Looks like this is just yet another scratch survey of a handful of the main identities expressed in an intermediary notation. Nothing to interesting here that isn't expressed elsewhere.

$$[\zeta_n(s)]^{*1}=\sum_{j=1}^{\lfloor n\rfloor} j^{-s} \qquad [\zeta_n(s)]^{*2}=\sum_{j=1}^{\lfloor n\rfloor}\sum_{k=1}^{\lfloor \frac{n}{j}\rfloor}(j\cdot k)^{-s} \qquad [\zeta_n(s)]^{*3}=\sum_{j=1}^{\lfloor n\rfloor}\sum_{k=1}^{\lfloor \frac{n}{j}\rfloor}\sum_{m=1}^{\lfloor \frac{n}{jk}\rfloor}(j\cdot k\cdot m)^{-s}$$

$$[\zeta_n(s)-1]^{*1}=\sum_{j=2}^{\lfloor n\rfloor} j^{-s} \qquad [\zeta_n(s)-1]^{*2}=\sum_{j=2}^{n}\sum_{k=2}^{\lfloor \frac{n}{j}\rfloor}(j\cdot k)^{-s} \qquad [\zeta_n(s)-1]^{*3}=\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor \frac{n}{j}\rfloor}\sum_{m=2}^{\lfloor \frac{n}{jk}\rfloor}(j\cdot k\cdot m)^{-s}$$

The limit of this as n approaches infinity, if $\Re(s)>1$, is

$$\lim_{n\to\infty}[\zeta_n(s)]^{*k}=\zeta(s)^k \quad \text{and} \quad \lim_{n\to\infty}[\zeta_n(s)-1]^{*k}=(\zeta(s)-1)^k$$

$$\binom{z}{k}=\frac{z(z-1)\ldots(z-k+1)}{k!}$$

$$d_z(n)=\prod_{p^\alpha|n}(-1)^\alpha\binom{-z}{\alpha}$$

$$[\zeta_n(s)]^{*z}=\sum_{j=1}^{n} j^{-s}d_z(j)$$

The limit of this as n approaches infinity, if $\Re(s)>1$, is

$$\lim_{n\to\infty}[\zeta_n(s)]^{*z}=\zeta(s)^z$$

$$\Pi(n)=\sum_{j=2}^{n}\frac{\Lambda(j)}{\log n}$$

$$\psi(n)=\sum_{j=2}^{n}\Lambda(j)$$

$$\Pi(n)=\lim_{s\to0}\lim_{z\to0}\frac{\partial}{\partial z}[\zeta_n(s)]^{*z}$$

$$\psi(n)=-\lim_{s\to0}\lim_{z\to0}\frac{\partial}{\partial s}\frac{\partial}{\partial z}[\zeta_n(s)]^{*z}$$

$$[\zeta_n(s)]^{*z}=\binom{z}{0}1+\binom{z}{1}\sum_{j=2}^{\lfloor n\rfloor} j^{-s}+\binom{z}{2}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor \frac{n}{j}\rfloor}(j\cdot k)^{-s}+\binom{z}{3}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor \frac{n}{j}\rfloor}\sum_{l=2}^{\lfloor \frac{n}{j\cdot k}\rfloor}(j\cdot k\cdot l)^{-s}+\ldots$$

$$[\zeta_n(s)]^{*z}=F_1(n) \text{ where } F_k(n)=1+\left(\frac{z+1}{k}-1\right)\sum_{j=2}^{\lfloor n\rfloor} j^{-s}F_{k+1}\left(\frac{n}{j}\right) \tag{2.3.1}$$

The limit of this as n approaches infinity, if $\Re(s)>1$, is

$$\zeta(s)^z=\binom{z}{0}1+\binom{z}{1}\sum_{j=2}^{\infty} j^{-s}+\binom{z}{2}\sum_{j=2}^{\infty}\sum_{k=2}^{\infty}(j\cdot k)^{-s}+\binom{z}{3}\sum_{j=2}^{\infty}\sum_{k=2}^{\infty}\sum_{l=2}^{\infty}(j\cdot k\cdot l)^{-s}+\ldots \tag{2.3.2}$$

which is to say,

$$\zeta(s)^z=\binom{z}{0}(\zeta(s)-1)^0+\binom{z}{1}(\zeta(s)-1)^1+\binom{z}{2}(\zeta(s)-1)^2+\binom{z}{3}(\zeta(s)-1)^3+\ldots=\sum_{k=0}\binom{z}{k}(\zeta(s)-1)^k \tag{2.3.2}$$

$$[\zeta_n(s)]^{*z}=\binom{z}{0}1+\binom{z}{1}\sum_{j=2}^{\lfloor n\rfloor} j^{-s}+\binom{z}{2}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor \frac{n}{j}\rfloor}(j\cdot k)^{-s}+\binom{z}{3}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor \frac{n}{j}\rfloor}\sum_{l=2}^{\lfloor \frac{n}{j\cdot k}\rfloor}(j\cdot k\cdot l)^{-s}+\ldots$$

$$?=\binom{z}{0}+\binom{z}{1}\int_1^n x^{-s}\,dx+\binom{z}{2}\int_1^n\int_1^{\frac{n}{x}}(x\cdot y)^{-s}\,dy\,dx+\binom{z}{3}\int_1^n\int_1^{\frac{n}{x}}\int_1^{\frac{n}{xy}}(x\cdot y\cdot z)^{-s}\,dz\,dy\,dx+\ldots$$

$$\sum_{k=0}^{\infty}\binom{z}{k}\frac{1}{(s-1)^k}\cdot\frac{\gamma(k,(s-1)\log n)}{\Gamma(k)}$$

$$\sum_{k=0}^{\infty}\binom{z}{k}\frac{1}{(0-1)^k}\cdot\frac{\gamma(k,-\log n)}{\Gamma(k)}=L_{-z}(\log n)$$

$$-\Gamma(0,(s-1)\log n)+\Gamma(0,s\log n)-\log((s-1)\log n)+\log(s\log n)$$