$$\sum_{k=0}^{\infty} \frac{z^{(k)}}{k!} = \left(\frac{1}{1-x^0}\right)^z$$

$$\sum_{k=0}^{\infty} {z \choose k} = (1+x^0)^z$$

$$\sum_{k=0}^{\infty} \frac{z^k}{k!} = e^{x^0 \cdot z}$$

...

$$(1+x^0)^z = (\frac{1-x^{2\cdot 0}}{1-x^0})^z$$

$$(1+x^{-s}+x^{-2s})^z = (\frac{1-x^{-3s}}{1-x^{-s}})^z$$

$$(1+x^{-s}+x^{-2s}+x^{-3s})^z = (\frac{1-x^{-4s}}{1-x^{-s}})^z$$

$$\left(\sum_{j=0}^{a-1} x^{-js}\right)^z = \left(\frac{1-x^{-as}}{1-x^{-s}}\right)^z$$

. . .

$$\sum_{k=0}^{\infty} \frac{z^{(k)}}{k!} \cdot x^{(-2s\,k)} = \left(\frac{1}{1-x^{-2\,s}}\right)^{z}$$

$$\sum_{k=0}^{\infty} \frac{z^{(k)}}{k!} \cdot x^{(-ask)} = \left(\frac{1}{1 - x^{-as}}\right)^{z}$$

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$$\lim_{s \to 0} \left(\frac{1 - x^{-as}}{1 - x^{-s}} \right)^z = \lim_{s \to 1} \left(\frac{1 - x^{-as}}{1 - x^{-s}} \right)^z = a^z$$

$$[(\frac{1}{1-x^0})^2]_n^+ = \sum_{j=0}^n \sum_{k=0}^{n-j} 1$$

$$[(\frac{1}{1-x^0})^3]_n^+ = \sum_{j=0}^n \sum_{k=0}^{n-j} \sum_{l=0}^{n-j-k} 0$$

$$[(\frac{1}{1-x^0})^3]_n^+ = \sum_{j=0}^n [(\frac{1}{1-x^0})^{k-1}]_{n-j}^+$$

$$[(\frac{1}{1-x^0})^3]_n^+ = \sum_{j=0}^n 1$$

$$[(\frac{1}{1-x^0})^3]_n^+ = \sum_{j=1}^n 1$$

$$[(\frac{1}{1-x^0}-1)^3]_n^+ = \sum_{j=1}^{n-j} \sum_{k=1}^{n-j} \sum_{k=1}^{n-j-1} 1$$

$$[(\frac{1}{1-x^0}-1)^3]_n^+ = \sum_{j=1}^{n-j-1} \sum_{k=1}^{n-j-1} \sum_{l=1}^{n-j-k} 1$$

$$[(\frac{1}{1-x^0}-1)^3]_n^+ = \sum_{j=1}^{n-k+1} [(\frac{1}{1-x^0}-1)^{k-1}]_{n-j}^+$$

$$[(\frac{1}{1-x^0}-1)^3]_n^+ = \sum_{j=1}^n \sum_{k=1}^{n-j-1} \sum_{k=1}^{n-j-1} \frac{1}{j}$$

$$[(\log(\frac{1}{1-x^0}))^3]_n^+ = \sum_{j=1}^{n-j-1} \sum_{k=1}^{n-j-1} \sum_{l=1}^{n-j-k} \frac{1}{j \cdot k}$$

$$[\log((\frac{1}{1-x^0}))^3]_n^+ = \sum_{j=1}^{n-j-1} \sum_{k=1}^{n-j-1} \sum_{l=1}^{n-j-k} \frac{1}{j \cdot k \cdot l}$$

$$[(\log((\frac{1}{1-x^0})))^3]_n^+ = \sum_{j=1}^{n-j-1} \sum_{k=1}^{n-j-1} \sum_{l=1}^{n-j-k} \frac{1}{j \cdot k \cdot l}$$

$$[(\log((\frac{1}{1-x^0})))^3]_n^+ = \sum_{j=1}^{n-j-1} \sum_{k=1}^{n-j-1} \sum_{l=1}^{n-j-k} \frac{1}{j \cdot k \cdot l}$$

$$[(\log((\frac{1}{1-x^0})))^3]_n^+ = \sum_{j=1}^{n-j-1} \sum_{k=1}^{n-j-1} \sum_{l=1}^{n-j-k} \frac{1}{j \cdot k \cdot l}$$

 $\left[\frac{1}{1-x^0}\right]^+ = \sum_{i=0}^n 1$

$$[(\log(\frac{1}{1-x^{0}}))^{s}]_{n}^{s} = \sum_{j=1}^{n-k+1} \frac{1}{j} \cdot [(\log(\frac{1}{1-x^{0}}))^{s-1}]_{n-j}^{s}$$

$$[(\log(\frac{1}{1-x^{0}}))^{s}]_{n}^{s} = \sum_{k=0}^{n-k+1} \frac{1}{j} \cdot [(\log(\frac{1}{1-x^{0}}))^{s-1}]_{n-j}^{s}$$

$$...$$

$$[(\frac{1}{1-x^{0}})^{s}]_{n}^{s} = \sum_{k=0}^{n-k+1} \frac{1}{k} \cdot [(\frac{1}{1-x^{0}})^{s-1}]_{n}^{s-1}$$

$$[(\frac{1}{1-x^{0}})^{s-1}]_{n}^{s} = \sum_{k=1}^{n-k} \frac{1}{k} \cdot [(\frac{1}{1-x^{0}})^{s-1}]_{n}^{s-1}$$

$$[\log(\frac{1}{1-x^{0}})]_{n}^{s} = \lim_{s \to 0} \frac{\partial}{\partial z} [(\frac{1}{1-x^{0}})^{s-1}]_{n}^{s-1}$$

$$[(\log(\frac{1}{1-x^{0}}))^{s-1}]_{n}^{s} = \lim_{s \to 0} \frac{\partial^{k}}{\partial z^{k}} [(\frac{1}{1-x^{0}})^{s-1}]_{n}^{s-1}$$

$$[(\frac{1}{1-x^{0}})^{s-1}]_{n}^{s} = \sum_{k=0}^{n-k+1} \frac{z^{k}}{k!} [(\log(\frac{1}{1-x^{0}}))^{s-1}]_{n}^{s-1}$$

$$[e]_{n}^{+} = \sum_{j=0}^{n} \frac{1}{j!}$$

$$[e^{2}]_{n}^{+} = \sum_{j=0}^{n} \sum_{k=0}^{n-j} \frac{1}{j! \cdot k!}$$

$$[e^{3}]_{n}^{+} = \sum_{j=0}^{n} \sum_{k=0}^{n-j} \sum_{l=0}^{n-j-k} \frac{1}{j! \cdot k! \cdot l!}$$

$$[e^{k}]_{n}^{+} = \sum_{j=0}^{n} \frac{1}{j!} \cdot [e^{(k-1) \cdot x^{0}}]_{n-j}^{+}$$

$$[e^{0}]_{n}^{+} = \mathbf{1}_{n \ge 0}(n)$$

$$[e^{2}]_{n}^{+} = \sum_{k=0}^{n} \frac{z^{k}}{k!}$$
...
$$[e-1]_{n}^{+} = \sum_{j=1}^{n-1} \sum_{k=1}^{n-j} \frac{1}{j! \cdot k!}$$

$$[(e-1)^{3}]_{n}^{+} = \sum_{j=1}^{n-2} \sum_{k=1}^{n-1} \sum_{l=1}^{n-j-k} \frac{1}{j! \cdot k! \cdot l!}$$

$$[(e-1)^{k}]_{n}^{+} = \sum_{j=1}^{n-k+1} \frac{1}{j!} \cdot [(e-1)^{k-1}]_{n-j}^{+}$$

$$[e^{0}]_{n}^{+} = \mathbf{1}_{n \ge 0}(n)$$
...
$$[\log e]_{n}^{+} = (n > 0)? 1: 0$$

$$[\log^{2} e]_{n}^{+} = (n > 1)? 1: 0$$

$$[\log^{3} e]_{n}^{+} = (n > 2)? 1: 0$$

$$[\log^{k} e]_{n}^{+} = 0 \text{ if } k > n$$

$$[\log^{k} e]_{n}^{+} = (n \ge k)? [\log^{k-1} e]_{n}^{+}: 0$$

$$[(\log(e^{x^{0}}))^{0}]_{n}^{+} = \mathbf{1}_{n \ge 0}(n)$$
...
$$[(\frac{1}{1-x^{0}})^{z}]_{n}^{+} = \sum_{k=0}^{n} (\frac{z}{k}) [(\frac{1}{1-x^{0}}-1)^{k-1}]_{n}^{+}$$

$$[(e-1)^{k}]_{n}^{+} = \sum_{j=0}^{k} (-1)^{k-j} {k \choose j} [e^{j}]_{n}^{+}$$

$$[\log(\frac{1}{1-x^{0}})]_{n}^{+} = \sum_{k=1}^{k} \frac{(-1)^{k+1}}{k} [(e-1)^{k}]_{n}^{+}$$

$$[\log e]_{n}^{+} = \lim_{z \to 0} \frac{\partial}{\partial z} [e^{z}]_{n}^{+}$$

$$[\log^{k} e]_{n}^{+} = \lim_{z \to 0} \frac{\partial^{k}}{\partial z^{k}} [e^{z}]_{n}^{+}$$

$$[e^{z}]_{n}^{+} = \sum_{k=0}^{k} \frac{z^{k}}{k!} [\log^{k} e]_{n}^{+}$$

$$\{I+n\}^{+\Sigma} = \sum_{j=0}^{n} 1$$

$$\{(I+n)^{2}\}^{+\Sigma} = \sum_{j=0}^{n} \sum_{k=0}^{n-j} 1$$

$$\{(I+n)^{3}\}^{+\Sigma} = \sum_{j=0}^{n} \sum_{k=0}^{n-j} \sum_{l=0}^{n-j-k} 1$$

$$\{(I+n)^{k}\}^{+\Sigma} = \sum_{j=0}^{n} \{(I+(n-j))^{k-1}\}^{+\Sigma}$$

$$\{(I+n)^{0}\}^{+\Sigma} = \mathbf{1}_{n\geq 0}(n)$$

$$\{(I+n)^{z}\}^{+\Sigma} = \sum_{j=0}^{n} \frac{z^{(k)}}{k!} = \frac{(z+1)^{(n)}}{n!}$$
...
$$\{n\}^{+\Sigma} = \sum_{j=1}^{n} \sum_{k=1}^{n-j-1} 1$$

$$\{n^{3}\}^{+\Sigma} = \sum_{j=1}^{n-j-1} \sum_{k=1}^{n-j-k} 1$$

$$\{n^{k}\}^{+\Sigma} = 0 \text{ if } k > n$$

$$\{n^{k}\}^{+\Sigma} = \sum_{j=1}^{n-k+1} \{(n-j)^{k-1}\}^{+\Sigma}$$

$$\{n^{0}\}^{+\Sigma} = \mathbf{1}_{n\geq 0}(n)$$

$$\{n^{z}\}^{+\Sigma} = (n)$$
...
$$\{\log^{2}(I+n)\}^{+\Sigma} = \sum_{j=1}^{n-j} \sum_{k=1}^{n-j-1} \frac{1}{j \cdot k}$$

$$\{\log^{3}(I+n)\}^{+\Sigma} = \sum_{j=1}^{n-2} \sum_{k=1}^{n-j-1} \sum_{l=1}^{n-j-k} \frac{1}{j \cdot k \cdot l}$$

 $\{\log^k(I+n)\}^{+\sum} = 0 \text{ if } k > n$

$$\{\log^{k}(I+n)\}^{+\Sigma} = \sum_{j=1}^{n-k+1} \frac{1}{j} \cdot \{\log^{k-1}(I+n-j)\}^{+\Sigma}$$

$$\{\log^{0}(I+n)\}^{+\Sigma} = \mathbf{1}_{n \ge 0}(n)$$
...
$$\{(I+n)^{z}\}^{+\Sigma} = \sum_{k=0}^{k} {z \choose k} \{n^{k}\}^{+\Sigma}$$

$$\{n^{k}\}^{+\Sigma} = \sum_{j=0}^{k} (-1)^{k-j} {k \choose j} \{(I+n)^{j}\}^{+\Sigma}$$

$$\{\log(I+n)\}^{+\Sigma} = \sum_{k=1}^{k} \frac{(-1)^{k+1}}{k} \{n^{k}\}^{+\Sigma}$$

$$\{\log(I+n)\}^{+\Sigma} = \lim_{z \to 0} \frac{\partial}{\partial z} \{(I+n)^{z}\}^{+\Sigma}$$

$$\{\log^{k}(I+n)\}^{+\Sigma} = \lim_{z \to 0} \frac{\partial^{k}}{\partial z^{k}} \{(I+n)^{z}\}^{+\Sigma}$$

$$\{(I+n)^{z}\}^{+\Sigma} = \sum_{k=0}^{k} \frac{z^{k}}{k!} \{\log^{k}(I+n)\}^{+\Sigma}$$

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$$\{I+e_n\}^{+\Sigma} = \sum_{j=0}^{n} \frac{1}{j!}$$

$$\{(I+e_n)^2\}^{+\Sigma} = \sum_{j=0}^{n} \sum_{k=0}^{n-j} \frac{1}{j!k!}$$

$$\{(I+e_n)^3\}^{+\Sigma} = \sum_{j=0}^{n} \sum_{k=0}^{n-j} \sum_{l=0}^{n-j-k} \frac{1}{j!k!l!}$$

$$\{(I+e_n)^k\}^{+\Sigma} = \sum_{j=0}^{n} \frac{1}{j!} \cdot \{(I+e_{n-j})^{k-1}\}^{+\Sigma}$$

$$\{(I+e_n)^0\}^{+\Sigma} = \mathbf{1}_{n\geq 0}(n)$$

$$\{(I+e_n)^c\}^{+\Sigma} = \sum_{j=1}^{n} \frac{1}{j!}$$

$$\{e_n^2\}^{+\Sigma} = \sum_{j=1}^{n-j} \sum_{k=1}^{n-j} \frac{1}{j!k!}$$

$$\{e_n^3\}^{+\Sigma} = \sum_{j=1}^{n} \sum_{k=1}^{n-j} \sum_{l=1}^{n-j-k} \frac{1}{j!k!l!}$$

$$\{e_n^k\}^{+\Sigma} = \sum_{j=1}^{n-k+1} \frac{1}{j!} \cdot \{e_{n-j}^{k-1}\}^{+\Sigma}$$

$$\{e_n^0\}^{+\Sigma} = \mathbf{1}_{n\geq 0}(n)$$
...
$$\{\log(I+e_n)\}^{+\Sigma} = (n>0)?1:0$$

$$\{\log^2(I+e_n)\}^{+\Sigma} = (n>0)?1:0$$

$$\{\log^3(I+e_n)\}^{+\Sigma} = (n>2)?1:0$$

$$\{\log^k(I+e_n)\}^{+\Sigma} = 0 \text{ if } k > n$$

$$\{\log^k(I+e_n)\}^{+\Sigma} = 0 \text{ if } k > n$$

$$\{\log^k(I+e_n)\}^{+\Sigma} = (n\geq k)?\{\log^{k-1}(I+e_n)\}^{+\Sigma}:0$$

$$\{\log^0(I+e_n)\}^{+\Sigma} = \mathbf{1}_{n\geq 0}(n)$$
...
$$[(\frac{1}{1-v^0})^{\frac{1}{2}}]^{\frac{1}{2}} = \sum_{k=0}^{n} \sum_{k=0}^{n} [(\frac{1}{1-v^0}-1)^{\frac{k}{2}}]^{\frac{1}{2}}$$

$$\{(I+e_{n})^{z}\}^{+\Sigma} = \sum_{k=0}^{\infty} {z \choose k} \{e_{n}^{k}\}^{+\Sigma}$$

$$\{e_{n}^{k}\}^{+\Sigma} = \sum_{j=0}^{k} {(-1)^{k-j} {k \choose j}} \{(I+e_{n})^{j}\}^{+\Sigma}$$

$$[\log(\frac{1}{1-x^{0}})]_{n}^{+} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \{e_{n}^{k}\}^{+\Sigma}$$

$$\{\log(I+e_{n})\}^{+\Sigma} = \lim_{z \to 0} \frac{\partial}{\partial z} \{(I+e_{n})^{z}\}^{+\Sigma}$$

$$\{\log^{k}(I+e_{n})\}^{+\Sigma} = \lim_{z \to 0} \frac{\partial^{k}}{\partial z^{k}} \{(I+e_{n})^{z}\}^{+\Sigma}$$

$$\{(I+e_{n})^{z}\}^{+\Sigma} = \sum_{k=0}^{\infty} \frac{z^{k}}{k!} \{\log^{k}(I+e_{n})\}^{+\Sigma}$$

$$\{e_{x}^{k}\} = \sum_{j=0}^{\infty} \frac{1}{j!} \cdot (\lim_{t \to 0} \frac{\partial^{j}}{\partial t^{j}} (e^{t}-1)^{k}) \{\log^{j}(I+e_{x})\}$$

$$\{e_{x}^{k}\} = \sum_{j=0}^{\infty} \frac{S(j,k) \cdot k!}{j!} \cdot \{\log^{j}(I+e_{x})\}$$
where $S(j,k)$ are Stirling numbers of the second kind.

$$\{\log(I+n)\}^{+\sum} = \sum_{k=1}^{\infty} \frac{1}{k} \{\log(I+e_{\lfloor \frac{n}{k} \rfloor})\}^{+\sum}$$

$$\{\log(I+e_n)\}^{+\sum} = \sum_{k=1}^{\infty} \frac{1}{k} \{\log(I+\lfloor \frac{n}{k} \rfloor)\}^{+\sum}$$

$$\{(I+n)^{z}\}^{+\sum} = \sum_{a+2b+3c+4d+...\leq n} \nabla \{e_{a}^{z}\} \cdot \nabla \{e_{b}^{\frac{z}{2}}\} \cdot \nabla \{e_{c}^{\frac{z}{3}}\} \cdot \nabla \{e_{d}^{\frac{z}{4}}\} \cdot \nabla \{e_{d}^{2}\} \cdot \nabla \{e_{d}^{\frac{z}{4}}\} \cdot \nabla \{e_{d}^{\frac{z$$

$$\left[\left(\frac{1}{1-x}\right)^{z}\right]_{n} = \sum_{a_{1}+2} \sum_{a_{2}+\ldots+k: a_{k} \leq n} \prod_{k} \nabla \left[e^{\frac{z}{k}}\right]_{a_{k}}$$

$$\left\{\left.e_{n}^{z}\right\}^{+\sum} = \sum_{a+2b+3c+4d+\ldots\leq n} \nabla \left\{\left(I+a\right)^{z}\right\}^{+\sum} \cdot \nabla \left\{\left(I+b\right)^{-\frac{z}{2}}\right\}^{+\sum} \cdot \nabla \left\{\left(I+b\right)^{-\frac{z}{2}}\right\}$$

$$[e^{z}]_{n} = \sum_{a_{1}+2 \ a_{2}+...+k \cdot a_{k} \le n} \prod_{k} \nabla \left[\left(\frac{1}{1-x} \right)^{\mu(k) \cdot \frac{z}{k}} \right]_{a_{k}}$$

$$\{(I+n)^z\}^{+\sum} = \sum_{a_1+2a_2+...+k\cdot a_k \le n} \prod_j \nabla \{(I+e_{a_j})^{\frac{z}{j}}\}^{+\sum}$$

$$\{(I+e_n)^z\}^{+\sum} = \sum_{a_1+2a_2+...+k\cdot a_k \le n} \prod_j \nabla \{(I+a_j)^{\mu(k)\cdot \frac{z}{j}}\}^{+\sum}$$