

$$\Pi(n)=li(n)+\lim_{x\rightarrow 1+}\sum_{k=1}^{\lfloor \frac{\log n}{\log x}\rfloor}\frac{x^k}{k}+\sum_{k=1}^{\lfloor \frac{\log n}{\log x}\rfloor}\frac{(-1)^{k-1}}{k}E_{k,x}'(n)$$

$$\Pi(n)=li(n)+\lim_{x\rightarrow 1+}E_{\log,x}(n)+\sum_{k=1}^{\lfloor \frac{\log n}{\log x}\rfloor}\frac{x^k}{k}$$

...

$$\Pi(n)=li(n)-\sum_{\mathfrak{p}}li(n^{\mathfrak{p}})-\log 2+\int_n^{\infty}\frac{dt}{t(t^2-1)\log t}+\frac{\Lambda(n)}{2\log n}$$

Thus

$$-\sum_{\mathfrak{p}}li(n^{\mathfrak{p}})-\log 2+\int_n^{\infty}\frac{dt}{t(t^2-1)\log t}+\frac{\Lambda(n)}{2\log n}=\lim_{x\rightarrow 1+}E_{\log,x}(n)+\sum_{k=1}^{\lfloor \frac{\log n}{\log x}\rfloor}\frac{x^k}{k}$$

...

$$E_{k,x}(n)=\sum_{j=1}^nE_{k-1,x}(\frac{n}{j})-x\cdot E_{k,x}(\frac{n}{j\cdot x})$$

$$E_{k,x}'(n)=\sum_{j=2}^nE_{k-1,x}'(\frac{n}{j})-x\cdot \sum_{j=1}^nE_{k,x}'(\frac{n}{j\cdot x})$$

$$E_{z,x}(n)=\sum_{k=0}^{\infty}\binom{z}{k}E_{k,x}'(n)$$

$$E_{k,x}'(n)=\sum_{j=0}^k(-1)^{k-j}\binom{k}{j}E_{j,x}(n)$$

$$E_{k,z}'(n)=\frac{\sin(\pi z)}{\pi}\sum_{k=0}^{\infty}\frac{(-1)^k}{z-k}E_{k,x}'(n)$$

...

$$E_{\log,x}(n)=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\cdot E_{k,x}'(n)$$

$$E_{\log,x}(n)=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}E_{z,x}(n)$$

$$E_{\log,x}(n)=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}E_{z,x}'(n)$$

...

$$E_{z,x}(n)=\sum_{j=0}^{\infty}(-1)^j\binom{z}{j}x^jD_z(\frac{n}{x^j})$$

$$D_z(n)=\sum_{j=0}^{\infty}(-1)^j\binom{-z}{j}x^jE_{z,x}(\frac{n}{x^j})$$

$$D_z(n)=\sum_{j=0}\sum_{k=0}(-1)^j\binom{-z}{j}\binom{z}{k}x^jE_{k,x}(\frac{n}{x^j})$$

$$\ldots$$

$$E_{k,x}{}'(n)=\sum_{j=0}^k\sum_{m=0}^j(-1)^j\binom{k}{j}\binom{j}{m}x^jD_{k-m}{}'(\frac{n}{x^j})$$

$$D_k{}'(n)=(-1)^k+\sum_{j=0}^k\sum_{m=0}^k\binom{k}{m}\binom{m+j-1}{k-1}x^jE_{m,x}{}'(\frac{n}{x^j})$$

$$\ldots$$

$$\begin{aligned}
& [ (1+x^{1-s} \cdot \zeta(s, 1+x^{-1}))^z ]_n = f_z(n, 1+\frac{1}{x}) \text{ where} \\
& f_z(n, j) = \begin{cases} \sum_{k=0}^{\lfloor \frac{\log n}{\log(x \cdot j)} \rfloor} \binom{z}{k} \cdot (x^{1-s} \cdot j^{-s})^k \cdot f_{z-k}(\frac{n}{(x \cdot j)^k}, 1+y) & \text{if } n \geq x \cdot j \\ 1 & \text{if } n < x \cdot j \end{cases}
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\end{aligned}$$

$$[ ((1-x^{1-s}) \zeta(s))^z ]_n = 1 + f_1(n, 1+\frac{1}{x_d}) \text{ where } f_k(n, j) = \begin{cases} t_x(j) \cdot \frac{1}{x_d} \cdot (\frac{j}{x_d})^{-s} \cdot (\frac{z+1}{k} - 1) (1 + f_{k+1}(\frac{n}{j}, 1+\frac{1}{x_d})) + f_k(n, j+\frac{1}{x_d}) & \text{if } n \geq j \\ 0 & \text{if } n < j \end{cases}$$

$$t_x(m) = x \cdot (\lfloor \frac{m}{x} \rfloor - \lfloor \frac{m-1}{x} \rfloor) - (x+1) \cdot (\lfloor \frac{m}{x+1} \rfloor - \lfloor \frac{m-1}{x+1} \rfloor)$$

$$\left[ \left( 1+x^{1-s} \cdot \zeta \left( s, 1+\frac{1}{x} \right) \right)_n^z \right] = 1 + f_1 \left( n, 1+\frac{1}{x} \right) \text{ where } f_k(n, j) = \begin{cases} \frac{1}{x} \cdot j^{-s} \cdot \left( \frac{z+1}{k} - 1 \right) \left( 1 + f_{k+1} \left( \frac{n}{j}, 1+\frac{1}{x} \right) \right) + f_k \left( n, j+\frac{1}{x} \right) & \text{if } n \geq j \\ 0 & \text{if } n < j \end{cases}$$

$$\left[ \left( 1+x^{1-s} \cdot \zeta \left( s, 1+\frac{1}{x} \right) \right)_n^z \right] = f_z \left( n, 1+\frac{1}{x} \right) \text{ where } f_z(n, j) = \begin{cases} \sum_{k=0}^{\left\lfloor \frac{\log n}{\log j} \right\rfloor} \binom{z}{k} \cdot x^{-k} \cdot j^{-s \cdot k} \cdot f_{z-k} \left( \frac{n}{j^k}, j+\frac{1}{x} \right) & \text{if } n \geq j \\ 1 & \text{if } n < j \end{cases}$$

...

$$t_x(n) = (\lfloor n \rfloor - \lfloor n - \frac{1}{x} \rfloor) - (1 + \frac{1}{x}) \cdot (\lfloor \frac{nx}{x+1} \rfloor - \lfloor \frac{nx-1}{x+1} \rfloor)$$

$$\left[ \left( \left( 1 - \left( 1 + \frac{1}{x} \right)^{1-s} \right) \zeta(s) \right)_n^z \right] = 1 + f_1 \left( n, 1+\frac{1}{x} \right) \text{ where } f_k(n, j) = \begin{cases} t_x(j) \cdot j^{-s} \cdot \left( \frac{z+1}{k} - 1 \right) \left( 1 + f_{k+1} \left( \frac{n}{j}, 1+\frac{1}{x} \right) \right) + f_k \left( n, j+\frac{1}{x} \right) & \text{if } n \geq j \\ 0 & \text{if } n < j \end{cases}$$

$$\left[ \left( \left( 1 - \left( 1 + \frac{1}{x} \right)^{1-s} \right) \zeta(s) \right)_n^z \right] = f_z \left( n, 1+\frac{1}{x} \right) \text{ where } f_z(n, j) = \begin{cases} \sum_{k=0}^{\left\lfloor \frac{\log n}{\log j} \right\rfloor} \binom{z}{k} \cdot t_x(j)^k \cdot j^{-s \cdot k} \cdot f_{z-k} \left( \frac{n}{j^k}, j+\frac{1}{x} \right) & \text{if } n \geq j \\ 1 & \text{if } n < j \end{cases}$$

$$[(1+x^{1-s} \cdot \zeta(s, 1+x^{-s}))^z]_n = 1 + f_1(n, 1+x) \text{ where } f_k(n, j) = \begin{cases} x \cdot j^{-s} \cdot (\frac{z+1}{k} - 1) (1 + f_{k+1}(\frac{n}{j}, 1+x)) + f_k(n, j+x) & \text{if } n \geq j \\ 0 & \text{if } n < j \end{cases}$$

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...  
maps

$$t_x(n) = (\lfloor n \rfloor - \lfloor n-x \rfloor) - (1+x) \cdot (\lfloor \frac{n}{1+x} \rfloor - \lfloor \frac{n-x}{1+x} \rfloor)$$

$$[((1-(1+x)^{1-s})\zeta(s))^z]_n = 1 + f_1(n, 1+x) \text{ where } f_k(n, j) = \begin{cases} t_x(j) \cdot j^{-s} \cdot (\frac{z+1}{k} - 1) (1 + f_{k+1}(\frac{n}{j}, 1+x)) + f_k(n, j+x) & \text{if } n \geq j \\ 0 & \text{if } n < j \end{cases}$$

$$[((1-(1+x)^{1-s})\zeta(s))^z]_n = f_z(n, 1+x) \text{ where } f_z(n, j) = \begin{cases} \sum_{k=0}^{\lfloor \frac{\log n}{\log j} \rfloor} \binom{z}{k} \cdot t_x(j)^k \cdot j^{-sk} \cdot f_{z-k}(\frac{n}{j^k}, j+x) & \text{if } n \geq j \\ 1 & \text{if } n < j \end{cases}$$

$$[(1-2^{1-s})\zeta(s)]_n=f_z(n,2) \text{ where } f_z(n,j)=\begin{cases} \sum_{k=0}^{\lfloor \frac{\log n}{\log j} \rfloor} \binom{z}{k} (-1)^{(j+1)k} \cdot j^{-sk} \cdot f_{z-k}(\frac{n}{j^k},j+1) & \text{if } n \geq j \\ 1 & \text{if } n < j \end{cases}$$