```
Limit[Sum[ (a^k - 1) / k, {k, 1, Log[a, 100]}], a → 1]
```

$$\text{Limit}\left[-\text{HarmonicNumber}\left[\frac{\text{Log}[100]}{\text{Log}[a]}\right] - 100\, a\, \text{LerchPhi}\left[a,\, 1,\, 1+\frac{\text{Log}[100]}{\text{Log}[a]}\right] - \text{Log}[1-a],\, a \to 1\right]$$

```
Limit[Sum[ (1 - Log[100]^-1)^k / k, {k, 1, Log[a, 100]}], a → 1]
```

$$\text{Limit}\left[-\frac{1}{\text{Log}[100]}\left(-\text{LerchPhi}\left[1-\frac{1}{\text{Log}[100]},\, 1,\, 1+\frac{\text{Log}[100]}{\text{Log}[a]}\right]\left(1-\frac{1}{\text{Log}[100]}\right)^{\frac{\text{Log}[100]}{\text{Log}[a]}} + \text{LerchPhi}\left[1-\frac{1}{\text{Log}[100]},\, 1,\, 1+\frac{\text{Log}[100]}{\text{Log}[a]}\right]\left(1-\frac{1}{\text{Log}[100]}\right)^{\frac{\text{Log}[100]}{\text{Log}[a]}} \text{Log}[100] - \text{Log}[100]\,\text{Log}[\text{Log}[100]]\right),\, a \to 1\right]$$

```
Limit[-HarmonicNumber[Log[100] / Log[a]] - Log[1 - a], a → 1]
```

$$-\text{EulerGamma} - i\pi - \text{Log}[\text{Log}[100]]$$

```
Limit[Sum[ (a^k - 1 + (1 - Log[100]^-1)^k) / k, {k, 1, Log[a, 100]}], a → 1]
```

$$\text{Limit}\left[-\frac{1}{\text{Log}[100]}\left(-\text{LerchPhi}\left[1-\frac{1}{\text{Log}[100]},\, 1,\, 1+\frac{\text{Log}[100]}{\text{Log}[a]}\right]\left(1-\frac{1}{\text{Log}[100]}\right)^{\frac{\text{Log}[100]}{\text{Log}[a]}} + \text{HarmonicNumber}\left[\frac{\text{Log}[100]}{\text{Log}[a]}\right]\text{Log}[100] + 100\, a\, \text{LerchPhi}\left[a,\, 1,\, 1+\frac{\text{Log}[100]}{\text{Log}[a]}\right]\text{Log}[100] + \text{LerchPhi}\left[1-\frac{1}{\text{Log}[100]},\, 1,\, 1+\frac{\text{Log}[100]}{\text{Log}[a]}\right]\left(1-\frac{1}{\text{Log}[100]}\right)^{\frac{\text{Log}[100]}{\text{Log}[a]}}\text{Log}[100] + \text{Log}[100]\,\text{Log}[1-a] - \text{Log}[100]\,\text{Log}[\text{Log}[100]]\right),\, a \to 1\right]$$

```
Limit[Sum[ (1 - 100^-1)^k / k, {k, 1, Log[a, 100]}], a → 1]
```

$$\text{Limit}\left[100^{-1-\frac{\text{Log}[100]}{\text{Log}[a]}}\left(-99^{1+\frac{\text{Log}[100]}{\text{Log}[a]}}\,\text{LerchPhi}\left[\frac{99}{100},\, 1,\, 1+\frac{\text{Log}[100]}{\text{Log}[a]}\right] + 100^{1+\frac{\text{Log}[100]}{\text{Log}[a]}}\,\text{Log}[100]\right),\, a \to 1\right]$$

```
fb[a_] := 100^(-1 - Log[100]/Log[a]) (-99^(1 + Log[100]/Log[a]) LerchPhi[99/100, 1, 1 + Log[100]/Log[a]] + 100^(1 + Log[100]/Log[a]) Log[100])
```

```
fb[1.01]
```

4.60344846361

```
N[Log[100]]
```

4.60517

```
fd[a_] :=
 1/Log[100] 100^(-1-Log[100]/Log[a]) (100^(1+Log[100]/Log[a]) LerchPhi[1-1/Log[100], 1, 1+Log[100]/Log[a]] (1-1/Log[100])^(Log[100]/Log[a]) -
  100^(1+Log[100]/Log[a]) HarmonicNumber[Log[100]/Log[a]] Log[100] - 99^(1+Log[100]/Log[a]) LerchPhi[99/100, 1, 1+Log[100]/Log[a]]
  Log[100] - 100^(2+Log[100]/Log[a]) a LerchPhi[a, 1, 1+Log[100]/Log[a]] Log[100] -
  100^(1+Log[100]/Log[a]) LerchPhi[1-1/Log[100], 1, 1+Log[100]/Log[a]] (1-1/Log[100])^(Log[100]/Log[a]) Log[100] +
  100^(1+Log[100]/Log[a]) Log[100]^2 - 100^(1+Log[100]/Log[a]) Log[100] Log[1-a] + 100^(1+Log[100]/Log[a]) Log[100] Log[Log[100]])

N[fd[1.00001]]

34.1542 + 5.26872 × 10^-10 i

Expand[
 1/Log[100] 100^(-1-Log[100]/Log[a]) (100^(1+Log[100]/Log[a]) LerchPhi[1-1/Log[100], 1, 1+Log[100]/Log[a]] (1-1/Log[100])^(Log[100]/Log[a]) -
  100^(1+Log[100]/Log[a]) HarmonicNumber[Log[100]/Log[a]] Log[100] - 99^(1+Log[100]/Log[a]) LerchPhi[99/100, 1, 1+Log[100]/Log[a]]
  Log[100] - 100^(2+Log[100]/Log[a]) a LerchPhi[a, 1, 1+Log[100]/Log[a]] Log[100] -
  100^(1+Log[100]/Log[a]) LerchPhi[1-1/Log[100], 1, 1+Log[100]/Log[a]] (1-1/Log[100])^(Log[100]/Log[a]) Log[100] +
  100^(1+Log[100]/Log[a]) Log[100]^2 - 100^(1+Log[100]/Log[a]) Log[100] Log[1-a] + 100^(1+Log[100]/Log[a]) Log[100] Log[Log[100]])]
```

```
FullSimplify[-HarmonicNumber[Log[100]/Log[a]] -
  (100/99)^(-1-Log[100]/Log[a]) LerchPhi[99/100, 1, 1 + Log[100]/Log[a]] - 100 a LerchPhi[a, 1, 1 + Log[100]/Log[a]] -
  LerchPhi[1 - 1/Log[100], 1, 1 + Log[100]/Log[a]] (1 - 1/Log[100])^(Log[100]/Log[a]) +
  (LerchPhi[1 - 1/Log[100], 1, 1 + Log[100]/Log[a]] (1 - 1/Log[100])^(Log[100]/Log[a]))/Log[100] + Log[100] - Log[1 - a] + Log[Log[100]]]
```

```
-HarmonicNumber[Log[100]/Log[a]] - (99/100)^(Log[100 a]/Log[a]) LerchPhi[99/100, 1, Log[100 a]/Log[a]] -
  100 a LerchPhi[a, 1, Log[100 a]/Log[a]] - 1/Log[100]
  LerchPhi[1 - 1/Log[100], 1, Log[100 a]/Log[a]] (1 - 1/Log[100])^(Log[100]/Log[a]) (-1 + Log[100]) +
  Log[100] - Log[1 - a] + Log[Log[100]]
```

```
fd2[a_] := -HarmonicNumber[Log[100]/Log[a]] - (99/100)^(Log[100 a]/Log[a]) LerchPhi[99/100, 1, Log[100 a]/Log[a]] -
  100 a LerchPhi[a, 1, Log[100 a]/Log[a]] - 1/Log[100] LerchPhi[1 - 1/Log[100], 1, Log[100 a]/Log[a]]
  (1 - 1/Log[100])^(Log[100]/Log[a]) (-1 + Log[100]) + Log[100] - Log[1 - a] + Log[Log[100]]
```

```
fd2[1.00001]
```

```
34.1542 + 3.90581 × 10^-11 i
```

```
N[LogIntegral[100]] + Log[100] - EulerGamma
```

```
34.1541
```

```
Expand[-1/Log[100] (-LerchPhi[1 - 1/Log[100], 1, 1 + Log[100]/Log[a]] (1 - 1/Log[100])^(Log[100]/Log[a]) +
    HarmonicNumber[Log[100]/Log[a]] Log[100] + 100 a LerchPhi[a, 1, 1 + Log[100]/Log[a]] Log[100] +
    LerchPhi[1 - 1/Log[100], 1, 1 + Log[100]/Log[a]] (1 - 1/Log[100])^(Log[100]/Log[a]) Log[100] +
    Log[100] Log[1 - a] - Log[100] Log[Log[100]])]
```

```
FullSimplify[-HarmonicNumber[Log[100]/Log[a]] - 100 a LerchPhi[a, 1, 1 + Log[100]/Log[a]] -
  LerchPhi[1 - 1/Log[100], 1, 1 + Log[100]/Log[a]] (1 - 1/Log[100])^(Log[100]/Log[a]) +
  (LerchPhi[1 - 1/Log[100], 1, 1 + Log[100]/Log[a]] (1 - 1/Log[100])^(Log[100]/Log[a]) -
    Log[1 - a] + Log[Log[100]])/Log[100]]

fe[a_] := -HarmonicNumber[Log[100]/Log[a]] - 100 a LerchPhi[a, 1, Log[100 a]/Log[a]] - 1/Log[100]
  LerchPhi[1 - 1/Log[100], 1, Log[100 a]/Log[a]] (1 - 1/Log[100])^(Log[100]/Log[a]) (-1 + Log[100]) -
  Log[1 - a] + Log[Log[100]]

fe[1.00001] + EulerGamma

30.1262 + 3.90581 × 10^-11 i

N[LogIntegral[100]]

30.1261

Limit[Sum[ (a^k + (1 - Log[100]^-1)^k) / k, {k, 1, Log[a, 100]}], a → 1]

Limit[-1/Log[100] (-LerchPhi[1 - 1/Log[100], 1, 1 + Log[100]/Log[a]] (1 - 1/Log[100])^(Log[100]/Log[a]) +
    100 a LerchPhi[a, 1, 1 + Log[100]/Log[a]] Log[100] + LerchPhi[1 - 1/Log[100], 1, 1 + Log[100]/Log[a]]
     (1 - 1/Log[100])^(Log[100]/Log[a]) Log[100] + Log[100] Log[1 - a] - Log[100] Log[Log[100]]), a → 1]

Limit[ 1 / n Sum[ Ceiling[n / k] - n / k, {k, 1, n}], {n → Infinity}]

{Limit[(Sum_{k=1}^{n} (-n/k + Ceiling[n/k]))/n, n → ∞]}

Sum[(1 - 1 / n)^k, {k, 1, Infinity}]

-1 + n

Sum[(1 - 1 / n)^k / k, {k, 1, Infinity}]

-Log[1/n]

Limit[Sum[(1 - (1 - 1 / a)^k) / k, {k, 1, Infinity}], {a → Infinity}]

{Limit[Sum_{k=1}^{∞} (1 - (1 - 1/a)^k)/k, a → ∞]}

Sum[(1 - (1 - 1 / n)^k) / k, {k, 1, t}]
```

```
Expand[1/n ((n HarmonicNumber[t] - ((-1 + n)/n)^t LerchPhi[(-1 + n)/n, 1, 1 + t] +
      ((-1 + n)/n)^t n LerchPhi[(-1 + n)/n, 1, 1 + t] + n Log[1/n]))]
```

```
FullSimplify[HarmonicNumber[t] +
   ((-1 + n)/n)^t LerchPhi[(-1 + n)/n, 1, 1 + t] - ((-1+n)/n)^t LerchPhi[(-1+n)/n, 1, 1 + t]/n + Log[1/n]]
```

$$\text{HarmonicNumber}[t]+\left(\frac{-1+n}{n}\right)^{1+t}\text{LerchPhi}\left[\frac{-1+n}{n},1,1+t\right]+\text{Log}\left[\frac{1}{n}\right]$$

```
Limit[HarmonicNumber[t] + ((-1 + n)/n)^(1+t) LerchPhi[(-1 + n)/n, 1, 1 + t] + Log[1/n], t -> Infinity]
```

```
Limit[HarmonicNumber[t] + ((-1 + n)/n)^(1+t) LerchPhi[(-1 + n)/n, 1, 1 + t] + Log[1/n], t -> ∞]
```

```
zz[a_] := Sum[(1 - (1 - 1 / a) ^k) / k, {k, 1, a}]
```

```
N[zz[10 000]]
```

```
0.796613
```

```
t[n_, a_] := 1 - (Mod[n, a] - Mod[n - 1, a])
```

```
Table[ (t[k, 7]) / k, {k, 1, 100}]
```

$$\left\{0,0,0,0,0,0,1,0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0,\frac{1}{3},0,0,0,\right.$$
$$0,0,0,\frac{1}{4},0,0,0,0,0,0,\frac{1}{5},0,0,0,0,0,0,\frac{1}{6},0,0,0,0,0,0,\frac{1}{7},0,$$
$$0,0,0,0,0,\frac{1}{8},0,0,0,0,0,0,\frac{1}{9},0,0,0,0,0,0,\frac{1}{10},0,0,0,0,0,0,$$
$$\left.\frac{1}{11},0,0,0,0,0,0,\frac{1}{12},0,0,0,0,0,0,\frac{1}{13},0,0,0,0,0,0,\frac{1}{14},0,0\right\}$$

```
N[Log[10]]
```

```
2.30259
```