FullSimplify@Integrate[j^-s, {j, 0, n}]
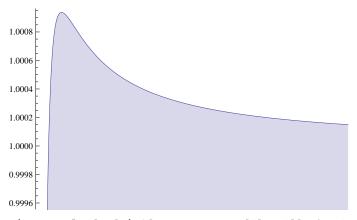
$\text{ConditionalExpression}\left[-\dfrac{n^{1-s}}{-1+s},\ \text{Re}[s]<1\right]$

pic[n_, s_] := Sum[j^-s, {j, 1, n}] $\Big/ \left(-\dfrac{n^{1-s}}{-1+s}\right)$
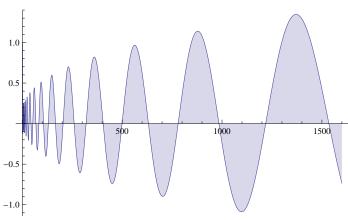
pic2[n_, s_] := Sum[j^-s, {j, 1, n}]

pic3[n_, s_] := $\left(\text{Sum[j^-s, {j, 1, n}]} \Big/ \left(-\dfrac{n^{1-s}}{-1+s}\right) - 1\right)\left(-\dfrac{n^{1-s}}{-1+s}\right)$

DiscretePlot[Re[pic[n, N@ZetaZero[1]]], {n, 1, 1600}]

DiscretePlot[Re[pic2[n, N@ZetaZero[1] + .1]], {n, 1, 1600}]

ple[n_, s_] := Sum[
 j^(-1 / 2) ((2 s Cosh[s Log[n / j]] - Sinh[s Log[n / j]]) / (2 s Cosh[s Log[n]] - Sinh[s Log[n]])),
 {j, 1, n}]

ple2[n_, s_] := Sum[j^(-1 / 2) ((2 s Cosh[s Log[n / j]] - Sinh[s Log[n / j]])), {j, 1, n}]

ple3[n_, s_] :=
 Sum[j^(-1 / 2) (Cos[s Log[j]] + Tan[s Log[n] + ArcCot[2 s]] Sin[s Log[j]]), {j, 1, n}]

N@ple[1 000 000., .2 + 50 I]

$0.186167 + 0.319876\ \mathbb{i}$

N@Zeta[.7 + 50 I]

$0.186429 + 0.323461\ \mathbb{i}$

ple2[1 000 000., -.5 + N@ZetaZero@1]

0. + 0.0141347 $\mathbb{i}$

N@ple3[10 000 000., 50 + .2 I]

0.187653 - 0.324221 $\mathbb{i}$

Integrate[j^-s, {j, 0, n}]
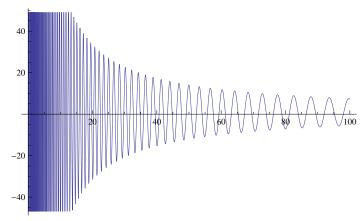
$\text{ConditionalExpression}\left[-\dfrac{n^{1-s}}{-1+s}, \text{Re[s]} < 1\right]$
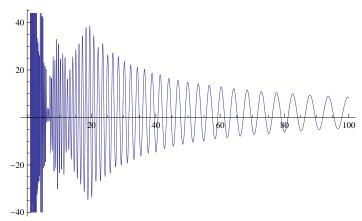
$-\dfrac{n^{1-s}}{-1+s}$ **/. s → 100 I**

$\left(\dfrac{1}{10\,001} + \dfrac{100\,\mathbb{i}}{10\,001}\right) n^{1-100\,\mathbb{i}}$

$\textbf{Plot}\left[1 + \textbf{Re}\left[\textbf{Zeta[100 I]}\right] \middle/ \left(\left(\dfrac{1}{10\,001} + \dfrac{100\,\mathbb{i}}{10\,001}\right) n^{1-100\,\mathbb{i}}\right)\right], \textbf{\{n, 0, 100\}}\right]$
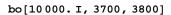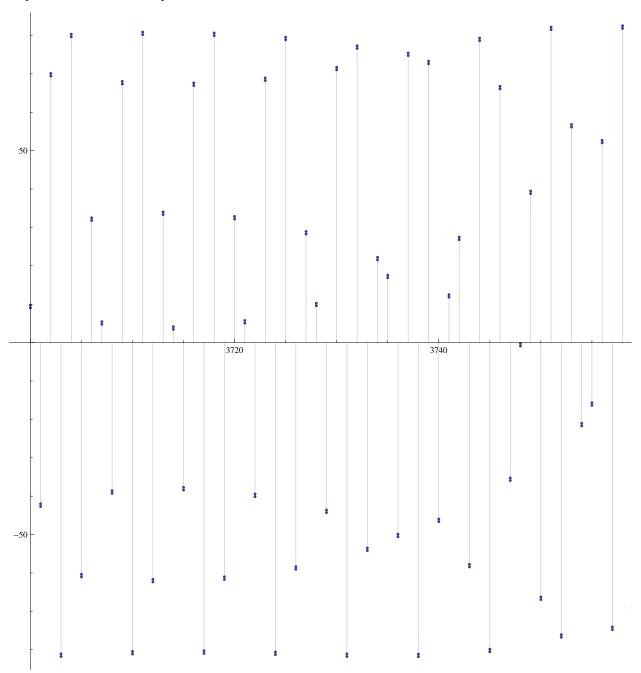
$\textbf{Plot}\left[1 + \textbf{Re}\left[\textbf{Sum[j^- (100 I), \{j, 1, n\}]}\right] \middle/ \left(\left(\dfrac{1}{10\,001} + \dfrac{100\,\mathbb{i}}{10\,001}\right) n^{1-100\,\mathbb{i}}\right)\right], \textbf{\{n, 0, 100\}}\right]$

$\textbf{bo[s\_, t0\_, t\_] := DiscretePlot}\Big[$

$\textbf{Re@}\left\{1 + \textbf{Zeta[s]} \middle/ \left(-\dfrac{n^{1-s}}{-1+s}\right), \textbf{(Sum[j^-s, \{j, 1, Floor[n]\}])} \middle/ \left(-\dfrac{n^{1-s}}{-1+s}\right)\right\}, \textbf{\{n, t0, t\}}\Big]$

bo[10 000. I, 3700, 3800]

$$1 \Big/ \left(-\frac{n^{1-s}}{-1+s} \right) \text{ /. } s \to A + f\, I \text{ /. } A \to .3 \text{ /. } f \to 10. \text{ /. } n \to 100$$

0.336311 + 0.214851 $\mathbb{i}$

$$1 \Big/ \left(-\frac{n^{1-s}}{-1+s} \right) \text{ /. } s \to A + f\, I$$

$$-(-1 + A + \mathbb{i}\, f)\, n^{-1+A+\mathbb{i}\, f} \text{ /. } A \to .3 \text{ /. } f \to 10. \text{ /. } n \to 100$$

0.336311 + 0.214851 $\mathbb{i}$

```
- (-1 + A + i f) n^(-1+A) E^ (f Log[n] I) /. A → .3 /. f → 10. /. n → 100
```

0.336311 + 0.214851 i

```
- (-1 + A + i f) n^(-1+A) (Cos[f Log[n]] + I Sin[f Log[n]]) /. A → .3 /. f → 10. /. n → 100
```

0.336311 + 0.214851 i

```
n^(-1+A) ((1 - A - i f) Cos[f Log[n]] + I (1 - A - i f) Sin[f Log[n]]) /. A → .3 /. f → 10. /. n → 100
```

0.336311 + 0.214851 i

```
n^(-1+A) ((1 - A) Cos[f Log[n]] + (-i f) Cos[f Log[n]] +
      I (1 - A) Sin[f Log[n]] + I (-i f) Sin[f Log[n]]) /. A → .3 /. f → 10. /. n → 100
```

0.336311 + 0.214851 i

```
n^(-1+A) ((1 - A) Cos[f Log[n]] + (-i f) Cos[f Log[n]] + I (1 - A) Sin[f Log[n]] + f Sin[f Log[n]]) /.
    A → .3 /. f → 10. /. n → 100
```

0.336311 + 0.214851 i

```
n^(-1+A) ((1 - A) Cos[f Log[n]] + f Sin[f Log[n]] + I ((- f) Cos[f Log[n]] + (1 - A) Sin[f Log[n]])) /.
    A → .3 /. f → 10. /. n → 100
```

0.336311 + 0.214851 i

```
n^(-1+A) ((1 - A) Cos[f Log[n]] + f Sin[f Log[n]]) +
     I n^(-1+A) ((1 - A) Sin[f Log[n]] - f Cos[f Log[n]]) /. A → .3 /. f → 10. /. n → 100
```

0.336311 + 0.214851 i

```
n^(-1+A) ((1 - A) Cos[f Log[n]] + f Sin[f Log[n]]) +
     I n^(-1+A) ((1 - A) Sin[f Log[n]] - f Cos[f Log[n]]) /. A → 0 /. f → 100.
```

$$\frac{i \,(-100.\, Cos[100.\, Log[n]] + Sin[100.\, Log[n]])}{n} + \frac{Cos[100.\, Log[n]] + 100.\, Sin[100.\, Log[n]]}{n}$$

```
Pow[a_, b_] := a^b
```

```
Pow[n, -1 + A] ((1 - A) Cos[f Log[n]] + f Sin[f Log[n]]) +
     I Pow[n, -1 + A] ((1 - A) Sin[f Log[n]] - f Cos[f Log[n]]) /. A → 0 /. f → 100.
```

$$\frac{i \,(-100.\, Cos[100.\, Log[n]] + Sin[100.\, Log[n]])}{n} + \frac{Cos[100.\, Log[n]] + 100.\, Sin[100.\, Log[n]]}{n}$$

```
CForm[Pow[n, -1 + A] ((1 - A) Sin[f Log[n]] - f Cos[f Log[n]])]
```

Power(n,-1 + A)*(-(f*Cos(f*Log(n))) + (1 - A)*Sin(f*Log(n)))

```
CForm[FullSimplify[ComplexExpand[Re[- (-1 + A + i f) n^(-1+A+i f)]], {n ∈ Integers, n > 0}]]
```

Power(n,-1 + A)*(-((-1 + A)*Cos(f*Log(n))) + f*Sin(f*Log(n)))

```
CForm[FullSimplify[ComplexExpand[Im[- (-1 + A + i f) n^(-1+A+i f)]], {n ∈ Integers, n > 0}]]
```

Power(n,-1 + A)*(-(f*Cos(f*Log(n))) - (-1 + A)*Sin(f*Log(n)))

```
CForm[FullSimplify[
   ComplexExpand[Re[(zr + zi I) / Integrate[j^-(A + f I), {j, 0, n}]]], {n ∈ Integers, n > 0}]]
```

ConditionalExpression(Power(n,-1 + A)*((f*zi + zr - A*zr)*Cos(f*Log(n)) + ((-1 + A)*zi + f*

```
FullSimplify[
 ComplexExpand[Re[(zr + zi I) / Integrate[j^- (A + f I), {j, 0, n}]]], {n ∈ Integers, n > 0}]
```

$\text{ConditionalExpression}\left[n^{-1+A} \left((f\,zi + zr - A\,zr) \cos[f\,\log[n]] + ((-1 + A)\,zi + f\,zr) \sin[f\,\log[n]]\right),\right.$
$\left.\text{Re}[A] < 1 + \text{Im}[f]\right] /. A \to .3 /. f \to 10. /.$
$n \to 100 /. zr \to \text{Re@Zeta}[100\,I] /. zi \to \text{Im@Zeta}[100\,I]$

2.15286

```
Zeta[100 I] / (0.3363108343676824` + 0.21485097066881187` i)
```

14.0065 − 8.40895 i

```
(zr + zi I) / Integrate[j^- (A + f I), {j, 0, n}] /. A → .3 /. f → 10. /. n → 100 /.
  zr → Re@Zeta[100 I] /. zi → Im@Zeta[100 I]
```

2.15286 + 1.4612 i

```
Zeta[100 I] / Integrate[j^- (A + f I), {j, 0, n}] /. A → .3 /. f → 10. /. n → 100
```

2.15286 + 1.4612 i

```
1 / Integrate[j^- (A + f I), {j, 0, n}] /. A → .3 /. f → 10. /. n → 100
```

0.336311 + 0.214851 i

```
FullSimplify[ComplexExpand[Re[n^- (A + f I)]], {n ∈ Integers, n > 0}]
```

$n^{-A} \cos[f\,\log[n]]$

```
N[Zeta[-10 I]]
```

1.75647 + 0.101512 i

```
N[Zeta[-1000 I]]
```

−8.46309098852087 − 8.34334485626739 i

```
N[Zeta[-10 000 I]]
```

14.3062 − 27.183 i

```
N[Zeta[-2 I]]
```

0.314726 + 0.23168 i

```
N[Zeta[-0 I]]
```

−0.5

```
CForm[FullSimplify[
  ComplexExpand[Re[(zr + zi I) / Integrate[j^- (A + f I), {j, 0, n}]]], {n ∈ Integers, n > 0}]]
```

ConditionalExpression(Power(n,-1 + A)*((f*zi + zr - A*zr)*Cos(f*Log(n)) + ((-1 + A)*zi + f*

```
CForm[FullSimplify[
  ComplexExpand[Im[(zr + zi I) / Integrate[j^- (A + f I), {j, 0, n}]]], {n ∈ Integers, n > 0}]]
```

ConditionalExpression(Power(n,-1 + A)*((zi - A*zi - f*zr)*Cos(f*Log(n)) + (f*zi + zr - A*zr

```
FullSimplify[E^ (t I) - E^ (-t I)]
```

2 i Sin[t]
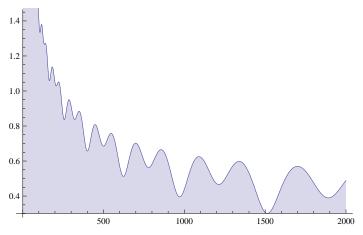
```
pa[n_, t_] := Sum[(-1) ^ (j + 1) / (j ^ (1 / 2)) 2 Sin[t Log[j]], {j, 1, n}]
paa[n_, t_] := Sum[1 / (j ^ (1 / 2)) × 2 Sin[t Log[j]], {j, 1, n}] -
   2 Sum[1 / ((2 j) ^ (1 / 2)) × 2 Sin[t Log[2 j]], {j, 1, Floor[n / 2]}]
paa1[n_, t_] := Sum[1 / (j ^ (1 / 2)) × 2 Sin[t Log[j]], {j, 1, n}]
paa2[n_, t_] := 2 Sum[1 / ((2 j) ^ (1 / 2)) × 2 Sin[t Log[2 j]], {j, 1, Floor[n / 2]}]
pla0[t_, n2_] := DiscretePlot[Re@pa[n, t], {n, 1, n2}]
pla[t_, n2_] := DiscretePlot[{paa1[n, t], paa2[n, t]}, {n, 1, n2}]

pla0[N@Im@ZetaZero@100, 400]
```



```
za[n_, x_] := Sum[(1 / j) ^ (1 / 2) (2 x Cos[x Log[j / n]] + Sin[x Log[j / n]]), {j, 1, n}]
za1[n_, x_] := Sum[1 / (j ^ (1 / 2)) (2 x Cos[x Log[j / n]]), {j, 1, n}]
za2[n_, x_] := Sum[1 / (j ^ (1 / 2)) (Sin[x Log[j / n]]), {j, 1, n}]
zla0[t_, n2_] := DiscretePlot[{Re@za[n, t]}, {n, 1, n2}]
zla0a[t_, fv_, n2_] := DiscretePlot[Re@za[n, t], {n, fv, n2}]
zla0aa[t_, fv_, n2_] := DiscretePlot[Abs@za[n, t], {n, fv, n2}]
zla0a2[t_, fv_, n2_] :=
 DiscretePlot[Re@(za[n, t] / (2 t Cos[t Log[n]] - Sin[t Log[n]])), {n, fv, n2}]
zla0a2i[t_, fv_, n2_] := DiscretePlot[
   Im@(za[n, t] / (2 t Cos[t Log[n]] - Sin[t Log[n]])), {n, fv, n2}]
zla0a2a[t_, fv_, n2_] := DiscretePlot[
   Abs@(za[n, t] / (2 t Cos[t Log[n]] - Sin[t Log[n]])), {n, fv, n2}]
zla[t_, n2_] := DiscretePlot[{Re@za1[n, t], Re@za2[n, t]}, {n, 1, n2}]
zla1[t_, n2_] := DiscretePlot[{Re@za1[n, t]}, {n, 1, n2}]

zaj[n_, x_, j_] := 1 / (j ^ (1 / 2)) (2 x Cos[x Log[j / n]] + Sin[x Log[j / n]])
za1j[n_, x_, j_] := 1 / (j ^ (1 / 2)) (2 x Cos[x Log[j / n]])
za2j[n_, x_, j_] := 1 / (j ^ (1 / 2)) (Sin[x Log[j / n]])
zla0j[t_, n2_] := DiscretePlot[{Re@zaj[n2, t, n]}, {n, 1, n2}]
zlaj[t_, n2_] := DiscretePlot[{Re@za1j[n2, t, n], Re@za2j[n2, t, n]}, {n, 1, n2}]

zak[n_, x_, k_] := Sum[1 / (j ^ (1 / 2)) (2 x Cos[x Log[j / n]] + Sin[x Log[j / n]]), {j, 1, k}]
za1k[n_, x_, k_] := Sum[1 / (j ^ (1 / 2)) (2 x Cos[x Log[j / n]]), {j, 1, k}]
za2k[n_, x_, k_] := Sum[1 / (j ^ (1 / 2)) (Sin[x Log[j / n]]), {j, 1, k}]
zla0k[t_, n2_] := DiscretePlot[{Re@zak[n2, t, n]}, {n, 1, n2}]
zlak[t_, n2_] := DiscretePlot[{Re@za1k[n2, t, n], -Re@za2k[n2, t, n]}, {n, 1, n2}]
```
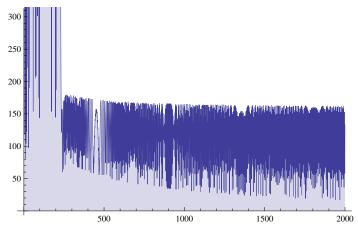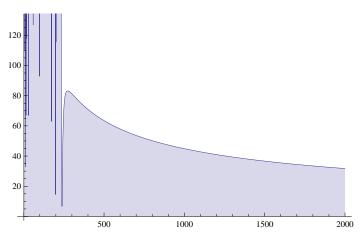
zla0aa[N@Im@ZetaZero@1 + .02 I, 1, 2000]

zla0aa[N@Im@ZetaZero@1000 + .02 I, 1, 2000]
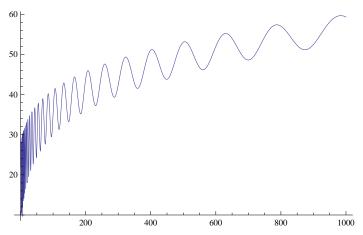

zla0aa[N@Im@ZetaZero@1000, 1, 2000]

Zeta[ZetaZero@1 + .2]

0.144977 + 0.0207464 i

zt[n_, t_] := 2 t Cos[t Log[n]] - Sin[t Log[n]]
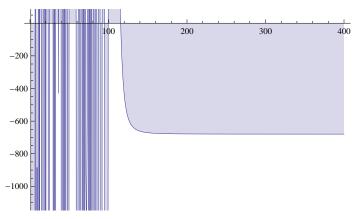
```
Plot[Abs@zt[n, N@Im@ZetaZero@1 + .2 I], {n, 1, 1000}]
```



```
rra[n_, t_] :=
 Sum[(n / j) ^ (1 / 2) ((1 / 2 - t I) (n / j) ^ (t I) - (1 / 2 + t I) (n / j) ^ (-t I)), {j, 1, n}]
rra0[t_, n2_] := DiscretePlot[Im@rra[n, t], {n, 1, n2}]
```

```
rra0[N@Im@ZetaZero@400, 400]
```
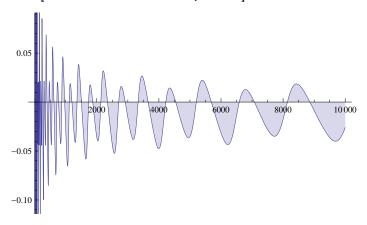


```
N@ZetaZero@400
```

```
0.5 + 679.742 i
```

```
ba[n_, x_] :=
 Sum[j ^ (-1 / 2) (Cos[x Log[j]] + Tan[x Log[n] + ArcCot[2 x]] Sin[x Log[j]]), {j, 1, n}]
Clear[dcos, dsin]
dcos[0, x_] := 0
dcos[j_, x_] := dcos[j, x] = j ^ (-1 / 2) (Cos[x Log[j]]) + dcos[j - 1, x]
dsin[0, x_] := 0
dsin[j_, x_] := dsin[j, x] = j ^ (-1 / 2) (Sin[x Log[j]]) + dsin[j - 1, x]
baa[n_, x_] := Sum[j ^ (-1 / 2) (Cos[x Log[j]]), {j, 1, n}] +
   Tan[x Log[n] + ArcCot[2 x]] Sum[j ^ (-1 / 2) (Sin[x Log[j]]), {j, 1, n}]
bab[n_, x_] := dcos[n, x] + Tan[x Log[n] + ArcCot[2 x]] dsin[n, x]
dbaa[t_, n2_] := DiscretePlot[Im@baa[n, t], {n, 1, n2}]
dbab[t_, n2_] := DiscretePlot[Im@bab[n, t], {n, 1, n2}]
```

dbab[N@Im@ZetaZero@1 + .1 I, 10 000]

baa[100 000, .4 I + N@Im@ZetaZero@1]

0.268843 − 0.0344886 𝕚
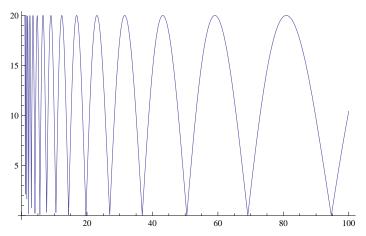
Zeta[N@ZetaZero@1 + .1]

0.0753346 + 0.0113729 𝕚

Sum[j^(-1/2) (Cos[x Log[j]]), {j, 1, n}] +
 Tan[x Log[n] + ArcCot[2 x]] Sum[j^(-1/2) (Sin[x Log[j]]), {j, 1, n}]

$$\sum_{j=1}^{n} \frac{\text{Cos}[x \, \text{Log}[j]]}{\sqrt{j}} + \left(\sum_{j=1}^{n} \frac{\text{Sin}[x \, \text{Log}[j]]}{\sqrt{j}} \right) \text{Tan}[\text{ArcCot}[2 \, x] + x \, \text{Log}[n]]$$

zt[n_, t_] := 2 t Cos[t Log[n]] − Sin[t Log[n]]
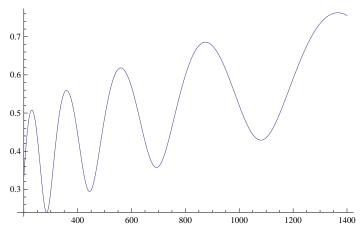
Plot[Abs@zt[n, 10], {n, 1, 100}]

Limit[HarmonicNumber[n, 1/2 + 10 I], n → Infinity]

ComplexInfinity

Plot[Abs[HarmonicNumber[n, ZetaZero@1 + .2]], {n, 200, 1400}]

N@ZetaZero[650]

0.5 + 1001.35 i