```
ClearAll["Global`*"]
```

```
E1x[n_, k_, b_] := Sum[ Binomial[k, j2] E2a[n, k - j2, b], {j2, 0, k}]
DDx[n_, k_, b_] := Sum[Binomial[k + j - 1, k - 1] b^j E1x[n / b^j, k, b], {j, 0, Log[b, n]}]
D2x[n_, k_, b_] := Sum[ (-1)^j Binomial[k, j] DDx[n, k - j, b], {j, 0, k}]

E2a[n_, k_, a_] :=
 E2a[n, k, a] = Sum[ E2a[n / j, k - 1, a], {j, 2, n}] - a Sum[ E2a[n / (a j), k - 1, a], {j, 1, n / a}];
E2a[n_, 0, a_] := 1
E1a[n_, k_, a_] :=
 E1a[n, k, a] = Sum[ E1a[n / j, k - 1, a], {j, 1, n}] - a Sum[ E1a[n / (a j), k - 1, a], {j, 1, n / a}];
E1a[n_, 0, a_] := 1

DDa[n_, k_] := DDa[n, k] = Sum[MoebiusMu[j] DDa[Floor[n / j], k - 1], {j, 1, n}]; DDa[n_, 0] := 1
D2a[n_, k_] := D2a[n, k] = Sum[MoebiusMu[j] D2a[Floor[n / j], k - 1], {j, 2, n}]; D2a[n_, 0] := 1

D2b[n_, k_] := Sum[(-1)^j Binomial[k, j] DDa[n, k - j], {j, 0, k}]
DDb[n_, k_] := Sum[Binomial[k, j] D2a[n, k - j], {j, 0, k}]
E2b[n_, k_, b_] := Sum[(-1)^j Binomial[k, j] E1a[n, k - j, b], {j, 0, k}]
E1b[n_, k_, b_] := Sum[ Binomial[k, j] E2a[n, k - j, b], {j, 0, k}]

DDd[n_, z_] := Sum[FactorialPower[z, a] / a! D2a[n, a], {a, 0, Log[2, n]}]

DDc[n_, k_, b_] := Sum[Binomial[k + j - 1, k - 1] b^j E1a[n / b^j, k, b], {j, 0, Log[b, n]}]
E1c[n_, k_, b_] := Sum[(-1)^j Binomial[k, j] b^j DDa[n / b^j, k], {j, 0, k}]
E2c[n_, k_, b_] :=
 Sum[(-1)^j b^j Binomial[k, j] Binomial[j, m] D2a[n / b^j, k - m], {j, 0, k}, {m, 0, j}]
D2E2[n_, k_, b_] := Sum[ (-1)^j b^j Binomial[k, j]
   Sum[ Binomial[j, m] If[n / b^j < 1, 0, D2a[n / b^j, k - m]], {m, 0, j}], {j, 0, k}]
E2D2[n_, k_, b_] := (-1)^k + Sum[b^a / ((k - 1)!) Binomial[k, j]
    Pochhammer[a - k + j + 1, k - 1] E2a[b^-a n, j, b], {a, 0, Log[b, n]}, {j, 0, k}]
```

```
{DDa[nn = 100, aa = 2], DDb[nn, aa], DDd[nn, aa]}
```

```
{19, 19, 19}
```

```
{D2a[nn = 100, aa = 3], D2b[nn, aa]}
```

```
{-8, -8}
```

```
f1[n_, a_] := Sum[ MoebiusMu[a j], {j, 1, n}]
```

```
f2[n_] := Sum[ (-1)^(j + 1) MoebiusMu[j], {j, 1, n}]
```

```
f1[100, 1]
```

```
1
```

```
f2[100]
```

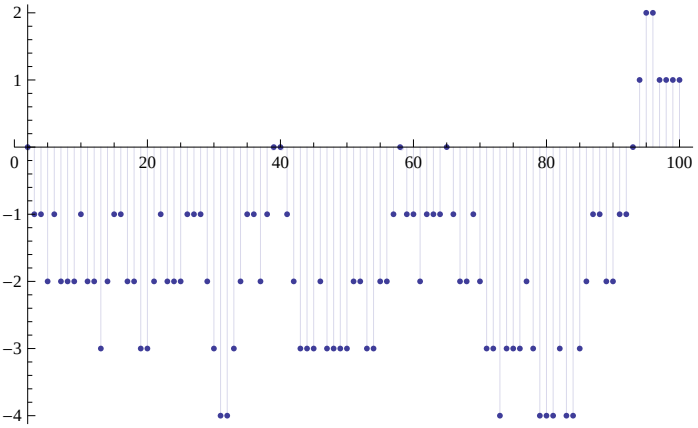
```
-15
```

```
f3[n_] := f1[n, 1] - 2 f1[n / 2, 2]
```

```
f3[100]
```

```
-15
```

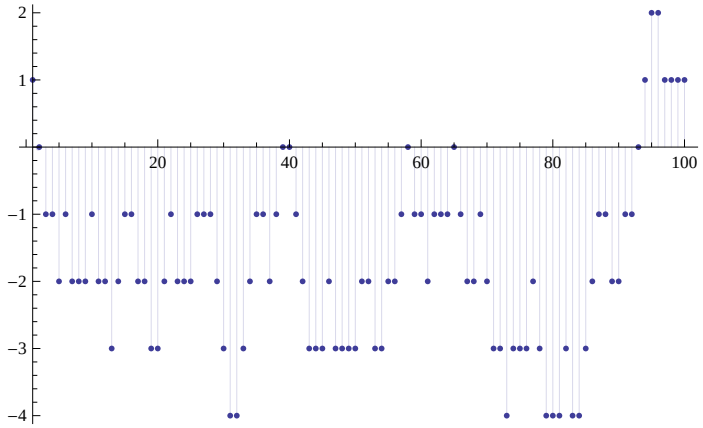
```
f4[n_, a_, c_] :=
 Sum[ (MoebiusMu[Floor[a j]]), {j, 1, n}] - c Sum[MoebiusMu[Floor[a c j]], {j, 1, n / c}]
f4a[n_, s_] := Sum[ s^k f4[n / (s^k), s^k, s], {k, 0, Log[s, n]}]

DiscretePlot[f4a[n, 2.1], {n, 2, 100}]
```



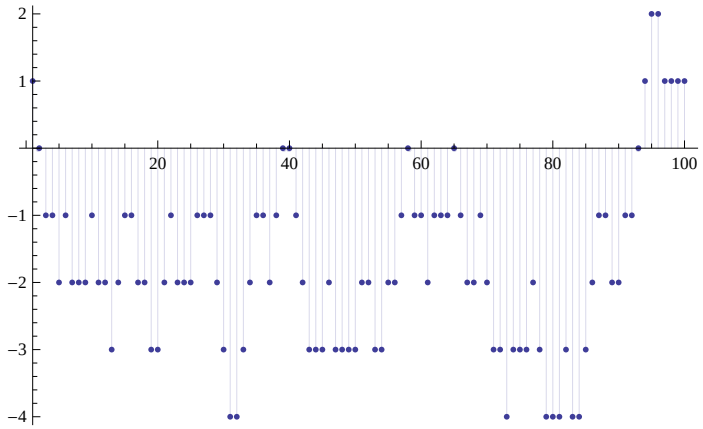

```
ref[n_, k_] := Sum[ (-1)^(j + 1) MoebiusMu[j] ref[Floor[n / j], k - 1], {j, 2, n}];
ref[n_, 0] := 1
tt[n_, k_] := Mod[n, k] - Mod[n - 1, k]

ref2[n_, k_, a_] := Sum[ tt[j, a] MoebiusMu[j] ref2[Floor[n / j], k - 1, a], {j, 2, n}];
ref2[n_, 0, a_] := 1

ref2a[n_, k_, a_] := Sum[ tt[j, a] MoebiusMu[j] ref2a[Floor[n / j], k - 1, a], {j, 1, n}];
ref2a[n_, 0, a_] := 1


ref7[n_, k_, a_, c_] := ref7[n, k, a, c] =
  Sum[ MoebiusMu[Floor[a j]] ref7[Floor[n / j], k - 1, a, c], {j, 2, n}] -
   c Sum[MoebiusMu[Floor[a j c]] ref7[Floor[n / (j c)], k - 1, a, c], {j, 1, n / c}]; ref7[
  n_, 0, a_, c_] := 1


ref8[n_, k_, a_, c_] :=
 Sum[ MoebiusMu[Floor[a j]] ref8[Floor[n / j], k - 1, a, c], {j, 1, n}] -
  c Sum[MoebiusMu[Floor[a j c]] ref8[Floor[n / (j c)], k - 1, a, c], {j, 1, n / c}]; ref8[
  n_, 0, a_, c_] := 1
ref8a[n_, s_] := Sum[ s^k ref8[n / (s^k), 1, s^k, s], {k, 0, Log[s, n]}]
ref8b[n_, k_, b_] :=
 Sum[Binomial[k + j - 1, k - 1] b^j ref8[n / b^j, k, b^j, b], {j, 0, Log[b, n]}]

test[n_, z_] := Sum[FactorialPower[z, a] / a! ref[n, a], {a, 0, Log[2, n]}]
test2[n_, z_, s_] := Sum[FactorialPower[z, a] / a! ref2[n, a, s], {a, 0, Log[2, n]}]
test3[n_, z_, s_] := Sum[FactorialPower[z, a] / a! ref7[n, a, 1, s], {a, 0, Log[2, n]}]
linm[n_, s_] := Sum[ (-1)^(k) / k ref7[n, k, 1, s], {k, 1, Log[If[s < 2, s, 2], n]}]



{ref2[1200, 3, 7], ref7[1200, 3, 1, 7]}
```

```
{-1840, -1840}
```

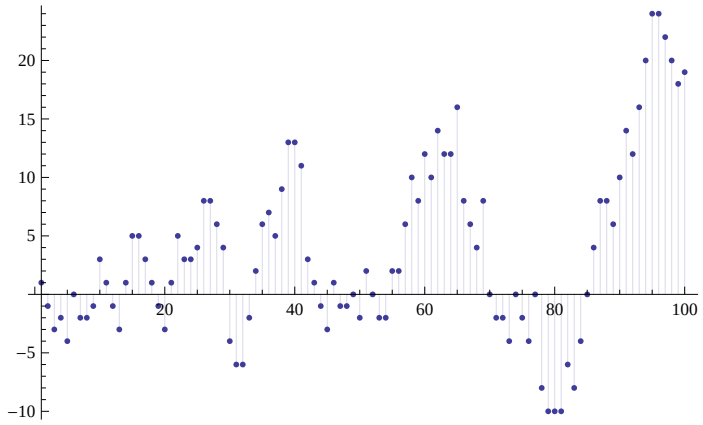
```
DiscretePlot[ref8a[n, 1.2], {n, 1, 100}]
```



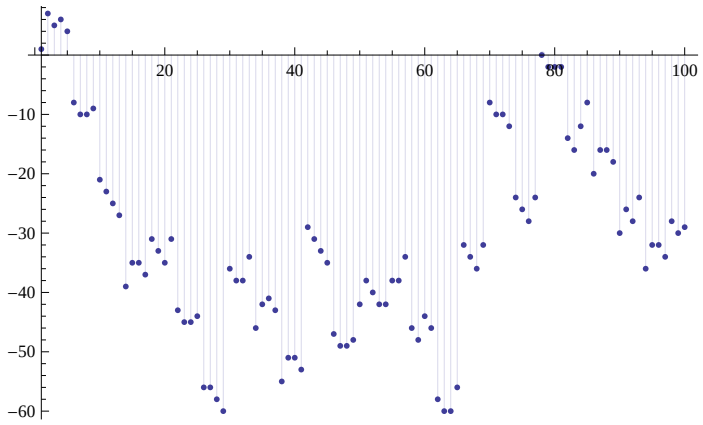

```
DiscretePlot[ref8b[n, 1, 2], {n, 1, 100}]
```



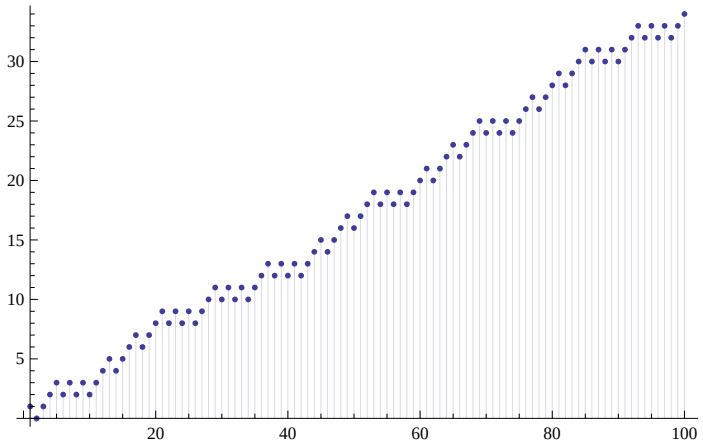

```
DiscretePlot[DDa[n, 2], {n, 1, 100}]
```



```
DiscretePlot[ref8b[n, 2, 2], {n, 1, 100}]
```




```
DiscretePlot[test3[n, -1, 2], {n, 1, 100}]
```




```
pp[n_] := pp[n] = Sum[ MangoldtLambda[j] / Log[j], {j, 2, n}]
DiscretePlot[linm[n, 3], {n, 1, 100}]
```