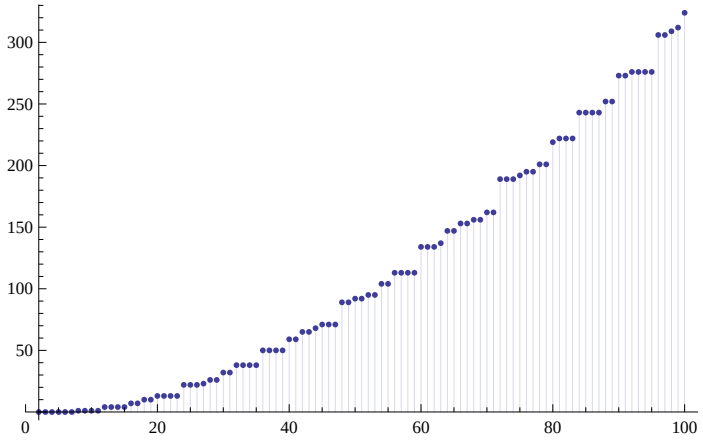
```
D3[n_] := -1 + Floor[n^(1/3)]^3 -
  3 Sum[Floor[n/k^2] + Floor[Sqrt[n/k]]^2 - 2 Sum[Floor[n/j/k], {j, k, Floor[(n/k)^.5]}], {k, 2, Floor[n^(1/3)]}]
DiscretePlot[D3[n], {n, 2, 100}]
```




```
DD[k_, a_, n_] :=
 Sum[(-1)^(j+1) Binomial[k, j] DD[k-j, m, Floor[n/(m^j)]], {m, a, n^(1/k)}, {j, 1, k}]
DD[1, a_, n_] := Floor[n] - a + 1
DD[0, a_, n_] := 1
DS[n_, k_] := DD[k, 2, n]
DDD[n_, k_] := Sum[DDD[n/j, k-1], {j, 2, n}]
DDD[n_, 0] := 1


D2a[n_] :=
 Sum[Binomial[3, 2] (Sum[Binomial[2, 1] Binomial[1, 0] Sum[1, {m, j, Floor[(n/(j k))]}] -
       Binomial[2, 0], {j, k, Floor[(n/k)^(1/2)]}]) - Binomial[3, 1]
    (Sum[1, {m, k, Floor[n/(k^2)]}]) + Binomial[3, 0], {k, 2, Floor[n^(1/3)]}]


D2a[1000]
```

11 217

```
DDD[1000, 3]
```

11 217

```
D2b[n_] :=
 Sum[3 (Sum[2 Sum[1, {m, j, Floor[(n/(j k))]}] - 1, {j, k, Floor[(n/k)^(1/2)]}]) -
   3 (Sum[1, {m, k, Floor[n/(k^2)]}]) + 1, {k, 2, Floor[n^(1/3)]}]
D2b[1000]
```

11 217

```
D2b1[n_] := Sum[3 (Sum[2 Sum[1, {m, j, Floor[(n/(j k))]}], {j, k, Floor[(n/k)^(1/2)]}]),
  {k, 2, Floor[n^(1/3)]}]
D2b2[n_] := Sum[3 (Sum[-1, {j, k, Floor[(n/k)^(1/2)]}]), {k, 2, Floor[n^(1/3)]}]
D2b3[n_] := Sum[-3 (Sum[1, {m, k, Floor[n/(k^2)]}]), {k, 2, Floor[n^(1/3)]}]
```

```
D2b4[n_] := Sum[ 1, {k, 2, Floor[n^(1 / 3)]}]
```

```
D2Ba[n_] := D2b1[n] + D2b2[n] + D2b3[n] + D2b4[n]
```

```
D2Ba[1000]
```

11 217

```
D2b1[100]
```

462

```
D2b1a[100]
```

462

```
D2b1a[n_] := Sum[ 3 (Sum[2 (Floor[(n / (j k))] - j + 1), {j, k, Floor[(n / k)^(1 / 2)]}]),
  {k, 2, Floor[n^(1 / 3)]}]
```

```
D2b1a1[n_] :=
 Sum[ 3 (Sum[2 (Floor[(n / (j k))]), {j, k, Floor[(n / k)^(1 / 2)]}]), {k, 2, Floor[n^(1 / 3)]}]
```

```
D2b1a2[n_] := Sum[ 3 (Sum[2 (-j + 1), {j, k, Floor[(n / k)^(1 / 2)]}]), {k, 2, Floor[n^(1 / 3)]}]
```

```
D2Bb[n_] := D2b1a1[n] + D2b1a2[n] + D2b2[n] + D2b3[n] + D2b4[n]
```

```
D2Bb[1000]
```

11 217

```
D2b1a2[n]
```

$$\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]} 3 \left(-1+k-\text{Floor}\left[\sqrt{\frac{n}{k}}\right]\right) \left(-2+k+\text{Floor}\left[\sqrt{\frac{n}{k}}\right]\right)$$

```
D2b1a2[n] + D2b2[n] + D2b3[n] + D2b4[n]
```

$$-1+\text{Floor}\left[n^{1/3}\right]+\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]} -3\left(1-k+\text{Floor}\left[\frac{n}{k^2}\right]\right)+$$

$$\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]} 3 \left(-1+k-\text{Floor}\left[\sqrt{\frac{n}{k}}\right]\right)+\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]} 3 \left(-1+k-\text{Floor}\left[\sqrt{\frac{n}{k}}\right]\right) \left(-2+k+\text{Floor}\left[\sqrt{\frac{n}{k}}\right]\right)$$

```
FFF[n_] := -1 + Floor[n^(1/3)] + Sum[ -3 (1 - k + Floor[n / k^2]), {k, 2, Floor[n^(1/3)]}] +
  Sum[ 3 (-1 + k - Floor[Sqrt[n / k]]), {k, 2, Floor[n^(1/3)]}] +
  Sum[ 3 (-1 + k - Floor[Sqrt[n / k]]) (-2 + k + Floor[Sqrt[n / k]]), {k, 2, Floor[n^(1/3)]}]
```

```
GGG[n_] := D2b1a1[n] + FFF[n]
```

```
GGG[1000]
```

11 217

```
Expand[-1 + Floor[n^(1/3)] + Sum[ -3 (1 - k + Floor[n / k^2]), {k, 2, Floor[n^(1/3)]}] +
  Sum[ 3 (-1 + k - Floor[Sqrt[n / k]]), {k, 2, Floor[n^(1/3)]}] +
  Sum[ 3 (-1 + k - Floor[Sqrt[n / k]]) (-2 + k + Floor[Sqrt[n / k]]), {k, 2, Floor[n^(1/3)]}]]
```

$$\text{FullSimplify}\left[-1+\text{Floor}\left[n^{1/3}\right]+\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}-3\left(1-k+\text{Floor}\left[\frac{n}{k^2}\right]\right)+\right.$$

$$\left.\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}3\left(-1+k-\text{Floor}\left[\sqrt{\frac{n}{k}}\right]\right)+\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}3\left(-1+k-\text{Floor}\left[\sqrt{\frac{n}{k}}\right]\right)\left(-2+k+\text{Floor}\left[\sqrt{\frac{n}{k}}\right]\right)\right]$$

$$-1+\text{Floor}\left[n^{1/3}\right]+\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}-3\left(1-k+\text{Floor}\left[\frac{n}{k^2}\right]\right)+$$

$$\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}3\left(-1+k-\text{Floor}\left[\sqrt{\frac{n}{k}}\right]\right)+\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}3\left(-1+k-\text{Floor}\left[\sqrt{\frac{n}{k}}\right]\right)\left(-2+k+\text{Floor}\left[\sqrt{\frac{n}{k}}\right]\right)$$

```
D2b1a1[n_] :=
 Sum[ 3 (Sum[2 (Floor[(n / (j k))]), {j, k, Floor[(n / k) ^ (1 / 2)]}]), {k, 2, Floor[n^(1 / 3)]}]
```

```
D2b1a1[n_] := 6 Sum[ Sum[Floor[n / j / k], {j, k, Floor[(n / k) ^ .5]}], {k, 2, Floor[n^(1 / 3)]}]
```

$$\text{FFF}[n\_] := -1+\text{Floor}\left[n^{1/3}\right]+\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}-3\left(1-k+\text{Floor}\left[\frac{n}{k^2}\right]\right)+$$

$$\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}3\left(-1+k-\text{Floor}\left[\sqrt{\frac{n}{k}}\right]\right)+\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}3\left(-1+k-\text{Floor}\left[\sqrt{\frac{n}{k}}\right]\right)\left(-2+k+\text{Floor}\left[\sqrt{\frac{n}{k}}\right]\right)$$

$$\text{FFF}[n\_] := -1+\text{Floor}\left[n^{1/3}\right]+\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}\left(-3+3k-3\,\text{Floor}\left[\frac{n}{k^2}\right]\right)+$$

$$\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}\left(-3+3k-3\,\text{Floor}\left[\sqrt{\frac{n}{k}}\right]\right)+\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}\left(6-9k+3k^2+3\,\text{Floor}\left[\sqrt{\frac{n}{k}}\right]-3\,\text{Floor}\left[\sqrt{\frac{n}{k}}\right]^2\right)$$

$$\text{FFF}[n\_] := -1+\text{Floor}\left[n^{1/3}\right]+\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}\left(-3k+3k^2-3\,\text{Floor}\left[\frac{n}{k^2}\right]-3\,\text{Floor}\left[\sqrt{\frac{n}{k}}\right]^2\right)$$

$$\text{FFF}[n\_] :=$$

$$-1+\text{Floor}\left[n^{1/3}\right]+\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}-3k+\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}3k^2+\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}-3\,\text{Floor}\left[\frac{n}{k^2}\right]+\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}-3\,\text{Floor}\left[\sqrt{\frac{n}{k}}\right]^2$$

$$\text{FFF}[n\_] := -1+\text{Floor}\left[n^{1/3}\right]^3+\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}-3\,\text{Floor}\left[\frac{n}{k^2}\right]+\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}-3\,\text{Floor}\left[\sqrt{\frac{n}{k}}\right]^2$$

```
AAA[n_] := 6 Sum[ Sum[Floor[n / j / k], {j, k, Floor[(n / k) ^ .5]}], {k, 2, Floor[n^(1 / 3)]}]
```

$$\text{FFF}[n\_] := -1+\text{Floor}\left[n^{1/3}\right]^3-3\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}\left(\text{Floor}\left[\frac{n}{k^2}\right]+\text{Floor}\left[\sqrt{\frac{n}{k}}\right]^2\right)$$

```
GGG[n_] := AAA[n] + FFF[n]
```

```
GGG[1000]
```

```
11 217
```

```
FullSimplify[
  -3 + 3 k - 3 Floor[n/k^2] - 3 + 3 k - 3 Floor[Sqrt[n/k]] + 6 - 9 k + 3 k^2 + 3 Floor[Sqrt[n/k]] - 3 Floor[Sqrt[n/k]]^2]
```

```
Expand[3 ((-1 + k) k - Floor[n/k^2] - Floor[Sqrt[n/k]]^2)]
```

$$-3\,k+3\,k^2-3\,\text{Floor}\left[\frac{n}{k^2}\right]-3\,\text{Floor}\left[\sqrt{\frac{n}{k}}\right]^2$$

$$-1+\text{Floor}\left[n^{1/3}\right]+\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}-3\,k+\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}3\,k^2+\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}-3\,\text{Floor}\left[\frac{n}{k^2}\right]+\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}-3\,\text{Floor}\left[\sqrt{\frac{n}{k}}\right]^2$$

```
FullSimplify[-1 + Floor[n^(1/3)] - 3/2 (-2 + Floor[n^(1/3)] + Floor[n^(1/3)]^2) +
  1/2 (-6 + Floor[n^(1/3)] + 3 Floor[n^(1/3)]^2 + 2 Floor[n^(1/3)]^3) +
  Sum[-3 Floor[n/k^2], {k, 2, Floor[n^(1/3)]}] + Sum[-3 Floor[Sqrt[n/k]]^2, {k, 2, Floor[n^(1/3)]}]]
```

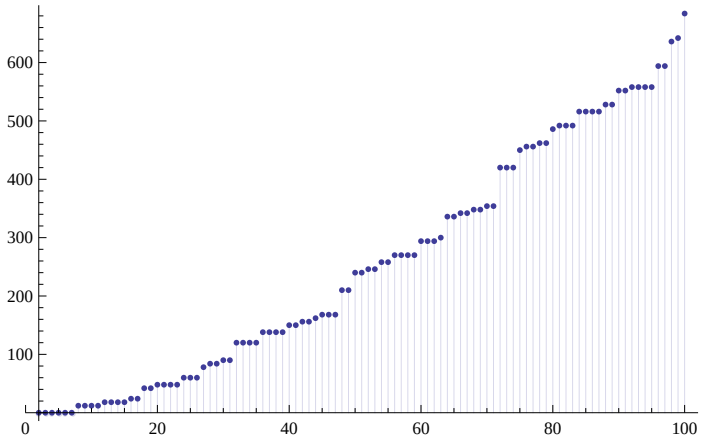
$$-1+\text{Floor}\left[n^{1/3}\right]^3+\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}-3\,\text{Floor}\left[\frac{n}{k^2}\right]+\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}-3\,\text{Floor}\left[\sqrt{\frac{n}{k}}\right]^2$$

```
AAA[n_] := 6 Sum[Sum[Floor[n / j / k], {j, k, Floor[(n / k)^.5]}], {k, 2, Floor[n^(1 / 3)]}]
FFF[n_] := -1 + Floor[n^(1/3)]^3 -
  3 Sum[(Floor[n/k^2] + Floor[Sqrt[n/k]]^2 - 2 Sum[Floor[n / j / k], {j, k, Floor[(n / k)^.5]}]), {k, 2, Floor[n^(1/3)]}]
GGG[n_] := AAA[n] + FFF[n]
D3[n_] := -1 + Floor[n^(1/3)]^3 -
  3 Sum[(Floor[n/k^2] + Floor[Sqrt[n/k]]^2 - 2 Sum[Floor[n / j / k], {j, k, Floor[(n / k)^.5]}]), {k, 2, Floor[n^(1/3)]}]
D3[1000]
```

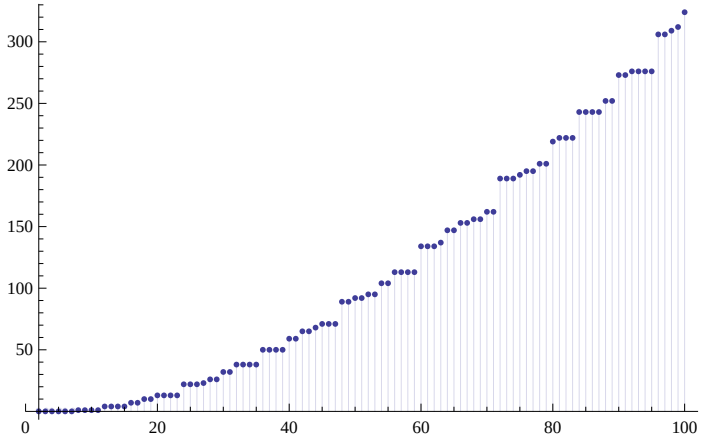
```
11217
```

$$\text{II}[n\_] := -1+\text{Floor}\left[n^{1/3}\right]^3-3\sum_{k=2}^{\text{Floor}\left[n^{1/3}\right]}\left(\text{Floor}\left[\frac{n}{k^2}\right]+\text{Floor}\left[\sqrt{\frac{n}{k}}\right]^2\right)$$

```
DiscretePlot[AAA[n], {n, 2, 100}]
```

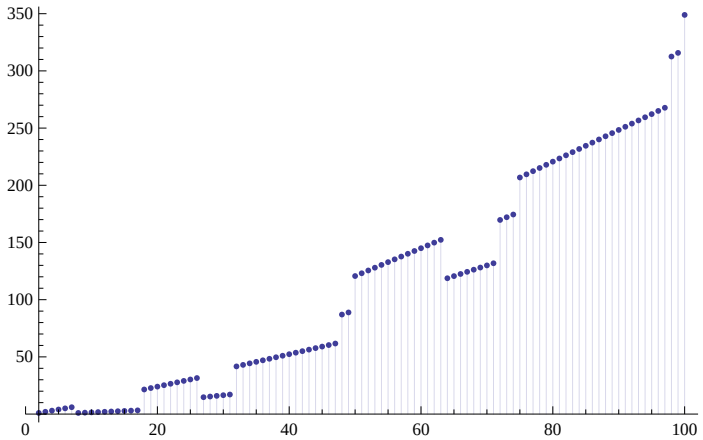



```
DiscretePlot[DDD[n, 3], {n, 2, 100}]
```



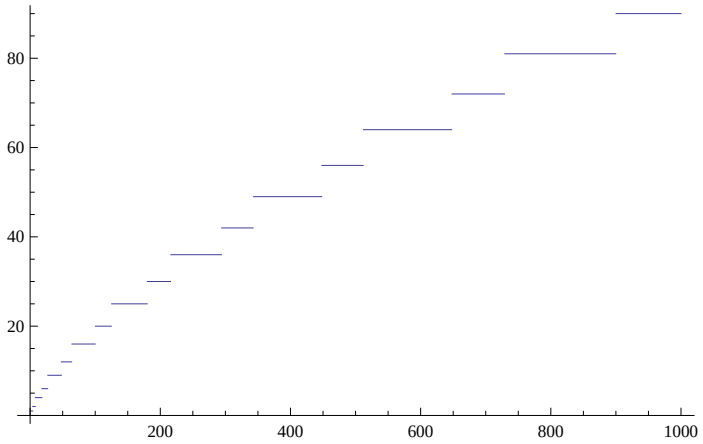

```
D3a[n_] := -1 + (n^(1/3))^3 - 3 Sum_(k=2)^(n^(1/3)) ((n/k^2) + (Sqrt[n/k])^2 - 2 Sum[n / j / k, {j, k, (n / k) ^ .5}])
```

```
DiscretePlot[D3a[n], {n, 2, 100}]
```




```
D3a[n_] := -1 + (n^(1/3))^3 - 3 Sum_(k=2)^(n^(1/3)) ((n/k^2) + (Sqrt[n/k])^2 - 2 Sum[n / j / k, {j, k, (n / k) ^ .5}])
```

```
Plot[Floor[ n^(1 / 3)] Floor[(n / Floor[ n^(1 / 3)])^(1 / 2)], {n, 1, 1000}]
```




```
AAB[n_] := Sum[ Sum[Floor[n / j / k], {j, k, Floor[(n / k)^.5]}], {k, 2, Floor[n^(1 / 3)]}]

AA[n_, a_, k_ ] := Sum[ AA[n / j, j, k - 1], {j, a, Floor[n^(1 / k)]}]

AA[n_, a_, 1 ] := Floor[n]

AA[64, 2, 3]
```

56

```
AAB[100]
```

114

```
BB[ n_ ] := Sum[ AA[n, 2, k] (k - 1)! (-1)^k, {k, 2, 20}]

BB[64]
```

260

```
Table[ AA[63, 2, k], {k, 1, 8}]
```

{63, 98, 50, 15, 3, 0, 0, 0}

```
Table[AA[64, 2, k], {k, 1, 8}]
```

{64, 108, 56, 20, 4, 2, 0, 0}

```
DiscretePlot[ BB[n], {n, 2, 1050}]
```