```
Residue[ ( -Zeta'[s] / Zeta[s]) x^s s^(-1), {s, 1}]
```

$$x$$

```
Residue[ ( (-Zeta'[s]) / Zeta[s]) x^s s^(-1), {s, 1}]
```

$$x$$

```
Residue[ (Zeta[s]^3) x^s s^(-1), {s, 1}]
```

$$\frac{1}{2}\left(2x - 6\,\text{EulerGamma}\,x + 6\,\text{EulerGamma}^2\,x - 2x\,\text{Log}[x] + 6\,\text{EulerGamma}\,x\,\text{Log}[x] + x\,\text{Log}[x]^2 - 6x\,\text{StieltjesGamma}[1]\right)$$

```
Residue[ ( (-Zeta'[s])^2 / Zeta[s]) x^s s^(-1), {s, 1}]
```

$$\frac{1}{2}\left(2x + 2\,\text{EulerGamma}\,x + 2\,\text{EulerGamma}^2\,x - 2x\,\text{Log}[x] - 2\,\text{EulerGamma}\,x\,\text{Log}[x] + x\,\text{Log}[x]^2 + 6x\,\text{StieltjesGamma}[1]\right)$$

```
Residue[ ( (Zeta''[s]) / Zeta[s]) x^s s^(-1), {s, 1}]
```

$$-2\,(x + \text{EulerGamma}\,x - x\,\text{Log}[x])$$

```
Residue[ ( (Zeta'''[s]) / Zeta[s]) x^s s^(-1), {s, 1}]
```

$$-3\left(2x + 2\,\text{EulerGamma}\,x + 2\,\text{EulerGamma}^2\,x - 2x\,\text{Log}[x] - 2\,\text{EulerGamma}\,x\,\text{Log}[x] + x\,\text{Log}[x]^2 + 2x\,\text{StieltjesGamma}[1]\right)$$

```
Residue[ ((1 / Zeta[s] - 1)^2) x^s s^(-1), {s, 1}]
```

$$0$$

```
Residue[ ((Zeta[s] - 1)^2) x^s s^(-1), {s, 1}]
```

$$-3x + 2\,\text{EulerGamma}\,x + x\,\text{Log}[x]$$

```
Residue[ ((Zeta[s])^2) x^s s^(-1), {s, 1}]
```

$$-x + 2\,\text{EulerGamma}\,x + x\,\text{Log}[x]$$

```
Residue[ ((Zeta[s] - 1)^3) x^s s^(-1), {s, 1}]
```

$$\frac{1}{2}\left(14x - 18\,\text{EulerGamma}\,x + 6\,\text{EulerGamma}^2\,x - 8x\,\text{Log}[x] + 6\,\text{EulerGamma}\,x\,\text{Log}[x] + x\,\text{Log}[x]^2 - 6x\,\text{StieltjesGamma}[1]\right)$$

```
Residue[ ((Zeta[s])^3) x^s s^(-1), {s, 1}]
```

$$\frac{1}{2}\left(2x - 6\,\text{EulerGamma}\,x + 6\,\text{EulerGamma}^2\,x - 2x\,\text{Log}[x] + 6\,\text{EulerGamma}\,x\,\text{Log}[x] + x\,\text{Log}[x]^2 - 6x\,\text{StieltjesGamma}[1]\right)$$

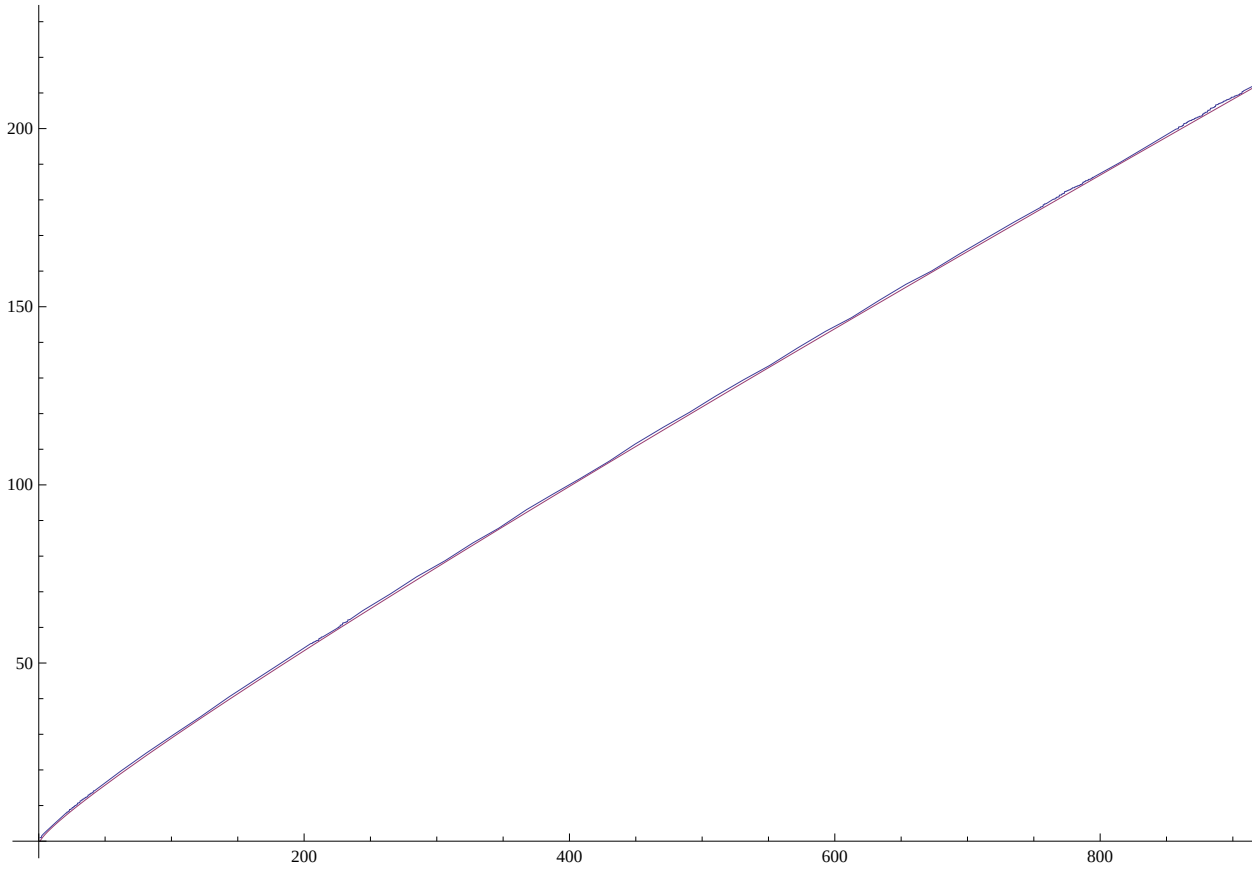
```
Residue[ ((Zeta[s])^(1 / 2)) x^s s^(-1), {s, 1}]
```

$$\text{Residue}\left[\frac{x^s\,\sqrt{\text{Zeta}[s]}}{s},\ \{s,\ 1\}\right]$$

```
CoefficientList[Series[(x + 1)^(1 / 2), {x, 0, 20}], x]
```

```
sq := sq = {1, 1/2, -1/8, 1/16, -5/128, 7/256, -21/1024, 33/2048, -429/32768,
    715/65536, -2431/262144, 4199/524288, -29393/4194304, 52003/8388608, -185725/33554432, 334305/67108864,
    -9694845/2147483648, 17678835/4294967296, -64822395/17179869184, 119409675/34359738368, -883631595/274877906944}
D2[n_, k_] := D2[n, k] = Sum[D2[Floor[n / j], k - 1], {j, 2, n}]; D2[n_, 0] := D2[n, 0] = 1
DD[n_, k_] := DD[n, k] = Sum[DD[Floor[n / j], k - 1], {j, 1, n}]; DD[n_, 0] := DD[n, 0] = 1
d[n_, z_] := Product[Pochhammer[z, a = p[[2]]] / a!, {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
Da[n_, z_] := Sum[ d[j, z], {j, 1, n}]
T1 := T1 = Table[ Residue[ (Zeta[s])^k x^s s^(-1), {s, 1}], {k, 1, 15}]
T2 := T2 = Table[ Residue[ (Zeta[s] - 1)^k x^s s^(-1), {s, 1}], {k, 1, 15}]
T2cal := T2cal = Table[T2[[k]] /. x → 1, {k, 1, 15}]
Ap[n_, k_] := (-1)^(k) (1 - Gamma[k, -Log[n]] / Gamma[k])
```

```
Plot[ {Da[n, .5], TT[n]}, {n, 1, 1000}]
```




```
D2e[n_, k_] := D2e[n, k] = (T2[[k]] /. x → n) - T2cal[[k]]
```

```
N[D2e[100, 15]]
```

```
0.174724
```

```
sq[[21]]
```

$$-\frac{883\,631\,595}{274\,877\,906\,944}$$

```
Series[(x + 1)^(1 / 2), {x, 0, 20}]
```

$$1+\frac{x}{2}-\frac{x^2}{8}+\frac{x^3}{16}-\frac{5\,x^4}{128}+\frac{7\,x^5}{256}-\frac{21\,x^6}{1024}+\frac{33\,x^7}{2048}-\frac{429\,x^8}{32\,768}+\frac{715\,x^9}{65\,536}-\frac{2431\,x^{10}}{262\,144}+\frac{4199\,x^{11}}{524\,288}-\frac{29\,393\,x^{12}}{4\,194\,304}+\frac{52\,003\,x^{13}}{8\,388\,608}-\frac{185\,725\,x^{14}}{33\,554\,432}+\frac{334\,305\,x^{15}}{67\,108\,864}-\frac{9\,694\,845\,x^{16}}{2\,147\,483\,648}+\frac{17\,678\,835\,x^{17}}{4\,294\,967\,296}-\frac{64\,822\,395\,x^{18}}{17\,179\,869\,184}+\frac{119\,409\,675\,x^{19}}{34\,359\,738\,368}-\frac{883\,631\,595\,x^{20}}{274\,877\,906\,944}+O[x]^{21}$$

```
TT[n_] := Sum[D2e[n, k] sq[[k + 1]], {k, 1, 15}]
```

```
N[TT[100]]
```

```
28.9118
```

```
Da[100, .5]
```

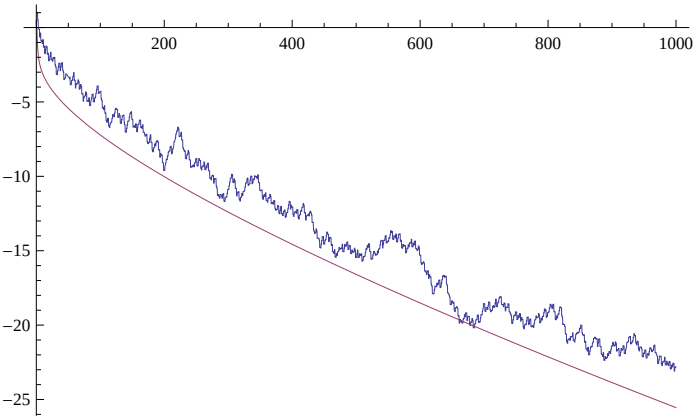
```
29.4385
```

```
CoefficientList[Series[(x + 1)^(-1 / 2), {x, 0, 20}], x]
```

```
sq2 := sq2 = {1, -1/2, 3/8, -5/16, 35/128, -63/256, 231/1024, -429/2048, 6435/32768,
  -12155/65536, 46189/262144, -88179/524288, 676039/4194304, -1300075/8388608, 5014575/33554432, -9694845/67108864,
  300540195/2147483648, -583401555/4294967296, 2268783825/17179869184, -4418157975/34359738368, 34461632205/274877906944}
```

```
TT2[n_] := Sum[D2e[n, k] sq2[[k + 1]], {k, 1, 15}]
```

```
Plot[{Da[n, -.5], TT2[n]}, {n, 1, 1000}]
```

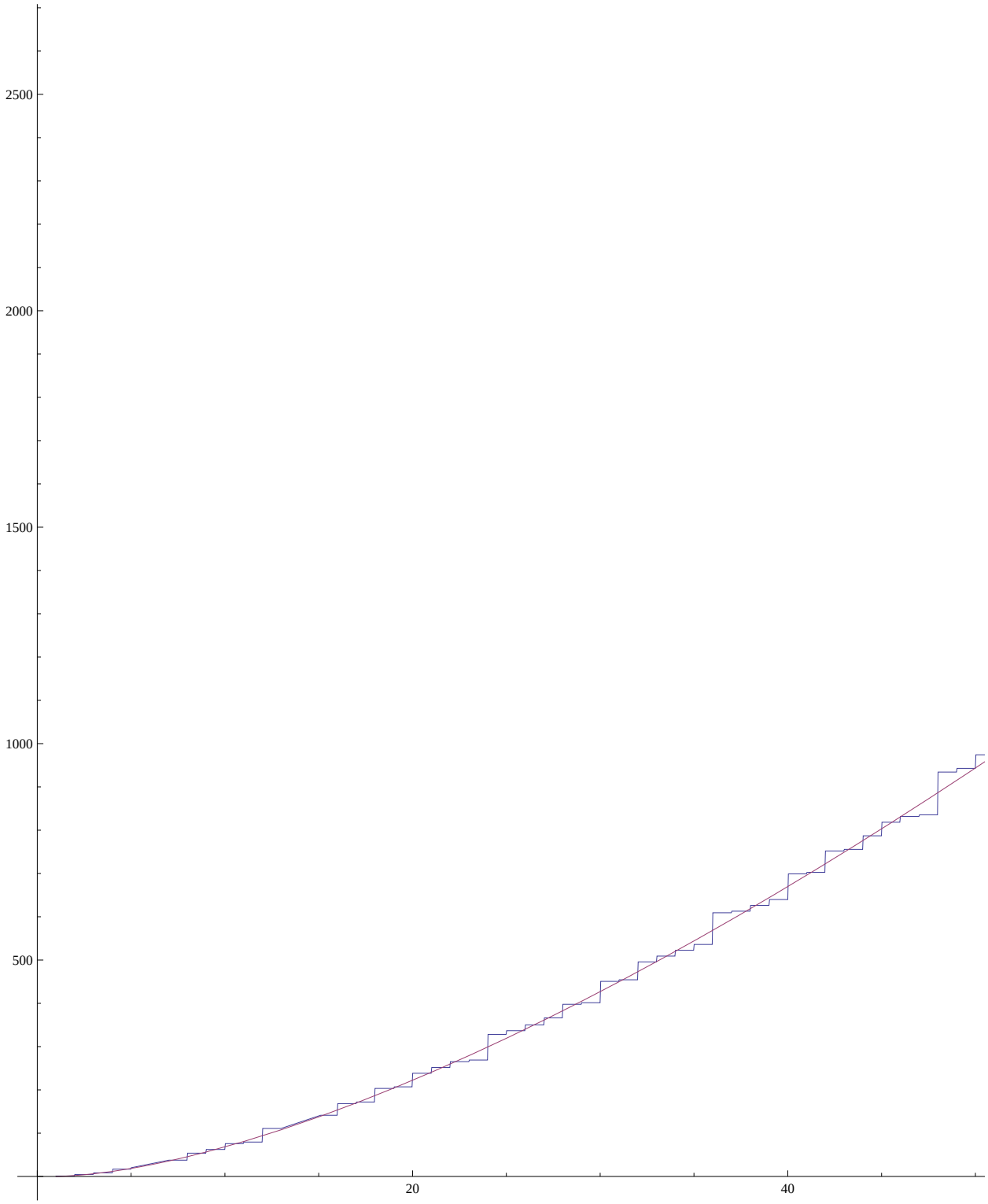



```
CoefficientList[Series[(x + 1)^(11 / 3), {x, 0, 20}], x]
```

```
sq4 := sq4 = {1, 11/3, 44/9, 220/81, 110/243, -22/729, 44/6561, -44/19683, 55/59049, -715/1594323,
  1144/4782969, -1976/14348907, 10868/129140163, -20900/387420489, 41800/1162261467, -259160/10460353203,
  550715/31381059609, -1198615/94143178827, 23972300/2541865828329, -54253100/7625597484987, 124782130/22876792454961}
```

```
TT3[n_] := Sum[ D2e[n, k] sq4[[k + 1]], {k, 1, 15}]
```

```
Plot[{Da[n, 11 / 3], TT3[n]}, {n, 1, 100}]
```




```
FullSimplify[TT3[100]]
```

```
$Aborted
```

```
Residue[ ((1 / Zeta[s] - 1)^3) x^s s^(-1), {s, ZetaZero[1]}]
```

$$\frac{1}{2\,\text{ZetaZero}[1]^3\,\text{Zeta}'[\text{ZetaZero}[1]]^5}\,x^{\text{ZetaZero}[1]}\,\Big(2\,\text{Zeta}'[\text{ZetaZero}[1]]^2 - 2\,\text{Log}[x]\,\text{ZetaZero}[1]\,\text{Zeta}'[\text{ZetaZero}[1]]^2 + \text{Log}[x]^2\,\text{ZetaZero}[1]^2\,\text{Zeta}'[\text{ZetaZero}[1]]^2 + 6\,\text{ZetaZero}[1]\,\text{Zeta}'[\text{ZetaZero}[1]]^3 - 6\,\text{Log}[x]\,\text{ZetaZero}[1]^2\,\text{Zeta}'[\text{ZetaZero}[1]]^3 + 6\,\text{ZetaZero}[1]^2\,\text{Zeta}'[\text{ZetaZero}[1]]^4 + 3\,\text{ZetaZero}[1]\,\text{Zeta}'[\text{ZetaZero}[1]]\,\text{Zeta}''[\text{ZetaZero}[1]] - 3\,\text{Log}[x]\,\text{ZetaZero}[1]^2\,\text{Zeta}'[\text{ZetaZero}[1]]\,\text{Zeta}''[\text{ZetaZero}[1]] + 6\,\text{ZetaZero}[1]^2\,\text{Zeta}'[\text{ZetaZero}[1]]^2\,\text{Zeta}''[\text{ZetaZero}[1]] + 3\,\text{ZetaZero}[1]^2\,\text{Zeta}''[\text{ZetaZero}[1]]^2 - \text{ZetaZero}[1]^2\,\text{Zeta}'[\text{ZetaZero}[1]]\,\text{Zeta}^{(3)}[\text{ZetaZero}[1]]\Big)$$

```
Expand[N[Residue[ (1 / Zeta[s] - 1)^5 x^s s^(-1), {s, -}]]]
```

$$-\frac{1.4336164867684953`*^6}{x^{12}} - \frac{1.000945863241807`*^6\,\text{Log}[x]}{x^{12}} - \frac{296294.1074337942`\,\text{Log}[x]^2}{x^{12}} - \frac{43931.56901950145`\,\text{Log}[x]^3}{x^{12}} - \frac{3424.5090584396494`\,\text{Log}[x]^4}{x^{12}} \;/.\; x \to 30$$

$$-1.9668 \times 10^{-11}$$

$$-\frac{2.6293790206287857`*^10}{x^4} + \frac{2.3362281125511642`*^10\,\text{Log}[x]}{x^4} - \frac{1.0492298802763643`*^10\,\text{Log}[x]^2}{x^4} + \frac{2.0378708460337496`*^9\,\text{Log}[x]^3}{x^4} - \frac{3.2112743289672685`*^8\,\text{Log}[x]^4}{x^4} \;/.\; x \to 30$$

$$-38\,275.6$$

$$\frac{6.323835876938515`*^8}{x^2} - \frac{4.285452073963622`*^8\,\text{Log}[x]}{x^2} + \frac{1.1648048622212459`*^8\,\text{Log}[x]^2}{x^2} - \frac{1.5116183750507852`*^7\,\text{Log}[x]^3}{x^2} + \frac{796035.2726571471`\,\text{Log}[x]^4}{x^2} \;/.\; x \to 30$$

$$37\,835.9$$

$$-\frac{1.3411724350748308`*^10}{x^8} - \frac{1.1006033172471167`*^10\,\text{Log}[x]}{x^8} - \frac{4.783694263329197`*^9\,\text{Log}[x]^2}{x^8} - \frac{8.32892837809015`*^8\,\text{Log}[x]^3}{x^8} - \frac{1.3094353790709832`*^8\,\text{Log}[x]^4}{x^8} \;/.\; x \to 30$$

$$-0.238497$$

$$\frac{1}{x^8}5.3471497797748664`*^13\,\Big(-0.00025082006121236776` - 0.00020582990239212187`\,\text{Log}[x] - 0.00008946250732349238`\,\text{Log}[x]^2 - 0.00001557638876994547`\,\text{Log}[x]^3 - 2.4488473916025503`*^-6\,\text{Log}[x]^4\Big) \;/.\; x \to 30$$

$$-0.238497$$

$$\frac{1}{x^8}2.958409010521708`*^10\,\Big(-0.001956045140633667` - 0.0020025789009419587`\,\text{Log}[x] - 0.0005729504086642285`\,\text{Log}[x]^2 - 0.0001472342287128506`\,\text{Log}[x]^3\Big) \;/.\; x \to 30$$

$$-0.000955393$$

```
N[Series[ (1 / (x + 1)) ^ (3 / 2), {x, 0, 20}]]
```

$$1. - 1.5\,(x + 0.) + 1.875\,(x + 0.)^2 - 2.1875\,(x + 0.)^3 + 2.46094\,(x + 0.)^4 - 2.70703\,(x + 0.)^5 + 2.93262\,(x + 0.)^6 - 3.14209\,(x + 0.)^7 + 3.33847\,(x + 0.)^8 - 3.52394\,(x + 0.)^9 + 3.70014\,(x + 0.)^{10} - 3.86833\,(x + 0.)^{11} + 4.02951\,(x + 0.)^{12} - 4.18449\,(x + 0.)^{13} + 4.33393\,(x + 0.)^{14} - 4.4784\,(x + 0.)^{15} + 4.61835\,(x + 0.)^{16} - 4.75418\,(x + 0.)^{17} + 4.88624\,(x + 0.)^{18} - 5.01483\,(x + 0.)^{19} + 5.1402\,(x + 0.)^{20} + O[x + 0.]^{21}$$

```
Sum[ (-1) ^ k (1 / Zeta[s] - 1) ^ k, {k, 0, Infinity}]
```

```
Zeta[s]
```

```
Eta[s_] := (1 - 2^(1 - s)) Zeta[s]
```

```
N[Eta[ZetaZero[3] + 1]]
```

```
0.810643 - 0.359653 i
```

```
pr[x_, t_] := Sum[N[Residue[ (1 / Zeta[s] - 1) ^ k x^s s^(-1), {s, -2 t}]], {k, 1, 16}]
```

```
pr[30, 1]
```

Sum::itraw : Raw object 1 cannot be used as an iterator. ≫

Sum::itraw : Raw object 1 cannot be used as an iterator. ≫

NSum::itraw : Raw object 1 cannot be used as an iterator. ≫

$$\text{NSum}\left[\text{Residue}\left[\frac{\left(\frac{1}{\text{Zeta}[s]} - 1\right)^1 30^s}{s}, \{s, -2\,1\}\right], \{1, 1, 16\}\right]$$