```
Clear[f1, f2, g]
bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k!
f1[f_, n_, k_] := f1[f, n, k] = Sum[ f[j] f1[f, Floor[n / j], k - 1], {j, 2, n}]
f1[f_, n_, 0] := UnitStep[n - 1]
f1z[f_, n_, z_] := Sum[ bin[z, k] f1[f, n, k], {k, 0, Log[2, n]}]
f2[f_, n_, k_] := f2[f, n, k] = Sum[ (-1)^(j + 1) f[j] f2[f, Floor[n / j], k - 1], {j, 2, n}]
f2[f_, n_, 0] := UnitStep[n - 1]
f2z[f_, n_, z_] := Sum[bin[z, k] f2[f, n, k], {k, 0, Log[2, n]}]
fx[ f_, n_] := Expand[FullSimplify[FullSimplify[Expand[D[f1z[f, n, z], z]] /. z → 0] -
    FullSimplify[Expand[D[f2z[f, n, z], z]] /. z → 0]]]
fx2[ f_, n_] := D[f1z[f, n, z] - f2z[f, n, z], z] /. z → 0
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, z_] := dz[n, z] = Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}]
g[n_] := dz[n, 1]
Table[fx[g, 2^n] - fx[g, 2^(n - 1)], {n, 1, 10}]
```

$$\left\{2, 2, \frac{8}{3}, 4, \frac{32}{5}, \frac{32}{3}, \frac{128}{7}, 32, \frac{512}{9}, \frac{512}{5}\right\}$$

```
FullSimplify[Expand[D[f1z[20, z], z]] /. z → 0]
```

$$\frac{f[2]^3}{3} - \frac{f[2]^4}{4} + f[3] + f[4] + f[5] + f[2]^2 \left(-\frac{1}{2} + f[3] + f[4] + f[5]\right) -$$
$$\frac{1}{2}(f[3] + f[4])(f[3] + f[4] + 2f[5]) + f[6] - f[3] f[6] + f[7] + f[8] + f[9] +$$
$$f[2](1 + (-1 + f[3]) f[3] - f[4] - f[5] - f[6] - f[7] - f[8] - f[9] - f[10]) +$$
$$f[10] + f[11] + f[12] + f[13] + f[14] + f[15] + f[16] + f[17] + f[18] + f[19] + f[20]$$

```
FullSimplify[Expand[D[f2z[20, z], z]] /. z → 0]
```

$$-\frac{1}{3} f[2]^3 - \frac{f[2]^4}{4} + f[3] - f[4] + f[5] + f[2]^2 \left(-\frac{1}{2} + f[3] - f[4] + f[5]\right) -$$
$$\frac{1}{2}(f[3] - f[4])(f[3] - f[4] + 2f[5]) - f[6] + f[3] f[6] + f[7] - f[8] + f[9] -$$
$$f[10] - f[2](1 + (-1 + f[3]) f[3] + f[4] - f[5] + f[6] - f[7] + f[8] - f[9] + f[10]) +$$
$$f[11] - f[12] + f[13] - f[14] + f[15] - f[16] + f[17] - f[18] + f[19] - f[20]$$

```
Expand[FullSimplify[FullSimplify[Expand[D[f1z[20, z], z]] /. z → 0] -
   FullSimplify[Expand[D[f2z[20, z], z]] /. z → 0]]]
```

$$2f[2] + \frac{2f[2]^3}{3} - 2f[2] f[3] + 2f[2] f[3]^2 + 2f[4] + 2f[2]^2 f[4] -$$
$$2f[3] f[4] - 2f[2] f[5] - 2f[4] f[5] + 2f[6] - 2f[3] f[6] - 2f[2] f[7] +$$
$$2f[8] - 2f[2] f[9] + 2f[10] + 2f[12] + 2f[14] + 2f[16] + 2f[18] + 2f[20]$$

$$\left\{0, 2, 4, \frac{20}{3}, \frac{32}{3}, \frac{256}{15}, \frac{416}{15}, \frac{4832}{105}, \frac{8192}{105}, \frac{42\,496}{315}, \frac{74\,752}{315}\right\}$$

```
Table[Sum[ 2^j / j , {j, 1, n}], {n, 1, 10}]
```

$$\left\{2, 4, \frac{20}{3}, \frac{32}{3}, \frac{256}{15}, \frac{416}{15}, \frac{4832}{105}, \frac{8192}{105}, \frac{42\,496}{315}, \frac{74\,752}{315}\right\}$$

```
4 / 1, 12 / 2, 40 / 3, 136 / 4, 464 / 5, 1584 / 6, 5408 / 7, 18 464 / 8, 63 040 / 9

1 / 1, 4 / 2, 10 / 3, 34 / 4, 116 / 5
```