```
Sum[ x^(-s k) / k!, {k, 0, Infinity}]
```

$e^{x^{-s}}$

```
N@ x^s/(-1 + x^s) /. x → 3 /. s → 3
```

1.03846

```
N@1 / (1 - x^-s) /. x → 3 /. s → 3
```

1.03846

```
Product[ (1 / (1 - x^-s))^(MoebiusMu[k] / k), {k, 1, Infinity}]
```

1

```
N@Table[ E^(2^-2 / k), {k, 1, 10}]
```

{1.28403, 1.13315, 1.0869, 1.06449, 1.05127, 1.04255, 1.03636, 1.03174, 1.02817, 1.02532}

```
N[1 / (1 - 2^-2)]
```

1.33333

```
Clear[l2mx]
l2mx[n_, k_] := l2mx[n, k] = Sum[ - (2^j - 1) / j l2mx[n - j, k - 1], {j, 1, n - 1}]
l2mx[n_, 1] := - (2^n - 1) / n
l2mx[n_, 0] := If[n == 0, 1, 0]
f2mx[n_, z_] := Sum[ z^k / k! l2mx[n, k], {k, 0, n}]


Table[f2mx[n, 1], {n, 0, 10}]
```

{1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1}

```
Series[2^(1) - 1 / (1 - x^-2), {x, 0, 20}]
```

$2 + x^2 + x^4 + x^6 + x^8 + x^{10} + x^{12} + x^{14} + x^{16} + x^{18} + x^{20} + O[x]^{21}$

```
Series[(2 x^2 - 1) / (x^2 - 1), {x, 0, 20}]
```

$1 - x^2 - x^4 - x^6 - x^8 - x^{10} - x^{12} - x^{14} - x^{16} - x^{18} - x^{20} + O[x]^{21}$

```
Series[(1 - 2 x^2) / (1 - x^2), {x, 0, 20}]
```

$1 - x^2 - x^4 - x^6 - x^8 - x^{10} - x^{12} - x^{14} - x^{16} - x^{18} - x^{20} + O[x]^{21}$

```
Series[(1 - 2 x^-s) / (1 - x^-s), {x, 0, 20}]
```

$$\frac{1 - 2\,x^{-s}}{1 - x^{-s}}$$

```
D[(1 - 2 x^-s)^z, z] /. z → 0
```

$\mathrm{Log}[1 - 2\,x^{-s}]$

```
Series[ 1 / (1 - 2 x^1), {x, 0, 20}]
```

$1 + 2x + 4x^2 + 8x^3 + 16x^4 + 32x^5 + 64x^6 + 128x^7 + 256x^8 + 512x^9 + 1024x^{10} + 2048x^{11} + 4096x^{12} + 8192x^{13} + 16384x^{14} + 32768x^{15} + 65536x^{16} + 131072x^{17} + 262144x^{18} + 524288x^{19} + 1048576x^{20} + O[x]^{21}$

```
Sum[ 1 / k! x^(-s k), {k, 0, Infinity}]
```

$e^{x^{-s}}$

```
Sum[ 1 / k! × 2^k x^(-s k), {k, 0, Infinity}]
```

$e^{2 x^{-s}}$

```
FullSimplify[e^(x^-s) / e^(2 x^-s)]
```

$e^{-x^{-s}}$

```
Sum[ (-1)^k / k! x^(-s k), {k, 0, Infinity}]
```

$e^{-x^{-s}}$

```
N[E^-(2^-3) Product[ E^(Prime[j]^-3), {j, 2, 160}]]
```

0.927523

```
N@Product[ (((1 - 2^(1 - 3)) Zeta[3 k]))^(MoebiusMu[k] / k), {k, 1, 300}]
```

1.19457

```
FullSimplify[((1 - 2^(1 - s)) / (1 - 2^-s)) Product[ 1 / (1 - Prime[j]^-s), {j, 2, Infinity}]]
```

$2^{-s} (-2 + 2^s)$ Zeta[s]

```
Expand[2^-s (-2 + 2^s)]
```

$1 - 2^{1-s}$

```
Series[1 / (1 - 2 x), {x, 0, 10}]
```

$1 + 2 x + 4 x^2 + 8 x^3 + 16 x^4 + 32 x^5 + 64 x^6 + 128 x^7 + 256 x^8 + 512 x^9 + 1024 x^{10} + O[x]^{11}$

```
Table[Integrate[ x^k, x] /. x → 1, {k, 0, 10}]
```

$\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}, \frac{1}{11}\right\}$

```
Table[Integrate[ 2^k x^k, x] /. x → 1, {k, 0, 10}]
```

$\left\{1, 1, \frac{4}{3}, 2, \frac{16}{5}, \frac{16}{3}, \frac{64}{7}, 16, \frac{256}{9}, \frac{256}{5}, \frac{1024}{11}\right\}$

```
Table[2^k / (k + 1), {k, 0, 10}]
```

$\left\{1, 1, \frac{4}{3}, 2, \frac{16}{5}, \frac{16}{3}, \frac{64}{7}, 16, \frac{256}{9}, \frac{256}{5}, \frac{1024}{11}\right\}$

```
A1[n_] := HarmonicNumber[Floor[n]]
A2[n_] := Sum[ 2^(k - 1) / (k), {k, 1, n}]
mA1[n_] := Sum[ MoebiusMu[k] / k A1[n / k], {k, 1, n}]
mA2[n_] := Sum[ MoebiusMu[k] / k A2[n / k], {k, 1, n}]
```

```
Table[mA2[n], {n, 1, 10}]
```

$\left\{1, \frac{3}{2}, \frac{5}{2}, 4, 7, \frac{23}{2}, \frac{41}{2}, \frac{71}{2}, \frac{127}{2}, 113\right\}$

```
Table[mA1[n], {n, 1, 10}]
```

{1, 1, 1, 1, 1, 1, 1, 1, 1, 1}

```
Table[A1[n], {n, 1, 10}]
```

$\left\{1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}, \frac{137}{60}, \frac{49}{20}, \frac{363}{140}, \frac{761}{280}, \frac{7129}{2520}, \frac{7381}{2520}\right\}$

```
Table[A2[n], {n, 1, 10}]
```

$$\left\{1, 2, \frac{10}{3}, \frac{16}{3}, \frac{128}{15}, \frac{208}{15}, \frac{2416}{105}, \frac{4096}{105}, \frac{21248}{315}, \frac{37376}{315}\right\}$$

```
pf[n_] := Sum[ Sum[ MoebiusMu[d] 2^(m / d - 1) / m, {d, Divisors[m]}], {m, 1, n}]
```

```
Table[pf[n], {n, 1, 10}]
```

$$\left\{1, \frac{3}{2}, \frac{5}{2}, 4, 7, \frac{23}{2}, \frac{41}{2}, \frac{71}{2}, \frac{127}{2}, 113\right\}$$

```
A2[2] - 1 / 2 A2[1]
```

$$\frac{3}{2}$$

```
A2[3]
```

$$\frac{10}{3}$$

```
Sum[Pochhammer[z, k] / k! x^k, {k, 0, Infinity}]
```

$(1 - x)^{-z}$

```
Sum[Pochhammer[z, k] / k! 2^k x^k, {k, 0, Infinity}]
```

$(1 - 2 x)^{-z}$

```
Sum[Pochhammer[z, k] / k! (-1)^k x^k, {k, 0, Infinity}]
```

$(1 + x)^{-z}$

```
Sum[ x^(2 k), {k, 0, Infinity}]
```

$$\frac{1}{1 - x^2}$$

```
Sum[Pochhammer[z, k] / k! x^(2 k), {k, 0, Infinity}]
```

$\left(1 - x^2\right)^{-z}$

```
CoefficientList[Series[Log[1 - x], {x, 0, 20}], x]
```

$$\left\{0, -1, -\frac{1}{2}, -\frac{1}{3}, -\frac{1}{4}, -\frac{1}{5}, -\frac{1}{6}, -\frac{1}{7}, -\frac{1}{8}, -\frac{1}{9}, -\frac{1}{10}, -\frac{1}{11}, -\frac{1}{12}, -\frac{1}{13}, -\frac{1}{14}, -\frac{1}{15}, -\frac{1}{16}, -\frac{1}{17}, -\frac{1}{18}, -\frac{1}{19}, -\frac{1}{20}\right\}$$

```
iv[n_] := Sum[ (-1)^(k + 1) / k, {k, 1, n}]
```

```
iv[1]
```

1

```
iv[2] - (1 / 2) iv[1]
```

0

```
InverseSeries[Series[Log[1 / (1 - x)], {x, 0, 10}]]
```

$$x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{24} + \frac{x^5}{120} - \frac{x^6}{720} + \frac{x^7}{5040} - \frac{x^8}{40320} + \frac{x^9}{362880} - \frac{x^{10}}{3628800} + O[x]^{11}$$

```
Clear[pp]
pp[n_, k_] := pp[n, k] = Sum[ (-1)^(j + 1) / j pp[Floor[n / j], k - 1], {j, 2, n}]
pp[n_, 0] := UnitStep[n - 1]
ppz[n_, z_] := Sum[bin[z, k] pp[n, k], {k, 0, Log2@n}]
Table[ppz[n, -1] - ppz[n - 1, -1], {n, 1, 10}]
```

$$\left\{1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{2}, -\frac{1}{5}, -\frac{1}{6}, -\frac{1}{7}, \frac{1}{2}, 0, -\frac{1}{10}\right\}$$

```
N@ppz[1 000 000, -1]
```

1.18562

```
2^(z 1.1856200038814326`) /. z → 1
```

2.27461

```
Sum[ (ppz[j, -1] - ppz[j - 1, -1]) ppz[100 / j, 1], {j, 1, 100}]
```

1

```
Log[2.]
```

0.693147

```
A1[n_] := Sum[ 1 / (k) x^k, {k, 1, n}]
A2[n_] := Sum[ 2^(k - 1) / (k) x^k, {k, 1, n}]
A2a[n_] := 2^(n - 1) A1[n] - Sum[ 2^(k - 1) A1[k], {k, 1, n - 1}]

A1[5]
```

$$x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5}$$

```
Expand[16 A1[5] - 8 A1[4] - 4 A1[3] - 2 A1[2] - A1[1]]
```

$$x + x^2 + \frac{4 x^3}{3} + 2 x^4 + \frac{16 x^5}{5}$$

```
A2[5]
```

$$x + x^2 + \frac{4 x^3}{3} + 2 x^4 + \frac{16 x^5}{5}$$

```
Expand[32 A1[6] - 16 A1[5] - 8 A1[4] - 4 A1[3] - 2 A1[2] - A1[1]]
```

$$x + x^2 + \frac{4 x^3}{3} + 2 x^4 + \frac{16 x^5}{5} + \frac{16 x^6}{3}$$

```
A2[6]
```

$$x + x^2 + \frac{4 x^3}{3} + 2 x^4 + \frac{16 x^5}{5} + \frac{16 x^6}{3}$$

```
Expand[A2a[6]]
```

$$x + x^2 + \frac{4 x^3}{3} + 2 x^4 + \frac{16 x^5}{5} + \frac{16 x^6}{3}$$

```
Expand[E^(2^-s) E^(-2^(1-s))]
```

$e^{-2^{1-s}+2^{-s}}$

```
N[E^(-2^(-s)) Product[E^(Prime[j]^-s), {j, 2, Infinity}] /. s → 3]
```

```
0.927523
```

```
N[E^(-2^(1-s)) Product[E^(Prime[j]^-s), {j, 1, Infinity}] /. s → 3]
```

```
0.927523
```

```
N@Product[ (((1 - 2^(k (1 - 3))) Zeta[3 k])) ^ (MoebiusMu[k] / k), {k, 1, 300}]
```

```
0.927523
```

```
N[Product[E^(Prime[j]^-s), {j, 1, Infinity}] /. s → 3]
```

```
1.19096
```

```
N@Product[ Zeta[3 k] ^ (MoebiusMu[k] / k), {k, 1, 300}]
```

```
1.19096
```

```
N[Product[ (1 / (1 - x^(k (-s)))) ^ (MoebiusMu[k] / k), {k, 1, 100}] /. x → 2 /. s → 3]
```

```
1.13315
```

```
N[E^(2^-3)]
```

```
1.13315
```

```
N[Product[ (1 / (1 - x^(k (1 - s)))) ^ (-MoebiusMu[k] / k), {k, 1, 100}] /. x → 2 /. s → 3]
```

```
0.778801
```

```
N[E^(-(2^(1 - 3)))]
```

```
0.778801
```

```
FullSimplify[E^(-(2^(1 - s))) E^(2^(-s))]
```

$e^{-2^{-s}}$

```
Sum[ 3^(-b s) / b!, {b, 0, Infinity}]
```

$e^{3^{-s}}$

```
Sum[(-1)^b 2^(-b s) / b!, {b, 0, Infinity}]
```

$e^{-2^{-s}}$

```
Clear[pd, ps]
ps[n_, s_] := ps[n, s] = N@Sum[Prime[j]^-s, {j, 1, PrimePi[n]}]
pr[n1_, n2_, s_] := ps[n2, s] - ps[n1, s] + 1
pd[n_, s_, k_] := pd[n, s, k] = If[n < Prime[k], 1,
   Sum[N[ Prime[k]^(-a s) / a!] pd[Floor[n / Prime[k]^a], s, k + 1], {a, 0, Log[Prime[k], n]}]]
pda[n_, s_, k_] := pda[n, s, k] = If[n < Prime[k], 1,
   If[n < Prime[k]^2, pr[Prime[k], n, s], Sum[
     N[ Prime[k]^(-a s) / a!] pda[Floor[n / Prime[k]^a], s, k + 1], {a, 0, Log[Prime[k], n]}]]]
pdb[n_, s_, k_] := If[n < Prime[k], 1, If[n < Prime[k]^2, pr[Prime[k], n, s],
   Sum[N[ (-1)^a Prime[k]^(-a s) / a!] pda[Floor[n / Prime[k]^a], s, k + 1],
    {a, 0, Log[Prime[k], n]}]]]
pdc[n_, s_, k_] := pdc[n, s, k] = If[n < Prime[k], 1, Sum[N[ (-1)^a Prime[k]^(-a s) / a!]
     pd[Floor[n / Prime[k]^a], s, k + 1], {a, 0, Log[Prime[k], n]}]]

pd[100 000, 2, 1]

1.57183

$RecursionLimit = 1 000 000

1 000 000

Zeta[2.]

1.64493

Prime[5]

11

pd[100, 2, 5]

124 020 778 002 215 450 777 436 066 242 054 922 242 650 889 014 310 897 895 124 981 018 510 /
 120 536 136 879 181 028 275 917 507 552 308 234 524 376 627 510 960 757 059 061 829 251 289

PrimePi[100] - PrimePi[10] + 1

22

ps[100, 2] - ps[10, 2] + 1

124 020 778 002 215 450 777 436 066 242 054 922 242 650 889 014 310 897 895 124 981 018 510 /
 120 536 136 879 181 028 275 917 507 552 308 234 524 376 627 510 960 757 059 061 829 251 289

pdb[1 000 000, N[ZetaZero[1]], 1]

9.7203 + 16.1394 ⅈ

pdc[10 000, N[ZetaZero[1]], 1]

$Aborted

Clear[dz]
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_] := dz[n] = Product[1 / (p[[2]]!), {p, FI[n]}]
sdz[n_, s_] := Sum[ dz[j] j^-s, {j, 1, n, 2}]
si[n_, s_] := Sum[N[ (-1)^a 2^(-a s) / a!] sdz[Floor[n / 2^a], s], {a, 0, Log[2, n]}]


pdc[100 000, .5, 1]

169.623
```

```
si[100000, 3]
```

```
0.927523
```

```
N@Product[(((1 - 2^(k (1 - 2))) Zeta[2 k]))^(MoebiusMu[k] / k), {k, 1, 300}]
```

```
0.95337
```

```
(((1 - 2^((1 - .5))) Zeta[.5]))
```

```
0.604899
```

```
N[E^(-2^(1 - s)) Product[E^(-Prime[j]^-s), {j, 1, Infinity}] /. s → .5]
```

Infinity::indet : Indeterminate expression $e^{\text{ComplexInfinity}}$ encountered. ≫

```
Indeterminate
```

```
Clear[oz]
oz[0, z_] := 1
oz[n_, z_] := oz[n, z] = Product[z^p[[2]] / (p[[2]]!), {p, FI[n]}]
dzo[n_, z_] := Product[Pochhammer[z, p[[2]]] / (p[[2]]!), {p, FI[n]}]
moz[n_, s_, k_] := Sum[ dzo[j, MoebiusMu[k] / k] j^(-s k) moz[Floor[n / j^k], s, k - 1],
  {j, 1, If[k == 1, n, Floor[Log[k, n]]]}]
moz[n_, s_, 0] := 1
mzz[n_, s_] := moz[n, s, Floor[Log[2, n]] + 1]
```

```
Table[mzz[n, 1] - mzz[n - 1, 1], {n, 2, 10}]
```

$$\left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{8}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{16}, \frac{1}{18}, \frac{1}{10}\right\}$$

```
oz[1, z]
```

```
1
```

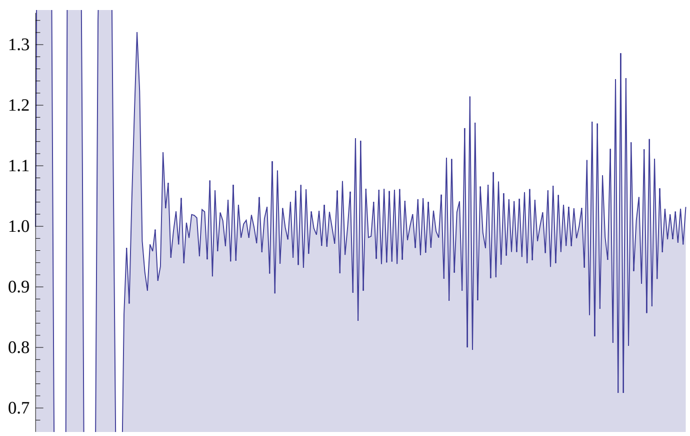
```
Log[2., 10]
```

```
3.32193
```

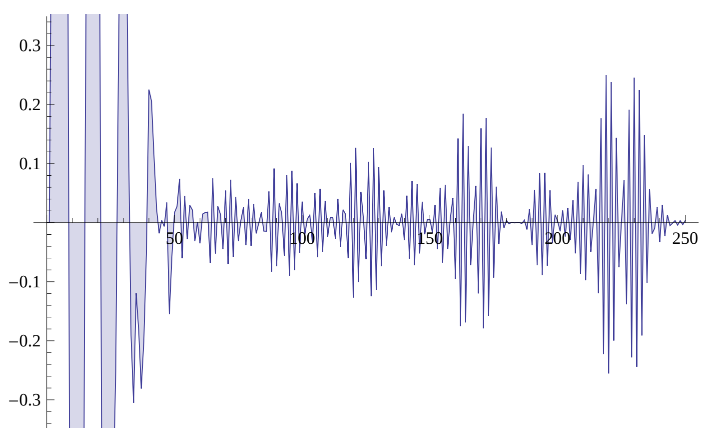
```
Clear[pe]
pe[n_, s_, k_] := pe[n, s, k] = N@Sum[ (-1)^(j+1) j^-s pe[n-j, s, k-1], {j, 1, n-1}]
pe[n_, s_, 1] := (-1)^(n+1) n^-s
pe[n_, s_, 0] := If[n == 0, 1, 0]
pa[n_, s_, z_] := Sum[ z^k / k! pe[n, s, k], {k, 0, n}]
pas[n_, s_, z_] := Sum[ pa[j, s, z], {j, 0, n}]
peo[n_, s_, k_] := peo[n, s, k] = N@Sum[ j^-s peo[n-j, s, k-1], {j, 1, n-1}]
peo[n_, s_, 1] := n^-s
peo[n_, s_, 0] := If[n == 0, 1, 0]
pao[n_, s_, z_] := Sum[ z^k / k! peo[n, s, k], {k, 0, n}]
Chop@Table[pa[n, 2, -1], {n, 0, 30}]
```

```
{1, -1, 0.75, -0.527778, 0.371528, -0.267083, 0.197157, -0.14943, 0.116041, -0.0920854,
 0.0744801, -0.0612542, 0.0511188, -0.0432119, 0.0369442, -0.0319041, 0.0277988,
 -0.0244159, 0.0215989, -0.0192309, 0.0172231, -0.0155073, 0.0140304, -0.0127507,
 0.0116352, -0.0106574, 0.00979569, -0.00903277, 0.00835423, -0.00774824, 0.00720492}
```

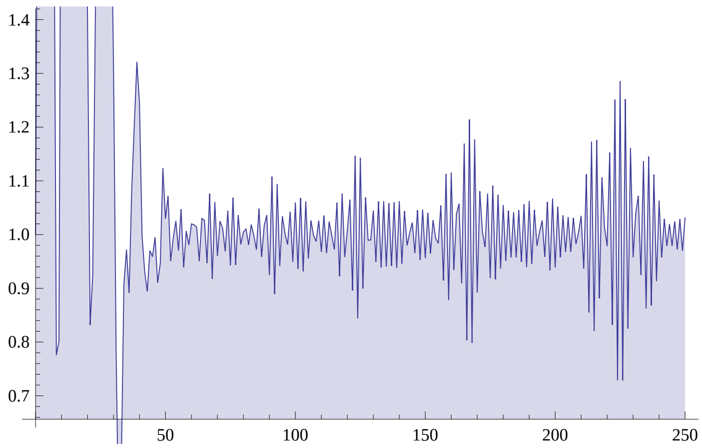
```
DiscretePlot[{Re[pas[n, .5 + 21.022 I, 1]]}, {n, 0, 250}]
```




```
DiscretePlot[{Im[pas[n, .5 + 21.022 I, 1]]}, {n, 0, 250}]
```



```
DiscretePlot[{Abs[pas[n, .5 + 21.022 I, 1]]}, {n, 0, 250}]
```



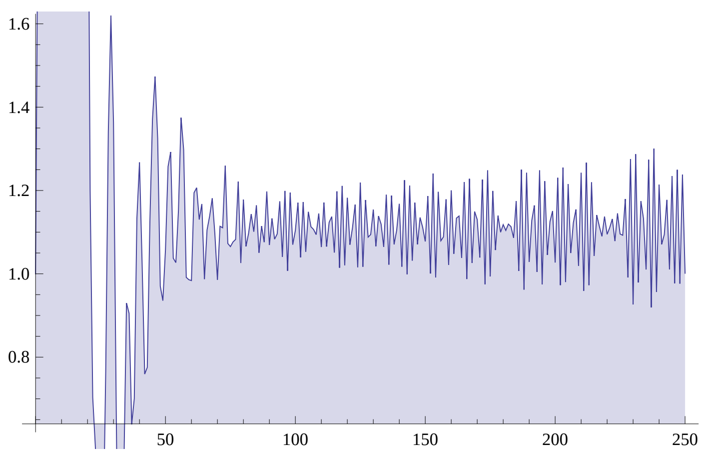

```
N[E^(Pi^2 / 12)]
```

2.27611

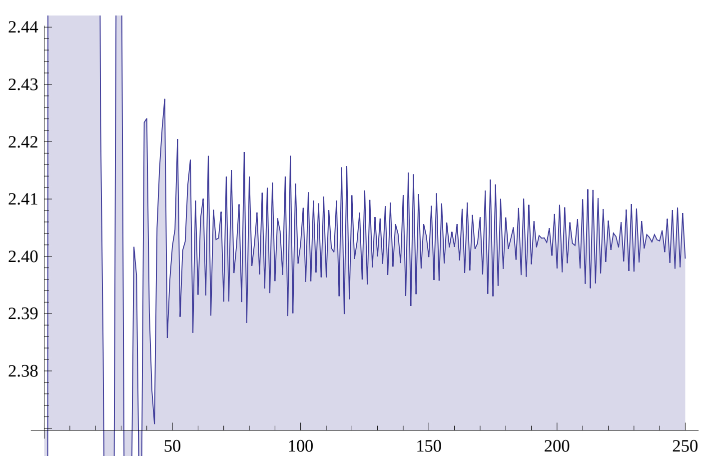
```
N@ZetaZero[2]
```

0.5 + 21.022 ⅈ

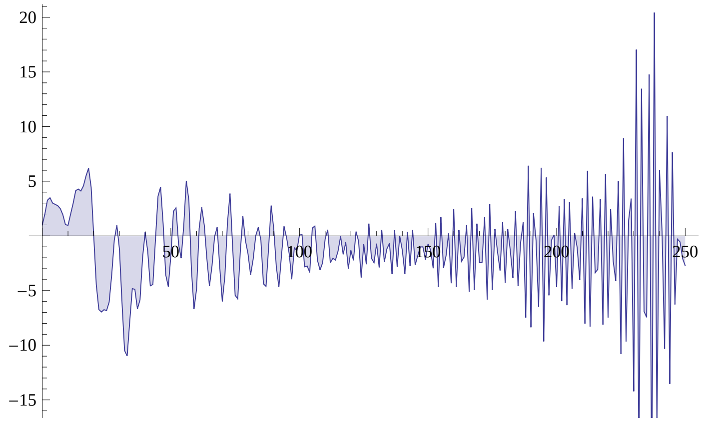
```
DiscretePlot[{Re[pas[n, .5 + 31 I, 1]]}, {n, 0, 250}]
```



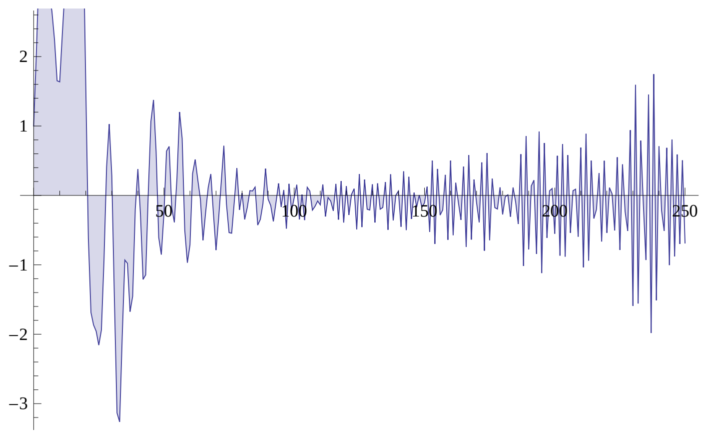

```
DiscretePlot[{Re[pas[n, .8 + 31 I, 1]]}, {n, 0, 250}]
```



DiscretePlot[{**Re**[**pas**[**n**, **.2 + 31 I**, **1**]]}, {**n**, **0**, **250**}]

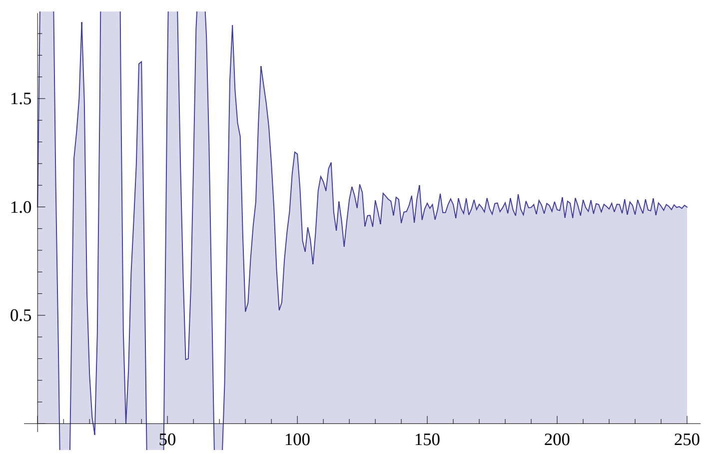


DiscretePlot[{**Re**[**pas**[**n**, **.35 + 31 I**, **1**]]}, {**n**, **0**, **250**}]



DiscretePlot[{**Re**[**pas**[**n**, **.2 + 31 I**, **1**]]}, {**n**, **0**, **250**}]

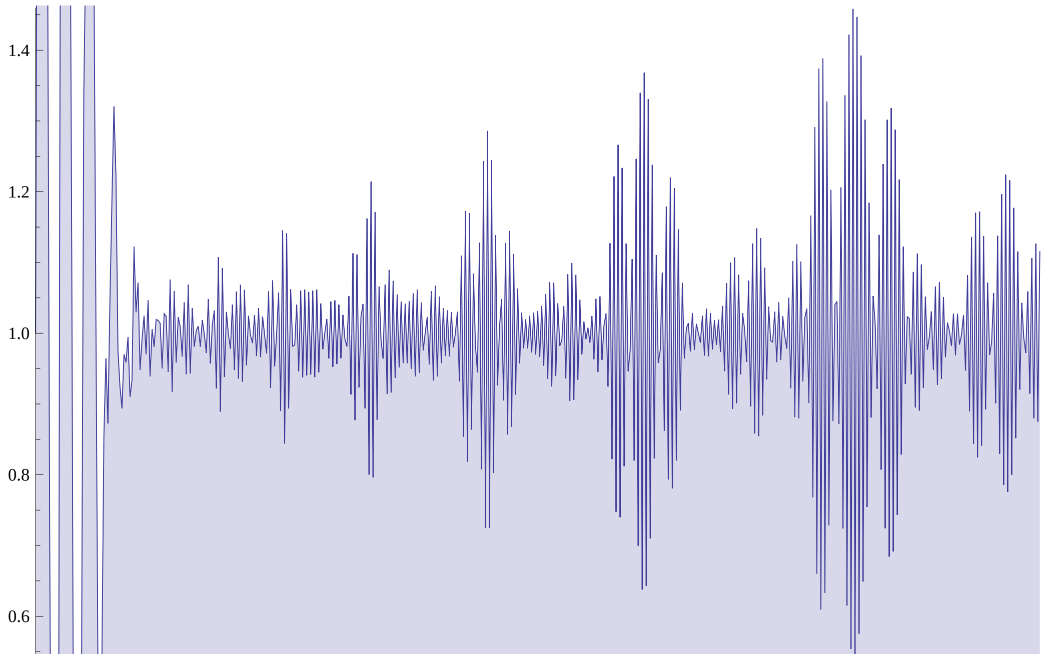
DiscretePlot[{**Re**[**pas**[**n**, **N**[**ZetaZero**[**12**]], **1**]]}, {**n**, **0**, **250**}]



N[**ZetaZero**[**12**]]

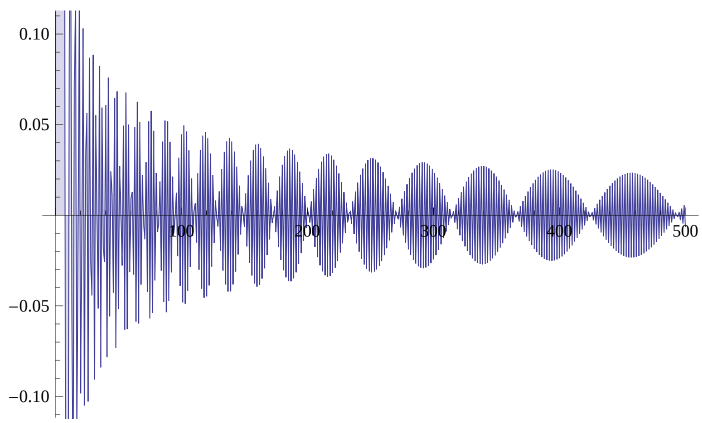
0.5 + 56.4462 i

```
DiscretePlot[{Re[pas[n, .5 + 21.022 I, 1]]}, {n, 0, 500}]
```




```
det[n_, s_] := Sum[ (-1)^(j + 1) j^-s, {j, 1, n}]
```

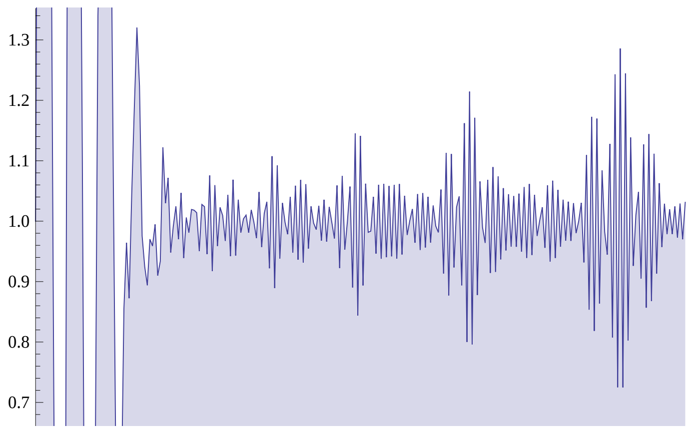
```
DiscretePlot[Re[det[n, .5 + 21.022 I]], {n, 0, 500}]
```



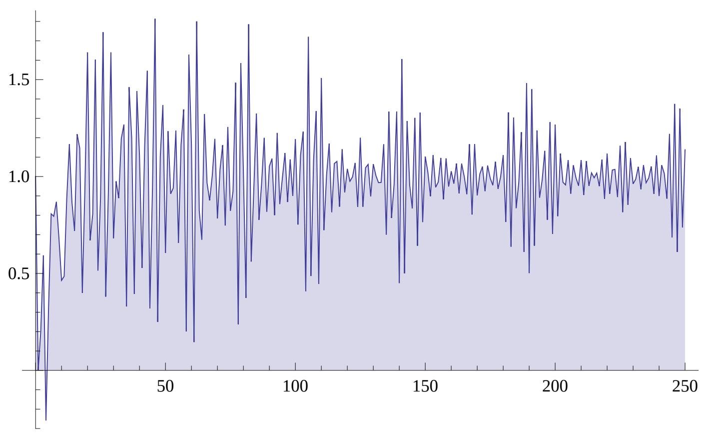

```
det[10, .5 + I]
```

0.74872 + 0.302933 i

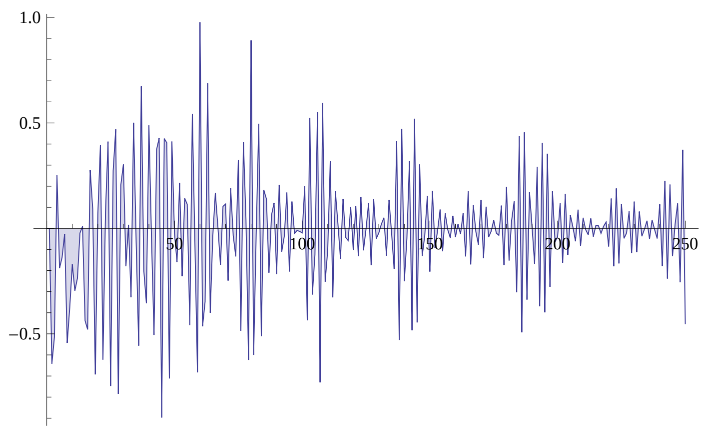
```
DiscretePlot[{Re[pas[n, .5 + 21.022 I, 1]]}, {n, 0, 250}]
```



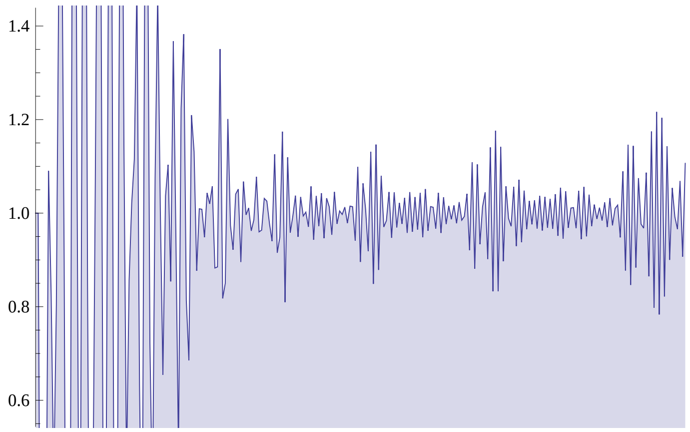

```
DiscretePlot[{Re[pas[n, .5 + 21.022 I, -1]]}, {n, 0, 250}]
```



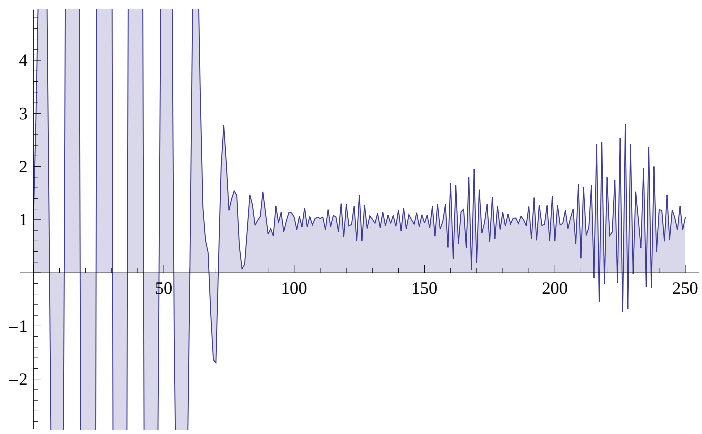

```
DiscretePlot[{Im[pas[n, .5 + 21.022 I, -1]]}, {n, 0, 250}]
```



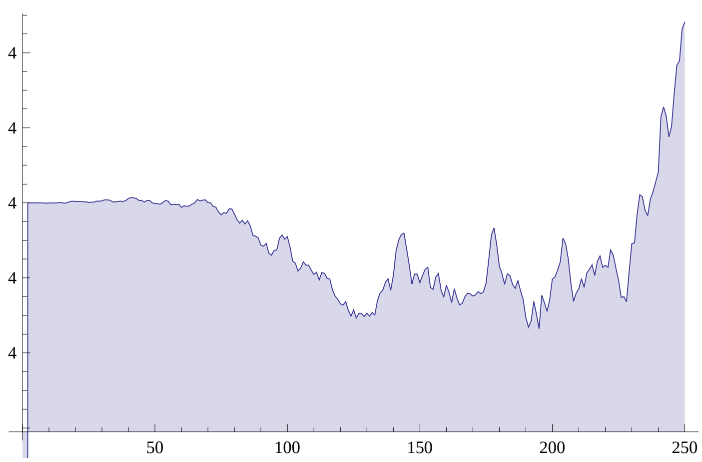
```
DiscretePlot[{Re[pas[n, .5 + 21.022 I, I]]}, {n, 0, 250}]
```




```
DiscretePlot[{Re[pas[n, .5 + 21.022 I, 2]]}, {n, 0, 250}]
```




```
DiscretePlot[{Re[pas[n, 1, 2]]}, {n, 0, 250}]
```



```
DiscretePlot[{Re[pas[n, 1, -1]]}, {n, 0, 50}]
```