```
fla[s2_] := Limit[(1/2) s (s-1) Pi^(-s/2) Gamma[s/2], s → s2]
```

$$\mathtt{so14y6[n\_,\ s\_]} := \frac{\mathrm{Gamma}\left[1+\frac{s}{2}\right]\,\mathrm{Gamma}\left[1+\frac{1-s}{2}\right]}{\mathrm{Gamma}\left[1+\frac{s}{2}\right]\,n^{1-s}\,\pi^{\frac{(1-s)}{2}}-\mathrm{Gamma}\left[1+\frac{1-s}{2}\right]\,\pi^{s/2}\,n\wedge s}$$
$$((1-s)\ n\wedge s\ \mathrm{HarmonicNumber}[n,\ s]-s\ n\wedge(1-s)\ \mathrm{HarmonicNumber}[n,\ 1-s])$$

```
zet14y[n_, s_] := so14y6[n, s] / fla[s]
```

$$\mathtt{zet0[n\_,\ s\_]} := \left(1+\frac{1}{2\wedge s\ s\wedge -1\ n^{2s-1}\,\pi^{s-1}\,\mathrm{Gamma}[2-s]\,\mathrm{Sin}\left[\frac{\pi s}{2}\right]-1}\right)\mathrm{HarmonicNumber}[n,\ s]+$$
$$\left(\frac{1}{n^{2s-1}\ (-1+s)\ /\ s+2\wedge(1-s)\ \pi^{-s}\,\mathrm{Cos}\left[\frac{\pi s}{2}\right]\,\mathrm{Gamma}[s]}\right)\mathrm{HarmonicNumber}[n,\ 1-s]$$

$$\mathtt{zet0a[n\_,\ s\_]} := \mathrm{Sum}\left[\left(1+\frac{1}{2\wedge s\ s\wedge -1\ n^{2s-1}\,\pi^{s-1}\,\mathrm{Gamma}[2-s]\,\mathrm{Sin}\left[\frac{\pi s}{2}\right]-1}\right)j\wedge -s+\right.$$
$$\left.\left(\frac{1}{n^{2s-1}\ (-1+s)\ /\ s+2\wedge(1-s)\ \pi^{-s}\,\mathrm{Cos}\left[\frac{\pi s}{2}\right]\,\mathrm{Gamma}[s]}\right)j\wedge(s-1),\ \{j,\ 1,\ n\}\right]$$

```
zet[n_, s_] := 1 / ((1/2) s (s-1) Pi^(-s/2) Gamma[s/2])
```

$$\frac{\mathrm{Gamma}\left[1+\frac{s}{2}\right]\,\mathrm{Gamma}\left[1+\frac{1-s}{2}\right]}{\mathrm{Gamma}\left[1+\frac{s}{2}\right]\,n^{1-s}\,\pi^{\frac{(1-s)}{2}}-\mathrm{Gamma}\left[1+\frac{1-s}{2}\right]\,\pi^{s/2}\,n\wedge s}$$
$$((1-s)\ n\wedge s\ \mathrm{HarmonicNumber}[n,\ s]-s\ n\wedge(1-s)\ \mathrm{HarmonicNumber}[n,\ 1-s])$$

$$\mathtt{zet2[n\_,\ s\_]} := \left(-\frac{1}{n^{s}\ (-1+s)+2\ n\wedge(1-s)\ (2\pi)^{-s}\,\mathrm{Cos}\left[\frac{\pi s}{2}\right]\,\mathrm{Gamma}[1+s]}\right)$$
$$\left(n^{s}\ (1-s)\ \mathrm{HarmonicNumber}[n,\ s]-n^{1-s}\ s\ \mathrm{HarmonicNumber}[n,\ 1-s]\right)$$

$$\mathtt{zet2a[n\_,\ s\_]} := \left(-\frac{1}{n^{s}\ (-1+s)}\right)(n^{s}\ (1-s)\ \mathrm{HarmonicNumber}[n,\ s])$$

$$\mathtt{zet2b[n\_,\ s\_]} := \left(-\frac{1}{2\ n\wedge(1-s)\ (2\pi)^{-s}\,\mathrm{Cos}\left[\frac{\pi s}{2}\right]\,\mathrm{Gamma}[1+s]}\right)\left(-n^{1-s}\ s\ \mathrm{HarmonicNumber}[n,\ 1-s]\right)$$

$$\mathtt{zet2bx[n\_,\ s\_]} := \frac{\mathrm{HarmonicNumber}[n,\ 1-s]}{2\ (2\pi)^{-s}\,\mathrm{Cos}\left[\frac{\pi s}{2}\right]\,\mathrm{Gamma}[s]}$$

$$\mathtt{zet3[n\_,\ s\_]} := \mathrm{Sum}\left[\left(-\frac{1}{n^{s}\ (-1+s)+2\ n\wedge(1-s)\ (2\pi)^{-s}\,\mathrm{Cos}\left[\frac{\pi s}{2}\right]\,\mathrm{Gamma}[1+s]}\right)\right.$$
$$\left.\left(n^{s}\ (1-s)\ j\wedge -s-n^{1-s}\ s\ j\wedge(s-1)\right),\ \{j,\ 1,\ n\}\right]$$

$$\mathtt{zet3a[n\_,\ s\_]} := \mathrm{Sum}\left[\left(-\frac{1}{n^{s}\ (-1+s)+2\ n\wedge(1-s)\ (2\pi)^{-s}\,\mathrm{Cos}\left[\frac{\pi s}{2}\right]\,\mathrm{Gamma}[1+s]}\right)\right.$$
$$\left.\left(n^{s}\ (1-s)\ j\wedge -s-n^{1-s}\ s\ j\wedge(s-1)\right),\ \{j,\ 1,\ n\}\right]$$

```
N@zet0a[10000000., .3 + 3 I]
```

0.494685 – 0.0632181 ⅈ

```
Zeta[.3 + 3 I]
```

0.49469 – 0.0632084 ⅈ

$$\text{FullSimplify}\Big[\; 1/((1/2)\, s\, (s-1)\, \text{Pi}^\wedge(-s/2)\, \text{Gamma}[s/2])\, \frac{\text{Gamma}\left[1+\frac{s}{2}\right]\text{Gamma}\left[1+\frac{1-s}{2}\right]}{\text{Gamma}\left[1+\frac{s}{2}\right] n^{1-s}\, \pi^{\frac{(1-s)}{2}} - \text{Gamma}\left[1+\frac{1-s}{2}\right] \pi^{s/2}\, n^\wedge s}\Big]$$

$$-\frac{n^s\, (2\pi)^s}{n^{2s}\, (2\pi)^s\, (-1+s) + 2\, n\, \text{Cos}\left[\frac{\pi s}{2}\right] \text{Gamma}[1+s]}$$

$$-\frac{1}{n^s\, (-1+s) + 2\, n^\wedge(1-s)\, (2\pi)^{-s}\, \text{Cos}\left[\frac{\pi s}{2}\right] \text{Gamma}[1+s]}$$

$$\text{FullSimplify}\Big[\text{Expand}\Big[\; 1/((1/2)\, s\, (s-1)\, \text{Pi}^\wedge(-s/2)\, \text{Gamma}[s/2])\, \frac{\text{Gamma}\left[1+\frac{s}{2}\right]\text{Gamma}\left[1+\frac{1-s}{2}\right]}{\text{Gamma}\left[1+\frac{s}{2}\right] n^{1-s}\, \pi^{\frac{(1-s)}{2}} - \text{Gamma}\left[1+\frac{1-s}{2}\right] \pi^{s/2}\, n^\wedge s}\Big]\Big]$$

$$-\frac{1}{n^s\, (-1+s) + 2\, n^\wedge(1-s)\, (2\pi)^{-s}\, \text{Cos}\left[\frac{\pi s}{2}\right] \text{Gamma}[1+s]}$$

$$\text{Limit}\Big[((1-s)\, n^\wedge s\, \text{HarmonicNumber}[n, s] - s\, n^\wedge(1-s)\, \text{HarmonicNumber}[n, 1-s]) \Big/ \left(\frac{2\, n^{1-s}\, \pi^{\frac{1-s}{2}}}{(1-s)\, \text{Gamma}\left[\frac{1}{2}-\frac{s}{2}\right]} - \frac{2\, n^s\, \pi^{s/2}}{s\, \text{Gamma}\left[\frac{s}{2}\right]}\right),\ s \to 1/2\Big]$$

$$-\Big(\text{Gamma}\left[\frac{5}{4}\right] \Big(-4\, \text{HurwitzZeta}\left[\frac{1}{2}, 1+n\right] (-2+\text{Log}[n]) + \left(-4+\pi+\text{Log}[4]+4\,\text{Log}[n]+2\,\text{Log}[\pi]-2\,\text{PolyGamma}\left[0, -\frac{1}{2}\right]\right) \text{Zeta}\left[\frac{1}{2}\right] - 4\, \text{HurwitzZeta}^{(1,0)}\left[\frac{1}{2}, 1+n\right]\Big)\Big) \Big/ \left(4\, \pi^{1/4} \left(2\, \text{Log}[n] + \text{Log}[\pi] - \text{PolyGamma}\left[0, \frac{5}{4}\right]\right)\right)$$

$$1 \Big/ \left(\frac{2\, n^{1-s}\, \pi^{\frac{1-s}{2}}}{(1-s)\, \text{Gamma}\left[\frac{1}{2}-\frac{s}{2}\right]} - \frac{2\, n^s\, \pi^{s/2}}{s\, \text{Gamma}\left[\frac{s}{2}\right]}\right) \text{ /. } s \to 1/2$$

ComplexInfinity

$$\text{Limit}\left[\frac{\frac{1}{2}\, n^{3/2}\, \text{HarmonicNumber}\left[n, \frac{3}{2}\right]}{\frac{n^{3/2}}{2} - \frac{3}{8\sqrt{n}\,\pi}},\ n \to \text{Infinity}\right]$$

$$\text{Zeta}\left[\frac{3}{2}\right]$$