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Expand@Integrate[1, {x, 1, n}, {y, 1, n/x}]

ConditionalExpression[1 - n + n Log[n], Re[n] ≥ 0 || n ∉ Reals]

Expand@Integrate[1, {x, 1, n}, {y, 1, (n/x)^(1/2)}]

ConditionalExpression[1 - 2√n + n, Re[n] ≥ 0 || n ∉ Reals]

Expand@Integrate[1, {x, 1, n}, {y, 1, (n/x)^(2/3)}]

ConditionalExpression[1 - 3n^{2/3} + 2n, Re[n] ≥ 0 || n ∉ Reals]

Expand@Integrate[1, {x, 1, n}, {y, 1, (n/x)^(3/4)}]

ConditionalExpression[1 - 4n^{3/4} + 3n, Re[n] ≥ 0 || n ∉ Reals]

Expand@Integrate[1, {x, 1, n}, {y, 1, (n/x)^(9/10)}]

ConditionalExpression[1 - 10n^{9/10} + 9n, Re[n] ≥ 0 || n ∉ Reals]

N[1 - 10n^{9/10} + 9n /. n → 100]

270.043

N[n Log[n] - n + 1 /. n → 100]

361.517

Expand@Integrate[1, {x, 1, n}, {y, 1, (n/x)^(99/100)}]

ConditionalExpression[1 - 100n^{99/100} + 99n, Re[n] ≥ 0 || n ∉ Reals]

N[1 - 100n^{99/100} + 99n /. n → 100]

351.074

Limit[1 - bn^((b-1)/b) + (b-1)n, b → Infinity]

1 - n + n Log[n]

Expand@Integrate[1, {x, 1, n^(2/3)}, {y, 1, (n/(x^(3/2)))^(1/2)}]

ConditionalExpression[1 - 4√n + 3n^{2/3}, Re[n^{2/3}] ≥ 0 || n^{2/3} ∉ Reals]

Expand@Integrate[1, {x, 1, n^(2/3)}, {y, 1, (n/(x^(3/2)))^(1/4)}]

ConditionalExpression[1 - $\frac{8n^{1/4}}{5} + \frac{3n^{2/3}}{5}$, Re[n^{2/3}] ≥ 0 || n^{2/3} ∉ Reals]

Expand@Integrate[1, {x, 1, n^(3/5)}, {y, 1, (n/(x^(5/3)))^(1/2)}]

ConditionalExpression[1 - 6√n + 5n^{3/5}, Re[n^{3/5}] ≥ 0 || n^{3/5} ∉ Reals]

Expand@Integrate[1, {x, 1, n^(a/b)}, {y, 1, (n/(x^(b/a)))^(c/d)}]

ConditionalExpression[$\frac{bc}{bc-ad} - \frac{ad}{bc-ad} - \frac{bcn^{a/b}}{bc-ad} + \frac{adn^{a/b}}{bc-ad} + \frac{adn^{c/d}}{bc-ad} - \frac{adn^{a/b} \left(n \left(n^{a/b} \right)^{-\frac{b}{a}} \right)^{c/d}}{bc-ad}$,
Re[n^{a/b}] ≥ 0 || n^{a/b} ∉ Reals]

$$\text{FullSimplify}\left[\frac{bc}{bc-ad} - \frac{ad}{bc-ad} - \frac{bc n^{a/b}}{bc-ad} + \frac{ad n^{a/b}}{bc-ad} + \frac{ad n^{c/d}}{bc-ad} - \frac{ad n^{a/b} \left(n \left(n^{a/b}\right)^{-\frac{b}{a}}\right)^{c/d}}{bc-ad}\right]$$

$$- \frac{bc \left(-1 + n^{a/b}\right) + ad \left(1 - n^{c/d} + n^{a/b} \left(-1 + \left(n \left(n^{a/b}\right)^{-\frac{b}{a}}\right)^{c/d}\right)\right)}{bc-ad}$$

Expand@Integrate[1, {x, 1, n^a}, {y, 1, (n / (x^(1/a)))^c}]

ConditionalExpression[

$$\frac{a}{a-c} - \frac{c}{a-c} - \frac{a n^a}{a-c} + \frac{c n^a}{a-c} - \frac{a n^c}{a-c} + \frac{a n^a \left(n \left(n^a\right)^{-1/a}\right)^c}{a-c}, \text{Re}[n^a] \geq 0 \mid n^a \notin \text{Reals}]$$

FullSimplify[

$$\frac{a}{a-c} - \frac{c}{a-c} - \frac{a n^a}{a-c} + \frac{c n^a}{a-c} - \frac{a n^c}{a-c} + \frac{a n^a \left(n \left(n^a\right)^{-1/a}\right)^c}{a-c}]$$

$$\frac{c \left(-1 + n^a\right) + a \left(1 - n^c + n^a \left(-1 + \left(n \left(n^a\right)^{-1/a}\right)^c\right)\right)}{a-c}$$

Expand@Integrate[1, {x, 1, n}, {y, 1, n/x},
{a, 1, (n / (xy))^(1/2)}, {b, 1, (n / (xya))^(1/2)}]

ConditionalExpression[$1 - 12\sqrt{n} + 32n^{3/4} - 21n + 3n \log[n]$, Re[n] ≥ 0 | n ∉ Reals]

Expand@Integrate[1, {x, 1, n}, {y, 1, n/x},
{a, 1, (n / (xy))^(1/3)}, {b, 1, (n / (xya))^(1/3)}]

ConditionalExpression[$1 - \frac{45n^{1/3}}{8} + \frac{243n^{5/9}}{32} - \frac{95n}{32} + \frac{5}{8}n \log[n]$, Re[n] ≥ 0 | n ∉ Reals]

Expand@Integrate[1, {x, 1, n}, {y, 1, n/x},
{a, 1, (n / (xy))^(1/4)}, {b, 1, (n / (xya))^(1/4)}]

ConditionalExpression[$1 - \frac{112n^{1/4}}{27} + \frac{1024n^{7/16}}{243} - \frac{259n}{243} + \frac{7}{27}n \log[n]$, Re[n] ≥ 0 | n ∉ Reals]

Expand@Integrate[1, {x, 1, n}, {y, 1, n/x},
{a, 1, (n / (xy))^(1/5)}, {b, 1, (n / (xya))^(1/5)}]

ConditionalExpression[$1 - \frac{225n^{1/5}}{64} + \frac{3125n^{9/25}}{1024} - \frac{549n}{1024} + \frac{9}{64}n \log[n]$, Re[n] ≥ 0 | n ∉ Reals]

Expand@Integrate[1, {x, 1, n}, {y, 1, n/x},
{a, 1, (n / (xy))^(2/3)}, {b, 1, (n / (xya))^(2/3)}]

ConditionalExpression[$1 - 36n^{2/3} + 243n^{8/9} - 208n + 16n \log[n]$, Re[n] ≥ 0 | n ∉ Reals]

Expand@Integrate[1, {x, 1, n}, {y, 1, n/x},
{a, 1, (n / (xy))^(3/4)}, {b, 1, (n / (xya))^(3/4)}]

ConditionalExpression[$1 - 80n^{3/4} + 1024n^{15/16} - 945n + 45n \log[n]$, Re[n] ≥ 0 | n ∉ Reals]

Expand@Integrate[1, {x, 1, n}, {y, 1, n/x},
 {a, 1, (n/(x y))^(4/5)}, {b, 1, (n/(x y a))^(4/5)}]

ConditionalExpression[1 - 150 n^{4/5} + 3125 n^{24/25} - 2976 n + 96 n Log[n], Re[n] ≥ 0 || n ∉ Reals]

$$\frac{a}{a-c} - \frac{c}{a-c} - \frac{a n^a}{a-c} + \frac{c n^a}{a-c} - \frac{a n^c}{a-c} + \frac{a n^a (n (n^a)^{-1/a})^c}{a-c}$$

$$\frac{a}{a-c} - \frac{c}{a-c} - \frac{a n^a}{a-c} + \frac{c n^a}{a-c} - \frac{a n^c}{a-c} + \frac{a n^a (n (n^a)^{-1/a})^c}{a-c}$$

Integrate[1, {x, 1, n^a}, {y, 1, (n/(x^(1/a)))^c}]

ConditionalExpression[$\frac{c(-1+n^a) + a(1-n^c + n^a(-1+(n(n^a)^{-1/a})^c))}{a-c}$, Re[n^a] ≥ 0 || n^a ∉ Reals]

Integrate[1, {x, 1, n^a}, {y, 1, (n/(x^(1/a)))^a},
 {a, 1, (n/(x^(1/a) y^(1/a)))^b}, {b, 1, (n/(x^(1/a) y^(1/a) a^(1/b)))^b}]

$$\int_1^{n^a} \int_1^{(n x^{-1/a})^a} \int_1^{(n x^{-1/a} y^{-1/a})^b} (-1 + (a^{-1/b} n x^{-1/a} y^{-1/a})^b) da dy dx$$

$$\text{Limit}\left[\frac{c(-1+n^a) + a(1-n^c + n^a(-1+(n(n^a)^{-1/a})^c))}{a-c}, a \rightarrow 1\right]$$

$$\frac{-1+c-cn+n^c}{-1+c}$$

$$\text{Limit}\left[\frac{-1+c-cn+n^c}{-1+c}, c \rightarrow 1\right]$$

$$1-n+n \log[n]$$

Expand@Integrate[1, {x, 1, n^a}]

$$-1+n^a$$

Expand@Integrate[1, {x, 1, n^a}, {y, 1, (n/(x^(1/a)))^c}]

Expand@Integrate[1, {x, 1, n}, {y, 1, n/x}]

ConditionalExpression[1 - n + n Log[n], Re[n] ≥ 0 || n ∉ Reals]

Integrate[1, {x, 1, n}]

Expand@Integrate[-1 + (n/y), {y, 1, n}]

ConditionalExpression[1 - n + n Log[n], Re[n] ≥ 0 || n ∉ Reals]

Expand@Integrate[1, {x, 1, n^a}, {y, 1, (n/(x^(1/a)))^c}]

$$\text{ConditionalExpression}\left[\frac{a}{a-c} - \frac{c}{a-c} - \frac{a n^a}{a-c} + \frac{c n^a}{a-c} - \frac{a n^c}{a-c} + \frac{a n^a (n (n^a)^{-1/a})^c}{a-c}, \text{Re}[n^a] \geq 0 \mid n^a \notin \text{Reals}\right]$$

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Integrate[1, {x, 1, n^a}]
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-1 + n^a
```

```
N[Integrate[-1 + ((n/y)^(1/a))^a, {y, 1, n^c}] /. {n -> 100, a -> 1/2, c -> 1/3}]
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-2.10653
```

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N@
$$\frac{a}{a-c} - \frac{c}{a-c} - \frac{a n^a}{a-c} + \frac{c n^a}{a-c} - \frac{a n^c}{a-c} + \frac{a n^a (n (n^a)^{-1/a})^c}{a-c}$$
 /. {n -> 100, a -> 1/2, c -> 1/3}
```

7.07523

```
Integrate[1, {x, 1, n}, {y, 1, (n/x)^(1/2)}]
```

```
ConditionalExpression[1 - 2 Sqrt[n] + n, Re[n] >= 0 || n < Real]
```

```
Integrate[(n/y^2) - 2 ((n/y^2)^(1/2)) + 1, {y, 1, (n)^(1/2)}]
```

```
-1 + n - Sqrt[n] Log[n]
```

```
Integrate[1, {y, 1, (n/x)^(1/2)}]
```

```
Integrate[-1 + Sqrt[n/x], {x, 1, n}]
```

```
ConditionalExpression[1 - 2 Sqrt[n] + n, Re[n] >= 0 || n < Real]
```

```
FullSimplify[a/(a-c) - c/(a-c)]
```

```
1
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```
Expand@Integrate[1, {x, 1, n}, {y, 1, (n/x)^(9/10)}]
```

```
ConditionalExpression[1 - 10 n^(9/10) + 9 n, Re[n] >= 0 || n < Real]
```

```
Expand@Integrate[1, {y, 1, n^(9/10)}, {x, 1, n/(y^(10/9))}]
```

```
ConditionalExpression[1 - 10 n^(9/10) + 9 n, Re[n^(9/10)] >= 0 || n^(9/10) < Real]
```

```
Expand@Integrate[1, {y, 1, n^(9/10)}, {x, 1, n/(y^(10/9))}]
```

```
-1 + n^(9/10)
```

$$N\left[\frac{a}{a-c} - \frac{c}{a-c} - \frac{an^a}{a-c} + \frac{cn^a}{a-c} - \frac{an^c}{a-c} + \frac{an^a (n (n^a)^{-1/a})^c}{a-c} \right] /. \{n \rightarrow 100, a \rightarrow 1/2, c \rightarrow 1/3\}$$

7.07523

$$N\left[1 - \frac{an^a}{a-c} + \frac{cn^a}{a-c} - \frac{an^c}{a-c} + \frac{an^a (n (n^a)^{-1/a})^c}{a-c} \right] /. \{n \rightarrow 100, a \rightarrow 1/2, c \rightarrow 1/3\}$$

7.07523

$$(n^a)^{-1/a} /. a \rightarrow 1/3 /. n \rightarrow 4$$

$$\frac{1}{4}$$

$$N\left[1 - \frac{an^a}{a-c} + \frac{cn^a}{a-c} - \frac{an^c}{a-c} + \frac{an^a (n (1/n))^c}{a-c} \right] /. \{n \rightarrow 100, a \rightarrow 1/2, c \rightarrow 1/3\}$$

7.07523

$$N\left[1 - \frac{an^a}{a-c} + \frac{cn^a}{a-c} - \frac{an^c}{a-c} + \frac{an^a (1)^c}{a-c} \right] /. \{n \rightarrow 100, a \rightarrow 1/2, c \rightarrow 1/3\}$$

7.07523

$$N\left[1 - \frac{an^a}{a-c} + \frac{cn^a}{a-c} - \frac{an^c}{a-c} + \frac{an^a}{a-c} \right] /. \{n \rightarrow 100, a \rightarrow 1/2, c \rightarrow 1/3\}$$

7.07523

$$(a-c) / (a-c) - \frac{an^a}{a-c} + \frac{cn^a}{a-c} - \frac{an^c}{a-c} + \frac{an^a}{a-c}$$

$$1 + \frac{cn^a}{a-c} - \frac{an^c}{a-c}$$

$$N\left[1 - \frac{an^a}{a-c} + \frac{cn^a}{a-c} - \frac{an^c}{a-c} + \frac{an^a}{a-c} \right] /. \{n \rightarrow 100, a \rightarrow 1/2, c \rightarrow 1/3\}$$

$$N\left[1 + \frac{cn^a}{a-c} - \frac{an^c}{a-c} \right] /. \{n \rightarrow 100, a \rightarrow 1/2, c \rightarrow 1/3\}$$

7.07523

$$N\left[1 + \frac{cn^a - an^c}{a-c} \right] /. \{n \rightarrow 100, a \rightarrow 1/2, c \rightarrow 1/3\}$$

7.07523

$$N\left[\frac{c (n^a - 1) - a (n^c - 1)}{a - c} /. \{n \rightarrow 100, a \rightarrow 1/2, c \rightarrow 1/3\}\right]$$

7.07523

FullSimplify $[n / (x^{(1/a)})]$

$n x^{-1/a}$

$n x^a - (a^{-1})$

$n x^{-1/a}$

FullSimplify $\left[\frac{c (n^a - 1) - a (n^c - 1)}{a - c} /. c \rightarrow 1\right]$

$$\frac{-1 + a - a n + n^a}{-1 + a}$$

Limit $\left[\frac{-1 + a - a n + n^a}{-1 + a}, a \rightarrow 1\right]$

$1 - n + n \log[n]$

D $\left[\frac{c (n^a - 1) - a (n^c - 1)}{a - c}, n\right]$

$$\frac{a c n^{-1+a} - a c n^{-1+c}}{a - c}$$

Integrate $[1, \{x, 1, n^a\}, \{y, 1, (n / (x^{(1/a)}))^a\}, \{a, 1, (n / (x^{(1/a)} y^{(1/a)}))^b\}, \{b, 1, (n / (x^{(1/a)} y^{(1/a)} a^{(1/b)}))^b\}]$

Integrate $[1, \{b, 1, (n / (x^{(1/a)} y^{(1/a)} a^{(1/b)}))^b\}]$

$$-1 + \left(a^{-1/b} n x^{-1/a} y^{-1/a}\right)^b$$

Integrate $[1, \{x, 1, n^a\}, \{y, 1, n/x\}, \{r, 1, (n / ((x y)^{(1/a)}))^b\}, \{s, 1, (n / ((x y)^{(1/a)}))^b / r\}]$

Integrate::pwr1: Unable to prove that integration limits $\{n^a\}$ are real. Adding assumptions may help. >>

\$Aborted

Integrate $[1, \{s, 1, (n / (x^{(1/a)} y^{(1/a)} r^{(1/b)}))^b\}]$

$$-1 + \left(n r^{-1/b} x^{-1/a} y^{-1/a}\right)^b$$

Integrate $\left[-1 + \left(n r^{-1/b} x^{-1/a} y^{-1/a}\right)^b, \{r, 1, (n / (x^{(1/a)} y^{(1/a)}))^b\}\right]$

\$Aborted

Integrate $\left[-1 + \left(n x^{-1/a} y^{-1/a}\right)^b / r, \{r, 1, (n / (x^{(1/a)} y^{(1/a)}))^b\}\right]$

ConditionalExpression $\left[1 + \left(n x^{-1/a} y^{-1/a}\right)^b \left(-1 + \log\left[\left(n x^{-1/a} y^{-1/a}\right)^b\right]\right), \right.$

$\left.\text{Re}\left[\left(n x^{-1/a} y^{-1/a}\right)^b\right] \geq 0 \mid \mid \left(n x^{-1/a} y^{-1/a}\right)^b \notin \text{Reals}\right]$

Expand $\left[1 + \left(n x^{-1/a} y^{-1/a}\right)^b \left(-1 + \text{Log}\left[\left(n x^{-1/a} y^{-1/a}\right)^b\right]\right)\right]$

$1 - \left(n x^{-1/a} y^{-1/a}\right)^b + \left(n x^{-1/a} y^{-1/a}\right)^b \text{Log}\left[\left(n x^{-1/a} y^{-1/a}\right)^b\right]$

Integrate $\left[1 - \left(n x^{-1/a} y^{-1/a}\right)^b + \left(n x^{-1/a} y^{-1/a}\right)^b \text{Log}\left[\left(n x^{-1/a} y^{-1/a}\right)^b\right], \{y, 1, n^a/x\}\right]$

ConditionalExpression $\left[\frac{1}{(a-b)^2 x}\right.$

$\left(a^2 n^a - 2 a b n^a + b^2 n^a - a^2 x + 2 a b x - b^2 x + a^2 n^b x - 2 a b n^b x - a (a-b) n^b x \text{Log}[n^b] + \right.$

$\left. a n^a \left(n (-n^a)^{-1/a} x^{-1/a}\right)^b \left(-a + 2 b + (a-b) \text{Log}\left[\left(n (-n^a)^{-1/a} x^{-1/a}\right)^b\right]\right)\right],$

$\left(\frac{n^a}{x} \notin \text{Reals} \mid \mid \text{Re}\left[\frac{n^a}{x}\right] \geq 0\right) \&\& \left(\left(\text{Im}[n^a] \neq \frac{\text{Im}[x] \text{Re}[n^a]}{\text{Re}[x]} \&\& \text{Re}[x] \neq 0\right) \mid \mid (\text{Re}[n^a] \geq 0 \&\& \text{Re}[x] > 0) \mid \mid \right.$

$\left. (\text{Re}[n^a] \leq 0 \&\& \text{Re}[x] < 0) \mid \mid (\text{Re}[x] = 0 \&\& (x \notin \text{Reals} \&\& \text{Re}[n^a] \neq 0) \mid \mid \right.$

$\left. (\text{Re}[n^a] = 0 \&\& ((\text{Im}[x] > 0 \&\& \text{Im}[n^a] \geq 0) \mid \mid (\text{Im}[x] < 0 \&\& \text{Im}[n^a] \leq 0)))\right)\right]$

Integrate $\left[\frac{1}{(a-b)^2 x}\right.$

$\left(a^2 n^a - 2 a b n^a + b^2 n^a - a^2 x + 2 a b x - b^2 x + a^2 n^b x - 2 a b n^b x - a (a-b) n^b x \text{Log}[n^b] + \right.$

$\left. a n^a \left(n (-n^a)^{-1/a} x^{-1/a}\right)^b \left(-a + 2 b + (a-b) \text{Log}\left[\left(n (-n^a)^{-1/a} x^{-1/a}\right)^b\right]\right)\right], \{x, 1, n^a\}]$

ConditionalExpression $\left[\frac{1}{(a-b)^2}\right.$

$\left(1/b(b^3 - 2 a^3 n^{a+b} + 2 a b^2 (-1 + n^b) + a^2 b (1 - n^b + 3 n^{a+b}) + a (a-b) n^b (b + a n^a) \text{Log}[n^b]) + \right.$

$\left. 1/b n^a (-a^2 b + 2 a b^2 - b^3 + a^2 b n^b - 2 a b^2 n^b + 2 a^3 (n (-n^{2a})^{-1/a})^b - 3 a^2 b (n (-n^{2a})^{-1/a})^b + \right.$

$\left. (a-b)^2 b \text{Log}[n^a] - a (a-b) b n^b \text{Log}[n^b] - a^3 (n (-n^{2a})^{-1/a})^b \text{Log}\left[\left(n (-n^{2a})^{-1/a}\right)^b\right] + \right.$

$\left. a^2 b (n (-n^{2a})^{-1/a})^b \text{Log}\left[\left(n (-n^{2a})^{-1/a}\right)^b\right]\right), \text{Re}[n^a] \geq 0 \mid \mid n^a \notin \text{Reals}]$

sa $[n_, a_, b_] := \frac{1}{(a-b)^2}$

$\left(1/b(b^3 - 2 a^3 n^{a+b} + 2 a b^2 (-1 + n^b) + a^2 b (1 - n^b + 3 n^{a+b}) + a (a-b) n^b (b + a n^a) \text{Log}[n^b]) + \right.$

$\left. 1/b n^a (-a^2 b + 2 a b^2 - b^3 + a^2 b n^b - 2 a b^2 n^b + 2 a^3 (n (-n^{2a})^{-1/a})^b - \right.$

$\left. 3 a^2 b (n (-n^{2a})^{-1/a})^b + (a-b)^2 b \text{Log}[n^a] - a (a-b) b n^b \text{Log}[n^b] - \right.$

$\left. a^3 (n (-n^{2a})^{-1/a})^b \text{Log}\left[\left(n (-n^{2a})^{-1/a}\right)^b\right] + a^2 b (n (-n^{2a})^{-1/a})^b \text{Log}\left[\left(n (-n^{2a})^{-1/a}\right)^b\right]\right)$

N@**sa** $[100, 1/2, 1/3]$

31.8371

$$\begin{aligned}
& \text{FullSimplify}\left[\frac{1}{(a-b)^2}\right. \\
& \left.(1/b(b^3 - 2a^3 n^{a+b} + 2ab^2(-1+n^b) + a^2b(1-n^b + 3n^{a+b}) + a(a-b)n^b(b+an^a)\text{Log}[n^b]) + \right. \\
& \left. 1/bn^a(-a^2b + 2ab^2 - b^3 + a^2bn^b - 2ab^2n^b + 2a^3(n(-n^{2a})^{-1/a})^b - \right. \\
& \left. 3a^2b(n(-n^{2a})^{-1/a})^b + (a-b)^2b\text{Log}[n^a] - a(a-b)bn^b\text{Log}[n^b] - \right. \\
& \left. a^3(n(-n^{2a})^{-1/a})^b\text{Log}[(n(-n^{2a})^{-1/a})^b] + a^2b(n(-n^{2a})^{-1/a})^b\text{Log}[(n(-n^{2a})^{-1/a})^b])\right) \\
& \frac{1}{(a-b)^2b} \left(a^2b - 2ab^2 + b^3 - a^2bn^a + 2ab^2n^a - b^3n^a - a^2bn^b + 2ab^2n^b - 2a^3n^{a+b} + \right. \\
& \left. 4a^2bn^{a+b} - 2ab^2n^{a+b} + (a-b)^2bn^a\text{Log}[n^a] + a(a-b)n^b(b+an^a - bn^a)\text{Log}[n^b] - \right. \\
& \left. a^2n^a(n(-n^{2a})^{-1/a})^b(-2a+3b+(a-b)\text{Log}[(n(-n^{2a})^{-1/a})^b]) \right) \\
& \text{Expand}\left[\frac{1}{(a-b)^2b} \left(a^2b - 2ab^2 + b^3 - a^2bn^a + 2ab^2n^a - b^3n^a - a^2bn^b + 2ab^2n^b - 2a^3n^{a+b} + \right. \right. \\
& \left. \left. 4a^2bn^{a+b} - 2ab^2n^{a+b} + (a-b)^2bn^a\text{Log}[n^a] + a(a-b)n^b(b+an^a - bn^a)\text{Log}[n^b] - \right. \right. \\
& \left. \left. a^2n^a(n(-n^{2a})^{-1/a})^b(-2a+3b+(a-b)\text{Log}[(n(-n^{2a})^{-1/a})^b]) \right) \right] \\
& N\left[\frac{a^2}{(a-b)^2} - \frac{2ab}{(a-b)^2} + \frac{b^2}{(a-b)^2} - \frac{a^2n^a}{(a-b)^2} + \frac{2abn^a}{(a-b)^2} - \frac{b^2n^a}{(a-b)^2} - \frac{a^2n^b}{(a-b)^2} + \frac{2abn^b}{(a-b)^2} + \frac{4a^2n^{a+b}}{(a-b)^2} - \right. \\
& \frac{2a^3n^{a+b}}{(a-b)^2b} - \frac{2abn^{a+b}}{(a-b)^2} - \frac{3a^2n^a(n(-n^{2a})^{-1/a})^b}{(a-b)^2} + \frac{2a^3n^a(n(-n^{2a})^{-1/a})^b}{(a-b)^2b} + \frac{a^2n^a\text{Log}[n^a]}{(a-b)^2} - \\
& \frac{2abn^a\text{Log}[n^a]}{(a-b)^2} + \frac{b^2n^a\text{Log}[n^a]}{(a-b)^2} + \frac{a^2n^b\text{Log}[n^b]}{(a-b)^2} - \frac{abn^b\text{Log}[n^b]}{(a-b)^2} - \frac{2a^2n^{a+b}\text{Log}[n^b]}{(a-b)^2} + \\
& \frac{a^3n^{a+b}\text{Log}[n^b]}{(a-b)^2b} + \frac{abn^{a+b}\text{Log}[n^b]}{(a-b)^2} + \frac{a^2n^a(n(-n^{2a})^{-1/a})^b\text{Log}[(n(-n^{2a})^{-1/a})^b]}{(a-b)^2} - \\
& \left. \frac{a^3n^a(n(-n^{2a})^{-1/a})^b\text{Log}[(n(-n^{2a})^{-1/a})^b]}{(a-b)^2b} \right] /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}
\end{aligned}$$

31.8371

$$\begin{aligned}
& N\left[\frac{a^2}{(a-b)^2} - \frac{2ab}{(a-b)^2} + \frac{b^2}{(a-b)^2} - \frac{a^2n^a}{(a-b)^2} + \frac{2abn^a}{(a-b)^2} - \frac{b^2n^a}{(a-b)^2} - \frac{a^2n^b}{(a-b)^2} + \frac{2abn^b}{(a-b)^2} + \frac{4a^2n^{a+b}}{(a-b)^2} - \right. \\
& \frac{2a^3n^{a+b}}{(a-b)^2b} - \frac{2abn^{a+b}}{(a-b)^2} - \frac{3a^2n^a(n(-n^{2a})^{-1/a})^b}{(a-b)^2} + \frac{2a^3n^a(n(-n^{2a})^{-1/a})^b}{(a-b)^2b} + \frac{a^2n^a\text{Log}[n^a]}{(a-b)^2} - \\
& \frac{2abn^a\text{Log}[n^a]}{(a-b)^2} + \frac{b^2n^a\text{Log}[n^a]}{(a-b)^2} + \frac{a^2n^b\text{Log}[n^b]}{(a-b)^2} - \frac{abn^b\text{Log}[n^b]}{(a-b)^2} - \frac{2a^2n^{a+b}\text{Log}[n^b]}{(a-b)^2} + \\
& \frac{a^3n^{a+b}\text{Log}[n^b]}{(a-b)^2b} + \frac{abn^{a+b}\text{Log}[n^b]}{(a-b)^2} + \frac{a^2n^a(n(-n^{2a})^{-1/a})^b\text{Log}[(n(-n^{2a})^{-1/a})^b]}{(a-b)^2} - \\
& \left. \frac{a^3n^a(n(-n^{2a})^{-1/a})^b\text{Log}[(n(-n^{2a})^{-1/a})^b]}{(a-b)^2b} \right] /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}
\end{aligned}$$

Integrate[$n^a / (x y)$, { x , 1, n^a }, { y , 1, $(n / x^{(1/a)})^b$ }]

ConditionalExpression $\left[\frac{n^a \left(a \operatorname{Log}[n^b]^2 - a \operatorname{Log}\left[\left(n (n^a)^{-1/a}\right)^b\right]^2\right)}{2 b}, \operatorname{Re}[n^a] \geq 0 \mid \mid n^a \notin \operatorname{Reals}\right]$

Integrate[$n^a b / (x^{(b/a)} y^{(b/a)})$, { x , 1, n^a }, { y , 1, $(n / x^{(1/a)})^b$ }]

ConditionalExpression $\left[\frac{a^2 n^b \left(b \left(-1 + (n^a)^{1-\frac{b}{a}}\right) + a (n^a)^{1-\frac{b}{a}} \left(-1 + \left((n^a)^{-1/a}\right)^b \left(\left((n^a)^{-1/a}\right)^b\right)^{-\frac{b}{a}}\right)\right)}{(a-b)^3}, \operatorname{Re}[n^a] \geq 0 \mid \mid n^a \notin \operatorname{Reals}\right]$

Integrate[$n^a / (x^{(a/a)} y^{(a/a)})$, { x , 1, n^a }, { y , 1, $(n / x^{(1/a)})^b$ }]

ConditionalExpression $\left[\frac{n^a \left(a \operatorname{Log}[n^b]^2 - a \operatorname{Log}\left[\left(n (n^a)^{-1/a}\right)^b\right]^2\right)}{2 b}, \operatorname{Re}[n^a] \geq 0 \mid \mid n^a \notin \operatorname{Reals}\right]$

N $\left[\frac{n^a \left(a \operatorname{Log}[n^b]^2 - a \operatorname{Log}\left[\left(n (n^a)^{-1/a}\right)^b\right]^2\right)}{2 b} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$

17.673

N $\left[\frac{n^a a b^2 \operatorname{Log}[n]^2}{2 b} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$

17.673

N $\left[\frac{n^a a b \operatorname{Log}[n]^2}{2} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$

17.673

N $\left[\frac{a^2 n^b \left(b \left(-1 + (n^a)^{1-\frac{b}{a}}\right) + a (n^a)^{1-\frac{b}{a}} \left(-1 + \left((n^a)^{-1/a}\right)^b \left(\left((n^a)^{-1/a}\right)^b\right)^{-\frac{b}{a}}\right)\right)}{(a-b)^3} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$

-11.6879

N $\left[\frac{a^2 n^b \left(b \left(-1 + (n^a)^{1-\frac{b}{a}}\right) + a (n^a)^{1-\frac{b}{a}} \left(-1 + (1/n)^b \left((1/n)^b\right)^{-\frac{b}{a}}\right)\right)}{(a-b)^3} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$

-11.6879

N $\left[\frac{a^2 n^b \left(b \left(-1 + n^{a-b}\right) + a n^{a-b} \left(-1 + (1/n)^b \left((1/n)^b\right)^{-\frac{b}{a}}\right)\right)}{(a-b)^3} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$

-11.6879

N $\left[\frac{a^2 n^b \left(b \left(-1 + n^{a-b}\right) + a n^{a-b} \left(-1 + (1/n)^b \left((1/n)^b\right)^{-\frac{b}{a}}\right)\right)}{(a-b)^3} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$

-11.6879

$$N\left[\frac{a^2 n^b \left(b (-1 + n^{a-b}) + a n^{a-b} \left(-1 + n^{-b} \left(n^{-b}\right)^{-\frac{b}{a}}\right)\right)}{(a-b)^3} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$$

-11.6879

$$N\left[\frac{a^2 n^b \left(b (-1 + n^{a-b}) + a n^{a-b} \left(-1 + n^{-b} \left(n^{b^2/a}\right)\right)\right)}{(a-b)^3} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$$

-11.6879

$$N\left[\frac{a^2 n^b \left(b (-1 + n^{a-b}) + a n^{a-b} \left(-1 + n^{-b} \left(n^{b^2/a}\right)\right)\right)}{(a-b)^3} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$$

-11.6879

$$N\left[\frac{\left(a^2 n^b b (-1 + n^{a-b}) + a^2 n^b a n^{a-b} \left(-1 + n^{-b} \left(n^{b^2/a}\right)\right)\right)}{(a-b)^3} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$$

-11.6879

$$N\left[\frac{\left(\left(-\left(a^2 n^b b\right) + a^2 n^b b n^{a-b}\right) + a^2 n^b a n^{a-b} \left(-1 + n^{-b} \left(n^{b^2/a}\right)\right)\right)}{(a-b)^3} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$$

-11.6879

$$N\left[\frac{\left(-\left(a^2 n^b b\right) + a^2 n^b b n^{a-b} + \left(-\left(a^2 n^b a n^{a-b}\right) + a^2 n^b a n^{a-b} n^{-b} \left(n^{b^2/a}\right)\right)\right)}{(a-b)^3} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$$

-11.6879

$$N\left[\frac{\left(-\left(a^2 n^b b\right) + a^2 b n^a + \left(-\left(a^2 n^b a n^{a-b}\right) + a^2 n^b a n^{a-b} n^{-b} \left(n^{b^2/a}\right)\right)\right)}{(a-b)^3} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$$

-11.6879

$$N\left[\frac{\left(-\left(a^2 n^b b\right) + a^2 b n^a + \left(-\left(a^2 a n^a\right) + a^2 n^b a n^{a-b} n^{-b} \left(n^{b^2/a}\right)\right)\right)}{(a-b)^3} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$$

-11.6879

$$N\left[\frac{\left(-\left(a^2 n^b b\right) + a^2 b n^a + \left(-\left(a^2 a n^a\right) + a^2 a n^{a-b} \left(n^{b^2/a}\right)\right)\right)}{(a-b)^3} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$$

-11.6879

$$N\left[\frac{\left(-\left(a^2 n^b b\right) + a^2 b n^a + \left(-\left(a^2 a n^a\right) + a^3 n^{a-b} n^{b^2/a}\right)\right)}{(a-b)^3} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$$

-11.6879

$$N\left[\frac{a^2 b \left(n^a - n^b\right) + -a^3 n^a + a^3 n^{a-b} n^{b^2/a}}{(a-b)^3} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$$

-11.6879

$$N\left[\frac{a^2 b (n^a - n^b) + a^3 n^a \left(-1 + n^{\frac{b(-a+b)}{a}}\right)}{(a-b)^3} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$$

-11.6879

$$\text{FullSimplify}[-a^2 n^b b + a^2 b n^a]$$

$$a^2 b (n^a - n^b)$$

$$\text{FullSimplify}[-a^3 n^a + a^3 n^{a-b} n^{b^2/a}]$$

$$a^3 n^a \left(-1 + n^{\frac{b(-a+b)}{a}}\right)$$

$$n^{\frac{b(-a+b)}{a}} /. \{n \rightarrow 10, b \rightarrow 4\}$$

$$10^{\frac{4(4-a)}{a}}$$

$$n^{\left(\frac{(b^2)}{a} - b\right)} /. \{n \rightarrow 10, b \rightarrow 4, a \rightarrow 3\}$$

$$10 \times 10^{1/3}$$

$$\text{FullSimplify}\left[\text{Expand}\left[a^3 n^a \left(-1 + n^{\left(\frac{(b^2)}{a} - b\right)}\right)\right]\right]$$

$$a^3 n^a \left(-1 + n^{\frac{b(-a+b)}{a}}\right)$$

$$\text{Expand}\left[\frac{a^2 b (n^a - n^b) + a^3 n^a \left(-1 + n^{\frac{b(-a+b)}{a}}\right)}{(a-b)^3}\right]$$

$$-\frac{a^3 n^a}{(a-b)^3} + \frac{a^2 b n^a}{(a-b)^3} - \frac{a^2 b n^b}{(a-b)^3} + \frac{a^3 n^{a+\frac{b(-a+b)}{a}}}{(a-b)^3}$$

$$\text{Integrate}[n^a / (x y^a (a/b)), \{x, 1, n^a\}, \{y, 1, (n/x^{(1/a)})^b\}]$$

$$\text{ConditionalExpression}\left[\frac{b n^a \left(a - a \left((n^a)^{-1/a}\right)^b \left(\left((n^a)^{-1/a}\right)^b\right)^{-\frac{a}{b}} + (a-b) \text{Log}[n^a]\right)}{(a-b)^2}, \text{Re}[n^a] \geq 0 \mid n^a \notin \text{Reals}\right]$$

$$\text{Integrate}[n^a b / (x^a (b/a) y), \{x, 1, n^a\}, \{y, 1, (n/x^{(1/a)})^b\}]$$

$$\text{ConditionalExpression}\left[\frac{a n^b \left(-b + (-a+b) \text{Log}[n^b] + (n^a)^{1-\frac{b}{a}} \left(b + (a-b) \text{Log}\left[\left(n (n^a)^{-1/a}\right)^b\right]\right)\right)}{(a-b)^2}, \text{Re}[n^a] \geq 0 \mid n^a \notin \text{Reals}\right]$$

$$N\left[\frac{b n^a \left(a - a \left((n^a)^{-1/a}\right)^b \left(\left((n^a)^{-1/a}\right)^b\right)^{-\frac{a}{b}} + (a-b) \text{Log}[n^a]\right)}{(a-b)^2} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$$

-23.2144

$$N\left[\frac{b n^a \left(a - a \left(1/n\right)^b \left(\left(1/n\right)^b\right)^{-\frac{a}{b}} + (a-b) \text{Log}[n^a]\right)}{(a-b)^2} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$$

-23.2144

$$N\left[\frac{b n^a (a - a n^{-b} n^a + (a - b) \operatorname{Log}[n^a])}{(a - b)^2} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$$

-23.2144

$$(n^{-b})^{-\frac{a}{b}} /. \{n \rightarrow 10, a \rightarrow 2, b \rightarrow 7\}$$

100

$$(n^{(-b) (-\frac{a}{b})}) /. \{n \rightarrow 10, a \rightarrow 3, b \rightarrow 5\}$$

1000

(* FIRST ONE *)

$$N\left[\frac{b a n^a - b a n^{2 a - b} + b n^a (a - b) \operatorname{Log}[n^a]}{(a - b)^2} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$$

-23.2144

$$\text{FullSimplify@Expand}\left[\frac{b n^a (a - a n^{-b} n^a + (a - b) \operatorname{Log}[n^a])}{(a - b)^2}\right]$$

$$n^{-b} n^a n^a$$

$$n^{2 a - b}$$

(* AND NOW THE SECOND ONE *)

$$\frac{a n^b \left(-b + (-a + b) \operatorname{Log}[n^b] + (n^a)^{1 - \frac{b}{a}} (b + (a - b) \operatorname{Log}[(n (n^a)^{-1/a})^b]) \right)}{(a - b)^2}$$

$$N\left[\frac{a n^b \left(-b + (-a + b) \operatorname{Log}[n^b] + (n^a)^{1 - \frac{b}{a}} (b + (a - b) \operatorname{Log}[(n (n^a)^{-1/a})^b]) \right)}{(a - b)^2} /. \{n \rightarrow 100, a \rightarrow 1/2, b \rightarrow 1/3\}\right]$$

10.7752

$$\text{Integrate}[1, \{x, 1, n\}, \{y, 1, (n/x)^a\}]$$

$$\text{ConditionalExpression}\left[\frac{-1 + a - a n + n^a}{-1 + a}, \operatorname{Re}[n] \geq 0 \mid \mid n \notin \text{Reals}\right]$$

$$\text{Limit}\left[\frac{-1 + a - a n + n^a}{-1 + a}, a \rightarrow 1\right]$$

$$1 - n + n \operatorname{Log}[n]$$

$$\text{Integrate}[1, \{x, 1, n\}, \{j, 1, n/x\}, \{y, 1, (n/(x j))^a\}, \{z, 1, (n/(x j y^{1/a}))^a\}]$$

$$\text{ConditionalExpression}\left[\frac{1}{2 (-1 + a)^3} \left(2 (-1 + 3 a^2 (-1 + n) - a^3 (-1 + n) + n^a - 3 a (-1 + n^a)) + (-1 + a) \right. \right. \\ \left. \left. \left(2 a^2 n \operatorname{Log}[n] + (-1 + a) a n \left(-1 + \left(\frac{1}{n} \right)^a n^a \right) \operatorname{Log}[n]^2 + 2 n^a \operatorname{Log}[n^a] \right) \right), \operatorname{Re}[n] \geq 0 \mid \mid n \notin \text{Reals}\right]$$

$$\text{FullSimplify}\left[\frac{1}{2(-1+a)^3}\left(2(-1+3a^2(-1+n)-a^3(-1+n)+n^a-3a(-1+n^a)) + (-1+a)\left(2a^2n\text{Log}[n] + (-1+a)an\left(-1+\left(\frac{1}{n}\right)^a n^a\right)\text{Log}[n]^2 + 2n^a\text{Log}[n^a]\right)\right)\right]$$

$$\frac{1}{2(-1+a)^3}\left(2(-1+3a^2(-1+n)-a^3(-1+n)+n^a-3a(-1+n^a)) + (-1+a)\left(2a^2n\text{Log}[n] + (-1+a)an\left(-1+\left(\frac{1}{n}\right)^a n^a\right)\text{Log}[n]^2 + 2n^a\text{Log}[n^a]\right)\right)$$

$$\text{N}\left[\frac{1}{2(-1+a)^3}\left(2(-1+3a^2(-1+n)-a^3(-1+n)+n^a-3a(-1+n^a)) + (-1+a)\left(2a^2n\text{Log}[n] + (-1+a)an\left(-1+\left(\frac{1}{n}\right)^a n^a\right)\text{Log}[n]^2 + 2n^a\text{Log}[n^a]\right)\right)\right] /. \{n \rightarrow 100, a \rightarrow 1/2\}]$$

93.6204

$$\text{N}\left[\frac{1}{2(-1+a)^3}\left(2(-1+3a^2(-1+n)-a^3(-1+n)+n^a-3a(-1+n^a)) + (-1+a)\left(2a^2n\text{Log}[n] + 2n^a\text{Log}[n^a]\right)\right)\right] /. \{n \rightarrow 100, a \rightarrow 1/2\}]$$

93.6204

$$\text{N}\left[\frac{1}{2(-1+a)^3}\left(2(-1+3a^2(-1+n)-a^3(-1+n)+n^a-3a(-1+n^a)) + (-1+a)\left(2a^2n\text{Log}[n] + 2n^a a\text{Log}[n]\right)\right)\right] /. \{n \rightarrow 100, a \rightarrow 1/2\}]$$

93.6204

$$\text{FullSimplify}\left[\frac{1}{2(-1+a)^3}\left(2(-1+3a^2(-1+n)-a^3(-1+n)+n^a-3a(-1+n^a)) + (-1+a)\left(2a^2n\text{Log}[n] + 2n^a a\text{Log}[n]\right)\right)\right]$$

$$\frac{1}{(-1+a)^3}\left(-1+3a^2(-1+n)-a^3(-1+n)+n^a-3a(-1+n^a) + (-1+a)a(an+n^a)\text{Log}[n]\right)$$

$$\text{N}\left[\frac{1}{(-1+a)^3}\left(-1+3a^2(-1+n)-a^3(-1+n)+n^a-3a(-1+n^a) + (-1+a)a(an+n^a)\text{Log}[n]\right)\right] /. \{n \rightarrow 100, a \rightarrow 1/2\}]$$

93.6204

$$\text{Expand}\left[\text{Limit}\left[\frac{1}{(-1+a)^3}\left(-1+3a^2(-1+n)-a^3(-1+n)+n^a-3a(-1+n^a) + (-1+a)a(an+n^a)\text{Log}[n]\right), a \rightarrow 1\right]\right]$$

$$1 - n + n\text{Log}[n] - \frac{1}{2}n\text{Log}[n]^2 + \frac{1}{6}n\text{Log}[n]^3$$

```

Integrate[ $\frac{-a (n / (xy^{(1/a)})) + (n / (xy^{(1/a)}))^a}{-1+a}$ , {x, 1, n}, {y, 1, (n/x)^a}]
ConditionalExpression[ $\frac{1}{(-1+a)^3}$ 
 $\left( a n \left( 1 + a \left( \left( \frac{1}{n} \right)^a \right)^{-1/a} + (-1+a) a \text{Log}[n] \right) - n^a (a + a^2 n (n^a)^{-1/a} - (-1+a) \text{Log}[n^a]) \right)$ ,
Re[n] ≥ 0 || n ∈ Reals]
N[ $\left( \frac{-1+a-an+n^a}{-1+a} + \frac{1}{(-1+a)^3} \left( a n \left( 1 + a \left( \left( \frac{1}{n} \right)^a \right)^{-1/a} + (-1+a) a \text{Log}[n] \right) - n^a (a + a^2 n (n^a)^{-1/a} - (-1+a) \text{Log}[n^a]) \right) \right) / .$ 
{n → 100, a → 1/2}]
-19706.4

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Integrate[1, {j, 1, n^a}, {k, 1, n^a/j}, {l, 1, n/(j k)^a}, {m, 1, n/((j k)^a l)}]
ConditionalExpression[
 $-\frac{1}{(-1+a)^3} (1 - 3a + 3a^2 - a^3 - n + 3an + n^{1+a} (n^{-a})^a - a n^{1+a} (n^{-a})^a + (n - an) \text{Log}[n] +$ 
 $(-1+a)^2 n^{1+a} (n^{-a})^a \text{Log}[n^{-a}]) + \frac{1}{2(-1+a)^3}$ 
 $(n^a)^{1-a} ((-1+a)^2 a n \text{Log}[n^{-a}]^2 + 2(-1+a)^2 a n \text{Log}[n^{-a}] \text{Log}[n^a] +$ 
 $2(2an + (n^a)^a - 3a(n^a)^a + 3a^2(n^a)^a - a^3(n^a)^a + (n - an) \text{Log}[n(n^a)^{-a}] +$ 
 $(-1+a)^2 \text{Log}[n^a] (n + (-1+a)(n^a)^a - n \text{Log}[n(n^a)^{-a}]))), \text{Re}[n^a] \geq 0 || n^a \notin \text{Reals}]$ 
N[ $-\frac{1}{(-1+a)^3} (1 - 3a + 3a^2 - a^3 - n + 3an + n^{1+a} (n^{-a})^a -$ 
 $a n^{1+a} (n^{-a})^a + (n - an) \text{Log}[n] + (-1+a)^2 n^{1+a} (n^{-a})^a \text{Log}[n^{-a}]) +$ 
 $\frac{1}{2(-1+a)^3} (n^a)^{1-a} ((-1+a)^2 a n \text{Log}[n^{-a}]^2 + 2(-1+a)^2 a n \text{Log}[n^{-a}] \text{Log}[n^a] +$ 
 $2(2an + (n^a)^a - 3a(n^a)^a + 3a^2(n^a)^a - a^3(n^a)^a + (n - an) \text{Log}[n(n^a)^{-a}] +$ 
 $(-1+a)^2 \text{Log}[n^a] (n + (-1+a)(n^a)^a - n \text{Log}[n(n^a)^{-a}])) / . \{n \rightarrow 100, a \rightarrow 1/2\}$ ]
-422.105

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Expand[Limit[ $\frac{1}{(-1+a)^3}$ 
 $(-1 + 3a^2(-1+n) - a^3(-1+n) + n^a - 3a(-1+n^a) + (-1+a)a(an+n^a)\text{Log}[n]), a \rightarrow 1]]$ 
 $1 - n + n \text{Log}[n] - \frac{1}{2} n \text{Log}[n]^2 + \frac{1}{6} n \text{Log}[n]^3$ 

```

Expand[$(-1 + a - a n + n^a) (a + 1)^2$]

$$-1 - a + a^2 + a^3 - a n - 2 a^2 n - a^3 n + n^a + 2 a n^a + a^2 n^a$$

$$(-1 - a + a^2 + a^3 - a n - 2 a^2 n - a^3 n + n^a + 2 a n^a + a^2 n^a) / (a - 1)^3$$

FullSimplify[

$$\frac{1}{(-1 + a)^3} (-1 + 3 a^2 (-1 + n) - a^3 (-1 + n) + n^a - 3 a (-1 + n^a) + (-1 + a) a (a n + n^a) \text{Log}[n]) -$$

$$(-1 - a + a^2 + a^3 - a n - 2 a^2 n - a^3 n + n^a + 2 a n^a + a^2 n^a) / (a - 1)^3]$$

$$\frac{a (4 - 4 a + n + 5 a n - (5 + a) n^a + (-1 + a) (a n + n^a) \text{Log}[n])}{(-1 + a)^3}$$

Expand@Limit[$\frac{1}{(-1 + a)^3}$

$$(-1 + 3 a^2 (-1 + n) - a^3 (-1 + n) + n^a - 3 a (-1 + n^a) + (-1 + a) a (a n + n^a) \text{Log}[n]), a \rightarrow 1]$$

$$1 - n + n \text{Log}[n] - \frac{1}{2} n \text{Log}[n]^2 + \frac{1}{6} n \text{Log}[n]^3$$

Expand@Limit[$\frac{1}{(-1 + a)^3}$

$$(-1 + 3 a^2 (-1 + n) - a^3 (-1 + n) + n^a - 3 a (-1 + n^a) + (-1 + a) a (a n + n^a) \text{Log}[n]), a \rightarrow 1]$$

$$1 - n + n \text{Log}[n] - \frac{1}{2} n \text{Log}[n]^2 + \frac{1}{6} n \text{Log}[n]^3$$

$$\frac{-1 + c - c n + n^c}{-1 + c} /. c \rightarrow a$$

$$\frac{-1 + a - a n + n^a}{-1 + a}$$

$$\frac{-1 + a - a n + n^a}{-1 + a}$$

$$1 - n + n \text{Log}[n]$$

Expand@

$$\text{Limit}\left[\frac{1}{(-1 + a)^3} (-1 + 3 a^2 (-1 + n) - a^3 (-1 + n) + n^a - 3 a (-1 + n^a) + (-1 + a) a (a n + n^a) \text{Log}[n]) -$$

$$\frac{-1 + a - a n + n^a}{-1 + a}, a \rightarrow 1\right]$$

$$-\frac{1}{2} n \text{Log}[n]^2 + \frac{1}{6} n \text{Log}[n]^3$$

$$\begin{aligned}
& \text{Expand@Limit} \left[\frac{1}{(-1+a)^3} \left(-1 + 3a^2(-1+n) - a^3(-1+n) + n^a - \right. \right. \\
& \quad \left. \left. 3a(-1+n^a) + (-1+a)a(an+n^a)\text{Log}[n] - (-1+a-an+n^a)(-1+a)^2 \right), a \rightarrow 1 \right] \\
& -\frac{1}{2} n \text{Log}[n]^2 + \frac{1}{6} n \text{Log}[n]^3 \\
& \text{Expand@Limit} \left[\frac{1}{(-1+a)^3} \left(-1 + 3a^2(-1+n) - a^3(-1+n) + n^a - \right. \right. \\
& \quad \left. \left. 3a(-1+n^a) + (-1+a)a(an+n^a)\text{Log}[n] - (-1+a-an+n^a)(-1+a)^2 \right), a \rightarrow 1 \right] \\
& \text{FullSimplify} \left[\text{Expand} \left[-1 + 3a^2(-1+n) - a^3(-1+n) + n^a - \right. \right. \\
& \quad \left. \left. 3a(-1+n^a) + (-1+a)a(an+n^a)\text{Log}[n] - (-1+a-an+n^a)(-1+a)^2 \right] \right] \\
& a(-1+a)(-n+n^a) + (-1+a)(an+n^a)\text{Log}[n] \\
& \text{Expand@Limit} \left[\frac{a(-1+a)(-n+n^a) + (-1+a)(an+n^a)\text{Log}[n]}{(-1+a)^3}, a \rightarrow 1 \right] \\
& -\frac{1}{2} n \text{Log}[n]^2 + \frac{1}{6} n \text{Log}[n]^3 \\
& \text{Expand} \left[\frac{-a(1+a)(-n+n^a) + a(-1+a)(an+n^a)\text{Log}[n]}{(-1+a)^3} \right] \\
& \frac{an}{(-1+a)^3} + \frac{a^2n}{(-1+a)^3} - \frac{an^a}{(-1+a)^3} - \frac{a^2n^a}{(-1+a)^3} - \frac{a^2n\text{Log}[n]}{(-1+a)^3} + \frac{a^3n\text{Log}[n]}{(-1+a)^3} - \frac{an^a\text{Log}[n]}{(-1+a)^3} + \frac{a^2n^a\text{Log}[n]}{(-1+a)^3} \\
& \text{FullSimplify} \left[-\frac{a^2n\text{Log}[n]}{(-1+a)^3} + \frac{a^3n\text{Log}[n]}{(-1+a)^3} - \frac{an^a\text{Log}[n]}{(-1+a)^3} + \frac{a^2n^a\text{Log}[n]}{(-1+a)^3} \right] \\
& \frac{a(an+n^a)\text{Log}[n]}{(-1+a)^2} \\
& \text{FullSimplify} \left[\frac{an}{(-1+a)^3} + \frac{a^2n}{(-1+a)^3} - \frac{an^a}{(-1+a)^3} - \frac{a^2n^a}{(-1+a)^3} \right] \\
& -\frac{a(1+a)(-n+n^a)}{(-1+a)^3} \\
& \text{Expand@Limit} \left[\frac{a(an+n^a)\text{Log}[n]}{(-1+a)^2} - \frac{a(1+a)(-n+n^a)}{(-1+a)^3}, a \rightarrow 1 \right] \\
& -\frac{1}{2} n \text{Log}[n]^2 + \frac{1}{6} n \text{Log}[n]^3 \\
& \text{Expand@Limit} \left[an \left(\frac{(a+n^{a-1})\text{Log}[n]}{(-1+a)^2} - \frac{(1+a)(-1+n^{a-1})}{(-1+a)^3} \right), a \rightarrow 1 \right] \\
& -\frac{1}{2} n \text{Log}[n]^2 + \frac{1}{6} n \text{Log}[n]^3 \\
& \text{Expand} \left[an \frac{(a+n^{a-1})\text{Log}[n]}{(-1+a)^2} - \frac{(1+a)(-1+n^{a-1})}{(-1+a)^3} \right]
\end{aligned}$$

$$\text{Expand@Limit}\left[\frac{1}{(-1+a)^3} + \frac{a}{(-1+a)^3} - \frac{n^{-1+a}}{(-1+a)^3} - \frac{a n^{-1+a}}{(-1+a)^3} + \frac{a^2 n \text{Log}[n]}{(-1+a)^2} + \frac{a n^a \text{Log}[n]}{(-1+a)^2}, a \rightarrow 1\right]$$

$$\text{DirectedInfinity}[(-1+n) \text{Log}[n]]$$

$$\text{Expand}\left[\frac{a (a n + n^a) \text{Log}[n]}{(-1+a)^2} - \frac{a (1+a) (-n + n^a)}{(-1+a)^3}\right]$$

$$\text{Expand}\left[\left(\frac{a n}{(-1+a)^3} + \frac{a^2 n}{(-1+a)^3} - \frac{a n^a}{(-1+a)^3} - \frac{a^2 n^a}{(-1+a)^3} + \frac{a^2 n \text{Log}[n]}{(-1+a)^2} + \frac{a n^a \text{Log}[n]}{(-1+a)^2}\right) / (a n)\right]$$

$$\frac{1}{(-1+a)^3} + \frac{a}{(-1+a)^3} - \frac{n^{-1+a}}{(-1+a)^3} - \frac{a n^{-1+a}}{(-1+a)^3} + \frac{a \text{Log}[n]}{(-1+a)^2} + \frac{n^{-1+a} \text{Log}[n]}{(-1+a)^2}$$

$$\text{Expand@Limit}\left[a n \left(\frac{1}{(-1+a)^3} + \frac{a}{(-1+a)^3} - \frac{n^{-1+a}}{(-1+a)^3} - \frac{a n^{-1+a}}{(-1+a)^3} + \frac{a \text{Log}[n]}{(-1+a)^2} + \frac{n^{-1+a} \text{Log}[n]}{(-1+a)^2}\right), a \rightarrow 1\right]$$

$$-\frac{1}{2} n \text{Log}[n]^2 + \frac{1}{6} n \text{Log}[n]^3$$

$$\text{Expand@Limit}\left[a n \left(\frac{2}{(-1+a)^3} + \frac{a-1}{(-1+a)^3} - \frac{n^{-1+a}}{(-1+a)^3} - \frac{a n^{-1+a}}{(-1+a)^3} + \frac{a \text{Log}[n]}{(-1+a)^2} + \frac{n^{-1+a} \text{Log}[n]}{(-1+a)^2}\right), a \rightarrow 1\right]$$

$$-\frac{1}{2} n \text{Log}[n]^2 + \frac{1}{6} n \text{Log}[n]^3$$

$$\text{Expand@Limit}\left[a n \left(\frac{2}{(-1+a)^3} - \frac{n^{-1+a}}{(-1+a)^3} - \frac{a n^{-1+a}}{(-1+a)^3} + \frac{a \text{Log}[n]}{(-1+a)^2} + \frac{n^{-1+a} \text{Log}[n]}{(-1+a)^2} + \frac{1}{(-1+a)^2}\right), a \rightarrow 1\right]$$

$$-\frac{1}{2} n \text{Log}[n]^2 + \frac{1}{6} n \text{Log}[n]^3$$

$$\text{FullSimplify}\left[\frac{a \text{Log}[n]}{(-1+a)^2} + \frac{n^{-1+a} \text{Log}[n]}{(-1+a)^2} + \frac{1}{(-1+a)^2}\right]$$

$$\frac{1 + (a + n^{a-1}) \text{Log}[n]}{(-1+a)^2}$$

$$\frac{1 + (a + n^{-1+a}) \text{Log}[n]}{(-1+a)^2}$$

$$\text{Expand@Limit}\left[a n \left(\frac{2}{(-1+a)^3} - \frac{n^{-1+a}}{(-1+a)^3} - \frac{a n^{-1+a}}{(-1+a)^3} + \frac{1 + (a + n^{-1+a}) \text{Log}[n]}{(-1+a)^2}\right), a \rightarrow 1\right]$$

$$-\frac{1}{2} n \text{Log}[n]^2 + \frac{1}{6} n \text{Log}[n]^3$$

$$\text{Expand@Limit}\left[a n \left(\frac{2}{(-1+a)^3} - \frac{n^{-1+a}}{(-1+a)^3} - \frac{a n^{-1+a}}{(-1+a)^3} + \frac{1 + (a + n^{-1+a}) \text{Log}[n]}{(-1+a)^2}\right), a \rightarrow 1\right]$$

$$\text{FullSimplify}\left[\frac{2}{(-1+a)^3} - \frac{n^{-1+a}}{(-1+a)^3} - \frac{a n^{-1+a}}{(-1+a)^3}\right]$$

$$\frac{2 - (1+a) n^{a-1}}{(-1+a)^3}$$

$$\frac{2 - (1+a) n^{-1+a}}{(-1+a)^3}$$

$$\frac{2 - (-1 + a) n^{a-1} + 2 n^a (a - 1)}{(-1 + a)^3}$$

$$\frac{2 + 2 n^{-1+a} - (-1 + a) n^{-1+a}}{(-1 + a)^3}$$

$$\frac{2 + 2 n^a (a - 1)}{(-1 + a)^3} + \frac{-n^{a-1}}{(-1 + a)^2}$$

$$\text{Expand@Limit}\left[a n \left(\frac{2 - (1 + a) n^{a-1}}{(-1 + a)^3} + \frac{1 + (a + n^{-1+a}) \text{Log}[n]}{(-1 + a)^2} \right), a \rightarrow 1\right]$$

$$-\frac{1}{2} n \text{Log}[n]^2 + \frac{1}{6} n \text{Log}[n]^3$$

$$\text{Expand@Limit}\left[a n \left(\frac{2 - 2 n^a (a - 1) - (-1 + a) n^{a-1}}{(-1 + a)^3} + \frac{1 + (a + n^{-1+a}) \text{Log}[n]}{(-1 + a)^2} \right), a \rightarrow 1\right]$$

$$-\frac{1}{2} n \text{Log}[n]^2 + \frac{1}{6} n \text{Log}[n]^3$$

$$\text{Expand@Limit}\left[a n \left(\frac{2 - 2 n^a (a - 1)}{(-1 + a)^3} - n^{a-1} / (a - 1)^2 + \frac{1 + (a + n^{-1+a}) \text{Log}[n]}{(-1 + a)^2} \right), a \rightarrow 1\right]$$

$$-\frac{1}{2} n \text{Log}[n]^2 + \frac{1}{6} n \text{Log}[n]^3$$

$$\text{FullSimplify}\left[-n^{a-1} / (a - 1)^2 + \frac{1 + (a + n^{-1+a}) \text{Log}[n]}{(-1 + a)^2}\right]$$

$$\frac{1 - n^{a-1} + n^{a-1} \text{Log}[n] + a \text{Log}[n]}{(-1 + a)^2}$$

$$\text{Expand@Limit}\left[a n \left(\frac{2 - 2 n^a (a - 1)}{(-1 + a)^3} + \frac{1 - n^{a-1} + n^{a-1} \text{Log}[n] + a \text{Log}[n]}{(-1 + a)^2} \right), a \rightarrow 1\right]$$

$$-\frac{1}{2} n \text{Log}[n]^2 + \frac{1}{6} n \text{Log}[n]^3$$

$$\text{Expand@Limit}\left[a \left(2 \frac{n - n^a}{(-1 + a)^3} + \frac{n - n^a + n^a \text{Log}[n] + a n \text{Log}[n]}{(-1 + a)^2} \right), a \rightarrow 1\right]$$

$$-\frac{1}{2} n \text{Log}[n]^2 + \frac{1}{6} n \text{Log}[n]^3$$

Expand@Integrate[1, {x, 1, n}, {y, 1, n/x}]

ConditionalExpression[1 - n + n Log[n], Re[n] ≥ 0 || n ∈ Reals]

Expand@Integrate[1, {x, 1, n}, {y, 1, n/x}, {z, 1, n/(xy)}]

ConditionalExpression[-1 + n - n Log[n] + $\frac{1}{2} n \text{Log}[n]^2$, Re[n] ≥ 0 || n ∈ Reals]

Expand@Integrate[1, {x, 1, n^a}, {y, 1, n^a/x}]

ConditionalExpression[1 - n^a + n^a Log[n^a], Re[n^a] ≥ 0 || n^a ∈ Reals]

Expand@Integrate[1, {x, 1, n^a}, {y, 1, (n / (x^(1 / a)))^a}]

ConditionalExpression[$1 - n^a + n^a (n (n^a)^{-1/a})^a \text{Log}[n^a]$, $\text{Re}[n^a] \geq 0 \mid n^a \notin \text{Reals}$]

$1 - n^a + n^a (n (n^a)^{-1/a})^a \text{Log}[n^a] /. \{n \rightarrow 100, a \rightarrow 1 / 2\}$

$-9 + 10 \text{Log}[10]$

$1 - n^a + n^a \text{Log}[n^a] /. \{n \rightarrow 100, a \rightarrow 1 / 2\}$

$-9 + 10 \text{Log}[10]$

$(n / (x^{(1 / a)}))^a /. \{n \rightarrow 100, a \rightarrow 1 / 2\}$

$10 \sqrt{\frac{1}{x^2}}$

$(n^a / x) /. \{n \rightarrow 100, a \rightarrow 1 / 2\}$

$\frac{10}{x}$

Integrate[1, {x, 1, n}, {y, 1, (n / x)^a}]

ConditionalExpression[$\frac{-1 + a - a n + n^a}{-1 + a}$, $\text{Re}[n] \geq 0 \mid n \notin \text{Reals}$]

Expand[$\frac{-1 + a - a n + n^a}{-1 + a}$]

$-\frac{1}{-1 + a} + \frac{a}{-1 + a} - \frac{a n}{-1 + a} + \frac{n^a}{-1 + a}$

Limit[$-\frac{a - 1}{-1 + a}$, $a \rightarrow 1$]

-1

Limit[$-\frac{a n}{-1 + a} + \frac{n^a}{-1 + a}$, $a \rightarrow 1$]

$n (-1 + \text{Log}[n])$

Limit[$\frac{n^a - a n}{-1 + a}$, $a \rightarrow 1$]

$n (-1 + \text{Log}[n])$

Limit[$\frac{n^a - a n}{-1 + a}$, $a \rightarrow 1$]

Expand[**Limit**[$\frac{(n^a (a^2)) - a n}{-1 + a}$, $a \rightarrow 1$]]

$-n + 2 n \text{Log}[n]$

Expand[**Limit**[$\frac{(n^a (a)) - a n}{-1 + a}$, $a \rightarrow 1$]]

$-n + n \text{Log}[n]$

$$\text{Expand}\left[\text{Limit}\left[\frac{(n^{(a^{(1/2)})}) - a n}{(-1 + a)}, a \rightarrow 1\right]\right]$$

$$-n + \frac{1}{2} n \log[n]$$

$$\text{Expand}\left[\text{Limit}\left[\frac{(n^{(a^2)}) - 2 (n^a) + a n}{(-1 + a)}, a \rightarrow 1\right]\right]$$

$$n$$

$$\text{Expand}\left[\text{Limit}\left[\frac{(n^{(a^2)}) - 2 (n^a) + a n}{(-1 + a)}, a \rightarrow 1\right]\right]$$

$$n$$

$$\text{Expand}\left[\text{Limit}\left[\frac{(n^{(a^5)}) - a n^{(a^2)}}{-1 + a}, a \rightarrow 1\right]\right]$$

$$-n + 3 n \log[n]$$

$$\text{Expand}\left[\text{Limit}\left[\frac{(n^{(a^5)}) - a n}{-1 + a}, a \rightarrow 1\right]\right]$$

$$-n + 5 n \log[n]$$

$$\text{Expand}\left[\text{Limit}\left[\frac{(n^{(a^5)}) - a^2 n}{-1 + a}, a \rightarrow 1\right]\right]$$

$$-2 n + 5 n \log[n]$$

$$\text{Expand}\left[\text{Limit}\left[\frac{n^{((1+a)^5)} - (a+1)^3 n}{a}, a \rightarrow 0\right]\right]$$

$$-3 n + 5 n \log[n]$$

$$\text{Limit}[(n^a - 1) / a, a \rightarrow 0]$$

$$\log[n]$$

$$\text{Expand}\left[\text{Limit}\left[\frac{-1 + a - a n + n^a}{-1 + a}, a \rightarrow 1\right]\right]$$

$$1 - n + n \log[n]$$

$$\text{Integrate}[(n / (x y))^a \log[(n / (x y))^a] - (n / (x y))^a + 1, \{x, 1, n\}, \{y, 1, n / x\}]$$

$$\text{ConditionalExpression}\left[\frac{1}{2 (-1 + a)^3} \left(2 (-1 + 3 a^2 (-1 + n) - a^3 (-1 + n) + n^a - 3 a (-1 + n^a)) + (-1 + a) \left(2 a^2 n \log[n] + (-1 + a) a n \left(-1 + \left(\frac{1}{n}\right)^a n^a\right) \log[n]^2 + 2 n^a \log[n^a]\right)\right), \text{Re}[n] \geq 0 \mid n \notin \text{Reals}\right]$$

Integrate[$a (n / (xy)) ^a \text{Log} [(n / (xy))] - (n / (xy)) ^a + 1, \{x, 1, n\}, \{y, 1, n / x\}$]

ConditionalExpression $\left[\frac{1}{(-1+a)^3} \left(-1+3a^2(-1+n)-a^3(-1+n)+n^a-3a(-1+n^a)+(-1+a)a(an+n^a)\text{Log}[n]\right), \text{Re}[n] \geq 0 \mid \mid n \notin \text{Reals}\right]$

Integrate[$a n^a x^a - a y^a - a (\text{Log}[n] - \text{Log}[x] - \text{Log}[y]) - n^a x^a - a y^a - a + 1, \{x, 1, n\}, \{y, 1, n / x\}$]

ConditionalExpression $\left[\frac{1}{(-1+a)^3} \left(-1+3a^2(-1+n)-a^3(-1+n)+n^a-3a(-1+n^a)+(-1+a)a(an+n^a)\text{Log}[n]\right), \text{Re}[n] \geq 0 \mid \mid n \notin \text{Reals}\right]$

Integrate[$(n^a x^a - a y^a - a) (a (\text{Log}[n] - \text{Log}[x] - \text{Log}[y]) - 1) + 1, \{x, 1, n\}, \{y, 1, n / x\}$]

ConditionalExpression $\left[\frac{1}{(-1+a)^3} \left(-1+3a^2(-1+n)-a^3(-1+n)+n^a-3a(-1+n^a)+(-1+a)a(an+n^a)\text{Log}[n]\right), \text{Re}[n] \geq 0 \mid \mid n \notin \text{Reals}\right]$

$n^a \text{Integrate} [(x^a - a y^a - a) (a (\text{Log}[n] - \text{Log}[x] - \text{Log}[y]) - 1), \{x, 1, n\}, \{y, 1, n / x\}] + \text{Integrate}[1, \{x, 1, n\}, \{y, 1, n / x\}]$

ConditionalExpression $\left[1+n(-1+\text{Log}[n]) + \frac{-(-1+3a)(-n+n^a)+(-1+a)((-1+2a)n+an^a)\text{Log}[n]}{(-1+a)^3}, \text{Re}[n] \geq 0 \mid \mid n \notin \text{Reals}\right]$

$N\left[\frac{1}{(-1+a)^3} \left(-1+3a^2(-1+n)-a^3(-1+n)+n^a-3a(-1+n^a)+(-1+a)a(an+n^a)\text{Log}[n]\right) /. \{n \rightarrow 100, a \rightarrow 1/2\}\right]$

93.6204

$N\left[1+n(-1+\text{Log}[n]) + \frac{-(-1+3a)(-n+n^a)+(-1+a)((-1+2a)n+an^a)\text{Log}[n]}{(-1+a)^3} /. \{n \rightarrow 100, a \rightarrow 1/2\}\right]$

93.6204

$n^a \text{Integrate} [(x^a - a y^a - a) (a (\text{Log}[n] - \text{Log}[x] - \text{Log}[y])), \{x, 1, n\}, \{y, 1, n / x\}] + n^a \text{Integrate} [(x^a - a y^a - a) (-1), \{x, 1, n\}, \{y, 1, n / x\}] + \text{Integrate}[1, \{x, 1, n\}, \{y, 1, n / x\}]$

ConditionalExpression $\left[1+n(-1+\text{Log}[n]) + \frac{n-n^a+(-1+a)n\text{Log}[n]}{(-1+a)^2} + \frac{a(2n-2n^a+(-1+a)(n+n^a)\text{Log}[n])}{(-1+a)^3}, \text{Re}[n] \geq 0 \mid \mid n \notin \text{Reals}\right]$

$$N\left[1 + n(-1 + \text{Log}[n]) + \frac{n - n^a + (-1 + a)n \text{Log}[n]}{(-1 + a)^2} + \frac{a(2n - 2n^a + (-1 + a)(n + n^a) \text{Log}[n])}{(-1 + a)^3} \right] /. \{n \rightarrow 100, a \rightarrow 1/2\}$$

93.6204

$$n^a a \text{Integrate}[(xy)^{-a} (\text{Log}[n] - \text{Log}[x] - \text{Log}[y]), \{x, 1, n\}, \{y, 1, n/x\}] - n^a \text{Integrate}[(xy)^{-a}, \{x, 1, n\}, \{y, 1, n/x\}] + \text{Integrate}[1, \{x, 1, n\}, \{y, 1, n/x\}]$$

$$\text{ConditionalExpression}\left[1 + n(-1 + \text{Log}[n]) + \frac{a(2n - 2n^a + (-1 + a)(n + n^a) \text{Log}[n])}{(-1 + a)^3} - \right.$$

$$\left. n^a \left(\frac{1}{(-1 + a)^2} - \frac{n^{1-a}(1 + (-1 + a) \text{Log}[n])}{(-1 + a)^2} \right), \text{Re}[n] \geq 0 \mid n \notin \text{Reals} \right]$$

$$\text{FullSimplify}\left[\frac{a(2n - 2n^a + (-1 + a)(n + n^a) \text{Log}[n])}{(-1 + a)^3}\right]$$

$$\frac{a(2n - 2n^a + (-1 + a)(n + n^a) \text{Log}[n])}{(-1 + a)^3}$$

ff[n_, a_, b_, c_, d_] :=

$$-a^3 n^{1/a} / ((a-b)(a-c)(a-d)) - b^3 n^{1/b} / ((b-a)(b-c)(b-d)) - c^3 n^{1/c} / ((c-b)(c-a)(c-d)) - d^3 n^{1/d} / ((d-b)(d-c)(d-a)) + 1$$

ff[100, .5, .5, e, f]

Power::infy: Infinite expression $\frac{1}{0}$ encountered. >>Power::infy: Infinite expression $\frac{1}{0}$ encountered. >>

Infinity::indet:

$$\text{Indeterminate expression } 1 - \frac{100^{1/e} e^3}{(-0.5 + e)^2 (e - f)} - \frac{100^{1/f} f^3}{(-0.5 + f)^2 (-e + f)} + \text{ComplexInfinity} + \text{ComplexInfinity encountered.} \gg$$

Indeterminate

$$-a^3 n^{1/a} / ((a-b)(a-c)(a-d)) - b^3 n^{1/b} / ((b-a)(b-c)(b-d)) - c^3 n^{1/c} / ((c-b)(c-a)(c-d)) - d^3 n^{1/d} / ((d-b)(d-c)(d-a)) + 1$$

$$\text{Limit}\left[\text{Limit}\left[1 - \frac{a^3 n^{\frac{1}{a}}}{(a-b)(a-c)(a-d)} - \frac{b^3 n^{\frac{1}{b}}}{(-a+b)(b-c)(b-d)} - \frac{c^3 n^{\frac{1}{c}}}{(-a+c)(-b+c)(c-d)} - \frac{d^3 n^{\frac{1}{d}}}{(-a+d)(-b+d)(-c+d)}, c \rightarrow a\right], d \rightarrow b\right]$$

$$1 + \frac{b n^{\frac{1}{b}} (b(-3a+b) + (a-b) \text{Log}[n])}{(a-b)^3} - \frac{a n^{\frac{1}{a}} (a(a-3b) + (-a+b) \text{Log}[n])}{(a-b)^3}$$

$$\text{Limit}\left[1 + \frac{b n^{\frac{1}{b}} (b(-3a+b) + (a-b) \text{Log}[n])}{(a-b)^3} - \frac{a n^{\frac{1}{a}} (a(a-3b) + (-a+b) \text{Log}[n])}{(a-b)^3}, a \rightarrow 1\right]$$

$$\text{Expand@Limit}\left[1 - \frac{b n^{\frac{1}{b}} ((-3 + b) b - (-1 + b) \text{Log}[n])}{(-1 + b)^3} + \frac{n (1 - 3 b + (-1 + b) \text{Log}[n])}{(-1 + b)^3}, b \rightarrow 1\right]$$

$$1 - n + n \text{Log}[n] - \frac{1}{2} n \text{Log}[n]^2 + \frac{1}{6} n \text{Log}[n]^3$$

$$N\left[1 - 800 \left(-\frac{1}{2} - \frac{\text{Log}[100]}{2}\right) + 40\,000 \left(-\frac{5}{4} + \frac{\text{Log}[100]}{2}\right)\right]$$

$$44\,346.5$$

$$1 - \frac{a n^{\frac{1}{a}}}{(a - b)} - \frac{b n^{\frac{1}{b}}}{(-a + b)}$$

$$1 - \frac{a n^{\frac{1}{a}}}{a - b} - \frac{b n^{\frac{1}{b}}}{-a + b}$$

$$\text{Limit}\left[1 - \frac{a n^{\frac{1}{a}}}{a - b} - \frac{b n^{\frac{1}{b}}}{-a + b}, a \rightarrow 1\right]$$

$$\text{Limit}\left[\frac{-1 + b + n - b n^{\frac{1}{b}}}{-1 + b}, b \rightarrow 1\right]$$

$$1 - n + n \text{Log}[n]$$

$$\text{FullSimplify}\left[1 - \frac{a n^{\frac{1}{a}}}{a - b} - \frac{b n^{\frac{1}{b}}}{-a + b}\right]$$

$$\frac{a - a n^{\frac{1}{a}} + b \left(-1 + n^{\frac{1}{b}}\right)}{a - b}$$

$$\frac{a \left(1 - n^{\frac{1}{a}}\right) + b \left(-1 + n^{\frac{1}{b}}\right)}{a - b}$$

$$\frac{a \left(1 - n^{\frac{1}{a}}\right) + b \left(-1 + n^{\frac{1}{b}}\right)}{a - b}$$

$$\text{Limit}[1 - b n^{\frac{1}{b}} ((b - 1) / b) + (b - 1) n, b \rightarrow \text{Infinity}]$$

$$\text{Limit}\left[\frac{a \left(1 - n^{\frac{1}{a}}\right) + b \left(-1 + n^{\frac{1}{b}}\right)}{a - b}, a \rightarrow 1\right]$$

$$\text{Limit}\left[\frac{-1 + b + n - b n^{\frac{1}{b}}}{-1 + b}, b \rightarrow 1\right]$$

$$1 - n + n \text{Log}[n]$$

```

FullSimplify[1 -  $\frac{a^3 n^{\frac{1}{a}}}{(a-b)(a-c)(a-d)}$  -
 $\frac{b^3 n^{\frac{1}{b}}}{(-a+b)(b-c)(b-d)}$  -  $\frac{c^3 n^{\frac{1}{c}}}{(-a+c)(-b+c)(c-d)}$  -  $\frac{d^3 n^{\frac{1}{d}}}{(-a+d)(-b+d)(-c+d)}$ ]
1 -  $\frac{a^3 n^{\frac{1}{a}}}{(a-b)(a-c)(a-d)}$  -  $\frac{b^3 n^{\frac{1}{b}}}{(-a+b)(b-c)(b-d)}$  -
 $\frac{c^3 n^{\frac{1}{c}}}{(-a+c)(-b+c)(c-d)}$  -  $\frac{d^3 n^{\frac{1}{d}}}{(-a+d)(-b+d)(-c+d)}$ 
cc = {1, 2, 3, 4}
{1, 2, 3, 4}
Length[cc]
4
fg[n_, aa_] :=
1 - Sum[Limit[(c - a) c^(Length[aa] - 1) n^(1/c) / Product[c - b, {b, aa}], c → a], {a, aa}]
fg[100, {1, 2, 3, 4}]
 $-\frac{67}{3} + \frac{27 \times 5^{2/3}}{2^{1/3}} - \frac{32 \sqrt{10}}{3}$ 
fh[n_, aa_] :=
1 - Sum[Limit[(c - a) c^(Length[aa] - 1) n^(1/c) / Product[c - b, {b, aa}], c → a], {a, aa}]

```