```
RiePrimeCnt[n_] := Sum[PrimePi[n^(1/j)]/j, {j, 1, Log[2, n]}]
RieExplicitForumla[x_, t_] :=
 Sum[
  MoebiusMu[j]/j ( LogIntegral[Floor[x^(1/j)]] -
     N[2 Re[Sum[ExpIntegralEi[ZetaZero[k] Log[Floor[x^(1/j)]]], {k, 1, t}]]] + NIntegrate[
      1/((y^3 - y) Log[y]), {y, Floor[x^(1/j)], Infinity}] - Log[2]), {j, 1, Log[2, x]}]
Table[{n, PrimePi[n], RieExplicitForumla[n, 200]}, {n, 2, 100}] // TableForm
```

2	1	1.00472
3	2	1.99549
4	2	1.99492
5	3	3.24489
6	3	3.25292
7	4	4.2518
8	4	4.24572
9	4	4.16752
10	4	4.41975
11	5	5.4074
12	5	5.44261
13	6	6.43031
14	6	6.41827
15	6	6.42828
16	6	6.17565
17	7	7.30004
18	7	7.28872
19	8	8.2884
20	8	8.29036
21	8	8.29422
22	8	8.296
23	9	9.28529
24	9	9.29562
25	9	9.16308
26	9	9.43152
27	9	9.23113
28	9	9.42433
29	10	10.4333
30	10	10.4311
31	11	11.4203
32	11	11.3908
33	11	11.5
34	11	11.5118
35	11	11.5188
36	11	11.2671
37	12	12.2476
38	12	12.2821
39	12	12.2883
40	12	12.2113
41	13	13.2913
42	13	13.2568
43	14	14.2533
44	14	14.304
45	14	14.248
46	14	14.2556
47	15	15.256
48	15	15.2729

49	15	15.2883
50	15	15.497
51	15	15.5019
52	15	15.5437
53	16	16.5189
54	16	16.4819
55	16	16.5075
56	16	16.5187
57	16	16.5344
58	16	16.5686
59	17	17.5349
60	17	17.5151
61	18	18.4937
62	18	18.4558
63	18	18.4878
64	18	18.0772
65	18	18.1631
66	18	18.192
67	19	19.1483
68	19	19.1097
69	19	19.1543
70	19	19.209
71	20	20.1998
72	20	20.2079
73	21	21.2094
74	21	21.1781
75	21	21.192
76	21	21.1749
77	21	21.1638
78	21	21.195
79	22	22.182
80	22	22.1717
81	22	22.106
82	22	22.2344
83	23	23.2118
84	23	23.2225
85	23	23.2763
86	23	23.2369
87	23	23.176
88	23	23.2056
89	24	24.2354
90	24	24.2449
91	24	24.2544
92	24	24.2135
93	24	24.2102
94	24	24.2494
95	24	24.2225
96	24	24.1697
97	25	25.2141
98	25	25.3034
99	25	25.2196
100	25	25.0293

```
RiePrimeCnt[n_] := Sum[PrimePi[n^(1/j)]/j, {j, 1, Log[2, n]}]
RieExplicitForumla[x_, t_] := Sum[MoebiusMu[j]/j (LogIntegral[a = Floor[x^(1/j)]] -
    N[2 Re[Sum[ExpIntegralEi[ZetaZero[k] Log[a]], {k, 1, t}]]] +
    NIntegrate[1/((y^3 - y) Log[y]), {y, a, Infinity}] - Log[2]), {j, 1, Log[2, x]}]
Timing[Table[{n, PrimePi[n], RieExplicitForumla[n, 200]}, {n, 2, 30}] // TableForm ]
```

{5.662,

2	1	0.504723
3	2	1.49549
4	2	1.99492
5	3	2.74489
6	3	3.25292
7	4	3.7518
8	4	4.24572
9	4	4.16752
10	4	4.41975
11	5	4.9074
12	5	5.44261
13	6	5.93031
14	6	6.41827
15	6	6.42828
16	6	6.17565
17	7	6.80004
18	7	7.28872
19	8	7.7884
20	8	8.29036
21	8	8.29422
22	8	8.296
23	9	8.78529
24	9	9.29562
25	9	9.16308
26	9	9.43152
27	9	9.23113
28	9	9.42433
29	10	9.93332
30	10	10.4311

}

```
RieExplicitForumla[x_, t_] := Sum[MoebiusMu[j] / j (LogIntegral[a = Floor[x^(1 / j)]] -
    N[2 Re[Sum[ExpIntegralEi[ZetaZero[k] Log[a]], {k, 1, t}]]] +
    NIntegrate[1 / ((y^3 - y) Log[y]), {y, a, Infinity}] - Log[2]), {j, 1, Log[2, x]}]
Timing[Table[{n, PrimePi[n], RieExplicitForumla[n, 100]}, {n, 2, 30}] // TableForm]
```

{2.184,

2	1	0.507407
3	2	1.49555
4	2	1.98613
5	3	2.74936
6	3	3.24187
7	4	3.74141
8	4	4.22353
9	4	4.17861
10	4	4.41403
11	5	4.90318
12	5	5.43601
13	6	5.92149
14	6	6.39555
15	6	6.42845
16	6	6.14532
17	7	6.80166
18	7	7.3174
19	8	7.80338
20	8	8.27543
21	8	8.27704
22	8	8.27938
23	9	8.79944
24	9	9.30825
25	9	9.1466
26	9	9.43973
27	9	9.22401
28	9	9.42297
29	10	9.91472
30	10	10.4079

}

```
RieExplicitForumla[x_, t_] := Sum[MoebiusMu[j] / j (LogIntegral[a = Floor[x^(1 / j)]] -
    N[2 Re[Sum[ExpIntegralEi[ZetaZero[k] Log[a]], {k, 1, t}]]] +
    NIntegrate[1 / ((y^3 - y) Log[y]), {y, a, Infinity}] - Log[2]), {j, 1, Log[2, x]}]
Timing[Table[{n, PrimePi[n], RieExplicitForumla[n, 400]}, {n, 2, 30}] // TableForm ]
```

{10.639,

2	1	0.507068
3	2	1.50275
4	2	1.99643
5	3	2.74307
6	3	3.246
7	4	3.74942
8	4	4.24375
9	4	4.16449
10	4	4.40947
11	5	4.91627
12	5	5.41431
13	6	5.91278
14	6	6.4083
15	6	6.41381
16	6	6.15792
17	7	6.79317
18	7	7.29052
19	8	7.78444
20	8	8.30194
21	8	8.28788
22	8	8.28964
23	9	8.79075
24	9	9.2955
25	9	9.15849
26	9	9.41954
27	9	9.25477
28	9	9.41877
29	10	9.90978
30	10	10.4171

}

```
RiePrimeCnt[n_] := Sum[PrimePi[n^(1/j)]/j, {j, 1, Log[2, n]}]
RieExplicitForumla[x_, t_] :=
 LogIntegral[a = x] - N[2 Re[Sum[ExpIntegralEi[ZetaZero[k] Log[a]], {k, 1, t}]]] +
  NIntegrate[1/((y^3 - y) Log[y]), {y, a, Infinity}] - Log[2]
Timing[Table[{n, RiePrimeCnt[n], RieExplicitForumla[n, 1000]}, {n, 2, 30}] // TableForm ]
```

{4.93,

2	1	0.507068
3	2	1.50275
4	$\frac{5}{2}$	2.24996
5	$\frac{7}{2}$	2.9966
6	$\frac{7}{2}$	3.49954
7	$\frac{9}{2}$	4.00295
8	$\frac{29}{6}$	4.66631
9	$\frac{16}{3}$	5.08489
10	$\frac{16}{3}$	5.32987
11	$\frac{19}{3}$	5.83667
12	$\frac{19}{3}$	6.33471
13	$\frac{22}{3}$	6.83317
14	$\frac{22}{3}$	7.32869
15	$\frac{22}{3}$	7.33421
16	$\frac{91}{12}$	7.45192
17	$\frac{103}{12}$	8.08718
18	$\frac{103}{12}$	8.58452
19	$\frac{115}{12}$	9.07844
20	$\frac{115}{12}$	9.59594
21	$\frac{115}{12}$	9.58189
22	$\frac{115}{12}$	9.58365
23	$\frac{127}{12}$	10.0848
24	$\frac{127}{12}$	10.5895
25	$\frac{133}{12}$	10.8258
26	$\frac{133}{12}$	11.0869
27	$\frac{137}{12}$	11.254
28	$\frac{137}{12}$	11.418
29	$\frac{149}{12}$	11.909
30	$\frac{149}{12}$	12.4164

}

```
RiePrimeCnt[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
RieExplicitForumla[x_, t_] :=
 LogIntegral[a = x] - N[2 Re[Sum[ExpIntegralEi[ZetaZero[k] Log[a]], {k, 1, t}]]] +
  Integrate[1 / ((y^3 - y) Log[y]), {y, a, Infinity}] - Log[2]
Timing[Table[{n, RiePrimeCnt[n], RieExplicitForumla[n, 1000]}, {n, 2, 30}] // TableForm ]
```

{14.04,

2	1	0.497791
3	2	1.49968
4	$\frac{5}{2}$	2.24767
5	$\frac{7}{2}$	3.00119
6	$\frac{7}{2}$	3.50193
7	$\frac{9}{2}$	4.00022
8	$\frac{29}{6}$	4.66657
9	$\frac{16}{3}$	5.08214
10	$\frac{16}{3}$	5.33126
11	$\frac{19}{3}$	5.83329
12	$\frac{19}{3}$	6.33554
13	$\frac{22}{3}$	6.83141
14	$\frac{22}{3}$	7.33526
15	$\frac{22}{3}$	7.33309
16	$\frac{91}{12}$	7.45547
17	$\frac{103}{12}$	8.08628
18	$\frac{103}{12}$	8.58299
19	$\frac{115}{12}$	9.08132
20	$\frac{115}{12}$	9.58852
21	$\frac{115}{12}$	9.58566
22	$\frac{115}{12}$	9.5841
23	$\frac{127}{12}$	10.0798
24	$\frac{127}{12}$	10.5894
25	$\frac{133}{12}$	10.8266
26	$\frac{133}{12}$	11.0794
27	$\frac{137}{12}$	11.2533
28	$\frac{137}{12}$	11.4294
29	$\frac{149}{12}$	11.9234
30	$\frac{149}{12}$	12.4094

}

```
RiePrimeCnt[n_] := Sum[PrimePi[n^(1/j)]/j, {j, 1, Log[2, n]}]
RieExplicitForumla[x_, t_] :=
 LogIntegral[a = x] - N[2 Re[Sum[ExpIntegralEi[ZetaZero[k] Log[a]], {k, 1, t}]]] +
  Integrate[1/((y^3 - y) Log[y]), {y, a, Infinity}] - Log[2]
Timing[Table[{n, N[RiePrimeCnt[n]], N[RieExplicitForumla[n, 1000] + If[PrimeQ[n], .5, 0]]},
   {n, 2, 30}] // TableForm ]
```

{109.918,

2	1	0.997791
3	2	1.99968
4	$\frac{5}{2}$	2.24767
5	$\frac{7}{2}$	3.50119
6	$\frac{7}{2}$	3.50193
7	$\frac{9}{2}$	4.50022
8	$\frac{29}{6}$	4.66657
9	$\frac{16}{3}$	5.08214
10	$\frac{16}{3}$	5.33126
11	$\frac{19}{3}$	6.33329
12	$\frac{19}{3}$	6.33554
13	$\frac{22}{3}$	7.33141
14	$\frac{22}{3}$	7.33526
15	$\frac{22}{3}$	7.33309
16	$\frac{91}{12}$	7.45547
17	$\frac{103}{12}$	8.58628
18	$\frac{103}{12}$	8.58299
19	$\frac{115}{12}$	9.58132
20	$\frac{115}{12}$	9.58852
21	$\frac{115}{12}$	9.58566
22	$\frac{115}{12}$	9.5841
23	$\frac{127}{12}$	10.5798
24	$\frac{127}{12}$	10.5894
25	$\frac{133}{12}$	10.8266
26	$\frac{133}{12}$	11.0794
27	$\frac{137}{12}$	11.2533
28	$\frac{137}{12}$	11.4294
29	$\frac{149}{12}$	12.4234
30	$\frac{149}{12}$	12.4094

}

```
RiePrimeCnt[n_] := Sum[PrimePi[n^(1/j)]/j, {j, 1, Log[2, n]}]
RieExplicitForumla[x_, t_] :=
 LogIntegral[a = x] - N[2 Re[Sum[ExpIntegralEi[ZetaZero[k] Log[a]], {k, 1, t}]]] +
  Integrate[1/((y^3 - y) Log[y]), {y, a, Infinity}] - Log[2]
Timing[Table[{n, b = N[RiePrimeCnt[n]],
    c = N[RieExplicitForumla[n, 1000] + MangoldtLambda[n]/Log[n]/2],
    b - c}, {n, 2, 30}] // TableForm ]
```

{117.376,

2	1.	0.997791	0.00220873
3	2.	1.99968	0.000322103
4	2.5	2.49767	0.00232789
5	3.5	3.50119	-0.00119439
6	3.5	3.50193	-0.00193282
7	4.5	4.50022	-0.000220549
8	4.83333	4.83323	0.0000984788
9	5.33333	5.33214	0.00119656
10	5.33333	5.33126	0.00206982
11	6.33333	6.33329	0.0000459831
12	6.33333	6.33554	-0.00220202
13	7.33333	7.33141	0.00192403
14	7.33333	7.33526	-0.00192993
15	7.33333	7.33309	0.000239337
16	7.58333	7.58047	0.0028664
17	8.58333	8.58628	-0.00295142
18	8.58333	8.58299	0.000347853
19	9.58333	9.58132	0.00201128
20	9.58333	9.58852	-0.00519076
21	9.58333	9.58566	-0.00232871
22	9.58333	9.5841	-0.000763641
23	10.5833	10.5798	0.0035651
24	10.5833	10.5894	-0.00603006
25	11.0833	11.0766	0.00674039
26	11.0833	11.0794	0.00394085
27	11.4167	11.42	-0.00334517
28	11.4167	11.4294	-0.0127103
29	12.4167	12.4234	-0.00673667
30	12.4167	12.4094	0.00727285

}

```
RiePrimeCnt[n_] := Sum[PrimePi[n^(1/j)]/j, {j, 1, Log[2, n]}]
RieExplicitForumla[x_, t_] := MangoldtLambda[x] / Log[x] / 2 +
  LogIntegral[a = x] - N[2 Re[Sum[ExpIntegralEi[ZetaZero[k] Log[a]], {k, 1, t}]]] +
  NIntegrate[1 / ((y^3 - y) Log[y]), {y, a, Infinity}] - Log[2]
Timing[Table[{n, b = N[RiePrimeCnt[n]], c = N[RieExplicitForumla[n, 100]], b - c},
   {n, 2, 30}] // TableForm ]
```

{0.608,

2	1.	1.00741	−0.00740725
3	2.	1.99555	0.00444929
4	2.5	2.48983	0.0101696
5	3.5	3.50306	−0.0030617
6	3.5	3.49558	0.00442225
7	4.5	4.49511	0.00488742
8	4.83333	4.81304	0.0202971
9	5.33333	5.34552	−0.0121882
10	5.33333	5.33094	0.00239409
11	6.33333	6.32009	0.0132406
12	6.33333	6.35292	−0.0195903
13	7.33333	7.3384	−0.00506654
14	7.33333	7.31246	0.0208769
15	7.33333	7.34536	−0.0120308
16	7.58333	7.55937	0.023963
17	8.58333	8.59071	−0.00738062
18	8.58333	8.60645	−0.0231169
19	9.58333	9.59243	−0.00910077
20	9.58333	9.56449	0.0188478
21	9.58333	9.56609	0.0172404
22	9.58333	9.56843	0.0149048
23	10.5833	10.5885	−0.00515306
24	10.5833	10.5973	−0.0139694
25	11.0833	11.0673	0.0160642
26	11.0833	11.1104	−0.0270621
27	11.4167	11.3907	0.0259435
28	11.4167	11.423	−0.00635205
29	12.4167	12.4148	0.00190341
30	12.4167	12.408	0.00870043

}

```
RiePrimeCnt[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
RieExplicitForumla[x_, t_] :=
 LogIntegral[a = x] - N[2 Re[Sum[ExpIntegralEi[ZetaZero[k] Log[a]], {k, 1, t}]]] +
  NIntegrate[1 / ((y^3 - y) Log[y]), {y, a, Infinity}] - Log[2]
Timing[Table[{n, b = N[RiePrimeCnt[n]],
    c = N[RieExplicitForumla[n, 1000] + MangoldtLambda[n] / Log[n] / 2],
    b - c}, {n, 2, 30}] // TableForm ]
```

{279.616,

2	1.	0.997791	0.00220873
3	2.	1.99968	0.000322103
4	2.5	2.49767	0.00232789
5	3.5	3.50119	−0.00119439
6	3.5	3.50193	−0.00193282
7	4.5	4.50022	−0.000220549
8	4.83333	4.83323	0.0000984788
9	5.33333	5.33214	0.00119656
10	5.33333	5.33126	0.00206982
11	6.33333	6.33329	0.0000459831
12	6.33333	6.33554	−0.00220202
13	7.33333	7.33141	0.00192403
14	7.33333	7.33526	−0.00192993
15	7.33333	7.33309	0.000239337
16	7.58333	7.58047	0.0028664
17	8.58333	8.58628	−0.00295142
18	8.58333	8.58299	0.000347853
19	9.58333	9.58132	0.00201128
20	9.58333	9.58852	−0.00519076
21	9.58333	9.58566	−0.00232871
22	9.58333	9.5841	−0.000763641
23	10.5833	10.5798	0.0035651
24	10.5833	10.5894	−0.00603006
25	11.0833	11.0766	0.00674039
26	11.0833	11.0794	0.00394085
27	11.4167	11.42	−0.00334517
28	11.4167	11.4294	−0.0127103
29	12.4167	12.4234	−0.00673667
30	12.4167	12.4094	0.00727285

}

```
RiePrimeCnt[n_] := Sum[PrimePi[n^(1/j)]/j, {j, 1, Log[2, n]}]
RieExplicitForumla[x_, t_] := MangoldtLambda[x]/Log[x]/2 +
  LogIntegral[a = x] - N[2 Re[Sum[ExpIntegralEi[ZetaZero[k] Log[a]], {k, 1, t}]]] +
  NIntegrate[1/((y^3 - y) Log[y]), {y, a, Infinity}] - Log[2]
P2[n_] := Sum[ MoebiusMu[k]/k RieExplicitForumla[ Floor[n^(1/k)], 100], {k, 1, Log[2, n]}]
Timing[Table[{n, b = N[PrimePi[n]], c = P2[n], b - c}, {n, 2, 30}] // TableForm ]
```

{2.184,

2	1.	1.00741	−0.00740725
3	2.	1.99555	0.00444929
4	2.	1.98613	0.0138732
5	3.	2.99936	0.000641931
6	3.	2.99187	0.00812588
7	4.	3.99141	0.00859105
8	4.	3.97353	0.0264698
9	4.	4.01194	−0.0119437
10	4.	3.99736	0.00263853
11	5.	4.98651	0.013485
12	5.	5.01935	−0.0193459
13	6.	6.00482	−0.0048221
14	6.	5.97888	0.0211214
15	6.	6.01179	−0.0117864
16	6.	5.97865	0.0213472
17	7.	7.01	−0.00999634
18	7.	7.02573	−0.0257326
19	8.	8.01172	−0.0117165
20	8.	7.98377	0.0162321
21	8.	7.98538	0.0146246
22	8.	7.98771	0.0122891
23	9.	9.00777	−0.00776879
24	9.	9.01659	−0.0165851
25	9.	8.97994	0.0200642
26	9.	9.02306	−0.0230621
27	9.	8.97401	0.0259913
28	9.	9.0063	−0.0063043
29	10.	9.99805	0.00195117
30	10.	9.99125	0.00874818

}

```
RiePrimeCnt[n_] := Sum[PrimePi[n^(1/j)]/j, {j, 1, Log[2, n]}]
RieExplicitForumla[x_, t_] := MangoldtLambda[x]/Log[x]/2 +
  LogIntegral[a = x] - N[2 Re[Sum[ExpIntegralEi[ZetaZero[k] Log[a]], {k, 1, t}]]] +
  NIntegrate[1/((y^3 - y) Log[y]), {y, a, Infinity}] - Log[2]
P2[n_] := Sum[ MoebiusMu[k]/k RieExplicitForumla[ Floor[n^(1/k)], 200], {k, 1, Log[2, n]}]
Timing[Table[{n, b = N[PrimePi[n]], c = P2[n], b - c}, {n, 2, 100}] // TableForm ]
```

2	1.	1.00472	−0.00472302
3	2.	1.99549	0.00450686
4	2.	1.99492	0.00507826
5	3.	2.99489	0.00511265
6	3.	3.00292	−0.00291906
7	4.	4.0018	−0.00180229
8	4.	3.99572	0.00427588
9	4.	4.00086	−0.000855443
10	4.	4.00309	−0.00308779
11	5.	4.99073	0.00926677
12	5.	5.02594	−0.0259391
13	6.	6.01365	−0.0136477
14	6.	6.0016	−0.00160182

{25.522,

15	6.	6.01161	−0.0116092
16	6.	6.00899	−0.00898592
17	7.	7.00838	−0.00837543
18	7.	6.99705	0.00294995
19	8.	7.99673	0.00327163
20	8.	7.9987	0.00130286
21	8.	8.00255	−0.00254987
22	8.	8.00433	−0.00433477
23	9.	8.99362	0.00637942
24	9.	9.00395	−0.00395471
25	9.	8.99641	0.00358963
26	9.	9.01485	−0.0148536
27	9.	8.98113	0.0188679
28	9.	9.00767	−0.00766742
29	10.	10.0167	−0.0166522
30	10.	10.0144	−0.0144087
31	11.	11.0036	−0.00363565
32	11.	10.9741	0.0258852
33	11.	10.9833	0.0167136
34	11.	10.9951	0.00488951
35	11.	11.0021	−0.0021361
36	11.	11.0004	−0.000425747
37	12.	11.9809	0.0190563
38	12.	12.0155	−0.0154791
39	12.	12.0216	−0.0216215
40	12.	11.9446	0.0554102
41	13.	13.0246	−0.0246447
42	13.	12.9902	0.00982405
43	14.	13.9866	0.0133718
44	14.	14.0373	−0.0372891
45	14.	13.9814	0.0186481
46	14.	13.989	0.0110335
47	15.	14.9893	0.0107121
48	15.	15.0063	−0.00625079
49	15.	15.0216	−0.0215996
50	15.	14.9804	0.0196474
51	15.	14.9852	0.0147936
52	15.	15.0271	−0.0270706
53	16.	16.0022	−0.00218705
54	16.	15.9652	0.0347976
55	16.	15.9909	0.00914771
56	16.	16.002	−0.00202138
57	16.	16.0177	−0.0176968
58	16.	16.052	−0.0519626
59	17.	17.0182	−0.0182332
60	17.	16.9985	0.00152566
61	18.	17.977	0.0230061
62	18.	17.9392	0.0608212
63	18.	17.9711	0.028854
64	18.	17.9772	0.0228242
65	18.	17.9798	0.020237
66	18.	18.0087	−0.00869786
67	19.	18.965	0.0350029
68	19.	18.9264	0.0736186
69	19.	18.971	0.0290068
70	19.	19.0257	−0.0256907
71	20.	20.0165	−0.016456

}

72	20.	20.0246	−0.024556
73	21.	21.0261	−0.0260694
74	21.	20.9948	0.005231
75	21.	21.0087	−0.00866794
76	21.	20.9916	0.00842441
77	21.	20.9804	0.0195726
78	21.	21.0116	−0.0116264
79	22.	21.9987	0.00131372
80	22.	21.9884	0.0116485
81	22.	22.006	−0.0059534
82	22.	22.0094	−0.00935346
83	23.	22.9868	0.0131561
84	23.	22.9975	0.00253978
85	23.	23.0513	−0.0513419
86	23.	23.0119	−0.0119051
87	23.	22.951	0.0490168
88	23.	22.9806	0.0194135
89	24.	24.0104	−0.010366
90	24.	24.0199	−0.0199162
91	24.	24.0294	−0.0293696
92	24.	23.9885	0.0114804
93	24.	23.9852	0.0148245
94	24.	24.0244	−0.0244445
95	24.	23.9975	0.00247514
96	24.	23.9447	0.0553084
97	25.	24.9891	0.0108721
98	25.	25.0784	−0.078394
99	25.	24.9946	0.00542122
100	25.	24.9293	0.0706925

```
pi[n_] := Sum[ MoebiusMu[k] / k (
    MangoldtLambda[m = Floor[n^(1 / k)]] / Log[m] / 2 + LogIntegral[m] -
     N[2 Re[Sum[ExpIntegralEi[ZetaZero[k] Log[m]], {k, 1, 200}]]] +
     NIntegrate[1 / ((y^3 - y) Log[y]), {y, m, Infinity}] - Log[2]), {k, 1, Log[2, n]}]
Table[{n, pi[n]}, {n, 2, 100}] // TableForm
```

2	1.	1.00472
3	2.	1.99549
4	2.	1.99492
5	3.	2.99489
6	3.	3.00292
7	4.	4.0018
8	4.	3.99572
9	4.	4.00086
10	4.	4.00309
11	5.	4.99073
12	5.	5.02594
13	6.	6.01365
14	6.	6.0016
15	6.	6.01161
16	6.	6.00899
17	7.	7.00838
18	7.	6.99705
19	8.	7.99673
20	8.	7.9987
21	8.	8.00255
22	8.	8.00433

{24.976,

23	9.	8.99362
24	9.	9.00395
25	9.	8.99641
26	9.	9.01485
27	9.	8.98113
28	9.	9.00767
29	10.	10.0167
30	10.	10.0144
31	11.	11.0036
32	11.	10.9741
33	11.	10.9833
34	11.	10.9951
35	11.	11.0021
36	11.	11.0004
37	12.	11.9809
38	12.	12.0155
39	12.	12.0216
40	12.	11.9446
41	13.	13.0246
42	13.	12.9902
43	14.	13.9866
44	14.	14.0373
45	14.	13.9814
46	14.	13.989
47	15.	14.9893
48	15.	15.0063
49	15.	15.0216
50	15.	14.9804
51	15.	14.9852
52	15.	15.0271
53	16.	16.0022
54	16.	15.9652
55	16.	15.9909
56	16.	16.002
57	16.	16.0177
58	16.	16.052
59	17.	17.0182
60	17.	16.9985
61	18.	17.977
62	18.	17.9392
63	18.	17.9711
64	18.	17.9772
65	18.	17.9798
66	18.	18.0087
67	19.	18.965
68	19.	18.9264
69	19.	18.971
70	19.	19.0257
71	20.	20.0165
72	20.	20.0246
73	21.	21.0261
74	21.	20.9948
75	21.	21.0087
76	21.	20.9916
77	21.	20.9804
78	21.	21.0116
79	22.	21.9987

}

80	22.	21.9884
81	22.	22.006
82	22.	22.0094
83	23.	22.9868
84	23.	22.9975
85	23.	23.0513
86	23.	23.0119
87	23.	22.951
88	23.	22.9806
89	24.	24.0104
90	24.	24.0199
91	24.	24.0294
92	24.	23.9885
93	24.	23.9852
94	24.	24.0244
95	24.	23.9975
96	24.	23.9447
97	25.	24.9891
98	25.	25.0784
99	25.	24.9946
100	25.	24.9293

```
m := {1, -1, -1, 0, -1, 1, -1, 0, 0, 1, -1, 0, -1, 1, 1, 0}
pi[n_] := Sum[m[[k]] / k (MangoldtLambda[a = Floor[n^(1 / k)]] / Log[a] / 2 +
      LogIntegral[a] - N[2 Re[Sum[ExpIntegralEi[ZetaZero[k] Log[a]], {k, 1, 200}]]] +
      NIntegrate[1 / ((y^3 - y) Log[y]), {y, a, Infinity}] - Log[2]), {k, 1, Log[2, n]}]
pi2[n_] := Sum[m[[k]] / k (MangoldtLambda[a = Floor[n^(1 / k)]] / Log[a] / 2 +
      - (Gamma[0, -Log[a]] + (Pi I)) -
      N[2 Re[Sum[-Gamma[0, -ZetaZero[k] Log[a]], {k, 1, 200}]]] +
      NIntegrate[1 / ((y^3 - y) Log[y]), {y, a, Infinity}] - Log[2]), {k, 1, Log[2, n]}]
Table[{n, pi[n], pi2[n]}, {n, 2, 100}] // TableForm
```

2	1.00472	1.00472 + 0. i
3	1.99549	1.99549 + 0. i
4	1.99492	1.99492 + 0. i
5	2.99489	2.99489 + 0. i
6	3.00292	3.00292 + 0. i
7	4.0018	4.0018 + 0. i
8	3.99572	3.99572 + 0. i
9	4.00086	4.00086 + 0. i
10	4.00309	4.00309 + 0. i
11	4.99073	4.99073 + 0. i
12	5.02594	5.02594 + 0. i
13	6.01365	6.01365 + 0. i
14	6.0016	6.0016 + 0. i
15	6.01161	6.01161 + 0. i
16	6.00899	6.00899 + 0. i
17	7.00838	7.00838 + 0. i
18	6.99705	6.99705 + 0. i
19	7.99673	7.99673 + 0. i
20	7.9987	7.9987 + 0. i
21	8.00255	8.00255 + 0. i
22	8.00433	8.00433 + 0. i

23	8.99362	8.99362 + 0. i
24	9.00395	9.00395 + 0. i
25	8.99641	8.99641 + 0. i
26	9.01485	9.01485 + 0. i
27	8.98113	8.98113 + 0. i
28	9.00767	9.00767 + 0. i
29	10.0167	10.0167 + 0. i
30	10.0144	10.0144 + 0. i
31	11.0036	11.0036 + 0. i
32	10.9741	10.9741 + 0. i
33	10.9833	10.9833 + 0. i
34	10.9951	10.9951 + 0. i
35	11.0021	11.0021 + 0. i
36	11.0004	11.0004 + 0. i
37	11.9809	11.9809 + 0. i
38	12.0155	12.0155 + 0. i
39	12.0216	12.0216 + 0. i
40	11.9446	11.9446 + 0. i
41	13.0246	13.0246 + 0. i
42	12.9902	12.9902 + 0. i
43	13.9866	13.9866 + 0. i
44	14.0373	14.0373 + 0. i
45	13.9814	13.9814 + 0. i
46	13.989	13.989 + 0. i
47	14.9893	14.9893 + 0. i
48	15.0063	15.0063 + 0. i
49	15.0216	15.0216 + 0. i
50	14.9804	14.9804 + 0. i
51	14.9852	14.9852 + 0. i
52	15.0271	15.0271 + 0. i
53	16.0022	16.0022 + 0. i
54	15.9652	15.9652 + 0. i
55	15.9909	15.9909 + 0. i
56	16.002	16.002 + 0. i
57	16.0177	16.0177 + 0. i
58	16.052	16.052 + 0. i
59	17.0182	17.0182 + 0. i
60	16.9985	16.9985 + 0. i
61	17.977	17.977 + 0. i
62	17.9392	17.9392 + 0. i
63	17.9711	17.9711 + 0. i
64	17.9772	17.9772 + 0. i
65	17.9798	17.9798 + 0. i
66	18.0087	18.0087 + 0. i
67	18.965	18.965 + 0. i
68	18.9264	18.9264 + 0. i
69	18.971	18.971 + 0. i
70	19.0257	19.0257 + 0. i
71	20.0165	20.0165 + 0. i
72	20.0246	20.0246 + 0. i
73	21.0261	21.0261 + 0. i
74	20.9948	20.9948 + 0. i
75	21.0087	21.0087 + 0. i
76	20.9916	20.9916 + 0. i
77	20.9804	20.9804 + 0. i
78	21.0116	21.0116 + 0. i

79	21.9987	21.9987 + 0. i
80	21.9884	21.9884 + 0. i
81	22.006	22.006 + 0. i
82	22.0094	22.0094 + 0. i
83	22.9868	22.9868 + 0. i
84	22.9975	22.9975 + 0. i
85	23.0513	23.0513 + 0. i
86	23.0119	23.0119 + 0. i
87	22.951	22.951 + 0. i
88	22.9806	22.9806 + 0. i
89	24.0104	24.0104 + 0. i
90	24.0199	24.0199 + 0. i
91	24.0294	24.0294 + 0. i
92	23.9885	23.9885 + 0. i
93	23.9852	23.9852 + 0. i
94	24.0244	24.0244 + 0. i
95	23.9975	23.9975 + 0. i
96	23.9447	23.9447 + 0. i
97	24.9891	24.9891 + 0. i
98	25.0784	25.0784 + 0. i
99	24.9946	24.9946 + 0. i
100	24.9293	24.9293 + 0. i