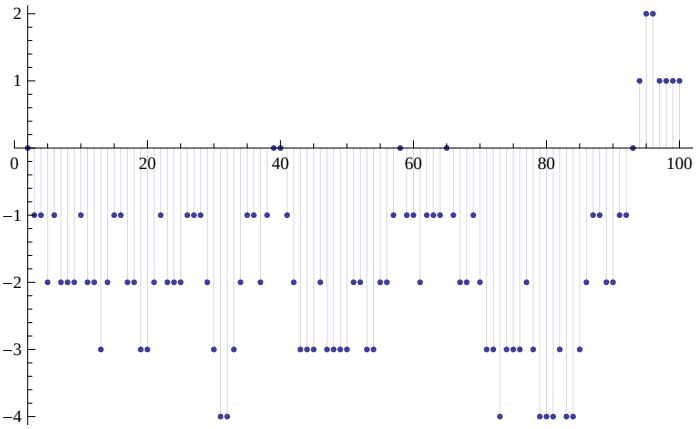
```
E2[n_, k_, b_] :=
 E2[n, k, b] = Sum[ E2[n / j, k - 1, b], {j, 2, n}] - b Sum[ E2[n / (b j), k - 1, b], {j, 1, n / b}];
E2[n_, 0, a_] := 1
D1[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j
   Sum[FactorialPower[k, a] / a! E2[n / b^j, a, b], {a, 0, Log[If[b > 2, 2, b], n / b^j]}],
  {j, 0, Log[b, n]}]
D1null[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j
   Sum[FactorialPower[k, a] / a! E2null[n / b^j, a], {a, 0, Log[If[b > 2, 2, b], n / b^j]}],
  {j, 0, Log[b, n]}]
```

```
DiscretePlot[ D1[ n, -1, 2], {n, 2, 100}]
```




```
D1null[ 100, -1, 6. / 5]
```

```
0. - 1.2 (E2null[83.3333, 0] - E2null[83.3333, 1] + E2null[83.3333, 2] -
     E2null[83.3333, 3] + E2null[83.3333, 4] - E2null[83.3333, 5] + E2null[83.3333, 6] -
     E2null[83.3333, 7] + E2null[83.3333, 8] - E2null[83.3333, 9] + E2null[83.3333, 10] -
     E2null[83.3333, 11] + E2null[83.3333, 12] - E2null[83.3333, 13] +
     E2null[83.3333, 14] - E2null[83.3333, 15] + E2null[83.3333, 16] - E2null[83.3333, 17] +
     E2null[83.3333, 18] - E2null[83.3333, 19] + E2null[83.3333, 20] - E2null[83.3333, 21] +
     E2null[83.3333, 22] - E2null[83.3333, 23] + E2null[83.3333, 24]) +
 1. (E2null[100., 0] - E2null[100., 1] + E2null[100., 2] - E2null[100., 3] + E2null[100., 4] -
     E2null[100., 5] + E2null[100., 6] - E2null[100., 7] + E2null[100., 8] - E2null[100., 9] +
     E2null[100., 10] - E2null[100., 11] + E2null[100., 12] - E2null[100., 13] +
     E2null[100., 14] - E2null[100., 15] + E2null[100., 16] - E2null[100., 17] +
     E2null[100., 18] - E2null[100., 19] + E2null[100., 20] - E2null[100., 21] +
     E2null[100., 22] - E2null[100., 23] + E2null[100., 24] - E2null[100., 25])
```

```
D1[ 100, -1, 6. / 5]
```

```
1.
```

```
(-1)^(j + 1) (-1)^(k + 1) (-1)^(m + 1)
```

$(-1)^{3+j+k+m}$

```
(-1)^(j + 1) (-1)^(k + 1) (-1)^(m + 1) (-1)^(o + 1)
```

$(-1)^{4+j+k+m+o}$

```
D1null[ 1000, 1, 7]
```

$$343\left(\text{E2null}\left[\frac{1000}{343},0\right]+\text{E2null}\left[\frac{1000}{343},1\right]\right)+49\left(\text{E2null}\left[\frac{1000}{49},0\right]+\text{E2null}\left[\frac{1000}{49},1\right]\right)+7\left(\text{E2null}\left[\frac{1000}{7},0\right]+\text{E2null}\left[\frac{1000}{7},1\right]\right)+\text{E2null}[1000,0]+\text{E2null}[1000,1]$$

```
D1null[ 40, 2, 3]
```

$$108\,\text{E2null}\left[\frac{40}{27},0\right]+27\left(\text{E2null}\left[\frac{40}{9},0\right]+2\,\text{E2null}\left[\frac{40}{9},1\right]+\text{E2null}\left[\frac{40}{9},2\right]\right)+6\left(\text{E2null}\left[\frac{40}{3},0\right]+2\,\text{E2null}\left[\frac{40}{3},1\right]+\text{E2null}\left[\frac{40}{3},2\right]\right)+\text{E2null}[40,0]+2\,\text{E2null}[40,1]+\text{E2null}[40,2]$$

```
Table[ {n, E2[n, 1, 2], E2[n, 2, 2], E2[n, 3, 2], E2[n, 4, 2],
   E2[n, 5, 2], E2[n, 6, 2], E2[n, 7, 2]}, {n, 900, 1000}] // TableForm
```

900	−1	5	12	56	55	−4	118
901	0	7	12	56	55	−4	118
902	−1	1	6	56	55	−4	118
903	0	7	12	56	55	−4	118
904	−1	9	15	52	55	−4	118
905	0	11	15	52	55	−4	118
906	−1	5	9	52	55	−4	118
907	0	5	9	52	55	−4	118
908	−1	5	12	52	55	−4	118
909	0	9	15	52	55	−4	118
910	−1	−5	−21	28	55	−4	118
911	0	−5	−21	28	55	−4	118
912	−1	1	−9	−4	40	26	118
913	0	3	−9	−4	40	26	118
914	−1	1	−9	−4	40	26	118
915	0	7	−3	−4	40	26	118
916	−1	7	0	−4	40	26	118
917	0	9	0	−4	40	26	118
918	−1	−5	−45	−56	20	26	118
919	0	−5	−45	−56	20	26	118
920	−1	−3	−30	−60	0	26	118
921	0	−1	−30	−60	0	26	118
922	−1	−3	−30	−60	0	26	118
923	0	−1	−30	−60	0	26	118
924	−1	−7	−9	24	60	26	118
925	0	−3	−6	24	60	26	118
926	−1	−5	−6	24	60	26	118
927	0	−1	−3	24	60	26	118
928	−1	5	−9	16	75	20	118
929	0	5	−9	16	75	20	118
930	−1	−9	−45	−8	75	20	118
931	0	−5	−42	−8	75	20	118
932	−1	−5	−39	−8	75	20	118
933	0	−3	−39	−8	75	20	118
934	−1	−5	−39	−8	75	20	118
935	0	1	−33	−8	75	20	118
936	−1	3	0	12	5	−40	118
937	0	3	0	12	5	−40	118
938	−1	−3	−6	12	5	−40	118

939	0	−1	−6	12	5	−40	118
940	−1	−3	3	24	5	−40	118
941	0	−3	3	24	5	−40	118
942	−1	−9	−3	24	5	−40	118
943	0	−7	−3	24	5	−40	118
944	−1	−3	−3	16	10	−40	118
945	0	11	42	68	30	−40	118
946	−1	5	36	68	30	−40	118
947	0	5	36	68	30	−40	118
948	−1	3	45	80	30	−40	118
949	0	5	45	80	30	−40	118
950	−1	−5	24	68	30	−40	118
951	0	−3	24	68	30	−40	118
952	−1	−1	39	64	10	−40	118
953	0	−1	39	64	10	−40	118
954	−1	−11	18	52	10	−40	118
955	0	−9	18	52	10	−40	118
956	−1	−9	21	52	10	−40	118
957	0	−3	27	52	10	−40	118
958	−1	−5	27	52	10	−40	118
959	0	−3	27	52	10	−40	118
960	−1	11	6	−8	135	−34	−1
961	0	12	6	−8	135	−34	−1
962	−1	6	0	−8	135	−34	−1
963	0	10	3	−8	135	−34	−1
964	−1	10	6	−8	135	−34	−1
965	0	12	6	−8	135	−34	−1
966	−1	−2	−30	−32	135	−34	−1
967	0	−2	−30	−32	135	−34	−1
968	−1	0	−21	−36	125	−34	−1
969	0	6	−15	−36	125	−34	−1
970	−1	0	−21	−36	125	−34	−1
971	0	0	−21	−36	125	−34	−1
972	−1	−4	−6	44	250	50	20
973	0	−2	−6	44	250	50	20
974	−1	−4	−6	44	250	50	20
975	0	6	15	56	250	50	20
976	−1	10	15	48	255	50	20
977	0	10	15	48	255	50	20
978	−1	4	9	48	255	50	20
979	0	6	9	48	255	50	20
980	−1	2	24	96	285	50	20
981	0	6	27	96	285	50	20
982	−1	4	27	96	285	50	20
983	0	4	27	96	285	50	20
984	−1	6	42	92	265	50	20
985	0	8	42	92	265	50	20
986	−1	2	36	92	265	50	20
987	0	8	42	92	265	50	20
988	−1	6	51	104	265	50	20
989	0	8	51	104	265	50	20
990	−1	−14	−42	−28	205	50	20
991	0	−14	−42	−28	205	50	20
992	−1	−8	−48	−36	220	44	20
993	0	−6	−48	−36	220	44	20
994	−1	−12	−54	−36	220	44	20

995	0	−10	−54	−36	220	44	20
996	−1	−12	−45	−24	220	44	20
997	0	−12	−45	−24	220	44	20
998	−1	−14	−45	−24	220	44	20
999	0	−8	−36	−20	220	44	20
1000	−1	−6	−19	−16	190	24	20

```
b = 7;
Table[{n, E2[n, 1, (b + 1) / b] - E2[n - 1, 1, (b + 1) / b], tt[n, b]}, {n, 1, 100}] // TableForm
```

1	0	0
2	$-\frac{1}{7}$	$-\frac{1}{7}$
3	$-\frac{1}{7}$	$-\frac{1}{7}$
4	$-\frac{1}{7}$	$-\frac{1}{7}$
5	$-\frac{1}{7}$	$-\frac{1}{7}$
6	$-\frac{1}{7}$	$-\frac{1}{7}$
7	$-\frac{1}{7}$	$-\frac{1}{7}$
8	$-\frac{1}{7}$	$-\frac{1}{7}$
9	1	1
10	$-\frac{1}{7}$	$-\frac{1}{7}$
11	$-\frac{1}{7}$	$-\frac{1}{7}$
12	$-\frac{1}{7}$	$-\frac{1}{7}$
13	$-\frac{1}{7}$	$-\frac{1}{7}$
14	$-\frac{1}{7}$	$-\frac{1}{7}$
15	$-\frac{1}{7}$	$-\frac{1}{7}$
16	$-\frac{1}{7}$	$-\frac{1}{7}$
17	1	1
18	$-\frac{1}{7}$	$-\frac{1}{7}$
19	$-\frac{1}{7}$	$-\frac{1}{7}$
20	$-\frac{1}{7}$	$-\frac{1}{7}$
21	$-\frac{1}{7}$	$-\frac{1}{7}$
22	$-\frac{1}{7}$	$-\frac{1}{7}$
23	$-\frac{1}{7}$	$-\frac{1}{7}$
24	$-\frac{1}{7}$	$-\frac{1}{7}$
25	1	1
26	$-\frac{1}{7}$	$-\frac{1}{7}$
27	$-\frac{1}{7}$	$-\frac{1}{7}$
28	$-\frac{1}{7}$	$-\frac{1}{7}$
29	$-\frac{1}{7}$	$-\frac{1}{7}$
30	$-\frac{1}{7}$	$-\frac{1}{7}$
31	$-\frac{1}{7}$	$-\frac{1}{7}$
32	$-\frac{1}{7}$	$-\frac{1}{7}$
33	1	1
34	$-\frac{1}{7}$	$-\frac{1}{7}$

35	$-\frac{1}{7}$	$-\frac{1}{7}$
36	$-\frac{1}{7}$	$-\frac{1}{7}$
37	$-\frac{1}{7}$	$-\frac{1}{7}$
38	$-\frac{1}{7}$	$-\frac{1}{7}$
39	$-\frac{1}{7}$	$-\frac{1}{7}$
40	$-\frac{1}{7}$	$-\frac{1}{7}$
41	1	1
42	$-\frac{1}{7}$	$-\frac{1}{7}$
43	$-\frac{1}{7}$	$-\frac{1}{7}$
44	$-\frac{1}{7}$	$-\frac{1}{7}$
45	$-\frac{1}{7}$	$-\frac{1}{7}$
46	$-\frac{1}{7}$	$-\frac{1}{7}$
47	$-\frac{1}{7}$	$-\frac{1}{7}$
48	$-\frac{1}{7}$	$-\frac{1}{7}$
49	1	1
50	$-\frac{1}{7}$	$-\frac{1}{7}$
51	$-\frac{1}{7}$	$-\frac{1}{7}$
52	$-\frac{1}{7}$	$-\frac{1}{7}$
53	$-\frac{1}{7}$	$-\frac{1}{7}$
54	$-\frac{1}{7}$	$-\frac{1}{7}$
55	$-\frac{1}{7}$	$-\frac{1}{7}$
56	$-\frac{1}{7}$	$-\frac{1}{7}$
57	1	1
58	$-\frac{1}{7}$	$-\frac{1}{7}$
59	$-\frac{1}{7}$	$-\frac{1}{7}$
60	$-\frac{1}{7}$	$-\frac{1}{7}$
61	$-\frac{1}{7}$	$-\frac{1}{7}$
62	$-\frac{1}{7}$	$-\frac{1}{7}$
63	$-\frac{1}{7}$	$-\frac{1}{7}$
64	$-\frac{1}{7}$	$-\frac{1}{7}$
65	1	1
66	$-\frac{1}{7}$	$-\frac{1}{7}$
67	$-\frac{1}{7}$	$-\frac{1}{7}$
68	$-\frac{1}{7}$	$-\frac{1}{7}$
69	$-\frac{1}{7}$	$-\frac{1}{7}$
70	$-\frac{1}{7}$	$-\frac{1}{7}$
71	$-\frac{1}{7}$	$-\frac{1}{7}$
72	$-\frac{1}{7}$	$-\frac{1}{7}$
73	1	1
74	$-\frac{1}{7}$	$-\frac{1}{7}$

75	$-\frac{1}{7}$	$-\frac{1}{7}$
76	$-\frac{1}{7}$	$-\frac{1}{7}$
77	$-\frac{1}{7}$	$-\frac{1}{7}$
78	$-\frac{1}{7}$	$-\frac{1}{7}$
79	$-\frac{1}{7}$	$-\frac{1}{7}$
80	$-\frac{1}{7}$	$-\frac{1}{7}$
81	1	1
82	$-\frac{1}{7}$	$-\frac{1}{7}$
83	$-\frac{1}{7}$	$-\frac{1}{7}$
84	$-\frac{1}{7}$	$-\frac{1}{7}$
85	$-\frac{1}{7}$	$-\frac{1}{7}$
86	$-\frac{1}{7}$	$-\frac{1}{7}$
87	$-\frac{1}{7}$	$-\frac{1}{7}$
88	$-\frac{1}{7}$	$-\frac{1}{7}$
89	1	1
90	$-\frac{1}{7}$	$-\frac{1}{7}$
91	$-\frac{1}{7}$	$-\frac{1}{7}$
92	$-\frac{1}{7}$	$-\frac{1}{7}$
93	$-\frac{1}{7}$	$-\frac{1}{7}$
94	$-\frac{1}{7}$	$-\frac{1}{7}$
95	$-\frac{1}{7}$	$-\frac{1}{7}$
96	$-\frac{1}{7}$	$-\frac{1}{7}$
97	1	1
98	$-\frac{1}{7}$	$-\frac{1}{7}$
99	$-\frac{1}{7}$	$-\frac{1}{7}$
100	$-\frac{1}{7}$	$-\frac{1}{7}$

```
tt[ n_, b_] := If[ n == 1, 0, (Mod[n + b - 1, b + 1] - Mod[n + b, b + 1]) / b]
```

```
N[Sum[ tt[n, 3] / n, {n, 2, 200 000}]]
```

```
-0.245355
```

```
Et[ n_, k_, b_] := Sum[ tt[ j, b] Et[ n / j, k - 1, b], {j, 2, n}]; Et[ n_, 0, b_] := 1
```

```
Et[100, 2, 7]
```

$$\frac{11}{49}$$

```
E2[ 100, 2, 8 / 7]
```

$$\frac{491}{49}$$

```
b = 2;
Table[{n, E2[n, 2, (b + 1) / b] - E2[n - 1, 2, (b + 1) / b],
    Et[4 n, 2, b] - Et[4 (n - 1), 2, b]}, {n, 1, 100}] // TableForm
```

1	0	$\frac{1}{4}$
2	0	$-\frac{1}{2}$
3	$-\frac{3}{4}$	$\frac{1}{4}$
4	1	1
5	$\frac{3}{2}$	-1
6	-4	$-\frac{1}{2}$
7	$\frac{9}{2}$	$\frac{9}{4}$
8	-1	-2
9	$-\frac{5}{4}$	$\frac{1}{4}$
10	2	1
11	-3	-2
12	$-\frac{1}{2}$	$\frac{1}{2}$
13	0	2
14	5	-1
15	-7	-1
16	$\frac{15}{2}$	$\frac{5}{4}$
17	-3	-3
18	-2	$\frac{9}{2}$
19	0	1
20	1	-5
21	$-\frac{1}{4}$	$-\frac{3}{4}$
22	2	3
23	0	1
24	-6	-2
25	$\frac{11}{2}$	3
26	-1	-3
27	$\frac{1}{2}$	$\frac{3}{2}$
28	4	1
29	-3	$-\frac{3}{2}$
30	$-\frac{9}{2}$	-2
31	0	$\frac{9}{4}$
32	4	-3
33	-7	$\frac{3}{2}$
34	11	$\frac{11}{2}$
35	-1	-5
36	$-\frac{11}{4}$	$\frac{1}{4}$

37	0	0
38	−4	$-\frac{7}{2}$
39	$-\frac{5}{2}$	1
40	6	5
41	$\frac{9}{2}$	−1
42	−9	$-\frac{1}{2}$
43	$\frac{9}{2}$	0
44	1	$-\frac{3}{2}$
45	$-\frac{7}{2}$	2
46	2	−1
47	−3	$-\frac{5}{2}$
48	2	$\frac{3}{2}$
49	1	$\frac{15}{4}$
50	4	−4
51	−7	−2
52	$\frac{17}{2}$	$\frac{11}{2}$
53	−9	−4
54	0	2
55	2	$\frac{11}{2}$
56	3	−6
57	$-\frac{1}{4}$	$-\frac{5}{4}$
58	2	0
59	0	$-\frac{1}{2}$
60	−11	$\frac{1}{2}$
61	9	$\frac{3}{2}$
62	−1	−3
63	$-\frac{7}{2}$	3
64	5	$\frac{7}{2}$
65	−1	−3
66	$-\frac{9}{2}$	$\frac{1}{2}$
67	0	$\frac{1}{2}$
68	7	−2
69	−7	−5
70	$\frac{21}{2}$	$\frac{13}{2}$
71	−3	−2
72	$-\frac{7}{2}$	3
73	0	$-\frac{5}{4}$
74	−4	−3
75	−2	−1
76	4	$\frac{13}{2}$
77	2	−5
78	−9	1
79	9	$\frac{5}{2}$
80	5	$-\frac{11}{2}$

81	$\frac{9}{4}$	$\frac{17}{4}$
82	2	$\frac{7}{2}$
83	-9	-4
84	$-\frac{13}{2}$	-4
85	2	4
86	2	-1
87	-7	0
88	15	$\frac{1}{2}$
89	-3	$-\frac{3}{2}$
90	-5	$\frac{3}{2}$
91	2	$\frac{13}{2}$
92	1	-5
93	$-\frac{5}{2}$	1
94	2	0
95	5	-4
96	-8	$-\frac{3}{2}$
97	$\frac{9}{2}$	4
98	-5	$-\frac{11}{2}$
99	$-\frac{7}{2}$	1
100	7	$\frac{15}{4}$

8. / 3

2.66667

4. / (3 / 2)

2.66667

N[4 / (9 / 4)]

1.77778

5. / (3 / 2)

3.33333

N[5 / (9 / 4)]

2.22222

6. / (3 / 2)

4.

6. / (9 / 4)

2.66667

N[5 / (3 / 2)]

3.33333

```
N[4 / (3 / 2)]
```

2.66667

```
400.^(1 / 3)
```

7.36806

```
200.^(1 / 2)
```

14.1421

```
133^(1 / 2.)
```

11.5326

```
6 * 7 * 8
```

336

```
7 * 8 * 9
```

504

```
(400. / 6)^(1 / 2)
```

8.16497

```
Table[{j + 1, Floor[400 / 6 / j]}, {j, 7, 8}] // TableForm
```

8 9
9 8

```
E2[33, 1, 2] - E2[7 - 1, 1, 2]
```

1

```
D1[120, 2, 3 / 2]
```

602

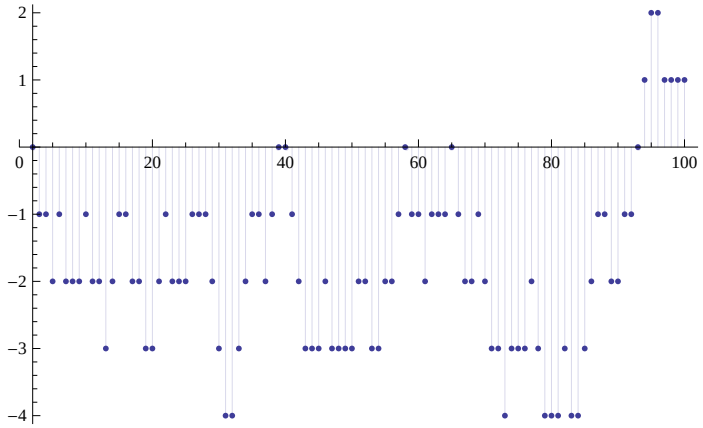
```
D2Alt[n_, a_, b_] := Sum[(j + 1) a^j / b^j
   (E2[n b^j / a^j, 0, a / b] + 2 E2[n b^j / a^j, 1, a / b] + E2[n b^j / a^j, 2, a / b]),
  {j, 0, Log[n] / (Log[a] - Log[b])}]
D2Alt2[n_, a_, b_] := Sum[(j + 1) a^j / b^j
   (1 + 2 E2[n b^j / a^j, 1, a / b] + E2[n b^j / a^j, 2, a / b]),
  {j, 0, Log[n] / (Log[a] - Log[b])}]
```

```
D2Alt2[120, 4, 3]
```

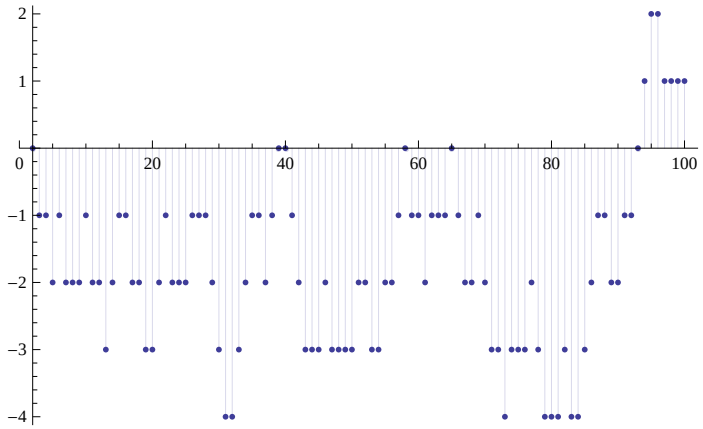
602

```
MertensAlt[n_, a_, b_] := Sum[
  (-1)^k (E2[n, k, a / b] - (a / b) E2[n b / a, k, a / b]), {k, 0, Log[n] / (Log[a] - Log[b])}]
```

```
DiscretePlot[ MAlt[n, 4, 3], {n, 2, 100}]
```




```
DiscretePlot[D1[n, -1, 4 / 3], {n, 2, 100}]
```




```
E2i[ n_, a_, b_] :=
 b^-1 If[ n < b + 1, 0, (b Floor[n b / b] - a Floor[n b / a]) - (b Floor[(b) / b] - a Floor[(b) / a])]
E2j[ n_, a_, b_] := b^-1 If[ n < b + 1, 0, (n b - a Floor[n b / a]) - (b - a Floor[b / a])]

E2[123, 1, 4 / 3]
```

$$-\frac{2}{3}$$

```
Grid[Table[ N[E2j[ n, b + 1, b]], {n, 10, 500, 10}, {b, 10, 100, 10}]]
```

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
−0.8	0.	0.	0.	0.	0.	0.	0.	0.	0.
−0.7	−0.4	0.	0.	0.	0.	0.	0.	0.	0.
−0.6	−0.9	−0.266667	0.	0.	0.	0.	0.	0.	0.
−0.5	−0.35	−0.6	−0.2	0.	0.	0.	0.	0.	0.
−0.4	−0.85	−0.933333	−0.45	−0.16	0.	0.	0.	0.	0.
−0.3	−0.3	−0.233333	−0.7	−0.36	−0.133333	0.	0.	0.	0.
−0.2	−0.8	−0.566667	−0.95	−0.56	−0.3	−0.114286	0.	0.	0.
−0.1	−0.25	−0.9	−0.175	−0.76	−0.466667	−0.257143	−0.1	0.	0.
0.	−0.75	−0.2	−0.425	−0.96	−0.633333	−0.4	−0.225	−0.0888889	0.
−1.	−0.2	−0.533333	−0.675	−0.14	−0.8	−0.542857	−0.35	−0.2	−0.08
−0.9	−0.7	−0.866667	−0.925	−0.34	−0.966667	−0.685714	−0.475	−0.311111	−0.18
−0.8	−0.15	−0.166667	−0.15	−0.54	−0.116667	−0.828571	−0.6	−0.422222	−0.28
−0.7	−0.65	−0.5	−0.4	−0.74	−0.283333	−0.971429	−0.725	−0.533333	−0.38
−0.6	−0.1	−0.833333	−0.65	−0.94	−0.45	−0.1	−0.85	−0.644444	−0.48
−0.5	−0.6	−0.133333	−0.9	−0.12	−0.616667	−0.242857	−0.975	−0.755556	−0.58
−0.4	−0.05	−0.466667	−0.125	−0.32	−0.783333	−0.385714	−0.0875	−0.866667	−0.68
−0.3	−0.55	−0.8	−0.375	−0.52	−0.95	−0.528571	−0.2125	−0.977778	−0.78
−0.2	0.	−0.1	−0.625	−0.72	−0.1	−0.671429	−0.3375	−0.0777778	−0.88
−0.1	−0.5	−0.433333	−0.875	−0.92	−0.266667	−0.814286	−0.4625	−0.188889	−0.98
0.	−1.	−0.766667	−0.1	−0.1	−0.433333	−0.957143	−0.5875	−0.3	−0.07
−1.	−0.45	−0.0666667	−0.35	−0.3	−0.6	−0.0857143	−0.7125	−0.411111	−0.17
−0.9	−0.95	−0.4	−0.6	−0.5	−0.766667	−0.228571	−0.8375	−0.522222	−0.27
−0.8	−0.4	−0.733333	−0.85	−0.7	−0.933333	−0.371429	−0.9625	−0.633333	−0.37
−0.7	−0.9	−0.0333333	−0.075	−0.9	−0.0833333	−0.514286	−0.075	−0.744444	−0.47
−0.6	−0.35	−0.366667	−0.325	−0.08	−0.25	−0.657143	−0.2	−0.855556	−0.57
−0.5	−0.85	−0.7	−0.575	−0.28	−0.416667	−0.8	−0.325	−0.966667	−0.67
−0.4	−0.3	0.	−0.825	−0.48	−0.583333	−0.942857	−0.45	−0.0666667	−0.77
−0.3	−0.8	−0.333333	−0.05	−0.68	−0.75	−0.0714286	−0.575	−0.177778	−0.87
−0.2	−0.25	−0.666667	−0.3	−0.88	−0.916667	−0.214286	−0.7	−0.288889	−0.97
−0.1	−0.75	−1.	−0.55	−0.06	−0.0666667	−0.357143	−0.825	−0.4	−0.06
0.	−0.2	−0.3	−0.8	−0.26	−0.233333	−0.5	−0.95	−0.511111	−0.16
−1.	−0.7	−0.633333	−0.025	−0.46	−0.4	−0.642857	−0.0625	−0.622222	−0.26
−0.9	−0.15	−0.966667	−0.275	−0.66	−0.566667	−0.785714	−0.1875	−0.733333	−0.36
−0.8	−0.65	−0.266667	−0.525	−0.86	−0.733333	−0.928571	−0.3125	−0.844444	−0.46
−0.7	−0.1	−0.6	−0.775	−0.04	−0.9	−0.0571429	−0.4375	−0.955556	−0.56
−0.6	−0.6	−0.933333	0.	−0.24	−0.05	−0.2	−0.5625	−0.0555556	−0.66
−0.5	−0.05	−0.233333	−0.25	−0.44	−0.216667	−0.342857	−0.6875	−0.166667	−0.76
−0.4	−0.55	−0.566667	−0.5	−0.64	−0.383333	−0.485714	−0.8125	−0.277778	−0.86
−0.3	0.	−0.9	−0.75	−0.84	−0.55	−0.628571	−0.9375	−0.388889	−0.96
−0.2	−0.5	−0.2	−1.	−0.02	−0.716667	−0.771429	−0.05	−0.5	−0.05
−0.1	−1.	−0.533333	−0.225	−0.22	−0.883333	−0.914286	−0.175	−0.611111	−0.15
0.	−0.45	−0.866667	−0.475	−0.42	−0.0333333	−0.0428571	−0.3	−0.722222	−0.25
−1.	−0.95	−0.166667	−0.725	−0.62	−0.2	−0.185714	−0.425	−0.833333	−0.35
−0.9	−0.4	−0.5	−0.975	−0.82	−0.366667	−0.328571	−0.55	−0.944444	−0.45
−0.8	−0.9	−0.833333	−0.2	0.	−0.533333	−0.471429	−0.675	−0.0444444	−0.55
−0.7	−0.35	−0.133333	−0.45	−0.2	−0.7	−0.614286	−0.8	−0.155556	−0.65
−0.6	−0.85	−0.466667	−0.7	−0.4	−0.866667	−0.757143	−0.925	−0.266667	−0.75
−0.5	−0.3	−0.8	−0.95	−0.6	−0.0166667	−0.9	−0.0375	−0.377778	−0.85
−0.4	−0.8	−0.1	−0.175	−0.8	−0.183333	−0.0285714	−0.1625	−0.488889	−0.95

```
Expand[(n (b - 1)) / b]
```

$$n - \frac{n}{b}$$

```
MertensReference[n_] := Sum[MoebiusMu[j], {j, 1, n}]
alpha[n_, a_, b_] := b (Floor[n / b] - Floor[(n - 1) / b]) - a (Floor[n / a] - Floor[(n - 1) / a])
E2[n_, k_, a_, b_] := E2[n, k, a, b] =
  (1 / b) Sum[If[alpha[j, a, b] == 0, 0, alpha[j, a, b] E2[(b n) / j, k - 1, a, b]], {j, b + 1, b n}];
E2[n_, 0, a_, b_] := 1
MertensAlt[n_, a_, b_] := Sum[(-1)^k (E2[n, k, a, b] - (a / b) E2[n b / a, k, a, b]),
  {k, 0, Floor[Log[n] / (Log[a] - Log[b])]}]
Grid[Table[{MertensReference[n], MertensAlt[n, b + 1, b]}, {n, 10, 100, 10}, {b, 1, 5}]]
```

```
{-1, -1}  {-1, -1}  {-1, -1}  {-1, -1}  {-1, -1}
{-3, -3}  {-3, -3}  {-3, -3}  {-3, -3}  {-3, -3}
{-3, -3}  {-3, -3}  {-3, -3}  {-3, -3}  {-3, -3}
  {0, 0}    {0, 0}    {0, 0}    {0, 0}    {0, 0}
{-3, -3}  {-3, -3}  {-3, -3}  {-3, -3}  {-3, -3}
{-1, -1}  {-1, -1}  {-1, -1}  {-1, -1}  {-1, -1}
{-2, -2}  {-2, -2}  {-2, -2}  {-2, -2}  {-2, -2}
{-4, -4}  {-4, -4}  {-4, -4}  {-4, -4}  {-4, -4}
{-2, -2}  {-2, -2}  {-2, -2}  {-2, -2}  {-2, -2}
  {1, 1}    {1, 1}    {1, 1}    {1, 1}    {1, 1}
```

```
d1[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
ReferenceD1[n_, z_] := Sum[d1[j, z], {j, 1, n}]
alpha[n_, a_, b_] := b (Floor[n / b] - Floor[(n - 1) / b]) - a (Floor[n / a] - Floor[(n - 1) / a])
E2[n_, k_, a_, b_] := E2[n, k, a, b] =
  (1 / b) Sum[If[alpha[j, a, b] == 0, 0, alpha[j, a, b] E2[(b n) / j, k - 1, a, b]], {j, b + 1, b n}];
E2[n_, 0, a_, b_] := 1
D1Alt[n_, z_, a_, b_] := Sum[(-1)^j Binomial[-z, j] Binomial[z, k] a^j / b^j
    E2[n b^j / a^j, k, a, b], {j, 0, Floor[Log[n] / (Log[a] - Log[b])]},
  {k, 0, Floor[(Log[n] + j Log[b] - j Log[a]) / (Log[a] - Log[b])]}]
Grid[Table[{ReferenceD1[123, j + 1 / 3], D1Alt[123, j + 1 / 3, b + 1, b]},
  {j, 1, 5}, {b, 1, 5}]]
```

$$\begin{array}{ccccc}
\left\{\frac{1496045}{6561}, \frac{1496045}{6561}\right\} & \left\{\frac{1496045}{6561}, \frac{1496045}{6561}\right\} & \left\{\frac{1496045}{6561}, \frac{1496045}{6561}\right\} & \left\{\frac{1496045}{6561}, \frac{1496045}{6561}\right\} & \left\{\frac{1496045}{6561}, \frac{1496045}{6561}\right\} \\
\left\{\frac{6095315}{6561}, \frac{6095315}{6561}\right\} & \left\{\frac{6095315}{6561}, \frac{6095315}{6561}\right\} & \left\{\frac{6095315}{6561}, \frac{6095315}{6561}\right\} & \left\{\frac{6095315}{6561}, \frac{6095315}{6561}\right\} & \left\{\frac{6095315}{6561}, \frac{6095315}{6561}\right\} \\
\left\{\frac{17459030}{6561}, \frac{17459030}{6561}\right\} & \left\{\frac{17459030}{6561}, \frac{17459030}{6561}\right\} & \left\{\frac{17459030}{6561}, \frac{17459030}{6561}\right\} & \left\{\frac{17459030}{6561}, \frac{17459030}{6561}\right\} & \left\{\frac{17459030}{6561}, \frac{17459030}{6561}\right\} \\
\left\{\frac{41166875}{6561}, \frac{41166875}{6561}\right\} & \left\{\frac{41166875}{6561}, \frac{41166875}{6561}\right\} & \left\{\frac{41166875}{6561}, \frac{41166875}{6561}\right\} & \left\{\frac{41166875}{6561}, \frac{41166875}{6561}\right\} & \left\{\frac{41166875}{6561}, \frac{41166875}{6561}\right\} \\
\left\{\frac{85424393}{6561}, \frac{85424393}{6561}\right\} & \left\{\frac{85424393}{6561}, \frac{85424393}{6561}\right\} & \left\{\frac{85424393}{6561}, \frac{85424393}{6561}\right\} & \left\{\frac{85424393}{6561}, \frac{85424393}{6561}\right\} & \left\{\frac{85424393}{6561}, \frac{85424393}{6561}\right\}
\end{array}$$

```
d1[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}];
FI[n_] := FactorInteger[n]; FI[1] := {}
ReferenceD1[n_, z_] := Sum[d1[j, z], {j, 1, n}]
alpha[n_, a_, b_] := b (Floor[n / b] - Floor[(n - 1) / b]) - a (Floor[n / a] - Floor[(n - 1) / a])
E2[n_, k_, a_, b_] := E2[n, k, a, b] =
  (1 / b) Sum[If[alpha[j, a, b] == 0, 0, alpha[j, a, b] E2[(b n) / j, k - 1, a, b]], {j, b + 1, b n}];
E2[n_, 0, a_, b_] := 1
DAlt[ n_, a_, b_] := Sum[ (j + 1) a^j / b^j
   ( E2[ n b^j / a^j, 0, a, b] + 2 E2[ n b^j / a^j, 1, a, b] + E2[ n b^j / a^j, 2, a, b] ),
  { j, 0, Log[n] / ( Log[a] - Log[b])}]
Grid[ Table[ {ReferenceD1[n, 2 ], DAlt[ n, b + 1, b]}, {n, 10, 100, 10}, {b, 1, 7}]]
```

{27, 27}	{27, 27}	{27, 27}	{27, 27}	{27, 27}	{27, 27}	{27, 27}
{66, 66}	{66, 66}	{66, 66}	{66, 66}	{66, 66}	{66, 66}	{66, 66}
{111, 111}	{111, 111}	{111, 111}	{111, 111}	{111, 111}	{111, 111}	{111, 111}
{158, 158}	{158, 158}	{158, 158}	{158, 158}	{158, 158}	{158, 158}	{158, 158}
{207, 207}	{207, 207}	{207, 207}	{207, 207}	{207, 207}	{207, 207}	{207, 207}
{261, 261}	{261, 261}	{261, 261}	{261, 261}	{261, 261}	{261, 261}	{261, 261}
{312, 312}	{312, 312}	{312, 312}	{312, 312}	{312, 312}	{312, 312}	{312, 312}
{368, 368}	{368, 368}	{368, 368}	{368, 368}	{368, 368}	{368, 368}	{368, 368}
{425, 425}	{425, 425}	{425, 425}	{425, 425}	{425, 425}	{425, 425}	{425, 425}
{482, 482}	{482, 482}	{482, 482}	{482, 482}	{482, 482}	{482, 482}	{482, 482}

```
alpha[n_, a_, b_] := b (Floor[n / b] - Floor[(n - 1) / b]) - a (Floor[n / a] - Floor[(n - 1) / a])
E1[n_, k_, a_, b_] := E1[n, k, a, b] =
  (1 / b) Sum[If[alpha[j, a, b] == 0, 0, alpha[j, a, b] E1[(b n) / j, k - 1, a, b]], {j, 1, b n}]; E1[
  n_, 0, a_, b_] := 1
E2[n_, k_, a_, b_] := E2[n, k, a, b] =
  (1 / b) Sum[If[alpha[j, a, b] == 0, 0, alpha[j, a, b] E2[(b n) / j, k - 1, a, b]], {j, b + 1, b n}];
E2[n_, 0, a_, b_] := 1
E1Alt[n_, z_, a_, b_] :=
 Sum[Binomial[z, k] E2[n, k, a, b], {k, 0, Floor[Log[n] / (Log[a] - Log[b])]}]
Grid[ Table[ {E1[n, 3, b + 1, b ], E1Alt[ n, 3, b + 1, b]}, {n, 10, 80, 10}, {b, 1, 6}]]
```

{-3, -3}	$\{\frac{59}{4}, \frac{59}{4}\}$	$\{-\frac{121}{27}, -\frac{121}{27}\}$	$\{-\frac{57}{16}, -\frac{57}{16}\}$	$\{-\frac{431}{125}, -\frac{431}{125}\}$	$\{-\frac{1159}{216}, -\frac{1159}{216}\}$
{-6, -6}	{8, 8}	$\{-\frac{524}{27}, -\frac{524}{27}\}$	$\{-\frac{1097}{64}, -\frac{1097}{64}\}$	$\{-\frac{1016}{125}, -\frac{1016}{125}\}$	$\{-\frac{1063}{108}, -\frac{1063}{108}\}$
{-14, -14}	$\{-\frac{33}{2}, -\frac{33}{2}\}$	$\{\frac{196}{27}, \frac{196}{27}\}$	$\{-\frac{2731}{64}, -\frac{2731}{64}\}$	$\{-\frac{1878}{125}, -\frac{1878}{125}\}$	$\{\frac{6619}{216}, \frac{6619}{216}\}$
{10, 10}	$\{\frac{27}{4}, \frac{27}{4}\}$	$\{-\frac{1190}{27}, -\frac{1190}{27}\}$	$\{-\frac{451}{16}, -\frac{451}{16}\}$	$\{-\frac{1548}{125}, -\frac{1548}{125}\}$	$\{-\frac{3053}{216}, -\frac{3053}{216}\}$
{-2, -2}	$\{\frac{123}{4}, \frac{123}{4}\}$	$\{\frac{820}{27}, \frac{820}{27}\}$	$\{\frac{611}{64}, \frac{611}{64}\}$	$\{\frac{1434}{125}, \frac{1434}{125}\}$	$\{\frac{1181}{24}, \frac{1181}{24}\}$
{-2, -2}	$\{-\frac{279}{8}, -\frac{279}{8}\}$	$\{-\frac{1118}{27}, -\frac{1118}{27}\}$	$\{-\frac{273}{4}, -\frac{273}{4}\}$	$\{-\frac{5058}{125}, -\frac{5058}{125}\}$	$\{\frac{901}{18}, \frac{901}{18}\}$
{-21, -21}	$\{\frac{181}{4}, \frac{181}{4}\}$	$\{-\frac{155}{9}, -\frac{155}{9}\}$	$\{-\frac{3599}{64}, -\frac{3599}{64}\}$	$\{\frac{6851}{125}, \frac{6851}{125}\}$	$\{-\frac{6671}{72}, -\frac{6671}{72}\}$
{0, 0}	$\{\frac{275}{8}, \frac{275}{8}\}$	$\{-\frac{220}{9}, -\frac{220}{9}\}$	$\{-\frac{757}{32}, -\frac{757}{32}\}$	$\{-\frac{3352}{125}, -\frac{3352}{125}\}$	$\{-\frac{589}{36}, -\frac{589}{36}\}$