

```

F[fn_, n_, k_, s_] := Sum[(fn[m]^(k-j)) Binomial[k, j] F[fn, n/(m^(k-j)), j, m+1],
  {m, s, n^(1/k)}, {j, 0, k-1}]
F[fn_, n_, 0, s_] := 1
f[fn_, n_, k_] := F[fn, n, k, 2] - F[fn, n-1, k, 2]
FAlt[fn_, n_, k_, t_] := Sum[fn[j] F[fn, n/j, k-1, 2], {j, t+1, n^(1/2)}] +
  Sum[Sum[fn[m], {m, Floor[n/(j+1)]+1, n/j}] F[fn, j, k-1, 2],
    {j, 1, n/Floor[n^(1/2)]-1}] +
  Sum[f[fn, j, k-1] Sum[fn[m], {m, 2, n/j}], {j, 2, t}] +
  Sum[fn[s] f[fn, j, m] F[fn, n/(j s), k-m-1, 2],
    {j, 2, t}, {s, Floor[t/j]+1, Floor[n/j]^(1/2)}, {m, 1, k-2}] +
  Sum[(Sum[fn[m], {m, Floor[n/(j(s+1))]+1, n/(j s)}])
    (Sum[f[fn, j, m] F[fn, s, k-m-1, 2], {m, 1, k-2}]),
    {j, 2, t}, {s, 1, Floor[n/j]/Floor[Floor[n/j]^(1/2)]-1}]
FAlt[fn_, n_, 1, t_] := Sum[fn[j], {j, 2, n}]

```

```

id[n_] := 1
F[MoebiusMu, 10, 3, 2]

```

```
-1
```

```
FAlt[MoebiusMu, 10, 3, 9]
```

```
-1
```

```
ClearAll["Global`*"]
```

```

F[fn_, n_, k_, s_] :=
  F[fn, n, k, s] = Sum[(fn[m]^(k-j)) Binomial[k, j] F[fn, n/(m^(k-j)), j, m+1],
    {m, s, n^(1/k)}, {j, 0, k-1}]
F[fn_, n_, 0, s_] := 1
F[fn_, n_, k_] := F[fn, n, k, 1]
f[fn_, n_, k_] := F[fn, n, k] - F[fn, n-1, k]
FAlt[fn_, n_, k_, t_] := Sum[fn[j] F[fn, n/j, k-1], {j, t+1, n^(1/2)}] +
  Sum[Sum[fn[m], {m, Floor[n/(j+1)]+1, n/j}] F[fn, j, k-1],
    {j, 1, n/Floor[n^(1/2)]-1}] +
  Sum[f[fn, j, k-1] Sum[fn[m], {m, 1, n/j}], {j, 1, t}] +
  Sum[fn[s] f[fn, j, m] F[fn, n/(j s), k-m-1],
    {j, 1, t}, {s, Floor[t/j]+1, Floor[n/j]^(1/2)}, {m, 1, k-2}] +
  Sum[(Sum[fn[m], {m, Floor[n/(j(s+1))]+1, n/(j s)}])
    (Sum[f[fn, j, m] F[fn, s, k-m-1], {m, 1, k-2}]),
    {j, 1, t}, {s, 1, Floor[n/j]/Floor[Floor[n/j]^(1/2)]-1}]
FAlt[fn_, n_, 1, t_] := Sum[fn[j], {j, 1, n}]
Grid[Table[{F[MoebiusMu, n, k, 1], FAlt[MoebiusMu, n, k, Floor[n^(1/3)]]},
  {n, 10, 500, 10}, {k, 1, 7}]]

```

{-1, -1}	{3, 3}	{12, 12}	{25, 25}	{41, 41}	{59, 59}	{78, 78}
{-3, -3}	{-3, -3}	{-9, -9}	{-30, -30}	{-74, -74}	{-148, -148}	{-258, -258}
{-3, -3}	{-4, -4}	{-19, -19}	{-60, -60}	{-134, -134}	{-243, -243}	{-384, -384}
{0, 0}	{13, 13}	{32, 32}	{64, 64}	{130, 130}	{264, 264}	{512, 512}
{-3, -3}	{-2, -2}	{-19, -19}	{-66, -66}	{-150, -150}	{-279, -279}	{-468, -468}
{-1, -1}	{12, 12}	{35, 35}	{94, 94}	{240, 240}	{543, 543}	{1086, 1086}
{-2, -2}	{0, 0}	{-16, -16}	{-42, -42}	{-45, -45}	{28, 28}	{246, 246}
{-4, -4}	{-10, -10}	{-55, -55}	{-162, -162}	{-360, -360}	{-704, -704}	{-1294, -1294}

{-2, -2}	{10, 10}	{32, 32}	{103, 103}	{285, 285}	{643, 643}	{1226, 1226}
{1, 1}	{19, 19}	{47, 47}	{127, 127}	{335, 335}	{772, 772}	{1562, 1562}
{-5, -5}	{-9, -9}	{-37, -37}	{-73, -73}	{-85, -85}	{-44, -44}	{64, 64}
{-3, -3}	{-7, -7}	{-58, -58}	{-193, -193}	{-490, -490}	{-1112, -1112}	{-2344, -2344}
{-2, -2}	{-2, -2}	{-41, -41}	{-139, -139}	{-345, -345}	{-781, -781}	{-1680, -1680}
{-4, -4}	{0, 0}	{1, 1}	{41, 41}	{165, 165}	{377, 377}	{602, 602}
{0, 0}	{18, 18}	{52, 52}	{171, 171}	{485, 485}	{1136, 1136}	{2310, 2310}
{0, 0}	{20, 20}	{67, 67}	{235, 235}	{680, 680}	{1622, 1622}	{3374, 3374}
{-2, -2}	{7, 7}	{10, 10}	{45, 45}	{155, 155}	{365, 365}	{686, 686}
{-3, -3}	{-5, -5}	{-59, -59}	{-215, -215}	{-605, -605}	{-1501, -1501}	{-3346, -3346}
{-4, -4}	{-21, -21}	{-119, -119}	{-355, -355}	{-860, -860}	{-1897, -1897}	{-3892, -3892}
{-8, -8}	{-28, -28}	{-116, -116}	{-311, -311}	{-730, -730}	{-1642, -1642}	{-3528, -3528}
{-1, -1}	{14, 14}	{37, 37}	{109, 109}	{220, 220}	{230, 230}	{-196, -196}
{4, 4}	{38, 38}	{107, 107}	{289, 289}	{665, 665}	{1290, 1290}	{2198, 2198}
{1, 1}	{29, 29}	{98, 98}	{313, 313}	{805, 805}	{1713, 1713}	{3206, 3206}
{-1, -1}	{19, 19}	{77, 77}	{313, 313}	{970, 970}	{2451, 2451}	{5446, 5446}
{-1, -1}	{11, 11}	{44, 44}	{237, 237}	{839, 839}	{2265, 2265}	{5222, 5222}
{-2, -2}	{5, 5}	{11, 11}	{101, 101}	{404, 404}	{1119, 1119}	{2611, 2611}
{-2, -2}	{1, 1}	{-31, -31}	{-99, -99}	{-256, -256}	{-621, -621}	{-1323, -1323}
{-3, -3}	{-3, -3}	{-46, -46}	{-159, -159}	{-466, -466}	{-1227, -1227}	{-2807, -2807}
{-8, -8}	{-36, -36}	{-157, -157}	{-425, -425}	{-1001, -1001}	{-2232, -2232}	{-4711, -4711}
{-5, -5}	{-22, -22}	{-127, -127}	{-405, -405}	{-1106, -1106}	{-2754, -2754}	{-6237, -6237}
{-2, -2}	{-4, -4}	{-58, -58}	{-205, -205}	{-636, -636}	{-1806, -1806}	{-4529, -4529}
{-4, -4}	{-8, -8}	{-67, -67}	{-241, -241}	{-776, -776}	{-2244, -2244}	{-5677, -5677}
{1, 1}	{18, 18}	{26, 26}	{29, 29}	{-101, -101}	{-729, -729}	{-2541, -2541}
{2, 2}	{24, 24}	{47, 47}	{109, 109}	{189, 189}	{177, 177}	{-105, -105}
{1, 1}	{32, 32}	{115, 115}	{369, 369}	{894, 894}	{1741, 1741}	{2933, 2933}
{-1, -1}	{18, 18}	{76, 76}	{353, 353}	{1189, 1189}	{3235, 3235}	{7721, 7721}
{-2, -2}	{9, 9}	{49, 49}	{303, 303}	{1119, 1119}	{3154, 3154}	{7637, 7637}
{-3, -3}	{13, 13}	{85, 85}	{423, 423}	{1384, 1384}	{3598, 3598}	{8211, 8211}
{-2, -2}	{25, 25}	{142, 142}	{607, 607}	{1849, 1849}	{4594, 4594}	{10115, 10115}
{1, 1}	{33, 33}	{127, 127}	{457, 457}	{1294, 1294}	{3127, 3127}	{6923, 6923}
{-2, -2}	{11, 11}	{40, 40}	{197, 197}	{634, 634}	{1639, 1639}	{3871, 3871}
{0, 0}	{11, 11}	{-17, -17}	{-119, -119}	{-441, -441}	{-1193, -1193}	{-2457, -2457}
{-2, -2}	{-13, -13}	{-107, -107}	{-339, -339}	{-871, -871}	{-1925, -1925}	{-3591, -3591}
{-7, -7}	{-41, -41}	{-206, -206}	{-619, -619}	{-1586, -1586}	{-3665, -3665}	{-7679, -7679}
{-7, -7}	{-38, -38}	{-203, -203}	{-655, -655}	{-1796, -1796}	{-4376, -4376}	{-9555, -9555}

{-5, -5}	{-30, -30}	{-185, -185}	{-635, -635}	{-1826, -1826}	{-4604, -4604}	{-10 297, -10 297}
{-7, -7}	{-30, -30}	{-176, -176}	{-635, -635}	{-1941, -1941}	{-5126, -5126}	{-11 837, -11 837}
{-6, -6}	{-28, -28}	{-167, -167}	{-591, -591}	{-1821, -1821}	{-4976, -4976}	{-12 089, -12 089}
{-4, -4}	{-17, -17}	{-122, -122}	{-447, -447}	{-1446, -1446}	{-4133, -4133}	{-10 381, -10 381}
{-6, -6}	{-21, -21}	{-113, -113}	{-387, -387}	{-1281, -1281}	{-3815, -3815}	{-9905, -9905}