

$$(x-1)^k = \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} x^j$$

$$\{(x-1)^k\} = \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} \{x^j\}$$

| | \int | Σ |
|---|--|--|
| + | $\frac{(x-1)^k}{k!} = \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} \cdot L_j(1-x)$ | $\binom{x-1}{k} = \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} \cdot \frac{x^{(j)}}{j!}$ |
| * | $(-1)^k \cdot \frac{\mathcal{Y}(k, -\log x)}{\Gamma(k)} = \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} \cdot L_{-z}(\log x)$ | $D_k'(x) = \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} D_j(x)$ |

| | \int | Σ |
|---|---|--|
| + | $\frac{(x-1)^{k-1}}{(k-1)!} = \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} \cdot L_{z-1}^{(1)}(1-x)$ | $\binom{x-2}{k-1} = \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} \cdot \frac{x^{(j-1)}}{(j-1)!}$ |
| * | $\frac{\log^{k-1} x}{(k-1)!} = \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} \cdot \left(-\frac{1}{x} \cdot L_{-j-1}^{(1)}(\log x)\right)$ | $d_k'(x) = \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} d_j(x)$ |

$$x^z = \sum_{k=0}^{\infty} \frac{z^k}{k!} \log^k x$$

$$\{x^z\} = \sum_{k=0}^{\infty} \frac{z^k}{k!} \{\log^k x\}$$

...

$$(x-1)^k = \sum_{j=0}^{\infty} \left(\lim_{t \rightarrow 0} \frac{\partial^j}{\partial t^j} (e^t - 1)^k \right) \log^j x$$

$$\{(x-1)^k\} = \sum_{j=0}^{\infty} \left(\lim_{t \rightarrow 0} \frac{\partial^j}{\partial t^j} (e^t - 1)^k \right) \{\log^j x\}$$

$$(x-1)^k = \sum_{j=0} \frac{1}{j!} \left(\lim_{t \rightarrow 0} \frac{\partial^j}{\partial t^j} \frac{t}{\log(1+t)} \right) \cdot (x-1)^{k-1+j} \cdot \log x$$

$$\{(x-1)^k\} = \sum_{j=0} \frac{1}{j!} \left(\lim_{t \rightarrow 0} \frac{\partial^j}{\partial t^j} \frac{t}{\log(1+t)} \right) \cdot \{(x-1)^{k-1+j} \cdot \log x\}$$

$$\log^k x = \sum_{j=1} \frac{(-1)^{j+1}}{j} (x-1)^j \cdot \log^{k-1} x$$

$$\{\log^k x\} = \sum_{j=1} \frac{(-1)^{j+1}}{j} \{(x-1)^j \cdot \log^{k-1} x\}$$

$$\log x = \sum_{k=0} \frac{B_k}{k!} (x-1) \cdot \log^k x$$

$$\{\log x\}=\sum_{k=0}\frac{B_k}{k!}\{(x-1)\cdot\log^k x\}$$

$$\log^ax=\sum_{k=0}\frac{B_k}{k!}(x-1)\cdot\log^{k+a}x$$

$$\{\log^ax\}=\sum_{k=0}\frac{B_k}{k!}\{(x-1)\cdot\log^{k+a}x\}$$

which is

$$\log x=\sum_{k=0}\frac{1}{k!}\cdot(\lim_{t\rightarrow 0}\frac{\partial^k}{\partial t^k}\frac{t}{e^t-1})\cdot(x-1)\cdot\log^k x$$

...

$$(x-1)^{a+b}=(x-1)^a\cdot(x-1)^b$$

$$\{(x-1)^{a+b}\}=\{(x-1)^a\cdot(x-1)^b\}$$

$$x^{y+z}=x^y\cdot x^z$$

$$\{x^{y+z}\}=\{x^y\cdot x^z\}$$

$$\log^{a+b}x=\log^ax\cdot\log^bx$$

$$\{\log^{a+b}x\}=\{\log^ax\cdot\log^bx\}$$

...

$$\log x=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}x^z$$

$$\{\log x\}=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}\{x^z\}$$

| | ∫ | Σ |
|---|---|--|
| + | $\Gamma(0,x-1)+\log(x-1)+\gamma=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}L_z(1-x)$ | $H_{x-1}=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}\frac{x^{(z)}}{z!}$ |
| * | $li(x)-\log\log x-\gamma=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}L_{-z}(\log x)$ | $\Pi(x)=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}D_z(x)$ |

| | ∫ | Σ |
|---|---|--|
| + | $\frac{1}{x-1}-\frac{e^{1-x}}{x-1}=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}L_{z-1}^{(1)}(1-x)$ | $\frac{1}{x-1}=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}\frac{x^{(z-1)}}{(z-1)!}$ |
| * | $\frac{1}{\log x}-\frac{1}{x\log x}=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}-\frac{1}{x}\cdot L_{-z-1}^{(1)}(\log x)$ | $\kappa(x)=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}d_z(x)$ |

$$\log x=\lim_{z\rightarrow 0}\frac{x^z-1}{z}$$

$$\{\log x\}=\lim_{z\rightarrow 0}\frac{\{x^z\}-1}{z}$$

$$\log^k x=\lim_{z\rightarrow 0}\frac{\partial^k}{\partial z^k}x^z$$

$$\{\log^k x\}=\lim_{z\rightarrow 0}\frac{\partial^k}{\partial z^k}\{x^z\}$$

...

$$\log x^z=z\log x$$

$$\{\log x^z\}=z\{\log x\}$$

$$\log a\cdot b=\log a+\log b$$

$$\{\log a\cdot b\}=\{\log a\}+\{\log b\}$$

$$\log \frac{a}{b}=\log a-\log b$$

$$\{\log \frac{a}{b}\}=\{\log a\}-\{\log b\}$$

...

$$t\cdot \log x=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}(x^{z\cdot t})$$

$$t\cdot \{\log x\}=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}\{x^{z\cdot t}\}$$

| | \int | Σ |
|---|---|---|
| + | $t\cdot (\Gamma(0,x-1)+\log(x-1)+\gamma)=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}L_{z\cdot t}(1-x)$ | $t\cdot H_{x-1}=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}\frac{x^{(z\cdot t)}}{(zt)!}$ |
| * | $t\cdot (li(x)-\log\log x-\gamma)=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}L_{-(z\cdot t)}(\log x)$ | $t\cdot \Pi(x)=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}D_{z\cdot t}(x)$ |

| | \int | Σ |
|---|---|---|
| + | $t\cdot (\frac{1}{x-1}-\frac{e^{1-x}}{x-1})=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}L_{t\cdot z-1}^{(1)}(1-x)$ | $t\cdot \frac{1}{x-1}=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}\frac{x^{(t\cdot z-1)}}{(t\cdot z-1)!}$ |
| * | $t(\frac{1}{\log x}-\frac{1}{x\log x})=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}-\frac{1}{x}\cdot L_{-t\cdot z-1}^{(1)}(\log x)$ | $t\cdot \kappa(x)=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}d_{t\cdot z}(x)$ |

...

$$\log n+\log m=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}(n^z\cdot m^z)$$

$$\{\log n\}+\{\log m\}=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}(\{n^z\}\cdot\{m^z\})$$

$$\log n-\log m=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}(\frac{n^z}{m^z})$$

$$\{\log n\}-\{\log m\}=\lim_{z\rightarrow 0}\frac{\partial}{\partial z}(\frac{\{n^z\}}{\{m^z\}})$$

$$\dots$$

$$\{(n\cdot m)^z\}=\sum_{\frac{\log j}{\log n}+\frac{\log k}{\log m}\leq 1}\nabla\{j^z\}\cdot\nabla\{k^z\}$$

$$\{(\frac{n}{m})^z\}=\sum_{\frac{\log j}{\log n}+\frac{\log k}{\log m}\leq 1}\nabla\{j^z\}\cdot\nabla\{k^{-z}\}$$

$$\dots$$

$$\log(n\cdot m)=\log n+\log m$$

$$\{\log(n\cdot m)\}=\{\log n\}+\{\log m\}$$

$$\log\frac{n}{m}=\log n-\log m$$

$$\{\log\frac{n}{m}\}=\{\log n\}-\{\log m\}$$

$$\{(x-1)^{a+b}\}=\{(x-1)^a.(x-1)^b\}$$

$$\frac{x^{a+b}}{(a+b)!}=\int_0^x\int_0^{x-t}\frac{t^{a-1}}{(a-1)!}.\frac{u^{b-1}}{(b-1)!}du\,dt$$

$$\binom{x}{a+b}=\sum_{t=1}^x\sum_{u=1}^{x-t}\binom{t-1}{a-1}.\binom{u-1}{b-1}$$

$$(-1)^{a+b}.\frac{\mathcal{Y}(a+b,-\log x)}{\Gamma(a+b)}=\int_1^x\int_1^{\frac{x}{t}}\frac{\log^{a-1}t}{(a-1)!}.\frac{\log^{b-1}u}{(b-1)!}u\,du\,dt$$

$$D_{a+b}'(x)=\sum_{t=2}^x\sum_{u=2}^{\frac{x}{t}}d_a'(t).d_b'(u)$$

$$\text{AND}$$

$$\{(x-1)^{a+b}\}=\{(x-1)^a.(x-1)^b\}$$

$$\frac{x^{a+b-1}}{(a+b-1)!}=\int_0^x\frac{t^{a-1}}{(a-1)!}.\frac{(x-t)^{b-1}}{(b-1)!}dt$$

$$\frac{\log^{a+b-1}t}{(a+b-1)!}=\int_1^x\frac{\log^{a-1}t}{(a-1)!}.\frac{\log^{b-1}\frac{x}{t}}{(b-1)!}dt\,\,\,???$$

$$d_{a+b}'(x)=\sum_{t+u=x}d_a'(t).d_b'(u)$$

$$\ldots$$

$$\{x^{y+z}\}=\{x^y\cdot x^z\}$$

$$\frac{x^{(z)}}{z!} \quad \frac{x^{(z-1)}}{(z-1)!}$$