```
ClearAll["Global`*"]
```

```
t[n_, a_] := Mod[n, a] - Mod[n - 1, a]
s1[n_, 0] := 1; s1[n_, k_] := s1[n, k] = s1[n, k] = Sum[ s1[Floor[n / j], k - 1], {j, 1, n}]
s1p[n_, k_] := s1p[n, k] = Sum[(-1)^(k - j) Binomial[k, j] s1[n, j], {j, 0, k}]

s2[n_, 0] := 1; s2[n_, k_] := s2[n, k] = s2[n, k] = Sum[t[j, 3] s2[Floor[n / j], k - 1], {j, 1, n}]
s2p[n_, k_] := s2p[n, k] = Sum[(-1)^(k - j) Binomial[k, j] s2[n, j], {j, 0, k}]
s2pa[n_, a_] := Sum[(-1)^j Binomial[a, j] 3^j s1p[Floor[n / 3^j], a], {j, 0, a}]
s2a[n_, a_] := Sum[(-1)^j Binomial[a, j] 3^j s1[Floor[n / 3^j], a], {j, 0, a}]
s2m[n_, a_, b_] :=
 s2m[n, a, b] = Sum[(-1)^j Binomial[a, j] b^j s1[Floor[n / b^j], a], {j, 0, a}]
s2mp[n_, a_, b_] := s2mp[n, a, b] = Sum[(-1)^(a - j) Binomial[a, j] s2m[n, j, b], {j, 0, a}]
s2p2[n_, k_] := s2p2[n, k] = Sum[ t[j, 3] s2p2[Floor[n / j], k - 1], {j, 2, n}]; s2p2[n_, 0] := 1

nmod[n_, b_] := s2m[n, 1, b] - s2m[n - 1, 1, b]
LAdd[n_, b_] := Sum[ b^k / k, {k, 1, Log[b, n]}]
lin[n_, b_] := Sum[(-1)^(k + 1) / k s2mp[n, k, b], {k, 1, Log[2, n]}]
lin2[n_] := Sum[(-1)^(k + 1) / k s2p[n, k], {k, 1, Log[2, n]}]
lin3[n_] := Sum[(-1)^(k + 1) / k s1p[n, k], {k, 1, Log[2, n]}]
```

```
s1p[100, 3]
```

```
324
```

```
s2p2[100, 5]
```

```
-9
```

```
s2p[100, 5]
```

```
-9
```

```
s2mp[100, 5, 3]
```

```
-9
```

```
s2pa[100, 5]
```

```
279
```

```
lin[100, 2] + LAdd[100, 2]
```

$$\frac{428}{15}$$

```
lin[100, 3] + LAdd[100, 3]
```

$$\frac{428}{15}$$

```
lin2[100] + LAdd[100, 3]
```

$$\frac{428}{15}$$

```
lin3[100]
```

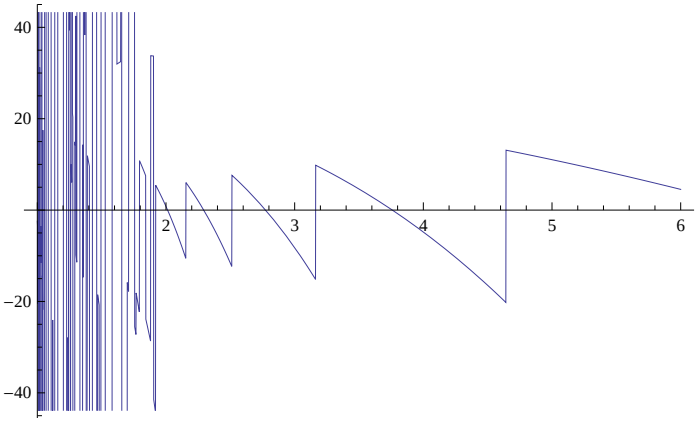
$$\frac{428}{15}$$

```
lin[100, 5 / 2] + LAdd[100, 5 / 2]
```

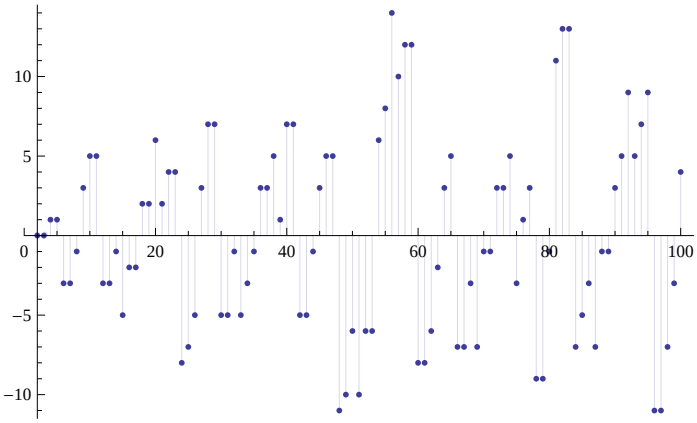
$$\frac{428}{15}$$

```
lin[100, 2.1]
```

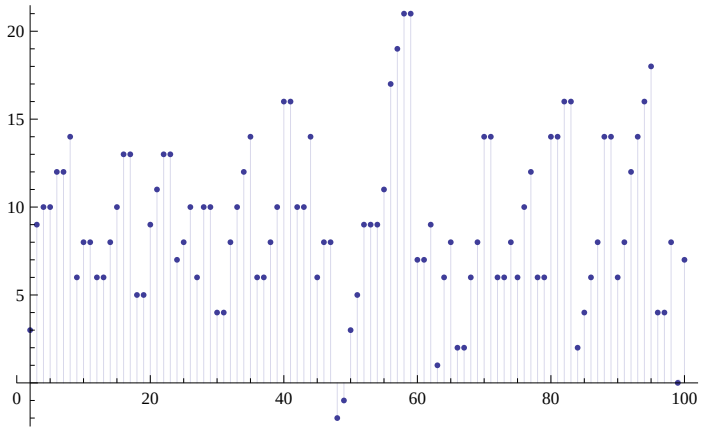
```
Plot[lin[100, n], {n, 1, 6}]
```



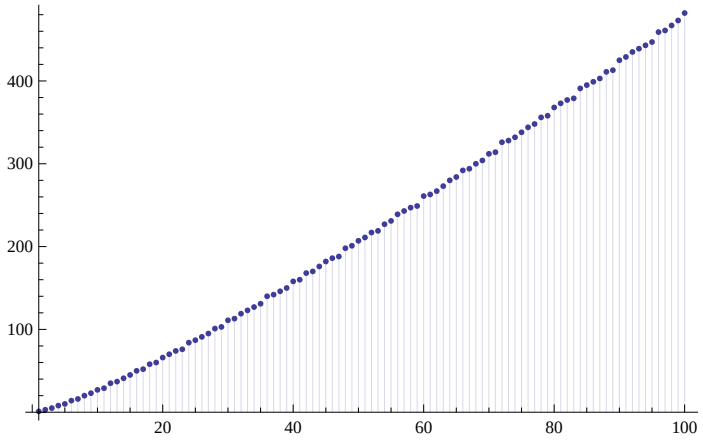

```
DiscretePlot[s2p[n, 2], {n, 2, 100}]
```



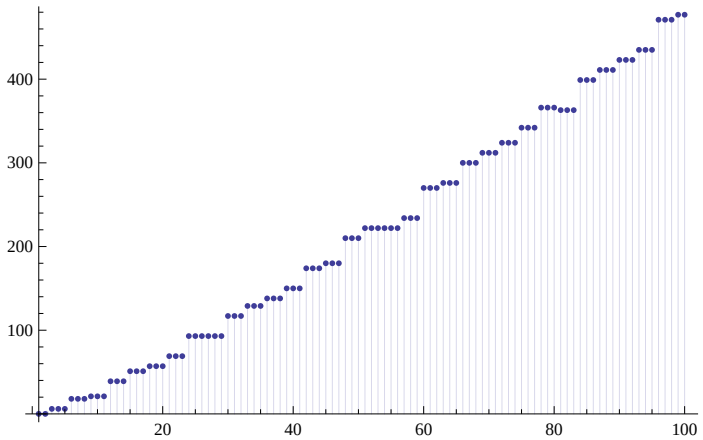
```
DiscretePlot[ s2pa[n, 2], {n, 2, 100}]
```




```
DiscretePlot[s1[n, 2], {n, 1, 100}]
```




```
DiscretePlot[s1[n, 2] - s2[n, 2], {n, 1, 100}]
```



```
f[n_] := (s1[2 n, 2] - s2[2 n, 2]) / 4

Table[{n, s2[n, 1], Sum[(-1)^j Binomial[1, j] 3^j s1[Floor[n / 3^j], 1], {j, 0, 2}]},
  {n, 1, 100}] // TableForm
```

1	1	1
2	2	2
3	0	0
4	1	1
5	2	2
6	0	0
7	1	1
8	2	2
9	0	0
10	1	1
11	2	2
12	0	0
13	1	1
14	2	2
15	0	0
16	1	1
17	2	2
18	0	0
19	1	1
20	2	2
21	0	0
22	1	1
23	2	2
24	0	0
25	1	1
26	2	2
27	0	0
28	1	1
29	2	2
30	0	0
31	1	1
32	2	2
33	0	0
34	1	1
35	2	2
36	0	0
37	1	1
38	2	2
39	0	0
40	1	1
41	2	2
42	0	0
43	1	1
44	2	2
45	0	0
46	1	1
47	2	2
48	0	0
49	1	1
50	2	2
51	0	0

```
52    1    1
53    2    2
54    0    0
55    1    1
56    2    2
57    0    0
58    1    1
59    2    2
60    0    0
61    1    1
62    2    2
63    0    0
64    1    1
65    2    2
66    0    0
67    1    1
68    2    2
69    0    0
70    1    1
71    2    2
72    0    0
73    1    1
74    2    2
75    0    0
76    1    1
77    2    2
78    0    0
79    1    1
80    2    2
81    0    0
82    1    1
83    2    2
84    0    0
85    1    1
86    2    2
87    0    0
88    1    1
89    2    2
90    0    0
91    1    1
92    2    2
93    0    0
94    1    1
95    2    2
96    0    0
97    1    1
98    2    2
99    0    0
100   1    1
```

```
Table[{n, s2[n, 2], Sum[(-1)^j Binomial[2, j] 3^j s1[Floor[n / 3^j], 2], {j, 0, 2}]},
  {n, 1, 100}] // TableForm
```

```
1     1     1
2     3     3
3     -1    -1
4     2     2
```

5	4	4
6	−4	−4
7	−2	−2
8	2	2
9	2	2
10	6	6
11	8	8
12	−4	−4
13	−2	−2
14	2	2
15	−6	−6
16	−1	−1
17	1	1
18	1	1
19	3	3
20	9	9
21	1	1
22	5	5
23	7	7
24	−9	−9
25	−6	−6
26	−2	−2
27	2	2
28	8	8
29	10	10
30	−6	−6
31	−4	−4
32	2	2
33	−6	−6
34	−2	−2
35	2	2
36	2	2
37	4	4
38	8	8
39	0	0
40	8	8
41	10	10
42	−6	−6
43	−4	−4
44	2	2
45	2	2
46	6	6
47	8	8
48	−12	−12
49	−9	−9
50	−3	−3
51	−11	−11
52	−5	−5
53	−3	−3
54	5	5
55	9	9
56	17	17
57	9	9
58	13	13
59	15	15
60	−9	−9

61	−7	−7
62	−3	−3
63	−3	−3
64	4	4
65	8	8
66	−8	−8
67	−6	−6
68	0	0
69	−8	−8
70	0	0
71	2	2
72	2	2
73	4	4
74	8	8
75	−4	−4
76	2	2
77	6	6
78	−10	−10
79	−8	−8
80	2	2
81	10	10
82	14	14
83	16	16
84	−8	−8
85	−4	−4
86	0	0
87	−8	−8
88	0	0
89	2	2
90	2	2
91	6	6
92	12	12
93	4	4
94	8	8
95	12	12
96	−12	−12
97	−10	−10
98	−4	−4
99	−4	−4
100	5	5

```
Expand[(x - 2)^3]
```

$-8 + 12\,x - 6\,x^2 + x^3$

```
Table[{n, s2[n, 3], Sum[(-1)^j Binomial[3, j] 3^j s1[Floor[n / 3^j], 3], {j, 0, 3}]},
  {n, 1, 100}] // TableForm
```

1	1	1
2	4	4
3	−2	−2
4	4	4
5	7	7
6	−11	−11
7	−8	−8
8	2	2
9	8	8

10	17	17
11	20	20
12	−16	−16
13	−13	−13
14	−4	−4
15	−22	−22
16	−7	−7
17	−4	−4
18	14	14
19	17	17
20	35	35
21	17	17
22	26	26
23	29	29
24	−31	−31
25	−25	−25
26	−16	−16
27	−6	−6
28	12	12
29	15	15
30	−39	−39
31	−36	−36
32	−15	−15
33	−33	−33
34	−24	−24
35	−15	−15
36	21	21
37	24	24
38	33	33
39	15	15
40	45	45
41	48	48
42	−6	−6
43	−3	−3
44	15	15
45	33	33
46	42	42
47	45	45
48	−45	−45
49	−39	−39
50	−21	−21
51	−39	−39
52	−21	−21
53	−18	−18
54	12	12
55	21	21
56	51	51
57	33	33
58	42	42
59	45	45
60	−63	−63
61	−60	−60
62	−51	−51
63	−33	−33
64	−5	−5
65	4	4

66	−50	−50
67	−47	−47
68	−29	−29
69	−47	−47
70	−20	−20
71	−17	−17
72	43	43
73	46	46
74	55	55
75	19	19
76	37	37
77	46	46
78	−8	−8
79	−5	−5
80	40	40
81	46	46
82	55	55
83	58	58
84	−50	−50
85	−41	−41
86	−32	−32
87	−50	−50
88	−20	−20
89	−17	−17
90	37	37
91	46	46
92	64	64
93	46	46
94	55	55
95	64	64
96	−62	−62
97	−59	−59
98	−41	−41
99	−23	−23
100	13	13

```
Expand[(x - 2)^4]
```

$16 - 32\,x + 24\,x^2 - 8\,x^3 + x^4$

```
Table[{n, s2[n, 4], Sum[(-1)^j Binomial[4, j] 3^j s1[Floor[n / 3^j], 4], {j, 0, 4}]},
  {n, 1, 100}] // TableForm
```

1	1	1
2	5	5
3	−3	−3
4	7	7
5	11	11
6	−21	−21
7	−17	−17
8	3	3
9	19	19
10	35	35
11	39	39
12	−41	−41
13	−37	−37

14	−21	−21
15	−53	−53
16	−18	−18
17	−14	−14
18	50	50
19	54	54
20	94	94
21	62	62
22	78	78
23	82	82
24	−78	−78
25	−68	−68
26	−52	−52
27	−44	−44
28	−4	−4
29	0	0
30	−128	−128
31	−124	−124
32	−68	−68
33	−100	−100
34	−84	−84
35	−68	−68
36	92	92
37	96	96
38	112	112
39	80	80
40	160	160
41	164	164
42	36	36
43	40	40
44	80	80
45	144	144
46	160	160
47	164	164
48	−116	−116
49	−106	−106
50	−66	−66
51	−98	−98
52	−58	−58
53	−54	−54
54	−22	−22
55	−6	−6
56	74	74
57	42	42
58	58	58
59	62	62
60	−258	−258
61	−254	−254
62	−238	−238
63	−174	−174
64	−90	−90
65	−74	−74
66	−202	−202
67	−198	−198
68	−158	−158
69	−190	−190

```
70    -126   -126
71    -122   -122
72    198    198
73    202    202
74    218    218
75    138    138
76    178    178
77    194    194
78    66     66
79    70     70
80    210    210
81    194    194
82    210    210
83    214    214
84    -106   -106
85    -90    -90
86    -74    -74
87    -106   -106
88    -26    -26
89    -22    -22
90    234    234
91    250    250
92    290    290
93    258    258
94    274    274
95    290    290
96    -158   -158
97    -154   -154
98    -114   -114
99    -50    -50
100   50     50
```

```
Table[{n, s1[n, 1], Sum[Binomial[k + 0, 0] 3^k s2[Floor[n / 3^k], 1], {k, 0, Log[3, n]}]},
  {n, 1, 100}] // TableForm
```

```
1     1     1
2     2     2
3     3     3
4     4     4
5     5     5
6     6     6
7     7     7
8     8     8
9     9     9
10    10    10
11    11    11
12    12    12
13    13    13
14    14    14
15    15    15
16    16    16
17    17    17
18    18    18
19    19    19
20    20    20
21    21    21
22    22    22
```

23	23	23
24	24	24
25	25	25
26	26	26
27	27	27
28	28	28
29	29	29
30	30	30
31	31	31
32	32	32
33	33	33
34	34	34
35	35	35
36	36	36
37	37	37
38	38	38
39	39	39
40	40	40
41	41	41
42	42	42
43	43	43
44	44	44
45	45	45
46	46	46
47	47	47
48	48	48
49	49	49
50	50	50
51	51	51
52	52	52
53	53	53
54	54	54
55	55	55
56	56	56
57	57	57
58	58	58
59	59	59
60	60	60
61	61	61
62	62	62
63	63	63
64	64	64
65	65	65
66	66	66
67	67	67
68	68	68
69	69	69
70	70	70
71	71	71
72	72	72
73	73	73
74	74	74
75	75	75
76	76	76
77	77	77
78	78	78

```
79     79     79
80     80     80
81     81     81
82     82     82
83     83     83
84     84     84
85     85     85
86     86     86
87     87     87
88     88     88
89     89     89
90     90     90
91     91     91
92     92     92
93     93     93
94     94     94
95     95     95
96     96     96
97     97     97
98     98     98
99     99     99
100    100    100
```

```
Sum[ 2^k s2[ Floor[100 / (2^k)], 1], {k, 0, Log[2, 100]}]
```

```
100
```

```
Expand[
  (x^0 + 2 x^1 + 4 x^2 + 8 x^3 + 16 x^4 + 32 x^5 + 64 x^6 + 128 x^7 + 256 x^8 + 512 x^9 + 1024 x^10)^2]
```

$$1 + 4\,x + 12\,x^2 + 32\,x^3 + 80\,x^4 + 192\,x^5 + 448\,x^6 + 1024\,x^7 + 2304\,x^8 + 5120\,x^9 + 11\,264\,x^{10} + 20\,480\,x^{11} + 36\,864\,x^{12} + 65\,536\,x^{13} + 114\,688\,x^{14} + 196\,608\,x^{15} + 327\,680\,x^{16} + 524\,288\,x^{17} + 786\,432\,x^{18} + 1\,048\,576\,x^{19} + 1\,048\,576\,x^{20}$$

```
ff[n_] := 2^(n - 1) n
```

```
ff[5]
```

```
80
```

```
Table[{n, s1[n, 2], Sum[ Binomial[k + 1, 1] 3^k s2[ Floor[n / 3^k], 2], {k, 0, Log[3, n]}]},
  {n, 1, 100}] // TableForm
```

```
1      1      1
2      3      3
3      5      5
4      8      8
5      10     10
6      14     14
7      16     16
8      20     20
9      23     23
10     27     27
11     29     29
12     35     35
13     37     37
14     41     41
15     45     45
16     50     50
```

17	52	52
18	58	58
19	60	60
20	66	66
21	70	70
22	74	74
23	76	76
24	84	84
25	87	87
26	91	91
27	95	95
28	101	101
29	103	103
30	111	111
31	113	113
32	119	119
33	123	123
34	127	127
35	131	131
36	140	140
37	142	142
38	146	146
39	150	150
40	158	158
41	160	160
42	168	168
43	170	170
44	176	176
45	182	182
46	186	186
47	188	188
48	198	198
49	201	201
50	207	207
51	211	211
52	217	217
53	219	219
54	227	227
55	231	231
56	239	239
57	243	243
58	247	247
59	249	249
60	261	261
61	263	263
62	267	267
63	273	273
64	280	280
65	284	284
66	292	292
67	294	294
68	300	300
69	304	304
70	312	312
71	314	314
72	326	326

```
73     328    328
74     332    332
75     338    338
76     344    344
77     348    348
78     356    356
79     358    358
80     368    368
81     373    373
82     377    377
83     379    379
84     391    391
85     395    395
86     399    399
87     403    403
88     411    411
89     413    413
90     425    425
91     429    429
92     435    435
93     439    439
94     443    443
95     447    447
96     459    459
97     461    461
98     467    467
99     473    473
100    482    482
```

```
Expand[
  (x^0 + 2 x^1 + 4 x^2 + 8 x^3 + 16 x^4 + 32 x^5 + 64 x^6 + 128 x^7 + 256 x^8 + 512 x^9 + 1024 x^10)^3]
```

$$1 + 6\,x + 24\,x^2 + 80\,x^3 + 240\,x^4 + 672\,x^5 + 1792\,x^6 + 4608\,x^7 + 11\,520\,x^8 + 28\,160\,x^9 + 67\,584\,x^{10} + 153\,600\,x^{11} + 335\,872\,x^{12} + 712\,704\,x^{13} + 1\,474\,560\,x^{14} + 2\,981\,888\,x^{15} + 5\,898\,240\,x^{16} + 11\,403\,264\,x^{17} + 21\,495\,808\,x^{18} + 39\,321\,600\,x^{19} + 69\,206\,016\,x^{20} + 115\,343\,360\,x^{21} + 188\,743\,680\,x^{22} + 301\,989\,888\,x^{23} + 469\,762\,048\,x^{24} + 704\,643\,072\,x^{25} + 1\,006\,632\,960\,x^{26} + 1\,342\,177\,280\,x^{27} + 1\,610\,612\,736\,x^{28} + 1\,610\,612\,736\,x^{29} + 1\,073\,741\,824\,x^{30}$$

```
Table[{n, s1[n, 3], Sum[Binomial[k + 2, 2] 3^k s2[Floor[n / 3^k], 3], {k, 0, Log[3, n]}]},
  {n, 1, 100}] // TableForm
```

```
1      1       1
2      4       4
3      7       7
4      13      13
5      16      16
6      25      25
7      28      28
8      38      38
9      44      44
10     53      53
11     56      56
12     74      74
13     77      77
14     86      86
15     95      95
16     110     110
17     113     113
```

18	131	131
19	134	134
20	152	152
21	161	161
22	170	170
23	173	173
24	203	203
25	209	209
26	218	218
27	228	228
28	246	246
29	249	249
30	276	276
31	279	279
32	300	300
33	309	309
34	318	318
35	327	327
36	363	363
37	366	366
38	375	375
39	384	384
40	414	414
41	417	417
42	444	444
43	447	447
44	465	465
45	483	483
46	492	492
47	495	495
48	540	540
49	546	546
50	564	564
51	573	573
52	591	591
53	594	594
54	624	624
55	633	633
56	663	663
57	672	672
58	681	681
59	684	684
60	738	738
61	741	741
62	750	750
63	768	768
64	796	796
65	805	805
66	832	832
67	835	835
68	853	853
69	862	862
70	889	889
71	892	892
72	952	952
73	955	955

```
74     964     964
75     982     982
76     1000    1000
77     1009    1009
78     1036    1036
79     1039    1039
80     1084    1084
81     1099    1099
82     1108    1108
83     1111    1111
84     1165    1165
85     1174    1174
86     1183    1183
87     1192    1192
88     1222    1222
89     1225    1225
90     1279    1279
91     1288    1288
92     1306    1306
93     1315    1315
94     1324    1324
95     1333    1333
96     1396    1396
97     1399    1399
98     1417    1417
99     1435    1435
100    1471    1471
```

```
Expand[(x^0 + 2 x^1 + 4 x^2 + 8 x^3 + 16 x^4 +
     32 x^5 + 64 x^6 + 128 x^7 + 256 x^8 + 512 x^9 + 1024 x^10)^4]
```

$$1 + 8\,x + 40\,x^2 + 160\,x^3 + 560\,x^4 + 1792\,x^5 + 5376\,x^6 + 15\,360\,x^7 + 42\,240\,x^8 + 112\,640\,x^9 + 292\,864\,x^{10} + 737\,280\,x^{11} + 1\,798\,144\,x^{12} + 4\,259\,840\,x^{13} + 9\,830\,400\,x^{14} + 22\,151\,168\,x^{15} + 48\,824\,320\,x^{16} + 105\,381\,888\,x^{17} + 222\,822\,400\,x^{18} + 461\,373\,440\,x^{19} + 934\,281\,216\,x^{20} + 1\,845\,493\,760\,x^{21} + 3\,565\,158\,400\,x^{22} + 6\,744\,440\,832\,x^{23} + 12\,499\,025\,920\,x^{24} + 22\,682\,796\,032\,x^{25} + 40\,265\,318\,400\,x^{26} + 69\,793\,218\,560\,x^{27} + 117\,843\,165\,184\,x^{28} + 193\,273\,528\,320\,x^{29} + 307\,090\,161\,664\,x^{30} + 472\,446\,402\,560\,x^{31} + 708\,669\,603\,840\,x^{32} + 1\,030\,792\,151\,040\,x^{33} + 1\,443\,109\,011\,456\,x^{34} + 1\,924\,145\,348\,608\,x^{35} + 2\,405\,181\,685\,760\,x^{36} + 2\,748\,779\,069\,440\,x^{37} + 2\,748\,779\,069\,440\,x^{38} + 2\,199\,023\,255\,552\,x^{39} + 1\,099\,511\,627\,776\,x^{40}$$

```
Table[{n, s1[n, 4], Sum[Binomial[k + 3, 3] 3^k s2[Floor[n / 3^k], 4], {k, 0, Log[3, n]}]},
  {n, 1, 100}] // TableForm
```

```
1      1       1
2      5       5
3      9       9
4      19      19
5      23      23
6      39      39
7      43      43
8      63      63
9      73      73
10     89      89
11     93      93
12     133     133
13     137     137
14     153     153
15     169     169
```

16	204	204
17	208	208
18	248	248
19	252	252
20	292	292
21	308	308
22	324	324
23	328	328
24	408	408
25	418	418
26	434	434
27	454	454
28	494	494
29	498	498
30	562	562
31	566	566
32	622	622
33	638	638
34	654	654
35	670	670
36	770	770
37	774	774
38	790	790
39	806	806
40	886	886
41	890	890
42	954	954
43	958	958
44	998	998
45	1038	1038
46	1054	1054
47	1058	1058
48	1198	1198
49	1208	1208
50	1248	1248
51	1264	1264
52	1304	1304
53	1308	1308
54	1388	1388
55	1404	1404
56	1484	1484
57	1500	1500
58	1516	1516
59	1520	1520
60	1680	1680
61	1684	1684
62	1700	1700
63	1740	1740
64	1824	1824
65	1840	1840
66	1904	1904
67	1908	1908
68	1948	1948
69	1964	1964
70	2028	2028
71	2032	2032

```
72    2232   2232
73    2236   2236
74    2252   2252
75    2292   2292
76    2332   2332
77    2348   2348
78    2412   2412
79    2416   2416
80    2556   2556
81    2591   2591
82    2607   2607
83    2611   2611
84    2771   2771
85    2787   2787
86    2803   2803
87    2819   2819
88    2899   2899
89    2903   2903
90    3063   3063
91    3079   3079
92    3119   3119
93    3135   3135
94    3151   3151
95    3167   3167
96    3391   3391
97    3395   3395
98    3435   3435
99    3475   3475
100   3575   3575
```

```
Expand[ (x - 2) ^ 4]
```

$16 - 32\,x + 24\,x^2 - 8\,x^3 + x^4$

```
Sum[(-1) ^j Binomial[4, j] 2^j st[Floor[n / 2^j], 4], {j, 0, 4}]
```

$$16\,\text{st}\left[\text{Floor}\left[\frac{n}{16}\right], 4\right] - 32\,\text{st}\left[\text{Floor}\left[\frac{n}{8}\right], 4\right] + 24\,\text{st}\left[\text{Floor}\left[\frac{n}{4}\right], 4\right] - 8\,\text{st}\left[\text{Floor}\left[\frac{n}{2}\right], 4\right] + \text{st}[\text{Floor}[n], 4]$$

```
Table[{n, s2[n, 4], s1[n, 4] - 8 s1[Floor[n / 2], 4] + 24 s1[Floor[n / 4], 4] -
    32 s1[Floor[n / 8], 4] + 16 s1[Floor[n / 16], 4]}, {n, 1, 100}] // TableForm
```

```
Table[{n, s2[n, 4],  Sum[(-1) ^j Binomial[4, j] 3^j s1[Floor[n / 3^j], 4], {j, 0, 4}]},
  {n, 1, 100}] // TableForm
```

```
1     1     1
2     5     5
3     -3    -3
4     7     7
5     11    11
6     -21   -21
7     -17   -17
8     3     3
9     19    19
10    35    35
```

11	39	39
12	−41	−41
13	−37	−37
14	−21	−21
15	−53	−53
16	−18	−18
17	−14	−14
18	50	50
19	54	54
20	94	94
21	62	62
22	78	78
23	82	82
24	−78	−78
25	−68	−68
26	−52	−52
27	−44	−44
28	−4	−4
29	0	0
30	−128	−128
31	−124	−124
32	−68	−68
33	−100	−100
34	−84	−84
35	−68	−68
36	92	92
37	96	96
38	112	112
39	80	80
40	160	160
41	164	164
42	36	36
43	40	40
44	80	80
45	144	144
46	160	160
47	164	164
48	−116	−116
49	−106	−106
50	−66	−66
51	−98	−98
52	−58	−58
53	−54	−54
54	−22	−22
55	−6	−6
56	74	74
57	42	42
58	58	58
59	62	62
60	−258	−258
61	−254	−254
62	−238	−238
63	−174	−174
64	−90	−90
65	−74	−74
66	−202	−202

67	−198	−198
68	−158	−158
69	−190	−190
70	−126	−126
71	−122	−122
72	198	198
73	202	202
74	218	218
75	138	138
76	178	178
77	194	194
78	66	66
79	70	70
80	210	210
81	194	194
82	210	210
83	214	214
84	−106	−106
85	−90	−90
86	−74	−74
87	−106	−106
88	−26	−26
89	−22	−22
90	234	234
91	250	250
92	290	290
93	258	258
94	274	274
95	290	290
96	−158	−158
97	−154	−154
98	−114	−114
99	−50	−50
100	50	50

s2[50, 5]

−168

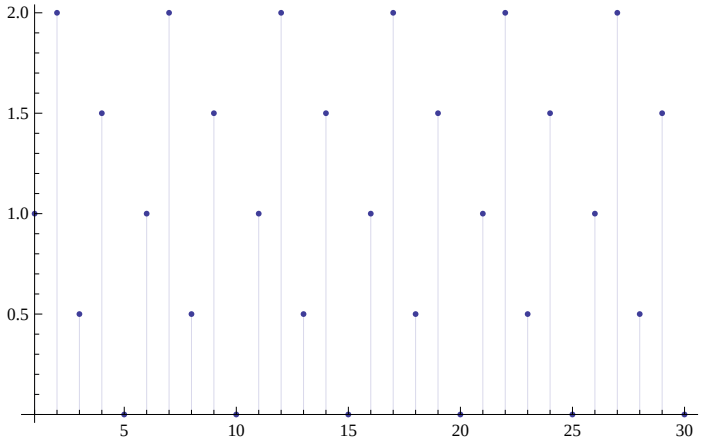
s2a[50, 5]

−168

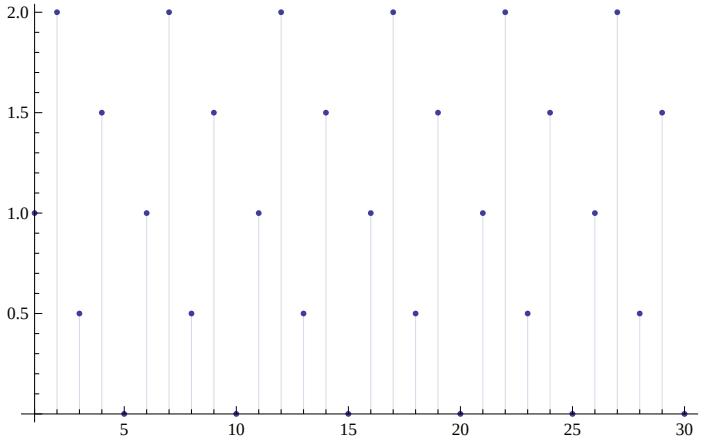
s2a[40, 1]

1

```
DiscretePlot[s2m[n, 1, 5 / 2], {n, 1, 30}]
```




```
DiscretePlot[Mod[n, 5 / 2], {n, 1, 30}]
```




```
Table[{n, aa = s2m[n, 1, 5 / 2], bb = Mod[n, 5 / 2], aa - bb}, {n, 1, 30, .3}] // TableForm
```

1.	1	1.	0.
1.3	1	1.3	−0.3
1.6	1	1.6	−0.6
1.9	1	1.9	−0.9
2.2	2	2.2	−0.2
2.5	$-\frac{1}{2}$	0.	−0.5
2.8	$-\frac{1}{2}$	0.3	−0.8
3.1	$\frac{1}{2}$	0.6	−0.1
3.4	$\frac{1}{2}$	0.9	−0.4
3.7	$\frac{1}{2}$	1.2	−0.7
4.	$\frac{3}{2}$	1.5	0.
4.3	$\frac{3}{2}$	1.8	−0.3
4.6	$\frac{3}{2}$	2.1	−0.6
4.9	$\frac{3}{2}$	2.4	−0.9
5.2	0	0.2	−0.2
5.5	0	0.5	−0.5
5.8	0	0.8	−0.8
6.1	1	1.1	−0.1

6.4	1	1.4	−0.4
6.7	1	1.7	−0.7
7.	2	2.	0.
7.3	2	2.3	−0.3
7.6	$-\frac{1}{2}$	0.1	−0.6
7.9	$-\frac{1}{2}$	0.4	−0.9
8.2	$\frac{1}{2}$	0.7	−0.2
8.5	$\frac{1}{2}$	1.	−0.5
8.8	$\frac{1}{2}$	1.3	−0.8
9.1	$\frac{3}{2}$	1.6	−0.1
9.4	$\frac{3}{2}$	1.9	−0.4
9.7	$\frac{3}{2}$	2.2	−0.7
10.	0	0.	0.
10.3	0	0.3	−0.3
10.6	0	0.6	−0.6
10.9	0	0.9	−0.9
11.2	1	1.2	−0.2
11.5	1	1.5	−0.5
11.8	1	1.8	−0.8
12.1	2	2.1	−0.1
12.4	2	2.4	−0.4
12.7	$-\frac{1}{2}$	0.2	−0.7
13.	$\frac{1}{2}$	0.5	0.
13.3	$\frac{1}{2}$	0.8	−0.3
13.6	$\frac{1}{2}$	1.1	−0.6
13.9	$\frac{1}{2}$	1.4	−0.9
14.2	$\frac{3}{2}$	1.7	−0.2
14.5	$\frac{3}{2}$	2.	−0.5
14.8	$\frac{3}{2}$	2.3	−0.8
15.1	0	0.1	−0.1
15.4	0	0.4	−0.4
15.7	0	0.7	−0.7
16.	1	1.	0.
16.3	1	1.3	−0.3
16.6	1	1.6	−0.6
16.9	1	1.9	−0.9
17.2	2	2.2	−0.2
17.5	$-\frac{1}{2}$	0.	−0.5
17.8	$-\frac{1}{2}$	0.3	−0.8
18.1	$\frac{1}{2}$	0.6	−0.1
18.4	$\frac{1}{2}$	0.9	−0.4
18.7	$\frac{1}{2}$	1.2	−0.7
19.	$\frac{3}{2}$	1.5	0.
19.3	$\frac{3}{2}$	1.8	−0.3
19.6	$\frac{3}{2}$	2.1	−0.6

19.9	$\frac{3}{2}$	2.4	−0.9
20.2	0	0.2	−0.2
20.5	0	0.5	−0.5
20.8	0	0.8	−0.8
21.1	1	1.1	−0.1
21.4	1	1.4	−0.4
21.7	1	1.7	−0.7
22.	2	2.	0.
22.3	2	2.3	−0.3
22.6	$-\frac{1}{2}$	0.1	−0.6
22.9	$-\frac{1}{2}$	0.4	−0.9
23.2	$\frac{1}{2}$	0.7	−0.2
23.5	$\frac{1}{2}$	1.	−0.5
23.8	$\frac{1}{2}$	1.3	−0.8
24.1	$\frac{3}{2}$	1.6	−0.1
24.4	$\frac{3}{2}$	1.9	−0.4
24.7	$\frac{3}{2}$	2.2	−0.7
25.	0	0.	0.
25.3	0	0.3	−0.3
25.6	0	0.6	−0.6
25.9	0	0.9	−0.9
26.2	1	1.2	−0.2
26.5	1	1.5	−0.5
26.8	1	1.8	−0.8
27.1	2	2.1	−0.1
27.4	2	2.4	−0.4
27.7	$-\frac{1}{2}$	0.2	−0.7
28.	$\frac{1}{2}$	0.5	0.
28.3	$\frac{1}{2}$	0.8	−0.3
28.6	$\frac{1}{2}$	1.1	−0.6
28.9	$\frac{1}{2}$	1.4	−0.9
29.2	$\frac{3}{2}$	1.7	−0.2
29.5	$\frac{3}{2}$	2.	−0.5
29.8	$\frac{3}{2}$	2.3	−0.8

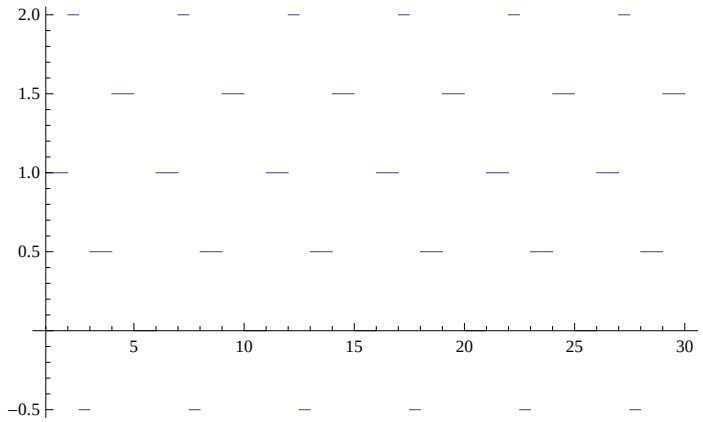
```
Table[{n, s2m[n, 1, a = 5 / 2], s2m[n - 1, 1, a], nmod[n, a], Mod[n, a], Mod[n - 1, a]},
  {n, 1, 30, .3}] // TableForm
```

1.	1	0	1	1.	0.
1.3	1	0	1	1.3	0.3
1.6	1	0	1	1.6	0.6
1.9	1	0	1	1.9	0.9
2.2	2	1	1	2.2	1.2
2.5	$-\frac{1}{2}$	1	$-\frac{3}{2}$	0.	1.5
2.8	$-\frac{1}{2}$	1	$-\frac{3}{2}$	0.3	1.8
3.1	$\frac{1}{2}$	2	$-\frac{3}{2}$	0.6	2.1
3.4	$\frac{1}{2}$	2	$-\frac{3}{2}$	0.9	2.4
3.7	$\frac{1}{2}$	$-\frac{1}{2}$	1	1.2	0.2

4.	$\frac{3}{2}$	$\frac{1}{2}$	1	1.5	0.5
4.3	$\frac{3}{2}$	$\frac{1}{2}$	1	1.8	0.8
4.6	$\frac{3}{2}$	$\frac{1}{2}$	1	2.1	1.1
4.9	$\frac{3}{2}$	$\frac{1}{2}$	1	2.4	1.4
5.2	0	$\frac{3}{2}$	$-\frac{3}{2}$	0.2	1.7
5.5	0	$\frac{3}{2}$	$-\frac{3}{2}$	0.5	2.
5.8	0	$\frac{3}{2}$	$-\frac{3}{2}$	0.8	2.3
6.1	1	0	1	1.1	0.1
6.4	1	0	1	1.4	0.4
6.7	1	0	1	1.7	0.7
7.	2	1	1	2.	1.
7.3	2	1	1	2.3	1.3
7.6	$-\frac{1}{2}$	1	$-\frac{3}{2}$	0.1	1.6
7.9	$-\frac{1}{2}$	1	$-\frac{3}{2}$	0.4	1.9
8.2	$\frac{1}{2}$	2	$-\frac{3}{2}$	0.7	2.2
8.5	$\frac{1}{2}$	$-\frac{1}{2}$	1	1.	0.
8.8	$\frac{1}{2}$	$-\frac{1}{2}$	1	1.3	0.3
9.1	$\frac{3}{2}$	$\frac{1}{2}$	1	1.6	0.6
9.4	$\frac{3}{2}$	$\frac{1}{2}$	1	1.9	0.9
9.7	$\frac{3}{2}$	$\frac{1}{2}$	1	2.2	1.2
10.	0	$\frac{3}{2}$	$-\frac{3}{2}$	0.	1.5
10.3	0	$\frac{3}{2}$	$-\frac{3}{2}$	0.3	1.8
10.6	0	$\frac{3}{2}$	$-\frac{3}{2}$	0.6	2.1
10.9	0	$\frac{3}{2}$	$-\frac{3}{2}$	0.9	2.4
11.2	1	0	1	1.2	0.2
11.5	1	0	1	1.5	0.5
11.8	1	0	1	1.8	0.8
12.1	2	1	1	2.1	1.1
12.4	2	1	1	2.4	1.4
12.7	$-\frac{1}{2}$	1	$-\frac{3}{2}$	0.2	1.7
13.	$\frac{1}{2}$	2	$-\frac{3}{2}$	0.5	2.
13.3	$\frac{1}{2}$	2	$-\frac{3}{2}$	0.8	2.3
13.6	$\frac{1}{2}$	$-\frac{1}{2}$	1	1.1	0.1
13.9	$\frac{1}{2}$	$-\frac{1}{2}$	1	1.4	0.4
14.2	$\frac{3}{2}$	$\frac{1}{2}$	1	1.7	0.7
14.5	$\frac{3}{2}$	$\frac{1}{2}$	1	2.	1.
14.8	$\frac{3}{2}$	$\frac{1}{2}$	1	2.3	1.3
15.1	0	$\frac{3}{2}$	$-\frac{3}{2}$	0.1	1.6
15.4	0	$\frac{3}{2}$	$-\frac{3}{2}$	0.4	1.9
15.7	0	$\frac{3}{2}$	$-\frac{3}{2}$	0.7	2.2
16.	1	0	1	1.	0.
16.3	1	0	1	1.3	0.3
16.6	1	0	1	1.6	0.6

16.9	1	0	1	1.9	0.9
17.2	2	1	1	2.2	1.2
17.5	$-\frac{1}{2}$	1	$-\frac{3}{2}$	0.	1.5
17.8	$-\frac{1}{2}$	1	$-\frac{3}{2}$	0.3	1.8
18.1	$\frac{1}{2}$	2	$-\frac{3}{2}$	0.6	2.1
18.4	$\frac{1}{2}$	2	$-\frac{3}{2}$	0.9	2.4
18.7	$\frac{1}{2}$	$-\frac{1}{2}$	1	1.2	0.2
19.	$\frac{3}{2}$	$\frac{1}{2}$	1	1.5	0.5
19.3	$\frac{3}{2}$	$\frac{1}{2}$	1	1.8	0.8
19.6	$\frac{3}{2}$	$\frac{1}{2}$	1	2.1	1.1
19.9	$\frac{3}{2}$	$\frac{1}{2}$	1	2.4	1.4
20.2	0	$\frac{3}{2}$	$-\frac{3}{2}$	0.2	1.7
20.5	0	$\frac{3}{2}$	$-\frac{3}{2}$	0.5	2.
20.8	0	$\frac{3}{2}$	$-\frac{3}{2}$	0.8	2.3
21.1	1	0	1	1.1	0.1
21.4	1	0	1	1.4	0.4
21.7	1	0	1	1.7	0.7
22.	2	1	1	2.	1.
22.3	2	1	1	2.3	1.3
22.6	$-\frac{1}{2}$	1	$-\frac{3}{2}$	0.1	1.6
22.9	$-\frac{1}{2}$	1	$-\frac{3}{2}$	0.4	1.9
23.2	$\frac{1}{2}$	2	$-\frac{3}{2}$	0.7	2.2
23.5	$\frac{1}{2}$	$-\frac{1}{2}$	1	1.	0.
23.8	$\frac{1}{2}$	$-\frac{1}{2}$	1	1.3	0.3
24.1	$\frac{3}{2}$	$\frac{1}{2}$	1	1.6	0.6
24.4	$\frac{3}{2}$	$\frac{1}{2}$	1	1.9	0.9
24.7	$\frac{3}{2}$	$\frac{1}{2}$	1	2.2	1.2
25.	0	$\frac{3}{2}$	$-\frac{3}{2}$	0.	1.5
25.3	0	$\frac{3}{2}$	$-\frac{3}{2}$	0.3	1.8
25.6	0	$\frac{3}{2}$	$-\frac{3}{2}$	0.6	2.1
25.9	0	$\frac{3}{2}$	$-\frac{3}{2}$	0.9	2.4
26.2	1	0	1	1.2	0.2
26.5	1	0	1	1.5	0.5
26.8	1	0	1	1.8	0.8
27.1	2	1	1	2.1	1.1
27.4	2	1	1	2.4	1.4
27.7	$-\frac{1}{2}$	1	$-\frac{3}{2}$	0.2	1.7
28.	$\frac{1}{2}$	2	$-\frac{3}{2}$	0.5	2.
28.3	$\frac{1}{2}$	2	$-\frac{3}{2}$	0.8	2.3
28.6	$\frac{1}{2}$	$-\frac{1}{2}$	1	1.1	0.1
28.9	$\frac{1}{2}$	$-\frac{1}{2}$	1	1.4	0.4
29.2	$\frac{3}{2}$	$\frac{1}{2}$	1	1.7	0.7
29.5	$\frac{3}{2}$	$\frac{1}{2}$	1	2.	1.

```
Plot[s2m[n, 1, 5 / 2], {n, 1, 30}]
```




```
Plot[Mod[n, 5 / 2], {n, 1, 30}]
```