```
delta = .01;
thetaAdd[x_, t_] := x - t
thetaMul[x_, t_] := x / t
thetaEq[x_, t_] := x
f[x_, 0, d_, fn_, I_] := UnitStep[x]
f[x_, k_, d_, fn_, I_] := Sum[d f[fn[x, d t + I], k - 1, d, fn, I], {t, 1, (x - I) / d}]
g[x_, j_, k_, d_, fn_, I_] :=
 If[k == 0, 1, If[x < j, 0, d g[fn[x, j], I + d, k - 1, d, fn, I] + g[x, j + d, k, d, fn, I]]]
```

```
f[100, 3, 1, thetaMul, 1]
```

324

```
g[100, 2, 3, 1, thetaMul, 1]
```

324

```
g[14, 1, 4, 1, thetaAdd, 0]
```

1001

```
Binomial[14, 4]
```

1001

```
FullSimplify[1 / Gamma[z] / Gamma[1 - z] Sum[ (-1) ^k / (z - k) Binomial[x, k], {k, 0, Infinity}]]
```

$$\frac{\text{Gamma}[1+x]}{\text{Gamma}[1+x-z]\,\text{Gamma}[1+z]}$$

```
Sum[ (-1) ^k / (z - k) Binomial[x, k], {k, 0, Infinity}]
```

$$\frac{\text{Gamma}[1+x]\,\text{Gamma}[1-z]}{z\,\text{Gamma}[1+x-z]}$$

```
Sum[(-1) ^k / (z - k) x^k / k!, {k, 0, Infinity}]
```

$$\frac{x^z\,(\text{Gamma}[1-z]+z\,\text{Gamma}[-z,x])}{z}$$

```
Gamma[1 - z] + z Gamma[-z, x] /. x → 3. /. z → 2.3
```

$3.32991 + 4.07797 \times 10^{-16}\,i$

```
(-z) Gamma[-z] + z Gamma[-z, x] /. x → 3. /. z → 2.3
```

$3.32991 + 4.07797 \times 10^{-16}\,i$

```
-z Gamma[-z, 0, x] /. x → 3. /. z → 2.3
```

$3.32991 + 4.07797 \times 10^{-16}\,i$

$$\frac{x^z\,(\text{Gamma}[1-z]+z\,\text{Gamma}[-z,x])}{z}\ \text{/. x → 3. /. z → 2.3}$$

$18.1169 + 2.21868 \times 10^{-15}\,i$

```
-x^z Gamma[-z, 0, x] /. x → 3. /. z → 2.3
```

$18.1169 + 2.21868 \times 10^{-15}\,i$

```
Sum[(-1)^k / (z - k) x^k / k!, {k, 0, Infinity}] /. x → 3. /. z → 2.3
```

$18.1169 + 2.21868 \times 10^{-15}\, i$

```
Sum[(-1)^k / (z - k) Log[x]^k / k!, {k, 0, Infinity}]
```

$$\frac{(\text{Gamma}[1-z] + z\,\text{Gamma}[-z, \text{Log}[x]])\,\text{Log}[x]^z}{z}$$

```
(Gamma[1 - z] + z Gamma[-z, Log[x]]) Log[x]^z / z /. x → 3. /. z → 2.3
```

$1.88563 + 2.30923 \times 10^{-16}\, i$

```
-Log[x]^z Gamma[-z, 0, Log[x]] /. x → 3. /. z → 2.3
```

$1.88563 + 2.30923 \times 10^{-16}\, i$

```
Sum[(-1)^k / (z - k) Hypergeometric1F1[k, k + 1, Log[x]] Log[x]^k / k!, {k, 0, Infinity}]
```

$$\sum_{k=0}^{\infty} \frac{(-1)^k\,(\text{Gamma}[1+k] - k\,\text{Gamma}[k, -\text{Log}[x]])\,(-\text{Log}[x])^{-k}\,\text{Log}[x]^k}{(-k+z)\,k!}$$

```
FullSimplify[Pochhammer[k, j] / Pochhammer[k + 1, j]]
```

$$\frac{k}{j+k}$$

```
Sum[k/(j + k) Log[x]^j / j!, {j, 0, Infinity}]
```

$(\text{Gamma}[1+k] - k\,\text{Gamma}[k, -\text{Log}[x]])\,(-\text{Log}[x])^{-k}$

```
Sum[(-1)^k / (z - k) Hypergeometric1F1[k, k + 1, Log[x]] Log[x]^k / k!, {k, 0, Infinity}]
(Gamma[1 + k] - k Gamma[k, -Log[x]]) (-Log[x])^-k /. x → 3. /. k → 2
```

$2.14729 - 5.25935 \times 10^{-16}\, i$

```
k (Gamma[k, 0, -Log[x]]) (-Log[x])^-k /. x → 3. /. k → 2
```

$2.14729 - 5.25935 \times 10^{-16}\, i$

```
N[Hypergeometric1F1[k, k + 1, Log[x]] Log[x]^k / k! /. x → 3 /. k → 2]
```

$1.29584 - 3.17388 \times 10^{-16}\, i$

```
N[k (Gamma[k, 0, -Log[x]]) (-Log[x])^-k Log[x]^k / k! /. x → 3 /. k → 2]
```

$1.29584 - 3.17388 \times 10^{-16}\, i$

```
N[k (Gamma[k, 0, -Log[x]]) (-1)^-k / k! /. x → 3 /. k → 2]
```

$1.29584 - 3.17388 \times 10^{-16}\, i$

```
Sum[1 / (z - k) (Gamma[k, 0, -Log[x]]) / Gamma[k], {k, 0, Infinity}]
```

$$\sum_{k=0}^{\infty} \frac{\text{Gamma}[k, 0, -\text{Log}[x]]}{(-k+z)\,\text{Gamma}[k]}$$

```
Sum[1 / (z - k) (GammaRegularized[k, 0, -Log[x]]), {k, 0, Infinity}]
```

$$\sum_{k=0}^{\infty} \frac{\text{GammaRegularized}[k, 0, -\text{Log}[x]]}{-k+z}$$

```
Sum[1 / (z - k) (GammaRegularized[k, 0, x]), {k, 0, Infinity}]
```

$$\sum_{k=0}^{\infty} \frac{\text{GammaRegularized}[k, 0, x]}{-k + z}$$

```
Integrate[ t^(z - 1) / Gamma[z], {t, 0, x}]
```

$$\text{ConditionalExpression}\left[\frac{x^z}{z\,\text{Gamma}[z]}, \text{Re}[z] > 0\right]$$

```
Integrate[Log[t]^(z - 1) / Gamma[z], {t, 1, x}]
```

$$\text{ConditionalExpression}\left[\frac{(\text{Gamma}[z] - \text{Gamma}[z, -\text{Log}[x]])\,(-\text{Log}[x])^{-z}\,\text{Log}[x]^z}{\text{Gamma}[z]}, \text{Re}[z] > 0\right]$$

```
Sum[(-1)^k / (z - k) Log[x]^(z - 1) / Gamma[z], {k, 0, Infinity}]
```

$$-\frac{\text{HurwitzLerchPhi}[-1, 1, -z]\,\text{Log}[x]^{-1+z}}{\text{Gamma}[z]}$$

$$\textbf{Integrate}\left[-\frac{\textbf{HurwitzLerchPhi}[-1, 1, -z]\,\textbf{Log}[x]^{-1+z}}{\textbf{Gamma}[z]}, \{x, 1, n\}\right]$$

$$\text{ConditionalExpression}\left[-\frac{1}{\text{Gamma}[z]}(\text{Gamma}[z] - \text{Gamma}[z, -\text{Log}[n]])\,\text{HurwitzLerchPhi}[-1, 1, -z]\,(-\text{Log}[n])^{-z}\,\text{Log}[n]^z, \text{Re}[z] > 0\right]$$

```
N[Sum[1 / (z - k) (GammaRegularized[k, 0, -Log[x]]), {k, 0, Infinity}] /. x → 3. /. z → 2.3]
```

3.43979

$$\textbf{N}\left[-\frac{1}{\textbf{Gamma}[z]}(\textbf{Gamma}[z] - \textbf{Gamma}[z, -\textbf{Log}[n]])\,\textbf{HurwitzLerchPhi}[-1, 1, -z]\,(-\textbf{Log}[n])^{-z}\,\textbf{Log}[n]^z \;/.\; n \to 3. \;/.\; z \to 2.3\right]$$

$4.15075 + 2.22045 \times 10^{-16}\, i$

```
Sum[(-1)^k / (z - k) Integrate[ Log[t]^(z - 1) / Gamma[z], {t, 1, x}], {k, 0, Infinity}]
```

$$\text{ConditionalExpression}\left[-\frac{1}{\text{Gamma}[z]}(\text{Gamma}[z] - \text{Gamma}[z, -\text{Log}[x]])\,\text{HurwitzLerchPhi}[-1, 1, -z]\,(-\text{Log}[x])^{-z}\,\text{Log}[x]^z, \text{Re}[z] > 0\right]$$

```
Sum[(-1)^k / (z - k) Integrate[ Log[t]^(k - 1) / Gamma[k], {t, 1, x}], {k, 0, Infinity}] /.
  x → 3. /. z → 2.3
```

$Aborted

```
N[Sum[(-1)^k / (2.3 - k) Hypergeometric1F1[k, k + 1, Log[3.]] Log[3.]^k / k!, {k, 0, Infinity}]]
```

3.87457

```
Integrate[ Log[t]^(k - 1) / Gamma[k], {t, 1, x}]
```

$$\text{ConditionalExpression}\left[\frac{(\text{Gamma}[k] - \text{Gamma}[k, -\text{Log}[x]])\,(-\text{Log}[x])^{-k}\,\text{Log}[x]^k}{\text{Gamma}[k]}, \text{Re}[k] > 0\right]$$

```
ta[x_, z_] := Sum[1 / (z - k) ((-Gamma[k, 0, -Log[x]])/Gamma[k]), {k, 0, Infinity}]
N@ta[3., 2.3]
```

Infinity::indet : Indeterminate expression 0. ∞ encountered. ≫

Infinity::indet : Indeterminate expression 0. ∞ encountered. ≫

NSum::nsnum : Summand (or its derivative) $-\frac{\text{Gamma}[k, 0, -1.09861]}{(2.3 - k)\,\text{Gamma}[k]}$ is not numerical at point k = 0. ≫

Infinity::indet : Indeterminate expression 0. ∞ encountered. ≫

General::stop : Further output of Infinity::indet will be suppressed during this calculation. ≫

NSum::nsnum : Summand (or its derivative) $-\frac{\text{Gamma}[k, 0, -1.09861]}{(2.3 - k)\,\text{Gamma}[k]}$ is not numerical at point k = 0. ≫

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General::stop : Further output of NSum::nsnum will be suppressed during this calculation. ≫

$$\text{NSum}\left[-\frac{\text{Gamma}[k, 0, -1.09861]}{(2.3 - k)\ \text{Gamma}[k]}, \{k, 0, \infty\}\right]$$

```
Integrate[ Sum[(-1)^k / (z - k) Log[t]^(k - 1) / Gamma[k], {k, 0, Infinity}], {t, 1, x}]
Sum[(-1)^k / (z - k) Log[t]^(k - 1) / Gamma[k], {k, 0, Infinity}]
```

$$(\text{Gamma}[1 - z] - \text{Gamma}[1 - z, \text{Log}[t]])\ \text{Log}[t]^{-1+z}$$

```
Integrate[ (Gamma[1 - z] - Gamma[1 - z, Log[t]]) Log[t]^(-1+z), {t, 1, x}]
```

$$\int_1^x (\text{Gamma}[1 - z] - \text{Gamma}[1 - z, \text{Log}[t]])\ \text{Log}[t]^{-1+z}\, dt$$

```
Integrate[ -Gamma[1 - z, 0, Log[t]] Log[t]^(-1+z), {t, 1, x}]
```

$$\int_1^x -\text{Gamma}[1 - z, 0, \text{Log}[t]]\ \text{Log}[t]^{-1+z}\, dt$$

```
Sum[(-1)^k / (z - k) t^(k - 1) / Gamma[k], {k, 0, Infinity}]
```

$$t^{-1+z}\ (\text{Gamma}[1 - z] - \text{Gamma}[1 - z, t])$$

```
Table[-Integrate[ Gamma[-z, 0, Log[t]] Log[t]^z, {t, 1, x}], {z, -6, 0}]
```

$$\left\{-\int_1^x \frac{\text{Gamma}[6, 0, \text{Log}[t]]}{\text{Log}[t]^6}\, dt,\ -\int_1^x \frac{\text{Gamma}[5, 0, \text{Log}[t]]}{\text{Log}[t]^5}\, dt,\ -\int_1^x \frac{\text{Gamma}[4, 0, \text{Log}[t]]}{\text{Log}[t]^4}\, dt,\ -\int_1^x \frac{\text{Gamma}[3, 0, \text{Log}[t]]}{\text{Log}[t]^3}\, dt,\ -\int_1^x \frac{\text{Gamma}[2, 0, \text{Log}[t]]}{\text{Log}[t]^2}\, dt,\ \text{ConditionalExpression}[\text{EulerGamma} + \text{Gamma}[0, -\text{Log}[x]] + \text{Log}[-\text{Log}[x]],\ \text{Im}[x] \neq 0 \,||\, \text{Re}[x] \geq 0],\ \text{ComplexInfinity}\right\}$$

```
ba[x_, z_] := -Integrate[ -Gamma[1 - z, 0, Log[t]] Log[t]^(-1+z), {t, 1, x}]
```

```
N@ba[13., 12.3 + I]
```

```
-0.201918 + 0.0156451 i
```

```
N[Sum[1 / (z - k) (GammaRegularized[k, 0, -Log[x]]), {k, 0, Infinity}] /. x → 13. /. z → 12.3 + I]
```

```
-0.201918 + 0.0156451 i
```

```
D[1 / (z - k) Gamma[k, 0, -Log[x]] / Gamma[k], z]
```

$$-\frac{\text{Gamma}[k, 0, -\text{Log}[x]]}{(-k + z)^2\, \text{Gamma}[k]}$$

```
br[x_, k_] := (-1) ^ (k - 1) (-1 + Sum[x (-Log[x]) ^ j / j !, {j, 0, k - 1}])
```

```
Table[br[x, k], {k, 0, 5}] // TableForm
```

1
$-1 + x$
$1 - x + x \text{Log}[x]$
$-1 + x - x \text{Log}[x] + \frac{1}{2} x \text{Log}[x]^2$
$1 - x + x \text{Log}[x] - \frac{1}{2} x \text{Log}[x]^2 + \frac{1}{6} x \text{Log}[x]^3$
$-1 + x - x \text{Log}[x] + \frac{1}{2} x \text{Log}[x]^2 - \frac{1}{6} x \text{Log}[x]^3 + \frac{1}{24} x \text{Log}[x]^4$

```
Table[
 Sum[1 / (z - k) ((-1) ^k ((-1) ^n x / ((n - 1) !) Log[x] ^ (n - 1))), {k, 0, Infinity}], {n, 0, 5}]
```

$$\Big\{0,\ x\, \text{HurwitzLerchPhi}[-1, 1, -z],\ -x\, \text{HurwitzLerchPhi}[-1, 1, -z]\, \text{Log}[x],\ \frac{1}{2} x\, \text{HurwitzLerchPhi}[-1, 1, -z]\, \text{Log}[x]^2,\ -\frac{1}{6} x\, \text{HurwitzLerchPhi}[-1, 1, -z]\, \text{Log}[x]^3,\ \frac{1}{24} x\, \text{HurwitzLerchPhi}[-1, 1, -z]\, \text{Log}[x]^4\Big\}$$

```
Sum[1 / (z - k) ((-1) ^k ( x / ((n - 1) !) Log[x] ^ (n - 1))), {k, 0, Infinity}]
```

$$-\frac{x\, \text{HurwitzLerchPhi}[-1, 1, -z]\, \text{Log}[x]^{-1+n}}{(-1 + n)!}$$

$$\textbf{Sum}\Big[-\frac{x\, \text{HurwitzLerchPhi}[-1, 1, -z]\, \text{Log}[x]^{-1+n}}{(-1 + n)!}, \{n, 0, \text{Infinity}\}\Big]$$

$$-x^2\, \text{LerchPhi}[-1, 1, -z]$$

$$-x^2\, \text{LerchPhi}[-1, 1, -z]\ /.\ x \to 3.\ /.\ z \to 2.3$$

```
-17.4689
```

```
Sum[1 / (z - k) ((-1) ^ (k + 1) x / 6 Log[x] ^3 ), {k, 4, Infinity}]
```

$$\frac{1}{6} x\, \text{HurwitzLerchPhi}[-1, 1, 4 - z]\, \text{Log}[x]^3$$

```
Expand@
 Limit[(-1)^(-z) Sin[Pi z] / Pi (-HurwitzLerchPhi[-1, 1, -z] - x HurwitzLerchPhi[-1, 1, 1 - z] -
     x HurwitzLerchPhi[-1, 1, 2 - z] Log[x] - 1/2 x HurwitzLerchPhi[-1, 1, 3 - z] Log[x]^2 -
     1/6 x HurwitzLerchPhi[-1, 1, 4 - z] Log[x]^3), z → 4]
```

$1 - x + x \operatorname{Log}[x] - \frac{1}{2} x \operatorname{Log}[x]^2 + \frac{1}{6} x \operatorname{Log}[x]^3$

```
Sum[1 / (z - k) ((-1)^(k + 1) x ), {k, 0, Infinity}]
```

x HurwitzLerchPhi[-1, 1, -z]

```
Sum[1 / (z - k) ((-1)^(k) x Log[x] ), {k, 0, Infinity}]
```

-x HurwitzLerchPhi[-1, 1, -z] Log[x]

```
N@Sum[1/k! x HurwitzLerchPhi[-1, 1, k + 1 - z] Log[x]^k, {k, 0, Infinity}] /. x → 3. /. z → 2.3
Table[Limit[(-1)^z Sin[Pi z] / Pi HurwitzLerchPhi[-1, 1, 4 - z], z → z2], {z2, 0, 9}]
```

{0, 0, 0, 0, -1, 1, -1, 1, -1, 1}

```
Sum[(-1)^(-z) Sin[Pi z] / Pi (1/k! x HurwitzLerchPhi[-1, 1, k + 1 - z] Log[x]^k), {k, 0, Infinity}]
```

$\sum_{k=0}^{\infty} \frac{(-1)^{-z} x \operatorname{HurwitzLerchPhi}[-1, 1, 1 + k - z] \operatorname{Log}[x]^k \operatorname{Sin}[\pi z]}{\pi k!}$

```
D[1 / (z - k) GammaRegularized[k, 0, -Log[x]], z]
```

$-\frac{\operatorname{GammaRegularized}[k, 0, -\operatorname{Log}[x]]}{(-k + z)^2}$

```
D[1 / (z - k) Hypergeometric1F1[k, k + 1, Log[x]] Log[x]^k / k!, z]
```

$-\frac{(\operatorname{Gamma}[1 + k] - k \operatorname{Gamma}[k, -\operatorname{Log}[x]]) (-\operatorname{Log}[x])^{-k} \operatorname{Log}[x]^k}{(-k + z)^2 k!}$

```
Sum[-GammaRegularized[k, 0, -Log[x]] / (-k + z)^2, {k, 0, Infinity}]
```

$\sum_{k=0}^{\infty} -\frac{\operatorname{GammaRegularized}[k, 0, -\operatorname{Log}[x]]}{(-k + z)^2}$

```
Integrate[1 / (z - k) GammaRegularized[k, 0, -Log[x]], z]
```

GammaRegularized[k, 0, -Log[x]] Log[-k + z]

```
Sum[GammaRegularized[k, 0, -Log[x]] Log[-k + z], {k, 0, Infinity}]
```

$\sum_{k=0}^{\infty} \operatorname{GammaRegularized}[k, 0, -\operatorname{Log}[x]] \operatorname{Log}[-k + z]$

```
Integrate[(-1)^k / (z - k) f[x, k], z]
```

$(-1)^k f[x, k] \operatorname{Log}[-k + z]$

```
Integrate[(-1)^k / (z - k) x^k / k!, z]
```

$$\frac{(-1)^k x^k \mathrm{Log}[-k+z]}{k!}$$

```
-Integrate[ Log[r]^(z - 1) t^-z E^-t, {r, 1, x}, {t, 0, Log[r]}]
```

```
$Aborted
```

```
-Integrate[ (Log[r] / t)^z / Log[r] E^-t, {r, 1, x}, {t, 0, Log[r]}]
```

$$-\int_1^x \left(-\mathrm{ExpIntegralE}[z, \mathrm{Log}[r]] + \mathrm{Gamma}[1-z]\,\mathrm{Log}[r]^{-1+z}\right) dr$$

```
FullSimplify[(Log[r] / t)^z / Log[r] E^-t]
```

$$\frac{e^{-t}\left(\frac{\mathrm{Log}[r]}{t}\right)^z}{\mathrm{Log}[r]}$$

$$-\int_1^x \left(-\mathrm{ExpIntegralE}[z, \mathrm{Log}[r]] + \mathrm{Gamma}[1-z]\,\mathrm{Log}[r]^{-1+z}\right) dr$$

$$-\int_1^x (-\mathrm{ExpIntegralE}[z, \mathrm{Log}[r]])\, dr - \mathrm{Integrate}\left[\mathrm{Gamma}[1-z]\,\mathrm{Log}[r]^{-1+z}, \{r, 1, x\}\right]$$

$$\mathrm{ConditionalExpression}\left[-\int_1^x -\mathrm{ExpIntegralE}[z, \mathrm{Log}[r]]\, dr - \mathrm{Gamma}[1-z]\,(\mathrm{Gamma}[z] - \mathrm{Gamma}[z, -\mathrm{Log}[x]])\,(-\mathrm{Log}[x])^{-z}\,\mathrm{Log}[x]^z,\ \mathrm{Re}[z] > 0\right]$$

```
-Integrate[ (Log[r] / t)^z / Log[r] E^-t, {r, 1, x}, {t, 0, Log[r]}] /. x → 3. /. z → 2.3
```

$-3.43979 + 2.15125 \times 10^{-16}\, i$

$$-\int_1^x (-\mathrm{ExpIntegralE}[z, \mathrm{Log}[r]])\, dr - \mathrm{Integrate}\left[\mathrm{Gamma}[1-z]\,\mathrm{Log}[r]^{-1+z}, \{r, 1, x\}\right] /.\ x \to 3.\ /.\ z \to 2.3$$

$-3.43979 - 7.6904 \times 10^{-17}\, i$

$$-\int_1^x -\mathrm{ExpIntegralE}[z, \mathrm{Log}[r]]\, dr - \mathrm{Gamma}[1-z]\,(\mathrm{Gamma}[z] - \mathrm{Gamma}[z, -\mathrm{Log}[x]])\,(-1)^{-z} /.\ x \to 3.\ /.\ z \to 2.3$$

$-3.43979 - 7.6904 \times 10^{-17}\, i$

$$\int_1^x \mathrm{ExpIntegralE}[z, \mathrm{Log}[r]]\, dr - \mathrm{Gamma}[1-z]\,(\mathrm{Gamma}[z, 0, -\mathrm{Log}[x]])\,(-1)^{-z} /.\ x \to 3.\ /.\ z \to 2.3$$

$-3.43979 - 7.6904 \times 10^{-17}\, i$

$$\int_1^x \mathrm{ExpIntegralE}[z, \mathrm{Log}[r]]\, dr /.\ x \to 3.\ /.\ z \to 2.3$$

$0.533382 - 2.98949 \times 10^{-16}\, i$

$$\int_1^x \mathrm{Integrate}[E\verb|^|(-\mathrm{Log}[r]\, t) / t\verb|^|z, \{t, 1, \mathrm{Infinity}\}]\, dr /.\ x \to 3.\ /.\ z \to 2.3$$

Integrate::pwrl : Unable to prove that integration limits {x} are real. Adding assumptions may help. ≫

$0.533382 + 5.55112 \times 10^{-17}\, i$

```
FullSimplify[E^(-Log[r] t) / t^z]
```

$r^{-t} t^{-z}$

```
Integrate[r^-t t^-z, {r, 1, x}, {t, 1, Infinity}]
```

$\int_1^x \text{ExpIntegralE}[z, \text{Log}[r]]\, dr$

```
Integrate[Integrate[r^-t, {r, 1, x}] t^-z, {t, 1, Infinity}]
```

$\int_1^\infty \text{ConditionalExpression}\left[\frac{t^{-z} x^{-t} \left(-x + x^t\right)}{-1 + t}, \text{Re}[x] \geq 0 \,||\, x \notin \text{Reals}\right] dt$

$\text{FullSimplify}\left[\frac{t^{-z} x^{-t} \left(-x + x^t\right)}{-1 + t}\right]$

$\frac{t^{-z} x^{-t} \left(-x + x^t\right)}{-1 + t}$

$\text{Integrate}\left[\frac{t^{-z}}{-1 + t} - \frac{t^{-z} x^{1-t}}{-1 + t}, \{t, 1, \text{Infinity}\}\right]$

$\int_1^\infty \left(\frac{t^{-z}}{-1 + t} - \frac{t^{-z} x^{1-t}}{-1 + t}\right) dt$

```
bl[x_, z_] := Sin[Pi z] / Pi ( Integrate[ExpIntegralE[z, Log[t]], {t, 1, x}] +
    Sum[1 / (z - k) GammaRegularized[k, 0, -Log[x]], {k, 0, Infinity}])
bll[x_, z_] := Sin[Pi z] / Pi ( Integrate[ExpIntegralE[z, Log[t]], {t, 1, x}])
```

```
N@bl[13., 2.3]
```

22.3656

```
(-1) ^-z GammaRegularized[z, 0, -Log[x]] /. x → 13. /. z → 2.3
```

$22.3656 + 1.77636 \times 10^{-15}\, i$

```
N@bl[13, .5]
```

5.86035

```
(-1) ^-z GammaRegularized[z, 0, -Log[x]] /. x → 13. /. z → .5
```

$5.86035 + 0.\, i$

```
N@bll[13, 3.000000001]
```

-7.49542×10^{-10}

```
Plot[bll[13., x], {x, -3, 5}]
```

$Aborted

```
FullSimplify@
 Expand[x^(z - 1) Gamma[1 - z] + Sum[ (-1) ^k / ((z - 1) - k) x^k / k !, {k, 0, Infinity}]]
```

ExpIntegralE[z, x]

```
FullSimplify@Expand[x^z Gamma[-z] + Sum[ (-1) ^k / (z - k) x^k / k !, {k, 0, Infinity}]]
```

ExpIntegralE[1 + z, x]

```
Integrate[ Log[x] ^k
```

```
FullSimplify@
 Expand[Log[x]^(z-1) Gamma[1-z] + Sum[ (-1)^k / ((z-1) - k) Log[x]^k / k!, {k, 0, Infinity}]]
```

ExpIntegralE[z, Log[x]]

```
FullSimplify@Integrate[ Log[t]^k / k!, {t, 1, x}]
```

ConditionalExpression$\left[\frac{(-k\,\text{Gamma}[k] + \text{Gamma}[1+k, -\text{Log}[x]])\,(-\text{Log}[x])^{-k}\,\text{Log}[x]^k}{k!}, \text{Re}[k] > -1\right]$

$\frac{(-k\,\text{Gamma}[k] + \text{Gamma}[1+k, -\text{Log}[x]])\,(-\text{Log}[x])^{-k}\,\text{Log}[x]^k}{k!}$ /. x → 4. /. k → 2.

$1.29845 - 1.59014 \times 10^{-16}\,i$

$\frac{\text{Gamma}[1+k, 0, -\text{Log}[x]]\,(-1)^{-k-1}}{k!}$ /. x → 4. /. k → 2.

$1.29845 - 4.77042 \times 10^{-16}\,i$

$(-1)^{-k-1}$ GammaRegularized[1 + k, 0, -Log[x]] /. x → 4. /. k → 2.

$1.29845 - 4.77042 \times 10^{-16}\,i$

```
FullSimplify@Integrate[ Log[t]^(z-1) Gamma[1-z], {t, 1, x}]
```

ConditionalExpression[
 Gamma[1 - z] (Gamma[z] - Gamma[z, -Log[x]]) $(-\text{Log}[x])^{-z}$ $\text{Log}[x]^z$, Re[z] > 0]

Gamma[1 - z] (Gamma[z] - Gamma[z, -Log[x]]) $(-\text{Log}[x])^{-z}$ $\text{Log}[x]^z$ /. x → 4. /. z → 2.3

$8.42242 + 0.\,i$

Gamma[1 - z] (Gamma[z, 0, -Log[x]]) $(-1)^{-z}$ /. x → 4. /. z → 2.3

$8.42242 + 0.\,i$

Gamma[1 - z] Gamma[z] (GammaRegularized[z, 0, -Log[x]]) $(-1)^{-z}$ /. x → 4. /. z → 2.3

$8.42242 - 8.88178 \times 10^{-16}\,i$

Pi / Sin[Pi z] (GammaRegularized[z, 0, -Log[x]]) $(-1)^{-z}$ /. x → 4. /. z → 2.3

$8.42242 + 0.\,i$