$$(E)^0(n)=1_{[1,\infty)}(n);\ (E)^k(n)=\sum_{j=1}(E)^{k-1}(\frac{n}{j})-x(E)^{k-1}(\frac{n}{jx})$$

$$(E-1)^0(n)=1_{[1,\infty)}(n);(E-1)^k(n)=-x(E-1)^{k-1}(\frac{n}{x})+\sum_{j=2}(E-1)^{k-1}(\frac{n}{j})-x(E-1)^{k-1}(\frac{n}{jx})$$

$$(E)^z(n)=\sum_{j=0}(-1)^j\binom{z}{j}c^j D_z(\frac{n}{c^j})$$

$$(E)^z(n)=\sum_{j=0}(-1)^j\binom{z}{j}c^j\sum_{k=1}^{\lfloor\frac{n}{c^j}\rfloor}d_z(k)$$

$$(E)^z(n)=\sum_{k=1}^{n}d_z(k)\sum_{j=0}^{\lfloor\frac{\log n-\log k}{\log c}\rfloor}(-1)^j\binom{z}{j}c^j$$

$$(E)^z(20)=\sum_{j=0}(-1)^j\binom{z}{j}\frac{3}{2}^j\sum_{k=1}^{\lfloor\frac{20}{(\frac{3}{2})^j}\rfloor}d_z(k)$$

$$f(a)=\sum_{j=0}^{a}(-1)^j\binom{z}{j}(\frac{3}{2})^j \rightarrow f(n,k,c)=\sum_{j=0}^{\lfloor\frac{\log n-\log k}{\log c}\rfloor}(-1)^j\binom{z}{j}(\frac{3}{2})^j$$

$$(E)^z(n)=\sum_{k=1}^{n}d_z(k)\sum_{j=0}^{\lfloor\frac{\log n-\log k}{\log c}\rfloor}(-1)^j\binom{z}{j}c^j$$

$$(E)^z(n)=\sum_{k=1}^{n}d_z(k)\sum_{j=0}^{\lfloor\frac{\log n-\log k}{\log c}\rfloor}(-1)^j\binom{z}{j}c^j$$

$\lim_{c\to 1}(E)^z(n)=d_z(n)$ if n is an integer, 0 otherwise … Right? For z > 0?

$$(-1)^j\binom{-z}{j}=\binom{z+j-1}{j}$$

$$\boxed{D_z(n)=\sum_{j=0}(-1)^j\binom{-z}{j}c^j(E)^z\left(\frac{n}{c^j}\right)}$$

$$\boxed{D_z(n)=\sum_{j=0}\binom{z+j-1}{j}c^j(E)^z\left(\frac{n}{c^j}\right)}$$

$$D_z(10)=\lim_{c\to 1}\sum_{j=0}(-1)^j\binom{-z}{j}c^j(E)^z\left(\frac{10}{c^j}\right)$$

when $c^j=\frac{10}{9}$, $(E)^z=d_z(9)$. j for this will be $\log_c\frac{10}{9}=\frac{\log 10-\log 9}{\log c}$

$$D_z(10)=\lim_{c\to 1}\sum_{k=1}^{10}(-1)^{\left(\frac{\log 10-\log k}{\log c}\right)}\binom{-z}{\frac{\log 10-\log k}{\log c}}\frac{10}{k}d_z(k)$$

$$D_z(10)=\lim_{c\to 1}\sum_{k=1}^{10}\binom{z+\frac{\log 10-\log k}{\log c}-1}{\frac{\log 10-\log k}{\log c}}\frac{10}{k}d_z(k)$$

$$(-1)^j\binom{-z}{j}=\binom{z+j-1}{j}$$

$$(E-1)^k(n)=\sum_{j=0}^{k}\sum_{m=0}^{j}(-1)^j\binom{k}{j}\binom{j}{m}b^j(D-1)^{k-m}\left(\frac{n}{b^j}\right)$$

$$(E-1)^k(n)=\sum_{j=0}^{k}\sum_{m=0}^{j}\binom{j-k-1}{j}\binom{j}{m}c^j(D-1)^{k-m}\left(\frac{n}{c^j}\right)$$

$$(E-1)^k(n)=\sum_{j=0}^{k}\sum_{m=0}^{j}(-1)^j\binom{k}{j}\binom{j}{m}b^j(D-1)^{k-m}\left(\frac{n}{b^j}\right)$$

$$(E-1)^k(n)=\sum_{j=0}^{k}\sum_{m=0}^{j}(-1)^j\binom{k}{j}\binom{j}{m}b^j\sum_{s=1}^{\lfloor\frac{n}{b^j}\rfloor}(d-1)^{k-m}(s)$$

$$(E-1)^k(n)=\sum_{m=0}^{k}\sum_{s=1}^{n}(d-1)^{k-m}(s)\cdot(...)$$

$$(C-1)^0(x,y)=1_{[1,\infty)}(n);\ (C-1)^k(x,y)=y^{-1}\sum_{j=1}(C-1)^{k-1}\left(\frac{x\,y}{j+y},y\right)$$

$$(C-1)^0(x)=1_{[1,\infty)}(n);\ (C-1)^k(x)=y^{-1}\sum_{j=1}(C-1)^{k-1}\left(\frac{x\,y}{j+y}\right)$$

$$(C-1)^0(x)=1_{[1,\infty)}(n);\ (C-1)^k(x)=\frac{1}{y}\sum_{j=1}(C-1)^{k-1}\left(x\left(1+\frac{j}{y}\right)^{-1}\right)$$

$$\boxed{(D-y)^0(x)=1_{[1,\infty)}(n);\ (D-y)^k(x)=\sum_{j=1}(D-y)^{k-1}\left(\frac{x}{j+y}\right)}$$

$$\sum_{j=1} f(j)(\log F)^k\left(\frac{n}{j}\right)=\sum_{j=0}\frac{1}{j!}(\log F)^{k+j}(n)$$

Compare this to $n(\log n)^k=\sum_{j=0}^{\infty}\frac{1}{j!}(\log n)^{k+j}$

```
{x Log[x]^k, Sum[ 1/(j!) Log[ x]^(k+j),{j,0,Infinity}]}
```

$$\Pi(n)=\sum_{k=0}\frac{B_k}{k!}\sum_{j=2}(\log D)^k\left(\frac{n}{j}\right)$$

$$\Pi(n)=\sum_{k=0}\frac{B_k}{k!}\sum_{j=2}(\log d)^k(j)(D-1)^1\left(\frac{n}{j}\right)$$

$$(\log D)^k(n)=\sum_{m=0}\frac{B_m}{m!}\sum_{j=2}(\log F)^{k-1+m}\left(\frac{n}{j}\right)$$

$$(\log F)^k(n)=\sum_{m=0}\frac{B_m}{m!}\sum_{j=1}(f-1)^1(j)(\log F)^{k-1+m}\left(\frac{n}{j}\right)$$
$$(\log F)^k(n)=\sum_{m=0}\frac{B_m}{m!}\sum_{j=1}(\log f)^{k-1+m}(j)(F-1)^1\left(\frac{n}{j}\right)$$

Compare this to $(\log n)^k=\sum_{m=0}^{\infty}\left(\lim_{x\to 0}\frac{\partial^m}{\partial x^m}\frac{x}{e^x-1}\right)\frac{1}{m!}(n-1)(\log n)^{k-1+m}$

```
Sum[ BernoulliB[ b]/b!(x-1) Log[x]^(b+k-1),{b,0,Infinity}]
```

TO DO

add these power series
add more interchapter headings
continue to unify syntax

$$[(1-x^{1-s})D^s](n)^{*k}=\sum_{j=1} j^{-s}[(1-x^{1-s})D^s](\frac{n}{j})^{*k-1}-x\cdot(jx)^{-s}[(1-x^{1-s})D^s](\frac{n}{jx})^{*k-1}$$

$$[(1-x^{1-s})D^s-1](n)^{*k}=\sum_{j=1}(j+1)^{-s}[(1-x^{1-s})D^s-1](\frac{n}{j+1})^{*k-1}-x\cdot(jx)^{-s}[(1-x^{1-s})D^s-1](\frac{n}{jx})^{*k-1}$$

$$\boxed{(D(s)-1)^{*k}(n)=\sum_{j=0}^{\infty}(\lim_{x\to 0}\frac{\partial^j}{\partial x^j}(e^x-1)^k)(\log D(s))^j(n)}$$

$$(\zeta(s)-1)^k=\sum_{j=0}^{\infty}(\lim_{x\to 0}\frac{\partial^j}{\partial x^j}(e^x-1)^k)(\log\zeta(s))^j$$

$$[D^s-1](n)^{*0}=1_{[1,\infty)}(n);\ [D^s-1](n)^{*k}=\sum_{j=2} j^{-s}[D^s-1](\frac{n}{j})^{*k-1}$$

$$[\frac{\partial}{\partial s}D^s](n)^{*z}=-\sum_{k=0}^{\infty}\frac{1}{k}\binom{z}{k}\lim_{s\to 0}\frac{\partial}{\partial s}[D^s-1](n)^{*k}$$

$$\psi(x)=[\frac{\partial}{\partial s}D^s](n)^{*-1}$$

$$\boxed{\psi(n)=\prod_{\rho}(1+\frac{1}{\rho})}$$

```
bin[z_,k_]:=Product[z-j,{j,0,k-1}]/k!
Dm1[n_,k_,s_]:=Sum[j^(-s)Dm1[n/j,k-1,s],{j,2,n}];Dm1[n_,0,s_]:=UnitStep[n-1]
dsDz[n_,z_]:=-Sum[1/k bin[z,k] D[Dm1[n,k,s],s]/.s->0,{k,1,Log[2,n]}]
zeros[n_]:=List@@NRoots[dsDz[n,z]==-1,z][[All,2]]
Table[{Chop[-1+Product[1-1/r,{r,zeros[n]}]-N[Sum[Log[j],{j,2,n}]]],Chop[1-Product[1+1/r,{r,zeros[n]}]-N[Sum[MangoldtLambda[j],
{j,2,n}]]]},{n,4,10}]//TableForm
```

$$(\log n)^a = (n-1)\sum_{k=0} \frac{B_k}{k!} \lim_{z\to 0} \frac{\partial^{k+a-1}}{\partial z^{k+a-1}} n^z$$

$$(\log x)^a = (x-1)\sum_{k=0} \frac{B_k}{k!} \lim_{n\to 0} (\log x)^{k+a-1}$$

$$\Pi(n) = \sum_{j=2} \lfloor \frac{n}{j} - 1 \rfloor \sum_{k=0} \frac{B_k}{k!} \lim_{z\to 0} \frac{\partial^k}{\partial z^k} [d^0](j)^{*z}$$

$$\Pi(n) = \sum_{j=2} \sum_{k=0} \frac{B_k}{k!} \lim_{z\to 0} \frac{\partial^k}{\partial z^k} [D^s](\frac{n}{j})^{*z}$$

$$\log n = (n-1)\sum_{k=0} \frac{B_k}{k!} \lim_{z\to 0} \frac{\partial^k}{\partial z^k} n^z$$

$$\Pi(n) = \sum_{j=2} \lfloor \frac{n}{j} - 1 \rfloor \sum_{k=0} \frac{B_k}{k!} \lim_{z\to 0} \frac{\partial^k}{\partial k} d_z(j)$$

$$\frac{\Lambda(n)}{\log n} = \sum_{j|n,\, 1<j<n} \sum_{k=0} \frac{B_k}{k!} \lim_{z\to 0} \frac{\partial^k}{\partial z^k} d_z(j)$$

$$\log \zeta(s) = (\zeta(s)-1)\sum_{k=0} \frac{B_k}{k!} \lim_{z\to 0} \frac{\partial^k}{\partial z^k} \zeta(s)^z$$

$$\lim_{z\to 0} \frac{\partial^a}{\partial z^a} n^z = (n-1)^1 \sum_{k=0} \frac{B_k}{k!} \lim_{z\to 0} \frac{\partial^{k+a-1}}{\partial z^{k+a-1}} n^z$$

$$\lim_{z\to 0} \frac{\partial^k}{\partial z^k} n^z = (n-1)\sum_{k=0} \frac{B_k}{k!} \lim_{z\to 0} \frac{\partial^{k+a-1}}{\partial z^{k+a-1}} n^z$$

If k is a negative integer,

$$\zeta_n(-k) = \sum_{j=0}^{k} \binom{k}{j} \frac{B_{k-j}}{j+1} n^{j+1}$$

$$\zeta(-k) = \frac{-B_{k+1}}{k+1}$$

$$[D^s-a](n)^{*k}=\sum_{j=1}(j+a)^{-s}[D^s-a](n(j+a)^{-1})^{*k-1}$$

$$[\log((D^s-a)+1)](n)^{*1}=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}[(D^s-a)](n)^{*k}$$

$$\Pi(n)=\int_1^{n+1}\frac{\partial}{\partial a}[\log((D^0-a)+1)](n)^{*1}\,da$$

$$\zeta(s,a)^k=\sum_{j=0}^{k}\binom{k}{j}a^{-sj}\zeta(s,a+1)^{k-j}$$

```
FullSimplify[Table[ Zeta[s,a]^k-Sum[a^(-s j)Binomial[k,j] Zeta[s, a+1]^(k-j), {j,0,k}],{k,1,5},{a,2,5},{s,2,4}]]
```

$$\zeta(s,a)^k=\sum_{j=0}^{k}(-1)^j\binom{k}{j}(a-1)^{-sj}\zeta(s,a-1)^{k-j}$$

```
FullSimplify[Table[ Zeta[s,a]^k-Sum[(-1)^j (a-1)^(-s j)Binomial[k,j] Zeta[s, a-1]^(k-j), {j,0,k}],{k,1,5},{a,2,5},{s,2,4}]]
```

$$\zeta(s,a)^z=\sum_{j=0}^{\infty}(-1)^j\binom{z}{j}(a-1)^{-sj}\zeta(s,a-1)^{z-j}$$

```
FullSimplify[Table[ Chop[Zeta[s,a]^z-Sum[(-1)^j (a-1)^(-s j)Binomial[z,j] Zeta[s, a-1]^(z-j), {j,0,Infinity}]],{z,2.5,5, .7},{a,2,5},{s,2,4}]]
```

$$\zeta(s,a)^k=\sum_{m=a+1}^{\infty}\sum_{j=1}^{k}\binom{k}{j}(m-1)^{-sj}\zeta(s,m)^{k-j}$$

$$\zeta(s,a)=\sum_{l=a}^{\infty}l^{-s}$$

$$\zeta(s,a)^2=\sum_{m=a+1}^{\infty}2(m-1)^{-s}\sum_{l=m}^{\infty}l^{-s}+(m-1)^{-2s}$$

$$\zeta(s,a)^2=2\sum_{m=a}^{\infty}\sum_{l=m+1}^{\infty}m^{-s}l^{-s}+\sum_{m=a}^{\infty}m^{-2s}$$

$$\zeta(s,a)^3=\sum_{m=a+1}^{\infty}3(m-1)^{-s}\zeta(s,m)^2+3(m-1)^{-2s}\zeta(s,m)+(m-1)^{-3s}$$

$$[x^{1-s}\cdot(D^s-1)](n)^{*k}=x\sum_{j=1}(jx+1)^{-s}[x^{1-s}\cdot(D^s-1)](n(jx+1)^{-1})^{*k-1}$$

$$[1+x^{1-s}\cdot(D^s-1)](n)^{*k}=[1+x^{1-s}\cdot(D^s-1)](n)^{*k-1}+x\sum_{j=1}(jx+1)^{-s}[1+x^{1-s}\cdot(D^s-1)](n(jx+1)^{-1})^{*k-1}$$

$$\lim_{x\to 0}(n^x-n^0)x^{-1}=(\log n)^1$$

$$\lim_{x\to\infty}(nx^{-1}+n^0)^x=e^x$$

$$\lim_{x\to 0}([D^0](n)^{*x}-[D^0](n)^{*0})x^{-1}=[\log D](n)^{*1}$$

$$[e^z D^s](n)^{*z}=\sum_{k=0}\frac{z}{k!}[D^s](n)^{*k}$$

$$[e^z D^s](n)^{*z}=e^z\sum_{k=0}\frac{z}{k!}[D^s-1](n)^{*k}$$

$$[e^z(D^s-1)](n)^{*z}=\sum_{k=0}\frac{z}{k!}[D^s-1](n)^{*k}$$

$$\Pi(n) = li(n) - \log\log n - \gamma + \int_0^1 \frac{\partial}{\partial y}[\log((D^0 - 1)\cdot y + 1)](n)^{*1}\,dy$$

$$\Pi(n)=li(n)-\log\log n-\gamma+\lim_{x\to 1+}[\log((1-x^{1-0})D^0)](n)^{*1}+H_{\lfloor\frac{\log n}{\log x}\rfloor}$$
$$\psi(n)=(n-1)+\lim_{x\to 1+}\lim_{s\to 0}\frac{\partial}{\partial s}[\log((1-x^{1-s})D^s)](n)^{*k}$$
$$\Pi(n)=li(n)-\log\log n-\gamma-\lim_{x\to 1+}\lim_{s\to 0}\lim_{z\to 0}\frac{\partial}{\partial z}[D^s-x^{1-s}D^s](n)^{*z}+H_{\lfloor\frac{\log n}{\log x}\rfloor}$$
$$\psi(n)=(n-1)+\lim_{x\to 1+}\lim_{s\to 0}\lim_{z\to 0}\frac{\partial}{\partial s}\frac{\partial}{\partial z}[D^s-x^{1-s}D^s](n)^{*z}$$

$$f_0(n)=1_{[1,\infty)}(n)$$
$$f_k(n)=\sum_{j=1}(j+1)^{-s}f_{k-1}(n\cdot(j+1)^{-1})-x\cdot(jx)^{-s}f_{k-1}(n\cdot(jx)^{-1})$$
$$g_z(n)=\sum_{k=0}\binom{z}{k}f_k(n)$$
$$\Pi(n)=li(n)-\log\log n-\gamma-\lim_{x\to 1+}\lim_{s\to 0}\lim_{z\to 0}\frac{\partial}{\partial z}g_z(n)+H_{\lfloor\frac{\log n}{\log x}\rfloor}$$
$$\psi(n)=(n-1)+\lim_{x\to 1+}\lim_{s\to 0}\lim_{z\to 0}\frac{\partial}{\partial s}\frac{\partial}{\partial z}g_z(n)$$

$$f_0(n)=1_{[1,\infty)}(n)$$
$$f_k(n)=\sum_{j=1}(j+1)^{-s}f_{k-1}(n\cdot(j+1)^{-1})-x\cdot(jx)^{-s}f_{k-1}(n\cdot(jx)^{-1})$$
$$g(n)=\sum_{k=1}\frac{(-1)^{k+1}}{k}f_k(n)$$
$$\Pi(n)=li(n)-\log\log n-\gamma-\lim_{x\to 1+}\lim_{s\to 0}(g(n)+H_{\lfloor\frac{\log n}{\log x}\rfloor})$$
$$\psi(n)=(n-1)-\lim_{x\to 1+}\lim_{s\to 0}(\frac{\partial}{\partial s}g(n))$$

$$f_k(n)=\sum_{j=2}^{\lfloor n\rfloor}j^{-s}(k^{-1}-f_{k+1}(n\cdot j^{-1}))-x\sum_{j=1}^{\lfloor nx^{-1}\rfloor}(jx)^{-s}(k^{-1}-f_{k-1}(n\cdot(jx)^{-1}))$$
$$\Pi(n)=li(n)-\log\log n-\gamma-\lim_{x\to 1+}\lim_{s\to 0}(f_1(n)+H_{\lfloor\frac{\log n}{\log x}\rfloor})$$
$$\psi(n)=(n-1)+\lim_{x\to 1+}\lim_{s\to 0}(\frac{\partial}{\partial s}f_1(n))$$

$$D_k(n,z)=1+(\frac{z+1}{k}-1)\sum_{j=2}^{\lfloor n\rfloor}j^{-s}D_{k+1}(\frac{n}{j},z)$$
$$\Pi(n)=\lim_{s\to 0}\lim_{z\to 0}\frac{\partial}{\partial z}D_1(n,z)$$
$$\psi(n)=-\lim_{s\to 0}\lim_{z\to 0}\frac{\partial}{\partial s}\frac{\partial}{\partial z}D_1(n,z)$$

$$f_k(n,s)=\sum_{j=2}^{\lfloor n\rfloor}j^{-s}(k^{-1}-f_{k+1}(\frac{n}{j},s))$$
$$\Pi(n)=\lim_{s\to 0}f_1(n,s)$$
$$\psi(n)=-\lim_{s\to 0}\frac{\partial}{\partial s}f_1(n,s)$$

$$[\log D^s](n)^{*1}=\sum_{k=1}\frac{1}{k}\left(x^{((1-s)k)}[(1-x^{1-s})D^s-1]\left(\frac{n}{x^k}\right)^{*0}+(-1)^{k+1}[(1-x^{1-s})D^s-1](n)^{*k}\right)$$

$$-\lim_{s\to 0}\frac{\partial}{\partial s}[\log D^s](n)^{*1}=-\lim_{s\to 0}\frac{\partial}{\partial s}\sum_{k=1}\frac{1}{k}\left(x^{(k(1-s))}[(1-x^{1-s})D^s-1]\left(\frac{n}{x^k}\right)^{*0}+(-1)^{k+1}[(1-x^{1-s})D^s-1](n)^{*k}\right)$$

$$[(1-y)L](n)^{*0}=0$$

$$[(1-y)L](n)^{*1}=\sum_{j=2}^{n}\log j-y\sum_{j=1}^{\lfloor\frac{n}{y}\rfloor}\log j\,y$$

$$[(1-y)L](n)^{*k}=\sum_{j=2}^{n}[(1-y)L]\left(\frac{n}{j}\right)^{*k-1}-y\sum_{j=1}^{\lfloor\frac{n}{y}\rfloor}[(1-y)L]\left(\frac{n}{j\,y}\right)^{*k-1}$$

$$[(1-x^{1-s})D^s-1](n)^{*k}=\sum_{j=1}(j+1)^{-s}[(1-x^{1-s})D^s-1](n\cdot(j+1)^{-1})^{*k-1}-x\cdot(j\,x)^{-s}[(1-x^{1-s})D^s-1](n\cdot(j\,x)^{-1})^{*k-1}$$

$$-\frac{1}{k}\lim_{s\to 0}\frac{\partial}{\partial s}[(1-x^{1-s})\zeta_n(s)-1]^{*k}=[(1-y)L](n)^{*k}$$

```
D1xD[n_,k_,x_, s_]:=D1xD[n,k,x,s]=Sum[(j+1)^-s D1xD[n/(j+1),k-1,x,s]-x (j x)^-s  D1xD[n/(x j),k-1,x,s],{j,1,n}]
D1xD[n_,0,x_,s_]:=UnitStep[n-1]
L2[n_,1,b_]:=L2[n,1,b]=Sum[Log[j],{j,2,n}]-b Sum[Log[j b],{j,1,n/b}]
L2[n_,k_,b_]:=Sum[L2[n/j,k-1,b],{j,2,n}]-b Sum[L2[n/(j b),k-1,b],{j,1,n}]
{N[D[D1xD[100,3,1.5,s],s]/.s->0], -3 N[L2[100,3,1.5]]}
```

$$\psi(n)=-\sum_{k=0}^{\lfloor\frac{\log n}{\log x}\rfloor}(-1)^k[(1-x)L](n)^{*k}+\sum_{k=1}^{\lfloor\frac{\log n}{\log x}\rfloor}x^k\cdot\log x$$

$$\psi(n)=-\sum_{k=1}^{\lfloor\frac{\log n}{\log x}\rfloor}\frac{(-1)^{k+1}}{k}\lim_{s\to 0}\frac{\partial}{\partial s}[(1-x^{1-s})\zeta_n(s)-1]^{*k}+\sum_{k=1}^{\lfloor\frac{\log n}{\log x}\rfloor}x^k\cdot\log x$$

```
chebyshev[n_]:=Sum[MangoldtLambda[j],{j,2,n}]
D1xD[n_,k_,x_, s_]:=D1xD[n,k,x,s]=Sum[(j+1)^-s D1xD[n/(j+1),k-1,x,s]-x (j x)^-s  D1xD[n/(x j),k-1,x,s],
{j,1,n}];D1xD[n_,0,x_,s_]:=UnitStep[n-1]
ChebAlt[n_,c_]:=Sum[(-1)^(k)/k  (D[D1xD[n,k,c,s],s]/.s->0),{k,1,Floor[Log[n]/Log[If[c<2,c,2]]]}]+Sum[c^k Log[c],
{k,1,Floor[Log[n]/Log[c]]}]
```

$$\psi(n)=(n-1)-\lim_{x\to 1+}\sum_{k=1}^{\lfloor\frac{\log n}{\log x}\rfloor}\frac{(-1)^{k+1}}{k}\lim_{s\to 0}\frac{\partial}{\partial s}[(1-x^{1-s})\zeta_n(s)-1]^{*k}$$

$$\psi(n)=(n-1)-\lim_{x\to 1+}\lim_{s\to 0}\frac{\partial}{\partial s}[\log((1-x^{1-s})\zeta_n(s))]^{*k}$$

$$\psi(n)=(n-1)-\lim_{x\to 1+}\lim_{s\to 0}\lim_{z\to 0}\frac{\partial}{\partial s}\frac{\partial}{\partial z}[\zeta_n(s)-x^{1-s}\zeta_n(s)]^{*z}$$

$$\psi(n)=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\lim_{s\to 0}\frac{\partial}{\partial s}[D^s-1](n)^{*k}$$

$$\psi(n)=-\lim_{s\to 0}\frac{\partial}{\partial s}[\log D^s](n)^{*1}$$

$$(L-1)^1(n)=\sum_{j=2}\log j\,;\ (L-1)^k(n)=\sum_{j=2}(L-1)^{k-1}(\frac{n}{j})$$

$$(L)^z(n)=\sum_{k=0}^{\infty}\binom{z}{k}(L-1)^k(n)$$

$$(L)^1(n)=\sum_{j=1}\log j\,;\ (L)^k(n)=\sum_{j=1}(L)^{k-1}(\frac{n}{j})$$

$$[(D^s-1)\cdot y](x)^{*k}=y\sum_{j=1}(1+j\,y)^{-s}[(D^s-1)\cdot y](x(j\,y+1)^{-1})^{*k-1}$$

$$\Pi(n)=li(n)-\log\log n-\gamma+\int_0^1\frac{\partial}{\partial y}\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}[(D^0-1)\cdot y](n)^{*k}\,dy$$

$$\Pi(n)=li(n)-\log\log n-\gamma+\int_0^1\frac{\partial}{\partial y}[\log((D^0-1)\cdot y+1)](n)^{*1}\,dy$$

$$f_k(n,s,y)=y\sum_{j=1}^{\lfloor(n-1)y^{-1}\rfloor}(j\,y+1)^{-s}(k^{-1}-f_{k+1}(n(j\,y+1)^{-1},s,y))$$

$$\Pi(n)=li(n)-\log\log n-\gamma+\int_0^1\frac{\partial}{\partial y}\lim_{s\to 0}f_1(n,s,y)\,dy$$

$$\psi(n)=n-\log n-1-\int_0^1\frac{\partial}{\partial y}\lim_{s\to 0}\frac{\partial}{\partial s}f_1(n,s,y)\,dy$$

$$f_k(n,s)=\sum_{j=1}^{\lfloor n-1\rfloor}(j+1)^{-s}(k^{-1}-f_{k+1}(n(j+1)^{-1},s))$$
$$\Pi(n)=\lim_{s\to 0} f_1(n,s)$$
$$\psi(n)=-\lim_{s\to 0}\frac{\partial}{\partial s} f_1(n,s)$$

$$f_k(n,s,x)=x\sum_{j=1}^{\lfloor (n-1)x^{-1}\rfloor}(jx+1)^{-s}(k^{-1}-f_{k+1}(n(jx+1)^{-1},s,x))$$
$$\Pi(n)=li(n)-\log\log n-\gamma+\int_0^1 \frac{\partial}{\partial x}\lim_{s\to 0} f_1(n,s,x)\,dx$$
$$\psi(n)=n-\log n-1-\int_0^1 \frac{\partial}{\partial x}\lim_{s\to 0}\frac{\partial}{\partial s} f_1(n,s,x)\,dx$$

$$f_k(n,s,x)=\sum_{j=1}^{\lfloor n-1\rfloor}(j+1)^{-s}(k^{-1}-f_{k+1}(n\cdot(j+1)^{-1}),s,x)-x\sum_{j=1}^{\lfloor nx^{-1}\rfloor}(jx)^{-s}(k^{-1}-f_{k+1}(n\cdot(jx)^{-1},s,x))$$
$$\Pi(n)=li(n)-\log\log n-\gamma-\lim_{x\to 1+}\lim_{s\to 0}(f_1(n,s,x)+H_{\lfloor\frac{\log n}{\log x}\rfloor})$$
$$\psi(n)=(n-1)+\lim_{x\to 1+}\lim_{s\to 0}(\frac{\partial}{\partial s} f_1(n,s,x))$$

$$\frac{\zeta(2s)}{\zeta(s)}=(\sum_{n=1} \frac{1}{n^{2s}})\cdot(\sum_{n=1} n^{s})$$

$$\sum_{j=1}^{n} \lambda(j)=\sum_{j=1}^{n} \mu(j)\lfloor(n j^{-1})^{\frac{1}{2}}\rfloor$$

$$\sum_{j=1}^{n} j^{-s}\lambda(j)=\sum_{j=1}^{n}[d^{s}](j)^{*-1}[D^{2s}](\lfloor(n j^{-1})^{\frac{1}{2}}\rfloor)^{*1}$$

$$\sum_{j=1}^{n} j^{-s}\lambda(j)=\sum_{j=1}^{\lfloor n^{\frac{1}{2}}\rfloor}[d^{2s}](j)^{*1}[D^{s}](n j^{-2})^{*-1}$$

$$\sum_{j=1}^{n} j^{-s}\mu(j)^{2}=\sum_{j=1}^{n}[d^{s}](j)^{*1}[D^{2s}](\lfloor(n j^{-1})^{\frac{1}{2}}\rfloor)^{*-1}$$

$$\sum_{j=1}^{n} j^{-s}\mu(j)^{2}=\sum_{j=1}^{\lfloor n^{\frac{1}{2}}\rfloor}[d^{2s}](j)^{*-1}[D^{s}](n j^{-2})^{*1}$$

$$\sum_{j|n} d(j^{2})=d(n)^{2}$$

$$\sum_{j|n} \mu(\frac{n}{j})d(j)^{2}=d(n^{2})$$

$$[\zeta_n(s)]^{*y} = 1 + \int_0^y \frac{\partial}{\partial z}[\zeta_n(s)]^{*z}\, dz$$

$$[\zeta_n(s)]^{*y} = 1 + \int_0^y \frac{\partial}{\partial z}[\zeta_n(s)]^{*z}\, dz$$

$$[\zeta_n(t)]^{*z} = 1 - \int_t^\infty \frac{\partial}{\partial s}[\zeta_n(s)]^{*z}\, ds$$

$$\Pi(n) = \int_0^\infty \frac{\partial}{\partial s}[\log \zeta_n(s)]^{*z}\, ds$$

$$[\zeta_n(t)]^{*z} - [\zeta_n(u)]^{*z} = \int_u^t \frac{\partial}{\partial s}[\zeta_n(s)]^{*z}\, ds$$

$$[\zeta_n(s)-1]^{*k} = \sum_{m=0} \frac{1}{m!}(\lim_{x\to 0} \frac{\partial^m}{\partial x^m}\frac{x}{\log(1+x)})[\zeta_n(s)-1]^{*k-1+m} * [\log \zeta_n(s)]^{*1}$$

$$[F_n]*[G_n] = \sum_{j=1} ([F_j]-[F_{j-1}])\cdot[G_{n j^{-1}}] = \sum_{j=1} ([G_j]-[G_{j-1}])\cdot[F_{n j^{-1}}]$$

$$\psi(n) = -[\zeta_n(0)]^{*-1} * (\lim_{s\to 0} \frac{\partial}{\partial s}[\zeta_n(s)]^{*1})$$

There is no z such that

$$\zeta(s)^z = 0$$

because $n^z = 0$ never happens.

There are, however, z's such that

$$[\zeta_n(s)]^{*z} = 0$$

So here is the question. Do those z's converge as n approaches infinity? What is their long-term behavior?

$$[\zeta(s)]^{*z} = 0$$

$$[\zeta(2)]^{*\rho} = 0$$

$$\frac{\pi^2}{6} = \prod_{\rho} \left(1 - \frac{1}{\rho}\right)$$

What might make more sense is to look at

$$\lim_{n \to \infty} [\eta_n(s)]^{*z} = 0$$

Here's the deal...

Given $[\eta(s)]^{*\rho} = 0$, $[\eta(s)]^{*1} = \prod_{\rho} \left(1 - \frac{1}{\rho}\right)$. For $\eta(s) = 0$, it MUST be the case that at least 1 $\rho = 1$.

If $\eta(s)^1 = 0$, then $\eta(s)^2 = 0$ and $\log \eta(s)$ is undefined.

This is not the case for the convolutions, though.