

```

ClearAll["Global`*"]
D3[n_] := Sum[1, {x, 1, n}, {y, 1, Floor[n/x]}, {z, 1, Floor[n/(xy)]]}
D3[100]
1471

d[n_, k_] := d[n, k] = Sum[d[j, k - 1] d[n/j, 1], {j, Divisors[n]}];
d[n_, 1] := d[n, 1] = 1; d[n_, 0] := 0; d[1, 0] := 1
DD[n_, k_] := DD[n, k] = Sum[d[j, k], {j, 1, Floor[n]}]

a1[n_] := -Sum[d[j, 2] FractionalPart[n/j], {j, 1, n}] + n Sum[d[j, 2]/j, {j, 1, n}]
N[a1[100]]
1471.

a0[n_] := Sum[d[j, 2] Floor[n/j], {j, 1, n}]
a0[100]
1471

a01[n_] := Sum[d[j, 2] (n/j - FractionalPart[n/j]), {j, 1, n}]
a01[100]
1471

a01[n_] := Sum[d[j, 2] (n/j), {j, 1, n}] - Sum[d[j, 2] FractionalPart[n/j], {j, 1, n}]
a01[100]
1471

a2[n_] := -Sum[d[j, 2] FractionalPart[n/j], {j, 1, n}] +
  DD[n, 2] + n Integrate[DD[Floor[u], 2]/u^2, {u, 1, n}]
N[a2[100]]
NIntegrate::slwcon :
  Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration
  is 0, highly oscillatory integrand, or WorkingPrecision too small. >>
NIntegrate::ncvb :
  NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in u near {u} = {2.1666}. NIntegrate
  obtained 11.595903097791236` and 0.05806940463647516` for the integral and error estimates. >>
1466.92

SS[n_] := n Integrate[DD[Floor[u], 2]/u^2, {u, 1, n}]
N[SS[10]]
NIntegrate::slwcon :
  Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration
  is 0, highly oscillatory integrand, or WorkingPrecision too small. >>
NIntegrate::ncvb :
  NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in u near {u} = {2.00181}. NIntegrate
  obtained 3.301665218344092` and 0.0021079831626377287` for the integral and error estimates. >>
33.0167

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a3[n_] := -Sum[d[j, 2] FractionalPart[n / j], {j, 1, n}] +
  DD[n, 2] + n Integrate[(1 - 2 Floor[u] + Floor[u] HarmonicNumber[Floor[u]] -
    Sum[FractionalPart[Floor[u] / x], {x, 1, Floor[u]}]) / u^2, {u, 1, n}]
N[a3[100]]
$Aborted

a3a[n_] := -Sum[d[j, 2] FractionalPart[n / j], {j, 1, n}] +
  DD[n, 2] + n Integrate[(1 - 2 Floor[u] + Floor[u] HarmonicNumber[Floor[u]] -
    Sum[FractionalPart[Floor[u] / x], {x, 1, Floor[u]}]) / u^2, {u, 1, n}]

n Integrate[(1 - 2 Floor[u] + Floor[u] HarmonicNumber[Floor[u]]) / u^2, {u, 1, n}]
$Aborted

```