```
zm[n_, s_, k_, y_] := Sum[j^-s zm[n / j, s, k - 1, y], {j, y, n}]
zm[n_, s_, 0, y_] := UnitStep[n - 1]
```

```
zm[100, -1, 1, 10]
```

```
5005
```

```
zm[100, -1, 1, 1]
```

```
5050
```

```
zm[100, -1, 1, 1] - zm[9, -1, 1, 1]
```

```
5005
```

```
Table[FullSimplify[ zm[n, s, 3, Floor[n^(1 / 3)]]], {n, 1, 40}] // TableForm
```

$$1$$
$$1+3\times 2^{-s}$$
$$1+3\times 2^{-s}+3^{1-s}$$
$$1+3\times 2^{1-2 s}+3\times 2^{-s}+3^{1-s}$$
$$1+3\times 2^{1-2 s}+3\times 2^{-s}+3^{1-s}+3\times 5^{-s}$$
$$60^{-s}\left(3^{1+s} 4^s+5^s\left(3\times 2^s\left(3+2^s\right)+3^s\left(6+3\times 2^s+4^s\right)\right)\right)$$
$$420^{-s}\left(70^s\left(9+3^{1+s}+6^s\right)+3\left(60^s+84^s+2\times 105^s+140^s\right)\right)$$
$$8^{-s}$$
$$8^{-s}$$
$$8^{-s}$$
$$8^{-s}$$
$$24^{-s}\left(3\times 2^s+3^s\right)$$
$$24^{-s}\left(3\times 2^s+3^s\right)$$
$$24^{-s}\left(3\times 2^s+3^s\right)$$
$$24^{-s}\left(3\times 2^s+3^s\right)$$
$$48^{-s}\left(3^{1+s}+3\times 4^s+6^s\right)$$
$$48^{-s}\left(3^{1+s}+3\times 4^s+6^s\right)$$
$$144^{-s}\left(9^s\left(3+2^s\right)+3\times 4^s\left(2^s+3^s\right)\right)$$
$$144^{-s}\left(9^s\left(3+2^s\right)+3\times 4^s\left(2^s+3^s\right)\right)$$
$$720^{-s}\left(3\times 40^s+90^s+3^{1+s}\left(12^s+15^s+20^s\right)\right)$$
$$720^{-s}\left(3\times 40^s+90^s+3^{1+s}\left(12^s+15^s+20^s\right)\right)$$
$$720^{-s}\left(3\times 40^s+90^s+3^{1+s}\left(12^s+15^s+20^s\right)\right)$$
$$720^{-s}\left(3\times 40^s+90^s+3^{1+s}\left(12^s+15^s+20^s\right)\right)$$
$$720^{-s}\left(3^{1+2 s} 4^s+5^s\left(3\times 8^s+2^s 3^{1+s}\left(3+2^s\right)+9^s\left(3+2^s\right)\right)\right)$$
$$720^{-s}\left(3^{1+2 s} 4^s+5^s\left(3\times 8^s+2^s 3^{1+s}\left(3+2^s\right)+9^s\left(3+2^s\right)\right)\right)$$
$$720^{-s}\left(3^{1+2 s} 4^s+5^s\left(3\times 8^s+2^s 3^{1+s}\left(3+2^s\right)+9^s\left(3+2^s\right)\right)\right)$$
$$3^{-3 s}$$
$$3^{-3 s}$$
$$3^{-3 s}$$
$$3^{-3 s}$$
$$3^{-3 s}$$
$$3^{-3 s}$$
$$3^{-3 s}$$
$$3^{-3 s}$$
$$108^{-s}\left(3^{1+s}+4^s\right)$$
$$108^{-s}\left(3^{1+s}+4^s\right)$$
$$108^{-s}\left(3^{1+s}+4^s\right)$$
$$108^{-s}\left(3^{1+s}+4^s\right)$$
$$108^{-s}\left(3^{1+s}+4^s\right)$$

```
FullSimplify[j^-s (n j^-1)^-s] /. s → -3
```

$$n^3$$

```
3^4 × 4^4
```

$$20\,736$$

```
12^4
```

```
20 736
```

```
Grid[Table[ 50 / Abs[(a + 5 I) (b + 5 I)], {a, 1, 10}, {b, 1, 10}]]
```

```
N[Log[s Log[n] ] - Log[(s - 1) Log[n]] /. {s → 2, n → 100}]
```

```
0.693147
```

```
N[Log[(s Log[n]) / ((s - 1) Log[n]) ] /. {s → 2, n → 100}]
```

```
0.693147
```

```
N[Log[s / (s - 1) ] /. {s → 2, n → 100}]
```

```
0.693147
```

```
N[Gamma[0, s Log[n] ] - Gamma[0, (s - 1) Log[n]] /. {s → 2, n → 100}]
```

```
-0.00181987
```

```
Integrate[ t^(k - 1) E^-t, {t, (s - 1) Log[n], s Log[n]}]
```

$$\text{ConditionalExpression}\left[\text{Gamma}[k, (-1+s)\,\text{Log}[n]] - \text{Gamma}[k, s\,\text{Log}[n]],\ \left(\left(\text{Arg}[n] \le 0 \&\& -i\,\text{Abs}[\text{Log}[n]]^2\,(s - \text{Re}[s]) \ge 0\right) \mid\mid \left(\text{Arg}[n] \ge 0 \&\& -i\,\text{Abs}[\text{Log}[n]]^2\,(s - \text{Re}[s]) \le 0\right)\right) \&\& (s \notin \text{Reals} \mid\mid \text{Re}[s] \ge 1 \mid\mid \text{Re}[s] \le 0)\right]$$

```
-(ExpIntegralEi[-s Log[n]] - ExpIntegralEi[-(s - 1) Log[n]])
```

$$\text{ExpIntegralEi}[(1-s)\,\text{Log}[n]] - \text{ExpIntegralEi}[-s\,\text{Log}[n]]$$

```
FullSimplify[Log[ Abs[ (1 - s) Log[n]]] - Log[Abs[-s Log[n]]] /. {s → -3 + I, n → 20}]
```

$$\frac{1}{2}\,\text{Log}\left[\frac{17}{10}\right]$$

```
FullSimplify[Log[ Abs[ (1 - s) Log[n]]] - Log[Abs[-s Log[n]]]]
```

$$\text{Log}\left[\frac{\text{Abs}[(-1+s)\,\text{Log}[n]]}{\text{Abs}[s\,\text{Log}[n]]}\right]$$

```
FullSimplify[Log[ Abs[ (1 - s)] Log[n]] - Log[Abs[-s] Log[n]]] /. s → 2
```

```
-Log[2]
```

```
Log[ Abs[ (s - 1) / s]] /. s → -3
```

$$\text{Log}\left[\frac{4}{3}\right]$$

```
FullSimplify[Log[ Abs[ (1 - s) / s]]]
```

$$\text{Log}\left[\text{Abs}\left[-1 + \frac{1}{s}\right]\right]$$

```
Sum[ ((1 - s)^k - (-s)^k ) (Log[n])^k / k / k!, {k, 1, Infinity}]
```

```
-Gamma[0, (-1 + s) Log[n]] + Gamma[0, s Log[n]] - Log[(-1 + s) Log[n]] + Log[s Log[n]]
```

```
ss[k_] := Expand[ Sum[(-1)^j Binomial[k, j] s^j , {j, 0, k}]] / k / k!
```

```
-(ExpIntegralEi[Log[n^-s]] - ExpIntegralEi[Log[n^(1 - s)]])
```

```
ExpIntegralEi[Log[n^(1-s)]] - ExpIntegralEi[Log[n^-s]]
```

```
N[Gamma[0, s Log[n]] - Gamma[0, (s - 1) Log[n]] /. {n → 3, s → 3}]
```

```
-0.0283455
```

```
N[Gamma[0, s / (s - 1)] /. {n → 3, s → 3}]
```

```
0.10002
```