It looks like here I was trying to enumerate the relationship between certain basic functions and then their arbitrary complex dirichlet convolutions based on the prime factorizatons of their inputs.

The prototypical example of this would be the divisor function, when it admits complex values, which is to say,

$$d_z(n)=\prod_{p^\alpha | n}(-1)^\alpha\binom{-z}{\alpha}$$

I have since become convinced that it is much more natural to write this with the Pochhammer symbol, which is to say, as

$$d_z(n)=\prod_{p^\alpha | n}\frac{z^{(\alpha)}}{\alpha!}$$

(granting for a second that the Pochammer notation itself is a bit frustrating)

I won't rehash the reasoning here, but the general thrust is covered in my notes about additive convolutions

$$f(n)=1$$
$$f_z(n)=\prod_{p^\alpha | n}(-1)^\alpha\binom{-z}{\alpha}$$

$$f(n)=\mu(n)$$
$$f_z(n)=\prod_{p^\alpha | n}(-1)^\alpha\binom{z}{\alpha}$$

$$f(n)=d_x(n)$$
$$f_z(n)=\prod_{p^\alpha | n}(-1)^\alpha\binom{-x\cdot z}{\alpha}$$

$$f(n)=n$$
$$f_z(n)=n\prod_{p^\alpha | n}(-1)^\alpha\binom{-z}{\alpha}$$

$$f(n)=d_x(n)\cdot n^p$$
$$f_z(n)=n^p\cdot\prod_{p^\alpha | n}(-1)^\alpha\binom{-x\cdot z}{\alpha}$$

This is the liouville lambda function

$$f(n)=\lambda(n)$$
$$f_z(n)=\prod_{p^\alpha | n}\binom{-z}{\alpha}$$

$$f(n)=\mu(n)^2$$

$$f_z(n)=\prod_{p^\alpha | n} \binom{z}{\alpha}$$

This is the prime, not prime power, function

$$f_z(n)=\prod_{p^\alpha | n} \frac{z^\alpha}{\alpha!}$$