```
Expand@Sum[1/(k!)(n-1)^k, \{k, 1, Infinity\}]
 Sum[BernoulliB[k] / (k!) (n-1) ^k, {k, 0, Infinity}]
-1 + n
Expand@Sum[1/(k!)(n-1)^(k), \{k, 1, Infinity\}]
 Sum[BernoulliB[k] / (k!) (n-1)^(k+a), \{k, 0, Infinity\}]
(-1 + n)^{1+a}
FullSimplify@Sum[1/(k!)(n-1)^(k), \{k, 1, Infinity\}]
-1 + e^{-1+n}
Expand@Sum[BernoulliB[k] / (k!) (n-1)^{(k+a)}, \{k, 0, Infinity\}]
(-1+n)^{1+a}
 -1 + e^{-1+n}
gg[n_, rr_] := (-1) ^ (rr) Gamma[rr, 0, -Log[n]] / Gamma[rr]
aa[n_{, k_{]}} := (-1)^{(k+1)} / (k-1)! Integrate[t^{(k-1)}E^{-t}, \{t, -Log[n], 0\}]
N[Sum[1/(k!)gg[20, k], {k, 1, 100}] Sum[BernoulliB[k]/(k!)gg[20, k], {k, 0, 120}]]
-254.939
Chop@N@gg[10, 10]
0.00952161
N[Sum[1/(k!)gg[20, k], \{k, 1, 100\}]]
49.535
Sum[1/(k!) aa[n, k], {k, 1, Infinity}]
$Aborted
N[aa[30, 4]]
96.2415
Chop@N[gg[30, 4]]
96.2415
Integrate [Sum[(-1)^{(k+1)}/(k) ((-1)^{(k+1)}/(k-1)! t^{(k-1)} E^{-t)}, \{k, 1, Infinity\}],
 {t, -Log[n], 0}]
\texttt{ConditionalExpression[-EulerGamma+ExpIntegralEi[Log[n]]-Log[Log[n]], Log[n]>0]}
Integrate[Sum[BernoulliB[k] / (k!) ((-1)^{(k+1)} / (k-1)! t^{(k-1)} E^{-t}),\\
  {k, 0, Infinity}], {t, -Log[n], 0}]
\int_{-Log[n]}^{0} \left( \sum_{k=0}^{\infty} \frac{(-1)^{1+k} e^{-t} t^{-1+k} BernoulliB[k]}{(-1+k) ! k !} \right) dt
aa[n_{k-1}, k_{k-1}] := (-1)^{(k+1)}/(k-1)! Integrate[t^(k-1)E^-t, {t, -Log[n], 0}]
ab[n_{-}, k_{-}] := (-1)^{(k+1)}/(k-1)! Integrate [(-t)^{(k-1)}E^{t}, \{t, 0, Log[n]\}]
ac[n_{-}, k_{-}] := 1 / (k-1) ! Integrate[ t^(k-1) E^t, {t, 0, Log[n]}]
Table[N@aa[79, k], {k, 1, 5}]
{78., 267.186, 486.951, 611.437, 588.4}
```

```
Table[N@ac[79, k], {k, 1, 5}]
{78., 267.186, 486.951, 611.437, 588.4}
\label{eq:simplify} \texttt{Simplify[-(-1)^k(t)^(k-1)]/.} \ \{t \rightarrow 3.3, \, k \rightarrow 3\}
10.89
Integrate [Sum[Binomial[z, k]] / (k-1)! t^(k-1) E^t, {k, 0, Infinity}],
 \{t, 0, (1-s) Log[n]\}
ConditionalExpression[-1 + LaguerreL[-z, Log[n] - s Log[n]],
 -1 \le \text{Re}[\text{Log}[n] - s \text{Log}[n]] \le 1 \mid \mid (-1 + s) \text{Log}[n] \notin \text{Reals}]
Integrate[Sum[ (-1)^{(k+1)}/k 1/(k-1)! t^{(k-1)} E^t, \{k, 1, Infinity\}], \{t, 0, Log[n]\}]
-EulerGamma + ExpIntegralEi[Log[n]] - Log[Log[n]]
N@Sum[Binomial[3, k] 1 / (3-1) k Gamma[k, 0, (3-1) Log[100]] / Gamma[k], {k, 1, 40}] + 1
3.37343
N@LaguerreL[-3, -1, (3-1) Log[100.]]
-2.2426 \times 10^6
Integrate [ E^{(t(1-0))} z Hypergeometric [1-z, 2, t], \{t, -Log[n], 0\}] /. \{n \rightarrow 100, z \rightarrow 3\}
\frac{1}{200} (1386 - 8 Log[100] - Log[100]<sup>2</sup>)
N@ = \frac{1}{200} (1386 - 8 \log[100] - \log[100]^2)
6.63976
N@LaguerreL[-3, Log[100.]]
2081.41
D[LaguerreL[-z, (1-s) Log[100]], s]
DiscretePlot[N[D[LaguerreL[-1-z, 1, (1-s) Log[n]] Log[n], z] /. \{s \to 0, z \to 0\}], \{n, 1, 100\}]
 -20
 -40
 -60
 -80
FullSimplify[Integrate[ (x) ^-s, {x, 1, n}]]
ConditionalExpression \left[\frac{n^{-s} (-n+n^s)}{-1+s}, \text{Re}[n] \ge 0 \mid \mid n \notin \text{Reals}\right]
```

```
Gamma[1, 0, (s-1) Log[n]] / Gamma[1]
1 - n^{1-s}
Sum[(-1)^k/(k!) Binomial[-z,k] (Log[x])^k, \{k, 0, Infinity\}]
Hypergeometric1F1[z, 1, Log[x]]
Hypergeometric1F1[3, 1, Log[100.]]
 2081.41
LaguerreL[-3, Log[100.]]
Table [Expand [ (Log[x] + Log[y])^k], {k, 1, 6}] // TableForm
Log[x] + Log[y]
Log[x]^2 + 2 Log[x] Log[y] + Log[y]^2
Log[x]^3 + 3 Log[x]^2 Log[y] + 3 Log[x] Log[y]^2 + Log[y]^3
Log[x]^4 + 4 Log[x]^3 Log[y] + 6 Log[x]^2 Log[y]^2 + 4 Log[x] Log[y]^3 + Log[y]^4
\log[x]^{5} + 5\log[x]^{4}\log[y] + 10\log[x]^{3}\log[y]^{2} + 10\log[x]^{2}\log[y]^{3} + 5\log[x]\log[y]^{4} + \log[y]^{5}
\log[x]^{6} + 6 \log[x]^{5} \log[y] + 15 \log[x]^{4} \log[y]^{2} + 20 \log[x]^{3} \log[y]^{3} + 15 \log[x]^{2} \log[y]^{4} + 6 \log[x] \log[x]^{2} \log[y]^{4} + 6 \log[x]^{2} \log[x]^{2} + 20 \log[x]
Grid@Table[(-1)^k / (k!) bins[-z, k] Log[x]^j Log[y]^(k-j), \{k, 0, 5\}, \{j, 0, k\}]
    bins[-z, 0]
  -bins[-z, 1] -bins[-z, 1]
    Log[y]
                                            Log[x]
  \frac{1}{2} bins [-z, 2]
                                          \frac{1}{2} bins [-z, 2]
                                                                                     \frac{1}{2} bins [-z, 2]
                                            Log[x] Log[y] Log[x]^2
                                                                                      -\frac{1}{6}
                                               6
    bins[-z, 3]
                                             bins[-z, 3]
                                                                                     bins[-z, 3]
                                                                                                                                 bins[-z, 3]
     Log[y]^3
                                                                                     Log[x]^2
                                                                                                                                 Log[x]^3
                                              Log[x]
                                              Log[y]^2
                                                                                     Log[y]
 \frac{1}{24} bins [-z, 4]
                                         \frac{1}{24} bins[-z, 4] \frac{1}{24} bins[-z, 4] \frac{1}{24} bins[-z, 4]
   Log[y]4
                                                                                     Log[x]^2
                                                                                                                               Log[x]^3
                                                                                                                                                                         Log[x]^4
                                             Log[x]
                                                                                     Log[y]^2
                                             Log[y]^3
                                                                                                                               Log[y]
  -\frac{1}{120}
                                                                                     -\frac{1}{120}
                                            -\frac{1}{120}
                                                                                                                               -\frac{1}{120}
                                                                                                                                                                         -\frac{1}{120}
     bins[-z, 5]
                                            bins[-z, 5]
                                                                                    bins[-z, 5] bins[-z, 5]
                                                                                                                                                                     bins[-z, 5]
                                                                                                                                                                                                                 bins[-z, 5]
     Log[y]^5
                                                                                     Log[x]^2
                                                                                                                               Log[x]^3
                                                                                                                                                                         Log[x]^4
                                              Log[x]
                                                                                                                                                                                                                     Log[x]^5
                                              Log[y]^4
                                                                                     Log[y]^3
                                                                                                                                Log[y]^2
                                                                                                                                                                           Log[y]
Sum[(-1)^k/(k!) 2^k Binomial[-z,k](Log[x])^k, \{k, 0, Infinity\}]
Hypergeometric1F1[z, 1, 2 Log[x]]
 Sum[ (Log[n]) ^ (2k) / ((2k)! (2k)), {k, 1, Infinity}]
 -EulerGamma + CoshIntegral[Log[n]] - Log[Log[n]]
 (LogIntegral[33.] - Log[Log[33.]] - EulerGamma) - 2 SinhIntegral[Log[33.]]
```

-1.83598

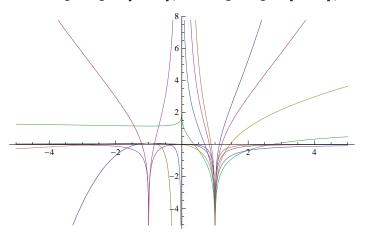
```
Re[(LogIntegral[1 / 33.] - Log[Log[1 / 33.]] - EulerGamma)]
-1.83598
\label{eq:logIntegral} $$N@LogIntegral[1/(33Log[33])] - Log[Log[1/(33Log[33])]] - EulerGamma $$N@LogIntegral[1/(33Log[33])] - EulerGamma $$N@LogIntegral[1/(33Log[33])] - Log[Log[1/(33Log[33])]] - Log[Log[1/(33Log[33])]] - EulerGamma $$N@LogIntegral[1/(33Log[33])] - Log[Log[1/(33Log[33])]] - EulerGamma $$N@LogIntegral[1/(33Log[33])] - Log[Log[1/(33Log[33])]] - EulerGamma $$N@LogIntegral[1/(33Log[33])] - Log[Log[1/(33Log[33])]] - EulerGamma $$N@LogIntegral[1/(33Log[33])]] - EulerGamma $$N@LogIntegral[1/(33Log[33])] - EulerGamma $$N@LogIntegral[1/(33Log[33])]] - EulerGamma $$N@LogIntegral[1/(33Log[33])] - EulerG
-2.13654 - 3.14159 i
LogIntegral[33.] - Log[Log[33.]] - EulerGamma
12.0634
FullSimplify[ (1/Log[n]) (1-1/n)]
n Log[n]
Log[n-1] - Log[n] - Log[Log[n]] /. n \rightarrow 12
Log[11] - Log[12] - Log[Log[12]]
ss[n_{x}] := Sum[(x^k-1)/k, \{k, 1, Log[x, n]\}]
ss[100, 1.00001] +ss2[100, 1.00001]
1243.34
LogIntegral[100.] - Log@Log@100. - EulerGamma
28.0217
ss[100, 1.00001]
ss2[100, 1.00001]
28.0218
1215.32
LogIntegral[10000.] - Log@Log@10000. - EulerGamma
1243.34
FullSimplify[x^k + (x+c)^k]
x^k + (c + x)^k
ss2[100, 1.001]
1214.86
 \texttt{D[Sum[Binomial[z,k]1/(s-1)^k Gamma[k,0,(s-1) Log[n]]/Gamma[k],\{k,1,Infinity\}],n]} 
n^{-s} z Hypergeometric1F1[1 - z, 2, -Log[n]] /. s \rightarrow 0
Integrate [ z Hypergeometric1F1[1-z, 2, -Log[n]], {n, 0, x}]
{\tt ConditionalExpression[LaguerreL[-z,Log[x]],Re[z]>0]}
Integrate [n z \ Hypergeometric1F1[1-z, 2, -Log[n]], \{n, 0, x\}]
 n z Hypergeometric1F1[1-z, 2, -Log[n]] dn
 \text{Limit}[(a-1) ^s (-1) ^s \text{Sum}[a^kk^* (s-1), \{k, 1, Log[a, n]\}] /. \{n \rightarrow 100, s \rightarrow 7/2\}, a \rightarrow 1]
```

```
N\left[-i\left(-1+a\right)^{7/2}\left(-100\text{ a LerchPhi}\left[a,-\frac{5}{2},1+\frac{\text{Log}\left[100\right]}{\text{Log}\left[a\right]}\right]+\text{PolyLog}\left[-\frac{5}{2},a\right]\right)\text{ /. }a\rightarrow1.000001\right]
2.60253 \times 10^{-10} - 2782.57 i
N[6-6n+6nLog[n]-3nLog[n]^2+nLog[n]^3/.n \rightarrow 100]
5573.28
Chop@Gamma[3.5, 0, -Log[100.]]
 0. - 2782.56 i
N@Sum[Re[n^k/(k!k)]/. {n \rightarrow Log[12000], z \rightarrow 5.5}, {k, 1, 25}]
 1458.28
gg[n_, k_] :=
     \textbf{If} \left[ \text{ $k = 0, Limit} \left[ \text{ $Gamma[kk, 0, -Log[n]] / $Gamma[kk], kk $\rightarrow k$} \right], \text{ $Gamma[k, 0, -Log[n]] / $Gamma[k]} \right] 
N@Sum[Re[(-1)^k(-1)^k(+1)/kgg[n,k]/.\{n \rightarrow 12000, z \rightarrow 5\}], {k, 1, 40}]
1458.28
FullSimplify[D[(-1)^k1/(k!) Binomial[-z,k], z] n^k]
 \frac{1}{-} \; (-1)^k \; n^k \; \texttt{Binomial}[\, -z \,, \, k] \; \; (\texttt{HarmonicNumber}[\, -k \, -z \,] \; - \; \texttt{HarmonicNumber}[\, -z \,] \; )
Limit[((-1)^k \text{k Gamma}[kk, 0, -\text{Log}[100]] / \text{Gamma}[kk] - 1) / \text{kk, kk} \rightarrow 0]
 i\pi - Gamma[0, -Log[100]]
pp[n_, z_] :=
     (((-1) \cdot z \cdot Gamma[z, 0, -Log[n]] / Gamma[z]) - ((-1) \cdot -z \cdot Gamma[-z, 0, -Log[n]] / Gamma[-z])) / (((-1) \cdot z \cdot Gamma[z, 0, -Log[n]] / Gamma[-z])) / (((-1) \cdot z \cdot Gamma[z, 0, -Log[n]]) / ((-1) \cdot z \cdot Gamma[z, 0
         (2z)
pp2[n_{,z_{|}} := (((-1)^z Gamma[z, 0, -Log[n]]/Gamma[z]) - 1)/z
pp[100, .000001]
30.1261 + 6.28319 i
D[ LogIntegral[n] - Log@Log@n - EulerGamma, n]
 Log[n] n Log[n]
N[D[LaguerreL[-z, Log[n]], n] /. \{n \rightarrow 100, z \rightarrow 3\}]
27.4193
 -N@Sum[(-1)^iBinomial[-z-1+1, -z-1-i]Log[n]^i/i!, {i, 0, -z-1}]/n/.
    \{z \rightarrow 3, n \rightarrow 100\}
 27.4193
D[ LaguerreL[-z, Log[n]], n]
      LaguerreL[-1-z, 1, Log[n]]
```

```
Sum[(-1)^iBinomial[-z-1+1, -z-1-i]Log[n]^i/i!, {i, 0, -z-1}]/n
 z Hypergeometric1F1[1 + z, 2, Log[n]]
D[(-1) ^z, z]
i (-1) z π
FullSimplify@D[(-1)^z Gamma[z, 0, -Log[n]]/Gamma[z], z]
$Aborted
-N@Gamma[0, -Log[100]]
30.1261 + 3.14159 i
D[ LogIntegral[n] - Log@Log@n - EulerGamma, n]
Log[n] nLog[n]
\label{eq:decomposition} D[LogIntegral[n] - Log@Log@n - EulerGamma, n] /. n \rightarrow 20
    19
20 Log[20]
FullSimplify@(1/Log[n])(1-1/n)/.n \rightarrow 20
20 Log[20]
Limit[ (1/Log[x]) (1-1/x), x \rightarrow 1]
\texttt{Limit}[1/\texttt{Log}[x], x \rightarrow 1]
Expand[(1/Log[x])(1-1/x)]
         1
Log[x] x Log[x]
D[1/Log[x], x]
   1
 x Log[x]^2
D[1-1/x,x]
1
LogIntegral[1]
Limit[LogIntegral[1+x]/x, x \rightarrow 0]
Integrate[(x-1)/(Log[x]),x]
ExpIntegralEi[2 Log[x]] - LogIntegral[x]
```

```
N@ExpIntegralEi[2Log[x]] - LogIntegral[x] /. x \rightarrow 10
23.9605
N@LogIntegral[100] - LogIntegral[10]
23.9605
Integrate [(1-1/x)(x^{(-1)})/(Log[x]), x]
-ExpIntegralEi[-Log[x]] + Log[Log[x]]
N@ExpIntegralEi[3 Log[x]] /. x \rightarrow 30
2984.42
N@Log@Log@30 - N@LogIntegral[30^-1]
1.23201
N@Integrate[ (1-1/x) (x^z/Log[x]), x]
-1. ExpIntegralEi[zLog[x]] + ExpIntegralEi[(1. + z) Log[x]]
Integrate [x^z / Log[x], x]
ExpIntegralEi[(1 + z) Log[x]]
Integrate [1-1/x, x]
x - Log[x]
Expand[ (1-1/x)(x^2/Log[x]), x]
   x^{-1+z}
Limit[LogIntegral[x^5] - Log[Log[x]], x \rightarrow 1]
EulerGamma + Log[5]
\label{eq:limit} \mbox{Limit[LogIntegral[x^2] - LogIntegral[x], $x \to 1$]}
Log[2]
Limit[LogIntegral[x^2] - LogIntegral[x^4], x \to 1]
\label{limit} \mbox{Limit[LogIntegral[x^5]-LogIntegral[x^-2],x} \rightarrow 1]
Log\left[\frac{5}{2}\right]
Limit[LogIntegral[x^5] - LogIntegral[x^3], x \rightarrow 1]
Log\left[\frac{5}{2}\right]
\label{limit} \mbox{Limit[LogIntegral[x^6]-LogIntegral[x^(1/10)],x \rightarrow 1]}
Log[60]
Limit[LogIntegral[x^2] - Log[Log[x]], x \rightarrow 1]
EulerGamma + Log[2]
Integrate[x^-1/Log[x], x]
Log[Log[x]]
```

 $\label{eq:continuous} Re@\ LogIntegral[x^-1],\ Re@\ LogIntegral[x^-2],\ Re@\ LogIntegral[x^-3]\},\ \{x,-5,5\}]$ 



D[LogIntegral[x^s], x]

$$\frac{s x^{-1+s}}{Log[x^s]}$$

$$\text{Limit}\left[\frac{\text{s}\,x^{-1+s}}{\text{Log}\left[x^{s}\right]}\,,\,\,\text{s}\to0\right]$$

$$\frac{1}{x \, \text{Log}[x]}$$

$$\texttt{Integrate}\Big[\,\frac{1}{\mathtt{x}\,\mathtt{Log}\,[\mathtt{x}]}\,,\,\mathtt{x}\Big]$$

Log[Log[x]]

Animate[Plot[{Re@LogIntegral[ $x^y$ ]}, {x, 29, 31}], {y, -1, 1}]

 $\label{eq:limit_logIntegral} \texttt{Limit[LogIntegral[x^z],z} \rightarrow .000000001] \ /.\ x \rightarrow 30$ 

-18.9219

$$\texttt{Integrate}\Big[\,\frac{\texttt{s}\, \texttt{x}^{-1+\texttt{s}}}{\texttt{Log}\,[\texttt{x}^{\texttt{s}}]}\,,\,\,\texttt{x}\Big]$$

LogIntegral[x<sup>s</sup>]

$$Limit\left[\frac{s x^{-1+s}}{Log[x^s]}, x \to 1\right]$$

$$\texttt{sDirectedInfinity}\Big[\frac{1}{\texttt{Sign[s]}}\Big]$$

$$\frac{s x^{-1+s}}{s Log[x]}$$

$$\texttt{Integrate}\Big[\,\frac{\mathbf{x}^{\texttt{-1+s}}}{\texttt{Log}\,[\mathbf{x}]}\,,\,\mathbf{x}\Big]$$

ExpIntegralEi[s Log[x]]

Integrate[x^-1 / (Log[x]), x]

Log[Log[x]]

Integrate  $[x^(s-1), x]$ 

D[LogIntegral[x^s], x]

$$\frac{s x^{-1+s}}{Log[x^s]}$$

Integrate  $[x^{(s)}x^{(t)}, x]$ 

Expand[Integrate[  $(x^{(s-1)}/Log[x])$   $(x^{(t-1)}/Log[x])$ ,  $\{x, 0, 20\}$ , PrincipalValue  $\rightarrow$  True]] /.  $\{s \rightarrow 4, t \rightarrow 4\}$ 

Integrate::idiv: Integral of  $\frac{x^6}{-\log[x]^2}$  does not converge on {0, 20}.  $\gg$ 

Integrate  $\left[\frac{x^6}{\text{Log}[x]^2}, \{x, 0, 20\}, \text{PrincipalValue} \rightarrow \text{True}\right]$ 

 $N[LogIntegral[x^4] LogIntegral[x^4] /.x \rightarrow 20]$ 

 $2.16967 \times 10^{8}$ 

$$N\left[7 \text{ ExpIntegralEi}\left[7 \log[x]\right] - \frac{x^7}{\log[x]}\right] / . x \to 20$$

 $2.26681 \times 10^{7}$ 

 $\texttt{Expand} \texttt{[Integrate[ (x^{(s-1) / Log[x]), x]] /. \{s \rightarrow 4\}}$ 

ExpIntegralEi[4 Log[x]]

 $N[LogIntegral[x^4] /. x \rightarrow 20]$ 

14729.8

 $N[ExpIntegralEi[4 Log[x]] /. x \rightarrow 20]$ 

14729

7 ExpIntegralEi[7 Log[x]] -  $\frac{x^7}{\text{Log}[x]}$  /.  $x \to 1$ 

Power::infy: Infinite expression  $\frac{1}{0}$  encountered.  $\gg$ 

Infinity::indet: Indeterminate expression ComplexInfinity +  $-\infty$  encountered.  $\gg$ 

Indeterminate

FullSimplify@Integrate[ $x^(s-1) / Log[x^2], x$ ]

$$\frac{1}{2} \; \mathbf{x^s} \; \left(\mathbf{x^2}\right)^{-s/2} \; \mathtt{ExpIntegralEi} \Big[\frac{1}{2} \; \mathbf{s} \; \mathtt{Log} \big[\mathbf{x^2}\big] \, \Big]$$

$$N\left[\frac{1}{2} x^{s} (x^{2})^{-s/2} \text{ ExpIntegralEi}\left[\frac{1}{2} s \text{ Log}\left[x^{2}\right]\right] /. \{x \rightarrow 100, s \rightarrow 1\}\right]$$

15.0631

$$N\left[\frac{1}{2} x^{s} (x^{2})^{-s/2} LogIntegral[x^{s}] /. \{x \rightarrow 100, s \rightarrow 2\}\right]$$

$$N\left[\frac{1}{2} \text{LogIntegral}[x^s] /. \{x \rightarrow 100, s \rightarrow 2\}\right]$$

$$N\Big[\frac{1}{2}\;x^{s}\;\left(x^{2}\right)^{-s/2}\;\text{ExpIntegralEi}\Big[\frac{1}{2}\;s\;\text{Log}\Big[x^{2}\Big]\Big]\;\text{/.}\;\left\{x\to30\,\text{,}\;s\to3\right\}\Big]$$

1492.21

N@LogIntegral[30^3]/2

1492.21

FullSimplify@Integrate[ $x^(s-1) / Log[x^3], x$ ]

$$\frac{1}{3} x^{s} (x^{3})^{-s/3} \text{ExpIntegralEi} \left[ \frac{1}{3} s Log[x^{3}] \right]$$

$$N\Big[\frac{1}{3}\;x^{s}\;\left(x^{3}\right)^{-s/3}\;\text{ExpIntegralEi}\Big[\frac{1}{3}\;s\;\text{Log}\Big[x^{3}\Big]\Big]\;\text{/.}\;\left\{x\to40\,\text{,}\;s\to4\right\}\Big]$$

62438.2

N@LogIntegral[40^4]/3

62438.2

Integrate  $[x^{(-1)} Log[x]^k, x]$ 

$$\frac{\text{Log}[x]^{1+k}}{1+k}$$

LogIntegral[x^s]/k

Integrate  $[x^-1 Log[x]^-1, x]$ 

Log[Log[x]]

Integrate[LogIntegral[x], x]

-ExpIntegralEi[2Log[x]] + xLogIntegral[x]

Integrate [LogIntegral  $[x^2]$ , x] /.  $x \rightarrow 10$ 

-ExpIntegralEi
$$\left[\frac{3 \log[100]}{2}\right]$$
 + 10 LogIntegral[100]

Integrate[LogIntegral[ $x^3$ ], x] /.  $x \rightarrow 10$ 

-ExpIntegralEi
$$\left[\frac{4 \log[1000]}{3}\right]$$
 + 10 LogIntegral[1000]

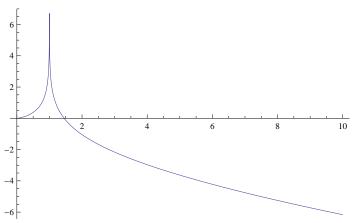
```
Integrate[Log[Log[x]], x]
x Log[Log[x]] - LogIntegral[x]
Integrate [LogIntegral [x^(1/500)], \{x, 0, 1\}]
-Log[501]
Integrate[Log[Log[x]], {x, 0, 1}]
-EulerGamma + i \pi
Integrate[Log[x], \{x, 0, 1\}]
p[n_{,k_{]}} = Sum[1/k-p[n/j,k-1], {j, 2, k}]
p[100, 1]
$RecursionLimit::reclim: Recursion depth of 256 exceeded. >>>
$RecursionLimit::reclim: Recursion depth of 256 exceeded. >>>
$RecursionLimit::reclim: Recursion depth of 256 exceeded. >>>
General::stop: Further output of $RecursionLimit::reclim will be suppressed during this calculation. ≫
$IterationLimit::itlim: Iteration limit of 4096 exceeded. >>>
$IterationLimit::itlim: Iteration limit of 4096 exceeded. >>>
$IterationLimit::itlim: Iteration limit of 4096 exceeded. >>>
General::stop: Further output of $IterationLimit::itlim will be suppressed during this calculation. ≫
```

```
D[n!, n]
Gamma[1+n] PolyGamma[0,1+n]
Grid[Table[Integrate[x^{(s-1)}/(Log[x]^k),x], \{k,-4,4\}, \{s,-4,4\}]]
```

Integrate[ $x^(s-1)(Log[x]^k)$ , x] /. { $s \rightarrow 1$ ,  $x \rightarrow 10$ }

```
-(-1)^{-1-k} Gamma[1+k, -Log[10]]
 Log[x]^{1+k} (-sLog[x])^{-1-k} /.x \rightarrow 30
(-s)^{-1-k}
Table[(-1)^k, \{k, -4, 4\}]
\{1, -1, 1, -1, 1, -1, 1, -1, 1\}
-(-s)^{-1+k} Gamma [1-k, -s Log[x]]
-(-s)^{-1+k} Gamma [1 - k, -s Log[x]]
(-1) ^k Gamma [1+k, -Log[x]]
(-1)^k Gamma [1+k, -Log[x]]
Integrate [1 / (Log[x]^k), x] /. \{x \rightarrow 20\}
(-1)^{k} Gamma [1-k, -Log[20]]
\label{eq:limit} \texttt{Limit[(-1)^(k) Integrate[(Log[x]^(k)), x], x \to 1]}
Gamma[1+k, 0]
N@Gamma[5, -Log[1.01]]
24. - 1.20448 \times 10^{-26} i
(4.5 + I)!
-2.21893 + 47.3558 i
\label{eq:limit} \mbox{Limit[ Integrate[(-1) $^k$ (Log[x] $^k$), $x], $x \to 1] $/.$ $k \to 7$}
5040
 -Integrate[(Log[1/x]^k), \{x, 1, n\}]
\texttt{ConditionalExpression} \left[ \texttt{Gamma} \left[ 1 + k \right] - \texttt{Gamma} \left[ 1 + k \right, - \texttt{Log} \left[ n \right] \right], \; \texttt{Re} \left[ k \right] \, > \, -1 \; \&\& \; \texttt{Log} \left[ n \right] \, > \, 0 \, \right]
N[-Integrate[(Log[1/x]^3), \{x, 1, n\}]/.n \rightarrow 100]
5573.28
Chop@Gamma[4, 0, -Log[100.]]
5573.28
```

Plot 
$$\left[-\text{ExpIntegralEi}\left[-\text{Log}\left[\frac{1}{x}\right]\right], \{x, 0, 10\}\right]$$



Integrate [1/Log[1/x], x]

-ExpIntegralEi
$$\left[-Log\left[\frac{1}{x}\right]\right]$$

Expand@Integrate[Log[x], {x, 1, n}]

ConditionalExpression $[1 - n + n Log[n], Re[n] \ge 0 \mid \mid n \notin Reals]$ 

FullSimplify@Integrate[Log[x]^z, {x, 1, n}]

 $\texttt{ConditionalExpression[(-z \, \texttt{Gamma}\, [\, z\, ]\, +\, \texttt{Gamma}\, [\, 1+z\, ,\, -\, \texttt{Log}\, [\, n\, ]\, ]\, )\, \, (-\, \texttt{Log}\, [\, n\, ]\, )^{\, -z}\,\, \texttt{Log}\, [\, n\, ]^{\, z}\, ,\,\, \texttt{Re}\, [\, z\, ]\, >\, -\, 1\, ]}$ 

FullSimplify@Integrate[ $(1/x)^z$ ,  $\{x, 1, n\}$ ]

$$\label{eq:conditional} Conditional Expression \Big[ -\frac{-1+\left(\frac{1}{n}\right)^{-1+z}}{-1+z} \text{ , } \text{Re}\left[n\right] \text{ } \geq \text{ } 0 \text{ } | \text{ } | \text{ } n \notin \text{Reals} \Big]$$

$$(-z \text{ Gamma}[z] + \text{Gamma}[1 + z, -\text{Log}[n]]) (-\text{Log}[n])^{-z} \text{Log}[n]^{z} /. n \rightarrow 100$$

 $(-1)^{-z}$  (-z Gamma[z] + Gamma[1 + z, -Log[100]])

FullSimplify 
$$\left[-\frac{-1+\left(\frac{1}{n}\right)^{-1+z}}{-1+z}\right]$$
 /.  $n \rightarrow 200$ 

$$-\frac{-1+200^{1-z}}{-1+z}$$

Integrate[Log[1/x]^z, x]

$$\operatorname{Gamma}\left[1+z, \operatorname{Log}\left[\frac{1}{x}\right]\right]$$

Integrate  $[x^-1 Log[x]^-1, x]$ 

Log[Log[x]]

Integrate[ $x^(s-1) Log[x]^(k-1)$ ,  $x]/. {x \rightarrow 10}$ 

$$-(-s)^{-k}$$
 Gamma[k,  $-s$  Log[10]]

Integrate[1/(xLog[x]), x]

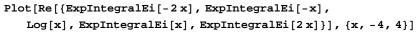
Log[Log[x]]

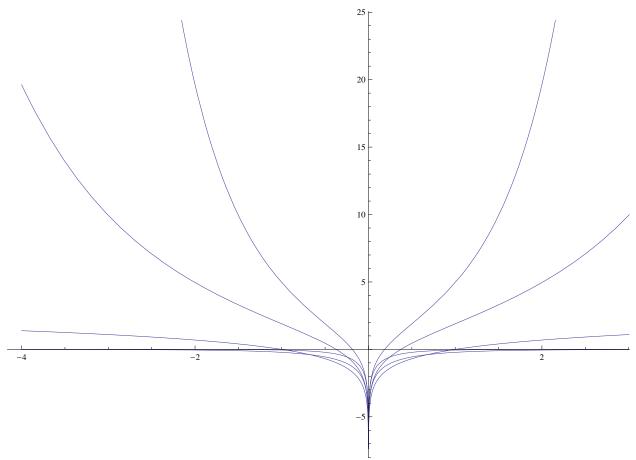
```
Integrate [x^{(k-1)}(E^x)^(s), x]
-e^{-s\,x}\,\left(e^{x}\right)^{s}\,x^{k}\,\left(-s\,x\right)^{-k}\,\text{Gamma}\left[\,k\,,\,-s\,x\,
ight]
FullSimplify \left[-e^{-sx}(e^x)^s x^k(-sx)^{-k} Gamma[k, -sx] / \cdot x \rightarrow 10\right]
-(-s)^{-k} Gamma[k, -10s]
Integrate [(E^x)^(0) \times (0-1), x]
Log[x]
-e^{-sx}(e^x)^s x^k(-sx)^{-k} Gamma[k, -sx]/.s \rightarrow -1
-Gamma[k, x]
\label{eq:limit} \mbox{Limit[-Gamma[k, x] + Gamma[k, 0], x $\rightarrow$ Infinity] /.k $\rightarrow$ 8}
 5040
 Integrate [x^-2 Log[x]^(k-1), \{x, 1, Infinity\}]
{\tt ConditionalExpression[Gamma[k], Re[k] > 0]}
Integrate[ (-Log[x])^k/(x^2), \{x, 1, n\}]
\texttt{ConditionalExpression}\left[ \left( -1 \right)^k \left( \mathsf{Gamma}\left[ 1+k \right] - \mathsf{Gamma}\left[ 1+k \right], \, \mathsf{Log}\left[ n \right] \right) \right), \, \mathsf{Re}\left[ k \right] > -1 \, \& \, \mathsf{Log}\left[ n \right] > 0 \, \mathsf{log}\left[ n \right] >
 Integrate[ (Log[x]^(k-1)/x^2) (Log[y])^(-k)/(y^2), \{x, 1, Infinity\}, \{y, 1, Infinity\}]
\label{eq:conditionalExpression} \begin{split} & \texttt{ConditionalExpression}[\texttt{Gamma}\,[\,1-k\,]\,\,\texttt{Gamma}\,[\,k\,]\,\,,\,\, \texttt{Re}\,[\,k\,]\,\,>\,0\,] \end{split}
Pi / Sin[ Pi 3.2]
 -5.3448
Expand [ (Log[x]^{(k-1)} / x^2) (Log[y])^{(-k)} / (y^2) ]
  Log[x]^{-1+k} Log[y]^{-k}
                                    x^2 y^2
Integrate[Log[x]^k/x^2, {x, 1, Infinity}]
Conditional \texttt{Expression} [ \texttt{Gamma} [ 1 + k ] \text{ , } \texttt{Re} [ k ] \text{ } > -1 ]
Integrate [(E^x)^s x^k (k-1), x]/.x \rightarrow 10
 -(-s)^{-k} Gamma[k, -10s]
```

#### $Grid[Table[Expand@Integrate[x^(s-1)Log[x]^(k-1),x], \{k,-2,2\}, \{s,-2,2\}]]$

# $\label{linear_cond} \mbox{Grid[Table[Expand@Integrate[ (E^x) ^s x^ (k-1) , x] , \{k, -2, 2\} , \{s, -2, 2\}]]}$

| $-\frac{e^{-2x}}{2x^2} + \frac{e^{-2x}}{x} + 2 \text{ ExpIntegralEi}[$ $-2x]$ | $-\frac{e^{-x}}{2x^2} + \frac{e^{-x}}{2x} + \frac{1}{2}$ ExpIntegralEi[-x] | $-\frac{1}{2 x^2}$ | $-\frac{e^{x}}{2 x^{2}} - \frac{e^{x}}{2 x} + \frac{\text{ExpIntegralEi}[x]}{2}$ | $-\frac{e^{2x}}{2x^{2}} - \frac{e^{2x}}{x} + 2 \text{ ExpIntegralEi}[$ $2x]$ |
|---|--|--------------------|--|--|
| $-\frac{e^{-2x}}{x}$  | $-\frac{e^{-x}}{x}$  | $-\frac{1}{x}$     | $-\frac{e^x}{x}$ +   | $-\frac{e^2x}{x}$ +  |
| 2 ExpIntegralEi[  | ExpIntegralEi[-x]  |                    | ExpIntegralEi[x]   | 2 ExpIntegralEi[   |
| -2x]  |  |                    |  | 2 x]   |
| ExpIntegralEi[-2x]  | ExpIntegralEi[-x]  | Log[x]             | ${\tt ExpIntegralEi}[{\tt x}]$   | ExpIntegralEi[2x]  |
| $-\frac{1}{2} e^{-2x}$  | $-\mathbb{e}^{-\mathbf{x}}$  | x                  | $e^{x}$  | $\frac{e^{2x}}{2}$   |
| $-\frac{1}{4} e^{-2x} - \frac{1}{2} e^{-2x} x$                                | $-\mathbb{G}_{-x}-\mathbb{G}_{-x}$ x                                       | $\frac{x^2}{2}$    | $-\mathbb{e}^{x}+\mathbb{e}^{x} x$   | $-\frac{e^{2x}}{4} + \frac{1}{2} e^{2x} x$                                   |





 $\label{linegrate} $$ \operatorname{Grid}[Table[Integrate[\ x^{(s-1)} \log[x]^{(k-1)}, \{x, 1, Infinity\}], \{k, 1, 6\}, \{s, -6, -1\}]] $$ $$ $$ $$$ 

| $\frac{1}{6}$    | <u>1</u><br>5  | $\frac{1}{4}$   | $\frac{1}{3}$ | $\frac{1}{2}$ | 1   |
|------------------|----------------|-----------------|---------------|---------------|-----|
| <u>1</u><br>36   | <u>1</u><br>25 | $\frac{1}{16}$  | $\frac{1}{9}$ | $\frac{1}{4}$ | 1   |
| 1 108            | 2<br>125       | 1<br>32         | 2<br>27       | $\frac{1}{4}$ | 2   |
| 1 216            | 6<br>625       | 3<br>128        | 2<br>27       | <u>3</u><br>8 | 6   |
| 324              | 24<br>3125     | $\frac{3}{128}$ | 8 81          | $\frac{3}{4}$ | 24  |
| $\frac{5}{1944}$ | 24<br>3125     | 15<br>512       | 40<br>243     | 15<br>8       | 120 |

# $Grid[Table[Gamma[k] (-s)^-k, \{k, 1, 6\}, \{s, -6, -1\}]]$

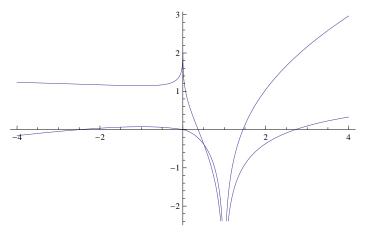
```
Integrate [x^{(s-1)} Log[x]^{(k-1)}, \{x, 1, Infinity\}]
```

 $\texttt{ConditionalExpression} \left[ (-s)^{-k} \, \texttt{Gamma[k], Re[s]} \, < \, 0 \, \&\& \, \text{Re[k]} \, > \, 0 \, \right]$ 

#### Integrate [ $Log[x]^z$ , {x, 1, n}]

 $\texttt{ConditionalExpression[(-z \, \texttt{Gamma} \, [z] + \texttt{Gamma} \, [1 + z, \, -\texttt{Log} \, [n]]) \, (-\texttt{Log} \, [n])^{-z} \, \texttt{Log} \, [n]^{z}, \, \texttt{Re} \, [z] > -1]}$ 

### Plot[Re[{Log@Log@x, LogIntegral[x]}], {x, -4, 4}]



#### FullSimplify@D[-(-s) $^-k$ Gamma[k, -sLog[x]], k]

 $(-s)^{-k}$  (Gamma[k, -sLog[x]] (Log[-s] - Log[-sLog[x]]) -MeijerG[ $\{\{\}, \{1, 1\}\}, \{\{0, 0, k\}, \{\}\}, -s Log[x]]$ )

#### FullSimplify@D[-(-s) $^-k$ Gamma[k, -sx], k]

 $(-s)^{-k}$  (Gamma[k, -sx] (Log[-s] - Log[-sx]) - MeijerG[{{}}, {1, 1}}, {{0, 0, k}, {}}, -sx])

#### -Integrate [Log $[1/x]^(z-1)x^(s-1), \{x, 1, n\}$ ]

 $\texttt{ConditionalExpression[-(-1)^z(-s)^{-z}(-Gamma[z]+Gamma[z,-sLog[n]]),Re[z]>0 \&\& Log[n]>0]}$ 

#### FullSimplify[ $(-1)^z(-s)^{-z}$ ] /. $z \rightarrow -4$

 $s^4$ 

#### $s^{-z}$ Gamma[z, 0, -s Log[n]]

 $s^{-z}$  Gamma[z, 0, -s Log[n]]

# -Integrate $[(-Log[x])^(z-1)x^(s-1), \{x, 1, n\}]$

 $\texttt{ConditionalExpression[-(-1)^z(-s)^{-z}(-Gamma[z]+Gamma[z,-sLog[n]]),Re[z]>0 \&\& Log[n]>0]}$ 

 $N[-(-1)^{z}(-s)^{-z}(-Gamma[z]+Gamma[z,-sLog[n]]) /. \{n \to 10, z \to 2, s \to 2\}]$ 

 $90.3793 - 1.10377 \times 10^{-14}$  i

 $\texttt{N[-Integrate[(-Log[x])^(z-1)x^(s-1),\{x,1,n\}]/.\{n\rightarrow 10,z\rightarrow 2,s\rightarrow 2\}]}$ 

 $90.3793 - 1.10377 \times 10^{-14}$  i

 $N[s^{-z} (Gamma[z, 0, -s Log[n]]) /. \{n \to 10, z \to 2, s \to 2\}]$ 

 $90.3793 - 1.10377 \times 10^{-14}$  i