```
FF[n_, r_] := Sum[FractionalPart[n / x], {x, r, n}]
```

```
FF[100, 51]
```

$$\frac{13\,119\,434\,949\,299\,548\,336\,211\,704\,572\,006\,746\,344\,217}{697\,203\,752\,297\,124\,771\,645\,338\,089\,353\,123\,035\,568}$$

```
FF2[n_, r_] := Sum[1 - ((x - r) / (n - r)), {x, r, n}]
```

```
FF2[100, 50]
```

$$\frac{51}{2}$$

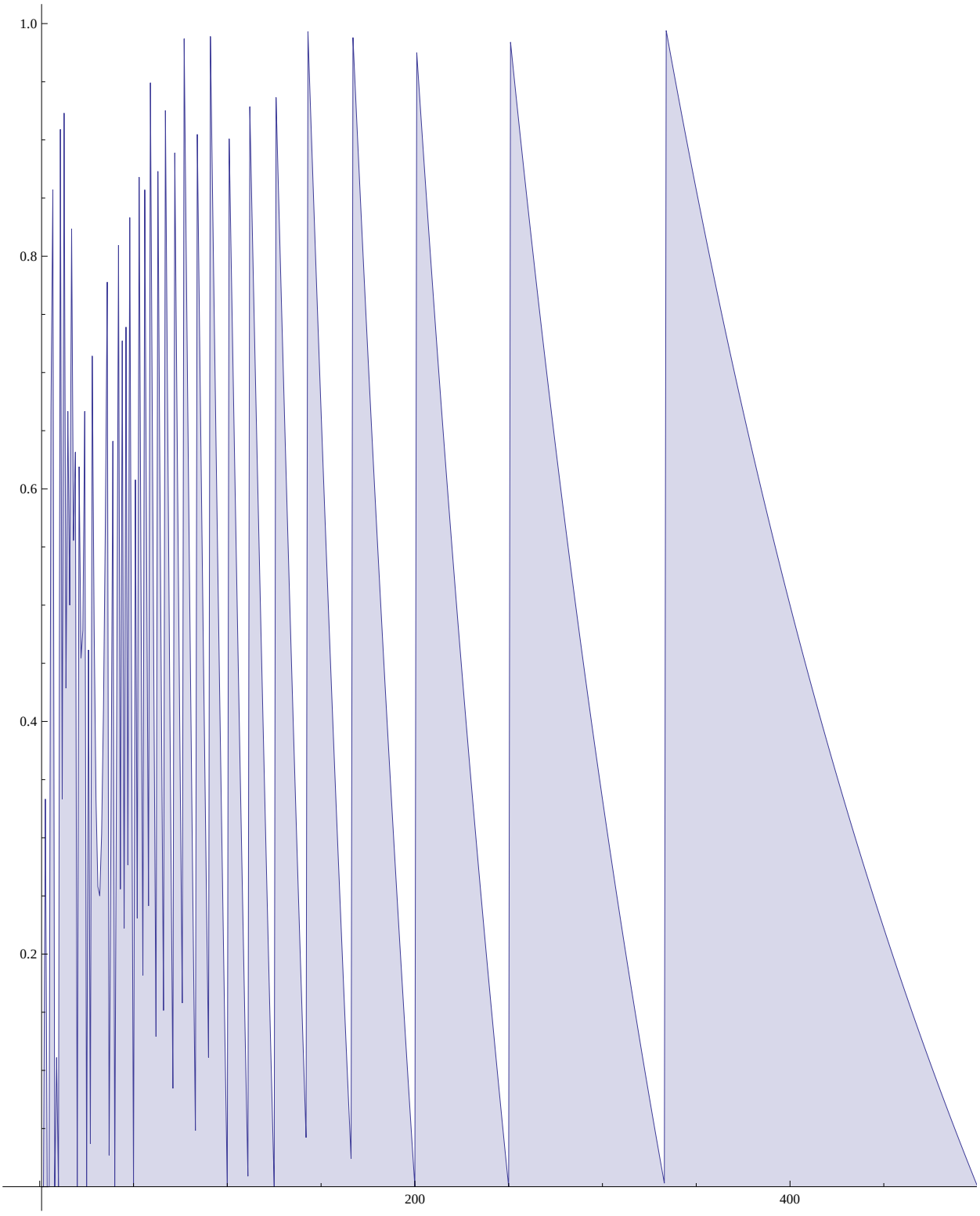
```
N[0 / 50]
```

```
0.
```

```
N[FractionalPart[100 / 51]]
```

```
0.960784
```

```
DiscretePlot[ FractionalPart[1000 / n], {n, 1, 1000}]
```




```
Sum[FractionalPart[n / x], {x, Floor[n / 2] + 1, n}]
```

$$-n+\text{Floor}\left[\frac{n}{2}\right]+n\,\text{PolyGamma}[0,\,1+n]-n\,\text{PolyGamma}\left[0,\,1+\text{Floor}\left[\frac{n}{2}\right]\right]$$

```
Sum[FractionalPart[n / x], {x, Floor[n / 3] + 1, n / 2}]
```

$$-n + 2\,\text{Floor}\left[\frac{n}{3}\right] + n\,\text{PolyGamma}\left[0,\ 1 + \frac{n}{2}\right] - n\,\text{PolyGamma}\left[0,\ 1 + \text{Floor}\left[\frac{n}{3}\right]\right]$$

```
FF[100, 51]
```

$$\frac{13\,119\,434\,949\,299\,548\,336\,211\,704\,572\,006\,746\,344\,217}{697\,203\,752\,297\,124\,771\,645\,338\,089\,353\,123\,035\,568}$$

```
FP[n_, a_] := Sum[FractionalPart[n / x], {x, Floor[n / (a + 1)] + 1, n / a}]
FP[100, 1]
```

$$\frac{13\,119\,434\,949\,299\,548\,336\,211\,704\,572\,006\,746\,344\,217}{697\,203\,752\,297\,124\,771\,645\,338\,089\,353\,123\,035\,568}$$

```
FF[99, 50]
```

$$\frac{2\,692\,903\,118\,875\,180\,604\,233\,495\,593\,953\,476\,599\,637}{140\,849\,242\,888\,308\,034\,675\,825\,876\,636\,994\,552\,640}$$

```
FP[99]
```

$$\frac{2\,692\,903\,118\,875\,180\,604\,233\,495\,593\,953\,476\,599\,637}{140\,849\,242\,888\,308\,034\,675\,825\,876\,636\,994\,552\,640}$$

```
FF2[n_, a_] := FF[n, Floor[n / (a + 1)] + 1, n / a]
FF[100, 51, 100]
```

$$\frac{13\,119\,434\,949\,299\,548\,336\,211\,704\,572\,006\,746\,344\,217}{697\,203\,752\,297\,124\,771\,645\,338\,089\,353\,123\,035\,568}$$

```
FF[n_, r1_, r2_] := Sum[FractionalPart[n / x], {x, r1, r2}]
FP[n_, a_] := Sum[FractionalPart[n / x], {x, Floor[n / (a + 1)] + 1, n / a}]
Table[ FP[100, n] - FF2[100, n], {n, 1, 99}]
```

```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

```
FF2[n, a]
```

$$\sum_{x=1+\text{Floor}\left[\frac{n}{1+a}\right]}^{\frac{n}{a}} \text{FractionalPart}\left[\frac{n}{x}\right]$$

```
FP[n, a]
```

$$\sum_{x=1+\text{Floor}\left[\frac{n}{1+a}\right]}^{\frac{n}{a}} \text{FractionalPart}\left[\frac{n}{x}\right]$$

```
Sum[FractionalPart[n / x], {x, Floor[n / (3 + 1)] + 1, n / 3}]
```

$$-n + 3\,\text{Floor}\left[\frac{n}{4}\right] + n\,\text{PolyGamma}\left[0,\ 1 + \frac{n}{3}\right] - n\,\text{PolyGamma}\left[0,\ 1 + \text{Floor}\left[\frac{n}{4}\right]\right]$$

```
Sum[FractionalPart[n / x], {x, Floor[n / (4 + 1)] + 1, Floor[n / 4]}]
```

$$\sum_{x=1+\text{Floor}\left[\frac{n}{5}\right]}^{\text{Floor}\left[\frac{n}{4}\right]} \text{FractionalPart}\left[\frac{n}{x}\right]$$

```
FR[n_, a_] := -n + a Floor[n / (a + 1)] + n PolyGamma[0, 1 + n / a] -
  n PolyGamma[0, 1 + Floor[n / (a + 1)]] - (n / a - Floor[n / a])
```

```
N[FR[100, 3]]
```

```
2.94071
```

```
N[FP[100, 3]]
```

```
3.284
```