```
Integrate[1 / (1 + x), {x, 0, 3}]
```

Log[4]

```
N[HarmonicNumber[10 000] - HarmonicNumber[10 000 / (E^2)]]
```

1.99968

```
f[a_] := Limit[HarmonicNumber[x] - HarmonicNumber[x / a], {x → Infinity}]
f[1.4423]
```

{0.366239}

```
f'[a]
```

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

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General::stop : Further output of $RecursionLimit::reclim will be suppressed during this calculation. ≫

$$\left\{\mathrm{Hold}\left[\partial_a \mathrm{Limit}\left[\mathrm{HarmonicNumber}[x] - \mathrm{HarmonicNumber}\left[\frac{x}{a}\right], x \to \infty\right]\right]\right\}$$

```
Log[1.4423]
```

0.366239

```
s[n_] := PolyGamma[n + 1] + EulerGamma
s[30] - HarmonicNumber[30]
```

0

```
HarmonicNumber[30]
```

$$\frac{9\,304\,682\,830\,147}{2\,329\,089\,562\,800}$$

```
f2[a_] := Limit[s[x] - s[x / a], {x → Infinity}]
f2[10 + 10 I]
```

$$\left\{-\mathrm{Log}\left[\frac{1}{20} - \frac{i}{20}\right]\right\}$$

```
Log[10 I]
```

$$\frac{i\,\pi}{2} + \mathrm{Log}[10]$$

$$\mathbf{N}\left[-\mathbf{Log}\left[\frac{1}{20} - \frac{i}{20}\right]\right]$$

$2.64916 + 0.785398\,i$

```
s2[n_] := PolyGamma[2, n + 1] + EulerGamma
f3[a_] := Limit[s2[x] - s2[x / a], {x → Infinity}]

Limit[PolyGamma[2, x] - PolyGamma[2, x / 81], {x → Infinity}]
```

{0}

```
Limit[PolyGamma[1, x] - PolyGamma[1, x / 81], {x → Infinity}]
```

{0}

```
Limit[PolyGamma[0, x] - PolyGamma[0, x / 81], {x → Infinity}]
```

{Log[81]}

```
Limit[Gamma'[x] / Gamma[x] - Gamma'[x / 81] / Gamma[x / 81], {x → Infinity}]
```

{Log[81]}

```
Limit[Gamma'[x] / Gamma[x] - Gamma'[x / c] / Gamma[x / c], {x → Infinity}]
```

$\left\{\text{Limit}\left[\text{PolyGamma}[0, x] - \text{PolyGamma}\left[0, \frac{x}{c}\right], x \to \infty\right]\right\}$

```
Integrate[ E^(-t) / t - E^(-z t) / (1 - E^(-t)), {t, 0, Infinity}]
```

$\text{ConditionalExpression}\left[-\frac{1}{z} + \text{PolyGamma}[0, 1 + z], \text{Re}[z] > 0\right]$

```
Integrate[-E^(-z t) / (1 - E^(-t)) + E^(-(z / 81) t) / (1 - E^(-t)), {t, 0, Infinity}]
```

$\text{ConditionalExpression}\left[-\text{PolyGamma}\left[0, \frac{z}{81}\right] + \text{PolyGamma}[0, z], \text{Re}[z] > 0\right]$

```
Integrate[(E^(-(z / 81) t) - E^(-z t)) / (1 - E^(-t)), {t, 0, Infinity}]
```

$\text{ConditionalExpression}\left[-\text{PolyGamma}\left[0, \frac{z}{81}\right] + \text{PolyGamma}[0, z], \text{Re}[z] > 0\right]$

```
Expand[(E^(-(z / 81) t) - E^(-z t))]
```

$-e^{-t z} + e^{-\frac{t z}{81}}$

```
PolyGamma[1 / 4]
```

$\text{PolyGamma}\left[0, \frac{1}{4}\right]$

```
t[n_, a_] := Mod[n, a] - Mod[n - 1, a]
```

```
t[5, 5 / 2]
```

$-\frac{3}{2}$

```
Table[Floor[(a + 1) (2 / 5)], {a, 1, 10}]
```

{0, 1, 1, 2, 2, 2, 3, 3, 4, 4}

```
fa[a_] := Floor[(a + 1) (2 / 5)]
```

```
fa[3]
```

1

```
fb := {0, -1 / 4, -1 / 4, 0, -3 / 3}
```

```
fb[[2]]
```

$-\frac{1}{4}$

```
Table[ fa[n] fb[[Mod[n - 1, 5] + 1]], {n, 1, 10}]
```

$\{0, -\frac{1}{4}, -\frac{1}{4}, 0, -2, 0, -\frac{3}{4}, -\frac{3}{4}, 0, -4\}$

```
Mod[1, 5]
```

1

```
1 - 1 / 6 + 1 / 4 - 3 / 10 + 1 / 6 - 1 / 42 - 1 / 24 + 1 / 9 - 3 / 20
```

$N\left[\frac{2131}{2520}\right]$

0.845635

```
N[Log[5 / 2]]
```

0.916291

```
1 - 1 / 6 + 1 / 4 - 3 / 10
```

$\frac{47}{60}$

```
1 + (1 / 2 - 1 / 2 × 1) + (1 / 3 - 1 / 2 × 1) + (1 / 4) + (1 / 5 - 1 × 1 / 2) +
  (1 / 6) + (1 / 7 - 1 / 2 × 1 / 3) + (1 / 8 - 1 / 2 × 1 / 3) + (1 / 9) + (1 / 10 - 1 / 4) +
  (1 / 11) + (1 / 12 - 1 / 2 × 1 / 5) + (1 / 13 - 1 / 2 × 1 / 5) + (1 / 14) + (1 / 15 - 1 / 6) +
  (1 / 16) + (1 / 17 - 1 / 2 × 1 / 7) + (1 / 18 - 1 / 2 × 1 / 7) + (1 / 19) + (1 / 20 - 1 / 8)
```

$N\left[\frac{68\,276\,701}{77\,597\,520}\right]$

0.879883

$N\left[\frac{62\,575}{72\,072}\right]$

0.868229

```
1 + (1 / 2 - 1 / 2 × 1) + (1 / 3 - 1 / 2 × 1) + (1 / 4) + (1 / 5 - 1 × 1 / 2) + (1 / 6) +
  (1 / 7 - 1 / 2 × 1 / 3) + (1 / 8 - 1 / 2 × 1 / 3) + (1 / 9) + (1 / 10 - 1 / 4) + (1 / 11) +
  (1 / 12 - 1 / 2 × 1 / 5) + (1 / 13 - 1 / 2 × 1 / 5) + (1 / 14) + (1 / 15 - 1 / 6) + (1 / 16) +
  (1 / 17 - 1 / 2 × 1 / 7) + (1 / 18 - 1 / 2 × 1 / 7) + (1 / 19) + (1 / 20 - 1 / 8) + (1 / 21) +
  (1 / 22 - 1 / 2 × 1 / 9) + (1 / 23 - 1 / 2 × 1 / 9) + (1 / 24) + (1 / 25 - 1 / 10) + (1 / 26) +
  (1 / 27 - 1 / 2 × 1 / 11) + (1 / 28 - 1 / 2 × 1 / 11) + (1 / 29) + (1 / 30 - 1 / 12)
```

$N\left[\frac{2\,077\,027\,228\,357}{2\,329\,089\,562\,800}\right]$

0.891776

```
1 + (1 / 2 - 1 / 2 × 1) + (1 / 3 - 1 / 2 × 1) + (1 / 4) + (1 / 5 - 1 × 1 / 2) + (1 / 6) +
  (1 / 7 - 1 / 2 × 1 / 3) + (1 / 8 - 1 / 2 × 1 / 3) + (1 / 9) + (1 / 10 - 1 / 4) + (1 / 11) +
  (1 / 12 - 1 / 2 × 1 / 5) + (1 / 13 - 1 / 2 × 1 / 5) + (1 / 14) + (1 / 15 - 1 / 6) + (1 / 16) +
  (1 / 17 - 1 / 2 × 1 / 7) + (1 / 18 - 1 / 2 × 1 / 7) + (1 / 19) + (1 / 20 - 1 / 8) + (1 / 21) +
  (1 / 22 - 1 / 2 × 1 / 9) + (1 / 23 - 1 / 2 × 1 / 9) + (1 / 24) + (1 / 25 - 1 / 10) + (1 / 26) +
  (1 / 27 - 1 / 2 × 1 / 11) + (1 / 28 - 1 / 2 × 1 / 11) + (1 / 29) + (1 / 30 - 1 / 12) +
  (1 / 31) + (1 / 32 - 1 / 2 × 1 / 13) + (1 / 33 - 1 / 2 × 1 / 13) + (1 / 34) + (1 / 35 - 1 / 14) +
  (1 / 36) + (1 / 37 - 1 / 2 × 1 / 15) + (1 / 38 - 1 / 2 × 1 / 15) + (1 / 39) + (1 / 40 - 1 / 16) +
  (1 / 41) + (1 / 42 - 1 / 2 × 1 / 17) + (1 / 43 - 1 / 2 × 1 / 17) + (1 / 44) + (1 / 45 - 1 / 18) +
  (1 / 46) + (1 / 47 - 1 / 2 × 1 / 19) + (1 / 48 - 1 / 2 × 1 / 19) + (1 / 49) + (1 / 50 - 1 / 20)
```

$$N\left[\frac{2793682265045051088509}{3099044504245996706400}\right]$$

0.901466

```
FF[n_] := (1 / (5 n + 1)) + (1 / (5 n + 2) - 1 / 2 × 1 / 1) +
  (1 / (5 n + 3) - 1 / 2 × 1 / 1) + (1 / (5 n + 4)) + (1 / (5 n + 5) - 1 / 2)
```

```
Floor[(5 n + 3) * 2 / 5] + 2
Floor[(5 n + 4) * 2 / 5] + 2
```

$$2 + \mathrm{Floor}\left[\frac{2}{5}(3 + 5 n)\right]$$

$$2 + \mathrm{Floor}\left[\frac{2}{5}(4 + 5 n)\right]$$

18

18

```
Floor[(5 n + 6) * 2 / 5] + 2
```

20

```
FF2[n_] := (1 / (5 n + 1)) + (1 / (5 n + 2) - 1 / 2 × 1 / (Floor[(5 n) * 2 / 5] + 1)) +
  (1 / (5 n + 3) - 1 / 2 × 1 / (Floor[(5 n) * 2 / 5] + 1)) +
  (1 / (5 n + 4)) + (1 / (5 n + 5) - 1 / (Floor[(5 n) * 2 / 5] + 2))
```

```
FF2[s]
```

$$\frac{1}{1+5 s}+\frac{1}{2+5 s}+\frac{1}{3+5 s}+\frac{1}{4+5 s}+\frac{1}{5+5 s}-\frac{1}{1+\mathrm{Floor}[2 s]}-\frac{1}{2+\mathrm{Floor}[2 s]}$$

```
1 + (1 / 2 - 1 / 2 × 1) + (1 / 3 - 1 / 2 × 1) + (1 / 4) + (1 / 5 - 1 × 1 / 2)
```

$$\frac{47}{60}$$

```
FF2[0] + FF2[1] + FF2[2] + FF2[3]
```

$$\frac{68276701}{77597520}$$

```
FF[0]
```

$$\frac{47}{60}$$

```
1 / 2 × 1 / (Floor[(5 × 0 + 3) * 2 / 5] + 2)
```

$$\frac{1}{6}$$

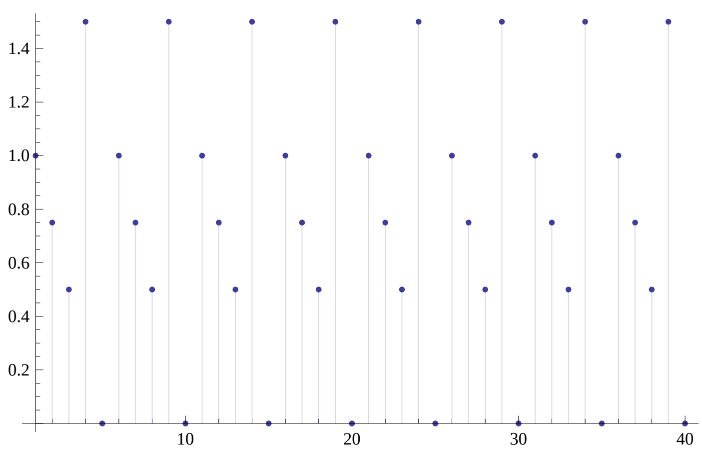
$$N\left[\frac{68276701}{77597520}\right]$$

0.879883

```
N[Sum[ FF2[k], {k, 0, 13000}]]
```

0.916279

```
N[Log[5 / 2]]

0.916291

FF3[n_] := Sum[(1 / (5 n + a)), {a, 1, 5}] - (1 / (2 n + 1)) - (1 / (2 n + 2))

N[Sum[ FF3[k], {k, 0, 13 000}]]

0.916279

(5 n) * 2 / 5

2 n

FF4[n_] := Sum[(1 / (5 n + a)), {a, 1, 5}] - Sum[(1 / (2 n + a)), {a, 1, 2}]

N[Sum[ FF4[k], {k, 0, 13 000}]]

0.916279

FF5[n_, b_, c_] := Sum[(1 / (b n + a)), {a, 1, b}] - Sum[(1 / (c n + a)), {a, 1, c}]

N[Sum[ FF5[k, 7, 3], {k, 0, 13 000}]]

0.847291

N[Log[7 / 3]]

0.847298

S2[ b_, c_] := N[Sum[ FF5[k, b, c], {k, 0, 13 000}]]

S2[1, 7]

-1.94588

N[Log[1 / 7]]

-1.94591

tt := {-3 / 2, 1, -1 / 4, -1 / 4, 1}

FF[n_] := Sum[ tt[[Mod[k, 5] + 1]], {k, 1, n}]

FF[5]

0

DiscretePlot[ FF[n], {n, 1, 40}]
```



```
tt[[Mod[5, 5] + 1]]
```

$-\frac{3}{2}$

```
FFa[n_] := Sum[ tt[[Mod[k, 5] + 1]] / k, {k, 1, n}]
```

```
FFa[5]
```

$\frac{89}{120}$

```
tt2 := {1, 0, 1 / 2, 1 / 2, 0}
```

```
FFa[n_] := Sum[ 1 / k - tt2[[Mod[k, 5] + 1]] / (Floor[2 (k) / 5] + 1), {k, 1, n}]
```

```
FFa[5]
```

$\frac{6}{5}$

```
Co[k_] :=  1 / k - tt2[[Mod[k, 5] + 1]] / (Floor[2 (k - 1) / 5] + 1)
```

```
Co[10]
```

$-\frac{3}{20}$

```
Table[ Co[n], {n, 1, 50}]
```

$$\left\{1, 0, -\frac{1}{6}, \frac{1}{4}, -\frac{3}{10}, \frac{1}{6}, -\frac{1}{42}, -\frac{1}{24}, \frac{1}{9}, -\frac{3}{20}, \frac{1}{11}, -\frac{1}{60}, -\frac{3}{130}, \frac{1}{14}, -\frac{1}{10}, \frac{1}{16}, -\frac{3}{238}, -\frac{1}{63}, \frac{1}{19}, -\frac{3}{40}, \frac{1}{21}, -\frac{1}{99}, -\frac{5}{414}, \frac{1}{24}, -\frac{3}{50}, \frac{1}{26}, -\frac{5}{594}, -\frac{3}{308}, \frac{1}{29}, -\frac{1}{20}, \frac{1}{31}, -\frac{3}{416}, -\frac{7}{858}, \frac{1}{34}, -\frac{3}{70}, \frac{1}{36}, -\frac{7}{1110}, -\frac{2}{285}, \frac{1}{39}, -\frac{3}{80}, \frac{1}{41}, -\frac{2}{357}, -\frac{9}{1462}, \frac{1}{44}, -\frac{1}{30}, \frac{1}{46}, -\frac{9}{1786}, -\frac{5}{912}, \frac{1}{49}, -\frac{3}{100}\right\}$$

```
N[Sum[ Co[k], {k, 1, 100 000}]]
```

0.916283

```
N[Log[5 / 2]]
```

0.916291

```
Co[100 004]
```

$\frac{1}{100\,004}$

```
Table[ Co[n], {n, 100 000, 100 000 + 25}]
```

$$\left\{-\frac{3}{200\,000}, \frac{1}{100\,001}, -\frac{5000}{2\,000\,090\,001}, -\frac{20\,001}{8\,000\,440\,006}, \frac{1}{100\,004}, -\frac{1}{66\,670}, \frac{1}{100\,006}, -\frac{20\,001}{8\,001\,160\,042}, -\frac{10\,001}{4\,000\,620\,024}, \frac{1}{100\,009}, -\frac{3}{200\,020}, \frac{1}{100\,011}, -\frac{10\,001}{4\,000\,980\,060}, -\frac{20\,003}{8\,002\,040\,130}, \frac{1}{100\,014}, -\frac{3}{200\,030}, \frac{1}{100\,016}, -\frac{20\,003}{8\,002\,760\,238}, -\frac{5001}{2\,000\,710\,063}, \frac{1}{100\,019}, -\frac{1}{66\,680}, \frac{1}{100\,021}, -\frac{5001}{2\,000\,890\,099}, -\frac{20\,005}{8\,003\,640\,414}, \frac{1}{100\,024}, -\frac{3}{200\,050}\right\}$$

```
tt3 := {1, 0, 0, 1, 0}
```

```
Co2[k_] := 1 / k - tt3[[Mod[k, 5] + 1]] / (Floor[2 (k - 1) / 5] + 1)
Table[ Co2[n], {n, 1, 50}]
```

$\left\{1, \frac{1}{2}, -\frac{2}{3}, \frac{1}{4}, -\frac{3}{10}, \frac{1}{6}, \frac{1}{7}, -\frac{5}{24}, \frac{1}{9}, -\frac{3}{20}, \frac{1}{11}, \frac{1}{12}, -\frac{8}{65}, \frac{1}{14}, -\frac{1}{10}, \frac{1}{16}, \frac{1}{17}, -\frac{11}{126}, \frac{1}{19}, -\frac{3}{40}, \frac{1}{21}, \frac{1}{22}, -\frac{14}{207}, \frac{1}{24}, -\frac{3}{50}, \frac{1}{26}, \frac{1}{27}, -\frac{17}{308}, \frac{1}{29}, -\frac{1}{20}, \frac{1}{31}, \frac{1}{32}, -\frac{20}{429}, \frac{1}{34}, -\frac{3}{70}, \frac{1}{36}, \frac{1}{37}, -\frac{23}{570}, \frac{1}{39}, -\frac{3}{80}, \frac{1}{41}, \frac{1}{42}, -\frac{26}{731}, \frac{1}{44}, -\frac{1}{30}, \frac{1}{46}, \frac{1}{47}, -\frac{29}{912}, \frac{1}{49}, -\frac{3}{100}\right\}$

```
N[Sum[ Co2[k], {k, 1, 100000}]]
```

0.916283