```
aa[f_, n_, s_] := n (1 / n Sum[(j / n) ^ (-1 / 2) f[s Log[j / n]], {j, 1, n}] -
    Integrate[f[s Log[x]] / x^(1 / 2), {x, 0, 1}])
```

```
DiscretePlot[aa[Sin, n, 10], {n, 1, 100}]
```

```
$Aborted
```

```
Integrate[Sin[s Log[x]] / x^(1 / 2), {x, 0, 1}]
```

$$\text{ConditionalExpression}\left[-\frac{4\,s}{1+4\,s^2},\ -\frac{1}{2}<\text{Im}[s]<\frac{1}{2}\right]$$

```
Integrate[Cos[s Log[x]] / x^(1 / 2), {x, 0, 1}]
```

$$\text{ConditionalExpression}\left[\frac{2}{1+4\,s^2},\ s\in\text{Reals}\right]$$

```
Integrate[Tan[s Log[x]] / x^(1 / 2), {x, 0, 1}]
```

```
$Aborted
```

```
Integrate[s Log[x] / x^(1 / 2), {x, 0, 1}]
```

$$-4\,s$$

```
Integrate[(s Log[x])^2 / x^(1 / 2), {x, 0, 1}]
```

$$16\,s^2$$

```
Integrate[Log[s Log[x]] / x^(1 / 2), {x, 0, 1}]
```

$$-2\,\text{EulerGamma}+2\,\text{Log}[-2\,s]$$

```
Integrate[Sinh[s Log[x]] / x^(1 / 2), {x, 0, 1}]
```

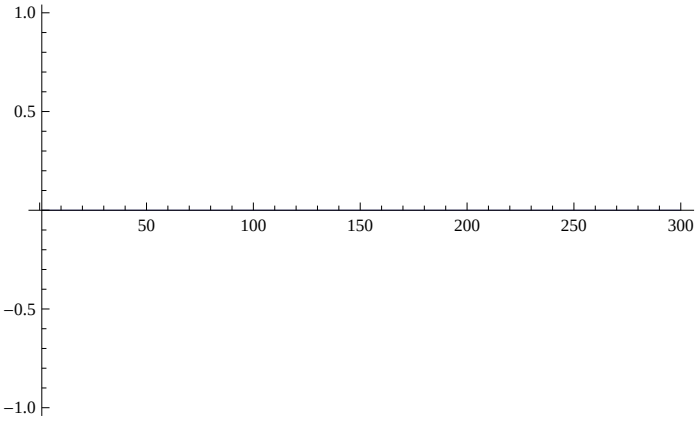
$$\text{ConditionalExpression}\left[\frac{4\,s}{-1+4\,s^2},\ -\frac{1}{2}<s<\frac{1}{2}\right]$$

```
Integrate[Exp[s Log[x]] / x^(1 / 2), {x, 0, 1}]
```

$$\text{ConditionalExpression}\left[\frac{2}{1+2\,s},\ \text{Re}[s]>-\frac{1}{2}\right]$$

```
tsin[n_, s_] := n (1 / n Sum[(j / n) ^ (-1 / 2) Sin[s Log[j / n]], {j, 1, n}] - (-4 s / (1 + 4 s^2)))

tcos[n_, s_] := n (1 / n Sum[(j / n) ^ (-1 / 2) Cos[s Log[j / n]], {j, 1, n}] - (2 / (1 + 4 s^2))) - .5

tid[n_, s_] := n (1 / n Sum[(j / n) ^ (-1 / 2) (s Log[j / n]), {j, 1, n}] - (-4 s))

tsq[n_, s_] := n (1 / n Sum[(j / n) ^ (-1 / 2) (s Log[j / n]) ^2, {j, 1, n}] - (16 s^2))

tlog[n_, s_] :=
 n (1 / n Sum[(j / n) ^ (-1 / 2) Log[s Log[j / n]], {j, 1, n}] - (-2 EulerGamma + 2 Log[-2 s]))

tsinh[n_, s_] := n (1 / n Sum[(j / n) ^ (-1 / 2) Sinh[s Log[j / n]], {j, 1, n}] - (4 s / (-1 + 4 s^2)))

texp[n_, s_] := n (1 / n Sum[(j / n) ^ (-1 / 2) Exp[s Log[j / n]], {j, 1, n}] - (2 / (1 + 2 s))) - .5

tboth[n_, s_] := n (1 / n Sum[(j / n) ^ (-1 / 2) Cos[s Log[j / n]], {j, 1, n}] - (2 / (1 + 4 s^2)) +
      I (1 / n Sum[(j / n) ^ (-1 / 2) Sin[s Log[j / n]], {j, 1, n}] - (-4 s / (1 + 4 s^2)))) - .5

tboth2[n_, s_] := n (1 / n Sum[(j / n)^(-1/2 + I s), {j, 1, n}] + (2 I / (-I + 2 s))) - .5

tdif[n_, s_] := tsin[n, s] + 2 s tcos[n, s]

tdif2[n_, s_] := (n (1 / n Sum[(j / n) ^ (-1 / 2) Sin[s Log[j / n]], {j, 1, n}] - (-4 s / (1 + 4 s^2)))) +
  2 s (n (1 / n Sum[(j / n) ^ (-1 / 2) Cos[s Log[j / n]], {j, 1, n}] - (2 / (1 + 4 s^2)))) - s

tdif3[n_, s_] := (n (1 / n Sum[(j / n) ^ (-1 / 2) Sin[s Log[j / n]], {j, 1, n}])) +
  (n (1 / n Sum[(j / n) ^ (-1 / 2) 2 s Cos[s Log[j / n]], {j, 1, n}])) - s

tdif4[n_, s_] :=
 (n (1 / n Sum[(j / n) ^ (-1 / 2) (Sin[s Log[j / n]] + 2 s Cos[s Log[j / n]]), {j, 1, n}])) - s

tdif5[n_, s_] := (Sqrt[2] Sqrt[s])
  ((n (1 / n Sum[(j / n) ^ (-1 / 2) ((1 / (Sqrt[2] Sqrt[s])) Sin[s Log[j / n]] + 2 s / (Sqrt[2] Sqrt[s])
          Cos[s Log[j / n]]), {j, 1, n}])) - s / (Sqrt[2] Sqrt[s]))

tdifx[n_, s_] := (n (1 / n Sum[(j / n) ^ (-1 / 2) (Sqrt[(1/2 - s) / (1/2 + s)]) Sin[s Log[j / n] + Pi / 4] +
      1 / Sqrt[(1/2 + s) / (1/2 - s)] Sin[s Log[j / n] + 3 Pi / 4], {j, 1, n}]))

tdif5[100, N@Im@ZetaZero@10 + .1]

96.6095

tdif[100, N@Im@ZetaZero@10 + .1]

96.6095
```

```
DiscretePlot[Re[tdifx[n, N@Im@ZetaZero@1]], {n, 1, 300}]
```




```
FullSimplify[(j / n)^(-1 / 2) Cos[s Log[j / n]] + I (j / n)^(-1 / 2) Sin[s Log[j / n]]]
```

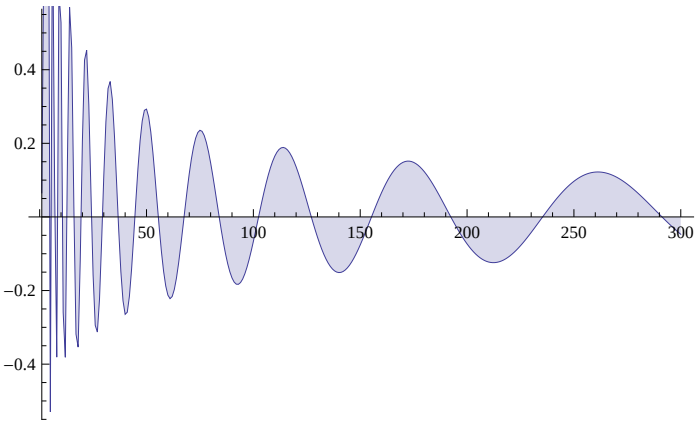
$$\left(\frac{j}{n}\right)^{-\frac{1}{2}+i\,s}$$

```
Integrate[Tan[s Log[x]] / x^(1 / 2), {x, 0, 1}]
```

```
$Aborted
```

```
tlogsin[n_, s_] := n (1 / n Sum[(j / n)^(-1 / 2) (j / n) Sin[s Log[j / n]], {j, 1, n}] - (-(4 s / (9 + 4 s^2))))
```

```
DiscretePlot[Re[tlogsin[n, N@Im@ZetaZero@1 + 1]], {n, 1, 300}]
```




```
FullSimplify[-(2 / (1 + 4 s^2)) + I (-(-(4 s / (1 + 4 s^2))))]
```

$$\frac{2\,i}{-i+2\,s}$$

```
Integrate[x^(-1 / 2 + s I), {x, 0, 1}]
```

$$\text{ConditionalExpression}\left[\frac{2\,i}{i-2\,s},\ \text{Im}[s] < \frac{1}{2}\right]$$

```
FullSimplify@Integrate[Sin[s Log[x] + c] / x^(1 / 2), {x, 0, 1}]
```

$$\frac{2\,(-2\,s\,\text{Cos}[c] + \text{Sin}[c])}{1 + 4\,s^2}$$

```
2 (-2 s Cos[c] + Sin[c]) /. c → Pi / 4
```

$$\frac{2\left(\frac{1}{\sqrt{2}} - \sqrt{2}\,s\right)}{1 + 4\,s^2}$$

```
2 (-2 s Cos[c] + Sin[c]) /. c → 3 Pi / 4
```

$$\frac{2\left(\frac{1}{\sqrt{2}} + \sqrt{2}\,s\right)}{1 + 4\,s^2}$$

```
2 (-2 s Cos[c] + Sin[c]) /. c → 0
```

$$-\frac{4\,s}{1 + 4\,s^2}$$

```
2 (-2 s Cos[c] + Sin[c]) /. c → Pi / 2
```

$$\frac{2}{1 + 4\,s^2}$$

```
cc[c_] := 2 (-2 s Cos[c] + Sin[c]) / (1 + 4 s^2)
cc[0] / cc[Pi / 2]
```

$$-2\,s$$

```
cc[Pi / 4] / cc[3 Pi / 4]
```

$$\frac{\frac{1}{\sqrt{2}} - \sqrt{2}\,s}{\frac{1}{\sqrt{2}} + \sqrt{2}\,s}$$

```
cc[3 Pi / 4] / cc[Pi / 4]
```

$$\frac{\frac{1}{\sqrt{2}} + \sqrt{2}\,s}{\frac{1}{\sqrt{2}} - \sqrt{2}\,s}$$

```
(** 2s and 1 vs 1 and 1/(2s)}
(2 s)^(1 / 2)
```

$$\sqrt{2}\,\sqrt{s}$$

```
(2 s) / (Sqrt[2] Sqrt[s])
```

$$\sqrt{2}\,\sqrt{s}$$

```
1/(Sqrt[2] Sqrt[s])
```

$$\frac{1}{\sqrt{2}\ \sqrt{s}}$$

```
FullSimplify[((1/Sqrt[2] + Sqrt[2] s)/(1/Sqrt[2] - Sqrt[2] s))^(1/2)]
```

$$\sqrt{\frac{1+2s}{1-2s}}$$

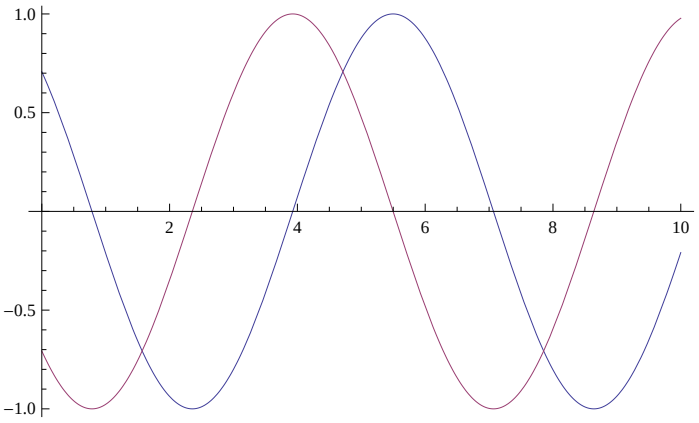
```
FullSimplify[((1/Sqrt[2] - Sqrt[2] s)/(1/Sqrt[2] + Sqrt[2] s)) * (Sqrt[(1 + 2 s)/(1 - 2 s)])]
```

$$\frac{1}{\sqrt{\frac{1+2s}{1-2s}}}$$

```
FullSimplify[((1/Sqrt[2] + Sqrt[2] s)/(1/Sqrt[2] - Sqrt[2] s)) * (Sqrt[(1 + 2 s)/(1 - 2 s)])]
```

$$-\frac{\sqrt{\frac{1}{1-2s}}\ (1+2s)^{3/2}}{-1+2s}$$

```
Plot[{Sin[x + 3 Pi/4], -Sin[x + Pi/4]}, {x, 0, 10}]
```




```
ach[s_] := {Sqrt[2] Sqrt[s], 1/(Sqrt[2] Sqrt[s])}
```

```
ach2[s_] := FullSimplify[{Sqrt[-1 + 2/(1 + 2 s)], 1/Sqrt[-1 + 2/(1 + 2 s)]}]
```

```
ach2[200]
```

$$\left\{i\sqrt{\frac{399}{401}},\ -i\sqrt{\frac{401}{399}}\right\}$$

```
FullSimplify[((1/Sqrt[2] - Sqrt[2] s)/(1/Sqrt[2] + Sqrt[2] s))^(1/2)]
```

$$\sqrt{-1+\frac{2}{1+2s}}$$

```
(-1 (1/2 + s) + 1)/(1/2 + s)
```

$$\frac{\frac{1}{2}-s}{\frac{1}{2}+s}$$

```
((1/2 - s)/(1/2 + s))^(1/2)
```

$$\sqrt{\frac{\frac{1}{2}-s}{\frac{1}{2}+s}}$$

```
Integrate[Sin[s Log[x] + Pi/4]/x^(1/2), {x, 0, 1}]
```

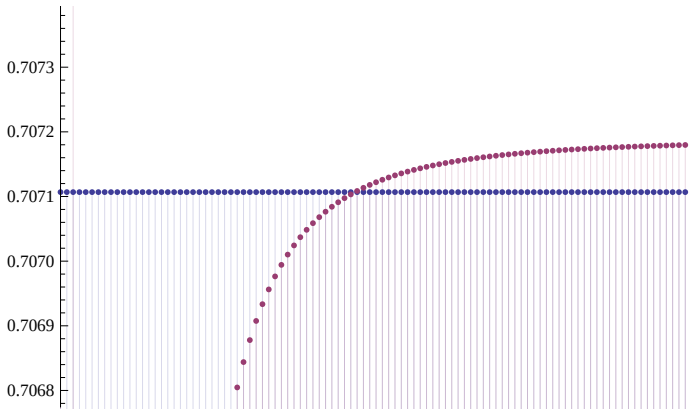
$$\text{ConditionalExpression}\left[\frac{\sqrt{2}\ (1-2s)}{1+4s^2},\ -\frac{1}{2}<\text{Im}[s]<\frac{1}{2}\right]$$

```
Integrate[Sin[s Log[x] - Pi/4]/x^(1/2), {x, 0, 1}]
```

$$\text{ConditionalExpression}\left[-\frac{\sqrt{2}\ (1+2s)}{1+4s^2},\ -\frac{1}{2}<\text{Im}[s]<\frac{1}{2}\right]$$

```
tsinp[n_, s_] :=
 n (1 / n Sum[(j / n) ^ (-1 / 2) Sin[s Log[j / n] + Pi / 4], {j, 1, n}] - (Sqrt[2] (1 - 2 s) / (1 + 4 s^2)))
tsinm[n_, s_] :=
 n (1 / n Sum[(j / n) ^ (-1 / 2) Sin[s Log[j / n] - Pi / 4], {j, 1, n}] - (-Sqrt[2] (1 + 2 s) / (1 + 4 s^2)))
tsinpm[n_, s_] := tsinp[n, s] + tsinm[n, s]
tsinpm2[n_, s_] :=
 (n (1 / n Sum[(j / n) ^ (-1 / 2) Sin[s Log[j / n] + Pi / 4], {j, 1, n}] - (Sqrt[2] (1 - 2 s) / (1 + 4 s^2)))) +
  (n (1 / n Sum[(j / n) ^ (-1 / 2) Sin[s Log[j / n] - Pi / 4], {j, 1, n}] - (-Sqrt[2] (1 + 2 s) / (1 + 4 s^2))))
tsinpm3[n_, s_] := (1 / Sqrt[-(1 / 2 - s) / (1 / 2 + s)])
   (n (1 / n Sum[(j / n) ^ (-1 / 2) Sin[s Log[j / n] + Pi / 4], {j, 1, n}])) -
  Sqrt[-(1 / 2 - s) / (1 / 2 + s)] (n (1 / n Sum[(j / n) ^ (-1 / 2) Sin[s Log[j / n] - Pi / 4], {j, 1, n}])) - 2^(1 / 2) / 2
tsinpm4[n_, s_] :=
 (n (1 / n Sum[(j / n) ^ (-1 / 2) (1 / Sqrt[-(1 / 2 - s) / (1 / 2 + s)]) Sin[s Log[j / n] + Pi / 4], {j, 1, n}])) -
  (n (1 / n Sum[(j / n) ^ (-1 / 2) Sqrt[-(1 / 2 - s) / (1 / 2 + s)] Sin[s Log[j / n] - Pi / 4], {j, 1, n}])) -
  2^(1 / 2) / 2
tsinpm5[n_, s_] := n (1 / n Sum[(j / n) ^ (-1 / 2) ((1 / Sqrt[-(1 / 2 - s) / (1 / 2 + s)]) Sin[s Log[j / n] + Pi / 4] -
       Sqrt[-(1 / 2 - s) / (1 / 2 + s)] Sin[s Log[j / n] - Pi / 4]), {j, 1, n}])
```

```
DiscretePlot[{2^(1 / 2) / 2, Re@tsinpm5[n, N@Im@ZetaZero@5]}, {n, 1, 100}]
```



$$\left(\left(\frac{\sqrt{2}\ (1-2\,s)}{1+4\,s^2}\right) \Big/ \left(-\frac{\sqrt{2}\ (1+2\,s)}{1+4\,s^2}\right)\right)$$

$$-\frac{1-2\,s}{1+2\,s}$$

$$\left(\left(-\frac{\sqrt{2}\ (1+2\,s)}{1+4\,s^2}\right) \Big/ \left(\frac{\sqrt{2}\ (1-2\,s)}{1+4\,s^2}\right)\right)$$

$$-\frac{1+2\,s}{1-2\,s}$$

$$\left(-\frac{1-2\,s}{1+2\,s}\right)^{(1/2)}$$

$$\sqrt{-\frac{1-2\,s}{1+2\,s}}$$

```
2^.5 / 2
```

```
0.707107
```

```
N@2^(1 / 2) / 2
```

```
0.707107
```

```
1 / N@Im@ZetaZero@1
```

```
0.0707477
```

```
((.5^2) + (1 / N@Im@ZetaZero@1)^2)^.5
```

```
0.50498
```

```
1 * Sin[Pi / 4.] + 21 / Im@ZetaZero@1 * Sin[-Pi / 4]
```

```
-0.343444
```

```
(** n
```

$$\left(1/n\ \text{Sum}\left[(j/n)^{(-1/2)}\left(\left(1\Big/\sqrt{-\tfrac{1/2-s}{1/2+s}}\right)\text{Sin}[s\ \text{Log}[j/n]+\text{Pi}/4]-\left(\sqrt{-\tfrac{1/2-s}{1/2+s}}\right)\text{Sin}[s\ \text{Log}[j/n]-\text{Pi}/4]\right),\right.$$

```
    {j,1,n}]) **)

tsinhalf[n_, s_] := 2 n (1 / n Sum[(j / n) ^ (-1 / 2) Sin[s Log[j / n] + Pi / 4], {j, 1, n}] -
      (Sqrt[2] (1 - 2 s) / (1 + 4 s^2))) - 1 / Sqrt[2]

tsinmhalf[n_, s_] := 2 n (1 / n Sum[(j / n) ^ (-1 / 2) Sin[s Log[j / n] - Pi / 4], {j, 1, n}] -
      (-Sqrt[2] (1 + 2 s) / (1 + 4 s^2))) - (-1 / Sqrt[2])

tsinthird[n_, s_] := 2 n (1 / n Sum[(j / n) ^ (-1 / 2) Sin[s Log[j / n] + Pi / 6], {j, 1, n}] -
      ((1 - 2 Sqrt[3] s) / (1 + 4 s^2))) - 1 / 2

tmid[n_, s_] := (2 n (1 / n Sum[(j / n) ^ (-1 / 2) Sin[s Log[j / n] + Pi / 4], {j, 1, n}] -
      (Sqrt[2] (1 - 2 s) / (1 + 4 s^2))) - 1 / Sqrt[2]) -
   (2 n (1 / n Sum[(j / n) ^ (-1 / 2) Sin[s Log[j / n] - Pi / 4], {j, 1, n}] - (-Sqrt[2] (1 + 2 s) / (1 + 4 s^2))) -
      (-1 / Sqrt[2]))

tmid2[n_, s_] := (2 n (1 / n Sum[(j / n) ^ (-1 / 2) Sin[s Log[j / n] + Pi / 4], {j, 1, n}] -
      (Sqrt[2] (1 - 2 s) / (1 + 4 s^2))) - 1 / Sqrt[2]) 1 / Sqrt[-(1 / 2 - s) / (1 / 2 + s)] -
   (2 n (1 / n Sum[(j / n) ^ (-1 / 2) Sin[s Log[j / n] - Pi / 4], {j, 1, n}] - (-Sqrt[2] (1 + 2 s) / (1 + 4 s^2))) -
      (-1 / Sqrt[2])) Sqrt[-(1 / 2 - s) / (1 / 2 + s)]

tmid3[n_, s_] := (2 n (1 / n Sum[(j / n) ^ (-1 / 2) Sin[s Log[j / n] + Pi / 4], {j, 1, n}]) - 1 / Sqrt[2])
   1 / Sqrt[-(1 / 2 - s) / (1 / 2 + s)] -
```

$$\left(2\,n\,(1/n\,\text{Sum}[(j/n)^{(-1/2)}\,\text{Sin}[s\,\text{Log}[j/n]-\text{Pi}/4],\{j,1,n\}])-\left(-\frac{1}{\sqrt{2}}\right)\right)\sqrt{-\frac{1/2-s}{1/2+s}}$$

```
tmid4[n_, s_] :=
```

$$\left(n\left(1/n\,\text{Sum}\left[(j/n)^{(-1/2)}\left(1\Big/\sqrt{-\frac{1/2-s}{1/2+s}}\right)\text{Sin}[s\,\text{Log}[j/n]+\text{Pi}/4],\{j,1,n\}\right]-\frac{1}{2\sqrt{2}}\Big/\sqrt{-\frac{1/2-s}{1/2+s}}\right)-\left(n\left(1/n\,\text{Sum}\left[(j/n)^{(-1/2)}\sqrt{-\frac{1/2-s}{1/2+s}}\,\text{Sin}[s\,\text{Log}[j/n]-\text{Pi}/4],\{j,1,n\}\right]-\left(-\frac{1}{2\sqrt{2}}\right)\sqrt{-\frac{1/2-s}{1/2+s}}\right)\right.$$

```
tmid5[n_, s_] :=
```

$$\left(n\left(1/n\,\text{Sum}\left[(j/n)^{(-1/2)}\left(1\Big/\sqrt{-\frac{1/2-s}{1/2+s}}\right)\text{Sin}[s\,\text{Log}[j/n]+\text{Pi}/4],\{j,1,n\}\right]\right)-\left(n\left(1/n\,\text{Sum}\left[(j/n)^{(-1/2)}\sqrt{-\frac{1/2-s}{1/2+s}}\,\text{Sin}[s\,\text{Log}[j/n]-\text{Pi}/4],\{j,1,n\}\right]\right)-\frac{1}{2\sqrt{2}}\Big/\sqrt{-\frac{1/2-s}{1/2+s}}+\left(-\frac{1}{2\sqrt{2}}\right)\sqrt{-\frac{1/2-s}{1/2+s}}\right.$$

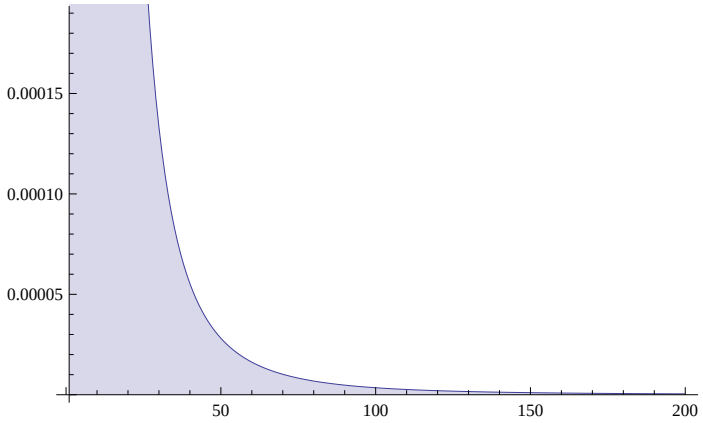
```
tmid6[n_, s_] :=
```

$$\left(\left(\text{Sum}\left[(j/n)^{(-1/2)}\left(1\Big/\sqrt{-\frac{1/2-s}{1/2+s}}\right)\text{Sin}[s\,\text{Log}[j/n]+\text{Pi}/4],\{j,1,n\}\right]\right)-\left(\left(\text{Sum}\left[(j/n)^{(-1/2)}\sqrt{-\frac{1/2-s}{1/2+s}}\,\text{Sin}[s\,\text{Log}[j/n]-\text{Pi}/4],\{j,1,n\}\right]\right)-\frac{1}{2\sqrt{2}}\Big/\sqrt{-\frac{1/2-s}{1/2+s}}+\left(-\frac{1}{2\sqrt{2}}\right)\sqrt{-\frac{1/2-s}{1/2+s}}\right.$$

```
tmid7[n_, s_] := (( Sum[(j/n)^(-1/2)
```

```
        (((1 / Sqrt[-((1/2 - s)/(1/2 + s))]) Sin[s Log[j / n] + Pi / 4] - Sqrt[-((1/2 - s)/(1/2 + s))] Sin[s Log[j / n] - Pi / 4]),
        {j, 1, n}])) - 1/(2 Sqrt[2]) / Sqrt[-((1/2 - s)/(1/2 + s))] + (-1/(2 Sqrt[2])) Sqrt[-((1/2 - s)/(1/2 + s))]

tmid8[n_, s_] := Sum[(j / n)^(-1/2) (((Sqrt[-((1/2 - s)/(1/2 + s))])^-1 Sin[s Log[j / n] + Pi / 4] -
        Sqrt[-((1/2 - s)/(1/2 + s))] Sin[s Log[j / n] - Pi / 4]), {j, 1, n}] - s/(Sqrt[-1/2 + s] Sqrt[(1/2 + s)] 2^(1/2))

tmid9[n_, s_] := Sum[(j / n)^(-1/2) ((-((1/2 - s)/(1/2 + s)))^(-1/2) Sin[s Log[j / n] + Pi / 4] -
        (-((1/2 - s)/(1/2 + s)))^(1/2) Sin[s Log[j / n] - Pi / 4]),
    {j, 1, n}] - s/(Sqrt[-1/2 + s] Sqrt[(1/2 + s)] 2^(1/2))

tmid10[n_, s_] := Sum[(j / n)^(-1/2) (((-((1/2 - s)/(1/2 + s)))^(-1/2) (1 / (2 I))
        (E^(I (s Log[j / n] + Pi / 4)) - E^(-I (s Log[j / n] + Pi / 4))) - (-((1/2 - s)/(1/2 + s)))^(1/2)
        (1 / (2 I)) (E^(I (s Log[j / n] - Pi / 4)) - E^(-I (s Log[j / n] - Pi / 4)))),
    {j, 1, n}] - s/(Sqrt[-1/2 + s] Sqrt[(1/2 + s)] 2^(1/2))

tmid11[n_, s_] := (1 / (2 I)) Sum[(j / n)^(-1/2)
        (((-((1/2 - s)/(1/2 + s)))^(-1/2) (E^(I (s Log[j / n] + Pi / 4)) - E^(-I (s Log[j / n] + Pi / 4))) -
        (-((1/2 - s)/(1/2 + s)))^(1/2) (E^(I (s Log[j / n] - Pi / 4)) - E^(-I (s Log[j / n] - Pi / 4)))),
    {j, 1, n}] - s/(Sqrt[-1/2 + s] Sqrt[(1/2 + s)] 2^(1/2))

tmid12[n_, s_] := (1 / (2 I)) Sum[(j / n)^(-1/2) (E^((-1/2) Log[(-((1/2 - s)/(1/2 + s)))])
        (E^(I (s Log[j / n] + Pi / 4)) - E^(-I (s Log[j / n] + Pi / 4))) -
        E^(1/2 Log[(-((1/2 - s)/(1/2 + s)))]) (E^(I (s Log[j / n] - Pi / 4)) - E^(-I (s Log[j / n] - Pi / 4)))),
```

```
{j, 1, n}] - s / (Sqrt[-1/2 + s] Sqrt[(1/2 + s)] 2^(1/2))

tmid13[n_, s_] := (1/(2 I)) Sum[(j/n)^(-1/2) (E^((-1/2) Log[(-(1/2 - s)/(1/2 + s))]) (E^(I (s Log[j/n] + Pi/4)) - E^(-I (s Log[j/n] + Pi/4))) - E^(1/2 Log[(-(1/2 - s)/(1/2 + s))]) (E^(I (s Log[j/n] - Pi/4)) - E^(-I (s Log[j/n] - Pi/4))),
{j, 1, n}] - s / (Sqrt[-1/2 + s] Sqrt[(1/2 + s)] 2^(1/2))

tmid14[n_, s_] := (1/(2 I)) Sum[(j/n)^(-1/2)
((E^(I (s Log[j/n] + Pi/4) + (-1/2) Log[(-(1/2 - s)/(1/2 + s))]) -
E^(-I (s Log[j/n] + Pi/4) + (-1/2) Log[(-(1/2 - s)/(1/2 + s))])) -
(E^(-1/2 Log[(-(1/2 - s)/(1/2 + s))] + I (s Log[j/n] - Pi/4)) -
E^(-1/2 Log[(-(1/2 - s)/(1/2 + s))] - I (s Log[j/n] - Pi/4))),
{j, 1, n}] - s / (Sqrt[-1/2 + s] Sqrt[(1/2 + s)] 2^(1/2))

DiscretePlot[Abs@tmid13[n, N@Im@ZetaZero@2], {n, 1, 200}]
```




```
FullSimplify[-1/(2 Sqrt[2]) / Sqrt[-(1/2 - s)/(1/2 + s)] + (-1/(2 Sqrt[2])) Sqrt[-(1/2 - s)/(1/2 + s)]]
```

$$-\frac{s}{\sqrt{-\frac{1}{2}+s}\ \sqrt{1+2s}}$$

```
N[(1/Sqrt[-(1/2 - s)/(1/2 + s)]) Sin[s Log[j / n] + Pi / 4] -
    Sqrt[-(1/2 - s)/(1/2 + s)] Sin[s Log[j / n] - Pi / 4] /. s → 1000000]
```

$$1.\ \text{Sin}\left[0.785398 - 1.\times 10^6\ \text{Log}\left[\frac{j}{n}\right]\right] + 1.\ \text{Sin}\left[0.785398 + 1.\times 10^6\ \text{Log}\left[\frac{j}{n}\right]\right]$$

```
(Sqrt[-(1/2 - s)/(1/2 + s)])^-1
```

$$\frac{1}{\sqrt{1-\frac{2}{1+2\,s}}}$$

$$-\frac{1}{2}\ \text{Log}[-1+2\,s] + \frac{1}{2}\ \text{Log}[1+2\,s]$$

```
TrigToExp[Sin[x]]
```

$$\frac{1}{2}\ i\ e^{-i\,x} - \frac{1}{2}\ i\ e^{i\,x}$$

```
Log[(1/2 - s)/(1/2 + s)] /. s → 4.3
```

$$-0.233615$$

```
Log[(1/2 - s)/(1/2 + s)] /. s → 4.3
```

$$-0.233615 + 3.14159\ i$$

```
Log[(1/2 - s)/(1/2 + s)] - Pi I /. s → 4.3
```

$$-0.233615 + 0.\ i$$

```
Log[1 / 2 - s] - Log[1 / 2 + s] - Pi I /. s → (1 / 4) t
```

$$-i\,\pi + \text{Log}\left[\frac{1}{2} - \frac{t}{4}\right] - \text{Log}\left[\frac{1}{2} + \frac{t}{4}\right]$$

```
TrigToExp[ArcTanh[s]]
```

$$-\frac{1}{2}\text{Log}[1-s]+\frac{1}{2}\text{Log}[1+s]$$

```
ss[n_, s_] :=
 Sum[j^(-1 / 2) Sin[s Log[n / j] - ArcTan[2 s]] / Sin[s Log[n] - ArcTan[2 s]], {j, 1, n}]
ssa[n_, s_] :=
 Sum[j^(-1 / 2) Sin[s Log[n] - s Log[j] - ArcTan[2 s]] / Sin[s Log[n] - ArcTan[2 s]], {j, 1, n}]
ssb[n_, s_] :=
 Sum[j^(-1 / 2) Cos[s Log[n] - s Log[j] + ArcCot[2 s]] / Cos[s Log[n] + ArcCot[2 s]], {j, 1, n}]
ssbd[n_, j_, s_] := j^(-1 / 2) Cos[s Log[n] - s Log[j] + ArcCot[2 s]] / Cos[s Log[n] + ArcCot[2 s]]
tt[n_, s_] :=
 Sum[j^(-1 / 2) (Cos[s Log[j]] + Tan[s Log[n] + ArcCot[2 s]] Sin[s Log[j]]), {j, 1, n}]
```

```
Chop@ssb[10 000, 1.5 I]
```

```
1.64493
```

```
Zeta[2.]
```

```
1.64493
```

```
Integrate[Sin[s Log[x] + ArcTan[2 s]] / x^(1 / 2), {x, 0, 1}]
```

$$\text{ConditionalExpression}\left[0, -\frac{1}{2}<\text{Im}[s]<\frac{1}{2}\right]$$

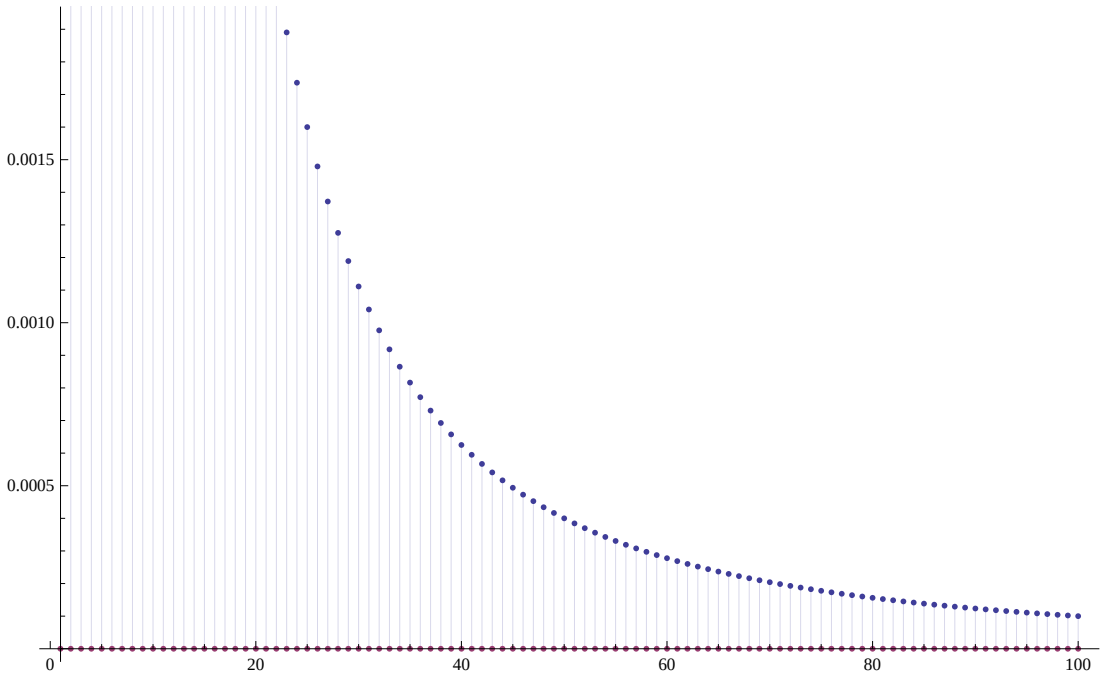
```
N@ArcTan[2 × 4 I ]
```

```
1.5708 + 0.125657 i
```

```
N@Pi / 2
```

```
1.5708
```

```
DiscretePlot[{Re[ssbd[4000, n, 1.5 I]], 0}, {n, 1, 100}]
```



```
e1[x_, t_] := 2 t Cos[t Log[x]] + Sin[t Log[x]]
e2[x_, t_] := (Sin[t Log[x] + ArcTan[2 t]]) (2 t)^(1 / 2)
```

```
N@e1[100, 2]
```

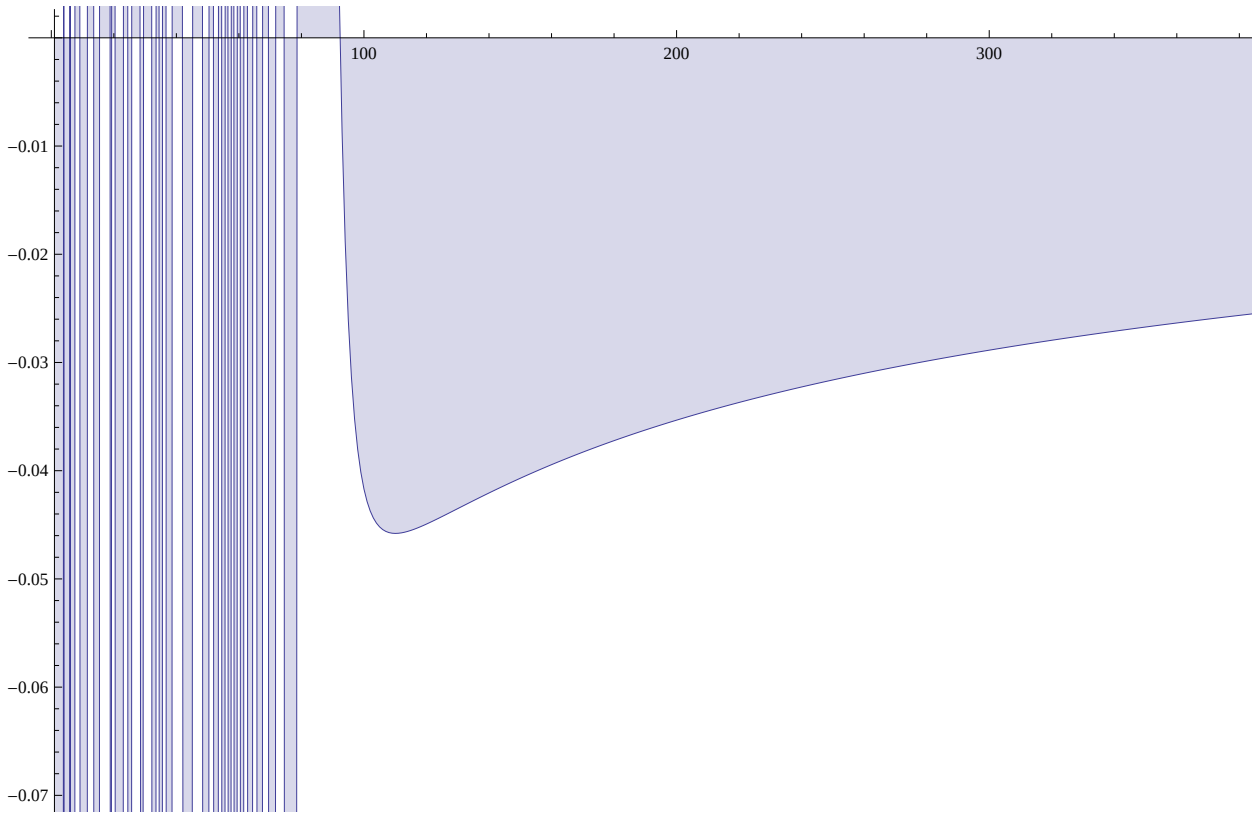
```
-3.69559
```

```
N@e2[100, 2]
```

```
-1.79262
```

```
qq[n_, s_] := Sum[j^(-1 / 2) Sin[s Log[n / j] - ArcTan[2 s]], {j, 1, n}]
qqa[n_, s_] := Sum[j^(-1 / 2) Sin[s Log[n / j]], {j, 1, n}]
qq2[n_, s_] := n^(1 / 2) ((1 / n) Sum[(n / j)^(1 / 2) Sin[s Log[n / j] - ArcTan[2 s]], {j, 1, n}])
```

```
DiscretePlot[Re@qq[n, Im@N@ZetaZero@300], {n, 1, 400}]
```



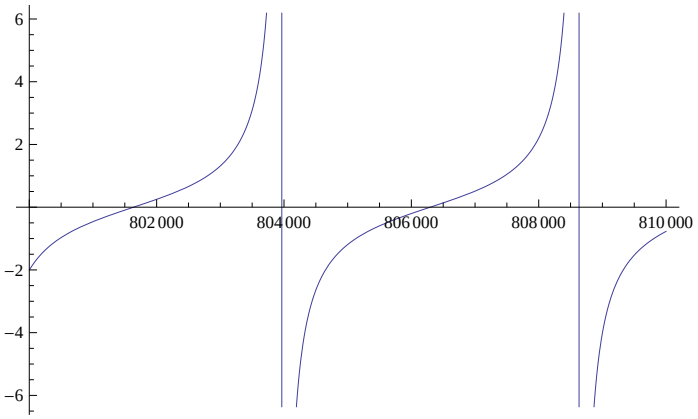

```
(Integrate[x^(-1 / 2) Sin[-s Log[x] - ArcTan[2 s]], {x, 0, 1}])
```

$$\text{ConditionalExpression}\left[0, -\frac{1}{2} < \text{Im}[s] < \frac{1}{2}\right]$$

```
Im@N@ZetaZero@300
```

```
541.847
```

```
Plot[Im@Tanh[s Log[n] - ArcTanh[1 / (2 s)]] /. s → Im@N@ZetaZero@300 I + .3 I,
 {n, 800 000, 810 000}]
```




```
-8 s / (1 + 4 s^2) / (2 s) ^ (1 / 2)
```

$$-\frac{4\sqrt{2}\ \sqrt{s}}{1+4\,s^2}$$

```
-4 / (1 + 4 s^2) (2 s) ^ (1 / 2)
```

$$-\frac{4\sqrt{2}\ \sqrt{s}}{1+4\,s^2}$$