```
D[ExpIntegralE[z + 1, -Log[100.]], z] /. z → 0
```

-9.70202 - 6.61115 ⅈ

```
-ExpIntegralE[0. + 1, -Log[100.]]
```

30.1261 + 3.14159 ⅈ

```
LogIntegral[100.]
```

30.1261

```
-Limit[x^z Gamma[-z] + Sum[(-1)^k / (z - k) x^k / k!, {k, 0, Infinity}] /. x → (-Log[x]), z → 0]
```

-Gamma[0, -Log[x]]

```
FullSimplify[x^z Gamma[-z] + Sum[(-1)^k / (z - k) x^k / k!, {k, 0, Infinity}]] /. z → 0
```

ExpIntegralE[1, x]

```
Sum[(-1)^k / (z - k) Binomial[x, k], {k, 0, Infinity}]
```

$$\frac{\text{Gamma}[1+x]\ \text{Gamma}[1-z]}{z\ \text{Gamma}[1+x-z]}$$

```
1 / Gamma[z] / Gamma[1 - z] Sum[(-1)^k / (z - k) x^k / k!, {k, 0, Infinity}]
```

$$\frac{x^z\ (\text{Gamma}[1-z] + z\ \text{Gamma}[-z, x])}{z\ \text{Gamma}[1-z]\ \text{Gamma}[z]}$$

$$\text{FullSimplify}\left[\frac{(-\ \text{Gamma}[-z] + \text{Gamma}[-z, x])}{\text{Gamma}[-z]\ \text{Gamma}[z]}\right]$$

$$\frac{-1 + \frac{\text{Gamma}[-z,x]}{\text{Gamma}[-z]}}{\text{Gamma}[z]}$$

```
-Limit[x^z Gamma[-z] + Sum[(-1)^k / (z - k) x^k / k!, {k, 0, Infinity}] /. x → (-x), z → 0]
```

-Gamma[0, -x]

```
FullSimplify[
 - (-Log[x])^z Gamma[-z] - Sum[(-1)^k / (z - k) (-Log[x])^k / k!, {k, 0, Infinity}]]
```

-ExpIntegralE[1 + z, -Log[x]]

```
FullSimplify[- (-Log[x])^z Gamma[-z] - Sum[1 / (z - k) Log[x]^k / k!, {k, 0, Infinity}]]
```

-ExpIntegralE[1 + z, -Log[x]]

```
Table[(-1)^k / (z - k) (-1)^k, {k, 0, 5}]
```

$$\left\{\frac{1}{z}, \frac{1}{-1+z}, \frac{1}{-2+z}, \frac{1}{-3+z}, \frac{1}{-4+z}, \frac{1}{-5+z}\right\}$$

```
Limit[- (-Log[x])^z Gamma[-z] - Sum[1 / (z - k) Log[x]^k / k!, {k, 0, Infinity}], z → 0]
```

-Gamma[0, -Log[x]]

```
Limit[-Gamma[-z] - Sum[1 / (z - k) Log[x]^k / k!, {k, 0, Infinity}], z → 0]
```

-Gamma[0, -Log[x]] - Log[-Log[x]]

```
-Sum[1 / k Log[x]^k / k!, {k, 1, Infinity}]
```

EulerGamma + Gamma[0, -Log[x]] + Log[-Log[x]]

```
-Sum[1 / k x^k / k!, {k, 1, Infinity}]
```

EulerGamma + Gamma[0, -x] + Log[-x]

```
D[1 / Gamma[z] / Gamma[1 - z] ( Integrate[ ExpIntegralE[z, Log[t]], {t, 1, x}] +
    Sum[ (-1)^k / (z - k) (f[k]), {k, 0, Infinity}]), z]
```

$$\left(\int_1^x \left(\text{Gamma}[1-z,\text{Log}[t]]\,\text{Log}[t]^{-1+z}\,\text{Log}[\text{Log}[t]]-\text{Log}[t]^{-1+z}\left(\text{Gamma}[1-z,\text{Log}[t]]\,\text{Log}[\text{Log}[t]]+\text{MeijerG}[\{\{\},\{1,1\}\},\{\{0,0,1-z\},\{\}\},\text{Log}[t]]\right)\right)dt+\sum_{k=0}^{\infty}-\frac{(-1)^k f[k]}{(-k+z)^2}\right)\Big/\left(\text{Gamma}[1-z]\,\text{Gamma}[z]\right)+\frac{\text{PolyGamma}[0,1-z]\left(\int_1^x \text{ExpIntegralE}[z,\text{Log}[t]]\,dt+\sum_{k=0}^{\infty}\frac{(-1)^k f[k]}{-k+z}\right)}{\text{Gamma}[1-z]\,\text{Gamma}[z]}-\frac{\text{PolyGamma}[0,z]\left(\int_1^x \text{ExpIntegralE}[z,\text{Log}[t]]\,dt+\sum_{k=0}^{\infty}\frac{(-1)^k f[k]}{-k+z}\right)}{\text{Gamma}[1-z]\,\text{Gamma}[z]}$$

```
D[Sin[Pi z] / Pi ( Integrate[ ExpIntegralE[z, Log[t]], {t, 1, x}]), z]
```

$$\text{Cos}[\pi z]\int_1^x \text{ExpIntegralE}[z,\text{Log}[t]]\,dt+\frac{1}{\pi}\left(\int_1^x \left(\text{Gamma}[1-z,\text{Log}[t]]\,\text{Log}[t]^{-1+z}\,\text{Log}[\text{Log}[t]]-\text{Log}[t]^{-1+z}\left(\text{Gamma}[1-z,\text{Log}[t]]\,\text{Log}[\text{Log}[t]]+\text{MeijerG}[\{\{\},\{1,1\}\},\{\{0,0,1-z\},\{\}\},\text{Log}[t]]\right)\right)dt\right)\text{Sin}[\pi z]$$

```
D[Sin[Pi z] / Pi ( Sum[ (-1)^k / (z - k) (f[k]), {k, 0, Infinity}]), z]
```

$$\frac{\text{Sin}[\pi z]\sum_{k=0}^{\infty}-\frac{(-1)^k f[k]}{(-k+z)^2}}{\pi}+\text{Cos}[\pi z]\sum_{k=0}^{\infty}\frac{(-1)^k f[k]}{-k+z}$$

$$\text{Cos}[\pi z]\int_1^x \text{ExpIntegralE}[z,\text{Log}[t]]\,dt+\frac{1}{\pi}\left(\int_1^x \left(\text{Gamma}[1-z,\text{Log}[t]]\,\text{Log}[t]^{-1+z}\,\text{Log}[\text{Log}[t]]-\text{Log}[t]^{-1+z}\left(\text{Gamma}[1-z,\text{Log}[t]]\,\text{Log}[\text{Log}[t]]+\text{MeijerG}[\{\{\},\{1,1\}\},\{\{0,0,1-z\},\{\}\},\text{Log}[t]]\right)\right)dt\right)\text{Sin}[\pi z]\ /.\ z\to 0$$

Integrate::idiv : Integral of $\frac{1}{t\,\text{Log}[t]}$ does not converge on $\{1, x\}$. ≫

$$\int_1^x \frac{1}{t\,\text{Log}[t]}\,dt$$

$$\frac{\mathrm{Sin}[\pi z] \sum_{k=0}^{\infty} -\frac{(-1)^k f[k]}{(-k+z)^2}}{\pi} + \mathrm{Cos}[\pi z] \sum_{k=0}^{\infty} \frac{(-1)^k f[k]}{-k+z} \ /.\ z \to 0$$

Power::infy : Infinite expression $\frac{1}{0^2}$ encountered. ≫

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

$$\sum_{k=0}^{\infty} -\frac{(-1)^k f[k]}{k}$$

$$\mathrm{Cos}[\pi z] \int_1^x \mathrm{ExpIntegralE}[z, \mathrm{Log}[t]] \, dt + \frac{1}{\pi} \left(\int_1^x \left(\mathrm{Gamma}[1-z, \mathrm{Log}[t]]\, \mathrm{Log}[t]^{-1+z} \mathrm{Log}[\mathrm{Log}[t]] - \mathrm{Log}[t]^{-1+z} (\mathrm{Gamma}[1-z, \mathrm{Log}[t]]\, \mathrm{Log}[\mathrm{Log}[t]] + \mathrm{MeijerG}[\{\{\}, \{1, 1\}\}, \{\{0, 0, 1-z\}, \{\}\}, \mathrm{Log}[t]])\right) dt\right) \mathrm{Sin}[\pi z] + \frac{\mathrm{Sin}[\pi z] \sum_{k=0}^{\infty} -\frac{(-1)^k f[k]}{(-k+z)^2}}{\pi} + \mathrm{Cos}[\pi z] \sum_{k=0}^{\infty} \frac{(-1)^k f[k]}{-k+z} \ /.\ z \to 0$$

Integrate::idiv : Integral of $\frac{1}{t\,\mathrm{Log}[t]}$ does not converge on {1, x}. ≫

Power::infy : Infinite expression $\frac{1}{0^2}$ encountered. ≫

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

$$\int_1^x \frac{1}{t\,\mathrm{Log}[t]} \, dt + \sum_{k=0}^{\infty} -\frac{(-1)^k f[k]}{k}$$

```
Clear[D2]
D2[n_, k_] := D2[n, k] = Sum[D2[Floor[n / j], k - 1], {j, 2, n}]
D2[n_, 0] := 1
D2z[n_, z2_] := Limit[Sin[Pi z] / Pi Sum[ (-1)^k / (z - k) D2[n, k], {k, 0, Log2@n}], z → z2]
```

```
Limit[D[D2z[100, z], z], z → 0]
```

$$\frac{428}{15}$$

```
D[D2z[100, z], z]
```

$$\left(\frac{7}{-6+z} - \frac{51}{-5+z} + \frac{184}{-4+z} - \frac{324}{-3+z} + \frac{283}{-2+z} - \frac{99}{-1+z} + \frac{1}{z}\right) \mathrm{Cos}[\pi z] + \frac{\left(-\frac{7}{(-6+z)^2} + \frac{51}{(-5+z)^2} - \frac{184}{(-4+z)^2} + \frac{324}{(-3+z)^2} - \frac{283}{(-2+z)^2} + \frac{99}{(-1+z)^2} - \frac{1}{z^2}\right) \mathrm{Sin}[\pi z]}{\pi}$$

$$\frac{\text{Sin}[\pi z] \sum_{k=0}^{\infty} -\frac{(-1)^k f[k]}{(-k+z)^2}}{\pi} + \text{Cos}[\pi z] \sum_{k=0}^{\infty} \frac{(-1)^k f[k]}{-k+z}$$

Limit[(1 / z) Cos[Pi z], z → 0]

∞

$$\text{Limit}\left[\frac{\left(-\frac{7}{(-6+z)^2} + \frac{51}{(-5+z)^2} - \frac{184}{(-4+z)^2} + \frac{324}{(-3+z)^2} - \frac{283}{(-2+z)^2} + \frac{99}{(-1+z)^2} - \frac{1}{z^2}\right) \text{Sin}[\pi z]}{\pi}, z \to 0\right]$$

$-\infty$

$$\text{Limit}\left[\left(\frac{7}{-6+z} - \frac{51}{-5+z} + \frac{184}{-4+z} - \frac{324}{-3+z} + \frac{283}{-2+z} - \frac{99}{-1+z} + \frac{1}{z}\right) \text{Cos}[\pi z], z \to 0\right]$$

∞

$$\text{Limit}\left[\frac{\text{Sin}[\pi z] \sum_{k=0}^{\infty} -\frac{(-1)^k}{(-k+z)^2}}{\pi}, z \to 0\right]$$

$-\infty$

Limit[- (99 / (z - 1)) Cos[Pi z] + Sin[Pi z] / Pi (99 / (z - 1) ^2), z → 0]

99

D[Sum[1 / (z - k) GammaRegularized[k, -Log[x]], {k, 0, Infinity}] , z]

$$\sum_{k=0}^{\infty} -\frac{\text{GammaRegularized}[k, -\text{Log}[x]]}{(-k+z)^2}$$

$$\sum_{k=0}^{\infty} -\frac{\text{GammaRegularized}[k, -\text{Log}[x]]}{(-k+z)^2} \text{ /. } x \to 100. \text{ /. } z \to 0$$

Power::infy : Infinite expression $\frac{1}{0^2}$ encountered. ≫

$$\sum_{k=0}^{\infty} -\frac{\text{GammaRegularized}[k, -4.60517]}{k^2}$$

$$\text{N}\left[\text{Limit}\left[\sum_{k=0}^{\infty} -\frac{\text{GammaRegularized}[k, -\text{Log}[x]]}{(-k+z)^2} \text{ /. } x \to 100., z \to 0\right]\right]$$

NSum::nsnum : Summand (or its derivative) $-\frac{\text{GammaRegularized}[k, -4.60517]}{(-k+z)^2}$ is not numerical at point k = 1. ≫

NSum::nsnum : Summand (or its derivative) $-\frac{\text{GammaRegularized}[k, -4.60517]}{(-k+z)^2}$ is not numerical at point k = 1. ≫

NSum::nsnum : Summand (or its derivative) $-\frac{\text{GammaRegularized}[k, -4.60517]}{(-k+z)^2}$ is not numerical at point k = 1. ≫

General::stop : Further output of NSum::nsnum will be suppressed during this calculation. ≫

$$\text{Limit}\left[\text{NSum}\left[-\frac{\text{GammaRegularized}[k, -4.60517]}{(-k+z)^2}, \{k, 0, \infty\}\right], z \to 0.\right]$$

Integrate[1 / Log[t], {t, 0, x}, PrincipalValue → True]

ConditionalExpression[LogIntegral[x], Im[x] ≠ 0 || Re[x] ≤ 1]

```
Integrate[1 / x, {x, -1, 2}, PrincipalValue → True]

Integrate[ 1 / (t Log[t]), {t, 1, x}, PrincipalValue → True]
```

Integrate::idiv : Integral of $\frac{1}{t\,\mathrm{Log}[t]}$ does not converge on {1, x}. ≫

$$\mathrm{Integrate}\left[\frac{1}{t\,\mathrm{Log}[t]},\ \{t,1,x\},\ \mathrm{PrincipalValue}\to\mathrm{True}\right]$$

```
Clear[pp, ppx]
pp[n_, j_, k_, z_, d_] :=
 pp[n, j, k, z, d] = If[n < j, 0, d (1 / (z - k) - pp[n / j, 1 + d, k + 1, z, d]) + pp[n, j + d, k, z, d]]
ppz[n_, z_, d_] := 1 / z - pp[n, 1 + d, 1, z, d]
ppx[n_, k_, z_, d_] :=
 ppx[n, k, z, d] = Expand[d Sum[1 / (z - k) - ppx[n / j, k + 1, z, d], {j, 1 + d, n, d}]]
ppxz[n_, z_, d_] := 1 / z - ppx[n, 1, z, d]

ppz[100, z, 1]
```

$$\frac{7}{-6+z}-\frac{51}{-5+z}+\frac{184}{-4+z}-\frac{324}{-3+z}+\frac{283}{-2+z}-\frac{99}{-1+z}+\frac{1}{z}$$

```
Expand@ppz[10, z, 1 / 10]

$Aborted

ppxz[2., z, .02]

(-1) ^ (-3.) GammaRegularized[3., 0, -Log[2.]]
```

$0.0941587 - 3.45933\times10^{-17}\,i$

```
Integrate[1, {t, 1, 3.}, {u, 1, 3. / t}, {v, 1, 3. / (t u)}]

0.514587

Sin[Pi 2] / Pi Integrate[ExpIntegralE[2., Log[t]], {t, 1, 3.}]

0. + 0. i
```

$$\mathrm{Limit}\left[\mathrm{Sin}[\mathrm{Pi}\,z]/\mathrm{Pi}\left(-\frac{0.08325600000000001\grave{}}{-3+z}\right),\ z\to 3\right]$$

```
0.083256

D[Sin[Pi z] / Pi (f[z] - g[z]), z]
```

$$\mathrm{Cos}[\pi z]\,(f[z]-g[z])+\frac{\mathrm{Sin}[\pi z]\,(f'[z]-g'[z])}{\pi}$$

```
Clear[pp, ppx, dppx, dpp]
pp[n_, j_, k_, z_, d_] :=
 pp[n, j, k, z, d] = If[n < j, 0, d (1 / (z - k) - pp[n / j, 1 + d, k + 1, z, d]) + pp[n, j + d, k, z, d]]
ppz[n_, z_, d_] := 1 / z - pp[n, 1 + d, 1, z, d]
ppx[n_, k_, z_, d_] :=
 ppx[n, k, z, d] = Expand[d Sum[1 / (z - k) - ppx[n / j, k + 1, z, d], {j, 1 + d, n, d}]]
ppxz[n_, z_, d_] := 1 / z - ppx[n, 1, z, d]
dpp[n_, j_, k_, z_, d_] := dpp[n, j, k, z, d] =
  If[n < j, 0, d (1 / (z - k) ^2 - dpp[n / j, 1 + d, k + 1, z, d]) + dpp[n, j + d, k, z, d]]
dppz[n_, z_, d_] := -1 / z^2 + dpp[n, 1 + d, 1, z, d]
dppx[n_, k_, z_, d_] :=
 dppx[n, k, z, d] = Expand[d Sum[1 / (z - k) ^2 - dppx[n / j, k + 1, z, d], {j, 1 + d, n, d}]]
dppxz[n_, z_, d_] := -1 / z^2 + dppx[n, 1, z, d]
```

```
Expand@D[ppz[20, z, 1 / 2], z]
```

$$\frac{1}{128\,(-7+z)^2}-\frac{13}{64\,(-6+z)^2}+\frac{61}{32\,(-5+z)^2}-\frac{81}{8\,(-4+z)^2}+\frac{99}{4\,(-3+z)^2}-\frac{33}{(-2+z)^2}+\frac{19}{(-1+z)^2}-\frac{1}{z^2}$$

```
dppxz[20, z, 1 / 2]
```

$$\frac{1}{128\,(-7+z)^2}-\frac{13}{64\,(-6+z)^2}+\frac{61}{32\,(-5+z)^2}-\frac{81}{8\,(-4+z)^2}+\frac{99}{4\,(-3+z)^2}-\frac{33}{(-2+z)^2}+\frac{19}{(-1+z)^2}-\frac{1}{z^2}$$

```
Expand@dppz[20, z, 1 / 2]
```

$$\frac{1}{128\,(-7+z)^2}-\frac{13}{64\,(-6+z)^2}+\frac{61}{32\,(-5+z)^2}-\frac{81}{8\,(-4+z)^2}+\frac{99}{4\,(-3+z)^2}-\frac{33}{(-2+z)^2}+\frac{19}{(-1+z)^2}-\frac{1}{z^2}$$

```
Table[Limit[(-1) ^k (f[x] / (z - k)) Cos[Pi z] - Sin[Pi z] / Pi (f[x] / (z - k) ^2), z → 0], {k, 0, 6}]
```

$$\left\{0,\ f[x],\ -\frac{f[x]}{2},\ \frac{f[x]}{3},\ -\frac{f[x]}{4},\ \frac{f[x]}{5},\ -\frac{f[x]}{6}\right\}$$

```
Limit[- (n / (z - 0)) Cos[Pi z] + Sin[Pi z] / Pi (n / (z - 0) ^2), z → 0]
```

0

```
Limit[(f[x] / (z - k)) Cos[Pi z] + Sin[Pi z] / Pi (f[x] / (z - k) ^2), z → 0]
```

$$-\frac{f[x]}{k}$$

D[Sin[Pi z] / Pi ppxz[4, z, .05], z]

$$\left(0.+\frac{3.72529\times10^{-37}}{-28+z}-\frac{2.08616\times10^{-34}}{-27+z}+\frac{5.63264\times10^{-32}}{-26+z}-\frac{9.76324\times10^{-30}}{-25+z}+\frac{1.22184\times10^{-27}}{-24+z}-\frac{1.1779\times10^{-25}}{-23+z}+\frac{9.3462\times10^{-24}}{-22+z}-\frac{6.89934\times10^{-22}}{-21+z}+\frac{4.51195\times10^{-20}}{-20+z}-\frac{3.24133\times10^{-18}}{-19+z}+\frac{1.30438\times10^{-16}}{-18+z}-\frac{3.98893\times10^{-15}}{-17+z}+\frac{1.05887\times10^{-13}}{-16+z}-\frac{2.51603\times10^{-12}}{-15+z}+\frac{5.4652\times10^{-11}}{-14+z}-\frac{1.08456\times10^{-9}}{-13+z}+\frac{1.89574\times10^{-8}}{-12+z}-\frac{2.7688\times10^{-7}}{-11+z}+\frac{3.41332\times10^{-6}}{-10+z}-\frac{0.0000364756}{-9+z}+\frac{0.000336757}{-8+z}-\frac{0.00260168}{-7+z}+\frac{0.016703}{-6+z}-\frac{0.0875238}{-5+z}+\frac{0.359725}{-4+z}-\frac{1.11738}{-3+z}+\frac{2.415}{-2+z}-\frac{3.}{-1+z}+\frac{1}{z}\right)\mathrm{Cos}[\pi z]+$$

$$\frac{1}{\pi}\left(-\frac{3.72529\times10^{-37}}{(-28+z)^2}+\frac{2.08616\times10^{-34}}{(-27+z)^2}-\frac{5.63264\times10^{-32}}{(-26+z)^2}+\frac{9.76324\times10^{-30}}{(-25+z)^2}-\frac{1.22184\times10^{-27}}{(-24+z)^2}+\frac{1.1779\times10^{-25}}{(-23+z)^2}-\frac{9.3462\times10^{-24}}{(-22+z)^2}+\frac{6.89934\times10^{-22}}{(-21+z)^2}-\frac{4.51195\times10^{-20}}{(-20+z)^2}+\frac{3.24133\times10^{-18}}{(-19+z)^2}-\frac{1.30438\times10^{-16}}{(-18+z)^2}+\frac{3.98893\times10^{-15}}{(-17+z)^2}-\frac{1.05887\times10^{-13}}{(-16+z)^2}+\frac{2.51603\times10^{-12}}{(-15+z)^2}-\frac{5.4652\times10^{-11}}{(-14+z)^2}+\frac{1.08456\times10^{-9}}{(-13+z)^2}-\frac{1.89574\times10^{-8}}{(-12+z)^2}+\frac{2.7688\times10^{-7}}{(-11+z)^2}-\frac{3.41332\times10^{-6}}{(-10+z)^2}+\frac{0.0000364756}{(-9+z)^2}-\frac{0.000336757}{(-8+z)^2}+\frac{0.00260168}{(-7+z)^2}-\frac{0.016703}{(-6+z)^2}+\frac{0.0875238}{(-5+z)^2}-\frac{0.359725}{(-4+z)^2}+\frac{1.11738}{(-3+z)^2}-\frac{2.415}{(-2+z)^2}+\frac{3.}{(-1+z)^2}-\frac{1}{z^2}\right)\mathrm{Sin}[\pi z]$$

ee[x_, z_] := Sum[(-1)^k / (z - k) GammaRegularized[k, 0, -Log[x]], {k, 0, Infinity}]

Limit[(-1)^k n Cos[Pi z] / (z - k) - n Sin[Pi z] / Pi / (z - k)^2 /. k → 0, z → 0]

0

Sum[(-1)^(k+1) / k (-1)^-k Gamma[k, 0, -Log[100.]] / Gamma[k], {k, 1, 30.}]

$28.0217 - 2.09386\times10^{-14}\,i$

Limit[((-1)^-z Gamma[z, 0, -Log[x]] / Gamma[z] - 1) / z, z → 0]

$-i\pi$ - Gamma[0, -Log[x]]

D[Hypergeometric1F1[z, z+1, Log[x]] Log[x]^z / z!, z] /. z → 0

-Gamma[0, -Log[x]] - Log[-Log[x]] + Log[Log[x]]

Limit[D[Log[x]^(z-1) / (z-1)!, z], z → 0]

$$\frac{1}{\mathrm{Log}[x]}$$

```
D[Sin[Pi z] / Pi Integrate[ ExpIntegralE[z, Log[t]], {t, 1, x}], z]
```

$$\cos[\pi z] \int_1^x \text{ExpIntegralE}[z, \text{Log}[t]]\, dt + \frac{1}{\pi} \left(\int_1^x \left(\text{Gamma}[1-z, \text{Log}[t]]\, \text{Log}[t]^{-1+z}\, \text{Log}[\text{Log}[t]] - \text{Log}[t]^{-1+z} (\text{Gamma}[1-z, \text{Log}[t]]\, \text{Log}[\text{Log}[t]] + \text{MeijerG}[\{\{\}, \{1, 1\}\}, \{\{0, 0, 1-z\}, \{\}\}, \text{Log}[t]])\right) dt\right) \sin[\pi z]$$

$$\text{Limit}\Big[\cos[\pi z] \int_1^x \text{ExpIntegralE}[z, \text{Log}[t]]\, dt + \frac{1}{\pi} \left(\int_1^x \left(\text{Gamma}[1-z, \text{Log}[t]]\, \text{Log}[t]^{-1+z}\, \text{Log}[\text{Log}[t]] - \text{Log}[t]^{-1+z} (\text{Gamma}[1-z, \text{Log}[t]]\, \text{Log}[\text{Log}[t]] + \text{MeijerG}[\{\{\}, \{1, 1\}\}, \{\{0, 0, 1-z\}, \{\}\}, \text{Log}[t]])\right) dt\right) \sin[\pi z], z \to 0\Big]$$

$Aborted

```
(-1) ^-z GammaRegularized[z, 0, -Log[x]] -
 Sin[Pi z] / Pi Sum[ (-1) ^k / (z - k) GammaRegularized[x, k], {k, 1, Infinity}]
```

$$(-1)^{-z}\, \text{GammaRegularized}[z, 0, -\text{Log}[x]] - \frac{\sin[\pi z] \sum_{k=1}^{\infty} \frac{(-1)^k\, \text{GammaRegularized}[x,k]}{-k+z}}{\pi}$$

```
Limit[D[(-1) ^-z GammaRegularized[z, 0, -Log[x]] -
   Sin[Pi z] / Pi Sum[ (-1) ^k / (z - k) GammaRegularized[x, k], {k, 1, Infinity}], z], z → 0]
```

$Aborted

```
D[(-1) ^-z GammaRegularized[z, 0, -Log[100.]], z] /. z → .00000001
```

$30.1261 + 8.88178 \times 10^{-16}\, i$

```
LogIntegral[100.]
```

30.1261

```
D[(-1) ^-z GammaRegularized[z, 0, -Log[x]] -
    Sin[Pi z] / Pi Sum[ (-1) ^k / (z - k) (-1) ^-k GammaRegularized[k, 0, -Log[x]], {k, 1, 50}],
   z] /. z → .0000001 /. x → 100.
```

$2.10439 + 2.13163 \times 10^{-14}\, i$

```
Log[Log[100.]] + EulerGamma
```

2.1044

```
N[D[(-1) ^-z GammaRegularized[z, 0, -Log[x]] -
     Sin[Pi z] / Pi Sum[ (-1) ^k / (z - k) GammaRegularized[k, 0, -Log[x]], {k, 1, 40}],
    z] /. x → 100. /. z → .00000001]
```

```
Integrate[ExpIntegralE[z, t], {t, 1, x}]
```

ConditionalExpression[ExpIntegralE[1 + z, 1] - ExpIntegralE[1 + z, x], Re[x] ≥ 0 || x ∉ Reals]

```
D[ExpIntegralE[1 + z, 1] - ExpIntegralE[1 + z, x], z]
```

$-x^z\, \text{Gamma}[-z, x]\, \text{Log}[x] - \text{MeijerG}[\{\{\}, \{1, 1\}\}, \{\{0, 0, -z\}, \{\}\}, 1] + x^z (\text{Gamma}[-z, x]\, \text{Log}[x] + \text{MeijerG}[\{\{\}, \{1, 1\}\}, \{\{0, 0, -z\}, \{\}\}, x])$