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binomial[z, k] := binomial[z, k] = Product[z - j, {j, 0, k - 1}] / k!
zetaHurwitz[n_, s_, y_, 0] := UnitStep[n - 1]
zetaHurwitz[n_, s_, y_, 1] := {2.48832, -1/2}, {1.728, -1/3}, {2.0736, -1/4},
zetaHurwitz[n, s, y, 1] = HarmonicNumber[Floor[n], s] - HarmonicNumber[y, s]
zetaHurwitz[n_, s_, y_, 2] := zetaHurwitz[n, s, y, 1] + {2.48832, -1/2}, {1.728, -1/3}, {2.0736, -1/4},
Sum[(m^(-2s)) + 2 (m^-s) (zetaHurwitz[Floor[n/m], s, m, 1]), {m, y + 1, Floor[n^(1/2)]]}
zetaHurwitz[n_, s_, y_, k_] := zetaHurwitz[n, s, y, k] + {2.48832, -1/2}, {1.728, -1/3}, {2.0736, -1/4},
Sum[(m^(-sk)) + k (m^(-s(k-1))) zetaHurwitz[Floor[n/(m^(k-1))], s, m, 1] +
Sum[binomial[k, j] (m^-s)^j zetaHurwitz[Floor[n/(m^j)], s, m, k - j], {j, 1, k - 2}],
{m, y + 1, Floor[n^(1/k)]]}
zeta[n_, s_, z_] := Expand@Sum[binomial[z, j] zetaHurwitz[n, s, 1, k], {k, 0, Log2@n}]
zetaAlt[n_, s_, x_, z_] := {2.48832, -1/2}, {1.728, -1/3}, {2.0736, -1/4},
Expand@Sum[(-1)^j binomial[z, j] x^j (j(1-s)) zeta[n/(x^j), s, z], {j, 0, Log[x, n]}]
zetaAlt2[n_, x_] := {2.48832, -1/2}, {1.728, -1/3}, {2.0736, -1/4},
D[Expand@Sum[(-1)^j binomial[z, j] x^j zeta[n/(x^j), 0, z], z] /. z -> 0, z] /. z -> 0
zetaAlt3[n_, x_] := {2.48832, -1/2}, {1.728, -1/3}, {2.0736, -1/4},
Sum[D[(-1)^j binomial[z, j] x^j zeta[n/(x^j), 0, z], z] /. z -> 0, {j, 0, Log[x, n]}]
zetaAlt4[n_, x_] := {2.48832, -1/2}, {1.728, -1/3}, {2.0736, -1/4},
Sum[D[(-1)^j binomial[z, j] x^j zeta[n/(x^j), 0, z], z] /. z -> 0, {j, 1, Log[x, n]}]
zetaAlt5[n_, x_] := {2.48832, -1/2}, {1.728, -1/3}, {2.0736, -1/4},
Sum[(-1)^j x^j D[binomial[z, j] zeta[n/(x^j), 0, z], z] /. z -> 0, {j, 1, Log[x, n]}]
zetaAlt5a[n_, x_] := Table[{x^j, D[(-1)^j binomial[z, j] zeta[n/(x^j), 0, z], z] /. z -> 0},
{5, 1, Log[x, n]}]
zetaAlt5b[n_, x_] := Table[{x^j, (D[(-1)^j binomial[z, j], z] zeta[n/(x^j), 0, z] /. z -> 0) +
((-1)^j binomial[z, j] D[zeta[n/(x^j), 0, z], z] /. z -> 0)}, {j, 1, Log[x, n]}]
zetaAlt5c[n_, x_] := Table[{x^j, (- (1/j) zeta[n/(x^j), 0, z] /. z -> 0) +
((-1)^j binomial[z, j] D[zeta[n/(x^j), 0, z], z] /. z -> 0)}, {j, 1, Log[x, n]}]
zetaAlt5d[n_, x_] := Table[{x^j, (- (1/j) +
((-1)^j binomial[z, j] D[zeta[n/(x^j), 0, z], z] /. z -> 0)}, {j, 1, Log[x, n]}]
zetaAlt5e[n_, x_] := Table[{x^j, (- (1/j))}, {j, 1, Log[x, n]}]

pb[n_, s_] := (D[zetaAlt[n, 0, s, z], z] /. z -> 0) + HarmonicNumber[Floor[Log[n] / Log[s]]]
pbz[n_, s_] := zetaAlt5[n, s] + HarmonicNumber[Floor[Log[n] / Log[s]]]
pb2[n_] :=
N@Sum[MangoldtLambda[j] / Log[j], {j, 2, n}] - (LogIntegral[n] - Log@Log@n - EulerGamma)
pb[13, 1.01]
1.47374
pb2[13.]
1.45594
pbz[13, 1.01]
1.47374

```

**zetaAlt5a[10, 1.2]**

$$\left\{ \{1.2, -1\}, \left\{1.44, -\frac{1}{2}\right\}, \left\{1.728, -\frac{1}{3}\right\}, \left\{2.0736, -\frac{1}{4}\right\}, \right. \\ \left. \left\{2.48832, -\frac{1}{5}\right\}, \left\{2.98598, -\frac{1}{6}\right\}, \left\{3.58318, -\frac{1}{7}\right\}, \left\{4.29982, -\frac{1}{8}\right\}, \right. \\ \left. \left\{5.15978, -\frac{1}{9}\right\}, \left\{6.19174, -\frac{1}{10}\right\}, \left\{7.43008, -\frac{1}{11}\right\}, \left\{8.9161, -\frac{1}{12}\right\} \right\}$$

**zetaAlt5b[10, 1.2]**

$$\left\{ \{1.2, -1\}, \left\{1.44, -\frac{1}{2}\right\}, \left\{1.728, -\frac{1}{3}\right\}, \left\{2.0736, -\frac{1}{4}\right\}, \right. \\ \left. \left\{2.48832, -\frac{1}{5}\right\}, \left\{2.98598, -\frac{1}{6}\right\}, \left\{3.58318, -\frac{1}{7}\right\}, \left\{4.29982, -\frac{1}{8}\right\}, \right. \\ \left. \left\{5.15978, -\frac{1}{9}\right\}, \left\{6.19174, -\frac{1}{10}\right\}, \left\{7.43008, -\frac{1}{11}\right\}, \left\{8.9161, -\frac{1}{12}\right\} \right\}$$

**zetaAlt5e[10, 1.2]**

$$\left\{ \{1.2, -1\}, \left\{1.44, -\frac{1}{2}\right\}, \left\{1.728, -\frac{1}{3}\right\}, \left\{2.0736, -\frac{1}{4}\right\}, \right. \\ \left. \left\{2.48832, -\frac{1}{5}\right\}, \left\{2.98598, -\frac{1}{6}\right\}, \left\{3.58318, -\frac{1}{7}\right\}, \left\{4.29982, -\frac{1}{8}\right\}, \right. \\ \left. \left\{5.15978, -\frac{1}{9}\right\}, \left\{6.19174, -\frac{1}{10}\right\}, \left\{7.43008, -\frac{1}{11}\right\}, \left\{8.9161, -\frac{1}{12}\right\} \right\}$$

**Table[(-1)^j binomial[z, j], z] /. z -> 0, {j, 1, 10}]**

$$\left\{ -1, -\frac{1}{2}, -\frac{1}{3}, -\frac{1}{4}, -\frac{1}{5}, -\frac{1}{6}, -\frac{1}{7}, -\frac{1}{8}, -\frac{1}{9}, -\frac{1}{10} \right\}$$

**Table[(-1)^j binomial[z, j] /. z -> 0, {j, 1, 10}]**

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$$