```
ClearAll["Global`*"]

E2a[n_, k_, a_] :=
 E2a[n, k, a] = Sum[ E2a[n / j, k - 1, a], {j, 2, n}] - a Sum[ E2a[n / (a j), k - 1, a], {j, 1, n / a}];
E2a[n_, 0, a_] := 1
D2a[n_, k_] := D2a[n, k] = Sum[D2a[Floor[n / j], k - 1], {j, 2, n}]; D2a[n_, 0] := 1
DD[n_, z_] := DD[n, z] = Sum[FactorialPower[z, a] / a! D2a[n, a], {a, 0, Log[2, n]}]
EE[n_, z_, b_] :=
 EE[n, z, b] = Sum[FactorialPower[z, a] / a! E2a[n, a, b], {a, 0, Log[If[b > 2, 2, b], n]}]
D1b[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j E1b[n / b^j, k, b], {j, 0, Log[b, n]}]
D1b2[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j E1[n / b^j, k, b], {j, 0, Log[b, n]}]
D1b2a[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j E1[n / b^j, k, b] / b, {j, 0, Log[b, n]}]
E1b[n_, k_, b_] := Sum[FactorialPower[k, a] / a! E2b[n, a, b], {a, 0, Log[If[b > 2, 2, b], n]}]
E2b[n_, k_, a_] :=
 E2b[n, k, a] = Sum[ E2b[n / j, k - 1, a], {j, 2, n}] - a Sum[ E2b[n / (a j), k - 1, a], {j, 1, n / a}];
E2b[n_, 0, a_] := 1
D1c[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j
    Sum[FactorialPower[k, a] / a! E2b[n / b^j, a, b], {a, 0, Log[If[b > 2, 2, b], n / b^j]}],
  {j, 0, Log[b, n]}]
D1d[n_, z_, b_] := Sum[
  Binomial[z + j - 1, z - 1] Binomial[z, k] b^j E2[n / b^j, k, b],
  {j, 0, Log[b, n]}, {k, 0, Log[If[b > 2, 2, b], n / b^j]}]
D1e[n_, k_, b_] := Grid[Table[
    Binomial[k + j - 1, k - 1] Binomial[k, a] b^j E2[n / b^j, a, b],
    {j, 0, Log[b, n]}, {a, 0, Log[If[b > 2, 2, b], n / b^j]}]]
D1e2[n_, k_, b_] := Grid[Table[
    Binomial[k + j - 1, k - 1] FactorialPower[k, a] / a! b^j E2[n / b^j, a, b] / k,
    {j, 0, Log[b, n]}, {a, 0, Log[If[b > 2, 2, b], n / b^j]}]]
D1c2[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j
    Sum[FactorialPower[k, a] / a! E2b[n / b^j, a, b], {a, 0, Log[If[b > 2, 2, b], n / b^j]}],
  {j, 0, Log[b, n]}]
lin[n_, b_] := Sum[(-1)^(k + 1) / k E2b[n, k, b], {k, 1, Log[2, n]}]
M2[n_, a_] := Sum[(-1)^k ( E2b[n, k, a] - a E2b[n / a, k, a]), {k, 0, Log[a, n]}]

EM2[n_, a_, b_] :=
 EM2[n, a, b] = Sum[(-1)^k Binomial[k - 1, k - a] E2a[n, k, b], {k, 1, Log[If[b < 2, b, 2], n]}];
EM2[n_, 0, b_] := 1
E2d[n_, a_, b_] :=
 E2d[n, a, b] = Sum[(-1)^k Binomial[k - 1, k - a] EM2[n, k, b], {k, 1, Log[If[b < 2, b, 2], n]}];
E2d[n_, 0, b_] := 1
E1e[n_, k_, b_] := Sum[FactorialPower[k, a] / a! E2d[n, a, b], {a, 0, Log[If[b > 2, 2, b], n]}]
D1g[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j E1e[n / b^j, k, b], {j, 0, Log[b, n]}]

EP2[n_, a_, b_] :=
 EP2[n, a, b] = Sum[SeriesCoefficient[Series[(Log[x + 1])^a, {x, 0, 30}], k] E2a[n, k, b],
   {k, 1, Log[If[b > 2, 2, b], n]}]
E1p[n_, a_, b_] := E1p[n, a, b] = 1 + Sum[a^k / k! EP2[n, k, b], {k, 1, Log[If[b > 2, 2, b], n]}]
D1f[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j E1p[n / b^j, k, b], {j, 0, Log[b, n]}]

E2d[100, 4, 1.5]
```

```
84.25
```

```
E2a[100, 4, 1.5]
```

84.25

```
D1g[300, -1, 1.5]
```

-5.

```
D1c[300, -1, 1.5]
```

-5.

```
E1p[300, 3, 1.5]
```

-48.

```
E1b[300, 3, 1.5]
```

-48.

```
D1f[300, -1, 1.5]
```

-5.

D1e2[200, .0000001, 2]

$1.\times 10^{7}$ E2[200, 0, 2]	1. E2[200, 1, 2]	−0.5 E2[200, 2, 2]	0.333333 E2[200, 3, 2]	−0.25 E2[200, 4, 2]	0.2 E2[200, 5, 2]	−0.166667 E2[200, 6, 2]	0.142857 E2[200, 7, 2]
2. E2[100, 0, 2]	$2.\times 10^{-7}$ E2[100, 1, 2]	$-1.\times 10^{-7}$ E2[100, 2, 2]	6.66667×10^{-8} E2[100, 3, 2]	$-5.\times 10^{-8}$ E2[100, 4, 2]	$4.\times 10^{-8}$ E2[100, 5, 2]	-3.33333×10^{-8} E2[100, 6, 2]	
2. E2[50, 0, 2]	$2.\times 10^{-7}$ E2[50, 1, 2]	$-1.\times 10^{-7}$ E2[50, 2, 2]	6.66667×10^{-8} E2[50, 3, 2]	$-5.\times 10^{-8}$ E2[50, 4, 2]	$4.\times 10^{-8}$ E2[50, 5, 2]		
2.66667 E2[25, 0, 2]	2.66667×10^{-7} E2[25, 1, 2]	-1.33333×10^{-7} E2[25, 2, 2]	8.88889×10^{-8} E2[25, 3, 2]	-6.66667×10^{-8} E2[25, 4, 2]			
4. E2[$\frac{25}{2}$, 0, 2]	$4.\times 10^{-7}$ E2[$\frac{25}{2}$, 1, 2]	$-2.\times 10^{-7}$ E2[$\frac{25}{2}$, 2, 2]	1.33333×10^{-7} E2[$\frac{25}{2}$, 3, 2]				
6.4 E2[$\frac{25}{4}$, 0, 2]	6.4×10^{-7} E2[$\frac{25}{4}$, 1, 2]	-3.2×10^{-7} E2[$\frac{25}{4}$, 2, 2]					
10.6667 E2[$\frac{25}{8}$, 0, 2]	1.06667×10^{-6} E2[$\frac{25}{8}$, 1, 2]						
18.2857 E2[$\frac{25}{16}$, 0, 2]							

D1e[900, -1 / 2, 2]

E2[900, 0, 2]	$-\frac{1}{2}$ E2[900, 1, 2]	$\frac{3}{8}$ E2[900, 2, 2]	$-\frac{5}{16}$ E2[900, 3, 2]	$\frac{35}{128}$ E2[900, 4, 2]	$-\frac{63}{256}$ E2[900, 5, 2]	(231 E2[900, 6, 2]) / 1024	−(429 E2[900, 7, 2]) / 2048	(6435 E2[900, 8, 2]) / 32 768	−(12 155 E2[900, 9, 2]) / 65 536
−E2[450, 0, 2]	$\frac{1}{2}$ E2[450, 1, 2]	$-\frac{3}{8}$ E2[450, 2, 2]	$\frac{5}{16}$ E2[450, 3, 2]	$-\frac{35}{128}$ E2[450, 4, 2]	$\frac{63}{256}$ E2[450, 5, 2]	−(231 E2[450, 6, 2]) / 1024	(429 E2[450, 7, 2]) / 2048	−(6435 E2[450, 8, 2]) / 32 768	

$$-\frac{1}{2}\,E2[225,0,2]\quad \frac{1}{4}\,E2[225,1,2]\quad -\frac{3}{16}\,E2[225,2,2]\quad \frac{5}{32}\,E2[225,3,2]\quad -\frac{35}{256}\,E2[225,4,2]\quad \frac{63}{512}\,E2[225,5,2]\quad -\frac{(231\,E2[225,6,2])}{2048}\quad \frac{(429\,E2[225,7,2])}{4096}$$

$$-\frac{1}{2}\,E2\left[\frac{225}{2},0,2\right]\quad \frac{1}{4}\,E2\left[\frac{225}{2},1,2\right]\quad -\frac{3}{16}\,E2\left[\frac{225}{2},2,2\right]\quad \frac{5}{32}\,E2\left[\frac{225}{2},3,2\right]\quad -\frac{35}{256}\,E2\left[\frac{225}{2},4,2\right]\quad \frac{63}{512}\,E2\left[\frac{225}{2},5,2\right]\quad -\frac{(231\,E2[\frac{225}{2},6,2])}{2048}$$

$$-\frac{5}{8}\,E2\left[\frac{225}{4},0,2\right]\quad \frac{5}{16}\,E2\left[\frac{225}{4},1,2\right]\quad -\frac{15}{64}\,E2\left[\frac{225}{4},2,2\right]\quad \frac{25}{128}\,E2\left[\frac{225}{4},3,2\right]\quad -\frac{(175\,E2[\frac{225}{4},4,2])}{1024}\quad \frac{(315\,E2[\frac{225}{4},5,2])}{2048}$$

$$-\frac{7}{8}\,E2\left[\frac{225}{8},0,2\right]\quad \frac{7}{16}\,E2\left[\frac{225}{8},1,2\right]\quad -\frac{21}{64}\,E2\left[\frac{225}{8},2,2\right]\quad \frac{35}{128}\,E2\left[\frac{225}{8},3,2\right]\quad -\frac{(245\,E2[\frac{225}{8},4,2])}{1024}$$

$$-\frac{21}{16}\,E2\left[\frac{225}{16},0,2\right]\quad \frac{21}{32}\,E2\left[\frac{225}{16},1,2\right]\quad -\frac{63}{128}\,E2\left[\frac{225}{16},2,2\right]\quad \frac{105}{256}\,E2\left[\frac{225}{16},3,2\right]$$

$$-\frac{33}{16}\,E2\left[\frac{225}{32},0,2\right]\quad \frac{33}{32}\,E2\left[\frac{225}{32},1,2\right]\quad -\frac{99}{128}\,E2\left[\frac{225}{32},2,2\right]$$

$$-\frac{429}{128}\,E2\left[\frac{225}{64},0,2\right]\quad \frac{429}{256}\,E2\left[\frac{225}{64},1,2\right]$$

$$-\frac{715}{128}\,E2\left[\frac{225}{128},0,2\right]$$

D1e[900, -1, 2]

E2[900, 0, 2]	−E2[900, 1, 2]	E2[900, 2, 2]	−E2[900, 3, 2]	E2[900, 4, 2]	−E2[900, 5, 2]	E2[900, 6, 2]	−E2[900, 7, 2]	E2[900, 8, 2]	−E2[900, 9, 2]
−2 E2[450, 0, 2]	2 E2[450, 1, 2]	−2 E2[450, 2, 2]	2 E2[450, 3, 2]	−2 E2[450, 4, 2]	2 E2[450, 5, 2]	−2 E2[450, 6, 2]	2 E2[450, 7, 2]	−2 E2[450, 8, 2]	
0	0	0	0	0	0	0	0		
0	0	0	0	0	0	0			
0	0	0	0	0	0				
0	0	0	0	0					
0	0	0	0						
0	0	0							
0	0								
0									

D1e[900, −2, 2]

E2[900, 0, 2]	−2 E2[900, 1, 2]	3 E2[900, 2, 2]	−4 E2[900, 3, 2]	5 E2[900, 4, 2]	−6 E2[900, 5, 2]	7 E2[900, 6, 2]	−8 E2[900, 7, 2]	9 E2[900, 8, 2]	−10 E2[900, 9, 2]
−4 E2[450, 0, 2]	8 E2[450, 1, 2]	−12 E2[450, 2, 2]	16 E2[450, 3, 2]	−20 E2[450, 4, 2]	24 E2[450, 5, 2]	−28 E2[450, 6, 2]	32 E2[450, 7, 2]	−36 E2[450, 8, 2]	
4 E2[225, 0, 2]	−8 E2[225, 1, 2]	12 E2[225, 2, 2]	−16 E2[225, 3, 2]	20 E2[225, 4, 2]	−24 E2[225, 5, 2]	28 E2[225, 6, 2]	−32 E2[225, 7, 2]		
0	0	0	0	0	0	0			
0	0	0	0	0	0				
0	0	0	0	0					
0	0	0	0						
0	0	0							
0	0								
0									

D1e[900, −3, 2]

E2[900, 0, 2]	−3 E2[900, 1, 2]	6 E2[900, 2, 2]	−10 E2[900, 3, 2]	15 E2[900, 4, 2]	−21 E2[900, 5, 2]	28 E2[900, 6, 2]	−36 E2[900, 7, 2]	45 E2[900, 8, 2]	−55 E2[900, 9, 2]
−6 E2[450, 0, 2]	18 E2[450, 1, 2]	−36 E2[450, 2, 2]	60 E2[450, 3, 2]	−90 E2[450, 4, 2]	126 E2[450, 5, 2]	−168 E2[450, 6, 2]	216 E2[450, 7, 2]	−270 E2[450, 8, 2]	
12 E2[225, 0, 2]	−36 E2[225, 1, 2]	72 E2[225, 2, 2]	−120 E2[225, 3, 2]	180 E2[225, 4, 2]	−252 E2[225, 5, 2]	336 E2[225, 6, 2]	−432 E2[225, 7, 2]		
−8 E2[$\frac{225}{2}$, 0, 2]	24 E2[$\frac{225}{2}$, 1, 2]	−48 E2[$\frac{225}{2}$, 2, 2]	80 E2[$\frac{225}{2}$, 3, 2]	−120 E2[$\frac{225}{2}$, 4, 2]	168 E2[$\frac{225}{2}$, 5, 2]	−224 E2[$\frac{225}{2}$, 6, 2]			
0	0	0	0	0	0				
0	0	0	0	0					
0	0	0	0						
0	0	0							
0	0								
0									

D1e[900, -4, 2]

E2[900, 0, 2]	$-4\,E2[900, 1, 2]$	$10\,E2[900, 2, 2]$	$-20\,E2[900, 3, 2]$	$35\,E2[900, 4, 2]$	$-56\,E2[900, 5, 2]$	$84\,E2[900, 6, 2]$	$-120\,E2[900, 7, 2]$	$165\,E2[900, 8, 2]$	$-220\,E2[900, 9, 2]$
$-8\,E2[450, 0, 2]$	$32\,E2[450, 1, 2]$	$-80\,E2[450, 2, 2]$	$160\,E2[450, 3, 2]$	$-280\,E2[450, 4, 2]$	$448\,E2[450, 5, 2]$	$-672\,E2[450, 6, 2]$	$960\,E2[450, 7, 2]$	$-1320\,E2[450, 8, 2]$	
$24\,E2[225, 0, 2]$	$-96\,E2[225, 1, 2]$	$240\,E2[225, 2, 2]$	$-480\,E2[225, 3, 2]$	$840\,E2[225, 4, 2]$	$-1344\,E2[225, 5, 2]$	$2016\,E2[225, 6, 2]$	$-2880\,E2[225, 7, 2]$		
$-32\,E2\left[\frac{225}{2}, 0, 2\right]$	$128\,E2\left[\frac{225}{2}, 1, 2\right]$	$-320\,E2\left[\frac{225}{2}, 2, 2\right]$	$640\,E2\left[\frac{225}{2}, 3, 2\right]$	$-1120\,E2\left[\frac{225}{2}, 4, 2\right]$	$1792\,E2\left[\frac{225}{2}, 5, 2\right]$	$-2688\,E2\left[\frac{225}{2}, 6, 2\right]$			
$16\,E2\left[\frac{225}{4}, 0, 2\right]$	$-64\,E2\left[\frac{225}{4}, 1, 2\right]$	$160\,E2\left[\frac{225}{4}, 2, 2\right]$	$-320\,E2\left[\frac{225}{4}, 3, 2\right]$	$560\,E2\left[\frac{225}{4}, 4, 2\right]$	$-896\,E2\left[\frac{225}{4}, 5, 2\right]$				
0	0	0	0	0					
0	0	0	0						
0	0	0							
0	0								
0									

D1e[900, 1 / 2, 2]

E2[900, 0, 2]	$\frac{1}{2}\,E2[900, 1, 2]$	$-\frac{1}{8}\,E2[900, 2, 2]$	$\frac{1}{16}\,E2[900, 3, 2]$	$-\frac{5}{128}\,E2[900, 4, 2]$	$\frac{7}{256}\,E2[900, 5, 2]$	$-(21\,E2[900, 6, 2]) / 1024$	$(33\,E2[900, 7, 2]) / 2048$	$-(429\,E2[900, 8, 2]) / 32\,768$	$(715\,E2[900, 9, 2]) / 65\,536$
E2[450, 0, 2]	$\frac{1}{2}\,E2[450, 1, 2]$	$-\frac{1}{8}\,E2[450, 2, 2]$	$\frac{1}{16}\,E2[450, 3, 2]$	$-\frac{5}{128}\,E2[450, 4, 2]$	$\frac{7}{256}\,E2[450, 5, 2]$	$-(21\,E2[450, 6, 2]) / 1024$	$(33\,E2[450, 7, 2]) / 2048$	$-(429\,E2[450, 8, 2]) / 32\,768$	
$\frac{3}{2}\,E2[225, 0, 2]$	$\frac{3}{4}\,E2[225, 1, 2]$	$-\frac{3}{16}\,E2[225, 2, 2]$	$\frac{3}{32}\,E2[225, 3, 2]$	$-\frac{15}{256}\,E2[225, 4, 2]$	$\frac{21}{512}\,E2[225, 5, 2]$	$-(63\,E2[225, 6, 2]) / 2048$	$(99\,E2[225, 7, 2]) / 4096$		
$\frac{5}{2}\,E2\left[\frac{225}{2}, 0, 2\right]$	$\frac{5}{4}\,E2\left[\frac{225}{2}, 1, 2\right]$	$-\frac{5}{16}\,E2\left[\frac{225}{2}, 2, 2\right]$	$\frac{5}{32}\,E2\left[\frac{225}{2}, 3, 2\right]$	$-\frac{25}{256}\,E2\left[\frac{225}{2}, 4, 2\right]$	$\frac{35}{512}\,E2\left[\frac{225}{2}, 5, 2\right]$	$-(105\,E2\left[\frac{225}{2}, 6, 2\right]) / 2048$			

$$\frac{35}{8}\,E2\left[\frac{225}{4},\,0,\,2\right]\quad \frac{35}{16}\,E2\left[\frac{225}{4},\,1,\,2\right]\quad -\frac{35}{64}\,E2\left[\frac{225}{4},\,2,\,2\right]\quad \frac{35}{128}\,E2\left[\frac{225}{4},\,3,\,2\right]\quad -\left(175\,E2\left[\frac{225}{4},\,4,\,2\right]\right)/1024\quad \left(245\,E2\left[\frac{225}{4},\,5,\,2\right]\right)/2048$$

$$\frac{63}{8}\,E2\left[\frac{225}{8},\,0,\,2\right]\quad \frac{63}{16}\,E2\left[\frac{225}{8},\,1,\,2\right]\quad -\frac{63}{64}\,E2\left[\frac{225}{8},\,2,\,2\right]\quad \frac{63}{128}\,E2\left[\frac{225}{8},\,3,\,2\right]\quad -\left(315\,E2\left[\frac{225}{8},\,4,\,2\right]\right)/1024$$

$$\frac{231}{16}\,E2\left[\frac{225}{16},\,0,\,2\right]\quad \frac{231}{32}\,E2\left[\frac{225}{16},\,1,\,2\right]\quad -\frac{231}{128}\,E2\left[\frac{225}{16},\,2,\,2\right]\quad \frac{231}{256}\,E2\left[\frac{225}{16},\,3,\,2\right]$$

$$\frac{429}{16}\,E2\left[\frac{225}{32},\,0,\,2\right]\quad \frac{429}{32}\,E2\left[\frac{225}{32},\,1,\,2\right]\quad -\frac{429}{128}\,E2\left[\frac{225}{32},\,2,\,2\right]$$

$$\frac{6435}{128}\,E2\left[\frac{225}{64},\,0,\,2\right]\quad \frac{6435}{256}\,E2\left[\frac{225}{64},\,1,\,2\right]$$

$$\frac{12\,155}{128}\,E2\left[\frac{225}{128},\,0,\,2\right]$$

D1e[900, 1, 2]

$E2[900, 0, 2]$	$E2[900, 1, 2]$	0	0	0	0	0	0	0	0
$2\,E2[450, 0, 2]$	$2\,E2[450, 1, 2]$	0	0	0	0	0	0	0	
$4\,E2[225, 0, 2]$	$4\,E2[225, 1, 2]$	0	0	0	0	0	0		
$8\,E2\left[\frac{225}{2}, 0, 2\right]$	$8\,E2\left[\frac{225}{2}, 1, 2\right]$	0	0	0	0	0			
$16\,E2\left[\frac{225}{4}, 0, 2\right]$	$16\,E2\left[\frac{225}{4}, 1, 2\right]$	0	0	0	0				
$32\,E2\left[\frac{225}{8}, 0, 2\right]$	$32\,E2\left[\frac{225}{8}, 1, 2\right]$	0	0	0					
$64\,E2\left[\frac{225}{16}, 0, 2\right]$	$64\,E2\left[\frac{225}{16}, 1, 2\right]$	0	0						
$128\,E2\left[\frac{225}{32}, 0, 2\right]$	$128\,E2\left[\frac{225}{32}, 1, 2\right]$	0							
$256\,E2\left[\frac{225}{64}, 0, 2\right]$	$256\,E2\left[\frac{225}{64}, 1, 2\right]$								
$512\,E2\left[\frac{225}{128}, 0, 2\right]$									

D1e[900, 2, 2]

$$\begin{array}{lll}
E2[900,0,2] & 2\,E2[900,1,2] & E2[900,2,2] \\
4\,E2[450,0,2] & 8\,E2[450,1,2] & 4\,E2[450,2,2] \\
12\,E2[225,0,2] & 24\,E2[225,1,2] & 12\,E2[225,2,2] \\
32\,E2\left[\frac{225}{2},0,2\right] & 64\,E2\left[\frac{225}{2},1,2\right] & 32\,E2\left[\frac{225}{2},2,2\right] \\
80\,E2\left[\frac{225}{4},0,2\right] & 160\,E2\left[\frac{225}{4},1,2\right] & 80\,E2\left[\frac{225}{4},2,2\right] \\
192\,E2\left[\frac{225}{8},0,2\right] & 384\,E2\left[\frac{225}{8},1,2\right] & 192\,E2\left[\frac{225}{8},2,2\right] \\
448\,E2\left[\frac{225}{16},0,2\right] & 896\,E2\left[\frac{225}{16},1,2\right] & 448\,E2\left[\frac{225}{16},2,2\right] \\
1024\,E2\left[\frac{225}{32},0,2\right] & 2048\,E2\left[\frac{225}{32},1,2\right] & 1024\,E2\left[\frac{225}{32},2,2\right] \\
2304\,E2\left[\frac{225}{64},0,2\right] & 4608\,E2\left[\frac{225}{64},1,2\right] & \\
5120\,E2\left[\frac{225}{128},0,2\right] & &
\end{array}$$

$$0\ 0\ 0\ 0\ 0\ 0\ 0$$
$$0\ 0\ 0\ 0\ 0\ 0$$
$$0\ 0\ 0\ 0\ 0$$
$$0\ 0\ 0\ 0$$
$$0\ 0\ 0$$
$$0\ 0$$
$$0$$

D1e[900, 4, 2]

$$\begin{array}{lllll}
E2[900,0,2] & 4\,E2[900,1,2] & 6\,E2[900,2,2] & 4\,E2[900,3,2] & E2[900,4,2] \\
8\,E2[450,0,2] & 32\,E2[450,1,2] & 48\,E2[450,2,2] & 32\,E2[450,3,2] & 8\,E2[450,4,2] \\
40\,E2[225,0,2] & 160\,E2[225,1,2] & 240\,E2[225,2,2] & 160\,E2[225,3,2] & 40\,E2[225,4,2] \\
160\,E2\left[\frac{225}{2},0,2\right] & 640\,E2\left[\frac{225}{2},1,2\right] & 960\,E2\left[\frac{225}{2},2,2\right] & 640\,E2\left[\frac{225}{2},3,2\right] & 160\,E2\left[\frac{225}{2},4,2\right] \\
560\,E2\left[\frac{225}{4},0,2\right] & 2240\,E2\left[\frac{225}{4},1,2\right] & 3360\,E2\left[\frac{225}{4},2,2\right] & 2240\,E2\left[\frac{225}{4},3,2\right] & 560\,E2\left[\frac{225}{4},4,2\right] \\
1792\,E2\left[\frac{225}{8},0,2\right] & 7168\,E2\left[\frac{225}{8},1,2\right] & 10\,752\,E2\left[\frac{225}{8},2,2\right] & 7168\,E2\left[\frac{225}{8},3,2\right] & 1792\,E2\left[\frac{225}{8},4,2\right] \\
5376\,E2\left[\frac{225}{16},0,2\right] & 21\,504\,E2\left[\frac{225}{16},1,2\right] & 32\,256\,E2\left[\frac{225}{16},2,2\right] & 21\,504\,E2\left[\frac{225}{16},3,2\right] & \\
15\,360\,E2\left[\frac{225}{32},0,2\right] & 61\,440\,E2\left[\frac{225}{32},1,2\right] & 92\,160\,E2\left[\frac{225}{32},2,2\right] & & \\
42\,240\,E2\left[\frac{225}{64},0,2\right] & 168\,960\,E2\left[\frac{225}{64},1,2\right] & & & \\
112\,640\,E2\left[\frac{225}{128},0,2\right] & & & &
\end{array}$$

$$0\ 0\ 0\ 0\ 0$$
$$0\ 0\ 0\ 0$$
$$0\ 0\ 0$$
$$0\ 0$$
$$0$$

```
D1e[900, 5, 2]
```

$$\begin{array}{llllll} E2[900,0,2] & 5\,E2[900,1,2] & 10\,E2[900,2,2] & 10\,E2[900,3,2] & 5\,E2[900,4,2] & E2[900,5,2] \\ 10\,E2[450,0,2] & 50\,E2[450,1,2] & 100\,E2[450,2,2] & 100\,E2[450,3,2] & 50\,E2[450,4,2] & 10\,E2[450,5,2] \\ 60\,E2[225,0,2] & 300\,E2[225,1,2] & 600\,E2[225,2,2] & 600\,E2[225,3,2] & 300\,E2[225,4,2] & 60\,E2[225,5,2] \\ 280\,E2\left[\frac{225}{2},0,2\right] & 1400\,E2\left[\frac{225}{2},1,2\right] & 2800\,E2\left[\frac{225}{2},2,2\right] & 2800\,E2\left[\frac{225}{2},3,2\right] & 1400\,E2\left[\frac{225}{2},4,2\right] & 280\,E2\left[\frac{225}{2},5,2\right] \\ 1120\,E2\left[\frac{225}{4},0,2\right] & 5600\,E2\left[\frac{225}{4},1,2\right] & 11\,200\,E2\left[\frac{225}{4},2,2\right] & 11\,200\,E2\left[\frac{225}{4},3,2\right] & 5600\,E2\left[\frac{225}{4},4,2\right] & 1120\,E2\left[\frac{225}{4},5,2\right] \\ 4032\,E2\left[\frac{225}{8},0,2\right] & 20\,160\,E2\left[\frac{225}{8},1,2\right] & 40\,320\,E2\left[\frac{225}{8},2,2\right] & 40\,320\,E2\left[\frac{225}{8},3,2\right] & 20\,160\,E2\left[\frac{225}{8},4,2\right] & \\ 13\,440\,E2\left[\frac{225}{16},0,2\right] & 67\,200\,E2\left[\frac{225}{16},1,2\right] & 134\,400\,E2\left[\frac{225}{16},2,2\right] & 134\,400\,E2\left[\frac{225}{16},3,2\right] & & \\ 42\,240\,E2\left[\frac{225}{32},0,2\right] & 211\,200\,E2\left[\frac{225}{32},1,2\right] & 422\,400\,E2\left[\frac{225}{32},2,2\right] & & & \\ 126\,720\,E2\left[\frac{225}{64},0,2\right] & 633\,600\,E2\left[\frac{225}{64},1,2\right] & & & & \\ 366\,080\,E2\left[\frac{225}{128},0,2\right] & & & & & \end{array}$$

```
Residue[ ((Zeta[s])) x^s s^(-1), {s, 1}]
```

$$x$$

```
Residue[ ((Zeta[s]^2)) x^s / s, {s, 1}]
```

$$-x + 2\,\text{EulerGamma}\,x + x\,\text{Log}[x]$$

```
FullSimplify[Residue[ ((Zeta[s]^3)) x^s / s, {s, 1}]]
```

$$\frac{1}{2}x\,(2 + 6\,(-1 + \text{EulerGamma})\,\text{EulerGamma} + \text{Log}[x]\,(-2 + 6\,\text{EulerGamma} + \text{Log}[x]) - 6\,\text{StieltjesGamma}[1])$$

```
FullSimplify[Residue[ ((Zeta[s]^4)) x^s / s, {s, 1}]]
```

$$\frac{1}{6}x\,\Big(3\,(-1 + 4\,\text{EulerGamma})\,\text{Log}[x]^2 + \text{Log}[x]^3 + 6\,\text{Log}[x]\,\big(1 - 4\,\text{EulerGamma} + 6\,\text{EulerGamma}^2 - 4\,\text{StieltjesGamma}[1]\big) + 6\,(-1 + 2\,\text{EulerGamma}\,(2 + \text{EulerGamma}\,(-3 + 2\,\text{EulerGamma}) - 6\,\text{StieltjesGamma}[1]) + 4\,\text{StieltjesGamma}[1] + 2\,\text{StieltjesGamma}[2])\Big)$$

```
Limit[ (a - 1)^2 Sum[k a^k , {k, 1, Log[a, x]}], a → 1]
```

$$1 - x + x\,\text{Log}[x]$$

```
Residue[ ((Zeta[s]^2)) x^s / s, {s, 1}]
```

$$-x + 2\,\text{EulerGamma}\,x + x\,\text{Log}[x]$$

```
D1e[100, 2, 2]
```

$$\begin{array}{cccccccc}
\text{E2}[100,0,2] & 2\,\text{E2}[100,1,2] & \text{E2}[100,2,2] & 0 & 0 & 0 & 0 \\
4\,\text{E2}[50,0,2] & 8\,\text{E2}[50,1,2] & 4\,\text{E2}[50,2,2] & 0 & 0 & 0 & \\
12\,\text{E2}[25,0,2] & 24\,\text{E2}[25,1,2] & 12\,\text{E2}[25,2,2] & 0 & 0 & & \\
32\,\text{E2}\left[\frac{25}{2},0,2\right] & 64\,\text{E2}\left[\frac{25}{2},1,2\right] & 32\,\text{E2}\left[\frac{25}{2},2,2\right] & 0 & & & \\
80\,\text{E2}\left[\frac{25}{4},0,2\right] & 160\,\text{E2}\left[\frac{25}{4},1,2\right] & 80\,\text{E2}\left[\frac{25}{4},2,2\right] & & & & \\
192\,\text{E2}\left[\frac{25}{8},0,2\right] & 384\,\text{E2}\left[\frac{25}{8},1,2\right] & & & & & \\
448\,\text{E2}\left[\frac{25}{16},0,2\right] & & & & & &
\end{array}$$

```
Series[(1 / (x + 1) - 1)^1, {x, 0, 20}]
```

$$-x + x^2 - x^3 + x^4 - x^5 + x^6 - x^7 + x^8 - x^9 + x^{10} - x^{11} + x^{12} - x^{13} + x^{14} - x^{15} + x^{16} - x^{17} + x^{18} - x^{19} + x^{20} + O[x]^{21}$$

```
Table[ (-1)^k Binomial[k - 1, k - 1] x^k, {k, 1, 20}]
```

$$\{-x,\ x^2,\ -x^3,\ x^4,\ -x^5,\ x^6,\ -x^7,\ x^8,\ -x^9,\ x^{10},\ -x^{11},\ x^{12},\ -x^{13},\ x^{14},\ -x^{15},\ x^{16},\ -x^{17},\ x^{18},\ -x^{19},\ x^{20}\}$$

```
Series[(1 / (x + 1) - 1)^2, {x, 0, 20}]
```

$$x^2 - 2\,x^3 + 3\,x^4 - 4\,x^5 + 5\,x^6 - 6\,x^7 + 7\,x^8 - 8\,x^9 + 9\,x^{10} - 10\,x^{11} + 11\,x^{12} - 12\,x^{13} + 13\,x^{14} - 14\,x^{15} + 15\,x^{16} - 16\,x^{17} + 17\,x^{18} - 18\,x^{19} + 19\,x^{20} + O[x]^{21}$$

```
Table[ (-1)^k Binomial[k - 1, k - 2] x^k, {k, 1, 20}]
```

$$\{0,\ x^2,\ -2\,x^3,\ 3\,x^4,\ -4\,x^5,\ 5\,x^6,\ -6\,x^7,\ 7\,x^8,\ -8\,x^9,\ 9\,x^{10},\ -10\,x^{11},\ 11\,x^{12},\ -12\,x^{13},\ 13\,x^{14},\ -14\,x^{15},\ 15\,x^{16},\ -16\,x^{17},\ 17\,x^{18},\ -18\,x^{19},\ 19\,x^{20}\}$$

```
Series[(1 / (x + 1) - 1)^3, {x, 0, 20}]
```

$$-x^3 + 3\,x^4 - 6\,x^5 + 10\,x^6 - 15\,x^7 + 21\,x^8 - 28\,x^9 + 36\,x^{10} - 45\,x^{11} + 55\,x^{12} - 66\,x^{13} + 78\,x^{14} - 91\,x^{15} + 105\,x^{16} - 120\,x^{17} + 136\,x^{18} - 153\,x^{19} + 171\,x^{20} + O[x]^{21}$$

```
Table[ (-1)^k Binomial[k - 1, k - 3] x^k, {k, 1, 20}]
```

$$\{0,\ 0,\ -x^3,\ 3\,x^4,\ -6\,x^5,\ 10\,x^6,\ -15\,x^7,\ 21\,x^8,\ -28\,x^9,\ 36\,x^{10},\ -45\,x^{11},\ 55\,x^{12},\ -66\,x^{13},\ 78\,x^{14},\ -91\,x^{15},\ 105\,x^{16},\ -120\,x^{17},\ 136\,x^{18},\ -153\,x^{19},\ 171\,x^{20}\}$$

```
Series[(1 / (x + 1) - 1)^4, {x, 0, 20}]
```

$$x^4 - 4\,x^5 + 10\,x^6 - 20\,x^7 + 35\,x^8 - 56\,x^9 + 84\,x^{10} - 120\,x^{11} + 165\,x^{12} - 220\,x^{13} + 286\,x^{14} - 364\,x^{15} + 455\,x^{16} - 560\,x^{17} + 680\,x^{18} - 816\,x^{19} + 969\,x^{20} + O[x]^{21}$$

```
Table[ (-1)^k Binomial[k - 1, k - 4] x^k, {k, 1, 20}]
```

$$\{0,\ 0,\ 0,\ x^4,\ -4\,x^5,\ 10\,x^6,\ -20\,x^7,\ 35\,x^8,\ -56\,x^9,\ 84\,x^{10},\ -120\,x^{11},\ 165\,x^{12},\ -220\,x^{13},\ 286\,x^{14},\ -364\,x^{15},\ 455\,x^{16},\ -560\,x^{17},\ 680\,x^{18},\ -816\,x^{19},\ 969\,x^{20}\}$$

```
M2[n_, a_] := Sum[(-1)^k Binomial[k - 1, k - a] D2a[n, k], {k, 1, Log[2, n]}]
```

```
M2[1000, 4]
```

199

```
MM[n_, k_] := Sum[ MoebiusMu[j] MM[Floor[n / j], k - 1], {j, 2, n}]; MM[n_, 0] := 1
```

```
MM[1000, 4]
```

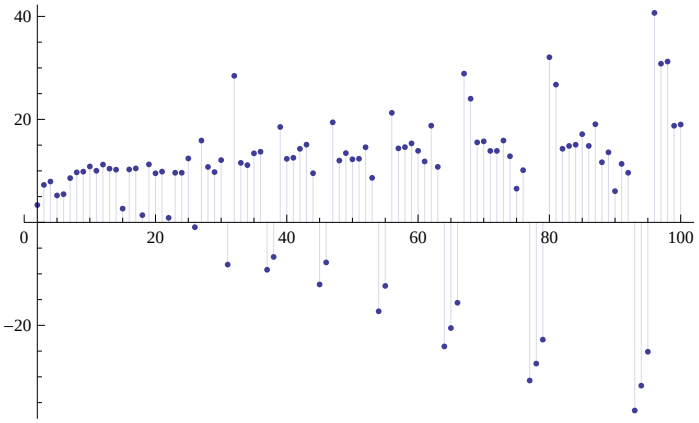
```
199
```

```
EM2[n_, a_, b_] :=
 EM2[n, a, b] = Sum[(-1)^k Binomial[k - 1, k - a] E2a[n, k, b], {k, 1, Log[If[b < 2, b, 2], n]}]
```

```
EM2[100, 2, 1.1]
```

```
108.295
```

```
DiscretePlot[EM2[n, 1, 1.2], {n, 2, 100}]
```




```
Series[(Log[x + 1])^(2), {x, 0, 20}]
```

$$x^2 - x^3 + \frac{11 x^4}{12} - \frac{5 x^5}{6} + \frac{137 x^6}{180} - \frac{7 x^7}{10} + \frac{363 x^8}{560} - \frac{761 x^9}{1260} + \frac{7129 x^{10}}{12\,600} - \frac{671 x^{11}}{1260} + \frac{83\,711 x^{12}}{166\,320} - \frac{6617 x^{13}}{13\,860} + \frac{1\,145\,993 x^{14}}{2\,522\,520} - \frac{1\,171\,733 x^{15}}{2\,702\,700} + \frac{1\,195\,757 x^{16}}{2\,882\,880} - \frac{143\,327 x^{17}}{360\,360} + \frac{42\,142\,223 x^{18}}{110\,270\,160} - \frac{751\,279 x^{19}}{2\,042\,040} + \frac{275\,295\,799 x^{20}}{775\,975\,200} + O[x]^{21}$$

```
SeriesCoefficient[Series[(Log[x + 1])^2, {x, 0, 20}], 4]
```

$$\frac{11}{12}$$

```
P2[n_, a_] :=
 Sum[SeriesCoefficient[Series[(Log[x + 1])^a, {x, 0, 30}], k] D2a[n, k], {k, 1, Log[2, n]}]
```

```
P2[100, 2]
```

$$\frac{16\,289}{180}$$

```
PP[n_, k_] := Sum[
  (FullSimplify[MangoldtLambda[j] / Log[j]]) PP[Floor[n / j], k - 1], {j, 2, n}]; PP[n_, 0] := 1
```

```
PP[100, 2]
```

$$\frac{16\,289}{180}$$

```
EP2[n_, a_, b_] :=
 EP2[n, a, b] = Sum[SeriesCoefficient[Series[(Log[x + 1])^a, {x, 0, 30}], k] E2a[n, k, b],
   {k, 1, Log[If[b > 2, 2, b], n]}]
```

```
DiscretePlot[EP2[n, 4, 1.2], {n, 2, 100}]
```