

This looks like some more scrap write up that is superseded by my major write ups. Nothing too interesting here.

$$[(1-x^{1-s})\zeta_n(s)]^{*z} = \sum_{j=0} (-1)^j \binom{z}{j} x^{j(1-s)} [\zeta_{n \cdot x^{-j}}(s)]^{*z}$$

$$[(1-2^{1-s})\zeta_n(s)]^{*z} = \sum_{j=0} (-1)^j \binom{z}{j} 2^{j(1-s)} [\zeta_{\frac{n}{2^j}}(s)]^{*z}$$

$$\begin{aligned} [(1-2^{1-s})\zeta_n(s)]^{*z} = & [\zeta_n(s)]^{*z} + \\ & -z2^{1-s}[\zeta_{\frac{n}{2}}(s)]^{*z} + \\ & \frac{z(z-1)}{2}4^{1-s}[\zeta_{\frac{n}{4}}(s)]^{*z} + \\ & -\frac{z(z-1)(z-2)}{6}8^{1-s}[\zeta_{\frac{n}{8}}(s)]^{*z} \\ & + \dots \end{aligned}$$

$$\begin{aligned} [(1-2^{1-s})\zeta_n(s)]^{*z} = & [\zeta_n(s)]^{*z} + \\ & -\binom{z}{1}2^{1-s}[\zeta_{\frac{n}{2}}(s)]^{*z} + \\ & \binom{z}{2}4^{1-s}[\zeta_{\frac{n}{4}}(s)]^{*z} + \\ & -\binom{z}{3}8^{1-s}[\zeta_{\frac{n}{8}}(s)]^{*z} \\ & + \dots \end{aligned}$$

$$[(1-x^{1-s})\zeta_{\Delta n}(s)]^{*z} = \sum_{j=0} (-1)^j \binom{z}{j} x^{j(1-s)} (\lfloor \frac{n}{x^j} \rfloor - \lfloor \frac{n-1}{x^j} \rfloor) \cdot \lfloor \frac{n}{x^j} \rfloor^{-s} \cdot d_z(\frac{n}{x^j})$$

$$[(1-x^{\{1-s\}})\zeta_{\Delta n}(s)]^{\{z\}} = \sum_{j=0} (-1)^j \binom{z}{j} x^{j(1-s)} (\lfloor \frac{n}{x^j} \rfloor - \lfloor \frac{n-1}{x^j} \rfloor) \cdot \lfloor \frac{n}{x^j} \rfloor^{-s} \cdot d_z(\frac{n}{x^j})$$

$$e_z(n, x) = \sum_{j=0}^{\log_2 n} (-1)^j \binom{z}{j} x^{j(1-s)} (\lfloor \frac{n}{x^j} \rfloor - \lfloor \frac{n-1}{x^j} \rfloor) \cdot d_z(\frac{n}{x^j})$$

$$\begin{aligned} e_z(n, 2) = & \binom{z}{0} \cdot d_z(n) + \\ & -\binom{z}{1} 2^{1-s} (\lfloor \frac{n}{2} \rfloor - \lfloor \frac{n-1}{2} \rfloor) \cdot d_z(\frac{n}{2}) + \\ & \binom{z}{2} 4^{1-s} (\lfloor \frac{n}{4} \rfloor - \lfloor \frac{n-1}{4} \rfloor) \cdot d_z(\frac{n}{4}) + \\ & \dots \end{aligned}$$

$$\begin{aligned}
e_z(n,x) = & \\
& \binom{z}{0} \cdot d_z(n) + \\
& - \binom{z}{1} x \left(\lfloor \frac{n}{x} \rfloor - \lfloor \frac{n-1}{x} \rfloor \right) \cdot d_z\left(\frac{n}{x}\right) + \\
& \binom{z}{2} x^2 \left(\lfloor \frac{n}{x^2} \rfloor - \lfloor \frac{n-1}{x^2} \rfloor \right) \cdot d_z\left(\frac{n}{x^2}\right) + \\
& \dots
\end{aligned}$$

$$\begin{aligned}
\kappa e(n,x) = & \\
& \kappa(n) + \\
& - \frac{1}{2} x \left(\lfloor \frac{n}{x} \rfloor - \lfloor \frac{n-1}{x} \rfloor \right) \cdot \kappa\left(\frac{n}{x}\right) + \\
& \frac{1}{3} x^2 \left(\lfloor \frac{n}{x^2} \rfloor - \lfloor \frac{n-1}{x^2} \rfloor \right) \cdot \kappa\left(\frac{n}{x^2}\right) + \\
& \dots
\end{aligned}$$

$$[\zeta_n(s, y)]^{*k} = \sum_{j=0}^k \binom{k}{j} [\zeta_{n \cdot y^{j-k}}(s, 1+y)]^{*j}$$

$$[\zeta_n(s, 1+y)]^{*k} = \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} [\zeta_{n \cdot y^{j-k}}(s, y)]^{*j}$$

$$// ?$$

$$[y^{s-1} \cdot \zeta_n(s, 1+y)]^{*k} = y^{k(s-1)} [\zeta_{n \cdot y^k}(s, 1+y)]^{*k}$$

$$[\zeta_n(s, 1+y)]^{*1} = \sum_{j=1}^{\lfloor n-y \rfloor} (y+j)^{-s}$$

$$[\zeta_n(s, 1+y)]^{*2} = \sum_{j=1}^{\lfloor n-y \rfloor} \sum_{k=1}^{\lfloor \frac{n}{j+y} - y \rfloor} ((y+j) \cdot (y+k))^{-s}$$

$$[\zeta_n(s, 1+y)]^{*3} = \sum_{j=1}^{\lfloor n-y \rfloor} \sum_{k=1}^{\lfloor \frac{n}{j+y} - y \rfloor} \sum_{l=1}^{\lfloor \frac{n}{(y+j)(y+k)} - y \rfloor} ((y+j) \cdot (y+k) \cdot (y+l))^{-s}$$

$$[\zeta_n(s, 1+y)]^{*1} = y^{-s} \sum_{j=1}^{\lfloor n-y \rfloor} \left(1 + \frac{j}{y}\right)^{-s}$$

$$[\zeta_n(s, 1+y)]^{*2} = y^{-2s} \sum_{j=1}^{\lfloor n-y \rfloor} \sum_{k=1}^{\lfloor \frac{n}{j+y} - y \rfloor} \left(\left(1 + \frac{j}{y}\right) \cdot \left(1 + \frac{k}{y}\right)\right)^{-s}$$

$$[\zeta_n(s, 1+y)]^{*3} = y^{-3s} \sum_{j=1}^{\lfloor n-y \rfloor} \sum_{k=1}^{\lfloor \frac{n}{j+y} - y \rfloor} \sum_{l=1}^{\lfloor \frac{n}{(y+j)(y+k)} - y \rfloor} \left(\left(1 + \frac{j}{y}\right) \cdot \left(1 + \frac{k}{y}\right) \cdot \left(1 + \frac{l}{y}\right)\right)^{-s}$$

$$[y^{s-1} \cdot \zeta_n(s, 1+y)]^{*1} = y^{-1} \sum_{j=1}^{(n-1) \cdot y} (y^{-1} \cdot (y+j))^{-s}$$

$$[y^{s-1} \cdot \zeta_n(s, 1+y)]^{*2} = y^{-2} \sum_{j=1}^{(n-1) \cdot y} \sum_{k=1}^{(n-1) \cdot y} ((y^{-1} \cdot (y+j)) \cdot (y^{-1} \cdot (y+k)))^{-s}$$

$$[y^{s-1} \cdot \zeta_n(s, 1+y)]^{*3} = y^{-3} \sum_{j=1}^{(n-1) \cdot y} \sum_{k=1}^{(n-1) \cdot y} \sum_{l=1}^{(n-1) \cdot y} ((y^{-1} \cdot (y+j)) \cdot (y^{-1} \cdot (y+k)) \cdot (y^{-1} \cdot (y+l)))^{-s}$$

// need to keep track of how n changes better here, as that is crucial.

$$[y^{s-1} \cdot \zeta_n(s, 1+y)]^{*1} = y^{s-1} \sum_{j=1}^{(n-1) \cdot y} (y+j)^{-s}$$

$$[y^{s-1} \cdot \zeta_n(s, 1+y)]^{*2} = y^{2(s-1)} \sum_{j=1}^{(n-1) \cdot y} \sum_{k=1}^{(n-1) \cdot y} ((y+j) \cdot (y+k))^{-s}$$

$$[y^{s-1} \cdot \zeta_n(s, 1+y)]^{*3} = y^{3(s-1)} \sum_{j=1}^{(n-1) \cdot y} \sum_{k=1}^{(n-1) \cdot y} \sum_{l=1}^{(n-1) \cdot y} ((y+j) \cdot (y+k) \cdot (y+l))^{-s}$$

$$\begin{aligned}
& [1+y^{s-1} \cdot \zeta_n(s, 1+y)]^* z = \\
& \binom{z}{0} y^0 \\
& + \binom{z}{1} y^{-1} \sum_{j=1}^{(n-1) \cdot y} \left(1 + \frac{j}{y}\right)^{-s} \\
& + \binom{z}{2} y^{-2} \sum_{j=1} \sum_{k=1} \left(\left(1 + \frac{j}{y}\right) \cdot \left(1 + \frac{k}{y}\right)\right)^{-s} \\
& + \binom{z}{3} y^{-3} \sum_{j=1} \sum_{k=1} \sum_{l=1} \left(\left(1 + \frac{j}{y}\right) \cdot \left(1 + \frac{k}{y}\right) \cdot \left(1 + \frac{l}{y}\right)\right)^{-s} \\
& + \dots
\end{aligned}$$

$$\begin{aligned}
& [\log(1+y^{s-1} \cdot \zeta_n(s, 1+y))]^* 1 = \\
& y^{-1} \sum_{j=1}^{(n-1) \cdot y} \left(1 + \frac{j}{y}\right)^{-s} \\
& - \frac{1}{2} y^{-2} \sum_{j=1} \sum_{k=1} \left(\left(1 + \frac{j}{y}\right) \cdot \left(1 + \frac{k}{y}\right)\right)^{-s} \\
& + \frac{1}{3} y^{-3} \sum_{j=1} \sum_{k=1} \sum_{l=1} \left(\left(1 + \frac{j}{y}\right) \cdot \left(1 + \frac{k}{y}\right) \cdot \left(1 + \frac{l}{y}\right)\right)^{-s} \\
& - \dots
\end{aligned}$$

$$\begin{aligned}
& [1+y^{s-1} \cdot \zeta_n(s, 1+y)]^* z = \\
& \binom{z}{0} y^0 \\
& + \binom{z}{1} y^{(s-1)} \sum_{j=1}^{(n-1) \cdot y} (y+j)^{-s} \\
& + \binom{z}{2} y^{2(s-1)} \sum_{j=1} \sum_{k=1} ((y+j) \cdot (y+k))^{-s} \\
& + \binom{z}{3} y^{3(s-1)} \sum_{j=1} \sum_{k=1} \sum_{l=1} ((y+j) \cdot (y+k) \cdot (y+l))^{-s} \\
& + \dots
\end{aligned}$$

$$\begin{aligned}
& [\log(1+y^{s-1} \cdot \zeta_n(s, 1+y))]^* 1 = \\
& + y^{(s-1)} \sum_{j=1}^{(n-1) \cdot y} (y+j)^{-s} \\
& - \frac{1}{2} y^{2(s-1)} \sum_{j=1} \sum_{k=1} ((y+j) \cdot (y+k))^{-s} \\
& + \frac{1}{3} y^{3(s-1)} \sum_{j=1} \sum_{k=1} \sum_{l=1} ((y+j) \cdot (y+k) \cdot (y+l))^{-s} \\
& - \dots
\end{aligned}$$

$$[\zeta_n(0)]^{*z}=L_{-z}(\log n)-\int_1^{\infty}\frac{\partial}{\partial y}[1+y^{-1}\cdot\zeta_n(0,1+y)]^{*z}dy$$

$$\Pi(n)=\\ li(n)-\log\log n-y-\int_1^{\infty}\frac{\partial}{\partial y}[\log(1+y^{-1}\cdot\zeta_n(0,1+y))]^{*1}dy$$

$$\begin{aligned} [1+y^{-1}\cdot\zeta_n(0,1+y)]^{*z}=\\ \binom{z}{0}y^0\\ +\binom{z}{1}y^{-1}\sum_{j=1}^{(n-1)\cdot y}1\\ +\binom{z}{2}y^{-2}\sum_{j=1}\sum_{k=1}1\\ +\binom{z}{3}y^{-3}\sum_{j=1}\sum_{k=1}\sum_{l=1}1\\ +\ldots \end{aligned}$$

$$\begin{aligned} [\log(1+y^{s-1}\cdot\zeta_n(s,1+y))]^{*1}=\\ +y^{(s-1)}\sum_{j=1}^{(n-1)\cdot y}(y+j)^{-s}\\ -\frac{1}{2}y^{2(s-1)}\sum_{j=1}\sum_{k=1}((y+j)\cdot(y+k))^{-s}\\ +\frac{1}{3}y^{3(s-1)}\sum_{j=1}\sum_{k=1}\sum_{l=1}((y+j)\cdot(y+k)\cdot(y+l))^{-s}\\ -\ldots \end{aligned}$$

$$\begin{aligned}
[1+y^{s-1}\cdot\zeta_n(s,1+y)]^{*k} = \\
[1+y^{s-1}\cdot\zeta_n(s,1+y)]^{*k-1} + \\
y^{s-1}\cdot\sum_{j=1}(j+y)^{-s}[1+y^{s-1}\cdot\zeta_{n,y(j+y)^{-1}}(s,1+y)]^{*k-1}
\end{aligned}$$

$$[1+y^{s-1}\cdot\zeta_n(s,1+y)]^{*1}=1+y^{s-1}\cdot\sum_{j=1}(j+y)^{-s}$$

$$[1+y^{s-1}\cdot\zeta_n(s,1+y)]^{*2}=1+2\cdot y^{s-1}\cdot\sum_{j=1}(j+y)^{-s}+y^{2(s-1)}\sum_{j=1}\sum_{k=1}((j+y)(k+y))^{-s}$$