```
FactInteger[n_] := If[n == 1, {}, FactorInteger[n]]
d[n_, z_] := Product[1 / (j[[2]] !) Pochhammer[z, j[[2]]], {j, FactInteger[n]}]


d[100, 3]

36

dl[n_, z_] := Sum[Log[1 / (j[[2]] !) Pochhammer[z, j[[2]]]], {j, FactInteger[n]}]

dl[30, 3]

3 Log[3]

N[Log[d[100, 3]]]

3.58352

FullSimplify[Log[1 / (a !) Pochhammer[z, a]]]
```

$$Log\left[\frac{Pochhammer[z, a]}{a!}\right]$$

```
FullSimplify[Log[Binomial[a, b]]]
```

Log[Binomial[a, b]]

```
DD[n_, z_] := Sum[d[j, z], {j, 1, n}]

DD[100, 2]

482

DM[n_, z_] := Sum[d[j, -1] DD[Floor[n / j], z + 1], {j, 1, n}]

DD[100, I]
```

$$-\frac{2881}{72} + \frac{65\,i}{8}$$

```
DM[100, I]
```

$$-\frac{2881}{72} + \frac{65\,i}{8}$$

```
Table[{n, MoebiusMu[n], d[n, -1]}, {n, 1, 100}] // TableForm
```

1	1	1
2	-1	-1
3	-1	-1
4	0	0
5	-1	-1
6	1	1
7	-1	-1
8	0	0
9	0	0
10	1	1
11	-1	-1
12	0	0
13	-1	-1
14	1	1
15	1	1
16	0	0
17	-1	-1

18	0	0
19	−1	−1
20	0	0
21	1	1
22	1	1
23	−1	−1
24	0	0
25	0	0
26	1	1
27	0	0
28	0	0
29	−1	−1
30	−1	−1
31	−1	−1
32	0	0
33	1	1
34	1	1
35	1	1
36	0	0
37	−1	−1
38	1	1
39	1	1
40	0	0
41	−1	−1
42	−1	−1
43	−1	−1
44	0	0
45	0	0
46	1	1
47	−1	−1
48	0	0
49	0	0
50	0	0
51	1	1
52	0	0
53	−1	−1
54	0	0
55	1	1
56	0	0
57	1	1
58	1	1
59	−1	−1
60	0	0
61	−1	−1
62	1	1
63	0	0
64	0	0
65	1	1
66	−1	−1
67	−1	−1
68	0	0
69	1	1
70	−1	−1
71	−1	−1
72	0	0
73	−1	−1

```
74      1      1
75      0      0
76      0      0
77      1      1
78     -1     -1
79     -1     -1
80      0      0
81      0      0
82      1      1
83     -1     -1
84      0      0
85      1      1
86      1      1
87      1      1
88      0      0
89     -1     -1
90      0      0
91      1      1
92      0      0
93      1      1
94      1      1
95      1      1
96      0      0
97     -1     -1
98      0      0
99      0      0
100     0      0
```

DM2[n_, z_, k_] := Sum[d[j, k] DD[Floor[n / j], z - k], {j, 1, n}]

DD[334, 2.4 I]

45.3394 - 765.029 i

DM2[334, 2.4 I, 7.2 + 3.2 I]

45.3394 - 765.029 i

DH[n_, k_, s_] := Sum[Binomial[k, k - j] DH[n / m^j, k - j, m + 1], {m, s, n^(1 / k)}, {j, 1, k}]
DH[n_, 0, s_] := 1

DH[100, 2, 1]

482

DDH[x_] := 2 Sum[Floor[x / k], {k, 1, Floor[x^(1 / 2)]}] - Floor[x^(1 / 2)]^2

DH[n, 1, 1]

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

General::stop : Further output of $RecursionLimit::reclim will be suppressed during this calculation. ≫

$IterationLimit::itlim : Iteration limit of 4096 exceeded. ≫

$IterationLimit::itlim : Iteration limit of 4096 exceeded. ≫

DDH[100]

482

DH[120, 1, 1]

120

DD[100, 4]

3575

DF[k_, n_, s_] :=
 If[s > n^(1/k), 0, Sum[Binomial[k, j] DF[k - j, Floor[n / s^j], s + 1], {j, 0, k}]]
DF[0, n_, s_] := 1

DF[3, 100, 1]

1471

DR[k_, n_, s_] := If[s > n^(1/k), 0, Sum[If[N[Binomial[k, k - j]] < .1,
    0, Binomial[k, k - j] DR[k - j, Floor[n / s^j], s + 1]], {j, 0, k}]]
DR[0, n_, s_] := 1

DR[3.5, 100, 1]

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

General::stop : Further output of $RecursionLimit::reclim will be suppressed during this calculation. ≫

$1. \left(0. + \right.$

$2.1875 \, \text{Binomial}[0.5, 0.5] \, \text{If}\left[\text{Hold}\left[128 > 12^{1/0.5} \right], 0, \sum_{j=0}^{0.5} \text{If}\left[N[\text{Binomial}[0.5, 0.5 - j]] < 0.1, \right.\right.$

$\left.\left. 0, \text{Binomial}[0.5, 0.5 - j] \, \text{DR}\left[0.5 - j, \text{Floor}\left[\frac{12}{128^j} \right], 128 + 1 \right] \right] \right] \right) + 3.5 \left(0. + \right.$

$2.5 \left(1. \left(0. + 1.5 \, \text{Binomial}[0.5, 0.5] \, \text{If}\left[\text{Hold}\left[128 > 12^{1/0.5} \right], 0, \sum_{j=0}^{0.5} \text{If}\left[N[\text{Binomial}[0.5, 0.5 - \right.\right.\right.\right.$

$\left.\left.\left. j]] < 0.1, 0, \text{Binomial}[0.5, 0.5 - j] \, \text{DR}\left[0.5 - j, \text{Floor}\left[\frac{12}{128^j} \right], 128 + 1 \right] \right] \right] \right) + $

$1.5 \, \text{Binomial}[0.5, 0.5] \, \text{If}\left[\text{Hold}\left[128 > 16^{1/0.5} \right], 0, \sum_{j=0}^{0.5} \text{If}\left[N[\text{Binomial}[0.5, 0.5 - j]] < 0.1, \right.\right.$

$\left.\left. 0, \text{Binomial}[0.5, 0.5 - j] \, \text{DR}\left[0.5 - j, \text{Floor}\left[\frac{16}{128^j} \right], 128 + 1 \right] \right] \right] \right) + $

$1.875 \, \text{Binomial}[0.5, 0.5] \, \text{If}\left[\text{Hold}\left[128 > 25^{1/0.5} \right], 0, \sum_{j=0}^{0.5} \text{If}\left[N[\text{Binomial}[0.5, 0.5 - j]] < 0.1, \right.\right.$

$$0, \text{Binomial}[0.5, 0.5 - j] \, \text{DR}\Big[0.5 - j, \text{Floor}\Big[\frac{25}{128^j}\Big], 128 + 1\Big]\Big]\Big]\Big) +$$

$$4.375 \Big(1. \Big(1. \Big(1. \Big(1. \Big(1. \Big(1. \Big(0. + 1.5 \, \text{Binomial}[0.5, 0.5] \, \text{If}\Big[\text{Hold}\Big[128 > 12^{1/0.5}\Big], 0,$$

$$\sum_{j=0}^{0.5} \text{If}\Big[\text{N}[\text{Binomial}[0.5, 0.5 - j]] < 0.1, 0, \text{Binomial}[0.5, 0.5 - j]$$

$$\text{DR}\Big[0.5 - j, \text{Floor}\Big[\frac{12}{128^j}\Big], 128 + 1\Big]\Big]\Big]\Big) + 1.5 \, \text{Binomial}[0.5, 0.5]$$

$$\text{If}\Big[\text{Hold}\Big[128 > 14^{1/0.5}\Big], 0, \sum_{j=0}^{0.5} \text{If}\Big[\text{N}[\text{Binomial}[0.5, 0.5 - j]] < 0.1,$$

$$0, \text{Binomial}[0.5, 0.5 - j] \, \text{DR}\Big[0.5 - j, \text{Floor}\Big[\frac{14}{128^j}\Big], 128 + 1\Big]\Big]\Big]\Big) +$$

$$1.5 \, \text{Binomial}[0.5, 0.5] \, \text{If}\Big[\text{Hold}\Big[128 > 16^{1/0.5}\Big], 0, \sum_{j=0}^{0.5} \text{If}\Big[$$

$$\text{N}[\text{Binomial}[0.5, 0.5 - j]] < 0.1, 0, \text{Binomial}[0.5, 0.5 - j]$$

$$\text{DR}\Big[0.5 - j, \text{Floor}\Big[\frac{16}{128^j}\Big], 128 + 1\Big]\Big]\Big]\Big) + 1.5 \, \text{Binomial}[0.5, 0.5]$$

$$\text{If}\Big[\text{Hold}\Big[128 > 20^{1/0.5}\Big], 0, \sum_{j=0}^{0.5} \text{If}\Big[\text{N}[\text{Binomial}[0.5, 0.5 - j]] < 0.1, 0,$$

$$\text{Binomial}[0.5, 0.5 - j] \, \text{DR}\Big[0.5 - j, \text{Floor}\Big[\frac{20}{128^j}\Big], 128 + 1\Big]\Big]\Big]\Big) +$$

$$1.5 \, \text{Binomial}[0.5, 0.5] \, \text{If}\Big[\text{Hold}\Big[128 > 25^{1/0.5}\Big], 0, \sum_{j=0}^{0.5} \text{If}\Big[\text{N}[\text{Binomial}[0.5, 0.5 - j]] <$$

$$0.1, 0, \text{Binomial}[0.5, 0.5 - j] \, \text{DR}\Big[0.5 - j, \text{Floor}\Big[\frac{25}{128^j}\Big], 128 + 1\Big]\Big]\Big]\Big) +$$

$$1.5 \, \text{Binomial}[0.5, 0.5] \, \text{If}\Big[\text{Hold}\Big[128 > 33^{1/0.5}\Big], 0, \sum_{j=0}^{0.5} \text{If}\Big[\text{N}[\text{Binomial}[0.5, 0.5 - j]] < 0.1,$$

$$0, \text{Binomial}[0.5, 0.5 - j] \, \text{DR}\Big[0.5 - j, \text{Floor}\Big[\frac{33}{128^j}\Big], 128 + 1\Big]\Big]\Big]\Big) +$$

$$1.5 \, \text{Binomial}[0.5, 0.5] \, \text{If}\Big[\text{Hold}\Big[128 > 50^{1/0.5}\Big],$$

$$0,$$

$$\sum_{j=0}^{0.5} \text{If}\Big[\text{N}[\text{Binomial}[0.5, 0.5 - j]] < 0.1, 0,$$

$$\text{Binomial}[0.5, 0.5 - j] \, \text{DR}\Big[0.5 - j, \text{Floor}\Big[\frac{50}{128^j}\Big], 128 + 1\Big]\Big]\Big]\Big) +$$

$$2.1875 \, \text{Binomial}[0.5, 0.5] \, \text{If}\Big[$$

$$\text{Hold}\Big[$$

$$128 >$$
$$100^{1/0.5} \Big],$$

$$0, \; \sum_{j=0}^{0.5} \text{If} \Big[N[\text{Binomial}[0.5, 0.5 - j]] < $$

$$0.1, \; 0,$$

$$\text{Binomial}[0.5, 0.5 - j] \; DR \Big[0.5 - j, \; \text{Floor} \Big[\frac{100}{128^{j}} \Big], \; 128 + 1 \Big] \Big] \Big]$$

DiscretePlot[Binomial[-1, n], {n, -1, 3}]
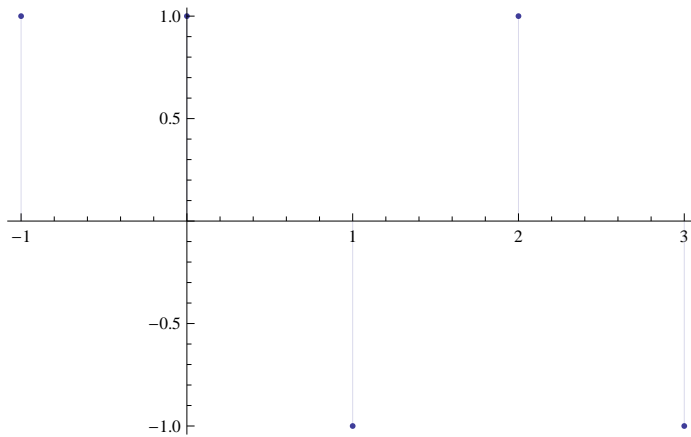
Binomial[-1, -2]

-1

Expand[(x + 1) ^ 6]

$1 + 6\,x + 15\,x^2 + 20\,x^3 + 15\,x^4 + 6\,x^5 + x^6$

Table[Binomial[6, j], {j, 0, 6}]

{1, 6, 15, 20, 15, 6, 1}

Expand[(x + 1) ^ (11 / 3)]

$(1 + x)^{2/3} + 3\,x\,(1 + x)^{2/3} + 3\,x^2\,(1 + x)^{2/3} + x^3\,(1 + x)^{2/3}$
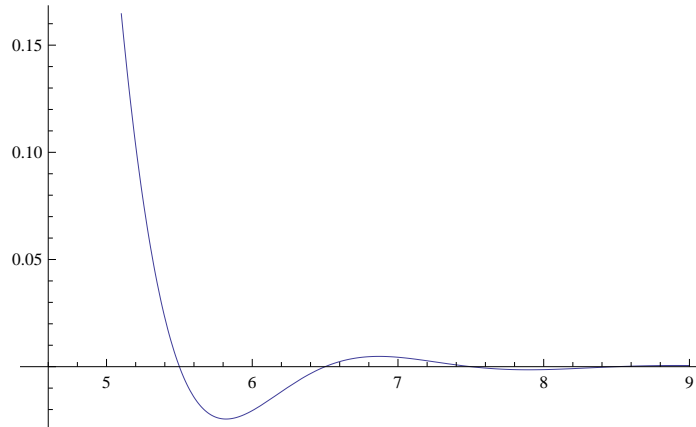
Plot[Binomial[4, n], {n, -4, 5}]
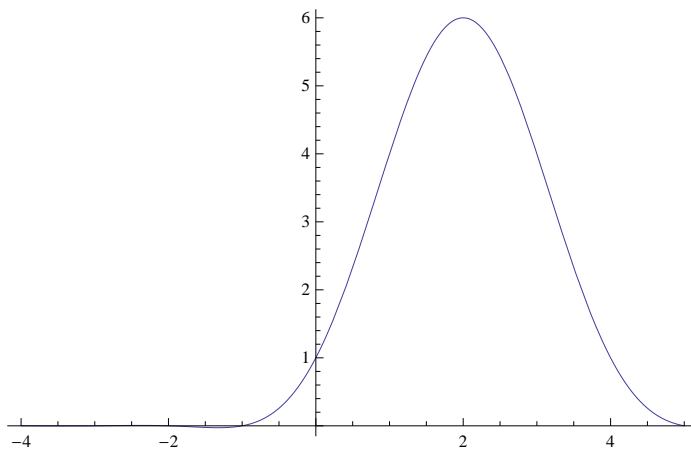
```
Bin[x_, y_] := Gamma[x + 1] / ( Gamma[y + 1] Gamma[x - y + 1] )
```

```
N[Bin[2.5, 2 I]]
```

$-7.1986 + 9.77265 \, i$

```
Plot[Binomial[4.5, n], {n, 4.5, 9}]
```



```
Binomial[4.5, 0]
```

1.

```
d[100, -15.5]
```

12 628.1

```
Binomial[5.5, 15.5]
```

0.

```
Sum[ d[10, 2.25 - k] Binomial[2.25, k] (-1) ^ (k - 1), {k, 0, 11 128}]
```

-0.223456