```
Clear[g]
g[n_, k_, a_] := Sum[ ((-1)^(m + 1))^(k - j) Binomial[k, j] g[Floor[n / m^(k - j)], j, m + 1],
   {m, a, n^(1 / k)}, {j, 0, k - 1}];
g[n_, 1, a_] := (1 / 2) ((-1)^(n + 1) + (-1)^(a + 1))
g[n_, 0, a_] := 1
LAdd[n_] := Sum[ 2^k / k, {k, 1, Log[2, n]}]
LinE[n_] := LAdd[n] + Sum[ (-1)^(k + 1) / k g[n, k, 2], {k, 1, Log[2, n]}]
LinE2[n_] :=
 LAdd[n] + Sum[ (-1)^(k + 1) / k Eta[n, k, 2, Floor[n^(1 / 2)]], {k, 1, Log[2, n]}]
Dhyp[n_, k_, a_] := Dhyp[n, k, a] = g[n, k, a]
dhyp[n_, k_, a_] := Dhyp[n, k, a] - Dhyp[n - 1, k, a]
Eta[n_, k_, a_, t_] := Sum[dhyp[j, 1, a] Dhyp[Floor[n / j], k - 1, a], {j, t + 1, n}] +
  Sum[dhyp[j, k - 1, a] Dhyp[Floor[n / j], 1, a], {j, a, t}] +
  Sum[dhyp[s, 1, a] dhyp[j, m, a] Dhyp[Floor[n / (j s)], k - m - 1, a],
   {j, a, t}, {s, Floor[t / j] + 1, Floor[n / j]}, {m, 1, k - 2}]
Eta[n_, 1, a_, t_] := (1 / 2) ((-1)^(n + 1) + (-1)^(a + 1))
Eta[n_, 0, a_, t_] := 1


Table[
  Grid[Table[{Dhyp[n, k, a], Eta[n, k, a, Floor[n^(1 / 3)]], Eta[n, k, a, Floor[n^(1 / 2)]]},
    {n, 7, 300, 21}, {k, 2, 6}]], {a, 1, 4}] // TableForm


LinE2[100]
```

$$\frac{428}{15}$$

```
D2Alt[10 000, 3, 2, 100]
```

```
-6
```

```
g[10 000, 3, 2]
```

```
-6
```

```
D2Alt[n_, k_, a_, t_] := Sum[dh[j, 1] DH[Floor[n / j], k - 1], {j, t + 1, n}] +
  Sum[dh[j, k - 1] DH[Floor[n / j], 1], {j, a, t}] +
  Sum[dh[s, 1] dh[j, m] DH[Floor[n / (j s)], k - m - 1],
   {j, a, t}, {s, Floor[t / j] + 1, Floor[n / j]}, {m, 1, k - 2}]
```

```
D2Alt[n, 2, 2, Floor[n^(1 / 2)]]
```

$$\sum_{j=2}^{\text{Floor}\left[\sqrt{n}\right]} \text{dh}[j, 1]\ \text{DH}\left[\text{Floor}\left[\frac{n}{j}\right], 1\right] + \sum_{j=1+\text{Floor}\left[\sqrt{n}\right]}^{n} \text{dh}[j, 1]\ \text{DH}\left[\text{Floor}\left[\frac{n}{j}\right], 1\right]$$

```
D2Alt[n, 3, 2, Floor[n^(1 / 2)]]
```

$$\sum_{j=2}^{\text{Floor}\left[\sqrt{n}\right]} \text{dh}[j, 2]\ \text{DH}\left[\text{Floor}\left[\frac{n}{j}\right], 1\right] + \sum_{j=1+\text{Floor}\left[\sqrt{n}\right]}^{n} \text{dh}[j, 1]\ \text{DH}\left[\text{Floor}\left[\frac{n}{j}\right], 2\right] +$$

$$\sum_{j=2}^{\text{Floor}\left[\sqrt{n}\right]} \sum_{s=1+\text{Floor}\left[\frac{\text{Floor}\left[\sqrt{n}\right]}{j}\right]}^{\text{Floor}\left[\frac{n}{j}\right]} \sum_{m=1}^{1} \text{dh}[j, m]\ \text{dh}[s, 1]\ \text{DH}\left[\text{Floor}\left[\frac{n}{j\,s}\right], 2 - m\right]$$

```
D2Alt[n, 4, 2, Floor[n^(1 / 2)]]
```

$$\sum_{j=2}^{\text{Floor}\left[\sqrt{n}\right]} \text{dh}[j, 3]\ \text{DH}\left[\text{Floor}\left[\frac{n}{j}\right], 1\right] + \sum_{j=1+\text{Floor}\left[\sqrt{n}\right]}^{n} \text{dh}[j, 1]\ \text{DH}\left[\text{Floor}\left[\frac{n}{j}\right], 3\right] +$$

$$\sum_{j=2}^{\text{Floor}\left[\sqrt{n}\right]} \sum_{s=1+\text{Floor}\left[\frac{\text{Floor}\left[\sqrt{n}\right]}{j}\right]}^{\text{Floor}\left[\frac{n}{j}\right]} \sum_{m=1}^{2} \text{dh}[j, m]\ \text{dh}[s, 1]\ \text{DH}\left[\text{Floor}\left[\frac{n}{j\,s}\right], 3 - m\right]$$

```
D2Alt[n, a, 2, Floor[n^(1 / 2)]]
```

$$\sum_{j=2}^{\text{Floor}\left[\sqrt{n}\right]} \text{dh}[j, -1 + a]\ \text{DH}\left[\text{Floor}\left[\frac{n}{j}\right], 1\right] + \sum_{j=1+\text{Floor}\left[\sqrt{n}\right]}^{n} \text{dh}[j, 1]\ \text{DH}\left[\text{Floor}\left[\frac{n}{j}\right], -1 + a\right] +$$

$$\sum_{j=2}^{\text{Floor}\left[\sqrt{n}\right]} \sum_{s=1+\text{Floor}\left[\frac{\text{Floor}\left[\sqrt{n}\right]}{j}\right]}^{\text{Floor}\left[\frac{n}{j}\right]} \sum_{m=1}^{-2+a} \text{dh}[j, m]\ \text{dh}[s, 1]\ \text{DH}\left[\text{Floor}\left[\frac{n}{j\,s}\right], -1 + a - m\right]$$

da1[n_, a_] :=

$$\sum_{j=2}^{\text{Floor}\left[\sqrt{n}\right]} \text{dh}[j, -1+a]\ \text{DH}\left[\text{Floor}\left[\frac{n}{j}\right], 1\right] + \sum_{j=1+\text{Floor}\left[\sqrt{n}\right]}^{n} \text{dh}[j, 1]\ \text{DH}\left[\text{Floor}\left[\frac{n}{j}\right], -1+a\right] +$$

$$\sum_{j=2}^{\text{Floor}\left[\sqrt{n}\right]} \sum_{s=1+\text{Floor}\left[\frac{\text{Floor}\left[\sqrt{n}\right]}{j}\right]}^{\text{Floor}\left[\frac{n}{j}\right]} \sum_{m=1}^{-2+a} \text{dh}[j, m]\ \text{dh}[s, 1]\ \text{DH}\left[\text{Floor}\left[\frac{n}{j\,s}\right], -1+a-m\right]$$

da2[n_, a_] :=

$$\sum_{j=2}^{\text{Floor}\left[\sqrt{n}\right]} \text{dh}[j, -1+a]\ \text{DH}\left[\text{Floor}\left[\frac{n}{j}\right], 1\right] + \sum_{j=1+\text{Floor}\left[\sqrt{n}\right]}^{n} \text{dh}[j, 1]\ \text{DH}\left[\text{Floor}\left[\frac{n}{j}\right], -1+a\right] +$$

$$\sum_{j=2}^{\text{Floor}\left[\sqrt{n}\right]} \sum_{s=1+\text{Floor}\left[\frac{\text{Floor}\left[\sqrt{n}\right]}{j}\right]}^{\text{Floor}\left[\frac{n}{j}\right]} \sum_{m=1}^{-2+a} \text{dh}[j, m]\ \text{dh}[s, 1]\ \text{DH}\left[\text{Floor}\left[\frac{n}{j\,s}\right], -1+a-m\right]$$