630, −2310, 6930, −18 018}	$\{-\frac{126}{5}, 252, -1386, 5544, -18\,018, \frac{252\,252}{5}\}$

```
Table[Binomial[z, k] Binomial[z - k, j] Binomial[z - k - j, l] /. z → -1,
  {k, 0, 5}, {j, 0, 5}, {l, 0, 5}] // Grid
```

{1, -1, 1, -1, 1, -1}	{-1, 2, -3, 4, -5, 6}	{1, -3, 6, -10, 15, -21}	{-1, 4, -10, 20, -35, 56}	{1, -5, 15, -35, 70, -126}	{-1, 6, -21, 56, -126, 252}
{-1, 2, -3, 4, -5, 6}	{2, -6, 12, -20, 30, -42}	{-3, 12, -30, 60, -105, 168}	{4, -20, 60, -140, 280, -504}	{-5, 30, -105, 280, -630, 1260}	{6, -42, 168, -504, 1260, -2772}
{1, -3, 6, -10, 15, -21}	{-3, 12, -30, 60, -105, 168}	{6, -30, 90, -210, 420, -756}	{-10, 60, -210, 560, -1260, 2520}	{15, -105, 420, -1260, 3150, -6930}	{-21, 168, -756, 2520, -6930, 16 632}
{-1, 4, -10, 20, -35, 56}	{4, -20, 60, -140, 280, -504}	{-10, 60, -210, 560, -1260, 2520}	{20, -140, 560, -1680, 4200, -9240}	{-35, 280, -1260, 4200, -11 550, 27 720}	{56, -504, 2520, -9240, 27 720, -72 072}
{1, -5, 15, -35, 70, -126}	{-5, 30, -105, 280, -630, 1260}	{15, -105, 420, -1260, 3150, -6930}	{-35, 280, -1260, 4200, -11 550, 27 720}	{70, -630, 3150, -11 550, 34 650, -90 090}	{-126, 1260, -6930, 27 720, -90 090, 252 252}
{-1, 6, -21, 56, -126, 252}	{6, -42, 168, -504, 1260, -2772}	{-21, 168, -756, 2520, -6930, 16 632}	{56, -504, 2520, -9240, 27 720, -72 072}	{-126, 1260, -6930, 27 720, -90 090, 252 252}	{252, -2772, 16 632, -72 072, 252 252, -756 756}

```
Table[Binomial[z, k] Binomial[z - k, j] Binomial[z - k - j, l] /. z → 1,
  {k, 0, 5}, {j, 0, 5}, {l, 0, 5}] // Grid
```

{1, 1, 0, 0, 0, 0}	{1, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}
{1, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}
{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}
{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}
{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}
{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0}

```
Binomial[z, 1] Binomial[z - 1, 1]
  Binomial[z - 2, 1] Binomial[z - 3, 1] Binomial[z - 4, 1] / z /. z → 0
```

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```
Binomial[z, 2] Binomial[z - 2, 2] / z /. z → 0
```

$-\frac{3}{2}$