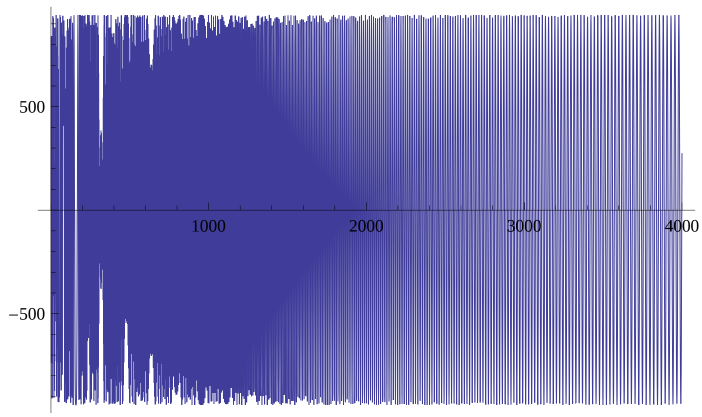
```
Zeta[-(1000. I + 1)]
```

-1575.360186805219 - 1109.537965641057 ⅈ

```
Zeta[-(1000. I)]
```

-8.46309098852087 - 8.34334485626739 ⅈ

$$\texttt{DiscretePlot}\left[\mathrm{Re}\left[1 + \mathrm{Zeta}[1 - 1000\, I] \Big/ \left(-\frac{i\, n^{1000\, i}}{1000}\right)\right], \{n, 1, 4000\}\right]$$



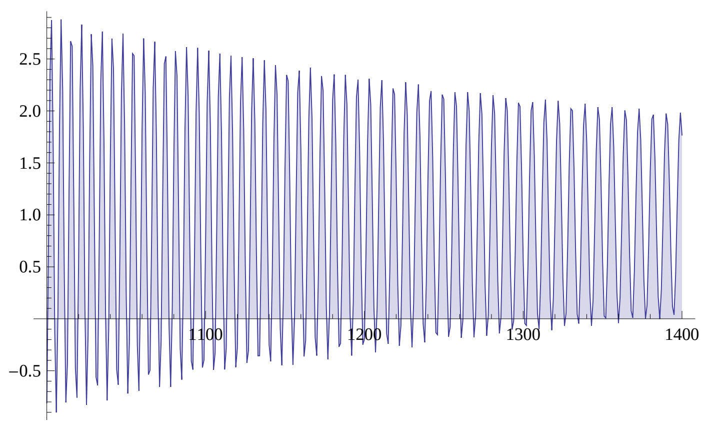

```
Integrate[j^(-1 + 1000 I), {j, 0, n}]
```

$$-\frac{i\, n^{1000\, i}}{1000}$$

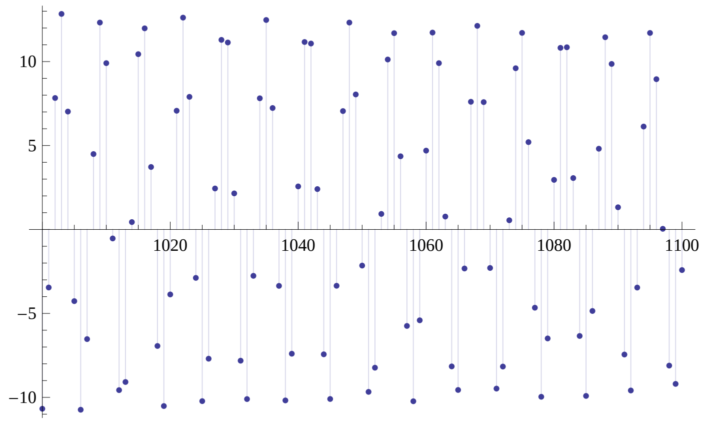
```
Integrate[j^(1 + 1000 I), {j, 0, n}]
```

$$\left(\frac{1}{500002} - \frac{250\, i}{250001}\right) n^{2+1000\, i}$$

$$\texttt{DiscretePlot}\left[\mathrm{Re}\left[1 + \mathrm{Zeta}[-1 - 1000\, I] \Big/ \left(\left(\frac{1}{500002} - \frac{250\, i}{250001}\right) n^{2+1000\, i}\right)\right], \{n, 1000, 1400\}\right]$$



DiscretePlot$\left[\mathrm{Re}\left[1+\mathrm{Zeta}[-1000\,I]\Big/\left(\left(\frac{1}{1\,000\,001}-\frac{1000\,i}{1\,000\,001}\right)n^{1+1000\,i}\right)\right],\ \{n,\ 1000,\ 1100\}\right]$



Integrate$[j\,\hat{}\,(1000\,I),\ \{j,\ 0,\ n\}]$

$$\left(\frac{1}{1\,000\,001}-\frac{1000\,i}{1\,000\,001}\right)n^{1+1000\,i}$$

Integrate$[j\,\hat{}\,(-1+1000\,I),\ \{j,\ 0,\ n\}]$

$$-\frac{i\,n^{1000\,i}}{1000}$$

$1+\mathrm{Zeta}[-1-1000\,I]\Big/\left(\left(\frac{1}{500\,002}-\frac{250\,i}{250\,001}\right)n^{2+1000\,i}\right)$

$1+(2+1000\,i)\,n^{-2-1000\,i}\,\mathrm{Zeta}[-1-1000\,i]$

Limit$\left[1+\mathrm{Zeta}[1-1000\,i]\Big/\left(-\frac{i\,n^{1000\,i}}{1000}\right),\ n\to\mathrm{Infinity}\right]$

$1+1000\,i\,e^{2\,i\,\mathrm{Interval}[\{0,\pi\}]}\,\mathrm{Zeta}[1-1000\,i]$

Integrate$[j\,\hat{}\,(s+t\,I),\ \{j,\ 0,\ n\}]$

ConditionalExpression$\left[\frac{n^{1+s+i\,t}}{1+s+i\,t},\ 1+\mathrm{Re}[s]>\mathrm{Im}[t]\right]\hat{}-1$

$\mathrm{ConditionalExpression}\left[n^{-1-s-i\,t}\,(1+s+i\,t),\ 1+\mathrm{Re}[s]>\mathrm{Im}[t]\right]$

Integrate$[j\,\hat{}\,(s-t\,I),\ \{j,\ 0,\ n\}]\hat{}-1$

$\mathrm{ConditionalExpression}\left[n^{-1-s+i\,t}\,(1+s-i\,t),\ \mathrm{Im}[t]+\mathrm{Re}[s]>-1\right]$

```
N[j^(s+t I) - j^(s-t I) /. j → 7 /. s → 3 /. t → 4]
```

0. + 684.304 i

```
N[j^s (j^(+t I) - j^(-t I)) /. j → 7 /. s → 3 /. t → 4]
```

0. + 684.304 i

```
N[j^s (E^(t Log[j] I) - E^(-t Log[j] I)) /. j → 7 /. s → 3 /. t → 4]
```

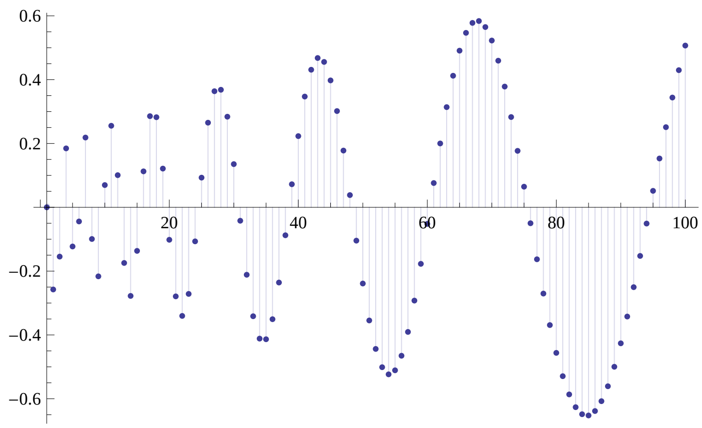
0. + 684.304 i

```
N[j^s 2 I Sin[t Log[j]] /. j -> 7 /. s -> 3 /. t -> 4]
```

0. + 684.304 i

```
bb[n_, s_, t_] := Sum[j^s Sin[t Log[j]], {j, 1, n}]

DiscretePlot[bb[n, -.5, Im@ZetaZero@1], {n, 1, 100}]
```



N[
$n^{-1-s-i t}$ (1 + s + i t) j^(s + t I) − $n^{-1-s+i t}$ (1 + s − i t) j^(s − t I) /. j → 7 /. s → 3 /. t → 4 /. n → 30]

0. + 0.00454257 i

N[n^(−1 − s) j^s ($n^{-i t}$ (1 + s + i t) j^(t I) − $n^{i t}$ (1 + s − i t) j^(−t I)) /. j → 7 /. s → 3 /. t → 4 /. n → 30]

−1.88053 × 10^{-19} + 0.00454257 i

N[n^(−1 − s) j^s ($(n / j)^{-i t}$ (1 + s + i t) − $(n / j)^{i t}$ (1 + s − i t)) /. j → 7 /. s → 3 /. t → 4 /. n → 30]

−1.88053 × 10^{-19} + 0.00454257 i

N[n^(−1 − s) j^s ($(n / j)^{-i t}$ (1 + s) − $(n / j)^{i t}$ (1 + s) + $(n / j)^{-i t}$ (i t) − $(n / j)^{i t}$ (−i t)) /. j → 7 /. s → 3 /. t → 4 /. n → 30]

−1.88053 × 10^{-19} + 0.00454257 i

N[n^(−1 − s) j^s ((1 + s) ($(n / j)^{-i t}$ − $(n / j)^{i t}$) + (i t) ($(n / j)^{-i t}$ + $(n / j)^{i t}$)) /. j → 7 /. s → 3 /. t → 4 /. n → 30]

0. + 0.00454257 i

N[n^(−1 − s) j^s ((1 + s) ($E^{-i Log[n/j] t}$ − $E^{i Log[n/j] t}$) + (i t) ($E^{-i Log[n/j] t}$ + $E^{i Log[n/j] t}$)) /. j → 7 /. s → 3 /. t → 4 /. n → 30]

0. + 0.00454257 i

N[n^(−1 − s) j^s (−(1 + s) 2 I Sin[Log[n / j] t] + (i t) 2 Cos[Log[n / j] t]) /. j → 7 /. s → 3 /. t → 4 /. n → 30]

0. + 0.00454257 i

N[n^−1 (n / j)^(−s) 2 I (t Cos[Log[n / j] t] − (1 + s) Sin[Log[n / j] t]) /. j → 7 /. s → 3 /. t → 4 /. n → 30]

0. + 0.00454257 i

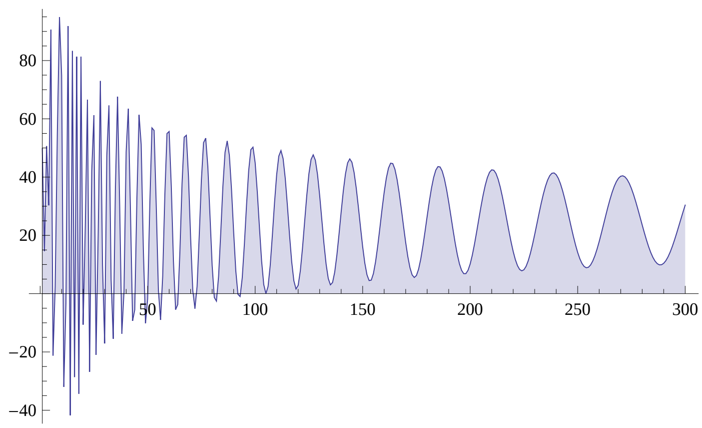
```
N[n^-1 (n / j) ^ (s) 2 I (t Cos[Log[n / j] t] - (1 - s) Sin[Log[n / j] t]) /. j → 7 /. s → -3 /. t → 4 /.
  n → 30]
```

0. + 0.00454257 i

```
bl[n_, s_, t_] := 2 I n^s Sum[ j^-s (t Cos[Log[n / j] t] - (1 - s) Sin[Log[n / j] t]), {j, 1, n}]
```

bl2[n_, A_, f_] := n^(A + 1 / 2) $\sum_{j=1}^{n} j^{-\frac{1}{2}-A} \left(f\, \mathrm{Cos}\left[f\, \mathrm{Log}\left[\frac{n}{j}\right]\right] - \left(\frac{1}{2} - A\right) \mathrm{Sin}\left[f\, \mathrm{Log}\left[\frac{n}{j}\right]\right]\right)$

```
DiscretePlot[{bl2[n, -1, N@Im@ZetaZero@10]}, {n, 1, 300}]
```




```
bl[100, -2, 0]
```

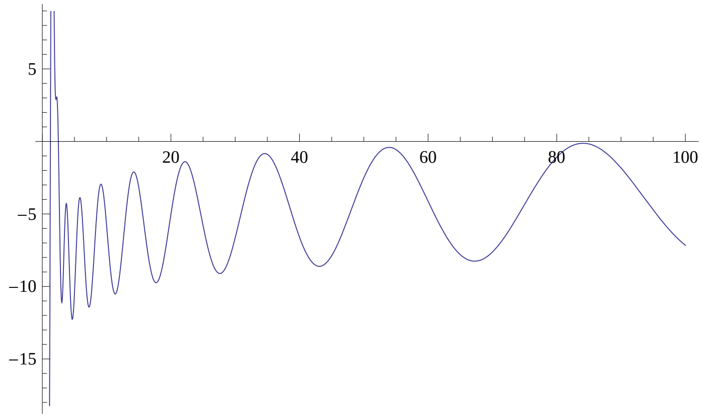
0

```
2 I n^s Sum[ j^-s (t Cos[Log[n / j] t] - (1 - s) Sin[Log[n / j] t]), {j, 1, n}] /. t → f /. s → A + 1 / 2
```

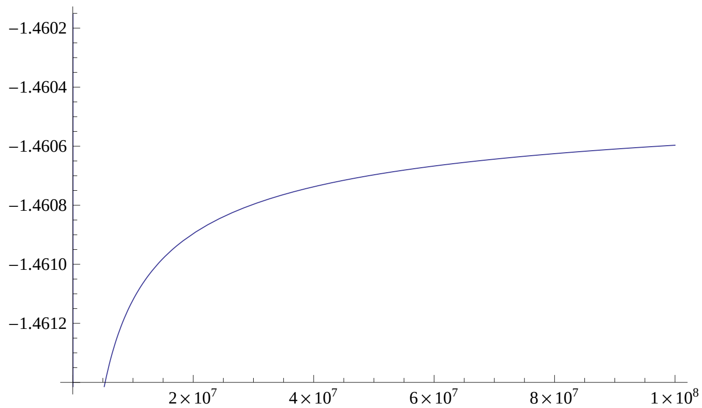
$$2\, i\, n^{\frac{1}{2}+A} \sum_{j=1}^{n} j^{-\frac{1}{2}-A} \left(f\, \mathrm{Cos}\left[f\, \mathrm{Log}\left[\frac{n}{j}\right]\right] - \left(\frac{1}{2} - A\right) \mathrm{Sin}\left[f\, \mathrm{Log}\left[\frac{n}{j}\right]\right]\right)$$

```
h1[n_, x_] :=
 (-1 / 2 - x) n^-x HarmonicNumber[n, 1 / 2 - x] - (-1 / 2 + x) n^x HarmonicNumber[n, 1 / 2 + x]
h2[n_, x_] := (-1 / 3 - x) n^-x HarmonicNumber[n, 1 / 3 - x] -
  (-1 / 3 + x) n^x HarmonicNumber[n, 1 / 3 + x]
h3[n_, s_, t_] := ((1 - s) n^s HarmonicNumber[n, s] - (1 - t) n^t HarmonicNumber[n, t]) /
  ((1 - s) n^s - (1 - t) n^t)
h3a[n_, s_, t_] := ((1 - t) n^t HarmonicNumber[n, t] - n) / ((1 - t) n^t - 1)
h3b[n_, s_, t_] := ((1 - t) n^t HarmonicNumber[n, t] - n)
```

```
Plot[Im@h2[n, N@Im@ZetaZero@1 I], {n, 1, 100}]
```




```
Plot[Re@h3a[n, -2, .5], {n, 1, 100 000 000}]
```




```
N@Zeta[.5]
```

-1.46035

```
((1 - s) n^s HarmonicNumber[n, s] - (1 - t) n^t HarmonicNumber[n, t]) /
  ((1 - s) n^s - (1 - t) n^t) /. s → 0
```

$$\frac{\text{n} - \text{n}^{\text{t}}\ (1 - \text{t})\ \text{HarmonicNumber}[\text{n}, \text{t}]}{1 - \text{n}^{\text{t}}\ (1 - \text{t})}$$

```
(1 - (1 - t) n^(t - 1) HarmonicNumber[n, t]) / (1 / n - (1 - t) n^(t - 1))
```

$$\text{Expand@}\frac{1 - \text{n}^{-1+\text{t}}\ (1 - \text{t})\ \text{HarmonicNumber}[\text{n}, \text{t}]}{\frac{1}{\text{n}} - \text{n}^{-1+\text{t}}\ (1 - \text{t})}$$

$$\frac{1}{\frac{1}{\text{n}} - \text{n}^{-1+\text{t}}\ (1 - \text{t})} - \frac{\text{n}^{-1+\text{t}}\ \text{HarmonicNumber}[\text{n}, \text{t}]}{\frac{1}{\text{n}} - \text{n}^{-1+\text{t}}\ (1 - \text{t})} + \frac{\text{n}^{-1+\text{t}}\ \text{t}\ \text{HarmonicNumber}[\text{n}, \text{t}]}{\frac{1}{\text{n}} - \text{n}^{-1+\text{t}}\ (1 - \text{t})}$$

$$\text{FullSimplify@}\frac{1}{\frac{1}{\text{n}} - \text{n}^{-1+\text{t}}\ (1 - \text{t})}$$

$$\frac{\text{n}}{1 + \text{n}^{\text{t}}\ (-1 + \text{t})}$$

$$\text{FullSimplify}\left[-\frac{n^{-1+t}\,\text{HarmonicNumber}[n,t]}{\frac{1}{n}-n^{-1+t}\,(1-t)}+\frac{n^{-1+t}\,t\,\text{HarmonicNumber}[n,t]}{\frac{1}{n}-n^{-1+t}\,(1-t)}\right]$$

$$\frac{n^t\,(-1+t)\,\text{HarmonicNumber}[n,t]}{1+n^t\,(-1+t)}$$

```
(n - (1 - t) n^t HarmonicNumber[n, t]) / (1 - (1 - t) n^t) /. t → 1 / 2 + 10 I
```

$$\frac{n-\left(\frac{1}{2}-10\,i\right)n^{\frac{1}{2}+10\,i}\,\text{HarmonicNumber}\left[n,\frac{1}{2}+10\,i\right]}{1-\left(\frac{1}{2}-10\,i\right)n^{\frac{1}{2}+10\,i}}$$

$$\text{Limit}\left[\frac{n-\left(\frac{1}{2}-10\,i\right)n^{\frac{1}{2}+10\,i}\,\text{HarmonicNumber}\left[n,\frac{1}{2}+10\,i\right]}{1-\left(\frac{1}{2}-10\,i\right)n^{\frac{1}{2}+10\,i}},\ n\to 1\,000\,000\,000.\right]$$

$1.54491 - 0.11533\,i$

```
Zeta[.5 + 10 I]
```

$1.5449 - 0.115336\,i$

```
((1 - t) n^t HarmonicNumber[n, t] - n) / ((1 - t) n^t - 1) /. t → s
```

$$\frac{-n+n^s\,(1-s)\,\text{HarmonicNumber}[n,s]}{-1+n^s\,(1-s)}$$

```
pil[n_, s_] := Sum[(1 - s) (n / j)^s - 1, {j, 1, n}] / ((1 - s) n^s - 1)
pil[10 000 000, .5 + 10 I]
```

$1.54476 - 0.11523\,i$

```
hx[n_, t_] := 2 ((1 - t) n^t HarmonicNumber[n, t] - n)
hx2[n_, t_] := ((1 - t) n^(t - 1) HarmonicNumber[n, t] - 1)
h3f[n_, t_] := ((1 - t) n^t HarmonicNumber[n, t] - n) / ((1 - t) n^t - 1)
h3ff[n_, t_] := ((1 - t) n^t HarmonicNumber[n, t] - n) / ((1 - t) n^t)
h3fa[n_, t_] := (-n) / ((1 - t) n^t - 1)
h3fax[n_, t_] := (-n) / ((1 - t) n^t)
h3fb[n_, t_] := ((1 - t) n^t HarmonicNumber[n, t]) / ((1 - t) n^t - 1)
hy[n_, t_] := 2 ((1 - t) n^t Sum[j^-t, {j, 1, n}] - n)
hy2[n_, s_] := 2 Sum[(1 - s) (n / j)^s - 1, {j, 1, n}]
gg[n_, s_] := 2 Sum[(1 + s) (j / n)^s - 1, {j, 1, n}]
```

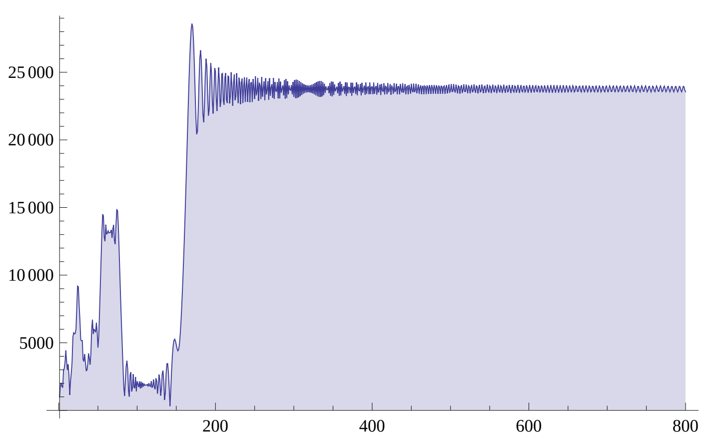
ggd[n_, s_] := $2\sum_{j=1}^{n}\left(\left(\frac{j}{n}\right)^{s}+\left(\frac{j}{n}\right)^{s}(1+s)\,\mathrm{Log}\left[\frac{j}{n}\right]\right)$

```
ggt[n_, s_] := 2 Table[(1 + s) (j / n)^s - 1, {j, 1, n}]
gggd[n2_, s_] := DiscretePlot[{Re@ggd[n, s], Im@ggd[n, s]}, {n, 1, n2}]
ggg2d[n2_, s_] := DiscretePlot[Abs@ggd[n, s], {n, 1, n2}]
ggg[n2_, s_] := DiscretePlot[{Re@gg[n, s], Im@gg[n, s]}, {n, 1, n2}]
ggg2[n2_, s_] := DiscretePlot[Abs@gg[n, s], {n, 1, n2}]
gga[n_, s_] := 2 ((1 + s) n^-s HarmonicNumber[n, -s] - n) - (1 + s)
```

dgga[n_, s_] := $2\left(-1-n^{-1-s}\,s\,(1+s)\,\mathrm{HarmonicNumber}[n,-s]-n^{-s}\,s\,(1+s)\,(-\mathrm{HarmonicNumber}[n,1-s]+\mathrm{Zeta}[1-s])\right)$

```
ggga[n2_, s_] := DiscretePlot[{Re@gga[n, s], Im@gga[n, s]}, {n, 1, n2}]
dggga[n2_, s_] := DiscretePlot[{Re@dgga[n, s], Im@dgga[n, s]}, {n, 1, n2}]
ggg2a[n2_, s_] := DiscretePlot[Abs@gga[n, s], {n, 1, n2}]
dggg2a[n2_, s_] := DiscretePlot[Abs@dgga[n, s], {n, 1, n2}]
ggg2as[n2_, s_] := Plot[Abs@gga[n, s], {n, 1, n2}]
gggas[n2_, s_] := Plot[{Re@gga[n, s], Im@gga[n, s]}, {n, 1, n2}]
```

```
ggg2a[800, 1000. I]
```




```
Abs[1 + N@Im@ZetaZero@1000]
```

1420.42

```
Integrate[j^-s, {j, 0, n}]^-1
```

ConditionalExpression$\left[-n^{-1+s}\,(-1+s),\ \mathrm{Re}[s]<1\right]$

```
-N@ZetaZero@1000
```

$-0.5-1419.42\,i$

```
Limit[gg[n, -1 / 10], n → Infinity]
```

$-\infty$

```
Abs[(1 + 14.134725141734695` I) + 1]
```

14.2755

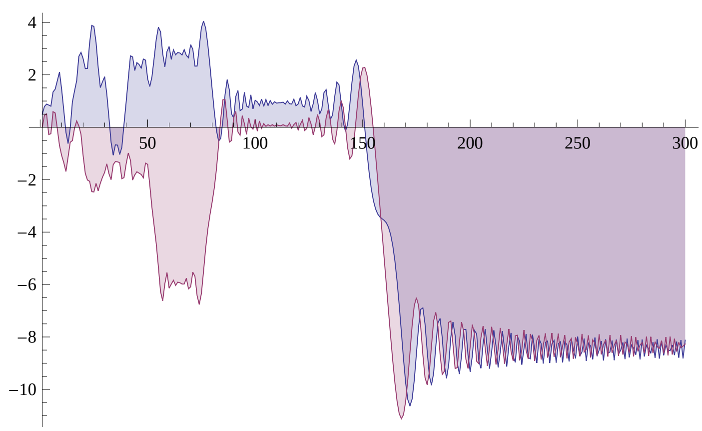
```
D[2 Sum[(1 + s) (j / n) ^s - 1, {j, 1, n}], s]
```

$$2 \sum_{j=1}^{n} \left(\left(\frac{j}{n}\right)^{s} + \left(\frac{j}{n}\right)^{s} (1+s) \operatorname{Log}\left[\frac{j}{n}\right] \right)$$

```
FullSimplify@ggt[30, -1 / 2 + Im@ZetaZero@1 I]

igg[n_, s_] := 2 n ((1 + s) n^- (s + 1) Sum[j^s, {j, 1, n}] - 1 - (1 + s) / (2 n))
iagg[n_, s_] := (2 (1 + s) n^- s Sum[j^s, {j, 1, n}] - 2 n - (1 + s))
igge[n_, s_] := Sum[j^s, {j, 1, n}] - n^(1 + s) / (1 + s) - n^s / 2
iggg[n2_, s_] := DiscretePlot[{Re@igge[n, s], Im@igge[n, s]}, {n, 1, n2}]
```

```
iggg[300, 1000. I]
```




```
720 / 12
```

60

```
(BernoulliB[4] / 4!)
```

$-\frac{1}{720}$

```
(BernoulliB[2] / 2!) / (BernoulliB[4] / 4!)
```

-60

```
(BernoulliB[4] / 4!) / (BernoulliB[6] / 6!)
```

-42

```
Table[Limit[D[x / (E^x - 1), {x, k}] / k !, x → 0] D[n^s, {n, k - 1}], {k, 1, 9}] // TableForm
```

$-\frac{n^s}{2}$

$\frac{1}{12} n^{-1+s} s$

0

$-\frac{1}{720} n^{-3+s} (-2+s)(-1+s) s$

0

$\frac{n^{-5+s} (-4+s)(-3+s)(-2+s)(-1+s) s}{30240}$

0

$-\frac{n^{-7+s} (-6+s)(-5+s)(-4+s)(-3+s)(-2+s)(-1+s) s}{1209600}$

0

```
Table[Limit[D[x / (E^x - 1), {x, k}] / k !, x → 0], {k, 0, 5}]
```

$\left\{1, -\frac{1}{2}, \frac{1}{12}, 0, -\frac{1}{720}, 0\right\}$

```
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k !
Table[bin[s, k] BernoulliB[k + 1] n^(s - k) / (k + 1), {k, 0, 5}] // TableForm
```

$-\frac{n^s}{2}$

$\frac{1}{12} n^{-1+s} s$

0

$-\frac{1}{720} n^{-3+s} (-2+s)(-1+s) s$

0

$\frac{n^{-5+s} (-4+s)(-3+s)(-2+s)(-1+s) s}{30240}$

```
D[2 ((1 + s) n^-s HarmonicNumber[n, -s] - n) - (1 + s), n]
```

$2\left(-1 - n^{-1-s} s (1+s) \text{HarmonicNumber}[n, -s] - n^{-s} s (1+s) (-\text{HarmonicNumber}[n, 1-s] + \text{Zeta}[1-s])\right)$

```
2 (1 + s) n^-s j^s - 2 n - (1 + s)
```

$-1 - s + 2 (-n + j^s n^{-s} (1+s))$

```
2 (1 + s) n^-s j^s - 2 n - (1 + s) /. s → A + f I
```

$-1 - A - i f - 2 n + 2 (1 + A + i f) j^{A+i f} n^{-A-i f}$

```
FullSimplify[ComplexExpand[Re[-1 - A - i f - 2 n + 2 (1 + A + i f) j^(A+i f) n^(-A-i f)]],
   {Element[j, Integers], Element[n, Integers]}] /.
  Arg[n] → 0 /. Arg[j] → 0 /. Abs[n] → n /. Abs[j] → j
```

$n^{-A}\left(-n^A (1+A+2n) + 2 j^A \left((1+A) \text{Cos}\left[\frac{1}{2} f \text{Log}\left[\frac{j^2}{n^2}\right]\right] - f \text{Sin}\left[\frac{1}{2} f \text{Log}\left[\frac{j^2}{n^2}\right]\right]\right)\right)$

$\mathbf{n^{-A}\left(-n^A (1+A+2n) + 2 j^A \left((1+A) \text{Cos}\left[f \text{Log}\left[\frac{j}{n}\right]\right] - f \text{Sin}\left[f \text{Log}\left[\frac{j}{n}\right]\right]\right)\right)}$

$\mathbf{\left(-(1+A+2n) + n^{-A} 2 j^A \left((1+A) \text{Cos}\left[f \text{Log}\left[\frac{j}{n}\right]\right] - f \text{Sin}\left[f \text{Log}\left[\frac{j}{n}\right]\right]\right)\right)}$

$-1 - A - 2n + 2 j^A n^{-A}\left((1+A) \text{Cos}\left[f \text{Log}\left[\frac{j}{n}\right]\right] - f \text{Sin}\left[f \text{Log}\left[\frac{j}{n}\right]\right]\right)$

```
FullSimplify[
  ComplexExpand[Re[-1 - A - ⅈ f + 2 (-n + (1 + A + ⅈ f) n^(-A-ⅈ f) HarmonicNumber[n, -A - ⅈ f])]],
  {Element[j, Integers], Element[n, Integers]}] /.
 Arg[n] → 0 /. Arg[j] → 0 /. Abs[n] → n /. Abs[j] → j
```

$$n^{-A}\left(-n^{A}(1+A+2n)+\left(\frac{1}{\text{Sign}[n]}\right)^{-i f}\left(2(1+A-i f)\,n^{i f}\,\text{Re}[\text{HarmonicNumber}[n,-A-i f]]\,\text{Sign}[n]^{A}+2 i\,\text{HarmonicNumber}[n,-A-i f]\left(f\,\text{Cos}\left[\frac{1}{2}f\,\text{Log}\left[n^{2}\right]\right]-(1+A)\,\text{Sin}\left[\frac{1}{2}f\,\text{Log}\left[n^{2}\right]-i A\,\text{Log}[\text{Sign}[n]]\right]\right)\right)\right)$$

```
2 ((1 + s) n^-s HarmonicNumber[n, -s] - n) - (1 + s) /. s → A + f I
```

$$-1-A-i f+2\left(-n+(1+A+i f)\,n^{-A-i f}\,\text{HarmonicNumber}[n,-A-i f]\right)$$

```
CForm[
 FullSimplify[ComplexExpand[Re[-1 - A - ⅈ f - 2 n + 2 (1 + A + ⅈ f) n^(-A-ⅈ f)]], {Element[j, Integers],
     Element[n, Integers]}] /. Arg[n] → 0 /. Arg[j] → 0 /. Abs[n] → n /. Abs[j] → j]
```

```
(-(Power(n,A)*(1 + A + 2*n)) + 2*((1 + A)*Cos((f*Log(Power(n,2)))/2.) + f*Sin((f*Log(Power(
```

```
( (1 + A + 2 n) + 2 n^-A ((1 + A) Cos[1/2 f Log[n^2]] + f Sin[1/2 f Log[n^2]]))
```

$$1+A+2n+2n^{-A}\left((1+A)\,\text{Cos}\left[\frac{1}{2}f\,\text{Log}\left[n^{2}\right]\right]+f\,\text{Sin}\left[\frac{1}{2}f\,\text{Log}\left[n^{2}\right]\right]\right)$$

```
CForm[
 FullSimplify[ComplexExpand[Im[-1 - A - ⅈ f - 2 n + 2 (1 + A + ⅈ f) n^(-A-ⅈ f)]], {Element[j, Integers],
     Element[n, Integers]}] /. Arg[n] → 0 /. Arg[j] → 0 /. Abs[n] → n /. Abs[j] → j]
```

```
(-(f*Power(n,A)) + 2*(f*Cos((f*Log(Power(n,2)))/2.) - (1 + A)*Sin((f*Log(Power(n,2)))/2.)))
```

```
(-f + 2 n^-A (f Cos[1/2 f Log[n^2]] - (1 + A) Sin[1/2 f Log[n^2]]))
```

$$-f+2n^{-A}\left(f\,\text{Cos}\left[\frac{1}{2}f\,\text{Log}\left[n^{2}\right]\right]-(1+A)\,\text{Sin}\left[\frac{1}{2}f\,\text{Log}\left[n^{2}\right]\right]\right)$$

```
1 + A + 2 n + 2 n^-A ((1 + A) Cos[f Log[n]] + f Sin[f Log[n]])
```

$$1+A+2n+2n^{-A}\left((1+A)\,\text{Cos}[f\,\text{Log}[n]]+f\,\text{Sin}[f\,\text{Log}[n]]\right)$$

```
-f + 2 n^-A (f Cos[ f Log[n]] - (1 + A) Sin[ f Log[n]])
```

$$-f+2n^{-A}\left(f\,\text{Cos}[f\,\text{Log}[n]]-(1+A)\,\text{Sin}[f\,\text{Log}[n]]\right)$$

```
CForm[1 + A + 2 n + 2 n^-A ((1 + A) Cos[f Log[n]] + f Sin[f Log[n]])]
```

```
1 + A + 2*n + (2*((1 + A)*Cos(f*Log(n)) + f*Sin(f*Log(n))))/Power(n,A)
```

```
CForm[-f + 2 n^-A (f Cos[f Log[n]] - (1 + A) Sin[f Log[n]])]
```

```
-f + (2*(f*Cos(f*Log(n)) - (1 + A)*Sin(f*Log(n))))/Power(n,A)
```

```
plo[n_, A_, f_] := (1 + A + 2 n + 2 n^-A ((1 + A) Cos[f Log[n]] + f Sin[f Log[n]])) Sum[j^A
```

```
Expand[2 ((1 + s) n^-s HarmonicNumber[n, -s] - n) - (1 + s) /. s → A + f I] /. n → 10 /. A -> .5 /.
 f → 10
```

-13.6572 - 4.89332 i

```
Expand[2 ((1 + s) n^-s Sum[j^s, {j, 1, n}] - n) - (1 + s) /. s → A + f I]
```

$-1 - A - i f - 2 n + 2 n^{-A-i f}$ HarmonicNumber[n, -A - i f] +
$2 A n^{-A-i f}$ HarmonicNumber[n, -A - i f] + $2 i f n^{-A-i f}$ HarmonicNumber[n, -A - i f]

```
2 ((1 + A + f I) n^-(A + f I) Sum[j^A (Cos[f Log[j]] + I Sin[f Log[j]]), {j, 1, n}] - n) -
    (1 + A + f I) /. n → 10 /. A -> .5 /. f → 10
```

-13.6572 - 4.89332 i

```
2 ((1 + A + f I) n^-(A + f I) ( Sum[j^A (Cos[f Log[j]]), {j, 1, n}] +
          I Sum[j^A ( Sin[f Log[j]]), {j, 1, n}]) - n) - (1 + A + f I) /. n → 10 /. A -> .5 /. f → 10
```

-13.6572 - 4.89332 i

```
- (1 + A) - f I - 2 n + 2 ((1 + A) + f I) n^-A E^(-f Log[n] I) ( Sum[j^A (Cos[f Log[j]]), {j, 1, n}] +
       I Sum[j^A ( Sin[f Log[j]]), {j, 1, n}]) /. n → 10 /. A -> .5 /. f → 10
```

-13.6572 - 4.89332 i

```
- (1 + A) - f I - 2 n + 2 ((1 + A) + f I) n^-A
     (Cos[f Log[n]] - I Sin[f Log[n]]) ( Sum[j^A (Cos[f Log[j]]), {j, 1, n}] +
       I Sum[j^A ( Sin[f Log[j]]), {j, 1, n}]) /. n → 10 /. A -> .5 /. f → 10
```

-13.6572 - 4.89332 i

```
- (1 + A) - f I - 2 n +
    2 ((1 + A) + f I) n^-A (Cos[f Log[n]] - I Sin[f Log[n]]) Sum[j^A (Cos[f Log[j]]), {j, 1, n}] +
    I 2 ((1 + A) + f I) n^-A (Cos[f Log[n]] - I Sin[f Log[n]])
     Sum[j^A ( Sin[f Log[j]]), {j, 1, n}] /. n → 10 /. A -> .5 /. f → 10
```

-13.6572 - 4.89332 i

```
- (1 + A) - f I - 2 n + 2 n^-A (((1 + A) + f I) Cos[f Log[n]] - I ((1 + A) + f I) Sin[f Log[n]])
     Sum[j^A (Cos[f Log[j]]), {j, 1, n}] +
    I 2 n^-A (((1 + A) + f I) Cos[f Log[n]] - I ((1 + A) + f I) Sin[f Log[n]])
     Sum[j^A ( Sin[f Log[j]]), {j, 1, n}] /. n → 10 /. A -> .5 /. f → 10
```

-13.6572 - 4.89332 i

```
- (1 + A) - f I - 2 n + 2 n^-A (((1 + A)) Cos[f Log[n]] - I (f I) Sin[f Log[n]] +
       (f I) Cos[f Log[n]] - I ((1 + A)) Sin[f Log[n]]) Sum[j^A (Cos[f Log[j]]), {j, 1, n}] +
    I 2 n^-A (((1 + A)) Cos[f Log[n]] - I (f I) Sin[f Log[n]] + (f I) Cos[f Log[n]] - I ((1 + A))
        Sin[f Log[n]]) Sum[j^A ( Sin[f Log[j]]), {j, 1, n}] /. n → 10 /. A -> .5 /. f → 10
```

-13.6572 - 4.89332 i

```
- (1 + A) - 2 n - f I +
    2 n^-A ((1 + A) Cos[f Log[n]] + f Sin[f Log[n]] + I (f Cos[f Log[n]] - (1 + A) Sin[f Log[n]]))
     Sum[j^A (Cos[f Log[j]]), {j, 1, n}] + I 2 n^-A
     (((1 + A)) Cos[f Log[n]] + f Sin[f Log[n]] + I (f Cos[f Log[n]] - (1 + A) Sin[f Log[n]]))
     Sum[j^A ( Sin[f Log[j]]), {j, 1, n}] /. n → 10 /. A -> .5 /. f → 10
```

-13.6572 - 4.89332 i

```
-(1 + A) - 2 n - f I +
    2 n^-A ((1 + A) Cos[f Log[n]] + f Sin[f Log[n]]) Sum[j^A (Cos[f Log[j]]), {j, 1, n}] +
    2 n^-A (I (f Cos[f Log[n]] - (1 + A) Sin[f Log[n]])) Sum[j^A (Cos[f Log[j]]), {j, 1, n}] +
    I 2 n^-A (((1 + A)) Cos[f Log[n]] + f Sin[f Log[n]]) Sum[j^A ( Sin[f Log[j]]), {j, 1, n}] +
    I 2 n^-A (I (f Cos[f Log[n]] - (1 + A) Sin[f Log[n]]))
     Sum[j^A ( Sin[f Log[j]]), {j, 1, n}] /. n → 10 /. A -> .5 /. f → 10
```

```
-13.6572 - 4.89332 i
```

```
-(1 + A) - 2 n - f I +
    2 n^-A ((1 + A) Cos[f Log[n]] + f Sin[f Log[n]]) Sum[j^A (Cos[f Log[j]]), {j, 1, n}] +
    I 2 n^-A (f Cos[f Log[n]] - (1 + A) Sin[f Log[n]]) Sum[j^A (Cos[f Log[j]]), {j, 1, n}] +
    I 2 n^-A ((1 + A) Cos[f Log[n]] + f Sin[f Log[n]]) Sum[j^A ( Sin[f Log[j]]), {j, 1, n}] +
    - 2 n^-A (f Cos[f Log[n]] - (1 + A) Sin[f Log[n]]) Sum[j^A ( Sin[f Log[j]]), {j, 1, n}] /.
   n → 10 /. A -> .5 /. f → 10
```

```
-13.6572 - 4.89332 i
```

```
-(1 + A) - 2 n +
    2 n^-A ((1 + A) Cos[f Log[n]] + f Sin[f Log[n]]) Sum[j^A (Cos[f Log[j]]), {j, 1, n}] +
    - 2 n^-A (f Cos[f Log[n]] - (1 + A) Sin[f Log[n]]) Sum[j^A ( Sin[f Log[j]]), {j, 1, n}] +
    -f I +
    I 2 n^-A (f Cos[f Log[n]] - (1 + A) Sin[f Log[n]]) Sum[j^A (Cos[f Log[j]]), {j, 1, n}] +
    I 2 n^-A ((1 + A) Cos[f Log[n]] + f Sin[f Log[n]]) Sum[j^A ( Sin[f Log[j]]), {j, 1, n}] /.
   n → 10 /. A -> .5 /. f → 10
```

```
-13.6572 - 4.89332 i
```

```
-(1 + A) - 2 n +
    2 n^-A ( ((1 + A) Cos[f Log[n]] + f Sin[f Log[n]]) Sum[j^A (Cos[f Log[j]]), {j, 1, n}] +
       - (f Cos[f Log[n]] - (1 + A) Sin[f Log[n]]) Sum[j^A ( Sin[f Log[j]]), {j, 1, n}]) +
    I (-f +
       2 n^-A ((f Cos[f Log[n]] - (1 + A) Sin[f Log[n]]) Sum[j^A (Cos[f Log[j]]), {j, 1, n}] +
          ((1 + A) Cos[f Log[n]] + f Sin[f Log[n]]) Sum[j^A ( Sin[f Log[j]]), {j, 1, n}])) /.
   n → 10 /. A -> .5 /. f → 10
```

```
-13.6572 - 4.89332 i
```

```
-(1 + A) - 2 n +
   2 n^-A ( ((1 + A) Cos[f Log[n]] + f Sin[f Log[n]]) Sum[j^A (Cos[f Log[j]]), {j, 1, n}] +
      - (f Cos[f Log[n]] - (1 + A) Sin[f Log[n]]) Sum[j^A ( Sin[f Log[j]]), {j, 1, n}]) +
   I (-f +
      2 n^-A ((f Cos[f Log[n]] - (1 + A) Sin[f Log[n]]) Sum[j^A (Cos[f Log[j]]), {j, 1, n}] +
         ((1 + A) Cos[f Log[n]] + f Sin[f Log[n]]) Sum[j^A ( Sin[f Log[j]]), {j, 1, n}])) /.
  Sum[j^A (Cos[f Log[j]]), {j, 1, n}] → CosSum /. Sum[j^A ( Sin[f Log[j]]), {j, 1, n}] → SinSum
```

```
-1 - A - 2 n + 2 n^-A (SinSum (-f Cos[f Log[n]] + (1 + A) Sin[f Log[n]]) +
    CosSum ((1 + A) Cos[f Log[n]] + f Sin[f Log[n]])) +
 i (-f + 2 n^-A (CosSum (f Cos[f Log[n]] - (1 + A) Sin[f Log[n]]) +
       SinSum ((1 + A) Cos[f Log[n]] + f Sin[f Log[n]])))
```

```
CForm[-1 - A - 2 n + 2 n^-A (SinSum (-f Cos[f Log[n]] + (1 + A) Sin[f Log[n]]) +
     CosSum ((1 + A) Cos[f Log[n]] + f Sin[f Log[n]]))]
```

```
-1 - A - 2*n + (2*(SinSum*(-(f*Cos(f*Log(n))) + (1 + A)*Sin(f*Log(n))) + CosSum*((1 + A)*Co
```

```
CForm[-f + 2 n^-A (CosSum (f Cos[f Log[n]] - (1 + A) Sin[f Log[n]]) +
    SinSum ((1 + A) Cos[f Log[n]] + f Sin[f Log[n]]))]
```

```
-f + (2*(CosSum*(f*Cos(f*Log(n)) - (1 + A)*Sin(f*Log(n))) + SinSum*((1 + A)*Cos(f*Log(n)) +
```

```
N@ZetaZero@100 000
```

```
0.5` + 74920.82749899419` ⅈ
```

$0.5 + 74\,920.8\, i$

```
0.5` + 1419.4224809459956` ⅈ
```

$0.5 + 1419.42\, i$

```
0.5` + 9877.782654005501` ⅈ
```

$0.5 + 9877.78\, i$

```
-(1 + A) - 2 n +
 2 n^-A ( ((1 + A) Cos[f Log[n]] + f Sin[f Log[n]]) Sum[j^A (Cos[f Log[j]]), {j, 1, n}] +
    - (f Cos[f Log[n]] - (1 + A) Sin[f Log[n]]) Sum[j^A ( Sin[f Log[j]]), {j, 1, n}]) +
 I (-f +
    2 n^-A ((f Cos[f Log[n]] - (1 + A) Sin[f Log[n]]) Sum[j^A (Cos[f Log[j]]), {j, 1, n}] +
       ((1 + A) Cos[f Log[n]] + f Sin[f Log[n]]) Sum[j^A ( Sin[f Log[j]]), {j, 1, n}]))
```

$$-1 - A - 2 n + 2 n^{-A} \left(((1 + A) \mathrm{Cos}[f \mathrm{Log}[n]] + f \mathrm{Sin}[f \mathrm{Log}[n]]) \sum_{j=1}^{n} j^A \mathrm{Cos}[f \mathrm{Log}[j]] + (-f \mathrm{Cos}[f \mathrm{Log}[n]] + (1 + A) \mathrm{Sin}[f \mathrm{Log}[n]]) \sum_{j=1}^{n} j^A \mathrm{Sin}[f \mathrm{Log}[j]] \right) + i \left(-f + 2 n^{-A} \left((f \mathrm{Cos}[f \mathrm{Log}[n]] - (1 + A) \mathrm{Sin}[f \mathrm{Log}[n]]) \sum_{j=1}^{n} j^A \mathrm{Cos}[f \mathrm{Log}[j]] + ((1 + A) \mathrm{Cos}[f \mathrm{Log}[n]] + f \mathrm{Sin}[f \mathrm{Log}[n]]) \sum_{j=1}^{n} j^A \mathrm{Sin}[f \mathrm{Log}[j]] \right) \right) \; /. \; n \to E\text{^}(2 \mathrm{Pi}\, n \,/\, f) \; /. \; \mathrm{Cos}\left[f \mathrm{Log}\left[e^{\frac{2 n \pi}{f}}\right]\right] \to 1 \; /. \; \mathrm{Sin}\left[f \mathrm{Log}\left[e^{\frac{2 n \pi}{f}}\right]\right] \to 0$$

$$-1 - A - 2 e^{\frac{2 n \pi}{f}} + 2 \left(e^{\frac{2 n \pi}{f}}\right)^{-A} \left((1 + A) \sum_{j=1}^{e^{\frac{2 n \pi}{f}}} j^A \mathrm{Cos}[f \mathrm{Log}[j]] - f \sum_{j=1}^{e^{\frac{2 n \pi}{f}}} j^A \mathrm{Sin}[f \mathrm{Log}[j]] \right) + i \left(-f + 2 \left(e^{\frac{2 n \pi}{f}}\right)^{-A} \left(f \sum_{j=1}^{e^{\frac{2 n \pi}{f}}} j^A \mathrm{Cos}[f \mathrm{Log}[j]] + (1 + A) \sum_{j=1}^{e^{\frac{2 n \pi}{f}}} j^A \mathrm{Sin}[f \mathrm{Log}[j]] \right) \right)$$

$$\mathrm{Sin}\left[f \mathrm{Log}\left[e^{\frac{2 n \pi}{f}}\right]\right] \; /. \; f \to 10 \; /. \; n \to 4$$

0

$$-1-A-2e^{\frac{2n\pi}{f}}+2\left(e^{\frac{2n\pi}{f}}\right)^{-A}\left((1+A)\sum_{j=1}^{e^{\frac{2n\pi}{f}}} j^A \mathrm{Cos}[f\,\mathrm{Log}[j]]-f\sum_{j=1}^{e^{\frac{2n\pi}{f}}} j^A \mathrm{Sin}[f\,\mathrm{Log}[j]]\right)+$$
$$i\left(-f+2\left(e^{\frac{2n\pi}{f}}\right)^{-A}\left(f\sum_{j=1}^{e^{\frac{2n\pi}{f}}} j^A \mathrm{Cos}[f\,\mathrm{Log}[j]]+(1+A)\sum_{j=1}^{e^{\frac{2n\pi}{f}}} j^A \mathrm{Sin}[f\,\mathrm{Log}[j]]\right)\right)\ /.\ A\to -1/2$$

$$-\frac{1}{2}-2e^{\frac{2n\pi}{f}}+i\left(-f+2\sqrt{e^{\frac{2n\pi}{f}}}\left(f\sum_{j=1}^{e^{\frac{2n\pi}{f}}}\frac{\mathrm{Cos}[f\,\mathrm{Log}[j]]}{\sqrt{j}}+\frac{1}{2}\sum_{j=1}^{e^{\frac{2n\pi}{f}}}\frac{\mathrm{Sin}[f\,\mathrm{Log}[j]]}{\sqrt{j}}\right)\right)+$$
$$2\sqrt{e^{\frac{2n\pi}{f}}}\left(\frac{1}{2}\sum_{j=1}^{e^{\frac{2n\pi}{f}}}\frac{\mathrm{Cos}[f\,\mathrm{Log}[j]]}{\sqrt{j}}-f\sum_{j=1}^{e^{\frac{2n\pi}{f}}}\frac{\mathrm{Sin}[f\,\mathrm{Log}[j]]}{\sqrt{j}}\right)$$

$$\mathbf{CForm}\left[-\frac{1}{2}-2e^{\frac{2n\pi}{f}}+\sqrt{e^{\frac{2n\pi}{f}}}\ (\mathrm{CosSum}-2f\,\mathrm{SinSum})\right]$$

```
-0.5 - 2*Power(E,(2*n*Pi)/f) + Sqrt(Power(E,(2*n*Pi)/f))*(CosSum - 2*f*SinSum)
```

$$\mathbf{CForm}\left[-f+\sqrt{e^{\frac{2n\pi}{f}}}\ (2f\,\mathrm{CosSum}+\mathrm{SinSum})\right]$$

```
-f + Sqrt(Power(E,(2*n*Pi)/f))*(2*CosSum*f + SinSum)
```

$$\mathbf{FullSimplify}\left[-f+2\sqrt{e^{\frac{2n\pi}{f}}}\left(f\,\mathrm{CosSum}+\frac{1}{2}\mathrm{SinSum}\right)\right]$$

$$-f+\sqrt{e^{\frac{2n\pi}{f}}}\ (2\,\mathrm{CosSum}\,f+\mathrm{SinSum})$$

$$\mathbf{FullSimplify}\left[-\frac{1}{2}-2e^{\frac{2n\pi}{f}}+2\sqrt{e^{\frac{2n\pi}{f}}}\left(\frac{1}{2}\mathrm{CosSum}-f\,\mathrm{SinSum}\right)\right]$$

$$-\frac{1}{2}-2e^{\frac{2n\pi}{f}}+\sqrt{e^{\frac{2n\pi}{f}}}\ (\mathrm{CosSum}-2f\,\mathrm{SinSum})$$

N@Pi

3.141592653589793`

N@E

2.718281828459045`

$$\mathbf{CForm}\left[e^{\frac{2n\pi}{f}}\right]$$

```
Power(E,(2*n*Pi)/f)
```

N@ZetaZero[10 000 000] / 10

0.05` + 499238.10140031786` i

0.05 + 499 238. i

0.5` + 4.992381014003178`*^6 i

```
0.5 + 4.99238 × 10^6 i

0.5` + 600269.6770124449` i

0.5 + 600 270. i

0.5` + 236.5242296658162` i

0.5 + 236.524 i

4.992381014003178`20 * 1 000 000

4.9923810140031779999999999999999999999973594`20.*^5

499 238.10140031780000

10^6

1 000 000

4 992 381.0140031786

2 ((1 + s) n^-s HarmonicNumber[n, -s] - n) - (1 + s) /. n → 100 /. s → .5 + 10 I

3.18945 - 2.37779 i

-(1 + A) - 2 n +
    2 n^-A ( ((1 + A) Cos[f Log[n]] + f Sin[f Log[n]]) Sum[j^A (Cos[f Log[j]]), {j, 1, n}] +
       - (f Cos[f Log[n]] - (1 + A) Sin[f Log[n]]) Sum[j^A ( Sin[f Log[j]]), {j, 1, n}]) +
    I (-f +
       2 n^-A ((f Cos[f Log[n]] - (1 + A) Sin[f Log[n]]) Sum[j^A (Cos[f Log[j]]), {j, 1, n}] +
          ((1 + A) Cos[f Log[n]] + f Sin[f Log[n]]) Sum[j^A ( Sin[f Log[j]]), {j, 1, n}])) /.
   n → 100 /. f → 10. /. A → .5

3.18945 - 2.37779 i

2 n ((1 + s) n^(-s - 1) HarmonicNumber[n, -s] - 1) - (1 + s) /. n → 100 /. s → .5 + 10 I

3.18945 - 2.37779 i

FullSimplify[2 n (1 + s) n^(-s - 1) (Sum[j^s, {j, 1, n}] - Integrate[j^s, {j, 0, n}]) - (1 + s)]

ConditionalExpression[-1 - 2 n - s + 2 n^-s (1 + s) HarmonicNumber[n, -s], Re[s] > -1]

10 000 000^.5

3162.28

CForm@Table[Prime[k], {k, 1, PrimePi[3163]}]

List(2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97,101,103,107,109
   293,307,311,313,317,331,337,347,349,353,359,367,373,379,383,389,397,401,409,419,421,431,
   641,643,647,653,659,661,673,677,683,691,701,709,719,727,733,739,743,751,757,761,769,773,
   1013,1019,1021,1031,1033,1039,1049,1051,1061,1063,1069,1087,1091,1093,1097,1103,1109,111
   1303,1307,1319,1321,1327,1361,1367,1373,1381,1399,1409,1423,1427,1429,1433,1439,1447,145
   1619,1621,1627,1637,1657,1663,1667,1669,1693,1697,1699,1709,1721,1723,1733,1741,1747,175
   1979,1987,1993,1997,1999,2003,2011,2017,2027,2029,2039,2053,2063,2069,2081,2083,2087,208
   2297,2309,2311,2333,2339,2341,2347,2351,2357,2371,2377,2381,2383,2389,2393,2399,2411,241
   2659,2663,2671,2677,2683,2687,2689,2693,2699,2707,2711,2713,2719,2729,2731,2741,2749,275
   2969,2971,2999,3001,3011,3019,3023,3037,3041,3049,3061,3067,3079,3083,3089,3109,3119,312
```

```
Table[Factorial[k], {k, 1, 40}]
```

```
{1, 2, 6, 24, 120, 720, 5040, 40 320, 362 880, 3 628 800, 39 916 800,
 479 001 600, 6 227 020 800, 87 178 291 200, 1 307 674 368 000, 20 922 789 888 000,
 355 687 428 096 000, 6 402 373 705 728 000, 121 645 100 408 832 000, 2 432 902 008 176 640 000,
 51 090 942 171 709 440 000, 1 124 000 727 777 607 680 000, 25 852 016 738 884 976 640 000,
 620 448 401 733 239 439 360 000, 15 511 210 043 330 985 984 000 000, 403 291 461 126 605 635 584 000 000,
 10 888 869 450 418 352 160 768 000 000, 304 888 344 611 713 860 501 504 000 000,
 8 841 761 993 739 701 954 543 616 000 000, 265 252 859 812 191 058 636 308 480 000 000,
 8 222 838 654 177 922 817 725 562 880 000 000, 263 130 836 933 693 530 167 218 012 160 000 000,
 8 683 317 618 811 886 495 518 194 401 280 000 000, 295 232 799 039 604 140 847 618 609 643 520 000 000,
 10 333 147 966 386 144 929 666 651 337 523 200 000 000,
 371 993 326 789 901 217 467 999 448 150 835 200 000 000,
 13 763 753 091 226 345 046 315 979 581 580 902 400 000 000,
 523 022 617 466 601 111 760 007 224 100 074 291 200 000 000,
 20 397 882 081 197 443 358 640 281 739 902 897 356 800 000 000,
 815 915 283 247 897 734 345 611 269 596 115 894 272 000 000 000}
```

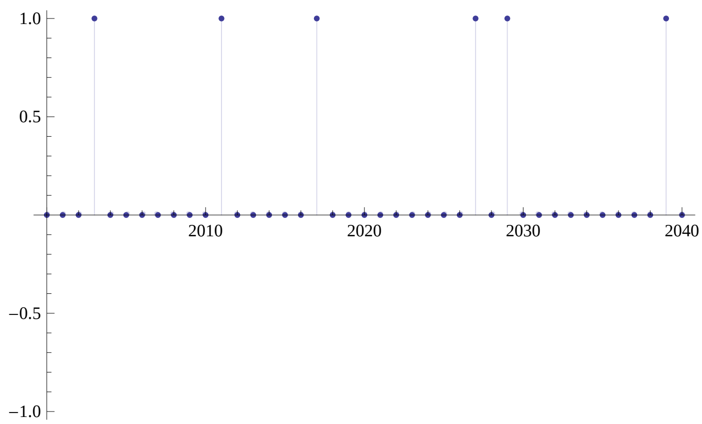
```
LogIntegral[1 000 000.]
```

78 627.5

```
N@Sum[PrimePi[100 000^(1/k)]/k, {k, 1, Log2@100 000}]
```

9633.77

```
DiscretePlot[If[PrimeQ[j], 1, 0], {j, 2000, 2040}]
```




```
Log[10 000 000.]
```

16.1181

```
N@EulerGamma
```

0.577216

```
gga[n_, s_] := 2 n ((1 + s) n^(-s - 1) HarmonicNumber[n, -s] - 1) - (1 + s)
ggaa[n_, s_] := gga[n, s] / (2 n^-s (1 + s))
gga1[n_, s_] := (2 ((1 + s) n^-s HarmonicNumber[n, -s] - n) - (1 + s)) / (2 n^-s (1 + s))
```

gga2[n_, s_] := HarmonicNumber[n, -s] $- \frac{n^{1+s}}{1+s} + \frac{n^{s}\,(-1-s)}{2\,(1+s)}$

gga3[n_, s_] := HarmonicNumber[n, -s] $- \frac{n^{1+s}}{1+s} - \frac{n^{s}}{2}$

```
gga[10 000 000 000, -N@ZetaZero[1]]
```

$-8.84041\times10^{-6}+0.000161622\,i$

```
Zeta[-(.5 + 200 I)]
```

$32.8715+49.3787\,i$

```
(2 ((1 + s) n^-s HarmonicNumber[n, -s] - n) - (1 + s)) / (2 n^-s (1 + s)) /. s → 0
```

$-\frac{1}{2}$

```
FullSimplify[(2 ((1 + s) n^-s HarmonicNumber[n, -s] - n) - (1 + s)) / (2 n^-s (1 + s))]
```

$-\frac{n^s\,(1+2\,n+s)}{2\,(1+s)}+\text{HarmonicNumber}[n,-s]$

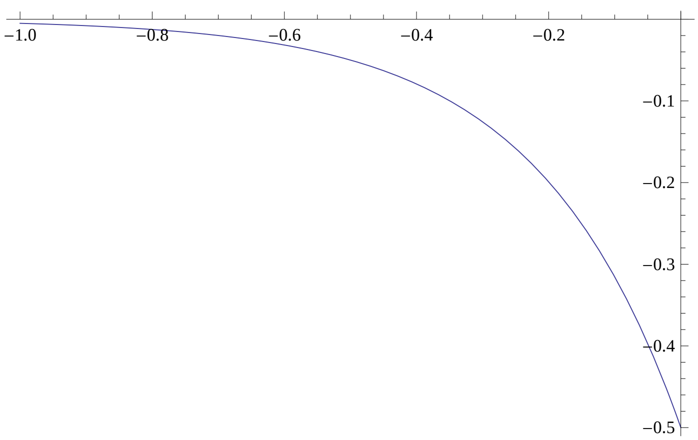
```
(2 (-n)) / (2 n^-s (1 + s))
```

$-\frac{n^{1+s}}{1+s}$

```
(-(1 + s)) / (2 n^-s (1 + s))
```

$\frac{n^s\,(-1-s)}{2\,(1+s)}$

Plot$\left[\frac{n^s\,(-1-s)}{2\,(1+s)}\ /.\ n\to100,\ \{s,-1,0\}\right]$



HarmonicNumber$[n,-s]-\frac{n^{1+s}}{1+s}+\frac{n^s\,(-1-s)}{2\,(1+s)}\ /.\ s\to-s$

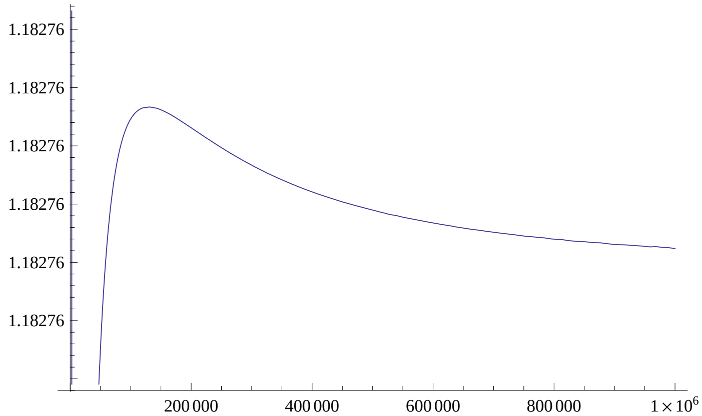
$-\frac{n^{1-s}}{1-s}+\frac{n^{-s}\,(-1+s)}{2\,(1-s)}+\text{HarmonicNumber}[n,s]$

```
ggo[n_, s_] := (1 + s) - 2 n ((1 + s) n^(-s - 1) HarmonicNumber[n, -s] - 1)
```

```
Limit[ggo[n, I], n → Infinity]
```

Indeterminate

```
Plot[Abs[ggo[n, I]], {n, 0, 1000000}]
```




```
Limit[Abs[ggo[n, I]], n → Infinity]
```

```
$Aborted
```

```
N@E
```

```
2.71828
```

```
N@Log[Pi]
```

```
1.14473
```

```
gga1[n_, s_] := ((1 + s) - 2 ((1 + s) n^-s HarmonicNumber[n, -s] - n)) / (-2 n^-s (1 + s))
gga1a[n_, s_] := 1 / (-2 n^-s (1 + s))
ggoo[n_, s_] := ((1 + s) - 2 n ((1 + s) n^(-s - 1) HarmonicNumber[n, -s] - 1))
```

```
Limit[ggoo[n, -ZetaZero[1]], n → Infinity]
```

```
0
```

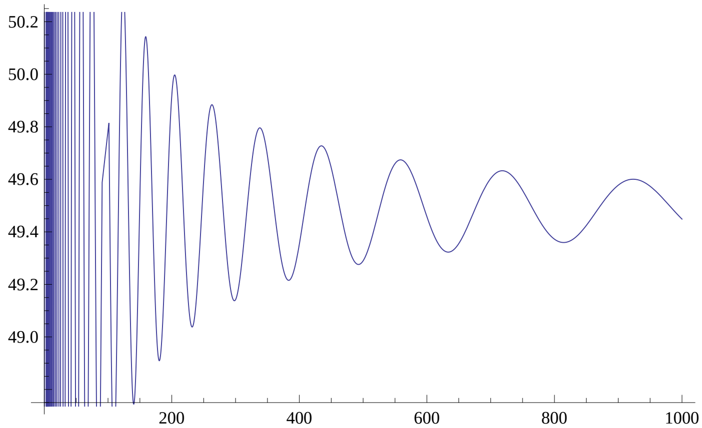
```
Zeta[-(.5 + 3 I)]
```

```
0.352914 - 0.012125 i
```

```
gga1[100000, .5 + 3 I]
```

```
0.352767 - 0.0129129 i
```

```
Plot[Abs[ggo[n, N@Im@ZetaZero@3 I]], {n, 0, 1000}]
```




```
hha1[n_, s_] := (2 ((1 + s) n^-s HarmonicNumber[n, -s] - n) - (1 + s)) / (2 n^-s (1 + s))
hha1a[n_, s_] := (2 ((1 + s) n^-s HarmonicNumber[n, -s] - n) - (1 + s))
hha2[n_, s_] := (((1 + s) n^-s HarmonicNumber[n, -s] - n) - (1 + s) / 2) / (n^-s (1 + s))
hha3[n_, s_] := (((1 + s) n^(1 - s) HarmonicNumber[n, -s] - n^2) - (1 + s) n / 2 + (1 + s) (-s) / 12) /
  (n^(1 - s) (1 + s))
hha3a[n_, s_] := (((1 + s) n^(1 - s) HarmonicNumber[n, -s] - n^2) - (1 + s) n / 2 + (1 + s) (-s) / 12)
hha5[n_, s_] := (((1 + s) n^(1 - s) HarmonicNumber[n, -s] - n^2) - (1 + s) n / 2 +
    (1 + s) (-s) / 12 - n^(-2) (1 + s) (-s) (1 - s) (2 - s) / 720) / (n^(1 - s) (1 + s))
hha5t[n_, s_] := (((1 + s) n^(3 - s) HarmonicNumber[n, -s] - n^4) - (1 + s) n^3 / 2 +
    n^2 (1 + s) (-s) / 12 - (1 + s) (-s) (1 - s) (2 - s) / 720) / (n^(3 - s) (1 + s))
hha5a[n_, s_] := (((1 + s) n^(3 - s) HarmonicNumber[n, -s] - n^4) -
   (1 + s) n^3 / 2 + n^2 (1 + s) (-s) / 12 - (1 + s) (-s) (1 - s) (2 - s) / 720)
hha5b[n_, s_] := (1 + s) ((n^(3 - s) HarmonicNumber[n, -s] - n^4 / (1 + s)) -
    n^3 / 2 + n^2 (-s) / 12 - (-s) (1 - s) (2 - s) / 720)
hha5c[n_, s_] := (1 + s) n^(3 - s) ( HarmonicNumber[n, -s] - n^(s + 1) / (1 + s) -
    n^s / 2 + n^(s - 1) (-s) / 12 - n^(s - 3) (-s) (1 - s) (2 - s) / 720)
hha5d[n_, s_] := (1 + s) n^(5 - s) ( HarmonicNumber[n, -s] - n^(s + 1) / (1 + s) -
    n^s / 2 + n^(s - 1) (-s) / 12 - n^(s - 3) (-s) (1 - s) (2 - s) / 720 +
    n^(s - 5) (-s) (1 - s) (2 - s) (3 - s) (4 - s) / 30 240)
```

```
hha5t[100, -N@ZetaZero@1]
```

$-2.69918\times10^{-12}+2.06203\times10^{-10}\,i$

```
Zeta[-(.5 + 10 I)]
```

$2.04226+0.0497166\,i$

```
Limit[hha5c[n, -ZetaZero@1], n → Infinity]
```

0

```
FullSimplify[(((1 + s) n^(3 - s) HarmonicNumber[n, -s] - n^4) -
   (1 + s) n^3 / 2 + n^2 (1 + s) (-s) / 12 - (1 + s) (-s) (1 - s) (2 - s) / 720)]
```

$-n^4-\frac{1}{2}n^3(1+s)-\frac{1}{12}n^2 s(1+s)+\frac{1}{720}(-2+s)(-1+s)s(1+s)+n^{3-s}(1+s)\,\text{HarmonicNumber}[n,-s]$

```
(1 + s) n^(3 - s) ((HarmonicNumber[n, -s] - n^(s + 1) / (1 + s)) -
   n^s / 2 + n^(s - 1) (-s) / 12 - n^(s - 3) (-s) (1 - s) (2 - s) / 720)
```

```
BernoulliB[6] / 6!
```

$$\frac{1}{30240}$$