```
FullSimplify@Integrate[1 / t, {t, 1, x}]
```

```
ConditionalExpression[Log[x], Re[x] ≥ 0 || x ∉ Reals]
```

```
FullSimplify@Integrate[1 / (-1 + t) - E^(1-t) / (-1 + t), {t, 1, x}]
```

```
ConditionalExpression[EulerGamma + Gamma[0, -1 + x] + Log[-1 + x], Re[x] > 1]
```

```
FullSimplify@Integrate[1 / Log[t] - 1 / (t Log[t]), {t, 1, x}]
```

```
ConditionalExpression[-EulerGamma - Gamma[0, -Log[x]] - Log[-Log[x]], Im[x] ≠ 0 || Re[x] ≥ 0]
```

```
N[Integrate[(1 / (-1 + t) - E^(1-t) / (-1 + t)) (1 / (-1 + u) - E^(1-u) / (-1 + u)), {t, 0, x}, {u, 0, x - t}] /. x → 9.]
```

```
NIntegrate::slwcon :
   Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration
     is 0, highly oscillatory integrand, or WorkingPrecision too small. ≫

NIntegrate::ncvb : NIntegrate failed to converge to prescribed accuracy after
     9 recursive bisections in t near {t} = {8.01549}. NIntegrate obtained 13.6164 + 24.1341 i
  and 0.0006251207415111835` for the integral and error estimates. ≫
```

```
13.6164 + 24.1341 i
```

```
N[D[LaguerreL[z, 1 - (10)], {z, 2}] /. z → 0]
```

```
6.28971
```

```
N[Integrate[(1 / (-1 + t) - E^(1-t) / (-1 + t)) (1 / (-1 + u) - E^(1-u) / (-1 + u)), {t, 0, x}, {u, 1, x - t}] /. x → 10.]
```

```
NIntegrate::slwcon :
   Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration
     is 0, highly oscillatory integrand, or WorkingPrecision too small. ≫

NIntegrate::ncvb : NIntegrate failed to converge to prescribed accuracy after
     9 recursive bisections in t near {t} = {9.00375}. NIntegrate obtained 15.9202 - 12.4868 i
  and 0.0004955320520802499` for the integral and error estimates. ≫
```

```
15.9202 - 12.4868 i
```

```
Sum[x^k / k! N[D[Pochhammer[n, z] / z!, {z, k}] /. z → 0 /. n → 12], {k, 0, 16}] /. x → -2.2 + 3 I
```

```
-1.11018 - 0.1562 i
```

```
{N[D[Pochhammer[n + 1, z] / z!, {z, 2}] /. n → 6 /. z → 0],
 N@Sum[1 / j × 1 / k, {j, 1, 6}, {k, 1, 6 - j}]}
```

```
{4.51111, 4.51111}
```

```
{N[D[Pochhammer[n + 1, z] / z!, {z, 1}] /. n → 6 /. z → 0], N@Sum[1 / j, {j, 1, 6}]}
```

```
{2.45, 2.45}
```

```
{N[D[Pochhammer[z + 1, n] / n!, {z, 2}] /. n → 6 /. z → 0],
 N@Sum[1 / j × 1 / k, {j, 1, 6}, {k, 1, 6 - j}]}
```

```
{4.51111, 4.51111}
```

```
{N[D[Pochhammer[n, z] / z!, {z, 3}] /. n → 17 /. z → 0],
 N@Sum[1 / j × 1 / k × 1 / l, {j, 1, 16}, {k, 1, 16 - j}, {l, 1, 16 - j - k}]}
```

```
{24.9712, 24.9712}
```

```
{N[Integrate[ (1 / Log[x] - 1 / (x Log[x])) (1 / Log[y] - 1 / (y Log[y])), {x, 1, n}, {y, 1, n / x}] /.
   n → 7], N[D[LaguerreL[-z, Log[n]], {z, 2}] /. z → 0 /. n → 7]}
```

```
{3.93891, 3.93891}
```

```
{N[Integrate[ ((x - 1) / (x Log[x])) ((y - 1) / (y Log[y])), {x, 1, n}, {y, 1, n / x}] /. n → 7],
 N[D[LaguerreL[-z, Log[n]], {z, 2}] /. z → 0 /. n → 7]}
```

```
{3.93891, 3.93891}
```

$$\left\{\mathbf{N}[\mathbf{D}[\mathbf{LaguerreL}[z, 1-7], z] /. z \to 0], \mathbf{N}\left[\mathbf{Integrate}\left[\frac{1}{-1+t} - \frac{e^{1-t}}{-1+t}, \{t, 1, 7\}\right]\right]\right\}$$

```
{2.36934, 2.36934}
```

$$\left\{\mathbf{N}[\mathbf{D}[\mathbf{LaguerreL}[z, 1-8], \{z, 2\}] /. z \to 0],\ \mathbf{N}\left[\mathbf{Integrate}\left[\left(\frac{1}{-1+t} - \frac{e^{1-t}}{-1+t}\right)\left(\frac{1}{-1+u} - \frac{e^{1-u}}{-1+u}\right), \{t, 1, 8\}, \{u, 1, 8-t+1\}\right]\right]\right\}$$

```
{5.03314, 5.03314 - 7.927 i}
```

```
D[EulerGamma + Gamma[0, -1 + x] + Log[-1 + x], x]
```

$$\frac{1}{-1+x} - \frac{e^{1-x}}{-1+x}$$

```
D[EulerGamma + Gamma[0, x] + Log[x], x]
```

$$\frac{1}{x} - \frac{e^{-x}}{x}$$

```
Integrate[ 1/x - e^-x/x, {x, 0, n - 1}]
```

```
ConditionalExpression[EulerGamma + Gamma[0, -1 + n] + Log[-1 + n], Re[n] > 1]
```

```
1/x - e^-x/x /. x → t
```

$$\frac{1}{t} - \frac{e^{-t}}{t}$$

```
1/x - e^-x/x /. x → u
```

$$\frac{1}{u} - \frac{e^{-u}}{u}$$

$$\left\{\mathbf{N}[\mathbf{D}[\mathbf{LaguerreL}[z, -7], z] /. z \to 0], \mathbf{N}\left[\mathbf{Integrate}\left[\left(\frac{1}{t} - \frac{e^{-t}}{t}\right), \{t, 0, 7\}\right]\right]\right\}$$

```
{2.52324, 2.52324}
```

$$\left\{\mathbf{N}[\mathbf{D}[\mathbf{LaguerreL}[z, -7], \{z, 2\}] /. z \to 0],\ \mathbf{N}\left[\mathbf{Integrate}\left[\left(\frac{1}{t} - \frac{e^{-t}}{t}\right)\left(\frac{1}{u} - \frac{e^{-u}}{u}\right), \{t, 0, 7\}, \{u, 0, 7-t\}\right]\right]\right\}$$

```
{5.03314, 5.03314}
```

```
{N[D[LaguerreL[z, -7], {z, 3}] /. z → 0],
 N[Integrate[(1/t - e^-t/t) (1/u - e^-u/u) (1/v - e^-v/v), {t, 0, 7}, {u, 0, 7 - t}, {v, 0, 7 - t - u}]]}
```

```
{8.19258, 8.19258}
```

```
pr[n_, z_] := Pochhammer[n + 1, z] / z!
pr2[n_, z_] := (n + z)! / (n! z!)
pr2a[n_, z_] := 1 / Beta[n + 1, z + 1] / (n + z + 1)
```

```
{N[D[pr2[n, z], {z, 2}] /. n → 6 /. z → 0], N@Sum[1 / j × 1 / k, {j, 1, 6}, {k, 1, 6 - j}]}
```

```
{4.51111, 4.51111}
```

```
{N[D[pr2[n, z], {z, 3}] /. n → 6 /. z → 0],
 N@Sum[1 / j × 1 / k × 1 / l, {j, 1, 6}, {k, 1, 6 - j}, {l, 1, 6 - j - k}]}
```

```
{6.125, 6.125}
```

```
FullSimplify[Integrate[(1 - E^(-t)) / (t), {t, 0, n}]]
```

```
ConditionalExpression[EulerGamma + Gamma[0, n] + Log[n], Re[n] > 0]
```

```
Clear[F, G, G2]
F[n_, j_, k_, z_, d_] := F[n, j, k, z, d] =
  If[n < j, 0, d ((z + 1) / k - 1) (1 + F[Floor[n / j], 1 + d, k + 1, z, d]) + F[n, j + 1, k, z, d]]
zetaalt[n_, z_, d_] := 1 + F[n, 1 + d, 1, z, d]
G[n_, j_, k_, z_, d_] :=
 G[n, j, k, z, d] = If[n < j, 0, d ((z + 1) / k - 1) (1 + G[n - j, d, k + 1, z, d]) + G[n, j + 1, k, z, d]]
betaalt[n_, z_, d_] := 1 + G[n, d, 1, z, d]
G2[n_, j_, k_, z_, d_] := G2[n, j, k, z, d] =
  If[n < j, 0, (-1)^(j + 1) d ((z + 1) / k - 1) (1 + G2[n - j, d, k + 1, z, d]) + G2[n, j + 1, k, z, d]]
beta2alt[n_, z_, d_] := 1 + G2[n, d, 1, z, d]
```

```
Expand@zetaalt[100, z, 1]
```

$$1 + \frac{428\,z}{15} + \frac{16\,289\,z^2}{360} + \frac{331\,z^3}{16} + \frac{611\,z^4}{144} + \frac{67\,z^5}{240} + \frac{7\,z^6}{720}$$

```
Expand@betaalt[10, z, 1]
```

$$1 + \frac{7381\,z}{2520} + \frac{177\,133\,z^2}{50\,400} + \frac{84\,095\,z^3}{36\,288} + \frac{341\,693\,z^4}{362\,880} + \frac{8591\,z^5}{34\,560} + \frac{7513\,z^6}{172\,800} + \frac{121\,z^7}{24\,192} + \frac{11\,z^8}{30\,240} + \frac{11\,z^9}{725\,760} + \frac{z^{10}}{3\,628\,800}$$

```
Expand@FullSimplify@Sum[(D[pr[10, z], {z, k}] /. z → 0) z^k / k!, {k, 0, 10}]
```

$$1 + \frac{7381\,z}{2520} + \frac{177\,133\,z^2}{50\,400} + \frac{84\,095\,z^3}{36\,288} + \frac{341\,693\,z^4}{362\,880} + \frac{8591\,z^5}{34\,560} + \frac{7513\,z^6}{172\,800} + \frac{121\,z^7}{24\,192} + \frac{11\,z^8}{30\,240} + \frac{11\,z^9}{725\,760} + \frac{z^{10}}{3\,628\,800}$$

```
N@Expand@betaalt[10, z] /. z → -1.5
```

```
-0.00927353
```

```
pr2[10, -1.5]
```

```
-0.00927353
```

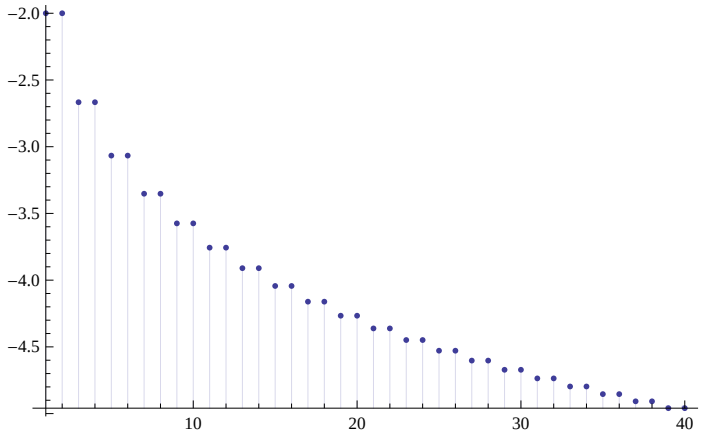
```
N@Sum[(D[pr[10, z], {z, k}] /. z → 0) z^k / k!, {k, 0, 10}] /. z → -1.5
```

```
-0.00927353
```

```
Expand@betaalt[10, z, 1]
```

$$1+\frac{7381\,z}{2520}+\frac{177\,133\,z^2}{50\,400}+\frac{84\,095\,z^3}{36\,288}+\frac{341\,693\,z^4}{362\,880}+\frac{8591\,z^5}{34\,560}+\frac{7513\,z^6}{172\,800}+\frac{121\,z^7}{24\,192}+\frac{11\,z^8}{30\,240}+\frac{11\,z^9}{725\,760}+\frac{z^{10}}{3\,628\,800}$$

```
Clear[s2o, sa2o]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
s2o[n_, 0] := UnitStep[n]
s2o[n_, k_] := s2o[n, k] = Sum[ (-1)^(j) s2o[n - j, k - 1], {j, 1, n}]
sz2[n_, z_] := Sum[ bin[z, k] s2o[n, k], {k, 0, n}]
sa2o[n_, 0] := UnitStep[n]
sa2o[n_, k_] := sa2o[n, k] = Sum[ sa2o[n - j, k - 1], {j, 1, n}]
saz2[n_, z_] := Sum[ bin[z, k] sa2o[n, k], {k, 0, n}]
DiscretePlot[D[sz2[n, z] - saz2[n, z], z] /. z → 0, {n, 1, 40}]
```




```
N@Log[1 / 2]
```

```
-0.693147
```

```
N@Integrate[1 / (j!), {j, 0, Infinity}]
```

```
2.26653
```

```
N@Integrate[1 / (j!) × 1 / (k!), {j, 0, Infinity}, {k, 0, Infinity}]
```

```
5.13718
```

```
N@Integrate[1 / (j!) × 1 / (k!) × 1 / (l!), {j, 0, Infinity}, {k, 0, Infinity}, {l, 0, Infinity}]
```

```
11.6436
```

```
N@Sum[1 / (j!), {j, 0, Infinity}]
```

```
2.71828
```

```
N@Sum[1 / (j!) × 1 / (k!), {j, 0, Infinity}, {k, 0, Infinity}]
```

```
7.38906
```

```
N[D[Sum[ z^k / (k!), {k, 0, n}], {z, 1}] /. z → 0 /. n → 10]
```

```
1.
```

```
D[Integrate[ z^k / k!, {k, 0, n}], z] /. z → 0
```

$$\int_0^n \frac{0^{-1+k}\, k}{k!}\, dk$$

```
D[Sum[ z^k / k!, {k, 0, 10}], z] /. z → 0
```

```
1
```