```
bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k!
```

```
Sum[ 1, {j, 0, n}, {k, 0, n - j}]
```

$$\frac{1}{2}(1+n)(2+n)$$

```
FullSimplify[Sum[ Pochhammer[2, k] / k! x^(-s k), {k, 0, n}]] -
 Sum[ x^(-s (j + k)), {j, 0, n}, {k, 0, n - j}]
```

0

```
Sum[ x^(-s (j + k)), {j, 0, n}, {k, 0, n - j}]
```

$$\frac{x^{-n s}\left(1+n-2x^{s}-n x^{s}+x^{(2+n) s}\right)}{(-1+x^{s})^{2}}$$

```
Expand[Sum[ x^(-s (j + k)), {j, 0, Infinity}, {k, 0, Infinity - j}]]
```

$$\frac{x^{2 s}}{(-1+x^{s})^{2}}$$

```
d2[n_, s_, k_] := Sum[ x^(-s j) d2[n - j, s, k - 1], {j, 1, n - k + 1}]
d2[n_, s_, 0] := UnitStep[n]
d2z[n_, s_, z_] := Sum[bin[z, k] d2[n, s, k], {k, 0, n}]
```

```
Table[{FullSimplify[D[(d2z[j, s, z] - d2z[j - 1, s, z]), z] /. z → 0], x^(-s j) / j},
  {j, 1, 7}] // TableForm
```

x^{-s}	x^{-s}
$\frac{x^{-2 s}}{2}$	$\frac{x^{-2 s}}{2}$
$\frac{x^{-3 s}}{3}$	$\frac{x^{-3 s}}{3}$
$\frac{x^{-4 s}}{4}$	$\frac{x^{-4 s}}{4}$
$\frac{x^{-5 s}}{5}$	$\frac{x^{-5 s}}{5}$
$\frac{x^{-6 s}}{6}$	$\frac{x^{-6 s}}{6}$
$\frac{x^{-7 s}}{7}$	$\frac{x^{-7 s}}{7}$

```
Sum[ x^(-s k) / k!, {k, 0, Infinity}]
```

$$e^{x^{-s}}$$

```
Sum[ (x^(-s j) / j!) (x^(-s k) / k!), {j, 1, Infinity}, {k, 1, Infinity}]
```

$$\left(-1+e^{x^{-s}}\right)^{2}$$

```
e2[n_, s_, k_] := Sum[ x^(-s j) / j! e2[n - j, s, k - 1], {j, 1, n - k + 1}]
e2[n_, s_, 0] := UnitStep[n]
e2z[n_, s_, z_] := Sum[bin[z, k] e2[n, s, k], {k, 0, n}]
```

```
FullSimplify[D[e2z[3, s, z], {z, 2}] /. z → 0]
```

$$x^{-2 s}$$

```
Sum[ z^k / k! 2^(-s k), {k, 0, Infinity}]
```

$$e^{2^{-s} z}$$

```
Sum[(-1)^k z^(2 k) / ((2 k)!) 2^(-2 s k), {k, 0, Infinity}]
```

$$\mathrm{Cos}[2^{-s} z]$$

```
Product[ Cos[Prime[p]^-s z], {p, 1, Infinity}]
```

$$\prod_{p=1}^{\infty} \text{Cos}[z\,\text{Prime}[p]^{-s}]$$

```
Product[ Sin[Prime[p]^-s z], {p, 1, Infinity}]
```

$$\prod_{p=1}^{\infty} \text{Sin}[z\,\text{Prime}[p]^{-s}]$$

```
Product[ Cos[Prime[p]^-s ], {p, 1, Infinity}]
```

$$\prod_{p=1}^{\infty} \text{Cos}[\text{Prime}[p]^{-s}]$$

```
Product[ Sin[Prime[p]^-s ], {p, 1, Infinity}]
```

$$\prod_{p=1}^{\infty} \text{Sin}[\text{Prime}[p]^{-s}]$$

```
Product[ Cos[Prime[p]^-2 ], {p, 1, Infinity}]
```

$$\prod_{p=1}^{\infty} \text{Cos}\left[\frac{1}{\text{Prime}[p]^{2}}\right]$$

```
Product[ Sin[Prime[p]^-2 ], {p, 1, Infinity}]
```

$$\prod_{p=1}^{\infty} \text{Sin}\left[\frac{1}{\text{Prime}[p]^{2}}\right]$$

```
Chop@N@Product[ Cos[Prime[p]^-2 ], {p, 1, 15 000}]
```

0.961904

```
Chop@N@Product[ Cos[Prime[p]^-3 ], {p, 1, 15 000}]
```

0.991481

```
Chop@N@Product[ Cos[Prime[p]^-1 ], {p, 1, 15 000}]
```

0.792194

```
Chop@N@Product[Cos[Prime[p]^0 ], {p, 1, 5000}]
```

0

```
N@Product[Sin[Prime[p]^1 ], {p, 1, 5000}]
```

$3.89443523080166 \times 10^{-1528}$

```
Chop@N@Product[ E^(Prime[p]^-I), {p, 1, 5000}]
```

0

```
Chop@N@Product[ E^(Prime[p]^-1), {p, 1, 15 000}]
```

15.5982

```
Chop@N@Product[ E^(Prime[p]^-2), {p, 1, 15 000}]
```

1.57184

```
Chop@N@Product[E^(Prime[p]^-3 I), {p, 1, 15000}]
```

0

```
Chop@N@Product[Cos[Prime[p]^-2], {p, 1, 5000}]
```

0.961904

```
Clear[sdz, cdz, edz]
coss := CoefficientList[Series[Cos[x], {x, 0, 40}], x]
sins := CoefficientList[Series[Sin[x], {x, 0, 40}], x]
es := CoefficientList[Series[E^x, {x, 0, 40}], x]
FI[n_] := FactorInteger[n]; FI[1] := {}
cdz[n_, z_] := cdz[n, z] = Product[z^p[[2]] coss[[1 + p[[2]]]], {p, FI[n]}]
sdz[n_, z_] := sdz[n, z] = If[n == 1, 0, Product[z^p[[2]] sins[[1 + p[[2]]]], {p, FI[n]}]]
edz[n_, z_] := edz[n, z] = Product[z^p[[2]] es[[1 + p[[2]]]], {p, FI[n]}]
```

```
es[[6]]
```

$\frac{1}{120}$

```
N@Sum[edz[n, 1] n^-2, {n, 1, 20000}]
```

1.5718

```
N@Sum[cdz[n, 1] n^-2, {n, 1, 20000}]
```

0.961904

```
N@Sum[sdz[n, 1] n^-2, {n, 1, 20000}]
```

0.516302

```
Table[I sdz[n, 1] + cdz[n, 1], {n, 1, 6}]
```

$\{1, i, i, -\frac{1}{2}, i, i\}$

```
Table[edz[n, I], {n, 1, 6}]
```

$\{1, i, i, -\frac{1}{2}, i, -1\}$

```
Table[cdz[n, 1], {n, 1, 6}]
```

$\{1, 0, 0, -\frac{1}{2}, 0, 0\}$

```
Table[sdz[n, 1], {n, 1, 6}]
```

{0, 1, 1, 0, 1, 1}

```
Table[edz[n, 1], {n, 1, 6}]
```

$\{1, 1, 1, \frac{1}{2}, 1, 1\}$

```
Table[edz[n, z I] - (cdz[n, z] + I sdz[n, z]), {n, 1, 10}] // TableForm
```

0
0
0
0
0
$(-1 - i) z^2$
0
0
0
$(-1 - i) z^2$

```
Expand[(1 / (1 - 2^-s)) ^z]
```

$$\left(\frac{1}{1-2^{-s}}\right)^z$$

```
d2[n, s, 1]
```

$$\begin{cases} x^{-s} & n == 1 \\ \frac{x^{-n s} (-1+x^{n s})}{-1+x^s} & \text{True} \end{cases}$$

$$\frac{x^{-n s} (-1+x^{n s})}{-1+x^s}$$

$$\frac{x^{-n s} (-1+x^{n s})}{-1+x^s}$$

```
d2[12, 2, 1] /. x → 5
```

$$\frac{2\,483\,526\,865\,641\,276}{59\,604\,644\,775\,390\,625}$$

$$\frac{x^{-n s} (-1+x^{n s})}{-1+x^s} \text{ /. n → 12 /. s → 2 /. x → 5}$$

$$\frac{2\,483\,526\,865\,641\,276}{59\,604\,644\,775\,390\,625}$$

$$\text{Limit}\left[\frac{x^{-n s} (-1+x^{n s})}{-1+x^s}, s \to 0\right]$$

n

$$\frac{x^{-n s} (-1+x^{n s})}{-1+x^s}$$

$$\frac{x^{-n s} (-1+x^{n s})}{-1+x^s}$$

```
Sum[ x^(-j s), {j, 1, n}]
```

$$\frac{x^{-n s}\left(-1+x^{n s}\right)}{-1+x^{s}}$$

```
FullSimplify@Sum[ x^(-j s), {j, 0, n}]
```

$$\frac{x^{-n s}\left(-1+x^{(1+n) s}\right)}{-1+x^{s}}$$

```
FullSimplify@Sum[x^(-s j) / j, {j, 1, n}]
```

$$-\left(x^{-s}\right)^{1+n} \text{LerchPhi}\left[x^{-s}, 1, 1+n\right]+\text{Log}\left[x^{s}\right]-\text{Log}\left[-1+x^{s}\right]$$

```
Clear[cs, cs2]
bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k!
cs[n_, s_, k_] :=
 cs[n, s, k] = Sum[ (-1)^j / ((2 j)!) x^(-2 j s) cs[n - 2 j, s, k - 1], {j, 0, Floor[n / 2]}]
cs[n_, s_, 0] := UnitStep[n]
cs2[n_, s_, k_] :=
 cs2[n, s, k] = Sum[ (-1)^j / ((2 j)!) x^(-2 j s) cs2[n - 2 j, s, k - 1], {j, 1, Floor[n / 2]}]
cs2[n_, s_, 0] := UnitStep[n]
csz[n_, s_, z_] := Sum[ bin[z, k] cs2[n, s, k], {k, 0, n}]
```

```
N@cs2[20, -1, 2] /. x → 2
```

2.00547

```
N[(Cos[1.2^2])]^2.5
```

0.00614315

```
N@csz[30, -2, 2.5] /. x → 1.2
```

0.00615117

```
Table[D[csz[n, 0, z] - csz[n - 1, 0, z], z] /. z → 0, {n, 1, 20}]
```

$$\left\{0, -\frac{1}{2}, 0, -\frac{1}{12}, 0, -\frac{1}{45}, 0, -\frac{17}{2520}, 0, -\frac{31}{14175}, 0, -\frac{691}{935550}, 0, -\frac{10922}{42567525}, 0, -\frac{929569}{10216206000}, 0, -\frac{3202291}{97692469875}, 0, -\frac{221930581}{18561569276250}\right\}$$