```
hzeta[n_, s_, y_] := Sum[ j^-s, {j, y, n}] -
  s / (1 - s) n (Sum[ j^ (-s - 1), {j, 1, Infinity}] - Sum[j^ (-s - 1), {j, 1, y + n}])
hzeta2[n_, s_, y_] := Sum[ j^-s, {j, y, n}] - s / (1 - s) n (Sum[ j^ (-s - 1), {j, y + n, Infinity}])
```

```
hzeta2[1 000 000, .5, 2]
```

-2.45835

```
Zeta[.5, 2]
```

-2.46035

```
FullSimplify[(D[-x^ (1 - s) Sum[ (j + y) ^-s, {j, 0, n / x}], {x, 1}] /. x → 1) / (1 - s)]
```

$$-\text{HurwitzZeta}[s,\ y] + \text{HurwitzZeta}[s,\ 1+n+y] - \frac{n\, s\, \text{HurwitzZeta}[1+s,\ 1+n+y]}{-1+s}$$

```
FullSimplify[(D[-x^ (1 - s) Sum[ (j + y) ^-s, {j, 0, n / x}], {x, 2}] /. x → 1) / (1 - s) / s]
```

$$\frac{1}{-1+s}\big((-1+s)\,\text{HurwitzZeta}[s,\ y] - (-1+s)\,\text{HurwitzZeta}[s,\ 1+n+y] + n\,(2\,s\,\text{HurwitzZeta}[1+s,\ 1+n+y] - n\,(1+s)\,\text{HurwitzZeta}[2+s,\ 1+n+y])\big)$$

```
FullSimplify[(D[-x^ (1 - s) Sum[ (j + y) ^-s, {j, 0, n / x}], {x, 3}] /. x → 1) / (1 - s) / s / (s + 1)]
```

$$\frac{1}{-1+s}\Big(-(-1+s)\,\text{HurwitzZeta}[s,\ y] + (-1+s)\,\text{HurwitzZeta}[s,\ 1+n+y] + n\,\big(-3\,s\,\text{HurwitzZeta}[1+s,\ 1+n+y] + 3\,n\,(1+s)\,\text{HurwitzZeta}[2+s,\ 1+n+y] - n^2\,(2+s)\,\text{HurwitzZeta}[3+s,\ 1+n+y]\big)\Big)$$

```
FullSimplify[
 (D[-x^ (1 - s) Sum[ (j + y) ^-s, {j, 0, n / x}], {x, 4}] /. x → 1) / (1 - s) / s / (s + 1) / (s + 2)]
```

$$\frac{1}{-1+s}\Big((-1+s)\,\text{HurwitzZeta}[s,\ y] - (-1+s)\,\text{HurwitzZeta}[s,\ 1+n+y] + n\,\big(4\,s\,\text{HurwitzZeta}[1+s,\ 1+n+y] - 6\,n\,(1+s)\,\text{HurwitzZeta}[2+s,\ 1+n+y] + 4\,n^2\,(2+s)\,\text{HurwitzZeta}[3+s,\ 1+n+y] - n^3\,(3+s)\,\text{HurwitzZeta}[4+s,\ 1+n+y]\big)\Big)$$

```
rs[n_, s_, y_, k_] :=
 Sum[ (-1) ^j Binomial[k, j] ((s - 1 + j) / (s - 1)) n^j Zeta[s + j, y + n], {j, 0, k}]
rst[n_, s_, y_, k_] :=
 Table[ (-1) ^j Binomial[k, j] ((s - 1 + j) / (s - 1)) n^j Zeta[s + j, y + n], {j, 0, k}]
rs2[n_, s_, y_, k_] := Sum[j^-s, {j, y, n + y}] -
  Sum[ (-1) ^j Binomial[k, j] ((s - 1 + j) / (s - 1)) n^j Zeta[s + j, y + n + 1], {j, 1, k}]
rs3[n_, s_, y_, k_] := Sum[j^-s, {j, y, n + y - 1}] -
  Sum[ (-1) ^j Binomial[k, j] ((s - 1 + j) / (s - 1)) n^j Zeta[s + j, y + n], {j, 1, k}]
rsa[n_, s_, y_, k_] := Sum[ (-1) ^j Binomial[k, j]
   ((s - 1 + 2 j) / (s - 1)) n^ (2 j) Zeta[s + 2 j, y + n], {j, 0, k}]
rsat[n_, s_, y_, k_] := Table[ (-1) ^j Binomial[k, j]
   ((s - 1 + 2 j) / (s - 1)) n^ (2 j) Zeta[s + 2 j, y + n], {j, 0, k}]
rsax[n_, s_, y_, k_, x_] := Sum[ (-1) ^j Binomial[k, j]
   ((s - 1 + x j) / (s - 1)) n^ (x j) Zeta[s + x j, y + n], {j, 0, k}]
rsat[n_, s_, y_, k_, x_] := Table[ (-1) ^j Binomial[k, j]
   ((s - 1 + x j) / (s - 1)) n^ (x j) Zeta[s + x j, y + n], {j, 0, k}]
(* !!!!!!!!!!!!!! *)
rsats[n_, s_, y_, k_, x_] :=
 Table[ (((s - 1 + x j) / (s - 1)) n^ (x j) Zeta[s + x j, y + n]), {j, 0, k}]
```

```
Chop@rs[1 000 000, -.5, 4, 4]
```

4.76837×10^{-7}

```
N[Chop@rs3[10 000, .5, 1, 2]]
```

-1.46035

```
N@Zeta[.5, 1]
```

-1.46035

```
Chop@N@rsa[100 000, .5 + I, 1, 3]
```

0

```
Chop@N@rsat[10 000 000, .5 + 4 I, 1, 4] // TableForm
```

```
-769.735 + 151.301 i
3078.94 - 605.205 i
-4618.41 + 907.808 i
3078.94 - 605.205 i
-769.735 + 151.301 i
```

```
Chop@N@rst[10 000 000, N[ZetaZero[1]], 1, 3] // TableForm
```

```
-222.581 + 21.1516 i
667.744 - 63.4549 i
-667.744 + 63.4549 i
222.581 - 21.1516 i
```

```
rsat[100 000, 1.5, 1, 7, 5] // TableForm
```

```
0.00632454
-0.0442707
0.132809
-0.221342
0.221337
-0.132799
0.0442651
-0.00632343
```

```
rsats[1 000 000, .5, 1, 3, 7] // TableForm
```

```
-2000.
-1999.99
-1999.99
-1999.98
```

```
rsats[1 000 000, N[ZetaZero[1]], 0, 1, 2] // TableForm
```

```
-36.0507 - 60.8221 i
-36.0507 - 60.8222 i
```

```
rsatsa[n_, s_, x_] :=
 FullSimplify[Table[(m[-n^(j x)]) (-1 + s + j x) (zetaa[s + x j, n]), {j, 0, 1}]]
rsatsb[n_, s_, x_] := FullSimplify[Table[ -n^(j x) (-1 + s + j x) (Zeta[s + x j, n]), {j, 0, 1}]]
rsatsc[n_, s_, x_] := Sum[ (-1)^j (-n^(j x)) (-1 + s + j x) (Zeta[s + x j, n]), {j, 0, 1}]
rsatsd[n_, s_, y_, x_] :=
 (-n^y) (-1 + s + y) (Zeta[s + y, n]) - (-n^x) (-1 + s + x) (Zeta[s + x , n])
rsatsda[n_, s_, y_, x_] :=
 { (-n^y) (-1 + s + y) (Zeta[s + y, n]), (-n^x) (-1 + s + x) (Zeta[s + x , n])}
rsatsda2[n_, s_, y_, x_] := ((-n^y) (Zeta[s + y, n])) / ( (-n^x) (Zeta[s + x , n]))
```

```
rsatsa[n, N[ZetaZero[1]], 0.` + 6.887314497036861` I]
```

{(-0.5 + 14.1347 i) m[-1. + 0. i] zetaa[0.5 + 14.1347 i, n],
 (-0.5 + 21.022 i) m[$-n^{0.+6.88731\, i}$] zetaa[0.5 + 21.022 i, n]}

```
N[1 - 2 ZetaZero[1]]
```

0. - 28.2695 i

```
N[1 - ZetaZero[1]]
```

0.5 - 14.1347 i

```
20^(-28.26945028346939` I)
```

-0.990861 - 0.134885 i

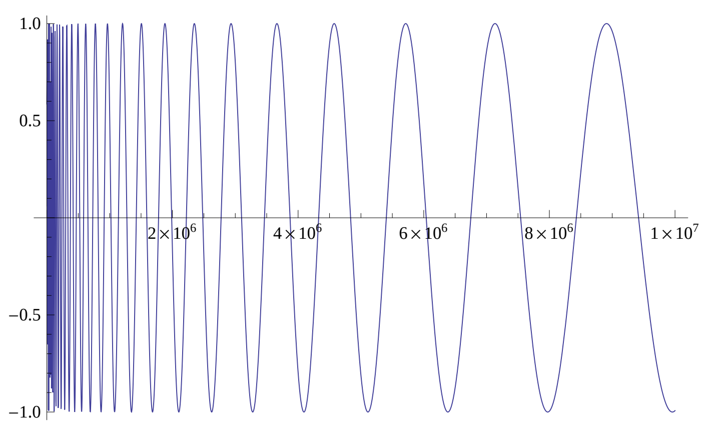
```
N[20^(1 - 2 ZetaZero[1])]
```

-0.990861 - 0.134885 i

```
1 - N[2 ZetaZero[1]]
```

0. - 28.2695 i

```
Plot[ Re[ x^(1 - 2 ZetaZero[1])], {x, 0, 10 000 000}]
```




```
rsatsa[1 000 000 000 000, sos = N[ZetaZero[1] + .1], 1 - 2 sos]
```

{(-0.4 + 14.1347 i) m[-1. + 0. i] zetaa[0.6 + 14.1347 i, 1 000 000 000 000],
 (-0.6 - 14.1347 i) m[0.00165236 + 0.00362196 i] zetaa[0.4 - 14.1347 i, 1 000 000 000 000]}

```
rsatsc[100 000 000 000, sos = N[ZetaZero[1] + .1], 1 - 2 sos]
```

$4.39337 \times 10^{-7} - 3.52503 \times 10^{-6}\, i$

```
rsatsb[100 000 000 000, sos = N[ZetaZero[1] + .1], 1 - 2 sos]
{-24 903.4 - 3283.19 i, -24 903.4 - 3283.19 i}
N[Zeta[ZetaZero[1] - .1]]
-0.0814815 - 0.013674 i
FullSimplify[(1 - s) (((s - 1 + x j) / (s - 1)) n^(x j))]
-n^(j x) (-1 + s + j x)
(-1)^j (-n^(j x)) (-1 + s + j x) /. j -> 1
n^x (-1 + s + x)
N[Zeta[s, n] / (n^x Zeta[s + x, n]) /. s -> 11 /. x -> 2 /. n -> 1 000 000 000 000 000]
1.2
N[1 + (2 / (11 - 1))]
1.2
rsatsd[100 000 000 000, 1 + I, -.5, .5]
4.90563 × 10^-12 - 9.72444 × 10^-13 i
rsatsda[100 000 000 000, 1 + I, -.2, .5]
{-0.980913 + 0.194448 i, -0.980913 + 0.194448 i}
n^y / n^x
n^(-x+y)
rsatsda2[1 000 000 000 000 000, 3, .3, .2]
0.956522
(3 - 1 + .2) / (3 - 1 + .3)
0.956522

N[ZetaZero[2] - ZetaZero[1]]
0. + 6.88731 i

FullSimplify[(-1 / 2 + x) / (-1 / 2 - x)] /. x -> 14 I
-783/785 - 56 i/785
N[n^(-2 * 14 I) /. n -> 10 000 000 000 000]
-0.78737 - 0.61648 i
N[1 / 5^(1 / 2 + 14 I)]
-0.383353 + 0.230305 i
N[10 000 000^(14 I)]
0.857023 - 0.515278 i
```

```
rsatsda[10, .5, -14 I, 14 I]
```

{-2.54891 + 2.21912 i, -2.54891 - 2.21912 i}

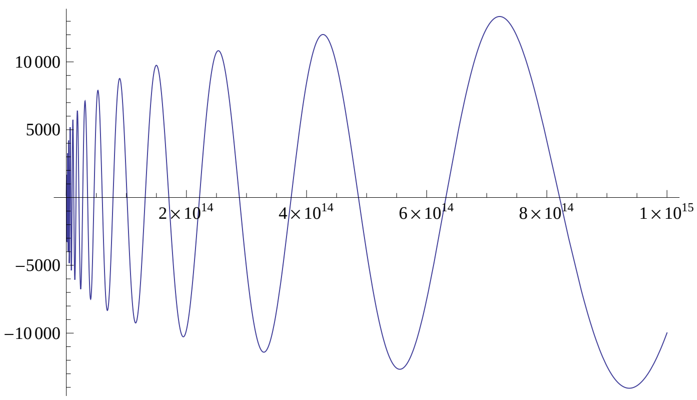
```
N[(10^800)^(-(.1 + 23 I))]
```

$9.98887 \times 10^{-81} - 4.71673 \times 10^{-82}\, i$

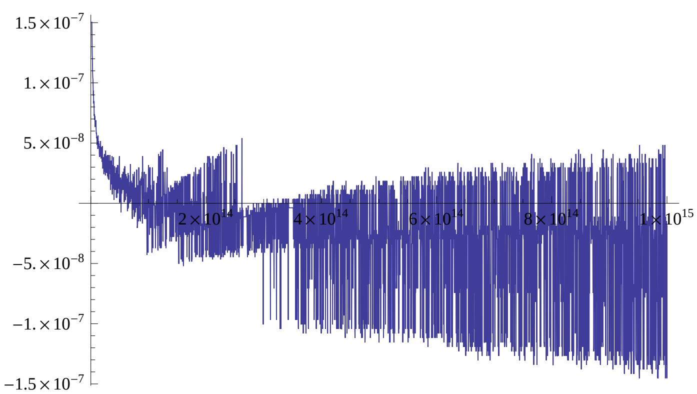
```
Sum[1/(j^(1/2 - ((.1 + 23 I)))), {j, 1, n}]
```

HarmonicNumber[n, 0.4 - 23. i]

```
Plot[(Abs[(-.5 - a) n^(-a) HarmonicNumber[n, .5 - a]] -
    Abs[(-.5 + a) n^(a) HarmonicNumber[n, .5 + a]]) /.
  a -> (-.2 + 12 I) /. b -> (-.1 + 12 I), {n, 1, 10^15}]
```




```
Plot[{Abs[(-.5 - a) n^(-a) (Zeta[.5 - a] - HarmonicNumber[n, .5 - a])] -
    Abs[(-.5 + a) n^(a) (Zeta[.5 + a] - HarmonicNumber[n, .5 + a])]} /.
  a -> (.2 + 11 I), {n, 1, 10^15}]
```




```
Expand[(-.5 - a) n^(-a) j^-(.5 - a) - (-.5 + a) n^(a) j^-(.5 + a)]
```

$-0.5\, j^{-0.5+a} n^{-a} - a\, j^{-0.5+a} n^{-a} + 0.5\, j^{-0.5-a} n^{a} - a\, j^{-0.5-a} n^{a}$

```
N[(-1 + s) Zeta[s, n] /. s → (1 - .1^10) /. n → 10^100]
```

1.

```
t2[x_, n_] := x n^x Sum[ j^(-x - 1), {j, 1, n}]
t3[n_, s_] := (s - 1) n^(s - 1) Sum[ j^(-s), {j, 1, n}]
t3a[n_, s_] := {(s - 1), n^(s - 1), Sum[ j^(-s), {j, 1, n}]}
```

```
t2[N@ZetaZero[1] - 1, 100 000]
```

$-1. + 0.0000706736\, i$

```
t3[10 000 000 000 000 000 000 000 000, 12.0]
```

1.10027×10^{276}

```
t3a[10 000 000 000 000 000 000, .1]
```

$\{-0.9,\ 7.94328 \times 10^{-18},\ 1.39881 \times 10^{17}\}$

```
n^-x / n^x
```

n^{-2x}

```
j^(1 / 2 - x) j^x
```

$\sqrt{j}$

```
(j^x n^-x (1 / 2 + x) - j^-x n^x (1 / 2 - x)) / j^(1 / 2)
```

$$\frac{-j^{-x} n^{x} \left(\frac{1}{2} - x\right) + j^{x} n^{-x} \left(\frac{1}{2} + x\right)}{\sqrt{j}}$$

```
j^x / j^(1 / 2)
```

$j^{-\frac{1}{2}+x}$

```
tg[n_, x_] := (-1 / 2 - x) n^-x (-Sum[ j^(-1 / 2 + x), {j, 1, n}]) -
   (-1 / 2 + x) n^x (-Sum[ j^(-1 / 2 - x), {j, 1, n}])
tg2[n_, x_] := (1 / 2 + x) n^-x (Sum[ j^(-1 / 2 + x), {j, 1, n}]) -
   (1 / 2 - x) n^x (Sum[ j^(-1 / 2 - x), {j, 1, n}])
tg3[n_, x_] := (Sum[ (1 / 2 + x) n^-x j^(-1 / 2 + x), {j, 1, n}]) -
   (Sum[(1 / 2 - x) n^x j^(-1 / 2 - x), {j, 1, n}])
tg4[n_, x_] := Sum[ n^-x (1 / 2 + x) j^(-1 / 2 + x) - n^x (1 / 2 - x) j^(-1 / 2 - x), {j, 1, n}]
tg4a[n_, x_, j_] := n^-x (1 / 2 + x) j^(-1 / 2 + x) - n^x (1 / 2 - x) j^(-1 / 2 - x)
```

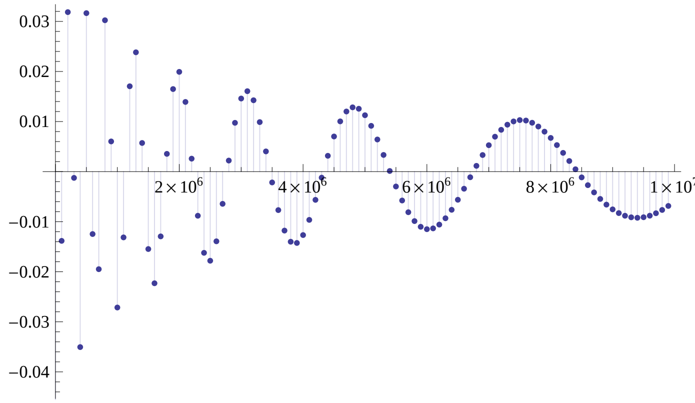
```
N@tg4[100 000 000 000 000, 21.022039638771556` i]
```

$0. + 2.7836 \times 10^{-6}\, i$

```
FullSimplify[tg4[n, x]]
```

$$\frac{1}{2} n^{-x} \left((1 + 2x)\, \text{HarmonicNumber}\left[n, \frac{1}{2} - x\right] + n^{2x} (-1 + 2x)\, \text{HarmonicNumber}\left[n, \frac{1}{2} + x\right]\right)$$

```
DiscretePlot[Im[ tg4a[1 000 000 000, 14.134725141734695` ⅈ, j]], {j, 1, 10 000 000, 100 000}]
```




```
D[tg4a[n, x, j], n]
```

$$-j^{-\frac{1}{2}-x}\, n^{-1+x} \left(\frac{1}{2}-x\right) x - j^{-\frac{1}{2}+x}\, n^{-1-x}\, x \left(\frac{1}{2}+x\right)$$

```
tg4b[n_, x_] :=
 Sum[ n^-x (1 / 2 + x) j^(-1 / 2 + x) - n^x (1 / 2 - x) j^(-1 / 2 - x), {j, 1, Infinity}] -
  Sum[ n^-x (1 / 2 + x) j^(-1 / 2 + x) - n^x (1 / 2 - x) j^(-1 / 2 - x), {j, n + 1, Infinity}]
```

```
N@tg4b[100 000 000 000 000, 21.022039638771556` ⅈ]
```

$Aborted

```
FullSimplify[s n^(1 - 2 s) / (1 - s)]
```

$$\frac{n^{1-2\,s}\, s}{1-s}$$

```
rsatsd[n_, s_, y_, x_] :=  (-n^y) (-1 + s + y) ( Zeta[s + y, n]) - (-n^x) (-1 + s + x) ( Zeta[s + x , n])
```

```
sa1[n_, s_] := Sum[ (-1) ^ (j + 1) j^-s, {j, 1, n}]
sa2[n_, s_] := Sum[ j^-s, {j, 1, n}] - (2^(1 - s)) Sum[ j^-s, {j, 1, n / 2}]
sa3[n_, s_, x_] := Sum[ j^-s, {j, 1, n}] - (x^(1 - s)) Sum[ j^-s, {j, 1, n / x}]
```

```
N@sa2[100, 2]
```

0.822418

```
D[sa3[n, s, x], x] /. x → 1
```

- (1 - s) HarmonicNumber[n, s] + n s (-HarmonicNumber[n, 1 + s] + Zeta[1 + s])

```
Table[(- (1 - s) HarmonicNumber[n, s] + n s (-HarmonicNumber[n, 1 + s] + Zeta[1 + s]) /. s → -2),
 {n, 1, 10}]
```

$$\left\{-\frac{5}{6}, -\frac{8}{3}, -\frac{11}{2}, -\frac{28}{3}, -\frac{85}{6}, -20, -\frac{161}{6}, -\frac{104}{3}, -\frac{87}{2}, -\frac{160}{3}\right\}$$

```
Table[ -n (6 n + 4) / 12, {n, 1, 10}]
```

$$\left\{-\frac{5}{6}, -\frac{8}{3}, -\frac{11}{2}, -\frac{28}{3}, -\frac{85}{6}, -20, -\frac{161}{6}, -\frac{104}{3}, -\frac{87}{2}, -\frac{160}{3}\right\}$$

```
Table[(-(1 - s) HarmonicNumber[n, s] + n s (-HarmonicNumber[n, 1 + s] + Zeta[1 + s]) /. s → -1),
 {n, 1, 10}]
```

$\left\{-\frac{1}{2}, -1, -\frac{3}{2}, -2, -\frac{5}{2}, -3, -\frac{7}{2}, -4, -\frac{9}{2}, -5\right\}$

```
Table[ -n / 2, {n, 1, 10}]
```

$\left\{-\frac{1}{2}, -1, -\frac{3}{2}, -2, -\frac{5}{2}, -3, -\frac{7}{2}, -4, -\frac{9}{2}, -5\right\}$

```
Table[(-(1 - s) HarmonicNumber[n, s] + n s (-HarmonicNumber[n, 1 + s] + Zeta[1 + s]) /. s → -3),
 {n, 1, 10}]
```

{-1, -6, -18, -40, -75, -126, -196, -288, -405, -550}

```
Table[ -n^2 (n + 1) / 2, {n, 1, 10}]
```

{-1, -6, -18, -40, -75, -126, -196, -288, -405, -550}

```
Table[
 ((-(1 - s) HarmonicNumber[n, s] + n s (-HarmonicNumber[n, 1 + s] + Zeta[1 + s]) /. s → -4)) 30,
 {n, 1, 10}]
```

{-31, -392, -1743, -5104, -11 855, -23 736, -42 847, -71 648, -112 959, -169 960}

```
ark0[n_, s_, y_, x_] := (-n^y) (-1 + s + y) (Zeta[s + y, n]) - (-n^x) (-1 + s + x) (Zeta[s + x , n])
ark[n_, s_, t_] := {(s - 1) (n^s Zeta[s] - Sum[ (n / j)^s, {j, 1, n}]),
  (t - 1) (n^t Zeta[t] - Sum[ (n / j)^t, {j, 1, n}])}
ark2[n_, s_, t_] := (s - 1) (n^s Zeta[s, n]) - (t - 1) (n^t Zeta[t, n])
ark4[n_, y_, x_] := {(n^y) (-1 + y) (Zeta[y, n]), (n^x) (-1 + x) (Zeta[x , n])}
tes[n_, s_] := (s - 1) (n^s Zeta[s] - Sum[ (n / j)^s, {j, 1, n}])
tes2[n_, s_] := (s - 1) n^s Zeta[s, n + 1]
tesa[n_, s_] := tes2[n, s] - n - (1 - s) / 2
tes4[n_, s_] := (s - 1) n^(s - 1) (Zeta[s] - Sum[ (1 / j)^s, {j, 1, n}])
```

```
ark2[100 000 000 000 000 000, .4 + 5 I, .7]
```

-16. + 4. i

```
ark[34 567, 3.3, 1.73]
```

{34 587.4, 34 566.6}

```
ark4[23 456, 3.3, 1.73]
```

{23 457.2, 23 456.4}

```
Table[{t, N[tes[scc = 16000, t] - scc - (1 - t) / 2], N[tes2[scc = 16000, t] - scc - (1 - t) / 2]},
  {t, -5, 5, 1 / 2}] // TableForm
```

Infinity::indet : Indeterminate expression 0 ComplexInfinity encountered. ≫

Infinity::indet : Indeterminate expression 0 ComplexInfinity encountered. ≫

-5	0.00015625	0.00015625
$-\frac{9}{2}$	0.000128906	0.000128906
-4	0.000104167	0.000104167
$-\frac{7}{2}$	0.0000820313	0.0000820312
-3	0.0000625	0.0000625
$-\frac{5}{2}$	0.0000455729	0.0000455729
-2	0.00003125	0.00003125
$-\frac{3}{2}$	0.0000195312	0.0000195313
-1	0.0000104167	0.0000104167
$-\frac{1}{2}$	3.90625×10^{-6}	3.90625×10^{-6}
0	0.	0.
$\frac{1}{2}$	-1.30208×10^{-6}	-1.30208×10^{-6}
1	Indeterminate	Indeterminate
$\frac{3}{2}$	3.90696×10^{-6}	3.90625×10^{-6}
2	0.0000103116	0.0000103663
$\frac{5}{2}$	0.000012435	0.0000195312
3	0.	0.00003125
$\frac{7}{2}$	-0.109787	0.0000455729
4	-38.5	23.1587
$\frac{9}{2}$	-2927.72	0.0000820312
5	1.03258×10^{6}	0.000104167

```
N[tes[1200, N[ZetaZero[1]]]]
```

1200.24 − 7.06736 ⅈ

```
Table[{t, N[tes4[scc = 16000, t]]}, {t, -5, 5, 1/2}] // TableForm
```

Infinity::indet : Indeterminate expression 0 ComplexInfinity encountered. ≫

-5	1.00019
$-\frac{9}{2}$	1.00017
-4	1.00016
$-\frac{7}{2}$	1.00014
-3	1.00013
$-\frac{5}{2}$	1.00011
-2	1.00009
$-\frac{3}{2}$	1.00008
-1	1.00006
$-\frac{1}{2}$	1.00005
0	1.00003
$\frac{1}{2}$	1.00002
1	Indeterminate
$\frac{3}{2}$	0.999984
2	0.999969
$\frac{5}{2}$	0.999953
3	0.999938
$\frac{7}{2}$	0.999921
4	1.00135
$\frac{9}{2}$	0.40265
5	0.