```
Clear[f]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
f[n_, y_, z_] := If[ n < y, 1, Sum[bin[z, k] f[n / y^k, y + 1, z - k], {k, 0, Log[y, n]}]]
f2[n_, y_, z_] :=
 If[ n < y, 1, f2[n, y + 1, z] + Sum[bin[z, k] f2[n / y^k, y + 1, z - k], {k, 1, Log[y, n]}]]
f3[n_, y_, z_] := If[ n < y, 1,
  f3[n, y + 1, z] + Sum[bin[z, k] f2[n / y^k, y + 1, z - k], {k, 1, Log[y, n]}]]

fx[n_, y_, z_, t_] := If[ n < y * y, pe[n, y, z], If[n < t, pp[n, y, z],
   If[ n < y, 1, Sum[bin[z, k] fx[Floor[n / y^k], y + 1, z - k, t], {k, 0, Log[y, n]}]]]]
fy[n_, y_, z_] := 1 + Sum[ bin[z, j] fy[n / a^j, a, z - j], {a, y + 1, n}, {j, 1, Log[a, n]}]
```

```
D[Expand@fx[100, 2, z, 10], z] /. z → 0
```

$$-\frac{1}{6}\,pe[1,3,-6]+2\,pe[1,4,-5]-\frac{5}{4}\,pe[1,4,-4]-\frac{9}{2}\,pe[1,5,-4]+\frac{1}{3}\,pe[1,5,-3]-$$
$$\frac{3}{2}\,pe[1,6,-4]+2\,pe[1,6,-3]+pe[1,7,-3]+pe[1,8,-3]-\frac{1}{2}\,pe[1,9,-2]-\frac{1}{2}\,pe[1,10,-2]-$$
$$\frac{1}{2}\,pe[1,11,-2]-\frac{3}{2}\,pe[2,4,-4]+pe[2,5,-3]+pe[2,6,-3]-\frac{1}{2}\,pe[2,7,-2]-$$
$$\frac{1}{2}\,pe[2,8,-2]+\frac{1}{5}\,pe[3,3,-5]+\frac{1}{3}\,pe[3,4,-3]+pe[3,5,-3]-pe[4,4,-4]+2\,pe[4,5,-3]-$$
$$\frac{1}{2}\,pe[4,6,-2]+pe[5,4,-3]+pe[5,6,-3]-pe[5,6,-2]-\frac{1}{4}\,pe[6,3,-4]+pe[6,5,-3]-$$
$$\frac{1}{2}\,pe[6,5,-2]-pe[6,6,-2]-pe[7,8,-2]+pe[8,4,-3]-pe[8,5,-2]-pe[8,7,-2]-$$
$$pe[10,6,-2]+pe[10,11,-1]-\frac{1}{2}\,pe[11,4,-2]+pe[11,10,-1]+\frac{1}{3}\,pe[12,4,-3]-$$
$$pe[12,5,-2]+pe[12,9,-1]+pe[14,8,-1]-pe[16,5,-2]+pe[16,7,-1]+pe[20,6,-1]-$$
$$\frac{1}{2}\,pe[25,6,-2]+pe[25,6,-1]+pe[33,6,-1]+pe[50,8,-1]+pe^{(0,0,1)}[100,11,0]$$

```
Sum[bin[z, k] f2[n / y^k, y + 1, z - k], {k, 1, Log[y, n]}]
```

```
bin[z, 1]
```

```
z
```

```
Expand@fy[100, 1, z]
```

$$1+\frac{428\,z}{15}+\frac{16\,289\,z^2}{360}+\frac{331\,z^3}{16}+\frac{611\,z^4}{144}+\frac{67\,z^5}{240}+\frac{7\,z^6}{720}$$