```
TestPrimePowerCount[ A_, n_] :=
 FullSimplify[ Sum[ MangoldtLambda[ j] / Log[j] j^A, {j, 2, n}]]
TestSumPrimes[A_, n_ ] :=
 Sum[ 1 / (j ) MoebiusMu[ j] TestPrimePowerCount[ j A, n^(1 / j]], {j, 1, Log[2, n]}]


StrictDivisors[A_, k_, n_] := Sum[ j^A StrictDivisors[A, k - 1, n / j], {j, 2, n}]
StrictDivisors[A_, 1, n_] := Sum[ j^A, {j, 2, n}]
SumPrimes[A_, n_ ] :=
 Sum[ (-1) ^ (k + 1) / (j k) MoebiusMu[ j] StrictDivisors[ j A, k, n^(1 / j)],
  {j, 1, Log[2, n]}, {k, 1, Log[2, (n^(1 / j))]}]


RecurseCount[ A_, k_, n_ ] := Sum[ j^A (1 / k - RecurseCount[ A, k + 1, n / j]), {j, 2, n}]
SumPrimesRecurse[A_, n_] :=
 Sum[ 1 / j MoebiusMu[ j] RecurseCount[ j A, 1, n^(1 / j)], {j, 1, Log[2, n]}]


StrictDivisorsHyperbola[ A_, k_, n_, s_] :=
 Sum[ ((m^A) ^ (k - j)) Binomial[ k, j] StrictDivisorsHyperbola[ A, j, n / (m^(k - j)), m + 1],
  {m, s, n^(1 / k)}, {j, 0, k - 1}]
StrictDivisorsHyperbola[A_, 1, n_, s_] := Sum[ j^A, { j, s, n}]
StrictDivisorsHyperbola[0, 1, n_, s_] := Floor[n] - s + 1
StrictDivisorsHyperbola[1, 1, n_, s_] := Floor[n] (Floor[n] + 1) / 2 - s (s - 1) / 2
StrictDivisorsHyperbola[2, 1, n_, s_] :=
 Floor[n] (Floor[n] + 1) (2 Floor[n] + 1) / 6 - (s - 1) s (2 s - 1) / 6
StrictDivisorsHyperbola[3, 1, n_, s_] := Floor[n]^2 (Floor[n] + 1) ^ 2 / 4 - s^2 (s - 1) ^ 2 / 4
StrictDivisorsHyperbola[A_, 0, n_, s_] := 1
SumPrimesHyperbola[A_, n_] :=
 Sum[ (-1) ^ (k + 1) / (j k) MoebiusMu[ j] StrictDivisorsHyperbola[ j A, k, n^(1 / j), 2],
  {j, 1, Log[2, n]}, {k, 1, Log[2, (n^(1 / j))]}]


Smalld[ A_, k_, n_] :=
 StrictDivisorsHyperbola[ A, k, n, 2] - StrictDivisorsHyperbola[ A, k, n - 1, 2]
StrictDivisorsReduced[ a_, A_, k_, n_ ] :=
 Sum[ Smalld[ A, 1, j] StrictDivisors[ A, k - 1, n / j], {j, a + 1, n}] +
  Sum[ Smalld[ A, k - 1, j] StrictDivisors[ A, 1, n / j], {j, 2, a}] +
  Sum[ Smalld[ A, 1, s] Smalld[ A, m, j] StrictDivisors[ A, k - m - 1, n / ( j s )],
   {j, 2, a}, {s, Floor[a / j] + 1, n / j}, {m, 1, k - 2}]
StrictDivisorsReduced[ a_, A_, 1, n_] := Sum[ j^A, {j, 2, n}]
SumPrimesReduced[A_, n_] :=
 Sum[ (-1) ^ (k + 1) / (j k) MoebiusMu[ j] StrictDivisorsReduced[ Floor[ n^(1 / 3)],
   j A, k, n^(1 / j)], {j, 1, Log[2, n]}, {k, 1, Log[2, (n^(1 / j))]}]


StrictDivisorsFullReduced[ A_, k_, n_] :=
 Sum[ j^A StrictDivisorsHyperbola[ A, k - 1, n / j, 2], {j, Floor[ n^(1 / 3)] + 1, n^(1 / 2)}] +
  Sum[ Sum[m^A, {m, Floor[ n / (j + 1)] + 1, n / j}] StrictDivisorsHyperbola[ A, k - 1, j, 2],
   {j, 1, n / Floor[n^(1 / 2)] - 1}] +
  Sum[ Smalld[ A, k - 1, j] Sum[m^A, {m, 2, n / j}], {j, 2, n^(1 / 3)}] +
  Sum[ s^A Smalld[ A, m, j] StrictDivisorsHyperbola[ A, k - m - 1, n / (j s), 2], {j, 2,
   n^(1 / 3)}, {s, Floor[ Floor[n^(1 / 3)] / j] + 1, Floor[ n / j] ^ (1 / 2)}, {m, 1, k - 2}] +
  Sum[ (Sum[m^A, {m, Floor[ n / (j (s + 1))] + 1, n / (j s)}] )
    (Sum[ Smalld[ A, m, j] StrictDivisorsHyperbola[ A, k - m - 1, s, 2], {m, 1, k - 2}]),
   {j, 2, n^(1 / 3)}, {s, 1, Floor[ n / j] / Floor[Floor[ n / j] ^ (1 / 2)] - 1}]
StrictDivisorsFullReduced[ A_, 1, n_] := Sum[ j^A, {j, 2, n}]
```