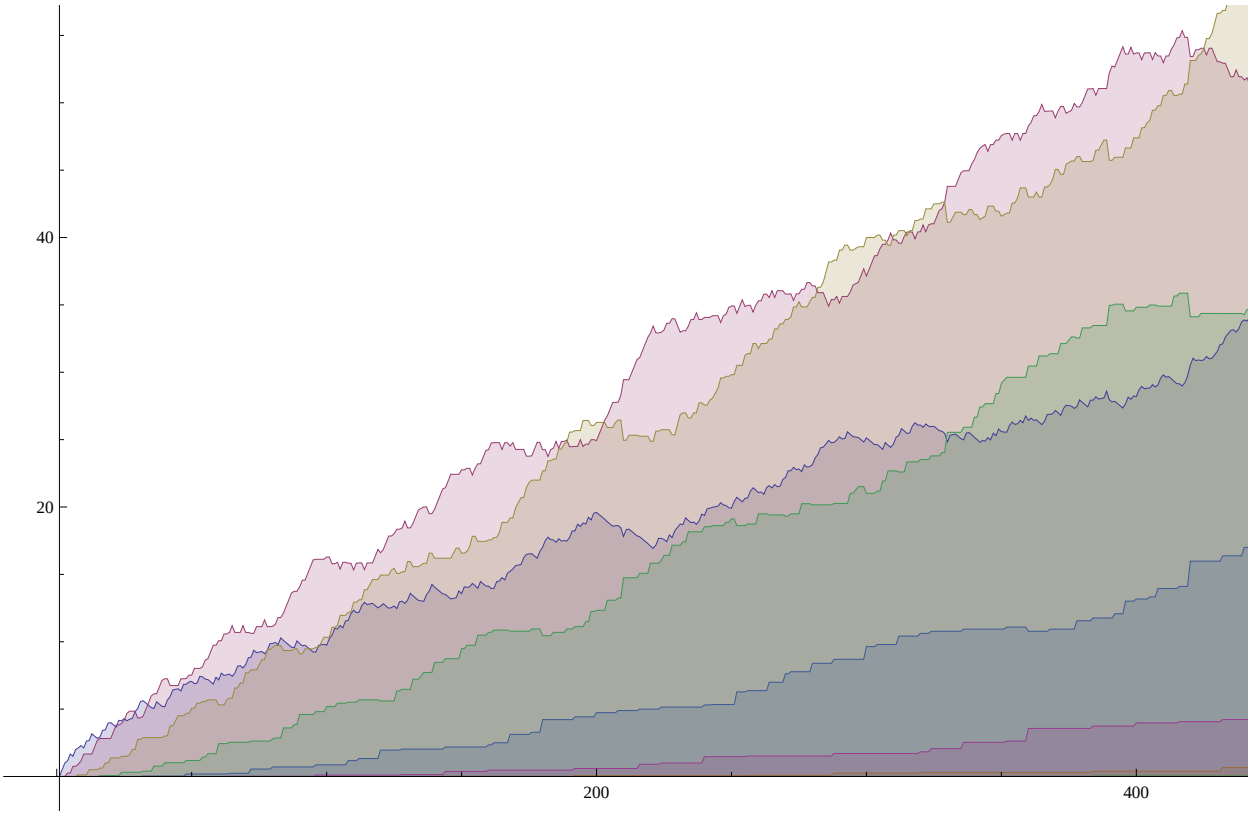
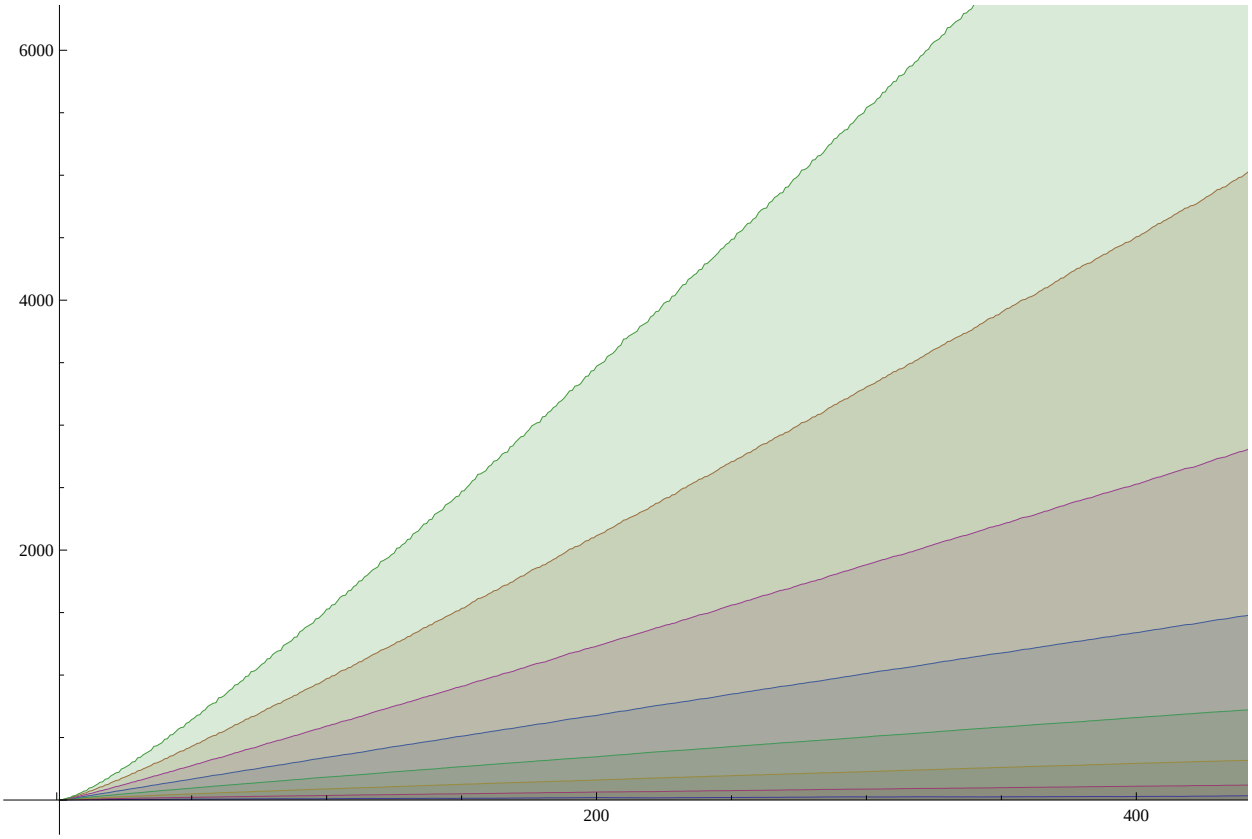
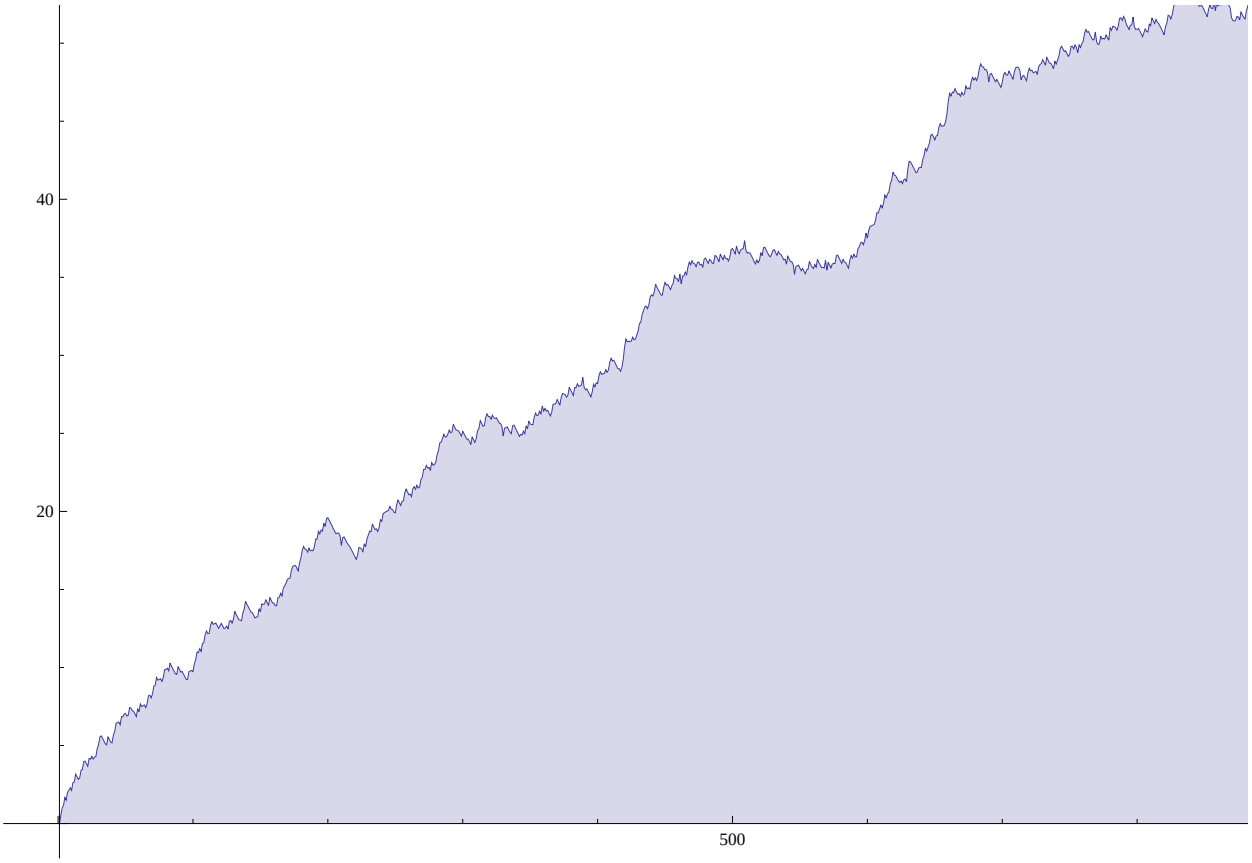
```
ClearAll["Global`*"]
d2[n_, k_] := d2[n, k] = Sum[d2[j, k - 1] d2[n / j, 1], {j, Divisors[n]}];
d2[n_, 1] := 1; d2[1, 1] := 0; d2[n_, 0] := 0; d2[1, 0] := 1
D2[n_, k_] := D2[n, k] = D2[n - 1, k] + d2[n, k]; D2[1, k_] := 0
K[n_] := K[n] = FullSimplify[MangoldtLambda[n] / Log[n]]
k2[n_, k_] := k2[n, k] = Sum[k2[j, k - 1] k2[n / j, 1], {j, Divisors[n]}];
k2[n_, 1] := K[n]; k2[1, 1] := 0; k2[n_, 0] := 0; k2[1, 0] := 1
K2[n_, k_] := K2[n, k] = K2[n - 1, k] + k2[n, k]; K2[1, k_] := 0

cc := cc = CoefficientList[Series[x / Log[1 + x], {x, 0, 30}], x]
e2[n_, 1] := e2[n, 1] = Sum[ cc[[k + 1]] k2[n, k], {k, 0, Log[2, n]}]; e2[1, 1] := 0;
e2[n_, k_] := Sum[e2[j, k - 1] e2[n / j, 1], {j, Divisors[n]}]; e2[n_, 0] := 0; e2[1, 0] := 1
E2[n_, k_] := E2[n, k] = E2[n - 1, k] + e2[n, k]; E2[1, k_] := 0
E1[n_, k_] := Sum[ Binomial[k, j] E2[n, j], {j, 0, k}]
DiscretePlot[
 {E2[n, 1], E2[n, 2], E2[n, 3], E2[n, 4], E2[n, 5], E2[n, 6], E2[n, 7], E2[n, 8]}, {n, 1, 1000}]
DiscretePlot[ {E1[n, 1], E1[n, 2], E1[n, 3], E1[n, 4],
  E1[n, 5], E1[n, 6], E1[n, 7], E1[n, 8]}, {n, 1, 1000}]
DiscretePlot[ {E1[n, 1]}, {n, 1, 2000}]
```


40
20
200
400

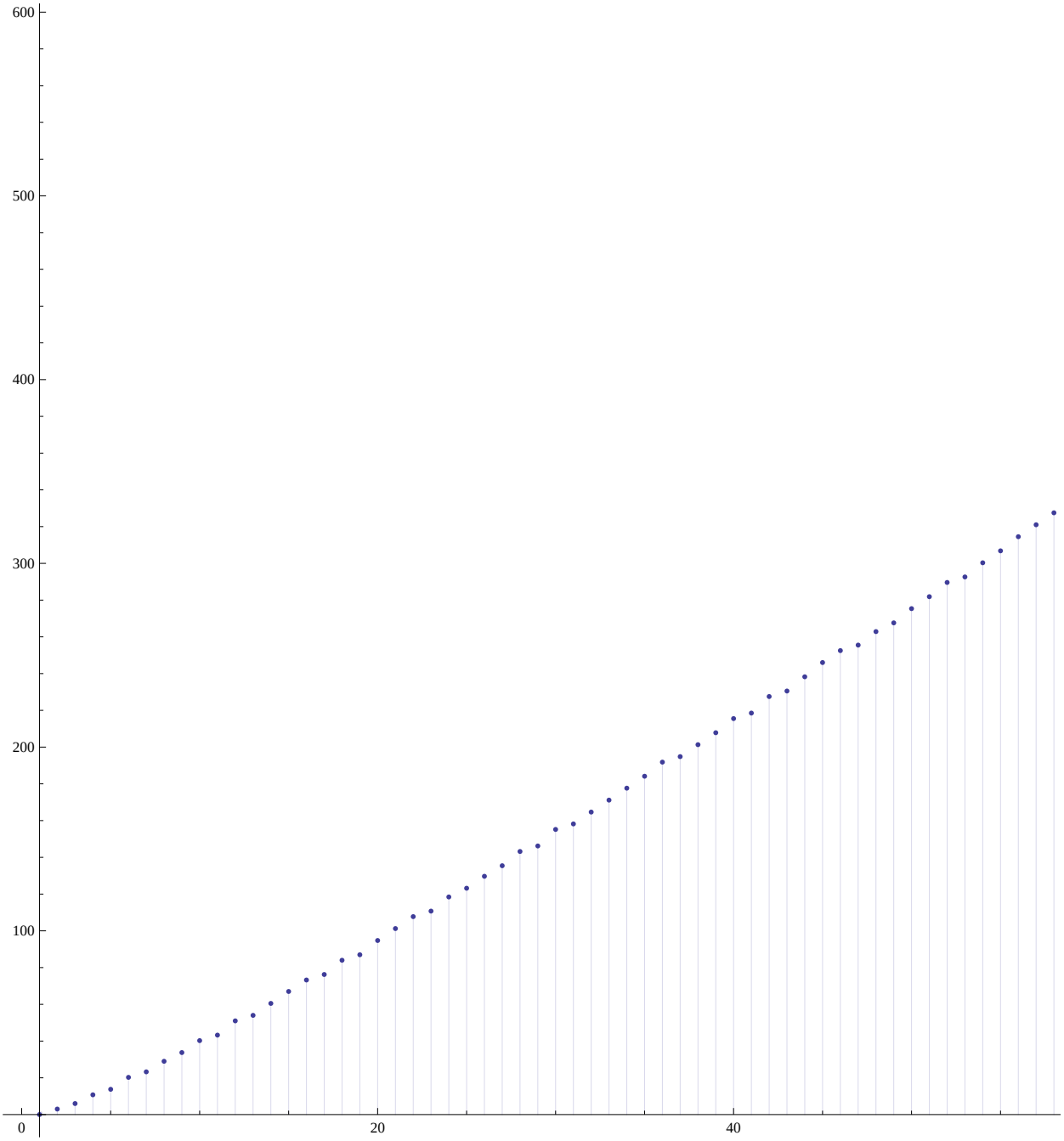

12 000
10 000
8000


6000
4000
2000
200
400


100
80
60




```
DiscretePlot[{E1[n, 6]}, {n, 1, 100}]
```




```
Table[{n, E2[n, 4]}, {n, 2, 100}] // TableForm
```

2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0

12	0
13	0
14	0
15	0
16	$\frac{1}{16}$
17	$\frac{1}{16}$
18	$\frac{1}{16}$
19	$\frac{1}{16}$
20	$\frac{1}{16}$
21	$\frac{1}{16}$
22	$\frac{1}{16}$
23	$\frac{1}{16}$
24	$\frac{5}{16}$
25	$\frac{5}{16}$
26	$\frac{5}{16}$
27	$\frac{5}{16}$
28	$\frac{5}{16}$
29	$\frac{5}{16}$
30	$\frac{5}{16}$
31	$\frac{5}{16}$
32	$\frac{19}{48}$
33	$\frac{19}{48}$
34	$\frac{19}{48}$
35	$\frac{19}{48}$
36	$\frac{37}{48}$
37	$\frac{37}{48}$
38	$\frac{37}{48}$
39	$\frac{37}{48}$
40	$\frac{49}{48}$
41	$\frac{49}{48}$
42	$\frac{49}{48}$
43	$\frac{49}{48}$
44	$\frac{49}{48}$
45	$\frac{49}{48}$
46	$\frac{49}{48}$
47	$\frac{49}{48}$
48	$\frac{19}{16}$
49	$\frac{19}{16}$
50	$\frac{19}{16}$
51	$\frac{19}{16}$

52 $\frac{19}{16}$

53 $\frac{19}{16}$

54 $\frac{23}{16}$

55 $\frac{23}{16}$

56 $\frac{27}{16}$

57 $\frac{27}{16}$

58 $\frac{27}{16}$

59 $\frac{27}{16}$

60 $\frac{39}{16}$

61 $\frac{39}{16}$

62 $\frac{39}{16}$

63 $\frac{39}{16}$

64 $\frac{61}{24}$

65 $\frac{61}{24}$

66 $\frac{61}{24}$

67 $\frac{61}{24}$

68 $\frac{61}{24}$

69 $\frac{61}{24}$

70 $\frac{61}{24}$

71 $\frac{61}{24}$

72 $\frac{21}{8}$

73 $\frac{21}{8}$

74 $\frac{21}{8}$

75 $\frac{21}{8}$

76 $\frac{21}{8}$

77 $\frac{21}{8}$

78 $\frac{21}{8}$

79 $\frac{21}{8}$

80 $\frac{67}{24}$

81 $\frac{137}{48}$

82 $\frac{137}{48}$

83 $\frac{137}{48}$

84 $\frac{173}{48}$

85 $\frac{173}{48}$

86 $\frac{173}{48}$

87 $\frac{173}{48}$

88 $\frac{185}{48}$

89 $\frac{185}{48}$

90 $\frac{221}{48}$

91 $\frac{221}{48}$

92 $\frac{221}{48}$

93 $\frac{221}{48}$

94 $\frac{221}{48}$

95 $\frac{221}{48}$

96 $\frac{77}{16}$

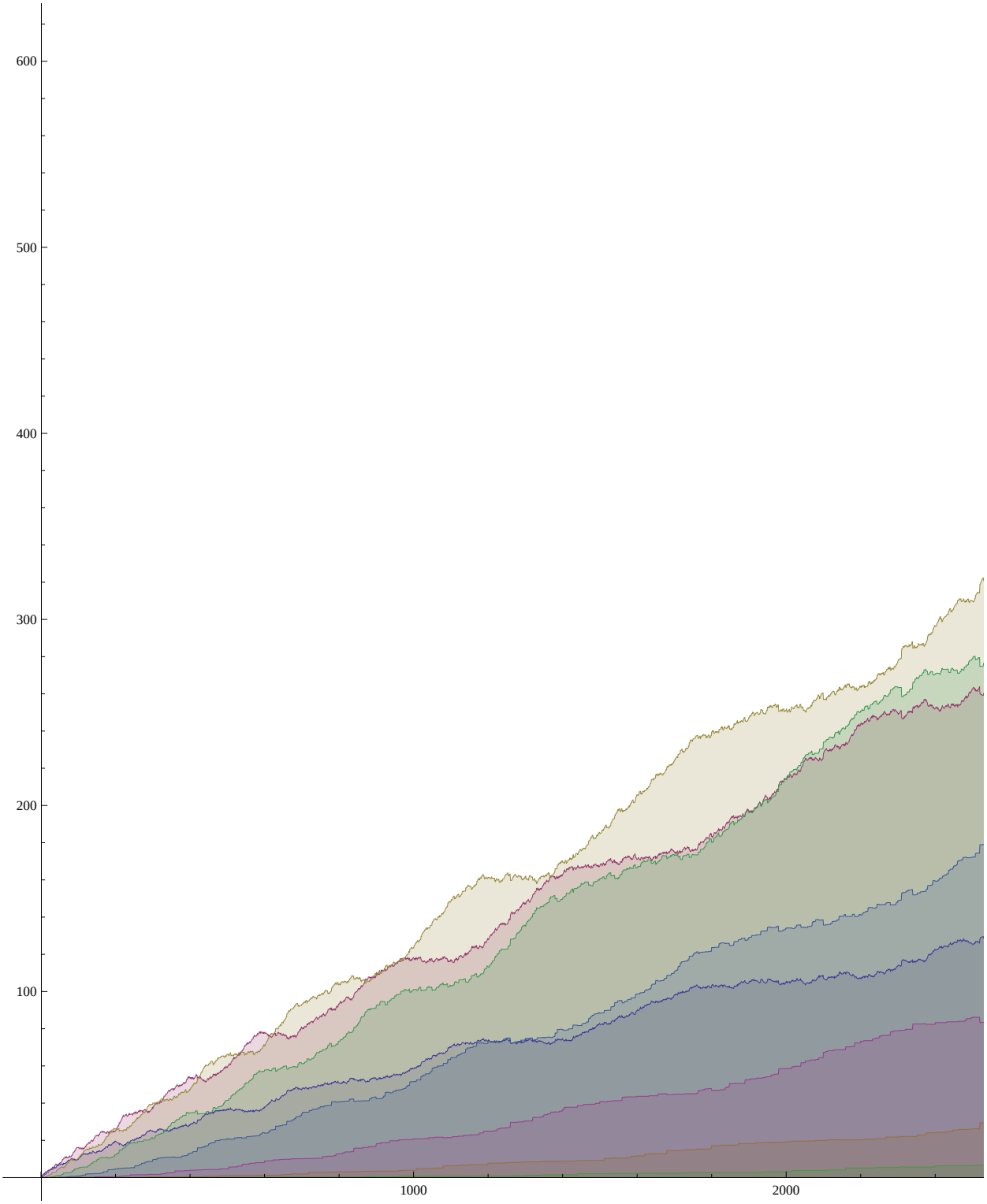
97 $\frac{77}{16}$

98 $\frac{77}{16}$

99 $\frac{77}{16}$

100 $\frac{83}{16}$

```
DiscretePlot[
 {E2[n, 1], E2[n, 2], E2[n, 3], E2[n, 4], E2[n, 5], E2[n, 6], E2[n, 7], E2[n, 8]}, {n, 1, 5000}]
```


600
500
400
300
200
100
1000
2000