```
binomial[z_, k_] := binomial[z, k] = Product[z - j, {j, 0, k - 1}] / k!
Ds[n_, 0, s_, a_] := UnitStep[n - 1]
Ds[n_, 1, s_, a_] := Ds[n, 1, s, a] = HarmonicNumber[Floor[n], s] - HarmonicNumber[a, s]
Ds[n_, 2, s_, a_] := Ds[n, 2, s, a] =
  Sum[(m^(-2 s)) + 2 (m^-s) (Ds[Floor[n / m], 1, s, m]), {m, a + 1, Floor[n^(1 / 2)]}]
Ds[n_, k_, s_, a_] := Ds[n, k, s, a] =
  Sum[(m^(-s k)) + k (m^(-s (k - 1))) Ds[Floor[n / (m^(k - 1))], 1, s, m] +
    Sum[binomial[k, j] (m^-s)^j Ds[Floor[n / (m^j)], k - j, s, m], {j, 1, k - 2}],
   {m, a + 1, Floor[n^(1 / k)]}]

Ddy[n_, s_, y_, k_] := y^(k (s - 1)) Ds[n y^k, k, s, y]
Ddy[0, s_, y_, k_] := 0

Dnsyz[n_, s_, y_, z_] := Expand@Sum[binomial[z, k] Ddy[n, s, y, k], {k, 0, Log[(y + 1) / y, n]}]
d2[n_, y_, k_] := Ddy[n, 0, y, k]
dd[n_, y_, z_] := Dnsyz[n, 0, y, z]

ltod[ n_, y_,  z_]  :=  Sum[z^k / k! D[dd[n, y, t], {t, k}] /. t → 0, {k, 0, Log[(y + 1) / y, n]}]


dd[100, 2, z]
```

$$1+\frac{202\,986\,703\,z}{7\,096\,320}+\frac{68\,602\,319\,z^{2}}{1\,612\,800}+\frac{622\,902\,011\,z^{3}}{29\,030\,400}+\frac{2\,091\,660\,979\,z^{4}}{371\,589\,120}+\frac{52\,801\,531\,z^{5}}{74\,317\,824}+\frac{21\,461\,041\,z^{6}}{353\,894\,400}+\frac{5\,689\,681\,z^{7}}{2\,477\,260\,800}+\frac{16\,259\,z^{8}}{247\,726\,080}+\frac{739\,z^{9}}{743\,178\,240}+\frac{37\,z^{10}}{7\,431\,782\,400}+\frac{z^{11}}{81\,749\,606\,400}$$

```
D[dd[100, 2, z], {z, 2}] /. z → 0
```

$$\frac{68\,602\,319}{806\,400}$$

```
ltod[10, 16, 3]
```

$$\frac{326\,425}{4096}$$

```
dd[10, 16, 3]
```

$$\frac{326\,425}{4096}$$

```
N@LaguerreL[-3, Log[10]]
```

82.5612

```
ff[n_,  z_]  :=  (-1)^z Gamma[z, 0, -Log[n]] / Gamma[z]
Chop@N@Integrate[ ff[10 / j, 3.3], {j, 1, 10}]
```

$-2.00791+6.17971\,i$

```
Chop@N@Gamma[4.3, 0, -Log[10]] / Gamma[4.3]
```

$3.81927+5.25678\,i$

```
Expand@Integrate[1, {x, 1, n}, {y, 1, n / x}]
```

ConditionalExpression[$1 - n + n \text{Log}[n]$, Re[n] ≥ 0 || n ∉ Reals]

```
Expand@Integrate[n / x - 1, {x, 1, n}]
```

ConditionalExpression[$1 - n + n \text{Log}[n]$, Re[n] ≥ 0 || n ∉ Reals]

```
Expand@Integrate[-Gamma[1, 0, -Log[n / x]] / Gamma[1], {x, 1, n}]
```

ConditionalExpression[$1 - n + n \text{Log}[n]$, Re[n] ≥ 0 || n ∉ Reals]

```
Expand@Integrate[-Gamma[2, 0, -Log[n / (x y)]] / Gamma[2], {x, 1, n}, {y, 1, n / x}]
```

ConditionalExpression$\left[-1 + n - n \text{Log}[n] + \frac{1}{2} n \text{Log}[n]^2 - \frac{1}{6} n \text{Log}[n]^3\right.$, Re[n] ≥ 0 || n ∉ Reals$\left.\right]$

```
Sum[ dd[100/j,2,2](dd[j,2,2]-dd[j-1,2,2]),{j,1,100}]
```

$\frac{70469}{16}$

```
dd[100, 2, 4]
```

$\frac{37027}{8}$

```
Sum[ (-1)^j (2 j)! / ((1 - 2 j) (j!)^2 (4^j)), {j, 0, Infinity}]
```

$\sqrt{2}$

```
Clear[d2]
ex[j_] := ex[j] = N[(-1)^(j - 1) (2 (j - 1))! / ((1 - 2 (j - 1)) ((j - 1)!)^2 (4^(j - 1)))]
d2[n_, k_] := d2[n, k] = Sum[ex[j] d2[Floor[n / j], k - 1], {j, 2, n}]
d2[n_, 0] := UnitStep[n - 1]
dz[n_, z_] := Sum[binomial[z, k] d2[n, k], {k, 0, Log2@n}]
DzRoots[n_] := If[(c = Exponent[f = dz[n, z], z]) == 0, {},
  If[c == 1, List@NRoots[f == 0, z][[2]], List @@ NRoots[f == 0, z][[All, 2]]]]
DzR[n_, z_] := Chop@Expand@Product[1 - z / rho, {rho, DzRoots[n]}]
```

```
Expand@N@dz[1000, z]
```

$1. + 0.346616 z + 0.0600399 z^2 + 0.00684002 z^3 + 0.000729602 z^4 - 0.0000264379 z^5 + 0.0000210949 z^6 - 2.63562 \times 10^{-6} z^7 + 2.13441 \times 10^{-7} z^8 - 6.72786 \times 10^{-9} z^9$

```
N@2^(1 / 2)
```

1.41421

```
N@Sum[ (-1)^j (2 j)! / ((1 - 2 j) (j!)^2 (4^j)), {j, 2, 100}]
```

-0.085927

```
N@(D[dz[1000, z], z] /. z → 0)
```

0.346616

```
N@Log[2] / 2
```

0.346574

```
DzRoots[1000]
```

{-3.77072 - 1.77362 i, -3.77072 + 1.77362 i, -1.82879 - 5.22551 i, -1.82879 + 5.22551 i, 2.28265 - 8.308 i, 2.28265 + 8.308 i, 8.83877 - 10.1876 i, 8.83877 + 10.1876 i, 20.6812}

```
DzR[1000, 4]
```

4.00002

```
N@Log[2^(1/2), 77]
```

12.5336

```
DzR[10 000, 12.533573081389804`]
```

76.9496

```
(2^(1/2))^z
```

$2^{z/2}$

```
Limit[ Sum[ (a - 1)^s (-1)^s a^k k^(s - 1), {k, 1, Infinity}] /. s → 1/2, a → 1]
```

$\sqrt{\pi}$

```
Gamma[1/2 + I]
```

$\text{Gamma}\left[\frac{1}{2} + i\right]$

```
Limit[Sum[ a - 1, {k, 0, Log[a, n]}], a → 1]
```

Log[n]

```
ff[n_, a_] := Sum[ a - 1, {k, 0, Log[a, n]}]
f2[s_, b_] := Limit[ Sum[ (a - 1)^s (-1)^s a^k k^(s - 1), {k, 0, Infinity}], a → b]
```

```
Plot[ f2[n, 1], {n, .2, 1}]
```

Power::infy : Infinite expression $\frac{1}{0^{0.799984}}$ encountered. ≫

Power::infy : Infinite expression $\frac{1}{0^{0.799984}}$ encountered. ≫

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General::stop : Further output of Power::infy will be suppressed during this calculation. ≫

NSum::nsnum : Summand (or its derivative) $\frac{(0.808987 + 0.587827\, i)(-1 + a)^{0.200016} a^k}{k^{0.799984}}$ is not numerical at point k = 46661. ≫

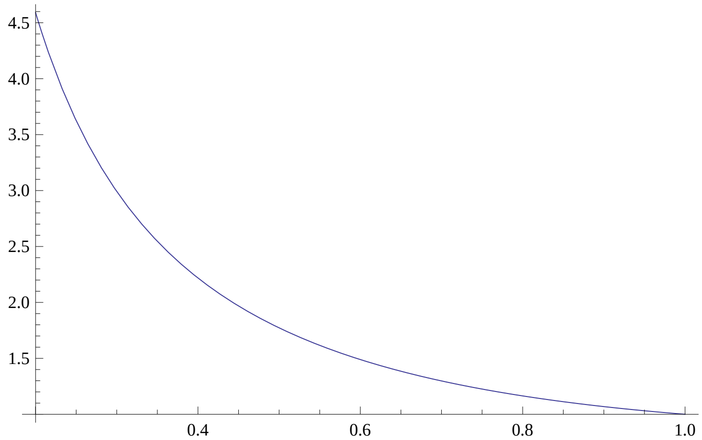
NSum::nsnum : Summand (or its derivative) $\frac{(0.808987 + 0.587827\, i)(-1 + a)^{0.200016} a^k}{k^{0.799984}}$ is not numerical at point k = 46661. ≫

NSum::nsnum : Summand (or its derivative) $\frac{(0.808987 + 0.587827\, i)(-1 + a)^{0.200016} a^k}{k^{0.799984}}$ is not numerical at point k = 46661. ≫

General::stop : Further output of NSum::nsnum will be suppressed during this calculation. ≫

$Aborted

```
Plot[Gamma[n], {n, .2, 1}]
```




```
Limit[(LaguerreL[-z, Log[100.]] - 1) / z, z → 0]
```

```
28.0217
```

```
f2[n_, z_, a_] := (LaguerreL[a - z, Log[n]] - LaguerreL[a + z, Log[n]]) / (2 z)
f3[n_, z_] := (f2[n, z, -z] - f2[n, z, z]) / (2 z)
```

```
f2[100, .001, -.1]
```

```
36.7743
```

```
f3[100, .00001]
```

```
80.5038
```

```
g1[n_, 0] := 1
g1[n_, a_] :=
 Sum[ (-1)^k Gamma[k, 0, -Log[n]] / Gamma[k] (D[Log[1 + x]^a, {x, k}] /. x → 0) / k!, {k, 1, 80}]
gz[ n_, z_] := Sum[ z^k / k! g1[n, k], {k, 0, 40}]
```

```
Table[ Chop@N@g1[100, j], {j, 0, 40}]
```

{1., 28.0217, 80.5038, 134.883, 162.645, 154.116, 120.57, 80.4139, 46.7678, 24.1221, 11.1796, 4.70479, 1.81335, 0.644718, 0.212737, 0.0654893, 0.0188938, 0.00512882, 0.00131459, 0.000319152, 0.0000735961, 0.0000161607, $3.38696 \times 10^{-6}$, $6.78903 \times 10^{-7}$, $1.30401 \times 10^{-7}$, $2.40431 \times 10^{-8}$, $4.26221 \times 10^{-9}$, $7.27547 \times 10^{-10}$, $1.19748 \times 10^{-10}$, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

```
Chop@N@gz[100, -3 + I]
```

4.28551 + 2.99377 i

```
N@LaguerreL[-(-3 + I), Log[100]]
```

4.28551 + 2.99377 i

```
N@Integrate[ LaguerreL[-3, Log[100 / x]], {x, 1, 100}]
```

4209.02

```
N@Integrate[
  D[Gamma[2, 0, -Log[j]] / Gamma[2], j] Gamma[2, 0, -Log[100 / j]] / Gamma[2], {j, 1, 100}]
```

928.88

```
Chop@Gamma[4, 0, -Log[100.]] / Gamma[4]
```

928.88

```
N@Integrate[
  D[Gamma[2, 0, -Log[j]] / Gamma[2], j] Gamma[3, 0, -Log[100 / j]] / Gamma[3], {j, 1, 100}]
```

-945.128

```
N@Integrate[
  D[Gamma[3, 0, -Log[j]] / Gamma[3], j] Gamma[2, 0, -Log[100 / j]] / Gamma[2], {j, 1, 100}]
```

-945.128

```
N@Integrate[ D[(-1)^1 Gamma[1, 0, -Log[j]] / Gamma[1], j]
   (-1)^4 Gamma[4, 0, -Log[100 / j]] / Gamma[4], {j, 1, 100}]
```

945.128

```
Chop@((-1)^5 Gamma[5, 0, -Log[100.]] / Gamma[5])
```

945.128

```
ff[n_, a_, b_] := Chop@N@Integrate[ D[(-1)^a Gamma[a, 0, -Log[j]] / Gamma[a], j]
     (-1)^b Gamma[b, 0, -Log[n / j]] / Gamma[b], {j, 1, n}]
f1[n_, a_, b_] := Chop@N@Integrate[ D[(-1)^a Gamma[a, 0, -Log[j]] / Gamma[a], j]
     D[(-1)^b Gamma[b, 0, -Log[k]] / Gamma[b], k], {j, 1, n}, {k, 1, n / j}]
f2[n_, a_, b_] := Chop@N@((-1)^(a + b) Gamma[a + b, 0, -Log[n]] / Gamma[a + b])
```

```
ff[30, 1, 2 + .3 I]
```

15.4926 + 0.524826 i

```
f2[30, 1, 2 + .3 I]
```

15.4926 + 0.524826 i

```
f1[30, 1, 2 + .3 I]
```

15.4926 + 0.524826 i

```
N@Integrate[ D[LaguerreL[-3, Log[j]], j] LaguerreL[-1, Log[100 / j]], {j, 1, 100}]
```

6190.43

```
N@LaguerreL[-4, Log[100]]
```

6290.43

```
D[ Gamma[1, 0, -Log[x]], x]
```

-1

```
-b Integrate[ Gamma[ b, 0, -Log[n / x]], {x, 1, n}]
```

$$\text{ConditionalExpression}\left[-b\left(-\text{Gamma}[b]+\text{Gamma}[b,-\text{Log}[n]]+\frac{n\,(-\text{Log}[n])^{b}}{b}\right),\ \text{Re}[b]>0\ \&\&\ (n\notin\text{Reals}\ ||\ \text{Re}[n]\geq 0)\right]$$

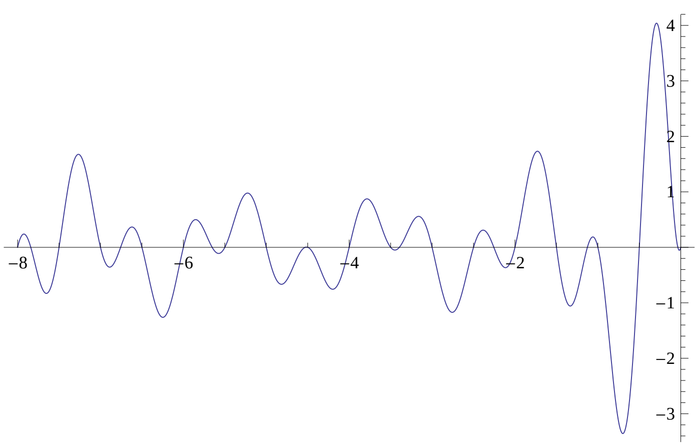
```
E^(-Log[n])
```

$$\frac{1}{n}$$

```
Limit[ Gamma[3, 0, -Log[n]] / Gamma[3], n → Infinity]
```

$-\infty$

```
gg[n_, z_] := (-1)^z Gamma[z, 0, -Log[n]] / Gamma[z]
Plot[Im[gg[100, n]], {n, -8, 0}]
```




```
gf[n_, a_, b_] := Chop@N@Integrate[ D[(-1)^(a + 1) (Gamma[a, -Log[j]] / Gamma[a]), j]
      D[(-1)^(b + 1) (Gamma[b, -Log[k]] / Gamma[b]), k], {j, 1, n}, {k, 1, n / j}]
gf2[n_, a_, b_] := N[(-1)^(a + b) / (Gamma[a] Gamma[b])
   Integrate[ D[ (Gamma[a, -Log[j]]), j] D[ (Gamma[b, -Log[k]]), k], {j, 1, n}, {k, 1, n / j}]]
g2[n_, a_, b_] := Chop@
  N@((-1)^(a + b) + (-1)^(a + b + 1) (Gamma[a + b, -Log[n]] / Gamma[a + b]))

{gf[100, 2.2, 3], gf2[100, 2.2, 3], g2[100, 2.2, 3]}
```

{285.438 + 878.488 i, 285.438 + 878.488 i, 285.438 + 878.488 i}

```
hf2[n_, a_, b_] := N[(1 / ((-1) ^ (a + b + 1) / Gamma[a + b]))
   (- (-1) ^ (a + b) + (-1) ^ (a + b) / (Gamma[a] Gamma[b]) Integrate[
      D[ (Gamma[a, -Log[j]]), j] D[ (Gamma[b, -Log[k]]), k], {j, 1, n}, {k, 1, n / j}])]
hf3[n_, a_, b_] := N[Gamma[a + b] - Gamma[a + b] / (Gamma[a] Gamma[b]) Integrate[
     D[ (Gamma[a, -Log[j]]), j] D[ (Gamma[b, -Log[k]]), k], {j, 1, n}, {k, 1, n / j}]]
h2[n_, a_, b_] := N@((Gamma[a + b, -Log[n]]))
```

```
{hf2[100, 2.2, 3], hf3[100, 2.2, 3], h2[100, 2.2, 3]}
```

{24 377.8 + 17 687.8 ⅈ, 24 377.8 + 17 687.8 ⅈ, 24 377.8 + 17 687.8 ⅈ}

```
Expand[ (1 / ((-1) ^ (a + b + 1) / Gamma[a + b]))
  (- (-1) ^ (a + b) + (-1) ^ (a + b) / (Gamma[a] Gamma[b]) VVV)]
```

$$\text{Gamma}[a+b]-\frac{\text{VVV Gamma}[a+b]}{\text{Gamma}[a]\ \text{Gamma}[b]}$$

```
gg[n_, z_] := (-1) ^ z Gamma[z, 0, -Log[n]] / Gamma[z]
hh[n_, z_] := (Log[n]) ^ (z - 1) / Gamma[z]
```

```
D[gg[n, z], n] /. z → 2
```

Log[n]

```
Expand@Integrate[ Log[a] Log[b], {a, 1, n}, {b, 1, n / a}]
```

$$\text{ConditionalExpression}\left[1-n+n\,\text{Log}[n]-\frac{1}{2}\,n\,\text{Log}[n]^2+\frac{1}{6}\,n\,\text{Log}[n]^3,\ \text{Re}[n]\geq 0\ ||\ n\notin\text{Reals}\right]$$

```
FullSimplify@Integrate[ hh[a, z], {a, 1, n}]
```

$$\text{ConditionalExpression}\left[\frac{(\text{Gamma}[z]-\text{Gamma}[z,-\text{Log}[n]])\ (-\text{Log}[n])^{-z}\ \text{Log}[n]^z}{\text{Gamma}[z]},\ \text{Re}[z]>0\right]$$

$$\frac{(\text{Gamma}[z]-\text{Gamma}[z,-\text{Log}[n]])\ (-\text{Log}[n])^{-z}\ \text{Log}[n]^z}{\text{Gamma}[z]}$$

$$\frac{(\text{Gamma}[z]-\text{Gamma}[z,-\text{Log}[n]])\ (-\text{Log}[n])^{-z}\ \text{Log}[n]^z}{\text{Gamma}[z]}$$

```
N@gg[100, 3.25]
```

$-7.95808\times 10^{-13}+772.957\,ⅈ$

```
tt[ n_, z_] := N@Sum[(-1) ^ (k) Binomial[z, k] LaguerreL[-k, Log[n]], {k, 0, 40}]
```

```
(-1) ^ (5) Chop@N@Gamma[5, 0, -Log[10]] / Gamma[5]
```

3.84941

```
tt[10, 5]
```

-3.84941

```
N@D[LaguerreL[-3, Log[n]], {n, 2}] /. n → 100
```

0.0760517

```
N@Sum[ D[ Binomial[ 3, k] (((-1) ^k Gamma[k, 0, -Log[n]] / Gamma[k])), {n, 2}], {k, 1, 4}] /.
 n → 100
```

0.0760517

```
FullSimplify@Sum[ Binomial[ a, k] Binomial[b, j - k], {k, 0, j}]
```

```
Binomial[a + b, j]
```

```
Binomial[a + b, 5]
```

$$\frac{1}{120}(-4+a+b)(-3+a+b)(-2+a+b)(-1+a+b)(a+b)$$

```
N@Integrate[
  D[ LaguerreL[-1, Log[a]], a] D[ LaguerreL[-4, Log[b]], b], {a, 1, 10}, {b, 1, 10 / a}]
```

```
166.307
```

```
LaguerreL[-5, Log[10.]]
```

```
354.26
```

```
aa[x_, z_] := x^z
```

```
D[aa[x, 1], x]
```

```
1
```

```
Integrate[ D[aa[x, 1.5 + I], x] D[aa[y, 1], y], {x, 0, 10}, {y, 0, 10}]
```

```
-211.304 + 235.267 i
```

```
aa[10, 3.5 + I]
```

```
-2113.04 + 2352.67 i
```

```
Integrate[ D[x^(1.5 + I), x] D[y^2, y], {x, 0, 10}, {y, 0, 10}]
```

```
-2113.04 + 2352.67 i
```

```
aff[n_, a_] := (-1)^a Gamma[a, 0, -Log[n]] / Gamma[a]
aff[n_, 0] := Limit[ (-1)^a Gamma[a, 0, -Log[n]] / Gamma[a], a → 0]
af[n_, s_, a_] := (-1)^a Gamma[a, 0, (s - 1) Log[n]] / Gamma[a]
a1[n_, s_, a_, b_] :=
 Chop@N@Integrate[ D[af[j, s, a], j] D[af[k, s, b], k], {j, 1, n}, {k, 1, n / j}]
a2[n_, s_, a_, b_] := Chop@N@(af[n, s, a + b])
aaa[n_, z_] := (-1)^z Sum[ (-1)^k Binomial[z, k] aff[n, k], {k, 0, 120}]
aaa2[n_, z_] := (-1)^z Sum[ (-1)^k Binomial[z, k] aff[n, k], {k, 0, Infinity}]
aab[n_, z_] := Chop@N[ Sum[ Binomial[z, k] aff[n, k], {k, 0, 120}]]
```

```
a1[100, 0, .5, 1.5]
```

```
361.517
```

```
a2[100, 0, 1, 1]
```

```
361.517
```

```
Table[ N@aaa[100, k] / k, {k, 1, 20}]
```

{98., 82.2585 - 2.20753 × 10^-14 i, -29.8962 - 1.29884 × 10^-14 i,
-23.117 + 1.99666 × 10^-14 i, 33.6365 - 2.93797 × 10^-14 i, -17.3605 + 9.18606 × 10^-14 i,
-3.59917 - 2.49128 × 10^-13 i, 16.4263 + 5.31436 × 10^-13 i, -18.1963 - 9.62693 × 10^-13 i,
11.9096 + 1.5656 × 10^-12 i, -2.48172 - 2.37613 × 10^-12 i, -5.8531 + 3.51323 × 10^-12 i,
10.655 - 5.41806 × 10^-12 i, -11.3375 + 9.48482 × 10^-12 i, 8.66831 - 1.94566 × 10^-11 i,
-4.10328 + 4.41531 × 10^-11 i, -0.804403 - 1.02326 × 10^-10 i,
4.80809 + 2.30686 × 10^-10 i, -7.16442 - 4.96401 × 10^-10 i, 7.65696 + 1.01559 × 10^-9 i}

```
N@LaguerreL[-10, Log[100]]
```

780182.

```
aaa[100, 0] + 4 aaa[100, 1] + 6 aaa[100, 2] + 4 aaa[100, 3] + aaa[100, 4]
```

928.88

```
N@aff[100, 4]
```

928.88 - 3.40898 × 10^-13 i

```
ff5[ n_, z_] :=
 Integrate[Sum[ Binomial[z, k] (-1) ^ (k + 1) / ((k - 1) !) t^ (k - 1) E^ (-t), {k, 0, Infinity}],
  {t, -Log[n], 0}]
```

```
ff5a[ n_, z_] :=
 Integrate[Sum[(-1) ^k Binomial[z, k] (-1) ^ (k + 1) / ((k - 1) !) t^ (k - 1) E^ (-t),
   {k, 0, Infinity}], {t, -Log[n], 0}]
```

```
LaguerreL[-2, Log[n]] /. n → 100
```

LaguerreL[-2, Log[100]]

```
N[((aff[100, bb]) - (aff[100, -bb])) / (2 bb) /. bb → .00001]
```

30.1261 + 6.28319 i

```
Gamma[0, -Log[100.]]
```

-30.1261 - 3.14159 i

```
Residue[ aff[100., z] / z^2, {z, 0}]
```

Residue$\left[\dfrac{(-1)^z \text{ Gamma}[z, 0, -4.60517]}{z^2 \text{ Gamma}[z]}, \{z, 0\}\right]$

```
N[((LaguerreL[-3, Log[100 + bb]]) - (LaguerreL[-3, Log[100 - bb]])) / (2 bb) /. bb → .000001]
```

27.4193

```
D[ LaguerreL[-3, Log[n]], n] /. n → 100.
```

27.4193

```
N@Integrate[ D[LaguerreL[-1, Log[j]], j] D[LaguerreL[-3, Log[k]], k],
   {j, 1, 10}, {k, 1, 10 / j}] + LaguerreL[-1, Log[10]] + LaguerreL[-3, Log[10]] - 1
```

178.953

```
N@Integrate[ D[LaguerreL[-2, Log[j]], j] D[LaguerreL[-2, Log[k]], k],
   {j, 1, 10}, {k, 1, 10 / j}] + LaguerreL[-2, Log[10]] + LaguerreL[-2, Log[10]] - 1
```

178.953

```
N@LaguerreL[-4, Log[10]]
```

178.953

```
N@Integrate[ D[LaguerreL[-2, Log[j]], j] D[LaguerreL[-3, Log[k]], k],
   {j, 1, 10}, {k, 1, 10 / j}] + LaguerreL[-2, Log[10]] + LaguerreL[-3, Log[10]] - 1
```

354.26

```
N@Integrate[ D[LaguerreL[-.75, Log[j]], j] D[LaguerreL[4.5, Log[k]], k],
   {j, 1, 10}, {k, 1, 10 / j}] + LaguerreL[-.75, Log[10]] + LaguerreL[4.5, Log[10]] - 1
```

0.591448


```
N@Integrate[ D[LaguerreL[-.75, Log[j]], j] D[LaguerreL[4.5, Log[k]], k],
   {j, 1, 10}, {k, 1, 10 / j}] + LaguerreL[-.75, Log[10]] + LaguerreL[4.5, Log[10]] - 1
```

0.591448


```
N@LaguerreL[4.5 - .75, Log[10]]
```

0.591448

```
N[D[LaguerreL[-1, Log[n]], n] /. n → 100]
```

1.

```
Gamma[3, 0, -Log[10]] / Gamma[3] -
 3 Gamma[2, 0, -Log[10]] / Gamma[2] + 3 Gamma[1, 0, -Log[10]] / Gamma[1] - 1
```

$-28 - 3\,\text{Gamma}[2, 0, -\text{Log}[10]] + \frac{1}{2}\,\text{Gamma}[3, 0, -\text{Log}[10]]$

```
N@Gamma[3, 0, -Log[10]]
```

$-24.9673 + 9.17283 \times 10^{-15}\,i$

```
N@-2 (10 Log[10]^2 / 2 - 10 Log[10] + 10 - 1)
```

-24.9673

```
N@Gamma[2, 0, -Log[10]]
```

$14.0259 - 1.59521 \times 10^{-15}\,i$

```
N@(10 Log[10] - 10 + 1)
```

14.0259

```
N@Gamma[1, 0, -Log[10]]
```

-9.

```
Sum[ Binomial[z, k] (-1)^(k + 1) (-Log[ x] )^(k - 1) / Gamma[k], {k, 0, Infinity}]
```

z Hypergeometric1F1[1 - z, 2, -Log[x]]

```
N[z Hypergeometric1F1[1 - z, 2, -Log[x]] /. {x → 100, z → 3}]
```

27.4193

```
D[ LaguerreL[-3, Log[n]], n] /. n → 100.
```

27.4193

```
Integrate[ (2 Hypergeometric1F1[1 - 2, 2, -Log[x]])
  (3 Hypergeometric1F1[1 - 3, 2, -Log[y]]), {x, 1, n}, {y, 1, n / x}]
```

ConditionalExpression$\left[\frac{1}{2}\left(2-2\,n+\frac{1}{12}\,n\,\mathrm{Log}[n]\,(24+\mathrm{Log}[n]\,(6+\mathrm{Log}[n])\,(10+\mathrm{Log}[n]))\right),\ \mathrm{Re}[n]\geq 0\,||\,n\notin\mathrm{Reals}\right]$

```
Integrate[ (4 Hypergeometric1F1[1 - 4, 2, -Log[x]])
  (3 Hypergeometric1F1[1 - 3, 2, -Log[y]]), {x, 1, n}, {y, 1, n / x}]
```

ConditionalExpression$\left[\frac{1}{12}\left(-12\,(-1+n)+\frac{1}{60}\,n\,\mathrm{Log}[n]\,(720+\mathrm{Log}[n]\,(6+\mathrm{Log}[n])\,(660+\mathrm{Log}[n]\,(270+\mathrm{Log}[n]\,(30+\mathrm{Log}[n]))))\right),\ \mathrm{Re}[n]\geq 0\,||\,n\notin\mathrm{Reals}\right]$

```
Integrate[ (5 Hypergeometric1F1[1 - 5, 2, -Log[x]])
  (2 Hypergeometric1F1[1 - 2, 2, -Log[y]]), {x, 1, n}, {y, 1, n / x}]
```

ConditionalExpression$\left[\frac{1}{24}\left(-24\,(-1+n)+\frac{1}{30}\,n\,\mathrm{Log}[n]\,(720+\mathrm{Log}[n]\,(3240+\mathrm{Log}[n]\,(1920+\mathrm{Log}[n]\,(420+\mathrm{Log}[n]\,(36+\mathrm{Log}[n])))))\right),\ \mathrm{Re}[n]\geq 0\,||\,n\notin\mathrm{Reals}\right]$

```
ar[n_, a_] := (-1) ^a Gamma[a, 0, -Log[n]] / Gamma[a]
ar2[n_, a_] := Gamma[a, 0, -Log[n]] / Gamma[a]
ar3[n_, a_] := Gamma[a, 0, Log[n]] / Gamma[a]
ar4[x_, z_] := Gamma[z, 0, (s - 1) Log[x]] / Gamma[z]
ar5[x_, z_] := (-1) ^z Gamma[z, 0, (s - 1) Log[x]] / Gamma[z]
```

```
FullSimplify[D[ar4[x, z], x]]
```

$$\frac{x^{-s}\,((-1+s)\,\mathrm{Log}[x])^{z}}{\mathrm{Gamma}[z]\,\mathrm{Log}[x]}$$

```
FullSimplify[D[ar5[x, z], x]]
```

$$\frac{(-1)^{z}\,x^{-s}\,((-1+s)\,\mathrm{Log}[x])^{z}}{\mathrm{Gamma}[z]\,\mathrm{Log}[x]}$$

```
D[ar[n, a], n] /. a → 1
```

1

```
FullSimplify[D[Gamma[ a, 0, -Log[x]], x]]
```

$$\frac{(-\mathrm{Log}[x])^{a}}{\mathrm{Log}[x]}$$

```
ff5[n_, z_] :=
 Integrate[Sum[Binomial[z, k] (-1)^(k+1) / ((k-1)!) t^(k-1) E^(-t), {k, 0, Infinity}],
  {t, -Log[n], 0}]
ff5b[n_, z_] := Integrate[Sum[z^k / k! (-1)^(k+1) / ((k-1)!) t^(k-1) E^(-t),
   {k, 0, Infinity}], {t, -Log[n], 0}]
ff5c[n_, z_] := Sum[z^k / k! (-1)^k (1 - Gamma[k, -Log[n]] / Gamma[k]), {k, 0, Infinity}]
ff5d[n_, z_] := Sum[(-1)^k z^(2 k) / (2 k)!
   (-1)^(2 k) (1 - Gamma[2 k, -Log[n]] / Gamma[2 k]), {k, 0, Infinity}]
ff5e[n_, z_, t_] := Sum[(-1)^k z^(2 k) / (2 k)! (-1)^(2 k)
   (1 - Gamma[2 k, -Log[n]] / Gamma[2 k]), {k, 0, t}]
```

```
ff5b[100, 1]
```

$$\int_{-\mathrm{Log}[100]}^{0} \frac{e^{-t}\,\mathrm{BesselJ}\left[1, 2\sqrt{t}\right]}{\sqrt{t}}\,dt$$

```
ff5c[100, 1]
```

$$\sum_{k=0}^{\infty} \frac{(-1)^k \left(1 - \frac{\mathrm{Gamma}[k, -\mathrm{Log}[100]]}{\mathrm{Gamma}[k]}\right)}{k!}$$

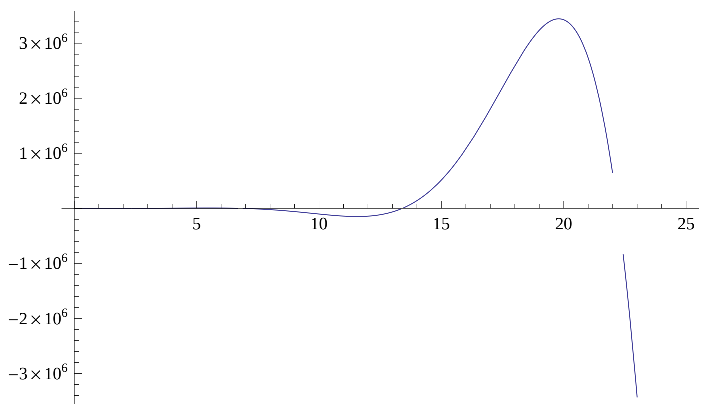
```
ff5d[100, 1]
```

$$\sum_{k=0}^{\infty} \frac{(-1)^{3k} \left(1 - \frac{\mathrm{Gamma}[2k, -\mathrm{Log}[100]]}{\mathrm{Gamma}[2k]}\right)}{(2k)!}$$

```
N@ff5e[10, 1, 30]
```

$-5.68744 + 6.38683 \times 10^{-16}\,i$

```
Plot[ ff5e[100, z, 30], {z, 0, 25}]
```




```
Integrate[ 1, {x, 1, n}]
```

$-1 + n$

```
Expand@Integrate[ 1, {x, 1, n}, {y, 1, n / x}]
```

ConditionalExpression[1 - n + n Log[n], Re[n] ≥ 0 || n ∉ Reals]

```
Expand@Integrate[1, {x, 1, n}, {y, 1, n / x}, {z, 1, n / (x y)}]
```

$$\text{ConditionalExpression}\left[-1 + n - n\,\text{Log}[n] + \frac{1}{2}\,n\,\text{Log}[n]^2,\ \text{Re}[n] \geq 0\ ||\ n \notin \text{Reals}\right]$$

```
N@(-1)^2 Gamma[2, 0, -Log[100]] / Gamma[2]
```

$361.517 - 4.41506 \times 10^{-14}\,i$

```
N[(n Log[n] - n + 1) /. n → 100]
```

361.517

```
FullSimplify@Sum[z^k / k! (-1)^k, {k, 1, Infinity}]
```

$-1 + e^{-z}$

```
FullSimplify@Sum[z^k / k! (-1)^(k+1) n, {k, 2, Infinity}]
```

$-n\,(-1 + e^{-z} + z)$

```
FullSimplify@Sum[z^k / k! (-1)^(k+0) n Log[n], {k, 3, Infinity}]
```

$$-\frac{1}{2}\,n\,\text{Log}[n]\,(2 + (-2 + z)\,z - 2\,\text{Cosh}[z] + 2\,\text{Sinh}[z])$$

```
FullSimplify@Sum[z^k / k! (-1)^(k+1) n Log[n]^2 / 2, {k, 4, Infinity}]
```

$$\frac{1}{12}\,e^{-z}\,n\,(-6 + e^{z}\,(6 - z\,(6 + (-3 + z)\,z)))\,\text{Log}[n]^2$$

```
FullSimplify@Sum[z^k / k! (-1)^(k+0) n Log[n]^3 / 6, {k, 5, Infinity}] /. z → 3
```

$$\frac{\left(24 - 33\,e^{3}\right)\,n\,\text{Log}[n]^3}{144\,e^{3}}$$

```
Table[Cos[Pi / 2 j], {j, 0, 10}]
```

{1, 0, -1, 0, 1, 0, -1, 0, 1, 0, -1}

```
Integrate[D[LaguerreL[-z, Log[x]], x], {x, 0, n}]
```

$\text{ConditionalExpression}[\text{LaguerreL}[-z, \text{Log}[n]],\ \text{Re}[z] > 0\ \&\&\ (n \notin \text{Reals}\ ||\ \text{Re}[n] \leq e)]$

```
Integrate[D[LaguerreL[-z, Log[x]], x], {x, 1, n}]
```

$\text{ConditionalExpression}[-1 + \text{Hypergeometric1F1}[z, 1, \text{Log}[n]],\ 0 \leq \text{Re}[n] \leq e\ ||\ n \notin \text{Reals}]$

```
N@Integrate[D[LaguerreL[-.75, Log[j]], j] D[LaguerreL[-4.5, Log[k]], k],
   {j, 1, 10}, {k, 1, 10 / j}] + LaguerreL[-.75, Log[10]] + LaguerreL[-4.5, Log[10]] - 1
```

415.707

```
N@Integrate[D[LaguerreL[-.75, Log[j]], j]
    Integrate[D[LaguerreL[-4.5, Log[k]], k], {k, 1, 10 / j}], {j, 1, 10}] +
  LaguerreL[-.75, Log[10]] + LaguerreL[-4.5, Log[10]] - 1
```

415.707

```
Integrate[ D[LaguerreL[z, Log[k]], k], {k, 1, n / j}]
```

$$\text{ConditionalExpression}\left[-1 + \text{LaguerreL}\left[z, \text{Log}\left[\frac{n}{j}\right]\right], \left(\frac{n}{j} \notin \text{Reals} \mid\mid \left(\text{Re}\left[\frac{n}{j}\right] \geq 0 \,\&\&\, \left(j^2 \neq j\, n \mid\mid \text{Re}\left[\frac{n}{j}\right] \leq 1\right)\right)\right) \,\&\&\, \left(\left(\text{Re}[j] \neq 0 \,\&\&\, \text{Im}[n] \neq \frac{\text{Im}[j]\, \text{Re}[n]}{\text{Re}[j]}\right) \mid\mid (\text{Re}[j] > 0 \,\&\&\, \text{Re}[n] \geq 0) \mid\mid (\text{Re}[j] < 0 \,\&\&\, \text{Re}[n] \leq 0) \mid\mid (\text{Re}[j] == 0 \,\&\&\, ((j \notin \text{Reals} \,\&\&\, \text{Re}[n] \neq 0) \mid\mid (\text{Re}[n] == 0 \,\&\&\, ((\text{Im}[j] > 0 \,\&\&\, \text{Im}[n] \geq 0) \mid\mid (\text{Im}[j] < 0 \,\&\&\, \text{Im}[n] \leq 0)))))\right)\right]$$

```
Integrate[ D[LaguerreL[z, Log[k]], k], {k, 0, n / j}]
```

$$\text{ConditionalExpression}\left[\text{LaguerreL}\left[z, \text{Log}\left[\frac{n}{j}\right]\right], \text{Re}[z] < 0\right]$$

```
Integrate[ D[LaguerreL[-2, Log[k]], k], {k, 0, 10 / j}]
```

$$\frac{10 \left(1 + \text{Log}[10] + \text{Log}\left[\frac{1}{j}\right]\right)}{j}$$

```
N@Integrate[ D[LaguerreL[-.75, Log[j]], j]
    Integrate[ D[LaguerreL[-4.5, Log[k]], k], {k, 1, 10 / j}], {j, 1, 10}] +
 LaguerreL[-.75, Log[10]] + LaguerreL[-4.5, Log[10]] - 1
```

415.707

```
N@Integrate[ D[LaguerreL[-.75, (-2 + I) Log[j]], j]
    (-1 + LaguerreL[-4.5, (-2 + I) Log[10 / j]]), {j, 1, 10}] +
 LaguerreL[-.75, (-2 + I) Log[10]] + LaguerreL[-4.5, (-2 + I) Log[10]] - 1
```

-0.0923407 - 0.0296361 ⅈ

```
LaguerreL[-.75 - 4.5, (-2 + I) Log[10]]
```

-0.0923407 - 0.0296361 ⅈ

```
s = .5 + I; t = .3 + 2 I; x = 20; l = 2 - I;
N[Gamma[s + t, 0, l Log[x]]]
N[Gamma[s + t] / (Gamma[s] Gamma[t])
  Integrate[ D[Gamma[s, 0, l Log[y]], y] (Gamma[t, 0, l Log[x / y]]), {y, 1, x}]]
```

0.0277866 + 0.0198951 ⅈ

0.0277866 + 0.0198951 ⅈ

```
s = .5 + I; t = .3 + 2 I; x = 20; l = 2 - I;
N[Gamma[s + t, l Log[x]]]
N[Gamma[s + t] - Gamma[s + t] / (Gamma[s] Gamma[t])
   Integrate[ D[Gamma[s, l Log[y]], y] (D[Gamma[t, l Log[z]], z]), {y, 1, x}, {z, 1, x / y}]]
```

-0.00524201 + 0.00178877 ⅈ

-0.00524201 + 0.00178877 ⅈ

```
Clear[x, a, b, y, n, z, u, t, l]
FullSimplify[Integrate[ D[Gamma[t, l Log[z]], z], {z, 1, n / y}]]
FullSimplify[Integrate[ D[Gamma[t, 0, l Log[z]], z], {z, 1, n / y}]]
```

ConditionalExpression$\left[-\text{Gamma}[t]+\text{Gamma}\left[t,\ l\,\text{Log}\left[\frac{n}{y}\right]\right],\ \text{Re}[t]>0\ \&\&\ \text{Log}\left[\frac{n}{y}\right]>0\right]$

ConditionalExpression$\left[\text{Gamma}[t]-\text{Gamma}\left[t,\ l\,\text{Log}\left[\frac{n}{y}\right]\right],\ \text{Re}[t]>0\ \&\&\ \text{Log}\left[\frac{n}{y}\right]>0\right]$

```
Clear[x, a, b, y, n, z, u, t]
Integrate[ D[y^t, y] D[z^u, z], {y, 0, x}, {z, 0, x}]
```

ConditionalExpression$\left[x^{t+u},\ \text{Re}[t]>0\right]$

**N**$\left[-\text{Gamma}[t]+\text{Gamma}\left[t,\ l\,\text{Log}\left[\frac{n}{y}\right]\right]\ /.\ \{t\to 3,\ l\to -1,\ n\to 100,\ y\to 1\}\right]$

$1397.73-3.42834\times 10^{-13}\,i$

**N**$\left[\text{Gamma}[t]-\text{Gamma}\left[t,\ l\,\text{Log}\left[\frac{n}{y}\right]\right]\ /.\ \{t\to 3,\ l\to -1,\ n\to 100,\ y\to 1\}\right]$

$-1397.73+3.42834\times 10^{-13}\,i$

```
Clear[x, b, n]
-b Integrate[ Gamma[ b, 0, -Log[n] + Log[x]], {x, 1, n}]
```

ConditionalExpression$\left[-b\left(-\text{Gamma}[b]+\text{Gamma}[b,\ -\text{Log}[n]]+\frac{n\,(-\text{Log}[n])^{b}}{b}\right),\right.$
$\left.\text{Re}[b]>-1\ \&\&\ (n\notin\text{Reals}\ ||\ \text{Re}[n]\geq 0)\right]$

```
Chop@N@Gamma[6, 0, -Log[20]]
```

1665.96

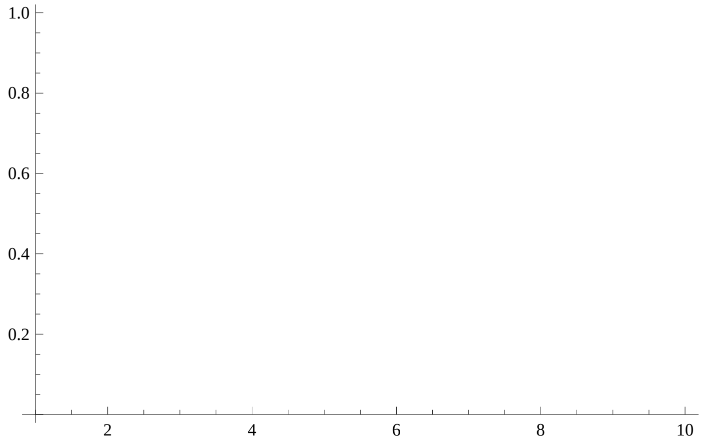
```
E^-(-Log[n])
```

n

```
Limit[D[Gamma[k, 0, -Log[n]], n], k → 0]
```

$\frac{1}{\text{Log}[n]}$

```
Plot[ Gamma[0, 0, -Log[n]], {n, 1, 10}]
```




```
N@Gamma[0, -Log[100]]

-30.1261 - 3.14159 i

Limit[(LaguerreL[s, -Log[n]] - 1) / s, s → 0]

LaguerreL^(1,0)[0, -Log[n]]

D[LaguerreL^(1,0)[0, -Log[n]], n]
```

$$-\frac{\text{LaguerreL}^{(1,1)}[0, -\text{Log}[n]]}{n}$$

```
D[ Gamma[k, 0, -Log[n]], n] /. k → 0
```

$$\frac{1}{\text{Log}[n]}$$

```
Sum[ Binomial[ z, k] (-1)^(k+1) (-Log[n])^(k-1) / Gamma[k], {k, 0, Infinity}]

z Hypergeometric1F1[1 - z, 2, -Log[n]]

N[D[LaguerreL[-z, Log[n]], n] /. {n → 10, z → -3}]

0.125681

N[D[LaguerreL[-z, Log[n]], z] /. {n → 100, z → 0}]

28.0217

D[ Gamma[k, -Log[10]], k]

Gamma[k, -Log[10]] (i π + Log[Log[10]]) + MeijerG[{{}, {1, 1}}, {{0, 0, k}, {}}, -Log[10]]

D[LaguerreL[-z, Log[n]], n]
```

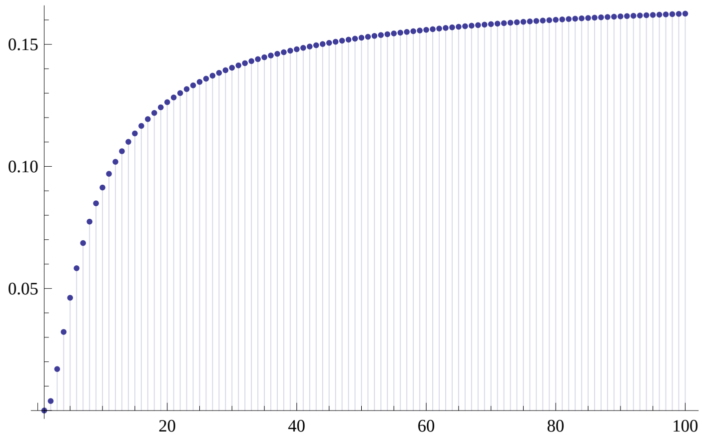
$$-\frac{\text{LaguerreL}[-1 - z, 1, \text{Log}[n]]}{n}$$

```
Integrate[ D[ LaguerreL[-z, Log[y]], y], {y, 0, x}]

ConditionalExpression[LaguerreL[-z, Log[x]], Re[z] > 0 && (x ∉ Reals || Re[x] ≤ e)]
```

```
Clear[ck]
ck[n_, s_, k_] := ck[n, s, k] = Sum[ j^-s ck[ n, s, k - 1], {j, 1, n}]
ck[n_, s_, 0] := 1
cz[n_, s_, z_] := Sum[ Binomial[ z, k] ck[n, s, k], {k, 0, Infinity}]
```

```
DiscretePlot[ ck[n, 2, 4], {n, 1, 100}]
```




```
ck[20, 0, 5]
```

3 200 000

```
Sum[ (ck[j, 0, 2] - ck[j - 1, 0, 2]) ck[20, 0, 3], {j, 1, 20}]
```

3 200 000

```
Sum[ (j k l) ^1, {j, 2, n}, {k, 2, n}, {l, 2, n}]
```

$$\frac{1}{8}\left(-2+n+n^2\right)^3$$

```
Integrate[ j^-s, {j, 1, x}]
```

$$\text{ConditionalExpression}\left[\frac{x^{-s}\left(-x+x^s\right)}{-1+s}, \text{Re}[x] \geq 0 \,||\, x \notin \text{Reals}\right]$$

```
Integrate[ (j k) ^-s, {j, 1, x}, {k, 1, x}]
```

$$\text{ConditionalExpression}\left[\frac{x^{-2s}\left(-x+x^s\right)^2}{(-1+s)^2}, \text{Re}[x] \geq 0 \,||\, x \notin \text{Reals}\right]$$

```
FullSimplify@Integrate[ (j k l) ^-s, {j, 1, x}, {k, 1, x}, {l, 1, x}]
```

$$\text{ConditionalExpression}\left[\frac{x^{-3s}\left(-x+x^s\right)^3}{(-1+s)^3}, \text{Re}[x] \geq 0 \,||\, x \notin \text{Reals}\right]$$

```
FullSimplify@Integrate[ (j k l m) ^-s, {j, 1, x}, {k, 1, x}, {l, 1, x}, {m, 1, x}]
```

$$\text{ConditionalExpression}\left[\frac{x^{-4s}\left(-x+x^s\right)^4}{(-1+s)^4}, \text{Re}[x] \geq 0 \,||\, x \notin \text{Reals}\right]$$

$$\texttt{FullSimplify@Sum}\left[\texttt{Binomial[z, k]}\,\frac{x^{-k s}\left(-x+x^s\right)^k}{(-1+s)^k}, \{k, 0, \texttt{Infinity}\}\right]$$

$$\left(\frac{x^{-s}\left(-x+s\,x^s\right)}{-1+s}\right)^z$$

$$\text{Expand}\left[\left(\frac{x^{-s}\,(-x+s\,x^{s})}{-1+s}\right)^{z}\right]$$

$$\left(\frac{x^{-s}\,(-x+s\,x^{s})}{-1+s}\right)^{z}$$

$$\text{Expand}[x^{-s}\,(-x+s\,x^{s})]$$

$$s-x^{1-s}$$

$$\text{FullSimplify}\left[\left(s-x^{1-s}\right)/(-1+s)\right]$$

$$\frac{s-x^{1-s}}{-1+s}$$

```
6^4

Clear[ee]
ee[n_, 1] := Sum[ PrimePi[ n^(1 / k)] / k, {k, 1, Log2@n}]
ee[n_, k_] := ee[n, k] = If[ n <= k, 0, ee[n, k - 1] - ee[n - 1, k - 1]]
ee[0, 1] := 0
ee[0, k_] := 0
Table[ ee[n, k], {n, 1, 12}, {k, 1, 12}] // TableForm
```

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\frac{5}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\frac{7}{2}$ | 1 | $\frac{1}{2}$ | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\frac{7}{2}$ | 0 | $-1$ | $-\frac{3}{2}$ | $-\frac{5}{2}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\frac{9}{2}$ | 1 | 1 | 2 | $\frac{7}{2}$ | 6 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\frac{29}{6}$ | $\frac{1}{3}$ | $-\frac{2}{3}$ | $-\frac{5}{3}$ | $-\frac{11}{3}$ | $-\frac{43}{6}$ | $-\frac{79}{6}$ | 0 | 0 | 0 | 0 | 0 |
| $\frac{16}{3}$ | $\frac{1}{2}$ | $\frac{1}{6}$ | $\frac{5}{6}$ | $\frac{5}{2}$ | $\frac{37}{6}$ | $\frac{40}{3}$ | $\frac{53}{2}$ | 0 | 0 | 0 | 0 |
| $\frac{16}{3}$ | 0 | $-\frac{1}{2}$ | $-\frac{2}{3}$ | $-\frac{3}{2}$ | $-4$ | $-\frac{61}{6}$ | $-\frac{47}{2}$ | $-50$ | 0 | 0 | 0 |
| $\frac{19}{3}$ | 1 | 1 | $\frac{3}{2}$ | $\frac{13}{6}$ | $\frac{11}{3}$ | $\frac{23}{3}$ | $\frac{107}{6}$ | $\frac{124}{3}$ | $\frac{274}{3}$ | 0 | 0 |
| $\frac{19}{3}$ | 0 | $-1$ | $-2$ | $-\frac{7}{2}$ | $-\frac{17}{3}$ | $-\frac{28}{3}$ | $-17$ | $-\frac{209}{6}$ | $-\frac{457}{6}$ | $-\frac{335}{2}$ | 0 |

```
Table[ ee[n, n - 1], {n, 2, 12}]
```

$$\left\{1, 1, -\frac{1}{2}, 1, -\frac{5}{2}, 6, -\frac{79}{6}, \frac{53}{2}, -50, \frac{274}{3}, -\frac{335}{2}\right\}$$

```
9 / 2 + 1 / 3
```

$$\frac{29}{6}$$

```
29 / 6 + 1 / 2
```

$$\frac{16}{3}$$

```
1 / 6 - (-2 / 3)
```

$$\frac{5}{6}$$

```
-1 / 2 - 1 / 6
```

$-\frac{2}{3}$

```
pp[x_, s_, z_] := ((s - x^(1-s))/(-1 + s))^z
```

```
pp[10, 0, -2]
```

$\frac{1}{100}$

```
D[pp[x, s, z], z] /. {z → 0, s → 0}
```

```
Log[x]
```

$\frac{s - x^{1-s}}{-1 + s}$

$\frac{s - x^{1-s}}{-1 + s}$ /. {x → 3, s → 2}

$\frac{5}{3}$

$\frac{-s + x^{1-s}}{1 - s}$

$\frac{-s + x^{1-s}}{1 - s}$

```
N[Integrate[ D[ LaguerreL[-2, Log[j]], j] D[LaguerreL[-3, Log[k]], k],
   {j, 1, 20}, {k, 1, 20 / j}] + LaguerreL[-2, Log[20]] + LaguerreL[-3, Log[20]] - 1]
```

```
1223.71
```

```
N[LaguerreL[-5, Log@20]]
```

```
1223.71
```

```
N[Integrate[ D[ LaguerreL[-2, Log[j]], j] D[LaguerreL[-3, Log[k]], k],
  {j, .125, 20}, {k, .125, 20 / j}]]
```

```
1608.3
```

```
Integrate[ D[ LaguerreL[-3 / 2, Log[j]], j], {j, 1, 20}]
```

$-1 + \text{LaguerreL}\left[-\frac{3}{2}, \text{Log}[20]\right]$