```
FindRoot[LogIntegral[x] - Log[Log[x]] - EulerGamma == 1, {x, 2}]
```

$\{x \to 2.23525\}$

```
LogIntegral[2.235248176511392`] - Log@Log@2.235248176511392` - EulerGamma
```

1.

```
Clear[bin, co, gg]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
co[a_, s_] := co[a, s] = Limit[ D[Log[x + 1]^a / s!, {x, s}], x → 0]
gg[n_, k_] := gg[n, k] = (-1)^k Gamma[ k, 0, -Log[n]] / Gamma[k]
gga[n_, k_] := (-1)^k gs[ k, ln] / Gamma[k]
```

```
Table[ co[2, s], {s, 0, 10}]
```

$$\left\{0, 0, 1, -1, \frac{11}{12}, -\frac{5}{6}, \frac{137}{180}, -\frac{7}{10}, \frac{363}{560}, -\frac{761}{1260}, \frac{7129}{12\,600}\right\}$$

```
Series[ Log[x + 1]^3, {x, 0, 10}]
```

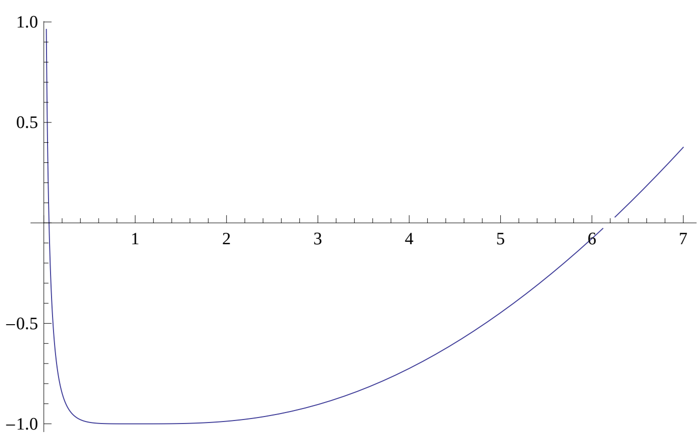
$$x^3 - \frac{3 x^4}{2} + \frac{7 x^5}{4} - \frac{15 x^6}{8} + \frac{29 x^7}{15} - \frac{469 x^8}{240} + \frac{29\,531 x^9}{15\,120} - \frac{1303 x^{10}}{672} + O[x]^{11}$$

```
pp[n_, a_, t_] := Sum[ co[a, s] gg[n, s], {s, 1, t}]
pa[n_, a_, t_] := 1 + Sum[ bin[z, s] gga[n, s], {s, 1, t}]
```

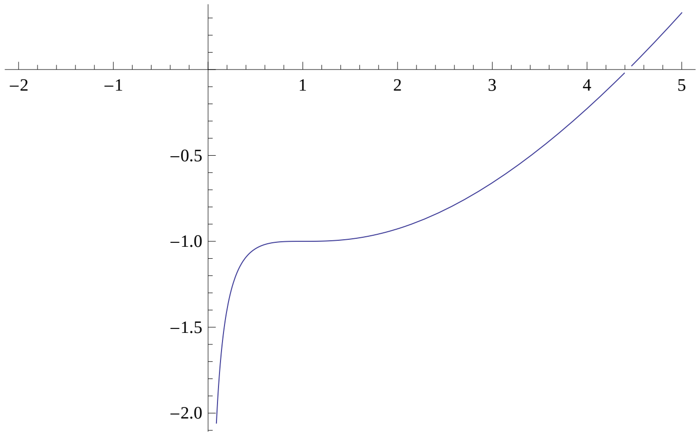
```
Chop@N@pp[2.235248176511392`, 1, 50.]
```

1.

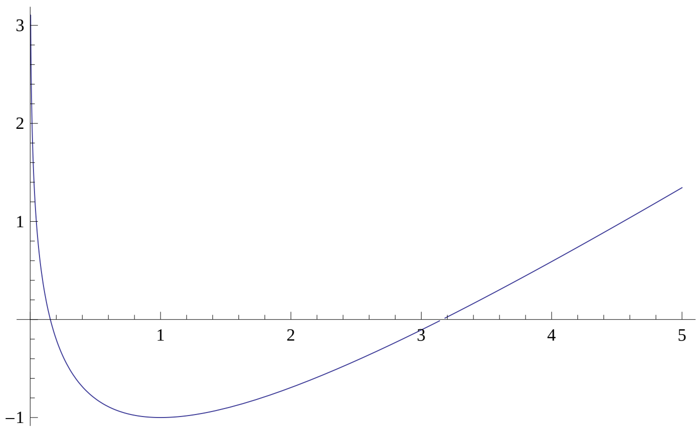
```
Plot[ pp[n, 4, 80.] - 1, {n, 0, 7}]
```



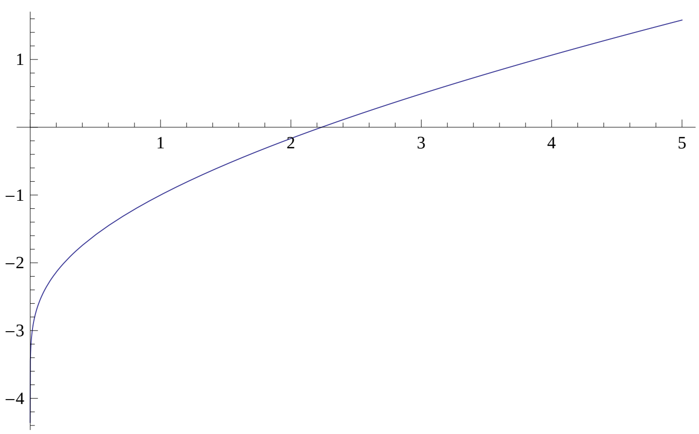
```
Plot[pp[n, 3, 80.] - 1, {n, -2, 5}]
```




```
Plot[pp[n, 2, 80.] - 1, {n, 0, 5}]
```




```
Plot[pp[n, 1, 80.] - 1, {n, 0, 5}]
```




```
N@D[LaguerreL[-z, Log[100]], {z, 1}] /. z → 0
```

28.0217

```
Table[{(-1)^k / (k!) Binomial[-z, k], Binomial[z + k - 1, k] / k!, Gamma[k + z]/(Gamma[1 + k]^2 Gamma[z])},
  {k, 0, 6}] /. {z → 5} // TableForm
```

1	1	1
5	5	5
$\frac{15}{2}$	$\frac{15}{2}$	$\frac{15}{2}$
$\frac{35}{6}$	$\frac{35}{6}$	$\frac{35}{6}$
$\frac{35}{12}$	$\frac{35}{12}$	$\frac{35}{12}$
$\frac{21}{20}$	$\frac{21}{20}$	$\frac{21}{20}$
$\frac{7}{24}$	$\frac{7}{24}$	$\frac{7}{24}$

```
FullSimplify[(z + k - 1)! / ((((z + k - 1) - (k))!) ((k)!) ((k)!))]
```

$$\frac{\text{Gamma}[k+z]}{\text{Gamma}[1+k]^2\,\text{Gamma}[z]}$$

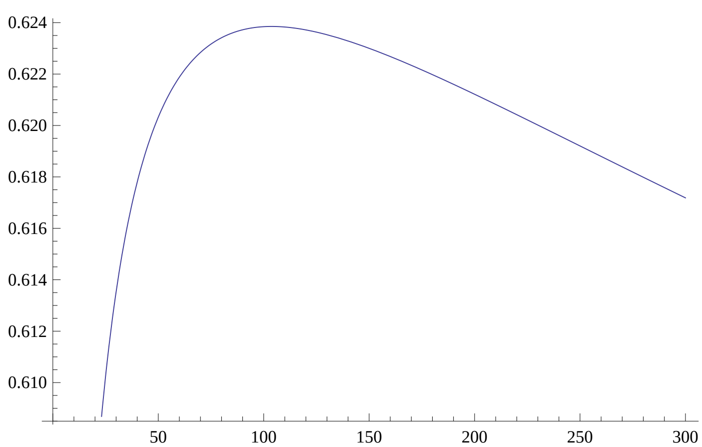
```
Expand@D[pa[n, 1, 7], z]
```

$$-\text{gs}[1,\text{ln}]-\frac{1}{2}\text{gs}[2,\text{ln}]+z\,\text{gs}[2,\text{ln}]-\frac{1}{6}\text{gs}[3,\text{ln}]+\frac{1}{2}z\,\text{gs}[3,\text{ln}]-\frac{1}{4}z^2\text{gs}[3,\text{ln}]-\frac{1}{24}\text{gs}[4,\text{ln}]+\frac{11}{72}z\,\text{gs}[4,\text{ln}]-\frac{1}{8}z^2\text{gs}[4,\text{ln}]+\frac{1}{36}z^3\text{gs}[4,\text{ln}]-\frac{1}{120}\text{gs}[5,\text{ln}]+\frac{5}{144}z\,\text{gs}[5,\text{ln}]-\frac{7}{192}z^2\text{gs}[5,\text{ln}]+\frac{1}{72}z^3\text{gs}[5,\text{ln}]-\frac{1}{576}z^4\text{gs}[5,\text{ln}]-\frac{1}{720}\text{gs}[6,\text{ln}]+\frac{137\,z\,\text{gs}[6,\text{ln}]}{21600}-\frac{1}{128}z^2\text{gs}[6,\text{ln}]+\frac{17\,z^3\text{gs}[6,\text{ln}]}{4320}-\frac{z^4\text{gs}[6,\text{ln}]}{1152}+\frac{z^5\text{gs}[6,\text{ln}]}{14400}-\frac{\text{gs}[7,\text{ln}]}{5040}+\frac{7\,z\,\text{gs}[7,\text{ln}]}{7200}-\frac{29\,z^2\text{gs}[7,\text{ln}]}{21600}+\frac{7\,z^3\text{gs}[7,\text{ln}]}{8640}-\frac{5\,z^4\text{gs}[7,\text{ln}]}{20736}+\frac{z^5\text{gs}[7,\text{ln}]}{28800}-\frac{z^6\text{gs}[7,\text{ln}]}{518400}$$

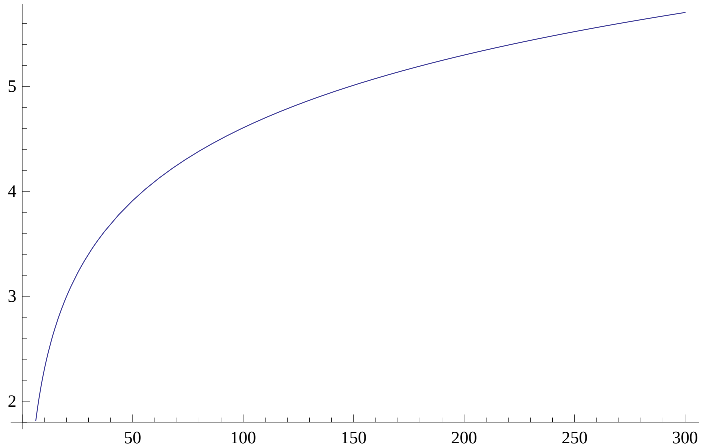
```
1 + Integrate[ Log[x] x^y, {y, 0, n}]
```

x^n

```
Plot[ pp[n, 2, 50] / pp[n, 1, 50.] / Log[n], {n, 1, 300}]
```



```
Plot[ Log[n], {n, 1, 300}]
```




```
D[Binomial[z, k], {z, 2}]
```

Binomial[z, k] (PolyGamma[0, 1 + z] - PolyGamma[0, 1 - k + z])^2 +
 Binomial[z, k] (PolyGamma[1, 1 + z] - PolyGamma[1, 1 - k + z])

```
Table[ Binomial[z, k] (PolyGamma[0, 1 + z] - PolyGamma[0, 1 - k + z])^2 + Binomial[z, k]
    (PolyGamma[1, 1 + z] - PolyGamma[1, 1 - k + z]) (-1)^k / k!, {k, 0, 5}] // TableForm
```

0

z (-PolyGamma[0, z] + PolyGamma[0, 1 + z])^2 - z (-PolyGamma[1, z] + PolyGamma[1, 1 + z])

$\frac{1}{2}$ (-1 + z) z (-PolyGamma[0, -1 + z] + PolyGamma[0, 1 + z])^2 + $\frac{1}{4}$ (-1 + z) z (-PolyGamma[1, -1 + z] + Poly

$\frac{1}{6}$ (-2 + z) (-1 + z) z (-PolyGamma[0, -2 + z] + PolyGamma[0, 1 + z])^2 - $\frac{1}{36}$ (-2 + z) (-1 + z) z (-PolyGamm

$\frac{1}{24}$ (-3 + z) (-2 + z) (-1 + z) z (-PolyGamma[0, -3 + z] + PolyGamma[0, 1 + z])^2 + $\frac{1}{576}$ (-3 + z) (-2 + z) (-1

$\frac{1}{120}$ (-4 + z) (-3 + z) (-2 + z) (-1 + z) z (-PolyGamma[0, -4 + z] + PolyGamma[0, 1 + z])^2 - $\frac{(-4+z)\ (-3+z)\ (-2+z}{}$

```
a Integrate[ D[y^(a - 1), y], {y, 0, x}]
```

ConditionalExpression[a x^{-1+a}, Re[a] > 1]

```
D[x^a, x]
```

a x^{-1+a}

```
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, z_] := Product[(-1)^p[[2]] bin[-z, p[[2]]], {p, FI[n]}]
Dz[n_, z_] := Sum[ dz[j, z], {j, 1, n}]
d2[n_, a_, b_] := Sum[dz[j, a] dz[k, b], {j, 1, n}, {k, 1, n / j}] -
  Sum[dz[j, a] dz[k, b], {j, 1, n - 1}, {k, 1, (n - 1) / j}]
d2a[n_, a_] := Sum[dz[j, a] dz[k, 1], {j, 1, n}, {k, 1, n / j}] -
  Sum[dz[j, a] dz[k, 1], {j, 1, n - 1}, {k, 1, (n - 1) / j}]
d2b[n_, a_] := Sum[dz[j, a], {j, 1, n}, {k, 1, n / j}] -
  Sum[dz[j, a], {j, 1, n - 1}, {k, 1, (n - 1) / j}]
d2c[n_, a_] := Sum[dz[j, a] Sum[ 1, {k, 1, n / j}], {j, 1, n}] -
  Sum[dz[j, a] Sum[ 1, {k, 1, (n - 1) / j}], {j, 1, n - 1}]
d2d[n_, a_] := Sum[dz[j, a] Floor[n / j], {j, 1, n}] -
  Sum[dz[j, a] Floor[(n - 1) / j], {j, 1, n - 1}]
d2e[n_, a_] := Sum[dz[j, a] Floor[n / j], {j, 1, n - 1}] +
  Sum[dz[j, a] Floor[n / j], {j, n, n}] - Sum[dz[j, a] Floor[(n - 1) / j], {j, 1, n - 1}]
d2f[n_, a_] := Sum[dz[j, a] Floor[n / j], {j, 1, n - 1}] +
  dz[n, a] - Sum[dz[j, a] Floor[(n - 1) / j], {j, 1, n - 1}]
d2g[n_, a_] := dz[n, a] + Sum[dz[j, a] Floor[n / j] - dz[j, a] Floor[(n - 1) / j], {j, 1, n - 1}]
d2h[n_, a_] := dz[n, a] + Sum[dz[j, a] (Floor[n / j] - Floor[(n - 1) / j]), {j, 1, n - 1}]
```

```
d2h[98, 4]
```

75

```
D[1, x]
```

0

```
FullSimplify@D[ Integrate[ D[ LaguerreL[-a, Log[x]], x], {x, 1, n}], n]
```

$$\text{ConditionalExpression}\left[\frac{a\,\text{Hypergeometric1F1}[1+a,2,\text{Log}[n]]}{n},\ 0\le \text{Re}[n]\le e\ ||\ n\notin \text{Reals}\right]$$

```
FullSimplify@D[ Integrate[ D[ LaguerreL[-b, Log[y]], y], {y, 1, n}], n]
```

$$\text{ConditionalExpression}\left[\frac{b\,\text{Hypergeometric1F1}[1+b,2,\text{Log}[n]]}{n},\ 0\le \text{Re}[n]\le e\ ||\ n\notin \text{Reals}\right]$$

```
FullSimplify@D[ Integrate[
   D[ LaguerreL[-a, Log[x]], x] D[ LaguerreL[-b, Log[y]], y], {x, 1, n}, {y, 1, n / x}], n]
```

$$\int_1^n \frac{1}{n\,x}\, a\, b\, \text{Hypergeometric1F1}[1+a,2,\text{Log}[x]]\ \text{Hypergeometric1F1}\left[1+b,2,\text{Log}\left[\frac{n}{x}\right]\right] dx$$

$$\text{N}\Big[\frac{a\,\text{Hypergeometric1F1}[1+a,2,\text{Log}[n]]}{n}+\frac{b\,\text{Hypergeometric1F1}[1+b,2,\text{Log}[n]]}{n}+\int_1^n \frac{1}{n\,x}\, a\, b\, \text{Hypergeometric1F1}[1+a,2,\text{Log}[x]]\ \text{Hypergeometric1F1}\left[1+b,2,\text{Log}\left[\frac{n}{x}\right]\right] dx\ /.\ \{a\to 3,\ b\to 2,\ n\to 10\}\Big]$$

65.88

```
N[D[LaguerreL[-3 - 2, Log[n]], n] /. n → 10]
```

65.88

```
Integrate[ D[ LaguerreL[-a, Log[x]], x], {x, 1, n}]
```

```
ConditionalExpression[-1 + Hypergeometric1F1[a, 1, Log[n]], 0 ≤ Re[n] ≤ ⅇ || n ∉ Reals
```

```
Integrate[ D[ LaguerreL[-a, Log[x]], x], {x, 1, n}]
```

```
ConditionalExpression[-1 + Hypergeometric1F1[a, 1, Log[n]], 0 ≤ Re[n] ≤ ⅇ || n ∉ Reals]
```

```
D[Integrate[ D[ LaguerreL[-1, Log[y]], y], {y, 1, n}], n]
```

1

```
D[Integrate[ D[ LaguerreL[-a, Log[x]], x] (n / x), {x, 1, n}], n]
```

$$\int_1^n -\frac{\text{LaguerreL}[-1-a,1,\text{Log}[x]]}{x^2}\,dx-\frac{\text{LaguerreL}[-1-a,1,\text{Log}[n]]}{n}$$

```
FullSimplify@D[ Integrate[D[ LaguerreL[-a, Log[x]], x] , {x, 1, n}, {y, 1, n / x}], n]
```

$$\int_1^n \frac{a\,\text{Hypergeometric1F1}[1+a,2,\text{Log}[x]]}{x^2}\,dx$$

$$\text{N}\left[1+\frac{a\,\text{Hypergeometric1F1}[1+a,2,\text{Log}[n]]}{n}+\int_1^n -\frac{\text{LaguerreL}[-1-a,1,\text{Log}[x]]}{x^2}\,dx\ /.\ \{a\to 4,\ n\to 10\}\right]$$

65.88

$$\text{N}\left[1+\int_1^n -\frac{\text{LaguerreL}[-1-a,1,\text{Log}[x]]}{x^2}\,dx-\frac{\text{LaguerreL}[-1-a,1,\text{Log}[n]]}{n}\ /.\ \{a\to 4,\ n\to 10\}\right]$$

65.88

$$\text{Integrate}\left[-\frac{a\,\text{Hypergeometric1F1}[1+a,2,\text{Log}[x]]}{x\char`^2},\ \{x,1,n\}\right]$$

$$\int_1^n -\frac{a\,\text{Hypergeometric1F1}[1+a,2,\text{Log}[x]]}{x^2}\,dx$$

```
n Integrate[ D[ LaguerreL[-a, Log[x]], x] / x, {x, 1, n}]
```

$$n\int_1^n -\frac{\text{LaguerreL}[-1-a,1,\text{Log}[x]]}{x^2}\,dx$$

```
D[ LaguerreL[-a, Log[x]], x]
```

$$-\frac{\text{LaguerreL}[-1-a,1,\text{Log}[x]]}{x}$$

```
D[ Gamma[k, 0, -Log[n]], n]
```

$$-(-\text{Log}[n])^{-1+k}$$

$$-(-\text{Log}[n])^{-1+k}\ /.\ \{k\to 5,\ n\to 20\}$$

$$-\text{Log}[20]^4$$

$$(-1)\char`^(k)\ (\text{Log}[n])^{-1+k}\ /.\ \{k\to 5,\ n\to 20\}$$

$$-\text{Log}[20]^4$$

```
Sum[ bin[z, k] Log[n]^(k - 1) / ((k - 1) !), {k, 0, Infinity}]
```

```
z Hypergeometric1F1[1 - z, 2, -Log[n]]
```

```
Sum[ (-1)^k / k Log[n]^(k - 1) / Gamma[k], {k, 1, Infinity}]
```

$$\frac{1-n}{n \operatorname{Log}[n]}$$

```
D[z Hypergeometric1F1[1 - z, 2, -Log[n]], {z, 1}] /. z → 0
```

$$\frac{-1+n}{n \operatorname{Log}[n]}$$

```
ff[z_] := Sum[ Binomial[z, k] Log[n]^(k - 1) / ((k - 1)!), {k, 1, z}]
ff[6]
```

$$6+15 \operatorname{Log}[n]+10 \operatorname{Log}[n]^2+\frac{5 \operatorname{Log}[n]^3}{2}+\frac{\operatorname{Log}[n]^4}{4}+\frac{\operatorname{Log}[n]^5}{120}$$

$$\mathrm{N}\left[-2\ \mathrm{Hypergeometric1F1}^{(1,0,0)}[1, 2, -\operatorname{Log}[n]] \ /.\ n \to 20\right]$$

0.851865

```
-2 D[Pochhammer[1 - z, k] (-Log[n])^k / (Pochhammer[2, k] k!), z] /. z → 0
```

$$\frac{2\,(-\operatorname{Log}[n])^k \operatorname{Pochhammer}[1, k]\,(\operatorname{EulerGamma}+\operatorname{PolyGamma}[0, 1+k])}{k!\ \operatorname{Pochhammer}[2, k]}$$

$$\frac{2\,(-\operatorname{Log}[n])^k \operatorname{Pochhammer}[1, k]\,(\operatorname{EulerGamma}+\operatorname{PolyGamma}[0, 1+k])}{k!\ \operatorname{Pochhammer}[2, k]}\ /.\ k \to 5$$

$$-\frac{137 \operatorname{Log}[n]^5}{21600}$$

$$\frac{2\,(-1)^k \operatorname{Pochhammer}[1, k]\,(\operatorname{EulerGamma}+\operatorname{PolyGamma}[0, 1+k])}{k!\ \operatorname{Pochhammer}[2, k]}\ /.\ k \to 5$$

$$-\frac{137}{21600}$$

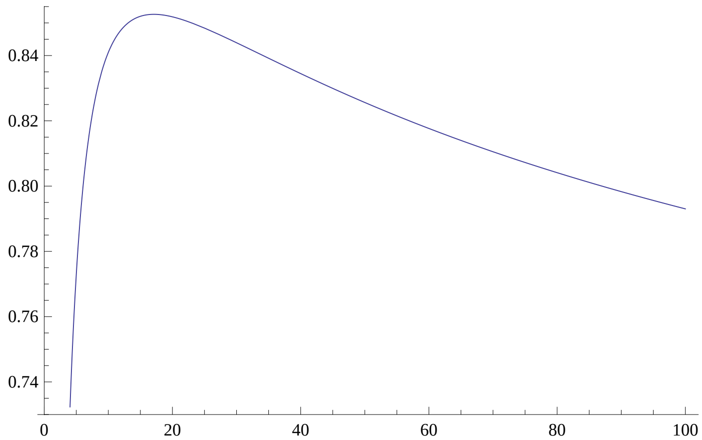
$$\operatorname{FullSimplify}\left[\frac{2\,(-1)^k \operatorname{Pochhammer}[1, k]\,(\operatorname{EulerGamma}+\operatorname{PolyGamma}[0, 1+k])}{k!\ \operatorname{Pochhammer}[2, k]}\right]$$

$$\frac{2\,(-1)^k \operatorname{HarmonicNumber}[k]}{\operatorname{Gamma}[2+k]}$$

$$-2\operatorname{Sum}\left[\frac{(-1)^k \operatorname{HarmonicNumber}[k]}{\operatorname{Gamma}[2+k]} \operatorname{Log}[n]\text{^}k,\ \{k, 0, \text{Infinity}\}\right]$$

$$-2\ \mathrm{Hypergeometric1F1}^{(1,0,0)}[1, 2, -\operatorname{Log}[n]]$$

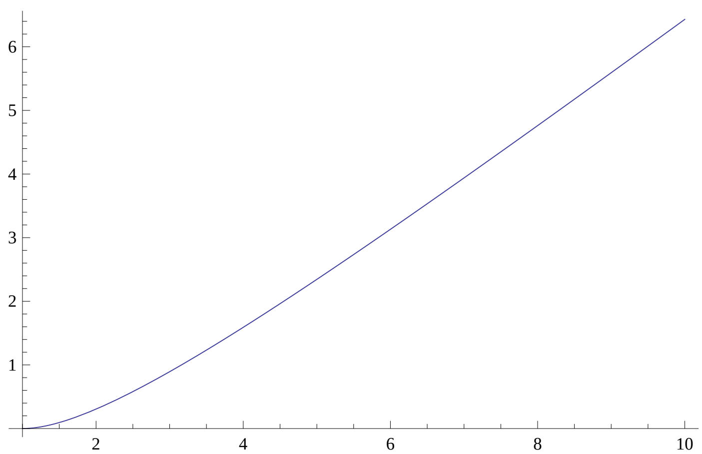
```
Plot[-2 Hypergeometric1F1^(1,0,0)[1, 2, -Log[n]], {n, 1, 100}]
```



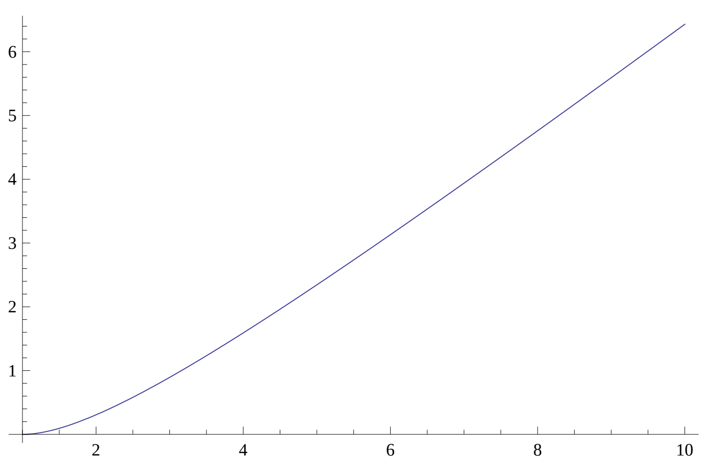

```
Integrate[-2 Hypergeometric1F1^(1,0,0)[1, 2, -Log[x]], {x, 0, n}]
```

$$\int_0^n -2\, \text{Hypergeometric1F1}^{(1,0,0)}[1, 2, -\text{Log}[x]]\, dx$$

```
Plot[∫_1^n -2 Hypergeometric1F1^(1,0,0)[1, 2, -Log[x]] dx, {n, 1, 10}]
```




```
Plot[pp[n, 2, 30.], {n, 1, 10}]
```



```
D[z Hypergeometric1F1[1 - z, 2, -Log[n]], {z, 1}] /. z → 0
```

$$\frac{-1+n}{n\,\mathrm{Log}[n]}$$

```
D[z^2 Hypergeometric1F1[1 - z, 2, -Log[n]], {z, 2}] /. z → 0
```

$$\frac{2\,(-1+n)}{n\,\mathrm{Log}[n]}$$

```
Integrate[ 1 / Log[n] - 1 / (n Log[n]), {n, 1, x}]
```

```
ConditionalExpression[-EulerGamma - Gamma[0, -Log[x]] - Log[-Log[x]], Im[x] ≠ 0 || Re[x] ≥ 0]
```

$$\mathrm{Limit}\left[\frac{2\,(-1+n)}{n\,\mathrm{Log}[n]},\ n \to 1\right]$$

2

$$\mathrm{FullSimplify}\left[\frac{2\,(-1+n)}{n\,\mathrm{Log}[n]} - (1\,/\,\mathrm{Log}[n]\ -\ 2\,/\,(n\,\mathrm{Log}[n])\ +\ 1\,/\,(n\hat{}2\,\mathrm{Log}[n]))\right]$$

$$\frac{-1+n^2}{n^2\,\mathrm{Log}[n]}$$

$$\mathrm{Limit}\left[\frac{-1+n^2}{n^2\,\mathrm{Log}[n]} - \left(\frac{-1+n}{n\,\mathrm{Log}[n]}\right),\ n \to 1\right]$$

1

$$\mathrm{Integrate}\left[\frac{-1+n^k}{n^k\,\mathrm{Log}[n]},\ n\right]$$

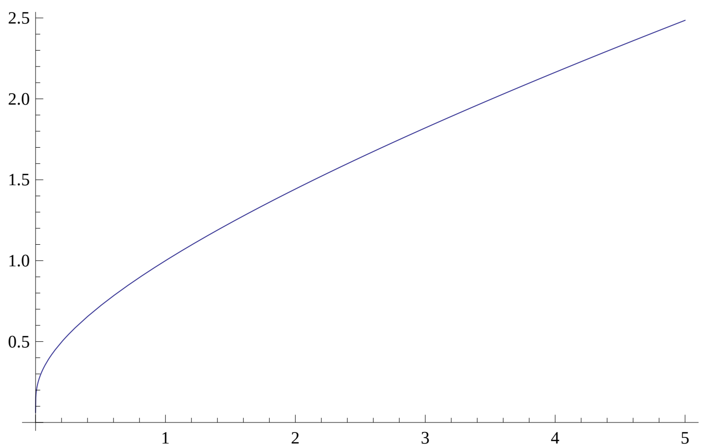
```
-ExpIntegralEi[-(-1 + k) Log[n]] + LogIntegral[n]
```

$$\mathrm{Limit}\left[\frac{-1+n^k}{n^k\,\mathrm{Log}[n]} - \frac{-1+n^{k-1}}{n^{k-1}\,\mathrm{Log}[n]},\ n \to 1\right]$$

1

$$\mathrm{ff}[n\_,\ k\_]\ :=\ \frac{-1+n^k}{n^k\,\mathrm{Log}[n]} - \frac{-1+n^{k-1}}{n^{k-1}\,\mathrm{Log}[n]}$$

```
Plot[ ff[n, 0], {n, 0, 5}]
```




```
FullSimplify[ff[n, 3]]
```

$$\frac{-1+n}{n^3\,\mathrm{Log}[n]}$$

```
D[x^z, {z, 2}] /. z → 0
```

$\mathrm{Log}[x]^2$

```
Integrate[ z Hypergeometric1F1[1 - z, 2, -Log[n]], {n, 1, x}]
```

$$\int_1^x z\,\mathrm{Hypergeometric1F1}[1-z, 2, -\mathrm{Log}[n]]\,dn$$

```
N[1 + Integrate[ z Hypergeometric1F1[1 - z, 2, -Log[n]], {n, 1, x}] /. {x → 10, z → 2}]
```

33.0259

```
N[LaguerreL[-2, Log[10]]]
```

33.0259

```
N[Hypergeometric1F1[2, 1, Log[10]]]
```

33.0259

```
FullSimplify[(1 + n) ! / (1 ! n !)]
```

$1+n$

```
N[ D[ LaguerreL[ -z, Log[n]], n] /. {n → 16, z → 3}]
```

15.1614

```
N[(-z / Log[n]) ( LaguerreL[ -z, Log[n]] - LaguerreL[-z - 1, Log[n]]) / n /. {n → 16, z → 3}]
```

15.1614

```
N@D[ LaguerreL[ z, Log[n]], n] /. {z → -3, n → 16}
```

15.1614

```
D[ fa[Log[x]], x]
```

$$\frac{fa'[\mathrm{Log}[x]]}{x}$$

```
FullSimplify[(-z / Log[n]) (LaguerreL[ -z, Log[n]] -  LaguerreL[-z - 1, Log[n]]) / n]
```

$$\frac{1}{n\,\mathrm{Log}[n]} z\,(-\mathrm{Hypergeometric1F1}[z, 1, \mathrm{Log}[n]] + \mathrm{Hypergeometric1F1}[1 + z, 1, \mathrm{Log}[n]])$$

```
(z / (n Log[n])) ( LaguerreL[-z - 1, Log[n]] - LaguerreL[ -z, Log[n]])
```

$$\frac{z\,(\mathrm{LaguerreL}[-1 - z, \mathrm{Log}[n]] - \mathrm{LaguerreL}[-z, \mathrm{Log}[n]])}{n\,\mathrm{Log}[n]}$$

```
N[
 1 + Integrate[ z / (x Log[x]) (LaguerreL[-z - 1,  Log[x]] - LaguerreL[-z, Log[x]]), {x, 1, n}] /.
  {z → 3, n → 12}]
```

108.686 + 0. i

```
N[LaguerreL[-3, Log[12]]]
```

108.686

```
N[1 + Integrate[ z Hypergeometric1F1[1 - z, 2, -Log[x]], {x, 1, n}] /. {z → 3, n → 12}]
```

108.686

```
LaguerreL[0, n]
```

1

```
N[Integrate[ D[LaguerreL[-z, Log[r]], r] /. r → x, {x, 1, n}] /. {z → 3, n → 12}]
```

Undefined

```
N@LaguerreL[-3, Log[12]]
```

108.686