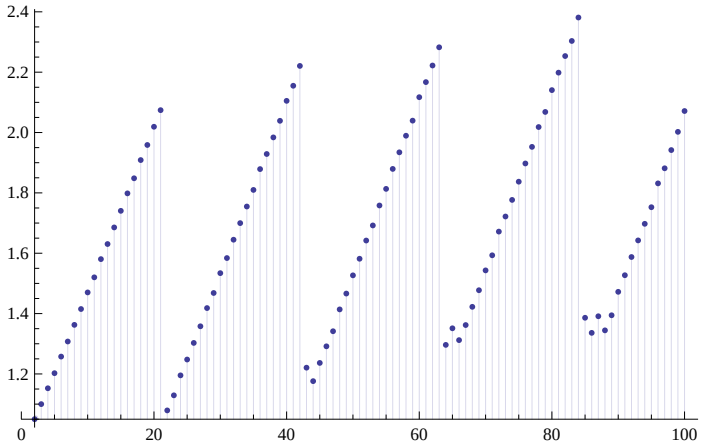
```
num[c_] := Numerator[c]; den[c_] := Denominator[c]
alpha[n_, c_] := alpha[n, c] = den[c] (Floor[n / den[c]] - Floor[(n - 1) / den[c]]) -
    num[c] (Floor[n / num[c]] - Floor[(n - 1) / num[c]])
F[n_, 0, s_, c_] := 1
F[n_, 1, s_, c_] := If[n < s, 0, (den[c] Floor[n / den[c]] - num[c] Floor[n / num[c]]) -
    (den[c] Floor[(s - 1) / den[c]] - num[c] Floor[(s - 1) / num[c]])]
F[n_, k_, s_, c_] := F[n, k, s, c] = Sum[If[alpha[m, c] == 0, 0, Binomial[k, j] alpha[m, c]^j
      F[Floor[n / (m^j)], k - j, m + 1, c]], {j, 1, k}, {m, s, Floor[n^(1 / k)]}]
E2Alt[n_, k_, c_] := den[c]^-k F[n den[c]^k, k, den[c] + 1, c]
E2[n_, k_, c_] :=
 E2[n, k, c] = (1 / den[c]) Sum[If[alpha[j, c] == 0, 0, alpha[j, c] E2[(den[c] n) / j, k - 1, c]],
     {j, den[c] + 1, den[c] n}]; E2[n_, 0, c_] := 1
E1[n_, z_, c_] := Sum[Binomial[z, k] E2Alt[n, k, c], {k, 0, Floor[Log[If[c < 2, c, 2], n]]}]
E2x[n_, k_, x_] :=
 E2x[n, k, x] = Sum[E2x[n / j, k - 1, x], {j, 2, n}] - x Sum[E2x[n / (x j), k - 1, x], {j, 1, n / x}];
E2x[n_, 0, x_] := 1
E1x[n_, z_, c_] := Sum[Binomial[z, k] E2x[n, k, c], {k, 0, Floor[Log[If[c < 2, c, 2], n]]}]
L2[n_, k_, c_] := L2[n, k, c] = (1 / den[c])
    Sum[If[alpha[j, c] == 0, 0, alpha[j, c] Log[j / den[c]] E2Alt[den[c] n / j, k - 1, c]],
     {j, den[c] + 1, den[c] n}]; L2[n_, 0, c_] := 1
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
L1[n_, z_, c_] :=
 L1[n, z, c] = Sum[bin[z, k] L2[n, k, c], {k, 1, Floor[Log[If[c < 2, c, 2], n]]}]
L2x[n_, 1, b_] := L2x[n, 1, b] =
  L2x[n, 1, b] = Sum[Log[j], {j, 2, n}] - b Sum[Log[j b], {j, 1, n / b}]
L2x[n_, k_, b_] := L2x[n, k, b] = Sum[L2x[n / j, k - 1, b], {j, 2, n}] -
    b Sum[L2x[n / (j b), k - 1, b], {j, 1, n}]
L1x[n_, z_, x_] := L1x[n, z, x] = Sum[bin[z, k] L2x[n, k, x],
    {k, 1, Floor[Log[If[x < 2, x, 2], n]]}]
```

```
DiscretePlot[D[ L1x[n, z, 1.05], {z, 1}] /. z → 0, {n, 2, 100}]
```

```
$Aborted
```

```
DiscretePlot[E1[n, -1, 1.05], {n, 2, 100}]
```



```
N[L1x[10, -1, 1.1]]
```

```
1.44615
```

```
ff[n_, c_] := -Sum[ Log[j] E1[ n / j, -1, c], {j, 2, n}] +
  c Sum[ Log[ j c ] E1[n / (j c), -1, c], {j, 1, Floor[n / c]}]
ff2[n_, c_] := -Sum[ Log[j] E1[ n / j, -1, c], {j, 2, n}] +
  Sum[ c Log[ j c ] E1[n / (j c), -1, c], {j, 2, Floor[n / c]}] + c Log[ c ] E1[n / (c), -1, c]
ff3[n_, c_] := -Sum[ Log[j] E1[ n / j, -1, c], {j, Floor[n / c] + 1, n}] -
  Sum[ Log[j] E1[ n / j, -1, c], {j, 2, Floor[n / c]}] +
  Sum[ c Log[ j c ] E1[n / (j c), -1, c], {j, 2, Floor[n / c]}] + c Log[ c ] E1[n / (c), -1, c]
ff4[n_, c_] := c Log[ c ] E1[n / (c), -1, c] +
  Sum[ c Log[ j c ] E1[n / (j c), -1, c] - Log[j] E1[ n / j, -1, c], {j, 2, Floor[n / c]}] -
  Sum[ Log[j] E1[ n / j, -1, c], {j, Floor[n / c] + 1, n}]
ff4a[n_, c_] := c Log[ c ] E1[n / (c), -1, c]
ff4b[n_, c_] :=
 Sum[ c Log[ j c ] E1[n / (j c), -1, c] - Log[j] E1[ n / j, -1, c], {j, 2, Floor[n / c]}]
ff4c[n_, c_] := -Sum[ Log[j] E1[ n / j, -1, c], {j, Floor[n / c] + 1, n}]
```

```
N[ff[ 10, 11 / 10]]
```

```
1.44615
```

```
N[ff4[ 10, 11 / 10]]
```

```
1.44615
```

```
N[ff4a[ 10, 11 / 10]]
```

```
2.56559
```

```
N[ff4b[ 10, 21 / 20]]
```

```
1.38471
```

```
N[ff4c[ 10, 21 / 20]]
```

```
-2.30259
```

```
$RecursionLimit = 10 000
xff[n_, c_] := -Sum[ Log[j] E1x[ n / j, -1, c], {j, 2, n}] +
  c Sum[ Log[ j c ] E1x[n / (j c), -1, c], {j, 1, Floor[n / c]}]
xff2[n_, c_] := -Sum[ Log[j] E1x[ n / j, -1, c], {j, 2, n}] +
  Sum[ c Log[ j c ] E1x[n / (j c), -1, c], {j, 2, Floor[n / c]}] + c Log[ c ] E1x[n / (c), -1, c]
xff3[n_, c_] := -Sum[ Log[j] E1x[ n / j, -1, c], {j, Floor[n / c] + 1, n}] -
  Sum[ Log[j] E1x[ n / j, -1, c], {j, 2, Floor[n / c]}] +
  Sum[ c Log[ j c ] E1x[n / (j c), -1, c], {j, 2, Floor[n / c]}] + c Log[ c ] E1x[n / (c), -1, c]
xff4[n_, c_] := c Log[ c ] E1x[n / (c), -1, c] +
  Sum[ c Log[ j c ] E1x[n / (j c), -1, c] - Log[j] E1x[ n / j, -1, c], {j, 2, Floor[n / c]}] -
  Sum[ Log[j] E1x[ n / j, -1, c], {j, Floor[n / c] + 1, n}]
xff4a[n_, c_] := c Log[ c ] E1x[n / (c), -1, c]
xff4b[n_, c_] :=
 Sum[ c Log[ j c ] E1x[n / (j c), -1, c] - Log[j] E1x[ n / j, -1, c], {j, 2, Floor[n / c]}]
xff4bt[n_, c_] := Table[ c Log[ j c ] E1x[n / (j c), -1, c] - Log[j] E1x[ n / j, -1, c],
   {j, 2, Floor[n / c]}] // TableForm
xff4c[n_, c_] := -Sum[ Log[j] E1x[ n / j, -1, c], {j, Floor[n / c] + 1, n}]
```

```
10 000
```

```
{N[xff4a[ nnn = 10, rrr = 1.02]], N[ xff4b[nnn, rrr]], N[xff4c[nnn, rrr]]}
```

```
{1.92275, 1.58203, -2.30259}
```

```
2.30259
```

```
E1x[10 / 1.02, -1, 1.02]
```

```
95.1919
```

```
Plot[ E1x[n, -1, 1.1], {n, 2, 10}]
```

```
{ N[ xff4b[nnn = 10, rrr = 1.003]]}
```

```
{1.56292}
```

```
E1x[10, -1, 1.1]
```

```
25.9184
```

```
E1x[9.5, -1, 1.1]
```

```
15.4068
```

```
N[xff4bt[10, 1.002]]
```

```
3.22147
2.65566
1.24994
0.997323
-1.12665
-1.51838
-1.8309
-2.08763
```

```
Em1[n_, y_] :=
 1 - Sum[ Em1[ n / j, y], {j, 2, n}] + y Sum[ Em1[ n / (j y), y], {j, 1, Floor[n / y]}]
Em1[n_, y_] := 1 - Sum[ Em1[ n / j, y], {j, 2, n}] +
  y Sum[ Em1[ n / (j y), y], {j, 2, Floor[n / y]}] + If[ n / y < 1, 0, y Em1[ n / y, y]]
```

```
E1[10, -1, 2]
```

```
3
```

```
Em1[10, 2]
```

```
3
```

```
Limit[ 1 + (n - 2) (1 - x) - x, x → 1]
```

```
0
```