```
Clear[d1]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
a1[x_, z_] := x^z
a2[x_, z_] := (x - 1)^z
g1[ x_, z_] := LaguerreL[-z, Log[x]]
g1l[x_] := LogIntegral[x] - Log[Log[x]] - EulerGamma
g2[x_, z_] := (-1)^z Gamma[ z, 0, -Log[x]] / Gamma[z]
g2l[x_] := LogIntegral[x]
d1[x_, z_, k_: 1] := d1[x, z, k] = 1 + ((z + 1) / k - 1) Sum[d1[x / j, z, k + 1], {j, 2, x}];
d1[0, z_, k_] := 0; d1[0, z_] := 0
d2[x_, z_] := d2[x, z] = Sum[ d2[x / j, z - 1], {j, 2, x}]; d2[x_, 0] := UnitStep[x - 1]
dd1[x_, z_] := d1[x, z] - d1[x - 1, z]
dd2[x_, z_] := d2[x, z] - d2[x - 1, z]
g1lk[ n_, z_, t_] := Sum[(D[Log[x + 1]^z, {x, k}] / k! /. x → 0)
    (-1)^k ( Gamma[ k, 0, -Log[n]] / Gamma[k]), {k, 1, t}]
g1m[ n_, m_] := -1 + g1[ n, 1] + g1[m, 1] +
  Integrate[ D[ g1[y, 1], y] D[g1[x, 1], x], {y, 1, n}, {x, 1, m^(1 - Log[n, y])}]
g1[x_, 1] := 1
g1[ x_, 2] := x (1 + Log[x])
g1[ x_, 3] := 1/2 x (2 + Log[x] (4 + Log[x]))
g1[x_, 4] := 1/6 x (6 + Log[x] (3 + Log[x]) (6 + Log[x]))
g1[x_, 5] := 1/24 x (24 + Log[x] (4 + Log[x]) (24 + Log[x] (12 + Log[x])))
g1[x_, 6] := 1/120 x (120 + Log[x] (20 + Log[x] (10 + Log[x])) (30 + Log[x] (15 + Log[x])))
g1d[x_, z_] := Sum[ Binomial[z, k] / ((k - 1)!) Log[x]^(k - 1), {k, 1, z}]


Sum[ Binomial[z, k] (-1)^k x^k, {k, 0, Infinity}]

(1 - x)^z

(1 - x)^z /. x → 4.3

(-3.3)^z

((-1) (x - 1))^z /. x → 4.3

(-3.3)^z

(-1)^z (x - 1)^z /. x → 4.3

(-3.3)^z

D[ x^z, x]

x^(-1+z) z

D[ (x - 1)^z, x]

(-1 + x)^(-1+z) z
```

```
D[ (-1)^z Gamma[ z, 0, -Log[x]] / Gamma[z], x]
```

$$-\frac{(-1)^z\ (-\text{Log}[x])^{-1+z}}{\text{Gamma}[z]}$$

```
D[ LaguerreL[-z, Log[x]], x]
```

$$-\frac{\text{LaguerreL}[-1-z,\ 1,\ \text{Log}[x]]}{x}$$

```
-(-1)^z (-Log[x])^(-1+z) / Gamma[z] /. {x → 12, z → 1.7}
```

$$-0.643144 - 1.97939\ i$$

```
-(-1)^z (-1)^(z-1) (Log[x])^(-1+z) / Gamma[z] /. {x → 12, z → 1.7}
```

$$-0.643144 - 1.97939\ i$$

```
(-1) (-1)^z (-1)^(z-1) (Log[x])^(-1+z) / Gamma[z] /. {x → 12, z → 1.7}
```

$$-0.643144 - 1.97939\ i$$

```
(-1) (-1)^z (-1)^(z-1) (Log[x])^(-1+z) / Gamma[z] /. {x → 12, z → 1.7}
```

```
FullSimplify[(-1) (-1)^z (-1)^(z-1)]
```

$$(-1)^{2z}$$

```
Clear[d1]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
a1[x_, z_] := x^z
a2[x_, z_] := (x - 1)^z
g1[ x_, z_] := LaguerreL[-z, Log[x]]
g2[x_, z_] := (-1)^z Gamma[ z, 0, -Log[x]] / Gamma[z]
d1[x_, z_, k_: 1] := d1[x, z, k] = 1 + ((z + 1) / k - 1) Sum[d1[x / j, z, k + 1], {j, 2, x}];
d1[0, z_, k_] := 0; d1[0, z_] := 0
d2[x_, z_] := d2[x, z] = Sum[ d2[x / j, z - 1], {j, 2, x}]; d2[x_, 0] := UnitStep[x - 1]
dd1[x_, z_] := d1[x, z] - d1[x - 1, z]
dd2[x_, z_] := d2[x, z] - d2[x - 1, z]
```

```
d1[0, 1]
```

$$1$$

```
Table[dd2[n, 1], {n, 1, 10}]
```

$$\{0, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$$

```
Limit[ g2[x, z], z → 1]
```

$$-1 + x$$

```
D[ (x - 1)^z, x]
```

$$(-1 + x)^{-1+z}\ z$$

```
FullSimplify[((x - 1)^(z + 1) - (x - 1)^z) z / ((x - 1) (x - 2))]
```

$(-1+x)^{-1+z}\,z$

```
N[D[ g2[ x, z], x] /. {x → 20, z → 3}]
```

4.48721

```
N[FullSimplify[Expand[z / ( (x - 1) Log[x - 1]) ( g2[ x, z + 1] - g2[x, z])]] /. {x → 20, z → 3}]
```

$-0.431277 - 1.09405 \times 10^{-16}\,i$

```
 Limit[ D[ a2[x, z], x] /. x → 1,  z → 1]
```

0

```
Limit[ D[ a1[x, z], x] /. x → 1,  z → 1]
```

1

```
 D[g1[x, z], x] /. x → 1 /. z → 3
```

3

```
 Limit[ D[ g2[x, z], x] /. x → 1, z → 1]
```

0

```
 D[ a2[x, z], x] /. z → 1
```

1

```
Integrate[ D[ a1[x, z], x], {x, 0, 1}]
```

ConditionalExpression[1, Re[z] > 0]

```
Integrate[ D[ a2[x, z], x], {x, 0, 1}]
```

ConditionalExpression[$-(-1)^z$, Re[z] > 0]

```
Integrate[ z / (x Log[x]) (g1[x, z + 1] - g1[x, z]), {x, 0, 1}]
```

1

```
Integrate[ D[ g2[x, z], x], {x, 0, 1}]
```

ConditionalExpression[$-(-1)^z$, Re[z] > 0]

```
z / (x Log[x]) (g1[x, z + 1] - g1[x, z])
```

```
Integrate[ D[ s^a, s] D[t^b, t], {s, 0, x}, {t, 0, x}]
```

ConditionalExpression$\left[x^{a+b}, \text{Re}[a] > 0\right]$

```
FullSimplify[
 Integrate[ D[ (s - 1)^a, s] D[(t - 1)^b, t], {s, 1, x}, {t, 1, x}] /. {x → 12, a → 2}]
```

11^{2+b}

```
Expand[(-1 + x^a) (-1 + x^b) + x^a + x^b - 1]
```

x^{a+b}

```
d1[100, 4.3]
```

4527.59

```
Sum[ dd1[j, 1.3] dd1[k, 3], {j, 1, 100}, {k, 1, 100 / j}]
```

4527.59

```
d1[0, 1]
```

0

```
d2[222, 4]
```

938

```
Sum[ dd2[j, 2] dd2[k, 2], {j, 2, 222}, {k, 2, 222 / j}]
```

938

```
Integrate[D[ g1[s, z], s], {s, 1, x}]
```

ConditionalExpression[-1 + Hypergeometric1F1[z, 1, Log[x]], 0 ≤ Re[x] ≤ ∈ || x ∉ Reals]

```
Integrate[D[ g2[s, z], s], {s, 1, x}]
```

$$\text{ConditionalExpression}\left[\frac{(-1)^z\,(\text{Gamma}[z]-\text{Gamma}[z,-\text{Log}[x]])}{\text{Gamma}[z]},\ \text{Re}[z]>0\ \&\&\ \text{Log}[x]>0\right]$$

```
Integrate[D[ a1[s, z], s], {s, 1, x}]
```

ConditionalExpression[-1 + x^z, Re[x] ≥ 0 || x ∉ Reals]

```
Integrate[D[ a2[s, z], s], {s, 1, x}]
```

ConditionalExpression[$(-1+x)^z$, Re[z] > 0]

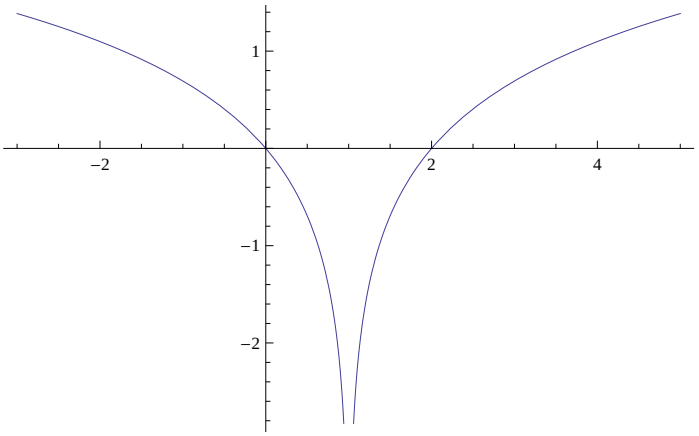
```
Sum[ dd1[j, 4], {j, 2, 100}]
```

3574

```
Limit[ D[ Log[ x - 1], x], x → 1]
```

∞

```
Plot[Re[ {Log[x - 1]}], {x, -3, 5}]
```




```
Limit[ Log[x] / x, x → 0]
```

-∞

```
D[Log[x - 1], x]
```

$$\frac{1}{-1+x}$$

```
ff[x_] := Log[x - 1]
```

```
D[ ff[x], x]
```

$$\frac{1}{-1+x}$$

```
FullSimplify[D[ ff[1 / x], x]] /. x → 3
```

$$\frac{1}{6}$$

```
FullSimplify[D[ff[x], x] / D[ff[x], x]]
```

$$\frac{1}{x}$$

```
D[ff[x], x] /. x → 7
```

$$\frac{1}{6}$$

```
D[ff[x], x] /. x → 1 / 7
```

$$-\frac{7}{6}$$

```
Table[ {- (D[ff[x], x] /. x → n) n, D[ff[x], x] /. x → (1 / n)}, {n, 2, 8}]
```

$$\left\{\{-2, -2\}, \left\{-\frac{3}{2}, -\frac{3}{2}\right\}, \left\{-\frac{4}{3}, -\frac{4}{3}\right\}, \left\{-\frac{5}{4}, -\frac{5}{4}\right\}, \left\{-\frac{6}{5}, -\frac{6}{5}\right\}, \left\{-\frac{7}{6}, -\frac{7}{6}\right\}, \left\{-\frac{8}{7}, -\frac{8}{7}\right\}\right\}$$

```
ff[x_] := Log[x]
Table[ {1 / (D[ff[x], x] /. x → n), D[ff[x], x] /. x → (1 / n)}, {n, 2, 8}]
```

$$\{\{2, 2\}, \{3, 3\}, \{4, 4\}, \{5, 5\}, \{6, 6\}, \{7, 7\}, \{8, 8\}\}$$

```
ff[x_] := LogIntegral[x] - Log[Log[x]] - EulerGamma
Table[ {n (D[ff[x], x] /. x → n), D[ff[x], x] /. x → (1 / n)}, {n, 2, 8}]
```

$$\left\{\left\{\frac{1}{\text{Log}[2]}, \frac{1}{\text{Log}[2]}\right\}, \left\{\frac{2}{\text{Log}[3]}, \frac{2}{\text{Log}[3]}\right\}, \left\{\frac{3}{\text{Log}[4]}, \frac{3}{\text{Log}[4]}\right\},\right.$$
$$\left.\left\{\frac{4}{\text{Log}[5]}, \frac{4}{\text{Log}[5]}\right\}, \left\{\frac{5}{\text{Log}[6]}, \frac{5}{\text{Log}[6]}\right\}, \left\{\frac{6}{\text{Log}[7]}, \frac{6}{\text{Log}[7]}\right\}, \left\{\frac{7}{\text{Log}[8]}, \frac{7}{\text{Log}[8]}\right\}\right\}$$

```
ff[x_] := LogIntegral[x]
Table[ {- (D[ff[x], x] /. x → n), D[ff[x], x] /. x → (1 / n)}, {n, 2, 8}]
```

$$\left\{\left\{-\frac{1}{\text{Log}[2]}, -\frac{1}{\text{Log}[2]}\right\}, \left\{-\frac{1}{\text{Log}[3]}, -\frac{1}{\text{Log}[3]}\right\}, \left\{-\frac{1}{\text{Log}[4]}, -\frac{1}{\text{Log}[4]}\right\},\right.$$
$$\left.\left\{-\frac{1}{\text{Log}[5]}, -\frac{1}{\text{Log}[5]}\right\}, \left\{-\frac{1}{\text{Log}[6]}, -\frac{1}{\text{Log}[6]}\right\}, \left\{-\frac{1}{\text{Log}[7]}, -\frac{1}{\text{Log}[7]}\right\}, \left\{-\frac{1}{\text{Log}[8]}, -\frac{1}{\text{Log}[8]}\right\}\right\}$$

```
ff[x_] := LogIntegral[x + 1] - Log[Log[x + 1]] - EulerGamma
Table[ {(D[ff[x], x] /. x → n), D[ff[x], x] /. x → (1 / n)}, {n, 2, 8}]
```

$$\left\{\left\{\frac{2}{3\,\text{Log}[3]}, \frac{1}{3\,\text{Log}\left[\frac{3}{2}\right]}\right\}, \left\{\frac{3}{4\,\text{Log}[4]}, \frac{1}{4\,\text{Log}\left[\frac{4}{3}\right]}\right\}, \left\{\frac{4}{5\,\text{Log}[5]}, \frac{1}{5\,\text{Log}\left[\frac{5}{4}\right]}\right\},\right.$$
$$\left.\left\{\frac{5}{6\,\text{Log}[6]}, \frac{1}{6\,\text{Log}\left[\frac{6}{5}\right]}\right\}, \left\{\frac{6}{7\,\text{Log}[7]}, \frac{1}{7\,\text{Log}\left[\frac{7}{6}\right]}\right\}, \left\{\frac{7}{8\,\text{Log}[8]}, \frac{1}{8\,\text{Log}\left[\frac{8}{7}\right]}\right\}, \left\{\frac{8}{9\,\text{Log}[9]}, \frac{1}{9\,\text{Log}\left[\frac{9}{8}\right]}\right\}\right\}$$

```
Expand[(D[ff[x], x] /. x → (1 / y)) - (D[ff[x], x] /. x → y)]
```

$$\frac{1}{\text{Log}\left[1+\frac{1}{y}\right]}-\frac{1}{\left(1+\frac{1}{y}\right)\text{Log}\left[1+\frac{1}{y}\right]}-\frac{1}{\text{Log}[1+y]}+\frac{1}{(1+y)\,\text{Log}[1+y]}$$

```
1 / (7 Log[7] - 7 Log[6])
```

$$\frac{1}{-7\,\text{Log}[6]+7\,\text{Log}[7]}$$

```
(1 / x) / (1 - (1 / x))
```

$$\frac{1}{\left(1-\frac{1}{x}\right)x}$$

```
1 / (1 - 1 / x) × 1 / x
```

$$\frac{1}{\left(1-\frac{1}{x}\right)x}$$

```
x / (x - 1) 1 / x
```

$$\frac{1}{-1+x}$$

```
(g2[100, z] - 1) / z /. z → .00001
```

$$30.1264+6.28508\,i$$

```
N@Gamma[0, -Log[4]]
```

$$-2.96759-3.14159\,i$$

```
Integrate[ 1, {s, 1, x}, {t, 1, x}]
```

$$(-1+x)^2$$

```
Sum[ (-1)^(k + 1) / k (x - 1)^k, {k, 1, Infinity}]
```

$$\text{Log}[x]$$

```
d2[100, 7]
```

$$0$$

```
D[ Log[x - 1], x]
```

$$\frac{1}{-1+x}$$

```
D[ LogIntegral[x] - Log[Log[x]] - EulerGamma, x]
```

$$\frac{1}{\text{Log}[x]}-\frac{1}{x\,\text{Log}[x]}$$

```
FullSimplify[D[ LogIntegral[x - 1] - Log[Log[x - 1]] - EulerGamma, x]]
```

$$\frac{-2+x}{(-1+x)\,\text{Log}[-1+x]}$$

```
Limit[Log[x] / (x - 1), x → 1]
```

$$1$$

```
Limit[ D[ Log[x - 1], x], x → 1]
```

∞

```
Limit[ D[LogIntegral[x], x], x → .9999999999]
```

$-1. \times 10^{10}$

```
D[ g1l[x], x]
```

```
Integrate[ (1/Log[s] - 1/(s Log[s])) (1/Log[t] - 1/(t Log[t])), {s, 1, x}, {t, 1, x / s}]
```

$$\int_1^x -\frac{(-1+s)\left(\text{EulerGamma}+\text{Log}\left[\text{Log}\left[\frac{x}{s}\right]\right]-\text{LogIntegral}\left[\frac{x}{s}\right]\right)}{s\,\text{Log}[s]}\,ds$$

```
N[∫_1^x -((-1 + s) (EulerGamma + Log[Log[x/s]] - LogIntegral[x/s]))/(s Log[s]) ds /. x → 100]
```

80.5038

```
Table[ D[Log[x + 1]^2, {x, k}] / k! /. x → 0, {k, 0, 6}]
```

$$\left\{0, 0, 1, -1, \frac{11}{12}, -\frac{5}{6}, \frac{137}{180}\right\}$$

```
Chop@N@g1lk[100, 3, 40]
```

134.883

```
N[Integrate[ D[g1l[s], s] D[g1l[t], t] D[g1l[u], u],
   {s, 1, x}, {t, 1, x / s}, {u, 1, x / (s t)}] /. x → 100]
```

134.883

```
N[D[ g1[100, z], {z, 3}] /. z → 0]
```

134.883

```
D[g1l[s], {s, 1}]
```

$$\frac{1}{\text{Log}[s]}-\frac{1}{s\,\text{Log}[s]}$$

```
Integrate[ (1/Log[s] - 1/(s Log[s])) (1/Log[t] - 1/(t Log[t])), {s, 1, x}, {t, 1, x / s}]
```

$Aborted

```
Table[N[ aa[10, .1^k]], {k, 1, 5}]
```

{0.840729, 0.840946, 0.840968, 0.84097, 0.84097}

```
N[D[ (LogIntegral[x] - Log[Log[x]] - EulerGamma)^2, x] /. x → 10]
```

3.71662

```
FullSimplify[(1/Log[s] - 1/(s Log[s])) (1/Log[t] - 1/(t Log[t]))]
```

$$\frac{(-1+s)\,(-1+t)}{s\,t\,\text{Log}[s]\,\text{Log}[t]}$$

```
Integrate[(1/Log[t] - 1/(t Log[t])), t]
```

```
Limit[LogIntegral[t] - Log[Log[t]], t → x / s]
```

$$-\text{Log}\left[\text{Log}\left[\frac{x}{s}\right]\right]+\text{LogIntegral}\left[\frac{x}{s}\right]$$

```
Integrate[
 (1 / Log[s] - 1 / (s Log[s])) (LogIntegral[x / s] - Log[Log[x / s]] - EulerGamma), {s, 1, x}]
```

$$\int_1^x \left(\frac{1}{\text{Log}[s]}-\frac{1}{s\,\text{Log}[s]}\right)\left(-\text{EulerGamma}-\text{Log}\left[\text{Log}\left[\frac{x}{s}\right]\right]+\text{LogIntegral}\left[\frac{x}{s}\right]\right)\,ds$$

```
Integrate[LogIntegral[x / s], {s, 1, x}]
```

```
$Aborted
```

```
Table[(-1)^(k + 1) / k D[g2[n, k], n], {k, 1, 7}]
```

$$\left\{1,\ -\frac{\text{Log}[n]}{2},\ \frac{\text{Log}[n]^2}{6},\ -\frac{1}{24}\text{Log}[n]^3,\ \frac{\text{Log}[n]^4}{120},\ -\frac{1}{720}\text{Log}[n]^5,\ \frac{\text{Log}[n]^6}{5040}\right\}$$

```
Expand[Sum[(-1)^(k + 1) / (k!) Log[n]^(k - 1), {k, 1, Infinity}]]
```

$$\frac{1}{\text{Log}[n]}-\frac{1}{n\,\text{Log}[n]}$$

```
Table[(-1)^(k + 1) / k D[g2[n, k], {n, 2}], {k, 1, 7}]
```

$$\left\{0,\ -\frac{1}{2\,n},\ \frac{\text{Log}[n]}{3\,n},\ -\frac{\text{Log}[n]^2}{8\,n},\ \frac{\text{Log}[n]^3}{30\,n},\ -\frac{\text{Log}[n]^4}{144\,n},\ \frac{\text{Log}[n]^5}{840\,n}\right\}$$

```
(-1)^(k + 1) / k D[g2[n, k], {n, 2}]
```

$$\frac{(-1)^{1+2k}\,(-1+k)\,(-\text{Log}[n])^{-2+k}}{k\,n\,\text{Gamma}[k]}$$

```
Sum[((-1)^(1 + 2 k) (-1 + k) (-Log[n])^(-2 + k))/(k n Gamma[k]), {k, 1, Infinity}]
```

$$-\frac{-1+n-\text{Log}[n]}{n^2\,\text{Log}[n]^2}$$

```
Integrate[-(-1 + n - Log[n])/(n^2 Log[n]^2), {n, 1, x}]
```

$$\text{ConditionalExpression}\left[-1+\frac{-1+x}{x\,\text{Log}[x]},\ \text{Im}[x]\neq 0\ ||\ \text{Re}[x]\geq 0\right]$$

```
Expand[ -(-1 + n - Log[n])/(n^2 Log[n]^2) ]
```

$$\frac{1}{n^2 \log[n]^2} - \frac{1}{n \log[n]^2} + \frac{1}{n^2 \log[n]}$$

```
Integrate[ D[ Log[s]^2, s] D[Log[t]^2, t], {s, 1, x}, {t, 1, x}]
```

```
ConditionalExpression[Log[x]^4, Re[x] ≥ 0 || x ∉ Reals]
```

```
D[ x^z, {z, 3}] /. z → 0
```

```
Log[x]^3
```

```
FullSimplify[D[ Log[x]^(a + b), x]]
```

$$\frac{(a+b)\,\log[x]^{-1+a+b}}{x}$$

```
Sum[ Log[x]^k / (k ! k), {k, 1, Infinity}]
```

```
-EulerGamma - Gamma[0, -Log[x]] - Log[-Log[x]]
```

```
Limit[ Sum[D[ Log[x]^k, x] / (k ! k), {k, 1, Infinity}], x → 1]
```

```
1
```

```
Limit[Sum[D[ Log[x]^k, {x, 2}] / (k ! k), {k, 1, Infinity}], x → 1]
```

$$-\frac{1}{2}$$

```
Limit[ Sum[D[ Log[x]^k, {x, 3}] / (k ! k), {k, 1, Infinity}], x → 1]
```

$$\frac{5}{6}$$

```
Limit[ Sum[D[ Log[x]^k, {x, 4}] / (k ! k), {k, 1, Infinity}], x → 1]
```

$$-\frac{9}{4}$$

```
Limit[ Sum[D[ Log[x]^k, {x, 5}] / (k ! k), {k, 1, Infinity}], x → 1]
```

$$\frac{251}{30}$$

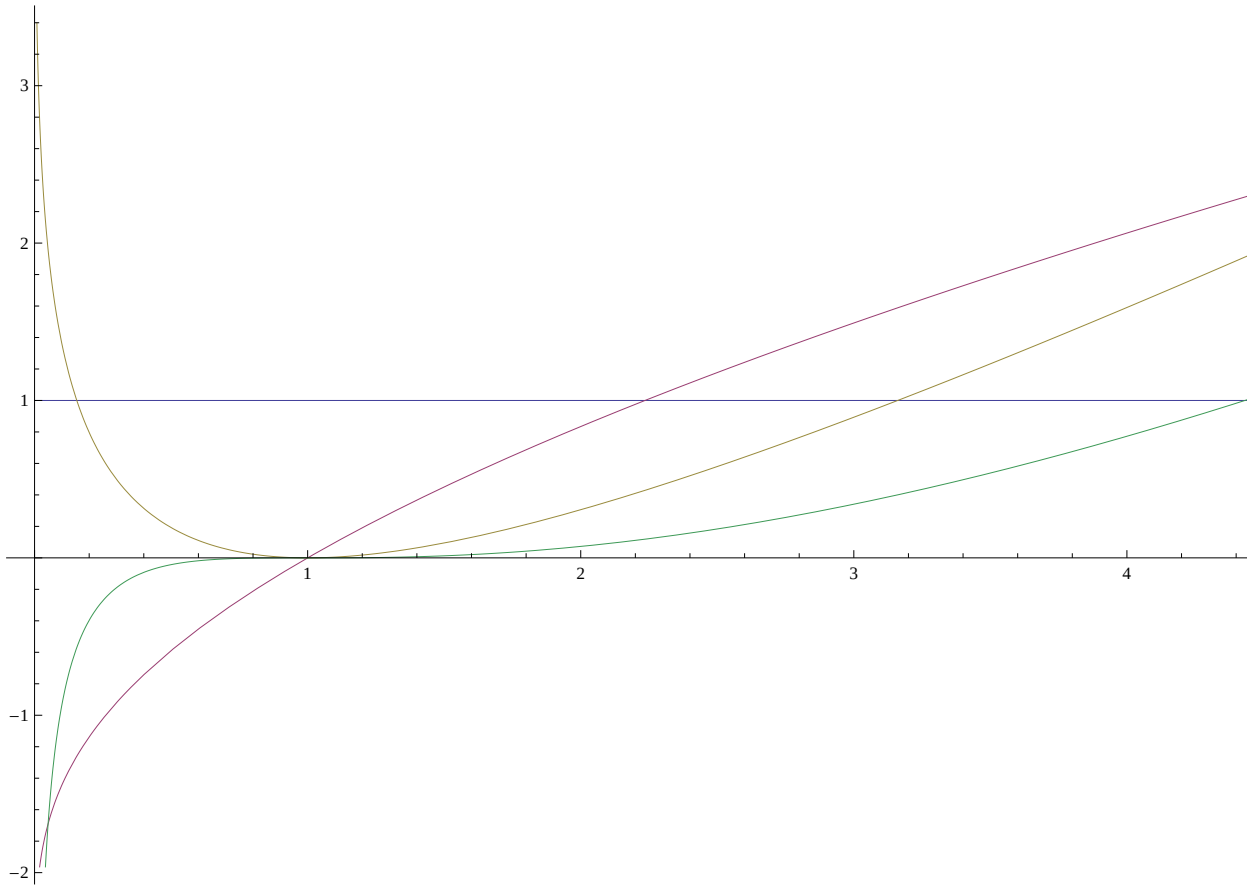
```
Limit[ Sum[D[ Log[x]^k, {x, 6}] / (k ! k), {k, 1, Infinity}], x → 1]
```

$$-\frac{475}{12}$$

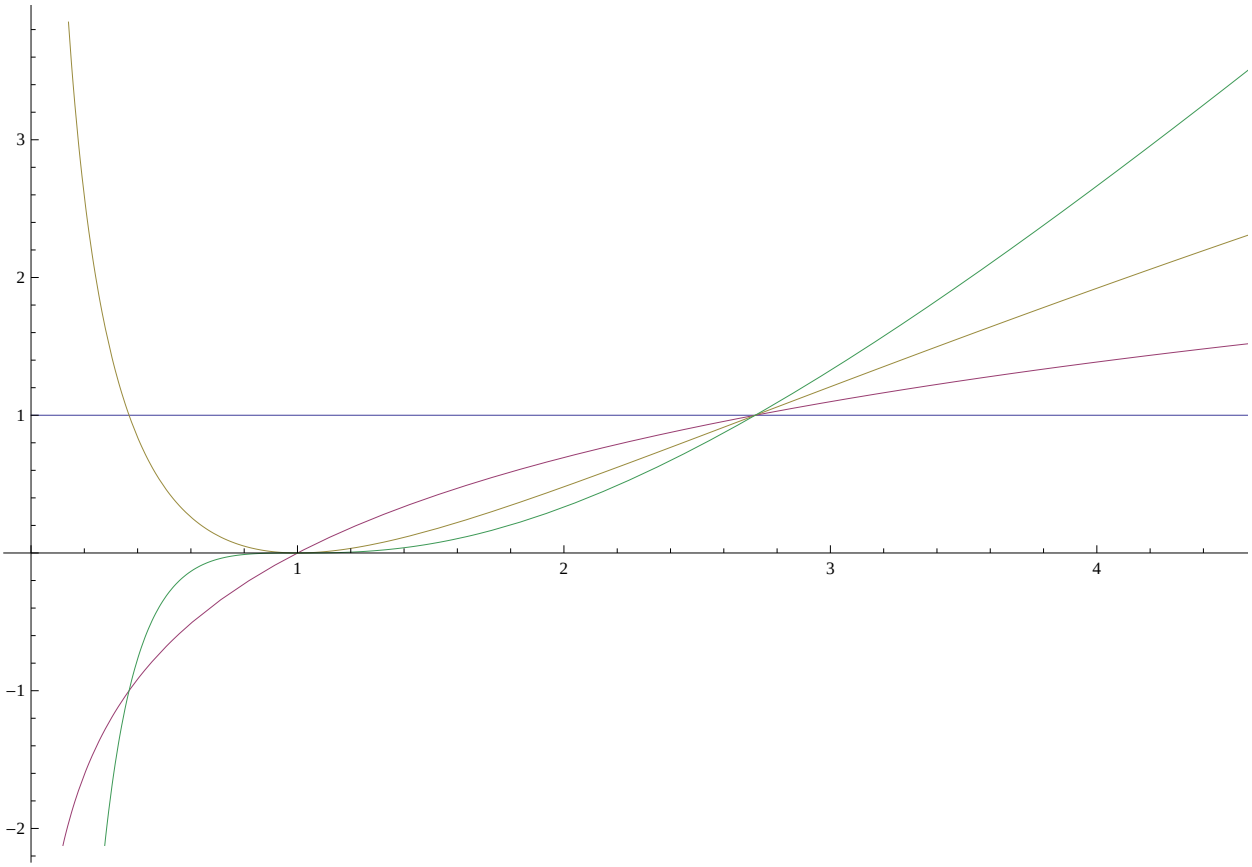
```
Table[ N[D[ a1[x, z], {z, k}] /. z → 0 /. x → 1], {k, 0, 8}]
```

```
{1., 0., 0., 0., 0., 0., 0., 0., 0.}
```

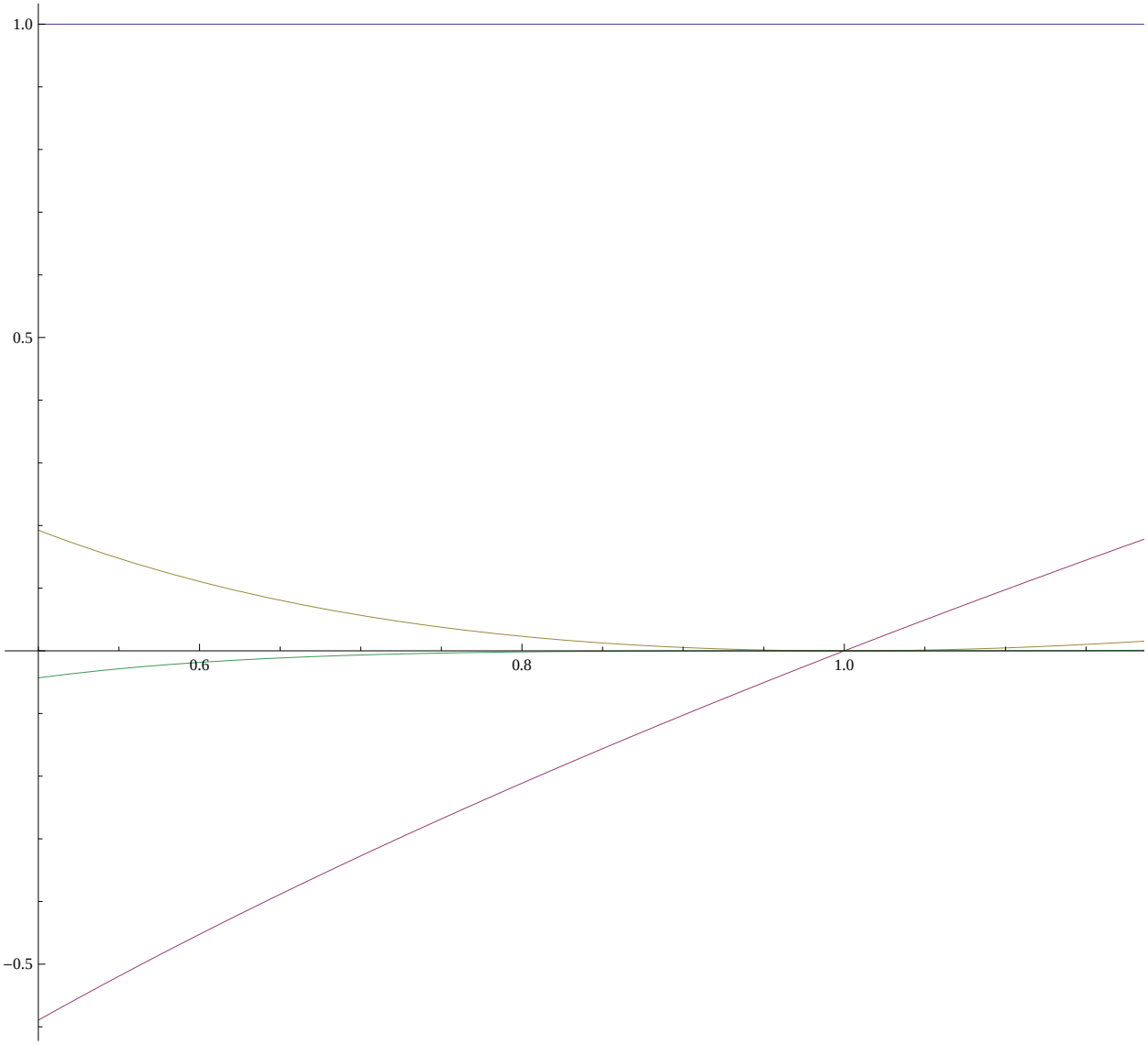
```
Plot[{1, D[ LaguerreL[-z, Log[x]], {z, 1}] /. z → 0,
  D[ LaguerreL[-z, Log[x]], {z, 2}] /. z → 0 ,
  D[ LaguerreL[-z, Log[x]], {z, 3}] /. z → 0}, {x, 0, 5}]
```



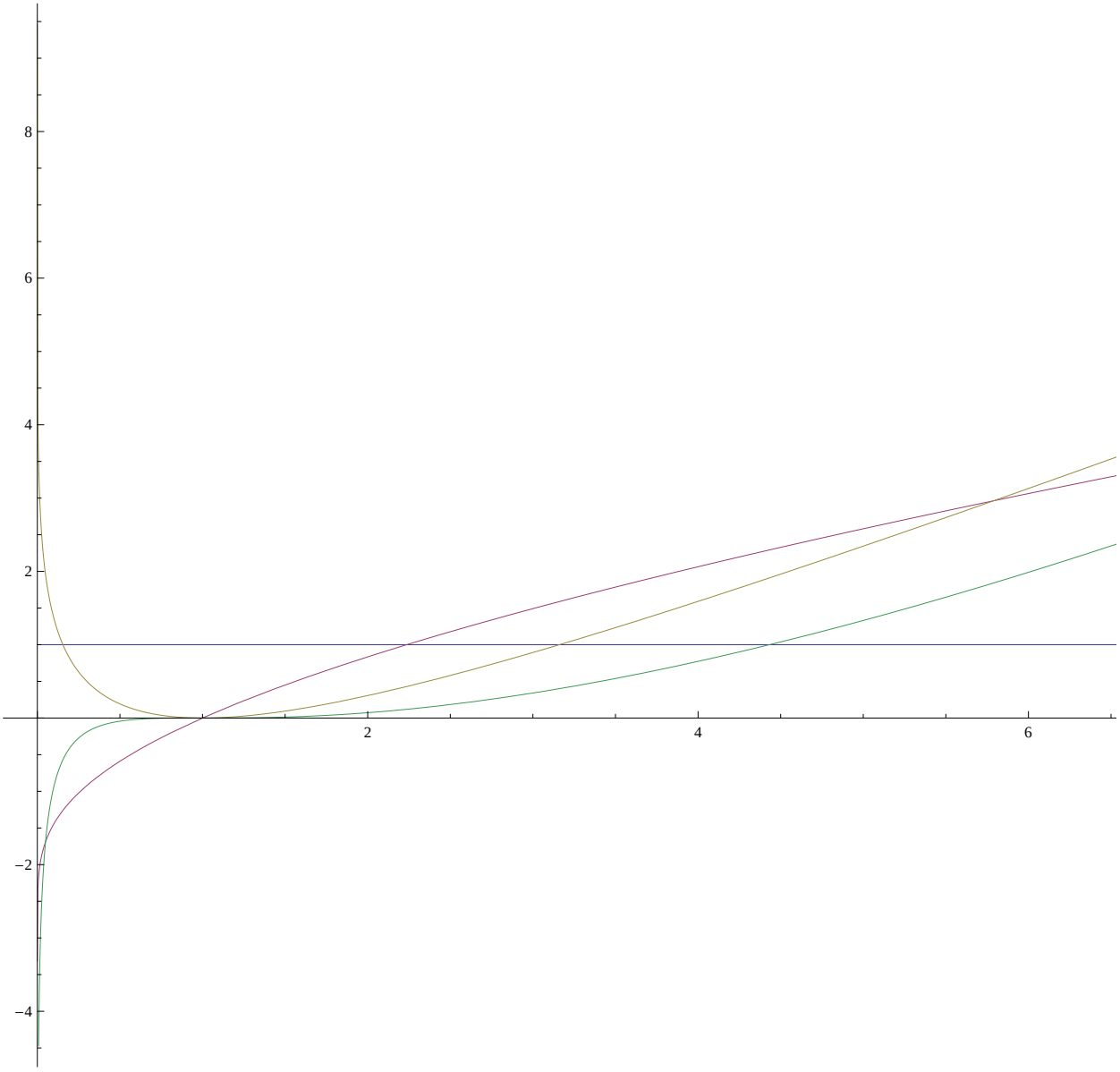
```
Plot[{1, D[ x^z, {z, 1}] /. z → 0,
  D[ x^z, {z, 2}] /. z → 0 , D[ x^z, {z, 3}] /. z → 0}, {x, 0, 5}]
```



```
Plot[{1, D[ LaguerreL[-z, Log[x]], {z, 1}] /. z → 0,
  D[ LaguerreL[-z, Log[x]], {z, 2}] /. z → 0 ,
  D[ LaguerreL[-z, Log[x]], {z, 3}] /. z → 0}, {x, .5, 1.5}]
```

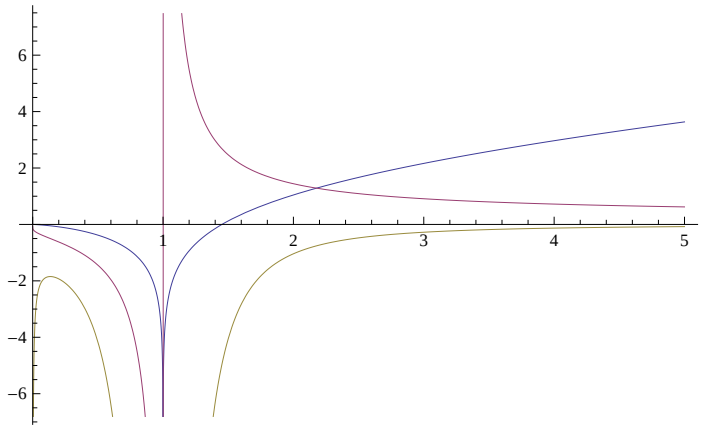


```
Plot[{1, D[ LaguerreL[-z, Log[x]], {z, 1}] /. z → 0,
  D[ LaguerreL[-z, Log[x]], {z, 2}] /. z → 0 ,
  D[ LaguerreL[-z, Log[x]], {z, 3}] /. z → 0}, {x, 0, 10}]
```




```
ss[x_] := -ExpIntegralEi[2 Log[x]] + x LogIntegral[x]
```

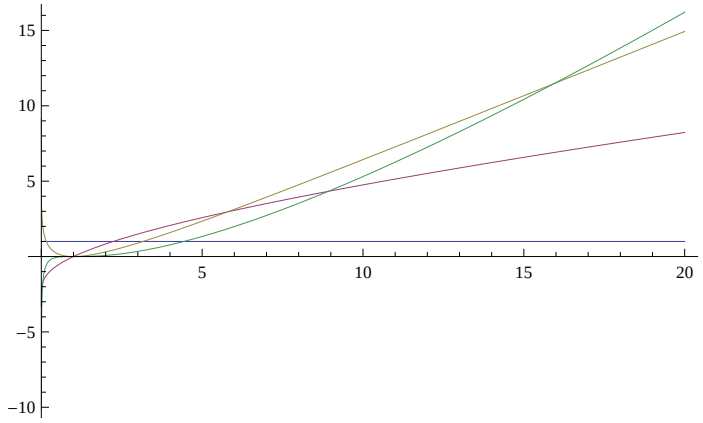
```
Plot[{LogIntegral[x], 1 / Log[x], -1 / (x Log[x]^2)}, {x, 0, 5}]
```



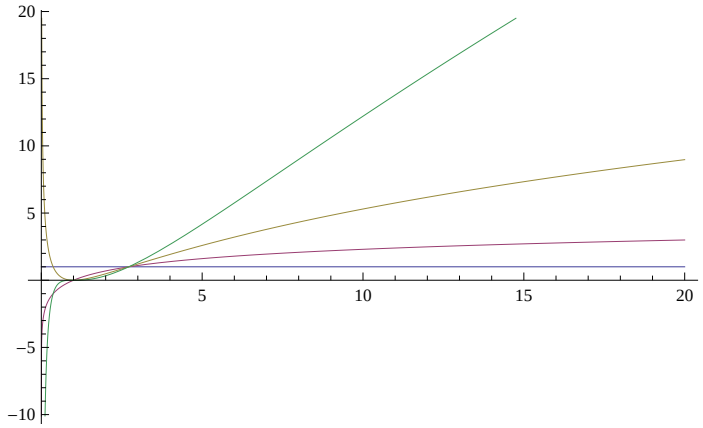

```
FullSimplify[D[ss[x] , {x, 4}]]
```

$$\frac{2 + \text{Log}[x]}{x^2 \, \text{Log}[x]^3}$$

```
Plot[{1, D[ LaguerreL[-z, Log[x]], {z, 1}] /. z → 0,
  D[ LaguerreL[-z, Log[x]], {z, 2}] /. z → 0 ,
  D[ LaguerreL[-z, Log[x]], {z, 3}] /. z → 0}, {x, 0, 20}]
```




```
Plot[{1, D[ x^z, {z, 1}] /. z → 0,
  D[ x^z, {z, 2}] /. z → 0 , D[ x^z, {z, 3}] /. z → 0}, {x, 0, 20}]
```



```
D[ Log[x]^(a + b), x]
```

$$\frac{(a+b)\,\text{Log}[x]^{-1+a+b}}{x}$$

```
D[Integrate[ D[ Log[s]^a, s] D[Log[t]^b, t], {s, 1, x}, {t, 1, x}], x]
```

$$\text{ConditionalExpression}\left[\frac{(a+b)\,\text{Log}[x]^{-1+a+b}}{x},\ (\text{Re}[x]\geq 0\ ||\ x\notin \text{Reals})\ \&\&\ \text{Re}[a]>0\right]$$

```
FullSimplify[ (a + b) Log[x]^(-1+a+b) / x ] /. x → 100 /. a → 3 /. b → 2
```

$$\frac{\text{Log}[100]^4}{20}$$

```
D[ Integrate[ D[f[s], s] D[g[t], t], {s, 1, x}, {t, 1, x}], x]
```

$$-(g[1]-g[x])\,f'[x]-(f[1]-f[x])\,g'[x]$$

```
D[Integrate[ D[ Log[s]^a, s] Integrate[ D[Log[t]^b, t], {t, 1, x}], {s, 1, x}], x]
```

$$\text{ConditionalExpression}\left[\frac{(a+b)\,\text{Log}[x]^{-1+a+b}}{x},\ (\text{Re}[x]\geq 0\ ||\ x\notin \text{Reals})\ \&\&\ \text{Re}[b]>0\ \&\&\ \text{Re}[a]>0\right]$$

```
Integrate[ D[Log[t]^b, t], {t, 1, x}]
```

$$\text{ConditionalExpression}\left[\text{Log}[x]^b,\ (\text{Re}[x]\geq 0\ ||\ x\notin \text{Reals})\ \&\&\ \text{Re}[b]>0\right]$$

```
D[Integrate[ D[ Log[s]^a, s] Log[x]^b, {s, 1, x}], x]
```

$$\text{ConditionalExpression}\left[\frac{(a+b)\,\text{Log}[x]^{-1+a+b}}{x},\ (\text{Re}[x]\geq 0\ ||\ x\notin \text{Reals})\ \&\&\ \text{Re}[a]>0\right]$$

```
D[Log[x]^b Integrate[ D[ Log[s]^a, s] , {s, 1, x}], x]
```

$$\text{ConditionalExpression}\left[\frac{(a+b)\,\text{Log}[x]^{-1+a+b}}{x},\ (\text{Re}[x]\geq 0\ ||\ x\notin \text{Reals})\ \&\&\ \text{Re}[a]>0\right]$$

```
Integrate[ D[ Log[s]^a, s] , {s, 1, x}]
```

$$\text{ConditionalExpression}[\text{Log}[x]^a,\ (\text{Re}[x]\geq 0\ ||\ x\notin \text{Reals})\ \&\&\ \text{Re}[a]>0]$$

```
D[Log[x]^b Log[x]^a, x]
```

$$\frac{(a+b)\,\text{Log}[x]^{-1+a+b}}{x}$$

```
D[ Log[s]^a, s] D[Log[t]^b, t] /. {s → x, t → x}
```

$$\frac{a\,b\,\text{Log}[x]^{-2+a+b}}{x^2}$$

```
D[ Integrate[ D[f[s], s] D[g[t], t], {s, 1, x}, {t, 1, x / s}], x]
```

$$\int_1^x \frac{f'[s]\,g'\left[\frac{x}{s}\right]}{s}\,\mathrm{d}s$$

```
ff[x_] := 1 / Log[x] - 1 / (x Log[x])
```

```
Sum[ z^k / k ! Log[x]^k, {k, 0, Infinity}]
```

$$x^z$$

```
Sum[ z^k / k ! Log[x]^(k + 1), {k, 0, Infinity}]
```

$$x^z \text{Log}[x]$$

```
D[ x^z, z]
```

$$x^z \text{Log}[x]$$

```
Integrate[ D[ Log[s], s] D[ t^z, t], {s, 1, x}, {t, 0, x}]
```

$$\text{ConditionalExpression}[x^z \text{Log}[x], \text{Re}[x] \geq 0 \mid\mid x \notin \text{Reals}]$$

```
1 + Integrate[ Log[x] x^y, {y, 0, z}]
```

$$x^z$$

```
1 + Integrate[ D[Log[s], s] Integrate[ x^y, {y, 0, z}], {s, 1, x}]
```

$$\text{ConditionalExpression}[x^z, \text{Re}[x] \geq 0 \mid\mid x \notin \text{Reals}]$$

```
D[ Log[s], s] D[ t^y, t]
```

$$\frac{t^{-1+y}\, y}{s}$$

```
Integrate[ t^(-1+y) y / s, {y, 0, z}]
```

$$\frac{1 - t^z + t^z z \text{Log}[t]}{s\, t \text{Log}[t]^2}$$

```
1 + Integrate[D[ Log[s], s] D[ t^y, t], {y, 0, z} , {s, 1, x}, {t, 0, x}]
```

$$x^z$$

$$-1 + x^z$$

```
1 + Integrate[D[ g1l[s], s] D[ g1[t, y], t], {y, 0, z} , {s, 1, x}, {t, 1, x}]
```

$$1 + \int_0^z -(-1 + \text{Hypergeometric1F1}[y, 1, \text{Log}[x]])\,(\text{EulerGamma} + \text{Log}[\text{Log}[x]] - \text{LogIntegral}[x])\, dy$$

```
N[ 1 + Integrate[ -(-1 + Hypergeometric1F1[y, 1, Log[x]])
      (EulerGamma + Log[Log[x]] - LogIntegral[x]) dy /. {x → 100, z → 2}]
```

8800.43

```
N@g1[ 100, 2]
```

560.517

```
Sum[ BernoulliB[k] / (k !) Log[x]^(k + z - 1) (x - 1), {k, 0, Infinity}]
```

$$\text{Log}[x]^z$$

```
Sum[ BernoulliB[k] / (k!) Integrate[ D[ Log[y]^(k + z - 1), y], {y, 1, x}]
  Integrate[ D[ (y - 1), y], {y, 1, x}], {k, 0, Infinity}]
```

$$\sum_{k=0}^{\infty} \text{ConditionalExpression}\left[\frac{(-1+x)\ \text{BernoulliB}[k]\ \text{Log}[x]^{-1+k+z}}{k!},\ (\text{Re}[x] \geq 0 \,||\, x \notin \text{Reals}) \&\& \text{Re}[k+z] > 1\right]$$

```
Integrate[ D[ Log[y]^(k + z - 1), y], {y, 1, x}] Integrate[ D[ (y - 1), y], {y, 1, x}]
```

$$\text{ConditionalExpression}\left[(-1+x)\ \text{Log}[x]^{-1+k+z},\ (\text{Re}[x] \geq 0 \,||\, x \notin \text{Reals}) \&\& \text{Re}[k+z] > 1\right]$$

```
Sum[ BernoulliB[k] / (k!) ((-1 + x) Log[x]^(-1+k+z)), {k, 0, Infinity}]
```

$$\text{Log}[x]^{z}$$

```
Integrate[ D[ Log[y]^(k + z - 1), y], {y, 1, x}] Integrate[ D[ (y - 1), y], {y, 1, x}]
```

$$\text{ConditionalExpression}\left[(-1+x)\ \text{Log}[x]^{-1+k+z},\ (\text{Re}[x] \geq 0 \,||\, x \notin \text{Reals}) \&\& \text{Re}[k+z] > 1\right]$$

```
1 + Integrate[ Log[x] x^y, {y, 0, z}]
```

$$x^{z}$$

```
Integrate[ D[ s^a, s], {s, 1, x}]
```

$$\text{ConditionalExpression}[-1 + x^{a},\ \text{Re}[x] \geq 0 \,||\, x \notin \text{Reals}]$$

```
Expand[Integrate[ D[ s^a, s] D[t^a, t], {s, 1, x}, {t, 1, x / s}]]
```

$$\text{ConditionalExpression}[1 - x^{a} + a\, x^{a}\, \text{Log}[x],\ \text{Re}[x] \geq 0 \,||\, x \notin \text{Reals}]$$

```
Expand[Integrate[ D[ s^a, s] D[t^a, t] D[u^a, u], {s, 1, x}, {t, 1, x / s}, {u, 1, x / (s t)}]]
```

$$\text{ConditionalExpression}\left[-1 + x^{a} - a\, x^{a}\, \text{Log}[x] + \frac{1}{2} a^{2}\, x^{a}\, \text{Log}[x]^{2},\ \text{Re}[x] \geq 0 \,||\, x \notin \text{Reals}\right]$$

```
N@((-1)^a Gamma[3, 0, -a Log[x]] / Gamma[3]) /. {x → 100, a → 4}
```

$$-1.5224 \times 10^{10} + 3.72881 \times 10^{-6}\ i$$

```
N[(-1 + x^a - a x^a Log[x] + 1/2 a^2 x^a Log[x]^2) /. {x → 100, a → 4}]
```

$$1.5224 \times 10^{10}$$

```
ffx[ n_, a_, t_] :=
 Sum[ (-1)^(k + 1) / k (-1)^k ( 1 - Gamma[ k, -a Log[n]] / Gamma[k]), {k, 1, t}]
Chop@N[ffx[100, 1, 30]]
```

$$28.0217$$

```
Chop@N[ffx[100, 2, 30]]
```

$$1243.32 - 1.13248 \times 10^{-9}\ i$$

```
FullSimplify[Integrate[ D[ s^a, s] D[t^a, t], {s, 1, x}, {t, 1, x}]]
```

$$\text{ConditionalExpression}\left[(-1+x^{a})^{2},\ \text{Re}[x] \geq 0 \,||\, x \notin \text{Reals}\right]$$

```
FullSimplify[Integrate[ D[ s^a, s] D[t^a, t] D[u^a, u], {s, 1, x}, {t, 1, x}, {u, 1, x}]]
```

$$\text{ConditionalExpression}\left[(-1+x^{a})^{3},\ \text{Re}[x] \geq 0 \,||\, x \notin \text{Reals}\right]$$

```
Sum[ (-1)^(k+1) / k (x^a-1)^k, {k, 1, Infinity}]
```

$$\text{Log}[x^a]$$

```
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}]

pp[n_, k_, z_] := Sum[ dz[j, z] (1 / k - pp[n / j, k+1, z]), {j, 2, n}]
pp[100, 1, 3] / 3
```

$$\frac{428}{15}$$

```
Limit[ (g1[ 100, a z] - 1) / z, z → 0]
```

$$-a\ \text{LaguerreL}^{(1,0)}[0, \text{Log}[100]]$$

```
Limit[ (g1[ 100, z] - 1) / z, z → 0]
```

$$-\text{LaguerreL}^{(1,0)}[0, \text{Log}[100]]$$

```
Limit[ (g1[ 100^a, z] - 1) / z, z → 0]
```

$$-\text{LaguerreL}^{(1,0)}[0, \text{Log}[100^a]]$$

```
FullSimplify[Integrate[ D[ s^a, s] D[t^a, t], {s, 1, x}, {t, 1, x}]]
```

$$\text{ConditionalExpression}\left[(-1+x^a)^2, \text{Re}[x] \geq 0 \,||\, x \notin \text{Reals}\right]$$

```
FullSimplify[Integrate[ D[ s, s] D[t, t], {s, 1, x^a}, {t, 1, x^a}]]
```

$$(-1+x^a)^2$$

```
FullSimplify[Integrate[ D[ s^a, s] D[t^a, t], {s, 1, x}, {t, 1, x / s}]]
```

$$\text{ConditionalExpression}[1+x^a\ (-1+a\ \text{Log}[x]), \text{Re}[x] \geq 0 \,||\, x \notin \text{Reals}]$$

```
FullSimplify[Integrate[ D[ s, s] D[t, t], {s, 1, x^a}, {t, 1, x^a / s}]]
```

$$\text{ConditionalExpression}[1+x^a\ (-1+\text{Log}[x^a]), \text{Re}[x^a] \geq 0 \,||\, x^a \notin \text{Reals}]$$

```
Integrate[ D[ s^a, s] D[t^a, t], {s, 0, 1}, {t, 0, 1}]
```

$$\text{ConditionalExpression}[1, \text{Re}[a] > 0]$$

```
FullSimplify[Integrate[ D[ s^a, s] D[t^a, t], {s, 1, x}, {t, 1, x / s}]]
```

$$\text{ConditionalExpression}[1+x^a\ (-1+a\ \text{Log}[x]), \text{Re}[x] \geq 0 \,||\, x \notin \text{Reals}]$$

```
FullSimplify[Integrate[ D[ s^a, s], {s, 1, x}]]
```

$$\text{ConditionalExpression}[-1+x^a, \text{Re}[x] \geq 0 \,||\, x \notin \text{Reals}]$$

```
Expand[1 + x^a (-1 + a Log[x]) + 2 (-1 + x^a) + 1]
```

$$x^a + a\ x^a\ \text{Log}[x]$$

```
N[LaguerreL[ -2, Log[x^a]] /. {x → 10, a → 2}]
```

$$560.517$$

```
N[x^a + a x^a Log[x] /. {x → 10, a → 2}]
```

$$560.517$$

```
a1[a1[4, 3], 2]
```

$$4096$$

```
a1[4, 6]
```

4096

```
N[g1[g1[13, 3], 2]]
```

711.167

```
N@g1[13, 6]
```

1102.22

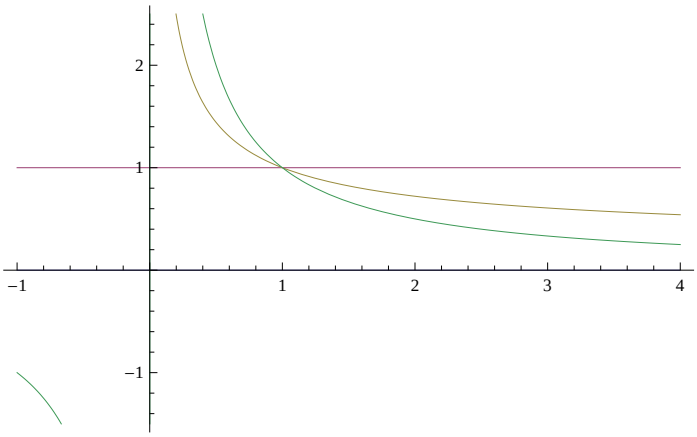
```
N[d1[d1[55, 3], 2]]
```

4187.

```
N@d1[55, 6]
```

4856.

```
Plot[{0, 1, 1/Log[x] - 1/(x Log[x]), 1 / x}, {x, -1, 4}]
```




```
N@(x D[g1l[x], x]) /. x → (1 / 100)
```

0.214976

```
FullSimplify[D[g1l[E^x], x]]
```

$$\frac{-1+e^x}{\text{Log}[e^x]}$$

```
(E^x - 1) / x
```

$$\frac{-1+e^x}{x}$$

```
Series[(-1 + e^x)/x, {x, 0, 10}]
```

$$1+\frac{x}{2}+\frac{x^2}{6}+\frac{x^3}{24}+\frac{x^4}{120}+\frac{x^5}{720}+\frac{x^6}{5040}+\frac{x^7}{40\,320}+\frac{x^8}{362\,880}+\frac{x^9}{3\,628\,800}+\frac{x^{10}}{39\,916\,800}+O[x]^{11}$$

```
Series[E^x, {x, 0, 10}]
```

$$1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+\frac{x^5}{120}+\frac{x^6}{720}+\frac{x^7}{5040}+\frac{x^8}{40\,320}+\frac{x^9}{362\,880}+\frac{x^{10}}{3\,628\,800}+O[x]^{11}$$

```
FullSimplify[D[g1l[x], x]]
```

$$\frac{-1+x}{x\,\text{Log}[x]}$$

```
D[(E^x - 1) / x, {x, 4}]
```

$$\frac{24\,(-1+e^x)}{x^5}-\frac{24\,e^x}{x^4}+\frac{12\,e^x}{x^3}-\frac{4\,e^x}{x^2}+\frac{e^x}{x}$$

```
(-1 + e^x) / x /. x → 3
```

$$\frac{1}{3}\left(-1+e^3\right)$$

```
(-1 + e^x) / x /. x → -3
```

$$\frac{1}{3}\left(1-\frac{1}{e^3}\right)$$

```
FullSimplify[D[g1l[x], x]]
```

$$\frac{-1+x}{x\,\text{Log}[x]}$$

```
Log[12, 31 * 17]
```

$$\frac{\text{Log}[527]}{\text{Log}[12]}$$

```
FullSimplify[Log[12, 31] + Log[12, 17]]
```

$$\frac{\text{Log}[527]}{\text{Log}[12]}$$

```
N@g1l[g1[1.7, 14]]
```

```
16.2968
```

```
Log[5, 5^7]
```

```
7
```

```
Log[E^7]
```

```
7
```

```
ff5[n_, z_] :=
 Integrate[Sum[Binomial[z, k] (-1)^(k + 1) / ((k - 1)!) t^(k - 1) E^(-t), {k, 0, Infinity}],
  {t, -Log[n], 0}]
ff6[n_, z_] := Integrate[Sum[z^k / (k!) (-1)^(k + 1) / ((k - 1)!) t^(k - 1) E^(-t),
   {k, 0, Infinity}], {t, -Log[n], 0}]
```

```
ff5[n, z]
```

$$\text{ConditionalExpression}\left[-1+\text{LaguerreL}[-z,\text{Log}[n]],-1\le\text{Re}[\text{Log}[n]]\le 1\ ||\ \text{Log}[n]\notin\text{Reals}\right]$$

```
N[ff6[E, N[LogIntegral[6] - Log[Log[6]] - EulerGamma]]]
```

```
12.0578
```

```
Sum[ z^k / (k!) (-1)^(k + 1) / ((k - 1)!) t^(k - 1) E^(-t), {k, 0, Infinity}]
```

$$\frac{e^{-t}\ \sqrt{z}\ \text{BesselJ}\left[1,\ 2\ \sqrt{t}\ \sqrt{z}\right]}{\sqrt{t}}$$

```
InverseFunction[LogIntegral -]
```

```
LogIntegral^(-1)
```

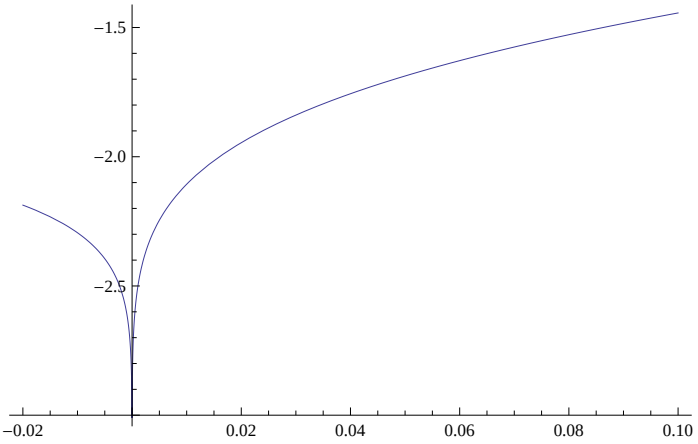
```
Solve[y == a x + b, x]
```

$$\left\{\left\{x \to \frac{-b + y}{a}\right\}\right\}$$

```
InverseFunction[Log]
```

```
Exp
```

```
Plot[ Re[LogIntegral[x] - Log[Log[x]] - EulerGamma], {x, -.02, .1}]
```




```
5 × 5!
```

```
600
```

```
Sum[ (Log[x])^k / k / k!, {k, 1, 12}]
```

$$\text{Log}[x] + \frac{\text{Log}[x]^2}{4} + \frac{\text{Log}[x]^3}{18} + \frac{\text{Log}[x]^4}{96} + \frac{\text{Log}[x]^5}{600} + \frac{\text{Log}[x]^6}{4320} + \frac{\text{Log}[x]^7}{35\,280} + \frac{\text{Log}[x]^8}{322\,560} + \frac{\text{Log}[x]^9}{3\,265\,920} + \frac{\text{Log}[x]^{10}}{36\,288\,000} + \frac{\text{Log}[x]^{11}}{439\,084\,800} + \frac{\text{Log}[x]^{12}}{5\,748\,019\,200}$$

```
Expand[1 / 4 (Log[x] + Log[x]^2/4 + Log[x]^3/18 + Log[x]^4/96 + Log[x]^5/600 + Log[x]^6/4320 + Log[x]^7/35280 + Log[x]^8/322560)^2]
```

$$\frac{\text{Log}[x]^2}{4}+\frac{\text{Log}[x]^3}{8}+\frac{25\,\text{Log}[x]^4}{576}+\frac{7\,\text{Log}[x]^5}{576}+\frac{1507\,\text{Log}[x]^6}{518\,400}+\frac{53\,\text{Log}[x]^7}{86\,400}+\frac{15\,787\,\text{Log}[x]^8}{135\,475\,200}+\frac{12\,317\,\text{Log}[x]^9}{609\,638\,400}+\frac{6943\,\text{Log}[x]^{10}}{2\,257\,920\,000}+\frac{289\,\text{Log}[x]^{11}}{677\,376\,000}+\frac{15\,557\,\text{Log}[x]^{12}}{292\,626\,432\,000}+\frac{143\,\text{Log}[x]^{13}}{24\,385\,536\,000}+\frac{107\,\text{Log}[x]^{14}}{191\,182\,602\,240}+\frac{\text{Log}[x]^{15}}{22\,759\,833\,600}+\frac{\text{Log}[x]^{16}}{416\,179\,814\,400}$$

```
f1[x] := Log[x] + Log[x]^2/4 + Log[x]^3/18 + Log[x]^4/96 + Log[x]^5/600 + Log[x]^6/4320 +
  Log[x]^7/35280 + Log[x]^8/322560 + Log[x]^9/3265920 + Log[x]^10/36288000 + Log[x]^11/439084800 + Log[x]^12/5748019200
f1[x] - Expand[(1 / 4) f1[x]^2] + Expand[(5 / 72) f1[x]^3] -
 Expand[(11 / 576) f1[x]^4] + Expand[(431 / 86400) f1[x]^5]
```

```
2! 3! 3!
```

72

```
2! × 3! × 4! × 5! × 6! × 6!
```

17 915 904 000

```
17 915 904 000 / 1 036 800
```

17 280

```
4 147 200 / 86 400
```

48

```
1241 × 17 280
```

21 444 480

```
Sum[(x)^k / k / k!, {k, 1, 12}]
```

$$x+\frac{x^2}{4}+\frac{x^3}{18}+\frac{x^4}{96}+\frac{x^5}{600}+\frac{x^6}{4320}+\frac{x^7}{35\,280}+\frac{x^8}{322\,560}+\frac{x^9}{3\,265\,920}+\frac{x^{10}}{36\,288\,000}+\frac{x^{11}}{439\,084\,800}+\frac{x^{12}}{5\,748\,019\,200}$$

```
f1[x] := x + x^2/4 + x^3/18 + x^4/96 + x^5/600 + x^6/4320 + x^7/35280 +
  x^8/322560 + x^9/3265920 + x^10/36288000 + x^11/439084800 + x^12/5748019200
Expand[f1[x] - (1 / 4) f1[x]^2 + (5 / 72) f1[x]^3 - (132 / 6912) f1[x]^4 +
  (20688 / 4147200) f1[x]^5 - (21444480 / 17915904000) f1[x]^6]
```

$$x-\frac{3055\,x^7}{12\,192\,768}-\frac{215\,867\,x^8}{541\,900\,800}-\frac{458\,392\,747\,x^9}{1\,316\,818\,944\,000}-\frac{413\,541\,517\,x^{10}}{1\,881\,169\,920\,000}-\frac{85\,420\,232\,869\,x^{11}}{764\,808\,442\,675\,200}-\frac{1\,055\,410\,491\,367\,x^{12}}{21\,851\,669\,790\,720\,000}-\frac{126\,628\,969\,850\,413\,x^{13}}{6\,883\,275\,984\,076\,800\,000}-\frac{51\,622\,810\,392\,787\,x^{14}}{8\,176\,497\,532\,600\,320\,000}-\frac{24\,088\,066\,977\,405\,451\,x^{15}}{12\,142\,098\,835\,911\,475\,200\,000}-\frac{50\,999\,322\,463\,996\,549\,x^{16}}{88\,306\,173\,352\,083\,456\,000\,000}-$$

$$\frac{763\,389\,979\,133\,343\,253\,x^{17}}{4\,856\,839\,534\,364\,590\,080\,000\,000} - \frac{10\,563\,123\,451\,990\,645\,139\,x^{18}}{262\,269\,334\,855\,687\,864\,320\,000\,000} - \frac{3\,417\,320\,410\,902\,077\,653\,x^{19}}{349\,692\,446\,474\,250\,485\,760\,000\,000} - \frac{500\,817\,403\,129\,821\,473\,x^{20}}{222\,026\,950\,142\,381\,260\,800\,000\,000} - \frac{15\,335\,389\,813\,412\,775\,362\,791\,x^{21}}{30\,842\,873\,779\,028\,892\,844\,032\,000\,000\,000} - \frac{62\,699\,227\,636\,446\,189\,380\,059\,x^{22}}{597\,118\,036\,361\,999\,365\,460\,459\,520\,000\,000} - \frac{4\,543\,102\,502\,230\,399\,150\,537\,x^{23}}{213\,256\,441\,557\,856\,916\,235\,878\,400\,000\,000} - \frac{182\,587\,187\,718\,254\,540\,072\,359\,x^{24}}{43\,869\,896\,549\,044\,851\,339\,952\,128\,000\,000\,000} - \frac{3\,904\,078\,316\,405\,013\,432\,934\,841\,x^{25}}{4\,975\,983\,636\,349\,994\,712\,170\,496\,000\,000\,000\,000} - \frac{25\,609\,439\,445\,175\,101\,986\,598\,157\,x^{26}}{179\,135\,410\,908\,599\,809\,638\,137\,856\,000\,000\,000\,000} - \frac{4\,356\,022\,931\,635\,351\,618\,763\,647\,x^{27}}{172\,737\,717\,661\,864\,102\,151\,061\,504\,000\,000\,000\,000} - \frac{8\,175\,273\,442\,776\,233\,537\,431\,984\,091\,x^{28}}{1\,895\,969\,189\,056\,620\,385\,210\,051\,067\,904\,000\,000\,000\,000} - \frac{226\,104\,426\,237\,460\,056\,487\,870\,213\,x^{29}}{315\,994\,864\,842\,770\,064\,201\,675\,177\,984\,000\,000\,000\,000} - \frac{8\,946\,884\,706\,021\,209\,379\,035\,991\,601\,x^{30}}{77\,562\,375\,915\,952\,652\,122\,229\,361\,868\,800\,000\,000\,000\,000} - \frac{192\,836\,206\,394\,489\,566\,938\,536\,011\,x^{31}}{10\,664\,826\,688\,443\,489\,666\,806\,537\,256\,960\,000\,000\,000\,000} - \frac{5\,020\,113\,801\,081\,632\,634\,102\,991\,411\,x^{32}}{1\,820\,130\,421\,494\,355\,569\,801\,649\,025\,187\,840\,000\,000\,000\,000} - \frac{152\,257\,060\,374\,299\,562\,476\,061\,868\,241\,x^{33}}{371\,647\,880\,438\,878\,727\,908\,874\,210\,330\,542\,080\,000\,000\,000\,000} - \frac{654\,121\,679\,169\,499\,461\,313\,234\,447\,x^{34}}{11\,032\,219\,085\,384\,155\,188\,389\,586\,948\,587\,520\,000\,000\,000\,000} - \frac{121\,870\,630\,710\,376\,604\,408\,981\,410\,174\,609\,x^{35}}{14\,568\,596\,913\,204\,046\,134\,027\,869\,044\,957\,249\,536\,000\,000\,000\,000\,000} - \frac{724\,421\,904\,025\,071\,453\,646\,974\,034\,026\,683\,x^{36}}{629\,363\,386\,650\,414\,792\,990\,003\,942\,742\,153\,179\,955\,200\,000\,000\,000\,000} - \frac{32\,415\,927\,097\,725\,954\,050\,829\,815\,301\,659\,x^{37}}{209\,787\,795\,550\,138\,264\,330\,001\,314\,247\,384\,393\,318\,400\,000\,000\,000\,000} - \frac{125\,828\,531\,086\,844\,128\,226\,335\,327\,073\,x^{38}}{6\,215\,934\,682\,967\,059\,683\,851\,890\,792\,515\,093\,135\,360\,000\,000\,000\,000} - \frac{1\,955\,038\,084\,336\,375\,700\,912\,177\,321\,791\,x^{39}}{755\,236\,063\,980\,497\,751\,588\,004\,731\,290\,583\,815\,946\,240\,000\,000\,000\,000} - \frac{\left(195\,264\,123\,788\,589\,015\,093\,036\,780\,574\,229\,x^{40}\right)/\left(604\,188\,851\,184\,398\,201\,270\,403\,785\,032\,467\,052\,756\,992\,000\,000\,000\,000\,000\right) - \frac{2\,380\,251\,621\,154\,507\,632\,525\,466\,292\,783\,x^{41}}{60\,418\,885\,118\,439\,820\,127\,040\,378\,503\,246\,705\,275\,699\,200\,000\,000\,000\,000} - \frac{\left(499\,763\,701\,860\,048\,526\,410\,311\,037\,779\,303\,x^{42}\right)/\left(106\,578\,913\,348\,927\,842\,704\,099\,227\,679\,727\,188\,106\,333\,388\,800\,000\,000\,000\,000\right) - \left(276\,582\,759\,193\,593\,821\,849\,926\,593\,983\,x^{43}\right)/\left(507\,518\,634\,994\,894\,489\,067\,139\,179\,427\,272\,324\,315\,873\,280\,000\,000\,000\,000\right) - \left(5\,523\,584\,363\,578\,214\,753\,721\,695\,121\,043\,x^{44}\right)/$$

$$89\,323\,279\,759\,101\,430\,075\,816\,495\,579\,199\,929\,079\,593\,697\,280\,000\,000\,000\,000 - \left(5\,678\,767\,436\,702\,113\,736\,893\,011\,494\,405\,371\,x^{45}\right) /$$
$$829\,031\,690\,264\,160\,147\,891\,171\,849\,594\,449\,341\,769\,979\,002\,880\,000\,000\,000\,000\,000 - \left(98\,232\,593\,461\,144\,257\,000\,053\,568\,519\,703\,x^{46}\right) /$$
$$132\,645\,070\,442\,265\,623\,662\,587\,495\,935\,111\,894\,683\,196\,640\,460\,800\,000\,000\,000\,000 - \left(1\,036\,271\,840\,620\,076\,511\,517\,774\,653\,289\,x^{47}\right) /$$
$$13\,264\,507\,044\,226\,562\,366\,258\,749\,593\,511\,189\,468\,319\,664\,046\,080\,000\,000\,000\,000 - \left(438\,704\,872\,317\,370\,827\,448\,425\,003\,069\,271\,x^{48}\right) /$$
$$54\,573\,971\,839\,103\,570\,878\,321\,712\,613\,303\,179\,526\,800\,903\,503\,872\,000\,000\,000\,000\,000 - \left(916\,931\,542\,353\,162\,104\,672\,307\,470\,971\,x^{49}\right) /$$
$$1\,136\,957\,746\,647\,991\,059\,965\,035\,679\,443\,816\,240\,141\,685\,489\,664\,000\,000\,000\,000\,000 - \left(256\,125\,784\,743\,372\,048\,563\,096\,893\,937\,x^{50}\right) /$$
$$3\,248\,450\,704\,708\,545\,885\,614\,387\,655\,553\,760\,686\,119\,101\,399\,040\,000\,000\,000\,000\,000 - \left(349\,172\,362\,032\,207\,523\,755\,321\,064\,457\,x^{51}\right) /$$
$$46\,511\,907\,817\,417\,816\,089\,478\,732\,340\,883\,391\,642\,159\,860\,940\,800\,000\,000\,000\,000\,000 - \left(1\,898\,160\,047\,118\,482\,821\,933\,365\,139\,811\,x^{52}\right) /$$
$$2\,728\,698\,591\,955\,178\,543\,916\,085\,630\,665\,158\,976\,340\,045\,175\,193\,600\,000\,000\,000\,000\,000 - \left(3\,054\,109\,987\,346\,644\,108\,351\,072\,453\,x^{53}\right) /$$
$$48\,726\,760\,570\,628\,188\,284\,215\,814\,833\,306\,410\,291\,786\,520\,985\,600\,000\,000\,000\,000\,000 - \left(37\,119\,794\,770\,101\,712\,227\,937\,667\,629\,x^{54}\right) /$$
$$6\,766\,058\,753\,521\,514\,144\,608\,253\,145\,424\,832\,971\,945\,214\,056\,857\,600\,000\,000\,000\,000\,000 - \left(1\,236\,244\,591\,274\,859\,992\,631\,178\,031\,053\,x^{55}\right) /$$
$$2\,653\,172\,113\,070\,704\,852\,075\,551\,117\,673\,534\,040\,387\,776\,067\,665\,920\,000\,000\,000\,000\,000\,000 - \left(651\,164\,395\,582\,185\,202\,142\,914\,533\,571\,x^{56}\right) /$$
$$16\,980\,301\,523\,652\,511\,053\,283\,527\,153\,110\,617\,858\,481\,766\,833\,061\,888\,000\,000\,000\,000\,000\,000 - \left(14\,583\,044\,165\,939\,892\,912\,814\,395\,163\,x^{57}\right) /$$
$$4\,775\,709\,803\,527\,268\,733\,735\,992\,011\,812\,361\,272\,697\,996\,921\,798\,656\,000\,000\,000\,000\,000\,000 - \left(10\,171\,750\,897\,721\,203\,243\,416\,251\,x^{58}\right) /$$
$$43\,317\,095\,723\,603\,344\,523\,682\,467\,227\,323\,004\,741\,024\,915\,390\,464\,000\,000\,000\,000\,000\,000 - \left(2\,638\,198\,025\,861\,236\,550\,540\,413\,x^{59}\right) /$$
$$151\,609\,835\,032\,611\,705\,832\,888\,635\,295\,630\,516\,593\,587\,203\,866\,624\,000\,000\,000\,000\,000\,000 - \left(676\,466\,006\,722\,995\,785\,415\,016\,477\,x^{60}\right) /$$
$$545\,795\,406\,117\,402\,140\,998\,399\,087\,064\,269\,859\,736\,913\,933\,919\,846\,400\,000\,000\,000\,000\,000\,000 - \left(128\,238\,193\,726\,920\,880\,332\,277\,x^{61}\right) /$$
$$1\,516\,098\,350\,326\,117\,058\,328\,886\,352\,956\,305\,165\,935\,872\,038\,666\,240\,000\,000\,000\,000\,000\,000 - \left(33\,414\,749\,902\,373\,535\,304\,177\,x^{62}\right) /$$
$$6\,064\,393\,401\,304\,468\,233\,315\,545\,411\,825\,220\,663\,743\,488\,154\,664\,960\,000\,000\,000\,000\,000\,000 - \left(159\,528\,956\,762\,588\,769\,877\,x^{63}\right) /$$
$$467\,824\,633\,814\,916\,120\,855\,770\,646\,055\,088\,451\,203\,069\,086\,217\,011\,200\,000\,000\,000\,000\,000 - \left(22\,102\,088\,800\,050\,784\,183\,x^{64}\right) /$$
$$1\,108\,917\,650\,524\,245\,619\,806\,271\,161\,019\,468\,921\,370\,237\,833\,995\,878\,400\,000\,000\,000\,000\,000 - \left(15\,134\,883\,208\,270\,559\,x^{65}\right) /$$
$$13\,861\,470\,631\,553\,070\,247\,578\,389\,512\,743\,361\,517\,127\,972\,924\,948\,480\,000\,000\,000\,000\,000 - \left(3\,350\,943\,683\,257\,699\,327\,x^{66}\right) /$$
$$60\,380\,566\,071\,045\,173\,998\,451\,464\,717\,510\,082\,768\,609\,450\,061\,075\,578\,880\,000\,000\,000\,000\,000 - \left(157\,417\,411\,200\,169\,x^{67}\right) /$$
$$60\,990\,470\,778\,833\,509\,089\,344\,913\,856\,070\,790\,675\,363\,080\,869\,773\,312\,000\,000\,000\,000\,000 - \left(1\,364\,276\,456\,267\,x^{68}\right) /$$
$$12\,673\,344\,577\,419\,949\,940\,643\,098\,983\,079\,644\,815\,659\,860\,959\,952\,896\,000\,000\,000\,000\,000 - \left(1\,012\,021\,849\,x^{69}\right) /$$

$$259\,227\,502\,719\,953\,521\,513\,154\,297\,381\,174\,553\,047\,588\,065\,089\,945\,600\,000\,000\,000\,000 - \left(6\,119\,371\,x^{70}\right) / 52\,369\,192\,468\,677\,479\,093\,566\,524\,723\,469\,606\,676\,280\,417\,189\,888\,000\,000\,000\,000\,000 - \left(1241\,x^{71}\right) / 476\,083\,567\,897\,067\,991\,759\,695\,679\,304\,269\,151\,602\,549\,247\,180\,800\,000\,000\,000\,000 - \left(1241\,x^{72}\right) / 37\,394\,200\,242\,096\,976\,807\,307\,006\,083\,535\,322\,453\,145\,686\,324\,019\,200\,000\,000\,000\,000$$

```
Product[k!, {k, 1, n}]
```

```
BarnesG[2 + n]
```

```
f1[x_] := ExpIntegralEi[x] - Log[x] - EulerGamma
f2[x_] := f1[x] - (1 / 4) f1[x]^2 + (5 / 72) f1[x]^3 - (132 / 6912) f1[x]^4 +
  (20 688 / 4 147 200) f1[x]^5 - (21 444 480 / 17 915 904 000) f1[x]^6
```

```
N[f2[.2]]
```

```
0.2
```

```
N@g1[2, 3]
```

```
5.25304
```

```
{N@g1[7, g1[2, 3]], N@g1[g1[7, 3], 2], N@g1[g1[7, 2], 3], N@g1[7, g1m[3, 2]]}
```

```
{208.207, 230.86, 239.868, 150.828}
```

```
{N@a1[7, 2 × 3], N@a1[a1[7, 3], 2], N@a1[a1[7, 2], 3]}
```

```
{117 649., 117 649., 117 649.}
```

```
g1m[N@FullSimplify[g1m[5, 5]], 5] x
```

```
26.5505
```

```
N@FullSimplify[g1m[3, 2]]
```

```
6.70951
```

```
N@g1[5, 3]
```

```
27.5701
```

```
N[g1[107, 4]]
```

```
6931.13
```

```
N[(x / 6 Log[x]^3 - x / 2 Log[x]^2 + x Log[x] - x + 1) +
   4 (x / 2 Log[x]^2 - x Log[x] + x - 1) + 6 (x Log[x] - x + 1) + 4 (x - 1) + 1 /. x → 107]
```

```
6931.13
```

```
g1[x_, 1] := 1
g1[ x_, 2] := x (1 + Log[x])
g1[ x_, 3] := 1/2 x (2 + Log[x] (4 + Log[x]))
g1[x_, 4] := 1/6 x (6 + Log[x] (3 + Log[x]) (6 + Log[x]))
g1[x_, 5] := 1/24 x (24 + Log[x] (4 + Log[x]) (24 + Log[x] (12 + Log[x])))
```

```
FullSimplify[(x Log[x] - x + 1) + 2 (x - 1) + 1]
```

$$x\,(1 + \text{Log}[x])$$

```
FullSimplify[(x / 2 Log[x]^2 - x Log[x] + x - 1) + 3 (x Log[x] - x + 1) + 3 (x - 1) + 1]
```

$$\frac{1}{2}\,x\,(2 + \text{Log}[x]\,(4 + \text{Log}[x]))$$

```
FullSimplify[(x / 6 Log[x]^3 - x / 2 Log[x]^2 + x Log[x] - x + 1) +
  4 (x / 2 Log[x]^2 - x Log[x] + x - 1) + 6 (x Log[x] - x + 1) + 4 (x - 1) + 1]
```

$$\frac{1}{6}\,x\,(6 + \text{Log}[x]\,(3 + \text{Log}[x])\,(6 + \text{Log}[x]))$$

```
FullSimplify[(x / 24 Log[x]^4 - x / 6 Log[x]^3 + x / 2 Log[x]^2 - x Log[x] + x - 1) +
  5 (x / 6 Log[x]^3 - x / 2 Log[x]^2 + x Log[x] - x + 1) +
  10 (x / 2 Log[x]^2 - x Log[x] + x - 1) + 10 (x Log[x] - x + 1) + 5 (x - 1) + 1]
```

$$\frac{1}{24}\,x\,(24 + \text{Log}[x]\,(4 + \text{Log}[x])\,(24 + \text{Log}[x]\,(12 + \text{Log}[x])))$$

```
N[1/24 x (24 + Log[x] (4 + Log[x]) (24 + Log[x] (12 + Log[x]))) /. x → 107]
```

18 520.1

```
Expand[D[ g1[x, 5], x]]
```

$$5 + 10\,\text{Log}[x] + 5\,\text{Log}[x]^2 + \frac{5\,\text{Log}[x]^3}{6} + \frac{\text{Log}[x]^4}{24}$$

```
N[Integrate[ 5 + 10 Log[x] + 5 Log[x]^2 + 5 Log[x]^3 / 6 + Log[x]^4 / 24, {x, 0, y}] /. y → 120]
```

22 074.2

```
N@g1[120, 6]
```

54 338.7

```
g1a[x_, z_] :=
 Sum[ Binomial[z, k] / ((k - 1) !) Integrate[ Log[y]^(k - 1), {y, 0, x}], {k, 1, z}]
g1b[x_, z_] := Integrate[
  Sum[ Binomial[z, k] / ((k - 1) !) Log[y]^(k - 1), {k, 1, z}], {y, 0, x}]
g1d[x_, z_] := Sum[ Binomial[z, k] / ((k - 1) !) Log[x]^(k - 1), {k, 1, z}]
```

```
FullSimplify[Expand[g1a[x, 6]]]
```

$$\frac{1}{120}\,x\,(120 + \text{Log}[x]\,(20 + \text{Log}[x]\,(10 + \text{Log}[x]))\,(30 + \text{Log}[x]\,(15 + \text{Log}[x])))$$

```
Table[ Binomial[5, k] / ((k - 1) !), {k, 1, 5}]
```

$$\left\{5,\ 10,\ 5,\ \frac{5}{6},\ \frac{1}{24}\right\}$$

```
Integrate[ Log[y]^(k - 1), {y, 0, x}]
```

```
ConditionalExpression[-(-1)^k Gamma[k] + (Gamma[k] - Gamma[k, -Log[x]]) (-Log[x])^-k Log[x]^k,
 Im[Log[x]] ≠ 0 || Log[x] < 0]
```

```
N@g1b[120, 6]
```

```
54 338.7
```

```
Sum[ Binomial[z, k] / ((k - 1) !) Log[y]^(k - 1), {k, 1, z}]
```

```
z Hypergeometric1F1[1 - z, 2, -Log[y]]
```

```
g1d[x, k]
```

$$\frac{x\ \text{Gamma}[k,\ \text{Log}[x]]}{\text{Gamma}[k]}$$

```
Expand[D[ g1[x, 3], x]]
```

$$3 + 3\ \text{Log}[x] + \frac{\text{Log}[x]^2}{2}$$

```
Expand[Integrate[ g1d[y, 1] g1d[s, 1], {y, 1, x}, {s, 1, x / y}]]
```

```
ConditionalExpression[1 - x + x Log[x], Re[x] ≥ 0 || x ∉ Reals]
```

```
Expand[Integrate[ g1d[y, 1] g1d[s, 1] g1d[t, 1] g1d[u, 1],
  {y, 1, x}, {s, 1, x / y}, {t, 1, x / (y s)}, {u, 1, x / (y s t)}]]
```

$$\text{ConditionalExpression}\left[1 - x + x\ \text{Log}[x] - \frac{1}{2} x\ \text{Log}[x]^2 + \frac{1}{6} x\ \text{Log}[x]^3,\ \text{Re}[x] \geq 0 \ ||\ x \notin \text{Reals}\right]$$

```
Expand[Integrate[ g1d[y, 2] g1d[s, 2], {y, 1, x}, {s, 1, x / y}]]
```

$$\text{ConditionalExpression}\left[1 - x + x\ \text{Log}[x] + \frac{3}{2} x\ \text{Log}[x]^2 + \frac{1}{6} x\ \text{Log}[x]^3,\ \text{Re}[x] \geq 0 \ ||\ x \notin \text{Reals}\right]$$

```
Integrate[ g1d[y, 2], {y, 1, x}]
```

```
Expand[g1d[y, 2]]
```

```
2 + Log[y]
```

```
Expand[Integrate[ 4 + 2 Log[s] + 2 Log[y] + Log[s] Log[y], {y, 1, x}, {s, 1, x / y}]]
```

$$\text{ConditionalExpression}\left[1 - x + x\ \text{Log}[x] + \frac{3}{2} x\ \text{Log}[x]^2 + \frac{1}{6} x\ \text{Log}[x]^3,\ \text{Re}[x] \geq 0 \ ||\ x \notin \text{Reals}\right]$$

```
Expand[Integrate[g1d[y, 2], {y, 1, x}]]
```

```
ConditionalExpression[-1 + x + x Log[x], Re[x] ≥ 0 || x ∉ Reals]
```

```
Expand[Integrate[ 2 + Log[y], {y, 1, x}]]
```

$$\text{ConditionalExpression}[-1+x+x\,\text{Log}[x],\ \text{Re}[x]\geq 0\ ||\ x\notin \text{Reals}]$$

```
Expand[Integrate[ g1d[y, 2] g1d[s, 2] g1d[t, 2], {y, 1, x}, {s, 1, x / y}, {t, 1, x / (y s)}]]
```

$$\text{ConditionalExpression}\left[-1+x-x\,\text{Log}[x]+\frac{1}{2}x\,\text{Log}[x]^2+\frac{7}{6}x\,\text{Log}[x]^3+\frac{5}{24}x\,\text{Log}[x]^4+\frac{1}{120}x\,\text{Log}[x]^5,\ \text{Re}[x]\geq 0\ ||\ x\notin \text{Reals}\right]$$

```
Expand[Integrate[ g1d[y, 2] g1d[s, 2] g1d[t, 2] g1d[u, 2],
  {y, 1, x}, {s, 1, x / y}, {t, 1, x / (y s)}, {u, 1, x / (y s t)}]]
```

$$\text{ConditionalExpression}\left[1-x+x\,\text{Log}[x]-\frac{1}{2}x\,\text{Log}[x]^2+\frac{1}{6}x\,\text{Log}[x]^3+\frac{5}{8}x\,\text{Log}[x]^4+\frac{17}{120}x\,\text{Log}[x]^5+\frac{7}{720}x\,\text{Log}[x]^6+\frac{x\,\text{Log}[x]^7}{5040},\ \text{Re}[x]\geq 0\ ||\ x\notin \text{Reals}\right]$$

```
Expand[Integrate[ g1d[y, 2] g1d[s, 2] g1d[t, 2] g1d[u, 2] g1d[v, 2], {y, 1, x},
  {s, 1, x / y}, {t, 1, x / (y s)}, {u, 1, x / (y s t)}, {v, 1, x / (y s t u)}]]
```

$$\text{ConditionalExpression}\left[-1+x-x\,\text{Log}[x]+\frac{1}{2}x\,\text{Log}[x]^2-\frac{1}{6}x\,\text{Log}[x]^3+\frac{1}{24}x\,\text{Log}[x]^4+\frac{31}{120}x\,\text{Log}[x]^5+\frac{49}{720}x\,\text{Log}[x]^6+\frac{31\,x\,\text{Log}[x]^7}{5040}+\frac{x\,\text{Log}[x]^8}{4480}+\frac{x\,\text{Log}[x]^9}{362\,880},\ \text{Re}[x]\geq 0\ ||\ x\notin \text{Reals}\right]$$

```
Expand[Integrate[ g1d[y, 2] g1d[s, 2] g1d[t, 2] g1d[u, 2] g1d[v, 2] g1d[w, 2], {y, 1, x},
  {s, 1, x / y}, {t, 1, x / (y s)}, {u, 1, x / (y s t)}, {v, 1, x / (y s t u)}, {w, 1, x / (y s t u v)}]]
```

$$\text{ConditionalExpression}\left[1-x+x\,\text{Log}[x]-\frac{1}{2}x\,\text{Log}[x]^2+\frac{1}{6}x\,\text{Log}[x]^3-\frac{1}{24}x\,\text{Log}[x]^4+\frac{1}{120}x\,\text{Log}[x]^5+\frac{7}{80}x\,\text{Log}[x]^6+\frac{43\,x\,\text{Log}[x]^7}{1680}+\frac{37\,x\,\text{Log}[x]^8}{13\,440}+\frac{7\,x\,\text{Log}[x]^9}{51\,840}+\frac{11\,x\,\text{Log}[x]^{10}}{3\,628\,800}+\frac{x\,\text{Log}[x]^{11}}{39\,916\,800},\ \text{Re}[x]\geq 0\ ||\ x\notin \text{Reals}\right]$$

```
Expand[Integrate[ g1d[y, 2] g1d[s, 2] g1d[t, 2] g1d[u, 2] g1d[v, 2] g1d[w, 2] g1d[r, 2],
  {y, 1, x}, {s, 1, x / y}, {t, 1, x / (y s)}, {u, 1, x / (y s t)},
  {v, 1, x / (y s t u)}, {w, 1, x / (y s t u v)}, {r, 1, x / (y s t u v w)}]]
$Aborted
```

```
Expand[
 Integrate[ g1d[y, 2] g1d[s, 2] g1d[t, 2] g1d[u, 2] g1d[v, 2] g1d[w, 2] g1d[r, 2] g1d[p, 2],
  {y, 1, x}, {s, 1, x / y}, {t, 1, x / (y s)}, {u, 1, x / (y s t)}, {v, 1, x / (y s t u)},
  {w, 1, x / (y s t u v)}, {r, 1, x / (y s t u v w)}, {p, 1, x / (y s t u v w r)}]]
```

$$\text{FullSimplify}\left[-1+x-x\,\text{Log}[x]+\frac{1}{2}x\,\text{Log}[x]^2+\frac{7}{6}x\,\text{Log}[x]^3+\frac{5}{24}x\,\text{Log}[x]^4+\frac{1}{120}x\,\text{Log}[x]^5\right]$$

$$-1+x+\frac{1}{120}x\,\text{Log}[x]\,(-120+\text{Log}[x]\,(60+\text{Log}[x]\,(140+\text{Log}[x]\,(25+\text{Log}[x]))))$$

$$\text{FullSimplify}\left[1-x+x\,\text{Log}[x]-\frac{1}{2}x\,\text{Log}[x]^2+\frac{1}{6}x\,\text{Log}[x]^3+\frac{5}{8}x\,\text{Log}[x]^4+\frac{17}{120}x\,\text{Log}[x]^5+\frac{7}{720}x\,\text{Log}[x]^6+\frac{x\,\text{Log}[x]^7}{5040}\right]$$

$$1-x+\frac{1}{5040}x\,\text{Log}[x]\,(5040+\text{Log}[x]\,(-2520+\text{Log}[x]\,(840+\text{Log}[x]\,(3150+\text{Log}[x]\,(714+\text{Log}[x]\,(49+\text{Log}[x]))))))$$

```
Table[Log[x]^k Binomial[7, k] / k!, {k, 0, 7}]
```

$$\left\{1,\ 7\,\text{Log}[x],\ \frac{21\,\text{Log}[x]^2}{2},\ \frac{35\,\text{Log}[x]^3}{6},\ \frac{35\,\text{Log}[x]^4}{24},\ \frac{7\,\text{Log}[x]^5}{40},\ \frac{7\,\text{Log}[x]^6}{720},\ \frac{\text{Log}[x]^7}{5040}\right\}$$

$$\text{Expand}\left[\left(x+x\,\text{Log}[x]-\frac{1}{2}x\,\text{Log}[x]^2+\frac{1}{6}x\,\text{Log}[x]^3+\frac{5}{8}x\,\text{Log}[x]^4+\frac{17}{120}x\,\text{Log}[x]^5+\frac{7}{720}x\,\text{Log}[x]^6+\frac{x\,\text{Log}[x]^7}{5040}\right)\Big/\,x\right]$$

$$1+\text{Log}[x]-\frac{\text{Log}[x]^2}{2}+\frac{\text{Log}[x]^3}{6}+\frac{5\,\text{Log}[x]^4}{8}+\frac{17\,\text{Log}[x]^5}{120}+\frac{7\,\text{Log}[x]^6}{720}+\frac{\text{Log}[x]^7}{5040}$$

$$1-x+x\,\text{Log}[x]+\frac{3}{2}x\,\text{Log}[x]^2+\frac{1}{6}x\,\text{Log}[x]^3$$

$$1-x+x\,\text{Log}[x]+\frac{3}{2}x\,\text{Log}[x]^2+\frac{1}{6}x\,\text{Log}[x]^3$$

```
aa[k_] := (-1)^(k + 1) + Sum[ (-1)^(k - j) Binomial[k, j] x Log[x]^j / j!, {j, 0, k}]
bb[k_] := (-1)^(k + 1) + Sum[ (-1)^(k - j) x Log[x]^j / j!, {j, 0, k}]
```

$$\left(1-x+x\,\text{Log}[x]+\frac{3}{2}x\,\text{Log}[x]^2+\frac{1}{6}x\,\text{Log}[x]^3\right)-\text{bb}[3]$$

$$2\,x\,\text{Log}[x]^2$$

$$\left(-1+x-x\,\text{Log}[x]+\frac{1}{2}x\,\text{Log}[x]^2+\frac{7}{6}x\,\text{Log}[x]^3+\frac{5}{24}x\,\text{Log}[x]^4+\frac{1}{120}x\,\text{Log}[x]^5\right)-\text{bb}[5]$$

$$-2+2\,x-2\,x\,\text{Log}[x]+x\,\text{Log}[x]^2+x\,\text{Log}[x]^3+\frac{1}{4}x\,\text{Log}[x]^4$$

$$\left(1 - x + x\,\text{Log}[x] - \frac{1}{2} x\,\text{Log}[x]^2 + \frac{1}{6} x\,\text{Log}[x]^3 + \frac{5}{8} x\,\text{Log}[x]^4 + \frac{17}{120} x\,\text{Log}[x]^5 + \frac{7}{720} x\,\text{Log}[x]^6 + \frac{x\,\text{Log}[x]^7}{5040}\right) - \text{bb}[7]$$

$$\frac{2}{3} x\,\text{Log}[x]^4 + \frac{2}{15} x\,\text{Log}[x]^5 + \frac{1}{90} x\,\text{Log}[x]^6$$

$$\text{ff1}[x\_] := -1 + x + x\,\text{Log}[x]$$

$$\text{ff2}[x\_] := 1 - x + x\,\text{Log}[x] + \frac{3}{2} x\,\text{Log}[x]^2 + \frac{1}{6} x\,\text{Log}[x]^3$$

$$\text{ff3}[x\_] := -1 + x - x\,\text{Log}[x] + \frac{1}{2} x\,\text{Log}[x]^2 + \frac{7}{6} x\,\text{Log}[x]^3 + \frac{5}{24} x\,\text{Log}[x]^4 + \frac{1}{120} x\,\text{Log}[x]^5$$

$$\text{ff4}[x\_] := 1 - x + x\,\text{Log}[x] - \frac{1}{2} x\,\text{Log}[x]^2 + \frac{1}{6} x\,\text{Log}[x]^3 + \frac{5}{8} x\,\text{Log}[x]^4 + \frac{17}{120} x\,\text{Log}[x]^5 + \frac{7}{720} x\,\text{Log}[x]^6 + \frac{x\,\text{Log}[x]^7}{5040}$$

$$\text{ff5}[x\_] := -1 + x - x\,\text{Log}[x] + \frac{1}{2} x\,\text{Log}[x]^2 - \frac{1}{6} x\,\text{Log}[x]^3 + \frac{1}{24} x\,\text{Log}[x]^4 + \frac{31}{120} x\,\text{Log}[x]^5 + \frac{49}{720} x\,\text{Log}[x]^6 + \frac{31\,x\,\text{Log}[x]^7}{5040} + \frac{x\,\text{Log}[x]^8}{4480} + \frac{x\,\text{Log}[x]^9}{362\,880}$$

$$\text{ff6}[x\_] := 1 - x + x\,\text{Log}[x] - \frac{1}{2} x\,\text{Log}[x]^2 + \frac{1}{6} x\,\text{Log}[x]^3 - \frac{1}{24} x\,\text{Log}[x]^4 + \frac{1}{120} x\,\text{Log}[x]^5 + \frac{7}{80} x\,\text{Log}[x]^6 + \frac{43\,x\,\text{Log}[x]^7}{1680} + \frac{37\,x\,\text{Log}[x]^8}{13\,440} + \frac{7\,x\,\text{Log}[x]^9}{51\,840} + \frac{11\,x\,\text{Log}[x]^{10}}{3\,628\,800} + \frac{x\,\text{Log}[x]^{11}}{39\,916\,800}$$

$$\text{ff7}[x\_] := -1 + x - x\,\text{Log}[x] + \frac{1}{2} x\,\text{Log}[x]^2 - \frac{1}{6} x\,\text{Log}[x]^3 + \frac{1}{24} x\,\text{Log}[x]^4 - \frac{1}{120} x\,\text{Log}[x]^5 + \frac{1}{720} x\,\text{Log}[x]^6 + \frac{127\,x\,\text{Log}[x]^7}{5040} + \frac{107\,x\,\text{Log}[x]^8}{13\,440} + \frac{13\,x\,\text{Log}[x]^9}{13\,440} + \frac{209\,x\,\text{Log}[x]^{10}}{3\,628\,800} + \frac{71\,x\,\text{Log}[x]^{11}}{39\,916\,800} + \frac{13\,x\,\text{Log}[x]^{12}}{479\,001\,600} + \frac{x\,\text{Log}[x]^{13}}{6\,227\,020\,800}$$

$$\text{ff8}[x\_] := 1 - x + x\,\text{Log}[x] - \frac{1}{2} x\,\text{Log}[x]^2 + \frac{1}{6} x\,\text{Log}[x]^3 - \frac{1}{24} x\,\text{Log}[x]^4 + \frac{1}{120} x\,\text{Log}[x]^5 - \frac{1}{720} x\,\text{Log}[x]^6 + \frac{x\,\text{Log}[x]^7}{5040} + \frac{17\,x\,\text{Log}[x]^8}{2688} + \frac{769\,x\,\text{Log}[x]^9}{362\,880} + \frac{341\,x\,\text{Log}[x]^{10}}{1\,209\,600} + \frac{769\,x\,\text{Log}[x]^{11}}{39\,916\,800} + \frac{13\,x\,\text{Log}[x]^{12}}{17\,740\,800} + \frac{97\,x\,\text{Log}[x]^{13}}{6\,227\,020\,800} + \frac{x\,\text{Log}[x]^{14}}{5\,811\,886\,080} + \frac{x\,\text{Log}[x]^{15}}{1\,307\,674\,368\,000}$$

$$\text{ff9}[x\_] := -1 + x - x\,\text{Log}[x] + \frac{1}{2} x\,\text{Log}[x]^2 - \frac{1}{6} x\,\text{Log}[x]^3 + \frac{1}{24} x\,\text{Log}[x]^4 - \frac{1}{120} x\,\text{Log}[x]^5 + \frac{1}{720} x\,\text{Log}[x]^6 - \frac{x\,\text{Log}[x]^7}{5040} + \frac{x\,\text{Log}[x]^8}{40\,320} + \frac{73\,x\,\text{Log}[x]^9}{51\,840} + \frac{1793\,x\,\text{Log}[x]^{10}}{3\,628\,800} + \frac{563\,x\,\text{Log}[x]^{11}}{7\,983\,360} + \frac{2561\,x\,\text{Log}[x]^{12}}{479\,001\,600} + \frac{1471\,x\,\text{Log}[x]^{13}}{6\,227\,020\,800} + \frac{109\,x\,\text{Log}[x]^{14}}{17\,435\,658\,240} + \frac{127\,x\,\text{Log}[x]^{15}}{1\,307\,674\,368\,000} + \frac{17\,x\,\text{Log}[x]^{16}}{20\,922\,789\,888\,000} + \frac{x\,\text{Log}[x]^{17}}{355\,687\,428\,096\,000}$$

$$\text{ff10}[x\_] := 1 - x + x\,\text{Log}[x] - \frac{1}{2} x\,\text{Log}[x]^2 + \frac{1}{6} x\,\text{Log}[x]^3 - \frac{1}{24} x\,\text{Log}[x]^4 + \frac{1}{120} x\,\text{Log}[x]^5 -$$

$$\frac{1}{720}\,x\,\text{Log}[x]^6+\frac{x\,\text{Log}[x]^7}{5040}-\frac{x\,\text{Log}[x]^8}{40\,320}+\frac{x\,\text{Log}[x]^9}{362\,880}+\frac{341\,x\,\text{Log}[x]^{10}}{1\,209\,600}+\frac{4097\,x\,\text{Log}[x]^{11}}{39\,916\,800}+$$
$$\frac{7423\,x\,\text{Log}[x]^{12}}{479\,001\,600}+\frac{7937\,x\,\text{Log}[x]^{13}}{6\,227\,020\,800}+\frac{5503\,x\,\text{Log}[x]^{14}}{87\,178\,291\,200}+\frac{197\,x\,\text{Log}[x]^{15}}{100\,590\,336\,000}+$$
$$\frac{799\,x\,\text{Log}[x]^{16}}{20\,922\,789\,888\,000}+\frac{23\,x\,\text{Log}[x]^{17}}{50\,812\,489\,728\,000}+\frac{19\,x\,\text{Log}[x]^{18}}{6\,402\,373\,705\,728\,000}+\frac{x\,\text{Log}[x]^{19}}{121\,645\,100\,408\,832\,000}$$

```
ff11[x_] := -1 + x - x Log[x] + 1/2 x Log[x]^2 - 1/6 x Log[x]^3 + 1/24 x Log[x]^4 -
  1/120 x Log[x]^5 + 1/720 x Log[x]^6 - x Log[x]^7/5040 + x Log[x]^8/40320 - x Log[x]^9/362880 + x Log[x]^10/3628800 +
  2047 x Log[x]^11/39916800 + 9217 x Log[x]^12/479001600 + 18943 x Log[x]^13/6227020800 + 23297 x Log[x]^14/87178291200 +
  18943 x Log[x]^15/1307674368000 + 85 x Log[x]^16/167382319104 + 4159 x Log[x]^17/355687428096000 + 1121 x Log[x]^18/6402373705728000 +
  199 x Log[x]^19/121645100408832000 + x Log[x]^20/115852476579840000 + x Log[x]^21/51090942171709440000
```

$$1-x+x\,\text{Log}[x]-\frac{1}{2}\,x\,\text{Log}[x]^2+\frac{1}{6}\,x\,\text{Log}[x]^3-\frac{1}{24}\,x\,\text{Log}[x]^4+\frac{1}{120}\,x\,\text{Log}[x]^5-$$
$$\frac{1}{720}\,x\,\text{Log}[x]^6+\frac{x\,\text{Log}[x]^7}{5040}-\frac{x\,\text{Log}[x]^8}{40\,320}+\frac{x\,\text{Log}[x]^9}{362\,880}-\frac{x\,\text{Log}[x]^{10}}{3\,628\,800}+\frac{x\,\text{Log}[x]^{11}}{39\,916\,800}+$$
$$\frac{13\,x\,\text{Log}[x]^{12}}{1\,520\,640}+\frac{6827\,x\,\text{Log}[x]^{13}}{2\,075\,673\,600}+\frac{2243\,x\,\text{Log}[x]^{14}}{4\,151\,347\,200}+\frac{65\,537\,x\,\text{Log}[x]^{15}}{1\,307\,674\,368\,000}+\frac{61\,183\,x\,\text{Log}[x]^{16}}{20\,922\,789\,888\,000}+$$
$$\frac{40\,193\,x\,\text{Log}[x]^{17}}{355\,687\,428\,096\,000}+\frac{18\,943\,x\,\text{Log}[x]^{18}}{6\,402\,373\,705\,728\,000}+\frac{6401\,x\,\text{Log}[x]^{19}}{121\,645\,100\,408\,832\,000}+\frac{31\,x\,\text{Log}[x]^{20}}{49\,651\,061\,391\,360\,000}+$$
$$\frac{241\,x\,\text{Log}[x]^{21}}{51\,090\,942\,171\,709\,440\,000}+\frac{23\,x\,\text{Log}[x]^{22}}{1\,124\,000\,727\,777\,607\,680\,000}+\frac{x\,\text{Log}[x]^{23}}{25\,852\,016\,738\,884\,976\,640\,000}$$

```
ff13[x_] := -1 + x - x Log[x] + 1/2 x Log[x]^2 - 1/6 x Log[x]^3 + 1/24 x Log[x]^4 -
  1/120 x Log[x]^5 + 1/720 x Log[x]^6 - x Log[x]^7/5040 + x Log[x]^8/40320 - x Log[x]^9/362880 + x Log[x]^10/3628800 -
  x Log[x]^11/39916800 + x Log[x]^12/479001600 + 8191 x Log[x]^13/6227020800 + 15019 x Log[x]^14/29059430400 + 12743 x Log[x]^15/145297152000 +
  178177 x Log[x]^16/20922789888000 + 187903 x Log[x]^17/355687428096000 + 141569 x Log[x]^18/6402373705728000 + 78079 x Log[x]^19/121645100408832000 +
  907 x Log[x]^20/69511485947904000 + 9439 x Log[x]^21/51090942171709440000 + 667 x Log[x]^22/374666909259202560000 +
  41 x Log[x]^23/3693145248412139520000 + x Log[x]^24/24817936069329577574400 + x Log[x]^25/15511210043330985984000000
```

$$1-x+x\,\text{Log}[x]-\frac{1}{2}\,x\,\text{Log}[x]^2+\frac{1}{6}\,x\,\text{Log}[x]^3-\frac{1}{24}\,x\,\text{Log}[x]^4+$$
$$\frac{1}{120}\,x\,\text{Log}[x]^5-\frac{1}{720}\,x\,\text{Log}[x]^6+\frac{x\,\text{Log}[x]^7}{5040}-\frac{x\,\text{Log}[x]^8}{40\,320}+\frac{x\,\text{Log}[x]^9}{362\,880}-\frac{x\,\text{Log}[x]^{10}}{3\,628\,800}+$$
$$\frac{x\,\text{Log}[x]^{11}}{39\,916\,800}-\frac{x\,\text{Log}[x]^{12}}{479\,001\,600}+\frac{x\,\text{Log}[x]^{13}}{6\,227\,020\,800}+\frac{5461\,x\,\text{Log}[x]^{14}}{29\,059\,430\,400}+\frac{19\,661\,x\,\text{Log}[x]^{15}}{261\,534\,873\,600}+$$

$$\frac{91\,477\,x\,\text{Log}[x]^{16}}{6\,974\,263\,296\,000}+\frac{471\,041\,x\,\text{Log}[x]^{17}}{355\,687\,428\,096\,000}+\frac{184\,661\,x\,\text{Log}[x]^{18}}{2\,134\,124\,568\,576\,000}+\frac{471\,041\,x\,\text{Log}[x]^{19}}{121\,645\,100\,408\,832\,000}+$$
$$\frac{297\,727\,x\,\text{Log}[x]^{20}}{2\,432\,902\,008\,176\,640\,000}+\frac{7451\,x\,\text{Log}[x]^{21}}{2\,688\,996\,956\,405\,760\,000}+\frac{50\,623\,x\,\text{Log}[x]^{22}}{1\,124\,000\,727\,777\,607\,680\,000}+$$
$$\frac{13\,441\,x\,\text{Log}[x]^{23}}{25\,852\,016\,738\,884\,976\,640\,000}+\frac{103\,x\,\text{Log}[x]^{24}}{24\,817\,936\,069\,329\,577\,574\,400}+\frac{337\,x\,\text{Log}[x]^{25}}{15\,511\,210\,043\,330\,985\,984\,000\,000}+$$
$$\frac{x\,\text{Log}[x]^{26}}{14\,936\,720\,782\,466\,875\,392\,000\,000}+\frac{x\,\text{Log}[x]^{27}}{10\,888\,869\,450\,418\,352\,160\,768\,000\,000}$$

```
ff15[x_] := -1 + x - x Log[x] + 1/2 x Log[x]^2 - 1/6 x Log[x]^3 + 1/24 x Log[x]^4 -
  1/120 x Log[x]^5 + 1/720 x Log[x]^6 - (x Log[x]^7)/5040 + (x Log[x]^8)/40320 - (x Log[x]^9)/362880 + (x Log[x]^10)/3628800 -
  (x Log[x]^11)/39916800 + (x Log[x]^12)/479001600 - (x Log[x]^13)/6227020800 + (x Log[x]^14)/87178291200 + (4681 x Log[x]^15)/186810624000 +
  (19363 x Log[x]^16)/1902071808000 + (647167 x Log[x]^17)/355687428096000 + (1216513 x Log[x]^18)/6402373705728000 + (1579007 x Log[x]^19)/121645100408832000 +
  (299213 x Log[x]^20)/486580401635328000 + (12547 x Log[x]^21)/601069907902464000 + (116173 x Log[x]^22)/224800145555521536000 +
  (48563 x Log[x]^23)/5170403347776995328000 + (5167 x Log[x]^24)/41363226782215962624000 + (6197 x Log[x]^25)/5170403347776995328000000 +
  (19 x Log[x]^26)/2358429597231611904000000 + x Log[x]^27/27848770972936962048000000 +
  (29 x Log[x]^28)/304888344611713860501504000000 + x Log[x]^29/8841761993739701954543616000000
```

$$\text{ff16}[x\_] := 1-x+x\,\text{Log}[x]-\frac{1}{2}x\,\text{Log}[x]^2+\frac{1}{6}x\,\text{Log}[x]^3-\frac{1}{24}x\,\text{Log}[x]^4+$$
$$\frac{1}{120}x\,\text{Log}[x]^5-\frac{1}{720}x\,\text{Log}[x]^6+\frac{x\,\text{Log}[x]^7}{5040}-\frac{x\,\text{Log}[x]^8}{40\,320}+\frac{x\,\text{Log}[x]^9}{362\,880}-\frac{x\,\text{Log}[x]^{10}}{3\,628\,800}+$$
$$\frac{x\,\text{Log}[x]^{11}}{39\,916\,800}-\frac{x\,\text{Log}[x]^{12}}{479\,001\,600}+\frac{x\,\text{Log}[x]^{13}}{6\,227\,020\,800}-\frac{x\,\text{Log}[x]^{14}}{87\,178\,291\,200}+\frac{x\,\text{Log}[x]^{15}}{1\,307\,674\,368\,000}+$$
$$\frac{4369\,x\,\text{Log}[x]^{16}}{1\,394\,852\,659\,200}+\frac{458\,753\,x\,\text{Log}[x]^{17}}{355\,687\,428\,096\,000}+\frac{1\,507\,327\,x\,\text{Log}[x]^{18}}{6\,402\,373\,705\,728\,000}+\frac{1\,026\,731\,x\,\text{Log}[x]^{19}}{40\,548\,366\,802\,944\,000}+$$
$$\frac{4\,374\,527\,x\,\text{Log}[x]^{20}}{2\,432\,902\,008\,176\,640\,000}+\frac{4\,571\,137\,x\,\text{Log}[x]^{21}}{51\,090\,942\,171\,709\,440\,000}+\frac{241\,937\,x\,\text{Log}[x]^{22}}{74\,933\,381\,851\,840\,512\,000}+$$
$$\frac{4691\,x\,\text{Log}[x]^{23}}{54\,425\,298\,397\,652\,582\,400}+\frac{12\,547\,x\,\text{Log}[x]^{24}}{7\,299\,392\,961\,567\,522\,816\,000}+\frac{15\,913\,x\,\text{Log}[x]^{25}}{620\,448\,401\,733\,239\,439\,360\,000}+$$
$$\frac{12\,743\,x\,\text{Log}[x]^{26}}{44\,810\,162\,347\,400\,626\,176\,000\,000}+\frac{8363\,x\,\text{Log}[x]^{27}}{3\,629\,623\,150\,139\,450\,720\,256\,000\,000}+$$
$$\frac{4031\,x\,\text{Log}[x]^{28}}{304\,888\,344\,611\,713\,860\,501\,504\,000\,000}+\frac{449\,x\,\text{Log}[x]^{29}}{8\,841\,761\,993\,739\,701\,954\,543\,616\,000\,000}+$$
$$\frac{31\,x\,\text{Log}[x]^{30}}{265\,252\,859\,812\,191\,058\,636\,308\,480\,000\,000}+\frac{x\,\text{Log}[x]^{31}}{8\,222\,838\,654\,177\,922\,817\,725\,562\,880\,000\,000}$$

```
ff17[x_] := -1 + x - x Log[x] + 1/2 x Log[x]^2 - 1/6 x Log[x]^3 + 1/24 x Log[x]^4 -
```

$$\frac{1}{120}\,x\,\text{Log}[x]^5+\frac{1}{720}\,x\,\text{Log}[x]^6-\frac{x\,\text{Log}[x]^7}{5040}+\frac{x\,\text{Log}[x]^8}{40\,320}-\frac{x\,\text{Log}[x]^9}{362\,880}+\frac{x\,\text{Log}[x]^{10}}{3\,628\,800}-\frac{x\,\text{Log}[x]^{11}}{39\,916\,800}+\frac{x\,\text{Log}[x]^{12}}{479\,001\,600}-\frac{x\,\text{Log}[x]^{13}}{6\,227\,020\,800}+\frac{x\,\text{Log}[x]^{14}}{87\,178\,291\,200}-\frac{x\,\text{Log}[x]^{15}}{1\,307\,674\,368\,000}+\frac{x\,\text{Log}[x]^{16}}{20\,922\,789\,888\,000}+\frac{131\,071\,x\,\text{Log}[x]^{17}}{355\,687\,428\,096\,000}+\frac{983\,041\,x\,\text{Log}[x]^{18}}{6\,402\,373\,705\,728\,000}+\frac{496\,201\,x\,\text{Log}[x]^{19}}{17\,377\,871\,486\,976\,000}+\frac{7\,667\,713\,x\,\text{Log}[x]^{20}}{2\,432\,902\,008\,176\,640\,000}+\frac{11\,829\,247\,x\,\text{Log}[x]^{21}}{51\,090\,942\,171\,709\,440\,000}+\frac{13\,516\,801\,x\,\text{Log}[x]^{22}}{1\,124\,000\,727\,777\,607\,680\,000}+\frac{11\,829\,247\,x\,\text{Log}[x]^{23}}{25\,852\,016\,738\,884\,976\,640\,000}+\frac{1\,617\,101\,x\,\text{Log}[x]^{24}}{124\,089\,680\,346\,647\,887\,872\,000}+\frac{872\,243\,x\,\text{Log}[x]^{25}}{3\,102\,242\,008\,666\,197\,196\,800\,000}+\frac{41\,381\,x\,\text{Log}[x]^{26}}{8\,962\,032\,469\,480\,125\,235\,200\,000}+\frac{627\,199\,x\,\text{Log}[x]^{27}}{10\,888\,869\,450\,418\,352\,160\,768\,000\,000}+\frac{10\,991\,x\,\text{Log}[x]^{28}}{20\,325\,889\,640\,780\,924\,033\,433\,600\,000}+\frac{33\,151\,x\,\text{Log}[x]^{29}}{8\,841\,761\,993\,739\,701\,954\,543\,616\,000\,000}+\frac{1643\,x\,\text{Log}[x]^{30}}{88\,417\,619\,937\,397\,019\,545\,436\,160\,000\,000}+\frac{73\,x\,\text{Log}[x]^{31}}{1\,174\,691\,236\,311\,131\,831\,103\,651\,840\,000\,000}+\frac{x\,\text{Log}[x]^{32}}{7\,973\,661\,725\,263\,440\,308\,097\,515\,520\,000\,000}+\frac{x\,\text{Log}[x]^{33}}{8\,683\,317\,618\,811\,886\,495\,518\,194\,401\,280\,000\,000}$$

```
xx[x_, z_] := 0
xx[x_, 1] := ff1[x]; xx[x_, 2] := ff2[x]; xx[x_, 3] := ff3[x]; xx[x_, 4] := ff4[x];
xx[x_, 5] := ff5[x]; xx[x_, 6] := ff6[x]; xx[x_, 7] := ff7[x]; xx[x_, 8] := ff8[x];
xx[x_, 9] := ff9[x]; xx[x_, 10] := ff10[x]; xx[x_, 11] := ff11[x];
xx[x_, 12] := ff12[x]; xx[x_, 13] := ff13[x]; xx[x_, 14] := ff14[x];
xx[x_, 15] := ff15[x]; xx[x_, 16] := ff16[x]; xx[x_, 17] := ff17[x]
```

```
N@xx[10, 0]
```

```
0.
```

```
N@Sum[ (-1)^(k + 1) / k xx[8, k], {k, 1, 17}]
```

```
7.88882
```

```
2 N[LogIntegral[8] - Log[Log[8]] - EulerGamma]
```

```
7.88881
```

```
Expand[Integrate[ g1d[y, 2] g1d[s, 2], {y, 1, x}, {s, 1, x / y}]]
```

$$\text{ConditionalExpression}\left[1-x+x\,\text{Log}[x]+\frac{3}{2}\,x\,\text{Log}[x]^2+\frac{1}{6}\,x\,\text{Log}[x]^3,\ \text{Re}[x]\geq 0\ ||\ x\notin\text{Reals}\right]$$

```
Expand[Integrate[ g1d[y, 2] g1d[s, 2] g1d[t, 2], {y, 1, x}, {s, 1, x / y}, {t, 1, x / (y s)}]]
```

$$\text{ConditionalExpression}\left[-1+x-x\,\text{Log}[x]+\frac{1}{2}\,x\,\text{Log}[x]^2+\frac{7}{6}\,x\,\text{Log}[x]^3+\frac{5}{24}\,x\,\text{Log}[x]^4+\frac{1}{120}\,x\,\text{Log}[x]^5,\ \text{Re}[x]\geq 0\ ||\ x\notin\text{Reals}\right]$$

```
Expand[Integrate[
  g1d[y, 2] Integrate[ g1d[s, 2] g1d[t, 2], {s, 1, x / y}, {t, 1, x / (y s)}], {y, 1, x}]]
```

Integrate::pwrl : Unable to prove that integration limits {x} are real. Adding assumptions may help. ≫

$$\int_1^x \text{ConditionalExpression}\left[\frac{\left(y+\frac{1}{6}x\left(-6+\text{Log}\left[\frac{x}{y}\right]\left(6+\text{Log}\left[\frac{x}{y}\right]\left(9+\text{Log}\left[\frac{x}{y}\right]\right)\right)\right)\right)(2+\text{Log}[y])}{y},\right.$$
$$\left(\frac{x}{y}\notin \text{Reals} \,||\, \left(\text{Re}\left[\frac{x}{y}\right]\geq 0 \,\&\&\, \left(\text{Re}\left[\frac{x}{y}\right]\leq 1 \,||\, x\,y\neq y^2\right)\right)\right)\,\&\&$$
$$\left(\left(\text{Im}[x]\neq \frac{\text{Im}[y]\,\text{Re}[x]}{\text{Re}[y]} \,\&\&\, \text{Re}[y]\neq 0\right) \,||\, (\text{Re}[x]\geq 0 \,\&\&\, \text{Re}[y]>0) \,||\right.$$
$$(\text{Re}[x]\leq 0 \,\&\&\, \text{Re}[y]<0) \,||\, (\text{Re}[y]==0 \,\&\&\, ((y\notin \text{Reals} \,\&\&\, \text{Re}[x]\neq 0) \,||$$
$$\left.\left.(\text{Re}[x]==0 \,\&\&\, ((\text{Im}[x]\geq 0 \,\&\&\, \text{Im}[y]>0) \,||\, (\text{Im}[x]\leq 0 \,\&\&\, \text{Im}[y]<0)))))\right)\Big] \,dy$$

```
Expand[Integrate[ g1d[s, 2] g1d[t, 2], {s, 1, x / y}, {t, 1, x / (y s)}]]
```

$$\text{ConditionalExpression}\left[1-\frac{x}{y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]}{y}+\frac{3\,x\,\text{Log}\left[\frac{x}{y}\right]^2}{2\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^3}{6\,y},\right.$$
$$\left(\frac{x}{y}\notin \text{Reals} \,||\, \left(\text{Re}\left[\frac{x}{y}\right]\geq 0 \,\&\&\, \left(\text{Re}\left[\frac{x}{y}\right]\leq 1 \,||\, x\,y\neq y^2\right)\right)\right) \,\&\&\, \left(\left(\text{Im}[x]\neq \frac{\text{Im}[y]\,\text{Re}[x]}{\text{Re}[y]} \,\&\&\, \text{Re}[y]\neq 0\right) \,||\right.$$
$$(\text{Re}[x]\geq 0 \,\&\&\, \text{Re}[y]>0) \,||\, (\text{Re}[x]\leq 0 \,\&\&\, \text{Re}[y]<0) \,||\, (\text{Re}[y]==0 \,\&\&\, ((y\notin \text{Reals} \,\&\&\, \text{Re}[x]\neq 0) \,||$$
$$\left.\left.(\text{Re}[x]==0 \,\&\&\, ((\text{Im}[x]\geq 0 \,\&\&\, \text{Im}[y]>0) \,||\, (\text{Im}[x]\leq 0 \,\&\&\, \text{Im}[y]<0)))))\right)\right]$$

$$\text{Expand}\left[\text{Integrate}\left[\text{g1d}[y,\,2]\left(1-\frac{x}{y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]}{y}+\frac{3\,x\,\text{Log}\left[\frac{x}{y}\right]^2}{2\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^3}{6\,y}\right),\,\{y,\,1,\,x\}\right]\right]$$

$$\text{ConditionalExpression}\left[-1+x-x\,\text{Log}[x]+\frac{1}{2}x\,\text{Log}[x]^2+\frac{7}{6}x\,\text{Log}[x]^3+\frac{5}{24}x\,\text{Log}[x]^4+\frac{1}{120}x\,\text{Log}[x]^5,\ \text{Re}[x]\geq 0 \,||\, x\notin \text{Reals}\right]$$

$$1-x+x\,\text{Log}[x]-\frac{1}{2}x\,\text{Log}[x]^2+\frac{1}{6}x\,\text{Log}[x]^3-\frac{1}{24}x\,\text{Log}[x]^4+\frac{1}{120}x\,\text{Log}[x]^5+\frac{7}{80}x\,\text{Log}[x]^6+$$
$$\frac{43\,x\,\text{Log}[x]^7}{1680}+\frac{37\,x\,\text{Log}[x]^8}{13\,440}+\frac{7\,x\,\text{Log}[x]^9}{51\,840}+\frac{11\,x\,\text{Log}[x]^{10}}{3\,628\,800}+\frac{x\,\text{Log}[x]^{11}}{39\,916\,800}\ /.\ x\to x/y$$

$$\text{Integrate}\left[\text{g1d}[y,\,2]\left(1-\frac{x}{y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]}{y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^2}{2\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^3}{6\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^4}{24\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^5}{120\,y}+\frac{7\,x\,\text{Log}\left[\frac{x}{y}\right]^6}{80\,y}+\right.\right.$$
$$\left.\left.\frac{43\,x\,\text{Log}\left[\frac{x}{y}\right]^7}{1680\,y}+\frac{37\,x\,\text{Log}\left[\frac{x}{y}\right]^8}{13\,440\,y}+\frac{7\,x\,\text{Log}\left[\frac{x}{y}\right]^9}{51\,840\,y}+\frac{11\,x\,\text{Log}\left[\frac{x}{y}\right]^{10}}{3\,628\,800\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{11}}{39\,916\,800\,y}\right),\,\{y,\,1,\,x\}\right]$$

```
ConditionalExpression[
 1/6 227 020 800 (-6 227 020 800 + 6 227 020 800 x - 6 227 020 800 x Log[x] + 3 113 510 400 x Log[x]^2 -
    1 037 836 800 x Log[x]^3 + 259 459 200 x Log[x]^4 - 51 891 840 x Log[x]^5 + 8 648 640 x Log[x]^6 +
    156 911 040 x Log[x]^7 + 49 575 240 x Log[x]^8 + 6 023 160 x Log[x]^9 + 358 644 x Log[x]^10 +
    11 076 x Log[x]^11 + 169 x Log[x]^12 + x Log[x]^13), Re[x] ≥ 0 || x ∉ Reals]
```

```
Expand[1/6 227 020 800 (-6 227 020 800 + 6 227 020 800 x - 6 227 020 800 x Log[x] +
    3 113 510 400 x Log[x]^2 - 1 037 836 800 x Log[x]^3 + 259 459 200 x Log[x]^4 -
    51 891 840 x Log[x]^5 + 8 648 640 x Log[x]^6 + 156 911 040 x Log[x]^7 + 49 575 240 x Log[x]^8 +
    6 023 160 x Log[x]^9 + 358 644 x Log[x]^10 + 11 076 x Log[x]^11 + 169 x Log[x]^12 + x Log[x]^13)]
```

$$-1+x-x\log[x]+\frac{1}{2}x\log[x]^2-\frac{1}{6}x\log[x]^3+\frac{1}{24}x\log[x]^4-\frac{1}{120}x\log[x]^5+\frac{1}{720}x\log[x]^6+\frac{127x\log[x]^7}{5040}+\frac{107x\log[x]^8}{13\,440}+\frac{13x\log[x]^9}{13\,440}+\frac{209x\log[x]^{10}}{3\,628\,800}+\frac{71x\log[x]^{11}}{39\,916\,800}+\frac{13x\log[x]^{12}}{479\,001\,600}+\frac{x\log[x]^{13}}{6\,227\,020\,800}$$

```
-1 + x - x Log[x] + 1/2 x Log[x]^2 - 1/6 x Log[x]^3 + 1/24 x Log[x]^4 -
  1/120 x Log[x]^5 + 1/720 x Log[x]^6 + 127 x Log[x]^7/5040 + 107 x Log[x]^8/13 440 +
  13 x Log[x]^9/13 440 + 209 x Log[x]^10/3 628 800 + 71 x Log[x]^11/39 916 800 +
  13 x Log[x]^12/479 001 600 + x Log[x]^13/6 227 020 800 /. x → x / y
```

```
Expand[ Integrate[
  g1d[y, 2] (-1 + x/y - x Log[x/y]/y + x Log[x/y]^2/(2 y) - x Log[x/y]^3/(6 y) + x Log[x/y]^4/(24 y) -
    x Log[x/y]^5/(120 y) + x Log[x/y]^6/(720 y) + 127 x Log[x/y]^7/(5040 y) + 107 x Log[x/y]^8/(13 440 y) +
    13 x Log[x/y]^9/(13 440 y) + 209 x Log[x/y]^10/(3 628 800 y) + 71 x Log[x/y]^11/(39 916 800 y) +
    13 x Log[x/y]^12/(479 001 600 y) + x Log[x/y]^13/(6 227 020 800 y)), {y, 1, x}]]
```

$$\text{ConditionalExpression}\left[1-x+x\log[x]-\frac{1}{2}x\log[x]^2+\frac{1}{6}x\log[x]^3-\frac{1}{24}x\log[x]^4+\frac{1}{120}x\log[x]^5-\frac{1}{720}x\log[x]^6+\frac{x\log[x]^7}{5040}+\frac{17x\log[x]^8}{2688}+\frac{769x\log[x]^9}{362\,880}+\frac{341x\log[x]^{10}}{1\,209\,600}+\frac{769x\log[x]^{11}}{39\,916\,800}+\frac{13x\log[x]^{12}}{17\,740\,800}+\frac{97x\log[x]^{13}}{6\,227\,020\,800}+\frac{x\log[x]^{14}}{5\,811\,886\,080}+\frac{x\log[x]^{15}}{1\,307\,674\,368\,000},\ \text{Re}[x]\geq 0\ ||\ x\notin\text{Reals}\right]$$

$$1 - x + x\,\text{Log}[x] - \frac{1}{2}x\,\text{Log}[x]^2 + \frac{1}{6}x\,\text{Log}[x]^3 - \frac{1}{24}x\,\text{Log}[x]^4 + \frac{1}{120}x\,\text{Log}[x]^5 - \frac{1}{720}x\,\text{Log}[x]^6 + \frac{x\,\text{Log}[x]^7}{5040} + \frac{17\,x\,\text{Log}[x]^8}{2688} + \frac{769\,x\,\text{Log}[x]^9}{362\,880} + \frac{341\,x\,\text{Log}[x]^{10}}{1\,209\,600} + \frac{769\,x\,\text{Log}[x]^{11}}{39\,916\,800} + \frac{13\,x\,\text{Log}[x]^{12}}{17\,740\,800} + \frac{97\,x\,\text{Log}[x]^{13}}{6\,227\,020\,800} + \frac{x\,\text{Log}[x]^{14}}{5\,811\,886\,080} + \frac{x\,\text{Log}[x]^{15}}{1\,307\,674\,368\,000} \text{ /. } x \to x/y$$

$$\text{Expand}\left[\text{Integrate}\left[\text{g1d}[y, 2]\left(1 - \frac{x}{y} + \frac{x\,\text{Log}\left[\frac{x}{y}\right]}{y} - \frac{x\,\text{Log}\left[\frac{x}{y}\right]^2}{2y} + \frac{x\,\text{Log}\left[\frac{x}{y}\right]^3}{6y} - \frac{x\,\text{Log}\left[\frac{x}{y}\right]^4}{24y} + \frac{x\,\text{Log}\left[\frac{x}{y}\right]^5}{120y} - \frac{x\,\text{Log}\left[\frac{x}{y}\right]^6}{720y} + \frac{x\,\text{Log}\left[\frac{x}{y}\right]^7}{5040y} + \frac{17\,x\,\text{Log}\left[\frac{x}{y}\right]^8}{2688y} + \frac{769\,x\,\text{Log}\left[\frac{x}{y}\right]^9}{362\,880y} + \frac{341\,x\,\text{Log}\left[\frac{x}{y}\right]^{10}}{1\,209\,600y} + \frac{769\,x\,\text{Log}\left[\frac{x}{y}\right]^{11}}{39\,916\,800y} + \frac{13\,x\,\text{Log}\left[\frac{x}{y}\right]^{12}}{17\,740\,800y} + \frac{97\,x\,\text{Log}\left[\frac{x}{y}\right]^{13}}{6\,227\,020\,800y} + \frac{x\,\text{Log}\left[\frac{x}{y}\right]^{14}}{5\,811\,886\,080y} + \frac{x\,\text{Log}\left[\frac{x}{y}\right]^{15}}{1\,307\,674\,368\,000y}\right), \{y, 1, x\}\right]\right]$$

$$\text{ConditionalExpression}\left[-1 + x - x\,\text{Log}[x] + \frac{1}{2}x\,\text{Log}[x]^2 - \frac{1}{6}x\,\text{Log}[x]^3 + \frac{1}{24}x\,\text{Log}[x]^4 - \frac{1}{120}x\,\text{Log}[x]^5 + \frac{1}{720}x\,\text{Log}[x]^6 - \frac{x\,\text{Log}[x]^7}{5040} + \frac{x\,\text{Log}[x]^8}{40\,320} + \frac{73\,x\,\text{Log}[x]^9}{51\,840} + \frac{1793\,x\,\text{Log}[x]^{10}}{3\,628\,800} + \frac{563\,x\,\text{Log}[x]^{11}}{7\,983\,360} + \frac{2561\,x\,\text{Log}[x]^{12}}{479\,001\,600} + \frac{1471\,x\,\text{Log}[x]^{13}}{6\,227\,020\,800} + \frac{109\,x\,\text{Log}[x]^{14}}{17\,435\,658\,240} + \frac{127\,x\,\text{Log}[x]^{15}}{1\,307\,674\,368\,000} + \frac{17\,x\,\text{Log}[x]^{16}}{20\,922\,789\,888\,000} + \frac{x\,\text{Log}[x]^{17}}{355\,687\,428\,096\,000}, \text{Re}[x] \geq 0 \,||\, x \notin \text{Reals}\right]$$

$$-1 + x - x\,\text{Log}[x] + \frac{1}{2}x\,\text{Log}[x]^2 - \frac{1}{6}x\,\text{Log}[x]^3 + \frac{1}{24}x\,\text{Log}[x]^4 - \frac{1}{120}x\,\text{Log}[x]^5 + \frac{1}{720}x\,\text{Log}[x]^6 - \frac{x\,\text{Log}[x]^7}{5040} + \frac{x\,\text{Log}[x]^8}{40\,320} + \frac{73\,x\,\text{Log}[x]^9}{51\,840} + \frac{1793\,x\,\text{Log}[x]^{10}}{3\,628\,800} + \frac{563\,x\,\text{Log}[x]^{11}}{7\,983\,360} + \frac{2561\,x\,\text{Log}[x]^{12}}{479\,001\,600} + \frac{1471\,x\,\text{Log}[x]^{13}}{6\,227\,020\,800} + \frac{109\,x\,\text{Log}[x]^{14}}{17\,435\,658\,240} + \frac{127\,x\,\text{Log}[x]^{15}}{1\,307\,674\,368\,000} + \frac{17\,x\,\text{Log}[x]^{16}}{20\,922\,789\,888\,000} + \frac{x\,\text{Log}[x]^{17}}{355\,687\,428\,096\,000} \text{ /. } x \to x/y$$

```
Expand[ Integrate[
  g1d[y, 2] (-1 + x/y - (x Log[x/y])/y + (x Log[x/y]^2)/(2 y) - (x Log[x/y]^3)/(6 y) + (x Log[x/y]^4)/(24 y) - (x Log[x/y]^5)/(120 y) + (x Log[x/y]^6)/(720 y) -
    (x Log[x/y]^7)/(5040 y) + (x Log[x/y]^8)/(40 320 y) + (73 x Log[x/y]^9)/(51 840 y) + (1793 x Log[x/y]^10)/(3 628 800 y) + (563 x Log[x/y]^11)/(7 983 360 y) +
    (2561 x Log[x/y]^12)/(479 001 600 y) + (1471 x Log[x/y]^13)/(6 227 020 800 y) + (109 x Log[x/y]^14)/(17 435 658 240 y) + (127 x Log[x/y]^15)/(1 307 674 368 000 y) +
    (17 x Log[x/y]^16)/(20 922 789 888 000 y) + (x Log[x/y]^17)/(355 687 428 096 000 y)), {y, 1, x}]]
```

```
ConditionalExpression[1 - x + x Log[x] - 1/2 x Log[x]^2 + 1/6 x Log[x]^3 - 1/24 x Log[x]^4 + 1/120 x Log[x]^5 -
  1/720 x Log[x]^6 + (x Log[x]^7)/5040 - (x Log[x]^8)/40 320 + (x Log[x]^9)/362 880 + (341 x Log[x]^10)/1 209 600 + (4097 x Log[x]^11)/39 916 800 +
  (7423 x Log[x]^12)/479 001 600 + (7937 x Log[x]^13)/6 227 020 800 + (5503 x Log[x]^14)/87 178 291 200 + (197 x Log[x]^15)/100 590 336 000 + (799 x Log[x]^16)/20 922 789 888 000 +
  (23 x Log[x]^17)/50 812 489 728 000 + (19 x Log[x]^18)/6 402 373 705 728 000 + (x Log[x]^19)/121 645 100 408 832 000, Re[x] ≥ 0 || x ∉ Reals]
```

```
1 - x + x Log[x] - 1/2 x Log[x]^2 + 1/6 x Log[x]^3 - 1/24 x Log[x]^4 + 1/120 x Log[x]^5 -
  1/720 x Log[x]^6 + (x Log[x]^7)/5040 - (x Log[x]^8)/40 320 + (x Log[x]^9)/362 880 + (341 x Log[x]^10)/1 209 600 + (4097 x Log[x]^11)/39 916 800 +
  (7423 x Log[x]^12)/479 001 600 + (7937 x Log[x]^13)/6 227 020 800 + (5503 x Log[x]^14)/87 178 291 200 + (197 x Log[x]^15)/100 590 336 000 + (799 x Log[x]^16)/20 922 789 888 000 +
  (23 x Log[x]^17)/50 812 489 728 000 + (19 x Log[x]^18)/6 402 373 705 728 000 + (x Log[x]^19)/121 645 100 408 832 000 /. x → x / y
```

```
Expand[ Integrate[
  g1d[y, 2] (1 - x/y + (x Log[x/y])/y - (x Log[x/y]^2)/(2 y) + (x Log[x/y]^3)/(6 y) - (x Log[x/y]^4)/(24 y) + (x Log[x/y]^5)/(120 y) - (x Log[x/y]^6)/(720 y) +
    (x Log[x/y]^7)/(5040 y) - (x Log[x/y]^8)/(40 320 y) + (x Log[x/y]^9)/(362 880 y) + (341 x Log[x/y]^10)/(1 209 600 y) + (4097 x Log[x/y]^11)/(39 916 800 y) + (7423 x Log[x/y]^12)/(479 001 600 y) +
    (7937 x Log[x/y]^13)/(6 227 020 800 y) + (5503 x Log[x/y]^14)/(87 178 291 200 y) + (197 x Log[x/y]^15)/(100 590 336 000 y) + (799 x Log[x/y]^16)/(20 922 789 888 000 y) +
    (23 x Log[x/y]^17)/(50 812 489 728 000 y) + (19 x Log[x/y]^18)/(6 402 373 705 728 000 y) + (x Log[x/y]^19)/(121 645 100 408 832 000 y)), {y, 1, x}]]
```

```
ConditionalExpression[-1 + x - x Log[x] + 1/2 x Log[x]^2 - 1/6 x Log[x]^3 + 1/24 x Log[x]^4 -
  1/120 x Log[x]^5 + 1/720 x Log[x]^6 - (x Log[x]^7)/5040 + (x Log[x]^8)/40320 - (x Log[x]^9)/362880 + (x Log[x]^10)/3628800 +
  (2047 x Log[x]^11)/39916800 + (9217 x Log[x]^12)/479001600 + (18943 x Log[x]^13)/6227020800 + (23297 x Log[x]^14)/87178291200 + (18943 x Log[x]^15)/1307674368000 +
  (85 x Log[x]^16)/167382319104 + (4159 x Log[x]^17)/355687428096000 + (1121 x Log[x]^18)/6402373705728000 + (199 x Log[x]^19)/121645100408832000 +
  (x Log[x]^20)/115852476579840000 + (x Log[x]^21)/51090942171709440000, Re[x] ≥ 0 || x ∉ Reals]
```

```
-1 + x - x Log[x] + 1/2 x Log[x]^2 - 1/6 x Log[x]^3 + 1/24 x Log[x]^4 -
  1/120 x Log[x]^5 + 1/720 x Log[x]^6 - (x Log[x]^7)/5040 + (x Log[x]^8)/40320 - (x Log[x]^9)/362880 + (x Log[x]^10)/3628800 +
  (2047 x Log[x]^11)/39916800 + (9217 x Log[x]^12)/479001600 + (18943 x Log[x]^13)/6227020800 + (23297 x Log[x]^14)/87178291200 +
  (18943 x Log[x]^15)/1307674368000 + (85 x Log[x]^16)/167382319104 + (4159 x Log[x]^17)/355687428096000 + (1121 x Log[x]^18)/6402373705728000 +
  (199 x Log[x]^19)/121645100408832000 + (x Log[x]^20)/115852476579840000 + (x Log[x]^21)/51090942171709440000 /. x → x / y
```

$$-1+\frac{x}{y}-\frac{x \log\left[\frac{x}{y}\right]}{y}+\frac{x \log\left[\frac{x}{y}\right]^2}{2\,y}-\frac{x \log\left[\frac{x}{y}\right]^3}{6\,y}+\frac{x \log\left[\frac{x}{y}\right]^4}{24\,y}-\frac{x \log\left[\frac{x}{y}\right]^5}{120\,y}+$$
$$\frac{x \log\left[\frac{x}{y}\right]^6}{720\,y}-\frac{x \log\left[\frac{x}{y}\right]^7}{5040\,y}+\frac{x \log\left[\frac{x}{y}\right]^8}{40\,320\,y}-\frac{x \log\left[\frac{x}{y}\right]^9}{362\,880\,y}+\frac{x \log\left[\frac{x}{y}\right]^{10}}{3\,628\,800\,y}+\frac{2047\,x \log\left[\frac{x}{y}\right]^{11}}{39\,916\,800\,y}+$$
$$\frac{9217\,x \log\left[\frac{x}{y}\right]^{12}}{479\,001\,600\,y}+\frac{18\,943\,x \log\left[\frac{x}{y}\right]^{13}}{6\,227\,020\,800\,y}+\frac{23\,297\,x \log\left[\frac{x}{y}\right]^{14}}{87\,178\,291\,200\,y}+\frac{18\,943\,x \log\left[\frac{x}{y}\right]^{15}}{1\,307\,674\,368\,000\,y}+$$
$$\frac{85\,x \log\left[\frac{x}{y}\right]^{16}}{167\,382\,319\,104\,y}+\frac{4159\,x \log\left[\frac{x}{y}\right]^{17}}{355\,687\,428\,096\,000\,y}+\frac{1121\,x \log\left[\frac{x}{y}\right]^{18}}{6\,402\,373\,705\,728\,000\,y}+$$
$$\frac{199\,x \log\left[\frac{x}{y}\right]^{19}}{121\,645\,100\,408\,832\,000\,y}+\frac{x \log\left[\frac{x}{y}\right]^{20}}{115\,852\,476\,579\,840\,000\,y}+\frac{x \log\left[\frac{x}{y}\right]^{21}}{51\,090\,942\,171\,709\,440\,000\,y}$$

```
Expand[
 Integrate[(2 + Log[y]) (-1 + x/y - (x Log[x/y])/y + (x Log[x/y]^2)/(2 y) - (x Log[x/y]^3)/(6 y) + (x Log[x/y]^4)/(24 y) - (x Log[x/y]^5)/(120 y) +
    (x Log[x/y]^6)/(720 y) - (x Log[x/y]^7)/(5040 y) + (x Log[x/y]^8)/(40 320 y) - (x Log[x/y]^9)/(362 880 y) + (x Log[x/y]^10)/(3 628 800 y) + (2047 x Log[x/y]^11)/(39 916 800 y) +
    (9217 x Log[x/y]^12)/(479 001 600 y) + (18 943 x Log[x/y]^13)/(6 227 020 800 y) + (23 297 x Log[x/y]^14)/(87 178 291 200 y) + (18 943 x Log[x/y]^15)/(1 307 674 368 000 y) +
    (85 x Log[x/y]^16)/(167 382 319 104 y) + (4159 x Log[x/y]^17)/(355 687 428 096 000 y) + (1121 x Log[x/y]^18)/(6 402 373 705 728 000 y) + (199 x Log[x/y]^19)/(121 645 100 408 832 000 y) +
    (x Log[x/y]^20)/(115 852 476 579 840 000 y) + (x Log[x/y]^21)/(51 090 942 171 709 440 000 y)), {y, 1, x}]]
```

```
ConditionalExpression[1 - x + x Log[x] - (1/2) x Log[x]^2 + (1/6) x Log[x]^3 - (1/24) x Log[x]^4 + (1/120) x Log[x]^5 -
 (1/720) x Log[x]^6 + (x Log[x]^7)/5040 - (x Log[x]^8)/40 320 + (x Log[x]^9)/362 880 - (x Log[x]^10)/3 628 800 + (x Log[x]^11)/39 916 800 + (13 x Log[x]^12)/1 520 640 +
 (6827 x Log[x]^13)/2 075 673 600 + (2243 x Log[x]^14)/4 151 347 200 + (65 537 x Log[x]^15)/1 307 674 368 000 + (61 183 x Log[x]^16)/20 922 789 888 000 + (40 193 x Log[x]^17)/355 687 428 096 000 +
 (18 943 x Log[x]^18)/6 402 373 705 728 000 + (6401 x Log[x]^19)/121 645 100 408 832 000 + (31 x Log[x]^20)/49 651 061 391 360 000 + (241 x Log[x]^21)/51 090 942 171 709 440 000 +
 (23 x Log[x]^22)/1 124 000 727 777 607 680 000 + (x Log[x]^23)/25 852 016 738 884 976 640 000, Re[x] ≥ 0 || x ∉ Reals]
```

```
1 - x + x Log[x] - (1/2) x Log[x]^2 + (1/6) x Log[x]^3 - (1/24) x Log[x]^4 + (1/120) x Log[x]^5 -
 (1/720) x Log[x]^6 + (x Log[x]^7)/5040 - (x Log[x]^8)/40 320 + (x Log[x]^9)/362 880 - (x Log[x]^10)/3 628 800 + (x Log[x]^11)/39 916 800 +
 (13 x Log[x]^12)/1 520 640 + (6827 x Log[x]^13)/2 075 673 600 + (2243 x Log[x]^14)/4 151 347 200 + (65 537 x Log[x]^15)/1 307 674 368 000 + (61 183 x Log[x]^16)/20 922 789 888 000 +
 (40 193 x Log[x]^17)/355 687 428 096 000 + (18 943 x Log[x]^18)/6 402 373 705 728 000 + (6401 x Log[x]^19)/121 645 100 408 832 000 + (31 x Log[x]^20)/49 651 061 391 360 000 +
 (241 x Log[x]^21)/51 090 942 171 709 440 000 + (23 x Log[x]^22)/1 124 000 727 777 607 680 000 + (x Log[x]^23)/25 852 016 738 884 976 640 000 /. x → x / y
```

```
Expand[ Integrate[
  (2 + Log[y]) (1 - x/y + (x Log[x/y])/y - (x Log[x/y]^2)/(2 y) + (x Log[x/y]^3)/(6 y) - (x Log[x/y]^4)/(24 y) + (x Log[x/y]^5)/(120 y) - (x Log[x/y]^6)/(720 y) +
   (x Log[x/y]^7)/(5040 y) - (x Log[x/y]^8)/(40 320 y) + (x Log[x/y]^9)/(362 880 y) - (x Log[x/y]^10)/(3 628 800 y) + (x Log[x/y]^11)/(39 916 800 y) +
   (13 x Log[x/y]^12)/(1 520 640 y) + (6827 x Log[x/y]^13)/(2 075 673 600 y) + (2243 x Log[x/y]^14)/(4 151 347 200 y) + (65 537 x Log[x/y]^15)/(1 307 674 368 000 y) +
   (61 183 x Log[x/y]^16)/(20 922 789 888 000 y) + (40 193 x Log[x/y]^17)/(355 687 428 096 000 y) + (18 943 x Log[x/y]^18)/(6 402 373 705 728 000 y) +
   (6401 x Log[x/y]^19)/(121 645 100 408 832 000 y) + (31 x Log[x/y]^20)/(49 651 061 391 360 000 y) + (241 x Log[x/y]^21)/(51 090 942 171 709 440 000 y) +
   (23 x Log[x/y]^22)/(1 124 000 727 777 607 680 000 y) + (x Log[x/y]^23)/(25 852 016 738 884 976 640 000 y)), {y, 1, x}]]

ConditionalExpression[-1 + x - x Log[x] + 1/2 x Log[x]^2 - 1/6 x Log[x]^3 + 1/24 x Log[x]^4 -
  1/120 x Log[x]^5 + 1/720 x Log[x]^6 - (x Log[x]^7)/5040 + (x Log[x]^8)/40 320 - (x Log[x]^9)/362 880 + (x Log[x]^10)/3 628 800 - (x Log[x]^11)/39 916 800 +
  (x Log[x]^12)/479 001 600 + (8191 x Log[x]^13)/6 227 020 800 + (15 019 x Log[x]^14)/29 059 430 400 + (12 743 x Log[x]^15)/145 297 152 000 + (178 177 x Log[x]^16)/20 922 789 888 000 +
  (187 903 x Log[x]^17)/355 687 428 096 000 + (141 569 x Log[x]^18)/6 402 373 705 728 000 + (78 079 x Log[x]^19)/121 645 100 408 832 000 + (907 x Log[x]^20)/69 511 485 947 904 000 +
  (9439 x Log[x]^21)/51 090 942 171 709 440 000 + (667 x Log[x]^22)/374 666 909 259 202 560 000 + (41 x Log[x]^23)/3 693 145 248 412 139 520 000 +
  (x Log[x]^24)/24 817 936 069 329 577 574 400 + (x Log[x]^25)/15 511 210 043 330 985 984 000 000, Re[x] ≥ 0 || x ∉ Reals]
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$$-1+x-x\,\text{Log}[x]+\frac{1}{2}x\,\text{Log}[x]^2-\frac{1}{6}x\,\text{Log}[x]^3+\frac{1}{24}x\,\text{Log}[x]^4-\frac{1}{120}x\,\text{Log}[x]^5+\frac{1}{720}x\,\text{Log}[x]^6-\frac{x\,\text{Log}[x]^7}{5040}+\frac{x\,\text{Log}[x]^8}{40\,320}-\frac{x\,\text{Log}[x]^9}{362\,880}+\frac{x\,\text{Log}[x]^{10}}{3\,628\,800}-\frac{x\,\text{Log}[x]^{11}}{39\,916\,800}+\frac{x\,\text{Log}[x]^{12}}{479\,001\,600}+\frac{8191\,x\,\text{Log}[x]^{13}}{6\,227\,020\,800}+\frac{15\,019\,x\,\text{Log}[x]^{14}}{29\,059\,430\,400}+\frac{12\,743\,x\,\text{Log}[x]^{15}}{145\,297\,152\,000}+\frac{178\,177\,x\,\text{Log}[x]^{16}}{20\,922\,789\,888\,000}+\frac{187\,903\,x\,\text{Log}[x]^{17}}{355\,687\,428\,096\,000}+\frac{141\,569\,x\,\text{Log}[x]^{18}}{6\,402\,373\,705\,728\,000}+\frac{78\,079\,x\,\text{Log}[x]^{19}}{121\,645\,100\,408\,832\,000}+\frac{907\,x\,\text{Log}[x]^{20}}{69\,511\,485\,947\,904\,000}+\frac{9439\,x\,\text{Log}[x]^{21}}{51\,090\,942\,171\,709\,440\,000}+\frac{667\,x\,\text{Log}[x]^{22}}{374\,666\,909\,259\,202\,560\,000}+\frac{41\,x\,\text{Log}[x]^{23}}{3\,693\,145\,248\,412\,139\,520\,000}+\frac{x\,\text{Log}[x]^{24}}{24\,817\,936\,069\,329\,577\,574\,400}+\frac{x\,\text{Log}[x]^{25}}{15\,511\,210\,043\,330\,985\,984\,000\,000}\;/.\;x\to x/y$$

$$\text{Expand}\left[\text{Integrate}\left[(2+\text{Log}[y])\left(-1+\frac{x}{y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]}{y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^2}{2\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^3}{6\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^4}{24\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^5}{120\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^6}{720\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^7}{5040\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^8}{40\,320\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^9}{362\,880\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{10}}{3\,628\,800\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{11}}{39\,916\,800\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{12}}{479\,001\,600\,y}+\frac{8191\,x\,\text{Log}\left[\frac{x}{y}\right]^{13}}{6\,227\,020\,800\,y}+\frac{15\,019\,x\,\text{Log}\left[\frac{x}{y}\right]^{14}}{29\,059\,430\,400\,y}+\frac{12\,743\,x\,\text{Log}\left[\frac{x}{y}\right]^{15}}{145\,297\,152\,000\,y}+\frac{178\,177\,x\,\text{Log}\left[\frac{x}{y}\right]^{16}}{20\,922\,789\,888\,000\,y}+\frac{187\,903\,x\,\text{Log}\left[\frac{x}{y}\right]^{17}}{355\,687\,428\,096\,000\,y}+\frac{141\,569\,x\,\text{Log}\left[\frac{x}{y}\right]^{18}}{6\,402\,373\,705\,728\,000\,y}+\frac{78\,079\,x\,\text{Log}\left[\frac{x}{y}\right]^{19}}{121\,645\,100\,408\,832\,000\,y}+\frac{907\,x\,\text{Log}\left[\frac{x}{y}\right]^{20}}{69\,511\,485\,947\,904\,000\,y}+\frac{9439\,x\,\text{Log}\left[\frac{x}{y}\right]^{21}}{51\,090\,942\,171\,709\,440\,000\,y}+\frac{667\,x\,\text{Log}\left[\frac{x}{y}\right]^{22}}{374\,666\,909\,259\,202\,560\,000\,y}+\frac{41\,x\,\text{Log}\left[\frac{x}{y}\right]^{23}}{3\,693\,145\,248\,412\,139\,520\,000\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{24}}{24\,817\,936\,069\,329\,577\,574\,400\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{25}}{15\,511\,210\,043\,330\,985\,984\,000\,000\,y}\right),\{y,1,x\}\right]\right]$$

```
ConditionalExpression[1 - x + x Log[x] - 1/2 x Log[x]^2 + 1/6 x Log[x]^3 - 1/24 x Log[x]^4 +
  1/120 x Log[x]^5 - 1/720 x Log[x]^6 + (x Log[x]^7)/5040 - (x Log[x]^8)/40320 + (x Log[x]^9)/362880 - (x Log[x]^10)/3628800 +
  (x Log[x]^11)/39916800 - (x Log[x]^12)/479001600 + (x Log[x]^13)/6227020800 + (5461 x Log[x]^14)/29059430400 + (19661 x Log[x]^15)/261534873600 +
  (91477 x Log[x]^16)/6974263296000 + (471041 x Log[x]^17)/355687428096000 + (184661 x Log[x]^18)/2134124568576000 + (471041 x Log[x]^19)/121645100408832000 +
  (297727 x Log[x]^20)/2432902008176640000 + (7451 x Log[x]^21)/2688996956405760000 + (50623 x Log[x]^22)/1124000727777607680000 +
  (13441 x Log[x]^23)/25852016738884976640000 + (103 x Log[x]^24)/24817936069329577574400 + (337 x Log[x]^25)/15511210043330985984000000 +
  (x Log[x]^26)/14936720782466875392000000 + (x Log[x]^27)/10888869450418352160768000000, Re[x] ≥ 0 || x ∉ Reals]
```

```
1 - x + x Log[x] - 1/2 x Log[x]^2 + 1/6 x Log[x]^3 - 1/24 x Log[x]^4 + 1/120 x Log[x]^5 -
  1/720 x Log[x]^6 + (x Log[x]^7)/5040 - (x Log[x]^8)/40320 + (x Log[x]^9)/362880 - (x Log[x]^10)/3628800 + (x Log[x]^11)/39916800 -
  (x Log[x]^12)/479001600 + (x Log[x]^13)/6227020800 + (5461 x Log[x]^14)/29059430400 + (19661 x Log[x]^15)/261534873600 + (91477 x Log[x]^16)/6974263296000 +
  (471041 x Log[x]^17)/355687428096000 + (184661 x Log[x]^18)/2134124568576000 + (471041 x Log[x]^19)/121645100408832000 +
  (297727 x Log[x]^20)/2432902008176640000 + (7451 x Log[x]^21)/2688996956405760000 + (50623 x Log[x]^22)/1124000727777607680000 +
  (13441 x Log[x]^23)/25852016738884976640000 + (103 x Log[x]^24)/24817936069329577574400 + (337 x Log[x]^25)/15511210043330985984000000 +
  (x Log[x]^26)/14936720782466875392000000 + (x Log[x]^27)/10888869450418352160768000000 /. x → x / y
```

$$\text{Expand}\left[\text{Integrate}\left[(2+\text{Log}[y])\left(1-\frac{x}{y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]}{y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^2}{2y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^3}{6y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^4}{24y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^5}{120y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^6}{720y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^7}{5040y}-\right.\right.\right.$$

$$\frac{x\,\text{Log}\left[\frac{x}{y}\right]^8}{40\,320y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^9}{362\,880y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{10}}{3\,628\,800y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{11}}{39\,916\,800y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{12}}{479\,001\,600y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{13}}{6\,227\,020\,800y}+$$

$$\frac{5461\,x\,\text{Log}\left[\frac{x}{y}\right]^{14}}{29\,059\,430\,400y}+\frac{19\,661\,x\,\text{Log}\left[\frac{x}{y}\right]^{15}}{261\,534\,873\,600y}+\frac{91\,477\,x\,\text{Log}\left[\frac{x}{y}\right]^{16}}{6\,974\,263\,296\,000y}+\frac{471\,041\,x\,\text{Log}\left[\frac{x}{y}\right]^{17}}{355\,687\,428\,096\,000y}+$$

$$\frac{184\,661\,x\,\text{Log}\left[\frac{x}{y}\right]^{18}}{2\,134\,124\,568\,576\,000y}+\frac{471\,041\,x\,\text{Log}\left[\frac{x}{y}\right]^{19}}{121\,645\,100\,408\,832\,000y}+\frac{297\,727\,x\,\text{Log}\left[\frac{x}{y}\right]^{20}}{2\,432\,902\,008\,176\,640\,000y}+$$

$$\frac{7451\,x\,\text{Log}\left[\frac{x}{y}\right]^{21}}{2\,688\,996\,956\,405\,760\,000y}+\frac{50\,623\,x\,\text{Log}\left[\frac{x}{y}\right]^{22}}{1\,124\,000\,727\,777\,607\,680\,000y}+\frac{13\,441\,x\,\text{Log}\left[\frac{x}{y}\right]^{23}}{25\,852\,016\,738\,884\,976\,640\,000y}+$$

$$\frac{103\,x\,\text{Log}\left[\frac{x}{y}\right]^{24}}{24\,817\,936\,069\,329\,577\,574\,400y}+\frac{337\,x\,\text{Log}\left[\frac{x}{y}\right]^{25}}{15\,511\,210\,043\,330\,985\,984\,000\,000y}+$$

$$\left.\left.\left.\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{26}}{14\,936\,720\,782\,466\,875\,392\,000\,000y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{27}}{10\,888\,869\,450\,418\,352\,160\,768\,000\,000y}\right),\{y,1,x\}\right]\right]$$

$$\text{ConditionalExpression}\left[-1+x-x\,\text{Log}[x]+\frac{1}{2}x\,\text{Log}[x]^2-\frac{1}{6}x\,\text{Log}[x]^3+\frac{1}{24}x\,\text{Log}[x]^4-\right.$$

$$\frac{1}{120}x\,\text{Log}[x]^5+\frac{1}{720}x\,\text{Log}[x]^6-\frac{x\,\text{Log}[x]^7}{5040}+\frac{x\,\text{Log}[x]^8}{40\,320}-\frac{x\,\text{Log}[x]^9}{362\,880}+\frac{x\,\text{Log}[x]^{10}}{3\,628\,800}-\frac{x\,\text{Log}[x]^{11}}{39\,916\,800}+$$

$$\frac{x\,\text{Log}[x]^{12}}{479\,001\,600}-\frac{x\,\text{Log}[x]^{13}}{6\,227\,020\,800}+\frac{x\,\text{Log}[x]^{14}}{87\,178\,291\,200}+\frac{4681\,x\,\text{Log}[x]^{15}}{186\,810\,624\,000}+\frac{19\,363\,x\,\text{Log}[x]^{16}}{1\,902\,071\,808\,000}+$$

$$\frac{647\,167\,x\,\text{Log}[x]^{17}}{355\,687\,428\,096\,000}+\frac{1\,216\,513\,x\,\text{Log}[x]^{18}}{6\,402\,373\,705\,728\,000}+\frac{1\,579\,007\,x\,\text{Log}[x]^{19}}{121\,645\,100\,408\,832\,000}+\frac{299\,213\,x\,\text{Log}[x]^{20}}{486\,580\,401\,635\,328\,000}+$$

$$\frac{12\,547\,x\,\text{Log}[x]^{21}}{601\,069\,907\,902\,464\,000}+\frac{116\,173\,x\,\text{Log}[x]^{22}}{224\,800\,145\,555\,521\,536\,000}+\frac{48\,563\,x\,\text{Log}[x]^{23}}{5\,170\,403\,347\,776\,995\,328\,000}+$$

$$\frac{5167\,x\,\text{Log}[x]^{24}}{41\,363\,226\,782\,215\,962\,624\,000}+\frac{6197\,x\,\text{Log}[x]^{25}}{5\,170\,403\,347\,776\,995\,328\,000\,000}+\frac{19\,x\,\text{Log}[x]^{26}}{2\,358\,429\,597\,231\,611\,904\,000\,000}+$$

$$\frac{x\,\text{Log}[x]^{27}}{27\,848\,770\,972\,936\,962\,048\,000\,000}+\frac{29\,x\,\text{Log}[x]^{28}}{304\,888\,344\,611\,713\,860\,501\,504\,000\,000}+$$

$$\left.\frac{x\,\text{Log}[x]^{29}}{8\,841\,761\,993\,739\,701\,954\,543\,616\,000\,000},\text{Re}[x]\geq 0\,||\,x\notin\text{Reals}\right]$$

$$-1+x-x\,\text{Log}[x]+\frac{1}{2}x\,\text{Log}[x]^2-\frac{1}{6}x\,\text{Log}[x]^3+\frac{1}{24}x\,\text{Log}[x]^4-\frac{1}{120}x\,\text{Log}[x]^5+$$
$$\frac{1}{720}x\,\text{Log}[x]^6-\frac{x\,\text{Log}[x]^7}{5040}+\frac{x\,\text{Log}[x]^8}{40\,320}-\frac{x\,\text{Log}[x]^9}{362\,880}+\frac{x\,\text{Log}[x]^{10}}{3\,628\,800}-\frac{x\,\text{Log}[x]^{11}}{39\,916\,800}+$$
$$\frac{x\,\text{Log}[x]^{12}}{479\,001\,600}-\frac{x\,\text{Log}[x]^{13}}{6\,227\,020\,800}+\frac{x\,\text{Log}[x]^{14}}{87\,178\,291\,200}+\frac{4681\,x\,\text{Log}[x]^{15}}{186\,810\,624\,000}+\frac{19\,363\,x\,\text{Log}[x]^{16}}{1\,902\,071\,808\,000}+$$
$$\frac{647\,167\,x\,\text{Log}[x]^{17}}{355\,687\,428\,096\,000}+\frac{1\,216\,513\,x\,\text{Log}[x]^{18}}{6\,402\,373\,705\,728\,000}+\frac{1\,579\,007\,x\,\text{Log}[x]^{19}}{121\,645\,100\,408\,832\,000}+$$
$$\frac{299\,213\,x\,\text{Log}[x]^{20}}{486\,580\,401\,635\,328\,000}+\frac{12\,547\,x\,\text{Log}[x]^{21}}{601\,069\,907\,902\,464\,000}+\frac{116\,173\,x\,\text{Log}[x]^{22}}{224\,800\,145\,555\,521\,536\,000}+$$
$$\frac{48\,563\,x\,\text{Log}[x]^{23}}{5\,170\,403\,347\,776\,995\,328\,000}+\frac{5167\,x\,\text{Log}[x]^{24}}{41\,363\,226\,782\,215\,962\,624\,000}+\frac{6197\,x\,\text{Log}[x]^{25}}{5\,170\,403\,347\,776\,995\,328\,000\,000}+$$
$$\frac{19\,x\,\text{Log}[x]^{26}}{2\,358\,429\,597\,231\,611\,904\,000\,000}+\frac{x\,\text{Log}[x]^{27}}{27\,848\,770\,972\,936\,962\,048\,000\,000}+$$
$$\frac{29\,x\,\text{Log}[x]^{28}}{304\,888\,344\,611\,713\,860\,501\,504\,000\,000}+\frac{x\,\text{Log}[x]^{29}}{8\,841\,761\,993\,739\,701\,954\,543\,616\,000\,000}\;/.\;x\to x/y$$

$$\text{Expand}\Big[\text{Integrate}\Big[(2+\text{Log}[y])\left(-1+\frac{x}{y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]}{y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^2}{2\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^3}{6\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^4}{24\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^5}{120\,y}+\right.$$
$$\frac{x\,\text{Log}\left[\frac{x}{y}\right]^6}{720\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^7}{5040\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^8}{40\,320\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^9}{362\,880\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{10}}{3\,628\,800\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{11}}{39\,916\,800\,y}+$$
$$\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{12}}{479\,001\,600\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{13}}{6\,227\,020\,800\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{14}}{87\,178\,291\,200\,y}+\frac{4681\,x\,\text{Log}\left[\frac{x}{y}\right]^{15}}{186\,810\,624\,000\,y}+\frac{19\,363\,x\,\text{Log}\left[\frac{x}{y}\right]^{16}}{1\,902\,071\,808\,000\,y}+$$
$$\frac{647\,167\,x\,\text{Log}\left[\frac{x}{y}\right]^{17}}{355\,687\,428\,096\,000\,y}+\frac{1\,216\,513\,x\,\text{Log}\left[\frac{x}{y}\right]^{18}}{6\,402\,373\,705\,728\,000\,y}+\frac{1\,579\,007\,x\,\text{Log}\left[\frac{x}{y}\right]^{19}}{121\,645\,100\,408\,832\,000\,y}+$$
$$\frac{299\,213\,x\,\text{Log}\left[\frac{x}{y}\right]^{20}}{486\,580\,401\,635\,328\,000\,y}+\frac{12\,547\,x\,\text{Log}\left[\frac{x}{y}\right]^{21}}{601\,069\,907\,902\,464\,000\,y}+\frac{116\,173\,x\,\text{Log}\left[\frac{x}{y}\right]^{22}}{224\,800\,145\,555\,521\,536\,000\,y}+$$
$$\frac{48\,563\,x\,\text{Log}\left[\frac{x}{y}\right]^{23}}{5\,170\,403\,347\,776\,995\,328\,000\,y}+\frac{5167\,x\,\text{Log}\left[\frac{x}{y}\right]^{24}}{41\,363\,226\,782\,215\,962\,624\,000\,y}+$$
$$\frac{6197\,x\,\text{Log}\left[\frac{x}{y}\right]^{25}}{5\,170\,403\,347\,776\,995\,328\,000\,000\,y}+\frac{19\,x\,\text{Log}\left[\frac{x}{y}\right]^{26}}{2\,358\,429\,597\,231\,611\,904\,000\,000\,y}+$$
$$\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{27}}{27\,848\,770\,972\,936\,962\,048\,000\,000\,y}+\frac{29\,x\,\text{Log}\left[\frac{x}{y}\right]^{28}}{304\,888\,344\,611\,713\,860\,501\,504\,000\,000\,y}+$$
$$\left.\left.\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{29}}{8\,841\,761\,993\,739\,701\,954\,543\,616\,000\,000\,y}\right),\{y,1,x\}\Big]\Big]$$

```
ConditionalExpression[
 1 - x + x Log[x] - 1/2 x Log[x]^2 + 1/6 x Log[x]^3 - 1/24 x Log[x]^4 + 1/120 x Log[x]^5 - 1/720 x Log[x]^6 +
  x Log[x]^7/5040 - x Log[x]^8/40320 + x Log[x]^9/362880 - x Log[x]^10/3628800 + x Log[x]^11/39916800 - x Log[x]^12/479001600 + x Log[x]^13/6227020800 -
  x Log[x]^14/87178291200 + x Log[x]^15/1307674368000 + 4369 x Log[x]^16/1394852659200 + 458753 x Log[x]^17/355687428096000 + 1507327 x Log[x]^18/6402373705728000 +
  1026731 x Log[x]^19/40548366802944000 + 4374527 x Log[x]^20/2432902008176640000 + 4571137 x Log[x]^21/51090942171709440000 +
  241937 x Log[x]^22/74933381851840512000 + 4691 x Log[x]^23/54425298397652582400 + 12547 x Log[x]^24/7299392961567522816000 +
  15913 x Log[x]^25/620448401733239439360000 + 12743 x Log[x]^26/44810162347400626176000000 +
  8363 x Log[x]^27/3629623150139450720256000000 + 4031 x Log[x]^28/304888344611713860501504000000 +
  449 x Log[x]^29/8841761993739701954543616000000 + 31 x Log[x]^30/265252859812191058636308480000000 +
  x Log[x]^31/8222838654177922817725562880000000, Re[x] ≥ 0 || x ∉ Reals]
```

```
1 - x + x Log[x] - 1/2 x Log[x]^2 + 1/6 x Log[x]^3 - 1/24 x Log[x]^4 +
  1/120 x Log[x]^5 - 1/720 x Log[x]^6 + x Log[x]^7/5040 - x Log[x]^8/40320 + x Log[x]^9/362880 - x Log[x]^10/3628800 +
  x Log[x]^11/39916800 - x Log[x]^12/479001600 + x Log[x]^13/6227020800 - x Log[x]^14/87178291200 + x Log[x]^15/1307674368000 +
  4369 x Log[x]^16/1394852659200 + 458753 x Log[x]^17/355687428096000 + 1507327 x Log[x]^18/6402373705728000 + 1026731 x Log[x]^19/40548366802944000 +
  4374527 x Log[x]^20/2432902008176640000 + 4571137 x Log[x]^21/51090942171709440000 + 241937 x Log[x]^22/74933381851840512000 +
  4691 x Log[x]^23/54425298397652582400 + 12547 x Log[x]^24/7299392961567522816000 + 15913 x Log[x]^25/620448401733239439360000 +
  12743 x Log[x]^26/44810162347400626176000000 + 8363 x Log[x]^27/3629623150139450720256000000 +
  4031 x Log[x]^28/304888344611713860501504000000 + 449 x Log[x]^29/8841761993739701954543616000000 +
  31 x Log[x]^30/265252859812191058636308480000000 + x Log[x]^31/8222838654177922817725562880000000 /. x → x / y
```

$$\text{Expand}\left[\text{Integrate}\left[(2+\text{Log}[y])\left(1-\frac{x}{y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]}{y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^2}{2\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^3}{6\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^4}{24\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^5}{120\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^6}{720\,y}+\right.\right.\right.$$

$$\frac{x\,\text{Log}\left[\frac{x}{y}\right]^7}{5040\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^8}{40\,320\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^9}{362\,880\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{10}}{3\,628\,800\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{11}}{39\,916\,800\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{12}}{479\,001\,600\,y}+$$

$$\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{13}}{6\,227\,020\,800\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{14}}{87\,178\,291\,200\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{15}}{1\,307\,674\,368\,000\,y}+\frac{4369\,x\,\text{Log}\left[\frac{x}{y}\right]^{16}}{1\,394\,852\,659\,200\,y}+$$

$$\frac{458\,753\,x\,\text{Log}\left[\frac{x}{y}\right]^{17}}{355\,687\,428\,096\,000\,y}+\frac{1\,507\,327\,x\,\text{Log}\left[\frac{x}{y}\right]^{18}}{6\,402\,373\,705\,728\,000\,y}+\frac{1\,026\,731\,x\,\text{Log}\left[\frac{x}{y}\right]^{19}}{40\,548\,366\,802\,944\,000\,y}+$$

$$\frac{4\,374\,527\,x\,\text{Log}\left[\frac{x}{y}\right]^{20}}{2\,432\,902\,008\,176\,640\,000\,y}+\frac{4\,571\,137\,x\,\text{Log}\left[\frac{x}{y}\right]^{21}}{51\,090\,942\,171\,709\,440\,000\,y}+\frac{241\,937\,x\,\text{Log}\left[\frac{x}{y}\right]^{22}}{74\,933\,381\,851\,840\,512\,000\,y}+$$

$$\frac{4691\,x\,\text{Log}\left[\frac{x}{y}\right]^{23}}{54\,425\,298\,397\,652\,582\,400\,y}+\frac{12\,547\,x\,\text{Log}\left[\frac{x}{y}\right]^{24}}{7\,299\,392\,961\,567\,522\,816\,000\,y}+$$

$$\frac{15\,913\,x\,\text{Log}\left[\frac{x}{y}\right]^{25}}{620\,448\,401\,733\,239\,439\,360\,000\,y}+\frac{12\,743\,x\,\text{Log}\left[\frac{x}{y}\right]^{26}}{44\,810\,162\,347\,400\,626\,176\,000\,000\,y}+$$

$$\frac{8363\,x\,\text{Log}\left[\frac{x}{y}\right]^{27}}{3\,629\,623\,150\,139\,450\,720\,256\,000\,000\,y}+\frac{4031\,x\,\text{Log}\left[\frac{x}{y}\right]^{28}}{304\,888\,344\,611\,713\,860\,501\,504\,000\,000\,y}+$$

$$\frac{449\,x\,\text{Log}\left[\frac{x}{y}\right]^{29}}{8\,841\,761\,993\,739\,701\,954\,543\,616\,000\,000\,y}+\frac{31\,x\,\text{Log}\left[\frac{x}{y}\right]^{30}}{265\,252\,859\,812\,191\,058\,636\,308\,480\,000\,000\,y}+$$

$$\left.\left.\left.\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{31}}{8\,222\,838\,654\,177\,922\,817\,725\,562\,880\,000\,000\,y}\right),\{y,1,x\}\right]\right]$$

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ConditionalExpression[
 -1 + x - x Log[x] + 1/2 x Log[x]^2 - 1/6 x Log[x]^3 + 1/24 x Log[x]^4 - 1/120 x Log[x]^5 + 1/720 x Log[x]^6 -
  x Log[x]^7/5040 + x Log[x]^8/40320 - x Log[x]^9/362880 + x Log[x]^10/3628800 - x Log[x]^11/39916800 + x Log[x]^12/479001600 - x Log[x]^13/6227020800 +
  x Log[x]^14/87178291200 - x Log[x]^15/1307674368000 + x Log[x]^16/20922789888000 + 131071 x Log[x]^17/355687428096000 + 983041 x Log[x]^18/6402373705728000 +
  496201 x Log[x]^19/17377871486976000 + 7667713 x Log[x]^20/2432902008176640000 + 11829247 x Log[x]^21/51090942171709440000 +
  13516801 x Log[x]^22/1124000727777607680000 + 11829247 x Log[x]^23/25852016738884976640000 + 1617101 x Log[x]^24/124089680346647887872000 +
  872243 x Log[x]^25/3102242008666197196800000 + 41381 x Log[x]^26/8962032469480125235200000 +
  627199 x Log[x]^27/10888869450418352160768000000 + 10991 x Log[x]^28/20325889640780924033433600000 +
  33151 x Log[x]^29/8841761993739701954543616000000 + 1643 x Log[x]^30/88417619937397019545436160000000 +
  73 x Log[x]^31/1174691236311131831103651840000000 + x Log[x]^32/7973661725263440308097515520000000 +
  x Log[x]^33/8683317618811886495518194401280000000, Re[x] ≥ 0 || x ∉ Reals]
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-1 + x - x Log[x] + 1/2 x Log[x]^2 - 1/6 x Log[x]^3 + 1/24 x Log[x]^4 - 1/120 x Log[x]^5 + 1/720 x Log[x]^6 -
 x Log[x]^7/5040 + x Log[x]^8/40320 - x Log[x]^9/362880 + x Log[x]^10/3628800 - x Log[x]^11/39916800 + x Log[x]^12/479001600 - x Log[x]^13/6227020800 +
 x Log[x]^14/87178291200 - x Log[x]^15/1307674368000 + x Log[x]^16/20922789888000 + 131071 x Log[x]^17/355687428096000 + 983041 x Log[x]^18/6402373705728000 +
 496201 x Log[x]^19/17377871486976000 + 7667713 x Log[x]^20/2432902008176640000 + 11829247 x Log[x]^21/51090942171709440000 +
 13516801 x Log[x]^22/1124000727777607680000 + 11829247 x Log[x]^23/25852016738884976640000 + 1617101 x Log[x]^24/124089680346647887872000 +
 872243 x Log[x]^25/3102242008666197196800000 + 41381 x Log[x]^26/8962032469480125235200000 +
 627199 x Log[x]^27/10888869450418352160768000000 + 10991 x Log[x]^28/20325889640780924033433600000 +
 33151 x Log[x]^29/8841761993739701954543616000000 + 1643 x Log[x]^30/88417619937397019545436160000000 +
 73 x Log[x]^31/1174691236311131831103651840000000 + x Log[x]^32/7973661725263440308097515520000000 +
 x Log[x]^33/8683317618811886495518194401280000000 /. x → x / y
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$$\text{Expand}\Big[\text{Integrate}\Big[(2+\text{Log}[y])\left(-1+\frac{x}{y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]}{y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^2}{2\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^3}{6\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^4}{24\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^5}{120\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^6}{720\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^7}{5040\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^8}{40\,320\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^9}{362\,880\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{10}}{3\,628\,800\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{11}}{39\,916\,800\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{12}}{479\,001\,600\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{13}}{6\,227\,020\,800\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{14}}{87\,178\,291\,200\,y}-\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{15}}{1\,307\,674\,368\,000\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{16}}{20\,922\,789\,888\,000\,y}+\frac{131\,071\,x\,\text{Log}\left[\frac{x}{y}\right]^{17}}{355\,687\,428\,096\,000\,y}+\frac{983\,041\,x\,\text{Log}\left[\frac{x}{y}\right]^{18}}{6\,402\,373\,705\,728\,000\,y}+\frac{496\,201\,x\,\text{Log}\left[\frac{x}{y}\right]^{19}}{17\,377\,871\,486\,976\,000\,y}+\frac{7\,667\,713\,x\,\text{Log}\left[\frac{x}{y}\right]^{20}}{2\,432\,902\,008\,176\,640\,000\,y}+\frac{11\,829\,247\,x\,\text{Log}\left[\frac{x}{y}\right]^{21}}{51\,090\,942\,171\,709\,440\,000\,y}+\frac{13\,516\,801\,x\,\text{Log}\left[\frac{x}{y}\right]^{22}}{1\,124\,000\,727\,777\,607\,680\,000\,y}+\frac{11\,829\,247\,x\,\text{Log}\left[\frac{x}{y}\right]^{23}}{25\,852\,016\,738\,884\,976\,640\,000\,y}+\frac{1\,617\,101\,x\,\text{Log}\left[\frac{x}{y}\right]^{24}}{124\,089\,680\,346\,647\,887\,872\,000\,y}+\frac{872\,243\,x\,\text{Log}\left[\frac{x}{y}\right]^{25}}{3\,102\,242\,008\,666\,197\,196\,800\,000\,y}+\frac{41\,381\,x\,\text{Log}\left[\frac{x}{y}\right]^{26}}{8\,962\,032\,469\,480\,125\,235\,200\,000\,y}+\frac{627\,199\,x\,\text{Log}\left[\frac{x}{y}\right]^{27}}{10\,888\,869\,450\,418\,352\,160\,768\,000\,000\,y}+\frac{10\,991\,x\,\text{Log}\left[\frac{x}{y}\right]^{28}}{20\,325\,889\,640\,780\,924\,033\,433\,600\,000\,y}+\frac{33\,151\,x\,\text{Log}\left[\frac{x}{y}\right]^{29}}{8\,841\,761\,993\,739\,701\,954\,543\,616\,000\,000\,y}+\frac{1643\,x\,\text{Log}\left[\frac{x}{y}\right]^{30}}{88\,417\,619\,937\,397\,019\,545\,436\,160\,000\,000\,y}+\frac{73\,x\,\text{Log}\left[\frac{x}{y}\right]^{31}}{1\,174\,691\,236\,311\,131\,831\,103\,651\,840\,000\,000\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{32}}{7\,973\,661\,725\,263\,440\,308\,097\,515\,520\,000\,000\,y}+\frac{x\,\text{Log}\left[\frac{x}{y}\right]^{33}}{8\,683\,317\,618\,811\,886\,495\,518\,194\,401\,280\,000\,000\,y}\right),\{y,1,x\}\Big]\Big]$$