```
m[n_, z_] := Pochhammer[z, n] / (n!)
bo2[z_, k_] := Sum[m[k - 2 j, z] m[j, -z], {j, 0, k / 2}]
bo3[z_, k_] := Sum[m[k - 2 j, z + 1] m[j, -z], {j, 0, k / 2}]
bo3a[z_, k_] := Sum[m[j, z] m[l, -z], {l, 0, k / 2}, {j, 0, k - 2 l}]
bo3b[z_, t_, k_] := Sum[m[j, z] m[l, -z], {l, 0, k / t}, {j, 0, k - t l}]
```

```
Table[bo3[k, j], {k, 0, 5}, {j, 0, k}] // Grid
```

```
1
1  2
1  3   4
1  4   7   8
1  5  11  15  16
1  6  16  26  31  32
```

```
Table[Sum[Binomial[k, n], {n, 0, j}], {k, 0, 5}, {j, 0, k}] // Grid
```

```
1
1  2
1  3   4
1  4   7   8
1  5  11  15  16
1  6  16  26  31  32
```

```
Table[bo3[7.3, j], {j, 0, 8}]
```

```
{1, 8.3, 31.295, 71.9195, 115.591, 144.414, 155.463, 157.515, 157.592}
```

```
Table[Sum[Binomial[7.3, n], {n, 0, j}], {j, 0, 8}]
```

```
{1., 8.3, 31.295, 71.9195, 115.591, 144.414, 155.463, 157.515, 157.592}
```

```
bo2[2.3, 2]
```

```
1.495
```

```
Binomial[2.3, 2]
```

```
1.495
```

```
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
neg[n_, t_] := 1 - t (Floor[n / t] - Floor[(n - 1) / t])
al[n_, t_, 0] := UnitStep[n]
al[n_, t_, k_] := al[n, t, k] = Sum[neg[j, t] / j al[n - j, t, k - 1], {j, 1, n}]
az[n_, t_, z_] := Sum[z^k / k! al[n, t, k], {k, 0, n}]
daz[n_, t_, z_] := az[n, t, z] - az[n - 1, t, z]
bn[z_, k_] := daz[k, 2, z]
```

```
Table[daz[n, 2, 3], {n, 0, 5}]
```

```
{1, 3, 3, 1, 0, 0}
```

```
Table[bn[7, n], {n, 0, 7}]
```

```
{1, 7, 21, 35, 35, 21, 7, 1}
```

```
(1 + x) / (1 + x / 2)
```

$$\frac{1 + x}{1 + \frac{x}{2}}$$

```
D[bo3b[z, 5, 20], z] /. z → 0
```

$$\frac{23\,502\,835}{15\,519\,504}$$

```
HarmonicNumber[20] - HarmonicNumber[Floor[20 / 5]]
```

$$\frac{23\,502\,835}{15\,519\,504}$$

```
D[az[20, 5, z], z] /. z → 0
```

$$\frac{23\,502\,835}{15\,519\,504}$$

```
al[20, 5, 1]
```

$$\frac{23\,502\,835}{15\,519\,504}$$

```
D[((1 + x) / (1 + x / 2))^z, z] /. z → 0
```

$$\mathrm{Log}\left[\frac{1+x}{1+\frac{x}{2}}\right]$$

```
pri[n_] := Sum[PrimePi[n^(1 / k)] / k, {k, 1, Log2@n}]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, z_] := Product[(-1)^p[[2]] bin[-z, p[[2]]], {p, FI[n]}]

po[n_, z_] := Sum[dz[j, -z] dz[k, z], {j, 1, n}, {k, 1, (n / j)^(1 / 2)}]
```

```
D[Expand@po[100, z], z] /. z → 0
```

$$-\frac{116}{5}$$

```
pri[10] - pri[100]
```

$$-\frac{116}{5}$$

```
lo[n_, 0] := UnitStep[n - 1]
lo[n_, k_] := lo[n, k] = Sum[Abs[MoebiusMu[j]] lo[Floor[n / j], k - 1], {j, 2, n}]
lz[n_, z_] := Sum[bin[z, k] lo[n, k], {k, 0, Log2@n}]
lzd[n_, z_] := Product[bin[z, p[[2]]], {p, FI[n]}]
lza[n_, z_] := Sum[lzd[j, z], {j, 1, n}]
```

```
Expand@lz[100, z]
```

$$1+\frac{116\,z}{5}+\frac{9389\,z^2}{360}+\frac{395\,z^3}{48}+\frac{347\,z^4}{144}+\frac{17\,z^5}{240}+\frac{7\,z^6}{720}$$

```
1 + Integrate[D[(1 + t)^z, t], {t, 0, x}] + Integrate[D[(1 + u)^-z, u], {u, 0, x / 2}] +
 Integrate[D[(1 + t)^z, t] D[(1 + u)^-z, u], {t, 0, x}, {u, 0, (x - t) / 2}]
```

$$\mathrm{ConditionalExpression}\left[1-\left(\frac{1}{z}-\frac{2^z\,(2+x)^{-z}}{z}\right)z-2^z\left(\frac{1}{3+x}\right)^z(3+x)^z\,z\,\mathrm{Beta}\left[\frac{2}{3+x},1-z,z\right]+(3+x)^{-z}\left(\frac{1}{6+2x}\right)^{-z}z\,\mathrm{Beta}\left[\frac{2+x}{3+x},1-z,z\right],\ \mathrm{Re}[x]\geq -1\ ||\ x\notin \mathrm{Reals}\right]$$

$$\mathbf{FullSimplify}\left[1-\left(\frac{1}{z}-\frac{2^z\,(2+x)^{-z}}{z}\right)z-2^z\left(\frac{1}{3+x}\right)^z(3+x)^z\,z\,\mathrm{Beta}\left[\frac{2}{3+x},1-z,z\right]+(3+x)^{-z}\left(\frac{1}{6+2x}\right)^{-z}z\,\mathrm{Beta}\left[\frac{2+x}{3+x},1-z,z\right]\right]\;/.\;x\to 3\;/.\;z\to 2$$

Infinity::indet : Indeterminate expression $\frac{4}{25}$ + ComplexInfinity + ComplexInfinity encountered. ≫

```
Indeterminate
```

```
ab[x_, z_] :=
 1 + Integrate[D[(1 + t)^z, t], {t, 0, x}] + Integrate[D[(1 + u)^-z, u], {u, 0, x / 2}] +
  Integrate[D[(1 + t)^z, t] D[(1 + u)^-z, u], {t, 0, x}, {u, 0, x / 2}]
ab2[x_, z_] := ((1 + x) / (1 + x / 2))^z
```

```
ab[6, .5]
```

```
1.32288
```

```
1 + Integrate[D[(1 + t)^z, t], {t, 0, x}] + Integrate[D[(1 + u)^-z, u], {u, 0, x / 2}] +
 Integrate[D[(1 + t)^z, t] D[(1 + u)^-z, u], {t, 0, x}, {u, 0, x / 2}]
```

$$\mathrm{ConditionalExpression}\left[(1+x)^z+(2+x)^{-z}\left(-1+(1+x)^z\right)\left(2^z-(2+x)^z\right)-\left(\frac{1}{z}-\frac{2^z\,(2+x)^{-z}}{z}\right)z,\ \mathrm{Re}[x]\geq -1\ ||\ x\notin \mathrm{Reals}\right]$$

$$\mathbf{FullSimplify}\left[(1+x)^z+(2+x)^{-z}\left(-1+(1+x)^z\right)\left(2^z-(2+x)^z\right)-\left(\frac{1}{z}-\frac{2^z\,(2+x)^{-z}}{z}\right)z\right]$$

$$2^z\left(\frac{1+x}{2+x}\right)^z\;/.\;x\to 113.3\;/.\;z\to 2.3$$

```
4.8269
```

```
((1 + x) / (1 + x / 2))^z /. x → 113.3 /. z → 2.3
```

```
4.8269
```

```
D[(1 + t)^z, t]
```

$$(1+t)^{-1+z}\,z$$

```
D[(1 + t)^-z, t] /. t → u
```

$$-(1+u)^{-1-z}\,z$$

```
FullSimplify[Integrate[LaguerreL[z - 1, 1, -t], {t, 0, x}]]
```

$$-1+\mathrm{LaguerreL}[z,-x]$$

```
FullSimplify@Integrate[LaguerreL[-z - 1, 1, -t], {t, 0, x / 2}]
```

$$\mathrm{ConditionalExpression}\left[-1+\mathrm{Hypergeometric1F1}\left[z,1,-\frac{x}{2}\right],\ \mathrm{Re}[x]\geq -2\ ||\ x\notin \mathrm{Reals}\right]$$

```
Integrate[LaguerreL[z - 1, 1, -t] LaguerreL[-z - 1, 1, -u], {t, 0, x}, {u, 0, (x - t) / 2}]
```

$$\int_0^x z\,\mathrm{Hypergeometric1F1}[1-z,2,-t]\left(-1+\mathrm{Hypergeometric1F1}\left[z,1,\frac{t-x}{2}\right]\right)dt$$

FullSimplify@Table[1 + (-1 + LaguerreL[z, -x]) + (-1 + Hypergeometric1F1[z, 1, -x/2]) + ∫_0^x z Hypergeometric1F1[1 - z, 2, -t] (-1 + Hypergeometric1F1[z, 1, (t - x)/2]) ⅆt, {z, -3, 3}] // TableForm

$\frac{1}{16} e^{-x} (14 + 2 e^{x} + (-10 + x) x)$

$\frac{1}{4} e^{-x} (3 + e^{x} - x)$

$\frac{1}{2} (1 + e^{-x})$

1

$2 - e^{-x/2}$

$4 - \frac{1}{2} e^{-x/2} (6 + x)$

$8 - \frac{1}{8} e^{-x/2} (56 + x (16 + x))$

FullSimplify@Integrate[D[(1 + t)^z, t], {t, 0, x}]

ConditionalExpression[-1 + (1 + x)^z, Re[x] ≥ -1 || x ∉ Reals]

FullSimplify@Integrate[D[(1 + u)^-z, u], {u, 0, x / 2}]

ConditionalExpression[-1 + 2^z (2 + x)^-z, Re[x] ≥ -2 || x ∉ Reals]

FullSimplify@Integrate[D[(1 + t)^z, t] D[(1 + u)^-z, u], {t, 0, x}, {u, 0, x / 2}]

ConditionalExpression[(2 + x)^-z (-1 + (1 + x)^z) (2^z - (2 + x)^z), Re[x] ≥ -1 || x ∉ Reals]

1 + Integrate[D[(1 + t)^z, t], {t, 0, x}] + Integrate[D[(1 + u)^-z, u], {u, 0, x / k}] + Integrate[D[(1 + t)^z, t] D[(1 + u)^-z, u], {t, 0, x}, {u, 0, x / k}]

$$\text{ConditionalExpression}\left[1 + \left(\frac{k+x}{k}\right)^{-z} (-1 + (1+x)^z) - \left(\frac{1}{z} - \frac{\left(\frac{k+x}{k}\right)^{-z}}{z}\right) z,\ (\text{Re}[x] \geq -1 \,||\, x \notin \text{Reals}) \,\&\&\, \left(\left(k \neq 0 \,\&\&\, x \neq 0 \,\&\&\, \text{Re}\left[\frac{k}{x}\right] \geq 0\right) \,||\, \text{Re}\left[\frac{k}{x}\right] \leq -1 \,||\, \frac{k}{x} \notin \text{Reals}\right)\right]$$

$$\textbf{FullSimplify}\left[1 + \left(\frac{k+x}{k}\right)^{-z} (-1 + (1+x)^z) - \left(\frac{1}{z} - \frac{\left(\frac{k+x}{k}\right)^{-z}}{z}\right) z\right] \text{ /. } x \to 8. \text{ /. } k \to 3. \text{ /. } z \to 2.3$$

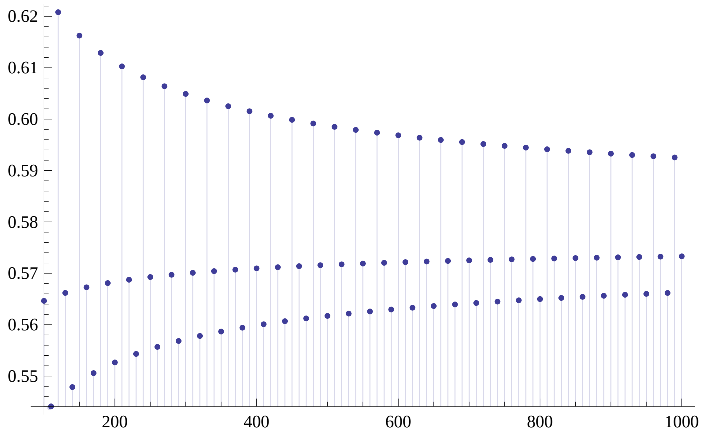
7.88739

((1 + x) / (1 + x / k))^z /. x → 8. /. k → 3. /. z → 2.3

7.88739

```
pz[x_, z_] := Pochhammer[z, x] / x!
pk[x_, z_, k_] := Sum[pz[t, z] pz[u, -z], {t, 0, x}, {u, 0, (x - t) / k}]
pka[x_, z_, k_] := Sum[pz[x - u k, z + 1] pz[u, -z], {u, 0, x / k}]
pkb[x_, z_, k_] := Sum[pz[x - u k, -z + 1] pz[u, z], {u, 0, x / k}]
```

```
DiscretePlot[pka[n, z, 3] /. z → -.5, {n, 100, 1000, 10}]
```



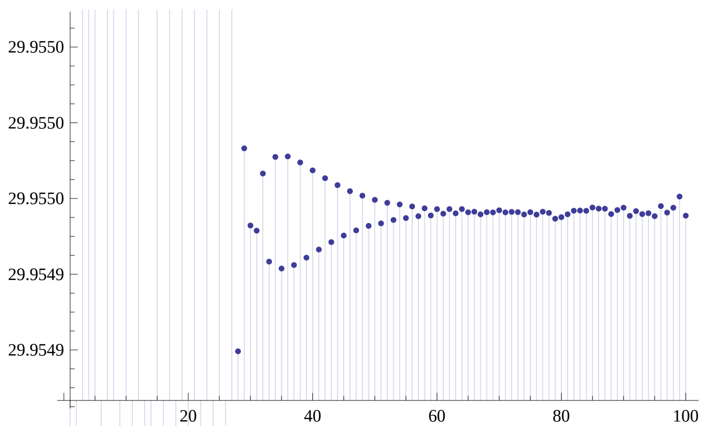

```
2^(5.1 + 3 I)
```

-16.7023 + 29.955 i

```
pka[30., 2.5, 2]
```

5.65685

```
DiscretePlot[Im@pka[n, z, 2] /. z → (5.1 + 3 I), {n, 1, 100}]
```



$$bb[x\_, z\_] := 1 + (-1 + \text{LaguerreL}[z, -x]) + \left(-1 + \text{Hypergeometric1F1}\left[z, 1, -\frac{x}{2}\right]\right) + \int_0^x z\,\text{Hypergeometric1F1}[1 - z, 2, -t]\left(-1 + \text{Hypergeometric1F1}\left[z, 1, \frac{t - x}{2}\right]\right) dt$$

```
N[bb[58., 2.5 + I]]
```

4.35147 + 3.61451 i

```
2^(2.5 + I)
```

4.35147 + 3.61451 i

```
Limit[((1 + x) / (1 + x / k))^z, x → Infinity]
```

k^z

```
FullSimplify[Integrate[LaguerreL[z - 1, 1, -Log[t]], {t, 1, x}]]
```

$$\int_1^x \text{LaguerreL}[-1+z, 1, -\text{Log}[t]]\, dt$$

```
FullSimplify@Integrate[LaguerreL[-z - 1, 1, -Log[u]], {u, 1, x^(1/2)}]
```

$$\int_1^{\sqrt{x}} -z\, \text{Hypergeometric1F1}[1+z, 2, -\text{Log}[u]]\, du$$

```
Integrate[LaguerreL[z - 1, 1, -Log[t]] LaguerreL[-z - 1, 1, -Log[u]],
 {t, 1, x}, {u, 1, (x / t)^(1 / 2)}]
```

$$\int_1^x \left(-1 + \text{LaguerreL}\left[z, \frac{1}{2}\text{Log}\left[\frac{x}{t}\right]\right]\right) \text{LaguerreL}[-1+z, 1, -\text{Log}[t]]\, dt$$

cc[x_, z_] :=

$$1 + \int_1^x \text{LaguerreL}[-1+z, 1, -\text{Log}[t]]\, dt + \int_1^{\sqrt{x}} -z\, \text{Hypergeometric1F1}[1+z, 2, -\text{Log}[u]]\, du + \int_1^x \left(-1 + \text{LaguerreL}\left[z, \frac{1}{2}\text{Log}\left[\frac{x}{t}\right]\right]\right) \text{LaguerreL}[-1+z, 1, -\text{Log}[t]]\, dt$$

```
FullSimplify@cc[x, 1]
```

$$\text{ConditionalExpression}\left[\frac{1+x}{2}, \text{Re}[x] \geq 0 \,||\, x \notin \text{Reals}\right]$$

```
FullSimplify@cc[x, 2]
```

$$\text{ConditionalExpression}\left[\frac{1}{4}(1 + 3x + x\,\text{Log}[x]), \text{Re}[x] \geq 0 \,||\, x \notin \text{Reals}\right]$$

```
FullSimplify@cc[x, 3]
```

$$\text{ConditionalExpression}\left[\frac{1}{16}(2 + 14x + x\,\text{Log}[x]\,(10 + \text{Log}[x])), \text{Re}[x] \geq 0 \,||\, x \notin \text{Reals}\right]$$

```
pkk[x_, z_, k_] :=
 Sum[pz[t, 2 z] pz[u, -z] pz[v, -z], {t, 0, x}, {u, 0, (x - t) / k}, {v, 0, (x - t - k u) / k}]
pkk[20., 2.6, 2]
```

36.7583

```
4^(2.6)
```

36.7583

```
FullSimplify[Sum[Pochhammer[-z, u] / u!, {u, 0, Floor[(x - t) / t]}]]
```

$$\frac{\text{Gamma}\left[-z + \text{Floor}\left[\frac{x}{t}\right]\right]}{\text{Gamma}[1-z]\,\text{Gamma}\left[\text{Floor}\left[\frac{x}{t}\right]\right]}$$

$$\text{Sum}\left[\frac{\text{Gamma}\left[-z + \text{Floor}\left[\frac{x}{t}\right]\right]}{\text{Gamma}[1-z]\,\text{Gamma}\left[\text{Floor}\left[\frac{x}{t}\right]\right]}, \{t, 0, x\}\right]$$

$$\sum_{t=0}^{x} \frac{\text{Gamma}\left[-z + \text{Floor}\left[\frac{x}{t}\right]\right]}{\text{Gamma}[1-z]\,\text{Gamma}\left[\text{Floor}\left[\frac{x}{t}\right]\right]}$$

Sum[Gamma[-z + x / t] / (Gamma[1 - z] Gamma[x / t]), {t, 0, x}]

$$\sum_{t=0}^{x} \frac{\text{Gamma}\left[\frac{x}{t} - z\right]}{\text{Gamma}\left[\frac{x}{t}\right] \text{Gamma}[1 - z]}$$

Expand@FullSimplify[(1 - x^4) / (1 - x)]

$1 + x + x^2 + x^3$

(1 - x^k) / (1 - x)

$$\frac{1 - x^k}{1 - x}$$

(1 + x) / (1 + x / k)

$$\frac{1 + x}{1 + \frac{x}{k}}$$

```
pz[x_, z_] := Pochhammer[z, x] / x!
pt[x_, z_, a_] := If[x / a < 1, 1, Sum[pz[j, z] pt[x - a j, z, a + 1], {j, 0, x / a}]]
```

D[Expand@pt[20, z, 1], z] /. z → 0

$$\frac{7\,257\,705\,647}{232\,792\,560}$$

Sum[PartitionsP[j], {j, 0, 20}]

2714

Sum[HarmonicNumber[Floor[20 / k]], {k, 1, 20}]

$$\frac{7\,257\,705\,647}{232\,792\,560}$$

FullSimplify@Expand[x / (1 - x - x^2) /. x → (1 + x)]

$$-\frac{1 + x}{1 + x\,(3 + x)}$$

Sum[Fibonacci[k] x^k, {k, 0, Infinity}]

$$-\frac{x}{-1 + x + x^2}$$

Table[D[x / (1 - x - x^2), {x, k}] / k! /. x → 0, {k, 0, 20}]

{0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765}

FullSimplify[1 - x - x^2]

$1 - x\,(1 + x)$

Sum[Binomial[z, k] (-1)^k x^k, {k, 0, Infinity}]

$(1 - x)^z$

```
fl[j_, k_] := 1 - k (Floor[j / k] - Floor[(j - 1) / k])
tri[z_, x_] := Sum[pz[x - 3 u, z] pz[u, -z], {u, 0, x / 3}]
```

Table[tri[4, k], {k, 0, 12}]

{1, 4, 10, 16, 19, 16, 10, 4, 1, 0, 0, 0, 0}

```
tt[z_] := Sum[tri[z, k] x^k, {k, 0, 10}]
```

```
tt[1]
```

$-1 + x - x^2$

```
(* http://oeis.org/A001350 *)
Clear[am, amm]
am[n_, 0] := UnitStep[n]
am[n_, k_] := am[n, k] = Sum[Fibonacci[j] am[n - j, k - 1], {j, 1, n}]
amz[n_, z_] := Sum[bin[z, k] am[n, k], {k, 0, n}]
damz[n_, z_] := amz[n, z] - amz[n - 1, z]
iv[n_] := Floor[n / 2] - Floor[(n + 1) / 2]
amm[n_, 0] := UnitStep[n]
amm[n_, k_] := amm[n, k] = Sum[iv[j] amm[n - j, k - 1], {j, 1, n}]
ammz[n_, z_] := Sum[bin[z, k] amm[n, k], {k, 0, n}]
dammz[n_, z_] := amz[n, z] - ammz[n - 1, z]

Table[D[amz[j, z], z] /. z → 0, {j, 1, 10}]
Table[D[damz[j, z], z] /. z → 0, {j, 1, 10}]
Table[damz[j, z] /. z → -1, {j, 1, 32}]
Table[ammz[j, z] /. z → -1, {j, 1, 32}]
```

$\left\{1, \frac{3}{2}, \frac{17}{6}, \frac{49}{12}, \frac{377}{60}, \frac{179}{20}, \frac{1833}{140}, \frac{5241}{280}, \frac{68449}{2520}, \frac{98941}{2520}\right\}$

$\left\{1, \frac{1}{2}, \frac{4}{3}, \frac{5}{4}, \frac{11}{5}, \frac{8}{3}, \frac{29}{7}, \frac{45}{8}, \frac{76}{9}, \frac{121}{10}\right\}$

{-1, 0, -1, 0, -1, 0, -1, 0, -1, 0, -1, 0, -1, 0,
 -1, 0, -1, 0, -1, 0, -1, 0, -1, 0, -1, 0, -1, 0, -1, 0, -1, 0}

{2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987,
 1597, 2584, 4181, 6765, 10 946, 17 711, 28 657, 46 368, 75 025, 121 393,
 196 418, 317 811, 514 229, 832 040, 1 346 269, 2 178 309, 3 524 578, 5 702 887}

```
damz[10, 1]
```

55

```
Fibonacci[10]
```

55

```
377 * 2
```

754

```
Table[iv[n], {n, 1, 10}]
```

{-1, 0, -1, 0, -1, 0, -1, 0, -1, 0}