```
pp[z_, k_] := Product[z - j, {j, 0, k - 1}]
Table[j ! pp[z, j + 1] Sum[(-1)^k / (z - k) StirlingS2[j, k] k ! / j !, {k, 0, Infinity}],
  {j, 0, 10}] // TableForm
```

1

$\frac{(-1+z)\,z}{1-z}$

z^2

$-z^2\,(1+z)$

$z^3\,(5+z)$

$-z^2\,\left(-4+11\,z+16\,z^2+z^3\right)$

$-z\,\left(42\,z^2-119\,z^3-42\,z^4-z^5\right)$

$-z\,\left(120\,z-398\,z^2+141\,z^3+757\,z^4+99\,z^5+z^6\right)$

$-z\,\left(-2160\,z^2+7250\,z^3-6189\,z^4-3721\,z^5-219\,z^6-z^7\right)$

$-z\,\left(-12\,096\,z+45\,624\,z^2-41\,186\,z^3-41\,171\,z^4+72\,976\,z^5+15\,706\,z^6+466\,z^7+z^8\right)$

$-z\,\left(332\,640\,z^2-1\,261\,788\,z^3+1\,594\,648\,z^4-371\,569\,z^5-595\,760\,z^6-60\,082\,z^7-968\,z^8-z^9\right)$

```
pp[z, 5] /. z → 2.5
```

1.40625

```
FactorialPower[z, 5] /. z → 2.5
```

1.40625

```
FullSimplify[1 / Gamma[z] / Gamma[1 - z]]
```

$\frac{\mathrm{Sin}[\pi\, z]}{\pi}$

```
Sum[StirlingS2[j, k] k ! / j ! Log[1 + x]^j, {j, 0, Infinity}] /. x → .307 /. k → 4
```

0.00888287

```
.307^4
```

0.00888287

```
FullSimplify@(1 / Gamma[z] / Gamma[1 - z]) Sum[(-1)^k / (z - k) x^k, {k, 0, Infinity}]
```

$-\frac{\mathrm{HurwitzLerchPhi}[-x,\,1,\,-z]\;\mathrm{Sin}[\pi\, z]}{\pi}$

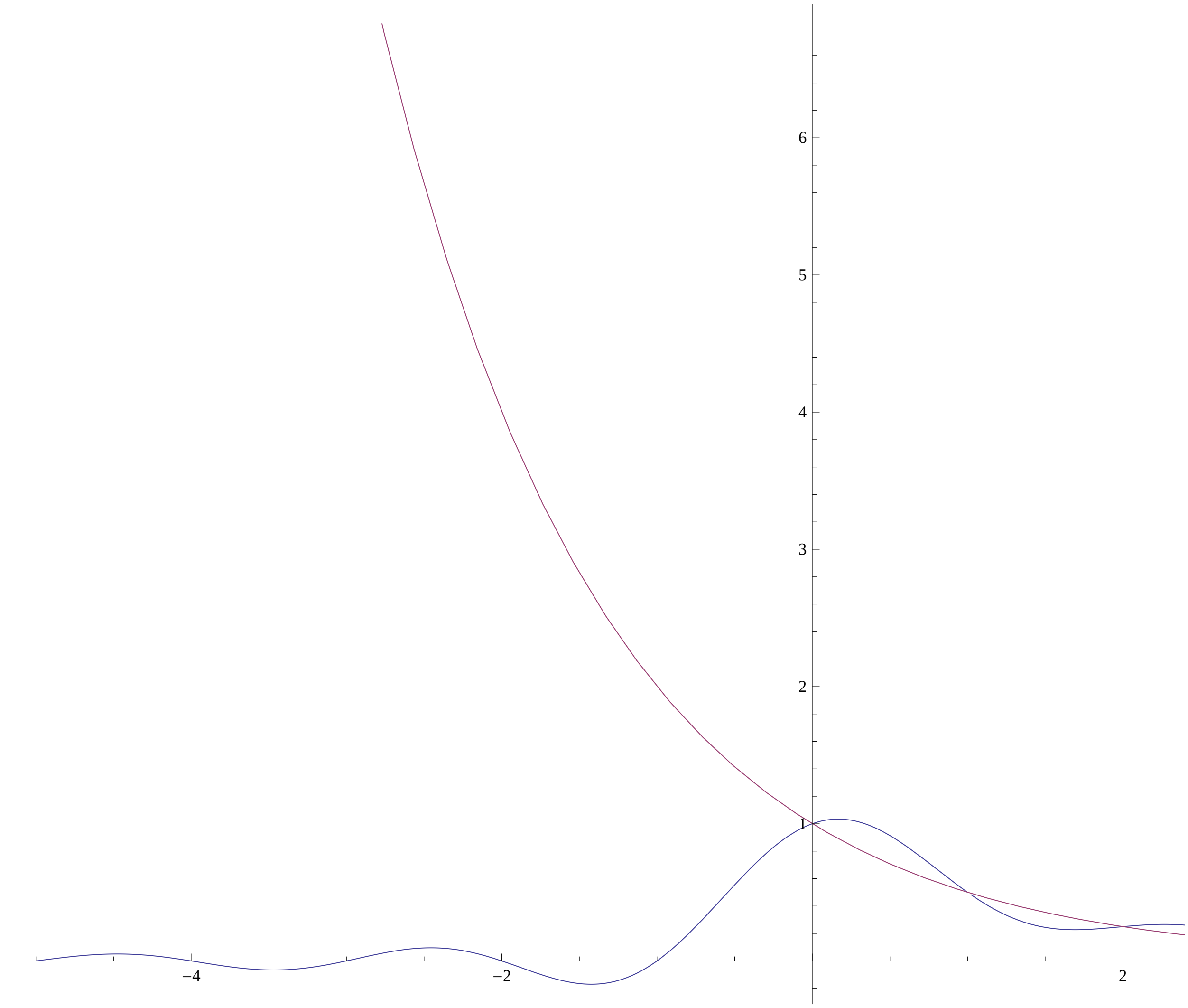
$$-\frac{\mathrm{HurwitzLerchPhi}[-x,\,1,\,-z]\;\mathrm{Sin}[\pi\, z]}{\pi}\ \text{/. z → 3.0000000001 /. x → 10}$$

999.999

```
10^1.5
```

31.6228

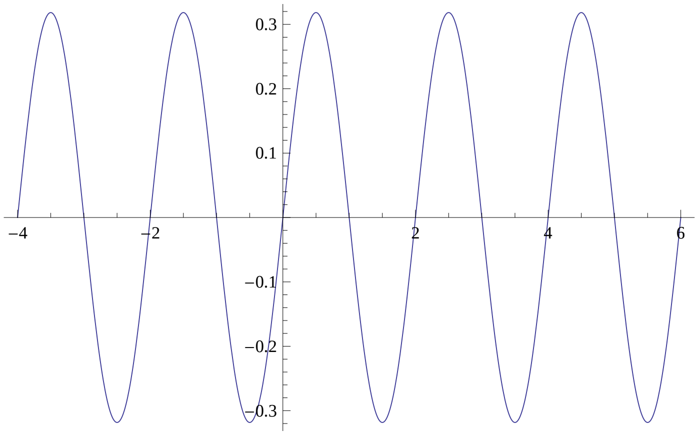
Plot$\left[\left\{-\frac{\text{HurwitzLerchPhi}[-.5,\,1,\,-z]\,\text{Sin}[\pi\, z]}{\pi},\ .5^{\wedge}(z)\right\},\ \{z,\,-5,\,5\}\right]$



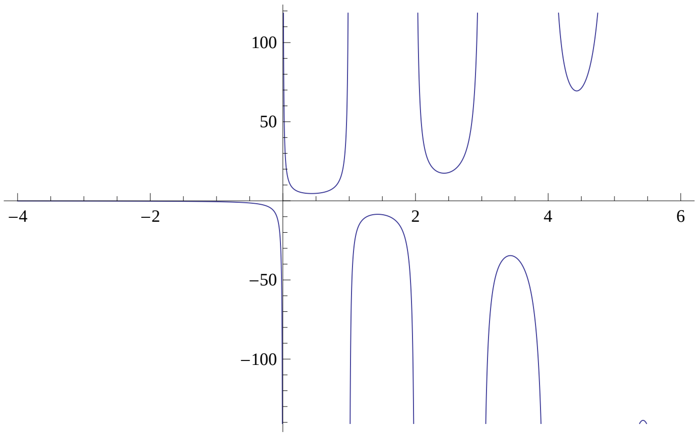
Series$\left[-\frac{\text{HurwitzLerchPhi}[-x,\,1,\,-z]\,\text{Sin}[\pi\, z]}{\pi},\ \{z,\,0,\,20\}\right]$

$$\text{HurwitzLerchPhi}[-x,1,-z]\left(-z+\frac{\pi^2 z^3}{6}-\frac{\pi^4 z^5}{120}+\frac{\pi^6 z^7}{5040}-\frac{\pi^8 z^9}{362\,880}+\frac{\pi^{10} z^{11}}{39\,916\,800}-\frac{\pi^{12} z^{13}}{6\,227\,020\,800}+\frac{\pi^{14} z^{15}}{1\,307\,674\,368\,000}-\frac{\pi^{16} z^{17}}{355\,687\,428\,096\,000}+\frac{\pi^{18} z^{19}}{121\,645\,100\,408\,832\,000}+O[z]^{21}\right)$$

```
Plot[{Sin[π z]/π}, {z, -4, 6}]
```




```
Plot[-HurwitzLerchPhi[-2, 1, -z] , {z, -4, 6}]
```




```
Sum[(1 / Gamma[z] / Gamma[1 - z]) (-1)^k / (z - k) x^k /. z → 3.5 /. x → .5, {k, 0, Infinity}]
```

```
0.033875
```

```
.5^3.5
```

```
0.0883883
```

```
pp[z_, k_] := Product[z - j, {j, 0, k - 1}]
Table[j ! pp[z, j + 1] Sum[(-1) ^k / (z - k) StirlingS2[j, k] k ! / j !, {k, 0, Infinity}],
  {j, 0, 10}] // TableForm
```

1

$\frac{(-1+z)\,z}{1-z}$

z^2

$-z^2\,(1+z)$

$z^3\,(5+z)$

$-z^2\,\left(-4+11\,z+16\,z^2+z^3\right)$

$-z\,\left(42\,z^2-119\,z^3-42\,z^4-z^5\right)$

$-z\,\left(120\,z-398\,z^2+141\,z^3+757\,z^4+99\,z^5+z^6\right)$

$-z\,\left(-2160\,z^2+7250\,z^3-6189\,z^4-3721\,z^5-219\,z^6-z^7\right)$

$-z\,\left(-12\,096\,z+45\,624\,z^2-41\,186\,z^3-41\,171\,z^4+72\,976\,z^5+15\,706\,z^6+466\,z^7+z^8\right)$

$-z\,\left(332\,640\,z^2-1\,261\,788\,z^3+1\,594\,648\,z^4-371\,569\,z^5-595\,760\,z^6-60\,082\,z^7-968\,z^8-z^9\right)$

```
FullSimplify@Sum[(-1) ^k StirlingS2[7, k] k ! / 7 ! / (z - k) / Gamma[z] / Gamma[1 - z], {k, 1, 7}]
```

$$-\frac{z\,(1+z)\,(120+z\,(-518+z\,(659+z\,(98+z))))\,\mathrm{Sin}[\pi\,z]}{5040\,\pi\,(-7+z)\,(-6+z)\,(-5+z)\,(-4+z)\,(-3+z)\,(-2+z)\,(-1+z)}$$

```
FullSimplify@
 Sum[7 ! / Gamma[z - 7] / Gamma[1 - z] (-1) ^k StirlingS2[7, k] k ! / 7 ! / (z - k), {k, 1, 7}]
```

$$-\frac{z\,(1+z)\,(120+z\,(-518+z\,(659+z\,(98+z))))\,\mathrm{Sin}[\pi\,z]}{\pi}$$

```
Sum[(-1) ^k / (z - k) StirlingS2[j, k] k ! / j !, {k, 0, 7}] /. j → 7
```

$$-\frac{1}{-7+z}+\frac{3}{-6+z}-\frac{10}{3\,(-5+z)}+\frac{5}{3\,(-4+z)}-\frac{43}{120\,(-3+z)}+\frac{1}{40\,(-2+z)}-\frac{1}{5040\,(-1+z)}$$

```
1 / pp[z - 1, 7] /. z → 3.3
```

0.0937767

```
Gamma[z - 7] / Gamma[z] /. z → 3.3
```

0.0937767

```
Table[Limit[Sum[(-1) ^k StirlingS2[j, k] k ! / j ! / (z - k) / Gamma[z] / Gamma[1 - z], {k, 0, j}],
   z → 3], {j, 0, 10}] // TableForm
```

0

0

0

1

$\frac{3}{2}$

$\frac{5}{4}$

$\frac{3}{4}$

$\frac{43}{120}$

$\frac{23}{160}$

$\frac{605}{12\,096}$

$\frac{311}{20\,160}$

```
Series[(E^x - 1)^(3), {x, 0, 10}]
```

$$x^3 + \frac{3\,x^4}{2} + \frac{5\,x^5}{4} + \frac{3\,x^6}{4} + \frac{43\,x^7}{120} + \frac{23\,x^8}{160} + \frac{605\,x^9}{12\,096} + \frac{311\,x^{10}}{20\,160} + O[x]^{11}$$

```
Gamma[3]
```

2

```
Sum[(-1)^k StirlingS2[j, k] k! / j! / (z - k) / Gamma[z] / Gamma[1 - z], {k, 0, j}]
```

$$\sum_{k=0}^{j} \frac{(-1)^k\, k!\, \mathrm{StirlingS2}[j,\, k]}{(-k + z)\, j!\, \mathrm{Gamma}[1 - z]\, \mathrm{Gamma}[z]}$$

```
Sum[(-1)^k (1 / k! Sum[(-1)^i Binomial[k, i] (k - i)^j, {i, 0, k}])
  k! / j! / (z - k) / Gamma[z] / Gamma[1 - z], {k, 0, j}]
```

$$\sum_{k=0}^{j} \frac{(-1)^k\, k!\, \mathrm{StirlingS2}[j,\, k]}{(-k + z)\, j!\, \mathrm{Gamma}[1 - z]\, \mathrm{Gamma}[z]}$$

```
Sum[(-1)^k (1 / k! Sum[(-1)^i Binomial[k, i] (k - i)^j, {i, 0, k}])
    k! / j! / (z - k) / Gamma[z] / Gamma[1 - z], {k, 0, j}] /. j → 3 /. z → 4.2
```

-0.0806151

```
Sum[(-1)^k (Sum[(-1)^i Binomial[k, i] (k - i)^j, {i, 0, k}]) / j! / (z - k) / Gamma[z] /
     Gamma[1 - z], {k, 0, j}] /. j → 3 /. z → 4.2
```

-0.0806151

```
Sum[(Sum[(-1)^k (-1)^i Binomial[k, i] (k - i)^j / j! / (z - k) / Gamma[z] / Gamma[1 - z],
     {i, 0, k}]), {k, 0, j}] /. j → 3 /. z → 4.2
```

-0.0806151

```
Sum[(-1)^k (-1)^i Binomial[k, i] (k - i)^j / j! / (z - k) / Gamma[z] / Gamma[1 - z],
   {k, 0, j}, {i, 0, k}] /. j → 3 /. z → 4.2
```

-0.0806151

```
Sum[(-1)^k (-1)^i Binomial[k, i] (k - i)^j / j! / (z - k) / Gamma[z] / Gamma[1 - z],
   {k, 0, j}, {i, 0, k}] /. j → 3 /. z → 4.2
Sum[(-1)^k (-1)^i Binomial[k, i] (k - i)^j / j! / (z - k) / Gamma[z] / Gamma[1 - z],
 {k, 0, j}, {i, 0, k}]
```

$$\sum_{k=0}^{j} \sum_{i=0}^{k} -\frac{(-1)^{i+k}\, (-i + k)^j\, \mathrm{Binomial}[k,\, i]\, \mathrm{Sin}[\pi z]}{\pi\, (k - z)\, j!}$$

```
Expand@FullSimplify@Table[j ! pp[z, j + 1]
    Sum[(-1) ^k / (z - k) StirlingS2[j, k] k ! / j !, {k, 0, 2 j}], {j, 0, 10}] // TableForm
```

1

$-z$

z^2

$-z^2 - z^3$

$5 z^3 + z^4$

$4 z^2 - 11 z^3 - 16 z^4 - z^5$

$-42 z^3 + 119 z^4 + 42 z^5 + z^6$

$-120 z^2 + 398 z^3 - 141 z^4 - 757 z^5 - 99 z^6 - z^7$

$2160 z^3 - 7250 z^4 + 6189 z^5 + 3721 z^6 + 219 z^7 + z^8$

$12\,096 z^2 - 45\,624 z^3 + 41\,186 z^4 + 41\,171 z^5 - 72\,976 z^6 - 15\,706 z^7 - 466 z^8 - z^9$

$-332\,640 z^3 + 1\,261\,788 z^4 - 1\,594\,648 z^5 + 371\,569 z^6 + 595\,760 z^7 + 60\,082 z^8 + 968 z^9 + z^{10}$

```
Sum[Sin[Pi z] / Pi (-1) ^k / (z - k) (-1) ^ (k - j) Binomial[k, j], {k, 0, Infinity}]
```

$$\frac{1}{\pi z} (-1)^{-j} \text{Binomial}[0, j]\ \text{HypergeometricPFQ}[\{1, 1, -z\}, \{1 - j, 1 - z\}, 1]\ \text{Sin}[\pi z]$$

$$\text{Limit}\left[\frac{1}{\pi z} (-1)^{-j} \text{Binomial}[0, j]\ \text{HypergeometricPFQ}[\{1, 1, -z\}, \{1 - j, 1 - z\}, 1]\ \text{Sin}[\pi z], z \to 3\right]$$

$$\text{Limit}\left[\frac{1}{\pi z} (-1)^{-j} \text{Binomial}[0, j]\ \text{HypergeometricPFQ}[\{1, 1, -z\}, \{1 - j, 1 - z\}, 1]\ \text{Sin}[\pi z], z \to 3\right]$$

```
N@Limit[
  Sum[Sin[Pi z] / Pi (-1) ^k / (z - k) (-1) ^ (k - j) Binomial[k, j], {k, 0, Infinity}] /. z → 3.3,
  j → 4]
```

$Aborted