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bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
Da[n_, 0, a_] := UnitStep[n - 1]; Da[n_, 1, a_] := Floor[n] - a
Da[n_, k_, a_] :=
  Sum[Binomial[k, j] Da[n / (m^(k - j)), j, m], {m, a + 1, n^(1 / k)}, {j, 0, k - 1}]
Daz[n_, z_, a_] := Sum[bin[z, k] Da[n, k, a], {k, 0, Log[a + 1, n]}]
lda[n_, a_] := Sum[(-1)^(k + 1) / k Da[n, k, a], {k, 1, Log[a + 1, n]}]
ldf[n_, a_] := lda[n, a] - lda[n, a + 1]
```

**Expand@Daz[100, z, 1] - Expand@Daz[100, z, 2]**

$$\frac{7z}{60} - \frac{572z^2}{45} + \frac{437z^3}{48} + \frac{605z^4}{144} + \frac{67z^5}{240} + \frac{7z^6}{720}$$

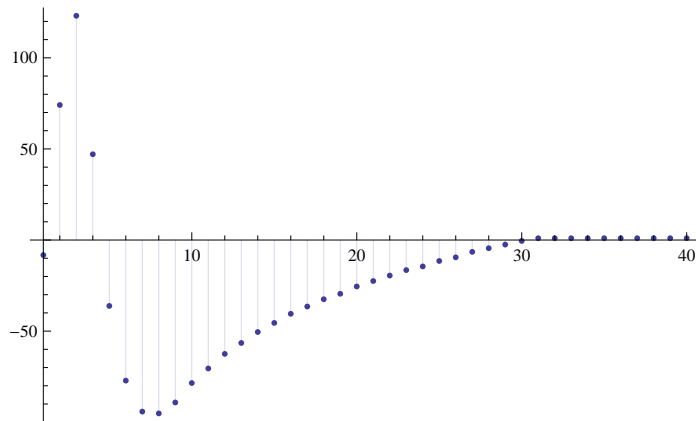
**Expand@Daz[100, z, 2] - Expand@Daz[100, z, 3]**

$$-\frac{113z}{12} + \frac{11z^2}{24} + \frac{119z^3}{12} + \frac{z^4}{24}$$

**Expand@Daz[100, z, 3] - Expand@Daz[100, z, 4]**

$$-\frac{103z}{6} + \frac{33z^2}{2} + \frac{5z^3}{3}$$

**DiscretePlot[ldf[1000, a], {a, 1, 40}]**



```

binomial[z_, k_] := binomial[z, k] = Product[z - j, {j, 0, k - 1}] / k!
Ds[n_, 0, s_, a_] := UnitStep[n - 1]
Ds[n_, 1, s_, a_] := Ds[n, 1, s, a] = HarmonicNumber[Floor[n], s] - HarmonicNumber[a, s]
Ds[n_, 2, s_, a_] := Ds[n, 2, s, a] =
  Sum[(m^(-2 s)) + 2 (m^-s) (Ds[Floor[n / m], 1, s, m]), {m, a + 1, Floor[n^(1 / 2)]}]
Ds[n_, k_, s_, a_] := Ds[n, k, s, a] =
  Sum[(m^(-s k)) + k (m^(-s (k - 1))) Ds[Floor[n / (m^(k - 1))], 1, s, m] +
    Sum[binomial[k, j] (m^-s)^j Ds[Floor[n / (m^j)], k - j, s, m], {j, 1, k - 2}],
    {m, a + 1, Floor[n^(1 / k)]}]

Dnka[n_, 0, a_] := UnitStep[n - 1]
Dnka[n_, 1, a_] := Dnka[n, 1, a] = Floor[n] - a
Dnka[n_, 2, a_] :=
  Dnka[n, 2, a] = Sum[1 + 2 (Dnka[Floor[n / m], 1, m]), {m, a + 1, Floor[n^(1 / 2)]}]
Dnka[n_, k_, a_] := Dnka[n, k, a] =
  Sum[1 + k Dnka[Floor[n / (m^(k - 1))], 1, m] + Sum[binomial[k, j]
    Dnka[Floor[n / (m^j)], k - j, m], {j, 1, k - 2}], {m, a + 1, Floor[n^(1 / k)]}]

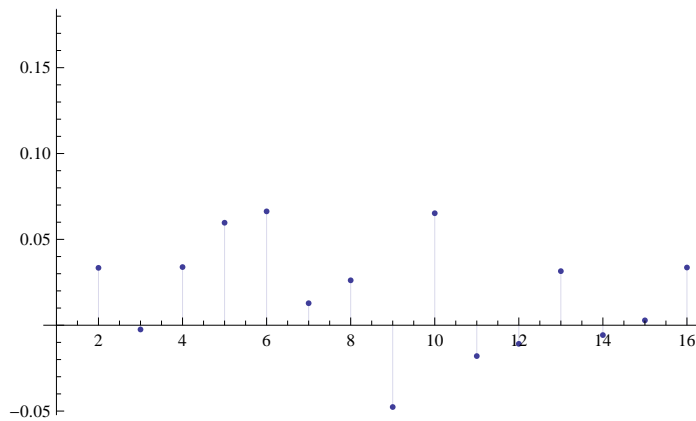
Ddy[n_, s_, y_, k_] := y^(k (s - 1)) Ds[n y^k, k, s, y]
Ddy2[n_, s_, y_, k_] := y^(k (s - 1)) Dnka[n y^k, k, y]

Dnsyz[n_, s_, y_, z_] := Expand@Sum[binomial[z, k] Ddy[n, s, y, k], {k, 0, Log[(y + 1) / y, n]}]
Dnsyz2[n_, s_, y_, z_] :=
  Expand@Sum[binomial[z, k] Ddy2[n, s, y, k], {k, 0, Log[(y + 1) / y, n]}]

Table[
  N[(D[Dnsyz2[20, 0, k, z], z] /. z -> 0) - (D[Dnsyz2[20, 0, k + 1, z], z] /. z -> 0)], {k, 1, 16}]
{1.01607, 0.0334186, -0.00247749, 0.0338736, 0.0596889,
  0.0663087, 0.012818, 0.0261579, -0.0476362, 0.0652243, -0.0179694,
  -0.0108422, 0.0314993, -0.00573718, 0.00283552, 0.0336085}

DiscretePlot[
  N[(D[Dnsyz2[20, 0, k, z], z] /. z -> 0) - (D[Dnsyz2[20, 0, k + 1, z], z] /. z -> 0)], {k, 1, 16}]

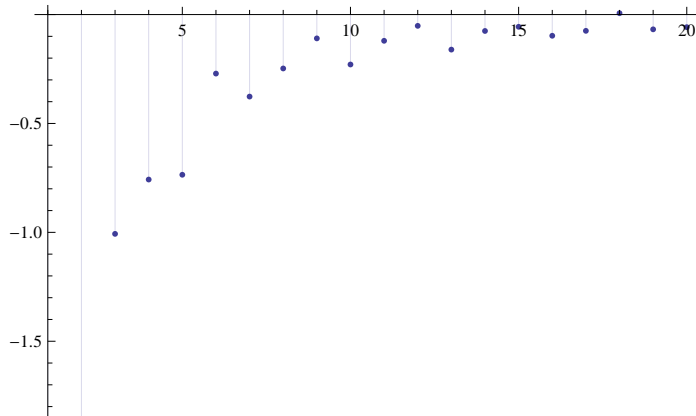
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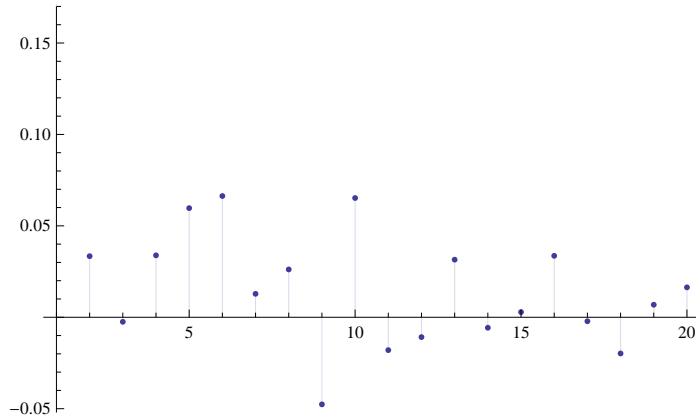
**Table[(Dnsyz2[20, 0, k, 2]) - (Dnsyz2[20, 0, k + 1, 2]), {k, 1, 20}]**

$$\left\{ -6, -\frac{23}{9}, -\frac{145}{144}, -\frac{303}{400}, -\frac{331}{450}, -\frac{239}{882}, -\frac{1181}{3136}, -\frac{1283}{5184}, -\frac{443}{4050}, -\frac{1387}{6050}, -\frac{2099}{17424}, \right. \\ \left. -\frac{97}{1872}, -\frac{205}{1274}, -\frac{1661}{22050}, -\frac{3233}{57600}, -\frac{7199}{73984}, -\frac{583}{7803}, -\frac{61}{9747}, -\frac{1959}{28880}, -\frac{2041}{35280} \right\}$$

**DiscretePlot[(Dnsyz2[20, 0, k, 2]) - (Dnsyz2[20, 0, k + 1, 2]), {k, 1, 20}]**



**DiscretePlot[D[(Dnsyz2[20, 0, k, z]) - (Dnsyz2[20, 0, k + 1, z]), z] /. z -> 0, {k, 1, 20}]**



**Dnsyz2[100, 0, 2, z] - Dnsyz2[100 - 1 / 10 000 000, 0, 2, z]**

$$\frac{3z}{128} + \frac{107z^2}{512} + \frac{599z^3}{3072} + \frac{197z^4}{3072} + \frac{25z^5}{3072} + \frac{z^6}{3072}$$

**Dnsyz2[100, 0, 1, z] - Dnsyz2[99, 0, 1, z]**

$$\frac{z^2}{4} + \frac{z^3}{2} + \frac{z^4}{4}$$

**N@Dnsyz2[100, 0, 12, 2]**

552.556

**N[Gamma[2, 0, -Log[100.]] / Gamma[2]]**

361.517 - 4.41506 × 10<sup>-14</sup> i

**N@LaguerreL[-2, Log[100.]]**

560.517