

```

K[n_] := If[n == 1, 0, FullSimplify[MangoldtLambda[n] / Log[n]]]
P[n_, k_] := P[n, k] = Sum[K[j] P[Floor[n / j], k - 1], {j, 2, n}]; P[n_, 0] := 1
D2[n_, k_] := D2[n, k] = Sum[D2[Floor[n / j], k - 1], {j, 2, n}]; D2[n_, 0] := 1
bins[z_, a_] := Product[(z - k), {k, 0, a - 1}] / a!
DD[n_, z_] := Sum[bins[z, a] D2[n, a], {a, 0, Log[2, n]}]
P[n_, k_, s_] := P[n, k, s] = Sum[j^(-s) K[j] P[Floor[n / j], k - 1, s], {j, 2, n}];
P[n_, 0, s_] := 1
DDa[n_, z_] := Sum[z^k / k! P[n, k], {k, 0, Log[2, n]}]
DDa[n_, z_, s_] := Sum[z^k / k! P[n, k, s], {k, 0, Log[2, n]}]
DDa2[n_, z_] := Sum[z^k / k! P[n, k] / z, {k, 0, Log[2, n]}]

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```
Expand[D[DD[100, z], z]]
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$$\frac{428}{15} + \frac{16289z}{180} + \frac{993z^2}{16} + \frac{611z^3}{36} + \frac{67z^4}{48} + \frac{7z^5}{120}$$

```
Roots[Expand[D[DD[10, z], z]] == 0, z]
```

$$z = -\frac{1}{3} \left(-21 - \sqrt{345} \right) \quad || \quad z = -\frac{1}{3} \left(-21 + \sqrt{345} \right)$$

```
Table[{n, Expand[Roots[Expand[D[DD[n, x], x]] == 0, x]]}, {n, 2, 16}] // TableForm
```

2	False
3	False
4	$x = -\frac{5}{2}$
5	$x = -\frac{7}{2}$
6	$x = -\frac{7}{6}$
7	$x = -\frac{3}{2}$
8	$x = -4 - \sqrt{\frac{19}{3}} \quad \quad x = -4 + \sqrt{\frac{19}{3}}$
9	$x = -5 - \sqrt{\frac{43}{3}} \quad \quad x = -5 + \sqrt{\frac{43}{3}}$
10	$x = -7 - \sqrt{\frac{115}{3}} \quad \quad x = -7 + \sqrt{\frac{115}{3}}$
11	$x = -7 - \sqrt{\frac{109}{3}} \quad \quad x = -7 + \sqrt{\frac{109}{3}}$
12	$x = -2 - \sqrt{\frac{5}{6}} \quad \quad x = -2 + \sqrt{\frac{5}{6}}$
13	$x = -2 - \frac{1}{\sqrt{3}} \quad \quad x = -2 + \frac{1}{\sqrt{3}}$
14	$x = -\frac{5}{2} - \frac{\sqrt{\frac{31}{3}}}{2} \quad \quad x = -\frac{5}{2} + \frac{\sqrt{\frac{31}{3}}}{2}$
15	$x = -3 - \frac{4}{\sqrt{3}} \quad \quad x = -3 + \frac{4}{\sqrt{3}}$
16	$x = -\frac{11}{2} + \frac{1}{6} \left(5184 - 3 \sqrt{2539353} \right)^{1/3} + \frac{\left(1728 + \sqrt{2539353} \right)^{1/3}}{2 \times 3^{2/3}} \quad \quad x = -\frac{11}{2} - \frac{1}{12} \left(5184 - 3 \sqrt{2539353} \right)^{1/3} - \frac{\left(1728 + \sqrt{2539353} \right)^{1/3}}{2 \times 3^{2/3}}$

```
Expand[D[DD[10, z], z]]
```

$$\frac{16}{3} + 7z + \frac{z^2}{2}$$

$$f[z_] := \frac{16}{3} + 7z + \frac{z^2}{2}$$

f[0]

$$\frac{16}{3}$$

$$f2[z_] := \text{FullSimplify}\left[\left(z - \left(-7 - \sqrt{\frac{115}{3}}\right)\right)\left(z - \left(-7 + \sqrt{\frac{115}{3}}\right)\right)\right]/2$$

f2[0]

$$\frac{16}{3}$$

$$\text{FullSimplify}\left[\left(-7 - \sqrt{\frac{115}{3}}\right)\left(-7 + \sqrt{\frac{115}{3}}\right)/2\right]$$

$$\frac{16}{3}$$

Expand[D[DD[13, z], z]]

$$\frac{22}{3} + 8z + 2z^2$$

$$f3[z_] := \frac{22}{3} + 8z + 2z^2$$

$$f4[z_] := \text{FullSimplify}\left[2\left(z - \left(-2 - \frac{1}{\sqrt{3}}\right)\right)\left(z - \left(-2 + \frac{1}{\sqrt{3}}\right)\right)\right]$$

f3[1]

$$\frac{52}{3}$$

f4[1]

$$\frac{52}{3}$$

Expand[D[DD[15, z], z]]

$$\frac{22}{3} + 12z + 2z^2$$

$$f5[z_] := \frac{22}{3} + 12z + 2z^2$$

$$f6[z_] := \text{FullSimplify}\left[2\left(z - \left(-3 - \frac{4}{\sqrt{3}}\right)\right)\left(z - \left(-3 + \frac{4}{\sqrt{3}}\right)\right)\right]$$

f5[-3]

$$-\frac{32}{3}$$

f6[-3]

$$-\frac{32}{3}$$

N[Roots[Expand[D[DD[100, x], x]] == 0, x]]

x == -0.425539 || x == -9.59257 - 10.0056 i ||
x == -9.59257 + 10.0056 i || x == -2.15895 - 1.14966 i || x == -2.15895 + 1.14966 i

f7[z_] :=

FullSimplify[(z - (-0.425539418089958)) (z - (-9.592566044154667 - 10.00556445690649 i))
(z - (-9.592566044154667 + 10.00556445690649 i))
(z - (-2.1589499610860696 - 1.1496632937798965 i))
(z - (-2.1589499610860696 + 1.1496632937798965 i)) (7 / 120)]

f7[1]

199.517 + 0. i

Expand[D[DD[100, z], z]] /. z -> 1.

199.517

(- (-0.425539418089958)) (- (-9.592566044154667 - 10.00556445690649 i))
(- (-9.592566044154667 + 10.00556445690649 i))
(- (-2.1589499610860696 - 1.1496632937798965 i))
(- (-2.1589499610860696 + 1.1496632937798965 i)) (7 / 120)

28.5333 + 0. i

Table[{n, N[Roots[Expand[D[DD[n, x], x]] == 0, x]]}, {n, 2, 110}] // TableForm

2	False
3	False
4	x == -2.5
5	x == -3.5
6	x == -1.16667
7	x == -1.5
8	x == -6.51661 x == -1.48339
9	x == -8.78594 x == -1.21406
10	x == -13.1914 x == -0.808608
11	x == -13.0277 x == -0.972286
12	x == -2.91287 x == -1.08713
13	x == -2.57735 x == -1.42265
14	x == -4.10728 x == -0.892725
15	x == -5.3094 x == -0.690599
16	x == -0.682041 x == -7.90898 + 2.0395 i x == -7.90898 - 2.0395 i
17	x == -0.791494 x == -7.85425 + 1.8378 i x == -7.85425 - 1.8378 i
18	x == -0.811453 - 1.77636 × 10 ⁻¹⁵ i x == -2.91482 + 1.77636 × 10 ⁻¹⁵ i x == -21.7737 - 3.33067 ×
19	x == -0.958823 - 1.77636 × 10 ⁻¹⁵ i x == -2.75229 + 2.66454 × 10 ⁻¹⁵ i x == -21.7889 - 4.44089 ×
20	x == -1.07047 + 1.77636 × 10 ⁻¹⁵ i x == -1.69249 + 0. i x == -31.737 - 1.11022 × 10 ⁻¹⁵ i
21	x == -0.757018 - 3.55271 × 10 ⁻¹⁵ i x == -2.42534 + 3.55271 × 10 ⁻¹⁵ i x == -31.3176 - 7.21645 ×
22	x == -0.622137 - 3.55271 × 10 ⁻¹⁵ i x == -2.99246 + 3.55271 × 10 ⁻¹⁵ i x == -30.8854 + 0. i
23	x == -0.709221 - 1.77636 × 10 ⁻¹⁵ i x == -2.89827 + 0. i x == -30.8925 - 3.33067 × 10 ⁻¹⁶ i
24	x == -0.713384 x == -3.99331 + 1.36234 i x == -3.99331 - 1.36234 i
25	x == -0.695008 x == -4.0025 + 1.76536 i x == -4.0025 - 1.76536 i

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26 x == -0.597522 || x == -4.05124 - 2.41786 i || x == -4.05124 + 2.41786 i
27 x == -0.591904 || x == -4.35405 + 2.04644 i || x == -4.35405 - 2.04644 i
28 x == -0.58734 + 0. i || x == -3.18184 + 8.88178 × 10-16 i || x == -7.33082 - 5.55112 × 10-16 i
29 x == -0.658618 - 3.55271 × 10-16 i || x == -3.06837 + 4.44089 × 10-16 i || x == -7.37301 - 1.11022 ×
30 x == -0.825848 - 3.55271 × 10-16 i || x == -1.45246 + 4.44089 × 10-16 i || x == -12.4217 - 1.66533 ×
31 x == -12.4311 || x == -1.13445 - 0.090378 i || x == -1.13445 + 0.090378 i
32 x == -12.8711 - 9.48345 i || x == -12.8711 + 9.48345 i || x == -1.12888 - 0.0646669 i || x == -1.1
33 x == -12.7505 - 9.34829 i || x == -12.7505 + 9.34829 i || x == -1.75333 || x == -0.745656
34 x == -12.6234 - 9.20877 i || x == -12.6234 + 9.20877 i || x == -2.12266 || x == -0.630585
35 x == -12.4889 - 9.06456 i || x == -12.4889 + 9.06456 i || x == -2.46574 || x == -0.556552
36 x == -3.47402 - 1.05319 i || x == -3.47402 + 1.05319 i || x == -44.4946 || x == -0.557345
37 x == -3.4447 - 0.962549 i || x == -3.4447 + 0.962549 i || x == -44.4943 || x == -0.616312
38 x == -3.46269 - 1.51527 i || x == -3.46269 + 1.51527 i || x == -44.5231 || x == -0.551514
39 x == -3.47306 - 1.90322 i || x == -3.47306 + 1.90322 i || x == -44.5519 || x == -0.502027
40 x == -2.75817 - 1.92535 i || x == -2.75817 + 1.92535 i || x == -61.9835 || x == -0.500208
41 x == -2.73552 - 1.8972 i || x == -2.73552 + 1.8972 i || x == -61.9833 || x == -0.545619
42 x == -2.37767 + 5.72545 × 10-17 i || x == -0.596711 + 5.72545 × 10-17 i || x == -60.6715 + 5.82925 ×
43 x == -2.2597 - 5.71533 × 10-17 i || x == -0.659723 - 5.71533 × 10-17 i || x == -60.6714 - 5.99237 ×
44 x == -1.82871 + 0. i || x == -0.659234 + 0. i || x == -59.9985 + 0. i || x == -5.51354 + 0. i
45 x == -1.5826 - 1.04521 × 10-15 i || x == -0.658671 + 8.7741 × 10-16 i || x == -59.3081 - 1.17059 × 10-16 i
46 x == -1.86433 - 8.31462 × 10-16 i || x == -0.578485 + 5.50011 × 10-16 i || x == -59.324 - 9.83954 × 10-16 i
47 x == -1.78668 - 8.29534 × 10-16 i || x == -0.638092 + 7.17018 × 10-16 i || x == -59.3239 + 6.05237 × 10-16 i
48 x == -6.45024 - 4.45679 i || x == -6.45024 + 4.45679 i || x == -1.79377 || x == -0.639094
49 x == -6.39966 - 4.4057 i || x == -6.39966 + 4.4057 i || x == -1.9033 || x == -0.630726
50 x == -6.51664 - 5.14835 i || x == -6.51664 + 5.14835 i || x == -1.67147 || x == -0.628589
51 x == -6.42459 - 5.07186 i || x == -6.42459 + 5.07186 i || x == -1.92114 || x == -0.563001
52 x == -6.52532 - 5.71546 i || x == -6.52532 + 5.71546 i || x == -1.72412 || x == -0.55858
53 x == -6.53097 - 5.7151 i || x == -6.53097 + 5.7151 i || x == -1.65964 || x == -0.611745
54 x == -7.86346 - 3.66074 i || x == -7.86346 + 3.66074 i || x == -1.66129 || x == -0.611796
55 x == -7.77276 - 3.51817 i || x == -7.77276 + 3.51817 i || x == -1.90227 || x == -0.552216
56 x == -1.93343 - 2.53528 × 10-16 i || x == -0.551142 + 2.28381 × 10-16 i || x == -12.3907 - 3.78971 × 10-16 i
57 x == -2.22686 - 6.58137 × 10-16 i || x == -0.505997 + 6.32169 × 10-16 i || x == -12.5085 - 1.43757 × 10-16 i
58 x == -2.57902 + 3.09676 × 10-16 i || x == -0.470109 - 7.43212 × 10-16 i || x == -12.6199 + 2.45899 × 10-16 i
59 x == -2.50106 + 1.41729 × 10-16 i || x == -0.505559 - 3.03362 × 10-16 i || x == -12.6156 + 8.08164 × 10-16 i
60 x == -2.16114 - 1.39556 i || x == -2.16114 + 1.39556 i || x == -23.8343 || x == -0.510129
61 x == -2.14245 - 1.37056 i || x == -2.14245 + 1.37056 i || x == -23.8339 || x == -0.54788
62 x == -2.15302 - 1.52188 i || x == -2.15302 + 1.52188 i || x == -23.8512 || x == -0.509437
63 x == -2.30762 - 1.33214 i || x == -2.30762 + 1.33214 i || x == -23.5462 || x == -0.505271
64 x == -0.504138 || x == -18.6969 - 19.151 i || x == -18.6969 + 19.151 i || x == -2.30101 - 1.33325 i
65 x == -0.472457 || x == -18.7068 - 19.1494 i || x == -18.7068 + 19.1494 i || x == -2.307 - 1.4903 i
66 x == -0.495454 || x == -18.3726 - 18.9112 i || x == -18.3726 + 18.9112 i || x == -2.62963 - 0.7071 i
67 x == -0.531673 || x == -18.3725 - 18.9114 i || x == -18.3725 + 18.9114 i || x == -2.61163 - 0.6433 i
68 x == -3.37209 || x == -2.19619 || x == -0.527566 || x == -18.2021 - 18.7908 i || x == -18.2021 + 18.7908 i
69 x == -0.491535 || x == -18.2128 - 18.7885 i || x == -18.2128 + 18.7885 i || x == -2.79144 - 0.3910 i
70 x == -4.60733 || x == -1.68842 || x == -0.518903 || x == -17.8427 - 18.5493 i || x == -17.8427 + 18.5493 i
71 x == -4.6265 || x == -1.62861 || x == -0.55977 || x == -17.8426 - 18.5495 i || x == -17.8426 + 18.5495 i
72 x == -79.7862 || x == -1.63197 || x == -0.560293 || x == -5.26079 - 3.25908 i || x == -5.26079 + 3.25908 i
73 x == -79.7862 || x == -1.56927 || x == -0.606045 || x == -5.26926 - 3.26151 i || x == -5.26926 + 3.26151 i
74 x == -79.7856 || x == -1.77373 || x == -0.555524 || x == -5.19257 - 3.17855 i || x == -5.19257 + 3.17855 i
75 x == -79.8186 || x == -1.6266 || x == -0.5518 || x == -5.25152 - 3.61886 i || x == -5.25152 + 3.61886 i
76 x == -79.8514 || x == -1.51343 || x == -0.547911 || x == -5.29361 - 3.99825 i || x == -5.29361 + 3.99825 i
77 x == -79.8509 || x == -1.67034 || x == -0.508542 || x == -5.23511 - 3.94397 i || x == -5.23511 + 3.94397 i
78 x == -79.9171 || x == -1.34684 || x == -0.53963 || x == -5.34824 - 4.64389 i || x == -5.34824 + 4.64389 i
79 x == -79.9171 || x == -1.29357 || x == -0.584458 || x == -5.35245 - 4.64392 i || x == -5.35245 + 4.64392 i

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80 x == -106.811 || x == -1.2894 || x == -0.584561 || x == -4.40755 - 4.27315 i || x == -4.40755 + 4.27315 i
81 x == -106.612 || x == -1.2918 || x == -0.581543 || x == -4.50722 - 4.23571 i || x == -4.50722 + 4.23571 i
82 x == -106.612 || x == -1.43203 || x == -0.534236 || x == -4.46087 - 4.20318 i || x == -4.46087 + 4.20318 i
83 x == -106.612 || x == -1.38251 || x == -0.574506 || x == -4.4655 - 4.20247 i || x == -4.4655 + 4.20247 i
84 x == -104.069 || x == -1.55652 || x == -0.579433 || x == -5.64734 - 1.42495 i || x == -5.64734 + 1.42495 i
85 x == -104.069 || x == -1.76055 || x == -0.536168 || x == -5.56708 - 1.19136 i || x == -5.56708 + 1.19136 i
86 x == -104.069 || x == -1.98671 || x == -0.502025 || x == -5.47119 - 0.861203 i || x == -5.47119 + 0.861203 i
87 x == -104.069 || x == -5.55982 || x == -5.13671 || x == -2.26101 || x == -0.473811
88 x == -103.202 || x == -8.05846 || x == -3.342 || x == -2.42485 || x == -0.472433
89 x == -103.202 || x == -8.05218 || x == -3.42946 || x == -2.31538 || x == -0.500712
90 x == -100.457 || x == -12.4435 || x == -0.504364 || x == -2.04775 - 1.02334 i || x == -2.04775 + 1.02334 i
91 x == -100.456 || x == -12.4695 || x == -0.477127 || x == -2.04855 - 1.15395 i || x == -2.04855 + 1.15395 i
92 x == -100.477 || x == -12.2358 || x == -0.473527 || x == -2.15661 - 1.01214 i || x == -2.15661 + 1.01214 i
93 x == -100.477 || x == -12.2632 || x == -0.45024 || x == -2.15468 - 1.14584 i || x == -2.15468 + 1.14584 i
94 x == -100.477 || x == -12.2904 || x == -0.429914 || x == -2.15141 - 1.26288 i || x == -2.15141 + 1.26288 i
95 x == -100.477 || x == -12.3172 || x == -0.411914 || x == -2.14711 - 1.36789 i || x == -2.14711 + 1.36789 i
96 x == -0.411442 || x == -9.60927 - 9.0899 i || x == -9.60927 + 9.0899 i || x == -2.1493 - 1.39183 i
97 x == -0.432477 || x == -9.60897 - 9.09033 i || x == -9.60897 + 9.09033 i || x == -2.13908 - 1.3742 i
98 x == -0.429214 || x == -9.49511 - 9.01607 i || x == -9.49511 + 9.01607 i || x == -2.25457 - 1.2506 i
99 x == -0.425929 || x == -9.37569 - 8.94212 i || x == -9.37569 + 8.94212 i || x == -2.37563 - 1.0943 i
100 x == -0.425539 || x == -9.59257 - 10.0056 i || x == -9.59257 + 10.0056 i || x == -2.15895 - 1.1496 i
101 x == -0.447488 || x == -9.59239 - 10.0059 i || x == -9.59239 + 10.0059 i || x == -2.14816 - 1.1287 i
102 x == -0.46202 || x == -9.37867 - 9.89563 i || x == -9.37867 + 9.89563 i || x == -2.35461 - 0.59242 i
103 x == -0.487258 || x == -9.37849 - 9.89603 i || x == -9.37849 + 9.89603 i || x == -2.34216 - 0.5414 i
104 x == -0.486061 || x == -9.48536 - 10.527 i || x == -9.48536 + 10.527 i || x == -2.23589 - 0.603318 i
105 x == -3.19822 || x == -1.65592 || x == -0.506616 || x == -9.28391 - 10.4354 i || x == -9.28391 + 10.4354 i
106 x == -2.98984 || x == -1.87231 || x == -0.479082 || x == -9.29367 - 10.4308 i || x == -9.29367 + 10.4308 i
107 x == -3.0291 || x == -1.80611 || x == -0.506244 || x == -9.29356 - 10.4312 i || x == -9.29356 + 10.4312 i
108 x == -3.48818 || x == -1.8028 || x == -0.506413 || x == -12.637 - 3.17055 i || x == -12.637 + 3.17055 i
109 x == -3.52364 || x == -1.74109 || x == -0.535694 || x == -12.6355 - 3.16928 i || x == -12.6355 + 3.16928 i
110 x == -4.79469 || x == -1.35426 || x == -0.56616 || x == -12.1782 - 1.84407 i || x == -12.1782 + 1.84407 i

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```
Table[{n, Expand[D[DD[n, z], z]]}, {n, 2, 110}] // TableForm
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```

2      1
3      2
4       $\frac{5}{2} + z$ 
5       $\frac{7}{2} + z$ 
6       $\frac{7}{2} + 3 z$ 
7       $\frac{9}{2} + 3 z$ 
8       $\frac{29}{6} + 4 z + \frac{z^2}{2}$ 
9       $\frac{16}{3} + 5 z + \frac{z^2}{2}$ 
10      $\frac{16}{3} + 7 z + \frac{z^2}{2}$ 
11      $\frac{19}{3} + 7 z + \frac{z^2}{2}$ 
12      $\frac{19}{3} + 8 z + 2 z^2$ 
13      $\frac{22}{3} + 8 z + 2 z^2$ 
14      $\frac{22}{3} + 10 z + 2 z^2$ 
15      $\frac{22}{3} + 12 z + 2 z^2$ 
16      $\frac{91}{12} + \frac{155 z}{12} + \frac{11 z^2}{4} + \frac{z^3}{6}$ 
17      $\frac{103}{12} + \frac{155 z}{12} + \frac{11 z^2}{4} + \frac{z^3}{6}$ 

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$$\begin{aligned}
18 & \frac{103}{12} + \frac{167z}{12} + \frac{17z^2}{4} + \frac{z^3}{6} \\
19 & \frac{115}{12} + \frac{167z}{12} + \frac{17z^2}{4} + \frac{z^3}{6} \\
20 & \frac{115}{12} + \frac{179z}{12} + \frac{23z^2}{4} + \frac{z^3}{6} \\
21 & \frac{115}{12} + \frac{203z}{12} + \frac{23z^2}{4} + \frac{z^3}{6} \\
22 & \frac{115}{12} + \frac{227z}{12} + \frac{23z^2}{4} + \frac{z^3}{6} \\
23 & \frac{127}{12} + \frac{227z}{12} + \frac{23z^2}{4} + \frac{z^3}{6} \\
24 & \frac{127}{12} + \frac{235z}{12} + \frac{29z^2}{4} + \frac{5z^3}{6} \\
25 & \frac{133}{12} + \frac{247z}{12} + \frac{29z^2}{4} + \frac{5z^3}{6} \\
26 & \frac{133}{12} + \frac{271z}{12} + \frac{29z^2}{4} + \frac{5z^3}{6} \\
27 & \frac{137}{12} + \frac{283z}{12} + \frac{31z^2}{4} + \frac{5z^3}{6} \\
28 & \frac{137}{12} + \frac{295z}{12} + \frac{37z^2}{4} + \frac{5z^3}{6} \\
29 & \frac{149}{12} + \frac{295z}{12} + \frac{37z^2}{4} + \frac{5z^3}{6} \\
30 & \frac{149}{12} + \frac{295z}{12} + \frac{49z^2}{4} + \frac{5z^3}{6} \\
31 & \frac{161}{12} + \frac{295z}{12} + \frac{49z^2}{4} + \frac{5z^3}{6} \\
32 & \frac{817}{60} + \frac{305z}{12} + \frac{105z^2}{8} + \frac{7z^3}{6} + \frac{z^4}{24} \\
33 & \frac{817}{60} + \frac{329z}{12} + \frac{105z^2}{8} + \frac{7z^3}{6} + \frac{z^4}{24} \\
34 & \frac{817}{60} + \frac{353z}{12} + \frac{105z^2}{8} + \frac{7z^3}{6} + \frac{z^4}{24} \\
35 & \frac{817}{60} + \frac{377z}{12} + \frac{105z^2}{8} + \frac{7z^3}{6} + \frac{z^4}{24} \\
36 & \frac{817}{60} + \frac{383z}{12} + \frac{117z^2}{8} + \frac{13z^3}{6} + \frac{z^4}{24} \\
37 & \frac{877}{60} + \frac{383z}{12} + \frac{117z^2}{8} + \frac{13z^3}{6} + \frac{z^4}{24} \\
38 & \frac{877}{60} + \frac{407z}{12} + \frac{117z^2}{8} + \frac{13z^3}{6} + \frac{z^4}{24} \\
39 & \frac{877}{60} + \frac{431z}{12} + \frac{117z^2}{8} + \frac{13z^3}{6} + \frac{z^4}{24} \\
40 & \frac{877}{60} + \frac{439z}{12} + \frac{129z^2}{8} + \frac{17z^3}{6} + \frac{z^4}{24} \\
41 & \frac{937}{60} + \frac{439z}{12} + \frac{129z^2}{8} + \frac{17z^3}{6} + \frac{z^4}{24} \\
42 & \frac{937}{60} + \frac{439z}{12} + \frac{153z^2}{8} + \frac{17z^3}{6} + \frac{z^4}{24} \\
43 & \frac{997}{60} + \frac{439z}{12} + \frac{153z^2}{8} + \frac{17z^3}{6} + \frac{z^4}{24} \\
44 & \frac{997}{60} + \frac{451z}{12} + \frac{165z^2}{8} + \frac{17z^3}{6} + \frac{z^4}{24} \\
45 & \frac{997}{60} + \frac{463z}{12} + \frac{177z^2}{8} + \frac{17z^3}{6} + \frac{z^4}{24} \\
46 & \frac{997}{60} + \frac{487z}{12} + \frac{177z^2}{8} + \frac{17z^3}{6} + \frac{z^4}{24} \\
47 & \frac{1057}{60} + \frac{487z}{12} + \frac{177z^2}{8} + \frac{17z^3}{6} + \frac{z^4}{24} \\
48 & \frac{1057}{60} + \frac{493z}{12} + \frac{47z^2}{2} + \frac{23z^3}{6} + \frac{z^4}{4} \\
49 & \frac{1087}{60} + \frac{505z}{12} + \frac{47z^2}{2} + \frac{23z^3}{6} + \frac{z^4}{4} \\
50 & \frac{1087}{60} + \frac{517z}{12} + 25z^2 + \frac{23z^3}{6} + \frac{z^4}{4} \\
51 & \frac{1087}{60} + \frac{541z}{12} + 25z^2 + \frac{23z^3}{6} + \frac{z^4}{4} \\
52 & \frac{1087}{60} + \frac{553z}{12} + \frac{53z^2}{2} + \frac{23z^3}{6} + \frac{z^4}{4}
\end{aligned}$$

$$\begin{aligned}
53 & \frac{1147}{60} + \frac{553z}{12} + \frac{53z^2}{2} + \frac{23z^3}{6} + \frac{z^4}{4} \\
54 & \frac{1147}{60} + \frac{187z}{4} + 28z^2 + \frac{9z^3}{2} + \frac{z^4}{4} \\
55 & \frac{1147}{60} + \frac{195z}{4} + 28z^2 + \frac{9z^3}{2} + \frac{z^4}{4} \\
56 & \frac{1147}{60} + \frac{593z}{12} + \frac{59z^2}{2} + \frac{31z^3}{6} + \frac{z^4}{4} \\
57 & \frac{1147}{60} + \frac{617z}{12} + \frac{59z^2}{2} + \frac{31z^3}{6} + \frac{z^4}{4} \\
58 & \frac{1147}{60} + \frac{641z}{12} + \frac{59z^2}{2} + \frac{31z^3}{6} + \frac{z^4}{4} \\
59 & \frac{1207}{60} + \frac{641z}{12} + \frac{59z^2}{2} + \frac{31z^3}{6} + \frac{z^4}{4} \\
60 & \frac{1207}{60} + \frac{641z}{12} + 31z^2 + \frac{43z^3}{6} + \frac{z^4}{4} \\
61 & \frac{1267}{60} + \frac{641z}{12} + 31z^2 + \frac{43z^3}{6} + \frac{z^4}{4} \\
62 & \frac{1267}{60} + \frac{665z}{12} + 31z^2 + \frac{43z^3}{6} + \frac{z^4}{4} \\
63 & \frac{1267}{60} + \frac{677z}{12} + \frac{65z^2}{2} + \frac{43z^3}{6} + \frac{z^4}{4} \\
64 & \frac{1277}{60} + \frac{2573z}{45} + \frac{535z^2}{16} + \frac{275z^3}{36} + \frac{17z^4}{48} + \frac{z^5}{120} \\
65 & \frac{1277}{60} + \frac{2663z}{45} + \frac{535z^2}{16} + \frac{275z^3}{36} + \frac{17z^4}{48} + \frac{z^5}{120} \\
66 & \frac{1277}{60} + \frac{2663z}{45} + \frac{583z^2}{16} + \frac{275z^3}{36} + \frac{17z^4}{48} + \frac{z^5}{120} \\
67 & \frac{1337}{60} + \frac{2663z}{45} + \frac{583z^2}{16} + \frac{275z^3}{36} + \frac{17z^4}{48} + \frac{z^5}{120} \\
68 & \frac{1337}{60} + \frac{2708z}{45} + \frac{607z^2}{16} + \frac{275z^3}{36} + \frac{17z^4}{48} + \frac{z^5}{120} \\
69 & \frac{1337}{60} + \frac{2798z}{45} + \frac{607z^2}{16} + \frac{275z^3}{36} + \frac{17z^4}{48} + \frac{z^5}{120} \\
70 & \frac{1337}{60} + \frac{2798z}{45} + \frac{655z^2}{16} + \frac{275z^3}{36} + \frac{17z^4}{48} + \frac{z^5}{120} \\
71 & \frac{1397}{60} + \frac{2798z}{45} + \frac{655z^2}{16} + \frac{275z^3}{36} + \frac{17z^4}{48} + \frac{z^5}{120} \\
72 & \frac{1397}{60} + \frac{2813z}{45} + \frac{675z^2}{16} + \frac{323z^3}{36} + \frac{37z^4}{48} + \frac{z^5}{120} \\
73 & \frac{1457}{60} + \frac{2813z}{45} + \frac{675z^2}{16} + \frac{323z^3}{36} + \frac{37z^4}{48} + \frac{z^5}{120} \\
74 & \frac{1457}{60} + \frac{2903z}{45} + \frac{675z^2}{16} + \frac{323z^3}{36} + \frac{37z^4}{48} + \frac{z^5}{120} \\
75 & \frac{1457}{60} + \frac{2948z}{45} + \frac{699z^2}{16} + \frac{323z^3}{36} + \frac{37z^4}{48} + \frac{z^5}{120} \\
76 & \frac{1457}{60} + \frac{2993z}{45} + \frac{723z^2}{16} + \frac{323z^3}{36} + \frac{37z^4}{48} + \frac{z^5}{120} \\
77 & \frac{1457}{60} + \frac{3083z}{45} + \frac{723z^2}{16} + \frac{323z^3}{36} + \frac{37z^4}{48} + \frac{z^5}{120} \\
78 & \frac{1457}{60} + \frac{3083z}{45} + \frac{771z^2}{16} + \frac{323z^3}{36} + \frac{37z^4}{48} + \frac{z^5}{120} \\
79 & \frac{1517}{60} + \frac{3083z}{45} + \frac{771z^2}{16} + \frac{323z^3}{36} + \frac{37z^4}{48} + \frac{z^5}{120} \\
80 & \frac{1517}{60} + \frac{6211z}{90} + \frac{793z^2}{16} + \frac{359z^3}{36} + \frac{47z^4}{48} + \frac{z^5}{120} \\
81 & \frac{383}{15} + \frac{12587z}{180} + \frac{805z^2}{16} + \frac{365z^3}{36} + \frac{47z^4}{48} + \frac{z^5}{120} \\
82 & \frac{383}{15} + \frac{12947z}{180} + \frac{805z^2}{16} + \frac{365z^3}{36} + \frac{47z^4}{48} + \frac{z^5}{120} \\
83 & \frac{398}{15} + \frac{12947z}{180} + \frac{805z^2}{16} + \frac{365z^3}{36} + \frac{47z^4}{48} + \frac{z^5}{120} \\
84 & \frac{398}{15} + \frac{12947z}{180} + \frac{829z^2}{16} + \frac{437z^3}{36} + \frac{47z^4}{48} + \frac{z^5}{120} \\
85 & \frac{398}{15} + \frac{13307z}{180} + \frac{829z^2}{16} + \frac{437z^3}{36} + \frac{47z^4}{48} + \frac{z^5}{120} \\
86 & \frac{398}{15} + \frac{13667z}{180} + \frac{829z^2}{16} + \frac{437z^3}{36} + \frac{47z^4}{48} + \frac{z^5}{120} \\
87 & \frac{398}{15} + \frac{14027z}{180} + \frac{829z^2}{16} + \frac{437z^3}{36} + \frac{47z^4}{48} + \frac{z^5}{120}
\end{aligned}$$

88	$\frac{398}{15} + \frac{14\,147\,z}{180} + \frac{853\,z^2}{16} + \frac{461\,z^3}{36} + \frac{47\,z^4}{48} + \frac{z^5}{120}$
89	$\frac{413}{15} + \frac{14\,147\,z}{180} + \frac{853\,z^2}{16} + \frac{461\,z^3}{36} + \frac{47\,z^4}{48} + \frac{z^5}{120}$
90	$\frac{413}{15} + \frac{14\,147\,z}{180} + \frac{877\,z^2}{16} + \frac{533\,z^3}{36} + \frac{47\,z^4}{48} + \frac{z^5}{120}$
91	$\frac{413}{15} + \frac{14\,507\,z}{180} + \frac{877\,z^2}{16} + \frac{533\,z^3}{36} + \frac{47\,z^4}{48} + \frac{z^5}{120}$
92	$\frac{413}{15} + \frac{14\,687\,z}{180} + \frac{901\,z^2}{16} + \frac{533\,z^3}{36} + \frac{47\,z^4}{48} + \frac{z^5}{120}$
93	$\frac{413}{15} + \frac{15\,047\,z}{180} + \frac{901\,z^2}{16} + \frac{533\,z^3}{36} + \frac{47\,z^4}{48} + \frac{z^5}{120}$
94	$\frac{413}{15} + \frac{15\,407\,z}{180} + \frac{901\,z^2}{16} + \frac{533\,z^3}{36} + \frac{47\,z^4}{48} + \frac{z^5}{120}$
95	$\frac{413}{15} + \frac{15\,767\,z}{180} + \frac{901\,z^2}{16} + \frac{533\,z^3}{36} + \frac{47\,z^4}{48} + \frac{z^5}{120}$
96	$\frac{413}{15} + \frac{15\,839\,z}{180} + \frac{921\,z^2}{16} + \frac{575\,z^3}{36} + \frac{67\,z^4}{48} + \frac{7\,z^5}{120}$
97	$\frac{428}{15} + \frac{15\,839\,z}{180} + \frac{921\,z^2}{16} + \frac{575\,z^3}{36} + \frac{67\,z^4}{48} + \frac{7\,z^5}{120}$
98	$\frac{428}{15} + \frac{16\,019\,z}{180} + \frac{945\,z^2}{16} + \frac{575\,z^3}{36} + \frac{67\,z^4}{48} + \frac{7\,z^5}{120}$
99	$\frac{428}{15} + \frac{16\,199\,z}{180} + \frac{969\,z^2}{16} + \frac{575\,z^3}{36} + \frac{67\,z^4}{48} + \frac{7\,z^5}{120}$
100	$\frac{428}{15} + \frac{16\,289\,z}{180} + \frac{993\,z^2}{16} + \frac{611\,z^3}{36} + \frac{67\,z^4}{48} + \frac{7\,z^5}{120}$
101	$\frac{443}{15} + \frac{16\,289\,z}{180} + \frac{993\,z^2}{16} + \frac{611\,z^3}{36} + \frac{67\,z^4}{48} + \frac{7\,z^5}{120}$
102	$\frac{443}{15} + \frac{16\,289\,z}{180} + \frac{1041\,z^2}{16} + \frac{611\,z^3}{36} + \frac{67\,z^4}{48} + \frac{7\,z^5}{120}$
103	$\frac{458}{15} + \frac{16\,289\,z}{180} + \frac{1041\,z^2}{16} + \frac{611\,z^3}{36} + \frac{67\,z^4}{48} + \frac{7\,z^5}{120}$
104	$\frac{458}{15} + \frac{16\,409\,z}{180} + \frac{1065\,z^2}{16} + \frac{635\,z^3}{36} + \frac{67\,z^4}{48} + \frac{7\,z^5}{120}$
105	$\frac{458}{15} + \frac{16\,409\,z}{180} + \frac{1113\,z^2}{16} + \frac{635\,z^3}{36} + \frac{67\,z^4}{48} + \frac{7\,z^5}{120}$
106	$\frac{458}{15} + \frac{16\,769\,z}{180} + \frac{1113\,z^2}{16} + \frac{635\,z^3}{36} + \frac{67\,z^4}{48} + \frac{7\,z^5}{120}$
107	$\frac{473}{15} + \frac{16\,769\,z}{180} + \frac{1113\,z^2}{16} + \frac{635\,z^3}{36} + \frac{67\,z^4}{48} + \frac{7\,z^5}{120}$
108	$\frac{473}{15} + \frac{16\,829\,z}{180} + \frac{1133\,z^2}{16} + \frac{683\,z^3}{36} + \frac{29\,z^4}{16} + \frac{7\,z^5}{120}$
109	$\frac{488}{15} + \frac{16\,829\,z}{180} + \frac{1133\,z^2}{16} + \frac{683\,z^3}{36} + \frac{29\,z^4}{16} + \frac{7\,z^5}{120}$
110	$\frac{488}{15} + \frac{16\,829\,z}{180} + \frac{1181\,z^2}{16} + \frac{683\,z^3}{36} + \frac{29\,z^4}{16} + \frac{7\,z^5}{120}$