```
N@Zeta[1000 I - .5]
```

```
-123.541 + 90.0001 i
```

```
N@Zeta[1000 I + .5]
```

```
0.356334 + 0.931998 i
```

```
0.3563343671948756`
```

```
N@Zeta[1000 I - 1]
```

```
-1575.360186805219 + 1109.537965641057 i
```

```
N@Zeta[1000 I + 2]
```

```
0.953262 - 0.110723 i
```

```
N@Zeta[1000 I - 1 / 4]
```

```
-33.61469020757686 + 26.75552974311933 i
```

```
N@ZetaZero[10 000]
```

```
0.5` + 9877.782654005501` i
```

```
0.5`
+14.134725141734695` i
```

```
0.5` + 49.7738324776723` i
```

```
0.5` +
 236.5242296658162` i
```

```
0.5` + 1419.4224809459956` i
```

```
N[Zeta[1 + 1000 I]]
```

```
0.9409368682928779` + 0.045226652072133965` i
```

```
x^(A + f I) / (A + f I) /. x → 100. /. A → -.5 /. f → Im@ZetaZero@1
```

```
0.00560796 + 0.00430588 i
```

```
x^A E^(Log[x] f I) / (A + f I) /. x → 100. /. A → -.5 /. f → Im@ZetaZero@1
```

```
0.00560796 + 0.00430588 i
```

```
(A - f I) x^A E^(Log[x] f I) / (A^2 + f^2) /. x → 100. /. A → -.5 /. f → Im@ZetaZero@1
```

```
0.00560796 + 0.00430588 i
```

```
(x^A / (A^2 + f^2)) (A - f I) (Cos[f Log[x]] + I Sin[f Log[x]]) /. x → 100. /. A → -.5 /.
 f → Im@ZetaZero@1
```

```
0.00560796 + 0.00430588 i
```

```
(x^A / (A^2 + f^2)) (A Cos[f Log[x]] - f I Cos[f Log[x]] + I A Sin[f Log[x]] + f Sin[f Log[x]]) /.
   x → 100. /. A → -.5 /. f → Im@ZetaZero@1
```

```
0.00560796 + 0.00430588 i
```

```
Expand[(A + f I) (A - f I)]
```

$A^2 + f^2$

```
(x^A / (A^2 + f^2)) (A Cos[f Log[x]] + f Sin[f Log[x]] + I (A Sin[f Log[x]] - f Cos[f Log[x]])) /.
   x → 100. /. A → -.5 /. f → Im@ZetaZero@1
```

$0.00560796 + 0.00430588\,i$

```
(x^A / (A^2 + f^2)) (A Cos[f Log[x]] + f Sin[f Log[x]]) /. x → 100. /. A → -.5 /.
 f → Im@ZetaZero@1
```

0.00560796

```
(x^A / (A^2 + f^2)) (I (A Sin[f Log[x]] - f Cos[f Log[x]])) /. x → 100. /. A → -.5 /.
 f → Im@ZetaZero@1
```

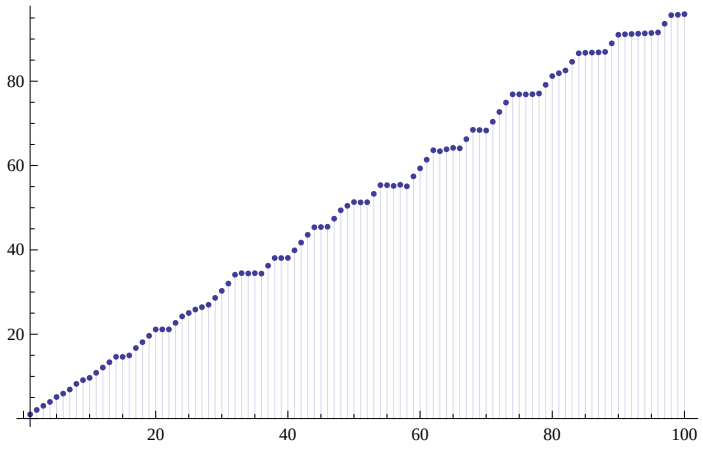
$0. + 0.00430588\,i$

```
FullSimplify[2 (x^A / (A^2 + f^2)) (A Cos[f Log[x]] + f Sin[f Log[x]]) /. A → (1 / 2)]
```

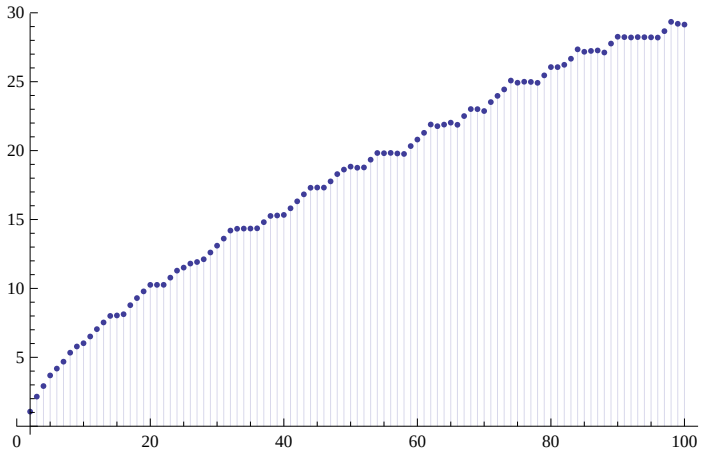
$$\frac{4\sqrt{x}\ (\text{Cos}[f\ \text{Log}[x]] + 2\,f\,\text{Sin}[f\ \text{Log}[x]])}{1 + 4\,f^2}$$

```
vl[x_, f_] := 4 √x (Cos[f Log[x]] + 2 f Sin[f Log[x]]) / (1 + 4 f^2)
sm[x_, t_] := Sum[vl[x, N@Im@ZetaZero@j], {j, 1, t}]
sm2[x_, t_] :=
 Sum[x^N[ZetaZero[j]] / N[ZetaZero[j]] + x^N[ZetaZero[-j]] / N[ZetaZero[-j]], {j, 1, t}]
sm3[x_, t_] := Sum[Re[ExpIntegralEi[N[ZetaZero[j]] Log[x]] +
    ExpIntegralEi[N[ZetaZero[-j]] Log[x]]], {j, 1, t}]
```

```
DiscretePlot[x - sm[x, 300], {x, 1, 100}]
```



```
DiscretePlot[LogIntegral[x] - sm3[x, 100], {x, 2, 100}]
```




```
N@vl[100, Im@ZetaZero@1]
```

1.05794

```
Re[2 x^(A + f I) / (A + f I)] /. x → 100. /. A → -.5 /. f → Im@ZetaZero@1
```

0.0112159

```
sm2[100, 1]
```

1.05794 + 0. i

```
x^N[ZetaZero[j]] / N[ZetaZero[j]] /. x → 100. /. j → 1
```

0.528969 + 0.469138 i

```
ExpIntegralEi[10. + I]
```

1568.28 + 1914.17 i

```
be[x_, t_] := EulerGamma + Log[Abs[x]] + Sum[x^k / k / k!, {k, 1, t}]
bea[x_, t_] := Table[x^k / k / k!, {k, 1, t}]
```

```
be[10. + 14 I, 20]
```

38 682. - 95 730.4 i

```
N@LogIntegral[100^ZetaZero[1]]
```

1.35421 + 6.31436 i

```
N@ExpIntegralEi[ZetaZero[1] Log[100.]]
```

0.116437 + 3.24171 i

```
Log[100.]
```

4.60517

```
be[ZetaZero[1] Log[100.], 10]
```

$-3.25919 \times 10^{10} + 1.88372 \times 10^{10}$ i

```
N@ExpIntegralEi[ZetaZero[1] Log[100.]]
```

$0.116437 + 3.24171\,i$

```
(D[LaguerreL[-z, ZetaZero[1] Log[100.]], z] /. z → 0) +
 Log[ZetaZero[1] Log[100.]] + EulerGamma
```

$0.116437 + 3.24171\,i$

```
N@LogIntegral[100^ZetaZero[1]]
```

$1.35421 + 6.31436\,i$

```
br[x_, t_] := Sum[(-1)^(k + 1) / k (-1)^k GammaRegularized[k, 0, -Log[x]], {k, 1, t}]
```

```
br[100., 50]
```

$28.0217 - 2.09386 \times 10^{-14}\,i$

```
FullSimplify[((A^2 + (f I)^2))^(1 / 2)]
```

$\sqrt{A^2 - f^2}$

```
be[x_, t_] := EulerGamma + Log[Abs[x]] + Sum[x^k / k / k!, {k, 1, t}]
be2[A_, f_, t_] := EulerGamma + Log[Abs[A + f I]] + Sum[(A + f I)^k / k / k!, {k, 1, t}]
be2a[A_, f_, t_] := ExpIntegralEi[A + f I]
be3[A_, f_, t_] := EulerGamma + (1 / 2) Log[A^2 + f^2] + Sum[(A + f I)^k / k / k!, {k, 1, t}]
be4[A_, f_, t_] := EulerGamma + (1 / 2) Log[A^2 + f^2] +
  Sum[Sum[Binomial[k, 2 j] (-1)^j A^(k - 2 j) f^(2 j), {j, 0, Floor[k / 2]}] / k / k!, {k, 1, t}]
be4b[A_, f_, t_] := Sum[
  Sum[Binomial[k, 2 j] (-1)^j A^(k - 2 j) f^(2 j), {j, 0, Floor[k / 2]}] / k / k!, {k, 1, t}]
be4a[A_, f_, t_] :=
 I Sum[Sum[Binomial[k, 2 j + 1] (-1)^j A^(k - 2 j - 1) f^(2 j + 1), {j, 0, Floor[k / 2]}] / k / k!,
   {k, 1, t}]
sm3[x_, t_] := N@Table[Re[ExpIntegralEi[ZetaZero[j] Log[x]] +
     ExpIntegralEi[ZetaZero[-j] Log[x]]], {j, 1, t}]
sm4[x_, t_, t2_] := N[2 Table[be4[Re[ZetaZero[j]] Log[x],
     Im[ZetaZero[j]] Log[x], t2], {j, 1, t}]]
```

```
N@be4[.5 Log[10.], Im@ZetaZero@1 Log[10.], 101.]
```

0.0885639

```
N@be[(.5 + Im@ZetaZero@1 I) Log[10], 100.]
```

$0.088171 + 1.56513\,i$

```
N@be[10 + I, 100.]
```

$1568.28 + 1914.07\,i$

```
Expand[(A + f I)^5]
```

$A^5 + 5\,i\,A^4\,f - 10\,A^3\,f^2 - 10\,i\,A^2\,f^3 + 5\,A\,f^4 + i\,f^5$

```
FullSimplify[Sum[Binomial[k, j] A^(k - j) I^j f^j, {j, 0, k}]]
```

$$A^k \left(1 + \frac{i\,f}{A}\right)^k$$

```
Sum[Binomial[k, 2 j] (-1)^j A^(k - 2 j) f^(2 j), {j, 0, Floor[k / 2]}]
```

$A^5 - 10\,A^3\,f^2 + 5\,A\,f^4$

```
sm3[3, 10]
```

```
{0.045795, -0.132346, 0.0914722, 0.0943251, -0.0955739,
 -0.0357901, 0.0631589, -0.0327046, 0.041109, -0.0604237}
```

```
sm4[3, 10, 2000]
```

```
{0.045795, -0.132346, 0.0914682, 0.0933917,
 -0.094645, -1.19346, -74.6775, -445.621, -149 764., 109 505.}
```

```
N@Sum[be4[Re[ZetaZero[j]] Log[10.], Im[ZetaZero[j]] Log[10.], 150], {j, 1, 1}]
```

```
0.0885639
```

```
ExpIntegralEi[ZetaZero[1] Log[10.]]
```

```
0.0880046 + 3.10063 i
```

```
2 N@be[ZetaZero[1] Log[10], 2000]
```

```
0.176055 + 3.13088 i
```

```
1 / (1 - 2^(1 - s)) /. s → .3 + 4 I
```

```
0.377123 - 0.0878555 i
```

```
1 / (1 - 2^(1 - A - f I)) /. A → .3 /. f -> 4
```

```
0.377123 - 0.0878555 i
```

```
1 / (1 - 2^(1 - A)  2^(-f I)) /. A → .3 /. f -> 4
```

```
0.377123 - 0.0878555 i
```

```
1 / (1 - 2^(1 - A) E^(-f Log[2] I )) /. A → .3 /. f -> 4
```

```
0.377123 - 0.0878555 i
```

```
1 / (1 - 2^(1 - A) (Cos[-f Log[2]] + I Sin[-f Log[2]])) /. A → .3 /. f -> 4
```

```
0.377123 - 0.0878555 i
```

```
1 / (1 - 2^(1 - A) Cos[-f Log[2]] - I 2^(1 - A) Sin[-f Log[2]]) /. A → .3 /. f -> 4
```

```
0.377123 - 0.0878555 i
```

```
(1 - 2^(1 - A) Cos[-f Log[2]] + I 2^(1 - A) Sin[-f Log[2]]) /
    ((1 - 2^(1 - A) Cos[-f Log[2]])^2 + (2^(1 - A) Sin[-f Log[2]])^2) /. A → .3 /. f -> 4
```

```
0.377123 - 0.0878555 i
```

```
(1 - 2^(1 - A) Cos[-f Log[2]] + I 2^(1 - A) Sin[-f Log[2]]) /
 ((1 - 2^(1 - A) Cos[-f Log[2]])^2 + (2^(1 - A) Sin[-f Log[2]])^2)
```

$$\frac{1 - 2^{1-A}\cos[f\log[2]] - i\,2^{1-A}\sin[f\log[2]]}{\left(1 - 2^{1-A}\cos[f\log[2]]\right)^2 + 2^{2-2A}\sin[f\log[2]]^2}$$

```
FullSimplify[((1 - 2^(1 - A) Cos[-f Log[2]])^2 + (2^(1 - A) Sin[-f Log[2]])^2)]
```

$$4^{-A}\left(4 + 4^A - 2^{2+A}\cos[f\log[2]]\right)$$

```
(1 - 2^(1 - A) Cos[-f Log[2]]) / (4^-A (4 + 4^A - 2^(2+A) Cos[f Log[2]])) +
    I (2^(1 - A) Sin[-f Log[2]]) / (4^-A (4 + 4^A - 2^(2+A) Cos[f Log[2]])) /. A → .3 /. f -> 4
```

```
0.377123 - 0.0878555 i
```

```
(1 - 2^(1 - A) Cos[-f Log[2]]) / (4^-A (4 + 4^A - 2^(2+A) Cos[f Log[2]])) +
 I (2^(1 - A) Sin[-f Log[2]]) / (4^-A (4 + 4^A - 2^(2+A) Cos[f Log[2]]))
```

```
1 / (1 - 2^(1 - s)) Sum[(-1)^(j + 1) / j^s, {j, 1, Infinity}] /. s → .3 + 4 I
```

```
0.575756 + 0.10773 i
```

```
Zeta[.3 + 4 I]
```

```
0.575756 + 0.10773 i
```

```
1 / (1 - 2^(1 - (A + f I))) Sum[(-1)^(j + 1) / j^(A + f I), {j, 1, Infinity}] /. A → .3 /. f -> 400
```

```
-1.74573 + 1.07588 i
```

```
((1 - 2^(1 - A) Cos[-f Log[2]]) / (4^-A (4 + 4^A - 2^(2+A) Cos[f Log[2]])) +
      I (2^(1 - A) Sin[-f Log[2]]) / (4^-A (4 + 4^A - 2^(2+A) Cos[f Log[2]])))
   Sum[(-1)^(j + 1) / j^(A + f I), {j, 1, Infinity}] /. A → .3 /. f → 1000
```

```
-0.920724 + 2.21155 i
```

```
((1 - 2^(1 - A) Cos[-f Log[2]]) / (4^-A (4 + 4^A - 2^(2+A) Cos[f Log[2]])) +
      I (2^(1 - A) Sin[-f Log[2]]) / (4^-A (4 + 4^A - 2^(2+A) Cos[f Log[2]])))
   Sum[(-1)^(j + 1) / j^(A + f I), {j, 1, 2000}] /. A → .3 /. f → 1000
```

```
-0.937577 + 2.19497 i
```

```
FullSimplify[D[Cos[100 Log[x]], x]]
```

$$-\frac{100 \sin[100 \log[x]]}{x}$$

```
FullSimplify[Sum[1 / n Cos[n], {n, 1, Infinity}]]
```

$$-\frac{1}{2} \log[2 - 2 \cos[1]]$$

```
Integrate[1 / n Cos[n], {n, 1, Infinity}]
```

```
-CosIntegral[1]
```

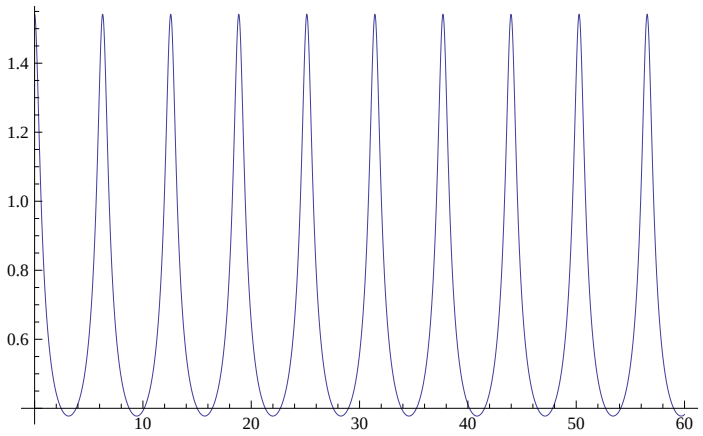
```
FullSimplify[Cos[100 t] + I Sin[100 t]]
```

$$e^{100 i t}$$

```
FullSimplify@Sum[E^(-s t), {t, 1, Infinity}]
```

$$\frac{1}{-1 + e^{s}}$$

```
Plot[Abs[1 / (-1 + E^(.5 + t I))], {t, 0, 60}]
```



```
Integrate[E^(-s t), {t, 0, x}] /. s → .3 + 5 I /. x → 12.
```

```
0.0106084 - 0.204568 i
```

```
(1 / (A + f I))
```

$$\frac{1}{A + i f}$$

```
(A - f I) / (A^2 - f^2)
```

$$\frac{A - i f}{A^2 - f^2}$$

```
A / (A^2 - f^2) - I (f) / (A^2 - f^2)
```

$$\frac{A}{A^2 - f^2} - \frac{i f}{A^2 - f^2}$$

```
Plot[(1 - e^(-s x))/s, {t, 0, 30}]
```

```
(1 - e^(-s x))/s /. s → .3 + 5 I /. x → 12.
```

```
0.0106084 - 0.204568 i
```

```
(1 - e^(-(A+f I) x))/(A + f I) /. A → .3 /. f -> 5 /. x → 12.
```

```
0.0106084 - 0.204568 i
```

```
(1 - e^(-A x) E^(-f x I))/(A + f I) /. A → .3 /. f -> 5 /. x → 12.
```

```
0.0106084 - 0.204568 i
```

```
((A - f I) (1 - e^(-A x) E^(-f x I)))/(A^2 + f^2) /. A → .3 /. f -> 5 /. x → 12.
```

```
0.0106084 - 0.204568 i
```

```
((A - f I) (1 - e^(-A x) (Cos[-f x] + I Sin[-f x])))/(A^2 + f^2) /. A → .3 /. f -> 5 /. x → 12.
```

```
0.0106084 - 0.204568 i
```

```
((A - f I) (1 - e^(-A x) Cos[-f x] - I e^(-A x) Sin[-f x]))/(A^2 + f^2) /. A → .3 /. f -> 5 /. x → 12.
```

```
0.0106084 - 0.204568 i
```

```
(((A - f I) - (A - f I) e^(-A x) Cos[-f x] - I (A - f I) e^(-A x) Sin[-f x]))/(A^2 + f^2) /. A → .3 /. f -> 5 /. x → 12.
```

```
0.0106084 - 0.204568 i
```

```
(A - A e^(-A x) Cos[-f x] - f e^(-A x) Sin[-f x])/(A^2 + f^2) + I (-f + f e^(-A x) Cos[-f x] - A e^(-A x) Sin[-f x])/(A^2 + f^2) /. A → .3 /.
  f -> 5 /. x → 12.
```

```
0.0106084 - 0.204568 i
```

$$\frac{A - e^{-A x} (A \cos[-f x] + f \sin[-f x])}{(A^2 + f^2)} - I \frac{f + e^{-A x} (A \sin[-f x] - f \cos[-f x])}{(A^2 + f^2)} /. A \to .3 /. f \to 5 /. x \to 12.$$

$$0.0106084 - 0.204568\, i$$

$$\frac{A - e^{-A x} (A \cos[f x] + f \sin[-f x])}{(A^2 + f^2)} - I \frac{f - e^{-A x} (A \sin[f x] + f \cos[f x])}{(A^2 + f^2)} /. A \to .3 /. f \to 5 /. x \to 12.$$

$$0.0106084 - 0.204568\, i$$

$$\frac{A - e^{-A x} (A \cos[f x] + f \sin[-f x])}{(A^2 + f^2)} /. A \to 0$$

$$\frac{\sin[f x]}{f}$$

$$\frac{f - e^{-A x} (A \sin[f x] + f \cos[f x])}{(A^2 + f^2)} /. A \to 0$$

$$\frac{f - f \cos[f x]}{f^2}$$

$$\frac{1}{-1 + e^{(A+f I)}} /. A \to .3 /. f \to 5 /. x \to 12.$$

$$-0.300099 + 0.629483\, i$$

$$\frac{1}{-1 + E^A e^{(f I)}} /. A \to .3 /. f \to 5 /. x \to 12.$$

$$-0.300099 + 0.629483\, i$$

$$\frac{1}{-1 + E^A (\cos[f] + I \sin[f])} /. A \to .3 /. f \to 5 /. x \to 12.$$

$$-0.300099 + 0.629483\, i$$

$$\frac{1}{-1 + E^A \cos[f] + I E^A \sin[f]} /. A \to .3 /. f \to 5 /. x \to 12.$$

$$-0.300099 + 0.629483\, i$$

$$\frac{-1 + E^A \cos[f] - I E^A \sin[f]}{(-1 + E^A \cos[f])^2 + (E^A \sin[f])^2} /. A \to .3 /. f \to 5 /. x \to 12.$$

$$-0.300099 + 0.629483\, i$$

$$\frac{-1 + E^A \cos[f] - I E^A \sin[f]}{(-1 + E^A \cos[f])^2 + (E^A \sin[f])^2} /. A \to .3 /. f \to 5 /. x \to 12.$$

$$\frac{-1 + E^A \cos[f]}{(-1 + E^A \cos[f])^2 + (E^A \sin[f])^2} /. A \to .3 /. f \to 5 /. x \to 12.$$

$$-0.300099$$

$$\frac{-1 + E^A \cos[f]}{(-1 + E^A \cos[f])^2 + (E^A \sin[f])^2}$$

```
FullSimplify[(-1 + E^A Cos[f])^2 + (E^A Sin[f])^2]
```

$$1 + e^{2A} - 2\,e^{A}\,\text{Cos}[f]$$

```
FullSimplify[(-1 + E^A Cos[f]) / (1 + e^(2 A) - 2 e^A Cos[f])]
```

$$\frac{-1 + e^{A}\,\text{Cos}[f]}{1 + e^{2A} - 2\,e^{A}\,\text{Cos}[f]}$$

```
N@Zeta[10 I]
```

```
1.756468592974971` - 0.1015119854361739` i
```

$$1.75647 - 0.101512\,i$$

```
N@Zeta[1000 I]
```

$$-8.46309098852087 + 8.34334485626739\,i$$

```
-10.4 - 2.7
```

$$-13.1$$

```
N@Zeta[10 000 I]
```

```
530 * 3
```

$$1590$$

```
6.517210426253009` + 0.18128842533808529` i
```

$$6.51721 + 0.181288\,i$$

```
bk[s_, t_] := Sum[s^k, {k, 0, t}]
bk2[s_, t_] := Sum[k^s, {k, 1, t}]
```

```
Zeta[.02 I]
```

```
-0.4995988886255318` - 0.018370767581264345` i
```

```
-0.4614111392162328` - 0.1760891180160563` i
```

$$-0.461411 - 0.176089\,i$$

```
0.31472576404209895` - 0.23167964875052027` i
```

```
Zeta[-.5]
```

$$-0.207886$$

```
Expand[(1 - x / 2) (1 - x / 3) (1 - x / 4 ) (1 - x / 5)]
```

$$1 - \frac{77\,x}{60} + \frac{71\,x^2}{120} - \frac{7\,x^3}{60} + \frac{x^4}{120}$$

```
Log[1 - 14 x + 71 x^2 - 154 x^3 + 120 x^4]
```

$$\text{Log}\left[1 - 14\,x + 71\,x^2 - 154\,x^3 + 120\,x^4\right]$$

```
Log[(1 - 2 x) (1 - 3 x) (1 - 4 x ) (1 - 5 x)]
```

```
N@Log[(1 - x / 5) (1 - x / 4) (1 - x / 3) (1 - x / 2)] /. x → 1
```

```
-1.60944
```

```
N@Sum[Log[1 - 1 / k], {k, 2, 5}]
```

```
-1.60944
```

```
Sum[1 / k x^k, {k, 1, Infinity}]
```

```
-Log[1 - x]
```

$$\mathbf{D}\left[1 - \frac{77\,x}{60} + \frac{71\,x^2}{120} - \frac{7\,x^3}{60} + \frac{x^4}{120}, x\right] /. x \to 0$$

$$-\frac{77}{60}$$

```
Sum[-1 / k, {k, 2, 5}]
```

$$-\frac{77}{60}$$

```
Product[1 - s / r, {r, ZetaZero}]
```

$$\frac{\text{Pochhammer}[1 - s, \text{ZetaZero}]}{\text{ZetaZero}!}$$

```
pe[x_, t_] := EulerGamma + Log@Log@x + x^(1 / 2)
   Sum[(-1)^(n - 1) (Log@x)^n / n! / 2^(n - 1)  1 / (2 k + 1), {n, 1, t}, {k, 0, Floor[(n - 1) / 2]}]
```

```
pe[1000., 15]
```

```
177.612
```

```
LogIntegral[1000.]
```

```
177.61
```

```
Table[ j!, {j, 0, 20}]
```

```
{1, 1, 2, 6, 24, 120, 720, 5040, 40 320, 362 880, 3 628 800, 39 916 800,
 479 001 600, 6 227 020 800, 87 178 291 200, 1 307 674 368 000, 20 922 789 888 000,
 355 687 428 096 000, 6 402 373 705 728 000, 121 645 100 408 832 000, 2 432 902 008 176 640 000}
```

```
N@EulerGamma
```

```
0.5772156649015329`
```

```
0.577216
```

```
Log[100.]^15
```

$$8.88557 \times 10^9$$

```
LogIntegral[4245.25]
```

```
594.827
```

```
FullSimplify[Zeta'[s] / Zeta[s] /. s → 0]
```

$$\text{Log}[2\,\pi]$$

```
FullSimplify@Log[1 - s / p]
```

$$\text{Log}\left[1 - \frac{s}{p}\right]$$

```
rr[s_, t_] := Pi^(s / 2) / 2 / (s - 1) / Gamma[1 + s / 2]
  Product[(1 - s / ZetaZero[j]) (1 - s / ZetaZero[-j]), {j, 1, t}]
```

```
N@rr[2 + 6 I, 1000]
```

```
0.951822 + 0.148437 i
```

```
N@Zeta[2 + 6 I]
```

```
0.926863 + 0.156867 i
```

```
rl[s_, t_] := Log[Pi^(s / 2) / 2 / (s - 1) / Gamma[1 + s / 2]
   Product[(1 - s / ZetaZero[j]) (1 - s / ZetaZero[-j]), {j, 1, t}]]
rla[s_, t_] := Log[Pi^(s / 2)] - Log[2] - Log[(s - 1)] - Log[Gamma[1 + s / 2]] +
  Log[Product[(1 - s / ZetaZero[j]) (1 - s / ZetaZero[-j]), {j, 1, t}]]
rlb[s_, t_] := (s / 2) Log[Pi] - Log[2] - Log[s - 1] - Log[Gamma[1 + s / 2]] +
  Sum[Log[(1 - s / ZetaZero[-j])], {j, 1, t}] + Sum[Log[(1 - s / ZetaZero[j])], {j, 1, t}]
rlc[s_, t_] := (s / 2) Log[Pi] - Log[2] - Log[s - 1] - Log[Gamma[1 + s / 2]] -
  Sum[Sum[(s / ZetaZero[-j])^k / k, {k, 1, Infinity}], {j, 1, t}] -
  Sum[Log[(1 - s / ZetaZero[j])], {j, 1, t}]
```

```
N@rlb[1.6, 40]
```

```
0.821794 + 0. i
```

```
N@Log[Zeta[1.6]]
```

```
0.826701
```

```
Expand@Log[1 - s / ZetaZero[1]]
```

$$\text{Log}\left[1-\frac{s}{\text{ZetaZero}[1]}\right]$$

```
FullSimplify@Sum[-1 / k (s / ZetaZero[1])^k, {k, 1, Infinity}]
```

$$\text{Log}\left[1-\frac{s}{\text{ZetaZero}[1]}\right]$$

```
Clear[ms, ps, ds, dds]
D2[n_, 0] := UnitStep[n - 1]
D2[n_, k_] := D2[n, k] = Sum[D2[Floor[n / j], k - 1], {j, 2, n}]
d2[n_, k_] := d2[n, k] = D2[n, k] - D2[n - 1, k]
ps[1, s_] := 0
ps[n_, s_] := ps[n, s] = FullSimplify[MangoldtLambda[n] / Log[n]] n^-s + ps[n - 1, s]
ms[1, s_] := 1
ms[n_, s_] := ms[n, s] = MoebiusMu[n] n^-s + ms[n - 1, s]
ds[1, s_] := 1
ds[n_, s_] := ds[n, s] = n^-s + ds[n - 1, s]
dds[1, s_, k_] := 0
dds[n_, s_, k_] := dds[n, s, k] = d2[n, k] n^-s + dds[n - 1, s, k]
```

```
FullSimplify@ds[10, s]
```

$$1+2^{-s}+3^{-s}+4^{-s}+5^{-s}+6^{-s}+7^{-s}+8^{-s}+9^{-s}+10^{-s}$$

```
DiscretePlot[Re@dds[n, 100 I, 2], {n, 1, 1000}]
```

```
N@1 / Zeta[100 I]
```

```
0.153321 - 0.00426492 i
```

```
N@Log@Log@6000.
```

2.16327

```
dds[10, s, 2]
```

$2^{1-3\,s} + 2^{1-s}\,3^{-s} + 4^{-s} + 2^{1-s}\,5^{-s} + 9^{-s}$

```
d2[2, 1]
```

1

```
DiscretePlot[Re@dds[n, 100 I, 3], {n, 1, 2000}]
```

```
DiscretePlot[Re@dds[n, 100 I, 2], {n, 1, 2000}]
```

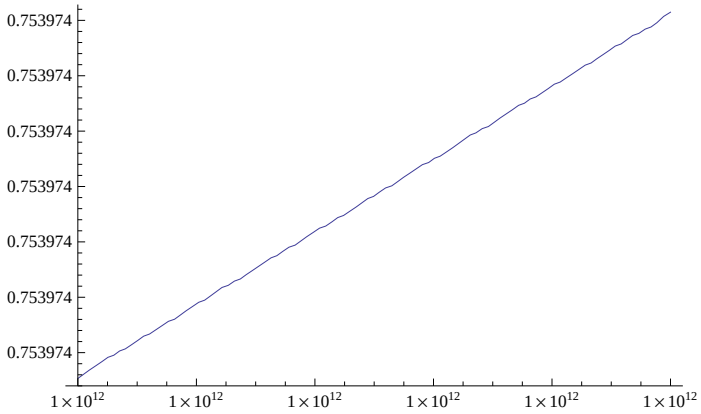
```
N@Sin[100 Log[x]] /. x → 1 000 000 000 000
```

−0.997454

```
N@Sin[10 000 Log[x]] /. x → 1 000 000 000 001
```

0.753974

```
Plot[Sin[10 000 Log[x]], {x, 1 000 000 000 000, 1 000 000 000 000 + 1}]
```




```
p1[n_, s_] := Sum[j^-s, {j, 1, n}]
p2[n_, s_] := Integrate[j^-s, {j, 0, n}]
```

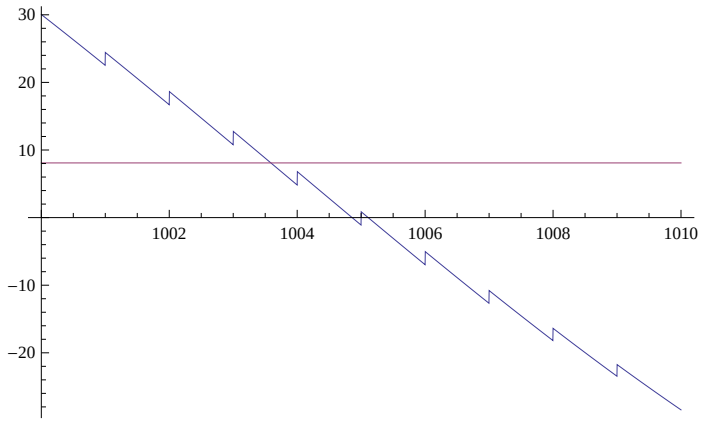
```
N@p1[1000, -.3 + 100 I] - N@p2[1000, -.3 + 100 I]
```

16.5331 + 1.88315 i

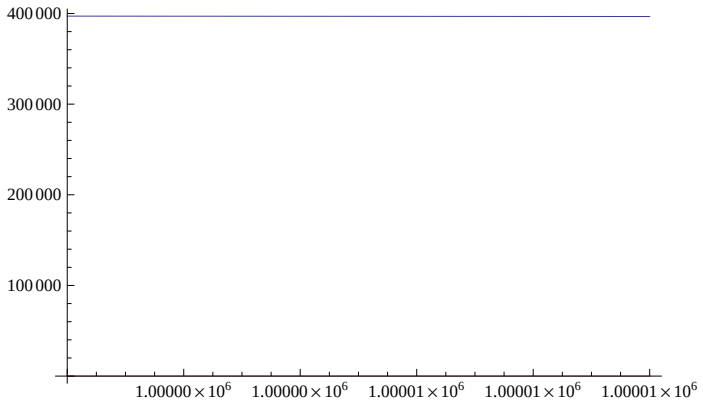
```
N@Zeta[-.3 + 100 I]
```

12.8128 + 0.490515 i

```
Plot[{Re@p1[n, -.1 + 100 I] - Re@p2[n, -.3 + 100 I], Re@N@Zeta[-.1 + 100 I]}, {n, 1000, 1010}]
```




```
Plot[{Re@p1[n, -.1 + 100 I] - Re@p2[n, -.3 + 100 I], Re@N@Zeta[-.1 + 100 I]},
 {n, 1 000 000, 1 000 010}]
```




```
Expand[(1 - x / -2) (1 - x / -1) (1 - x / 3) (1 - x / 4)]
```

$$1 + \frac{11\,x}{12} - \frac{7\,x^2}{24} - \frac{x^3}{6} + \frac{x^4}{24}$$

```
ex[s_] := Pi^(s / 2) / (2 (s - 1) Gamma[1 + s / 2])
ex2[s_, t_] := Product[(1 - s / ZetaZero[j]) (1 - s / ZetaZero[-j]), {j, 1, t}]
ex3[s_, t_] := ex[s] ex2[s, t]

N@ex[10 I]
```

22.2245 + 5.37482 i

```
N@ex2[10 I, 60]
```

0.112627 - 0.0291961 i

```
N@ex3[10 I, 1000]
```

1.88826 - 0.0955008 i

```
N@Zeta[10 I]
```

1.75647 - 0.101512 i

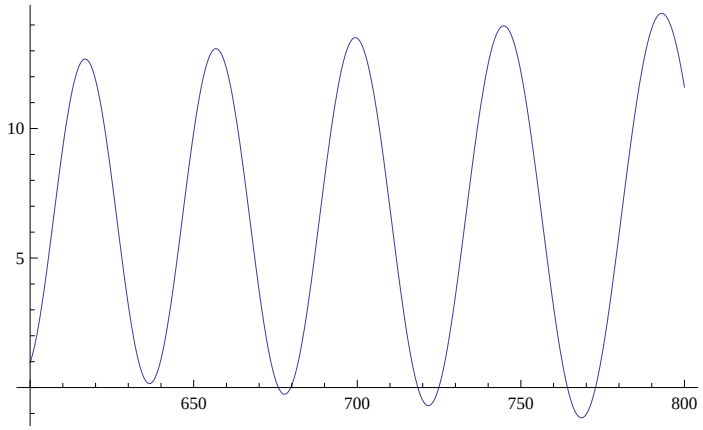
```
N@ex3[10 I, 1000]
```

1.88826 - 0.0955008 i

```
N@Zeta[2 I]
```

```
0.314726 - 0.23168 i
```

```
Plot[, {x, 600, 800}]
```




```
Plot[{ Im[Sum[j^-(100 I), {j, 1, x}]],
  Im[Zeta[100 I] + x^(1 - 100 I) / (1 - 100 I)]}, {x, 600, 800}]
```


5
650
−5

```
N@Zeta[-1 + 2 I]
```

```
0.168916 - 0.070516 i
```

```
-123.54067467712193` + 90.0000774946647` i
```

```
-123.541 + 90.0001 i
```

```
0.3563343671948756` + 0.9319978312333586` i
```

```
-123.54067467712193` + 90.0000774946647` i
```

```
14.306224558410431` + 27.183025808206345` i
```

```
1 / 12.
```

```
0.0833333
```

```
Integrate[t^(-3 - 1 / 10 I), {t, 0, x}]
```

$$\left(-\frac{200}{401}+\frac{10\,i}{401}\right) x^{-2-\frac{i}{10}}$$

```
Sum[E^(s t), {t, 0, Infinity}]
```

$$-\frac{1}{-1+e^{s}}$$