```
Residue[ (( 1 / Zeta[s])) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z
```

$$\frac{x^z}{z \, \text{Zeta}'[z]}$$

```
Residue[ ((Zeta'[s] / Zeta[s])) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z
```

$$\frac{x^z}{z}$$

```
FullSimplify[Residue[ (( 1 / Zeta[s]^2)) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]
```

$$\frac{x^z \, ((-1 + z \, \text{Log}[x]) \, \text{Zeta}'[z] - z \, \text{Zeta}''[z])}{z^2 \, \text{Zeta}'[z]^3}$$

```
Residue[ (( Zeta'[s] / Zeta[s]^2)) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z
```

$$\frac{x^z \, (-1 + z \, \text{Log}[x])}{z^2 \, \text{Zeta}'[z]}$$

```
FullSimplify[
 Residue[ (( Zeta'[s]^2 / Zeta[s]^2)) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]
```

$$\frac{x^z \, ((-1 + z \, \text{Log}[x]) \, \text{Zeta}'[z] + z \, \text{Zeta}''[z])}{z^2 \, \text{Zeta}'[z]}$$

```
Residue[ (( Zeta'[s]^2 / Zeta[s])) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z
```

$$\frac{x^z \, \text{Zeta}'[z]}{z}$$

```
FullSimplify[
 Residue[ (( Zeta''[s] / Zeta[s])) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]
```

$$\frac{x^z \, \text{Zeta}''[z]}{z \, \text{Zeta}'[z]}$$

```
FullSimplify[Residue[ ((1 / (s Zeta[s] ))) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]
```

$$\frac{x^z}{z^2 \, \text{Zeta}'[z]}$$

```
FullSimplify[
 Residue[ ((Log''[s] / Zeta[s])) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]
```

$$-\frac{x^z}{z^3 \, \text{Zeta}'[z]}$$

```
Residue[ ((s / (Zeta[s]))) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z
```

$$\frac{x^z}{\text{Zeta}'[z]}$$

```
Residue[ ((s Zeta'[s] / (Zeta[s]))) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z
```

$$x^z$$

```
FullSimplify[
 Residue[ (s^1 Zeta'[s] / Zeta[s]^2) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]
```

$$\frac{x^z \, \text{Log}[x]}{\text{Zeta}'[z]}$$

```
FullSimplify[
 Residue[ (s^1 Zeta'[s] / Zeta[s]^3) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]
```

$$\frac{x^z \, \text{Log}[x] \, (\text{Log}[x] \, \text{Zeta}'[z] - \text{Zeta}''[z])}{2 \, \text{Zeta}'[z]^3}$$

```
FullSimplify[Residue[ (1 / Zeta[s]^2) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]
```

$$\frac{x^z \, ((-1 + z \, \text{Log}[x]) \, \text{Zeta}'[z] - z \, \text{Zeta}''[z])}{z^2 \, \text{Zeta}'[z]^3}$$

```
FullSimplify[Residue[ (s / Zeta[s]^2) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]
```

$$\frac{x^z \, (\text{Log}[x] \, \text{Zeta}'[z] - \text{Zeta}''[z])}{\text{Zeta}'[z]^3}$$

```
FullSimplify[Residue[ (s^2 / Zeta[s]^2) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]
```

$$\frac{x^z \, ((1 + z \, \text{Log}[x]) \, \text{Zeta}'[z] - z \, \text{Zeta}''[z])}{\text{Zeta}'[z]^3}$$

```
FullSimplify[
 Residue[ (s Zeta'[s] / Zeta[s]^2) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]
```

$$\frac{x^z \, \text{Log}[x]}{\text{Zeta}'[z]}$$

```
FullSimplify[
 Residue[ (s Zeta''[s] / Zeta[s]^2) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]
```

$$\frac{x^z \left(-\text{Zeta}''[z]^2 + \text{Zeta}'[z] \left(\text{Log}[x] \, \text{Zeta}''[z] + \text{Zeta}^{(3)}[z]\right)\right)}{\text{Zeta}'[z]^3}$$

```
Residue[ (1 / Zeta[s]) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z
```

$$\frac{x^z}{z \, \text{Zeta}'[z]}$$

```
FullSimplify[Residue[ (1 / Zeta[s]^2) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]
```

$$\frac{x^z \, ((-1 + z \, \text{Log}[x]) \, \text{Zeta}'[z] - z \, \text{Zeta}''[z])}{z^2 \, \text{Zeta}'[z]^3}$$

```
FullSimplify[Residue[ (1 / Zeta[s]^3) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]
```

$$\frac{1}{2 \, z^3 \, \text{Zeta}'[z]^5} \, x^z \left((2 + z \, \text{Log}[x] \, (-2 + z \, \text{Log}[x])) \, \text{Zeta}'[z]^2 + \right.$$
$$\left. 3 \, z^2 \, \text{Zeta}''[z]^2 - z \, \text{Zeta}'[z] \left(3 \, (-1 + z \, \text{Log}[x]) \, \text{Zeta}''[z] + z \, \text{Zeta}^{(3)}[z]\right)\right)$$

```
FullSimplify[Residue[ (1 / Zeta[s]^4) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]
```

$$\frac{1}{6 \, z^4 \, \text{Zeta}'[z]^7} \, x^z \left((-6 + z \, \text{Log}[x] \, (6 + z \, \text{Log}[x] \, (-3 + z \, \text{Log}[x]))) \, \text{Zeta}'[z]^3 - 15 \, z^3 \, \text{Zeta}''[z]^3 + \right.$$
$$5 \, z^2 \, \text{Zeta}'[z] \, \text{Zeta}''[z] \left(3 \, (-1 + z \, \text{Log}[x]) \, \text{Zeta}''[z] + 2 \, z \, \text{Zeta}^{(3)}[z]\right) - z \, \text{Zeta}'[z]^2$$
$$\left. \left(6 \, (2 + z \, \text{Log}[x] \, (-2 + z \, \text{Log}[x])) \, \text{Zeta}''[z] + z \left(4 \, (-1 + z \, \text{Log}[x]) \, \text{Zeta}^{(3)}[z] + z \, \text{Zeta}^{(4)}[z]\right)\right)\right)$$

```
FullSimplify[Residue[ (1 / Zeta[s]^2) x^s s^ (-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]
FullSimplify[
 Residue[ (Zeta'[s]^2 / Zeta[s]^2) x^s s^ (-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]
```

$$\frac{x^z \left(\left(-1 + z \, \text{Log}[x]\right) \text{Zeta}'[z] - z \, \text{Zeta}''[z]\right)}{z^2 \, \text{Zeta}'[z]^3}$$

$$\frac{x^z \left(\left(-1 + z \, \text{Log}[x]\right) \text{Zeta}'[z] + z \, \text{Zeta}''[z]\right)}{z^2 \, \text{Zeta}'[z]}$$

```
FullSimplify[Residue[ (1 / Zeta[s]^3) x^s s^ (-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]
FullSimplify[
 Residue[ (Zeta'[s]^3 / Zeta[s]^3) x^s s^ (-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]
```

$$\frac{1}{2 \, z^3 \, \text{Zeta}'[z]^5} x^z \left(\left(2 + z \, \text{Log}[x] \, (-2 + z \, \text{Log}[x])\right) \text{Zeta}'[z]^2 + \right.$$
$$\left. 3 \, z^2 \, \text{Zeta}''[z]^2 - z \, \text{Zeta}'[z] \left(3 \, (-1 + z \, \text{Log}[x]) \, \text{Zeta}''[z] + z \, \text{Zeta}^{(3)}[z]\right)\right)$$

$$\frac{1}{2 \, z^3 \, \text{Zeta}'[z]}$$
$$x^z \left(\left(2 + z \, \text{Log}[x] \, (-2 + z \, \text{Log}[x])\right) \text{Zeta}'[z] + z \left(3 \, (-1 + z \, \text{Log}[x]) \, \text{Zeta}''[z] + 2 \, z \, \text{Zeta}^{(3)}[z]\right)\right)$$

```
Residue[ ((1 / Zeta[s] - 1)) x^s s^ (-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z
```

$$\frac{x^z}{z \, \text{Zeta}'[z]}$$

```
FullSimplify[Residue[ ((1 / Zeta[s] - 1)^2) x^s s^ (-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]
```

$$-\frac{x^z \left(\text{Zeta}'[z] \left(1 - z \, \text{Log}[x] + 2 \, z \, \text{Zeta}'[z]\right) + z \, \text{Zeta}''[z]\right)}{z^2 \, \text{Zeta}'[z]^3}$$

```
FullSimplify[Residue[ ((1 / Zeta[s]^2)) x^s s^ (-1), {s, ZetaZero[1]}] -
   Residue[ ((2 / Zeta[s])) x^s s^ (-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]
```

$$-\frac{x^z \left(\text{Zeta}'[z] \left(1 - z \, \text{Log}[x] + 2 \, z \, \text{Zeta}'[z]\right) + z \, \text{Zeta}''[z]\right)}{z^2 \, \text{Zeta}'[z]^3}$$

```
Expand[Residue[ ((1 / Zeta[s] - 1)^1) x^s s^ (-1), {s, -22}]]
```

$$-\frac{1}{22 \, x^{22} \, \text{Zeta}'[-22]}$$

Expand[Residue[((1 / Zeta[s])^2) x^s s^(-1), {s, -22}]]

$$-\frac{1}{484\, x^{22}\, \text{Zeta}'[-22]^2} - \frac{\text{Log}[x]}{22\, x^{22}\, \text{Zeta}'[-22]^2} + \frac{\text{Zeta}''[-22]}{22\, x^{22}\, \text{Zeta}'[-22]^3}$$

Expand[Residue[((1 / Zeta[s])^3) x^s s^(-1), {s, -22}]]

$$-\frac{1}{10\,648\, x^{22}\, \text{Zeta}'[-22]^3} - \frac{\text{Log}[x]}{484\, x^{22}\, \text{Zeta}'[-22]^3} - \frac{\text{Log}[x]^2}{44\, x^{22}\, \text{Zeta}'[-22]^3} +$$

$$\frac{3\, \text{Zeta}''[-22]}{968\, x^{22}\, \text{Zeta}'[-22]^4} + \frac{3\, \text{Log}[x]\, \text{Zeta}''[-22]}{44\, x^{22}\, \text{Zeta}'[-22]^4} - \frac{3\, \text{Zeta}''[-22]^2}{44\, x^{22}\, \text{Zeta}'[-22]^5} + \frac{\text{Zeta}^{(3)}[-22]}{44\, x^{22}\, \text{Zeta}'[-22]^4}$$

Expand[Residue[((1 / Zeta[s])^4) x^s s^(-1), {s, -22}]]

$$-\frac{1}{234\,256\, x^{22}\, \text{Zeta}'[-22]^4} - \frac{\text{Log}[x]}{10\,648\, x^{22}\, \text{Zeta}'[-22]^4} - \frac{\text{Log}[x]^2}{968\, x^{22}\, \text{Zeta}'[-22]^4} -$$

$$\frac{\text{Log}[x]^3}{132\, x^{22}\, \text{Zeta}'[-22]^4} + \frac{\text{Zeta}''[-22]}{5324\, x^{22}\, \text{Zeta}'[-22]^5} + \frac{\text{Log}[x]\, \text{Zeta}''[-22]}{242\, x^{22}\, \text{Zeta}'[-22]^5} + \frac{\text{Log}[x]^2\, \text{Zeta}''[-22]}{22\, x^{22}\, \text{Zeta}'[-22]^5} -$$

$$\frac{5\, \text{Zeta}''[-22]^2}{968\, x^{22}\, \text{Zeta}'[-22]^6} - \frac{5\, \text{Log}[x]\, \text{Zeta}''[-22]^2}{44\, x^{22}\, \text{Zeta}'[-22]^6} + \frac{5\, \text{Zeta}''[-22]^3}{44\, x^{22}\, \text{Zeta}'[-22]^7} + \frac{\text{Zeta}^{(3)}[-22]}{726\, x^{22}\, \text{Zeta}'[-22]^5} +$$

$$\frac{\text{Log}[x]\, \text{Zeta}^{(3)}[-22]}{33\, x^{22}\, \text{Zeta}'[-22]^5} - \frac{5\, \text{Zeta}''[-22]\, \text{Zeta}^{(3)}[-22]}{66\, x^{22}\, \text{Zeta}'[-22]^6} + \frac{\text{Zeta}^{(4)}[-22]}{132\, x^{22}\, \text{Zeta}'[-22]^5}$$

Eta[s_] := (1 - 2^(1 - s)) Zeta[s]

Et3[s_] := (1 - 3^(1 - s)) Zeta[s]

Et[k_, s_] := (1 - k^(1 - s)) Zeta[s]

Expand[Residue[((1 / Eta[s])) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]

$$\frac{2^z\, x^z}{(-2 + 2^z)\, z\, \text{Zeta}'[z]}$$

Expand[Residue[((1 / Zeta[s])) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]

$$\frac{x^z}{z\, \text{Zeta}'[z]}$$

FullSimplify[Residue[((1 / Et[2, s])) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]

$$\frac{2^z\, x^z}{(-2 + 2^z)\, z\, \text{Zeta}'[z]}$$

Expand[Residue[((Eta'[s] / Eta[s])) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]

$$\frac{x^z}{z}$$

FullSimplify[Residue[((Eta'[s] / Et3[s])) x^s s^(-1), {s, ZetaZero[1]}] /. ZetaZero[1] → z]

$$\frac{\left(\frac{3}{2}\right)^z (-2 + 2^z)\, x^z}{(-3 + 3^z)\, z}$$

```
Residue[ (( Zeta[s])^2) x^s / s, {s, 1}]
```

$-x + 2 \, \text{EulerGamma} \, x + x \, \text{Log}[x]$

```
Limit[D[(Zeta[s])^2 / Zeta[2 s] x^s / s (s - 1)^2, s], s → 1]
```

$$\frac{6 \, x \, \left((-1 + 2 \, \text{EulerGamma}) \, \pi^2 + \pi^2 \, \text{Log}[x] - 12 \, \text{Zeta}'[2] \right)}{\pi^4}$$

```
Limit[D[(Zeta[s])^2 x^s / s (s - 1)^2, s], s → 1]
```

$x \, (-1 + 2 \, \text{EulerGamma} + \text{Log}[x])$

```
Limit[D[(Zeta[s])^2 x^s / s (s - 1)^2, s], s → 1]
```

$x \, (-1 + 2 \, \text{EulerGamma} + \text{Log}[x])$

```
Residue[ (( Zeta[s])^3) x^s / s, {s, 1}]
```

$\frac{1}{2} \left(2 \, x - 6 \, \text{EulerGamma} \, x + 6 \, \text{EulerGamma}^2 \, x - \right.$
$\left. 2 \, x \, \text{Log}[x] + 6 \, \text{EulerGamma} \, x \, \text{Log}[x] + x \, \text{Log}[x]^2 - 6 \, x \, \text{StieltjesGamma}[1] \right)$

```
Limit[D[(Zeta[s])^3 x^s / s (s - 1)^3 / 2, {s, 2}], s → 1]
```

$\frac{1}{2} \, x \left(2 - 6 \, \text{EulerGamma} + 6 \, \text{EulerGamma}^2 + (-2 + 6 \, \text{EulerGamma}) \, \text{Log}[x] + \text{Log}[x]^2 - 6 \, \text{StieltjesGamma}[1] \right)$

```
Limit[D[(Zeta[s] - 1)^3 x^s / s (s - 1)^3 / 2, {s, 2}], s → 1]
```

$\frac{1}{2} \, x$
$\left((-8 + 6 \, \text{EulerGamma}) \, \text{Log}[x] + \text{Log}[x]^2 + 2 \left(7 - 9 \, \text{EulerGamma} + 3 \, \text{EulerGamma}^2 - 3 \, \text{StieltjesGamma}[1] \right) \right)$

```
f[k_] := Zeta[v]^k
```

```
f'[0]
```

$\text{Log}[\text{Zeta}[v]]$

```
et[s_, a_] := (1 - a^(1 - s)) Zeta[s]
et2[s_, a_] := et[s, a] / (1 - a^(1 - s))
Residue[ (( 1 / et[s, a])) x^s s^(-1), {s, ZetaZero[1]}]
```
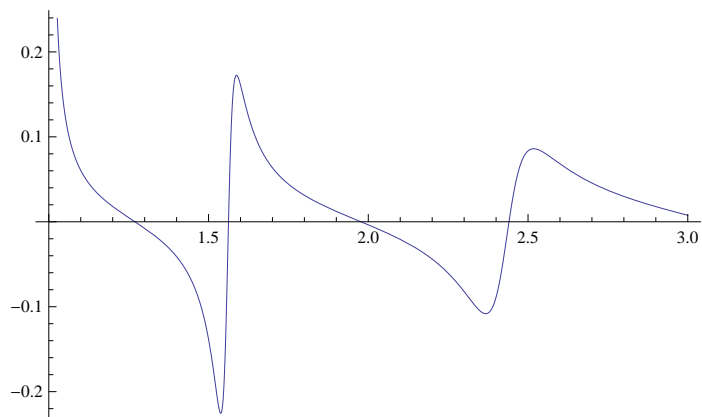
$$\frac{a^{\text{ZetaZero}[1]} \, x^{\text{ZetaZero}[1]}}{\left(-a + a^{\text{ZetaZero}[1]} \right) \text{ZetaZero}[1] \, \text{Zeta}'[\text{ZetaZero}[1]]}$$

```
N[Residue[ (( 1 / et[s, 1.1])) x^s s^(-1), {s, ZetaZero[1]}]]
```

$(-0.0604593 - 0.034716 \, \mathbb{i}) \, x^{0.5 + 14.1347 \, \mathbb{i}}$

Residue[((1 / et[s, a])) x^s s^(-1), {s, ZetaZero[1]}]

$$\frac{a^{ZetaZero[1]}\, x^{ZetaZero[1]}}{\left(-a + a^{ZetaZero[1]}\right) ZetaZero[1]\, Zeta'[ZetaZero[1]]}$$

N$\left[\dfrac{1}{\text{ZetaZero[1] Zeta'[ZetaZero[1]]}}\right]$

$-0.0108939 - 0.0884734\, \mathbb{i}$

Plot$\left[\left\{\text{Re}\left[\dfrac{-a^{ZetaZero[aa=1]}}{\left(-a + a^{ZetaZero[aa]}\right) ZetaZero[aa]\, Zeta'[ZetaZero[aa]]}\right]\right\}, \{a, 1, 3\}\right]$

Residue[(((a - 1) / et[s, a])) x^s s^(-1), {s, ZetaZero[1]}]

Limit$\left[\dfrac{(-1 + a)\, a^{ZetaZero[1]}\, x^{ZetaZero[1]}}{\left(-a + a^{ZetaZero[1]}\right) ZetaZero[1]\, Zeta'[ZetaZero[1]]}, a \to 0\right]$

$$-\frac{x^{ZetaZero[1]}}{ZetaZero[1]\, Zeta'[ZetaZero[1]]}$$

Limit$\left[\dfrac{1}{2\left(1 + a + a^2\right) x^2\, Zeta'[-2]}, a \to 1\right]$

$$\frac{1}{6\, x^2\, Zeta'[-2]}$$

$$\frac{1}{6\, x^2\, Zeta'[-2]}$$

Residue[((1 / Zeta[s])) x^s s^(-1), {s, ZetaZero[1]}]

$$\frac{x^{ZetaZero[1]}}{ZetaZero[1]\, Zeta'[ZetaZero[1]]}$$

D[et[s, a], a] /. a → 1

$-(1-s)\,\text{Zeta}[s]$

Limit[et2[s, k], k → 1]

$\text{Zeta}[s]$

Residue[((D[et[s, a], s] / et[s, a])) x^s s^(-1), {s, ZetaZero[1]}]

$$\frac{x^{\text{ZetaZero}[1]}}{\text{ZetaZero}[1]}$$

Residue[((D[et[s, a], s] / et[s, a])) x^s s^(-1), {s, -2}]

$$-\frac{1}{2\,x^2}$$

Residue[(Zeta'[s] / Zeta[s]) x^s s^(-1), {s, -2}]

$$-\frac{1}{2\,x^2}$$

Attributes::ssle : Symbol, string, or HoldPattern[symbol] expected at position 1 in Attributes[Zeta′]. ≫

Residue[Zeta[s] x^s s^(-1), {s, 1}]

x

Limit[Residue[et[s, a] x^s s^(-1), {s, 0}], a → 1]

0

Residue[(Zeta[s])^2 x^s s^(-1), {s, 1}]

$-x + 2\,\text{EulerGamma}\,x + x\,\text{Log}[x]$

Residue[(et2[s, a])^2 x^s s^(-1), {s, 1}]

$-x + 2\,\text{EulerGamma}\,x + x\,\text{Log}[x]$

Limit[et2[s, a], a → 1]

$\text{Zeta}[s]$

f[k_] := Zeta[s]^k

Residue[((f'[t])) x^s s^(-1), {s, ZetaZero[1]}]

$$\text{Residue}\left[\frac{x^s\,\text{Log}[\text{Zeta}[s]]\,\text{Zeta}[s]^t}{s},\ \{s,\ \text{ZetaZero}[1]\}\right]$$

Residue[1 / ((s - 1) Zeta[s]) x^s s^-1, {s, ZetaZero[1]}] /. ZetaZero[1] → z

$$\frac{x^z}{(-1+z)\,z\,\text{Zeta}'[z]}$$

Limit[(s - 1) Zeta[s], s → 1]

1

Residue[1 / (Zeta[s]) x^s s^-1, {s, ZetaZero[1]}] /. ZetaZero[1] → z

$$\frac{x^z}{z\,\text{Zeta}'[z]}$$