```
t[n_, a_] := Mod[n, a] - Mod[n - 1, a]

ll[n_, a_] := Sum[ t[k, a] / k n^k, {k, 1, 20 000}]

N[ll[1, 2]]
```

0.693097

```
N[Log[1 + 1]]
```

0.693147

```
N[ll[1, 3]]
```

1.09861

```
N[Log[3]]
```

1.09861

```
E^ll[.5, 3]
```

1.75

```
E^0.5596157879354224`
```

1.75

```
E^ll[.5, 4]
```

1.875

```
E^ll[.5, 5]
```

1.9375

```
E^ll[.5, 6]
```

1.96875

```
E^ll[.5, 7]
```

1.98437

```
E^ll[.5, 8]
```

1.99219

```
E^ll[.5, 12]
```

1.99951

```
E^ll[.5, 20]
```

2.

```
E^ll[.1, 20]
```

1.11111

```
E^ll[.25, 20]
```

1.33333

```
E^ll[-.25, 20]
```

0.8

```
E^ll[-.5, 20]
```

0.666666

```
E^ll[-.75, 50]
```

0.571428

```
N[E^ll[-1, 50]]
```

0.00014033

```
E^ll[.5, 20]
```

```
E^ll[-.1, 20]
```

0.909091

```
Table[{n, ss = ll[n, 50], E^ss, 1 / (E^ss)}, {n, -1, 1, .05}] // TableForm
```

-1.	-7.26305	0.000700966	1426.6
-0.95	-0.747896	0.473362	2.11255
-0.9	-0.647021	0.523603	1.90984
-0.85	-0.615481	0.540381	1.85055
-0.8	-0.587801	0.555548	1.80003
-0.75	-0.559616	0.571428	1.75
-0.7	-0.530628	0.588235	1.7
-0.65	-0.500775	0.606061	1.65
-0.6	-0.470004	0.625	1.6
-0.55	-0.438255	0.645161	1.55
-0.5	-0.405465	0.666667	1.5
-0.45	-0.371564	0.689655	1.45
-0.4	-0.336472	0.714286	1.4
-0.35	-0.300105	0.740741	1.35
-0.3	-0.262364	0.769231	1.3
-0.25	-0.223144	0.8	1.25
-0.2	-0.182322	0.833333	1.2
-0.15	-0.139762	0.869565	1.15
-0.1	-0.0953102	0.909091	1.1
-0.05	-0.0487902	0.952381	1.05
0.	0.	1.	1.
0.05	0.0512933	1.05263	0.95
0.1	0.105361	1.11111	0.9
0.15	0.162519	1.17647	0.85
0.2	0.223144	1.25	0.8
0.25	0.287682	1.33333	0.75
0.3	0.356675	1.42857	0.7
0.35	0.430783	1.53846	0.65
0.4	0.510826	1.66667	0.6
0.45	0.597837	1.81818	0.55
0.5	0.693147	2.	0.5
0.55	0.798508	2.22222	0.45
0.6	0.916291	2.5	0.4
0.65	1.04982	2.85714	0.35
0.7	1.20397	3.33333	0.3
0.75	1.38629	4.	0.25
0.8	1.60942	4.99993	0.200003
0.85	1.89682	6.66469	0.150044
0.9	2.29742	9.94846	0.100518
0.95	2.91567	18.4611	0.054168
1.	3.9108	49.9388	0.0200245

```
Table[{n, ss = ll[n, 2], E^ss, 1 / (E^ss)}, {n, -1, 1, .05}] // TableForm
```

-1.	-10.4807	0.0000280723	35622.3
-0.95	-2.99573	0.05	20.
-0.9	-2.30259	0.1	10.
-0.85	-1.89712	0.15	6.66667
-0.8	-1.60944	0.2	5.
-0.75	-1.38629	0.25	4.
-0.7	-1.20397	0.3	3.33333
-0.65	-1.04982	0.35	2.85714
-0.6	-0.916291	0.4	2.5
-0.55	-0.798508	0.45	2.22222
-0.5	-0.693147	0.5	2.
-0.45	-0.597837	0.55	1.81818
-0.4	-0.510826	0.6	1.66667
-0.35	-0.430783	0.65	1.53846
-0.3	-0.356675	0.7	1.42857
-0.25	-0.287682	0.75	1.33333
-0.2	-0.223144	0.8	1.25
-0.15	-0.162519	0.85	1.17647
-0.1	-0.105361	0.9	1.11111
-0.05	-0.0512933	0.95	1.05263
0.	0.	1.	1.
0.05	0.0487902	1.05	0.952381
0.1	0.0953102	1.1	0.909091
0.15	0.139762	1.15	0.869565
0.2	0.182322	1.2	0.833333
0.25	0.223144	1.25	0.8
0.3	0.262364	1.3	0.769231
0.35	0.300105	1.35	0.740741
0.4	0.336472	1.4	0.714286
0.45	0.371564	1.45	0.689655
0.5	0.405465	1.5	0.666667
0.55	0.438255	1.55	0.645161
0.6	0.470004	1.6	0.625
0.65	0.500775	1.65	0.606061
0.7	0.530628	1.7	0.588235
0.75	0.559616	1.75	0.571429
0.8	0.587787	1.8	0.555556
0.85	0.615186	1.85	0.540541
0.9	0.641854	1.9	0.526316
0.95	0.667829	1.95	0.512821
1.	0.693122	1.99995	0.500012

```
Table[{n, ss = ll[n, 3], E^ss, 1 / (E^ss)}, {n, -1, 1, .05}] // TableForm
```

-1.	-0.0000500025	0.99995	1.00005
-0.95	-0.0486652	0.9525	1.04987
-0.9	-0.0943107	0.91	1.0989
-0.85	-0.136393	0.8725	1.14613
-0.8	-0.174353	0.84	1.19048
-0.75	-0.207639	0.8125	1.23077
-0.7	-0.235722	0.79	1.26582
-0.65	-0.258123	0.7725	1.2945
-0.6	-0.274437	0.76	1.31579
-0.55	-0.284354	0.7525	1.3289
-0.5	-0.287682	0.75	1.33333
-0.45	-0.284354	0.7525	1.3289
-0.4	-0.274437	0.76	1.31579
-0.35	-0.258123	0.7725	1.2945
-0.3	-0.235722	0.79	1.26582
-0.25	-0.207639	0.8125	1.23077
-0.2	-0.174353	0.84	1.19048
-0.15	-0.136393	0.8725	1.14613
-0.1	-0.0943107	0.91	1.0989
-0.05	-0.0486652	0.9525	1.04987
0.	0.	1.	1.
0.05	0.0511683	1.0525	0.950119
0.1	0.10436	1.11	0.900901
0.15	0.159138	1.1725	0.852878
0.2	0.215111	1.24	0.806452
0.25	0.271934	1.3125	0.761905
0.3	0.329304	1.39	0.719424
0.35	0.386962	1.4725	0.679117
0.4	0.444686	1.56	0.641026
0.45	0.502289	1.6525	0.605144
0.5	0.559616	1.75	0.571429
0.55	0.616536	1.8525	0.539811
0.6	0.672944	1.96	0.510204
0.65	0.728756	2.0725	0.482509
0.7	0.783902	2.19	0.456621
0.75	0.838329	2.3125	0.432432
0.8	0.891998	2.44	0.409836
0.85	0.944878	2.5725	0.388727
0.9	0.996949	2.71	0.369004
0.95	1.0482	2.8525	0.35057
1.	1.09866	3.00015	0.333317

```
Table[{n, ss = ll[n, 4], E^ss, 1 / (E^ss)}, {n, -1, 1, .05}] // TableForm
```

-1.	-9.78763	0.0000561417	17812.1
-0.95	-2.35256	0.095125	10.5125
-0.9	-1.70926	0.181	5.52486
-0.85	-1.35334	0.258375	3.87034
-0.8	-1.11474	0.328	3.04878
-0.75	-0.940007	0.390625	2.56
-0.7	-0.805197	0.447	2.23714
-0.65	-0.697406	0.497875	2.00854
-0.6	-0.608806	0.544	1.83824
-0.55	-0.534222	0.586125	1.70612
-0.5	-0.470004	0.625	1.6
-0.45	-0.413434	0.661375	1.512
-0.4	-0.362406	0.696	1.43678
-0.35	-0.315225	0.729625	1.37057
-0.3	-0.270497	0.763	1.31062
-0.25	-0.227057	0.796875	1.2549
-0.2	-0.183923	0.832	1.20192
-0.15	-0.140268	0.869125	1.15058
-0.1	-0.0954102	0.909	1.10011
-0.05	-0.0487964	0.952375	1.05001
0.	0.	1.	1.
0.05	0.051287	1.05263	0.950006
0.1	0.105261	1.111	0.90009
0.15	0.162013	1.17588	0.850431
0.2	0.221542	1.248	0.801282
0.25	0.283768	1.32812	0.752941
0.3	0.348542	1.417	0.705716
0.35	0.415663	1.51538	0.659903
0.4	0.484892	1.624	0.615764
0.45	0.555966	1.74363	0.573518
0.5	0.628609	1.875	0.533333
0.55	0.70254	2.01888	0.495325
0.6	0.777488	2.176	0.459559
0.65	0.853191	2.34713	0.426053
0.7	0.929404	2.533	0.394789
0.75	1.0059	2.73437	0.365714
0.8	1.08248	2.952	0.338753
0.85	1.15896	3.18663	0.313812
0.9	1.23518	3.439	0.290782
0.95	1.311	3.70987	0.269551
1.	1.38622	3.9997	0.250019

```
N[Log[3] + Log[3]]
```

2.19722

```
N[Log[9]]
```

2.19722

```
2 Sum[ t[n, 3] / n, {n, 1, 9}]
```

$$\frac{2509}{1260}$$

```
N[Sum[t[n, 9] / n, {n, 1, ss = 27 * 9}] - 2 Sum[ t[n, 3] / n, {n, 1, ss}]]
```

-0.00814015

```
2 Table[ t[n, 3] / n, {n, 1, 30}]
```

$$\left\{2, 1, -\frac{4}{3}, \frac{1}{2}, \frac{2}{5}, -\frac{2}{3}, \frac{2}{7}, \frac{1}{4}, -\frac{4}{9}, \frac{1}{5}, \frac{2}{11}, -\frac{1}{3}, \frac{2}{13}, \frac{1}{7}, -\frac{4}{15}, \frac{1}{8}, \frac{2}{17}, -\frac{2}{9}, \frac{2}{19}, \frac{1}{10}, -\frac{4}{21}, \frac{1}{11}, \frac{2}{23}, -\frac{1}{6}, \frac{2}{25}, \frac{1}{13}, -\frac{4}{27}, \frac{1}{14}, \frac{2}{29}, -\frac{2}{15}\right\}$$

```
Table[ t[n, 9] / n, {n, 1, 30}]
```

$$\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, -\frac{8}{9}, \frac{1}{10}, \frac{1}{11}, \frac{1}{12}, \frac{1}{13}, \frac{1}{14}, \frac{1}{15}, \frac{1}{16}, \frac{1}{17}, -\frac{4}{9}, \frac{1}{19}, \frac{1}{20}, \frac{1}{21}, \frac{1}{22}, \frac{1}{23}, \frac{1}{24}, \frac{1}{25}, \frac{1}{26}, -\frac{8}{27}, \frac{1}{28}, \frac{1}{29}, \frac{1}{30}\right\}$$

```
Table[ t[n, 2] / n, {n, 1, 30}] + Table[ t[n, 3] / n, {n, 1, 30}]
```

$$\left\{2, 0, -\frac{1}{3}, 0, \frac{2}{5}, -\frac{1}{2}, \frac{2}{7}, 0, -\frac{1}{9}, 0, \frac{2}{11}, -\frac{1}{4}, \frac{2}{13}, 0, -\frac{1}{15}, 0, \frac{2}{17}, -\frac{1}{6}, \frac{2}{19}, 0, -\frac{1}{21}, 0, \frac{2}{23}, -\frac{1}{8}, \frac{2}{25}, 0, -\frac{1}{27}, 0, \frac{2}{29}, -\frac{1}{10}\right\}$$

```
Table[ t[n, 6] / n, {n, 1, 30}]
```

$$\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, -\frac{5}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}, \frac{1}{11}, -\frac{5}{12}, \frac{1}{13}, \frac{1}{14}, \frac{1}{15}, \frac{1}{16}, \frac{1}{17}, -\frac{5}{18}, \frac{1}{19}, \frac{1}{20}, \frac{1}{21}, \frac{1}{22}, \frac{1}{23}, -\frac{5}{24}, \frac{1}{25}, \frac{1}{26}, \frac{1}{27}, \frac{1}{28}, \frac{1}{29}, -\frac{1}{6}\right\}$$

```
cs := CoefficientList[Series[Tan[x], {x, 0, 120}], x]
```

```
cs[[4]]
```

$$\frac{1}{3}$$

```
FC[x_, a_] := Sum[ cs[[k + 1]] t[Floor[k / 2 + 1], a] x^k, {k, 0, 119}]
```

```
N[FC[Pi / 4, 2 000 000]]
```

```
1.
```

```
N[FC[Pi / 4, 2]]
```

```
0.655794
```

```
Series[Tan[x], {x, 0, 20}]
```

$$x + \frac{x^3}{3} + \frac{2\,x^5}{15} + \frac{17\,x^7}{315} + \frac{62\,x^9}{2835} + \frac{1382\,x^{11}}{155\,925} + \frac{21\,844\,x^{13}}{6\,081\,075} + \frac{929\,569\,x^{15}}{638\,512\,875} + \frac{6\,404\,582\,x^{17}}{10\,854\,718\,875} + \frac{443\,861\,162\,x^{19}}{1\,856\,156\,927\,625} + O[x]^{21}$$

```
Series[Tanh[x], {x, 0, 20}]
```

$$x - \frac{x^3}{3} + \frac{2\,x^5}{15} - \frac{17\,x^7}{315} + \frac{62\,x^9}{2835} - \frac{1382\,x^{11}}{155\,925} + \frac{21\,844\,x^{13}}{6\,081\,075} - \frac{929\,569\,x^{15}}{638\,512\,875} + \frac{6\,404\,582\,x^{17}}{10\,854\,718\,875} - \frac{443\,861\,162\,x^{19}}{1\,856\,156\,927\,625} + O[x]^{21}$$

```
N[Tanh[Pi / 4]]
```

```
0.655794
```

```
Tan[Pi / 4 I]
```

$$i \tanh\left[\frac{\pi}{4}\right]$$

```
N[x - x^3/3 + 2 x^5/15 - 17 x^7/315 + 62 x^9/2835 - 1382 x^11/155925 + 21844 x^13/6081075 -
    929569 x^15/638512875 + 6404582 x^17/10854718875 - 443861162 x^19/1856156927625 /. x → I Pi / 4]
```

$$0. + 0.999999\, i$$

```
N[FC[I Pi / 4, 20000000]]
```

$$0. + 0.655794\, i$$

```
N[FC[I Pi / 4, 3]]
```

$$0. + 0.538092\, i$$

```
N[FC[ Pi / 4, 3]]
```

$$0.878566$$

```
Table[{n, s1 = N[FC[ Pi / 4, n]], s2 = N[FC[I Pi / 4, n]], (s1^2 + Abs[s2]^2)^(1 / 2)},
  {n, 2, 16}] // TableForm
```

2	0.655794	$0. + 1.\, i$	1.19585
3	0.878566	$0. + 0.538092\, i$	1.03025
4	0.960049	$0. + 0.695745\, i$	1.18565
5	0.987554	$0. + 0.643372\, i$	1.17864
6	0.996269	$0. + 0.659525\, i$	1.19479
7	0.998912	$0. + 0.654706\, i$	1.19435
8	0.999689	$0. + 0.656105\, i$	1.19576
9	0.999913	$0. + 0.655707\, i$	1.19573
10	0.999976	$0. + 0.655818\, i$	1.19585
11	0.999993	$0. + 0.655788\, i$	1.19584
12	0.999998	$0. + 0.655796\, i$	1.19585
13	1.	$0. + 0.655794\, i$	1.19585
14	1.	$0. + 0.655794\, i$	1.19585
15	1.	$0. + 0.655794\, i$	1.19585
16	1.	$0. + 0.655794\, i$	1.19585

```
Table[{n, N[FC[ n I, 3]]}, {n, 0, Pi, Pi / 64}] // TableForm
```

Power::indet : Indeterminate expression 0^0 encountered. ≫

0	Indeterminate
$\frac{\pi}{64}$	$0. + 0.0490479\, i$
$\frac{\pi}{32}$	$0. + 0.0978569\, i$
$\frac{3\pi}{64}$	$0. + 0.146179\, i$
$\frac{\pi}{16}$	$0. + 0.193748\, i$
$\frac{5\pi}{64}$	$0. + 0.240268\, i$
$\frac{3\pi}{32}$	$0. + 0.285407\, i$
$\frac{7\pi}{64}$	$0. + 0.328782\, i$
$\frac{\pi}{8}$	$0. + 0.36995\, i$
$\frac{9\pi}{64}$	$0. + 0.408396\, i$

$\frac{5\pi}{32}$ $0. + 0.44352\,i$

$\frac{11\pi}{64}$ $0. + 0.474629\,i$

$\frac{3\pi}{16}$ $0. + 0.500922\,i$

$\frac{13\pi}{64}$ $0. + 0.52149\,i$

$\frac{7\pi}{32}$ $0. + 0.535309\,i$

$\frac{15\pi}{64}$ $0. + 0.541249\,i$

$\frac{\pi}{4}$ $0. + 0.538092\,i$

$\frac{17\pi}{64}$ $0. + 0.524565\,i$

$\frac{9\pi}{32}$ $0. + 0.499396\,i$

$\frac{19\pi}{64}$ $0. + 0.461385\,i$

$\frac{5\pi}{16}$ $0. + 0.409509\,i$

$\frac{21\pi}{64}$ $0. + 0.34304\,i$

$\frac{11\pi}{32}$ $0. + 0.261682\,i$

$\frac{23\pi}{64}$ $0. + 0.165705\,i$

$\frac{3\pi}{8}$ $0. + 0.056054\,i$

$\frac{25\pi}{64}$ $0. - 0.0655899\,i$

$\frac{13\pi}{32}$ $0. - 0.196817\,i$

$\frac{27\pi}{64}$ $0. - 0.334585\,i$

$\frac{7\pi}{16}$ $0. - 0.475405\,i$

$\frac{29\pi}{64}$ $0. - 0.615583\,i$

$\frac{15\pi}{32}$ $0. - 0.751093\,i$

$\frac{31\pi}{64}$ $0. - 0.854546\,i$

$\frac{\pi}{2}$ $0. + 0.275012\,i$

$\frac{33\pi}{64}$ $0. + 54.5483\,i$

$\frac{17\pi}{32}$ $0. + 2143.29\,i$

$\frac{35\pi}{64}$ $0. + 73\,486.1\,i$

$\frac{9\pi}{16}$ $0. + 2.25707 \times 10^{6}\,i$

$\frac{37\pi}{64}$ $0. + 6.2573 \times 10^{7}\,i$

$\frac{19\pi}{32}$ $0. + 1.57584 \times 10^{9}\,i$

$\frac{39\pi}{64}$ $0. + 3.62611 \times 10^{10}\,i$

$\frac{5\pi}{8}$ $0. + 7.66435 \times 10^{11}\,i$

$\frac{41\pi}{64}$ $0. + 1.49524 \times 10^{13}\,i$

$\frac{21\pi}{32}$ $0. + 2.70437 \times 10^{14}\,i$

$\frac{43\pi}{64}$ $0. + 4.55301 \times 10^{15}\,i$

$\frac{11\pi}{16}$ $0. + 7.16177 \times 10^{16}\,i$

$\frac{45\pi}{64}$ $0. + 1.05613 \times 10^{18}\,i$

$\frac{23\pi}{32}$ $0. + 1.46471 \times 10^{19}\,i$

$\frac{47\pi}{64}$ $0. + 1.91599 \times 10^{20}\,i$

$\frac{3\pi}{4}$ $0. + 2.37034 \times 10^{21}\,i$

$\frac{49\,\pi}{64}$ $0. + 2.78027 \times 10^{22}\,\mathrm{i}$

$\frac{25\,\pi}{32}$ $0. + 3.09907 \times 10^{23}\,\mathrm{i}$

$\frac{51\,\pi}{64}$ $0. + 3.28992 \times 10^{24}\,\mathrm{i}$

$\frac{13\,\pi}{16}$ $0. + 3.33292 \times 10^{25}\,\mathrm{i}$

$\frac{53\,\pi}{64}$ $0. + 3.22829 \times 10^{26}\,\mathrm{i}$

$\frac{27\,\pi}{32}$ $0. + 2.99498 \times 10^{27}\,\mathrm{i}$

$\frac{55\,\pi}{64}$ $0. + 2.66572 \times 10^{28}\,\mathrm{i}$

$\frac{7\,\pi}{8}$ $0. + 2.27987 \times 10^{29}\,\mathrm{i}$

$\frac{57\,\pi}{64}$ $0. + 1.87638 \times 10^{30}\,\mathrm{i}$

$\frac{29\,\pi}{32}$ $0. + 1.48816 \times 10^{31}\,\mathrm{i}$

$\frac{59\,\pi}{64}$ $0. + 1.13884 \times 10^{32}\,\mathrm{i}$

$\frac{15\,\pi}{16}$ $0. + 8.41977 \times 10^{32}\,\mathrm{i}$

$\frac{61\,\pi}{64}$ $0. + 6.02104 \times 10^{33}\,\mathrm{i}$

$\frac{31\,\pi}{32}$ $0. + 4.16926 \times 10^{34}\,\mathrm{i}$

$\frac{63\,\pi}{64}$ $0. + 2.79848 \times 10^{35}\,\mathrm{i}$

π $0. + 1.82261 \times 10^{36}\,\mathrm{i}$

cs