```
Expand[ ((a + 1) (a + 2) - 6 (a + 1) + 6) / 2]
```

$$1 - \frac{3a}{2} + \frac{a^2}{2}$$

```
Expand[ (a + 1) (a + 2) (a + 3) / 6 - 2 (a + 1) (a + 2) + 6 (a + 1) - 4]
```

$$-1 + \frac{11a}{6} - a^2 + \frac{a^3}{6}$$

```
SS[k_] := Product[ a + j, {j, 1, k - 1}] / ((k - 1) !)
SS[6]
```

$$\frac{1}{120} (1 + a) (2 + a) (3 + a) (4 + a) (5 + a)$$

```
SSS[k_] := Sum[(-1)^(k - j) Binomial[ k, j ] SS[j], {j, 1, k}]
Expand[SSS[4]]
```

$$-1 + \frac{11a}{6} - a^2 + \frac{a^3}{6}$$

```
Expand[(a - 1) (a - 2) (a - 3) / 6]
```

$$-1 + \frac{11a}{6} - a^2 + \frac{a^3}{6}$$

$$\text{KK[a\_, k\_]} := -\frac{(-1)^k (-1 - a + k)!}{(-a)! (-1 + k)!}$$

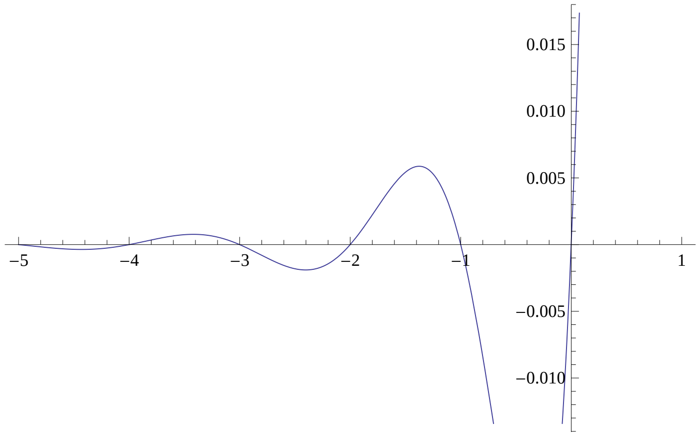
```
JJ[a_, k_] := (a - 1) ! / ( (k - 1) ! (a - k) ! )
KK[3, 3]
```

Infinity::indet : Indeterminate expression 0 ComplexInfinity encountered. ≫

```
Indeterminate
JJ[9, 3]
28
DD[n_, k_] := Sum[ DD[n / j, k - 1], {j, 2, n}]
DD[n_, 0] := 1
DDD[n_, k_ ] := DD[n, k] - DD[n - 1, k]
DDD[2^8, 3]
21
FullSimplify[JJ[a, k]]
```

$$\frac{\text{Gamma}[a]}{\text{Gamma}[1 + a - k]\ \text{Gamma}[k]}$$

```
Plot[Gamma[5] / (Gamma[1 + 5 - k] Gamma[k]), {k, -5, 1}]
```




```
FF[n_, k_] :=
 Product[Binomial[FactorInteger[n][[j]][[2]] + k - 1, FactorInteger[n][[j]][[2]]],
  {j, 1, Length[FactorInteger[n]]}]

FF2[n_, k_] := FF[n, k] / k

FF3[n_, k_] := Sum[FF2[j, k], {j, 2, n}]

GG[n_, k_, a_] := Sum[(k MangoldtLambda[j]) / (a Log[j]) (1 + GG[n / j, k, a + 1]), {j, 2, n}]

XX[n_, k_] :=
 Product[(FactorInteger[n][[j]][[2]] - 1)! / ((FactorInteger[n][[j]][[2]] - k)! (k - 1)!),
  {j, 1, Length[FactorInteger[n]]}]

XX[2^2 * 3, 3]
```

0

```
DDD[2^2 * 3, 3]
```

3

```
(2 + 1) (2 + 2) (1 + 1) (1 + 2) / 4 - 3 (2 + 1) (1 + 1) + 3
```

3

```
Expand[(a + 1) (b + 1) - 2]
```

$-1 + a + b + a b$

```
Expand[(a + 1) (a + 2) (b + 1) (b + 2) / (2 × 2) - 3 (a + 1) (b + 1) + 3]
```

$$1 - \frac{3 a}{2} + \frac{a^2}{2} - \frac{3 b}{2} - \frac{3 a b}{4} + \frac{3 a^2 b}{4} + \frac{b^2}{2} + \frac{3 a b^2}{4} + \frac{a^2 b^2}{4}$$

```
Expand[(a - 1) (a - 2) (b + 1) (b + 2) / 4]
```

$$1-\frac{3\,a}{2}+\frac{a^2}{2}+\frac{3\,b}{2}-\frac{9\,a\,b}{4}+\frac{3\,a^2\,b}{4}+\frac{b^2}{2}-\frac{3\,a\,b^2}{4}+\frac{a^2\,b^2}{4}$$

```
Simplify[3 a + 3 b + 3 a b]
```

$$3\,(a+b+a\,b)$$

```
Expand[(a - 1) (a - 2) (b - 1) (b - 2) / 4] -
 Expand[(a + 1) (a + 2) (b + 1) (b + 2) / (2 × 2) - 3 (a + 1) (b + 1) + 3]
```

$$3\,a\,b-\frac{3\,a^2\,b}{2}-\frac{3\,a\,b^2}{2}$$

```
Expand[(a + 1) (a + 2) (b + 1) (b + 2) / 4] -
 Expand[(a + 1) (a + 2) (b + 1) (b + 2) / (2 × 2) - 3 (a + 1) (b + 1) + 3]
```

$$3\,a+3\,b+3\,a\,b$$

```
SR[k_] := Product[ a + j, {j, 1, k - 1}] / ((k - 1) !) Product[ b + j, {j, 1, k - 1}] / ((k - 1) !)
```

```
SR[4]
```

$$\frac{1}{36}\,(1+a)\,(2+a)\,(3+a)\,(1+b)\,(2+b)\,(3+b)$$

$$\frac{\text{Pochhammer}[1+a,\,-1+k]\;\text{Pochhammer}[1+b,\,-1+k]}{((-1+k)\,!)^2}$$

$$\frac{\text{Pochhammer}[1+a,\,-1+k]\;\text{Pochhammer}[1+b,\,-1+k]}{((-1+k)\,!)^2}$$

```
SSR[k_] := Sum[(-1)^(k - j) Binomial[ k, j ] SR[j], {j, 1, k}]
```

```
SSR[3]
```

$$3-3\,(1+a)\,(1+b)+\frac{1}{4}\,(1+a)\,(2+a)\,(1+b)\,(2+b)$$

```
SR2[k_, a_, b_] :=
 Product[ a + j, {j, 1, k - 1}] / ((k - 1) !) Product[ b + j, {j, 1, k - 1}] / ((k - 1) !)
```

```
SSR2[k_, a_, b_] := Sum[(-1)^(j) Binomial[ k, k - j ] SR2[k - j, a, b], {j, 0, 4 k}]
```

```
SSR2[5.1, a, b]
```

$$3.70237\times 10^{15}-9.86502\,(1+a)\,(1+b)+2.1648\,(1+a)\,(2+a)\,(1+b)\,(2+b)-0.109886\,(1+a)\,(2+a)\,(3+a)\,(1+b)\,(2+b)\,(3+b)+0.00128175\,(1+a)\,(2+a)\,(3+a)\,(4+a)\,(1+b)\,(2+b)\,(3+b)\,(4+b)$$

```
Plot[SSR2,
```

```
ST[k_] := Product[ a + j, {j, 1, k - 1}] / ((k - 1) !)
  Product[ b + j, {j, 1, k - 1}] / ((k - 1) !) Product[ c + j, {j, 1, k - 1}] / ((k - 1) !)
```

```
ST[k]
```

$$\frac{1}{((-1+k)\,!)^3}\,\text{Pochhammer}[1+a,\,-1+k]\;\text{Pochhammer}[1+b,\,-1+k]\;\text{Pochhammer}[1+c,\,-1+k]$$

```
SST[k_] := Sum[(-1)^(k - j) Binomial[ k, j ] ST[j], {j, 1, k}]
```

```
FullSimplify[SST[k]]
```

$$-(-1)^k\,k\,\text{HypergeometricPFQ}[\{1+a,\,1+b,\,1+c,\,1-k\},\,\{1,\,1,\,2\},\,1]$$

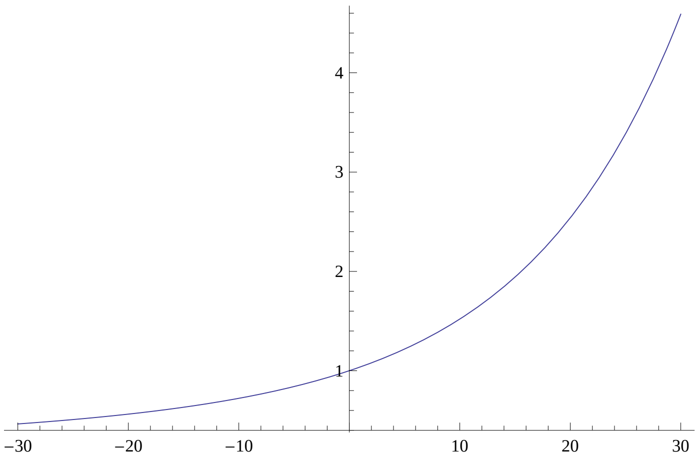
SST[k]

$-(-1)^k$ k HypergeometricPFQ[{1 + a, 1 + b, 1 + c, 1 - k}, {1, 1, 2}, 1]

$-(-1)^k$ k HypergeometricPFQ[{1 + a, 1 - k}, {2}, 1]

$-(-1)^k$ k HypergeometricPFQ[{1 + a, 1 - k}, {2}, 1]

Plot[HypergeometricPFQ[{1, 1}, {3, 3, 3}, x], {x, -30, 30}]



$-(-1)^3$ 3 HypergeometricPFQ[{1 + a, 1 + b, 1 - 3}, {1, 2}, 1]

Expand$\left[3\left(1-(1+a)\,(1+b)+\frac{1}{12}\,(1+a)\,(2+a)\,(1+b)\,(2+b)\right)\right]$

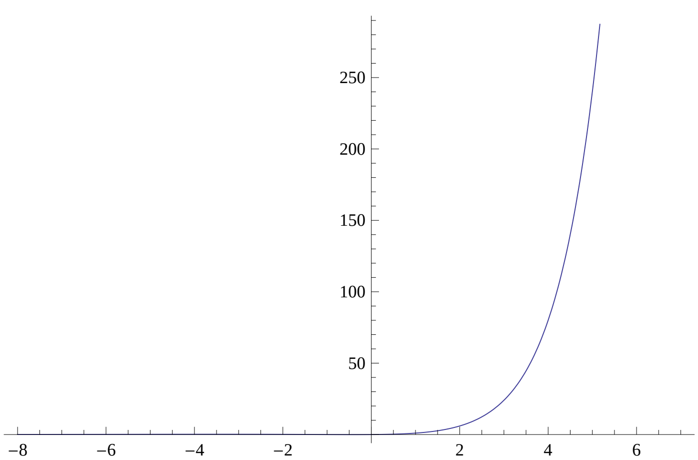
$$1-\frac{3\,a}{2}+\frac{a^2}{2}-\frac{3\,b}{2}-\frac{3\,a\,b}{4}+\frac{3\,a^2\,b}{4}+\frac{b^2}{2}+\frac{3\,a\,b^2}{4}+\frac{a^2\,b^2}{4}$$

SSR[3]

Expand$\left[3-3\,(1+a)\,(1+b)+\frac{1}{4}\,(1+a)\,(2+a)\,(1+b)\,(2+b)\right]$

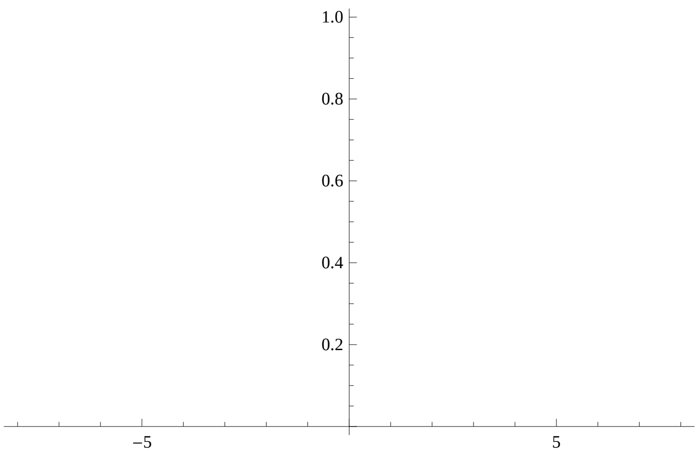
$$1-\frac{3\,a}{2}+\frac{a^2}{2}-\frac{3\,b}{2}-\frac{3\,a\,b}{4}+\frac{3\,a^2\,b}{4}+\frac{b^2}{2}+\frac{3\,a\,b^2}{4}+\frac{a^2\,b^2}{4}$$

Plot[k HypergeometricPFQ[{1 + 1, 1 + 1, 1 - k}, {1, 2}, -1], {k, -8, 7}]



AA[a_, b_, k_] := $-(-1)^k$ k HypergeometricPFQ[{1 + a, 1 + b, 1 - k}, {1, 2}, 1]

```
AA[4, 2, 3]

48

BB[a_, b_, k_] := k HypergeometricPFQ[{1 + a, 1 + b, 1 - k}, {1, 2}, 1]

BB[4, 2, 2]

ComplexInfinity

DDD[2^7 × 3^3, 3]

267

Plot[ AA[n, 1, 1.5], {n, -8, 8}]
```




```
Binomial[4, 0]

1

Pochhammer[4, -1]

1
—
3

Binomial[4.5, -600.5]

8.530219409113667`*^-15

Product[ j, {j, 8, 2}]

1

TT[a_, k_ ] := Product[ a + j, {j, 1, k}]

TT[1, -4]

1

TT2[ k_, j_, a_, b_] :=
 (-1) ^j Binomial[ k, k - j] TT[ a, k - j - 1] TT[b, k - j - 1] / (((k - j - 1) !) ^2)

TT2[4.5, 0, a, b]

0.00739114 (1 + a) (2 + a) (3 + a) (1 + b) (2 + b) (3 + b)

TT3[ n_, a_, b_ ] := Sum[ TT2[n, j, a, b], {j, 0, 30}]
```

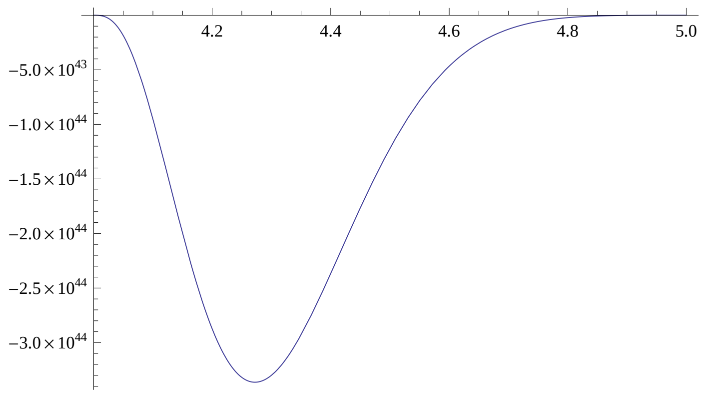
```
TT2[4.2, 15, a, b]
```

-2.23076 × 10^8

```
DDD[2^3 × 3^2, 4]
```

28

```
Plot[ TT3[n, 3, 2], {n, 4, 5}]
```




```
TT3[4.1, 3, 2]
```

-9.51784 × 10^43

```
Binomial[4.1, 4.1 - 15]
```

6.02929 × 10^-6

```
(-1)^15 Binomial[ 4.1, 4.1 - 15] TT[ a, 4.1 - 15 - 1] TT[b, 4.1 - 15 - 1] / (((4.1 - 15 - 1)!)^2)
```

-5.70761 × 10^7

```
(((4.1 - 15 - 1)!)^2)
```

1.05636 × 10^-13

```
Gamma[4.1 - 15]
```

-3.25017 × 10^-7

```
TT[ a, 4.1 - 15 - 1]
```

1

```
SS[n_, k_, a_] := Sum[ a (1 / k - SS[n / j, k + 1, a]), {j, 2, n}]
N[SS[100, 1, 2]]
```

11.7333

```
RP[n_] := N[MangoldtLambda[n] / Log[n]]
RR[ n_ ] := Sum[ RP[j], {j, 2, n}]

RR[1000]
```

176.696

```
RP2[n_] := Sum[ RP[j] RR[n / j], {j, 2, n}]
```

```
RP[1]
```

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

Infinity::indet : Indeterminate expression 0 ComplexInfinity encountered. ≫

```
Indeterminate
```

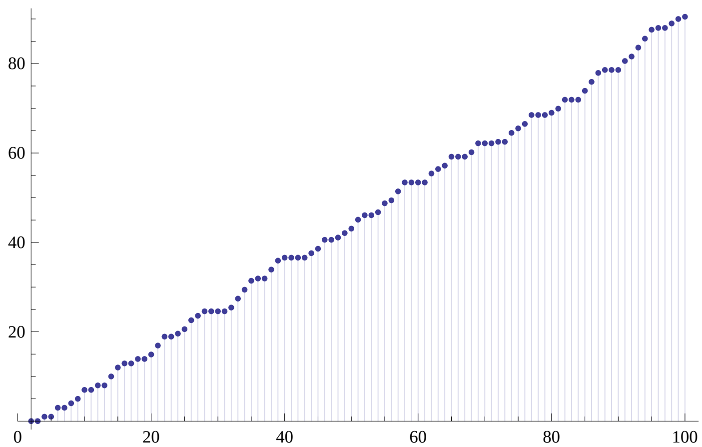
```
N[SS[100, 1, 2]]
```

```
11.7333
```

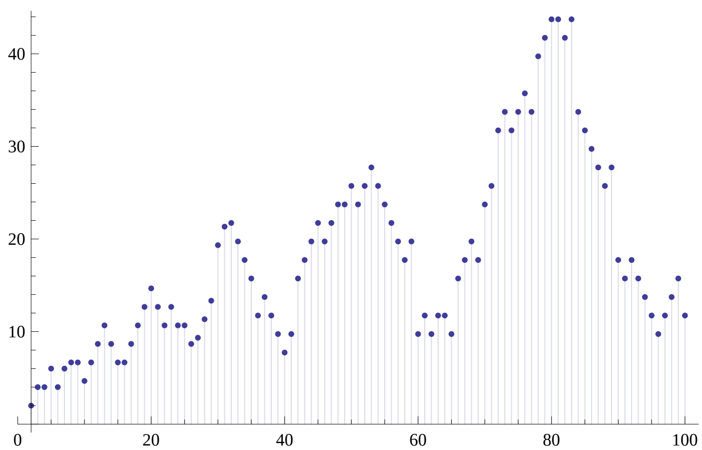
```
RR[100]
```

```
28.5333
```

```
DiscretePlot[RP2[n], {n, 2, 100}]
```

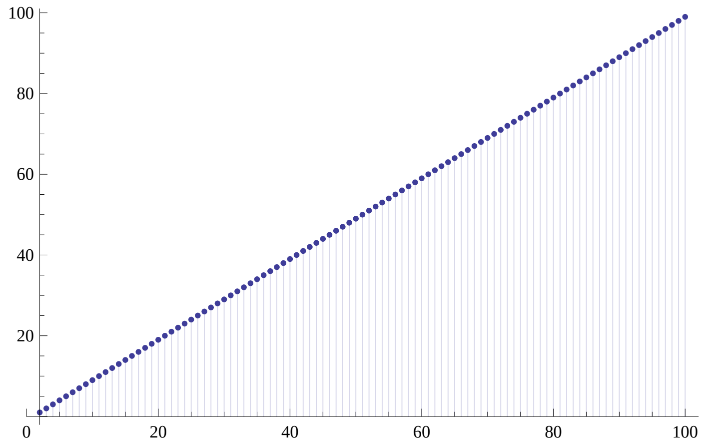



```
DiscretePlot[SS[n, 1, 2], {n, 2, 100}]
```




```
XX[n_, k_, a_] := Sum[ a RP[j] (1 / (k!) + XX[n / j, k + 1, a]), {j, 2, n}]
```

```
DiscretePlot[XX[n, 1, 1], {n, 2, 100}]
```




```
Log[1.1]
```

```
0.0953102
```

```
FF[n_, k_] :=
 Product[Binomial[FactorInteger[n][[j]][[2]] + k - 1, FactorInteger[n][[j]][[2]]],
  {j, 1, Length[FactorInteger[n]]}]
FF[1, k_] := 1
FF2[n_, k_] := FF[n, k] / k

FF3[n_, k_] := Sum[FF2[j, k], {j, 2, n}]

GG[n_, k_, a_] := Sum[(k MangoldtLambda[j]) / (a Log[j]) (1 + GG[n / j, k, a + 1]), {j, 2, n}]

FF2[1, .000001]
```

$1. \times 10^{6}$

```
FFX[ k_] := FF2[1, k] Log[1 + k]
```