```
Expand[(x Log[n]) + (-(1/2 + x) Log[j])]
```

$$-\frac{\mathrm{Log}[j]}{2} - x\,\mathrm{Log}[j] + x\,\mathrm{Log}[n]$$

```
Expand[(-x Log[n]) + (-(1/2 - x) Log[j])]
```

$$-\frac{\mathrm{Log}[j]}{2} + x\,\mathrm{Log}[j] - x\,\mathrm{Log}[n]$$

```
E^(-Log[j]/2)
```

$$\frac{1}{\sqrt{j}}$$

```
Expand[(-1/2 - x) (j/n)^x - (-1/2 + x) (j/n)^-x]
```

$$\frac{1}{2}\left(\frac{j}{n}\right)^{-x} - \frac{1}{2}\left(\frac{j}{n}\right)^{x} - \left(\frac{j}{n}\right)^{-x} x - \left(\frac{j}{n}\right)^{x} x$$

```
FullSimplify[j^(-1/2) (FullSimplify[3 I ((j/n)^3 I + (j/n)^-3 I)] +
      FullSimplify[(1/2) ((j/n)^3 I - (j/n)^-3 I)])] /. n → 10 /. j → 1
```

$$-\frac{3\,000\,003}{1000} - \frac{999\,999\,i}{2000}$$

```
FullSimplify[(1/2) ((j/n)^3 I - (j/n)^-3 I)]
```

$$\frac{i\left(j^{6} - n^{6}\right)}{2\,j^{3}\,n^{3}}$$

```
pt[n_, x_] := Sum[ j^(-1/2 + x) n^-x (1/2 + x) - j^(-1/2 - x) n^x (1/2 - x), {j, 1, n}]
pt2[n_, x_] := Sum[ j^(-1/2) (j^x n^-x (1/2 + x) - j^(-x) n^x (1/2 - x)), {j, 1, n}]
pt3[n_, x_] := Sum[ j^(-1/2) ((j/n)^x (1/2 + x) - (j/n)^(-x) (1/2 - x)), {j, 1, n}]
pt4[n_, x_] :=
 Sum[ j^(-1/2) (x ((j/n)^x + (j/n)^-x ) + 1/2 ((j/n)^x  - (j/n)^-x )), {j, 1, n}]
pt5[n_, x_] := Sum[ j^(-1/2) (x (E^(x Log[j/n]) + E^(-x Log[j/n])) +
     1/2 (E^(x Log[j/n]) - E^(-x Log[j/n]))), {j, 1, n}]
pt6[n_, x_] := Sum[ j^(-1/2) (2 x Cosh[x Log[j/n]] + Sinh[x Log[j/n]]), {j, 1, n}]
pt7[n_, x_] := Sum[ j^(-1/2) (2 x Cos[x Log[j/n] I] - I Sin[x Log[j/n] I]), {j, 1, n}]
```

```
pt[100 000, 1 + 14.134725141734695` I]
```

$$418\,355. + 646\,413.\,i$$

```
pt6[100 000, 1 + 14.134725141734695` I]
```

$$418\,355. + 646\,413.\,i$$

```
pt7[100 000, 1 + 14.134725141734695` I]
```

$$418\,355. + 646\,413.\,i$$

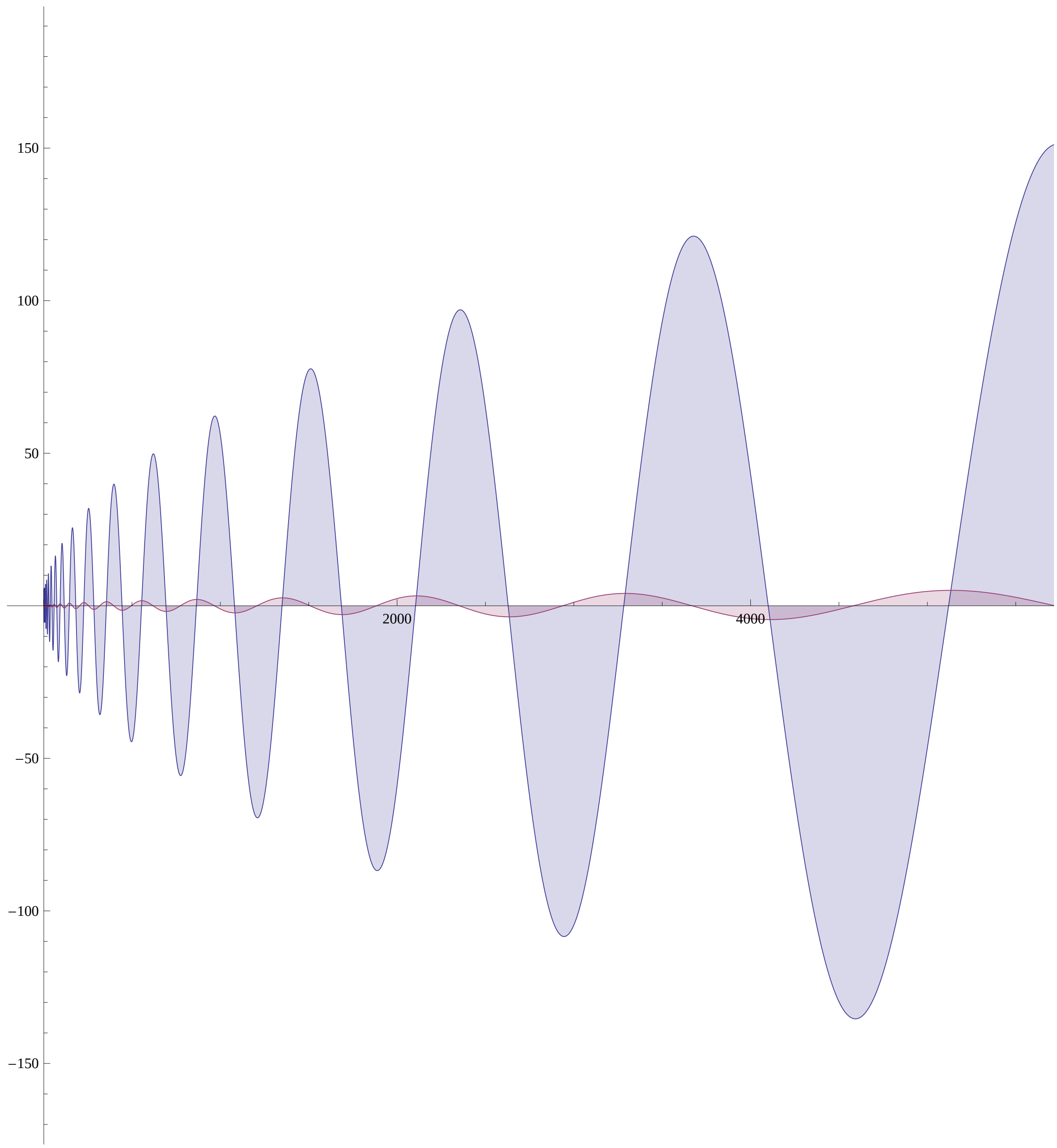
```
et[x_, n_] := Table[ (E^(x Log[j/n]) + E^(-x Log[j/n])), {j, 1, n}]
et2[x_, n_] := Table[2 Cosh[x Log[j/n]], {j, 1, n}]
```

```
N@et[3 I, 8]
```

$$\{1.99799 + 0.\,i,\ -1.05135 + 0.\,i,\ -1.96049 + 0.\,i,\ -0.973989 + 0.\,i,\ 0.320187 + 0.\,i,\ 1.30025 + 0.\,i,\ 1.84166 + 0.\,i,\ 2.\}$$

```
N@et2[3 I, 8]

{1.99799, -1.05135, -1.96049, -0.973989, 0.320187, 1.30025, 1.84166, 2.}

pt6a[n_, x_] := Sum[ j^(-1/2) (2 x I Cos[x Log[j / n]] + I Sin[x Log[j / n]]), {j, 1, n}]
pt6b[n_, x_] := Sum[ j^(-1/2) (2 x Cos[x Log[j / n]] + Sin[x Log[j / n]]), {j, 1, n}]
pt6bx[n_, x_] := {Sum[ j^(-1/2) (2 x Cos[x Log[j / n]]), {j, 1, n}],
  Sum[ j^(-1/2) (Sin[x Log[j / n]]), {j, 1, n}]}
pt6by[n_, x_] := Table[{ j^(-1/2) (2 x Cos[x Log[j / n]]),
   j^(-1/2) (Sin[x Log[j / n]])}, {j, 1, n}]
pt6c[n_, x_] := DiscretePlot[ j^(-1/2) (2 x Cos[x Log[j / n]] + Sin[x Log[j / n]]), {j, 1, n}]
pt6e[n_, x_] :=
 DiscretePlot[ {j^(-1/2) (2 x Cos[x Log[j / n]]), j^(-1/2) (Sin[x Log[j / n]])}, {j, 1, n}]
pt6e2[n_, x_] := DiscretePlot[
  {j^(-1/2) (2 x Cos[x Log[j / n]]), j^(-1/2) (Sin[x Log[j / n]])}, {j, 1, n}]
pt6e2r[n_, x_] := DiscretePlot[ {Re[j^(-1/2) (2 x Cos[x Log[j / n]])],
   Re[j^(-1/2) (Sin[x Log[j / n]])]}, {j, 1, n}]
pt6e2i[n_, x_] := DiscretePlot[ {Im[j^(-1/2) (2 x Cos[x Log[j / n]])],
   Im[j^(-1/2) (Sin[x Log[j / n]])]}, {j, 1, n}]
pt6e4[n_, x_] := DiscretePlot[ Re[{-Sum[ j^(-1/2) (2 x Cos[x Log[j / n]]), {j, 1, j2}],
    Sum[j^(-1/2) (Sin[x Log[j / n]]), {j, 1, j2}]}], {j2, 1, n}]
pt6e4s[n_, x_] := DiscretePlot[ Im[{-Sum[ j^(-1/2) (2 x Cos[x Log[j / n]]), {j, 1, j2}],
    Sum[j^(-1/2) (Sin[x Log[j / n]]), {j, 1, j2}]}], {j2, 1, n}]
pt6e4a[n_, x_] := DiscretePlot[ {-Sum[ j^(-1/2) (2 x Cos[x Log[j / n]]) / j2, {j, 1, j2}],
    Sum[j^(-1/2) (Sin[x Log[j / n]]), {j, 1, j2}]} / j2, {j2, 1, n}]
pt6e5[n_, x_] := DiscretePlot[ {-Re[Sum[ j^(-1/2) (2 x Cos[x Log[j / n]]), {j, 1, j2}]],
   Re[Sum[j^(-1/2) (Sin[x Log[j / n]]), {j, 1, j2}]]}, {j2, 1, n}]
pt6e6[n_, x_] := DiscretePlot[ {-Im[Sum[ j^(-1/2) (2 x Cos[x Log[j / n]]), {j, 1, j2}]],
   Im[Sum[j^(-1/2) (Sin[x Log[j / n]]), {j, 1, j2}]]}, {j2, 1, n}]
pt6e4b[n_, x_] := DiscretePlot[ Sum[j^(-1/2) (Sin[x Log[j / n]]), {j, 1, j2}], {j2, 1, n}]
pt6e4b2[n_, x_] := DiscretePlot[ Sum[j^(-1/2) (Sin[x Log[j / n]]), {j, 1, j2}], {j2, 1, 100}]
pt6e4b2d[n_, x_] := DiscretePlot[j^(-1/2) (Sin[x Log[j / n]]), {j, 1, 100}]
pt6f[n_, x_] := Table[j^(-1/2) (Sin[x Log[j / n]]), {j, 1, 50}]

pt6bx[10000, 14.134725141734695` ]

{7.20728, -7.06593}
```

pt6e4[10 000, 14.134725141734695`]



N@ZetaZero[10]

0.5 + 49.7738 ⅈ

```
Sin[(14.134725141734695`) Log[1.0 / 1 000 000]]
```

```
-0.479162
```

```
Animate[pt6e4b2d[x, 24.134725141734695` ], {x, 500, 3000}]
```

```
N@pt6f[100 000 000, x ] // TableForm
```

```
-1. Sin[18.4207 x]
-0.707107 Sin[17.7275 x]
-0.57735 Sin[17.3221 x]
-0.5 Sin[17.0344 x]
-0.447214 Sin[16.8112 x]
-0.408248 Sin[16.6289 x]
-0.377964 Sin[16.4748 x]
-0.353553 Sin[16.3412 x]
-0.333333 Sin[16.2235 x]
-0.316228 Sin[16.1181 x]
-0.301511 Sin[16.0228 x]
-0.288675 Sin[15.9358 x]
-0.27735 Sin[15.8557 x]
-0.267261 Sin[15.7816 x]
-0.258199 Sin[15.7126 x]
-0.25 Sin[15.6481 x]
-0.242536 Sin[15.5875 x]
-0.235702 Sin[15.5303 x]
-0.229416 Sin[15.4762 x]
-0.223607 Sin[15.4249 x]
-0.218218 Sin[15.3762 x]
-0.213201 Sin[15.3296 x]
-0.208514 Sin[15.2852 x]
-0.204124 Sin[15.2426 x]
-0.2 Sin[15.2018 x]
-0.196116 Sin[15.1626 x]
-0.19245 Sin[15.1248 x]
-0.188982 Sin[15.0885 x]
-0.185695 Sin[15.0534 x]
-0.182574 Sin[15.0195 x]
-0.179605 Sin[14.9867 x]
-0.176777 Sin[14.9549 x]
-0.174078 Sin[14.9242 x]
-0.171499 Sin[14.8943 x]
-0.169031 Sin[14.8653 x]
-0.166667 Sin[14.8372 x]
-0.164399 Sin[14.8098 x]
-0.162221 Sin[14.7831 x]
-0.160128 Sin[14.7571 x]
-0.158114 Sin[14.7318 x]
-0.156174 Sin[14.7071 x]
-0.154303 Sin[14.683 x]
-0.152499 Sin[14.6595 x]
-0.150756 Sin[14.6365 x]
-0.149071 Sin[14.614 x]
-0.147442 Sin[14.592 x]
-0.145865 Sin[14.5705 x]
-0.144338 Sin[14.5495 x]
-0.142857 Sin[14.5289 x]
-0.141421 Sin[14.5087 x]
```

```
pt6b[n_, x_] := Sum[ j^(-1/2) (2 x Cos[x Log[j / n]] + Sin[x Log[j / n]]), {j, 1, n}]
pt7[n_, x_] := (2 x Sin[x Log[n]] + Cos[x Log[n]]) Sum[ j^(-1/2) Sin[x Log[j]], {j, 1, n}] +
  (2 x Cos[x Log[n]] - Sin[x Log[n]]) Sum[ j^(-1/2) Cos[x Log[j]], {j, 1, n}]
pt7a[n_, x_] := {(2 x Sin[x Log[n]] + Cos[x Log[n]]) Sum[ j^(-1/2) Sin[x Log[j]], {j, 1, n}],
  (2 x Cos[x Log[n]] - Sin[x Log[n]]) Sum[ j^(-1/2) Cos[x Log[j]], {j, 1, n}]}
pt7b[n_, x_] := {(2 x Sin[x Log[n]] + Cos[x Log[n]]), Sum[ j^(-1/2) Sin[x Log[j]], {j, 1, n}],
  (2 x Cos[x Log[n]] - Sin[x Log[n]]), Sum[ j^(-1/2) Cos[x Log[j]], {j, 1, n}]}
pt7c[n_, x_] := {(2 x Sin[x Log[n]] + Cos[x Log[n]]), (2 x Cos[x Log[n]] - Sin[x Log[n]])}
pt7d[j_, x_] := {j^(-1/2) Sin[x Log[j]], j^(-1/2) Cos[x Log[j]]}
pt7e[n_, x_] :=
 {Sum[j^(-1/2) Sin[x Log[j]], {j, 1, n}], Sum[ j^(-1/2) Cos[x Log[j]], {j, 1, n}]}
pt7ex[n_, x_] := {Sum[j^(-1/2) (-1)^(j) Sin[x Log[j]], {j, 1, n}],
  Sum[ j^(-1/2) (-1)^(j) Cos[x Log[j]], {j, 1, n}]}
pt7exa[n_, x_] := Sum[j^(-1/2) (-1)^(j) Sin[x Log[j]], {j, 1, n}]
pt7ex2[n_, x_, s_] := {Sum[j^(-1/2) (-1)^(j s) Sin[x Log[j]], {j, 1, n}],
  Sum[ j^(-1/2) (-1)^(j s) Cos[x Log[j]], {j, 1, n}]}
pt7e1[n_, x_] := Sum[j^(-1/2) Sin[x Log[j]], {j, 1, n}]
pt7e2[n_, x_] := Sum[ j^(-1/2) Cos[x Log[j]], {j, 1, n}]
pt7t1[n_, x_] := Table[j^(-1/2) Sin[x Log[j]], {j, 1, n}]
pt7t2[n_, x_] := Table[ j^(-1/2) Cos[x Log[j]], {j, 1, n}]
```

```
pt6b[1 000 000, 14.134725141734695` ]
```

0.0141347

```
pt7[1 000 000, 14.134725141734695` ]
```

0.0141347

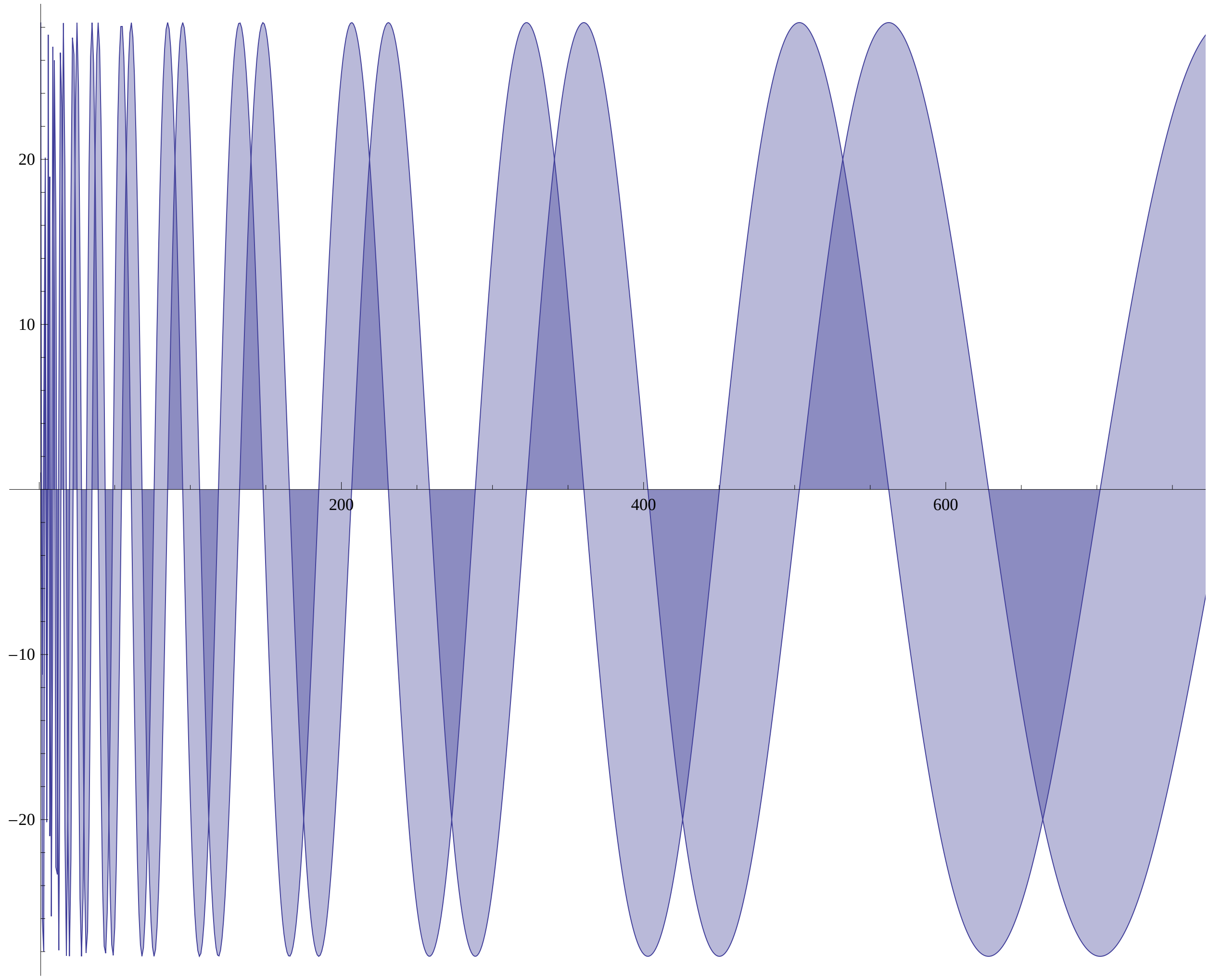
```
pt7a[1 000 000, 14.134725141734695` ]
```

{-877.254, 877.268}

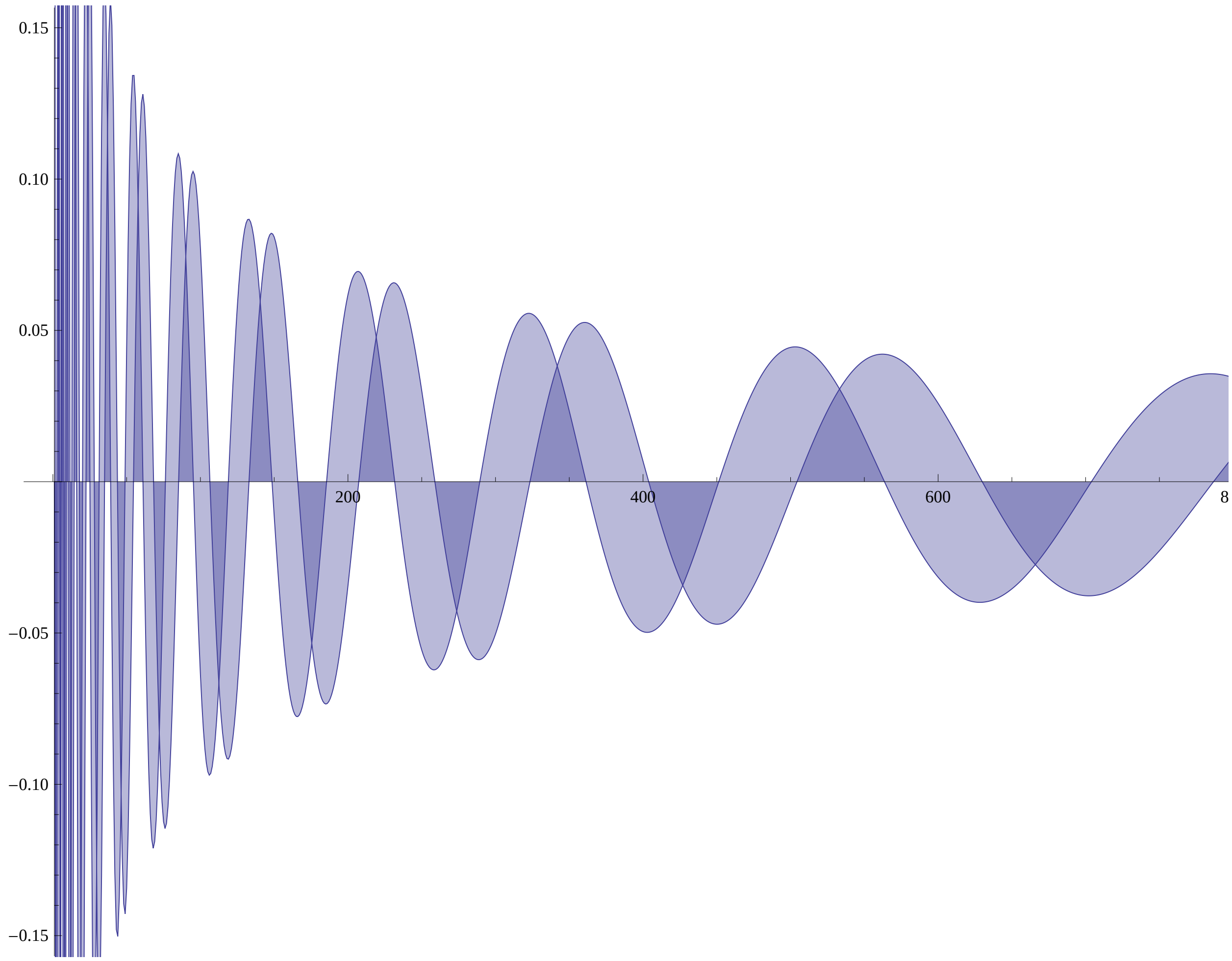
```
pt7b[1 000 000, 14.134725141734695` ]
```

{14.4234, -60.8217, 24.3337, 36.0516}

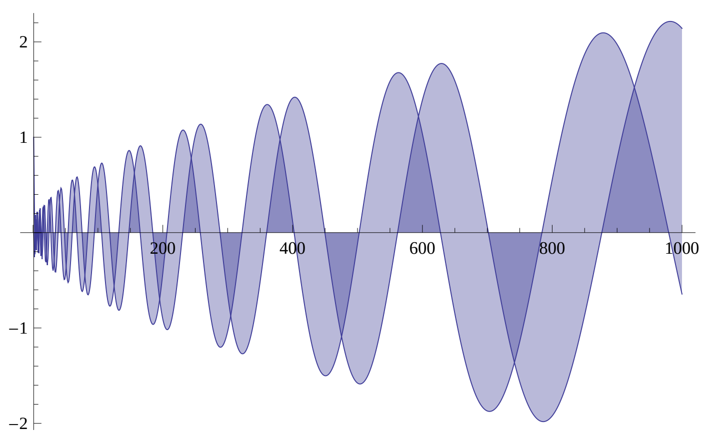
```
DiscretePlot[pt7c[n, 14.134725141734695`], {n, 1, 1000}]
```



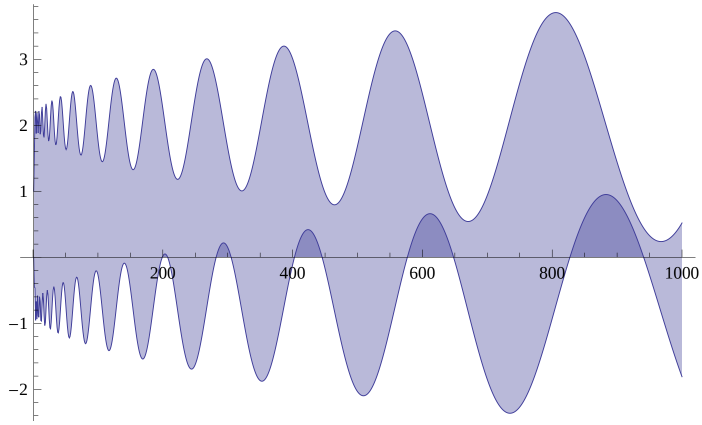
```
DiscretePlot[pt7d[n, 14.134725141734695`], {n, 1, 1000}]
```



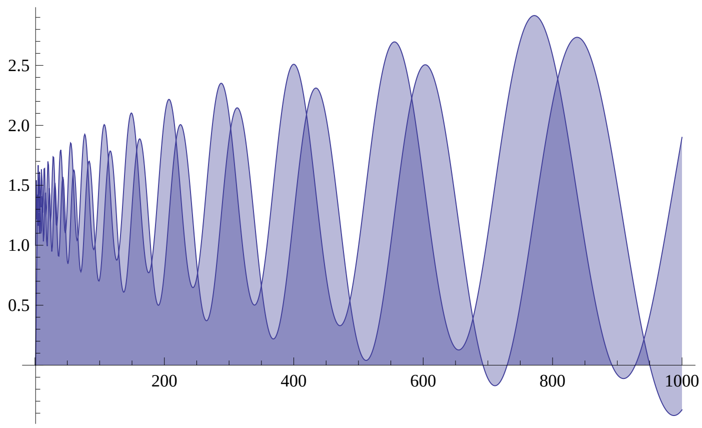

```
DiscretePlot[pt7e[n, 14.134725141734695`], {n, 1, 1000}]
```



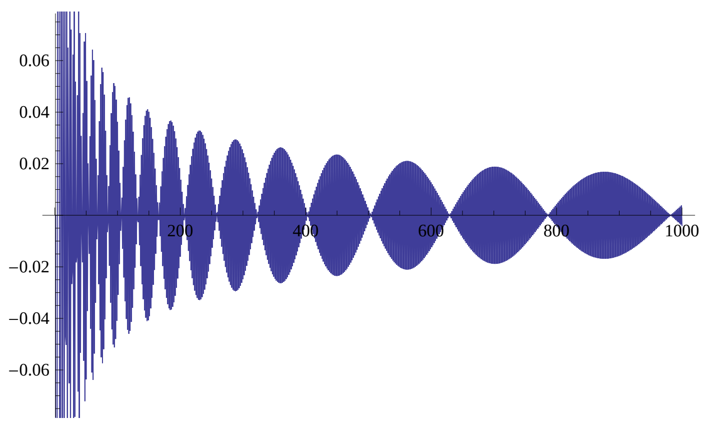
```
DiscretePlot[ pt7e[n, 3 + 14.134725141734695`], {n, 1, 1000}]
```



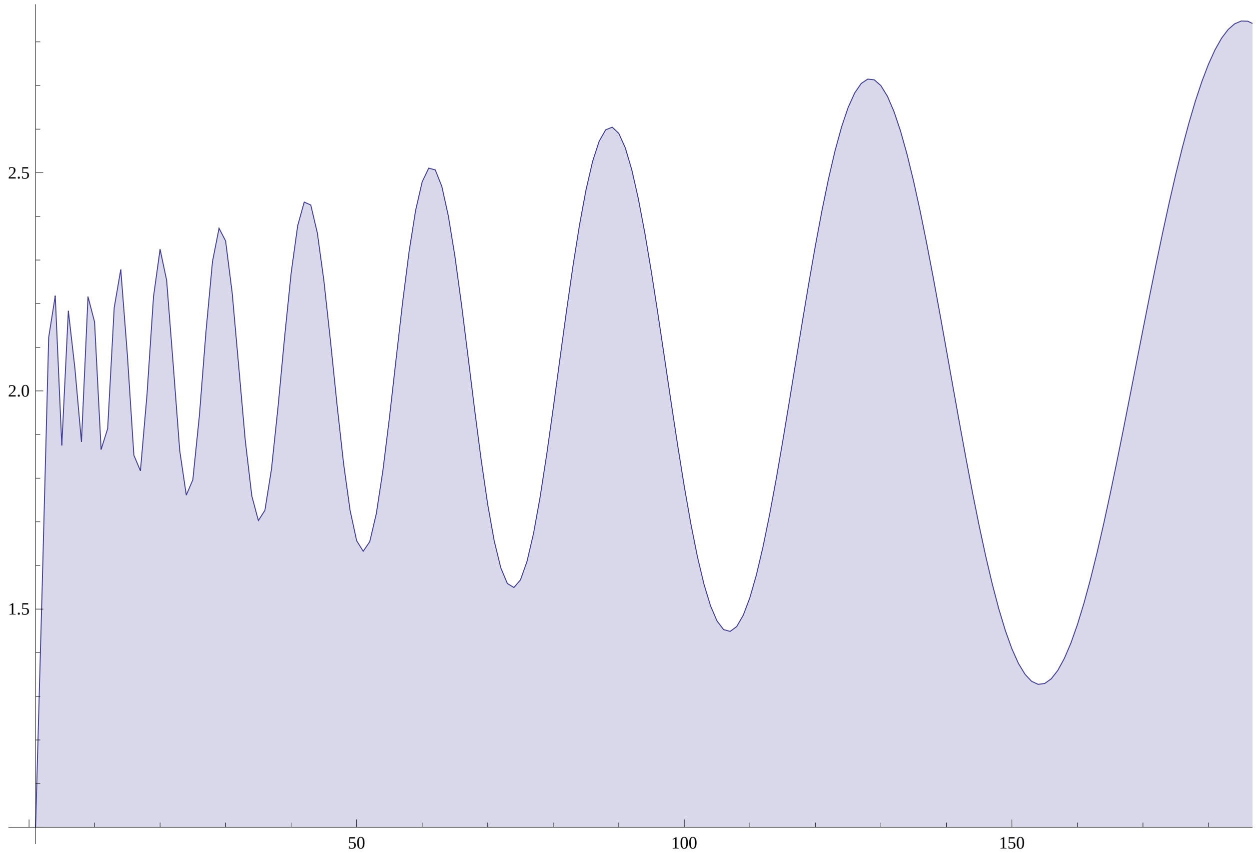

```
DiscretePlot[ pt7e[n, 5 + 14.134725141734695`], {n, 1, 1000}]
```




```
DiscretePlot[pt7exa[n, N@Im@ZetaZero@1], {n, 1, 1000}]
```



```
DiscretePlot[ pt7e2[n, 3 + 14.134725141734695`], {n, 1, 200}]
```

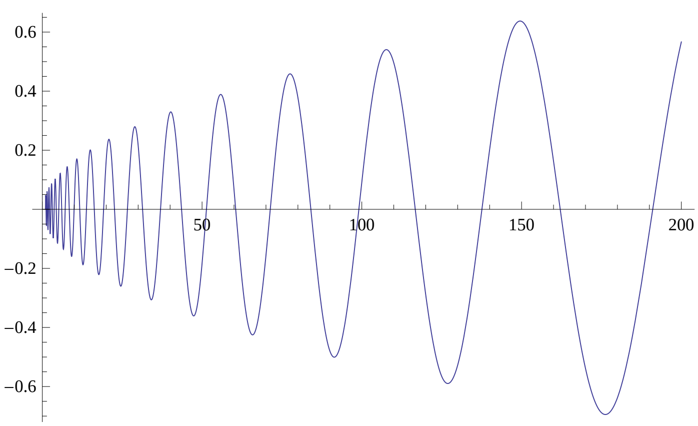



```
FullSimplify[Integrate[ 1 / (j ^ (1 / 2)) Cos[ x Log[j]], {j, 1, n}]]
```

ConditionalExpression$\left[\frac{-2+2\sqrt{n}\ (\text{Cos}[x\ \text{Log}[n]]+2\ x\ \text{Sin}[x\ \text{Log}[n]])}{1+4\ x^2}, \text{Re}[n]\geq 0\ ||\ n\notin \text{Reals}\right]$

$$\text{tm}[n\_, x\_] := \frac{-2+2\sqrt{n}\ (\text{Cos}[x\ \text{Log}[n]]+2\ x\ \text{Sin}[x\ \text{Log}[n]])}{1+4\ x^2}$$

```
Plot[ tm[n, 5 + 14.134725141734695`], {n, 1, 200}]
```



```
FullSimplify[Integrate[1 / (j^(1 / 2)) Sin[x Log[j]], {j, 1, n}]]
```

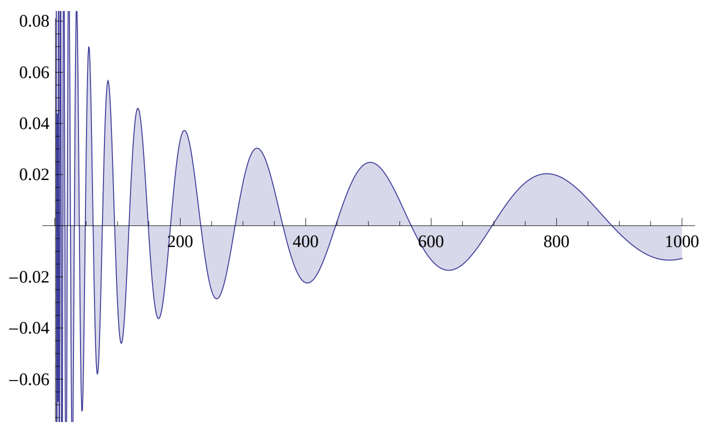
$$\text{ConditionalExpression}\left[\frac{4\,x + 2\sqrt{n}\,\left(-2\,x\,\text{Cos}[x\,\text{Log}[n]] + \text{Sin}[x\,\text{Log}[n]]\right)}{1 + 4\,x^2},\ \text{Re}[n] \geq 0\ ||\ n \notin \text{Reals}\right]$$

```
pt7f1[n_, x_] :=
```

$$\text{Sum}[j^{\wedge}(-1/2)\ \text{Sin}[x\,\text{Log}[j]],\ \{j, 1, n\}] + \frac{4\,x + 2\sqrt{n}\,\left(-2\,x\,\text{Cos}[x\,\text{Log}[n]] + \text{Sin}[x\,\text{Log}[n]]\right)}{1 + 4\,x^2}$$

$$\text{pt7f2}[n\_, x\_] := \text{Sum}[\,j^{\wedge}(-1/2)\ \text{Cos}[x\,\text{Log}[j]],\ \{j, 1, n\}] - \frac{-2 + 2\sqrt{n}\,\left(\text{Cos}[x\,\text{Log}[n]] + 2\,x\,\text{Sin}[x\,\text{Log}[n]]\right)}{1 + 4\,x^2}$$

```
DiscretePlot[ pt7f2[n, 14.134725141734695`], {n, 1, 1000}]
```

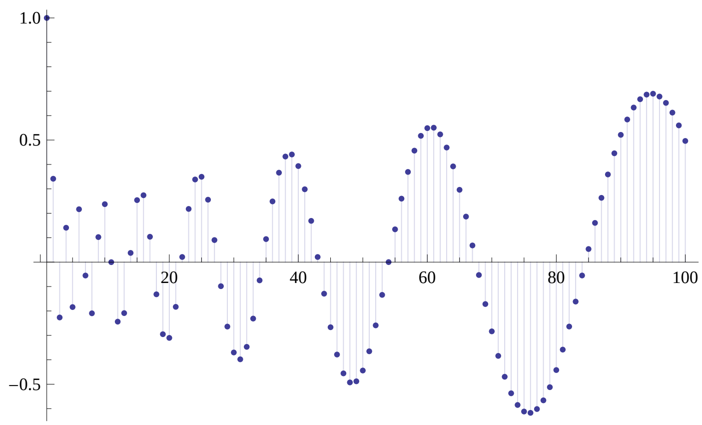



```
N@pt7t2[80, x] // TableForm
```

```
1.
0.707107 Cos[0.693147 x]
0.57735 Cos[1.09861 x]
0.5 Cos[1.38629 x]
0.447214 Cos[1.60944 x]
0.408248 Cos[1.79176 x]
0.377964 Cos[1.94591 x]
0.353553 Cos[2.07944 x]
0.333333 Cos[2.19722 x]
0.316228 Cos[2.30259 x]
0.301511 Cos[2.3979 x]
0.288675 Cos[2.48491 x]
0.27735 Cos[2.56495 x]
0.267261 Cos[2.63906 x]
0.258199 Cos[2.70805 x]
0.25 Cos[2.77259 x]
0.242536 Cos[2.83321 x]
0.235702 Cos[2.89037 x]
0.229416 Cos[2.94444 x]
0.223607 Cos[2.99573 x]
0.218218 Cos[3.04452 x]
0.213201 Cos[3.09104 x]
0.208514 Cos[3.13549 x]
0.204124 Cos[3.17805 x]
0.2 Cos[3.21888 x]
0.196116 Cos[3.2581 x]
```

```
0.19245 Cos[3.29584 x]
0.188982 Cos[3.3322 x]
0.185695 Cos[3.3673 x]
0.182574 Cos[3.4012 x]
0.179605 Cos[3.43399 x]
0.176777 Cos[3.46574 x]
0.174078 Cos[3.49651 x]
0.171499 Cos[3.52636 x]
0.169031 Cos[3.55535 x]
0.166667 Cos[3.58352 x]
0.164399 Cos[3.61092 x]
0.162221 Cos[3.63759 x]
0.160128 Cos[3.66356 x]
0.158114 Cos[3.68888 x]
0.156174 Cos[3.71357 x]
0.154303 Cos[3.73767 x]
0.152499 Cos[3.7612 x]
0.150756 Cos[3.78419 x]
0.149071 Cos[3.80666 x]
0.147442 Cos[3.82864 x]
0.145865 Cos[3.85015 x]
0.144338 Cos[3.8712 x]
0.142857 Cos[3.89182 x]
0.141421 Cos[3.91202 x]
0.140028 Cos[3.93183 x]
0.138675 Cos[3.95124 x]
0.137361 Cos[3.97029 x]
0.136083 Cos[3.98898 x]
0.13484 Cos[4.00733 x]
0.133631 Cos[4.02535 x]
0.132453 Cos[4.04305 x]
0.131306 Cos[4.06044 x]
0.130189 Cos[4.07754 x]
0.129099 Cos[4.09434 x]
0.128037 Cos[4.11087 x]
0.127 Cos[4.12713 x]
0.125988 Cos[4.14313 x]
0.125 Cos[4.15888 x]
0.124035 Cos[4.17439 x]
0.123091 Cos[4.18965 x]
0.122169 Cos[4.20469 x]
0.121268 Cos[4.21951 x]
0.120386 Cos[4.23411 x]
0.119523 Cos[4.2485 x]
0.118678 Cos[4.26268 x]
0.117851 Cos[4.27667 x]
0.117041 Cos[4.29046 x]
0.116248 Cos[4.30407 x]
0.11547 Cos[4.31749 x]
0.114708 Cos[4.33073 x]
0.113961 Cos[4.34381 x]
0.113228 Cos[4.35671 x]
0.112509 Cos[4.36945 x]
0.111803 Cos[4.38203 x]
```

```
DiscretePlot[ pt7e2[n, 14.134725141734695`], {n, 1, 100}]
```




```
Animate[DiscretePlot[ Re[pt7e[n, s]], {n, 1, 100}, PlotRange → 5], {s, 10, 260}]
```

```
pt6b[n_, x_] := Sum[ j^(-1/2) (2 x Cos[x Log[j/n]] + Sin[x Log[j/n]]), {j, 1, n}]
pt6b1[n_, x_, c_] := Sum[ j^(-1/2) (2 x Cos[x Log[j/n] + c] + Sin[x Log[j/n] + c]), {j, 1, n}]
pt6e4r[n_, x_, c_] := DiscretePlot[ {-Sum[ j^(-1/2) (2 x Cos[c + x Log[j/n]]), {j, 1, j2}],
   Sum[j^(-1/2) (Sin[c + x Log[j/n]]), {j, 1, j2}]}, {j2, 1, n}]
```

```
pt6b[100 000, N@Im[ZetaZero[1]]]
```

0.0446979

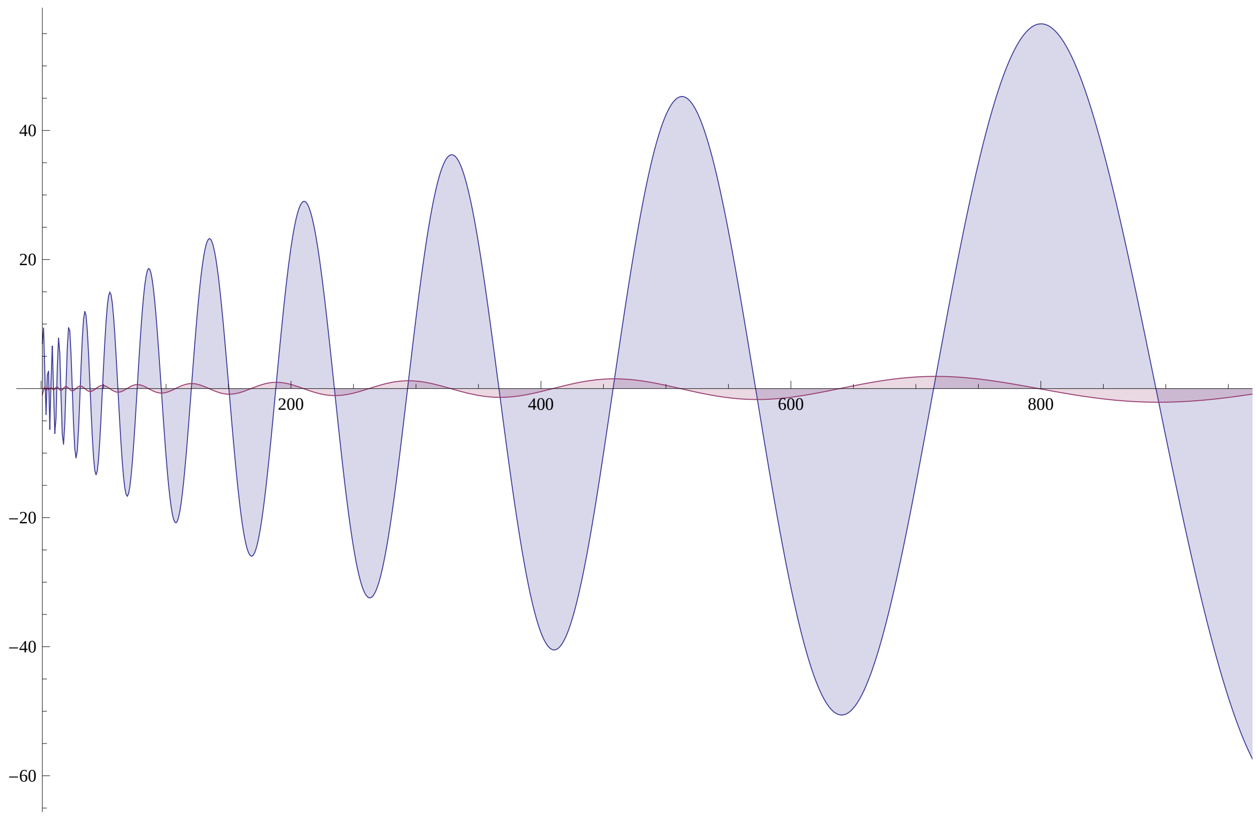
```
pt6b1[100 000, N@Im[ZetaZero[1]], 0]
```

0.0446979

```
pt6b1[100 000, N@Im[ZetaZero[1]], N@Pi/2]
```

632.457

```
pt6e4r[1000, 14.134725141734695` , N@Pi / 2]
```




```
pt7r[n_, x_, c_] :=
 (2 x Sin[c + x Log[n]] + Cos[c + x Log[n]]) Sum[ j^(-1 / 2) Sin[c + x Log[j]], {j, 1, n}] +
  (2 x Cos[c + x Log[n]] - Sin[c + x Log[n]]) Sum[ j^(-1 / 2) Cos[c + x Log[j]], {j, 1, n}]
pt7ar[n_, x_, c_] := {(2 x Sin[c + x Log[n]] + Cos[c + x Log[n]])
   Sum[ j^(-1 / 2) Sin[c + x Log[j]], {j, 1, n}],
  (2 x Cos[c + x Log[n]] - Sin[c + x Log[n]]) Sum[ j^(-1 / 2) Cos[c + x Log[j]], {j, 1, n}]}
pt7cr[n_, x_, c_] := {(2 x Sin[c + x Log[n]] + Cos[c + x Log[n]]),
  (2 x Cos[c + x Log[n]] - Sin[c + x Log[n]])}
pt7dr[j_, x_, c_] := {j^(-1 / 2) Sin[c + x Log[j]], j^(-1 / 2) Cos[c + x Log[j]]}
pt7er[n_, x_, c_] :=
 {Sum[j^(-1 / 2) Sin[c + x Log[j]], {j, 1, n}], Sum[ j^(-1 / 2) Cos[c + x Log[j]], {j, 1, n}]}
```

```
pt7r[10 000 000, 14.134725141734695` , 1 ]
```

0.00446979

```
pt7r[1 000 000, 14.134725141734695` , 2 + I ]
```

$0.0141347 - 6.86668 \times 10^{-11}\, i$

```
pt7r[1 000 000, .3 + 14.134725141734695` , 0 ]
```

7.04051

```
pt7r[1 000 000, .3 + 14.134725141734695` , 2 + I ]
```

$7.04051 + 7.59428 \times 10^{-11}\, i$

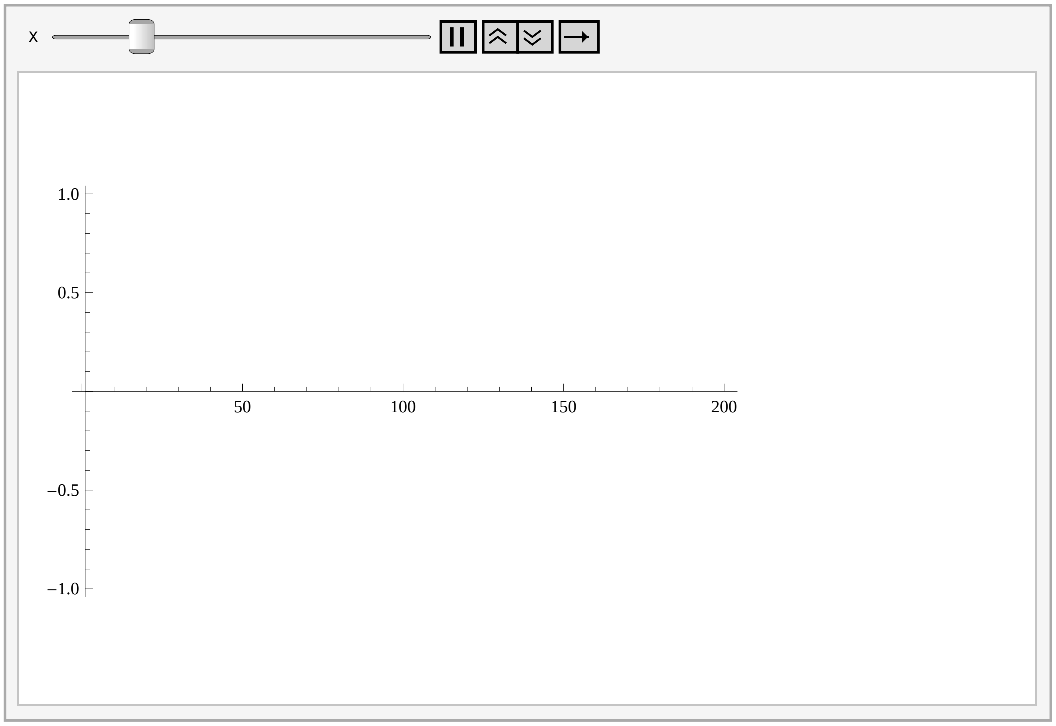
```
pt7r2[1 000 000, .3 + 14.134725141734695` , 1, 1 ]
```

7.04051

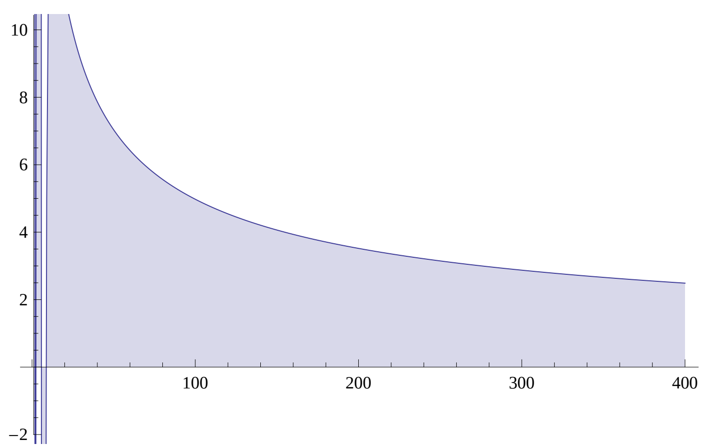
```
pt7s[1 000 000, 14.134725141734695` , 0]
```

-70.6593

```
Animate[DiscretePlot[ pt7ar[n, N@Im@ZetaZero[100], x], {n, 1, 200}], {x, 0, 6.28}]
```




```
DiscretePlot[ pt7r[n, N@Im@ZetaZero@10, 1], {n, 1, 400}]
```




```
D[(1 - x^(1 - s)) Zeta[s], x]
```

$-(1-s)\, x^{-s}\, \text{Zeta}[s]$

```
FullSimplify[Gamma[1 - s] / Gamma[-s - 1]]
```

$s\,(1+s)$

```
D[Sum[ j^-s - (j + x n)^-s, {j, 1, Infinity}], x]
```

n s HurwitzZeta[1 + s, 1 + n x]

```
FullSimplify[Gamma[1 - s] / Gamma[1 - s - k]]
```

$$\frac{\text{Gamma}[1-s]}{\text{Gamma}[1-k-s]}$$

```
3!
```

6

```
D[n^z (s - 1 + z) (Zeta[s + z] - Sum[j^(-(s + z)), {j, 1, n}]), z]
```

```
D[n^z (s - 1 + z) Zeta[s + z], z]
```

n^z Zeta[s + z] + n^z (-1 + s + z) Log[n] Zeta[s + z] + n^z (-1 + s + z) Zeta′[s + z]

```
D[n^z (s - 1 + z) (-j^(-(s + z))), z]
```

$-j^{-s-z} n^z + j^{-s-z} n^z$ (-1 + s + z) Log[j] $- j^{-s-z} n^z$ (-1 + s + z) Log[n]

FullSimplify[n^z ((1 + (-1 + s + z) Log[n]) Zeta[s + z] + (-1 + s + z) Zeta′[s + z]) +
Sum[$j^{-s-z} n^z$ (-1 + (-1 + s + z) Log[j] - (-1 + s + z) Log[n]), {j, 1, Infinity}]]

0

n^z ((1 + (-1 + s + z) Log[n]) Zeta[s + z] + (-1 + s + z) Zeta′[s + z]) +
Sum[$j^{-s-z} n^z$ (-1 + (-1 + s + z) Log[j] - (-1 + s + z) Log[n]), {j, 1, Infinity}]

$-n^z$ (Zeta[s + z] - Log[n] Zeta[s + z] + s Log[n] Zeta[s + z] +
z Log[n] Zeta[s + z] - Zeta′[s + z] + s Zeta′[s + z] + z Zeta′[s + z]) +
n^z ((1 + (-1 + s + z) Log[n]) Zeta[s + z] + (-1 + s + z) Zeta′[s + z])

```
D[n^z (s - 1 + z) (Zeta[s + z] - Sum[j^(-(s + z)), {j, 1, n}]), z]
```

n^z (-HarmonicNumber[n, s + z] + Zeta[s + z]) +
n^z (-1 + s + z) Log[n] (-HarmonicNumber[n, s + z] + Zeta[s + z]) +
n^z (-1 + s + z) (Zeta′[s + z] - HarmonicNumber$^{(0,1)}$[n, s + z])

FullSimplify[n^z Zeta[s + z] + n^z (-1 + s + z) Log[n] Zeta[s + z] + n^z (-1 + s + z) Zeta′[s + z] +
Sum[$-j^{-s-z} n^z + j^{-s-z} n^z$ (-1 + s + z) Log[j] $- j^{-s-z} n^z$ (-1 + s + z) Log[n], {j, 1, Infinity}]]

0

```
D[n^z (s - 1 + z) (fn[a] - fx[b]), z]
```

n^z (fn[a] - fx[b]) + n^z (-1 + s + z) (fn[a] - fx[b]) Log[n]

```
D[fr[c] (Zeta[s + z] - Sum[j^(-(s + z)), {j, 1, n}]), z]
```

fr[c] (Zeta′[s + z] - HarmonicNumber$^{(0,1)}$[n, s + z])

```
Expand[x^(1 - s) / (1 - x^(1 - s)) j^-s]
```

$$\frac{j^{-s}\, x^{1-s}}{1 - x^{1-s}}$$

```
Expand[(j^-s - (j + n x)^-s) / (1 - x^(1 - s))]
```

$$\frac{j^{-s}}{1 - x^{1-s}} - \frac{(j + n\,x)^{-s}}{1 - x^{1-s}}$$

```
Expand[x^(1 - s) / (1 - x^(1 - s)) (j^-s - (j + n)^-s)]
```

$$\frac{j^{-s}\, x^{1-s}}{1 - x^{1-s}} - \frac{(j + n)^{-s}\, x^{1-s}}{1 - x^{1-s}}$$

Expand[((j^-s - (j + n x)^-s) / (1 - x^(1 - s))) - (x^(1 - s) / (1 - x^(1 - s)) (j^-s - (j + n)^-s))]

$$\frac{j^{-s}}{1-x^{1-s}}-\frac{j^{-s}\,x^{1-s}}{1-x^{1-s}}+\frac{(j+n)^{-s}\,x^{1-s}}{1-x^{1-s}}-\frac{(j+n\,x)^{-s}}{1-x^{1-s}}$$

FullSimplify[
((j^-s - (j + n x)^-s) / (1 - x^(1 - s))) - (x^(1 - s) / (1 - x^(1 - s)) (j^-s - (j + n)^-s))]

$$\frac{-(j+n)^{-s}\,x+x^{s}\,(j+n\,x)^{-s}+j^{-s}\,(x-x^{s})}{x-x^{s}}$$

$$\texttt{Limit}\left[\frac{-(j+n)^{-s}\,x+x^{s}\,(j+n\,x)^{-s}+j^{-s}\,(x-x^{s})}{x-x^{s}},\ x\to 1\right]$$

$$-\frac{-(j+n)^{-s}-j^{-s}\,(-1+s)+j\,(j+n)^{-1-s}\,s}{-1+s}$$

$$\texttt{Limit}\left[-\frac{-(j+n)^{-s}-j^{-s}\,(-1+s)+j\,(j+n)^{-1-s}\,s}{-1+s},\ n\to \texttt{Infinity}\right]$$

$$\texttt{Limit}\left[-\frac{-(j+n)^{-s}-j^{-s}\,(-1+s)+j\,(j+n)^{-1-s}\,s}{-1+s},\ n\to\infty\right]$$

$$\texttt{Expand}\left[-\frac{-(j+n)^{-s}-j^{-s}\,(-1+s)+j\,(j+n)^{-1-s}\,s}{-1+s}\right]$$

$$\texttt{ra[n\_, s\_] := Sum}\left[\left\{-\frac{j^{-s}}{-1+s},\ +\frac{(j+n)^{-s}}{-1+s},\ +\frac{j^{-s}\,s}{-1+s},\ -\frac{j\,(j+n)^{-1-s}\,s}{-1+s}\right\},\ \{j,1,n\}\right]$$

$$\texttt{ra2[n\_, s\_] := Sum}\left[-\frac{-(j+n)^{-s}-j^{-s}\,(-1+s)+j\,(j+n)^{-1-s}\,s}{-1+s},\ \{j,1,n\}\right]$$

N@ra[2000000, .5]

{5653.93, -2343.15, -2826.97, 343.146}

N@ra2[2000000, .5]

826.967

fa1[n_, s_, x_] := Sum[j^-s - (j + n x)^-s, {j, 1, Infinity}] / (1 - x^(1 - s)) -
(x^(1 - s)) / (1 - x^(1 - s)) Sum[j^-s, {j, 1, n}]

fa2[n_, s_, x_] := Sum[(j^-s - (j + n x)^-s) / (1 - x^(1 - s)), {j, 1, Infinity}] -
Sum[(j^-s - (j + n)^-s) (x^(1 - s)) / (1 - x^(1 - s)), {j, 1, Infinity}]

$$\texttt{fa3[n\_, s\_, x\_] := Sum}\left[\frac{x^{s}\,(j^{-s}-(j+n\,x)^{-s})}{-x+x^{s}},\ \{j,1,\texttt{Infinity}\}\right]-$$
$$\texttt{Sum}\left[-\frac{(j^{-s}-(j+n)^{-s})\,x}{x-x^{s}},\ \{j,1,\texttt{Infinity}\}\right]$$

$$\texttt{fa4[n\_, s\_, x\_] := Sum}\left[-\frac{-(j+n)^{-s}-j^{-s}\,(-1+s)+j\,(j+n)^{-1-s}\,s}{-1+s},\ \{j,1,\texttt{Infinity}\}\right]$$

$$\texttt{fa5[n\_, s\_] := Sum}\left[\left((j+n)^{-s}+j^{-s}\,(-1+s)-j\,(j+n)^{-1-s}\,s\right)\big/(s-1),\ \{j,1,\texttt{Infinity}\}\right]$$

$$\texttt{fa6[n\_, s\_, t\_] := Sum}\left[j^{-s}+\left((j+n)^{-s}-j\,(j+n)^{-1-s}\,s\right)\big/(s-1),\ \{j,1,t\}\right]$$

$$\texttt{fa7[n\_, s\_, t\_] := Sum}\left[j^{-s}+(j+n)^{-s}/(s-1)-\left(j\,(j+n)^{-1-s}\,s\right)\big/(s-1),\ \{j,1,t\}\right]$$

$$\texttt{fa8[n\_, s\_, t\_] :=}$$
$$\texttt{Sum}\left[j^{-s}-(j+n)^{-s}+(1-(1-s)\verb|^|{-1})\,(j+n)^{-s}-\left(j\,(j+n)^{-1-s}\,s\right)\big/(s-1),\ \{j,1,t\}\right]$$

$$\texttt{fa9[n\_, s\_] := Sum}[j^{-s},\ \{j,1,n\}]\ +$$
$$\texttt{Sum}\left[(1-(1-s)\verb|^|{-1})\,(j+n)^{-s}-\left(j\,(j+n)^{-1-s}\,s\right)\big/(s-1),\ \{j,1,\texttt{Infinity}\}\right]$$

```
N@fa2[10000, .5 + I, 2]
```

$Aborted

```
Zeta[.6]
```

-1.95266

```
FullSimplify[
 (j^-s - (j + n x)^-s) / (1 - x^(1 - s)) - (j^-s - (j + n)^-s) (x^(1 - s)) / (1 - x^(1 - s))]
```

$$\frac{-(j+n)^{-s}\,x + x^{s}\,(j+n\,x)^{-s} + j^{-s}\,(x - x^{s})}{x - x^{s}}$$

```
FullSimplify[(j^-s - (j + n x)^-s) / (1 - x^(1 - s))]
```

$$\frac{x^{s}\,(j^{-s} - (j+n\,x)^{-s})}{-x + x^{s}}$$

```
FullSimplify[(j^-s - (j + n)^-s) (x^(1 - s)) / (1 - x^(1 - s))]
```

$$-\frac{(j^{-s} - (j+n)^{-s})\,x}{x - x^{s}}$$

$$\mathbf{FullSimplify}\left[\frac{x^{s}\,(j^{-s} - (j+n\,x)^{-s})}{-x + x^{s}} - \left(-\frac{(j^{-s} - (j+n)^{-s})\,x}{x - x^{s}}\right)\right]$$

$$\frac{-(j+n)^{-s}\,x + x^{s}\,(j+n\,x)^{-s} + j^{-s}\,(x - x^{s})}{x - x^{s}}$$

$$\mathbf{Limit}\left[\frac{-(j+n)^{-s}\,x + x^{s}\,(j+n\,x)^{-s} + j^{-s}\,(x - x^{s})}{x - x^{s}},\ x \to 1\right]$$

$$-\frac{-(j+n)^{-s} - j^{-s}\,(-1+s) + j\,(j+n)^{-1-s}\,s}{-1+s}$$

$$\mathbf{FullSimplify}\left[-\frac{-(j+n)^{-s} - j^{-s}\,(-1+s) + j\,(j+n)^{-1-s}\,s}{-1+s}\right]$$

$$j^{-s} + \frac{(j+n)^{-1-s}\,(j+n-j\,s)}{-1+s}$$

$$\mathbf{FullSimplify}\left[j\text{^}-s + \frac{(j+n)^{-1-s}\,(j+n-j\,s)}{-1+s}\right]$$

$$j^{-s} + \frac{(j+n)^{-1-s}\,(j+n-j\,s)}{-1+s}$$

```
D[Sum[j^-s, {j, 1, n}] - (s / (1 - s) Zeta[s] - Sum[s / (1 - s) 1 / (j^(s + 1)), {j, 1, n}]), s]
```

$$\frac{\text{HarmonicNumber}[n,\ 1+s]}{1-s} + \frac{s\,\text{HarmonicNumber}[n,\ 1+s]}{(1-s)^{2}} - \frac{\text{Zeta}[s]}{1-s} - \frac{s\,\text{Zeta}[s]}{(1-s)^{2}} - \frac{s\,\text{Zeta}'[s]}{1-s} + \text{HarmonicNumber}^{(0,1)}[n,\ s] + \frac{s\,\text{HarmonicNumber}^{(0,1)}[n,\ 1+s]}{1-s}$$

$$\mathbf{FullSimplify}\left[-\left(-(j+n)^{-s} - j^{-s}\,(-1+s) + j\,(j+n)^{-1-s}\,s\right)\right]$$

$$(j+n)^{-s} + j^{-s}\,(-1+s) - j\,(j+n)^{-1-s}\,s$$

```
FullSimplify[((j + n)^-s + j^-s (-1 + s) - j (j + n)^(-1-s) s) / (s - 1)]
```

$$j^{-s} + \frac{(j+n)^{-1-s} (j+n-j s)}{-1+s}$$

```
FullSimplify[(1 - s) j^-s - (j + n - j s) / (j + n)^(s + 1)]
```

$$-j^{-s} (-1+s) - (j+n)^{-1-s} (j+n-j s)$$

```
FullSimplify[((j + n)^-s - j (j + n)^(-1-s) s) / (s - 1)]
```

$$\frac{(j+n)^{-1-s} (j+n-j s)}{-1+s}$$

```
(1 / (s + 1)) (j + n - j s) / (j + n)^(s + 1)

pt6b[n_, x_] := Sum[ j^(-1 / 2) (2 x Cos[x Log[j / n]] + Sin[x Log[j / n]]), {j, 1, n}]

Animate[DiscretePlot[ pt7ar[n, N@Im@ZetaZero[100], x], {n, 1, 200}], {x, 0, 6.28}]
```

$$\text{FullSimplify}\left[\text{Limit}\left[\frac{-(j+n)^{-s} x + x^s (j+n x)^{-s} + j^{-s} (x - x^s)}{x - x^s}, x \to 1\right]\right]$$

$$j^{-s} + \frac{(j+n)^{-1-s} (j+n-j s)}{-1+s}$$

$$\text{Limit}\left[\frac{-(j+n)^{-s} x + x^s (j+n x)^{-s}}{x - x^s}, x \to 1\right]$$

$$\frac{(j+n)^{-1-s} (j+n-j s)}{-1+s}$$

```
Sum[ j^-s - 1 / (1 - s) (j + n)^-s + s n / (1 - s) (j + n)^(-s - 1), {j, 1, Infinity}] /. s → .5 /.
  n → 10 000
```

598.545

$$\frac{1}{-1+s} (-\text{HurwitzZeta}[s, 1+n] + n\, s\, \text{HurwitzZeta}[1+s, 1+n] - \text{Zeta}[s] + s\, \text{Zeta}[s]) \text{ /. } s \to .5 \text{ /. } n \to 10\,000\,000$$

-18 975.1

```
N[Zeta[.5]]
```

-1.46035

```
fa5[n_, s_] := Sum[((j + n)^-s + j^-s (-1 + s) - j (j + n)^(-1-s) s) / (s - 1), {j, 1, Infinity}]
fa5t[n_, s_, t_] := Sum[((j + n)^-s + j^-s (-1 + s) - j (j + n)^(-1-s) s) / (s - 1), {j, 1, t}]
fa5a[n_, s_] := (1 / (s - 1)) Sum[((j + n)^-s + j^-s (-1 + s) - j (j + n)^(-1-s) s), {j, 1, n}]
```

```
N@fa5[10000, .45]
```

NIntegrate::ncvb :
NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in j near {j} = $\{6.3887\times10^{56}\}$. NIntegrate obtained -6.07199×10^{238} and 6.071990919400807`*^238for the integral and error estimates. ≫

-6.07199×10^{238}

```
Zeta[.4 + I]
```

0.0948136 - 0.65653 i

```
fa1[n_, s_, x_] := Sum[ j^-s - (j + n x) ^-s, {j, 1, Infinity}] / (1 - x^ (1 - s)) -
  (x^ (1 - s)) / (1 - x^ (1 - s)) Sum[ j^-s, {j, 1, n}]
```

```
Sum[ j^-s - (j + n x) ^-s, {j, 1, Infinity}] / (1 - x^ (1 - s))
```

$$\frac{-\mathrm{HurwitzZeta}[s, 1+n x]+\mathrm{Zeta}[s]}{1-x^{1-s}}$$

```
(x^ (1 - s)) / (1 - x^ (1 - s)) Sum[ j^-s, {j, 1, n}]
```

$$\frac{x^{1-s}\,\mathrm{HarmonicNumber}[n, s]}{1-x^{1-s}}$$

```
Sum[j^-s - (j + n x) ^-s, {j, 1, Infinity}] / (1 - x^ (1 - s)) -
 (x^ (1 - s)) / (1 - x^ (1 - s)) Sum[ j^-s, {j, 1, n}]
```

$$-\frac{x^{1-s}\,\mathrm{HarmonicNumber}[n, s]}{1-x^{1-s}}+\frac{-\mathrm{HurwitzZeta}[s, 1+n x]+\mathrm{Zeta}[s]}{1-x^{1-s}}$$

```
fa1[n, s, x] /. n → 10000 /. s → N@ZetaZero@1 /. x → 2
```

-0.0000537764 + 0.00148849 i

$$\mathrm{FullSimplify}\left[-\frac{x^{1-s}\,\mathrm{HarmonicNumber}[n, s]}{1-x^{1-s}}+\frac{-\mathrm{HurwitzZeta}[s, 1+n x]+\mathrm{Zeta}[s]}{1-x^{1-s}}\right]$$

$$\frac{x\,\mathrm{HarmonicNumber}[n, s]+x^{s}\,(\mathrm{HurwitzZeta}[s, 1+n x]-\mathrm{Zeta}[s])}{x-x^{s}}$$

$$\mathrm{Limit}\left[\frac{x\,\mathrm{HarmonicNumber}[n, s]+x^{s}\,(\mathrm{HurwitzZeta}[s, 1+n x]-\mathrm{Zeta}[s])}{x-x^{s}}, x\to 1\right]$$

$$\frac{1}{-1+s}\,(-(-1+s)\,\mathrm{HurwitzZeta}[s, 1+n]+n\,s\,\mathrm{HurwitzZeta}[1+s, 1+n]+(-1+s)\,\mathrm{Zeta}[s])$$

$$\mathrm{FullSimplify}\left[\frac{1}{-1+s}\,(-(-1+s)\,\mathrm{HurwitzZeta}[s, 1+n]+n\,s\,\mathrm{HurwitzZeta}[1+s, 1+n]+(-1+s)\,\mathrm{Zeta}[s])\right]$$

$$\frac{1}{-1+s}\,(-(-1+s)\,\mathrm{HurwitzZeta}[s, 1+n]+n\,s\,\mathrm{HurwitzZeta}[1+s, 1+n]+(-1+s)\,\mathrm{Zeta}[s])$$

$$\frac{1}{-1+s}\,(-(-1+s)\,\mathrm{HurwitzZeta}[s, 1+n]+n\,s\,\mathrm{HurwitzZeta}[1+s, 1+n]+(-1+s)\,\mathrm{Zeta}[s])$$

```
N@Im@ZetaZero@100
```

```
236.524
```

```
`
77.1448400688748`
236.5242296658162`
```