```
Clear[rb]
bin2[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
rb[n_, k_, f_] := rb[n, k, f] = Sum[f[j] rb[Floor[n / j], k - 1, f], {j, 2, n}]
rb[n_, 0, f_] := UnitStep[n - 1]
lrb[n_, f_] := Sum[(-1)^(k + 1) / k rb[n, k, f], {k, 1, Log2@n}]
rbz[n_, z_, f_] := Sum[bin2[z, k] rb[n, k, f], {k, 0, Log2@n}]
lrz[n_, z_, f_] := Sin[Pi z] / Pi Sum[ (-1)^k / (z - k) rb[n, k, f], {k, 0, Log2@n}]
id[n_] := 1

Limit[lrz[100, z, id], z → 1]
```

99

```
Integrate[ lrz[100, z, id], {z, 0, Infinity}]
```

$Aborted

$Aborted

```
D[x^z / z!, z] /. z → 0
```

EulerGamma + Log[x]

```
D[Hypergeometric1F1[z, z + 1, Log[x]] Log[x]^z / (z!), z] /. z → 0
```

- Gamma[0, - Log[x]] - Log[- Log[x]] + Log[Log[x]]

```
D[FactorialPower[x, z] / z!, z] /. z → 0
```

EulerGamma + PolyGamma[0, 1 + x]

```
Limit[D[lrz[100, z, id], z], z → 0]
```

$\frac{428}{15}$

```
D[1 / z!, z] /. z → 0
```

EulerGamma

```
D[x^z, z] /. z → 0
```

Log[x]

```
D[Hypergeometric1F1[z, z + 1, Log[x]] Log[x]^z, z] /. z → 0
```

- EulerGamma - Gamma[0, - Log[x]] - Log[- Log[x]] + Log[Log[x]]

```
D[FactorialPower[x, z], z] /. z → 0
```

PolyGamma[0, 1 + x]

```
Limit[D[lrz[100, z, id] z!, z], z → 0]
```

$\frac{428}{15}$ - EulerGamma

```
Limit[D[z!, z], z → 0]
```

- EulerGamma

```
FullSimplify[D[(x + z) ! / x ! / z !, z] /. z → 0]
```

```
HarmonicNumber[x]
```

```
FullSimplify[D[(x + z) ! / x !, z] /. z → 0]
```

```
PolyGamma[0, 1 + x]
```

```
FI[n_] := FactorInteger[n]; FI[1] := {}
bin2[z_, k_] := Product[z - j, {j, 0, k - 1}] / k !
dzeta[j_, s_, z_] := j^-s Product[(-1)^p[[2]] bin2[-z, p[[2]]], {p, FI[j]}]
dza[j_, z_] := z ! Product[(-1)^p[[2]] bin2[-z, p[[2]]], {p, FI[j]}]
zeta[n_, s_, z_] := Sum[dzeta[j, s, z], {j, 1, n}]
```

```
D[Expand[zeta[100, 0, z] z !], z] /. z → 0
```

$$\frac{428}{15} - \text{EulerGamma}$$

```
dza[4, z]
```

$$-\frac{1}{2}\,(-1 - z)\, z\, z\,!$$

$$\text{D}\left[-\frac{1}{2}\,(-1 - z)\, z\, z\,!,\ z\right]\ /.\ z \to 0$$

$$\frac{1}{2}$$

```
z ! /. z → 0
```

```
1
```

```
Integrate[Log[x]^(z - 1) / (z - 1) !, {x, 1, n}]
```

$$\text{ConditionalExpression}\left[\frac{(\text{Gamma}[z] - \text{Gamma}[z, -\text{Log}[n]])\,(-\text{Log}[n])^{-z}\,\text{Log}[n]^{z}}{(-1 + z)\,!},\ \text{Re}[z] > 0\right]$$

$$\text{FullSimplify}\left[\frac{(\text{Gamma}[z] - \text{Gamma}[z, -\text{Log}[n]])\,(-\text{Log}[n])^{-z}\,\text{Log}[n]^{z}}{(-1 + z)\,!}\right]$$

$$\text{p1}[n\_, z\_] := \frac{(\text{Gamma}[z] - \text{Gamma}[z, -\text{Log}[n]])\,(-\text{Log}[n])^{-z}\,\text{Log}[n]^{z}}{\text{Gamma}[z]}$$

$$\text{p1a}[n\_, z\_] := \frac{(\text{Gamma}[z, 0, -\text{Log}[n]])\,(-\text{Log}[n])^{-z}\,\text{Log}[n]^{z}}{\text{Gamma}[z]}$$

$$\text{p1b}[n\_, z\_] := (-1)\text{\^{}}(-z)\ \frac{(\text{Gamma}[z, 0, -\text{Log}[n]])}{\text{Gamma}[z]}$$

```
p1c[n_, z_] := (-1)^(-z) (GammaRegularized[z, 0, -Log[n]])
p2[n_, z_] := Hypergeometric1F1[z, z + 1, Log[n]] Log[n]^z / z !
```

```
p1c[33, 2.7]
```

$$111.431 + 7.10543 \times 10^{-15}\ i$$

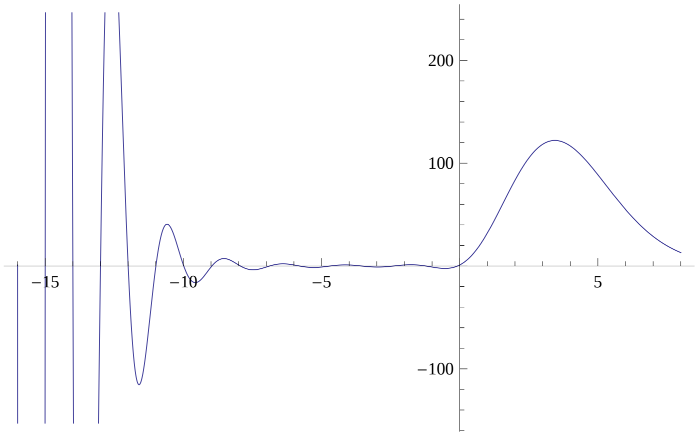
```
p2[33, 2.7]
```

```
111.431
```

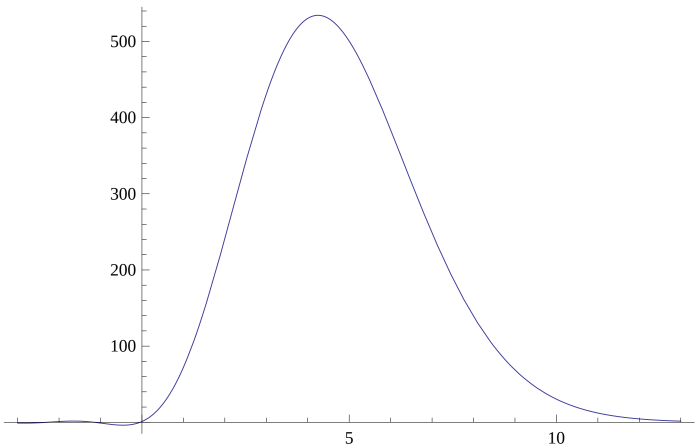
$$\text{FullSimplify}[(-1)\text{\^{}}(-z)\ (\text{Log}[n])^{-z}\ \text{Log}[n]^{z}]$$

$$(-1)^{-z}$$

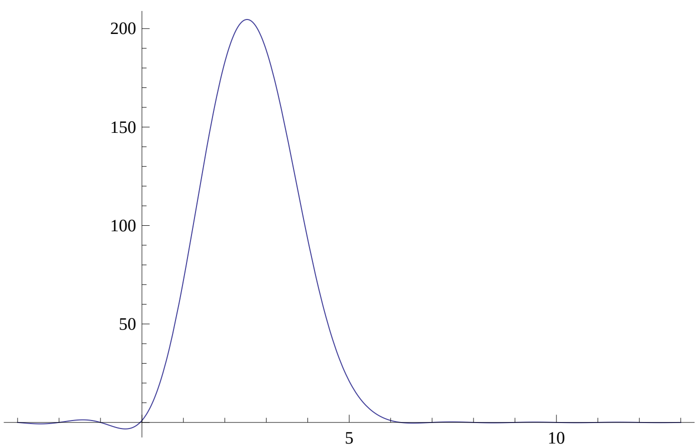
```
Plot[p1c[33, z], {z, -16, 8}]
```



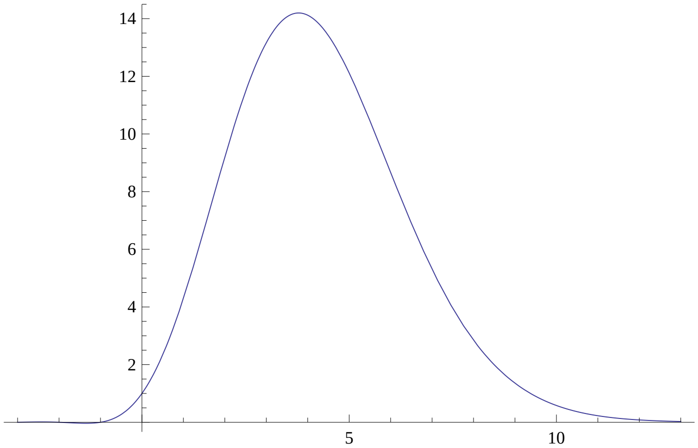

```
Plot[(-1)^(-z) GammaRegularized[z, 0, -Log[73.]], {z, -3, 13}]
```




```
Plot[lrz[73., z, id], {z, -3, 13}]
```



```
Plot[Log[73]^z / z!, {z, -3, 13}]
```




```
Sum[Binomial[z, k] x^k, {k, 0, Infinity}]
```

$$(1+x)^z$$

```
Sum[Binomial[z, k] Binomial[x, k], {k, 0, Infinity}]
```

$$\frac{\text{Gamma}[1+x+z]}{\text{Gamma}[1+x]\ \text{Gamma}[1+z]}$$

```
Integrate[lrz[20, 2 E^(z I), id], {z, 0, 2 Pi}]
```

$$2\pi$$

```
N@E^(0 I)
```

1.

```
Integrate[E^(z I), {z, 0, 2 Pi}]
```

0

```
FullSimplify@lrz[100, z, id]
```

$$\frac{\left(\frac{7}{-6+z}-\frac{51}{-5+z}+\frac{184}{-4+z}-\frac{324}{-3+z}+\frac{283}{-2+z}-\frac{99}{-1+z}+\frac{1}{z}\right)\text{Sin}[\pi z]}{\pi}$$

```
Series[((7/(-6+z) - 51/(-5+z) + 184/(-4+z) - 324/(-3+z) + 283/(-2+z) - 99/(-1+z) + 1/z) Sin[π z])/π, {z, 0, 20}]
```

$$1+\frac{428 z}{15}+\frac{1}{450}\left(24568-75\pi^2\right)z^2+\frac{\left(1974391-128400\pi^2\right)z^3}{27000}+$$
$$\frac{\left(\frac{137165317\pi}{1620000}-\frac{6142\pi^3}{675}+\frac{\pi^5}{120}\right)z^4}{\pi}+\frac{\left(\frac{8876820679\pi}{97200000}-\frac{1974391\pi^3}{162000}+\frac{107\pi^5}{450}\right)z^5}{\pi}+$$
$$\frac{\left(\frac{553927801273\pi}{5832000000}-\frac{137165317\pi^3}{9720000}+\frac{3071\pi^5}{6750}-\frac{\pi^7}{5040}\right)z^6}{\pi}+\frac{\left(\frac{33916558662151\pi}{349920000000}-\frac{8876820679\pi^3}{583200000}+\frac{1974391\pi^5}{3240000}-\frac{107\pi^7}{18900}\right)z^7}{\pi}+$$
$$\frac{\left(\frac{2056295769648937\pi}{20995200000000}-\frac{553927801273\pi^3}{34992000000}+\frac{137165317\pi^5}{194400000}-\frac{3071\pi^7}{283500}+\frac{\pi^9}{362880}\right)z^8}{\pi}+\frac{1}{\pi}$$
$$\left(\frac{124035085696334119\pi}{1259712000000000}-\frac{33916558662151\pi^3}{2099520000000}+\frac{8876820679\pi^5}{11664000000}-\frac{1974391\pi^7}{136080000}+\frac{107\pi^9}{1360800}\right)z^9+$$

$$\frac{1}{\pi}\left(\frac{7462202539264212553\,\pi}{75582720000000000}-\frac{2056295769648937\,\pi^{3}}{125971200000000}+\frac{553927801273\,\pi^{5}}{699840000000}-\frac{137165317\,\pi^{7}}{8164800000}+\frac{3071\,\pi^{9}}{20412000}-\frac{\pi^{11}}{39916800}\right)z^{10}+\frac{1}{\pi}$$

$$\left(\frac{448342799392293597511\,\pi}{4534963200000000000}-\frac{124035085696334119\,\pi^{3}}{7558272000000000}+\frac{33916558662151\,\pi^{5}}{41990400000000}-\frac{8876820679\,\pi^{7}}{489888000000}+\frac{1974391\,\pi^{9}}{9797760000}-\frac{107\,\pi^{11}}{149688000}\right)z^{11}+\frac{1}{\pi}$$

$$\left(\frac{26919044642424368873257\,\pi}{272097792000000000000}-\frac{7462202539264212553\,\pi^{3}}{453496320000000000}+\frac{2056295769648937\,\pi^{5}}{2519424000000000}-\frac{79132543039\,\pi^{7}}{4199040000000}+\frac{137165317\,\pi^{9}}{587865600000}-\frac{3071\,\pi^{11}}{2245320000}+\frac{\pi^{13}}{6227020800}\right)z^{12}+\frac{1}{\pi}$$

$$\left(\frac{1615700166389595112025959\,\pi}{16325867520000000000000}-\frac{448342799392293597511\,\pi^{3}}{27209779200000000000}+\frac{124035085696334119\,\pi^{5}}{151165440000000000}-\frac{33916558662151\,\pi^{7}}{1763596800000000}+\frac{8876820679\,\pi^{9}}{35271936000000}-\frac{1974391\,\pi^{11}}{1077753600000}+\frac{107\,\pi^{13}}{23351328000}\right)z^{13}+$$

$$\frac{1}{\pi}\left(\frac{96958799007822681627514633\,\pi}{979552051200000000000000}-\frac{26919044642424368873257\,\pi^{3}}{1632586752000000000000}+\frac{7462202539264212553\,\pi^{5}}{9069926400000000000}-\frac{2056295769648937\,\pi^{7}}{105815808000000000}+\frac{79132543039\,\pi^{9}}{302330880000000}-\frac{137165317\,\pi^{11}}{64665216000000}+\frac{3071\,\pi^{13}}{350269920000}-\frac{\pi^{15}}{1307674368000}\right)z^{14}+\frac{1}{\pi}$$

$$\left(\frac{5818032908672660283778222471\,\pi}{58773123072000000000000000}-\frac{1615700166389595112025959\,\pi^{3}}{97955205120000000000000}+\frac{448342799392293597511\,\pi^{5}}{544195584000000000000}-\frac{124035085696334119\,\pi^{7}}{6348948480000000000}+\frac{33916558662151\,\pi^{9}}{126978969600000000}-\frac{8876820679\,\pi^{11}}{3879912960000000}+\frac{1974391\,\pi^{13}}{168129561600000}-\frac{107\,\pi^{15}}{4903778880000}\right)z^{15}+\frac{1}{\pi}$$

$$\left(\frac{349097149662302744969059372777\,\pi}{3526387384320000000000000000}-\frac{96958799007822681627514633\,\pi^{3}}{5877312307200000000000000}+\frac{26919044642424368873257\,\pi^{5}}{32651735040000000000000}-\frac{7462202539264212553\,\pi^{7}}{380936908800000000000}+\frac{2056295769648937\,\pi^{9}}{7618738176000000000}-\frac{7193867549\,\pi^{11}}{3023308800000000}+\frac{137165317\,\pi^{13}}{10087773696000000}-\frac{3071\,\pi^{15}}{73556683200000}+\frac{\pi^{17}}{355687428096000}\right)z^{16}+$$

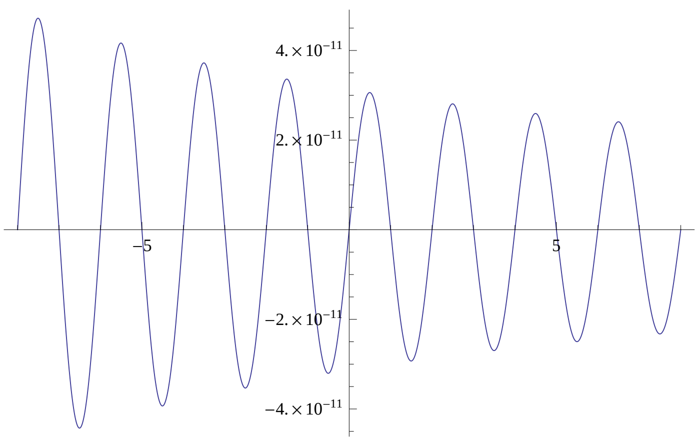
$$\frac{1}{\pi}\left(\frac{20946284758123742763084523020199\,\pi}{211583243059200000000000000000}-\frac{5818032908672660283778222471\,\pi^{3}}{352638738432000000000000000}+\frac{1615700166389595112025959\,\pi^{5}}{1959104102400000000000000}-\frac{448342799392293597511\,\pi^{7}}{22856214528000000000000}+\frac{124035085696334119\,\pi^{9}}{457124290560000000000}-\frac{3083323514741\,\pi^{11}}{1269789696000000000}+\frac{8876820679\,\pi^{13}}{605266421760000000}-\frac{1974391\,\pi^{15}}{35307207936000000}+\frac{107\,\pi^{17}}{1333827855360000}\right)z^{17}+\frac{1}{\pi}$$

$$\left(\frac{1 256 790 769 354 858 960 508 851 434 445 513\,\pi}{12 694 994 583 552 000 000 000 000 000 000} - \frac{349 097 149 662 302 744 969 059 372 777\,\pi^3}{21 158 324 305 920 000 000 000 000 000} + \frac{96 958 799 007 822 681 627 514 633\,\pi^5}{117 546 246 144 000 000 000 000 000} - \frac{3 845 577 806 060 624 124 751\,\pi^7}{195 910 410 240 000 000 000 000} + \frac{7 462 202 539 264 212 553\,\pi^9}{27 427 457 433 600 000 000 000} - \frac{2 056 295 769 648 937\,\pi^{11}}{838 061 199 360 000 000 000} + \frac{7 193 867 549\,\pi^{13}}{471 636 172 800 000 000} - \frac{137 165 317\,\pi^{15}}{2 118 432 476 160 000 000} + \frac{3071\,\pi^{17}}{20 007 417 830 400 000} - \frac{\pi^{19}}{121 645 100 408 832 000}\right) z^{18} + \frac{1}{\pi}$$
$$\left(\frac{75 407 856 888 133 723 986 383 774 586 393 031\,\pi}{761 699 675 013 120 000 000 000 000 000 000} - \frac{20 946 284 758 123 742 763 084 523 020 199\,\pi^3}{1 269 499 458 355 200 000 000 000 000 000} + \frac{5 818 032 908 672 660 283 778 222 471\,\pi^5}{7 052 774 768 640 000 000 000 000 000} - \frac{1 615 700 166 389 595 112 025 959\,\pi^7}{82 282 372 300 800 000 000 000 000} + \frac{448 342 799 392 293 597 511\,\pi^9}{1 645 647 446 016 000 000 000 000} - \frac{124 035 085 696 334 119\,\pi^{11}}{50 283 671 961 600 000 000 000} + \frac{3 083 323 514 741\,\pi^{13}}{198 087 192 576 000 000 000} - \frac{8 876 820 679\,\pi^{15}}{127 105 948 569 600 000 000} + \frac{1 974 391\,\pi^{17}}{9 603 560 558 592 000 000} - \frac{107\,\pi^{19}}{456 169 126 533 120 000}\right) z^{19} +$$
$$\frac{1}{\pi}\left(\frac{4 524 483 739 317 223 324 740 768 655 632 419 497\,\pi}{45 701 980 500 787 200 000 000 000 000 000 000} - \frac{1 256 790 769 354 858 960 508 851 434 445 513\,\pi^3}{76 169 967 501 312 000 000 000 000 000 000} + \frac{349 097 149 662 302 744 969 059 372 777\,\pi^5}{423 166 486 118 400 000 000 000 000 000} - \frac{96 958 799 007 822 681 627 514 633\,\pi^7}{4 936 942 338 048 000 000 000 000 000} + \frac{3 845 577 806 060 624 124 751\,\pi^9}{14 105 549 537 280 000 000 000 000} - \frac{678 382 049 024 019 323\,\pi^{11}}{274 274 574 336 000 000 000 000} + \frac{2 056 295 769 648 937\,\pi^{13}}{130 737 547 100 160 000 000 000} - \frac{7 193 867 549\,\pi^{15}}{99 043 596 288 000 000 000} + \frac{137 165 317\,\pi^{17}}{576 213 633 515 520 000 000} - \frac{3071\,\pi^{19}}{6 842 536 897 996 800 000} + \frac{\pi^{21}}{51 090 942 171 709 440 000}\right) z^{20} + O[z]^{21}$$

```
FullSimplify@
 Expand[1 / Gamma[z] / Gamma[1 - z] Sum[ (-1)^k / (z - k) ((x)^k) / k!, {k, 0, Infinity}]]
```

$$\frac{x^z}{\text{Gamma}[1+z]} + \frac{\text{ExpIntegralE}[1+z,\,x]\ \text{Sin}[\pi\,z]}{\pi}$$

$$\text{Plot}\left[\frac{\text{ExpIntegralE}[1+z,\,20.]\ \text{Sin}[\pi\,z]}{\pi},\ \{z,\,-8,\,8\}\right]$$



```
FullSimplify@Expand[ Sum[ 1 / (z - k ) x^k / k !, {k, 0, Infinity}]]
```

ExpIntegralE[1 + z, - x] - (- x)^{z} Gamma[- z]

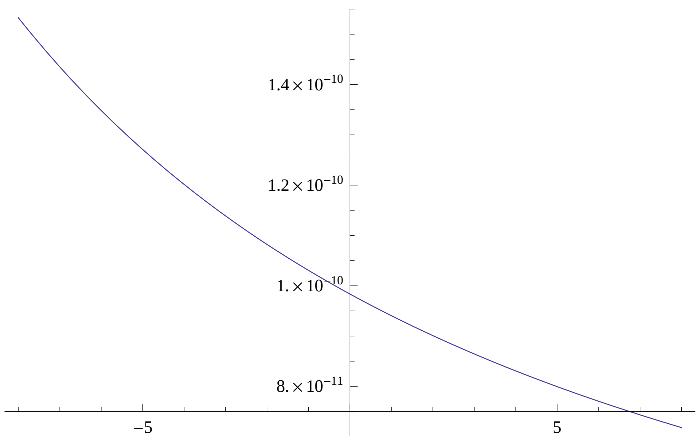
```
FullSimplify@Expand[ Sum[ (-1) ^k / (k ) ((x) ^k) / k !, {k, 1, Infinity}]]
```

- EulerGamma - Gamma[0, x] - Log[x]

```
FullSimplify@
 Expand[1 / Gamma[z] / Gamma[1 - z] Integrate[ (-1) ^k / (z - k) ((x) ^k) / k !, {k, 0, Infinity}]]
```

$$\frac{\left(\int_0^\infty \frac{(-1)^k x^k}{(-k+z)\, k!}\, dk\right) \mathrm{Sin}[\pi z]}{\pi}$$

```
Plot[ExpIntegralE[1 + z, 20.] , {z, -8, 8}]
```




```
1.4 + 1.7 + 1.7
```

4.8

```
FullSimplify[1 / Gamma[z] / Gamma[1 - z] Sum[(-1) ^k / (z - k) Binomial[x, k], {k, 0, Infinity}]]
```

$$\frac{\mathrm{Gamma}[1 + x]}{\mathrm{Gamma}[1 + x - z]\ \mathrm{Gamma}[1 + z]}$$

```
FullSimplify@Series[Sin[Pi z] / Pi (-1)^k / (z - k) x^k / k!, {z, 0, 20}]
```

$$-\frac{(-1)^k x^k z}{k^2\,\text{Gamma}[k]}-\frac{(-1)^k x^k z^2}{k^3\,\text{Gamma}[k]}+\frac{(-1)^k\left(-6+k^2\pi^2\right)x^k z^3}{6\,k^4\,\text{Gamma}[k]}+\frac{(-1)^k\left(-6+k^2\pi^2\right)x^k z^4}{6\,k^5\,\text{Gamma}[k]}-$$

$$\frac{\left((-1)^k\left(120-20\,k^2\pi^2+k^4\pi^4\right)x^k\right)z^5}{120\left(k^6\,\text{Gamma}[k]\right)}-\frac{\left((-1)^k\left(120-20\,k^2\pi^2+k^4\pi^4\right)x^k\right)z^6}{120\left(k^7\,\text{Gamma}[k]\right)}+$$

$$\frac{(-1)^k\left(-5040+840\,k^2\pi^2-42\,k^4\pi^4+k^6\pi^6\right)x^k z^7}{5040\,k^8\,\text{Gamma}[k]}+\frac{(-1)^k\left(-5040+840\,k^2\pi^2-42\,k^4\pi^4+k^6\pi^6\right)x^k z^8}{5040\,k^9\,\text{Gamma}[k]}-$$

$$\frac{\left((-1)^k\left(362\,880-60\,480\,k^2\pi^2+3024\,k^4\pi^4-72\,k^6\pi^6+k^8\pi^8\right)x^k\right)z^9}{362\,880\left(k^9\,k!\right)}-$$

$$\frac{\left((-1)^k\left(362\,880-60\,480\,k^2\pi^2+3024\,k^4\pi^4-72\,k^6\pi^6+k^8\pi^8\right)x^k\right)z^{10}}{362\,880\left(k^{11}\,\text{Gamma}[k]\right)}+$$

$$\left((-1)^k\left(-39\,916\,800+k^2\pi^2\left(6\,652\,800-332\,640\,k^2\pi^2+7920\,k^4\pi^4-110\,k^6\pi^6+k^8\pi^8\right)\right)x^k z^{11}\right)\Big/\left(39\,916\,800\,k^{12}\,\text{Gamma}[k]\right)+$$

$$\left((-1)^k\left(-39\,916\,800+k^2\pi^2\left(6\,652\,800-332\,640\,k^2\pi^2+7920\,k^4\pi^4-110\,k^6\pi^6+k^8\pi^8\right)\right)x^k z^{12}\right)\Big/\left(39\,916\,800\,k^{13}\,\text{Gamma}[k]\right)-\Big(\Big((-1)^k\Big(6\,227\,020\,800+k^2\pi^2\left(-1\,037\,836\,800+k^2\pi^2\left(51\,891\,840-1\,235\,520\,k^2\pi^2+17\,160\,k^4\pi^4-156\,k^6\pi^6+k^8\pi^8\right)\right)\Big)x^k\Big)z^{13}\Big)\Big/\left(6\,227\,020\,800\left(k^{14}\,\text{Gamma}[k]\right)\right)-\Big(\Big((-1)^k\Big(6\,227\,020\,800+k^2\pi^2\left(-1\,037\,836\,800+k^2\pi^2\left(51\,891\,840-1\,235\,520\,k^2\pi^2+17\,160\,k^4\pi^4-156\,k^6\pi^6+k^8\pi^8\right)\right)\Big)x^k\Big)z^{14}\Big)\Big/\left(6\,227\,020\,800\left(k^{15}\,\text{Gamma}[k]\right)\right)+$$

$$\frac{1}{\pi\,k!}(-1)^k\left(-\frac{\pi}{k^{15}}+\frac{\pi^3}{6\,k^{13}}-\frac{\pi^5}{120\,k^{11}}+\frac{\pi^7}{5040\,k^9}-\frac{\pi^9}{362\,880\,k^7}+\frac{\pi^{11}}{39\,916\,800\,k^5}-\frac{\pi^{13}}{6\,227\,020\,800\,k^3}+\frac{\pi^{15}}{1\,307\,674\,368\,000\,k}\right)x^k z^{15}+$$

$$\Big((-1)^k\Big(-1\,307\,674\,368\,000+k^2\pi^2\Big(217\,945\,728\,000+k^2\pi^2\Big(-10\,897\,286\,400+k^2\pi^2\left(259\,459\,200-3\,603\,600\,k^2\pi^2+32\,760\,k^4\pi^4-210\,k^6\pi^6+k^8\pi^8\right)\Big)\Big)\Big)x^k z^{16}\Big)\Big/\left(1\,307\,674\,368\,000\,k^{17}\,\text{Gamma}[k]\right)+$$

$$\frac{1}{\pi\,k!}(-1)^k\left(-\frac{\pi}{k^{17}}+\frac{\pi^3}{6\,k^{15}}-\frac{\pi^5}{120\,k^{13}}+\frac{\pi^7}{5040\,k^{11}}-\frac{\pi^9}{362\,880\,k^9}+\frac{\pi^{11}}{39\,916\,800\,k^7}-\frac{\pi^{13}}{6\,227\,020\,800\,k^5}+\frac{\pi^{15}}{1\,307\,674\,368\,000\,k^3}-\frac{\pi^{17}}{355\,687\,428\,096\,000\,k}\right)x^k z^{17}-$$

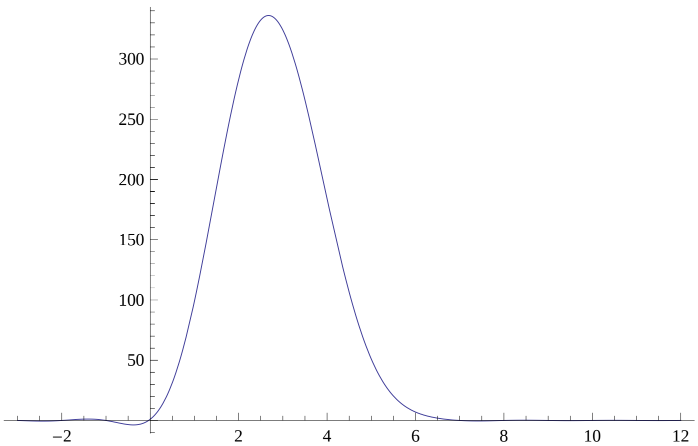
$$\Big(\Big((-1)^k\Big(355\,687\,428\,096\,000+k^2\pi^2\Big(-59\,281\,238\,016\,000+k^2\pi^2\Big(2\,964\,061\,900\,800+k^2\pi^2\Big(-70\,572\,902\,400+k^2\pi^2\left(980\,179\,200-8\,910\,720\,k^2\pi^2+57\,120\,k^4\pi^4-272\,k^6\pi^6+k^8\pi^8\right)\Big)\Big)\Big)\Big)x^k\Big)z^{18}\Big)\Big/\left(355\,687\,428\,096\,000\left(k^{19}\,\text{Gamma}[k]\right)\right)+$$

$$\frac{1}{\pi\,k!}(-1)^k\left(-\frac{\pi}{k^{19}}+\frac{\pi^3}{6\,k^{17}}-\frac{\pi^5}{120\,k^{15}}+\frac{\pi^7}{5040\,k^{13}}-\frac{\pi^9}{362\,880\,k^{11}}+\frac{\pi^{11}}{39\,916\,800\,k^9}-\frac{\pi^{13}}{6\,227\,020\,800\,k^7}+\frac{\pi^{15}}{1\,307\,674\,368\,000\,k^5}-\frac{\pi^{17}}{355\,687\,428\,096\,000\,k^3}+\frac{\pi^{19}}{121\,645\,100\,408\,832\,000\,k}\right)x^k z^{19}+$$

$$\frac{1}{\pi\,k!}(-1)^k\left(-\frac{\pi}{k^{20}}+\frac{\pi^3}{6\,k^{18}}-\frac{\pi^5}{120\,k^{16}}+\frac{\pi^7}{5040\,k^{14}}-\frac{\pi^9}{362\,880\,k^{12}}+\frac{\pi^{11}}{39\,916\,800\,k^{10}}-\frac{\pi^{13}}{6\,227\,020\,800\,k^8}+\frac{\pi^{15}}{1\,307\,674\,368\,000\,k^6}-\frac{\pi^{17}}{355\,687\,428\,096\,000\,k^4}+\frac{\pi^{19}}{121\,645\,100\,408\,832\,000\,k^2}\right)x^k z^{20}+O[z]^{21}$$

```
Clear[pp, qq]
pp[n_, j_, k_, z_] :=
 pp[n, j, k, z] = If[n < j, 0, 1 / (z - k) - pp[Floor[n / j], 2, k + 1, z] + pp[n, j + 1, k, z]]
ppx[n_, z_] := Sin[Pi z] / Pi (1 / z - pp[n, 2, 1, z])
ppz[n_, z_] := Limit[ppx[n, z2], z2 → z]
qq[n_, j_, k_, z_] :=
 qq[n, j, k, z] = If[n < j, 0, 1 / (z - k) - qq[n - j, 1, k + 1, z] + qq[n, j + 1, k, z]]
qqx[n_, z_] := Sin[Pi z] / Pi (1 / z - qq[n, 1, 1, z])
qqz[n_, z_] := Limit[qqx[n, z2], z2 → z]
```

```
Plot[ppx[100, z], {z, -3, 12}]
```




```
qqx[9, z]
```

$$\frac{\left(-\frac{1}{-9+z}+\frac{9}{-8+z}-\frac{36}{-7+z}+\frac{84}{-6+z}-\frac{126}{-5+z}+\frac{126}{-4+z}-\frac{84}{-3+z}+\frac{36}{-2+z}-\frac{9}{-1+z}+\frac{1}{z}\right)\text{Sin}[\pi z]}{\pi}$$

```
ppz[100, 1 / 2]
```

$$\frac{113\,678}{1155\,\pi}$$

```
Limit[D[ppx[100, z], z], z → 0]
```

$$\frac{428}{15}$$

```
Sum[(-1)^k / (z - k) x^k, {k, 0, Infinity}]
```

-HurwitzLerchPhi[-x, 1, -z]

```
Sum[(-1)^n / (a - n) z^n, {n, 0, Infinity}]
```

-HurwitzLerchPhi[-z, 1, -a]

```
FullSimplify@
 Expand[1 / Gamma[z] / Gamma[1 - z] Sum[(-1)^k / (z - k) x^k / Gamma[k + 1], {k, 0, Infinity}]]
```

$$\frac{x^z}{\text{Gamma}[1+z]}+\frac{\text{ExpIntegralE}[1+z,\ x]\ \text{Sin}[\pi z]}{\pi}$$

```
Gamma[1 - z] /. z → .3
```

1.29806

```
Gamma[-z] (-z) /. z → .3
```

1.29806

$$\frac{x^z\,\text{Gamma}[1-z]}{z} + x^z\,\text{Gamma}[-z, x] \;/.\; z \to .3 \;/.\; x \to 1.4$$

$4.89096 + 5.98969 \times 10^{-16}\, i$

$$-x^z\,\text{Gamma}[-z] + x^z\,\text{Gamma}[-z, x] \;/.\; z \to .3 \;/.\; x \to 1.4$$

$4.89096 + 5.98969 \times 10^{-16}\, i$

$$-x^z\,\text{Gamma}[-z, 0, x] \;/.\; z \to .3 \;/.\; x \to 1.4$$

$4.89096 + 5.98969 \times 10^{-16}\, i$

$$\text{Expand@FullSimplify@Expand}[(-x^z\,\text{Gamma}[-z] + x^z\,\text{Gamma}[-z, x]) / \text{Gamma}[z] / \text{Gamma}[1-z]]$$

$$\frac{\text{ExpIntegralE}[1+z, x]\,\text{Sin}[\pi z]}{\pi} - \frac{x^z\,\text{Gamma}[-z]\,\text{Sin}[\pi z]}{\pi}$$

$$\text{D}\left[\frac{x^z}{\text{Gamma}[1+z]} + \frac{\text{ExpIntegralE}[1+z, x]\,\text{Sin}[\pi z]}{\pi}, z\right] /.\; z \to 0$$

```
EulerGamma + ExpIntegralE[1, x] + Log[x]
```

```
Sin[.5 Pi] / Pi Sum[(-1)^k / (.5 - k) 10.^k / Gamma[k + 1], {k, 0, Infinity}]
```

3.56825

```
10^.5 / (.5)!
```

3.56825

```
D[x^z / z!, z] /. z → 0
```

```
EulerGamma + Log[x]
```

```
D[Binomial[x, z], z] /. z → 0
```

```
EulerGamma + PolyGamma[0, 1 + x]
```

```
D[x^z, z] /. z → 0
```

```
Log[x]
```

```
D[Binomial[x, z] z!, z] /. z → 0
```

```
PolyGamma[0, 1 + x]
```

```
Expand@FullSimplify[ppx[100, z] / (Sin[Pi z] / Pi) Product[(z - k), {k, 0, 6}]] /. z → 2
```

13584

```
po[n_] := List @@
  NRoots[Expand@FullSimplify[ppx[n, z] / (Sin[Pi z] / Pi) Product[z - k, {k, 0, Log2@n}]] == 0,
    z][[All, 2]]
```

```
Limit[FullSimplify[
  Expand[6 ! Product[1 - z / r, {r, po[100]}] / Product[z - k, {k, 0, 6}] Sin[Pi z] / Pi]], z → 2]
```

$283. + 0.\, i$

```
Limit[FullSimplify[
  Expand[6! Product[1 - z / r, {r, po[100]}] / Product[z - k, {k, 0, 6}] Sin[Pi z] / Pi]], z → 3]
```

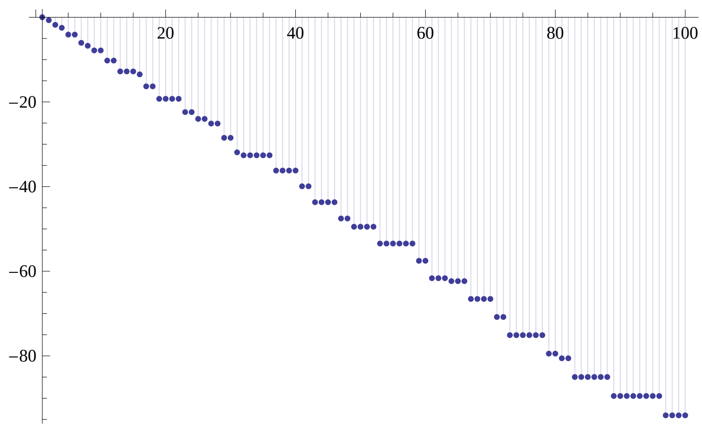
324. + 0. i

```
Sum[-1 / r, {r, po[100]}]
```

26.0833 + 0. i

```
pps[n_, j_, k_, z_, s_] := pps[n, j, k, z, s] =
  If[n < j, 0, j^-s (1 / (z - k) - pps[Floor[n / j], 2, k + 1, z, s]) + pps[n, j + 1, k, z, s]]
ppsx[n_, z_, s_] := Sin[Pi z] / Pi  (1 / z - pps[n, 2, 1, z, s])
ppsz[n_, z_, s_] := Limit[ppsx[n, z2, s], z2 → z]
```

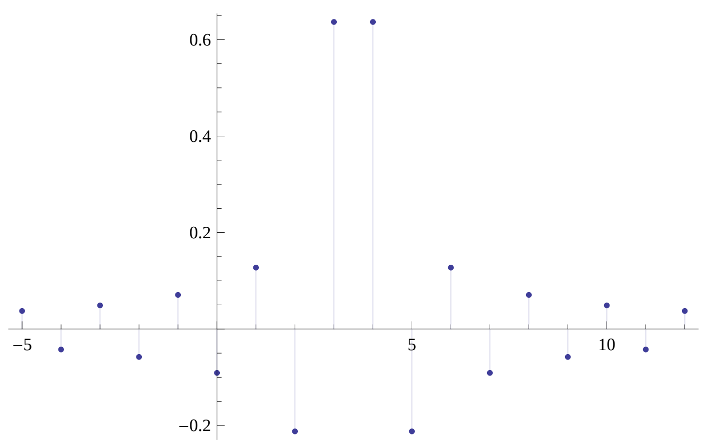
```
DiscretePlot[N[D[D[ppsx[n, z, s], s] /. s → 0, z] /. z → 0], {n, 1, 100}]
```




```
FullSimplify[1 / (z - 2) / Gamma[z] / Gamma[1 - z]]
```

$$\frac{\text{Sin}[\pi z]}{\pi(-2+z)}$$

```
DiscretePlot[Re[(-1)^k Sin[Pi z] / Pi / (z - k)] /. z → 3.5, {k, -5, 12}]
```




```
Sum[1 / Gamma[z] / Gamma[1 - z] (-1)^k / (z - k) x^k / k!, {k, -1000, Infinity}]
```

$$\frac{x^z\,(\text{Gamma}[1-z] + z\,\text{Gamma}[-z, x])}{z\,\text{Gamma}[1-z]\,\text{Gamma}[z]}$$

Table[Limit[Sin[Pi - 3] / Pi (-1)^k / (-3 - k) x^k / k!, k → k2], {k2, -15, 15}]

$$\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{2\,\mathrm{Sin}[3]}{\pi\, x^3}, 0, 0, -\frac{\mathrm{Sin}[3]}{3\,\pi}, \frac{x\,\mathrm{Sin}[3]}{4\,\pi}, -\frac{x^2\,\mathrm{Sin}[3]}{10\,\pi}, \frac{x^3\,\mathrm{Sin}[3]}{36\,\pi}, -\frac{x^4\,\mathrm{Sin}[3]}{168\,\pi}, \frac{x^5\,\mathrm{Sin}[3]}{960\,\pi}, -\frac{x^6\,\mathrm{Sin}[3]}{6480\,\pi}, \frac{x^7\,\mathrm{Sin}[3]}{50400\,\pi}, -\frac{x^8\,\mathrm{Sin}[3]}{443520\,\pi}, \frac{x^9\,\mathrm{Sin}[3]}{4354560\,\pi}, -\frac{x^{10}\,\mathrm{Sin}[3]}{47174400\,\pi}, \frac{x^{11}\,\mathrm{Sin}[3]}{558835200\,\pi}, -\frac{x^{12}\,\mathrm{Sin}[3]}{7185024000\,\pi}, \frac{x^{13}\,\mathrm{Sin}[3]}{99632332800\,\pi}, -\frac{x^{14}\,\mathrm{Sin}[3]}{1482030950400\,\pi}, \frac{x^{15}\,\mathrm{Sin}[3]}{23538138624000\,\pi}\right\}$$

(-3)!

ComplexInfinity

FullSimplify@Expand[1 / Gamma[z] / Gamma[1 - z] Sum[(-1)^k / (z - k) 1 / k!, {k, 0, Infinity}]]

$$\frac{1}{\mathrm{Gamma}[1+z]} + \frac{\mathrm{ExpIntegralE}[1+z, 1]\,\mathrm{Sin}[\pi\, z]}{\pi}$$

FullSimplify@
Expand[1 / Gamma[z] / Gamma[1 - z] Sum[(-1)^k / (z - k) Pochhammer[x, k] / k!, {k, 0, Infinity}]]

$$\frac{\mathrm{Hypergeometric2F1}[x, -z, 1-z, -1]\,\mathrm{Sin}[\pi\, z]}{\pi\, z}$$

$$\frac{\mathrm{Hypergeometric2F1}[x, -z, 1-z, -1]\,\mathrm{Sin}[\pi\, z]}{\pi\, z}\ /.\ x \to 7.\ /.\ z \to 3.3$$

112.366

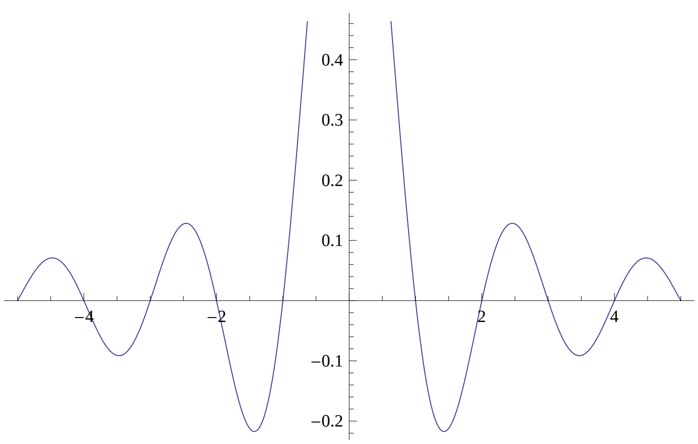
Pochhammer[7, 3.3] / (3.3)!

112.366

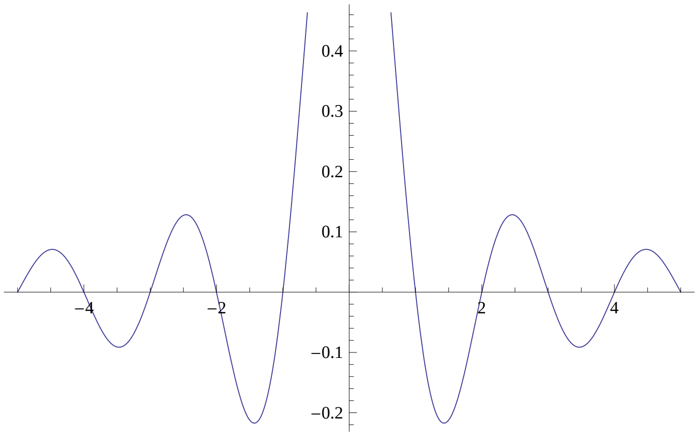
FullSimplify@
Expand[1 / Gamma[z] / Gamma[1 - z] Sum[(-1)^k / (z - k) Binomial[x, k], {k, 0, Infinity}]]

$$\frac{\mathrm{Gamma}[1+x]}{\mathrm{Gamma}[1+x-z]\,\mathrm{Gamma}[1+z]}$$

Plot[Sin[Pi z] / (Pi (z)), {z, -5, 5}]



```
Plot[Binomial[0, z], {z, -5, 5}]
```




```
FactorialPower[x, 3] /. x → 5
```

60

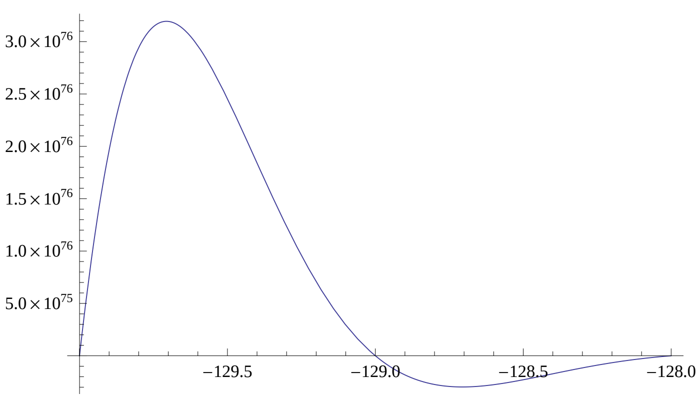
$5 \times 4 \times 3$

60

```
Expand[1 / Gamma[z] / Gamma[1 - z] Sum[ (-1) ^k / (z - k) x^k / (k !), {k, 0, Infinity}]] /. x → n
```

$$\frac{n^z}{z\,\text{Gamma}[z]} + \frac{n^z\,\text{Gamma}[-z, n]}{\text{Gamma}[1-z]\,\text{Gamma}[z]}$$

$$\text{Plot}\left[\frac{\text{ExpIntegralE}[1+z, 12]\,\text{Sin}[\pi z]}{\pi}, \{z, -130, -128\}\right]$$



$$\frac{n^z\,\text{Gamma}[-z, n]}{\text{Gamma}[1-z]\,\text{Gamma}[z]} \text{ /. } n \to -3.3 \text{ /. } z \to 1.3$$

3.11237 + 3.27388 i

$$-\frac{n^z\,\text{Gamma}[-z, n]}{\text{Gamma}[-z]\,\text{Gamma}[z+1]} \text{ /. } n \to -3.3 \text{ /. } z \to 1.3$$

3.11237 + 3.27388 i

$$-\frac{n^z \text{GammaRegularized}[-z, n]}{\text{Gamma}[z+1]} \text{ /. } n \to -3.3 \text{ /. } z \to 1.3$$

3.11237 + 3.27388 i

$$\frac{n^z}{z \text{ Gamma}[z]} + \frac{n^z \text{ Gamma}[-z, n]}{\text{Gamma}[1-z] \text{ Gamma}[z]} \text{ /. } n \to 13.3 \text{ /. } z \to 1.3$$

24.7767

$$\frac{n^z}{\text{Gamma}[z+1]} (1 - \text{GammaRegularized}[-z, n]) \text{ /. } n \to 13.3 \text{ /. } z \to 1.3$$

24.7767

$$\frac{n^z}{\text{Gamma}[z+1]} (\text{GammaRegularized}[-z, 0, n]) \text{ /. } n \to 13.3 \text{ /. } z \to 1.3$$

24.7767

```
FullSimplify[1 / Gamma[z] / Gamma[1 - z] / GammaRegularized[-z, 0, x]
  Sum[ (-1)^k / (z - k) x^k / (k!), {k, 0, Infinity}]]
```

$$-\frac{x^z \text{ Gamma}[-z] \text{ Sin}[\pi z]}{\pi}$$

```
17^(5 / 3) / Gamma[5 / 3 + 1]
```

$$\frac{17 \times 17^{2/3}}{\text{Gamma}\left[\frac{8}{3}\right]}$$

```
FullSimplify[Gamma[-z] / Gamma[z] / z / Gamma[-z] / Gamma[-z, 0, x]]
```

$$\frac{1}{z \text{ Gamma}[z] \text{ Gamma}[-z, 0, x]}$$

$$-\frac{x^z \text{ Gamma}[-z] \text{ Sin}[\pi z]}{\pi} \text{ /. } x \to 12.3 \text{ /. } z \to 2.2$$

103.101

```
12.3^2.2 / (2.2!)
```

103.101

```
FullSimplify[z Gamma[z] Gamma[-z, 0, x]]
```

z Gamma[z] Gamma[-z, 0, x]

```
FullSimplify[-1 / Gamma[z + 1] / Gamma[-z, 0, x] / GammaRegularized[-z, 0, x]
  Sum[ (-1)^k / (z - k) x^k / (k!), {k, 0, Infinity}]]
```

$$\frac{x^z}{\text{Gamma}[1+z] \text{ GammaRegularized}[-z, 0, x]}$$

$$\frac{x^z}{\text{Gamma}[1+z] \text{ GammaRegularized}[-z, 0, x]} \text{ /. } x \to 12.3 \text{ /. } z \to 2.2$$

103.101

$$-\frac{x^z \text{ Gamma}[-z] \text{ Sin}[\pi z]}{\pi} \text{ /. } z \to -40.2 \text{ /. } x \to 62.3$$

5.79129×10^{-27}

```
FullSimplify@Expand[1 / Gamma[z] / Gamma[1 - z] / GammaRegularized[-z, 0, x]
   Sum[ (-1)^k / (z - k) x^k / (k!), {k, 0, Infinity}]]
```

$$-\frac{x^z \, \text{Gamma}[-z] \, \text{Sin}[\pi z]}{\pi}$$

$$\text{Limit}\left[-\frac{\text{Gamma}[-z] \, \text{Sin}[\pi z]}{\pi}, z \to 1/2\right]$$

$$\frac{2}{\sqrt{\pi}}$$

```
1 / (1 / 2)!
```

$$\frac{2}{\sqrt{\pi}}$$

```
FullSimplify@Expand[Gamma[-z] / Gamma[z] / Gamma[-z] / -z / Gamma[-z, 0, x]
   Sum[ (-1)^k / (z - k) x^k / (k!), {k, 0, Infinity}]]
```

$$\frac{x^z}{\text{Gamma}[1+z]}$$

```
FullSimplify@
 Expand[1 / Gamma[z] / -z / Gamma[-z, 0, x] Sum[ (-1)^k / (z - k) x^k / (k!), {k, 0, Infinity}]]
```

$$\frac{x^z}{\text{Gamma}[1+z]}$$

```
FullSimplify@
 Expand[-1 / (Gamma[1 + z] Gamma[-z, 0, x]) Sum[ (-1)^k / (z - k) x^k / (k!), {k, 0, Infinity}]]
```

$$\frac{x^z}{\text{Gamma}[1+z]}$$

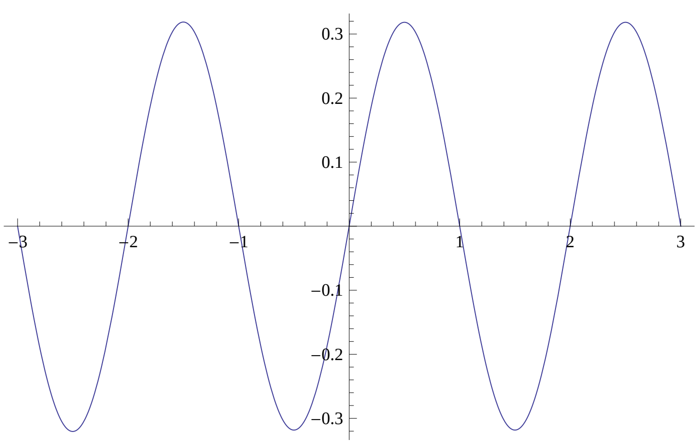
```
FullSimplify[Sin[Pi z] / Pi / GammaRegularized[-z, 0, x]
    Sum[ (-1)^k / (z - k) x^k / (k!), {k, 0, Infinity}]] /. x → 13.3 /. z → 4.2
```

1611.58

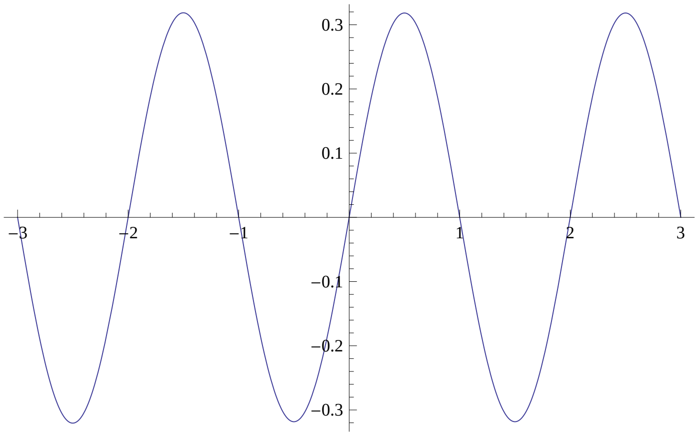
```
FullSimplify@Expand[-1 / z! / ( Gamma[-z, 0, x])
     Sum[ (-1)^k / (z - k) x^k / (k!), {k, 0, Infinity}]] /. x → 13.3 /. z → 4.2
```

1611.58

```
Plot[ -1 / z! / Gamma[-z, 0, 8.], {z, -3, 3}]
```



```
Plot[Sin[Pi z] / Pi / GammaRegularized[-z, 0, 8.], {z, -3, 3}]
```



```
FullSimplify@Table[(-1)^k / (z - k) 17.^k / k!, {k, 0, 55}]
```

$$\left\{\frac{1.}{z}, -\frac{17.}{-1.+z}, \frac{144.5}{-2.+z}, -\frac{818.833}{-3.+z}, \frac{3480.04}{-4.+z}, -\frac{11832.1}{-5.+z}, \frac{33524.4}{-6.+z}, -\frac{81416.4}{-7.+z}, \frac{173010.}{-8.+z}, -\frac{326796.}{-9.+z}, \frac{555554.}{-10.+z}, -\frac{858583.}{-11.+z}, \frac{1.21633\times10^{6}}{-12.+z}, -\frac{1.59058\times10^{6}}{-13.+z}, \frac{1.93142\times10^{6}}{-14.+z}, -\frac{2.18894\times10^{6}}{-15.+z}, \frac{2.32575\times10^{6}}{-16.+z}, -\frac{2.32575\times10^{6}}{-17.+z}, \frac{2.19654\times10^{6}}{-18.+z}, -\frac{1.96533\times10^{6}}{-19.+z}, \frac{1.67053\times10^{6}}{-20.+z}, -\frac{1.35233\times10^{6}}{-21.+z}, \frac{1.04498\times10^{6}}{-22.+z}, -\frac{772380.}{-23.+z}, \frac{547102.}{-24.+z}, -\frac{372029.}{-25.+z}, \frac{243250.}{-26.+z}, -\frac{153157.}{-27.+z}, \frac{92988.4}{-28.+z}, -\frac{54510.5}{-29.+z}, \frac{30889.3}{-30.+z}, -\frac{16939.3}{-31.+z}, \frac{8998.99}{-32.+z}, -\frac{4635.84}{-33.+z}, \frac{2317.92}{-34.+z}, -\frac{1125.85}{-35.+z}, \frac{531.65}{-36.+z}, -\frac{244.272}{-37.+z}, \frac{109.279}{-38.+z}, -\frac{47.6346}{-39.+z}, \frac{20.2447}{-40.+z}, -\frac{8.39415}{-41.+z}, \frac{3.39763}{-42.+z}, -\frac{1.34325}{-43.+z}, \frac{0.518983}{-44.+z}, -\frac{0.19606}{-45.+z}, \frac{0.0724571}{-46.+z}, -\frac{0.0262079}{-47.+z}, \frac{0.00928195}{-48.+z}, -\frac{0.00322027}{-49.+z}, \frac{0.00109489}{-50.+z}, -\frac{0.000364964}{-51.+z}, \frac{0.000119315}{-52.+z}, -\frac{0.0000382709}{-53.+z}, \frac{0.0000120482}{-54.+z}, -\frac{3.724\times10^{-6}}{-55.+z}\right\}$$

```
-1 / Gamma[-1 / 2, 0, 17.]
```

0.282095

```
Integrate[ Log[x]^(z - 1) / (z - 1)!, {x, 1, n}]
```

$$\text{ConditionalExpression}\left[\frac{(\text{Gamma}[z] - \text{Gamma}[z, -\text{Log}[n]])\,(-\text{Log}[n])^{-z}\,\text{Log}[n]^{z}}{(-1+z)!}, \text{Re}[z] > 0\right]$$

$$\text{FullSimplify}[(\text{GammaRegularized}[z, 0, -\text{Log}[n]])\,(-1)\text{^}-z\,(\text{Log}[n])^{-z}\,\text{Log}[n]^{z}]$$

$$(-1)^{-z}\,\text{GammaRegularized}[z, 0, -\text{Log}[n]]$$

$$\text{N}[(-1)^{-z}\,\text{GammaRegularized}[z, 0, -\text{Log}[n]] \;/.\; n \to 100 \;/.\; z \to 2.5]$$

$532.148 + 0.\,i$

```
Hypergeometric1F1[z, z + 1, Log[n]] Log[n]^z / z! /. n → 100. /. z → 2.5
```

$532.148 + 3.22168\times10^{-17}\,i$

```
N[(-1)^-z GammaRegularized[z, 0, -n] /. n → 3. /. z → 2.5]
```

47.5073 + 0. ⅈ

```
n^z / z! /. n → 3. /. z → 2.5
```

4.69058

```
Sin[Pi z] / Pi / GammaRegularized[-z, 0, x]
   Sum[ (-1)^k / (z - k) x^k / (k!), {k, 0, Infinity}] /. x → 13.3 /. z → 4.2
```

1611.58

```
x^z / z! /. x → 13.3 /. z → 4.2
```

1611.58

```
bb[x_, z_] := Sin[Pi z] / Pi / GammaRegularized[z, 0, x]
  Sum[ (-1)^k / (z - k) Hypergeometric1F1[k, k + 1, Log[x]] Log[x]^k / k!, {k, 0, 80}]
bb2[x_, z_] := (-1)^-z GammaRegularized[z, 0, -Log[x]]
```

```
bb[131.1, 7.2]
```

961.377

```
bb2[131.1, 7.2]
```

$961.298 + 5.68434 \times 10^{-14}\, ⅈ$

```
Expand[Sum[ 1 / Gamma[z] / Gamma[1 - z] (-1)^k / (z - k)
   Hypergeometric1F1[k, k + 1, Log[x]] Log[x]^k / k!, {k, 0, Infinity}]]
```

$$\sum_{k=0}^{\infty} \frac{(-1)^k\,(\text{Gamma}[1+k] - k\,\text{Gamma}[k, -\text{Log}[x]])\,(-\text{Log}[x])^{-k}\,\text{Log}[x]^k}{(-k+z)\,k!\,\text{Gamma}[1-z]\,\text{Gamma}[z]}$$

```
D[-1 / (Gamma[1 + z] Gamma[-z, 0, x]) Sum[ (-1)^k / (z - k) x^k / (k!), {k, 0, Infinity}], x]
```

$$\frac{e^{-x}\,(\text{Gamma}[1-z] + z\,\text{Gamma}[-z, x])}{x\,z\,\text{Gamma}[1+z]\,\text{Gamma}[-z, 0, x]^2} + \frac{e^{-x}}{x\,\text{Gamma}[1+z]\,\text{Gamma}[-z, 0, x]} - \frac{x^{-1+z}\,(\text{Gamma}[1-z] + z\,\text{Gamma}[-z, x])}{\text{Gamma}[1+z]\,\text{Gamma}[-z, 0, x]}$$

$$\text{Integrate}\left[\frac{e^{-x}\,(\text{Gamma}[1-z] + z\,\text{Gamma}[-z, x])}{x\,z\,\text{Gamma}[1+z]\,\text{Gamma}[-z, 0, x]^2} + \frac{e^{-x}}{x\,\text{Gamma}[1+z]\,\text{Gamma}[-z, 0, x]} - \frac{x^{-1+z}\,(\text{Gamma}[1-z] + z\,\text{Gamma}[-z, x])}{\text{Gamma}[1+z]\,\text{Gamma}[-z, 0, x]}, x\right]$$

$$-\frac{x^z\,(\text{Gamma}[1-z] + z\,\text{Gamma}[-z, x])}{z\,\text{Gamma}[1+z]\,(\text{Gamma}[-z] - \text{Gamma}[-z, x])}$$

```
D[-1 / z! / Gamma[-z, 0, x] Sum[(-1)^k / (z - k) x^k / k!, {k, 0, Infinity}], x] /. x → 13.3 /.
 z → 3.2
```

122.447

```
x^(z - 1) / (z - 1)! /. x → 13.3 /. z → 3.2
```

122.447

```
1 / Gamma[z] / Gamma[1 - z] /. z → 13.3
```

-0.257518

```
1 / Gamma[z + 1] / Gamma[-z] / (-1) /. z → 13.3
```

-0.257518

```
Gamma[-3.3]
```

0.438517

```
Gamma[-3.3]
```

0.438517

```
Sum[Gamma[k, 0, -Log[x]] / Gamma[k] / (z - k), {k, 0, Infinity}]
```

$$\sum_{k=0}^{\infty} \frac{\text{Gamma}[k, 0, -\text{Log}[x]]}{(-k + z)\ \text{Gamma}[k]}$$

```
Integrate[ 1, {x, 1, n}]
```

$-1 + n$

```
Expand@Integrate[1, {x, 1, n}, {y, 1, n / x}]
```

ConditionalExpression$[1 - n + n\ \text{Log}[n],\ \text{Re}[n] \geq 0\ ||\ n \notin \text{Reals}]$

```
Expand@Integrate[1, {x, 1, n}, {y, 1, n / x}, {z, 1, n / (x y)}]
```

$$\text{ConditionalExpression}\left[-1 + n - n\ \text{Log}[n] + \frac{1}{2}\ n\ \text{Log}[n]^2,\ \text{Re}[n] \geq 0\ ||\ n \notin \text{Reals}\right]$$

$$-1 + n - n\ \text{Log}[n] + \frac{1}{2}\ n\ \text{Log}[n]^2\ /.\ n \to 13.3$$

22.4146

```
(-1) ^ (-3) GammaRegularized[3, 0, -Log[13.3]]
```

$22.4146 - 8.235 \times 10^{-15}\ i$

```
pk[x_, k_] := 1 - x Sum[(-Log[x]) ^ j / j !, {j, 0, k - 1}]
```

```
Expand@pk[n, 4]
```

$$1 - n + n\ \text{Log}[n] - \frac{1}{2}\ n\ \text{Log}[n]^2 + \frac{1}{6}\ n\ \text{Log}[n]^3$$

```
Gamma[k, 0, -Log[x]] / Gamma[k] /. x → 13. /. k → 4
```

$15.1429 - 7.4179 \times 10^{-15}\ i$

```
Table[Expand[pk[n, k] / (z - k)], {k, 0, 5}] // TableForm
```

$$\frac{1}{z}$$
$$\frac{1}{-1+z} - \frac{n}{-1+z}$$
$$\frac{1}{-2+z} - \frac{n}{-2+z} + \frac{n\ \text{Log}[n]}{-2+z}$$
$$\frac{1}{-3+z} - \frac{n}{-3+z} + \frac{n\ \text{Log}[n]}{-3+z} - \frac{n\ \text{Log}[n]^2}{2\ (-3+z)}$$
$$\frac{1}{-4+z} - \frac{n}{-4+z} + \frac{n\ \text{Log}[n]}{-4+z} - \frac{n\ \text{Log}[n]^2}{2\ (-4+z)} + \frac{n\ \text{Log}[n]^3}{6\ (-4+z)}$$
$$\frac{1}{-5+z} - \frac{n}{-5+z} + \frac{n\ \text{Log}[n]}{-5+z} - \frac{n\ \text{Log}[n]^2}{2\ (-5+z)} + \frac{n\ \text{Log}[n]^3}{6\ (-5+z)} - \frac{n\ \text{Log}[n]^4}{24\ (-5+z)}$$

```
FullSimplify@Sum[pk[n, k] / (z - k), {k, 0, Infinity}]
```

$$\sum_{k=0}^{\infty} \frac{1 - \frac{\text{Gamma}[k, -\text{Log}[n]]}{\text{Gamma}[k]}}{-k + z}$$

```
((-1)^(-z) Gamma[z, 0, -Log[x]] / Gamma[z]) /
 Sum[(-1)^k / (z - k) (-1)^(-k) Gamma[k, 0, -Log[x]] / Gamma[k], {k, 0, Infinity}]
```

$$\frac{(-1)^{-z}\,\text{Gamma}[z, 0, -\text{Log}[x]]}{\text{Gamma}[z] \sum_{k=0}^{\infty} \frac{\text{Gamma}[k,0,-\text{Log}[x]]}{(-k+z)\,\text{Gamma}[k]}}$$