```
Dhyp[n_, k_, a_] := Dhyp[n, k, a] =
  Sum[Binomial[k, j] Dhyp[Floor[n / (m^(k - j))], j, m + 1], {m, a, n^(1 / k)}, {j, 0, k - 1}]
Dhyp[n_, 1, a_] := If[n < a, 0, Floor[n] - a + 1]; Dhyp[n_, 0, a_] := 1
dh[n_, k_, a_] := a^-k Dhyp[n a^k, k, a + 1]
bins[z_, a_] := Product[(z - k), {k, 0, a - 1}] / a!
Dc[n_, z_, b_] := Expand[Sum[bins[z, a] dh[n, a, b], {a, 0, 30}]]
gg[n_, z_, b_] := Expand[FullSimplify[(Dc[n, z + 1, b] - 1) / (z + 1)]]


N[Dc[100, z, 2]]
```

$$1. + 28.6045\,z + 42.5362\,z^2 + 21.4569\,z^3 + 5.62896\,z^4 + 0.710483\,z^5 + 0.0606425\,z^6 + 0.00229676\,z^7 + 0.000065633\,z^8 + 9.94378\times10^{-7}\,z^9 + 4.97862\times10^{-9}\,z^{10} + 1.22325\times10^{-11}\,z^{11}$$

```
N[gg[100, z, 1]]
```

$$99. + 100.517\,z + 35.1889\,z^2 + 5.45694\,z^3 + 0.327778\,z^4 + 0.00972222\,z^5$$

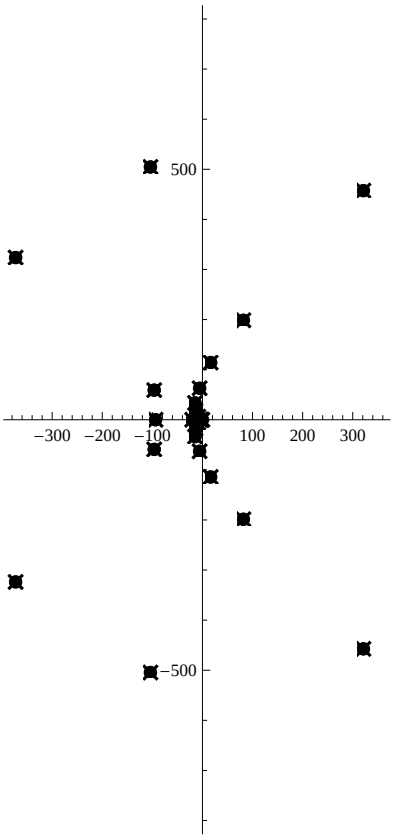
```
N[Dc[100, z, 6]]
```

$$1. + 28.2024\,z + 41.0485\,z^2 + 22.156\,z^3 + 6.36986\,z^4 + 1.08782\,z^5 + 0.124978\,z^6 + 0.00988025\,z^7 + 0.000579927\,z^8 + 0.0000251671\,z^9 + 8.52108\times10^{-7}\,z^{10} + 2.23652\times10^{-8}\,z^{11} + 4.67164\times10^{-10}\,z^{12} + 7.87744\times10^{-12}\,z^{13} + 1.06331\times10^{-13}\,z^{14} + 1.18335\times10^{-15}\,z^{15} + 1.05916\times10^{-17}\,z^{16} + 8.33157\times10^{-20}\,z^{17} + 4.49621\times10^{-22}\,z^{18} + 2.66349\times10^{-24}\,z^{19} + 1.01452\times10^{-26}\,z^{20} + 2.98173\times10^{-29}\,z^{21} + 7.56172\times10^{-32}\,z^{22} + 2.46312\times10^{-34}\,z^{23} + 4.11198\times10^{-37}\,z^{24} + 2.60536\times10^{-40}\,z^{25} + 2.95225\times10^{-43}\,z^{26} + 2.83712\times10^{-46}\,z^{27} + 1.94947\times10^{-49}\,z^{28} + 9.20865\times10^{-53}\,z^{29}$$

```
Table[{n, N[Roots[ Expand[Dc[n, x, 6]] == 0, x]]}, {n, 2, 16}] // TableForm
```

```
2     x == -41.1436 - 41.3085 i || x == -41.1436 + 41.3085 i || x == -6.24826 || x == -1.46448
3     x == -0.756967 || x == -111.626 - 19.2615 i || x == -111.626 + 19.2615 i || x == -18.9898 - 52.1172
4     x == -3.48734 || x == -0.585923 || x == -57.7458 - 26.6503 i || x == -57.7458 + 26.6503 i || x == -1
5     x == -403.314 || x == -0.446297 || x == -46.3139 - 281.284 i || x == -46.3139 + 281.284 i || x == -1
6     x == -43.0461 || x == -29.3354 || x == -0.372971 || x == -289.517 - 833.521 i || x == -289.517 + 83
7     x == -2.86963 || x == -0.329815 || x == -343.053 - 1052.15 i || x == -343.053 + 1052.15 i || x == -5
8     x == -855.761 || x == -45.7615 || x == -22.5823 || x == -2.64453 || x == -0.295273 || x == -9.84125
9     x == -58.854 || x == -12.0168 || x == -2.65441 || x == -0.263642 || x == -543.589 - 226.221 i || x =
10    x == -89.3305 || x == -4.16338 || x == -3.30637 || x == -0.238588 || x == -232.633 - 202.185 i || x
11    x == -456.762 || x == -92.4693 || x == -35.3893 || x == -2.30789 || x == -0.221275 || x == -334.626
12    x == -480.946 || x == -54.7808 || x == -19.2603 || x == -0.203318 || x == -516.457 - 352.808 i || x
13    x == -2.59165 || x == -0.190187 || x == -1318.33 - 1790.56 i || x == -1318.33 + 1790.56 i || x == -1
14    x == -610.415 || x == -71.5486 || x == -23.8455 || x == -2.47791 || x == -0.17924 || x == -564.72 -
15    x == -694.591 || x == -119.408 || x == -95.0493 || x == -2.33376 || x == -0.169764 || x == -366.951
16    x == -301.29 || x == -51.6073 || x == -23.066 || x == -2.00212 || x == -0.162876 || x == -285.003 -
```

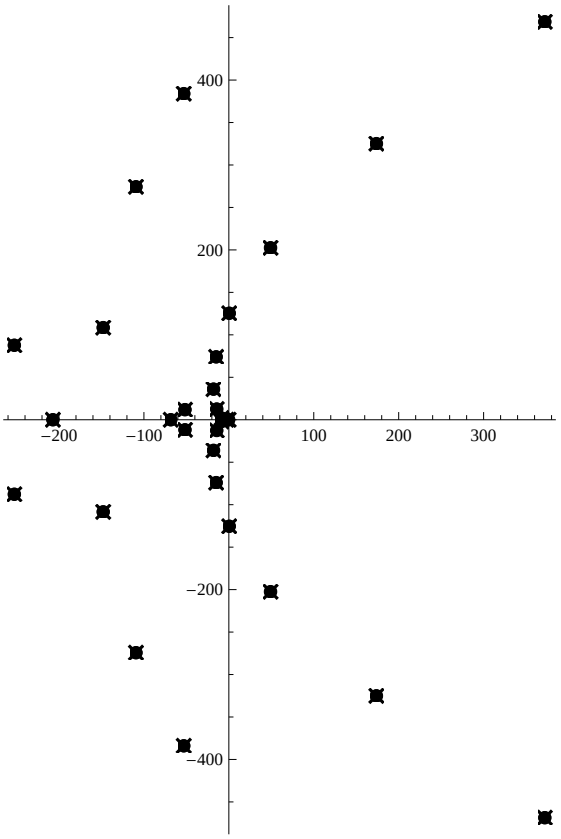
```
RootLocusPlot[1 / Expand[Dc[20, x, 10]], {k, 0, 1}, FeedbackType → None]
```




```
N[Dc[20, z, 20]]
```

$$1. + 8.30161\, z + 7.47988\, z^2 + 2.65334\, z^3 + 0.501508\, z^4 + 0.0587419\, z^5 + 0.00464604\, z^6 + 0.000263783\, z^7 + 0.0000112158\, z^8 + 3.68743 \times 10^{-7}\, z^9 + 9.62241 \times 10^{-9}\, z^{10} + 2.03064 \times 10^{-10}\, z^{11} + 3.52862 \times 10^{-12}\, z^{12} + 5.11048 \times 10^{-14}\, z^{13} + 6.24844 \times 10^{-16}\, z^{14} + 6.50933 \times 10^{-18}\, z^{15} + 5.82928 \times 10^{-20}\, z^{16} + 4.52016 \times 10^{-22}\, z^{17} + 3.05628 \times 10^{-24}\, z^{18} + 1.80951 \times 10^{-26}\, z^{19} + 9.44432 \times 10^{-29}\, z^{20} + 4.36614 \times 10^{-31}\, z^{21} + 1.78872 \times 10^{-33}\, z^{22} + 6.54569 \times 10^{-36}\, z^{23} + 2.1361 \times 10^{-38}\, z^{24} + 6.29146 \times 10^{-41}\, z^{25} + 1.67211 \times 10^{-43}\, z^{26} + 3.81266 \times 10^{-46}\, z^{27} + 9.4746 \times 10^{-49}\, z^{28} + 8.55293 \times 10^{-52}\, z^{29} + 6.02983 \times 10^{-54}\, z^{30}$$

```
RootLocusPlot[1 / Expand[Dc[10, x, 22]], {k, 0, 1}, FeedbackType → None]
```



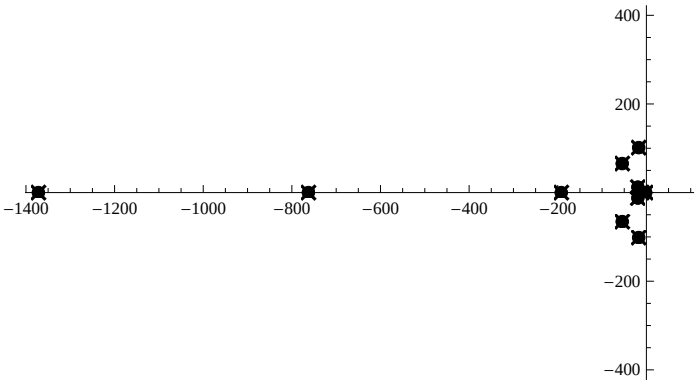

```
(-1 / List @@ NRoots[ Dc[10, x, 22] == 0, x][[All, 2]])
```

```
{0.00353273 - 0.00122686 i, 0.00353273 + 0.00122686 i, 0.00482654, 0.00439654 - 0.00322286 i,
 0.00439654 + 0.00322286 i, 0.00125389 - 0.00314563 i, 0.00125389 + 0.00314563 i, 0.0146241,
 0.000353284 - 0.00255564 i, 0.000353284 + 0.00255564 i, 0.0183872 - 0.00422387 i,
 0.0183872 + 0.00422387 i, 0.0111077 - 0.0221723 i, 0.0111077 + 0.0221723 i,
 0.0026152 - 0.012975 i, 0.0026152 + 0.012975 i, 0.0393803 - 0.0347825 i,
 0.0393803 + 0.0347825 i, 0.111253 - 0.0261541 i, 0.111253 + 0.0261541 i,
 0.344096, 4.05819, -0.000029247 - 0.00797395 i, -0.000029247 + 0.00797395 i,
 -0.00113104 - 0.00466471 i, -0.00113104 + 0.00466471 i, -0.00127865 - 0.00239264 i,
 -0.00127865 + 0.00239264 i, -0.00103925 - 0.00130892 i, -0.00103925 + 0.00130892 i}
```

```
N[LogIntegral[10] - Log[Log[10]] - EulerGamma]
```

```
4.75435
```

```
RootLocusPlot[1 / Expand[Dc[2, x, 22]], {k, 0, 1}, FeedbackType → None]
```




```
RootLocusPlot[1 / Expand[Dc[3, x, 22]], {k, 0, 1}, FeedbackType → None]
```

```
RootLocusPlot[1 / Expand[Dc[4, x, 22]], {k, 0, 1}, FeedbackType → None]
```

```
RootLocusPlot[1 / Expand[Dc[5, x, 22]], {k, 0, 1}, FeedbackType → None]
```

```
RootLocusPlot[1 / Expand[Dc[6, x, 22]], {k, 0, 1}, FeedbackType → None]
```

```
RootLocusPlot[1 / Expand[Dc[7, x, 22]], {k, 0, 1}, FeedbackType → None]
```

```
RootLocusPlot[1 / Expand[gg[10, x, 22]], {k, 0, 1}, FeedbackType → None]
```