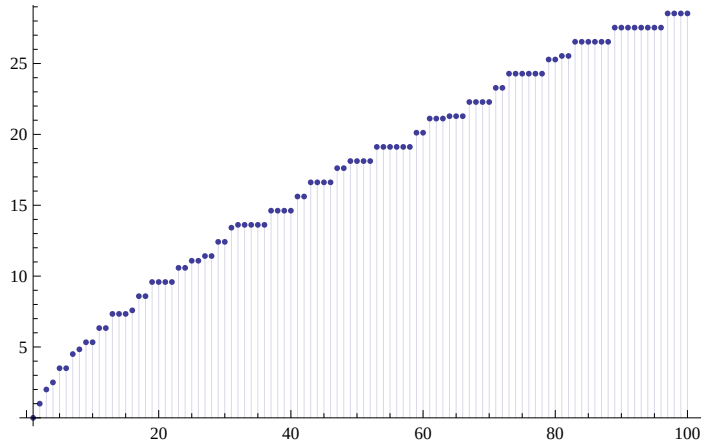
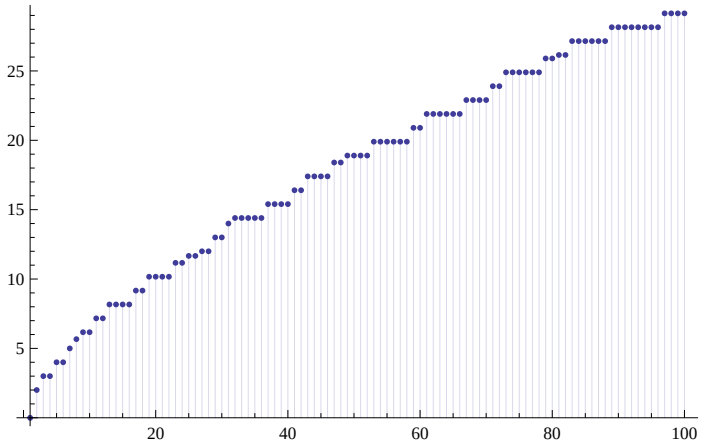
```
ClearAll["Global`*"]
```

ClearAll::clloc : Cannot clear local variable j. ≫

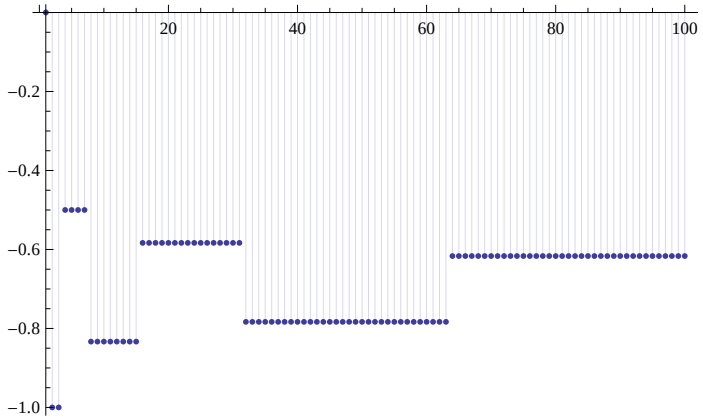
```
E2a[n_, k_, a_] :=
 E2a[n, k, a] = Sum[ E2a[n / j, k - 1, a], {j, 2, n}] - a Sum[ E2a[n / (a j), k - 1, a], {j, 1, n / a}];
E2a[n_, 0, a_] := 1
D2a[n_, k_] := D2a[n, k] = Sum[D2a[Floor[n / j], k - 1], {j, 2, n}]; D2a[n_, 0] := 1
DD[n_, z_] := DD[n, z] = Sum[FactorialPower[z, a] / a! D2a[n, a], {a, 0, Log[2, n]}]
EE[n_, z_, b_] :=
 EE[n, z, b] = Sum[FactorialPower[z, a] / a! E2a[n, a, b], {a, 0, Log[If[b > 2, 2, b], n]}]
D1b[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j E1b[n / b^j, k, b], {j, 0, Log[b, n]}]
E1b[n_, k_, b_] := Sum[FactorialPower[k, a] / a! E2b[n, a, b], {a, 0, Log[If[b > 2, 2, b], n]}]
E2b[n_, k_, a_] :=
 E2b[n, k, a] = Sum[ E2b[n / j, k - 1, a], {j, 2, n}] + Sum[ E2b[n / (a j), k - 1, a], {j, 1, n / a}];
E2b[n_, 0, a_] := 1
D1c[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j (-1)^j
    Sum[FactorialPower[k, a] / a! E2b[n / b^j, a, b], {a, 0, Log[If[b > 2, 2, b], n / b^j]}],
  {j, 0, Log[b, n]}]
D1d[n_, z_, b_] := Sum[
  Binomial[z + j - 1, z - 1] Binomial[z, k] b^j (-1)^j E2[n / b^j, k, b],
  {j, 0, Log[b, n]}, {k, 0, Log[If[b > 2, 2, b], n / b^j]}]
D1e[n_, k_, b_] := Grid[Table[
    Binomial[k + j - 1, k - 1] Binomial[k, a] b^j (-1)^j E2[n / b^j, a, b],
    {j, 0, Log[b, n]}, {a, 0, Log[If[b > 2, 2, b], n / b^j]}]]
D1e2[n_, k_, b_] := Grid[Table[
    Binomial[k + j - 1, k - 1] FactorialPower[k, a] / a! b^j (-1)^j E2[n / b^j, a, b] / k,
    {j, 0, Log[b, n]}, {a, 0, Log[If[b > 2, 2, b], n / b^j]}]]
D1c2[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j (-1)^j
    Sum[FactorialPower[k, a] / a! E2b[n / b^j, a, b], {a, 0, Log[If[b > 2, 2, b], n / b^j]}],
  {j, 0, Log[b, n]}]
lin[n_, b_] := Sum[(-1)^(k + 1) / k E2b[n, k, b], {k, 1, Log[b, n]}]
M2[n_, a_] := Sum[(-1)^k ( E2b[n, k, a] - a E2b[n / a, k, a]), {k, 0, Log[a, n]}]

E2b[n_, k_, a_, b_] := E2b[n, k, a, b] = Sum[ E2b[n / j, k - 1, a, b], {j, 2, n}] -
    a Sum[ E2b[n / (a j), k - 1, a, b], {j, 1, n / a}]; E2b[n_, 0, a_, b_] := 1
lin[n_, b_, a_] := Sum[(-1)^(k + 1) / k E2b[n, k, b, a], {k, 1, Log[b, n]}]

DiscretePlot[ (DD[n, .000001] - 1) / .000001, {n, 1, 100}]
```



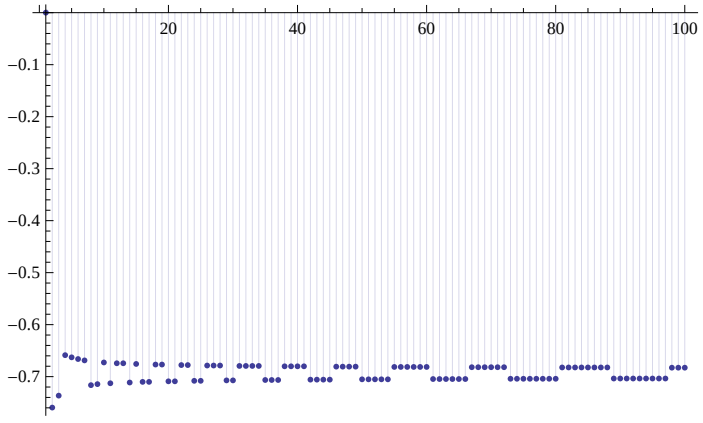
```
DiscretePlot[ lin[n, 2], {n, 1, 100}]
```




```
DiscretePlot[ (DD[n, .000001] - 1) / .000001 - lin[n, 2], {n, 1, 100}]
```




```
DiscretePlot[ (DD[n, .000001] - 1) / .000001 - lin[n, 1.1], {n, 1, 100}]
```




```
N[-Log[2]]
```

```
-0.693147
```

```
fdif[n_, a_] := Sum[ (-1)^(k) a^(0 k) / k, {k, 1, Log[a, n]}]
fdif[100, 1.000001]
```

```
-0.693147 + 1.08508 × 10^-7 i
```

```
Limit[ Sum[ (-1)^(k) a^(0 k) / k, {k, 1, Log[a, n]}], a → 1]
```

$$\text{Limit}\left[(-1)^{\frac{\text{Log}[n]}{\text{Log}[a]}} \text{LerchPhi}\left[-1, 1, 1+\frac{\text{Log}[n]}{\text{Log}[a]}\right] - \text{Log}[2], a \to 1\right]$$

```
Limit[ Sum[ (-1)^(k) a^(1 k) / k, {k, 1, Log[a, n]}], a → 1]
```

$$\text{Limit}\left[(-1)^{\frac{\text{Log}[n]}{\text{Log}[a]}} a\, n\, \text{LerchPhi}\left[-a, 1, 1+\frac{\text{Log}[n]}{\text{Log}[a]}\right] - \text{Log}[1+a], a \to 1\right]$$

```
Limit[ Sum[ (-1)^(k) a^(2 k) / k, {k, 1, Log[a, n]}], a → 1]
```

$$-\text{Log}[2]$$

```
Limit[ Sum[ a^(0 k) / k, {k, 1, Log[a, n]}], a → 1]
```

$$\text{Limit}\left[\text{HarmonicNumber}\left[\frac{\text{Log}[n]}{\text{Log}[a]}\right], a \to 1\right]$$

```
Limit[ Sum[ (a^((1 / 2) k) - 1) / k, {k, 1, Log[a, 100]}], a → 1]
```

$$\text{Limit}\left[\frac{1}{2}\left(2\,\text{ArcTanh}\left[\sqrt{a}\right] - 2\,\text{HarmonicNumber}\left[\frac{\text{Log}[100]}{\text{Log}[a]}\right] - 10\sqrt{a}\,\text{LerchPhi}\left[a, 1, \frac{1}{2}\left(1+\frac{\text{Log}[100]}{\text{Log}[a]}\right)\right] - 10\,a\,\text{LerchPhi}\left[a, 1, \frac{1}{2}\left(2+\frac{\text{Log}[100]}{\text{Log}[a]}\right)\right] - \text{Log}[1-a]\right), a \to 1\right]$$

```
Limit[1/2 (2 ArcTanh[Sqrt[a]] - 2 HarmonicNumber[Log[100]/Log[a]] - Log[1 - a]), a → 1]
```

$$-\text{EulerGamma} - i\pi - \text{Log}[\text{Log}[10]]$$

```
Limit[ Sum[ (a^((-1) k) - 1) / k, {k, 1, Log[a, n]}], a → 1]
```

$$\text{Limit}\left[-\frac{1}{a}\left(a\,\text{HarmonicNumber}\left[\frac{\text{Log}[n]}{\text{Log}[a]}\right] + \left(\frac{1}{a}\right)^{\frac{\text{Log}[n]}{\text{Log}[a]}} \text{LerchPhi}\left[\frac{1}{a}, 1, 1+\frac{\text{Log}[n]}{\text{Log}[a]}\right] + a\,\text{Log}\left[\frac{-1+a}{a}\right]\right), a \to 1\right]$$

```
Limit[-(a HarmonicNumber[Log[100]/Log[a]] + a Log[(-1+a)/a])/a, a → 1]
```

$$-\text{EulerGamma} - \text{Log}[\text{Log}[100]]$$

```
Limit[ Sum[ a^(1 k) / k, {k, 1, Log[a, n]}], a → 1]
```

$$\text{Limit}\left[-a\,n\,\text{LerchPhi}\left[a, 1, 1+\frac{\text{Log}[n]}{\text{Log}[a]}\right] - \text{Log}[1-a], a \to 1\right]$$

```
Limit[ Sum[ a^(2 k) / k, {k, 1, Log[a, n]}], a → 1]
```

$$\text{Limit}\left[-\left(a^2\right)^{1+\frac{\text{Log}[n]}{\text{Log}[a]}} \text{LerchPhi}\left[a^2, 1, 1+\frac{\text{Log}[n]}{\text{Log}[a]}\right] - \text{Log}\left[1-a^2\right], a \to 1\right]$$

```
Limit[Sum[(a^(1 k) - 1) / k, {k, 1, Log[a, n]}], a → 1]
```

$$\text{Limit}\left[-\text{HarmonicNumber}\left[\frac{\text{Log}[n]}{\text{Log}[a]}\right] - a\, n\, \text{LerchPhi}\left[a, 1, 1+\frac{\text{Log}[n]}{\text{Log}[a]}\right] - \text{Log}[1-a], a \to 1\right]$$

```
Limit[Sum[(a^(2 k) - 1) / k, {k, 1, Log[a, n]}], a → 1]
```

$$\text{Limit}\left[-\text{HarmonicNumber}\left[\frac{\text{Log}[n]}{\text{Log}[a]}\right] - \left(a^2\right)^{1+\frac{\text{Log}[n]}{\text{Log}[a]}}\, \text{LerchPhi}\left[a^2, 1, 1+\frac{\text{Log}[n]}{\text{Log}[a]}\right] - \text{Log}\left[1-a^2\right], a \to 1\right]$$

$$\text{Limit}\left[-\text{HarmonicNumber}\left[\frac{\text{Log}[100]}{\text{Log}[a]}\right] - \text{Log}[1-a], a \to 1\right]$$

$$-\text{EulerGamma} - i\,\pi - \text{Log}[\text{Log}[100]]$$

$$\text{Limit}\left[-\text{HarmonicNumber}\left[\frac{\text{Log}[100]}{\text{Log}[a]}\right] - \text{Log}\left[1-a^2\right], a \to 1\right]$$

$$-\text{EulerGamma} - i\,\pi - \text{Log}[\text{Log}[10\,000]]$$

```
fa[n_, a_] := -(a^2)^(1+Log[n]/Log[a]) LerchPhi[a^2, 1, 1+Log[n]/Log[a]]
```

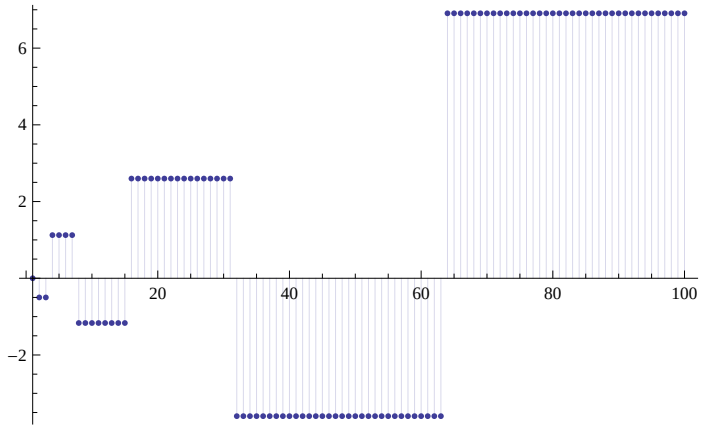
```
fa[100, 1.0001]
```

$$1246.25 + 3.14159\, i$$

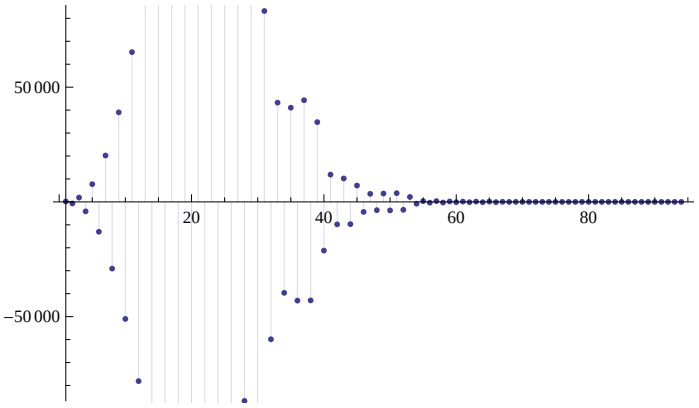
```
N[-Gamma[0, -2 Log[100]]]
```

$$1246.14 + 3.14159\, i$$

```
DiscretePlot[((D1c2[n, .0000001, 2] - 1) / .0000001) - lin[n, 2, -1], {n, 1, 100}]
```



```
DiscretePlot[(-1)^(k+1) / k E2b[100, k, 1.05], {k, 1, Log[1.05, 100]}]
```




```
(D1c2[100, .0000001, 2] - 1) / .0000001
```

28.5333

```
lin[100, 2]
```

$$\frac{344}{15}$$

```
D1e2[100, .0000001, 2]
```

$1.\times 10^{7}\,E2[100, 0, 2]$	$1.\,E2[100, 1, 2]$	$-0.5\,E2[100, 2, 2]$	$0.333333\,E2[100, 3, 2]$	$-0.25\,E2[100, 4, 2]$	$0.2\,E2[100, 5, 2]$	$-0.166667\,E2[100, 6, 2]$
$-2.\,E2[50, 0, 2]$	$-2.\times 10^{-7}\,E2[50, 1, 2]$	$1.\times 10^{-7}\,E2[50, 2, 2]$	$-6.66667\times 10^{-8}\,E2[50, 3, 2]$	$5.\times 10^{-8}\,E2[50, 4, 2]$	$-4.\times 10^{-8}\,E2[50, 5, 2]$	
$2.\,E2[25, 0, 2]$	$2.\times 10^{-7}\,E2[25, 1, 2]$	$-1.\times 10^{-7}\,E2[25, 2, 2]$	$6.66667\times 10^{-8}\,E2[25, 3, 2]$	$-5.\times 10^{-8}\,E2[25, 4, 2]$		
$-2.66667\,E2\left[\frac{25}{2}, 0, 2\right]$	$-2.66667\times 10^{-7}\,E2\left[\frac{25}{2}, 1, 2\right]$	$1.33333\times 10^{-7}\,E2\left[\frac{25}{2}, 2, 2\right]$	$-8.88889\times 10^{-8}\,E2\left[\frac{25}{2}, 3, 2\right]$			
$4.\,E2\left[\frac{25}{4}, 0, 2\right]$	$4.\times 10^{-7}\,E2\left[\frac{25}{4}, 1, 2\right]$	$-2.\times 10^{-7}\,E2\left[\frac{25}{4}, 2, 2\right]$				
$-6.4\,E2\left[\frac{25}{8}, 0, 2\right]$	$-6.4\times 10^{-7}\,E2\left[\frac{25}{8}, 1, 2\right]$					
$10.6667\,E2\left[\frac{25}{16}, 0, 2\right]$						

D1e[100, -1, 2]

E2[100, 0, 2]	-E2[100, 1, 2]	E2[100, 2, 2]	-E2[100, 3, 2]	E2[100, 4, 2]	-E2[100, 5, 2]	E2[100, 6, 2]
2 E2[50, 0, 2]	-2 E2[50, 1, 2]	2 E2[50, 2, 2]	-2 E2[50, 3, 2]	2 E2[50, 4, 2]	-2 E2[50, 5, 2]	
0	0	0	0	0		
0	0	0	0			
0	0	0				
0	0					
0						

D1e[900, 1, 2]

E2[900, 0, 2]	E2[900, 1, 2]	0 0 0 0 0 0 0 0
-2 E2[450, 0, 2]	-2 E2[450, 1, 2]	0 0 0 0 0 0 0
4 E2[225, 0, 2]	4 E2[225, 1, 2]	0 0 0 0 0 0
-8 E2[225/2, 0, 2]	-8 E2[225/2, 1, 2]	0 0 0 0 0
16 E2[225/4, 0, 2]	16 E2[225/4, 1, 2]	0 0 0 0
-32 E2[225/8, 0, 2]	-32 E2[225/8, 1, 2]	0 0 0
64 E2[225/16, 0, 2]	64 E2[225/16, 1, 2]	0 0
-128 E2[225/32, 0, 2]	-128 E2[225/32, 1, 2]	0
256 E2[225/64, 0, 2]	256 E2[225/64, 1, 2]	
-512 E2[225/128, 0, 2]		

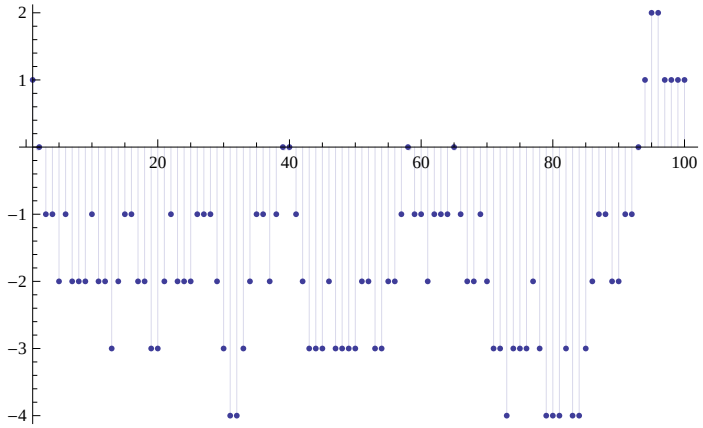
D1e[900, 2, 2]

E2[900, 0, 2]	2 E2[900, 1, 2]	E2[900, 2, 2]	0 0 0 0 0 0 0
-4 E2[450, 0, 2]	-8 E2[450, 1, 2]	-4 E2[450, 2, 2]	0 0 0 0 0 0
12 E2[225, 0, 2]	24 E2[225, 1, 2]	12 E2[225, 2, 2]	0 0 0 0 0
-32 E2[225/2, 0, 2]	-64 E2[225/2, 1, 2]	-32 E2[225/2, 2, 2]	0 0 0 0
80 E2[225/4, 0, 2]	160 E2[225/4, 1, 2]	80 E2[225/4, 2, 2]	0 0 0
-192 E2[225/8, 0, 2]	-384 E2[225/8, 1, 2]	-192 E2[225/8, 2, 2]	0 0
448 E2[225/16, 0, 2]	896 E2[225/16, 1, 2]	448 E2[225/16, 2, 2]	0
-1024 E2[225/32, 0, 2]	-2048 E2[225/32, 1, 2]	-1024 E2[225/32, 2, 2]	
2304 E2[225/64, 0, 2]	4608 E2[225/64, 1, 2]		
-5120 E2[225/128, 0, 2]			

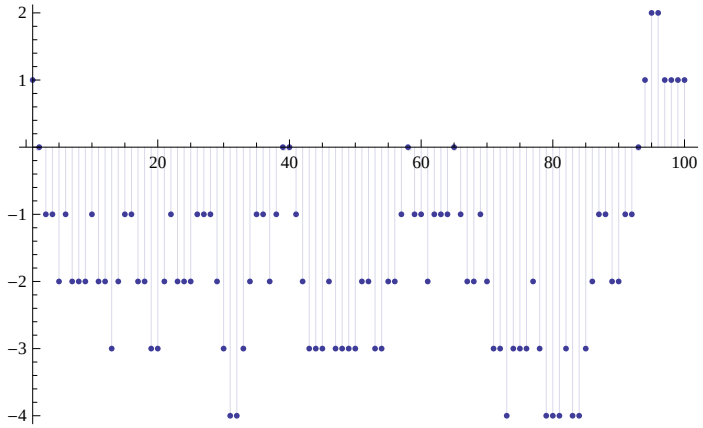
DD[100, -1]

1

```
DiscretePlot[ DD[n, -1], {n, 1, 100}]
```




```
DiscretePlot[ EE[n, -1, 200], {n, 1, 100}]
```




```
Animate[DiscretePlot[ EE[n, 7, z], {n, 1, 100}], {z, 2, 100}]
```

```
Animate[DiscretePlot[ EE[n, -1, z], {n, 1, 100}], {z, 2, 100}]
```

```
Animate[DiscretePlot[( EE[n, .0001, z] - 1) / .0001, {n, 1, 100}], {z, 2, 100}]
```

D1d[1000, -3 / 2, 2]

$$-\frac{143}{128}\,E2\left[\frac{125}{64},0,2\right]+\frac{99}{128}\,E2\left[\frac{125}{32},0,2\right]-\frac{297}{256}\,E2\left[\frac{125}{32},1,2\right]-\frac{9}{16}\,E2\left[\frac{125}{16},0,2\right]+$$
$$\frac{27}{32}\,E2\left[\frac{125}{16},1,2\right]-\frac{135}{128}\,E2\left[\frac{125}{16},2,2\right]+\frac{7}{16}\,E2\left[\frac{125}{8},0,2\right]-\frac{21}{32}\,E2\left[\frac{125}{8},1,2\right]+$$
$$\frac{105}{128}\,E2\left[\frac{125}{8},2,2\right]-\frac{245}{256}\,E2\left[\frac{125}{8},3,2\right]-\frac{3}{8}\,E2\left[\frac{125}{4},0,2\right]+\frac{9}{16}\,E2\left[\frac{125}{4},1,2\right]-$$
$$\frac{45}{64}\,E2\left[\frac{125}{4},2,2\right]+\frac{105}{128}\,E2\left[\frac{125}{4},3,2\right]-\frac{945\,E2\left[\frac{125}{4},4,2\right]}{1024}+\frac{3}{8}\,E2\left[\frac{125}{2},0,2\right]-\frac{9}{16}\,E2\left[\frac{125}{2},1,2\right]+$$
$$\frac{45}{64}\,E2\left[\frac{125}{2},2,2\right]-\frac{105}{128}\,E2\left[\frac{125}{2},3,2\right]+\frac{945\,E2\left[\frac{125}{2},4,2\right]}{1024}-\frac{2079\,E2\left[\frac{125}{2},5,2\right]}{2048}-$$
$$\frac{1}{2}\,E2[125,0,2]+\frac{3}{4}\,E2[125,1,2]-\frac{15}{16}\,E2[125,2,2]+\frac{35}{32}\,E2[125,3,2]-\frac{315}{256}\,E2[125,4,2]+$$
$$\frac{693}{512}\,E2[125,5,2]-\frac{3003\,E2[125,6,2]}{2048}+\frac{3}{2}\,E2[250,0,2]-\frac{9}{4}\,E2[250,1,2]+\frac{45}{16}\,E2[250,2,2]-$$
$$\frac{105}{32}\,E2[250,3,2]+\frac{945}{256}\,E2[250,4,2]-\frac{2079}{512}\,E2[250,5,2]+\frac{9009\,E2[250,6,2]}{2048}-$$
$$\frac{19\,305\,E2[250,7,2]}{4096}+3\,E2[500,0,2]-\frac{9}{2}\,E2[500,1,2]+\frac{45}{8}\,E2[500,2,2]-$$
$$\frac{105}{16}\,E2[500,3,2]+\frac{945}{128}\,E2[500,4,2]-\frac{2079}{256}\,E2[500,5,2]+\frac{9009\,E2[500,6,2]}{1024}-$$
$$\frac{19\,305\,E2[500,7,2]}{2048}+\frac{328\,185\,E2[500,8,2]}{32\,768}+E2[1000,0,2]-\frac{3}{2}\,E2[1000,1,2]+$$
$$\frac{15}{8}\,E2[1000,2,2]-\frac{35}{16}\,E2[1000,3,2]+\frac{315}{128}\,E2[1000,4,2]-\frac{693}{256}\,E2[1000,5,2]+$$
$$\frac{3003\,E2[1000,6,2]}{1024}-\frac{6435\,E2[1000,7,2]}{2048}+\frac{109\,395\,E2[1000,8,2]}{32\,768}-\frac{230\,945\,E2[1000,9,2]}{65\,536}$$

D1e[900, -1, 2]

E2[900, 0, 2]	-E2[900, 1, 2]	E2[900, 2, 2]	-E2[900, 3, 2]	E2[900, 4, 2]	-E2[900, 5, 2]	E2[900, 6, 2]	-E2[900, 7, 2]	E2[900, 8, 2]	-E2[900, 9, 2]
2 E2[450, 0, 2]	-2 E2[450, 1, 2]	2 E2[450, 2, 2]	-2 E2[450, 3, 2]	2 E2[450, 4, 2]	-2 E2[450, 5, 2]	2 E2[450, 6, 2]	-2 E2[450, 7, 2]	2 E2[450, 8, 2]	
0	0	0	0	0	0	0	0		
0	0	0	0	0	0	0			
0	0	0	0	0	0				
0	0	0	0	0					
0	0	0	0						
0	0	0							
0	0								
0									

```
D1f[n_, k_, b_] := Sum[
  Binomial[k + j - 1, k - 1] Binomial[k, a] b^j E2a[n / b^j, a, b],
  {j, 0, Log[b, n]}, {a, 0, Log[If[b > 2, 2, b], n / b^j]}]


ME2[n_, b_] := Sum[ (-1)^k E2a[n, k, b], {k, 0, Log[If[b < 2, b, 2], n]}]
ME2a[n_, b_] := ME2[n, b] - b ME2[n / b, b]
ME2b[n_, a_] := Sum[ (-1)^k (E2a[n, k, a] - a E2a[n / a, k, a]), {k, 0, Log[If[a < 2, a, 2], n]}]
```

```
ME2[100, 2]
```

```
-13
```

```
M2[n_, a_] := Sum[ (-1)^k (k + 1) (E2a[n, k, a] - 2 a E2a[n / a, k, a] + a^2 E2a[n / (a^2), k, a]),
  {k, 0, Log[If[a < 2, a, 2], n]}]


M2[100, 4]
```

```
19
```

```
DD[100, -2]
```

```
19
```

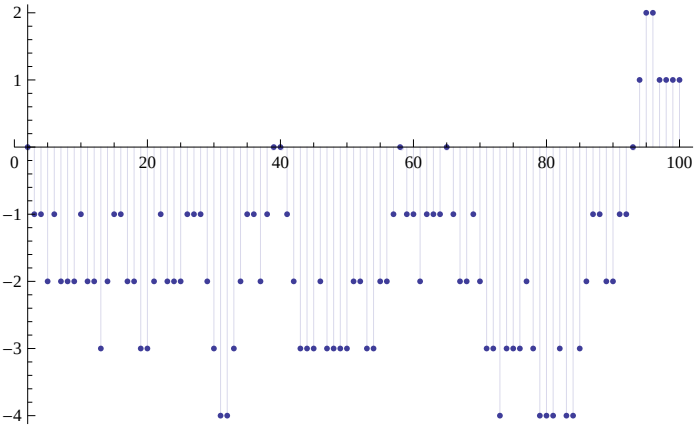
```
D1c[100, -2, 2]
```

```
19
```

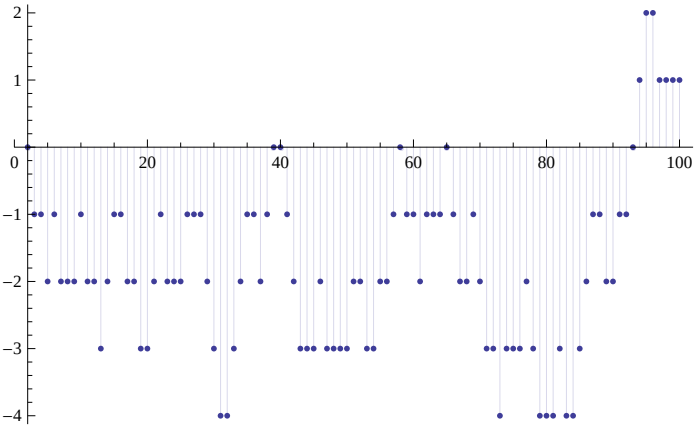
```
D1e[2400, -2, 3]
```

```
E2[2400, 0, 3] - 2 E2[2400, 1, 3] + 3 E2[2400, 2, 3] - 4 E2[2400, 3, 3] + 5 E2[2400, 4, 3] - 6 E2[2400, 5, 3] + 7 E2[2400, 6, 3] - 8 E2[2400, 7, 3] + 9 E2[2400, 8, 3] - 10 E2[2400, 9, 3] + 11 E2[2400, 10, 3] - 12 E2[2400, 11, 3] +
6 E2[800, 0, 3] - 12 E2[800, 1, 3] + 18 E2[800, 2, 3] - 24 E2[800, 3, 3] + 30 E2[800, 4, 3] - 36 E2[800, 5, 3] + 42 E2[800, 6, 3] - 48 E2[800, 7, 3] + 54 E2[800, 8, 3] - 60 E2[800, 9, 3] +
9 E2[800 / 3, 0, 3] - 18 E2[800 / 3, 1, 3] + 27 E2[800 / 3, 2, 3] - 36 E2[800 / 3, 3, 3] + 45 E2[800 / 3, 4, 3] - 54 E2[800 / 3, 5, 3] + 63 E2[800 / 3, 6, 3] - 72 E2[800 / 3, 7, 3] + 81 E2[800 / 3, 8, 3] +
0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0
```

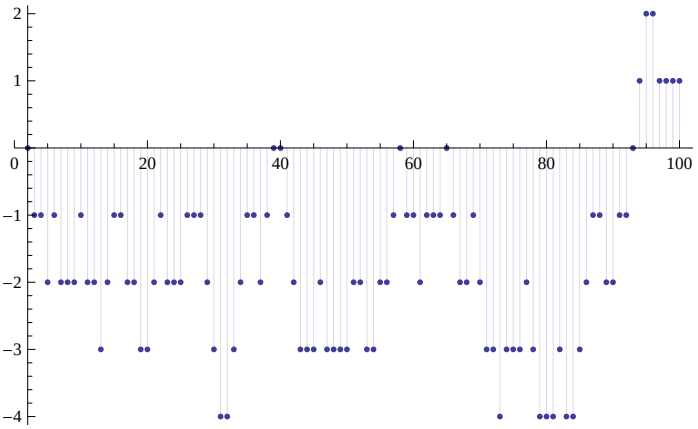
```
DiscretePlot[ME2b[n, 2], {n, 2, 100}]
```



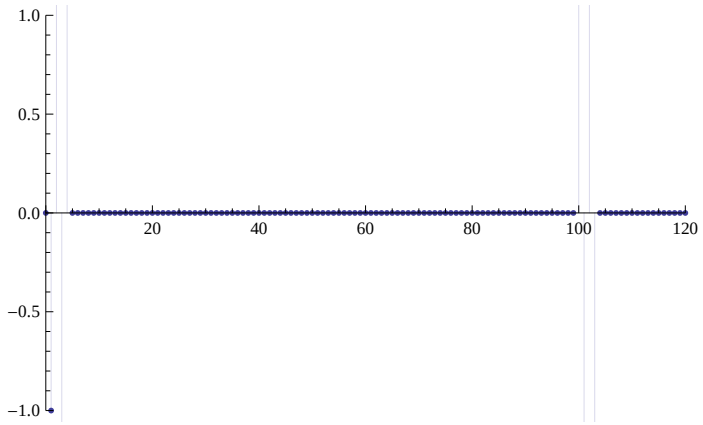

```
DiscretePlot[D1f[n, -1, 2], {n, 2, 100}]
```




```
DiscretePlot[ME2[n, (3 / 2)] - (3 / 2) ME2[n / (3 / 2), (3 / 2)], {n, 2, 100}]
```



```
DiscretePlot[(-1)^k (E2a[100, k, 1.0001] - 1.0001 E2a[100 / 1.0001, k, 1.0001]),
 {k, 0, 120}, PlotRange → {{0, 120}, {-1, 1}}]
```




```
D1e[2400, -2, 7]
```

```
E2[ -2 E2[ 3 E2[ -4 E2[ 5 E2[ -6 E2[ 7 E2[ -8 E2[ 9 E2[ -10 11 E2[ -12
 2400,  2400  2400  2400  2400  2400  2400  2400  2400  E2[  2400  E2[
 0, 7]  ,     ,     ,     ,     ,     ,     ,     ,     2400  ,     2400
        1,    2,    3,    4,    5,    6,    7,    8,    ,     10,   ,
        7]    7]    7]    7]    7]    7]    7]    7]    9,    7]    11,
                                                         7]          7]

14 E2[ -28  42 E2[ -56  70 E2[ -84  98 E2[ -112 126
  2400  E2[    2400  E2[    2400  E2[    2400  E2[    E2[
  /     2400   /     2400   /     2400   /     2400   2400
  7,    /      7,    /      7,    /      7,    /      /
  0,    7,     2,    7,     4,    7,     6,    7,     7,
  7]    1,     7]    3,     7]    5,     7]    7,     8,
        7]           7]           7]           7]     7]

49 E2[ -98  147  -196  245  -294
  2400  E2[   E2[   E2[   E2[   E2[
  /     2400  2400  2400  2400  2400
  49,   /     /     /     /     /
  0,    49,   49,   49,   49,   49,
  7]    1,    2,    3,    4,    5,
        7]    7]    7]    7]    7]
   0     0     0
```

D1e[1200, -3, 2]

```
E2[ -3 E2[  6 E2[ -10 E2[ 15 E2[ -21 E2[ 28 E2[ -36 E2[ 45 E2[ -55 E2[ 66 E2[
 1200,   1200   1200   1200   1200   1200   1200   1200   1200   1200   1200
 0, 2]     ,      ,      ,      ,      ,      ,      ,      ,      ,      ,
         1, 2]  2, 2]  3, 2]  4, 2]  5, 2]  6, 2]  7, 2]  8, 2]  9, 2]  10,
                                                                          2]
6 E2[  -18 E2[ 36 E2[ -60 E2[ 90 E2[ -126   168 E2[ -216   270 E2[ -330
  600,    600,   600,   600,   600,   E2[     600,   E2[     600,   E2[
  0, 2]   1, 2]  2, 2]  3, 2]  4, 2]   600,   6, 2]   600,   8, 2]   600,
                                       5, 2]          7, 2]          9, 2]
12 E2[  -36 E2[ 72 E2[ -120   180 E2[ -252   336 E2[ -432   540 E2[
  300,    300,   300,   E2[     300,   E2[     300,   E2[     300,
  0, 2]   1, 2]  2, 2]   300,   4, 2]   300,   6, 2]   300,   8, 2]
                        3, 2]          5, 2]          7, 2]
8 E2[  -24 E2[ 48 E2[ -80 E2[ 120 E2[ -168   224 E2[ -288
  150,    150,   150,   150,   150,   E2[     150,   E2[
  0, 2]   1, 2]  2, 2]  3, 2]  4, 2]   150,   6, 2]   150,
                                       5, 2]          7, 2]
  0       0      0      0      0       0      0
  0       0      0      0      0       0
  0       0      0      0      0
  0       0      0      0
  0       0      0
  0       0
  0
```

$RecursionLimit = 10 000

10 000

Table[{1.0001^k, (-1)^k (E2a[100, k, 1.0001] - 1.0001 E2a[100 / 1.0001, k, 1.0001])}, {k, 0, 200}] // TableForm

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

General::stop : Further output of $RecursionLimit::reclim will be suppressed during this calculation. ≫

$Aborted

```
N[Table[{k, (-1)^k (E2a[1000, k, ss = 2] + ss E2a[1000 / ss, k, ss])}, {k, 0, 20}] // TableForm]
```

```
0.    3.
1.    3.
2.    -8.
3.    25.
4.    -130.
5.    -260.
6.    22.
7.    -60.
8.    -49.
9.    -8.
10.   0.
11.   0.
12.   0.
13.   0.
14.   0.
15.   0.
16.   0.
17.   0.
18.   0.
19.   0.
20.   0.
```

```
N[Table[{k, (-1)^k (E2a[10 000, k, ss = 2] + ss E2a[1000 / ss, k, ss])}, {k, 0, 20}] // TableForm]
```

```
0.    3.
1.    3.
2.    3.
3.    12.
4.    297.
5.    -1209.
6.    -796.
7.    1213.
8.    -182.
9.    1018.
10.   939.
11.   320.
12.   67.
13.   1.
14.   0.
15.   0.
16.   0.
17.   0.
18.   0.
19.   0.
20.   0.
```

```
Binomial[z, 0]
```

```
1
```

```
Limit[ Sum[ Binomial[z + j - 1,  z - 1] a^j, {j, 0, Log[a, n]}] /. z → 1, a → 1]
```

```
DirectedInfinity[-1 + n]
```

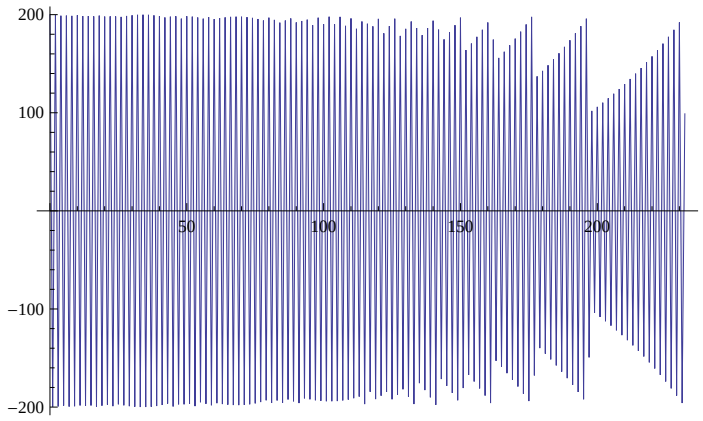
```
D1e[100, -3, 2]
```

```
E2[100, 0, 2]   -3 E2[100, 1, 2]   6 E2[100, 2, 2]   -10 E2[100, 3, 2]   15 E2[100, 4, 2]   -21 E2[100, 5, 2]   28 E2[100, 6, 2]
6 E2[50, 0, 2]   -18 E2[50, 1, 2]   36 E2[50, 2, 2]   -60 E2[50, 3, 2]   90 E2[50, 4, 2]   -126 E2[50, 5, 2]
12 E2[25, 0, 2]   -36 E2[25, 1, 2]   72 E2[25, 2, 2]   -120 E2[25, 3, 2]   180 E2[25, 4, 2]
8 E2[25/2, 0, 2]   -24 E2[25/2, 1, 2]   48 E2[25/2, 2, 2]   -80 E2[25/2, 3, 2]
0   0   0
0   0
0
```

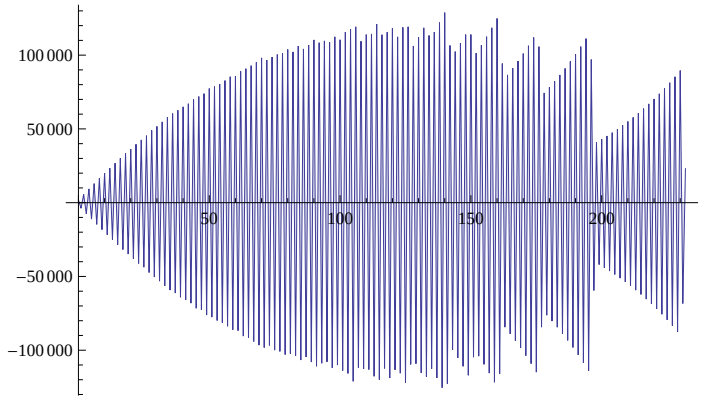
```
$RecursionLimit = 1 000 000
dr[n_, k_, b_] := DiscretePlot[ Binomial[k + j - 1, k - 1] b^j (-1)^j
    Sum[FactorialPower[k, a] / a! E2b[n / b^j, a, b], {a, 0, Log[If[b > 2, 2, b], n / b^j]}],
  {j, 0, Log[b, n]}]
```

```
1 000 000
```

```
dr[100, 1, 1.02]
```

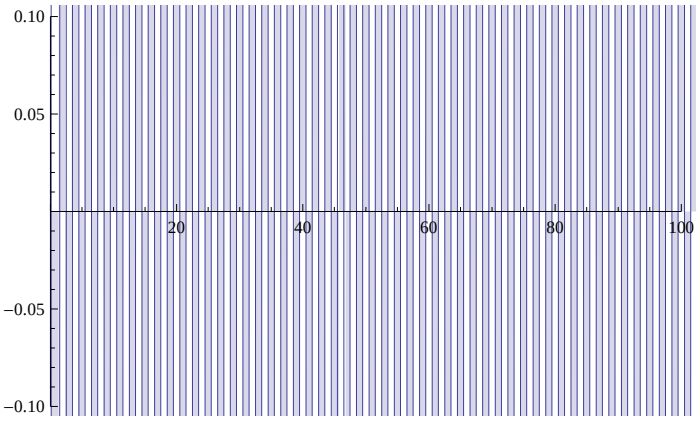



```
dr[100, 2, 1.02]
```



```
Dr2[n_, k_, b_] := DiscretePlot[Sum[ Binomial[k + j - 1, k - 1] b^j (-1)^j
     Sum[FactorialPower[k, a] / a! E2b[n / b^j, a, b], {a, s, s}], {j, 0, Log[b, n]}],
  {s, 0, Log[b, n]}, PlotRange → {{0, 100}, {-.1, .1}}]
```

```
Dr2[5, -1, 1.005]
```




```
Dr3[n_, k_, b_] := Table[{b^s, Sum[ Binomial[k + j - 1, k - 1] b^j
       Sum[FactorialPower[k, a] / a! E2b[n / b^j, a, b], {a, s, s}], {j, 0, Log[b, n]}]},
   {s, 0, Log[b, n]}] // TableForm
```

```
Dr3[12, -1, 1.02]
```

1.	−0.02
1.02	−0.5756
1.0404	2.50217
1.06121	0.444801
1.08243	−2.06826
1.10408	5.22111
1.12616	−16.5302
1.14869	24.2717
1.17166	−34.1435
1.19509	−52.2886
1.21899	172.532
1.24337	−63.0479
1.26824	−87.5836
1.29361	110.314
1.31948	4.31786
1.34587	−137.993
1.37279	−31.8071
1.40024	229.224
1.42825	−273.9
1.45681	324.691
1.48595	1951.99
1.51567	−6959.36
1.54598	7414.56
1.5769	−2529.08
1.60844	−113.67
1.64061	125.599
1.67342	−138.353
1.70689	104.961
1.74102	−116.76
1.77584	179.896

1.81136	−196.214
1.84759	213.547
1.88454	−231.939
1.92223	251.433
1.96068	−272.076
1.99989	−2204.95
2.03989	4949.83
2.08069	−2542.92
2.1223	−129.849
2.16474	139.241
2.20804	−149.13
2.2522	159.536
2.29724	−170.479
2.34319	181.979
2.39005	−86.7932
2.43785	94.8425
2.48661	−215.515
2.53634	229.279
2.58707	−243.709
2.63881	258.832
2.69159	−274.674
2.74542	291.261
2.80033	−308.621
2.85633	326.784
2.91346	−345.778
2.97173	−4302.36
3.03117	9216.11
3.09179	−4728.86
3.15362	−14.5524
3.2167	15.0983
3.28103	−15.6602
3.34665	16.2385
3.41358	−16.8337
3.48186	17.4461
3.55149	−18.0764
3.62252	18.7249
3.69497	−19.3921
3.76887	20.0785
3.84425	−20.7846
3.92114	21.5109
3.99956	263.311
4.07955	−272.415
4.16114	−11.9498
4.24436	12.3569
4.32925	−12.7755
4.41584	13.2059
4.50415	−13.6485
4.59424	14.1034
4.68612	−14.5711
4.77984	15.0518
4.87544	−15.546
4.97295	16.0538
5.07241	−16.5758
5.17386	17.1123
5.27733	−17.6635
5.38288	18.23

```
5.49054    -18.8121
5.60035    19.4101
5.71235    -20.0246
5.8266     20.6559
5.94313    524.275
6.062      -540.704
6.18324    -0.123665
6.3069     0.126138
6.43304    -0.128661
6.5617     0.131234
6.69293    -0.133859
6.82679    0.136536
6.96333    -0.139267
7.10259    0.142052
7.24465    -0.144893
7.38954    0.147791
7.53733    -0.150747
7.68808    0.153762
7.84184    -0.156837
7.99867    0.159973
8.15865    -0.163173
8.32182    0.166436
8.48826    -0.169765
8.65802    0.17316
8.83118    -0.176624
9.00781    0.180156
9.18796    -0.183759
9.37172    0.187434
9.55916    -0.191183
9.75034    0.195007
9.94535    -0.198907
10.1443    0.202885
10.3471    -0.206943
10.5541    0.211082
10.7652    -0.215303
10.9805    0.219609
11.2001    -0.224002
11.4241    0.228482
11.6526    -0.233051
11.8856    -11.8856
```

```
DD[5, -1]
```

```
-2
```

```
DD[6, -1]
```

```
-1
```

```
$RecursionLimit = 10 000
```

```
10 000
```

```
Table[{k, (-1)^k (E2b[5, k, 1.01] - 1.01 E2b[5 / 1.01, k, 1.01]),
    (-1)^(k + 1) / k E2b[5, k, 1.01]}, {k, 1, Log[1.01, 5] + 10}] // TableForm
```

```
1      -0.9296       8.04
2      -0.172408     -8.6204
3      0.317254      10.5751
```

4	−0.506193	−12.6548
5	0.740504	14.8101
6	−1.02149	−17.0249
7	1.35049	19.2927
8	−1.72885	−21.6106
9	2.15796	23.9773
10	−2.63922	−26.3922
11	3.17408	28.8553
12	−3.764	−31.3667
13	4.41048	33.9268
14	−5.11504	−36.536
15	5.87922	39.1948
16	−6.70462	−41.9039
17	7.59284	44.6638
18	−8.54553	−47.4752
19	9.56436	50.3387
20	−10.651	−53.2552
21	11.8073	56.2252
22	305.027	−59.2496
23	−654.173	47.2432
24	342.316	−19.4209
25	1.28891	5.15563
26	−1.35335	−5.20519
27	1.41895	5.25538
28	−1.48573	−5.30619
29	1.55371	5.35761
30	−1.62289	−5.40963
31	1.6933	5.46227
32	−1.76496	−5.5155
33	1.83788	5.56934
34	−1.91209	−5.62379
35	1.98759	5.67883
36	−2.06442	−5.73449
37	2.14258	5.79075
38	−2.22209	−5.84762
39	2.30299	5.9051
40	−2.38528	−5.96319
41	2.46898	6.02191
42	−2.55412	−6.08125
43	2.64072	6.14121
44	−2.72879	−6.2018
45	2.81836	6.26303
46	−2.90945	−6.3249
47	3.00208	6.38741
48	−3.09627	−6.45057
49	3.19205	6.51438
50	−3.28943	−6.57885
51	−82.1737	6.64399
52	84.6231	−5.03211
53	1.80419	3.40413
54	−1.85629	−3.43757
55	1.90925	3.47136
56	−1.96309	−3.50551
57	2.01781	3.54001
58	−2.07343	−3.57487
59	2.12996	3.6101

60	−2.18741	−3.64568
61	2.2458	3.68164
62	−2.30514	−3.71797
63	2.36544	3.75467
64	−2.42672	−3.79174
65	2.48898	3.8292
66	−2.55225	−3.86705
67	2.61653	3.90528
68	−2.68185	−3.9439
69	2.74821	3.98291
70	−2.81563	−4.02233
71	2.88412	4.06214
72	−2.9537	−4.10236
73	3.02438	4.14299
74	−3.09619	−4.18404
75	3.16912	4.2255
76	−3.24321	−4.26738
77	3.31846	4.30968
78	−3.39489	−4.35242
79	3.47251	4.39559
80	−3.55135	−4.43919
81	3.63142	4.48324
82	−3.71274	−4.52773
83	3.79532	4.57267
84	−3.87918	−4.61807
85	3.96433	4.66392
86	−4.05081	−4.71024
87	4.13861	4.75702
88	−4.22777	−4.80428
89	4.31829	4.85201
90	−4.41021	−4.90023
91	4.50353	4.94893
92	227.502	−4.99812
93	−232.275	2.52498
94	−0.0254806	−0.027107
95	0.0257354	0.0270899
96	−0.0259927	−0.0270758
97	0.0262527	0.0270646
98	−0.0265152	−0.0270563
99	0.0267803	0.0270508
100	−0.0270481	−0.0270481
101	0.0273186	0.0270481
102	−0.0275918	−0.0270508
103	0.0278677	0.027056
104	−0.0281464	−0.0270638
105	0.0284279	0.0270742
106	−0.0287121	−0.0270869
107	0.0289993	0.0271021
108	−0.0292893	−0.0271197
109	0.0295822	0.0271396
110	−0.029878	−0.0271618
111	0.0301768	0.0271863
112	−0.0304785	−0.027213
113	0.0307833	0.0272419
114	−0.0310911	−0.0272729
115	0.031402	0.0273061

116	−0.0317161	−0.0273414
117	0.0320332	0.0273788
118	−0.0323536	−0.0274183
119	0.0326771	0.0274597
120	−0.0330039	−0.0275032
121	0.0333339	0.0275487
122	−0.0336672	−0.0275961
123	0.0340039	0.0276455
124	−0.034344	−0.0276967
125	0.0346874	0.0277499
126	−0.0350343	−0.027805
127	0.0353846	0.0278619
128	−0.0357385	−0.0279207
129	0.0360958	0.0279813
130	−0.0364568	−0.0280437
131	0.0368214	0.0281079
132	−0.0371896	−0.0281739
133	0.0375615	0.0282417
134	−0.0379371	−0.0283113
135	0.0383165	0.0283826
136	−0.0386996	−0.0284556
137	0.0390866	0.0285304
138	−0.0394775	−0.0286069
139	0.0398723	0.0286851
140	−0.040271	−0.028765
141	0.0406737	0.0288466
142	−0.0410804	−0.0289299
143	0.0414912	0.0290149
144	−0.0419062	−0.0291015
145	0.0423252	0.0291898
146	−0.0427485	−0.0292798
147	0.043176	0.0293714
148	−0.0436077	−0.0294647
149	0.0440438	0.0295596
150	−0.0444842	−0.0296562
151	0.0449291	0.0297544
152	−0.0453784	−0.0298542
153	0.0458321	0.0299557
154	−0.0462905	−0.0300587
155	0.0467534	0.0301635
156	−0.0472209	−0.0302698
157	0.0476931	0.0303778
158	−0.04817	−0.0304874
159	0.0486517	0.0305986
160	−0.0491383	−0.0307114
161	−4.96296	0.0308259
162	0.	0.
163	0.	0.
164	0.	0.
165	0.	0.
166	0.	0.
167	0.	0.
168	0.	0.
169	0.	0.
170	0.	0.
171	0.	0.

```
-E2b[5, 161, 1.01] - 1.01 E2b[5 / 1.01, 161, 1.01]
```

```
4.96296
```

```
(-1)^159 ( E2b[5, 159, 1.01] - 1.01 E2b[5 / 1.01, 159, 1.01])
```

```
-0.0486517
```