

```

f[n_, k_] := Sum[Abs[MoebiusMu[j]] (1/k - f[Floor[n/j], k+1]), {j, 2, n}]

f[100, 1]


$$\frac{116}{5}$$


{n^(a+b), -1+n^a+n^b+Integrate[D[x^a, x] D[y^b, y], {x, 1, n}, {y, 1, n}]}

{n^a+b, ConditionalExpression[-1+n^a+n^b+(-1+n^a) (-1+n^b), Re[n] ≥ 0 || n ∉ Reals]}

FullSimplify[Expand[-1+n^a+n^b+(-1+n^a) (-1+n^b)]]

n^a+b

Sum[1, {j, 1, 100}, {k, 1, (100/j)^(1/2)}]

153

N[Log[100]/Log[10]]

2.

Expand[Log[n, k^(Log[m, n])]] /. k -> 10


$$\frac{\text{Log}\left[10^{\frac{\text{Log}[n]}{\text{Log}[m]}}\right]}{\text{Log}[n]}$$


Sum[1, {j, 1, 100}, {k, 1, (100/j)^(1/Log[10, 100])}]

153

Sum[1, {j, 1, E^Log[100]}, {k, 1, (E/j^(1/Log[100]))^Log[10]}]

Sum::itflrw :
Warning: In evaluating Floor[5 × 21 -  $\frac{2 \text{Log}[10]}{\text{Log}[100]}$ }] to find the number of iterations to use for Sum, $MaxExtraPrecision =
50.` was encountered. An upper estimate will be used for the number of iterations. >>

Sum::itflrw :
Warning: In evaluating Floor[2 × 51 -  $\frac{2 \text{Log}[10]}{\text{Log}[100]}$ }] to find the number of iterations to use for Sum, $MaxExtraPrecision =
50.` was encountered. An upper estimate will be used for the number of iterations. >>

153

FullSimplify[j^(1/Log[n]) k^(1/Log[m])]


$$j^{\frac{1}{\text{Log}[n]}} k^{\frac{1}{\text{Log}[m]}}$$


Grid[Table[If[1 - Log[j]/Log[100] - Log[k]/Log[10] > 0, 1, 0], {j, 1, 100}, {k, 1, 10}]]

N::meprec : Internal precision limit $MaxExtraPrecision = 50.` reached while evaluating  $-1 + \frac{\text{Log}[5]}{\text{Log}[10]} + \frac{\text{Log}[4]}{\text{Log}[100]}$ . >>

N::meprec : Internal precision limit $MaxExtraPrecision = 50.` reached while evaluating  $-1 + \frac{\text{Log}[2]}{\text{Log}[10]} + \frac{\text{Log}[25]}{\text{Log}[100]}$ . >>

Expand[1 + (m-1) + (n-1) + Integrate[1, {j, 1, m}, {k, 1, n}]]

m n

l[n_, z_] := LaguerreL[-z, Log[n]]

```

**D[1[n, 1], n]**

$$-\frac{\text{LaguerreL}[-2, 1, \text{Log}[n]]}{n}$$

$$\mathbf{N}\left[-\frac{\text{LaguerreL}[-2, 1, \text{Log}[n]]}{n} /. n \rightarrow 8\right]$$

1.

**Expand[1 + (m - 1) + (n - 1) + Integrate[1, {j, 1, n}, {k, 1, (n / j) ^ (1 / Log[m, n])}]]**

$$\text{ConditionalExpression}\left[-1 + m + n + \frac{\text{Log}[m]}{\text{Log}[m] - \text{Log}[n]} - \frac{n \text{Log}[m]}{\text{Log}[m] - \text{Log}[n]} - \frac{\text{Log}[n]}{\text{Log}[m] - \text{Log}[n]} + \frac{m \text{Log}[n]}{\text{Log}[m] - \text{Log}[n]}, \text{Re}[n] \geq 0 \mid \mid n \notin \text{Reals}\right]$$

$$\mathbf{FullSimplify}\left[-1 + m + n + \frac{\text{Log}[m]}{\text{Log}[m] - \text{Log}[n]} - \frac{n \text{Log}[m]}{\text{Log}[m] - \text{Log}[n]} - \frac{\text{Log}[n]}{\text{Log}[m] - \text{Log}[n]} + \frac{m \text{Log}[n]}{\text{Log}[m] - \text{Log}[n]}\right]$$

$$\frac{m \text{Log}[m] - n \text{Log}[n]}{\text{Log}[m] - \text{Log}[n]}$$

**Log[3, 2]**

$\frac{\text{Log}[2]}{\text{Log}[3]}$

$\frac{1}{\text{Log}[2, 3]}$

$\frac{\text{Log}[2]}{\text{Log}[3]}$

$\frac{\text{Log}[2]}{\text{Log}[3]}$

$\frac{\text{Log}[2]}{\text{Log}[3]}$

**(n / j) ^ Log[n, m]**

$$\left(\frac{n}{j}\right)^{\frac{\text{Log}[m]}{\text{Log}[n]}}$$

**n ^ Log[n, m]**

m

**Integrate[1, {x, 1, n}, {y, 1, m / (x ^ Log[n, m])}]**

$$\text{ConditionalExpression}\left[\frac{\text{Log}[m] - n \text{Log}[m] - \text{Log}[n] + m \text{Log}[n]}{\text{Log}[m] - \text{Log}[n]}, \text{Re}[n] \geq 0 \mid \mid n \notin \text{Reals}\right]$$

$$\mathbf{FullSimplify}\left[\frac{\text{Log}[m] - n \text{Log}[m] - \text{Log}[n] + m \text{Log}[n]}{\text{Log}[m] - \text{Log}[n]}\right]$$

$$\frac{-(-1 + n) \text{Log}[m] + (-1 + m) \text{Log}[n]}{\text{Log}[m] - \text{Log}[n]}$$

$$n + m - 1 + \frac{\text{Log}[m] - n \text{Log}[m] - \text{Log}[n] + m \text{Log}[n]}{\text{Log}[m] - \text{Log}[n]}$$

$$\mathbf{FullSimplify}\left[\mathbf{Expand}\left[-1 + m + n + \frac{\text{Log}[m] - n \text{Log}[m] - \text{Log}[n] + m \text{Log}[n]}{\text{Log}[m] - \text{Log}[n]}\right]\right]$$

$$\frac{m \operatorname{Log}[m] - n \operatorname{Log}[n]}{\operatorname{Log}[m] - \operatorname{Log}[n]}$$

Table[FullSimplify[Limit[ $\frac{m \operatorname{Log}[m] - n \operatorname{Log}[n]}{\operatorname{Log}[m] - \operatorname{Log}[n]}$ ,  $m \rightarrow ma$ ]], {ma, 1, 10}, {n, 1, 10}] // Grid

1	2	3	4	5	6	7	8	9	10
2	$2 + \operatorname{Log}[4]$	$\operatorname{Log}\left[\frac{27}{4}\right] / \operatorname{Log}\left[\frac{3}{2}\right]$	6	$\operatorname{Log}\left[\frac{3125}{4}\right] / \operatorname{Log}\left[\frac{5}{2}\right]$	$\operatorname{Log}\left[\frac{11664}{11}\right] / \operatorname{Log}[3]$	$\operatorname{Log}\left[\frac{823543}{4}\right] / \operatorname{Log}\left[\frac{7}{2}\right]$	11	$(2 \operatorname{Log}\left[\frac{196}{83}\right] + \operatorname{Log}\left[\frac{2}{2}\right]) / \operatorname{Log}\left[\frac{9}{2}\right]$	$10 + (8 \operatorname{Log}\left[\frac{10}{2}\right] - \operatorname{Log}[27]) / \operatorname{Log}\left[\frac{10}{3}\right]$
3	$\operatorname{Log}\left[\frac{27}{4}\right] / \operatorname{Log}\left[\frac{3}{2}\right]$	$3 + \operatorname{Log}[27]$	$\operatorname{Log}\left[\frac{256}{27}\right] / \operatorname{Log}\left[\frac{4}{3}\right]$	$\operatorname{Log}\left[\frac{3125}{27}\right] / \operatorname{Log}\left[\frac{5}{3}\right]$	$\operatorname{Log}\left[\frac{1728}{1728}\right] / \operatorname{Log}[2]$	$\operatorname{Log}\left[\frac{823543}{27}\right] / \operatorname{Log}\left[\frac{7}{3}\right]$	$(3 \operatorname{Log}\left[\frac{256}{3}\right] + \operatorname{Log}\left[\frac{8}{3}\right]) / \operatorname{Log}\left[\frac{10}{3}\right]$	15	$(10 \operatorname{Log}[10] - \operatorname{Log}[27]) / \operatorname{Log}\left[\frac{10}{3}\right]$
4	6	$\operatorname{Log}\left[\frac{256}{27}\right] / \operatorname{Log}\left[\frac{4}{3}\right]$	$4 + \operatorname{Log}[256]$	$\operatorname{Log}\left[\frac{3125}{256}\right] / \operatorname{Log}\left[\frac{5}{4}\right]$	$\operatorname{Log}\left[\frac{729}{4}\right] / \operatorname{Log}\left[\frac{3}{2}\right]$	$\operatorname{Log}\left[\frac{823543}{256}\right] / \operatorname{Log}\left[\frac{7}{4}\right]$	16	$\operatorname{Log}\left[\frac{19683}{16}\right] / \operatorname{Log}\left[\frac{3}{2}\right]$	$(2 \operatorname{Log}\left[\frac{6250}{5}\right] + \operatorname{Log}\left[\frac{5}{2}\right]) / \operatorname{Log}\left[\frac{5}{2}\right]$
5	$\operatorname{Log}\left[\frac{3125}{4}\right] / \operatorname{Log}\left[\frac{5}{2}\right]$	$\operatorname{Log}\left[\frac{3125}{27}\right] / \operatorname{Log}\left[\frac{5}{3}\right]$	$\operatorname{Log}\left[\frac{3125}{256}\right] / \operatorname{Log}\left[\frac{5}{4}\right]$	$5 (1 + \operatorname{Log}[5])$	$(-5 \operatorname{Log}[5] + 6 \operatorname{Log}[6]) / \operatorname{Log}\left[\frac{6}{5}\right]$	$(-5 \operatorname{Log}[5] + 7 \operatorname{Log}[7]) / \operatorname{Log}\left[\frac{7}{5}\right]$	$(-5 \operatorname{Log}[5] + 8 \operatorname{Log}[8]) / \operatorname{Log}\left[\frac{8}{5}\right]$	$(-5 \operatorname{Log}[5] + 9 \operatorname{Log}[9]) / \operatorname{Log}\left[\frac{9}{5}\right]$	$(5 \operatorname{Log}[20] - \operatorname{Log}[2]) / \operatorname{Log}[2]$
6	$\operatorname{Log}\left[\frac{11664}{11}\right] / \operatorname{Log}[3]$	$\operatorname{Log}\left[\frac{1728}{1728}\right] / \operatorname{Log}[2]$	$\operatorname{Log}\left[\frac{729}{4}\right] / \operatorname{Log}\left[\frac{3}{2}\right]$	$(-5 \operatorname{Log}[5] + 6 \operatorname{Log}[6]) / \operatorname{Log}\left[\frac{6}{5}\right]$	$6 (1 + \operatorname{Log}[6])$	$(-6 \operatorname{Log}[6] + 7 \operatorname{Log}[7]) / \operatorname{Log}\left[\frac{7}{6}\right]$	$(-6 \operatorname{Log}[6] + 8 \operatorname{Log}[8]) / \operatorname{Log}\left[\frac{4}{3}\right]$	$(6 \operatorname{Log}\left[\frac{9}{2}\right] + \operatorname{Log}\left[\frac{3}{2}\right]) / \operatorname{Log}\left[\frac{5}{3}\right]$	$(-6 \operatorname{Log}[6] + 10 \operatorname{Log}[10]) / \operatorname{Log}\left[\frac{5}{3}\right]$
7	$\operatorname{Log}\left[\frac{823543}{4}\right] / \operatorname{Log}\left[\frac{7}{2}\right]$	$\operatorname{Log}\left[\frac{823543}{27}\right] / \operatorname{Log}\left[\frac{7}{3}\right]$	$\operatorname{Log}\left[\frac{823543}{256}\right] / \operatorname{Log}\left[\frac{7}{4}\right]$	$(-5 \operatorname{Log}[5] + 7 \operatorname{Log}[7]) / \operatorname{Log}\left[\frac{7}{5}\right]$	$(-6 \operatorname{Log}[6] + 7 \operatorname{Log}[7]) / \operatorname{Log}\left[\frac{7}{6}\right]$	$7 (1 + \operatorname{Log}[7])$	$(-7 \operatorname{Log}[7] + 8 \operatorname{Log}[8]) / \operatorname{Log}\left[\frac{8}{7}\right]$	$(-7 \operatorname{Log}[7] + 9 \operatorname{Log}[9]) / \operatorname{Log}\left[\frac{9}{7}\right]$	$(-7 \operatorname{Log}[7] + 10 \operatorname{Log}[10]) / \operatorname{Log}\left[\frac{10}{7}\right]$
8	11	$(3 \operatorname{Log}\left[\frac{256}{3}\right] + \operatorname{Log}\left[\frac{8}{3}\right]) / \operatorname{Log}\left[\frac{8}{3}\right]$	16	$(-5 \operatorname{Log}[5] + 8 \operatorname{Log}[8]) / \operatorname{Log}\left[\frac{8}{5}\right]$	$(-6 \operatorname{Log}[6] + 8 \operatorname{Log}[8]) / \operatorname{Log}\left[\frac{4}{3}\right]$	$(-7 \operatorname{Log}[7] + 8 \operatorname{Log}[8]) / \operatorname{Log}\left[\frac{8}{7}\right]$	$8 (1 + \operatorname{Log}[8])$	$(6 \operatorname{Log}\left[\frac{27}{16}\right] + \operatorname{Log}\left[\frac{9}{8}\right]) / \operatorname{Log}\left[\frac{9}{8}\right]$	$(2 \operatorname{Log}\left[\frac{3125}{128}\right] + \operatorname{Log}\left[\frac{5}{4}\right]) / \operatorname{Log}\left[\frac{5}{4}\right]$

$$\begin{aligned}
 & 9 \left( 2 \operatorname{Log}\left[\frac{196}{83}\right] / \operatorname{Log}\left[\frac{9}{2}\right] \right) / \operatorname{Log}\left[\frac{9}{2}\right] \\
 & 15 \operatorname{Log}\left[\frac{19683}{16}\right] / \operatorname{Log}\left[\frac{3}{2}\right] \\
 & (-5 \operatorname{Log}[5] + 9 \operatorname{Log}\left[\frac{9}{2}\right]) / \operatorname{Log}\left[\frac{9}{5}\right] \\
 & (6 \operatorname{Log}\left[\frac{9}{2}\right] / \operatorname{Log}\left[\frac{3}{2}\right] - 7 \operatorname{Log}[7] + 9 \operatorname{Log}\left[\frac{9}{2}\right]) / \operatorname{Log}\left[\frac{9}{7}\right] \\
 & (6 \operatorname{Log}\left[\frac{27}{16}\right] / \operatorname{Log}\left[\frac{9}{8}\right] - 9 (1 + \operatorname{Log}[9]) - 9 \operatorname{Log}[9] + 10 \operatorname{Log}[10]) / \operatorname{Log}\left[\frac{10}{9}\right] \\
 & 10 (10 + (8 \operatorname{Log}[2] / \operatorname{Log}[5]) / \operatorname{Log}\left[\frac{10}{3}\right] - (10 \operatorname{Log}[10] - 6250) / \operatorname{Log}\left[\frac{5}{2}\right] - (2 \operatorname{Log}[6250] / \operatorname{Log}[2] - 5 \operatorname{Log}[20]) / \operatorname{Log}[2] - 6 \operatorname{Log}[6] + 10 \operatorname{Log}[10]) / \operatorname{Log}\left[\frac{5}{3}\right] - 7 \operatorname{Log}[7] + 10 \operatorname{Log}[10]) / \operatorname{Log}\left[\frac{10}{7}\right] - (2 \operatorname{Log}\left[\frac{3125}{128}\right] / \operatorname{Log}\left[\frac{5}{4}\right] - 9 \operatorname{Log}[9] + 10 \operatorname{Log}[10]) / \operatorname{Log}\left[\frac{10}{9}\right] - 10 (1 + \operatorname{Log}[10]) / \operatorname{Log}[10])
 \end{aligned}$$

`mm[ma_, n_] := FullSimplify[Limit[ $\frac{m \operatorname{Log}[m] - n \operatorname{Log}[n]}{\operatorname{Log}[m] - \operatorname{Log}[n]}$ , m → ma]]`

`N@mm[2, 6]`

8.52372

`N@mm[3, 4]`

7.81884

`N[m^Log[m, n] /. {m → 12, n → 17}]`

17.

`N@Log[32, 64]`

1.2

**Integrate**[ 1, {x, 1, n}, {y, 1, m / (x^Log[n, m])}, {z, 1, o / (x^Log[n, o] y^Log[m, o])}]

**ConditionalExpression**[

$$\begin{aligned}
 & -\frac{n \operatorname{Log}[m] \operatorname{Log}[o]}{(\operatorname{Log}[m] - \operatorname{Log}[n]) (\operatorname{Log}[n] - \operatorname{Log}[o])} - \left( -(-1+m) \operatorname{Log}[n] (\operatorname{Log}[n] - \operatorname{Log}[o]) \operatorname{Log}[o] - \right. \\
 & \quad \left. \operatorname{Log}[m]^2 ((-1+o) \operatorname{Log}[n] + \operatorname{Log}[o]) + \operatorname{Log}[m] ((-1+o) \operatorname{Log}[n]^2 + \operatorname{Log}[o]^2) \right) / \\
 & \quad ((\operatorname{Log}[m] - \operatorname{Log}[n]) (\operatorname{Log}[m] - \operatorname{Log}[o]) (\operatorname{Log}[n] - \operatorname{Log}[o])), \\
 & (n \notin \text{Reals} \mid \mid \operatorname{Re}[n] \geq 0) \&\& \left( \operatorname{Im}[n] \operatorname{Re}\left[\frac{1}{-1+n}\right] + \operatorname{Im}\left[\frac{1}{-1+n}\right] (-1 + \operatorname{Re}[n]) \neq 0 \mid \mid \right. \\
 & \quad \left. \operatorname{Im}\left[\frac{1}{-1+n}\right] \operatorname{Im}[n] \geq \operatorname{Re}\left[\frac{1}{-1+n}\right] (-1 + \operatorname{Re}[n]) \mid \mid 1 + \operatorname{Im}\left[\frac{1}{-1+n}\right] \operatorname{Im}[n] \leq \operatorname{Re}\left[\frac{1}{-1+n}\right] (-1 + \operatorname{Re}[n]) \right) \&\& \\
 & \quad \left( (\operatorname{Re}[n] < 0 \&\& n \notin \text{Reals}) \mid \mid 0 \leq \operatorname{Re}[n] < 1 \mid \mid \left( \operatorname{Re}[n] = 1 \&\& \left( -\frac{1}{2} \leq \operatorname{Im}[n] < 0 \mid \mid \operatorname{Im}[n] > 0 \right) \right) \mid \mid \right. \\
 & \quad \left. \operatorname{Re}[n] > 1 \right) \&\& \left( \operatorname{Re}\left[\frac{(2+i) - 2 \operatorname{Re}[n]}{2 - 2n}\right] = 0 \mid \mid \operatorname{Re}\left[\frac{(2+i) - 2 \operatorname{Re}[n]}{2 - 2n}\right] = 1 \mid \mid \right. \\
 & \quad \left. \frac{(2+i) - 2 \operatorname{Re}[n]}{2 - 2n} \notin \text{Reals} \mid \mid \operatorname{Re}\left[\frac{(2+i) - 2 \operatorname{Re}[n]}{2 - 2n}\right] \geq 1 \mid \mid \operatorname{Re}\left[\frac{(2+i) - 2 \operatorname{Re}[n]}{2 - 2n}\right] \leq 0 \right) \&\& \\
 & \quad \left( \frac{(2+i) - 2 \operatorname{Re}[n]}{2 - 2n} \notin \text{Reals} \mid \mid \operatorname{Re}\left[\frac{(2+i) - 2 \operatorname{Re}[n]}{2 - 2n}\right] = 0 \mid \mid \operatorname{Re}\left[\frac{(2+i) - 2 \operatorname{Re}[n]}{2 - 2n}\right] \leq 0 \mid \mid \right. \\
 & \quad \left. \frac{(2+i) - 2 \operatorname{Re}[n]}{-1+n} \notin \text{Reals} \mid \mid \operatorname{Re}\left[\frac{(2+i) - 2 \operatorname{Re}[n]}{-1+n}\right] = -2 \mid \mid \operatorname{Re}\left[\frac{(2+i) - 2 \operatorname{Re}[n]}{-1+n}\right] \leq -2 \right) \Big]
 \end{aligned}$$

**FullSimplify**@

$$\begin{aligned}
 & \text{Expand}\left[ -\frac{n \operatorname{Log}[m] \operatorname{Log}[o]}{(\operatorname{Log}[m] - \operatorname{Log}[n]) (\operatorname{Log}[n] - \operatorname{Log}[o])} - \left( -(-1+m) \operatorname{Log}[n] (\operatorname{Log}[n] - \operatorname{Log}[o]) \operatorname{Log}[o] - \right. \right. \\
 & \quad \left. \left. \operatorname{Log}[m]^2 ((-1+o) \operatorname{Log}[n] + \operatorname{Log}[o]) + \operatorname{Log}[m] ((-1+o) \operatorname{Log}[n]^2 + \operatorname{Log}[o]^2) \right) / \right. \\
 & \quad \left. ((\operatorname{Log}[m] - \operatorname{Log}[n]) (\operatorname{Log}[m] - \operatorname{Log}[o]) (\operatorname{Log}[n] - \operatorname{Log}[o])) \right] \\
 & \quad \left( (-1+m) \operatorname{Log}[n] (\operatorname{Log}[n] - \operatorname{Log}[o]) \operatorname{Log}[o] + \right. \\
 & \quad \left. \operatorname{Log}[m]^2 ((-1+o) \operatorname{Log}[n] - (-1+n) \operatorname{Log}[o]) + \operatorname{Log}[m] ((-1+o) \operatorname{Log}[n]^2 + (-1+n) \operatorname{Log}[o]^2) \right) / \\
 & \quad ((\operatorname{Log}[m] - \operatorname{Log}[n]) (\operatorname{Log}[m] - \operatorname{Log}[o]) (\operatorname{Log}[n] - \operatorname{Log}[o])) \\
 & \frac{\operatorname{Log}[m] - n \operatorname{Log}[m] - \operatorname{Log}[n] + m \operatorname{Log}[n]}{\operatorname{Log}[m] - \operatorname{Log}[n]} /. m \rightarrow o \\
 & \frac{-\operatorname{Log}[n] + o \operatorname{Log}[n] + \operatorname{Log}[o] - n \operatorname{Log}[o]}{-\operatorname{Log}[n] + \operatorname{Log}[o]} \\
 & \frac{\operatorname{Log}[m] - n \operatorname{Log}[m] - \operatorname{Log}[n] + m \operatorname{Log}[n]}{\operatorname{Log}[m] - \operatorname{Log}[n]} /. n \rightarrow o \\
 & \frac{\operatorname{Log}[m] - o \operatorname{Log}[m] - \operatorname{Log}[o] + m \operatorname{Log}[o]}{\operatorname{Log}[m] - \operatorname{Log}[o]}
 \end{aligned}$$

$$\begin{aligned}
& 1 + (n-1) + (m-1) + (o-1) + \frac{\text{Log}[m] - n \text{Log}[m] - \text{Log}[n] + m \text{Log}[n]}{\text{Log}[m] - \text{Log}[n]} + \\
& \frac{-\text{Log}[n] + o \text{Log}[n] + \text{Log}[o] - n \text{Log}[o]}{-\text{Log}[n] + \text{Log}[o]} + \frac{\text{Log}[m] - o \text{Log}[m] - \text{Log}[o] + m \text{Log}[o]}{\text{Log}[m] - \text{Log}[o]} + \\
& \left( (-1+m) \text{Log}[n] (\text{Log}[n] - \text{Log}[o]) \text{Log}[o] + \right. \\
& \quad \left. \text{Log}[m]^2 ((-1+o) \text{Log}[n] - (-1+n) \text{Log}[o]) + \text{Log}[m] ((-1+o) \text{Log}[n]^2 + (-1+n) \text{Log}[o]^2) \right) / \\
& ((\text{Log}[m] - \text{Log}[n]) (\text{Log}[m] - \text{Log}[o]) (\text{Log}[n] - \text{Log}[o])) \\
& - 2 + m + n + o + \frac{\text{Log}[m] - n \text{Log}[m] - \text{Log}[n] + m \text{Log}[n]}{\text{Log}[m] - \text{Log}[n]} + \\
& \frac{\text{Log}[m] - o \text{Log}[m] - \text{Log}[o] + m \text{Log}[o]}{\text{Log}[m] - \text{Log}[o]} + \frac{-\text{Log}[n] + o \text{Log}[n] + \text{Log}[o] - n \text{Log}[o]}{-\text{Log}[n] + \text{Log}[o]} + \\
& \left( (-1+m) \text{Log}[n] (\text{Log}[n] - \text{Log}[o]) \text{Log}[o] + \text{Log}[m]^2 ((-1+o) \text{Log}[n] - (-1+n) \text{Log}[o]) + \right. \\
& \quad \left. \text{Log}[m] ((1-o) \text{Log}[n]^2 + (-1+n) \text{Log}[o]^2) \right) / \\
& ((\text{Log}[m] - \text{Log}[n]) (\text{Log}[m] - \text{Log}[o]) (\text{Log}[n] - \text{Log}[o]))
\end{aligned}$$

FullSimplify@

$$\begin{aligned}
& \text{Expand} \left[ -2 + m + n + o + \frac{\text{Log}[m] - n \text{Log}[m] - \text{Log}[n] + m \text{Log}[n]}{\text{Log}[m] - \text{Log}[n]} + \frac{\text{Log}[m] - o \text{Log}[m] - \text{Log}[o] + m \text{Log}[o]}{\text{Log}[m] - \text{Log}[o]} + \right. \\
& \quad \frac{-\text{Log}[n] + o \text{Log}[n] + \text{Log}[o] - n \text{Log}[o]}{-\text{Log}[n] + \text{Log}[o]} + \left. \left( (-1+m) \text{Log}[n] (\text{Log}[n] - \text{Log}[o]) \text{Log}[o] + \right. \right. \\
& \quad \left. \left. \text{Log}[m]^2 ((-1+o) \text{Log}[n] - (-1+n) \text{Log}[o]) + \text{Log}[m] ((1-o) \text{Log}[n]^2 + (-1+n) \text{Log}[o]^2) \right) \right] / \\
& ((\text{Log}[m] - \text{Log}[n]) (\text{Log}[m] - \text{Log}[o]) (\text{Log}[n] - \text{Log}[o]))
\end{aligned}$$

$$\begin{aligned}
& \frac{(m-o) \text{Log}[m]^2}{\text{Log}[m] - \text{Log}[o]} + \frac{(-n+o) \text{Log}[n]^2}{\text{Log}[n] - \text{Log}[o]} \\
& o + \frac{\text{Log}[m] - \text{Log}[n]}{\text{Log}[m] - \text{Log}[n]}
\end{aligned}$$

$$\text{Limit} \left[ o + \frac{\frac{(m-o) \text{Log}[m]^2}{\text{Log}[m] - \text{Log}[o]} + \frac{(-n+o) \text{Log}[n]^2}{\text{Log}[n] - \text{Log}[o]}}{\text{Log}[m] - \text{Log}[n]}, m \rightarrow n \right]$$

$$\frac{n \text{Log}[n]^3 - 2 n \text{Log}[n] \text{Log}[o] + o \text{Log}[o]^2 + \text{Log}[n]^2 (n - n \text{Log}[o])}{(\text{Log}[n] - \text{Log}[o])^2}$$

$$\text{Limit} \left[ \frac{n \text{Log}[n]^3 - 2 n \text{Log}[n] \text{Log}[o] + o \text{Log}[o]^2 + \text{Log}[n]^2 (n - n \text{Log}[o])}{(\text{Log}[n] - \text{Log}[o])^2}, o \rightarrow n \right]$$

$$\text{Expand} \left[ \frac{1}{2} n (2 + 4 \text{Log}[n] + \text{Log}[n]^2) \right]$$

$$n + 2 n \text{Log}[n] + \frac{1}{2} n \text{Log}[n]^2$$

$$\text{N} \left[ n + 2 n \text{Log}[n] + \frac{1}{2} n \text{Log}[n]^2 /. n \rightarrow 33 \right]$$

465.491

LaguerreL[-3, Log[33.]]

465.491

$$(-a n^{(1/a)} / (a - m) - m n^{(1/m)} / (m - a) + 1) / . a \rightarrow 1$$

$$1 - \frac{n}{1-m} - \frac{m n^{\frac{1}{m}}}{-1+m}$$

$$\text{Expand}[1 + (x-1) + (y-1) + (z-1) + (x-1)(y-1) + (x-1)(z-1) + (y-1)(z-1) + (x-1)(y-1)(z-1)]$$

$$x y z$$

$$o + \frac{\frac{(m-o) \text{Log}[m]^2}{\text{Log}[m]-\text{Log}[o]} + \frac{(-n+o) \text{Log}[n]^2}{\text{Log}[n]-\text{Log}[o]}}{\text{Log}[m] - \text{Log}[n]}$$

$$\text{Expand}\left[o + \frac{\frac{(m-o) \text{Log}[m]^2}{\text{Log}[m]-\text{Log}[o]} + \frac{(-n+o) \text{Log}[n]^2}{\text{Log}[n]-\text{Log}[o]}}{\text{Log}[m] - \text{Log}[n]}\right]$$

$$o + \frac{\frac{(m-o) \text{Log}[m]^2}{\text{Log}[m]-\text{Log}[o]} + \frac{(-n+o) \text{Log}[n]^2}{\text{Log}[n]-\text{Log}[o]}}{\text{Log}[m] - \text{Log}[n]}$$

$$o + \frac{m \text{Log}[m]^2}{(\text{Log}[m] - \text{Log}[n]) (\text{Log}[m] - \text{Log}[o])} - \frac{o \text{Log}[m]^2}{(\text{Log}[m] - \text{Log}[n]) (\text{Log}[m] - \text{Log}[o])} - \frac{n \text{Log}[n]^2}{(\text{Log}[m] - \text{Log}[n]) (\text{Log}[n] - \text{Log}[o])} + \frac{o \text{Log}[n]^2}{(\text{Log}[m] - \text{Log}[n]) (\text{Log}[n] - \text{Log}[o])}$$

$$\text{mul3}[n_, m2_, o2_] :=$$

$$\text{Limit}\left[\text{Limit}\left[o + \frac{m \text{Log}[m]^2}{(\text{Log}[m] - \text{Log}[n]) (\text{Log}[m] - \text{Log}[o])} - \frac{o \text{Log}[m]^2}{(\text{Log}[m] - \text{Log}[n]) (\text{Log}[m] - \text{Log}[o])} - \frac{n \text{Log}[n]^2}{(\text{Log}[m] - \text{Log}[n]) (\text{Log}[n] - \text{Log}[o])} + \frac{o \text{Log}[n]^2}{(\text{Log}[m] - \text{Log}[n]) (\text{Log}[n] - \text{Log}[o])}, m \rightarrow m2\right], o \rightarrow o2\right]$$

$$\text{mm}[ma_, n_] := \text{FullSimplify}\left[\text{Limit}\left[\frac{m \text{Log}[m] - n \text{Log}[n]}{\text{Log}[m] - \text{Log}[n]}, m \rightarrow ma\right]\right]$$

$$\text{N@mul3}[3, 3, 3]$$

$$11.4021$$

$$\text{N@mm}[3, 3]$$

$$6.29584$$

$$\text{mm}[3, 6.295836866004329`]$$

$$11.1804$$

$$\text{N@mm}[3, \text{mm}[3, 3]]$$

$$11.1804$$

$$\text{D}[1[n, -1], n]$$

$$-\frac{1}{n}$$

```

Expand[1 + Integrate[-1 / j, {j, 1, m}] + (n - 1) +
  Integrate[-1 / k, {j, 1, n}, {k, 1, (n / j)^(1 / Log[m, n])}]]
ConditionalExpression[n +  $\frac{\text{Log}[m]}{\text{Log}[n]}$  -  $\frac{n \text{Log}[m]}{\text{Log}[n]}$ , (Re[m] ≥ 0 || m ∈ Reals) && (Re[n] ≥ 0 || n ∈ Reals)]
dv[n_, m_] := n +  $\frac{\text{Log}[m]}{\text{Log}[n]}$  -  $\frac{n \text{Log}[m]}{\text{Log}[n]}$ 
mm[3, 5]

$$\frac{\text{Log}\left[\frac{3125}{27}\right]}{\text{Log}\left[\frac{5}{3}\right]}$$

N[dv[ $\frac{\text{Log}\left[\frac{3125}{27}\right]}{\text{Log}\left[\frac{5}{3}\right]}$ , 5]]
3.3105
Expand[1 + Integrate[D[1[j, -1], j], {j, 1, m}] + (n - 1) +
  Integrate[D[1[k, -1], k], {j, 1, n}, {k, 1, (n / j)^(1 / Log[m, n])}]]
ConditionalExpression[n +  $\frac{\text{Log}[m]}{\text{Log}[n]}$  -  $\frac{n \text{Log}[m]}{\text{Log}[n]}$ , (Re[m] ≥ 0 || m ∈ Reals) && (Re[n] ≥ 0 || n ∈ Reals)]
FullSimplify[n +  $\frac{\text{Log}[m]}{\text{Log}[n]}$  -  $\frac{n \text{Log}[m]}{\text{Log}[n]}$ ]
n -  $\frac{(-1 + n) \text{Log}[m]}{\text{Log}[n]}$ 
Grid[Table[dv[n, m], {n, 1, 10}, {m, 1, 10}]]
Power::infy: Infinite expression  $\frac{1}{0}$  encountered. >>
Infinity::indet: Indeterminate expression 0 ComplexInfinity encountered. >>
Power::infy: Infinite expression  $\frac{1}{0}$  encountered. >>
Infinity::indet: Indeterminate expression 0 ComplexInfinity encountered. >>
Power::infy: Infinite expression  $\frac{1}{0}$  encountered. >>
General::stop: Further output of Power::infy will be suppressed during this calculation. >>
Infinity::indet: Indeterminate expression 1 + ComplexInfinity + ComplexInfinity encountered. >>
General::stop: Further output of Infinity::indet will be suppressed during this calculation. >>

```



Indete\ rmina\ te	Indete\ rmina\ te	Indete\ rmina\ te	Indete\ rmina\ te	Indete\ rmina\ te	Indete\ rmina\ te	Indete\ rmina\ te	Indete\ rmina\ te	Indete\ rmina\ te	Indete\ rmina\ te
2	1	2 - Log[ 3] / Log[ 2]	2 - Log[ 4] / Log[ 2]	2 - Log[ 5] / Log[ 2]	2 - Log[ 6] / Log[ 2]	2 - Log[ 7] / Log[ 2]	2 - Log[ 8] / Log[ 2]	2 - Log[ 9] / Log[ 2]	2 - Log[ 10] / Log[ 2]
3	3 - (2 Log[ 2]) / Log[ 3]	1	3 - (2 Log[ 4]) / Log[ 3]	3 - (2 Log[ 5]) / Log[ 3]	3 - (2 Log[ 6]) / Log[ 3]	3 - (2 Log[ 7]) / Log[ 3]	3 - (2 Log[ 8]) / Log[ 3]	3 - (2 Log[ 9]) / Log[ 3]	3 - (2 Log[ 10]) / Log[ 3]
4	4 - (3 Log[ 2]) / Log[ 4]	4 - (3 Log[ 3]) / Log[ 4]	1	4 - (3 Log[ 5]) / Log[ 4]	4 - (3 Log[ 6]) / Log[ 4]	4 - (3 Log[ 7]) / Log[ 4]	4 - (3 Log[ 8]) / Log[ 4]	4 - (3 Log[ 9]) / Log[ 4]	4 - (3 Log[ 10]) / Log[ 4]
5	5 - (4 Log[ 2]) / Log[ 5]	5 - (4 Log[ 3]) / Log[ 5]	5 - (4 Log[ 4]) / Log[ 5]	1	5 - (4 Log[ 6]) / Log[ 5]	5 - (4 Log[ 7]) / Log[ 5]	5 - (4 Log[ 8]) / Log[ 5]	5 - (4 Log[ 9]) / Log[ 5]	5 - (4 Log[ 10]) / Log[ 5]
6	6 - (5 Log[ 2]) / Log[ 6]	6 - (5 Log[ 3]) / Log[ 6]	6 - (5 Log[ 4]) / Log[ 6]	6 - (5 Log[ 5]) / Log[ 6]	1	6 - (5 Log[ 7]) / Log[ 6]	6 - (5 Log[ 8]) / Log[ 6]	6 - (5 Log[ 9]) / Log[ 6]	6 - (5 Log[ 10]) / Log[ 6]
7	7 - (6 Log[ 2]) / Log[ 7]	7 - (6 Log[ 3]) / Log[ 7]	7 - (6 Log[ 4]) / Log[ 7]	7 - (6 Log[ 5]) / Log[ 7]	7 - (6 Log[ 6]) / Log[ 7]	1	7 - (6 Log[ 8]) / Log[ 7]	7 - (6 Log[ 9]) / Log[ 7]	7 - (6 Log[ 10]) / Log[ 7]
8	8 - (7 Log[ 2]) / Log[ 8]	8 - (7 Log[ 3]) / Log[ 8]	8 - (7 Log[ 4]) / Log[ 8]	8 - (7 Log[ 5]) / Log[ 8]	8 - (7 Log[ 6]) / Log[ 8]	8 - (7 Log[ 7]) / Log[ 8]	1	8 - (7 Log[ 9]) / Log[ 8]	8 - (7 Log[ 10]) / Log[ 8]
9	9 - (8 Log[ 2]) / Log[ 9]	9 - (8 Log[ 3]) / Log[ 9]	9 - (8 Log[ 4]) / Log[ 9]	9 - (8 Log[ 5]) / Log[ 9]	9 - (8 Log[ 6]) / Log[ 9]	9 - (8 Log[ 7]) / Log[ 9]	9 - (8 Log[ 8]) / Log[ 9]	1	9 - (8 Log[ 10]) / Log[ 9]
10	10 - (9 Log[ 2]) / Log[ 10]	10 - (9 Log[ 3]) / Log[ 10]	10 - (9 Log[ 4]) / Log[ 10]	10 - (9 Log[ 5]) / Log[ 10]	10 - (9 Log[ 6]) / Log[ 10]	10 - (9 Log[ 7]) / Log[ 10]	10 - (9 Log[ 8]) / Log[ 10]	10 - (9 Log[ 9]) / Log[ 10]	1

```

fg[n_, aa_] :=
  1 - Sum[ Limit[ (c - a) c^ (Length[aa] - 1) n^ (1 / c) / Product[ c - b, {b, aa}], c → a], {a, aa}]
fg[100, {1, 2, 3}]

- 9 -  $\frac{9 \times 5^{2/3}}{2^{1/3}}$ 

m1[a_] :=  $n^{\frac{1}{a}} - 1$ 

m2[a_, b_] :=  $1 - \frac{a n^{\frac{1}{a}}}{(a - b)} - \frac{b n^{\frac{1}{b}}}{(-a + b)}$ 

m3[a_, b_, c_] :=  $1 - \frac{a^2 n^{\frac{1}{a}}}{(a - b)(a - c)} - \frac{b^2 n^{\frac{1}{b}}}{(-a + b)(b - c)} - \frac{c^2 n^{\frac{1}{c}}}{(-a + c)(-b + c)}$ 

m4[a_, b_, c_, d_] :=  $1 - \frac{a^3 n^{\frac{1}{a}}}{(a - b)(a - c)(a - d)} -$ 
 $\frac{b^3 n^{\frac{1}{b}}}{(-a + b)(b - c)(b - d)} - \frac{c^3 n^{\frac{1}{c}}}{(-a + c)(-b + c)(c - d)} - \frac{d^3 n^{\frac{1}{d}}}{(-a + d)(-b + d)(-c + d)}$ 

m1[2]

- 1 +  $\sqrt{n}$ 

FullSimplify[Expand[Limit[FullSimplify[1 + m1[a] + m1[b] + m2[a, b]], a → n]]] /. n → a

 $\frac{-a^{1+\frac{1}{b}} + a^{\frac{1}{a}} b}{-a + b}$ 

```