```
Clear[g]
e2[n_, k_] := e2[n, k] = Sum[e2[j, k - 1] e2[n / j, 1], {j, Divisors[n]}];
e2[n_, 1] := (-1)^(n + 1); e2[1, 1] := 0; e2[n_, 0] := 0; e2[1, 0] := 1
E2[n_, k_] := E2[n, k] = Sum[ e2[j, k], {j, 2, n}]
g[n_, k_, a_] := Sum[ ((-1)^(m + 1))^(k - j) Binomial[k, j] g[Floor[n / m^(k - j)], j, m + 1],
    {m, a, n^(1 / k)}, {j, 0, k - 1}];
g[n_, 1, a_] := (1 / 2) ((-1)^(n + 1) + (-1)^(a + 1))
g[n_, 0, a_] := 1
LAdd[n_] := Sum[ 2^k / k, {k, 1, Log[2, n]}]
LinE[n_] := LAdd[n] + Sum[ (-1)^(k + 1) / k g[n, k, 2], {k, 1, Log[2, n]}]


ES2[n_, a_] := Sum[ (-1)^(j+Floor[n/j]), {j, a, Floor[n^(1 / 2)]}]

ES2[1450, 2]
```

7

```
E2[1450, 2]
```

7

```
E2[1000, 3]
```

-19

```
g[1000, 3, 2]
```

-19

```
g2[n_, k_, a_] := Sum[ ((-1)^(m + 1))^(k - j) Binomial[k, j] g[Floor[n / m^(k - j)], j, m + 1],
  {m, a, n^(1 / k)}, {j, 0, k - 1}]
g2[1000, 3, 2]
```

-19

```
g3[n_, a_] := Sum[ ((-1)^(m + 1))^(3 - j) Binomial[3, j] g[Floor[n / m^(3 - j)], j, m + 1],
  {m, a, n^(1 / 3)}, {j, 0, 3 - 1}]
g3[1000, 2]
```

-19

```
g4[n_, a_] := Sum[

  ((-1)^(m + 1))^(3 - 0) Binomial[3, 0] g[Floor[n / m^(3 - 0)], 0, m + 1] +
   ((-1)^(m + 1))^(3 - 1) Binomial[3, 1] g[Floor[n / m^(3 - 1)], 1, m + 1] +
   ((-1)^(m + 1))^(3 - 2) Binomial[3, 2] g[Floor[n / m^(3 - 2)], 2, m + 1]

  , {m, a, n^(1 / 3)}]
g4[1000, 2]
```

-19

```
g5[n_, a_] := Sum[

  ((-1)^(m+1))^3 g[Floor[n / m^3], 0, m+1] +
   ((-1)^(m+1))^2 3 g[Floor[n / m^2], 1, m+1] +
   ((-1)^(m+1)) 3 g[Floor[n / m^1], 2, m+1]

  , {m, a, n^(1/3)}]
```

```
g5[1000, 2]
```

```
-19
```

```
g6[n_, a_] := Sum[

  ((-1)^(m+1))^3 +
   ((-1)^(m+1))^2 3 g[Floor[n / m^2], 1, m+1] +
   ((-1)^(m+1)) 3 g[Floor[n / m^1], 2, m+1]

  , {m, a, n^(1/3)}]
```

```
g6[1000, 2]
```

```
-19
```

```
g[Floor[n / m^2], 1, m+1]
```

$$\frac{1}{2}\left((-1)^{2+m}+(-1)^{1+\text{Floor}\left[\frac{n}{m^2}\right]}\right)$$

```
(1/2) ((-1)^(2+m) + (-1)^(1+Floor[n / m^2]))
```

```
g7[n_, a_] := Sum[

  ((-1)^(m+1))^3 + ((-1)^(m+1))^2 3 ((1/2) ((-1)^(2+m) + (-1)^(1+Floor[n / m^2]))) +
   ((-1)^(m+1)) 3 g[Floor[n / m^1], 2, m+1]

  , {m, a, n^(1/3)}]
```

```
g7[1000, 2]
```

```
-19
```

```
Expand[
  ((-1)^(m+1))^3 + ((-1)^(m+1))^2 3 ((1/2) ((-1)^(2+m) + (-1)^(1+Floor[n / m^2])))]
```

$$(-1)^{3+3m}+\frac{3}{2}(-1)^{4+3m}+\frac{3}{2}(-1)^{3+2m+\text{Floor}\left[\frac{n}{m^2}\right]}$$

```
g8[n_, a_] := Sum[

  (-1)^(3+3m) + 3/2 (-1)^(4+3m) + 3/2 (-1)^(3+2m+Floor[n/m^2]) +
   ((-1)^(m+1)) 3 g[Floor[n / m^1], 2, m+1]

  , {m, a, n^(1/3)}]
```

```
g8[1000, 2]
```

```
-19
```

```
g9[n_, a_] :=

 Sum[ (-1)^(3+3 m), {m, a, n^(1 / 3)}] +

  Sum[ 3/2 (-1)^(4+3 m), {m, a, n^(1 / 3)}] +

  Sum[ 3/2 (-1)^(3+2 m+Floor[n/m^2]), {m, a, n^(1 / 3)}] +

  Sum[ ((-1)^(m + 1)) 3 g[Floor[n / m^1], 2, m + 1], {m, a, n^(1 / 3)}]
```

```
g9[1000, 2]
```

```
-19
```

```
g10[n_, a_] :=

 -1/2 (-1)^(3 a) - 1/2 (-1)^(3 n^(1/3)) +

  Sum[ 3/2 (-1)^(4+3 m), {m, a, n^(1 / 3)}] +

  Sum[ 3/2 (-1)^(3+2 m+Floor[n/m^2]), {m, a, n^(1 / 3)}] +

  Sum[ ((-1)^(m + 1)) 3 g[Floor[n / m^1], 2, m + 1], {m, a, n^(1 / 3)}]
```

```
Sum[ (-1)^(3+3 m), {m, a, n^(1 / 3)}]
```

```
Expand[1/2 (-(-1)^(3 a) - (-1)^(3 n^(1/3)))]
```

```
g10[1000, 2]
```

```
-19
```

```
g11[n_, a_] :=

  -1/2 (-1)^(3 a) - 1/2 (-1)^(3 n^(1/3)) +

  3/4 (-1)^(3 a) + 3/4 (-1)^(3 n^(1/3)) +

  Sum[ 3/2 (-1)^(3+2 m+Floor[n/m^2]), {m, a, n^(1/3)}] +

  Sum[ ((-1)^(m+1)) 3 g[Floor[n/m^1], 2, m+1], {m, a, n^(1/3)}]

Sum[ 3/2 (-1)^(4+3 m), {m, a, n^(1/3)}]

Expand[3/4 ((-1)^(3 a) + (-1)^(3 n^(1/3)))]

3/4 (-1)^(3 a) + 3/4 (-1)^(3 n^(1/3))

g11[1000, 2]

-19

g12[n_, a_] :=

  -1/2 (-1)^(3 a) - 1/2 (-1)^(3 n^(1/3)) +

  3/4 (-1)^(3 a) + 3/4 (-1)^(3 n^(1/3)) +

  Sum[ 3/2 (-1)^(3+2 m+Floor[n/m^2]), {m, a, n^(1/3)}] +

  Sum[ ((-1)^(m+1)) 3 g[Floor[n/m^1], 2, m+1], {m, a, n^(1/3)}]

Sum[ 3/2 (-1)^(3+2 m+Floor[n/m^2]), {m, a, n^(1/3)}]

Sum_{m=a}^{n^(1/3)} 3/2 (-1)^(3+2 m+Floor[n/m^2])

g12[1000, 2]

-19
```

```
g13[n_, a_] :=

  -1/2 (-1)^(3 a) - 1/2 (-1)^(3 n^(1/3)) +

  3/4 (-1)^(3 a) + 3/4 (-1)^(3 n^(1/3)) +

  Sum[ 3/2 (-1)^(3+2 m+Floor[n/m^2]), {m, a, n^(1/3)}] +

  3 Sum[ ((-1)^(m+1)) g[Floor[n/m], 2, m+1], {m, a, n^(1/3)}]

g13[1000, 2]
```

```
-19
```

```
g14[n_, a_] :=

  -1/2 (-1)^(3 a) - 1/2 (-1)^(3 n^(1/3)) +

  3/4 (-1)^(3 a) + 3/4 (-1)^(3 n^(1/3)) +

  Sum[ 3/2 (-1)^(3+2 m+Floor[n/m^2]), {m, a, n^(1/3)}] +

  3 Sum[ ((-1)^(m+1)) Sum[(-1)^(j+Floor[Floor[n/m]/j]), {j, 1+m, Floor[Sqrt[Floor[n/m]]]}], {m, a, n^(1/3)}]

ES2[Floor[n/m], m+1]
```

$$\sum_{j=1+m}^{\text{Floor}\left[\sqrt{\text{Floor}\left[\frac{n}{m}\right]}\right]} (-1)^{j+\text{Floor}\left[\frac{\text{Floor}\left[\frac{n}{m}\right]}{j}\right]}$$

```
g14[1000, 2]
```

```
-19
```

```
g15[n_, a_] :=

 -1/2 (-1)^(3 a) - 1/2 (-1)^(3 n^(1/3)) +

 3/4 (-1)^(3 a) + 3/4 (-1)^(3 n^(1/3)) +

 3/2 Sum[ (-1)^(3+2 m+Floor[n/m^2]), {m, a, n^(1/3)}] +

 3 Sum[ ((-1)^(m+1)) Sum_(j=1+m)^(Floor[Sqrt[Floor[n/m]]]) (-1)^(j+Floor[Floor[n/m]/j]), {m, a, n^(1/3)}]
```

```
g15[1000, 2]
```

```
-19
```

```
g16[n_, a_] :=

 1/4 ((-1)^(3 a) + (-1)^(3 n^(1/3))) +

 3/2 Sum[ (-1)^(3+2 m+Floor[n/m^2]), {m, a, n^(1/3)}] +

 3 Sum[ ((-1)^(m+1)) Sum_(j=1+m)^(Floor[Sqrt[Floor[n/m]]]) (-1)^(j+Floor[Floor[n/m]/j]), {m, a, n^(1/3)}]
```

```
FullSimplify[Expand[-1/2 (-1)^(3 a) - 1/2 (-1)^(3 n^(1/3)) +

   3/4 (-1)^(3 a) + 3/4 (-1)^(3 n^(1/3))]]
```

$$\frac{1}{4}\left((-1)^{3a}+(-1)^{3n^{1/3}}\right)$$

```
g16[1000, 2]
```

```
-19
```

```
g17[n_, a_] :=

 1/4 ((-1)^(3 a) + (-1)^(3 Floor[n^(1/3)])) +

  3/2 Sum[ (-1)^(3+2 m+Floor[n/m^2]), {m, a, n^(1/3)}] +

  3 Sum[ ((-1)^(m+1)) Sum_{j=1+m}^{Floor[Sqrt[Floor[n/m]]]} (-1)^(j+Floor[Floor[n/m]/j]), {m, a, n^(1/3)}]


pp[n_, a_] := 1/4 ((-1)^(3 a) + (-1)^(3 Floor[n^(1/3)]))
pp2[n_, a_] := 3/2 Sum[ (-1)^(3+2 m+Floor[n/m^2]), {m, a, n^(1/3)}]
pp3[n_, a_] := 3 Sum[ ((-1)^(m+1)) Sum_{j=1+m}^{Floor[Sqrt[Floor[n/m]]]} (-1)^(j+Floor[Floor[n/m]/j]), {m, a, n^(1/3)}]
pp3a[n_, a_] := 3 Sum[ Sum_{j=1+m}^{Floor[Sqrt[Floor[n/m]]]} (-1)^(m+1+j+Floor[n/(m j)]), {m, a, Floor[n^(1/3)]}]
pp3b[n_, a_] :=
 3 Sum[ (-1)^(m+1+j+Floor[n/(m j)]), {m, a, Floor[n^(1/3)]}, {j, m+1, Floor[ Floor[n/m]^(1/2)]}]
Table[{n, pp3[n, 2], pp3a[n, 2], pp3b[n, 2]}, {n, 1000, 1030}] // TableForm
```

1000	−15	−15	−15
1001	−9	−9	−9
1002	−15	−15	−15
1003	−15	−15	−15
1004	−15	−15	−15
1005	−9	−9	−9
1006	−9	−9	−9
1007	−9	−9	−9
1008	30	30	30
1009	30	30	30
1010	24	24	24
1011	24	24	24
1012	30	30	30
1013	30	30	30
1014	9	9	9
1015	15	15	15
1016	21	21	21
1017	21	21	21
1018	21	21	21
1019	21	21	21
1020	39	39	39
1021	39	39	39
1022	33	33	33
1023	39	39	39
1024	12	12	12
1025	12	12	12
1026	−30	−30	−30
1027	−30	−30	−30
1028	−30	−30	−30
1029	−24	−24	−24
1030	−30	−30	−30

```
g18[n_, a_] :=

 1/4 ((-1)^(3 a) + (-1)^(3 Floor[n^(1/3)])) +

  3/2 Sum[ (-1)^(3+2 m+Floor[n/m^2]), {m, a, n^(1/3)}] +

  3 Sum[ (-1)^(m+1+j+Floor[n/(m j)]),
    {m, a, Floor[n^(1/3)]}, {j, m + 1, Floor[ Floor[n / m]^(1/2)]}]
```

```
g18[1000, 2]
```

```
-19
```

$$\frac{3}{2}\sum_{m=2}^{n^{1/3}}(-1)^{3+2m+\text{Floor}\left[\frac{n}{m^2}\right]}$$

```
3 Sum[ (-1)^(m+1+j+Floor[n/(m j)]), {m, 2, Floor[n^(1 / 3)]}, {j, m + 1, Floor[ Floor[n / m]^(1 / 2)]}]
```

$$3 \sum_{m=2}^{\text{Floor}\left[n^{1/3}\right]} \sum_{j=1+m}^{\text{Floor}\left[\sqrt{\text{Floor}\left[\frac{n}{m}\right]}\right]} (-1)^{1+j+m+\text{Floor}\left[\frac{n}{j\,m}\right]}$$

```
ps1[n_, a_] := 3/2 Sum[ (-1)^(3+2 m+Floor[n/m^2]), {m, a, n^(1 / 3)}]

ps2[n_, a_] := 3/2 Sum[ (-1)^(1+Floor[n/m^2]), {m, a, n^(1 / 3)}]

ps1[103 300, 2]
```

$$\frac{33}{2}$$

```
ps2[103 300, 2]
```

$$\frac{33}{2}$$

```
g19[n_, a_] :=

 1/4 ((-1)^(3 a) + (-1)^(3 Floor[n^(1/3)])) +

  3/2 Sum[ (-1)^(1+Floor[n/m^2]), {m, a, n^(1 / 3)}] +

  3 Sum[ (-1)^(m+1+j+Floor[n/(m j)]),
    {m, a, Floor[n^(1 / 3)]}, {j, m + 1, Floor[ Floor[n / m]^(1 / 2)]}]
```

```
3 Sum[ (-1)^(m+1+j+Floor[n/(m j)]), {j, m + 1, Floor[ Floor[n / m]^(1 / 2)]}]
```

$$\text{PP}[n\_, m\_] := 3 \sum_{j=1+m}^{\text{Floor}\left[\sqrt{\text{Floor}\left[\frac{n}{m}\right]}\right]} (-1)^{1+j+m+\text{Floor}\left[\frac{n}{j\,m}\right]} /. \; m \to 2$$

```
PP[100, 2]
```

$$-3$$

```
PP[100, 3]
```

$$0$$

```
Table[{n, PP[n, 2]}, {n, 2, 40}] // TableForm
```

2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0
16	0
17	0
18	−3
19	−3
20	−3
21	−3
22	−3
23	−3
24	3
25	3
26	3
27	3
28	3
29	3
30	−3
31	−3
32	−6
33	−6
34	−6
35	−6
36	0
37	0
38	0
39	0
40	6

```
g20[n_, a_] :=

 1/4 ((-1)^(3 a) + (-1)^(3 Floor[n^(1/3)])) +

  3/2 Sum[ (-1)^(1+Floor[n/m^2]), {m, a, n^(1/3)}] +

  3 Sum[ (-1)^(m+1+j+Floor[n/(m j)]),
    {m, a, Floor[n^(1/3)]}, {j, m+1, Floor[ Floor[n/m]^(1/2)]}]
```

```
g20[1000, 2]
```

```
-19
```

$$\text{Expand}\left[\frac{1}{4}\left(1+(-1)^{3\,\text{Floor}\left[n^{1/3}\right]}\right)+\frac{3}{2}\sum_{m=2}^{n^{1/3}}(-1)^{1+\text{Floor}\left[\frac{n}{m^2}\right]}+3\sum_{m=2}^{\text{Floor}\left[n^{1/3}\right]}\sum_{j=1+m}^{\text{Floor}\left[\sqrt{\text{Floor}\left[\frac{n}{m}\right]}\,\right]}(-1)^{1+j+m+\text{Floor}\left[\frac{n}{j\,m}\right]}\right]$$

$$\frac{1}{4}+\frac{1}{4}\,(-1)^{3\,\text{Floor}\left[n^{1/3}\right]}+\frac{3}{2}\sum_{m=2}^{n^{1/3}}(-1)^{1+\text{Floor}\left[\frac{n}{m^2}\right]}+3\sum_{m=2}^{\text{Floor}\left[n^{1/3}\right]}\sum_{j=1+m}^{\text{Floor}\left[\sqrt{\text{Floor}\left[\frac{n}{m}\right]}\,\right]}(-1)^{1+j+m+\text{Floor}\left[\frac{n}{j\,m}\right]}$$

$$\text{PD}[n\_,\ a\_]\ :=\ \sum_{m=a}^{\text{Floor}\left[n^{1/3}\right]}\sum_{j=1+m}^{\text{Floor}\left[\sqrt{\text{Floor}\left[\frac{n}{m}\right]}\,\right]}(-1)^{1+j+m+\text{Floor}\left[\frac{n}{j\,m}\right]}$$

$$\text{PD2}[n\_,\ a\_]\ :=\ \sum_{m=a}^{\text{Floor}\left[n^{1/3}\right]}\sum_{j=1+m}^{\text{Floor}\left[\sqrt{\text{Floor}\left[\frac{n}{m}\right]}\,\right]}\text{Mo}\left[1+j+m+\text{Floor}\left[\frac{n}{j\,m}\right]\right]$$

$$\text{PD2a}[n\_,\ a\_]\ :=\ \sum_{m=a}^{\text{Floor}\left[n^{1/3}\right]}\sum_{j=1+m}^{\text{Floor}\left[\sqrt{\text{Floor}\left[\frac{n}{m}\right]}\,\right]}\text{Mo}[m,\ j]$$

```
PD2[1000, 2]
```

```
5 Mo[31] + 3 Mo[32] + 3 Mo[33] + 4 Mo[34] + 2 Mo[35] + 3 Mo[36] + 4 Mo[37] + Mo[38] + Mo[39] +
  4 Mo[40] + 4 Mo[41] + Mo[42] + Mo[43] + Mo[44] + 2 Mo[45] + 4 Mo[47] + 3 Mo[48] + Mo[49] +
  2 Mo[50] + Mo[51] + 2 Mo[52] + Mo[53] + Mo[54] + Mo[56] + Mo[58] + Mo[59] + Mo[60] + Mo[63] +
  Mo[65] + Mo[67] + Mo[73] + Mo[75] + Mo[81] + Mo[91] + Mo[92] + Mo[108] + Mo[132] + Mo[172]
```

```
PD2[10 000, 2]
```

```
3 Mo[65] + 9 Mo[66] + 12 Mo[67] + 8 Mo[68] + 7 Mo[69] + 9 Mo[70] + 6 Mo[71] + 10 Mo[72] + 5 Mo[73] +
 8 Mo[74] + 5 Mo[75] + 7 Mo[76] + 7 Mo[77] + 5 Mo[78] + 6 Mo[79] + 6 Mo[80] + 5 Mo[81] + 5 Mo[82] +
 6 Mo[83] + 6 Mo[84] + 4 Mo[85] + 4 Mo[86] + 5 Mo[87] + 4 Mo[88] + 6 Mo[89] + 5 Mo[90] + 4 Mo[91] +
 3 Mo[92] + 3 Mo[93] + 3 Mo[94] + 6 Mo[95] + 5 Mo[96] + 4 Mo[97] + 3 Mo[98] + 5 Mo[99] +
 3 Mo[100] + Mo[101] + 3 Mo[102] + 3 Mo[103] + Mo[104] + 9 Mo[105] + 4 Mo[106] + 4 Mo[107] +
 4 Mo[108] + 2 Mo[109] + 2 Mo[110] + 2 Mo[111] + 3 Mo[112] + 4 Mo[113] + Mo[114] + 2 Mo[115] +
 Mo[116] + Mo[117] + 4 Mo[118] + 5 Mo[119] + 4 Mo[120] + 3 Mo[121] + 5 Mo[122] + 2 Mo[123] +
 3 Mo[124] + 2 Mo[125] + 2 Mo[126] + 3 Mo[127] + Mo[128] + Mo[129] + 3 Mo[130] + 2 Mo[131] +
 Mo[132] + 3 Mo[133] + Mo[134] + Mo[135] + 2 Mo[136] + Mo[138] + Mo[139] + 4 Mo[140] +
 Mo[142] + 7 Mo[144] + 6 Mo[145] + 2 Mo[146] + 4 Mo[147] + 4 Mo[148] + Mo[149] + 2 Mo[150] +
 2 Mo[151] + Mo[152] + Mo[153] + 3 Mo[154] + 2 Mo[155] + 2 Mo[156] + 2 Mo[157] + Mo[158] +
 Mo[159] + 2 Mo[160] + Mo[161] + 3 Mo[162] + Mo[164] + Mo[165] + Mo[166] + Mo[168] + 2 Mo[169] +
 Mo[170] + Mo[171] + 2 Mo[172] + 2 Mo[175] + 3 Mo[177] + Mo[180] + 2 Mo[183] + 2 Mo[184] +
 Mo[186] + Mo[187] + Mo[190] + Mo[191] + Mo[194] + Mo[195] + Mo[197] + 2 Mo[198] + Mo[199] +
 Mo[201] + Mo[204] + Mo[207] + Mo[209] + Mo[210] + Mo[215] + Mo[216] + Mo[217] + Mo[221] +
 Mo[223] + Mo[225] + 2 Mo[228] + Mo[235] + Mo[237] + Mo[241] + 2 Mo[243] + 2 Mo[252] + Mo[256] +
 Mo[262] + Mo[264] + Mo[265] + 2 Mo[273] + Mo[285] + Mo[291] + Mo[293] + 2 Mo[298] + Mo[314] +
 Mo[318] + Mo[325] + Mo[331] + Mo[345] + Mo[347] + Mo[351] + Mo[369] + Mo[374] + Mo[383] +
 Mo[400] + Mo[427] + Mo[428] + Mo[431] + Mo[468] + Mo[487] + Mo[510] + Mo[513] + Mo[565] +
 Mo[567] + Mo[636] + Mo[675] + Mo[724] + Mo[841] + Mo[842] + Mo[1008] + Mo[1257] + Mo[1672]
```

```
Table[{n, Floor[10 000 / (2 n)]}, {n, 2, 70}] // TableForm
```

2	2500
3	1666
4	1250
5	1000
6	833
7	714
8	625
9	555
10	500
11	454
12	416
13	384
14	357
15	333
16	312
17	294
18	277
19	263
20	250
21	238
22	227
23	217
24	208
25	200
26	192
27	185
28	178
29	172
30	166
31	161
32	156
33	151

34	147
35	142
36	138
37	135
38	131
39	128
40	125
41	121
42	119
43	116
44	113
45	111
46	108
47	106
48	104
49	102
50	100
51	98
52	96
53	94
54	92
55	90
56	89
57	87
58	86
59	84
60	83
61	81
62	80
63	79
64	78
65	76
66	75
67	74
68	73
69	72
70	71