```
Integrate[ 1 / x^s, {x, 1, Infinity}]
```

$$\text{ConditionalExpression}\left[\frac{1}{-1+s}, \text{Re}[s] > 1\right]$$

```
Integrate[ 1 / x^s, {x, 1 / 10 000, Infinity}]
```

$$\text{ConditionalExpression}\left[\frac{10\,000^{-1+s}}{-1+s}, \text{Re}[s] > 1\right]$$

```
l[n_, k_, x_] := l[n, k, x] = Sum[ l[n / (j + x), k - 1, x], {j, 0, n - x}];
l[n_, 1, x_] := l[n, 1, x] = Sum[ Log[(j + x) / (x - 1)], {j, 0, n - x}]; l[n_, 0, x_] := 1
N[l[100, 1, 1]]
```

363.739

```
cl[ n_, k_, x_] := cl[n, k, x] = x^-k l[n x^k, k, x + 1]
clt[ n_, x_, t_] := Sum[ (-1) ^k cl[n, k, x], {k, 1, t}]
```

```
N[clt[100, 1, 10]]
```

−94.0453

```
N[cl[100, 1, 3000]]
```

361.518

```
N[Integrate[ Log[n], {n, 1, 100}]]
```

361.517

```
N[Sum[Log[n], {n, 2, 100}]]
```

363.739

```
N[Sum[Log[n], {k, 2, 100}, {n, 2, 100 / k}]]
```

557.102

```
N[Integrate[ Log[n], {k, 1, 100}, {n, 1, 100 / k}]]
```

698.863

```
N[cl[100, 2, 80]]
```

696.619

```
Expand[Integrate[ Log[x], {x, 1, n}]]
```

$$\text{ConditionalExpression}[1 - n + n\,\text{Log}[n], \text{Re}[n] \geq 0 \,||\, n \notin \text{Reals}]$$

```
Expand[Integrate[ Log[x], {k, 1, n}, {x, 1, n / k}]]
```

$$\text{ConditionalExpression}\left[-1 + n - n\,\text{Log}[n] + \frac{1}{2} n\,\text{Log}[n]^2, \text{Re}[n] \geq 0 \,||\, n \notin \text{Reals}\right]$$

```
Expand[Integrate[ Log[k], {k, 1, n}, {x, 1, n / k}]]
```

$$\text{ConditionalExpression}\left[-1+n-n\,\text{Log}[n]+\frac{1}{2}\,n\,\text{Log}[n]^2,\ \text{Re}[n]\geq 0\ ||\ n\notin \text{Reals}\right]$$

```
Integrate[ Log[x], {x, 1, n}] - Integrate[ Log[x], {k, 1, n}, {x, 1, n / k}] +
 Integrate[ Log[x], {k, 1, n}, {j, 1, n / k}, {x, 1, n / (j k)}] -
 Integrate[ Log[x], {k, 1, n}, {j, 1, n / k}, {m, 1, n / (j k)}, {x, 1, n / (j k m)}]
```

$$\text{ConditionalExpression}\Big[4-3\,n+n\,(-1+\text{Log}[n])-\frac{1}{2}\,n\,(-2+\text{Log}[n])\,\text{Log}[n]+\frac{1}{6}\,n\,\text{Log}[n]\,(6+(-3+\text{Log}[n])\,\text{Log}[n])-\frac{1}{24}\,n\,\text{Log}[n]\,(-24+\text{Log}[n]\,(12+(-4+\text{Log}[n])\,\text{Log}[n])),\ \text{Re}[n]\geq 0\ ||\ n\notin\text{Reals}\Big]$$

$$\textbf{Expand}\Big[4-3\,n+n\,(-1+\text{Log}[n])-\frac{1}{2}\,n\,(-2+\text{Log}[n])\,\text{Log}[n]+\frac{1}{6}\,n\,\text{Log}[n]\,(6+(-3+\text{Log}[n])\,\text{Log}[n])-\frac{1}{24}\,n\,\text{Log}[n]\,(-24+\text{Log}[n]\,(12+(-4+\text{Log}[n])\,\text{Log}[n]))\Big]$$

$$4-4\,n+4\,n\,\text{Log}[n]-\frac{3}{2}\,n\,\text{Log}[n]^2+\frac{1}{3}\,n\,\text{Log}[n]^3-\frac{1}{24}\,n\,\text{Log}[n]^4$$

```
Integrate[ Log[x], {k, 1, n}, {j, 1, n / k}, {m, 1, n / (j k)}, {x, 1, n / (j k m)}] /. n → 100
```

```
Sum[ (-1)^k ((-1)^k (1 - Gamma[ k, -Log[10]] / Gamma[k])), {k, 0, Infinity}]
```

$$\sum_{k=0}^{\infty}(-1)^{2k}\left(1-\frac{\text{Gamma}[k,-\text{Log}[10]]}{\text{Gamma}[k]}\right)$$

$$\textbf{N}\left[\sum_{k=0}^{\infty}(-1)^{2k}\left(1-\frac{\text{Gamma}[k,-\text{Log}[10]]}{\text{Gamma}[k]}\right)\right]$$

$$-1.30259+2.9772\times 10^{-17}\,i$$

```
N[1 - Log[10]]
```

-1.30259

```
Integrate[ Log[x] (1 - Log[n / x]), {x, 1, n}]
```

$$\text{ConditionalExpression}[-1+n-\text{Log}[n],\ \text{Re}[n]\geq 0\ ||\ n\notin\text{Reals}]$$

```
N[Integrate[ Log[x], {k, 1, n}, {j, 1, n / k}, {m, 1, n / (j k)}, {x, 1, n / (j k m)}] /. n → 100]
```

945.128

```
N[Integrate[ Log[x], {k, 1, n}, {j, 1, n / k}, {m, 1, n / (j k)},
   {a, 1, n / (j k m)}, {b, 1, n / (j k m a)}, {c, 1, n / (j k m a b)},
   {d, 1, n / (j k m a b c)}, {x, 1, n / (j k m a b c d)}] /. n → 100]
```

$Aborted

```
g[x_, k_, z_ ] := (x^(k (s - 1))) Zeta[s, x + 1]^k
```

```
D[g[x, k, z], x]
```

$$k\,(-1+s)\,x^{-1+k\,(-1+s)}\,\text{Zeta}[s,\,1+x]^{k}-k\,s\,x^{k\,(-1+s)}\,\text{Zeta}[s,\,1+x]^{-1+k}\,\text{Zeta}[1+s,\,1+x]$$

```
Sum[ Binomial[z, k] D[g[x, k, z], x], {k, 0, Infinity}]
```

$$-\left(x^{-1+s}\,z\left(\frac{x+x^{s}\,\text{Zeta}[s,\,1+x]}{x}\right)^{z}(\text{Zeta}[s,\,1+x]-s\,\text{Zeta}[s,\,1+x]+s\,x\,\text{Zeta}[1+s,\,1+x])\right)\Big/\left(x+x^{s}\,\text{Zeta}[s,\,1+x]\right)$$

```
FullSimplify[
 -(x^(-1+s) z ((x + x^s Zeta[s, 1+x])/x)^z (Zeta[s, 1+x] - s Zeta[s, 1+x] + s x Zeta[1+s, 1+x]))/
   (x + x^s Zeta[s, 1+x])]
```

$$-\left(z\left(1+x^{-1+s}\,\text{Zeta}[s,\,1+x]\right)^{z}(-(-1+s)\,\text{Zeta}[s,\,1+x]+s\,x\,\text{Zeta}[1+s,\,1+x])\right)\Big/\left(x^{2-s}+x\,\text{Zeta}[s,\,1+x]\right)$$

```
Integrate[ -(z (1 + x^(-1+s) Zeta[s, 1+x])^z (-(-1+s) Zeta[s, 1+x] + s x Zeta[1+s, 1+x]))/
   (x^(2-s) + x Zeta[s, 1+x]), {x, 1, Infinity}]
```

$$\int_{1}^{\infty}-\left(z\left(1+x^{-1+s}\,\text{Zeta}[s,\,1+x]\right)^{z}((1-s)\,\text{Zeta}[s,\,1+x]+s\,x\,\text{Zeta}[1+s,\,1+x])\right)\Big/\left(x^{2-s}+x\,\text{Zeta}[s,\,1+x]\right)\,dx$$

```
Sum[ (-1)^(k-1) / k D[g[x, k, z], x], {k, 1, Infinity}]
```

$$-\frac{x^{-1+s}\,(\text{Zeta}[s,\,1+x]-s\,\text{Zeta}[s,\,1+x]+s\,x\,\text{Zeta}[1+s,\,1+x])}{x+x^{s}\,\text{Zeta}[s,\,1+x]}$$

```
Integrate[ -(x^(-1+s) (Zeta[s, 1+x] - s Zeta[s, 1+x] + s x Zeta[1+s, 1+x]))/(x + x^s Zeta[s, 1+x]), {x, 1, Infinity}]
```

$$\int_{1}^{\infty}-\frac{x^{-1+s}\,(\text{Zeta}[s,\,1+x]-s\,\text{Zeta}[s,\,1+x]+s\,x\,\text{Zeta}[1+s,\,1+x])}{x+x^{s}\,\text{Zeta}[s,\,1+x]}\,dx$$

```
-(x^(-1+s) (Zeta[s, 1+x] - s Zeta[s, 1+x] + s x Zeta[1+s, 1+x]))/(x + x^s Zeta[s, 1+x]) /. s → 2
```

$$-\frac{x\,(-\text{Zeta}[2,\,1+x]+2\,x\,\text{Zeta}[3,\,1+x])}{x+x^{2}\,\text{Zeta}[2,\,1+x]}$$