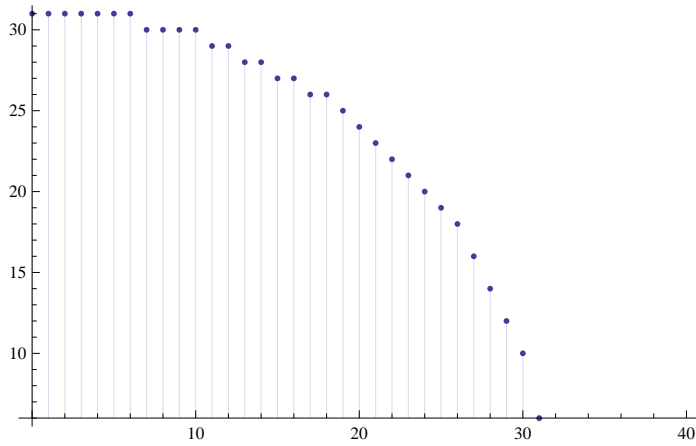


```
DiscretePlot[Floor[(1000 - j^2)^(1/2)], {j, 0, 40}]
```



```
Integrate[1, {t, 0, x^(1/2)}]
```

$$\sqrt{x}$$

```
Integrate[1, {t, 0, x^(1/2)}, {u, 0, (x - t^2)^(1/2)}]
```

$$\frac{\pi x}{4}$$

```
Integrate[1, {t, 0, x^(1/2)}, {u, 0, (x - t^2)^(1/2)}, {v, 0, (x - t^2 - u^2)^(1/2)}]
```

$$\frac{1}{6} \pi x^{3/2}$$

```
Integrate[1, {t, 0, x^(1/2)}, {u, 0, (x - t^2)^(1/2)}, {v, 0, (x - t^2 - u^2)^(1/2)}, {w, 0, (x - t^2 - u^2 - v^2)^(1/2)}]
```

$$\frac{\pi^2 x^2}{32}$$

```
Integrate[1, {t, 0, x^(1/2)}, {u, 0, (x - t^2)^(1/2)}, {v, 0, (x - t^2 - u^2)^(1/2)}, {w, 0, (x - t^2 - u^2 - v^2)^(1/2)}, {y, 0, (x - t^2 - u^2 - v^2 - w^2)^(1/2)}]
```

$$\frac{1}{60} \pi^2 x^{5/2}$$

```
Table[Gamma[j/2], {j, 1, 10}]
```

$$\left\{ \sqrt{\pi}, 1, \frac{\sqrt{\pi}}{2}, 1, \frac{3\sqrt{\pi}}{4}, 2, \frac{15\sqrt{\pi}}{8}, 6, \frac{105\sqrt{\pi}}{16}, 24 \right\}$$

```
val[r_, n_] := Pi^(n/2) / Gamma[1 + n/2] r^n
```

```
val2[r_, n_] := Pi^(n/2) / Gamma[1 + n/2] r^(n/2) / 2^n
```

```
val3[n_, k_] := (2^-k n^(k/2) Pi^(k/2)) / Gamma[1 + k/2]
```

```
val4[n_, k_] := (2^-k (Pi n)^(k/2)) / (k/2)!
```

```
val5[n_, k_] := ((n Pi / 4)^k) / k!
```

```
val6[n_, k_] := ((Pi n) / 4)^(k/2) / (k/2)!
```

```
val[n^(1/2), 3] / 2^3
```

$$\frac{1}{6} n^{3/2} \pi$$

```
val6[n, 6]
```

$$\frac{n^3 \pi^3}{384}$$

```
Table[val5[n, k], {k, 0, 6}]
```

$$\left\{1, \frac{n \pi}{4}, \frac{n^2 \pi^2}{32}, \frac{n^3 \pi^3}{384}, \frac{n^4 \pi^4}{6144}, \frac{n^5 \pi^5}{122880}, \frac{n^6 \pi^6}{2949120}\right\}$$

```
Sum[Binomial[z, k] ((\pi n) / 4)^{k/2}, {k, 0, Infinity}]
```

$$\sqrt{n} z \text{HypergeometricPFQ}\left[\left\{\frac{1}{2} - \frac{z}{2}, 1 - \frac{z}{2}\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, \frac{n \pi}{4}\right] +$$

$$\text{HypergeometricPFQ}\left[\left\{\frac{1}{2} - \frac{z}{2}, -\frac{z}{2}\right\}, \left\{\frac{1}{2}, 1\right\}, \frac{n \pi}{4}\right]$$

```
FullSimplify@Sum[(-1)^(k+1) / k ((\pi n) / 4)^{k/2}, {k, 1, Infinity}]
```

$$\frac{1}{2} \left(\text{EulerGamma} + \text{Gamma}\left[0, -\frac{n \pi}{4}\right] + 2 \sqrt{n} \text{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, \frac{n \pi}{4}\right] + \text{Log}\left[-\frac{n \pi}{4}\right] \right)$$

```
Sum[Binomial[z, k] (n \pi / 4)^k, {k, 0, Infinity}]
```

$$\text{Hypergeometric1F1}\left[-z, 1, -\frac{n \pi}{4}\right]$$

```
FullSimplify@Sum[(-1)^(k+1) / k (n \pi / 4)^k, {k, 1, Infinity}]
```

$$\text{EulerGamma} + \text{Gamma}\left[0, \frac{n \pi}{4}\right] + \text{Log}\left[\frac{n \pi}{4}\right]$$

```
Clear[C2]
```

```
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
```

```
C2[n_, 0] := UnitStep[n]
```

```
C2[n_, k_] := C2[n, k] = Sum[C2[n - j^2, k - 1], {j, 1, Floor[n^(1/2)]}]
```

```
C2f[n_, k_] := C2[n, 2 k]
```

```
dc2f[n_, k_] := C2[n, 2 k] - C2[n - 1, 2 k]
```

```
Cz[n_, z_] := Sum[bin[z, k] C2[n, 2 k], {k, 0, n/2 + 1}]
```

```
Cza[n_, z_] := Sum[bin[z, k] C2[n, k], {k, 0, n}]
```

```
aprox[n_, k_] := (n \pi / 4)^k / k!
```

```
aproxz[n_, z_] := Hypergeometric1F1[-z, 1, -\frac{n \pi}{4}]
```

```
Table[{n, C2f[n, 1]}, {n, 1, 40}] // TableForm
```

C2f[18., 3]

93

aprox[18., 3]

470.908

Cz[18., 3]

259

aproxz[18., 3]

814.109

$$D\left[\frac{(n\pi/4)^k}{k!}, k\right] /. k \rightarrow 0$$

$$\text{EulerGamma} + \text{Log}[n] + \text{Log}\left[\frac{\pi}{4}\right]$$

$$D\left[\frac{(n\pi/4)^k}{k!}, n\right]$$

$$\frac{k n^{-1+k} \left(\frac{\pi}{4}\right)^k}{k!}$$

Table[{n, n/2 D[Cz[n, z] - Cz[n - 1, z], z] /. z -> 0}, {n, 1, 64}] // TableForm

Table[{n, FullSimplify[Cz[n, 1] - Cz[n - 1, 1]]}, {n, 1, 40}] // TableForm

```
Table[{n, N@ $\left(\frac{\pi}{4}\right)$ , N@C2f[n, 1] / n}, {n, 13501, 13540}] // TableForm
```

13501	0.785398	0.776831
13502	0.785398	0.776774
13503	0.785398	0.776716
13504	0.785398	0.776659
13505	0.785398	0.777194
13506	0.785398	0.777136
13507	0.785398	0.777079
13508	0.785398	0.777021
13509	0.785398	0.776964
13510	0.785398	0.776906
13511	0.785398	0.776848
13512	0.785398	0.776791
13513	0.785398	0.776882
13514	0.785398	0.77712
13515	0.785398	0.777063
13516	0.785398	0.777005
13517	0.785398	0.776948
13518	0.785398	0.77689
13519	0.785398	0.776833
13520	0.785398	0.777219
13521	0.785398	0.777161
13522	0.785398	0.777252
13523	0.785398	0.777194
13524	0.785398	0.777137
13525	0.785398	0.777523
13526	0.785398	0.777466
13527	0.785398	0.777408
13528	0.785398	0.777351
13529	0.785398	0.777293
13530	0.785398	0.777236
13531	0.785398	0.777178
13532	0.785398	0.777121
13533	0.785398	0.777063
13534	0.785398	0.777006
13535	0.785398	0.776949
13536	0.785398	0.776891
13537	0.785398	0.776982
13538	0.785398	0.776924
13539	0.785398	0.776867
13540	0.785398	0.777105

```
D[aprox[x, 1], x]
```

$$\frac{\pi}{4}$$

```
Table[{n, dC2f[n, 1] / n}, {n, 1, 40}] // TableForm
```

```
(** http://oeis.org/A162552 **)
```

```
Table[{n, nD[Cza[n, z] - Cza[n - 1, z], z] /. z -> 0}, {n, 1, 64}] // TableForm
```

1	1
2	-1
3	1

4	3
5	- 4
6	5
7	- 6
8	3
9	10
10	- 16
11	23
12	- 27
13	14
14	6
15	- 34
16	83
17	- 101
18	86
19	- 37
20	- 72
21	204
22	- 309
23	346
24	- 243
25	- 29
26	454
27	- 908
28	1214
29	- 1130
30	470
31	776
32	- 2413
33	3884
34	- 4421
35	3244
36	162
37	- 5438
38	11 285
39	- 15 352
40	14 688
41	- 6887
42	- 8640
43	29 241
44	- 48 353
45	56 270
46	- 42 850
47	1834
48	63 965
49	- 138 529
50	192 709
51	- 189 566
52	97 022
53	94 182
54	- 353 398
55	600 728
56	- 714 970
57	565 916
58	- 68 934
59	- 749 653

```

60      1 699 518
61      -2 418 832
62      2 441 404
63      -1 350 570
64      -999 405

```

```
D[(Pi / 4 x) ^ (k / 2) / (k / 2) !, x]
```

$$\frac{2^{-1-k} k \pi^{k/2} x^{-1+\frac{k}{2}}}{\frac{k}{2} !}$$

```
D[x ^ (1 / 2), x]
```

$$\frac{1}{2 \sqrt{x}}$$

```
val7[n_, k_] := ((Pi n) / 4) ^ k/2
                (k / 2) !
```

```
cz[n_, z_] := Sum[Binomial[z, k] val7[n, k], {k, 0, Infinity}]
```

```
cz[10., 3.]
```

```
50.6064
```

```
LaguerreL[1.5, -Pi / 4 10.]
```

```
21.3774
```

```
cz[x, z]
```

$$\sqrt{x} z \text{HypergeometricPFQ}\left[\left\{\frac{1}{2} - \frac{z}{2}, 1 - \frac{z}{2}\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, \frac{\pi x}{4}\right] +$$

$$\text{HypergeometricPFQ}\left[\left\{\frac{1}{2} - \frac{z}{2}, -\frac{z}{2}\right\}, \left\{\frac{1}{2}, 1\right\}, \frac{\pi x}{4}\right]$$

```
CoefficientList[Series[Log[Sum[x ^ (n ^ 2), {n, 0, Infinity}]], {x, 0, 20}], x]
```

$$\left\{0, 1, -\frac{1}{2}, \frac{1}{3}, \frac{3}{4}, -\frac{4}{5}, \frac{5}{6}, -\frac{6}{7}, \frac{3}{8}, \frac{10}{9}, -\frac{8}{5}, \frac{23}{11}, -\frac{9}{4}, \frac{14}{13}, \frac{3}{7}, -\frac{34}{15}, \frac{83}{16}, -\frac{101}{17}, \frac{43}{9}, -\frac{37}{19}, -\frac{18}{5}\right\}$$

```
Table[ D[Cza[n, z] - Cza[n - 1, z], z] /. z -> 0, {n, 0, 20}]
```

$$\left\{0, 1, -\frac{1}{2}, \frac{1}{3}, \frac{3}{4}, -\frac{4}{5}, \frac{5}{6}, -\frac{6}{7}, \frac{3}{8}, \frac{10}{9}, -\frac{8}{5}, \frac{23}{11}, -\frac{9}{4}, \frac{14}{13}, \frac{3}{7}, -\frac{34}{15}, \frac{83}{16}, -\frac{101}{17}, \frac{43}{9}, -\frac{37}{19}, -\frac{18}{5}\right\}$$

```
Table[ Cza[n, z] - Cza[n - 1, z] /. z -> 2, {n, 0, 20}]
```

```
{1, 2, 1, 0, 2, 2, 0, 0, 1, 2, 2, 0, 0, 2, 0, 0, 2, 2, 1, 0, 2}
```

```
CoefficientList[Series[Sum[x ^ ((n) ^ 2), {n, 0, Infinity}] ^ 2, {x, 0, 20}], x]
```

```
{1, 2, 1, 0, 2, 2, 0, 0, 1, 2, 2, 0, 0, 2, 0, 0, 2, 2, 1, 0, 2}
```

```
Table[ Cza[n, z] - Cza[n - 1, z] /. z -> -1, {n, 0, 20}]
```

```
{1, -1, 1, -1, 0, 1, -2, 3, -3, 1, 2, -6, 10, -11, 8, 0, -14, 29, -39, 38, -18}
```

```
CoefficientList[Series[Sum[x ^ ((n) ^ 2), {n, 0, Infinity}] ^ -1, {x, 0, 20}], x]
```

```
{1, -1, 1, -1, 0, 1, -2, 3, -3, 1, 2, -6, 10, -11, 8, 0, -14, 29, -39, 38, -18}
```

```

CoefficientList[Series[Log[Sum[x^((n)^2), {n, 0, Infinity}]], {x, 0, 20}], x]
{0, 1, -1/2, 1/3, 3/4, -4/5, 5/6, -6/7, 3/8, 10/9, -8/5, 23/11, -9/4, 14/13, 3/7, -34/15, 83/16, -101/17, 43/9, -37/19, -18/5}

Table[D[Cz[n, z] - Cz[n - 1, z], z] /. z -> 0, {n, 0, 20}]
{0, 0, 1, 0, -1/2, 2, 1/3, -2, 3/4, 2, -4/5, -2, 17/6, 2, -34/7, -4/3, 51/8, 0, -80/9, 6, 62/5}

Table[Cz[n, z] - Cz[n - 1, z] /. z -> 1, {n, 1, 20}]
{0, 1, 0, 0, 2, 0, 0, 1, 0, 2, 0, 0, 2, 0, 0, 0, 2, 1, 0, 2}

CoefficientList[Series[Sum[(x)^(n^2), {n, 0, Infinity}]], {x, 0, 20}], x]
{1, 1, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1}

Table[sq[n], {n, 1, 20}]
{0, 1, 0, 0, 2, 0, 0, 1, 0, 2, 0, 0, 2, 0, 0, 0, 2, 1, 0, 2}

Table[C2f[n, 1] - C2f[n - 1, 1], {n, 1, 20}]
{0, 1, 0, 0, 2, 0, 0, 1, 0, 2, 0, 0, 2, 0, 0, 0, 2, 1, 0, 2}

SquaresR[1, 9]
2

sq[n_] := SquaresR[2, n] / 4 - SquaresR[1, n] / 2

Series[Sum[SquaresR[k, n] x^n, {n, 0, Infinity}], {x, 0, 20}]

$$\sum_{n=0}^{\infty} x^n \text{SquaresR}[k, n]$$


CoefficientList[Series[EllipticTheta[2, 3, x], {x, 0, 20}], x]
General::poly: 2 x^(1/4) Cos[3] + 2 x^(9/4) Cos[9] + 2 x^(25/4) Cos[15] + 2 x^(49/4) Cos[21] is not a polynomial. >>
{2 x^(1/4) Cos[3] + 2 x^(9/4) Cos[9] + 2 x^(25/4) Cos[15] + 2 x^(49/4) Cos[21]}

Sum[x^(n^2), {n, 0, Infinity}]
1/2 (1 + EllipticTheta[3, 0, x])

```