```
E2a[n_, k_, a_] :=
 E2a[n, k, a] = Sum[ E2a[n / j, k - 1, a], {j, 2, n}] - a Sum[ E2a[n / (a j), k - 1, a], {j, 1, n / a}]];
E2a[n_, 0, a_] := 1
EE[n_, z_, b_] :=
 EE[n, z, b] = Sum[FactorialPower[z, a] / a! E2a[n, a, b], {a, 0, Log[If[b > 2, 2, b], n]}]
EEa[n_, z_, b_] := EEa[n, z, b] =
  Sum[Binomial[z, a] E2a[n, a, b], {a, 0, Log[If[b > 2, 2, b], n]}]
bins[ z_, a_] := Product[ (z - k), {k, 0, a - 1}] / a!
bins2[ z_, a_] := Product[ (z - k), {k, 1, a - 1}] / a!
EEb[n_, z_, b_] :=
 EEb[n, z, b] = Sum[bins[z, a] E2a[n, a, b], {a, 0, Log[If[b > 2, 2, b], n]}]
EEc[n_, z_, b_] := Expand[Sum[bins[z, a] E2a[n, a, b], {a, 0, Log[If[b > 2, 2, b], n]}]]
gg[n_, z_, b_] := Expand[FullSimplify[(EEc[n, z + 1, b] - 1) / (z + 1)]]
```

EE[100, z, 2]

$$1 - z + \frac{3}{2} \text{FactorialPower}[z, 2] - \frac{2}{3} \text{FactorialPower}[z, 3] -$$
$$\frac{1}{3} \text{FactorialPower}[z, 4] + \frac{3}{40} \text{FactorialPower}[z, 5] - \frac{1}{144} \text{FactorialPower}[z, 6]$$

EEa[100, z, 2]

$$1 - z + \frac{3}{2} (-1 + z) z - \frac{2}{3} (-2 + z) (-1 + z) z - \frac{1}{3} (-3 + z) (-2 + z) (-1 + z) z +$$
$$\frac{3}{40} (-4 + z) (-3 + z) (-2 + z) (-1 + z) z - 5 \text{Binomial}[z, 6]$$

EEb[100, z, 2]

$$1 - z + \frac{3}{2} (-1 + z) z - \frac{2}{3} (-2 + z) (-1 + z) z - \frac{1}{3} (-3 + z) (-2 + z) (-1 + z) z +$$
$$\frac{3}{40} (-4 + z) (-3 + z) (-2 + z) (-1 + z) z - \frac{1}{144} (-5 + z) (-4 + z) (-3 + z) (-2 + z) (-1 + z) z$$

EEc[100, z, 2]

$$1 + \frac{4 z}{5} - \frac{419 z^2}{72} + \frac{265 z^3}{48} - \frac{241 z^4}{144} + \frac{43 z^5}{240} - \frac{z^6}{144}$$

Limit[(EE[100, z, 2] - 1) / z, {z → 0}]

$$\left\{ \frac{4}{5} \right\}$$

f[z_] := $1 + \frac{4 z}{5} - \frac{419 z^2}{72} + \frac{265 z^3}{48} - \frac{241 z^4}{144} + \frac{43 z^5}{240} - \frac{z^6}{144}$

Limit[(f[z] - 1) / z, {z → 0}]

$$\left\{ \frac{4}{5} \right\}$$

Table[{n, Expand[Roots[EEc[n, x, 2] == 0, x]]}, {n, 2, 40}] // TableForm

```
2     x == 1
3     False
4     x == 1 || x == 2
```

5      $x == \frac{1}{2} - \frac{i\sqrt{7}}{2} \;||\; x == \frac{1}{2} + \frac{i\sqrt{7}}{2}$

6      $x == -2 \;||\; x == 1$

7      $x == -1 \;||\; x == 2$

8      $x == 1 \;||\; x == 2 \;||\; x == 3$

9      $x == 3 + \frac{1}{3}\left(486 - 3\sqrt{5667}\right)^{1/3} + \frac{\left(162 + \sqrt{5667}\right)^{1/3}}{3^{2/3}} \;||\; x == 3 - \frac{1}{6}\left(486 - 3\sqrt{5667}\right)^{1/3} - \frac{i\left(486 - 3\sqrt{5667}\right)^{1/3}}{2\sqrt{3}} - \frac{\left(162\ldots\right.}{\ldots}$

10      $x == 1 - i\sqrt{5} \;||\; x == 1 + i\sqrt{5} \;||\; x == 1$

11      $x == i\sqrt{2} \;||\; x == -i\sqrt{2} \;||\; x == 3$

12      $x == -1 \;||\; x == 1 \;||\; x == 3$

13      $x == 1 - \left(\frac{2}{3\left(27 - \sqrt{717}\right)}\right)^{1/3} - \frac{\left(\frac{1}{2}\left(27 - \sqrt{717}\right)\right)^{1/3}}{3^{2/3}} \;||\; x == 1 + \frac{\left(\frac{1}{2}\left(27 - \sqrt{717}\right)\right)^{1/3}}{2 \times 3^{2/3}} + \frac{1}{2^{2/3}\,3^{1/3}\left(27 - \sqrt{717}\right)^{1/3}} - \frac{i\,3^{1/6}}{2^{2/3}\left(27 - \sqrt{717}\right)^{1}}$

14      $x == \frac{5}{2} - \frac{\sqrt{37}}{2} \;||\; x == \frac{5}{2} + \frac{\sqrt{37}}{2} \;||\; x == 1$

15      $x == 1 - \left(\frac{2}{3\left(27 - \sqrt{717}\right)}\right)^{1/3} - \frac{\left(\frac{1}{2}\left(27 - \sqrt{717}\right)\right)^{1/3}}{3^{2/3}} \;||\; x == 1 + \frac{\left(\frac{1}{2}\left(27 - \sqrt{717}\right)\right)^{1/3}}{2 \times 3^{2/3}} + \frac{1}{2^{2/3}\,3^{1/3}\left(27 - \sqrt{717}\right)^{1/3}} - \frac{i\,3^{1/6}}{2^{2/3}\left(27 - \sqrt{717}\right)^{1}}$

16      $x == 3 - \frac{23}{3^{1/3}\left(-324 + \sqrt{141\,477}\right)^{1/3}} + \frac{\left(-324 + \sqrt{141\,477}\right)^{1/3}}{3^{2/3}} \;||\; x == 3 + \frac{23}{2 \times 3^{1/3}\left(-324 + \sqrt{141\,477}\right)^{1/3}} - \frac{23\,i\,3^{1/6}}{2\left(-324 + \sqrt{141\,477}\right)^{1/3}} - \frac{\left(-\ldots\right.}{\ldots}$

17      $x == \frac{5}{2} + \frac{1}{2}\sqrt{\frac{1}{3}\left(-43 + \frac{2269}{\left(87\,803 + 90\,i\sqrt{490\,403}\right)^{1/3}} + \left(87\,803 + 90\,i\sqrt{490\,403}\right)^{1/3}\right)} - \frac{1}{2}\sqrt{-\frac{86}{3} - \frac{2269}{3\left(87\,803 + 90\,i\sqrt{49\ldots}\right.}}$

18      $x == 7 + \frac{1}{3}\left(7128 - 33\sqrt{2733}\right)^{1/3} + \frac{\left(11\left(216 + \sqrt{2733}\right)\right)^{1/3}}{3^{2/3}} \;||\; x == 7 - \frac{1}{6}\left(7128 - 33\sqrt{2733}\right)^{1/3} - \frac{i\left(7128 - 33\sqrt{273\ldots}\right)}{2\sqrt{3}}$

19      $x == \frac{20}{3} + \frac{1}{3}\left(7532 - 9\sqrt{28\,285}\right)^{1/3} + \frac{1}{3}\left(7532 + 9\sqrt{28\,285}\right)^{1/3} \;||\; x == \frac{20}{3} - \frac{1}{6}\left(7532 - 9\sqrt{28\,285}\right)^{1/3} - \frac{i}{\ldots}$

20      $x == 3 + \frac{1}{3}\left(972 - 3\sqrt{58\,101}\right)^{1/3} + \frac{\left(324 + \sqrt{58\,101}\right)^{1/3}}{3^{2/3}} \;||\; x == 3 - \frac{1}{6}\left(972 - 3\sqrt{58\,101}\right)^{1/3} - \frac{i\left(972 - 3\sqrt{58\,101}\right)^{1/3}}{2\sqrt{3}}$

21      $x == \frac{5}{2} + \frac{1}{2}\sqrt{\frac{1}{3}\left(5 + \frac{733}{\left(13\,211 + 18\,i\sqrt{676\,859}\right)^{1/3}} + \left(13\,211 + 18\,i\sqrt{676\,859}\right)^{1/3}\right)} - \frac{1}{2}\sqrt{\frac{10}{3} - \frac{733}{3\left(13\,211 + 18\,i\sqrt{676\,859}\right.}}$

22      $x == 3 + \frac{1}{3}\left(972 - 3\sqrt{58\,101}\right)^{1/3} + \frac{\left(324 + \sqrt{58\,101}\right)^{1/3}}{3^{2/3}} \;||\; x == 3 - \frac{1}{6}\left(972 - 3\sqrt{58\,101}\right)^{1/3} - \frac{i\left(972 - 3\sqrt{58\,101}\right)^{1/3}}{2\sqrt{3}}$

23      $x == \frac{8}{3} + \frac{1}{3}\left(854 - 9\sqrt{2917}\right)^{1/3} + \frac{1}{3}\left(854 + 9\sqrt{2917}\right)^{1/3} \;||\; x == \frac{8}{3} - \frac{1}{6}\left(854 - 9\sqrt{2917}\right)^{1/3} - \frac{i\left(854 - 9\sqrt{2917}\right.}{2\sqrt{3}}$

24      $x == \frac{17}{6} - \frac{\sqrt{433}}{6} \;||\; x == \frac{17}{6} + \frac{\sqrt{433}}{6} \;||\; x == 1 \;||\; x == 2$

25      $x == \frac{13}{6} - \frac{1}{6}\sqrt{103 - \frac{185 \times 3^{2/3}}{\left(-44\,037 + 2\sqrt{489\,563\,061}\right)^{1/3}} + \left(3\left(-44\,037 + 2\sqrt{489\,563\,061}\right)\right)^{1/3}} - \frac{1}{2}\sqrt{\frac{206}{9} - \frac{\left(-44\,037 + 2\sqrt{\ldots}\right.}{3 \times \ldots}}$

26      $x == \frac{7}{3} - \frac{\sqrt{73}}{3} \;||\; x == \frac{7}{3} + \frac{\sqrt{73}}{3} \;||\; x == 1 \;||\; x == 3$

27      $x == \frac{5}{2} - \frac{1}{2}\sqrt{15 + \frac{\left(-2223 + 2\sqrt{1\,239\,117}\right)^{1/3}}{3^{2/3}} - \frac{17}{\left(3\left(-2223 + 2\sqrt{1\,239\,117}\right)\right)^{1/3}}} - \frac{1}{2}\sqrt{30 - \frac{\left(-2223 + 2\sqrt{1\,239\,117}\right)^{1/3}}{3^{2/3}} + \frac{\ldots}{\left(3\left(-2223\ldots\right.\right.}}$

28     $x == \frac{13}{3} + \frac{127^{2/3}}{3\left(10+3\,\mathbb{i}\,\sqrt{3}\right)^{1/3}} + \frac{1}{3}\left(127\left(10+3\,\mathbb{i}\,\sqrt{3}\right)\right)^{1/3} \ || \ x == \frac{13}{3} - \frac{127^{2/3}}{6\left(10+3\,\mathbb{i}\,\sqrt{3}\right)^{1/3}} - \frac{\mathbb{i}\,127^{2/3}}{2\sqrt{3}\left(10+3\,\mathbb{i}\,\sqrt{3}\right)^{1/3}} - \frac{1}{6}\left(1\right.$

29     $x == \frac{7}{2} - \frac{1}{2}\sqrt{31 - \frac{5}{\left(551-2\sqrt{75869}\right)^{1/3}} - \left(551-2\sqrt{75869}\right)^{1/3}} - \frac{1}{2}\sqrt{62 + \frac{5}{\left(551-2\sqrt{75869}\right)^{1/3}} + \left(551-2\sqrt{75869}\right)^{1/3}}$

30     $x == \frac{5}{3} - \frac{41}{3\left(-478+3\sqrt{33045}\right)^{1/3}} + \frac{1}{3}\left(-478+3\sqrt{33045}\right)^{1/3} \ || \ x == \frac{5}{3} + \frac{41}{6\left(-478+3\sqrt{33045}\right)^{1/3}} + \frac{41\,\mathbb{i}}{2\sqrt{3}\left(-478+3\sqrt{33045}\right.}$

31     $x == \frac{3}{2} + \frac{1}{2}\sqrt{-9 + \frac{1}{3}\left(14067-6\sqrt{5126901}\right)^{1/3} + \frac{\left(4689+2\sqrt{5126901}\right)^{1/3}}{3^{2/3}}} - \frac{1}{2}\sqrt{-18 - \frac{1}{3}\left(14067-6\sqrt{5126\,9}\right.}$

32     $x == \frac{7}{2} + \frac{1}{2}\sqrt{-65 + \frac{1}{3}\left(2578635-162\sqrt{18581677}\right)^{1/3} + \left(3\left(31835+2\sqrt{18581677}\right)\right)^{1/3}} - \frac{1}{2}\sqrt{-130\,\cdot}$

33     $x == \text{Root}\left[-120+414\,\#1-585\,\#1^2+185\,\#1^3-15\,\#1^4+\#1^5 \ \&, \ 1\right] \ || \ x == \text{Root}\left[-120+414\,\#1-585\,\#1^2+\right.$

34     $x == \frac{7}{2} + \frac{1}{2}\sqrt{-65 + \frac{1}{3}\left(2578635-162\sqrt{18581677}\right)^{1/3} + \left(3\left(31835+2\sqrt{18581677}\right)\right)^{1/3}} - \frac{1}{2}\sqrt{-130\,\cdot}$

35     $x == \text{Root}\left[-120+414\,\#1-585\,\#1^2+185\,\#1^3-15\,\#1^4+\#1^5 \ \&, \ 1\right] \ || \ x == \text{Root}\left[-120+414\,\#1-585\,\#1^2+\right.$

36     $x == 14 + \frac{157}{\left(1955+2\,\mathbb{i}\,\sqrt{11967}\right)^{1/3}} + \left(1955+2\,\mathbb{i}\,\sqrt{11967}\right)^{1/3} \ || \ x == 14 - \frac{157}{2\left(1955+2\,\mathbb{i}\,\sqrt{11967}\right)^{1/3}} - \frac{157\,\mathbb{i}\,\sqrt{3}}{2\left(1955+2\,\mathbb{i}\,\sqrt{11967}\right.}$

37     $x == \text{Root}\left[-120+294\,\#1-495\,\#1^2+245\,\#1^3-45\,\#1^4+\#1^5 \ \&, \ 1\right] \ || \ x == \text{Root}\left[-120+294\,\#1-495\,\#1^2+\right.$

38     $x == 11 - \frac{1}{2}\sqrt{350 + \frac{1}{3}\left(3872475-108\sqrt{709335247}\right)^{1/3} + \left(143425+4\sqrt{709335247}\right)^{1/3}} - \frac{1}{2}\sqrt{700-}$

39     $x == \text{Root}\left[-120+294\,\#1-495\,\#1^2+245\,\#1^3-45\,\#1^4+\#1^5 \ \&, \ 1\right] \ || \ x == \text{Root}\left[-120+294\,\#1-495\,\#1^2+\right.$

40     $x == 6 - \frac{175}{\sqrt{3\left(350-\frac{9407}{\left(463175+36\sqrt{807848783}\right)^{1/3}}+\left(463175+36\sqrt{807848783}\right)^{1/3}\right)}} + \frac{9407}{2\left(463175+36\sqrt{807848783}\right)^{1/3}\sqrt{3\left(350-\frac{9407}{\left(463175+36\sqrt{807\,8}\right.}\right)}}$

Limit[(EE[8, z, 2] - 1) / z, {z → 0}]

$\left\{-\frac{11}{6}\right\}$

- (1 / 1 + 1 / 2 + 1 / 3)

$-\frac{11}{6}$

Limit[(EE[26, z, 2] - 1) / z, {z → 0}]

$\left\{\frac{5}{12}\right\}$

```
-FullSimplify[(7/3 - √73/3)^-1 + (7/3 + √73/3)^-1 + (1)^-1 + (3)^-1]
```

$$\frac{5}{12}$$

```
$RecursionLimit = 10 000
```

```
10 000
```

```
Table[{n, N[Roots[EEc[n, x, 1.01] == 0, x]]}, {n, 2, 16}] // TableForm
```

```
N[Roots[EEc[2, x, 101 / 100] == 0, x]]
```

```
x == 70.824 || x == 0.209281 - 0.338999 i || x == 0.209281 + 0.338999 i ||
 x == 0.393948 - 1.68916 i || x == 0.393948 + 1.68916 i || x == 1.00944 - 3.33336 i ||
 x == 1.00944 + 3.33336 i || x == 1.94453 - 5.06556 i || x == 1.94453 + 5.06556 i ||
 x == 3.15762 - 6.83009 i || x == 3.15762 + 6.83009 i || x == 4.62473 - 8.58908 i ||
 x == 4.62473 + 8.58908 i || x == 6.32725 - 10.3115 i || x == 6.32725 + 10.3115 i ||
 x == 8.24826 - 11.9706 i || x == 8.24826 + 11.9706 i || x == 10.3711 - 13.5424 i ||
 x == 10.3711 + 13.5424 i || x == 12.6789 - 15.0058 i || x == 12.6789 + 15.0058 i ||
 x == 15.1537 - 16.3417 i || x == 15.1537 + 16.3417 i || x == 17.7771 - 17.5334 i ||
 x == 17.7771 + 17.5334 i || x == 20.5297 - 18.5665 i || x == 20.5297 + 18.5665 i ||
 x == 23.3913 - 19.4287 i || x == 23.3913 + 19.4287 i || x == 26.341 - 20.1098 i ||
 x == 26.341 + 20.1098 i || x == 29.3574 - 20.6018 i || x == 29.3574 + 20.6018 i ||
 x == 32.4184 - 20.8993 i || x == 32.4184 + 20.8993 i || x == 35.5018 - 20.9986 i ||
 x == 35.5018 + 20.9986 i || x == 38.5852 - 20.8987 i || x == 38.5852 + 20.8987 i ||
 x == 41.6462 - 20.6008 i || x == 41.6462 + 20.6008 i || x == 44.6626 - 20.1082 i ||
 x == 44.6626 + 20.1082 i || x == 47.6123 - 19.4266 i || x == 47.6123 + 19.4266 i ||
 x == 50.4738 - 18.5638 i || x == 50.4738 + 18.5638 i || x == 53.2264 - 17.53 i ||
 x == 53.2264 + 17.53 i || x == 55.8498 - 16.3375 i || x == 55.8498 + 16.3375 i ||
 x == 58.3246 - 15.0008 i || x == 58.3246 + 15.0008 i || x == 60.6322 - 13.5365 i ||
 x == 60.6322 + 13.5365 i || x == 62.755 - 11.9634 i || x == 62.755 + 11.9634 i ||
 x == 64.6757 - 10.3027 i || x == 64.6757 + 10.3027 i || x == 66.3778 - 8.57808 i ||
 x == 66.3778 + 8.57808 i || x == 67.8441 - 6.81575 i || x == 67.8441 + 6.81575 i ||
 x == 69.0557 - 5.04529 i || x == 69.0557 + 5.04529 i || x == 69.988 - 3.29996 i ||
 x == 69.988 + 3.29996 i || x == 70.6014 - 1.61453 i || x == 70.6014 + 1.61453 i
```

```
N[Roots[EEc[2, x, 1.003] == 0, x]]
```

```
x == -83.1205 + 304.614 i || x == -82.1545 - 263.302 i || x == -79.8459 + 277.923 i ||
 x == -79.4643 + 245.856 i || x == -79.1961 - 236.716 i || x == -78.6374 - 322.324 i ||
 x == -75.0903 - 362.774 i || x == -74.7427 + 347.027 i || x == -73.4258 - 296.033 i ||
 x == -73.1365 + 218.501 i || x == -71.8672 + 196.228 i || x == -71.0665 - 188.251 i ||
 x == -70.3673 - 409.916 i || x == -68.9662 + 185.574 i || x == -68.3749 - 209.477 i ||
 x == -67.4865 - 168.626 i || x == -67.4466 + 278.338 i || x == -67.3308 + 166.83 i ||
 x == -65.3902 + 386.573 i || x == -63.8926 + 434.737 i || x == -63.6043 + 266.618 i ||
 x == -61.8043 + 150.05 i || x == -61.7195 - 287.879 i || x == -60.9488 - 150.593 i ||
 x == -57.7587 - 210.741 i || x == -55.5722 + 135.734 i || x == -55.3143 - 420.627 i ||
 x == -52.2003 + 122.431 i || x == -51.6783 - 134.059 i || x == -48.0369 - 117.183 i ||
 x == -45.9521 + 111.603 i || x == -44.1531 - 104.153 i || x == -42.6659 + 100.771 i ||
 x == -41.5963 + 486.924 i || x == -41.0189 - 475.084 i || x == -40.2402 - 93.8738 i ||
 x == -39.8561 + 90.4992 i || x == -36.8917 - 135.984 i || x == -36.36 - 85.3614 i ||
 x == -35.5484 + 83.6038 i || x == -32.6304 + 79.4037 i || x == -32.4899 - 77.8037 i ||
 x == -29.5076 + 72.493 i || x == -28.2219 - 70.2707 i || x == -26.9913 + 66.8528 i ||
```

x == – 26.2007 – 66.1979 i || x == – 25.6691 – 119.715 i || x == – 24.1304 + 62.4062 i ||
x == – 23.2981 – 61.4563 i || x == – 21.9851 + 58.1735 i || x == – 21.1298 – 56.0447 i ||
x == – 20.2965 + 54.1582 i || x == – 18.9039 + 51.0049 i || x == – 18.8405 – 179.253 i ||
x == – 17.2475 – 51.1383 i || x == – 15.9083 + 47.2141 i || x == – 14.7896 – 43.6307 i ||
x == – 14.2497 + 43.243 i || x == – 14.1739 – 537.125 i || x == – 12.8802 – 45.7433 i ||
x == – 11.4244 + 38.4832 i || x == – 11.156 – 39.9245 i || x == – 10.7198 + 39.0787 i ||
x == – 10.4242 – 134.933 i || x == – 9.46536 – 52.1957 i || x == – 9.33408 – 36.5598 i ||
x == – 8.75001 – 34.2091 i || x == – 8.65283 + 35.436 i || x == – 8.32364 + 32.9157 i ||
x == – 7.86841 – 59.7958 i || x == – 6.9138 – 31.3889 i || x == – 6.21645 – 81.8803 i ||
x == – 6.14782 + 29.0334 i || x == – 5.38427 – 28.4583 i || x == – 4.61119 + 25.7814 i ||
x == – 4.10178 – 25.2204 i || x == – 2.99261 + 23.0041 i || x == – 2.75354 + 541.236 i ||
x == – 2.60602 – 22.0723 i || x == – 2.14022 – 21.2011 i || x == – 1.47124 + 20.5303 i ||
x == – 0.64622 + 17.7315 i || x == – 0.144576 – 18.5764 i || x == 0.0965352 – 16.4394 i ||
x == 0.183201 – 0.286683 i || x == 0.183201 + 0.286683 i || x == 0.234071 – 1.31412 i ||
x == 0.234071 + 1.31412 i || x == 0.509072 – 2.56297 i || x == 0.509072 + 2.56297 i ||
x == 0.62226 + 15.8168 i || x == 0.925907 – 3.89807 i || x == 0.925907 + 3.89807 i ||
x == 1.28344 + 14.0056 i || x == 1.29664 – 17.7612 i || x == 1.45681 – 5.29441 i ||
x == 1.45681 + 5.29441 i || x == 1.57655 + 13.0099 i || x == 1.6901 – 13.547 i ||
x == 2.0883 – 6.73913 i || x == 2.08839 + 6.73912 i || x == 2.44348 + 11.3369 i ||
x == 2.54321 + 10.1932 i || x == 2.58725 – 11.4954 i || x == 2.81188 + 8.21959 i ||
x == 2.83786 – 8.2348 i || x == 2.95252 + 21.422 i || x == 3.04058 – 9.72693 i ||
x == 3.05003 – 22.0656 i || x == 3.28491 + 8.5825 i || x == 3.66036 – 15.233 i ||
x == 4.26071 – 8.41345 i || x == 4.58918 – 30.6125 i || x == 5.08178 + 7.77372 i ||
x == 5.1935 – 53.9584 i || x == 5.28105 + 46.1693 i || x == 5.51699 – 7.93166 i ||
x == 5.82855 + 7.41882 i || x == 6.78111 – 6.61963 i || x == 7.10834 + 6.52437 i ||
x == 7.41854 – 5.46567 i || x == 8.17447 + 5.10261 i || x == 8.65516 – 3.70716 i ||
x == 8.87668 + 3.27586 i || x == 9.21966 + 1.93298 i || x == 9.34598 + 0.96125 i ||
x == 9.63484 + 30.2585 i || x == 9.75456 – 22.2475 i || x == 9.81864 – 1.70114 i ||
x == 11.1471 – 14.794 i || x == 11.4984 – 7.16362 i || x == 11.903 – 1.88207 i ||
x == 12.4422 – 597.976 i || x == 12.8658 + 2.1233 i || x == 14.0034 – 4.09188 i ||
x == 14.0621 + 0.210409 i || x == 15.6664 + 10.9671 i || x == 15.8205 + 7.1282 i ||
x == 15.8873 – 29.3746 i || x == 16.4865 + 21.5533 i || x == 17.3469 – 45.3892 i ||
x == 17.6198 – 35.4267 i || x == 20.8479 – 128.327 i || x == 21.5014 + 7.98687 i ||
x == 23.2941 – 12.4399 i || x == 24.2288 + 6.10464 i || x == 26.1876 + 198.656 i ||
x == 27.3958 + 124.451 i || x == 28.9973 + 4.79162 i || x == 29.0401 – 54.3195 i ||
x == 29.3442 + 618.011 i || x == 30.668 + 18.7703 i || x == 33.46 – 10.5677 i ||
x == 36.2701 – 6.43921 i || x == 40.1699 – 5.20476 i || x == 42.1088 – 5.21194 i ||
x == 47.0643 – 57.3624 i || x == 48.6414 – 54.0289 i || x == 48.9184 + 47.7406 i ||
x == 51.894 + 1.0873 i || x == 52.0207 – 337.868 i || x == 57.3304 + 54.154 i ||
x == 59.0486 – 0.268606 i || x == 59.0959 – 655.577 i || x == 61.1004 + 62.2465 i ||
x == 71.449 – 149.685 i || x == 72.7347 + 124.021 i || x == 86.7166 – 153.743 i ||
x == 86.7812 – 61.7248 i || x == 92.4704 – 57.0161 i || x == 93.5872 + 678.113 i ||
x == 95.0818 – 92.4863 i || x == 95.2341 – 14.9264 i || x == 97.2021 – 121.958 i ||
x == 100.012 + 52.7037 i || x == 100.086 + 6.82001 i || x == 102.594 + 134.708 i ||
x == 104.147 + 7.18284 i || x == 111.897 – 99.4839 i || x == 113.648 + 19.5883 i ||
x == 115.682 – 208.9 i || x == 116.094 – 241.274 i || x == 135.658 – 732.723 i ||
x == 137.99 – 5.18387 i || x == 144.723 – 49.0362 i || x == 146.186 – 12.4762 i ||
x == 147.557 – 558.842 i || x == 148.065 + 128.631 i || x == 149.475 + 754.715 i ||
x == 161.285 – 96.5686 i || x == 189.141 – 393.492 i || x == 190.18 + 772.593 i ||
x == 192.223 + 293.443 i || x == 211.743 – 29.5178 i || x == 239.785 – 793.392 i ||
x == 240.161 + 103.718 i || x == 249.851 – 0.654296 i || x == 251.905 + 43.5166 i ||
x == 280.653 + 0.784738 i || x == 288.708 + 398.254 i || x == 294.954 + 810.204 i ||
x == 337.297 + 337.415 i || x == 356.844 + 360.746 i || x == 359.54 + 19.8412 i ||
x == 383.912 – 862.09 i || x == 405.642 + 468.058 i || x == 412.344 + 8.25429 i ||
x == 419.453 + 861.082 i || x == 438.434 – 316.751 i || x == 450.396 – 894.763 i ||
x == 462.267 – 666.399 i || x == 467.123 – 583.065 i || x == 499.759 + 515.869 i ||

x == 506.282 − 134.061 i || x == 560.823 − 882.429 i || x == 572.913 + 895.511 i ||
x == 655.595 + 218.982 i || x == 715.35 − 905.384 i || x == 765.835 + 901.113 i ||
x == 921.07 + 307.296 i || x == 951.762 + 773.105 i || x == 980.479 − 800.726 i ||
x == 1080.12 − 511.467 i || x == 1088.72 − 712.586 i || x == 1154.58 + 655.614 i ||
x == 1236.95 − 617.295 i || x == 1362.38 + 470.091 i || x == 1395.17 + 325.07 i ||
x == 1405.41 − 407.585 i || x == 1453.42 − 143.677 i || x == 1481.49 + 110.897 i

```
N[Roots[EEc[100, x, 1.05] == 0, x]]
```

x == −8.15188 || x == −7.4818 + 59.6922 i || x == −7.4818 − 59.6922 i || x == −2.04037 ||
x == −1.53261 − 9.95097 i || x == −1.53261 + 9.95097 i || x == −1.40468 − 4.81095 i ||
x == −1.40468 + 4.81095 i || x == −0.805147 − 1.58531 i || x == −0.805147 + 1.58531 i ||
x == −0.41015 + 43.4838 i || x == −0.410139 − 43.4838 i || x == −0.00663915 − 33.6786 i ||
x == −0.00663787 + 33.6786 i || x == 0.160227 − 8.00765 i || x == 0.160227 + 8.00765 i ||
x == 0.5493 − 2.07253 i || x == 0.5493 + 2.07253 i || x == 0.598941 − 0.175866 i ||
x == 0.598941 + 0.175866 i || x == 0.744024 − 16.6642 i || x == 0.744024 + 16.6642 i ||
x == 0.777657 − 0.750947 i || x == 0.777657 + 0.750947 i || x == 1.41752 − 24.6592 i ||
x == 1.41752 + 24.6592 i || x == 3.67945 + 4.86714 i || x == 3.67945 − 4.86714 i ||
x == 5.24862 − 10.7911 i || x == 5.24862 + 10.7911 i || x == 6.67698 + 2.48217 i ||
x == 6.67698 − 2.48217 i || x == 11.1657 − 20.0877 i || x == 11.1667 + 20.0873 i ||
x == 12.5377 + 11.1633 i || x == 12.5377 − 11.1633 i || x == 17.2299 || x == 18.0297 ||
x == 18.422 || x == 18.632 || x == 18.8962 || x == 19.0708 || x == 19.0855 || x == 19.1484 ||
x == 20.3514 || x == 21.3113 || x == 21.3172 || x == 21.4505 || x == 21.5236 ||
x == 22.0756 || x == 22.1042 || x == 23.9305 || x == 26.6548 || x == 26.7842 ||
x == 27.183 || x == 27.2134 || x == 27.6447 || x == 27.9489 || x == 30.6535 || x == 34.2157 ||
x == 35.3123 || x == 35.4905 || x == 35.7919 || x == 38.7926 || x == 39.3352 ||
x == 39.9035 || x == 42.0279 || x == 44.3045 || x == 44.6598 || x == 55.3283 ||
x == 56.3555 || x == 57.0564 || x == 58.9898 || x == 67.6019 || x == 71.2165 || x == 75.4629 ||
x == 81.1907 || x == 86.9248 || x == 92.9711 || x == 97.8484 || x == 109.382 || x == 126.833 ||
x == 139.033 || x == 168.103 || x == 169. || x == 174.549 || x == 178.245 || x == 209.158 ||
x == 213.264 || x == 234.183 || x == 238.718 || x == 239.202 || x == 240.877 || x == 250.928

`RootLocusPlot[1 / Expand[EEc[100, x, 1.1]], {k, 0, 1}, FeedbackType → None]`
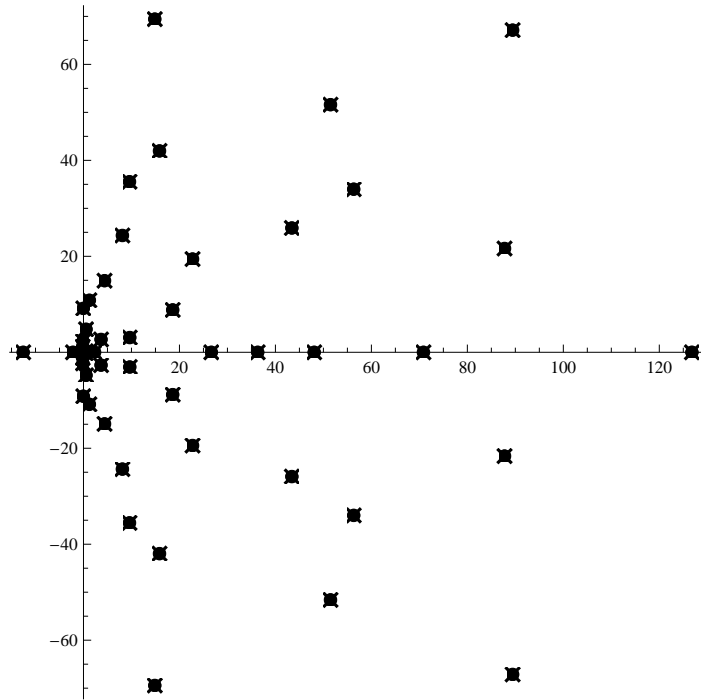
`Table[{n, Expand[Roots[EEc[n, x, 2] == 0, x]]}, {n, 41, 64}] // TableForm`

41     $x == \text{Root}\left[-120 + 174\,\#1 - 215\,\#1^2 + 65\,\#1^3 - 25\,\#1^4 + \#1^5\,\&,\,1\right] \;||\; x == \text{Root}\left[-120 + 174\,\#1 - 215\,\#1^2 + 6\right.$

42     $x == 6 + \frac{1}{2}\sqrt{\frac{1}{3}\left(110 + \frac{23\,473}{\left(3\,511\,295+36\,i\,\sqrt{466\,050\,577}\right)^{1/3}} + \left(3\,511\,295 + 36\,i\,\sqrt{466\,050\,577}\right)^{1/3}\right)} - \frac{1}{2}\sqrt{\frac{220}{3} - \frac{}{3\,(3\,\text{!}}}$

43     $x == \text{Root}\left[-120 + 54\,\#1 - 215\,\#1^2 + 185\,\#1^3 - 25\,\#1^4 + \#1^5\,\&,\,1\right] \;||\; x == \text{Root}\left[-120 + 54\,\#1 - 215\,\#1^2 + 18\right.$

44     $x == 6 - \frac{115}{\sqrt{3\left(230+\left(2\,305\,835-36\,\sqrt{703\,377\,158}\right)^{1/3}+\left(2\,305\,835+36\,\sqrt{703\,377\,158}\right)^{1/3}\right)}} - \frac{\left(2\,305\,835-36\,\sqrt{703\,377}\right.}{2\sqrt{3\left(230+\left(2\,305\,835-36\,\sqrt{703\,377\,158}\right)^{1/3}+\left(\right.\right.}}$

45     $x == \text{Root}\left[-120 + 54\,\#1 - 95\,\#1^2 + 65\,\#1^3 - 25\,\#1^4 + \#1^5\,\&,\,1\right] \;||\; x == \text{Root}\left[-120 + 54\,\#1 - 95\,\#1^2 + 65\,\#1\right.$

46     $x == 6 - \frac{175}{\sqrt{3\left(350+\left(1\,175\,975-36\,\sqrt{690\,521\,723}\right)^{1/3}+\left(1\,175\,975+36\,\sqrt{690\,521\,723}\right)^{1/3}\right)}} - \frac{\left(1\,175\,975-36\,\sqrt{690\,521}\right.}{2\sqrt{3\left(350+\left(1\,175\,975-36\,\sqrt{690\,521\,723}\right)^{1/3}+\left(\right.\right.}}$

47     $x == \frac{23}{4} - \frac{1435}{4\sqrt{3\left(1435+4\left(571\,780-828\,\sqrt{243\,597}\right)^{1/3}+4\times 2^{2/3}\left(23\left(6215+9\,\sqrt{243\,597}\right)\right)^{1/3}\right)}} - \frac{\left(571\,780-828\,\sqrt{}\right.}{\sqrt{3\left(1435+4\left(571\,780-828\,\sqrt{243\,597}\right)^{1/3}\right.}}$

48     $x == \frac{61}{16} + \frac{1915}{16\sqrt{3\left(1915+16\left(10\,216\,180-9\,\sqrt{256\,605\,466\,893}\right)^{1/3}+16\left(10\,216\,180+9\,\sqrt{256\,605\,466\,893}\right)^{1/3}\right)}} + \frac{}{\sqrt{3\left(1915+16\left(10\,216\,180-9\right.\right.}}$

49     $x == \text{Root}\left[120 + 126\,\#1 - 415\,\#1^2 + 350\,\#1^3 - 65\,\#1^4 + 4\,\#1^5\,\&,\,1\right] \;||\; x == \text{Root}\left[120 + 126\,\#1 - 415\,\#1^2 + 3\right.$

50    $x == \frac{61}{16} - \dfrac{3835}{16\sqrt{3\left(3835 - \frac{26\,608}{\left(2\,257\,730 - 9\sqrt{62\,873\,402\,613}\right)^{1/3}} - 16\left(2\,257\,730 - 9\sqrt{62\,873\,402\,613}\right)^{1/3}\right)}} + \dfrac{}{\left(2\,257\,730 - 9\sqrt{62\,873\,402\,613}\right)^{1/3}\sqrt{3\left(3835\right.}}$

51    $x == \frac{45}{16} - \dfrac{3995}{16\sqrt{3\left(3995 - \frac{15\,632}{\left(-503\,110 + 9\sqrt{3\,136\,447\,493}\right)^{1/3}} + 16\left(-503\,110 + 9\sqrt{3\,136\,447\,493}\right)^{1/3}\right)}} + \dfrac{}{\left(-503\,110 + 9\sqrt{3\,136\,447\,493}\right)^{1/3}\sqrt{3\left(3995 - \frac{}{\left(-\right.}\right.}}$

52    $x == \frac{61}{16} + \dfrac{1915}{16\sqrt{3\left(1915 + 16\left(6\,856\,030 - 117\sqrt{869\,402\,037}\right)^{1/3} + 16\left(6\,856\,030 + 117\sqrt{869\,402\,037}\right)^{1/3}\right)}} + \dfrac{\left(6\,856\right.}{\sqrt{3\left(1915 + 16\left(6\,856\,030 - 117\sqrt{8}\right.\right.}}$

53    $x == \text{Root}\left[120 + 246\,\#1 - 535\,\#1^2 + 350\,\#1^3 - 65\,\#1^4 + 4\,\#1^5\,\&,\,1\right] \;||\; x == \text{Root}\left[120 + 246\,\#1 - 535\,\#1^2 + 3\right.$

54    $x == \frac{81}{16} - \dfrac{12\,995}{16\sqrt{3\left(12\,995 - \frac{816\,272}{\left(-18\,536\,410 + 9\sqrt{5\,881\,261\,616\,173}\right)^{1/3}} + 16\left(-18\,536\,410 + 9\sqrt{5\,881\,261\,616\,173}\right)^{1/3}\right)}} + \dfrac{}{\left(-18\,536\,410 + 9\sqrt{5\,881\,261\,616\,17}\right.}$

55    $x == \text{Root}\left[120 + 246\,\#1 - 455\,\#1^2 + 290\,\#1^3 - 85\,\#1^4 + 4\,\#1^5\,\&,\,1\right] \;||\; x == \text{Root}\left[120 + 246\,\#1 - 455\,\#1^2 + 2\right.$

56    $x == \frac{101}{16} - \frac{1}{16}\sqrt{6265 - \dfrac{34\,672}{\left(520\,530 - \sqrt{260\,775\,489\,437}\right)^{1/3}} - 16\left(520\,530 - \sqrt{260\,775\,489\,437}\right)^{1/3}} - \frac{1}{2}\sqrt{\frac{6265}{32} + \dfrac{}{4\left(52\right.}}$

57    $x == \text{Root}\left[120 + 246\,\#1 - 615\,\#1^2 + 470\,\#1^3 - 105\,\#1^4 + 4\,\#1^5\,\&,\,1\right] \;||\; x == \text{Root}\left[120 + 246\,\#1 - 615\,\#1^2 + \right.$

58    $x == \frac{101}{16} - \frac{1}{16}\sqrt{6265 - \dfrac{34\,672}{\left(520\,530 - \sqrt{260\,775\,489\,437}\right)^{1/3}} - 16\left(520\,530 - \sqrt{260\,775\,489\,437}\right)^{1/3}} - \frac{1}{2}\sqrt{\frac{6265}{32} + \dfrac{}{4\left(52\right.}}$

59    $x == \text{Root}\left[120 + 366\,\#1 - 735\,\#1^2 + 470\,\#1^3 - 105\,\#1^4 + 4\,\#1^5\,\&,\,1\right] \;||\; x == \text{Root}\left[120 + 366\,\#1 - 735\,\#1^2 + \right.$

60    $x == \frac{41}{16} + \frac{1}{16}\sqrt{-975 - \dfrac{6288}{\left(275\,750 + \sqrt{76\,098\,760\,957}\right)^{1/3}} + 16\left(275\,750 + \sqrt{76\,098\,760\,957}\right)^{1/3}} - \frac{1}{2}\sqrt{-\frac{975}{32} + \dfrac{}{4\left(275\,7\right.}}$

61    $x == \text{Root}\left[120 + 486\,\#1 - 735\,\#1^2 + 290\,\#1^3 - 45\,\#1^4 + 4\,\#1^5\,\&,\,1\right] \;||\; x == \text{Root}\left[120 + 486\,\#1 - 735\,\#1^2 + 2\right.$

62    $x == \frac{41}{16} + \frac{1}{16}\sqrt{-975 - \dfrac{32\,528}{\left(333\,650 + \sqrt{119\,724\,892\,437}\right)^{1/3}} + 16\left(333\,650 + \sqrt{119\,724\,892\,437}\right)^{1/3}} - \frac{1}{2}\sqrt{-\frac{975}{32} + \dfrac{}{4\left(33\right.}}$

63    $x == \text{Root}\left[120 + 486\,\#1 - 795\,\#1^2 + 350\,\#1^3 - 45\,\#1^4 + 4\,\#1^5\,\&,\,1\right] \;||\; x == \text{Root}\left[120 + 486\,\#1 - 795\,\#1^2 + 3\right.$

64    $x == \text{Root}\left[-720 + 3924\,\#1 - 1780\,\#1^2 + 395\,\#1^3 - 20\,\#1^4 + \#1^5\,\&,\,1\right] \;||\; x == \text{Root}\left[-720 + 3924\,\#1 - 1780\,\right.$

```
EE1[n_, z_, b_] :=
 Sum[FactorialPower[z, a] / a! (E2a[n, a, b]) / z + 1 / a, {a, 1, Log[If[b > 2, 2, b], n]}]
Limit[( EE1[100, z, 2]), {z → 0}] + Sum[ ((2)^k - 1) / k, {k, 1, Log[2, 100]}]
```

$$\left\{\frac{428}{15}\right\}$$

```
EE1a[n_, z_, b_] :=
 Sum[FactorialPower[z, a] / a! (E2aa[n, a, b]) / z + 1 / a, {a, 1, Log[If[b > 2, 2, b], n]}]
```

Expand[Limit[(EE1a[100, z, 2]), {z → 0}]]

$\left\{\frac{49}{20} + \text{E2aa}[100, 1, 2] - \frac{1}{2} \text{E2aa}[100, 2, 2] + \right.$

$\left. \frac{1}{3} \text{E2aa}[100, 3, 2] - \frac{1}{4} \text{E2aa}[100, 4, 2] + \frac{1}{5} \text{E2aa}[100, 5, 2] - \frac{1}{6} \text{E2aa}[100, 6, 2]\right\}$

EE1b[n_, z_, b_] := Sum[bins[z, a] E2a[n, a, b] / z + 1 / a, {a, 1, Log[If[b > 2, 2, b], n]}]
Limit[(EE1b[100, z, 2]), {z → 0}] + Sum[((2) ^k − 1) / k, {k, 1, Log[2, 100]}]

$\left\{\frac{428}{15}\right\}$

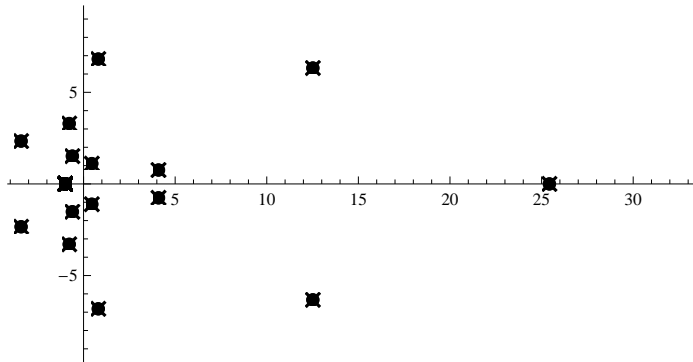**(1 − 2 / List @@ NRoots[EEc[94, x, 2] == 0, x][[All, 2]])**

$\{2.10101 - 1.89751\,\mathbb{i}, 2.10101 + 1.89751\,\mathbb{i}, -1.,$
$0.720918 - 0.364325\,\mathbb{i}, 0.720918 + 0.364325\,\mathbb{i}, 0.956155\}$

EEc[100, 1, 2]

0

RootLocusPlot[1 / Expand[gg[20, x, 1.1]], {k, 0, 1}, FeedbackType → None]

EEc[99, 1, 2]

1

(-1 / List @@ NRoots[gg[101, x, 2] == 0, x][[All, 2]])

Power::infy : Infinite expression $\frac{1}{0.}$ encountered. ≫

$\{2.25303, \text{ComplexInfinity}, -0.376268, -0.0861804 - 0.0429715\,\mathbb{i}, -0.0861804 + 0.0429715\,\mathbb{i}\}$

ff[z_] := z (n − 1) (1 − (z − 1) / a) (1 − (z − 1) / b) (1 − (z − 1) / c) (1 − (z − 1) / d) (1 − (z − 1) / e)
ffp[z_] := (n − 1) (1 + 1 / a) (1 + 1 / b) (1 + 1 / c) (1 + 1 / d) (1 + 1 / e)

FullSimplify[Expand[ff[p]] / Expand[ff[q]]]

$$\frac{(1 + a - p)\ (1 + b - p)\ (1 + c - p)\ (1 + d - p)\ (1 + e - p)\ p}{(1 + a - q)\ q\ (-1 - b + q)\ (-1 - c + q)\ (-1 - d + q)\ (-1 - e + q)}$$

FullSimplify[ff[p] / ff[q]]

$$\frac{(1 + a - p)\ p\ (-1 - b + p)\ (-1 - c + p)\ (-1 - d + p)\ (-1 - e + p)}{(1 + a - q)\ q\ (-1 - b + q)\ (-1 - c + q)\ (-1 - d + q)\ (-1 - e + q)}$$

FullSimplify[ff[2] / ffp[q]]

$$\frac{2\ (-1 + a)\ (-1 + b)\ (-1 + c)\ (-1 + d)\ (-1 + e)}{(1 + a)\ (1 + b)\ (1 + c)\ (1 + d)\ (1 + e)}$$

FullSimplify[ff[-1] / ffp[q]]

$$-\frac{(2 + a)\ (2 + b)\ (2 + c)\ (2 + d)\ (2 + e)}{(1 + a)\ (1 + b)\ (1 + c)\ (1 + d)\ (1 + e)}$$

FullSimplify[ff[5] / ff[2]]

$$\frac{5\ (-4 + a)\ (-4 + b)\ (-4 + c)\ (-4 + d)\ (-4 + e)}{2\ (-1 + a)\ (-1 + b)\ (-1 + c)\ (-1 + d)\ (-1 + e)}$$

FullSimplify[ff[2] / ffp[q]]

$$\frac{2\ (-1 + a)\ (-1 + b)\ (-1 + c)\ (-1 + d)\ (-1 + e)}{(1 + a)\ (1 + b)\ (1 + c)\ (1 + d)\ (1 + e)}$$