

```

d2[n_, k_] := Sum[d2[j, k - 1] d2[n / j, 1], {j, Divisors[n]}];
d2[n_, 1] := 1; d2[1, 1] := 0; d2[n_, 0] := 0; d2[1, 0] := 1
dd[n_, z_] := Sum[FactorialPower[z, a] / a! d2[n, a], {a, 0, Log[2, n]}]
PrimeKappa[n_, 0] := If[n == 1, 1, 0]
PrimeKappa[n_, 1] := If[n == 1, 0, FullSimplify[MangoldtLambda[n] / Log[n]]]
PrimeKappa[n_, k_] := Sum[PrimeKappa[j, k - 1] PrimeKappa[n / j, 1], {j, Divisors[n]}]
dda[n_, z_] := Sum[z^k / k! PrimeKappa[n, k], {k, 0, Log[2, n]}]

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Table[{n, Expand[Roots[Expand[dda[n, x]] == 0, x]]}, {n, 2, 40}] // TableForm

```

```

2      x == 0
3      x == 0
4      x == 0 || x == -1
5      x == 0
6      x == 0 || x == 0
7      x == 0
8      x == 0 || x == -2 || x == -1
9      x == 0 || x == -1
10     x == 0 || x == 0
11     x == 0
12     x == 0 || x == 0 || x == -1
13     x == 0
14     x == 0 || x == 0
15     x == 0 || x == 0
16     x == 0 || x == -3 || x == -2 || x == -1
17     x == 0
18     x == 0 || x == 0 || x == -1
19     x == 0
20     x == 0 || x == 0 || x == -1
21     x == 0 || x == 0
22     x == 0 || x == 0
23     x == 0
24     x == 0 || x == 0 || x == -2 || x == -1
25     x == 0 || x == -1
26     x == 0 || x == 0
27     x == 0 || x == -2 || x == -1
28     x == 0 || x == 0 || x == -1
29     x == 0
30     x == 0 || x == 0 || x == 0
31     x == 0
32     x == 0 || x == -4 || x == -3 || x == -2 || x == -1
33     x == 0 || x == 0
34     x == 0 || x == 0
35     x == 0 || x == 0
36     x == 0 || x == 0 || x == -1 || x == -1
37     x == 0
38     x == 0 || x == 0
39     x == 0 || x == 0
40     x == 0 || x == 0 || x == -2 || x == -1

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```

E2a[n_, k_, a_] :=
  E2a[n, k, a] = Sum[E2a[n / j, k - 1, a], {j, 2, n}] - a Sum[E2a[n / (a j), k - 1, a], {j, 1, n / a}];
E2a[n_, 0, a_] := 1
EE[n_, z_, b_] :=
  EE[n, z, b] = Sum[FactorialPower[z, a] / a! E2a[n, a, b], {a, 0, Log[If[b > 2, 2, b], n]}]
EEa[n_, z_, b_] := EEa[n, z, b] =
  Sum[Binomial[z, a] E2a[n, a, b], {a, 0, Log[If[b > 2, 2, b], n]}]
bins[z_, a_] := Product[(z - k), {k, 0, a - 1}] / a!
EEb[n_, z_, b_] :=
  EEb[n, z, b] = Sum[bins[z, a] E2a[n, a, b], {a, 0, Log[If[b > 2, 2, b], n]}]
EEc[n_, z_, b_] := Expand[Sum[bins[z, a] E2a[n, a, b], {a, 0, Log[If[b > 2, 2, b], n]}]]
ee[n_, z_, b_] := EEc[n, z, b] - EEc[n - 1, z, b]

Table[{n, ee[n, z, 2]}, {n, 2, 100}] // TableForm

```

2	$-z$
3	$z$
4	$-\frac{3z}{2} + \frac{z^2}{2}$
5	$z$
6	$-z^2$
7	$z$
8	$-\frac{7z}{3} + \frac{3z^2}{2} - \frac{z^3}{6}$
9	$\frac{z}{2} + \frac{z^2}{2}$
10	$-z^2$
11	$z$
12	$-\frac{3z^2}{2} + \frac{z^3}{2}$
13	$z$
14	$-z^2$
15	$z^2$
16	$-\frac{15z}{4} + \frac{83z^2}{24} - \frac{3z^3}{4} + \frac{z^4}{24}$
17	$z$
18	$-\frac{z^2}{2} - \frac{z^3}{2}$
19	$z$
20	$-\frac{3z^2}{2} + \frac{z^3}{2}$
21	$z^2$
22	$-z^2$
23	$z$
24	$-\frac{7z^2}{3} + \frac{3z^3}{2} - \frac{z^4}{6}$
25	$\frac{z}{2} + \frac{z^2}{2}$
26	$-z^2$
27	$\frac{z}{3} + \frac{z^2}{2} + \frac{z^3}{6}$
28	$-\frac{3z^2}{2} + \frac{z^3}{2}$
29	$z$
30	$-z^3$
31	$z$
32	$-\frac{31z}{5} + \frac{29z^2}{4} - \frac{55z^3}{24} + \frac{z^4}{4} - \frac{z^5}{120}$
33	$z^2$
34	$-z^2$

$$\begin{array}{ll}
35 & z^2 \\
36 & -\frac{3z^2}{4} - \frac{z^3}{2} + \frac{z^4}{4} \\
37 & z \\
38 & -z^2 \\
39 & z^2 \\
40 & -\frac{7z^2}{3} + \frac{3z^3}{2} - \frac{z^4}{6} \\
41 & z \\
42 & -z^3 \\
43 & z \\
44 & -\frac{3z^2}{2} + \frac{z^3}{2} \\
45 & \frac{z^2}{2} + \frac{z^3}{2} \\
46 & -z^2 \\
47 & z \\
48 & -\frac{15z^2}{4} + \frac{83z^3}{24} - \frac{3z^4}{4} + \frac{z^5}{24} \\
49 & \frac{z}{2} + \frac{z^2}{2} \\
50 & -\frac{z^2}{2} - \frac{z^3}{2} \\
51 & z^2 \\
52 & -\frac{3z^2}{2} + \frac{z^3}{2} \\
53 & z \\
54 & -\frac{z^2}{3} - \frac{z^3}{2} - \frac{z^4}{6} \\
55 & z^2 \\
56 & -\frac{7z^2}{3} + \frac{3z^3}{2} - \frac{z^4}{6} \\
57 & z^2 \\
58 & -z^2 \\
59 & z \\
60 & -\frac{3z^3}{2} + \frac{z^4}{2} \\
61 & z \\
62 & -z^2 \\
63 & \frac{z^2}{2} + \frac{z^3}{2} \\
64 & -\frac{21z}{2} + \frac{5237z^2}{360} - \frac{95z^3}{16} + \frac{137z^4}{144} - \frac{z^5}{16} + \frac{z^6}{720} \\
65 & z^2 \\
66 & -z^3 \\
67 & z \\
68 & -\frac{3z^2}{2} + \frac{z^3}{2} \\
69 & z^2 \\
70 & -z^3 \\
71 & z \\
72 & -\frac{7z^2}{6} - \frac{5z^3}{12} + \frac{2z^4}{3} - \frac{z^5}{12} \\
73 & z \\
74 & -z^2 \\
75 & \frac{z^2}{2} + \frac{z^3}{2} \\
76 & -\frac{3z^2}{2} + \frac{z^3}{2} \\
77 & z^2 \\
78 & -z^3
\end{array}$$

$$\begin{aligned}
79 & \quad z \\
80 & \quad -\frac{15 z^2}{4} + \frac{83 z^3}{24} - \frac{3 z^4}{4} + \frac{z^5}{24} \\
81 & \quad \frac{z}{4} + \frac{11 z^2}{24} + \frac{z^3}{4} + \frac{z^4}{24} \\
82 & \quad -z^2 \\
83 & \quad z \\
84 & \quad -\frac{3 z^3}{2} + \frac{z^4}{2} \\
85 & \quad z^2 \\
86 & \quad -z^2 \\
87 & \quad z^2 \\
88 & \quad -\frac{7 z^2}{3} + \frac{3 z^3}{2} - \frac{z^4}{6} \\
89 & \quad z \\
90 & \quad -\frac{z^3}{2} - \frac{z^4}{2} \\
91 & \quad z^2 \\
92 & \quad -\frac{3 z^2}{2} + \frac{z^3}{2} \\
93 & \quad z^2 \\
94 & \quad -z^2 \\
95 & \quad z^2 \\
96 & \quad -\frac{31 z^2}{5} + \frac{29 z^3}{4} - \frac{55 z^4}{24} + \frac{z^5}{4} - \frac{z^6}{120} \\
97 & \quad z \\
98 & \quad -\frac{z^2}{2} - \frac{z^3}{2} \\
99 & \quad \frac{z^2}{2} + \frac{z^3}{2} \\
100 & \quad -\frac{3 z^2}{4} - \frac{z^3}{2} + \frac{z^4}{4}
\end{aligned}$$

**Table[ {n, Expand[Roots[ Expand[ee[n, x, 3 / 2]] == 0, x]]}, {n, 2, 40}] // TableForm**

$$\begin{aligned}
2 & \quad x == 0 \\
3 & \quad x == -\frac{1}{3} \mid \mid x == 0 \\
4 & \quad x == 0 \mid \mid x == \frac{35}{18} - \frac{\sqrt{865}}{18} \mid \mid x == \frac{35}{18} + \frac{\sqrt{865}}{18} \\
5 & \quad x == 0 \mid \mid x == \frac{7}{6} - \frac{\sqrt{17}}{6} \mid \mid x == \frac{7}{6} + \frac{\sqrt{17}}{6} \\
6 & \quad x == 0 \mid \mid x == \frac{86}{27} + \frac{1}{81} \left( 8454672 - 243 \sqrt{133594341} \right)^{1/3} + \frac{1}{27} \left( 313136 + 9 \sqrt{133594341} \right)^{1/3} \mid \mid x == \frac{86}{27} \\
7 & \quad x == 0 \mid \mid x == \frac{5}{3} + \frac{1}{27} \left( 43011 - 1458 \sqrt{321} \right)^{1/3} + \frac{1}{3} \left( 59 + 2 \sqrt{321} \right)^{1/3} \mid \mid x == \frac{5}{3} - \frac{1}{54} \left( 43011 - 1458 \sqrt{321} \right) \\
8 & \quad x == 0 \mid \mid x == \frac{5}{2} + \frac{1}{54} \sqrt{2495 + \frac{14551957}{\left( 48038673185 + 162 i \sqrt{29484847057714747} \right)^{1/3}}} + \left( 48038673185 + 162 i \sqrt{29484848} \right) \\
9 & \quad x == 0 \mid \mid x == \frac{8}{9} + \frac{67}{9 \left( 224 + 3 i \sqrt{27843} \right)^{1/3}} + \frac{1}{9} \left( 224 + 3 i \sqrt{27843} \right)^{1/3} \mid \mid x == \frac{8}{9} - \frac{67}{18 \left( 224 + 3 i \sqrt{27843} \right)^{1/3}} - \frac{1}{6 \sqrt{3}} \left( 2 \right) \\
10 & \quad x == 0 \mid \mid x == 0 \\
11 & \quad x == 0 \mid \mid x == \frac{13}{6} - \frac{1}{6} \sqrt{55 - \frac{185 \times 3^{2/3}}{\left( -5861 + 2 \sqrt{13336549} \right)^{1/3}}} + \left( 3 \left( -5861 + 2 \sqrt{13336549} \right) \right)^{1/3} - \frac{1}{2} \sqrt{\frac{110}{9} - \frac{\left( -5861 + 2 \right)}{6}} \\
12 & \quad x == 0 \mid \mid x == \text{Root} \left[ -9720 + 16434 \#1 - 13745 \#1^2 + 5605 \#1^3 - 1215 \#1^4 + 81 \#1^5 \&, 1 \right] \mid \mid x == \text{Root} \left[ -9 \right. \\
13 & \quad x == 0
\end{aligned}$$

$$\begin{aligned}
14 \quad & x = 0 \mid \mid x = 0 \mid \mid x = 2 + \frac{19}{3 \left( 48 + i \sqrt{4555} \right)^{1/3}} + \frac{1}{3} \left( 48 + i \sqrt{4555} \right)^{1/3} \mid \mid x = 2 - \frac{19}{6 \left( 48 + i \sqrt{4555} \right)^{1/3}} - \frac{19 i}{2 \sqrt{3} \left( 48 + i \sqrt{4555} \right)^{1/3}} \\
15 \quad & x = \frac{2}{3} \mid \mid x = 0 \mid \mid x = 0 \\
16 \quad & x = 0 \mid \mid x = \text{Root} \left[ -960 + 13252 \#1 - 26340 \#1^2 + 13205 \#1^3 - 3240 \#1^4 + 243 \#1^5 \&, 1 \right] \mid \mid x = \text{Root} \left[ - \right. \\
17 \quad & x = 0 \mid \mid x = 1 - \frac{5}{3 \left( -12 + \sqrt{269} \right)^{1/3}} + \frac{1}{3} \left( -12 + \sqrt{269} \right)^{1/3} \mid \mid x = 1 + \frac{5}{6 \left( -12 + \sqrt{269} \right)^{1/3}} + \frac{5 i}{2 \sqrt{3} \left( -12 + \sqrt{269} \right)^{1/3}} - \frac{1}{6} \left( - \right. \\
18 \quad & x = 0 \mid \mid x = \text{Root} \left[ 174960 - 437612 \#1 + 425992 \#1^2 - 151725 \#1^3 + 29085 \#1^4 - 5103 \#1^5 + 243 \#1^6 \&, \right. \\
19 \quad & x = 0 \\
20 \quad & x = 2 \mid \mid x = 0 \mid \mid x = 0 \\
21 \quad & x = 0 \mid \mid x = 0 \mid \mid x = \frac{31}{12} - \frac{1}{12} \sqrt{329 - \frac{984}{\left( 5626 - \sqrt{29791009} \right)^{1/3}} - 8 \left( 5626 - \sqrt{29791009} \right)^{1/3}} - \frac{1}{2} \sqrt{\frac{329}{18} + \frac{1}{3 \left( 5626 - \sqrt{29791009} \right)^{1/3}}} \\
22 \quad & x = 0 \mid \mid x = 0 \\
23 \quad & x = 0 \mid \mid x = \text{Root} \left[ 5120 - 17496 \#1 + 24954 \#1^2 - 23805 \#1^3 + 10125 \#1^4 - 1539 \#1^5 + 81 \#1^6 \&, 1 \right] \mid \mid x = \\
24 \quad & x = 0 \mid \mid x = 0 \mid \mid x = -\frac{37}{9} - \frac{2017}{9 \left( 93421 - 18 \sqrt{1610322} \right)^{1/3}} - \frac{1}{9} \left( 93421 - 18 \sqrt{1610322} \right)^{1/3} \mid \mid x = -\frac{37}{9} + \frac{1}{18 \left( 93421 - 18 \sqrt{1610322} \right)^{1/3}} \\
25 \quad & x = 0 \mid \mid x = \frac{5}{18} - \frac{i \sqrt{119}}{18} \mid \mid x = \frac{5}{18} + \frac{i \sqrt{119}}{18} \\
26 \quad & x = 0 \mid \mid x = \text{Root} \left[ -3674160 + 7501612 \#1 - 6912108 \#1^2 + 3483081 \#1^3 - 1186920 \#1^4 + 234738 \#1^5 - 27 \right. \\
27 \quad & x = 0 \mid \mid x = \frac{7}{4} - \frac{1}{2} \sqrt{\frac{67}{12} + \frac{1}{3} 2^{2/3} \left( \frac{1}{3} \left( 521 + i \sqrt{361295} \right) \right)^{1/3} + \frac{52}{3^{2/3} \left( \frac{1}{2} \left( 521 + i \sqrt{361295} \right) \right)^{1/3}}} - \frac{1}{2} \sqrt{\frac{67}{6} - \frac{1}{3} 2^{2/3} \left( \frac{1}{3} \left( 521 + i \sqrt{361295} \right) \right)^{1/3} + \frac{52}{3^{2/3} \left( \frac{1}{2} \left( 521 + i \sqrt{361295} \right) \right)^{1/3}}} \\
28 \quad & x = 0 \mid \mid x = 0 \mid \mid x = -1 \\
29 \quad & x = \frac{2}{3} \mid \mid x = 0 \\
30 \quad & x = 0 \mid \mid x = 0 \mid \mid x = \frac{11}{12} - \frac{i \sqrt{95}}{12} \mid \mid x = \frac{11}{12} + \frac{i \sqrt{95}}{12} \\
31 \quad & x = 0 \mid \mid x = \text{Root} \left[ -2560 + 3384 \#1 + 414 \#1^2 - 8595 \#1^3 + 5985 \#1^4 - 1269 \#1^5 + 81 \#1^6 \&, 1 \right] \mid \mid x = \text{Root} \left[ - \right. \\
32 \quad & x = 0 \mid \mid x = -\frac{145}{4} + \frac{1}{4} \sqrt{\frac{1}{3} \left( 65315 + \frac{113876 \times 2^{2/3}}{\left( -2864615 + 9 i \sqrt{468411316953} \right)^{1/3}} + 4 \left( 2 \left( -2864615 + 9 i \sqrt{468411316953} \right)^{1/3} \right) \right)} \\
33 \quad & x = \frac{2}{3} \mid \mid x = 0 \mid \mid x = 0 \\
34 \quad & x = 0 \mid \mid x = 0 \mid \mid x = \frac{5}{3} + \frac{1}{27} \left( 43011 - 1458 \sqrt{321} \right)^{1/3} + \frac{1}{3} \left( 59 + 2 \sqrt{321} \right)^{1/3} \mid \mid x = \frac{5}{3} - \frac{1}{54} \left( 43011 - 1458 \sqrt{321} \right)^{1/3} \\
35 \quad & x = 0 \mid \mid x = 0 \mid \mid x = \text{Root} \left[ 346880 - 739368 \#1 + 649782 \#1^2 - 274995 \#1^3 + 59535 \#1^4 - 6237 \#1^5 + 27 \right. \\
36 \quad & x = 0 \mid \mid x = 0 \mid \mid x = -\frac{25}{24} + \frac{1}{24} \sqrt{3761 - \frac{51512}{\left( 4296178 - 57 \sqrt{5598701085} \right)^{1/3}} - 8 \left( 4296178 - 57 \sqrt{5598701085} \right)^{1/3}} \\
37 \quad & x = 0 \\
38 \quad & x = 0 \mid \mid x = 0 \mid \mid x = \frac{5}{2} + \frac{1}{6} \sqrt{\frac{1}{3} \left( -115 + \frac{35293}{\left( 3312395 + 6 i \sqrt{916357046187} \right)^{1/3}} + \left( 3312395 + 6 i \sqrt{916357046187} \right)^{1/3} \right)} \\
39 \quad & x = 0 \mid \mid x = \text{Root} \left[ 9797760 - 26342192 \#1 + 29564292 \#1^2 - 16350012 \#1^3 + 5455107 \#1^4 - 110224 \#1^5 \&, 1 \right] \mid \mid x = \\
40 \quad & x = 0 \mid \mid x = 0 \mid \mid x = -2 \mid \mid x = -1
\end{aligned}$$