```
F1[n_, k_] := Sum[ (-1)^(a + 1) / a Binomial[a, a], {a, 1, Log[k, n]}]
F2[n_, k_, l_] := Sum[ (-1)^(a + b + 1) / (a + b) Binomial[a + b, a] Binomial[b, b],
  {a, 1, Log[k, n]}, {b, 1, Log[l, Floor[n / (k^a)]]}]
F3[n_, k_, l_, m_] := Sum[ (-1)^(a + b + c + 1) / (a + b + c)
   Binomial[a + b + c, a] Binomial[b + c, b] Binomial[c, c], {a, 1, Log[k, n]},
  {b, 1, Log[l, Floor[n / (k^a)]]}, {c, 1, Log[m, Floor[n / ( k^a l^b)]]}]
Table[{n = 2^m, N[F1[n, 2]], N[F2[n, 2, 3]], N[F3[n, 2, 3, 4]]}, {m, 1, 62}] // TableForm
```

2	1.	0.	0.
4	0.5	0.	0.
8	0.833333	−1.	0.
16	0.583333	0.	0.
32	0.783333	0.	2.
64	0.616667	−1.5	−1.
128	0.759524	1.5	−3.
256	0.634524	−2.33333	8.
512	0.745635	1.16667	−6.
1024	0.645635	−0.333333	−3.
2048	0.736544	−2.75	14.
4096	0.653211	6.25	−16.5
8192	0.730134	−12.6167	9.
16 384	0.658705	20.55	−0.833333
32 768	0.725372	−34.2	9.16667
65 536	0.662872	55.3	−42.3333
131 072	0.721695	−92.1167	117.667
262 144	0.66614	150.25	−251.333
524 288	0.718771	−239.476	371.167
1 048 576	0.668771	360.69	−298.5
2 097 152	0.71639	−508.818	60.5
4 194 304	0.670936	652.349	−146.
8 388 608	0.714414	−725.435	996.483
16 777 216	0.672747	589.204	−2017.18
33 554 432	0.712747	7.70437	1507.73
67 108 864	0.674286	−1520.62	2068.23
134 217 728	0.711323	4730.96	−8586.35
268 435 456	0.675609	−10 955.5	15 969.1
536 870 912	0.710091	22 318.6	−22 482.8
1 073 741 824	0.676758	−42 127.4	31 945.1
2 147 483 648	0.709016	75 295.1	−57 815.1
4 294 967 296	0.677766	−128 814.	120 493.
8 589 934 592	0.708069	212 204.	−236 775.
17 179 869 184	0.678657	−338 019.	402 680.
34 359 738 368	0.707229	522 609.	−579 903.
68 719 476 736	0.679451	−787 975.	729 946.
137 438 953 472	0.706478	1.16655×10^6	−929 368.
274 877 906 944	0.680162	-1.71305×10^6	1.45737×10^6
549 755 813 888	0.705803	2.53137×10^6	-2.65034×10^6
1 099 511 627 776	0.680803	-3.83204×10^6	4.57908×10^6
2 199 023 255 552	0.705194	6.04775×10^6	-7.0148×10^6
4 398 046 511 104	0.681384	-1.00552×10^7	1.00849×10^7
8 796 093 022 208	0.70464	1.75835×10^7	-1.554×10^7
17 592 186 044 416	0.681913	-3.1939×10^7	2.82789×10^7
35 184 372 088 832	0.704135	5.92447×10^7	-5.74291×10^7
70 368 744 177 664	0.682396	-1.10484×10^8	1.15764×10^8

140 737 488 355 328	0.703672	2.04748×10^8	-2.18145×10^8
281 474 976 710 656	0.682839	-3.74166×10^8	3.85081×10^8
562 949 953 421 312	0.703247	6.71028×10^8	-6.59132×10^8
1 125 899 906 842 624	0.683247	-1.17737×10^9	1.13206×10^9
2 251 799 813 685 248	0.702855	2.01655×10^9	-1.96546×10^9
4 503 599 627 370 496	0.683624	-3.36466×10^9	3.38078×10^9
9 007 199 254 740 992	0.702492	5.45565×10^9	-5.60347×10^9
18 014 398 509 481 984	0.683974	-8.56696×10^9	8.78448×10^9
36 028 797 018 963 968	0.702155	1.29589×10^{10}	-1.29819×10^{10}
72 057 594 037 927 936	0.684298	-1.8717×10^{10}	1.82466×10^{10}
144 115 188 075 855 872	0.701842	2.54061×10^{10}	-2.45664×10^{10}
288 230 376 151 711 744	0.684601	-3.13766×10^{10}	3.1057×10^{10}
576 460 752 303 423 488	0.70155	3.24538×10^{10}	-3.38318×10^{10}
1 152 921 504 606 846 976	0.684883	-1.95621×10^{10}	2.25999×10^{10}
2 305 843 009 213 693 952	0.701277	-2.54444×10^{10}	2.34529×10^{10}
4 611 686 018 427 387 904	0.685148	1.36921×10^{11}	-1.40508×10^{11}

```
F1A[n_, k_] := Sum[ (-1)^(a + 1) / a Binomial[a, a], {a, 1, Infinity}]
F2A[n_, k_, l_] := Sum[ (-1)^(a + b + 1) / (a + b) Binomial[a + b, a] Binomial[b, b],
  {a, 1, Infinity}, {b, 1, Infinity}]
F3A[n_, k_, l_, m_] := Sum[ (-1)^(a + b + c + 1) / (a + b + c) Binomial[a + b + c, a]
   Binomial[b + c, b] Binomial[c, c], {a, 1, Infinity}, {b, 1, Infinity}, {c, 1, Infinity}]
F4A[n_, k_, l_, m_, o_] := Sum[ (-1)^(a + b + c + d + 1) / (a + b + c + d)
   Binomial[a + b + c + d, a] Binomial[b + c + d, b] Binomial[c + d, c] Binomial[d, d],
  {a, 1, Infinity}, {b, 1, Infinity}, {c, 1, Infinity}, {d, 1, Infinity}]
```

```
F4A[1000, 2, 3, 4, 5]
```

```
-6 Log[4] + Log[3645]
```

```
F1M[n_, k_] := Sum[ (-1)^(a) Binomial[a, a], {a, 1, Log[k, n]}]
F2M[n_, k_, l_] := Sum[ (-1)^(a + b) Binomial[a + b, a] Binomial[b, b],
  {a, 1, Log[k, n]}, {b, 1, Log[l, Floor[n / (k^a)]]}]
F3M[n_, k_, l_, m_] := Sum[ (-1)^(a + b + c) Binomial[a + b + c, a]
   Binomial[b + c, b] Binomial[c, c], {a, 1, Log[k, n]},
  {b, 1, Log[l, Floor[n / (k^a)]]}, {c, 1, Log[m, Floor[n / ( k^a l^b)]]}]
```

```
Table[{n = 2^m, N[F1M[n, 2]], N[F2M[n, 2, 3]], N[F3M[n, 2, 3, 4]]}, {m, 1, 26}] // TableForm
```

2	−1.	0.	0.
4	0.	0.	0.
8	−1.	2.	0.
16	0.	−1.	0.
32	−1.	0.	−6.
64	0.	5.	6.
128	−1.	−9.	10.
256	0.	14.	−40.
512	−1.	−13.	48.
1024	0.	6.	−6.
2048	−1.	15.	−79.
4096	0.	−55.	151.
8192	−1.	123.	−156.
16 384	0.	−231.	116.
32 768	−1.	409.	−140.
65 536	0.	−714.	398.
131 072	−1.	1250.	−1290.
262 144	0.	−2169.	3324.
524 288	−1.	3662.	−5811.
1 048 576	0.	−5893.	6139.
2 097 152	−1.	8892.	−3403.
4 194 304	0.	−12 309.	3403.
8 388 608	−1.	15 022.	−14 514.
16 777 216	0.	−14 407.	33 193.
33 554 432	−1.	5104.	−35 043.
67 108 864	0.	23 168.	−15 681.

```
F1Ma[n_, k_] := Sum[ (-1)^(a) Binomial[a, a], {a, 1, Infinity}]
F2Ma[n_, k_, l_] := Sum[ (-1)^(a + b) Binomial[a + b, a] Binomial[b, b],
  {a, 1, Infinity}, {b, 1, Log[l, Infinity]}]
F3Ma[n_, k_, l_, m_] := Sum[ (-1)^(a + b + c) Binomial[a + b + c, a] Binomial[b + c, b]
   Binomial[c, c], {a, 1, Infinity}, {b, 1, Infinity}, {c, 1, Infinity}]
```

```
F1A[100, 2]
F2A[100, 2, 3]
F3A[100, 2, 3, 4]
```

```
Log[2]
```

$$-\text{Log}\left[\frac{4}{3}\right]$$

$$\text{Log}\left[\frac{32}{27}\right]$$

```
N[-Log[4/3]]
```

```
-0.287682
```

```
N[Log[32/27]]
```

```
0.169899
```

```
N[-6 Log[4] + Log[3645]]
```

```
-0.116655
```

```
Bin2[n_, k_] := Gamma[n + 1] / (Gamma[k + 1] (Gamma[n - k + 1]))

F1I[n_, k_] := Integrate[ (-1)^(a + 1) / a Binomial[a, a], {a, 1, Log[k, n]}]
F2I[n_, k_, l_] := Integrate[ (-1)^(a + b + 1) / (a + b) Binomial[a + b, a] Binomial[b, b],
  {a, 1, Log[k, n]}, {b, 1, Log[l, n / (k^a)]}]
F3I[n_, k_, l_, m_] := Integrate[ (-1)^(a + b + c + 1) / (a + b + c)
   Binomial[a + b + c, a] Binomial[b + c, b] Binomial[c, c],
  {a, 1, Log[k, n]}, {b, 1, Log[l, n / (k^a)]}, {c, 1, Log[m, n / ( k^a l^b)]}]

Table[{n = 2^m, N[F1I[n, 2]], N[F2I[n, 2, 3]], N[F3I[n, 2, 3, 4]]}, {m, 1, 8}] // TableForm
```

```
$Aborted
```

```
Table[{n = 2^m, N[F1I[n, 2]], N[F2I[n, 2, 3]]}, {m, 1, 12}] // TableForm
```

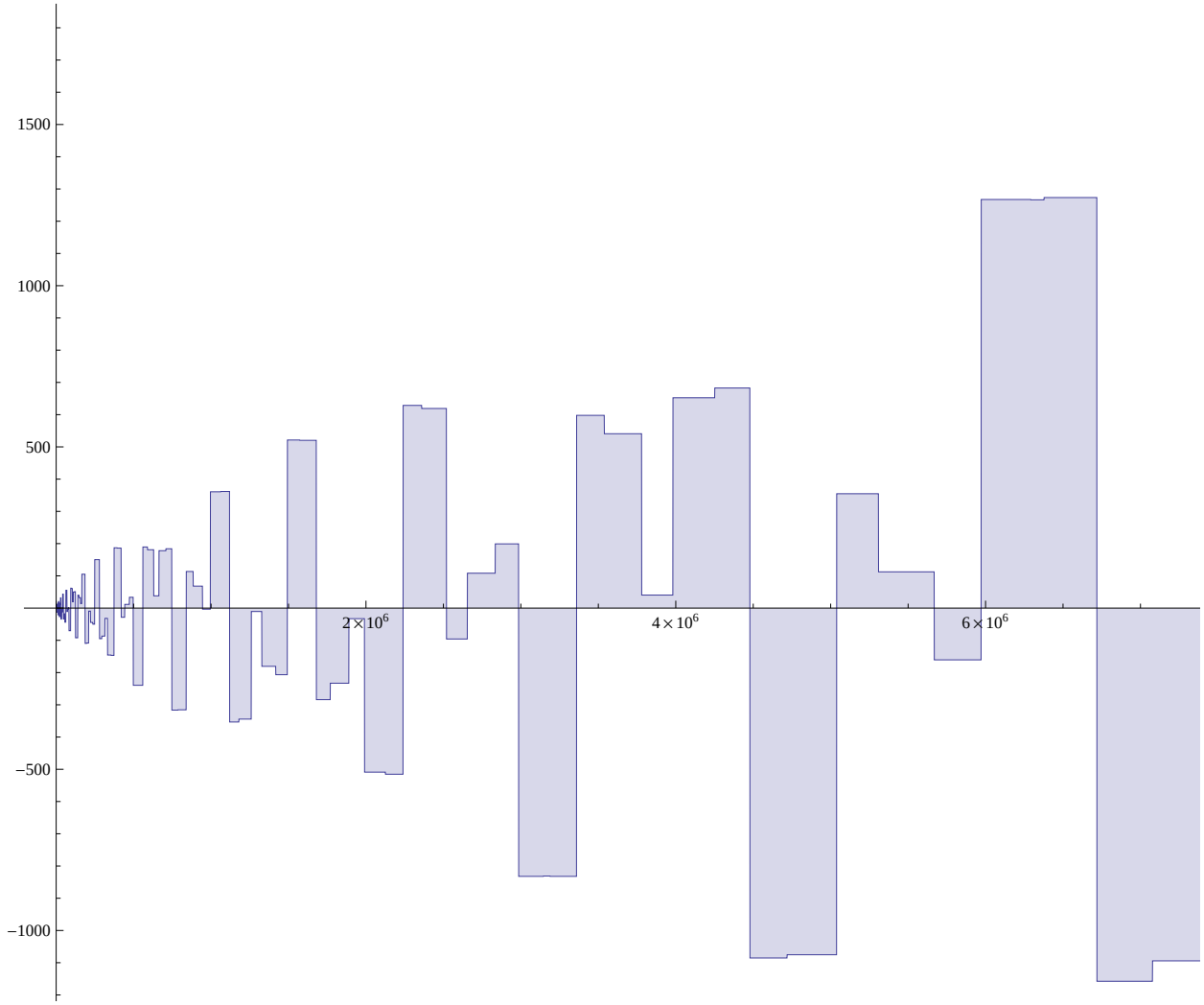
```
2       0.                      0.
4       0.0962286 + 0.433785 i  0.308033 + 0.218235 i
8       0.0630477 + 0.177175 i  -0.344312 - 0.156246 i
16      0.0797846 + 0.359776 i  0.613216 - 0.15015 i
32      0.0697067 + 0.217972 i  -0.172515 + 0.471386 i
64      0.0764373 + 0.333903 i  -0.105383 - 0.496747 i
128     0.0716248 + 0.235852 i  0.675675 + 0.145189 i
256     0.0752365 + 0.320806 i  -0.504954 + 0.406999 i
512     0.0724262 + 0.24586 i   0.335155 - 0.844099 i
1024    0.0746751 + 0.312908 i  0.433607 + 0.98041 i
2048    0.0728347 + 0.252251 i  -0.841503 - 0.758871 i
4096    -2.48491 + 0.30763 i    1.51332 + 0.190476 i
```

```
Table[{n = 2^m, N[F1[n, 2]], N[F2[n, 2, 3]] + N[F1[n, 6]],
   N[F3[n, 2, 3, 4]] + N[F2[n, 2, 6]] + N[F2[n, 3, 4]] + N[F1[n, 12]]}, {m, 1, 62}] // TableForm
```

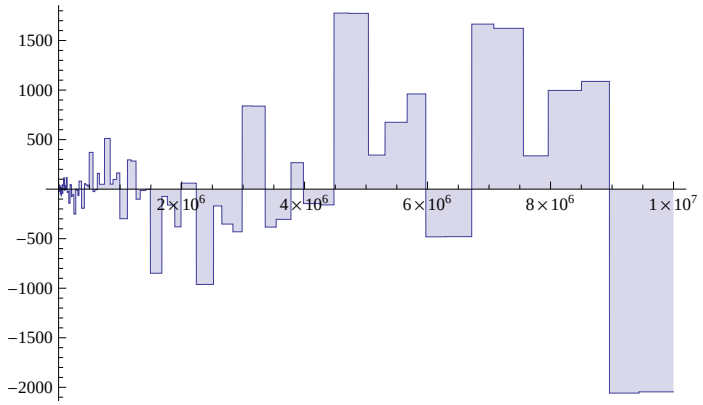
```
2                       1.          0.              0.
4                       0.5         0.              0.
8                       0.833333    0.              0.
16                      0.583333    1.              -1.
32                      0.783333    1.              2.
64                      0.616667    -1.             0.
128                     0.759524    2.              -1.
256                     0.634524    -1.5            4.5
512                     0.745635    2.              -4.5
1024                    0.645635    0.5             -1.
2048                    0.736544    -2.16667        11.1667
4096                    0.653211    6.83333         -17.3333
8192                    0.730134    -11.8333        13.6667
16 384                  0.658705    21.3333         4.16667
32 768                  0.725372    -33.4167        -12.5833
65 536                  0.662872    55.9167         -25.75
131 072                 0.721695    -91.5           134.75
262 144                 0.66614     150.867         -270.45
524 288                 0.718771    -238.717        331.133
1 048 576               0.668771    361.45          -247.7
2 097 152               0.71639     -508.183        144.217
4 194 304               0.670936    652.983         -345.95
8 388 608               0.714414    -724.8          1098.57
16 777 216              0.672747    589.95          -2020.6
33 554 432              0.712747    8.45            1803.23
```

67 108 864	0.674286	−1519.97	1226.96
134 217 728	0.711323	4731.61	−7568.04
268 435 456	0.675609	−10 954.9	15 370.9
536 870 912	0.710091	22 319.3	−22 088.3
1 073 741 824	0.676758	−42 126.6	30 292.9
2 147 483 648	0.709016	75 295.9	−54 161.6
4 294 967 296	0.677766	−128 813.	117 842.
8 589 934 592	0.708069	212 205.	−238 795.
17 179 869 184	0.678657	−338 018.	404 581.
34 359 738 368	0.707229	522 610.	−572 264.
68 719 476 736	0.679451	−787 974.	720 238.
137 438 953 472	0.706478	1.16655×10^6	−938 806.
274 877 906 944	0.680162	-1.71305×10^6	1.47565×10^6
549 755 813 888	0.705803	2.53137×10^6	-2.63198×10^6
1 099 511 627 776	0.680803	-3.83204×10^6	4.52549×10^6
2 199 023 255 552	0.705194	6.04775×10^6	-6.99008×10^6
4 398 046 511 104	0.681384	-1.00552×10^7	1.00736×10^7
8 796 093 022 208	0.70464	1.75835×10^7	-1.54024×10^7
17 592 186 044 416	0.681913	-3.1939×10^7	2.79582×10^7
35 184 372 088 832	0.704135	5.92447×10^7	-5.70044×10^7
70 368 744 177 664	0.682396	-1.10484×10^8	1.15169×10^8
140 737 488 355 328	0.703672	2.04748×10^8	-2.17171×10^8
281 474 976 710 656	0.682839	-3.74166×10^8	3.83648×10^8
562 949 953 421 312	0.703247	6.71028×10^8	-6.56916×10^8
1 125 899 906 842 624	0.683247	-1.17737×10^9	1.12835×10^9
2 251 799 813 685 248	0.702855	2.01655×10^9	-1.96073×10^9
4 503 599 627 370 496	0.683624	-3.36466×10^9	3.37643×10^9
9 007 199 254 740 992	0.702492	5.45565×10^9	-5.59696×10^9
18 014 398 509 481 984	0.683974	-8.56696×10^9	8.77029×10^9
36 028 797 018 963 968	0.702155	1.29589×10^{10}	-1.29641×10^{10}
72 057 594 037 927 936	0.684298	-1.8717×10^{10}	1.82372×10^{10}
144 115 188 075 855 872	0.701842	2.54061×10^{10}	-2.45564×10^{10}
288 230 376 151 711 744	0.684601	-3.13766×10^{10}	3.10167×10^{10}
576 460 752 303 423 488	0.70155	3.24538×10^{10}	-3.37714×10^{10}
1 152 921 504 606 846 976	0.684883	-1.95621×10^{10}	2.25757×10^{10}
2 305 843 009 213 693 952	0.701277	-2.54444×10^{10}	2.34527×10^{10}
4 611 686 018 427 387 904	0.685148	1.36921×10^{11}	-1.40591×10^{11}

```
DiscretePlot[ F2[n, 2, 3], {n, 1, 10 000 000, 1000}]
```




```
DiscretePlot[F3[n, 2, 3, 4], {n, 1, 10 000 000, 1000}]
```




```
F2I[n, k, l]
```

$$\int_1^{\frac{\text{Log}[4\,611\,686\,018\,427\,387\,904]}{\text{Log}[k]}} \int_1^{\frac{\text{Log}[4\,611\,686\,018\,427\,387\,904\,k^{-a}]}{\text{Log}[l]}} \frac{(-1)^{1+a+b}\,\text{Binomial}[a+b,\,a]}{a+b}\,\mathrm{d}l\,\mathrm{d}a$$

```
Unprotect[Power]; Power[0, 0] = 1; Protect[Power];
K[n_] := K[n] = FullSimplify[ MangoldtLambda[n] / Log[n]]
PP[n_, k_, a_] := PP[n, k, a] =
  If[n < a^k, 0, Sum[ K[a]^j Binomial[k, j] PP[Floor[n / a^j], k - j, a + 1], {j, 0, k}]]
PP[n_, 0, a_] := 1

PP[100, 1, 2]
```

$$\frac{428}{15}$$