

```

bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k!
pp[f_, n_, 0] := UnitStep[n - 1]
pp[f_, n_, k_] := pp[f, n, k] = Sum[f[j] pp[f, Floor[n / j], k - 1], {j, 2, n}]
ppa[f_, n_, 0, a_] := UnitStep[n - 1]
ppa[f_, n_, 1, a_] := ppa[f, n, 1, a] = N[Sum[f[m], {m, a + 1, n}]]
ppa[f_, n_, k_, a_] :=
  ppa[f, n, k, a] = N[Sum[bin[k, j] f[m]^j ppa[f, Floor[n / (m^j)], k - j, m],
    {j, 1, k}, {m, a + 1, Floor[n^(1 / k)]]}]
pz[f_, n_, z_] := Sum[bin[z, k] pp[f, n, k], {k, 0, Log[2, n]}]
pza[f_, n_, z_] := Sum[bin[z, k] ppa[f, n, k, 1], {k, 0, Log[2, n]}]
pz2[f_, n_, z_] := Sum[z^k f[k + 1], {k, 0, n}]
pzeros[f_, n_] := List@@NRoots[pz[f, n, z] == 0, z][[All, 2]]
pzerosa[f_, n_] := List@@NRoots[pza[f, n, z] == 0, z][[All, 2]]
pzeros2[f_, n_] := List@@NRoots[pz2[f, n, z] == 0, z][[All, 2]]
zeta[0, s_, z_, k_] := 0
zeta[n_, s_, z_, 0] := UnitStep[n - 1]
zeta[n_, s_, z_, k_] :=
  zeta[n, s, z, k] = 1 + ((z + 1) / k - 1) Sum[j^-s zeta[Floor[n / j], s, z, k + 1], {j, 2, n}]
dzeta[n_, s_, z_] := zeta[n, s, z, 1] - zeta[n - 1, s, z, 1]

```

```

lg[n_, k_] := 1 - k (Floor[n / k] - Floor[(n - 1) / k])
fe[n_] := 1
fx[n_] := n^-s
ff[n_] := N[1 / ((n - 1) !)]
ff2[n_] := N[1 / (n !)]
fr[n_] := fs[n - 1]
fg[n_] := (-1)^(n - 1) 1 / ((n - 1) !)
fh[n_] := (I)^(n - 1) 1 / ((n - 1) !)
gg[n_] := gg[n] = D[Cos[x], {x, n}] /. x -> 0
hh[n_] := hh[n] = D[Cos[x], {x, n - 1}] /. x -> 0
cs[n_] := cs[n] = (D[Cos[x], {x, n - 1}] /. x -> 0) / ((n - 1) !)
sn[n_] := sn[n] = (D[Sin[x], {x, n - 1}] /. x -> 0) / ((n - 1) !)
exp[n_] := exp[n] = 1 / ((n - 1) !)
jj[n_] := (-1)^(n + 1) / n
fo[n_] := 0; fo[1] := 7; fo[2] := 3; fo[3] := 12;
fo[4] := 55; fo[5] := -3 + I; fo[6] := -7

```

ff[8]

$\frac{1}{5040}$

Expand[pza[jj, 100 000, z]]

$1 - 0.359995 z + 0.0388798 z^2 + 0.0391955 z^3 - 0.0401951 z^4 + 0.0205268 z^5 - 0.00635971 z^6 +$
 $0.00122432 z^7 - 0.000144154 z^8 + 0.0000101242 z^9 - 4.1536 \times 10^{-7} z^{10} + 8.50103 \times 10^{-9} z^{11} -$
 $6.97606 \times 10^{-11} z^{12} + 3.37341 \times 10^{-12} z^{13} - 1.36292 \times 10^{-13} z^{14} + 1.44886 \times 10^{-15} z^{15} - 7.04981 \times 10^{-18} z^{16}$

Expand[pz[ff, 60, 0, z]]

$$1 + \frac{(2931003346949718170578224494139352409419032200378117270318906218274370849923909z) / 346707796364224589344847549300973515864757191931718581352053237350400000000000}{668802103698132768433689196979z^2} + \frac{573482765z^3}{6974263296} + \frac{11z^4}{2160} + \frac{7z^5}{240}$$

N[pz[ff, 60, 0, -1]]

0.804729

N[Expand[pz[ff, 500, 0, z]]]

$$1. + 1.22792z - 0.119284z^2 + 0.822859z^3 - 0.31065z^4 + 0.109187z^5 - 0.0123753z^6 + 0.000501268z^7 + 0.000124008z^8$$

N[D[Expand[pz[ff, 400, 0, z]], z] /. z -> 0]

0.78388

pzeros[ff, 500, 0]

$$\{-15.2276, -0.597394, 0.0086062 - 1.44375i, 0.0086062 + 1.44375i, 1.54627 - 3.01933i, 1.54627 + 3.01933i, 4.33653 - 4.26038i, 4.33653 + 4.26038i\}$$

Product[1 - 1/j, {j, pzeros[ff, 500, 0]}]

2.71828 + 0. i

Table[{80 n, N[pz[ff, 80 n, 0, (2 Pi I)]]}, {n, 1, 40}] // TableForm

80	-301.288 + 227.97 i
160	-138.791 - 510.916 i
240	284.677 - 567.874 i
320	604.564 + 131.935 i
400	206.287 - 36.9285 i
480	568.929 + 702.92 i
560	649.614 + 548.514 i
640	-413.519 + 399.338 i
720	-213.391 + 917.398 i
800	136.375 + 512.984 i
880	-975.691 + 152.597 i
960	-1145.89 + 121.769 i
1040	-1085.99 + 320.45 i
1120	-1044.63 + 439.849 i
1200	-20.4109 - 309.537 i
1280	9.89149 - 344.706 i
1360	-734.927 - 682.984 i
1440	-1015.82 - 773.154 i
1520	-1009.46 - 749.508 i
1600	-1001.85 - 228.975 i
1680	-988.189 - 180.024 i
1760	560.751 - 954.867 i
1840	566.834 - 941.031 i
1920	739.42 - 1067.78 i
2000	432.623 - 1240.57 i
2080	366.752 - 1281.14 i
2160	119.354 - 1395.07 i
2240	119.209 - 1393.71 i
2320	-175.583 + 23.916 i
2400	-199.085 + 16.1965 i
2480	-196.683 + 26.1896 i
2560	-195.595 + 71.2173 i
2640	1208. - 399.653 i
2720	1215.95 - 395.839 i
2800	1215.96 - 395.801 i
2880	1607.88 - 588.019 i
2960	1529.4 - 638.43 i
3040	1521.44 - 641.391 i
3120	1206.86 - 1013.91 i
3200	1214.17 - 1019.19 i

N[E^ (2 Pi I)]

1.

Product[1 + .5 / j, {j, pzeros[ff, 3200, 0]}]

-0.0386673 - 8.67362 × 10⁻¹⁹ i

Table[gg[n], {n, 0, 10}]

{1, 0, -1, 0, 1, 0, -1, 0, 1, 0, -1}

Product[1 - 3 / j, {j, pzeros[hh, 3200, 0]}]

-50. + 0. i

N[Expand[pz[hh, 3200, 0, z]]]

$1. + 0.940476 z - 3.80278 z^2 + 3.38611 z^3 - 1.67361 z^4 + 0.173611 z^5 - 0.0236111 z^6 - 0.000198413 z^7$

Table[{80 n, N[pz[ff, 80 n, 0, 2]]}, {n, 1, 40}] // TableForm

80	7.38906
160	7.38906
240	7.38906
320	7.38906
400	7.38906
480	7.38906
560	7.38906
640	7.38906
720	7.38906
800	7.38906
880	7.38906
960	7.38906
1040	7.38906
1120	7.38906
1200	7.38906
1280	7.38906
1360	7.38906
1440	7.38906
1520	7.38906
1600	7.38906
1680	7.38906
1760	7.38906
1840	7.38906
1920	7.38906
2000	7.38906
2080	7.38906
2160	7.38906
2240	7.38906
2320	7.38906
2400	7.38906
2480	7.38906
2560	7.38906
2640	7.38906
2720	7.38906
2800	7.38906
2880	7.38906
2960	7.38906
3040	7.38906
3120	7.38906
3200	7.38906

N[Expand[pz[jj, 3200, 0, z]] /. z → 1]

0.692991

```
Table[{80 n, N[pz[jj, 80 n, 0, -2]]}, {n, 1, 40}] // TableForm
```

80	2.58404
160	2.69477
240	0.669176
320	2.76322
400	0.807427
480	0.489065
560	3.67904
640	2.83425
720	2.54238
800	0.687572
880	0.495279
960	0.362281
1040	3.59252
1120	3.78876
1200	3.95564
1280	2.91309
1360	3.24324
1440	2.61706
1520	2.6304
1600	0.542107
1680	0.268508
1760	0.324877
1840	-0.213788
1920	0.218701
2000	0.162965
2080	3.72384
2160	4.01147
2240	3.94546
2320	4.32104
2400	4.10289
2480	3.94164
2560	2.98383
2640	2.83006
2720	3.33652
2800	3.31052
2880	2.63429
2960	2.56583
3040	2.67806
3120	0.329998
3200	0.430759

```
N[Log[2] ^ -1]
```

```
1.4427
```

```
Table[ {80 n, N[pz[fg, 80 n, 0, 2]]}, {n, 1, 40}] // TableForm
```

80	0.135335
160	0.135335
240	0.135335
320	0.135335
400	0.135335
480	0.135335
560	0.135335
640	0.135335
720	0.135335
800	0.135335
880	0.135335
960	0.135335
1040	0.135335
1120	0.135335
1200	0.135335
1280	0.135335
1360	0.135335
1440	0.135335
1520	0.135335
1600	0.135335
1680	0.135335
1760	0.135335
1840	0.135335
1920	0.135335
2000	0.135335
2080	0.135335
2160	0.135335
2240	0.135335
2320	0.135335
2400	0.135335
2480	0.135335
2560	0.135335
2640	0.135335
2720	0.135335
2800	0.135335
2880	0.135335
2960	0.135335
3040	0.135335
3120	0.135335
3200	0.135335

```
N[E^-2]
```

```
0.135335
```

```
Table[ {80 n, N[pz[fh, 80 n, 0, 1]]}, {n, 1, 40}] // TableForm
```

```
80      0.540302 + 0.841471 i
160     0.540302 + 0.841471 i
240     0.540302 + 0.841471 i
320     0.540302 + 0.841471 i
400     0.540302 + 0.841471 i
480     0.540302 + 0.841471 i
560     0.540302 + 0.841471 i
640     0.540302 + 0.841471 i
720     0.540302 + 0.841471 i
800     0.540302 + 0.841471 i
880     0.540302 + 0.841471 i
960     0.540302 + 0.841471 i
1040    0.540302 + 0.841471 i
1120    0.540302 + 0.841471 i
1200    0.540302 + 0.841471 i
1280    0.540302 + 0.841471 i
1360    0.540302 + 0.841471 i
1440    0.540302 + 0.841471 i
1520    0.540302 + 0.841471 i
1600    0.540302 + 0.841471 i
1680    0.540302 + 0.841471 i
1760    0.540302 + 0.841471 i
1840    0.540302 + 0.841471 i
1920    0.540302 + 0.841471 i
2000    0.540302 + 0.841471 i
2080    0.540302 + 0.841471 i
2160    0.540302 + 0.841471 i
2240    0.540302 + 0.841471 i
2320    0.540302 + 0.841471 i
2400    0.540302 + 0.841471 i
2480    0.540302 + 0.841471 i
2560    0.540302 + 0.841471 i
2640    0.540302 + 0.841471 i
2720    0.540302 + 0.841471 i
2800    0.540302 + 0.841471 i
2880    0.540302 + 0.841471 i
2960    0.540302 + 0.841471 i
3040    0.540302 + 0.841471 i
3120    0.540302 + 0.841471 i
3200    0.540302 + 0.841471 i
```

```
pzeros[ff, 3200, 0]
```

```
{-9.25681 + 3.73518 i, -0.301298 + 0.323107 i, -0.193189 - 1.6486 i,
 1.02888 + 1.10365 i, 2.7018 + 1.49628 i, 4.39838 - 10.7166 i, 4.42626 + 1.50155 i,
 6.04342 + 1.10359 i, 7.39576 + 0.168091 i, 8.34917 - 1.66614 i, 8.52496 - 5.21208 i}
```

```
Product[1 - 1/j, {j, pzeros[fh, 3200, 0]}]
```

```
0.540302 + 0.841471 i
```

N[E^I]

0.540302 + 0.841471 i

D[N[Expand[pz[fh, 3200, 0, z]]], z] /. z -> 0

0.240314 + 1.6822 i

Table[{80 n, D[N[Expand[pz[fh, 3200, 0, z]]], z] /. z -> 0}, {n, 1, 40}] // TableForm

80	0.240314 + 1.6822 i
160	0.240314 + 1.6822 i
240	0.240314 + 1.6822 i
320	0.240314 + 1.6822 i
400	0.240314 + 1.6822 i
480	0.240314 + 1.6822 i
560	0.240314 + 1.6822 i
640	0.240314 + 1.6822 i
720	0.240314 + 1.6822 i
800	0.240314 + 1.6822 i
880	0.240314 + 1.6822 i
960	0.240314 + 1.6822 i
1040	0.240314 + 1.6822 i
1120	0.240314 + 1.6822 i
1200	0.240314 + 1.6822 i
1280	0.240314 + 1.6822 i
1360	0.240314 + 1.6822 i
1440	0.240314 + 1.6822 i
1520	0.240314 + 1.6822 i
1600	0.240314 + 1.6822 i
1680	0.240314 + 1.6822 i
1760	0.240314 + 1.6822 i
1840	0.240314 + 1.6822 i
1920	0.240314 + 1.6822 i
2000	0.240314 + 1.6822 i
2080	0.240314 + 1.6822 i
2160	0.240314 + 1.6822 i
2240	0.240314 + 1.6822 i
2320	0.240314 + 1.6822 i
2400	0.240314 + 1.6822 i
2480	0.240314 + 1.6822 i
2560	0.240314 + 1.6822 i
2640	0.240314 + 1.6822 i
2720	0.240314 + 1.6822 i
2800	0.240314 + 1.6822 i
2880	0.240314 + 1.6822 i
2960	0.240314 + 1.6822 i
3040	0.240314 + 1.6822 i
3120	0.240314 + 1.6822 i
3200	0.240314 + 1.6822 i

-Sum[1 / j, {j, pzeros[fe, 100, 0]}]

28.5333 + 0. i

Product[1 - 1 / j, {j, pzeros[fe, 100, 0]}]

100. + 1.77636 $\times 10^{-15}$ i

Sum[Log[1 - 1 / j], {j, pzeros[fe, 100, 0]}]

4.60517 + 0. i

N[Log[100]]

4.60517

-Sum[j^-k / k, {k, 1, Infinity}]

$\text{Log}\left[\frac{-1 + j}{j}\right]$

pzeros[fe, 100, 0]

{-11.1997 - 12.3982 i, -11.1997 + 12.3982 i,
-2.67195 - 1.86184 i, -2.67195 + 1.86184 i, -0.933809, -0.0372047}

Sum[Log[(j - 1)] - Log[j], {j, pzeros[fe, 100, 0]}]

4.60517 + 0. i

N[Expand[pz[ff, 512, 0, z]]]

1. + 1.23839 z - 0.15349 z² + 0.869054 z³ - 0.344641 z⁴ +
0.12422 z⁵ - 0.0165045 z⁶ + 0.00119599 z⁷ + 0.0000578704 z⁸ + 2.75573 $\times 10^{-6}$ z⁹

pzeros[ff, 512]

{-16.0782 - 21.5378 i, -16.0782 + 21.5378 i, -0.5848,
0.0131878 - 1.42824 i, 0.0131878 + 1.42824 i, 1.53338 - 3.00638 i,
1.53338 + 3.00638 i, 4.32402 - 4.27461 i, 4.32402 + 4.27461 i}

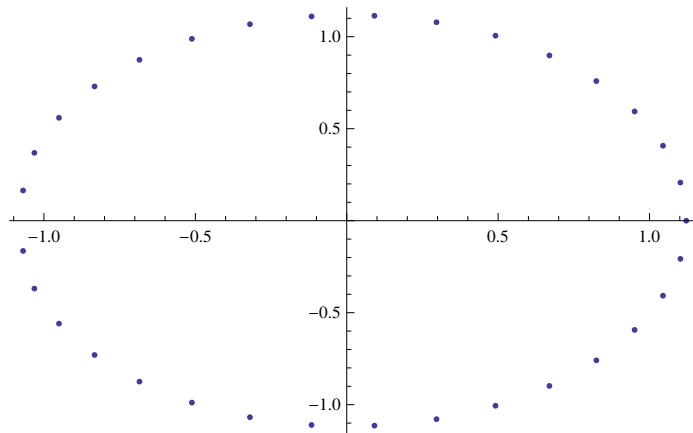
N[pz2[ff, 512, z]]

1. + z + 0.5 z² + 0.166667 z³ + 0.0416667 z⁴ + 0.00833333 z⁵ +
0.00138889 z⁶ + 0.000198413 z⁷ + 0.0000248016 z⁸ + 2.75573 $\times 10^{-6}$ z⁹

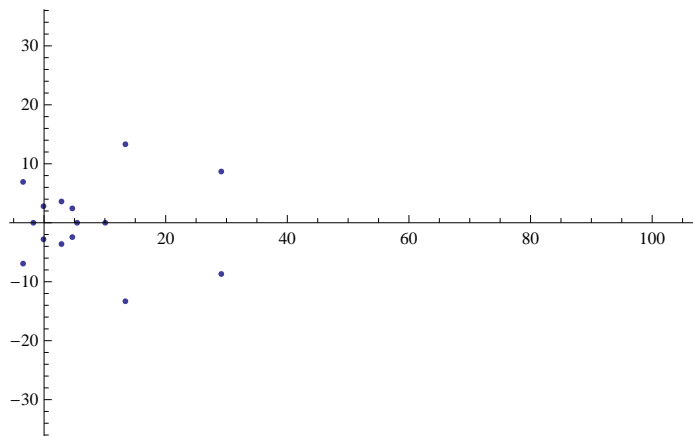
pzeros2[ff, 512]

{-3.33355, -3.03865 - 1.5868 i, -3.03865 + 1.5868 i, -2.11084 - 3.08991 i, -2.11084 + 3.08991 i,
-0.38107 - 4.38464 i, -0.38107 + 4.38464 i, 2.69733 - 5.18416 i, 2.69733 + 5.18416 i}

```
ListPlot[Table[{Re[n], Im[n]}, {n, pzeros2[jj, 10 000 000 000]}]]
```



```
ListPlot[Table[{Re[n], Im[n]}, {n, pzerosa[jj, 1 000 000]}]]
```



```
Expand[pza[ff, 10 000, z]]
```

$$1 + 0.56733 z + 1.81209 z^2 - 1.47941 z^3 + 1.18578 z^4 - 0.485266 z^5 + 0.140903 z^6 - 0.0263209 z^7 + 0.00346853 z^8 - 0.000311406 z^9 + 0.0000218201 z^{10} - 1.13957 \times 10^{-6} z^{11} + 4.07097 \times 10^{-8} z^{12} + 1.6059 \times 10^{-10} z^{13}$$

```
Expand[pza[ff, 100 000, z]]
```

```
$Aborted
```

```
N[Expand[Sum[bin[z, k] 3^k, {k, 0, 3}]]] /. z -> 1
```

```
4.
```

```
pzerosa[jj, 1 000 000]
```

```
{-3.45042 - 6.92631 i, -3.45042 + 6.92631 i, -1.75739,
-0.0911848 - 2.80665 i, -0.0911848 + 2.80665 i, 2.863 - 3.61048 i,
2.863 + 3.61048 i, 4.64524 - 2.43395 i, 4.64524 + 2.43395 i, 5.4378, 10.0628,
13.374 - 13.3145 i, 13.374 + 13.3145 i, 29.1437 - 8.68903 i, 29.1437 + 8.68903 i,
42.1878 - 52.3997 i, 42.1878 + 52.3997 i, 319.776 - 189.885 i, 319.776 + 189.885 i}
```

```
pzeros[fo, 10]
```

```
{-57.4386 - 0.665461 i, 0.00504456 - 0.0000508451 i, 0.766848 - 0.00115505 i}
```

```
Sum[fo[n], {n, 1, 20}]
```

```
67 + i
```

```
Expand[pz[fo, 10, z]]
```

$$1 - \left(\frac{399}{2} + 2i \right) z + (255 + 3i) z^2 + \frac{9 z^3}{2}$$

```
Product[1 - 1 / j, {j, pzeros[fo, 10]}]
```

```
61. + 1. i
```

```
Sum[ (z Log[Zeta[s]]) ^k / k! , {k, 0, Infinity}]
```

```
Zeta[s]^z
```

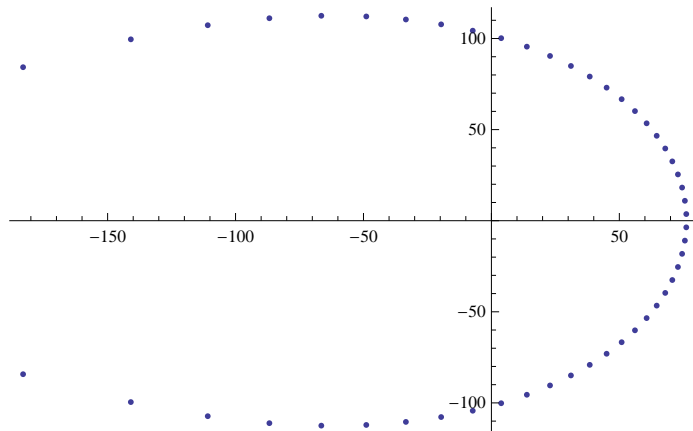
```
px[s_, z_, t_] := Sum[ (z Log[ (1 - 2^(1 - s)) Zeta[s]]) ^k / k! , {k, 0, t}]
```

```
pxzeros[s_, t_] := List@@NRoots[px[s, z, t] == 0, z][[All, 2]]
```

```
1 - 1 / pxzeros[2, 20] // TableForm
```

```
1.01046 - 0.00893346 i
1.01046 + 0.00893346 i
1.00877 - 0.0148524 i
1.00877 + 0.0148524 i
1.00491 - 0.0195277 i
1.00491 + 0.0195277 i
0.999559 - 0.0226156 i
0.999559 + 0.0226156 i
0.993336 - 0.0238402 i
0.993336 + 0.0238402 i
0.986871 - 0.023075 i
0.986871 + 0.023075 i
0.980785 - 0.0203657 i
0.980785 + 0.0203657 i
0.97565 - 0.0159293 i
0.97565 + 0.0159293 i
0.971939 - 0.0101363 i
0.971939 + 0.0101363 i
0.969994 - 0.00347788 i
0.969994 + 0.00347788 i
```

```
ListPlot[Table[{Re[n], Im[n]}, {n, pxzeros[2, 50]}]]
```



```
Product[1 - 1 / j, {j, pxzeros[2, 50]}]
```

```
0.822467 + 3.46945 × 10-18 i
```

```
-Sum[1 / j, {j, pxzeros[2, 50]}]
```

```
-0.195446 + 0. i
```

```
N[Log[Pi ^ 2 / 12]]
```

```
-0.195447
```

```
N[Pi ^ 2 / 12]
```

```
0.822467
```

```
Expand[px[2, z, 20]]
```

$$\begin{aligned}
 &1 + z \operatorname{Log}\left[\frac{\pi^2}{12}\right] + \frac{1}{2} z^2 \operatorname{Log}\left[\frac{\pi^2}{12}\right]^2 + \frac{1}{6} z^3 \operatorname{Log}\left[\frac{\pi^2}{12}\right]^3 + \frac{1}{24} z^4 \operatorname{Log}\left[\frac{\pi^2}{12}\right]^4 + \frac{1}{120} z^5 \operatorname{Log}\left[\frac{\pi^2}{12}\right]^5 + \\
 &\frac{1}{720} z^6 \operatorname{Log}\left[\frac{\pi^2}{12}\right]^6 + \frac{z^7 \operatorname{Log}\left[\frac{\pi^2}{12}\right]^7}{5040} + \frac{z^8 \operatorname{Log}\left[\frac{\pi^2}{12}\right]^8}{40320} + \frac{z^9 \operatorname{Log}\left[\frac{\pi^2}{12}\right]^9}{362880} + \frac{z^{10} \operatorname{Log}\left[\frac{\pi^2}{12}\right]^{10}}{3628800} + \frac{z^{11} \operatorname{Log}\left[\frac{\pi^2}{12}\right]^{11}}{39916800} + \\
 &\frac{z^{12} \operatorname{Log}\left[\frac{\pi^2}{12}\right]^{12}}{479001600} + \frac{z^{13} \operatorname{Log}\left[\frac{\pi^2}{12}\right]^{13}}{6227020800} + \frac{z^{14} \operatorname{Log}\left[\frac{\pi^2}{12}\right]^{14}}{87178291200} + \frac{z^{15} \operatorname{Log}\left[\frac{\pi^2}{12}\right]^{15}}{1307674368000} + \frac{z^{16} \operatorname{Log}\left[\frac{\pi^2}{12}\right]^{16}}{20922789888000} + \\
 &\frac{z^{17} \operatorname{Log}\left[\frac{\pi^2}{12}\right]^{17}}{355687428096000} + \frac{z^{18} \operatorname{Log}\left[\frac{\pi^2}{12}\right]^{18}}{6402373705728000} + \frac{z^{19} \operatorname{Log}\left[\frac{\pi^2}{12}\right]^{19}}{121645100408832000} + \frac{z^{20} \operatorname{Log}\left[\frac{\pi^2}{12}\right]^{20}}{2432902008176640000}
 \end{aligned}$$

```
gx[s_, z_, t_] := Sum[(z Log[Zeta[s]])^k / k!, {k, 0, t}]
```

```
gxzeros[s_, t_] := List@@NRoots[gx[s, z, t] == 0, z][[All, 2]]
```

```
N[gx[4, z, 20]]
```

$$\begin{aligned}
 &1. + 0.0791099 z + 0.00312919 z^2 + 0.0000825165 z^3 + 1.63197 \times 10^{-6} z^4 + 2.58209 \times 10^{-8} z^5 + \\
 &3.40449 \times 10^{-10} z^6 + 3.84755 \times 10^{-12} z^7 + 3.80474 \times 10^{-14} z^8 + 3.34436 \times 10^{-16} z^9 + 2.64572 \times 10^{-18} z^{10} + \\
 &1.90275 \times 10^{-20} z^{11} + 1.25439 \times 10^{-22} z^{12} + 7.63341 \times 10^{-25} z^{13} + 4.31342 \times 10^{-27} z^{14} + 2.27489 \times 10^{-29} z^{15} + \\
 &1.12479 \times 10^{-31} z^{16} + 5.23423 \times 10^{-34} z^{17} + 2.30044 \times 10^{-36} z^{18} + 9.5783 \times 10^{-39} z^{19} + 3.78869 \times 10^{-41} z^{20}
 \end{aligned}$$

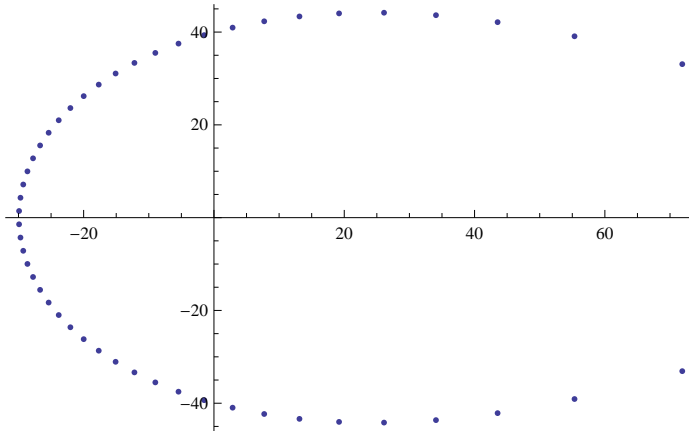
```
gxzeros[4, 20]
```

```
{-81.2453 - 9.41692 i, -81.2453 + 9.41692 i, -77.8803 - 28.1321 i, -77.8803 + 28.1321 i,
-71.0529 - 46.4806 i, -71.0529 + 46.4806 i, -60.5528 - 64.1806 i, -60.5528 + 64.1806 i,
-46.0198 - 80.8834 i, -46.0198 + 80.8834 i, -26.8683 - 96.1202 i, -26.8683 + 96.1202 i,
-2.12877 - 109.201 i, -2.12877 + 109.201 i, 29.9239 - 118.991 i, 29.9239 + 118.991 i,
72.8409 - 123.316 i, 72.8409 + 123.316 i, 136.577 - 116.663 i, 136.577 + 116.663 i}
```

```
Product[1 - 1/j, {j, gxzeros[2, 30]}]
```

```
1.64493 - 1.38778 × 10-17 i
```

```
ListPlot[Table[{Re[n], Im[n]}, {n, gxzeros[2, 50]}]]
```



```
gam[t_] := E^(-EulerGamma t) / t Product[(1 + t/n)^(-1 E^(t/n)), {n, 1, Infinity}]
```

```
lgam[t_] := Log[E^(-EulerGamma t) / t Product[(1 + t/n)^(-1 E^(t/n)), {n, 1, Infinity}]]
```

```
lgam2[t_] :=
```

```
(-EulerGamma t) - Log[t] + Log[Product[(1 + t/n)^(-1 E^(t/n)), {n, 1, Infinity}]]
```

```
lgam3[t_] := (-EulerGamma t) - Log[t] + Sum[Log[(1 + t/n)^(-1 E^(t/n))], {n, 1, Infinity}]
```

```
lgam4[t_] := (-EulerGamma t) - Log[t] + Sum[-Log[(1 + t/n)] + t/n, {n, 1, Infinity}]
```

```
N[lgam4[6.2 + I]]
```

```
5.04524 + 1.74679 i
```

```
Log[Gamma[6.2 + I]]
```

```
5.04524 + 1.74679 i
```

```
D[Expand[pza[fe, 10, z]], z] /. z -> 0
```

```
5.33333
```

```
D[Expand[pza[ff, 10, z]], z] /. z -> 0
```

```
0.718282
```

```
Table[{n, D[pz[fr, n, z], z] /. z -> 0}, {n, 1, 10}] // TableForm
```

```
1      0
2      fs[1]
3      fs[1] + fs[2]
4      fs[1] -  $\frac{fs[1]^2}{2}$  + fs[2] + fs[3]
5      fs[1] -  $\frac{fs[1]^2}{2}$  + fs[2] + fs[3] + fs[4]
6      fs[1] + fs[2] +  $\frac{1}{2}$  (-fs[1] fs[2] - fs[1] (fs[1] + fs[2])) + fs[3] + fs[4] + fs[5]
7      fs[1] + fs[2] +  $\frac{1}{2}$  (-fs[1] fs[2] - fs[1] (fs[1] + fs[2])) + fs[3] + fs[4] + fs[5] + fs[6]
8      fs[1] +  $\frac{fs[1]^3}{3}$  + fs[2] + fs[3] +  $\frac{1}{2}$  (-fs[1] fs[2] - fs[1] fs[3] - fs[1] (fs[1] + fs[2] + fs[3])) + f
9      fs[1] +  $\frac{fs[1]^3}{3}$  + fs[2] + fs[3] +  $\frac{1}{2}$  (-fs[2] (fs[1] + fs[2]) - fs[1] fs[3] - fs[1] (fs[1] + fs[2] + f
10     fs[1] +  $\frac{fs[1]^3}{3}$  + fs[2] + fs[3] + fs[4] +  $\frac{1}{2}$  (-fs[2] (fs[1] + fs[2]) - fs[1] fs[3] - fs[1] fs[4] - fs
```

```
oo[n_, k_] := Sum[1 / ((j - 1)!) (1 / k - oo[n / j, k + 1]), {j, 2, n}]
```

```
N[oo[10, 1]]
```

```
0.718282
```

```
pza[ff, 100 000, 10]
```

```
22 025.1
```

```
N[E^10]
```

```
22 026.5
```

```
N[1 / 10 000!]
```

```
3.513382867714318 × 10-35 660
```

```
Sum[(-1)^(k + 1) / k (E - 1)^k, {k, 1, Infinity}]
```

```
Sum::div: Sum does not converge. >>
```

$$\sum_{k=1}^{\infty} \frac{(-1)^{1+k} (-1 + e)^k}{k}$$

```
bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k!
```

```
e2[n_, k_] := e2[n, k] = Sum[1 / ((j - 1)!) e2[Floor[n / j], k - 1], {j, 2, n}]
```

```
e2[n_, 0] := UnitStep[n - 1]
```

```
ez[n_, z_] := Sum[bin[z, k] e2[n, k], {k, 0, Log[2, n]}]
```

```
dez[n_, z_] := ez[n, z] - ez[n - 1, z]
```

```
ldez[n_, k_] := D[dez[n, z], {z, k}] /. z -> 0
```

```
ezl[n_, k_] := ezl[n, k] = D[ez[n, z], {z, k}] /. z -> 0
```

```
ezlz[n_, z_] := Sum[z^k / (k!) ezl[n, k], {k, 0, Log[2, n]}]
```

```
cosz[n_, z_] :=
```

```
cosz[n, z] = Sum[(D[Cos[x], {x, k}] /. x -> 0) z^k / (k!) ezl[n, k], {k, 0, Log[2, n]}]
```

```
sinz[n_, z_] := sinz[n, z] =
```

```
Sum[(D[Sin[x], {x, k}] /. x -> 0) z^k / (k!) ezl[n, k], {k, 0, Log[2, n]}]
```

```
dcosz[n_, z_] := cosz[n, z] - cosz[n - 1, z]
```

```
dsinz[n_, z_] := sinz[n, z] - sinz[n - 1, z]
```

```
Grid[Table[(D[Expand[ez[n, z] - ez[n - 1, z]], {z, k}] /. z -> 0), {k, 1, 4}, {n, 2, 20}]]
```

1	$\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{24}$	-59	$\frac{1}{720}$	$\frac{841}{5040}$	-5\	-1\	1 /	16 \	1 /	-8 \	-1 \	-1 \	1 /	87 \	1 /	42 \	
			/				0 \	5 \	3 \	2 \	4 \	6 \	8 \	2 \	2 \	4 \	6 \	2 \	
			1 \				3 \	1 \	6 \	9 \	7 \	4 \	1 \	7 \	0 \	3 \	4 \	3 \	
			2 \				9 /	1 \	2 \	9 \	9 \	8 \	6 \	3 \	9 \	1 \	0 \	4 \	
			0				4 \	9 /	8 \	3 \	0 \	6 \	2 \	9 \	2 \	0 \	2 \	5 \	
							0 \	3 \	8 \	6 \	0 \	3 \	1 \	4 \	2 \	0 \	3 \	2 \	
							3 \	6 \	0 \	1 /	1 \	9 /	4 \	4 \	7 \	4 \	7 \	9 \	
							2 \	2 \	0	3 \	6 \	6 \	3 \	6 \	8 \	4 \	3 \	8 \	
							0	8 \		9 \	0 \	2 \	9 \	7 \	9 \	6 \	7 \	7 \	
								8 \		9 \	0	2 \	9 /	1 \	8 \	0 \	0 \	3 \	
								0		1 \		7 \	8 \	9 \	8 \	8 \	5 \	5 \	
										6 \		0 \	7 \	9 /	8 \	0 \	7 \	3 \	
										8 \		2 \	1 \	1 \	0 \	1 /	2 \	6 \	
										0 \		0 \	7 \	3 \	0 \	3 \	8 \	0 \	
										0		8 \	8 \	0 \	0	5 \	0 \	1 /	
												0 \	2 \	7 \		5 \	0 \	1 \	
												0	9 \	6 \		6 \	0	2 \	
													1 \	7 \		8 \		1 \	
													2 \	4 \		7 \		6 \	
													0 \	3 \		4 \		4 \	
													0	6 \		2 \		5 \	
														8 \		8 \		1 \	
														0 \		0 \		0 \	
														0 \		9 \		0 \	
														0		6 \		4 \	
																0 \		0 \	
																0 \		8 \	
																0		8 \	
																		3 \	
																		2 \	
																		0 \	
																		0 \	
																		0	
0	0	1	0	1	0	$-\frac{2}{3}$	$\frac{1}{4}$	$\frac{1}{12}$	0	$-\frac{79}{60}$	0	$\frac{1}{360}$	$\frac{1}{24}$	$\frac{1121}{2520}$	0	-1 \	0	-2 \	
																	4 \		0 \
																	9 \		1 \
																	5 \		5 \
																	1 /		9 /
																	2 \		1 \
																	0 \		8 \
																	1 \		1 \
																	6 \		4 \
																	0		4 \
																			0
0	0	0	0	0	0	1	0	0	0	$\frac{3}{2}$	0	0	0	-1	0	$\frac{3}{4}$	0	$\frac{1}{8}$	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	

N[Sum[dez[j, 2] ez[240 / j, 3 + I], {j, 1, 240}]]
 82.2577 + 124.894 i
N[ez[240, 5 + I]]
 82.2577 + 124.894 i

Expand[pza[ff, 40 000, z]]

$$1 + 0.634243 z + 1.73374 z^2 - 1.6071 z^3 + 1.49646 z^4 - 0.755491 z^5 + 0.273172 z^6 - 0.0672299 z^7 + 0.0118127 z^8 - 0.0014458 z^9 + 0.00012414 z^{10} - 7.33435 \times 10^{-6} z^{11} + 3.22744 \times 10^{-7} z^{12} - 1.13705 \times 10^{-8} z^{13} + 2.99195 \times 10^{-10} z^{14} + 7.64716 \times 10^{-13} z^{15}$$

pza[fh, 4000, 1]

$$0.540302 + 0.841471 i$$

N[E^(1 I)]

$$0.540302 + 0.841471 i$$

pz2[ff, 5, z]

$$1. + 1. z + 0.5 z^2 + 0.166667 z^3 + 0.0416667 z^4 + 0.00833333 z^5$$

N[pz2[ff, 20, 2] pz2[ff, 20, 3 + I]]

$$80.188 + 124.885 i$$

N[pz2[ff, 20, 5 + I]]

$$80.188 + 124.885 i$$

Expand[N[cosz[100, z] + I sinz[100, z]]]

$$1. + (0. + 0.937597 i) z - 0.627118 z^2 - (0. + 0.0689598 i) z^3 + 0.0894681 z^4 - (0. + 0.0104167 i) z^5 - 0.00555556 z^6$$

Expand[N[ez[100, z I]]]

$$1. + (0. + 0.937597 i) z - (0.627118 + 0. i) z^2 - (0. + 0.0689598 i) z^3 + (0.0894681 + 0. i) z^4 - (0. + 0.0104167 i) z^5 - (0.00555556 + 0. i) z^6$$

N[cosz[1000, Pi / 2]]

$$0.685114$$

N[sinz[1000, -Pi / 2]]

$$-1.8145$$

N[sinz[100, 2 + 3 I]]

$$3.77651 + 8.4109 i$$

N[Sum[dsinz[j, 2] cosz[100 / j, 3 I], {j, 1, 100}] + Sum[dcosz[j, 2] sinz[100 / j, 3 I], {j, 1, 100}]]

$$3.77651 + 8.4109 i$$

N[cosz[100, 2 + 3 I]]

$$-17.8167 - 13.6616 i$$

N[Sum[dcosz[j, 2] cosz[100 / j, 3 I], {j, 1, 100}] - Sum[dsinz[j, 2] sinz[100 / j, 3 I], {j, 1, 100}]]

$$-17.8167 - 13.6616 i$$

N[sinz[100, 2 (2 + 3 I)]]

$$-11.5391 + 199.66 i$$

2 N[Sum[dsinz[j, 2 + 3 I] cosz[100 / j, 2 + 3 I], {j, 1, 100}]]

$$-11.5391 + 199.66 i$$


```
N[cosz[100, 2 (2 + 3 I)]]
```

```
-880.36 + 92.5196 i
```

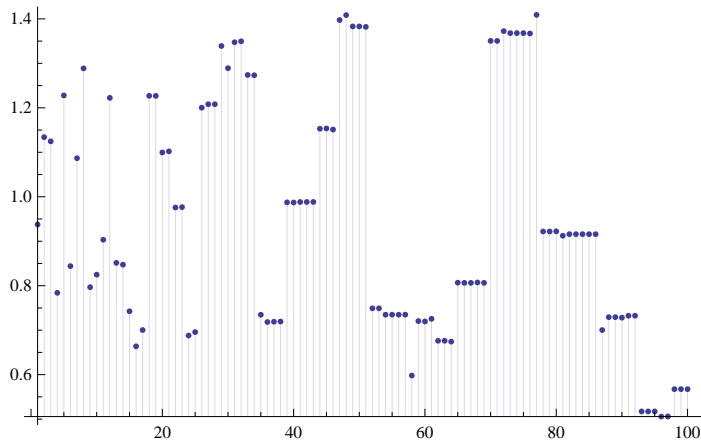
```
N[Sum[dcosz[j, 2 + 3 I] cosz[100 / j, 2 + 3 I], {j, 1, 100}] -  
Sum[dsinz[j, 2 + 3 I] sinz[100 / j, 2 + 3 I], {j, 1, 100}]]
```

```
-880.36 + 92.5196 i
```

```
N[Sum[dcosz[j, 2 + 3 I] cosz[100 / j, 2 + 3 I], {j, 1, 100}] +  
Sum[dsinz[j, 2 + 3 I] sinz[100 / j, 2 + 3 I], {j, 1, 100}]]
```

```
1.
```

```
DiscretePlot[D[Expand[N[pza[ff, 100 n, z]]], z] /. z -> 0, {n, 1, 100}]
```



```
N[Expand[ez[100, z]]]
```

```
1. + 0.937597 z + 0.627118 z2 + 0.0689598 z3 + 0.0894681 z4 - 0.0104167 z5 + 0.00555556 z6
```

```
Expand[pza[ff, 10 000, z]]
```

```
1 + 0.56733 z + 1.81209 z2 - 1.47941 z3 + 1.18578 z4 -  
0.485266 z5 + 0.140903 z6 - 0.0263209 z7 + 0.00346853 z8 - 0.000311406 z9 +  
0.0000218201 z10 - 1.13957 × 10-6 z11 + 4.07097 × 10-8 z12 + 1.6059 × 10-10 z13
```

```
Grid[Table[ldez[n, k], {k, 1, 4}, {n, 2, 20}]]
```

1	$\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{24}$	-59	$\frac{1}{720}$	$\frac{841}{5040}$	-5\	-1\	1 /	16\	1 /	-8\	-1\	-1\	1 /	87\	1 /	42\
			/				0\	5\	3\	2\	4\	6\	8\	2\	2\	4\	6\	2\
			1\				3\	1\	6\	9\	7\	4\	1\	7\	0\	3\	4\	3\
			2\				9 /	1\	2\	9\	9\	8\	6\	3\	9\	1\	0\	4\
			0				4\	9 /	8\	3\	0\	6\	2\	9\	2\	0\	2\	5\
							0\	3\	8\	6\	0\	3\	1\	4\	2\	0\	3\	2\
							3\	6\	0\	1 /	1\	9 /	4\	4\	7\	4\	7\	9\
							2\	2\	0	3\	6\	6\	3\	6\	8\	4\	3\	8\
							0	8\		9\	0\	2\	9\	7\	9\	6\	7\	7\
								8\		9\	0	2\	9 /	1\	8\	0\	0\	3\
								0		1\		7\	8\	9\	8\	8\	5\	5\
										6\		0\	7\	9 /	8\	0\	7\	3\
										8\		2\	1\	1\	0\	1 /	2\	6\
										0\		0\	7\	3\	0\	3\	8\	0\
										0		8\	8\	0\	0	5\	0\	1 /
												0\	2\	7\		5\	0\	1\
												0	9\	6\		6\	0	2\
													1\	7\		8\		1\
													2\	4\		7\		6\
													0\	3\		4\		4\
													0	6\		2\		5\
														8\		8\		1\
														0\		0\		0\
														0\		9\		0\
														0		6\		4\
																0\		0\
																0\		8\
																0		8\
																		3\
																		2\
																		0\
																		0\
																		0

0	0	1	0	1	0	$-\frac{2}{3}$	$\frac{1}{4}$	$\frac{1}{12}$	0	$-\frac{79}{60}$	0	$\frac{1}{360}$	$\frac{1}{24}$	$\frac{1121}{2520}$	0	-1\	0	-2\
																4\		0\
																9\		1\
																5\		5\
																1 /		9 /
																2\		1\
																0\		8\
																1\		1\
																6\		4\
																0		4\
																		0

0	0	0	0	0	0	1	0	0	0	$\frac{3}{2}$	0	0	0	-1	0	$\frac{3}{4}$	0	$\frac{1}{8}$
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0

```
Table[ldez[2^n, 1], {n, 1, 5}]
```

```
{1, -\frac{1}{3}, \frac{841}{5040}, -\frac{127394467199}{1307674368000}, \frac{503866402976241324312641187840001}{8222838654177922817725562880000000}}
```

```
DiscretePlot[D[ez[n, z], z] /. z -> 0, {n, 2, 100}]
```

```

Sum[1 / (x + 1) ^ k / k, {k, 1, Infinity}]

-Log[ $\frac{x}{1+x}$ ]

Sum[-(1 - x) ^ k / k, {k, 1, Infinity}]

Log[x]

Sum[-(1 - x) ^ k / k, {k, 1, Infinity}]

Sum[(-1) ^ (k + 1) x ^ k / k, {k, 1, Infinity}]

Log[1 + x]

Sum[(-1) ^ (k + 1) (x - 1) ^ k / k, {k, 1, Infinity}]

Log[x]

Sum[z ^ k Log[x] ^ k / k!, {k, 0, Infinity}]

xz

lx[n_, x_, k_] := lx[n, x, k] = Sum[(-j ^ -1) (1 - x) ^ j lx[Floor[n / j], x, k - 1], {j, 1, n}]
lx[n_, x_, 0] := UnitStep[n - 1]
xz[n_, x_, z_] := Sum[z ^ k / (k!) lx[n, x, k], {k, 0, 70}]
lx[3000, 1.5, 16]

4.61208 × 10-7

N[Log[1.5]] ^ 16

5.33649 × 10-7

Expand[xz[500, N[1 / E], z]] /. z → -1

2.71826

1 / (N[1 / E] ^ 2)

7.38906

N[ez1[500, 2]]

-0.238567

lx[500, N[1 / E], 2]

1.

Log[(a - 1) !] + Log[(b - 1) !]

-Log[(-1 + a) !] - Log[(-1 + b) !]

-Log[(-1 + a) !] - Log[(-1 + b) !]

Expand[pza[ff, 10 000, z]]

1 + 0.56733 z + 1.81209 z2 - 1.47941 z3 + 1.18578 z4 -
0.485266 z5 + 0.140903 z6 - 0.0263209 z7 + 0.00346853 z8 - 0.000311406 z9 +
0.0000218201 z10 - 1.13957 × 10-6 z11 + 4.07097 × 10-8 z12 + 1.6059 × 10-10 z13

```

Grid[Table[pza[ff, 10 000, a + b I], {a, -3, 3, .6}, {b, -3, 3, .6}]]

-118.18	-125.12	-77.768	-10.142	45.6188	66.9091	45.6188	-10.142	-77.768	-125.12	-1
4 -	12.8	1 +	8 +	52.9	0. i	52.9	8 -	1 -	12.8	
120.835 i	964 i	58.4	78.8	852 i		852 i	78.8	58.4	964 i	1
		357 i	621 i				621 i	357 i		
-36.24	-55.09	-40.465	-10.549	16.3285	26.89	16.3285	-10.549	-40.465	-55.09	-3
69 -	39 -	+	+	+	39 +	-	-	-	39 +	
77.57	24.00	16.26	31.90	23.28	0. i	23.28	31.90	16.26	24.00	7
18 i	24 i	36 i	49 i	45 i		45 i	49 i	36 i	24 i	
-1.725	-19.60	-18.44	-7.023	4.82234	9.683	4.82234	-7.023	-18.44	-19.60	-1
48 -	81 -	04 +	62 +	+	09 +	-	62 -	04 -	81 +	
42.49	19.76	0.730	10.79	9.117	0. i	9.117	10.79	0.730	19.76	4
49 i	64 i	831 i	69 i	78 i		78 i	69 i	831 i	64 i	
8.48289	-4.0194	-6.816	-3.493	1.12322	3.151	1.12322	-3.493	-6.816	-4.0194	8.
-	-	31 -	48 +	+	13 +	-	48 -	31 +	+	+
19.11	12.16	3.1756	2.405	2.929	0. i	2.929	2.405	3.1756	12.16	1
68 i	51 i	i	58 i	87 i		87 i	58 i	i	51 i	
8.11545	1.23447	-1.509	-1.079	0.4427	1.193	0.4427	-1.079	-1.509	1.23447	8.
-	-	01 -	52 -	85 +	63 +	85 -	52 +	01 +	+	+
6.1614	6.033	3.007	0.362	0.517	0. i	0.517	0.362	3.007	6.033	6
i	18 i	89 i	778 i	466 i		466 i	778 i	89 i	18 i	
4.57207	1.91479	0.4434	0.2895	0.7398	1. +	0.7398	0.2895	0.4434	1.91479	4.
-	-	61 -	98 -	46 -	0. i	46 +	98 +	61 +	+	+
0.653	2.564	2.122	1.057	0.316		0.316	1.057	2.122	2.564	0
437 i	37 i	44 i	79 i	688 i		688 i	79 i	44 i	37 i	
1.22423	1.13168	0.9384	1.04758	1.31444	1.448	1.31444	1.04758	0.9384	1.13168	1.
+	-	13 -	-	-	11 +	+	+	13 +	+	-
0.417	1.3962	1.7147	1.277	0.633	0. i	0.633	1.277	1.7147	1.3962	0
724 i	i	i	47 i	04 i		04 i	47 i	i	i	
-0.899	0.2922	1.03856	1.62654	2.05396	2.214	2.05396	1.62654	1.03856	0.2922	-0
336 -	79 -	-	-	-	79 +	+	+	+	79 +	
0.654	1.7566	2.022	1.642	0.895	0. i	0.895	1.642	2.022	1.7566	0
071 i	i	04 i	54 i	945 i		945 i	54 i	04 i	i	
-1.948	-0.174	1.24698	2.34978	3.07146	3.324	3.07146	2.34978	1.24698	-0.174	-1
96 -	666 -	-	-	-	97 +	+	+	+	666 +	
2.497	3.016	2.983	2.3607	1.294	0. i	1.294	2.3607	2.983	3.016	2
49 i	09 i	73 i	i	59 i		59 i	i	73 i	09 i	
-2.512	-0.309	1.75635	3.45009	4.56651	4.956	4.56651	3.45009	1.75635	-0.309	-2
49 -	393 -	-	-	-	94 +	+	+	+	393 +	
4.575	4.888	4.565	3.531	1.922	0. i	1.922	3.531	4.565	4.888	4
38 i	49 i	12 i	14 i	89 i		89 i	14 i	12 i	49 i	
-3.203	-0.300	2.64111	5.13698	6.80385	7.389	6.80385	5.13698	2.64111	-0.300	-3
13 -	157 -	-	-	-	06 +	+	+	+	157 +	
6.974	7.448	6.891	5.295	2.874	0. i	2.874	5.295	6.891	7.448	6
38 i	97 i	84 i	81 i	56 i		56 i	81 i	84 i	97 i	

Grid[Table[E^(a + b I), {a, -2, 2, .4}, {b, -2, 2, .4}]]

-0.05\	-0.00\	0.049\	0.094\	0.124\	0.135\	0.124\	0.094\	0.049\	-0.00\	-0
63\	39\	0398 -	289 -	652 -	335 +	652 +	289 +	0398 +	39\	
19\	51\	0.12\	0.09\	0.05\	0. i	0.05\	0.09\	0.12\	51\	
3 -	73 -	613\	708\	270\		270\	708\	613\	73 +	
0.12\	0.13\	8 i	36 i	2 i		2 i	36 i	8 i	0.13\	0
306 i	527\								527\	
	8 i								8 i	
-0.084\	-0.005\	0.0731\	0.1406\	0.1859\	0.201\	0.1859\	0.1406\	0.0731\	-0.005\	-0
018\	89\	588 -	63 -	59 -	897 +	59 +	63 +	588 +	89\	
6 -	52\	0.188\	0.144\	0.078\	0. i	0.078\	0.144\	0.188\	52\	
0.183\	8 -	175 i	832 i	6222		6222	832 i	175 i	8 +	0
584 i	0.201\			i		i			0.201\	
	81 i								81 i	
-0.125\	-0.008\	0.10914	0.2098\	0.2774\	0.301\	0.2774\	0.2098\	0.10914	-0.008\	-0
341 -	79\	-	44 -	18 -	194 +	18 +	44 +	+	79\	
0.273\	47\	0.280\	0.216\	0.117\	0. i	0.117\	0.216\	0.280\	47\	0
875 i	3 -	725 i	064 i	291 i		291 i	064 i	725 i	3 +	
	0.301\								0.301\	
	066 i								066 i	
-0.186\	-0.013\	0.1628\	0.3130\	0.4138\	0.449\	0.4138\	0.3130\	0.1628\	-0.013\	-0
987 -	120\	18 -	51 -	59 -	329 +	59 +	51 +	18 +	120\	
0.408\	2 -	0.418\	0.322\	0.174\	0. i	0.174\	0.322\	0.418\	2 +	0
574 i	0.449\	792 i	329 i	977 i		977 i	329 i	792 i	0.449\	
	137 i								137 i	
-0.278\	-0.019\	0.2428\	0.4670\	0.6174\	0.670\	0.6174\	0.4670\	0.2428\	-0.019\	-0
952 -	573 -	96 -	16 -	06 -	32 +	06 +	16 +	96 +	573 +	
0.609\	0.670\	0.624\	0.480\	0.261\	0. i	0.261\	0.480\	0.624\	0.670\	0
52 i	034 i	764 i	858 i	035 i		035 i	858 i	764 i	034 i	
-0.416\	-0.029\	0.3623\	0.6967\	0.9210\	1.	0.9210\	0.6967\	0.3623\	-0.029\	-0
147 -	199\	58 -	07 -	61 -		61 +	07 +	58 +	199\	
0.909\	5 -	0.932\	0.717\	0.389\		0.389\	0.717\	0.932\	5 +	0
297 i	0.999\	039 i	356 i	418 i		418 i	356 i	039 i	0.999\	
	574 i								574 i	
-0.620\	-0.043\	0.5405\	1.03936	1.37406	1.491\	1.37406	1.03936	0.5405\	-0.043\	-0
818 -	560\	74 -	-	-	82 +	+	+	74 +	560\	
1.356\	6 -	1.390\	1.070\	0.580\	0. i	0.580\	1.070\	1.390\	6 +	1
51 i	1.491\	44 i	17 i	944 i		944 i	17 i	44 i	1.491\	
	19 i								19 i	
-0.926\	-0.064\	0.8064\	1.55055	2.04986	2.225\	2.04986	1.55055	0.8064\	-0.064\	-0
152 -	984\	42 -	-	-	54 +	+	+	42 +	984\	
2.023\	7 -	2.074\	1.596\	0.866\	0. i	0.866\	1.596\	2.074\	7 +	2
68 i	2.224\	29 i	51 i	666 i		666 i	51 i	29 i	2.224\	
	59 i								59 i	
-1.381\	-0.096\	1.20307	2.31315	3.05803	3.320\	3.05803	2.31315	1.20307	-0.096\	-1
66 -	945\	-	-	-	12 +	+	+	+	945\	
3.018\	8 -	3.094\	2.381\	1.292\	0. i	1.292\	2.381\	3.094\	8 +	3
97 i	3.3187	48 i	71 i	91 i		91 i	71 i	48 i	3.3187	
	i								i	
-2.061\	-0.144\	1.79477	3.45081	4.56204	4.953\	4.56204	3.45081	1.79477	-0.144\	-2
19 -	626 -	-	-	-	03 +	+	+	+	626 +	
4.503\	4.950\	4.616\	3.553\	1.9288	0. i	1.9288	3.553\	4.616\	4.950\	4
78 i	92 i	42 i	09 i	i		i	09 i	42 i	92 i	

```

-3.074\ -0.215\ 2.67748 5.148 - 6.80577 7.389\ 6.80577 5.148 + 2.67748 -0.215\ -3
93 - 757 - - 5.300\ - 06 + + 5.300\ + 757 +
6.718\ 7.385\ 6.886\ 58 i 2.877\ 0. i 2.877\ 58 i 6.886\ 7.385\ 6
85 i 91 i 89 i 43 i 43 i 89 i 91 i

```

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Grid[Table[pza[ff, 10 000, a + b I] - E^ (a + b I), {a, -3, 3, .6}, {b, -3, 3, .6}]]
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-21.5\ -761.\ -735.\ -261.\ 248.3\ 460.1\ 248.3\ -261.\ -735.\
45\ 33\ 82\ 17 + 71 + 49 + 71 - 17 - 82\
7 - 1 - 4 + 512.\ 419.\ 0. i 419.\ 512.\ 4 -
1497\ 693.\ 109.\ 894 i 476 i 476 i 894 i 109.\
.35 i 846 i 399 i 399 i
271.723 -182.9\ -281.1\ -135.0\ 64.4098 152.2\ 64.4098 -135.0\ -281.1\
- 45 - 23 - 14 + + 15 + - 14 - 23 +
616.9\ 387.9\ 52.37\ 154.8\ 151.5\ 0. i 151.5\ 154.8\ 52.37\
89 i 28 i 15 i 42 i 39 i 39 i 42 i 15 i
236.752 7.09808 -83.73\ -56.95\ 10.2796 42.78\ 10.2796 -56.95\ -83.73\
- - 96 - 25 + + 63 + - 25 - 96 +
177.3\ 170.4\ 58.50\ 32.82\ 46.95\ 0. i 46.95\ 32.82\ 58.50\
01 i 86 i 93 i 13 i 11 i 11 i 13 i 93 i
127.856 39.1537 -13.38\ -18.54\ -0.849\ 9.381\ -0.849\ -18.54\ -13.38\
- - 05 - 96 + 82 + 89 + 82 - 96 - 05 +
3.726\ 52.58\ 31.34\ 1.011\ 11.43\ 0. i 11.43\ 1.011\ 31.34\
18 i 12 i 15 i 56 i 36 i 36 i 56 i 15 i
42.494 + 24.4338 3.53499 -3.783\ -1.191\ 1.281\ -1.191\ -3.783\ 3.53499
35.74\ - - 93 - 23 + 48 + 23 - 93 + +
35 i 4.751\ 10.31\ 2.810\ 1.703\ 0. i 1.703\ 2.810\ 10.31\
89 i 86 i 13 i 93 i 93 i 13 i 86 i
0.5012\ 7.3952 + 3.38592 0.0811\ -0.330\ 0. + -0.330\ 0.0811\ 3.38592
34 + 6.235\ - 037 - 528 - 0. i 528 + 037 + +
24.778 65 i 1.056\ 1.1904 0.058\ 0.058\ 1.1904 1.056\
i 24 i i 3084 i i 3084 i 24 i
-9.505\ -0.687\ 0.8437\ 0.3361\ 0.00686\ -0.02\ 0.00686\ 0.3361\ 0.8437\
95 + 161 + 27 + 75 - 235 - 70\ 235 + 75 + 27 -
6.755\ 3.934\ 0.920\ 0.082\ 0.0813\ 85\ 0.0813\ 0.082\ 0.920\
16 i 57 i 956 i 5363 i 466 i 6 + 466 i 5363 i 956 i
0. i
-5.272\ -1.7586 -0.256\ 0.04390\ 0.02389\ 0.004\ 0.02389\ 0.04390\ -0.256\
74 - + 666 + 96 + 52 + 8550\ 52 - 96 - 666 -
2.673\ 0.394\ 0.422\ 0.1107\ 0.0043\ 2 + 0. i 0.0043\ 0.1107\ 0.422\
32 i 219 i 273 i 46 i 8912 i 8912 i 46 i 273 i
0.2886\ -0.431\ -0.205\ -0.0443\ -0.0024\ 0.00175\ -0.0024\ -0.0443\ -0.205\
06 - 86 - 613 - 002 + 5115 + 243 + 0. i 5115 - 002 - 613 +
3.268\ 0.793\ 0.083\ 0.0188\ 0.0087\ 0.0087\ 0.0188\ 0.083\
22 i 026 i 4832 i 182 i 1033 i 1033 i 182 i 4832 i
1.97668 0.3951\ 0.03639\ -0.0073\ -0.0035\ -0.001189\ -0.0035\ -0.0073\ 0.03639\
- 98 - 11 - 13 - 108 - 97 + 108 + 13 + 11 +
0.403\ 0.316\ 0.10687 0.0210\ 0.0019\ 0. i 0.0019\ 0.0210\ 0.10687
477 i 94 i i 753 i 2363 i 2363 i 753 i i
0.55134 0.2208\ 0.06053\ 0.01140\ 0.00123\ 0. + 0. i 0.00123\ 0.01140\ 0.06053\
+ 67 + 37 + 39 - 102 - 102 + 39 + 37 -
1.275\ 0.232\ 0.0233\ 0.0020\ 0.0013\ 0.0013\ 0.0020\ 0.0233\
22 i 239 i 881 i 4373 i 5544 i 5544 i 4373 i 881 i

```

Grid[Table[pza[ff, 100 000, a + b I] - E^(a + b I), {a, -3, 3, .6}, {b, -3, 3, .6}]]

1738.\%	538.0\%	-309.\%	-358.\%	21.05\%	237.3\%	21.05\%	-358.\%	-309.\%
61 +	47 -	50\%	54\%	68 +	17 +	68 -	54\%	50\%
64.7\%	824.\%	6 -	1 +	267.\%	0. i	267.\%	1 -	6 +
318 i	394 i	492.\%	93.1\%	194 i		194 i	93.1\%	492.\%
		703 i	447 i				447 i	703 i
506.075	303.968	0.6491\%	-89.76\%	-12.52\%	44.88\%	-12.52\%	-89.76\%	0.6491\%
+	-	41 -	96 -	27 +	22 +	27 -	96 +	41 +
401.9\%	122.4\%	169.7\%	20.53\%	54.38\%	0. i	54.38\%	20.53\%	169.7\%
26 i	66 i	79 i	04 i	37 i		37 i	04 i	79 i
37.9632	100.306	31.7413	-12.60\%	-6.504\%	4.926\%	-6.504\%	-12.60\%	31.7413
+	+	-	12 -	29 +	46 +	29 -	12 +	+
234.9\%	37.11\%	35.422	16.645	6.165\%	0. i	6.165\%	16.645	35.422
39 i	14 i	i	i	33 i		33 i	i	i
-56.39\%	11.8716	13.4481	1.49004	-1.382\%	-0.25\%	-1.382\%	1.49004	13.4481
34 +	+	+	-	97 -	47\%	97 +	+	-
68.37\%	33.20\%	0.849\%	4.612\%	0.605\%	04 +	0.605\%	4.612\%	0.849\%
61 i	04 i	633 i	76 i	015 i	0. i	015 i	76 i	633 i
-33.95\%	-7.405\%	1.73994	1.23971	0.0197\%	-0.17\%	0.0197\%	1.23971	1.73994
42 -	91 +	+	-	328 -	48\%	328 +	+	-
2.6219	9.546\%	3.638\%	0.146\%	0.367\%	9 +	0.367\%	0.146\%	3.638\%
i	04 i	18 i	436 i	843 i	0. i	843 i	436 i	18 i
-5.784\%	-4.300\%	-0.890\%	0.13445	0.0797\%	0. +	0.0797\%	0.13445	-0.890\%
18 -	54 -	751 +	+	037 -	0. i	037 +	-	751 -
12.45\%	0.915\%	0.951\%	0.323\%	0.009\%		0.009\%	0.323\%	0.951\%
36 i	183 i	901 i	351 i	2558\%		2558\%	351 i	901 i
				8 i		8 i		
3.85278	-0.239\%	-0.408\%	-0.096\%	0.00204\%	0.007\%	0.00204\%	-0.096\%	-0.408\%
-	241 -	213 -	475\%	784 +	5113\%	784 -	475\%	213 +
4.449\%	1.709\%	0.220\%	2 +	0.0235\%	2 + 0. i	0.0235\%	2 -	0.220\%
13 i	64 i	944 i	0.056\%	42 i		42 i	0.056\%	944 i
			406 i				406 i	
2.52351	0.6788\%	0.0596\%	-0.022\%	-0.0079\%	-0.0015\%	-0.0079\%	-0.022\%	0.0596\%
+	58 -	424 -	157\%	326\%	5843 +	326\%	157\%	424 +
0.972\%	0.2909	0.174\%	3 -	5 -	0. i	5 +	3 +	0.174\%
416 i	i	347 i	0.034\%	0.0005\%		0.0005\%	0.034\%	347 i
			6435 i	4708 i		4708 i	6435 i	
-0.112\%	0.1970\%	0.0796\%	0.01476\%	0.00058\%	-0.0006\%	0.00058\%	0.01476\%	0.0796\%
804 +	32 +	028 +	89 -	7594 -	2498\%	7594 +	89 +	028 -
1.332\%	0.283\%	0.018\%	0.0091\%	0.0031\%	1 +	0.0031\%	0.0091\%	0.018\%
69 i	924 i	3708 i	1813 i	8053 i	0. i	8053 i	1813 i	3708 i
-0.703\%	-0.127\%	-0.0066\%	0.00398\%	0.00143\%	0.00046\%	0.00143\%	0.00398\%	-0.0066\%
708 +	822 +	8977 +	417 +	266 +	9755 +	266 -	417 -	8977 -
0.106\%	0.121\%	0.0397\%	0.0072\%	0.0006\%	0. i	0.0006\%	0.0072\%	0.0397\%
475 i	987 i	296 i	8907 i	00265 i		00265 i	8907 i	296 i
-0.139\%	-0.076\%	-0.0217\%	-0.0040\%	-0.00042\%	0. + 0. i	-0.00042\%	-0.0040\%	-0.0217\%
118 -	261\%	953 -	5502 +	9422 +		9422 -	5502 -	953 +
0.381\%	9 -	0.0029\%	0.0018\%	0.00066\%		0.00066\%	0.0018\%	0.0029\%
321 i	0.062\%	0354 i	5249 i	4949 i		4949 i	5249 i	0354 i
	0886 i							

Grid[Table[pza[ff, 100 000, a + b I] - E^(a + b I), {a, -5, 5, 1}, {b, -5, 5, 1}]]

64 281\	70 012.	3673.\	-22 23\	-2000\	14 197\	-2000\	-22 23\	3673.\	70
.9 +	-	47 -	6.\	.4\	.5	.4\	6.\	47 +	
127 0\	8226\	37 45\	2 -	6 +		6 -	2 +	37 45\	
47. i	.66 i	3.5 i	3892\	15 90\		15 90\	3892\	3.5 i	
			.39 i	9. i		9. i	.39 i		
-14 225\	15 203.3	5842.11	-3358.\	-1073\	2229.\	-1073\	-3358.\	5842.11	1
.7 +	+	-	73 -	.7\	55	.7\	73 +	+	
41 643\	11 057\	6220.\	2736.\	8 +		8 -	2736.\	6220.\	
.5 i	.5 i	47 i	16 i	2446\		2446\	16 i	47 i	
				.19 i		.19 i			
-12 667\	103.254	1738.61	-115.2\	-248.82	237.3\	-248.82	-115.2\	1738.61	1
.4 +	+	+	17 -	+	17	-	17 +	-	
3816.\	4731.\	64.73\	679.2\	211.7\		211.7\	679.2\	64.73\	
66 i	59 i	18 i	15 i	66 i		66 i	15 i	18 i	
-2685.\	-1040.\	135.005	72.4984	-26.64\	11.39\	-26.64\	72.4984	135.005	-
24 -	63 +	+	-	98 -	75	98 +	+	-	
2929.\	519.5\	303.2\	59.35\	3.9075		3.9075	59.35\	303.2\	
32 i	82 i	56 i	62 i	i		i	62 i	56 i	
660.709	-183.1\	-53.88\	8.79261	0.4429\	-0.34\	0.4429\	8.79261	-53.88\	-
-	35 -	89 +	+	75 -	835\	75 +	-	89 -	
1043.\	229.2\	34.80\	9.026\	2.043\	8	2.043\	9.026\	34.80\	
56 i	68 i	73 i	96 i	01 i		01 i	96 i	73 i	
364.891	63.5883	-5.784\	-1.765\	0.1585\	0.	0.1585\	-1.765\	-5.784\	6
+	-	18 -	72 +	57 +		57 -	72 -	18 +	
192.2\	49.01\	12.45\	0.882\	0.151\		0.151\	0.882\	12.45\	
53 i	15 i	36 i	825 i	955 i		955 i	825 i	36 i	
-83.68\	10.4709	3.47243	0.0594\	-0.033\	4.440\	-0.033\	0.0594\	3.47243	1
97 +	+	-	098 -	041\	89 x	041\	098 +	+	
127.9\	22.60\	0.167\	0.390\	9 -	10 ⁻¹⁶	9 +	0.390\	0.167\	
35 i	89 i	265 i	343 i	0.005\		0.005\	343 i	265 i	
				1542\		1542\			
				7 i		7 i			
-43.349	-8.989\	-0.538\	0.0595\	0.00746\	0.	0.00746\	0.0595\	-0.538\	-
-	19 +	972 +	94 +	396 -		396 +	94 -	972 -	
47.52\	0.346\	0.930\	0.086\	0.0020\		0.0020\	0.086\	0.930\	
25 i	529 i	314 i	1939 i	5494 i		5494 i	1939 i	314 i	
29.2173	1.79776	-0.139\	-0.0347\	-0.0020\	0.	-0.0020\	-0.0347\	-0.139\	1
-	-	118 -	66 -	8239 +		8239 -	66 +	118 +	
10.39\	3.325\	0.381\	0.0114\	0.0014\		0.0014\	0.0114\	0.381\	
33 i	44 i	321 i	166 i	7848 i		7848 i	166 i	321 i	
-3.5957	0.7336\	0.1673\	0.0158 -	0.00067\	0.	0.00067\	0.0158 +	0.1673\	0
+	46 +	74 +	0.0049\	9721 -		9721 +	0.0049\	74 -	
16.70\	1.6229	0.070\	3253 i	0.0009\		0.0009\	3253 i	0.070\	
29 i	i	1458 i		40981 i		40981 i		1458 i	
-5.881\	-0.754\	-0.0784\	-0.0055\	-0.00020\	-2.84\	-0.00020\	-0.0055\	-0.0784\	-
07 -	944 -	859 +	2141 +	667 +	217 x	667 -	2141 -	859 -	
9.169\	0.290\	0.0453\	0.0082\	0.00070\	10 ⁻¹⁴	0.00070\	0.0082\	0.0453\	
63 i	649 i	717 i	3736 i	1551 i		1551 i	3736 i	717 i	

Limit[(pza[ff, 100 000, z] - 1) / z, z → 0]

1.0943


```

Table[ D[pza[ff, 1000 n, z], {z, 1}] /. z -> 0, {n, 1, 35}]
{0.824825, 1.09966, 1.28911, 0.987097, 1.38306, 0.71955, 1.3505, 0.922266, 0.728064,
 0.56733, 1.33983, 1.23492, 1.06204, 0.531632, 0.52117, 1.17214, 1.18722, 1.36354,
 1.54283, 1.43169, 0.587194, 0.554, 0.554792, 0.84169, 0.823726, 1.11178, 1.09761,
 1.60824, 1.64054, 1.59375, 1.62317, 0.721691, 0.693366, 0.693199, 0.585877}

zet2x[n_, s_, k_, x_] := Sum[ x^j j^s - s zet2x[ Floor[n / j], s, k - 1, x], {j, 2, n}]
zet2x[n_, s_, 0, x_] := UnitStep[n - 1]
zetzx[n_, s_, z_, x_] := Sum[ bin[ z, k] zet2x[n, s, k, x], {k, 0, Log[2, n]}]

N[zetzx[300, 0, 3, 1 / 3]]
1.58796

2. × 13 / 12.

2.16667

2 × 7. / 6.

2.33333

Sum[ x^j, {j, 2, Infinity}]

$$-\frac{x^2}{-1+x}$$

FullSimplify[1 -  $\frac{x^2}{-1+x}$ ]

$$1 - \frac{x^2}{-1+x}$$

Sum[ x^j x^k, {j, 2, Infinity}, {k, 2, Infinity}]

$$\frac{x^4}{(-1+x)^2}$$

Sum[ x^j x^k x^l, {j, 2, Infinity}, {k, 2, Infinity}, {l, 2, Infinity}]

$$-\frac{x^6}{(-1+x)^3}$$

ft[t_] := (-1)^t x^(2 t) / (x - 1)^t
ft[0]
1
Sum[Binomial[z, n] (-1)^n x^(2 n) / (x - 1)^n, {n, 0, Infinity}]

$$\left( \frac{-1+x-x^2}{-1+x} \right)^z$$

Sum[ (-1)^n x^(2 n) / (x - 1)^n, {n, 0, 3}]

$$1 - \frac{x^2}{-1+x} + \frac{x^4}{(-1+x)^2} - \frac{x^6}{(-1+x)^3}$$

FullSimplify[ $\frac{-1+x-x^2}{-1+x}$ ]

$$\frac{1}{1-x} - x$$


```

$$\left(\frac{-1 + x - x^2}{-1 + x} \right)^3$$

$$\frac{(-1 + x - x^2)^3}{(-1 + x)^3}$$

$$N\left[\left(\frac{1}{1-x} - x\right)^3 /. x \rightarrow 1/3\right]$$

$$1.58796$$

$$N\left[\left(\frac{1}{1-x} - (x - x^2) / (1-x)\right)^3 /. x \rightarrow 1/3\right]$$

$$1.58796$$

$$\text{FullSimplify}\left[\frac{-1 + x - x^2}{-1 + x}\right]$$

$$\frac{1}{1-x} - x$$

$$\text{Sum}[(-1)^j x^j, \{j, 2, \text{Infinity}\}]$$

$$\frac{x^2}{1+x}$$

$$\text{Sum}[(-1)^{k+j} x^j x^k, \{j, 2, \text{Infinity}\}, \{k, 2, \text{Infinity}\}]$$

$$\frac{x^4}{(1+x)^2}$$

$$\text{Sum}[x^j, \{j, 2, \text{Infinity}\}]$$

$$-\frac{x^2}{-1+x}$$

$$\text{Sum}[x^j x^k, \{j, 2, \text{Infinity}\}, \{k, 2, \text{Infinity}\}]$$

$$\frac{x^4}{(-1+x)^2}$$

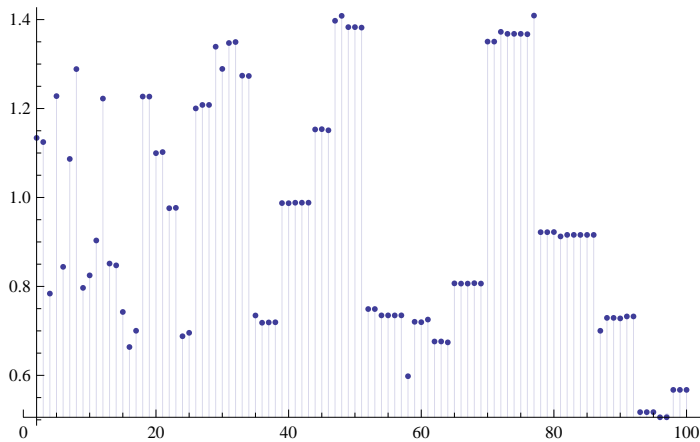
$$\text{Sum}[\text{Binomial}[z, n] x^{2n} / (x+1)^n, \{n, 0, \text{Infinity}\}]$$

$$\left(\frac{1+x+x^2}{1+x} \right)^z$$

$$\text{Sum}[\text{Binomial}[z, n] (-1)^n x^{2n} / (x-1)^n, \{n, 0, \text{Infinity}\}]$$

$$\left(\frac{-1+x-x^2}{-1+x} \right)^z$$

DiscretePlot[D[pza[ff, 100 n, z], z] /. z → 0, {n, 2, 100}]



Table[{n, D[pza[ff, n, z] - pza[ff, n - 1, z], z] /. z → 0}, {n, 2, 100}]

```
{ {2, 1.}, {3, 0.5}, {4, -0.333333}, {5, 0.0416667}, {6, -0.491667}, {7, 0.00138889},
  {8, 0.166865}, {9, -0.124975}, {10, -0.0416639}, {11, 2.75573 × 10-7}, {12, 0.408333},
  {13, 2.08768 × 10-9}, {14, -0.00138889}, {15, -0.0208333}, {16, -0.0974206},
  {17, 4.77396 × 10-14}, {18, 0.245809}, {19, 2.22045 × 10-16}, {20, 0.0347195},
  {21, -0.000694444}, {22, -2.75573 × 10-7}, {23, 0}, {24, -0.326488}, {25, -0.000868056},
  {26, -2.08768 × 10-9}, {27, 0.0416543}, {28, 0.00115741}, {29, 0}, {30, 0.0413181},
  {31, 0}, {32, 0.0612765}, {33, -1.37787 × 10-7}, {34, -4.77396 × 10-14},
  {35, -0.0000578704}, {36, -0.325014}, {37, 0}, {38, 0}, {39, -1.04384 × 10-9},
  {40, -0.0277837}, {41, 0}, {42, 0.00137731}, {43, 0}, {44, 2.29644 × 10-7},
  {45, 0.0104156}, {46, 0}, {47, 0}, {48, 0.25853}, {49, -9.64506 × 10-7},
  {50, 0.001736}, {51, -2.37588 × 10-14}, {52, 1.73973 × 10-9}, {53, 0}, {54, -0.122892},
  {55, -1.14822 × 10-8}, {56, -0.000926201}, {57, 0}, {58, 0}, {59, 0}, {60, -0.0548584},
  {61, 0}, {62, 0}, {63, 0.000347188}, {64, -0.0402558}, {65, -8.69864 × 10-11},
  {66, 2.73277 × 10-7}, {67, 0}, {68, 3.9746 × 10-14}, {69, 0}, {70, 0.000115737}, {71, 0},
  {72, 0.363991}, {73, 0}, {74, 0}, {75, 0.000868056}, {76, 0}, {77, -3.82741 × 10-10},
  {78, 2.07028 × 10-9}, {79, 0}, {80, 0.0220054}, {81, -0.0156188}, {82, 0}, {83, 0},
  {84, -0.0018287}, {85, -2.22045 × 10-15}, {86, 0}, {87, 0}, {88, -1.8377 × 10-7},
  {89, 0}, {90, -0.0309}, {91, -2.89968 × 10-12}, {92, 0}, {93, 0}, {94, 0}, {95, 0},
  {96, -0.204196}, {97, 0}, {98, 1.92901 × 10-6}, {99, 6.88865 × 10-8}, {100, -0.00231459} }
```

FI[n_] := FactorInteger[n]; FI[1] := {}

dzeta[j_, s_, z_] := j^(-s) Product[(-1)^p[[2]] bin[-z, p[[2]]], {p, FI[j]}]

zeta[n_, s_, z_] := Sum[dzeta[j, s, z], {j, 1, n}]

D[Expand[zeta[100, 0, z]], z]

$$\frac{428}{15} + \frac{16289z}{180} + \frac{993z^2}{16} + \frac{611z^3}{36} + \frac{67z^4}{48} + \frac{7z^5}{120}$$

Expand[Sum[(D[dzeta[j, 0, z], z] /. z -> 0) zeta[100 / j, 0, z], {j, 1, 100}]]

$$\frac{428}{15} + \frac{16\,289\,z}{180} + \frac{993\,z^2}{16} + \frac{611\,z^3}{36} + \frac{67\,z^4}{48} + \frac{7\,z^5}{120}$$

Expand[Sum[(D[zeta[100 / j, 0, z], z] /. z -> 0) dzeta[j, 0, z], {j, 1, 100}]]

$$\frac{428}{15} + \frac{16\,289\,z}{180} + \frac{993\,z^2}{16} + \frac{611\,z^3}{36} + \frac{67\,z^4}{48} + \frac{7\,z^5}{120}$$

D[Zeta[s]^z, z]

Log[Zeta[s]] Zeta[s]^z

Integrate[Log[Zeta[s]] Zeta[s]^z, z]

Zeta[s]^z

D[Zeta[s]^z, {z, 2}]

Log[Zeta[s]]^2 Zeta[s]^z

1 + Integrate[

Expand[Sum[(D[zeta[100 / j, 0, z], z] /. z -> 0) dzeta[j, 0, z], {j, 1, 100}]], {z, 0, t}]

$$1 + \frac{428\,t}{15} + \frac{16\,289\,t^2}{360} + \frac{331\,t^3}{16} + \frac{611\,t^4}{144} + \frac{67\,t^5}{240} + \frac{7\,t^6}{720}$$

Integrate[dzeta[9, 0, z], {z, 0, 1}]

$$\frac{5}{12}$$

D[D[Zeta[s]^z, z], s] /. {s -> 0}

$$-(-1)^{-1+z} 2^{-z} \text{Log}[2\pi] - (-1)^{-1+z} 2^{-z} z (\text{i}\pi - \text{Log}[2]) \text{Log}[2\pi]$$

FullSimplify[D[Log[Zeta[s]] Zeta[s]^z, s] /. s -> 0]

$$\left(-\frac{1}{2}\right)^z (1 + \text{i}\pi z - z \text{Log}[2]) \text{Log}[2\pi]$$

D[Zeta[s]^z, s]

z Zeta[s]^{-1+z} Zeta'[s]

D[Log[Zeta[s]], s]

$\frac{\text{Zeta}'[s]}{\text{Zeta}[s]}$

Integrate[- $\frac{\text{Zeta}'[t]}{\text{Zeta}[t]}$, {t, s, Infinity}]

Log[Zeta[s]]

FullSimplify[Sum[j^s - s k^s, {j, 1, 10}, {k, 1, 10}]] /. s -> 2

$$\frac{3\,874\,319\,052\,241}{1\,613\,103\,206\,400}$$

FullSimplify[Sum[j^s - s, {j, 1, 10}]]^2 /. s -> 2

$$\frac{3\,874\,319\,052\,241}{1\,613\,103\,206\,400}$$

```

Sum[ j^-s k^-s, {j, 1, 10}, {k, 1, 10 / j}] /. s -> 2
301801
132300
Sum[ j^-s k^-s, {j, 1, 10}, {k, 1, 10}] /. s -> 2
3874319052241
1613103206400
(Sum[ j^-s k^-s, {j, 1, 10}, {k, 1, 10}] -
Sum[ j^-s k^-s, {j, 1, 10}, {k, Floor[10 / j] + 1, 10}]) /. s -> 2
301801
132300
Sum[ j^-s k^-s l^-s, {j, 1, 10}, {k, 1, 10 / j}, {l, 1, 10 / (j k)}] /. s -> 2
3397339
1058400
(Sum[ j^-s k^-s l^-s, {j, 1, 10}, {k, 1, 10}, {l, 1, 10}] -
Sum[ j^-s k^-s l^-s, {j, 1, 10}, {k, 1, 10}, {l, Floor[10 / (j k)] + 1, 10}]) /. s -> 2
3397339
1058400
Integrate[ D[Zeta[s]^z, s], {s, a, b}]
ConditionalExpression[-Zeta[a]^z + Zeta[b]^z, Zeta[a] >= 0 && Zeta[b] >= 0]
zeta[n_, s_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[ j^-s zeta[n / j, s, z, k + 1], {j, 2, n}]

Integrate[ D[zeta[100, s, 1, 1], s], {s, 0, -1}]
4950
zeta[100, -1, 1, 1] - zeta[100, 0, 1, 1]
4950
D[zeta[100, s, 1, 1], s]
-2^-s Log[2] - 3^-s Log[3] - 4^-s Log[4] - 5^-s Log[5] - 6^-s Log[6] - 7^-s Log[7] -
8^-s Log[8] - 9^-s Log[9] - 10^-s Log[10] - 11^-s Log[11] - 12^-s Log[12] - 13^-s Log[13] -
14^-s Log[14] - 15^-s Log[15] - 16^-s Log[16] - 17^-s Log[17] - 18^-s Log[18] -
19^-s Log[19] - 20^-s Log[20] - 21^-s Log[21] - 22^-s Log[22] - 23^-s Log[23] -
24^-s Log[24] - 25^-s Log[25] - 26^-s Log[26] - 27^-s Log[27] - 28^-s Log[28] - 29^-s Log[29] -
30^-s Log[30] - 31^-s Log[31] - 32^-s Log[32] - 33^-s Log[33] - 34^-s Log[34] - 35^-s Log[35] -
36^-s Log[36] - 37^-s Log[37] - 38^-s Log[38] - 39^-s Log[39] - 40^-s Log[40] - 41^-s Log[41] -
42^-s Log[42] - 43^-s Log[43] - 44^-s Log[44] - 45^-s Log[45] - 46^-s Log[46] - 47^-s Log[47] -
48^-s Log[48] - 49^-s Log[49] - 50^-s Log[50] - 51^-s Log[51] - 52^-s Log[52] - 53^-s Log[53] -
54^-s Log[54] - 55^-s Log[55] - 56^-s Log[56] - 57^-s Log[57] - 58^-s Log[58] - 59^-s Log[59] -
60^-s Log[60] - 61^-s Log[61] - 62^-s Log[62] - 63^-s Log[63] - 64^-s Log[64] - 65^-s Log[65] -
66^-s Log[66] - 67^-s Log[67] - 68^-s Log[68] - 69^-s Log[69] - 70^-s Log[70] - 71^-s Log[71] -
72^-s Log[72] - 73^-s Log[73] - 74^-s Log[74] - 75^-s Log[75] - 76^-s Log[76] - 77^-s Log[77] -
78^-s Log[78] - 79^-s Log[79] - 80^-s Log[80] - 81^-s Log[81] - 82^-s Log[82] - 83^-s Log[83] -
84^-s Log[84] - 85^-s Log[85] - 86^-s Log[86] - 87^-s Log[87] - 88^-s Log[88] - 89^-s Log[89] -
90^-s Log[90] - 91^-s Log[91] - 92^-s Log[92] - 93^-s Log[93] - 94^-s Log[94] - 95^-s Log[95] -
96^-s Log[96] - 97^-s Log[97] - 98^-s Log[98] - 99^-s Log[99] - 100^-s Log[100]

```

Expand[D[zeta[10, s, 2, 1], s]]

$-2^{1-3s} \log[2] - 2^{1-2s} \log[2] - 2^{1-s} \log[2] - 2^{1-s} 3^{-s} \log[2] - 2^{1-s} 5^{-s} \log[2] - 2 \times 3^{-2s} \log[3] -$
 $2 \times 3^{-s} \log[3] - 2^{1-s} 3^{-s} \log[3] - 2^{1-3s} \log[4] - 2^{1-2s} \log[4] - 2 \times 5^{-s} \log[5] - 2^{1-s} 5^{-s} \log[5] -$
 $2^{1-s} 3^{-s} \log[6] - 2 \times 7^{-s} \log[7] - 2^{1-3s} \log[8] - 2 \times 9^{-s} \log[9] - 2^{1-s} 5^{-s} \log[10]$

1 - Integrate[D[zeta[100, s, 1, 1], s], {s, 0, Infinity}]

100

1 - Integrate[D[zeta[100, s, 2, 1], s], {s, 0, Infinity}]

482

1 - Integrate[D[zeta[100, s, 1, 1], s], {s, 0, Infinity}]

100

Integrate[Expand[D[D[zeta[100, s, z, 1], z] /. z -> 0, s]], {s, 0, Infinity}]

\$Aborted

Expand[D[D[zeta[100, s, z, 1], z] /. z -> 0, s]]

\$Aborted

FullSimplify[D[Expand[D[zeta[10, s, z, 1], z] /. z -> 0], s]]

$2520^{-s} (-315^s (1 + 2^s + 4^s) \log[2] - 8^s (35^s (1 + 3^s) \log[3] + 63^s \log[5] + 45^s \log[7]))$

Integrate[2520^{-s} (-315^s (1 + 2^s + 4^s) Log[2] - 8^s (35^s (1 + 3^s) Log[3] + 63^s Log[5] + 45^s Log[7])),
{s, 0, Infinity}]

$-\frac{16}{3}$

Expand[zeta[10, -1, z, 1]]

$1 + \frac{157z}{6} + \frac{53z^2}{2} + \frac{4z^3}{3}$

1 - Integrate[D[zeta[100, s, -2, 1], s], {s, -1, Infinity}]

\$Aborted

zeta[100, -1, -2, 1]

1 - Integrate[D[Zeta[s]^z, s], {s, t, Infinity}]

Zeta[t]^z

-Integrate[D[D[Zeta[s]^z, s], z] /. z -> 0, {s, t, Infinity}]

Log[Zeta[t]]

D[Zeta[s], s]

Zeta'[s]

1 - Integrate[Zeta'[s], {s, t, Infinity}]

Zeta[t]

-Integrate[Zeta'[s] / Zeta[s], {s, t, Infinity}]

Log[Zeta[t]]

$12 / 2^{(1-5)}$

192

$12 / 0.7071067811865476^{\sim}$

16.9706

$12 / 2^{(-2)}$

48

$\text{oo}[n_, a_, z_, k_] := n a^{(k-z)} / (a+1)^k$

$\text{oo2}[n_, a_, z_, k_] := \text{Floor}[\text{Log}[a+1, n a^{(k-z)}] - k]$

$\text{Table}[\text{If}[\text{oo}[12, 2, 1, k] \geq 1, 1, 0], \{k, 0, 10\}]$

{1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0}

$\text{Table}[\text{oo2}[12, 2, 1, k], \{k, 0, 10\}]$

{1, 1, 0, 0, 0, -1, -1, -1, -2, -2, -3}

$\text{Solve}[12 \times 2^{(k-2)} / (2+1)^k \geq 0, k]$

Solve::nsmet: This system cannot be solved with the methods available to Solve. >>

$\text{Solve}[2^k 3^{1-k} \geq 0, k]$

$n (y+1)^{(z-2)}$

$n (1+y)^{-2+z}$

$n (y+1)^z / (y+1) / (y+1)$

$n (1+y)^{-2+z}$

$\text{dm}[n_, s_, z_, y_, k_] :=$

$1 + (z+1/k-1) \text{Sum}[(j+y)^{-s} \text{dm}[n (j+y)^{-1} (y+1)^{-1}, z, k+1], \{j, 1, \text{Floor}[n/(y+1)]\}]$

$\text{dmz}[n_, s_, z_, y_] := \text{dm}[n (y)^z, s, z, y, 1]$

$\text{dmz}[100, 0, 1, 1]$

\$RecursionLimit::reclim: Recursion depth of 256 exceeded. >>

\$RecursionLimit::reclim: Recursion depth of 256 exceeded. >>

\$RecursionLimit::reclim: Recursion depth of 256 exceeded. >>

General::stop: Further output of \$RecursionLimit::reclim will be suppressed during this calculation. >>

\$IterationLimit::itlim: Iteration limit of 4096 exceeded. >>

\$IterationLimit::itlim: Iteration limit of 4096 exceeded. >>

\$IterationLimit::itlim: Iteration limit of 4096 exceeded. >>

General::stop: Further output of \$IterationLimit::itlim will be suppressed during this calculation. >>

\$Aborted

```

zeta[n_, s_, z_, k_] :=
  zeta[n, s, z, k] = 1 + ((z + 1) / k - 1) Sum[j^(-s zeta[Floor[n / j], s, z, k + 1]), {j, 2, n}]
zeros[n_, s_] := List@@NRoots[zeta[n, s, z, 1] == 0, z][[All, 2]]
dzeros[n_] := List@@NRoots[(D[zeta[n, s, z, 1], s] /. s -> 0) - 1 == 0, z][[All, 2]]

```

```
Expand[zeta[100, 1, z, 1]]
```

$$1 + \frac{292\,149\,953\,504\,274\,361\,788\,974\,787\,095\,433\,526\,022\,627\,z}{139\,440\,750\,459\,424\,954\,329\,067\,617\,870\,624\,607\,113\,600} + \frac{29\,826\,319\,283\,165\,261\,943\,577\,z^2}{19\,402\,713\,417\,887\,979\,379\,200} + \frac{5\,744\,488\,816\,811\,z^3}{11\,993\,472\,691\,200} + \frac{34\,450\,699\,z^4}{479\,001\,600} + \frac{553\,z^5}{138\,240} + \frac{z^6}{9216}$$

```
zeros[100, 1]
```

```
{-13.8863 - 13.5466 i, -13.8863 + 13.5466 i, -3.33053 - 2.26209 i,
-3.33053 + 2.26209 i, -1.21647 - 0.175924 i, -1.21647 + 0.175924 i}
```

```
Expand[N[D[zeta[100, s, z, 1], s] /. s -> 0]]
```

$$-94.0453 z - 169.15 z^2 - 81.6195 z^3 - 17.6846 z^4 - 1.19616 z^5 - 0.0438125 z^6$$

```
dzeros[100]
```

```
{-10.6971 - 12.1993 i, -10.6971 + 12.1993 i,
-2.54005 - 1.8272 i, -2.54005 + 1.8272 i, -0.816685, -0.0108436}
```

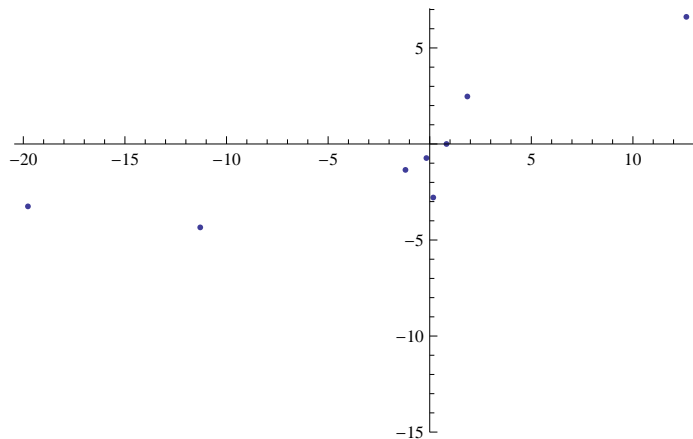
```
Sum[-j^(-1), {j, dzeros[100]}]
```

$$94.0453 + 0. i$$

```
FullSimplify[zeros[10, 0] / zeros[10, 1]]
```

```
$Aborted
```

```
ListPlot[Table[{Re[n], Im[n]}, {n, zeros[1000, .5 + 30 I]}]]
```



```
Expand[N[zeta[100, 3 I, z, 1]]]
```

$$1. + (5.76904 + 1.2356 i) z + (14.6072 - 0.934103 i) z^2 + (8.3492 - 0.107725 i) z^3 + (2.296 - 0.623888 i) z^4 + (0.191317 - 0.0770772 i) z^5 + (0.00496385 - 0.00740045 i) z^6$$


```

Clear[f, fr]
f[n_, 0, s_, a_] := 1
fr[n_, s_] := fr[n, s] = Sum[N[m^-s], {m, 1, n}]
f[n_, 1, s_, a_] := f[n, 1, s, a] = fr[Floor[n], s] - fr[a, s]
f[n_, k_, s_, a_] :=
  f[n, k, s, a] = N[Sum[Binomial[k, j] (m^-s)^j f[Floor[n / (m^j)], k - j, s, m],
    {j, 1, k}], {m, a + 1, Floor[n^(1 / k)]]}]

Timing[f[1 000 000, 4, N[3 + 7 I], 0]]
{14.414, 1.00108 + 0.397512 i}

Clear[f]
f[n_, 0, s_, a_] := 1
f[n_, 1, s_, a_] := f[n, 1, s, a] = N[HarmonicNumber[n, s]] - N[HarmonicNumber[a, s]]
f[n_, k_, s_, a_] :=
  f[n, k, s, a] = N[Sum[Binomial[k, j] (N[m]^-s)^j f[Floor[n / (N[m]^j)], k - j, s, m],
    {j, 1, k}], {m, a + 1., Floor[N[n]^(1. / k)]]}]

HarmonicNumber[10, I] // N
0.0418976 - 7.84548 i
N[Sum[j^I, {j, 1, 10}]]
0.0418976 + 7.84548 i
Timing[f[1 000 000, 4, N[3 + 7 I], 0]]
{4.976, 1.00108 + 0.397512 i}

Log[Zeta[s]] Integrate[Zeta[s]^z, z]
Zeta[s]^z
D[Zeta[s]^z, z] / Log[Zeta[s]]
Zeta[s]^z
D[Zeta[s]^z, z] / Zeta[s]^z
Log[Zeta[s]]
Zeta[s]^z / Integrate[Zeta[s]^z, z]
Log[Zeta[s]]

Clear[zeta]
Integrate[zeta[100, 0, z, 1], z]

$$z + \frac{214 z^2}{15} + \frac{16\,289 z^3}{1080} + \frac{331 z^4}{64} + \frac{611 z^5}{720} + \frac{67 z^6}{1440} + \frac{z^7}{720}$$

Expand[D[zeta[100, 0, z, 1], z]]

$$\frac{428}{15} + \frac{16\,289 z}{180} + \frac{993 z^2}{16} + \frac{611 z^3}{36} + \frac{67 z^4}{48} + \frac{7 z^5}{120}$$


```

Integrate[Zeta[s] ^ z, z] / Zeta[s] ^ z

$$\frac{1}{\text{Log}[\text{Zeta}[s]]}$$

Zeta[s] ^ z / **D**[Zeta[s] ^ z, z]

$$\frac{1}{\text{Log}[\text{Zeta}[s]]}$$

D[Zeta[s] ^ z, z] / Zeta[s] ^ z

$$\text{Log}[\text{Zeta}[s]]$$

Expand[Sum[

(zeta[j, 0, -z, 1] - zeta[j - 1, 0, -z, 1]) **D**[zeta[Floor[100 / j], 0, z, 1], z], {j, 1, 100}]]

$$\frac{428}{15}$$

$$\frac{428}{15}$$

Expand[

Sum[(**D**[zeta[j, 0, z, 1] - zeta[j - 1, 0, z, 1], z]) zeta[Floor[100 / j], 0, -z, 1], {j, 1, 100}]]

$$\frac{428}{15}$$

$$\frac{428}{15}$$

Expand[Sum[Integrate[(zeta[j, 0, z, 1] - zeta[j - 1, 0, z, 1]), z]

zeta[Floor[100 / j], 0, -z, 1], {j, 1, 100}]]

$$z - \frac{214 z^2}{15} + \frac{16289 z^3}{1080} - \frac{331 z^4}{64} + \frac{611 z^5}{720} - \frac{67 z^6}{1440} + \frac{z^7}{720}$$

D[Zeta[s] ^ z, {z, 1}] / Zeta[s] ^ z

D[Zeta[s] ^ z, {z, 2}] / Zeta[s] ^ z

D[Zeta[s] ^ z, {z, 3}] / Zeta[s] ^ z

$$\text{Log}[\text{Zeta}[s]]$$

$$\text{Log}[\text{Zeta}[s]]^2$$

$$\text{Log}[\text{Zeta}[s]]^3$$

Zeta[s] ^ z / **Integrate**[Zeta[s] ^ z, z]

D[Zeta[s] ^ z, {z, 3}] / **D**[Zeta[s] ^ z, {z, 2}]

$$\text{Log}[\text{Zeta}[s]]$$

Expand[Sum[(**D**[zeta[j, 0, z, 1] - zeta[j - 1, 0, z, 1], {z, 2}])

zeta[Floor[100 / j], 0, -z, 1], {j, 1, 100}]]

$$\frac{16289}{180}$$

$$\frac{16289}{180}$$

D[zeta[100, 0, z, 1], {z, 2}] /. z -> 0

$$\frac{16289}{180}$$

$$\frac{16289}{180}$$

Limit[(Zeta[s] ^ z - 1) / z, z -> 0]

D[Zeta[s] ^ z, {z, 2}] / Zeta[s] ^ z

$$\text{Log}[\text{Zeta}[s]]^2$$

Log[Zeta[s]]

Log[Zeta[s]]

-D[N[Expand[Sum[D[dzeta[j, s, z], s] zeta[100 / j, s, -z, 1], {j, 1, 100}]] /. s → 1], z]

-D[N[Expand[Sum[D[zeta[100 / j, s, z, 1], s] dzeta[j, s, -z], {j, 1, 100}]] /. s → 1], z]

3.98562

3.98562

chebyshev[n_] := Sum[MangoldtLambda[j], {j, 2, n}]

dz[n_, z_, s_] := (n^-s) Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];

FI[n_] := FactorInteger[n]; FI[1] := {}

Dz[n_, z_, s_] := Sum[dz[j, z, s], {j, 1, n}]

Table[Chop[(-N[Sum[dz[j, -1, 0] (D[Dz[n / j, 1, s], s] /. s → 0), {j, 1, n}]]],
{n, 10, 100, 10}]

Table[Chop[(-N[Sum[(D[Dz[n / j, 1, s], s] /. s → 0) dz[j, -1, 0], {j, 1, n}]]],
{n, 10, 100, 10}]

{7.83201, 19.2657, 28.4765, 36.2146, 49.4854, 57.5332, 66.5419, 79.4645, 89.4706, 94.0453}

{7.83201, 19.2657, 28.4765, 36.2146, 49.4854, 57.5332, 66.5419, 79.4645, 89.4706, 94.0453}

Table[Chop[(-N[Sum[dzeta[j, -1, 0] (D[zeta[n / j, s, 1], s] /. s → 0), {j, 1, n}]]],
{n, 10, 100, 10}]

Table[Chop[(-N[Sum[(D[Dz[n / j, 1, s], s] /. s → 0) dzeta[j, -1, 0], {j, 1, n}]]],
{n, 10, 100, 10}]

Table[FullSimplify[(-D[D[zeta[n, s, z, 1], z] /. z → 0, s]) -
(-D[D[zeta[n - 1, s, z, 1], z] /. z → 0, s])], {n, 2, 20}] // TableForm

$2^{-s} \text{Log}[2]$

$3^{-s} \text{Log}[3]$

$4^{-s} \text{Log}[2]$

$5^{-s} \text{Log}[5]$

0

$7^{-s} \text{Log}[7]$

$8^{-s} \text{Log}[2]$

$9^{-s} \text{Log}[3]$

0

$11^{-s} \text{Log}[11]$

0

$13^{-s} \text{Log}[13]$

0

0

$16^{-s} \text{Log}[2]$

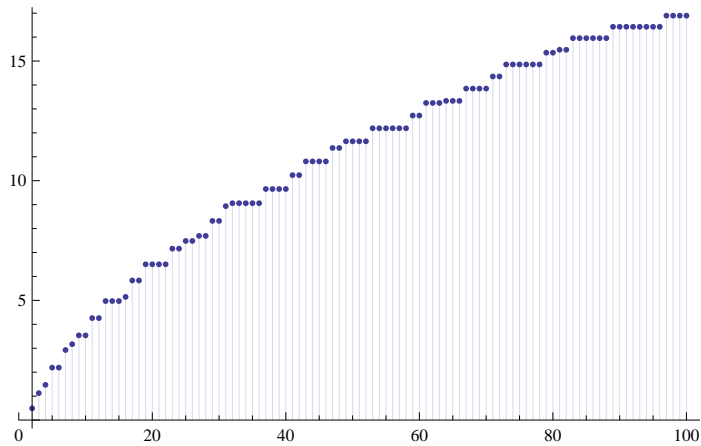
$17^{-s} \text{Log}[17]$

0

$19^{-s} \text{Log}[19]$

0

```
DiscretePlot[-N[D[D[zeta[n, s, z, 1], z] /. z -> 0, s]] /. s -> 1/2, {n, 2, 100}]
```



```
Expand[pz[fe, 100, z]]
```

$$1 + \frac{428 z}{15} + \frac{16289 z^2}{360} + \frac{331 z^3}{16} + \frac{611 z^4}{144} + \frac{67 z^5}{240} + \frac{7 z^6}{720}$$

```
fe2[n_, s_] := (s - 1)^-1 (n / (n + 1)^s - (n - s) / n^s)
```

```
fe20[n_] := fe2[n, 0]
```

```
Expand[pz[fe20, 100, z]]
```

```
Sum[fe2[n, 2], {n, 1, 10}]
```

$$\frac{250868609}{153679680}$$

$$\frac{1968329}{1270080}$$

```
Sum[j^-2, {j, 1, 10}]
```

$$\frac{1968329}{1270080}$$

$$\frac{1968329}{1270080}$$

```
Sum[fe2[j, s], {j, 1, Infinity}]
```

$$\sum_{j=1}^{\infty} \frac{j(1+j)^{-s} - j^{-s}(j-s)}{-1+s}$$

```
N[Integrate[(x^(3-1)/(E^x-1)), {x, 0, Log[10]}]]
```

$$1.17035$$

```
N[Sum[j^-3, {j, 1, 10}]]
```

$$1.19753$$

```
zeta[100, 1, 1, 1]
```

$$\frac{14466636279520351160221518043104131447711}{2788815009188499086581352357412492142272}$$

$$\frac{14466636279520351160221518043104131447711}{2788815009188499086581352357412492142272}$$

```
zeroh[n_, s_] := List@@NRoots[zeta[n, s, z, 1] == 0, z][[All, 2]]
```

```
Table[zeroh[n, 1], {n, 4, 10}] // TableForm
```

```
-6.4207      -1.24597
-8.30318     -0.963486
-2.70299     -1.26843
-3.47442     -0.986804
-11.9371     -4.07645      -0.986414
-15.548      -3.13336      -0.985271
-21.4404     -1.73867      -1.28763
```

```
Sum[BernoulliB[k] / k! (Zeta[s] - 1) Log[Zeta[s]] ^ (k), {k, 0, Infinity}]
```

```
Log[Zeta[s]]
```

```
Sum[BernoulliB[k] / k! Sum[D[zeta[100 / j, 0, z, 1], {z, k}] /. z -> 0, {j, 2, 100}],
{k, 0, Log[2, 100]}]
```

```
428
-----
15
```

```
D[zeta[100, 0, z, 1], {z, -1}] /. z -> 0
```

```
D::dvar:
```

Multiple derivative specifier {z, -1} does not have the form {variable, n}, where n is a non-negative machine integer. >>

```
D::dvar:
```

Multiple derivative specifier {0, -1} does not have the form {variable, n}, where n is a non-negative machine integer. >>

```
 $\partial_{\{0, -1\}} 1$ 
```

```
Sum[BernoulliB[k] / k! Sum[D[zeta[100 / j, 0, z, 1], {z, k - 1}] /. z -> 0, {j, 2, 100}],
{k, 1, Log[2, 100]}] +
1 / Sum[BernoulliB[k] / k! Sum[D[zeta[100 / j, 0, z, 1], z] /. z -> 0, {j, 2, 100}], {k, 0, 0}]
3 138 807 413
-----
88 637 760
```

```
Sum[BernoulliB[k] / k! (Zeta[s] - 1) D[Zeta[s] ^ z, {z, k}] / Zeta[s] ^ z, {k, 0, Infinity}]
```

```
$Aborted
```

```
(D[Zeta[s] ^ z, z] / Zeta[s] ^ z)
```

```
Log[Zeta[s]]
```

```
Residue[(Zeta'[s] / Zeta[s]) x^s s^(-1), {s, ZetaZero[1]}]
```

```
 $\frac{x^{\text{ZetaZero}[1]}}{\text{ZetaZero}[1]}$ 
```

```
((D[Zeta[s] ^ z, z] / Zeta[s] ^ z) x^s s^(-1)
```

```
 $\frac{x^s \text{Log}[Zeta[s]]}{s}$ 
```

```
((D[Zeta[s] ^ z, z] / Zeta[s] ^ z) x^s s^(-1)
```

```
Log[Zeta[s]] ^ (3 / 2)
```

```
Log[Zeta[s]] ^ (3 / 2)
```

$$-1 + \text{Zeta}[s]$$

```

Sum[1/(q+j)^s,{j,1,Infinity}]^2
HurwitzZeta[s,1+q]^2
(1/3+1/4+1/5+1/6)(1/3+1/4+1/5+1/6)
dd[n_,s_,1,q_]:=If[n>=q,n^-s,0]
dd[n_,s_,k_,q_]:=dd[n,s,k,q]=
  Sum[If[dd[a,s,1,q]==0,0,dd[a,s,1,q]dd[n/a,s,k-1,q]],{a,Divisors[n]}]
Table[dd[16,2,3,n],{n,1,10}]
{15/256,3/256,0,0,0,0,0,0,0,0}
N[Sum[dd[n,2,2,7],{n,1,100000}]]
0.0234884
N[Zeta[2,7]^2]
0.0235761

```