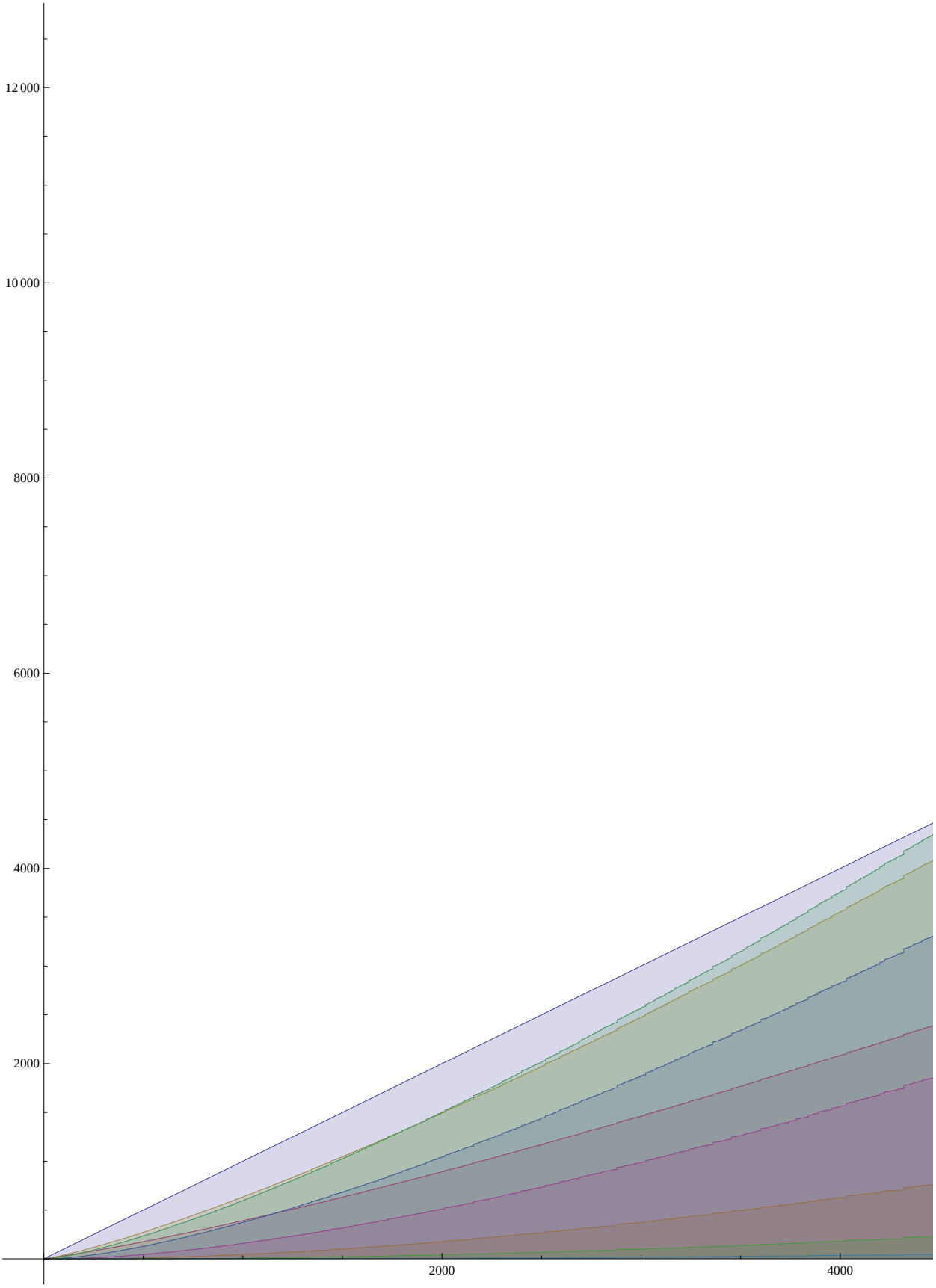
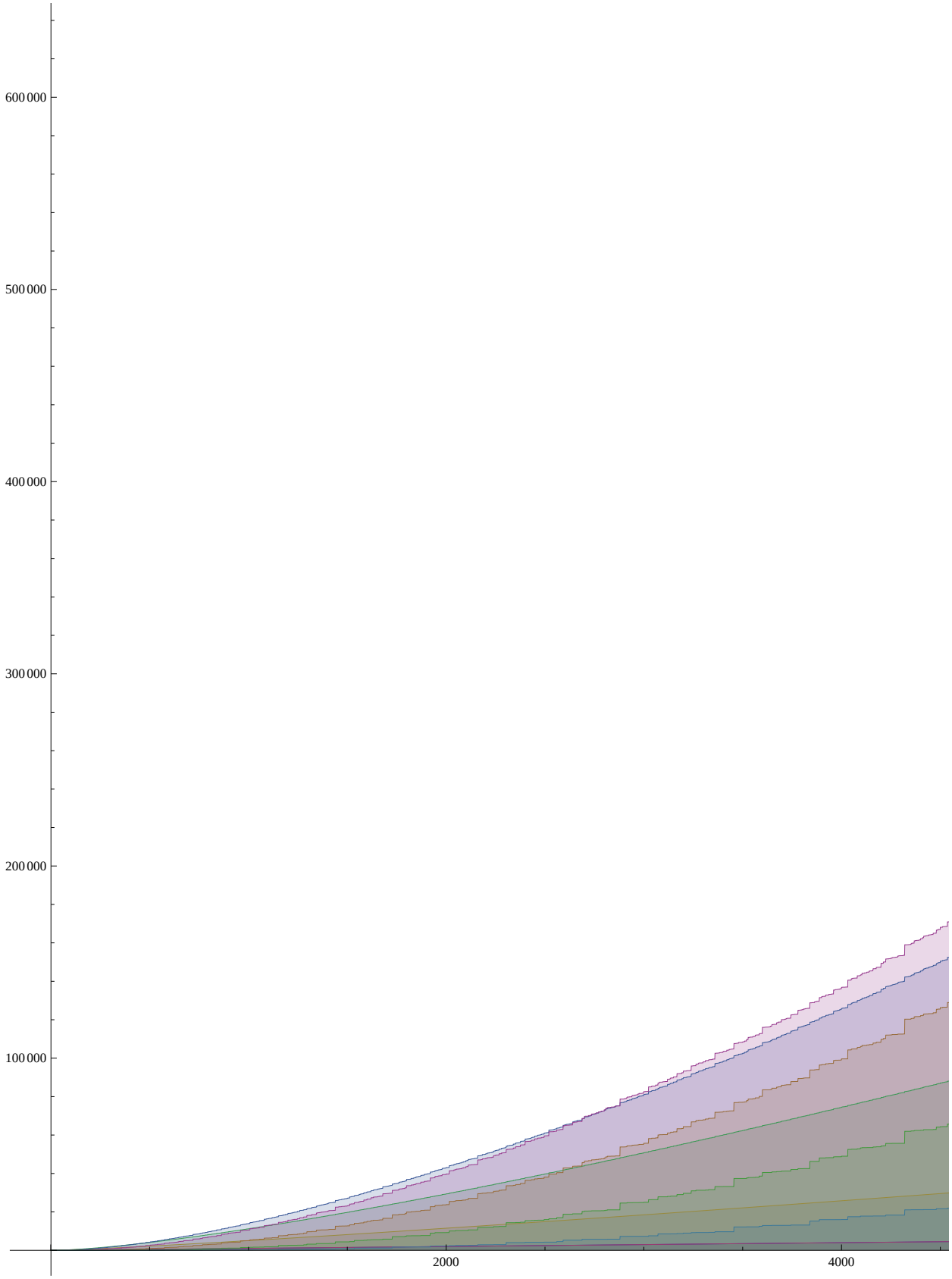
```
ClearAll["Global`*"]
$RecursionLimit = 10 000 000
```

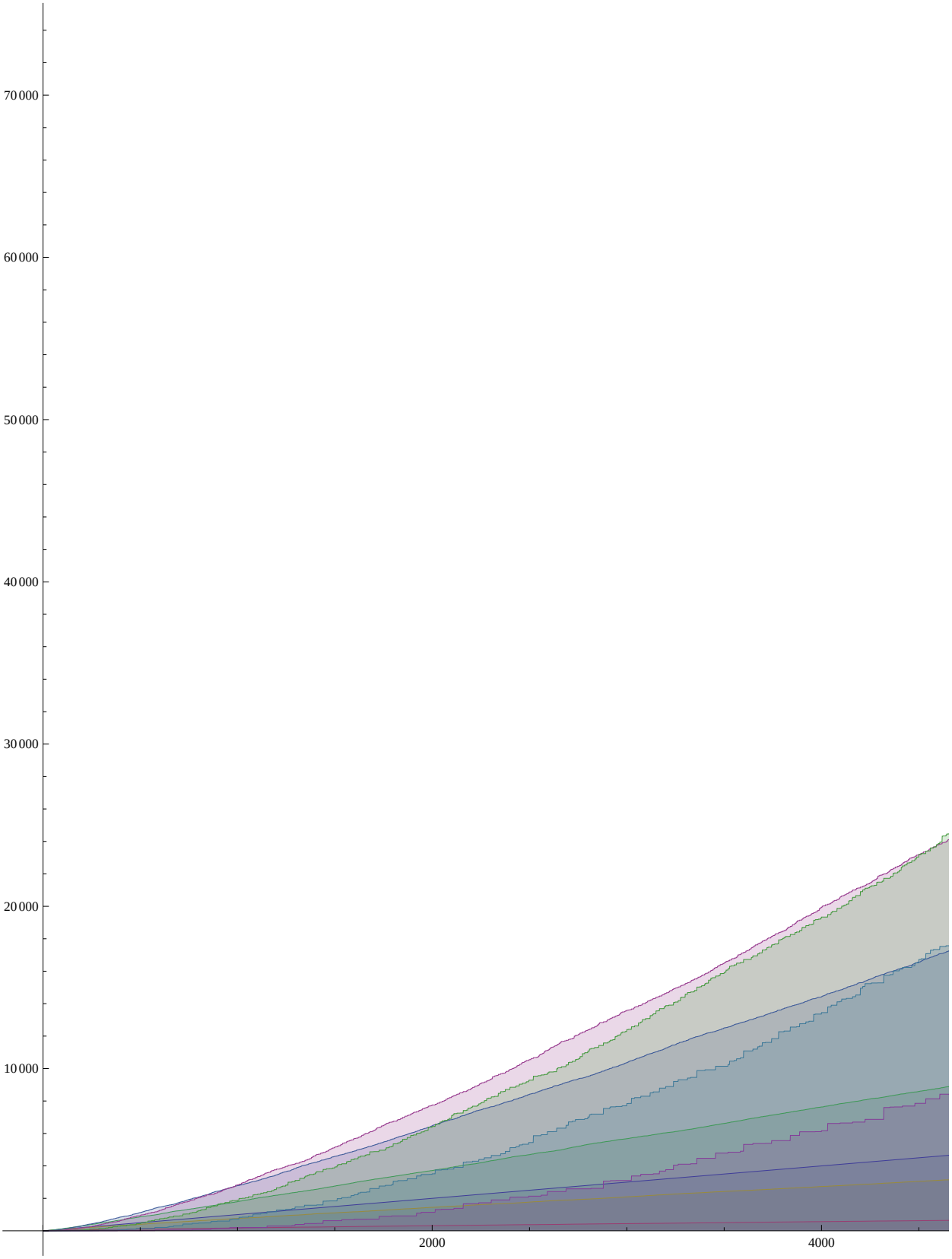
```
10 000 000
```

```
cc := cc = CoefficientList[Series[ x / Log[1 - x], {x, 0, 20}], x]
d2[n_, k_] := d2[n, k] = Sum[d2[j, k - 1] d2[n / j, 1], {j, Divisors[n]}];
d2[n_, 1] := 1; d2[1, 1] := 0; d2[n_, 0] := 0; d2[1, 0] := 1
D2[n_, k_] := D2[n, k] = D2[n - 1, k] + d2[n, k]; D2[1, k_] := 0
K[n_] := K[n] = FullSimplify[MangoldtLambda[n] / Log[n]]
k2[n_, k_] := k2[n, k] = Sum[k2[j, k - 1] k2[n / j, 1], {j, Divisors[n]}];
k2[n_, 1] := K[n]; k2[1, 1] := 0; k2[n_, 0] := 0; k2[1, 0] := 1
K2[n_, k_] := K2[n, k] = K2[n - 1, k] + k2[n, k]; K2[1, k_] := 0
e2[n_, 1] := e2[n, 1] = Sum[ cc[[k + 1]] k2[n, k], {k, 0, Log[2, n]}]; e2[1, 1] := 0;
e2[n_, k_] := Sum[e2[j, k - 1] e2[n / j, 1], {j, Divisors[n]}]; e2[n_, 0] := 0; e2[1, 0] := 1
E2[n_, k_] := E2[n, k] = E2[n - 1, k] + e2[n, k]; E2[1, k_] := 0
```

```
DiscretePlot[ {n, E2[n, 1], E2[n, 2], E2[n, 3],
  E2[n, 4], E2[n, 5], E2[n, 6], E2[n, 7], E2[n, 8]}, {n, 1, 10 000}]
DiscretePlot[ {n, D2[n, 1], D2[n, 2], D2[n, 3], D2[n, 4],
  D2[n, 5], D2[n, 6], D2[n, 7], D2[n, 8]}, {n, 1, 10 000}]
DiscretePlot[ {n, K2[n, 1], K2[n, 2], K2[n, 3], K2[n, 4],
  K2[n, 5],  , K2[n, 6], K2[n, 7], K2[n, 8]}, {n, 1, 10 000}]
```


12 000
10 000
8000
6000
4000
2000
2000
4000


600 000
500 000
400 000
300 000
200 000
100 000
2000
4000


70 000
60 000
50 000
40 000
30 000
20 000
10 000
2000
4000

```
Series[ Log[x + 1] / Log[1 - Log[x + 1]], {x, 0, 20}]
```

$$-1+\frac{x}{2}-\frac{x^2}{6}+\frac{x^3}{8}-\frac{61\,x^4}{720}+\frac{5\,x^5}{72}-\frac{3379\,x^6}{60\,480}+\frac{829\,x^7}{17\,280}-\frac{37\,501\,x^8}{907\,200}+$$
$$\frac{265\,723\,x^9}{7\,257\,600}-\frac{15\,650\,779\,x^{10}}{479\,001\,600}+\frac{16\,097\,x^{11}}{544\,320}-\frac{23\,499\,108\,071\,x^{12}}{871\,782\,912\,000}+\frac{1\,994\,072\,953\,x^{13}}{80\,472\,268\,800}-$$
$$\frac{179\,760\,855\,523\,x^{14}}{7\,846\,046\,208\,000}+\frac{27\,293\,443\,733\,x^{15}}{1\,280\,987\,136\,000}-\frac{212\,398\,886\,231\,569\,x^{16}}{10\,670\,622\,842\,880\,000}+\frac{325\,618\,047\,593\,x^{17}}{17\,435\,658\,240\,000}-$$
$$\frac{898\,455\,354\,657\,922\,519\,x^{18}}{51\,090\,942\,171\,709\,440\,000}+\frac{89\,350\,786\,286\,831\,273\,x^{19}}{5\,377\,993\,912\,811\,520\,000}-\frac{132\,709\,572\,856\,375\,410\,013\,x^{20}}{8\,430\,005\,458\,332\,057\,600\,000}+O[x]^{21}$$

```
FullSimplify[Log[x + 1] / Log[1 - Log[x + 1]]]
```

$$\frac{\text{Log}[1+x]}{\text{Log}[1-\text{Log}[1+x]]}$$

```
Series[ x / Log[1 - x], {x, 0, 20}]
```

$$-1+\frac{x}{2}+\frac{x^2}{12}+\frac{x^3}{24}+\frac{19\,x^4}{720}+\frac{3\,x^5}{160}+\frac{863\,x^6}{60\,480}+\frac{275\,x^7}{24\,192}+\frac{33\,953\,x^8}{3\,628\,800}+$$
$$\frac{8183\,x^9}{1\,036\,800}+\frac{3\,250\,433\,x^{10}}{479\,001\,600}+\frac{4671\,x^{11}}{788\,480}+\frac{13\,695\,779\,093\,x^{12}}{2\,615\,348\,736\,000}+\frac{2\,224\,234\,463\,x^{13}}{475\,517\,952\,000}+$$
$$\frac{132\,282\,840\,127\,x^{14}}{31\,384\,184\,832\,000}+\frac{2\,639\,651\,053\,x^{15}}{689\,762\,304\,000}+\frac{111\,956\,703\,448\,001\,x^{16}}{32\,011\,868\,528\,640\,000}+\frac{50\,188\,465\,x^{17}}{15\,613\,165\,568}+$$
$$\frac{2\,334\,028\,946\,344\,463\,x^{18}}{786\,014\,494\,949\,376\,000}+\frac{301\,124\,035\,185\,049\,x^{19}}{109\,285\,437\,800\,448\,000}+\frac{12\,365\,722\,323\,469\,980\,029\,x^{20}}{4\,817\,145\,976\,189\,747\,200\,000}+O[x]^{21}$$

```
tt := tt = CoefficientList[Series[ Log[x + 1] / Log[1 - Log[x + 1]], {x, 0, 20}], x]
```

```
tt[[4]]
```

$$\frac{1}{8}$$

```
E2[100, 1]
```

$$\frac{262\,613}{8640}$$

```
Sum[ tt[[k + 1]] D2[100, k], {k, 0, Log[2, 100]}]
```

$$\frac{262\,613}{8640}$$

```
t2 := t2 = CoefficientList[Series[ (Log[x + 1] / Log[1 - Log[x + 1]]), {x, 0, 20}], x]
```

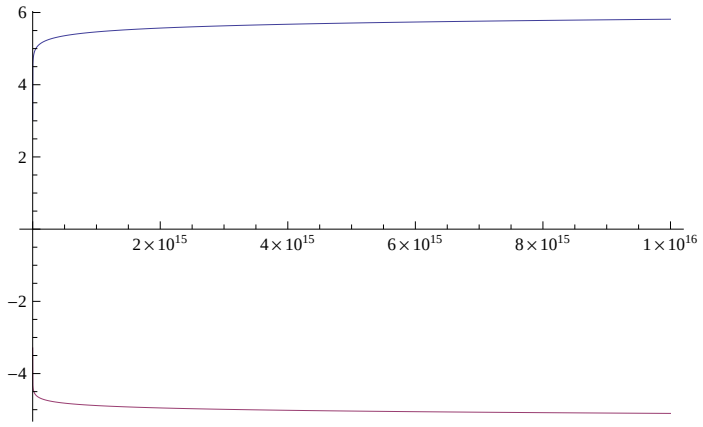
```
Sum[ t2[[k + 1]] D2[100, k], {k, 0, Log[2, 100]}]
```

$$\frac{262\,613}{8640}$$

```
E2[100, 2]
```

$$\frac{338\,761}{8640}$$

```
Plot[{Re[Log[x + 1] / Log[1 - Log[x + 1]]], Im[Log[x + 1] / Log[1 - Log[x + 1]]]},
 {x, 1, 10 000 000 000 000 000}]
```




```
N[Log[1 + x] / Log[1 - Log[1 + x]] /. x → 4]
```

$$-0.0787973 - 0.499879\,i$$

```
CoefficientList[Series[(x / Log[1 - x]), {x, 0, 20}], x]
```

$$\left\{-1, \frac{1}{2}, \frac{1}{12}, \frac{1}{24}, \frac{19}{720}, \frac{3}{160}, \frac{863}{60\,480}, \frac{275}{24\,192}, \frac{33\,953}{3\,628\,800}, \frac{8183}{1\,036\,800}, \frac{3\,250\,433}{479\,001\,600}, \frac{4671}{788\,480}, \frac{13\,695\,779\,093}{2\,615\,348\,736\,000}, \frac{2\,224\,234\,463}{475\,517\,952\,000}, \frac{132\,282\,840\,127}{31\,384\,184\,832\,000}, \frac{2\,639\,651\,053}{689\,762\,304\,000}, \frac{111\,956\,703\,448\,001}{32\,011\,868\,528\,640\,000}, \frac{50\,188\,465}{15\,613\,165\,568}, \frac{2\,334\,028\,946\,344\,463}{786\,014\,494\,949\,376\,000}, \frac{301\,124\,035\,185\,049}{109\,285\,437\,800\,448\,000}, \frac{12\,365\,722\,323\,469\,980\,029}{4\,817\,145\,976\,189\,747\,200\,000}\right\}$$

```
CoefficientList[Series[(x / Log[1 - x] + 1)^2, {x, 0, 20}], x]
```

$$\left\{0, 0, \frac{1}{4}, \frac{1}{12}, \frac{7}{144}, \frac{1}{30}, \frac{43}{1728}, \frac{79}{4032}, \frac{717}{44\,800}, \frac{3481}{259\,200}, \frac{100\,189}{8\,709\,120}, \frac{533\,077}{53\,222\,400}, \frac{1\,777\,722\,593}{201\,180\,672\,000}, \frac{156\,155\,179}{19\,813\,248\,000}, \frac{74\,216\,302\,403}{10\,461\,394\,944\,000}, \frac{15\,537\,618\,841}{2\,414\,168\,064\,000}, \frac{11\,069\,240\,202\,341}{1\,883\,051\,089\,920\,000}, \frac{5\,762\,870\,563\,187}{1\,067\,062\,284\,288\,000}, \frac{2\,682\,308\,717\,818\,019}{537\,799\,391\,281\,152\,000}, \frac{927\,089\,189\,292\,457}{200\,356\,635\,967\,488\,000}, \frac{3\,726\,882\,116\,303\,677\,517}{864\,615\,944\,444\,313\,600\,000}\right\}$$

```
CoefficientList[Series[(x / Log[1 - x] + 1)^3, {x, 0, 20}], x]
```

$$\left\{0, 0, 0, \frac{1}{8}, \frac{1}{16}, \frac{1}{24}, \frac{133}{4320}, \frac{139}{5760}, \frac{4759}{241\,920}, \frac{19\,951}{1\,209\,600}, \frac{51\,263}{3\,628\,800}, \frac{357\,151}{29\,030\,400}, \frac{312\,087\,533}{28\,740\,096\,000}, \frac{162\,473\,587}{16\,765\,056\,000}, \frac{6\,523\,044\,373}{747\,242\,496\,000}, \frac{165\,820\,399\,033}{20\,922\,789\,888\,000}, \frac{4\,547\,062\,307\,503}{627\,683\,696\,640\,000}, \frac{278\,721\,337\,231}{41\,845\,579\,776\,000}, \frac{41\,383\,527\,816\,151}{6\,722\,492\,391\,014\,400}, \frac{25\,299\,094\,414\,249}{4\,426\,332\,438\,528\,000}, \frac{1\,814\,880\,475\,713\,196\,207}{340\,606\,281\,144\,729\,600\,000}\right\}$$

```
t2 := t2 = CoefficientList[Series[(Log[x + 1] / Log[1 - Log[x + 1]] + 1)^2, {x, 0, 20}], x]
```

```
t2 := t2 = CoefficientList[Series[(Log[x + 1] / Log[1 - Log[x + 1]] + 1)^2, {x, 0, 20}], x]
t3 := t3 = CoefficientList[Series[(Log[x + 1] / Log[1 - Log[x + 1]] + 1)^3, {x, 0, 20}], x]
```

```
Sum[tx[2][[k + 1]] D2[100, k], {k, 0, Log[2, 100]}]
```

$$\frac{338\,761}{8640}$$

```
E2[100, 2]
```

$$\frac{338\,761}{8640}$$

```
Sum[ t3[[k + 1]] D2[100, k], {k, 0, Log[2, 100]}]
```

$$\frac{202\,967}{8640}$$

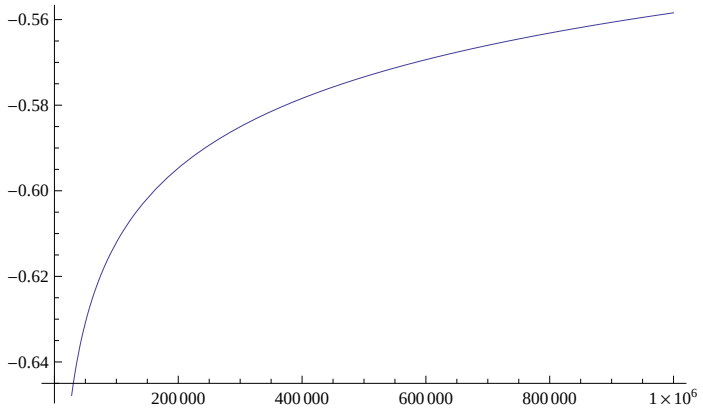
```
E2[100, 3]
```

$$\frac{202\,967}{8640}$$

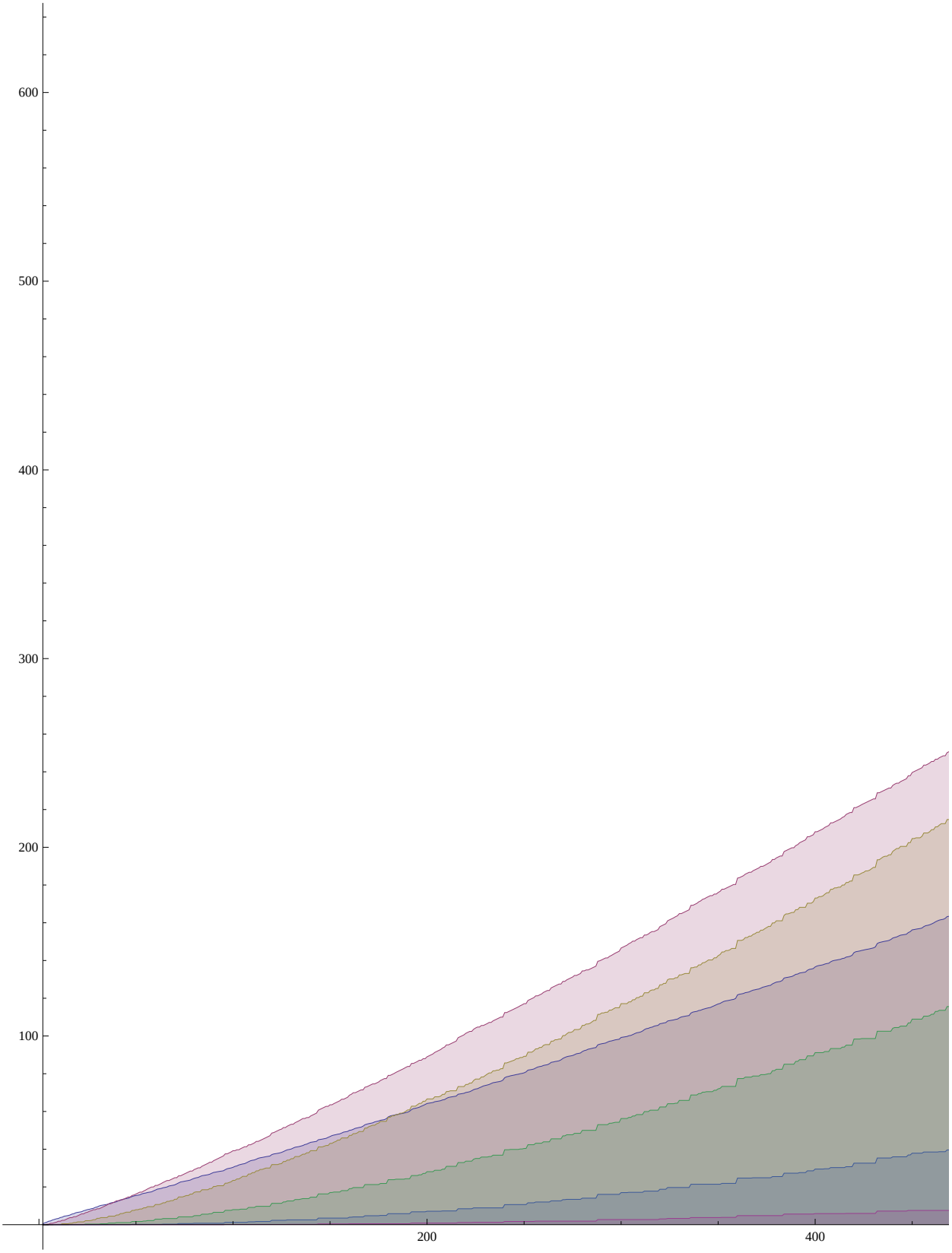
```
Series[Log[1 - Log[x + 1]] / Log[1 + Log[1 - Log[x + 1]]], {x, 0, 20}]
```

$$1 - \frac{x}{2} - \frac{x^2}{12} - \frac{x^3}{8} - \frac{x^4}{30} - \frac{19\,x^5}{288} - \frac{37\,x^6}{1680} - \frac{803\,x^7}{17\,280} - \frac{69\,509\,x^8}{3\,628\,800} - \frac{23\,141\,x^9}{604\,800} - \frac{27\,113\,x^{10}}{1\,425\,600} - \frac{3\,015\,113\,x^{11}}{87\,091\,200} - \frac{4\,383\,292\,801\,x^{12}}{217\,945\,728\,000} - \frac{6\,708\,412\,793\,x^{13}}{201\,180\,672\,000} - \frac{14\,375\,213\,173\,x^{14}}{653\,837\,184\,000} - \frac{702\,739\,034\,477\,x^{15}}{20\,922\,789\,888\,000} - \frac{9\,691\,804\,656\,587\,x^{16}}{395\,208\,253\,440\,000} - \frac{2\,928\,977\,233\,757\,x^{17}}{83\,691\,159\,552\,000} - \frac{472\,335\,337\,449\,947\,611\,x^{18}}{17\,030\,314\,057\,236\,480\,000} - \frac{246\,832\,926\,868\,303\,x^{19}}{6\,590\,678\,814\,720\,000} - \frac{356\,283\,474\,579\,591\,300\,341\,x^{20}}{11\,240\,007\,277\,776\,076\,800\,000} + O[x]^{21}$$

```
Plot[Re[Log[1 - Log[x - 1]] / Log[1 - Log[1 - Log[x - 1]]]], {x, 2, 1 000 000}]
```



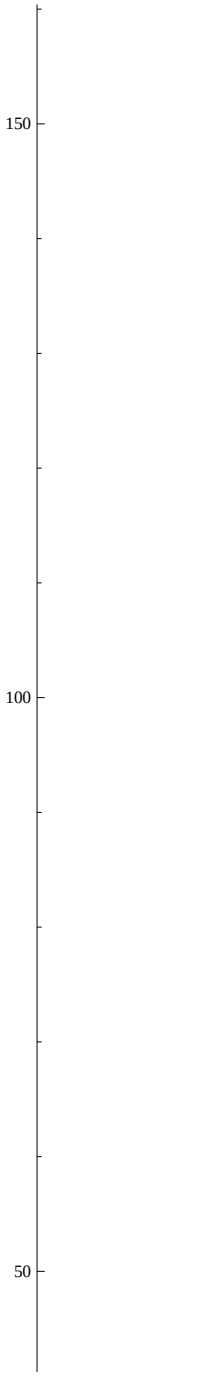

```
tx[p_] := CoefficientList[Series[ (Log[x + 1] / Log[1 - Log[x + 1]] + 1) ^ p, {x, 0, 20}], x]
DiscretePlot[ {Sum[ tx[1][[k + 1]] D2[n, k], {k, 0, Log[2, n]}],
  Sum[ tx[2][[k + 1]] D2[n, k], {k, 0, Log[2, n]}],
  Sum[ tx[3][[k + 1]] D2[n, k], {k, 0, Log[2, n]}],
  Sum[ tx[4][[k + 1]] D2[n, k], {k, 0, Log[2, n]}], Sum[ tx[5][[k + 1]] D2[n, k],
   {k, 0, Log[2, n]}], Sum[ tx[6][[k + 1]] D2[n, k], {k, 0, Log[2, n]}]}, {n, 2, 1000}]
```

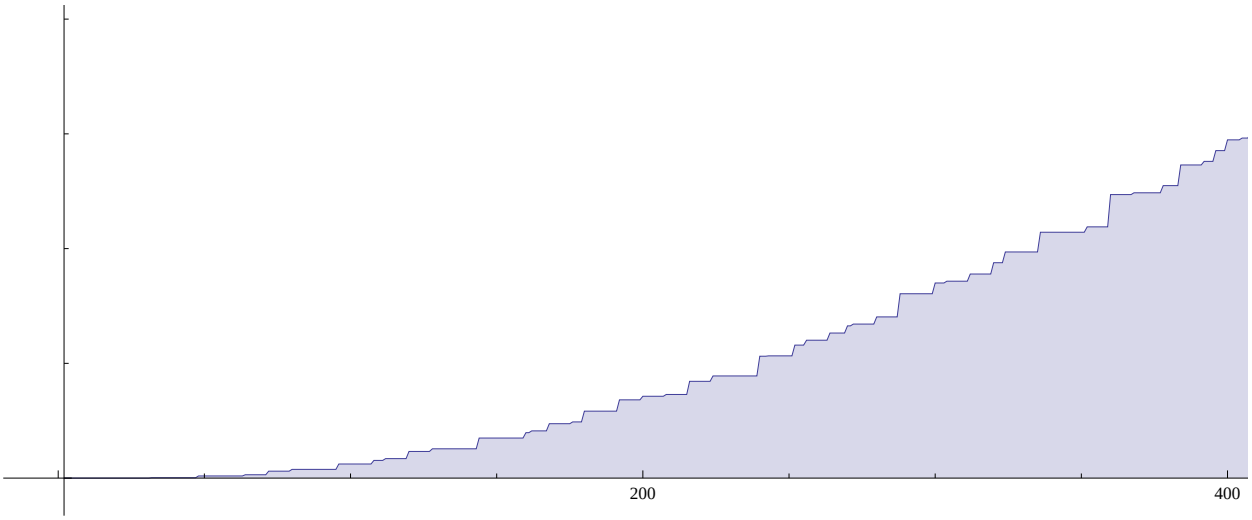

600
500
400
300
200
100
200
400

```
tx[5]
```

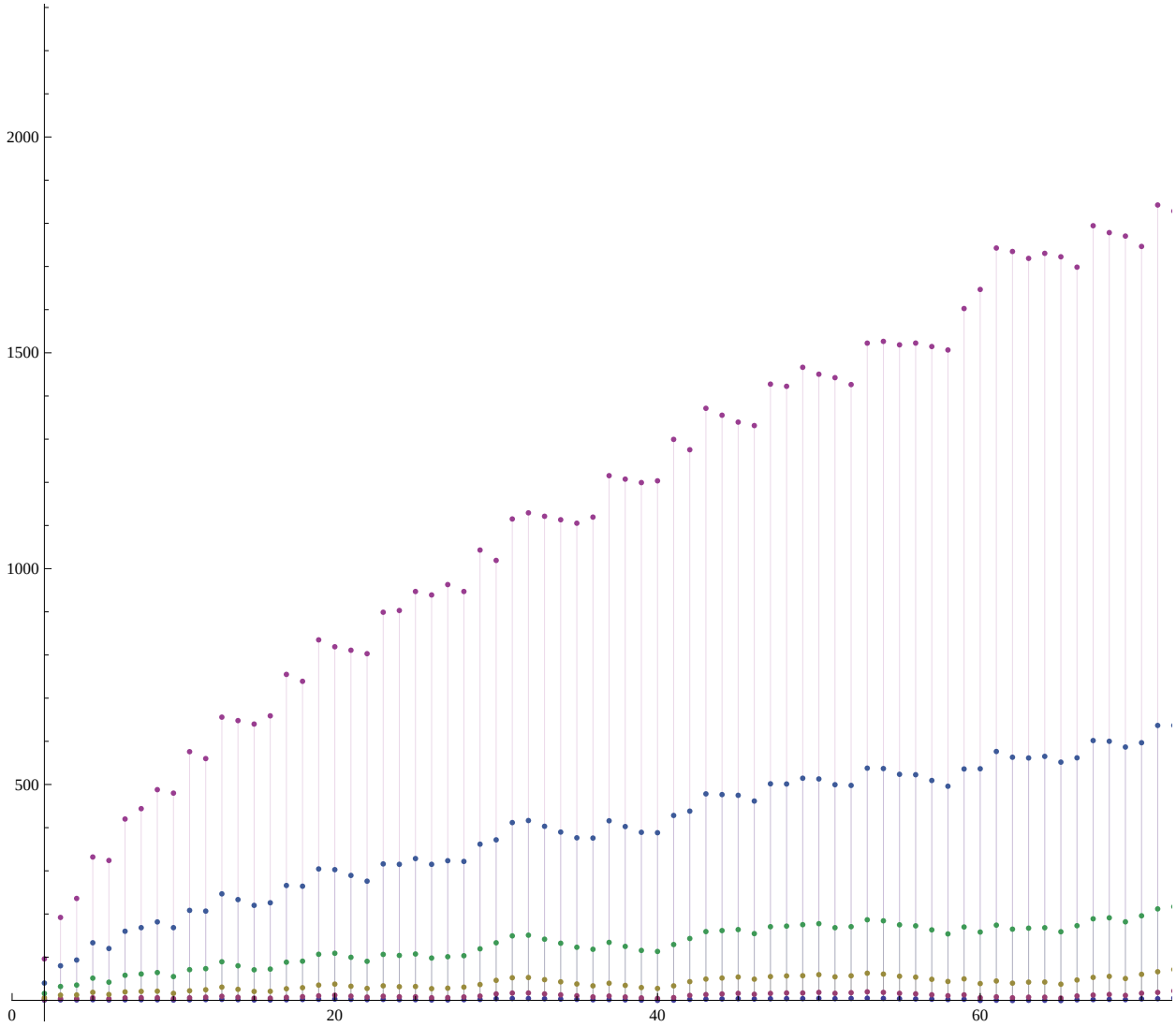
$$\left\{0, 0, 0, 0, 0, \frac{1}{32}, -\frac{5}{96}, \frac{85}{1152}, -\frac{623}{6912}, \frac{2167}{20\,736}, -\frac{201\,995}{1\,741\,824}, \frac{2\,189\,227}{17\,418\,240}, -\frac{6\,984\,499}{52\,254\,720}, \frac{2\,445\,181}{17\,418\,240}, -\frac{1\,118\,775\,773}{7\,664\,025\,600}, \frac{145\,519\,551\,643}{965\,667\,225\,600}, -\frac{529\,483\,907\,749}{3\,423\,729\,254\,400}, \frac{5\,409\,067\,633\,031}{34\,237\,292\,544\,000}, -\frac{242\,225\,756\,635\,337}{1\,506\,440\,871\,936\,000}, \frac{163\,854\,164\,309\,879}{1\,004\,293\,914\,624\,000}, -\frac{2\,220\,202\,095\,423\,620\,309}{13\,444\,984\,782\,028\,800\,000}\right\}$$

```
tx[p_] := CoefficientList[Series[ (Log[x + 1] / Log[1 - Log[x + 1]] + 1)^p, {x, 0, 20}], x]
DiscretePlot[ {Sum[ tx[5][[k + 1]] D2[n, k], {k, 0, Log[2, n]}]}, {n, 2, 1000}]
```

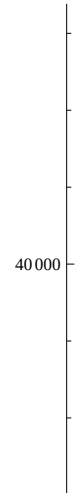


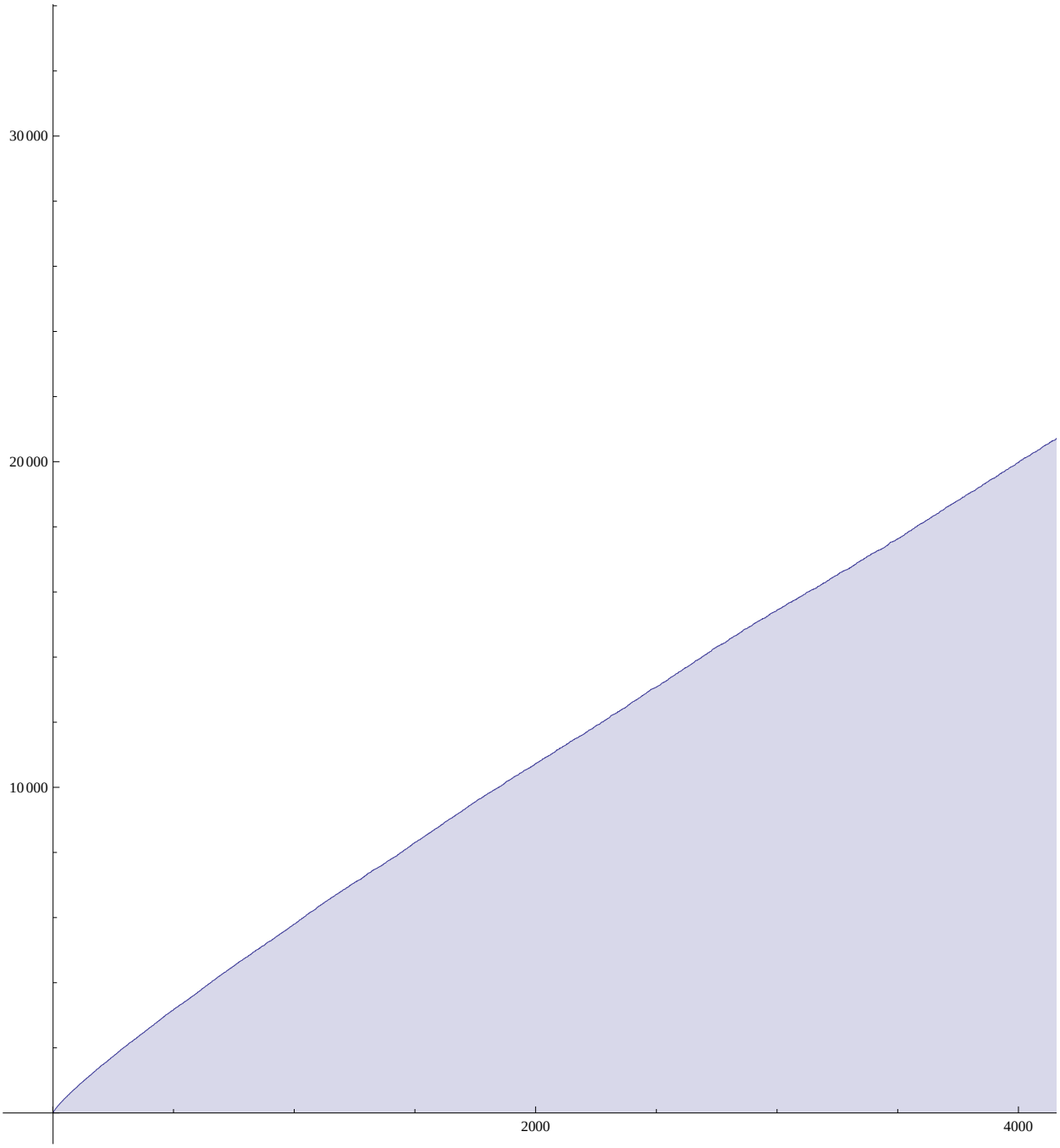



```
tx[p_] := CoefficientList[
  Series[ (1 + Log[1 + Log[1 + x]] / Log[1 + Log[1 + Log[1 + x]]])^p, {x, 0, 20}], x]
DiscretePlot[ {Sum[ tx[1][[k + 1]] D2[n, k], {k, 0, Log[2, n]}],
  Sum[ tx[2][[k + 1]] D2[n, k], {k, 0, Log[2, n]}],
  Sum[ tx[3][[k + 1]] D2[n, k], {k, 0, Log[2, n]}],
  Sum[ tx[4][[k + 1]] D2[n, k], {k, 0, Log[2, n]}], Sum[ tx[5][[k + 1]] D2[n, k],
   {k, 0, Log[2, n]}], Sum[ tx[6][[k + 1]] D2[n, k], {k, 0, Log[2, n]}]}, {n, 2, 100}]
```




```
tx[p_] := CoefficientList[Series[ (Log[x + 1] / Log[1 + Log[x + 1]] + 1)^p, {x, 0, 20}], x]
DiscretePlot[ {Sum[ tx[4][[k + 1]] D2[n, k], {k, 0, Log[2, n]}]}, {n, 1, 10 000}]
```






```
CoefficientList[Series[(Log[x + 1] / Log[1 + Log[x + 1]] - 1)^4, {x, 0, 20}], x]
```

$$\left\{0, 0, 0, 0, \frac{1}{16}, -\frac{1}{6}, \frac{5}{16}, -\frac{2207}{4320}, \frac{40\,519}{51\,840}, -\frac{20\,959}{18\,144}, \frac{36\,504\,023}{21\,772\,800}, -\frac{13\,130\,779}{5\,443\,200}, \frac{1\,808\,060\,263}{522\,547\,200},\right.$$
$$-\frac{7\,926\,004\,517}{1\,596\,672\,000}, \frac{1\,435\,964\,301\,661}{201\,180\,672\,000}, -\frac{161\,590\,944\,260\,929}{15\,692\,092\,416\,000}, \frac{28\,085\,912\,543\,626\,837}{1\,883\,051\,089\,920\,000},$$
$$\left.-\frac{464\,199\,970\,130\,951}{21\,398\,307\,840\,000}, \frac{786\,750\,180\,754\,421}{24\,831\,442\,944\,000}, -\frac{120\,130\,716\,762\,724\,379}{2\,585\,573\,996\,544\,000}, \frac{5\,517\,399\,076\,532\,628\,998\,957}{80\,669\,908\,692\,172\,800\,000}\right\}$$

```
tx[p_] := CoefficientList[Series[ (Tan[x])^p, {x, 0, 20}], x]
DiscretePlot[ {Sum[ tx[1][[k + 1]] D2[n, k], {k, 0, Log[2, n]}],
  Sum[ tx[2][[k + 1]] D2[n, k], {k, 0, Log[2, n]}],
  Sum[ tx[3][[k + 1]] D2[n, k], {k, 0, Log[2, n]}],
  Sum[ tx[4][[k + 1]] D2[n, k], {k, 0, Log[2, n]}], Sum[ tx[5][[k + 1]] D2[n, k],
   {k, 0, Log[2, n]}], Sum[ tx[6][[k + 1]] D2[n, k], {k, 0, Log[2, n]}]}, {n, 2, 1000}]
```