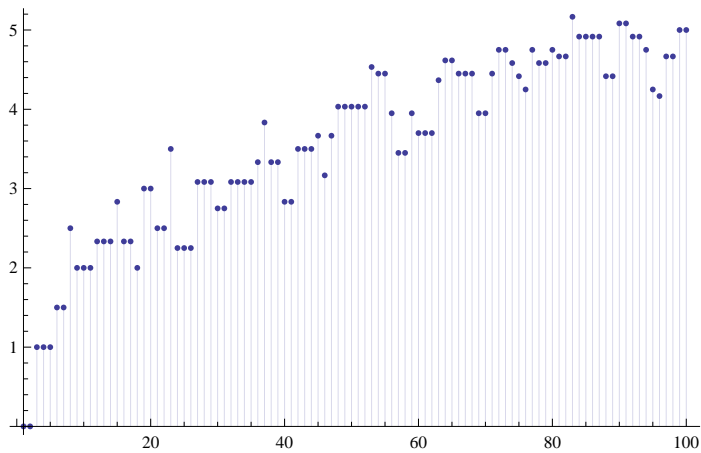```
sq[n_, k_] := Floor[n^k] - Floor[(n - 1) ^k]

Table[sq[n, 2 / 3], {n, 1, 30}]

{1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0}

Clear[dd, aa]
dd[n_, t_, k_] := dd[n, t, k] = Sum[dd[n / (j^(1 / t)), t, k - 1], {j, 2, n^t}]
dd[n_, t_, 0] := UnitStep[n - 1]
ld[n_, t_] := Sum[(-1) ^ (k + 1) / k dd[n, t, k], {k, 1, Log2@n}]
aa[n_, t_, k_] :=
 aa[n, t, k] = Sum[aa[Floor[n / j], t, k - 1], {j, 1, Floor[n^t]}] - aa[n, t, k - 1]
aa[n_, t_, 0] := UnitStep[n - 1]
la[n_, t_] := Sum[(-1) ^ (k + 1) / k aa[n, t, k], {k, 1, Log2@n}]

DiscretePlot[la[n, 2 / 3], {n, 1, 100}]
```
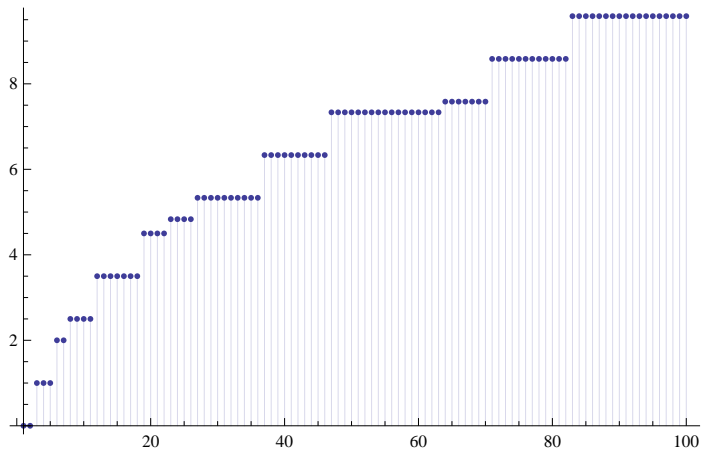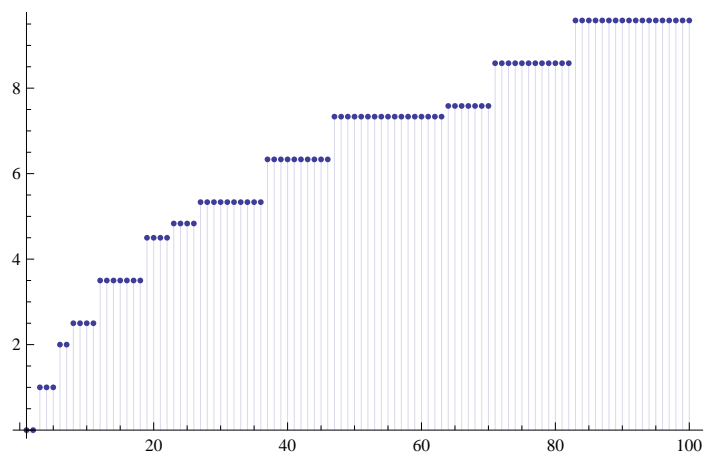


```
DiscretePlot[{ld[n, 2 / 3]}, {n, 1, 100}]
```

DiscretePlot[{ld[Floor[n^(2/3)], 1]}, {n, 1, 100}]

dd[100, 2/3, 2]

27

Sum[1, {j, 2, Floor[100^(2/3)]}, {k, 2, Floor[100^(2/3)/j]}]

29

dd[100, 2/3, 2]

27

dd[Floor[100^(2/3)], 1, 2]

29

Floor[100^(2/3)]

21

Grid@Table[1, {j, 2, 21}, {k, 2, 21/j}]

```
1  1  1  1  1  1  1  1  1
1  1  1  1  1  1
1  1  1  1
1  1  1
1  1
1  1
1
1
1
```

```
16 * 6
```

96

```
sq[15, 2 / 3]
```

1

```
sq[6, 2 / 3]
```

1

```
Table[ sq[j, 2 / 3], {j, 1, 16}]
```

{1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0}


```
dd[16, 2 / 3, 1]
```

5

```
dd[21 / 3, 1, 1]
```

6

```
N[16 ^ (2 / 3)]
```

6.3496

```
Floor[100 ^ (2 / 3)] / 3
```

7

```
Floor[30 ^ (2 / 3)]
```

9

```
1 ^ (3 / 2) × 9 ^ (3 / 2)
```

27

```
Floor[(30) ^ (2 / 3) / 3]
```

3

```
N[3 ^ (3 / 2) × 3 ^ (3 / 2)]
```

27.

```
Sum[ 1, {j, 2, Floor[100 ^ (2 / 3)]}, {k, 2, (100 / j ^ (3 / 2)) ^ (2 / 3)}]
```

29

```
Clear[dd]
dd[n_, p_, q_, k_] := dd[n, p, q, k] = Sum[ dd[n / (j ^ (1 / p) j2 ^ (1 / q)), p, q, k - 1],
    {j, 1, Floor[n^p]}, {j2, 1, Floor[(n / j ^ (1 / p)) ^ q]}]
dd[n_, p_, q_, 0] := UnitStep[n - 1]
d2[n_, p_, q_, k_] := Sum[ (-1) ^ (k - j) Binomial[k, j] dd[n, p, q, j], {j, 0, k}]
ld[n_, p_, q_] :=
 Sum[ (-1) ^ (k + 1) / k d2[n, N@Log[p] / Log[n], N@Log[q] / Log[n], k], {k, 1, Log2@n}]
pr[n_] := Sum[ PrimePi[ n ^ (1 / k)] / k, {k, 1, Log2@n}]
```


```
ld[100, 70, 60]
```

$$\frac{212}{5}$$

```
pr[ 70] + pr[60]
```

$$\frac{212}{5}$$

```
dd[100, 1, 1, 1]
```

```
482
```

```
Sum[ 1, {j, 2, 100}, {k, 2, 100 / j}, {l, 2, 100 / (j k)}]
```

```
324
```

```
d2[100, 1, 1, 2]
```

```
2612
```

```
0.922549
```

```
Sum[ 1, {j, 1, 100}, {k, 1, (100 / j) ^ Log[100, 70]}]
```

```
388
```

```
Sum[ 1, {j, 1, 70}, {k, 1, (70 / j) ^ Log[70, 100]}]
```

```
388
```

```
Clear[dz]
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, z_] := dz[n, z] = Product[(-1) ^ p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}]
Dz[n_, z_] := Sum[dz[j, z], {j, 1, n}]
```

```
dd[100, 1, N@Log[70] / Log[100], 1]
```

```
387
```

```
Sum[ 1, {j, 1, 100^1}, {j2, 1, Floor[(100 / j^ (1 / 1)) ^ (N@Log[70] / Log[100])]}]
```

```
387
```

```
(* Recursive Form *)
Clear[dd, d2a]
dd[n_, q_, k_] :=
 dd[n, q, k] = Sum[ dd[n / (j j2^ (1 / q)), q, k - 1], {j, 1, Floor[n]}, {j2, 1, Floor[(n / j) ^ q]}]
dd[n_, q_, 0] := UnitStep[n - 1]
d2[n_, q_, k_] := Sum[ (-1) ^ (k - j) Binomial[k, j] dd[n, q, j], {j, 0, k}]
d2a[n_, q_, k_] := d2a[n, q, k] = Sum[d2a[n / (j j2^ (1 / q)), q, k - 1],
     {j, 1, Floor[n]}, {j2, 1, Floor[(n / j) ^ q]}] - d2a[n, q, k - 1]
d2a[n_, q_, 0] := UnitStep[n - 1]
ld[n_, q_] := Sum[ (-1) ^ (k + 1) / k d2a[n, N@Log[q] / Log[n], k], {k, 1, Log2@n}]
pr[n_] := Sum[ PrimePi[ n^ (1 / k)] / k, {k, 1, Log2@n}]
```

```
ld[100, 70]
```

$$\frac{3049}{60}$$

```
pr[100] + pr[70]
```

$\dfrac{3049}{60}$

```
Sum[1, {a, 1, 100}, {b, 1, Floor[100 / a]}, {c, 1, Floor[(100 / (a b)) ^ (N@Log[100, 70])]},
 {d, 1, Floor[(100 / (a b c ^ (N@Log[70, 100]))) ^ N@ (Log[100, 70])]}]
```

2702

```
dd[100, N@Log[100, 70], 2]
```

2699

```
Sum[1, {a, 1, 100}, {c, 1, Floor[(100 / (a )) ^ (N@Log[100, 70])]}]
```

387

```
Sum[1, {a, 1, Floor[100]}, {c, 1, Floor[(100 / a) ^ (ll = N@Log[100, 70])]},
 {b, 1, Floor[100 / (a c ^ (1 / ll))]}, {d, 1, Floor[(100 / (a b c ^ (1 / ll))) ^ ll]}]
```

2699

```
Sum[1, {a, 1, Floor[100]}, {b, 1, Floor[100 / a]},
 {c, 1, Floor[(100 / (a b)) ^ (ll = N@Log[100, 70])]}, {d, 1, Floor[(100 / (a b c ^ (1 / ll))) ^ ll]}]
```

2699

```
Sum[ dz[a, 2], {a, 1, Floor[100]},
 {c, 1, Floor[(100 / (a)) ^ (ll = N@Log[100, 70])]}, {d, 1, Floor[(100 / (a c ^ (1 / ll))) ^ ll]}]
```

2699

```
Sum[ dz[a, 2] dz[c, 2], {a, 1, Floor[100]}, {c, 1, Floor[(100 / a) ^ (N@Log[100, 70])]}]
```

2699

```
Sum[ dz[a, 3] dz[c, 3], {a, 1, Floor[100]}, {c, 1, Floor[(100 / a) ^ (N@Log[100, 70])]}]
```

10 319

```
dd[100, N@Log[100, 70], 3]
```
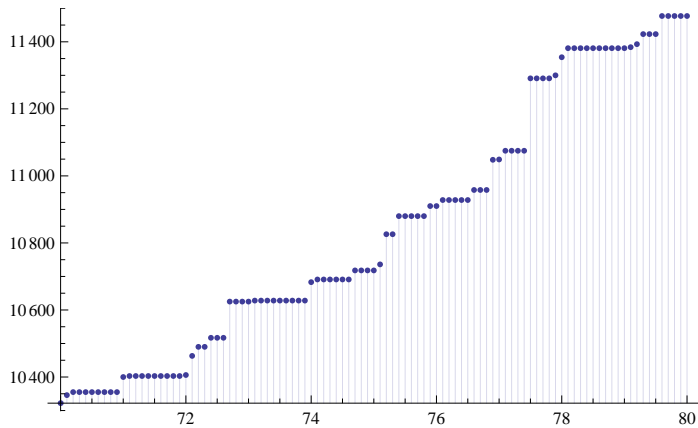
10 319

```
Sum[ dz[a, 5] dz[c, 5], {a, 1, Floor[100]}, {c, 1, Floor[(100 / a) ^ (N@Log[100, 70])]}]
```
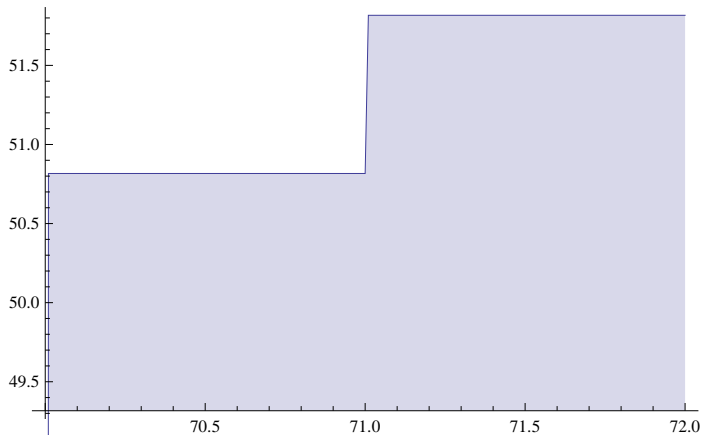
69 103

```
dd[100, N@Log[100, 70], 5]
```

69 103

DiscretePlot[dd[100, N@Log[100, y], 3], {y, 70, 80, .1}]

DiscretePlot[ld[100, y], {y, 70, 72, .01}]


```
(* Form relying on dz *)
Clear[dd]
Clear[dz]
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, z_] := dz[n, z] = Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}]
Dz[n_, z_] := Sum[dz[j, z], {j, 1, n}]
dd[n_, q_, z_] :=
  dd[n, q, z] = Sum[dz[a, z] dz[b, z], {a, 1, Floor[n]}, {b, 1, Floor[(n / a)^q]}]
d2[n_, q_, k_] := Sum[ (-1)^(k - j) Binomial[k, j] dd[n, q, j], {j, 0, k}]
ld[n_, q_] := Sum[ (-1)^(k + 1) / k d2[n, N@Log[q] / Log[n], k], {k, 1, Log2@n}]
pr[n_] := Sum[ PrimePi[ n^(1 / k)] / k, {k, 1, Log2@n}]
```

ld[100, 70]

$$\frac{3049}{60}$$

pr[100] + pr[70]

$$\frac{3049}{60}$$

```
(* Inverse? *)
Clear[dd]
Clear[dz]
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, z_] := dz[n, z] = Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}]
Dz[n_, z_] := Sum[dz[j, z], {j, 1, n}]
dd[n_, q_, z_] :=
 dd[n, q, z] = Sum[dz[a, z] dz[b, -z], {a, 1, Floor[n]}, {b, 1, Floor[(n / a)^q]}]
d2[n_, q_, k_] := Sum[ (-1)^(k - j) Binomial[k, j] dd[n, q, j], {j, 0, k}]
ld[n_, q_] := Sum[ (-1)^(k + 1) / k d2[n, N@Log[q] / Log[n], k], {k, 1, Log2@n}]
pr[n_] := Sum[ PrimePi[ n^(1 / k)] / k, {k, 1, Log2@n}]
```

```
ld[1000, 170]
```

$$\frac{47\,767}{360}$$

```
(* Inverse!!!!!! *)
pr[1000] - pr[170]
```

$$\frac{47\,767}{360}$$

```
(* Generalize? *)
Clear[dd]
Clear[dz]
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, z_] := dz[n, z] = Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}]
Dz[n_, z_] := Sum[dz[j, z], {j, 1, n}]
dd[n_, q_, z_, s_, t_] :=
 dd[n, q, z, s, t] = Sum[dz[a, z s] dz[b, z t], {a, 1, Floor[n]}, {b, 1, Floor[(n / a)^q]}]
d2[n_, q_, k_, s_, t_] := Sum[ (-1)^(k - j) Binomial[k, j] dd[n, q, j, s, t], {j, 0, k}]
ld[n_, q_, s_, t_] := Sum[ (-1)^(k + 1) / k d2[n, N@Log[q] / Log[n], k, s, t], {k, 1, Log2@n}]
pr[n_] := Sum[ PrimePi[ n^(1 / k)] / k, {k, 1, Log2@n}]
```

```
ld[1000, 170, 3, 4]
```

$$\frac{5649}{8}$$

```
(* Generalize!!! *)
3 pr[1000] + 4 pr[170]
```

$$\frac{5649}{8}$$

```
(* j^-s Form? *)
Clear[dda]
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, z_] := dz[n, z] = Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}]
Dz[n_, z_] := Sum[dz[j, z], {j, 1, n}]
dda[n_, s_, q_, z_] := dda[n, s, q, z] =
  Sum[a^-s b^-s dz[a, z] dz[b, z], {a, 1, Floor[n]}, {b, 1, Floor[(n / a)^q]}]
dd[n_, s_, q_, k_] := dd[n, s, q, k] = Sum[ j^-s j2^-s dd[n / (j j2^(1 / q)), s, q, k - 1],
    {j, 1, Floor[n]}, {j2, 1, Floor[(n / j)^q]}]
dd[n_, s_, q_, 0] := UnitStep[n - 1]
d2[n_, s_, q_, k_] := Sum[ (-1)^(k - j) Binomial[k, j] dda[n, s, q, j], {j, 0, k}]
ld[n_, s_, q_] := Sum[ (-1)^(k + 1) / k d2[n, s, N@Log[q] / Log[n], k], {k, 1, Log2@n}]
pr[n_, s_] := Sum[ FullSimplify[MangoldtLambda[j] / Log[j]] j^-s, {j, 2, n}]
ch[n_] := -Sum[MangoldtLambda[j], {j, 2, n}]
(* j^-s Form! *)


ld[100, 0, 70]
```

$$\frac{3049}{60}$$

```
pr[100, 0] + pr[70, 0]
```

$$\frac{3049}{60}$$

```
N[D[ld[100, s, 70], s] /. s → 0]
```

-160.587

```
N[D[pr[100, s] + pr[70, s], s] /. s → 0]
```

-160.587

```
N[ch[100] + ch[70]]
```

-160.587

```
Clear[dd, d2a, d2b]
dd[n_, q_, k_] :=
 dd[n, q, k] = Sum[dd[n / (j j2^(1 / q)), q, k - 1], {j, 1, Floor[n]}, {j2, 1, Floor[(n / j)^q]}]
dd[n_, q_, 0] := UnitStep[n - 1]
d2a[n_, q_, k_] := d2a[n, q, k] = Sum[d2a[n / (j j2^(1 / q)), q, k - 1],
     {j, 1, Floor[n]}, {j2, 1, Floor[(n / j)^q]}] - d2a[n, q, k - 1]
d2a[n_, q_, 0] := UnitStep[n - 1]
d2b[n_, q_, k_] := d2b[n, q, k] =
   Sum[d2b[n / (j j2^(1 / q)), q, k - 1], {j, 2, Floor[n]}, {j2, 2, Floor[(n / j)^q]}] +
    Sum[d2b[n / j, q, k - 1], {j, 2, Floor[n]}] +
    Sum[d2b[n / j2^(1 / q), q, k - 1], {j2, 2, Floor[n^q]}]
d2b[n_, q_, 0] := UnitStep[n - 1]
d2[n_, q_, k_] := Sum[(-1)^(k - j) Binomial[k, j] dd[n, q, j], {j, 0, k}]
ld[n_, q_] := Sum[(-1)^(k + 1) / k d2[n, N@Log[q] / Log[n], k], {k, 1, Log2@n}]
pr[n_] := Sum[PrimePi[n^(1 / k)] / k, {k, 1, Log2@n}]
```

```
d2b[100, N[Log[100, 70]], 3]
```

3382

```
d2[100, N[Log[100, 70]], 3]
```

3382

```
d2a[100, N[Log[100, 70]], 3]
```

3382

```
p q - 1
```

$-1 + p q$

```
bb = p q - 1
```

$-1 + p q$

```
Expand[aa - bb]
```

$2 - p - q$

```
Expand[(p - 1) (q - 1) + (p - 1) + (q - 1)]
```

$-1 + p q$