```
d2[n_, k_] := Sum[d2[j, k - 1] d2[n / j, 1], {j, Divisors[n]}];
d2[n_, 1] := 1; d2[1, 1] := 0; d2[n_, 0] := 0; d2[1, 0] := 1

d[n_, k_] := Sum[d[j, k - 1] d[n / j, 1], {j, Divisors[n]}];
d[n_, 1] := 1; d[n_, 0] := 0; d[1, 0] := 1

K[n_, 0] := If[n == 1, 1, 0]
K[n_, 1] := If[n == 1, 0, FullSimplify[MangoldtLambda[n] / Log[n]]]
K[n_, k_] := Sum[K[j, k - 1] K[n / j, 1], {j, Divisors[n]}]
K1[n_, k_] := K1[n, k] = Sum[Binomial[k, j] K[n, k - j], {j, 0, k}]

sc[f_, k_, t_] := SeriesCoefficient[Series[f[x], {x, 0, Floor[t]}], k]

q2[b_, f_, n_, 0] := q2[b, f, n, 0] = 1
q2[b_, f_, n_, 1] :=
 q2[b, f, n, 1] = Sum[ b[n, k] sc[f, k, N[Floor[Log[2, n]]], {k, 0, N[Log[2, n]]}]
q2[b_, f_, n_, k_] := q2[b, f, n, k] =
  Sum[q2[b, f, n / j, k - 1] q2[b, f, j, 1], {j, Divisors[n]}]

q1[b_, f_, n_, 0] := q1[b, f, n, 0] = 1
q1[b_, f_, n_, 1] := q1[b, f, n, 1] = Sum[ b[n, k] sc[f, k, 20], {k, 0, 20}]
q1[b_, f_, n_, k_] :=
 q1[b, f, n, k] = Sum[q1[b, f, n / j, k - 1] q1[b, f, j, 1], {j, Divisors[n]}]


Mcos[x_] := -Cos[x]
Msin[x_] := -Sin[x]
Expd[x_] := E^x
Lg1[x_] := Log[x + 1]
Lg2[x_] := Log[1 - x]
lg1[n_, k_] := q2[d2, Lg1, n, k]
lg2[n_, k_] := q2[d2, Lg2, n, k]
lg2d[n_, k_] := q1[d, Lg2, n, k]
expd[n_, k_] := q1[d, Expd, n, k]
expd2[n_, k_] := q2[d2, Expd, n, k]
expk[n_, k_] := q2[K, Expd, n, k]
sind[n_, k_] := q1[K1, Sin, n, k]
cosd[n_, k_] := q1[K1, Cos, n, k]
mcosd[n_, k_] := q1[K1, Mcos, n, k]
msind[n_, k_] := q1[K1, Msin, n, k]
tand[n_, k_] := q2[d2, Tan, n, k]
asinsind[n_, k_] := q2[sind, ArcSin, n, k]
atantand[n_, k_] := q2[tand, ArcTan, n, k]

Table[{n, mcosd[n, 2], msind[n, 2]}, {n, 1, 100}] // TableForm
```

```
1     1     0
2     0     0
3     0     0
4     -1    1
5     0     0
6     -2    2
7     0     0
```

8	-1	1
9	-1	1
10	-2	2
11	0	0
12	-1	1
13	0	0
14	-2	2
15	-2	2
16	$-\frac{7}{12}$	$\frac{7}{12}$
17	0	0
18	-1	1
19	0	0
20	-1	1
21	-2	2
22	-2	2
23	0	0
24	$\frac{2}{3}$	$-\frac{2}{3}$
25	-1	1
26	-2	2
27	-1	1
28	-1	1
29	0	0
30	0	0
31	0	0
32	$-\frac{1}{6}$	$\frac{1}{6}$
33	-2	2
34	-2	2
35	-2	2
36	$\frac{3}{2}$	$-\frac{3}{2}$
37	0	0
38	-2	2
39	-2	2
40	$\frac{2}{3}$	$-\frac{2}{3}$
41	0	0
42	0	0
43	0	0
44	-1	1
45	-1	1
46	-2	2
47	0	0
48	$\frac{3}{2}$	$-\frac{3}{2}$
49	-1	1
50	-1	1
51	-2	2
52	-1	1
53	0	0
54	$\frac{2}{3}$	$-\frac{2}{3}$
55	-2	2
56	$\frac{2}{3}$	$-\frac{2}{3}$
57	-2	2
58	-2	2
59	0	0
60	4	-4

61	0	0
62	−2	2
63	−1	1
64	$\frac{5}{36}$	$-\frac{5}{36}$
65	−2	2
66	0	0
67	0	0
68	−1	1
69	−2	2
70	0	0
71	0	0
72	$\frac{7}{3}$	$-\frac{7}{3}$
73	0	0
74	−2	2
75	−1	1
76	−1	1
77	−2	2
78	0	0
79	0	0
80	$\frac{3}{2}$	$-\frac{3}{2}$
81	$-\frac{7}{12}$	$\frac{7}{12}$
82	−2	2
83	0	0
84	4	−4
85	−2	2
86	−2	2
87	−2	2
88	$\frac{2}{3}$	$-\frac{2}{3}$
89	0	0
90	4	−4
91	−2	2
92	−1	1
93	−2	2
94	−2	2
95	−2	2
96	$\frac{5}{3}$	$-\frac{5}{3}$
97	0	0
98	−1	1
99	−1	1
100	$\frac{3}{2}$	$-\frac{3}{2}$

```
Table[{n, mcosd[n, 1], cosd[n, 1], msind[n, 1], sind[n, 1]}, {n, 1, 10}] // TableForm
```

1	-1	$\frac{1\,314\,502\,564\,969\,066\,301}{2\,432\,902\,008\,176\,640\,000}$	0	$\frac{102\,360\,822\,438\,075\,317}{121\,645\,100\,408\,832\,000}$
2	0	$-\frac{102\,360\,822\,438\,075\,317}{121\,645\,100\,408\,832\,000}$	-1	$\frac{691\,843\,455\,246\,877}{1\,280\,474\,741\,145\,600}$
3	0	$-\frac{102\,360\,822\,438\,075\,317}{121\,645\,100\,408\,832\,000}$	-1	$\frac{691\,843\,455\,246\,877}{1\,280\,474\,741\,145\,600}$
4	$\frac{1}{2}$	$-\frac{21\,010\,743\,835\,816\,079}{30\,411\,275\,102\,208\,000}$	$-\frac{1}{2}$	$-\frac{275\,456\,347\,290\,391}{1\,829\,249\,630\,208\,000}$
5	0	$-\frac{102\,360\,822\,438\,075\,317}{121\,645\,100\,408\,832\,000}$	-1	$\frac{691\,843\,455\,246\,877}{1\,280\,474\,741\,145\,600}$
6	1	$-\frac{691\,843\,455\,246\,877}{1\,280\,474\,741\,145\,600}$	0	$-\frac{23\,023\,126\,954\,133}{27\,360\,571\,392\,000}$
7	0	$-\frac{102\,360\,822\,438\,075\,317}{121\,645\,100\,408\,832\,000}$	-1	$\frac{691\,843\,455\,246\,877}{1\,280\,474\,741\,145\,600}$
8	$\frac{1}{2}$	$-\frac{23\,041\,246\,706\,418\,097}{56\,143\,892\,496\,384\,000}$	$-\frac{1}{6}$	$-\frac{6\,351\,508\,922\,783\,491}{19\,207\,121\,117\,184\,000}$
9	$\frac{1}{2}$	$-\frac{21\,010\,743\,835\,816\,079}{30\,411\,275\,102\,208\,000}$	$-\frac{1}{2}$	$-\frac{275\,456\,347\,290\,391}{1\,829\,249\,630\,208\,000}$
10	1	$-\frac{691\,843\,455\,246\,877}{1\,280\,474\,741\,145\,600}$	0	$-\frac{23\,023\,126\,954\,133}{27\,360\,571\,392\,000}$

```
Table[{n, lg1[n, 1], lg2[n, 1]}, {n, 1, 100}] // TableForm
```

1	0	0
2	1	-1
3	1	-1
4	$\frac{1}{2}$	$-\frac{3}{2}$
5	1	-1
6	0	-2
7	1	-1
8	$\frac{1}{3}$	$-\frac{7}{3}$
9	$\frac{1}{2}$	$-\frac{3}{2}$
10	0	-2
11	1	-1
12	0	-4
13	1	-1
14	0	-2
15	0	-2
16	$\frac{1}{4}$	$-\frac{15}{4}$
17	1	-1
18	0	-4
19	1	-1
20	0	-4
21	0	-2
22	0	-2
23	1	-1
24	0	-8
25	$\frac{1}{2}$	$-\frac{3}{2}$
26	0	-2
27	$\frac{1}{3}$	$-\frac{7}{3}$
28	0	-4
29	1	-1
30	0	-6
31	1	-1
32	$\frac{1}{5}$	$-\frac{31}{5}$
33	0	-2
34	0	-2
35	0	-2
36	0	-10
37	1	-1

38	0	-2
39	0	-2
40	0	-8
41	1	-1
42	0	-6
43	1	-1
44	0	-4
45	0	-4
46	0	-2
47	1	-1
48	0	-16
49	$\frac{1}{2}$	$-\frac{3}{2}$
50	0	-4
51	0	-2
52	0	-4
53	1	-1
54	0	-8
55	0	-2
56	0	-8
57	0	-2
58	0	-2
59	1	-1
60	0	-16
61	1	-1
62	0	-2
63	0	-4
64	$\frac{1}{6}$	$-\frac{21}{2}$
65	0	-2
66	0	-6
67	1	-1
68	0	-4
69	0	-2
70	0	-6
71	1	-1
72	0	-24
73	1	-1
74	0	-2
75	0	-4
76	0	-4
77	0	-2
78	0	-6
79	1	-1
80	0	-16
81	$\frac{1}{4}$	$-\frac{15}{4}$
82	0	-2
83	1	-1
84	0	-16
85	0	-2
86	0	-2
87	0	-2
88	0	-8
89	1	-1
90	0	-16
91	0	-2
92	0	-4

93	0	−2
94	0	−2
95	0	−2
96	0	−32
97	1	−1
98	0	−4
99	0	−4
100	0	−10