

```

zb[t_, s_] := E^((t + ArcTan[2 s]) I) Zeta[1 / 2 - s I] + E^(-(t + ArcTan[2 s]) I) Zeta[1 / 2 + s I]
zb1[s_] := zb[0, s]
zb2[s_] := zb[-Pi / 2, s]
zt[s_] := (zb1[s] + I zb2[s]) / 2
zt2[s_] := -(zb1[s] - I zb2[s]) / (2 I)
rat[s_] := zb1[s] / zb2[s]

```

$$\text{rata}[s_] := i - \frac{2 i}{1 - \frac{e^{2 i \text{ArcTan}[2 s]} \text{Zeta}\left[\frac{1}{2} - i s\right]}{\text{Zeta}\left[\frac{1}{2} + i s\right]}}$$

```

zc[t_, s_] := E^((t) I) Zeta[1 / 2 - s I] + E^(-(t) I) Zeta[1 / 2 + s I]
zc1[s_] := zc[0, s]
zc2[s_] := zc[-Pi / 2, s]
ztc[s_] := (zc1[s] + I zc2[s]) / 2
ztc2[s_] := -(zc1[s] - I zc2[s]) / (2 I)

```

```
FullSimplify[zb1[s] + I zb2[s]]
```

$$2 e^{i \text{ArcTan}[2 s]} \text{Zeta}\left[\frac{1}{2} - i s\right]$$

```
FullSimplify[zt[s]]
```

$$e^{i \text{ArcTan}[2 s]} \text{Zeta}\left[\frac{1}{2} - i s\right]$$

```
FullSimplify[zb1[s] / zb2[s]]
```

$$i - \frac{2 i}{1 - \frac{e^{2 i \text{ArcTan}[2 s]} \text{Zeta}\left[\frac{1}{2} - i s\right]}{\text{Zeta}\left[\frac{1}{2} + i s\right]}}$$

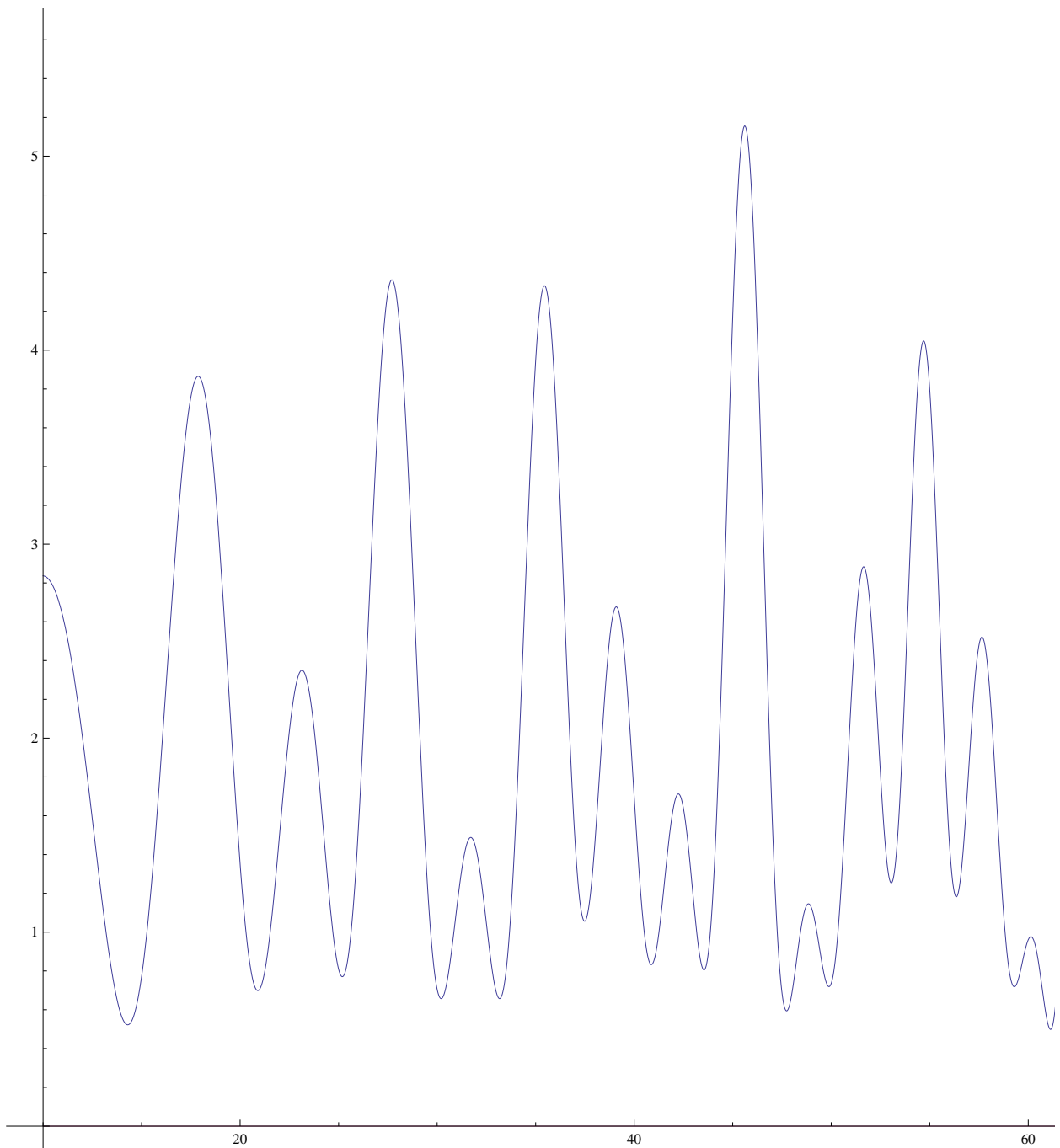
$$\text{FullSimplify}\left[i - \frac{2 i}{1 - e^{2 i \text{ArcTan}[2 s]}}\right]$$

$$\frac{1}{2 s}$$

```
rat[N@Im@ZetaZero@3]
```

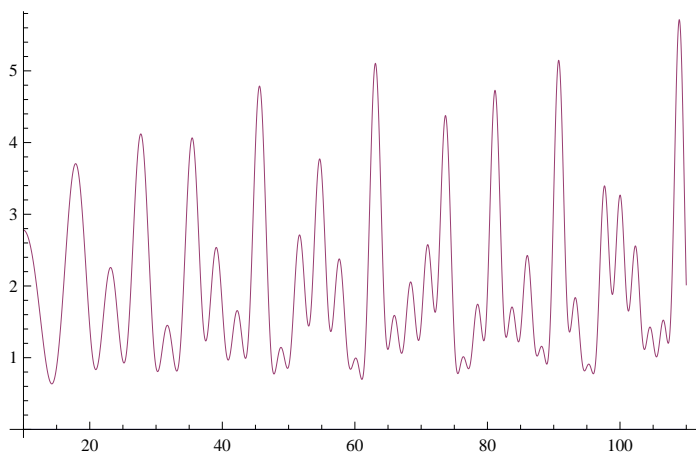
$$0.271911 + 0. i$$

```
Plot[{Im@zb1[s + .4 I] + Re@zb2[s + .4 I], 0}, {s, 10, 100}]
```



```
(* ??? This doesn't seem to cross the line 0 ever. *)
```

```
Plot[{0, Im@zb1[s + .5 I] + Re@zb2[s + .5 I]}, {s, 10, 110}]
```



```
Integrate[Sin[s x] / x, {x, 0, n}]
```

```
SinIntegral[n s]
```

```
Integrate[Cos[s x] / x, {x, 2, n}]
```

```
ConditionalExpression[-CosIntegral[2 s] + CosIntegral[n s], Re[n] ≥ 0 || n ∈ Reals]
```

```
Sum[Sin[s x] / x, {x, 1, n}]
```

$$\frac{1}{2} i e^{-i s} \left(- \left(e^{-i s} \right)^n \text{LerchPhi} \left[e^{-i s}, 1, 1+n \right] + e^{2 i s} \left(e^{i s} \right)^n \text{LerchPhi} \left[e^{i s}, 1, 1+n \right] + e^{i s} \text{Log} \left[1 - e^{i s} \right] - e^{i s} \text{Log} \left[e^{-i s} \left(-1 + e^{i s} \right) \right] \right)$$

```
Sum[Cos[s x] / x, {x, 1, n}]
```

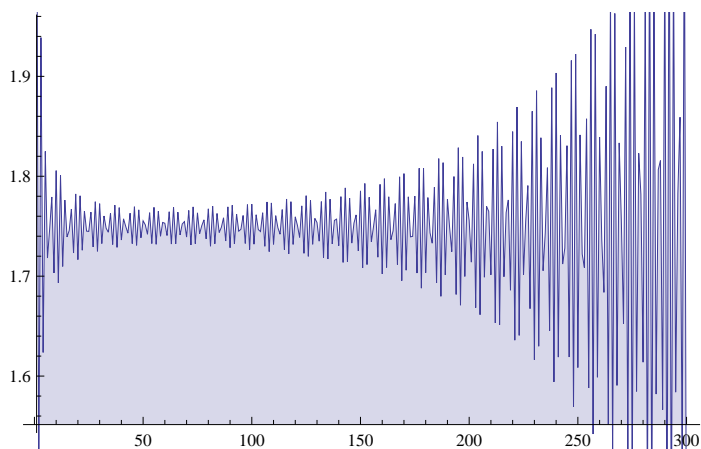
$$-\frac{1}{2} e^{-i s} \left(\left(e^{-i s} \right)^n \text{LerchPhi} \left[e^{-i s}, 1, 1+n \right] + e^{2 i s} \left(e^{i s} \right)^n \text{LerchPhi} \left[e^{i s}, 1, 1+n \right] + e^{i s} \text{Log} \left[1 - e^{i s} \right] + e^{i s} \text{Log} \left[e^{-i s} \left(-1 + e^{i s} \right) \right] \right)$$

```
pa[n_, s_] := Sum[Sin[s x] / x, {x, 1, n}] - Integrate[Sin[s x] / x, {x, 0, n}]
```

```
pa2[n_, s_] := Sum[Sin[s x] / x, {x, 1, n}] - SinIntegral[n s]
```

```
ca[n_, s_] := Integrate[(Cos[s Ceiling[x]] - Cos[s x]) / x, {x, 2, n}]
```

```
DiscretePlot[Re@pa2[n, -3.5 - .02 I], {n, 1, 300}]
```



Pi / 4.

0.785398

Limit $\left[\left(\frac{1}{2} i e^{-i s} \left(-\left(e^{-i s}\right)^n \text{LerchPhi}\left[e^{-i s}, 1, 1+n\right]+e^{2 i s}\left(e^{i s}\right)^n \text{LerchPhi}\left[e^{i s}, 1, 1+n\right]+e^{i s} \text{Log}\left[1-e^{i s}\right]-e^{i s} \text{Log}\left[e^{-i s}\left(-1+e^{i s}\right)\right]\right)\right)-\text{SinIntegral}[n s], n \rightarrow \text{Infinity}\right]$

Limit $\left[\frac{1}{2} i e^{-i s} \left(-\left(e^{-i s}\right)^n \text{LerchPhi}\left[e^{-i s}, 1, 1+n\right]+e^{2 i s}\left(e^{i s}\right)^n \text{LerchPhi}\left[e^{i s}, 1, 1+n\right]+e^{i s} \text{Log}\left[1-e^{i s}\right]-e^{i s} \text{Log}\left[e^{-i s}\left(-1+e^{i s}\right)\right]\right)-\text{SinIntegral}[n s], n \rightarrow \infty\right]$

Integrate[(Sin[s x] - Sin[Ceiling[s] x]) / x, {x, 0, n}]

SinIntegral[n s] - SinIntegral[n Ceiling[s]]

Limit[SinIntegral[n s] - SinIntegral[n Ceiling[s]], n → Infinity]

Limit[SinIntegral[n s] - SinIntegral[n Ceiling[s]], n → ∞]

ca[100., s]

\$Aborted

Cos[s Log[j] + ArcTan[2 s]] /. s → 22 + .1 I /. j → 7

0.947901 - 0.0719083 i

Cos[s Log[j] + Log[E^ArcTan[2 s]]] /. s → 22 + .1 I /. j → 7

0.947901 - 0.0719083 i

Cos[s Log[j] + s Log[(E^ArcTan[2 s])^(1/s)]] /. s → 22 + .1 I /. j → 7

0.947901 - 0.0719083 i

Cos[Log[j^s] + Log[(E^ArcTan[2 s])]] /. s → 22 + .1 I /. j → 7

0.947901 - 0.0719083 i

Cos[Log[j^s E^ArcTan[2 s]]] /. s → 22 + .1 I /. j → 7

0.947901 - 0.0719083 i

Cos[Log[j^s E^ArcTan[2 s]]]

Cos[Log[e^ArcTan[2 s] j^s]]

FullSimplify@TrigToExp[e^ArcTan[2 s]]

$(1 - 2 i s)^{\frac{i}{2}} (1 + 2 i s)^{-\frac{i}{2}}$

Cos $\left[\text{Log}\left[j^s \left(\left(\frac{1}{2} - i s\right) / \left(\frac{1}{2} + i s\right)\right)^{\frac{i}{2}}\right]\right] /. s \rightarrow 22 + .1 I /. j \rightarrow 7$

0.947901 - 0.0719083 i

Cos $\left[s \text{Log}\left[\left(\frac{\frac{1}{2} - i s}{\frac{1}{2} + i s}\right)^{\frac{i}{2 s}} j\right]\right] /. s \rightarrow 22 + .1 I /. j \rightarrow 7$

0.947901 - 0.0719083 i

