```
faxc[n_, t_] := Sum[(n / j) ^ (1 / 2) (t (E ^ (t Log[n / j] I) + E ^ (-t Log[n / j] I) ) -
      (1 / (2 I)) ((n / j) ^ (t I) - (n / j) ^ (-t I))), {j, 1, n}]

fa[n_, t_] := Sum[j ^ (-1 / 2) (2 t Cos[t Log[n / j]] - Sin[t Log[n / j]]), {j, 1, n}]
fa2[n_, t_] := fa[n, t] / ((-t I + 1 / 2) n ^ (t I ) + (-t I - 1 / 2) n ^ (-t I)) / I
fa3[n_, t_] := fa[n, t] / (2 t Cos[t Log[n]] - Sin[t Log[n]])
fa4[n_, s_] := Sum[j ^ (-1 / 2)
    ((s + 1 / 2) (n / j) ^ -s + (s - 1 / 2) (n / j) ^s) / ((s + 1 / 2) n ^ -s + (s - 1 / 2) n ^s), {j, 1, n}]
fa4a[n_, s_] := Sum[j ^ (-1 / 2) ((s I + 1 / 2) (n / j) ^ - (s I) + (s I - 1 / 2) (n / j) ^ (s I)) /
      ((s I + 1 / 2) n ^ - (s I) + (s I - 1 / 2) n ^ (s I)), {j, 1, n}]
fa4b[n_, s_] := -I Sum[j ^ (-1 / 2) ((s I + 1 / 2) (n / j) ^ - (s I) + (s I - 1 / 2) (n / j) ^ (s I)),
    {j, 1, n}]
fa2a[n_, t_] := fa[n, t] / (((t I + 1 / 2) n ^ - (t I) + (t I - 1 / 2) n ^ ( t I)) / I)
```

fa2b[n_, t_] := -fa[n, t] $\Big/ \Big(n^{-\dot{\imath}\,t} \Big(\frac{1}{2} \dot{\imath} - t \Big) \Big)$

```
fax[n_, t_] := Sum[(n / j) ^ (1 / 2) (2 t Cos[t Log[n / j]] - Sin[t Log[n / j]]), {j, 1, n}]
fax1[n_, t_] := Sum[(n / j) ^ (1 / 2) (2 t Cos[t Log[n / j]]), {j, 1, n}]
fax2[n_, t_] := Sum[(n / j) ^ (1 / 2) (Sin[t Log[n / j]]), {j, 1, n}]
```

fa2c[n_, t_] := -fax[n, t] $\Big/ \Big(n^{1/2-\dot{\imath}\,t} \Big(\frac{1}{2} \dot{\imath} - t \Big) \Big)$

fa2ca[n_, t_] := -1 $\Big/ \Big(n^{1/2-\dot{\imath}\,t} \Big(\frac{1}{2} \dot{\imath} - t \Big) \Big)$

fa2[100 000, 30 + .2 I]

0.136341 + 0.538241 $\dot{\imath}$

Zeta[.7 + 30 I]

0.145667 - 0.547036 $\dot{\imath}$

fa4[100 000, .2 + 30 I]

0.136341 - 0.538241 $\dot{\imath}$

fa4a[100 000, .2 I - 30]

0.136341 - 0.538241 $\dot{\imath}$

fa2a[100 000, 30 + .2 I]

0.136341 + 0.538241 $\dot{\imath}$

fa3[100 000, .2 I - 30]

0.136341 - 0.538241 $\dot{\imath}$

fa2b[100 000, 30 + .2 I]

0.139429 + 0.542857 $\dot{\imath}$

FullSimplify[(((t I + 1 / 2) n ^ - (t I)) I)]

$\frac{1}{2} n^{-\dot{\imath}\,t} (\dot{\imath} - 2 t)$

fa2c[100 000, 30 + .2 I]

0.139429 + 0.542857 $\dot{\imath}$

2

NB 2017-01-03 00-48-10 Trying to think about convergence for that trig sum that goes to zeta zeroes and doesnt converge otherwise seeming
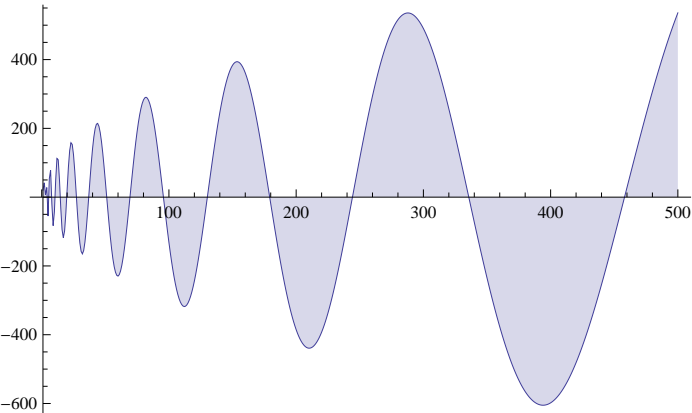
fa2c[100 000, N@Im@ZetaZero@1]

0.00261645 − 0.00177248 i

fax[100 000, N@Im@ZetaZero@1]

14.1347

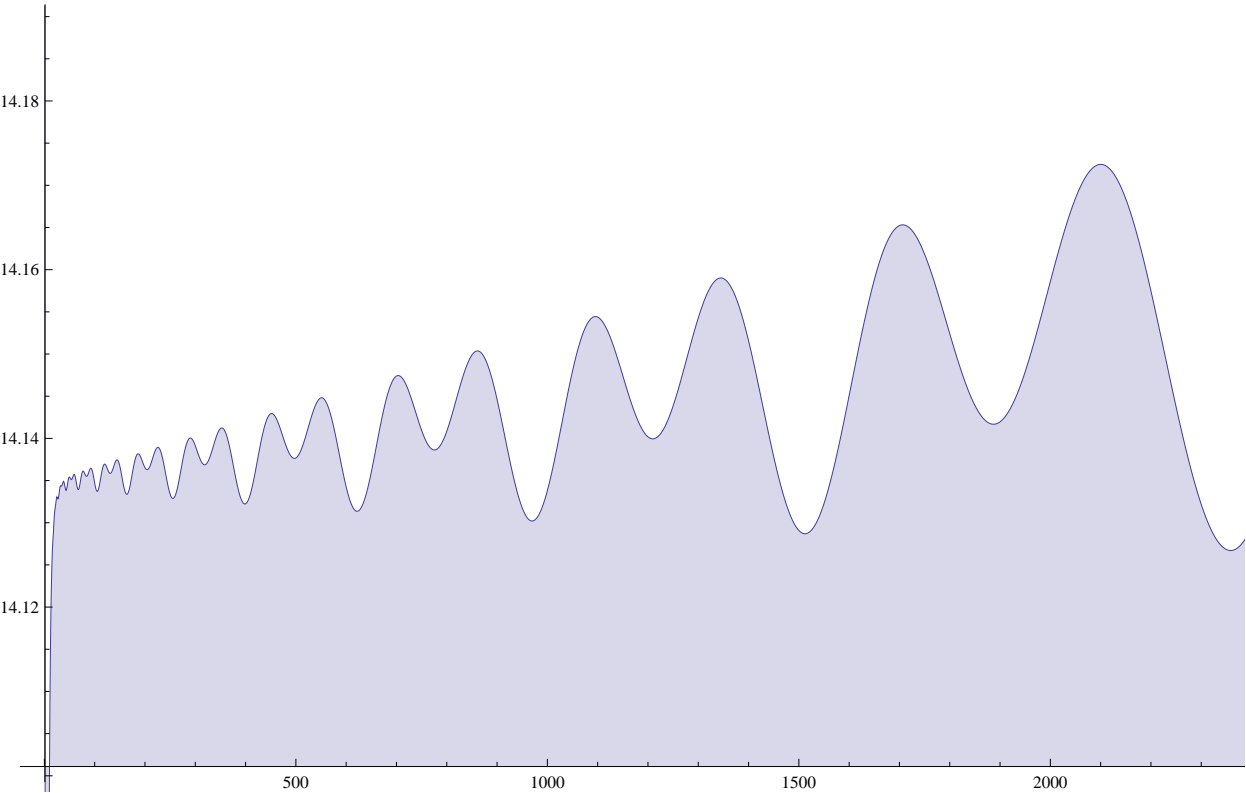**DiscretePlot[Re@fax[n, 10], {n, 1, 500}]**

Zeta[.5 + 10 I]

1.5449 − 0.115336 i

DiscretePlot[Abs@fax[n, N@Im@ZetaZero@1 + .001 I], {n, 1, 2400}]

N@Im@ZetaZero@1

14.1347

NB 2017-01-03 00-48-10 Trying to think about convergence for that trig sum that goes to zeta zeroes and doesnt converge otherwise seemingly.nb

| 3

```mathematica
DiscretePlot[Re@fax1[n, N@Im@ZetaZero@10 000], {n, 1, 2400}]
```

```mathematica
DiscretePlot[Re@fax2[n, N@Im@ZetaZero@10 000], {n, 1, 2400}]
```

```mathematica
fax2[240 000, N@Im@ZetaZero@1 + .001 I]
```

16 958.2 - 0.890911 ⅈ

```mathematica
fax1[240 000, N@Im@ZetaZero@1 + .001 I]
```

16 972.5 + 5.58088 ⅈ

```mathematica
fax[240 000, N@Im@ZetaZero@1 + .001 I]
```

14.2402 + 6.47179 ⅈ

```mathematica
faxa[n_, t_] := Sum[(n / j) ^ (1 / 2) (2 t Cos[t Log[n / j]] - Sin[t Log[n / j]]), {j, 1, n}]
faxb[n_, t_] := Sum[(n / j) ^ (1 / 2) (t (E ^ (t Log[n / j] I) + E ^ (-t Log[n / j] I)) -
      (1 / (2 I)) (E ^ (t Log[n / j] I) - E ^ (-t Log[n / j] I))), {j, 1, n}]
faxc[n_, t_] := Sum[(n / j) ^ (1 / 2) (t (E ^ (t Log[n / j] I) + E ^ (-t Log[n / j] I)) -
      (1 / (2 I)) ((n / j) ^ (t I) - (n / j) ^ (-t I))), {j, 1, n}]
faxd[n_, t_] := Sum[(n / j) ^ (1 / 2) (t ((n / j) ^ (t I) + (n / j) ^ (-t I)) -
      (1 / (2 I)) ((n / j) ^ (t I) - (n / j) ^ (-t I))), {j, 1, n}]
faxe[n_, t_, a_] := Sum[(n / j) ^ (1 / 2) ((t + a I) ((n / j) ^ ((t + a I) I) + (n / j) ^ (- (t + a I) I)) -
      (1 / (2 I)) ((n / j) ^ ((t + a I) I) - (n / j) ^ (- (t + a I) I))), {j, 1, n}]
faxf[n_, t_, a_] := Sum[(n / j) ^ (1 / 2) ((t + a I) ((n / j) ^ (t I - a) + (n / j) ^ (-t I + a)) -
      (1 / (2 I)) ((n / j) ^ (t I - a) - (n / j) ^ (-t I + a))), {j, 1, n}]
```

$$faxg[n\_, t\_, a\_] := Sum\left[-\frac{1}{2} \, ⅈ \, \left(\frac{n}{j}\right)^{\frac{1}{2}+a-ⅈ\,t} + ⅈ\,a\,\left(\frac{n}{j}\right)^{\frac{1}{2}+a-ⅈ\,t} + \left(\frac{n}{j}\right)^{\frac{1}{2}+a-ⅈ\,t} t + \right.$$

$$\left.\frac{1}{2} \, ⅈ \, \left(\frac{n}{j}\right)^{\frac{1}{2}-a+ⅈ\,t} + ⅈ\,a\,\left(\frac{n}{j}\right)^{\frac{1}{2}-a+ⅈ\,t} + \left(\frac{n}{j}\right)^{\frac{1}{2}-a+ⅈ\,t} t, \{j, 1, n\}\right]$$

$$faxh[n\_, t\_, a\_] := Sum\left[\frac{1}{2}\left(\frac{n}{j}\right)^{\frac{1}{2}+a-ⅈ\,t} (-ⅈ + 2\,ⅈ\,a + 2\,t) + \frac{1}{2}\left(\frac{n}{j}\right)^{\frac{1}{2}-a+ⅈ\,t} (ⅈ + 2\,ⅈ\,a + 2\,t), \{j, 1, n\}\right]$$

$$faxi[n\_, t\_, a\_] := Sum\left[\frac{1}{2}\left(\frac{n}{j}\right)^{\frac{1}{2}} \left(\left(\frac{n}{j}\right)^{a-ⅈ\,t} (-ⅈ + 2\,ⅈ\,a + 2\,t) + \left(\frac{n}{j}\right)^{-a+ⅈ\,t} (ⅈ + 2\,ⅈ\,a + 2\,t)\right), \{j, 1, n\}\right]$$

```mathematica
faxj[n_, s_] := (-1 / 2 - s) n ^ (1 / 2 - s) HarmonicNumber[n, 1 / 2 - s] -
   (-1 / 2 + s) n ^ (1 / 2 + s) HarmonicNumber[n, 1 / 2 + s]
```

$$faxk[n\_, A\_, f\_] := \left(-\frac{1}{2} - A - ⅈ\,f\right) n^{\frac{1}{2}-A-ⅈ\,f} HarmonicNumber\left[n, \frac{1}{2} - A - ⅈ\,f\right] -$$

$$\left(-\frac{1}{2} + A + ⅈ\,f\right) n^{\frac{1}{2}+A+ⅈ\,f} HarmonicNumber\left[n, \frac{1}{2} + A + ⅈ\,f\right]$$

$$faxl[n\_, A\_, f\_] := Sum\left[\left(-\frac{1}{2} - A - ⅈ\,f\right) (n / j)^{\frac{1}{2}-A-ⅈ\,f} - \left(-\frac{1}{2} + A + ⅈ\,f\right) (n / j)^{\frac{1}{2}+A+ⅈ\,f}, \{j, 1, n\}\right]$$

$$faxm[n\_, A\_, f\_] :=$$

$$Sum\left[(n / j) ^ (1 / 2) \left(\left(-\frac{1}{2} - A - ⅈ\,f\right) (n / j)^{-A-ⅈ\,f} - \left(-\frac{1}{2} + A + ⅈ\,f\right) (n / j)^{A+ⅈ\,f}\right), \{j, 1, n\}\right]$$

NB 2017-01-03 00-48-10 Trying to think about convergence for that trig sum that goes to zeta zeroes and doesnt converge otherwise seeming

faxm[100, .2, 12]

$-284.609 + 104.381\,\dot{\mathbb{i}}$

faxj[100, .2 + 12 I]

$-284.609 + 104.381\,\dot{\mathbb{i}}$

faxa[100, 12. + .2 I]

$-104.381 + 284.609\,\dot{\mathbb{i}}$

faxj[100, .2 + 12 I]

$-284.609 + 104.381\,\dot{\mathbb{i}}$

(-1 / 2 - s) n ^ (1 / 2 - s) HarmonicNumber[n, 1 / 2 - s] -
(-1 / 2 + s) n ^ (1 / 2 + s) HarmonicNumber[n, 1 / 2 + s] /. s → A + f I

$$\left(-\frac{1}{2} - A - \dot{\mathbb{i}}\,f\right) n^{\frac{1}{2}-A-\dot{\mathbb{i}}\,f}\, \mathrm{HarmonicNumber}\left[n, \frac{1}{2} - A - \dot{\mathbb{i}}\,f\right] -$$

$$\left(-\frac{1}{2} + A + \dot{\mathbb{i}}\,f\right) n^{\frac{1}{2}+A+\dot{\mathbb{i}}\,f}\, \mathrm{HarmonicNumber}\left[n, \frac{1}{2} + A + \dot{\mathbb{i}}\,f\right]$$

faxj2[n_, s_] := -n^{1-s} s HarmonicNumber[n, 1 - s] - n^s (-1 + s) HarmonicNumber[n, s]

faxk2[n_, s_] := Sum$\left[- (n / j)^{1-s}\, s - (n / j)^s\, (-1 + s)\, , \{j, 1, n\}\right]$

faxl2[n_, s_] := Sum$\left[(n / j) ^ (1 / 2)\, \left(- (n / j)^{1/2-s}\, s - (n / j)^{s-1/2}\, (-1 + s)\right)\, , \{j, 1, n\}\right]$

faxm2[n_, s_] := -Sum$\left[(n / j) ^ (1 / 2)\, \left(s\, (n / j)^{-(s-1/2)}\, + (s - 1)\, (n / j)^{s-1/2}\right)\, , \{j, 1, n\}\right]$

faxn2[n_, s_] :=
-Sum$\left[(n / j) ^ (1 / 2)\, \left(s\, \left(E^{-(\mathrm{Log}[n/j]\,(s-1/2))}\, + E^{\mathrm{Log}[n/j]\,(s-1/2)}\right) - (n / j)^{s-1/2}\right)\, , \{j, 1, n\}\right]$

faxo2[n_, s_] := -Sum$\left[(n / j) ^ (1 / 2)\, \left(2\, s\, \mathrm{Cosh}[(s - 1 / 2)\, \mathrm{Log}[n / j]] - (n / j)^{s-1/2}\right)\, , \{j, 1, n\}\right]$

faxo2[100, .7 + 12 I]

$-284.609 + 104.381\,\dot{\mathbb{i}}$

faxj2[100, .7 + 12 I]

$-284.609 + 104.381\,\dot{\mathbb{i}}$