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b[x_, z_] := (x + z)! / (x! z!)
bb[x2_, z_] := Limit[(x + z)! / (x! z!), x → x2]
bn[x_, z_] := b[x - z, z]
px[x_, z_] := b[x - 1, z]
py[x_, z_] := Pochhammer[x + 1, z] / z!

D[Pochhammer[x, z] / z!, {z, 3}] /. z → 0

EulerGamma^3 -  $\frac{\text{EulerGamma} \pi^2}{2}$  + 3  $\left( \text{EulerGamma}^2 - \frac{\pi^2}{6} \right)$  PolyGamma[0, x] +
PolyGamma[0, x]^3 + 3 PolyGamma[0, x] PolyGamma[1, x] +
3 EulerGamma (PolyGamma[0, x]^2 + PolyGamma[1, x]) - PolyGamma[2, 1] + PolyGamma[2, x]

D[Pochhammer[x, z] / z!, z] /. z → 0

EulerGamma + PolyGamma[0, x]

D[LaguerreL[z, 1 - x], {z, 2}] /. z → 0

LaguerreL(2,0)[0, 1 - x]

Sum[D[Binomial[z, k] (x - 1)^k / k!, {z, 2}] /. z → 0, {k, 0, Infinity}]

 $\sum_{k=0}^{\infty} \frac{1}{k!} (-1 + x)^k \left( \text{Binomial}[0, k] (-\text{EulerGamma} - \text{PolyGamma}[0, 1 - k])^2 + \right.$ 
 $\left. \text{Binomial}[0, k] \left( \frac{\pi^2}{6} - \text{PolyGamma}[1, 1 - k] \right) \right)$ 

Sum[Binomial[a, j] Binomial[b, k - j], {j, 0, k}]

Binomial[a + b, k]

tt[x_, z_] := Sum[Pochhammer[z + 1, x - 2 k] / ((x - 2 k)!) Pochhammer[-z, k] / (k!), {k, 0, x / 2}]
tt2[x_, z_, m_] :=
Sum[Pochhammer[z + 1, x - m k] / ((x - m k)!) Pochhammer[-z, k] / (k!), {k, 0, x / m}]
tt3[x_, z_, m_] := Sum[b[z, x - m k] bb[-z - 1, k], {k, 0, x / m}]
tt4[x_, z_, m_] := Sum[b[z - 1, x - m k] bb[-z - 1, k], {k, 0, x / m}]
tt4a[x_, z_] := Sum[bb[x - 1, z - 2 k] bb[-x - 1, k], {k, 0, z / 2}]
tt4b[x_, z_] := bb[x - z, z]
tt4c[x_, z_, m_] := Sum[bb[x - 1, z - m k] bb[-x - 1, k], {k, 0, z / m}]

Table[tt4a[11, a], {a, 0, 11}]

{1, 11, 55, 165, 330, 462, 462, 330, 165, 55, 11, 1}

Table[b[11 - k, k], {k, 0, 11}]

{1, 11, 55, 165, 330, 462, 462, 330, 165, 55, 11, 1}

tt4a[11, 4]

330

tt4b[11, 4]

330

Table[tt4c[6, a, 3], {a, 0, 12}]

{1, 6, 21, 50, 90, 126, 141, 126, 90, 50, 21, 6, 1}

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```
Table[b[k, 11 - k], {k, 0, 11}]
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```
{1, 11, 55, 165, 330, 462, 462, 330, 165, 55, 11, 1}
```

```
ot[z_, n_] :=
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Sum[Pochhammer[z + 1, n - 2 k] / ((n - 2 k)!) Pochhammer[-z, k] / (k!), {k, 0, Floor[n / 2]}]
```

```
ota[z2_, n_] := Sum[Limit[(z + n - 2 k)! / (z! (n - 2 k)!) (-z - 1 + k)! / ((-z - 1)! (k!)), z → z2],  
{k, 0, Floor[n / 2]}]
```

```
otb[z_, n_] := Sum[- $\frac{\text{Gamma}[k - z] \text{Gamma}[1 - 2 k + n + z] \text{Sin}[\pi z]}{\pi k! (-2 k + n)!}$ , {k, 0, Floor[n / 2]}]
```

```
ot2[z_, n_] := Sum[Binomial[z, k], {k, 0, n}]
```

```
ov[x_] := Sum[(-1)^k b[x, k], {k, 0, x}]
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```
Table[ov[j], {j, 1, 10}]
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{-1, 4, -13, 46, -166, 610, -2269, 8518, -32206, 122464}
```

```
Table[Sum[(-1)^a tt4c[k, a, 7], {a, 0, 6 k}], {k, 1, 10}]
```

```
{1, 1, 1, 1, 1, 1, 1, 1, 1, 1}
```

```
Table[tt4c[4, a, 7], {a, 0, 6 × 4}]
```

```
{1, 4, 10, 20, 35, 56, 84, 116, 149, 180, 206,  
224, 231, 224, 206, 180, 149, 116, 84, 56, 35, 20, 10, 4, 1}
```

```
Table[Sum[tt4c[k, a, 7], {a, 0, 6 k}], {k, 1, 10}]
```

```
{7, 49, 343, 2401, 16807, 117649, 823543, 5764801, 40353607, 282475249}
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```
Table[7^k, {k, 1, 10}]
```

```
{7, 49, 343, 2401, 16807, 117649, 823543, 5764801, 40353607, 282475249}
```

```

otm[z_, n_, m_] :=
  Sum[Pochhammer[z, n - m k] / ((n - m k)!) Pochhammer[-z, k] / (k!), {k, 0, Floor[n / m]}]
Table[FullSimplify@Pochhammer[z, k] / k!, {k, 1, 7}]
Table[FullSimplify@otm[z, k, 2], {k, 1, 7}]
Table[FullSimplify@otm[z, k, 3], {k, 1, 7}]

```

$$\left\{ z, \frac{1}{2} z (1+z), \frac{1}{6} z (1+z) (2+z), \frac{1}{24} z (1+z) (2+z) (3+z), \frac{1}{120} z (1+z) (2+z) (3+z) (4+z), \right.$$

$$\left. \frac{1}{720} z (1+z) (2+z) (3+z) (4+z) (5+z), \frac{z (1+z) (2+z) (3+z) (4+z) (5+z) (6+z)}{5040} \right\}$$

$$\left\{ z, \frac{1}{2} (-1+z) z, \frac{1}{6} (-2+z) (-1+z) z, \frac{1}{24} (-3+z) (-2+z) (-1+z) z, \right.$$

$$\left. \frac{1}{120} (-4+z) (-3+z) (-2+z) (-1+z) z, \frac{1}{720} (-5+z) (-4+z) (-3+z) (-2+z) (-1+z) z, \right.$$

$$\left. \frac{(-6+z) (-5+z) (-4+z) (-3+z) (-2+z) (-1+z) z}{5040} \right\}$$

$$\left\{ z, \frac{1}{2} z (1+z), \frac{1}{6} (-1+z) z (4+z), \frac{1}{24} (-1+z) z (-6+z (7+z)), \right.$$

$$\left. \frac{1}{120} (-2+z) (-1+z) z (1+z) (12+z), \frac{1}{720} (-2+z) (-1+z) z (-120+z (1+z) (17+z)), \right.$$

$$\left. \frac{(-3+z) (-2+z) (-1+z) z (-120+z (116+z (27+z)))}{5040} \right\}$$

```

otr[z_, n_, m_] :=
  Sum[Pochhammer[z + 1, n - m k] / ((n - m k)!) Pochhammer[-z, k] / (k!), {k, 0, Floor[n / m]}]
Table[FullSimplify@Pochhammer[z + 1, k] / k!, {k, 1, 7}]
Table[FullSimplify@otr[z, k, 2], {k, 1, 7}]
Table[FullSimplify@otr[z, k, 3], {k, 1, 7}]

{1 + z,  $\frac{1}{2} (1 + z) (2 + z)$ ,  $\frac{1}{6} (1 + z) (2 + z) (3 + z)$ ,
 $\frac{1}{24} (1 + z) (2 + z) (3 + z) (4 + z)$ ,  $\frac{1}{120} (1 + z) (2 + z) (3 + z) (4 + z) (5 + z)$ ,
 $\frac{1}{720} (1 + z) (2 + z) (3 + z) (4 + z) (5 + z) (6 + z)$ ,  $\frac{(1 + z) (2 + z) (3 + z) (4 + z) (5 + z) (6 + z) (7 + z)}{5040}$ }

{1 + z,  $\frac{1}{2} (2 + z + z^2)$ ,  $\frac{1}{6} (6 + 5 z + z^3)$ ,
 $\frac{1}{24} (24 + z (1 + z) (14 + (-3 + z) z))$ ,  $\frac{1}{120} (1 + z) (120 + (-1 + z) z (26 + (-5 + z) z))$ ,
 $\frac{1}{720} (720 + z (1 + z) (444 + z (-140 + z (65 + (-10 + z) z))))$ ,
 $\frac{(1 + z) (5040 + (-1 + z) z (1212 + z (-364 + z (113 + (-14 + z) z))))}{5040}$ }

{1 + z,  $\frac{1}{2} (1 + z) (2 + z)$ ,  $\frac{1}{6} (6 + z (1 + z) (5 + z))$ ,
 $\frac{1}{24} (1 + z) (24 + z (2 + z) (9 + z))$ ,  $\frac{1}{120} (1 + z) (2 + z) (60 + (-1 + z) z (13 + z))$ ,
 $\frac{1}{720} (720 + z (1 + z) (684 + z (-20 + z (35 + z (20 + z))))$ ,
 $\frac{(1 + z) (5040 + z (468 + z (2164 + z (-225 + z (85 + z (27 + z))))))}{5040}$ }

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