```
ClearAll["Global`*"]

L[n_, k_, a_] := L[n, k, a] = Sum[L[n / (j + a), k - 1, a], {j, 0, Floor[n - a]}];
L[n_, 1, a_] := L[n, 1, a] = Sum[Log[(j + a) / (a - 1)], {j, 0, Floor[n - a]}]
LI[n_, a_] := Sum[(-1)^(k + 1) a^-k L[n a^k, k, a + 1], {k, 1, 10}]

N[LI[10, 3]]
```

7.16879

```
10 - Log[10.] - 1
```

6.69741

```
L[100, 2, 3]
```

$131 \, \text{Log}\left[\frac{3}{2}\right] + 98 \, \text{Log}[2] + 78 \, \text{Log}\left[\frac{5}{2}\right] + 64 \, \text{Log}[3] + 55 \, \text{Log}\left[\frac{7}{2}\right] + 48 \, \text{Log}[4] + 42 \, \text{Log}\left[\frac{9}{2}\right] + 38 \, \text{Log}[5] +$

$34 \, \text{Log}\left[\frac{11}{2}\right] + 31 \, \text{Log}[6] + 28 \, \text{Log}\left[\frac{13}{2}\right] + 26 \, \text{Log}[7] + 24 \, \text{Log}\left[\frac{15}{2}\right] + 23 \, \text{Log}[8] + 21 \, \text{Log}\left[\frac{17}{2}\right] +$

$20 \, \text{Log}[9] + 19 \, \text{Log}\left[\frac{19}{2}\right] + 18 \, \text{Log}[10] + 17 \, \text{Log}\left[\frac{21}{2}\right] + 16 \, \text{Log}[11] + 15 \, \text{Log}\left[\frac{23}{2}\right] + 14 \, \text{Log}[12] +$

$14 \, \text{Log}\left[\frac{25}{2}\right] + 13 \, \text{Log}[13] + 12 \, \text{Log}\left[\frac{27}{2}\right] + 12 \, \text{Log}[14] + 11 \, \text{Log}\left[\frac{29}{2}\right] + 11 \, \text{Log}[15] + 10 \, \text{Log}\left[\frac{31}{2}\right] +$

$10 \, \text{Log}[16] + 10 \, \text{Log}\left[\frac{33}{2}\right] + 9 \, \text{Log}[17] + 9 \, \text{Log}\left[\frac{35}{2}\right] + 9 \, \text{Log}[18] + 8 \, \text{Log}\left[\frac{37}{2}\right] + 8 \, \text{Log}[19] +$

$8 \, \text{Log}\left[\frac{39}{2}\right] + 8 \, \text{Log}[20] + 7 \, \text{Log}\left[\frac{41}{2}\right] + 7 \, \text{Log}[21] + 7 \, \text{Log}\left[\frac{43}{2}\right] + 7 \, \text{Log}[22] + 6 \, \text{Log}\left[\frac{45}{2}\right] + 6 \, \text{Log}[23] +$

$6 \, \text{Log}\left[\frac{47}{2}\right] + 6 \, \text{Log}[24] + 6 \, \text{Log}\left[\frac{49}{2}\right] + 6 \, \text{Log}[25] + 5 \, \text{Log}\left[\frac{51}{2}\right] + 5 \, \text{Log}[26] + 5 \, \text{Log}\left[\frac{53}{2}\right] + 5 \, \text{Log}[27] +$

$5 \, \text{Log}\left[\frac{55}{2}\right] + 5 \, \text{Log}[28] + 5 \, \text{Log}\left[\frac{57}{2}\right] + 4 \, \text{Log}[29] + 4 \, \text{Log}\left[\frac{59}{2}\right] + 4 \, \text{Log}[30] + 4 \, \text{Log}\left[\frac{61}{2}\right] + 4 \, \text{Log}[31] +$

$4 \, \text{Log}\left[\frac{63}{2}\right] + 4 \, \text{Log}[32] + 4 \, \text{Log}\left[\frac{65}{2}\right] + 4 \, \text{Log}[33] + 3 \, \text{Log}\left[\frac{67}{2}\right] + 3 \, \text{Log}[34] + 3 \, \text{Log}\left[\frac{69}{2}\right] + 3 \, \text{Log}[35] +$

$3 \, \text{Log}\left[\frac{71}{2}\right] + 3 \, \text{Log}[36] + 3 \, \text{Log}\left[\frac{73}{2}\right] + 3 \, \text{Log}[37] + 3 \, \text{Log}\left[\frac{75}{2}\right] + 3 \, \text{Log}[38] + 3 \, \text{Log}\left[\frac{77}{2}\right] + 3 \, \text{Log}[39] +$

$3 \, \text{Log}\left[\frac{79}{2}\right] + 3 \, \text{Log}[40] + 2 \, \text{Log}\left[\frac{81}{2}\right] + 2 \, \text{Log}[41] + 2 \, \text{Log}\left[\frac{83}{2}\right] + 2 \, \text{Log}[42] + 2 \, \text{Log}\left[\frac{85}{2}\right] +$

$2 \, \text{Log}[43] + 2 \, \text{Log}\left[\frac{87}{2}\right] + 2 \, \text{Log}[44] + 2 \, \text{Log}\left[\frac{89}{2}\right] + 2 \, \text{Log}[45] + 2 \, \text{Log}\left[\frac{91}{2}\right] + 2 \, \text{Log}[46] +$

$2 \, \text{Log}\left[\frac{93}{2}\right] + 2 \, \text{Log}[47] + 2 \, \text{Log}\left[\frac{95}{2}\right] + 2 \, \text{Log}[48] + 2 \, \text{Log}\left[\frac{97}{2}\right] + 2 \, \text{Log}[49] + 2 \, \text{Log}\left[\frac{99}{2}\right] + 2 \, \text{Log}[50] +$

$\text{Log}\left[\frac{101}{2}\right] + \text{Log}[51] + \text{Log}\left[\frac{103}{2}\right] + \text{Log}[52] + \text{Log}\left[\frac{105}{2}\right] + \text{Log}[53] + \text{Log}\left[\frac{107}{2}\right] + \text{Log}[54] +$

$\text{Log}\left[\frac{109}{2}\right] + \text{Log}[55] + \text{Log}\left[\frac{111}{2}\right] + \text{Log}[56] + \text{Log}\left[\frac{113}{2}\right] + \text{Log}[57] + \text{Log}\left[\frac{115}{2}\right] + \text{Log}[58] +$

$\text{Log}\left[\frac{117}{2}\right] + \text{Log}[59] + \text{Log}\left[\frac{119}{2}\right] + \text{Log}[60] + \text{Log}\left[\frac{121}{2}\right] + \text{Log}[61] + \text{Log}\left[\frac{123}{2}\right] + \text{Log}[62] +$

$\text{Log}\left[\frac{125}{2}\right] + \text{Log}[63] + \text{Log}\left[\frac{127}{2}\right] + \text{Log}[64] + \text{Log}\left[\frac{129}{2}\right] + \text{Log}[65] + \text{Log}\left[\frac{131}{2}\right] + \text{Log}[66] + \text{Log}\left[\frac{133}{2}\right]$