

```

Clear[pxa, pza, pxi, po, pzam]
t[n_, a_, b_] := b (Floor[n/b] - Floor[(n-1)/b]) - a (Floor[n/a] - Floor[(n-1)/a])
pxa[n_, a_, b_, k_] := pxa[n, a, b, k] = Sum[t[j, a, b] / j pxa[n-j, a, b, k-1], {j, 1, n-1}]
pxa[n_, a_, b_, 1] := t[n, a, b] / n
pxa[n_, a_, b_, 0] := If[n == 0 && a == b, 1, 0]
pza[n_, a_, b_, z_] := Sum[z^k / k! pxa[n, a, b, k], {k, 0, n}]
pza[0, a_, b_, z_] := 1
pzaf[n_, a_, b_, z_] := Sum[pza[j, a, b, z], {j, 0, n}]

```

```

pxam[n_, r_, a_, b_, k_] :=
  pxam[n, r, a, b, k] = Sum[r t[j, a, b] / j pxam[n-j, r, a, b, k-1], {j, 1, n-1}]
pxam[n_, r_, a_, b_, 1] := t[n, a, b] / n
pxam[n_, r_, a_, b_, 0] := If[n == 0 && a == b, 1, 0]
pzam[n_, r_, a_, b_, z_] := Sum[z^k / k! pxam[n, r, a, b, k], {k, 0, n}]
pzam[0, r_, a_, b_, z_] := 1

```

```

padd[n_, a_, b_, z_] := Sum[pza[n-j, a, b, z-1], {j, 0, a-1}]
psa[n_, a_, b_, z_] := Sum[pza[j, a, b, z], {j, 0, n}]

```

```

pxi[n_, k_] := pxi[n, k] = Sum[1 / j pxi[n-j, k-1], {j, 1, n-1}]
pxi[n_, 1] := 1 / n
pxi[n_, 0] := 0
pzi[n_, z_] := Sum[z^k / k! pxi[n, k], {k, 0, n}]
pzi[0, z_] := 1
po[z_, n_] := po[z, n] = Pochhammer[z, n] / (n!)
po2[z_, n_] := (-1)^n Binomial[-z, n]
mul[m_, z_, n_] := Sum[Expand[po[z+1, n-m k] po[-z, k]], {k, 0, n/m}]

```

```

Table[pxa[n, 1, 2, 1], {n, 1, 10}]

```

$$\left\{-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, -\frac{1}{5}, \frac{1}{6}, -\frac{1}{7}, \frac{1}{8}, -\frac{1}{9}, \frac{1}{10}\right\}$$

```

Table[pxa[n, 2, 1, 1], {n, 1, 10}]

```

$$\left\{1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \frac{1}{7}, -\frac{1}{8}, \frac{1}{9}, -\frac{1}{10}\right\}$$

```

Table[pxa[n, 3, 1, 1], {n, 1, 10}]

```

$$\left\{1, \frac{1}{2}, -\frac{2}{3}, \frac{1}{4}, \frac{1}{5}, -\frac{1}{3}, \frac{1}{7}, \frac{1}{8}, -\frac{2}{9}, \frac{1}{10}\right\}$$

```

Table[pxa[n, 1, 3, 1], {n, 1, 10}]

```

$$\left\{-1, -\frac{1}{2}, \frac{2}{3}, -\frac{1}{4}, -\frac{1}{5}, \frac{1}{3}, -\frac{1}{7}, -\frac{1}{8}, \frac{2}{9}, -\frac{1}{10}\right\}$$

```

Table[pxa[n, 2, 2, 1], {n, 1, 10}]

```

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$$

```
Table[pza[n, k, 1, 1], {k, 1, 6}, {n, 0, 20}] // TableForm
```

1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0

```
Table[pza[n, 1, k, 1], {k, 1, 6}, {n, 0, 20}] // TableForm
```

1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
0	-1	0	1	-1	0	1	-1	0	1	-1	0	1	-1	0	1	-1	0	1	-1
0	-1	0	0	1	-1	0	0	1	-1	0	0	1	-1	0	0	1	-1	0	0
0	-1	0	0	0	1	-1	0	0	0	1	-1	0	0	0	1	-1	0	0	0
0	-1	0	0	0	0	1	-1	0	0	0	0	1	-1	0	0	0	0	0	0

```
pza[7, 3, 1, z]
```

$$\frac{z}{7} - \frac{2z^2}{5} + \frac{131z^3}{360} - \frac{5z^4}{48} - \frac{z^5}{144} + \frac{z^6}{240} + \frac{z^7}{5040}$$

```
Sum[pza[k, 2, 1, z], {k, 0, 100}] /. z -> 2.3
```

```
3.92458
```

```
2^2.3 - 1
```

```
3.92458
```

```
pza[0, 2, 1, z]
```

```
1
```

```
D[pza[32, 2, 1, z], z] /. z -> 0
```

$$-\frac{1}{32}$$

```
Expand[D[padd[32, 2, 1, z], z]] /. z -> 0
```

$$-\frac{1}{32}$$

```
D[N@Sum[t[j, 3, 1] pza[j, 3, 1, z], {j, 0, 100}], z] /. z -> 0
```

```
9.27618
```

```
psa[5, 2, 1, 12]
```

```
1586
```

```
Sum[pza[j, 2, 1, 12], {j, 0, 5}]
```

```
1586
```

```
Sum[pza[12 - j, 2, 1, 12], {j, 0, 5}]
```

```
1586
```

```

Sum[ pza[12 - j, 2, 1, 12], {j, 7, 12}]
1586
psa[12, 2, 1, 12] - psa[6, 2, 1, 12]
1586
psa[5, 3, 1, 12]
5096
psa[24, 3, 1, 12] - psa[12, 3, 1, 12]
228 826
Sum[ pza[j, 3, 1, 12], {j, 0, 5}]
5096
Sum[ pza[24 - j, 3, 1, 12], {j, 0, 5}]
5096
Sum[ pza[j, 3, 1, 12], {j, 19, 24}]
5096
psa[24, 3, 1, 12] - psa[18, 3, 1, 12]
5096
psa[8, 3, 1, 3]
27
psa[8, 3, 1, 2] + psa[7, 3, 1, 2] + psa[6, 3, 1, 2]
27
pzi[4, z]

$$\frac{z}{4} + \frac{11 z^2}{24} + \frac{z^3}{4} + \frac{z^4}{24}$$

Expand[Pochhammer[z, 4] / 4!]

$$\frac{z}{4} + \frac{11 z^2}{24} + \frac{z^3}{4} + \frac{z^4}{24}$$

Expand[Sum[ pzi[j, z] pzi[k, -z], {j, 0, 8}, {k, 0, (8 - j) / 3}]]

$$1 + \frac{341 z}{280} + \frac{799 z^2}{2016} + \frac{47 z^3}{160} + \frac{63 z^4}{640} - \frac{z^5}{80} + \frac{z^6}{192} + \frac{z^7}{1120} + \frac{z^8}{40320}$$

Expand@Sum[psa[k, 3, 1, z], {k, 0, 8}]

$$1 + \frac{341 z}{280} + \frac{799 z^2}{2016} + \frac{47 z^3}{160} + \frac{63 z^4}{640} - \frac{z^5}{80} + \frac{z^6}{192} + \frac{z^7}{1120} + \frac{z^8}{40320}$$

Expand[Sum[ pzi[j, z] pzi[k, -z], {k, 0, 8 / 3}, {j, 0, 8 - 3 k}]]

$$1 + \frac{341 z}{280} + \frac{799 z^2}{2016} + \frac{47 z^3}{160} + \frac{63 z^4}{640} - \frac{z^5}{80} + \frac{z^6}{192} + \frac{z^7}{1120} + \frac{z^8}{40320}$$

Expand[Sum[pzi[8 - 3 k, z + 1] pzi[k, -z], {k, 0, 8 / 3}]]

$$1 + \frac{341 z}{280} + \frac{799 z^2}{2016} + \frac{47 z^3}{160} + \frac{63 z^4}{640} - \frac{z^5}{80} + \frac{z^6}{192} + \frac{z^7}{1120} + \frac{z^8}{40320}$$


```

Expand[Sum[pzi[j, z] pzi[k, -z], {j, 0, 11}, {k, 0, (11 - j) / 4}]]

$$1 + \frac{42131z}{27720} + \frac{38873z^2}{50400} + \frac{58067z^3}{129600} + \frac{5465z^4}{24192} + \frac{20777z^5}{725760} + \frac{7z^6}{3200} + \frac{4111z^7}{1209600} + \frac{5z^8}{8064} + \frac{z^9}{20736} + \frac{z^{10}}{604800} + \frac{z^{11}}{39916800}$$

Expand@Sum[pza[k, 4, 1, z], {k, 0, 11}]

$$1 + \frac{42131z}{27720} + \frac{38873z^2}{50400} + \frac{58067z^3}{129600} + \frac{5465z^4}{24192} + \frac{20777z^5}{725760} + \frac{7z^6}{3200} + \frac{4111z^7}{1209600} + \frac{5z^8}{8064} + \frac{z^9}{20736} + \frac{z^{10}}{604800} + \frac{z^{11}}{39916800}$$

Expand[Sum[pzi[11 - 4 k, z] pzi[k, -z], {k, 0, Floor[11 / 4]}]]

$$\frac{z}{11} - \frac{109z^2}{2520} - \frac{5737z^3}{50400} + \frac{10001z^4}{181440} + \frac{8383z^5}{362880} - \frac{419z^6}{34560} - \frac{37z^7}{172800} + \frac{31z^8}{120960} + \frac{z^9}{30240} + \frac{z^{10}}{725760} + \frac{z^{11}}{39916800}$$

j + 4 k = 11 : (11, 0), (7, 1), (3, 2)

pza[11, 4, 1, z]

$$\frac{z}{11} - \frac{109z^2}{2520} - \frac{5737z^3}{50400} + \frac{10001z^4}{181440} + \frac{8383z^5}{362880} - \frac{419z^6}{34560} - \frac{37z^7}{172800} + \frac{31z^8}{120960} + \frac{z^9}{30240} + \frac{z^{10}}{725760} + \frac{z^{11}}{39916800}$$

FullSimplify@pzi[3, -z]

$$-\frac{1}{6}(-2+z)(-1+z)z$$

Expand[Sum[pzi[11 - 4 k, z] pzi[k, -z], {k, 0, Floor[11 / 4]}]]

$$\frac{z}{11} - \frac{109z^2}{2520} - \frac{5737z^3}{50400} + \frac{10001z^4}{181440} + \frac{8383z^5}{362880} - \frac{419z^6}{34560} - \frac{37z^7}{172800} + \frac{31z^8}{120960} + \frac{z^9}{30240} + \frac{z^{10}}{725760} + \frac{z^{11}}{39916800}$$

pza[11, 4, 1, z]

$$\frac{z}{11} - \frac{109z^2}{2520} - \frac{5737z^3}{50400} + \frac{10001z^4}{181440} + \frac{8383z^5}{362880} - \frac{419z^6}{34560} - \frac{37z^7}{172800} + \frac{31z^8}{120960} + \frac{z^9}{30240} + \frac{z^{10}}{725760} + \frac{z^{11}}{39916800}$$

`D[Sum[pzi[11 - 4 k, z] pzi[k, -z], {k, 0, 11 / 4}], {z, 2}] /. z -> 0`

$$-\frac{109}{1260}$$

`pxa[11, 4, 1, 2]`

$$-\frac{109}{1260}$$

`mul[.7, 3.3 + I, 20]`

`0.288794 - 0.1076 i`

`(.7) ^ (3.3 + I)`

`0.288797 - 0.107609 i`

`80 / 11.7`

`6.83761`

`ff[n_] := Sum[(-1) ^ (j + 1) / j, {j, 1, n}]`

`gg[n_] := HarmonicNumber[n]`

`gg[100] - gg[50]`

`47 979 622 564 155 786 918 478 609 039 662 898 122 617`
`69 720 375 229 712 477 164 533 808 935 312 303 556 800`

`ff[100]`

`47 979 622 564 155 786 918 478 609 039 662 898 122 617`
`69 720 375 229 712 477 164 533 808 935 312 303 556 800`

`Sum[ff[100 / 2 ^ k], {k, 0, Floor[Log2@100]}]`

`14 466 636 279 520 351 160 221 518 043 104 131 447 711`
`2 788 815 009 188 499 086 581 352 357 412 492 142 272`

`HarmonicNumber[100]`

`14 466 636 279 520 351 160 221 518 043 104 131 447 711`
`2 788 815 009 188 499 086 581 352 357 412 492 142 272`

`Table[pza[k, 3, 1, k], {k, 0, 10}]`

`{1, 1, 3, 7, 19, 51, 141, 393, 1107, 3139, 8953}`

`Sum[n / (3 × 2 ^ k), {k, 0, Infinity}]`

$$\frac{2 n}{3}$$

Product[(n / (2^k)) / (n / (3 × 2^k)), {k, 0, Infinity}]

Product::div : Product does not converge. >>

FullSimplify[Sum[pza[j, 2, 1, z] pza[k, 2, 1, z] pza[l, 2, 1, z] pza[m, 2, 1, z] pza[n, 2, 1, z],
{j, 0, 5}, {k, 0, (5 - j) / 2}, {l, 0, (5 - j - 2 k) / 4},
{m, 0, (5 - j - 2 k - 4 l) / 8}, {n, 0, (5 - j - 2 k - 4 l - 8 m) / 16}]]

$$\frac{1}{120} (1 + z) (2 + z) (3 + z) (4 + z) (5 + z)$$

FullSimplify[Sum[pza[j, 2, 1, z] pza[k, 2, 1, z] pza[l, 2, 1, z],
{j, 0, 5}, {k, 0, (5 - j) / 2}, {l, 0, (5 - j - 2 k) / 4}]]

$$\frac{1}{120} (1 + z) (2 + z) (3 + z) (4 + z) (5 + z)$$

FullSimplify[Sum[pza[j, 3, 1, z] pza[k, 3, 1, z] pza[l, 3, 1, z],
{j, 0, 5}, {k, 0, (5 - j) / 3}, {l, 0, (5 - j - 3 k) / 9}]]

$$\frac{1}{120} (1 + z) (2 + z) (3 + z) (4 + z) (5 + z)$$

FullSimplify[Sum[pzi[6 - 2 k, z] pzi[k, -z], {k, 0, 6 / 2}]]

$$\frac{1}{720} (-5 + z) (-4 + z) (-3 + z) (-2 + z) (-1 + z) z$$

FullSimplify[Sum[Log[2] / 2^k, {k, 0, n}] - Sum[Log[2] / (3 × 2^k), {k, 0, n}]]

$$\frac{2}{3} (2 - 2^{-n}) \text{Log}[2]$$

$$N\left[\frac{2}{3} (2 - 2^{-n}) \text{Log}[2] /. n \rightarrow 100\right]$$

$$0.924196$$

D[pza[8, 2, 1, z] + pza[4, 2, 1, z] + pza[2, 2, 1, z] + pza[1, 2, 1, z], z] /. z → 0

$$\frac{1}{8}$$

D[pzi[8, z], z] /. z → 0

$$\frac{1}{8}$$

D[pzi[8, z] - pzi[Floor[8 / 3], z], z] /. z → 0

$$-\frac{3}{8}$$

D[pza[8, 3, 1, z], z] /. z → 0

$$\frac{1}{8}$$

D[pzaf[8, 2, 1, z] + pzaf[4, 2, 1, z], z] /. z → 0

$$\frac{341}{280}$$

D[pzaf[8, 4, 1, z], z] /. z → 0

$$\frac{341}{280}$$

D[pzaf[8, 2, 1, z] + pzaf[4, 2, 1, z] + pzaf[2, 2, 1, z], z] /. z → 0

$$\frac{481}{280}$$

D[pzaf[8, 8, 1, z], z] /. z → 0

$$\frac{481}{280}$$

Expand[Sum[pza[11 - 3 k, 3, 1, z] pza[k, 3, 1, z], {k, 0, Floor[11 / 3]}]]

$$\frac{z}{11} - \frac{589 z^2}{2520} - \frac{9097 z^3}{50400} + \frac{7645 z^4}{36288} + \frac{31063 z^5}{362880} + \frac{781 z^6}{34560} + \frac{683 z^7}{172800} + \frac{11 z^8}{24192} + \frac{z^9}{30240} + \frac{z^{10}}{725760} + \frac{z^{11}}{39916800}$$

pza[11, 9, 1, z]

$$\frac{z}{11} - \frac{589 z^2}{2520} - \frac{9097 z^3}{50400} + \frac{7645 z^4}{36288} + \frac{31063 z^5}{362880} + \frac{781 z^6}{34560} + \frac{683 z^7}{172800} + \frac{11 z^8}{24192} + \frac{z^9}{30240} + \frac{z^{10}}{725760} + \frac{z^{11}}{39916800}$$

**FullSimplify[Sum[pza[j, 4, 1, z] pza[k, 4, 1, -z] pza[l, 4, 1, z] pza[m, 4, 1, z],
{j, 0, 5}, {k, 0, (5 - j) / 2}, {l, 0, (5 - j - 2 k) / 4}, {m, 0, (5 - j - 2 k - 4 l) / 8}]]**

$$\frac{1}{120} (1 + z) (120 + (-1 + z) z (26 + (-5 + z) z))$$

FullSimplify@pzaf[5, 2, 1, z]

$$\frac{1}{120} (1 + z) (120 + (-1 + z) z (26 + (-5 + z) z))$$

**FullSimplify[Sum[pza[j, 2, 1, z] pza[k, 2, 1, -z] pza[l, 2, 1, z] pza[m, 2, 1, z],
{j, 0, 5}, {k, 0, (5 - j) / 3}, {l, 0, (5 - j - 3 k) / 2}, {m, 0, (5 - j - 3 k - 2 l) / 6}]]**

$$\frac{1}{120} (1 + z) (4 + z) (30 + z (-29 + z (10 + z)))$$

FullSimplify@pza[5, 3, 1, z]

$$\frac{1}{120} (-2 + z) (-1 + z) z (1 + z) (12 + z)$$

Sum[3^z / 3^k, {k, 0, Infinity}]

$$\frac{3^{1+z}}{2}$$

N@mul[1.5, 3, 6]

3.375

1.5^3

3.375

pza[5, 2, 1, z]

$$\frac{z}{5} - \frac{5z^2}{12} + \frac{7z^3}{24} - \frac{z^4}{12} + \frac{z^5}{120}$$

pzam[5, 2, 2, 1, z]

$$\frac{z}{5} - \frac{5z^2}{6} + \frac{7z^3}{6} - \frac{2z^4}{3} + \frac{2z^5}{15}$$

Expand@pza[4, 10 000, 1, z]

$$\frac{z}{4} + \frac{11z^2}{24} + \frac{z^3}{4} + \frac{z^4}{24}$$

Expand@pzam[4, 2, 10 000, 1, z]

$$\frac{z}{4} + \frac{11z^2}{12} + z^3 + \frac{z^4}{3}$$

Expand@Sum[pza[k, 10 000, 1, z + 1], {k, 0, 4}]

$$5 + \frac{77z}{12} + \frac{71z^2}{24} + \frac{7z^3}{12} + \frac{z^4}{24}$$

Sum[po2[z, k], {k, 0, n}]

$$-\frac{(-1)^n (1+n) \text{Binomial}[-z, 1+n]}{z}$$

$$\text{Sum}\left[-\frac{(-1)^n (1+n) \text{Binomial}[-z, 1+n]}{z}, \{n, 0, k\}\right] /. k \rightarrow n$$

$$\frac{(-1)^n (1+n) (2+n) \text{Binomial}[-z, 2+n]}{z (1+z)}$$

FullSimplify[Sum[Pochhammer[z, k] / k!, {k, 0, n}]]

$$\frac{\text{Gamma}[1+n+z]}{\text{Gamma}[1+n] \text{Gamma}[1+z]}$$

$$\text{FullSimplify}\left[\text{Sum}\left[\frac{\text{Gamma}[1+n+z]}{\text{Gamma}[1+n] \text{Gamma}[1+z]}, \{n, 0, k\}\right] /. k \rightarrow n\right]$$

$$\frac{\text{Gamma}[2+n+z]}{\text{Gamma}[1+n] \text{Gamma}[2+z]}$$

$$\text{FullSimplify}\left[\text{Sum}\left[\frac{\text{Gamma}[2+n+z]}{\text{Gamma}[1+n] \text{Gamma}[2+z]}, \{n, 0, k\}\right] /. k \rightarrow n\right]$$

$$\frac{\text{Gamma}[3+n+z]}{\text{Gamma}[1+n] \text{Gamma}[3+z]}$$

Pochhammer[(z + 1), n] / n! /. {z -> 3, n -> 14}

680


```

Gamma[1 + n + z]
----- /. {z -> 3, n -> 14}
Gamma[1 + n] Gamma[1 + z]
680

Pochhammer[(z + 2), n] / n! /. {z -> 3, n -> 14}
3060

Gamma[2 + n + z]
----- /. {z -> 3, n -> 14}
Gamma[1 + n] Gamma[2 + z]
3060

Pochhammer[(z + 3), n] / n! /. {z -> 3.7, n -> 14}
27 337.4

Gamma[3 + n + z]
----- /. {z -> 3.7, n -> 14}
Gamma[1 + n] Gamma[3 + z]
27 337.4

Sum[Pochhammer[3.4, n] / n!, {n, 0, 12}]
810.357

Pochhammer[4.4, 12] / (12!)
810.357

pza[5, 2, 1, z]
z 5 z^2 7 z^3 z^4 z^5
-- - + - - +
5 12 24 12 120
Expand@Sum[pza[n, 2, 1, z - 1], {n, 0, 5}]
23 z 11 z^2 11 z^3 z^4 z^5
-- - + - - +
15 12 24 12 120
Table[po[z + 1, 10 - 2 k] po[-z, k], {k, 0, 5}, {z, 0, 5}] // TableForm
1 11 66 286 1001 3003
0 -9 -90 -495 -1980 -6435
0 0 28 252 1260 4620
0 0 0 -35 -280 -1260
0 0 0 0 15 105
0 0 0 0 0 -1
Expand@Sum[(Log[2] - Log[2] / 3) / 2^k, {k, 0, Infinity}]
4 Log[2]
3

ft[n_] := Sum[(-1)^(j + 1) / j, {j, 1, n}]
ft2[n1_, n2_] := Sum[(-1)^(j + 1) / j, {j, n1, n2}]
fta[n1_, n2_, a_] := Sum[t[j, a, 1] / j, {j, n1, n2}]
tt[n_] := Sum[ft[n / 2^k] - ft[n / (3 * 2^k)], {k, 0, Log2@n}]
tt2[n_] := Sum[ft2[Floor[n / (3 * 2^k)] + 1, n / 2^k], {k, 0, Log2@n}]
tta[n_, a_, b_] := Sum[fta[Floor[n / (b a^k)] + 1, n / a^k, a], {k, 0, Log[a, n]}}
N@tt2[1000.]
1.09861

```

```

N@tta[16 000., 4, 2]

0.693116

Log[2.]

0.693147


FullSimplify[(1 - x^10) / (1 - x)]


$$\frac{1 - x^{10}}{1 - x}$$


Table[pza[n, 2, 1, 1], {n, 0, 6}]

{1, 1, 0, 0, 0, 0, 0}

Table[pzi[n, 1], {n, 1, 6}]

{1, 1, 1, 1, 1, 1}

Sum[pza[j, 2, 1, 3] pza[6 - j, 2, 1, 4], {j, 0, 6}]

7

pza[6, 2, 1, 7]

7

Sum[pzi[j, 3] pzi[6 - j, 4], {j, 0, 6}]

924

pzi[6, 7]

924

CoefficientList[Series[1 / (1 + x), {x, 0, 10}], x]

{1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1}

Clear[pk]
pk[0, 0] := 1
pk[n_, 0] := 1
pk[0, 1] := 1
pk[n_, 1] := 0
pk[0, 2] := 1
pk[1, 2] := 0
pk[2, 2] := -1
pk[n_, 2] := 0
pk[n_, k_] := pk[n, k] = Sum[pk[j, 1] pk[n - j, k - 1], {j, 0, n}]
Table[pk[j, 3], {j, 0, 3}]

{1, 0, -1, 0}

Table[pk[k, j], {j, 0, 12}, {k, 0, j}] // TableForm

```

```

1
1 0
1 0 -1
1 0 -1 0
1 0 -1 0 0
1 0 -1 0 0 0
1 0 -1 0 0 0 0
1 0 -1 0 0 0 0 0
1 0 -1 0 0 0 0 0 0
1 0 -1 0 0 0 0 0 0 0
1 0 -1 0 0 0 0 0 0 0 0
1 0 -1 0 0 0 0 0 0 0 0 0
1 0 -1 0 0 0 0 0 0 0 0 0 0

```

```
pk[2, 1]
```

```
0
```

```
CoefficientList[Series[(1 + x^2)^4, {x, 0, 10}], x]
```

```
{1, 0, 4, 0, 6, 0, 4, 0, 1}
```

```
CoefficientList[Series[Log[(1 + x + x^2)], {x, 0, 10}], x]
```

```
{0, 1, 1/2, -1/3, 1/4, 1/5, -1/3, 1/7, 1/8, -2/9, 1/10}
```

```
CoefficientList[Series[Log[(1 - x^3) / (1 - x)], {x, 0, 10}], x]
```

```
{0, 1, 1/2, -1/3, 1/4, 1/5, -1/3, 1/7, 1/8, -2/9, 1/10}
```

```
Limit[Log[(1 - x^3) / (1 - x)], x -> 1]
```

```
Log[3]
```

```
CoefficientList[Series[Log[(1 - x) / (1 - x^5)], {x, 0, 10}], x]
```

```
{0, -1, -1/2, -1/3, -1/4, 1/5, -1/6, -1/7, -1/8, -1/9, 2/5}
```

```
Limit[Log[(1 - x) / (1 - x^3)], x -> 1]
```

```
-Log[3]
```

```
CoefficientList[Series[Log[(1 - x^3) / (1 - x^5)], {x, 0, 10}], x]
```

```
{0, 0, 0, -1, 0, 1, -1/2, 0, 0, -1/3, 1/2}
```

```
Limit[Log[(1 - x^5) / (1 - x^3)], x -> 1]
```

```
Log[5/3]
```

```
Limit[Log[(1 - x^4) / (1 - x^2)], x -> 1]
```

```
Log[2]
```

```
Expand[FullSimplify@Expand[(1 - x^2) / (1 - x) (1 - x^4) / (1 - x^2)]]
```

```
1 + x + x^2 + x^3
```

```
FullSimplify[(1 - x^6) / (1 - x^3)]
```

```
1 + x^3
```

```
FullSimplify[(1 - x^6) / (1 - x^2)]
```

```
1 + x^2 + x^4
```

```
Product[(1 + x^(2^k)), {k, 0, Infinity}]
```

```
$Aborted
```

```
Expand[(1 + x + x^2) (1 + x^3 + x^6) (1 + x^9 + x^18) (1 + x^27 + x^54)]
```

```
1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^10 + x^11 + x^12 + x^13 + x^14 + x^15 + x^16 + x^17 + x^18 + x^19 + x^20 + x^21 + x^22 +
x^23 + x^24 + x^25 + x^26 + x^27 + x^28 + x^29 + x^30 + x^31 + x^32 + x^33 + x^34 + x^35 + x^36 + x^37 + x^38 + x^39 + x^40 + x^41 +
x^42 + x^43 + x^44 + x^45 + x^46 + x^47 + x^48 + x^49 + x^50 + x^51 + x^52 + x^53 + x^54 + x^55 + x^56 + x^57 + x^58 + x^59 + x^60 +
x^61 + x^62 + x^63 + x^64 + x^65 + x^66 + x^67 + x^68 + x^69 + x^70 + x^71 + x^72 + x^73 + x^74 + x^75 + x^76 + x^77 + x^78 + x^79 + x^80
```

```
Sum[Binomial[z, k] ((-s) / (1 - x) - 1)^k, {k, 0, Infinity}]
```

```

$$\left( \frac{s}{-1 + x} \right)^z$$

```

```
FullSimplify[(-s / (1 - x)) / (-s / (1 - x^2))]
```

```
1 + x
```