```
Clear[cc]
cc[n_, s_, 0] := 1
cc[0, s_, k_] := 0
cc[n_, s_, k_] := cc[n, s, k] = Sum[ j^-s cc[n, s, k - 1], {j, 1, n}]
dc[n_, s_, k_] := cc[n, s, k] - cc[n - 1, s, k]
Clear[ee]
ee[n_, 1] := dc[n, 0, 2]
ee[n_, k_] := ee[n, k] = If[ n <= k, 0, ee[n, k - 1] - ee[n - 1, k - 1]]
ee[0, 1] := 0
ee[0, k_] := 0
dif[ x_, n_, k_] := Sum[ (-1)^i Binomial[ k, i] (x - i)^6, {i, 0, k}]



Clear[cc0, B2]
binomial[z_, k_] := binomial[z, k] = Product[z - j, {j, 0, k - 1}] / k!
cc0[n_, 0] := 1
cc0[0, k_] := 0
cc0[n_, k_] := cc0[n, k] = Sum[ j^2 cc0[n, k - 1], {j, 1, n}]
dc0[n_, k_] := cc0[n, k] - cc0[n - 1, k]
ds0[n_, k_] := Sum[ Binomial[k, j] cc0[n - 1, j], {j, 0, k - 1}]
ds0a[n_, k_] := n^k - (n - 1)^k
ds0z[n_, z_] := n^z - If[n == 1, 0, (n - 1)^z]
dsm1z[n_, z_] := (n (n + 1) / 2)^z - If[n == 1, 0, (n (n - 1) / 2)^z]
dsm2z[n_, z_] := (n (n + 1) (2 n + 1) / 6)^z - If[n == 1, 0, ((n - 1) n (2 n - 1) / 6)^z]
B[n_, s_, z_] := If[ n == 0, 0, Sum[ j^-s, {j, 1, n}]^z]
b[n_, s_, z_] := B[n, s, z] - B[n - 1, s, z]
B2[n_, s_, k_] := B2[n, s, k] = Sum[ j^-s B2[n, s, k - 1], {j, 2, n}]
B2[n_, s_, 0] := 1
B2z[n_, s_, z_] := Sum[ Binomial[ z, k] B2[n, s, k], {k, 0, z}]
B2t[n_, s_, z_, t_] := Sum[ binomial[ z, k] B2[n, s, k], {k, 0, t}]

Table[ FullSimplify@ee[n, 1], {n, 1, 10}] // TableForm
```

```
1
3
5
7
9
11
13
15
17
19
```

```
Table[ee[n, k], {n, 1, 12}, {k, 1, 12}] // TableForm
```

1	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0
5	2	0	0	0	0	0	0	0	0	0	0
7	2	0	0	0	0	0	0	0	0	0	0
9	2	0	0	0	0	0	0	0	0	0	0
11	2	0	0	0	0	0	0	0	0	0	0
13	2	0	0	0	0	0	0	0	0	0	0
15	2	0	0	0	0	0	0	0	0	0	0
17	2	0	0	0	0	0	0	0	0	0	0
19	2	0	0	0	0	0	0	0	0	0	0
21	2	0	0	0	0	0	0	0	0	0	0
23	2	0	0	0	0	0	0	0	0	0	0

```
Table[dif[n, 1, 0], {n, 1, 12}] // TableForm
```

1
64
729
4096
15 625
46 656
117 649
262 144
531 441
1 000 000
1 771 561
2 985 984

```
dc[100, -1, 3]
```

7 500 250 000

```
3 × 100 cc[99, -1, 2] + 3 × 100^2 cc[99, -1, 1] + 100^3 cc[99, -1, 0]
```

7 500 250 000

```
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
dk[n_, s_, k_] := Sum[ Binomial[k, j] n^(-s (k - j)) cc[n - 1, s, j], {j, 0, k - 1}]
ds0[n_, k_] := Sum[ Binomial[k, j] cc[n - 1, 0, j], {j, 0, k - 1}]
```

```
dk[100, 1, 4]
```

47 170 088 538 039 346 024 993 687 016 198 208 885 341 961 652 897 437 934 688 066 906 484 013 393 825 301 ↘
423 546 153 070 666 526 044 644 602 689 597 753 123 388 367 /
8 472 647 847 886 561 981 082 043 136 480 502 696 466 773 828 864 323 778 666 520 172 493 782 146 353 512 ↘
253 887 868 783 096 147 331 569 582 517 869 260 800 000 000

```
dc[100, 1, 4]
```

47 170 088 538 039 346 024 993 687 016 198 208 885 341 961 652 897 437 934 688 066 906 484 013 393 825 301 ↘
423 546 153 070 666 526 044 644 602 689 597 753 123 388 367 /
8 472 647 847 886 561 981 082 043 136 480 502 696 466 773 828 864 323 778 666 520 172 493 782 146 353 512 ↘
253 887 868 783 096 147 331 569 582 517 869 260 800 000 000

```
ds0[100, 4]
```

3 940 399

```
dc[100, 0, 4]
```

3 940 399

```
dc0[n, 1]
```

$$-\frac{1}{6}(-1+n)\,n\,(-1+2n)+\frac{1}{6}n\,(1+n)\,(1+2n)$$

```
FullSimplify@(dc0[n, 5] - n^5)
```

$$-(-1+n)^5$$

```
Expand@ds0a[n, 5]
```

$$1-5n+10n^2-10n^3+5n^4$$

```
D[ds0z[n, z], z] /. z → 0
```

$$-\text{Log}[-1+n]+\text{Log}[n]$$

```
D[dsm1z[n, z], z] /. z → 0
```

$$-\text{Log}[(-1+n)\,n]+\text{Log}[n\,(1+n)]$$

```
D[dsm2z[n, z], z] /. z → 0
```

$$-\text{Log}[(-1+n)\,n\,(-1+2n)]+\text{Log}[n\,(1+n)\,(1+2n)]$$

```
FullSimplify[-(-1+n) n+n (1+n)]
```

$2n$

```
FullSimplify[-(-1+n)^2 n^2+n^2 (1+n)^2]
```

$4n^3$

```
FullSimplify[-(-1+n)^3 n^3+n^3 (1+n)^3]
```

$2\left(n^3+3n^5\right)$

```
-(-1+n)^3 n^3+n^3 (1+n)^3 /. n → 17
```

8 528 968

```
n^3 (-(-1+n)^3+(1+n)^3) /. n → 17
```

8 528 968

```
N[-Log[(-1+n) n]+Log[n (1+n)] /. n → 17]
```

0.117783

```
N[Log[(n (1+n)) / (n (n-1))] /. n → 17]
```

0.117783

```
N[Log[(n+1) / (n-1)] /. n → 17]
```

0.117783

```
N[-Log[-1 + n] + Log[n] /. n → 17]
```

0.0606246

```
N[Log[n / (n - 1)] /. n → 17]
```

0.0606246

```
N[(-Log[(-1 + n) n (-1 + 2 n)] + Log[n (1 + n) (1 + 2 n)]) /. n → 17]
```

0.176624

```
N[(Log[(n (1 + n) (1 + 2 n)) / ((-1 + n) n (-1 + 2 n))]) /. n → 17]
```

0.176624

```
N[(Log[( (1 + n) (1 + 2 n)) / ((-1 + n) n (-1 + 2 n))]) /. n → 17]
```

0.176624

```
N[(Log[( (1 + n) (1 + 2 n)) / ((-1 + n) (-1 + 2 n))]) /. n → 17]
```

0.176624

```
( (1 + n) (1 + 2 n)) / ((-1 + n) (-1 + 2 n))
```

$$\frac{(1+n)\,(1+2\,n)}{(-1+n)\,(-1+2\,n)}$$

```
Sum[ dsm1z[n, 1.3 - I], {n, 1, 100}]
```

-40 652.6 - 50 980.6 i

```
Chop@Sum[ dsm1z[j, -1.1 + 3 I] dsm1z[k, 1.3 - I + 1.1 - 3 I], {j, 1, 100}, {k, 1, 100}]
```

-40 652.6 - 50 980.6 i

```
dsm2z[100, 1.3 - I]
```

515 182. - 537 136. i

```
b[100, -2, 1.3 - I]
```

515 182. - 537 136. i

```
Chop@Sum[b[j, -1, -1.1 + 3 I] b[k, -1, 1.3 - I + 1.1 - 3 I], {j, 1, 100}, {k, 1, 100}]
```

-40 652.6 - 50 980.6 i

```
D[b[100, 0, z], z] /. z → 0
```

-Log[99] + Log[100]

```
b[100, 0, z]
```

$-99^z + 100^z$

```
Expand@Sum[ (D[b[j, 0, z], z] /. z → 0) (D[b[k, 0, r], r] /. r → 0), {j, 1, 100}, {k, 1, 100}]
```

$\text{Log}[100]^2$

```
bb[n_, z_] := Sum[ (z^k) / (k!) (Log[n]^k - Log[n - 1]^k), {k, 0, Infinity}]
```

```
bb[100, z]
```

$-99^z + 100^z$

```
D[b[100, 0, r], {r, 3}] /. r → 0
```

$-\text{Log}[99]^3 + \text{Log}[100]^3$

```
b[100, 0, z]
```

$$-99^z + 100^z$$

```
b[n, s, 1]
```

$$-\text{If}\left[-1+n == 0,\ 0,\ \left(\sum_{j=1}^{-1+n} j^{-s}\right)^1\right] + \text{If}\left[n == 0,\ 0,\ \left(\sum_{j=1}^{n} j^{-s}\right)^1\right]$$

```
FullSimplify[Sum[ j^-s, {j, 1, n}]^1 - Sum[ j^-s, {j, 1, n - 1}]^1]
```

$$n^{-s}$$

```
B[100, -1, 3]
```

128 787 625 000

```
B2z[100, -1, 3]
```

128 787 625 000

```
B[100, 1.5, .5]
```

1.55334

```
B2t[100, 1.5, .5, 10]
```

1.39236