```
ClearAll["Global`*"]

Dhyp[n_, k_, a_] := Dhyp[n, k, a] =
   Sum[Binomial[k, j] Dhyp[n / (m^(k - j)), j, m + 1], {m, a, n^(1 / k)}, {j, 0, k - 1}]
Dhyp[n_, 1, a_] := Floor[n] - a + 1; Dhyp[n_, 0, a_] := 1
DA[n_, k_, a_] := Sum[ DA[n / j, k - 1, a], {j, a, n}]; DA[n_, 0, a_] := 1

da[n_, k_, c_] := c Sum[da[n / (j c), k - 1, c], {j, 1 + 1 / c, n / c^k}]; da[n_, 0, c_] := 1
dc[n_, k_, c_] := c^-k Dhyp[n c^k, k, c + 1]
de[n_, k_, c_] := c^-k DA[n c^k, k, c + 1]

df[n_, k_, c_] := c^-k Sum[ DA[n c^k j^-1, k - 1, c + 1], {j, c + 1, n c^k}]
db[n_, k_, c_] := c^-1 Sum[db[n c^1 j^-1, k - 1, c],    {j, c + 1, n c^k}];
db[n_, 0, c_] := 1
dbb[n_, k_, c_] := Sum[dbb[n c^1 j^-1, k - 1, c],    {j, c + 1, n c^k}]; dbb[n_, 0, c_] := 1

$RecursionLimit = 1 000 000
Dh[n_, k_, a_] := Dh[n, k, a] =
   If[n < a^k, 0, Sum[Binomial[k, j] Dh[n / a^j, k - j, a + 1], {j, 0, k}]]; Dh[n_, 0, a_] := 1
dhl[n_, k_, c_]  := c^-k Dh[n c^k, k, c + 1]



1 000 000

{db[nn = 200, kk = 2, cc = 4], da[nn, kk, 1 / cc],
 dc[nn, kk, cc], de[nn, kk, cc], df[nn, kk, cc], dg[nn, kk, cc]}

{6513/8, 6513/8, 6513/8, 6513/8, 6513/8, 328167/256}

di[n_, k_, c_] :=
 Sum[ (c^-1 db[n c^1 j^-1, k - 1, c]) - (c^-k DA[n c^k j^-1, k - 1, c + 1]), {j, c + 1, n c^k}]

di[200, 3, 4]

0

di2[n_, k_, c_, j_] := (c^-1 db[n c^1 j^-1, k, c]) - (c^-(k+1) DA[n c^(k+1) j^-1, k, c + 1])

di2[200, 2, 2, 4]

0

di3[n_, k_, c_] := (c^-1 db[n c^1 , k, c]) - (c^-(k+1) DA[n c^(k+1), k, c + 1])

di3[120, 3, 2]

0

di4[n_, k_, c_] := (db[n, k, c]) - (c^-k DA[n c^k, k, c + 1])

di4[160, 4, 2]

0

di5[n_, k_, c_] := (c^-2 db[n c^2 , k, c]) - (c^-(k+2) DA[n c^(k+2), k, c + 1])

di5[60, 3, 2]

0
```

```
di6[n_, k_, c_, a_] := (c^-a db[n c^a , k, c]) - (c^-(k + a) DA[n c^(k + a), k, c + 1])

di6[160, 3, 2, -3]

0

di7[n_, k_, c_] := (c^k db[n c^-k , k, c]) - DA[n, k, c + 1]

di7[160, 3, 2]

0

di8[n_, k_, c_] := (c^k db[n c^-k , k, c])

di8[100, 2, 1]

283

di9[n_, k_, c_] := ((c - 1)^k db[n (c - 1)^-k , k, (c - 1)])

di9[100, 2, 2]

283

Dhyp[100, 2, 3]

186

N[4 Sum[ 1, {j, 2, (100 / 4)}, {k, 2, ((100) / (2 j)) / 2}]]

152.

di9[100, 2, 3]

186

4 db[100 / 4 , 2, 2]

186

db[100 , 2, 1]

283

9 db[100 / 9 , 2, 3]

125

Dhyp[100, 2, 4]

125

db2[n_, c_] := Sum[ 1, {j, 1 + c, n c^2}, {k, 1 + c, (c n / j) c}]

db2[25, 2]

186

db2[100 / (3^2), 3]

125

db2[100 / (s^2), s] /. s → 2

186

dbb[100 / 9 , 2, 3]

125
```

N[db[30, 2, 50]]

72.4572

N[Gamma[2, 0, -Log[100]] / Gamma[2]]

$361.517 - 4.41506 \times 10^{-14}$ i

1 000 000

1 000 000

N[dhl[100, 2, 30]]

358.271

dha[n_, k_, a_] := Sum[(-1)^j Binomial[k, j] dha[n / (a - 1)^j, k - j, a - 1], {j, 0, k}];
dha[n_, 0, a_] := 1
dha[n_, k_, 2] := d2[n, k]
dhla[n_, k_, c_] := c^-k dha[n c^k, k, c + 1]

dha[n, 2, 5]

$3 - 2\left(-2 + d2\left[\frac{n}{4}, 1\right]\right) - 2\left(-1 + d2\left[\frac{n}{3}, 1\right]\right) - 2\,d2\left[\frac{n}{2}, 1\right] + d2[n, 2]$

Table[{j, dhla[n, 2, j]}, {j, 1, 10}] // TableForm

1   d2[n, 2]

2    $\frac{1}{4}$ (1 - 2 d2[2 n, 1] + d2[4 n, 2])

3    $\frac{1}{9}\left(2 - 2(-1 + d2[3\,n, 1]) - 2\,d2\left[\frac{9\,n}{2}, 1\right] + d2[9\,n, 2]\right)$

4    $\frac{1}{16}\left(3 - 2(-2 + d2[4\,n, 1]) - 2\left(-1 + d2\left[\frac{16\,n}{3}, 1\right]\right) - 2\,d2[8\,n, 1] + d2[16\,n, 2]\right)$

5    $\frac{1}{25}\left(4 - 2(-3 + d2[5\,n, 1]) - 2\left(-2 + d2\left[\frac{25\,n}{4}, 1\right]\right) - 2\left(-1 + d2\left[\frac{25\,n}{3}, 1\right]\right) - 2\,d2\left[\frac{25\,n}{2}, 1\right] + d2[25\,n, 2]\right.$

6    $\frac{1}{36}\left(5 - 2(-4 + d2[6\,n, 1]) - 2\left(-3 + d2\left[\frac{36\,n}{5}, 1\right]\right) - 2(-2 + d2[9\,n, 1]) - 2(-1 + d2[12\,n, 1]) - 2\,d2[\right.$

7    $\frac{1}{49}\left(6 - 2(-5 + d2[7\,n, 1]) - 2\left(-4 + d2\left[\frac{49\,n}{6}, 1\right]\right) - 2\left(-3 + d2\left[\frac{49\,n}{5}, 1\right]\right) - 2\left(-2 + d2\left[\frac{49\,n}{4}, 1\right]\right) - 2\left(-\right.\right.$

8    $\frac{1}{64}\left(7 - 2(-6 + d2[8\,n, 1]) - 2\left(-5 + d2\left[\frac{64\,n}{7}, 1\right]\right) - 2\left(-4 + d2\left[\frac{32\,n}{3}, 1\right]\right) - 2\left(-3 + d2\left[\frac{64\,n}{5}, 1\right]\right) - 2(-\right.$

9    $\frac{1}{81}\left(8 - 2(-7 + d2[9\,n, 1]) - 2\left(-6 + d2\left[\frac{81\,n}{8}, 1\right]\right) - 2\left(-5 + d2\left[\frac{81\,n}{7}, 1\right]\right) - 2\left(-4 + d2\left[\frac{27\,n}{2}, 1\right]\right) - 2(-\right.$

10    $\frac{1}{100}\left(9 - 2(-8 + d2[10\,n, 1]) - 2\left(-7 + d2\left[\frac{100\,n}{9}, 1\right]\right) - 2\left(-6 + d2\left[\frac{25\,n}{2}, 1\right]\right) - 2\left(-5 + d2\left[\frac{100\,n}{7}, 1\right]\right) - 2\right.$

dha1[n_, k_, a_] := Sum[(-1)^j Binomial[k, j] dha1[n / (a - 1)^j, k - j, a - 1], {j, 0, k}];
dha1[n_, 0, a_] := 1
dha1[n_, k_, 1] := d1[n, k]
dhla1[n_, k_, c_] := c^-k dha1[n c^k, k, c + 1]

```
Table[{j, Expand[dhla1[n, 2, j]]}, {j, 1, 10}] // TableForm
```

1    $1 - 2\,d1[n, 1] + d1[n, 2]$

2    $1 - \frac{1}{2}\,d1[2\,n, 1] - \frac{1}{2}\,d1[4\,n, 1] + \frac{1}{4}\,d1[4\,n, 2]$

3    $1 - \frac{2}{9}\,d1[3\,n, 1] - \frac{2}{9}\,d1\left[\frac{9\,n}{2}, 1\right] - \frac{2}{9}\,d1[9\,n, 1] + \frac{1}{9}\,d1[9\,n, 2]$

4    $1 - \frac{1}{8}\,d1[4\,n, 1] - \frac{1}{8}\,d1\left[\frac{16\,n}{3}, 1\right] - \frac{1}{8}\,d1[8\,n, 1] - \frac{1}{8}\,d1[16\,n, 1] + \frac{1}{16}\,d1[16\,n, 2]$

5    $1 - \frac{2}{25}\,d1[5\,n, 1] - \frac{2}{25}\,d1\left[\frac{25\,n}{4}, 1\right] - \frac{2}{25}\,d1\left[\frac{25\,n}{3}, 1\right] - \frac{2}{25}\,d1\left[\frac{25\,n}{2}, 1\right] - \frac{2}{25}\,d1[25\,n, 1] + \frac{1}{25}\,d1[25\,n, 2]$

6    $1 - \frac{1}{18}\,d1[6\,n, 1] - \frac{1}{18}\,d1\left[\frac{36\,n}{5}, 1\right] - \frac{1}{18}\,d1[9\,n, 1] - \frac{1}{18}\,d1[12\,n, 1] - \frac{1}{18}\,d1[18\,n, 1] - \frac{1}{18}\,d1[36\,n, 1]$

7    $1 - \frac{2}{49}\,d1[7\,n, 1] - \frac{2}{49}\,d1\left[\frac{49\,n}{6}, 1\right] - \frac{2}{49}\,d1\left[\frac{49\,n}{5}, 1\right] - \frac{2}{49}\,d1\left[\frac{49\,n}{4}, 1\right] - \frac{2}{49}\,d1\left[\frac{49\,n}{3}, 1\right] - \frac{2}{49}\,d1\left[\frac{49\,n}{2}, 1\right] -$

8    $1 - \frac{1}{32}\,d1[8\,n, 1] - \frac{1}{32}\,d1\left[\frac{64\,n}{7}, 1\right] - \frac{1}{32}\,d1\left[\frac{32\,n}{3}, 1\right] - \frac{1}{32}\,d1\left[\frac{64\,n}{5}, 1\right] - \frac{1}{32}\,d1[16\,n, 1] - \frac{1}{32}\,d1\left[\frac{64\,n}{3}, 1\right] -$

9    $1 - \frac{2}{81}\,d1[9\,n, 1] - \frac{2}{81}\,d1\left[\frac{81\,n}{8}, 1\right] - \frac{2}{81}\,d1\left[\frac{81\,n}{7}, 1\right] - \frac{2}{81}\,d1\left[\frac{27\,n}{2}, 1\right] - \frac{2}{81}\,d1\left[\frac{81\,n}{5}, 1\right] - \frac{2}{81}\,d1\left[\frac{81\,n}{4}, 1\right] -$

10   $1 - \frac{1}{50}\,d1[10\,n, 1] - \frac{1}{50}\,d1\left[\frac{100\,n}{9}, 1\right] - \frac{1}{50}\,d1\left[\frac{25\,n}{2}, 1\right] - \frac{1}{50}\,d1\left[\frac{100\,n}{7}, 1\right] - \frac{1}{50}\,d1\left[\frac{50\,n}{3}, 1\right] - \frac{1}{50}\,d1[20\,n, $

```
dha1a[n_, k_, a_] := Sum[(-1)^j Binomial[k, j] dha1a[n / (a - 1)^j, k - j, a - 1], {j, 0, k}];
dha1a[n_, 0, a_] := 1
dha1a[n_, k_, 1] := Dhyp[n, k, 1]
dhla1a[n_, k_, c_] := c^-k dha1a[n c^k, k, c + 1]


N[dhla1a[100, 2, 4]]
```

338.625

```
-N[Gamma[3, 0, -Log[100]] / Gamma[3]]
```

$698.863 - 1.71417 \times 10^{-13}\,\mathbb{i}$

```
$RecursionLimit = 1 000 000
Dh[n_, k_, a_] := Dh[n, k, a] =
   If[n < a^k, 0, Sum[Binomial[k, j] Dh[n / a^j, k - j, a + 1], {j, 0, k}]]; Dh[n_, 0, a_] := 1
dhl[n_, k_, c_] := c^-k Dh[n c^k, k, c + 1]


N[Dh[100, 2, 4]]
```

125.

```
DH1[n_, k_, a_] := Sum[(-1)^j Binomial[k, j] Dh[n / ((a - 1)^j), k - j, a - 1], {j, 0, k}]

DH1[100, 2, 5]
```

82

```
Dhyp[100, 2, 5]
```

82

```
ap[n_, c_] := c^-2 ( Dhyp[ c^2 n, 2, 1] - 2 Sum[ Dhyp[c^2 n j^-1, 1, 1] - j + 1, {j, 1, c}] + c )
ap2[n_, c_] := c^-2 ( d1[ c^2 n, 2] - 2 Sum[ d1[c^2 n j^-1, 1] - j + 1, {j, 1, c}] + c )
ap3[n_, c_] := c^-2 ( d1[ c^2 n, 2] - 2 Sum[ d1[c^2 n j^-1, 1], {j, 1, c}] + c^2 )


N[ap[100, 4]]
```

338.625

```
d11[n_, k_] := Dhyp[n, k, 1]
```

$$N\left[\frac{1}{16}\left(4 - 2\,(-3 + d1[4\,n, 1]) - 2\left(-2 + d1\left[\frac{16\,n}{3}, 1\right]\right) - 2\,(-1 + d1[8\,n, 1]) - 2\,d1[16\,n, 1] + d1[16\,n, 2]\right) /.\ \{d1 \to d11, n \to 100\}\right]$$

338.625

```
Expand[ap2[100, 10]]
```

$$1 - \frac{1}{50}\,d1[1000, 1] - \frac{1}{50}\,d1\left[\frac{10\,000}{9}, 1\right] - \frac{1}{50}\,d1[1250, 1] -$$
$$\frac{1}{50}\,d1\left[\frac{10\,000}{7}, 1\right] - \frac{1}{50}\,d1\left[\frac{5000}{3}, 1\right] - \frac{1}{50}\,d1[2000, 1] - \frac{1}{50}\,d1[2500, 1] -$$
$$\frac{1}{50}\,d1\left[\frac{10\,000}{3}, 1\right] - \frac{1}{50}\,d1[5000, 1] - \frac{1}{50}\,d1[10\,000, 1] + \frac{1}{100}\,d1[10\,000, 2]$$

$$Expand\left[\frac{1}{16}\left(4 - 2\,(-3 + d1[4\,n, 1]) - 2\left(-2 + d1\left[\frac{16\,n}{3}, 1\right]\right) - 2\,(-1 + d1[8\,n, 1]) - 2\,d1[16\,n, 1] + d1[16\,n, 2]\right) /.\ \{n \to 100\}\right]$$

$$1 - \frac{1}{8}\,d1[400, 1] - \frac{1}{8}\,d1\left[\frac{1600}{3}, 1\right] - \frac{1}{8}\,d1[800, 1] - \frac{1}{8}\,d1[1600, 1] + \frac{1}{16}\,d1[1600, 2]$$

```
Expand[ap3[100, 4]]
```

$$1 - \frac{1}{8}\,d1[400, 1] - \frac{1}{8}\,d1\left[\frac{1600}{3}, 1\right] - \frac{1}{8}\,d1[800, 1] - \frac{1}{8}\,d1[1600, 1] + \frac{1}{16}\,d1[1600, 2]$$

```
N[ap[100, 100]]
```

360.5354`

```
ap4[n_, c_] := c^-2 (Dhyp[c^2 n, 2, 1]) - 2 c^-2 Sum[Dhyp[c^2 n j^-1, 1, 1], {j, 1, c}] + 1
N[ap4[100, 10]]
```

351.92

```
ap5[n_, c_] := c^-2 (Dhyp[c^2 n, 2, 1]) - 2 c^-2 Sum[Floor[c^2 n j^-1], {j, 1, c}] + 1
N[ap5[100^2, 60]]
```

81 938.3

```
N[Gamma[2, 0, -2 Log[100]] / Gamma[2]]
```

$82\,104.4 - 1.00548 \times 10^{-11}\,\mathbb{i}$

```
dha[n_, k_, a_] := Sum[(-1)^j Binomial[k, j] dha[n / (a - 1)^j, k - j, a - 1], {j, 0, k}];
dha[n_, 0, a_] := 1
dha[n_, k_, 2] := d2[n, k]
dhla[n_, k_, c_] := c^-k dha[n c^k, k, c + 1]
```

```
Table[{j, Expand[dhla[n, 2, j]]}, {j, 1, 10}] // TableForm
```

1   d2[n, 2]

2     $\frac{1}{4} - \frac{1}{2}$ d2[2 n, 1] + $\frac{1}{4}$ d2[4 n, 2]

3     $\frac{4}{9} - \frac{2}{9}$ d2[3 n, 1] $- \frac{2}{9}$ d2$\left[\frac{9 n}{2}, 1\right]$ + $\frac{1}{9}$ d2[9 n, 2]

4     $\frac{9}{16} - \frac{1}{8}$ d2[4 n, 1] $- \frac{1}{8}$ d2$\left[\frac{16 n}{3}, 1\right]$ $- \frac{1}{8}$ d2[8 n, 1] + $\frac{1}{16}$ d2[16 n, 2]

5     $\frac{16}{25} - \frac{2}{25}$ d2[5 n, 1] $- \frac{2}{25}$ d2$\left[\frac{25 n}{4}, 1\right]$ $- \frac{2}{25}$ d2$\left[\frac{25 n}{3}, 1\right]$ $- \frac{2}{25}$ d2$\left[\frac{25 n}{2}, 1\right]$ + $\frac{1}{25}$ d2[25 n, 2]

6     $\frac{25}{36} - \frac{1}{18}$ d2[6 n, 1] $- \frac{1}{18}$ d2$\left[\frac{36 n}{5}, 1\right]$ $- \frac{1}{18}$ d2[9 n, 1] $- \frac{1}{18}$ d2[12 n, 1] $- \frac{1}{18}$ d2[18 n, 1] + $\frac{1}{36}$ d2[36 n, 2]

7     $\frac{36}{49} - \frac{2}{49}$ d2[7 n, 1] $- \frac{2}{49}$ d2$\left[\frac{49 n}{6}, 1\right]$ $- \frac{2}{49}$ d2$\left[\frac{49 n}{5}, 1\right]$ $- \frac{2}{49}$ d2$\left[\frac{49 n}{4}, 1\right]$ $- \frac{2}{49}$ d2$\left[\frac{49 n}{3}, 1\right]$ $- \frac{2}{49}$ d2$\left[\frac{49 n}{2}, 1\right]$

8     $\frac{49}{64} - \frac{1}{32}$ d2[8 n, 1] $- \frac{1}{32}$ d2$\left[\frac{64 n}{7}, 1\right]$ $- \frac{1}{32}$ d2$\left[\frac{32 n}{3}, 1\right]$ $- \frac{1}{32}$ d2$\left[\frac{64 n}{5}, 1\right]$ $- \frac{1}{32}$ d2[16 n, 1] $- \frac{1}{32}$ d2$\left[\frac{64 n}{3}, 1\right]$

9     $\frac{64}{81} - \frac{2}{81}$ d2[9 n, 1] $- \frac{2}{81}$ d2$\left[\frac{81 n}{8}, 1\right]$ $- \frac{2}{81}$ d2$\left[\frac{81 n}{7}, 1\right]$ $- \frac{2}{81}$ d2$\left[\frac{27 n}{2}, 1\right]$ $- \frac{2}{81}$ d2$\left[\frac{81 n}{5}, 1\right]$ $- \frac{2}{81}$ d2$\left[\frac{81 n}{4}, 1\right]$

10     $\frac{81}{100} - \frac{1}{50}$ d2[10 n, 1] $- \frac{1}{50}$ d2$\left[\frac{100 n}{9}, 1\right]$ $- \frac{1}{50}$ d2$\left[\frac{25 n}{2}, 1\right]$ $- \frac{1}{50}$ d2$\left[\frac{100 n}{7}, 1\right]$ $- \frac{1}{50}$ d2$\left[\frac{50 n}{3}, 1\right]$ $- \frac{1}{50}$ d2[20 n

```
a2p[n_, c_] :=
 c^-2 ( Dhyp[ c^2 n, 2, 2] - 2 Sum[ Dhyp[c^2 n j^-1, 1, 2], {j, 2, c}] ) + (c - 1) ^2 / c^2
```

```
N[a2p[100, 30]]
```

358.271

```
Table[{j, Expand[dhla[n, 3, j]]}, {j, 1, 10}] // TableForm
```

1   d2[n, 3]

2     $-\frac{1}{8} + \frac{3}{8}$ d2[2 n, 1] $- \frac{3}{8}$ d2[4 n, 2] + $\frac{1}{8}$ d2[8 n, 3]

3     $-\frac{8}{27} + \frac{1}{9}$ d2[3 n, 1] + $\frac{2}{9}$ d2$\left[\frac{9 n}{2}, 1\right]$ + $\frac{1}{9}$ d2$\left[\frac{27 n}{4}, 1\right]$ $- \frac{1}{9}$ d2[9 n, 2] $- \frac{1}{9}$ d2$\left[\frac{27 n}{2}, 2\right]$ + $\frac{1}{27}$ d2[27 n, 3]

4     $-\frac{27}{64} + \frac{3}{64}$ d2[4 n, 1] + $\frac{3}{32}$ d2$\left[\frac{16 n}{3}, 1\right]$ + $\frac{3}{64}$ d2$\left[\frac{64 n}{9}, 1\right]$ + $\frac{3}{32}$ d2[8 n, 1] + $\frac{3}{32}$ d2$\left[\frac{32 n}{3}, 1\right]$ + $\frac{3}{64}$ d2[16 n, 1

5     $-\frac{64}{125} + \frac{3}{125}$ d2[5 n, 1] + $\frac{6}{125}$ d2$\left[\frac{25 n}{4}, 1\right]$ + $\frac{3}{125}$ d2$\left[\frac{125 n}{16}, 1\right]$ + $\frac{6}{125}$ d2$\left[\frac{25 n}{3}, 1\right]$ + $\frac{6}{125}$ d2$\left[\frac{125 n}{12}, 1\right]$ + $\frac{6}{125}$ d2

6     $-\frac{125}{216} + \frac{1}{72}$ d2[6 n, 1] + $\frac{1}{36}$ d2$\left[\frac{36 n}{5}, 1\right]$ + $\frac{1}{72}$ d2$\left[\frac{216 n}{25}, 1\right]$ + $\frac{1}{36}$ d2[9 n, 1] + $\frac{1}{36}$ d2$\left[\frac{54 n}{5}, 1\right]$ + $\frac{1}{36}$ d2[12 n,

7     $-\frac{216}{343} + \frac{3}{343}$ d2[7 n, 1] + $\frac{6}{343}$ d2$\left[\frac{49 n}{6}, 1\right]$ + $\frac{3}{343}$ d2$\left[\frac{343 n}{36}, 1\right]$ + $\frac{6}{343}$ d2$\left[\frac{49 n}{5}, 1\right]$ + $\frac{6}{343}$ d2$\left[\frac{343 n}{30}, 1\right]$ + $\frac{6}{343}$ d2

8     $-\frac{343}{512} + \frac{3}{512}$ d2[8 n, 1] + $\frac{3}{256}$ d2$\left[\frac{64 n}{7}, 1\right]$ + $\frac{3}{512}$ d2$\left[\frac{512 n}{49}, 1\right]$ + $\frac{3}{256}$ d2$\left[\frac{32 n}{3}, 1\right]$ + $\frac{3}{256}$ d2$\left[\frac{256 n}{21}, 1\right]$ + $\frac{3}{256}$ d2

9     $-\frac{512}{729} + \frac{1}{243}$ d2[9 n, 1] + $\frac{2}{243}$ d2$\left[\frac{81 n}{8}, 1\right]$ + $\frac{1}{243}$ d2$\left[\frac{729 n}{64}, 1\right]$ + $\frac{2}{243}$ d2$\left[\frac{81 n}{7}, 1\right]$ + $\frac{2}{243}$ d2$\left[\frac{729 n}{56}, 1\right]$ + $\frac{2}{243}$ d2

10     $-\frac{729}{1000} + \frac{3 \, d2[10 n, 1]}{1000} + \frac{3}{500}$ d2$\left[\frac{100 n}{9}, 1\right]$ + $\frac{3 \, d2\left[\frac{1000 n}{81}, 1\right]}{1000}$ + $\frac{3}{500}$ d2$\left[\frac{25 n}{2}, 1\right]$ + $\frac{3}{500}$ d2$\left[\frac{125 n}{9}, 1\right]$ + $\frac{3}{500}$ d2$\left[\frac{100 n}{7}, \right.$

```
Sum[1, {j, 2, c}, {k, 2, c}]
```

$(-1 + c)^2$

```
Sum[ 1, {j, 2, 10}, {k, 2, ((10^2) 100) / j}] /. {c → 10, n → 100}
```

19 279

```
Sum[ 1, {j, 2, c}, {k, 2, ((c^2) n) / j}] /. {c → 10, n → 100}
```

$\frac{1\,214\,683}{63}$

```
Sum[1, {j, 2, (c^2 n) / (c n)}, {k, 2, (c^2) n / j}] /. {c → 10, n → 100}
```

$\frac{1\,214\,683}{63}$

```
Sum[1, {k, 2, (10^2) 100}, {j, 2, 10}]
```

89 991

```
f1[n_, c_] := Sum[1, {j, 2, (c^2 n) / (c n)}, {k, 2, (c^2) n / j}]
f2[n_, c_] := Sum[1, {k, 2, (c^2) n}, {j, 2, (c^2 n) / (c n) / k}]

f1[100, 10]
```

19 279

```
f2[100, 10]
```

8

```
a3p[n_, c_] := c^-3 (Sum[1, {j, 2, c^3 n}, {k, 2, c^3 n / j}, {m, 2, c^3 n / (j k)}] -
    3 Sum[1, {j, 2, c}, {k, 2, c^3 n / j}, {m, 2, c^3 n / (j k)}] +
    3 Sum[1, {j, 2, c}, {k, 2, c}, {m, 2, c^3 n / (j k)}] - Sum[1, {j, 2, c}, {k, 2, c}, {m, 2, c}])

N[a3p[100, 20]]
```

672.244

```
Dhyp[100, 3, 2]
```

324

```
N[-Gamma[3, 0, -Log[100]] / Gamma[3]]
```

$698.863 - 1.71417 \times 10^{-13}$ i

```
a13p[n_, c_] := c^-3 (Sum[1, {j, 1, c^3 n}, {k, 1, c^3 n / j}, {m, 1, c^3 n / (j k)}] -
    3 Sum[1, {j, 1, c}, {k, 1, c^3 n / j}, {m, 1, c^3 n / (j k)}] +
    3 Sum[1, {j, 1, c}, {k, 1, c}, {m, 1, c^3 n / (j k)}] - Sum[1, {j, 1, c}, {k, 1, c}, {m, 1, c}])

N[a13p[100, 10]]
```

646.722

```
a1kp[n_, c_, r_] := c^-3 (Sum[1, {j, r, c^3 n}, {k, r, c^3 n / j}, {m, r, c^3 n / (j k)}] -
    3 Sum[1, {j, r, c}, {k, r, c^3 n / j}, {m, r, c^3 n / (j k)}] +
    3 Sum[1, {j, r, c}, {k, r, c}, {m, r, c^3 n / (j k)}] - Sum[1, {j, r, c}, {k, r, c}, {m, r, c}])

N[a1kp[100, 20, 10]]
```

672.244

```
N[a1kp[100, 20, 7]]
```

672.244

```
aa1kp[n_, c_, r_] := c^-3 (Sum[1, {j, r, c^3 n}, {k, r, c^3 n / j}, {m, r, c^3 n / (j k)}] -
    Sum[1, {j, r, c}, {k, r, c}, {m, r, c}])

N[aa1kp[100, 20, 1]]
```
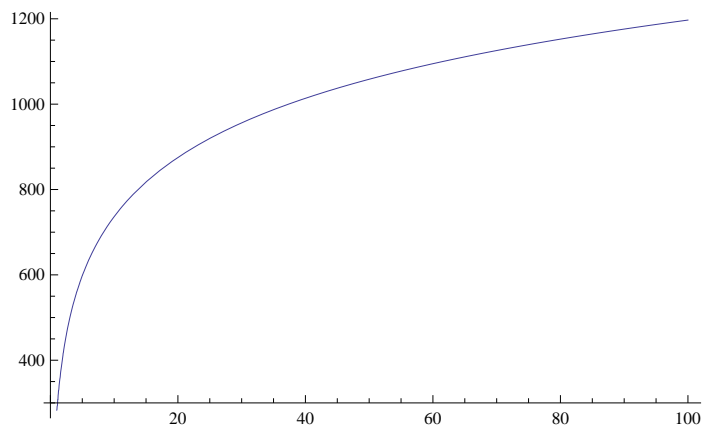
10 279.8
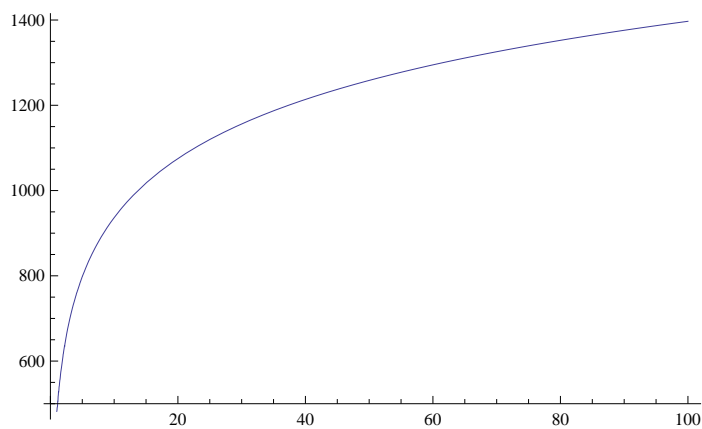
```
Limit[ (x - 1)^2 / x^2, {x -> Infinity}]
```

{1}

Plot[Dhyp[100 c^2, 2, 2] / c^2, {c, 1, 100}]

Plot[Dhyp[100 c^2, 2, 1] / c^2, {c, 1, 100}]

Limit[c^-3 (Sum[1, {j, 1, c^3 n}, {k, 1, c^3 n / j}, {m, 1, c^3 n / (j k)}] -
    3 Sum[1, {j, 1, c}, {k, 1, c^3 n / j}, {m, 1, c^3 n / (j k)}] +
    3 Sum[1, {j, 1, c}, {k, 1, c}, {m, 1, c^3 n / (j k)}] -
    Sum[1, {j, 1, c}, {k, 1, c}, {m, 1, c}]), {c → Infinity}]

$Aborted

Table[{j, Expand[dhla[n, 1, j]]}, {j, 1, 10}] // TableForm

1	d2[n, 1]
2	$-\frac{1}{2} + \frac{1}{2}$ d2[2 n, 1]
3	$-\frac{2}{3} + \frac{1}{3}$ d2[3 n, 1]
4	$-\frac{3}{4} + \frac{1}{4}$ d2[4 n, 1]
5	$-\frac{4}{5} + \frac{1}{5}$ d2[5 n, 1]
6	$-\frac{5}{6} + \frac{1}{6}$ d2[6 n, 1]
7	$-\frac{6}{7} + \frac{1}{7}$ d2[7 n, 1]
8	$-\frac{7}{8} + \frac{1}{8}$ d2[8 n, 1]
9	$-\frac{8}{9} + \frac{1}{9}$ d2[9 n, 1]
10	$-\frac{9}{10} + \frac{1}{10}$ d2[10 n, 1]