```
(y^-1)^-s /. s → 3

y^3

y^s /. s → 3

y^3

(y^-1) (y^s)

y^(-1+s)

(y^-3) (y^{3 s})

{y^(-3+3 s)}

Dy[n_, s_, y_, k_] :=
 y^-1 Sum[( (1 + j / y))^-s Dy[n (1 + j / y)^-1, s, y, k - 1], {j, 1, (n - 1) y}]
Dy[n_, s_, y_, 0] := UnitStep[n - 1]
Dd[n_, s_, y_, k_] := Sum[(j + y)^-s Dd[n (y + j)^-1, s, y, k - 1], {j, 0, n - y}]
Dd[n_, s_, y_, 0] := UnitStep[n - 1]
Cc[n_, s_, y_, k_] := y^(k (s - 1)) Dd[n y^k, s, y + 1, k]

N[Dy[100, 0, 2, 2]]

318.

N[Cc[100, 0, 2, 2]]

318.

D2a[n_, k_, s_] := D2a[n, k, s] = Sum[j^(-s) D2a[Floor[n / j], k - 1, s], {j, 2, n}]
D2a[n_, 0, s_] := 1
E2a[n_, k_, a_, s_] := E2a[n, k, a, s] = Sum[ j^(-s) E2a[n / j, k - 1, a, s], {j, 2, n}] -
   a Sum[ (j a)^(-s) E2a[n / (a j), k - 1, a, s], {j, 1, n / a}]
E2a[n_, 0, a_, s_] := 1
D2E2[n_, s_, k_, b_] := Sum[ (-1)^j b^(j (1 - s)) Binomial[k, j]
   Sum[ Binomial[j, m] If[n / b^j < 1, 0, D2a[n / b^j, k - m, s]], {m, 0, j}], {j, 0, k}]
E2D2[n_, s_, k_, b_] := (-1)^k + Sum[b^(a (1 - s)) / ((k - 1)!) Binomial[k, j]
    Pochhammer[a - k + j + 1, k - 1] E2a[b^-a n, j, b, s], {a, 0, Log[b, n]}, {j, 0, k}]


E2D2[100, 0, 2, 2]

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D2a[n_, k_, s_] := Sum[j^(-s) D2a[Floor[n / j], k - 1, s], {j, 2, n}]
D2a[n_, 0, s_] := UnitStep[n - 1]
E2a[n_, k_, x_, s_] := Sum[ j^(-s) E2a[n / j, k - 1, x, s], {j, 2, n}] -
  x Sum[ (j x)^(-s) E2a[n / (x j), k - 1, x, s], {j, 1, n / x}]
E2a[n_, 0, x_, s_] := UnitStep[n - 1]
D2E2[n_, s_, k_, x_] := Sum[ (-1)^j x^(j (1 - s)) Binomial[k, j]
   Sum[ Binomial[j, m] If[n / x^j < 1, 0, D2a[n / x^j, k - m, s]], {m, 0, j}], {j, 0, k}]
E2D2[n_, s_, k_, x_] := (-1)^k + Sum[x^(a (1 - s)) / ((k - 1)!) Binomial[k, j]
    Pochhammer[a - k + j + 1, k - 1] E2a[x^-a n, j, x, s], {a, 0, Log[x, n]}, {j, 0, k}]
{E2D2[100, -1, 2, 2], D2a[100, 2, -1], D2E2[100, -1, 2, 2], E2a[100, 2, 2, -1]}

{16 780, 16 780, 276, 276}
```

```
D2a[n_, k_, s_] := Sum[j^(-s) D2a[Floor[n j^-1], k - 1, s], {j, 2, n}]
D2a[n_, 0, s_] := UnitStep[n - 1]
E2a[n_, k_, x_, s_] := Sum[ j^(-s) E2a[n j^-1, k - 1, x, s], {j, 2, n}] -
  x Sum[ (j x)^(-s) E2a[n (x j)^-1, k - 1, x, s], {j, 1, n x^-1}]
E2a[n_, 0, x_, s_] := UnitStep[n - 1]
D2E2[n_, s_, k_, x_] := Sum[ (-1)^j x^(j (1 - s)) Binomial[k, j]
   Sum[ Binomial[j, m] D2a[n x^-j, k - m, s], {m, 0, j}], {j, 0, k}]
E2D2[n_, s_, k_, x_] := (-1)^k + Sum[x^(a (1 - s)) / ((k - 1)!) Binomial[k, j]
    Pochhammer[a - k + j + 1, k - 1] E2a[x^-a n, j, x, s], {a, 0, Log[x, n]}, {j, 0, k}]
{E2D2[100, -1, 2, 2], D2a[100, 2, -1], D2E2[100, -1, 2, 2], E2a[100, 2, 2, -1]}

{16 780, 16 780, 276, 276}

D2a[n_, s_, k_] := Sum[j^(-s) D2a[Floor[n j^-1], s, k - 1], {j, 2, n}]
D2a[n_, s_, 0] := UnitStep[n - 1]
E2a[n_, s_, k_, x_] := Sum[ j^(-s) E2a[n j^-1, s, k - 1, x], {j, 2, n}] -
  x Sum[ (j x)^(-s) E2a[n (x j)^-1, s, k - 1, x], {j, 1, n x^-1}]
E2a[n_, s_, 0, x_] := UnitStep[n - 1]
D2E2[n_, s_, k_, x_] := Sum[ (-1)^j x^(j (1 - s))
   Binomial[k, j] Binomial[j, m] D2a[n x^-j, s, k - m], {j, 0, k}, {m, 0, j}]
E2D2[n_, s_, k_, x_] := (-1)^k + Sum[x^(j (1 - s)) Binomial[k, m]
    Binomial[m + j - 1, k - 1] E2a[n x^-j, s, m, x], {j, 0, Log[x, n]}, {m, 0, k}]
{E2D2[100, -1, 4, 2], D2a[100, -1, 4], D2E2[100, -1, 4, 2], E2a[100, -1, 4, 2]}

{13 441, 13 441, -799, -799}

D2a[n_, s_, k_] := Sum[j^(-s) D2a[Floor[n j^-1], s, k - 1], {j, 2, n}]
D2a[n_, s_, 0] := UnitStep[n - 1]
E2a[n_, s_, k_, x_] := Sum[ j^(-s) E2a[n j^-1, s, k - 1, x], {j, 2, n}] -
  x Sum[ (j x)^(-s) E2a[n (x j)^-1, s, k - 1, x], {j, 1, n x^-1}]
E2a[n_, s_, 0, x_] := UnitStep[n - 1]
D2E2[n_, s_, k_, x_] := Sum[ (-1)^j x^(j (1 - s))
   Binomial[k, j] Binomial[j, m] D2a[n x^-j, s, k - m], {j, 0, k}, {m, 0, j}]
E2D2[n_, s_, k_, x_] := (-1)^k + Sum[x^(j (1 - s)) Binomial[k, m]
    Binomial[m + j - 1, k - 1] E2a[n x^-j, s, m, x], {j, 0, Log[x, n]}, {m, 0, k}]
{E2D2[100, -1, 4, 2], D2a[100, -1, 4], D2E2[100, -1, 4, 2], E2a[100, -1, 4, 2]}

{13 441, 13 441, -799, -799}

Table[ {Expand[(-1)^j Binomial[-z, j]] - Expand[Binomial[z + j - 1, j]]}, {j, 0, 5}] //
 TableForm

0
0
0
0
0
0
```

```
D2a[n_, s_, k_] := Sum[j^(-s) D2a[Floor[n j^-1], s, k - 1], {j, 2, n}]
D2a[n_, s_, 0] := UnitStep[n - 1]
E2a[n_, s_, k_, x_] := Sum[ j^(-s) E2a[n j^-1, s, k - 1, x], {j, 2, n}] -
  x Sum[ (j x)^(-s) E2a[n (x j)^-1, s, k - 1, x], {j, 1, n x^-1}]
E2a[n_, s_, 0, x_] := UnitStep[n - 1]
D2E2[n_, s_, k_, x_] := Sum[ (-1)^j x^(j (1 - s))
   Binomial[k, j] Binomial[j, m] D2a[n x^-j, s, k - m], {j, 0, k}, {m, 0, j}]
E2D2[n_, s_, k_, x_] := (-1)^k + Sum[(-1)^(k - 1) x^(j (1 - s)) Binomial[k, k - m]
    Binomial[m - j - 1, k - 1] E2a[n x^-j, s, k - m, x], {j, 0, Log[x, n]}, {m, 0, k}]
{E2D2[120, 2, 4, 3 / 2], D2a[120, 2, 4], D2E2[100, -1, 4, 2], E2a[100, -1, 4, 2]}
```

$$\left\{\frac{518\,378\,950\,339\,973}{8\,414\,884\,558\,080\,000}, \frac{518\,378\,950\,339\,973}{8\,414\,884\,558\,080\,000}, -799, -799\right\}$$

```
th[n_, k_] := FullSimplify[MangoldtLambda[ n ] / Log[n]] -
  n / Log[k, n] ( 1 + Floor[ Floor[ Log[k, n]] - Log[k, n]])

ts[n_, s_, 0] := UnitStep[n - 1]
ts[n_, s_, k_] := Sum[ th[j, 2] j^-s ts[ n / j, s, k - 1], {j, 2, n}]

ts[100, -1, 1]
```

$$\frac{10\,301}{60}$$

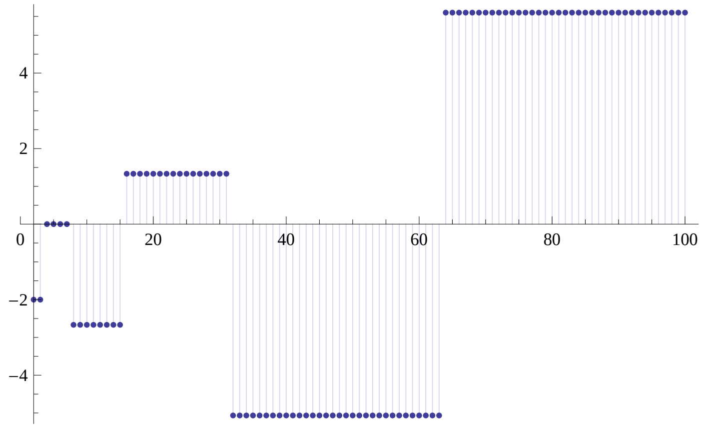
```
th[n_, k_, b_] :=
 (Floor[n / b] - Floor[(n - 1) / b]) FullSimplify[MangoldtLambda[ n / b ] / Log[n / b]] -
  n / b / Log[k, n / b] ( 1 + Floor[ Floor[ Log[k, n / b]] - Log[k, n / b]])

Table[{n / 2, th[n, 2, 2]}, {n, 3, 20}]
```

$$\left\{\left\{\frac{3}{2}, 0\right\}, \{2, -1\}, \left\{\frac{5}{2}, 0\right\}, \{3, 1\}, \left\{\frac{7}{2}, 0\right\}, \left\{4, -\frac{3}{2}\right\}, \left\{\frac{9}{2}, 0\right\}, \{5, 1\}, \left\{\frac{11}{2}, 0\right\}, \{6, 0\}, \left\{\frac{13}{2}, 0\right\}, \{7, 1\}, \left\{\frac{15}{2}, 0\right\}, \left\{8, -\frac{7}{3}\right\}, \left\{\frac{17}{2}, 0\right\}, \left\{9, \frac{1}{2}\right\}, \left\{\frac{19}{2}, 0\right\}, \{10, 0\}\right\}$$

```
bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k!
aa[f_, n_, k_] := aa[f, n, k] = Sum[ f[j] aa[f, n / j, k - 1], {j, 2, n}]
aa[f_, n_, 0] := UnitStep[n - 1]
aaz[f_, n_, z_] := Sum[bin[z, k] aa[f, n, k], {k, 0, Log[2, n]}]
laaz[f_, n_] := D[aaz[f, n, z], z] /. z → 0
bb[f_, n_, k_] := bb[f, n, k] = Sum[(-1)^(j + 1) f[j] bb[f, n / j, k - 1], {j, 2, n}]
bb[f_, n_, 0] := UnitStep[n - 1]
bbz[f_, n_, z_] := Sum[bin[z, k] bb[f, n, k], {k, 0, Log[2, n]}]
lbbz[f_, n_] := D[bbz[f, n, z], z] /. z → 0
```

```
DiscretePlot[laaz[LiouvilleLambda, n] - lbbz[LiouvilleLambda, n], {n, 2, 100}]
```




```
Table[{(laaz[EulerPhi, n] - lbbz[EulerPhi, n]), Sum[(3^j - 1) / j, {j, 1, Log[2, n]}]},
  {n, 2, 20}] // TableForm
```

2	2
2	2
6	6
6	6
6	6
6	6
$\frac{44}{3}$	$\frac{44}{3}$
$\frac{44}{3}$	$\frac{44}{3}$
$\frac{44}{3}$	$\frac{44}{3}$
$\frac{44}{3}$	$\frac{44}{3}$
$\frac{44}{3}$	$\frac{44}{3}$
$\frac{44}{3}$	$\frac{44}{3}$
$\frac{44}{3}$	$\frac{44}{3}$
$\frac{44}{3}$	$\frac{44}{3}$
$\frac{104}{3}$	$\frac{104}{3}$
$\frac{104}{3}$	$\frac{104}{3}$
$\frac{104}{3}$	$\frac{104}{3}$
$\frac{104}{3}$	$\frac{104}{3}$
$\frac{104}{3}$	$\frac{104}{3}$

```
Table[{laaz[LiouvilleLambda, n] - lbbz[LiouvilleLambda, n],
   Sum[ ((-2)^j) / j, {j, 1, Log[2, n]}]}, {n, 120, 130}] // TableForm
```

$\frac{28}{5}$	$\frac{28}{5}$
$\frac{28}{5}$	$\frac{28}{5}$
$\frac{28}{5}$	$\frac{28}{5}$
$\frac{28}{5}$	$\frac{28}{5}$
$\frac{28}{5}$	$\frac{28}{5}$
$\frac{28}{5}$	$\frac{28}{5}$
$\frac{28}{5}$	$\frac{28}{5}$
$\frac{28}{5}$	$\frac{28}{5}$
$-\frac{444}{35}$	$-\frac{444}{35}$
$-\frac{444}{35}$	$-\frac{444}{35}$
$-\frac{444}{35}$	$-\frac{444}{35}$

```
idd[n_] := 1
Table[
  {2^n, (laaz[ff = MoebiusMu, jj = 2^n] - lbbz[ff, jj]) - (laaz[ff, jj - 1] - lbbz[ff, jj - 1]),
   -2 / n (1 - Floor[n / 2] + Floor[(n - 1) / 2])}, {n, 1, 10}] // TableForm
```

2	-2	-2
4	0	0
8	$-\frac{2}{3}$	$-\frac{2}{3}$
16	0	0
32	$-\frac{2}{5}$	$-\frac{2}{5}$
64	0	0
128	$-\frac{2}{7}$	$-\frac{2}{7}$
256	0	0
512	$-\frac{2}{9}$	$-\frac{2}{9}$
1024	0	0

```
Table[{2^n, (laaz[ff = EulerPhi, jj = 2^n] - lbbz[ff, jj]) -
    (laaz[ff, jj - 1] - lbbz[ff, jj - 1]), (3^n - 1) / n}, {n, 1, 10}] // TableForm
```

2	2	2
4	4	4
8	$\frac{26}{3}$	$\frac{26}{3}$
16	20	20
32	$\frac{242}{5}$	$\frac{242}{5}$
64	$\frac{364}{3}$	$\frac{364}{3}$
128	$\frac{2186}{7}$	$\frac{2186}{7}$
256	820	820
512	$\frac{19682}{9}$	$\frac{19682}{9}$
1024	$\frac{29524}{5}$	$\frac{29524}{5}$

```
idc[n_] := DivisorSigma[1, n]
Table[{2^n, (laaz[ff = idc, jj = 2^n] - lbbz[ff, jj]) - (laaz[ff, jj - 1] - lbbz[ff, jj - 1]),
   (2^n) / n}, {n, 1, 10}] // TableForm
```

2	6	2
4	14	2
8	48	$\frac{8}{3}$
16	188	4
32	$\frac{3936}{5}$	$\frac{32}{5}$
64	$\frac{10\,304}{3}$	$\frac{32}{3}$
128	$\frac{107\,904}{7}$	$\frac{128}{7}$
256	70 624	32
512	328 704	$\frac{512}{9}$
1024	$\frac{7\,745\,024}{5}$	$\frac{512}{5}$

```
ide[n_] := 1
Table[{2^n, (laaz[ff = ide, jj = 2^n] - lbbz[ff, jj]) - (laaz[ff, jj - 1] - lbbz[ff, jj - 1]),
   (2^n) / n}, {n, 1, 10}] // TableForm
```

2	2	2
4	2	2
8	$\frac{8}{3}$	$\frac{8}{3}$
16	4	4
32	$\frac{32}{5}$	$\frac{32}{5}$
64	$\frac{32}{3}$	$\frac{32}{3}$
128	$\frac{128}{7}$	$\frac{128}{7}$
256	32	32
512	$\frac{512}{9}$	$\frac{512}{9}$
1024	$\frac{512}{5}$	$\frac{512}{5}$