```
P[n_, k_, j_] := (-1)^(k + 1) / k + P[Floor[n / j], k + 1, Floor[n / j]] + P[n, k, j - 1]
P[n_, k_, 1] := 0
```

```
P[100, 1, 100]
```

$$\frac{428}{15}$$

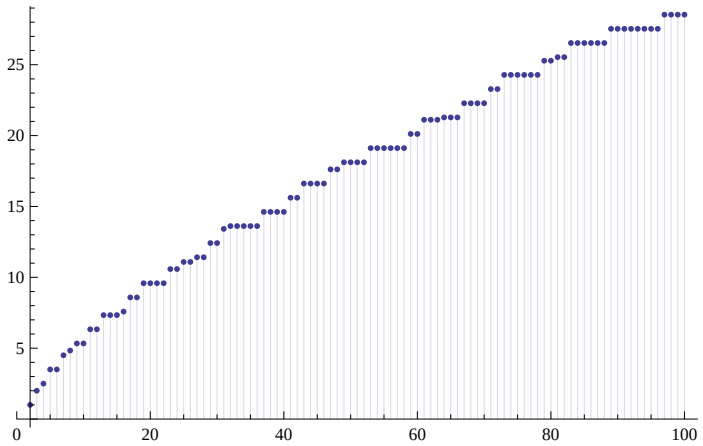
```
P[n, 1, n]
```

```
$Aborted
```

```
P[100, 1, 50]
```

$$-\frac{322}{15}$$

```
DiscretePlot[ Abs[P[n, 1, n]], {n, 2, 100}]
```




```
P[101, k, 101]
```

$$\frac{100\,(-1)^{1+k}}{k}+\frac{283\,(-1)^{2+k}}{1+k}+\frac{324\,(-1)^{3+k}}{2+k}+\frac{184\,(-1)^{4+k}}{3+k}+\frac{51\,(-1)^{5+k}}{4+k}+\frac{7\,(-1)^{6+k}}{5+k}$$

```
S[k_] := P[100, k, 100] - P[99, k, 99]
```

```
S[1.000001]
```

$$-2.08333\times10^{-7}-6.53609\times10^{-13}\,i$$

```
P[100, k, 100]
```

$$\frac{99\,(-1)^{1+k}}{k}+\frac{283\,(-1)^{2+k}}{1+k}+\frac{324\,(-1)^{3+k}}{2+k}+\frac{184\,(-1)^{4+k}}{3+k}+\frac{51\,(-1)^{5+k}}{4+k}+\frac{7\,(-1)^{6+k}}{5+k}$$

```
P[99, k, 99]
```

$$\frac{98\,(-1)^{1+k}}{k}+\frac{276\,(-1)^{2+k}}{1+k}+\frac{312\,(-1)^{3+k}}{2+k}+\frac{178\,(-1)^{4+k}}{3+k}+\frac{51\,(-1)^{5+k}}{4+k}+\frac{7\,(-1)^{6+k}}{5+k}$$

```
T[n_] := P[n, k, n]
```

```
T[99]
```

$$\frac{98\,(-1)^{1+k}}{k}+\frac{276\,(-1)^{2+k}}{1+k}+\frac{312\,(-1)^{3+k}}{2+k}+\frac{178\,(-1)^{4+k}}{3+k}+\frac{51\,(-1)^{5+k}}{4+k}+\frac{7\,(-1)^{6+k}}{5+k}$$

```
T[120]
```

$$\frac{119\,(-1)^{1+k}}{k}+\frac{363\,(-1)^{2+k}}{1+k}+\frac{453\,(-1)^{3+k}}{2+k}+\frac{284\,(-1)^{4+k}}{3+k}+\frac{86\,(-1)^{5+k}}{4+k}+\frac{7\,(-1)^{6+k}}{5+k}$$

```
V[n_, k_] := Sum[((-1)^(k+1))/k + V[n/j, k+1], {j, 2, n}]
```

```
V[100, k]
```

$$\frac{99\,(-1)^{1+k}}{k}+\frac{283\,(-1)^{2+k}}{1+k}+\frac{324\,(-1)^{3+k}}{2+k}+\frac{184\,(-1)^{4+k}}{3+k}+\frac{51\,(-1)^{5+k}}{4+k}+\frac{7\,(-1)^{6+k}}{5+k}$$

```
3+3 I
```

$3+3\,i$

```
Re[V[100, .5]]
```

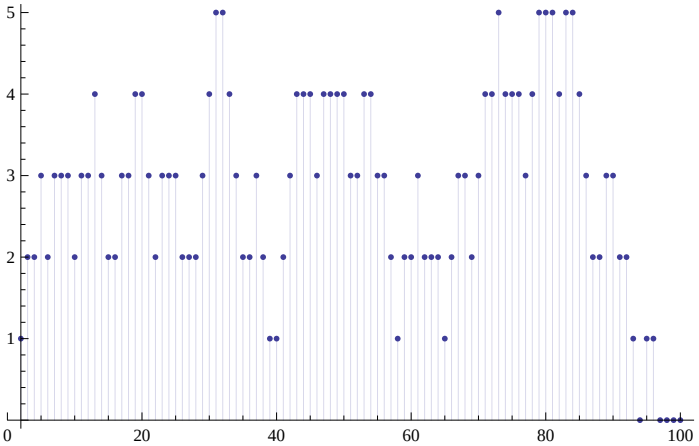
-3.42012×10^{-14}

```
Table[N[Im[P[n, 1 I, n] - P[n-1, 1 I, n-1]]], {n, 2, 100}]
```

$$\{0.0432139,\ 0.0432139,\ 0.021607,\ 0.0432139,\ -1.38778\times10^{-17},\ 0.0432139,\ 0.00864278,\ 0.021607,\ -1.38778\times10^{-17},\ 0.0432139,\ -0.0172856,\ 0.0432139,\ -1.38778\times10^{-17},\ -1.38778\times10^{-17},\ -1.04083\times10^{-17},\ 0.0432139,\ -0.0172856,\ 0.0432139,\ -0.0172856,\ -1.38778\times10^{-17},\ -1.38778\times10^{-17},\ 0.0432139,\ -0.0259284,\ 0.021607,\ -1.38778\times10^{-17},\ 0.00864278,\ -0.0172856,\ 0.0432139,\ -0.0345711,\ 0.0432139,\ -0.00610079,\ -1.38778\times10^{-17},\ -1.38778\times10^{-17},\ -1.38778\times10^{-17},\ -0.0302497,\ 0.0432139,\ -1.38778\times10^{-17},\ -1.38778\times10^{-17},\ -0.0259284,\ 0.0432139,\ -0.0345711,\ 0.0432139,\ -0.0172856,\ -0.0172856,\ -1.38778\times10^{-17},\ 0.0432139,\ -0.0305039,\ 0.021607,\ -0.0172856,\ -1.38778\times10^{-17},\ -0.0172856,\ 0.0432139,\ -0.0259284,\ -1.38778\times10^{-17},\ -0.0259284,\ -1.38778\times10^{-17},\ -1.38778\times10^{-17},\ 0.0432139,\ -0.0432139,\ 0.0432139,\ -1.38778\times10^{-17},\ -0.0172856,\ -0.0105591,\ -1.38778\times10^{-17},\ -0.0345711,\ 0.0432139,\ -0.0172856,\ -1.38778\times10^{-17},\ -0.0345711,\ 0.0432139,\ -0.0350795,\ 0.0432139,\ -1.38778\times10^{-17},\ -0.0172856,\ -0.0172856,\ -1.38778\times10^{-17},\ -0.0345711,\ 0.0432139,\ -0.0305039,\ -1.04083\times10^{-17},\ -1.38778\times10^{-17},\ 0.0432139,\ -0.0432139,\ -1.38778\times10^{-17},\ -1.38778\times10^{-17},\ -1.38778\times10^{-17},\ -0.0259284,\ 0.0432139,\ -0.0432139,\ -1.38778\times10^{-17},\ -0.0172856,\ -1.38778\times10^{-17},\ -1.38778\times10^{-17},\ -1.38778\times10^{-17},\ -0.0328504,\ 0.0432139,\ -0.0172856,\ -0.0172856,\ -0.0302497\}$$

```
M[n_, k_, j_] := (-1)^(k+1) + M[n/j, k+1, Floor[n/j]] + M[n, k, j-1]
M[n_, k_, 1] := 0
```

```
DiscretePlot[Abs[M[n, 2.5, n]], {n, 2, 100}]
```



```
N[M[88, 1 + I, 88]]
```

$$0.0864278 + 3.70453 \times 10^{-17}\, i$$

```
P2[n_, k_, j_] := j ((-1)^(k + 1) / k + P2[Floor[n / j], k + 1, Floor[n / j]]) + P2[n, k, j - 1]
P2[n_, k_, 1] := 0
```

```
P2[100, 1, 100]
```

$$\frac{292\,149\,953\,504\,274\,361\,788\,974\,787\,095\,433\,526\,022\,627}{139\,440\,750\,459\,424\,954\,329\,067\,617\,870\,624\,607\,113\,600}$$

```
P3[n_, k_] := Sum[j ((-1)^(k + 1) / k + P3[Floor[n / j], k + 1]), {j, 2, n}]
```

```
P3[100, 1]
```

$$\frac{69\,389}{60}$$

```
P3[99, 1]
```

$$\frac{69\,389}{60}$$

```
Expand[P2[100, k, 100]]
```

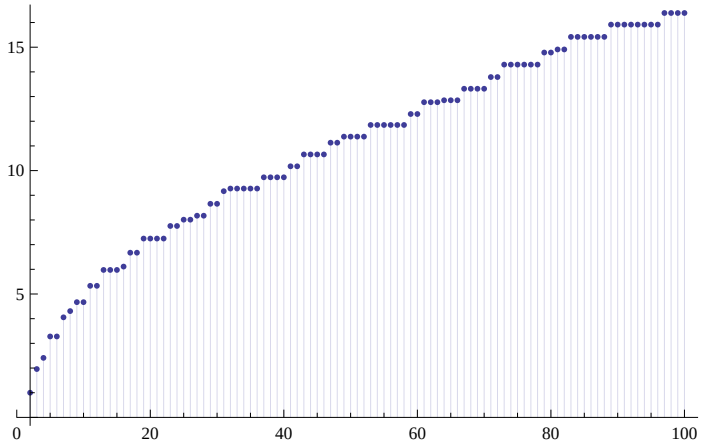
$$\frac{5049\,(-1)^{1+k}}{k} + \frac{16\,780\,(-1)^{2+k}}{1+k} + \frac{21\,426\,(-1)^{3+k}}{2+k} + \frac{13\,441\,(-1)^{4+k}}{3+k} + \frac{4112\,(-1)^{5+k}}{4+k} + \frac{640\,(-1)^{6+k}}{5+k}$$

```
P4[n_, k_, j_, a_] :=
 j^a ((-1)^(k + 1) / k + P4[Floor[n / j], k + 1, Floor[n / j], a]) + P4[n, k, j - 1, a]
P4[n_, k_, 1, a_] := 0
```

```
N[P4[100, 1, 100, I]]
```

$$-15.2228 - 6.06391\, i$$

```
DiscretePlot[Abs[P4[n, 1, n, I]], {n, 2, 100}]
```



```
Table[N[Abs[P4[n, 1, n, I]] - Abs[P4[n - 1, 1, n - 1, I]]], {n, 2, 100}]
```

$$\{1.,\ 0.95904,\ 0.452594,\ 0.867604,\ 0.,\ 0.775158,\ 0.253034,\ 0.361793,\ 8.88178\times10^{-16},\ 0.662151,\ -8.88178\times10^{-16},\ 0.64353,\ 8.88178\times10^{-16},\ 8.88178\times10^{-16},\ 0.135927,\ 0.56273,\ 8.88178\times10^{-16},\ 0.572853,\ -8.88178\times10^{-16},\ -8.88178\times10^{-16},\ 1.77636\times10^{-15},\ 0.509893,\ 0.,\ 0.25518,\ -1.77636\times10^{-15},\ 0.160927,\ 0.,\ 0.482207,\ -3.55271\times10^{-15},\ 0.511384,\ 0.106546,\ 0.,\ 0.,\ 0.,\ 3.55271\times10^{-15},\ 0.455764,\ 0.,\ 3.55271\times10^{-15},\ -3.55271\times10^{-15},\ 0.446087,\ -1.77636\times10^{-15},\ 0.481048,\ 7.10543\times10^{-15},\ -3.55271\times10^{-15},\ -1.77636\times10^{-15},\ 0.475359,\ -3.55271\times10^{-15},\ 0.245451,\ -3.55271\times10^{-15},\ 3.55271\times10^{-15},\ 0.,\ 0.472611,\ 3.55271\times10^{-15},\ 0.,\ 0.,\ 0.,\ 0.,\ 0.443995,\ -8.88178\times10^{-15},\ 0.478449,\ -3.55271\times10^{-15},\ 3.55271\times10^{-15},\ 0.0786341,\ 3.55271\times10^{-15},\ -7.10543\times10^{-15},\ 0.466536,\ -3.55271\times10^{-15},\ 0.,\ 0.,\ 0.473867,\ -7.10543\times10^{-15},\ 0.504785,\ 3.55271\times10^{-15},\ -8.88178\times10^{-15},\ 0.,\ 5.32907\times10^{-15},\ -3.55271\times10^{-15},\ 0.488939,\ -5.32907\times10^{-15},\ 0.124856,\ 0.,\ 0.509364,\ -7.10543\times10^{-15},\ 7.10543\times10^{-15},\ -5.32907\times10^{-15},\ -1.77636\times10^{-15},\ 7.10543\times10^{-15},\ 0.497493,\ -1.77636\times10^{-15},\ 3.55271\times10^{-15},\ -1.77636\times10^{-15},\ -3.55271\times10^{-15},\ -5.32907\times10^{-15},\ 3.55271\times10^{-15},\ 5.32907\times10^{-15},\ 0.470298,\ 0.,\ -3.55271\times10^{-15},\ 0.\}$$

```
PS[n_, k_, j_] := 1 / k - PS[Floor[n / j], k + 1, Floor[n / j]] + PS[n, k, j - 1]
PS[n_, k_, 1] := 0

PS[100, 1, 100]
```

$$\frac{428}{15}$$

```
PS[n, 1, n] - PS[n - 1, 1, n - 1]
```

```
$Aborted
```

```
PS[n, 1, n]
```

```
$Aborted
```

```
F[n_] := PS[n, 1, n] - PS[n - 1, 1, n - 1]

F[101]
```

```
1
```

```
F2[n_] := PS[n, 1, n - 1] - PS[n - 1, 1, n - 2]

F2[1]
```

```
0
```

```
Table[F2[n], {n, 3, 100}]
```

$$\{1,\ \frac{1}{2},\ 1,\ 0,\ 1,\ \frac{1}{3},\ \frac{1}{2},\ 0,\ 1,\ 0,\ 1,\ 0,\ 0,\ \frac{1}{4},\ 1,\ 0,\ 1,\ 0,\ 0,\ 0,\ 1,\ 0,\ \frac{1}{2},\ 0,\ \frac{1}{3},\ 0,\ 1,\ 0,\ 1,\ \frac{1}{5},\ 0,\ 0,\ 0,\ 0,\ 1,\ 0,\ 0,\ 0,\ 1,\ 0,\ 1,\ 0,\ 0,\ 0,\ 1,\ 0,\ \frac{1}{2},\ 0,\ 0,\ 0,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0,\ 1,\ 0,\ 1,\ 0,\ 0,\ \frac{1}{6},\ 0,\ 0,\ 1,\ 0,\ 0,\ 0,\ 1,\ 0,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0,\ 1,\ 0,\ \frac{1}{4},\ 0,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 1,\ 0,\ 0,\ 0\}$$

```
P[100, k, 100] - P[100, k + 1, 100]
```

$$\frac{99\ (-1)^{1+k}}{k}+\frac{184\ (-1)^{2+k}}{1+k}+\frac{41\ (-1)^{3+k}}{2+k}-\frac{140\ (-1)^{4+k}}{3+k}-\frac{133\ (-1)^{5+k}}{4+k}-\frac{44\ (-1)^{6+k}}{5+k}-\frac{7\ (-1)^{7+k}}{6+k}$$