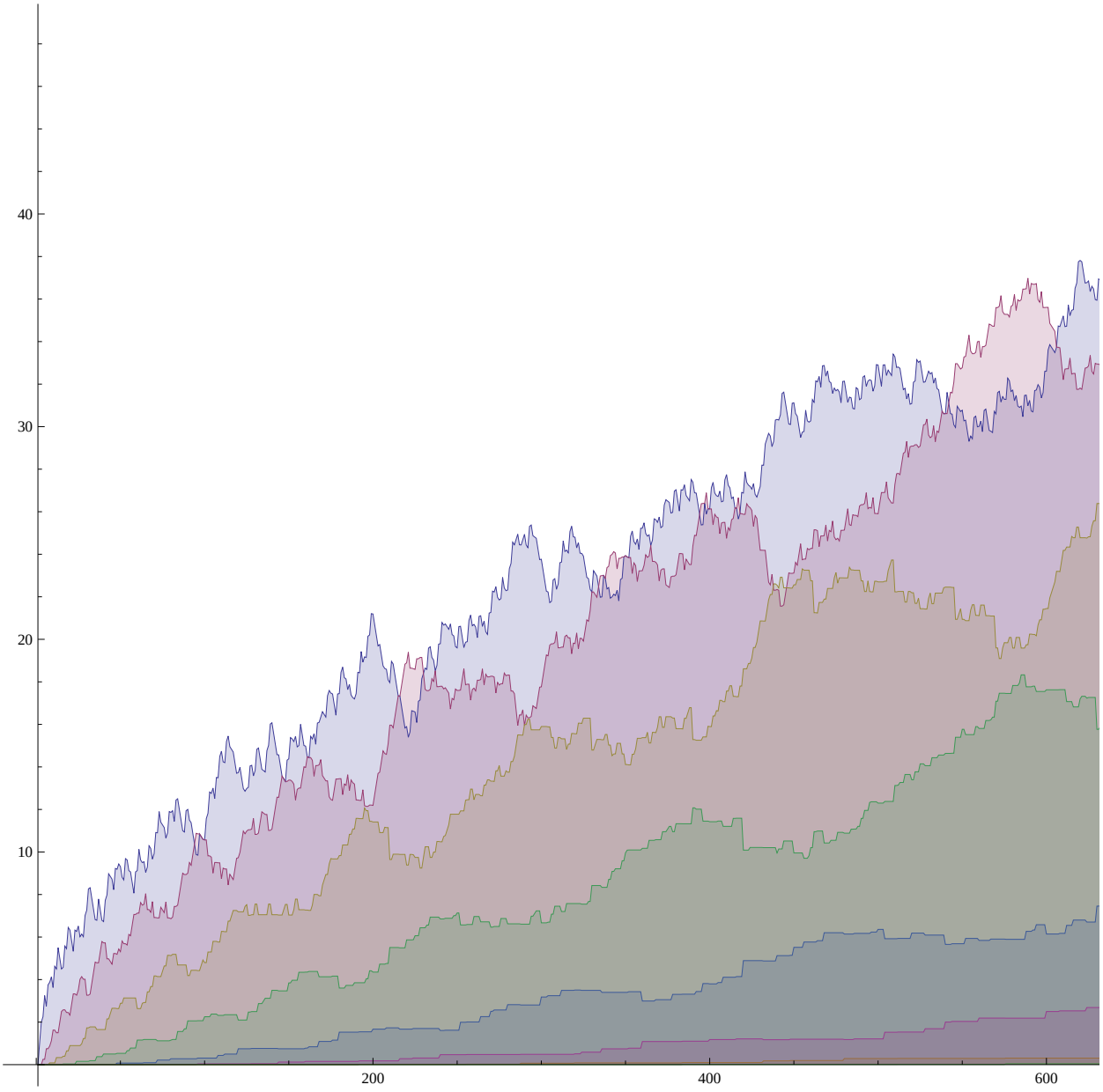
```
ClearAll["Global`*"]
```

```
d2[n_, k_] := d2[n, k] = Sum[d2[j, k - 1] d2[n / j, 1], {j, Divisors[n]}];
d2[n_, 1] := 1; d2[1, 1] := 0; d2[n_, 0] := 0; d2[1, 0] := 1
D2[n_, k_] := D2[n, k] = Sum[d2[j, k], {j, 2, n}]
d[n_, z_] := d[n, z] = Product[1 / (p[[2]]!) Pochhammer[z, p[[2]]], {p, FI[n]}];
FI[n_] := If[n == 1, {}, FactorInteger[n]]
DD[n_, k_] := DD[n, k] = Sum[d[j, k], {j, 1, n}]
Li[n_, a_, k_] :=
 Re[Li[n, a, k] = (-1)^(k + 1) / k Sum[(-1)^(k - j) Binomial[k, j] DD[n, a j], {j, 0, k}] / a]
Li2[n_, a_] := Sum[ Li[n, a, k], {k, 1, Log[2, n]}]

DiscretePlot[{ Li[n, ss = -.5, 1], Li[n, ss, 2], Li[n, ss, 3], Li[n, ss, 4],
  Li[n, ss, 5], Li[n, ss, 6], Li[n, ss, 7], Li[n, ss, 8]}, {n, 1, 1000}]
```



```
Li2[100, -I]
```

$$\frac{428}{15}$$

```
Li[100, I, 1]
```

$$\frac{65}{8} + \frac{2953\,i}{72}$$

```
Li[100, 1, 1]
```

$$99$$

```
Li[100, -1, 1]
```

$$0$$

```
Li[100, -I, 1]
```

$$\frac{65}{8} - \frac{2953\,i}{72}$$

```
Table[{n, d[n, -1 / 2]}, {n, 1, 100}] // TableForm
```

1	1
2	$-\frac{1}{2}$
3	$-\frac{1}{2}$
4	$-\frac{1}{8}$
5	$-\frac{1}{2}$
6	$\frac{1}{4}$
7	$-\frac{1}{2}$
8	$-\frac{1}{16}$
9	$-\frac{1}{8}$
10	$\frac{1}{4}$
11	$-\frac{1}{2}$
12	$\frac{1}{16}$
13	$-\frac{1}{2}$
14	$\frac{1}{4}$
15	$\frac{1}{4}$
16	$-\frac{5}{128}$
17	$-\frac{1}{2}$
18	$\frac{1}{16}$
19	$-\frac{1}{2}$
20	$\frac{1}{16}$
21	$\frac{1}{4}$
22	$\frac{1}{4}$
23	$-\frac{1}{2}$
24	$\frac{1}{32}$
25	$-\frac{1}{8}$

26 $\frac{1}{4}$

27 $-\frac{1}{16}$

28 $\frac{1}{16}$

29 $-\frac{1}{2}$

30 $-\frac{1}{8}$

31 $-\frac{1}{2}$

32 $-\frac{7}{256}$

33 $\frac{1}{4}$

34 $\frac{1}{4}$

35 $\frac{1}{4}$

36 $\frac{1}{64}$

37 $-\frac{1}{2}$

38 $\frac{1}{4}$

39 $\frac{1}{4}$

40 $\frac{1}{32}$

41 $-\frac{1}{2}$

42 $-\frac{1}{8}$

43 $-\frac{1}{2}$

44 $\frac{1}{16}$

45 $\frac{1}{16}$

46 $\frac{1}{4}$

47 $-\frac{1}{2}$

48 $\frac{5}{256}$

49 $-\frac{1}{8}$

50 $\frac{1}{16}$

51 $\frac{1}{4}$

52 $\frac{1}{16}$

53 $-\frac{1}{2}$

54 $\frac{1}{32}$

55 $\frac{1}{4}$

56 $\frac{1}{32}$

57 $\frac{1}{4}$

58 $\frac{1}{4}$

59 $-\frac{1}{2}$

60 $-\frac{1}{32}$

61 $-\frac{1}{2}$

62 $\frac{1}{4}$

63 $\frac{1}{16}$

64 $-\frac{21}{1024}$

65 $\frac{1}{4}$

66 $-\frac{1}{8}$

67 $-\frac{1}{2}$

68 $\frac{1}{16}$

69 $\frac{1}{4}$

70 $-\frac{1}{8}$

71 $-\frac{1}{2}$

72 $\frac{1}{128}$

73 $-\frac{1}{2}$

74 $\frac{1}{4}$

75 $\frac{1}{16}$

76 $\frac{1}{16}$

77 $\frac{1}{4}$

78 $-\frac{1}{8}$

79 $-\frac{1}{2}$

80 $\frac{5}{256}$

81 $-\frac{5}{128}$

82 $\frac{1}{4}$

83 $-\frac{1}{2}$

84 $-\frac{1}{32}$

85 $\frac{1}{4}$

86 $\frac{1}{4}$

87 $\frac{1}{4}$

88 $\frac{1}{32}$

89 $-\frac{1}{2}$

90 $-\frac{1}{32}$

91 $\frac{1}{4}$

92 $\frac{1}{16}$

93 $\frac{1}{4}$

94 $\frac{1}{4}$

95 $\frac{1}{4}$

96 $\frac{7}{512}$

97 $-\frac{1}{2}$

98 $\frac{1}{16}$

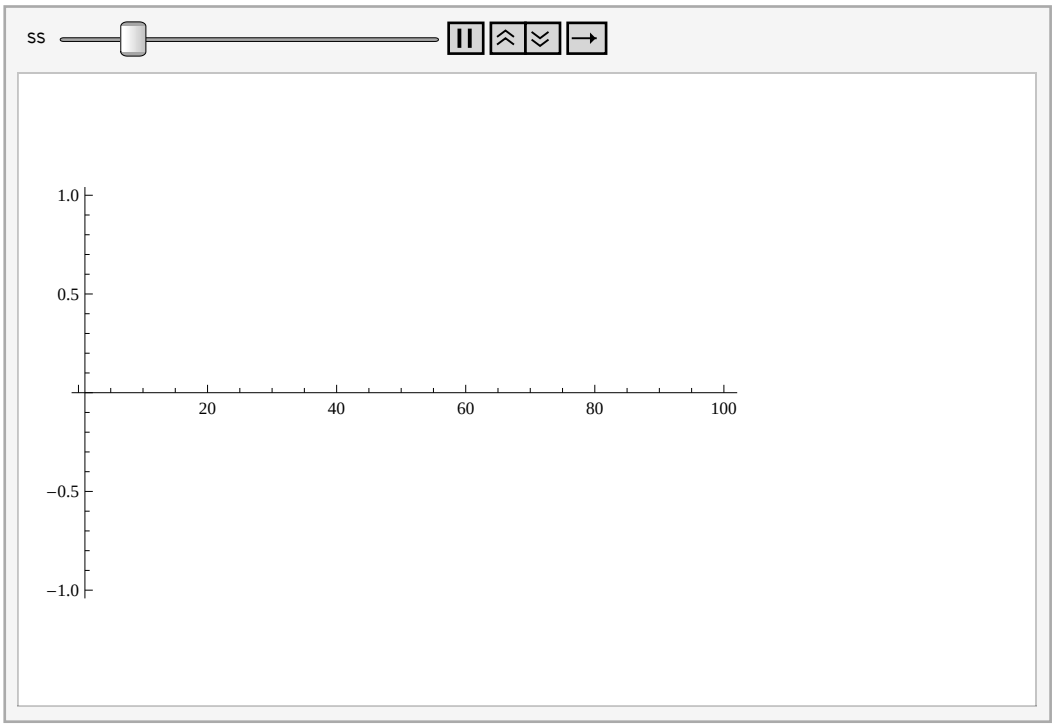
99 $\frac{1}{16}$

100 $\frac{1}{64}$

```
Li[1000, 1 / 2, 2]
```

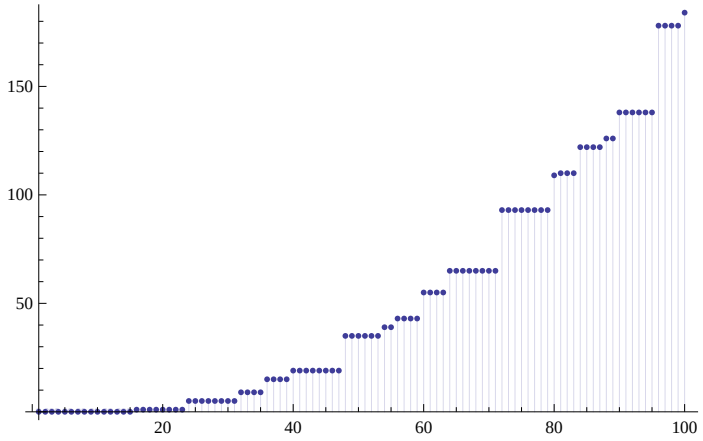
$$-\frac{4\,431\,721}{8192}$$

```
Animate[DiscretePlot[{ Li[n, ss, 1]}, {n, 1, 100}], {ss, -2, 2}]
```




```
Lia[n_, a_, k_] := Re[Lia[n, a, k] = Sum[(-1)^(k - j) Binomial[k, j] DD[n, a j], {j, 0, k}]]
```

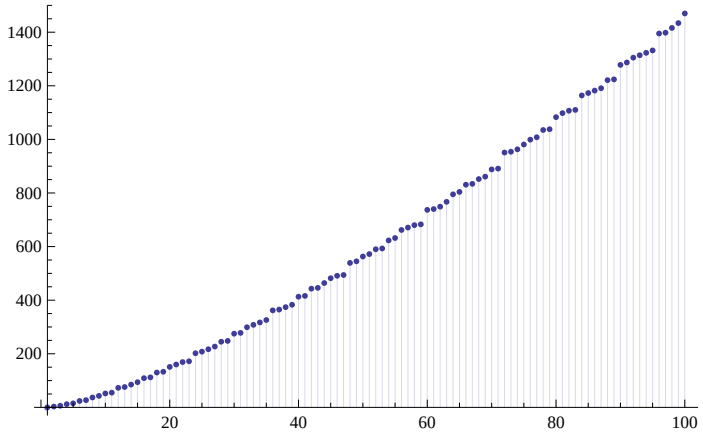
```
DiscretePlot[Lia[n, 1, 4], {n, 1, 100}]
```




```
Sum[ d[j, 1 / 2] DD[Floor[1000 / j], 1 / 2], {j, 1, 1000}]
```

```
1000
```

```
DiscretePlot[Re[Lia[n, 3 , 1]], {n, 1, 100}]
```




```
Series[(x^3 + 1)^3, {x, 0, 20}]
```

$$1 + 3\,x^3 + 3\,x^6 + x^9 + O[x]^{21}$$

```
DX[n_, z_] := Sum[FactorialPower[z, a] / a ! Da[n, a], {a, 0, Log[2, n]}]
Lia2[n_, a_, k_] := Lia[n, a, k] = Sum[(-1)^(k - j) Binomial[k, j] DX[n, a j], {j, 0, k}]
Table[ {a, Expand[Lia2[10 000, ss = 2, a]]}, {a, 1, 16}] // TableForm
```

1	2 Da[10 000, 1] + Da[10 000, 2]
2	4 Da[10 000, 2] + 4 Da[10 000, 3] + Da[10 000, 4]
3	8 Da[10 000, 3] + 12 Da[10 000, 4] + 6 Da[10 000, 5] + Da[10 000, 6]
4	16 Da[10 000, 4] + 32 Da[10 000, 5] + 24 Da[10 000, 6] + 8 Da[10 000, 7] + Da[10 000, 8]
5	32 Da[10 000, 5] + 80 Da[10 000, 6] + 80 Da[10 000, 7] + 40 Da[10 000, 8] + 10 Da[10 000, 9] + Da[1(
6	64 Da[10 000, 6] + 192 Da[10 000, 7] + 240 Da[10 000, 8] + 160 Da[10 000, 9] + 60 Da[10 000, 10] + 1
7	128 Da[10 000, 7] + 448 Da[10 000, 8] + 672 Da[10 000, 9] + 560 Da[10 000, 10] + 280 Da[10 000, 11
8	256 Da[10 000, 8] + 1024 Da[10 000, 9] + 1792 Da[10 000, 10] + 1792 Da[10 000, 11] + 1120 Da[10 0(
9	512 Da[10 000, 9] + 2304 Da[10 000, 10] + 4608 Da[10 000, 11] + 5376 Da[10 000, 12] + 4032 Da[10 (
10	1024 Da[10 000, 10] + 5120 Da[10 000, 11] + 11 520 Da[10 000, 12] + 15 360 Da[10 000, 13]
11	2048 Da[10 000, 11] + 11 264 Da[10 000, 12] + 28 160 Da[10 000, 13]
12	4096 Da[10 000, 12] + 24 576 Da[10 000, 13]
13	8192 Da[10 000, 13]
14	0
15	0
16	0

```
Expand[Liax[10 000, 2, 3]]
```

$$8\,x^3 + 12\,x^4 + 6\,x^5 + x^6$$

```
DX[n_, z_] := Sum[FactorialPower[z, a] / a ! Da[n, a], {a, 0, Log[2, n]}]
Lia2[n_, a_, k_] := Lia[n, a, k] = Sum[(-1)^(k - j) Binomial[k, j] DX[n, a j], {j, 0, k}]
Sum[ Expand[(-1)^(a + 1) / a Lia2[10 000, ss = 4, a] / ss], {a, 1, 16}]
```

$$\text{Da}[10\,000, 1] - \frac{1}{2}\text{Da}[10\,000, 2] + \frac{1}{3}\text{Da}[10\,000, 3] - \frac{1}{4}\text{Da}[10\,000, 4] + \frac{1}{5}\text{Da}[10\,000, 5] - \frac{1}{6}\text{Da}[10\,000, 6] + \frac{1}{7}\text{Da}[10\,000, 7] - \frac{1}{8}\text{Da}[10\,000, 8] + \frac{1}{9}\text{Da}[10\,000, 9] - \frac{1}{10}\text{Da}[10\,000, 10] + \frac{1}{11}\text{Da}[10\,000, 11] - \frac{1}{12}\text{Da}[10\,000, 12] + \frac{1}{13}\text{Da}[10\,000, 13]$$

Liax

Liax

```
Liay[n_, a_, k_] := Expand[Sum[(-1)^(k - j) Binomial[k, j] (x + 1)^(a j), {j, 0, k}]]
```

```
Liay[10 000, 2, 6]
```

$$64\,x^6 + 192\,x^7 + 240\,x^8 + 160\,x^9 + 60\,x^{10} + 12\,x^{11} + x^{12}$$

d

```
Series[Log[x^2], {x, 0, 30}]
```

$$2\,\text{Log}[x] + O[x]^{31}$$

```
Sum[ Expand[(-1)^(a) Lia2[10 000, ss = 3, a]], {a, 1, 16}]
```

$$-3\,\text{Da}[10\,000, 1] + 6\,\text{Da}[10\,000, 2] - 10\,\text{Da}[10\,000, 3] + 15\,\text{Da}[10\,000, 4] - 21\,\text{Da}[10\,000, 5] + 28\,\text{Da}[10\,000, 6] - 36\,\text{Da}[10\,000, 7] + 45\,\text{Da}[10\,000, 8] - 55\,\text{Da}[10\,000, 9] + 66\,\text{Da}[10\,000, 10] - 78\,\text{Da}[10\,000, 11] + 91\,\text{Da}[10\,000, 12] - 105\,\text{Da}[10\,000, 13]$$