```
Expand[(a - 1)^s (a^k - 1) (k^(s - 1))]
```

$-(-1+a)^{s} k^{-1+s}+(-1+a)^{s} a^{k} k^{-1+s}$

```
FullSimplify[(a - 1)^s (a^k - 1) (k^(s - 1)) /. s → 0]
```

$$\frac{-1+a^{k}}{k}$$

```
FullSimplify[(a - 1)^s (a^k - 1) (k^(s - 1)) /. s → 1]
```

$(-1+a)\left(-1+a^{k}\right)$

```
FullSimplify[(a - 1)^s (a^k - 1) (k^(s - 1)) /. s → 2]
```

$(-1+a)^{2}\left(-1+a^{k}\right) k$

```
FullSimplify[(a - 1)^s (a^k - 1) (k^(s - 1)) /. s → 3]
```

$(-1+a)^{3}\left(-1+a^{k}\right) k^{2}$

```
FullSimplify[(a - 1)^s (a^k - 1) (k^(s - 1)) /. s → 4]
```

$(-1+a)^{4}\left(-1+a^{k}\right) k^{3}$

```
FullSimplify[(a - 1)^s (a^k - 1) (k^(s - 1)) /. s → 5]
```

$(-1+a)^{5}\left(-1+a^{k}\right) k^{4}$

```
Limit[ Sum[ ((a^2)^k - 1) / k, {k, 1, Log[a, 100]}], {a → 1}]
```

$$\left\{\mathrm{Limit}\left[-\mathrm{HarmonicNumber}\left[\frac{\mathrm{Log}[100]}{\mathrm{Log}[a]}\right]-\left(a^{2}\right)^{1+\frac{\mathrm{Log}[100]}{\mathrm{Log}[a]}} \mathrm{LerchPhi}\left[a^{2}, 1, 1+\frac{\mathrm{Log}[100]}{\mathrm{Log}[a]}\right]-\mathrm{Log}\left[1-a^{2}\right], a \to 1\right]\right\}$$

```
se[n_, a_, s_] := Sum[ ((a^(1 - s))^k - 1) / k, {k, 1, Log[a, n]}]
se2[n_, a_, t_] := Sum[ ((a^(1 - t))^k - 1) (k^(-1)), {k, 1, Log[a, n]}]
se1[n_, a_, t_] := Sum[ (-1 + a) (-1 + (a^(1 - t))^k), {k, 1, Log[a, n]}]
see[n_, a_, s_, t_] :=
 Sum[ (a^(1 - t) - 1)^s ((a^(1 - t))^k - 1) (k^(s - 1)), {k, 1, Log[a, n]}]
seeadd[n_, s_, t_] := (-1)^(s + 1) ((1 - t) Log[n])^s / s + Gamma[s]
```

```
s = 0; t = N[ZetaZero[1]]; {-see[nn = 10, 1.00001, 0, t] - EulerGamma - Log[(1 - t) Log[nn]],
 N[Gamma[s, -(1 - t) Log[nn]] + Pi I]}
```

$\{-0.0880018+3.10056\, i, -0.0880046+3.10063\, i\}$

```
s = 1; t = ZetaZero[1];
{see[nn = 100, 1.000001, s, t] + (1 - t) Log[nn] + 1, N[Gamma[s, -(1 - t) Log[nn]]]}
```

$\{-6.3663-7.71127\, i, -6.36665-7.71141\, i\}$

```
s = 2; t = -1;
{-see[nn = 100, 1.000005, s, t] - ((1 - t) Log[nn])^2 / 2 + 1, N[Gamma[s, -(1 - t) Log[nn]]]}
```

$\left\{-82104.4, -82103.4+1.00548\times 10^{-11}\, i\right\}$

```
s = 3; t = -1;
{see[nn = 100, 1.00001, s, t] + (1 - t)^3 Log[nn]^3 / 3 + 2, N[Gamma[s, -(1 - t) Log[nn]]]}
```

$\left\{684120., 684097.-1.67555\times 10^{-10}\, i\right\}$

```
s = 3; t = -1;
{see[nn = 100, 1.000005, s, t] + ((1 - t) Log[nn]) ^3 / 3 + 2, N[Gamma[s, -(1 - t) Log[nn]]]}
```

{684109., 684097. - 1.67555 × 10^-10 i}

```
s = 3; t = -1;
{see[nn = 100, 1.00001, s, t] + ((1 - t) Log[nn]) ^3 / 3 + 2, N[Gamma[s, -(1 - t) Log[nn]]]}
```

{684120., 684097. - 1.67555 × 10^-10 i}

```
s = 3; t = -1;
{see[nn = 100, 1.000004, s, t] + ((1 - t) Log[nn]) ^3 / 3 + 2, N[Gamma[s, -(1 - t) Log[nn]]]}
```

$Aborted

```
s = 3; t = ZetaZero[1];
{see[nn = 100, 1.00001, s, t] + (1 - t) ^3 Log[nn] ^3 / 3 + 2, N[Gamma[s, -(1 - t) Log[nn]]]}
```

{25640. + 33724. i, 25651.1 + 33732.7 i}

```
s = 3; t = ZetaZero[1];
{see[nn = 100, 1.000005, s, t] + (1 - t) ^3 Log[nn] ^3 / 3 + 2, N[Gamma[s, -(1 - t) Log[nn]]]}
```

{25645.5 + 33728.4 i, 25651.1 + 33732.7 i}

```
s = 2; t = ZetaZero[1];
{(-1) ^ (s + 1) see[nn = 100, 1.00001, s, t] + seeadd[nn, s, t], N[Gamma[s, -(1 - t) Log[nn]]]}
```

{510.153 - 404.167 i, 510.25 - 404.378 i}

```
s = 4; t = -1;
{(-1) ^ (s + 1) see[nn = 100, 1.00001, s, t] + seeadd[nn, s, t], N[Gamma[s, -(1 - t) Log[nn]]]}
```

{-5.76113 × 10^6, -5.76088 × 10^6 + 2.11651 × 10^-9 i}

```
se[n_, k_, s_, a_] := Gamma[k] + (-1) ^ (k + 1) ( ((1 - s) Log[n]) ^k / k +
     Sum[(a^ (1 - s) - 1) ^k ((a^ (1 - s)) ^j - 1) (j^ (k - 1)), {j, 1, Log[a, n]}] )
se[100, 2, -1, 1.000005]
```

-82104.4

```
se2[n_, k_, s_, a_] := (-1) ^k ( ((1 - s) Log[n]) ^k / k +
    Sum[(a^ (1 - s) - 1) ^k ((a^ (1 - s)) ^j - 1) (j^ (k - 1)), {j, 1, Log[a, n]}] )
{se2[100, 3, 0, 1.000005], N[Gamma[3, 0, -(1 - (0)) Log[100]]]}
```

{-1397.74, -1397.73 + 3.42834 × 10^-13 i}