$$\sum_{j=1} f(j)(\log F)^k(\frac{n}{j})=\sum_{j=0}\frac{1}{j!}(\log F)^{k+j}(n)$$

Compare this to $n(\log n)^k=\sum_{j=0}^{\infty}\frac{1}{j!}(\log n)^{k+j}$

`{x Log[x]^k, Sum[ 1/(j!) Log[ x]^(k+j),{j,0,Infinity}]}`

$$\Pi(n)=\sum_{k=0}\frac{B_k}{k!}\sum_{j=2}(\log D)^k(\frac{n}{j})$$

$$\Pi(n)=\sum_{k=0}\frac{B_k}{k!}\sum_{j=2}(\log d)^k(j)(D-1)^1(\frac{n}{j})$$

$$(\log D)^k(n)=\sum_{m=0}\frac{B_m}{m!}\sum_{j=2}(\log F)^{k-1+m}(\frac{n}{j})$$

$$(\log F)^k(n)=\sum_{m=0}\frac{B_m}{m!}\sum_{j=1}(f-1)^1(j)(\log F)^{k-1+m}(\frac{n}{j})$$

$$(\log F)^k(n)=\sum_{m=0}\frac{B_m}{m!}\sum_{j=1}(\log f)^{k-1+m}(j)(F-1)^1(\frac{n}{j})$$

Compare this to $(\log n)^k=\sum_{m=0}^{\infty}(\lim_{x\to 0}\frac{\partial^m}{\partial x^m}\frac{x}{e^x-1})\frac{1}{m!}(n-1)(\log n)^{k-1+m}$

`Sum[ BernoulliB[ b]/b!(x-1) Log[x]^(b+k-1),{b,0,Infinity}]`

TO DO

add these power series
add more interchapter headings
continue to unify syntax

$$\Pi(n)=\sum_{j=2}\lfloor\frac{n}{j}-1\rfloor\sum_{k=0}\frac{B_k}{k!}\lim_{z\to 0}\frac{\partial^k}{\partial z^k}[d^0](j)^{*z}$$

$$\Pi(n)=\sum_{j=2}\sum_{k=0}\frac{B_k}{k!}\lim_{z\to 0}\frac{\partial^k}{\partial z^k}[D^s](\frac{n}{j})^{*z}$$

$$[\log\zeta_n(s)]^{*1}=\sum_{k=0}\frac{B_k}{k!}[\zeta_n(s)-1]^{*1}*\lim_{z\to 0}\frac{\partial^k}{\partial z^k}[\zeta_n(s)]^{*z}$$

$$[\log\zeta_n(s)]^{*1}=\sum_{k=0}\frac{B_k}{k!}[\zeta_n(s)-1]^{*1}*[\log\zeta_n(s)]^{*k}$$

$$[\log\zeta_n(s)]^{*j}=\sum_{k=0}\frac{B_k}{k!}[\zeta_n(s)-1]^{*1}*[\log\zeta_n(s)]^{*k+j}$$

```
 Sum[ BernoulliB[k]/k!Sum[  D[zeta[100/j,0,z,1],{z,k}]/.z->0,{j,2,100}],
{k,0,Log[2,100]}]
```

$$\log\zeta(s)=\sum_{k=0}\frac{B_k}{k!}(\zeta_n(s)-1)\log\zeta(s)^k$$

$$\log\zeta(s)^j=\sum_{k=0}\frac{B_k}{k!}(\zeta_n(s)-1)\log\zeta(s)^{k+j}$$

$$\Pi(n)=\sum_{j=2}\lfloor\frac{n}{j}-1\rfloor\sum_{k=0}\frac{B_k}{k!}\lim_{z\to 0}\frac{\partial^k}{\partial k}d_z(j)$$

$$\zeta_n(-k)=\sum_{j=0}^{k}\binom{k}{j}\frac{B_{k-j}}{j+1}n^{j+1}$$

$$\zeta(-k)=\frac{-B_{k+1}}{k+1}$$

$$[D^s - a](n)^{*k} = \sum_{j=1} (j+a)^{-s} [D^s - a](n(j+a)^{-1})^{*k-1}$$

$$[\log((D^s - a)+1)](n)^{*1} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} [(D^s - a)](n)^{*k}$$

$$\Pi(n) = \int_1^{n+1} \frac{\partial}{\partial a} [\log((D^0 - a)+1)](n)^{*1} \, da$$

$$\zeta(s,a)^k = \sum_{j=0}^{k} \binom{k}{j} a^{-sj} \zeta(s,a+1)^{k-j}$$

```
FullSimplify[Table[ Zeta[s,a]^k-Sum[a^(-s j)Binomial[k,j] Zeta[s, a+1]^(k-j), {j,0,k}],{k,1,5},{a,2,5},{s,2,4}]]
```

$$\zeta(s,a)^k = \sum_{j=0}^{k} (-1)^j \binom{k}{j} (a-1)^{-sj} \zeta(s,a-1)^{k-j}$$

```
FullSimplify[Table[ Zeta[s,a]^k-Sum[(-1)^j (a-1)^(-s j)Binomial[k,j] Zeta[s, a-1]^(k-j), {j,0,k}],{k,1,5},{a,2,5},{s,2,4}]]
```

$$\zeta(s,a)^z = \sum_{j=0}^{\infty} (-1)^j \binom{z}{j} (a-1)^{-sj} \zeta(s,a-1)^{z-j}$$

```
FullSimplify[Table[ Chop[Zeta[s,a]^z-Sum[(-1)^j (a-1)^(-s j)Binomial[z,j] Zeta[s, a-1]^(z-j), {j,0,Infinity}]],{z,2.5,5, .7},{a,2,5},{s,2,4}]]
```

$$\zeta(s,a)^k = \sum_{m=a+1}^{\infty} \sum_{j=1}^{k} \binom{k}{j} (m-1)^{-sj} \zeta(s,m)^{k-j}$$

$$\zeta(s,a) = \sum_{l=a}^{\infty} l^{-s}$$

$$\zeta(s,a)^2 = \sum_{m=a+1}^{\infty} 2(m-1)^{-s} \sum_{l=m}^{\infty} l^{-s} + (m-1)^{-2s}$$

$$\zeta(s,a)^2 = 2 \sum_{m=a}^{\infty} \sum_{l=m+1}^{\infty} m^{-s} l^{-s} + \sum_{m=a}^{\infty} m^{-2s}$$

$$\zeta(s,a)^3 = \sum_{m=a+1}^{\infty} 3(m-1)^{-s} \zeta(s,m)^2 + 3(m-1)^{-2s} \zeta(s,m) + (m-1)^{-3s}$$

$$[e^z D^s](n)^{*z} = \sum_{k=0} \frac{z}{k!}[D^s](n)^{*k}$$

$$[e^z D^s](n)^{*z} = e^z \sum_{k=0} \frac{z}{k!}[D^s - 1](n)^{*k}$$

$$[e^z (D^s - 1)](n)^{*z} = \sum_{k=0} \frac{z}{k!}[D^s - 1](n)^{*k}$$

$$\Pi(n) = li(n) - \log\log n - \gamma + \int_0^1 \frac{\partial}{\partial y}[\log((D^0 - 1)\cdot y + 1)](n)^{*1}\, dy$$

$$\Pi(n)=li(n)-\log\log n-\gamma+\lim_{x\to 1+}[\log((1-x^{1-0})D^0)](n)^{*1}+H_{\lfloor\frac{\log n}{\log x}\rfloor}$$
$$\psi(n)=(n-1)+\lim_{x\to 1+}\lim_{s\to 0}\frac{\partial}{\partial s}[\log((1-x^{1-s})D^s)](n)^{*k}$$
$$\Pi(n)=li(n)-\log\log n-\gamma-\lim_{x\to 1+}\lim_{s\to 0}\lim_{z\to 0}\frac{\partial}{\partial z}[D^s-x^{1-s}D^s](n)^{*z}+H_{\lfloor\frac{\log n}{\log x}\rfloor}$$
$$\psi(n)=(n-1)+\lim_{x\to 1+}\lim_{s\to 0}\lim_{z\to 0}\frac{\partial}{\partial s}\frac{\partial}{\partial z}[D^s-x^{1-s}D^s](n)^{*z}$$

$$f_0(n)=1_{[1,\infty)}(n)$$
$$f_k(n)=\sum_{j=1}(j+1)^{-s}f_{k-1}(n\cdot(j+1)^{-1})-x\cdot(jx)^{-s}f_{k-1}(n\cdot(jx)^{-1})$$
$$g_z(n)=\sum_{k=0}\binom{z}{k}f_k(n)$$
$$\Pi(n)=li(n)-\log\log n-\gamma-\lim_{x\to 1+}\lim_{s\to 0}\lim_{z\to 0}\frac{\partial}{\partial z}g_z(n)+H_{\lfloor\frac{\log n}{\log x}\rfloor}$$
$$\psi(n)=(n-1)+\lim_{x\to 1+}\lim_{s\to 0}\lim_{z\to 0}\frac{\partial}{\partial s}\frac{\partial}{\partial z}g_z(n)$$

$$f_0(n)=1_{[1,\infty)}(n)$$
$$f_k(n)=\sum_{j=1}(j+1)^{-s}f_{k-1}(n\cdot(j+1)^{-1})-x\cdot(jx)^{-s}f_{k-1}(n\cdot(jx)^{-1})$$
$$g(n)=\sum_{k=1}\frac{(-1)^{k+1}}{k}f_k(n)$$
$$\Pi(n)=li(n)-\log\log n-\gamma-\lim_{x\to 1+}\lim_{s\to 0}(g(n)+H_{\lfloor\frac{\log n}{\log x}\rfloor})$$
$$\psi(n)=(n-1)-\lim_{x\to 1+}\lim_{s\to 0}(\frac{\partial}{\partial s}g(n))$$

$$f_k(n)=\sum_{j=2}^{\lfloor n\rfloor}j^{-s}(k^{-1}-f_{k+1}(n\cdot j^{-1}))-x\sum_{j=1}^{\lfloor nx^{-1}\rfloor}(jx)^{-s}(k^{-1}-f_{k-1}(n\cdot(jx)^{-1}))$$
$$\Pi(n)=li(n)-\log\log n-\gamma-\lim_{x\to 1+}\lim_{s\to 0}(f_1(n)+H_{\lfloor\frac{\log n}{\log x}\rfloor})$$
$$\psi(n)=(n-1)+\lim_{x\to 1+}\lim_{s\to 0}(\frac{\partial}{\partial s}f_1(n))$$

$$D_k(n,z)=1+(\frac{z+1}{k}-1)\sum_{j=2}^{\lfloor n\rfloor}j^{-s}D_{k+1}(\frac{n}{j},z)$$
$$\Pi(n)=\lim_{s\to 0}\lim_{z\to 0}\frac{\partial}{\partial z}D_1(n,z)$$
$$\psi(n)=-\lim_{s\to 0}\lim_{z\to 0}\frac{\partial}{\partial s}\frac{\partial}{\partial z}D_1(n,z)$$

$$f_k(n,s)=\sum_{j=2}^{\lfloor n\rfloor}j^{-s}(k^{-1}-f_{k+1}(\frac{n}{j},s))$$
$$\Pi(n)=\lim_{s\to 0}f_1(n,s)$$
$$\psi(n)=-\lim_{s\to 0}\frac{\partial}{\partial s}f_1(n,s)$$

$$[\log D^s](n)^{*1}=\sum_{k=1}\frac{1}{k}\left(x^{((1-s)k)}[(1-x^{1-s})D^s-1]\left(\frac{n}{x^k}\right)^{*0}+(-1)^{k+1}[(1-x^{1-s})D^s-1](n)^{*k}\right)$$

$$-\lim_{s\to 0}\frac{\partial}{\partial s}[\log D^s](n)^{*1}=-\lim_{s\to 0}\frac{\partial}{\partial s}\sum_{k=1}\frac{1}{k}\left(x^{(k(1-s))}[(1-x^{1-s})D^s-1]\left(\frac{n}{x^k}\right)^{*0}+(-1)^{k+1}[(1-x^{1-s})D^s-1](n)^{*k}\right)$$

$$[(1-y)L](n)^{*0}=0$$

$$[(1-y)L](n)^{*1}=\sum_{j=2}^{n}\log j-y\sum_{j=1}^{\lfloor\frac{n}{y}\rfloor}\log jy$$

$$[(1-y)L](n)^{*k}=\sum_{j=2}^{n}[(1-y)L]\left(\frac{n}{j}\right)^{*k-1}-y\sum_{j=1}^{\lfloor\frac{n}{y}\rfloor}[(1-y)L]\left(\frac{n}{jy}\right)^{*k-1}$$

$$[(1-x^{1-s})D^s-1](n)^{*k}=\sum_{j=1}(j+1)^{-s}[(1-x^{1-s})D^s-1](n\cdot(j+1)^{-1})^{*k-1}-x\cdot(jx)^{-s}[(1-x^{1-s})D^s-1](n\cdot(jx)^{-1})^{*k-1}$$

$$-\frac{1}{k}\lim_{s\to 0}\frac{\partial}{\partial s}[(1-x^{1-s})\zeta_n(s)-1]^{*k}=[(1-y)L](n)^{*k}$$

```
D1xD[n_,k_,x_, s_]:=D1xD[n,k,x,s]=Sum[(j+1)^-s D1xD[n/(j+1),k-1,x,s]-x (j x)^-s  D1xD[n/(x j),k-1,x,s],{j,1,n}]
D1xD[n_,0,x_,s_]:=UnitStep[n-1]
L2[n_,1,b_]:=L2[n,1,b]=Sum[Log[j],{j,2,n}]-b Sum[Log[j b],{j,1,n/b}]
L2[n_,k_,b_]:=Sum[L2[n/j,k-1,b],{j,2,n}]-b Sum[L2[n/(j b),k-1,b],{j,1,n}]
{N[D[D1xD[100,3,1.5,s],s]/.s->0], -3 N[L2[100,3,1.5]]}
```

$$\psi(n)=-\sum_{k=0}^{\lfloor\frac{\log n}{\log x}\rfloor}(-1)^k[(1-x)L](n)^{*k}+\sum_{k=1}^{\lfloor\frac{\log n}{\log x}\rfloor}x^k\cdot\log x$$

$$\psi(n)=-\sum_{k=1}^{\lfloor\frac{\log n}{\log x}\rfloor}\frac{(-1)^{k+1}}{k}\lim_{s\to 0}\frac{\partial}{\partial s}[(1-x^{1-s})\zeta_n(s)-1]^{*k}+\sum_{k=1}^{\lfloor\frac{\log n}{\log x}\rfloor}x^k\cdot\log x$$

```
chebyshev[n_]:=Sum[MangoldtLambda[j],{j,2,n}]
D1xD[n_,k_,x_, s_]:=D1xD[n,k,x,s]=Sum[(j+1)^-s D1xD[n/(j+1),k-1,x,s]-x (j x)^-s  D1xD[n/(x j),k-1,x,s],
{j,1,n}];D1xD[n_,0,x_,s_]:=UnitStep[n-1]
ChebAlt[n_,c_]:=Sum[(-1)^(k)/k  (D[D1xD[n,k,c,s],s]/.s->0),{k,1,Floor[Log[n]/Log[If[c<2,c,2]]]}]+Sum[c^k Log[c],
{k,1,Floor[Log[n]/Log[c]]}]
```

$$\psi(n)=(n-1)-\lim_{x\to 1+}\sum_{k=1}^{\lfloor\frac{\log n}{\log x}\rfloor}\frac{(-1)^{k+1}}{k}\lim_{s\to 0}\frac{\partial}{\partial s}[(1-x^{1-s})\zeta_n(s)-1]^{*k}$$

$$\psi(n)=(n-1)-\lim_{x\to 1+}\lim_{s\to 0}\frac{\partial}{\partial s}[\log((1-x^{1-s})\zeta_n(s))]^{*k}$$

$$\psi(n)=(n-1)-\lim_{x\to 1+}\lim_{s\to 0}\lim_{z\to 0}\frac{\partial}{\partial s}\frac{\partial}{\partial z}[\zeta_n(s)-x^{1-s}\zeta_n(s)]^{*z}$$

$$\psi(n)=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\lim_{s\to 0}\frac{\partial}{\partial s}[D^s-1](n)^{*k}$$

$$\psi(n) = -\lim_{s\to 0} \frac{\partial}{\partial s}[\log D^s](n)^{*1}$$

$$(L-1)^1(n) = \sum_{j=2} \log j;\ (L-1)^k(n) = \sum_{j=2} (L-1)^{k-1}\left(\frac{n}{j}\right)$$

$$(L)^z(n) = \sum_{k=0}^{\infty} \binom{z}{k} (L-1)^k(n)$$

$$(L)^1(n) = \sum_{j=1} \log j;\ (L)^k(n) = \sum_{j=1} (L)^{k-1}\left(\frac{n}{j}\right)$$

$$[(D^s-1)\cdot y](x)^{*k}=y\sum_{j=1}(1+jy)^{-s}[(D^s-1)\cdot y](x(jy+1)^{-1})^{*k-1}$$

$$\Pi(n)=li(n)-\log\log n-\gamma+\int_0^1\frac{\partial}{\partial y}\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}[(D^0-1)\cdot y](n)^{*k}\,dy$$

$$\Pi(n)=li(n)-\log\log n-\gamma+\int_0^1\frac{\partial}{\partial y}[\log((D^0-1)\cdot y+1)](n)^{*1}\,dy$$

$$f_k(n,s,y)=y\sum_{j=1}^{\lfloor(n-1)y^{-1}\rfloor}(jy+1)^{-s}(k^{-1}-f_{k+1}(n(jy+1)^{-1},s,y))$$

$$\Pi(n)=li(n)-\log\log n-\gamma+\int_0^1\frac{\partial}{\partial y}\lim_{s\to 0}f_1(n,s,y)\,dy$$

$$\psi(n)=n-\log n-1-\int_0^1\frac{\partial}{\partial y}\lim_{s\to 0}\frac{\partial}{\partial s}f_1(n,s,y)\,dy$$

$$f_k(n,s) = \sum_{j=1}^{\lfloor n-1 \rfloor} (j+1)^{-s} (k^{-1} - f_{k+1}(n(j+1)^{-1,} s))$$

$$\Pi(n) = \lim_{s \to 0} f_1(n,s)$$

$$\psi(n) = -\lim_{s \to 0} \frac{\partial}{\partial s} f_1(n,s)$$

$$f_k(n,s,x) = x \sum_{j=1}^{\lfloor (n-1) x^{-1} \rfloor} (jx+1)^{-s} (k^{-1} - f_{k+1}(n(jx+1)^{-1}, s, x))$$

$$\Pi(n) = li(n) - \log\log n - \gamma + \int_0^1 \frac{\partial}{\partial x} \lim_{s \to 0} f_1(n,s,x)\, dx$$

$$\psi(n) = n - \log n - 1 - \int_0^1 \frac{\partial}{\partial x} \lim_{s \to 0} \frac{\partial}{\partial s} f_1(n,s,x)\, dx$$

$$f_k(n,s,x) = \sum_{j=1}^{\lfloor n-1 \rfloor} (j+1)^{-s} (k^{-1} - f_{k+1}(n \cdot (j+1)^{-1}), s, x) - x \sum_{j=1}^{\lfloor n x^{-1} \rfloor} (jx)^{-s} (k^{-1} - f_{k+1}(n \cdot (jx)^{-1}, s, x))$$

$$\Pi(n) = li(n) - \log\log n - \gamma - \lim_{x \to 1+} \lim_{s \to 0} (f_1(n,s,x) + H_{\lfloor \frac{\log n}{\log x} \rfloor})$$

$$\psi(n) = (n-1) + \lim_{x \to 1+} \lim_{s \to 0} (\frac{\partial}{\partial s} f_1(n,s,x))$$

$$\frac{\zeta(2s)}{\zeta(s)}=(\sum_{n=1}\frac{1}{n^{2s}})\cdot(\sum_{n=1}n^{s})$$

$$\sum_{j=1}^{n}\lambda(j)=\sum_{j=1}^{n}\mu(j)\lfloor(n\,j^{-1})^{\frac{1}{2}}\rfloor$$

$$\sum_{j=1}^{n}j^{-s}\lambda(j)=\sum_{j=1}^{n}[d^{s}](j)^{*-1}[D^{2s}](\lfloor(n\,j^{-1})^{\frac{1}{2}}\rfloor)^{*1}$$

$$\sum_{j=1}^{n}j^{-s}\lambda(j)=\sum_{j=1}^{\lfloor n^{\frac{1}{2}}\rfloor}[d^{2s}](j)^{*1}[D^{s}](n\,j^{-2})^{*-1}$$

$$\sum_{j=1}^{n}j^{-s}\mu(j)^{2}=\sum_{j=1}^{n}[d^{s}](j)^{*1}[D^{2s}](\lfloor(n\,j^{-1})^{\frac{1}{2}}\rfloor)^{*-1}$$

$$\sum_{j=1}^{n}j^{-s}\mu(j)^{2}=\sum_{j=1}^{\lfloor n^{\frac{1}{2}}\rfloor}[d^{2s}](j)^{*-1}[D^{s}](n\,j^{-2})^{*1}$$

$$\sum_{j|n}d(j^{2})=d(n)^{2}$$

$$\sum_{j|n}\mu(\frac{n}{j})d(j)^{2}=d(n^{2})$$

$$[\zeta_n(s)]^{*y} = 1 + \int_0^y \frac{\partial}{\partial z}[\zeta_n(s)]^{*z}\, dz$$

$$[\zeta_n(t)]^{*z} = 1 - \int_t^\infty \frac{\partial}{\partial s}[\zeta_n(s)]^{*z}\, ds$$

$$\Pi(n) = \int_0^\infty \frac{\partial}{\partial s}([\zeta_n(s)]^{*1} * [\zeta_n(s)]^{*-1})\, ds$$

$$\frac{\partial}{\partial s}([\zeta_n(s)]^{*1} * [\zeta_n(s)]^{*-1})$$

$$[\zeta_n(t)]^{*z} - [\zeta_n(u)]^{*z} = \int_u^t \frac{\partial}{\partial s}[\zeta_n(s)]^{*z}\, ds$$

$$[\zeta_n(s)-1]^{*k} = \sum_{m=0} \frac{1}{m!}(\lim_{x \to 0} \frac{\partial^m}{\partial x^m} \frac{x}{\log(1+x)})[\zeta_n(s)-1]^{*k-1+m} * [\log \zeta_n(s)]^{*1}$$

$$\psi(n) = -[\zeta_n(0)]^{*-1} * (\lim_{s \to 0} \frac{\partial}{\partial s}[\zeta_n(s)]^{*1})$$

$$\log \zeta(s) = s \int_1^\infty [\log \zeta_x(0)]^{*1} x^{-s-1}\, dx$$

$$[\log \zeta_x(0)]^{*1} = \lim_{t \to \infty} (2\pi i)^{-1} \int_{2-it}^{2+it} x^s s^{-1} \log \zeta(s)\, ds$$

There is no z such that

$$\zeta(s)^z = 0$$

because $n^z = 0$ never happens.

There are, however, z's such that

$$[\zeta_n(s)]^{*z} = 0$$

So here is the question. Do those z's converge as n approaches infinity? What is their long-term behavior?

$$[\zeta(s)]^{*z} = 0$$

$$[\zeta(2)]^{*\rho} = 0$$

$$\frac{\pi^2}{6} = \prod_{\rho} (1 - \frac{1}{\rho})$$

What might make more sense is to look at

$$\lim_{n \to \infty} [\eta_n(s)]^{*z} = 0$$

Here's the deal...

Given $[\eta(s)]^{*\rho} = 0$, $[\eta(s)]^{*1} = \prod_{\rho} (1 - \frac{1}{\rho})$. For $\eta(s) = 0$, it MUST be the case that at least 1 $\rho = 1$.

If $\eta(s)^1 = 0$, then $\eta(s)^2 = 0$, and $\log \eta(s)$ and $\eta(s)^{-1}$ are undefined.

This is not the case for the convolutions, though.

So, if $[\eta(s)]^{*1} = 0$, $[\eta(s)]^{*2}$ and $[\log \eta(s)]^{*1}$ and $[\eta(s)]^{*-1}$ all have definable values...?

$$f(n,k)=\sum_{j=2}^{\lfloor n\rfloor}\left(\frac{1}{k}-f\left(\frac{n}{j}\right),k+1\right)-x\sum_{j=1}^{\lfloor\frac{n}{x}\rfloor}\left(\frac{1}{k}-f\left(\frac{n}{j\cdot x},k+1\right)\right)$$

$$\Pi(n)=li(n)-\log\log n-\gamma-\lim_{x\to 1+}\left(f_1(n)+H_{\lfloor\frac{\log n}{\log x}\rfloor}\right)$$

$$p(n,j,k)=\begin{cases}\left(\left(\lfloor\frac{j}{b}\rfloor-\lfloor\frac{j-1}{b}\rfloor\right)-\frac{b+1}{b}\cdot\left(\lfloor\frac{j}{b+1}\rfloor-\lfloor\frac{j-1}{b+1}\rfloor\right)\right)\left(\frac{1}{k}-p\left(\frac{nb}{j},1+b,k+1\right)\right)+p(n,j+1,k) & \text{if } nb\geq j\\ 0 & \text{if } nb<j\end{cases}$$

$$\Pi(n)=li(n)-\log\log n-\gamma-\lim_{b\to\infty}\left(p(n,1+b,1)+H_{\lfloor\frac{\log n}{\log(b+1)-\log b}\rfloor}\right)$$

$$n-1=\Pi(n)+\frac{1}{2}\sum_{j=2}^{\lfloor n\rfloor}\Pi\left(\frac{n}{j}\right)-\frac{1}{12}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}\Pi\left(\frac{n}{j\cdot k}\right)+\frac{1}{24}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}\sum_{l=2}^{\lfloor\frac{n}{j\cdot k}\rfloor}\Pi\left(\frac{n}{j\cdot k\cdot l}\right)-\frac{19}{720}\sum_{j=2}^{\lfloor n\rfloor}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}\sum_{l=2}^{\lfloor\frac{n}{j\cdot k}\rfloor}\sum_{m=2}^{\lfloor\frac{n}{j\cdot k\cdot l}\rfloor}\Pi\left(\frac{n}{j\cdot k\cdot l\cdot m}\right)+\ldots$$

$$n-1=\Pi(n)+\frac{1}{2}\sum_{1<j\leq n}\Pi\left(\frac{n}{j}\right)-\frac{1}{12}\sum_{1<j\cdot k\leq n}\Pi\left(\frac{n}{j\cdot k}\right)+\frac{1}{24}\sum_{1<j\cdot k\cdot l\leq n}\Pi\left(\frac{n}{j\cdot k\cdot l}\right)-\frac{19}{720}\sum_{1<j\cdot k\cdot l\cdot m\leq n}\Pi\left(\frac{n}{j\cdot k\cdot l\cdot m}\right)+\ldots$$

$$n-1=\sum_{j=2}^{n}\kappa(j)\cdot\sum_{k=0}(-1)^k C_k D_k\left(\frac{n}{j}\right)$$

$$[(1-x^{1-s})\zeta_n(s)]^{*k}=\sum_{j=1} j^{-s}[(1-x^{1-s})\zeta_{n j^{-1}}(s)]^{*k-1}-x\cdot(j x)^{-s}[(1-x^{1-s})\zeta_{n(j x)^{-1}}(s)]^{*k-1}$$

$$[(1-x^{1-1})\zeta_n(1)]^{*k}=\sum_{j=1} j^{-1}[(1-x^{1-1})\zeta_{n j^{-1}}(1)]^{*k-1}-x\cdot(j x)^{-1}[(1-x^{1-1})\zeta_{n(j x)^{-1}}(1)]^{*k-1}$$

$$\lim_{n\to\infty}\sum_{j=1} j^{-1}[(1-x^0)\zeta_{n j^{-1}}(1)]^{*0}-j^{-1}[(1-x^0)\zeta_{n(j x)^{-1}}(1)]^{*0}$$

$$\lim_{n\to\infty}\sum_{j=1}^{n} j^{-1}-\sum_{j=1}^{n x^{-1}} j^{-1}=\log x$$

$$\lim_{n\to\infty}\sum_{j=\lfloor n x^{-1}\rfloor+1}^{n} j^{-1}=\log x$$

$$f_0(n,s)=1 \; if \; n \geq 1, 0 \; otherwise$$
$$f_k(n,s)=\sum_{j=2}^{\lfloor n \rfloor} (-1)^{j+1} j^{-s} f_{k-1}(n j^{-1}, s)$$

$$f_k(n,s)=0 \; if \; n<2^k$$

$$\eta_n(s)^{*z}=\sum_{k=0} \binom{z}{k} f_k(n,s)$$

$$\lim_{n \to \infty} \eta_n(s)^{*z}=\eta(s)^z \; for \; \Re s>0$$

$$\eta(s)=\sum_{j=1}^{\infty} (-1)^{j+1} j^{-s}$$

$$\eta_n(s)^{*\rho}=0$$
$$\eta_n(s)^{*z}=\prod_{\rho} (1-\frac{z}{\rho})$$

$$[f](n)^{*0}=1_{[1,\infty)}(|n|)$$

$$[\zeta_n(s)]^{*k}=\sum_{j=1} j^{-s}[\zeta_{n j^{-1}}(s)]^{*k-1}=[1+\zeta_n(s,2)]^{*k}$$

$$[\zeta_n(s)-1]^{*k}=\sum_{j=1}(j+1)^{-s}[\zeta_{n(j+1)^{-1}}(s)-1]^{*k-1}$$

$$[\zeta_n(s,a+1)]^{*k}=\sum_{j=1}(j+a)^{-s}[\zeta_{n(j+a)^{-1}}(s,a+1)]^{*k-1}$$

$$[1+\zeta_n(s,a+1)]^{*k}=[\zeta_n(s,a+1)]^{*k-1}+\sum_{j=1}(j+a)^{-s}[\zeta_{n(j+a)^{-1}}(s,a+1)]^{*k-1}$$

$$[x^{1-s}\zeta_n(s)]^{*k}=x\sum_{j=1}(jx)^{-s}[x^{1-s}\cdot\zeta_{n(jx)^{-1}}(s)]^{*k-1}$$

$$[1+x^{1-s}\zeta_n(s)]^{*k}=[1+x^{1-s}\cdot\zeta_n(s)]^{*k-1}+x\sum_{j=1}(jx)^{-s}[x^{1-s}\cdot\zeta_{n(jx)^{-1}}(s)]^{*k-1}$$

$$[x^{1-s}\cdot\zeta_n(s,a+1)]^{*k}=x\sum_{j=1}(jx+a)^{-s}[x^{1-s}\cdot\zeta_{n(jx+a)^{-1}}(s,a+1)]^{*k-1}$$

$$[1+x^{1-s}\cdot\zeta_n(s,a+1)]^{*k}=[1+x^{1-s}\cdot\zeta_n(s,a+1)]^{*k-1}+x\sum_{j=1}(jx+a)^{-s}[1+x^{1-s}\cdot\zeta_{n(jx+a)^{-1}}(s,a+1)]^{*k-1}$$

$$[(1-x^{1-s})\zeta_n(s)-1]^{*k}=\sum_{j=1}(j+1)^{-s}[(1-x^{1-s})\zeta_{n(j+1)^{-1}}(s)-1]^{*k-1}-x\cdot(jx)^{-s}[(1-x^{1-s})\zeta_{n(jx)^{-1}}(s)]^{*k-1}$$

$$[(1-x^{1-s})\zeta_n(s)]^{*k}=\sum_{j=1} j^{-s}[(1-x^{1-s})\zeta_{n j^{-1}}(s)]^{*k-1}-x\cdot(jx)^{-s}[(1-x^{1-s})\zeta_{n(jx)^{-1}}(s)]^{*k-1}$$

$$[\zeta_n(s)^z-1]^{*k}=\sum_{j=1} d_z(j+1)(j+1)^{-s}[\zeta_{n(j+1)^{-1}}(s)^{-1}-1]^{*k-1}$$

$$[\zeta_n(s)-\zeta_y(s)]^{*k}=\sum_{j=1}(j+y)^{-s}[\zeta_{n(j+y)^{-1}}(s)-\zeta_y(s)]^{*k-1}$$

$$[f](n)^{*0}=1_{[1,\infty)}(|n|)$$

$$[f_n]^{*k}=\sum_{j=1} f(j)[f_{n j^{-1}}]^{*k-1}=[1+f_n(2)]^{*k}$$

$$[f_n-1]^{*k}=\sum_{j=1} f(j+1)[f_{n(j+1)^{-1}}-1]^{*k-1}$$

$$[f_n(a+1)]^{*k}=\sum_{j=1} f(j+a)[f_{n(j+a)^{-1}}(a+1)]^{*k-1}$$

$$[1+f_n(a+1)]^{*k}=[f_n(a+1)]^{*k-1}+\sum_{j=1} f(j+a)[f_{n(j+a)^{-1}}(a+1)]^{*k-1}$$

$$[x\,\zeta_n(s)]^{*k}=x\sum_{j=1} f(j\,x)[x\cdot f_{n(j\,x)^{-1}}]^{*k-1}$$

$$[1+x\,f_n]^{*k}=[1+x\cdot f_n]^{*k-1}+x\sum_{j=1} f(j\,x)[x\cdot f_{n(j\,x)^{-1}}]^{*k-1}$$

$$[x\cdot\zeta_n(a+1)]^{*k}=x\sum_{j=1} f(j\,x+a)[x\cdot f_{n(j\,x+a)^{-1}}(a+1)]^{*k-1}$$

$$[1+x\cdot f_n(a+1)]^{*k}=[1+x\cdot f_n(a+1)]^{*k-1}+x\sum_{j=1} f(j\,x+a)[1+x\cdot f_{n(j\,x+a)^{-1}}(a+1)]^{*k-1}$$

$$[(1-x)f_n(s)-1]^{*k}=\sum_{j=1} f(j+1)[(1-x)f_{n(j+1)^{-1}}-1]^{*k-1}-x\cdot f(j\,x)[(1-x)f_{n(j\,x)^{-1}}]^{*k-1}$$

$$[(1-x)f_n]^{*k}=\sum_{j=1} f(j)[(1-x)f_{n j^{-1}}]^{*k-1}-x\cdot f(j\,x)[(1-x)f_{n(j\,x)^{-1}}]^{*k-1}$$

$$[f_n^z-1]^{*k}=\sum_{j=1} d_z(j+1)\,f(j+1)[f_{n(j+1)^{-z}}^{-1}-1]^{*k-1}$$

$$[f_n-f_y]^{*k}=\sum_{j=1} f(j+y)[f_{n(j+y)^{-1}}-f_y]^{*k-1}$$

$$[f](n)^{*0}=1_{[1,\infty)}(|n|)$$

$$[f_n]^{*k}=\sum_{j=1} f(j)[@_{n j^{-1}}]^{*k-1}$$

$$[f_n-1]^{*k}=\sum_{j=1} f(j+1)[@_{n(j+1)^{-1}}]^{*k-1}=-f(1)[@_n]^{*k-1}+\sum_{j=1} f(j)[@_{n j^{-1}}]^{*k-1}$$

$$[f_n(a+1)]^{*k}=\sum_{j=1} f(j+a)[@_{n(j+a)^{-1}}]^{*k-1}$$

$$[1+f_n(a+1)]^{*k}=[@_n]^{*k-1}+\sum_{j=1} f(j+a)[@_{n(j+a)^{-1}}]^{*k-1}$$

$$[x f_n]^{*k}=\sum_{j=1} x\cdot f(j x)[@_{n(j x)^{-1}}]^{*k-1}$$

$$[1+x f_n]^{*k}=f(1)[@_n]^{*k-1}+\sum_{j=1} x\cdot f(j x)[@_{n(j x)^{-1}}]^{*k-1}$$

$$[x\cdot f_n(a+1)]^{*k}=\sum_{j=1} x\cdot f(j x+a)[@_{n(j x+a)^{-1}}]^{*k-1}$$

$$[1+x\cdot f_n(a+1)]^{*k}=f(1)[@_n]^{*k-1}+\sum_{j=1} x\cdot f(j x+a)[@_{n(j x+a)^{-1}}]^{*k-1}$$

$$[(1-x) f_n-1]^{*k}=\sum_{j=1} f(j+1)[@_{n(j+1)^{-1}}]^{*k-1}-x\cdot f(j x)[@_{n(j x)^{-1}}]^{*k-1}$$

$$[(1-x) f_n-1]^{*k}=-f(1)[@_n]^{*k-1}+\sum_{j=1} f(j)[@_{n j^{-1}}]^{*k-1}-x\cdot f(j x)[@_{n(j x)^{-1}}]^{*k-1}$$

$$[(1-x) f_n]^{*k}=\sum_{j=1} f(j)[@_{n j^{-1}}]^{*k-1}-x\cdot f(j x)[@_{n(j x)^{-1}}]^{*k-1}$$

$$[f_n^z-1]^{*k}=\sum_{j=1} d_z(j+1) f(j+1)[@^{-z}_{n(j+1)^{-1}}]^{*k-1}$$

$$[f_n-f_y]^{*k}=\sum_{j=1} f(j+y)[@_{n(j+y)^{-1}}]^{*k-1}$$

$$[\log f_n]^{*k}=\sum_{j=1} \kappa(j) f(j)[@_{n j^{-1}}]^{*k-1}$$

$$[1+\log f_n]^{*k}=f(1)[@_n]^{*k-1}+\sum_{j=1} \kappa(j) f(j)[@_{n j^{-1}}]^{*k-1}$$

$$[f](n)^{*0}=1_{[1,\infty)}(|n|)$$

$$[f_n]^{*3}=\sum_{j=1}^{\lfloor n\cdot 1^{-1}\cdot 1^{-1}\rfloor}\sum_{k=1}^{\lfloor n\cdot 1^{-1}\cdot j^{-1}\rfloor}\sum_{l=1}^{\lfloor n\cdot j^{-1}k^{-1}\rfloor} f(j)\cdot f(k)\cdot f(l)$$

$$[f_n]^{*3}=f(1)[f_n]^{*2}+\sum_{j=1}^{\lfloor n\cdot(1+1)^{-1}\cdot(1+1)^{-1}\rfloor}\sum_{k=1}^{\lfloor n\cdot(1+1)^{-1}(j+1)^{-1}\rfloor}\sum_{l=1}^{\lfloor n(j+1)^{-1}(k+1)^{-1}\rfloor} f(j+1)\cdot f(k+1)\cdot f(l+1)$$

$$[f_n-1]^{*3}=\sum_{j=1}^{\lfloor n\cdot(1+1)^{-1}\cdot(1+1)^{-1}\rfloor}\sum_{k=1}^{\lfloor n\cdot(1+1)^{-1}(j+1)^{-1}\rfloor}\sum_{l=1}^{\lfloor n(j+1)^{-1}(k+1)^{-1}\rfloor} f(j+1)\cdot f(k+1)\cdot f(l+1)$$

$$[f_n-1]^{*3}=-f(1)[f_n-1]^{*2}+\sum_{j=1}^{\lfloor n\cdot 1^{-1}\cdot 1^{-1}\rfloor}\sum_{k=1}^{\lfloor n\cdot 1^{-1}\cdot j^{-1}\rfloor}\sum_{l=1}^{\lfloor n\cdot j^{-1}k^{-1}\rfloor} f(j)\cdot f(k)\cdot f(l)$$

$$[f_n(a+1)]^{*3}=\sum_{j=1}^{\lfloor n\cdot(1+a)^{-1}\cdot(1+a)^{-1}\rfloor}\sum_{k=1}^{\lfloor n\cdot(1+a)^{-1}(j+a)^{-1}\rfloor}\sum_{l=1}^{\lfloor n(j+a)^{-1}(k+a)^{-1}\rfloor} f(j+a)\cdot f(k+a)\cdot f(l+a)$$

$$[1+f_n(a+1)]^{*3}=f(1)[1+f_n(a+1)]^{*2}+\sum_{j=1}^{\lfloor n\cdot(1+a)^{-1}\cdot(1+a)^{-1}\rfloor}\sum_{k=1}^{\lfloor n\cdot(1+a)^{-1}(j+a)^{-1}\rfloor}\sum_{l=1}^{\lfloor n(j+a)^{-1}(k+a)^{-1}\rfloor} f(j+a)\cdot f(k+a)\cdot f(l+a)$$

$$[x f_n]^{*3}=\sum_{j=1}^{\lfloor n\cdot(x)^{-1}\cdot(x)^{-1}\rfloor}\sum_{k=1}^{\lfloor n\cdot(x)^{-1}\cdot(jx)^{-1}\rfloor}\sum_{l=1}^{\lfloor n\cdot(jx)^{-1}\cdot(kx)^{-1}\rfloor} x\cdot f(jx)\cdot x\cdot f(kx)\cdot x\cdot f(lx)$$

$$[1+x f_n]^{*3}=f(1)[1+x f_n]^{*2}+\sum_{j=1}^{\lfloor n\cdot(x)^{-1}\cdot(x)^{-1}\rfloor}\sum_{k=1}^{\lfloor n\cdot(x)^{-1}\cdot(jx)^{-1}\rfloor}\sum_{l=1}^{\lfloor n\cdot(jx)^{-1}\cdot(kx)^{-1}\rfloor} x\cdot f(jx)\cdot x\cdot f(kx)\cdot x\cdot f(lx)$$

$$[x\cdot f_n(a+1)]^{*3}=\sum_{j=1}^{\lfloor n\cdot(x+a)^{-1}\cdot(x+a)^{-1}\rfloor}\sum_{k=1}^{\lfloor n\cdot(x+a)^{-1}(jx+a)^{-1}\rfloor}\sum_{l=1}^{\lfloor n(jx+a)^{-1}(kx+a)^{-1}\rfloor} f(jx+a)\cdot f(kx+a)\cdot f(lx+a)$$

$$[1+x\cdot f_n(a+1)]^{*3}=f(1)[1+x\cdot f_n(a+1)]^{*2}+\sum_{j=1}^{\lfloor n\cdot(x+a)^{-1}\cdot(x+a)^{-1}\rfloor}\sum_{k=1}^{\lfloor n\cdot(x+a)^{-1}(jx+a)^{-1}\rfloor}\sum_{l=1}^{\lfloor n(jx+a)^{-1}(kx+a)^{-1}\rfloor} f(jx+a)\cdot f(kx+a)\cdot f(lx+a)$$

$$[(1-x)f_n-1]^{*k}=\sum_{j=1} f(j+1)[@_{n(j+1)^{-1}}]^{*k-1}-x\cdot f(jx)[@_{n(jx)^{-1}}]^{*k-1}$$

$$[(1-x)f_n-1]^{*k}=-f(1)[@_n]^{*k-1}+\sum_{j=1} f(j)[@_{nj^{-1}}]^{*k-1}-x\cdot f(jx)[@_{n(jx)^{-1}}]^{*k-1}$$

$$[(1-x)f_n]^{*k}=\sum_{j=1} f(j)[@_{nj^{-1}}]^{*k-1}-x\cdot f(jx)[@_{n(jx)^{-1}}]^{*k-1}$$

$$[f_n^z-1]^{*k}=\sum_{j=1} d_z(j+1)\,f(j+1)[@^{-z}_{n(j+1)^{-1}}]^{*k-1}$$

$$[f_n-f_y]^{*k}=\sum_{j=1} f(j+y)[@_{n(j+y)^{-1}}]^{*k-1}$$

$$[\log f_n]^{*3}=\sum_{j=1}^{\lfloor n\cdot(1+1)^{-1}\cdot(1+1)^{-1}\rfloor}\sum_{k=1}^{\lfloor n\cdot(1+1)^{-1}(j+1)^{-1}\rfloor}\sum_{l=1}^{\lfloor n(j+1)^{-1}(k+1)^{-1}\rfloor} \kappa(j+1)\cdot f(j+1)\cdot\kappa(k+1)\cdot f(k+1)\cdot\kappa(l+1)\cdot f(l+1)$$

$$[1+\log f_n]^{*k} = f(1)[@_n]^{*k-1} + \sum_{j=1} \kappa(j) f(j)[@_{n j^{-1}}]^{*k-1}$$

[x cdot f_n(a+1)]^{"*"3} = sum from j = 1 to lfloor n cdot(x+a)^-1 cdot (x+a)^-1 rfloor sum from k = 1 to lfloor n cdot (x+a)^-1 (j x+a)^-1 rfloor sum from l = 1 to lfloor n (j x+a)^-1 (k x+a)^-1 rfloor f(j x+a) cdot f(k x+a) cdot f(l x+a)

$$(f * f)(m) = \sum_{j|m} f(j) f(m j^{-1})$$

$$[f_n]^{*2} = \sum_{m=1}^{n} (f * f)(m)$$

$$[f_n]^{*2} = \sum_{m=1}^{n} \sum_{j|m} f(j) f(m j^{-1})$$

$$[f_n]^{*2} = \sum_{j=1}^{\lfloor n \cdot 1^{-1} \cdot 1^{-1} \rfloor} \sum_{k=1}^{\lfloor n \cdot 1^{-1} \cdot j^{-1} \rfloor} f(j) \cdot f(k)$$

$$[f_n(a+1)]^{*2} = \sum_{j=1}^{\lfloor n \cdot (1+a)^{-1} \cdot (1+a)^{-1} \rfloor} \sum_{k=1}^{\lfloor n \cdot (1+a)^{-1} (j+a)^{-1} \rfloor} f(j+a) \cdot f(k+a)$$

What $[f_n]^{*2}$ represents:

Given all solutions (j,k) to $|n \cdot j^{-1} \cdot k^{-1}| \geq 1$, the sum of $f(j) \cdot f(k)$.

Notably, every $j \cdot k$ is an integer.

What $[f_n(a+1)]^{*2}$ represents:

Given all solutions (j,k) to $|n \cdot (j+a)^{-1} \cdot (k+a)^{-1}| \geq 1$, the sum of $f(j+a) \cdot f(k+a)$.

Notably, $(j+a) \cdot (k+a)$ is only an integer if a is an integer.

Let's suppose we wanted to somehow split up the calculation of $[f_{10}(\frac{1}{2}+1)]^{*2}$. We would have

$(\frac{1}{2}+1) \cdot (\frac{1}{2}+1) = \frac{9}{4}$, $(\frac{1}{2}+1) \cdot (\frac{1}{2}+2) = \frac{15}{4}$, $(\frac{1}{2}+1) \cdot (\frac{1}{2}+3) = \frac{21}{4}$, $(\frac{1}{2}+1) \cdot (\frac{1}{2}+4) = \frac{27}{4}$, …, $(\frac{1}{2}+1) \cdot (\frac{1}{2}+6) = \frac{39}{4}$
$(\frac{1}{2}+2) \cdot (\frac{1}{2}+1) = \frac{15}{4}$

So, for $[f_{10}(\frac{1}{2}+1)]^{*2}$, the smallest possible unit of change is $\frac{1}{4}$. More generally, if we have $[f_n(a+1)]^{*k}$ where a is a rational fraction of the form $\frac{b}{c}$, then the smallest unit of change will be c^{-k}. If we have $[1+f_n(a+1)]^{*k}$, the smallest unit will be $c^{-\log_2 n}$.

$$\Pi(n)=$$
$$li(n)-\log\log n-\gamma-\int_1^{\infty}\frac{\partial}{\partial y}\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}[y^{-1}\cdot\zeta_n(0,1+y)]^{*k}\,dy$$

$$[1+x\cdot f_n(a+1)]^{*3}=f(1)[1+x\cdot f_n(a+1)]^{*2}+\sum_{j=1}^{\lfloor n\cdot(x+a)^{-1}\cdot(x+a)^{-1}\rfloor}\sum_{k=1}^{\lfloor n\cdot(x+a)^{-1}(jx+a)^{-1}\rfloor}\sum_{l=1}^{\lfloor n(jx+a)^{-1}(kx+a)^{-1}\rfloor}f(jx+a)\cdot f(kx+a)\cdot f(lx+a)$$

$$[y^{s-1}\cdot\zeta_n(s,1+y)]^{*k}=y^{k(s-1)}\zeta_{n\,y^k}(s,y+1)$$

$$[1+y^{s-1}\cdot\zeta_n(s,1+y)]^{*z}=\sum_{k=0}^{\infty}\binom{z}{k}[y^{s-1}\cdot\zeta_n(s,1+y)]^{*k}$$

$$[\zeta_n(s,y)]^{*k}=\sum_{j=0}^{k}\binom{k}{j}[\zeta_{n\cdot y^{j-k}}(s,y+1)]^{*j}$$

$$[\zeta_n(s,y+1)]^{*k}=\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}[\zeta_{n\,y^{j-k}}(s,y)]^{*j}$$

$$D_z(n,s,y)=1+\binom{z}{1}y^{(s-1)}\sum_{j=y+1}^{\lfloor n\,y\rfloor}j^{-s}+\binom{z}{2}y^{2(s-1)}\sum_{j=y+1}^{\lfloor\frac{n\,y^2}{y+1}\rfloor}\sum_{k=y+1}^{\lfloor\frac{n\,y^2}{j}\rfloor}(j\cdot k)^{-s}+\binom{z}{3}y^{3(s-1)}\sum_{j=y+1}^{\lfloor\frac{n\,y^3}{(y+1)^2}\rfloor}\sum_{k=y+1}^{\lfloor\frac{n\,y^3}{j(y+1)}\rfloor}\sum_{l=y+1}^{\lfloor\frac{n\,y^3}{j\,k}\rfloor}(j\,k\,l)^{-s}+\ldots$$

$$D_z(n,s,1)=\binom{z}{0}1+\binom{z}{1}\sum_{j=2}^{\lfloor n\rfloor}j^{-s}+\binom{z}{2}\sum_{j=2}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}(j\cdot k)^{-s}+\binom{z}{3}\sum_{j=2}\sum_{k=2}\sum_{l=2}^{\lfloor\frac{n}{j\cdot k}\rfloor}(j\cdot k\cdot l)^{-s}+\ldots$$

$$D_z(n,s,2)=1+\binom{z}{1}2^{(s-1)}\sum_{j=3}^{\lfloor 2n\rfloor}j^{-s}+\binom{z}{2}4^{s-1}\sum_{j=3}\sum_{k=3}^{\lfloor\frac{4n}{j}\rfloor}(j\cdot k)^{-s}+\binom{z}{3}8^{s-1}\sum_{j=3}\sum_{k=3}\sum_{l=3}^{\lfloor\frac{8n}{jk}\rfloor}(j\,k\,l)^{-s}+\ldots$$

$$D_z(n,s,3)=1+\binom{z}{1}3^{(s-1)}\sum_{j=4}^{\lfloor 3n\rfloor}j^{-s}+\binom{z}{2}9^{s-1}\sum_{j=4}\sum_{k=4}^{\lfloor\frac{9n}{j}\rfloor}(j\cdot k)^{-s}+\binom{z}{3}27^{s-1}\sum_{j=4}\sum_{k=4}\sum_{l=4}^{\lfloor\frac{27n}{j\,k}\rfloor}(j\,k\,l)^{-s}+\ldots$$

$$D_z(n,s,y)=\sum_{k=0}^{\infty}\binom{z}{k}y^{k(s-1)}\zeta_{n\,y^k}(s,y+1)$$

$$D_z(n,s,1)=\sum_{k=0}^{\infty}\binom{z}{k}\zeta_n(s,1+1)$$

$$D_z(n,s,2)=\sum_{k=0}^{\infty}\binom{z}{k}2^{k(s-1)}\zeta_{2^k n}(s,2+1)$$

$$D_z(n,s,3)=\sum_{k=0}^{\infty}\binom{z}{k}3^{k(s-1)}\zeta_{3^k n}(s,3+1)$$

$$D_{z,\lambda}(n,s)=\sum_{j=1}^{\lfloor n^{\frac{1}{2}}\rfloor}\left(D_z(j,2s)-D_z(j-1,2s)\right)\cdot D_{-z}\left(\frac{n}{j^2},s\right)$$

$$D_{z,\lambda}(n,s)=D_{-z}(n,s)+\sum_{j=2}^{\lfloor n^{\frac{1}{2}}\rfloor}d_z(j,2s)\cdot D_{-z}\left(\frac{n}{j^2},s\right)$$