```
ts[n_, s_, x_] :=
 Sum[ j^-s, {j, 1, n}] + n^x (s - 1 + x) / (s - 1) (Zeta[s + x] - Sum[ j^- (s + x), {j, 1, n}])
ts2[n_, s_, x_] := {Sum[ j^-s, {j, 1, n}],
  n^x (s - 1 + x) / (s - 1) (Zeta[s + x] - Sum[ j^- (s + x), {j, 1, n}])}
ts3[n_, s_, x_] := {Sum[ j^-s, {j, 1, n}], n^x (s - 1 + x) / (s - 1),
  (Zeta[s + x] - Sum[ j^- (s + x), {j, 1, n}])}
```

```
ts2[1 000 000, .5, .500000001]
```

{1998.54, -2000.}

```
Zeta[.5]
```

-1.46035

```
ts[1 000 000, .5, .500000001]
```

-1.45963

```
Limit[ (s - 1) Sum[j^-s, {j, 1, n}], s → 1]
```

0

```
Limit[ (s - 1) Zeta[s], s → 1]
```

1

```
sr[n_, x_] := 1 - n^(x - 1) (1 - x) ( Sum[ 1 / j^x, {j, 1, n}])
```

```
sr[100 000, N@ZetaZero[1]]
```

$-2.49833 \times 10^{-6} + 0.0000706736\, i$

```
sr[10 000 000, N@ZetaZero[2] + 3 I]
```

$0.00191011 - 0.00812102\, i$

```
Table[ D[- (j + x n) ^-t f[j], {x, k}], {k, 0, 7}] // TableForm
```

$-(j+n x)^{-t} f[j]$
$n\, t\, (j+n x)^{-1-t} f[j]$
$n^2 (-1-t)\, t\, (j+n x)^{-2-t} f[j]$
$n^3 (-2-t) (-1-t)\, t\, (j+n x)^{-3-t} f[j]$
$n^4 (-3-t) (-2-t) (-1-t)\, t\, (j+n x)^{-4-t} f[j]$
$n^5 (-4-t) (-3-t) (-2-t) (-1-t)\, t\, (j+n x)^{-5-t} f[j]$
$n^6 (-5-t) (-4-t) (-3-t) (-2-t) (-1-t)\, t\, (j+n x)^{-6-t} f[j]$
$n^7 (-6-t) (-5-t) (-4-t) (-3-t) (-2-t) (-1-t)\, t\, (j+n x)^{-7-t} f[j]$

```
Table[-n^k Product[ -t - j, {j, 0, k - 1}] (j + n x) ^ (-t - k) f[j], {k, 0, 7}] // TableForm
```

$-(j+n x)^{-t} f[j]$
$n\, t\, (j+n x)^{-1-t} f[j]$
$n^2 (-1-t)\, t\, (j+n x)^{-2-t} f[j]$
$n^3 (-2-t) (-1-t)\, t\, (j+n x)^{-3-t} f[j]$
$n^4 (-3-t) (-2-t) (-1-t)\, t\, (j+n x)^{-4-t} f[j]$
$n^5 (-4-t) (-3-t) (-2-t) (-1-t)\, t\, (j+n x)^{-5-t} f[j]$
$n^6 (-5-t) (-4-t) (-3-t) (-2-t) (-1-t)\, t\, (j+n x)^{-6-t} f[j]$
$n^7 (-6-t) (-5-t) (-4-t) (-3-t) (-2-t) (-1-t)\, t\, (j+n x)^{-7-t} f[j]$

```
 Product[ -t - j, {j, 0, k - 1}]
```

$(-1)^k\, t\, \text{Pochhammer}[1+t, -1+k]$

```
Table[{FullSimplify[(-1)^k t Pochhammer[1 + t, -1 + k]],
   FactorialPower[-t, k], Gamma[1 - t] / Gamma[1 - t - k]}, {k, 0, 7}] // TableForm
```

1	1	1
$-t$	$-t$	$\frac{\text{Gamma}[1-t]}{\text{Gamma}[-t]}$
$t\,(1+t)$	FactorialPower[-t, 2]	$\frac{\text{Gamma}[1-t]}{\text{Gamma}[-1-t]}$
$-t\,(1+t)\,(2+t)$	FactorialPower[-t, 3]	$\frac{\text{Gamma}[1-t]}{\text{Gamma}[-2-t]}$
$t\,(1+t)\,(2+t)\,(3+t)$	FactorialPower[-t, 4]	$\frac{\text{Gamma}[1-t]}{\text{Gamma}[-3-t]}$
$-t\,(1+t)\,(2+t)\,(3+t)\,(4+t)$	FactorialPower[-t, 5]	$\frac{\text{Gamma}[1-t]}{\text{Gamma}[-4-t]}$
$t\,(1+t)\,(2+t)\,(3+t)\,(4+t)\,(5+t)$	FactorialPower[-t, 6]	$\frac{\text{Gamma}[1-t]}{\text{Gamma}[-5-t]}$
$-t\,(1+t)\,(2+t)\,(3+t)\,(4+t)\,(5+t)\,(6+t)$	FactorialPower[-t, 7]	$\frac{\text{Gamma}[1-t]}{\text{Gamma}[-6-t]}$

```
Table[D[-x^(1 - t) f[n], {x, k}], {k, 0, 7}] // TableForm
```

$-x^{1-t}\,f[n]$

$-(1-t)\,x^{-t}\,f[n]$

$(1-t)\,t\,x^{-1-t}\,f[n]$

$(-1-t)\,(1-t)\,t\,x^{-2-t}\,f[n]$

$(-2-t)\,(-1-t)\,(1-t)\,t\,x^{-3-t}\,f[n]$

$(-3-t)\,(-2-t)\,(-1-t)\,(1-t)\,t\,x^{-4-t}\,f[n]$

$(-4-t)\,(-3-t)\,(-2-t)\,(-1-t)\,(1-t)\,t\,x^{-5-t}\,f[n]$

$(-5-t)\,(-4-t)\,(-3-t)\,(-2-t)\,(-1-t)\,(1-t)\,t\,x^{-6-t}\,f[n]$

```
Table[-Product[-t - j + 1, {j, 0, k - 1}] x^(-t - k + 1) f[n], {k, 0, 7}] // TableForm
```

$-x^{1-t}\,f[n]$

$(-1+t)\,x^{-t}\,f[n]$

$(1-t)\,t\,x^{-1-t}\,f[n]$

$(-1-t)\,(1-t)\,t\,x^{-2-t}\,f[n]$

$(-2-t)\,(-1-t)\,(1-t)\,t\,x^{-3-t}\,f[n]$

$(-3-t)\,(-2-t)\,(-1-t)\,(1-t)\,t\,x^{-4-t}\,f[n]$

$(-4-t)\,(-3-t)\,(-2-t)\,(-1-t)\,(1-t)\,t\,x^{-5-t}\,f[n]$

$(-5-t)\,(-4-t)\,(-3-t)\,(-2-t)\,(-1-t)\,(1-t)\,t\,x^{-6-t}\,f[n]$

```
Product[-t - j + 1, {j, 0, k - 1}]
```

$(-1)^k\,(-1+t)$ Pochhammer[t, -1 + k]

```
Table[{(-1)^k (-1 + t) Pochhammer[t, -1 + k], (1 - t) FactorialPower[-t, k - 1],
   (1 - t) Gamma[1 - t] / Gamma[2 - t - k]}, {k, 0, 7}] // TableForm
```

1	1	$\frac{(1-t)\,\text{Gamma}[1-t]}{\text{Gamma}[2-t]}$
$1-t$	$1-t$	$1-t$
$(-1+t)\,t$	$-(1-t)\,t$	$\frac{(1-t)\,\text{Gamma}[1-t]}{\text{Gamma}[-t]}$
$-(-1+t)\,t\,(1+t)$	$(1-t)$ FactorialPower[-t, 2]	$\frac{(1-t)\,\text{Gamma}[1-t]}{\text{Gamma}[-1-t]}$
$(-1+t)\,t\,(1+t)\,(2+t)$	$(1-t)$ FactorialPower[-t, 3]	$\frac{(1-t)\,\text{Gamma}[1-t]}{\text{Gamma}[-2-t]}$
$-(-1+t)\,t\,(1+t)\,(2+t)\,(3+t)$	$(1-t)$ FactorialPower[-t, 4]	$\frac{(1-t)\,\text{Gamma}[1-t]}{\text{Gamma}[-3-t]}$
$(-1+t)\,t\,(1+t)\,(2+t)\,(3+t)\,(4+t)$	$(1-t)$ FactorialPower[-t, 5]	$\frac{(1-t)\,\text{Gamma}[1-t]}{\text{Gamma}[-4-t]}$
$-(-1+t)\,t\,(1+t)\,(2+t)\,(3+t)\,(4+t)\,(5+t)$	$(1-t)$ FactorialPower[-t, 6]	$\frac{(1-t)\,\text{Gamma}[1-t]}{\text{Gamma}[-5-t]}$

```
d1[k_] := D[(1 - x^(1 - t)) Zeta[t], {x, k}]
d2[k_] :=
 D[Sum[ j^-t - (j + n x)^-t, {j, 1, Infinity}] - x^(1 - t) Sum[ j^-t, {j, 1, n}], {x, k}]
```

```
d1[7]
```

(-5 - t) (-4 - t) (-3 - t) (-2 - t) (-1 - t) (1 - t) t x^{-6-t} Zeta[t]

```
d2[7]
```

(-5 - t) (-4 - t) (-3 - t) (-2 - t) (-1 - t) (1 - t) t x^{-6-t} HarmonicNumber[n, t] +
 n^7 t (1 + t) (2 + t) (3 + t) (4 + t) (5 + t) (6 + t) HurwitzZeta[7 + t, 1 + n x]

```
FullSimplify[d2[7] / d1[7]]
```

$$\frac{1}{(-1+t)\,\text{Zeta}[t]}\left((-1+t)\,\text{HarmonicNumber}[n,t]+n^7\,(6+t)\,x^{6+t}\,\text{HurwitzZeta}[7+t,1+n\,x]\right)$$

```
(Gamma[1 - t] / Gamma[1 - t - k]) / ((1 - t) Gamma[1 - t] / Gamma[2 - t - k]) /. k → 7
```

$$\frac{\text{Gamma}[-5-t]}{(1-t)\,\text{Gamma}[-6-t]}$$

```
FullSimplify[(Gamma[1 - t] / Gamma[1 - t - k]) / (Gamma[1 - t] / Gamma[2 - t - k]) / (1 - t)] /. t → s
```

$$1+\frac{k}{-1+s}$$

```
FullSimplify[(Gamma[1 - t] / Gamma[1 - t - k]) / (Gamma[1 - t] / Gamma[2 - t - k])] /. t → s
```

1 - k - s

```
Expand[a^(1 - s) - (a + 1)^(1 - s)]
```

$a^{1-s} - (1+a)^{1-s}$

```
tt[x_] := x^2
```

```
Limit[FullSimplify[(tt[(n a + 1) / (n b)] - tt[n a / ( n b )]) (n b)], n → Infinity]
```

$$\frac{2\,a}{b}$$

```
N@77 / 60
```

1.28333

```
N@(tt[77 / 60 + 1 / 60] - tt[77 / 60]) 60
```

2.58333

```
D[tt[x], x] /. x → 77 / 60
```

$$\frac{77}{30}$$

```
Limit[ HarmonicNumber[ x n, s] - x^(1 - s) HarmonicNumber[ n, s], n → Infinity]
```

Limit$\left[-x^{1-s}\,\text{HarmonicNumber}[n,s]+\text{HarmonicNumber}[n\,x,s],\ n\to\infty\right]$

```
{x^(1 - s) Zeta[s, n + 1], Zeta[s, n x + 1]} /. n → 100 /. s → 1.5 /. x → 2.2
```

{0.134504, 0.134687}

```
x^(1 - s) Zeta[s, n + 1] - Zeta[s, n x + 1]
```

x^{1-s} Zeta[s, 1 + n] - Zeta[s, 1 + n x]

```
Sum[x^(1 - s) (j + n)^-s - (j + n x)^-s, {j, 1, Infinity}]
```

$-x^{-s}\left(-x\,\text{HurwitzZeta}[s, 1+n] + x^s\,\text{HurwitzZeta}[s, 1+n\,x]\right)$

```
Limit[x^(1 - s) (j + n)^-s - (j + n x)^-s, n → Infinity] /. s → I + 1 / 100
```

0

```
Limit[x (j x + n x)^-s - (j + n x)^-s, n → Infinity] /. s → 2
```

0

```
x (j x + n x)^-s - (j + n x)^-s /. s → 1 / 2 /. n → 1 000 000 000 000 000 /. j → 1200 /. x → 8
```

$$\frac{1}{5\sqrt{5\,000\,000\,000\,006}} - \frac{1}{20\sqrt{20\,000\,000\,000\,003}}$$

```
Limit[x (j x + n x)^-s, n → Infinity] /. x → 3 I /. s → .1
```

0. + 0. i