```
num[c_] := Numerator[c]; den[c_] := Denominator[c]
alpha[n_, c_] := alpha[n, c] = den[c] (Floor[n / den[c]] - Floor[(n - 1) / den[c]]) -
    num[c] (Floor[n / num[c]] - Floor[(n - 1) / num[c]])
L2[n_, 1, c_] := L2[n, 1, c] = (1 / den[c])
    Sum[alpha[j, c] Log[j / den[c]], {j, den[c] + 1, den[c] n}]; L2[n_, 0, c_] := 0
L2[n_, k_, c_] := L2[n, k, c] = (1 / den[c])
    Sum[If[alpha[j, c] == 0, 0, alpha[j, c] L2[den[c] n / j, k - 1, c]], {j, den[c] + 1, den[c] n}]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
L1[n_, z_, c_] := Sum[bin[z, k] L2[n, k, c], {k, 0, Floor[Log[n] / Log[c]]}]
L1Alt[n_, z_, c_] :=
 den[c]^-1 Sum[E1[n den[c] / j, z, c] N[alpha[j, c] Log[j / den[c]]], {j, 1, n den[c]}]
E2[n_, k_, c_] := E2[n, k, c] =
  (1 / den[c]) Sum[If[alpha[j, c] == 0, 0, alpha[j, c] E2[(den[c] n) / j, k - 1, c]],
    {j, den[c] + 1, den[c] n}]; E2[n_, 0, c_] := 1
E1[n_, z_, c_] := Sum[Binomial[z, k] E2[n, k, c], {k, 0, Floor[Log[If[c < 2, c, 2], n]]}]
```