```
f[n_, s_, z_] := 1 + Integrate[E^(t (s - 1)) z Hypergeometric1F1[1 - z, 2, t], {t, -Log[n], 0}]
f2[n_, s_, z_] := 1 + Integrate[E^(t (s - 1)) LaguerreL[z - 1, 1, t], {t, -Log[n], 0}]
```

$$\texttt{f3[n\_, s\_, z\_] := 1 + } \int_1^n t^{-s}\, \texttt{z Hypergeometric1F1[1 - z, 2, -Log[t]]}\, dt$$

$$\texttt{f4[n\_, s\_, z\_] := 1 + } \int_1^n t^{-s}\, \texttt{LaguerreL[z - 1, 1, -Log[t]]}\, dt$$

```
Limit[f4[n, 2 I + ZetaZero[100], 1], n → Infinity]
```

ComplexInfinity

```
f[100, 1 / 2 + I, -1]
```

$$1 - \left(\frac{1}{25} - \frac{2\,i}{25}\right)\left(10 - 10^{-2\,i}\right)$$

```
Hypergeometric1F1[1 - z, 2, t]

z Hypergeometric1F1[1 - z, 2, t]
```

z Hypergeometric1F1[1 - z, 2, t]

```
LaguerreL[z - 1, t]


1 + Integrate[E^(t (s - 1)) LaguerreL[z - 1, 1, t], {t, -Log[n], 0}]
```

$$1 + \int_{-\mathrm{Log}[n]}^{0} e^{(-1+s)\,t}\, \mathrm{LaguerreL}[-1 + z, 1, t]\, dt$$

```
pa[n_, s_, k_] := (s - 1)^(k) Gamma[k, 0, (s - 1) Log[n]] / Gamma[k]


FullSimplify@D[pa[n, s, k], n]
```

$$\frac{n^{-s}\,(-1+s)^{k}\,((-1+s)\,\mathrm{Log}[n])^{k}}{\mathrm{Gamma}[k]\,\mathrm{Log}[n]}$$

$$\frac{n^{-s}\,(-1+s)^{-k}\,((-1+s)\,\mathrm{Log}[n])^{k}}{\mathrm{Gamma}[k]\,\mathrm{Log}[n]} \;/.\; s \to 2$$

$$\frac{\mathrm{Log}[n]^{-1+k}}{n^{2}\,\mathrm{Gamma}[k]}$$

$$\mathrm{Integrate}\left[\frac{9\verb|^|k\; n\verb|^|2\; \mathrm{Log}[n]^{k-1}}{(k-1)\,!},\ \{n, 1, x\}\right]$$

$$\mathrm{ConditionalExpression}\left[\frac{3^{k}\,(\mathrm{Gamma}[k] - \mathrm{Gamma}[k, -3\,\mathrm{Log}[x]])\,(-\mathrm{Log}[x])^{-k}\,\mathrm{Log}[x]^{k}}{(-1+k)\,!},\ \mathrm{Re}[k] > 0\right]$$

$$\mathrm{Integrate}\left[\frac{n^{-s}\,\mathrm{Log}[n]^{k-1}}{(k-1)\,!},\ \{n, 1, x\}\right]$$

$$\mathrm{ConditionalExpression}\left[\frac{(-1+s)^{-k}\,(\mathrm{Gamma}[k] - \mathrm{Gamma}[k, (-1+s)\,\mathrm{Log}[x]])}{(-1+k)\,!},\ \mathrm{Re}[k] > 0\ \&\&\ \mathrm{Log}[x] > 0\right]$$

```
pa[100., 2, 2]
```

0.943948

$$\frac{(-1+s)^{-k}\,(\mathrm{Gamma}[k]-\mathrm{Gamma}[k,\,(-1+s)\,\mathrm{Log}[x]])}{(-1+k)\,!}\ \text{/. } x\to 100 \text{ /. } s\to 2 \text{ /. } k\to 3.$$

0.83791

$$\frac{(-1+s)^{-k}\,(\mathrm{Gamma}[k,\,0,\,(-1+s)\,\mathrm{Log}[x]])}{(-1+k)\,!}\ \text{/. } x\to 100 \text{ /. } s\to 2 \text{ /. } k\to 3.$$

0.83791

$$(-1+s)^{-k}\,(\mathrm{GammaRegularized}[k,\,0,\,(-1+s)\,\mathrm{Log}[x]])\ \text{/. } x\to 100 \text{ /. } s\to 2 \text{ /. } k\to 3.$$

0.83791

$$\mathrm{Sum}\Big[\mathrm{Binomial}[z,\,k]\left(\frac{n^{-s}\,\mathrm{Log}[n]^{k-1}}{(k-1)\,!}\right),\,\{k,\,0,\,\mathrm{Infinity}\}\Big]$$

$$n^{-s}\,z\,\mathrm{Hypergeometric1F1}[1-z,\,2,\,-\mathrm{Log}[n]]$$

$$\mathrm{Integrate}[n^{-s}\,z\,\mathrm{Hypergeometric1F1}[1-z,\,2,\,-\mathrm{Log}[n]],\,\{n,\,1,\,x\}]\ \text{/. } n\to t \text{ /. } x\to n$$

$Aborted

$$1+\int_1^x n^{-s}\,z\,\mathrm{Hypergeometric1F1}[1-z,\,2,\,-\mathrm{Log}[n]]\,dn\ \text{/. } n\to t$$

$Aborted

$$\mathrm{FullSimplify}\Big[1+\int_1^n t^{-s}\,\mathrm{LaguerreL}[z-1,\,1,\,-\mathrm{Log}[t]]\,dt,\,\mathrm{Element}[n,\,\mathrm{Integers}]\Big]$$

$$1+\int_1^n t^{-s}\,\mathrm{LaguerreL}[-1+z,\,1,\,-\mathrm{Log}[t]]\,dt$$

```
Integrate[t^s, {t, 0, x}]
```

$$\mathrm{ConditionalExpression}\Big[\frac{x^{1+s}}{1+s},\,\mathrm{Re}[s]>-1\Big]$$

```
Integrate[(t + u)^s, {t, 0, x}, {u, 0, x - t}]
```

$$\mathrm{ConditionalExpression}\Big[\frac{x^{2+s}}{2+s},\,\mathrm{Re}[s]>-2\Big]$$

```
FullSimplify@Integrate[(t + u + v)^s, {t, 0, x}, {u, 0, x - t}, {v, 0, x - t - u}]
```

$$\mathrm{ConditionalExpression}\Big[\frac{x^{3+s}}{6+2\,s},\,\mathrm{Re}[s]>-3\Big]$$

```
FullSimplify@
 Integrate[(t + u + v + w)^s, {t, 0, x}, {u, 0, x - t}, {v, 0, x - t - u}, {w, 0, x - t - u - v}]
```

$$\mathrm{ConditionalExpression}\Big[\frac{x^{4+s}}{24+6\,s},\,\mathrm{Re}[s]>-4\Big]$$

```
FullSimplify@Integrate[(t + u + v + w + y)^s, {t, 0, x},
  {u, 0, x - t}, {v, 0, x - t - u}, {w, 0, x - t - u - v}, {y, 0, x - t - u - v - w}]
```

$$\mathrm{ConditionalExpression}\Big[\frac{\left(\frac{1}{x}\right)^{-5-s}\left(6+s+s^2-\frac{24\,\left(\frac{1}{x}\right)^{s}x^{s}}{5+s}\right)}{24\,(1+s)\,(2+s)\,(3+s)},\,\mathrm{Re}[s]>-5\Big]$$

```
FullSimplify@Sum[Binomial[z, k] x^(k + s) / ((k + s) (k - 1) !), {k, 0, Infinity}]
```

x^{1+s} z Gamma[1 + s] HypergeometricPFQRegularized[{1 + s, 1 - z}, {2, 2 + s}, -x]

```
FullSimplify@D[x^(k + s) / (k ! + (k - 1) ! s), x]
```

$$\frac{x^{-1+k+s}}{\text{Gamma}[k]}$$

$$\text{Sum}\left[\text{Binomial}[z,k]\left(\frac{x^{-1+k+s}}{(k-1)\,!}\right),\{k,0,\text{Infinity}\}\right]$$

x^{s} z Hypergeometric1F1[1 - z, 2, -x]

$$\text{Integrate}\left[\frac{x^{-1+k+s}}{(k-1)\,!},\{x,0,n\}\right]$$

$$\text{ConditionalExpression}\left[\frac{n^{k+s}}{(k+s)\,(-1+k)\,!},\ \text{Re}[k+s]>0\right]$$

$$\text{Limit}\left[D\left[\frac{x^{-1+k+s}}{(k-1)\,!},k\right],k\to 0\right]$$

x^{-1+s}

```
Sum[t^s, {t, 1, x}]
```

HarmonicNumber[x, -s]

```
Sum[(t + u)^s, {t, 1, x}, {u, 1, x - t}] /. s → -2 /. x → 1
```

$$\frac{1}{6}\left(6\left(\frac{3}{2}-\text{EulerGamma}\right)+6\,\text{EulerGamma}-\pi^2+12\left(-\frac{5}{4}+\frac{\pi^2}{6}\right)-6\left(-1+\frac{\pi^2}{6}\right)\right)$$