```
d[n_, 0, a_] := 1
d[n_, 1, a_] := n - a + 1
d[n_, k_, a_] :=
 Sum[Binomial[k, j] d[Floor[n / (m^j)], k - j, m + 1], {j, 1, k}, {m, a, n^(1 / k)}]

D32Unrolled[ n_ ] := -1 + Floor[ n^(1 / 3)]^3 +
  3 Sum[ Floor[ n / (m^2)] - Floor[ Floor[n / m]^(1 / 2)]^2 + 2 Sum[ Floor[Floor[n / m] / j],
       {j, m + 1, Floor[Floor[n / m]^(1 / 2)]}], {m, 2, Floor[n^(1 / 3)]}]

D22Unrolled[ n_ ] := 1 - Floor[ n^(1 / 2)]^2 + 2 Sum [ Floor[ n / m], {m, 2, Floor[n^(1 / 2)]}]
d0[n_, a_Integer] := 1
d1[n_, a_Integer] := n - a + 1
d2a[n_, a_Integer] :=
 Sum[ Binomial[ 2, 2 ] d0[Floor[n / (m^2)], m + 1] , {m, a, Floor[n^(1 / 2)]}] +
  Sum[Binomial[ 2, 1 ] d1[Floor[n / (m^1)], m + 1], {m, a, Floor[n^(1 / 2)]}]
d2[n_, a_Integer] :=
 1 - 2 a + a^2 - Floor[ n^(1 / 2)]^2 + 2 Sum [ Floor[ n / m], {m, a, Floor[n^(1 / 2)]}]
d2[0, a_Integer] := 0
d3a[n_, a_Integer] :=
 Sum[ Binomial[ 3, 3 ] d0[Floor[n / (m^3)], m + 1] , {m, a, Floor[n^(1 / 3)]}] +
  Sum[Binomial[ 3, 2 ] d1[Floor[n / (m^2)], m + 1], {m, a, Floor[n^(1 / 3)]}] +
  Sum[Binomial[ 3, 1 ] d2a[Floor[n / (m^1)], m + 1], {m, a, Floor[n^(1 / 3)]}]

d3b[n_, a_Integer] := Sum[1 , {m, a, Floor[n^(1 / 3)]}] +
  Sum[3 ( Floor[n / (m^2)] - m), {m, a, Floor[n^(1 / 3)]}] +
  Sum[3 d2[Floor[n / m], m + 1], {m, a, Floor[n^(1 / 3)]}]

d3b[n, 2]
```

$$-1 + \text{Floor}\left[n^{1/3}\right] + \sum_{m=2}^{\text{Floor}\left[n^{1/3}\right]} 3\, d2\left[\text{Floor}\left[\frac{n}{m}\right], 1 + m\right] + \sum_{m=2}^{\text{Floor}\left[n^{1/3}\right]} 3\left(-m + \text{Floor}\left[\frac{n}{m^2}\right]\right)$$

```
d[100, 4, 2]
```

184

```
d2e[n_, a_Integer] :=
 (1 - a)^2 - Floor[ n^(1 / 2)]^2 + 2 Sum [ Floor[ n / m], {m, a, Floor[n^(1 / 2)]}]
d2e[123 210, 4]
```

1 011 514

```
d[123 210, 2, 4]
```

1 011 514

```
dd3[n_, a_ ] :=
 Sum[ 3 (Floor[n / (s^2)] - s) + 1 + 3 (1 - 2 (s + 1) + (s + 1)^2 - Floor[Floor[n / s]^(1 / 2)]^2 +
      2 Sum[ Floor[n / m / s], {m, s + 1, Floor[Floor[n / s]^(1 / 2)]}]), {s, a, Floor[n^(1 / 3)]}]

dd3[100, 3]
```

71

```
d[100, 3, 3]
```

71

```
d3c[100, 2]
```

```
324
```

```
d3c[n_, a_Integer] :=
 Sum[1 + 3 ( Floor[n / (m^2)] - m), {m, a, Floor[n^(1 / 3)]}] +
  Sum[3 d2[Floor[n / m], m + 1], {m, a, Floor[n^(1 / 3)]}]
```

```
dd3a[n_, a_ ] :=
 Sum[ 3 (Floor[n / (s^2)] - s) + 1 + 3 (1 - 2 (s + 1) + (s + 1)^2 - Floor[Floor[n / s]^(1 / 2)]^2 +
      2 Sum[ Floor[n / m / s], {m, s + 1, Floor[Floor[n / s]^(1 / 2)]}]), {s, a, Floor[n^(1 / 3)]}]
```

```
dd3a[200, 2]
```

```
1027
```

```
d[200, 3, 2]
```

```
1027
```

```
Expand[FullSimplify[
  3 (Floor[n / (s^2)] - s) + 1 + 3 (1 - 2 (s + 1) + (s + 1)^2 - Floor[Floor[n / s]^(1 / 2)]^2)]]
```

$$1 - 3\,s + 3\,s^2 + 3\,\text{Floor}\left[\frac{n}{s^2}\right] - 3\,\text{Floor}\left[\sqrt{\text{Floor}\left[\frac{n}{s}\right]}\,\right]^2$$

```
dd3b[n_, a_ ] := Sum[ 1 - 3 s + 3 s^2 + 3 Floor[n / s^2] - 3 Floor[Sqrt[Floor[n / s]]]^2 +
   6 Sum[ Floor[n / m / s], {m, s + 1, Floor[Floor[n / s]^(1 / 2)]}], {s, a, Floor[n^(1 / 3)]}]
```

```
dd3b[200, 2]
```

```
1027
```

```
dd3c[n_, a_ ] :=
 Sum[ 1 - 3 s + 3 s^2, {s, a, Floor[n^(1 / 3)]}] + Sum[3 Floor[n / s^2] - 3 Floor[Sqrt[Floor[n / s]]]^2 +
    6 Sum[ Floor[n / m / s], {m, s + 1, Floor[Floor[n / s]^(1 / 2)]}], {s, a, Floor[n^(1 / 3)]}]
```

```
dd3c[200, 2]
```

```
1027
```

```
Expand[FullSimplify[Sum[1 - 3 s + 3 s^2, {s, a, Floor[n^(1 / 3)]}]]]
```

$$1 - 3\,a + 3\,a^2 - a^3 + \text{Floor}\left[n^{1/3}\right]^3$$

```
dd3d[n_, a_ ] := 1 - 3 a + 3 a^2 - a^3 + Floor[n^(1/3)]^3 + Sum[3 Floor[n / s^2] - 3 Floor[Sqrt[Floor[n / s]]]^2 +
    6 Sum[ Floor[n / m / s], {m, s + 1, Floor[Floor[n / s]^(1 / 2)]}], {s, a, Floor[n^(1 / 3)]}]
```

```
dd3d[300, 3]
```

```
709
```

```
dd3e[n_, a_ ] := (1 - a)^3 + Floor[n^(1/3)]^3 + Sum[3 Floor[n / s^2] - 3 Floor[Sqrt[Floor[n / s]]]^2 +
    6 Sum[ Floor[n / m / s], {m, s + 1, Floor[Floor[n / s]^(1 / 2)]}], {s, a, Floor[n^(1 / 3)]}]
```

```
dd3e[300, 3]
```

```
709
```

```
d[300, 3, 3]
```

```
709
```

```
d4[x_, b_] := Sum[ d[Floor[x / (u^4)], 0, u + 1] + 4 d[Floor[x / (u^3)], 1, u + 1] +
   6 d[Floor[x / (u^2)], 2, u + 1] + 4 d[Floor[x / (u^1)], 3, u + 1], {u, b, Floor[x^(1 / 4)]}]
```

```
d4[100, 2]
```

```
184
```

```
d[10 000, 4, 3]
```

```
171 994
```

```
Floor[(100)^(1 / 4)]
```

```
3
```

```
d4a[x_, b_] := Sum[ 1 + 4 (Floor[x / (u^3)] - u) +
   6 d[Floor[x / (u^2)], 2, u + 1] + 4 d[Floor[x / (u^1)], 3, u + 1], {u, b, Floor[x^(1 / 4)]}]
```

```
d4a[10 000, 3]
```

```
171 994
```

```
d4b[x_, b_] := Sum[ 1 + 4 (Floor[x / (u^3)] - u)

   + 6 ( 1 - 2 (u + 1) + (u + 1)^2 - Floor[ Floor[x / (u^2)]^(1 / 2)]^2 +
      2 Sum [ Floor[ Floor[x / (u^2)] / m], {m, (u + 1), Floor[Floor[x / (u^2)]^(1 / 2)]}])

   + 4 d[Floor[x / (u^1)], 3, u + 1], {u, b, Floor[x^(1 / 4)]}]
```

```
d4b[22 200, 3]
```

```
605 878
```

```
# a = (u + 1) # n = Floor[x / u]
```

```
d4c[x_, b_] := Sum[ 1 + 4 (Floor[x / (u^3)] - u)

   + 6 ( 1 - 2 (u + 1) + (u + 1)^2 - Floor[ Floor[x / (u^2)]^(1 / 2)]^2 +
       2 Sum [ Floor[ Floor[x / (u^2)] / m], {m, (u + 1), Floor[Floor[x / (u^2)]^(1 / 2)]}])

   + 4 ( 1 - 3 (u + 1) + 3 (u + 1)^2 - (u + 1)^3 + Floor[Floor[x / u]^(1/3)]^3 +

       Sum[3 Floor[ Floor[x / u] / s^2 ] - 3 Floor[ Sqrt[ Floor[ Floor[x / u] / s ] ] ]^2 +
         6 Sum[ Floor[Floor[x / u] / m / s], {m, s + 1, Floor[Floor[Floor[x / u] / s]^(1 / 2)]}],
        {s, (u + 1), Floor[Floor[x / u]^(1 / 3)]}] )

  , {u, b, Floor[x^(1 / 4)]}]
d4c[22 200, 3]
605 878
d[12 200, 4, 4]
95 137
d4d[x_, b_] :=
 Sum[1 + 6 ( 1 - 2 (u + 1) + (u + 1)^2 ) + 4 ( 1 - 3 (u + 1) + 3 (u + 1)^2 - (u + 1)^3 - u),
   {u, b, Floor[x^(1 / 4)]}] +
  Sum[
    4 Floor[x / (u^3)] + 6 (-Floor[ Floor[x / (u^2)]^(1 / 2)]^2 +
       2 Sum [ Floor[ Floor[x / (u^2)] / m], {m, (u + 1), Floor[Floor[x / (u^2)]^(1 / 2)]}]) +
    4 ( Floor[Floor[x / u]^(1/3)]^3 + Sum[3 Floor[ Floor[x / u] / s^2 ] - 3 Floor[ Sqrt[ Floor[ Floor[x / u] / s ] ] ]^2 +
         6 Sum[ Floor[Floor[x / u] / m / s], {m, s + 1, Floor[Floor[Floor[x / u] / s]^(1 / 2)]}],
        {s, (u + 1), Floor[Floor[x / u]^(1 / 3)]}] )

   , {u, b, Floor[x^(1 / 4)]}]
d4d[12 200, 4]
95 137
```

```
FullSimplify[
 Expand[Sum[1 + 6 (1 - 2 (u + 1) + (u + 1)^2) + 4 (1 - 3 (u + 1) + 3 (u + 1)^2 - (u + 1)^3 - u),
   {u, b, Floor[x^(1 / 4)]}]]]
```

$$(-1+b)^4-\text{Floor}\left[x^{1/4}\right]^4$$

```
Expand[Sum[1 + 6 (1 - 2 (u + 1) + (u + 1)^2) + 4 (1 - 3 (u + 1) + 3 (u + 1)^2 - (u + 1)^3 - u),
  {u, b, Floor[x^(1 / 4)]}]]
```

$$1-4\,b+6\,b^2-4\,b^3+b^4-\text{Floor}\left[x^{1/4}\right]^4$$

```
d4e[x_, b_] :=
 (-1 + b)^4 - Floor[x^(1/4)]^4 +
  Sum[
    4 Floor[x / (u^3)] + 6 (-Floor[ Floor[x / (u^2)]^(1 / 2)]^2 +
       2 Sum [ Floor[ Floor[x / (u^2)] / m], {m, (u + 1), Floor[Floor[x / (u^2)]^(1 / 2)]}]) +
     4 ( Floor[Floor[x / u]^(1/3)]^3 + Sum[3 Floor[ Floor[x / u] / s^2 ] - 3 Floor[ Sqrt[ Floor[ Floor[x / u] / s ] ] ]^2 +
          6 Sum[ Floor[Floor[x / u] / m / s], {m, s + 1, Floor[Floor[Floor[x / u] / s]^(1 / 2)]}],
         {s, (u + 1), Floor[Floor[x / u]^(1 / 3)]}] )
   , {u, b, Floor[x^(1 / 4)]}]
```

```
d4e[72200, 2]
```

7 624 011

```
d[72200, 4, 2]
```

7 624 011

```
Sum[
  4 Floor[x / (u^3)] + 6 (-Floor[ Floor[x / (u^2)]^(1 / 2)]^2 +
     2 Sum [ Floor[ Floor[x / (u^2)] / m], {m, (u + 1), Floor[Floor[x / (u^2)]^(1 / 2)]}]) +
   4 ( Floor[Floor[x / u]^(1/3)]^3 + Sum[3 Floor[ Floor[x / u] / s^2 ] - 3 Floor[ Sqrt[ Floor[ Floor[x / u] / s ] ] ]^2 +
        6 Sum[ Floor[Floor[x / u] / m / s], {m, s + 1, Floor[Floor[Floor[x / u] / s]^(1 / 2)]}],
       {s, (u + 1), Floor[Floor[x / u]^(1 / 3)]}] )
 , {u, b, Floor[x^(1 / 4)]}]
```

```
d4f[x_, b_] :=
 (-1 + b)^4 - Floor[x^(1/4)]^4 +
  Sum[
   4 Floor[x / (u^3)]
    + 6 (
      -Floor[ Floor[x / (u^2)]^(1 / 2)]^2
       + 2 Sum [ Floor[ Floor[x / (u^2)] / m], {m, (u + 1), Floor[Floor[x / (u^2)]^(1 / 2)]}]
     )
    + 4 (
      Floor[Floor[x / u]^(1/3)]^3
       + 3 Sum[
              Floor[ Floor[x / u] / s^2 ]
               - Floor[ Sqrt[ Floor[ Floor[x / u] / s ] ] ]^2
               +
           2 Sum[ Floor[Floor[x / u] / m / s], {m, s + 1, Floor[Floor[Floor[x / u] / s]^(1 / 2)]}
            ], {s, (u + 1), Floor[Floor[x / u]^(1 / 3)]}] ),
   {u, b, Floor[x^(1 / 4)]}]
d4g[72 200, 2]
```

```
7 624 011
```

```
d4g[x_, b_] :=
 (-1 + b)^4 - Floor[x^(1/4)]^4 +
  Sum[4 Floor[x / (u^3)], {u, b, Floor[x^(1 / 4)]}] +
  -6 Sum[Floor[ Floor[x / (u^2)]^(1 / 2)]^2 , {u, b, Floor[x^(1 / 4)]}] +
  Sum[
    6 (
       2 Sum [ Floor[ Floor[x / (u^2)] / m], {m, (u + 1), Floor[Floor[x / (u^2)]^(1 / 2)]}]
      )
     + 4 (
       Floor[Floor[x / u]^(1/3)]^3
        + 3 Sum[
                Floor[ Floor[x / u] / s^2 ]
                 - Floor[ Sqrt[ Floor[ Floor[x / u] / s ] ] ]^2
                 +
            2 Sum[ Floor[Floor[x / u] / m / s], {m, s + 1, Floor[Floor[Floor[x / u] / s]^(1 / 2)]}
             ], {s, (u + 1), Floor[Floor[x / u]^(1 / 3)]}] ),
   {u, b, Floor[x^(1 / 4)]}]

d4h[72 200, 2]
```

```
7 624 011
```

```
d4i[x_, b_] :=
 (-1 + b)^4 - Floor[x^(1/4)]^4 +
  Sum[4 Floor[x / (u^3)], {u, b, Floor[x^(1 / 4)]}] +
  -6 Sum[Floor[ Floor[x / (u^2)]^(1 / 2)]^2 , {u, b, Floor[x^(1 / 4)]}] +
  12 Sum[ Sum [ Floor[ Floor[x / (u^2)] / m], {m, (u + 1), Floor[Floor[x / (u^2)]^(1 / 2)]}],
    {u, b, Floor[x^(1 / 4)]}] +
  4 Sum[Floor[Floor[x / u]^(1/3)]^3, {u, b, Floor[x^(1 / 4)]}] +
  Sum[
   4 ( 3 Sum[
            Floor[ Floor[x / u] / s^2 ]
             - Floor[ Sqrt[ Floor[ Floor[x / u] / s ] ] ]^2
             +
          2 Sum[ Floor[Floor[x / u] / m / s], {m, s + 1, Floor[Floor[Floor[x / u] / s]^(1 / 2)]}
           ], {s, (u + 1), Floor[Floor[x / u]^(1 / 3)]}] ),
    {u, b, Floor[x^(1 / 4)]}]
d4i[12200, 2]
```

648 367

```
d[12200, 4, 2]
```

648 367

```
d4j[x_, b_] :=
 (-1 + b)^4 - Floor[x^(1/4)]^4 +
  Sum[4 Floor[x / (u^3)], {u, b, Floor[x^(1 / 4)]}] +
  -6 Sum[Floor[ Floor[x / (u^2)]^(1 / 2)]^2 , {u, b, Floor[x^(1 / 4)]}] +
  12 Sum[ Sum [ Floor[ Floor[x / (u^2)] / m], {m, (u + 1), Floor[Floor[x / (u^2)]^(1 / 2)]}],
    {u, b, Floor[x^(1 / 4)]}] +
  4 Sum[Floor[Floor[x / u]^(1/3)]^3, {u, b, Floor[x^(1 / 4)]}] +
  +12 Sum[ Sum[ Floor[ Floor[x / u] / s^2 ], {s, (u + 1), Floor[Floor[x / u]^(1 / 3)]}],
    {u, b, Floor[x^(1 / 4)]}] +
  Sum[ Sum[
        -12 Floor[ Sqrt[ Floor[ Floor[x / u] / s ] ] ]^2
          + 24 Sum[ Floor[Floor[x / u] / m / s], {m, s + 1, Floor[Floor[Floor[x / u] / s]^(1 / 2)]}
      ]
    , {s, (u + 1), Floor[Floor[x / u]^(1 / 3)]}], {u, b, Floor[x^(1 / 4)]}]
d4j[12 200, 2]
```

```
648 367
```

```
d4k[x_, b_] :=
 (-1 + b)^4 +
  -Floor[x^(1/4)]^4 +
  4 Sum[ Floor[x / (u^3)], {u, b, Floor[x^(1 / 4)]}] +
  -6 Sum[Floor[ Floor[x / (u^2)]^(1 / 2)]^2 , {u, b, Floor[x^(1 / 4)]}] +
  12 Sum[ Sum [ Floor[ Floor[x / (u^2)] / m], {m, (u + 1), Floor[Floor[x / (u^2)]^(1 / 2)]}],
    {u, b, Floor[x^(1 / 4)]}] +
  4 Sum[Floor[Floor[x / u]^(1/3)]^3, {u, b, Floor[x^(1 / 4)]}] +
  12 Sum[ Sum[ Floor[ Floor[x / u] / s^2 ], {s, (u + 1), Floor[Floor[x / u]^(1 / 3)]}],
    {u, b, Floor[x^(1 / 4)]}] +
  -12 Sum[ Sum[Floor[ Sqrt[ Floor[ Floor[x / u] / s ] ] ]^2 , {s, (u + 1), Floor[Floor[x / u]^(1 / 3)]}],
    {u, b, Floor[x^(1 / 4)]}] +
  24 Sum[ Sum[ Sum[ Floor[Floor[x / u] / m / s],
      {m, s + 1, Floor[Floor[Floor[x / u] / s]^(1 / 2)]}],
     {s, (u + 1), Floor[Floor[x / u]^(1 / 3)]}], {u, b, Floor[x^(1 / 4)]}]
d4k[16 553, 5]
```

```
70 313
```

```
d[16 553, 4, 5]
```

```
70 313
```

```
d4l[x_, b_] :=
 24
  Sum[ Sum[ Sum[ Floor[Floor[x / u] / m / s], {m, s + 1, Floor[Floor[Floor[x / u] / s]^(1 / 2)]}],
    {s, (u + 1), Floor[Floor[x / u]^(1 / 3)]}], {u, b, Floor[x^(1 / 4)]}]
```

```
f[n_] := Sum[ (Floor[n^(1 / j)]^j - 1) / j, {j, 1, 50}]
```

```
GG[x_, b_] :=
 Sum[ Sum[ Sum[ Floor[Floor[x / u] / m / s], {m, s + 1, Floor[Floor[Floor[x / u] / s]^(1 / 2)]}],
   {s, (u + 1), Floor[Floor[x / u]^(1 / 3)]}], {u, b, Floor[x^(1 / 4)]}]
```

```
GG[100 000, 2]
```

536 683

```
HH[x_, b_] := Sum[ Sum[ Sum[ Floor[x / u / m / s], {m, s + 1, Floor[x / u / s]^(1 / 2)}],
   {s, u + 1, (x / u)^(1 / 3)}], {u, b, x^(1 / 4)}]
```

```
HH[100 000, 2]
```

536 683

```
II[x_, b_] := Sum[ Floor[x / (u m s)], {u, b, x^(1 / 4)},
  {s, u + 1, (x / u)^(1 / 3)}, {m, s + 1, (x / (u s))^(1 / 2)}]
```

```
II[100 000, 2]
```

536 683

```
d4m[x_, b_] :=
 (-1 + b)^4 +
  -Floor[x^(1/4)]^4 +
  4 Sum[ Floor[x / (u^3)], {u, b, x^(1 / 4)}] +
  -6 Sum[Floor[ Floor[x / (u^2)]^(1 / 2)]^2 , {u, b, x^(1 / 4)}] +
  12 Sum [ Floor[x / (u^2 m)],
    {u, b, x^(1 / 4)}, {m, (u + 1), Floor[Floor[x / (u^2)]^(1 / 2)]}] +
  4 Sum[Floor[Floor[x / u]^(1/3)]^3, {u, b, x^(1 / 4)}] +
  12 Sum[ Floor[x / ( u s^2)], {u, b, x^(1 / 4)}, {s, (u + 1), Floor[Floor[x / u]^(1 / 3)]}] +
  -12 Sum[ Floor[Floor[x / (u s)]^(1 / 2)]^2, {u, b, x^(1 / 4)}, {s, (u + 1), (x / u)^(1 / 3)}] +
  24 Sum[ Floor[x / (u m s)], {u, b, x^(1 / 4)},
    {s, u + 1, (x / u)^(1 / 3)}, {m, s + 1, (x / (u s))^(1 / 2)}]
```

```
d4k[100 000, 2]
```

11 796 070

```
d4m[100 000, 2]
```

11 796 070

```
JJ[x_, b_] := Sum[ Sum[Floor[Sqrt[Floor[Floor[x / u] / s]]]^2,
   {s, (u + 1), Floor[Floor[x / u]^(1 / 3)]}], {u, b, Floor[x^(1 / 4)]}]
```

```
JJ[100 000, 2]
```

364 490

```
KK[x_, b_] :=
 Sum[ Floor[Floor[x / (u s)]^(1 / 2)]^2 , {u, b, x^(1 / 4)}, {s, (u + 1), (x / u)^(1 / 3)}]
```

```
KK[100 000, 2]
```

364 490