```
Clear[ee]
bin[z_, k_] := bin[z, k] = Product[z - j, {j, 0, k - 1}] / k!
ee[n_, k_] := ee[n, k] = Sum[ DivisorSigma[1, j] / j ee[n - j, k - 1], {j, 1, n}]
ee[n_, 0] := 1
ez[n_, z_] := Sum[ z^k / k! ee[n, k], {k, 0, Log2@n}]
eez[n_, z_] := ez[n, z] - ez[n - 1, z]
```

```
Table[ee[10, n], {n, 0, 20}]
```

$$\left\{1, \frac{7583}{504}, \frac{750\,731}{8400}, \frac{8\,720\,689}{30\,240}, \frac{1\,708\,153}{3024}, \frac{201\,641}{288}, \frac{133\,993}{240}, \frac{6779}{24}, \frac{260}{3}, \frac{29}{2}, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}$$

```
eez[10, 1]
```

$$\frac{931\,447}{33\,600}$$

```
Clear[pp, pe, pa]
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, z_] := dz[n, z] = Product[(-1)^p[[2]] bin[-z, p[[2]]], {p, FI[n]}]
pp[n_, k_] := pp[n, k] = Sum[ PartitionsP[j] pp[n - j, k - 1], {j, 1, n - 1}]
pp[n_, 1] := PartitionsP[n]
pe[n_, k_] := pe[n, k] = Sum[ DivisorSigma[1, j] / j pe[n - j, k - 1], {j, 1, n - 1}]
pe[n_, 1] := DivisorSigma[1, n] / n
pe[n_, 0] := UnitStep[n]
pa[z_, 0] := 1
pa[n_, z_] := Sum[ z^k / k! pe[n, k], {k, 0, n}]
ps[n_, z_] := Sum[ bin[z, k] pe[n, k], {k, 0, n}]
pss[n_, z_] := Sum[ bin[z, k] pp[n, k], {k, 0, n}]
roots[n_] := If[(c = Exponent[f = pa[n, z], z]) == 0, {},
  If[c == 1, List@NRoots[f == 0, z][[2]], List @@ NRoots[f == 0, z][[All, 2]]]]
rootsa[n_] := If[(c = Exponent[f = pa[n, z], z]) == 0, {},
  If[c == 1, List@Roots[f == 0, z][[2]], List @@ Roots[f == 0, z][[All, 2]]]]
pes[fn_, n_, k_] := pes[fn, n, k] = Sum[ fn[j] pes[fn, n - j, k - 1], {j, 1, n - 1}]
pes[fn_, n_, 1] := fn[n]
paz[fn_, n_, z_] := Sum[ z^k / k! pes[fn, n, k], {k, 1, n}]
```

```
pa[20, z]
```

$$\frac{21\,z}{10} + \frac{36\,090\,792\,041\,z^2}{1\,551\,950\,400} + \frac{19\,440\,485\,960\,947\,z^3}{205\,837\,632\,000} + \frac{270\,511\,021\,952\,161\,z^4}{1\,543\,782\,240\,000} + \frac{61\,124\,886\,023\,017\,z^5}{348\,713\,164\,800} + \frac{1\,094\,874\,497\,862\,469\,z^6}{10\,461\,394\,944\,000} + \frac{1\,251\,336\,411\,115\,663\,z^7}{31\,384\,184\,832\,000} + \frac{9\,546\,917\,708\,780\,929\,z^8}{941\,525\,544\,960\,000} + \frac{2\,287\,136\,081\,711\,z^9}{1\,287\,556\,300\,800} + \frac{8\,463\,269\,445\,643\,z^{10}}{38\,626\,689\,024\,000} + \frac{1\,255\,560\,697\,z^{11}}{65\,028\,096\,000} + \frac{236\,767\,459\,153\,z^{12}}{193\,133\,445\,120\,000} + \frac{108\,621\,773\,z^{13}}{1\,931\,334\,451\,200} + \frac{1\,399\,967\,489\,z^{14}}{753\,220\,435\,968\,000} + \frac{73\,327\,z^{15}}{1\,673\,823\,191\,040} + \frac{906\,977\,z^{16}}{1\,255\,367\,393\,280\,000} + \frac{z^{17}}{123\,986\,903\,040} + \frac{491\,z^{18}}{8\,536\,498\,274\,304\,000} + \frac{z^{19}}{4\,268\,249\,137\,152\,000} + \frac{z^{20}}{2\,432\,902\,008\,176\,640\,000}$$

```
roots[10]
```

```
{-58.1801, -32.1652, -17.6741, -13.0545,
 -5.46205 - 0.708088 i, -5.46205 + 0.708088 i, -1.71036, -1., -0.291568, 0.}
```

```
Table[D[Expand@ps[n, z], z] /. z → 0, {n, 1, 20}]
```

$$\left\{1, 1, \frac{1}{6}, \frac{13}{24}, -\frac{4}{15}, \frac{283}{360}, -\frac{1037}{2520}, \frac{2561}{6720}, \frac{12\,881}{45\,360}, -\frac{30\,869}{151\,200}, \frac{6649}{831\,600}, \frac{690\,467}{997\,920}, -\frac{55\,064\,743}{129\,729\,600}, -\frac{79\,788\,679}{454\,053\,600}, \frac{19\,899\,122\,311}{27\,243\,216\,000}, -\frac{564\,628\,819}{5\,660\,928\,000}, -\frac{7\,889\,326\,717}{22\,054\,032\,000}, \frac{775\,467\,764\,119}{2\,381\,835\,456\,000}, \frac{4\,337\,536\,994\,443}{105\,594\,705\,216\,000}, \frac{52\,988\,658\,881\,489}{162\,453\,392\,640\,000}\right\}$$

```
pq[n_] := DivisorSigma[1, n] / n
pqe[n_] := DivisorSigma[1, n]
peh[n_] := 1
poh[n_] := 1 / n
poh2[n_] := DivisorSigma[0, n] / n
pohm1[n_] := dz[n, -1] / n
pah[n_] := 1 / n^2
pzh[n_] := n
pwh[n_] := PartitionsP[n] / n
pqa[n_] := EulerPhi[n] / n
pqaa[n_] := EulerPhi[n]
pqb[n_] := DivisorSigma[2, n] / n
pqc[n_] := DivisorSigma[3, n] / n
```

```
Table[paz[pq, n, 1], {n, 1, 10}]
```

```
{1, 2, 3, 5, 7, 11, 15, 22, 30, 42}
```

```
Table[PartitionsP[n], {n, 1, 10}]
```

```
{1, 2, 3, 5, 7, 11, 15, 22, 30, 42}
```

```
(* https://oeis.org/A010815 *)
Table[paz[pq, n, -1], {n, 1, 20}]
```

```
{-1, -1, 0, 0, 1, 0, 1, 0, 0, 0, 0, -1, 0, 0, -1, 0, 0, 0, 0, 0}
```

```
ae[n_] := SeriesCoefficient[Product[1 - x^k, {k, n}], {x, 0, n}]; Table[ae[n], {n, 1, 20}]
```

```
{-1, -1, 0, 0, 1, 0, 1, 0, 0, 0, 0, -1, 0, 0, -1, 0, 0, 0, 0, 0}
```

```
Table[paz[peh, n, 1], {n, 1, 10}]
```

$$\left\{1, \frac{3}{2}, \frac{13}{6}, \frac{73}{24}, \frac{167}{40}, \frac{4051}{720}, \frac{37\,633}{5040}, \frac{43\,817}{4480}, \frac{4\,596\,553}{362\,880}, \frac{58\,941\,091}{3\,628\,800}\right\}$$

```
Table[paz[poh, n, 1], {n, 1, 10}]
```

```
{1, 1, 1, 1, 1, 1, 1, 1, 1, 1}
```

```
Table[paz[pah, n, 1], {n, 1, 10}]
```

$$\left\{1, \frac{3}{4}, \frac{19}{36}, \frac{107}{288}, \frac{641}{2400}, \frac{51\,103}{259\,200}, \frac{1\,897\,879}{12\,700\,800}, \frac{7\,860\,361}{67\,737\,600}, \frac{505\,249\,081}{5\,486\,745\,600}, \frac{40\,865\,339\,743}{548\,674\,560\,000}\right\}$$

```
Table[paz[pzh, n, 1], {n, 1, 10}]
```

$$\left\{1, \frac{5}{2}, \frac{31}{6}, \frac{241}{24}, \frac{2261}{120}, \frac{24\,781}{720}, \frac{61\,967}{1008}, \frac{4\,342\,241}{40\,320}, \frac{67\,308\,841}{362\,880}, \frac{1\,141\,960\,501}{3\,628\,800}\right\}$$

```
Table[paz[pqa, n, 1], {n, 1, 10}]
```

$$\left\{1, 1, \frac{4}{3}, \frac{19}{12}, \frac{131}{60}, \frac{433}{180}, \frac{1009}{315}, \frac{38\,399}{10\,080}, \frac{415\,199}{90\,720}, \frac{2\,426\,923}{453\,600}\right\}$$

```
Table[paz[pqaa, n, 1], {n, 1, 10}]
```

$$\left\{1, \frac{3}{2}, \frac{19}{6}, \frac{121}{24}, \frac{387}{40}, \frac{9931}{720}, \frac{124\,363}{5040}, \frac{514\,043}{13\,440}, \frac{21\,594\,961}{362\,880}, \frac{335\,083\,411}{3\,628\,800}\right\}$$

```
(* https://oeis.org/A000219 Planar Partitions*)
Table[paz[pqb, n, 1], {n, 1, 10}]
```

```
{1, 3, 6, 13, 24, 48, 86, 160, 282, 500}
```

```
(* https://oeis.org/A023871 *)
Table[paz[pqc, n, 1], {n, 1, 10}]
```

```
{1, 5, 14, 40, 101, 266, 649, 1593, 3765, 8813}
```

```
(* https://oeis.org/A028342 *)
Table[paz[poh2, n, 1], {n, 1, 10}]
```

$$\left\{1, \frac{3}{2}, \frac{11}{6}, \frac{59}{24}, \frac{113}{40}, \frac{2629}{720}, \frac{20\,677}{5040}, \frac{67\,363}{13\,440}, \frac{2\,066\,201}{362\,880}, \frac{24\,322\,931}{3\,628\,800}\right\}$$

```
nmax = 10; CoefficientList[
 Series[Product[1 / (1 - x^k)^(1 / k), {k, 1, nmax}], {x, 0, nmax}], x]
```

$$\left\{1, 1, \frac{3}{2}, \frac{11}{6}, \frac{59}{24}, \frac{113}{40}, \frac{2629}{720}, \frac{20\,677}{5040}, \frac{67\,363}{13\,440}, \frac{2\,066\,201}{362\,880}, \frac{24\,322\,931}{3\,628\,800}\right\}$$

```
Table[paz[pwh, n, 1], {n, 1, 10}]
```

$$\left\{1, \frac{3}{2}, \frac{13}{6}, \frac{79}{24}, \frac{193}{40}, \frac{5209}{720}, \frac{7621}{720}, \frac{29\,917}{1920}, \frac{1\,174\,783}{51\,840}, \frac{17\,067\,613}{518\,400}\right\}$$

```
Table[paz[pohm1, n, 1], {n, 1, 10}]
```

$$\left\{1, 0, -\frac{2}{3}, -\frac{5}{12}, -\frac{3}{20}, \frac{7}{45}, \frac{13}{126}, -\frac{33}{1120}, -\frac{155}{2592}, \frac{7181}{56\,700}\right\}$$

```
Clear[pres]
pres[n_, k_] := pres[n, k] = Sum[ 1 / j pres[n - j, k - 1], {j, 1, n - 1}]
pres[n_, 1] := 1 / n
praz[n_, z_] := Sum[ z^k / k ! pres[n, k], {k, 1, n}]
rootspr[n_] := If[(c = Exponent[f = praz[n, z], z]) == 0, {},
  If[c == 1, List@NRoots[f == 0, z][[2]], List @@ NRoots[f == 0, z][[All, 2]]]]


Table[praz[n, 3], {n, 0, 10}]
```

```
{0, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66}
```

```
Table[Binomial[n + 2, n], {n, 0, 10}]
```

```
{1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66}
```

```
Table[Pochhammer[3, n] / n !, {n, 0, 10}]
```

```
{1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66}
```

```
(* Binomial[n+3,n] *)
Table[praz[n, 4], {n, 0, 10}]
```

$$\{0, 4, 10, 20, 35, 56, 84, 120, 165, 220, 286\}$$

```
Table[Binomial[n + 3, n], {n, 0, 10}]
```

$$\{1, 4, 10, 20, 35, 56, 84, 120, 165, 220, 286\}$$

```
Table[D[praz[n, z], z] /. z → 0, {n, 0, 10}]
```

$$\left\{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}\right\}$$

```
(* https://oeis.org/A000712 *)
Table[paz[pq, n, 2], {n, 1, 10}]
```

$$\{2, 5, 10, 20, 36, 65, 110, 185, 300, 481\}$$

```
CoefficientList[Series[Product[1 / (1 - x^n)^2, {n, 20}], {x, 0, 20}], x]
```

$$\{1, 2, 5, 10, 20, 36, 65, 110, 185, 300, 481, 752,\\ 1165, 1770, 2665, 3956, 5822, 8470, 12\,230, 17\,490, 24\,842\}$$

```
praz[10, z]
```

$$\frac{z}{10} + \frac{7129\, z^2}{25\,200} + \frac{1303\, z^3}{4032} + \frac{4523\, z^4}{22\,680} + \frac{19\, z^5}{256} + \frac{3013\, z^6}{172\,800} + \frac{z^7}{384} + \frac{29\, z^8}{120\,960} + \frac{z^9}{80\,640} + \frac{z^{10}}{3\,628\,800}$$

```
rootspr[8]
```

$$\{-7., -6., -5., -4., -3., -2., -1., 0.\}$$

```
CoefficientList[Series[Product[(1 - x^n), {n, 20}], {x, 0, 20}], x]
```

$$\{1, -1, -1, 0, 0, 1, 0, 1, 0, 0, 0, 0, -1, 0, 0, -1\}$$

```
CoefficientList[Series[Product[1 / (1 - x^n), {n, 20}], {x, 0, 20}], x]
```

$$\{1, 1, 2, 3, 5, 7, 11, 15, 22, 30, 42, 56, 77, 101, 135, 176, 231, 297, 385, 490, 627\}$$

```
CoefficientList[Series[Product[1 / (1 - x^n)^2, {n, 20}], {x, 0, 20}], x]
```

$$\{1, 2, 5, 10, 20, 36, 65, 110, 185, 300, 481, 752,\\ 1165, 1770, 2665, 3956, 5822, 8470, 12\,230, 17\,490, 24\,842\}$$

```
Table[D[paz[pq, n, z], z] /. z → 0, {n, 1, 10}]
```

$$\left\{1, \frac{3}{2}, \frac{4}{3}, \frac{7}{4}, \frac{6}{5}, 2, \frac{8}{7}, \frac{15}{8}, \frac{13}{9}, \frac{9}{5}\right\}$$

```
Table[ DivisorSigma[1, n] / n, {n, 1, 10}]
```

$$\left\{1, \frac{3}{2}, \frac{4}{3}, \frac{7}{4}, \frac{6}{5}, 2, \frac{8}{7}, \frac{15}{8}, \frac{13}{9}, \frac{9}{5}\right\}$$

```
Table[roots[n], {n, 1, 20}] // TableForm
```

```
0.
-3.          0.
-8.          -1.          0.
-14.         -3.          -1.          0.
-20.6119     -6.          -3.          -0.388126    0.
-27.64       -10.         -5.39416     -1.          -0.96583                 0.
-34.9715     -14.7966     -8.          -3.          -2.                      -0.231903
-42.5351     -20.1783     -10.8349     -6.          -3.                      -1.
-50.2827     -26.         -14.         -9.4652      -4.                      -3.
-58.1801     -32.1652     -17.6741     -13.0545     -5.46205 - 0.708088 i    -5.46205 + 0.708088 i
-66.2025     -38.6075     -21.9966     -16.5203     -8.                      -7.54367
-74.3306     -45.279      -26.9155     -19.8473     -12.121                  -8.58295
-82.5497     -52.1437     -32.2872     -23.1509     -16.2505                 -10.
-90.8481     -59.174      -37.9982     -26.5663     -20.5515                 -11.4437 - 0.50959 i
-99.2161     -66.3482     -43.9762     -30.262      -24.8648                 -14.3077
-107.646     -73.6486     -50.1736     -34.4387     -28.9777                 -18.7081
-116.131     -81.0611     -56.5574     -39.1911     -32.784                  -23.075
-124.666     -88.5735     -63.1029     -44.4289     -36.3715                 -27.6415
-133.247     -96.176      -69.7907     -50.0164     -39.8936                 -32.3573
-141.868     -103.86      -76.6054     -55.8603     -43.502                  -37.1266
```

```
Table[pss[n, 2], {n, 1, 10}]
```

```
{2, 5, 10, 20, 36, 65, 110, 185, 300, 481}
```

```
Table[pp[n, 2], {n, 1, 10}]
```

```
{0, 1, 4, 10, 22, 43, 80, 141, 240, 397}
```

```
Table[pa[n, 2], {n, 1, 10}]
```

```
{2, 5, 10, 20, 36, 65, 110, 185, 300, 481}
```

```
Product[ 1 - If[ j ≠ 0, 1 / j, 0], {j, roots[20]}]
```

```
298.571 + 0. i
```

```
pa[10, z]
```

$$\frac{9\,z}{5}+\frac{252\,019\,z^2}{25\,200}+\frac{64\,193\,z^3}{4032}+\frac{59\,453\,z^4}{5670}+\frac{7457\,z^5}{2304}+\frac{88\,453\,z^6}{172\,800}+\frac{49\,z^7}{1152}+\frac{221\,z^8}{120\,960}+\frac{z^9}{26\,880}+\frac{z^{10}}{3\,628\,800}$$

```
Clear[pres]
pres[n_, s_, k_] := pres[n, k] = Sum[j^-s pres[n - j, s, k - 1], {j, 1, n}]
pres[n_, s_, 0] := UnitStep[n]
praz[n_, s_, z_] := Sum[ bin[z, k] pres[n, s, k], {k, 0, n}]
```

```
Table[D[praz[n, 1, z] - praz[n - 1, 1, z], z] /. z → 0, {n, 1, 10}]
```

$$\left\{1,\ 0,\ \frac{1}{6},\ \frac{1}{24},\ \frac{1}{15},\ \frac{13}{360},\ \frac{97}{2520},\ \frac{571}{20\,160},\ \frac{1217}{45\,360},\ \frac{3391}{151\,200}\right\}$$

```
Table[pres[10, 0, n], {n, 0, 12}]
```

```
{1, 10, 45, 120, 210, 252, 210, 120, 45, 10, 1, 0, 0}
```

```
Table[N@praz[n, 2, 1], {n, 0, 15}]
```

```
{1., 2., 2.25, 2.36111, 2.42361, 2.46361, 2.49139, 2.5118,
 2.52742, 2.53977, 2.54977, 2.55803, 2.56498, 2.57089, 2.576, 2.58044}
```

```
Zeta[2.]
```

```
1.64493
```

```
Sum[ j^-s k^-s, {j, 1, Infinity}, {k, 1, Infinity - j}]
```

```
Zeta[s]^2
```

```
Sum[ z^k / k! (Log[1 + Zeta[s]])^k, {k, 0, Infinity}]
```

```
(1 + Zeta[s])^z
```

```
D[ (1 + Zeta[s])^z, z] /. z → 0
```

```
Log[1 + Zeta[s]]
```

```
D[ (1 + Zeta[s])^z, {z, 3}] /. z → 0
```

```
Log[1 + Zeta[s]]^3
```

```
Clear[cr, lcr]
cr[fn_, n_, k_] := cr[n, k] = Sum[fn[j] cr[fn, n - j, k - 1], {j, 1, n - 1}]
cr[fn_, n_, 1] := fn[n]
crp1z[fn_, n_, z_] := Sum[ bin[z, k] cr[fn, n, k], {k, 0, n}]
f1[n_] := Log[n]
f1a[n_] := Log[n] / n
f2[n_] := PartitionsP[n]
f2a[n_] := DivisorSigma[1, n] / n
f3[n_] := PrimeNu[n]
f3a[n_] := PrimeNu[n] / n
f4[n_] := PrimeOmega[n]
f4a[n_] := PrimeOmega[n] / n
lcr[fn_, n_, k_] := lcr[fn, n, k] = Sum[ fn[j] lcr[fn, n - j, k - 1], {j, 1, n - 1}]
lcr[fn_, n_, 1] := fn[n]
crp1za[fn_, n_, z_] := Sum[ z^k / k! lcr[fn, n, k], {k, 1, n}]
```

```
Table[{n, D[ Expand@crp1za[f2, n, z], z] /. z → 0}, {n, 1, 10}] // TableForm
```

1	1
2	2
3	3
4	5
5	7
6	11
7	15
8	22
9	30
10	42

roots[20]

```
{-141.868, -103.86, -76.6054, -55.8603, -43.502, -37.1266, -26., -22.7689, -15.9744,
 -14.3239, -8.6201 - 0.464361 i, -8.6201 + 0.464361 i, -5.4618, -3.53606, -2.81615, -1.77376,
 -0.6842 - 0.364405 i, -0.6842 + 0.364405 i, 0.0429813 - 0.197428 i, 0.0429813 + 0.197428 i}
```

Product[1 - 1 / r, {r, roots[20]}]

```
628. + 0. i
```

pa[40, z]

$$1+\frac{9\,z}{4}+\frac{10\,906\,249\,596\,554\,029\,957\,z^2}{213\,717\,258\,282\,528\,000}+\frac{3\,311\,332\,130\,252\,150\,702\,631\,039\,031\,z^3}{7\,107\,398\,238\,824\,413\,770\,240\,000}+$$
$$\frac{4\,601\,466\,647\,479\,933\,530\,174\,591\,644\,889\,997\,z^4}{2\,241\,069\,275\,674\,920\,027\,963\,225\,600\,000}+\frac{5\,100\,050\,698\,134\,682\,569\,341\,998\,909\,638\,191\,z^5}{988\,728\,670\,480\,498\,692\,717\,772\,800\,000}+$$
$$\frac{16\,975\,111\,345\,995\,360\,587\,507\,359\,194\,030\,174\,953\,z^6}{2\,076\,330\,208\,009\,047\,254\,707\,322\,880\,000\,000}+$$
$$\frac{545\,670\,212\,324\,283\,649\,935\,259\,455\,168\,638\,595\,919\,z^7}{62\,289\,906\,240\,271\,417\,641\,219\,686\,400\,000\,000}+$$
$$\frac{24\,909\,826\,023\,335\,410\,344\,764\,528\,414\,602\,582\,410\,377\,z^8}{3\,737\,394\,374\,416\,285\,058\,473\,181\,184\,000\,000\,000}+$$
$$\frac{262\,700\,183\,459\,124\,516\,334\,784\,383\,436\,171\,366\,483\,z^9}{70\,350\,952\,930\,188\,895\,218\,318\,704\,640\,000\,000}+$$
$$\frac{2\,229\,066\,270\,921\,060\,495\,911\,868\,781\,251\,474\,650\,597\,z^{10}}{1\,407\,019\,058\,603\,777\,904\,366\,374\,092\,800\,000\,000}+$$
$$\frac{1\,183\,032\,294\,365\,751\,271\,278\,046\,727\,755\,918\,349\,z^{11}}{2\,273\,595\,941\,247\,351\,931\,168\,358\,400\,000\,000}+$$
$$\frac{900\,219\,289\,350\,030\,881\,436\,026\,443\,643\,402\,657\,491\,z^{12}}{6\,684\,372\,067\,267\,214\,677\,634\,973\,696\,000\,000\,000}+$$
$$\frac{385\,440\,360\,682\,144\,500\,918\,130\,500\,023\,561\,621\,z^{13}}{13\,829\,735\,311\,587\,340\,712\,348\,221\,440\,000\,000}+$$
$$\frac{42\,601\,056\,604\,962\,083\,337\,434\,078\,277\,300\,770\,783\,z^{14}}{9\,127\,625\,305\,647\,644\,870\,149\,826\,150\,400\,000\,000}+$$
$$\frac{43\,192\,602\,555\,717\,427\,366\,863\,480\,139\,617\,191\,z^{15}}{67\,612\,039\,301\,093\,665\,704\,813\,527\,040\,000\,000}+$$
$$\frac{171\,029\,511\,842\,859\,041\,512\,765\,317\,585\,166\,064\,501\,z^{16}}{2\,373\,182\,579\,468\,387\,666\,238\,954\,799\,104\,000\,000\,000}+$$
$$\frac{1\,564\,222\,580\,690\,909\,597\,710\,802\,971\,760\,267\,z^{17}}{231\,812\,706\,175\,178\,282\,416\,503\,521\,280\,000\,000}+\frac{2\,702\,790\,736\,193\,859\,007\,633\,708\,376\,829\,971\,z^{18}}{5\,124\,280\,873\,346\,046\,242\,891\,130\,470\,400\,000\,000}+$$
$$\frac{2\,488\,815\,159\,576\,410\,776\,517\,750\,149\,354\,937\,z^{19}}{71\,962\,727\,047\,424\,910\,280\,601\,527\,910\,400\,000\,000}+$$
$$\frac{8\,245\,875\,355\,177\,058\,075\,315\,285\,293\,628\,623\,z^{20}}{4\,317\,763\,622\,845\,494\,616\,836\,091\,674\,624\,000\,000\,000}+\frac{14\,764\,203\,489\,121\,959\,389\,036\,786\,527\,z^{21}}{165\,732\,341\,078\,917\,975\,191\,688\,367\,308\,800\,000}+$$
$$\frac{524\,834\,980\,812\,629\,675\,613\,463\,741\,427\,z^{22}}{149\,159\,106\,971\,026\,177\,672\,519\,530\,577\,920\,000\,000}+\frac{2\,447\,876\,434\,804\,055\,690\,439\,293\,z^{23}}{20\,768\,463\,794\,350\,623\,457\,605\,058\,560\,000\,000}+$$
$$\frac{7\,396\,020\,255\,683\,272\,037\,662\,789\,481\,z^{24}}{2\,206\,856\,962\,787\,697\,248\,605\,113\,522\,585\,600\,000\,000}+\frac{6\,563\,928\,432\,381\,490\,989\,053\,z^{25}}{81\,134\,447\,161\,312\,398\,845\,776\,232\,448\,000\,000}+$$

$$\frac{4\,033\,252\,698\,616\,346\,913\,707\,z^{26}}{2\,434\,033\,414\,839\,371\,965\,373\,286\,973\,440\,000\,000} + \frac{155\,537\,187\,999\,119\,482\,223\,z^{27}}{5\,408\,963\,144\,087\,493\,256\,385\,082\,163\,200\,000\,000} + \frac{5\,338\,925\,207\,906\,623\,707\,391\,z^{28}}{12\,656\,973\,757\,164\,734\,219\,941\,092\,261\,888\,000\,000\,000} + \frac{402\,805\,901\,582\,083\,z^{29}}{77\,270\,902\,058\,392\,760\,805\,501\,173\,760\,000\,000} + \frac{16\,948\,630\,976\,848\,201\,z^{30}}{313\,719\,862\,357\,074\,608\,870\,334\,765\,465\,600\,000\,000} + \frac{59\,892\,691\,473\,553\,z^{31}}{128\,339\,943\,691\,530\,521\,810\,591\,494\,963\,200\,000\,000} + \frac{795\,668\,688\,879\,353\,z^{32}}{238\,712\,295\,266\,246\,770\,567\,700\,180\,631\,552\,000\,000\,000} + \frac{287\,022\,371\,z^{33}}{14\,735\,326\,868\,286\,837\,689\,364\,208\,680\,960\,000\,000} + \frac{24\,115\,804\,931\,z^{34}}{262\,583\,524\,792\,871\,447\,624\,470\,198\,694\,707\,200\,000\,000} + \frac{4\,858\,033\,z^{35}}{14\,171\,174\,353\,900\,998\,760\,685\,693\,262\,888\,960\,000\,000} + \frac{1\,544\,999\,z^{36}}{1\,566\,287\,691\,746\,952\,494\,602\,102\,939\,582\,464\,000\,000\,000} + \frac{313\,z^{37}}{148\,797\,330\,715\,960\,486\,987\,199\,779\,260\,334\,080\,000\,000} + \frac{1031\,z^{38}}{330\,330\,074\,189\,432\,281\,111\,583\,509\,957\,941\,657\,600\,000\,000} + \frac{z^{39}}{348\,681\,744\,977\,734\,074\,506\,671\,482\,733\,382\,860\,800\,000\,000} + \frac{z^{40}}{815\,915\,283\,247\,897\,734\,345\,611\,269\,596\,115\,894\,272\,000\,000\,000}$$

```
PartitionsP[40] - 1
```

37 337

```
Sum[-1 / r, {r, roots[40]}]
```

$2.25006 - 1.11378 \times 10^{-12}\,i$

```
N@(DivisorSigma[1, 40]) / 40
```

2.25

```
1 / roots[40]
```

```
{-0.00312885, -0.00373402, -0.00438121, -0.0051149, -0.00597085, -0.00698917,
 -0.00821703, -0.00957805, -0.0099307, -0.0115984, -0.0131675, -0.0145961,
 -0.0173821, -0.0183391, -0.0222501, -0.0222674, -0.0281348, -0.0301333,
 -0.0351275, -0.039339, -0.0440606, -0.0540093, -0.0586939, -0.0687219,
 -0.0809013, -0.0978023, -0.109839, -0.137531, -0.166765, -0.196692, -0.27266,
 -0.322421, -0.399344, -0.758371 - 0.120633 i, -0.758371 + 0.120633 i, -1.,
 -1.86697 + 1.69499 i, -1.86697 - 1.69499 i, 3.15973 + 7.77127 i, 3.15973 - 7.77127 i}
```

```
pa[5, z]
```

$$1+\frac{6\,z}{5}+\frac{15\,z^2}{4}+\frac{43\,z^3}{24}+\frac{z^4}{4}+\frac{z^5}{120}$$

```
N@Sum[ -1 / r, {r, rootsa[52]}]
```

$$1.88462-3.44234\times10^{-18}\,i$$

```
N@DivisorSigma[1, 52] / 52
```

$$1.88462$$

```
Length@rootsa[42]
```

$$42$$

```
pa[42, z]
```

```
AccountingForm@N@Product[1 - 1 / r, {r, rootsa[112]}]
```

$$761002157.+0.0000000720834\,i$$

```
1 + PartitionsP[112]
```

$$761\,002\,157$$

```
Clear[pp, pe, pa]
pp[n_, k_] := pp[n, k] = Sum[ PartitionsP[j] pp[n - j, k - 1], {j, 1, n - 1}]
pp[n_, 1] := PartitionsP[n]
pe[n_, k_] := pe[n, k] = Sum[ DivisorSigma[1, j] / j pe[n - j, k - 1], {j, 1, n - 1}]
pe[n_, 1] := DivisorSigma[1, n] / n
pe[n_, 0] := 0
pa[z_, 0] := 1
pa[n_, z_] := Sum[ z^k / k ! pe[n, k], {k, 0, n}]
pss[n_, z_] := Sum[ bin[z, k] pp[n, k], {k, 0, n}]
ppe[n_, k_] :=
 ppe[n, k] = Sum[ (-1)^(j + 1) DivisorSigma[1, j] / j ppe[n - j, k - 1], {j, 1, n - 1}]
ppe[n_, 1] := (-1)^(n + 1) DivisorSigma[1, n] / n
ppe[n_, 0] := 0
ppa[n_, z_] := Sum[ z^k / k ! ppe[n, k], {k, 0, n}]
```

```
Table[ppa[n, -1], {n, 1, 20}]
```

$$\{-1,\ 2,\ -3,\ 5,\ -7,\ 11,\ -15,\ 22,\ -30,\ 42,\ -56,\ 77,\ -101,\ 135,\ -176,\ 231,\ -297,\ 385,\ -490,\ 627\}$$