

Expand[Integrate[j, {j, 1, n}]]

$$-\frac{1}{2} + \frac{n^2}{2}$$

Expand[Integrate[1, {j, 1, n}, {k, 1, n/j}]]

ConditionalExpression[1 - n + n Log[n], Re[n] ≥ 0 || n ∈ Reals]

Expand[Integrate[j k, {j, 1, n}, {k, 1, n/j}]]

$$\text{ConditionalExpression}\left[\frac{1}{4} - \frac{n^2}{4} + \frac{1}{2} n^2 \text{Log}[n], \text{Re}[n] \geq 0 \mid \mid n \in \text{Reals}\right]$$

Expand[Integrate[j k l, {j, 1, n}, {k, 1, n/j}, {l, 1, n/(j k)}]]

$$\text{ConditionalExpression}\left[-\frac{1}{8} + \frac{n^2}{8} - \frac{1}{4} n^2 \text{Log}[n] + \frac{1}{4} n^2 \text{Log}[n]^2, \text{Re}[n] \geq 0 \mid \mid n \in \text{Reals}\right]$$

Expand[Integrate[j k l m, {j, 1, n}, {k, 1, n/j}, {l, 1, n/(j k)}, {m, 1, n/(j k l)}]]

$$\text{ConditionalExpression}\left[\frac{1}{16} - \frac{n^2}{16} + \frac{1}{8} n^2 \text{Log}[n] - \frac{1}{8} n^2 \text{Log}[n]^2 + \frac{1}{12} n^2 \text{Log}[n]^3, \text{Re}[n] \geq 0 \mid \mid n \in \text{Reals}\right]$$

**Expand[Integrate[j k l m o, {j, 1, n}, {k, 1, n/j},
{l, 1, n/(j k)}, {m, 1, n/(j k l)}, {o, 1, n/(j k l m)}]]**

$$\text{ConditionalExpression}\left[-\frac{1}{32} + \frac{n^2}{32} - \frac{1}{16} n^2 \text{Log}[n] + \frac{1}{16} n^2 \text{Log}[n]^2 - \frac{1}{24} n^2 \text{Log}[n]^3 + \frac{1}{48} n^2 \text{Log}[n]^4, \text{Re}[n] \geq 0 \mid \mid n \in \text{Reals}\right]$$

$$-\frac{1}{2} + \frac{n^2}{2}$$

$$\frac{1}{4} - \frac{n^2}{4} + \frac{1}{2} n^2 \text{Log}[n]$$

$$-\frac{1}{8} + \frac{n^2}{8} - \frac{1}{4} n^2 \text{Log}[n] + \frac{1}{4} n^2 \text{Log}[n]^2$$

$$\frac{1}{16} - \frac{n^2}{16} + \frac{1}{8} n^2 \text{Log}[n] - \frac{1}{8} n^2 \text{Log}[n]^2 + \frac{1}{12} n^2 \text{Log}[n]^3$$

$$-\frac{1}{32} + \frac{n^2}{32} - \frac{1}{16} n^2 \text{Log}[n] + \frac{1}{16} n^2 \text{Log}[n]^2 - \frac{1}{24} n^2 \text{Log}[n]^3 + \frac{1}{48} n^2 \text{Log}[n]^4$$

**f[j_] := (-1)^(j+1) (1/2)^(j+1) +
Sum[n^2 / k! (Log[n])^k (-1)^(j-k) (1/2)^(j-k+1), {k, 0, j}]**

f[0]

$$-\frac{1}{2} + \frac{n^2}{2}$$

**f2[k_] := Expand[
(-1)^(k+1) ((1/2)^(k+1) - n^2/2 Sum[1/j! (-Log[n])^j (1/2)^(k-j), {j, 0, k}])]**

f2[3]

$$\frac{1}{16} - \frac{n^2}{16} + \frac{1}{8} n^2 \text{Log}[n] - \frac{1}{8} n^2 \text{Log}[n]^2 + \frac{1}{12} n^2 \text{Log}[n]^3$$

Expand[Integrate[j^2, {j, 1, n}]]

$$-\frac{1}{3} + \frac{n^3}{3}$$

Expand[Integrate[j^2 k^2, {j, 1, n}, {k, 1, n/j}]]

$$\text{ConditionalExpression}\left[\frac{1}{9} - \frac{n^3}{9} + \frac{1}{3} n^3 \text{Log}[n], \text{Re}[n] \geq 0 \mid n \notin \text{Reals}\right]$$

Expand[Integrate[j^2 k^2 l^2, {j, 1, n}, {k, 1, n/j}, {l, 1, n/(j k)}]]

$$\text{ConditionalExpression}\left[-\frac{1}{27} + \frac{n^3}{27} - \frac{1}{9} n^3 \text{Log}[n] + \frac{1}{6} n^3 \text{Log}[n]^2, \text{Re}[n] \geq 0 \mid n \notin \text{Reals}\right]$$

Expand[

Integrate[j^2 k^2 l^2 m^2, {j, 1, n}, {k, 1, n/j}, {l, 1, n/(j k)}, {m, 1, n/(j k l)}]]

ConditionalExpression[

$$\frac{1}{81} - \frac{n^3}{81} + \frac{1}{27} n^3 \text{Log}[n] - \frac{1}{18} n^3 \text{Log}[n]^2 + \frac{1}{18} n^3 \text{Log}[n]^3, \text{Re}[n] \geq 0 \mid n \notin \text{Reals}]$$

Expand[Integrate[j^2 k^2 l^2 m^2 o^2, {j, 1, n},

{k, 1, n/j}, {l, 1, n/(j k)}, {m, 1, n/(j k l)}, {o, 1, n/(j k l m)}]]

\$Aborted

$$-\frac{1}{3} + \frac{n^3}{3}$$

$$\frac{1}{9} - \frac{n^3}{9} + \frac{1}{3} n^3 \text{Log}[n]$$

$$-\frac{1}{27} + \frac{n^3}{27} - \frac{1}{9} n^3 \text{Log}[n] + \frac{1}{6} n^3 \text{Log}[n]^2$$

$$\frac{1}{81} - \frac{n^3}{81} + \frac{1}{27} n^3 \text{Log}[n] - \frac{1}{18} n^3 \text{Log}[n]^2 + \frac{1}{18} n^3 \text{Log}[n]^3$$

f3[k_] := Expand[

(-1)^(k+1) ((1/3)^(k+1) - n^3/3 Sum[1/j! (-Log[n])^j (1/3)^(k-j), {j, 0, k}])]

f3[3]

$$\frac{1}{81} - \frac{n^3}{81} + \frac{1}{27} n^3 \text{Log}[n] - \frac{1}{18} n^3 \text{Log}[n]^2 + \frac{1}{18} n^3 \text{Log}[n]^3$$

fs[n_, k_, s_] := Expand[(-1)^(k+1) ((1/(1-s))^(k+1) -

n^(1-s)/(1-s) Sum[1/j! (-Log[n])^j (1/(1-s))^(k-j), {j, 0, k}])]

fs[n, 3, 0]

$$1 - n + n \text{Log}[n] - \frac{1}{2} n \text{Log}[n]^2 + \frac{1}{6} n \text{Log}[n]^3$$

```
ps[n_, s_] := Sum[ (-1) ^ (k + 1) / k N[fs[n, k, s]], {k, 1, 100}]
```

```
fs[n, 4, ZetaZero[1]]
```

$$-\frac{1}{(1 - \text{ZetaZero}[1])^5} + \frac{n^{1-\text{ZetaZero}[1]}}{(1 - \text{ZetaZero}[1])^5} - \frac{n^{1-\text{ZetaZero}[1]} \text{Log}[n]}{(1 - \text{ZetaZero}[1])^4} + \frac{n^{1-\text{ZetaZero}[1]} \text{Log}[n]^2}{2 (1 - \text{ZetaZero}[1])^3} - \frac{n^{1-\text{ZetaZero}[1]} \text{Log}[n]^3}{6 (1 - \text{ZetaZero}[1])^2} + \frac{n^{1-\text{ZetaZero}[1]} \text{Log}[n]^4}{24 (1 - \text{ZetaZero}[1])}$$

```
ee[n_] := (-0.0049864938890432945` + 0.00035322521742155925` i) +  
  (0.0049864938890432945` - 0.00035322521742155925` i) n0.5` - 14.134725141734695` i +  
  (0.0024994944168615697` + 0.07065933315127745` i) n0.5` - 14.134725141734695` i Log[n]
```

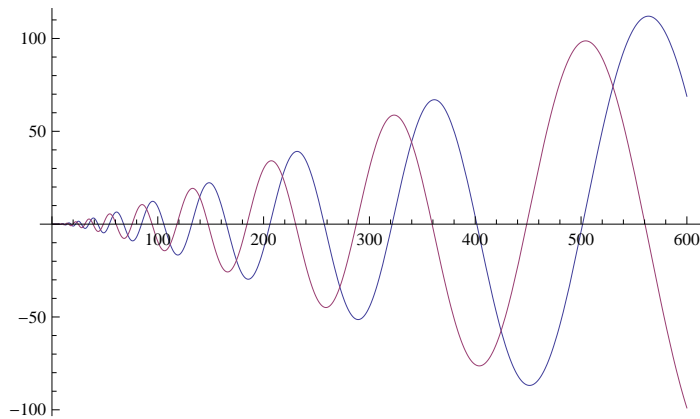
```
ps[100, 0]
```

```
182.601
```

```
N[ZetaZero[1]]
```

```
0.5 + 14.1347 i
```

```
Plot[{Re[fs[n, cc = 4, dc = 1 / 2 + 14.134725141734695` I]], Im[fs[n, cc, dc]]}, {n, 1, 600}]
```



```
fs[n, 4, 0]
```

$$-1 + n - n \text{Log}[n] + \frac{1}{2} n \text{Log}[n]^2 - \frac{1}{6} n \text{Log}[n]^3 + \frac{1}{24} n \text{Log}[n]^4$$

```
fs[n, 3, 0]
```

$$1 - n + n \text{Log}[n] - \frac{1}{2} n \text{Log}[n]^2 + \frac{1}{6} n \text{Log}[n]^3$$

```
N[fs[100, 3, 0]]
```

```
928.88
```

```
N[Gamma[4, 0, -Log[100]] / Gamma[4]]
```

```
928.88 - 3.40898 × 10-13 i
```

```
N[Integrate[t ^ (4 - 1) E ^ (-t), {t, 0, -Log[100]}] / Gamma[4]]
```

```
928.88
```

fs[n, 3, -1]

$$\frac{1}{16} - \frac{n^2}{16} + \frac{1}{8} n^2 \operatorname{Log}[n] - \frac{1}{8} n^2 \operatorname{Log}[n]^2 + \frac{1}{12} n^2 \operatorname{Log}[n]^3$$

N[fs[100, 3, -1]]

60 009.2

N[Integrate[t^(4-1) E^(-2 t), {t, 0, -Log[100]}] / Gamma[4]]

60 009.2

N[fs[100, 3, -2]]

4.40583×10^6

N[Integrate[t^(4-1) E^(-3 t), {t, 0, -Log[100]}] / Gamma[4]]

4.40583×10^6

Integrate[t^(a-1) E^(-t), {t, 0, -Log[n]}] / Gamma[a]

ConditionalExpression $\left[\frac{\operatorname{Gamma}[a] - \operatorname{Gamma}[a, -\operatorname{Log}[n]]}{\operatorname{Gamma}[a]}, \operatorname{Re}[a] > 0\right]$

Integrate[t^(a-1) E^(-2 t), {t, 0, -Log[n]}] / Gamma[a]

ConditionalExpression $\left[\frac{2^{-a} (\operatorname{Gamma}[a] - \operatorname{Gamma}[a, -2 \operatorname{Log}[n]])}{\operatorname{Gamma}[a]}, \operatorname{Re}[a] > 0\right]$

Integrate[t^(a-1) E^(-3 t), {t, 0, -Log[n]}] / Gamma[a]

ConditionalExpression $\left[\frac{3^{-a} (\operatorname{Gamma}[a] - \operatorname{Gamma}[a, -3 \operatorname{Log}[n]])}{\operatorname{Gamma}[a]}, \operatorname{Re}[a] > 0\right]$

Integrate[t^(a-1) E^(1 t), {t, 0, -Log[n]}] / Gamma[a]

ConditionalExpression $\left[\frac{(\operatorname{Gamma}[a] - \operatorname{Gamma}[a, \operatorname{Log}[n]]) (-\operatorname{Log}[n])^a \operatorname{Log}[n]^{-a}}{\operatorname{Gamma}[a]}, \operatorname{Re}[a] > 0\right]$

Integrate[t^(a-1) E^(-s t), {t, 0, -Log[n]}] / Gamma[a]

ConditionalExpression $\left[\frac{(\operatorname{Gamma}[a] - \operatorname{Gamma}[a, -s \operatorname{Log}[n]]) (-\operatorname{Log}[n])^a (-s \operatorname{Log}[n])^{-a}}{\operatorname{Gamma}[a]}, \operatorname{Re}[a] > 0\right]$

$(-\operatorname{Log}[n])^a (-s \operatorname{Log}[n])^{-a} /. s \rightarrow -2 + i$

$(-\operatorname{Log}[n])^a ((2 - i) \operatorname{Log}[n])^{-a}$

-N[Integrate[t^(2-1) E^(2 t), {t, 0, -Log[100]}]]

-0.249745

Expand $\left[\frac{(\operatorname{Gamma}[a] - \operatorname{Gamma}[a, -s \operatorname{Log}[n]]) (-\operatorname{Log}[n])^a (-s \operatorname{Log}[n])^{-a}}{\operatorname{Gamma}[a]}\right]$

Fa1[n_, a_, s_] :=

$$(-\text{Log}[n])^a (- (1 - s) \text{Log}[n])^{-a} - \frac{\text{Gamma}[a, - (1 - s) \text{Log}[n]] (-\text{Log}[n])^a (- (1 - s) \text{Log}[n])^{-a}}{\text{Gamma}[a]}$$

Fa2[n_, a_, s_] := $\frac{1}{\text{Gamma}[a]} (\text{Gamma}[a] - \text{Gamma}[a, - (1 - s) \text{Log}[n]]) (-\text{Log}[n])^a (- (1 - s) \text{Log}[n])^{-a}$

Fa3[n_, a_, s_] := $\frac{(\text{Gamma}[a, 0, - (1 - s) \text{Log}[n]]) (1 - s)^{-a}}{\text{Gamma}[a]}$

N[{Fa1[a0 = 140, a1 = 2, a2 = 0], Fa2[a0, a1, a2], Fa3[a0, a1, a2]}]

{552.83 - 6.75797 × 10⁻¹⁴ i, 552.83 - 6.75797 × 10⁻¹⁴ i, 552.83 - 6.75797 × 10⁻¹⁴ i}

FullSimplify[$\frac{(\text{Gamma}[a, 0, -s \text{Log}[n]]) (-\text{Log}[n])^a (-s \text{Log}[n])^{-a}}{\text{Gamma}[a]}$]

$\frac{\text{Gamma}[a, 0, -s \text{Log}[n]] (-\text{Log}[n])^a (-s \text{Log}[n])^{-a}}{\text{Gamma}[a]} /. s \rightarrow 0$

$\frac{0^{-a} \text{Gamma}[a, 0, 0] (-\text{Log}[n])^a}{\text{Gamma}[a]}$

FullSimplify[$(-\text{Log}[n])^a (-s \text{Log}[n])^{-a}$]

e1[n_, a_, s_] := $(-\text{Log}[n])^a (-s \text{Log}[n])^{-a}$

e1[23, 2, 3]

$\frac{1}{9}$

e2[n_, a_, s_] := $(-\text{Log}[n])^a / (-s \text{Log}[n])^a$

e2[23, 2, 3]

$\frac{1}{9}$

e3[n_, a_, s_] := $((\text{Log}[n]) / (s \text{Log}[n]))^a$

e3[23, 2, 3]

$\frac{1}{9}$

e4[n_, a_, s_] := s^{-a}

e4[23, 2, 3]

$\frac{1}{9}$

Sum[$t^{(j-1)} / (j!)$, {j, 1, Infinity}]

$\frac{-1 + e^t}{t}$