```
num[c_] := Numerator[c]; den[c_] := Denominator[c]
alpha[n_{c}, c_{c}] := den[c] (Floor[n/den[c]] - Floor[(n-1)/den[c]]) -
      num[c] (Floor[n / num[c]] - Floor[(n - 1) / num[c]])
L2[n_{1}, 1, c_{1}] := L2[n, 1, c] = (1/den[c])
         Sum[alpha[j, c] Log[j/den[c]], {j, den[c] + 1, den[c] n}]; L2[n_, 0, c_] := 0
L2[n_{,k_{,c}]} := L2[n,k,c] = (1/den[c])
         Sum[If[alpha[j, c] = 0, 0, alpha[j, c] L2[den[c] n / j, k - 1, c]], {j, den[c] + 1, den[c] n}]
bin[z_{,k_{]}} := Product[z-j, {j, 0, k-1}]/k!
L1[n_{z}, z_{c}] := Sum[bin[z, k] L2[n, k, c], \{k, 0, Floor[Log[c, n]]\}]
zeros[n_{, c_{]}} := List@@Roots[L1[n, z, c] - 1 == 0, z][[All, 2]]
\texttt{L1Alt[} \; n\_, \; z\_, \; c\_] \; := \; 1 - \texttt{Product[} \; 1 - z \, / \, r, \; \{r, \, zeros[n, \, c] \}]
Llm[n_{, c_{]}} := 1 - Product[1 + r^{-1}, \{r, zeros[n, c]\}]
Expand[N[L1[100, z, 4/3]]]
1. -3.84024 z +6.34645 z<sup>2</sup> -6.32933 z<sup>3</sup> +2.83562 z<sup>4</sup> +0.0783242 z<sup>5</sup> +0.14543 z<sup>6</sup> -0.150889 z<sup>7</sup> +
  0.0386438\ z^{8}-0.00777589\ z^{9}+0.00118332\ z^{10}-0.000115508\ z^{11}+7.29511\times 10^{-6}\ z^{12}-0.00118332\ z^{10}-0.000115508\ z^{11}+0.00118332\ z^{11}+0.0011832\ z^{11}+0.001182\ z^{11}+
   2.95732 \times 10^{-7} \; z^{13} \; + \; 8.80027 \times 10^{-9} \; z^{14} \; - \; 1.81087 \times 10^{-10} \; z^{15} \; + \; 1.37187 \times 10^{-12} \; z^{16}
N[Log[2, 100]]
6.64386
N[Log[4/3, 100]]
N[L1Alt[100, -1, 3/2]]
9.95352 - 6.57473 \times 10^{-16} i
N[L1[100, -1, 3/2]]
9.95352
N[L1m[100, 3/2]]
9.95352 - 6.57473 \times 10^{-16} i
p[n_{-}, j_{-}, k_{-}] := If[n < j, 0, 1/k - p[n/j, 2, k+1] + p[n, j+1, k]]
pi[n_{j}] := Sum[MoebiusMu[j] j^{-1}p[n^{(1/j)}, 2, 1], {j, 1, Log[2, n]}]
pi[100]
25
DiscretePlot[pi[n], {n, 1, 100}]
25
20
15
```

100

(-1) ^k Binomial[-1, k] /. k  $\rightarrow$  3

1

 $\label{eq:def:def:def:def:DiscretePlot} DiscretePlot[\,FullSimplify[D[\,L1[\,n,\,\,z,\,\,3]\,,\,\{z,\,2\}]\,]\,,\,\,\{n,\,2,\,70\}]\,\,//\,\,TableForm$ 

