$$\lim_{z \to 0} \frac{(D_z(n)-1)}{z} = \Pi(n)$$

$$D_j(n) \approx (-1)^j + n \sum_{k=0}^{j-1} \frac{(-\log n)^k}{k!}$$

$$D_j(n) \approx (-1)^j - \frac{(-1)^j \Gamma(j, -\log n)}{\Gamma(j)}$$

$$\lim_{z \to 0} \frac{1}{z} (((-1)^z - \frac{(-1)^z \Gamma(z, -\log n)}{\Gamma(z)}) - 1) \approx \Pi(n)$$

$$\lim_{z \to 0} \frac{1}{z} \cdot -1 \frac{\Gamma(z, -\log n)}{\Gamma(z)} \approx \Pi(n)$$

$$\lim_{z \to 0} -\Gamma(z, -\log n) \approx \Pi(n)$$

$$-\Gamma(0, -\log n) = li(n)$$

$$\sum_{z=1} \frac{(-1)^{z+1}}{z} (-1)^z (1 - \frac{\Gamma(z, -\log n)}{\Gamma(z)})$$

$$\sum_{j=1} (\frac{\Gamma(j, -\log n) - \Gamma(j)}{\Gamma(j+1)})$$

$$\Gamma(j, n) + \gamma(j, n) = \Gamma(j)$$
$$\gamma(j, n) = \Gamma(j) - \Gamma(j, n)$$

$$\sum_{j=1}^{\infty} - \frac{\gamma(j, -\log n)}{j!}$$

$$\gamma(j, n) = \int_0^n t^{a-1} e^{-t} \, dt$$

$$\sum_{j=1}^{\infty} - \frac{\gamma(j, -\log n)}{j!}$$

$$\sum_{j=1}^{\infty} - \frac{1}{j!} \int_{-\log n}^{0} t^{j-1} e^{-t} \, dt$$

$$- \int_{-\log n}^{0} \sum_{j=1}^{\infty} \frac{t^{j-1}}{j!} e^{-t} \, dt$$

$$- \int_{-\log n}^{0} \frac{-1 + e^t}{t} e^{-t} \, dt$$

$$- \int_{-\log n}^{0} \frac{-e^{-t} + 1}{t} \, dt$$

$$\boxed{D_j'(n) \approx (-1)^j \left(1 - n\sum_{k=0}^{j-1} \frac{(-\log n)^k}{k!}\right)}$$

This approximation can be made to handle real values for *j*. The lower incomplete gamma function $\gamma(j,n)$ can be written as $\gamma(j,n) = \Gamma(j)\left(1 - e^{-n}\sum_{k=0}^{j-1} \frac{n^k}{k!}\right)$ and thus $\gamma(j,-\log n) = \Gamma(j)\left(1 - n\sum_{k=0}^{j-1} \frac{(-\log n)^k}{k!}\right)$ Thus, we can rewrite our approximation as

$$\boxed{D_z'(n) \approx (-1)^z \frac{\gamma(z,-\log n)}{\Gamma(z)}}$$

$$\Pi(n) \approx \lim_{z\to 0} \frac{1}{z}\left(\frac{(-1)^z \gamma(z,-\log n)}{\Gamma(z)} - 1\right) = \lim_{z\to 0} -\frac{\Gamma(0,-\log n)}{z\cdot\Gamma(z)} = \lim_{z\to 0} -\frac{\Gamma(0,-\log n)}{\Gamma(z+1)} =$$