

```

D2approx[n_, j_] := (-1)^j (1 - (Gamma[j, -Log[n]] / Gamma[j]))
D2sapprox[n_, k_, s_] := (-1)^k  $\frac{(\text{Gamma}[k, 0, -(1-s) \text{Log}[n]]) (1-s)^{-k}}{\text{Gamma}[k]}$ 
Dl2sapprox[n_, j_, s_] :=
  If[j == 0, Limit[ $(-1)^k \frac{(\text{Gamma}[k, 0, -(1-s) \text{Log}[n]]) (1-s)^{-k}}{\text{Gamma}[k]}$ , {k -> j}],
    (-1)^j  $\frac{(\text{Gamma}[j, 0, -(1-s) \text{Log}[n]]) (1-s)^{-j}}{\text{Gamma}[j]}$ ]
Dapprox[n_, z_] := Sum[FactorialPower[z, a] / a! D2approx[n, a], {a, 0, 2000}]
Dsapprox[n_, z_, s_] := Sum[FactorialPower[z, a] / a! D2sapprox[n, a, s], {a, 1, 2000}]
EtaApprox[n_, z_] := (1 - 2^(1 - z)) Dapprox[n, z]
EtaApprox2[n_, k_, b_, s_] :=
  Sum[(-1)^j Binomial[k, j] b^(j (1 - s)) Dsapprox[n / b^j, k, s], {j, 0, k}]

```

```

N[EtaApprox[100, -1]]

```

```

10.8155 - 5.11208 × 10-13 i

```

```

EtaApprox2[100, 1, 2, ZetaZero[1]]

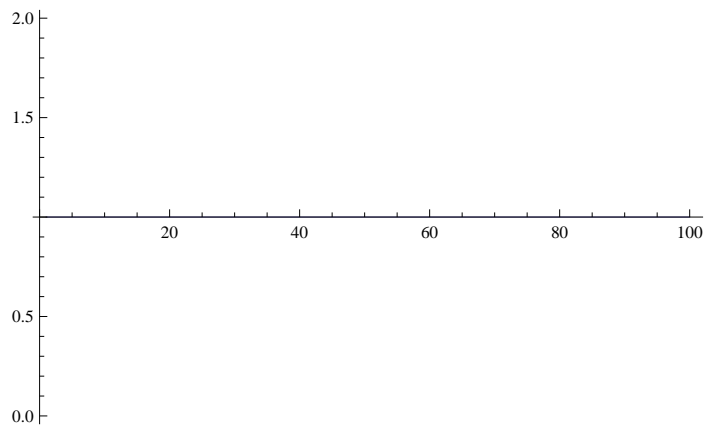
```

$$\frac{2^{1-\text{ZetaZero}[1]} \left(1 - 50^{1-\text{ZetaZero}[1]}\right)}{1 - \text{ZetaZero}[1]} - \frac{1 - 100^{1-\text{ZetaZero}[1]}}{1 - \text{ZetaZero}[1]}$$

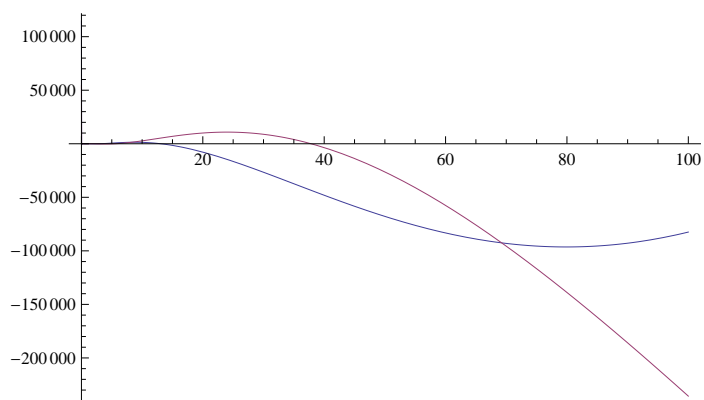
```

Plot[EtaApprox2[n, 1, 2, 0], {n, 1, 100}]

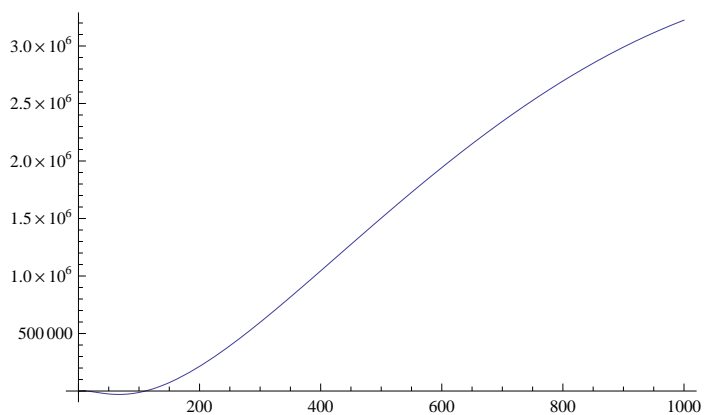
```



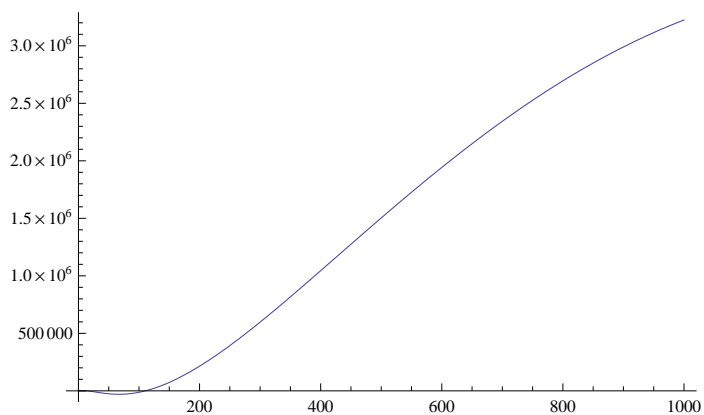
```
Plot[{Re[EtaApprox[n, ZetaZero[1]]], Im[EtaApprox[n, ZetaZero[1]]], {n, 0, 100}]
```



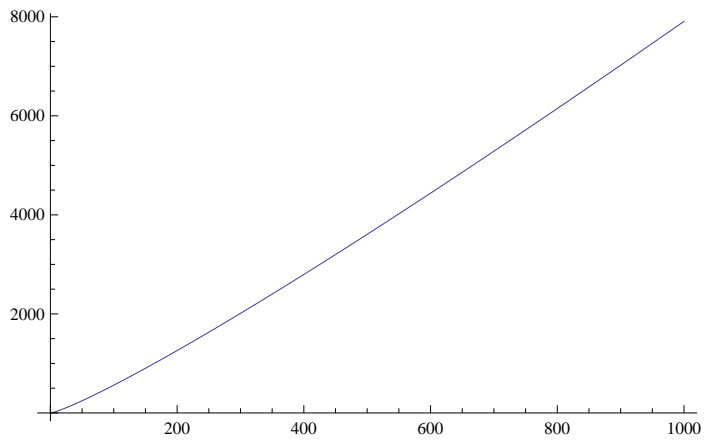
```
Plot[{Re[Dapprox[n, 1 - ZetaZero[1]]], {n, 0, 1000}]
```



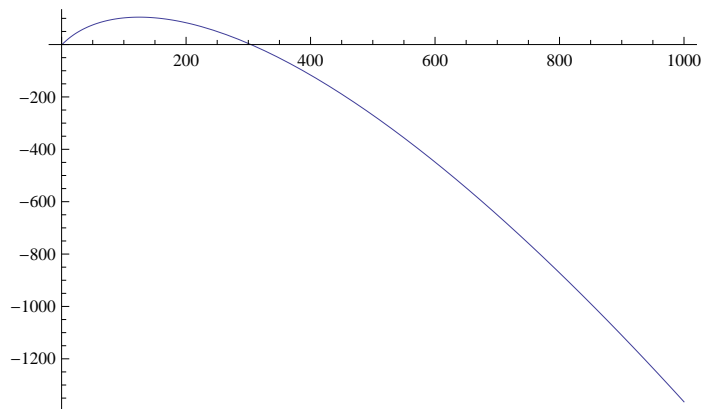
```
Plot[{Re[Dapprox[n, ZetaZero[1]]], {n, 0, 1000}]
```



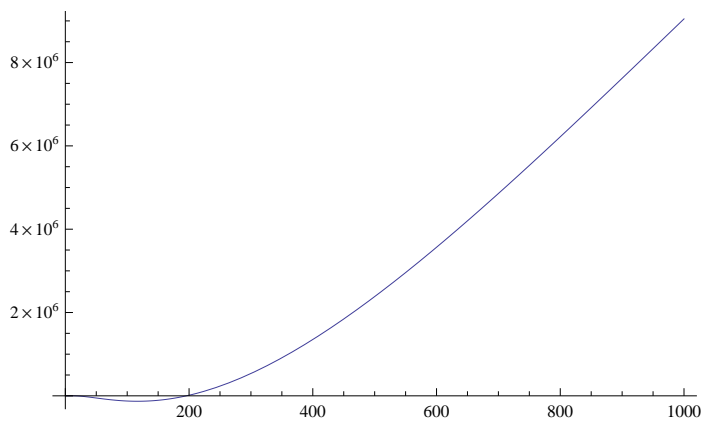
```
Plot[{Re[Dapprox[n, 2]]}, {n, 0, 1000}]
```



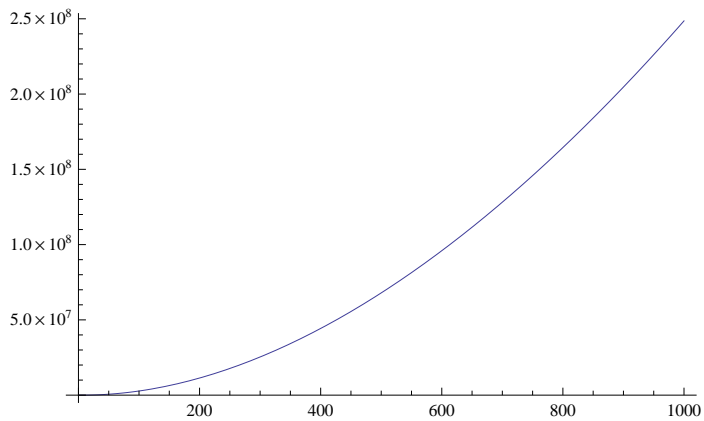
```
Plot[{Re[Dapprox[n, 2 + I]]}, {n, 0, 1000}]
```



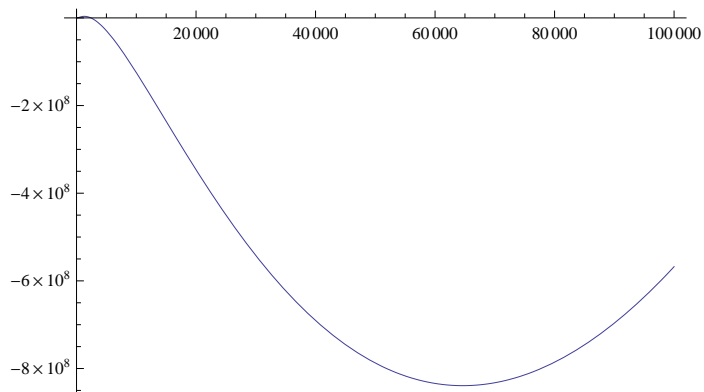
```
Plot[{Re[Dapprox[n, 2 + 14 I]]}, {n, 0, 1000}]
```



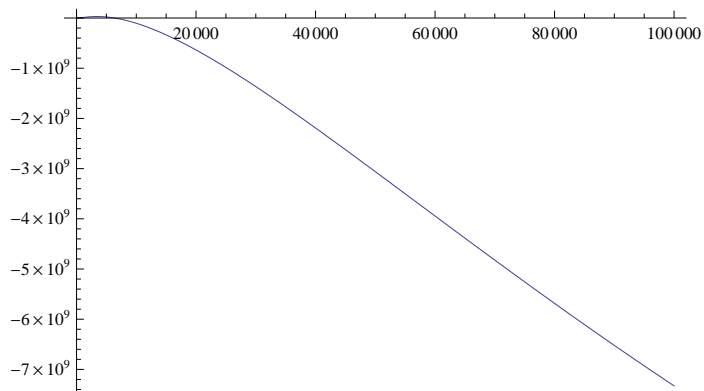
`Plot[{Re[Dapprox[n, 12]]}, {n, 0, 1000}]`



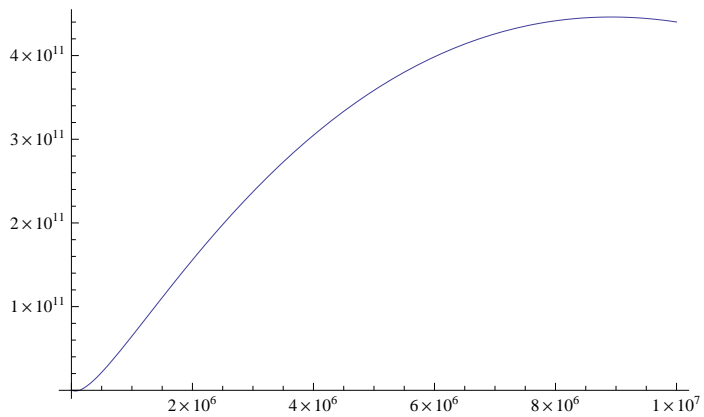
`Plot[{Re[Dapprox[n, ZetaZero[1]]]}, {n, 0, 100 000}]`



`Plot[{Re[Dapprox[n, 2 + 14 I]]}, {n, 0, 100 000}]`



```
Plot[{Re[Dapprox[n, ZetaZero[1]]]}, {n, 0, 10 000 000}]
```



```
Plot[{Re[Dapprox[n, 1 / 2 + 13 I]]}, {n, 0, 10 000 000}]
```

```
$Aborted
```

```
t[s_] := N[Sum[(-1)^(j+1) / j^s, {j, 1, 1 000 000}]]
```

```
t[ZetaZero[2]]
```

```
-0.0000831614 + 0.000493036 i
```

```
Sum[(-1)^(j+1) / j^ZetaZero[2], {j, 1, Infinity}]
```

```
0
```

```
N[Pi^2 / 12]
```

```
0.822467
```