

```

Expand@Integrate[1, {s, 1, x}, {t, 1, x/s}]
ConditionalExpression[1 - x + x Log[x], Re[x] ≥ 0 || x ∈ Reals]
FullSimplify@Integrate[1, {s, 1, x},
  {t, 1, y^(1 - Log[s] / Log[x])}, {u, 1, z^(1 - Log[s] / Log[x] - Log[t] / Log[y])}]
$Aborted
FullSimplify[E^(Log[b] (1 - Log[s] / Log[a]))]

$$b^{1 - \frac{\text{Log}[s]}{\text{Log}[a]}}$$

Integrate[1, {s, 0, 1}]
1
Integrate[1, {s, 0, 2}, {t, 0, 1 (1 - s / 2)}]
1
Integrate[1, {s, 0, 3}, {t, 0, 2 (1 - s / 3)}, {u, 0, 1 (1 - s / 3 - t / 2)}]
1
Integrate[1, {s, 0, 4}, {t, 0, 3 (1 - s / 4)},
  {u, 0, 2 (1 - s / 4 - t / 3)}, {v, 0, 1 (1 - s / 4 - t / 3 - u / 2)}]
1
Integrate[1, {s, 0, 5}, {t, 0, 4 (1 - s / 5)}, {u, 0, 3 (1 - s / 5 - t / 4)},
  {v, 0, 2 (1 - s / 5 - t / 4 - u / 3)}, {w, 0, 1 (1 - s / 5 - t / 4 - u / 3 - v / 2)}]
1
Integrate[1, {s, 0, 6}, {t, 0, 5 (1 - s / 6)},
  {u, 0, 4 (1 - s / 6 - t / 5)}, {v, 0, 3 (1 - s / 6 - t / 5 - u / 4)},
  {w, 0, 2 (1 - s / 6 - t / 5 - u / 4 - v / 3)}, {x, 0, 1 (1 - s / 6 - t / 5 - u / 4 - v / 3 - w / 2)}]
1
Sum[1, {s, 0, 1}]
2
Sum[1, {s, 0, 2}, {t, 0, 1 (1 - s / 2)}]
4
Sum[1, {s, 0, 3}, {t, 0, 2 (1 - s / 3)}, {u, 0, 1 (1 - s / 3 - t / 2)}]
8
Sum[1, {s, 0, 4}, {t, 0, 3 (1 - s / 4)},
  {u, 0, 2 (1 - s / 4 - t / 3)}, {v, 0, 1 (1 - s / 4 - t / 3 - u / 2)}]
17
Sum[1, {s, 0, 5}, {t, 0, 4 (1 - s / 5)}, {u, 0, 3 (1 - s / 5 - t / 4)},
  {v, 0, 2 (1 - s / 5 - t / 4 - u / 3)}, {w, 0, 1 (1 - s / 5 - t / 4 - u / 3 - v / 2)}]
37
Sum[1, {s, 0, 6}, {t, 0, 5 (1 - s / 6)},
  {u, 0, 4 (1 - s / 6 - t / 5)}, {v, 0, 3 (1 - s / 6 - t / 5 - u / 4)},
  {w, 0, 2 (1 - s / 6 - t / 5 - u / 4 - v / 3)}, {x, 0, 1 (1 - s / 6 - t / 5 - u / 4 - v / 3 - w / 2)}]
86

```

```
Sum[1, {s, 0, 7}, {t, 0, 6 (1 - s / 7)}, {u, 0, 5 (1 - s / 7 - t / 6)},
  {v, 0, 4 (1 - s / 7 - t / 6 - u / 5)}, {w, 0, 3 (1 - s / 7 - t / 6 - u / 5 - v / 4)},
  {x, 0, 2 (1 - s / 7 - t / 6 - u / 5 - v / 4 - w / 3)}, {y, 0, 1 (1 - s / 7 - t / 6 - u / 5 - v / 4 - w / 3 - x / 2)}]
```

199

```
(** A212658 Number of multisets {1^k1, 2^k2, ..., n^kn},
ki ≥ 0, with the sum of reciprocals ≤ 1. **)
```

```
Integrate[1, {s, 0, a}, {t, 0, b (1 - s / a)}]
```

$$\frac{a b}{2}$$

```
Integrate[1, {s, 0, a}, {t, 0, b (1 - s / a)}, {u, 0, c (1 - s / a - t / b)}]
```

$$\frac{a b c}{6}$$

```
Integrate[1, {s, 0, a}, {t, 0, b (1 - s / a)},
  {u, 0, c (1 - s / a - t / b)}, {v, 0, d (1 - s / a - t / b - u / c)}]
```

$$\frac{1}{24} a b c d$$

```
FullSimplify@Integrate[1, {s, 1, a}, {t, 1, b^ (1 - Log[s] / Log[a])}]
```

$$\text{ConditionalExpression}\left[\frac{-(-1+b) \log[a] + (-1+a) \log[b]}{\log[a] - \log[b]}, \text{Re}[a] \geq 0 \mid \mid a \notin \text{Reals}\right]$$

```
FullSimplify@Integrate[1, {s, 1, a},
  {t, 1, b^ (1 - Log[s] / Log[a])}, {u, 1, c^ (1 - Log[s] / Log[a] - Log[t] / Log[b])}]
```

$$\text{ConditionalExpression}\left[\frac{\left((-1+a) \log[b] (\log[b] - \log[c]) \log[c] + \log[a]^2 ((-1+c) \log[b] - (-1+b) \log[c]) + \log[a] (-(-1+c) \log[b]^2 + (-1+b) \log[c]^2)\right)}{((\log[a] - \log[b]) (\log[a] - \log[c]) (\log[b] - \log[c]))}, \text{Re}[a] \geq 0 \mid \mid a \notin \text{Reals}\right]$$

```
Sum[1, {s, 1, 1}]
```

1

```
Sum[1, {s, 1, 2}, {t, 1, 1^ (1 - Log[s] / Log[2])}]
```

2

```
Sum[1, {s, 1, 3}, {t, 1, 2^ (1 - Log[s] / Log[3])},
  {u, 1, 1^ (1 - Log[s] / Log[3] - Log[t] / Log[2])}]
```

4

```
Sum[1, {s, 1, 4}, {t, 1, 3^ (1 - Log[s] / Log[4])},
  {u, 1, 2^ (1 - Log[s] / Log[4] - Log[t] / Log[3])},
  {v, 1, 1^ (1 - Log[s] / Log[4] - Log[t] / Log[3] - Log[u] / Log[2])}]
```

7

```
Sum[1, {s, 1, 5}, {t, 1, 4^ (1 - Log[s] / Log[5])},
  {u, 1, 3^ (1 - Log[s] / Log[5] - Log[t] / Log[4])},
  {v, 1, 2^ (1 - Log[s] / Log[5] - Log[t] / Log[4] - Log[u] / Log[3])},
  {w, 1, 1^ (1 - Log[s] / Log[5] - Log[t] / Log[4] - Log[u] / Log[3] - Log[v] / Log[2])}]
```

12

```
Sum[1, {s, 1, 6^(1)}, {t, 1, 5^(1 - Log[s] / Log[6])},
  {u, 1, 4^(1 - Log[s] / Log[6] - Log[t] / Log[5])},
  {v, 1, 3^(1 - Log[s] / Log[6] - Log[t] / Log[5] - Log[u] / Log[4])},
  {w, 1, 2^(1 - Log[s] / Log[6] - Log[t] / Log[5] - Log[u] / Log[4] - Log[v] / Log[3])}, {x, 1,
    1^(1 - Log[s] / Log[6] - Log[t] / Log[5] - Log[u] / Log[4] - Log[v] / Log[3] - Log[w] / Log[2])}]
```

19

```
Clear[pr, pa]
pr[n_, v_] := pr[n, v] = If[n == 1, 1, Sum[pr[n - 1, v - Log[n, s]], {s, 1, n^v}]]
pa[n_, v_] := pa[n, v] = If[n == 0, 1, Sum[pa[n - 1, v - s / n], {s, 0, n v}]]
pf[n_, v_] := pf[n, v] = If[n == 0, 1, Sum[pf[n - 1, v], {s, 0, n v}]]

Table[pr[n, 1], {n, 1, 30}]

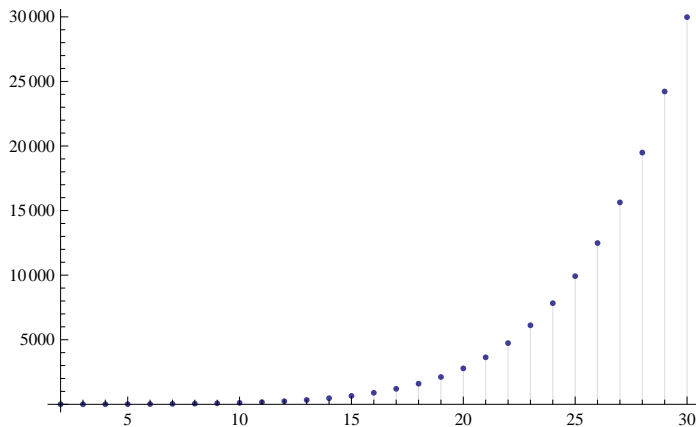
{1, 2, 4, 7, 12, 19, 32, 49, 75, 112, 165, 237, 335, 470, 652, 892, 1199, 1598,
  2114, 2781, 3638, 4736, 6119, 7826, 9919, 12486, 15635, 19491, 24220, 29977}

{1, 2, 4, 7, 12, 19, 32, 49, 75, 112, 165, 237, 335, 470, 652, 892, 1199, 1598,
  2114, 2781, 3638, 4736, 6119, 7826, 9919, 12486, 15635, 19491, 24220, 29977}
```

```
Table[pa[n, 1], {n, 0, 20}]

{1, 2, 4, 8, 17, 37, 86, 199, 475, 1138, 2769, 6748, 16613,
  40904, 101317, 251401, 624958, 1555940, 3882708, 9701790, 24276866}
```

```
DiscretePlot[pr[n, 1], {n, 2, 30}]
```



$1 \times 3 / 2$

$\frac{3}{2}$

$1 \times 2 / 2$

1

$2 \times 3 / 2$

3

$$(1 \times 2 \times 3) / 6 + (1 \times 2 + 1 \times 3 + 2 \times 3) / 2 + (1 + 2 + 3) + 1$$

$$\frac{27}{2}$$

$$(1 \times 2) / 2 + (1 + 2) + 1$$

$$5$$

$$(1 \times 2 \times 3 \times 4) / 24 + (1 \times 2 \times 3 + 1 \times 2 \times 4 + 1 \times 3 \times 4 + 2 \times 3 \times 4) / 6 + (1 \times 2 + 1 \times 3 + 1 \times 4 + 2 \times 3 + 2 \times 4 + 3 \times 4) / 2 + (1 + 2 + 3 + 4) + 1$$

$$\frac{227}{6}$$

```
po[n_, 1_] := If[n < 1, 1, po[n - 1, 1] + n / 1 po[n - 1, 1 + 1]]
```

```
Table[po[n, 1], {n, 1, 15}]
```

$$\left\{ 2, 5, \frac{27}{2}, \frac{227}{6}, \frac{1301}{12}, \frac{9461}{30}, \frac{22255}{24}, \frac{1385435}{504}, \frac{20666521}{2520}, \frac{41316049}{1680}, \frac{248844017}{3360}, \frac{372261390253}{1663200}, \frac{6770504800583}{9979200}, \frac{66872883541291}{32432400}, \frac{232790311796099}{37065600} \right\}$$

$$27 \star 3$$

$$81$$

$$227 \times 4$$

$$908$$

```
Clear[pr, pa]
```

```
pr[n_, v_] := pr[n, v] = If[n == 1, 1, Sum[pr[n - 1, v - Log[n, s]], {s, 1, n^v}]]
```

```
pa[n_, v_] := pa[n, v] = If[n == 0, 1, Sum[pa[n - 1, v - s / n], {s, 0, nv}]]
```

```
pf[n_, v_] := pf[n, v] = If[n == 0, 1, Sum[pf[n - 1, v], {s, 0, nv}]]
```

```
1 + N@Sum[1 / pa[n, 1], {n, 0, 20}]
```

$$2.9811$$

```
N@E
```

$$2.71828$$

```
1 + N@Sum[1 / pr[n, 1], {n, 1, 30}]
```

$$3.12423$$

```
Expand@Sum[1, {s, 0, a}, {t, 0, Floor[b (1 - s / a)]]]
```

$$\sum_{s=0}^a \sum_{t=0}^{\text{Floor}\left[b\left(1-\frac{s}{a}\right)\right]} 1$$

```
Clear[a, b, c]
```

```
Expand@Sum[1, {s, 0, a}, {t, 0, Floor[b (1 - s / a)]}, {u, 0, Floor[c (1 - s / a - t / b)]]]
```

$$\sum_{s=0}^a \sum_{t=0}^{\text{Floor}\left[b\left(1-\frac{s}{a}\right)\right]} \sum_{u=0}^{\text{Floor}\left[c\left(1-\frac{s}{a}-\frac{t}{b}\right)\right]} 1$$

```

am[n_, m_] := Sum[1, {j, 1, n}, {k, 1, Floor[m (1 - j / n)]]]
aml[n_, m_] := Sum[1, {j, 0, n}, {k, 0, Floor[m (1 - j / n)]]]
mm[n_, m_] :=
  If[n == 1, If[m == 1, 0, mm[m, n]], Sum[1, {j, 2, n}, {k, 2, Floor[m^(1 - Log[n, j])]}]]
mml[n_, m_] := If[n == 1, If[m == 1, 1, mml[m, n]],
  Sum[1, {j, 1, n}, {k, 1, Floor[m^(1 - Log[n, j])]}]]]
Table[mm[m, n], {m, 1, 16}, {n, 1, 16}] // Grid

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1
0 0 0 1 1 1 1 1 2 2 2 2 2 2 2 3
0 0 0 1 1 1 2 2 3 3 3 4 4 4 4 4
0 0 0 1 1 3 3 3 3 4 4 4 4 5 5 5
0 0 1 1 2 3 3 3 4 4 4 5 7 7 7 7
0 0 1 1 2 3 3 5 5 5 6 7 7 7 8 8
0 0 1 2 3 3 4 5 6 6 7 7 7 9 9 10
0 0 1 2 3 4 4 5 6 8 8 8 9 9 10 11
0 0 1 2 3 4 4 6 7 8 8 8 10 11 11 13
0 0 1 2 4 4 5 7 7 8 8 12 12 12 13 13
0 0 1 2 4 4 7 7 7 9 10 12 12 12 13 13
0 0 1 2 4 5 7 7 9 9 11 12 12 14 14 15
0 0 1 2 4 5 7 8 9 10 11 13 13 14 16 16
0 0 1 3 4 5 7 8 10 11 13 13 13 15 16 19

FullSimplify[(x + 1)! / (x!)]

1 + x

Table[Pochhammer[z, x] / (x!) /. z -> -1, {x, 0, 10}]

{1, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0}

Table[Pochhammer[z, x] / (x!) /. z -> -1, {x, 0, 10}]

{1, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0}

```