```
ClearAll["Global`*"]

str := 2
K[n_] := If[ n == 1, 0, FullSimplify[MangoldtLambda[n] / Log[n]]]
K2[n_] := If[Floor[n^(1/str)] == n^(1/str), K[n^(1/str)], 0]
K7[n_] := K2[n] - If[ Floor[Log[2, n]] == Log[2, n], n^(1/str) / Log[2, n], 0]
P[n_, 0] = 1;
P[n_, k_] := P[n, k] = Sum[ K7[j] P[Floor[n/j], k - 1], {j, 2, n}]
p[n_, k_] := p[n, k] = P[n, k] - P[n - 1, k]
En[n_] := En[n] = Sum[ 1/(k!) P[n, k], {k, 0, Log[2, n]}]
En[n_, z_] := En[n, z] = Sum[ (z^k) / (k!) P[n, k], {k, 0, Log[2, n]}]
En[n_, 0] := 1
en[n_] := Sum[ 1/(k!) p[n, k], {k, 0, Log[2, n]}]
en[n_, z_] := En[n, z] - En[n - 1, z]
LAdd[n_] := Sum[ (2^(1/str))^k/k, {k, 1, Log[str, n]}]
PP[n_, k_] := PP[n, k] = Sum[ 1/k - PP[Floor[n/j], k + 1], {j, 2, n}]
En2[n_, k_] := En2[n, k] = Sum[(-1)^j Binomial[k, j] En[n, k - j], {j, 0, k}]
Lin[n_] := FullSimplify[Sum[ (-1)^(k + 1) / k En2[n, k], {k, 1, Log[2, n]}]]

b1[n_] := b1[n] = Floor[n^(1/str)] - Floor[(n^(1/str)) / (2^(1/str))] (2^(1/str))
bd[n_] := bd[n] = b1[n] - b1[n - 1]

b1[n_] := b1[n] = Floor[n^(1/str)] - Floor[(n^(1/str)) / (2^(1/str))] (2^(1/str))
bd[n_] := bd[n] = b1[n] - b1[n - 1]
s2[n_] := Sum[ bd[j] bd[k], {j, 1, n}, {k, 1, Floor[n/j]}]
s2d[n_] := s2[n] - s2[n - 1]
s2[n_, k_] := s2[n, k] = Sum[ bd[j] s2[Floor[n/j], k - 1], {j, 1, n}]
s2[n_, 0] := 1
s2d[n_, k_] := s2d[n, k] = FullSimplify[s2[n, k] - s2[n - 1, k]]
D1[n_, k_] := D1[n, k] = Sum[ D1[Floor[n/j], k - 1], {j, 1, n}]; D1[n_, 0] := 1
E1[n_, k_] := E1[n, k] = Sum[ (-1)^(j + 1) E1[Floor[n/j], k - 1], {j, 1, n}]; E1[n_, 0] := 1
S22[n_, k_] := S22[n, k] = Sum[(-1)^j Binomial[k, j] s2[n, k - j], {j, 0, k}]
LinS2[n_] := FullSimplify[Sum[ (-1)^(k + 1) / k S22[n, k], {k, 1, Log[2, n]}]]


d2[n_, k_] := d2[n, k] = Sum[d2[j, k - 1] d2[n/j, 1], {j, Divisors[n]}];
d2[n_, 1] := 1; d2[1, 1] := 0; d2[n_, 0] := 0; d2[1, 0] := 1

t1[n_, a_] := Sum[ 1, {j, 1, n}, {k, 1, Floor[n/j]}]
t2[n_, a_] := Sum[ ((2^(1/a))), {j, 1, n}, {k, 1, Floor[ n/(j (2^(1/a)))]}]
t3[n_, a_] :=
 Sum[ 2^(1/a)  2^(1/a), {j, 1, n/(2^(1/a))}, {k, 1, n/(2^(1/a)  2^(1/a) j )}]
ta[n_, a_] := t1[n^(1/a), a] - 2 t2[n^(1/a), a] + t3[n^(1/a), a]


t1[n_, a_] := t1[n, a] = Sum[ 1, {j, 1, n}, {k, 1, Floor[n/j]}]
t2[n_, a_] := t2[n, a] = Sum[ a, {j, 1, n}, {k, 1, Floor[ n/(j a)]}]
t3[n_, a_] := t3[n, a] = Sum[ a a, {j, 1, n/a}, {k, 1, n/(a a j )}]
ta[n_, a_] := ta[n, a] = t1[n, a] - 2 t2[n, a] + t3[n, a]
```

```
t21[n_, a_] := t21[n, a] = Sum[ 1, {j, 2, n}, {k, 2, Floor[n / j]}]
t22[n_, a_] := t22[n, a] = Sum[ a, {j, 2, n}, {k, 1, Floor[ n / (j a)]}]
t23[n_, a_] := t23[n, a] = Sum[ a a, {j, 1, n / a}, {k, 1, n / (a a j )}]
t2a[n_, a_] := t2a[n, a] = t21[n, a] - 2 t22[n, a] + t23[n, a]
d1a[n_] := d1a[n] = Sum[1, {j, 1, n}, {k, 1, Floor[n / j]}]
d2a[n_] := d2a[n] = Sum[1, {j, 2, n}, {k, 2, Floor[n / j]}]


t31[n_, a_] := t31[n, a] = Sum[ 1, {j, 2, n}, {k, 2, Floor[n / j]}, {m, 2, Floor[n / (j k)]}]
t32[n_, a_] := t32[n, a] = Sum[ a, {j, 2, n}, {k, 2, Floor[ n / j]}, {m, 1, Floor[ n / (j k a)]}]
t33[n_, a_] :=
 t33[n, a] = Sum[ a a, {j, 2, n}, {k, 1, Floor[n / (a j )]}, {m, 1, Floor[ n / (j k a a)]}]
t34[n_, a_] := t34[n, a] =
  Sum[ a a a, {j, 1, n / a}, {k, 1, Floor[n / (a a j )]}, {m, 1, Floor[n / (a a a j k )]}]
t3a[n_, a_] := t3a[n, a] = t31[n, a] - 3 t32[n, a] + 3 t33[n, a] - t34[n, a]
d31a[n_] := d31a[n] = Sum[1, {j, 1, n}, {k, 1, Floor[n / j]}, {m, 1, Floor[n / (j k)]}]
d32a[n_] := d32a[n] = Sum[1, {j, 2, n}, {k, 2, Floor[n / j]}, {m, 2, Floor[n / (j k)]}]


t31[n_, a_] := t31[n, a] = Sum[ 1, {j, 2, n}, {k, 2, Floor[n / j]}, {m, 2, Floor[n / (j k)]}]
t32[n_, a_] := t32[n, a] = Sum[ a, {j, 2, n}, {k, 2, Floor[ n / j]}, {m, 1, Floor[ n / (j k a)]}]
t33[n_, a_] :=
 t33[n, a] = Sum[ a a, {j, 2, n}, {k, 1, Floor[n / (a j )]}, {m, 1, Floor[ n / (j k a a)]}]
t34[n_, a_] := t34[n, a] =
  Sum[ a a a, {j, 1, n / a}, {k, 1, Floor[n / (a a j )]}, {m, 1, Floor[n / (a a a j k )]}]
t3a[n_, a_] := t3a[n, a] = t31[n, a] - 3 t32[n, a] + 3 t33[n, a] - t34[n, a]
t21[n_, a_] := t21[n, a] = Sum[ 1, {j, 2, n}, {k, 2, Floor[n / j]}]
t22[n_, a_] := t22[n, a] = Sum[ a, {j, 2, n}, {k, 1, Floor[ n / (j a)]}]
t23[n_, a_] := t23[n, a] = Sum[ a a, {j, 1, n / a}, {k, 1, n / (a a j )}]
t11[n_, a_] := t21[n, a] = Sum[ 1, {j, 2, n}]
t12[n_, a_] := t23[n, a] = Sum[ a , {j, 1, n / a}]
t1a[n_, a_] := t11[n, a] - t12[n, a]
t2a[n_, a_] := t2a[n, a] = t21[n, a] - 2 t22[n, a] + t23[n, a]


tk[n_, k_, a_] :=
 tk[n, k, a] = Sum[ tk[n / j, k - 1, a], {j, 2, n}] - a Sum[ tk[n / (a j), k - 1, a], {j, 1, n / a}];
tk[n_, 0, a_] := 1
tk1[n_, k_, a_] :=
 tk1[n, k, a] = Sum[ tk1[n / j, k - 1, a], {j, 1, n}] - a Sum[ tk1[n / (a j), k - 1, a], {j, 1, n / a}];
tk1[n_, 0, a_] := 1
tk2[n_, k_, a_] := tk2[n, k, a] = Sum[(-1) ^j Binomial[k, j] tk1[n, k - j, a], {j, 0, k}]


Lina[n_, a_] := FullSimplify[Sum[ (-1) ^ (k + 1) / k tk[n, k, a], {k, 1, Log[2, n^2]}]]
LAdda[n_, a_] := Sum[ a^k / k, {k, 1, Log[a, n]}]

LAdda2[n_, a_] := Sum[ (a^k - 1) / k, {k, 1, Log[a, n]}]
```

```
P[100^str, 1] + LAdd[100^str]
```

$$\frac{428}{15}$$

```
P[100, 1] + LAdd[100]
```

$$\frac{16}{3}$$

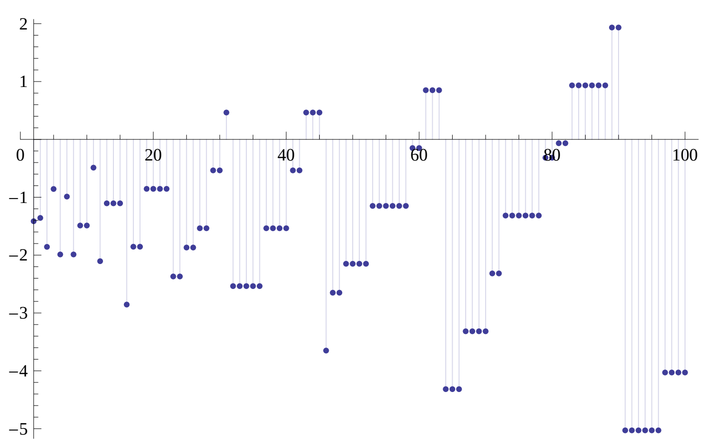
```
Lin[100] + LAdd[100]
```

$$\frac{16}{3}$$

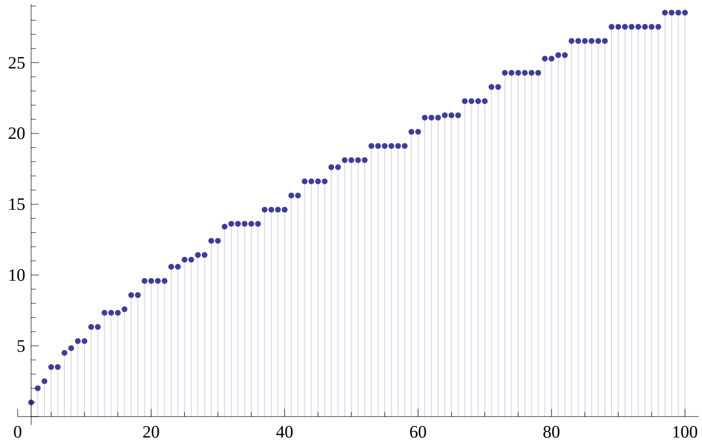
```
Lin[10 000]
```

$$\frac{44}{3} - \frac{595\,471\,\sqrt{2}}{45\,045}$$

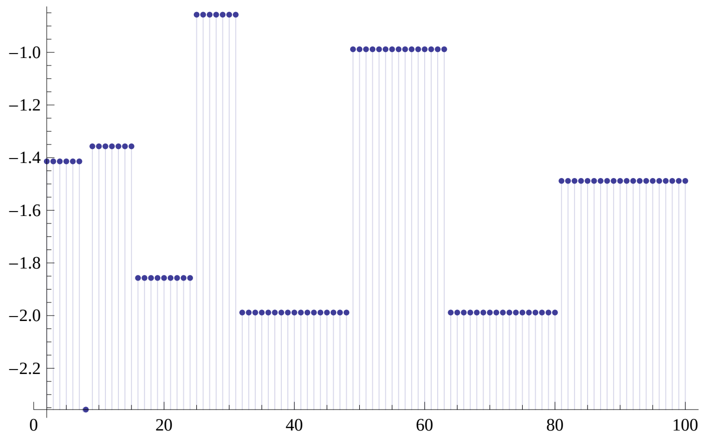
```
DiscretePlot[{Lin[n^str]}, {n, 2, 100}]
```



```
DiscretePlot[{P[n^str, 1] + LAdd[n^str]}, {n, 2, 100}]
```




```
DiscretePlot[{P[n, 1]}, {n, 2, 100}]
```

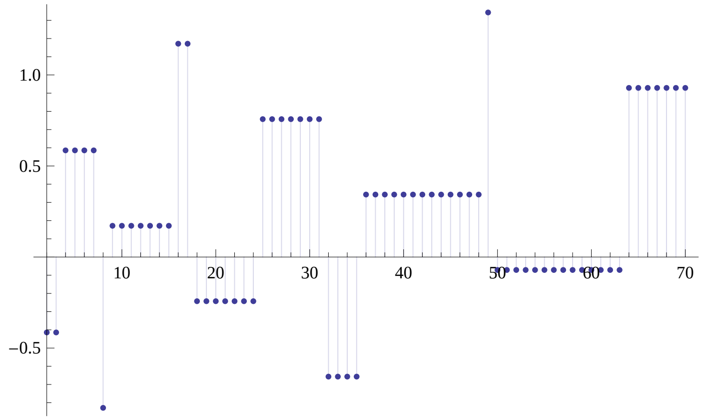



```
Table[{n^(1/2), FullSimplify[en[n]], bd[n]}, {n, 2, 50}] // TableForm
```

$\sqrt{2}$	$-2^{1/3}$	$-2^{1/3}$
$\sqrt{3}$	0	0
2	0	0
$\sqrt{5}$	0	0
$\sqrt{6}$	0	0
$\sqrt{7}$	0	0
$2\sqrt{2}$	1	1
3	0	0
$\sqrt{10}$	0	0
$\sqrt{11}$	0	0
$2\sqrt{3}$	0	0
$\sqrt{13}$	0	0
$\sqrt{14}$	0	0
$\sqrt{15}$	0	0
4	$-2^{1/3}$	$-2^{1/3}$
$\sqrt{17}$	0	0

$3\sqrt{2}$	0	0
$\sqrt{19}$	0	0
$2\sqrt{5}$	0	0
$\sqrt{21}$	0	0
$\sqrt{22}$	0	0
$\sqrt{23}$	0	0
$2\sqrt{6}$	0	0
5	0	0
$\sqrt{26}$	0	0
$3\sqrt{3}$	1	1
$2\sqrt{7}$	0	0
$\sqrt{29}$	0	0
$\sqrt{30}$	0	0
$\sqrt{31}$	0	0
$4\sqrt{2}$	0	0
$\sqrt{33}$	0	0
$\sqrt{34}$	0	0
$\sqrt{35}$	0	0
6	0	0
$\sqrt{37}$	0	0
$\sqrt{38}$	0	0
$\sqrt{39}$	0	0
$2\sqrt{10}$	0	0
$\sqrt{41}$	0	0
$\sqrt{42}$	0	0
$\sqrt{43}$	0	0
$2\sqrt{11}$	0	0
$3\sqrt{5}$	0	0
$\sqrt{46}$	0	0
$\sqrt{47}$	0	0
$4\sqrt{3}$	0	0
7	0	0
$5\sqrt{2}$	0	0

```
DiscretePlot[En[n], {n, 2, 70}]
```




```
Table[ {n^(1/2), FullSimplify[En[n]],
   Floor[n^(1/2)] - Floor[(n^(1/2)) / (2^(1/2))] (2^(1/2))}, {n, 1, 100}] // TableForm
```

1	1	1
$\sqrt{2}$	$1-\sqrt{2}$	$1-\sqrt{2}$
$\sqrt{3}$	$1-\sqrt{2}$	$1-\sqrt{2}$
2	$2-\sqrt{2}$	$2-\sqrt{2}$
$\sqrt{5}$	$2-\sqrt{2}$	$2-\sqrt{2}$
$\sqrt{6}$	$2-\sqrt{2}$	$2-\sqrt{2}$
$\sqrt{7}$	$2-\sqrt{2}$	$2-\sqrt{2}$
$2\sqrt{2}$	$2-2\sqrt{2}$	$2-2\sqrt{2}$
3	$3-2\sqrt{2}$	$3-2\sqrt{2}$
$\sqrt{10}$	$3-2\sqrt{2}$	$3-2\sqrt{2}$
$\sqrt{11}$	$3-2\sqrt{2}$	$3-2\sqrt{2}$
$2\sqrt{3}$	$3-2\sqrt{2}$	$3-2\sqrt{2}$
$\sqrt{13}$	$3-2\sqrt{2}$	$3-2\sqrt{2}$
$\sqrt{14}$	$3-2\sqrt{2}$	$3-2\sqrt{2}$
$\sqrt{15}$	$3-2\sqrt{2}$	$3-2\sqrt{2}$
4	$4-2\sqrt{2}$	$4-2\sqrt{2}$
$\sqrt{17}$	$4-2\sqrt{2}$	$4-2\sqrt{2}$
$3\sqrt{2}$	$4-3\sqrt{2}$	$4-3\sqrt{2}$
$\sqrt{19}$	$4-3\sqrt{2}$	$4-3\sqrt{2}$
$2\sqrt{5}$	$4-3\sqrt{2}$	$4-3\sqrt{2}$
$\sqrt{21}$	$4-3\sqrt{2}$	$4-3\sqrt{2}$
$\sqrt{22}$	$4-3\sqrt{2}$	$4-3\sqrt{2}$
$\sqrt{23}$	$4-3\sqrt{2}$	$4-3\sqrt{2}$
$2\sqrt{6}$	$4-3\sqrt{2}$	$4-3\sqrt{2}$
5	$5-3\sqrt{2}$	$5-3\sqrt{2}$
$\sqrt{26}$	$5-3\sqrt{2}$	$5-3\sqrt{2}$

$3\sqrt{3}$	$5-3\sqrt{2}$	$5-3\sqrt{2}$
$2\sqrt{7}$	$5-3\sqrt{2}$	$5-3\sqrt{2}$
$\sqrt{29}$	$5-3\sqrt{2}$	$5-3\sqrt{2}$
$\sqrt{30}$	$5-3\sqrt{2}$	$5-3\sqrt{2}$
$\sqrt{31}$	$5-3\sqrt{2}$	$5-3\sqrt{2}$
$4\sqrt{2}$	$5-4\sqrt{2}$	$5-4\sqrt{2}$
$\sqrt{33}$	$5-4\sqrt{2}$	$5-4\sqrt{2}$
$\sqrt{34}$	$5-4\sqrt{2}$	$5-4\sqrt{2}$
$\sqrt{35}$	$5-4\sqrt{2}$	$5-4\sqrt{2}$
6	$6-4\sqrt{2}$	$6-4\sqrt{2}$
$\sqrt{37}$	$6-4\sqrt{2}$	$6-4\sqrt{2}$
$\sqrt{38}$	$6-4\sqrt{2}$	$6-4\sqrt{2}$
$\sqrt{39}$	$6-4\sqrt{2}$	$6-4\sqrt{2}$
$2\sqrt{10}$	$6-4\sqrt{2}$	$6-4\sqrt{2}$
$\sqrt{41}$	$6-4\sqrt{2}$	$6-4\sqrt{2}$
$\sqrt{42}$	$6-4\sqrt{2}$	$6-4\sqrt{2}$
$\sqrt{43}$	$6-4\sqrt{2}$	$6-4\sqrt{2}$
$2\sqrt{11}$	$6-4\sqrt{2}$	$6-4\sqrt{2}$
$3\sqrt{5}$	$6-4\sqrt{2}$	$6-4\sqrt{2}$
$\sqrt{46}$	$6-4\sqrt{2}$	$6-4\sqrt{2}$
$\sqrt{47}$	$6-4\sqrt{2}$	$6-4\sqrt{2}$
$4\sqrt{3}$	$6-4\sqrt{2}$	$6-4\sqrt{2}$
7	$7-4\sqrt{2}$	$7-4\sqrt{2}$
$5\sqrt{2}$	$7-5\sqrt{2}$	$7-5\sqrt{2}$
$\sqrt{51}$	$7-5\sqrt{2}$	$7-5\sqrt{2}$
$2\sqrt{13}$	$7-5\sqrt{2}$	$7-5\sqrt{2}$
$\sqrt{53}$	$7-5\sqrt{2}$	$7-5\sqrt{2}$
$3\sqrt{6}$	$7-5\sqrt{2}$	$7-5\sqrt{2}$
$\sqrt{55}$	$7-5\sqrt{2}$	$7-5\sqrt{2}$
$2\sqrt{14}$	$7-5\sqrt{2}$	$7-5\sqrt{2}$
$\sqrt{57}$	$7-5\sqrt{2}$	$7-5\sqrt{2}$
$\sqrt{58}$	$7-5\sqrt{2}$	$7-5\sqrt{2}$
$\sqrt{59}$	$7-5\sqrt{2}$	$7-5\sqrt{2}$
$2\sqrt{15}$	$7-5\sqrt{2}$	$7-5\sqrt{2}$
$\sqrt{61}$	$7-5\sqrt{2}$	$7-5\sqrt{2}$
$\sqrt{62}$	$7-5\sqrt{2}$	$7-5\sqrt{2}$
$3\sqrt{7}$	$7-5\sqrt{2}$	$7-5\sqrt{2}$
8	$8-5\sqrt{2}$	$8-5\sqrt{2}$
$\sqrt{65}$	$8-5\sqrt{2}$	$8-5\sqrt{2}$
$\sqrt{66}$	$8-5\sqrt{2}$	$8-5\sqrt{2}$
$\sqrt{67}$	$8-5\sqrt{2}$	$8-5\sqrt{2}$

$2\sqrt{17}$	$8-5\sqrt{2}$	$8-5\sqrt{2}$
$\sqrt{69}$	$8-5\sqrt{2}$	$8-5\sqrt{2}$
$\sqrt{70}$	$8-5\sqrt{2}$	$8-5\sqrt{2}$
$\sqrt{71}$	$8-5\sqrt{2}$	$8-5\sqrt{2}$
$6\sqrt{2}$	$8-6\sqrt{2}$	$8-6\sqrt{2}$
$\sqrt{73}$	$8-6\sqrt{2}$	$8-6\sqrt{2}$
$\sqrt{74}$	$8-6\sqrt{2}$	$8-6\sqrt{2}$
$5\sqrt{3}$	$8-6\sqrt{2}$	$8-6\sqrt{2}$
$2\sqrt{19}$	$8-6\sqrt{2}$	$8-6\sqrt{2}$
$\sqrt{77}$	$8-6\sqrt{2}$	$8-6\sqrt{2}$
$\sqrt{78}$	$8-6\sqrt{2}$	$8-6\sqrt{2}$
$\sqrt{79}$	$8-6\sqrt{2}$	$8-6\sqrt{2}$
$4\sqrt{5}$	$8-6\sqrt{2}$	$8-6\sqrt{2}$
9	$9-6\sqrt{2}$	$9-6\sqrt{2}$
$\sqrt{82}$	$9-6\sqrt{2}$	$9-6\sqrt{2}$
$\sqrt{83}$	$9-6\sqrt{2}$	$9-6\sqrt{2}$
$2\sqrt{21}$	$9-6\sqrt{2}$	$9-6\sqrt{2}$
$\sqrt{85}$	$9-6\sqrt{2}$	$9-6\sqrt{2}$
$\sqrt{86}$	$9-6\sqrt{2}$	$9-6\sqrt{2}$
$\sqrt{87}$	$9-6\sqrt{2}$	$9-6\sqrt{2}$
$2\sqrt{22}$	$9-6\sqrt{2}$	$9-6\sqrt{2}$
$\sqrt{89}$	$9-6\sqrt{2}$	$9-6\sqrt{2}$
$3\sqrt{10}$	$9-6\sqrt{2}$	$9-6\sqrt{2}$
$\sqrt{91}$	$9-6\sqrt{2}$	$9-6\sqrt{2}$
$2\sqrt{23}$	$9-6\sqrt{2}$	$9-6\sqrt{2}$
$\sqrt{93}$	$9-6\sqrt{2}$	$9-6\sqrt{2}$
$\sqrt{94}$	$9-6\sqrt{2}$	$9-6\sqrt{2}$
$\sqrt{95}$	$9-6\sqrt{2}$	$9-6\sqrt{2}$
$4\sqrt{6}$	$9-6\sqrt{2}$	$9-6\sqrt{2}$
$\sqrt{97}$	$9-6\sqrt{2}$	$9-6\sqrt{2}$
$7\sqrt{2}$	$9-7\sqrt{2}$	$9-7\sqrt{2}$
$3\sqrt{11}$	$9-7\sqrt{2}$	$9-7\sqrt{2}$
10	$10-7\sqrt{2}$	$10-7\sqrt{2}$

```
Table[{n^(1/2), FullSimplify[en[n, 1]]}, {n, 1, 100}] // TableForm
```

1	1
$\sqrt{2}$	$-\sqrt{2}$
$\sqrt{3}$	0
2	1
$\sqrt{5}$	0

$\sqrt{6}$	0
$\sqrt{7}$	0
$2\sqrt{2}$	$-\sqrt{2}$
3	1
$\sqrt{10}$	0
$\sqrt{11}$	0
$2\sqrt{3}$	0
$\sqrt{13}$	0
$\sqrt{14}$	0
$\sqrt{15}$	0
4	1
$\sqrt{17}$	0
$3\sqrt{2}$	$-\sqrt{2}$
$\sqrt{19}$	0
$2\sqrt{5}$	0
$\sqrt{21}$	0
$\sqrt{22}$	0
$\sqrt{23}$	0
$2\sqrt{6}$	0
5	1
$\sqrt{26}$	0
$3\sqrt{3}$	0
$2\sqrt{7}$	0
$\sqrt{29}$	0
$\sqrt{30}$	0
$\sqrt{31}$	0
$4\sqrt{2}$	$-\sqrt{2}$
$\sqrt{33}$	0
$\sqrt{34}$	0
$\sqrt{35}$	0
6	1
$\sqrt{37}$	0
$\sqrt{38}$	0
$\sqrt{39}$	0
$2\sqrt{10}$	0
$\sqrt{41}$	0
$\sqrt{42}$	0
$\sqrt{43}$	0
$2\sqrt{11}$	0
$3\sqrt{5}$	0
$\sqrt{46}$	0
$\sqrt{47}$	0

$4\sqrt{3}$	0
7	1
$5\sqrt{2}$	$-\sqrt{2}$
$\sqrt{51}$	0
$2\sqrt{13}$	0
$\sqrt{53}$	0
$3\sqrt{6}$	0
$\sqrt{55}$	0
$2\sqrt{14}$	0
$\sqrt{57}$	0
$\sqrt{58}$	0
$\sqrt{59}$	0
$2\sqrt{15}$	0
$\sqrt{61}$	0
$\sqrt{62}$	0
$3\sqrt{7}$	0
8	1
$\sqrt{65}$	0
$\sqrt{66}$	0
$\sqrt{67}$	0
$2\sqrt{17}$	0
$\sqrt{69}$	0
$\sqrt{70}$	0
$\sqrt{71}$	0
$6\sqrt{2}$	$-\sqrt{2}$
$\sqrt{73}$	0
$\sqrt{74}$	0
$5\sqrt{3}$	0
$2\sqrt{19}$	0
$\sqrt{77}$	0
$\sqrt{78}$	0
$\sqrt{79}$	0
$4\sqrt{5}$	0
9	1
$\sqrt{82}$	0
$\sqrt{83}$	0
$2\sqrt{21}$	0
$\sqrt{85}$	0
$\sqrt{86}$	0
$\sqrt{87}$	0
$2\sqrt{22}$	0
$\sqrt{89}$	0

$3\sqrt{10}$	0
$\sqrt{91}$	0
$2\sqrt{23}$	0
$\sqrt{93}$	0
$\sqrt{94}$	0
$\sqrt{95}$	0
$4\sqrt{6}$	0
$\sqrt{97}$	0
$7\sqrt{2}$	$-\sqrt{2}$
$3\sqrt{11}$	0
10	1

```
N[10 / (2^(1 / 2))]
```

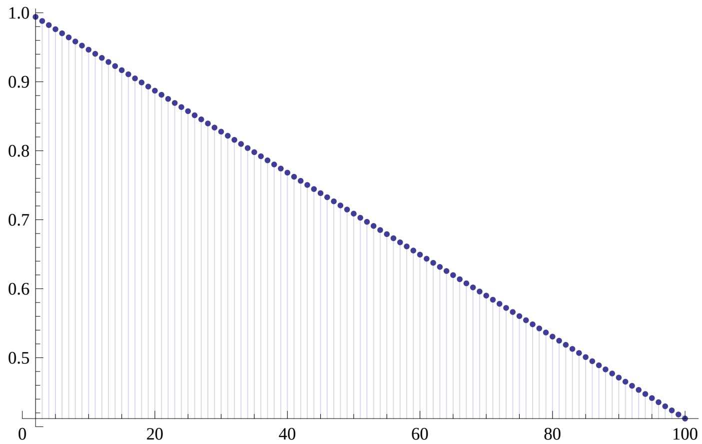
7.07107

```
Table[ {n, FullSimplify[En[n^2]] , (n - Floor[n / (2^(1 / 2))] (2^(1 / 2)))}, {n, 1, 30}] //
 TableForm
```

1	1	1
2	$2 - \sqrt{2}$	$2 - \sqrt{2}$
3	$3 - 2\sqrt{2}$	$3 - 2\sqrt{2}$
4	$4 - 2\sqrt{2}$	$4 - 2\sqrt{2}$
5	$5 - 3\sqrt{2}$	$5 - 3\sqrt{2}$
6	$6 - 4\sqrt{2}$	$6 - 4\sqrt{2}$
7	$7 - 4\sqrt{2}$	$7 - 4\sqrt{2}$
8	$8 - 5\sqrt{2}$	$8 - 5\sqrt{2}$
9	$9 - 6\sqrt{2}$	$9 - 6\sqrt{2}$
10	$10 - 7\sqrt{2}$	$10 - 7\sqrt{2}$
11	$11 - 7\sqrt{2}$	$11 - 7\sqrt{2}$
12	$12 - 8\sqrt{2}$	$12 - 8\sqrt{2}$
13	$13 - 9\sqrt{2}$	$13 - 9\sqrt{2}$
14	$14 - 9\sqrt{2}$	$14 - 9\sqrt{2}$
15	$15 - 10\sqrt{2}$	$15 - 10\sqrt{2}$
16	$16 - 11\sqrt{2}$	$16 - 11\sqrt{2}$
17	$17 - 12\sqrt{2}$	$17 - 12\sqrt{2}$
18	$18 - 12\sqrt{2}$	$18 - 12\sqrt{2}$
19	$19 - 13\sqrt{2}$	$19 - 13\sqrt{2}$
20	$20 - 14\sqrt{2}$	$20 - 14\sqrt{2}$
21	$21 - 14\sqrt{2}$	$21 - 14\sqrt{2}$
22	$22 - 15\sqrt{2}$	$22 - 15\sqrt{2}$
23	$23 - 16\sqrt{2}$	$23 - 16\sqrt{2}$
24	$24 - 16\sqrt{2}$	$24 - 16\sqrt{2}$
25	$25 - 17\sqrt{2}$	$25 - 17\sqrt{2}$
26	$26 - 18\sqrt{2}$	$26 - 18\sqrt{2}$
27	$27 - 19\sqrt{2}$	$27 - 19\sqrt{2}$
28	$28 - 19\sqrt{2}$	$28 - 19\sqrt{2}$
29	$29 - 20\sqrt{2}$	$29 - 20\sqrt{2}$
30	$30 - 21\sqrt{2}$	$30 - 21\sqrt{2}$

```
Table[ {n, N[(n - Floor[n / (2^(1 / 1155))] (2^(1 / 1155)))]}, {n, 1, 100}] // TableForm
```

```
DiscretePlot[(n - Floor[n / (2^(1 / 117))] (2^(1 / 117))), {n, 2, 100}]
```




```
Table[ {n^(1 / 2), FullSimplify[en[n, 2]], s2d[n], s2d[n, 2]}, {n, 1, 100}] // TableForm
```

1	1	1	1
$\sqrt{2}$	$-2\sqrt{2}$	$-2\sqrt{2}$	$-2\sqrt{2}$
$\sqrt{3}$	0	0	0
2	4	4	4
$\sqrt{5}$	0	0	0
$\sqrt{6}$	0	0	0
$\sqrt{7}$	0	0	0
$2\sqrt{2}$	$-4\sqrt{2}$	$-4\sqrt{2}$	$-4\sqrt{2}$
3	2	2	2
$\sqrt{10}$	0	0	0
$\sqrt{11}$	0	0	0
$2\sqrt{3}$	0	0	0
$\sqrt{13}$	0	0	0
$\sqrt{14}$	0	0	0
$\sqrt{15}$	0	0	0
4	7	7	7
$\sqrt{17}$	0	0	0
$3\sqrt{2}$	$-4\sqrt{2}$	$-4\sqrt{2}$	$-4\sqrt{2}$
$\sqrt{19}$	0	0	0
$2\sqrt{5}$	0	0	0
$\sqrt{21}$	0	0	0
$\sqrt{22}$	0	0	0
$\sqrt{23}$	0	0	0
$2\sqrt{6}$	0	0	0
5	2	2	2
$\sqrt{26}$	0	0	0

$3\sqrt{3}$	0	0	0
$2\sqrt{7}$	0	0	0
$\sqrt{29}$	0	0	0
$\sqrt{30}$	0	0	0
$\sqrt{31}$	0	0	0
$4\sqrt{2}$	$-6\sqrt{2}$	$-6\sqrt{2}$	$-6\sqrt{2}$
$\sqrt{33}$	0	0	0
$\sqrt{34}$	0	0	0
$\sqrt{35}$	0	0	0
6	8	8	8
$\sqrt{37}$	0	0	0
$\sqrt{38}$	0	0	0
$\sqrt{39}$	0	0	0
$2\sqrt{10}$	0	0	0
$\sqrt{41}$	0	0	0
$\sqrt{42}$	0	0	0
$\sqrt{43}$	0	0	0
$2\sqrt{11}$	0	0	0
$3\sqrt{5}$	0	0	0
$\sqrt{46}$	0	0	0
$\sqrt{47}$	0	0	0
$4\sqrt{3}$	0	0	0
7	2	2	2
$5\sqrt{2}$	$-4\sqrt{2}$	$-4\sqrt{2}$	$-4\sqrt{2}$
$\sqrt{51}$	0	0	0
$2\sqrt{13}$	0	0	0
$\sqrt{53}$	0	0	0
$3\sqrt{6}$	0	0	0
$\sqrt{55}$	0	0	0
$2\sqrt{14}$	0	0	0
$\sqrt{57}$	0	0	0
$\sqrt{58}$	0	0	0
$\sqrt{59}$	0	0	0
$2\sqrt{15}$	0	0	0
$\sqrt{61}$	0	0	0
$\sqrt{62}$	0	0	0
$3\sqrt{7}$	0	0	0
8	10	10	10
$\sqrt{65}$	0	0	0
$\sqrt{66}$	0	0	0
$\sqrt{67}$	0	0	0
$2\sqrt{17}$	0	0	0

$\sqrt{69}$	0	0	0
$\sqrt{70}$	0	0	0
$\sqrt{71}$	0	0	0
$6\sqrt{2}$	$-8\sqrt{2}$	$-8\sqrt{2}$	$-8\sqrt{2}$
$\sqrt{73}$	0	0	0
$\sqrt{74}$	0	0	0
$5\sqrt{3}$	0	0	0
$2\sqrt{19}$	0	0	0
$\sqrt{77}$	0	0	0
$\sqrt{78}$	0	0	0
$\sqrt{79}$	0	0	0
$4\sqrt{5}$	0	0	0
9	3	3	3
$\sqrt{82}$	0	0	0
$\sqrt{83}$	0	0	0
$2\sqrt{21}$	0	0	0
$\sqrt{85}$	0	0	0
$\sqrt{86}$	0	0	0
$\sqrt{87}$	0	0	0
$2\sqrt{22}$	0	0	0
$\sqrt{89}$	0	0	0
$3\sqrt{10}$	0	0	0
$\sqrt{91}$	0	0	0
$2\sqrt{23}$	0	0	0
$\sqrt{93}$	0	0	0
$\sqrt{94}$	0	0	0
$\sqrt{95}$	0	0	0
$4\sqrt{6}$	0	0	0
$\sqrt{97}$	0	0	0
$7\sqrt{2}$	$-4\sqrt{2}$	$-4\sqrt{2}$	$-4\sqrt{2}$
$3\sqrt{11}$	0	0	0
10	8	8	8

```
Table[{n^(1/2), FullSimplify[en[n, 4]], s2d[n, 4]}, {n, 1, 100}] // TableForm
```

1	1	1
$\sqrt{2}$	$-4\sqrt{2}$	$-4\sqrt{2}$
$\sqrt{3}$	0	0
2	16	16
$\sqrt{5}$	0	0
$\sqrt{6}$	0	0
$\sqrt{7}$	0	0
$2\sqrt{2}$	$-24\sqrt{2}$	$-24\sqrt{2}$
3	4	4

$\sqrt{10}$	0	0
$\sqrt{11}$	0	0
$2\sqrt{3}$	0	0
$\sqrt{13}$	0	0
$\sqrt{14}$	0	0
$\sqrt{15}$	0	0
4	62	62
$\sqrt{17}$	0	0
$3\sqrt{2}$	$-16\sqrt{2}$	$-16\sqrt{2}$
$\sqrt{19}$	0	0
$2\sqrt{5}$	0	0
$\sqrt{21}$	0	0
$\sqrt{22}$	0	0
$\sqrt{23}$	0	0
$2\sqrt{6}$	0	0
5	4	4
$\sqrt{26}$	0	0
$3\sqrt{3}$	0	0
$2\sqrt{7}$	0	0
$\sqrt{29}$	0	0
$\sqrt{30}$	0	0
$\sqrt{31}$	0	0
$4\sqrt{2}$	$-72\sqrt{2}$	$-72\sqrt{2}$
$\sqrt{33}$	0	0
$\sqrt{34}$	0	0
$\sqrt{35}$	0	0
6	64	64
$\sqrt{37}$	0	0
$\sqrt{38}$	0	0
$\sqrt{39}$	0	0
$2\sqrt{10}$	0	0
$\sqrt{41}$	0	0
$\sqrt{42}$	0	0
$\sqrt{43}$	0	0
$2\sqrt{11}$	0	0
$3\sqrt{5}$	0	0
$\sqrt{46}$	0	0
$\sqrt{47}$	0	0
$4\sqrt{3}$	0	0
7	4	4
$5\sqrt{2}$	$-16\sqrt{2}$	$-16\sqrt{2}$
$\sqrt{51}$	0	0

$2\sqrt{13}$	0	0
$\sqrt{53}$	0	0
$3\sqrt{6}$	0	0
$\sqrt{55}$	0	0
$2\sqrt{14}$	0	0
$\sqrt{57}$	0	0
$\sqrt{58}$	0	0
$\sqrt{59}$	0	0
$2\sqrt{15}$	0	0
$\sqrt{61}$	0	0
$\sqrt{62}$	0	0
$3\sqrt{7}$	0	0
8	156	156
$\sqrt{65}$	0	0
$\sqrt{66}$	0	0
$\sqrt{67}$	0	0
$2\sqrt{17}$	0	0
$\sqrt{69}$	0	0
$\sqrt{70}$	0	0
$\sqrt{71}$	0	0
$6\sqrt{2}$	$-96\sqrt{2}$	$-96\sqrt{2}$
$\sqrt{73}$	0	0
$\sqrt{74}$	0	0
$5\sqrt{3}$	0	0
$2\sqrt{19}$	0	0
$\sqrt{77}$	0	0
$\sqrt{78}$	0	0
$\sqrt{79}$	0	0
$4\sqrt{5}$	0	0
9	10	10
$\sqrt{82}$	0	0
$\sqrt{83}$	0	0
$2\sqrt{21}$	0	0
$\sqrt{85}$	0	0
$\sqrt{86}$	0	0
$\sqrt{87}$	0	0
$2\sqrt{22}$	0	0
$\sqrt{89}$	0	0
$3\sqrt{10}$	0	0
$\sqrt{91}$	0	0
$2\sqrt{23}$	0	0
$\sqrt{93}$	0	0

$\sqrt{94}$	0	0
$\sqrt{95}$	0	0
$4\sqrt{6}$	0	0
$\sqrt{97}$	0	0
$7\sqrt{2}$	$-16\sqrt{2}$	$-16\sqrt{2}$
$3\sqrt{11}$	0	0
10	64	64

```
Table[{n^(1/2), FullSimplify[s2[n, 2]], ta[n, 2]}, {n, 1, 100}] // TableForm
```

1	s2[1, 2]	1
$\sqrt{2}$	s2[2, 2]	$1-2\sqrt{2}$
$\sqrt{3}$	s2[3, 2]	$1-2\sqrt{2}$
2	s2[4, 2]	$5-2\sqrt{2}$
$\sqrt{5}$	s2[5, 2]	$5-2\sqrt{2}$
$\sqrt{6}$	s2[6, 2]	$5-2\sqrt{2}$
$\sqrt{7}$	s2[7, 2]	$5-2\sqrt{2}$
$2\sqrt{2}$	s2[8, 2]	$5-6\sqrt{2}$
3	s2[9, 2]	$7-6\sqrt{2}$
$\sqrt{10}$	s2[10, 2]	$7-6\sqrt{2}$
$\sqrt{11}$	s2[11, 2]	$7-6\sqrt{2}$
$2\sqrt{3}$	s2[12, 2]	$7-6\sqrt{2}$
$\sqrt{13}$	s2[13, 2]	$7-6\sqrt{2}$
$\sqrt{14}$	s2[14, 2]	$7-6\sqrt{2}$
$\sqrt{15}$	s2[15, 2]	$7-6\sqrt{2}$
4	s2[16, 2]	$14-6\sqrt{2}$
$\sqrt{17}$	s2[17, 2]	$14-6\sqrt{2}$
$3\sqrt{2}$	s2[18, 2]	$14-10\sqrt{2}$
$\sqrt{19}$	s2[19, 2]	$14-10\sqrt{2}$
$2\sqrt{5}$	s2[20, 2]	$14-10\sqrt{2}$
$\sqrt{21}$	s2[21, 2]	$14-10\sqrt{2}$
$\sqrt{22}$	s2[22, 2]	$14-10\sqrt{2}$
$\sqrt{23}$	s2[23, 2]	$14-10\sqrt{2}$
$2\sqrt{6}$	s2[24, 2]	$14-10\sqrt{2}$
5	s2[25, 2]	$16-10\sqrt{2}$
$\sqrt{26}$	s2[26, 2]	$16-10\sqrt{2}$
$3\sqrt{3}$	s2[27, 2]	$16-10\sqrt{2}$
$2\sqrt{7}$	s2[28, 2]	$16-10\sqrt{2}$
$\sqrt{29}$	s2[29, 2]	$16-10\sqrt{2}$
$\sqrt{30}$	s2[30, 2]	$16-10\sqrt{2}$
$\sqrt{31}$	s2[31, 2]	$16-10\sqrt{2}$
$4\sqrt{2}$	s2[32, 2]	$16-16\sqrt{2}$

$\sqrt{33}$	s2[33, 2]	$16-16\sqrt{2}$
$\sqrt{34}$	s2[34, 2]	$16-16\sqrt{2}$
$\sqrt{35}$	s2[35, 2]	$16-16\sqrt{2}$
6	s2[36, 2]	$24-16\sqrt{2}$
$\sqrt{37}$	s2[37, 2]	$24-16\sqrt{2}$
$\sqrt{38}$	s2[38, 2]	$24-16\sqrt{2}$
$\sqrt{39}$	s2[39, 2]	$24-16\sqrt{2}$
$2\sqrt{10}$	s2[40, 2]	$24-16\sqrt{2}$
$\sqrt{41}$	s2[41, 2]	$24-16\sqrt{2}$
$\sqrt{42}$	s2[42, 2]	$24-16\sqrt{2}$
$\sqrt{43}$	s2[43, 2]	$24-16\sqrt{2}$
$2\sqrt{11}$	s2[44, 2]	$24-16\sqrt{2}$
$3\sqrt{5}$	s2[45, 2]	$24-16\sqrt{2}$
$\sqrt{46}$	s2[46, 2]	$24-16\sqrt{2}$
$\sqrt{47}$	s2[47, 2]	$24-16\sqrt{2}$
$4\sqrt{3}$	s2[48, 2]	$24-16\sqrt{2}$
7	s2[49, 2]	$26-16\sqrt{2}$
$5\sqrt{2}$	s2[50, 2]	$26-20\sqrt{2}$
$\sqrt{51}$	s2[51, 2]	$26-20\sqrt{2}$
$2\sqrt{13}$	s2[52, 2]	$26-20\sqrt{2}$
$\sqrt{53}$	s2[53, 2]	$26-20\sqrt{2}$
$3\sqrt{6}$	s2[54, 2]	$26-20\sqrt{2}$
$\sqrt{55}$	s2[55, 2]	$26-20\sqrt{2}$
$2\sqrt{14}$	s2[56, 2]	$26-20\sqrt{2}$
$\sqrt{57}$	s2[57, 2]	$26-20\sqrt{2}$
$\sqrt{58}$	s2[58, 2]	$26-20\sqrt{2}$
$\sqrt{59}$	s2[59, 2]	$26-20\sqrt{2}$
$2\sqrt{15}$	s2[60, 2]	$26-20\sqrt{2}$
$\sqrt{61}$	s2[61, 2]	$26-20\sqrt{2}$
$\sqrt{62}$	s2[62, 2]	$26-20\sqrt{2}$
$3\sqrt{7}$	s2[63, 2]	$26-20\sqrt{2}$
8	s2[64, 2]	$36-20\sqrt{2}$
$\sqrt{65}$	s2[65, 2]	$36-20\sqrt{2}$
$\sqrt{66}$	s2[66, 2]	$36-20\sqrt{2}$
$\sqrt{67}$	s2[67, 2]	$36-20\sqrt{2}$
$2\sqrt{17}$	s2[68, 2]	$36-20\sqrt{2}$
$\sqrt{69}$	s2[69, 2]	$36-20\sqrt{2}$
$\sqrt{70}$	s2[70, 2]	$36-20\sqrt{2}$
$\sqrt{71}$	s2[71, 2]	$36-20\sqrt{2}$
$6\sqrt{2}$	s2[72, 2]	$36-28\sqrt{2}$
$\sqrt{73}$	s2[73, 2]	$36-28\sqrt{2}$

$\sqrt{74}$	s2[74, 2]	$36-28\sqrt{2}$
$5\sqrt{3}$	s2[75, 2]	$36-28\sqrt{2}$
$2\sqrt{19}$	s2[76, 2]	$36-28\sqrt{2}$
$\sqrt{77}$	s2[77, 2]	$36-28\sqrt{2}$
$\sqrt{78}$	s2[78, 2]	$36-28\sqrt{2}$
$\sqrt{79}$	s2[79, 2]	$36-28\sqrt{2}$
$4\sqrt{5}$	s2[80, 2]	$36-28\sqrt{2}$
9	s2[81, 2]	$39-28\sqrt{2}$
$\sqrt{82}$	s2[82, 2]	$39-28\sqrt{2}$
$\sqrt{83}$	s2[83, 2]	$39-28\sqrt{2}$
$2\sqrt{21}$	s2[84, 2]	$39-28\sqrt{2}$
$\sqrt{85}$	s2[85, 2]	$39-28\sqrt{2}$
$\sqrt{86}$	s2[86, 2]	$39-28\sqrt{2}$
$\sqrt{87}$	s2[87, 2]	$39-28\sqrt{2}$
$2\sqrt{22}$	s2[88, 2]	$39-28\sqrt{2}$
$\sqrt{89}$	s2[89, 2]	$39-28\sqrt{2}$
$3\sqrt{10}$	s2[90, 2]	$39-28\sqrt{2}$
$\sqrt{91}$	s2[91, 2]	$39-28\sqrt{2}$
$2\sqrt{23}$	s2[92, 2]	$39-28\sqrt{2}$
$\sqrt{93}$	s2[93, 2]	$39-28\sqrt{2}$
$\sqrt{94}$	s2[94, 2]	$39-28\sqrt{2}$
$\sqrt{95}$	s2[95, 2]	$39-28\sqrt{2}$
$4\sqrt{6}$	s2[96, 2]	$39-28\sqrt{2}$
$\sqrt{97}$	s2[97, 2]	$39-28\sqrt{2}$
$7\sqrt{2}$	s2[98, 2]	$39-32\sqrt{2}$
$3\sqrt{11}$	s2[99, 2]	$39-32\sqrt{2}$
10	s2[100, 2]	$47-32\sqrt{2}$

1	1	1
$\sqrt{2}$	$1-2\sqrt{2}$	$1-2\sqrt{2}$
$\sqrt{3}$	$1-2\sqrt{2}$	$1-2\sqrt{2}$
2	$5-2\sqrt{2}$	$5-2\sqrt{2}$
$\sqrt{5}$	$5-2\sqrt{2}$	$5-2\sqrt{2}$
$\sqrt{6}$	$5-2\sqrt{2}$	$5-2\sqrt{2}$
$\sqrt{7}$	$5-2\sqrt{2}$	$5-2\sqrt{2}$
$2\sqrt{2}$	$5-6\sqrt{2}$	$5-6\sqrt{2}$
3	$7-6\sqrt{2}$	$7-6\sqrt{2}$
$\sqrt{10}$	$7-6\sqrt{2}$	$7-6\sqrt{2}$
$\sqrt{11}$	$7-6\sqrt{2}$	$7-6\sqrt{2}$
$2\sqrt{3}$	$7-6\sqrt{2}$	$7-6\sqrt{2}$
$\sqrt{13}$	$7-6\sqrt{2}$	$7-6\sqrt{2}$

$\sqrt{14}$	$7-6\sqrt{2}$	$7-6\sqrt{2}$
$\sqrt{15}$	$7-6\sqrt{2}$	$7-6\sqrt{2}$
4	$14-6\sqrt{2}$	$14-6\sqrt{2}$
$\sqrt{17}$	$14-6\sqrt{2}$	$14-6\sqrt{2}$
$3\sqrt{2}$	$14-10\sqrt{2}$	$14-10\sqrt{2}$
$\sqrt{19}$	$14-10\sqrt{2}$	$14-10\sqrt{2}$
$2\sqrt{5}$	$14-10\sqrt{2}$	$14-10\sqrt{2}$
$\sqrt{21}$	$14-10\sqrt{2}$	$14-10\sqrt{2}$
$\sqrt{22}$	$14-10\sqrt{2}$	$14-10\sqrt{2}$
$\sqrt{23}$	$14-10\sqrt{2}$	$14-10\sqrt{2}$
$2\sqrt{6}$	$14-10\sqrt{2}$	$14-10\sqrt{2}$
5	$16-10\sqrt{2}$	$16-10\sqrt{2}$
$\sqrt{26}$	$16-10\sqrt{2}$	$16-10\sqrt{2}$
$3\sqrt{3}$	$16-10\sqrt{2}$	$16-10\sqrt{2}$
$2\sqrt{7}$	$16-10\sqrt{2}$	$16-10\sqrt{2}$
$\sqrt{29}$	$16-10\sqrt{2}$	$16-10\sqrt{2}$
$\sqrt{30}$	$16-10\sqrt{2}$	$16-10\sqrt{2}$
$\sqrt{31}$	$16-10\sqrt{2}$	$16-10\sqrt{2}$
$4\sqrt{2}$	$-16\left(-1+\sqrt{2}\right)$	$16-16\sqrt{2}$
$\sqrt{33}$	$-16\left(-1+\sqrt{2}\right)$	$16-16\sqrt{2}$
$\sqrt{34}$	$-16\left(-1+\sqrt{2}\right)$	$16-16\sqrt{2}$
$\sqrt{35}$	$-16\left(-1+\sqrt{2}\right)$	$16-16\sqrt{2}$
6	$24-16\sqrt{2}$	$24-16\sqrt{2}$
$\sqrt{37}$	$24-16\sqrt{2}$	$24-16\sqrt{2}$
$\sqrt{38}$	$24-16\sqrt{2}$	$24-16\sqrt{2}$
$\sqrt{39}$	$24-16\sqrt{2}$	$24-16\sqrt{2}$
$2\sqrt{10}$	$24-16\sqrt{2}$	$24-16\sqrt{2}$
$\sqrt{41}$	$24-16\sqrt{2}$	$24-16\sqrt{2}$
$\sqrt{42}$	$24-16\sqrt{2}$	$24-16\sqrt{2}$
$\sqrt{43}$	$24-16\sqrt{2}$	$24-16\sqrt{2}$
$2\sqrt{11}$	$24-16\sqrt{2}$	$24-16\sqrt{2}$
$3\sqrt{5}$	$24-16\sqrt{2}$	$24-16\sqrt{2}$
$\sqrt{46}$	$24-16\sqrt{2}$	$24-16\sqrt{2}$
$\sqrt{47}$	$24-16\sqrt{2}$	$24-16\sqrt{2}$
$4\sqrt{3}$	$24-16\sqrt{2}$	$24-16\sqrt{2}$
7	$26-16\sqrt{2}$	$26-16\sqrt{2}$
$5\sqrt{2}$	$26-20\sqrt{2}$	$26-20\sqrt{2}$
$\sqrt{51}$	$26-20\sqrt{2}$	$26-20\sqrt{2}$
$2\sqrt{13}$	$26-20\sqrt{2}$	$26-20\sqrt{2}$
$\sqrt{53}$	$26-20\sqrt{2}$	$26-20\sqrt{2}$
$3\sqrt{6}$	$26-20\sqrt{2}$	$26-20\sqrt{2}$

$\sqrt{55}$	$26 - 20\sqrt{2}$	$26 - 20\sqrt{2}$
$2\sqrt{14}$	$26 - 20\sqrt{2}$	$26 - 20\sqrt{2}$
$\sqrt{57}$	$26 - 20\sqrt{2}$	$26 - 20\sqrt{2}$
$\sqrt{58}$	$26 - 20\sqrt{2}$	$26 - 20\sqrt{2}$
$\sqrt{59}$	$26 - 20\sqrt{2}$	$26 - 20\sqrt{2}$
$2\sqrt{15}$	$26 - 20\sqrt{2}$	$26 - 20\sqrt{2}$
$\sqrt{61}$	$26 - 20\sqrt{2}$	$26 - 20\sqrt{2}$
$\sqrt{62}$	$26 - 20\sqrt{2}$	$26 - 20\sqrt{2}$
$3\sqrt{7}$	$26 - 20\sqrt{2}$	$26 - 20\sqrt{2}$
8	$36 - 20\sqrt{2}$	$36 - 20\sqrt{2}$
$\sqrt{65}$	$36 - 20\sqrt{2}$	$36 - 20\sqrt{2}$
$\sqrt{66}$	$36 - 20\sqrt{2}$	$36 - 20\sqrt{2}$
$\sqrt{67}$	$36 - 20\sqrt{2}$	$36 - 20\sqrt{2}$
$2\sqrt{17}$	$36 - 20\sqrt{2}$	$36 - 20\sqrt{2}$
$\sqrt{69}$	$36 - 20\sqrt{2}$	$36 - 20\sqrt{2}$
$\sqrt{70}$	$36 - 20\sqrt{2}$	$36 - 20\sqrt{2}$
$\sqrt{71}$	$36 - 20\sqrt{2}$	$36 - 20\sqrt{2}$
$6\sqrt{2}$	$36 - 28\sqrt{2}$	$36 - 28\sqrt{2}$
$\sqrt{73}$	$36 - 28\sqrt{2}$	$36 - 28\sqrt{2}$
$\sqrt{74}$	$36 - 28\sqrt{2}$	$36 - 28\sqrt{2}$
$5\sqrt{3}$	$36 - 28\sqrt{2}$	$36 - 28\sqrt{2}$
$2\sqrt{19}$	$36 - 28\sqrt{2}$	$36 - 28\sqrt{2}$
$\sqrt{77}$	$36 - 28\sqrt{2}$	$36 - 28\sqrt{2}$
$\sqrt{78}$	$36 - 28\sqrt{2}$	$36 - 28\sqrt{2}$
$\sqrt{79}$	$36 - 28\sqrt{2}$	$36 - 28\sqrt{2}$
$4\sqrt{5}$	$36 - 28\sqrt{2}$	$36 - 28\sqrt{2}$
9	$39 - 28\sqrt{2}$	$39 - 28\sqrt{2}$
$\sqrt{82}$	$39 - 28\sqrt{2}$	$39 - 28\sqrt{2}$
$\sqrt{83}$	$39 - 28\sqrt{2}$	$39 - 28\sqrt{2}$
$2\sqrt{21}$	$39 - 28\sqrt{2}$	$39 - 28\sqrt{2}$
$\sqrt{85}$	$39 - 28\sqrt{2}$	$39 - 28\sqrt{2}$
$\sqrt{86}$	$39 - 28\sqrt{2}$	$39 - 28\sqrt{2}$
$\sqrt{87}$	$39 - 28\sqrt{2}$	$39 - 28\sqrt{2}$
$2\sqrt{22}$	$39 - 28\sqrt{2}$	$39 - 28\sqrt{2}$
$\sqrt{89}$	$39 - 28\sqrt{2}$	$39 - 28\sqrt{2}$
$3\sqrt{10}$	$39 - 28\sqrt{2}$	$39 - 28\sqrt{2}$
$\sqrt{91}$	$39 - 28\sqrt{2}$	$39 - 28\sqrt{2}$
$2\sqrt{23}$	$39 - 28\sqrt{2}$	$39 - 28\sqrt{2}$
$\sqrt{93}$	$39 - 28\sqrt{2}$	$39 - 28\sqrt{2}$
$\sqrt{94}$	$39 - 28\sqrt{2}$	$39 - 28\sqrt{2}$
$\sqrt{95}$	$39 - 28\sqrt{2}$	$39 - 28\sqrt{2}$

$4\sqrt{6}$	$39-28\sqrt{2}$	$39-28\sqrt{2}$
$\sqrt{97}$	$39-28\sqrt{2}$	$39-28\sqrt{2}$
$7\sqrt{2}$	$39-32\sqrt{2}$	$39-32\sqrt{2}$
$3\sqrt{11}$	$39-32\sqrt{2}$	$39-32\sqrt{2}$
10	$47-32\sqrt{2}$	$47-32\sqrt{2}$

FullSimplify[bd[n]]

$$-\text{Floor}\left[\sqrt{-1+n}\right]+\sqrt{2}\,\text{Floor}\left[\frac{\sqrt{-1+n}}{\sqrt{2}}\right]+\text{Floor}\left[\sqrt{n}\right]-\sqrt{2}\,\text{Floor}\left[\frac{\sqrt{n}}{\sqrt{2}}\right]$$

Table[{n^(1/2), FullSimplify[s2[n, 2]], ta[n^(1/2), 2^(1/2)]}, {n, 1, 100}] // TableForm

1	s2[1, 2]	1
$\sqrt{2}$	s2[2, 2]	$1-2\sqrt{2}$
$\sqrt{3}$	s2[3, 2]	$1-2\sqrt{2}$
2	s2[4, 2]	$5-2\sqrt{2}$
$\sqrt{5}$	s2[5, 2]	$5-2\sqrt{2}$
$\sqrt{6}$	s2[6, 2]	$5-2\sqrt{2}$
$\sqrt{7}$	s2[7, 2]	$5-2\sqrt{2}$
$2\sqrt{2}$	s2[8, 2]	$5-6\sqrt{2}$
3	s2[9, 2]	$7-6\sqrt{2}$
$\sqrt{10}$	s2[10, 2]	$7-6\sqrt{2}$
$\sqrt{11}$	s2[11, 2]	$7-6\sqrt{2}$
$2\sqrt{3}$	s2[12, 2]	$7-6\sqrt{2}$
$\sqrt{13}$	s2[13, 2]	$7-6\sqrt{2}$
$\sqrt{14}$	s2[14, 2]	$7-6\sqrt{2}$
$\sqrt{15}$	s2[15, 2]	$7-6\sqrt{2}$
4	s2[16, 2]	$14-6\sqrt{2}$
$\sqrt{17}$	s2[17, 2]	$14-6\sqrt{2}$
$3\sqrt{2}$	s2[18, 2]	$14-10\sqrt{2}$
$\sqrt{19}$	s2[19, 2]	$14-10\sqrt{2}$
$2\sqrt{5}$	s2[20, 2]	$14-10\sqrt{2}$
$\sqrt{21}$	s2[21, 2]	$14-10\sqrt{2}$
$\sqrt{22}$	s2[22, 2]	$14-10\sqrt{2}$
$\sqrt{23}$	s2[23, 2]	$14-10\sqrt{2}$
$2\sqrt{6}$	s2[24, 2]	$14-10\sqrt{2}$
5	s2[25, 2]	$16-10\sqrt{2}$
$\sqrt{26}$	s2[26, 2]	$16-10\sqrt{2}$
$3\sqrt{3}$	s2[27, 2]	$16-10\sqrt{2}$
$2\sqrt{7}$	s2[28, 2]	$16-10\sqrt{2}$
$\sqrt{29}$	s2[29, 2]	$16-10\sqrt{2}$
$\sqrt{30}$	s2[30, 2]	$16-10\sqrt{2}$
$\sqrt{31}$	s2[31, 2]	$16-10\sqrt{2}$
$4\sqrt{2}$	s2[32, 2]	$16-16\sqrt{2}$

$\sqrt{33}$	s2[33, 2]	$16 - 16\sqrt{2}$
$\sqrt{34}$	s2[34, 2]	$16 - 16\sqrt{2}$
$\sqrt{35}$	s2[35, 2]	$16 - 16\sqrt{2}$
6	s2[36, 2]	$24 - 16\sqrt{2}$
$\sqrt{37}$	s2[37, 2]	$24 - 16\sqrt{2}$
$\sqrt{38}$	s2[38, 2]	$24 - 16\sqrt{2}$
$\sqrt{39}$	s2[39, 2]	$24 - 16\sqrt{2}$
$2\sqrt{10}$	s2[40, 2]	$24 - 16\sqrt{2}$
$\sqrt{41}$	s2[41, 2]	$24 - 16\sqrt{2}$
$\sqrt{42}$	s2[42, 2]	$24 - 16\sqrt{2}$
$\sqrt{43}$	s2[43, 2]	$24 - 16\sqrt{2}$
$2\sqrt{11}$	s2[44, 2]	$24 - 16\sqrt{2}$
$3\sqrt{5}$	s2[45, 2]	$24 - 16\sqrt{2}$
$\sqrt{46}$	s2[46, 2]	$24 - 16\sqrt{2}$
$\sqrt{47}$	s2[47, 2]	$24 - 16\sqrt{2}$
$4\sqrt{3}$	s2[48, 2]	$24 - 16\sqrt{2}$
7	s2[49, 2]	$26 - 16\sqrt{2}$
$5\sqrt{2}$	s2[50, 2]	$26 - 20\sqrt{2}$
$\sqrt{51}$	s2[51, 2]	$26 - 20\sqrt{2}$
$2\sqrt{13}$	s2[52, 2]	$26 - 20\sqrt{2}$
$\sqrt{53}$	s2[53, 2]	$26 - 20\sqrt{2}$
$3\sqrt{6}$	s2[54, 2]	$26 - 20\sqrt{2}$
$\sqrt{55}$	s2[55, 2]	$26 - 20\sqrt{2}$
$2\sqrt{14}$	s2[56, 2]	$26 - 20\sqrt{2}$
$\sqrt{57}$	s2[57, 2]	$26 - 20\sqrt{2}$
$\sqrt{58}$	s2[58, 2]	$26 - 20\sqrt{2}$
$\sqrt{59}$	s2[59, 2]	$26 - 20\sqrt{2}$
$2\sqrt{15}$	s2[60, 2]	$26 - 20\sqrt{2}$
$\sqrt{61}$	s2[61, 2]	$26 - 20\sqrt{2}$
$\sqrt{62}$	s2[62, 2]	$26 - 20\sqrt{2}$
$3\sqrt{7}$	s2[63, 2]	$26 - 20\sqrt{2}$
8	s2[64, 2]	$36 - 20\sqrt{2}$
$\sqrt{65}$	s2[65, 2]	$36 - 20\sqrt{2}$
$\sqrt{66}$	s2[66, 2]	$36 - 20\sqrt{2}$
$\sqrt{67}$	s2[67, 2]	$36 - 20\sqrt{2}$
$2\sqrt{17}$	s2[68, 2]	$36 - 20\sqrt{2}$
$\sqrt{69}$	s2[69, 2]	$36 - 20\sqrt{2}$
$\sqrt{70}$	s2[70, 2]	$36 - 20\sqrt{2}$
$\sqrt{71}$	s2[71, 2]	$36 - 20\sqrt{2}$
$6\sqrt{2}$	s2[72, 2]	$36 - 28\sqrt{2}$
$\sqrt{73}$	s2[73, 2]	$36 - 28\sqrt{2}$

$\sqrt{74}$	s2[74, 2]	$36 - 28\sqrt{2}$
$5\sqrt{3}$	s2[75, 2]	$36 - 28\sqrt{2}$
$2\sqrt{19}$	s2[76, 2]	$36 - 28\sqrt{2}$
$\sqrt{77}$	s2[77, 2]	$36 - 28\sqrt{2}$
$\sqrt{78}$	s2[78, 2]	$36 - 28\sqrt{2}$
$\sqrt{79}$	s2[79, 2]	$36 - 28\sqrt{2}$
$4\sqrt{5}$	s2[80, 2]	$36 - 28\sqrt{2}$
9	s2[81, 2]	$39 - 28\sqrt{2}$
$\sqrt{82}$	s2[82, 2]	$39 - 28\sqrt{2}$
$\sqrt{83}$	s2[83, 2]	$39 - 28\sqrt{2}$
$2\sqrt{21}$	s2[84, 2]	$39 - 28\sqrt{2}$
$\sqrt{85}$	s2[85, 2]	$39 - 28\sqrt{2}$
$\sqrt{86}$	s2[86, 2]	$39 - 28\sqrt{2}$
$\sqrt{87}$	s2[87, 2]	$39 - 28\sqrt{2}$
$2\sqrt{22}$	s2[88, 2]	$39 - 28\sqrt{2}$
$\sqrt{89}$	s2[89, 2]	$39 - 28\sqrt{2}$
$3\sqrt{10}$	s2[90, 2]	$39 - 28\sqrt{2}$
$\sqrt{91}$	s2[91, 2]	$39 - 28\sqrt{2}$
$2\sqrt{23}$	s2[92, 2]	$39 - 28\sqrt{2}$
$\sqrt{93}$	s2[93, 2]	$39 - 28\sqrt{2}$
$\sqrt{94}$	s2[94, 2]	$39 - 28\sqrt{2}$
$\sqrt{95}$	s2[95, 2]	$39 - 28\sqrt{2}$
$4\sqrt{6}$	s2[96, 2]	$39 - 28\sqrt{2}$
$\sqrt{97}$	s2[97, 2]	$39 - 28\sqrt{2}$
$7\sqrt{2}$	s2[98, 2]	$39 - 32\sqrt{2}$
$3\sqrt{11}$	s2[99, 2]	$39 - 32\sqrt{2}$
10	s2[100, 2]	$47 - 32\sqrt{2}$

```
Table[{n, ta[n, 2^(1/2)]}, {n, 1, 100}] // TableForm
```

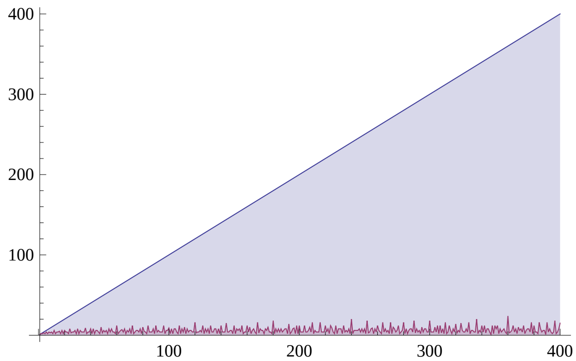
1	1
2	$5 - 2\sqrt{2}$
3	$7 - 6\sqrt{2}$
4	$14 - 6\sqrt{2}$
5	$16 - 10\sqrt{2}$
6	$24 - 16\sqrt{2}$
7	$26 - 16\sqrt{2}$
8	$36 - 20\sqrt{2}$
9	$39 - 28\sqrt{2}$
10	$47 - 32\sqrt{2}$
11	$49 - 32\sqrt{2}$
12	$63 - 40\sqrt{2}$
13	$65 - 46\sqrt{2}$

14	$73 - 46\sqrt{2}$
15	$77 - 54\sqrt{2}$
16	$90 - 58\sqrt{2}$
17	$92 - 70\sqrt{2}$
18	$104 - 70\sqrt{2}$
19	$106 - 74\sqrt{2}$
20	$120 - 82\sqrt{2}$
21	$124 - 82\sqrt{2}$
22	$132 - 90\sqrt{2}$
23	$134 - 100\sqrt{2}$
24	$154 - 100\sqrt{2}$
25	$157 - 104\sqrt{2}$
26	$165 - 116\sqrt{2}$
27	$169 - 120\sqrt{2}$
28	$183 - 120\sqrt{2}$
29	$185 - 132\sqrt{2}$
30	$201 - 140\sqrt{2}$
31	$203 - 140\sqrt{2}$
32	$219 - 148\sqrt{2}$
33	$223 - 152\sqrt{2}$
34	$231 - 168\sqrt{2}$
35	$235 - 168\sqrt{2}$
36	$256 - 174\sqrt{2}$
37	$258 - 182\sqrt{2}$
38	$266 - 182\sqrt{2}$
39	$270 - 190\sqrt{2}$
40	$290 - 202\sqrt{2}$
41	$292 - 202\sqrt{2}$
42	$308 - 206\sqrt{2}$
43	$310 - 222\sqrt{2}$
44	$324 - 226\sqrt{2}$
45	$330 - 226\sqrt{2}$
46	$338 - 238\sqrt{2}$
47	$340 - 246\sqrt{2}$
48	$366 - 246\sqrt{2}$
49	$369 - 254\sqrt{2}$
50	$381 - 262\sqrt{2}$
51	$385 - 280\sqrt{2}$
52	$399 - 280\sqrt{2}$
53	$401 - 284\sqrt{2}$
54	$417 - 292\sqrt{2}$

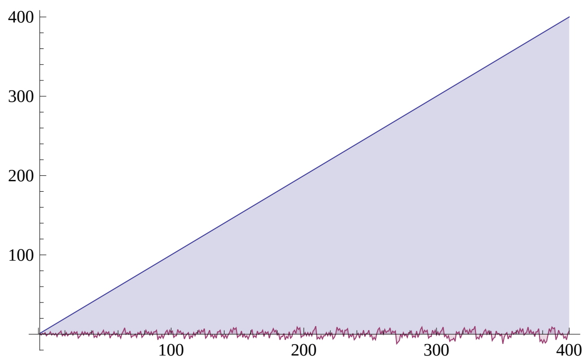
55	$421 - 292\sqrt{2}$
56	$441 - 300\sqrt{2}$
57	$445 - 316\sqrt{2}$
58	$453 - 320\sqrt{2}$
59	$455 - 320\sqrt{2}$
60	$483 - 336\sqrt{2}$
61	$485 - 340\sqrt{2}$
62	$493 - 340\sqrt{2}$
63	$499 - 352\sqrt{2}$
64	$518 - 364\sqrt{2}$
65	$522 - 364\sqrt{2}$
66	$538 - 372\sqrt{2}$
67	$540 - 376\sqrt{2}$
68	$554 - 396\sqrt{2}$
69	$558 - 396\sqrt{2}$
70	$574 - 402\sqrt{2}$
71	$576 - 414\sqrt{2}$
72	$606 - 414\sqrt{2}$
73	$608 - 422\sqrt{2}$
74	$616 - 434\sqrt{2}$
75	$622 - 438\sqrt{2}$
76	$636 - 438\sqrt{2}$
77	$640 - 454\sqrt{2}$
78	$656 - 462\sqrt{2}$
79	$658 - 462\sqrt{2}$
80	$684 - 478\sqrt{2}$
81	$689 - 486\sqrt{2}$
82	$697 - 486\sqrt{2}$
83	$699 - 494\sqrt{2}$
84	$727 - 498\sqrt{2}$
85	$731 - 522\sqrt{2}$
86	$739 - 522\sqrt{2}$
87	$743 - 526\sqrt{2}$
88	$763 - 534\sqrt{2}$
89	$765 - 534\sqrt{2}$
90	$789 - 546\sqrt{2}$
91	$793 - 560\sqrt{2}$
92	$807 - 568\sqrt{2}$
93	$811 - 568\sqrt{2}$
94	$819 - 584\sqrt{2}$
95	$823 - 588\sqrt{2}$

96	$855 - 588\sqrt{2}$
97	$857 - 600\sqrt{2}$
98	$869 - 608\sqrt{2}$
99	$875 - 624\sqrt{2}$
100	$896 - 624\sqrt{2}$

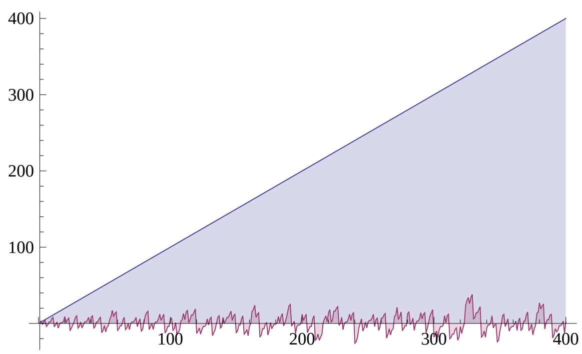
```
DiscretePlot[{n, ta[n, 1.00000001]}, {n, 1, 400}] // TableForm
```



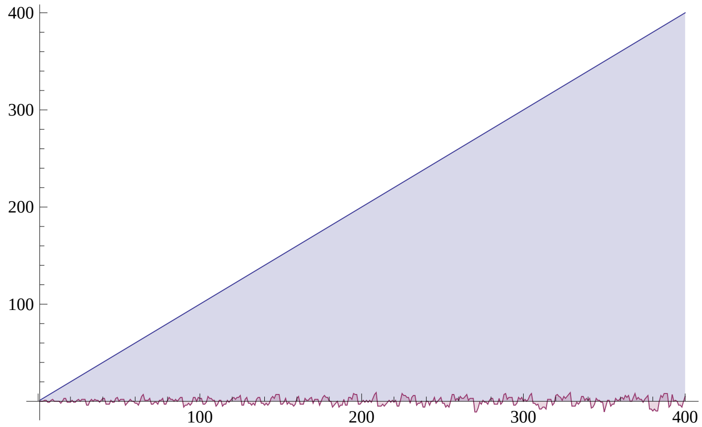

```
DiscretePlot[{n, ta[n, 2]}, {n, 1, 400}] // TableForm
```

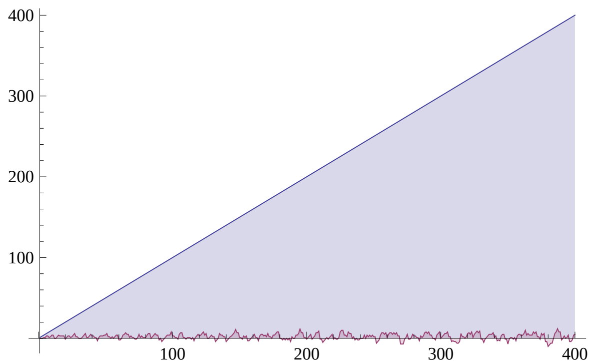



```
DiscretePlot[{n, ta[n, 3]}, {n, 1, 400}] // TableForm
```

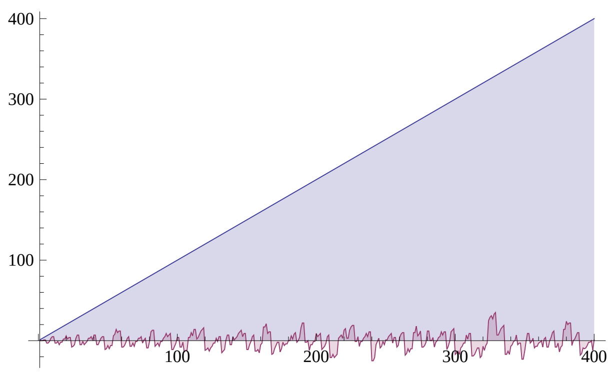


```
DiscretePlot[{n, t2a[n, 2]}, {n, 1, 400}] // TableForm
```

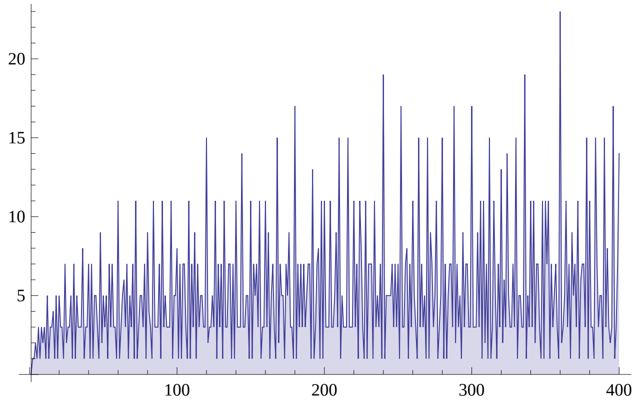




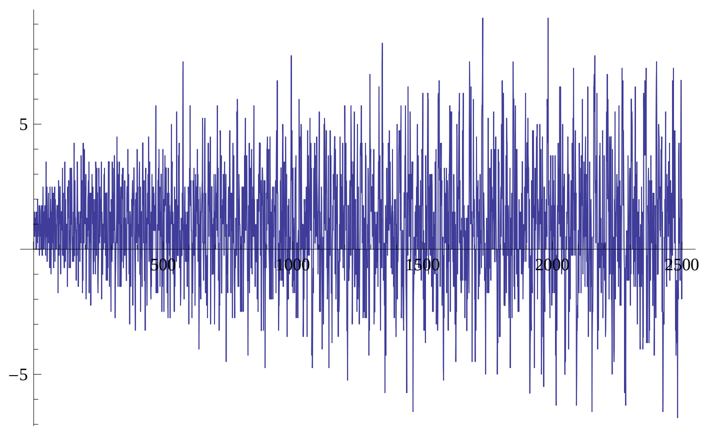

```
DiscretePlot[{n, t2a[n, 3]}, {n, 1, 400}] // TableForm
```



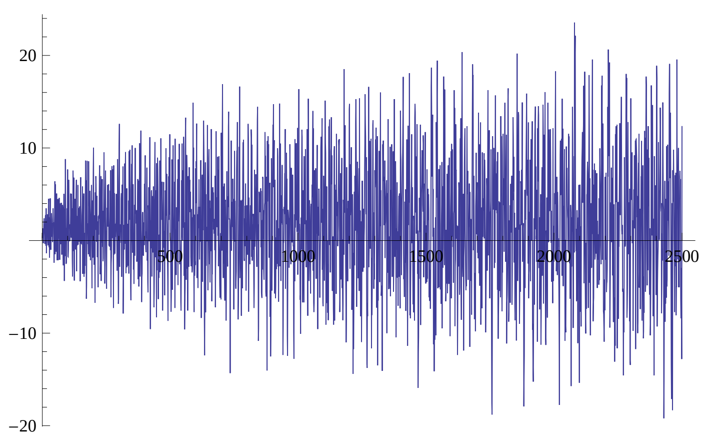
```
DiscretePlot[ {t2a[n, 1.00000001]}, {n, 1, 400}] // TableForm
```



```
DiscretePlot[ {t2a[n, .5] - n / 2}, {n, 1, 2500}] // TableForm
```



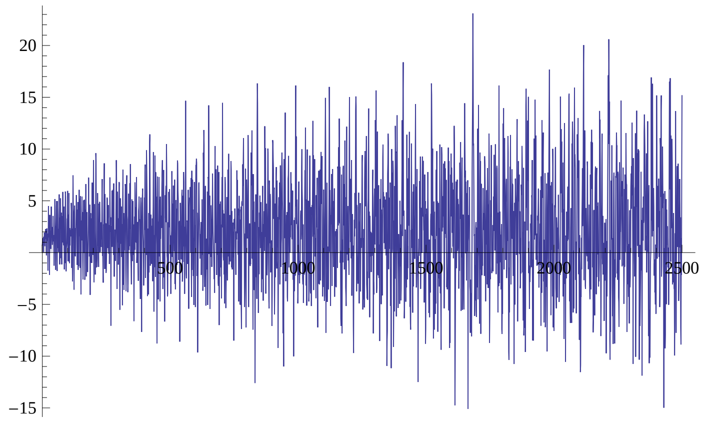
```
DiscretePlot[ {t2a[n, .6] - n * .4}, {n, 1, 2500}] // TableForm
```



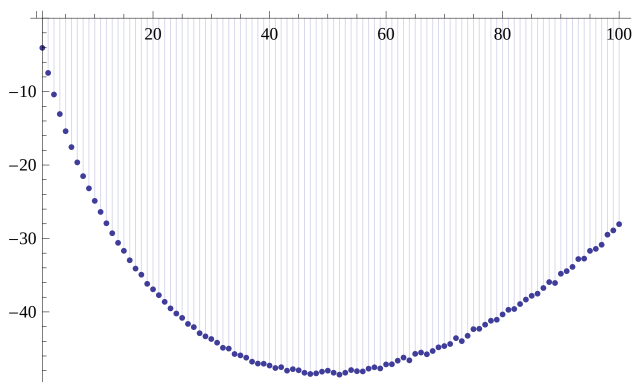
```
N[1750 / 2500]
```

0.7

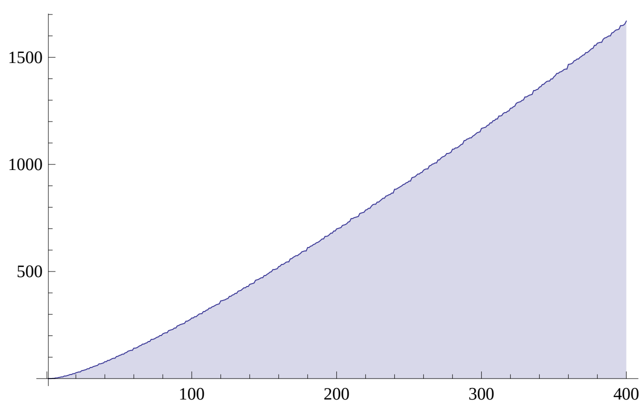
```
DiscretePlot[ {t2a[n, .4] - n * .7}, {n, 1, 2500}] // TableForm
```



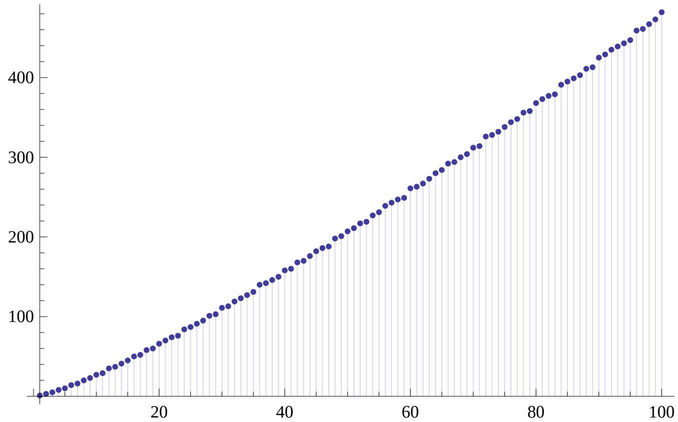

```
DiscretePlot[ {d2a[n] - t2a[n, 1 / 32]}, {n, 1, 100}] // TableForm
```



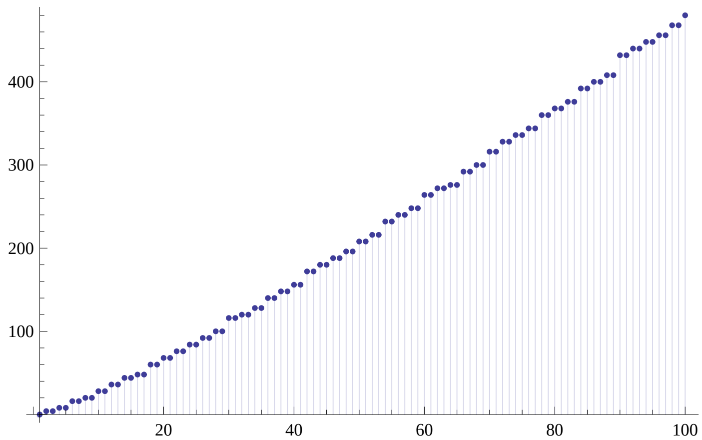

```
DiscretePlot[ {d2a[n]}, {n, 1, 400}] // TableForm
```



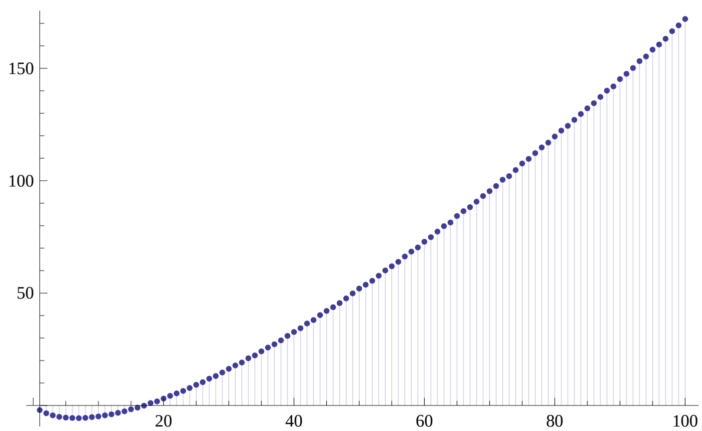
```
DiscretePlot[ {d1a[n] - ta[n, 1]}, {n, 1, 100}] // TableForm
```



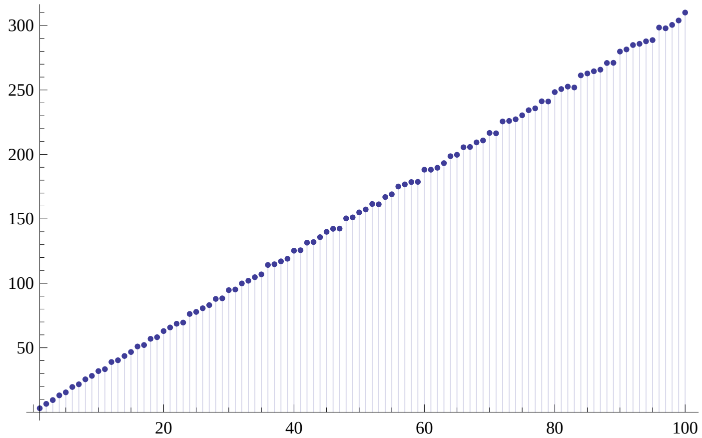

```
DiscretePlot[ {d1a[n] - ta[n, 2]}, {n, 1, 100}] // TableForm
```




```
DiscretePlot[ {d1a[n] - ta[n, 1 / 32]}, {n, 1, 100}] // TableForm
```

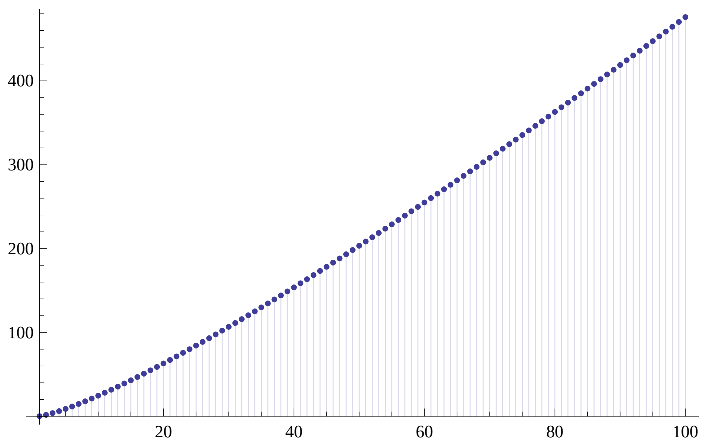


```
DiscretePlot[ {ta[n, 1 / 32]}, {n, 1, 100}] // TableForm
```

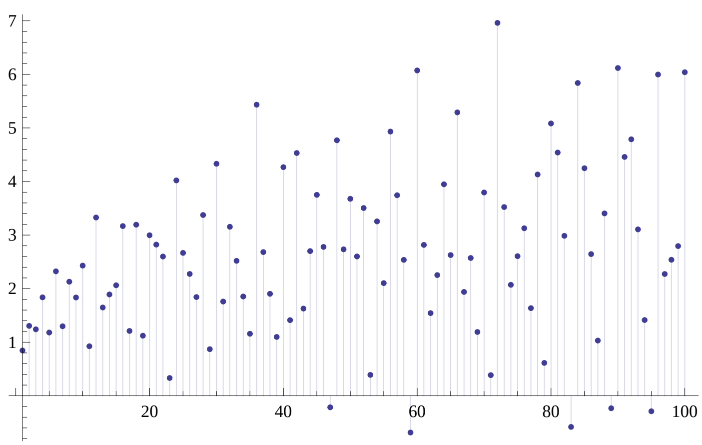


```
bs[n_] := n Log[n] + n (2 EulerGamma - 1)
```

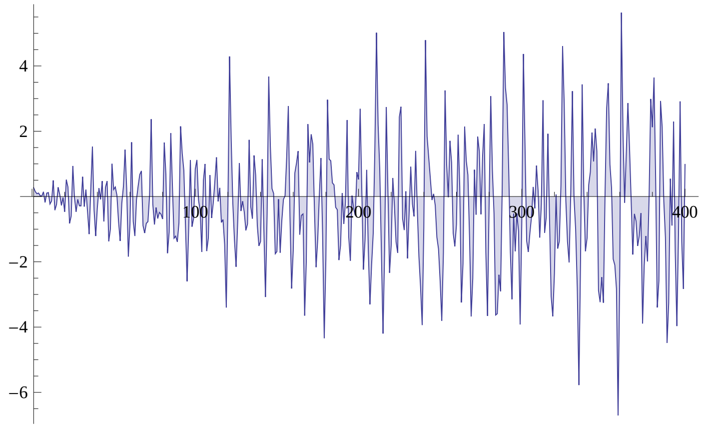
```
DiscretePlot[ bs[n], {n, 1, 100}]
```



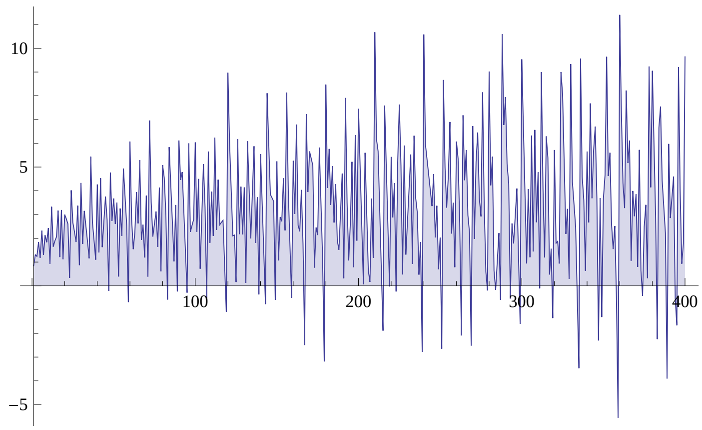
```
DiscretePlot[ {d1a[n] - ta[n, 1] - bs[n]}, {n, 1, 100}] // TableForm
```



```
DiscretePlot[{d1a[n] - EulerGamma ta[n, 1 + .0000000001] - bs[n]}, {n, 1, 400}] // TableForm
```




```
DiscretePlot[{d1a[n] - bs[n]}, {n, 1, 400}] // TableForm
```



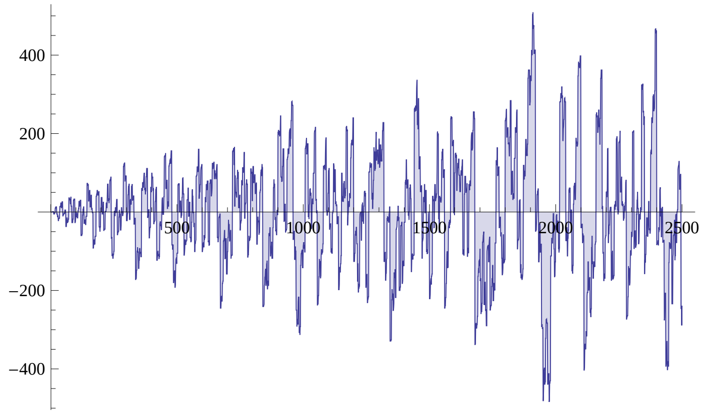

```
DiscretePlot[{ta[n, 1 / 64]}, {n, 1, 40}] // TableForm
DiscretePlot[{ta[n, 1 / 128]}, {n, 1, 40}] // TableForm
DiscretePlot[{ta[n, 1 / 256]}, {n, 1, 40}] // TableForm
```

```
Animate[DiscretePlot[{600, -100, ta[n, Sin[s] * 49 + 50]}, {n, 1, 100}], {s, 0, 2 Pi}]
```

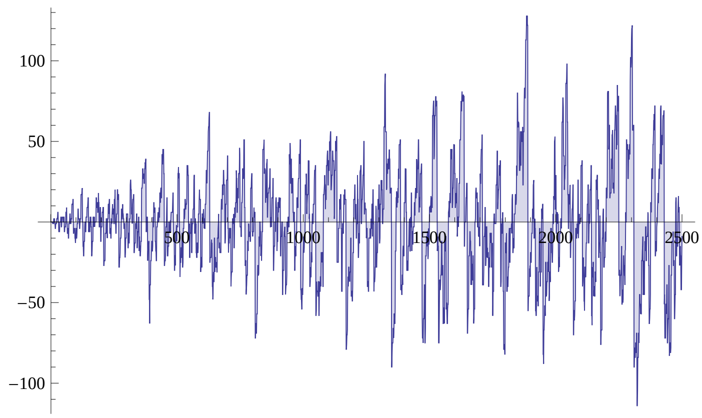
```
Sum[.5^k / k, {k, 1, Log[.5, 10]}]
```

0

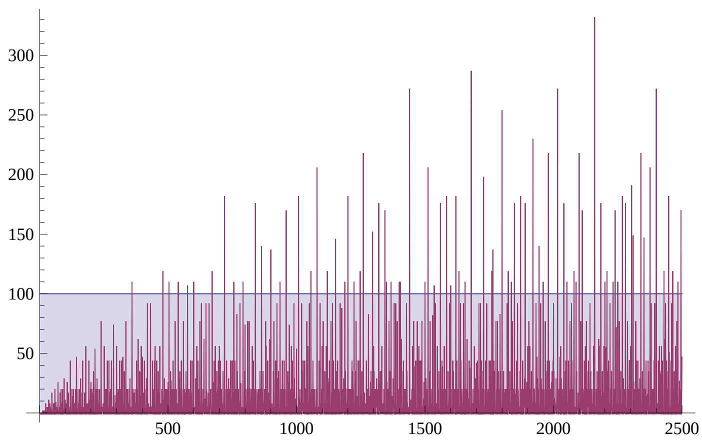
```
DiscretePlot[{t3a[n, 3]}, {n, 1, 2500}]
```



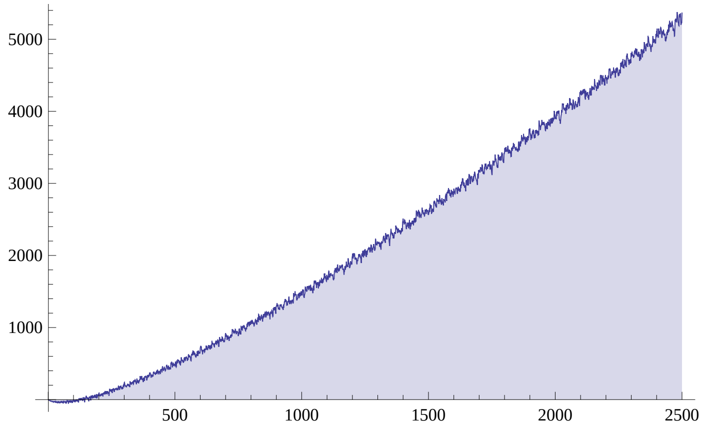

```
DiscretePlot[{t3a[n, 2]}, {n, 1, 2500}]
```



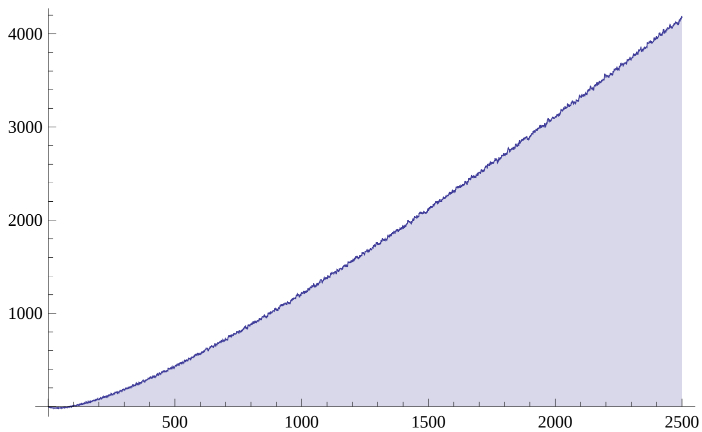

```
DiscretePlot[{100, t3a[n, 1.0000000001]}, {n, 1, 2500}]
```



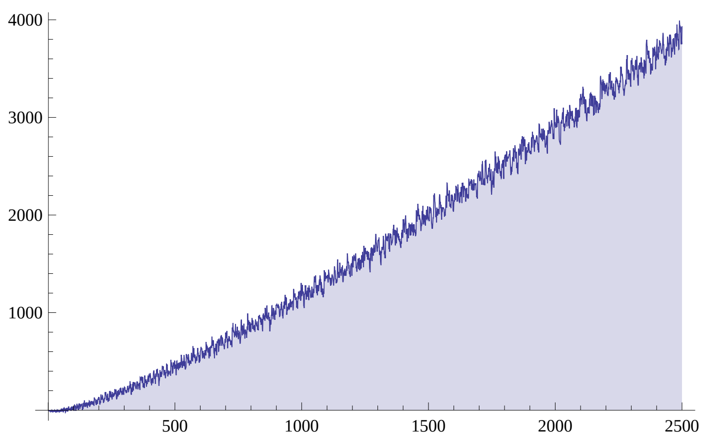
```
DiscretePlot[{t3a[n, .4]}, {n, 1, 2500}]
```



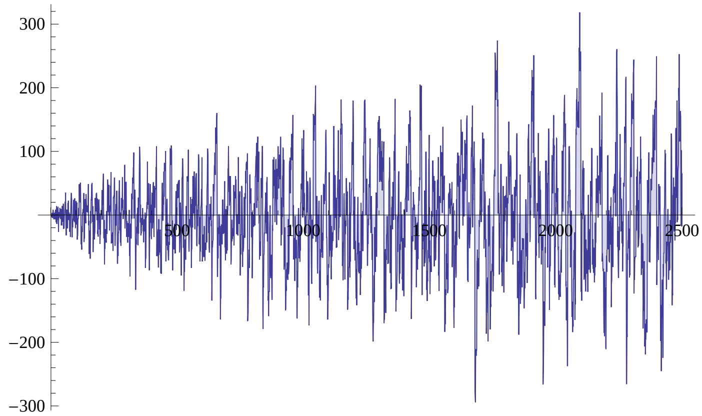

```
DiscretePlot[{t3a[n, .5]}, {n, 1, 2500}]
```



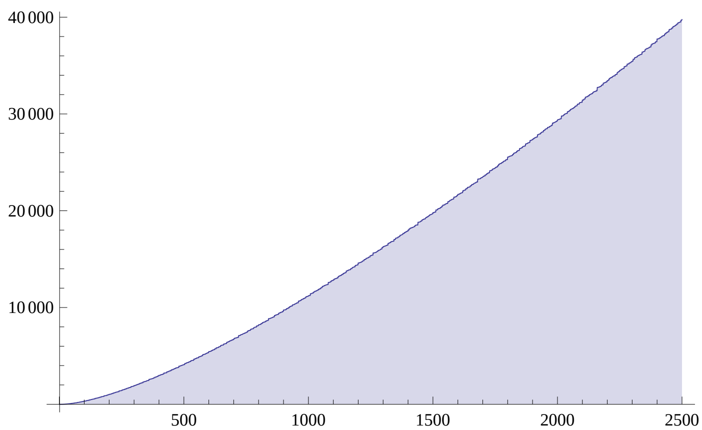

```
DiscretePlot[{t3a[n, .6]}, {n, 1, 2500}]
```



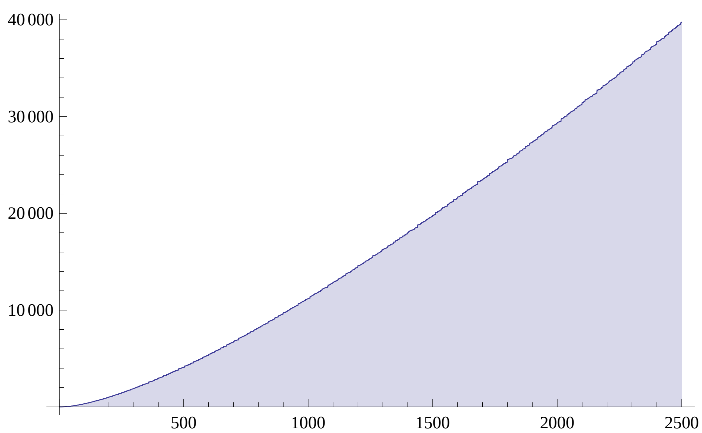
```
DiscretePlot[{t3a[n, 1.5]}, {n, 1, 2500}]
```




```
DiscretePlot[{t3a[n, 999]}, {n, 1, 2500}]
```




```
DiscretePlot[{d32a[n]}, {n, 1, 2500}]
```




```
Sum[1.000001^k / k, {k, 1, Log[1.000001, 100]}] -
 Sum[1.000001^k / k, {k, 1, Log[1.000001, 1.45136923488338105]}]
```

$30.1262 - 2.46016 \times 10^{-10}\, i$

```
N[LogIntegral[100]]
```

30.1261

```
Table[{n, t3a[n, 1.0000000001], d32a[n] - d32a[n - 1]}, {n, 1, 100}] // TableForm
```

1	0	0
2	−1.	0
3	−1.	0
4	−1.	0
5	−1.	0
6	−1.	0
7	−1.	0
8	-2.09999×10^{-9}	1
9	−1.	0
10	−1.	0
11	−1.	0
12	2.	3
13	−1.	0
14	−1.	0
15	−1.	0
16	2.	3
17	−1.	0
18	2.	3
19	−1.	0
20	2.	3
21	−1.	0
22	−1.	0
23	−1.	0
24	8.	9
25	−1.	0
26	−1.	0
27	-7.7998×10^{-9}	1
28	2.	3
29	−1.	0
30	5.	6
31	−1.	0
32	5.	6
33	−1.	0
34	−1.	0
35	−1.	0
36	11.	12
37	−1.	0
38	−1.	0
39	−1.	0
40	8.	9
41	−1.	0
42	5.	6
43	−1.	0
44	2.	3
45	2.	3
46	−1.	0
47	−1.	0
48	17.	18
49	−1.	0
50	2.	3
51	−1.	0
52	2.	3
53	−1.	0
54	8.	9
55	−1.	0
56	8.	9

57	-1.	0
58	-1.	0
59	-1.	0
60	20.	21
61	-1.	0
62	-1.	0
63	2.	3
64	9.	10
65	-1.	0
66	5.	6
67	-1.	0
68	2.	3
69	-1.	0
70	5.	6
71	-1.	0
72	26.	27
73	-1.	0
74	-1.	0
75	2.	3
76	2.	3
77	-1.	0
78	5.	6
79	-1.	0
80	17.	18
81	2.	3
82	-1.	0
83	-1.	0
84	20.	21
85	-1.	0
86	-1.	0
87	-1.	0
88	8.	9
89	-1.	0
90	20.	21
91	-1.	0
92	2.	3
93	-1.	0
94	-1.	0
95	-1.	0
96	29.	30
97	-1.	0
98	2.	3
99	2.	3
100	11.	12

```
Table[{n, t2a[n, 1.0000000001], d2a[n] - d2a[n - 1]}, {n, 1, 100}] // TableForm
```

1	0	0
2	1.	0
3	1.	0
4	2.	1
5	1.	0
6	3.	2
7	1.	0
8	3.	2
9	2.	1

10	3.	2
11	1.	0
12	5.	4
13	1.	0
14	3.	2
15	3.	2
16	4.	3
17	1.	0
18	5.	4
19	1.	0
20	5.	4
21	3.	2
22	3.	2
23	1.	0
24	7.	6
25	2.	1
26	3.	2
27	3.	2
28	5.	4
29	1.	0
30	7.	6
31	1.	0
32	5.	4
33	3.	2
34	3.	2
35	3.	2
36	8.	7
37	1.	0
38	3.	2
39	3.	2
40	7.	6
41	1.	0
42	7.	6
43	1.	0
44	5.	4
45	5.	4
46	3.	2
47	1.	0
48	9.	8
49	2.	1
50	5.	4
51	3.	2
52	5.	4
53	1.	0
54	7.	6
55	3.	2
56	7.	6
57	3.	2
58	3.	2
59	1.	0
60	11.	10
61	1.	0
62	3.	2
63	5.	4
64	6.	5
65	3.	2

66	7.	6
67	1.	0
68	5.	4
69	3.	2
70	7.	6
71	1.	0
72	11.	10
73	1.	0
74	3.	2
75	5.	4
76	5.	4
77	3.	2
78	7.	6
79	1.	0
80	9.	8
81	4.	3
82	3.	2
83	1.	0
84	11.	10
85	3.	2
86	3.	2
87	3.	2
88	7.	6
89	1.	0
90	11.	10
91	3.	2
92	5.	4
93	3.	2
94	3.	2
95	3.	2
96	11.	10
97	1.	0
98	5.	4
99	5.	4
100	8.	7

```
t2a[100, 2^(1/2)]
```

$499 - 346\sqrt{2}$

```
FullSimplify[tk1[100, 3, 2^(1/2)]]
```

$4855 - 3321\sqrt{2}$

```
FullSimplify[En[100^2, 3]]
```

$4855 - 3321\sqrt{2}$

```
FullSimplify[ta[100, 2^(1/2)]]
```

$896 - 624\sqrt{2}$

```
FullSimplify[Lina[100, 2^(1/2)]] + LAdda[100, 2^(1/2)]
```

$$\frac{428}{15}$$

```
$Aborted
```

```
Lin[10 000] + LAdd[10 000]
```

$$\frac{428}{15}$$

```
Table[ {n, a = FullSimplify[En2[10 000, n]],
   b = FullSimplify[tk2[100, n, 2^(1 / 2)]], a - b}, {n, 1, 12}] // TableForm
```

```
$Aborted
```

```
FullSimplify[tk[100, 1, 2^(1 / 2)]]
```

$99 - 69\sqrt{2}$

```
Table[{n, FullSimplify[En2[n^2, 3]], a = FullSimplify[tk[n, 3, 2^(1/2)]],
   b = FullSimplify[tk2[n, 3, 2^(1/2)]], a - b}, {n, 1, 30}] // TableForm
```

1	0	0	0	0
2	0	0	0	0
3	$-2\sqrt{2}$	$-2\sqrt{2}$	$-2\sqrt{2}$	0
4	$6-2\sqrt{2}$	$6-2\sqrt{2}$	$6-2\sqrt{2}$	0
5	$6-2\sqrt{2}$	$6-2\sqrt{2}$	$6-2\sqrt{2}$	0
6	$12-11\sqrt{2}$	$12-11\sqrt{2}$	$12-11\sqrt{2}$	0
7	$12-11\sqrt{2}$	$12-11\sqrt{2}$	$12-11\sqrt{2}$	0
8	$31-11\sqrt{2}$	$31-11\sqrt{2}$	$31-11\sqrt{2}$	0
9	$31-23\sqrt{2}$	$31-23\sqrt{2}$	$31-23\sqrt{2}$	0
10	$37-23\sqrt{2}$	$37-23\sqrt{2}$	$37-23\sqrt{2}$	0
11	$37-23\sqrt{2}$	$37-23\sqrt{2}$	$37-23\sqrt{2}$	0
12	$70-44\sqrt{2}$	$70-44\sqrt{2}$	$70-44\sqrt{2}$	0
13	$70-47\sqrt{2}$	$70-47\sqrt{2}$	$70-47\sqrt{2}$	0
14	$76-47\sqrt{2}$	$76-47\sqrt{2}$	$76-47\sqrt{2}$	0
15	$76-59\sqrt{2}$	$76-59\sqrt{2}$	$76-59\sqrt{2}$	0
16	$115-59\sqrt{2}$	$115-59\sqrt{2}$	$115-59\sqrt{2}$	0
17	$115-98\sqrt{2}$	$115-98\sqrt{2}$	$115-98\sqrt{2}$	0
18	$136-98\sqrt{2}$	$136-98\sqrt{2}$	$136-98\sqrt{2}$	0
19	$136-98\sqrt{2}$	$136-98\sqrt{2}$	$136-98\sqrt{2}$	0
20	$169-110\sqrt{2}$	$169-110\sqrt{2}$	$169-110\sqrt{2}$	0
21	$169-110\sqrt{2}$	$169-110\sqrt{2}$	$169-110\sqrt{2}$	0
22	$175-116\sqrt{2}$	$175-116\sqrt{2}$	$175-116\sqrt{2}$	0
23	$175-154\sqrt{2}$	$175-154\sqrt{2}$	$175-154\sqrt{2}$	0
24	$256-154\sqrt{2}$	$256-154\sqrt{2}$	$256-154\sqrt{2}$	0
25	$256-154\sqrt{2}$	$256-154\sqrt{2}$	$256-154\sqrt{2}$	0
26	$262-187\sqrt{2}$	$262-187\sqrt{2}$	$262-187\sqrt{2}$	0
27	$263-187\sqrt{2}$	$263-187\sqrt{2}$	$263-187\sqrt{2}$	0
28	$296-187\sqrt{2}$	$296-187\sqrt{2}$	$296-187\sqrt{2}$	0
29	$296-226\sqrt{2}$	$296-226\sqrt{2}$	$296-226\sqrt{2}$	0
30	$332-232\sqrt{2}$	$332-232\sqrt{2}$	$332-232\sqrt{2}$	0

```
Table[{k, FullSimplify[tk[100, k, 2^(1/2)]]}, {k, 1, 15}] // TableForm
```

1	$99 - 70\sqrt{2}$
2	$697 - 484\sqrt{2}$
3	$2466 - 1659\sqrt{2}$
4	$5780 - 3652\sqrt{2}$
5	$9971 - 5844\sqrt{2}$
6	$13393 - 7376\sqrt{2}$
7	$14140 - 7743\sqrt{2}$
8	$11464 - 6832\sqrt{2}$
9	$6864 - 4984\sqrt{2}$
10	$48\left(59 - 60\sqrt{2}\right)$
11	$704 - 1264\sqrt{2}$
12	$64 - 384\sqrt{2}$
13	$-64\sqrt{2}$
14	0
15	0

```
Table[{k, FullSimplify[tk[100, k, 2]]}, {k, 1, Log[2, 100^2]}] // TableForm
```

1	−1
2	3
3	−4
4	−8
5	9
6	−5
7	0
8	0
9	0
10	0
11	0
12	0
13	0

```
Table[{k, FullSimplify[tk[100, k, 2^(1/3)]]}, {k, 1, Log[2^(1/3), 100] + 1}] // TableForm
```

1	$99 - 79 \times 2^{1/3}$
2	$283 - 558 \times 2^{1/3} + 267 \times 2^{2/3}$
3	$3\left(-268 - 402 \times 2^{1/3} + 483 \times 2^{2/3}\right)$
4	$-5288 + 436 \times 2^{1/3} + 2802 \times 2^{2/3}$
5	$-9849 + 6340 \times 2^{1/3} + 468 \times 2^{2/3}$
6	$-3673 + 10\,878 \times 2^{1/3} - 6915 \times 2^{2/3}$
7	$15\,386 + 4133 \times 2^{1/3} - 10\,878 \times 2^{2/3}$
8	$-4\left(-6692 + 2727 \times 2^{1/3} + 811 \times 2^{2/3}\right)$
9	$4\left(2632 - 4041 \times 2^{1/3} + 1971 \times 2^{2/3}\right)$
10	$8\left(-2105 - 549 \times 2^{1/3} + 1080 \times 2^{2/3}\right)$
11	$16\left(-1320 + 539 \times 2^{1/3} + 49 \times 2^{2/3}\right)$
12	$-16\left(181 - 528 \times 2^{1/3} + 234 \times 2^{2/3}\right)$
13	$16\left(533 + 25 \times 2^{1/3} - 117 \times 2^{2/3}\right)$
14	$16\left(273 - 224 \times 2^{1/3} + 29 \times 2^{2/3}\right)$
15	$16\left(-62 - 105 \times 2^{1/3} + 30 \times 2^{2/3}\right)$
16	$32\left(-32 + 17 \times 2^{1/3}\right)$
17	$-32\, 2^{1/3}\left(-17 + 2^{1/3}\right)$
18	64
19	$-64\, 2^{1/3}$
20	0

```
Table[{k, FullSimplify[tk[100, k, 2^(1/4)]]}, {k, 1, Log[2^(1/4), 100] + 1}] // TableForm
```

1	$99 - 84 \times 2^{1/4}$
2	$283 - 614 \times 2^{1/4} + 312\sqrt{2}$
3	$324 - 1401 \times 2^{1/4} + 1731\sqrt{2} - 684 \times 2^{3/4}$
4	$-4\left(-670 + 365 \times 2^{1/4} - 843\sqrt{2} + 836 \times 2^{3/4}\right)$
5	$11841 - 4344 \times 2^{1/4} + 3030\sqrt{2} - 6060 \times 2^{3/4}$
6	$21307 - 15894 \times 2^{1/4} + 5392\sqrt{2} - 5200 \times 2^{3/4}$
7	$18760 - 25627 \times 2^{1/4} + 16667\sqrt{2} - 6631 \times 2^{3/4}$
8	$-4\left(-4904 + 4760 \times 2^{1/4} - 6237\sqrt{2} + 4368 \times 2^{3/4}\right)$
9	$4\left(11394 - 3955 \times 2^{1/4} + 4494\sqrt{2} - 5994 \times 2^{3/4}\right)$
10	$4\left(15930 - 8456 \times 2^{1/4} + 3641\sqrt{2} - 3660 \times 2^{3/4}\right)$
11	$40920 - 40040 \times 2^{1/4} + 29128\sqrt{2} - 9596 \times 2^{3/4}$
12	$-4\left(-6085 + 4840 \times 2^{1/4} - 8844\sqrt{2} + 4644 \times 2^{3/4}\right)$
13	$4\left(10894 - 2771 \times 2^{1/4} + 4862\sqrt{2} - 4524 \times 2^{3/4}\right)$
14	$43680 - 21392 \times 2^{1/4} + 11348\sqrt{2} - 5824 \times 2^{3/4}$
15	$-56\left(-260 + 315 \times 2^{1/4} - 360\sqrt{2} + 58 \times 2^{3/4}\right)$
16	$16\left(441 - 280 \times 2^{1/4} + 1140\sqrt{2} - 408 \times 2^{3/4}\right)$
17	$-16\left(-901 + 205 \times 2^{1/4} - 340\sqrt{2} + 204 \times 2^{3/4}\right)$
18	$16\left(459 - 378 \times 2^{1/4} + 208\sqrt{2}\right)$
19	$-48\, 2^{1/4}\left(57 - 133 \times 2^{1/4} + 13\sqrt{2}\right)$
20	$32\left(41 + 95\sqrt{2} - 20 \times 2^{3/4}\right)$
21	$64\left(21 - 11 \times 2^{1/4}\right)$
22	$32 \times 2^{1/4}\left(-22 + 23 \times 2^{1/4}\right)$
23	$-32\sqrt{2}\left(-23 + 2^{1/4}\right)$
24	64
25	$-64\, 2^{1/4}$
26	$64\sqrt{2}$
27	0

```
Lina[n_, a_] := FullSimplify[Sum[(-1)^(k+1) / k tk[n, k, a], {k, 1, Log[a, n]}]]
LAdda[n_, a_] := Sum[a^k / k, {k, 1, Log[a, n]}]
Lina[100, 2^(1/3)] + LAdda[100, 2^(1/3)]
```

$$\frac{428}{15}$$

```
Lina[100, 2^(1/4)] + LAdda[100, 2^(1/4)]
```

$$\frac{428}{15}$$

```
Lina[100, 2^(1/5)] + LAdda[100, 2^(1/5)]
```

$$\frac{428}{15}$$

```
Lina[100, 2^(1/6)] + LAdda[100, 2^(1/6)]
```

$\frac{428}{15}$

```
Lina[100, 2^(1/7)] + LAdda[100, 2^(1/7)]
```

$\frac{428}{15}$

```
N[LAdda[100, 2^(1/7)]]
```

32.3667

```
N[Lina[100, 2^(1/7)]]
```

-3.83334

```
Lina[100, 2^(1/8)] + LAdda[100, 2^(1/8)]
```

$\frac{428}{15}$

```
N[2^(1/8)]
```

1.09051

```
Table[{k, FullSimplify[tk[100, k, 2^(1/8)]]}, {k, 1, Log[2^(1/8), 100] + 1}] // TableForm
```

$Aborted

```
Lina[100, 1.1] + LAdda[100, 1.1]
```

28.5333

```
Lina[100, 1.05] + LAdda[100, 1.05]
```

28.5333

```
$RecursionLimit = 1000000
```

1 000 000

```
Lina[100, 1.01] + LAdda[100, 1.01]
```

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

General::stop : Further output of $RecursionLimit::reclim will be suppressed during this calculation. ≫

$Aborted

```
N[2^(1/4)]
```

1.18921

```
Table[{k, N[(-1)^(k+1)/k tk[100, k, 1.000000001]]}, {k, 1, 60}] // TableForm
```

1	-9.9×10^{-8}
2	-4.
3	3.66667
4	-1.75
5	-0.2

6	−0.166667
7	−0.142857
8	−0.125
9	−0.111111
10	−0.1
11	−0.0909092
12	−0.0833334
13	−0.0769232
14	−0.0714287
15	−0.0666668
16	−0.0625001
17	−0.0588236
18	−0.0555557
19	−0.0526317
20	−0.0500001
21	−0.0476191
22	−0.0454546
23	−0.0434784
24	−0.0416668
25	−0.0400001
26	−0.0384616
27	−0.0370371
28	−0.0357144
29	−0.0344829
30	−0.0333334
31	−0.0322582
32	−0.0312501
33	−0.0303031
34	−0.0294119
35	−0.0285715
36	−0.0277779
37	−0.0270271
38	−0.0263159
39	−0.0256411
40	−0.0250001
41	−0.0243903
42	−0.0238096
43	−0.0232559
44	−0.0227274
45	−0.0222223
46	−0.0217392
47	−0.0212767
48	−0.0208334
49	−0.0204083
50	−0.0200001
51	−0.0196079
52	−0.0192309
53	−0.018868
54	−0.0185186
55	−0.0181819
56	−0.0178572
57	−0.017544
58	−0.0172415
59	−0.0169493
60	−0.0166668

```
Table[{k, N[ (-1)^(k + 1) / k tk[100, k, 1.000000001] ] - (1.000000001^k) / k}, {k, 1, 60}] //
 TableForm
```

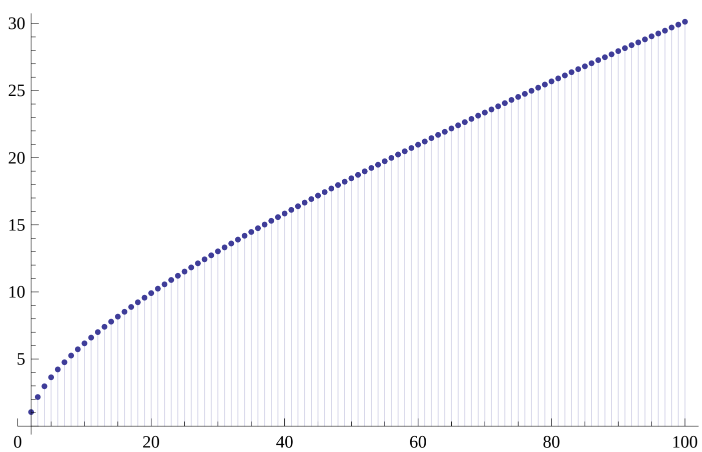
1	-1.
2	-4.5
3	3.33333
4	-2.
5	-0.4
6	-0.333333
7	-0.285714
8	-0.25
9	-0.222222
10	-0.2
11	-0.181818
12	-0.166667
13	-0.153846
14	-0.142857
15	-0.133333
16	-0.125
17	-0.117647
18	-0.111111
19	-0.105263
20	-0.1
21	-0.0952382
22	-0.0909092
23	-0.0869566
24	-0.0833334
25	-0.0800001
26	-0.0769232
27	-0.0740742
28	-0.0714287
29	-0.0689656
30	-0.0666668
31	-0.0645162
32	-0.0625001
33	-0.0606062
34	-0.0588236
35	-0.057143
36	-0.0555557
37	-0.0540542
38	-0.0526317
39	-0.0512822
40	-0.0500001
41	-0.0487806
42	-0.0476191
43	-0.0465117
44	-0.0454546
45	-0.0444445
46	-0.0434784
47	-0.0425533
48	-0.0416668
49	-0.0408164
50	-0.0400001

51	-0.0392158
52	-0.0384616
53	-0.0377359
54	-0.0370371
55	-0.0363637
56	-0.0357144
57	-0.0350878
58	-0.0344829
59	-0.0338984
60	-0.0333334

```
Table[{k, N[ (-1) ^ (k + 1) / k tk[10, k, 1 / 2]]}, {k, 1, 14}] // TableForm
```

1	-1.
2	-0.75
3	-0.916667
4	-1.15625
5	-1.675
6	-1.21875
7	1.23884
8	4.10986
9	1.26584
10	-12.8259
11	-31.5639
12	-23.5638
13	48.3149
14	163.709

```
DiscretePlot[LAdda[n, 1.001] - LAdda[1.45, 1.001], {n, 2, 100}]
```




```
Table[{k, N[ (-1) ^ (k + 1) / k tk[100, k, 2^ (1 / 8)]]},
  {k, 1, Log[2^ (1 / 8), 100] + 1}] // TableForm
```

1	-0.236204
2	-5.39838
3	24.1214
4	-62.4699
5	-42.037
6	548.661
7	-1325.21
8	1603.75
9	-280.997
10	-3187.62
11	6504.76
12	-5112.89
13	-792.106
14	6150.91
15	-9740.41
16	10 301.8
17	-2080.73
18	-10 566.4
19	12 884.3
20	-4667.05
21	-1368.73
22	5085.55
23	-10 691.7
24	9816.99
25	-304.917
26	-5405.04
27	2863.91
28	-107.805
29	-159.677
30	2208.43
31	-4094.19
32	1900.08
33	562.25
34	-537.039
35	10.8383
36	-8.99013
37	-9.87027
38	513.882
39	-474.455
40	-6.11174
41	28.2526
42	-4.06444
43	-4.40933
44	-4.78449
45	-5.1926
46	48.1808
47	-1.24869
48	-1.33333
49	-1.42434
50	-1.52219
51	-1.62741
52	-1.74057
53	-1.86229
54	0.

```
N[LAdda2[100, 1.000001]]
N[LAdda[100, 1.000001]]
```

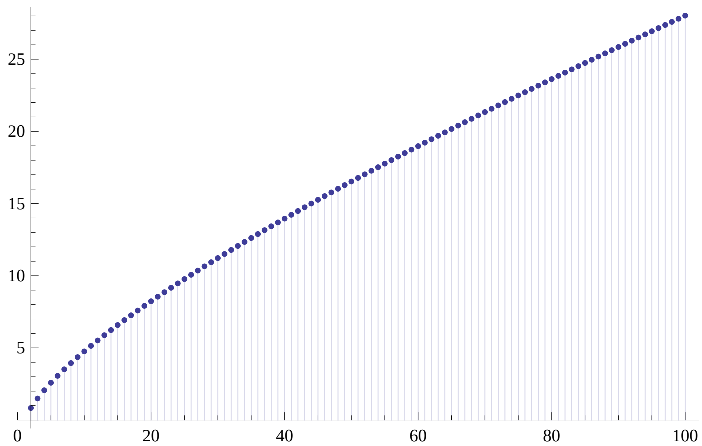
$28.0218 - 2.46016 \times 10^{-10}\, i$

$43.9417 - 2.46016 \times 10^{-10}\, i$

```
N[LogIntegral[100]]
```

30.1261

```
DiscretePlot[ LAdda2[n, 1.0001], {n, 2, 100}]
```



Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫