```
bins[ z_, a_] := Product[ (z - k), {k, 0, a - 1}] / a!
L2[n_, k_ ] := L2[n, k] = Sum[ L2[Floor[n / j], k - 1], {j, 2, n}];
L2[n_, 1] := Sum[ Log[j], {j, 2, n}]; L2[n_, 0] := 1
L1[n_, z_] := Sum[bins[z - 1, a] L2[n, a + 1], {a, 0, Log[2, n]}]
L1a[n_, z_] := Sum[bins[z, a] L2[n, a], {a, 0, Log[2, n]}]
L1b[n_, z_] := (Sum[bins[z, a] L2[n, a], {a, 0, Log[2, n]}] - 1) / z
```

```
N[L1a[100, 1]]
```

363.739

```
N[L1[100, 0]]
```

94.0453

```
Expand[N[(L1a[100., z])]]
```

$$192.184\,z + 129.802\,z^2 + 36.4475\,z^3 + 5.0375\,z^4 + 0.261138\,z^5 + 0.00730208\,z^6$$

```
(List @@ NRoots[ (L1a[100, x] / x) == 0, x][[All, 2]])
```

$$\{-12.9799 - 15.0426\,i,\ -12.9799 + 15.0426\,i,\ -3.68065,\ -3.06086 - 2.95722\,i,\ -3.06086 + 2.95722\,i\}$$

```
Expand[N[L1[100., z]]]
```

$$94.0453 + 169.15\,z + 81.6195\,z^2 + 17.6846\,z^3 + 1.19616\,z^4 + 0.0438125\,z^5$$

```
Roots[ L1[10, x] == 0, x]
```

$$x == \frac{1}{2\,\text{Log}[2]}\left(-5\,\text{Log}[2] - 4\,\text{Log}[3] - 2\,\text{Log}[4] - 2\,\text{Log}[5] - \sqrt{\left((5\,\text{Log}[2] + 4\,\text{Log}[3] + 2\,\text{Log}[4] + 2\,\text{Log}[5])^2 - 4\,\text{Log}[2]\,(-4\,\text{Log}[2] - 2\,\text{Log}[3] + 2\,\text{Log}[6] + 2\,\text{Log}[7] + 2\,\text{Log}[8] + 2\,\text{Log}[9] + 2\,\text{Log}[10])\right)}\right) ||$$

$$x == \frac{1}{2\,\text{Log}[2]}\left(-5\,\text{Log}[2] - 4\,\text{Log}[3] - 2\,\text{Log}[4] - 2\,\text{Log}[5] + \sqrt{\left((5\,\text{Log}[2] + 4\,\text{Log}[3] + 2\,\text{Log}[4] + 2\,\text{Log}[5])^2 - 4\,\text{Log}[2]\,(-4\,\text{Log}[2] - 2\,\text{Log}[3] + 2\,\text{Log}[6] + 2\,\text{Log}[7] + 2\,\text{Log}[8] + 2\,\text{Log}[9] + 2\,\text{Log}[10])\right)}\right)$$

```
Expand[L1[10, z]]
```

$$-2\,\text{Log}[2] + \frac{5}{2}\,z\,\text{Log}[2] + \frac{1}{2}\,z^2\,\text{Log}[2] - \text{Log}[3] + 2\,z\,\text{Log}[3] + z\,\text{Log}[4] + z\,\text{Log}[5] + \text{Log}[6] + \text{Log}[7] + \text{Log}[8] + \text{Log}[9] + \text{Log}[10]$$

```
f1[z_] := 7.8320141805054675` + 6.925824802290074` z + 0.34657359027997264` z^2
```

```
f1[1]
```

15.1044

```
f2[z_] := (z - (-18.780408851506877`)) (z - (-1.2032973411524708`)) (Log[2] / 2)
```

```
f2[0]
```

7.83201

```
f1[0]
```

7.83201

```
Expand[ ( z - (-18.780408851506877`)) (z - (-1.2032973411524708`))]
```

```
Expand[(22.59841603677455` + 19.983706192659348` z + z^2) * 0.34657359027997264`]
```

```
7.83201 + 6.92582 z + 0.346574 z^2
```

```
.5 Log[2]
```

```
0.346574
```

```
N[Roots[ L1[10, x] == 0, x]]
```

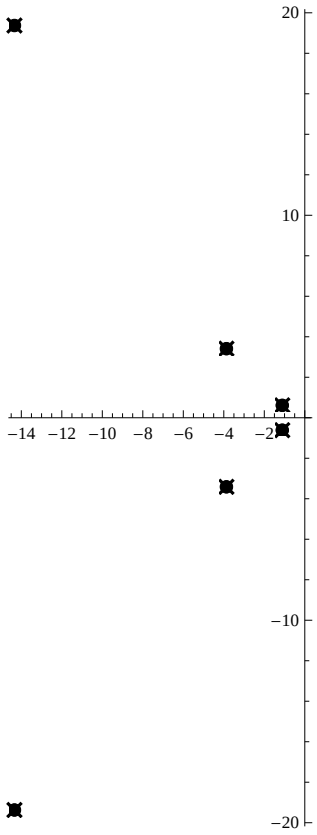
```
x == -18.7804 || x == -1.2033
```

```
N[Roots[Expand[ DDa[10, x]] == 0, x]]
```

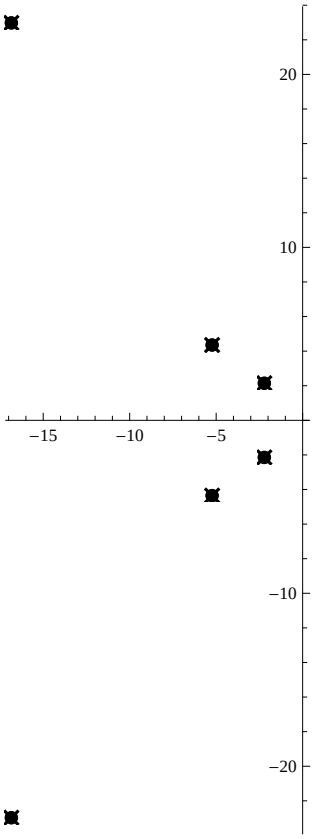
```
x == -0.218507 + 3.55271 × 10^-15 i ||
  x == -1.41809 - 3.55271 × 10^-15 i || x == -19.3634 + 1.66533 × 10^-16 i
```

```
K[n_] := If[n == 1, 0, FullSimplify[MangoldtLambda[n] / Log[n]]]
P[n_, k_] := P[n, k] = Sum[ K[j] P[Floor[n / j], k - 1], {j, 2, n}]; P[n_, 0] := 1
D2[n_, k_] := D2[n, k] = Sum[D2[Floor[n / j], k - 1], {j, 2, n}]; D2[n_, 0] := 1
DD[n_, z_] := Sum[FactorialPower[z, a] / a! D2[n, a], {a, 0, Log[2, n]}]
P[n_, k_, s_] := P[n, k, s] = Sum[ j^-s K[j] P[Floor[n / j], k - 1, s], {j, 2, n}];
P[n_, 0, s_] := 1
DDa[n_, z_] := Sum[ z^k / k! P[n, k], {k, 0, Log[2, n]}]
DDa[n_, z_, s_] := Sum[ z^k / k! P[n, k, s], {k, 0, Log[2, n]}]
DDa2[n_, z_] := Sum[ z^k / k! P[n, k] / z, {k, 0, Log[2, n]}]
```

```
RootLocusPlot[1 / Expand[L1[200, x]], {k, 0, 1}, FeedbackType → None]
```



```
RootLocusPlot[1 / Expand[L1a[200, x] / x], {k, 0, 1}, FeedbackType → None]
```




```
(List @@ NRoots[ (L1a[100, x] / x) == 0, x][[All, 2]])
```

```
{-12.9799 - 15.0426 i, -12.9799 + 15.0426 i,
 -3.68065, -3.06086 - 2.95722 i, -3.06086 + 2.95722 i}
```

```
Expand[N[L1a[100., z] / z]]
```

$$192.184 + 129.802\,z + 36.4475\,z^2 + 5.0375\,z^3 + 0.261138\,z^4 + 0.00730208\,z^5$$

```
vv := {-12.979912222112542` - 15.042599906951901` i,
  -12.979912222112542` + 15.042599906951901` i, -3.6806513834840673`,
  -3.060857811799053` - 2.957220916607718` i, -3.060857811799053` + 2.957220916607718` i}
```

```
[ 1 + 1 / j, {j, vv}]
```

```
94.0453 + 0. i
```

```
Expand[N[(L1[100., z + 1])]]
```

$$363.739 + 390.447\,z + 142.289\,z^2 + 22.9074\,z^3 + 1.41522\,z^4 + 0.0438125\,z^5$$

```
(List @@ NRoots[ (L1[100, x + 1]) == 0, x][[All, 2]])
```

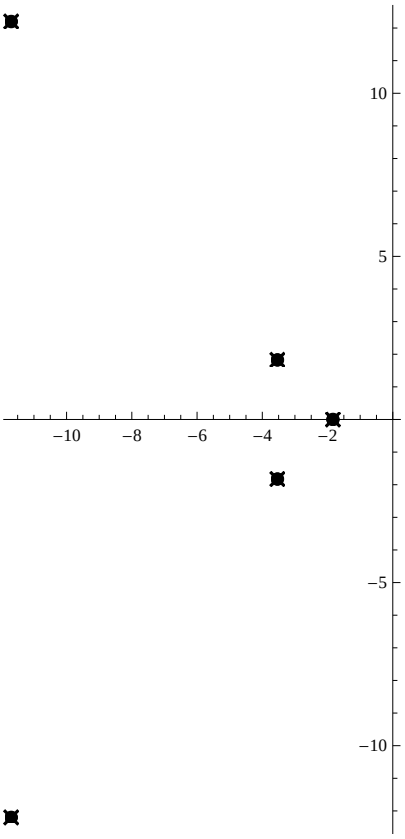
```
{-11.6971 - 12.1994 i, -11.6971 + 12.1994 i,
 -3.53645 - 1.82621 i, -3.53645 + 1.82621 i, -1.83471}
```

```
v2 := {-11.697109591508216` - 12.199354854259528` i,
  -11.697109591508216` + 12.199354854259528` i, -3.5364503360079835` - 1.8262103341135596` i,
  -3.5364503360079835` + 1.8262103341135596` i, -1.8347063543903144`}
```

```
N[Log[100!]] Product[ 1 + 1 / j, {j, v2}]
```

$$94.0453 - 2.52395 \times 10^{-15}\, i$$

```
RootLocusPlot[1 / Expand[L1[100, x + 1]], {k, 0, 1}, FeedbackType → None]
```




```
{-11.697109591508216` - 12.199354854259528` i,
 -11.697109591508216` + 12.199354854259528` i, -3.5364503360079835` - 1.8262103341135596` i,
 -3.5364503360079835` + 1.8262103341135596` i, -1.8347063543903144`}
```

```
{-11.199685576035792` - 12.398224487807212` i,
 -11.199685576035792` + 12.398224487807212` i, -2.6719503346754907` - 1.8618449055430242` i,
 -2.6719503346754907` + 1.8618449055430242` i, -0.9338092178222006`, -0.03720467504094745`}
```

```
(List @@ NRoots[ (L1a[100, x] / x) == 0, x][[All, 2]])
```

$$\{-12.9799 - 15.0426\, i, -12.9799 + 15.0426\, i,$$
$$-3.68065, -3.06086 - 2.95722\, i, -3.06086 + 2.95722\, i\}$$

```
v3 := {-12.979912222112542` - 15.042599906951901` i,
  -12.979912222112542` + 15.042599906951901` i, -3.6806513834840673`,
  -3.060857811799053` - 2.957220916607718` i, -3.060857811799053` + 2.957220916607718` i}
```

```
Log[100!] Product[ 1 + 2 / (j - 1), {j, v3}]
```

$$94.0453 + 0.\, i$$

```
(List @@ NRoots[ (L1a[100, x]) == 0, x][[All, 2]])
```

```
{-12.9799 - 15.0426 i, -12.9799 + 15.0426 i,
 -3.68065, -3.06086 - 2.95722 i, -3.06086 + 2.95722 i, 0.}
```

```
(List @@ NRoots[ (L1a[100, x] / x) == 0, x][[All, 2]])
```

```
{-12.9799 - 15.0426 i, -12.9799 + 15.0426 i,
 -3.68065, -3.06086 - 2.95722 i, -3.06086 + 2.95722 i}
```

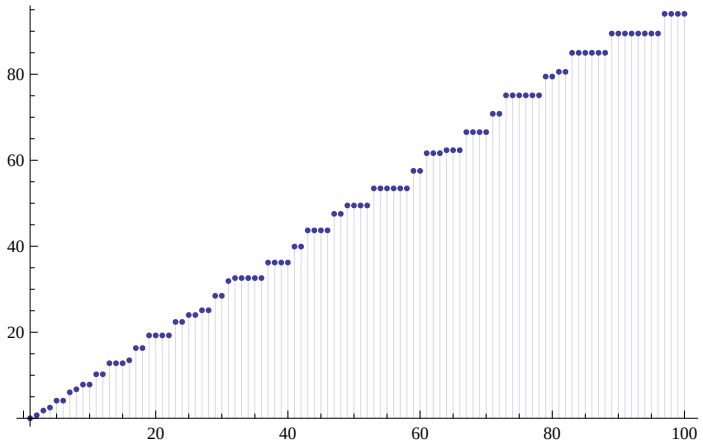
```
(List @@ NRoots[ (L1[100, x + 1]) == 0, x][[All, 2]])
```

```
{-11.6971 - 12.1994 i, -11.6971 + 12.1994 i,
 -3.53645 - 1.82621 i, -3.53645 + 1.82621 i, -1.83471}
```

```
N[Expand[L1a[100, z]]]
```

```
1. + 192.184 z + 129.802 z^2 + 36.4475 z^3 + 5.0375 z^4 + 0.261138 z^5 + 0.00730208 z^6
```

```
DiscretePlot[1 - L1a[n, -1], {n, 1, 100}]
```




```
(List @@ NRoots[ (L1a[100, x]) == 0, x][[All, 2]])
```

```
{-12.9799 - 15.0426 i, -12.9799 + 15.0426 i, -3.66756,
 -3.06482 - 2.95324 i, -3.06482 + 2.95324 i, -0.00522175}
```

```
v4 :=
 {-12.979886712505683` - 15.04256868566443` i, -12.979886712505683` + 15.04256868566443` i,
  -3.667560789512249`, -3.0648177458685923` - 2.953236749181686` i,
  -3.0648177458685923` + 2.953236749181686` i, -0.005221745046451817`}
```

```
Product[ 1 - 1 / j, {j, v4}] - 1
```

```
363.739 + 0. i
```

```
N[Log[100!]]
```

```
363.739
```

```
N[L1a[100, 1]] - 1
```

```
363.739
```

```
N[ Sum[ MangoldtLambda[ j ], {j, 1, 100}]]
```

```
94.0453
```

```
1 - N[L1a[100, -1]]
```

```
94.0453
```

```
1 - Product[ 1 + 1 / j, {j, v4}]
```

```
94.0453 + 0. i
```

```
(1 + 1 / List @@ NRoots[ (L1a[100, x]) == 0, x][[All, 2]])
```

```
{0.967119 + 0.038106 i, 0.967119 - 0.038106 i,
 0.727339, 0.830811 + 0.16303 i, 0.830811 - 0.16303 i, -190.507}
```

```
Sum[ -1 / j, {j, v4}]
```

```
192.184 + 0. i
```

```
(1 + 1 / List @@ NRoots[ (L1a[100, x]) == 0, x][[All, 2]])
```

```
{0.967119 + 0.038106 i, 0.967119 - 0.038106 i,
 0.727339, 0.830811 + 0.16303 i, 0.830811 - 0.16303 i, -190.507}
```