

```

f1[y_, s_] := Log[y]^s Integrate[y^x x^(s-1), {x, 0, 1}]
f2[y_, s_] := Abs[Gamma[s, 0, -Log[y]]]

N[f1[5, 3]]

4.85707

N[f2[5, 3]]

4.85707

f1a[y_, s_, n_] := Log[y]^s Integrate[y^(nx) (nx)^(s-1), {x, 0, 1}]
f2a[y_, s_, n_] := Abs[Gamma[s, 0, -n Log[y]]] / n

f1b[y_, s_, n_] := (n)^(s-1) Log[y]^s Integrate[y^(nx) (x)^(s-1), {x, 0, 1}]
f2b[y_, s_, n_] := Abs[Gamma[s, 0, -n Log[y]]] / n

N[f1b[5, 4, 3]]

2735.03

N[f2b[5, 4, 3]]

2735.03

f1c[y_, s_, n_] := Log[y]^s Integrate[y^(nx) x^(s-1), {x, 0, 1}]
f2c[y_, s_, n_] := Abs[Gamma[s, 0, -n Log[y]]] / n / n^(s-1)

N[f1c[5, 4, 3]]

101.297

N[f2c[5, 4, 3]]

101.297

f1d[y_, s_, n_] := Log[y]^s Integrate[y^(nx) x^(s-1), {x, 0, 1}]
f2d[y_, s_, n_] := Abs[Gamma[s, 0, -n Log[y]]] / n^s

N[f1d[3, 4, 3]]

5.73693

N[f2d[3, 4, 3]]

5.73693

f1e[y_, s_] := Log[y]^s Integrate[Sum[y^(nx), {n, 1, Infinity}] x^(s-1), {x, 0, 1}]
f2e[y_, s_] := Sum[Abs[Gamma[s, 0, -n Log[y]]] / n^s, {n, 1, 100 000}]

N[f1e[.5, 2]]

0.582241

N[f2e[.5, 2]]

0.582231

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```

f1f[y_, s_] := Log[y]^s Integrate[ $\left(-\frac{y^x}{-1+y^x}\right) x^{(s-1)}, \{x, 0, 1\}]$ 
f2f[y_, s_] := Sum[Abs[Gamma[s, 0, -n Log[y]]] / n^s, {n, 1, 100 000}]
N[f1f[.5, 2]]
0.582241
N[f2f[.5, 2]]
0.582231
Sum[y^(n x), {n, 1, Infinity}]

$$-\frac{y^x}{-1+y^x}$$


N[Log[100]^s Integrate[100^x x^(s-1), {x, 0, 1}] /. s -> 2]
361.517 - 4.41506 × 10-14 i
N[Abs[Gamma[2, 0, -Log[100]]]]
361.517
FullSimplify[Log[100]^s Integrate[100^-x x^(s-1), {x, 0, 1}] /. s -> 2]
1 - Gamma[2, Log[100]]
Log[100]^s Integrate[100^x (-x)^(s-1), {x, 0, 1}] /. s -> 5
-24 + Gamma[5, -2 Log[10]]

```