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ClearAll["Global`*"]
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lo[n_, k_] := Sum[ (-1)^(j+1) lo[Floor[n/j], k-1], {j, 1, n}];
lo[n_, 1] := Sum[ (-1)^(j+1) Log[j], {j, 1, n}]
t[n_, a_] := Mod[n, a] - Mod[n-1, a]
lp[n_, k_, b_] := Sum[ t[j, b] lp[Floor[n/j], k-1, b], {j, 1, n}];
lp[n_, 1, b_] := Sum[ t[j, b] Log[j], {j, 1, n}]
fa[n_, k_] := Sum[ 2^j Binomial[k, j] (-1)^j l1[n/2^j, k], {j, 0, k}] +
  Sum[ 2^j Binomial[k-1, j-1] (-1)^j Log[2] d1[n/2^j, k], {j, 1, k}]

L1[n_, k_] := Sum[ L1[Floor[n/j], k-1], {j, 1, n}];
L1[n_, 1] := Sum[ Log[j], {j, 1, n}]; L1[n_, 0] := 1
L2[n_, k_] := Sum[ L2[Floor[n/j], k-1], {j, 2, n}]; L2[n_, 1] := Sum[ Log[j], {j, 2, n}]
D1[n_, k_] := Sum[ D1[Floor[n/j], k-1], {j, 1, n}]; D1[n_, 0] := 1
L2toL1[n_, z_] := Sum[ FactorialPower[z-1, a] / a! L2[n, a+1], {a, 0, Log[2, n]}]
L2toL1x[n_, z_] := Sum[ Binomial[z-1, a] L2[n, a+1], {a, 0, Log[2, n]}]
L1toL2[n_, k_] := Sum[ (-1)^(k-j) Binomial[k-1, j-1] L1[n, j], {j, 1, k}]

EL[n_, k_, b_] :=
  EL[n, k, b] = Sum[ EL[n/j, k-1, b], {j, 1, n}] - b Sum[ EL[n/(j b), k-1, b], {j, 1, n}];
EL[n_, 1, b_] := EL[n, 1, b] = Sum[ Log[j], {j, 1, n}] - b Sum[ Log[j b], {j, 1, n/b}]
LtoEL[n_, k_, b_] := Sum[ b^j Binomial[k, j] (-1)^j L1[n/b^j, k], {j, 0, k}] +
  Sum[ b^j Binomial[k-1, j-1] (-1)^j Log[b] D1[n/b^j, k], {j, 1, k}]

EL1toL1[n_, b_] := Sum[ b^j EL[n/b^j, 1, b], {j, 0, Log[b, n]}] +
  Log[b] Sum[ b^j D1[n/b^j, 1], {j, 1, Log[b, n]}]

EL2[n_, k_, b_] :=
  EL2[n, k, b] = Sum[ EL2[n/j, k-1, b], {j, 2, n}] + b Sum[ EL2[n/(j b), k-1, b], {j, 1, n}];
EL2[n_, 1, b_] := EL2[n, 1, b] = Sum[ Log[j], {j, 2, n}] + b Sum[ Log[j b], {j, 1, n/b}]
EL2toEL1[n_, z_, b_] :=
  Sum[ FactorialPower[z-1, a] / a! EL2[n, a+1, b], {a, 0, Log[If[b < 2, b, 2], n]}]
EL1toEL2[n_, k_, b_] := Sum[ (-1)^(k-j) Binomial[k-1, j-1] EL[n, j, b], {j, 1, k}]

```

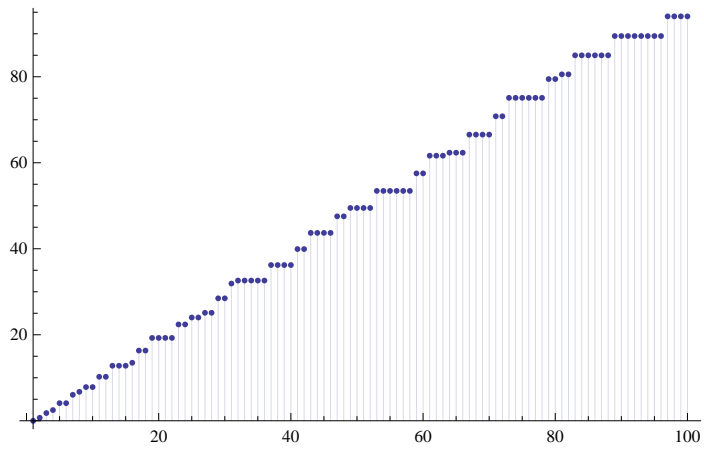
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N[L2toL1x[100, 0]]
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94.0453
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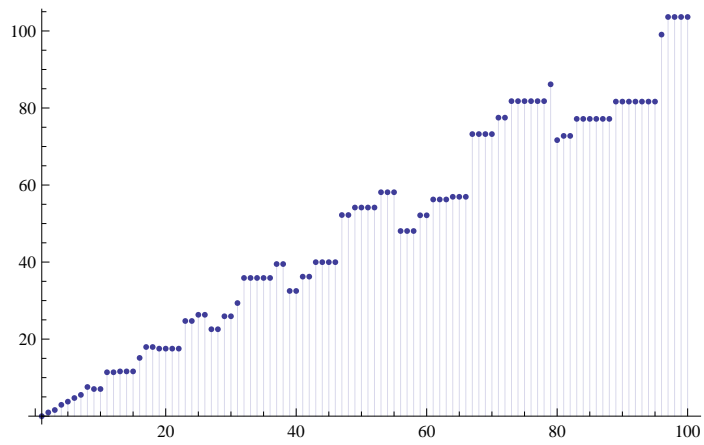
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N[EL2toEL1[100, 0, 101]]
```

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94.0453
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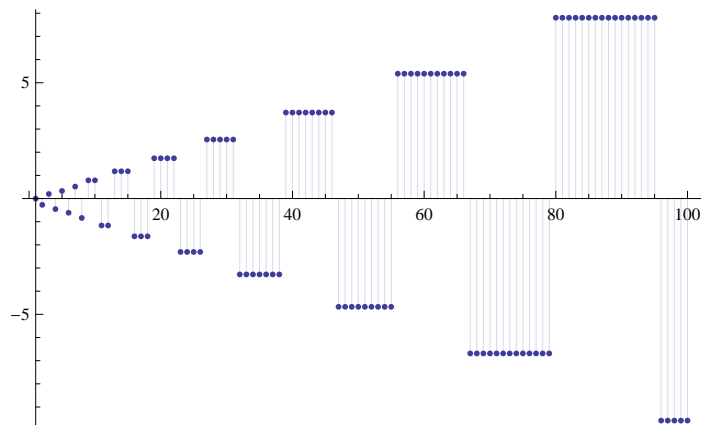
DiscretePlot[L2toL1[n, 0], {n, 1, 100}]



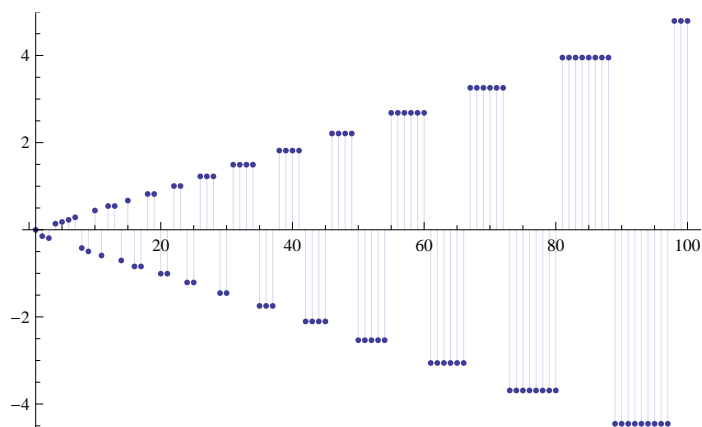
DiscretePlot[EL2toEL1[n, 0, 1.2], {n, 1, 100}]



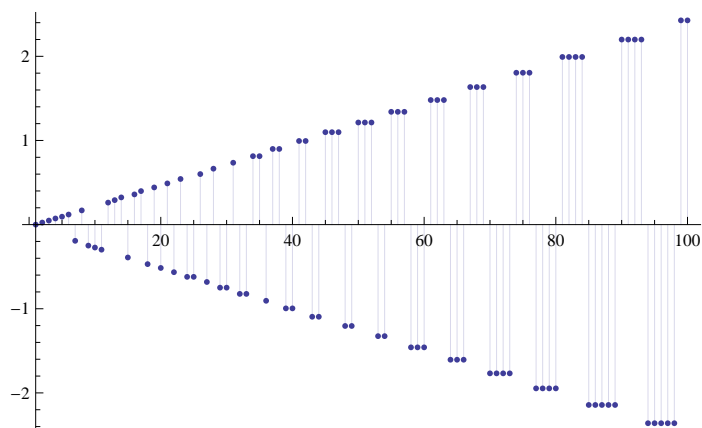
DiscretePlot[L2toL1[n, 0] - EL2toEL1[n, 0, 1.2], {n, 1, 100}]



```
DiscretePlot[L2toL1[n, 0] - EL2toEL1[n, 0, 1.1], {n, 1, 100}]
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DiscretePlot[L2toL1[n, 0] - EL2toEL1[n, 0, 1.05], {n, 1, 100}]
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N[Log[2 Pi]]
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1.83788
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fdif[n_, b_] := Sum[ (-1)^j Log[b^(b^j)], {j, 1, Log[b, n]}]
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N[fdif[100, 1.00001]]
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-0.000504998
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