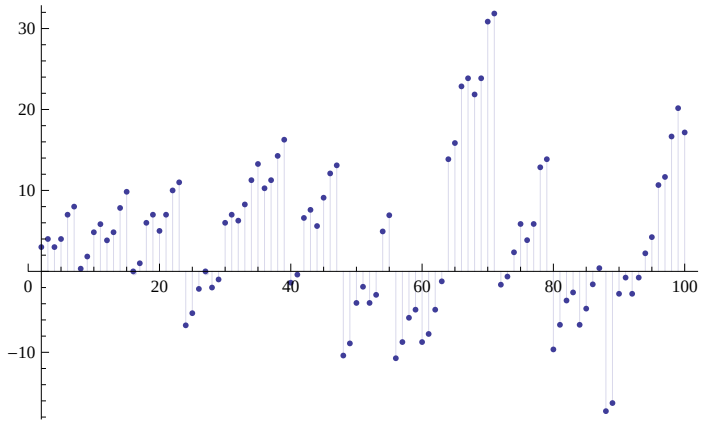
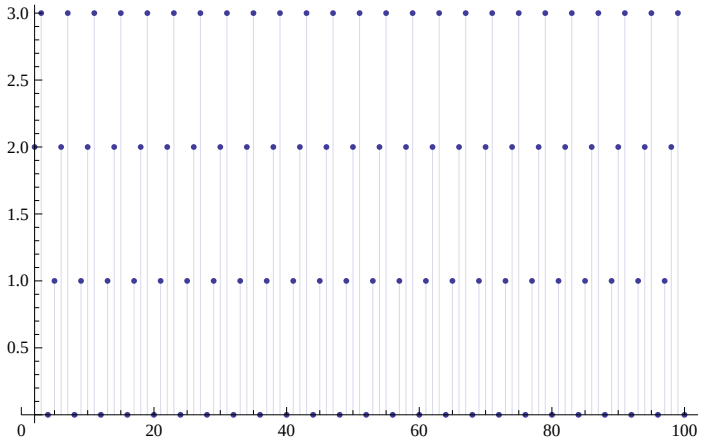
```
ClearAll["Global`*"]
```

```
F1[n_, 1] := F1[n, 1] = Mod[n, 4]; F1[1, 1] := 0
f1[n_, 1] := f1[n, 1] = F1[n, 1] - F1[n - 1, 1]
f1[n_, k_] := f1[n, k] = F1[n, k] - F1[n - 1, k]
f1[1, 0] := 1
F1[n_, 0] := F1[n, 0] = 1
F1[n_, k_] := F1[n, k] = Sum[ f1[j, 1] F1[Floor[n / j], k - 1], {j, 2, n}]
expf[n_] := Sum[ 1 / (k!) f1[n, k], {k, 0, Log[2, n]}]
Ex1[n_, 0] := 1;
Ex1[n_, k_] := Ex1[n, k] = Sum[ expf[j] Ex1[ Floor[n / j], k - 1], {j, 1, n}]
ex1[n_, k_] := ex1[n, k] = Ex1[n, k] - Ex1[n - 1, k]
Ex2[n_, k_] := Ex2[n, k] = Sum[(-1)^j Binomial[k, j] Ex1[n, k - j], {j, 0, k}]
EF2[n_] := Sum[ (-1)^(k + 1) / k Ex2[n, k], {k, 1, Log[2, n]}]
```

```
DiscretePlot[{Ex1[n, 1]}, {n, 2, 100}]
```




```
DiscretePlot[EF2[n], {n, 2, 100}]
```




```
Table[ {n, F1[n, 1], EF2[n]}, {n, 1, 100}] // TableForm
```

1	0	0
2	2	2
3	3	3
4	0	0
5	1	1

6	2	2
7	3	3
8	0	0
9	1	1
10	2	2
11	3	3
12	0	0
13	1	1
14	2	2
15	3	3
16	0	0
17	1	1
18	2	2
19	3	3
20	0	0
21	1	1
22	2	2
23	3	3
24	0	0
25	1	1
26	2	2
27	3	3
28	0	0
29	1	1
30	2	2
31	3	3
32	0	0
33	1	1
34	2	2
35	3	3
36	0	0
37	1	1
38	2	2
39	3	3
40	0	0
41	1	1
42	2	2
43	3	3
44	0	0
45	1	1
46	2	2
47	3	3
48	0	0
49	1	1
50	2	2
51	3	3
52	0	0
53	1	1
54	2	2
55	3	3
56	0	0
57	1	1
58	2	2
59	3	3
60	0	0
61	1	1

62	2	2
63	3	3
64	0	0
65	1	1
66	2	2
67	3	3
68	0	0
69	1	1
70	2	2
71	3	3
72	0	0
73	1	1
74	2	2
75	3	3
76	0	0
77	1	1
78	2	2
79	3	3
80	0	0
81	1	1
82	2	2
83	3	3
84	0	0
85	1	1
86	2	2
87	3	3
88	0	0
89	1	1
90	2	2
91	3	3
92	0	0
93	1	1
94	2	2
95	3	3
96	0	0
97	1	1
98	2	2
99	3	3
100	0	0

```
Table[{n, Ex1[n, 1]}, {n, 1, 100}] // TableForm
```

1	1
2	3
3	4
4	3
5	4
6	7
7	8
8	$\frac{1}{3}$
9	$\frac{11}{6}$
10	$\frac{29}{6}$
11	$\frac{35}{6}$
12	$\frac{23}{6}$
13	$\frac{29}{6}$

14	$\frac{47}{6}$
15	$\frac{59}{6}$
16	0
17	1
18	6
19	7
20	5
21	7
22	10
23	11
24	$-\frac{20}{3}$
25	$-\frac{31}{6}$
26	$-\frac{13}{6}$
27	0
28	-2
29	-1
30	6
31	7
32	$\frac{94}{15}$
33	$\frac{124}{15}$
34	$\frac{169}{15}$
35	$\frac{199}{15}$
36	$\frac{154}{15}$
37	$\frac{169}{15}$
38	$\frac{214}{15}$
39	$\frac{244}{15}$
40	$-\frac{7}{5}$
41	$-\frac{2}{5}$
42	$\frac{33}{5}$
43	$\frac{38}{5}$
44	$\frac{28}{5}$
45	$\frac{91}{10}$
46	$\frac{121}{10}$
47	$\frac{131}{10}$
48	$-\frac{52}{5}$
49	$-\frac{89}{10}$
50	$-\frac{39}{10}$
51	$-\frac{19}{10}$
52	$-\frac{39}{10}$
53	$-\frac{29}{10}$
54	$\frac{74}{15}$
55	$\frac{104}{15}$
56	$-\frac{161}{15}$

57 $-\frac{131}{15}$

58 $-\frac{86}{15}$

59 $-\frac{71}{15}$

60 $-\frac{131}{15}$

61 $-\frac{116}{15}$

62 $-\frac{71}{15}$

63 $-\frac{37}{30}$

64 $\frac{1247}{90}$

65 $\frac{1427}{90}$

66 $\frac{2057}{90}$

67 $\frac{2147}{90}$

68 $\frac{1967}{90}$

69 $\frac{2147}{90}$

70 $\frac{2777}{90}$

71 $\frac{2867}{90}$

72 $-\frac{74}{45}$

73 $-\frac{29}{45}$

74 $\frac{106}{45}$

75 $\frac{527}{90}$

76 $\frac{347}{90}$

77 $\frac{527}{90}$

78 $\frac{1157}{90}$

79 $\frac{1247}{90}$

80 $-\frac{434}{45}$

81 $-\frac{2377}{360}$

82 $-\frac{1297}{360}$

83 $-\frac{937}{360}$

84 $-\frac{2377}{360}$

85 $-\frac{1657}{360}$

86 $-\frac{577}{360}$

87 $\frac{143}{360}$

88 $-\frac{6217}{360}$

89 $-\frac{5857}{360}$

90 $-\frac{997}{360}$

91 $-\frac{277}{360}$

92 $-\frac{997}{360}$

93 $-\frac{277}{360}$

94 $\frac{803}{360}$

95 $\frac{1523}{360}$

96	$\frac{3839}{360}$
97	$\frac{4199}{360}$
98	$\frac{5999}{360}$
99	$\frac{7259}{360}$
100	$\frac{6179}{360}$