$$\lim_{n\to\infty}(1-s)(\zeta(s)-\sum_{j=1}^{n}\frac{1}{j^{s}})+n^{x}(s-1+x)(\zeta(s+x)-\sum_{j=1}^{n}\frac{1}{j^{s+x}})=0$$

$$\lim_{n\to\infty}n^{y}(s-1+y)(\zeta(s+y)-\sum_{j=1}^{n}\frac{1}{j^{s+y}})-n^{x}(s-1+x)(\zeta(s+x)-\sum_{j=1}^{n}\frac{1}{j^{s+x}})=0$$

$$\zeta(\frac{1}{2}+s)=\lim_{n\to\infty}\sum_{j=1}^{n}\frac{1}{\sqrt{j}}\cdot\frac{2s\cosh(s\cdot\log\frac{n}{j})-\sinh(s\cdot\log\frac{n}{j})}{2s\cosh(s\cdot\log n)-\sinh(s\cdot\log n)}$$ for re(s) > 0

$$\zeta(\frac{1}{2}+t\,i)=\lim_{n\to\infty}\sum_{j=1}^{n}\frac{1}{\sqrt{j}}\cdot\frac{2t\cos(t\cdot\log\frac{n}{j})-\sin(t\cdot\log\frac{n}{j})}{2t\cos(t\cdot\log n)-\sin(t\cdot\log n)}$$ for im(t) > 0

$$\zeta(\frac{1}{2}+t\cdot i)=\lim_{n\to\infty}\sum_{j=1}^{n}\frac{1}{\sqrt{j}}\cdot(\cos(t\log j)+\tan(t\log n+\cot^{-1}(2t))\cdot\sin(t\log j))$$ re(t) > 0

$$\lim_{n\to\infty}\sum_{j=1}^{n}\frac{1}{\sqrt{j}}\cdot(x(e^{x\cdot\log j/n}+e^{-x\cdot\log j/n})+\frac{1}{2}(e^{x\cdot\log j/n}-e^{-x\cdot\log j/n}))$$

$$\zeta(s-\frac{1}{2})=\lim_{n\to\infty}\sum_{j=1}^{n}\frac{1}{\sqrt{j}}\frac{\sinh(s\log\frac{n}{j}-\operatorname{arctanh}(2s))}{\sinh(s\log n-\operatorname{arctanh}(2s))}$$

$$\zeta(s-\frac{1}{2})=\lim_{n\to\infty}\sum_{j=1}^{n}\frac{\cosh(s\log j)}{\sqrt{j}}-\tanh(s\log n-\operatorname{arctanh}(\frac{1}{2s}))\cdot\sum_{j=1}^{n}\frac{\sinh(s\log j)}{\sqrt{j}}$$

$$\lim_{n\to\infty}\sum_{j=1}^{n}\frac{1}{\sqrt{j}}\cdot(2x\cos(x\cdot\log\frac{j}{n})+\sin(x\cdot\log\frac{j}{n}))$$

$$\lim_{n\to\infty}(2x\sin(x\log n)+\cos(x\log n))\cdot(\sum_{j=1}^{n}\frac{1}{\sqrt{j}}\cdot\sin(x\log j))+(2x\cos(x\log n)-\sin(x\log n))\cdot(\sum_{j=1}^{n}\frac{1}{\sqrt{j}}\cdot\cos(x\log j))=0$$

Another one! For Re(s) > 0

$$\zeta(s)=\lim_{n\to\infty}((1-s)n^{s}-1)^{-1}\cdot\sum_{j=1}^{n}(1-s)(\frac{n}{j})^{s}-1$$

$$\zeta(s)=\lim_{n\to\infty}\frac{n^{s}\cdot(1-s)(\sum_{j=1}^{n}j^{-s})-n}{n^{s}\cdot(1-s)-1}$$

Okay. So here's yet another useful variant. For

$$f(n,s)=2\cdot n^{s}\cdot(1-s)(\sum_{j=1}^{n}j^{-s})-n$$ which is $$f(n,s)=2n\cdot(\frac{\sum_{j=1}^{n}j^{-s}}{\int_{0}^{n}j^{-s}\,dj}-1)$$

if s is a zeta zero,

$$\boxed{\lim_{n \to \infty} f(n,s) = -s}$$

Otherwise seems not to converge (?)

$$\boxed{f(n,s)=2\cdot(\sum_{j=1}^{n}(1-s)(\frac{n}{j})^{s}-1)}$$

Slight variant:

$$\boxed{\zeta(-s)=\lim_{n\to\infty}\frac{(1+s)-2n(n^{-s-1}\cdot(1+s)(\sum_{j=1}^{n}j^{s})-1)}{-2n^{-s}\cdot(1+s)}}$$

NOTE. This converges for re(s) < 1, which is a bit more than before.

This can be rewritten as

$$\boxed{\zeta(-s)=\sum_{j=1}^{n}j^{s}-\frac{n^{1+s}}{1+s}-\frac{n^{s}}{2}}$$

This is just the first few terms of the euler maclurin formula.

Then we take the top part of that fraction.

...

$$\boxed{g(s)=\lim_{n\to\infty}(1+s)-2n(n^{-s-1}\cdot(1+s)(\sum_{j=1}^{n}j^{s})-1)}$$

SO. Some notes on g(s).

$$\mathit{if}\ \Re(s)>0,\ g(s)=0$$

$$\mathit{if}\ \Re(s)=0 \text{ and } \Im(s)\neq 0, |g(s)| \text{ converges to some value}$$

$$g(0)=1$$

$$\mathit{if}\ \Re(s)<0 \text{ and } \Im(s)=0,\ g(s)=\infty$$

$$\mathit{if}\ \Re(s)<0 \text{ and } \Im(s)\neq 0,\ g(s)=\text{complex infinity}$$

UNLESS

s is a zeta zero *-1, then $g(s)=0$

(this split is reminding me of something – half the plane is infinity, the other half is 0. But what? My generalization of binomial being used for 2^z comes to mind, but I'm not sure that's it).

. . .

As for the bottom part (the way I'm writing this, $\boxed{\zeta(-s)=g(s)\cdot h(s)}$)

$$\boxed{h(s)=\lim_{n\to\infty}\frac{1}{-2n^{-s}\cdot(1+s)}}$$

$\mathit{if}\ \Re(s)>0,\ g(s)=-\infty$ or complex infinity

$$g(0)=-\frac{1}{2}$$

$$if\ \Re(s)<0\,,\ g(s)=0$$

……

$$\boxed{\zeta(-s)=\lim_{n\to\infty}\frac{(1+s)-2n(n^{-s-1}\cdot(1+s)(\sum_{j=1}^{n}j^{s})-1)}{-2n^{-s}\cdot(1+s)}}$$

vs

$$\zeta(s)=\lim_{n\to\infty}(\sum_{j=1}^{n}j^{-s}-\frac{n^{1-s}}{1-s}-\frac{n^{-s}}{2}+\frac{n^{-s-1}\cdot s}{12}-\frac{n^{-s-3}\cdot s(s+1)(s+2)}{720}+\ldots)$$

vs

$$\zeta(-s)=\lim_{n\to\infty}\frac{n(n^{-s-1}\cdot(1+s)(\sum_{j=1}^{n}j^{s})-1)-\frac{(1+s)}{2}}{n^{-s}\cdot(1+s)}$$

$$\zeta(-s)=\lim_{n\to\infty}\frac{n^{-s}\cdot(1+s)(\sum_{j=1}^{n}j^{s})-n-\frac{(1+s)}{2}}{n^{-s}\cdot(1+s)}$$

$$\zeta(-s)=\lim_{n\to\infty}\frac{n^{-s}\cdot(\sum_{j=1}^{n}j^{s})-\frac{n}{1+s}-\frac{1}{2}}{n^{-s}}$$

$$\zeta(-s)=\lim_{n\to\infty}\sum_{j=1}^{n}j^{s}-\frac{n^{1+s}}{1+s}-\frac{n^{s}}{2}$$

...

$$\zeta(-s)=\lim_{n\to\infty}(\sum_{j=1}^{n}j^{s}-\frac{n^{1+s}}{1+s}-\frac{n^{s}}{2}+\frac{n^{s-1}\cdot(0-s)}{12}-\frac{n^{s-3}\cdot(0-s)(1-s)(2-s)}{720}+\ldots)$$

$$\zeta(-s)=\lim_{n\to\infty}\frac{n(n^{-s-1}\cdot(1+s)(\sum_{j=1}^{n}j^{s})-1)-\frac{(1+s)}{2}}{n^{-s}\cdot(1+s)}$$

$$\zeta(-s)=\lim_{n\to\infty}\frac{n^{-s}\cdot(1+s)\sum_{j=1}^{n}j^{s}-n-\frac{(1+s)}{2}}{n^{-s}\cdot(1+s)}$$

$$\zeta(-s)=\lim_{n\to\infty}\frac{n^{-s}\cdot(1+s)\sum_{j=1}^{n}j^{s}-n-\frac{(1+s)}{2}+\frac{n^{-1}\cdot(1+s)(0-s)}{12}}{n^{-s}\cdot(1+s)}$$

$$\zeta(-s)=\lim_{n\to\infty}\frac{n^{1-s}\cdot(1+s)\sum_{j=1}^{n} j^{s}-n^{2}-\frac{(1+s)n}{2}+\frac{(1+s)(0-s)}{12}}{n^{1-s}\cdot(1+s)}$$

$$\zeta(-s)=\lim_{n\to\infty}\frac{(1+s)-2n(n^{-s-1}\cdot(1+s)(\sum_{j=1}^{n} j^{s})-1)}{-2n^{-s}\cdot(1+s)}$$

$$\zeta(-s)=\lim_{n\to\infty}\frac{n^{-s}\cdot(\sum_{j=1}^{n} j^{s})-\frac{n}{1+s}-\frac{1}{2}}{n^{-s}}$$

$$g(s)=\lim_{n\to\infty}\frac{n^{-s}\cdot(1+s)(\sum_{j=1}^{n} j^{s})-n-\frac{1+s}{2}}{|s|}$$

$$g(s)=\lim_{n\to\infty}\frac{n^{-s}\cdot(1+s)(\sum_{j=1}^{n} j^{s})-n-\frac{1+s}{2}}{|s|}$$

$$g(s)=\lim_{n\to\infty}(1+s)(\sum_{j=1}^{n}(\frac{j}{n})^{s})-n-\frac{1+s}{2}$$

$$g(s)=\lim_{n\to\infty}(1+s)\sum_{j=1}^{n}(\frac{j}{n})^{s}-(1+s)\int_{0}^{n}(\frac{j}{n})^{s}\,dj-\frac{1+s}{2}$$

$$g(s)=\lim_{n\to\infty}(1+s)(\sum_{j=1}^{n}(\frac{j}{n})^{s}-\int_{0}^{n}(\frac{j}{n})^{s}\,dj-\frac{1}{2})$$

Why this, compared to the simpler $\zeta(-s)=\lim_{n\to\infty}\sum_{j=1}^{n} j^{s}-\int_{0}^{n} j^{s}\,dj$?

(For reference, this is $\lim_{n\to\infty}\frac{(1+s)}{n^{s}}\cdot\zeta(-s)=\lim_{n\to\infty}(1+s)(\sum_{j=1}^{n}(\frac{j}{n})^{s}-\int_{0}^{n}(\frac{j}{n})^{s}\,dj-\frac{1}{2})$)

Basically, for visual purposes, this doesn't converge if re(s) < 0 except at zeta zeros.

$$g(s)=\lim_{n\to\infty}(1+s)\left(\sum_{j=1}^{n}\left(\frac{j}{n}\right)^{s}\right)-n-\frac{1+s}{2}$$

$$\lim_{n\to\infty}\frac{(1+s)}{n^{s}}\cdot\zeta(-s)=\lim_{n\to\infty}(1+s)\left(\sum_{j=1}^{n}\left(\frac{j}{n}\right)^{s}\right)-n-\frac{1+s}{2}$$

$$\lim_{n\to\infty}-n+(1+s)\left(\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{-s}-n^{-s}\cdot\zeta(-s)\right)=\frac{1+s}{2}$$

$$\lim_{n\to\infty}-n+\left(\frac{1}{2}+s\right)\left(\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}-s}-n^{\frac{1}{2}-s}\cdot\zeta\left(\frac{1}{2}-s\right)\right)=\frac{\frac{1}{2}+s}{2}$$

$$\lim_{n\to\infty}-n+\left(\frac{1}{2}-s\right)\left(\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}+s}-n^{\frac{1}{2}+s}\cdot\zeta\left(\frac{1}{2}+s\right)\right)=\frac{\frac{1}{2}-s}{2}$$

...

$$\lim_{n\to\infty}\frac{(1+s)}{n^{s}}\cdot\zeta(-s)-\frac{(1+t)}{n^{t}}\cdot\zeta(-t)=\lim_{n\to\infty}(1+s)\left(\sum_{j=1}^{n}\left(\frac{j}{n}\right)^{s}\right)-n-\frac{1+s}{2}-(1+t)\left(\sum_{j=1}^{n}\left(\frac{j}{n}\right)^{t}\right)+n+\frac{1+t}{2}$$

$$\lim_{n\to\infty}(1+s)n^{-s}\cdot\zeta(-s)-(1+t)n^{-t}\cdot\zeta(-t)=\lim_{n\to\infty}(1+s)\sum_{j=1}^{n}\left(\frac{j}{n}\right)^{s}-\frac{1+s}{2}-(1+t)\sum_{j=1}^{n}\left(\frac{j}{n}\right)^{t}+\frac{1+t}{2}$$

$$\lim_{n\to\infty}\left(1+\left(s-\frac{1}{2}\right)\right)n^{-\left(s-\frac{1}{2}\right)}\cdot\zeta\left(-\left(s-\frac{1}{2}\right)\right)-\left(1+\left(-s-\frac{1}{2}\right)\right)n^{-\left(-s-\frac{1}{2}\right)}\cdot\zeta\left(-\left(-s-\frac{1}{2}\right)\right)=\lim_{n\to\infty}\left(1+\left(s-\frac{1}{2}\right)\right)\sum_{j=1}^{n}\left(\frac{j}{n}\right)^{\left(s-\frac{1}{2}\right)}-\frac{1+\left(s-\frac{1}{2}\right)}{2}-\left(1+\left(-s-\frac{1}{2}\right)\right)\sum_{j=1}^{n}\left(\frac{j}{n}\right)$$

$$\lim_{n\to\infty}\left(\frac{1}{2}+s\right)n^{\frac{1}{2}-s}\cdot\zeta\left(\frac{1}{2}-s\right)-\left(\frac{1}{2}-s\right)n^{\frac{1}{2}+s}\cdot\zeta\left(\frac{1}{2}+s\right)=\lim_{n\to\infty}\left(1+\left(s-\frac{1}{2}\right)\right)\sum_{j=1}^{n}\left(\frac{j}{n}\right)^{\left(s-\frac{1}{2}\right)}-\frac{1+\left(s-\frac{1}{2}\right)}{2}-\left(1+\left(-s-\frac{1}{2}\right)\right)\sum_{j=1}^{n}\left(\frac{j}{n}\right)^{\left(-s-\frac{1}{2}\right)}+\frac{1+\left(-s-\frac{1}{2}\right)}{2}$$

$$\lim_{n\to\infty}\left(\frac{1}{2}+s\right)n^{\frac{1}{2}-s}\cdot\zeta\left(\frac{1}{2}-s\right)-\left(\frac{1}{2}-s\right)n^{\frac{1}{2}+s}\cdot\zeta\left(\frac{1}{2}+s\right)=\lim_{n\to\infty}\left(\frac{1}{2}+s\right)\sum_{j=1}^{n}\left(\frac{j}{n}\right)^{-\frac{1}{2}+s}-\frac{\frac{1}{2}+s}{2}-\left(\frac{1}{2}-s\right)\sum_{j=1}^{n}\left(\frac{j}{n}\right)^{\left(-\frac{1}{2}-s\right)}+\frac{\frac{1}{2}-s}{2}$$

$$\lim_{n\to\infty}\left(\frac{1}{2}+s\right)n^{\frac{1}{2}-s}\cdot\zeta\left(\frac{1}{2}-s\right)-\left(\frac{1}{2}-s\right)n^{\frac{1}{2}+s}\cdot\zeta\left(\frac{1}{2}+s\right)=\lim_{n\to\infty}\left(\frac{1}{2}+s\right)\sum_{j=1}^{n}\left(\frac{j}{n}\right)^{-\frac{1}{2}+s}-\left(\frac{1}{2}-s\right)\sum_{j=1}^{n}\left(\frac{j}{n}\right)^{\left(-\frac{1}{2}-s\right)}-s$$

$$\lim_{n\to\infty}n^{\frac{1}{2}}\left(\left(\frac{1}{2}+s\right)n^{-s}\cdot\zeta\left(\frac{1}{2}-s\right)-\left(\frac{1}{2}-s\right)n^{s}\cdot\zeta\left(\frac{1}{2}+s\right)\right)=\lim_{n\to\infty}\left(\frac{1}{2}+s\right)\sum_{j=1}^{n}\left(\frac{j}{n}\right)^{-\frac{1}{2}+s}-\left(\frac{1}{2}-s\right)\sum_{j=1}^{n}\left(\frac{j}{n}\right)^{\left(-\frac{1}{2}-s\right)}-s$$

$$\lim_{n\to\infty}n^{\frac{1}{2}}\left(\left(\frac{1}{2}+s\right)n^{-s}\cdot\zeta\left(\frac{1}{2}-s\right)-\left(\frac{1}{2}-s\right)n^{s}\cdot\zeta\left(\frac{1}{2}+s\right)\right)=\lim_{n\to\infty}\sum_{j=1}^{n}\left(\frac{j}{n}\right)^{-\frac{1}{2}}\left(\frac{1}{2}+s\right)\left(\frac{j}{n}\right)^{s}-\left(\frac{1}{2}-s\right)\left(\frac{j}{n}\right)^{-s}-s$$

(revisit)

$$\lim_{n\to\infty} n^{\frac{1}{2}}((\frac{1}{2}+s)n^{-s}\cdot\zeta(\frac{1}{2}-s)-(\frac{1}{2}-s)n^{s}\cdot\zeta(\frac{1}{2}+s))=\lim_{n\to\infty} n^{\frac{1}{2}}((\frac{1}{2}+s)\sum_{j=1}^{n} j^{-\frac{1}{2}}(\frac{j}{n})^{s}-(\frac{1}{2}-s)\sum_{j=1}^{n} j^{-\frac{1}{2}}(\frac{j}{n})^{-s})-s$$

$$\lim_{n\to\infty} n^{\frac{1}{2}}((\frac{1}{2}+s)n^{-s}\cdot\zeta(\frac{1}{2}-s)-(\frac{1}{2}-s)n^{s}\cdot\zeta(\frac{1}{2}+s)-(\frac{1}{2}+s)\sum_{j=1}^{n} j^{-\frac{1}{2}}(\frac{j}{n})^{s}+(\frac{1}{2}-s)\sum_{j=1}^{n} j^{-\frac{1}{2}}(\frac{j}{n})^{-s})=-s$$

$$\lim_{n\to\infty} n^{\frac{1}{2}}((\frac{1}{2}+s)n^{-s}\cdot\zeta(\frac{1}{2}-s)-(\frac{1}{2}+s)\sum_{j=1}^{n} j^{-\frac{1}{2}}(\frac{j}{n})^{s}-(\frac{1}{2}-s)n^{s}\cdot\zeta(\frac{1}{2}+s)+(\frac{1}{2}-s)\sum_{j=1}^{n} j^{-\frac{1}{2}}(\frac{j}{n})^{-s})=-s$$

$$\lim_{n\to\infty} n^{\frac{1}{2}}((\frac{1}{2}+s)n^{-s}\cdot\zeta(\frac{1}{2}-s)-(\frac{1}{2}+s)\sum_{j=1}^{n} j^{-\frac{1}{2}}(\frac{j}{n})^{s}-(\frac{1}{2}-s)n^{s}\cdot\zeta(\frac{1}{2}+s)+(\frac{1}{2}-s)\sum_{j=1}^{n} j^{-\frac{1}{2}}(\frac{j}{n})^{-s})=-s$$

HERE, THIS.

$$\boxed{\lim_{n\to\infty} (\frac{1}{2}+s)(n^{\frac{1}{2}-s}\cdot\zeta(\frac{1}{2}-s)-\sum_{j=1}^{n}(\frac{n}{j})^{\frac{1}{2}-s})-(\frac{1}{2}-s)(n^{\frac{1}{2}+s}\cdot\zeta(\frac{1}{2}+s)-\sum_{j=1}^{n}(\frac{n}{j})^{\frac{1}{2}+s})=-s}$$

$$\boxed{\lim_{n\to\infty} -2n+(\frac{1}{2}+s)(n^{\frac{1}{2}-s}\cdot\zeta(\frac{1}{2}-s)-\sum_{j=1}^{n}(\frac{n}{j})^{\frac{1}{2}-s})+(\frac{1}{2}-s)(n^{\frac{1}{2}+s}\cdot\zeta(\frac{1}{2}+s)-\sum_{j=1}^{n}(\frac{n}{j})^{\frac{1}{2}+s})=-\frac{1}{2}}$$

NOT DONE

…

$$\boxed{\lim_{n\to\infty}\sum_{j=1}^{n} j^{s}-\int_{0}^{n} j^{s}\,dj=\zeta(-s)}$$

$$\boxed{\lim_{n\to\infty}\sum_{j=1}^{n} j^{s}-\frac{n^{1+s}}{1+s}=\zeta(-s)}$$

$$\boxed{\lim_{n\to\infty}\sum_{j=1}^{n} j^{s}-\frac{n^{1+s}}{1+s}-\zeta(-s)=0}$$

$$\boxed{\lim_{n\to\infty}(1+s)n^{-1-s}\sum_{j=1}^{n} j^{s}-1-n^{-1-s}(1+s)\zeta(-s)=0}$$

$$\boxed{\lim_{n\to\infty}\frac{1+s}{n}\sum_{j=1}^{n}\left(\frac{j}{n}\right)^{s}-n^{-1-s}(1+s)\zeta(-s)=1}$$

$$\boxed{\lim_{n\to\infty}\frac{\frac{1}{2}+s}{n}\sum_{j=1}^{n}\left(\frac{j}{n}\right)^{-\frac{1}{2}+s}-n^{-\frac{1}{2}-s}\left(\frac{1}{2}+s\right)\zeta\left(\frac{1}{2}-s\right)=1}$$

$$\boxed{\lim_{n\to\infty}\frac{\frac{1}{2}-s}{n}\sum_{j=1}^{n}\left(\frac{j}{n}\right)^{-\frac{1}{2}-s}-n^{-\frac{1}{2}+s}\left(\frac{1}{2}-s\right)\zeta\left(\frac{1}{2}+s\right)=1}$$

$$\boxed{\lim_{n\to\infty}\frac{1}{n}\left(\left(\frac{1}{2}-s\right)\left(\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}+s}-n^{\frac{1}{2}+s}\zeta\left(\frac{1}{2}+s\right)\right)\right)=1}$$

$$\boxed{\lim_{n\to\infty}\frac{1}{n}\left(\left(\frac{1}{2}+s\right)\left(\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}-s}-n^{\frac{1}{2}-s}\zeta\left(\frac{1}{2}-s\right)\right)\right)=1}$$

$$\boxed{\lim_{n\to\infty}\left(\frac{1}{2}+s\right)\left(n^{\frac{1}{2}-s}\cdot\zeta\left(\frac{1}{2}-s\right)-\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}-s}\right)-\left(\frac{1}{2}-s\right)\left(n^{\frac{1}{2}+s}\cdot\zeta\left(\frac{1}{2}+s\right)+\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}+s}\right)=-s}$$

$$\boxed{\lim_{n\to\infty}-2n+\left(\frac{1}{2}+s\right)\left(n^{\frac{1}{2}-s}\cdot\zeta\left(\frac{1}{2}-s\right)-\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}-s}\right)+\left(\frac{1}{2}-s\right)\left(n^{\frac{1}{2}+s}\cdot\zeta\left(\frac{1}{2}+s\right)+\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}+s}\right)=-\frac{1}{2}}$$

…

$$\left(\frac{1}{2}+s\right)\left(n^{\frac{1}{2}-s}\cdot\zeta\left(\frac{1}{2}-s\right)-\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}-s}\right)-\left(\frac{1}{2}-s\right)\left(n^{\frac{1}{2}+s}\cdot\zeta\left(\frac{1}{2}+s\right)+\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}+s}\right)$$

$$\left(\frac{1}{2}+s\right)\left(n^{\frac{1}{2}-s}\cdot\zeta\left(\frac{1}{2}-s\right)-\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}-s}\right)+\left(\frac{1}{2}-s\right)\left(n^{\frac{1}{2}+s}\cdot\zeta\left(\frac{1}{2}+s\right)+\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}+s}\right)$$

…

$$\left(\frac{1}{2}+s\right)\left(n^{\frac{1}{2}-s}\cdot\zeta\left(\frac{1}{2}-s\right)\right)-\left(\frac{1}{2}-s\right)\left(n^{\frac{1}{2}+s}\cdot\zeta\left(\frac{1}{2}+s\right)\right)-\left(\frac{1}{2}+s\right)\left(\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}-s}\right)+\left(\frac{1}{2}-s\right)\left(\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}+s}\right)$$

$$n^{\frac{1}{2}}\left(\left(\frac{1}{2}+s\right)n^{-s}\cdot\zeta\left(\frac{1}{2}-s\right)-\left(\frac{1}{2}-s\right)n^{s}\cdot\zeta\left(\frac{1}{2}+s\right)\right)+\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\cdot\left(\left(\frac{1}{2}-s\right)\left(\frac{n}{j}\right)^{s}-\left(\frac{1}{2}+s\right)\left(\frac{n}{j}\right)^{-s}\right)$$

…

Do these 4 permutations:

$$\left(\frac{1}{2}+s\right)\left(n^{\frac{1}{2}-s}\cdot\zeta\left(\frac{1}{2}-s\right)\right)-\left(\frac{1}{2}-s\right)\left(n^{\frac{1}{2}+s}\cdot\zeta\left(\frac{1}{2}+s\right)\right)$$

$$\left(\frac{1}{2}+s\right)\left(n^{\frac{1}{2}-s}\cdot\zeta\left(\frac{1}{2}-s\right)\right)+\left(\frac{1}{2}-s\right)\left(n^{\frac{1}{2}+s}\cdot\zeta\left(\frac{1}{2}+s\right)\right)$$

$$\left(\frac{1}{2}+s\right)\left(\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}-s}\right)-\left(\frac{1}{2}-s\right)\left(\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}+s}\right)$$

$$\left(\frac{1}{2}+s\right)\left(\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}-s}\right)+\left(\frac{1}{2}-s\right)\left(\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}+s}\right)$$

……………………

$$\xi(s)=\frac{1}{2}\cdot s(s-1)\pi^{(-s/2)}\Gamma(\frac{1}{2}\cdot s)\cdot\zeta(s)$$

$$\frac{2\pi^{(s/2)}}{s(s-1)\cdot\Gamma(\frac{1}{2}\cdot s)}\xi(s)=\zeta(s)$$

$$\frac{2\pi^{\frac{s}{2}+\frac{1}{4}}}{(s+\frac{1}{2})(s-\frac{1}{2})\cdot\Gamma(\frac{1}{4}\cdot\frac{s}{2})}\xi(\frac{1}{2}+s)=\zeta(\frac{1}{2}+s)$$

$$\frac{2\pi^{-\frac{s}{2}+\frac{1}{4}}}{(-s+\frac{1}{2})(-s-\frac{1}{2})\cdot\Gamma(\frac{1}{4}\cdot-\frac{s}{2})}\xi(\frac{1}{2}-s)=\zeta(\frac{1}{2}-s)$$

………………..

$$(\frac{1}{2}+s)(\sum_{j=1}^{n}(\frac{n}{j})^{\frac{1}{2}-s})-(\frac{1}{2}-s)(\sum_{j=1}^{n}(\frac{n}{j})^{\frac{1}{2}+s})$$

$$\sum_{j=1}^{n}(\frac{n}{j})^{\frac{1}{2}}((\frac{1}{2}+s)(\frac{n}{j})^{-s}-(\frac{1}{2}-s)(\frac{n}{j})^{s})$$

$$\sum_{j=1}^{n}(\frac{n}{j})^{\frac{1}{2}}(\frac{1}{2}(\frac{n}{j})^{-s}+s(\frac{n}{j})^{-s}-\frac{1}{2}(\frac{n}{j})^{s}+s(\frac{n}{j})^{s})$$

$$\sum_{j=1}^{n}(\frac{n}{j})^{\frac{1}{2}}(\frac{1}{2}((\frac{n}{j})^{-s}-(\frac{n}{j})^{s})+s((\frac{n}{j})^{-s}+(\frac{n}{j})^{s}))$$

$$\sum_{j=1}^{n}(\frac{n}{j})^{\frac{1}{2}}(2s\cosh(s\log\frac{n}{j})-\sinh(s\log\frac{n}{j}))$$

….

(save this! It's the plus version: $\sum_{j=1}^{n}(\frac{n}{j})^{\frac{1}{2}}(\frac{1}{2}(\frac{n}{j})^{-s}+s(\frac{n}{j})^{-s}+\frac{1}{2}(\frac{n}{j})^{s}-s(\frac{n}{j})^{s})$)

$$\sum_{j=1}^{n}(\frac{n}{j})^{\frac{1}{2}}(\cosh(s\log\frac{n}{j})-2s\sinh(s\log\frac{n}{j}))$$

BLAH BLAH BLAH

Finally, if s is a real-valued variable that is a zeta 0 frequency, then

$$\boxed{\lim_{n\to\infty} \sum_{j=1}^{n} \left(\frac{n}{j}\right)^{\frac{1}{2}} \left(2s\cos\left(s\log\frac{n}{j}\right) - \sin\left(s\log\frac{n}{j}\right)\right) = s}$$

$$\boxed{\lim_{n\to\infty} -2n + \sum_{j=1}^{n} \left(\frac{n}{j}\right)^{\frac{1}{2}} \left(\cos\left(s\log\frac{n}{j}\right) + 2s\sin\left(s\log\frac{n}{j}\right)\right) = \frac{1}{2}}$$

…

$$\boxed{\lim_{n\to\infty} 2s\sum_{j=1}^{n} \left(\frac{n}{j}\right)^{\frac{1}{2}} \cos\left(s\log\frac{n}{j}\right) - \sum_{j=1}^{n} \left(\frac{n}{j}\right)^{\frac{1}{2}} \sin\left(s\log\frac{n}{j}\right) = s}$$

$$\boxed{\lim_{n\to\infty} -2n + \sum_{j=1}^{n} \left(\frac{n}{j}\right)^{\frac{1}{2}} \cos\left(s\log\frac{n}{j}\right) + 2s\sum_{j=1}^{n} \left(\frac{n}{j}\right)^{\frac{1}{2}} \sin\left(s\log\frac{n}{j}\right) = \frac{1}{2}}$$

$$\boxed{\lim_{n\to\infty} 2s\,A(n) - B(n) = s}$$

$$\boxed{\lim_{n\to\infty} -2n + A(n) + 2s\,B(n) = \frac{1}{2}}$$

$$\boxed{\lim_{n\to\infty} -2n + A(n) + 2s\,B(n) - A(n) + \frac{1}{2s}\cdot B(n) = 0}$$

$$\boxed{\lim_{n\to\infty} -2n + \left(2s + \frac{1}{2s}\right) B(n) = 0}$$

$$\boxed{\lim_{n\to\infty} -2n + \left(2s + \frac{1}{2s}\right) \sum_{j=1}^{n} \left(\frac{n}{j}\right)^{\frac{1}{2}} \sin\left(s\log\frac{n}{j}\right) = 0}$$

…

$$\boxed{\lim_{n\to\infty} A(n) - \frac{1}{2s}\cdot B(n) = \frac{1}{2}}$$

$$\boxed{\lim_{n\to\infty} -\frac{2n}{2s} + \frac{1}{2s}\cdot A(n) + B(n) = \frac{1}{4s}}$$

$$\boxed{\lim_{n\to\infty} -\frac{2n}{2s} + \left(2s + \frac{1}{2s}\right) A(n) = s + \frac{1}{4s}}$$

...

$$\boxed{\lim_{n\to\infty} -2n + \left(2s + \frac{1}{2s}\right) \sum_{j=1}^{n} \left(\frac{n}{j}\right)^{\frac{1}{2}} \sin\left(s\log\frac{n}{j}\right) = 0}$$

$$\boxed{\lim_{n\to\infty} -\frac{2n}{2s} - s - \frac{1}{4s} + \left(2s + \frac{1}{2s}\right) \sum_{j=1}^{n} \left(\frac{n}{j}\right)^{\frac{1}{2}} \cos\left(s\log\frac{n}{j}\right) = 0}$$

(I should rework out what this looks like with the zeta components kept in, obviously.)

$$\lim_{n\to\infty} -n+\left(\frac{1}{2}+s\right)\left(\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}-s}-n^{\frac{1}{2}-s}\cdot\zeta\left(\frac{1}{2}-s\right)\right)=\frac{\frac{1}{2}+s}{2}$$

$$\lim_{n\to\infty} -n+\left(\frac{1}{2}-s\right)\left(\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}+s}-n^{\frac{1}{2}+s}\cdot\zeta\left(\frac{1}{2}+s\right)\right)=\frac{\frac{1}{2}-s}{2}$$

$$\lim_{n\to\infty}\left(\frac{1}{2}+s\right)\left(n^{\frac{1}{2}-s}\cdot\zeta\left(\frac{1}{2}-s\right)-\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}-s}\right)-\left(\frac{1}{2}-s\right)\left(n^{\frac{1}{2}+s}\cdot\zeta\left(\frac{1}{2}+s\right)+\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}+s}\right)=-s$$

$$\lim_{n\to\infty}-2n+\left(\frac{1}{2}+s\right)\left(n^{\frac{1}{2}-s}\cdot\zeta\left(\frac{1}{2}-s\right)-\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}-s}\right)+\left(\frac{1}{2}-s\right)\left(n^{\frac{1}{2}+s}\cdot\zeta\left(\frac{1}{2}+s\right)+\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}+s}\right)=-\frac{1}{2}$$

…

$$\lim_{n\to\infty}\left(\frac{1}{2}+s\right)\left(n^{\frac{1}{2}-s}\cdot\zeta\left(\frac{1}{2}-s\right)-\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}-s}\right)-\left(\frac{1}{2}-s\right)\left(n^{\frac{1}{2}+s}\cdot\zeta\left(\frac{1}{2}+s\right)+\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}+s}\right)=-s$$

$$\lim_{n\to\infty}\frac{\left(\frac{1}{2}+s\right)}{2s}\left(n^{\frac{1}{2}-s}\cdot\zeta\left(\frac{1}{2}-s\right)-\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}-s}\right)-\frac{\left(\frac{1}{2}-s\right)}{2s}\left(n^{\frac{1}{2}+s}\cdot\zeta\left(\frac{1}{2}+s\right)+\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}+s}\right)=-\frac{1}{2}$$

$$\lim_{n\to\infty}\left(\frac{1}{4s}+\frac{1}{2}\right)\left(n^{\frac{1}{2}-s}\cdot\zeta\left(\frac{1}{2}-s\right)-\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}-s}\right)+\left(\frac{-1}{4s}+\frac{1}{2}\right)\left(n^{\frac{1}{2}+s}\cdot\zeta\left(\frac{1}{2}+s\right)+\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}+s}\right)=-\frac{1}{2}$$

…

$$\lim_{n\to\infty}-2n+\left(s-\frac{1}{4s}\right)\left(n^{\frac{1}{2}-s}\cdot\zeta\left(\frac{1}{2}-s\right)-\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}-s}\right)-\left(s-\frac{1}{4s}\right)\left(n^{\frac{1}{2}+s}\cdot\zeta\left(\frac{1}{2}+s\right)+\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}+s}\right)=0$$

$$\lim_{n\to\infty}-2n+\left(s-\frac{1}{4s}\right)\left(n^{\frac{1}{2}-s}\cdot\zeta\left(\frac{1}{2}-s\right)-\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}-s}-n^{\frac{1}{2}+s}\cdot\zeta\left(\frac{1}{2}+s\right)-\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}+s}\right)=0$$

$$\lim_{n\to\infty}\frac{8s}{1-4s^2}n+\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}-s}-\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}+s}-n^{\frac{1}{2}-s}\cdot\zeta\left(\frac{1}{2}-s\right)+n^{\frac{1}{2}+s}\cdot\zeta\left(\frac{1}{2}+s\right)=0$$

$$\boxed{\lim_{n\to\infty}-\frac{8s}{1-4s^2}n+2\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\sinh\left(s\log\frac{n}{j}\right)+n^{\frac{1}{2}-s}\cdot\zeta\left(\frac{1}{2}-s\right)-n^{\frac{1}{2}+s}\cdot\zeta\left(\frac{1}{2}+s\right)=0}$$

…and...

$$\boxed{\lim_{n\to\infty}-\frac{4}{1-4s^2}n-1+2\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\cosh\left(s\log\frac{n}{j}\right)-n^{\frac{1}{2}-s}\cdot\zeta\left(\frac{1}{2}-s\right)-n^{\frac{1}{2}+s}\cdot\zeta\left(\frac{1}{2}+s\right)=0}$$

These can be shifted over by ½ and rewritten as

$$\boxed{\lim_{n\to\infty}-\frac{1-2s}{s(1-s)}n+2\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\sinh\left(\left(\frac{1}{2}-s\right)\log\frac{n}{j}\right)+n^{s}\cdot\zeta(s)-n^{1-s}\cdot\zeta(1-s)=0}$$

$$\boxed{\lim_{n\to\infty}-\frac{1}{s(1-s)}n-1+2\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\cosh\left(\left(\frac{1}{2}-s\right)\log\frac{n}{j}\right)-n^{1-s}\cdot\zeta(1-s)-n^{s}\cdot\zeta(s)=0}$$

which is

$$\boxed{\lim_{d\to 0}-\frac{1-2s}{s(1-s)}d^{-1}+2\sum_{t=1}^{d^{-1}}(d\cdot t)^{-\frac{1}{2}}\sinh\left(-\left(\frac{1}{2}-s\right)\log d\cdot t\right)+d^{-s}\cdot\zeta(-s)-d^{s-1}\cdot\zeta(1-s)=0}$$

$$\boxed{\lim_{d\to 0}-\frac{1}{s(1-s)}d^{-1}-1+2\sum_{t=1}^{d^{-1}}(d\cdot t)^{-\frac{1}{2}}\cosh\left(-\left(\frac{1}{2}-s\right)\log d\cdot t\right)-d^{s-1}\cdot\zeta(1-s)-d^{-s}\cdot\zeta(s)=0}$$

Or rotated instead, to get more familiar trig functions:

$$\boxed{\lim_{n\to\infty} -\frac{8s}{1+4s^2}n+2\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\sin\left(s\log\frac{n}{j}\right)+i\left(n^{\frac{1}{2}+si}\cdot\zeta\left(\frac{1}{2}+si\right)-n^{\frac{1}{2}-si}\cdot\zeta\left(\frac{1}{2}-si\right)\right)=0}$$

…and...

$$\boxed{\lim_{n\to\infty} -\frac{4}{1+4s^2}n-1+2\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\cos\left(s\log\frac{n}{j}\right)-\left(n^{\frac{1}{2}+si}\cdot\zeta\left(\frac{1}{2}+si\right)+n^{\frac{1}{2}-si}\cdot\zeta\left(\frac{1}{2}-si\right)\right)=0}$$

which is

$$\boxed{\lim_{d\to 0} -\frac{8s}{1+4s^2}d^{-1}+2\sum_{t=1}^{d^{-1}}\frac{\sin(-s\log d\cdot t)}{\sqrt{d\cdot t}}+i\left(d^{-\frac{1}{2}-si}\cdot\zeta\left(\frac{1}{2}+si\right)-d^{-\frac{1}{2}+si}\cdot\zeta\left(\frac{1}{2}-si\right)\right)=0}$$

…and...

$$\boxed{\lim_{d\to 0} -\frac{4}{1+4s^2}d^{-1}-1+2\sum_{t=1}^{d^{-1}}\frac{\cos(-s\log d\cdot t)}{\sqrt{d\cdot t}}-\left(d^{-\frac{1}{2}-si}\cdot\zeta\left(\frac{1}{2}+si\right)+d^{-\frac{1}{2}+si}\cdot\zeta\left(\frac{1}{2}-si\right)\right)=0}$$

Based on this, zeta zeros must satisfy the following

$$\boxed{\lim_{n\to\infty} -\frac{4s}{1+4s^2}\cdot n+\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\sin\left(s\log\frac{n}{j}\right)=0}$$

$$\boxed{\lim_{n\to\infty} -\frac{2}{1+4s^2}\cdot n-\frac{1}{2}+\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\cos\left(s\log\frac{n}{j}\right)=0}$$

$$\boxed{\lim_{n\to\infty} 2s\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\cos\left(s\log\frac{n}{j}\right)-\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\sin\left(s\log\frac{n}{j}\right)=s}$$

$$\boxed{\lim_{n\to\infty} -2n+\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\cos\left(s\log\frac{n}{j}\right)+2s\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\sin\left(s\log\frac{n}{j}\right)=\frac{1}{2}}$$

...

The following must be true for zeta zeros.

$$\lim_{n\to\infty}\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\cos\left(s\log\frac{n}{j}\right)-\frac{1}{2s}\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\sin\left(s\log\frac{n}{j}\right)-\frac{1}{2}=0$$

$$\lim_{n\to\infty} -n^{\frac{1}{2}}+\frac{1}{2}\sum_{j=1}^{n}\left(\frac{1}{j}\right)^{\frac{1}{2}}\cos\left(s\log\frac{n}{j}\right)+s\sum_{j=1}^{n}\left(\frac{1}{j}\right)^{\frac{1}{2}}\sin\left(s\log\frac{n}{j}\right)-n^{-\frac{1}{2}}=0$$

$$\lim_{n\to\infty} -n^{\frac{1}{2}}-n^{-\frac{1}{2}}+\frac{1}{2}\sum_{j=1}^{n} j^{-\frac{1}{2}}\cos\left(s\log\frac{n}{j}\right)+s\sum_{j=1}^{n} j^{-\frac{1}{2}}\sin\left(s\log\frac{n}{j}\right)=0$$

$$\boxed{\lim_{n\to\infty} -2\cdot\cosh\left(\frac{1}{2}\log n\right)+\sum_{j=1}^{n} j^{-\frac{1}{2}}\left(\frac{1}{2}\cos\left(s\log\frac{n}{j}\right)+s\sin\left(s\log\frac{n}{j}\right)\right)=0}$$

...

$$\lim_{n \to \infty} \sum_{j=1}^{n} \left(\frac{n}{j}\right)^{\frac{1}{2}} \left(2\, s \cos\left(s \log \frac{n}{j}\right) - \sin\left(s \log \frac{n}{j}\right)\right) = s$$

$$\lim_{n \to \infty} -2\, n + \sum_{j=1}^{n} \left(\frac{n}{j}\right)^{\frac{1}{2}} \left(\cos\left(s \log \frac{n}{j}\right) + 2\, s \sin\left(s \log \frac{n}{j}\right)\right) = \frac{1}{2}$$

$$\cdots$$

$$\lim_{n \to \infty} n \cdot \frac{1}{n} \cdot \sum_{j=1}^{n} \left(\frac{n}{j}\right)^{\frac{1}{2}} \left(2\, s \cos\left(s \log \frac{n}{j}\right) - \sin\left(s \log \frac{n}{j}\right)\right) = s$$

$$\lim_{n\to\infty} -\frac{8s}{1+4s^2}n+2\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\sin\left(s\log\frac{n}{j}\right)+i\left(n^{\frac{1}{2}+si}\cdot\zeta\left(\frac{1}{2}+si\right)-n^{\frac{1}{2}-si}\cdot\zeta\left(\frac{1}{2}-si\right)\right)=0$$

…and...

$$\lim_{n\to\infty} -\frac{4}{1+4s^2}n-1+2\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\cos\left(s\log\frac{n}{j}\right)-i\left(n^{\frac{1}{2}+si}\cdot\zeta\left(\frac{1}{2}+si\right)+n^{\frac{1}{2}-si}\cdot\zeta\left(\frac{1}{2}-si\right)\right)=0$$

…..

$$\xi(s)=\frac{1}{2}\cdot s(s-1)\pi^{(-s/2)}\Gamma\left(\frac{1}{2}\cdot s\right)\cdot\zeta(s)$$

$$\frac{2\pi^{(s/2)}}{s(s-1)\cdot\Gamma\left(\frac{1}{2}\cdot s\right)}\xi(s)=\zeta(s)$$

$$\frac{2\pi^{\frac{s}{2}+\frac{1}{4}}}{\left(s+\frac{1}{2}\right)\left(s-\frac{1}{2}\right)\cdot\Gamma\left(\frac{1}{4}\cdot\frac{s}{2}\right)}\xi\left(\frac{1}{2}+s\right)=\zeta\left(\frac{1}{2}+s\right)$$

$$\frac{2\pi^{-\frac{s}{2}+\frac{1}{4}}}{\left(-s+\frac{1}{2}\right)\left(-s-\frac{1}{2}\right)\cdot\Gamma\left(\frac{1}{4}\cdot-\frac{s}{2}\right)}\xi\left(\frac{1}{2}-s\right)=\zeta\left(\frac{1}{2}-s\right)$$

….

At zeta zeros,

$$\lim_{n\to\infty} -\frac{8s}{1+4s^2}\cdot n+2\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\sin\left(s\log\frac{n}{j}\right)=0$$

$$\lim_{n\to\infty} -\frac{4}{1+4s^2}\cdot n+2\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\cos\left(s\log\frac{n}{j}\right)=1$$

…

$$\lim_{d\to 0} d\cdot\sum_{t=1}^{\frac{1}{d}} f(d\cdot t)=\int_0^1 f(t)\,dt$$

Thus:

$$\lim_{d\to 0}\int_0^1 f(t)\,dt-d\cdot\sum_{t=1}^{\frac{1}{d}} f(d\cdot t)=0$$

Question – what do you think happens if we divide both sides by d?

$$\lim_{d\to 0}\frac{1}{d}\cdot\int_0^1 f(t)\,dt-\sum_{t=1}^{\frac{1}{d}} f(d\cdot t)=?$$

Restated, for all s is it the case that

$$\lim_{d\to 0}\int_0^1 \frac{\sin(s\log(x))}{\sqrt{x}}dx-d\cdot\sum_{t=1}^{d^{-1}}\frac{\sin(s\log(d\,t))}{\sqrt{dt}}=0$$

$$\lim_{d\to 0}\int_0^1 \frac{\cos(s\log(x))}{\sqrt{x}}dx-d\cdot\sum_{t=1}^{d^{-1}}\frac{\cos(s\log(d\,t))}{\sqrt{dt}}=0$$

$$\lim_{d\to 0}\int_0^1 \frac{\sin(s\log x+\theta)}{\sqrt{x}}dx-d\cdot\sum_{t=1}^{d^{-1}}\frac{\sin(s\log(d\cdot t)+\theta)}{\sqrt{d\cdot t}}=0$$

$$\lim_{d\to 0}\int_0^1 \frac{\sin(s\log x+\arctan 2s)}{\sqrt{x}}dx-d\cdot\sum_{t=1}^{d^{-1}}\frac{\sin(s\log(d\cdot t)+\arctan 2s)}{\sqrt{d\cdot t}}=0$$

but, if we divide both sides by d, only at zeta zeros is it the case that

$$\lim_{d\to 0}\frac{1}{d}\cdot\int_0^1 \frac{\sin(s\log(x))}{\sqrt{x}}dx-\sum_{t=1}^{d^{-1}}\frac{\sin(s\log(d\,t))}{\sqrt{dt}}=0$$

$$\lim_{d\to 0}\frac{1}{d}\cdot\int_0^1 \frac{\cos(s\log(x))}{\sqrt{x}}dx-\sum_{t=1}^{d^{-1}}\frac{\cos(s\log(d\,t))}{\sqrt{dt}}=\frac{1}{2}$$

$$\lim_{d\to 0}\frac{1}{d}\cdot\left(\int_0^1 \frac{\sin(s\log x+\theta)}{\sqrt{x}}dx\right)-\sum_{t=1}^{d^{-1}}\frac{\sin(s\log(d\cdot t)+\theta)}{\sqrt{d\cdot t}}=\frac{\sin(\theta)}{2}$$

$$\lim_{d\to 0}\frac{1}{d}\cdot\left(\int_0^1 \frac{\sin(s\log x+\arctan 2s)}{\sqrt{x}}dx\right)-\sum_{t=1}^{d^{-1}}\frac{\sin(s\log(d\cdot t)+\arctan 2s)}{\sqrt{d\cdot t}}=\frac{\sin(\arctan 2s)}{2}$$

which simplifies to

$$\lim_{d\to 0}\frac{1}{d}\cdot\frac{-4s}{1+4s^2}-\sum_{t=1}^{d^{-1}}\frac{\sin(s\log(d\cdot t))}{\sqrt{d\cdot t}}=0$$

$$\lim_{d\to 0}\frac{1}{d}\cdot\frac{2}{1+4s^2}-\sum_{t=1}^{d^{-1}}\frac{\cos(s\log(d\cdot t))}{\sqrt{d\cdot t}}=\frac{1}{2}$$

$$\lim_{d\to 0}\frac{1}{d}\cdot\frac{2\sin\theta-4s\cos\theta}{1+4s^2}-\sum_{t=1}^{d^{-1}}\frac{\sin(s\log(d\cdot t)+\theta)}{\sqrt{d\cdot t}}=\frac{\sin(\theta)}{2}$$

$$\lim_{d\to 0}-\sum_{t=1}^{d^{-1}}\frac{\sin(s\log(d\cdot t)+\arctan 2s)}{\sqrt{d\cdot t}}=\frac{\sin(\arctan 2s)}{2}$$

$$\lim_{d\to 0}\frac{1}{d}\cdot\int_0^1\frac{\sin(s\log(x))}{\sqrt{x}}dx-\sum_{t=1}^{d^{-1}}\frac{\sin(s\log(d\,t))}{\sqrt{dt}}=0$$

$$\lim_{d\to 0}\frac{1}{d}\cdot\int_0^1\frac{\cos(s\log(x))}{\sqrt{x}}dx-\sum_{t=1}^{d^{-1}}\frac{\cos(s\log(d\,t))}{\sqrt{dt}}=\frac{1}{2}$$

...

$$\lim_{d\to 0}\frac{1}{d}\cdot\int_0^1\frac{\cos(s\log(x))+i\sin(s\log(x))}{\sqrt{x}}dx-\sum_{t=1}^{d^{-1}}\frac{\cos(s\log(d\cdot t))+i\sin(s\log(d\cdot t))}{\sqrt{d\cdot t}}=\frac{1}{2}$$

$$\lim_{d\to 0}\frac{1}{d}\cdot\int_0^1\frac{x^{si}}{\sqrt{x}}dx-\sum_{t=1}^{d^{-1}}\frac{(d\cdot t)^{si}}{\sqrt{d\cdot t}}=\frac{1}{2}$$

$$\lim_{d\to 0}\frac{1}{d}\cdot\int_0^1 x^{si-\frac{1}{2}}dx-\sum_{t=1}^{d^{-1}}(d\cdot t)^{si-\frac{1}{2}}=\frac{1}{2}$$

$$\lim_{d\to 0}\frac{1}{d}\cdot\left(\frac{2}{1+2\cdot s\cdot i}\right)-\sum_{t=1}^{d^{-1}}(d\cdot t)^{si-\frac{1}{2}}=\frac{1}{2}$$

… reindex.

$$\lim_{d\to 0}\frac{1}{d}\cdot\int_0^1 x^{-s}dx-\sum_{t=1}^{d^{-1}}(d\cdot t)^{-s}=\frac{1}{2}$$

Then a sequence of fairly obvious transformations

$$\lim_{d\to 0}\frac{1}{d}\cdot\int_0^1 x^{-s}dx-\sum_{t=1}^{d^{-1}}(d\cdot t)^{-s}=\frac{1}{2}$$

$$\lim_{n\to\infty}n\cdot\int_0^1 x^{-s}dx-\sum_{t=1}^{n}\left(\frac{t}{n}\right)^{-s}=\frac{1}{2}$$

$$\lim_{n\to\infty}n\cdot\int_0^1 x^{-s}dx-n^{s}\cdot\sum_{t=1}^{n}t^{-s}=\frac{1}{2}$$

$$\lim_{n\to\infty}n^{1-s}\cdot\int_0^1 x^{-s}dx-\sum_{t=1}^{n}t^{-s}-\frac{1}{2}\cdot n^{-s}=0$$

$$\lim_{n\to\infty} -\frac{8s}{1+4s^2}n+2\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\sin\left(s\log\frac{n}{j}\right)+i\,n^{\frac{1}{2}+si}\cdot\zeta\left(\frac{1}{2}+si\right)-i\,n^{\frac{1}{2}-si}\cdot\zeta\left(\frac{1}{2}-si\right)=0$$

…and...

$$\lim_{n\to\infty} -\frac{4}{1+4s^2}n+2\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\cos\left(s\log\frac{n}{j}\right)-n^{\frac{1}{2}+si}\cdot\zeta\left(\frac{1}{2}+si\right)-n^{\frac{1}{2}-si}\cdot\zeta\left(\frac{1}{2}-si\right)=1$$

…

AND!

$$\boxed{\lim_{n\to\infty} 2\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\sin\left(s\log\frac{n}{j}-\tan^{-1}2s\right)+i\left(e^{-i\tan^{-1}2s}n^{\frac{1}{2}+si}\cdot\zeta\left(\frac{1}{2}+si\right)-e^{i\tan^{-1}2s}n^{\frac{1}{2}-si}\cdot\zeta\left(\frac{1}{2}-si\right)\right)=-\sin\left(\tan^{-1}2s\right)}$$

which is

$$\boxed{\lim_{d\to 0} -2\sum_{t=1}^{d^{-1}}(d\cdot t)^{-\frac{1}{2}}\sin\left(s\log(d\cdot t)-\tan^{-1}2s\right)+i\left(e^{-i\tan^{-1}2s}\cdot d^{-\frac{1}{2}-si}\cdot\zeta\left(-\left(-\frac{1}{2}-si\right)\right)-e^{i\tan^{-1}2s}\cdot d^{-\frac{1}{2}+si}\cdot\zeta\left(-\left(-\frac{1}{2}+si\right)\right)\right)=-\sin\left(\tan^{-1}2s\right)}$$

and

$$\boxed{\lim_{n\to\infty} 2\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\sin\left(s\log\frac{n}{j}-\tan^{-1}2s\right)+i\left(e^{-i\tan^{-1}2s}n^{\frac{1}{2}+si}\cdot\zeta\left(\frac{1}{2}+si\right)-e^{i\tan^{-1}2s}n^{\frac{1}{2}-si}\cdot\zeta\left(\frac{1}{2}-si\right)\right)=-\sin\left(\tan^{-1}2s\right)}$$

which is also except where convergence is a problem.

$$\boxed{\lim_{n\to\infty} 2\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\sin\left(s\log\frac{n}{j}-\frac{1}{2}i\log\left(\frac{\frac{1}{2}-is}{\frac{1}{2}+is}\right)\right)+i\left(\frac{\sqrt{\frac{1}{2}-is}}{\sqrt{\frac{1}{2}+is}}n^{\frac{1}{2}+si}\cdot\zeta\left(\frac{1}{2}+si\right)-\frac{\sqrt{\frac{1}{2}+is}}{\sqrt{\frac{1}{2}-is}}n^{\frac{1}{2}-si}\cdot\zeta\left(\frac{1}{2}-si\right)\right)=-\frac{s}{\sqrt{\left(\frac{1}{2}-is\right)\left(\frac{1}{2}+is\right)}}}$$

generalize -

$$\boxed{\lim_{n\to\infty} \frac{4\sin\theta-8s\cos\theta}{1+4s^2}n+2\sum_{j=1}^{n}\left(\frac{n}{j}\right)^{\frac{1}{2}}\sin\left(s\log\frac{n}{j}-\theta\right)+i\left(e^{-i\theta}n^{\frac{1}{2}+si}\cdot\zeta\left(\frac{1}{2}+si\right)-e^{i\theta}n^{\frac{1}{2}-si}\cdot\zeta\left(\frac{1}{2}-si\right)\right)=-\sin(\theta)}$$

which is

$$\boxed{\lim_{d\to 0} \frac{2\sin\theta-4s\cos\theta}{1+4s^2}\cdot d^{-1}-\sum_{t=1}^{d^{-1}}\frac{\sin\left(s\log(d\cdot t)+\theta\right)}{\sqrt{d\cdot t}}+\frac{1}{2}\cdot i\left(e^{-i\theta}\cdot d^{-\frac{1}{2}-si}\cdot\zeta\left(\frac{1}{2}+si\right)-e^{i\theta}\cdot d^{-\frac{1}{2}+si}\cdot\zeta\left(\frac{1}{2}-si\right)\right)-\frac{\sin(\theta)}{2}=0}$$

$$\boxed{\lim_{d\to 0} \left(\int_0^1\frac{\sin\left(s\log x+\theta\right)}{\sqrt{x}}dx\right)\cdot d^{-1}-\sum_{t=1}^{d^{-1}}\frac{\sin\left(s\log(d\cdot t)+\theta\right)}{\sqrt{d\cdot t}}+\frac{1}{2}\cdot i\left(e^{-i\theta}\cdot d^{-\frac{1}{2}-si}\cdot\zeta\left(\frac{1}{2}+si\right)-e^{i\theta}\cdot d^{-\frac{1}{2}+si}\cdot\zeta\left(\frac{1}{2}-si\right)\right)-\frac{\sin(\theta)}{2}=0}$$

Once again...

$$\lim_{d\to 0} \frac{4\sin\theta - 8s\cos\theta}{1+4s^2} d^{-1} + -2\sum_{t=1}^{d^{-1}} (d\cdot t)^{-\frac{1}{2}} \sin(s\log(d\cdot t)-\theta) + i(e^{-i\theta}\cdot d^{-\frac{1}{2}-si}\cdot\zeta(\frac{1}{2}+si) - e^{i\theta}\cdot d^{-\frac{1}{2}+si}\cdot\zeta(\frac{1}{2}-si)) = -\sin(\theta)$$

Revisit that special case

$$\lim_{d\to 0} -2\sum_{t=1}^{d^{-1}} (d\cdot t)^{-\frac{1}{2}} \sin(s\log(d\cdot t)-\tan^{-1}2s) + i(e^{-i\tan^{-1}2s}\cdot d^{-\frac{1}{2}-si}\cdot\zeta(-(-\frac{1}{2}-si)) - e^{i\tan^{-1}2s}\cdot d^{-\frac{1}{2}+si}\cdot\zeta(-(-\frac{1}{2}+si))) = -\sin(\tan^{-1}2s)$$

which means that, for zeta to be 0 for some value of -1/2+s i, that the following must be true

$$\lim_{d\to 0} -2\sum_{t=1}^{d^{-1}} (d\cdot t)^{-\frac{1}{2}} \sin(s\log(d\cdot t)-\tan^{-1}2s) = -\sin(\tan^{-1}2s)$$

which, divided, gets right back to

$$\lim_{d\to 0} \sum_{t=1}^{d^{-1}} (d\cdot t)^{-\frac{1}{2}} (\cos(s\log(d\,t)) - \frac{1}{2s}\sin(s\log(d\cdot t))) = \frac{1}{2}$$

Empirically, this doesn't converge, at all, when s has an imaginary component. It appears, in fact, that the center of the absolute value of the wave is a horizontal line if s is purely real, and it is a line with an increasing value if s has an imaginary component. If the center part is thusly increasing, there is certainly no way for the function to converge to sin(tan^-1 2s).

$$\lim_{n\to\infty} -n+(\frac{1}{2}+s)(\sum_{j=1}^{n}(\frac{n}{j})^{\frac{1}{2}-s}-n^{\frac{1}{2}-s}\cdot\zeta(\frac{1}{2}-s))=\frac{\frac{1}{2}+s}{2}$$

$$\lim_{n\to\infty} -n+(\frac{1}{2}-s)(\sum_{j=1}^{n}(\frac{n}{j})^{\frac{1}{2}+s}-n^{\frac{1}{2}+s}\cdot\zeta(\frac{1}{2}+s))=\frac{\frac{1}{2}-s}{2}$$

…

$$\lim_{n\to\infty} -n+(\frac{1}{2}+s)(\sum_{j=1}^{n}(\frac{n}{j})^{\frac{1}{2}-s}-n^{\frac{1}{2}-s}\cdot\zeta(\frac{1}{2}-s))=\frac{\frac{1}{2}+s}{2}$$

$$\lim_{d\to 0} -d^{-1}+(\frac{1}{2}-s)(\sum_{t=1}^{d^{-1}}(d\cdot t)^{-\frac{1}{2}-s}-d^{-\frac{1}{2}-s}\cdot\zeta(\frac{1}{2}+s))=\frac{\frac{1}{2}-s}{2}$$

…

What I really need to do – I don't understand the difference between the arctan stuff and the 2s A + B stuff. I need to get that clear in my head. $(2s)^{\frac{1}{2}}$ vs $(\frac{\sqrt{\frac{1}{2}-s}}{\sqrt{\frac{1}{2}+s}})^{\frac{1}{2}}$

…

$$\lim_{d\to 0}\frac{4\sin\theta-8s\cos\theta}{1+4s^2}d^{-1}+-2\sum_{t=1}^{d^{-1}}(d\cdot t)^{-\frac{1}{2}}\sin(s\log(d\cdot t)-\theta)+i(e^{-i\theta}\cdot d^{-\frac{1}{2}-si}\cdot\zeta(\frac{1}{2}+si)-e^{i\theta}\cdot d^{-\frac{1}{2}+si}\cdot\zeta(\frac{1}{2}-si))=-\sin(\theta)$$

$$\lim_{d\to 0}-2\sum_{t=1}^{d^{-1}}(d\cdot t)^{-\frac{1}{2}}\sin(s\log(d\cdot t)-\tan^{-1}2s)+i(e^{-i\tan^{-1}2s}\cdot d^{-\frac{1}{2}-si}\cdot\zeta(-(-\frac{1}{2}-si))-e^{i\tan^{-1}2s}\cdot d^{-\frac{1}{2}+si}\cdot\zeta(-(-\frac{1}{2}+si)))=-\sin(\tan^{-1}2s)$$

$$\lim_{n\to\infty}-\frac{8s}{1+4s^2}n+2\sum_{j=1}^{n}(\frac{n}{j})^{\frac{1}{2}}\sin(s\log\frac{n}{j})+in^{\frac{1}{2}+si}\cdot\zeta(\frac{1}{2}+si)-in^{\frac{1}{2}-si}\cdot\zeta(\frac{1}{2}-si)=0$$

$$\lim_{n\to\infty}-\frac{4}{1+4s^2}n+2\sum_{j=1}^{n}(\frac{n}{j})^{\frac{1}{2}}\cos(s\log\frac{n}{j})-n^{\frac{1}{2}+si}\cdot\zeta(\frac{1}{2}+si)-n^{\frac{1}{2}-si}\cdot\zeta(\frac{1}{2}-si)=1$$

vs

$$\lim_{n\to\infty}(\frac{1}{2}+s)(n^{\frac{1}{2}-s}\cdot\zeta(\frac{1}{2}-s)-\sum_{j=1}^{n}(\frac{n}{j})^{\frac{1}{2}-s})-(\frac{1}{2}-s)(n^{\frac{1}{2}+s}\cdot\zeta(\frac{1}{2}+s)-\sum_{j=1}^{n}(\frac{n}{j})^{\frac{1}{2}+s})=-s$$

(but see!)

$$\lim_{n\to\infty}2\sum_{j=1}^{n}(\frac{n}{j})^{\frac{1}{2}}\sin(s\log\frac{n}{j}-\frac{1}{2}i\log(\frac{\frac{1}{2}-is}{\frac{1}{2}+is}))+i(\frac{\sqrt{\frac{1}{2}-is}}{\sqrt{\frac{1}{2}+is}}n^{\frac{1}{2}+si}\cdot\zeta(\frac{1}{2}+si)-\frac{\sqrt{\frac{1}{2}+is}}{\sqrt{\frac{1}{2}-is}}n^{\frac{1}{2}-si}\cdot\zeta(\frac{1}{2}-si))=-\frac{s}{\sqrt{(\frac{1}{2}-is)(\frac{1}{2}+is)}}$$

$$\lim_{n\to\infty}-2n+(\frac{1}{2}+s)(n^{\frac{1}{2}-s}\cdot\zeta(\frac{1}{2}-s)-\sum_{j=1}^{n}(\frac{n}{j})^{\frac{1}{2}-s})+(\frac{1}{2}-s)(n^{\frac{1}{2}+s}\cdot\zeta(\frac{1}{2}+s)-\sum_{j=1}^{n}(\frac{n}{j})^{\frac{1}{2}+s})=-\frac{1}{2}$$

$$\zeta(-s)=\lim_{n\to\infty}\sum_{j=1}^{n} j^{s}-\frac{n^{1+s}}{1+s}-\frac{n^{s}}{2}$$

$$\zeta\left(\frac{1}{2}-s\,i\right)=\lim_{n\to\infty}\sum_{j=1}^{n} j^{-\frac{1}{2}+s\,i}-\frac{n^{\frac{1}{2}-s\,i}}{\frac{1}{2}-s\,i}-\frac{n^{-\frac{1}{2}+s\,i}}{2}$$

$$\zeta\left(\frac{1}{2}+s\,i\right)=\lim_{n\to\infty}\sum_{j=1}^{n} j^{-\frac{1}{2}-s\,i}-\frac{n^{\frac{1}{2}+s\,i}}{\frac{1}{2}+s\,i}-\frac{n^{-\frac{1}{2}-s\,i}}{2}$$

…

$$\zeta\left(\frac{1}{2}-s\,i\right)-\zeta\left(\frac{1}{2}+s\,i\right)=\lim_{n\to\infty}\sum_{j=1}^{n} j^{-\frac{1}{2}+s\,i}-\sum_{j=1}^{n} j^{-\frac{1}{2}-s\,i}-\frac{n^{\frac{1}{2}-s\,i}}{\frac{1}{2}-s\,i}+\frac{n^{\frac{1}{2}+s\,i}}{\frac{1}{2}+s\,i}-\frac{n^{-\frac{1}{2}+s\,i}}{2}+\frac{n^{-\frac{1}{2}-s\,i}}{2}$$

$$\zeta\left(\frac{1}{2}-s\,i\right)-\zeta\left(\frac{1}{2}+s\,i\right)=\lim_{n\to\infty}\sum_{j=1}^{n} j^{-\frac{1}{2}}\left(j^{s\,i}-j^{-s\,i}\right)-\left(\frac{n^{\frac{1}{2}-s\,i}}{\frac{1}{2}-s\,i}-\frac{n^{\frac{1}{2}+s\,i}}{\frac{1}{2}+s\,i}\right)-\left(\frac{n^{-\frac{1}{2}+s\,i}}{2}-\frac{n^{-\frac{1}{2}-s\,i}}{2}\right)$$

$$\zeta\left(\frac{1}{2}-s\,i\right)-\zeta\left(\frac{1}{2}+s\,i\right)=\lim_{n\to\infty}\sum_{j=1}^{n} j^{-\frac{1}{2}}\left(j^{s\,i}-j^{-s\,i}\right)-n^{\frac{1}{2}}\left(\frac{n^{-s\,i}}{\frac{1}{2}-s\,i}-\frac{n^{s\,i}}{\frac{1}{2}+s\,i}\right)-\left(\frac{n^{-\frac{1}{2}+s\,i}}{2}-\frac{n^{-\frac{1}{2}-s\,i}}{2}\right)$$

$$\zeta\left(\frac{1}{2}-s\,i\right)-\zeta\left(\frac{1}{2}+s\,i\right)=\lim_{n\to\infty}\sum_{j=1}^{n} j^{-\frac{1}{2}}\left(j^{s\,i}-j^{-s\,i}\right)-n^{\frac{1}{2}}\left(\frac{n^{-s\,i}}{\frac{1}{2}-s\,i}-\frac{n^{s\,i}}{\frac{1}{2}+s\,i}\right)-\frac{n^{-\frac{1}{2}}}{2}\left(n^{s\,i}-n^{-s\,i}\right)$$

$$\zeta\left(\frac{1}{2}-s\,i\right)-\zeta\left(\frac{1}{2}+s\,i\right)=\lim_{n\to\infty} 2\,i\sum_{j=1}^{n}\frac{\sin(s\log j)}{\sqrt{j}}-n^{\frac{1}{2}}\left(\frac{n^{-s\,i}}{\frac{1}{2}-s\,i}-\frac{n^{s\,i}}{\frac{1}{2}+s\,i}\right)-i\,\frac{\sin(n\log j)}{\sqrt{n}}$$

got signs and some constants messed up in here somewhere… :/ fixed in post!

$$\zeta\left(\frac{1}{2}-s\,i\right)-\zeta\left(\frac{1}{2}+s\,i\right)=\lim_{n\to\infty} 2\,i\sum_{j=1}^{n}\frac{\sin(s\log j)}{\sqrt{j}}-4\,i\,n^{\frac{1}{2}}\cdot\cos(\arctan 2\,s)\cdot\sin(s\log n+\arctan 2\,s)-i\,\frac{\sin(n\log j)}{\sqrt{n}}$$

$$\boxed{\frac{1}{2\,i}\cdot\left(\zeta\left(\frac{1}{2}-s\,i\right)-\zeta\left(\frac{1}{2}+s\,i\right)\right)=\lim_{n\to\infty}\sum_{j=1}^{n}\frac{\sin(s\log j)}{\sqrt{j}}-2\sqrt{n}\cdot\cos(\arctan 2\,s)\cdot\sin(s\log n-\arctan 2\,s)-\frac{\sin(s\log n)}{2\sqrt{n}}}$$

…

$$\zeta\left(\frac{1}{2}-s\,i\right)+\zeta\left(\frac{1}{2}+s\,i\right)=\lim_{n\to\infty}\sum_{j=1}^{n} j^{-\frac{1}{2}}\left(j^{s\,i}+j^{-s\,i}\right)-\left(\frac{n^{\frac{1}{2}-s\,i}}{\frac{1}{2}-s\,i}+\frac{n^{\frac{1}{2}+s\,i}}{\frac{1}{2}+s\,i}\right)-\left(\frac{n^{-\frac{1}{2}+s\,i}}{2}+\frac{n^{-\frac{1}{2}-s\,i}}{2}\right)$$

$$\zeta\left(\frac{1}{2}-s\,i\right)+\zeta\left(\frac{1}{2}+s\,i\right)=\lim_{n\to\infty} 2\sum_{j=1}^{n}\frac{\cos(s\log j)}{\sqrt{j}}-n^{\frac{1}{2}}\left(\frac{n^{-s\,i}}{\frac{1}{2}-s\,i}+\frac{n^{s\,i}}{\frac{1}{2}+s\,i}\right)-\frac{\cos(n\log j)}{\sqrt{n}}$$

$$\boxed{\frac{1}{2}\cdot\left(\zeta\left(\frac{1}{2}-s\,i\right)+\zeta\left(\frac{1}{2}+s\,i\right)\right)=\lim_{n\to\infty}\sum_{j=1}^{n}\frac{\cos(s\log j)}{\sqrt{j}}-2\sqrt{n}\cdot\cos(\arctan 2\,s)\cdot\cos(s\log n-\arctan 2\,s)-\frac{\cos(s\log n)}{2\sqrt{n}}}$$

And now! Generalize!

$$\cos(\theta)(\zeta(\frac{1}{2}-si)+\zeta(\frac{1}{2}+si))+i\sin(\theta)(\zeta(\frac{1}{2}-si)-\zeta(\frac{1}{2}+si))=$$
$$\lim_{n\to\infty} 2\sum_{j=1}^{n}\frac{\cos(s\log j+\theta)}{\sqrt{j}}-4\sqrt{n}\cdot\cos(\arctan 2s)\cdot\cos(s\log n+\theta-\arctan 2s)-\frac{\cos(s\log n+\theta)}{\sqrt{n}}$$

[FILL IN THE NEXT TWO]

IF $\theta=\arctan 2s$, with some simplification, this is

$$\cos(\theta)(\zeta(\frac{1}{2}-si)+\zeta(\frac{1}{2}+si))+i\sin(\theta)(\zeta(\frac{1}{2}-si)-\zeta(\frac{1}{2}+si))=$$
$$\lim_{n\to\infty} 2\sum_{j=1}^{n}\frac{\cos(s\log j+\theta)}{\sqrt{j}}-4\sqrt{n}\cdot\cos(\arctan 2s)\cdot\cos(s\log n+\theta-\arctan 2s)-\frac{\cos(s\log n+\theta)}{\sqrt{n}}$$

IF $\theta=\arctan 2s-s\log n+\frac{\pi}{2}$, this is

$$\sin(\arctan 2s-s\log n)(\zeta(\frac{1}{2}-si)+\zeta(\frac{1}{2}+si))-i\cos(\arctan 2s-s\log n)(\zeta(\frac{1}{2}-si)-\zeta(\frac{1}{2}+si))=$$
$$\lim_{n\to\infty} -2\sum_{j=1}^{n}\frac{1}{\sqrt{j}}\sin(s\log\frac{j}{n}+\arctan 2s)-\frac{2s}{\sqrt{n}\cdot\sqrt{1+4s^2}}$$

and… what about the earlier 2n thing?

NOTE NOTE NOTE – this only converges when the imaginary part of s is small enough – say -1 i < im(s) < 1 i

Slightly different? Not really.

$$\boxed{\begin{aligned}&e^{(\theta+\arctan 2s)i}\cdot\zeta\left(\frac{1}{2}-si\right)+e^{-(\theta+\arctan 2s)i}\cdot\zeta\left(\frac{1}{2}+si\right)=\\&\qquad\lim_{n\to\infty}2\sum_{j=1}^{n}\frac{\cos(s\log j+\theta+\arctan 2s)}{\sqrt{j}}-4\sqrt{n}\cdot\cos(\arctan 2s)\cdot\cos(s\log n+\theta)-\frac{\cos(s\log n+\theta+\arctan 2s)}{\sqrt{n}}\end{aligned}}$$

IF $\theta=-s\log n$, and divide by 2 cos(arctan 2s), this is

$$\boxed{\left(\frac{1}{2}+si\right)\cdot n^{-si}\cdot\zeta\left(\frac{1}{2}-si\right)+\left(\frac{1}{2}-si\right)\cdot n^{si}\cdot\zeta\left(\frac{1}{2}+si\right)=\lim_{n\to\infty}2\cdot\sqrt{\frac{1}{2}+s^2}\cdot\sum_{j=1}^{n}\frac{1}{\sqrt{j}}\cdot\cos\left(s\log\frac{j}{n}+\arctan 2s\right)-2\sqrt{n}-\frac{1}{2\sqrt{n}}}$$

IF $\theta=-s\log n-\frac{\pi}{2}$, and divide by 2 cos(arctan 2s), this is

$$\boxed{-i\cdot\left(\frac{1}{2}+si\right)\cdot n^{-si}\cdot\zeta\left(\frac{1}{2}-si\right)+i\left(\frac{1}{2}-si\right)\cdot n^{si}\cdot\zeta\left(\frac{1}{2}+si\right)=\lim_{n\to\infty}2\cdot\sqrt{\frac{1}{2}+s^2}\cdot\sum_{j=1}^{n}\frac{1}{\sqrt{j}}\cdot\sin\left(s\log\frac{j}{n}+\arctan 2s\right)-\frac{s}{\sqrt{n}}}$$

.....

$$\xi(s)=\frac{1}{2}\cdot s(s-1)\pi^{(-s/2)}\Gamma(\frac{1}{2}\cdot s)\cdot\zeta(s)$$

$$\frac{2\pi^{(s/2)}}{s(s-1)\cdot\Gamma(\frac{1}{2}\cdot s)}\xi(s)=\zeta(s)$$

$$\frac{2\pi^{\frac{s}{2}+\frac{1}{4}}}{(s+\frac{1}{2})(s-\frac{1}{2})\cdot\Gamma(\frac{1}{4}\cdot\frac{s}{2})}\xi(\frac{1}{2}+s)=\zeta(\frac{1}{2}+s)$$

$$\frac{2\pi^{-\frac{s}{2}+\frac{1}{4}}}{(-s+\frac{1}{2})(-s-\frac{1}{2})\cdot\Gamma(\frac{1}{4}\cdot-\frac{s}{2})}\xi(\frac{1}{2}-s)=\zeta(\frac{1}{2}-s)$$

....

for s to be a zeta zero, this must be true:

$$\lim_{d\to 0}\frac{1}{d}\cdot\int_0^1\frac{\cos(s\log(x))}{\sqrt{x}}dx-\sum_{t=1}^{d^{-1}}\frac{\cos(s\log(d\,t))}{\sqrt{dt}}=\frac{1}{2}$$

$$\lim_{d\to 0}\frac{1}{d}\cdot\frac{2}{1+4s^2}-\sum_{t=1}^{d^{-1}}\frac{\cos(s\log(d\cdot t))}{\sqrt{d\cdot t}}=\frac{1}{2}$$

$$\lim_{d\to 0}\frac{1}{d}\cdot\frac{2}{1+4(f+Ai)^2}-\sum_{t=1}^{d^{-1}}\frac{\cos((f+Ai)\log(d\cdot t))}{\sqrt{d\cdot t}}=\frac{1}{2}$$

$$\lim_{d\to 0}\frac{1}{d}\cdot\frac{2}{1+4(f+Ai)^2}-\sum_{t=1}^{d^{-1}}\frac{\cos(f\log(d\cdot t)+i\,A\log(d\cdot t))}{\sqrt{d\cdot t}}=\frac{1}{2}$$

$$\lim_{d\to 0}\frac{1}{d}\cdot\frac{2}{1+4(f+Ai)^2}-\sum_{t=1}^{d^{-1}}\frac{\cos(f\log(d\cdot t))\cosh(A\log(d\cdot t))}{\sqrt{d\cdot t}}-i\cdot\frac{\sin(f\log(d\cdot t))\sinh(A\log(d\cdot t))}{\sqrt{d\cdot t}}=\frac{1}{2}$$

$$\lim_{d\to 0}\frac{1}{d}\cdot\frac{2}{1+4(f+Ai)^2}-\sum_{t=1}^{d^{-1}}\frac{\cos(f\log(d\cdot t))\cosh(A\log(d\cdot t))}{\sqrt{d\cdot t}}+i\cdot\sum_{t=1}^{d^{-1}}\frac{\sin(f\log(d\cdot t))\sinh(A\log(d\cdot t))}{\sqrt{d\cdot t}}=\frac{1}{2}$$

$$\lim_{d\to 0}\frac{1}{d}\cdot\frac{2}{1+4(f+Ai)^2}-\sum_{t=1}^{d^{-1}}\frac{\cos(f\log(d\cdot t))\cosh(A\log(d\cdot t))}{\sqrt{d\cdot t}}+i\cdot\sum_{t=1}^{d^{-1}}\frac{\sin(f\log(d\cdot t))\sinh(A\log(d\cdot t))}{\sqrt{d\cdot t}}=\frac{1}{2}$$

$$\lim_{d\to 0}\frac{1}{d}\cdot\frac{2}{1+4(f+Ai)^2}-\sum_{t=1}^{d^{-1}}\frac{\cos(f\log(d\cdot t))\cosh(A\log(d\cdot t))}{\sqrt{d\cdot t}}+i\cdot\sum_{t=1}^{d^{-1}}\frac{\sin(f\log(d\cdot t))\sinh(A\log(d\cdot t))}{\sqrt{d\cdot t}}=\frac{1}{2}$$

$$\lim_{d\to 0}\frac{1}{d}\cdot\frac{2}{1+4(f+Ai)^2}-\sum_{t=1}^{d^{-1}}\frac{\cos(f\log(d\cdot t))\cosh(A\log(d\cdot t))}{\sqrt{d\cdot t}}+i\cdot\sum_{t=1}^{d^{-1}}\frac{\sin(f\log(d\cdot t))\sinh(A\log(d\cdot t))}{\sqrt{d\cdot t}}=\frac{1}{2}$$

$$\lim_{d\to 0}\frac{1}{d}\cdot\frac{2+8f-8A^2}{((1+4f^2)+4A^2)^2-16A^2}-\sum_{t=1}^{d^{-1}}\frac{\cos(f\log(d\cdot t))\cosh(A\log(d\cdot t))}{\sqrt{d\cdot t}}=\frac{1}{2}$$

AND

$$\lim_{d\to 0}\frac{1}{d}\cdot\frac{16Af}{((1+4f^2)+4A^2)^2-16A^2}+\sum_{t=1}^{d^{-1}}\frac{\sin(f\log(d\cdot t))\sinh(A\log(d\cdot t))}{\sqrt{d\cdot t}}=0$$

and if A is 0, this simplifies to

$$\lim_{d\to 0}\frac{1}{d}\cdot\frac{2}{1+4f^2}-\sum_{t=1}^{d^{-1}}\frac{\cos(f\log(d\cdot t))}{\sqrt{d\cdot t}}=\frac{1}{2}$$

$$\frac{1}{2i}\cdot\left(\zeta\left(\frac{1}{2}-si\right)-\zeta\left(\frac{1}{2}+si\right)\right)=\lim_{n\to\infty}\sum_{j=1}^{n}\frac{\sin(s\log j)}{\sqrt{j}}-2\sqrt{n}\cdot\cos(\arctan 2s)\cdot\sin(s\log n-\arctan 2s)-\frac{\sin(s\log n)}{2\sqrt{n}}$$

$$\frac{1}{2}\cdot\left(\zeta\left(\frac{1}{2}-si\right)+\zeta\left(\frac{1}{2}+si\right)\right)=\lim_{n\to\infty}\sum_{j=1}^{n}\frac{\cos(s\log j)}{\sqrt{j}}-2\sqrt{n}\cdot\cos(\arctan 2s)\cdot\cos(s\log n-\arctan 2s)-\frac{\cos(s\log n)}{2\sqrt{n}}$$

$$\cos(\theta)\left(\zeta\left(\frac{1}{2}-si\right)+\zeta\left(\frac{1}{2}+si\right)\right)+i\sin(\theta)\left(\zeta\left(\frac{1}{2}-si\right)-\zeta\left(\frac{1}{2}+si\right)\right)=$$
$$\lim_{n\to\infty}2\sum_{j=1}^{n}\frac{\cos(s\log j+\theta)}{\sqrt{j}}-4\sqrt{n}\cdot\cos(\arctan 2s)\cdot\cos(s\log n+\theta-\arctan 2s)-\frac{\cos(s\log n+\theta)}{\sqrt{n}}$$

…

$$Ez^{si}=\zeta\left(\frac{1}{2}+si\right)$$

$$Ez^{-si}=\zeta\left(\frac{1}{2}-si\right)$$

$$Cz(s)=\frac{1}{2}\left(Ez^{si}+Ez^{-si}\right)$$

$$Sz(s)=\frac{1}{2i}\left(Ez^{si}-Ez^{-si}\right)$$

…

$$Cz(s)=\lim_{n\to\infty}\sum_{j=1}^{n}\frac{\cos(s\log j)}{\sqrt{j}}-2\sqrt{n}\cdot\cos(\arctan 2s)\cdot\cos(s\log n-\arctan 2s)-\frac{\cos(s\log n)}{2\sqrt{n}}$$

$$Sz(s)=\lim_{n\to\infty}\sum_{j=1}^{n}\frac{\sin(s\log j)}{\sqrt{j}}-2\sqrt{n}\cdot\cos(\arctan 2s)\cdot\sin(s\log n-\arctan 2s)-\frac{\sin(s\log n)}{2\sqrt{n}}$$

$$\cos(\theta)\,Cz(s)-i\sin(\theta)\,Sz(s)=\lim_{n\to\infty}\sum_{j=1}^{n}\frac{\cos(s\log j+\theta)}{\sqrt{j}}-2\sqrt{n}\cdot\cos(\arctan 2s)\cdot\cos(s\log n+\theta-\arctan 2s)-\frac{\cos(s\log n+\theta)}{2\sqrt{n}}$$

…

$$Cz(s)+i\,Sz(s)=Ez^{si}$$

$$Cz(s)-i\,Sz(s)=Ez^{-si}$$

$$\frac{Cz(s)+i\,Sz(s)}{Cz(s)-i\,Sz(s)}=\frac{Ez^{si}}{Ez^{-si}}$$

…

$$Cz(s)=Cz(-s)$$

$$Sz(s)=-Sz(-s)$$

OH HO!

$$\boxed{Ez(z)=\lim_{n\to\infty}\sum_{j=1}^{n}\frac{j^z}{\sqrt{j}}-\int_0^n\frac{x^z}{\sqrt{x}}dx}$$

$$\boxed{Cz(z)=\lim_{n\to\infty}\sum_{j=1}^{n}\frac{\cos(z\log j)}{\sqrt{j}}-\int_0^n\frac{\cos(z\log x)}{\sqrt{x}}dx}$$

$$\boxed{Sz(z)=\lim_{n\to\infty}\sum_{j=1}^{n}\frac{\sin(z\log j)}{\sqrt{j}}-\int_0^n\frac{\sin(z\log x)}{\sqrt{x}}dx}$$

$$\boxed{Cz(z)=\lim_{n\to\infty}\sum_{j=1}^{n}\frac{\cos(z\log j+\theta)}{\sqrt{j}}-\int_0^n\frac{\cos(z\log x+\theta)}{\sqrt{x}}dx}$$

...

$$\boxed{Z^k=\lim_{n\to\infty}\sum_{j=2}^{n}\frac{\log^k j}{\sqrt{j}}-\int_1^n\frac{\log^k x}{\sqrt{x}}dx}$$

$$\boxed{Z^k=\lim_{z\to 0}\frac{\partial^k}{\partial z^k}(\zeta(\frac{1}{2}-z)-1+\frac{1}{\frac{1}{2}+z})}$$

...

$$\boxed{Ez(z)=1-\frac{1}{\frac{1}{2}+z}+\sum_{k=0}^{\infty}\frac{z^k}{k!}\cdot Z^k}$$

$$\boxed{Cz(z)=1-\frac{2}{1+4z^2}+\sum_{k=0}^{\infty}(-1)^k\cdot\frac{z^{2k}}{(2k)!}\cdot Z^{2k}}$$

$$\boxed{Sz(z)=\frac{4z}{1+4z^2}+\sum_{k=0}^{\infty}(-1)^k\cdot\frac{z^{2k+1}}{(2k+1)!}\cdot Z^{2k+1}}$$

$$\boxed{SCz(z,\theta)=\cos\theta-\frac{2(\cos\theta+2z\sin\theta)}{1+4z^2}+\sum_{k=0}^{\infty}\cos(\theta+\frac{\pi k}{2})\cdot\frac{z^k}{k!}\cdot Z^k}$$

...

$$\boxed{E_z(z)=\zeta(\frac{1}{2}-z)}$$

$$\boxed{Cz(z)=\frac{1}{2}\cdot(\zeta(\frac{1}{2}-zi)+\zeta(\frac{1}{2}+zi))}$$

$$\boxed{Sz(z)=\frac{1}{2i}\cdot(\zeta(\frac{1}{2}-zi)-\zeta(\frac{1}{2}+zi))}$$

$$\boxed{SCz(z,\theta)=\frac{1}{2}(e^{\theta i}\cdot\zeta(\frac{1}{2}-si)+e^{-\theta i}\cdot\zeta(\frac{1}{2}+si))}$$

…

$$\boxed{Ez(z)=Cz(z)+i\,Sz(z)}$$

$$\boxed{Cz(z)=\frac{1}{2}\cdot(Ez(z)+Ez(-z))}$$

$$\boxed{Sz(z)=\frac{1}{2i}\cdot(Ez(z)-Ez(-z))}$$

$$\cos(\theta)((a+bi)+(a-bi))+i\sin(\theta)((a+bi)-(a-bi))$$

$$\cos(\theta)(a)+i\sin(\theta)(bi)$$

$$\cos(\theta)\cdot a-\sin(\theta)\cdot b$$