```
$RecursionLimit = 10 000
Clear[ds, dw, pt, id1]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k !
bina[z_, k_, a_] := Product[z - j a, {j, 0, k - 1}] / k !
FI[n_] := FactorInteger[n]; FI[1] := {}
pt[n_, z_] := pt[n, z] = Product[z^p[[2]] / (p[[2]]!), {p, FI[n]}]
dz[n_, z_] := dz[n, z] = Product[(-1)^p[[2]] bin[-z, p[[2]]], {p, FI[n]}]
dza[n_, z_, a_] := dza[n, z, a] = Product[(-1)^p[[2]] bina[-z, p[[2]], a], {p, FI[n]}]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k !
ds[fn_, n_, s_, y_, z_] :=
 ds[fn, n, s, y, z] = If[ n < y, 1, Sum[ bin[z, k] If[ fn[y] == 0 && k == 0, 1, fn[y]^k
      (y^(-s k)) ds[fn, Floor[n / y^k] , s, y + 1, z - k], {k, 0, Log[y, n]}]]
dw[fn_, n_, s_, k_] := dw[fn, n, s, k] = Sum[ fn[j] j^-s dw[fn, Floor[n / j], s, k - 1],
    {j, 2, n}]
dw[fn_, n_, s_, 0] := UnitStep[n - 1]
dwz[fn_, n_, s_, z_] := Sum[ bin[z, k] dw[fn, n, s, k], {k, 0, Log2@n}]
dif[fn_, n_, s_, z_] := dwz[fn, n, s, z] - dwz[fn, n - 1, s, z]
dif2[fn_, n_, s_, z_] := ds[fn, n, s, 2, z] - ds[fn, n - 1, s, 2, z]
id0[n_] := 1
id1[n_] := Abs[MoebiusMu[n]]
id2[n_] := EulerPhi[n]
id3a[n_] := DivisorSigma[0, n]
id3[n_] := DivisorSigma[1, n]
id32[n_] := DivisorSigma[2, n]
id4[n_] := DivisorSigma[3, n]
id5[n_] := FiniteAbelianGroupCount[n]
id6[n_] := pt[n, 1]
id7[n_] := LiouvilleLambda[n]
id8[n_] := pt[n, 3]
id9[n_] := 3 / 2
```

10 000

Expand@dwz[id9, 200, -1, z]

$$1 + \frac{233\,795\,921\,827\,z}{6720} - \frac{507\,858\,113\,893\,z^2}{5760} + \frac{494\,992\,863\,833\,z^3}{5760} -$$
$$\frac{49\,035\,152\,777\,z^4}{1152} + \frac{67\,988\,575\,109\,z^5}{5760} - \frac{5\,208\,126\,911\,z^6}{2880} + \frac{1\,443\,215\,891\,z^7}{10\,080}$$

Expand@ds[id9, 200, -1, 2, z]

$$1 + \frac{233\,795\,921\,827\,z}{6720} - \frac{507\,858\,113\,893\,z^2}{5760} + \frac{494\,992\,863\,833\,z^3}{5760} -$$
$$\frac{49\,035\,152\,777\,z^4}{1152} + \frac{67\,988\,575\,109\,z^5}{5760} - \frac{5\,208\,126\,911\,z^6}{2880} + \frac{1\,443\,215\,891\,z^7}{10\,080}$$

Table[D[pt[n, z], z] /. z → 0, {n, 2, 10}]

{1, 1, 0, 1, 0, 1, 0, 0, 0}

```
Table[ D[dza[n, z, 3], z] /. z → 0, {n, 1, 100}]
```

$$\{0, 1, 1, \frac{3}{2}, 1, 0, 1, 3, \frac{3}{2}, 0, 1, 0, 1, 0, 0, \frac{27}{4}, 1, 0, 1, 0, 0, 0,$$

$$1, 0, \frac{3}{2}, 0, 3, 0, 1, 0, 1, \frac{81}{5}, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0,$$

$$\frac{3}{2}, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, \frac{81}{2}, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0,$$

$$0, 0, 0, 0, 1, 0, \frac{27}{4}, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0\}$$

```
Expand[{dif[aa = id0, 27, 0, z] dif[aa, 16, 0, z], Expand@dif[aa, 27 × 16, 0, z]}]
```

$$\left\{\frac{z^2}{12} + \frac{5 z^3}{18} + \frac{17 z^4}{48} + \frac{31 z^5}{144} + \frac{z^6}{16} + \frac{z^7}{144}, \frac{z^2}{12} + \frac{5 z^3}{18} + \frac{17 z^4}{48} + \frac{31 z^5}{144} + \frac{z^6}{16} + \frac{z^7}{144}\right\}$$

```
Table[{FullSimplify@dif[id0, 2^k, 0, z], Pochhammer[z, k] / k!}, {k, 0, 5}] // TableForm
```

1	1
z	z
$\frac{1}{2}$ z (1 + z)	$\frac{1}{2}$ z (1 + z)
$\frac{1}{6}$ z (1 + z) (2 + z)	$\frac{1}{6}$ z (1 + z) (2 + z)
$\frac{1}{24}$ z (1 + z) (2 + z) (3 + z)	$\frac{1}{24}$ z (1 + z) (2 + z) (3 + z)
$\frac{1}{120}$ z (1 + z) (2 + z) (3 + z) (4 + z)	$\frac{1}{120}$ z (1 + z) (2 + z) (3 + z) (4 + z)

```
Table[{FullSimplify@dif[id1, 2^k, 0, z], (-1)^k Pochhammer[-z, k] / k!}, {k, 0, 5}] //
 TableForm
```

1	1
z	z
$\frac{1}{2}$ (-1 + z) z	$-\frac{1}{2}$ (1 - z) z
$\frac{1}{6}$ (-2 + z) (-1 + z) z	$\frac{1}{6}$ (1 - z) (2 - z) z
$\frac{1}{24}$ (-3 + z) (-2 + z) (-1 + z) z	$-\frac{1}{24}$ (1 - z) (2 - z) (3 - z) z
$\frac{1}{120}$ (-4 + z) (-3 + z) (-2 + z) (-1 + z) z	$\frac{1}{120}$ (1 - z) (2 - z) (3 - z) (4 - z) z

```
Table[{dif[id3a, 2^k, 0, z]}, {k, 1, 5}] // TableForm
```

2 z

3 z + 2 (-1 + z) z

4 z + 6 (-1 + z) z + $\frac{4}{3}$ (-2 + z) (-1 + z) z

5 z + $\frac{25}{2}$ (-1 + z) z + 6 (-2 + z) (-1 + z) z + $\frac{2}{3}$ (-3 + z) (-2 + z) (-1 + z) z

6 z + 22 (-1 + z) z + 17 (-2 + z) (-1 + z) z + 4 (-3 + z) (-2 + z) (-1 + z) z + $\frac{4}{15}$ (-4 + z) (-3 + z) (-2 + z) (-

```
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, z_] := dz[n, z] = Product[(-1)^p[[2]] bin[-z, p[[2]]], {p, FI[n]}]
dzr[n_, z_] := Product[Pochhammer[z, p[[2]]] / (p[[2]]!), {p, FI[n]}]
daz[n_, a_, z_] := Product[(-1)^p[[2]] bin[-(a z), p[[2]]], {p, FI[n]}]
rise[n_, z_] := (-1)^z Pochhammer[-n, z]
dsz[n_, a_, z_] :=
 Product[Pochhammer[z, p[[2]]] / (p[[2]]!) (p[[1]]^(a p[[2]]) + 1) , {p, FI[n]}]
```

```
dz[4, 2]
```

3

```
D[dsz[4, 1, z], z] /. z -> 0
```

$\dfrac{5}{2}$

```
dsz[32, 1, 1]
```

33

```
FullSimplify@dif[id3, 32, 0, 1]
```

63

```
daz[2^7, 2, 7]
```

77 520

```
dsz[32, 0, 3] + dsz[32, 0, 3]
```

12

```
2^6
```

64


```
(Pochhammer[2, 2] Pochhammer[3, 2])
```

72

```
Pochhammer[6, 2]
```

42

```
FactorialPower[z, 2]
```

FactorialPower[z, 2]

```
rise[z, 2]
```

$-(1-z)z$

```
sig[n_, a_] := Product[(p[[1]]^((p[[2]] + 1) a) - 1) / (p[[1]]^a - 1), {p, FI[n]}]
sig2[n_, a_] := Product[Sum[ p[[1]]^(a j), {j, 0, p[[2]]}], {p, FI[n]}]
```

```
sig2[100, 2]
```

13 671

```
DivisorSigma[2, 100]
```

13 671

```
Table[ D[dif[id32, n, 0, z], z] /. z → 0, {n, 1, 100}]
```

$$\left\{0, 5, 10, \frac{17}{2}, 26, 0, 50, \frac{65}{3}, 41, 0, 122, 0, 170, 0, 0, \frac{257}{4}, 290, 0, 362, 0, 0, 0, 530, 0, 313, 0, \right.$$

$$\frac{730}{3}, 0, 842, 0, 962, 205, 0, 0, 0, 0, 1370, 0, 0, 0, 1682, 0, 1850, 0, 0, 0, 2210, 0, 1201,$$

$$0, 0, 0, 2810, 0, 0, 0, 0, 0, 3482, 0, 3722, 0, 0, \frac{4097}{6}, 0, 0, 4490, 0, 0, 0, 5042, 0, 5330,$$

$$\left. 0, 0, 0, 0, 0, 6242, 0, \frac{3281}{2}, 0, 6890, 0, 0, 0, 0, 0, 7922, 0, 0, 0, 0, 0, 0, 9410, 0, 0, 0 \right\}$$

```
dif[id32, 100, 0, 2]
```

132 526

```
sig2a[n_, a_] := Product[Sum[ p[[1]]^(a j), {j, 0, p[[2]]}], {p, FI[n]}]
```

```
sig2a[100, 2]
```

13 671

```
DivisorSigma[3, 640]
```

301 989 870

```
Sum[ Pochhammer[z, i] / (i !) p^((a - s) i)
   Pochhammer[z, k - i] / ((k - i) !) p^(-s (k - i) ) , {i, 0, k}]
```

$$\frac{p^{-k\,s} \, \text{Gamma}[k + z] \, \text{Hypergeometric2F1}[-k, z, 1 - k - z, p^a]}{\text{Gamma}[1 + k] \, \text{Gamma}[z]}$$

```
Sum[ Binomial[z + i - 1, i] p^((a - s) i) Binomial[z + k - i - 1, k - i] p^(-s (k - i) ) , {i, 0, k}]
```

$$p^{-k\,s} \, \text{Binomial}[-1 + k + z, k] \, \text{Hypergeometric2F1}[-k, z, 1 - k - z, p^a]$$

```
p^-k s Binomial[-1 + k + z, k] Hypergeometric2F1[-k, z, 1 - k - z, p^a] /.
  {z → 1, p → 7, a → 1, k → 3, s → 0}
```

400

```
D[p^-k s Binomial[-1 + k + z, k] Hypergeometric2F1[-k, z, 1 - k - z, p^a] /.
    {p → 7, a → 2, k → 3, s → 0}, z] /. z → 0
```

$$\frac{117\,650}{3}$$

```
p^-k s Binomial[-1 + k + z, k] Hypergeometric2F1[-k, z, 1 - k - z, p^a] /.
  {z → 1, p → 7, a → 1, k → 3, s → 0}
```


```
dsig[n_, s_, a_, z_] := Product[ p[[1]]^-p[[2]] s Binomial[-1 + p[[2]] + z, p[[2]]]
    Hypergeometric2F1[-p[[2]], z, 1 - p[[2]] - z, p[[1]]^a], {p, FI[n]}]
```

```
dsig[640, 0, 3, 1]
```

301 989 870

```
Sum[ Binomial[z + i - 1, i] p^((a - s) i) Binomial[-z + k - i - 1, k - i] p^(-s (k - i) ) , {i, 0, k}]
```

$$p^{-k\,s} \, \text{Binomial}[-1 + k - z, k] \, \text{Hypergeometric2F1}[-k, z, 1 - k + z, p^a]$$

```
deul[n_, s_, a_, z_] := Product[ p[[1]]^-p[[2]] s Binomial[-1 + p[[2]] - z, p[[2]]]
    Hypergeometric2F1[-p[[2]], z, 1 - p[[2]] + z, p[[1]]^a], {p, FI[n]}]
```

```
Limit[deul[96, 0, 1, z], z → 1]
```

32

```
EulerPhi[96]
```

32

```
FullSimplify[p^((a - s) i) p^(-s (k - i))]
```

$p^{a\,i-k\,s}$

```
Clear[apz, a2]
a[n_] := FiniteAbelianGroupCount[n]
a2[n_, k_] := a2[n, k] = Sum[a[j] a2[Floor[n / j], k - 1], {j, 2, n}]
a2[n_, 0] := UnitStep[n - 1]
az[n_, z_] := Sum[bin[z, k] a2[n, k], {k, 0, Log2@n}]
aaz[n_, z_] := az[n, z] - az[n - 1, z]
Pz[n_, k_] := Pz[n, k] = D[az[n, z], {z, k}] /. z → 0
apz[n_, k_] := apz[n, k] = D[aaz[n, z], {z, k}] /. z → 0
```

```
Table[aaz[2^k, z], {k, 0, 9}] // TableForm
```

1

z

$2 z + \frac{1}{2} (-1 + z) z$

$3 z + 2 (-1 + z) z + \frac{1}{6} (-2 + z) (-1 + z) z$

$5 z + 5 (-1 + z) z + (-2 + z) (-1 + z) z + \frac{1}{24} (-3 + z) (-2 + z) (-1 + z) z$

$7 z + 11 (-1 + z) z + \frac{7}{2} (-2 + z) (-1 + z) z + \frac{1}{3} (-3 + z) (-2 + z) (-1 + z) z + \frac{1}{120} (-4 + z) (-3 + z) (-2 + z) (-$

$11 z + \frac{43}{2} (-1 + z) z + \frac{59}{6} (-2 + z) (-1 + z) z + \frac{3}{2} (-3 + z) (-2 + z) (-1 + z) z + \frac{1}{12} (-4 + z) (-3 + z) (-2 + z) ($

$15 z + 40 (-1 + z) z + 24 (-2 + z) (-1 + z) z + \frac{31}{6} (-3 + z) (-2 + z) (-1 + z) z + \frac{11}{24} (-4 + z) (-3 + z) (-2 + z)$

$22 z + \frac{141}{2} (-1 + z) z + \frac{107}{2} (-2 + z) (-1 + z) z + \frac{181}{12} (-3 + z) (-2 + z) (-1 + z) z + \frac{15}{8} (-4 + z) (-3 + z) (-2 + z$

$30 z + 120 (-1 + z) z + \frac{223}{2} (-2 + z) (-1 + z) z + \frac{118}{3} (-3 + z) (-2 + z) (-1 + z) z + \frac{51}{8} (-4 + z) (-3 + z) (-2 +$

```
Table[{k, D[aaz[2^k, z], z] /. z → 0, D[aaz[2^k, z], {z, 2}] /. z → 0,
   D[aaz[2^k, z], {z, 3}] /. z → 0, D[aaz[2^k, z], {z, 4}] /. z → 0,
   D[aaz[2^k, z], {z, 5}] /. z → 0, D[aaz[2^k, z], {z, 6}] /. z → 0}, {k, 1, 10}] // TableForm
```

1	1	0	0	0	0	0
2	$\frac{3}{2}$	1	0	0	0	0
3	$\frac{4}{3}$	3	1	0	0	0
4	$\frac{7}{4}$	$\frac{59}{12}$	$\frac{9}{2}$	1	0	0
5	$\frac{6}{5}$	$\frac{15}{2}$	$\frac{43}{4}$	6	1	0
6	2	$\frac{1697}{180}$	$\frac{165}{8}$	$\frac{113}{6}$	$\frac{15}{2}$	1
7	$\frac{8}{7}$	$\frac{184}{15}$	$\frac{2021}{60}$	$\frac{89}{2}$	$\frac{175}{6}$	9
8	$\frac{15}{8}$	$\frac{8147}{560}$	$\frac{4049}{80}$	$\frac{21127}{240}$	$\frac{165}{2}$	$\frac{167}{4}$
9	$\frac{13}{9}$	$\frac{7019}{420}$	$\frac{1083899}{15120}$	$\frac{3097}{20}$	$\frac{27589}{144}$	138
10	$\frac{9}{5}$	$\frac{252019}{12600}$	$\frac{64193}{672}$	$\frac{237812}{945}$	$\frac{37285}{96}$	$\frac{88453}{240}$

```
Sum[ apz[2^5, k] / (k!), {k, 0, Log2@32}]
```

7

```
PartitionsP[5]
```

7

```
Pz[100, 2]
```

$$\frac{31\,949}{180}$$

```
Sum[ apz[j, 1] apz[k, 1], {j, 1, 100}, {k, 1, 100 / j}]
```

$$\frac{31\,949}{180}$$

```
apz[32, 2]
```

$$\frac{15}{2}$$

```
Sum[ apz[2^k, 1] apz[2^(5 - k), 1], {k, 0, 5}]
```

$$\frac{15}{2}$$

```
1 + 1 / 2 + 1 / 4 + 1 / 8
```

$$\frac{15}{8}$$

```
1 + 1 / 2 + 1 / 5 + 1 / 10
```

$$\frac{9}{5}$$

```
dzz[n_, s_, z_] := Product[(-1)^p[[2]] bin[-z, p[[2]]] p[[1]]^(-p[[2]] s), {p, FI[n]}]
```

```
dzz[10, 1, 2]
```

$$\frac{2}{5}$$


```
1 + 2 + 5 + 10
```

18

```
DivisorSigma[1, 10]
```

18

```
dsig[n_, s_, a_, z_] := If[n == 0, 0, Product[p[[1]]^(-p[[2]] s) Binomial[-1 + p[[2]] + z, p[[2]]]
     Hypergeometric2F1[-p[[2]], z, 1 - p[[2]] - z, p[[1]]^a], {p, FI[n]}]]
```

```
dsig[10, 1, 1, 1]
```

$$\frac{9}{5}$$

```
Table[{k, D[aaz[2^k, z], z] /. z → 0, dsig[k, 1, 1, 1]}, {k, 1, 10}] // TableForm
```

1	1	1
2	$\frac{3}{2}$	$\frac{3}{2}$
3	$\frac{4}{3}$	$\frac{4}{3}$
4	$\frac{7}{4}$	$\frac{7}{4}$
5	$\frac{6}{5}$	$\frac{6}{5}$
6	2	2
7	$\frac{8}{7}$	$\frac{8}{7}$
8	$\frac{15}{8}$	$\frac{15}{8}$
9	$\frac{13}{9}$	$\frac{13}{9}$
10	$\frac{9}{5}$	$\frac{9}{5}$

```
Table[{k, apz[2^k, 1], dsig[k, 1, 1, 1]}, {k, 1, 10}] // TableForm
```

1	1	1
2	$\frac{3}{2}$	$\frac{3}{2}$
3	$\frac{4}{3}$	$\frac{4}{3}$
4	$\frac{7}{4}$	$\frac{7}{4}$
5	$\frac{6}{5}$	$\frac{6}{5}$
6	2	2
7	$\frac{8}{7}$	$\frac{8}{7}$
8	$\frac{15}{8}$	$\frac{15}{8}$
9	$\frac{13}{9}$	$\frac{13}{9}$
10	$\frac{9}{5}$	$\frac{9}{5}$

```
Sum[ dsig[k, 1, 1, 1] dsig[5 - k, 1, 1, 1], {k, 1, 4}]
```

$$\frac{15}{2}$$

```
apz[2^5, 2]
```

$$\frac{15}{2}$$

```
Sum[ apz[2^k, 1] apz[2^(5 - k), 1], {k, 1, 4}]
```

$$\frac{15}{2}$$

```
apz[2^5, 3]
```

$$\frac{43}{4}$$

```
Sum[ apz[2^k, 1] apz[2^l, 1] apz[2^(5 - k - l), 1], {k, 1, 4}, {l, 1, 4 - k}]
```

$$\frac{43}{4}$$

```
Sum[dsig[k, 1, 1, 1] dsig[l, 1, 1, 1] dsig[(5 - k - l), 1, 1, 1], {k, 1, 4}, {l, 1, 4 - k}]
```

$\dfrac{43}{4}$

```
dsiga[n_] := If[n == 0, 0, Product[p[[1]]^-p[[2]] Binomial[-1 + p[[2]] + 1, p[[2]]]
    Hypergeometric2F1[-p[[2]], 1, 1 - p[[2]] - 1, p[[1]]], {p, FI[n]}]]
dsigb[n_] := (1 / n) Product[ Binomial[p[[2]], p[[2]]]
    Hypergeometric2F1[-p[[2]], 1, 1 - p[[2]] - 1, p[[1]]], {p, FI[n]}]
dsigc[n_] := (1 / n) Product[ Hypergeometric2F1[-p[[2]], 1, -p[[2]], p[[1]]], {p, FI[n]}]
dsigd[n_] := (1 / n) Product[ Sum[p[[1]]^k, {k, 0, p[[2]]}], {p, FI[n]}]
dsige[n_] := (1 / n) DivisorSigma[1, n]
```

```
Sum[dsige[k] dsige[l] dsige[(5 - k - l)], {k, 1, 4}, {l, 1, 4 - k}]
```

$\dfrac{43}{4}$

```
hef[p1_, p2_] := Hypergeometric2F1[-p2, 1, -p2, p1]
```

```
Table[ hef[p1, p2], {p1, 1, 10}, {p2, 1, 10}] // Grid
```

2	3	4	5	6	7	8	9	10	11
3	7	15	31	63	127	255	511	1023	2047
4	13	40	121	364	1093	3280	9841	29 524	88 573
5	21	85	341	1365	5461	21 845	87 381	349 525	1 398 101
6	31	156	781	3906	19 531	97 656	488 281	2 441 406	12 207 031
7	43	259	1555	9331	55 987	335 923	2 015 539	12 093 235	72 559 411
8	57	400	2801	19 608	137 257	960 800	6 725 601	47 079 208	329 554 457
9	73	585	4681	37 449	299 593	2 396 745	19 173 961	153 391 689	1 227 133 513
10	91	820	7381	66 430	597 871	5 380 840	48 427 561	435 848 050	3 922 632 451
11	111	1111	11 111	111 111	1 111 111	11 111 111	111 111 111	1 111 111 111	11 111 111 111

```
hef[x, 12]
```

$1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + x^{11} + x^{12}$

```
Clear[Laz, dlz, a2]
a[n_] := FiniteAbelianGroupCount[n]
a2[n_, k_] := a2[n, k] = Sum[a[j] a2[Floor[n / j], k - 1], {j, 2, n}]
a2[n_, 0] := UnitStep[n - 1]
az[n_, z_] := Sum[ bin[z, k] a2[n, k], {k, 0, Log2@n}]
daz[n_, z_] := az[n, z] - az[n - 1, z]
Laz[n_, k_] := Laz[n, k] = D[az[n, z], {z, k}] /. z -> 0
dlz[n_, k_] := dlz[n, k] = D[daz[n, z], {z, k}] /. z -> 0
prt[p_] := Sum[ dlz[2^p, k] / (k!), {k, 0, Log2@(2^p)}]
```

```
Table[{D[daz[2^k, z], z] /. z → 0, Expand@daz[2^k, z]}, {k, 0, 11}] // TableForm
```

0	1
1	z
$\frac{3}{2}$	$\frac{3z}{2} + \frac{z^2}{2}$
$\frac{4}{3}$	$\frac{4z}{3} + \frac{3z^2}{2} + \frac{z^3}{6}$
$\frac{7}{4}$	$\frac{7z}{4} + \frac{59z^2}{24} + \frac{3z^3}{4} + \frac{z^4}{24}$
$\frac{6}{5}$	$\frac{6z}{5} + \frac{15z^2}{4} + \frac{43z^3}{24} + \frac{z^4}{4} + \frac{z^5}{120}$
2	$2z + \frac{1697z^2}{360} + \frac{55z^3}{16} + \frac{113z^4}{144} + \frac{z^5}{16} + \frac{z^6}{720}$
$\frac{8}{7}$	$\frac{8z}{7} + \frac{92z^2}{15} + \frac{2021z^3}{360} + \frac{89z^4}{48} + \frac{35z^5}{144} + \frac{z^6}{80} + \frac{z^7}{5040}$
$\frac{15}{8}$	$\frac{15z}{8} + \frac{8147z^2}{1120} + \frac{4049z^3}{480} + \frac{21127z^4}{5760} + \frac{11z^5}{16} + \frac{167z^6}{2880} + \frac{z^7}{480} + \frac{z^8}{40320}$
$\frac{13}{9}$	$\frac{13z}{9} + \frac{7019z^2}{840} + \frac{1083899z^3}{90720} + \frac{3097z^4}{480} + \frac{27589z^5}{17280} + \frac{23z^6}{120} + \frac{97z^7}{8640} + \frac{z^8}{3360} + \frac{z^9}{362880}$
$\frac{9}{5}$	$\frac{9z}{5} + \frac{252019z^2}{25200} + \frac{64193z^3}{4032} + \frac{59453z^4}{5670} + \frac{7457z^5}{2304} + \frac{88453z^6}{172800} + \frac{49z^7}{1152} + \frac{221z^8}{120960} + \frac{z^9}{26880} + \frac{z^{10}}{3628800}$
$\frac{12}{11}$	$\frac{12z}{11} + \frac{163z^2}{15} + \frac{524729z^3}{25200} + \frac{192907z^4}{12096} + \frac{270239z^5}{45360} + \frac{2713z^6}{2304} + \frac{22223z^7}{172800} + \frac{z^8}{128} + \frac{31z^9}{120960} + \frac{z^{10}}{241920} + \frac{z^{11}}{39916800}$

```
Sum[ PartitionsP[k] PartitionsP[8 - k], {k, 1, 7}] / 2
```

$\frac{141}{2}$

```
Sum[ PartitionsP[k] PartitionsP[l] PartitionsP[8 - k - l], {k, 1, 7}, {l, 1, 7 - k}] / 6
```

$\frac{107}{2}$

```
Sum[ PartitionsP[k] PartitionsP[l] PartitionsP[m] PartitionsP[8 - k - l - m],
  {k, 1, 7}, {l, 1, 7 - k}, {m, 1, 7 - k - l}] / 24
```

$\frac{181}{12}$

```
Clear[pp, pe]
pp[n_, k_] := pp[n, k] = Sum[ PartitionsP[j] pp[n - j, k - 1], {j, 1, n - 1}]
pp[n_, 1] := PartitionsP[n]
pe[n_, k_] := pe[n, k] = Sum[DivisorSigma[1, j] / j pe[n - j, k - 1], {j, 1, n - 1}]
pe[n_, 1] := DivisorSigma[1, n] / n
pa[z_, 0] := 1
pa[z_, k_] := Sum[ z^j / j! pe[k, j], {j, 1, k}]
```

```
pp[4, 3]
```

1

```
Binomial[z, 3]
```

$\frac{1}{6} (-2 + z) (-1 + z) z$

```
Table[po[1, k], {k, 1, 24}]
```

{1, 2, 3, 5, 7, 11, 15, 22, 30, 42, 56, 77, 101,
 135, 176, 231, 297, 385, 490, 627, 792, 1002, 1255, 1575}

```
pa[1, 11]
```

56