

Not really sure what I was trying to see here... I'm pretty sure it was something like the following:

As is very well-known, the value $s=1/2$ plays a really important role with the riemann zeta function. Its values at s and $1-s$ are related to each other through the reflection formula.

I think I was looking, briefly, here if there was some relationship between s and $1-s$ in the partial sum case. But I wasn't looking especially hard.

$$H_n = 1 + \sum_{j=2}^n \frac{\kappa(j)}{j} + \frac{1}{2} \sum_{j=2}^n \sum_{k=2}^{\lfloor \frac{n}{j} \rfloor} \frac{\kappa(j)}{j} \frac{\kappa(k)}{k} + \frac{1}{6} \sum_{j=2}^n \sum_{k=2}^{\lfloor \frac{n}{j} \rfloor} \sum_{l=2}^{\lfloor \frac{n}{jk} \rfloor} \frac{\kappa(j)}{j} \frac{\kappa(k)}{k} \frac{\kappa(l)}{l} + \frac{1}{24} \dots$$

$s=1$

$$H_{n,2} = 1 + \sum_{j=2}^n \frac{\kappa(j)}{j^2} + \frac{1}{2} \sum_{j=2}^n \sum_{k=2}^{\lfloor \frac{n}{j} \rfloor} \frac{\kappa(j)}{j^2} \frac{\kappa(k)}{k^2} + \frac{1}{6} \sum_{j=2}^n \sum_{k=2}^{\lfloor \frac{n}{j} \rfloor} \sum_{l=2}^{\lfloor \frac{n}{jk} \rfloor} \frac{\kappa(j)}{j^2} \frac{\kappa(k)}{k^2} \frac{\kappa(l)}{l^2} + \frac{1}{24} \dots$$

$s=2$

$$H_{\infty,2} = \frac{\pi^2}{6}$$

$$D_1(n) = 1 + \sum_{j=2}^n \kappa(j) + \frac{1}{2} \sum_{j=2}^n \sum_{k=2}^{\lfloor \frac{n}{j} \rfloor} \kappa(j) \kappa(k) + \frac{1}{6} \sum_{j=2}^n \sum_{k=2}^{\lfloor \frac{n}{j} \rfloor} \sum_{l=2}^{\lfloor \frac{n}{jk} \rfloor} \kappa(j) \kappa(k) \kappa(l) + \frac{1}{24} \dots$$

$s=0$

$$N_n^1 = 1 + \sum_{j=2}^n j \kappa(j) + \frac{1}{2} \sum_{j=2}^n \sum_{k=2}^{\lfloor \frac{n}{j} \rfloor} j \kappa(j) k \kappa(k) + \frac{1}{6} \sum_{j=2}^n \sum_{k=2}^{\lfloor \frac{n}{j} \rfloor} \sum_{l=2}^{\lfloor \frac{n}{jk} \rfloor} j \kappa(j) k \kappa(k) l \kappa(l) + \frac{1}{24} \dots$$

$s=-1$

$$N^1(n) = \frac{(n)(n+1)}{2}$$

Relationship ought to be between $(1-s)$ and s . So, between 1 and 0, and between 2 and -1. And .5 should be its own special thing.

Over in the land of zeta, dirichlet eta converges at $s \geq 0$, while zeta converges only for $s > 1$. Here, that maps to using a different alternating series sort of idea to multiply values by. Might be interesting to think about, especially at $s=1/2$.