```
zeta[n_, s_, z_, k_] := 1 + ((z + 1) / k - 1) Sum[j^-s zeta[n / j, s, z, k + 1], {j, 2, n}]

Expand[D[zeta[100, 0, z, 1], {z, 0}]] /. z → 0
```

1

```
N[Expand[D[zeta[100, 0, z, 1], {z, 1}]]]
```

$28.5333 + 90.4944\,z + 62.0625\,z^2 + 16.9722\,z^3 + 1.39583\,z^4 + 0.0583333\,z^5$

```
zeros[n_, s_, d_] := List @@ Roots[D[zeta[n, s, z, 1], {z, d}] == 0, z][[All, 2]]

N[Sum[-1 / j, {j, zeros[100, 0, 1]}]] * N[Sum[-1 / j, {j, zeros[100, 0, 0]}]]
```

$90.4944 + 4.40139 \times 10^{-17}\,i$

```
N[Sum[-1 / j, {j, zeros[100, 0, 0]}] * Sum[-1 / j, {j, zeros[100, 0, 1]}]]
```

$90.4944 + 4.40139 \times 10^{-17}\,i$

```
90.49444444444444` / 3.1715342679127714`
```

28.5333

```
N[Sum[-1 / j, {j, zeros[100, 0, 0]}] *
   Sum[-1 / j, {j, zeros[100, 0, 1]}] * Sum[-1 / j, {j, zeros[100, 0, 2]}]] / 2
```

$62.0625 - 3.10947 \times 10^{-15}\,i$

```
N[Sum[-1 / j, {j, zeros[100, 0, 0]}] * Sum[-1 / j, {j, zeros[100, 0, 1]}] *
   Sum[-1 / j, {j, zeros[100, 0, 2]}] * Sum[-1 / j, {j, zeros[100, 0, 3]}]] / 6
```

$16.9722 - 8.50347 \times 10^{-16}\,i$

```
N[zeros[10, 0, 0]]
```

$\{-0.218507 + 3.55271 \times 10^{-15}\,i,\ -1.41809 - 3.55271 \times 10^{-15}\,i,\ -19.3634 + 1.66533 \times 10^{-16}\,i\}$

```
N[Product[1 - 1 / j, {j, zeros[10, 0, 0]}]]
```

$10. + 1.23078 \times 10^{-13}\,i$

```
N[Product[1 - 3 / j, {j, zeros[10, 0, 0]}]]
```

$53. + 7.13127 \times 10^{-13}\,i$

```
zeta[10, 0, 2, 1]
```

27

```
N[Log[2, 10]]
```

3.32193

```
(1 - 1 / -0.2185067738296822`) * (1 - 1 / -1.4180930358230457`) * (1 - 1 / -19.363400190347274)
```

10.

```
(1 - 2 / -0.2185067738296822`) * (1 - 2 / -1.4180930358230457`) * (1 - 2 / -19.363400190347274)
```

27.

```
(1 - 1 / a) (1 - 1 / b) (1 - 1 / c) = 10
(1 - 2 / a) (1 - 2 / b) (1 - 2 / c) = 27
```

Set::write : Tag Times in $\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)$ is Protected. ≫

```
10
```

Set::write : Tag Times in $\left(1-\frac{2}{a}\right)\left(1-\frac{2}{b}\right)\left(1-\frac{2}{c}\right)$ is Protected. ≫

```
27
```

```
FullSimplify[Expand[(1 - 1 / a) (1 - 1 / b) (1 - 1 / c)]]
```

$$\frac{(-1+a)\ (-1+b)\ (-1+c)}{a\ b\ c}$$

```
FullSimplify[Expand[(1 - 2 / a) (1 - 2 / b) (1 - 2 / c)]]
```

$$\frac{(-2+a)\ (-2+b)\ (-2+c)}{a\ b\ c}$$

```
Solve[(-1 + a) (-1 + b) (-1 + c) / (a b c) == 10]
```

Set::write : Tag Times in $\frac{(-1+a)(-1+b)(-1+c)}{a\,b\,c}$ is Protected. ≫

Solve::naqs : 10 is not a quantified system of equations and inequalities. ≫

```
Solve[10]
```

```
N[zeros[100, 0, 0]]
```

```
{-0.933809, -0.0372047, -11.1997 - 12.3982 i,
 -11.1997 + 12.3982 i, -2.67195 - 1.86184 i, -2.67195 + 1.86184 i}
```

```
N[zeros[100, 0, 1]]
```

```
{-0.425539, -9.59257 - 10.0056 i,
 -9.59257 + 10.0056 i, -2.15895 - 1.14966 i, -2.15895 + 1.14966 i}
```

```
N[zeros[100, 0, 2]]
```

```
{-7.98665 - 7.5895 i, -7.98665 + 7.5895 i, -1.58478 + 0.210915 i, -1.58478 - 0.210915 i}
```

```
N[zeros[100, 0, 3]]
```

```
{-1.58767, -6.38474 - 5.1232 i, -6.38474 + 5.1232 i}
```

```
N[zeros[100, 0, 4]]
```

```
{-4.78571 - 2.48841 i, -4.78571 + 2.48841 i}
```

```
N[zeros[100, 0, 5]]
```

Part::partd : Part specification $\left(z == -\frac{67}{14}\right)$[[All, 2]] is longer than depth of object. ≫

```
{z == -4.78571, All, 2.}
```

```
zm1[n_, s_, k_] := Sum[j^-s zm1[n / j, s, k - 1], {j, 2, n}]; zm1[n_, s_, 0] := UnitStep[n - 1]
ez[n_, s_, z_] := Sum[ z^k / k! zm1[n, s, k], {k, 0, Log[2, n]}];
ez[n_, s_, 0] := UnitStep[n - 1]
ezeros[n_, s_, d_] := List @@ Roots[D[ez[n, s, z], {z, d}] == 0, z][[All, 2]]
```

```
zm1[100, 0, 4]
```

184

```
ez[100, 0, 0]
```

1

```
N[Sum[ -1 / j, {j, ezeros[100, 0, 0]}]]
```

99. + 0. i

```
N[Sum[ -1 / j, {j, ezeros[100, 0, 0]}]] * N[Sum[ -1 / j, {j, ezeros[100, 0, 1]}]]
```

283. + 0. i

```
N[Sum[ -1 / j, {j, ezeros[100, 0, 0]}]] *
 N[Sum[ -1 / j, {j, ezeros[100, 0, 1]}]] * N[Sum[ -1 / j, {j, ezeros[100, 0, 2]}]]
```

324. + 0. i

```
N[Sum[ -1 / j, {j, ezeros[100, 0, 0]}]] * N[Sum[ -1 / j, {j, ezeros[100, 0, 1]}]] *
 N[Sum[ -1 / j, {j, ezeros[100, 0, 2]}]] * N[Sum[ -1 / j, {j, ezeros[100, 0, 3]}]]
```

184. + 0. i

```
N[Sum[ -1 / j, {j, ezeros[100, 0, 0]}]] *
 N[Sum[ -1 / j, {j, ezeros[100, 0, 1]}]] * N[Sum[ -1 / j, {j, ezeros[100, 0, 2]}]] *
 N[Sum[ -1 / j, {j, ezeros[100, 0, 3]}]] * N[Sum[ -1 / j, {j, ezeros[100, 0, 4]}]]
```

51. + 0. i

```
N[ezeros[100, 0, 0]]
```

{-6.53113, -3.7489, -1.06267, -0.0102506, -16.1807 - 11.1281 i, -16.1807 + 11.1281 i}

```
N[ezeros[100, 0, 1]]
```

{-5.43817, -2.47076, -0.461366, -14.0291 - 8.77247 i, -14.0291 + 8.77247 i}

```
N[ezeros[100, 0, 2]]
```

{-11.8598 - 6.3866 i, -11.8598 + 6.3866 i, -4.12772, -1.29553}

```
N[ezeros[100, 0, 3]]
```

{-2.5666, -9.64527 + 3.89511 i, -9.64527 - 3.89511 i}

```
N[ezeros[100, 0, 4]]
```

{-6.57143, -8.}

```
N[Sum[ -1 / j, {j, ezeros[100, 0, 1]}]]
```

2.85859 + 0. i

```
-1 / N[ezeros[100, 0, 3]]
```

{0.38962, 0.0891404 + 0.0359982 i, 0.0891404 - 0.0359982 i}

```
N[Sum[ -1 / j, {j, ezeros[100, 0, 1]}]]
```

2.85859 + 0. i

```
D[ez[100, 0, z], {z, 1}]
```

$$99 + 283\,z + 162\,z^2 + \frac{92\,z^3}{3} + \frac{17\,z^4}{8} + \frac{7\,z^5}{120}$$

$$(1)\ 283\,z + (-1/2)\ 162\,z^2 + (1/3)\ \frac{92\,z^3}{3} + (-1/4)\ \frac{17\,z^4}{8} + (1/5)\ \frac{7\,z^5}{120}$$

$$283\,z - 81\,z^2 + \frac{92\,z^3}{9} - \frac{17\,z^4}{32} + \frac{7\,z^5}{600}$$

```
D[ez[100, 0, z], {z, 0}]
```

$$1 + 99\,z + \frac{283\,z^2}{2} + 54\,z^3 + \frac{23\,z^4}{3} + \frac{17\,z^5}{40} + \frac{7\,z^6}{720}$$

```
N[Sum[-1 / j, {j, zeros[100, 0, 1]}]]
```

$$3.17153 + 0.\,i$$

```
N[zeros[100, 0, 1]]
```

$$\{-0.425539,\ -9.59257 - 10.0056\,i,\ -9.59257 + 10.0056\,i,\ -2.15895 - 1.14966\,i,\ -2.15895 + 1.14966\,i\}$$

```
N[Expand[D[zeta[100, 0, z, 1], {z, 1}] / (428 / 15)]]
```

$$1. + 3.17153\,z + 2.17509\,z^2 + 0.594821\,z^3 + 0.0489194\,z^4 + 0.00204439\,z^5$$