

```

Clear[p, eep, dd, ddm]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
p[fn_, n_, k_] := p[fn, n, k] = Sum[fn[j] p[fn, Floor[n / j], k - 1], {j, 2, n}]
p[fn_, n_, 0] := UnitStep[n - 1]
ep[fn_, n_, z_] := Sum[z^k / k! p[fn, n, k], {k, 0, Log2@n}]
ep[fn_, n_, 0] := UnitStep[n - 1]
e2[fn_, n_, k_] := Sum[bin[k, j] (-1)^(k - j) ep[fn, n, j], {j, 0, k}]
e2z[fn_, n_, z_, k_] := Sum[bin[k, j] (-1)^(k - j) ep[fn, n, z j], {j, 0, k}]
eep[fn_, n_, z_] := eep[fn, n, z] = ep[fn, n, z] - ep[fn, n - 1, z]
el[fn_, n_] := Sum[(-1)^(k + 1) / k e2[fn, n, k], {k, 1, Log2@n}]
elz[fn_, n_, z_] := Sum[(-1)^(k + 1) / k e2z[fn, n, z, k], {k, 1, Log2@n}]
dd[fn_, n_, q_, z_] :=
  dd[n, q, z] = Sum[N[eep[fn, a, z] eep[fn, b, z]], {a, 1, Floor[n]}, {b, 1, Floor[(n / a)^q]}]
d2[fn_, n_, q_, k_] := Sum[(-1)^(k - j) Binomial[k, j] dd[fn, n, q, j], {j, 0, k}]
ld[fn_, n_, q_] := Sum[(-1)^(k + 1) / k d2[fn, n, N@Log[q] / Log[n], k], {k, 1, Log2@n}]
ddm[fn_, n_, q_, z_] := ddm[n, q, z] =
  Sum[N[eep[fn, a, z] eep[fn, b, -z]], {a, 1, Floor[n]}, {b, 1, Floor[(n / a)^q]}]
d2m[fn_, n_, q_, k_] := Sum[(-1)^(k - j) Binomial[k, j] ddm[fn, n, q, j], {j, 0, k}]
ldm[fn_, n_, q_] := Sum[(-1)^(k + 1) / k d2m[fn, n, N@Log[q] / Log[n], k], {k, 1, Log2@n}]

ff[n_] := n + Sin[3 n] / n^2
N@p[ff, 100, 1]
5048.96
N@el[ff, 100]
5048.96
N@elz[ff, 100, -2] / -2
5048.96
ld[ff, 200, 53]
21528.9
N[p[ff, 200, 1] + p[ff, 53, 1]]
21528.9
ldm[ff, 200, 53]
18669.
N[p[ff, 200, 1] - p[ff, 53, 1]]
18669.

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