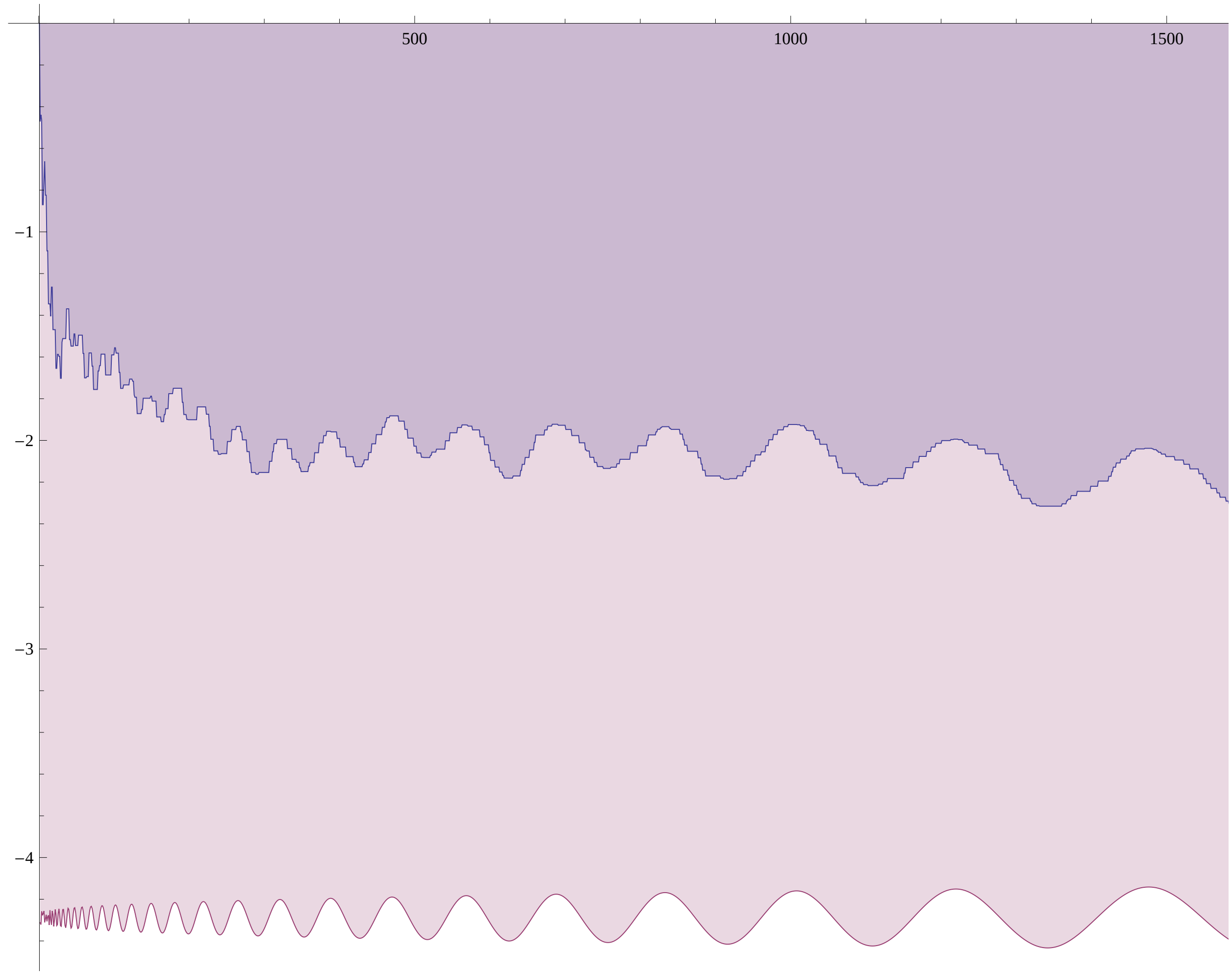
```
binomial[z_, k_] := binomial[z, k] = Product[z - j, {j, 0, k - 1}] / k!
zetaHurwitz[n_, s_, y_, 0] := UnitStep[n - 1]
zetaHurwitz[n_, s_, y_, 1] :=
 zetaHurwitz[n, s, y, 1] = HarmonicNumber[Floor[n], s] - HarmonicNumber[y, s]
zetaHurwitz[n_, s_, y_, 2] := zetaHurwitz[n, s, y, 2] =
  Sum[(m^(-2 s)) + 2 (m^-s) (zetaHurwitz[Floor[n / m], s, m, 1]), {m, y + 1, Floor[n^(1 / 2)]}]
zetaHurwitz[n_, s_, y_, k_] := zetaHurwitz[n, s, y, k] =
  Sum[(m^(-s k)) + k (m^(-s (k - 1))) zetaHurwitz[Floor[n / (m^(k - 1))], s, m, 1] +
    Sum[binomial[k, j] (m^-s)^j zetaHurwitz[Floor[n / (m^j)], s, m, k - j], {j, 1, k - 2}],
   {m, y + 1, Floor[n^(1 / k)]}]

zeta[n_, s_, 1] := Expand@Sum[binomial[1, k] zetaHurwitz[n, s, 1, k], {k, 0, 1}]
zeta[n_, s_, z_] := Expand@Sum[binomial[z, k] zetaHurwitz[n, s, 1, k], {k, 0, Log2@n}]

ka[n_] := ka[n] = FullSimplify[MangoldtLambda[n] / Log[n]]


zeta[200, N@ZetaZero@1, 1]
-0.42408 + 0.907121 i
```

```
DiscretePlot[{Re[D[zeta[n, .01 + N@ZetaZero@5, z], z] /. z → 0],
  Re[Log[Zeta[.01 + N@ZetaZero@5]] + ppp[n, .01 + N@ZetaZero@5]]}, {n, 1, 2000}]
```

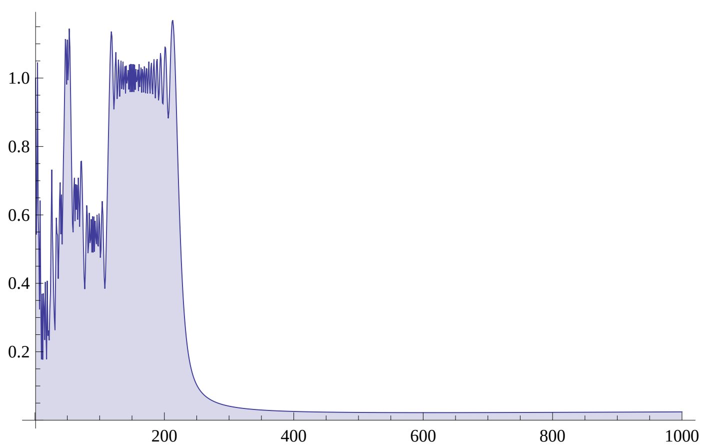
Infinity::indet : Indeterminate expression $(9.21686\times10^{-6} - 0.0303604\,i) + -\infty + \infty$ encountered. ≫



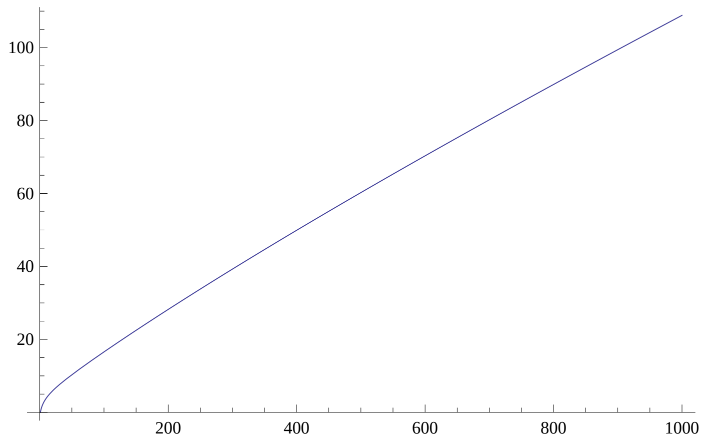

```
Log[Zeta[1 + 4. I]]
```

−0.38203 + 0.080295 i

```
DiscretePlot[Abs[HarmonicNumber[n, N@ZetaZero@1000]], {n, 1, 1000}]
```



```
ppp[n_, s_] := -Gamma[0, (s - 1) Log[n]] + Gamma[0, s Log[n]] + Log[s / (s - 1)]
Plot[Abs[ppp[n, 1. I]], {n, 1, 1000}]
```




```
Log[Zeta[ZetaZero@1]]
```

$-\infty$