```
referenceChebyshev[n_] := Sum[MangoldtLambda[j], {j, 2, n}]
num[c_] := Numerator[c]; den[c_] := Denominator[c]
alpha[n_, c_] := alpha[n, c] = den[c] (Floor[n / den[c]] - Floor[(n - 1) / den[c]]) -
    num[c] (Floor[n / num[c]] - Floor[(n - 1) / num[c]])
F[n_, 0, s_, c_] := 1
F[n_, 1, s_, c_] := If[n < s, 0, (den[c] Floor[n / den[c]] - num[c] Floor[n / num[c]]) -
    (den[c] Floor[(s - 1) / den[c]] - num[c] Floor[(s - 1) / num[c]])]
F[n_, k_, s_, c_] := F[n, k, s, c] = Sum[If[alpha[m, c] == 0, 0, Binomial[k, j] alpha[m, c]^j
     F[Floor[n / (m^j)], k - j, m + 1, c]], {j, 1, k}, {m, s, Floor[n^(1 / k)]}]
E1Alt[n_, k_, c_] := den[c]^-k F[n den[c]^k, k, 1, c]
E2Alt[n_, k_, c_] := den[c]^-k F[n den[c]^k, k, den[c] + 1, c]
E2[n_, k_, c_] :=
 E2[n, k, c] = (1 / den[c]) Sum[If[alpha[j, c] == 0, 0, alpha[j, c] E2[(den[c] n) / j, k - 1, c]],
    {j, den[c] + 1, den[c] n}]; E2[n_, 0, c_] := 1
E1[n_, z_, c_] := Sum[Binomial[z, k] E2[n, k, c], {k, 0, Floor[Log[If[c < 2, c, 2], n]]}]
e1[n_, z_, c_] := E1[n, z, c] - E1[n - 1, z, c]
L2[n_, k_, c_] := L2[n, k, c] = (1 / den[c])
   Sum[If[alpha[j, c] == 0, 0, alpha[j, c] Log[j / den[c]] E2Alt[den[c] n / j, k - 1, c]],
    {j, den[c] + 1, den[c] n}]; L2[n_, 0, c_] := 1
ChebAlt[n_, c_] := 1 + Sum[(-1)^(k - 1) L2[n, k, c],
   {k, 0, Floor[Log[n] / Log[If[c < 2, c, 2]]]}] +
  Sum[c^k Log[c], {k, 1, Floor[Log[n] / Log[c]]}]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
L1[n_, z_, c_] :=
 L1[n, z, c] = Sum[bin[z, k] L2[n, k, c], {k, 1, Floor[Log[If[c < 2, c, 2], n]]}]
l1[n_, z_, c_] := FullSimplify[L1[n, z, c] - L1[n - 1, z, c]]
L2a[n_, 1, c_] :=
 L2a[n, 1, c] = (1 / den[c]) Sum[alpha[j, c] Log[j / den[c]], {j, den[c] + 1, den[c] n}]
L2x[n_, 1, b_] := L2x[n, 1, b] = Sum[Log[j], {j, 2, n}] - b Sum[Log[j b], {j, 1, n / b}]
L2x[n_, k_, b_] := Sum[L2x[n / j, k - 1, b], {j, 2, n}] - b Sum[L2x[n / (j b), k - 1, b], {j, 1, n}]
E2x[n_, k_, x_] :=
 E2x[n, k, x] = Sum[E2x[n / j, k - 1, x], {j, 2, n}] - x Sum[E2x[n / (x j), k - 1, x], {j, 1, n / x}];
E2x[n_, 0, x_] := 1
E1x[n_, z_, c_] := Sum[Binomial[z, k] E2x[n, k, c], {k, 0, Floor[Log[If[c < 2, c, 2], n]]}]
e1x[n_, z_, c_] := E1x[n, z, c] - E1x[n - 1, z, c]
L1x[n_, z_, x_] :=
 L1x[n, z, x] = Sum[bin[z, k] L2x[n, k, x], {k, 1, Floor[Log[If[x < 2, x, 2], n]]}]
l1x[n_, z_, x_] := l1x[n, z, x] = L1x[n, z, x] - L1x[n - 1, z, x]
L1y[n_, 0, c_] := 0
L1y[n_, k_, c_] := den[c]^-1
  Sum[alpha[j, c] (Log[j / den[c]] + L1y[n den[c] / j, k - 1, c]), {j, 1, den[c] n}]
l1y[n_, k_, c_] := den[c]^-1 Sum[
    alpha[j, c] (Log[j / den[c]] + L1y[n den[c] / j, k - 1, c]), {j, 1, den[c] n}] - den[c]^-1
   Sum[alpha[j, c] (Log[j / den[c]] + L1y[(n den[c] - 1) / j, k - 1, c]), {j, 1, den[c] n - 1}]
L1w[n_, 0, b_] := 0
L1w[n_, k_, b_] := Sum[Log[j] + L1w[n / j, k - 1, b], {j, 1, n}] -
  b Sum[Log[j b] + L1w[n / (j b), k - 1, b], {j, 1, n}]
```

```
num[c_] := Numerator[c]; den[c_] := Denominator[c]
alpha[n_, c_] := den[c] (Floor[n / den[c]] - Floor[(n - 1) / den[c]]) -
  num[c] (Floor[n / num[c]] - Floor[(n - 1) / num[c]])
ChebAlt2[n_, c_] := Sum[(-1)^(k) L2[n, k, c], {k, 0, Floor[Log[n] / Log[If[c < 2, c, 2]]]}]
Lm1[ n_, c_ ] :=
 (1 / den[c]) Sum[If[alpha[j, c] == 0, 0, alpha[j, c] ( Log[j / den[c]] - Lm1[den[c] n / j, c])],
   {j, den[c] + 1, den[c] n}]
```

```
1 - N[Lm1[ 100, 3 / 2]]
```

```
10.953519285239476`
```

```
N[ChebAlt2[100, 3 / 2]]
```

```
10.9535
```

```
Sum[ N[Lm1[100 / j, 2]], {j, 1, 100}]
```

```
d - 18.877868113526315`
```

```
Sum[ N[ (-1)^(j + 1) Log[j]], {j, 1, 100}]
```

```
-2.53088
```

```
{Sum[ e1[ j, -1, 2 ] N[L1[Floor[100 / j], 1, 2]], {j, 1, 100}], -N[L1[ 100, -1, 2]]}
```

```
{6.70877, 6.70877}
```

```
{Sum[ e1[ j, 1, 2 ] N[L1[Floor[100 / j], -1, 2]], {j, 1, 100}], -N[L1[ 100, 1, 2]]}
```

```
{2.53088, 2.53088}
```

```
{Sum[ E1[ 100 / j, -1, 2 ] N[alpha[ j, 2] Log[j]], {j, 1, 100}], -N[L1[ 100, -1, 2]]}
```

```
{6.70877, 6.70877}
```

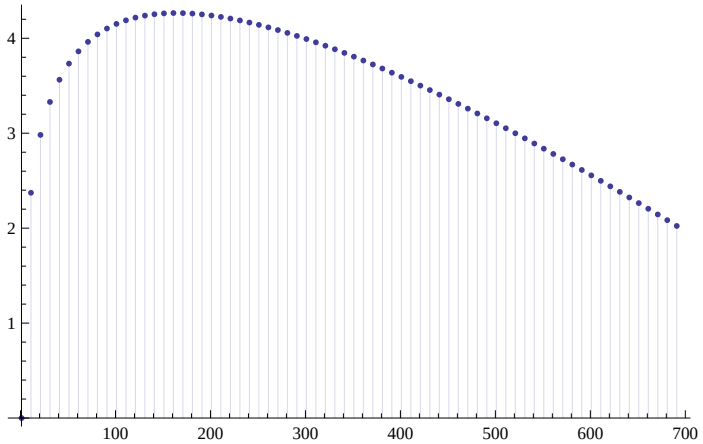
```
{Sum[ e1x[ j, -1, 2 ] N[L1x[Floor[100 / j], 1, 2]], {j, 1, 100}], -N[L1[ 100, -1, 2]]}
```

```
{6.70877, 6.70877}
```

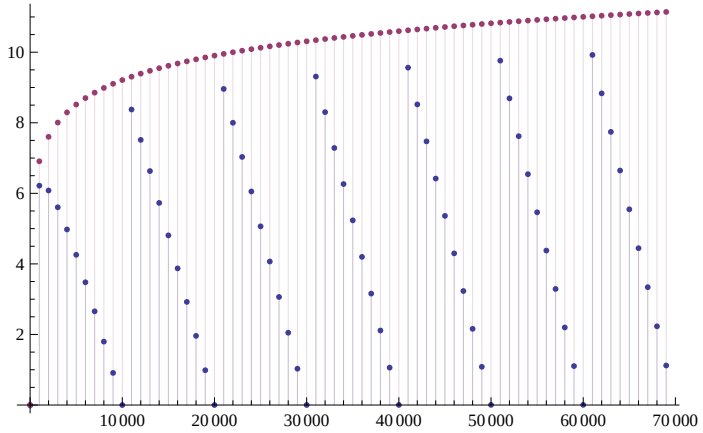
```
{Sum[ E1x[ 100 / j, -1, 2 ] N[l1x[j, 1, 2]], {j, 1, 100}], -N[L1[ 100, -1, 2]]}
```

```
{6.70877, 6.70877}
```

```
L2b[n_, 1, c_] := (1 / den[c]) Sum[alpha[j, c] Log[j / den[c]], {j, den[c] + 1, den[c] n}]
DiscretePlot[ L2a[ n, 1, 1001 / 1000], {n, 1, 700, 10}]
```



```
DiscretePlot[{L2x[n, 1, 10 001 / 10 000], Log[n]}, {n, 1, 70 000, 1000}]
```




```
{(1 / 2) Sum[E1[(2 × 10) / j, -1, 3 / 2] N[alpha[j, 3 / 2] Log[j / 2]], {j, 1, 2 × 10}],
 -N[L1[10, -1, 3 / 2]]}
```

```
{-0.188592, -0.188592}
```

```
{(1 / 3) Sum[E1[(3 × 100) / j, -1, 4 / 3] N[alpha[j, 4 / 3] Log[j / 3]], {j, 1, 3 × 100}],
 -N[L1[100, -1, 4 / 3]]}
```

```
{-19.6174, -19.6174}
```

```
ld[n_, c_] :=
 den[c]^-1 Sum[E1[n den[c] / j, -1, c] N[alpha[j, c] Log[j / den[c]]], {j, 1, n den[c]}]
ldx[n_, c_] := den[c]^-1
  Sum[E1x[n den[c] / j, -1, c] N[alpha[j, c] Log[j / den[c]]], {j, 1, n den[c]}]
```

```
{ldx[200, 11 / 10], -N[L1[200, -1, 11 / 10]]}
```

```
{8.9824, 8.9824}
```

```
{(1 / 2) Sum[E1[(2 × 100) / j, -1, 3 / 2] N[alpha[j, 3 / 2] Log[j / 2]], {j, 1, 2 × 100}],
 -N[L1[100, -1, 3 / 2]]}
```

```
L2y[n_, 1, b_] := Sum[Log[j], {j, 2, n}] - b Sum[Log[j b], {j, 1, n / b}]
{(1 / 2) Sum[e1[j, -1, 3 / 2] N[L2y[Floor[2 × 10 / j], 1, 3 / 2]], {j, 1, 2 × 10}],
 -N[L1[10, -1, 3 / 2]]}
```

```
{-2.27449, -0.188592}
```

```
{(1 / 2) Sum[e1[j, -1, 3 / 2] N[Sum[alpha[m, 3 / 2] Log[m / 2], {m, 1, Floor[2 × 100 / j]}]],
   {j, 1, 2 × 100}], -N[L1[100, -1, 3 / 2]]}
```

```
{-16.1766, -9.95352}
```

```
Table[alpha[j, 3 / 2], {j, 1, 20}]
```

```
{0, 2, -3, 2, 0, -1, 0, 2, -3, 2, 0, -1, 0, 2, -3, 2, 0, -1, 0, 2}
```

```
N[(1 / 2) Sum[
   Sum[ e1[j, -1, 3 / 2], {j, 1, Floor[10 × 2 / k]}] alpha[k, 3 / 2] Log[ k / 2], {k, 1, 2 × 10}]]
```

```
-1.96394
```

```
N[(1 / 2) Sum[E1[10 × 2 / k, -1, 3 / 2] alpha[k, 3 / 2] Log[ k / 2], {k, 1, 2 × 10}]]
```

```
-0.188592
```

```
N[(1 / 2) Sum[E1[Floor[10 × 2 / k], -1, 3 / 2] alpha[k, 3 / 2] Log[ k / 2], {k, 1, 2 × 10}]]
```

```
-1.96394
```

```
N[(1 / 2) Sum[ e1[ j, -1, 3 / 2] Sum[ alpha[k, 3 / 2] Log[ k / 2], {k, 1, 2 × 10 / j}], {j, 1, 2 × 10}]]
```

```
-1.96394
```

```
{(1 / 2) Sum[ E1[ (2 × 10) / j, -1, 3 / 2 ] N[alpha[ j, 3 / 2] Log[j / 2]], {j, 1, 2 × 10}],
 -N[L1[ 10, -1, 3 / 2]]}
```

```
{-0.188592, -0.188592}
```

```
E1[20, -1, 3 / 2]
```

$$\frac{597}{128}$$

```
Sum[ e1[ j, -1, 3 / 2], {j, 1, 20}]
```

$$\frac{597}{128}$$

```
{Sum[ E1[ 100 / j, 1, 2 ] N[l1[j, -1, 2]], {j, 1, 100}], -N[L1[ 100, 1, 2]]}
```

```
{2.53088, 2.53088}
```

```
{Sum[ E1[ 100 / j, -1, 2 ] N[l1[j, 1, 2]], {j, 1, 100}], -N[L1[ 100, -1, 2]]}
```

```
{6.70877, 6.70877}
```

```
{Sum[ L1[ 100 / j, 1, 2 ] N[e1[j, -1, 2]], {j, 1, 100}], -N[L1[ 100, -1, 2]]}
```

```
{6.70877, 6.70877}
```

```
{Sum[ L1[ 100 / j, -1, 2 ] N[e1[j, 1, 2]], {j, 1, 100}], -N[L1[ 100, 1, 2]]}
```

```
{2.53088, 2.53088}
```

```
ld[ n_, c_] :=
 den[c]^-1 Sum[ E1[ n den[c] / j, -1, c ] N[alpha[ j, c] Log[j / den[c]]], {j, 1, n den[c]}]
```

```
{(1 / 2) Sum[ E1[ 2 × 10 / j, 1, 3 / 2 ] N[l1[j, -1, 3 / 2]], {j, 1, 2 × 10}], -N[L1[ 10, 1, 3 / 2]]}
```

```
{-0.528475, -1.58635}
```

```
{(1 / 2) Sum[ E1[ 2 × 10 / j, -1, 3 / 2 ] N[l1[j, 1, 3 / 2]], {j, 1, 2 × 10}], -N[L1[ 10, -1, 3 / 2]]}
```

```
{-1.00113, -0.188592}
```

```
ld[10, 3 / 2]
```

```
-0.188592
```

```
{(1 / 2) Sum[ E1[ 2 × 10 / j, 1, 3 / 2 ] N[l1y[10, -1, 3 / 2]], {j, 1, 2 × 10}],
 -N[L1[ 10, 1, 3 / 2]]}
```

```
FullSimplify[L1[20, 1, 3 / 2] - L1[19, 1, 3 / 2]]
```

$$-\frac{3}{2}\,\text{Log}\left[\frac{39}{2}\right]+\text{Log}[20]$$

$$\text{Log}\left[\frac{20}{3}\right]$$

```
N[L1y[100, 2, 3 / 2]]
```

```
7.75993
```

```
N[L1[100, 2, 3 / 2]]
```

```
7.75993
```

```
l1dif[n_, 1, c_] := den[c]^-1 alpha[den[c] n, c] Log[n]
{FullSimplify[l1dif[20, 1, 3 / 2]], alpha[20, 3 / 2] Log[20 / 2]}
```

```
{Log[20], 2 Log[10]}
```

```
FullSimplify[l1y[20, 1, 3 / 2]]
```

```
Log[20]
```

```
lm1[j_, c_] :=
 N[alpha[j, c] Log[j / den[c]]] - Sum[If[k == 1 || k == j, 0, lm1[k, c]], {k, Divisors[j]}]
lm1[10, 3 / 2]
```

```
3.21888
```