

```

F2[f_, n_, k_] := F2[f, n, k] = Sum[f[j] F2[f, n/j, k-1], {j, 2, Floor[n]}];
F2[f_, n_, 0] := 1
bin[z_, k_] := Product[z - j, {j, 0, k-1}] / k!
F1[f_, n_, z_] := Sum[bin[z, k] F2[f, n, k], {k, 0, Log[2, n]}]
LF[f_, n_, k_] := D[F1[f, n, z], {z, k}] /. z -> 0

id3[n_] := 1/n
N[LF[id3, 98, 3]]
27.625

LFAlt[f_, n_, k_] :=
  Sum[FullSimplify[MangoldtLambda[j] / Log[j]] f[j] LFAlt[f, n/j, k-1], {j, 2, n}];
LFAlt[f_, n_, 0] := 1
N[LFAlt[id3, 98, 3]]
1.125

F2[f_, n_, k_] := F2[f, n, k] = Sum[f[j] F2[f, n/j, k-1], {j, 2, Floor[n]}]; F2[f_, n_, 0] := 1
bin[z_, k_] := Product[z - j, {j, 0, k-1}] / k!
F1[f_, n_, z_] := Sum[bin[z, k] F2[f, n, k], {k, 0, Log[2, n]}]
LF[f_, n_, k_] := D[F1[f, n, z], {z, k}] /. z -> 0
zeros[f_, n_] := List@@Roots[F1[f, n, z] == 0, z][[All, 2]]

zeros[LiouvilleLambda, 100]

{Root[720 - 16704 #1 + 18778 #1^2 - 5925 #1^3 + 1735 #1^4 - 51 #1^5 + 7 #1^6 &, 1],
 Root[720 - 16704 #1 + 18778 #1^2 - 5925 #1^3 + 1735 #1^4 - 51 #1^5 + 7 #1^6 &, 2],
 Root[720 - 16704 #1 + 18778 #1^2 - 5925 #1^3 + 1735 #1^4 - 51 #1^5 + 7 #1^6 &, 3],
 Root[720 - 16704 #1 + 18778 #1^2 - 5925 #1^3 + 1735 #1^4 - 51 #1^5 + 7 #1^6 &, 4],
 Root[720 - 16704 #1 + 18778 #1^2 - 5925 #1^3 + 1735 #1^4 - 51 #1^5 + 7 #1^6 &, 5],
 Root[720 - 16704 #1 + 18778 #1^2 - 5925 #1^3 + 1735 #1^4 - 51 #1^5 + 7 #1^6 &, 6]}

FullSimplify[-Sum[1/r, {r, zeros[id3, 100]}]]
58453230825931443183516468688731809305711
27888150091884990865813523574124921422720
LF[id3, 100, 1]
58453230825931443183516468688731809305711
27888150091884990865813523574124921422720
F1Alt[f_, n_, z_] := Product[1 - z/r, {r, zeros[f, n]}]
LF21Alt[f_, n_] := -Sum[1/r, {r, zeros[f, n]}]
F1[id3, 100, 1]
14466636279520351160221518043104131447711
2788815009188499086581352357412492142272
FullSimplify[F1Alt[LiouvilleLambda, 600, -3/2]]
60178863
65536

```

```
F1[LiouvilleLambda, 600, -3 / 2]
```

```
60178863
```

---

```
65536
```

```
266353
```

---

```
2520
```

```
l1[n_] := N[LaguerreL[2, n]]
```

```
N[{LF21Alt[l1, 100], LF[l1, 100, 1]}]
```

```
{164406. + 0. i, 164406.}
```