

f[n_, t_] := Sum[(-1)^(k+1) / k (-1)^k (1 - Gamma[k, -Log[n]] / Gamma[k]), {k, 1, t}]

N[f[100, 10 000]]

28.0217 - 2.09386 × 10⁻¹⁴ i

N[-Gamma[0, -Log[100]] - Log[Log[100]] - EulerGamma - I Pi

28.0217 + 0. i

N[Limit[((-1)^z - (-1)^z Gamma[z, -Log[100]] / Gamma[z] - 1) / z, z → 0]]

30.1261 + 6.28319 i

Integrate[(E^t - 1) / t, {t, 0, Log[n]}]

-EulerGamma + ExpIntegralEi[Log[n]] - Log[Log[n]]

Integrate[(E^t - 1) t^(1/2), {t, 0, Log[n]}]

$-\frac{1}{2} \sqrt{\pi} \operatorname{Erfi}\left[\sqrt{\operatorname{Log}[n]}\right] + \frac{1}{3} (3 n - 2 \operatorname{Log}[n]) \sqrt{\operatorname{Log}[n]}$

Integrate[(E^t) t^-1, {t, 0, Log[n]}]

Integrate::idiv: Integral of $\frac{e^t}{t}$ does not converge on {0, Log[n]}. >>

$\int_0^{\operatorname{Log}[n]} \frac{e^t}{t} dt$

Expand[Integrate[(E^t - 1) t^0, {t, 0, Log[n]}]]

-1 + n - Log[n]

Expand[Integrate[(E^t) t^0, {t, 0, Log[n]}]]

-1 + n

Expand[Integrate[(E^t - 1) t^1, {t, 0, Log[n]}]]

$1 - n + n \operatorname{Log}[n] - \frac{\operatorname{Log}[n]^2}{2}$

Expand[Integrate[(E^t) t^1, {t, 0, Log[n]}]]

1 - n + n Log[n]

Expand[Integrate[(E^t - 1) t^2, {t, 0, Log[n]}]]

$-2 + 2 n - 2 n \operatorname{Log}[n] + n \operatorname{Log}[n]^2 - \frac{\operatorname{Log}[n]^3}{3}$

Expand[Integrate[(E^t) t^2, {t, 0, Log[n]}]]

$-2 + 2 n - 2 n \operatorname{Log}[n] + n \operatorname{Log}[n]^2$

Expand[Integrate[(E^t - 1) t^3, {t, 0, Log[n]}]]

$6 - 6 n + 6 n \operatorname{Log}[n] - 3 n \operatorname{Log}[n]^2 + n \operatorname{Log}[n]^3 - \frac{\operatorname{Log}[n]^4}{4}$

Expand[Integrate[(E^t) t^3, {t, 0, Log[n]}]]

$6 - 6 n + 6 n \operatorname{Log}[n] - 3 n \operatorname{Log}[n]^2 + n \operatorname{Log}[n]^3$

Expand[Integrate[(E^t - 1) t^4, {t, 0, Log[n]}]]

$$-24 + 24n - 24n \operatorname{Log}[n] + 12n \operatorname{Log}[n]^2 - 4n \operatorname{Log}[n]^3 + n \operatorname{Log}[n]^4 - \frac{\operatorname{Log}[n]^5}{5}$$

Expand[Integrate[(E^t) t^4, {t, 0, Log[n]}]]

$$-24 + 24n - 24n \operatorname{Log}[n] + 12n \operatorname{Log}[n]^2 - 4n \operatorname{Log}[n]^3 + n \operatorname{Log}[n]^4$$

Expand[Integrate[(E^t - 1) t^4 / Gamma[5], {t, 0, Log[n]}]]

$$-1 + n - n \operatorname{Log}[n] + \frac{1}{2} n \operatorname{Log}[n]^2 - \frac{1}{6} n \operatorname{Log}[n]^3 + \frac{1}{24} n \operatorname{Log}[n]^4 - \frac{\operatorname{Log}[n]^5}{120}$$

Expand[Integrate[(E^t) t^(5 - 1) / Gamma[5], {t, 0, Log[n]}]]

$$-1 + n - n \operatorname{Log}[n] + \frac{1}{2} n \operatorname{Log}[n]^2 - \frac{1}{6} n \operatorname{Log}[n]^3 + \frac{1}{24} n \operatorname{Log}[n]^4$$

Expand[Integrate[(E^-t) t^(5 - 1) / Gamma[5], {t, 0, -Log[n]}]]

$$1 - n + n \operatorname{Log}[n] - \frac{1}{2} n \operatorname{Log}[n]^2 + \frac{1}{6} n \operatorname{Log}[n]^3 - \frac{1}{24} n \operatorname{Log}[n]^4$$

Expand[Integrate[((E^-t) t^(s - 1)) / Gamma[s], {t, 0, -Log[n]}]]

$$\text{ConditionalExpression}\left[1 - \frac{\operatorname{Gamma}[s, -\operatorname{Log}[n]]}{\operatorname{Gamma}[s]}, \operatorname{Re}[s] > 0\right]$$

Expand[Integrate[((E^-t) t^(s - 1)) / Gamma[s], {t, -Log[n], 0}]]

$$\text{ConditionalExpression}\left[-1 + \frac{\operatorname{Gamma}[s, -\operatorname{Log}[n]]}{\operatorname{Gamma}[s]}, \operatorname{Re}[s] > 0\right]$$

Expand[Integrate[(E^-t) t^(5 - 1), {t, 0, -Log[n]}]]

$$24 - 24n + 24n \operatorname{Log}[n] - 12n \operatorname{Log}[n]^2 + 4n \operatorname{Log}[n]^3 - n \operatorname{Log}[n]^4$$

Expand[Integrate[(E^-t) t^(5 - 1), {t, -Log[n], Infinity}]]

$$24n - 24n \operatorname{Log}[n] + 12n \operatorname{Log}[n]^2 - 4n \operatorname{Log}[n]^3 + n \operatorname{Log}[n]^4$$

Expand[Integrate[(E^-t) t^(5 - 1), {t, 0, Infinity}]]

$$24$$

Integrate[1, {j, 1, n}, {k, 1, n / j}, {l, 1, n / (j k)}, {m, 1, n / (j k l)}, {w, 1, n / (j k l m)}]

Expand[ConditionalExpression[

$$-1 + n + \frac{1}{24} n \operatorname{Log}[n] (-24 + \operatorname{Log}[n] (12 + (-4 + \operatorname{Log}[n]) \operatorname{Log}[n])), \operatorname{Re}[n] \geq 0 \mid n \notin \operatorname{Reals}]$$

ConditionalExpression[

$$-1 + n - n \operatorname{Log}[n] + \frac{1}{2} n \operatorname{Log}[n]^2 - \frac{1}{6} n \operatorname{Log}[n]^3 + \frac{1}{24} n \operatorname{Log}[n]^4, \operatorname{Re}[n] \geq 0 \mid n \notin \operatorname{Reals}]$$