```
Etx[s_, k_, t_] := Sum[ (Mod[n, k] - Mod[n - 1, k]) / n^s, {n, 1, t}]

{N[Etx[1, 2, 10000]], N[Log[2]]}

{0.693097, 0.693147}

{N[Etx[1, 3, 10000]], N[Log[3]]}

{1.09861, 1.09861}

{N[Etx[1, 4, 10000]], N[Log[4]]}

{1.38614, 1.38629}

{ N[Etx[2, 2, 10000]], N[Pi^2 / 12]}

{0.822467, 0.822467}

{N[Etx[2, 3, 10000]], Pi^2 / 9.}

{1.09662, 1.09662}

{N[Etx[2, 4, 10000.]], N[Pi^2 / 8]}

{1.2337, 1.2337}

{N[Etx[2, 5, 10000.]], N[2 Pi^2 / 15]}

{1.31595, 1.31595}

Table[{ 1 / (N[Etx[2, x, 10000.]] / Pi^2) - 6, N[6 / (x - 1)]}, {x, 2, 20}] // TableForm
```

6.	6.
3.	3.
2.	2.
1.5	1.5
1.2	1.2
1.	1.
0.857143	0.857143
0.75	0.75
0.666667	0.666667
0.6	0.6
0.545455	0.545455
0.5	0.5
0.461539	0.461538
0.428571	0.428571
0.4	0.4
0.375	0.375
0.352941	0.352941
0.333333	0.333333
0.31579	0.315789

```
Table[{ (N[Etx[2, x, 10 000.]] / Pi^2), 1 / (6 + N[6 / (x - 1)])}, {x, 2, 20}] // TableForm
```

0.0833333	0.0833333
0.111111	0.111111
0.125	0.125
0.133333	0.133333
0.138889	0.138889
0.142857	0.142857
0.145833	0.145833
0.148148	0.148148
0.15	0.15
0.151515	0.151515
0.152778	0.152778
0.153846	0.153846
0.154762	0.154762
0.155556	0.155556
0.15625	0.15625
0.156863	0.156863
0.157407	0.157407
0.157895	0.157895
0.158333	0.158333

```
Table[{ N[Etx[2, x, 10 000.]], Pi^2 / (6 + N[6 / (x - 1)])}, {x, 2, 20}] // TableForm
```

0.822467	0.822467
1.09662	1.09662
1.2337	1.2337
1.31595	1.31595
1.37078	1.37078
1.40994	1.40994
1.43932	1.43932
1.46216	1.46216
1.48044	1.48044
1.49539	1.49539
1.50786	1.50786
1.5184	1.5184
1.52744	1.52744
1.53527	1.53527
1.54213	1.54213
1.54817	1.54817
1.55355	1.55355
1.55836	1.55836
1.56269	1.56269

```
Expand[FullSimplify[Pi^2 / (6 + 6 / (x - 1))]]
```

$$\frac{\pi^2}{6} - \frac{\pi^2}{6\,x}$$

```
Etx3[s_, k_, t_] := Sum[ (Mod[n, k] - Mod[n - 1, k]) / (2 n - 1)^s, {n, 1, t}]
Table[{x, 1 / (N[Etx3[1, x, 40000.]] / Pi) - 8 / Log[2, x]}, {x, 2, 20000, 700}] // TableForm
```

2	-3.99997
702	-0.0554991
1402	-0.0374499
2102	-0.0275986
2802	-0.0235444
3502	-0.0208536
4202	-0.0189372
4902	-0.0141768
5602	-0.012092
6302	-0.0123538
7002	-0.0148935
7702	-0.00822575
8402	-0.0139329
9102	-0.00854646
9802	-0.0036429
10 502	-0.0131258
11 202	-0.00898143
11 902	-0.00514026
12 602	-0.00156385
13 302	0.00177957
14 002	-0.0127372
14 702	-0.00978447
15 402	-0.0069969
16 102	-0.00435826
16 802	-0.00185452
17 502	0.00052653
18 202	0.00279553
18 902	0.00496183
19 602	0.00703369

```
Table[{x, 1 / (N[Etx3[1, x, 240 000.]] / Pi) - 8 / Log[2, x]}, {x, 2, 20 000, 700}] // TableForm
```

2	-3.99999
702	-0.0547677
1402	-0.037207
2102	-0.029273
2802	-0.0248164
3502	-0.0213757
4202	-0.0183862
4902	-0.0174555
5602	-0.0157306
6302	-0.0132038
7002	-0.0123168
7702	-0.0110586
8402	-0.0109362
9102	-0.00972696
9802	-0.00926402
10 502	-0.00951427
11 202	-0.0078721
11 902	-0.00662958
12 602	-0.00575708
13 302	-0.00523297
14 002	-0.00504248
14 702	-0.00517696
15 402	-0.0056335
16 102	-0.00641497
16 802	-0.00391124
17 502	-0.00514929
18 202	-0.00288031
18 902	-0.00456009
19 602	-0.00248824

```
Etx3[s_, k_, t_] := Sum[ (Mod[n, k] - Mod[n - 1, k]) / (2 n - 1) ^s, {n, 1, t}]
Table[{x, 1 / (N[Etx3[1, x, 40 000.]] / Pi)}, {x, 2, 20}] // TableForm
```

2	4.00003	4.
3	2.89362	2.52372
4	2.46811	2.
5	2.22968	1.72271
6	2.0723	1.54741
7	1.95852	1.42483
8	1.87119	1.33333
9	1.80125	1.26186
10	1.7437	1.20412
11	1.69504	1.15626
12	1.65327	1.11577
13	1.61676	1.08095
14	1.5847	1.0506
15	1.55593	1.02383
16	1.53021	1.
17	1.50667	0.978602
18	1.48542	0.95925
19	1.46581	0.941636
20	1.44777	0.925513

```
Table[{n, N[(Etx[4, x, 10000.])], N[ZZ[x, 4]], ZZ[x, 4]}, {x, 2, 20}] // TableForm
```

n	0.947033	0.947033	$\frac{7\pi^4}{720}$
n	1.04224	1.04224	$\frac{13\pi^4}{1215}$
n	1.06541	1.06541	$\frac{7\pi^4}{640}$
n	1.07366	1.07366	$\frac{62\pi^4}{5625}$
n	1.07731	1.07731	$\frac{43\pi^4}{3888}$
n	1.07917	1.07917	$\frac{19\pi^4}{1715}$
n	1.08021	1.08021	$\frac{511\pi^4}{46080}$
n	1.08084	1.08084	$\frac{364\pi^4}{32805}$
n	1.08124	1.08124	$\frac{111\pi^4}{10000}$
n	1.08151	1.08151	$\frac{133\pi^4}{11979}$
n	1.0817	1.0817	$\frac{1727\pi^4}{155520}$
n	1.08183	1.08183	$\frac{122\pi^4}{10985}$
n	1.08193	1.08193	$\frac{2743\pi^4}{246960}$
n	1.082	1.082	$\frac{1687\pi^4}{151875}$
n	1.08206	1.08206	$\frac{91\pi^4}{8192}$
n	1.0821	1.0821	$\frac{2456\pi^4}{221085}$
n	1.08214	1.08214	$\frac{5831\pi^4}{524880}$
n	1.08217	1.08217	$\frac{381\pi^4}{34295}$
n	1.08219	1.08219	$\frac{7999\pi^4}{720000}$

```
N[242 / 229635]
```

0.00105385

```
ZZ[a_, s_] := (1 - a^(1 - s)) Zeta[s]
```

```
ZZ[3, 4]
```

$$\frac{13\pi^4}{1215}$$

```
z2[x_] := Expand[FullSimplify[Pi^2 / (6 + 6 / (x - 1))]]
```

```
z2[9]
```

$$\frac{4\pi^2}{27}$$

```
ZZ2[a_, s_] := (1 - a^(1 - s))
```

```
ZZ2[3, 3] / ZZ2[2, 3]
```

$$\frac{32}{27}$$

ZZ[a, 4]

$$\frac{1}{90}\left(1-\frac{1}{a^3}\right)\pi^4$$

ZZ[a, 6]

$$\frac{1}{945}\left(1-\frac{1}{a^5}\right)\pi^6$$

ZZ[a, 8]

$$\frac{\left(1-\frac{1}{a^7}\right)\pi^8}{9450}$$

ZZ[a, k]

$$\left(1-a^{1-k}\right)\text{Zeta}[k]$$

Sum[(-1)^(k+1) 1/((2k-1)^3), {k, 1, Infinity}]

$$\frac{\pi^3}{32}$$

N[Pi^3/32]

0.968946

Sum[1/((2k-1)^3), {k, 1, Infinity}]

$$\frac{7\,\text{Zeta}[3]}{8}$$

Sum[(-1)^(k+1) 1/((2k-1)^3), {k, 1, Infinity}]

$$\mathbf{N}\left[\frac{\pi^3}{32}\right]$$

0.968946

$$\mathbf{N}\left[\frac{\pi^3}{27}\right]$$

1.14838

Sum[(Mod[k, 2] - Mod[k-1, 2]) 1/((2k-1)^3), {k, 1, Infinity}]

$$1\Big/\ \mathbf{N}\left[\left(\frac{1}{128}\left(2\pi^3+\text{PolyGamma}\left[2,\frac{3}{4}\right]+56\,\text{Zeta}[3]\right)\right)\Big/\ \text{Pi}^\wedge 3\right]$$

32.

Sum[(Mod[k, 3] - Mod[k-1, 3]) 1/((2k-1)^3), {k, 1, Infinity}]

$$\mathbf{N}\left[\frac{1}{432}\left(-\text{PolyGamma}\left[2,\frac{1}{6}\right]-\text{PolyGamma}\left[2,\frac{1}{2}\right]+2\,\text{PolyGamma}\left[2,\frac{5}{6}\right]\right)\right]$$

1.02432

$$N\left[\frac{-\text{PolyGamma}\left[2, \frac{2}{9}\right] - \text{PolyGamma}\left[2, \frac{5}{9}\right] + 2\,\text{PolyGamma}\left[2, \frac{8}{9}\right]}{1458}\right]$$

0.12994

$$N\left[\frac{1}{432}\left(-\text{PolyGamma}\left[2, \frac{1}{6}\right] - \text{PolyGamma}\left[2, \frac{1}{2}\right] + 2\,\text{PolyGamma}\left[2, \frac{5}{6}\right]\right)\right]$$

1.02432

$$\text{Expand}\left[\frac{1}{128}\left(2\,\pi^3 + \text{PolyGamma}\left[2, \frac{3}{4}\right] + 56\,\text{Zeta}[3]\right)\right]$$

$$\frac{\pi^3}{64} + \frac{1}{128}\,\text{PolyGamma}\left[2, \frac{3}{4}\right] + \frac{7\,\text{Zeta}[3]}{16}$$

$$N\left[\frac{1}{128}\left(2\,\pi^3 + \text{PolyGamma}\left[2, \frac{3}{4}\right] + 56\,\text{Zeta}[3]\right)\right]$$

0.968946

$$\text{Expand}\left[\frac{1}{128}\left(2\,\pi^3 + \text{PolyGamma}\left[2, \frac{3}{4}\right] + 56\,\text{Zeta}[3]\right)\right]$$

$$\frac{\pi^3}{64} + \frac{1}{128}\,\text{PolyGamma}\left[2, \frac{3}{4}\right] + \frac{7\,\text{Zeta}[3]}{16}$$

$$N\left[\frac{\pi^3}{64}\right]$$

0.484473

$$N\left[\frac{1}{128}\,\text{PolyGamma}\left[2, \frac{3}{4}\right]\right]$$

-0.0414268

$$N\left[\frac{7\,\text{Zeta}[3]}{16}\right]$$

0.5258998951323225`

$$\frac{\pi^3}{64} + \frac{1}{128}\,\text{PolyGamma}\left[2, \frac{3}{4}\right] + \frac{7\,\text{Zeta}[3]}{16} - \frac{\pi^3}{32}$$

$$-\frac{\pi^3}{64} + \frac{1}{128}\,\text{PolyGamma}\left[2, \frac{3}{4}\right] + \frac{7\,\text{Zeta}[3]}{16}$$

$$\text{Expand}\left[\frac{\pi^3}{64} + \frac{1}{128}\,\text{PolyGamma}\left[2, \frac{3}{4}\right] + \frac{7\,\text{Zeta}[3]}{16} - \frac{\pi^3}{32}\right]$$

$$-\frac{\pi^3}{64} + \frac{1}{128}\,\text{PolyGamma}\left[2, \frac{3}{4}\right] + \frac{7\,\text{Zeta}[3]}{16}$$

$$N\left[-\left(-\frac{\pi^3}{64} + \frac{1}{128}\,\text{PolyGamma}\left[2, \frac{3}{4}\right]\right) 16 / 7\right]$$

1.20206

N[Zeta[3]]

1.20206

$$\mathbf{N}\left[\mathbf{PolyGamma}\left[2, \frac{3}{4}\right]\right]$$

$$-5.30263$$

$$\mathbf{Expand}\left[-\left(-\frac{\pi^3}{64} + \frac{1}{128}\,\mathbf{PolyGamma}\left[2, \frac{3}{4}\right]\right) 16 / 7\right]$$

$$\mathbf{N}\left[\frac{\pi^3}{28} - \frac{1}{56}\,\mathbf{PolyGamma}\left[2, \frac{3}{4}\right]\right]$$

$$1.20206$$

```
N[-PolyGamma[2, 1] / 2]
```

$$1.20206$$

$$\mathbf{N}\left[-\frac{1}{56}\,\mathbf{PolyGamma}\left[2, \frac{3}{4}\right]\right]$$

$$0.0946899$$

```
Sum[(Mod[k, 2] - Mod[k - 1, 2]) 1 / ((2 k - 1) ^3), {k, 1, Infinity}]
```

$$\frac{1}{128}\left(2\,\pi^3 + \text{PolyGamma}\left[2, \frac{3}{4}\right] + 56\,\text{Zeta}[3]\right)$$

```
Sum[(Mod[k, 3] - Mod[k - 1, 3]) 1 / ((2 k - 1) ^3), {k, 1, Infinity}]
```

$$\frac{1}{432}\left(-\text{PolyGamma}\left[2, \frac{1}{6}\right] - \text{PolyGamma}\left[2, \frac{1}{2}\right] + 2\,\text{PolyGamma}\left[2, \frac{5}{6}\right]\right)$$

$$\mathbf{Expand}\left[\frac{1}{432}\left(-\mathbf{PolyGamma}\left[2, \frac{1}{6}\right] - \mathbf{PolyGamma}\left[2, \frac{1}{2}\right] + 2\,\mathbf{PolyGamma}\left[2, \frac{5}{6}\right]\right)\right]$$

$$-\frac{1}{432}\,\text{PolyGamma}\left[2, \frac{1}{6}\right] - \frac{1}{432}\,\text{PolyGamma}\left[2, \frac{1}{2}\right] + \frac{1}{216}\,\text{PolyGamma}\left[2, \frac{5}{6}\right]$$

```
Sum[(Mod[k, 4] - Mod[k - 1, 4]) 1 / ((2 k - 1) ^3), {k, 1, Infinity}]
```

$$\frac{1}{1024}\left(-\text{PolyGamma}\left[2, \frac{1}{8}\right] - \text{PolyGamma}\left[2, \frac{3}{8}\right] - \text{PolyGamma}\left[2, \frac{5}{8}\right] + 3\,\text{PolyGamma}\left[2, \frac{7}{8}\right]\right)$$

```
Sum[(-1) ^ (k + 1) 1 / ((2 k - 1) ^5), {k, 1, Infinity}]
```

$$\frac{5\,\pi^5}{1536}$$

```
Sum[(Mod[k, 2] - Mod[k - 1, 2]) 1 / ((2 k - 1)^5), {k, 1, Infinity}]
```

$$\frac{40\,\pi^5 + \text{PolyGamma}\left[4, \frac{3}{4}\right] + 11\,904\,\text{Zeta}[5]}{24\,576}$$

```
Sum[(-1)^(k + 1) 1 / ((2 k - 1)^1), {k, 1, Infinity}]
```

$$\frac{\pi}{4}$$

```
Sum[(Mod[k, 2] - Mod[k - 1, 2]) 1 / ((2 k - 1)^1), {k, 1, Infinity}]
```

$$\sum_{k=1}^{\infty} \frac{-\text{Mod}[-1+k, 2] + \text{Mod}[k, 2]}{-1 + 2\,k}$$

```
Sum[(Mod[k, 2]) 1 / ((2 k - 1)^3), {k, 1, Infinity}] -
 Sum[(Mod[k - 1, 2]) 1 / ((2 k - 1)^3), {k, 1, Infinity}]
```

$$\frac{1}{128}\,\text{PolyGamma}\left[2, \frac{3}{4}\right] + \frac{1}{64}\left(\pi^3 + 28\,\text{Zeta}[3]\right)$$

```
Sum[(Mod[k, 2]) 1 / ((2 k - 1)^3), {k, 1, Infinity}]
```

$$\frac{1}{64}\left(\pi^3 + 28\,\text{Zeta}[3]\right)$$

```
Sum[(Mod[k - 1, 2]) 1 / ((2 k - 1)^3), {k, 1, Infinity}]
```

$$-\frac{1}{128}\,\text{PolyGamma}\left[2, \frac{3}{4}\right]$$

```
Sum[(Mod[k, 2]) 1 / ((2 k - 1)^4), {k, 1, Infinity}]
```

$$\frac{8\,\pi^4 + 3\,\text{Zeta}\left[4, \frac{1}{4}\right] - 3\,\text{Zeta}\left[4, \frac{3}{4}\right]}{1536}$$

```
Sum[(Mod[k, 2] - Mod[k - 1, 2]) 1 / ((2 k - 1)^4), {k, 1, Infinity}]
```

$$\frac{8\,\pi^4 - \text{PolyGamma}\left[3, \frac{3}{4}\right] + 3\,\text{Zeta}\left[4, \frac{1}{4}\right] - 3\,\text{Zeta}\left[4, \frac{3}{4}\right]}{1536}$$

```
Zeta[4, 1 / 4]
```

$$\text{Zeta}\left[4, \frac{1}{4}\right]$$

```
Sum[(-1)^(k + 1) 1 / ((2 k - 1)^5), {k, 1, Infinity}]
Sum[(Mod[k, 2]) 1 / ((2 k - 1)^5), {k, 1, Infinity}] -
 Sum[(Mod[k - 1, 2]) 1 / ((2 k - 1)^5), {k, 1, Infinity}]
```

$$\frac{5\,\pi^5}{1536}$$

$$\frac{\text{PolyGamma}\left[4, \frac{3}{4}\right]}{24\,576} + \frac{5\,\pi^5 + 1488\,\text{Zeta}[5]}{3072}$$

```
Sum[(-1)^(k+1) 1/((2 k-1)^7), {k, 1, Infinity}]
Sum[(Mod[k, 2]) 1/((2 k-1)^7), {k, 1, Infinity}] -
 Sum[(Mod[k-1, 2]) 1/((2 k-1)^7), {k, 1, Infinity}]
```

$$\frac{61\,\pi^7}{184\,320}$$

$$\frac{\text{PolyGamma}\left[6, \frac{3}{4}\right]}{11\,796\,480} + \frac{61\,\pi^7 + 182\,880\,\text{Zeta}[7]}{368\,640}$$

```
Sum[(-1)^(k+1) 1/((2 k-1)^2), {k, 1, Infinity}]
Sum[(Mod[k, 2]) 1/((2 k-1)^2), {k, 1, Infinity}] -
 Sum[(Mod[k-1, 2]) 1/((2 k-1)^2), {k, 1, Infinity}]
```

Catalan

$$\frac{1}{16}\left(8\,\text{Catalan} + \pi^2\right) - \frac{1}{16}\,\text{PolyGamma}\left[1, \frac{3}{4}\right]$$

```
Sum[(-1)^(k+1) 1/((2 k-1)^4), {k, 1, Infinity}]
Sum[(Mod[k, 2]) 1/((2 k-1)^4), {k, 1, Infinity}] -
 Sum[(Mod[k-1, 2]) 1/((2 k-1)^4), {k, 1, Infinity}]
```

$$\frac{1}{256}\left(\text{Zeta}\left[4, \frac{1}{4}\right] - \text{Zeta}\left[4, \frac{3}{4}\right]\right)$$

$$-\frac{\text{PolyGamma}\left[3, \frac{3}{4}\right]}{1536} + \frac{8\,\pi^4 + 3\,\text{Zeta}\left[4, \frac{1}{4}\right] - 3\,\text{Zeta}\left[4, \frac{3}{4}\right]}{1536}$$

```
Sum[(-1)^(k+1) 1/((2 k-1)^6), {k, 1, Infinity}]
Sum[(Mod[k, 2]) 1/((2 k-1)^6), {k, 1, Infinity}] -
 Sum[(Mod[k-1, 2]) 1/((2 k-1)^6), {k, 1, Infinity}]
```

$$\frac{\text{Zeta}\left[6, \frac{1}{4}\right] - \text{Zeta}\left[6, \frac{3}{4}\right]}{4096}$$

$$-\frac{\text{PolyGamma}\left[5, \frac{3}{4}\right]}{491\,520} + \frac{64\,\pi^6 + 15\,\text{Zeta}\left[6, \frac{1}{4}\right] - 15\,\text{Zeta}\left[6, \frac{3}{4}\right]}{122\,880}$$

```
Sum[(-1)^(k+1) 1/((2 k-1)^8), {k, 1, Infinity}]
Sum[(Mod[k, 2]) 1/((2 k-1)^8), {k, 1, Infinity}] -
 Sum[(Mod[k-1, 2]) 1/((2 k-1)^8), {k, 1, Infinity}]
```

$$\frac{\text{Zeta}\left[8, \frac{1}{4}\right] - \text{Zeta}\left[8, \frac{3}{4}\right]}{65\,536}$$

$$-\frac{\text{PolyGamma}\left[7, \frac{3}{4}\right]}{330\,301\,440} + \frac{2176\,\pi^8 + 315\,\text{Zeta}\left[8, \frac{1}{4}\right] - 315\,\text{Zeta}\left[8, \frac{3}{4}\right]}{41\,287\,680}$$

```
Sum[(-1)^(k+1) 1/((2k-1)^1), {k, 1, Infinity}]
Sum[(Mod[k, 2]) 1/((2k-1)^1), {k, 1, Infinity}] -
 Sum[(Mod[k-1, 2]) 1/((2k-1)^1), {k, 1, Infinity}]
```

$$\frac{\pi}{4}$$

Sum::div : Sum does not converge. ≫

Sum::div : Sum does not converge. ≫

$$-\sum_{k=1}^{\infty}\frac{\text{Mod}[-1+k,2]}{-1+2k}+\sum_{k=1}^{\infty}\frac{\text{Mod}[k,2]}{-1+2k}$$

```
N[Catalan]
```

0.915966

```
Sum[(-1)^(k+1) 1/((2k-0)^2), {k, 1, Infinity}]
```

$$\frac{\pi^2}{48}$$

```
Sum[(-1)^(k+1) 1/((k)^2), {k, 1, Infinity}]
```

$$\frac{\pi^2}{12}$$

```
Sum[(Mod[k, 2]) 1/((2k)^2), {k, 1, Infinity}]
```

$$\frac{\pi^2}{32}$$

```
Sum[(Mod[k-1, 2]) 1/((2k)^2), {k, 1, Infinity}]
```

$$\frac{\pi^2}{96}$$

```
Sum[(1)^(k+1) 1/((2k-1)^2), {k, 1, Infinity}]
Sum[(Mod[k, 2]) 1/((2k-1)^2), {k, 1, Infinity}] +
 Sum[(Mod[k-1, 2]) 1/((2k-1)^2), {k, 1, Infinity}]
```

$$\frac{\pi^2}{8}$$

$$\text{N}\left[\frac{1}{16}\left(8\,\text{Catalan}+\pi^2\right)+\frac{1}{16}\,\text{PolyGamma}\left[1,\frac{3}{4}\right]\right]$$

1.2337

$$\text{N}\left[\frac{\pi^2}{8}\right]$$

1.2337

```
Sum[(1)^(k+1) 1/(k^2), {k, 1, Infinity}]
Sum[(Mod[k, 2]) 1/(k^2), {k, 1, Infinity}]
Sum[(Mod[k-1, 2]) 1/(k^2), {k, 1, Infinity}]
```

$$\frac{\pi^2}{6}$$

$$\frac{\pi^2}{8}$$

$$\frac{\pi^2}{24}$$

```
Sum[(Mod[k-1, 2]) 1/(k^2), {k, 1, Infinity}]
```

$$\frac{\pi^2}{24}$$

```
Sum[(Mod[k, 8] - Mod[k-1, 8]) 1/(k^2), {k, 1, Infinity}]
```

$$\frac{1}{192}\left(-2\pi^2 + 3\,\text{PolyGamma}\left[1, \frac{1}{8}\right] + 3\,\text{PolyGamma}\left[1, \frac{1}{4}\right] + 3\,\text{PolyGamma}\left[1, \frac{3}{8}\right] + 3\,\text{PolyGamma}\left[1, \frac{5}{8}\right] + 3\,\text{PolyGamma}\left[1, \frac{3}{4}\right] + 3\,\text{PolyGamma}\left[1, \frac{7}{8}\right]\right)$$

```
Sum[(Mod[k, 18] - Mod[k-1, 18]) 1/(k^2), {k, 1, Infinity}]
```

$$\frac{1}{972}\left(-7\pi^2 + 3\,\text{PolyGamma}\left[1, \frac{1}{18}\right] + 3\,\text{PolyGamma}\left[1, \frac{1}{9}\right] + 3\,\text{PolyGamma}\left[1, \frac{1}{6}\right] + 3\,\text{PolyGamma}\left[1, \frac{2}{9}\right] + 3\,\text{PolyGamma}\left[1, \frac{5}{18}\right] + 3\,\text{PolyGamma}\left[1, \frac{1}{3}\right] + 3\,\text{PolyGamma}\left[1, \frac{7}{18}\right] + 3\,\text{PolyGamma}\left[1, \frac{4}{9}\right] + 3\,\text{PolyGamma}\left[1, \frac{5}{9}\right] + 3\,\text{PolyGamma}\left[1, \frac{11}{18}\right] + 3\,\text{PolyGamma}\left[1, \frac{2}{3}\right] + 3\,\text{PolyGamma}\left[1, \frac{13}{18}\right] + 3\,\text{PolyGamma}\left[1, \frac{7}{9}\right] + 3\,\text{PolyGamma}\left[1, \frac{5}{6}\right] + 3\,\text{PolyGamma}\left[1, \frac{8}{9}\right] + 3\,\text{PolyGamma}\left[1, \frac{17}{18}\right]\right)$$

$$\textbf{Expand}\left[\frac{1}{16}\left(8\,\text{Catalan} + \pi^2\right) + \frac{1}{16}\,\text{PolyGamma}\left[1, \frac{3}{4}\right] - \text{Pi}^\wedge 2/8\right]$$

$$\textbf{Expand}\left[2\left(\frac{\text{Catalan}}{2} - \frac{\pi^2}{16} + \frac{1}{16}\,\text{PolyGamma}\left[1, \frac{3}{4}\right]\right)\right]$$

$$\text{Catalan} - \frac{\pi^2}{8} + \frac{1}{8}\,\text{PolyGamma}\left[1, \frac{3}{4}\right]$$

$$-\left(-\frac{\pi^2}{8} + \frac{1}{8}\,\text{PolyGamma}\left[1, \frac{3}{4}\right]\right)$$

$$\frac{\pi^2}{8} - \frac{1}{8}\,\text{PolyGamma}\left[1, \frac{3}{4}\right]$$

```
Sum[1/((2k)^2), {k, 1, Infinity}]
```

$$\frac{\pi^2}{24}$$

```
Expand[Sum[1/((2k+1)^2), {k, 1, Infinity}]]
```

$$-1+\frac{\pi^2}{8}$$

```
Expand[Sum[1/((2k+2)^2), {k, 1, Infinity}]]
```

$$-\frac{1}{4}+\frac{\pi^2}{24}$$

```
Sum[1/((3k)^2), {k, 1, Infinity}]
```

$$\frac{\pi^2}{54}$$

```
Sum[1/((4k)^2), {k, 1, Infinity}]
```

$$\frac{\pi^2}{96}$$

```
Sum[1/((3k+1)^2), {k, 1, Infinity}]
```

$$\frac{1}{9}\,\mathrm{PolyGamma}\left[1,\frac{4}{3}\right]$$

```
Sum[1/((4k+1)^2), {k, 1, Infinity}]
```

$$\frac{1}{16}\left(-16+8\,\mathrm{Catalan}+\pi^2\right)$$

```
Sum[1/((3k+2)^2), {k, 1, Infinity}]
```

$$\frac{1}{9}\,\mathrm{PolyGamma}\left[1,\frac{5}{3}\right]$$

```
Sum[1/((4k+2)^2), {k, 1, Infinity}]
```

$$\frac{1}{32}\left(-8+\pi^2\right)$$

```
Sum[1/((4k+3)^2), {k, 1, Infinity}]
```

$$\frac{1}{144}\left(-16-72\,\mathrm{Catalan}+9\,\pi^2\right)$$

```
Sum[(-1)^(k+1) 1/((3k)^2), {k, 1, Infinity}]
```

$$\frac{\pi^2}{108}$$

```
Sum[(-1)^(k+1) 1/((3k)^3), {k, 1, Infinity}]
```

$$\frac{\mathrm{Zeta}[3]}{36}$$

```
Sum[(-1)^(k+1) 1/((3k+1)^2), {k, 1, Infinity}]
```

$$\frac{1}{36}\left(\mathrm{Zeta}\left[2,\frac{2}{3}\right]-\mathrm{Zeta}\left[2,\frac{7}{6}\right]\right)$$

```
Sum[(-1)^(k+1) 1/((2k-1)^3), {k, 1, Infinity}]
```

```
N[π^3/32]
```

0.968946

```
N[1 / (Sum[ (Mod[k, 2] - Mod[k - 1, 2]) 1 / ((2 k - 1) ^3), {k, 1, Infinity}] / Pi^3)]
```

32.

$$N\left[\frac{1}{128}\left(2\pi^3 + \text{PolyGamma}\left[2, \frac{3}{4}\right] + 56\,\text{Zeta}[3]\right)\right]$$

0.968946

```
N[1 / (Sum[ (Mod[k, 2] - Mod[k - 1, 2]) 1 / ((2 k - 1) ^3), {k, 1, Infinity}] / Pi^3)]
```

32.

```
t[n_, a_] := Mod[n, a] - Mod[n - 1, a]
Sum[(-1) ^ (k + 1) / k, {k, 1, Infinity}]
```

Log[2]

```
Sum[(Mod[k, 2] - Mod[k - 1, 2]) / k, {k, 1, Infinity}]
```

$$\sum_{k=1}^{\infty} \frac{-\text{Mod}[-1 + k, 2] + \text{Mod}[k, 2]}{k}$$

```
Sum[1 / (2 k - 1) - 1 / (2 k), {k, 1, Infinity}]
```

Log[2]

```
Sum[1 / (3 k) + 1 / (3 k + 1) - 2 / (3 k + 2), {k, 1, Infinity}]
```

$$\frac{1}{6}\left(\sqrt{3}\,\pi - 3\,\text{Log}[3]\right)$$

```
Sum[1 / (3 k - 2) + 1 / (3 k - 1) - 2 / (3 k), {k, 1, Infinity}]
```

Log[3]

$$N\left[\frac{1}{6}\left(\sqrt{3}\,\pi - 3\,\text{Log}[3]\right)\right]$$

0.357594

```
N[Log[3]]
```

1.09861

```
Sum[1 / (2 k) - 1 / (2 k + 1), {k, 1, Infinity}]
```

1 - Log[2]

```
Sum[1 / (3 k) + 1 / (3 k + 1) - 2 / (3 k + 2), {k, 1, Infinity}]
```

$$\frac{1}{6}\left(\sqrt{3}\,\pi - 3\,\text{Log}[3]\right)$$

```
Sum[1 / (4 k) + 1 / (4 k + 1) + 1 / (4 k + 2) - 3 / (4 k + 3), {k, 1, Infinity}]
```

$$\frac{1}{2}\left(-1+\pi-\text{Log}[4]\right)$$

```
Sum[1 / (3 k - 2) ^2 + 1 / (3 k - 1) ^2 - 2 / (3 k) ^2, {k, 1, Infinity}]
```

$$\frac{\pi^2}{9}$$

```
Sum[1 / (3 k - 2) ^3 + 1 / (3 k - 1) ^3 - 2 / (3 k) ^3, {k, 1, Infinity}]
```

$$\frac{8\,\text{Zeta}[3]}{9}$$

```
Sum[1 / (3 k - 2) ^4 + 1 / (3 k - 1) ^4 - 2 / (3 k) ^4, {k, 1, Infinity}]
```

$$\frac{13\,\pi^4}{1215}$$

```
Sum[ (-1) ^ (k + 1) 1 / ((2 k - 1) ^3), {k, 1, Infinity}]
```

$$\frac{\pi^3}{32}$$

```
Sum[ 1 / ((4 k - 3) ^3) - 1 / ((4 k - 1) ^3), {k, 1, Infinity}]
```

$$\frac{\pi^3}{32}$$

```
Sum[ 1 / ((4 k - 3) ^3), {k, 1, Infinity}]
Sum[ 1 / ((4 k - 2) ^3), {k, 1, Infinity}]
Sum[ 1 / ((4 k - 1) ^3), {k, 1, Infinity}]
Sum[ 1 / ((4 k - 0) ^3), {k, 1, Infinity}]
```

$$\frac{1}{64}\left(\pi^3+28\,\text{Zeta}[3]\right)$$

$$\frac{7\,\text{Zeta}[3]}{64}$$

$$\frac{1}{64}\left(-\pi^3+28\,\text{Zeta}[3]\right)$$

$$\frac{\text{Zeta}[3]}{64}$$

```
Sum[1/((8k-7)^3), {k, 1, Infinity}]
Sum[1/((8k-6)^3), {k, 1, Infinity}]
Sum[1/((8k-5)^3), {k, 1, Infinity}]
Sum[1/((8k-4)^3), {k, 1, Infinity}]
Sum[1/((8k-3)^3), {k, 1, Infinity}]
Sum[1/((8k-2)^3), {k, 1, Infinity}]
Sum[1/((8k-1)^3), {k, 1, Infinity}]
Sum[1/((8k-0)^3), {k, 1, Infinity}]
```

$$-\frac{\text{PolyGamma}\left[2, \frac{1}{8}\right]}{1024}$$

$$\frac{1}{512}\left(\pi^3 + 28\,\text{Zeta}[3]\right)$$

$$-\frac{\text{PolyGamma}\left[2, \frac{3}{8}\right]}{1024}$$

$$\frac{7\,\text{Zeta}[3]}{512}$$

$$-\frac{\text{PolyGamma}\left[2, \frac{5}{8}\right]}{1024}$$

$$\frac{1}{512}\left(-\pi^3 + 28\,\text{Zeta}[3]\right)$$

$$-\frac{\text{PolyGamma}\left[2, \frac{7}{8}\right]}{1024}$$

$$\frac{\text{Zeta}[3]}{512}$$

```
Expand[Sum[1/((8k-2)^3), {k, 1, Infinity}] - Sum[1/((8k-6)^3), {k, 1, Infinity}]]
```

$$-\frac{\pi^3}{256}$$

```
Sum[1/((6k-5)^3) + 1/((6k-3)^3) - 2/((6k-1)^3), {k, 1, Infinity}]
```

$$\frac{1}{36}\left(\sqrt{3}\,\pi^3 - 14\,\text{Zeta}[3]\right)$$

```
Sum[I/((4k-3)^3) - I/((4k-1)^3), {k, 1, Infinity}]
```

$$\frac{i\,\pi^3}{32}$$

```
Sum[1/((2k-1)^3) - 1/((2k)^3), {k, 1, Infinity}]
```

$$\frac{3\,\text{Zeta}[3]}{4}$$

```
Sum[1/((6k-5)^3) + 1/((6k-3)^3) - 3/((6k-1)^3), {k, 1, Infinity}]
```

$$\frac{1}{216}\left(8\sqrt{3}\,\pi^3 - 175\,\text{Zeta}[3]\right)$$

```
Sum[1/((6k-5)^3) + 1/((6k-3)^3) - 1/((6k-1)^3), {k, 1, Infinity}]
```

$$\frac{1}{216}\left(4\sqrt{3}\,\pi^3 + 7\,\text{Zeta}[3]\right)$$

```
Sum[1/((6k-5)^3) + 1/((6k-3)^3), {k, 1, Infinity}]
```

$$\frac{1}{108}\left(\sqrt{3}\,\pi^3 + 49\,\text{Zeta}[3]\right)$$

```
Sum[1/((6k-5)^3) + 1/((6k-2)^3), {k, 1, Infinity}]
```

$$\frac{1}{243}\left(2\sqrt{3}\,\pi^3 + 117\,\text{Zeta}[3]\right)$$

```
Sum[1/((6k-5)^3) - 1/((6k-2)^3), {k, 1, Infinity}]
```

$$\frac{1}{972}\left(10\sqrt{3}\,\pi^3 + 351\,\text{Zeta}[3]\right)$$

```
Sum[1/((6k-3)^3) + 1/((6k)^3), {k, 1, Infinity}]
```

$$\frac{\text{Zeta}[3]}{27}$$

```
Sum[1/((6k-4)^3) + 1/((6k-2)^3) - 2/((6k)^3), {k, 1, Infinity}]
```

$$\frac{\text{Zeta}[3]}{9}$$

```
Sum[1/((6k-4)^3) + 1/((6k-2)^3) - 2/((6k)^3), {k, 1, Infinity}]
Sum[1/((8k-7)^3) + 1/((8k-5)^3) + 1/((8k-3)^3) - 3/((8k-1)^3), {k, 1, Infinity}]
```

$$\frac{1}{256}\left(\text{PolyGamma}\left[2, \frac{7}{8}\right] + 224\,\text{Zeta}[3]\right)$$

```
Table[1/((6 k - 5)^3) + 1/((6 k - 3)^3) - 2/((6 k - 1)^3), {r, 1, 10}]
```

$$\Big\{\frac{1}{(-5+6k)^3}+\frac{1}{(-3+6k)^3}-\frac{2}{(-1+6k)^3},\ \frac{1}{(-5+6k)^3}+\frac{1}{(-3+6k)^3}-\frac{2}{(-1+6k)^3},$$
$$\frac{1}{(-5+6k)^3}+\frac{1}{(-3+6k)^3}-\frac{2}{(-1+6k)^3},\ \frac{1}{(-5+6k)^3}+\frac{1}{(-3+6k)^3}-\frac{2}{(-1+6k)^3},$$
$$\frac{1}{(-5+6k)^3}+\frac{1}{(-3+6k)^3}-\frac{2}{(-1+6k)^3},\ \frac{1}{(-5+6k)^3}+\frac{1}{(-3+6k)^3}-\frac{2}{(-1+6k)^3},$$
$$\frac{1}{(-5+6k)^3}+\frac{1}{(-3+6k)^3}-\frac{2}{(-1+6k)^3},\ \frac{1}{(-5+6k)^3}+\frac{1}{(-3+6k)^3}-\frac{2}{(-1+6k)^3},$$
$$\frac{1}{(-5+6k)^3}+\frac{1}{(-3+6k)^3}-\frac{2}{(-1+6k)^3},\ \frac{1}{(-5+6k)^3}+\frac{1}{(-3+6k)^3}-\frac{2}{(-1+6k)^3}\Big\}$$

```
Table[1/((4 k - 3)^3) - 1/((4 k - 1)^3), {r, 1, 10}]
```

$$\Big\{\frac{1}{(-3+4k)^3}-\frac{1}{(-1+4k)^3},\ \frac{1}{(-3+4k)^3}-\frac{1}{(-1+4k)^3},$$
$$\frac{1}{(-3+4k)^3}-\frac{1}{(-1+4k)^3},\ \frac{1}{(-3+4k)^3}-\frac{1}{(-1+4k)^3},$$
$$\frac{1}{(-3+4k)^3}-\frac{1}{(-1+4k)^3},\ \frac{1}{(-3+4k)^3}-\frac{1}{(-1+4k)^3},\ \frac{1}{(-3+4k)^3}-\frac{1}{(-1+4k)^3},$$
$$\frac{1}{(-3+4k)^3}-\frac{1}{(-1+4k)^3},\ \frac{1}{(-3+4k)^3}-\frac{1}{(-1+4k)^3},\ \frac{1}{(-3+4k)^3}-\frac{1}{(-1+4k)^3}\Big\}$$

```
Expand[Sum[1/((8 k - 2)^3), {k, 1, Infinity}] - Sum[1/((8 k - 6)^3), {k, 1, Infinity}]]
```

$$-\frac{\pi^3}{256}$$

```
Expand[Sum[1/((8 k - 0)^3), {k, 1, Infinity}] - Sum[1/((8 k - 4)^3), {k, 1, Infinity}]]
```

$$-\frac{3\,\text{Zeta}[3]}{256}$$

```
Expand[Sum[1/((8 k - 0)^3), {k, 1, Infinity}] - Sum[1/((8 k - 4)^3), {k, 1, Infinity}]]
```

```
Expand[Sum[1/((4 k - 3)^3), {k, 1, Infinity}] - Sum[1/((4 k - 1)^3), {k, 1, Infinity}]]
```

$$\frac{\pi^3}{32}$$

```
Sum[1/((6k-5)^3), {k, 1, Infinity}]
Sum[1/((6k-4)^3), {k, 1, Infinity}]
Sum[1/((6k-3)^3), {k, 1, Infinity}]
Sum[1/((6k-2)^3), {k, 1, Infinity}]
Sum[1/((6k-1)^3), {k, 1, Infinity}]
Sum[1/((6k-0)^3), {k, 1, Infinity}]
```

$$\frac{1}{216}\left(2\sqrt{3}\,\pi^3 + 91\,\text{Zeta}[3]\right)$$

$$\frac{2\sqrt{3}\,\pi^3 + 117\,\text{Zeta}[3]}{1944}$$

$$\frac{7\,\text{Zeta}[3]}{216}$$

$$\frac{-2\sqrt{3}\,\pi^3 + 117\,\text{Zeta}[3]}{1944}$$

$$\frac{1}{216}\left(-2\sqrt{3}\,\pi^3 + 91\,\text{Zeta}[3]\right)$$

$$\frac{\text{Zeta}[3]}{216}$$

```
Expand[Sum[1/((6k-5)^3), {k, 1, Infinity}] - Sum[1/((6k-1)^3), {k, 1, Infinity}]]
```

$$\frac{\pi^3}{18\sqrt{3}}$$

```
Expand[Sum[1/((6k-4)^3), {k, 1, Infinity}] - Sum[1/((6k-2)^3), {k, 1, Infinity}]]
```

$$\frac{\pi^3}{162\sqrt{3}}$$