

$$\text{Limit}\left[\text{Sum}\left[\frac{(a^k - 1)}{k}, \{k, 1, \text{Log}[a, 100]\}\right], a \rightarrow 1\right]$$

$$\text{Limit}\left[-\text{HarmonicNumber}\left[\frac{\text{Log}[100]}{\text{Log}[a]}\right] - 100 a \text{LerchPhi}\left[a, 1, 1 + \frac{\text{Log}[100]}{\text{Log}[a]}\right] - \text{Log}[1 - a], a \rightarrow 1\right]$$

$$\text{Sum}\left[-(1 - \text{Log}[n])^k / k, \{k, 1, \text{Infinity}\}\right]$$

$$\text{Log}[\text{Log}[n]]$$

$$\text{Limit}\left[-\text{HarmonicNumber}\left[\frac{\text{Log}[100]}{\text{Log}[a]}\right] - \text{Log}[1 - a], a \rightarrow 1\right]$$

$$-\text{EulerGamma} - i\pi - \text{Log}[\text{Log}[100]]$$

$$\text{Limit}\left[\text{Sum}\left[\frac{(a^k - 1)}{k} - (1 - \text{Log}[100])^k / k, \{k, 1, \text{Log}[a, 100]\}\right], a \rightarrow 1\right]$$

$$\text{Limit}\left[-\text{HarmonicNumber}\left[\frac{\text{Log}[100]}{\text{Log}[a]}\right] - 100 a \text{LerchPhi}\left[a, 1, 1 + \frac{\text{Log}[100]}{\text{Log}[a]}\right] +$$

$$\text{LerchPhi}\left[1 - \text{Log}[100], 1, 1 + \frac{\text{Log}[100]}{\text{Log}[a]}\right] (1 - \text{Log}[100])^{\frac{\text{Log}[100]}{\text{Log}[a]}} -$$

$$\text{LerchPhi}\left[1 - \text{Log}[100], 1, 1 + \frac{\text{Log}[100]}{\text{Log}[a]}\right] (1 - \text{Log}[100])^{\frac{\text{Log}[100]}{\text{Log}[a]}} \text{Log}[100] -$$

$$\text{Log}[1 - a] + \text{Log}[\text{Log}[100]], a \rightarrow 1\right]$$

$$\text{Limit}\left[\text{Sum}\left[-(1 - \text{Log}[100])^k / k, \{k, 1, \text{Log}[a, 100]\}\right], a \rightarrow 1\right]$$

$$\text{Limit}\left[\text{LerchPhi}\left[1 - \text{Log}[100], 1, 1 + \frac{\text{Log}[100]}{\text{Log}[a]}\right] (1 - \text{Log}[100])^{\frac{\text{Log}[100]}{\text{Log}[a]}} -$$

$$\text{LerchPhi}\left[1 - \text{Log}[100], 1, 1 + \frac{\text{Log}[100]}{\text{Log}[a]}\right] (1 - \text{Log}[100])^{\frac{\text{Log}[100]}{\text{Log}[a]}} \text{Log}[100] + \text{Log}[\text{Log}[100]], a \rightarrow 1\right]$$

$$\text{ff}[a_]:= \text{LerchPhi}\left[1 - \text{Log}[100], 1, 1 + \frac{\text{Log}[100]}{\text{Log}[a]}\right] (1 - \text{Log}[100])^{\frac{\text{Log}[100]}{\text{Log}[a]}} -$$

$$\text{LerchPhi}\left[1 - \text{Log}[100], 1, 1 + \frac{\text{Log}[100]}{\text{Log}[a]}\right] (1 - \text{Log}[100])^{\frac{\text{Log}[100]}{\text{Log}[a]}} \text{Log}[100]$$

$$\text{Limit}\left[\text{Sum}\left[(1 - 1 / \text{Log}[100])^k / k, \{k, 1, \text{Log}[a, 100]\}\right], a \rightarrow 1\right]$$

$$\text{Limit}\left[-\frac{1}{\text{Log}[100]}\right.$$

$$\left(-\text{LerchPhi}\left[1 - \frac{1}{\text{Log}[100]}, 1, 1 + \frac{\text{Log}[100]}{\text{Log}[a]}\right] \left(1 - \frac{1}{\text{Log}[100]}\right)^{\frac{\text{Log}[100]}{\text{Log}[a]}} + \text{LerchPhi}\left[1 - \frac{1}{\text{Log}[100]},$$

$$1, 1 + \frac{\text{Log}[100]}{\text{Log}[a]}\right] \left(1 - \frac{1}{\text{Log}[100]}\right)^{\frac{\text{Log}[100]}{\text{Log}[a]}} \text{Log}[100] - \text{Log}[100] \text{Log}[\text{Log}[100]]\right), a \rightarrow 1\right]$$

$$\text{Sum}\left[(1 - 1 / (\text{Log}[100]))^k / k, \{k, 1, \text{Infinity}\}\right]$$

$$\text{Log}[\text{Log}[100]]$$

$$\text{N}[\text{Log}[100]]$$

$$4.60517$$

$$\text{Sum}\left[(1 - 100^{-1})^k / k, \{k, 1, \text{Infinity}\}\right]$$

$$\text{Log}[100]$$

```

Sum[ (1 - 1 / Log[100]) ^ k / k, {k, 1, Infinity}]
Log[Log[100]]
Limit[Sum[ (1 - 1 / Log[100]) ^ k / k, {k, 1, a}], a → Infinity]
Limit[ -  $\frac{1}{\text{Log}[100]}$   $\left( -\text{LerchPhi}\left[1 - \frac{1}{\text{Log}[100]}, 1, 1 + a\right] \left(1 - \frac{1}{\text{Log}[100]}\right)^a + \right.$ 
       $\left. \text{LerchPhi}\left[1 - \frac{1}{\text{Log}[100]}, 1, 1 + a\right] \left(1 - \frac{1}{\text{Log}[100]}\right)^a \text{Log}[100] - \text{Log}[100] \text{Log}[\text{Log}[100]] \right), a \rightarrow \infty ]$ 
Sum[ (1 - 1 / Log[100]) ^ k / k, {k, 1, Infinity}]
Log[Log[100]]
Limit[Sum[ a - 1, {k, 1, Log[a, n]}], {a → 1}]
{Log[n]}
Expand[- (a - 1) (-1)]
-1 + a
Limit[Sum[1 / k, {k, 1, a}] - Log[a], {a → Infinity}]
{EulerGamma}
Limit[Sum[1 / k, {k, 1, a}] + Sum[ (1 - a^-1) ^ k / k, {k, 1, a}], {a → Infinity}]
{Limit[ HarmonicNumber[a] -  $\frac{1}{a} \left( - \left( \frac{-1 + a}{a} \right)^a \text{LerchPhi}\left[\frac{-1 + a}{a}, 1, 1 + a\right] + \right.$ 
       $\left. \left( \frac{-1 + a}{a} \right)^a a \text{LerchPhi}\left[\frac{-1 + a}{a}, 1, 1 + a\right] + a \text{Log}\left[\frac{1}{a}\right] \right), a \rightarrow \infty ]}$ 
ff[a_] := HarmonicNumber[a] -  $\frac{1}{a}$ 
       $\left( - \left( \frac{-1 + a}{a} \right)^a \text{LerchPhi}\left[\frac{-1 + a}{a}, 1, 1 + a\right] + \left( \frac{-1 + a}{a} \right)^a a \text{LerchPhi}\left[\frac{-1 + a}{a}, 1, 1 + a\right] + a \text{Log}\left[\frac{1}{a}\right] \right)$ 
N[ff[100]]
9.57683
Sum[ (1 - a^-1) ^ k / k, {k, 1, Infinity}] /. a → 10
Log[10]
Limit[Sum[1 / k, {k, 1, a}] - Log[a], a → Infinity]
EulerGamma
Sum[ (1 - a^-1) ^ k / k, {k, 1, Infinity}]
-Log[ $\frac{1}{a}$ ]
Limit[Sum[ a - 1, {k, 1, Log[a, Log[n]]}], {a → 1}]
{Log[Log[n]]}

```

Limit[**Sum**[**a** - 1 - 1 / **k**, {**k**, 1, **Log**[**a**, **n**]}], {**a** → 1}]

$$\left\{ \text{Limit} \left[\frac{-\text{HarmonicNumber} \left[\frac{\text{Log}[n]}{\text{Log}[a]} \right] \text{Log}[a] - \text{Log}[n] + a \text{Log}[n]}{\text{Log}[a]}, a \rightarrow 1 \right] \right\}$$

$$\text{fa}[n_ , a_] := \frac{-\text{HarmonicNumber} \left[\frac{\text{Log}[n]}{\text{Log}[a]} \right] \text{Log}[a] - \text{Log}[n] + a \text{Log}[n]}{\text{Log}[a]}$$

N[**fa**[100, 1.001]]

-4.40529

Limit[**Sum**[**k**^**a** (**a** - 1) ^**k**, {**k**, 1, **Log**[**a**, **n**]}], **a** → 1]

$$\text{Limit} \left[(-1 + a)^{\frac{\text{Log}[n]}{\text{Log}[a]}} \text{LerchPhi} \left[-1 + a, -a, 1 + \frac{\text{Log}[n]}{\text{Log}[a]} \right] - \right. \\ \left. (-1 + a)^{\frac{\text{Log}[n]}{\text{Log}[a]}} a \text{LerchPhi} \left[-1 + a, -a, 1 + \frac{\text{Log}[n]}{\text{Log}[a]} \right] + \text{PolyLog}[-a, -1 + a], a \rightarrow 1 \right]$$

pp[**s**_, **a**_] := (-1) ^**s** (**a** - 1) ^**s** **LerchPhi**[**a**, 1 - **s**, 0]

pp2[**s**_, **a**_] := **LerchPhi**[**a**, 1 - **s**, 50]

pp[4, 1.00000001]

6.

N[**Gamma**[-1, 0]]

ComplexInfinity

pp[4, 1.0000001]

6.0018

Limit[**Sum**[(**a** - 1) ^**s** (-1) ^**s** **a**^**n** / **n**^(1 - **s**), {**n**, 0, Infinity}], {**a** → 1}]

$$\{ \text{Limit} [(-1)^s (-1 + a)^s \text{HurwitzLerchPhi}[a, 1 - s, 0], a \rightarrow 1] \}$$

ff[**s**_] := **Limit**[**Sum**[(**a** - 1) ^**s** (-1) ^**s** **a**^**n** **n**^(**s** - 1), {**n**, 0, Infinity}], {**a** → 1}]

ff[7 / 2]

$$\left\{ \frac{15 \sqrt{\pi}}{8} \right\}$$

-0.0003593672291619696^

fg[**z**_, **s**_, **a**_] := **Sum**[**z**^**n** / (**n** + **a**) ^**s**, {**n**, 0, Infinity}]

fg[**z**, **s**, **a**]

HurwitzLerchPhi[**z**, **s**, **a**]

Limit[**Sum**[(**a** - 1) ^**s** (-1) ^**s** **a**^**n** / **n**^(1 - **s**), {**n**, 0, Infinity}], {**a** → 1}]

$$\{ \text{Limit} [(-1)^s (-1 + a)^s \text{HurwitzLerchPhi}[a, 1 - s, 0], a \rightarrow 1] \}$$

Expand[(**a** - 1) ^**s** (-1) ^**s** **a**^**n** / **n**^(1 - **s**)]

$$(-1)^s (-1 + a)^s a^n n^{-1+s}$$

ffa[**s**_] :=

$$(-1)^s \text{Limit} [\text{Sum} [E^{(s \text{Log}[a - 1] + n \text{Log}[a] + (s - 1) \text{Log}[n])}, \{n, 0, \text{Infinity}\}], \{a \rightarrow 1\}]$$

ffa[1]

{1}

FullSimplify[E^(s Log[a - 1] + n Log[a] + (s - 1) Log[n])]

$(-1 + a)^s a^n n^{-1+s}$

ffb[s_] :=

$(-1)^s \text{Limit}[\text{Sum}[E^{(s \text{Log}[a - 1] + n \text{Log}[a] + (s - 1) \text{Log}[n])}, \{n, 0, \text{Infinity}\}], \{a \rightarrow 1\}]$

ffb[5 / 2]

$\left\{ \frac{3 \sqrt{\pi}}{4} \right\}$

Limit[Sum[a^s (-1)^s (a + 1)^n / n^(1 - s), {n, 0, Infinity}], {a → 0}]

$\{\text{Limit}[(-1)^s a^s \text{HurwitzLerchPhi}[1 + a, 1 - s, 0], a \rightarrow 0]\}$

Limit[Sum[(a - 1) (-1) a^n, {n, 0, Infinity}], {a → 1}]

{1}

Limit[Sum[(a - 1)^2 a^n n, {n, 0, Infinity}], {a → 1}]

{1}

Limit[Sum[(a - 1)^3 (-1) a^n n^2, {n, 0, Infinity}], {a → 1}]

{2}

Limit[Sum[(a - 1)^4 a^n n^3, {n, 0, Infinity}], {a → 1}]

{6}

Limit[Sum[(a - 1)^5 (-1) a^n n^4, {n, 0, Infinity}], {a → 1}]

{24}

Limit[Sum[(a - 1)^6 a^n n^5, {n, 0, Infinity}], {a → 1}]

{120}

ffx[n_, s_] := Limit[Sum[(a - 1)^s (-1)^s a^k k^(s - 1), {k, Log[a, n], Infinity}], {a → 1}]

ffx[n, 3]

$\{2n - 2n \text{Log}[n] + n \text{Log}[n]^2\}$

ffv[n_, s_] := Limit[Sum[(a - 1)^s (-1)^s a^k k^(s - 1), {k, 1, Log[a, n]}], {a → 1}]

ffv[n, 3]

$\{2 - 2n + 2n \text{Log}[n] - n \text{Log}[n]^2\}$

N[ffx[100, 3 / 2]]

$$\left\{ \text{Limit} \left[(0. - 100. i) \left(-1. \sqrt{-1. + a} \text{HurwitzLerchPhi} \left[a, -0.5, \frac{4.60517}{\text{Log}[a]} \right] + \sqrt{-1. + a} a \text{HurwitzLerchPhi} \left[a, -0.5, \frac{4.60517}{\text{Log}[a]} \right] \right), a \rightarrow 1. \right] \right\}$$

ffx2[n_, s_] :=

Limit[Sum[(a - 1) ^ s (-1) ^ s a ^ k k ^ (s - 1), {k, Log[a, n], Infinity}], a → 1.0001]

ffx2[n_, s_, a_] := Sum[(a - 1) ^ s (-1) ^ s a ^ k k ^ (s - 1), {k, 1, Log[a, n]}]

N[ffx2[100, .5]]

1.7725 - 54.7289 i

Gamma[.5, -Log[100]]

1.77245 - 54.7298 i

Gamma[.5]

1.77245

$$1 / ((a - 1)^s ((-1)^s (a^n (n^{s-1})))$$

$$(-1)^{-s} (-1 + a)^{-s} a^{-n} n^{1-s}$$

gg[s_] := Limit[Sum[(a - 1) ^ s (-1) ^ s a ^ n / n ^ (1 - s), {n, 1, Infinity}], {a → 1}]

gg[s + 1] / gg[s]

$$\left\{ \frac{\text{Limit}[-(-1)^s (-1 + a)^{1+s} \text{PolyLog}[-s, a], a \rightarrow 1]}{\text{Limit}[(-1)^s (-1 + a)^s \text{PolyLog}[1 - s, a], a \rightarrow 1]} \right\}$$

$$\text{ff}[s_] := \frac{\text{Limit}[-(-1)^s a^{1+s} \text{HurwitzLerchPhi}[1 + a, -s, 0], a \rightarrow 0]}{\text{Limit}[(-1)^s a^s \text{HurwitzLerchPhi}[1 + a, 1 - s, 0], a \rightarrow 0]}$$

inv[s_] := Pi / Sin[Pi s] / gg[s]

inv[1 / 2]

$$\{\sqrt{\pi}\}$$

Gamma[1 - 1 / 2]

$$\sqrt{\pi}$$

gg'[2]

\$RecursionLimit::reclim : Recursion depth of 256 exceeded. >>

\$RecursionLimit::reclim : Recursion depth of 256 exceeded. >>

\$RecursionLimit::reclim : Recursion depth of 256 exceeded. >>

General::stop : Further output of \$RecursionLimit::reclim will be suppressed during this calculation. >>

$\{\text{Hold}[\partial_2 \text{Limit}[(-1)^2 (-1 + a)^2 \text{PolyLog}[1 - 2, a], a \rightarrow 1]]\}$

Limit[gg[s] - 1 / s, {s → 0}]

$\{-\infty\}$

N[Limit[- $\frac{1}{s}$ + Limit[$(-1)^s (-1 + a)^s \text{HurwitzLerchPhi}[a, 1 - s, 0], a \rightarrow 1], s \rightarrow 0]]$

gg[s]

$\{\text{Limit}[(-1)^s (-1 + a)^s \text{HurwitzLerchPhi}[a, 1 - s, 0], a \rightarrow 1]\}$

HurwitzLerchPhi[1, 1, 0]

ComplexInfinity

gg[s]

$\{\text{Limit}[(-1)^s (-1 + a)^s \text{HurwitzLerchPhi}[a, 1 - s, 0], a \rightarrow 1]\}$

gg[1.1 + I]

$\{0.517481 - 0.103723 i\}$

N[Gamma[1 - 1.1 + I]]

$-0.252912 - 0.4489 i$

inv[1.1 + I]

$\{-0.252912 + 0.4489 i\}$

gg2[s_] := Limit[Sum[$(-1)^s (a - 1)^s a^n n^{(s - 1)}$, {n, 1, Infinity}], {a → 1}]

gg2[5 / 3]

$\{\text{Gamma}\left[\frac{5}{3}\right]\}$

gg2[1 / 2]

$\{\sqrt{\pi}\}$

$(-1)^s (a - 1)^s a^n n^{(s - 1)} /. \{n \rightarrow 6, s \rightarrow 1 / 2\}$

$\frac{i \sqrt{-1 + a} a^6}{\sqrt{6}}$

gg2[t] - 1 / t /. t → .00001

$\{-0.577205 + 1.42979 \times 10^{-11} i\}$

gg2[0]

$\{\infty\}$

gg2[c]

$\{\text{Limit}[(-1)^c (-1+a)^c \text{PolyLog}[1-c, a], a \rightarrow 1]\}$

$\text{Limit}[(-1)^c (-1+a)^c \text{PolyLog}[1-c, a] /. c \rightarrow 1/2, a \rightarrow 1]$

$\sqrt{\pi}$

$\text{Limit}[(-1)^c (a)^c \text{PolyLog}[1-c, 1+a] /. c \rightarrow 1/2, a \rightarrow 0]$

$\sqrt{\pi}$

FullSimplify $[(-1)^c (a)^c]$

$(-1)^c a^c$

$\text{Limit}[(-a)^c \text{PolyLog}[1-c, 1+a] /. c \rightarrow 1/2, a \rightarrow 0]$

$\sqrt{\pi}$

gg2a[s_] := $\text{Limit}[\text{Sum}[(-1)^s (a-1)^s a^n n^{s-1}, \{n, 1, \text{Infinity}\}], \{a \rightarrow 1\}]$

gg2a[1/2]

$\{\sqrt{\pi}\}$

gg2b[s_] := $\text{Limit}[\text{Sum}[(-1)^s a^{-s} (a^{-1}+1)^k k^{s-1}, \{k, 1, \text{Infinity}\}], \{a \rightarrow \text{Infinity}\}]$

gg2b[6]

$\{120\}$

gg2c[s_] := $\text{Limit}[(-1)^s / a^s \text{Sum}[(1/a+1)^k k^{s-1}, \{k, 1, \text{Infinity}\}], \{a \rightarrow \text{Infinity}\}]$

gg2c[6]

$\{120\}$