$$\boxed{\lim_{x \to 1^+} \sum_{k=1}^{\lfloor \frac{\log n}{\log x} \rfloor} \frac{x^{k(1-s)}-1}{k} = li\left(n^{1-s}\right) - \log\log n^{1-s} - \gamma}$$

$$\boxed{\int_1^n \frac{x^{-s}}{\log x} - \frac{1}{x \log x}\, dx = li\left(n^{1-s}\right) - \log\log n^{1-s} - \gamma}$$

$$[\log((1-x^{1-s})\zeta(s))]_n = -\sum_{k=1}^{\lfloor \frac{\log n}{\log x} \rfloor} \frac{x^{k(1-s)}}{k} + [\log\zeta(s)]_n$$

$$[\log((1-x^{1-s})\zeta(s))]_n + \sum_{k=1}^{\lfloor \frac{\log n}{\log x} \rfloor} \frac{x^{k(1-s)}}{k} = [\log\zeta(s)]_n$$

$$\lim_{x \to 1^+} \sum_{k=1}^{\lfloor \frac{\log n}{\log x} \rfloor} \frac{x^k-1}{k} = li(n) - \log\log n - \gamma$$

$$\lim_{x \to 1^+} \sum_{k=0}^{\lfloor \log_x n - \log_x t \rfloor} \frac{x^{k+\log_x t}}{k+\log_x t} = li(n)$$

$$\lim_{x \to 1^+} \sum_{k=1.4513680}^{\lfloor \frac{\log n}{\log x} \rfloor} \frac{x^k}{k} = li(n)$$

$$\lim_{x \to 1^+} \sum_{k=1}^{\lfloor \frac{\log n}{\log x} \rfloor} \frac{x^k}{k} = \infty$$