```
Limit[(a - 1) Sum[ a^k , {k, Log[a, n], Infinity}], a → 1]
Integrate[ n^s Log[n], {s, 1, Infinity}]
```

$-n$

ConditionalExpression$[-n, \text{Re}[\text{Log}[n]] < 0]$

```
Limit[(a - 1) Sum[ a^-k , {k, Log[a, n], Infinity}], a → 1]
Integrate[ n^-s Log[n], {s, 1, Infinity}]
```

$\frac{1}{n}$

ConditionalExpression$\left[\frac{1}{n}, \text{Re}[\text{Log}[n]] > 0\right]$

```
Limit[(a - 1) Sum[ a^(2 k) , {k, Log[a, n], Infinity}], a → 1]
Integrate[ n^(2 s) Log[n], {s, 1, Infinity}]
```

$-\frac{n^2}{2}$

ConditionalExpression$\left[-\frac{n^2}{2}, \text{Re}[\text{Log}[n]] < 0\right]$

```
Limit[(a - 1) Sum[ a^(3 k) , {k, Log[a, n], Infinity}], a → 1]
Integrate[ n^(3 s) Log[n], {s, 1, Infinity}]
```

$-\frac{n^3}{3}$

ConditionalExpression$\left[-\frac{n^3}{3}, \text{Re}[\text{Log}[n]] < 0\right]$

```
Limit[(a - 1) Sum[ a^(c k) , {k, Log[a, n], Infinity}], a → 1]
Integrate[ n^(c s) Log[n], {s, 1, Infinity}]
```

$-\frac{n^c}{c}$

ConditionalExpression$\left[-\frac{n^c}{c}, \text{Re}[c\,\text{Log}[n]] < 0\right]$

$\frac{-1 + n^c}{c}$ /. c → 2

$\frac{1}{2}\left(-1 + n^2\right)$

```
Limit[ (a - 1) Sum[ 1, {k, Log[a, n], Infinity}], a → 1]
Integrate[ Log[n], {s, 1, Infinity}]
```

Sum::div : Sum does not converge. ≫

Sum::div : Sum does not converge. ≫

Sum::div : Sum does not converge. ≫

General::stop : Further output of Sum::div will be suppressed during this calculation. ≫

$$\text{Limit}\left[(-1+a)\sum_{k=\frac{\text{Log}[n]}{\text{Log}[a]}}^{\infty} 1,\ a\to 1\right]$$

$\infty\ \text{Log}[n]$

```
Limit[ (a - 1) ^2 Sum[k , {k, Log[a, n], Infinity}], a → 1]
Integrate[ s Log[n] ^2, {s, 1, Infinity}]
```

Sum::div : Sum does not converge. ≫

Sum::div : Sum does not converge. ≫

Sum::div : Sum does not converge. ≫

General::stop : Further output of Sum::div will be suppressed during this calculation. ≫

$$\text{Limit}\left[(-1+a)^2\sum_{k=\frac{\text{Log}[n]}{\text{Log}[a]}}^{\infty} k,\ a\to 1\right]$$

Integrate::idiv : Integral of s does not converge on {1, ∞}. ≫

$$\int_1^{\infty} s\ \text{Log}[n]^2\, ds$$

```
Limit[ (a - 1) Sum[ a^k, {k, Log[a, n], Infinity}], a → 1]
Integrate[ n^s Log[n], {s, 1, Infinity}]
```

$-n$

ConditionalExpression[-n, Re[Log[n]] < 0]

```
Limit[ (a - 1) ^2 Sum[k a^k , {k, Log[a, n], Infinity}], a → 1]
Integrate[ n^s s Log[n] ^2, {s, 1, Infinity}]
```

$n - n\ \text{Log}[n]$

ConditionalExpression[n - n Log[n], Re[Log[n]] < 0]

```
Limit[ (a - 1) ^3 Sum[k^2 a^k , {k, Log[a, n], Infinity}], a → 1]
Integrate[ n^s s^2 Log[n] ^3, {s, 1, Infinity}]
```

$-n\left(2 - 2\,\text{Log}[n] + \text{Log}[n]^2\right)$

ConditionalExpression[-n (2 + (-2 + Log[n]) Log[n]), Re[Log[n]] < 0]

```
Limit[ (a - 1) ^4 Sum[k^3 a^k , {k, Log[a, n], Infinity}], a → 1]
Expand[Integrate[ n^s s^3 Log[n] ^4, {s, 1, Infinity}]]
```

$$-n\left(-6+6\,\text{Log}[n]-3\,\text{Log}[n]^2+\text{Log}[n]^3\right)$$

$$\text{ConditionalExpression}\left[6\,n-6\,n\,\text{Log}[n]+3\,n\,\text{Log}[n]^2-n\,\text{Log}[n]^3,\ \text{Re}[\text{Log}[n]]<0\right]$$

```
Limit[ (a - 1) ^5 Sum[k^4 a^k , {k, Log[a, n], Infinity}], a → 1]
Expand[Integrate[ n^s s^4 Log[n] ^5, {s, 1, Infinity}]]
```

$$-n\left(24-24\,\text{Log}[n]+12\,\text{Log}[n]^2-4\,\text{Log}[n]^3+\text{Log}[n]^4\right)$$

$$\text{ConditionalExpression}\left[-24\,n+24\,n\,\text{Log}[n]-12\,n\,\text{Log}[n]^2+4\,n\,\text{Log}[n]^3-n\,\text{Log}[n]^4,\ \text{Re}[\text{Log}[n]]<0\right]$$

```
Limit[ (a - 1) ^m Sum[k^ (m - 1) a^k , {k, Log[a, n], Infinity}], a → 1]
Expand[Integrate[ n^s s^ (m - 1) Log[n] ^m, {s, 1, Infinity}]]
```

$$\text{Limit}\left[(-1+a)^m\,n\,\text{HurwitzLerchPhi}\left[a,\ 1-m,\ \frac{\text{Log}[n]}{\text{Log}[a]}\right],\ a\to 1\right]$$

$$\text{ConditionalExpression}[\text{ExpIntegralE}[1-m,\ -\text{Log}[n]]\,\text{Log}[n]^m,\ \text{Re}[\text{Log}[n]]<0]$$

```
Limit[ (a - 1) ^m Sum[k^ (m - 1) a^-k , {k, Log[a, n], Infinity}], a → 1]
Expand[Integrate[ n^-s s^ (m - 1) Log[n] ^m, {s, 1, Infinity}]]
```

$$\text{Limit}\left[\frac{(-1+a)^m\,\text{HurwitzLerchPhi}\left[\frac{1}{a},\ 1-m,\ \frac{\text{Log}[n]}{\text{Log}[a]}\right]}{n},\ a\to 1\right]$$

$$\text{ConditionalExpression}[\text{ExpIntegralE}[1-m,\ \text{Log}[n]]\,\text{Log}[n]^m,\ \text{Re}[\text{Log}[n]]>0]$$

```
Limit[ (a - 1) ^m Sum[k^ (m - 1) a^-k , {k, Log[a, n], Infinity}] /. m → 4, a → 1]
Limit[ (a - 1) ^m Sum[k^ (m - 1) a^-k , {k, 0, Log[a, n]}] /. m → 4, a → 1]
Limit[ (a - 1) ^m Sum[k^ (m - 1) a^-k , {k, 0, Infinity}] /. m → 4, a → 1]
```

$$\frac{6+6\,\text{Log}[n]+3\,\text{Log}[n]^2+\text{Log}[n]^3}{n}$$

$$-\frac{6-6\,n+6\,\text{Log}[n]+3\,\text{Log}[n]^2+\text{Log}[n]^3}{n}$$

$$6$$

```
Limit[ (a - 1) ^2 Sum[k a^ (2 k) , {k, Log[a, n], Infinity}], a → 1]
Integrate[ n^ (2 s) s Log[n] ^2, {s, 1, Infinity}]
```

$$-\frac{1}{4}\,n^2\,(-1+2\,\text{Log}[n])$$

$$\text{ConditionalExpression}\left[\frac{1}{4}\,n^2\,(1-2\,\text{Log}[n]),\ \text{Re}[\text{Log}[n]]<0\right]$$

```
Limit[ (a - 1) ^0 Sum[k^-1 a^k , {k, Log[a, n], Infinity}], a → 1]
```

$$\text{Limit}\left[n\ \text{HurwitzLerchPhi}\left[a, 1, \frac{\text{Log}[n]}{\text{Log}[a]}\right], a \to 1\right]$$

```
Limit[ (a - 1) ^ (4) Sum[k^ ((4) - 1) a^k , {k, Log[a, n], Infinity}], a → 1]
Expand[Integrate[ n^s s^ (4 - 1) Log[n] ^4, {s, 1, Infinity}]]
```

$$-n\left(-6 + 6\,\text{Log}[n] - 3\,\text{Log}[n]^2 + \text{Log}[n]^3\right)$$

$$\text{ConditionalExpression}\left[6\,n - 6\,n\,\text{Log}[n] + 3\,n\,\text{Log}[n]^2 - n\,\text{Log}[n]^3, \text{Re}[\text{Log}[n]] < 0\right]$$

```
Limit[Integrate[ s^ (a - 1) Log[n] ^a, {s, 1, Infinity}], a → 2]
```

```
Undefined
```

```
Limit[Integrate[ n^s s^ (a - 1) Log[n] ^a, {s, 1, Infinity}], a → 2]
```

```
ConditionalExpression[Gamma[2, -Log[n]], Re[Log[n]] < 0]
```

```
Limit[Integrate[ n^s s^ (a - 1) Log[n] ^a, {s, 0, Infinity}], a → c]
```

$$\text{ConditionalExpression}\left[\text{Gamma}[c]\,(-\text{Log}[n])^{-c}\,\text{Log}[n]^c, \text{Re}[\text{Log}[n]] < 0\ \&\&\ \text{Re}[c] \geq 0\right]$$

```
Limit[Integrate[ n^s s^ (a - 1) Log[n] ^a, {s, 1, Infinity}], a → 4]
```

```
ConditionalExpression[Gamma[4, -Log[n]], Re[Log[n]] < 0]
```

```
Limit[Integrate[ n^s s^ (a - 1) Log[n] ^a, {s, 1, Infinity}], a → 1]
```

```
ConditionalExpression[-n, Re[Log[n]] < 0]
```

```
Integrate[ n^s s^ (a - 1) Log[n] ^a, {s, 1, Infinity}]
```

$$\text{ConditionalExpression}\left[\text{ExpIntegralE}[1 - a, -\text{Log}[n]]\,\text{Log}[n]^a, \text{Re}[\text{Log}[n]] < 0\right]$$

```
Integrate[ n^s s^ (a - 1) Log[n] ^a, {s, 0, Infinity}]
```

$$\text{ConditionalExpression}\left[\text{Gamma}[a]\,(-\text{Log}[n])^{-a}\,\text{Log}[n]^a, \text{Re}[\text{Log}[n]] < 0\ \&\&\ \text{Re}[a] > 0\right]$$

```
Limit[Integrate[ n^s s^ (a - 1) Log[n] ^a, {s, 1, Infinity}], a → c]
```

$$\text{ConditionalExpression}\left[\text{Gamma}[c, -\text{Log}[n]]\,(-\text{Log}[n])^{-c}\,\text{Log}[n]^c, \text{Re}[\text{Log}[n]] < 0\right]$$

```
N[Integrate[ 100^s s^(7/4 - 1) Log[100]^(7/4), {s, 1, Infinity}]]
```

Integrate::idiv : Integral of $100^s s^{3/4}$ does not converge on $\{1, \infty\}$. ≫

NIntegrate::inumri :

The integrand $100^s s^{3/4} \text{Log}[100]^{7/4}$ has evaluated to Overflow, Indeterminate, or Infinity for all sampling points in the region with boundaries $\{\{1., 4.64782 \times 10^{14}\}\}$. ≫

NIntegrate::inumri :

The integrand $100^s s^{3/4} \text{Log}[100]^{7/4}$ has evaluated to Overflow, Indeterminate, or Infinity for all sampling points in the region with boundaries $\{\{1., 4.64782 \times 10^{14}\}\}$. ≫

NIntegrate::inumri :

The integrand $100^s s^{3/4} \text{Log}[100]^{7/4}$ has evaluated to Overflow, Indeterminate, or Infinity for all sampling points in the region with boundaries $\{\{1., 4.64782 \times 10^{14}\}\}$. ≫

General::stop : Further output of NIntegrate::inumri will be suppressed during this calculation. ≫

$\text{NIntegrate}\left[100^s s^{3/4} \text{Log}[100]^{7/4}, \{s, 1, \infty\}\right]$

```
Abs[N[Gamma[7/4, 0, -Log[100]]]]
```

259.651

```
Integrate[ Log[1/t]^(k - 1), {t, 1, Infinity}]
```

Integrate::idiv : Integral of $(-\text{Log}[t])^{-1+k}$ does not converge on $\{1, \infty\}$. ≫

$$\int_1^\infty \text{Log}\left[\frac{1}{t}\right]^{-1+k} dt$$