```
Clear[zx, dd, zetaHurwitz, zeta0V3Hurwitz]
delta = .01;
thetaAdd[x_, t_] := x - t
thetaMul[x_, t_] := x / t
thetaEq[x_, t_] := x
f[x_, z_, k_, d_, fn_, I_] := f[x, z, k, d, fn, I] =
  1 + d ((z + 1) / k - 1) Sum[f[fn[x, d t + I], z, k + 1, d, fn, I], {t, 1, (x - I) / d}]
binomial[z_, k_] := binomial[z, k] = Product[z - j, {j, 0, k - 1}] / k!
zx[n_, y_, z_, d_] := zx[n, y, z, d] =
  If[n < y, 1, Sum[d^k binomial[z, k] zx[n - k y, y + d, z - k, d], {k, 0, n / y / d}]]
zx3[n_, z_, d_] := zx[n, d, z, d]
dd[n_, 0, d_] := UnitStep[n - 1]
dd[n_, k_, d_] := dd[n, k, d] = d Sum[ dd[n / j, k - 1, d], {j, 1 + d, n / d, d}]
ddz[n_, z_, d_] := Sum[binomial[z, k] dd[n, k, d], {k, 0, Log[1 + d, n]}]
zetaHurwitz[n_, y_, 0] := UnitStep[n - 1]
zetaHurwitz[n_, y_, k_] :=
 zetaHurwitz[n, y, k] = Sum[binomial[k, j] zetaHurwitz[Floor[n / (m^j)], m, k - j],
   {j, 1, k}, {m, y + 1, Floor[n^(1 / k)]}]
zetaMinus1Scaled[n_, y_, k_] := y^k zetaHurwitz[n y^-k, y^-1, k]
zetaScaled[n_, y_, z_] :=
 Expand@Sum[binomial[z, k] zetaMinus1Scaled[n, y, k], {k, 0, Log[y + 1, n]}]
zeta0V3Hurwitz[n_, y_, z_, d_] := zeta0V3Hurwitz[n, y, z, d] = If[n < y, 1,
   Sum[d^k binomial[z, k] zeta0V3Hurwitz[n / y^k, y + d, z - k, d], {k, 0, Log[y, n]}]]
zeta0V3[n_, z_, d_] := zeta0V3Hurwitz[n, 1 + d, z, d]
```

```
Expand@ddz[40, z, 1 / 2]
```

$$1+\frac{9\,523\,289\,z}{645\,120}+\frac{6\,038\,543\,z^2}{368\,640}+\frac{309\,147\,221\,z^3}{46\,448\,640}+\frac{533\,603\,z^4}{491\,520}+\frac{994\,717\,z^5}{8\,847\,360}+\frac{863\,z^6}{245\,760}+\frac{3751\,z^7}{30\,965\,760}+\frac{z^8}{1\,474\,560}+\frac{z^9}{185\,794\,560}$$

```
Expand@zeta0V3[40, z, 1 / 2]
```

$$1+\frac{9\,523\,289\,z}{645\,120}+\frac{6\,038\,543\,z^2}{368\,640}+\frac{309\,147\,221\,z^3}{46\,448\,640}+\frac{533\,603\,z^4}{491\,520}+\frac{994\,717\,z^5}{8\,847\,360}+\frac{863\,z^6}{245\,760}+\frac{3751\,z^7}{30\,965\,760}+\frac{z^8}{1\,474\,560}+\frac{z^9}{185\,794\,560}$$

```
Expand@f[20, z, 1, 1 / 2, thetaMul, 1]
```

$$1+\frac{14\,393\,z}{1680}+\frac{92\,689\,z^2}{11\,520}+\frac{96\,251\,z^3}{46\,080}+\frac{5269\,z^4}{18\,432}+\frac{1099\,z^5}{92\,160}+\frac{23\,z^6}{92\,160}+\frac{z^7}{645\,120}$$

```
zetaScaled[20, 0, 1 / 2, z]
```

$$1+\frac{14\,393\,z}{1680}+\frac{92\,689\,z^2}{11\,520}+\frac{96\,251\,z^3}{46\,080}+\frac{5269\,z^4}{18\,432}+\frac{1099\,z^5}{92\,160}+\frac{23\,z^6}{92\,160}+\frac{z^7}{645\,120}$$

```
N[(3 / 2)^5]
```

```
7.59375
```

```
(7. / 2)^2
```

12.25

```
N@5 / 2 * 3
```

7.5

```
f[s_] := Limit[Sum[j^-s, {j, 1, x}] - Integrate[j^-s, {j, 0, x}], x → Infinity]
f2[s_] := Limit[Integrate[Floor[j]^-s - j^-s, {j, 0, x}], x → Infinity]
```

```
f[2 + I]
```

Zeta[2 + i]

```
Integrate[Floor[j]^-s - j^-s, {j, 0, Infinity}]
```

$$\int_0^\infty \left(-j^{-s}+\text{Floor}[j]^{-s}\right)\,dj$$

```
N@Zeta[-.5 + 100 I]
```

20.8635 + 0.859043 i

```
Integrate[Cos[1000 Log[j]], {j, 0, x}]
```

$$\frac{x\left(\text{Cos}[1000\,\text{Log}[x]]+1000\,\text{Sin}[1000\,\text{Log}[x]]\right)}{1\,000\,001}$$

```
Integrate[Cos[0 Log[j]], {j, 0, x}]
```

x

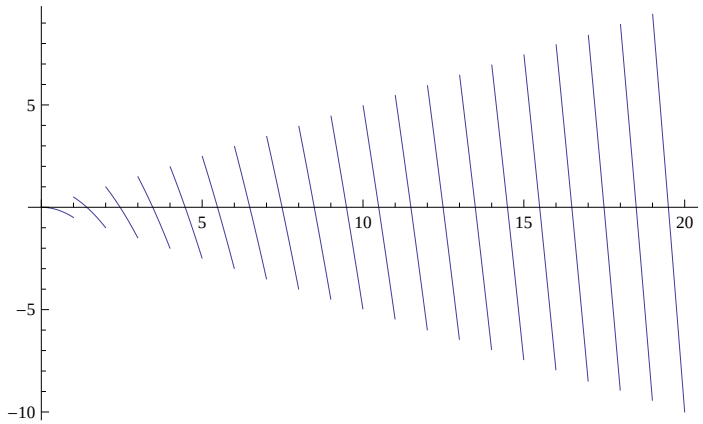
```
N@Zeta[10 I]
```

1.75647 - 0.101512 i

```
FullSimplify[Sum[j, {j, 1, Floor[x]}] - Integrate[j, {j, 0, x}]]
```

$$\frac{1}{2}\left(-x^2+\text{Floor}[x]+\text{Floor}[x]^2\right)$$

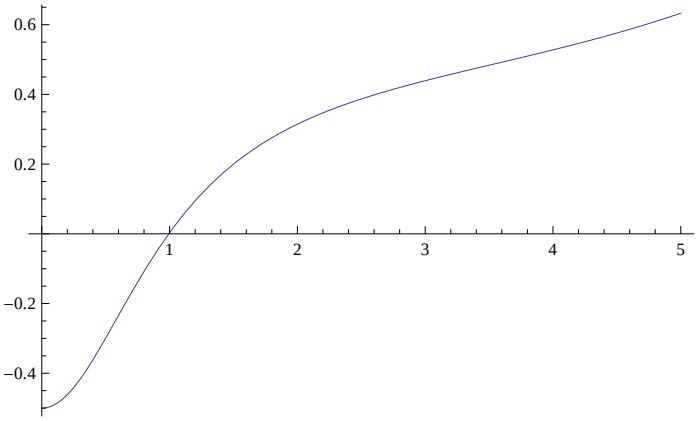
```
Plot[1/2 (-x^2 + Floor[x] + Floor[x]^2), {x, 0, 20}]
```




```
N@Zeta[2 I]
```

0.314726 - 0.23168 i

```
Plot[Re@Zeta[s I], {s, 0, 5}]
```




```
Integrate[Sin[1000 Log[t]], {t, 0, x}]
```

$$\frac{x\,(-1000\,\text{Cos}[1000\,\text{Log}[x]]+\text{Sin}[1000\,\text{Log}[x]])}{1\,000\,001}$$

```
Integrate[ Sin[1000 Log[t]], t]
```

$$-\frac{1000\,t\,\text{Cos}[1000\,\text{Log}[t]]}{1\,000\,001}+\frac{t\,\text{Sin}[1000\,\text{Log}[t]]}{1\,000\,001}$$

```
(1000 Log[5.] - 1000 Log[4.]) / (2 Pi)
```

```
35.5144
```

```
N@Table[{x, (1000 Log[x + 1] - 1000 Log[x]) / (2 Pi)}, {x, 2, 80}] // TableForm
```

```
2.     64.5318
3.     45.786
4.     35.5144
5.     29.0174
6.     24.5338
7.     21.2522
8.     18.7458
9.     16.7686
10.    15.1691
11.    13.8483
12.    12.7392
13.    11.7947
14.    10.9806
15.    10.2716
16.    9.64871
17.    9.09704
18.    8.60507
19.    8.16358
20.    7.7652
21.    7.40389
22.    7.07472
23.    6.77357
24.    6.49702
```

```
25.   6.24217
26.   6.00656
27.   5.78809
28.   5.58496
29.   5.3956
30.   5.21866
31.   5.05296
32.   4.89746
33.   4.75125
34.   4.61351
35.   4.48353
36.   4.36068
37.   4.24438
38.   4.13413
39.   4.02945
40.   3.92995
41.   3.83524
42.   3.74499
43.   3.6589
44.   3.57667
45.   3.49805
46.   3.42282
47.   3.35075
48.   3.28166
49.   3.21536
50.   3.15169
51.   3.09048
52.   3.03161
53.   2.97495
54.   2.92036
55.   2.86773
56.   2.81698
57.   2.76798
58.   2.72066
59.   2.67494
60.   2.63072
61.   2.58794
62.   2.54653
63.   2.50643
64.   2.46757
65.   2.42989
66.   2.39335
67.   2.35789
68.   2.32347
69.   2.29004
70.   2.25755
71.   2.22598
72.   2.19528
73.   2.16541
74.   2.13634
75.   2.10804
76.   2.08049
77.   2.05364
78.   2.02748
79.   2.00198
80.   1.97711
```

```
E^(Log[x] + 8 Pi / 1000)
```

$e^{\pi/125}\, x$

```
N[1 / (E^(8 Pi / 1000) - 1)]
```

```
39.2908
```

```
FullSimplify[1 / (E^((2 Pi s) / f) - 1)]
```

$$\frac{1}{2}\left(-1+\coth\left[\frac{\pi s}{f}\right]\right)$$

```
Table[ N[1/2 (-1 + Coth[(Pi s)/f])] /. f -> 10 000, {s, 0, 10, 1 / 2}]
```

```
{ComplexInfinity, 3182.6, 1591.05, 1060.53, 795.275, 636.12,
 530.017, 454.229, 397.388, 353.178, 317.81, 288.873, 264.759, 244.354,
 226.865, 211.707, 198.444, 186.742, 176.339, 167.032, 158.655}
```

```
N@Table[{x, (1000 Log[x + 1] - 1000 Log[x]) / (2 Pi)}, {x, 147, 172}] // TableForm
```

```
147.    1.07902
148.    1.07175
149.    1.06459
150.    1.05751
151.    1.05053
152.    1.04364
153.    1.03684
154.    1.03013
155.    1.02351
156.    1.01697
157.    1.01051
158.    1.00414
159.    0.99784
160.    0.991623
161.    0.985483
162.    0.979418
163.    0.973428
164.    0.96751
165.    0.961664
166.    0.955888
167.    0.950182
168.    0.944542
169.    0.93897
170.    0.933463
171.    0.92802
172.    0.92264
```

```
N@Zeta[50 000 I]
```

```
82.6139 + 69.0695 i
```