```
d2[n_{,k_{|}} := Sum[d2[n/j,k-1],{j,2,n}];d2[n_{,0}] := 1
d2b[n_{-}, k_{-}, a_{-}] := d2b[n, k, a] = Sum[a d2b[n/(ja), k-1, a], \{j, 2, n/a^k\}];
d2b[n_{,0,a_{]}:=1
d1[n_{,k_{-}}] := Sum[d1[n/j,k-1],{j,1,n}];d1[n_{,0}] := 1
dla[n_{,k_{,a}]} := Sum[adla[n_{,j,k_{-1,a}]}, {j,a,n_{,a,k_{-1}}}; dla[n_{,0,a_{-1}}] := 1
dlb[n_{-}, k_{-}, a_{-}] := Sum[adlb[n/(ja), k-1, a], {j, 1, n/(a^k)}]; dlb[n_{-}, 0, a_{-}] := 1
d22[n_{,k_{,a}] := a^-k d2b[na^k, k, a]
dlc[n_{,k_{,a_{,l}}} := a^kdl[na^-k, k]
d11[n_, k_, a_] := a^-kd1b[na^k, k, a]
d2bb[n_{,k_{,a}} = d2bb[n, k, a] = Sum[a d2bb[n / (ja), k-1, a], {j, 1+1/a, n / (a^k)}];
d2bb[n_, 0, a_] := 1
d2bb2[n_, k_, a_] :=
 d2bb2[n, k, a] = Sum[a d2bb2[n/((j+1/a)a), k-1, a], {j, 1, Floor[n/(a^k) - 1/a]}];
d2bb2[n_{,0,a_{]}:=1
d2ap[n_{j}] := (-1)^{j} (1 - (Gamma[j, -Log[n])) / Gamma[j])
dh[n_, k_, a_] :=
 Sum[Binomial[k, j] dh[n/(m^(k-j)), j, m+1], \{m, a, n^(1/k)\}, \{j, 0, k-1\}]
dh[n_, 1, a_] := Floor[n] - a + 1; dh[n_, 0, a_] := 1
fe[n_{,k_{,a}]} := (1/(a^k)) (dh[n*(a^k),k,a+1])
lin[n_{, a_{]}} := Sum[(-1)^{(k+1)}/kfe[n, k, a], \{k, 1, 100\}]
Table [n, aa = d1b[n, k = 3, a = 1/3], bb = (a^k) d1[n/(a^k), k], aa - bb, {n, 1, 60}] //
 TableForm
      76
              76
1
                      0
      9
              9
      208
              208
2
                      0
      1099
              1099
3
                      Ω
              1633
      1633
                      Ω
4
      2.7
               2.7
      2186
              2186
5
                      0
      27
      2810
              2810
6
      3413
              3413
7
                      0
      27
               27
      458
              458
8
                      0
      1598
              1598
9
                      0
      1835
              1835
10
      2069
              2069
11
                      0
      2332
              2332
                      0
12
      7711
              7711
13
                      0
      27
              27
      8518
              8518
14
                      0
      27
               27
      9295
              9295
15
                      0
      10153
              10 153
```

ClearAll["Global`*"]

2.7

17	10 903	10 903	0
18	11 791 27	11 791 27	0
19	12629	12 629 27	0
20	13 517 27	13 517 27	0
21	14 303	14 303	0
22	27 15 212 27	$ \begin{array}{r} 27 \\ 15 212 \\ \hline 27 \end{array} $	0
23	16 070 27	16 070 27	0
24	17 042 27	17 042 27	0
25	17 927 27	17 927 27	0
26	18 833 27	18 833 27	0
27	6592	6592	0
28	6908 9	6908	0
29	7202	7202	0
30	2509	2509	0
31	7820	7820	0
32	8183	8183	0
33	8477	8477	0
34	8804	8804	0
35	3046	3046	0
36	9482	9482	0
37	9775 9	9775	0
38	30 451 27	30 451 27	0
39	31 381 27	31 381 27	0
40	32 536 27	32 536 27	0
41	33 460	33 460	0
42	34 489 27	34 489 27	0
43	35 533 27	35 533 27	0
44	36 571 27	36 571 27	0
45	37 531 27	37 531 27	0
46	38 629	38 629	0
47	39 667	39 667	0
48	40 825	40 825	0
49	41 842	41 842	0
50	42 947 27	42 947 27	0
51	43 940	43 940	0
52	45 101 27	45 101 27	0
53	46 106 27	46 106 27	0
54	47 273 27	47 273 27	0
55	48 323 27	48 323	0

```
49 493
                 49 49 3
56
       50 468
                  50 468
57
                            0
       51 626
                 51626
58
                            0
        27
                  27
       52664
                 52664
59
                           0
        27
                   27
       53 912
                 53 912
60
                            0
         27
                   27
```

 $N[Table[n, aa = dlb[n, k = 2, a = 1/10], bb = (a^k) dl[n/(a^k), k], aa - bb], {n, 50, 60}] //$ TableForm]

50. 433.76 433.76 0. 51. 443.41 443.41 0. 52. 453.08 453.08 0. 53. 462.7 462.7 0. 54. 472.65 472.65 0. 55. 482.28 482.28 0. 56. 492.12 492.12 0. 57. 501.99 501.99 0. 58. 511.74 511.74 0. 59. 521.46 521.46 0. 60. 531.41 531.41 0.

 $N[Table[{n, aa = dla[n (a^k), k = 2, a = 1/4] / (a^k), bb = dl[n, k], aa - bb}, {n, 1, 50}] // TableForm]$

```
1.
               1.
                       7.
2.
       3.
               3.
                       0.
3.
       5.
               5.
                       0.
4.
       8.
               8.
                       0.
5.
       10.
               10.
                       0.
6.
       14.
               14.
                       0.
7.
       16.
               16.
                       0.
8.
       20.
               20.
                       0.
9.
       23.
               23.
                       0.
10.
       27.
               27.
                       0.
11.
       29.
               29.
                      0.
12.
       35.
               35.
                      0.
13.
       37.
               37.
                       0.
14.
       41.
               41.
                      0.
15.
       45.
               45.
                      0.
16.
       50.
               50.
                       0.
17.
       52.
               52.
                       0.
18.
       58.
               58.
                       0.
19.
       60.
               60.
                       0.
20.
       66.
               66.
                       0.
21.
       70.
               70.
                       0.
22.
       74.
               74.
                       0.
23.
       76.
               76.
                      0.
24.
               84.
       84.
                      0.
25.
       87.
               87.
                      0.
      91.
               91.
26.
                       0.
27.
       95.
               95.
                       0.
28.
       101.
               101.
                       0.
29.
      103.
               103.
                       0.
30.
      111.
               111.
                       0.
31.
      113.
               113.
                       0.
32.
      119.
               119.
                       0.
               123.
33.
      123.
                       0.
34.
      127.
               127.
                      0.
35.
      131.
               131.
                      0.
36.
      140.
               140.
37.
      142.
               142.
                      0.
38.
       146.
               146.
                      0.
39.
       150.
               150.
                       0.
40.
       158.
               158.
                       0.
41.
      160.
               160.
                       0.
42.
       168.
               168.
                       0.
43.
       170.
               170.
                       0.
44.
       176.
               176.
                       0.
45.
       182.
               182.
                       0.
46.
      186.
               186.
                      0.
47.
      188.
               188.
                      0.
48.
      198.
               198.
                       0.
49.
       201.
               201.
                       0.
50.
       207.
               207.
                      0.
```

```
Table [n, aa = (d1a[n, k = 2, a = .27] - d1a[n - 1, k, a]), bb =
      ((a^k) d1[n/(a^k), k] - (a^k) d1[(n-1)/(a^k), k]), aa - bb, \{n, 1, 40\}] // TableForm
                               1.33227 \times 10^{-15}
1
       2.6973
                   2.6973
                               -4.44089 \times 10^{-15}
2
       4.2282
                   4.2282
                               1.06581 \times 10^{-14}
3
       4.7385
                   4.7385
                              3.55271 \times 10^{-15}
4
       4.8843
                   4.8843
                               7.10543 \times 10^{-15}
5
       5.3217
                   5.3217
                               -3.55271 \times 10^{-15}
6
       5.6133
                   5.6133
                               -7.10543 \times 10^{-14}
7
       5.9778
                   5.9778
                               -1.20792 \times 10^{-13}
8
       5.1759
                   5.1759
9
       6.0507
                   6.0507
                               0.
                               -1.42109 \times 10^{-14}
10
       6.1236
                   6.1236
                               -1.42109 \times 10^{-14}
       6.0507
                   6.0507
11
12
       6.1236
                  6.1236
                               5.18696 \times 10^{-13}
13
       6.3423
                   6.3423
                               -8.52651 \times 10^{-14}
14
       6.9984
                   6.9984
                               1.42109 \times 10^{-14}
       5.7591
                   5.7591
15
                               8.52651 \times 10^{-14}
       6.4152
                   6.4152
16
                               4.26326 \times 10^{-14}
                   6.7797
17
       6.7797
                               -1.42109 \times 10^{-14}
       6.561
                   6.561
18
                               1.42109 \times 10^{-14}
19
       6.9255
                   6.9255
                               1.42109 \times 10^{-14}
20
       6.7068
                   6.7068
21
       7.4358
                   7.4358
                               -4.26326 \times 10^{-14}
                               2.84217 \times 10^{-14}
22
       6.1965
                   6.1965
23
       6.9984
                   6.9984
                               2.84217 \times 10^{-13}
24
       6.6339
                   6.6339
                               6.25278 \times 10^{-13}
25
       7.1442
                   7.1442
                               3.41061 \times 10^{-13}
26
       6.7068
                   6.7068
                               -1.98952 \times 10^{-13}
                   7.3629
27
       7.3629
                               -1.13687 \times 10^{-13}
28
       7.4358
                  7.4358
29
       6.561
                   6.561
                               0.
                               1.7053 \times 10^{-13}
                   7.2171
30
       7.2171
                               2.84217 \times 10^{-14}
31
       7.4358
                   7.4358
                               5.68434 \times 10^{-14}
                   6.7068
32
       6.7068
33
       7.6545
                   7.6545
                               2.84217 \times 10^{-14}
                               1.13687 \times 10^{-13}
                   7.29
       7.29
34
                               2.84217 \times 10^{-14}
35
       8.019
                   8.019
                               8.52651 \times 10^{-14}
36
       6.1965
                   6.1965
                               5.68434 \times 10^{-14}
37
       7.8732
                   7.8732
                               1.13687 \times 10^{-13}
38
       7.29
                   7.29
                               8.52651 \times 10^{-14}
39
       7.3629
                   7.3629
40
       7.29
                   7.29
                               8.52651 \times 10^{-14}
Table[{n, Floor[5/n*3]/9, dlas[5,1/3,n]}, {n,1/3,15,1/3}] // TableForm
       5
              5
<u>2</u>
3
       22
              22
       9
       5
1
       11
```

5 3 2 7 3 8	1 7 9 2 3 5	1 7 9 2 3 5
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	7 9 2 3 5 9 5 9 4 9 4 9 1 3 1 3 1 3 1 3 2 9 2 9 2 9 2 9 2 9 2 9 1	7 9 2 3 3 5 9 5 9 4 9 1 3 1 3 1 3 2 9 2 9 2 9 2 9 2 9 1 9 1 9 1 9 1 9 1 9
3 14 3 5 16 3 17 3	3 1 3 1 3 2 9 2	3 1 3 1 3 2 9 2
$ \begin{array}{c} 6 \\ $	2 9 2 9 2 9 2 9 2 9	2 9 2 9 2 9 2 9 2 9
8 25 3 26 3	1 9 1 9 1 9	1 9 1 9 1 9 1
31	_	
$ \begin{array}{r} 3 \\ \hline 32 \\ \hline 3 \end{array} $ 11 $ \begin{array}{r} 34 \\ \hline 3 \\ \hline 35 \\ \hline 3 \end{array} $	9 1 9 1 9 1 9	9 1 9 1 9 1 9
$ \begin{array}{r} \frac{35}{3} \\ 12 \\ \frac{37}{3} \\ \frac{38}{3} \\ 13 \\ \frac{40}{3} \end{array} $	1 9 1 9 1 9 1 9 1 9 1 9 1 9 1 9 1 9 1 9	9 1 9 1 9 1 9 1 9 1 9 1 9 1 9 1 9 1 9 1
$ \begin{array}{r} 3 \\ 41 \\ \hline 3 \end{array} $ 14 $ \begin{array}{r} 43 \\ \hline 3 \end{array} $	9 1 9 1 9 1 9	9 1 9 1 9 1 9

 8.52651×10^{-14}

 8.52651×10^{-14}

 8.52651×10^{-14}

 $\texttt{5.68434} \times \texttt{10}^{-14}$

 1.13687×10^{-13}

 8.52651×10^{-14}

 8.52651×10^{-14}

 $\text{1.42109} \times \text{10}^{\text{-13}}$

 5.68434×10^{-14}

 1.13687×10^{-13}

35

36

37

38

39

40

41

42

43

44

157.032

158.976

170.64

173.232

189.432

192.672

196.56

207.576

221.832

216.

157.032

158.976

170.64

173.232

189.432

192.672

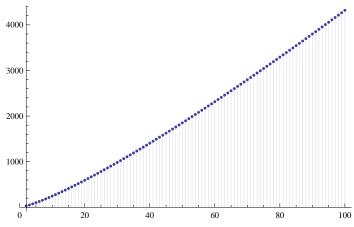
196.56

207.576

221.832

216.

DiscretePlot[d2b[n, 3, 1 / 10], {n, 2, 100}]



d2c[100, 3, 2]

32

d2b[100, 3, 2]

32

d22[100, 2, 3]

283

1 / (3 ^ 3)

1 27

3 ^ - 3

1 27

```
d11[100, 3, .3]
1471.
d1[100, 3]
1471
3^(-3)
 1
27
3 ^ - 3
 27
d2b[n, 2, a]
ss[n_{,a_{]}} := a^2 - 2n + n Harmonic Number \left[\frac{n}{a^2}\right]
N[ss[100, 3]]
111.949
ss[n_, a_] := a^2 Sum[1, {j, 2, Floor[na^-2]}, {k, 2, Floor[n/(ja^2)]}]
ss[1100, 4]
2640
d2b[1100, 2, 4]
2640
\texttt{Limit[ss[n, a], \{a \rightarrow 0\}]}
\left\{ \text{Limit} \left[ a^2 \sum_{j=2}^{\lceil Floor \left[ \frac{n}{a^2 \, j} \right]} \sum_{k=2}^{\lceil n \, a \, j \, j \right]} 1, \, a \to 0 \right] \right\}
s2[n_{,a_{,j}} := a Sum[1, {j, 2, Floor[na^-1]}]
\texttt{Limit[s2[n, a], \{a \rightarrow 0\}]}
\left\{ \texttt{Limit} \left[ \texttt{a} \left( -1 + \texttt{Floor} \left[ \frac{n}{a} \right] \right), \; \texttt{a} \to 0 \, \right] \right\}
N[s2[100, 1/32]]
99.9688
N[ss[100, 1/128]]
```

1246.37

```
N[d2bb[100, 2, 1/80]]
360.287
d2bb2[100, 2, 1/2]
318
N[d2bb[100, 2, 1/3]]
331.667
N[d2ap[100, 2]]
361.517 - 4.41506 \times 10^{-14} i
N[d2ap[100, 3]]
698.863 - 1.71417 \times 10^{-13} i
N[d2bb[100, 3, 1]]
324.
N[d2bb[100, 3, 1/2]]
475.25
N[d2bb[100, 3, 1/4]]
575.656
N[d2bb[100, 3, 1 / 8]]
634.17
N[d2bb[100, 3, 1/16]]
665.794
Table[{n, d2bb2[n, 2, 1/2], d2bb2[n, 2, 1]}, {n, 2, 50}] // TableForm
      0
            0
2
3
             0
      \frac{3}{2}
4
            1
5
           1
6
      4
            3
      21
7
           3
      27
8
           5
      9
9
           6
10 \frac{21}{2}
           8
11
      12
      29
12
           12
      65
13
           12
      \frac{75}{4}
14
           14
      \frac{85}{4}
15
           16
16
      23
           19
```

17	25	19
18	57 2	23
19	30	23
20	33	27
21	$\frac{143}{4}$	29
22	$\frac{151}{4}$	31
23	$\frac{163}{4}$	31
24	$\frac{175}{4}$	37
25	93	38
26	$\frac{97}{2}$	40
27	52	42
28	55	46
29	57	46
30	123	52
31	$\frac{251}{4}$	52
32	$\frac{265}{4}$	56
33	$\frac{279}{4}$	58
34	$\frac{291}{4}$	60
35	$\frac{303}{4}$	62
36	$\frac{159}{2}$	69
37	$\frac{163}{2}$	69
38	$\frac{169}{2}$	71
39	89	73
40	183 2	79
41	94	79
42	$\frac{197}{2}$	85
43	$\frac{405}{4}$	85
44	$\frac{419}{4}$	89
45	435	93
46	$\frac{445}{4}$	95
47	455	95
48	$\frac{475}{4}$	103
49	$\frac{243}{2}$	104
50	251	108

```
Table[\{j,\, a\, d2bb[n\,/\,\,(j\,a)\,,\, k\,-\,1,\, a]\}\,,\, \{j,\, 1\,+\,1\,/\,\,a,\, n\,/\,\,(a^{\,\wedge}k)\,\}]\,\,/\,.
    \{n \rightarrow 5, k \rightarrow 2, a \rightarrow 3/5\} // TableForm
Table::iterb : Iterator \left\{j, 1 + \frac{1}{-}, a^{-k} n\right\} does not have appropriate bounds. \gg
Table::iterb : Iterator \left\{j, 1 + \frac{1}{-}, a^{-k} n\right\} does not have appropriate bounds. \gg
\begin{array}{c} \frac{8}{3} \\ \frac{11}{3} \\ \frac{11}{3} \\ \frac{14}{3} \\ \frac{3}{17} \\ \frac{20}{3} \\ \frac{23}{3} \\ \frac{26}{3} \\ \frac{29}{3} \\ \frac{3}{35} \\ \frac{3}{3} \\ \frac{3}{3} \\ \frac{41}{3} \\ \end{array}
           25
           18
          \frac{9}{25}
          0
          0
          0
          0
          0
           0
N[d2bb[5, 2, 1/2]]
2.5
(1/4) (d2[20, 2] - 2d2[10, 1] + d2[5, 0])
5
d2[20, 2]
27
d2[20,1]
N[d2bb[25, 2, 1/2]]
N[(1/4)(d2[25*4,2]-2d2[25*2,1]+d2[25,0])]
46.5
N[d2bb[25, 3, 1/2]]
40.5
N[(1/8)(d2[25*8,3]-3d2[25*4,2]+3d2[25*2,1]-d2[25,0])]
40.5
N[(1/8) (dh[25*8,3,3])]
40.5
```

```
N[d2bb[10, 2, 1/3]]
11.5556
N[(1/9)(d2[10*9,2]-2d2[10*3,1]+d2[10,0])]
21.
N[(1/8) (dh[25*8,3,2]-3dh[25*4,2,2]+3dh[25*2,1,2]-dh[25,0,2])]
40.5
N[d2bb[10, 3, 1/3]]
6.77778
N[(1/27) (dh[10 * 27, 3, 4])]
6.77778
N[d2bb[10, 2, 1/3]]
11.5556
N[(1/9) (dh[10*9, 2, 4])]
11.5556
N[d2bb[100, 4, 1/3]]
556.296
N[(1/81) (dh[100 * 81, 4, 4])]
556.296
N[d2bb[100, 4, 1/6]]
721.063
N[(1/(6^4))(dh[100*(6^4),4,7])]
721.063
N[d2ap[100, 2]]
361.517 - 4.41506 \times 10^{-14} i
N[fe[100, 2, 2265]]
361.473
N[d2ap[100, 3]]
698.863 - 1.71417 \times 10^{-13} \ \text{i}
fe[100, 3, 165]
695.583
```

N[d2ap[100, 4]]

 $928.88 - 3.40898 \times 10^{-13} i$

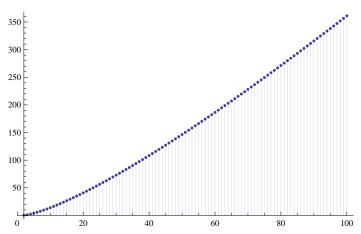
fe[100, 4, 75]

910.402

fe[100, 2, 1]

283.

DiscretePlot[fe[n, 2, 240], {n, 2, 100}]



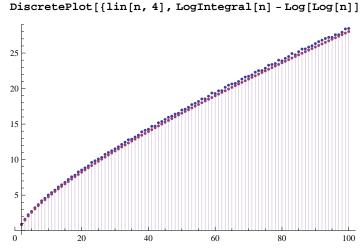
100 × 240 ^ 2

5 760 000

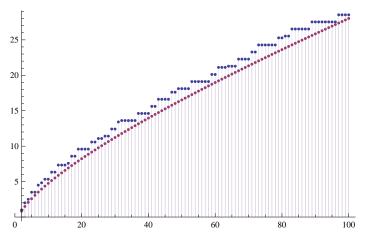
N[1/240]

0.00416667

 $\label{eq:log_log_log_log_log_log} \begin{tabular}{ll} DiscretePlot[\{lin[n,4],LogIntegral[n]-Log[Log[n]]-EulerGamma\},\{n,2,100\}] \end{tabular}$



 $\label{eq:log_log_log_log_log_log} \mbox{DiscretePlot}[\{\mbox{lin}[n,1],\mbox{LogIntegral}[n]-\mbox{Log[Log}[n]]-\mbox{EulerGamma}\}, \{n,2,100\}]$



 $\label{eq:log_log_log_n_log_log_n_log_log_n} Discrete Plot[\{lin[n, 10], LogIntegral[n] - Log[Log[n]] - Euler Gamma\}, \{n, 2, 100\}]$

\$Aborted

d2bbalt2[100, 3, 1 / 2]

d2bbalt[100, 3, 1 / 2]

d2bb2[100, 3, 1 / 2]

 $FullSimplify[\,(1\,/\,(\,(1\,/\,a)\,^{\,\,}k)\,)\,\,(dhf\,[\,n\,\star\,(\,(1\,/\,a)\,^{\,\,}k)\,,\,k\,,\,\,(1\,/\,a)\,+1]\,)\,]$

$$\left(\frac{1}{a}\right)^{-k} dhf\left[\left(\frac{1}{a}\right)^{k} n, k, 1 + \frac{1}{a}\right]$$

$$\begin{pmatrix} 1 \\ - \\ a \end{pmatrix}^{-1}$$

$$\left(\frac{1}{2}\right)^{-1}$$

FullSimplify[(a^-1)^-k]

$$\left(\frac{1}{a}\right)^{-k}$$

343

7 ^ 3

343

 $Plot[{0, x/Log[x+1]}, {x, 0, 50}]$