```
d2[n_, k_] := Sum[d2[j, k - 1] d2[n / j, 1], {j, Divisors[n]}];
d2[n_, 1] := 1; d2[1, 1] := 0; d2[n_, 0] := 0; d2[1, 0] := 1

d[n_, k_] := Sum[d[j, k - 1] d[n / j, 1], {j, Divisors[n]}];
d[n_, 1] := 1; d[n_, 0] := 0; d[1, 0] := 1

K[n_, 0] := If[n == 1, 1, 0]
K[n_, 1] := If[n == 1, 0, FullSimplify[MangoldtLambda[n] / Log[n]]]
K[n_, k_] := Sum[K[j, k - 1] K[n / j, 1], {j, Divisors[n]}]
K1[n_, k_] := K1[n, k] = Sum[Binomial[k, j] K[n, k - j], {j, 0, k}]

sc[f_, k_, t_] := SeriesCoefficient[Series[f[x], {x, 0, Floor[t]}], k]

q2[b_, f_, n_, 0] := q2[b, f, n, 0] = 1
q2[b_, f_, n_, 1] :=
 q2[b, f, n, 1] = Sum[ b[n, k] sc[f, k, N[Floor[Log[2, n]]]], {k, 0, N[Log[2, n]]}]
q2[b_, f_, n_, k_] := q2[b, f, n, k] =
  Sum[q2[b, f, n / j, k - 1] q2[b, f, j, 1], {j, Divisors[n]}]

q1[b_, f_, n_, 0] := q1[b, f, n, 0] = 1
q1[b_, f_, n_, 1] := q1[b, f, n, 1] = Sum[ b[n, k] sc[f, k, 14], {k, 0, 14}]
q1[b_, f_, n_, k_] :=
 q1[b, f, n, k] = Sum[q1[b, f, n / j, k - 1] q1[b, f, j, 1], {j, Divisors[n]}]


Mcos[x_] := -Cos[x]
Msin[x_] := -Sin[x]
Expd[x_] := E^x
expd[n_, k_] := q1[d, Expd, n, k]
expd2[n_, k_] := q2[d2, Expd, n, k]
expk[n_, k_] := q2[K, Expd, n, k]
expk1[n_, k_] := q1[K1, Expd, n, k]
sind[n_, k_] := q2[K, Sin, n, k]
cosd[n_, k_] := q2[K, Cos, n, k]
mcosd[n_, k_] := q2[K, Mcos, n, k]
msind[n_, k_] := q2[K, Msin, n, k]
tand[n_, k_] := q2[d2, Tan, n, k]
asinsind[n_, k_] := q2[sind, ArcSin, n, k]
atantand[n_, k_] := q2[tand, ArcTan, n, k]
```

```
Table[{n, N[expd[n, 1] / E], expd2[n, 1], expk[n, 1], N[expk1[n, 1] / E]}, {n, 1, 10}] //
 TableForm
```

1	1.	1	1	1.
2	1.	1	1	1.
3	1.	1	1	1.
4	1.5	$\frac{3}{2}$	1	1.
5	1.	1	1	1.
6	2.	2	1	1.
7	1.	1	1	1.
8	2.16667	$\frac{13}{6}$	1	1.
9	1.5	$\frac{3}{2}$	1	1.
10	2.	2	1	1.

```
Table[{n, mcosd[n, 1], cosd[n, 1], msind[n, 1], sind[n, 1]}, {n, 1, 100}] // TableForm
```

1	-1	1	0	0
2	0	0	-1	1
3	0	0	-1	1
4	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$
5	0	0	-1	1
6	1	-1	0	0
7	0	0	-1	1
8	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{6}$
9	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$
10	1	-1	0	0
11	0	0	-1	1
12	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
13	0	0	-1	1
14	1	-1	0	0
15	1	-1	0	0
16	$\frac{5}{12}$	$-\frac{5}{12}$	0	0
17	0	0	-1	1
18	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
19	0	0	-1	1
20	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
21	1	-1	0	0
22	1	-1	0	0
23	0	0	-1	1
24	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{2}$
25	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$
26	1	-1	0	0
27	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{6}$
28	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
29	0	0	-1	1
30	0	0	1	-1
31	0	0	-1	1
32	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{12}$	$-\frac{1}{12}$
33	1	-1	0	0
34	1	-1	0	0
35	1	-1	0	0

36	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
37	0	0	-1	1
38	1	-1	0	0
39	1	-1	0	0
40	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{2}$
41	0	0	-1	1
42	0	0	1	-1
43	0	0	-1	1
44	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
45	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
46	1	-1	0	0
47	0	0	-1	1
48	0	0	$\frac{5}{12}$	$-\frac{5}{12}$
49	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$
50	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
51	1	-1	0	0
52	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
53	0	0	-1	1
54	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{2}$
55	1	-1	0	0
56	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{2}$
57	1	-1	0	0
58	1	-1	0	0
59	0	0	-1	1
60	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
61	0	0	-1	1
62	1	-1	0	0
63	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
64	$\frac{19}{72}$	$-\frac{19}{72}$	$\frac{1}{8}$	$-\frac{1}{8}$
65	1	-1	0	0
66	0	0	1	-1
67	0	0	-1	1
68	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
69	1	-1	0	0
70	0	0	1	-1
71	0	0	-1	1
72	$-\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	$-\frac{1}{3}$
73	0	0	-1	1
74	1	-1	0	0
75	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
76	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
77	1	-1	0	0
78	0	0	1	-1
79	0	0	-1	1
80	0	0	$\frac{5}{12}$	$-\frac{5}{12}$
81	$\frac{5}{12}$	$-\frac{5}{12}$	0	0
82	1	-1	0	0
83	0	0	-1	1

84	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
85	1	−1	0	0
86	1	−1	0	0
87	1	−1	0	0
88	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{2}$
89	0	0	−1	1
90	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
91	1	−1	0	0
92	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
93	1	−1	0	0
94	1	−1	0	0
95	1	−1	0	0
96	$-\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{3}$	$-\frac{1}{3}$
97	0	0	−1	1
98	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
99	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
100	0	0	$\frac{1}{2}$	$-\frac{1}{2}$

```
Series[Sin[x]^2, {x, 0, 20}]
```

$$x^2 - \frac{x^4}{3} + \frac{2x^6}{45} - \frac{x^8}{315} + \frac{2x^{10}}{14175} - \frac{2x^{12}}{467775} + \frac{4x^{14}}{42567525} - \frac{x^{16}}{638512875} + \frac{2x^{18}}{97692469875} - \frac{2x^{20}}{9280784638125} + O[x]^{21}$$

```
Series[Cos[x]^2, {x, 0, 20}]
```

$$1 - x^2 + \frac{x^4}{3} - \frac{2x^6}{45} + \frac{x^8}{315} - \frac{2x^{10}}{14175} + \frac{2x^{12}}{467775} - \frac{4x^{14}}{42567525} + \frac{x^{16}}{638512875} - \frac{2x^{18}}{97692469875} + \frac{2x^{20}}{9280784638125} + O[x]^{21}$$

```
Series[1 / Sin[x], {x, 0, 20}]
```

$$\frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360} + \frac{31x^5}{15120} + \frac{127x^7}{604800} + \frac{73x^9}{3421440} + \frac{1414477x^{11}}{653837184000} + \frac{8191x^{13}}{37362124800} + \frac{16931177x^{15}}{762187345920000} + \frac{5749691557x^{17}}{2554547108585472000} + \frac{91546277357x^{19}}{401428831349145600000} + O[x]^{21}$$

```
K[n_, 0] := K[n, 0] = If[n == 1, 1, 0]
K[n_, 1] := K[n, 1] = If[n == 1, 0, FullSimplify[MangoldtLambda[n] / Log[n]]]
K[n_, k_] := K[n, k] = Sum[K[j, k - 1] K[n / j, 1], {j, Divisors[n]}]
K1[n_, k_] := K1[n, k] = Sum[Binomial[k, j] K[n, k - j], {j, 0, k}]
dv2[n_, k_] := Sum[k^j / (j!) K[n, j], {j, 0, N[Log[n] / Log[2]]}]
PK[n_, k_] := PK[n, k] = Sum[ K[j, k], {j, 1, n}]
PK1[n_, k_] := PK1[n, k] = Sum[ K1[j, k], {j, 1, n}]
dv2[n_, k_] := Sum[k^j / (j!) K[n, j], {j, 0, N[Log[n] / Log[2]]}]
dv1[n_, k_] := Sum[k^j / (j!) K1[n, j], {j, 0, 17}]
Dv2[n_, k_] := Sum[k^j / (j!) PK[n, j], {j, 0, N[Log[n] / Log[2]]}]
Dv1[n_, k_] := Sum[k^j / (j!) PK1[n, j], {j, 0, 17}]
```

```
dv1[97, 2 Pi I]
```

$$2\,i\,\pi - 4\,\pi^2 - 4\,i\,\pi^3 + \frac{8\,\pi^4}{3} + \frac{4\,i\,\pi^5}{3} - \frac{8\,\pi^6}{15} - \frac{8\,i\,\pi^7}{45} + \frac{16\,\pi^8}{315} + \frac{4\,i\,\pi^9}{315} - \frac{8\,\pi^{10}}{2835} - \frac{8\,i\,\pi^{11}}{14\,175} + \frac{16\,\pi^{12}}{155\,925} + \frac{8\,i\,\pi^{13}}{467\,775} - \frac{16\,\pi^{14}}{6\,081\,075}$$

```
N[Dv1[197, 1] / E]
```

197.

```
N[Dv2[197, 2]]
```

1072.

```
DiscretePlot[Im[Dv2[100 000, n / 100 I]], {n, -1000, 1000}] // TableForm
```

$Aborted

```
N[Dv2[100, ZetaZero[1]]]
```

104 593. + 90 831.3 i

```
Series[Sin[x], {x, 0, 20}]
```

$$x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \frac{x^9}{362\,880} - \frac{x^{11}}{39\,916\,800} + \frac{x^{13}}{6\,227\,020\,800} - \frac{x^{15}}{1\,307\,674\,368\,000} + \frac{x^{17}}{355\,687\,428\,096\,000} - \frac{x^{19}}{121\,645\,100\,408\,832\,000} + O[x]^{21}$$

```
Series[Cos[x], {x, 0, 20}]
```

$$1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40\,320} - \frac{x^{10}}{3\,628\,800} + \frac{x^{12}}{479\,001\,600} - \frac{x^{14}}{87\,178\,291\,200} + \frac{x^{16}}{20\,922\,789\,888\,000} - \frac{x^{18}}{6\,402\,373\,705\,728\,000} + \frac{x^{20}}{2\,432\,902\,008\,176\,640\,000} + O[x]^{21}$$

Dv2[100, 1]

100

Dv2[100, 1 + 2 Pi I]

$$1+\frac{428}{15}(1+2\,i\,\pi)+\frac{16289}{360}(1+2\,i\,\pi)^2+\frac{331}{16}(1+2\,i\,\pi)^3+\frac{611}{144}(1+2\,i\,\pi)^4+\frac{67}{240}(1+2\,i\,\pi)^5+\frac{7}{720}(1+2\,i\,\pi)^6$$

Dv2[100, 1 + 4 / 3 Pi I]

$$1+\frac{428}{15}\left(1+\frac{4\,i\,\pi}{3}\right)+\frac{16289}{360}\left(1+\frac{4\,i\,\pi}{3}\right)^2+\frac{331}{16}\left(1+\frac{4\,i\,\pi}{3}\right)^3+\frac{611}{144}\left(1+\frac{4\,i\,\pi}{3}\right)^4+\frac{67}{240}\left(1+\frac{4\,i\,\pi}{3}\right)^5+\frac{7}{720}\left(1+\frac{4\,i\,\pi}{3}\right)^6$$