```
e2[n_, k_] := e2[n, k] = Sum[e2[j, k - 1] e2[n / j, 1], {j, Divisors[n]}];
e2[n_, 1] := (-1)^(n + 1); e2[1, 1] := 0; e2[n_, 0] := 0; e2[1, 0] := 1
E2[n_, k_] := E2[n, k] = Sum[ e2[j, k], {j, 2, n}]
l2[n_] := l2[n] = Sum[(-1)^(k + 1) / k e2[n, k], {k, 1, Log[2, n]}]
L2[n_] := L2[n] = Sum[(-1)^(k + 1) / k E2[n, k], {k, 1, Log[2, n]}]
P2[n_, 0] := 1
P2[n_, k_] := P2[n, k] = Sum[ l2[j] P2[Floor[n / j], k - 1], {j, 2, n}]
Ex2[n_, z_] := Sum[ z^k / (k!) P2[n, k], {k, 0, Log[2, n]}]
```

```
E2[100, 1]
```

```
-1
```

```
DE[n_, 0] := 1
DE[n_, k_] := -Sum[ DE[Floor[n / (2 j)], k - 1], {j, 1, Floor[(n) / 2]}] +
  Sum[ DE[Floor[n / (2 j + 1)], k - 1], {j, 1, Floor[(n - 1) / 2]}]
DF[n_, 0] := 1
DF[n_, k_] := Sum[ DF[Floor[n / (2 j)], k - 1], {j, 1, Floor[(n) / 2]}] +
  Sum[ DF[Floor[n / (2 j + 1)], k - 1], {j, 1, Floor[(n - 1) / 2]}]
```

```
DE[100, 2]
```

```
3
```

```
Table[ {n, E2[n, 1], DE[n, 1], S1a[n], S1a2[n], S1a3[n]}, {n, 2, 100}] // TableForm
```

```
S1[n_ ] := Sum[1, {j, 1, Floor[(n - 1) / 2]}] - Sum[ 1, {j, 1, Floor[(n) / 2]}]
```

```
Expand[S1[n]]
```

```
S1a[n_] := Floor[1/2 (-1 + n)] - Floor[n/2]
```

```
S1a[100]
```

```
-1
```

```
S1b[n_] := (1/2 (-1 + n)) - (n/2)
```

```
Expand[S1b[n]]
```

$$-\frac{1}{2}$$

```
S1a2[n_] := -FractionalPart[1/2 (-1 + n)] + FractionalPart[n/2] - (1 / 2)
```

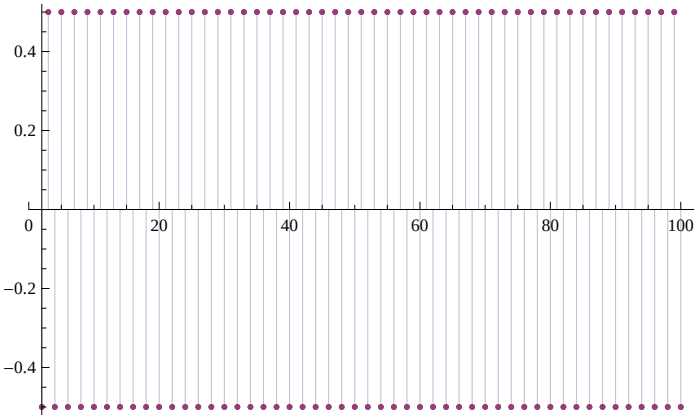
```
S1a3[n_] := ( (-1)^(n + 1) / 2) - (1 / 2)
```

```
S1a3[100]
```

```
-1
```

```
Expand[-FractionalPart[1/2 (-1 + n)] + FractionalPart[n/2]]
```

```
DiscretePlot[
 {-FractionalPart[1/2 (-1 + n)] + FractionalPart[n/2], ((-1)^(n + 1) / 2)}, {n, 2, 100}]
```




```
Expand[((-1)^(n + 1) / 2) - (1 / 2)]
```

$$-\frac{1}{2}+\frac{1}{2}\,(-1)^{1+n}$$

```
SF1a[n_] := Floor[1/2 (-1 + n)] + Floor[n/2]
SF1b[n_] := n - FractionalPart[n] - 1
SF1a[100]
```

```
99
```

```
Sum[1, {j, 1, Floor[(n - 1) / 2]}, {k, 1, Floor[(Floor[n / j] - 1) / 2]}]
```

```
$Aborted
```

```
f1[n_] := Sum[1, {j, 1, Floor[(n - 1) / 2]}, {k, 1, Floor[(Floor[n / j]) / 2]}]
f2[n_] := Sum[1, {j, 1, Floor[(n) / 2]}, {k, 1, Floor[(Floor[n / j] - 1) / 2]}]
f3[n_] := Sum[1, {j, 1, Floor[(n - 1) / 2]}, {k, 1, Floor[(Floor[n / j] - 1) / 2]}]
f4[n_] := Sum[1, {j, 1, Floor[(n) / 2]}, {k, 1, Floor[(Floor[n / j]) / 2]}]
f1[100]
```

```
206
```

```
f2[100]
```

```
175
```

```
f3[100] + f4[100] - f1[100] - f2[100]
```

```
1
```

```
DE[100, 2]
```

```
3
```

```
DF[100, 2]
```

```
283
```

```
f3[100] + f4[100] + f1[100] + f2[100]
```

763

```
DE[n, 2]
```

$$-\sum_{j=1}^{\text{Floor}\left[\frac{n}{2}\right]}\left(\text{Floor}\left[\frac{1}{2}\left(-1+\text{Floor}\left[\frac{n}{2\,j}\right]\right)\right]-\text{Floor}\left[\frac{1}{2}\,\text{Floor}\left[\frac{n}{2\,j}\right]\right]\right)+$$

$$\sum_{j=1}^{\text{Floor}\left[\frac{1}{2}(-1+n)\right]}\left(\text{Floor}\left[\frac{1}{2}\left(-1+\text{Floor}\left[\frac{n}{1+2\,j}\right]\right)\right]-\text{Floor}\left[\frac{1}{2}\,\text{Floor}\left[\frac{n}{1+2\,j}\right]\right]\right)$$

```
DF[n, 1]
```

$$\text{Floor}\left[\frac{1}{2}\,(-1+n)\right]+\text{Floor}\left[\frac{n}{2}\right]$$

```
FF[n_] := -Sum[(Floor[1/2 (-1 + Floor[n/(2 j)])] - Floor[1/2 Floor[n/(2 j)]]), {j, 1, Floor[n/2]}] +
  Sum[(Floor[1/2 (-1 + Floor[n/(1 + 2 j)])] - Floor[1/2 Floor[n/(1 + 2 j)]]), {j, 1, Floor[1/2 (-1 + n)]}]
```

```
FF[100]
```

3

```
DF[n, 2]
```

$$\sum_{j=1}^{\text{Floor}\left[\frac{n}{2}\right]}\left(\text{Floor}\left[\frac{1}{2}\left(-1+\text{Floor}\left[\frac{n}{2\,j}\right]\right)\right]+\text{Floor}\left[\frac{1}{2}\,\text{Floor}\left[\frac{n}{2\,j}\right]\right]\right)+$$

$$\sum_{j=1}^{\text{Floor}\left[\frac{1}{2}(-1+n)\right]}\left(\text{Floor}\left[\frac{1}{2}\left(-1+\text{Floor}\left[\frac{n}{1+2\,j}\right]\right)\right]+\text{Floor}\left[\frac{1}{2}\,\text{Floor}\left[\frac{n}{1+2\,j}\right]\right]\right)$$

```
FG[n_] := Sum[(Floor[1/2 (-1 + Floor[n/(2 j)])] + Floor[1/2 Floor[n/(2 j)]]), {j, 1, Floor[n/2]}] +
  Sum[(Floor[1/2 (-1 + Floor[n/(1 + 2 j)])] + Floor[1/2 Floor[n/(1 + 2 j)]]), {j, 1, Floor[1/2 (-1 + n)]}]
```

```
FG[100]
```

283

```
FF[n_] := -Sum[Floor[1/2 (-1 + Floor[n/(2 j)])] - Floor[1/2 Floor[n/(2 j)]], {j, 1, Floor[n/2]}] +
  Sum[Floor[1/2 (-1 + Floor[n/(1 + 2 j)])] - Floor[1/2 Floor[n/(1 + 2 j)]], {j, 1, Floor[1/2 (-1 + n)]}]

FF1[n_] := -Sum[-1/2 + (1/2) (-1)^(1 + Floor[n/(2 j)]), {j, 1, Floor[n/2]}] +
  Sum[Floor[1/2 (-1 + Floor[n/(1 + 2 j)])] - Floor[1/2 Floor[n/(1 + 2 j)]], {j, 1, Floor[1/2 (-1 + n)]}]

FF2[n_] :=
 -Sum[-1/2 + (1/2) (-1)^(1 + Floor[n/(2 j)]), {j, 1, Floor[n/2]}] + Sum[-1/2 + 1/2 (-1)^(1 + Floor[n/(1 + 2 j)]), {j, 1, Floor[1/2 (-1 + n)]}]

FF[1000]
```

-6

```
FF1[1000]
```

-6

```
FF2[1000]
```

-6

```
pp[n_, j_] := Expand[Floor[1/2 (-1 + Floor[n/(2 j)])] - Floor[1/2 Floor[n/(2 j)]]]

pp1[n_, j_] := -1/2 + (1/2) (-1)^(1 + Floor[n/(2 j)])
```

```
Table[{n, pp[n, 1], pp1[n, 1]}, {n, 1, 30}] // TableForm
```

1	−1	−1
2	0	0
3	0	0
4	−1	−1
5	−1	−1
6	0	0
7	0	0
8	−1	−1
9	−1	−1
10	0	0
11	0	0
12	−1	−1
13	−1	−1
14	0	0
15	0	0
16	−1	−1
17	−1	−1
18	0	0
19	0	0
20	−1	−1
21	−1	−1
22	0	0
23	0	0
24	−1	−1
25	−1	−1
26	0	0
27	0	0
28	−1	−1
29	−1	−1
30	0	0

```
Expand[ -1 / 2 + (1 / 2) (-1)^(1 + Floor[n / (2 j)])]
```

$$-\frac{1}{2}+\frac{1}{2}\ (-1)^{1+\text{Floor}\left[\frac{n}{2\,j}\right]}$$

```
px[n_, j_] := Floor[1/2 (-1 + Floor[n/(1 + 2 j)])] - Floor[1/2 Floor[n/(1 + 2 j)]]
```

```
px1[n_, j_] := -1 / 2 + (1 / 2) (-1)^(1 + Floor[(n) / (2 j + 1)])
```

```
Table[{n, px[n, 3], px1[n, 3]}, {n, 1, 30}] // TableForm
```

1	−1	−1
2	−1	−1
3	−1	−1
4	−1	−1
5	−1	−1
6	−1	−1
7	0	0
8	0	0
9	0	0
10	0	0
11	0	0
12	0	0
13	0	0
14	−1	−1
15	−1	−1
16	−1	−1
17	−1	−1
18	−1	−1
19	−1	−1
20	−1	−1
21	0	0
22	0	0
23	0	0
24	0	0
25	0	0
26	0	0
27	0	0
28	−1	−1
29	−1	−1
30	−1	−1

```
-1 / 2 + (1 / 2) (-1) ^ (1 + Floor[(n) / (2 j + 1)])
```

$$-\frac{1}{2}+\frac{1}{2}\ (-1)^{1+\text{Floor}\left[\frac{n}{1+2\,j}\right]}$$

```
FF2[n_] :=
```

$$-\sum_{j=1}^{\text{Floor}\left[\frac{n}{2}\right]}\left(-1/2+(1/2)\ (-1)^{\wedge}(1+\text{Floor}[n/(2\,j)])\right)+\sum_{j=1}^{\text{Floor}\left[\frac{1}{2}\,(-1+n)\right]}\left(-\frac{1}{2}+\frac{1}{2}\ (-1)^{1+\text{Floor}\left[\frac{n}{1+2\,j}\right]}\right)$$

```
FF3[n_] :=
```

$$\sum_{j=1}^{\text{Floor}\left[\frac{n}{2}\right]} (1/2 - (1/2)\,(-1)^{\wedge}(1+\text{Floor}[n/(2j)])) - \sum_{j=1}^{\text{Floor}\left[\frac{1}{2}(-1+n)\right]} \left(\frac{1}{2} - \frac{1}{2}\,(-1)^{1+\text{Floor}\left[\frac{n}{1+2j}\right]}\right)$$

```
FF4[n_] :=
```

$$\sum_{j=1}^{\text{Floor}\left[\frac{n}{2}\right]} (1/2 + (1/2)\,(-1)^{\wedge}(\text{Floor}[n/(2j)])) - \sum_{j=1}^{\text{Floor}\left[\frac{1}{2}(-1+n)\right]} \left(\frac{1}{2} + \frac{1}{2}\,(-1)^{\text{Floor}\left[\frac{n}{1+2j}\right]}\right)$$

```
FF5[n_] := (1/2)
```

$$\left(\sum_{j=1}^{\text{Floor}\left[\frac{n}{2}\right]} (1 + (-1)^{\wedge}(\text{Floor}[n/(2j)])) - \sum_{j=1}^{\text{Floor}\left[\frac{1}{2}(-1+n)\right]} \left(1 + (-1)^{\text{Floor}\left[\frac{n}{1+2j}\right]}\right)\right)$$

```
FF6[n_] :=
```

$$(1/2)\left((1/2)\,(1 + (-1)^{\wedge}n) + \sum_{j=1}^{\text{Floor}\left[\frac{n}{2}\right]} ((-1)^{\wedge}(\text{Floor}[n/(2j)])) - \sum_{j=1}^{\text{Floor}\left[\frac{1}{2}(-1+n)\right]} \left((-1)^{\text{Floor}\left[\frac{n}{1+2j}\right]}\right)\right)$$

```
FF7[n_] := (1/2)
```

$$\left((1/2)\,(1 + (-1)^{\wedge}n) + \sum_{j=1}^{\text{Floor}\left[\frac{n}{2}\right]} ((-1)^{\wedge}(\text{Floor}[n/(2j)])) - \sum_{j=1}^{\text{Floor}\left[\frac{1}{2}(-1+n)\right]} \left((-1)^{\text{Floor}\left[\frac{n}{1+2j}\right]}\right)\right)$$

$$\sum_{j=1}^{\text{Floor}\left[\frac{n}{2}\right]} (-1/2 + (1/2)\,(-1)^{\wedge}(1+\text{Floor}[n/(2j)]))$$

$$\sum_{j=1}^{\text{Floor}\left[\frac{n}{2}\right]} \left(-\frac{1}{2} + \frac{1}{2}\,(-1)^{1+\text{Floor}\left[\frac{n}{2j}\right]}\right)$$

```
TT[n_, j_] := ((-1)^(Floor[n/(2j)])) - ((-1)^(Floor[n/(1+2j)]))
```

```
TT[n, 1]
```

$$-(-1)^{\text{Floor}\left[\frac{n}{3}\right]} + (-1)^{\text{Floor}\left[\frac{n}{2}\right]}$$

```
FF2[1000]
```

$$-6$$

```
FF3[1000]
```

$$-6$$

```
FF4[1000]
```

$$-6$$

```
FF5[1330]
```

$$-7$$

```
Table[{n, FF5[n], FF6[n], FF5[n] - FF6[n]}, {n, 320, 340}] // TableForm
```

320	6	6	0
321	8	8	0
322	2	2	0
323	4	4	0
324	1	1	0
325	5	5	0
326	3	3	0
327	5	5	0
328	7	7	0
329	9	9	0
330	−5	−5	0
331	−5	−5	0
332	−5	−5	0
333	−1	−1	0
334	−3	−3	0
335	−1	−1	0
336	5	5	0
337	5	5	0
338	1	1	0
339	3	3	0
340	1	1	0

```
FH1[n_] := Sum[((-1)^(Floor[n / (2 j)])), {j, 1, Floor[n/2]}] - Sum[(-1)^Floor[n/(1+2 j)], {j, 1, Floor[1/2 (-1+n)]}]
```

```
FH1[1000]
```

```
-13
```

```
FH2[n_] := Sum[(-1)^(j + Floor[n / j]), {j, 2, n}]
```

```
FH2[1000]
```

```
-13
```

```
g[n_, k_, a_] := If[n < a^k, 0,
  Sum[(-1)^(k - j) Binomial[k, j] g[n / a^j, k - j, a + 1], {j, 0, k}]]; g[n_, 0, a_] := 1
{$RecursionLimit = 10000};
```

```
Table[{n, g[n, 2, 2], E2[n, 2]}, {n, 250, 270}] // TableForm
```

250	−2	−2
251	−2	−2
252	−6	−6
253	−4	−4
254	−6	−6
255	0	0
256	7	7
257	7	7
258	1	1
259	3	3
260	1	1
261	5	5
262	3	3
263	3	3
264	5	5
265	7	7
266	1	1
267	3	3
268	3	3
269	3	3
270	−11	−11

```
E2[2, 1]
```

```
LAdd[n_] := Sum[2^k / k, {k, 1, Log[2, n]}]
```

```
LinE[n_] := LAdd[n] - Sum[1 / k g[n, k, 2], {k, 1, Log[2, n]}]
```

```
LinE[100]
```

$$\frac{428}{15}$$

```
-g[100, 5, 2]
```

```
9
```

```
E2[100, 5]
```

```
9
```

```
DHyp[n_, k_, a_] := Sum[(-1)^(k - j + m - 1) Binomial[k, j] DHyp[n / (m^(k - j)), j, m + 1],
  {m, a, n^(1 / k)}, {j, 0, k - 1}]
DHyp[n_, 0, a_] := 1
```

```
DHyp[1000, 3, 2]
```

```
13
```

```
E2[1000, 3]
```

```
-19
```

```
FF[n_] := -Sum[(Floor[1/2 (-1 + Floor[n/(2 j)])] - Floor[1/2 Floor[n/(2 j)]]), {j, 1, Floor[n/2]}] +
  Sum[(Floor[1/2 (-1 + Floor[n/(1 + 2 j)])] - Floor[1/2 Floor[n/(1 + 2 j)]]), {j, 1, Floor[1/2 (-1 + n)]}]
```

```
FFI[n_] := -Sum[((1/2 (-1 + (n/(2 j)))) - (1/2 (n/(2 j)))), {j, 1, n/2}] + Sum[((1/2 (-1 + (n/(1 + 2 j)))) - (1/2 (n/(1 + 2 j)))), {j, 1, 1/2 (-1 + n)}]
```

```
FF[1000]
```

```
-6
```

```
FFI[20]
```

$$\frac{1}{2}$$

```
Expand[FFI[n]]
```

$$\frac{1}{4}$$

```
Expand[((1/2 (-1 + (n/(2 j)))) - (1/2 (n/(2 j))))]
```

$$-\frac{1}{2}$$

```
Expand[((1/2 (-1 + (n/(1 + 2 j)))) - (1/2 (n/(1 + 2 j))))]
```

$$-\frac{1}{2}$$

```
FFI[n]
```

$$\frac{1-n}{4}+\frac{n}{4}$$

```
FFI2[n_] := -Sum[((1/2 (-1 + (n/(2 j)))) - (1/2 (n/(2 j)))), {j, 1, n/2}] + Sum[((1/2 (-1 + (n/(1 + 2 j)))) - (1/2 (n/(1 + 2 j)))), {j, 1, 1/2 (-1 + n)}]
```

```
FFI3[n_] := -Integrate[((1/2 (-1 + (n/(2 j)))) - (1/2 (n/(2 j)))), {j, 1, n/2}] +
  Integrate[((1/2 (-1 + (n/(1 + 2 j)))) - (1/2 (n/(1 + 2 j)))), {j, 1, (n - 1)/2}]
```

```
FFI3[n]
```

$$\frac{1}{4}$$

```
-Integrate[((1/2 (-1 + (n/(2 j)))) - (1/2 (n/(2 j)))), {j, 1, n/2}]
```

$$-\frac{1}{2}+\frac{n}{4}$$

```
Integrate[(((1/2) (-1 + (n/(1 + 2 j)))) - ((1/2) (n/(1 + 2 j)))), {j, 1, (n - 1) / 2}]
```

$$\frac{3}{4} - \frac{n}{4}$$

```
FF7[n_] :=
 (1 / 2) ((1 / 2) (1 + (-1)^n) + Sum[((-1)^(Floor[n / (2 j)])), {j, 1, Floor[n/2]}] - Sum[(-1)^Floor[n/(1+2 j)], {j, 1, Floor[(1/2) (-1+n)]}])
```

```
FF7[100]
```

3

```
DE[100, 2]
```

3

```
Table[{n, FF7[n], DE[n, 2]}, {n, 250, 270}] // TableForm
```

250	−2	−2
251	−2	−2
252	−6	−6
253	−4	−4
254	−6	−6
255	0	0
256	7	7
257	7	7
258	1	1
259	3	3
260	1	1
261	5	5
262	3	3
263	3	3
264	5	5
265	7	7
266	1	1
267	3	3
268	3	3
269	3	3
270	−11	−11

```
TT[n_, j_] := ((-1)^(Floor[n / (2 j)])) - ((-1)^Floor[n/(1+2 j)])
```

```
TT[n, 1]
```

$$-(-1)^{\text{Floor}\left[\frac{n}{3}\right]} + (-1)^{\text{Floor}\left[\frac{n}{2}\right]}$$

```
TX[n_] := Sum[(-1)^(j + Floor[n / j]), {j, 2, n}]
```

```
TY[n_] := Sum[((-1)^(Floor[n / (2 j)])), {j, 1, Floor[n/2]}] - Sum[(-1)^Floor[n/(1+2 j)], {j, 1, Floor[(1/2) (-1+n)]}]
```

```
TX[1000]
```

−13

```
TY[1000]
```

```
-13
```

```
TX[n_] := Sum[ (-1)^(j + Floor[n / j]), {j, 2, n}]
```

```
bbb = 2
TX2[n_] := Sum[ (-1)^(j + Floor[n / j]), {j, Floor[n / (bbb + 1)] + 1, Floor[n / bbb]}]
Table[ {n, TX2[n], (1 / 2) ((-1)^(bbb + 1))
     (- (-1)^(Floor[n / bbb]) - (-1)^(Floor[n / (bbb + 1)] + 1))}, {n, 50, 100}] // TableForm
```

```
2
```

50	−1	−1
51	0	0
52	1	1
53	1	1
54	−1	−1
55	−1	−1
56	0	0
57	1	1
58	0	0
59	0	0
60	0	0
61	0	0
62	−1	−1
63	0	0
64	1	1
65	1	1
66	−1	−1
67	−1	−1
68	0	0
69	1	1
70	0	0
71	0	0
72	0	0
73	0	0
74	−1	−1
75	0	0
76	1	1
77	1	1
78	−1	−1
79	−1	−1
80	0	0
81	1	1
82	0	0
83	0	0
84	0	0
85	0	0
86	−1	−1
87	0	0
88	1	1
89	1	1
90	−1	−1
91	−1	−1
92	0	0
93	1	1
94	0	0
95	0	0
96	0	0
97	0	0
98	−1	−1
99	0	0
100	1	1

```
(1 / 2) ((-1)^(ccc)) ((-1)^(Floor[n / ccc]) - (-1)^(Floor[n / (ccc + 1)]))
```

$$\frac{1}{2}\,(-1)^{ccc}\left((-1)^{\text{Floor}\left[\frac{n}{ccc}\right]}-(-1)^{\text{Floor}\left[\frac{n}{1+ccc}\right]}\right)$$

```
u = 5
TX2[n_] := Sum[ (-1)^(j + Floor[n / j]), {j, Floor[n / (u + 1)] + 1, Floor[n / u]}]
Table[ {n, TX2[n], (1 / 2) ((-1)^(u)) ((-1)^(Floor[n / u]) - (-1)^(Floor[n / (u + 1)]))},
  {n, 50, 100}] // TableForm
```

5

```
50    0    0
51    0    0
52    0    0
53    0    0
54    -1   -1
55    0    0
56    0    0
57    0    0
58    0    0
59    0    0
60    0    0
61    0    0
62    0    0
63    0    0
64    0    0
65    1    1
66    0    0
67    0    0
68    0    0
69    0    0
70    -1   -1
71    -1   -1
72    0    0
73    0    0
74    0    0
75    1    1
76    1    1
77    1    1
78    0    0
79    0    0
80    -1   -1
81    -1   -1
82    -1   -1
83    -1   -1
84    0    0
85    1    1
86    1    1
87    1    1
88    1    1
89    1    1
90    -1   -1
91    -1   -1
92    -1   -1
93    -1   -1
94    -1   -1
95    0    0
96    1    1
97    1    1
98    1    1
99    1    1
100   0    0
```

```
u = 2
TX2[n_] := Sum[ (-1)^(j + Floor[n / j]), {j, Floor[n / (u + 2)] + 1, Floor[n / u]}]
Table[ {n, TX2[n],
    (1 / 2) ((-1)^(u)) ((-1)^(Floor[n / u]) - (-1)^(Floor[n / (u + 1)])) + (1 / 2) ((-1)^(u + 1))
      ((-1)^(Floor[n / (u + 1)]) - (-1)^(Floor[n / (u + 2)]))}, {n, 50, 100}] // TableForm
```

2

50	−1	−1
51	1	1
52	1	1
53	1	1
54	−2	−2
55	−2	−2
56	0	0
57	2	2
58	1	1
59	1	1
60	−1	−1
61	−1	−1
62	−2	−2
63	0	0
64	2	2
65	2	2
66	−1	−1
67	−1	−1
68	−1	−1
69	1	1
70	0	0
71	0	0
72	0	0
73	0	0
74	−1	−1
75	1	1
76	1	1
77	1	1
78	−2	−2
79	−2	−2
80	0	0
81	2	2
82	1	1
83	1	1
84	−1	−1
85	−1	−1
86	−2	−2
87	0	0
88	2	2
89	2	2
90	−1	−1
91	−1	−1
92	−1	−1
93	1	1
94	0	0
95	0	0
96	0	0
97	0	0
98	−1	−1
99	1	1
100	1	1

```
u = 2
Expand[(1 / 2) ((-1)^(u)) ((-1)^(Floor[n / u]) - (-1)^(Floor[n / (u + 1)])) +
  (1 / 2) ((-1)^(u + 1)) ((-1)^(Floor[n / (u + 1)]) - (-1)^(Floor[n / (u + 2)]))]
```

2

$$\frac{1}{2}\ (-1)^{\text{Floor}\left[\frac{n}{4}\right]} - (-1)^{\text{Floor}\left[\frac{n}{3}\right]} + \frac{1}{2}\ (-1)^{\text{Floor}\left[\frac{n}{2}\right]}$$

```
u = 2
TX2[n_] := Sum[ (-1)^(j + Floor[n / j]), {j, Floor[n / (u + 3)] + 1, Floor[n / u]}]
Table[ {n, TX2[n], (1 / 2) ((-1)^(u)) ((-1)^(Floor[n / u]) - (-1)^(Floor[n / (u + 1)]))
    + (1 / 2) ((-1)^(u + 1)) ((-1)^(Floor[n / (u + 1)]) - (-1)^(Floor[n / (u + 2)]))
    + (1 / 2) ((-1)^(u + 2)) ((-1)^(Floor[n / (u + 2)]) - (-1)^(Floor[n / (u + 3)]))},
  {n, 50, 100}] // TableForm
```

2

50	−1	−1
51	1	1
52	0	0
53	0	0
54	−3	−3
55	−2	−2
56	1	1
57	3	3
58	2	2
59	2	2
60	−2	−2
61	−2	−2
62	−3	−3
63	−1	−1
64	2	2
65	3	3
66	0	0
67	0	0
68	−1	−1
69	1	1
70	−1	−1
71	−1	−1
72	0	0
73	0	0
74	−1	−1
75	2	2
76	1	1
77	1	1
78	−2	−2
79	−2	−2
80	0	0
81	2	2
82	1	1
83	1	1
84	−2	−2
85	−1	−1
86	−2	−2
87	0	0
88	3	3
89	3	3
90	−1	−1
91	−1	−1
92	−2	−2
93	0	0
94	−1	−1
95	0	0
96	1	1
97	1	1
98	0	0
99	2	2
100	0	0

```
(1 / 2) ((-1)^(u)) ((-1)^(Floor[n / u]) - (-1)^(Floor[n / (u + 1)]))
+ (1 / 2) ((-1)^(u + 1)) ((-1)^(Floor[n / (u + 1)]) - (-1)^(Floor[n / (u + 2)]))
+ (1 / 2) ((-1)^(u + 2)) ((-1)^(Floor[n / (u + 2)]) - (-1)^(Floor[n / (u + 3)]))
```

$$\frac{1}{2}\left(-(-1)^{\text{Floor}\left[\frac{n}{5}\right]}+(-1)^{\text{Floor}\left[\frac{n}{4}\right]}\right)+\frac{1}{2}\left((-1)^{\text{Floor}\left[\frac{n}{4}\right]}-(-1)^{\text{Floor}\left[\frac{n}{3}\right]}\right)+\frac{1}{2}\left(-(-1)^{\text{Floor}\left[\frac{n}{3}\right]}+(-1)^{\text{Floor}\left[\frac{n}{2}\right]}\right)$$

```
u = 2
Expand[ (1 / 2) ((-1)^(u)) ((-1)^(Floor[n / u]) - (-1)^(Floor[n / (u + 1)]))
   + (1 / 2) ((-1)^(u + 1)) ((-1)^(Floor[n / (u + 1)]) - (-1)^(Floor[n / (u + 2)]))
   + (1 / 2) ((-1)^(u + 2)) ((-1)^(Floor[n / (u + 2)]) - (-1)^(Floor[n / (u + 3)]))]
```

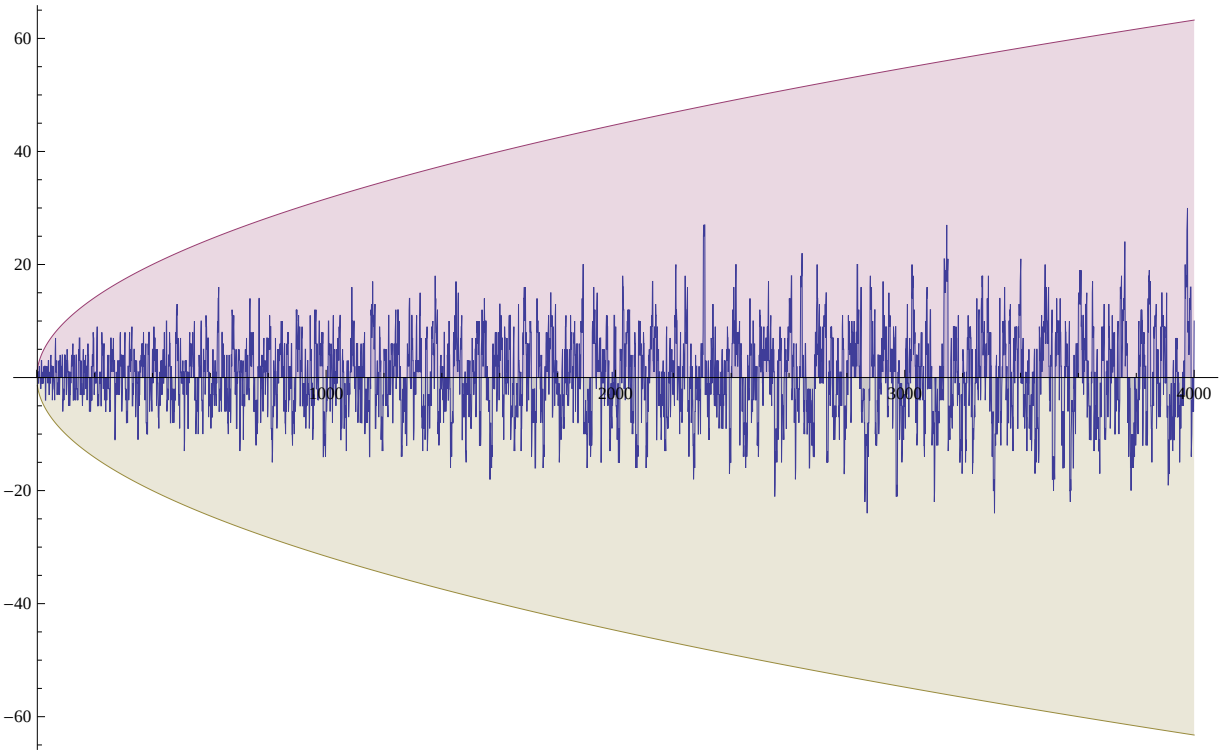
```
2
```

$$-\frac{1}{2}(-1)^{\text{Floor}\left[\frac{n}{5}\right]}+(-1)^{\text{Floor}\left[\frac{n}{4}\right]}-(-1)^{\text{Floor}\left[\frac{n}{3}\right]}+\frac{1}{2}(-1)^{\text{Floor}\left[\frac{n}{2}\right]}$$

```
e2[100, 5]
```

```
0
```

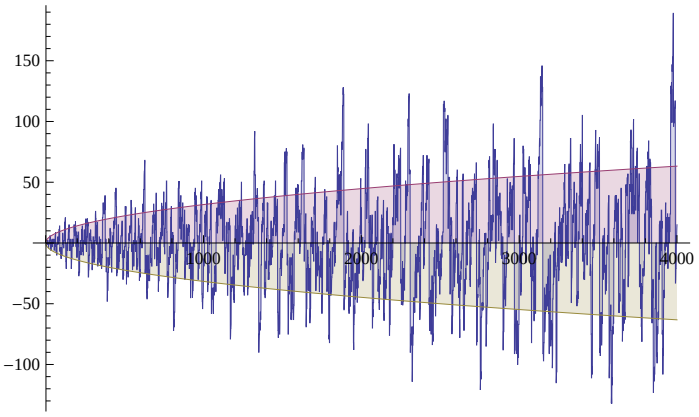
```
DiscretePlot[ {E2[n, 2], n^(1 / 2), -(n^(1 / 2))}, {n, 2, 4000}]
```



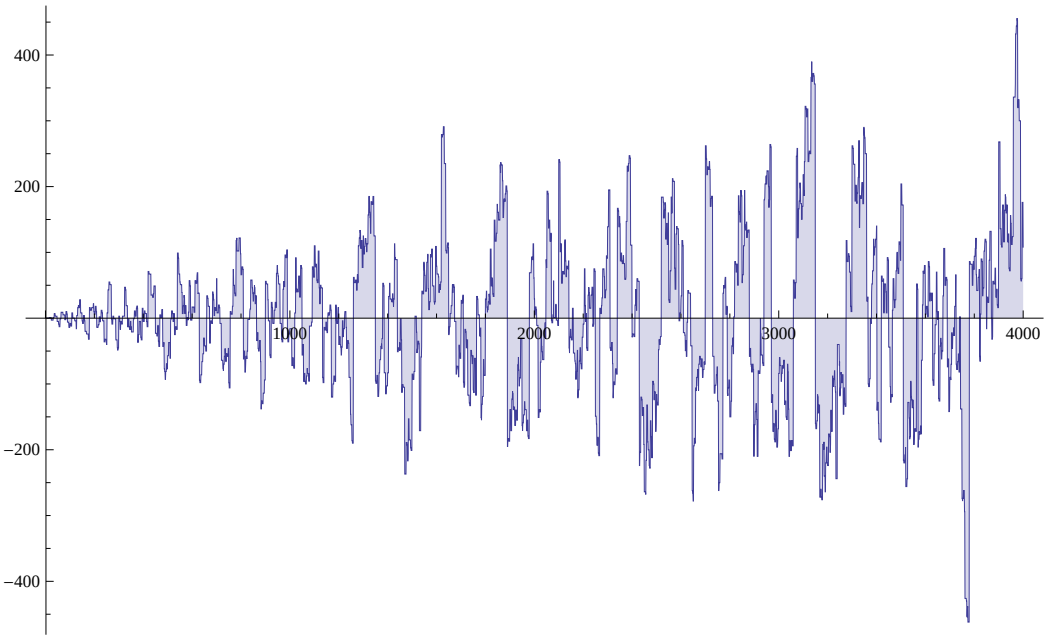

```
1000^(.25)
```

```
5.62341
```

```
DiscretePlot[{E2[n, 3], n^(1 / 2), -(n^(1 / 2))}, {n, 2, 4000}]
```

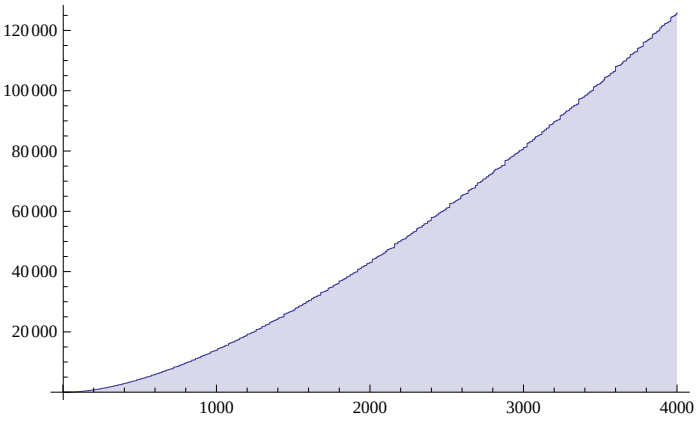



```
DiscretePlot[{E2[n, 4]}, {n, 2, 4000}]
```




```
D2Alt[n_, k_] := D2Alt[n, k] = Sum[D2Alt[Floor[n / j], k - 1], {j, 2, n}]; D2Alt[n_, 0] := 1
```

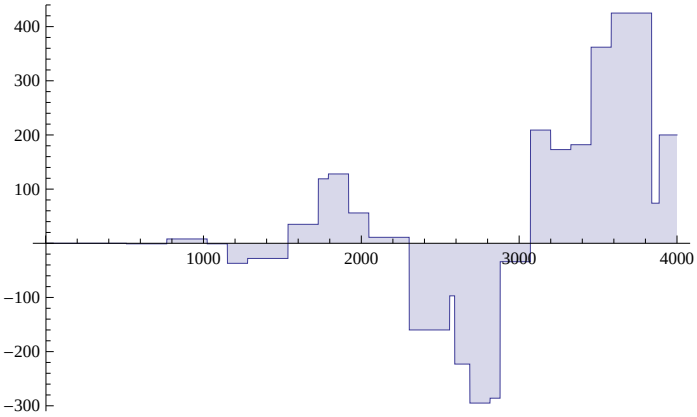
```
DiscretePlot[{D2Alt[n, 4]}, {n, 2, 4000}]
```



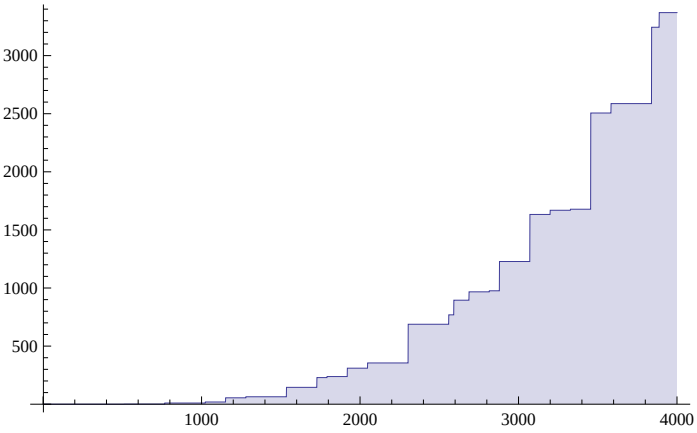
```
PrimePi[4000]
```

```
550
```

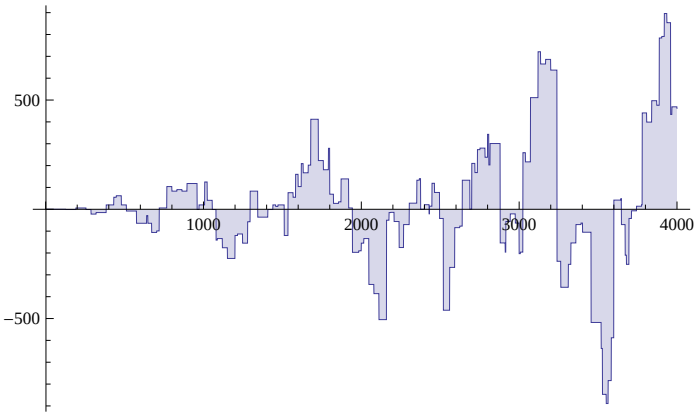
```
DiscretePlot[{E2[n, 9]}, {n, 2, 4000}]
```



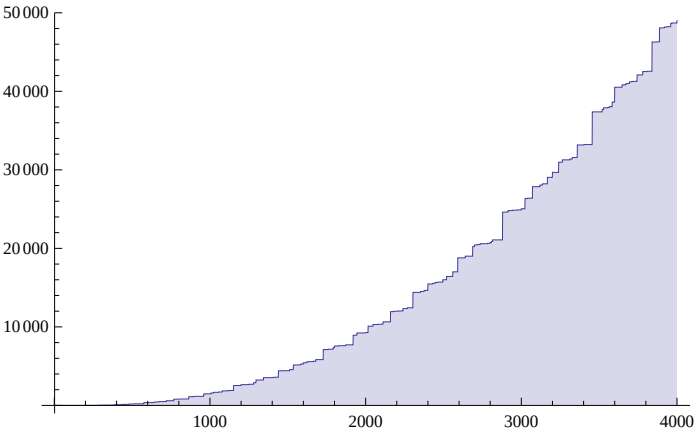

```
DiscretePlot[{D2Alt[n, 9]}, {n, 2, 4000}]
```




```
DiscretePlot[{E2[n, 7]}, {n, 2, 4000}]
```

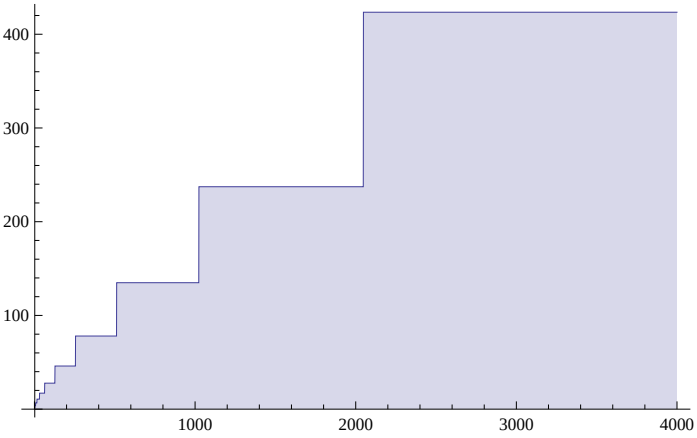


```
DiscretePlot[{D2Alt[n, 7]}, {n, 2, 4000}]
```




```
LAdd[n_] := Sum[2^k / k, {k, 1, Log[2, n]}]
```

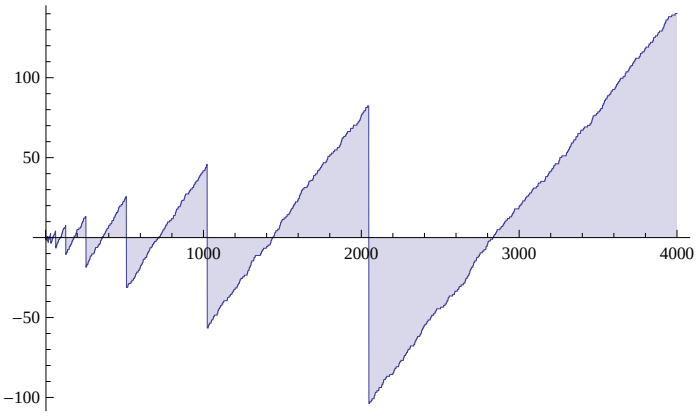
```
DiscretePlot[{LAdd[n]}, {n, 2, 4000}]
```



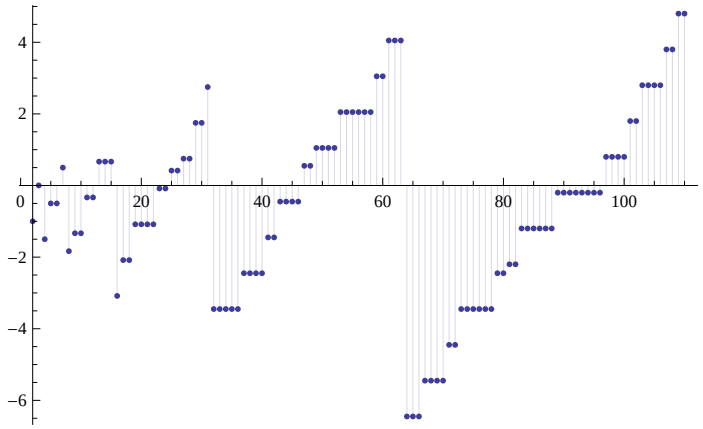

```
LAdd[100] + L2[100]
```

$$\frac{428}{15}$$

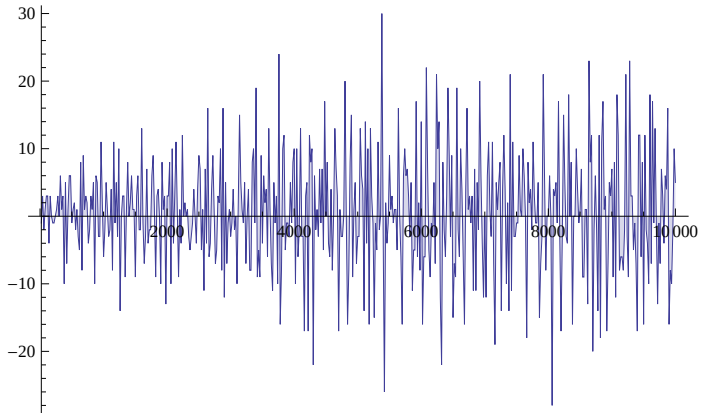
```
DiscretePlot[{L2[n]}, {n, 2, 4000}]
```



```
DiscretePlot[{L2[n]}, {n, 2, 110}]
```

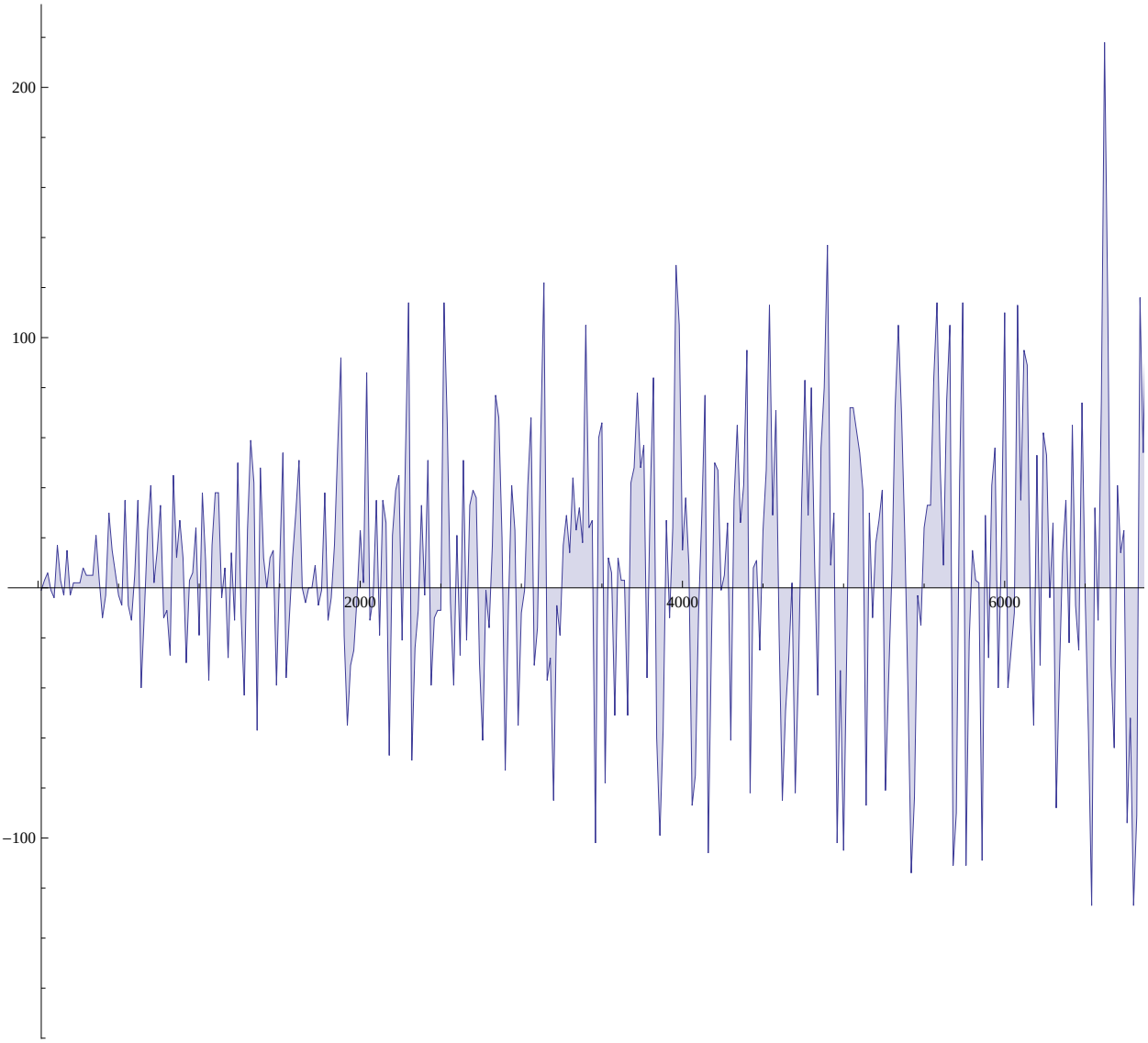



```
DiscretePlot[{E2[n, 2]}, {n, 20, 10 000, 20}]
```

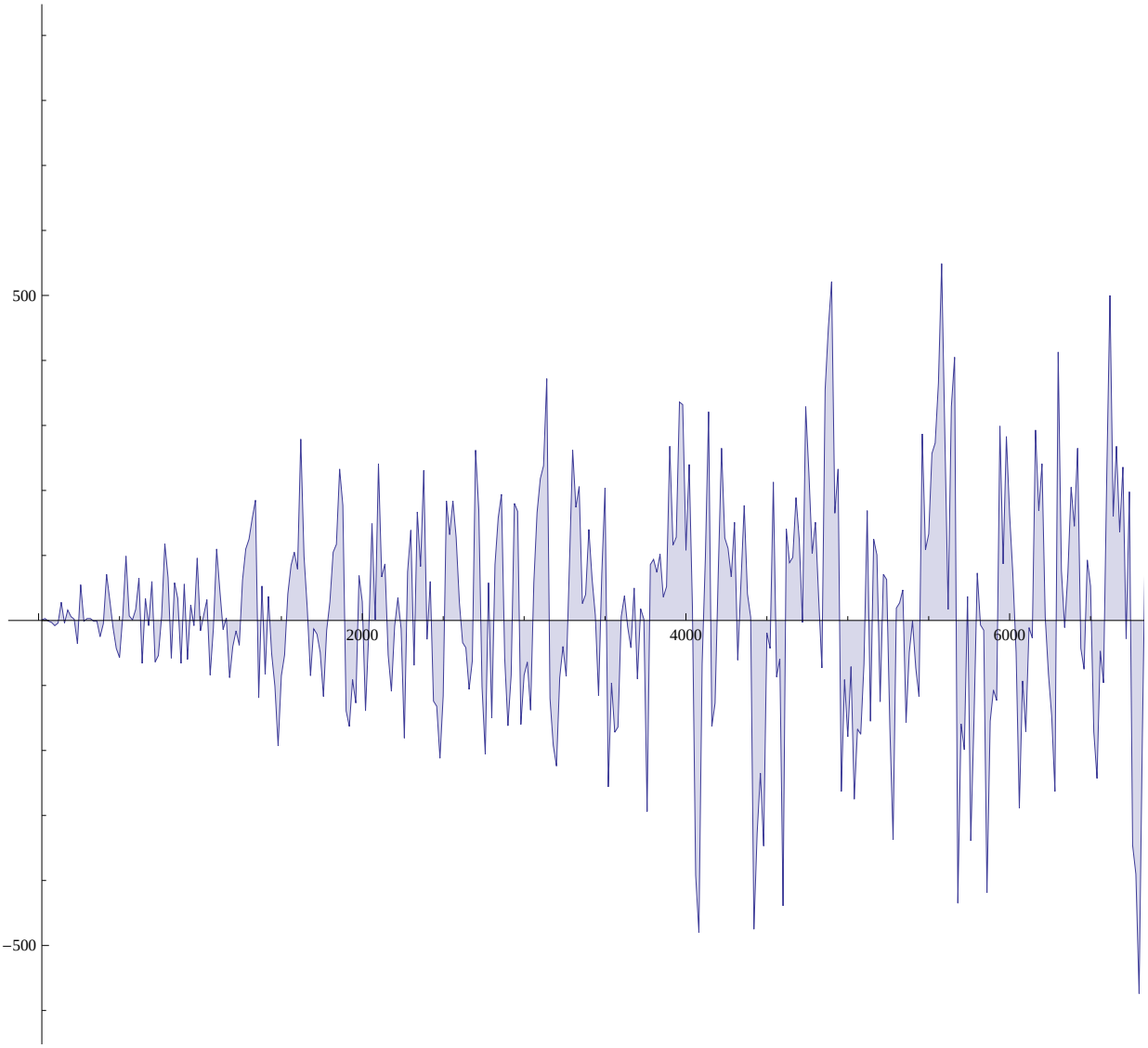



```
DiscretePlot[{E2[n, 3]}, {n, 20, 10 000, 20}] / 3
DiscretePlot[{E2[n, 4]}, {n, 20, 10 000, 20}] / -4
DiscretePlot[{E2[n, 5]}, {n, 20, 10 000, 20}] / 5
DiscretePlot[{E2[n, 6]}, {n, 20, 10 000, 20}] / -6
DiscretePlot[{E2[n, 7]}, {n, 20, 10 000, 20}] / 7
DiscretePlot[{E2[n, 8]}, {n, 20, 10 000, 20}] / -8
DiscretePlot[{E2[n, 9]}, {n, 20, 10 000, 20}] / 9
```

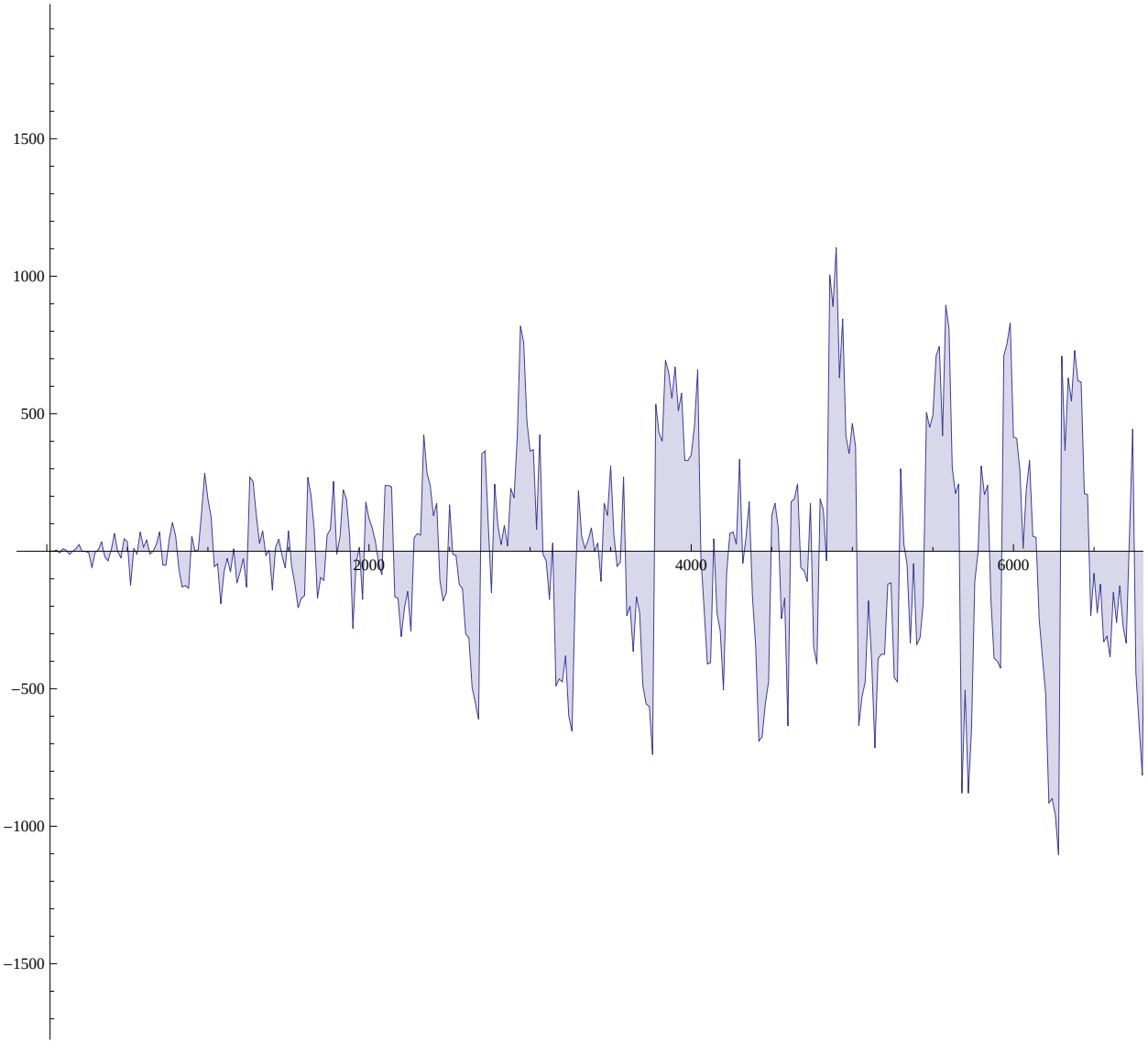
$$\frac{1}{3}$$



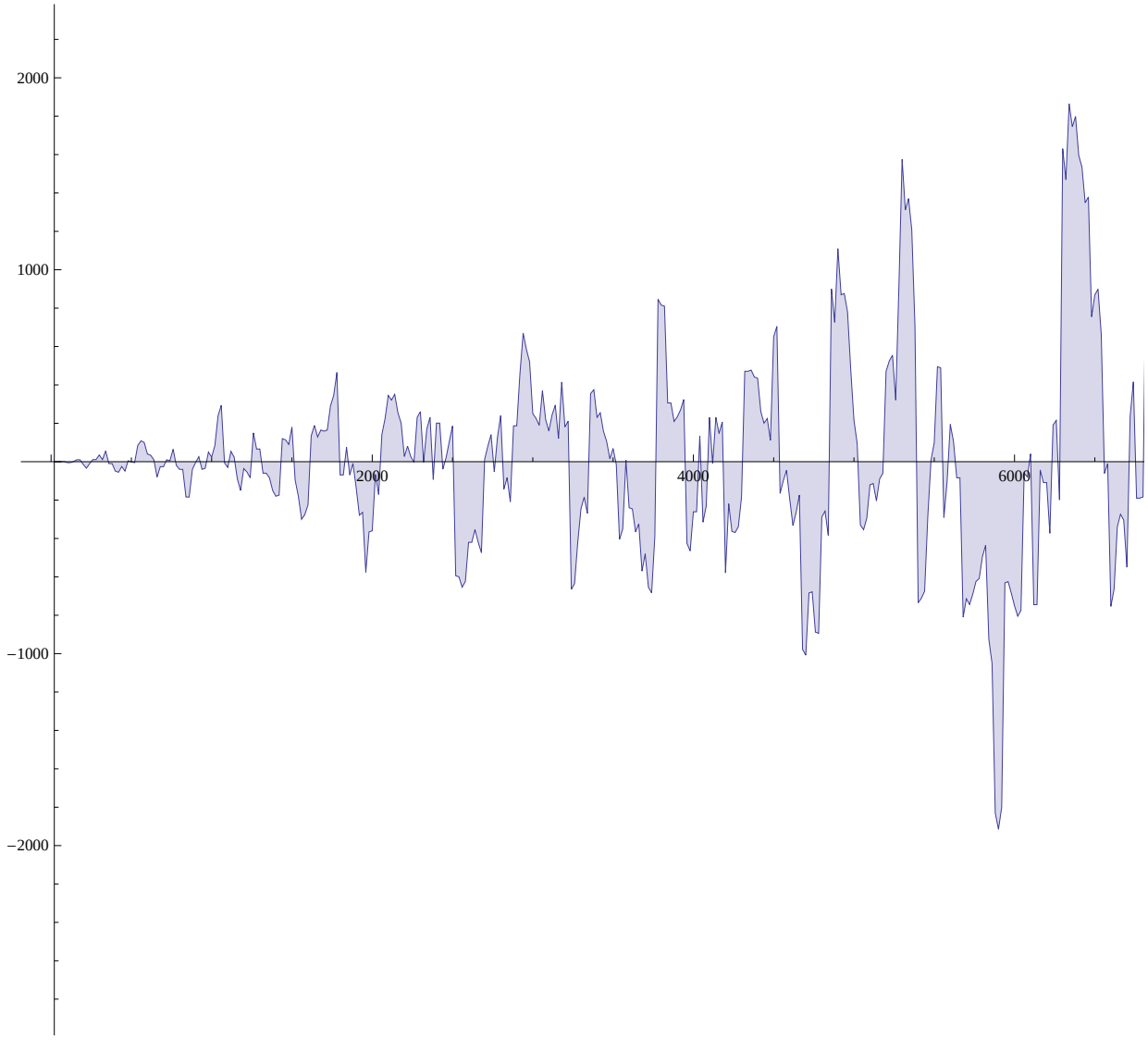
$$-\frac{1}{4}$$



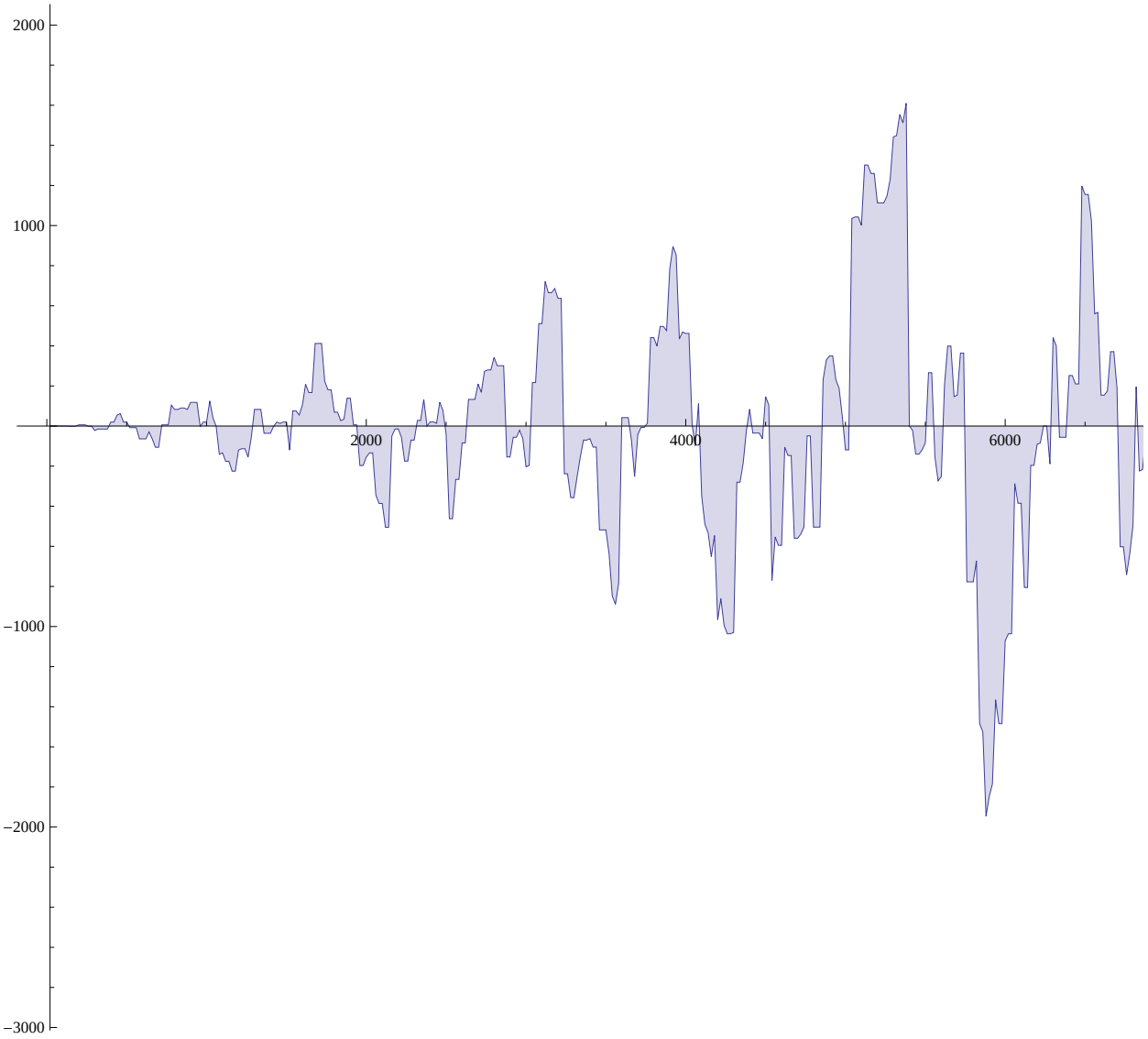
$$\frac{1}{5}$$



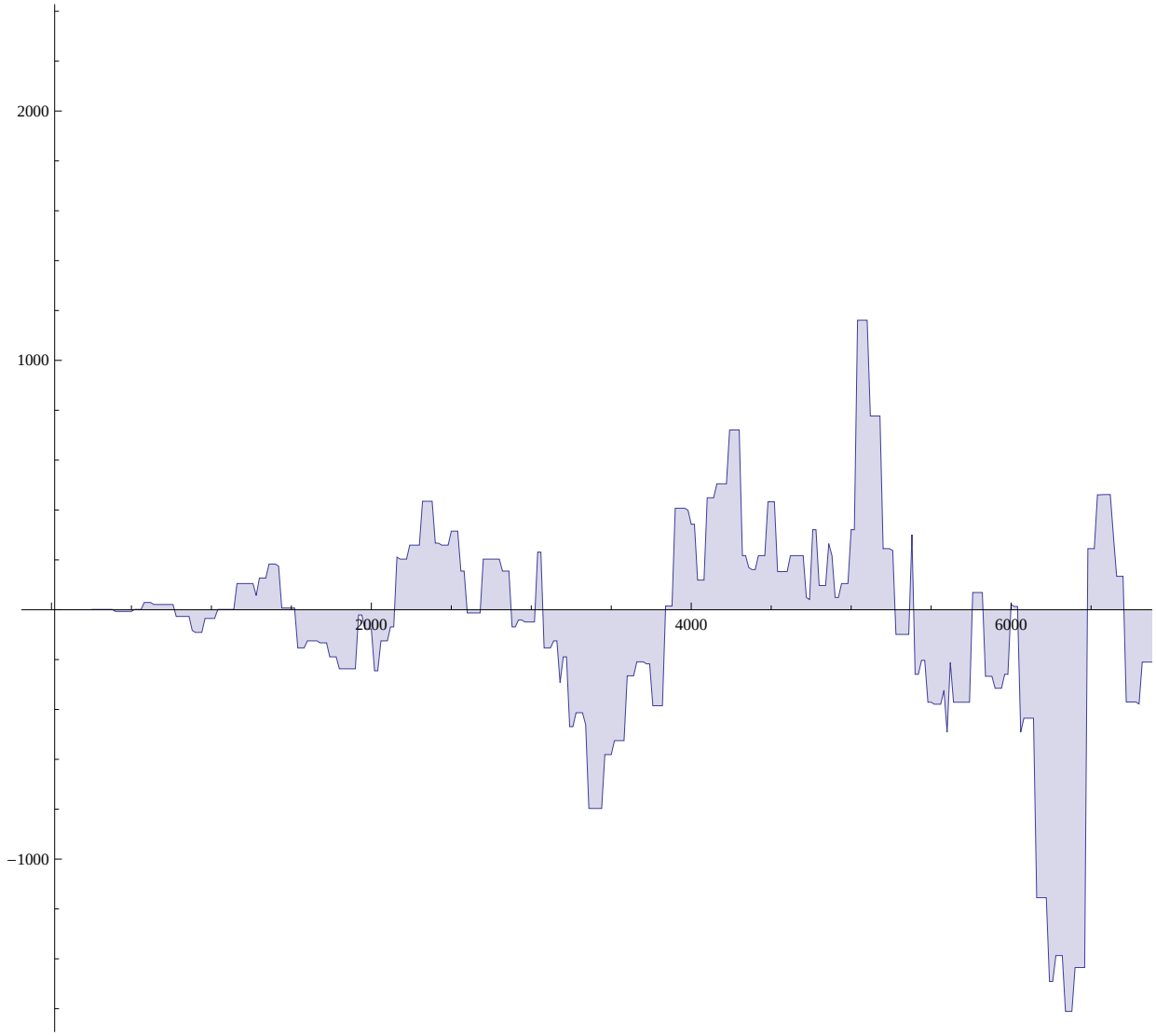
$$-\frac{1}{6}$$



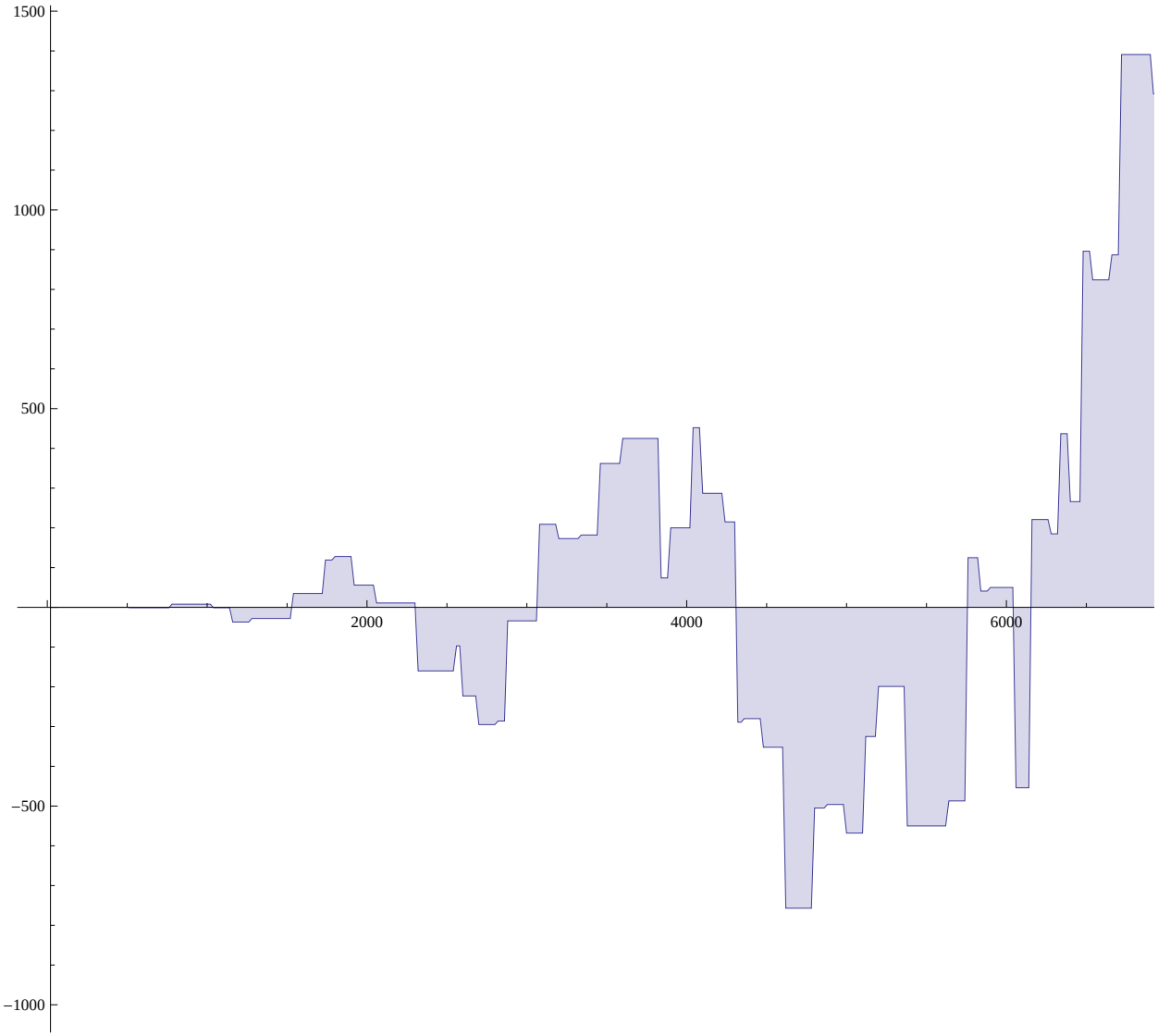
$$\frac{1}{7}$$



$$-\frac{1}{8}$$



$\frac{1}{9}$




```
Table[{n, l2[2^n], pk[2^n]}, {n, 1, 8}] // TableForm
```

1	-1	-1
2	$-\frac{3}{2}$	$-\frac{3}{2}$
3	$-\frac{7}{3}$	$-\frac{7}{3}$
4	$-\frac{15}{4}$	$-\frac{15}{4}$
5	$-\frac{31}{5}$	$-\frac{31}{5}$
6	$-\frac{21}{2}$	$-\frac{21}{2}$
7	$-\frac{127}{7}$	$-\frac{127}{7}$
8	$-\frac{255}{8}$	$-\frac{255}{8}$

```
E2[100, 2]
```

3

```
FF7[n_] :=
```

$$(1/2)\left((1/2)(1+(-1)^{n})+\sum_{j=1}^{\text{Floor}\left[\frac{n}{2}\right]}((-1)^{(\text{Floor}[n/(2j)])})-\sum_{j=1}^{\text{Floor}\left[\frac{1}{2}(-1+n)\right]}\left((-1)^{\text{Floor}\left[\frac{n}{1+2j}\right]}\right)\right)$$

```
FF8[n_] := (1 / 2) ( ( 1 / 2) (1 + (-1)^n ) + Sum[ (-1)^(j + Floor[n / j]), {j, 2, n}])

FF9[n_] :=
 (1 / 2) ( ( 1 / 2) (1 + (-1)^n ) + Sum[ (-1)^(j + Floor[n / j]), {j, 2, Floor[n^(1 / 2)]}] +
    Sum[ (-1)^(j + Floor[n / j]), {j, 1, Floor[n / Floor[n^(1 / 2)] - 1]}] )

Table[ {n, FF7[n], FF9[n]}, {n, 80, 100}] // TableForm
```

80	1	$\frac{1}{2}$
81	4	4
82	2	$\frac{3}{2}$
83	2	2
84	0	$-\frac{1}{2}$
85	2	2
86	0	$-\frac{1}{2}$
87	2	2
88	4	$\frac{7}{2}$
89	4	4
90	-6	-6
91	-4	$-\frac{7}{2}$
92	-4	-4
93	-2	$-\frac{3}{2}$
94	-4	-4
95	-2	$-\frac{3}{2}$
96	4	4
97	4	$\frac{9}{2}$
98	0	0
99	4	4
100	3	$\frac{5}{2}$

```
FF8[100]
```

3

-6

```
FF9[100]
```

$$\frac{5}{2}$$

```
u = 2
Expand[ (1 / 2) ((-1) ^ (u)) ((-1) ^ (Floor[n / u]) - (-1) ^ (Floor[n / (u + 1)]))
  + (1 / 2) ((-1) ^ (u + 1)) ((-1) ^ (Floor[n / (u + 1)]) - (-1) ^ (Floor[n / (u + 2)]))
  + (1 / 2) ((-1) ^ (u + 2)) ((-1) ^ (Floor[n / (u + 2)]) - (-1) ^ (Floor[n / (u + 3)]))]
```

2

$$-\frac{1}{2}\ (-1)^{\text{Floor}\left[\frac{n}{5}\right]}+(-1)^{\text{Floor}\left[\frac{n}{4}\right]}-(-1)^{\text{Floor}\left[\frac{n}{3}\right]}+\frac{1}{2}\ (-1)^{\text{Floor}\left[\frac{n}{2}\right]}$$

```
fe1[n_] := Sum[ (-1) ^ (j + Floor[n / j]), {j, 1, Floor[n / Floor[n^(1 / 2)] - 1]}]
fe1[1000]
```

-7

```
ee1[n_] := Sum[ (-1) ^j (-1) ^k, {j, 2, n}, {k, 2, Floor[n / j]}]
ee1[100]
```

3

```
ee1[1000]
```

-6

```
ee1a[n_] := 2 Sum[ (-1) ^j (-1) ^k, {j, 2, Floor[n^(1 / 2)]}, {k, 2, Floor[n / j]}] -
  Sum[ (-1) ^j (-1) ^k, {j, 2, Floor[n^(1 / 2)]}, {k, 2, Floor[n^(1 / 2)]}]
ee1a[1000]
```

-6

```
ee1b[n_] :=
 2 Sum[ (-1) ^j (-1) ^k, {j, 2, Floor[n^(1 / 2)]}, {k, 2, Floor[n / j]}] - 1/2 (1 + (-1) ^Floor[Sqrt[n]])
ee1b[1000]
```

-6

```
ee1a1[n_] := Sum[ (-1) ^j (-1) ^k, {j, 2, Floor[n^(1 / 2)]}, {k, 2, Floor[n^(1 / 2)]}]
```

```
Table[{n, ee1a1[n], 1 / 2 + (1 / 2) (-1)^(Floor[n^(1 / 2)])}, {n, 2, 50}] // TableForm
```

2	0	0
3	0	0
4	1	1
5	1	1
6	1	1
7	1	1
8	1	1
9	0	0
10	0	0
11	0	0
12	0	0
13	0	0
14	0	0
15	0	0
16	1	1
17	1	1
18	1	1
19	1	1
20	1	1
21	1	1
22	1	1
23	1	1
24	1	1
25	0	0
26	0	0
27	0	0
28	0	0
29	0	0
30	0	0
31	0	0
32	0	0
33	0	0
34	0	0
35	0	0
36	1	1
37	1	1
38	1	1
39	1	1
40	1	1
41	1	1
42	1	1
43	1	1
44	1	1
45	1	1
46	1	1
47	1	1
48	1	1
49	0	0
50	0	0

```
FullSimplify[Expand[(1 / 2 + (1 / 2) (-1)^(Floor[n^(1 / 2)]))]]
```

$$\frac{1}{2}\left(1+(-1)^{\text{Floor}\left[\sqrt{n}\right]}\right)$$

```
Sum[ (-1)^j (-1)^k, {j, 2, Floor[n^(1 / 2)]}, {k, 2, Floor[n / j]}]
```

```
$Aborted
```

```
Sum[ (-1)^j , {j, 2, n}]
```

$$\frac{1}{2}\,(1+(-1)^{n})$$

```
ee1b[n_] :=
  2 Sum[ (-1)^j (-1)^k, {j, 2, Floor[n^(1 / 2)]}, {k, 2, Floor[n / j]}] - 1/2 (1 + (-1)^Floor[Sqrt[n]])
```

```
ee1c[n_] :=
  2 Sum[ (-1)^j (1/2 (1 + (-1)^Floor[n/j])), {j, 2, Floor[n^(1 / 2)]}] - 1/2 (1 + (-1)^Floor[Sqrt[n]])
```

```
ee1c[100]
```

```
3
```

```
ee1d[n_] := 2 Sum[ (-1)^j / 2 + 1/2 (-1)^(j+Floor[n/j]), {j, 2, Floor[n^(1 / 2)]}] - 1/2 (1 + (-1)^Floor[Sqrt[n]])
```

```
ee1e[n_] := 2 Sum[ (-1)^j / 2, {j, 2, Floor[n^(1 / 2)]}] +
   2 Sum[ 1/2 (-1)^(j+Floor[n/j]), {j, 2, Floor[n^(1 / 2)]}] - 1/2 (1 + (-1)^Floor[Sqrt[n]])
```

```
ee1e[1000]
```

```
-6
```

```
ee1f[n_] := Sum[ (-1)^j, {j, 2, Floor[n^(1 / 2)]}] +
   Sum[ (-1)^(j+Floor[n/j]), {j, 2, Floor[n^(1 / 2)]}] - 1/2 (1 + (-1)^Floor[Sqrt[n]])
```

```
ee1f[1000]
```

```
-6
```

```
ee1g[n_] :=
  1/2 (1 + (-1)^Floor[Sqrt[n]]) + Sum[ (-1)^(j+Floor[n/j]), {j, 2, Floor[n^(1 / 2)]}] - 1/2 (1 + (-1)^Floor[Sqrt[n]])
```

```
ee1g[1000]
```

```
-6
```

```
ee1h[n_] := Sum[ (-1)^(j+Floor[n/j]), {j, 2, Floor[n^(1 / 2)]}]
```

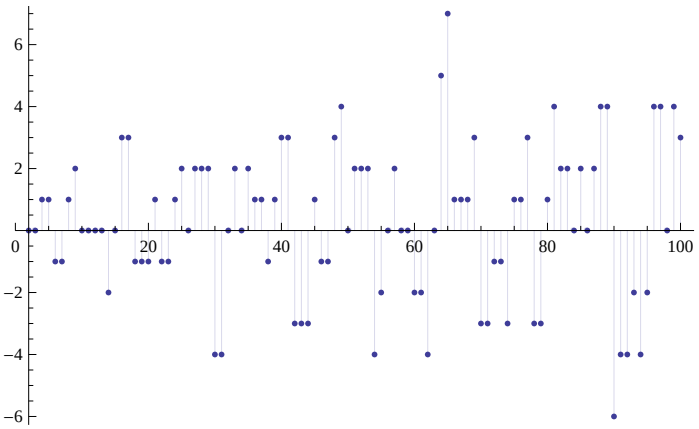
```
ee1h[1000]
```

```
-6
```

```
Sum[ (-1)^(j+Floor[n/j]), {j, 2, Floor[n^(1 / 2)]}]
```

$$\sum_{j=2}^{\text{Floor}\left[\sqrt{n}\right]} (-1)^{j+\text{Floor}\left[\frac{n}{j}\right]}$$

```
DiscretePlot[ee1h[n], {n, 2, 100}]
```




```
Sum[(-1)^j, {j, m, n}]
```

$$\frac{1}{2}\left((-1)^{m}+(-1)^{n}\right)$$