```
bin[z_{,k_{]} := bin[z,k] = Product[z-j, {j, 0, k-1}] / k!
FI[n_] := FactorInteger[n]; FI[1] := {}
bin[z_{,k_{]} := bin[z,k] = Product[z-j, {j, 0, k-1}] / k!
pp[n_{,s_{,k_{,j}}} := pp[n,s,k] = Sum[m^{(-sj)}pp[n-j,s,k-1], {j,1,n-1}]
pp[n_{, s_{, 1}}] := m^{(-s_{, 1})}
pp[n_{-}, s_{-}, 0] := If[n = 0, 1, 0]
pss[n_, s_, z_] := Sum[bin[z, k] pp[n, s, k], \{k, 0, n\}]
dez[n_, z_] := Product[z^p[[2]] / (p[[2]]!), {p, FI[n]}]
Expand@Table[D[pss[n, -1, z], z] /. z \rightarrow 0, {n, 1, 10}]
\big\{m\,,\,\,\frac{m^2}{2}\,,\,\,\frac{m^3}{3}\,,\,\,\frac{m^4}{4}\,,\,\,\frac{m^5}{5}\,,\,\,\frac{m^6}{6}\,,\,\,\frac{m^7}{7}\,,\,\,\frac{m^8}{8}\,,\,\,\frac{m^9}{9}\,,\,\,\frac{m^{10}}{10}\,\big\}
Table[pss[n, -s, 3], {n, 1, 10}]
\{3\,\mathrm{m^s}, 6\,\mathrm{m^{2}}^{\mathrm{s}}, 10\,\mathrm{m^{3}}^{\mathrm{s}}, 15\,\mathrm{m^{4}}^{\mathrm{s}}, 21\,\mathrm{m^{5}}^{\mathrm{s}}, 28\,\mathrm{m^{6}}^{\mathrm{s}}, 36\,\mathrm{m^{7}}^{\mathrm{s}}, 45\,\mathrm{m^{8}}^{\mathrm{s}}, 55\,\mathrm{m^{9}}^{\mathrm{s}}, 66\,\mathrm{m^{10}}^{\mathrm{s}}\}
FullSimplify@Table[pss[n, s, z], {n, 1, 10}] // TableForm
m^{-s}z
\frac{1}{2} m^{-2} s z (1 + z)
\frac{1}{6} \text{ m}^{-3 \text{ s}} \text{ z } (1 + \text{z}) (2 + \text{z})
\frac{1}{24} \, m^{-4} \, s \, z \, (1+z) \, (2+z) \, (3+z)
\frac{1}{120} \text{ m}^{-5 \text{ s}} \text{ z } (1+\text{z}) (2+\text{z}) (3+\text{z}) (4+\text{z})
\frac{1}{720} m<sup>-6 s</sup> z (1 + z) (2 + z) (3 + z) (4 + z) (5 + z)
\,m^{-7\,\,s}\,\,z\,\,\left(\,1\!+\!z\,\right)\,\,\left(\,2\!+\!z\,\right)\,\,\left(\,3\!+\!z\,\right)\,\,\left(\,4\!+\!z\,\right)\,\,\left(\,5\!+\!z\,\right)\,\,\left(\,6\!+\!z\,\right)
m^{-8\;\text{s}}\;z\;\left(1\!+\!z\right)\;\left(2\!+\!z\right)\;\left(3\!+\!z\right)\;\left(4\!+\!z\right)\;\left(5\!+\!z\right)\;\left(6\!+\!z\right)\;\left(7\!+\!z\right)
                          40 320
m^{-9\;s}\;z\;\left(1\!+\!z\right)\;\left(2\!+\!z\right)\;\left(3\!+\!z\right)\;\left(4\!+\!z\right)\;\left(5\!+\!z\right)\;\left(6\!+\!z\right)\;\left(7\!+\!z\right)\;\left(8\!+\!z\right)
                             362880
{\tt m^{-10\,s}\;z\;(1+z)\;(2+z)\;(3+z)\;(4+z)\;(5+z)\;(6+z)\;(7+z)\;(8+z)\;(9+z)}
                                 3 628 800
Sum[Binomial[z, k], {k, 0, Infinity}]
2^{z}
Sum[Binomial[z, k] x^(-sk), {k, 0, Infinity}]
(1 + x^{-s})^{z}
Sum[Pochhammer[z, k] / k! x^(-sk), {k, 0, Infinity}]
Sum[z^k/k! x^(-sk), {k, 0, Infinity}]
Sum[Pochhammer[z, k] / k! x^(-sk), \{k, 0, Infinity\}]
(1 - x^{-s})^{-z}
(1 - x^{-s})^{-z} / . x \rightarrow 5 / . z \rightarrow 1 / . s \rightarrow 2
25
24
```

Clear[pp, dez]

```
Sum[z^k/k! x^(-sk), \{k, 0, Infinity\}]
e^{x^{-s} z}
e^{x^{-s}z} /. z \rightarrow 1 /. x \rightarrow 5 /. s \rightarrow 2
e<sup>1/25</sup>
N@Product[E^(1/Prime[j]^2), {j, 1, 100 000}]
1.57184
N@Pi^2/6
1.64493
N@Product[E^(1/Prime[j]^2), {j, 1, 1000000}]
$Aborted
N@Product[E^{(1/Prime[j]^3), {j, 1, 100000}]
1.19096
Table[N@Pi^3/n, {n, 1, 50}]
{31.0063, 15.5031, 10.3354, 7.75157, 6.20126, 5.16771, 4.42947, 3.87578,
 3.44514, 3.10063, 2.81875, 2.58386, 2.3851, 2.21473, 2.06709, 1.93789, 1.8239,
 1.72257, 1.63191, 1.55031, 1.47649, 1.40938, 1.3481, 1.29193, 1.24025, 1.19255,
 1.14838, 1.10737, 1.06918, 1.03354, 1.0002, 0.968946, 0.939584, 0.911949,
 0.885894, 0.861285, 0.838007, 0.815955, 0.795033, 0.775157, 0.756251, 0.738245,
 0.721076,\, 0.704688,\, 0.689028,\, 0.674049,\, 0.659708,\, 0.645964,\, 0.632781,\, 0.620126\}
N@Product[E^(1/Prime[j]), {j, 1, 10000}]
15.0181
N@Product[E^(1/Prime[j]), {j, 1, 100 000}]
18.2862
N@Product[Zeta[2k]^(MoebiusMu[k]/k), \{k, 1, 400\}]
1.57184
N@Product[Zeta[2k], {k, 1, Infinity}]
1.82102
N@Product[Zeta[3k]^(MoebiusMu[k]/k), \{k, 1, 1000\}]
1.19096
N[Zeta[3]]
1.20206
\label{eq:new_product} $$N@Product[Zeta[ZetaZero[2] / 6 k]^(MoebiusMu[k] / k), \{k, 1, 100\}]$$
N@Sum[MoebiusMu[k]/k, {k, 1, 100}]
0.0311315
Zeta[1] ^-1
0
```

```
N@Product[Zeta[4k]^(MoebiusMu[k]/k), \{k, 1, 400\}]
1.08003
Zeta[4.] / (Pi^4)
 0.0111111
1.0800346670613987\(Pi^4)
 0.0110876
1.5718408053876343 / (Pi^2)
 0.159261
Zeta[2.] / (Pi^2)
 0.166667
Table[dez[n, 1], {n, 1, 12}]
 \left\{1, 1, 1, \frac{1}{2}, 1, 1, 1, \frac{1}{6}, \frac{1}{2}, 1, 1, \frac{1}{2}\right\}
Clear[de]
de[n_{-}, s_{-}, k_{-}] := de[n, s, k] = Sum[dez[j, 1] j^{-}(-s) de[Floor[n/j], s, k-1], {j, 2, n}]
de[n_{-}, s_{-}, 0] := UnitStep[n-1]
N@Table[edz[n, 2, 1], {n, 1, 20}]
   {1., 1.25, 1.36111, 1.39236, 1.43236, 1.46014, 1.48055, 1.48315, 1.48932, 1.49932, 1.50759,
         1.51106, 1.51698, 1.52208, 1.52652, 1.52669, 1.53015, 1.53169, 1.53446, 1.53571}
Table[N@ ZetaZero[1] / k, {k, 1, 50}]
   \{0.5+14.1347\,\text{i}\,,\,0.25+7.06736\,\text{i}\,,\,0.166667+4.71158\,\text{i}\,,\,0.125+3.53368\,\text{i}\,,\,0.1+2.82695\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i}\,,\,0.144.1347\,\text{i
         0.0833333 + 2.35579 \pm 0.0714286 + 2.01925 \pm 0.0625 + 1.76684 \pm 0.0555556 + 1.57053 \pm 0.0625 + 1.0684 \pm 0.0625 + 1.0684 \pm 0.0684 \pm 0.0684
         0.05 + 1.41347 \pm ,\ 0.0454545 + 1.28498 \pm ,\ 0.0416667 + 1.17789 \pm ,\ 0.0384615 + 1.08729 \pm ,
          0.0357143 + 1.00962\,\dot{\text{i}}, 0.03333333 + 0.942315\,\dot{\text{i}}, 0.03125 + 0.88342\,\dot{\text{i}}, 0.0294118 + 0.831454\,\dot{\text{i}},
         0.0277778 + 0.785263 \, i, 0.0263158 + 0.743933 \, i, 0.025 + 0.706736 \, i, 0.0238095 + 0.673082 \, i,
         0.0227273 + 0.642488\,\dot{\text{i}}, 0.0217391 + 0.614553\,\dot{\text{i}}, 0.0208333 + 0.588947\,\dot{\text{i}},
         0.02 + 0.565389\,\,\dot{\text{i}}, 0.0192308 + 0.543643\,\dot{\text{i}}, 0.0185185 + 0.523508\,\dot{\text{i}}, 0.0178571 + 0.504812\,\dot{\text{i}},
         0.0172414 + 0.487404 \pm 0.0166667 + 0.471158 \pm 0.016129 + 0.455959 \pm 0.015625 + 0.44171 \pm 0.016129 + 0.455959 \pm 0.016129 + 0.455959 \pm 0.015625 + 0.44171 \pm 0.016129 + 0.455959 \pm 0.015625 + 0.44171 \pm 0.016129 + 0.455959 \pm 0.015625 + 0.44171 \pm 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016129 + 0.016
         0.0151515 + 0.428325 \pm ,\ 0.0147059 + 0.415727 \pm ,\ 0.0142857 + 0.403849 \pm ,
         0.0138889 + 0.392631\,\dot{\mathrm{i}}\,,\, 0.0135135 + 0.38202\,\dot{\mathrm{i}}\,,\, 0.0131579 + 0.371966\,\dot{\mathrm{i}}\,,\, 0.0128205 + 0.362429\,\dot{\mathrm{i}}\,,\, 0.0128205 + 0.362429\,\dot{\mathrm{i}}\,
         0.0125 + 0.353368\,\dot{\mathtt{i}}\,,\, 0.0121951 + 0.344749\,\dot{\mathtt{i}}\,,\, 0.0119048 + 0.336541\,\dot{\mathtt{i}}\,,\, 0.0116279 + 0.328715\,\dot{\mathtt{i}}\,,\, 0.0116279 + 0.0016279\,\dot{\mathtt{i}}\,,\, 0.0016279 + 0.0016279\,\dot{\mathtt{i}}\,,\, 0.0016279\,\dot{\mathtt{i}}\,,\, 0.0016279\,\dot{\mathtt{i}}\,,\, 0.0016279\,\dot{\mathtt{i}}\,,\, 0.0016279\,\dot{\mathtt{i}}\,,\, 0.0016279\,\dot{\mathtt{i}}\,,\, 0.00
         0.0113636 + 0.321244 \pm, \ 0.01111111 + 0.314105 \pm, \ 0.0108696 + 0.307277 \pm,
          0.0106383 + 0.300739 \pm, \ 0.0104167 + 0.294473 \pm, \ 0.0102041 + 0.288464 \pm, \ 0.01 + 0.282695 \pm \}
\label{eq:new_product} $$N@Product[(Zeta[kZetaZero[1]/100])^(MoebiusMu[k]/k), \{k, 1, 400\}]$$
  Power::indet: Indeterminate expression 0<sup>0</sup> encountered. ≫
 Indeterminate
N[ZetaZero[1] / 100]
```

0.005 + 0.141347 i

```
data = Flatten@Table[ZetaZero[n] / k, {n, 1, 500}, {k, 1, 20}];
p = ListPlot[{Re[#], Im[#]} & /@data, AxesOrigin \rightarrow {0, 0},
   PlotRange \rightarrow \{\{0, 1\}, \{0, 100\}\}, \text{ImagePadding} \rightarrow 40, \text{AspectRatio} \rightarrow 1,
   Frame → True, FrameLabel → {{Im, None}, {Re, "complex plane"}},
   PlotStyle → Directive [Red, PointSize[.002]]];
Show[p, Graphics@Circle[{0, 0}, .01]]
Clear[de, ede, edeo, alt21]
rootsa[n_, s_] := If[(c = Exponent[f = deee[n, s, z], z]) == 0, {},
  If[c == 1, List@Roots[f == 0, z][[2]], List@@Roots[f == 0, z][[All, 2]]]]
\mathtt{de}[\texttt{n\_, s\_, k\_}] := \mathtt{de}[\texttt{n, s, k}] = \mathtt{Sum}[\mathtt{dez}[\texttt{j, 1}] \ \texttt{j^(-s)} \ \mathtt{de}[\mathtt{Floor}[\texttt{n / j}], \texttt{s, k - 1}], \ \{\texttt{j, 2, n}\}]
de[n_{-}, s_{-}, 0] := UnitStep[n-1]
ede[n_, s_, k_] :=
 ede[n, s, k] = Sum[(-1)^{(j+1)} dez[j, 1] j^{(-s)} ede[Floor[n/j], s, k-1], {j, 2, n}]
ede[n_, s_, 0] := UnitStep[n-1]
eedz[n_{s}, s_{s}] := Sum[bin[z, k] ede[n, s, k], \{k, 0, Log2@n\}]
edeo[n_, s_, k_] :=
 edeo[n, s, k] = Sum[(-1)^{(j+1)}^{(-s)}] edeo[Floor[n/j], s, k-1], {j, 2, n}]
edeo[n_s, s_s, 0] := UnitStep[n-1]
is2[j_] := is2[j] = FullSimplify[If[Log[2, j] == Floor[Log[2, j]]],
     2^(1/(MangoldtLambda[j]/Log[j])) (MangoldtLambda[j]/Log[j]), 0]]
alt2l[n_{,s_{,k_{,j}}} := alt2l[n,s,k] = Sum[(Floor[MangoldtLambda[j]/Log[j]] - is2[j])
     alt21[Floor[n/j], s, k-1], {j, 2, n}]
alt21[n_, s_, 0] := UnitStep[n-1]
Table [D[eedz[n, 0, z] - eedz[n-1, 0, z], z] /. z \rightarrow 0, {n, 2, 100}]
1, 0, 0, 0, 0, 1, 0, 1, -\frac{5}{4}, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 
 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, -\frac{541}{360}, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0
 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}
Table[D[eedz[2^n, 0, z] - eedz[2^n-1, 0, z], z] /. z \rightarrow 0, {n, 1, 12}]
\left\{-1\,,\,-1\,,\,-1\,,\,-\frac{13}{12}\,,\,-\frac{5}{4}\,,\,-\frac{541}{360}\,,\,-\frac{223}{120}\,,\,-\frac{47\,293}{20\,160}\,,\,-\frac{36\,389}{12\,096}\,,\,-\frac{7\,087\,261}{1\,814\,400}\,,\,-\frac{3\,098\,411}{604\,800}\,,\,-\frac{1\,622\,632\,573}{239\,500\,800}\right\}
(* https://oeis.org/A000629 *)
Table \left[ \; \left( \text{D[eedz[2^n, 0, z] - eedz[2^n - 1, 0, z], z] /. z \to 0) (n!) \;, \; \{n, 1, 12\} \right] \right]
\{-1, -2, -6, -26, -150, -1082, -9366,
 -94586, -1091670, -14174522, -204495126, -3245265146}
```

Table[-PolyLog[-n+1, 1/2]/n!, {n, 1, 12}]

$$\left\{-1\,,\,\,-1\,,\,\,-1\,,\,\,-\frac{13}{12}\,,\,\,-\frac{5}{4}\,,\,\,-\frac{541}{360}\,,\,\,-\frac{223}{120}\,,\,\,-\frac{47\,293}{20\,160}\,,\,\,-\frac{36\,389}{12\,096}\,,\,\,-\frac{7\,087\,261}{1\,814\,400}\,,\,\,-\frac{3\,098\,411}{604\,800}\,,\,\,-\frac{1\,622\,632\,573}{239\,500\,800}\right\}$$

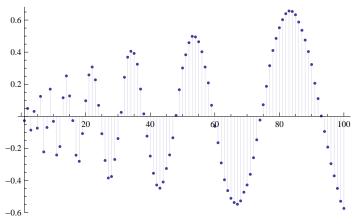
Table [D[eedz[2^n, -1, z] - eedz[2^n-1, -1, z], z] /.  $z \to 0$ , {n, 1, 12}]

$$\left\{ -2\,,\, -4\,,\, -8\,,\, -\frac{52}{3}\,,\, -40\,,\, -\frac{4328}{45}\,,\, -\frac{3568}{15}\,, \right. \\ \left. -\frac{189\,172}{315}\,,\, -\frac{291\,112}{189}\,,\, -\frac{56\,698\,088}{14\,175}\,,\, -\frac{49\,574\,576}{4725}\,,\, -\frac{12\,981\,060\,584}{467\,775} \right.$$

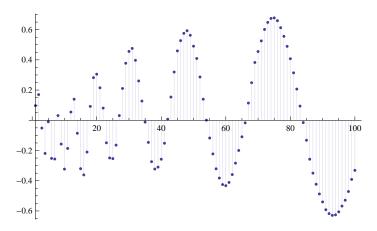
Table  $[-2^n PolyLog[-n+1, 1/2]/n!, \{n, 1, 12\}]$ 

$$\left\{-2, -4, -8, -\frac{52}{3}, -40, -\frac{4328}{45}, -\frac{3568}{15}, -\frac{189172}{315}, -\frac{291112}{189}, -\frac{56698088}{14175}, -\frac{49574576}{4725}, -\frac{12981060584}{467775} \right\}$$

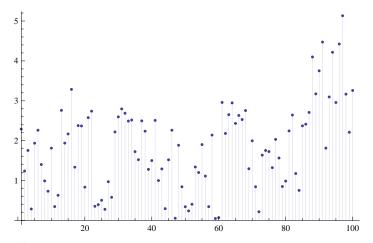
DiscretePlot[Re[ eedz[40 n, N[ZetaZero[1]], 1]], {n, 1, 100}]



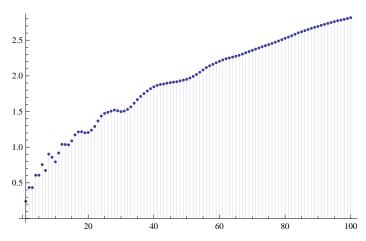
DiscretePlot[Im[ eedz[40 n, N[ZetaZero[1]], 1]], {n, 1, 100}]



# $\label{eq:decomposition} DiscretePlot[Abs[altz[30\,n,\,N[ZetaZero[1]]\,,\,1]],\,\{n,\,1,\,100\}]$



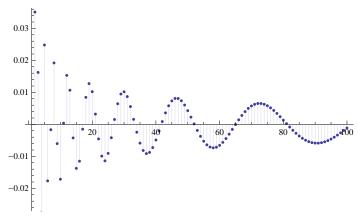
 $\label{eq:decomposition} DiscretePlot[Abs[dee[30\,n,\,N[ZetaZero[1]],\,1]],\,\{n,\,1,\,100\}]$ 



edz[10000, N[ZetaZero[1]], 1]

-5.10324 - 0.798884 i

 $\label{eq:decomposition} DiscretePlot[Re[eedoz[80\,n,\ N[ZetaZero[1]],\ 1]],\ \{n,1,100\}]$ 



N@ ((eedz[10000, 0, z] + eedz[10000, 0, -z]) / 2) /. z  $\rightarrow$  I + 2 Pi I 57 548.2 + 0. i

 $Log[e^{x^{-s}z}]$   $Log[e^{x^{-s}z}]$ 

N@dee[1000000., 1.1, 1]

6.40688

 $\label{eq:full-simplify} FullSimplify[Expand[(1 / (s - 1)) (n / (n + 1) ^s - (n - s) / n ^s)]]$ 

$$\frac{n \ (1+n)^{\,-s} + n^{-s} \ (-n+s)}{-1+s}$$

Table[dez[n, 1], {n, 1, 100}]

$$\left\{ 1,\,1,\,1,\,\frac{1}{2},\,1,\,1,\,1,\,\frac{1}{6},\,\frac{1}{2},\,1,\,1,\,\frac{1}{2},\,1,\,1,\,\frac{1}{24},\,1,\,\frac{1}{2},\,1,\,\frac{1}{2},\,1,\,1,\,1,\,\frac{1}{6},\,\frac{1}{2},\, \\ 1,\,\frac{1}{6},\,\frac{1}{2},\,1,\,1,\,1,\,\frac{1}{120},\,1,\,1,\,1,\,\frac{1}{4},\,1,\,1,\,1,\,\frac{1}{6},\,1,\,1,\,1,\,\frac{1}{2},\,\frac{1}{2},\,1,\,1,\,\frac{1}{24},\,\frac{1}{2},\,\frac{1}{2},\, \\ 1,\,\frac{1}{2},\,1,\,\frac{1}{6},\,1,\,\frac{1}{6},\,1,\,1,\,1,\,\frac{1}{2},\,1,\,1,\,\frac{1}{2},\,\frac{1}{720},\,1,\,1,\,1,\,\frac{1}{2},\,1,\,1,\,1,\,\frac{1}{12},\,1,\,1,\,\frac{1}{2},$$

#### PrimeQ[11]

True

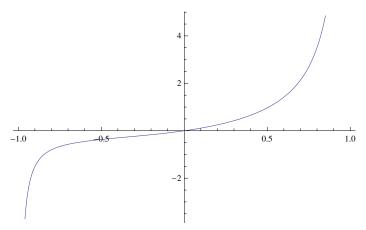
is2[2]

FullSimplify[is2[32]]

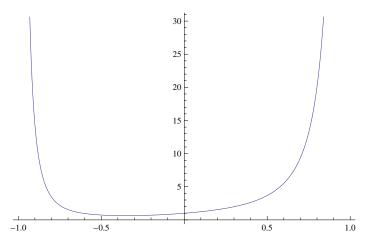
32 --

```
N@D[dee[10000, 0, z], {z, 0}]
1. +1229.z + 2612.5z^2 + 2181.83z^3 + 957.167z^4 + 263.317z^5 + 45.5958z^6 + 5.49762z^7 + 0.4313z^8 + 263.317z^5 + 45.5958z^6 + 5.49762z^7 + 0.4313z^8 + 263.317z^7 + 263.31
      0.0216049\ z^9 + 0.000857859\ z^{10} + 9.11897\times 10^{-6}\ z^{11} + 1.64926\times 10^{-7}\ z^{12} + 1.6059\times 10^{-10}\ z^{13}
PrimePi[10000]
1229
D[E^(z PrimeZetaP[s]), {z, 4}]
e^{z \operatorname{PrimeZetaP}[s]} \operatorname{PrimeZetaP}[s]^4
ff[z_, s_] := (E^z PrimeZetaP[s] + E^-z PrimeZetaP[s]) / 2
N@ff[2PiI + 2I, 2]
-0.188201 + 0.i
N@ (dee[10\ 000, 2, 2\ I + 2\ Pi\ I] + dee[10\ 000, 2, -2\ I - 2\ Pi\ I]) / 2
 -0.827469 - 1.66533 \times 10^{-16} i
N@ (dee[1000000, 2, 2I])
0.618082 + 0.786113 i
N@E^((I+2PiI) PrimeZetaP[2])
-0.988439 - 0.151622 i
N@Expand@deee[1000000, 2, z]
1. + 0.452247 z + 0.102264 z^2 + 0.015416 z^3 + 0.00174283 z^4 + 0.000157587 z^5 + 0.0000118633 z^6 + 0.000118633 z^6 + 0.00011863 z^6 + 0.00011863 z^6 + 0.00011863 z^6 + 0.00011863 z^6 + 0.00011862 z^6
      7.63386 \times 10^{-7} \text{ z}^7 + 4.26769 \times 10^{-8} \text{ z}^8 + 2.08783 \times 10^{-9} \text{ z}^9 + 8.90975 \times 10^{-11} \text{ z}^{10} + 3.28881 \times 10^{-12} \text{ z}^{11} + 3.28881 
       1.02193 \times 10^{-13} \; z^{12} + 2.64664 \times 10^{-15} \; z^{13} + 5.2449 \times 10^{-17} \; z^{14} + 8.31974 \times 10^{-19} \; z^{15} + 10.02193 \times 10^{-10} \; z^{10} + 10.
       9.13858 \times 10^{-21}~z^{16} + 6.50551 \times 10^{-23}~z^{17} + 3.11317 \times 10^{-25}~z^{18} + 2.8245 \times 10^{-28}~z^{19}
  (deee[1000000, 2, I] + deee[1000000, 2, -I]) / 2
 $Aborted
v100[z] := 1.^+ 0.4522473522653741^z + 0.10226367184134524^z^2 +
               0.015415962433582885\ \ z^{3} + 0.001742831515174532\ \ z^{4} + 0.00015758654321969503\ \ z^{5} +
               0.000011863275725862476 z^6 + 7.633863778094078 **-7 z^7 + 4.267692925500261 **-8 z^8 +
               2.087828092466002^*^-9z^9 + 8.909746884969071^*^-11z^{10} + 3.288807673239177^*^-12z^{11} +
               1.0219293646832407^*^-13 z^{12} + 2.6466413005396524^*^-15 z^{13} +
               6.505508494609741^**^{-23} z^{17} + 3.113170534545376^**^{-25} z^{18} + 2.8245024763158775^**^{-28} z^{19}
 (v100[I] + v100[-I]) / 2
 0.899467 + 0.i
  (v100[I + 2 Pi I] + v100[-I - 2 Pi I]) / 2
 -0.988635 + 0.i
N[PrimeZetaP[2]]
 0.452247
gf[x_{-}, t_{-}] := Sum[dez[k, 1] x^k, \{k, 1, t\}]
gf2[x_{-}, t_{-}] := \frac{x(-1+x^{t})}{-1+x}
```

### Plot[gf[x, 10000], {x, -.99, .99}]



Plot[D[gf[x, 10000], x] /.  $x \rightarrow y$ , {y, -.99, .99}]



Table[N@D[gf[x, 10000], x] /.  $x \rightarrow .5^k$ , {k, 1, 10}]

 $\{3.67685,\,1.74605,\,1.30221,\,1.13729,\,1.0655,\,1.03199,\,1.01581,\,1.00786,\,1.00392,\,1.00196\}$ 

### Table[gf[.5^k, 10000], {k, 1, 10}]

 $\{0.964379, 0.331366, 0.142735, 0.066659, 0.0322576,$ 0.015873, 0.00787401, 0.00392157, 0.00195695, 0.000977517}

### $Sum[x^k, \{k, 1, t\}]$

$$\frac{x\left(-1+x^{t}\right)}{-1+x}$$

rootsa[1000000, 2]

```
\{-877.729, -71.1618, -37.3299 - 96.7985 i, -37.3299 + 96.7985 i, -37.3290 + 96.7985 i, -37.3290 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 96.7980 + 
   -11.2549, -10.813 -4.36228 i, -10.813 +4.36228 i, -9.53977 -8.79402 i,
    -9.53977 + 8.79402\,\dot{\mathtt{i}}\,,\, -8.22574 - 43.3573\,\dot{\mathtt{i}}\,,\, -8.22574 + 43.3573\,\dot{\mathtt{i}}\,,
    -7.71922-13.0623\,\dot{\mathtt{i}}\,,\,-7.71922+13.0623\,\dot{\mathtt{i}}\,,\,-4.11337-16.6111\,\dot{\mathtt{i}}\,,\,-4.11337+16.6111\,\dot{\mathtt{i}}\,,
    -1.44157 - 19.9597 \, \mathrm{ii} \, , \, -1.44157 + 19.9597 \, \mathrm{ii} \, , \, 8.15454 - 17.9539 \, \mathrm{ii} \, , \, 8.15454 + 17.9539 \, \mathrm{ii} \, \}
deee[1000, 2, z]
1. \ + \ 0.45212 \ z + 0.101999 \ z^2 + 0.0151986 \ z^3 + 0.00165117 \ z^4 + 0.000133422 \ z^5 + 0.00165117 \ z^5 + 0.000133422 \ z^5 + 0.00165117 \ z^5 + 0.000133422 \ z^5 + 0.000165117 \ z^
    7.92725 \times 10^{-6} \text{ z}^6 + 2.84793 \times 10^{-7} \text{ z}^7 + 7.91626 \times 10^{-9} \text{ z}^8 + 5.25614 \times 10^{-11} \text{ z}^9
deee[1000, 1, z]
1. \ + \ 2.19808 \ z + 1.81743 \ z^2 + 0.769749 \ z^3 + 0.191547 \ z^4 + 0.0288456 \ z^5 \ +
    0.00280736 z^6 + 0.000129318 z^7 + 5.40495 \times 10^{-6} z^8 + 3.7676 \times 10^{-8} z^9
deee[1000, 0, z]
1. + 168 z + 293.5 z^2 + 189.667 z^3 + 64.7083 z^4 + 12.225 z^5 +
    1.48333 z^6 + 0.0710317 z^7 + 0.00409226 z^8 + 0.0000275573 z^9
deee[1000, -1, z]
1. +76127. z + 144479. z^2 + 99082. 3z^3 + 36544. 8z^4 +
    7368.42 z^{5} + 979.729 z^{6} + 44.1754 z^{7} + 3.32063 z^{8} + 0.0204586 z^{9}
E^(p^1z) / E^(p^2z)
ep z-p² z
pz[s_{-}, t_{-}] := Product[Zeta[ks]^(MoebiusMu[k]/k), \{k, 1, t\}]
Table [ N@pz[-3.17 + I, 2^k], \{k, 1, 8\}]
 Power::indet: Indeterminate expression (0. + 0. i)^0
   encountered. >>
 9.541 \times 10^{-9} - 4.45087 \times 10^{-9} \, \text{i}, 106.345 - 101.955 \, \text{i}, 0.0128624 - 0.0138756 \, \text{i}, Indeterminate
\label{lem:table_zeta} \mbox{Table[Zeta[k (-3.17+I)]^(MoebiusMu[k]/k), $\{k, 200, 200\}$] // TableForm}
 Power::indet: Indeterminate expression (0. + 0.i)^0
 encountered. >>
Indeterminate
Expand[E^(a^b)]
2 ^ - 2
```

## Table[deee[n, 0, 1] / n, {n, 1000, 100000, 1000}]

 $\{0.730658, 0.730059, 0.729808, 0.729799, 0.729808, 0.729516, 0.729492, 0.729547, 0.729519, 0.7$ 0.729636, 0.729457, 0.729394, 0.729465, 0.729433, 0.729482, 0.729436, 0.72952,  $0.729457,\, 0.729452,\, 0.72944,\, 0.729441,\, 0.729472,\, 0.72939,\, 0.729468,\, 0.729368,\, 0.729434,\, 0.7294444,\, 0.7294444,\,$ 0.729311, 0.729377, 0.729346, 0.729398, 0.729415, 0.729333, 0.72932, 0.729401,0.729363, 0.729359, 0.729358, 0.729299, 0.729297, 0.729303, 0.729322, 0.729321,  $0.729378,\, 0.72941,\, 0.729384,\, 0.729389,\, 0.729425,\, 0.729437,\, 0.729396,\, 0.72936,\, 0.729382,\, 0.729282,\, 0.729282,\, 0.729282,\, 0.729282,\, 0.729282,\, 0.729282,\,$ 0.729385, 0.729369, 0.729335, 0.72936, 0.729341, 0.729336, 0.729337, 0.729303,0.729318, 0.729376, 0.729376, 0.729354, 0.729338, 0.729333, 0.72934, 0.729334, 0.729363, 0.729316, 0.729339, 0.729346, 0.729352, 0.729346, 0.729309, 0.729313, 0.729292, 0.729298, 0.729315, 0.729321, 0.729291, 0.72929, 0.729264, 0.729299, 0.729295, 0.72931, 0.729302, 0.729304, 0.729322, 0.729342, 0.729313, 0.729312,0.729328, 0.729348, 0.729348, 0.72934, 0.729331, 0.729323, 0.729318, 0.729303, 0.72931

#### N[dee[1000000, 0, 1] / 1000000]

0.729276

### N[deee[10000000, 0, 1] / 10000000]

#### N[deee[100000000, 0, 1] / 100000000]

0.729266

### Integrate $[E^{(x)}, x]$

 $e^{x}$ 

 $D[E^x, x]$ 

ex

### Table[dez[n, z], {n, 1, 40}]

$$\left\{1,\,z,\,z,\,\frac{z^2}{2},\,z,\,z^2,\,z,\,\frac{z^3}{6},\,\frac{z^2}{2},\,z^2,\,z,\,\frac{z^3}{2},\,z,\,z^2,\,z^2,\,\frac{z^4}{24},\,z,\,\frac{z^3}{2},\,z,\,\frac{z^3}{2},\,\\ z^2,\,z^2,\,z,\,\frac{z^4}{6},\,\frac{z^2}{2},\,z^2,\,\frac{z^3}{6},\,\frac{z^3}{2},\,z,\,z^3,\,z,\,\frac{z^5}{120},\,z^2,\,z^2,\,z^2,\,\frac{z^4}{4},\,z,\,z^2,\,z^2,\,\frac{z^4}{6}\right\}$$

dee[100, 0, z]

$$1 + 25 z + 32 z^{2} + \frac{77 z^{3}}{6} + \frac{35 z^{4}}{12} + \frac{7 z^{5}}{40} + \frac{7 z^{6}}{720}$$

 $Expand@Sum[dez[j, 1]dee[100 / j, 0, z-1], {j, 1, 100}]$ 

$$1 + 25 z + 32 z^{2} + \frac{77 z^{3}}{6} + \frac{35 z^{4}}{12} + \frac{7 z^{5}}{40} + \frac{7 z^{6}}{720}$$

 ${\tt Expand@Sum[\,dez[j,\,1]\,dez[k,\,z-1]\,,\,\{j,\,1,\,100\}\,,\,\{k,\,1,\,100\,/\,j\}]}$ 

$$1 + 25 z + 32 z^{2} + \frac{77 z^{3}}{6} + \frac{35 z^{4}}{12} + \frac{7 z^{5}}{40} + \frac{7 z^{6}}{720}$$