$$C_j = \left(\lim_{t\to 0} \frac{\partial^j}{\partial t^j} \frac{t}{\log(1+t)}\right)$$

$$x^k = \sum_{j=0}^{\infty} \frac{C_j}{j!} \cdot x^{k+j-1} \cdot \log(1+x)$$

$$\boldsymbol{x}^{\boldsymbol{k}} = \sum_{j=0}^{\infty} \frac{C_j}{j!} \cdot \boldsymbol{x}^{\boldsymbol{k+j-1}} \cdot \log(1+\boldsymbol{x})$$

$$\boldsymbol{x}^{\boldsymbol{k}} =$$

| | $\int$ | $\sum$ |
|---|---|---|
| + | $\sum_{j=0}^{\infty} \frac{C_j}{j!} \cdot \int_0^x \int_0^{x-t} \frac{\partial}{\partial t} \boldsymbol{t}^{\boldsymbol{k+j-1}} \cdot \frac{\partial}{\partial u} \log(1+\boldsymbol{u})\, du\, dt$ | $\sum_{j=0}^{\infty} \frac{C_j}{j!} \cdot \sum_{t=1}^{x} \sum_{u=1}^{x-t} \nabla_t \boldsymbol{t}^{\boldsymbol{k+j-1}} \cdot \nabla_u \log(1+\boldsymbol{u})$ |
| * | $\sum_{j=0}^{\infty} \frac{C_j}{j!} \cdot \int_1^x \int_1^{\frac{x}{t}} \frac{\partial}{\partial t} \boldsymbol{t}^{\boldsymbol{k+j-1}} \cdot \frac{\partial}{\partial u} \log(1+\boldsymbol{u})\, du\, dt$ | $\sum_{j=0}^{\infty} \frac{C_j}{j!} \cdot \sum_{t=2}^{x} \sum_{u=2}^{\lfloor \frac{x}{t} \rfloor} \nabla_t \boldsymbol{t}^{\boldsymbol{k+j-1}} \cdot \nabla_u \log(1+\boldsymbol{u})$ |

$$\boldsymbol{x}^{\boldsymbol{k}} =$$

| | $\int$ | $\sum$ |
|---|---|---|
| + | $\frac{x^k}{k!} = \sum_{j=0}^{\infty} \frac{C_j}{j!} \cdot \int_0^x \int_0^{x-t} \frac{t^{k+j-2}}{(k+j-2)!} \cdot \left(\frac{1}{u} - \frac{e^{-u}}{u}\right) du\, dt$ | $\binom{x}{k} = \sum_{j=0}^{\infty} \frac{C_j}{j!} \cdot \sum_{t=1}^{x} \sum_{u=1}^{x-t} \binom{t-1}{k+j-2} \cdot \frac{1}{u}$ |
| * | $(-1)^{-k} P(k, -\log x) = \sum_{j=0}^{\infty} \frac{C_j}{j!} \cdot \int_1^x \int_1^{\frac{x}{t}} \frac{\log^{k+j-2} t}{(k+j-2)} \cdot \left(\frac{1}{\log u} - \frac{1}{u \log u}\right) du\, dt$ | $D_k'(x) = \sum_{j=0}^{\infty} \frac{C_j}{j!} \cdot \sum_{t=2}^{x} \sum_{u=2}^{\lfloor \frac{x}{t} \rfloor} d_{k+j-1}'(t) \cdot \kappa(u)$ |

$$\sum_{k=1} \frac{(-1)^{k+1}}{k} x^{k+a} = x^a \cdot \log(I + x)$$

$$\sum_{k=1} \frac{(-1)^{k+1}}{k} \boldsymbol{x}^{\boldsymbol{k}+\boldsymbol{a}} = \boldsymbol{x}^{\boldsymbol{a}} \cdot \log(1 + \boldsymbol{x})$$

| | $\int$ | $\sum$ |
|---|---|---|
| + | $\sum_{k=1} \frac{(-1)^{k+1}}{k} \boldsymbol{x}^{\boldsymbol{k}+\boldsymbol{a}} = \int_0^x \int_0^{x-t} \frac{\partial}{\partial t} \boldsymbol{t}^{\boldsymbol{a}} \cdot \frac{\partial}{\partial u} \log(1 + \boldsymbol{u})\, du\, dt$ | $\sum_{k=1} \frac{(-1)^{k+1}}{k} \boldsymbol{x}^{\boldsymbol{k}+\boldsymbol{a}} = \sum_{t=1}^{x} \sum_{u=1}^{x-t} \nabla_t \boldsymbol{t}^{\boldsymbol{a}} \cdot \nabla_u \log(1 + \boldsymbol{u})$ |
| * | $\sum_{k=1} \frac{(-1)^{k+1}}{k} \boldsymbol{x}^{\boldsymbol{k}+\boldsymbol{a}} = \int_1^x \int_1^{\frac{x}{t}} \frac{\partial}{\partial t} \boldsymbol{t}^{\boldsymbol{a}} \cdot \frac{\partial}{\partial u} \log(1 + \boldsymbol{u})\, du\, dt$ | $\sum_{k=1} \frac{(-1)^{k+1}}{k} \boldsymbol{x}^{\boldsymbol{k}+\boldsymbol{a}} = \sum_{t=2}^{x} \sum_{u=2}^{\lfloor \frac{x}{t} \rfloor} \nabla_t \boldsymbol{t}^{\boldsymbol{a}} \cdot \nabla_u \log(1 + \boldsymbol{u})$ |

| | $\int$ | $\sum$ |
|---|---|---|
| + | $\sum_{k=1} \frac{(-1)^{k+1}}{k} \cdot \frac{x^{k+a}}{(k+a)!} = \int_0^x \int_0^{x-t} \frac{t^{a-1}}{(a-1)!} \cdot (\frac{1}{u} - \frac{e^{-u}}{u})\, du\, dt$ | $\sum_{k=1} \frac{(-1)^{k+1}}{k} \binom{x}{k+a} = \sum_{t=1}^{x} \sum_{u=1}^{x-t} \binom{t-1}{a-1} \cdot \frac{1}{u}$ |
| * | $\sum_{k=1} \frac{(-1)^{k+1}}{k} (-1)^{-(k+a)} P(k+a, -\log x) =$ $\int_1^x \int_1^{\frac{x}{t}} \frac{\log^{a-1} t}{(a-1)!} \cdot (\frac{1}{\log u} - \frac{1}{u \log u})\, du\, dt$ | $\sum_{k=1} \frac{(-1)^{k+1}}{k} D_{k+a}{}'(x) = \sum_{t=2}^{x} \sum_{u=2}^{\lfloor \frac{x}{t} \rfloor} d_a{}'(t) \cdot \kappa(u)$ |

$$\log^{a}(1+x)=\sum_{k=1}\frac{(-1)^{k+1}}{k}x^{k}\cdot\log^{a-1}(1+x)$$

$$\log^{a}(1+x)=\sum_{k=0}\frac{B_k}{k!}x\cdot\log^{k+a-1}(1+x)$$

$$\log(1+x)=\frac{x}{1+x}+\sum_{k=2}\frac{(-1)^{k}}{k(k-1)}\cdot\frac{x^{k}}{1+x}$$

$$\log(1+x)=\sum_{k=1}(-1)^{k+1}\cdot H_k\cdot x^{k}\cdot(1+x)$$

$$\log(1+x)=\sum_{k=1}(-1)^{k+1}\cdot H_k\cdot x^k\cdot(1+x)$$

$$\log(1+\boldsymbol{x})=\sum_{k=1}(-1)^{k+1}\cdot H_k\cdot \boldsymbol{x}^{\boldsymbol{k}}\cdot(1+\boldsymbol{x})$$

$$\log(1+\boldsymbol{x})=\sum_{k=1}(-1)^{k+1}\cdot H_k\cdot \boldsymbol{x}^{\boldsymbol{k}}+\boldsymbol{x}^{\boldsymbol{k+1}}$$

| | $\int$ | $\sum$ |
|---|---|---|
| + | | $\sum_{k=0}^{\infty}(-1)^{k+1}\cdot H_k\cdot\sum_{t=0}^{x}\sum_{u=1}^{x-t}\nabla_t \boldsymbol{t}^{\boldsymbol{k}}\cdot\nabla_u(1+\boldsymbol{u})$ |
| * | | $\sum_{k=0}^{\infty}(-1)^{k+1}\cdot H_k\cdot\sum_{t=1}^{x}\sum_{u=2}^{\lfloor\frac{x}{t}\rfloor}\nabla_t \boldsymbol{t}^{\boldsymbol{k}}\cdot\nabla_u(1+\boldsymbol{u})$ |

| | $\int$ | $\sum$ |
|---|---|---|
| + | | $\sum_{k=0}^{\infty}(-1)^{k+1}\cdot H_k\cdot\sum_{t=0}^{x}\sum_{u=1}^{x-t}\binom{t-1}{k-1}\cdot u$ |
| * | | $\sum_{k=0}^{\infty}(-1)^{k+1}\cdot H_k\cdot\sum_{t=2}^{x}\sum_{u=1}^{\lfloor\frac{x}{t}\rfloor}d_k'(t)$ |

| | $\int$ | $\sum$ |
|---|---|---|
| + | | $\sum_{k=0}^{\infty}(-1)^{k+1}\cdot H_k\cdot\sum_{t=0}^{x}\binom{t-1}{k-1}\cdot\binom{t-x}{2}$ |
| * | | $\sum_{k=0}^{\infty}(-1)^{k+1}\cdot H_k\cdot\sum_{t=2}^{x}\lfloor\frac{x}{t}\rfloor\cdot d_k'(t)$ |

| | $\int$ | $\sum$ |
|---|---|---|
| + | | $H_x=\sum_{k=0}^{\infty}(-1)^{k+1}\cdot H_k\cdot(\binom{x}{k}+\binom{x}{k+1})$ |
| * | | $\Pi(x)=\sum_{k=0}^{\infty}(-1)^{k+1}\cdot H_k\cdot(D_k'(x)+D_{k+1}'(x))$ |

$$(\log \zeta(s))^a = \sum_{k=1} \frac{(-1)^{k+1}}{k} (\zeta(s)-1)^k (\log \zeta(s))^{a-1}$$

$$\log \zeta(s) = \sum_{k=0} \frac{B_k}{k!} (\zeta(s)-1) \cdot \lim_{z \to 0} \frac{\partial^k}{\partial z^k} \zeta(s)^z$$

$$\lim_{z \to 0} \frac{\partial^a}{\partial z^a} \zeta(s)^z = \sum_{k=0} \frac{B_k}{k!} (\zeta(s)-1) \cdot \lim_{z \to 0} \frac{\partial^{k+a-1}}{\partial z^{k+a-1}} \zeta(s)^z$$

$$(1+x)^z=\sum_{k=0}\binom{z}{k}x^k$$

$$(1-x)^z=\sum_{k=0}(-1)^k\cdot\binom{z}{k}\cdot x^k$$

*Note! The following two actually converge for arbitrary z! Neat!*

$$(1-x)^z=\sum_{k=0}(-1)^k\cdot\binom{z}{k}\cdot 2^{z-k}\cdot(1+x)^k$$

$$(1+x)^z=\sum_{k=0}(-1)^k\cdot\binom{z}{k}\cdot 2^{z-k}\cdot(1-x)^k$$

$$\log(1+x)=\lim_{z\to 0}\frac{\partial}{\partial z}\sum_{k=0}(-1)^k\cdot\binom{z}{k}\cdot 2^{z-k}\cdot(1-x)^k$$

$$\log(1+x)=\log 2-\sum_{k=1}\frac{1}{2^k\cdot k}\cdot(1-x)^k$$

$$\log(1+\boldsymbol{x})=\log 2-\sum_{k=1}\frac{1}{2^k\cdot k}\cdot(1-\boldsymbol{x})^k$$

$$\Pi(x)=\lim_{z\to 0}\frac{\partial}{\partial z}\sum_{k=0}(-1)^k\cdot\binom{z}{k}\cdot 2^{z-k}\cdot(1-\boldsymbol{x})^{k}{}^{*\Sigma}$$

$$\Pi(x)=\log 2-\sum_{k=1}\frac{1}{2^k\cdot k}\cdot(1-\boldsymbol{x})^{k}{}^{*\Sigma}$$

$$\log(1-x)=-\sum_{k=1}\frac{x^k}{k}$$

$$(1-\boldsymbol{x})^z=\sum_{k=0}(-1)^k\cdot\binom{z}{k}\cdot\boldsymbol{x}^{\boldsymbol{k}}$$

$$(1-x)^z=\sum_{k=0}(-1)^k\cdot\binom{z}{k}\cdot x^k$$

....

$$(1+bx)^z=\sum_{k=0}\binom{z}{k}b^k\cdot x^k$$

$$(a+x)^z=\sum_{k=0}\binom{z}{k}a^{z-k}\cdot x^k$$

$$(a+bx)^z=\sum_{k=0}\binom{z}{k}a^{z-k}\cdot b^k\cdot x^k$$

$$(a+bx)^z=b^z\cdot\left(\frac{a}{b}+x\right)^z=a^z\cdot\left(1+\frac{b}{a}x\right)^z$$

$$(a+b\boldsymbol{x})^z=\sum_{k=0}\binom{z}{k}a^{z-k}\cdot b^k\cdot\boldsymbol{x}^{\boldsymbol{k}}$$

$$\log(a+a\,\boldsymbol{x})=\log a+\log(1+\boldsymbol{x})$$

*Revisit an updated version of* $\frac{1-x^k}{1-x} = 1 + x + x^2 + \ldots + x^k$

$$\nabla[2^z]_n = \binom{z}{n} = \sum_{k=0}^{\frac{n}{2}} \nabla[\boldsymbol{\infty}^z]_{n-2k} \cdot \nabla[\boldsymbol{\infty}^{-z}]_k$$

$$\binom{z}{n} = \sum_{k=0}^{\frac{n}{2}} \left(\binom{z}{n-2k}\right) \cdot \left(\binom{-z}{k}\right)$$

$$\sum_{j=0}^{n} \binom{z}{j} = \sum_{k=0}^{\frac{n}{2}} \left(\binom{z+1}{n-2k}\right) \cdot \left(\binom{-z}{k}\right)$$

$$\sum_{j=0}^{n} \binom{z}{j} = \sum_{k=0}^{\frac{n}{2}} \sum_{j=0}^{n-2k} \left(\binom{z}{j}\right) \cdot \left(\binom{-z}{k}\right)$$

$$\sum_{j=0}^{n} \binom{z}{j} = \sum_{j+2k \le n} \nabla_j (1+\boldsymbol{j})^{z+\Sigma} \cdot \nabla_k (1+\boldsymbol{k})^{-z+\Sigma}$$

$$\sum_{j=0}^{x} \binom{z}{j} = \left(\frac{1+\boldsymbol{x}}{1+\frac{\boldsymbol{x}}{\boldsymbol{2}}}\right)^{z+\Sigma}$$

$$\binom{z}{x} = \nabla_x \cdot \left(\frac{1+\boldsymbol{x}}{1+\frac{\boldsymbol{x}}{\boldsymbol{2}}}\right)^{z+\Sigma}$$

$$\boldsymbol{z}^{x+\Sigma} = \nabla_x \cdot \left(\frac{1+\boldsymbol{x}}{1+\frac{\boldsymbol{x}}{\boldsymbol{2}}}\right)^{z+\Sigma}$$

$$\log\left(\frac{1+\boldsymbol{x}}{1+\frac{\boldsymbol{x}}{\boldsymbol{2}}}\right) = \log(1+\boldsymbol{x}) - \log\left(1+\frac{\boldsymbol{x}}{\boldsymbol{2}}\right)$$

$$\log\left(\frac{1+\boldsymbol{x}}{1+\frac{\boldsymbol{x}}{\boldsymbol{2}}}\right)^{+\Sigma} = H_x - H_{\lfloor \frac{x}{2} \rfloor}$$

$$(1+\boldsymbol{x}) \cdot (1+\boldsymbol{y})^{+\Sigma} = \sum_{\frac{t_1}{x}+\frac{t_2}{y} \le 1} 1$$

$$(1+\boldsymbol{n}) \cdot \left(1+\frac{\boldsymbol{n}}{\boldsymbol{2}}\right)^{+\Sigma} = \sum_{j+2k \le n} 1$$

...

$$\sum_{j=0}^{n} \lambda(j)^{\{z\}} = \sum_{j+k^2 \le n} \nabla_j (1+\boldsymbol{j})^{-z*\Sigma} \cdot \nabla_k (1+\boldsymbol{k})^{z*\Sigma}$$

$$\log\left(\frac{1+\boldsymbol{x}^{\frac{1}{2}}}{1+\boldsymbol{x}}\right)=\log\left(1+\boldsymbol{x}^{\frac{1}{2}}\right)-\log\left(1+\boldsymbol{x}\right)$$

$$\nabla_x\left(1+\boldsymbol{x}\right)^{z\ *\Sigma}=\prod_{p^k|x}\nabla_k\left(1+\boldsymbol{k}\right)^{z\ +\Sigma}$$

$$\left(1+\boldsymbol{x}\right)^{z\ *\Sigma}=\sum_{j=1}^{n}\sum_{p^k|j}\nabla_k\left(1+\boldsymbol{k}\right)^{z\ +\Sigma}$$

$$\left(\frac{1+\boldsymbol{x}}{1+\boldsymbol{x}^{\frac{1}{2}}}\right)^{z\ *\Sigma}=\sum_{j=1}^{n}\sum_{p^q|j}\nabla_k\left(\frac{1+\boldsymbol{k}}{1+\frac{k}{2}}\right)^{z\ +\Sigma}$$

…

$$\lim_{x\to\infty}\left(\frac{1+x}{1+\frac{x}{k}}\right)^{z}=k^{z}$$

and also

$$\lim_{x\to\infty}\left(\frac{1+\boldsymbol{x}}{1+\frac{\boldsymbol{x}}{\boldsymbol{k}}}\right)^{z\ *\Sigma}=k^{z}$$

and also

$$\lim_{x\to\infty}\left(\frac{1+\boldsymbol{x}}{1+\frac{\boldsymbol{x}}{\boldsymbol{k}}}\right)^{z\ *\Sigma}=k^{z}$$

...

$$\left[\left(\frac{\zeta_{1/2}(0)}{\zeta(0)}\right)\right]_n\ \sum_{j=1}^{n}\lambda(j)$$

$$\left[\left(\frac{\zeta_{1/2}(0)}{\zeta(0)}\right)^{-1}\right]_n\ \sum_{j=1}^{n}\mu(j)$$

...

$$\left[\prod_{k=1}\zeta_{1/k}(0)\right]_n\ \sum_{j=1}^{n}a(j)$$

$$\left\{\prod_{k=1}\left(I+\frac{x}{k}\right)^{z}\right\}$$

$$\sum_{a+2b+3c+\ldots\leq x}\nabla_a\left(1+\boldsymbol{a}\right)^{z}\cdot\nabla_b\left(1+\boldsymbol{b}\right)^{z}\cdot\nabla_c\left(1+\boldsymbol{c}\right)^{z}\cdot\ldots^{+\Sigma}$$

$$\sum_{a=0}^{x}t_z(a)\cdot\sum_{b=0}^{\frac{x-a}{2}}t_z(b)\cdot\sum_{c=0}^{\frac{x-a-2b}{3}}t_z(c)\cdot\ldots$$

...

$$\prod_{k=1}(1+\frac{\boldsymbol{x}}{\boldsymbol{k}})^{\frac{z\cdot\mu(k)}{k}}$$

$$\sum_{a+2b+3c+\ldots\leq x}\nabla_a(1+\boldsymbol{a})^z\cdot\nabla_b(1+\boldsymbol{b})^{-\frac{z}{2}}\cdot\nabla_c(1+\boldsymbol{c})^{-\frac{z}{c}}\cdot\ldots{}^{+}\Sigma$$

...

$$\frac{1+\boldsymbol{x}}{1+\frac{\boldsymbol{x}}{\boldsymbol{k}}}{}^{*}\Sigma=\frac{1-x^k}{1-x}=1+x+x^2+\ldots+x^k$$

$$\prod_{k=1}(1+\frac{\boldsymbol{x}}{\boldsymbol{k}})=\prod_{k=1}\frac{1}{1-x^k}$$

$$=\frac{x}{1-x-x^2}=\frac{x}{(x-\frac{1}{2}(1-\sqrt{5}))(x-\frac{1}{2}(1+\sqrt{5}))}$$

$$=\frac{1}{1-x-x^2}=\sum_{k=0}^{\infty}(x+x^2)^k$$

(so the additive log delta of fibonacci sequence is this:)

http://oeis.org/A001350

```
am[n_,0]:=UnitStep[n]
am[n_,k_]:=Sum[Fibonacci[j]am[n-j,k-1],{j,1,n}]
amz[n_,z_]:=Sum[bin[z,k]am[n,k],{k,0,n}]
damz[n_,z_]:=amz[n,z]-amz[n-1,z]
Table[D[damz[j,z],z]/.z->0,{j,1,10}]
Out[745]= {1,1/2,4/3,5/4,11/5,8/3,29/7,45/8,76/9,121/10}

(* http://oeis.org/A001350 *)
Clear[am,amm]
am[n_,0]:=UnitStep[n]
am[n_,k_]:=am[n,k]=Sum[Fibonacci[j]am[n-j,k-1],{j,1,n}]
amz[n_,z_]:=Sum[bin[z,k]am[n,k],{k,0,n}]
damz[n_,z_]:=amz[n,z]-amz[n-1,z]
iv[n_]:=Floor[n/2]-Floor[(n+1)/2]
amm[n_,0]:=UnitStep[n]
amm[n_,k_]:=amm[n,k]=Sum[iv[j]amm[n-j,k-1],{j,1,n}]
ammz[n_,z_]:=Sum[bin[z,k]amm[n,k],{k,0,n}]
dammz[n_,z_]:=amz[n,z]-ammz[n-1,z]
```

```
Table[D[amz[j,z],z]/.z->0,{j,1,10}]
Table[D[damz[j,z],z]/.z->0,{j,1,10}]
Table[damz[j,z]/.z->-1,{j,1,32}]
Table[ammz[j,z]/.z->-1,{j,1,32}]
```

$$1-\binom{z}{1}\sum_{(2j+1)\leq n}1+\binom{z}{2}\sum_{(2j+1)+(2k+1)\leq n}1-\binom{z}{3}\sum_{(2j+1)+(2k+1)+(2l+1)\leq n}1+\binom{z}{4}\ldots$$

$$1-\binom{z}{1}\sum_{(2j+1)\leq n}1+\binom{z}{2}\sum_{j+k\leq(\frac{n}{2}-1)}1-\binom{z}{3}\sum_{j+k+l\leq\frac{n-3}{2}}1+\binom{z}{4}\ldots$$

blah blah blah. And then…

$$F_n=\sum_{k=0}^{n}\frac{\lfloor\frac{n-k}{2}\rfloor^{(k)}}{k!}$$

$$F_n=\sum_{k=0}^{n}\binom{\lfloor\frac{n+k-2}{2}\rfloor}{k}$$

MEH. Known. Obvious. Already in my notes.

$$\left(\frac{1-x^2}{1-x}\right)^z \text{ -> } {z \atop n} = \sum_{k=0}^{\lfloor\frac{n}{2}\rfloor} \left(\binom{z}{n-2k}\right)\cdot\left(\binom{-z}{k}\right) = \nabla_x \frac{(1+\boldsymbol{x})^z}{\left(1+\frac{\boldsymbol{x}}{\boldsymbol{2}}\right)^z} {}^{+}\Sigma = \frac{(1+\boldsymbol{x})^{z-1}}{\left(1+\frac{\boldsymbol{x}}{\boldsymbol{2}}\right)^z} {}^{+}\Sigma$$

$$\frac{(1-x^2)^z}{(1-x)^{z+1}} \text{ -> } \sum_{j=0}^{n} \binom{z}{j} = \sum_{j=0}^{n} \binom{n-j}{n-j}\binom{z}{j} = \sum_{k=0}^{\lfloor\frac{n}{2}\rfloor} \left(\binom{z+1}{n-2k}\right)\cdot\left(\binom{-z}{k}\right) = \frac{(1+\boldsymbol{x})^z}{\left(1+\frac{\boldsymbol{x}}{\boldsymbol{2}}\right)^z} {}^{+}\Sigma$$

$$\frac{(1-x^2)^z}{(1-x)^{z+2}} \text{ -> } \sum_{j=0}^{n} \sum_{k=0}^{j} \binom{z}{k} = \sum_{j=0}^{n} \binom{n-j+1}{n-j}\binom{z}{j} = \sum_{k=0}^{\lfloor\frac{n}{2}\rfloor} \left(\binom{z+2}{n-2k}\right)\cdot\left(\binom{-z}{k}\right) = \frac{(1+\boldsymbol{x})^{z+1}}{\left(1+\frac{\boldsymbol{x}}{\boldsymbol{2}}\right)^z} {}^{+}\Sigma$$

$$\frac{(1-x^2)^z}{(1-x)^{z+3}} \text{ -> } \sum_{j=0}^{n} \sum_{k=0}^{j} \sum_{l=0}^{k} \binom{z}{l} = \sum_{j=0}^{n} \binom{n-j+2}{n-j}\binom{z}{j} = \sum_{k=0}^{\lfloor\frac{n}{2}\rfloor} \left(\binom{z+3}{n-2k}\right)\cdot\left(\binom{-z}{k}\right) = \frac{(1+\boldsymbol{x})^{z+2}}{\left(1+\frac{\boldsymbol{x}}{\boldsymbol{2}}\right)^z} {}^{+}\Sigma$$

$$\frac{(1-x^2)^z}{(1-x)^{z+a+1}} \text{ -> } \sum_{j=0}^{n} \binom{n-j+a}{n-j}\binom{z}{j} = \sum_{k=0}^{\lfloor\frac{n}{2}\rfloor} \left(\binom{z+a+1}{n-2k}\right)\cdot\left(\binom{-z}{k}\right) = \sum_{j=0}^{n} \sum_{k=0}^{\lfloor\frac{n-j}{2}\rfloor} \left(\binom{z+a}{j}\right)\cdot\left(\binom{-z}{k}\right) = \frac{(1+\boldsymbol{x})^{z+a}}{\left(1+\frac{\boldsymbol{x}}{\boldsymbol{2}}\right)^z} {}^{+}\Sigma$$

...

$$\sum_{j=0}^{n} \sum_{k=0}^{\lfloor\frac{n-j}{2}\rfloor} \left(\binom{z}{j}\right)\cdot\left(\binom{-z}{k}\right) = \sum_{k=0}^{\lfloor\frac{n}{2}\rfloor} \left(\binom{z+1}{n-2k}\right)\cdot\left(\binom{-z}{k}\right) = \sum_{k=0}^{n} \left(\binom{z}{k}\right)\cdot\left(\binom{-z+1}{\lfloor\frac{n-k}{2}\rfloor}\right)$$

…

$$\nabla_x \frac{(1+\boldsymbol{x})^z}{\left(1+\frac{\boldsymbol{x}}{\boldsymbol{2}}\right)^z} {}^{+}\Sigma = \frac{(1+\boldsymbol{x})^{z-1}}{\left(1+\frac{\boldsymbol{x}}{\boldsymbol{2}}\right)^z} {}^{+}\Sigma = \boldsymbol{x}^{\boldsymbol{k}} {}^{+}\Sigma$$

$$p(x,z)=\frac{(x+z)!}{x!\,z!}$$

$$\sum_{j=0}^{z} p(x,j)=p(x+1,z)$$

$$\sum_{k=0}^{z}\sum_{j=0}^{k} p(x,j)=p(x+2,z)$$

$$\sum_{k=0}^{z}\sum_{l=0}^{k}\sum_{j=0}^{l} p(x,j)=p(x+3,z)$$

$$\sum_{k=0}^{z}\sum_{l=0}^{k}\sum_{j=0}^{l} p(x,j)=\sum_{j=0}^{z} p(z-j,2)\cdot p(x,j)=p(x+3,z)$$

$$\sum_{j=0}^{z} p(z-j,a-1)\cdot p(x,j)=p(x+a,z)$$

$$\frac{(1-x^2)^z}{(1-x)^z}=(1+x)^z$$

$$\prod_{k=0}(1+x^{2^k})^z=(1-x)^z$$

$$\prod_{k=1}(1+x^{2^k})^z=(1-x^2)^z$$

$$\frac{(1-\boldsymbol{x}^{\mathbf{2}})^z}{(1-\boldsymbol{x})^z}=(1+\boldsymbol{x})^z$$

$$\prod_{k=0}(1+\boldsymbol{x}^{\mathbf{2}^k})^z=(1-\boldsymbol{x})^z$$

$$\log(1+\boldsymbol{x})=\log(1-\boldsymbol{x}^{\mathbf{2}})-\log(1-\boldsymbol{x})$$

$$\sum_{k=0}\log(1+\boldsymbol{x}^{\mathbf{2}^k})=\log(1-\boldsymbol{x})$$

$$\frac{(x+z)!}{x!\,z!}=\sum_{k=0}\binom{z}{k}\binom{x}{k}$$

$$\frac{(-x-z)!}{(-x)!\,(-z)!}=\sum_{k=0}\frac{z^{(k)}}{k!}\cdot\frac{x^{(k)}}{k!}$$

$$F_z(n,2) \text{ where } F_z(n,y)=\begin{cases} \sum_{k=0}^{\lfloor \frac{\log n}{\log y} \rfloor} \binom{z}{k} \cdot F_{z-k}\left(\frac{n}{y^k}, y+1\right) & \text{if } n \geq y \\ 1 & \text{if } n < y \end{cases}$$

$$F_z(n,y)=\sum_{j=1}^{n} f_z(j,y)$$

$$f_z(n,y)=F_z(n,y)-F_z(n-1,y)$$

$$\boxed{f_z(n,y+1)=\sum_{k=0}^{y^k|n} (-1)^k \binom{z}{k} \cdot \left(f_{z-k}\left(\frac{n}{y^k}, y\right)\right)}$$

$$\boxed{f_z(n,2)=\prod_{p^a|n} \left(\binom{z}{a}\right)}$$