```
D2[n_, k_, s_] := D2[n, k, s] = Sum[ j^-s D2[Floor[n / j], k - 1, s], {j, 2, n}];
D2[n_, 0, s_] := 1
bins[ z_, a_] := Product[ (z - k), {k, 0, a - 1}] / a!
DD[n_, z_, s_] := Expand[Sum[bins[z, a] D2[n, a, s], {a, 0, Log[2, n]}]]
```

```
N[Limit[D[(DD[100, z, s] - 1) / z, {s, 8}] /. s → 0, z → 0]]
```

1.47956×10^{6}

```
ll[n_, t_] := Limit[ (DD[n, z, 0] - 1) / z, z → 0] +
  N[Sum[ t^k / k! Limit[D[(DD[n, z, s] - 1) / z, {s, k}] /. s → 0, z → 0], {k, 1, 16}]]
```

```
ll[100, -1]
```

1156.48

```
1
```

```
N[Limit[ (DD[100, z, -1] - 1) / z, z → 0]]
```

1156.48

```
l1[n_, t_] := DD[n, 1, 0] + N[Sum[ t^k / k! D[(DD[n, 1, s]), {s, k}] /. s → 0, {k, 1, 48}]]
```

```
l1[100, t]
```

$$100 - 363.739\,t + 704.165\,t^2 - 937.029\,t^3 + 954.512\,t^4 - 789.543\,t^5 + 550.514\,t^6 - 332.037\,t^7 + 176.549\,t^8 - 83.9678\,t^9 + 36.134\,t^{10} - 14.201\,t^{11} + 5.13657\,t^{12} - 1.72104\,t^{13} + 0.537133\,t^{14} - 0.156899\,t^{15} + 0.0430744\,t^{16} - 0.0111551\,t^{17} + 0.00273403\,t^{18} - 0.000636014\,t^{19} + 0.0001408\,t^{20} - 0.0000297332\,t^{21} + 6.00224 \times 10^{-6}\,t^{22} - 1.16057 \times 10^{-6}\,t^{23} + 2.15323 \times 10^{-7}\,t^{24} - 3.83967 \times 10^{-8}\,t^{25} + 6.59084 \times 10^{-9}\,t^{26} - 1.09055 \times 10^{-9}\,t^{27} + 1.74169 \times 10^{-10}\,t^{28} - 2.68814 \times 10^{-11}\,t^{29} + 4.01403 \times 10^{-12}\,t^{30} - 5.80522 \times 10^{-13}\,t^{31} + 8.13951 \times 10^{-14}\,t^{32} - 1.10746 \times 10^{-14}\,t^{33} + 1.46348 \times 10^{-15}\,t^{34} - 1.87992 \times 10^{-16}\,t^{35} + 2.34923 \times 10^{-17}\,t^{36} - 2.85803 \times 10^{-18}\,t^{37} + 3.38742 \times 10^{-19}\,t^{38} - 3.91401 \times 10^{-20}\,t^{39} + 4.41164 \times 10^{-21}\,t^{40} - 4.85362 \times 10^{-22}\,t^{41} + 5.21516 \times 10^{-23}\,t^{42} - 5.47577 \times 10^{-24}\,t^{43} + 5.62113 \times 10^{-25}\,t^{44} - 5.64444 \times 10^{-26}\,t^{45} + 5.54682 \times 10^{-27}\,t^{46} - 5.33693 \times 10^{-28}\,t^{47} + 5.02983 \times 10^{-29}\,t^{48}$$

```
(List @@ NRoots[ l1[100, x] == 0, x][[All, 2]])
```

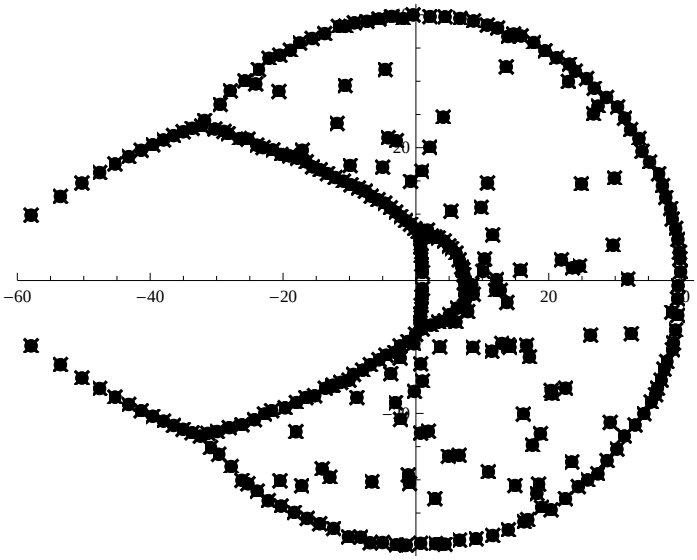
$$\{-7.55469 - 3.52856\,i,\ -7.55469 + 3.52856\,i,\ -5.7879 - 4.16192\,i,\ -5.7879 + 4.16192\,i,\ -4.52745 - 4.47878\,i,\ -4.52745 + 4.47878\,i,\ -3.51819 - 4.63165\,i,\ -3.51819 + 4.63165\,i,\ -2.66855 - 4.68076\,i,\ -2.66855 + 4.68076\,i,\ -1.93343 - 4.65728\,i,\ -1.93343 + 4.65728\,i,\ -1.28672 - 4.57985\,i,\ -1.28672 + 4.57985\,i,\ -0.711766 - 4.46068\,i,\ -0.711766 + 4.46068\,i,\ -0.196773 - 4.30889\,i,\ -0.196773 + 4.30889\,i,\ 0.272047 - 4.12977\,i,\ 0.272047 + 4.12977\,i,\ 0.715356 - 3.87949\,i,\ 0.715356 + 3.87949\,i,\ 0.852106 - 3.71314\,i,\ 0.852106 + 3.71314\,i,\ 0.906216 - 2.45483\,i,\ 0.906216 + 2.45483\,i,\ 0.974373 - 1.21578\,i,\ 0.974373 + 1.21578\,i,\ 1.34332 - 3.71213\,i,\ 1.34332 + 3.71213\,i,\ 1.8183 - 3.52108\,i,\ 1.8183 + 3.52108\,i,\ 2.26188 - 3.26388\,i,\ 2.26188 + 3.26388\,i,\ 2.6682 - 2.95038\,i,\ 2.6682 + 2.95038\,i,\ 3.03016 - 2.58663\,i,\ 3.03016 + 2.58663\,i,\ 3.34159 - 2.17892\,i,\ 3.34159 + 2.17892\,i,\ 3.59726 - 1.73425\,i,\ 3.59726 + 1.73425\,i,\ 3.79289 - 1.26017\,i,\ 3.79289 + 1.26017\,i,\ 3.92517 - 0.764739\,i,\ 3.92517 + 0.764739\,i,\ 3.99187 - 0.25636\,i,\ 3.99187 + 0.25636\,i\}$$

```
l1a[n_, t_, m_] := DD[n, 1, 0] + N[Sum[ t^k / k! D[(DD[n, 1, s]), {s, k}] /. s → 0, {k, 1, m}]]
```

```
(List @@ NRoots[ l1a[100, x, 50] == 0, x][[All, 2]])
```

```
{-7.93002 - 3.61438 i, -7.93002 + 3.61438 i, -6.12639 - 4.27727 i,
 -6.12639 + 4.27727 i, -4.83788 - 4.61665 i, -4.83788 + 4.61665 i, -3.80461 - 4.78825 i,
 -3.80461 + 4.78825 i, -2.93332 - 4.85358 i, -2.93332 + 4.85358 i, -2.17807 - 4.8445 i,
 -2.17807 + 4.8445 i, -1.51226 - 4.78002 i, -1.51226 + 4.78002 i, -0.9189 - 4.67257 i,
 -0.9189 + 4.67257 i, -0.386556 - 4.53108 i, -0.386556 + 4.53108 i,
 0.0945205 - 4.36426 i, 0.0945205 + 4.36426 i, 0.548861 - 4.17545 i, 0.548861 + 4.17545 i,
 0.807763 - 3.72871 i, 0.807763 + 3.72871 i, 0.906216 - 2.45483 i, 0.906216 + 2.45483 i,
 0.974373 - 1.21578 i, 0.974373 + 1.21578 i, 1.07501 - 3.96142 i, 1.07501 + 3.96142 i,
 1.58506 - 3.80414 i, 1.58506 + 3.80414 i, 2.05064 - 3.58588 i, 2.05064 + 3.58588 i,
 2.4836 - 3.30966 i, 2.4836 + 3.30966 i, 2.87839 - 2.98122 i, 2.87839 + 2.98122 i,
 3.2289 - 2.60604 i, 3.2289 + 2.60604 i, 3.52969 - 2.19009 i, 3.52969 + 2.19009 i,
 3.77611 - 1.73991 i, 3.77611 + 1.73991 i, 3.96435 - 1.26256 i, 3.96435 + 1.26256 i,
 4.0915 - 0.765506 i, 4.0915 + 0.765506 i, 4.15556 - 0.256503 i, 4.15556 + 0.256503 i}
```

```
RootLocusPlot[1 / Expand[l1a[100, x, 300]], {k, 0, 1}, FeedbackType → None]
```




```
l1b[n_, t_] :=
 DD[n, 1, 0] + N[Sum[ t^k / k ! D[(DD[n, 1, s]), {s, k}] /. s → 0, {k, 1, Infinity}]]
```

```
l1c[n_, t_, m_] := DD[n, 1, 0] + Sum[ t^k / k ! D[(DD[n, 1, s]), {s, k}] /. s → 0, {k, 1, m}]
```

```
RootLocusPlot[1 / Expand[l1c[100, x, 300]], {k, 0, 1}, FeedbackType → None]
```