This looks like an early mention of my exploring the partial sum equivalent of the log of dirichlet eta function. This later leads (once I figure out how to generalize it) to one of my two techniques for showing the difference between the logarithmic integral and the riemann prime counting function. My general sequence here was first noticing the identities I list below, then slowly trying to figure out how to generalize it, and then finally seeing the connection with the logarithmic integral.

The function written here as $D_{\eta,k}{}'(n)$ *I later write as* $[((1-2^{1-(0)})\zeta(0)-1)^k]_n$
The function written here as $\Pi_\eta(n)$ *I later write as* $[\log((1-2^{1-(0)})\zeta(0))]_n$
The function written here as $\Pi(n)$ *I later write as* $[\log\zeta(0)]_n$

Thus, $\Pi_\eta(n)+\sum_{k=1}^{\lfloor\log_2 n\rfloor}\frac{2^k}{k}=\Pi(n)$ *, in my more recent notation,*

$$[\log((1-2^{1-(0)})\zeta(0))]_n+\sum_{k=1}^{\lfloor\log_2 n\rfloor}\frac{2^k}{k}=[\log\zeta(0)]_n$$

Now, it's well known that $\sum_{k=1}^{\infty}\frac{x^k}{k}=-\log(1-x)$ *, for values of x that converge.*

If the partial sum equivalence is notated $\sum_{k=1}^{\lfloor\frac{\log n}{\log x}\rfloor}\frac{x^k}{k}=-[\log(1-x)]_n$ *, which obviously has no convergence issues, then that previous line becomes*

$$[\log((1-2^{1-(0)})\zeta(0))]_n-[\log(1-2^{1-(0)})]_n=[\log\zeta(0)]_n$$

Which is to say, the identity I write down below is something like a dirichlet convolution partial sum equivalent of

$$\log(a\cdot b)-\log a=\log b$$

$$D_{\eta,k}{}'(n)=\sum_{j=2}^{n}(-1)^{j+1}D_{\eta,k-1}{}'\left(\frac{n}{j}\right)$$
$$D_{\eta,0}{}'(n)=1$$
$$\Pi_\eta(n)=\sum_{k=1}^{\lfloor\log_2 n\rfloor}\frac{(-1)^{k+1}}{k}D_{\eta,k}{}'(n)$$

$$\Pi_\eta(n)+\sum_{k=1}^{\lfloor\log_2 n\rfloor}\frac{2^k}{k}=\Pi(n)$$

Assertion: $\Pi(n) - \sum_{k=1}^{\lfloor \log_2 n \rfloor} \frac{2^k}{k} \approx 0 \; when \; n = 2^{j+\frac{1}{2}}$

$$\Pi(n) \approx \sum_{k=1}^{\lfloor \log_2 n \rfloor} \frac{2^k}{k} \text{ when } n = 2^{j+\frac{1}{2}}$$