

```
ClearAll["Global`*"]
```

```

lo[n_, k_] := Sum[ (-1)^(j+1) lo[Floor[n/j], k-1], {j, 1, n}];
lo[n_, 1] := Sum[ (-1)^(j+1) Log[j], {j, 1, n}]
t[n_, a_] := Mod[n, a] - Mod[n-1, a]
lp[n_, k_, b_] := Sum[ t[j, b] lp[Floor[n/j], k-1, b], {j, 1, n}];
lp[n_, 1, b_] := Sum[ t[j, b] Log[j], {j, 1, n}]
fa[n_, k_] := Sum[ 2^j Binomial[k, j] (-1)^j l1[n/2^j, k], {j, 0, k}] +
  Sum[ 2^j Binomial[k-1, j-1] (-1)^j Log[2] d1[n/2^j, k], {j, 1, k}]

L1[n_, k_] := Sum[ L1[Floor[n/j], k-1], {j, 1, n}];
L1[n_, 1] := Sum[ Log[j], {j, 1, n}]; L1[n_, 0] := 1
L2[n_, k_] := Sum[ L2[Floor[n/j], k-1], {j, 2, n}]; L2[n_, 1] := Sum[ Log[j], {j, 2, n}]
D1[n_, k_] := Sum[ D1[Floor[n/j], k-1], {j, 1, n}]; D1[n_, 0] := 1
L2toL1[n_, z_] := Sum[ FactorialPower[z-1, a] / a! L2[n, a+1], {a, 0, Log[2, n]}]
L2toL1x[n_, z_] := Sum[ Binomial[z-1, a] L2[n, a+1], {a, 0, Log[2, n]}]
L1toL2[n_, k_] := Sum[ (-1)^(k-j) Binomial[k-1, j-1] L1[n, j], {j, 1, k}]

EL[n_, k_, b_] :=
  EL[n, k, b] = Sum[ EL[n/j, k-1, b], {j, 1, n}] - b Sum[ EL[n/(j b), k-1, b], {j, 1, n}];
EL[n_, 1, b_] := EL[n, 1, b] = Sum[ Log[j], {j, 1, n}] - b Sum[ Log[j b], {j, 1, n/b}]
LtoEL[n_, k_, b_] := Sum[ b^j Binomial[k, j] (-1)^j L1[n/b^j, k], {j, 0, k}] +
  Sum[ b^j Binomial[k-1, j-1] (-1)^j Log[b] D1[n/b^j, k], {j, 1, k}]

EL1toL1[n_, b_] := Sum[ b^j EL[n/b^j, 1, b], {j, 0, Log[b, n]}] +
  Log[b] Sum[ b^j D1[n/b^j, 1], {j, 1, Log[b, n]}]

EL2[n_, k_, b_] :=
  EL2[n, k, b] = Sum[ EL2[n/j, k-1, b], {j, 2, n}] - b Sum[ EL2[n/(j b), k-1, b], {j, 1, n}];
EL2[n_, 1, b_] := EL2[n, 1, b] = Sum[ Log[j], {j, 2, n}] - b Sum[ Log[j b], {j, 1, n/b}]
EL2toEL1[n_, z_, b_] :=
  Sum[ FactorialPower[z-1, a] / a! EL2[n, a+1, b], {a, 0, Log[If[b < 2, b, 2], n]}]
EL1toEL2[n_, k_, b_] := Sum[ (-1)^(k-j) Binomial[k-1, j-1] EL[n, j, b], {j, 1, k}]

```

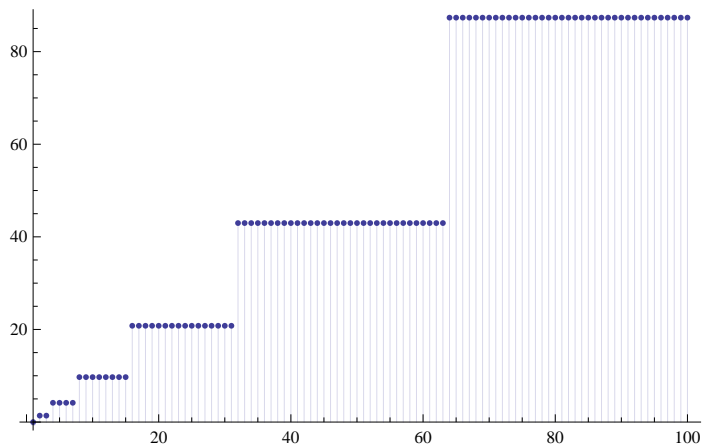
```
N[L2toL1x[100, 0]]
```

```
94.0453
```

```
N[EL2toEL1[100, 0, 101]]
```

```
94.0453
```

```
DiscretePlot[L2toL1[n, 0] - EL2toEL1[n, 0, 2], {n, 1, 100}]
```



```
N[EL1toEL2[160, 3, 2.5]]
```

```
-138.321
```

```
N[LtoEL[100, 2, 4.3]]
```

```
24.3806
```

```
N[EL[100, 2, 4.3]]
```

```
24.3806
```

```
{N[L2toL1[100, 2]], N[L1[100, 2]], N[L2[100, 2] + L2[100, 1]]}
```

```
{920.841, 920.841, 920.841}
```

```
{N[L2toL1[100, 3]], N[L1[100, 3]], N[L2[100, 3] + 2 L2[100, 2] + L2[100, 1]]}
```

```
{1921.09, 1921.09, 1921.09}
```

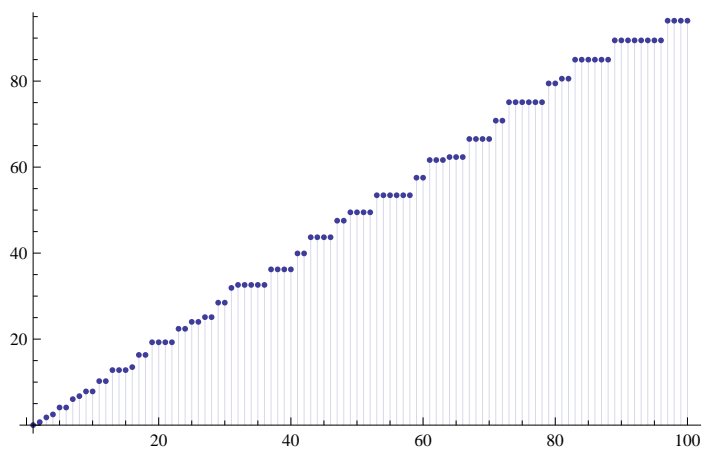
```
{N[L2toL1[100, 4]], N[L1[100, 4]], N[L2[100, 4] + 3 L2[100, 3] + 3 L2[100, 2] + L2[100, 1]]}
```

```
{3559.46, 3559.46, 3559.46}
```

```
N[L2toL1[100, 0]]
```

```
94.0453
```

```
DiscretePlot[L2toL1[n, 0], {n, 1, 100}]
```



```
f1[n_] := Sum[ (-1) ^ (j+k) Log[k], {j, 1, n}, {k, 1, n / j}]
```

```
N[f1[100]]
```

```
3.9172
```

```
f2[n_] := Sum[ (-1) ^ (k+1) Log[k], {j, 1, n}, {k, 1, n / j}] -  
2 Sum[ (-1) ^ (k+1) Log[k], {j, 2, n, 2}, {k, 1, n / j}]
```

```
N[f2[100]]
```

```
3.9172
```

```
f3[n_] := Sum[ Log[k], {j, 1, n}, {k, 1, n / j}] - 2 Sum[Log[k], {j, 1, n}, {k, 2, n / j, 2}] -  
2 Sum[ Log[k], {j, 2, n, 2}, {k, 1, n / j}] + 4 Sum[ Log[k], {j, 2, n, 2}, {k, 2, n / j, 2}]
```

```
N[f3[100]]
```

```
3.9172
```

```
f4[n_] := Sum[ Log[k], {j, 1, n}, {k, 1, n / j}] - 2 Sum[Log[2 k], {j, 1, n}, {k, 1, n / (2 j)}] -  
2 Sum[ Log[k], {j, 1, n / 2}, {k, 1, n / (2 j)}] + 4 Sum[ Log[2 k], {j, 1, n / 2}, {k, 1, n / (4 j)}]
```

```
N[f4[100]]
```

```
3.9172
```

```
f5[n_] :=  
Sum[ Log[k], {j, 1, n}, {k, 1, n / j}] - 2 Sum[Log[2] + Log[k], {j, 1, n}, {k, 1, n / (2 j)}] -  
2 Sum[ Log[k], {j, 1, n / 2}, {k, 1, n / (2 j)}] +  
4 Sum[ Log[2] + Log[k], {j, 1, n / 2}, {k, 1, n / (4 j)}]
```

```
N[f5[100]]
```

```
3.9172
```

```
f6[n_] := Sum[ Log[k], {j, 1, n}, {k, 1, n / j}] -  
2 Sum[Log[k], {j, 1, n}, {k, 1, n / (2 j)}] - 2 Sum[ Log[k], {j, 1, n / 2}, {k, 1, n / (2 j)}] +  
4 Sum[ Log[k], {j, 1, n / 2}, {k, 1, n / (4 j)}] -  
Log[2] 2 Sum[1, {j, 1, n}, {k, 1, n / (2 j)}] +  
Log[2] 4 Sum[ 1, {j, 1, n / 2}, {k, 1, n / (4 j)}]
```

```
N[f6[100]]
```

```
3.9172
```

```
f7[n_] := Sum[ Log[k], {j, 1, n}, {k, 1, n / j}] - 4 Sum[Log[k], {j, 1, n / 2}, {k, 1, n / (2 j)}] +  
4 Sum[ Log[k], {j, 1, n / 4}, {k, 1, n / (4 j)}] -  
2 Log[2] Sum[1, {j, 1, n / 2}, {k, 1, n / (2 j)}] +  
4 Log[2] Sum[ 1, {j, 1, n / 4}, {k, 1, n / (4 j)}]
```

```
N[f7[100]]
```

```
3.9172
```

```
g1[n_] := Sum[ (-1) ^ (j+k+m+1) Log[m], {j, 1, n}, {k, 1, n / j}, {m, 1, n / (j k)}]
```

```
N[g1[100]]
```

```
4.38425
```

```

g2[n_] := Sum[Log[m], {j, 1, n}, {k, 1, n/j}, {m, 1, n/(jk)}] -
  2 Sum[Log[m], {j, 2, n, 2}, {k, 1, n/j}, {m, 1, n/(jk)}] -
  2 Sum[Log[m], {j, 1, n}, {k, 2, n/j, 2}, {m, 1, n/(jk)}] -
  2 Sum[Log[m], {j, 1, n}, {k, 1, n/j}, {m, 2, n/(jk), 2}] +
  4 Sum[Log[m], {j, 2, n, 2}, {k, 2, n/j, 2}, {m, 1, n/(jk)}] +
  4 Sum[Log[m], {j, 2, n, 2}, {k, 1, n/j}, {m, 2, n/(jk), 2}] +
  4 Sum[Log[m], {j, 1, n}, {k, 2, n/j, 2}, {m, 2, n/(jk), 2}] -
  8 Sum[Log[m], {j, 2, n, 2}, {k, 2, n/j, 2}, {m, 2, n/(jk), 2}]

```

```
N[g2[100]]
```

```
4.38425
```

```

g3[n_] := Sum[Log[m], {j, 1, n}, {k, 1, n/j}, {m, 1, n/(jk)}] -
  2 Sum[Log[m], {j, 1, n/2}, {k, 1, n/(2j)}, {m, 1, n/(2jk)}] -
  2 Sum[Log[m], {j, 1, n}, {k, 1, n/(2j)}, {m, 1, n/(2jk)}] -
  2 Sum[Log[2m], {j, 1, n}, {k, 1, n/j}, {m, 1, n/(2jk)}] +
  4 Sum[Log[m], {j, 1, n/2}, {k, 1, n/(2j)}, {m, 1, n/(4jk)}] +
  4 Sum[Log[2m], {j, 1, n/2}, {k, 1, n/(2j)}, {m, 1, n/(4jk)}] +
  4 Sum[Log[2m], {j, 1, n}, {k, 1, n/(2j)}, {m, 1, n/(4jk)}] -
  8 Sum[Log[2m], {j, 1, n/2}, {k, 1, n/(4j)}, {m, 1, n/(8jk)}]

```

```
N[g3[100]]
```

```
4.38425
```

```

g4[n_] := Sum[Log[m], {j, 1, n}, {k, 1, n/j}, {m, 1, n/(jk)}] -
  2 Sum[Log[m], {j, 1, n/2}, {k, 1, n/(2j)}, {m, 1, n/(2jk)}] -
  2 Sum[Log[m], {j, 1, n}, {k, 1, n/(2j)}, {m, 1, n/(2jk)}] -
  2 Sum[Log[m], {j, 1, n}, {k, 1, n/j}, {m, 1, n/(2jk)}] +
  4 Sum[Log[m], {j, 1, n/2}, {k, 1, n/(2j)}, {m, 1, n/(4jk)}] +
  4 Sum[Log[m], {j, 1, n/2}, {k, 1, n/(2j)}, {m, 1, n/(4jk)}] +
  4 Sum[Log[m], {j, 1, n}, {k, 1, n/(2j)}, {m, 1, n/(4jk)}] -
  8 Sum[Log[m], {j, 1, n/2}, {k, 1, n/(4j)}, {m, 1, n/(8jk)}] -
  2 Sum[Log[2], {j, 1, n}, {k, 1, n/j}, {m, 1, n/(2jk)}] +
  4 Sum[Log[2], {j, 1, n/2}, {k, 1, n/(2j)}, {m, 1, n/(4jk)}] +
  4 Sum[Log[2], {j, 1, n}, {k, 1, n/(2j)}, {m, 1, n/(4jk)}] -
  8 Sum[Log[2], {j, 1, n/2}, {k, 1, n/(4j)}, {m, 1, n/(8jk)}]

```

```
N[g4[100]]
```

```
4.38425
```

```

g5[n_] := Sum[Log[m], {j, 1, n}, {k, 1, n/j}, {m, 1, n/(jk)}] -
  6 Sum[Log[m], {j, 1, n/2}, {k, 1, n/(2j)}, {m, 1, n/(2jk)}] +
  12 Sum[Log[m], {j, 1, n/2}, {k, 1, n/(2j)}, {m, 1, n/(4jk)}] -
  8 Sum[Log[m], {j, 1, n/2}, {k, 1, n/(4j)}, {m, 1, n/(8jk)}] -
  2 Sum[Log[2], {j, 1, n}, {k, 1, n/j}, {m, 1, n/(2jk)}] +
  4 Sum[Log[2], {j, 1, n/2}, {k, 1, n/(2j)}, {m, 1, n/(4jk)}] +
  4 Sum[Log[2], {j, 1, n}, {k, 1, n/(2j)}, {m, 1, n/(4jk)}] -
  8 Sum[Log[2], {j, 1, n/2}, {k, 1, n/(4j)}, {m, 1, n/(8jk)}]

```

```
N[g5[100]]
```

```
4.38425
```

```

g6[n_] := Sum[Log[m], {j, 1, n}, {k, 1, n/j}, {m, 1, n/(j k)}] -
  6 Sum[Log[m], {j, 1, n/2}, {k, 1, n/(2 j)}, {m, 1, n/(2 j k)}] +
  12 Sum[Log[m], {j, 1, n/2}, {k, 1, n/(2 j)}, {m, 1, n/(4 j k)}] -
  8 Sum[Log[m], {j, 1, n/2}, {k, 1, n/(4 j)}, {m, 1, n/(8 j k)}] -
  2 Log[2] Sum[1, {j, 1, n}, {k, 1, n/j}, {m, 1, n/(2 j k)}] +
  4 Log[2] Sum[1, {j, 1, n/2}, {k, 1, n/(2 j)}, {m, 1, n/(4 j k)}] +
  4 Log[2] Sum[1, {j, 1, n}, {k, 1, n/(2 j)}, {m, 1, n/(4 j k)}] -
  8 Log[2] Sum[1, {j, 1, n/2}, {k, 1, n/(4 j)}, {m, 1, n/(8 j k)}]

```

```
N[g6[100]]
```

```
4.38425
```

```

g7[n_] := Sum[Log[m], {j, 1, n}, {k, 1, n/j}, {m, 1, n/(j k)}] -
  6 Sum[Log[m], {j, 1, n/2}, {k, 1, n/(2 j)}, {m, 1, n/(2 j k)}] +
  12 Sum[Log[m], {j, 1, n/2}, {k, 1, n/(2 j)}, {m, 1, n/(4 j k)}] -
  8 Sum[Log[m], {j, 1, n/2}, {k, 1, n/(4 j)}, {m, 1, n/(8 j k)}] -
  2 Log[2] Sum[1, {j, 1, n/2}, {k, 1, n/(2 j)}, {m, 1, n/(2 j k)}] +
  8 Log[2] Sum[1, {j, 1, n/4}, {k, 1, n/(4 j)}, {m, 1, n/(4 j k)}] -
  8 Log[2] Sum[1, {j, 1, n/8}, {k, 1, n/(8 j)}, {m, 1, n/(8 j k)}]

```

```
N[g7[100]]
```

```
4.38425
```

```

h1[n_] :=
  Sum[(-1)^(j+k+m+o) Log[o], {j, 1, n}, {k, 1, n/j}, {m, 1, n/(j k)}, {o, 1, n/(j k m)}]

```

```
N[h1[100]]
```

```
-10.3864
```

```

h2[n_] := Sum[Log[m], {j, 1, n}, {k, 1, n/j}, {o, 1, n/(j k)}, {m, 1, n/(j k o)}] -
  8 Sum[Log[m], {j, 1, n/2}, {k, 1, n/(2 j)}, {o, 1, n/(2 j k)}, {m, 1, n/(2 j k o)}] +
  24 Sum[Log[m], {j, 1, n/4}, {k, 1, n/(4 j)}, {o, 1, n/(4 j k)}, {m, 1, n/(4 j k o)}] -
  32 Sum[Log[m], {j, 1, n/8}, {k, 1, n/(8 j)}, {o, 1, n/(8 j k)}, {m, 1, n/(8 j k o)}] +
  16 Sum[Log[m], {j, 1, n/16}, {k, 1, n/(16 j)}, {o, 1, n/(16 j k)}, {m, 1, n/(16 j k o)}] -
  2 Log[2] Sum[1, {j, 1, n/2}, {k, 1, n/(2 j)}, {o, 1, n/(2 j k)}, {m, 1, n/(2 j k o)}] +
  12 Log[2] Sum[1, {j, 1, n/4}, {k, 1, n/(4 j)}, {o, 1, n/(4 j k)}, {m, 1, n/(4 j k o)}] -
  24 Log[2] Sum[1, {j, 1, n/8}, {k, 1, n/(8 j)}, {o, 1, n/(8 j k)}, {m, 1, n/(8 j k o)}] +
  16 Log[2] Sum[1, {j, 1, n/16}, {k, 1, n/(16 j)}, {o, 1, n/(16 j k)}, {m, 1, n/(16 j k o)}]

```

```
N[h2[100]]
```

```
-10.3864
```

```

l1[n_, k_] := Sum[l1[Floor[n/j], k-1], {j, 1, n}]; l1[n_, 1] := Sum[Log[j], {j, 1, n}]
d1[n_, k_] := Sum[d1[Floor[n/j], k-1], {j, 1, n}]; d1[n_, 0] := 1

```

```

f7[n_] := Sum[Log[k], {j, 1, n}, {k, 1, n/j}] - 4 Sum[Log[k], {j, 1, n/2}, {k, 1, n/(2 j)}] +
  4 Sum[Log[k], {j, 1, n/4}, {k, 1, n/(4 j)}] -
  2 Log[2] Sum[1, {j, 1, n/2}, {k, 1, n/(2 j)}] +
  4 Log[2] Sum[1, {j, 1, n/4}, {k, 1, n/(4 j)}]

```

```

N[f7[100]]
3.9172
N[l1[100, 2] - 4 l1[50, 2] + 4 l1[25, 2] - 2 Log[2] d1[50, 2] + 4 Log[2] d1[25, 2]]
3.9172
fa[n_, k_] := Sum[ 2^j Binomial[k, j] (-1)^j l1[n / 2^j, k], {j, 0, k}] +
  Sum[ 2^j Binomial[k - 1, j - 1] (-1)^j Log[2] d1[n / 2^j, k], {j, 1, k}]
N[fa[100, 2]]
3.9172
N[f1[1000]]
-25.1378
N[fa[1000, 2]]
-25.1378
N[g1[1000]]
-90.1202
N[fa[1000, 3]]
-90.1202
N[h1[100]]
-10.3864
N[fa[100, 4]]
-10.3864
lo[n_, k_] := Sum[ (-1)^(j+1) lo[Floor[n / j], k - 1], {j, 1, n}];
lo[n_, 1] := Sum[ (-1)^(j+1) Log[j], {j, 1, n}]
N[lo[1000, 5]]
-200.591
N[fa[1000, 5]]
-200.591
N[L1[100, 1]]
363.739
LtoELa[n_, k_, b_] := Sum[ b^j Binomial[k, j] (-1)^j L1a[n / b^j, k], {j, 0, k}] +
  Sum[ b^j Binomial[k - 1, j - 1] (-1)^j Log[b] D1a[n / b^j, k], {j, 1, k}]
LtoELa[n, 1, b]
L1a[n, 1] - b L1a[ $\frac{n}{b}$ , 1] - b D1a[ $\frac{n}{b}$ , 1] Log[b]
t1[n_, b_] := EL[n, 1, b] + b Log[b] D1[n / b, 1] + b L1[n / b, 1]
N[t1[100, 2]]
363.739
t2[n_, b_] :=
  EL[n, 1, b] + b Log[b] D1[n / b, 1] + b (EL[n / b, 1, b] + b Log[b] D1[n / b^2, 1] + b L1[n / b^2, 1])

```

N[t2[100, 2]]

363.739

t3[n_, b_] := EL[n, 1, b] + b Log[b] D1[n / b, 1] +
b EL[n / b, 1, b] + b^2 Log[b] D1[n / b^2, 1] + b^2 L1[n / b^2, 1]

N[t3[100, 2]]

363.739

t4[n_, b_] := EL[n, 1, b] + b Log[b] D1[n / b, 1] + b EL[n / b, 1, b] +
b^2 Log[b] D1[n / b^2, 1] + b^2 (EL[n / b^2, 1, b] + b Log[b] D1[n / b^3, 1] + b L1[n / b^3, 1])

N[t4[100, 2]]

363.739

t5[n_, b_] := EL[n, 1, b] + b Log[b] D1[n / b, 1] + b EL[n / b, 1, b] + b^2 Log[b] D1[n / b^2, 1] +
b^2 EL[n / b^2, 1, b] + b^3 Log[b] D1[n / b^3, 1] + b^3 L1[n / b^3, 1]

N[t5[100, 2]]

363.739

t6[n_, b_] := EL[n, 1, b] + b EL[n / b, 1, b] + b^2 EL[n / b^2, 1, b] + b Log[b] D1[n / b, 1] +
b^2 Log[b] D1[n / b^2, 1] + b^3 Log[b] D1[n / b^3, 1] + b^3 L1[n / b^3, 1]

N[t6[100, 2]]

363.739

EL1toL1[n_, b_] := Sum[b^j EL[n / b^j, 1, b], {j, 0, Log[b, n]}] +
Log[b] Sum[b^j D1[n / b^j, 1], {j, 1, Log[b, n]}]

N[EL1toL1[100, 1.0001]]

363.739

LtoELA[n, 2, b]

$L1a[n, 2] + b^2 L1a\left[\frac{n}{b^2}, 2\right] - 2 b L1a\left[\frac{n}{b}, 2\right] + b^2 D1a\left[\frac{n}{b^2}, 2\right] \text{Log}[b] - b D1a\left[\frac{n}{b}, 2\right] \text{Log}[b]$

ts[n_, b_] := Log[b] Sum[b^j D1[n / b^j, 1], {j, 1, Log[b, n]}]

tsa[n_, b_] := Sum[b^j D1[n / b^j, 1], {j, 1, Log[b, n]}]

tsb[n_, b_] := Log[b] Sum[b^j Floor[n / b^j], {j, 1, Log[b, n]}]

ts2[n_, b_] := Sum[b^j EL[n / b^j, 1, b], {j, 0, Log[b, n]}]

Limit[tsb[100, a], a → 1]

\$Aborted

Plot[ts[100, n], {n, 1, 10}]

Plot[ts2[100, n], {n, 1, 10}]

```

tso[n_, b_] := Table[{b^j, Log[b] b^j D1[n/b^j, 1]}, {j, 1, Log[b, n]}]
tsp[n_, b_] := DiscretePlot[Log[b] b^j D1[n/b^j, 1], {j, 1, Log[b, n]}]
tsp2[n_, b_] := DiscretePlot[Log[b] b^j Floor[n/b^j], {j, 1, Log[b, n]}]
tsp3[n_, b_] := DiscretePlot[b^j EL[n/b^j, 1, b], {j, 0, Log[b, n]}]
tso[100, 1.01]

```

```
tsp[100, 1.002]
```

```
D1[n/b^j, 1]
```

```
 $b^{-j} n$ 
```

```
D1[100.9, 1]
```

```
100
```

```

Table[{n, If[Log[2, n] == Floor[Log[2, n]], N[Log[2^n]], 0],
      N[(L2toL1[n, 0] - EL2toEL1[n, 0, 2]) - (L2toL1[n - 1, 0] - EL2toEL1[n - 1, 0, 2])]}, {n,
      1, 100}] // TableForm

```

1	0.693147	0.
2	1.38629	1.38629
3	0	0.
4	2.77259	2.77259
5	0	0.
6	0	0.
7	0	0.
8	5.54518	5.54518
9	0	0.
10	0	-1.77636×10^{-15}
11	0	0.
12	0	8.88178×10^{-16}
13	0	0.
14	0	3.55271×10^{-15}
15	0	0.
16	11.0904	11.0904
17	0	0.
18	0	-7.10543×10^{-15}
19	0	0.
20	0	6.21725×10^{-15}
21	0	0.
22	0	0.
23	0	0.
24	0	-1.77636×10^{-15}
25	0	0.
26	0	-7.10543×10^{-15}
27	0	0.
28	0	1.77636×10^{-15}

29	0	0.
30	0	1.77636×10^{-15}
31	0	0.
32	22.1807	22.1807
33	0	0.
34	0	-3.55271×10^{-15}
35	0	0.
36	0	5.32907×10^{-15}
37	0	0.
38	0	-3.55271×10^{-15}
39	0	0.
40	0	-1.24345×10^{-14}
41	0	0.
42	0	3.55271×10^{-15}
43	0	0.
44	0	-1.33227×10^{-14}
45	0	0.
46	0	1.77636×10^{-14}
47	0	0.
48	0	2.75335×10^{-14}
49	0	0.
50	0	-3.73035×10^{-14}
51	0	0.
52	0	7.99361×10^{-15}
53	0	0.
54	0	2.66454×10^{-15}
55	0	0.
56	0	2.84217×10^{-14}
57	0	0.
58	0	-2.4869×10^{-14}
59	0	0.
60	0	3.19744×10^{-14}
61	0	0.
62	0	-3.90799×10^{-14}
63	0	0.
64	44.3614	44.3614
65	0	0.
66	0	-1.77636×10^{-14}
67	0	0.
68	0	4.44089×10^{-15}
69	0	0.
70	0	-7.10543×10^{-15}
71	0	0.
72	0	-1.42109×10^{-14}
73	0	0.
74	0	3.55271×10^{-15}
75	0	0.
76	0	5.77316×10^{-14}
77	0	0.
78	0	-4.61853×10^{-14}
79	0	0.
80	0	6.03961×10^{-14}

81	0	0.
82	0	6.03961×10^{-14}
83	0	0.
84	0	-4.26326×10^{-14}
85	0	0.
86	0	-2.4869×10^{-14}
87	0	0.
88	0	-6.03961×10^{-14}
89	0	0.
90	0	7.01661×10^{-14}
91	0	0.
92	0	3.64153×10^{-14}
93	0	0.
94	0	-1.38556×10^{-13}
95	0	0.
96	0	7.4607×10^{-14}
97	0	0.
98	0	1.45661×10^{-13}
99	0	0.
100	0	-1.54543×10^{-13}

```
fdif[n_, b_] := Sum[Log[b^(b^j)], {j, 1, Log[b, n]}]
```

```
fdif2[n_, b_] := Log[b] Sum[(b^j), {j, 1, Log[b, n]}]
```

```
Table[{n, N[fdif2[n, 2]], N[(L2toL1[n, 0] - EL2toEL1[n, 0, 2])]}, {n, 1, 100}] // TableForm
```

1	0.	0.
2	1.38629	1.38629
3	1.38629	1.38629
4	4.15888	4.15888
5	4.15888	4.15888
6	4.15888	4.15888
7	4.15888	4.15888
8	9.70406	9.70406
9	9.70406	9.70406
10	9.70406	9.70406
11	9.70406	9.70406
12	9.70406	9.70406
13	9.70406	9.70406
14	9.70406	9.70406
15	9.70406	9.70406
16	20.7944	20.7944
17	20.7944	20.7944
18	20.7944	20.7944
19	20.7944	20.7944
20	20.7944	20.7944
21	20.7944	20.7944
22	20.7944	20.7944
23	20.7944	20.7944
24	20.7944	20.7944
25	20.7944	20.7944

26	20.7944	20.7944
27	20.7944	20.7944
28	20.7944	20.7944
29	20.7944	20.7944
30	20.7944	20.7944
31	20.7944	20.7944
32	42.9751	42.9751
33	42.9751	42.9751
34	42.9751	42.9751
35	42.9751	42.9751
36	42.9751	42.9751
37	42.9751	42.9751
38	42.9751	42.9751
39	42.9751	42.9751
40	42.9751	42.9751
41	42.9751	42.9751
42	42.9751	42.9751
43	42.9751	42.9751
44	42.9751	42.9751
45	42.9751	42.9751
46	42.9751	42.9751
47	42.9751	42.9751
48	42.9751	42.9751
49	42.9751	42.9751
50	42.9751	42.9751
51	42.9751	42.9751
52	42.9751	42.9751
53	42.9751	42.9751
54	42.9751	42.9751
55	42.9751	42.9751
56	42.9751	42.9751
57	42.9751	42.9751
58	42.9751	42.9751
59	42.9751	42.9751
60	42.9751	42.9751
61	42.9751	42.9751
62	42.9751	42.9751
63	42.9751	42.9751
64	87.3365	87.3365
65	87.3365	87.3365
66	87.3365	87.3365
67	87.3365	87.3365
68	87.3365	87.3365
69	87.3365	87.3365
70	87.3365	87.3365
71	87.3365	87.3365
72	87.3365	87.3365
73	87.3365	87.3365
74	87.3365	87.3365
75	87.3365	87.3365
76	87.3365	87.3365
77	87.3365	87.3365
78	87.3365	87.3365
79	87.3365	87.3365
80	87.3365	87.3365
81	87.3365	87.3365

```

82      87.3365      87.3365
83      87.3365      87.3365
84      87.3365      87.3365
85      87.3365      87.3365
86      87.3365      87.3365
87      87.3365      87.3365
88      87.3365      87.3365
89      87.3365      87.3365
90      87.3365      87.3365
91      87.3365      87.3365
92      87.3365      87.3365
93      87.3365      87.3365
94      87.3365      87.3365
95      87.3365      87.3365
96      87.3365      87.3365
97      87.3365      87.3365
98      87.3365      87.3365
99      87.3365      87.3365
100     87.3365      87.3365

```

```
bb[b_, j_] := Log[b^ (b^ j)]
```

```
N[bb[3, 3.2]]
```

```
36.9515
```

```
bc[b_, j_] := (b^ j) Log[b]
```

```
N[bc[3, 3.2]]
```

```
36.9515
```

```
Log[b] Sum[ (b^ j), {j, 1, Floor[Log[b, n]]}]
```

```

bd[n_, b_] := 
$$\frac{b \left( -1 + b^{\text{Floor}\left[\frac{\text{Log}[n]}{\text{Log}[b]}\right]} \right) \text{Log}[b]}{-1 + b}$$


```

```
bd[40, 3.3]
```

```
59.8478
```

```
fdif2[40, 3.3]
```

```
59.8478
```

```

FullSimplify[
$$\frac{b \left( -1 + b^{\left(\frac{\text{Log}[n]}{\text{Log}[b]}\right)} \right) \text{Log}[b]}{-1 + b}$$
]

```

```


$$\frac{b (-1 + n) \text{Log}[b]}{-1 + b}$$


```

```

Limit[
$$\frac{b \left( -1 + b^{\text{Floor}\left[\frac{\text{Log}[n]}{\text{Log}[b]}\right]} \right) \text{Log}[b]}{-1 + b}, b \rightarrow 1]$$


```

```
$Aborted
```

```

Limit[
$$\frac{b \left( -1 + b^{\left(\frac{\text{Log}[n]}{\text{Log}[b]}\right)} \right) \text{Log}[b]}{-1 + b}, b \rightarrow 1]$$


```

```
-1 + n
```

```
Limit[Log[b] Sum[ (b^j), {j, 1, Log[b, n]}], b -> 1]
```

```
-1 + n
```

```
Limit[Log[b] Sum[ (b^j), {j, 1, Floor[Log[b, n]]}], b -> 1]
```

```
$Aborted
```

```
EL[n_, k_, b_] :=
```

```
EL[n, k, b] = Sum[ EL[ n / j, k - 1, b], {j, 1, n}] - b Sum[ EL[ n / (j b), k - 1, b], {j, 1, n}];
```

```
EL[n_, 1, b_] := EL[n, 1, b] = Sum[ Log[j], {j, 1, n}] - b Sum[ Log[j b], {j, 1, n / b}]
```

```
N[EL[100, 2, 2]]
```

```
3.9172
```

```
ELA[n_, k_, b_] :=
```

```
Sum[ ELA[ n / j, k - 1, b], {j, 1, n}] - b Sum[ ELA[ n / (j b), k - 1, b], {j, 1, n}];
```

```
ELA[n_, 1, b_] := Sum[ Log[j], {j, 1, n}] - b Sum[ Log[j], {j, 1, n / b}] - b Log[b] Floor[ $\frac{n}{b}$ ]
```

```
N[ELA[100, 2, 2]]
```

```
3.9172
```

```
EL2[n_, k_, b_] :=
```

```
EL2[n, k, b] = Sum[ EL2[ n / j, k - 1, b], {j, 2, n}] - b Sum[ EL2[ n / (j b), k - 1, b], {j, 1, n}];
```

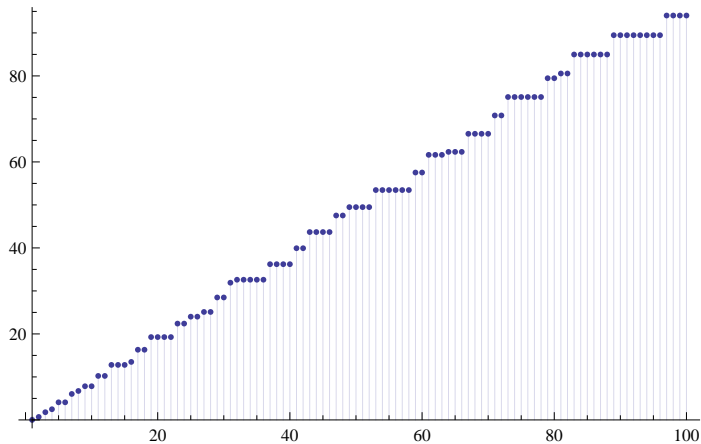
```
EL2[n_, 1, b_] := EL2[n, 1, b] = Sum[ Log[j], {j, 2, n}] - b Sum[ Log[j b], {j, 1, n / b}]
```

```
EL2toEL1[n_, z_, b_] :=
```

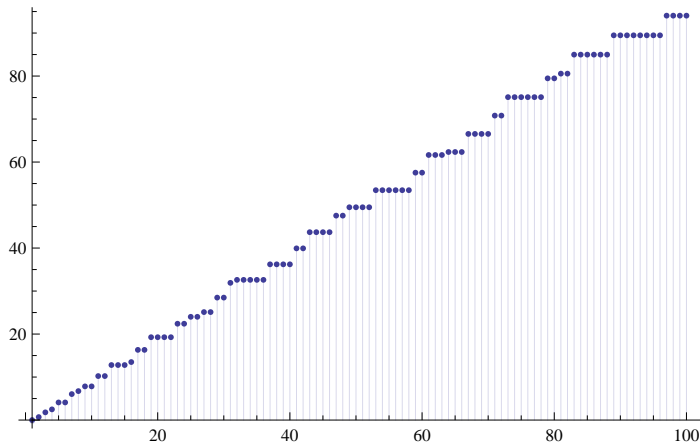
```
Sum[FactorialPower[z - 1, a] / a! EL2[n, a + 1, b], {a, 0, Log[If[ b < 2, b, 2], n]}]
```

```
fdif2[n_, b_] := Log[b] Sum[ (b^j), {j, 1, Log[b, n]}]
```

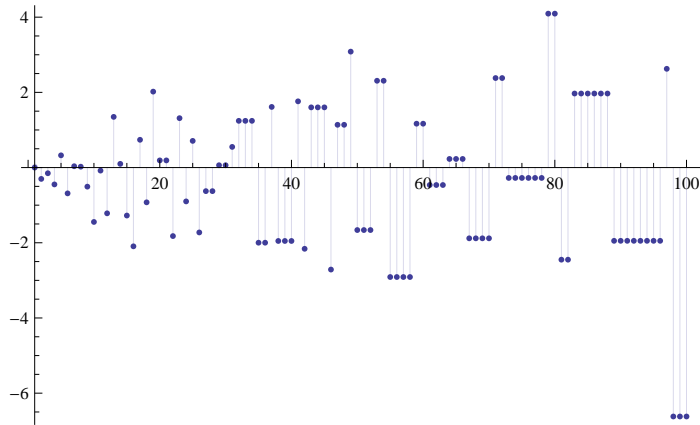
```
DiscretePlot[EL2toEL1[n, 0, aa = 1.5] + fdif2[n, aa], {n, 1, 100}]
```



```
DiscretePlot[EL2toEL1[n, 0, aa = 2] + fdif2[n, aa], {n, 1, 100}]
```



```
DiscretePlot[EL2toEL1[n, 0, aa = 1.1], {n, 1, 100}]
```



```
$RecursionLimit = 10 000
```

```
10 000
```

```
cheb[n_, z_, b_] := FactorialPower[z - 1, 0] / 0! EL2[n, 1, b] + Sum[
  Log[b] (b^a) + FactorialPower[z - 1, a] / a! EL2[n, a + 1, b], {a, 1, Log[If[b < 2, b, 2], n]}]
cheb2[n_, b_] := EL2[n, 1, b] + Sum[ Log[b] (b^a) + FactorialPower[-1, a] / a! EL2[n, a + 1, b],
  {a, 1, Log[If[b < 2, b, 2], n]}]
```

```
EL2a[n_, k_, b_] := EL2a[n, k, b] =
```

```
Sum[ EL2a[n / j, k - 1, b], {j, 2, n}] - b Sum[ EL2a[n / (j b), k - 1, b], {j, 1, n}]
```

```
EL2a[n_, 1, b_] := EL2a[n, 1, b] = Sum[ Log[j], {j, 2, n}] - b Sum[ Log[j b], {j, 1, n / b}]
```

```
cheb3[n_, b_] :=
```

```
EL2a[n, 1, b] + Sum[ Log[b] b^a + (-1)^a EL2a[n, a + 1, b], {a, 1, Log[If[b < 2, b, 2], n]}]
```

```
cheb3a[n_, b_] := Sum[ (-1)^a EL2a[n, a + 1, b], {a, 0, Log[If[b < 2, b, 2], n]}]
```

```
cheb3b[n_, b_] := Sum[ Log[b] b^a, {a, 1, Log[If[b < 2, b, 2], n]}]
```

```
cheb4[n_, b_] := Sum[ Log[b] b^a, {a, 1, Log[If[b < 2, b, 2], n]}] +
```

```
Sum[ (-1)^a EL2a[n, a + 1, b], {a, 0, Log[If[b < 2, b, 2], n]}]
```

```
c2[n_] := Sum[ MangoldtLambda[j], {j, 1, n}]
```

```
cheb4[100, 1.1]
```

```
94.0453
```

```
cheb3b[100, 1.000001]
```

```
99.
```

```
N[c2[100.]]
```

```
94.0453
```

```
EL2a[100, 1, 1.000001]
```

```
4.60471
```

```
EL2b[n_, k_, b_] :=
```

```
Sum[EL2b[n / j, k - 1, b], {j, 2, n}] - b Sum[EL2b[n / (j b), k - 1, b], {j, 1, n}]
```

```
EL2b[n_, 1, b_] := Sum[Log[j], {j, 2, n}] - b Sum[Log[j b], {j, 1, n / b}]
```

```
EL2c[n_, 1, b_] :=
```

```
Sum[Log[j], {j, 2, n}] - b Sum[Log[j], {j, 1, n / b}] - b Log[b] Floor[n / b]
```

```
N[EL2b[12, 3, 1.0000001]]
```

```
2.4849
```

```
N[Log[4] + Log[3]]
```

```
2.48491
```

```
N[Log[3]]
```

```
1.09861
```

```
Table[{a, (-1)^a EL2a[n, a + 1, b]}, {a, 1, 3 + Log[If[b < 2, b, 2], n]}] /. {n -> 5, b -> 1.02} //  
TableForm
```

Table::iterb : Iterator $\left\{a, 1, 3 + \frac{\text{Log}[n]}{\text{Log}[\text{If}[b < 2, b, 2]]}\right\}$ does not have appropriate bounds. >>

Table::iterb : Iterator $\left\{a, 1, 3 + \frac{\text{Log}[n]}{\text{Log}[\text{If}[b < 2, b, 2]]}\right\}$ does not have appropriate bounds. >>

```
1      -0.149136
```

```
2      -0.154058
```

```
3      -0.159125
```

```
4      -0.164342
```

```
5      -0.169712
```

```
6      -0.17524
```

```
7      -0.180931
```

```
8      -0.186788
```

```
9      -0.192816
```

```
10     -0.19902
```

```
11     13.1815
```

```
12     -12.1015
```

```
13     -0.138065
```

```
14     -0.141871
```

```
15     -0.145775
```

```
16     -0.149778
```

```
17     -0.153882
```

```
18     -0.158091
```

19	-0.162407
20	-0.166832
21	-0.171369
22	-0.176021
23	-0.180791
24	-0.185681
25	2.50933
26	-0.108032
27	-0.110869
28	-0.113776
29	-0.116754
30	-0.119807
31	-0.122935
32	-0.12614
33	-0.129424
34	-0.132789
35	-0.136237
36	-0.139769
37	-0.143389
38	-0.147097
39	-0.150896
40	-0.154789
41	-0.158777
42	-0.162862
43	-0.167047
44	-0.171335
45	-0.175727
46	3.93847
47	-0.0512308
48	-0.0522554
49	-0.0533005
50	-0.0543665
51	-0.0554539
52	-0.0565629
53	-0.0576942
54	-0.0588481
55	-0.060025
56	-0.0612255
57	-0.06245
58	-0.063699
59	-0.064973
60	-0.0662725
61	-0.0675979
62	-0.0689499
63	-0.0703289
64	-0.0717355
65	-0.0731702
66	-0.0746336
67	-0.0761263
68	-0.0776488
69	-0.0792018
70	-0.0807858
71	-0.0824015
72	-0.0840495
73	-0.0857305
74	-0.0874451

75	-0.089194
76	-0.0909779
77	-0.0927975
78	-0.0946534
79	-0.0965465
80	-0.0984774
81	0.
82	0.
83	0.
84	0.