

$$\frac{1}{2}(\zeta(\frac{1}{2}-t\cdot i)+\zeta(\frac{1}{2}+t\cdot i))=\lim_{n\rightarrow\infty}(\sum_{j=1}^n\frac{1}{\sqrt{j}}\cdot\cos(t\log j))-\frac{2n^{\frac{1}{2}}}{1+4t^2}\cdot(\cos(t\log n)+2t\sin(t\log n))$$

$$\frac{1}{2i}(\zeta(\frac{1}{2}-t\cdot i)-\zeta(\frac{1}{2}+t\cdot i))=\lim_{n\rightarrow\infty}(\sum_{j=1}^n\frac{1}{\sqrt{j}}\cdot\sin(t\log j))-\frac{2n^{\frac{1}{2}}}{1+4t^2}\cdot(\sin(t\log n)-2t\cos(t\log n))$$

(although fun to poke, this seems pointless)

vs

$$\zeta(\frac{1}{2}+t\cdot i)=\lim_{n\rightarrow\infty}\sum_{j=1}^n\frac{1}{\sqrt{j}}\cdot\frac{2t\cos(t\cdot\log\frac{n}{j})-\sin(t\cdot\log\frac{n}{j})}{2t\cos(t\cdot\log n)-\sin(t\cdot\log n)}\text{ for }\mathrm{im}(t)>0$$

$$\zeta(\frac{1}{2}+t\cdot i)=\lim_{n\rightarrow\infty}\sum_{j=1}^n\frac{1}{\sqrt{j}}\cdot(\cos(t\log j)+\tan(t\log n+\cot^{-1}(2t))\cdot\sin(t\log j))\quad \mathrm{re}(t)>0$$

$$\lim_{n\rightarrow\infty}\sum_{j=1}^n(\frac{n}{j})^{\frac{1}{2}}\cdot(2x\cos(x\cdot\log\frac{j}{n})+\sin(x\cdot\log\frac{j}{n}))$$

$$\lim_{n\rightarrow\infty}(-\frac{1}{2}-x)\cdot n^{\frac{1}{2}-x}\cdot H_n^{(\frac{1}{2}-x)}-(-\frac{1}{2}+x)\cdot n^{\frac{1}{2}+x}\cdot H_n^{(\frac{1}{2}+x)}$$