```
et[n_, k_] := et[n, k] = Sum[et[j, k - 1] et[n / j, 1], {j, Divisors[n]}];
et[n_, 1] := (-1) ^ (n + 1); et[n_, 0] := 0; et[1, 0] := 1
et2[n_, k_] := et2[n, k] = Sum[(-1) ^j Binomial[k, j] et[n, k - j], {j, 0, k}]
ET2[n_, k_]  := ET2[n, k] = Sum[ et2[j, k], {j, 2, n}]
lin[n_] := Sum[(-1) ^ (k + 1) / k et2[n, k], {k, 1, Log[2, n]}]
Lin[n_] := Lin[n] = Sum[(-1) ^ (k + 1) / k ET2[n, k], {k, 1, Log[2, n]}]
LAdd[n_]  := Sum[ 2^k / k, {k, 1, Log[2, n]}]
Table[ {n, et2[n, 1], et2[n, 2], et2[n, 3], et2[n, 4],  lin[n]}, {n, 1, 100}] // TableForm
```

1	0	0	0	0	0
2	−1	0	0	0	−1
3	1	0	0	0	1
4	−1	1	0	0	$-\frac{3}{2}$
5	1	0	0	0	1
6	−1	−2	0	0	0
7	1	0	0	0	1
8	−1	2	−1	0	$-\frac{7}{3}$
9	1	1	0	0	$\frac{1}{2}$
10	−1	−2	0	0	0
11	1	0	0	0	1
12	−1	0	3	0	0
13	1	0	0	0	1
14	−1	−2	0	0	0
15	1	2	0	0	0
16	−1	3	−3	1	$-\frac{15}{4}$
17	1	0	0	0	1
18	−1	−4	−3	0	0
19	1	0	0	0	1
20	−1	0	3	0	0
21	1	2	0	0	0
22	−1	−2	0	0	0
23	1	0	0	0	1
24	−1	2	3	−4	0
25	1	1	0	0	$\frac{1}{2}$
26	−1	−2	0	0	0
27	1	2	1	0	$\frac{1}{3}$
28	−1	0	3	0	0
29	1	0	0	0	1
30	−1	−6	−6	0	0
31	1	0	0	0	1
32	−1	4	−6	4	$-\frac{31}{5}$
33	1	2	0	0	0
34	−1	−2	0	0	0
35	1	2	0	0	0
36	−1	−1	6	6	0
37	1	0	0	0	1
38	−1	−2	0	0	0
39	1	2	0	0	0
40	−1	2	3	−4	0
41	1	0	0	0	1
42	−1	−6	−6	0	0
43	1	0	0	0	1

44	−1	0	3	0	0
45	1	4	3	0	0
46	−1	−2	0	0	0
47	1	0	0	0	1
48	−1	4	0	−8	0
49	1	1	0	0	$\frac{1}{2}$
50	−1	−4	−3	0	0
51	1	2	0	0	0
52	−1	0	3	0	0
53	1	0	0	0	1
54	−1	−6	−9	−4	0
55	1	2	0	0	0
56	−1	2	3	−4	0
57	1	2	0	0	0
58	−1	−2	0	0	0
59	1	0	0	0	1
60	−1	−2	9	12	0
61	1	0	0	0	1
62	−1	−2	0	0	0
63	1	4	3	0	0
64	−1	5	−10	10	$-\frac{21}{2}$
65	1	2	0	0	0
66	−1	−6	−6	0	0
67	1	0	0	0	1
68	−1	0	3	0	0
69	1	2	0	0	0
70	−1	−6	−6	0	0
71	1	0	0	0	1
72	−1	2	9	−4	0
73	1	0	0	0	1
74	−1	−2	0	0	0
75	1	4	3	0	0
76	−1	0	3	0	0
77	1	2	0	0	0
78	−1	−6	−6	0	0
79	1	0	0	0	1
80	−1	4	0	−8	0
81	1	3	3	1	$\frac{1}{4}$
82	−1	−2	0	0	0
83	1	0	0	0	1
84	−1	−2	9	12	0
85	1	2	0	0	0
86	−1	−2	0	0	0
87	1	2	0	0	0
88	−1	2	3	−4	0
89	1	0	0	0	1
90	−1	−10	−21	−12	0
91	1	2	0	0	0
92	−1	0	3	0	0
93	1	2	0	0	0
94	−1	−2	0	0	0
95	1	2	0	0	0
96	−1	6	−6	−8	0
97	1	0	0	0	1
98	−1	−4	−3	0	0

99	1	4	3	0	0
100	−1	−1	6	6	0

```
Table[{n, et2[n, 1], et2[n, 2], et2[n, 3], et2[n, 4], lin[n]}, {n, 1, 100}] // TableForm
```

1	0	0	0	0	0
2	−1	0	0	0	−1
3	1	0	0	0	1
4	−1	1	0	0	$-\frac{3}{2}$
5	1	0	0	0	1
6	−1	−2	0	0	0
7	1	0	0	0	1
8	−1	2	−1	0	$-\frac{7}{3}$
9	1	1	0	0	$\frac{1}{2}$
10	−1	−2	0	0	0
11	1	0	0	0	1
12	−1	0	3	0	0
13	1	0	0	0	1
14	−1	−2	0	0	0
15	1	2	0	0	0
16	−1	3	−3	1	$-\frac{15}{4}$
17	1	0	0	0	1
18	−1	−4	−3	0	0
19	1	0	0	0	1
20	−1	0	3	0	0
21	1	2	0	0	0
22	−1	−2	0	0	0
23	1	0	0	0	1
24	−1	2	3	−4	0
25	1	1	0	0	$\frac{1}{2}$
26	−1	−2	0	0	0
27	1	2	1	0	$\frac{1}{3}$
28	−1	0	3	0	0
29	1	0	0	0	1
30	−1	−6	−6	0	0
31	1	0	0	0	1
32	−1	4	−6	4	$-\frac{31}{5}$
33	1	2	0	0	0
34	−1	−2	0	0	0
35	1	2	0	0	0
36	−1	−1	6	6	0
37	1	0	0	0	1
38	−1	−2	0	0	0
39	1	2	0	0	0
40	−1	2	3	−4	0
41	1	0	0	0	1
42	−1	−6	−6	0	0
43	1	0	0	0	1
44	−1	0	3	0	0
45	1	4	3	0	0
46	−1	−2	0	0	0
47	1	0	0	0	1
48	−1	4	0	−8	0
49	1	1	0	0	$\frac{1}{2}$

50	−1	−4	−3	0	0
51	1	2	0	0	0
52	−1	0	3	0	0
53	1	0	0	0	1
54	−1	−6	−9	−4	0
55	1	2	0	0	0
56	−1	2	3	−4	0
57	1	2	0	0	0
58	−1	−2	0	0	0
59	1	0	0	0	1
60	−1	−2	9	12	0
61	1	0	0	0	1
62	−1	−2	0	0	0
63	1	4	3	0	0
64	−1	5	−10	10	$-\frac{21}{2}$
65	1	2	0	0	0
66	−1	−6	−6	0	0
67	1	0	0	0	1
68	−1	0	3	0	0
69	1	2	0	0	0
70	−1	−6	−6	0	0
71	1	0	0	0	1
72	−1	2	9	−4	0
73	1	0	0	0	1
74	−1	−2	0	0	0
75	1	4	3	0	0
76	−1	0	3	0	0
77	1	2	0	0	0
78	−1	−6	−6	0	0
79	1	0	0	0	1
80	−1	4	0	−8	0
81	1	3	3	1	$\frac{1}{4}$
82	−1	−2	0	0	0
83	1	0	0	0	1
84	−1	−2	9	12	0
85	1	2	0	0	0
86	−1	−2	0	0	0
87	1	2	0	0	0
88	−1	2	3	−4	0
89	1	0	0	0	1
90	−1	−10	−21	−12	0
91	1	2	0	0	0
92	−1	0	3	0	0
93	1	2	0	0	0
94	−1	−2	0	0	0
95	1	2	0	0	0
96	−1	6	−6	−8	0
97	1	0	0	0	1
98	−1	−4	−3	0	0
99	1	4	3	0	0
100	−1	−1	6	6	0

```
Table[{n, ET2[n, 1], ET2[n, 2], ET2[n, 3], ET2[n, 4], Lin[n]}, {n, 1, 100}] // TableForm
```

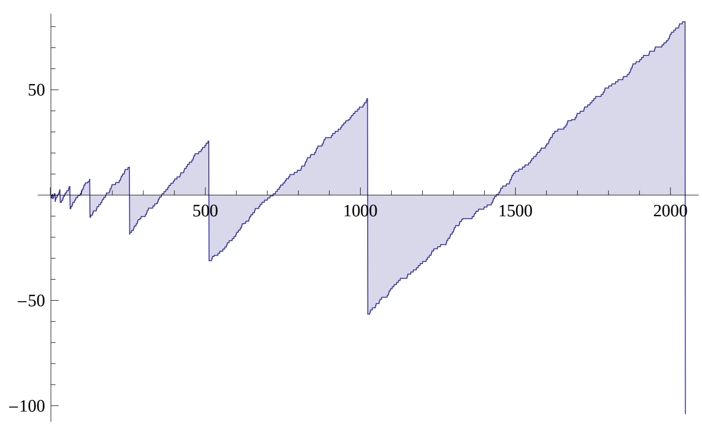
1	0	0	0	0	0
2	−1	0	0	0	−1
3	0	0	0	0	0

4	-1	1	0	0	$-\frac{3}{2}$
5	0	1	0	0	$-\frac{1}{2}$
6	-1	-1	0	0	$-\frac{1}{2}$
7	0	-1	0	0	$\frac{1}{2}$
8	-1	1	-1	0	$-\frac{11}{6}$
9	0	2	-1	0	$-\frac{4}{3}$
10	-1	0	-1	0	$-\frac{4}{3}$
11	0	0	-1	0	$-\frac{1}{3}$
12	-1	0	2	0	$-\frac{1}{3}$
13	0	0	2	0	$\frac{2}{3}$
14	-1	-2	2	0	$\frac{2}{3}$
15	0	0	2	0	$\frac{2}{3}$
16	-1	3	-1	1	$-\frac{37}{12}$
17	0	3	-1	1	$-\frac{25}{12}$
18	-1	-1	-4	1	$-\frac{25}{12}$
19	0	-1	-4	1	$-\frac{13}{12}$
20	-1	-1	-1	1	$-\frac{13}{12}$
21	0	1	-1	1	$-\frac{13}{12}$
22	-1	-1	-1	1	$-\frac{13}{12}$
23	0	-1	-1	1	$-\frac{1}{12}$
24	-1	1	2	-3	$-\frac{1}{12}$
25	0	2	2	-3	$\frac{5}{12}$
26	-1	0	2	-3	$\frac{5}{12}$
27	0	2	3	-3	$\frac{3}{4}$
28	-1	2	6	-3	$\frac{3}{4}$
29	0	2	6	-3	$\frac{7}{4}$
30	-1	-4	0	-3	$\frac{7}{4}$
31	0	-4	0	-3	$\frac{11}{4}$
32	-1	0	-6	1	$-\frac{69}{20}$
33	0	2	-6	1	$-\frac{69}{20}$
34	-1	0	-6	1	$-\frac{69}{20}$
35	0	2	-6	1	$-\frac{69}{20}$
36	-1	1	0	7	$-\frac{69}{20}$
37	0	1	0	7	$-\frac{49}{20}$
38	-1	-1	0	7	$-\frac{49}{20}$
39	0	1	0	7	$-\frac{49}{20}$
40	-1	3	3	3	$-\frac{49}{20}$
41	0	3	3	3	$-\frac{29}{20}$
42	-1	-3	-3	3	$-\frac{29}{20}$

43	0	-3	-3	3	$-\frac{9}{20}$
44	-1	-3	0	3	$-\frac{9}{20}$
45	0	1	3	3	$-\frac{9}{20}$
46	-1	-1	3	3	$-\frac{9}{20}$
47	0	-1	3	3	$\frac{11}{20}$
48	-1	3	3	-5	$\frac{11}{20}$
49	0	4	3	-5	$\frac{21}{20}$
50	-1	0	0	-5	$\frac{21}{20}$
51	0	2	0	-5	$\frac{21}{20}$
52	-1	2	3	-5	$\frac{21}{20}$
53	0	2	3	-5	$\frac{41}{20}$
54	-1	-4	-6	-9	$\frac{41}{20}$
55	0	-2	-6	-9	$\frac{41}{20}$
56	-1	0	-3	-13	$\frac{41}{20}$
57	0	2	-3	-13	$\frac{41}{20}$
58	-1	0	-3	-13	$\frac{41}{20}$
59	0	0	-3	-13	$\frac{61}{20}$
60	-1	-2	6	-1	$\frac{61}{20}$
61	0	-2	6	-1	$\frac{81}{20}$
62	-1	-4	6	-1	$\frac{81}{20}$
63	0	0	9	-1	$\frac{81}{20}$
64	-1	5	-1	9	$-\frac{129}{20}$
65	0	7	-1	9	$-\frac{129}{20}$
66	-1	1	-7	9	$-\frac{129}{20}$
67	0	1	-7	9	$-\frac{109}{20}$
68	-1	1	-4	9	$-\frac{109}{20}$
69	0	3	-4	9	$-\frac{109}{20}$
70	-1	-3	-10	9	$-\frac{109}{20}$
71	0	-3	-10	9	$-\frac{89}{20}$
72	-1	-1	-1	5	$-\frac{89}{20}$
73	0	-1	-1	5	$-\frac{69}{20}$
74	-1	-3	-1	5	$-\frac{69}{20}$
75	0	1	2	5	$-\frac{69}{20}$
76	-1	1	5	5	$-\frac{69}{20}$
77	0	3	5	5	$-\frac{69}{20}$
78	-1	-3	-1	5	$-\frac{69}{20}$
79	0	-3	-1	5	$-\frac{49}{20}$
80	-1	1	-1	-3	$-\frac{49}{20}$
81	0	4	2	-2	$-\frac{11}{5}$

82	-1	2	2	-2	$-\frac{11}{5}$
83	0	2	2	-2	$-\frac{6}{5}$
84	-1	0	11	10	$-\frac{6}{5}$
85	0	2	11	10	$-\frac{6}{5}$
86	-1	0	11	10	$-\frac{6}{5}$
87	0	2	11	10	$-\frac{6}{5}$
88	-1	4	14	6	$-\frac{6}{5}$
89	0	4	14	6	$-\frac{1}{5}$
90	-1	-6	-7	-6	$-\frac{1}{5}$
91	0	-4	-7	-6	$-\frac{1}{5}$
92	-1	-4	-4	-6	$-\frac{1}{5}$
93	0	-2	-4	-6	$-\frac{1}{5}$
94	-1	-4	-4	-6	$-\frac{1}{5}$
95	0	-2	-4	-6	$-\frac{1}{5}$
96	-1	4	-10	-14	$-\frac{1}{5}$
97	0	4	-10	-14	$\frac{4}{5}$
98	-1	0	-13	-14	$\frac{4}{5}$
99	0	4	-10	-14	$\frac{4}{5}$
100	-1	3	-4	-8	$\frac{4}{5}$

```
DiscretePlot[ Lin[n], {n, 2, 64 * 32}]
```




```
FG[n_, k_] := Sum[ ((-1) ^ (j + 1)) / k - FG[Floor[n / j], k + 1], {j, 2, n}]
```

```
FG[100, 1]
```

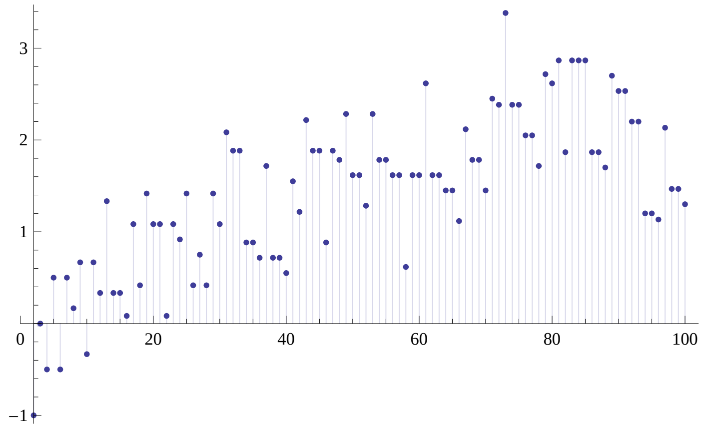
$\frac{13}{10}$

```
Lin[100]
```

$\frac{4}{5}$

```
DiscretePlot[FF[n, 1], {n, 2, 100}]
```

Lin[100, 1]



```
Table[ {2^n, lin[2^n], - (2^n - 1) / n}, {n, 1, 14}] // TableForm
```

2	-1	-1
4	$-\frac{3}{2}$	$-\frac{3}{2}$
8	$-\frac{7}{3}$	$-\frac{7}{3}$
16	$-\frac{15}{4}$	$-\frac{15}{4}$
32	$-\frac{31}{5}$	$-\frac{31}{5}$
64	$-\frac{21}{2}$	$-\frac{21}{2}$
128	$-\frac{127}{7}$	$-\frac{127}{7}$
256	$-\frac{255}{8}$	$-\frac{255}{8}$
512	$-\frac{511}{9}$	$-\frac{511}{9}$
1024	$-\frac{1023}{10}$	$-\frac{1023}{10}$
2048	$-\frac{2047}{11}$	$-\frac{2047}{11}$
4096	$-\frac{1365}{4}$	$-\frac{1365}{4}$
8192	$-\frac{8191}{13}$	$-\frac{8191}{13}$
16 384	$-\frac{16\,383}{14}$	$-\frac{16\,383}{14}$

```
Table[ {n, ET2[n, 1], ET2[n, 2], ET2[n, 3], ET2[n, 4], ET2[n, 5], ET2[n, 6], ET2[n, 7],
   ET2[n, 8], ET2[n, 9], ET2[n, 10], ET2[n, 11], Lin[n]}, {n, 100, 10 000, 100}] // TableForm
```

100	-1	3	-4	-8	9	-5	0	0	0	0
200	-1	-1	-3	6	24	-14	6	0	0	0
300	-1	0	5	3	-5	35	-22	1	0	0
400	-1	5	-12	-5	5	-49	20	-7	0	0
500	-1	-1	-3	-57	35	-1	20	-7	0	0
600	-1	-3	5	17	15	40	-64	29	-1	0
700	-1	1	41	60	70	-26	-106	21	-1	0
800	-1	3	-9	66	55	-40	83	-27	8	0
900	-1	5	12	56	55	-4	118	-91	8	0

1000	−1	−6	−19	−16	190	24	20	−35	8	0	0
1100	−1	−2	38	110	−76	−30	−134	1	−1	1	0
1200	−1	5	14	−40	−76	−36	−120	105	−37	1	0
1300	−1	3	23	125	134	66	83	127	−28	1	0
1400	−1	0	12	−83	−141	−180	−36	183	−28	1	0
1500	−1	−9	9	−85	74	180	20	7	−28	1	0
1600	−1	13	30	79	−161	−225	104	−125	35	−9	0
1700	−1	−4	0	−13	−96	159	412	−133	35	−9	0
1800	−1	1	−13	29	−11	−69	69	−237	128	−9	0
1900	−1	−10	−19	−139	−281	−135	139	−237	128	−9	0
2000	−1	3	23	29	119	−360	−155	−77	56	−9	0
2100	−1	−4	35	241	239	345	−386	−125	11	1	−1
2200	−1	1	21	−9	−311	27	−14	203	11	1	−1
2300	−1	0	114	139	64	261	−70	259	11	1	−1
2400	−1	−1	−3	−29	129	201	21	267	−160	46	−1
2500	−1	9	−9	−116	169	186	−42	315	−160	46	−1
2600	−1	7	21	28	−301	−420	−84	−13	−223	36	−1
2700	−1	4	39	262	354	12	210	203	−295	36	−1
2800	−1	3	−16	−150	94	241	343	203	−295	36	−1
2900	−1	−12	−73	−162	194	187	−154	−69	−34	36	−1
3000	−1	−3	−10	−84	364	250	−203	−49	−34	36	−1
3100	−1	−10	−16	218	−36	160	511	−153	209	−44	10
3200	−1	−1	−85	−224	−475	181	637	−189	173	−44	10
3300	−1	−8	14	262	220	−245	−357	−413	173	−44	10
3400	−1	19	105	140	0	231	−63	−797	182	−44	10
3500	−1	−4	66	204	310	69	−518	−581	362	−164	10
3600	−1	13	12	6	−235	−240	42	−265	425	−174	10
3700	−1	−1	48	−90	−490	−480	−42	−209	425	−174	10
3800	−1	−8	36	94	430	816	441	−385	425	−174	10
3900	−1	−1	27	268	670	234	784	407	200	−84	10
4000	−1	10	15	108	350	−261	462	343	200	−84	10
4100	−1	13	−10	−67	−410	231	−350	449	287	−29	−1
4200	−1	5	50	77	−505	−579	−966	505	287	−29	−1
4300	−1	−22	−61	151	335	−183	−1029	721	215	−29	−1
4400	−1	7	95	5	−345	435	84	161	−280	−29	−1
4500	−1	0	23	−19	130	651	147	433	−352	−29	−1
4600	−1	−8	−19	−439	−635	−195	−595	153	−352	−29	−1
4700	−1	−17	−82	125	−70	−1008	−560	217	−757	231	−5
4800	−1	20	80	151	190	−288	−504	97	−505	231	−5

4900	−1	15	137	521	1105	1110	350	49	−496	231	−5
5000	−1	−3	−105	−179	465	216	−119	321	−568	231	−5
5100	−1	−14	54	−67	−180	−120	1001	1161	−568	231	−5
5200	−1	13	18	−125	−375	471	1113	245	−199	361	−4
5300	−1	−5	6	19	300	1575	1442	−99	−199	361	−4
5400	−1	−2	−42	−1	−340	−735	0	−259	−550	451	−4
5500	−1	9	24	133	495	99	−84	−371	−550	451	−4
5600	−1	1	48	281	810	195	−252	−491	−550	451	−4
5700	−1	−16	−90	−159	−505	−714	154	−371	−487	441	−4
5800	−1	3	15	73	310	−495	−777	69	125	81	−4
5900	−1	−5	−28	−107	−400	−1915	−1848	−315	50	81	−4
6000	−1	14	110	163	415	−751	−1071	21	50	81	−4
6100	−1	7	35	−171	330	41	−385	−435	−454	81	−4
6200	−1	5	53	241	−520	−109	−91	−1155	221	−269	54
6300	−1	−12	26	413	710	1631	441	−1387	185	−269	54
6400	−1	3	−22	145	620	1595	252	−1435	266	−224	54
6500	−1	−15	−4	53	−80	869	1155	245	896	−224	54
6600	−1	−6	74	220	−385	−754	154	462	824	−224	54
6700	−1	4	41	236	−335	−550	189	134	887	−234	54
6800	−1	3	−127	−574	−815	−184	−497	−378	1391	−234	54
6900	−1	−2	225	502	470	644	−49	−210	1391	−234	54
7000	−1	0	33	−210	−430	−520	−406	638	1292	−624	21
7100	−1	−3	12	36	555	395	427	1274	1220	−624	21
7200	−1	1	−66	−64	−40	629	840	−6	707	−704	23
7300	−1	12	−36	−194	−425	−31	623	154	635	−704	23
7400	−1	21	21	−452	−680	443	1043	154	635	−704	23
7500	−1	−1	−9	−224	75	1379	1106	−118	896	−704	23
7600	−1	10	153	238	115	−811	−1918	−1246	896	−704	23
7700	−1	2	−6	502	620	−1525	−1148	−902	23	−174	12
7800	−1	−1	39	184	−296	−1882	−644	−986	−607	−426	12
7900	−1	−3	−12	72	−81	−1576	−707	−1106	−607	−426	12
8000	−1	0	−34	−64	−596	−1966	28	−370	−514	−426	12
8100	−1	3	80	610	1914	599	532	490	98	−786	12
8200	−1	−17	−172	−194	−36	1271	−196	946	−397	−566	54
8300	−1	−4	−4	−332	−1091	−91	−504	1282	−397	−566	54
8400	−1	0	35	−216	−781	665	966	2170	−469	−566	54
8500	−1	1	35	184	−231	−316	252	2290	−820	−476	54
8600	−1	1	65	376	19	−784	−252	2338	−820	−476	54
8700	−1	−20	−64	−374	−176	680	−588	378	−568	364	54

8800	−1	12	20	−382	−216	1694	154	−70	−469	354	54
8900	−1	3	104	380	1659	3200	728	146	−541	354	54
9000	−1	7	−43	−208	324	800	−763	−182	−892	444	54
9100	−1	12	92	30	−116	−4	133	1546	−262	444	54
9200	−1	−2	74	54	1144	2000	581	1266	−262	444	54
9300	−1	3	21	624	674	−266	595	770	−775	1219	−3
9400	−1	−17	105	896	529	−236	1848	1602	−775	1219	−3
9500	−1	−16	−90	−204	−1191	−1454	1036	1330	−514	1219	−3
9600	−1	18	−102	−568	−1081	−1154	399	1266	−406	859	−3
9700	−1	−1	45	−116	−381	−605	532	1266	−406	859	−3
9800	−1	3	−3	−412	−1481	−2885	−2912	−1190	−595	849	−3
9900	−1	−16	−27	444	1399	−56	−1827	−974	−667	849	−3
10 000	−1	5	−6	411	1139	−794	−1253	−168	−1018	939	−3

LAdd[n_] := Sum[2^k / k, {k, 1, Log[2, n]}]

N[Lin[1000] + LAdd[1000]]

176.696

N[Sum[MangoldtLambda[n] / Log[n], {n, 2, 1000}]]

176.696

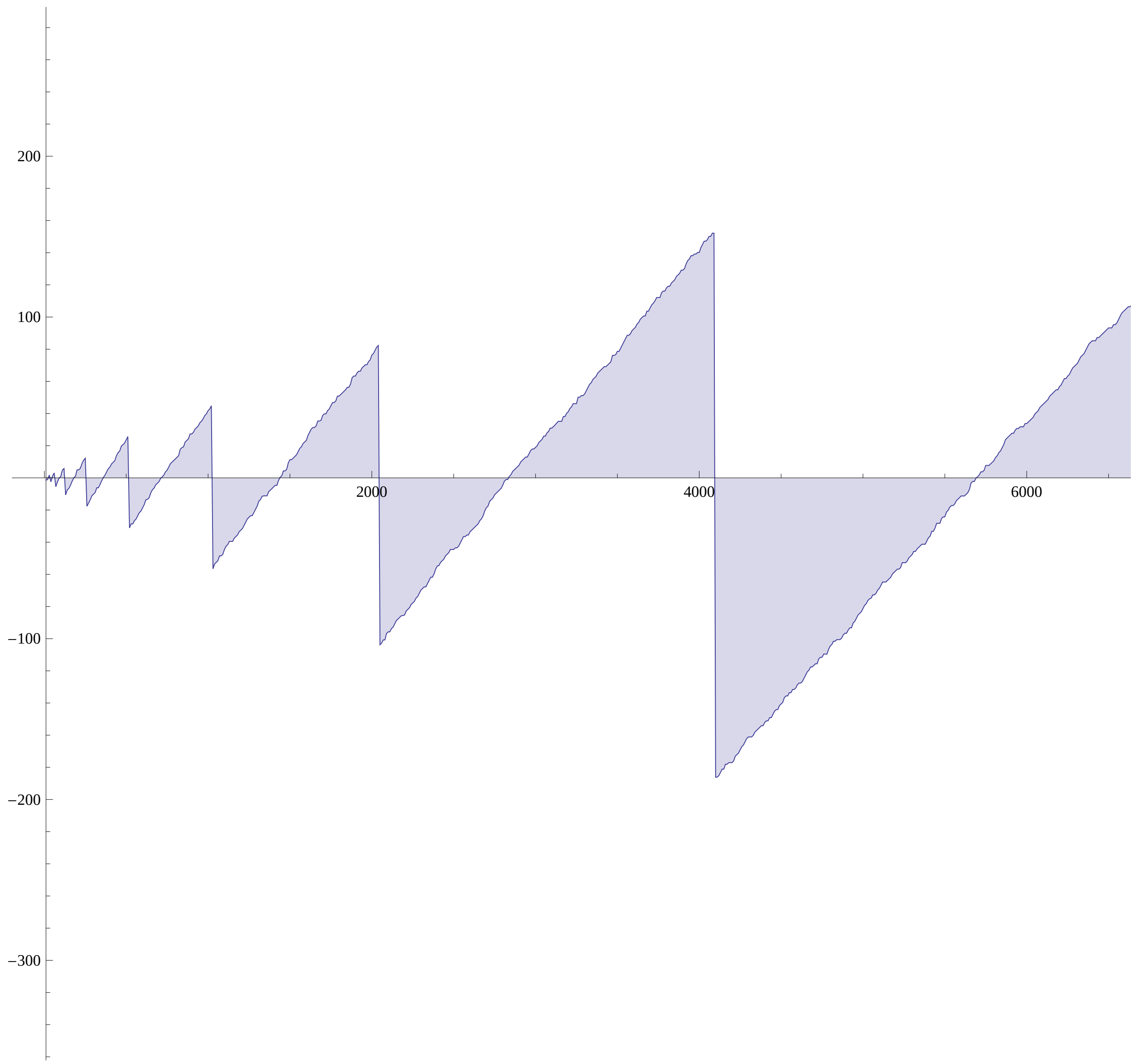
Expand[(2^k - 1) / k + 1 / k]

$$\frac{2^k}{k}$$

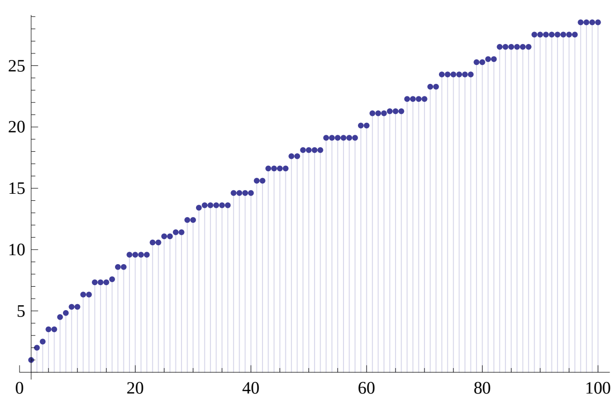
Expand[Sum[2^k / k, {k, 1, Log[2, n]}]]

$$-i\pi - 2\, n\, \text{LerchPhi}\left[2, 1, 1 + \frac{\text{Log}[n]}{\text{Log}[2]}\right]$$

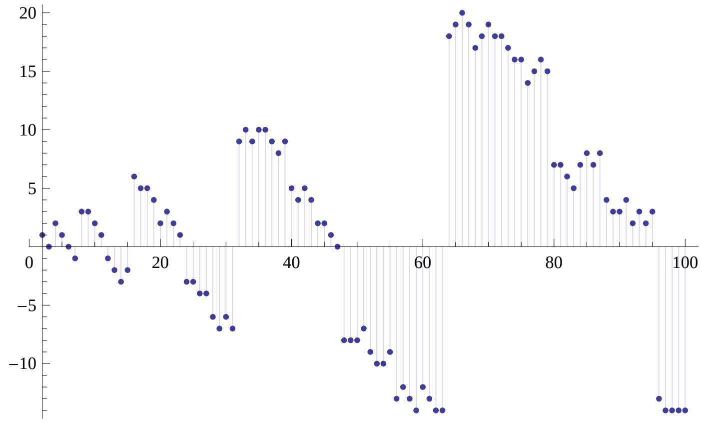
```
DiscretePlot[ Lin[n], {n, 10, 10 000, 10}]
```



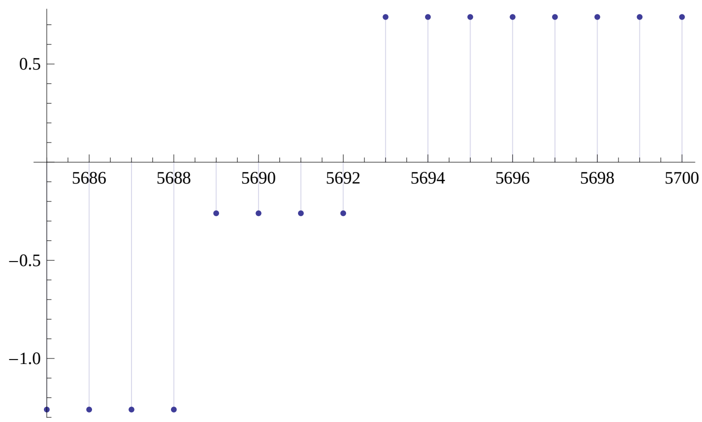

```
DiscretePlot[ Lin[n] + LAdd[n], {n, 2, 100}]
```



DiscretePlot[MLin[n], {n, 2, 100}]



DiscretePlot[Lin[n], {n, 5685, 5700}]



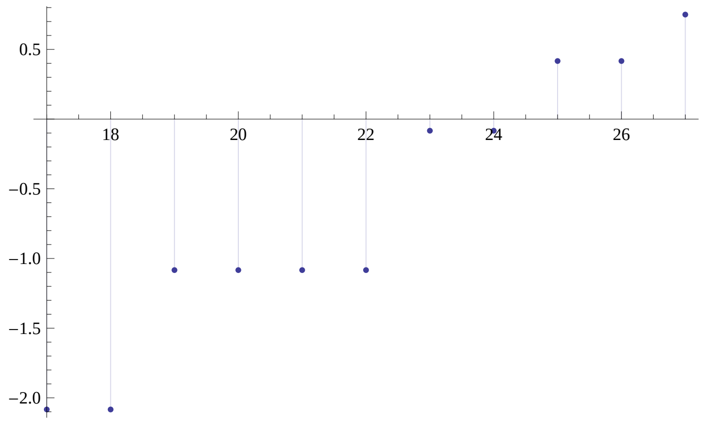
N[Lin[5693]]

0.739646

N[(5693 - 4096) / 4096]

0.389893

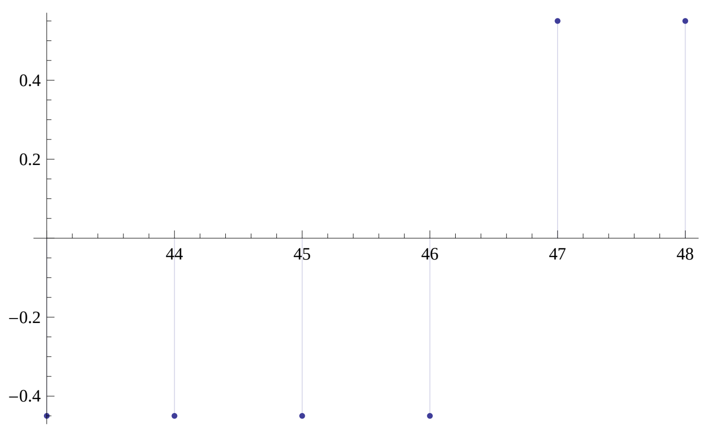
```
DiscretePlot[ Lin[n], {n, Floor[ (2^4) * 1.1], Floor[(2^4) * 1.7]}]
```




```
Lin[24]
```

$-\frac{1}{12}$

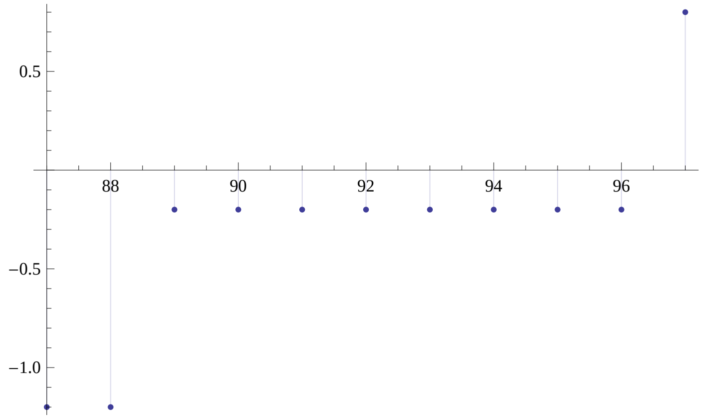
```
DiscretePlot[ Lin[n], {n, Floor[ (2^5) * 1.37], Floor[(2^5) * 1.52]}]
```




```
Lin[46]
```

$-\frac{9}{20}$

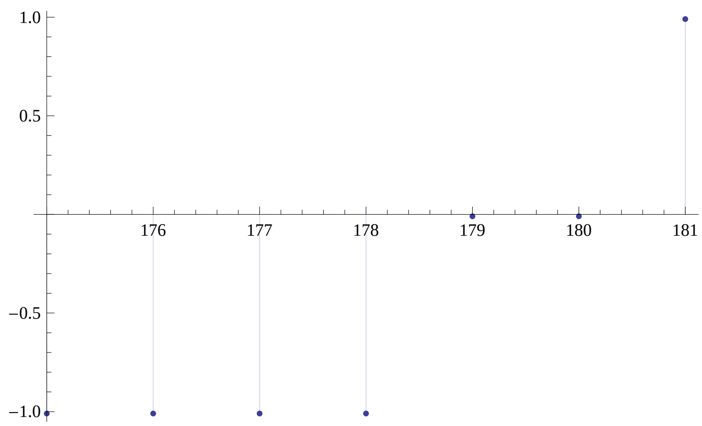
```
DiscretePlot[ Lin[n], {n, Floor[ (2^6) * 1.37], Floor[(2^6) * 1.52]}]
```




```
Lin[ 96]
```

$-\frac{1}{5}$

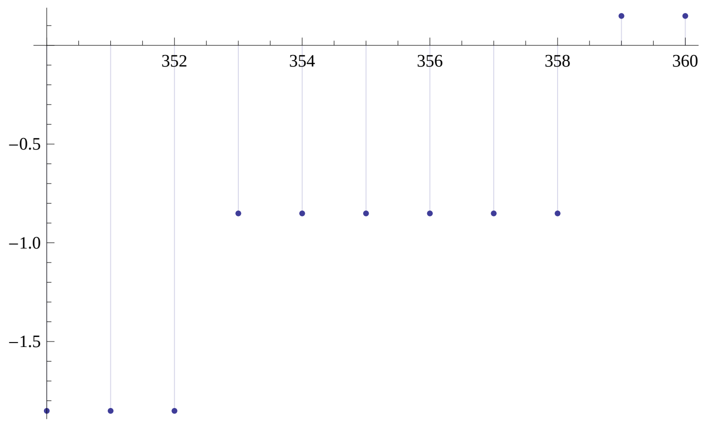
```
DiscretePlot[ Lin[n], {n, Floor[ (2^7) * 1.37], Floor[(2^7) * 1.42]}]
```




```
Lin[ 180]
```

$-\frac{1}{105}$

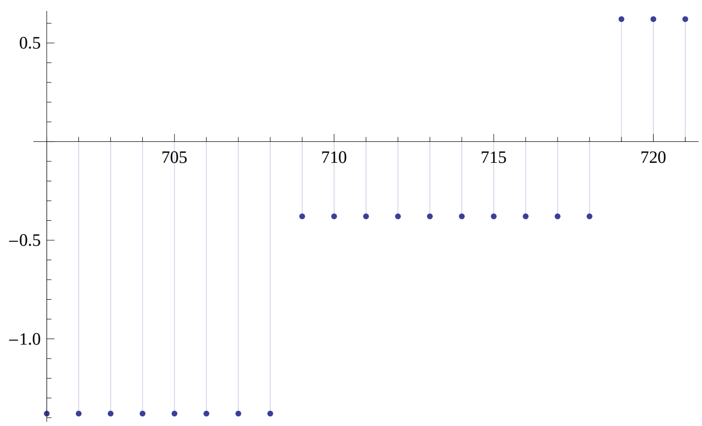
```
DiscretePlot[ Lin[n], {n, Floor[ (2^8) * 1.37], Floor[(2^8) * 1.41]}]
```




```
Lin[358]
```

$-\frac{143}{168}$

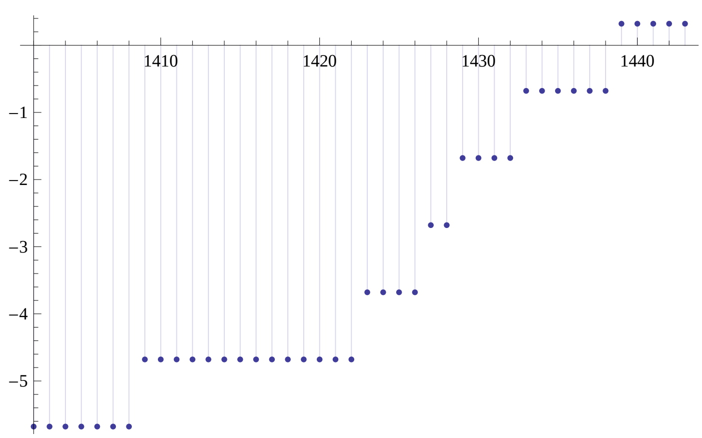
```
DiscretePlot[ Lin[n], {n, Floor[ (2^9) * 1.37], Floor[(2^9) * 1.41]}]
```




```
N[Lin[ 718]]
```

-0.378968

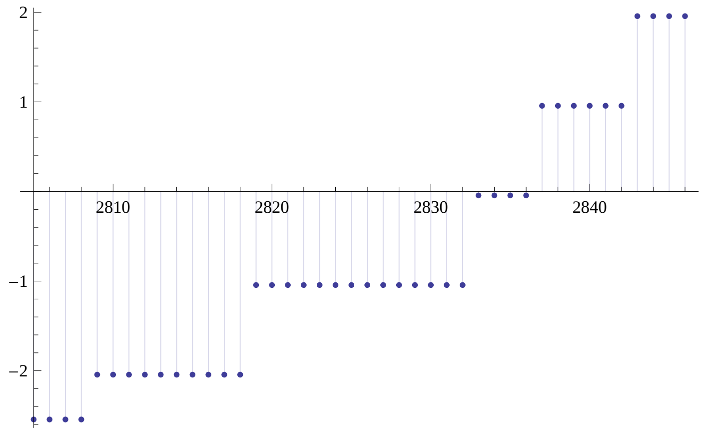
```
DiscretePlot[ Lin[n], {n, Floor[ (2^10) * 1.37], Floor[(2^10) * 1.41]}]
```



```
N[Lin[1438]]
```

-0.678968

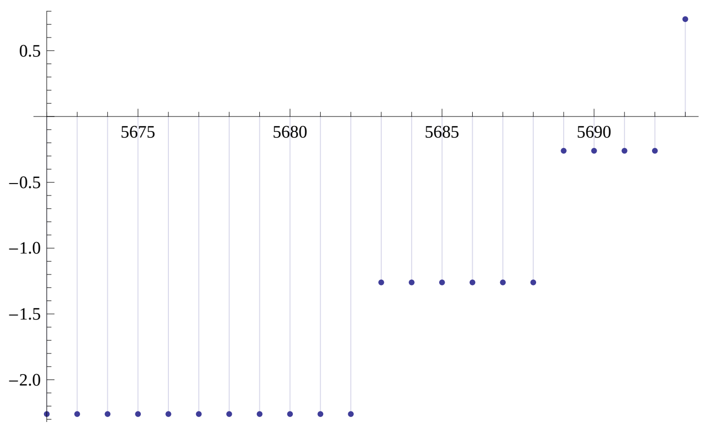
```
DiscretePlot[Lin[n], {n, Floor[(2^11) * 1.37], Floor[(2^11) * 1.39]}]
```




```
N[Lin[2836]]
```

-0.0436869

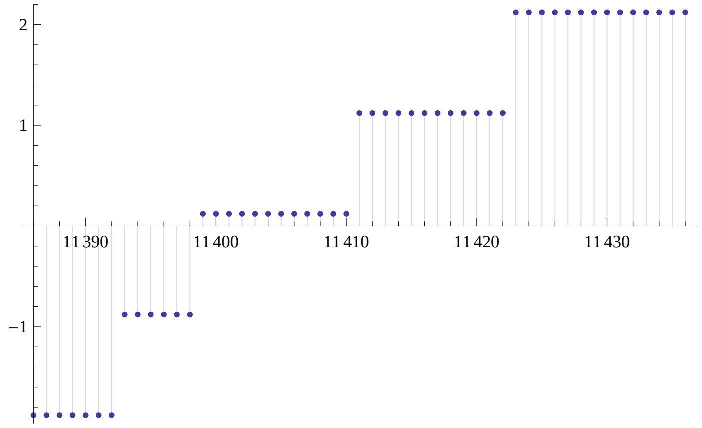
```
DiscretePlot[Lin[n], {n, Floor[(2^12) * 1.385], Floor[(2^12) * 1.39]}]
```




```
N[Lin[5692]]
```

-0.260354

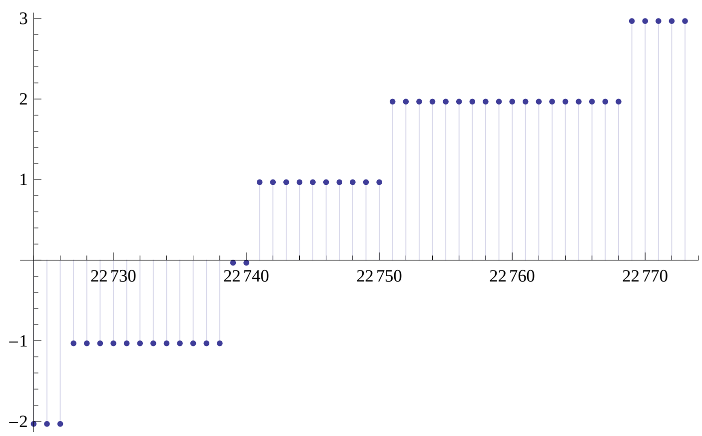
```
DiscretePlot[ Lin[n], {n, Floor[ (2^13) * 1.39], Floor[(2^13) * 1.396]}]
```




```
N[Lin[11 398]]
```

```
-0.878943
```

```
DiscretePlot[ Lin[n], {n, Floor[ (2^14) * 1.387], Floor[(2^14) * 1.39]}]
```




```
N[Lin[22 740]]
```

```
-0.0321179
```

```
DiscretePlot[ Lin[n], {n, Floor[ (2^15) * 1.385], Floor[(2^15) * 1.4]}]
```

```
$Aborted
```

```
dif := {24, 46, 96, 180, 358, 718, 1438, 2836, 5692, 11 398, 22 740}
N[Table[ {j, dif[[j]], dif[[j]] / (2^(j + 3.5)), dif[[j + 1]] / dif[[j]]},
   {j, 1, Length[dif] - 1}] // TableForm]
```

1.	24.	1.06066	1.91667
2.	46.	1.01647	2.08696
3.	96.	1.06066	1.875
4.	180.	0.994369	1.98889
5.	358.	0.988845	2.00559
6.	718.	0.991607	2.00279
7.	1438.	0.992988	1.97218
8.	2836.	0.979177	2.00705
9.	5692.	0.98263	2.00246
10.	11 398.	0.983838	1.99509

```
aba = Floor[2^(19 + (1 / 2))] * .983
N[LAdd[ aba]]
N[LogIntegral[aba]]
N[PP[aba]]
```

728 850.

58 713.6

58 785.

58 784.9

```
PP[n_] := N[Sum[ 1 / k PrimePi[ n^(1 / k)], {k, 1, n}]]
```

```
PP[10 000]
```

1247.1

```
N[Table[2^(j + 1 / 2), {j, 2, 18}]]
```

{5.65685, 11.3137, 22.6274, 45.2548, 90.5097, 181.019, 362.039, 724.077, 1448.15, 2896.31, 5792.62, 11 585.2, 23 170.5, 46 341., 92 681.9, 185 364., 370 728.}

```
LI1[n_] := Sum[ 2^k / k, {k, 1, Log[2, n]}]
```

```
LI1[n]
```

$$-i \left(\pi - 2\, i\, n\, \mathrm{LerchPhi}\left[2, 1, 1 + \frac{\mathrm{Log}[n]}{\mathrm{Log}[2]}\right]\right)$$

```
LI2[n_] := Integrate[ 2^k / k, {k, 1, Log[2, n]}]
```

```
LI2[n]
```

ConditionalExpression[
 Log[Log[2]] - Log[Log[n]] - LogIntegral[2] + LogIntegral[n], Im[Log[n]] ≠ 0 || Re[Log[n]] ≥ 0]

```
Integrate[ 2^k / k, {k, 1, Log[2, n]}]
```

ConditionalExpression[
 Log[Log[2]] - Log[Log[n]] - LogIntegral[2] + LogIntegral[n], Im[Log[n]] ≠ 0 || Re[Log[n]] ≥ 0]

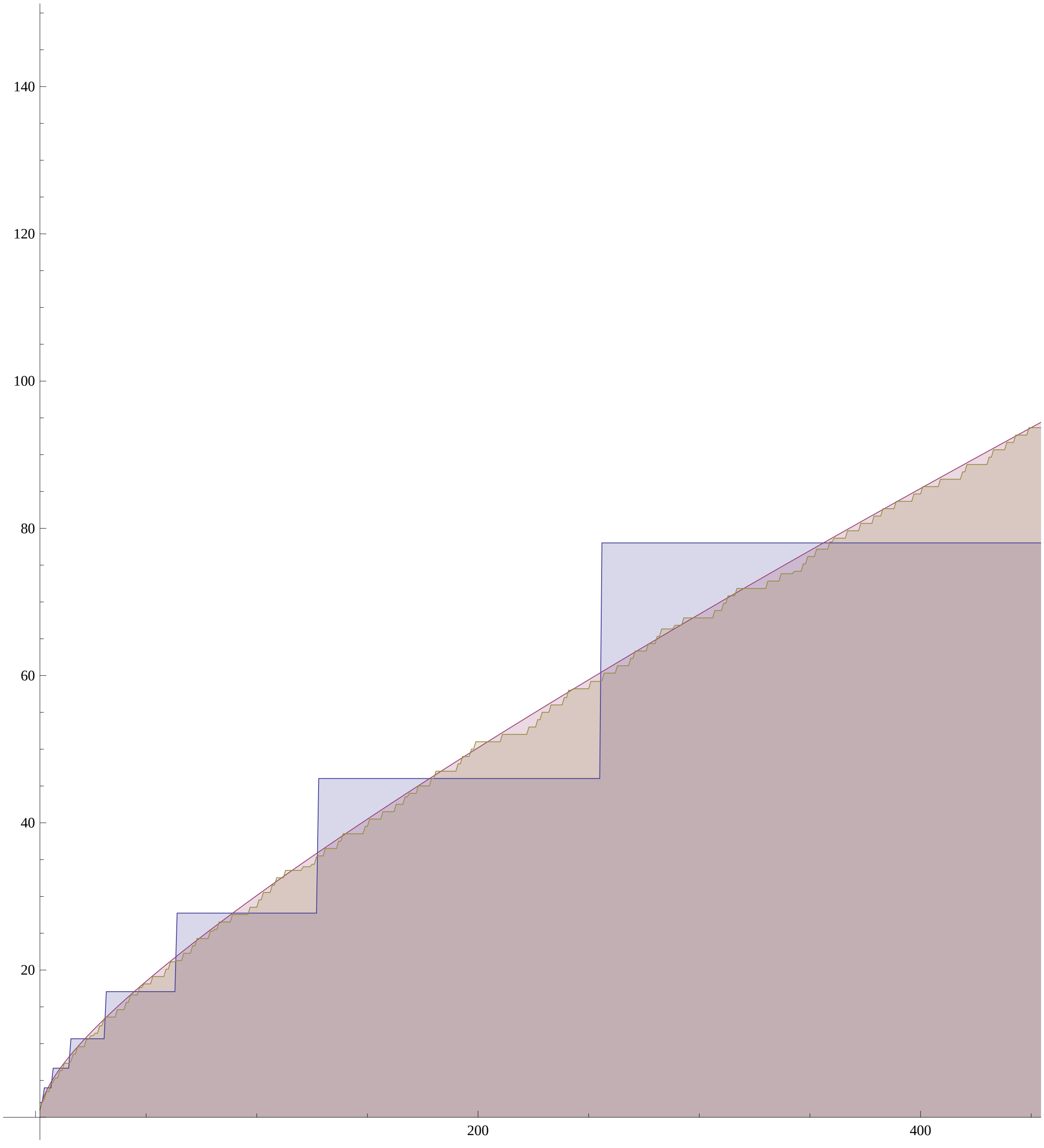
```
Integrate[ 3^k / k, {k, 1, Log[3, n]}]
```

ConditionalExpression[
 Log[Log[3]] - Log[Log[n]] - LogIntegral[3] + LogIntegral[n], Im[Log[n]] ≠ 0 || Re[Log[n]] ≥ 0]

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

```
Integrate[ 2^k / k, {k, 1, Log[2, n]}]

ConditionalExpression[
 Log[Log[2]] - Log[Log[n]] - LogIntegral[2] + LogIntegral[n], Im[Log[n]] ≠ 0 || Re[Log[n]] ≥ 0]

tt[n_] := Log[Log[2]] - Log[Log[n]] - LogIntegral[2] + LogIntegral[n]

N[tt[10 000]]

1242.51

N[LogIntegral[10 000]]

1246.14

N[LI1[10 000]]

1394.98


LogIntegral[100]


LAdd[n_, v_] := Sum[ v^k / k, {k, 1, Log[v, n]}]
```

```
DiscretePlot[ { LAdd[n], LogIntegral[n], PP[n]}, {n, 2, 800}]
```




```
Integrate[ 3^k / k, {k, 1, Log[3, n]}]
```

```
ConditionalExpression[
 Log[Log[3]] - Log[Log[n]] - LogIntegral[3] + LogIntegral[n], Im[Log[n]] ≠ 0 || Re[Log[n]] ≥ 0]
```

```
Integrate[ 8^k / k, {k, 1, Log[8, n]}]
```

```
ConditionalExpression[
 Log[Log[8]] - Log[Log[n]] - LogIntegral[8] + LogIntegral[n], Im[Log[n]] ≠ 0 || Re[Log[n]] ≥ 0]
```

```
Integrate[ (1.5)^k / k, {k, 1, Log[(1.5), n]}]
```

```
ConditionalExpression[(-1.02779 + 0. i) - 1. Gamma[0, -1. Log[n]] - 1. Log[-1. Log[n]],
 Im[Log[n]] ≠ 0 || Re[Log[n]] ≥ 0]
```

```
LAdd[n_, k_] := Sum[ k^j / j, {j, 1, Log[k, n]}]
Table[ { n, LAdd[n, 1.00001] - LAdd[2, 1.00001] + LogIntegral[2], N[LogIntegral[n]]},
  {n, 10, 2000, 10}] // TableForm
```

10	6.16558	6.1656
20	9.90527	9.9053
30	13.0226	13.0226
40	15.8395	15.8395
50	18.4687	18.4687
60	20.9654	20.9654
70	23.3618	23.3618
80	25.6785	25.6786
90	27.9299	27.9299
100	30.1262	30.1261
110	32.2751	32.2751
120	34.3828	34.3828
130	36.454	36.4541
140	38.4928	38.4928
150	40.5024	40.5023
160	42.485	42.4852
170	44.4438	44.4438
180	46.3801	46.38
190	48.2958	48.2957
200	50.1922	50.1922
210	52.0709	52.0709
220	53.9329	53.9329
230	55.7792	55.7793
240	57.6109	57.6109
250	59.4286	59.4287
260	61.2332	61.2334
270	63.0254	63.0256
280	64.8059	64.806
290	66.575	66.5752
300	68.3338	68.3336
310	70.082	70.0818
320	71.8199	71.8202
330	73.5493	73.5491
340	75.2691	75.2691
350	76.9805	76.9804
360	78.6835	78.6834
370	80.3785	80.3783
380	82.0658	82.0656
390	83.7452	83.7453
400	85.4179	85.4179
410	87.0833	87.0835

420	88.7423	88.7423
430	90.3946	90.3947
440	92.0407	92.0407
450	93.6803	93.6805
460	95.3143	95.3145
470	96.9427	96.9426
480	98.5649	98.5651
490	100.182	100.182
500	101.794	101.794
510	103.401	103.4
520	105.002	105.002
530	106.598	106.599
540	108.191	108.19
550	109.777	109.777
560	111.36	111.36
570	112.938	112.938
580	114.511	114.512
590	116.081	116.081
600	117.647	117.647
610	119.208	119.208
620	120.765	120.765
630	122.319	122.318
640	123.868	123.868
650	125.414	125.414
660	126.956	126.956
670	128.495	128.494
680	130.029	130.029
690	131.561	131.561
700	133.089	133.089
710	134.613	134.614
720	136.135	136.135
730	137.653	137.654
740	139.169	139.169
750	140.681	140.681
760	142.19	142.19
770	143.696	143.696
780	145.199	145.199
790	146.699	146.699
800	148.197	148.197
810	149.691	149.691
820	151.183	151.183
830	152.672	152.672
840	154.159	154.159
850	155.643	155.642
860	157.124	157.124
870	158.602	158.602
880	160.078	160.079
890	161.552	161.552
900	163.023	163.024
910	164.492	164.492
920	165.959	165.959
930	167.423	167.423
940	168.885	168.885
950	170.344	170.345
960	171.802	171.802
970	173.258	173.257

980	174.71	174.71
990	176.16	176.161
1000	177.609	177.61
1010	179.057	179.056
1020	180.501	180.501
1030	181.943	181.943
1040	183.384	183.384
1050	184.823	184.822
1060	186.258	186.259
1070	187.693	187.693
1080	189.127	189.126
1090	190.556	190.557
1100	191.986	191.986
1110	193.413	193.413
1120	194.837	194.838
1130	196.261	196.261
1140	197.682	197.683
1150	199.103	199.103
1160	200.521	200.521
1170	201.937	201.937
1180	203.352	203.352
1190	204.765	204.765
1200	206.176	206.176
1210	207.586	207.585
1220	208.994	208.993
1230	210.4	210.4
1240	211.805	211.804
1250	213.208	213.208
1260	214.61	214.609
1270	216.009	216.009
1280	217.408	217.408
1290	218.805	218.805
1300	220.2	220.2
1310	221.593	221.594
1320	222.987	222.986
1330	224.378	224.377
1340	225.768	225.767
1350	227.155	227.155
1360	228.541	228.542
1370	229.927	229.927
1380	231.311	231.311
1390	232.693	232.693
1400	234.075	234.074
1410	235.455	235.454
1420	236.832	236.832
1430	238.209	238.209
1440	239.585	239.585
1450	240.96	240.96
1460	242.332	242.333
1470	243.705	243.704
1480	245.075	245.075
1490	246.444	246.444
1500	247.812	247.812
1510	249.18	249.179
1520	250.545	250.545
1530	251.908	251.909

1540	253.272	253.272
1550	254.633	254.634
1560	255.994	255.995
1570	257.353	257.354
1580	258.712	258.712
1590	260.069	260.07
1600	261.425	261.426
1610	262.78	262.78
1620	264.133	264.134
1630	265.487	265.487
1640	266.837	266.838
1650	268.188	268.189
1660	269.537	269.538
1670	270.886	270.886
1680	272.233	272.233
1690	273.58	273.579
1700	274.925	274.924
1710	276.267	276.268
1720	277.61	277.61
1730	278.952	278.952
1740	280.292	280.293
1750	281.632	281.633
1760	282.971	282.971
1770	284.31	284.309
1780	285.645	285.646
1790	286.98	286.981
1800	288.315	288.316
1810	289.648	289.649
1820	290.981	290.982
1830	292.313	292.314
1840	293.644	293.644
1850	294.973	294.974
1860	296.302	296.303
1870	297.632	297.631
1880	298.958	298.958
1890	300.282	300.284
1900	301.608	301.609
1910	302.932	302.933
1920	304.255	304.256
1930	305.579	305.578
1940	306.901	306.9
1950	308.221	308.22
1960	309.539	309.54
1970	310.858	310.858
1980	312.178	312.176
1990	313.492	313.493
2000	314.81	314.809

```
Limit[ Sum[ v^k / k, {k, 1, Log[v, n]}], {v → 1}]
```

$$\left\{\text{Limit}\left[-n\,v\,\text{LerchPhi}\left[v,1,1+\frac{\text{Log}[n]}{\text{Log}[v]}\right]-\text{Log}[1-v],\ v\to 1\right]\right\}$$

```
Sum[ v^k / k, {k, 1, Log[v, n]}]
```

```
LAdd[n_, v_] := Sum[ v^k / k, {k, 1, Log[v, n]}]

LAdd[80, 1.000001] - LAdd[2, 1.000001] + N[LogIntegral[2]]
```

$25.6786 - 2.49006 \times 10^{-10}\, i$

```
N[LogIntegral[80]]
```

25.6786

```
LAdd[ 2, 1.00000001]
```

$19.4658 + 1.01851 \times 10^{-8}\, i$

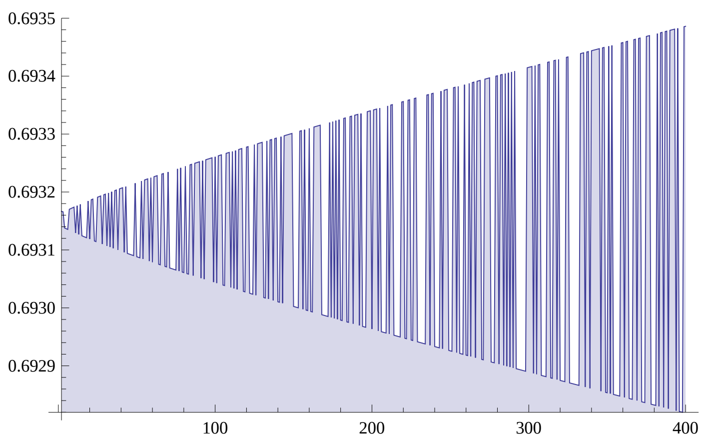
```
N[LogIntegral[2]]
```

1.04516

```
LAdd[ 2, 1 - .000001]
```

0

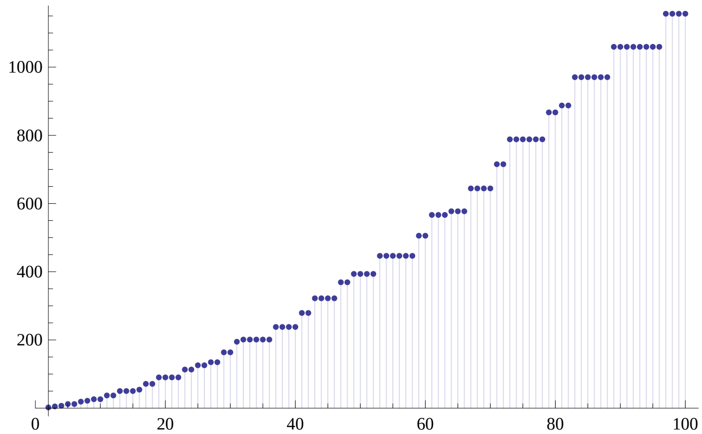
```
LAdd2[n_, v_] := Sum[ (-1)^(k + 1) v^k / k, {k, 1, Log[v, n]}]

DiscretePlot[ { LAdd2[n, 1.00001]}, {n, 2, 400}]
```




```
N[Log[2]]
```

0.693147

```
LAdd[n_, k_] := Sum[ k^j / j, {j, 1, Log[k, n]}]
L2[n_] := Sum[ LAdd[n, Prime[k]], {k, 1, 200}]
DiscretePlot[L2[n], {n, 2, 100}]
```



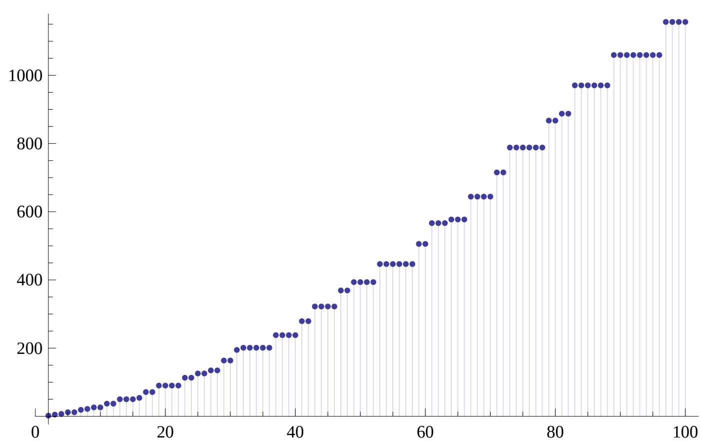

```
PX[n_, k_] := PX[n, k] = Sum[ j (1 / k - PX[Floor[n / j], k + 1]), {j, 2, n}]
PX[100, 1]
```

$$\frac{69389}{60}$$

```
L2[100]
```

$$\frac{69389}{60}$$

```
DiscretePlot[ PX[n, 1], {n, 2, 100}]
```



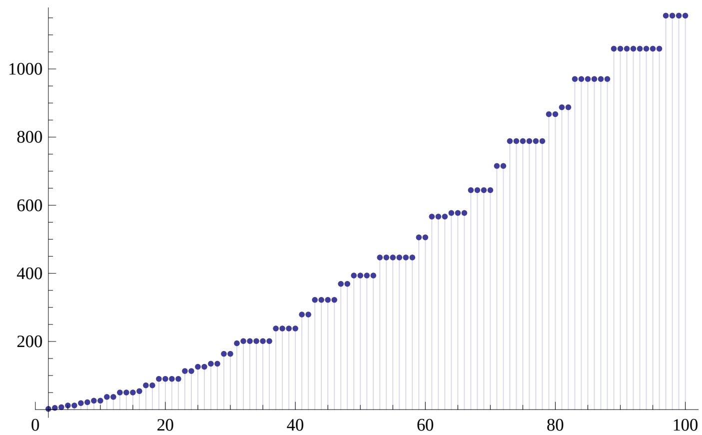

```
Integrate[ k^j / j, {j, 1, Log[k, n]}]
ConditionalExpression[-LogIntegral[k] + LogIntegral[n], Log[n] < Log[k] < 0]
Integrate[ k^j / j, {j, 1, n}]
ConditionalExpression[ExpIntegralEi[n Log[k]] - LogIntegral[k], Re[Log[k]] < 0 && n > 1]
```

Integrate::idiv : Integral of $\frac{k^j}{j}$ does not converge on $\left\{0, \frac{\text{Log}[n]}{\text{Log}[k]}\right\}$. ≫

```
LAddX[n_, k_] := Integrate[ k^j / j, {j, 1, Log[k, n]}]
L2X[n_] := Sum[ LAddX[n, Prime[k]], {k, 1, 200}]
DiscretePlot[L2X[n], {n, 2, 100}]
```

```
$Aborted
```

```
LAddX[n_, k_] := Sum[ k^j / j, {j, 1, Log[k, n]}]
L2X[n_] := Sum[ LAddX[n, Prime[k]], {k, 1, 200}]
DiscretePlot[L2X[n], {n, 2, 100}]
```




```
Integrate[2^j / j, {j, 1, Log[2, 100]}]
```

```
-ExpIntegralEi[Log[2]] + ExpIntegralEi[Log[100]]
```

```
N[-ExpIntegralEi[Log[2]] + ExpIntegralEi[Log[100]]]
```

```
29.081
```

```
N[LogIntegral[100] - LogIntegral[2]]
```

```
29.081
```

```
Soldner := 1.4513692348
lt[n_, k_] := Sum[ k^j / j, {j, Log[k, Soldner], Log[k, n]}]
Table[ { n, lt[n, 1.00001], N[LogIntegral[n]]}, {n, 120, 123}] // TableForm
```

120	34.3828	34.3828
121	34.5915	34.5915
122	34.7998	34.7998
123	35.0077	35.0078

```
Integrate[ k^j / j, {j, 1, Log[k, n]}]
```

```
ConditionalExpression[-LogIntegral[k] + LogIntegral[n], Log[n] < Log[k] < 0]
```

```
-LogIntegral[1.45136923488] + LogIntegral[n]
```

9.07639×10^{-12} + LogIntegral[n]

```
Integrate[ Soldner^j / j, {j, 1, Log[Soldner, 100]}]
```

$30.1261 - 1.15463\times 10^{-14}\, i$

```
N[LogIntegral[100]]
```

30.1261

```
Integrate[ Soldner^j / j, {j, Log[Soldner, 50], Log[Soldner, 100]}]
```

$11.6574 - 5.77316\times 10^{-15}\, i$

```
N[LogIntegral[100] - LogIntegral[50]]
```

11.6574

```
lt2[n_, b_, k_] := Sum[ k^j / j, {j, Log[k, b], Log[k, n]}]
Table[ { n, lt2[n, 2, 1.00001], N[LogIntegral[n] - LogIntegral[2]]}, {n, 120, 123}] //
 TableForm
```

120	33.3377	33.3376
121	33.5464	33.5463
122	33.7547	33.7547
123	33.9626	33.9627

```
N[LogIntegral[30]]
```

13.0226

```
N[ExpIntegralEi[Log[30]]]
```

13.0226

```
Expand[Log[a, E^x]]
```

$$\frac{x}{\mathrm{Log}[a]}$$

$$\frac{x}{\mathrm{Log}[a]}$$

```
Soldner := 1.4513692348
lt3[n_, k_] := Sum[ k^j / j, {j, Log[k, Soldner], n / Log[k]}]
Table[ { n, lt3[n, 1.00001], N[ExpIntegralEi[n]]}, {n, 4, 6, .3}] // TableForm
```

4.	19.631	19.6309
4.3	24.2274	24.2274
4.6	30.0141	30.0141
4.9	37.3325	37.3325
5.2	46.6248	46.6249
5.5	58.4654	58.4655
5.8	73.6005	73.6008

```
ff[x_] := Log[e^x]
```

```
ff[3]
```

3

```
Integrate[ Soldner^j / j, {j, 1, 12 / Log[Soldner]}]
```

$14\,959.5 - 1.99307\times 10^{-11}\, i$

```
N[ExpIntegralEi[12]]
```

14 959.5

```
Integrate[ a^j / j, {j, 1, Log[a, n]}]
```

```
ConditionalExpression[-LogIntegral[a] + LogIntegral[n], Log[n] < Log[a] < 0]
```

```
N[Integrate[ E^j / j, {j, 1, Log[E, 100]}]]
```

```
28.231
```

```
N[LogIntegral[100] - LogIntegral[E]]
```

```
28.231
```

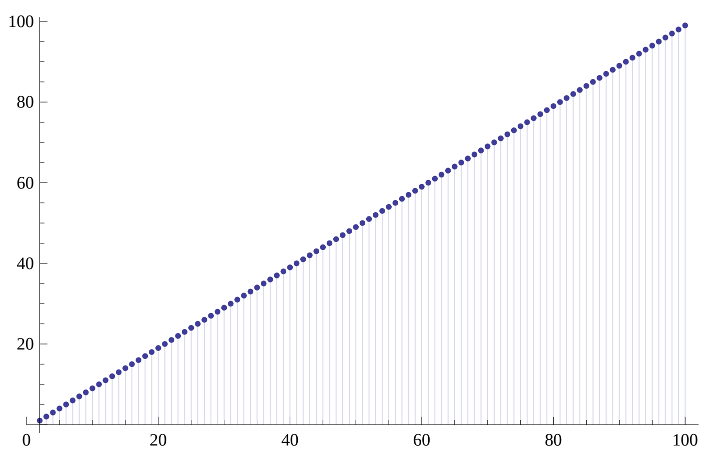
```
N[Integrate[ E^j / j, {j, 1, Log[1000]}]]
N[LogIntegral[1000] - LogIntegral[E]]
```

```
175.715

175.715
```

```
Integrate[ E^j / j, {j, 1, n}]
```

```
ConditionalExpression[
 CoshIntegral[n] - ExpIntegralEi[1] - Log[n] + SinhIntegral[n], Im[n] ≠ 0 || Re[n] ≥ 0]
```

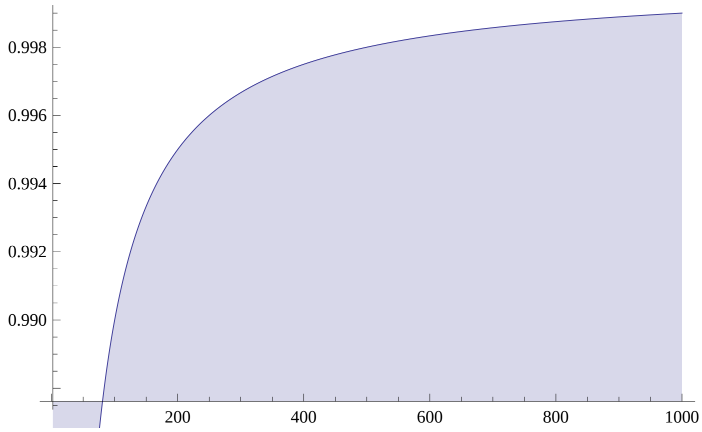
```
LAdd2a[n_, v_] := Sum[ v^k, {k, 1, Log[v, n]}]
DiscretePlot[ { LAdd2a[n, 1 + .000001] * .000001}, {n, 2, 100}]
```




```
LAdd2a[100, 1.0000001]
```

9.9×10^8

```
LAdd2a[n_, v_] := Sum[v^(-k), {k, 1, Log[v, n]}]
DiscretePlot[{LAdd2a[n, 1 + .000001] * .000001}, {n, 2, 1000}]
```




```
LAdd2a[n_, v_] := Sum[v^(-k) / k, {k, 1, Log[v, n]}]
DiscretePlot[{LAdd2a[n, 1 + .00001]}, {n, 2, 100}]
```