```
t[n_, a_] := Mod[n, a] - Mod[n - 1, a]
Sum[ (-1)^(k) 1 / (2 k - 1), {k, 0, Infinity}]
```

$$\frac{1}{4}(-4-\pi)$$

```
Sum[ t[k, 2] 1 / (2 k - 1), {k, 0, Infinity}]
```

$$\sum_{k=0}^{\infty}\frac{-\text{Mod}[-1+k,2]+\text{Mod}[k,2]}{-1+2k}$$

```
Series[Tan[x], {x, 0, 20}]
```

$$x+\frac{x^3}{3}+\frac{2x^5}{15}+\frac{17x^7}{315}+\frac{62x^9}{2835}+\frac{1382x^{11}}{155925}+\frac{21844x^{13}}{6081075}+\frac{929569x^{15}}{638512875}+\frac{6404582x^{17}}{10854718875}+\frac{443861162x^{19}}{1856156927625}+O[x]^{21}$$

```
Series[ArcTan[x], {x, 0, 20}]
```

$$x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\frac{x^9}{9}-\frac{x^{11}}{11}+\frac{x^{13}}{13}-\frac{x^{15}}{15}+\frac{x^{17}}{17}-\frac{x^{19}}{19}+O[x]^{21}$$

```
Tan[Pi / 10]
```

$$\sqrt{1-\frac{2}{\sqrt{5}}}$$

```
Tan[Pi / 6]
```

$$\frac{1}{\sqrt{3}}$$

```
Tan[Pi / 12]
```

$$2-\sqrt{3}$$

```
Tan[Pi / 3]
```

$$\sqrt{3}$$

```
Tan[Pi / 5]
```

$$\sqrt{5-2\sqrt{5}}$$

```
N[Pi / 4]
0.785398
N[Sum[ t[k, 2] 1 / (2 k - 1), {k, 1, Infinity}]]
0.785398
N[Sum[ t[k, 3] 1 / (2 k - 1), {k, 1, Infinity}]]
$Aborted
N[Sum[ t[k, 4] 1 / (2 k - 1), {k, 1, Infinity}]]
```

```
Sum[ (-1)^(k) 1 / (k!), {k, 0, Infinity}]
```

$$\frac{1}{e}$$

```
Sum[ t[k, 2] 1 / (k!), {k, 0, Infinity}]
```

$$\text{FullSimplify}\left[\frac{-1 + e^2 - e\sqrt{2\pi}\ \text{BesselI}\left[-\frac{1}{2}, 1\right]}{2\,e}\right]$$

$$-\frac{1}{e}$$

```
Sum[ t[k, 3] 1 / (k!), {k, 0, Infinity}]
```

$$\text{FullSimplify}\left[-\frac{2\,\text{Cos}\left[\frac{\sqrt{3}}{2}\right]}{\sqrt{e}}\right]$$

$$-\frac{2\,\text{Cos}\left[\frac{\sqrt{3}}{2}\right]}{\sqrt{e}}$$

```
Sum[ t[k, 4] 1 / (k!), {k, 0, Infinity}]
```

$$\text{FullSimplify}\left[\frac{1}{2}\left(-\sqrt{2\pi}\ \text{BesselI}\left[-\frac{1}{2}, 1\right] + \sqrt{2\pi}\ \text{BesselI}\left[\frac{1}{2}, 1\right] - 2\sqrt{2\pi}\ \text{BesselJ}\left[-\frac{1}{2}, 1\right]\right)\right]$$

$$-\frac{1}{e} - 2\,\text{Cos}[1]$$

```
Sum[ t[k, 5] 1 / (k!), {k, 0, Infinity}]
```

$$\text{FullSimplify}\left[\frac{1}{24}\left(-96\,\text{HypergeometricPFQ}\left[\{\}, \left\{\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}\right\}, \frac{1}{3125}\right] + \right.\right.$$
$$24\,\text{HypergeometricPFQ}\left[\{\}, \left\{\frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{6}{5}\right\}, \frac{1}{3125}\right] +$$
$$12\,\text{HypergeometricPFQ}\left[\{\}, \left\{\frac{3}{5}, \frac{4}{5}, \frac{6}{5}, \frac{7}{5}\right\}, \frac{1}{3125}\right] +$$
$$4\,\text{HypergeometricPFQ}\left[\{\}, \left\{\frac{4}{5}, \frac{6}{5}, \frac{7}{5}, \frac{8}{5}\right\}, \frac{1}{3125}\right] +$$
$$\left.\left.\text{HypergeometricPFQ}\left[\{\}, \left\{\frac{6}{5}, \frac{7}{5}, \frac{8}{5}, \frac{9}{5}\right\}, \frac{1}{3125}\right]\right)\right]$$

$$\text{Expand}\left[-e^{-(-1)^{1/5}} - e^{(-1)^{2/5}} - e^{-(-1)^{3/5}} - e^{(-1)^{4/5}}\right]$$

$$-e^{-(-1)^{1/5}} - e^{(-1)^{2/5}} - e^{-(-1)^{3/5}} - e^{(-1)^{4/5}}$$

```
FullSimplify[Sum[ t[k, 6] 1 / (k!), {k, 0, Infinity}]]
```

$$-\frac{1}{e} - 4\,\text{Cos}\left[\frac{\sqrt{3}}{2}\right]\,\text{Cosh}\left[\frac{1}{2}\right]$$

```
FullSimplify[Sum[ t[k, 7] 1 / (k!), {k, 0, Infinity}]]
```

$$-e^{-(-1)^{1/7}} - e^{(-1)^{2/7}} - e^{-(-1)^{3/7}} - e^{(-1)^{4/7}} - e^{-(-1)^{5/7}} - e^{(-1)^{6/7}}$$

```
FullSimplify[Sum[ t[k, 8] 1 / (k!), {k, 0, Infinity}]]
```

$$-\frac{1}{e} - 2\left(\text{Cos}[1] + 2\,\text{Cos}\left[\frac{1}{\sqrt{2}}\right]\,\text{Cosh}\left[\frac{1}{\sqrt{2}}\right]\right)$$

```
FullSimplify[Sum[ t[k, 9] 1 / (k!), {k, 0, Infinity}]]
```

$Aborted

```
FullSimplify[Sum[ t[k, 10] 1 / (k!), {k, 0, Infinity}]]
```

```
1 + 1 / 3 - 2 / 5 + 1 / 7 + 1 / 9 - 2 / 11 + 1 / 13 + 1 / 15 - 2 / 17 +
1 - 1 / 3 + 1 / 5 - 1 / 7 + 1 / 9 - 1 / 11 + 1 / 13 - 1 / 15 + 1 / 17 -
2 / 3 - 3 / 5 + 2 / 7 - 3 / 11 + 2 / 15 - 3 / 17
2 - 1 / 5 + 2 / 9 - 3 / 11 + 2 / 13 - 1 / 17
Expand[Sum[ 1 / (4 k + 1) - 1 / (4 k + 3), {k, 0, Infinity}]]
```

$$\frac{\pi}{4}$$

```
FullSimplify[Sum[ 1 / (6 k + 1) + 1 / (6 k + 3) - 2 / (6 k + 5), {k, 0, Infinity}]]
```

$$\frac{1}{4}\left(\sqrt{3}\,\pi - \text{Log}[3]\right)$$

```
Expand[Sum[ 1 / (8 k + 1) + 1 / (8 k + 3) + 1 / (8 k + 5) - 3 / (8 k + 7), {k, 0, Infinity}]]
```

$$\textbf{FullSimplify}\left[\frac{\pi}{4} + \frac{\pi}{2\sqrt{2}} - \frac{\text{Log}[2]}{2\sqrt{2}} + \frac{\text{Log}\left[2-\sqrt{2}\right]}{\sqrt{2}}\right]$$

$$\frac{1}{4}\left(\pi + \sqrt{2}\,\pi - \sqrt{2}\,\text{Log}\left[3+2\sqrt{2}\right]\right)$$

```
Expand[Sum[1 / (8 k + 1) + 1 / (8 k + 3) - 3 / (8 k + 5) + 1 / (8 k + 7), {k, 0, Infinity}]]
```

$$-\frac{\pi}{4} + \frac{\pi}{2\sqrt{2}} - \frac{\text{Log}[2]}{2\sqrt{2}} + \frac{\text{Log}\left[2+\sqrt{2}\right]}{\sqrt{2}}$$

```
Expand[
 Sum[ 1 / (10 k + 1) + 1 / (10 k + 3) + 1 / (10 k + 5) + 1 / (10 k + 7) - 4 / (10 k + 9), {k, 0, Infinity}]]
```

$$\textbf{FullSimplify}\left[\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,\pi}{2\left(-1+\sqrt{5}\right)} + \frac{\text{Log}[4]}{2\left(-1+\sqrt{5}\right)} - \frac{\sqrt{5}\,\text{Log}[4]}{2\left(-1+\sqrt{5}\right)} + \frac{3\,\text{Log}\left[\frac{1}{8}\left(5-\sqrt{5}\right)\right]}{4\left(-1+\sqrt{5}\right)} - \frac{\sqrt{5}\,\text{Log}\left[\frac{1}{8}\left(5-\sqrt{5}\right)\right]}{4\left(-1+\sqrt{5}\right)} + \frac{\text{Log}\left[-1+\sqrt{5}\right]}{-1+\sqrt{5}} - \frac{3\,\text{Log}\left[1+\sqrt{5}\right]}{2\left(-1+\sqrt{5}\right)} + \frac{\sqrt{5}\,\text{Log}\left[1+\sqrt{5}\right]}{2\left(-1+\sqrt{5}\right)} - \frac{\text{Log}\left[\frac{1}{8}\left(5+\sqrt{5}\right)\right]}{2\left(-1+\sqrt{5}\right)}\right]$$

$$\frac{1}{16}\left(1+\sqrt{5}\right)\left(\sqrt{2\left(5+\sqrt{5}\right)}\,\pi+\sqrt{5}\,\mathrm{Log}\left[1+\frac{2}{\sqrt{5}}\right]+\mathrm{Log}\left[1525-682\sqrt{5}\right]\right)$$

```
FullSimplify[Sum[ 1 / (12 k + 1) + 1 / (12 k + 3) +
   1 / (12 k + 5) + 1 / (12 k + 7) + 1 / (12 k + 9) - 5 / (12 k + 11), {k, 0, Infinity}]]
```

$$\frac{1}{4}\left(\left(2+\sqrt{3}\right)\pi-4\sqrt{3}\,\mathrm{ArcCoth}\left[\sqrt{3}\right]-\mathrm{Log}[3]\right)$$

```
FullSimplify[Sum[ 1 / (14 k + 1) + 1 / (14 k + 3) + 1 / (14 k + 5) +
   1 / (14 k + 7) + 1 / (14 k + 9) + 1 / (14 k + 11) - 6 / (14 k + 13), {k, 0, Infinity}]]
```

$$\frac{1}{4}\pi\,\mathrm{Cot}\left[\frac{\pi}{14}\right]+\mathrm{Cos}\left[\frac{\pi}{7}\right]\mathrm{Log}\left[\mathrm{Tan}\left[\frac{\pi}{14}\right]\right]+\mathrm{Log}\left[\mathrm{Tan}\left[\frac{3\pi}{14}\right]\right]\mathrm{Sin}\left[\frac{\pi}{14}\right]+\mathrm{Log}\left[\mathrm{Tan}\left[\frac{\pi}{7}\right]\right]\mathrm{Sin}\left[\frac{3\pi}{14}\right]$$

```
FullSimplify[Sum[ 1 / (12 k + 1) + 1 / (12 k + 3) + 1 / (12 k + 5) -
   1 / (12 k + 7) - 1 / (12 k + 9) - 1 / (12 k + 11), {k, 0, Infinity}]]
```

$$\frac{5\pi}{12}$$

```
FullSimplify[Sum[ 1 / (16 k + 1) + 1 / (16 k + 3) + 1 / (16 k + 5) + 1 / (16 k + 7) -
   1 / (16 k + 9) - 1 / (16 k + 11) - 1 / (16 k + 13) - 1 / (16 k + 15), {k, 0, Infinity}]]
```

$$\frac{1}{4}\sqrt{2+\sqrt{2}}\,\pi$$

```
FullSimplify[Sum[ 1 / (12 k + 1) ^3 + 1 / (12 k + 3) ^3 + 1 / (12 k + 5) ^3 -
   1 / (12 k + 7) ^3 - 1 / (12 k + 9) ^3 - 1 / (12 k + 11) ^3, {k, 0, Infinity}]]
```

$$\frac{29\pi^3}{864}$$

```
FullSimplify[Sum[ 1 / (16 k + 1) ^3 + 1 / (16 k + 3) ^3 + 1 / (16 k + 5) ^3 + 1 / (16 k + 7) ^3 -
   1 / (16 k + 9) ^3 - 1 / (16 k + 11) ^3 - 1 / (16 k + 13) ^3 - 1 / (16 k + 15) ^3, {k, 0, Infinity}]]
```

$$\frac{1}{512}\sqrt{274+17\sqrt{2}}\,\pi^3$$

```
FullSimplify[Sum[ 2 / (12 k + 1) ^3 + 1 / (12 k + 3) ^3 + 1 / (12 k + 5) ^3 -
   1 / (12 k + 7) ^3 - 1 / (12 k + 9) ^3 - 2 / (12 k + 11) ^3, {k, 0, Infinity}]]
```

$$\frac{1}{864}\left(43+8\sqrt{3}\right)\pi^3$$

```
FullSimplify[Sum[4 / (12 k + 1) + 2 / (12 k + 3) + 1 / (12 k + 5) -
   1 / (12 k + 7) - 2 / (12 k + 9) - 4 / (12 k + 11), {k, 0, Infinity}]]
```

$$\frac{1}{4}\left(4+\sqrt{3}\right)\pi$$

```
FullSimplify[Sum[ 6 / (12 k + 1) + 3 / (12 k + 3) + 1 / (12 k + 5) -
   1 / (12 k + 7) - 3 / (12 k + 9) - 6 / (12 k + 11), {k, 0, Infinity}]]
```

$$\frac{1}{12}\left(17+5\sqrt{3}\right)\pi$$

```
FullSimplify[Sum[ 6 / (12 k + 1) - 3 / (12 k + 3) + 1 / (12 k + 5) -
   1 / (12 k + 7) + 3 / (12 k + 9) - 6 / (12 k + 11), {k, 0, Infinity}]]
```

$$\frac{1}{12}\left(11+5\sqrt{3}\right)\pi$$

```
FullSimplify[Sum[ 2 / (12 k + 2) + 1 / (12 k + 4) + 1 / (12 k + 6) -
   1 / (12 k + 8) - 1 / (12 k + 10) - 2 / (12 k + 12), {k, 0, Infinity}]]
```

$$\frac{1}{72}\left(11\sqrt{3}\,\pi+9\,\mathrm{Log}[3]+12\,\mathrm{Log}[4]\right)$$

```
Sum[ (-1) ^ (k + 1) 1 / (2 k - 1) ^3, {k, 1, Infinity}]
```

$$\frac{\pi^3}{32}$$

```
Sum[ 1 / (4 k - 3) ^3 - 1 / (4 k - 1) ^3, {k, 1, Infinity}]
```

$$\frac{\pi^3}{32}$$

```
Sum[1 / (6 k - 5)^3 + 1 / (4 k - 1)^3, {k, 1, Infinity}]
Sum[ 1 / (6 k + 1)^3 + 1 / (6 k + 3)^3 - 2 / (6 k + 5)^3, {k, 0, Infinity}]
```

$$\frac{1}{36}\left(\sqrt{3}\,\pi^3 - 14\,\mathrm{Zeta}[3]\right)$$

```
Sum[ 1 / (6 k + 1) + 1 / (6 k + 3) - 2 / (6 k + 5), {k, 0, Infinity}]
```

$$\frac{1}{4}\left(\sqrt{3}\,\pi - \mathrm{Log}[3]\right)$$

```
Sum[1 / (3 k + 1) + 1 / (3 k + 2) - 2 / (3 k + 3), {k, 0, Infinity}]
Log[3]
Sum[1 / (3 k + 1)^2 + 1 / (3 k + 2)^2 - 2 / (3 k + 3)^2, {k, 0, Infinity}]
```

$$\frac{\pi^2}{9}$$

```
Sum[1 / (3 k + 1)^3 + 1 / (3 k + 2)^3 - 2 / (3 k + 3)^3, {k, 0, Infinity}]
```

$$\frac{8\,\mathrm{Zeta}[3]}{9}$$

```
Sum[1 / (3 k + 1)^4 + 1 / (3 k + 2)^4 - 2 / (3 k + 3)^4, {k, 0, Infinity}]
```

$$\frac{13\,\pi^4}{1215}$$

```
Sum[1 / ((3 k + 1)) + 1 / ((3 k + 2)) - 2 / ((3 k + 3)), {k, 0, Infinity}]
```

$\mathrm{Log}[3]$

```
Sum[1 / ((3 k + 1)) - 2 / ((3 k + 2)) + 1 / ((3 k + 3)), {k, 0, Infinity}]
```

$$\frac{1}{6}\left(\sqrt{3}\,\pi - \mathrm{Log}[27]\right)$$

```
Sum[-2 / ((3 k + 1)) + 1 / ((3 k + 2)) + 1 / ((3 k + 3)), {k, 0, Infinity}]
```

$$\frac{1}{6}\left(-\sqrt{3}\,\pi - 3\,\mathrm{Log}[3]\right)$$

```
Sum[-3 / ((3 k + 1)) + 2 / ((3 k + 2)) + 1 / ((3 k + 3)), {k, 0, Infinity}]
```

$$\frac{1}{18}\left(-5\sqrt{3}\,\pi - 9\,\mathrm{Log}[3]\right)$$

```
Sum[-4 / ((3 k + 1)) + 3 / ((3 k + 2)) + 1 / ((3 k + 3)), {k, 0, Infinity}]
```

$$\frac{1}{18}\left(-7\sqrt{3}\,\pi - 9\,\mathrm{Log}[3]\right)$$

```
Sum[-1 / ((3 k + 1)) + 1 / ((3 k + 2)), {k, 0, Infinity}]
```

$$-\frac{\pi}{3\sqrt{3}}$$

```
Sum[-1 / ((3 k + 1)) + 1 / ((3 k + 3)), {k, 0, Infinity}]
```

$$\frac{1}{18}\left(-\sqrt{3}\,\pi - 9\,\mathrm{Log}[3]\right)$$

```
Sum[-1 / ((3 k + 1))^3 + 1 / ((3 k + 2))^3, {k, 0, Infinity}]
```

$$-\frac{4\,\pi^3}{81\,\sqrt{3}}$$

```
Sum[-1/((4 k+3))^3 + 1/((4 k+1))^3, {k, 0, Infinity}]
```

$$\frac{\pi^3}{32}$$

```
Sum[-1/((5 k+4))^3 + 1/((5 k+1))^3, {k, 0, Infinity}]
```

$$-\frac{8\,\sqrt{\frac{1}{5}\left(5+2\,\sqrt{5}\right)}\,\pi^3}{125\,\left(-5+\sqrt{5}\right)}$$

```
Sum[-1/((5 k+4)) + 1/((5 k+1)), {k, 0, Infinity}]
```

$$\frac{1}{5}\,\sqrt{\frac{1}{5}\left(5+2\,\sqrt{5}\right)}\,\pi$$

```
Expand[Sum[1/(8 k+1) + 1/(8 k+3) - 1/(8 k+5) - 1/(8 k+7), {k, 0, Infinity}]]
```

$$\frac{\pi}{2\,\sqrt{2}}$$

```
Expand[Sum[1/(8 k+1)^3 + 1/(8 k+3)^3 - 1/(8 k+5)^3 - 1/(8 k+7)^3, {k, 0, Infinity}]]
```

$$\frac{3\,\pi^3}{64\,\sqrt{2}}$$

```
Expand[Sum[1/(8 k+1) - 0/(8 k+5) - 1/(8 k+7), {k, 0, Infinity}]]
```

$$\frac{\pi}{8}+\frac{\pi}{4\,\sqrt{2}}$$

```
Expand[Sum[1/(8 k+1)^3 - 1/(8 k+7)^3, {k, 0, Infinity}]]
```

$$\frac{\sqrt{2+\sqrt{2}}\,\pi^3}{128\,\left(2-\sqrt{2}\right)^{3/2}}$$

```
Expand[Sum[1/(10 k+1) - 1/(10 k+9), {k, 0, Infinity}]]
```

$$\frac{1}{10}\,\sqrt{5+2\,\sqrt{5}}\,\pi$$

```
Expand[Sum[1/(10 k+1)^3 - 1/(10 k+9)^3, {k, 0, Infinity}]]
```

$$\frac{1}{100}\,\sqrt{\frac{1}{5}\left(5+2\,\sqrt{5}\right)}\,\pi^3+\frac{3}{500}\,\sqrt{5+2\,\sqrt{5}}\,\pi^3$$

```
Expand[Sum[1/(12 k+1) - 1/(12 k+11), {k, 0, Infinity}]]
```

$$\frac{\pi}{6}+\frac{\pi}{4\,\sqrt{3}}$$

Expand[Sum[1 / (20 k + 1) - 1 / (20 k + 19), {k, 0, Infinity}]]

$$\frac{\pi}{10\sqrt{2}\left(\sqrt{2}+\sqrt{10}-2\sqrt{5-\sqrt{5}}\right)}+\frac{\pi}{2\sqrt{10}\left(\sqrt{2}+\sqrt{10}-2\sqrt{5-\sqrt{5}}\right)}+\frac{\sqrt{5-\sqrt{5}}\,\pi}{10\left(\sqrt{2}+\sqrt{10}-2\sqrt{5-\sqrt{5}}\right)}$$

Expand[Sum[1 / (20 k + 1) ^3 - 1 / (20 k + 19) ^3, {k, 0, Infinity}]]

$$\frac{\sqrt{\frac{2}{5}}\,\pi^3}{25\left(\sqrt{2}+\sqrt{10}-2\sqrt{5-\sqrt{5}}\right)^3}+\frac{\sqrt{2}\,\pi^3}{125\left(\sqrt{2}+\sqrt{10}-2\sqrt{5-\sqrt{5}}\right)^3}+\frac{2\sqrt{5-\sqrt{5}}\,\pi^3}{125\left(\sqrt{2}+\sqrt{10}-2\sqrt{5-\sqrt{5}}\right)^3}$$

Expand[Sum[1 / (22 k + 1) - 1 / (22 k + 21), {k, 0, Infinity}]]

$$\frac{1}{22}\,\pi\,\mathrm{Cot}\left[\frac{\pi}{22}\right]$$

Expand[Sum[1 / (20 k + 1) ^5 - 1 / (20 k + 19) ^5, {k, 0, Infinity}]]

$$\frac{4\sqrt{\frac{2}{5}}\,\pi^5}{625\left(\sqrt{2}+\sqrt{10}-2\sqrt{5-\sqrt{5}}\right)^5}+\frac{2\sqrt{2}\,\pi^5}{1875\left(\sqrt{2}+\sqrt{10}-2\sqrt{5-\sqrt{5}}\right)^5}+$$

$$\frac{\sqrt{\frac{1}{5}\left(5-\sqrt{5}\right)}\,\pi^5}{1875\left(\sqrt{2}+\sqrt{10}-2\sqrt{5-\sqrt{5}}\right)^5}+\frac{23\sqrt{5-\sqrt{5}}\,\pi^5}{9375\left(\sqrt{2}+\sqrt{10}-2\sqrt{5-\sqrt{5}}\right)^5}$$

Sum[1 / 3 / ((3 k + 1)) - 2 / 3 / ((3 k + 2)) + 1 / 3 / ((3 k + 3)), {k, 0, Infinity}]

$$\frac{1}{18}\left(\sqrt{3}\,\pi-3\,\mathrm{Log}[3]\right)$$

Sum[1 / ((3 k + 1)) - 3 / ((3 k + 2)) + 2 / ((3 k + 3)), {k, 0, Infinity}]

$$\frac{1}{9}\left(2\sqrt{3}\,\pi-9\,\mathrm{Log}[3]\right)$$

Sum[1 / ((3 k + 1)) + 1 / ((3 k + 2)) - 2 / ((3 k + 3)), {k, 0, Infinity}]

Log[3]

Sum[1 / ((3 k + 1)) + 1 / ((3 k + 2)) - 2 / ((3 k + 3)), {k, 0, Infinity}]

Log[3]

Sum[3 / ((3 k + 1)) + 4 / ((3 k + 2)) - 7 / ((3 k + 3)), {k, 0, Infinity}]

$$\frac{1}{18}\left(-\sqrt{3}\,\pi+21\,\mathrm{Log}[27]\right)$$

```
Sum[ 2 / ((3 k + 1)) + 2 / ((3 k + 2)) - 4 / ((3 k + 3)), {k, 0, Infinity}]
```

Log[9]

```
Sum[ 0 / ((3 k + 1)) + 1 / ((3 k + 2)) - 1 / ((3 k + 3)), {k, 0, Infinity}]
```

$$\frac{1}{18}\left(-\sqrt{3}\,\pi + 9\,\mathrm{Log}[3]\right)$$

```
Sum[ -2 / ((3 k + 1)) + 2 / ((3 k + 2)) - 0 / ((3 k + 3)), {k, 0, Infinity}]
```

$$-\frac{2\pi}{3\sqrt{3}}$$

```
FullSimplify[Sum[ 1 / (16 k + 2) ^3 + 1 / (16 k + 4) ^3 + 1 / (16 k + 6) ^3 + 1 / (16 k + 8) ^3 -
   1 / (16 k + 10) ^3 - 1 / (16 k + 12) ^3 - 1 / (16 k + 14) ^3 - 1 / (16 k + 16) ^3, {k, 0, Infinity}]]
```

$$\frac{\left(1 + 6\sqrt{2}\right)\pi^3 + 3\,\mathrm{Zeta}[3]}{2048}$$

```
FullSimplify[Sum[ 1 / (16 k + 2) + 1 / (16 k + 4) + 1 / (16 k + 6) + 1 / (16 k + 8) +
   1 / (16 k + 10) + -7 / (16 k + 12) + 1 / (16 k + 14) + 1 / (16 k + 16), {k, 0, Infinity}]]
```

$$\frac{\pi}{4}$$

```
FullSimplify[Sum[ 1 / (16 k + 2) ^3 + 1 / (16 k + 4) ^3 + 1 / (16 k + 6) ^3 + 1 / (16 k + 8) ^3 +
   1 / (16 k + 10) ^3 + -7 / (16 k + 12) ^3 + 1 / (16 k + 14) ^3 + 1 / (16 k + 16) ^3, {k, 0, Infinity}]]
```

Power::infy : Infinite expression $\frac{1}{0^3}$ encountered. ≫

```
FullSimplify[Sum[ 1 / (16 k + 2) + 1 / (16 k + 4) + 1 / (16 k + 6) - 7 / (16 k + 8) +
   1 / (16 k + 10) + +1 / (16 k + 12) + 1 / (16 k + 14) + 1 / (16 k + 16), {k, 0, Infinity}]]
```

$$\frac{\mathrm{Log}[2]}{2}$$

```
FullSimplify[Sum[ 1 / (16 k + 2) - 7 / (16 k + 4) + 1 / (16 k + 6) + 1 / (16 k + 8) +
   1 / (16 k + 10) + +1 / (16 k + 12) + 1 / (16 k + 14) + 1 / (16 k + 16), {k, 0, Infinity}]]
```

$$-\frac{\pi}{4}$$

```
FullSimplify[Sum[ 1 / (16 k + 2) + 1 / (16 k + 4) + 1 / (16 k + 6) + 1 / (16 k + 8) +
   1 / (16 k + 10) + +1 / (16 k + 12) + 1 / (16 k + 14) - 7 / (16 k + 16), {k, 0, Infinity}]]
```

$$\frac{\mathrm{Log}[8]}{2}$$

```
FullSimplify[Sum[ 1 / (8 k + 1) - 7 / (8 k + 2) + 1 / (8 k + 3) + 1 / (8 k + 4) +
   1 / (8 k + 5) + +1 / (8 k + 6) + 1 / (8 k + 7) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$-\frac{\pi}{2}$

```
FullSimplify[Sum[ 1 / (8 k + 1) + 1 / (8 k + 2) + 1 / (8 k + 3) + 1 / (8 k + 4) +
   1 / (8 k + 5) - 7 / (8 k + 6) + 1 / (8 k + 7) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$\frac{\pi}{2}$

```
FullSimplify[Sum[ 1 / (4 k + 1) - 3 / (4 k + 2) + 1 / (4 k + 3) + 1 / (4 k + 4), {k, 0, Infinity}]]
```

0

```
FullSimplify[
 Sum[ 1 / (4 k + 1) ^3 - 3 / (4 k + 2) ^3 + 1 / (4 k + 3) ^3 + 1 / (4 k + 4) ^3, {k, 0, Infinity}]]
```

Power::infy : Infinite expression $\frac{1}{0^3}$ encountered. ≫

```
N[Sum[ 1 / (4 k + 1) ^3 - 3 / (4 k + 2) ^3 + 1 / (4 k + 3) ^3 + 1 / (4 k + 4) ^3, {k, 0, 10 000}]]
```

0.6761570080272795

```
N[Sum[ 1 / (4 k + 1) - 3 / (4 k + 2) + 1 / (4 k + 3) + 1 / (4 k + 4), {k, 0, 100 000}]]
```

1.24998×10^{-6}

```
FullSimplify[Sum[ 1 / (8 k + 1) ^3 + 1 / (8 k + 2) ^3 + 1 / (8 k + 3) ^3 + 1 / (8 k + 4) ^3 +
   1 / (8 k + 5) ^3 - 7 / (8 k + 6) ^3 + 1 / (8 k + 7) ^3 + 1 / (8 k + 8) ^3, {k, 0, Infinity}]]
```

```
N[Sum[ 1 / (8 k + 1) ^3 + 1 / (8 k + 2) ^3 + 1 / (8 k + 3) ^3 + 1 / (8 k + 4) ^3 +
   1 / (8 k + 5) ^3 - 7 / (8 k + 6) ^3 + 1 / (8 k + 7) ^3 + 1 / (8 k + 8) ^3, {k, 0, 30 000}]]
```

1.16063

Power::infy : Infinite expression $\frac{1}{0^3}$ encountered. ≫

1.1606300811540349`

1.1606300811569534`

```
N[Sum[ 1 / (8 k + 1) ^3 - 7 / (8 k + 2) ^3 + 1 / (8 k + 3) ^3 + 1 / (8 k + 4) ^3 +
   1 / (8 k + 5) ^3 + +1 / (8 k + 6) ^3 + 1 / (8 k + 7) ^3 + 1 / (8 k + 8) ^3, {k, 0, 30 000}]]
```

0.191684

31.006276680299816` / 0.19168393489758725`

161.757

```
1 / N[Pi^3]^(1 / 2)
```

0.179587

```
1 / 31.006276680299816`
```

0.0322515

```
FullSimplify[Sum[-7 / (8 k + 1)^2 + 1 / (8 k + 2)^2 + 1 / (8 k + 3)^2 + 1 / (8 k + 4)^2 +
   1 / (8 k + 5)^2 + 1 / (8 k + 6)^2 + 1 / (8 k + 7)^2 + 1 / (8 k + 8)^2, {k, 0, Infinity}]]
```

$$\frac{\pi^2}{6} - \frac{1}{8}\,\text{PolyGamma}\left[1, \frac{1}{8}\right]$$

```
Sum[1 / (8 k + 1)^3 + 1 / (8 k + 2)^3 + 1 / (8 k + 3)^3 + 1 / (8 k + 4)^3 +
  1 / (8 k + 5)^3 + 1 / (8 k + 6)^3 + 1 / (8 k + 7)^3 - 7 / (8 k + 8)^3, {k, 0, Infinity}]
```

$$\frac{63\,\text{Zeta}[3]}{64}$$

```
Sum[-7 / (8 k + 1)^3 + 1 / (8 k + 2)^3 + 1 / (8 k + 3)^3 + 1 / (8 k + 4)^3 +
  1 / (8 k + 5)^3 + 1 / (8 k + 6)^3 + 1 / (8 k + 7)^3 + 1 / (8 k + 8)^3, {k, 0, Infinity}]
```

$$\frac{1}{128}\left(\text{PolyGamma}\left[2, \frac{1}{8}\right] + 128\,\text{Zeta}[3]\right)$$

```
N[Sum[ 1 / (8 k + 1) + 1 / (8 k + 2) + 1 / (8 k + 3) + 1 / (8 k + 4) +
   1 / (8 k + 5) - 7 / (8 k + 6) + 1 / (8 k + 7) + 1 / (8 k + 8), {k, 0, 1000}]]
```

1.57061

```
N[Pi / 2]
```

1.5708

```
FullSimplify[Sum[ 1 / (8 k + 2) + 2 / (8 k + 3) +
   1 / (8 k + 4) - 7 / (8 k + 6) + 2 / (8 k + 7) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$$\frac{\pi}{4}$$

```
FullSimplify[Sum[ 2 / (8 k + 1) + 1 / (8 k + 2) +
   1 / (8 k + 4) + 2 / (8 k + 5) - 7 / (8 k + 6) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$$\frac{3\pi}{4}$$

```
FullSimplify[Sum[-7 / (8 k + 2) + 2 / (8 k + 3) +
   1 / (8 k + 4) + 1 / (8 k + 6) + 2 / (8 k + 7) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$$-\frac{3\pi}{4}$$

```
FullSimplify[Sum[ 2 / (8 k + 1) - 7 / (8 k + 2) +
   1 / (8 k + 4) + 2 / (8 k + 5) + +1 / (8 k + 6) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$$-\frac{\pi}{4}$$

```
FullSimplify[Sum[ -7 / (8 k + 2) ^3 + 2 / (8 k + 3) ^3 + 1 / (8 k + 4) ^3 +
   1 / (8 k + 6) ^3 + 2 / (8 k + 7) ^3 + 1 / (8 k + 8) ^3, {k, 0, Infinity}]]
```

$$-\frac{3}{64}\left(\pi^3 - 12\,\mathrm{Zeta}[3]\right)$$

```
FullSimplify[Sum[ 2 / (8 k + 1) ^2 + 1 / (8 k + 2) ^2 + 1 / (8 k + 4) ^2 +
   2 / (8 k + 5) ^2 - 7 / (8 k + 6) ^2 + 1 / (8 k + 8) ^2, {k, 0, Infinity}]]
```

Power::infy : Infinite expression $\frac{1}{0^2}$ encountered. ≫

```
FullSimplify[Sum[ -8 / (8 k + 4) ^2, {k, 0, Infinity}]]
```

$$-\frac{\pi^2}{16}$$

```
FullSimplify[Sum[ 1 / (8 k + 4) ^2, {k, 0, Infinity}]]
```

$$\frac{\pi^2}{128}$$

```
FullSimplify[
 Sum[ 1 / (4 k + 1) ^2 - 3 / (4 k + 2) ^2 + 1 / (4 k + 3) ^2 + 1 / (4 k + 4) ^2, {k, 0, Infinity}]]
```

Power::infy : Infinite expression $\frac{1}{0^2}$ encountered. ≫

```
FullSimplify[
  Sum[ 1 / (4 k + 1) ^2 + 1 / (4 k + 2) ^2 + 1 / (4 k + 3) ^2 + 1 / (4 k + 4) ^2, {k, 0, Infinity}]] +
 FullSimplify[Sum[ -4 / (4 k + 2) ^2, {k, 0, Infinity}]]
```

$$\frac{\pi^2}{24}$$

```
FullSimplify[
  Sum[ 1 / (4 k + 1) ^3 + 1 / (4 k + 2) ^3 + 1 / (4 k + 3) ^3 + 1 / (4 k + 4) ^3, {k, 0, Infinity}]] +
 FullSimplify[Sum[ -4 / (4 k + 2) ^3, {k, 0, Infinity}]]
```

$$\frac{9\,\mathrm{Zeta}[3]}{16}$$

```
N[
 FullSimplify[Sum[ 1 / (4 k + 1) ^3 - 3 / (4 k + 2) ^3 + 1 / (4 k + 3) ^3 + 1 / (4 k + 4) ^3, {k, 0, 1000}]]]
```

0.676157

```
N[ 9 Zeta[3] / 16]
```

0.676157

```
FullSimplify[Sum[ 1 / (4 k + 2) ^2, {k, 0, Infinity}]]
```

$\frac{\pi^2}{32}$

```
FullSimplify[Sum[ 1 / (4 k + 4) ^2, {k, 0, Infinity}]]
```

$\frac{\pi^2}{96}$

```
FullSimplify[Sum[ 1 / (6 k + 3) ^2, {k, 0, Infinity}]]
```

$\frac{\pi^2}{72}$

```
FullSimplify[Sum[ 1 / (6 k + 6) ^2, {k, 0, Infinity}]]
```

$\frac{\pi^2}{216}$

```
FullSimplify[Sum[ 2 / (8 k + 1) + 1 / (8 k + 2) + 2 / (8 k + 3) + 1 / (8 k + 4) +
   2 / (8 k + 5) - 11 / (8 k + 6) + 2 / (8 k + 7) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$\frac{1}{4}$ (3 π + Log[4])

```
FullSimplify[Sum[ 1 / (8 k + 1) + 2 / (8 k + 2) + 1 / (8 k + 3) + 2 / (8 k + 4) +
   1 / (8 k + 5) - 10 / (8 k + 6) + 1 / (8 k + 7) + 2 / (8 k + 8), {k, 0, Infinity}]]
```

$\frac{1}{4}$ (3 π - Log[4])

```
FullSimplify[Sum[ 1 / (8 k + 1) + 3 / (8 k + 2) + 1 / (8 k + 3) + 1 / (8 k + 4) +
   1 / (8 k + 5) - 9 / (8 k + 6) + 1 / (8 k + 7) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$\frac{3\pi}{4}$

```
FullSimplify[Sum[ 1 / (8 k + 1) + 5 / (8 k + 2) + 1 / (8 k + 3) + 1 / (8 k + 4) +
   1 / (8 k + 5) - 11 / (8 k + 6) + 1 / (8 k + 7) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

π

```
FullSimplify[Sum[ 1 / (8 k + 1) - 1 / (8 k + 2) + 1 / (8 k + 3) + 1 / (8 k + 4) +
   1 / (8 k + 5) - 5 / (8 k + 6) + 1 / (8 k + 7) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$\frac{\pi}{4}$

```
FullSimplify[Sum[ 1 / (8 k + 1) - 3 / (8 k + 2) + 1 / (8 k + 3) + 1 / (8 k + 4) +
   1 / (8 k + 5) - 3 / (8 k + 6) + 1 / (8 k + 7) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

0

```
FullSimplify[Sum[ 1 / (8 k + 1) - 11 / (8 k + 2) + 1 / (8 k + 3) + 1 / (8 k + 4) +
   1 / (8 k + 5) + 5 / (8 k + 6) + 1 / (8 k + 7) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$-\pi$

```
FullSimplify[Sum[ 1 / (8 k + 1) ^3 + 5 / (8 k + 2) ^3 + 1 / (8 k + 3) ^3 + 1 / (8 k + 4) ^3 +
   1 / (8 k + 5) ^3 - 11 / (8 k + 6) ^3 + 1 / (8 k + 7) ^3 + 1 / (8 k + 8) ^3, {k, 0, Infinity}]]
```

Power::infy : Infinite expression $\frac{1}{0^3}$ encountered. ≫

```
FullSimplify[Sum[6 / (8 k + 2) ^3 - 12 / (8 k + 6) ^3, {k, 0, Infinity}]] +
 FullSimplify[Sum[ 1 / (8 k + 1) ^3 + 1 / (8 k + 2) ^3 + 1 / (8 k + 3) ^3 + 1 / (8 k + 4) ^3 +
    1 / (8 k + 5) ^3 + 1 / (8 k + 6) ^3 + 1 / (8 k + 7) ^3 + 1 / (8 k + 8) ^3, {k, 0, Infinity}]]
```

$\frac{3}{256}\left(3\,\pi^3 - 28\,\text{Zeta}[3]\right) + \text{Zeta}[3]$

```
Expand[Sum[ 1 / (4 k + 1) - 1 / (4 k + 3), {k, 0, Infinity}]]
```

$\frac{\pi}{4}$

```
FullSimplify[Sum[ 1 / (8 k + 1) + 1 / (8 k + 2) + 1 / (8 k + 3) + 1 / (8 k + 4) +
   1 / (8 k + 5) - 7 / (8 k + 6) + 1 / (8 k + 7) + 1 / (8 k + 8), {k, 0, Infinity}]]
```

$\frac{\pi}{2}$

```
Expand[Sum[ 1 / (8 k + 2) - 1 / (8 k + 6), {k, 0, Infinity}]]
```

$\frac{\pi}{8}$

```
FullSimplify[Sum[1 / (8 k + 1) ^2 + 1 / (8 k + 2) ^2 + 1 / (8 k + 3) ^2 + 1 / (8 k + 4) ^2 +
    1 / (8 k + 5) ^2 + 1 / (8 k + 6) ^2 + 1 / (8 k + 7) ^2 + 1 / (8 k + 8) ^2, {k, 0, Infinity}]] -
 FullSimplify[Sum[-4 / (8 k + 2) ^2 - 4 / (8 k + 6) ^2, {k, 0, Infinity}]]
```

$$\frac{7\pi^2}{24}$$

```
FullSimplify[
 Sum[1 / (16 k + 1) + 1 / (16 k + 2) + 1 / (16 k + 3) + 1 / (16 k + 4) + 1 / (16 k + 5) + -7 / (16 k + 6) +
   1 / (16 k + 7) + 1 / (16 k + 8) + 1 / (16 k + 9) + 1 / (16 k + 10) + 1 / (16 k + 11) + 1 / (16 k + 12) +
   1 / (16 k + 13) - 7 / (16 k + 14) + 1 / (16 k + 15) + 1 / (16 k + 16), {k, 0, Infinity}]]
```

$$\frac{\pi}{2}$$

```
FullSimplify[
 Sum[1 / (16 k + 1) - 7 / (16 k + 2) + 1 / (16 k + 3) + 1 / (16 k + 4) + 1 / (16 k + 5) + 1 / (16 k + 6) +
   1 / (16 k + 7) + 1 / (16 k + 8) + 1 / (16 k + 9) - 7 / (16 k + 10) + 1 / (16 k + 11) + 1 / (16 k + 12) +
   1 / (16 k + 13) + 1 / (16 k + 14) + 1 / (16 k + 15) + 1 / (16 k + 16), {k, 0, Infinity}]]
```

$$-\frac{\pi}{2}$$