Yuck.

So this was clearly another intermediary attempt at notation.

Using my later notation,

$$d_k^s{}'(n)=\nabla[(\zeta(s)-1)^k]_n$$

$$d_k^s(n)=\nabla[\zeta(s)^k]_n$$

$$\Pi^s(n)=[\log\zeta(s)]_n$$

$$D_k^s{}'(n)=[(\zeta(s)-1)^k]_n$$

$$D_k^s(n)=[\zeta(s)^k]_n$$

This actually makes a nice demonstration of why I changed notation - for the case of Re(s)>1, dropping the outer brackets and trailing subscript gives the limit as n goes to infinity. For example,

$$\lim_{n\to\infty}[\log\zeta(s)]_n=\log\zeta(s)$$

This same relationship is also true in this intermediary notation, but it's certainly less obvious:

$$\lim_{n\to\infty}\Pi^s(n)=\log\zeta(s)$$

$$d_k^s{}'(n)=\sum_{a_1\cdot a_2\cdot\ldots\cdot a_k=n\,;\,a_i>1} n^s$$

$$d_k^s{}'(n)=\sum_{j|n} d_{k-1}^s{}'(j)\,d_1^s{}'\left(\frac{n}{j}\right) \qquad d_1^s{}'(n)=n^s\, if\ n>1, 0\ otherwise \qquad d_0^s{}'(n)=1\ if\ n=1, 0\ otherwise$$

$$d_k^s{}'(n)=n^s d_k{}'(n)$$

$$\kappa(n)n^s=d_1^s{}'(n)-\frac{1}{2}d_2^s{}'(n)+\frac{1}{3}d_3^s{}'(n)-\frac{1}{4}d_4^s{}'(n)+\frac{1}{5}\ldots$$

$$d_k^s(n)=\sum_{a_1\cdot a_2\cdot\ldots\cdot a_k=n} n^s \qquad d_k^s(n)=\sum_{j|n} d_{k-1}^s(j)\,d_1^s\left(\frac{n}{j}\right) \qquad d_1^s(n)=n^s \qquad d_0^s(n)=1\ if\ n=1, 0\ otherwise$$

$$d_k^s(n)=n^s d_k(n)$$

$$d_z^s(n)=\prod_{p^a|n}\frac{p^s z(z+1)\ldots(z+a-1)}{a!}$$

$$\kappa(n)n^s=\lim_{z\to 0}\frac{d_z^s(n)}{z}$$

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$$\Pi^s(n)=\sum_{j=1}^{n}\kappa(n)n^s$$

$$\pi^s(n)=\sum \text{primes}^s \le n$$
$$\Pi^s(n)=\sum_{k=1}^{\lfloor \log_2 n\rfloor}\frac{1}{k}\pi(n^{\frac{1}{k}})$$
//fixme – not done yet.

$$\pi^s(n)=\sum_{k=1}^{\lfloor \log_2 n\rfloor}\frac{\mu(k)}{k}\Pi^{k\cdot s}(n^{\frac{1}{k}})$$

$$D_k^s{}'(n)=\sum_{j=2}^{n}d_k^s{}'(j) \qquad D_k^s(n)=\sum_{j=1}^{n}d_k^s(j)$$

$$D_k^s{}'(n)=\sum_{j=2}^{n}D_{k-1}^s{}'(\lfloor\frac{n}{j}\rfloor) \qquad D_k^s(n)=\sum_{j=1}^{n}D_{k-1}^s(\lfloor\frac{n}{j}\rfloor)$$
$$D_0^s{}'(n)=1 \qquad D_0^s(n)=1$$

$$D_k^s{}'(n)=\sum_{j=0}^{k}(-1)^{(k-j)}\binom{k}{j}D_j^s(n) \qquad D_k^s(n)=\sum_{j=0}^{k}\binom{k}{j}D_j^s{}'(n)$$

$$\Pi^s(n)=D_1^s{}'(n)-\frac{1}{2}D_2^s{}'(n)+\frac{1}{3}D_3^s{}'(n)-\frac{1}{4}D_4^s{}'(n)+\frac{1}{5}\ldots$$

$$\Pi^s(n)=\lim_{z\to 0}\frac{D_z^s(n)-1}{z}$$

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$$\Pi^s(n)=\sum_{j=2}^{n}j^s-\frac{1}{2}\sum_{j=2}^{n}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}j^s\cdot k^s+\frac{1}{3}\sum_{j=2}^{n}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}\sum_{l=2}^{\lfloor\frac{n}{j\cdot k}\rfloor}j^s\cdot k^s\cdot l^s-\frac{1}{4}\sum_{j=2}^{n}\sum_{k=2}^{\lfloor\frac{n}{j}\rfloor}\sum_{l=2}^{\lfloor\frac{n}{j\cdot k}\rfloor}\sum_{m=2}^{\lfloor\frac{n}{j\cdot k\cdot l}\rfloor}j^s\cdot k^s\cdot l^s\cdot m^s+\frac{1}{5}\ldots$$

$$\Pi_k^s(n)=\sum_{j=2}^{n}j^s(\frac{1}{k}-\Pi_{k+1}^s(\lfloor\frac{n}{j}\rfloor))$$
$$\Pi^s(n)=\Pi_1^s(n)$$

$$\Pi^s(n,j,k)=\frac{1}{k}-\Pi^s(\lfloor\frac{n}{j}\rfloor,\lfloor\frac{n}{j}\rfloor,k+1)+\Pi^s(n,j-1,k)$$
$$\Pi^s(n,1,k)=0$$
$$\Pi^s(n)=\Pi^s(n,n,1)$$

$$D^s_{k,a}(n)=\sum_{j=1}^{k}\binom{k}{j}\sum_{m=a}^{\lfloor n^{\frac{1}{k}}\rfloor} m^{sj} D^s_{k-j,m+1}\left(\frac{n}{m^j}\right)$$
$$D^s_{1,a}(n)=\sum_{j=a}^{n} j^s$$
$$D^s_{0,a}(n)=1$$
$$\Pi^s(n)=D^s_1(n)-\frac{1}{2}D^s_{2,2}(n)+\frac{1}{3}D^s_{3,2}(n)-\frac{1}{4}D^s_{4,2}(n)+\frac{1}{5}\ldots$$

$$\begin{aligned}
&D^s_k{}'(n)=\\
&\quad\sum_{j=a+1}^{n} d^s_1{}'(j)\,D^s_{k-1}{}'\left(\lfloor\frac{n}{j}\rfloor\right)\\
&\quad+\sum_{j=2}^{a} d^s_{k-1}{}'(j)\,D^s_1{}'\left(\lfloor\frac{n}{j}\rfloor\right)\\
&\quad+\sum_{j=2}^{a}\sum_{r=\frac{a}{j}+1}^{\frac{n}{j}}\sum_{m=1}^{k-2} d^s_1{}'(r)\,d^s_m{}'(j)\,D^s_{k-m-1}{}'\left(\lfloor\frac{n}{jr}\rfloor\right)
\end{aligned}$$

$$\begin{aligned}
\Pi(n)=D_1{}'(n)&+\sum_{j=\lfloor n^{\frac{1}{3}}\rfloor+1}^{\lfloor n^{\frac{1}{2}}\rfloor}\sum_{k=2}^{\lfloor\log_2 n\rfloor}\frac{-1^{k+1}}{k}D_{k-1}{}'\left(\lfloor\frac{n}{j}\rfloor\right)+\sum_{j=1}^{\lfloor n^{\frac{1}{2}}\rfloor}\left(D_1{}'\left(\lfloor\frac{n}{j}\rfloor\right)-D_1{}'\left(\lfloor\frac{n}{j+1}\rfloor\right)\right)\sum_{k=2}^{\lfloor\log_2 n\rfloor}\frac{-1^{k+1}}{k}D_{k-1}{}'(j)\\
&+\sum_{j=2}^{\lfloor n^{\frac{1}{3}}\rfloor}\sum_{k=2}^{\lfloor\log_2 n\rfloor}\frac{-1^{k+1}}{k}d_{k-1}{}'(j)\,D_1{}'\left(\lfloor\frac{n}{j}\rfloor\right)+\sum_{j=2}^{\lfloor n^{\frac{1}{3}}\rfloor}\sum_{s=\lfloor\frac{\lfloor n^{\frac{1}{3}}\rfloor}{j}\rfloor+1}^{\lfloor\frac{n}{j}\rfloor}\sum_{k=2}^{\lfloor\log_2 n\rfloor}\frac{-1^{k+1}}{k}\sum_{m=1}^{k-2}d_m{}'(j)\,D_{k-m-1}{}'\left(\lfloor\frac{n}{js}\rfloor\right)\\
&+\sum_{j=2}^{\lfloor n^{\frac{1}{3}}\rfloor}\sum_{s=1}^{\lfloor\lfloor\frac{n}{j}\rfloor^{\frac{1}{2}}\rfloor-1}\left(D_1{}'\left(\lfloor\frac{n}{js}\rfloor\right)-D_1{}'\left(\lfloor\frac{n}{j(s+1)}\rfloor\right)\right)\cdot\sum_{k=2}^{\lfloor\log_2 n\rfloor}\frac{-1^{k+1}}{k}\sum_{m=1}^{k-2}d_m{}'(j)\,D_{k-m-1}{}'(s)
\end{aligned}$$