```
D2Cache[n_, k_, s_] :=
 Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1 / k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]

LinnikSumFast[n_] := D2Cache[n, 1, 2] +

  Sum[(-1)^(k + 1) / k (

     Sum[D2Cache[n / j, k - 1, 2], {j, Floor[n^(1 / 3)] + 1, n^(1 / 2)}] +

      Sum[(Floor[n / j] - (Floor[n / (j + 1)])) D2Cache[j, k - 1, 2],
       {j, 1, n / Floor[n^(1 / 2)] - 1}] +

      Sum[d2cache[j, k - 1] (Floor[n / j] - 1), {j, 2, n^(1 / 3)}] +

      Sum[d2cache[j, m] D2Cache[n / (j s), k - m - 1, 2], {j, 2, n^(1 / 3)},
       {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[n / j]^(1 / 2)}, {m, 1, k - 2}] +

      Sum[(Sum[1, {m, Floor[n / (j (s + 1))] + 1, n / (j s)}])
        (Sum[d2cache[j, m] D2Cache[s, k - m - 1, 2], {m, 1, k - 2}]),
       {j, 2, n^(1 / 3)}, {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]

    ), {k, 2, Log[2, n]}]
RiePrimeCnt[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Table[{n, LinnikSumFast[n], RiePrimeCnt[n]}, {n, 1, 100}] // TableForm
```

1	0	0
2	1	1
3	2	2
4	$\frac{5}{2}$	$\frac{5}{2}$
5	$\frac{7}{2}$	$\frac{7}{2}$
6	$\frac{7}{2}$	$\frac{7}{2}$
7	$\frac{9}{2}$	$\frac{9}{2}$
8	$\frac{29}{6}$	$\frac{29}{6}$
9	$\frac{16}{3}$	$\frac{16}{3}$
10	$\frac{16}{3}$	$\frac{16}{3}$
11	$\frac{19}{3}$	$\frac{19}{3}$
12	$\frac{19}{3}$	$\frac{19}{3}$
13	$\frac{22}{3}$	$\frac{22}{3}$
14	$\frac{22}{3}$	$\frac{22}{3}$
15	$\frac{22}{3}$	$\frac{22}{3}$
16	$\frac{91}{12}$	$\frac{91}{12}$
17	$\frac{103}{12}$	$\frac{103}{12}$
18	$\frac{103}{12}$	$\frac{103}{12}$
19	$\frac{115}{12}$	$\frac{115}{12}$

20	$\frac{115}{12}$	$\frac{115}{12}$
21	$\frac{115}{12}$	$\frac{115}{12}$
22	$\frac{115}{12}$	$\frac{115}{12}$
23	$\frac{127}{12}$	$\frac{127}{12}$
24	$\frac{127}{12}$	$\frac{127}{12}$
25	$\frac{133}{12}$	$\frac{133}{12}$
26	$\frac{133}{12}$	$\frac{133}{12}$
27	$\frac{137}{12}$	$\frac{137}{12}$
28	$\frac{137}{12}$	$\frac{137}{12}$
29	$\frac{149}{12}$	$\frac{149}{12}$
30	$\frac{149}{12}$	$\frac{149}{12}$
31	$\frac{161}{12}$	$\frac{161}{12}$
32	$\frac{817}{60}$	$\frac{817}{60}$
33	$\frac{817}{60}$	$\frac{817}{60}$
34	$\frac{817}{60}$	$\frac{817}{60}$
35	$\frac{817}{60}$	$\frac{817}{60}$
36	$\frac{817}{60}$	$\frac{817}{60}$
37	$\frac{877}{60}$	$\frac{877}{60}$
38	$\frac{877}{60}$	$\frac{877}{60}$
39	$\frac{877}{60}$	$\frac{877}{60}$
40	$\frac{877}{60}$	$\frac{877}{60}$
41	$\frac{937}{60}$	$\frac{937}{60}$
42	$\frac{937}{60}$	$\frac{937}{60}$
43	$\frac{997}{60}$	$\frac{997}{60}$
44	$\frac{997}{60}$	$\frac{997}{60}$
45	$\frac{997}{60}$	$\frac{997}{60}$
46	$\frac{997}{60}$	$\frac{997}{60}$
47	$\frac{1057}{60}$	$\frac{1057}{60}$
48	$\frac{1057}{60}$	$\frac{1057}{60}$
49	$\frac{1087}{60}$	$\frac{1087}{60}$
50	$\frac{1087}{60}$	$\frac{1087}{60}$
51	$\frac{1087}{60}$	$\frac{1087}{60}$
52	$\frac{1087}{60}$	$\frac{1087}{60}$
53	$\frac{1147}{60}$	$\frac{1147}{60}$
54	$\frac{1147}{60}$	$\frac{1147}{60}$
55	$\frac{1147}{60}$	$\frac{1147}{60}$
56	$\frac{1147}{60}$	$\frac{1147}{60}$
57	$\frac{1147}{60}$	$\frac{1147}{60}$
58	$\frac{1147}{60}$	$\frac{1147}{60}$

59	$\frac{1207}{60}$	$\frac{1207}{60}$
60	$\frac{1207}{60}$	$\frac{1207}{60}$
61	$\frac{1267}{60}$	$\frac{1267}{60}$
62	$\frac{1267}{60}$	$\frac{1267}{60}$
63	$\frac{1267}{60}$	$\frac{1267}{60}$
64	$\frac{1277}{60}$	$\frac{1277}{60}$
65	$\frac{1277}{60}$	$\frac{1277}{60}$
66	$\frac{1277}{60}$	$\frac{1277}{60}$
67	$\frac{1337}{60}$	$\frac{1337}{60}$
68	$\frac{1337}{60}$	$\frac{1337}{60}$
69	$\frac{1337}{60}$	$\frac{1337}{60}$
70	$\frac{1337}{60}$	$\frac{1337}{60}$
71	$\frac{1397}{60}$	$\frac{1397}{60}$
72	$\frac{1397}{60}$	$\frac{1397}{60}$
73	$\frac{1457}{60}$	$\frac{1457}{60}$
74	$\frac{1457}{60}$	$\frac{1457}{60}$
75	$\frac{1457}{60}$	$\frac{1457}{60}$
76	$\frac{1457}{60}$	$\frac{1457}{60}$
77	$\frac{1457}{60}$	$\frac{1457}{60}$
78	$\frac{1457}{60}$	$\frac{1457}{60}$
79	$\frac{1517}{60}$	$\frac{1517}{60}$
80	$\frac{1517}{60}$	$\frac{1517}{60}$
81	$\frac{383}{15}$	$\frac{383}{15}$
82	$\frac{383}{15}$	$\frac{383}{15}$
83	$\frac{398}{15}$	$\frac{398}{15}$
84	$\frac{398}{15}$	$\frac{398}{15}$
85	$\frac{398}{15}$	$\frac{398}{15}$
86	$\frac{398}{15}$	$\frac{398}{15}$
87	$\frac{398}{15}$	$\frac{398}{15}$
88	$\frac{398}{15}$	$\frac{398}{15}$
89	$\frac{413}{15}$	$\frac{413}{15}$
90	$\frac{413}{15}$	$\frac{413}{15}$
91	$\frac{413}{15}$	$\frac{413}{15}$
92	$\frac{413}{15}$	$\frac{413}{15}$
93	$\frac{413}{15}$	$\frac{413}{15}$
94	$\frac{413}{15}$	$\frac{413}{15}$
95	$\frac{413}{15}$	$\frac{413}{15}$
96	$\frac{413}{15}$	$\frac{413}{15}$
97	$\frac{428}{15}$	$\frac{428}{15}$

98	$\frac{428}{15}$	$\frac{428}{15}$
99	$\frac{428}{15}$	$\frac{428}{15}$
100	$\frac{428}{15}$	$\frac{428}{15}$

```
D2Cache[n_, k_, s_] :=
 Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1 / k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]

LinnikSumFast[n_] := D2Cache[n, 1, 2] +
  Sum[(-1)^(k + 1) / k (Sum[D2Cache[n / j, k - 1, 2], {j, Floor[n^(1 / 3)] + 1, n^(1 / 2)}]),
   {k, 2, Log[2, n]}] +
  Sum[(-1)^(k + 1) / k (Sum[(Floor[n / j] - (Floor[n / (j + 1)])) D2Cache[j, k - 1, 2],
      {j, 1, n / Floor[n^(1 / 2)] - 1}]), {k, 2, Log[2, n]}] +
  Sum[(-1)^(k + 1) / k (Sum[d2cache[j, k - 1] (Floor[n / j] - 1), {j, 2, n^(1 / 3)}]),
   {k, 2, Log[2, n]}] +
  Sum[(-1)^(k + 1) / k (Sum[d2cache[j, m] D2Cache[n / (j s), k - m - 1, 2],
      {j, 2, n^(1 / 3)}, {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[n / j]^(1 / 2)},
      {m, 1, k - 2}]), {k, 2, Log[2, n]}] +
  Sum[(-1)^(k + 1) / k (Sum[(Sum[1, {m, Floor[n / (j (s + 1))] + 1, n / (j s)}])
       (Sum[d2cache[j, m] D2Cache[s, k - m - 1, 2], {m, 1, k - 2}]), {j, 2, n^(1 / 3)},
      {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]), {k, 2, Log[2, n]}]
RiePrimeCnt[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Table[{n, LinnikSumFast[n], RiePrimeCnt[n]}, {n, 1, 100}] // TableForm
```

1	0	0
2	1	1
3	2	2
4	$\frac{5}{2}$	$\frac{5}{2}$
5	$\frac{7}{2}$	$\frac{7}{2}$
6	$\frac{7}{2}$	$\frac{7}{2}$
7	$\frac{9}{2}$	$\frac{9}{2}$
8	$\frac{29}{6}$	$\frac{29}{6}$
9	$\frac{16}{3}$	$\frac{16}{3}$
10	$\frac{16}{3}$	$\frac{16}{3}$
11	$\frac{19}{3}$	$\frac{19}{3}$
12	$\frac{19}{3}$	$\frac{19}{3}$
13	$\frac{22}{3}$	$\frac{22}{3}$
14	$\frac{22}{3}$	$\frac{22}{3}$
15	$\frac{22}{3}$	$\frac{22}{3}$
16	$\frac{91}{12}$	$\frac{91}{12}$
17	$\frac{103}{12}$	$\frac{103}{12}$
18	$\frac{103}{12}$	$\frac{103}{12}$
19	$\frac{115}{12}$	$\frac{115}{12}$
20	$\frac{115}{12}$	$\frac{115}{12}$

21	$\frac{115}{12}$	$\frac{115}{12}$
22	$\frac{115}{12}$	$\frac{115}{12}$
23	$\frac{127}{12}$	$\frac{127}{12}$
24	$\frac{127}{12}$	$\frac{127}{12}$
25	$\frac{133}{12}$	$\frac{133}{12}$
26	$\frac{133}{12}$	$\frac{133}{12}$
27	$\frac{137}{12}$	$\frac{137}{12}$
28	$\frac{137}{12}$	$\frac{137}{12}$
29	$\frac{149}{12}$	$\frac{149}{12}$
30	$\frac{149}{12}$	$\frac{149}{12}$
31	$\frac{161}{12}$	$\frac{161}{12}$
32	$\frac{817}{60}$	$\frac{817}{60}$
33	$\frac{817}{60}$	$\frac{817}{60}$
34	$\frac{817}{60}$	$\frac{817}{60}$
35	$\frac{817}{60}$	$\frac{817}{60}$
36	$\frac{817}{60}$	$\frac{817}{60}$
37	$\frac{877}{60}$	$\frac{877}{60}$
38	$\frac{877}{60}$	$\frac{877}{60}$
39	$\frac{877}{60}$	$\frac{877}{60}$
40	$\frac{877}{60}$	$\frac{877}{60}$
41	$\frac{937}{60}$	$\frac{937}{60}$
42	$\frac{937}{60}$	$\frac{937}{60}$
43	$\frac{997}{60}$	$\frac{997}{60}$
44	$\frac{997}{60}$	$\frac{997}{60}$
45	$\frac{997}{60}$	$\frac{997}{60}$
46	$\frac{997}{60}$	$\frac{997}{60}$
47	$\frac{1057}{60}$	$\frac{1057}{60}$
48	$\frac{1057}{60}$	$\frac{1057}{60}$
49	$\frac{1087}{60}$	$\frac{1087}{60}$
50	$\frac{1087}{60}$	$\frac{1087}{60}$
51	$\frac{1087}{60}$	$\frac{1087}{60}$
52	$\frac{1087}{60}$	$\frac{1087}{60}$
53	$\frac{1147}{60}$	$\frac{1147}{60}$
54	$\frac{1147}{60}$	$\frac{1147}{60}$
55	$\frac{1147}{60}$	$\frac{1147}{60}$
56	$\frac{1147}{60}$	$\frac{1147}{60}$
57	$\frac{1147}{60}$	$\frac{1147}{60}$
58	$\frac{1147}{60}$	$\frac{1147}{60}$
59	$\frac{1207}{60}$	$\frac{1207}{60}$

60	$\frac{1207}{60}$	$\frac{1207}{60}$
61	$\frac{1267}{60}$	$\frac{1267}{60}$
62	$\frac{1267}{60}$	$\frac{1267}{60}$
63	$\frac{1267}{60}$	$\frac{1267}{60}$
64	$\frac{1277}{60}$	$\frac{1277}{60}$
65	$\frac{1277}{60}$	$\frac{1277}{60}$
66	$\frac{1277}{60}$	$\frac{1277}{60}$
67	$\frac{1337}{60}$	$\frac{1337}{60}$
68	$\frac{1337}{60}$	$\frac{1337}{60}$
69	$\frac{1337}{60}$	$\frac{1337}{60}$
70	$\frac{1337}{60}$	$\frac{1337}{60}$
71	$\frac{1397}{60}$	$\frac{1397}{60}$
72	$\frac{1397}{60}$	$\frac{1397}{60}$
73	$\frac{1457}{60}$	$\frac{1457}{60}$
74	$\frac{1457}{60}$	$\frac{1457}{60}$
75	$\frac{1457}{60}$	$\frac{1457}{60}$
76	$\frac{1457}{60}$	$\frac{1457}{60}$
77	$\frac{1457}{60}$	$\frac{1457}{60}$
78	$\frac{1457}{60}$	$\frac{1457}{60}$
79	$\frac{1517}{60}$	$\frac{1517}{60}$
80	$\frac{1517}{60}$	$\frac{1517}{60}$
81	$\frac{383}{15}$	$\frac{383}{15}$
82	$\frac{383}{15}$	$\frac{383}{15}$
83	$\frac{398}{15}$	$\frac{398}{15}$
84	$\frac{398}{15}$	$\frac{398}{15}$
85	$\frac{398}{15}$	$\frac{398}{15}$
86	$\frac{398}{15}$	$\frac{398}{15}$
87	$\frac{398}{15}$	$\frac{398}{15}$
88	$\frac{398}{15}$	$\frac{398}{15}$
89	$\frac{413}{15}$	$\frac{413}{15}$
90	$\frac{413}{15}$	$\frac{413}{15}$
91	$\frac{413}{15}$	$\frac{413}{15}$
92	$\frac{413}{15}$	$\frac{413}{15}$
93	$\frac{413}{15}$	$\frac{413}{15}$
94	$\frac{413}{15}$	$\frac{413}{15}$
95	$\frac{413}{15}$	$\frac{413}{15}$
96	$\frac{413}{15}$	$\frac{413}{15}$
97	$\frac{428}{15}$	$\frac{428}{15}$
98	$\frac{428}{15}$	$\frac{428}{15}$

99	$\frac{428}{15}$	$\frac{428}{15}$
100	$\frac{428}{15}$	$\frac{428}{15}$

```
D2Cache[n_, k_, s_] :=
 Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1 / k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]

LinnikSumFast[n_] := D2Cache[n, 1, 2] +
  Sum[(-1)^(k + 1) / k (Sum[D2Cache[n / j, k - 1, 2], {j, Floor[n^(1 / 3)] + 1, n^(1 / 2)}]),
   {k, 2, Log[2, n]}] +
  Sum[(-1)^(k + 1) / k (Sum[(Floor[n / j] - (Floor[n / (j + 1)])) D2Cache[j, k - 1, 2],
      {j, 1, n / Floor[n^(1 / 2)] - 1}]), {k, 2, Log[2, n]}] +
  Sum[(-1)^(k + 1) / k (Sum[d2cache[j, k - 1] (Floor[n / j] - 1), {j, 2, n^(1 / 3)}]),
   {k, 2, Log[2, n]}] +
  Sum[(-1)^(k + 1) / k (Sum[d2cache[j, m] D2Cache[n / (j s), k - m - 1, 2],
      {j, 2, n^(1 / 3)}, {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[n / j]^(1 / 2)},
      {m, 1, k - 2}]), {k, 2, Log[2, n]}] +
  Sum[(-1)^(k + 1) / k (Sum[(Sum[1, {m, Floor[n / (j (s + 1))] + 1, n / (j s)}])
       (Sum[d2cache[j, m] D2Cache[s, k - m - 1, 2], {m, 1, k - 2}]), {j, 2, n^(1 / 3)},
      {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]), {k, 2, Log[2, n]}]
RiePrimeCnt[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Table[{n, LinnikSumFast[n], RiePrimeCnt[n]}, {n, 1, 100}]
```

$$\{\{1, 0, 0\}, \{2, 1, 1\}, \{3, 2, 2\}, \{4, \frac{5}{2}, \frac{5}{2}\}, \{5, \frac{7}{2}, \frac{7}{2}\}, \{6, \frac{7}{2}, \frac{7}{2}\}, \{7, \frac{9}{2}, \frac{9}{2}\},$$
$$\{8, \frac{29}{6}, \frac{29}{6}\}, \{9, \frac{16}{3}, \frac{16}{3}\}, \{10, \frac{16}{3}, \frac{16}{3}\}, \{11, \frac{19}{3}, \frac{19}{3}\}, \{12, \frac{19}{3}, \frac{19}{3}\}, \{13, \frac{22}{3}, \frac{22}{3}\},$$
$$\{14, \frac{22}{3}, \frac{22}{3}\}, \{15, \frac{22}{3}, \frac{22}{3}\}, \{16, \frac{91}{12}, \frac{91}{12}\}, \{17, \frac{103}{12}, \frac{103}{12}\}, \{18, \frac{103}{12}, \frac{103}{12}\},$$
$$\{19, \frac{115}{12}, \frac{115}{12}\}, \{20, \frac{115}{12}, \frac{115}{12}\}, \{21, \frac{115}{12}, \frac{115}{12}\}, \{22, \frac{115}{12}, \frac{115}{12}\}, \{23, \frac{127}{12}, \frac{127}{12}\},$$
$$\{24, \frac{127}{12}, \frac{127}{12}\}, \{25, \frac{133}{12}, \frac{133}{12}\}, \{26, \frac{133}{12}, \frac{133}{12}\}, \{27, \frac{137}{12}, \frac{137}{12}\}, \{28, \frac{137}{12}, \frac{137}{12}\},$$
$$\{29, \frac{149}{12}, \frac{149}{12}\}, \{30, \frac{149}{12}, \frac{149}{12}\}, \{31, \frac{161}{12}, \frac{161}{12}\}, \{32, \frac{817}{60}, \frac{817}{60}\}, \{33, \frac{817}{60}, \frac{817}{60}\},$$
$$\{34, \frac{817}{60}, \frac{817}{60}\}, \{35, \frac{817}{60}, \frac{817}{60}\}, \{36, \frac{817}{60}, \frac{817}{60}\}, \{37, \frac{877}{60}, \frac{877}{60}\}, \{38, \frac{877}{60}, \frac{877}{60}\},$$
$$\{39, \frac{877}{60}, \frac{877}{60}\}, \{40, \frac{877}{60}, \frac{877}{60}\}, \{41, \frac{937}{60}, \frac{937}{60}\}, \{42, \frac{937}{60}, \frac{937}{60}\}, \{43, \frac{997}{60}, \frac{997}{60}\},$$
$$\{44, \frac{997}{60}, \frac{997}{60}\}, \{45, \frac{997}{60}, \frac{997}{60}\}, \{46, \frac{997}{60}, \frac{997}{60}\}, \{47, \frac{1057}{60}, \frac{1057}{60}\}, \{48, \frac{1057}{60}, \frac{1057}{60}\},$$
$$\{49, \frac{1087}{60}, \frac{1087}{60}\}, \{50, \frac{1087}{60}, \frac{1087}{60}\}, \{51, \frac{1087}{60}, \frac{1087}{60}\}, \{52, \frac{1087}{60}, \frac{1087}{60}\},$$
$$\{53, \frac{1147}{60}, \frac{1147}{60}\}, \{54, \frac{1147}{60}, \frac{1147}{60}\}, \{55, \frac{1147}{60}, \frac{1147}{60}\}, \{56, \frac{1147}{60}, \frac{1147}{60}\},$$
$$\{57, \frac{1147}{60}, \frac{1147}{60}\}, \{58, \frac{1147}{60}, \frac{1147}{60}\}, \{59, \frac{1207}{60}, \frac{1207}{60}\}, \{60, \frac{1207}{60}, \frac{1207}{60}\},$$
$$\{61, \frac{1267}{60}, \frac{1267}{60}\}, \{62, \frac{1267}{60}, \frac{1267}{60}\}, \{63, \frac{1267}{60}, \frac{1267}{60}\}, \{64, \frac{1277}{60}, \frac{1277}{60}\},$$
$$\{65, \frac{1277}{60}, \frac{1277}{60}\}, \{66, \frac{1277}{60}, \frac{1277}{60}\}, \{67, \frac{1337}{60}, \frac{1337}{60}\}, \{68, \frac{1337}{60}, \frac{1337}{60}\},$$
$$\{69, \frac{1337}{60}, \frac{1337}{60}\}, \{70, \frac{1337}{60}, \frac{1337}{60}\}, \{71, \frac{1397}{60}, \frac{1397}{60}\}, \{72, \frac{1397}{60}, \frac{1397}{60}\},$$
$$\{73, \frac{1457}{60}, \frac{1457}{60}\}, \{74, \frac{1457}{60}, \frac{1457}{60}\}, \{75, \frac{1457}{60}, \frac{1457}{60}\}, \{76, \frac{1457}{60}, \frac{1457}{60}\},$$
$$\{77, \frac{1457}{60}, \frac{1457}{60}\}, \{78, \frac{1457}{60}, \frac{1457}{60}\}, \{79, \frac{1517}{60}, \frac{1517}{60}\}, \{80, \frac{1517}{60}, \frac{1517}{60}\},$$
$$\{81, \frac{383}{15}, \frac{383}{15}\}, \{82, \frac{383}{15}, \frac{383}{15}\}, \{83, \frac{398}{15}, \frac{398}{15}\}, \{84, \frac{398}{15}, \frac{398}{15}\}, \{85, \frac{398}{15}, \frac{398}{15}\},$$
$$\{86, \frac{398}{15}, \frac{398}{15}\}, \{87, \frac{398}{15}, \frac{398}{15}\}, \{88, \frac{398}{15}, \frac{398}{15}\}, \{89, \frac{413}{15}, \frac{413}{15}\}, \{90, \frac{413}{15}, \frac{413}{15}\},$$
$$\{91, \frac{413}{15}, \frac{413}{15}\}, \{92, \frac{413}{15}, \frac{413}{15}\}, \{93, \frac{413}{15}, \frac{413}{15}\}, \{94, \frac{413}{15}, \frac{413}{15}\}, \{95, \frac{413}{15}, \frac{413}{15}\},$$
$$\{96, \frac{413}{15}, \frac{413}{15}\}, \{97, \frac{428}{15}, \frac{428}{15}\}, \{98, \frac{428}{15}, \frac{428}{15}\}, \{99, \frac{428}{15}, \frac{428}{15}\}, \{100, \frac{428}{15}, \frac{428}{15}\}\}$$

```
D2Cache[n_, k_, s_] :=
 Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1 / k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]

LinnikSumFast[n_] := D2Cache[n, 1, 2] +
  Sum[(Sum[(-1)^(k + 1) / k D2Cache[n / j, k - 1, 2], {j, Floor[n^(1 / 3)] + 1, n^(1 / 2)}]),
   {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k (Floor[n / j] - (Floor[n / (j + 1)])) D2Cache[j, k - 1, 2],
     {j, 1, n / Floor[n^(1 / 2)] - 1}]), {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k d2cache[j, k - 1] (Floor[n / j] - 1), {j, 2, n^(1 / 3)}]),
   {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k d2cache[j, m] D2Cache[n / (j s), k - m - 1, 2],
     {j, 2, n^(1 / 3)}, {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[n / j]^(1 / 2)},
     {m, 1, k - 2}]), {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k (Sum[1, {m, Floor[n / (j (s + 1))] + 1, n / (j s)}])
      (Sum[d2cache[j, m] D2Cache[s, k - m - 1, 2], {m, 1, k - 2}]), {j, 2, n^(1 / 3)},
     {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]), {k, 2, Log[2, n]}]
RiePrimeCnt[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Table[{n, LinnikSumFast[n], RiePrimeCnt[n]}, {n, 1, 100}]
```

$$\{\{1, 0, 0\}, \{2, 1, 1\}, \{3, 2, 2\}, \{4, \frac{5}{2}, \frac{5}{2}\}, \{5, \frac{7}{2}, \frac{7}{2}\}, \{6, \frac{7}{2}, \frac{7}{2}\}, \{7, \frac{9}{2}, \frac{9}{2}\},$$
$$\{8, \frac{29}{6}, \frac{29}{6}\}, \{9, \frac{16}{3}, \frac{16}{3}\}, \{10, \frac{16}{3}, \frac{16}{3}\}, \{11, \frac{19}{3}, \frac{19}{3}\}, \{12, \frac{19}{3}, \frac{19}{3}\}, \{13, \frac{22}{3}, \frac{22}{3}\},$$
$$\{14, \frac{22}{3}, \frac{22}{3}\}, \{15, \frac{22}{3}, \frac{22}{3}\}, \{16, \frac{91}{12}, \frac{91}{12}\}, \{17, \frac{103}{12}, \frac{103}{12}\}, \{18, \frac{103}{12}, \frac{103}{12}\},$$
$$\{19, \frac{115}{12}, \frac{115}{12}\}, \{20, \frac{115}{12}, \frac{115}{12}\}, \{21, \frac{115}{12}, \frac{115}{12}\}, \{22, \frac{115}{12}, \frac{115}{12}\}, \{23, \frac{127}{12}, \frac{127}{12}\},$$
$$\{24, \frac{127}{12}, \frac{127}{12}\}, \{25, \frac{133}{12}, \frac{133}{12}\}, \{26, \frac{133}{12}, \frac{133}{12}\}, \{27, \frac{137}{12}, \frac{137}{12}\}, \{28, \frac{137}{12}, \frac{137}{12}\},$$
$$\{29, \frac{149}{12}, \frac{149}{12}\}, \{30, \frac{149}{12}, \frac{149}{12}\}, \{31, \frac{161}{12}, \frac{161}{12}\}, \{32, \frac{817}{60}, \frac{817}{60}\}, \{33, \frac{817}{60}, \frac{817}{60}\},$$
$$\{34, \frac{817}{60}, \frac{817}{60}\}, \{35, \frac{817}{60}, \frac{817}{60}\}, \{36, \frac{817}{60}, \frac{817}{60}\}, \{37, \frac{877}{60}, \frac{877}{60}\}, \{38, \frac{877}{60}, \frac{877}{60}\},$$
$$\{39, \frac{877}{60}, \frac{877}{60}\}, \{40, \frac{877}{60}, \frac{877}{60}\}, \{41, \frac{937}{60}, \frac{937}{60}\}, \{42, \frac{937}{60}, \frac{937}{60}\}, \{43, \frac{997}{60}, \frac{997}{60}\},$$
$$\{44, \frac{997}{60}, \frac{997}{60}\}, \{45, \frac{997}{60}, \frac{997}{60}\}, \{46, \frac{997}{60}, \frac{997}{60}\}, \{47, \frac{1057}{60}, \frac{1057}{60}\}, \{48, \frac{1057}{60}, \frac{1057}{60}\},$$
$$\{49, \frac{1087}{60}, \frac{1087}{60}\}, \{50, \frac{1087}{60}, \frac{1087}{60}\}, \{51, \frac{1087}{60}, \frac{1087}{60}\}, \{52, \frac{1087}{60}, \frac{1087}{60}\},$$
$$\{53, \frac{1147}{60}, \frac{1147}{60}\}, \{54, \frac{1147}{60}, \frac{1147}{60}\}, \{55, \frac{1147}{60}, \frac{1147}{60}\}, \{56, \frac{1147}{60}, \frac{1147}{60}\},$$
$$\{57, \frac{1147}{60}, \frac{1147}{60}\}, \{58, \frac{1147}{60}, \frac{1147}{60}\}, \{59, \frac{1207}{60}, \frac{1207}{60}\}, \{60, \frac{1207}{60}, \frac{1207}{60}\},$$
$$\{61, \frac{1267}{60}, \frac{1267}{60}\}, \{62, \frac{1267}{60}, \frac{1267}{60}\}, \{63, \frac{1267}{60}, \frac{1267}{60}\}, \{64, \frac{1277}{60}, \frac{1277}{60}\},$$
$$\{65, \frac{1277}{60}, \frac{1277}{60}\}, \{66, \frac{1277}{60}, \frac{1277}{60}\}, \{67, \frac{1337}{60}, \frac{1337}{60}\}, \{68, \frac{1337}{60}, \frac{1337}{60}\},$$
$$\{69, \frac{1337}{60}, \frac{1337}{60}\}, \{70, \frac{1337}{60}, \frac{1337}{60}\}, \{71, \frac{1397}{60}, \frac{1397}{60}\}, \{72, \frac{1397}{60}, \frac{1397}{60}\},$$
$$\{73, \frac{1457}{60}, \frac{1457}{60}\}, \{74, \frac{1457}{60}, \frac{1457}{60}\}, \{75, \frac{1457}{60}, \frac{1457}{60}\}, \{76, \frac{1457}{60}, \frac{1457}{60}\},$$
$$\{77, \frac{1457}{60}, \frac{1457}{60}\}, \{78, \frac{1457}{60}, \frac{1457}{60}\}, \{79, \frac{1517}{60}, \frac{1517}{60}\}, \{80, \frac{1517}{60}, \frac{1517}{60}\},$$
$$\{81, \frac{383}{15}, \frac{383}{15}\}, \{82, \frac{383}{15}, \frac{383}{15}\}, \{83, \frac{398}{15}, \frac{398}{15}\}, \{84, \frac{398}{15}, \frac{398}{15}\}, \{85, \frac{398}{15}, \frac{398}{15}\},$$
$$\{86, \frac{398}{15}, \frac{398}{15}\}, \{87, \frac{398}{15}, \frac{398}{15}\}, \{88, \frac{398}{15}, \frac{398}{15}\}, \{89, \frac{413}{15}, \frac{413}{15}\}, \{90, \frac{413}{15}, \frac{413}{15}\},$$
$$\{91, \frac{413}{15}, \frac{413}{15}\}, \{92, \frac{413}{15}, \frac{413}{15}\}, \{93, \frac{413}{15}, \frac{413}{15}\}, \{94, \frac{413}{15}, \frac{413}{15}\}, \{95, \frac{413}{15}, \frac{413}{15}\},$$
$$\{96, \frac{413}{15}, \frac{413}{15}\}, \{97, \frac{428}{15}, \frac{428}{15}\}, \{98, \frac{428}{15}, \frac{428}{15}\}, \{99, \frac{428}{15}, \frac{428}{15}\}, \{100, \frac{428}{15}, \frac{428}{15}\}\}$$

```
D2Cache[n_, k_, s_] :=
 Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1 / k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]

LinnikSumFast[n_] := D2Cache[n, 1, 2] +
  Sum[(Sum[(-1)^(k + 1) / k D2Cache[n / j, k - 1, 2], {j, Floor[n^(1 / 3)] + 1, n^(1 / 2)}]),
   {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k (Floor[n / j] - (Floor[n / (j + 1)])) D2Cache[j, k - 1, 2],
     {j, 1, n / Floor[n^(1 / 2)] - 1}]), {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k d2cache[j, k - 1] (Floor[n / j] - 1), {j, 2, n^(1 / 3)}]),
   {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k d2cache[j, m] D2Cache[n / (j s), k - m - 1, 2],
     {j, 2, n^(1 / 3)}, {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[n / j]^(1 / 2)},
     {m, 1, k - 2}]), {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k (     Sum[1, {m, Floor[n / (j (s + 1))] + 1, n / (j s)}]    )
      (Sum[d2cache[j, m] D2Cache[s, k - m - 1, 2], {m, 1, k - 2}]), {j, 2, n^(1 / 3)},
     {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]), {k, 2, Log[2, n]}]
RiePrimeCnt[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Table[{n, LinnikSumFast[n], RiePrimeCnt[n]}, {n, 1, 100}]
```

$$\{\{1, 0, 0\}, \{2, 1, 1\}, \{3, 2, 2\}, \{4, \frac{5}{2}, \frac{5}{2}\}, \{5, \frac{7}{2}, \frac{7}{2}\}, \{6, \frac{7}{2}, \frac{7}{2}\}, \{7, \frac{9}{2}, \frac{9}{2}\},$$
$$\{8, \frac{29}{6}, \frac{29}{6}\}, \{9, \frac{16}{3}, \frac{16}{3}\}, \{10, \frac{16}{3}, \frac{16}{3}\}, \{11, \frac{19}{3}, \frac{19}{3}\}, \{12, \frac{19}{3}, \frac{19}{3}\}, \{13, \frac{22}{3}, \frac{22}{3}\},$$
$$\{14, \frac{22}{3}, \frac{22}{3}\}, \{15, \frac{22}{3}, \frac{22}{3}\}, \{16, \frac{91}{12}, \frac{91}{12}\}, \{17, \frac{103}{12}, \frac{103}{12}\}, \{18, \frac{103}{12}, \frac{103}{12}\},$$
$$\{19, \frac{115}{12}, \frac{115}{12}\}, \{20, \frac{115}{12}, \frac{115}{12}\}, \{21, \frac{115}{12}, \frac{115}{12}\}, \{22, \frac{115}{12}, \frac{115}{12}\}, \{23, \frac{127}{12}, \frac{127}{12}\},$$
$$\{24, \frac{127}{12}, \frac{127}{12}\}, \{25, \frac{133}{12}, \frac{133}{12}\}, \{26, \frac{133}{12}, \frac{133}{12}\}, \{27, \frac{137}{12}, \frac{137}{12}\}, \{28, \frac{137}{12}, \frac{137}{12}\},$$
$$\{29, \frac{149}{12}, \frac{149}{12}\}, \{30, \frac{149}{12}, \frac{149}{12}\}, \{31, \frac{161}{12}, \frac{161}{12}\}, \{32, \frac{817}{60}, \frac{817}{60}\}, \{33, \frac{817}{60}, \frac{817}{60}\},$$
$$\{34, \frac{817}{60}, \frac{817}{60}\}, \{35, \frac{817}{60}, \frac{817}{60}\}, \{36, \frac{817}{60}, \frac{817}{60}\}, \{37, \frac{877}{60}, \frac{877}{60}\}, \{38, \frac{877}{60}, \frac{877}{60}\},$$
$$\{39, \frac{877}{60}, \frac{877}{60}\}, \{40, \frac{877}{60}, \frac{877}{60}\}, \{41, \frac{937}{60}, \frac{937}{60}\}, \{42, \frac{937}{60}, \frac{937}{60}\}, \{43, \frac{997}{60}, \frac{997}{60}\},$$
$$\{44, \frac{997}{60}, \frac{997}{60}\}, \{45, \frac{997}{60}, \frac{997}{60}\}, \{46, \frac{997}{60}, \frac{997}{60}\}, \{47, \frac{1057}{60}, \frac{1057}{60}\}, \{48, \frac{1057}{60}, \frac{1057}{60}\},$$
$$\{49, \frac{1087}{60}, \frac{1087}{60}\}, \{50, \frac{1087}{60}, \frac{1087}{60}\}, \{51, \frac{1087}{60}, \frac{1087}{60}\}, \{52, \frac{1087}{60}, \frac{1087}{60}\},$$
$$\{53, \frac{1147}{60}, \frac{1147}{60}\}, \{54, \frac{1147}{60}, \frac{1147}{60}\}, \{55, \frac{1147}{60}, \frac{1147}{60}\}, \{56, \frac{1147}{60}, \frac{1147}{60}\},$$
$$\{57, \frac{1147}{60}, \frac{1147}{60}\}, \{58, \frac{1147}{60}, \frac{1147}{60}\}, \{59, \frac{1207}{60}, \frac{1207}{60}\}, \{60, \frac{1207}{60}, \frac{1207}{60}\},$$
$$\{61, \frac{1267}{60}, \frac{1267}{60}\}, \{62, \frac{1267}{60}, \frac{1267}{60}\}, \{63, \frac{1267}{60}, \frac{1267}{60}\}, \{64, \frac{1277}{60}, \frac{1277}{60}\},$$
$$\{65, \frac{1277}{60}, \frac{1277}{60}\}, \{66, \frac{1277}{60}, \frac{1277}{60}\}, \{67, \frac{1337}{60}, \frac{1337}{60}\}, \{68, \frac{1337}{60}, \frac{1337}{60}\},$$
$$\{69, \frac{1337}{60}, \frac{1337}{60}\}, \{70, \frac{1337}{60}, \frac{1337}{60}\}, \{71, \frac{1397}{60}, \frac{1397}{60}\}, \{72, \frac{1397}{60}, \frac{1397}{60}\},$$
$$\{73, \frac{1457}{60}, \frac{1457}{60}\}, \{74, \frac{1457}{60}, \frac{1457}{60}\}, \{75, \frac{1457}{60}, \frac{1457}{60}\}, \{76, \frac{1457}{60}, \frac{1457}{60}\},$$
$$\{77, \frac{1457}{60}, \frac{1457}{60}\}, \{78, \frac{1457}{60}, \frac{1457}{60}\}, \{79, \frac{1517}{60}, \frac{1517}{60}\}, \{80, \frac{1517}{60}, \frac{1517}{60}\},$$
$$\{81, \frac{383}{15}, \frac{383}{15}\}, \{82, \frac{383}{15}, \frac{383}{15}\}, \{83, \frac{398}{15}, \frac{398}{15}\}, \{84, \frac{398}{15}, \frac{398}{15}\}, \{85, \frac{398}{15}, \frac{398}{15}\},$$
$$\{86, \frac{398}{15}, \frac{398}{15}\}, \{87, \frac{398}{15}, \frac{398}{15}\}, \{88, \frac{398}{15}, \frac{398}{15}\}, \{89, \frac{413}{15}, \frac{413}{15}\}, \{90, \frac{413}{15}, \frac{413}{15}\},$$
$$\{91, \frac{413}{15}, \frac{413}{15}\}, \{92, \frac{413}{15}, \frac{413}{15}\}, \{93, \frac{413}{15}, \frac{413}{15}\}, \{94, \frac{413}{15}, \frac{413}{15}\}, \{95, \frac{413}{15}, \frac{413}{15}\},$$
$$\{96, \frac{413}{15}, \frac{413}{15}\}, \{97, \frac{428}{15}, \frac{428}{15}\}, \{98, \frac{428}{15}, \frac{428}{15}\}, \{99, \frac{428}{15}, \frac{428}{15}\}, \{100, \frac{428}{15}, \frac{428}{15}\}\}$$

```
D2Cache[n_, k_, s_] :=
 Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1 / k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]

LinnikSumFast[n_] := D2Cache[n, 1, 2] +
  Sum[(Sum[(-1)^(k + 1) / k D2Cache[n / j, k - 1, 2], {j, Floor[n^(1 / 3)] + 1, n^(1 / 2)}]),
   {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k (Floor[n / j] - (Floor[n / (j + 1)])) D2Cache[j, k - 1, 2],
     {j, 1, n / Floor[n^(1 / 2)] - 1}]), {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k d2cache[j, k - 1] (Floor[n / j] - 1), {j, 2, n^(1 / 3)}]),
   {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k d2cache[j, m] D2Cache[n / (j s), k - m - 1, 2],
     {j, 2, n^(1 / 3)}, {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[n / j]^(1 / 2)},
     {m, 1, k - 2}]), {k, 2, Log[2, n]}] +
  Sum[(Sum[(-1)^(k + 1) / k ( Floor[n / (j s)] - Floor[n / (j (s + 1))])
      (Sum[d2cache[j, m] D2Cache[s, k - m - 1, 2], {m, 1, k - 2}]), {j, 2, n^(1 / 3)},
     {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]), {k, 2, Log[2, n]}]
RiePrimeCnt[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Table[{n, LinnikSumFast[n], RiePrimeCnt[n]}, {n, 1, 100}]
```

$$\{\{1, 0, 0\}, \{2, 1, 1\}, \{3, 2, 2\}, \{4, \frac{5}{2}, \frac{5}{2}\}, \{5, \frac{7}{2}, \frac{7}{2}\}, \{6, \frac{7}{2}, \frac{7}{2}\}, \{7, \frac{9}{2}, \frac{9}{2}\},$$
$$\{8, \frac{29}{6}, \frac{29}{6}\}, \{9, \frac{16}{3}, \frac{16}{3}\}, \{10, \frac{16}{3}, \frac{16}{3}\}, \{11, \frac{19}{3}, \frac{19}{3}\}, \{12, \frac{19}{3}, \frac{19}{3}\}, \{13, \frac{22}{3}, \frac{22}{3}\},$$
$$\{14, \frac{22}{3}, \frac{22}{3}\}, \{15, \frac{22}{3}, \frac{22}{3}\}, \{16, \frac{91}{12}, \frac{91}{12}\}, \{17, \frac{103}{12}, \frac{103}{12}\}, \{18, \frac{103}{12}, \frac{103}{12}\},$$
$$\{19, \frac{115}{12}, \frac{115}{12}\}, \{20, \frac{115}{12}, \frac{115}{12}\}, \{21, \frac{115}{12}, \frac{115}{12}\}, \{22, \frac{115}{12}, \frac{115}{12}\}, \{23, \frac{127}{12}, \frac{127}{12}\},$$
$$\{24, \frac{127}{12}, \frac{127}{12}\}, \{25, \frac{133}{12}, \frac{133}{12}\}, \{26, \frac{133}{12}, \frac{133}{12}\}, \{27, \frac{137}{12}, \frac{137}{12}\}, \{28, \frac{137}{12}, \frac{137}{12}\},$$
$$\{29, \frac{149}{12}, \frac{149}{12}\}, \{30, \frac{149}{12}, \frac{149}{12}\}, \{31, \frac{161}{12}, \frac{161}{12}\}, \{32, \frac{817}{60}, \frac{817}{60}\}, \{33, \frac{817}{60}, \frac{817}{60}\},$$
$$\{34, \frac{817}{60}, \frac{817}{60}\}, \{35, \frac{817}{60}, \frac{817}{60}\}, \{36, \frac{817}{60}, \frac{817}{60}\}, \{37, \frac{877}{60}, \frac{877}{60}\}, \{38, \frac{877}{60}, \frac{877}{60}\},$$
$$\{39, \frac{877}{60}, \frac{877}{60}\}, \{40, \frac{877}{60}, \frac{877}{60}\}, \{41, \frac{937}{60}, \frac{937}{60}\}, \{42, \frac{937}{60}, \frac{937}{60}\}, \{43, \frac{997}{60}, \frac{997}{60}\},$$
$$\{44, \frac{997}{60}, \frac{997}{60}\}, \{45, \frac{997}{60}, \frac{997}{60}\}, \{46, \frac{997}{60}, \frac{997}{60}\}, \{47, \frac{1057}{60}, \frac{1057}{60}\}, \{48, \frac{1057}{60}, \frac{1057}{60}\},$$
$$\{49, \frac{1087}{60}, \frac{1087}{60}\}, \{50, \frac{1087}{60}, \frac{1087}{60}\}, \{51, \frac{1087}{60}, \frac{1087}{60}\}, \{52, \frac{1087}{60}, \frac{1087}{60}\},$$
$$\{53, \frac{1147}{60}, \frac{1147}{60}\}, \{54, \frac{1147}{60}, \frac{1147}{60}\}, \{55, \frac{1147}{60}, \frac{1147}{60}\}, \{56, \frac{1147}{60}, \frac{1147}{60}\},$$
$$\{57, \frac{1147}{60}, \frac{1147}{60}\}, \{58, \frac{1147}{60}, \frac{1147}{60}\}, \{59, \frac{1207}{60}, \frac{1207}{60}\}, \{60, \frac{1207}{60}, \frac{1207}{60}\},$$
$$\{61, \frac{1267}{60}, \frac{1267}{60}\}, \{62, \frac{1267}{60}, \frac{1267}{60}\}, \{63, \frac{1267}{60}, \frac{1267}{60}\}, \{64, \frac{1277}{60}, \frac{1277}{60}\},$$
$$\{65, \frac{1277}{60}, \frac{1277}{60}\}, \{66, \frac{1277}{60}, \frac{1277}{60}\}, \{67, \frac{1337}{60}, \frac{1337}{60}\}, \{68, \frac{1337}{60}, \frac{1337}{60}\},$$
$$\{69, \frac{1337}{60}, \frac{1337}{60}\}, \{70, \frac{1337}{60}, \frac{1337}{60}\}, \{71, \frac{1397}{60}, \frac{1397}{60}\}, \{72, \frac{1397}{60}, \frac{1397}{60}\},$$
$$\{73, \frac{1457}{60}, \frac{1457}{60}\}, \{74, \frac{1457}{60}, \frac{1457}{60}\}, \{75, \frac{1457}{60}, \frac{1457}{60}\}, \{76, \frac{1457}{60}, \frac{1457}{60}\},$$
$$\{77, \frac{1457}{60}, \frac{1457}{60}\}, \{78, \frac{1457}{60}, \frac{1457}{60}\}, \{79, \frac{1517}{60}, \frac{1517}{60}\}, \{80, \frac{1517}{60}, \frac{1517}{60}\},$$
$$\{81, \frac{383}{15}, \frac{383}{15}\}, \{82, \frac{383}{15}, \frac{383}{15}\}, \{83, \frac{398}{15}, \frac{398}{15}\}, \{84, \frac{398}{15}, \frac{398}{15}\}, \{85, \frac{398}{15}, \frac{398}{15}\},$$
$$\{86, \frac{398}{15}, \frac{398}{15}\}, \{87, \frac{398}{15}, \frac{398}{15}\}, \{88, \frac{398}{15}, \frac{398}{15}\}, \{89, \frac{413}{15}, \frac{413}{15}\}, \{90, \frac{413}{15}, \frac{413}{15}\},$$
$$\{91, \frac{413}{15}, \frac{413}{15}\}, \{92, \frac{413}{15}, \frac{413}{15}\}, \{93, \frac{413}{15}, \frac{413}{15}\}, \{94, \frac{413}{15}, \frac{413}{15}\}, \{95, \frac{413}{15}, \frac{413}{15}\},$$
$$\{96, \frac{413}{15}, \frac{413}{15}\}, \{97, \frac{428}{15}, \frac{428}{15}\}, \{98, \frac{428}{15}, \frac{428}{15}\}, \{99, \frac{428}{15}, \frac{428}{15}\}, \{100, \frac{428}{15}, \frac{428}{15}\}\}$$

```
D2Cache[n_, k_, s_] :=
 Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1 / k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]

LinnikSumFast[n_] := D2Cache[n, 1, 2] +
  Sum[( Sum[(-1)^(k + 1) / k D2Cache[n / j, k - 1, 2], {j, Floor[n^(1 / 3)] + 1, n^(1 / 2)}]),
   {k, 2, Log[2, n]}] +
  Sum[( Sum[(-1)^(k + 1) / k (Floor[n / j] - (Floor[n / (j + 1)])) D2Cache[j, k - 1, 2],
     {j, 1, n / Floor[n^(1 / 2)] - 1}]), {k, 2, Log[2, n]}] +
  Sum[( Sum[(-1)^(k + 1) / k d2cache[j, k - 1] (Floor[n / j] - 1), {j, 2, n^(1 / 3)}]),
   {k, 2, Log[2, n]}] +
  Sum[( Sum[(-1)^(k + 1) / k d2cache[j, m] D2Cache[n / (j s), k - m - 1, 2],
     {j, 2, n^(1 / 3)}, {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[n / j]^(1 / 2)},
     {m, 1, k - 2}]), {k, 2, Log[2, n]}] +
  Sum[( Sum[(-1)^(k + 1) / k ( Floor[n / (j s)] - Floor[n / (j (s + 1))])
      (Sum[d2cache[j, m] D2Cache[s, k - m - 1, 2], {m, 1, k - 2}]), {j, 2, n^(1 / 3)},
     {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]), {k, 2, Log[2, n]}]
RiePrimeCnt[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Table[{n, LinnikSumFast[n], RiePrimeCnt[n]}, {n, 1, 100}]
```

$$\{\{1, 0, 0\}, \{2, 1, 1\}, \{3, 2, 2\}, \{4, \frac{5}{2}, \frac{5}{2}\}, \{5, \frac{7}{2}, \frac{7}{2}\}, \{6, \frac{7}{2}, \frac{7}{2}\}, \{7, \frac{9}{2}, \frac{9}{2}\},$$
$$\{8, \frac{29}{6}, \frac{29}{6}\}, \{9, \frac{16}{3}, \frac{16}{3}\}, \{10, \frac{16}{3}, \frac{16}{3}\}, \{11, \frac{19}{3}, \frac{19}{3}\}, \{12, \frac{19}{3}, \frac{19}{3}\}, \{13, \frac{22}{3}, \frac{22}{3}\},$$
$$\{14, \frac{22}{3}, \frac{22}{3}\}, \{15, \frac{22}{3}, \frac{22}{3}\}, \{16, \frac{91}{12}, \frac{91}{12}\}, \{17, \frac{103}{12}, \frac{103}{12}\}, \{18, \frac{103}{12}, \frac{103}{12}\},$$
$$\{19, \frac{115}{12}, \frac{115}{12}\}, \{20, \frac{115}{12}, \frac{115}{12}\}, \{21, \frac{115}{12}, \frac{115}{12}\}, \{22, \frac{115}{12}, \frac{115}{12}\}, \{23, \frac{127}{12}, \frac{127}{12}\},$$
$$\{24, \frac{127}{12}, \frac{127}{12}\}, \{25, \frac{133}{12}, \frac{133}{12}\}, \{26, \frac{133}{12}, \frac{133}{12}\}, \{27, \frac{137}{12}, \frac{137}{12}\}, \{28, \frac{137}{12}, \frac{137}{12}\},$$
$$\{29, \frac{149}{12}, \frac{149}{12}\}, \{30, \frac{149}{12}, \frac{149}{12}\}, \{31, \frac{161}{12}, \frac{161}{12}\}, \{32, \frac{817}{60}, \frac{817}{60}\}, \{33, \frac{817}{60}, \frac{817}{60}\},$$
$$\{34, \frac{817}{60}, \frac{817}{60}\}, \{35, \frac{817}{60}, \frac{817}{60}\}, \{36, \frac{817}{60}, \frac{817}{60}\}, \{37, \frac{877}{60}, \frac{877}{60}\}, \{38, \frac{877}{60}, \frac{877}{60}\},$$
$$\{39, \frac{877}{60}, \frac{877}{60}\}, \{40, \frac{877}{60}, \frac{877}{60}\}, \{41, \frac{937}{60}, \frac{937}{60}\}, \{42, \frac{937}{60}, \frac{937}{60}\}, \{43, \frac{997}{60}, \frac{997}{60}\},$$
$$\{44, \frac{997}{60}, \frac{997}{60}\}, \{45, \frac{997}{60}, \frac{997}{60}\}, \{46, \frac{997}{60}, \frac{997}{60}\}, \{47, \frac{1057}{60}, \frac{1057}{60}\}, \{48, \frac{1057}{60}, \frac{1057}{60}\},$$
$$\{49, \frac{1087}{60}, \frac{1087}{60}\}, \{50, \frac{1087}{60}, \frac{1087}{60}\}, \{51, \frac{1087}{60}, \frac{1087}{60}\}, \{52, \frac{1087}{60}, \frac{1087}{60}\},$$
$$\{53, \frac{1147}{60}, \frac{1147}{60}\}, \{54, \frac{1147}{60}, \frac{1147}{60}\}, \{55, \frac{1147}{60}, \frac{1147}{60}\}, \{56, \frac{1147}{60}, \frac{1147}{60}\},$$
$$\{57, \frac{1147}{60}, \frac{1147}{60}\}, \{58, \frac{1147}{60}, \frac{1147}{60}\}, \{59, \frac{1207}{60}, \frac{1207}{60}\}, \{60, \frac{1207}{60}, \frac{1207}{60}\},$$
$$\{61, \frac{1267}{60}, \frac{1267}{60}\}, \{62, \frac{1267}{60}, \frac{1267}{60}\}, \{63, \frac{1267}{60}, \frac{1267}{60}\}, \{64, \frac{1277}{60}, \frac{1277}{60}\},$$
$$\{65, \frac{1277}{60}, \frac{1277}{60}\}, \{66, \frac{1277}{60}, \frac{1277}{60}\}, \{67, \frac{1337}{60}, \frac{1337}{60}\}, \{68, \frac{1337}{60}, \frac{1337}{60}\},$$
$$\{69, \frac{1337}{60}, \frac{1337}{60}\}, \{70, \frac{1337}{60}, \frac{1337}{60}\}, \{71, \frac{1397}{60}, \frac{1397}{60}\}, \{72, \frac{1397}{60}, \frac{1397}{60}\},$$
$$\{73, \frac{1457}{60}, \frac{1457}{60}\}, \{74, \frac{1457}{60}, \frac{1457}{60}\}, \{75, \frac{1457}{60}, \frac{1457}{60}\}, \{76, \frac{1457}{60}, \frac{1457}{60}\},$$
$$\{77, \frac{1457}{60}, \frac{1457}{60}\}, \{78, \frac{1457}{60}, \frac{1457}{60}\}, \{79, \frac{1517}{60}, \frac{1517}{60}\}, \{80, \frac{1517}{60}, \frac{1517}{60}\},$$
$$\{81, \frac{383}{15}, \frac{383}{15}\}, \{82, \frac{383}{15}, \frac{383}{15}\}, \{83, \frac{398}{15}, \frac{398}{15}\}, \{84, \frac{398}{15}, \frac{398}{15}\}, \{85, \frac{398}{15}, \frac{398}{15}\},$$
$$\{86, \frac{398}{15}, \frac{398}{15}\}, \{87, \frac{398}{15}, \frac{398}{15}\}, \{88, \frac{398}{15}, \frac{398}{15}\}, \{89, \frac{413}{15}, \frac{413}{15}\}, \{90, \frac{413}{15}, \frac{413}{15}\},$$
$$\{91, \frac{413}{15}, \frac{413}{15}\}, \{92, \frac{413}{15}, \frac{413}{15}\}, \{93, \frac{413}{15}, \frac{413}{15}\}, \{94, \frac{413}{15}, \frac{413}{15}\}, \{95, \frac{413}{15}, \frac{413}{15}\},$$
$$\{96, \frac{413}{15}, \frac{413}{15}\}, \{97, \frac{428}{15}, \frac{428}{15}\}, \{98, \frac{428}{15}, \frac{428}{15}\}, \{99, \frac{428}{15}, \frac{428}{15}\}, \{100, \frac{428}{15}, \frac{428}{15}\}\}$$

```
D2Cache[n_, k_, s_] :=
 Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1 / k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]

LinnikSumFast[n_] := D2Cache[n, 1, 2] +
  ( Sum[(-1)^(k + 1) / k D2Cache[n / j, k - 1, 2],
    {j, Floor[n^(1 / 3)] + 1, n^(1 / 2)}, {k, 2, Log[2, n]}]) +
  ( Sum[(-1)^(k + 1) / k (Floor[n / j] - (Floor[n / (j + 1)])) D2Cache[j, k - 1, 2],
    {j, 1, n / Floor[n^(1 / 2)] - 1}, {k, 2, Log[2, n]}]) +
  ( Sum[(-1)^(k + 1) / k d2cache[j, k - 1] (Floor[n / j] - 1),
    {j, 2, n^(1 / 3)}, {k, 2, Log[2, n]}]) +
  Sum[( Sum[(-1)^(k + 1) / k d2cache[j, m] D2Cache[n / (j s), k - m - 1, 2],
     {j, 2, n^(1 / 3)}, {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[n / j]^(1 / 2)},
     {m, 1, k - 2}]), {k, 2, Log[2, n]}] +
  Sum[( Sum[(-1)^(k + 1) / k ( Floor[n / (j s)] - Floor[n / (j (s + 1))])
      (Sum[d2cache[j, m] D2Cache[s, k - m - 1, 2], {m, 1, k - 2}]), {j, 2, n^(1 / 3)},
     {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]), {k, 2, Log[2, n]}]
RiePrimeCnt[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Table[{n, LinnikSumFast[n], RiePrimeCnt[n]}, {n, 1, 100}]
```

$$\{\{1, 0, 0\}, \{2, 1, 1\}, \{3, 2, 2\}, \{4, \frac{5}{2}, \frac{5}{2}\}, \{5, \frac{7}{2}, \frac{7}{2}\}, \{6, \frac{7}{2}, \frac{7}{2}\}, \{7, \frac{9}{2}, \frac{9}{2}\},$$
$$\{8, \frac{29}{6}, \frac{29}{6}\}, \{9, \frac{16}{3}, \frac{16}{3}\}, \{10, \frac{16}{3}, \frac{16}{3}\}, \{11, \frac{19}{3}, \frac{19}{3}\}, \{12, \frac{19}{3}, \frac{19}{3}\}, \{13, \frac{22}{3}, \frac{22}{3}\},$$
$$\{14, \frac{22}{3}, \frac{22}{3}\}, \{15, \frac{22}{3}, \frac{22}{3}\}, \{16, \frac{91}{12}, \frac{91}{12}\}, \{17, \frac{103}{12}, \frac{103}{12}\}, \{18, \frac{103}{12}, \frac{103}{12}\},$$
$$\{19, \frac{115}{12}, \frac{115}{12}\}, \{20, \frac{115}{12}, \frac{115}{12}\}, \{21, \frac{115}{12}, \frac{115}{12}\}, \{22, \frac{115}{12}, \frac{115}{12}\}, \{23, \frac{127}{12}, \frac{127}{12}\},$$
$$\{24, \frac{127}{12}, \frac{127}{12}\}, \{25, \frac{133}{12}, \frac{133}{12}\}, \{26, \frac{133}{12}, \frac{133}{12}\}, \{27, \frac{137}{12}, \frac{137}{12}\}, \{28, \frac{137}{12}, \frac{137}{12}\},$$
$$\{29, \frac{149}{12}, \frac{149}{12}\}, \{30, \frac{149}{12}, \frac{149}{12}\}, \{31, \frac{161}{12}, \frac{161}{12}\}, \{32, \frac{817}{60}, \frac{817}{60}\}, \{33, \frac{817}{60}, \frac{817}{60}\},$$
$$\{34, \frac{817}{60}, \frac{817}{60}\}, \{35, \frac{817}{60}, \frac{817}{60}\}, \{36, \frac{817}{60}, \frac{817}{60}\}, \{37, \frac{877}{60}, \frac{877}{60}\}, \{38, \frac{877}{60}, \frac{877}{60}\},$$
$$\{39, \frac{877}{60}, \frac{877}{60}\}, \{40, \frac{877}{60}, \frac{877}{60}\}, \{41, \frac{937}{60}, \frac{937}{60}\}, \{42, \frac{937}{60}, \frac{937}{60}\}, \{43, \frac{997}{60}, \frac{997}{60}\},$$
$$\{44, \frac{997}{60}, \frac{997}{60}\}, \{45, \frac{997}{60}, \frac{997}{60}\}, \{46, \frac{997}{60}, \frac{997}{60}\}, \{47, \frac{1057}{60}, \frac{1057}{60}\}, \{48, \frac{1057}{60}, \frac{1057}{60}\},$$
$$\{49, \frac{1087}{60}, \frac{1087}{60}\}, \{50, \frac{1087}{60}, \frac{1087}{60}\}, \{51, \frac{1087}{60}, \frac{1087}{60}\}, \{52, \frac{1087}{60}, \frac{1087}{60}\},$$
$$\{53, \frac{1147}{60}, \frac{1147}{60}\}, \{54, \frac{1147}{60}, \frac{1147}{60}\}, \{55, \frac{1147}{60}, \frac{1147}{60}\}, \{56, \frac{1147}{60}, \frac{1147}{60}\},$$
$$\{57, \frac{1147}{60}, \frac{1147}{60}\}, \{58, \frac{1147}{60}, \frac{1147}{60}\}, \{59, \frac{1207}{60}, \frac{1207}{60}\}, \{60, \frac{1207}{60}, \frac{1207}{60}\},$$
$$\{61, \frac{1267}{60}, \frac{1267}{60}\}, \{62, \frac{1267}{60}, \frac{1267}{60}\}, \{63, \frac{1267}{60}, \frac{1267}{60}\}, \{64, \frac{1277}{60}, \frac{1277}{60}\},$$
$$\{65, \frac{1277}{60}, \frac{1277}{60}\}, \{66, \frac{1277}{60}, \frac{1277}{60}\}, \{67, \frac{1337}{60}, \frac{1337}{60}\}, \{68, \frac{1337}{60}, \frac{1337}{60}\},$$
$$\{69, \frac{1337}{60}, \frac{1337}{60}\}, \{70, \frac{1337}{60}, \frac{1337}{60}\}, \{71, \frac{1397}{60}, \frac{1397}{60}\}, \{72, \frac{1397}{60}, \frac{1397}{60}\},$$
$$\{73, \frac{1457}{60}, \frac{1457}{60}\}, \{74, \frac{1457}{60}, \frac{1457}{60}\}, \{75, \frac{1457}{60}, \frac{1457}{60}\}, \{76, \frac{1457}{60}, \frac{1457}{60}\},$$
$$\{77, \frac{1457}{60}, \frac{1457}{60}\}, \{78, \frac{1457}{60}, \frac{1457}{60}\}, \{79, \frac{1517}{60}, \frac{1517}{60}\}, \{80, \frac{1517}{60}, \frac{1517}{60}\},$$
$$\{81, \frac{383}{15}, \frac{383}{15}\}, \{82, \frac{383}{15}, \frac{383}{15}\}, \{83, \frac{398}{15}, \frac{398}{15}\}, \{84, \frac{398}{15}, \frac{398}{15}\}, \{85, \frac{398}{15}, \frac{398}{15}\},$$
$$\{86, \frac{398}{15}, \frac{398}{15}\}, \{87, \frac{398}{15}, \frac{398}{15}\}, \{88, \frac{398}{15}, \frac{398}{15}\}, \{89, \frac{413}{15}, \frac{413}{15}\}, \{90, \frac{413}{15}, \frac{413}{15}\},$$
$$\{91, \frac{413}{15}, \frac{413}{15}\}, \{92, \frac{413}{15}, \frac{413}{15}\}, \{93, \frac{413}{15}, \frac{413}{15}\}, \{94, \frac{413}{15}, \frac{413}{15}\}, \{95, \frac{413}{15}, \frac{413}{15}\},$$
$$\{96, \frac{413}{15}, \frac{413}{15}\}, \{97, \frac{428}{15}, \frac{428}{15}\}, \{98, \frac{428}{15}, \frac{428}{15}\}, \{99, \frac{428}{15}, \frac{428}{15}\}, \{100, \frac{428}{15}, \frac{428}{15}\}\}$$

```
D2Cache[n_, k_, s_] :=
 Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1 / k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]

LinnikSumFast[n_] := D2Cache[n, 1, 2] +
  ( Sum[(-1)^(k + 1) / k D2Cache[n / j, k - 1, 2],
    {j, Floor[n^(1 / 3)] + 1, n^(1 / 2)}, {k, 2, Log[2, n]}]) +
  ( Sum[(-1)^(k + 1) / k (Floor[n / j] - (Floor[n / (j + 1)])) D2Cache[j, k - 1, 2],
    {j, 1, n / Floor[n^(1 / 2)] - 1}, {k, 2, Log[2, n]}]) +
  ( Sum[(-1)^(k + 1) / k d2cache[j, k - 1] (Floor[n / j] - 1),
    {j, 2, n^(1 / 3)}, {k, 2, Log[2, n]}]) +
  ( Sum[(-1)^(k + 1) / k d2cache[j, m] D2Cache[n / (j s), k - m - 1, 2], {j, 2, n^(1 / 3)}, {s,
     Floor[Floor[n^(1 / 3)] / j] + 1, Floor[n / j]^(1 / 2)}, {k, 2, Log[2, n]}, {m, 1, k - 2}]) +
  ( Sum[(-1)^(k + 1) / k ( Floor[n / (j s)] - Floor[n / (j (s + 1))])
     (Sum[d2cache[j, m] D2Cache[s, k - m - 1, 2], {m, 1, k - 2}]), {j, 2, n^(1 / 3)},
    {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}, {k, 2, Log[2, n]}])
RiePrimeCnt[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Table[{n, LinnikSumFast[n], RiePrimeCnt[n]}, {n, 1, 100}]
```

$$\{\{1, 0, 0\}, \{2, 1, 1\}, \{3, 2, 2\}, \{4, \frac{5}{2}, \frac{5}{2}\}, \{5, \frac{7}{2}, \frac{7}{2}\}, \{6, \frac{7}{2}, \frac{7}{2}\}, \{7, \frac{9}{2}, \frac{9}{2}\},$$
$$\{8, \frac{29}{6}, \frac{29}{6}\}, \{9, \frac{16}{3}, \frac{16}{3}\}, \{10, \frac{16}{3}, \frac{16}{3}\}, \{11, \frac{19}{3}, \frac{19}{3}\}, \{12, \frac{19}{3}, \frac{19}{3}\}, \{13, \frac{22}{3}, \frac{22}{3}\},$$
$$\{14, \frac{22}{3}, \frac{22}{3}\}, \{15, \frac{22}{3}, \frac{22}{3}\}, \{16, \frac{91}{12}, \frac{91}{12}\}, \{17, \frac{103}{12}, \frac{103}{12}\}, \{18, \frac{103}{12}, \frac{103}{12}\},$$
$$\{19, \frac{115}{12}, \frac{115}{12}\}, \{20, \frac{115}{12}, \frac{115}{12}\}, \{21, \frac{115}{12}, \frac{115}{12}\}, \{22, \frac{115}{12}, \frac{115}{12}\}, \{23, \frac{127}{12}, \frac{127}{12}\},$$
$$\{24, \frac{127}{12}, \frac{127}{12}\}, \{25, \frac{133}{12}, \frac{133}{12}\}, \{26, \frac{133}{12}, \frac{133}{12}\}, \{27, \frac{137}{12}, \frac{137}{12}\}, \{28, \frac{137}{12}, \frac{137}{12}\},$$
$$\{29, \frac{149}{12}, \frac{149}{12}\}, \{30, \frac{149}{12}, \frac{149}{12}\}, \{31, \frac{161}{12}, \frac{161}{12}\}, \{32, \frac{817}{60}, \frac{817}{60}\}, \{33, \frac{817}{60}, \frac{817}{60}\},$$
$$\{34, \frac{817}{60}, \frac{817}{60}\}, \{35, \frac{817}{60}, \frac{817}{60}\}, \{36, \frac{817}{60}, \frac{817}{60}\}, \{37, \frac{877}{60}, \frac{877}{60}\}, \{38, \frac{877}{60}, \frac{877}{60}\},$$
$$\{39, \frac{877}{60}, \frac{877}{60}\}, \{40, \frac{877}{60}, \frac{877}{60}\}, \{41, \frac{937}{60}, \frac{937}{60}\}, \{42, \frac{937}{60}, \frac{937}{60}\}, \{43, \frac{997}{60}, \frac{997}{60}\},$$
$$\{44, \frac{997}{60}, \frac{997}{60}\}, \{45, \frac{997}{60}, \frac{997}{60}\}, \{46, \frac{997}{60}, \frac{997}{60}\}, \{47, \frac{1057}{60}, \frac{1057}{60}\}, \{48, \frac{1057}{60}, \frac{1057}{60}\},$$
$$\{49, \frac{1087}{60}, \frac{1087}{60}\}, \{50, \frac{1087}{60}, \frac{1087}{60}\}, \{51, \frac{1087}{60}, \frac{1087}{60}\}, \{52, \frac{1087}{60}, \frac{1087}{60}\},$$
$$\{53, \frac{1147}{60}, \frac{1147}{60}\}, \{54, \frac{1147}{60}, \frac{1147}{60}\}, \{55, \frac{1147}{60}, \frac{1147}{60}\}, \{56, \frac{1147}{60}, \frac{1147}{60}\},$$
$$\{57, \frac{1147}{60}, \frac{1147}{60}\}, \{58, \frac{1147}{60}, \frac{1147}{60}\}, \{59, \frac{1207}{60}, \frac{1207}{60}\}, \{60, \frac{1207}{60}, \frac{1207}{60}\},$$
$$\{61, \frac{1267}{60}, \frac{1267}{60}\}, \{62, \frac{1267}{60}, \frac{1267}{60}\}, \{63, \frac{1267}{60}, \frac{1267}{60}\}, \{64, \frac{1277}{60}, \frac{1277}{60}\},$$
$$\{65, \frac{1277}{60}, \frac{1277}{60}\}, \{66, \frac{1277}{60}, \frac{1277}{60}\}, \{67, \frac{1337}{60}, \frac{1337}{60}\}, \{68, \frac{1337}{60}, \frac{1337}{60}\},$$
$$\{69, \frac{1337}{60}, \frac{1337}{60}\}, \{70, \frac{1337}{60}, \frac{1337}{60}\}, \{71, \frac{1397}{60}, \frac{1397}{60}\}, \{72, \frac{1397}{60}, \frac{1397}{60}\},$$
$$\{73, \frac{1457}{60}, \frac{1457}{60}\}, \{74, \frac{1457}{60}, \frac{1457}{60}\}, \{75, \frac{1457}{60}, \frac{1457}{60}\}, \{76, \frac{1457}{60}, \frac{1457}{60}\},$$
$$\{77, \frac{1457}{60}, \frac{1457}{60}\}, \{78, \frac{1457}{60}, \frac{1457}{60}\}, \{79, \frac{1517}{60}, \frac{1517}{60}\}, \{80, \frac{1517}{60}, \frac{1517}{60}\},$$
$$\{81, \frac{383}{15}, \frac{383}{15}\}, \{82, \frac{383}{15}, \frac{383}{15}\}, \{83, \frac{398}{15}, \frac{398}{15}\}, \{84, \frac{398}{15}, \frac{398}{15}\}, \{85, \frac{398}{15}, \frac{398}{15}\},$$
$$\{86, \frac{398}{15}, \frac{398}{15}\}, \{87, \frac{398}{15}, \frac{398}{15}\}, \{88, \frac{398}{15}, \frac{398}{15}\}, \{89, \frac{413}{15}, \frac{413}{15}\}, \{90, \frac{413}{15}, \frac{413}{15}\},$$
$$\{91, \frac{413}{15}, \frac{413}{15}\}, \{92, \frac{413}{15}, \frac{413}{15}\}, \{93, \frac{413}{15}, \frac{413}{15}\}, \{94, \frac{413}{15}, \frac{413}{15}\}, \{95, \frac{413}{15}, \frac{413}{15}\},$$
$$\{96, \frac{413}{15}, \frac{413}{15}\}, \{97, \frac{428}{15}, \frac{428}{15}\}, \{98, \frac{428}{15}, \frac{428}{15}\}, \{99, \frac{428}{15}, \frac{428}{15}\}, \{100, \frac{428}{15}, \frac{428}{15}\}\}$$

```
D2Cache[n_, k_, s_] :=
 Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1 / k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]

LinnikSumFast[n_] := D2Cache[n, 1, 2] +
  ( Sum[(-1)^(k + 1) / k D2Cache[n / j, k - 1, 2],
    {j, Floor[n^(1 / 3)] + 1, n^(1 / 2)}, {k, 2, Log[2, n]}]) +
  ( Sum[(-1)^(k + 1) / k (Floor[n / j] - (Floor[n / (j + 1)])) D2Cache[j, k - 1, 2],
    {j, 1, n / Floor[n^(1 / 2)] - 1}, {k, 2, Log[2, n]}]) +
  ( Sum[(-1)^(k + 1) / k d2cache[j, k - 1] (Floor[n / j] - 1),
    {j, 2, n^(1 / 3)}, {k, 2, Log[2, n]}]) +
  ( Sum[(-1)^(k + 1) / k d2cache[j, m] D2Cache[n / (j s), k - m - 1, 2], {j, 2, n^(1 / 3)}, {s,
     Floor[Floor[n^(1 / 3)] / j] + 1, Floor[n / j]^(1 / 2)}, {k, 2, Log[2, n]}, {m, 1, k - 2}]) +
  ( Sum[(-1)^(k + 1) / k ( Floor[n / (j s)] - Floor[n / (j (s + 1))])
     (Sum[d2cache[j, m] D2Cache[s, k - m - 1, 2], {m, 1, k - 2}]), {j, 2, n^(1 / 3)},
    {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}, {k, 2, Log[2, n]}])
RiePrimeCnt[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Table[{n, LinnikSumFast[n], RiePrimeCnt[n]}, {n, 1, 100}]
```

$$\{\{1, 0, 0\}, \{2, 1, 1\}, \{3, 2, 2\}, \{4, \frac{5}{2}, \frac{5}{2}\}, \{5, \frac{7}{2}, \frac{7}{2}\}, \{6, \frac{7}{2}, \frac{7}{2}\}, \{7, \frac{9}{2}, \frac{9}{2}\},$$
$$\{8, \frac{29}{6}, \frac{29}{6}\}, \{9, \frac{16}{3}, \frac{16}{3}\}, \{10, \frac{16}{3}, \frac{16}{3}\}, \{11, \frac{19}{3}, \frac{19}{3}\}, \{12, \frac{19}{3}, \frac{19}{3}\}, \{13, \frac{22}{3}, \frac{22}{3}\},$$
$$\{14, \frac{22}{3}, \frac{22}{3}\}, \{15, \frac{22}{3}, \frac{22}{3}\}, \{16, \frac{91}{12}, \frac{91}{12}\}, \{17, \frac{103}{12}, \frac{103}{12}\}, \{18, \frac{103}{12}, \frac{103}{12}\},$$
$$\{19, \frac{115}{12}, \frac{115}{12}\}, \{20, \frac{115}{12}, \frac{115}{12}\}, \{21, \frac{115}{12}, \frac{115}{12}\}, \{22, \frac{115}{12}, \frac{115}{12}\}, \{23, \frac{127}{12}, \frac{127}{12}\},$$
$$\{24, \frac{127}{12}, \frac{127}{12}\}, \{25, \frac{133}{12}, \frac{133}{12}\}, \{26, \frac{133}{12}, \frac{133}{12}\}, \{27, \frac{137}{12}, \frac{137}{12}\}, \{28, \frac{137}{12}, \frac{137}{12}\},$$
$$\{29, \frac{149}{12}, \frac{149}{12}\}, \{30, \frac{149}{12}, \frac{149}{12}\}, \{31, \frac{161}{12}, \frac{161}{12}\}, \{32, \frac{817}{60}, \frac{817}{60}\}, \{33, \frac{817}{60}, \frac{817}{60}\},$$
$$\{34, \frac{817}{60}, \frac{817}{60}\}, \{35, \frac{817}{60}, \frac{817}{60}\}, \{36, \frac{817}{60}, \frac{817}{60}\}, \{37, \frac{877}{60}, \frac{877}{60}\}, \{38, \frac{877}{60}, \frac{877}{60}\},$$
$$\{39, \frac{877}{60}, \frac{877}{60}\}, \{40, \frac{877}{60}, \frac{877}{60}\}, \{41, \frac{937}{60}, \frac{937}{60}\}, \{42, \frac{937}{60}, \frac{937}{60}\}, \{43, \frac{997}{60}, \frac{997}{60}\},$$
$$\{44, \frac{997}{60}, \frac{997}{60}\}, \{45, \frac{997}{60}, \frac{997}{60}\}, \{46, \frac{997}{60}, \frac{997}{60}\}, \{47, \frac{1057}{60}, \frac{1057}{60}\}, \{48, \frac{1057}{60}, \frac{1057}{60}\},$$
$$\{49, \frac{1087}{60}, \frac{1087}{60}\}, \{50, \frac{1087}{60}, \frac{1087}{60}\}, \{51, \frac{1087}{60}, \frac{1087}{60}\}, \{52, \frac{1087}{60}, \frac{1087}{60}\},$$
$$\{53, \frac{1147}{60}, \frac{1147}{60}\}, \{54, \frac{1147}{60}, \frac{1147}{60}\}, \{55, \frac{1147}{60}, \frac{1147}{60}\}, \{56, \frac{1147}{60}, \frac{1147}{60}\},$$
$$\{57, \frac{1147}{60}, \frac{1147}{60}\}, \{58, \frac{1147}{60}, \frac{1147}{60}\}, \{59, \frac{1207}{60}, \frac{1207}{60}\}, \{60, \frac{1207}{60}, \frac{1207}{60}\},$$
$$\{61, \frac{1267}{60}, \frac{1267}{60}\}, \{62, \frac{1267}{60}, \frac{1267}{60}\}, \{63, \frac{1267}{60}, \frac{1267}{60}\}, \{64, \frac{1277}{60}, \frac{1277}{60}\},$$
$$\{65, \frac{1277}{60}, \frac{1277}{60}\}, \{66, \frac{1277}{60}, \frac{1277}{60}\}, \{67, \frac{1337}{60}, \frac{1337}{60}\}, \{68, \frac{1337}{60}, \frac{1337}{60}\},$$
$$\{69, \frac{1337}{60}, \frac{1337}{60}\}, \{70, \frac{1337}{60}, \frac{1337}{60}\}, \{71, \frac{1397}{60}, \frac{1397}{60}\}, \{72, \frac{1397}{60}, \frac{1397}{60}\},$$
$$\{73, \frac{1457}{60}, \frac{1457}{60}\}, \{74, \frac{1457}{60}, \frac{1457}{60}\}, \{75, \frac{1457}{60}, \frac{1457}{60}\}, \{76, \frac{1457}{60}, \frac{1457}{60}\},$$
$$\{77, \frac{1457}{60}, \frac{1457}{60}\}, \{78, \frac{1457}{60}, \frac{1457}{60}\}, \{79, \frac{1517}{60}, \frac{1517}{60}\}, \{80, \frac{1517}{60}, \frac{1517}{60}\},$$
$$\{81, \frac{383}{15}, \frac{383}{15}\}, \{82, \frac{383}{15}, \frac{383}{15}\}, \{83, \frac{398}{15}, \frac{398}{15}\}, \{84, \frac{398}{15}, \frac{398}{15}\}, \{85, \frac{398}{15}, \frac{398}{15}\},$$
$$\{86, \frac{398}{15}, \frac{398}{15}\}, \{87, \frac{398}{15}, \frac{398}{15}\}, \{88, \frac{398}{15}, \frac{398}{15}\}, \{89, \frac{413}{15}, \frac{413}{15}\}, \{90, \frac{413}{15}, \frac{413}{15}\},$$
$$\{91, \frac{413}{15}, \frac{413}{15}\}, \{92, \frac{413}{15}, \frac{413}{15}\}, \{93, \frac{413}{15}, \frac{413}{15}\}, \{94, \frac{413}{15}, \frac{413}{15}\}, \{95, \frac{413}{15}, \frac{413}{15}\},$$
$$\{96, \frac{413}{15}, \frac{413}{15}\}, \{97, \frac{428}{15}, \frac{428}{15}\}, \{98, \frac{428}{15}, \frac{428}{15}\}, \{99, \frac{428}{15}, \frac{428}{15}\}, \{100, \frac{428}{15}, \frac{428}{15}\}\}$$

```
ClearAll["Global`*"]
d[n_, z_] := Product[Pochhammer[z, p[[2]]] / p[[2]]!, {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
dCache := dCache = Table[ d[n, z], {n, 1, 500}, {z, 0, Log[2, 500]}]
d2[n_, k_] := Sum[(-1)^(k - j) Binomial[k, j] dCache[[n]][[j + 1]], {j, 0, k}]
d2Cache := d2Cache = Table[ d2[n, z], {n, 1, 500}, {z, 1, Log[2, 500]}]
D2[n_, k_] := D2[n - 1, k] + d2Cache[[n]][[k]]; D2[1, k_] := 0
D2Cache := D2Cache = Table[ D2[n, z], {n, 1, 500}, {z, 1, Log[2, 500]}]
```

```
D2[100, 2]
```

324

```
D2Cache[[99]]
```

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

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General::stop : Further output of $RecursionLimit::reclim will be suppressed during this calculation. ≫

{98, 276, 312, 178, 51, 7, 0, 0}

```
8 - 3
```

5