$$\boxed{[(1+\zeta(0,y))^z]_n = \sum_{k=0} \binom{z}{k} \cdot [1+\zeta(0,y+1)^{z-k}]_{n/y^k}}$$

$$[\zeta(0)^z] = [(1+\zeta(0,2))^z]_n$$

(up to $k \le \log_y n$)

$$\boxed{F(n,y+1,z)=\sum_{k=0}^{\log_y n}(-1)^k\binom{z}{k}\cdot F\left(\frac{n}{y},y,z-k\right)}$$

$$\boxed{f(n,y+1,z)=\sum_{k=0}^{y^k|n}(-1)^k\binom{z}{k}\cdot f\left(\frac{n}{y^k},y,z-k\right)}$$

$$\boxed{f(n,2,z)=\prod_{p^a|n}\frac{z^{(a)}}{a!}}$$

```
bin[z_, k_] := Product[z - j, {j, 0, k - 1}]/k!
dd[n_, s_, y_, k_] := dd[n, s, y, k] = Sum[ j^-s dd[Floor[n/j], s, y, k - 1], {j, y, n}]
dd[n_, s_, y_, 0] := UnitStep[n - 1]
dz[n_, s_, y_, z_] := Sum[ bin[z, k] dd[n, s, y, k], {k, 0, Log[y, n]}]
ddz[n_, s_, y_, z_] := dz[n, s, y, z] - dz[n - 1, s, y, z]
de[n_, k_, y_, z_] := bin[z, k] ddz[n, 0, y - 1, z - k] - If[ Mod[n, y - 1] == 0, de[n/(y - 1), k + 1, y, z], 0]
```

$$\boxed{F(n,y,z)=\sum_{j=1}^{n}f(j,y,z)}$$

$$\boxed{f(n,y,z)=F(n,y,z)-F(n-1,y,z)}$$

$$\boxed{F(n,y,z)=1\ \mathit{if}\ n<y}$$

$$\boxed{F(n,y,z)=1+(n-y+1)\cdot z\ \mathit{if}\ n<y^2}$$

$$\boxed{F(n,y,z)=1+(n-y+1)\cdot z+\frac{z(z-1)}{2}\cdot\sum_{a=y}^{n}\left\lfloor\frac{n}{a}\right\rfloor-y+1\ \mathit{if}\ n<y^3}$$

$$\boxed{F(n,y,z)-F(n,y+1,z)=z+\frac{z(z-1)}{2}\cdot\left(1+2\left\lfloor\frac{n}{y}-y\right\rfloor\right)\mathit{if}\ n<y^3}$$