$$\sum_{k=0}^{\infty} \frac{z^{(k)}}{k!} \cdot x^{(-sk)} = \left(\frac{1}{1-x^{-s}}\right)^{z}$$

$$\sum_{k=0}^{\infty} \binom{z}{k} \cdot x^{(-sk)} = (1+x^{-s})^{z}$$

$$\sum_{k=0}^{\infty} \frac{z^{k}}{k!} \cdot x^{(-sk)} = e^{x^{-s} \cdot z}$$

...

$$(1+x^{-s})^{z} = \left(\frac{1-x^{-2s}}{1-x^{-s}}\right)^{z}$$

$$(1+x^{-s}+x^{-2s})^{z} = \left(\frac{1-x^{-3s}}{1-x^{-s}}\right)^{z}$$

$$(1+x^{-s}+x^{-2s}+x^{-3s})^{z} = \left(\frac{1-x^{-4s}}{1-x^{-s}}\right)^{z}$$

$$\left(\sum_{j=0}^{a-1} x^{-js}\right)^{z} = \left(\frac{1-x^{-as}}{1-x^{-s}}\right)^{z}$$

...

$$\sum_{k=0}^{\infty} \frac{z^{(k)}}{k!} \cdot x^{(-2sk)} = \left(\frac{1}{1-x^{-2s}}\right)^{z}$$

$$\sum_{k=0}^{\infty} \frac{z^{(k)}}{k!} \cdot x^{(-ask)} = \left(\frac{1}{1-x^{-as}}\right)^{z}$$

...

$$\lim_{s \to 0} \left(\frac{1-x^{-as}}{1-x^{-s}}\right)^{z} = \lim_{x \to 1} \left(\frac{1-x^{-as}}{1-x^{-s}}\right)^{z} = a^{z}$$

$$\left[\frac{1}{1-x^{-s}}\right]^{+}_{n}=\sum_{j=0}^{n} x^{-s\cdot j}$$

$$\left[\left(\frac{1}{1-x^{-s}}\right)^{2}\right]^{+}_{n}=\sum_{j=0}^{n}\sum_{k=0}^{n-j} x^{-s(j+k)}$$

$$\left[\left(\frac{1}{1-x^{-s}}\right)^{3}\right]^{+}_{n}=\sum_{j=0}^{n}\sum_{k=0}^{n-j}\sum_{l=0}^{n-j-k} x^{-s(j+k+l)}$$

$$\left[\left(\frac{1}{1-x^{-s}}\right)^{k}\right]^{+}_{n}=\sum_{j=0}^{n} x^{-sj}\cdot\left[\left(\frac{1}{1-x^{-s}}\right)^{k-1}\right]^{+}_{n-j}$$

$$\left[\left(\frac{1}{1-x^{-s}}\right)^{0}\right]^{+}_{n}=\mathbf{1}_{\boldsymbol{n\geq 0}}(n)$$

...

$$\left[\frac{1}{1-x^{-s}}-1\right]^{+}_{n}=\sum_{j=1}^{n} x^{-sj}$$

$$\left[\left(\frac{1}{1-x^{-s}}-1\right)^{2}\right]^{+}_{n}=\sum_{j=1}^{n-1}\sum_{k=1}^{n-j} x^{-s(j+k)}$$

$$\left[\left(\frac{1}{1-x^{-s}}-1\right)^{3}\right]^{+}_{n}=\sum_{j=1}^{n-2}\sum_{k=1}^{n-j-1}\sum_{l=1}^{n-j-k} x^{-s(j+k+l)}$$

$$\left[\left(\frac{1}{1-x^{-s}}-1\right)^{k}\right]^{+}_{n}=0\ if\ k>n$$

$$\left[\left(\frac{1}{1-x^{-s}}-1\right)^{k}\right]^{+}_{n}=\sum_{j=1}^{n-k+1} x^{-sj}\cdot\left[\left(\frac{1}{1-x^{-s}}-1\right)^{k-1}\right]^{+}_{n-j}$$

$$\left[\left(\frac{1}{1-x^{-s}}-1\right)^{0}\right]^{+}_{n}=\mathbf{1}_{\boldsymbol{n\geq 0}}(n)$$

...

$$\left[\log\left(\frac{1}{1-x^{-s}}\right)\right]^{+}_{n}=\sum_{j=1}^{n}\frac{x^{-sj}}{j}$$

$$\left[\left(\log\left(\frac{1}{1-x^{-s}}\right)\right)^{2}\right]^{+}_{n}=\sum_{j=1}^{n-1}\sum_{k=1}^{n-j}\frac{x^{-s(j+k)}}{j\cdot k}$$

$$\left[\log\left(\left(\frac{1}{1-x^{-s}}\right)\right)^{3}\right]^{+}_{n}=\sum_{j=1}^{n-2}\sum_{k=1}^{n-j-1}\sum_{l=1}^{n-j-k}\frac{x^{-s(j+k+l)}}{j\cdot k\cdot l}$$

$$\left[\left(\log\left(\frac{1}{1-x^{-s}}\right)\right)^{k}\right]^{+}_{n}=0\ if\ k>n$$

$$\left[\left(\log\left(\frac{1}{1-x^{-s}}\right)\right)^{k}\right]^{+}_{n}=\sum_{j=1}^{n-k+1}\frac{x^{-sj}}{j}\cdot\left[\left(\log\left(\frac{1}{1-x^{-s}}\right)\right)^{k-1}\right]^{+}_{n-j}$$

$$\left[\left(\log\left(\frac{1}{1-x^{-s}}\right)\right)^{0}\right]^{+}_{n}=\mathbf{1}_{\boldsymbol{n\geq 0}}(n)$$

...

$$\left[\left(\frac{1}{1-x^{-s}}\right)^{z}\right]^{+}_{n}=\sum_{k=0}\binom{z}{k}\left[\left(\frac{1}{1-x^{-s}}-1\right)^{k}\right]^{+}_{n}$$

$$\left[\left(\frac{1}{1-x^{-s}}-1\right)^{k}\right]^{+}_{n}=\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}\left[\left(\frac{1}{1-x^{-s}}\right)^{j}\right]^{+}_{n}$$

$$\left[\log\left(\frac{1}{1-x^{-s}}\right)\right]^{+}_{n}=\sum_{k=1}\frac{(-1)^{k+1}}{k}\left[\left(\frac{1}{1-x^{-s}}-1\right)^{k}\right]^{+}_{n}$$

$$\left[\log\left(\frac{1}{1-x^{-s}}\right)\right]^{+}_{n}=\lim_{z\to 0}\frac{\partial}{\partial z}\left[\left(\frac{1}{1-x^{-s}}\right)^{z}\right]^{+}_{n}$$

$$\left[\left(\log\left(\frac{1}{1-x^{-s}}\right)\right)^{k}\right]^{+}_{n}=\lim_{z\to 0}\frac{\partial^{k}}{\partial z^{k}}\left[\left(\frac{1}{1-x^{-s}}\right)^{z}\right]^{+}_{n}$$

$$\left[\left(\frac{1}{1-x^{-s}}\right)^{z}\right]^{+}_{n}=\sum_{k=0}\frac{z^{k}}{k!}\left[\left(\log\left(\frac{1}{1-x^{-s}}\right)\right)^{k}\right]^{+}_{n}$$

...

$$[e^{x^{-s}}]_n^+ = \sum_{j=0}^{n} \frac{x^{-s\cdot j}}{j!}$$

$$[e^{2x^{-s}}]_n^+ = \sum_{j=0}^{n} \sum_{k=0}^{n-j} \frac{x^{-s(j+k)}}{j!\cdot k!}$$

$$[e^{3x^{-s}}]_n^+ = \sum_{j=0}^{n} \sum_{k=0}^{n-j} \sum_{l=0}^{n-j-k} \frac{x^{-s(j+k+l)}}{j!\cdot k!\cdot l!}$$

$$[e^{k\cdot x^{-s}}]_n^+ = \sum_{j=0}^{n} \frac{x^{-sj}}{j!} \cdot [e^{(k-1)\cdot x^{-s}}]_{n-j}^+$$

$$[e^{0\cdot x^{-s}}]_n^+ = \mathbf{1}_{n\geq 0}(n)$$

$$[e^{z\cdot x^{-s}}]_n^+ = \sum_{k=0}^{n} \frac{z^k}{k!} \cdot x^{-s\cdot k}$$

$$\ldots$$

$$[e^{x^{-s}} - 1]_n^+ = \sum_{j=1}^{n} \frac{x^{-s\cdot j}}{j!}$$

$$[(e^{x^{-s}} - 1)^2]_n^+ = \sum_{j=1}^{n-1} \sum_{k=1}^{n-j} \frac{x^{-s(j+k)}}{j!\cdot k!}$$

$$[(e^{x^{-s}} - 1)^3]_n^+ = \sum_{j=1}^{n-2} \sum_{k=1}^{n-1} \sum_{l=1}^{n-j-k} \frac{x^{-s(j+k+l)}}{j!\cdot k!\cdot l!}$$

$$[(e^{x^{-s}} - 1)^k]_n^+ = \sum_{j=1}^{n-k+1} \frac{x^{-sj}}{j!} \cdot [(e^{x^{-s}} - 1)^{(k-1)}]_{n-j}^+$$

$$[e^{0\cdot x^{-s}}]_n^+ = \mathbf{1}_{n\geq 0}(n)$$

$$\ldots$$

$$[\log(e^{x^{-s}})]_n^+ = (n>0)\, ?\, x^{-s} : 0$$

$$[(\log(e^{x^{-s}}))^2]_n^+ = (n>1)\, ?\, x^{-2s} : 0$$

$$[\log(e^{x^{-s}})^3]_n^+ = (n>2)\, ?\, x^{-3s} : 0$$

$$[(\log(e^{x^{-s}}))^k]_n^+ = 0 \; if \; k>n$$

$$[(\log(e^{x^{-s}}))^k]_n^+ = (n\geq k)\, ?\, x^{-s} \cdot [(\log(e^{x^{-s}}))^{k-1}]_n^+ : 0$$

$$[(\log(e^{x^{-s}}))^0]_n^+ = \mathbf{1}_{n\geq 0}(n)$$

$$\ldots$$

$$[(\frac{1}{1-x^{-s}})^z]_n^+ = \sum_{k=0} \binom{z}{k} [(\frac{1}{1-x^{-s}} - 1)^k]_n^+$$

$$[(e^{x^{-s}} - 1)^k]_n^+ = \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} [e^{j\cdot x^{-s}}]_n^+$$

$$[\log(\frac{1}{1-x^{-s}})]_n^+ = \sum_{k=1} \frac{(-1)^{k+1}}{k} [(e^{x^{-s}} - 1)^k]_n^+$$

$$[\log(e^{x^{-s}})]_n^+ = \lim_{z\to 0} \frac{\partial}{\partial z} [e^{z\cdot x^{-s}}]_n^+$$

$$[(\log(e^{x^{-s}}))^k]_n^+ = \lim_{z\to 0} \frac{\partial^k}{\partial z^k} [e^{z\cdot x^{-s}}]_n^+$$

$$[e^{z\cdot x^{-s}}]_n^+ = \sum_{k=0} \frac{z^k}{k!} [(\log e^{x^{-s}})^k]_n^+$$

$$[\cos x^{-s}]_n^+ = \sum_{j=0}^{\lfloor n/2 \rfloor} \frac{(-1)^j}{(2j)!} \cdot x^{-s \cdot 2j}$$

$$[(\cos x^{-s})^2]_n^+ = \sum_{j=0}^{\lfloor n/2 \rfloor} \sum_{k=0}^{\lfloor (n-2j)/2 \rfloor} \left(\frac{(-1)^{j+k}}{(2j)!(2k)!} \cdot x^{-s \cdot 2(j+k)} \right)$$

$$[(\cos x^{-s})^k]_n^+ = \sum_{j=0}^{\lfloor n/2 \rfloor} \frac{(-1)^j}{(2j)!} x^{-2j} \cdot [(\cos x^{-s})^{k-1}]_{n-2j}^+$$

$$[(\cos x^{-s})^0]_n^+ = \mathbf{1}_{n \geq 0}(n)$$

…

$$[\cos x^{-s} - 1]_n^+ = \sum_{j=1}^{\lfloor n/2 \rfloor} \frac{(-1)^j}{(2j)!} \cdot x^{-s \cdot 2j}$$

$$[(\cos x^{-s} - 1)^k]_n^+ = \sum_{j=1}^{\lfloor n/2 \rfloor} \frac{(-1)^j}{(2j)!} x^{-2j} [(\cos x^{-s} - 1)^{k-1}]_{n-2j}^+$$

…

LOG?

Can't tell if this is interesting or a dead end.