```
Sum[ (-1)^(k + 1) / k (a b - 1)^k, {k, 1, Infinity}]
```

```
Log[a b]
```

```
dd[n_, m_] := Sum[ 1, {j, 1, n}, {k, 1, m^(1 - Log[n, j])}]
dd[0, m_] := 0
dd[n_, 0] := 0
dd[1, m_] := m
dd2[n_, m_] := Sum[ 1, {j, 2, n}, {k, 2, m^(1 - Log[n, j])}]
ddd[m_, n_, o_] :=
 Sum[ 1, {j, 1, m}, {k, 1, n^(1 - Log[m, j])}, {l, 1, o^(1 - Log[m, j] - Log[n, k])}]
ddd2[m_, n_, o_] := Sum[ 1, {j, 2, m}, {k, 2, n^(1 - Log[m, j])},
  {l, 2, o^(1 - Log[m, j] - Log[n, k])}]
dddd[m_, n_, o_, p_] := Sum[ 1, {j, 1, m}, {k, 1, n^(1 - Log[m, j])},
  {l, 1, o^(1 - Log[m, j] - Log[n, k])}, {s, 1, p^(1 - Log[m, j] - Log[n, k] - Log[o, l])}]
dddd2[m_, n_, o_, p_] := Sum[ 1, {j, 2, m}, {k, 2, n^(1 - Log[m, j])},
  {l, 2, o^(1 - Log[m, j] - Log[n, k])}, {s, 2, p^(1 - Log[m, j] - Log[n, k] - Log[o, l])}]
```

```
Table[{n, dd2[n, n] - dd2[n + 1, n - 1]}, {n, 2, 40}] // TableForm
```

2	0
3	0
4	1
5	0
6	1
7	0
8	1
9	1
10	1
11	0
12	2
13	0
14	1
15	1
16	2
17	0
18	2
19	0
20	2
21	1
22	1
23	0
24	3
25	1
26	1
27	1
28	2
29	0
30	3
31	0
32	2
33	1
34	1
35	1
36	4
37	0
38	1
39	1
40	3

```
n^(1 - Log[n, j])
```

$$\frac{n}{j}$$

```
dd2[101, 99]
```

279

```
ddd[5, 5, 1]
```

10

```
ddd[0, 0, 5]
```

0

```
dddd[100, 100, 100, 100]
```

3575

```
dddd2[47, 47, 70, 70] + 2 dd2[47, 70] + 1
```

302

```
dddd[47, 47, 70, 70]
```

1514

```
Sum[ 1,  {k, 1, Floor[m^(1 - Log[n, j])]}]
```

$$\text{Floor}\left[m^{1-\frac{\text{Log}[j]}{\text{Log}[n]}}\right]$$

```
Sum[ Floor[m^(1 - Log[j]/Log[n])], {j, 1, n}]
```

$$\sum_{j=1}^{n} \text{Floor}\left[m^{1-\frac{\text{Log}[j]}{\text{Log}[n]}}\right]$$

```
Grid[Table[ dd[n, m], {n, 1, 20}, {m, 1, 20}]]
```

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
3	4	5	6	7	8	10	11	12	13	14	15	16	17	18	19	20	21	22	24
4	5	6	8	9	10	11	12	14	15	16	17	18	19	20	22	23	24	25	26
5	6	7	9	10	11	13	14	16	17	18	20	21	22	23	24	26	27	28	29
6	7	8	10	11	14	15	16	17	19	20	21	22	24	25	26	27	29	31	32
7	8	10	11	13	15	16	17	19	20	21	23	26	27	28	29	31	32	33	34
8	9	11	12	14	16	17	20	21	22	24	26	27	28	30	31	32	33	36	37
9	10	12	14	16	17	19	21	23	24	26	27	28	31	32	34	35	37	38	40
10	11	13	15	17	19	20	22	24	27	28	29	31	32	34	36	38	39	40	42
11	12	14	16	18	20	21	24	26	28	29	30	33	35	36	39	40	41	43	45
12	13	15	17	20	21	23	26	27	29	30	35	36	37	39	40	41	44	45	46
13	14	16	18	21	22	26	27	28	31	33	36	37	38	40	41	43	46	47	49
14	15	17	19	22	24	27	28	31	32	35	37	38	41	42	44	47	48	50	52
15	16	18	20	23	25	28	30	32	34	36	39	40	42	45	46	48	50	51	54
16	17	19	22	24	26	29	31	34	36	39	40	41	44	46	50	51	52	54	56
17	18	20	23	26	27	31	32	35	38	40	41	43	47	48	51	52	53	56	58
18	19	21	24	27	29	32	33	37	39	41	44	46	48	50	52	53	58	59	60
19	20	22	25	28	31	33	36	38	40	43	45	47	50	51	54	56	59	60	61
20	21	24	26	29	32	34	37	40	42	45	46	49	52	54	56	58	60	61	66

```
Grid[Table[ dd2[n, m], {n, 1, 20}, {m, 1, 20}]]
```

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	2
0	0	0	1	1	1	1	1	2	2	2	2	2	2	2	3	3	3	3	3
0	0	0	1	1	1	2	2	3	3	3	4	4	4	4	4	5	5	5	5
0	0	0	1	1	3	3	3	3	4	4	4	4	5	5	5	5	6	7	7
0	0	1	1	2	3	3	3	4	4	4	5	7	7	7	7	8	8	8	8
0	0	1	1	2	3	3	5	5	5	6	7	7	7	8	8	8	8	10	10
0	0	1	2	3	3	4	5	6	6	7	7	7	9	9	10	10	11	11	12
0	0	1	2	3	4	4	5	6	8	8	8	9	9	10	11	12	12	12	13
0	0	1	2	3	4	4	6	7	8	8	8	10	11	11	13	13	13	14	15
0	0	1	2	4	4	5	7	7	8	8	12	12	12	13	13	13	15	15	15
0	0	1	2	4	4	7	7	7	9	10	12	12	12	13	13	14	16	16	17
0	0	1	2	4	5	7	7	9	9	11	12	12	14	14	15	17	17	18	19
0	0	1	2	4	5	7	8	9	10	11	13	13	14	16	16	17	18	18	20
0	0	1	3	4	5	7	8	10	11	13	13	13	15	16	19	19	19	20	21
0	0	1	3	5	5	8	8	10	12	13	13	14	17	17	19	19	19	21	22
0	0	1	3	5	6	8	8	11	12	13	15	16	17	18	19	19	23	23	23
0	0	1	3	5	7	8	10	11	12	14	15	16	18	18	20	21	23	23	23
0	0	2	3	5	7	8	10	12	13	15	15	17	19	20	21	22	23	23	27

```
Grid[Table[ (dd2[n, m] + (n - 1) + (m - 1) + 1) - dd[n, m], {n, 1, 20}, {m, 1, 20}]]
```

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

```
Grid[Table[ (dd[n, m] - (n) - (m) + 1) - dd2[n, m], {n, 1, 20}, {m, 1, 20}]]
```

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

```
D[n n, n]
```

```
2 n
```

```
n^(1 + Log[n, m])
```

```
m n
```

```
D[n^(1 + Log[n, m]), n]
```

```
m
```

```
Table[ Floor[Expand[12 / 20 (20 - n)]], {n, 1, 20}]
```

```
{11, 10, 10, 9, 9, 8, 7, 7, 6, 6, 5, 4, 4, 3, 3, 2, 1, 1, 0, 0}
```

```
Floor[m / n (n - s)]
```

```
Expand[m / n (n - s)]
```

```
dif[n_, m_, s_] := dd[ n - s, Floor[m - (m s)/n]] - dd[ n - s - 1, Floor[m - (m (s + 1))/n]]
```

```
Sum[ dif[12, 7, s], {s, 0, 12}]
```

```
23
```

```
dd[12, 7]
```

```
23
```

```
Clear[ep, eo, ex, ee, exp]
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}]
ee[n_, m_] := Table[ dd[k, Floor[k m / n]] - dd[(k - 1), Floor[(k - 1) m / n]], {k, 1, n}]
exo[k_] := dd[k, Floor[k / 2]] - dd[ Floor[(k - 1) / 2], (k - 1)]
exp[n_] := exp[n] = dd[n, n / 2] - dd[n - (1 / 2), (n - 1 / 2) / 2]
ep[n_, k_] :=
 ep[n, k] = (1 / 2) Sum[ exp[ j ] (1 / k - ep[n / j, k + 1]), {j, (2^k + 1) / (2^k), n, 1 / (2^k)}]
eo[n_, k_] := eo[n, k] = Sum[ 1 / k - eo[Floor[n / j], k + 1], {j, 2, n}]
ed[n_, k_, z_] := eo[n, k] = Sum[ dz[j, z] (1 / k - ed[Floor[n / j], k + 1, z]), {j, 2, n}]
```

```
Table[ exp[n, 6], {n, 1, 12}]
```

```
{6, 1, 1, 2, 1, 3, 1, 1, 1, 2, 1, 1}
```

```
ep[4, 1]
```

```
$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

General::stop : Further output of $RecursionLimit::reclim will be suppressed during this calculation. ≫

$IterationLimit::itlim : Iteration limit of 4096 exceeded. ≫

$IterationLimit::itlim : Iteration limit of 4096 exceeded. ≫

$IterationLimit::itlim : Iteration limit of 4096 exceeded. ≫

General::stop : Further output of $IterationLimit::itlim will be suppressed during this calculation. ≫
```

```
eo[4, 1] + eo[2, 1]
```

$$\frac{7}{2}$$

```
eo[100, 1]
```

$$\frac{428}{15}$$

```
Table[{n, dd[n, n / 2] - dd[n - (1 / 2), (n - 1 / 2) / 2], exp[n]}, {n, 2, 20, 1 / 2}] // TableForm
```

2	2	2
$\frac{5}{2}$	0	0
3	1	1
$\frac{7}{2}$	0	0
4	2	2
$\frac{9}{2}$	0	0
5	1	1
$\frac{11}{2}$	0	0
6	2	2
$\frac{13}{2}$	1	1
7	1	1
$\frac{15}{2}$	0	0
8	2	2
$\frac{17}{2}$	1	1
9	1	1
$\frac{19}{2}$	0	0
10	3	3
$\frac{21}{2}$	0	0
11	2	2
$\frac{23}{2}$	0	0
12	2	2
$\frac{25}{2}$	0	0
13	2	2
$\frac{27}{2}$	2	2
14	2	2
$\frac{29}{2}$	0	0
15	1	1
$\frac{31}{2}$	1	1
16	2	2
$\frac{33}{2}$	0	0
17	2	2
$\frac{35}{2}$	2	2
18	2	2
$\frac{37}{2}$	1	1
19	1	1
$\frac{39}{2}$	0	0
20	3	3