```
-2 N[Integrate[ Gamma[2, 0, -Log[100 / j]], {j, 1, 100}]]
N[Gamma[3, 0, -Log[100]]]
```

-1397.73

$-1397.73 + 3.42834 \times 10^{-13}\, i$

```
-3 N[Integrate[ Gamma[3, 0, -Log[100 / j]], {j, 1, 100}]]
N[Gamma[4, 0, -Log[100]]]
```

5573.28

$5573.28 - 2.04539 \times 10^{-12}\, i$

```
-4 N[Integrate[ Gamma[4, 0, -Log[100 / j]], {j, 1, 100}]]
N[Gamma[5, 0, -Log[100]]]
```

$-22\,683.1$

$-22\,683.1 + 1.38894 \times 10^{-11}\, i$

```
-3.5 N[Integrate[ Gamma[3.5, 0, -Log[100 / j]], {j, 1, 100}]]
N[Gamma[4.5, 0, -Log[100]]]
```

$0. + 11\,219.5\, i$

$6.18298 \times 10^{-12} + 11\,219.5\, i$

```
-3.15 N[Integrate[ Gamma[3.15, 0, -Log[100 / j]], {j, 1, 100}]]
N[Gamma[4.15, 0, -Log[100]]]
```

$6122.62 + 3119.63\, i$

$6122.62 + 3119.63\, i$

```
-(N[Integrate[ Gamma[4.55, -Log[100 / j]] / Gamma[4.55] - 1, {j, 1, 100}]] - 1) Gamma[5.55]
N[Gamma[5.55, -Log[100]]]
```

$-7673.31 + 48\,805.7\, i$

$-7673.31 + 48\,805.7\, i$

```
-(N[Integrate[ Gamma[4.55, -Log[100 / j]] / Gamma[4.55] - 1, {j, 1, 100}]] - 1) Gamma[5.55]
N[Gamma[5.55, -Log[100]]]
```

$-7673.31 + 48\,805.7\, i$

$-7673.31 + 48\,805.7\, i$

```
-(N[Integrate[ Gamma[.5, -Log[100 / j]] / Gamma[.5] - 1, {j, 1, 100}]] - 1) Gamma[1.5]
N[Gamma[1.5, -Log[100]]]
```

$0.886227 + 187.232\, i$

$0.886227 + 187.232\, i$

```
-(N[Integrate[Gamma[.1, -Log[100 / j]] / Gamma[.1] - 1, {j, 1, 100}]] - 1) Gamma[1.1]
N[Gamma[1.1, -Log[100]]]
```

```
107.658 + 34.671 i
```

```
107.658 + 34.671 i
```

```
-(N[Integrate[Gamma[.01, -Log[100 / j]] / Gamma[.01] - 1, {j, 1, 100}]] - 1) Gamma[1.01]
N[Gamma[1.01, -Log[100]]]
```

```
101.185 + 3.14862 i
```

```
101.185 + 3.14862 i
```

```
-(N[Integrate[Gamma[.0001, -Log[100 / j]] / Gamma[.0001] - 1, {j, 1, 100}]] - 1) Gamma[1.0001]
N[Gamma[1.0001, -Log[100]]]
```

```
100.012 + 0.0311056 i
```

```
100.012 + 0.0311056 i
```

```
N[Integrate[Gamma[0, -Log[100 / j]] / Gamma[0], {j, 1, 100}]]
```

```
0.
```

```
Limit[
 -(N[Integrate[Gamma[a - 1, -Log[100 / j]] / Gamma[a - 1] - 1, {j, 1, 100}]] - 1) Gamma[a], a → 0]
```

$$\text{Limit}\left[\text{ConditionalExpression}\left[\text{Gamma}[a]\left(0. - \frac{100.\ (-1.)^{a}\ 4.60517^{-1.+a}}{\text{Gamma}[a]} + \frac{1.\ \text{Gamma}[-1.+a,\ -4.60517]}{\text{Gamma}[-1.+a]}\right),\ \text{Re}[a] > 0.\right],\ a \to 0\right]$$

```
Limit[
 -(N[Integrate[Gamma[a - 1, -Log[100 / j]] / Gamma[a - 1] - 1, {j, 1, 100}]] - 1) Gamma[a], a → .5]
```

```
1.77245 - 54.7298 i
```

```
-2 N[Integrate[Gamma[2, 0, -Log[100 / j]], {j, 1, 100}]]
N[Gamma[3, 0, -Log[100]]]
```

```
-1397.73
```

```
-1397.73 + 3.42834 × 10^-13 i
```

```
ff[a_] := ((-2 N[Integrate[Gamma[2, 0, -Log[(100 + a) / j]], {j, 1, 100 + a}]]) +
     (-2 N[Integrate[Gamma[2, 0, -Log[(100 - a) / j]], {j, 1, 100 - a}]])) / 2
```

```
ff[.01]
```

```
-1397.73 + 0. i
```

```
ggx[n_] := Gamma[3, 0, n]
```

```
N[ggx'[-Log[100]] / ggx[-Log[100]]]
```

```
-1.51729 - 3.72161 × 10^-16 i
```

```
N[Gamma[2, 0, -Log[100]]]
```

```
361.517 - 4.41506 × 10^-14 i
```

```
-2 N[Integrate[ Gamma[2, 0, -Log[100 / j]], {j, 1, 100}]]
N[Gamma[3, 0, -Log[100]]]
```

-1397.73

$-1397.73 + 3.42834 \times 10^{-13}\,i$

```
N[Gamma[3, 0, -Log[100]]]
```

$-1397.73 + 3.42834 \times 10^{-13}\,i$

```
N[2 Gamma[2, 0, -Log[100]] - (-Log[100])^2 100]
```

$-1397.73 - 8.83012 \times 10^{-14}\,i$

```
N[(Gamma[3, 0, -Log[100]] + (-Log[100])^2 100) / Gamma[3]]
N[Gamma[2, 0, -Log[100]] ]
```

$361.517 + 1.71417 \times 10^{-13}\,i$

$361.517 - 4.41506 \times 10^{-14}\,i$

```
fb[s_, x_] := (Gamma[s, 0, -Log[x]] + (-Log[x])^(s - 1) x) / Gamma[s]
fb[3, n]
```

$$\frac{1}{2}\left(\text{Gamma}[3, 0, -\text{Log}[n]] + n\,\text{Log}[n]^2\right)$$

$$-2\,N\left[\text{Integrate}\left[\frac{1}{2}\left(\text{Gamma}[3, 0, -\text{Log}[100/j]] + (100/j)\,\text{Log}[(100/j)]^2\right), \{j, 1, 100\}\right]\right]$$

```
N[Gamma[3, 0, -Log[100]]]
```

-1397.73

$-1397.73 + 3.42834 \times 10^{-13}\,i$

$$-N\left[\text{Integrate}\left[\left(\text{Gamma}[3, 0, -\text{Log}[100/j]] + (100/j)\,\text{Log}[(100/j)]^2\right), \{j, 1, 100\}\right]\right]$$

-1397.73

```
(* Line of thinking: dh(n,2,2) = dh(n,2,3) + 2 dh(n,1,3) + dh(n,0,3)
       Is there an analogous relationship for inc gamma?
 *)


N[Gamma[2, 0, -Log[100]]] / Gamma[2]
```

$361.517 - 4.41506 \times 10^{-14}\,i$

```
N[2 Integrate[ Gamma[1, 0, -Log[100 / j]], {j, 1, 10}] / Gamma[1] -
  Limit[1 Integrate[ Gamma[a, 0, -Log[100 / j]], {j, 1, 10}] / Gamma[a], a → 0]]
```

451.517

```
N[Gamma[3, 0, -Log[100]]] / Gamma[3]
```

$-698.863 + 1.71417 \times 10^{-13}\,i$

```
N[-3 Integrate[ Gamma[2, 0, -Log[100 / j]], {j, 1, 100^(1 / 3)}] / Gamma[2]] +
 N[3 Integrate[ Gamma[1, 0, -Log[100 / j]], {j, 1, 100^(1 / 3)}] / Gamma[1]] +
 N[Limit[-1 Integrate[ Gamma[a, 0, -Log[100 / j]], {j, 1, 100^(1 / 3)}] / Gamma[a], a → 0]]
```

-1770.94

```
N[Gamma[2, -Log[100]]] / Gamma[2]
```

$-360.517 + 4.41506 \times 10^{-14}\, i$

```
N[-2 Integrate[ Gamma[1, -Log[100 / j]], {j, 1, 10}] / Gamma[1]]
```

-460.517

```
N[Gamma[3, -Log[100]]] / Gamma[3]
```

$699.863 - 1.71417 \times 10^{-13}\, i$

```
N[-3 Integrate[ Gamma[2, -Log[100 / j]], {j, 1, 100^(1 / 3)}] / Gamma[2]] +
 N[3 Integrate[ Gamma[1, -Log[100 / j]], {j, 1, 100^(1 / 3)}] / Gamma[1]] +
 N[Limit[-1 Integrate[ Gamma[a, -Log[100 / j]], {j, 1, 100^(1 / 3)}] / Gamma[a], a → 0]]
```

$1406.78 + 4.41506 \times 10^{-14}\, i$

```
{N[Gamma[2, -Log[10]]] / Gamma[2],
 N[-2 Integrate[ Gamma[1, -Log[10 / j^2]], {j, 1, 10^(1 / 2)}] / Gamma[1]]}
```

$\left\{-13.0259 + 1.59521 \times 10^{-15}\, i, -13.6754\right\}$

```
{N[Gamma[2, -Log[23]]] / Gamma[2],
 N[-2 Integrate[ Gamma[1, -Log[23 / j^2]], {j, 1, 23^(1 / 2)}] / Gamma[1]]}
```

$\left\{-49.1164 + 6.01502 \times 10^{-15}\, i, -36.4083\right\}$

```
Dhyp[n_, k_, a_] :=
 Sum[Binomial[k, j] Dhyp[n / (m^(k - j)), j, m + 1], {m, a, n^(1 / k)}, {j, 0, k - 1}]
Dhyp[n_, 1, a_] := Floor[n] - a + 1; Dhyp[n_, 0, a_] := 1
```

```
f[n_, k_, b_] := b^-k Dhyp[n b^k, k, b + 1]
f2[n_, b_] := (
  b^-2 Sum[2 Dhyp[(n b^2) / (m), 1, m + 1], {m, b + 1, b n^(1 / 2)}] +
   b^-2 Sum[ 1, {m, b + 1, b n^(1 / 2)}]
 )
f3[n_, b_] := (
  b^-2 Sum[2 Dhyp[(n b^2) / m, 1, m + 1], {m, b + 1, b n^(1 / 2)}] +
   b^-2 Sum[ 1, {m, b + 1, b n^(1 / 2)}]
 )
b^-2 Sum[ 1, {m, b + 1, Floor[b n^(1 / 2)]}]
```

$$\text{Limit}\left[\frac{-b + \text{Floor}\left[b\sqrt{n}\right]}{b^2}, b \to \text{Infinity}\right]$$

```
f[100, 2, 5] - f[100, 2, 4]
```

$\frac{827}{200}$

```
N[f[100, 2, 2500]]
```

361.477

```
N[Gamma[2, 0, -Log[100]]]
```

$361.517 - 4.41506 \times 10^{-14}\, i$

```
f3[100, 2000.]
```

361.468

```
N[Gamma[2, 0, -Log[100]]]
```

$361.517 - 4.41506 \times 10^{-14}\, i$

```
N[100 Log[100] - 100 + 1]
```

361.517

```
N[200 Log[10] - 100 + 1]
```

361.517

```
-N[Integrate[ Gamma[1, 0, -Log[100 / j]], {j, 1, 100}]]
```

361.517

```
-N[2 Integrate[ Gamma[1, 0, -Log[100 / j]], {j, 1, 10}]] - 80 - 1
```

361.517

```
N[100 / 2 (Log[100])^2]
```

1060.38

```
N[100 / 2 × 4 ( Log[10]^2)]
```

1060.38

```
N[100 / 2 × 9 ( Log[100^(1 / 3)])^2]
```

1060.38

```
-2 N[Integrate[ Gamma[2, 0, -Log[100 / j]], {j, 1, 100}]]
N[Gamma[3, 0, -Log[100]]]
```

-1397.73

$-1397.73 + 3.42834 \times 10^{-13}\, i$

```
-2 N[Sum[Integrate[ Gamma[2, 0, -Log[100 / j]], {j, k, k + 1}], {k, 1, 99}]]
```

$Aborted

```
FullSimplify[Integrate[ Gamma[2, 0, -Log[100 / j]], {j, k, k + 1}]]
```

$$\text{ConditionalExpression}\left[1+50\log\left[\frac{1}{k}\right]\left(-2+\log[10\,000]+\log\left[\frac{1}{k}\right]\right)-50\log\left[\frac{1}{1+k}\right]\left(-2+\log[10\,000]+\log\left[\frac{1}{1+k}\right]\right),\ \mathrm{Re}[k]>0\ ||\ \mathrm{Re}[k]\le -1\ ||\ k\notin\text{Reals}\right]$$

```
FullSimplify[Integrate[ Gamma[2, 0, -Log[n / j]], {j, k, k + 1}]]
```

$$\text{ConditionalExpression}\left[\frac{1}{2}\left(2+n\left(\log\left[\frac{n}{k}\right]-\log\left[\frac{n}{1+k}\right]\right)\left(-2+\log\left[\frac{n}{k}\right]+\log\left[\frac{n}{1+k}\right]\right)\right),\ \mathrm{Re}[k]>0\ ||\ \mathrm{Re}[k]\le -1\ ||\ k\notin\text{Reals}\right]$$

```
Expand[Integrate[ Gamma[3, 0, -Log[n / j]], {j, k, k + 1}]]
```

$$\text{ConditionalExpression}\left[2-2n\log\left[\frac{n}{k}\right]+n\log\left[\frac{n}{k}\right]^2-\frac{1}{3}n\log\left[\frac{n}{k}\right]^3+2n\log\left[\frac{n}{1+k}\right]-n\log\left[\frac{n}{1+k}\right]^2+\frac{1}{3}n\log\left[\frac{n}{1+k}\right]^3,\ \mathrm{Re}[k]>0\ ||\ \mathrm{Re}[k]\le -1\ ||\ k\notin\text{Reals}\right]$$

```
Expand[Integrate[ Gamma[5 / 2, 0, -Log[n / j]], {j, k, k + 1}]]
```

$$\text{ConditionalExpression}\left[\frac{3}{2}n\sqrt{-\log\left[\frac{n}{k}\right]}-\frac{3k\sqrt{\pi}\ \mathrm{Erfi}\left[\sqrt{\log\left[\frac{n}{k}\right]}\right]\sqrt{-\log\left[\frac{n}{k}\right]}}{4\sqrt{\log\left[\frac{n}{k}\right]}}-n\sqrt{-\log\left[\frac{n}{k}\right]}\log\left[\frac{n}{k}\right]+\frac{2}{5}n\sqrt{-\log\left[\frac{n}{k}\right]}\log\left[\frac{n}{k}\right]^2-\frac{3}{2}n\sqrt{-\log\left[\frac{n}{1+k}\right]}+\frac{3\sqrt{\pi}\ \mathrm{Erfi}\left[\sqrt{\log\left[\frac{n}{1+k}\right]}\right]\sqrt{-\log\left[\frac{n}{1+k}\right]}}{4\sqrt{\log\left[\frac{n}{1+k}\right]}}+\frac{3k\sqrt{\pi}\ \mathrm{Erfi}\left[\sqrt{\log\left[\frac{n}{1+k}\right]}\right]\sqrt{-\log\left[\frac{n}{1+k}\right]}}{4\sqrt{\log\left[\frac{n}{1+k}\right]}}+n\sqrt{-\log\left[\frac{n}{1+k}\right]}\log\left[\frac{n}{1+k}\right]-\frac{2}{5}n\sqrt{-\log\left[\frac{n}{1+k}\right]}\log\left[\frac{n}{1+k}\right]^2,\ (k\neq 0\ \&\&\ \mathrm{Re}[k]>0)\ ||\ \mathrm{Re}[k]\le -1\ ||\ k\notin\text{Reals}\right]$$

```
Table[{k, f[100, 2, k] - f[100, 2, k - 1], f[100, 3, k] - f[100, 3, k - 1]}, {k, 2, 50}] //
 TableForm
```

2	35	$\frac{605}{4}$
3	$\frac{41}{3}$	$\frac{7081}{108}$
4	$\frac{167}{24}$	$\frac{30\,103}{864}$
5	$\frac{827}{200}$	$\frac{89\,791}{4000}$
6	$\frac{81}{25}$	$\frac{16\,521}{1000}$

7	$\frac{101}{49}$	$\frac{28\,829}{2744}$
8	$\frac{613}{392}$	$\frac{1\,587\,345}{175\,616}$
9	$\frac{11}{8}$	$\frac{2\,527\,297}{373\,248}$
10	$\frac{23}{25}$	$\frac{2\,107\,169}{364\,500}$
11	$\frac{2967}{3025}$	$\frac{2\,946\,509}{665\,500}$
12	$\frac{713}{1089}$	$\frac{9\,240\,659}{2\,299\,968}$
13	$\frac{829}{1521}$	$\frac{12\,046\,859}{3\,796\,416}$
14	$\frac{8643}{16\,562}$	$\frac{4\,080\,799}{1\,507\,142}$
15	$\frac{3167}{7350}$	$\frac{651\,673}{257\,250}$
16	$\frac{3763}{9600}$	$\frac{1\,699\,661}{768\,000}$
17	$\frac{13\,735}{36\,992}$	$\frac{17\,749\,909}{10\,061\,824}$
18	$\frac{8917}{23\,409}$	$\frac{53\,176\,945}{28\,652\,616}$
19	$\frac{5483}{29\,241}$	$\frac{57\,079\,957}{40\,001\,688}$
20	$\frac{18\,047}{72\,200}$	$\frac{1\,539\,021}{1\,097\,440}$
21	$\frac{25\,793}{88\,200}$	$\frac{1\,832\,581}{1\,481\,760}$
22	$\frac{19\,651}{106\,722}$	$\frac{461\,731}{407\,484}$
23	$\frac{22\,017}{128\,018}$	$\frac{543\,395}{535\,348}$
24	$\frac{30\,331}{152\,352}$	$\frac{28\,453\,133}{28\,032\,768}$
25	$\frac{28\,901}{180\,000}$	$\frac{31\,681\,757}{36\,000\,000}$
26	$\frac{25\,999}{211\,250}$	$\frac{208\,253\,717}{274\,625\,000}$
27	$\frac{32\,917}{246\,402}$	$\frac{270\,560\,689}{345\,948\,408}$
28	$\frac{41\,009}{285\,768}$	$\frac{292\,243\,741}{432\,081\,216}$
29	$\frac{33\,135}{329\,672}$	$\frac{342\,483\,077}{535\,387\,328}$
30	$\frac{24\,076}{189\,225}$	$\frac{3\,387\,217}{5\,268\,024}$
31	$\frac{19\,004}{216\,225}$	$\frac{3\,529\,309}{6\,434\,856}$
32	$\frac{26\,039}{246\,016}$	$\frac{131\,983\,815}{244\,047\,872}$
33	$\frac{3265}{30\,976}$	$\frac{49\,555\,373}{98\,131\,968}$
34	$\frac{5493}{69\,938}$	$\frac{213\,851\,117}{470\,822\,616}$
35	$\frac{59\,091}{708\,050}$	$\frac{157\,327\,943}{337\,031\,800}$
36	$\frac{71\,669}{793\,800}$	$\frac{86\,424\,049}{200\,037\,600}$
37	$\frac{51\,163}{887\,112}$	$\frac{450\,021\,677}{1\,181\,633\,184}$
38	$\frac{23\,610}{494\,209}$	$\frac{508\,475\,461}{1\,389\,715\,708}$
39	$\frac{18\,236}{183\,027}$	$\frac{45\,720\,253}{125\,190\,468}$
40	$\frac{13\,643}{405\,600}$	$\frac{12\,720\,661}{36\,504\,000}$
41	$\frac{87\,231}{1\,344\,800}$	$\frac{181\,085\,313}{551\,368\,000}$
42	$\frac{15\,313}{211\,806}$	$\frac{1\,617\,691\,351}{5\,106\,219\,048}$
43	$\frac{8207}{232\,974}$	$\frac{1\,615\,010\,827}{5\,890\,514\,616}$
44	$\frac{26\,735}{447\,458}$	$\frac{1\,951\,772\,809}{6\,772\,724\,288}$
45	$\frac{24\,173}{490\,050}$	$\frac{2\,142\,282\,161}{7\,762\,392\,000}$

46	$\frac{68\,249}{1\,071\,225}$	$\frac{2\,187\,398\,081}{8\,869\,743\,000}$
47	$\frac{38\,195}{1\,168\,561}$	$\frac{2\,481\,795\,413}{10\,105\,715\,528}$
48	$\frac{6399}{141\,376}$	$\frac{982\,007\,311}{3\,827\,331\,072}$
49	$\frac{4801}{153\,664}$	$\frac{955\,981\,039}{4\,337\,012\,736}$
50	$\frac{2664}{60\,025}$	$\frac{3\,243\,973\,821}{14\,706\,125\,000}$