```
ClearAll["Global`*"]
```

```
K[n_] := FullSimplify[MangoldtLambda[n] / Log[n]] / n
P[n_, 0] := 1
P[n_, k_] := P[n, k] = Sum[ K[j] P[Floor[n / j], k - 1], {j, 2, n}]
DD[n_, z_] := Sum[ z^k / (k!) P[n, k], {k, 0, Log[2, n]}]
```

```
N[DD[1000, 1]]
```

7.48547

```
N[HarmonicNumber[1000]]
```

7.48547

```
DD[10, 1]
```

$$\frac{7381}{2520}$$

```
DD[10, -1]
```

$$\frac{19}{210}$$

```
Table[{n, DD[n, -1] - DD[n - 1, -1]}, {n, 2, 20}] // TableForm
```

2	$\frac{1}{2}$
3	$-\frac{1}{3}$
4	$\frac{1}{4}$
5	$-\frac{1}{5}$
6	$-\frac{1}{6}$
7	$-\frac{1}{7}$
8	$\frac{1}{8}$
9	0
10	$-\frac{1}{10}$
11	$-\frac{1}{11}$
12	$-\frac{1}{12}$
13	$-\frac{1}{13}$
14	$-\frac{1}{14}$
15	$\frac{1}{15}$
16	$\frac{1}{16}$
17	$-\frac{1}{17}$
18	0
19	$-\frac{1}{19}$
20	$-\frac{1}{20}$

```
K[n_] := MangoldtLambda[n] / Log[n]
```

```
N[Sum[ K[n] / n^2, {n, 2, 278700}]]
```

```
Pi^3 / 0.4977000362024152`
```

62.2991

```
N[Sum[ K[n] / n^2, {n, 2, 478 700}]]
```

$Aborted

```
N[Log[Pi^2 / 6]]
```

0.4977

```
d2[n_, k_] := d2[n, k] = Sum[d2[j, k - 1] d2[n / j, 1], {j, Divisors[n]}];
d2[n_, 1] := 1; d2[1, 1] := 0; d2[n_, 0] := 0; d2[1, 0] := 1
```

```
N[Sum[d2[n, 2] / n, {n, 2, 100}]]
```

7.0819

```
N[Sum[d2[n, 2] / n, {n, 2, 1000}]]
```

18.3475

```
N[Sum[d2[n, 2] / n, {n, 2, 10 000}]]
```

34.9535

```
N[Sum[d2[n, 2] / n, {n, 2, 100 000}]]
```

56.8633

```
N[Sum[d2[n, 3] / n, {n, 2, 100}]]
```

6.12163

```
N[Sum[d2[n, 3] / n, {n, 2, 1000}]]
```

29.6216

```
N[Sum[d2[n, 3] / n, {n, 2, 10 000}]]
```

83.1143

```
N[Sum[d2[n, 3] / n, {n, 2, 100 000}]]
```

178.695

```
N[Sum[d2[n, 4] / n, {n, 2, 100}]]
```

2.85547

```
N[Sum[d2[n, 4] / n, {n, 2, 1000}]]
```

29.9257

```
N[Sum[d2[n, 4] / n, {n, 2, 10 000}]]
```

131.372

```
N[Sum[d2[n, 4] / n, {n, 2, 100 000}]]
```

384.626

```
ClearAll["Global`*"]
ta := {0, 1, 0, -1}
```

```
pp[n_] := ta[[Mod[n, 4] + 1]] / n
```

```
pp[5]
```

$$\frac{1}{5}$$

```
DL[n_, 0] := 1;
DL[n_, k_] := DL[n, k] = Sum[ pp[j] DL[Floor[n / j], k - 1], {j, 1, n}]
Dl2[n_, k_] := Sum[(-1)^j Binomial[k, j] DL[n, k - j], {j, 0, k}]
Lin[n_] := Sum[(-1)^(k + 1) / k Dl2[n, k], {k, 1, Log[2, n]}]
```

```
0.0460214
```

```
N[Pi / 4 - 1]
```

```
-0.214602
```

```
N[Lin[10 000]]
```

```
-0.241173
```

```
N[Log[Pi / 4]]
```

```
-0.241564
```

```
Expand[(Pi / 4 - 1)^2]
```

$$N\left[1 - \frac{\pi}{2} + \frac{\pi^2}{16}\right]$$

```
N[Dl2[100 000, 2]]
```

```
0.0460539
```

```
0.0460214
```

```
FullSimplify[Expand[(Pi / 4 - 1)^2]]
```

$$\frac{1}{16}(-4+\pi)^2$$

```
Expand[(Pi / 4 - 1)^3]
N[Expand[(Pi / 4 - 1)^3]]
N[Dl2[400 000, 3]]
```

$$-1 + \frac{3\pi}{4} - \frac{3\pi^2}{16} + \frac{\pi^3}{64}$$

```
-0.00988326
```

```
$Aborted
```

```
N[DL[100 000, 3]]
```

```
0.485593
```

```
N[Pi^3 / 64]
```

```
0.484473
```

```
d[n_, k_] := d[n, k] = Sum[d[j, k - 1] d[n / j, 1], {j, Divisors[n]}];
d[n_, 1] := pp[n]; d[n_, 0] := 0; d[1, 0] := 1
d2[n_, k_] := d2[n, k] = Sum[(-1)^j Binomial[k, j] d[n, k - j], {j, 0, k}]
dz[n_, z_] := dz[n, z] = Sum[FactorialPower[z, a] / a! d2[n, a], {a, 0, Log[2, n]}]
ln[n_] := Sum[(-1)^(k + 1) / k d2[n, k], {k, 1, Log[2, n]}]
```

```
d2[7, 1]
```

$$-\frac{1}{7}$$

```
Sum[d[n, 5], {n, 1, 100}]
```

$$-\frac{109\,815\,795\,307\,530\,848\,181\,049\,380\,367\,132\,029\,191}{217\,876\,172\,592\,851\,491\,139\,168\,152\,922\,850\,948\,615}$$

```
N[DL[10 000, 2]]
```

```
0.614428
```

```
(Pi / 4)^2
```

```
N[π^2 / 16]
```

```
0.61685
```

```
Table[d[n, 2], {n, 1, 100}]
```

$$\left\{1, 0, -\frac{2}{3}, 0, \frac{2}{5}, 0, -\frac{2}{7}, 0, \frac{1}{3}, 0, -\frac{2}{11}, 0, \frac{2}{13}, 0, -\frac{4}{15}, 0, \frac{2}{17}, 0, -\frac{2}{19}, 0,\right.$$
$$\frac{4}{21}, 0, -\frac{2}{23}, 0, \frac{3}{25}, 0, -\frac{4}{27}, 0, \frac{2}{29}, 0, -\frac{2}{31}, 0, \frac{4}{33}, 0, -\frac{4}{35}, 0, \frac{2}{37}, 0, -\frac{4}{39}, 0,$$
$$\frac{2}{41}, 0, -\frac{2}{43}, 0, \frac{2}{15}, 0, -\frac{2}{47}, 0, \frac{3}{49}, 0, -\frac{4}{51}, 0, \frac{2}{53}, 0, -\frac{4}{55}, 0, \frac{4}{57}, 0, -\frac{2}{59}, 0,$$
$$\frac{2}{61}, 0, -\frac{2}{21}, 0, \frac{4}{65}, 0, -\frac{2}{67}, 0, \frac{4}{69}, 0, -\frac{2}{71}, 0, \frac{2}{73}, 0, -\frac{2}{25}, 0, \frac{4}{77}, 0, -\frac{2}{79}, 0,$$
$$\left.\frac{5}{81}, 0, -\frac{2}{83}, 0, \frac{4}{85}, 0, -\frac{4}{87}, 0, \frac{2}{89}, 0, -\frac{4}{91}, 0, \frac{4}{93}, 0, -\frac{4}{95}, 0, \frac{2}{97}, 0, -\frac{2}{33}, 0\right\}$$

```
N[DL[100 000, 3]]
```

```
0.485593
```

```
N[π^3 / 64]
```

```
0.484473
```

Table[d[n, 3], {n, 1, 100}]

$$\left\{1, 0, -1, 0, \frac{3}{5}, 0, -\frac{3}{7}, 0, \frac{2}{3}, 0, -\frac{3}{11}, 0, \frac{3}{13}, 0, -\frac{3}{5}, 0, \frac{3}{17}, 0, -\frac{3}{19}, 0,\right.$$
$$\frac{3}{7}, 0, -\frac{3}{23}, 0, \frac{6}{25}, 0, -\frac{10}{27}, 0, \frac{3}{29}, 0, -\frac{3}{31}, 0, \frac{3}{11}, 0, -\frac{9}{35}, 0, \frac{3}{37}, 0, -\frac{3}{13}, 0,$$
$$\frac{3}{41}, 0, -\frac{3}{43}, 0, \frac{2}{5}, 0, -\frac{3}{47}, 0, \frac{6}{49}, 0, -\frac{3}{17}, 0, \frac{3}{53}, 0, -\frac{9}{55}, 0, \frac{3}{19}, 0, -\frac{3}{59}, 0,$$
$$\frac{3}{61}, 0, -\frac{2}{7}, 0, \frac{9}{65}, 0, -\frac{3}{67}, 0, \frac{3}{23}, 0, -\frac{3}{71}, 0, \frac{3}{73}, 0, -\frac{6}{25}, 0, \frac{9}{77}, 0, -\frac{3}{79}, 0,$$
$$\left.\frac{5}{27}, 0, -\frac{3}{83}, 0, \frac{9}{85}, 0, -\frac{3}{29}, 0, \frac{3}{89}, 0, -\frac{9}{91}, 0, \frac{3}{31}, 0, -\frac{9}{95}, 0, \frac{3}{97}, 0, -\frac{2}{11}, 0\right\}$$

dz[5, 3]

$$\frac{3}{5}$$

N[Sum[dz[n, -1], {n, 1, 12000}]]

1.27523

N[4 / Pi]

1.27324

Table[dz[n, -1], {n, 1, 100}]

$$\left\{1, 0, \frac{1}{3}, 0, -\frac{1}{5}, 0, \frac{1}{7}, 0, 0, 0, \frac{1}{11}, 0, -\frac{1}{13}, 0, -\frac{1}{15}, 0, -\frac{1}{17}, 0, \frac{1}{19}, 0, \frac{1}{21}, 0, \frac{1}{23}, 0, 0, 0, 0,\right.$$
$$0, -\frac{1}{29}, 0, \frac{1}{31}, 0, \frac{1}{33}, 0, -\frac{1}{35}, 0, -\frac{1}{37}, 0, -\frac{1}{39}, 0, -\frac{1}{41}, 0, \frac{1}{43}, 0, 0, 0, \frac{1}{47}, 0, 0, 0, -\frac{1}{51},$$
$$0, -\frac{1}{53}, 0, -\frac{1}{55}, 0, \frac{1}{57}, 0, \frac{1}{59}, 0, -\frac{1}{61}, 0, 0, 0, \frac{1}{65}, 0, \frac{1}{67}, 0, \frac{1}{69}, 0, \frac{1}{71}, 0, -\frac{1}{73}, 0, 0, 0,$$
$$\left.\frac{1}{77}, 0, \frac{1}{79}, 0, 0, 0, \frac{1}{83}, 0, \frac{1}{85}, 0, -\frac{1}{87}, 0, -\frac{1}{89}, 0, -\frac{1}{91}, 0, \frac{1}{93}, 0, -\frac{1}{95}, 0, -\frac{1}{97}, 0, 0, 0\right\}$$

N[Sum[ln[n], {n, 1, 12000}]]

-0.242182

N[Log[Pi / 4]]

-0.241564

Table[ln[n], {n, 1, 100}]

$$\left\{0, 0, -\frac{1}{3}, 0, \frac{1}{5}, 0, -\frac{1}{7}, 0, \frac{1}{18}, 0, -\frac{1}{11}, 0, \frac{1}{13}, 0, 0, 0, \frac{1}{17}, 0, -\frac{1}{19}, 0, 0, 0, -\frac{1}{23}, 0, \frac{1}{50},\right.$$
$$0, -\frac{1}{81}, 0, \frac{1}{29}, 0, -\frac{1}{31}, 0, 0, 0, 0, 0, \frac{1}{37}, 0, 0, 0, \frac{1}{41}, 0, -\frac{1}{43}, 0, 0, 0, -\frac{1}{47}, 0, \frac{1}{98},$$
$$0, 0, 0, \frac{1}{53}, 0, 0, 0, 0, 0, -\frac{1}{59}, 0, \frac{1}{61}, 0, 0, 0, 0, 0, -\frac{1}{67}, 0, 0, 0, -\frac{1}{71}, 0, \frac{1}{73}, 0,$$
$$\left.0, 0, 0, 0, -\frac{1}{79}, 0, \frac{1}{324}, 0, -\frac{1}{83}, 0, 0, 0, 0, 0, \frac{1}{89}, 0, 0, 0, 0, 0, 0, 0, \frac{1}{97}, 0, 0, 0\right\}$$