```
Clear[Dz]
Dz[n_, z_, k_] := Dz[n, z, k] = 1 + ((z + 1) / k - 1) Sum[Dz[Floor[n / j], z, k + 1], {j, 2, n}]
```

```
Expand@Dz[100, z a, 1]
```

$$1 + \frac{428\,a\,z}{15} + \frac{16\,289\,a^2\,z^2}{360} + \frac{331\,a^3\,z^3}{16} + \frac{611\,a^4\,z^4}{144} + \frac{67\,a^5\,z^5}{240} + \frac{7\,a^6\,z^6}{720}$$

```
D[Expand@Dz[100, z, 1], z] /. z → 0
```

$$\frac{428}{15}$$

```
D[Expand@Dz[100, z a, 1], z] /. z → 0
```

$$\frac{428\,a}{15}$$

```
D[x^(a z), z] /. z → 0
```

```
a Log[x]
```

```
N[D[ LaguerreL[-(a z), Log[200]], z] /. z → 0]
```

```
47.9476 a
```

```
ff[n_, z_] := LaguerreL[-z, Log[n]]
fg[n_, z_] := n^z
fh[n_, z_] := (-1)^z Gamma[z, 0, -Log[n]] / Gamma[z]
```

```
Sum[ (-1)^(k + 1) fh[k, -1], {k, 1, Infinity}]
```

```
Infinity::indet : Indeterminate expression 0 ComplexInfinity encountered. ≫

Sum::div : Sum does not converge. ≫

∑_{k=1}^{∞} Indeterminate
```

```
N@Sum[ (-1)^(k + 1) fg[k, -3], {k, 1, Infinity}]
```

```
0.901543
```

```
Expand[FullSimplify[n / ((m + 1) ((x - 1) / (n - 1)) + 1)]]
```

$$-\frac{n}{-2 + n + m\,(-1 + x) + x} + \frac{n^2}{-2 + n + m\,(-1 + x) + x}$$

```
FullSimplify@Log[n, n^(1 / 2)]
```

$$\frac{1}{2}$$

```
E^(Log[j] / Log[n])
```

$$j^{\frac{1}{\text{Log}[n]}}$$

```
Table[ If[ Log[j] / Log[12] + Log[k] / Log[12] ≤ 1, 1, 0], {j, 1, 16}, {k, 1, 16}] // TableForm
```

1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

```
Table[ If[ N[j k] ≤ 12, 1, 0], {j, 1, 16}, {k, 1, 16}] // TableForm
```

1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

```
E^((Log[j] + Log[k]) / Log[n]) /. {n → 30, k → 2, j → 3}
```

$e^{\frac{\text{Log}[2]+\text{Log}[3]}{\text{Log}[30]}}$

```
E^((Log[j k]) / Log[n]) /. {n → 30, k → 2, j → 3}
```

$6^{\frac{1}{\text{Log}[30]}}$

```
N[Log[n, n] - Log[n, j] /. {n → 30, j → 5}]
```

0.526803

```
N[Log[n, n / j] /. {n → 30, j → 5}]
```

0.526803

```
Log[n, n / j]
```

$\frac{\text{Log}\left[\frac{n}{j}\right]}{\text{Log}[n]}$

```
m^Log[m, k]
```

k

```
N[FullSimplify[m^Log[n, n / j]] /. {n → 30, j → 3, m → 40}]
```

12.1502

```
N[m^(1 - Log[j] / Log[n]) /. {n → 30, j → 3, m → 40}]
```

12.1502

```
m * m^(-Log[j] / Log[n])
```

$$m^{1-\frac{\text{Log}[j]}{\text{Log}[n]}}$$

```
Log[x, 1]
```

0

```
Integrate[ D[LaguerreL[-1, Log[x]], x] D[LaguerreL[-1, Log[y]], y],
 {x, 1, n}, {y, 1, m^(1 - Log[n, x])}]
```

$$\text{ConditionalExpression}\left[\frac{\text{Log}[m]-n\,\text{Log}[m]-\text{Log}[n]+m\,\text{Log}[n]}{\text{Log}[m]-\text{Log}[n]},\ \text{Re}[n]\geq 0\ ||\ n\notin \text{Reals}\right]$$

```
FullSimplify[ (Log[m] - n Log[m] - Log[n] + m Log[n]) / (Log[m] - Log[n]) ]
```

$$\frac{-(-1+n)\,\text{Log}[m]+(-1+m)\,\text{Log}[n]}{\text{Log}[m]-\text{Log}[n]}$$

```
Integrate[ D[ LaguerreL[-1, Log[x]], x], {x, 1, n}]
```

$-1+n$

```
Integrate[ D[ LaguerreL[-1, Log[x]], x], {x, 1, m}]
```

$-1+m$

```
FullSimplify[ (Log[m] - n Log[m] - Log[n] + m Log[n]) / (Log[m] - Log[n]) + (-1 + n) + (-1 + m) + 1]
```

$$\frac{m\,\text{Log}[m]-n\,\text{Log}[n]}{\text{Log}[m]-\text{Log}[n]}$$

```
N[LaguerreL[-1, Log[m]] /. {m → 8, n → 7}]
```

$8.$

```
(m Log[m] - n Log[n]) / (Log[m] - Log[n]) /. {m → 8, n → 7}
```

$$\frac{-7\,\text{Log}[7]+8\,\text{Log}[8]}{-\text{Log}[7]+\text{Log}[8]}$$

```
FullSimplify[Integrate[ D[LaguerreL[-1, Log[x]], x] D[LaguerreL[-1, Log[y]], y], {x, 1, n},
   {y, 1, m^(1 - Log[x] / Log[n])}] + Integrate[ D[ LaguerreL[-1, Log[x]], x], {x, 1, n}] +
  Integrate[ D[ LaguerreL[-1, Log[x]], x], {x, 1, m}] + 1]
```

$$\text{ConditionalExpression}\left[\frac{m\,\text{Log}[m]-n\,\text{Log}[n]}{\text{Log}[m]-\text{Log}[n]},\ \text{Re}[n]\geq 0\ ||\ n\notin \text{Reals}\right]$$

```
N[D[ LaguerreL[-1, Log[n]], n] /. n → 6]
```

1.

```
FullSimplify[Integrate[1, {x, 1, n}, {y, 1, m^(1 - Log[x] / Log[n])}] +
  Integrate[1, {x, 1, n}] + Integrate[1, {x, 1, m}] + 1]
```

$$\text{ConditionalExpression}\left[\frac{m \log[m] - n \log[n]}{\log[m] - \log[n]}, \text{Re}[n] \geq 0 \mid\mid n \notin \text{Reals}\right]$$

```
Log[m^m / n^n] / Log[m / n]
```

$$\frac{\log[m^m n^{-n}]}{\log\left[\frac{m}{n}\right]}$$

```
N[(m Log[m] - n Log[n]) / (Log[m] - Log[n])] /. {m → 9, n → 7}
```

24.4859

```
N[Log[m^m / n^n] / Log[m / n] /. {m → 9, n → 7}]
```

24.4859

```
N[Log[m / n, m^m / n^n] /. {m → 9, n → 7}]
```

24.4859

```
Log[m / n, m^m / n^n]
```

$$\frac{\log[m^m n^{-n}]}{\log\left[\frac{m}{n}\right]}$$

```
Log[9 / 7, 9^9 / 7^7]
```

$$\frac{\log\left[\frac{387\,420\,489}{823\,543}\right]}{\log\left[\frac{9}{7}\right]}$$

```
Log[9 / 7, 9^2 × 9^7 / 7^7]
```

$$\frac{\log\left[\frac{387\,420\,489}{823\,543}\right]}{\log\left[\frac{9}{7}\right]}$$

```
Log[9 / 7, 9^2] + Log[9 / 7, 9^7 / 7^7]
```

$$7 + \frac{\log[81]}{\log\left[\frac{9}{7}\right]}$$

```
N[n + Log[m / n, m^(m - n)] /. {m → 9, n → 7}]
```

24.4859

```
n + (m - n) Log[m / n, m] /. {m → 9, n → 7}
```

$$7 + \frac{2 \log[9]}{\log\left[\frac{9}{7}\right]}$$

```
N[n + (m - n) (Log[m] / (Log[m] - Log[n])) /. {m → 9, n → 7}]
```

24.4859

```
N[Log[m^m / n^n] / Log[m / n] /. {m → 9, n → 7}]
```

24.4859

```
N[Log[ m^m ] / Log[ m / n] - Log[ n^n] / Log[ m / n] /. {m → 9, n → 7}]
```

```
24.4859
```

```
N[m Log[ m / n, m ] - n Log[ m / n, n] /. {m → 9, n → 7}]
```

```
24.4859
```

```
Integrate[1, {x, 1, n}, {y, 1, m^(1 - Log[x] / Log[n])}]
```

$$\text{ConditionalExpression}\left[\frac{\text{Log}[m]-n\,\text{Log}[m]-\text{Log}[n]+m\,\text{Log}[n]}{\text{Log}[m]-\text{Log}[n]},\ \text{Re}[n]\geq 0\ ||\ n\notin \text{Reals}\right]$$

```
Integrate[1, {x, 1, n}, {y, 1, m^(1 - Log[x] / Log[n])},
 {z, 1, o^(1 - Log[x] / Log[n] - Log[y] / Log[m])}]
```

$$\text{ConditionalExpression}\Big[\Big((-1+m)\,\text{Log}[n]\,(\text{Log}[n]-\text{Log}[o])\,\text{Log}[o]+\text{Log}[m]^2\,((-1+o)\,\text{Log}[n]-(-1+n)\,\text{Log}[o])+\text{Log}[m]\,\left(-(-1+o)\,\text{Log}[n]^2+(-1+n)\,\text{Log}[o]^2\right)\Big)\Big/\big((\text{Log}[m]-\text{Log}[n])\,(\text{Log}[m]-\text{Log}[o])\,(\text{Log}[n]-\text{Log}[o])\big),\ \text{Re}[n]\geq 0\ ||\ n\notin \text{Reals}\Big]$$

```
Expand@Integrate[1, {x, 1, n}, {y, 1, m^(1 - Log[x] / Log[n])},
  {z, 1, o^(1 - Log[x] / Log[n] - Log[y] / Log[m])}]
```

$$\text{ConditionalExpression}\left[-\frac{\text{Log}[m]^2\,\text{Log}[n]}{(\text{Log}[m]-\text{Log}[n])\,(\text{Log}[m]-\text{Log}[o])\,(\text{Log}[n]-\text{Log}[o])}+\frac{o\,\text{Log}[m]^2\,\text{Log}[n]}{(\text{Log}[m]-\text{Log}[n])\,(\text{Log}[m]-\text{Log}[o])\,(\text{Log}[n]-\text{Log}[o])}+\frac{\text{Log}[m]\,\text{Log}[n]^2}{(\text{Log}[m]-\text{Log}[n])\,(\text{Log}[m]-\text{Log}[o])\,(\text{Log}[n]-\text{Log}[o])}-\frac{o\,\text{Log}[m]\,\text{Log}[n]^2}{(\text{Log}[m]-\text{Log}[n])\,(\text{Log}[m]-\text{Log}[o])\,(\text{Log}[n]-\text{Log}[o])}+\frac{\text{Log}[m]^2\,\text{Log}[o]}{(\text{Log}[m]-\text{Log}[n])\,(\text{Log}[m]-\text{Log}[o])\,(\text{Log}[n]-\text{Log}[o])}-\frac{n\,\text{Log}[m]^2\,\text{Log}[o]}{(\text{Log}[m]-\text{Log}[n])\,(\text{Log}[m]-\text{Log}[o])\,(\text{Log}[n]-\text{Log}[o])}-\frac{\text{Log}[n]^2\,\text{Log}[o]}{(\text{Log}[m]-\text{Log}[n])\,(\text{Log}[m]-\text{Log}[o])\,(\text{Log}[n]-\text{Log}[o])}+\frac{m\,\text{Log}[n]^2\,\text{Log}[o]}{(\text{Log}[m]-\text{Log}[n])\,(\text{Log}[m]-\text{Log}[o])\,(\text{Log}[n]-\text{Log}[o])}-\frac{\text{Log}[m]\,\text{Log}[o]^2}{(\text{Log}[m]-\text{Log}[n])\,(\text{Log}[m]-\text{Log}[o])\,(\text{Log}[n]-\text{Log}[o])}+\frac{n\,\text{Log}[m]\,\text{Log}[o]^2}{(\text{Log}[m]-\text{Log}[n])\,(\text{Log}[m]-\text{Log}[o])\,(\text{Log}[n]-\text{Log}[o])}+\frac{\text{Log}[n]\,\text{Log}[o]^2}{(\text{Log}[m]-\text{Log}[n])\,(\text{Log}[m]-\text{Log}[o])\,(\text{Log}[n]-\text{Log}[o])}-\frac{m\,\text{Log}[n]\,\text{Log}[o]^2}{(\text{Log}[m]-\text{Log}[n])\,(\text{Log}[m]-\text{Log}[o])\,(\text{Log}[n]-\text{Log}[o])},\ \text{Re}[n]\geq 0\ ||\ n\notin\text{Reals}\right]$$

```
N@D[LaguerreL[-z, Log[10]], z] /. z → 0
```

4.75435

```
N@D[LaguerreL[-z, Log[10^2]], z] /. z → 0
```

28.0217

```
28.021746293370207` / 4.754351394637798`
```

5.89392

```
E^5.893915692679883`
```

362.823

```
N@D[LaguerreL[-z, 2], z] /. z → 0
```

3.68387

```
4.754351394637798`^3.683871510540406`
```

312.117

```
N[Log[10]]
```

```
2.30259
```

```
N[Log[10^2]]
```

```
4.60517
```

```
Table[ N@D[LaguerreL[-z, Log[n]], z] /. z → 0, {n, 1, 10}]
```

$$\{3.4683 \times 10^{-14},\ 0.834461,\ 1.49233,\ 2.06374,\ 2.58149,\ 3.06181,\ 3.51411,\ 3.9444,\ 4.35683,\ 4.75435\}$$

```
Limit[ (x^( a z) - 1) / z, z → 0]
```

```
a Log[x]
```

```
Sum[ (-1)^(k + 1) / k (x^a - 1)^( k), {k, 1, Infinity}]
```

```
Log[x^a]
```

```
Integrate[ D[x^2, x], {x, 1, n}]
```

```
-1 + n^2
```

```
Sum[ (-1)^(k + 1) / k (Integrate[ D[x^2, x], {x, 1, n}])^( k), {k, 1, Infinity}]
```

```
Log[n^2]
```

```
Integrate[ D[ x^a, x], {x, 1, n}]
```

```
ConditionalExpression[-1 + n^a, Re[n] ≥ 0 || n ∉ Reals]
```

```
Expand@Integrate[ D[ x^a, x] D[y^a, y], {x, 1, n}, {y, 1, n / x}]
```

```
ConditionalExpression[1 - n^a + a n^a Log[n], Re[n] ≥ 0 || n ∉ Reals]
```

```
Expand@Integrate[ D[ x^a, x] D[y^a, y] D[z^a, z], {x, 1, n}, {y, 1, n / x}, {z, 1, n / (x y)}]
```

$$\text{ConditionalExpression}\left[-1 + n^a - a\, n^a \text{Log}[n] + \frac{1}{2} a^2 n^a \text{Log}[n]^2,\ \text{Re}[n] \geq 0 \ ||\ n \notin \text{Reals}\right]$$

```
d
```