```
d[n_, 0, a_] := 1
d[n_, 1, a_] := n - a + 1
d[n_, k_, a_] :=
 Sum[Binomial[k, j] d[Floor[n / (m^j)], k - j, m + 1], {j, 1, k}, {m, a, n^(1 / k)}]


D32Unrolled[ n_ ] := -1 + Floor[ n^(1 / 3)]^3 +
  3 Sum[ Floor[ n / (m^2)] - Floor[ Floor[n / m]^(1 / 2)]^2 + 2 Sum[ Floor[Floor[n / m] / j],
      {j, m + 1, Floor[Floor[n / m]^(1 / 2)]}], {m, 2, Floor[n^(1 / 3)]}]


D22Unrolled[ n_ ] := 1 - Floor[ n^(1 / 2)]^2 + 2 Sum [ Floor[ n / m], {m, 2, Floor[n^(1 / 2)]}]
d0[n_, a_Integer] := 1
d1[n_, a_Integer] := n - a + 1
d2a[n_, a_Integer] :=
 Sum[ Binomial[ 2, 2 ] d0[Floor[n / (m^2)], m + 1] , {m, a, Floor[n^(1 / 2)]}] +
  Sum[Binomial[ 2, 1 ] d1[Floor[n / (m^1)], m + 1], {m, a, Floor[n^(1 / 2)]}]
d2[n_, a_Integer] :=
 1 - 2 a + a^2 - Floor[n^(1 / 2)]^2 + 2 Sum [ Floor[ n / m], {m, a, Floor[n^(1 / 2)]}]
d2[0, a_Integer] := 0
d3a[n_, a_Integer] :=
 Sum[ Binomial[ 3, 3 ] d0[Floor[n / (m^3)], m + 1] , {m, a, Floor[n^(1 / 3)]}] +
  Sum[Binomial[ 3, 2 ] d1[Floor[n / (m^2)], m + 1], {m, a, Floor[n^(1 / 3)]}] +
  Sum[Binomial[ 3, 1 ] d2a[Floor[n / (m^1)], m + 1], {m, a, Floor[n^(1 / 3)]}]

d3b[n_, a_Integer] := Sum[1 , {m, a, Floor[n^(1 / 3)]}] +
  Sum[3 ( Floor[n / (m^2)] - m), {m, a, Floor[n^(1 / 3)]}] +
  Sum[3 d2[Floor[n / m], m + 1], {m, a, Floor[n^(1 / 3)]}]

d3b[n, 2]
```

$$-1 + \text{Floor}\left[n^{1/3}\right] + \sum_{m=2}^{\text{Floor}\left[n^{1/3}\right]} 3 \, d2\left[\text{Floor}\left[\frac{n}{m}\right], 1 + m\right] + \sum_{m=2}^{\text{Floor}\left[n^{1/3}\right]} 3 \left(-m + \text{Floor}\left[\frac{n}{m^2}\right]\right)$$

```
d[100, 4, 2]
```

184


```
d2e[n_, a_Integer] :=
 (1 - a)^2 - Floor[ n^(1 / 2)]^2 + 2 Sum [ Floor[ n / m], {m, a, Floor[n^(1 / 2)]}]
d2e[123 210, 4]
```

1 011 514

```
d[123 210, 2, 4]
```

1 011 514

```
dd3[n_, a_] :=
 Sum[ 3 (Floor[n / (s^2)] - s) + 1 + 3 (1 - 2 (s + 1) + (s + 1)^2 - Floor[Floor[n / s]^(1 / 2)]^2 +
     2 Sum[ Floor[n / m / s], {m, s + 1, Floor[Floor[n / s]^(1 / 2)]}]), {s, a, Floor[n^(1 / 3)]}]
dd3[100, 3]
```

71

```
d[100, 3, 3]
```

71

```
d3c[100, 2]
```

```
324
```

```
d3c[n_, a_Integer] :=
 Sum[1 + 3 ( Floor[n / (m^2)] - m), {m, a, Floor[n^ (1 / 3)]}] +
   Sum[3 d2[Floor[n / m], m + 1], {m, a, Floor[n^ (1 / 3)]}]
```

```
dd3a[n_, a_] :=
 Sum[ 3 (Floor[n / (s^2)] - s) + 1 + 3 (1 - 2 (s + 1) + (s + 1)^2 - Floor[Floor[n / s]^ (1 / 2)]^2 +
       2 Sum[ Floor[n / m / s], {m, s + 1, Floor[Floor[n / s]^ (1 / 2)]}]), {s, a, Floor[n^ (1 / 3)]}]
```

```
dd3a[200, 2]
```

```
1027
```

```
d[200, 3, 2]
```

```
1027
```

```
Expand[FullSimplify[
   3 (Floor[n / (s^2)] - s) + 1 + 3 (1 - 2 (s + 1) + (s + 1)^2 - Floor[Floor[n / s]^ (1 / 2)]^2)]]
```

$$1 - 3\,s + 3\,s^2 + 3\,\text{Floor}\left[\frac{n}{s^2}\right] - 3\,\text{Floor}\left[\sqrt{\text{Floor}\left[\frac{n}{s}\right]}\,\right]^2$$

```
dd3b[n_, a_] := Sum[ 1 - 3 s + 3 s^2 + 3 Floor[ n/s^2 ] - 3 Floor[ Sqrt[ Floor[ n/s ] ] ]^2 +
    6 Sum[ Floor[n / m / s], {m, s + 1, Floor[Floor[n / s]^ (1 / 2)]}], {s, a, Floor[n^ (1 / 3)]}]
```

```
dd3b[200, 2]
```

```
1027
```

```
dd3c[n_, a_] :=
 Sum[ 1 - 3 s + 3 s^2, {s, a, Floor[n^ (1 / 3)]}] + Sum[ 3 Floor[ n/s^2 ] - 3 Floor[ Sqrt[ Floor[ n/s ] ] ]^2 +
    6 Sum[ Floor[n / m / s], {m, s + 1, Floor[Floor[n / s]^ (1 / 2)]}], {s, a, Floor[n^ (1 / 3)]}]
```

```
dd3c[200, 2]
```

```
1027
```

```
Expand[FullSimplify[Sum[1 - 3 s + 3 s^2, {s, a, Floor[n^ (1 / 3)]}]]]
```

$$1 - 3\,a + 3\,a^2 - a^3 + \text{Floor}\left[n^{1/3}\right]^3$$

```
dd3d[n_, a_] := 1 - 3 a + 3 a^2 - a^3 + Floor[ n^{1/3} ]^3 + Sum[ 3 Floor[ n/s^2 ] - 3 Floor[ Sqrt[ Floor[ n/s ] ] ]^2 +
    6 Sum[ Floor[n / m / s], {m, s + 1, Floor[Floor[n / s]^ (1 / 2)]}], {s, a, Floor[n^ (1 / 3)]}]
```

```
dd3d[300, 3]
```

```
709
```

```
dd3e[n_, a_] := (1 - a)^3 + Floor[ n^{1/3} ]^3 + Sum[ 3 Floor[ n/s^2 ] - 3 Floor[ Sqrt[ Floor[ n/s ] ] ]^2 +
    6 Sum[ Floor[n / m / s], {m, s + 1, Floor[Floor[n / s]^ (1 / 2)]}], {s, a, Floor[n^ (1 / 3)]}]
```

```
dd3e[300, 3]
```

709

```
d[300, 3, 3]
```

709

```
d4[x_, b_] := Sum[d[Floor[x / (u^4)], 0, u + 1] + 4 d[Floor[x / (u^3)], 1, u + 1] +
    6 d[Floor[x / (u^2)], 2, u + 1] + 4 d[Floor[x / (u^1)], 3, u + 1], {u, b, Floor[x^(1 / 4)]}]
```

```
d4[100, 2]
```

184

```
d[10 000, 4, 3]
```

171 994

```
Floor[(100)^(1 / 4)]
```

3

```
d4a[x_, b_] := Sum[1 + 4 (Floor[x / (u^3)] - u) +
    6 d[Floor[x / (u^2)], 2, u + 1] + 4 d[Floor[x / (u^1)], 3, u + 1], {u, b, Floor[x^(1 / 4)]}]
```

```
d4a[10 000, 3]
```

171 994

```
d4b[x_, b_] := Sum[1 + 4 (Floor[x / (u^3)] - u)

    + 6 (1 - 2 (u + 1) + (u + 1)^2 - Floor[Floor[x / (u^2)]^(1 / 2)]^2 +
        2 Sum[Floor[Floor[x / (u^2)] / m], {m, (u + 1), Floor[Floor[x / (u^2)]^(1 / 2)]}])

    + 4 d[Floor[x / (u^1)], 3, u + 1], {u, b, Floor[x^(1 / 4)]}]
```

```
d4b[22 200, 3]
```

605 878

```
#a = (u + 1) #n = Floor[x / u]
```

```
d4c[x_, b_] := Sum[ 1 + 4 (Floor[x / (u^3)] - u)

    + 6 ( 1 - 2 (u + 1) + (u + 1)^2 - Floor[ Floor[x / (u^2)] ^ (1 / 2)] ^2 +
       2 Sum [ Floor[ Floor[x / (u^2)] / m], {m, (u + 1), Floor[Floor[x / (u^2)] ^ (1 / 2)]}])
```

$$+ 4 \left(1 - 3 (u + 1) + 3 (u + 1)^2 - (u + 1)^3 + \text{Floor}\left[\text{Floor}[x / u]^{1/3}\right]^3 + \right.$$

$$\text{Sum}\left[3 \text{Floor}\left[\frac{\text{Floor}[x / u]}{s^2}\right] - 3 \text{Floor}\left[\sqrt{\text{Floor}\left[\frac{\text{Floor}[x / u]}{s}\right]}\, \right]^2 + \right.$$

```
       6 Sum[ Floor[Floor[x / u] / m / s], {m, s + 1, Floor[Floor[Floor[x / u] / s] ^ (1 / 2)]}],
```

$$\left. \{s, (u + 1), \text{Floor}[\text{Floor}[x / u] ^ (1 / 3)]\} \right)$$

```
    , {u, b, Floor[x^ (1 / 4)]}]

d4c[22 200, 3]

605 878

d[12 200, 4, 4]

95 137

d4d[x_, b_] :=
  Sum[1 + 6 ( 1 - 2 (u + 1) + (u + 1)^2 ) + 4 ( 1 - 3 (u + 1) + 3 (u + 1)^2 - (u + 1)^3 - u),
    {u, b, Floor[x^ (1 / 4)]}] +
  Sum[
    4 Floor[x / (u^3)] + 6 (-Floor[ Floor[x / (u^2)] ^ (1 / 2)] ^2 +
       2 Sum [ Floor[ Floor[x / (u^2)] / m], {m, (u + 1), Floor[Floor[x / (u^2)] ^ (1 / 2)]}]) +
```

$$4 \left(\text{Floor}\left[\text{Floor}[x / u]^{1/3}\right]^3 + \text{Sum}\left[3 \text{Floor}\left[\frac{\text{Floor}[x / u]}{s^2}\right] - 3 \text{Floor}\left[\sqrt{\text{Floor}\left[\frac{\text{Floor}[x / u]}{s}\right]}\, \right]^2 + \right. \right.$$

```
       6 Sum[ Floor[Floor[x / u] / m / s], {m, s + 1, Floor[Floor[Floor[x / u] / s] ^ (1 / 2)]}],
```

$$\left. \{s, (u + 1), \text{Floor}[\text{Floor}[x / u] ^ (1 / 3)]\} \right)$$

```
    , {u, b, Floor[x^ (1 / 4)]}]

d4d[12 200, 4]

95 137
```

```
FullSimplify[
 Expand[Sum[1 + 6 ( 1 - 2 (u + 1) + (u + 1) ^ 2 ) + 4 ( 1 - 3 (u + 1) + 3 (u + 1)² - (u + 1)³ - u),
   {u, b, Floor[x^(1 / 4)]}]]]
```

$(-1 + b)^4 - \text{Floor}\left[x^{1/4}\right]^4$

```
Expand[Sum[1 + 6 ( 1 - 2 (u + 1) + (u + 1) ^ 2 ) + 4 ( 1 - 3 (u + 1) + 3 (u + 1)² - (u + 1)³ - u),
   {u, b, Floor[x^(1 / 4)]}]]
```

$1 - 4 b + 6 b^2 - 4 b^3 + b^4 - \text{Floor}\left[x^{1/4}\right]^4$

```
d4e[x_, b_ ] :=
  (-1 + b)⁴ - Floor[x^(1/4)]⁴ +
  Sum[
    4 Floor[x / (u^3)] + 6 (-Floor[ Floor[x / (u^2)] ^ (1 / 2)] ^ 2 +
      2 Sum [ Floor[ Floor[x / (u^2)] / m], {m, (u + 1), Floor[Floor[x / (u^2)] ^ (1 / 2)]}]) +
    4 ( Floor[Floor[x / u]^(1/3)]³ + Sum[3 Floor[ Floor[x / u] / s² ] - 3 Floor[ √(Floor[ Floor[x / u] / s ]) ]² +

      6 Sum[ Floor[Floor[x / u] / m / s], {m, s + 1, Floor[Floor[Floor[x / u] / s] ^ (1 / 2)]}],

      {s, (u + 1), Floor[Floor[x / u] ^ (1 / 3)]}]

      )

    , {u, b, Floor[x^(1 / 4)]}]

d4e[72 200, 2]

7 624 011

d[72 200, 4, 2]

7 624 011

Sum[
  4 Floor[x / (u^3)] + 6 (-Floor[ Floor[x / (u^2)] ^ (1 / 2)] ^ 2 +
    2 Sum [ Floor[ Floor[x / (u^2)] / m], {m, (u + 1), Floor[Floor[x / (u^2)] ^ (1 / 2)]}]) +
  4 ( Floor[Floor[x / u]^(1/3)]³ + Sum[3 Floor[ Floor[x / u] / s² ] - 3 Floor[ √(Floor[ Floor[x / u] / s ]) ]² +

    6 Sum[ Floor[Floor[x / u] / m / s], {m, s + 1, Floor[Floor[Floor[x / u] / s] ^ (1 / 2)]}],

    {s, (u + 1), Floor[Floor[x / u] ^ (1 / 3)]}]

    )

  , {u, b, Floor[x^(1 / 4)]}]
```

```
d4f[x_, b_] :=
  (-1 + b)^4 - Floor[x^(1/4)]^4 +
   Sum[
     4 Floor[x / (u^3)]
       + 6 (
         - Floor[ Floor[x / (u^2)]^(1/2)]^2
         + 2 Sum [ Floor[ Floor[x / (u^2)] / m], {m, (u + 1), Floor[Floor[x / (u^2)]^(1/2)]}]
       )

     + 4 (

       Floor[Floor[x / u]^(1/3)]^3
         + 3 Sum[

             Floor[ Floor[x / u] / s^2 ]

             - Floor[ Sqrt[ Floor[ Floor[x / u] / s ] ] ]^2

           +
           2 Sum[ Floor[Floor[x / u] / m / s], {m, s + 1, Floor[Floor[Floor[x / u] / s]^(1/2)]}]

           ], {s, (u + 1), Floor[Floor[x / u]^(1/3)]}] ),

     {u, b, Floor[x^(1/4)]}]

d4g[72 200, 2]
```

7 624 011

```
d4g[x_, b_] :=
  (-1 + b)^4 - Floor[x^(1/4)]^4 +
   Sum[4 Floor[x / (u^3)], {u, b, Floor[x^(1/4)]}] +
   -6 Sum[Floor[ Floor[x / (u^2)]^(1/2)]^2 , {u, b, Floor[x^(1/4)]}] +
   Sum[
     6 (
         2 Sum [ Floor[ Floor[x / (u^2)] / m], {m, (u+1), Floor[Floor[x / (u^2)]^(1/2)]}]
       )

     + 4 (

       Floor[Floor[x / u]^(1/3)]^3
         + 3 Sum[

               Floor[ Floor[x / u] / s^2 ]

               - Floor[ Sqrt[ Floor[ Floor[x / u] / s ] ] ]^2

           +
           2 Sum[ Floor[Floor[x / u] / m / s], {m, s+1, Floor[Floor[Floor[x / u] / s]^(1/2)]}]

           ], {s, (u+1), Floor[Floor[x / u]^(1/3)]}]),

     {u, b, Floor[x^(1/4)]}]
```

```
d4h[72 200, 2]
```

```
7 624 011
```

```mathematica
d4i[x_, b_] :=
 (-1 + b)^4 - Floor[x^(1/4)]^4 +
  Sum[4 Floor[x / (u^3)], {u, b, Floor[x^(1/4)]}] +
  -6 Sum[Floor[ Floor[x / (u^2)]^(1/2)]^2, {u, b, Floor[x^(1/4)]}] +
  12 Sum[ Sum [ Floor[ Floor[x / (u^2)] / m], {m, (u + 1), Floor[Floor[x / (u^2)]^(1/2)]}],
     {u, b, Floor[x^(1/4)]}] +
  4 Sum[Floor[Floor[x / u]^(1/3)]^3, {u, b, Floor[x^(1/4)]}] +
  Sum[

   4 ( 3 Sum[

        Floor[ Floor[x / u] / s^2 ]

          - Floor[ Sqrt[ Floor[ Floor[x / u] / s ]] ]^2

        +
        2 Sum[ Floor[Floor[x / u] / m / s], {m, s + 1, Floor[Floor[Floor[x / u] / s]^(1/2)]}]

        ], {s, (u + 1), Floor[Floor[x / u]^(1/3)]}] ),

     {u, b, Floor[x^(1/4)]}]

d4i[12 200, 2]

648 367

d[12 200, 4, 2]

648 367
```

```
d4j[x_, b_] :=
 (-1 + b)^4 - Floor[x^(1/4)]^4 +
  Sum[4 Floor[x / (u^3)], {u, b, Floor[x^(1/4)]}] +
  -6 Sum[Floor[ Floor[x / (u^2)]^(1/2)]^2, {u, b, Floor[x^(1/4)]}] +
  12 Sum[ Sum [ Floor[ Floor[x / (u^2)] / m], {m, (u + 1), Floor[Floor[x / (u^2)]^(1/2)]}],
    {u, b, Floor[x^(1/4)]}] +
  4 Sum[Floor[Floor[x / u]^(1/3)]^3, {u, b, Floor[x^(1/4)]}] +
  +12 Sum[ Sum[ Floor[ Floor[x / u] / s^2 ], {s, (u + 1), Floor[Floor[x / u]^(1/3)]}],
    {u, b, Floor[x^(1/4)]}] +
  Sum[ Sum[

          -12 Floor[ Sqrt[ Floor[ Floor[x / u] / s ] ] ]^2

            + 24 Sum[ Floor[Floor[x / u] / m / s], {m, s + 1, Floor[Floor[Floor[x / u] / s]^(1/2)]}]
         ]

        , {s, (u + 1), Floor[Floor[x / u]^(1/3)]}], {u, b, Floor[x^(1/4)]}]

d4j[12 200, 2]

648 367

d4k[x_, b_] :=
 (-1 + b)^4 +
  -Floor[x^(1/4)]^4 +
  4 Sum[ Floor[x / (u^3)], {u, b, Floor[x^(1/4)]}] +
  -6 Sum[Floor[ Floor[x / (u^2)]^(1/2)]^2, {u, b, Floor[x^(1/4)]}] +
  12 Sum[ Sum [ Floor[ Floor[x / (u^2)] / m], {m, (u + 1), Floor[Floor[x / (u^2)]^(1/2)]}],
    {u, b, Floor[x^(1/4)]}] +
  4 Sum[Floor[Floor[x / u]^(1/3)]^3, {u, b, Floor[x^(1/4)]}] +
  12 Sum[ Sum[ Floor[ Floor[x / u] / s^2 ], {s, (u + 1), Floor[Floor[x / u]^(1/3)]}],
    {u, b, Floor[x^(1/4)]}] +
  -12 Sum[ Sum[Floor[ Sqrt[ Floor[ Floor[x / u] / s ] ] ]^2, {s, (u + 1), Floor[Floor[x / u]^(1/3)]}],
    {u, b, Floor[x^(1/4)]}] +
  24 Sum[ Sum[ Sum[ Floor[Floor[x / u] / m / s],
      {m, s + 1, Floor[Floor[Floor[x / u] / s]^(1/2)]}],
     {s, (u + 1), Floor[Floor[x / u]^(1/3)]}], {u, b, Floor[x^(1/4)]}]

d4k[16 553, 5]

70 313

d[16 553, 4, 5]

70 313
```

```
d4l[x_, b_] :=
 24
  Sum[ Sum[ Sum[ Floor[Floor[x / u] / m / s], {m, s + 1, Floor[Floor[Floor[x / u] / s]^(1 / 2)]}],
    {s, (u + 1), Floor[Floor[x / u]^(1 / 3)]}], {u, b, Floor[x^(1 / 4)]}]
```

```
f[n_] := Sum[ (Floor[n^(1 / j)]^j - 1 ) / j, {j, 1, 50}]
```

```
GG[x_, b_] :=
 Sum[ Sum[ Sum[ Floor[Floor[x / u] / m / s], {m, s + 1, Floor[Floor[Floor[x / u] / s]^(1 / 2)]}],
    {s, (u + 1), Floor[Floor[x / u]^(1 / 3)]}], {u, b, Floor[x^(1 / 4)]}]
```

```
GG[100 000, 2]
```

536 683

```
HH[x_, b_] := Sum[ Sum[ Sum[ Floor[x / u / m / s], {m, s + 1, Floor[x / u / s]^(1 / 2)}],
    {s, u + 1, (x / u)^(1 / 3)}], {u, b, x^(1 / 4)}]
```

```
HH[100 000, 2]
```

536 683

```
II[x_, b_] := Sum[ Floor[x / (u m s)], {u, b, x^(1 / 4)},
   {s, u + 1, (x / u)^(1 / 3)}, {m, s + 1, (x / (u s))^(1 / 2)}]
```

```
II[100 000, 2]
```

536 683

```
d4m[x_, b_] :=
 (-1 + b)^4 +
  -Floor[x^(1/4)]^4 +
  4 Sum[ Floor[x / (u^3)], {u, b, x^(1 / 4)}] +
  -6 Sum[Floor[ Floor[x / (u^2)]^(1 / 2)]^2 , {u, b, x^(1 / 4)}] +
  12 Sum [ Floor[x / (u^2 m)],
    {u, b, x^(1 / 4)}, {m, (u + 1), Floor[Floor[x / (u^2)]^(1 / 2)]}] +
  4 Sum[Floor[Floor[x / u]^(1/3)]^3, {u, b, x^(1 / 4)}] +
  12 Sum[ Floor[x / (u s^2)], {u, b, x^(1 / 4)}, {s, (u + 1), Floor[Floor[x / u]^(1 / 3)]}] +
  -12 Sum[ Floor[Floor[x / (u s)]^(1 / 2)]^2 , {u, b, x^(1 / 4)}, {s, (u + 1), (x / u)^(1 / 3)}] +
  24 Sum[ Floor[x / (u m s)], {u, b, x^(1 / 4)},
    {s, u + 1, (x / u)^(1 / 3)}, {m, s + 1, (x / (u s))^(1 / 2)}]
```

```
d4k[100 000, 2]
```

11 796 070

```
d4m[100 000, 2]
```

11 796 070

```
JJ[x_, b_] := Sum[ Sum[Floor[Sqrt[Floor[Floor[x / u] / s]]]^2 ,
    {s, (u + 1), Floor[Floor[x / u]^(1 / 3)]}], {u, b, Floor[x^(1 / 4)]}]
```

```
JJ[100 000, 2]
```

364 490

```
KK[x_, b_] :=
 Sum[ Floor[Floor[x / (u s)] ^ (1 / 2)]^2 , {u, b, x^(1 / 4)}, {s, (u + 1), (x / u) ^ (1 / 3)}]
```

```
KK[100 000, 2]
```

364 490

```
d4m2[x_ ] :=
 1 +
  -Floor[x^{1/4}]^4 +

  4 Sum[ Floor[x / (u^3)] + Floor[Floor[x / u]^{1/3}]^3, {u, 2, x^(1 / 4)}] +


  -6 Sum[Floor[ Floor[x / (u^2)]^(1 / 2)]^2 , {u, 2, x^(1 / 4)}] +

  12 Sum [ Floor[x / (u^2 m)],
     {u, 2, x^(1 / 4)}, {m, (u + 1), Floor[Floor[x / (u^2)]^(1 / 2)]}] +
  12 Sum[ Floor[x / ( u s^2)], {u, 2, x^(1 / 4)}, {s, (u + 1), Floor[Floor[x / u]^(1 / 3)]}] +
  -12 Sum[ Floor[Floor[x / (u s)]^(1 / 2)]^2 , {u, 2, x^(1 / 4)}, {s, (u + 1), (x / u) ^ (1 / 3)}] +

  24 Sum[ Floor[x / (u m s)], {u, 2, x^(1 / 4)},
     {s, u + 1, (x / u) ^ (1 / 3)}, {m, s + 1, (x / (u s)) ^ (1 / 2)}]
```

```
d4m2[1000]
```

13 952

```
d[1000, 4, 2]
```

```
d4m3[1000]
```
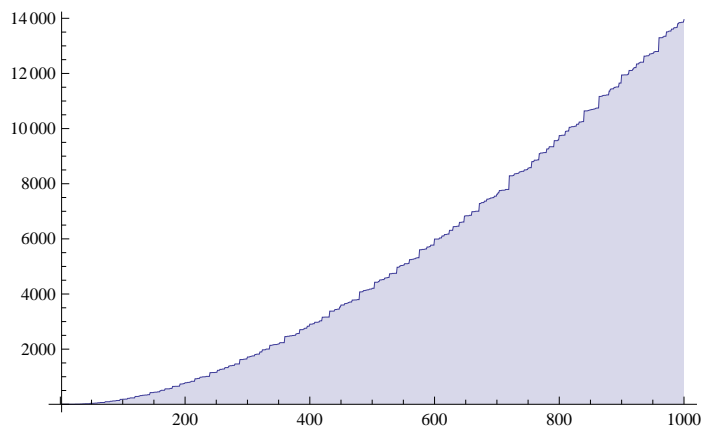
13 952

```
d4m3[x_ ] :=
 1 +
  - Floor[x^(1/4)]^4 +
  Sum[
   4 (Floor[x / (u^3)] + Floor[Floor[x / u]^(1/3)]^3)
    - 6 Floor[ Floor[x / (u^2)] ^ (1 / 2)] ^ 2 +

   12 Sum [ Floor[x / (u^2 m)], {m, (u + 1), Floor[Floor[x / (u^2)] ^ (1 / 2)]}] +
   12 Sum[ Floor[x / ( u s^2)], {s, (u + 1), Floor[Floor[x / u] ^ (1 / 3)]}] +
   - 12 Sum[ Floor[Floor[x / (u s)] ^ (1 / 2)]^2 , {s, (u + 1), (x / u) ^ (1 / 3)}] +

   24 Sum[ Floor[x / (u m s)], {s, u + 1, (x / u) ^ (1 / 3)}, {m, s + 1, (x / (u s)) ^ (1 / 2)}]
   , {u, 2, x^(1 / 4)}]


d4m4[x_ ] :=
 1 +
  - Floor[x^(1/4)]^4 +
  Sum[
   4 (Floor[x / (u^3)] + Floor[Floor[x / u]^(1/3)]^3)
    - 6 Floor[ Floor[x / (u^2)] ^ (1 / 2)] ^ 2 +
   12 (Sum [ Floor[x / (u^2 s)], {s, (u + 1), Floor[Floor[x / (u^2)] ^ (1 / 2)]}] +
       Sum[ Floor[x / ( u s^2)], {s, (u + 1), Floor[Floor[x / u] ^ (1 / 3)]}] +
       - Sum[ Floor[Floor[x / (u s)] ^ (1 / 2)]^2 , {s, (u + 1), (x / u) ^ (1 / 3)}]) +
   24 Sum[ Floor[x / (u m s)], {s, u + 1, (x / u) ^ (1 / 3)}, {m, s + 1, (x / (u s)) ^ (1 / 2)}]
   , {u, 2, x^(1 / 4)}]

d4m4[1000]

13 952


d4m5[x_ ] :=
 1 - Floor[x^(1/4)]^4 +
  Sum[
   4 (Floor[x / (u^3)] + Floor[Floor[x / u]^(1/3)]^3)
    - 6 Floor[ Floor[x / (u^2)] ^ (1 / 2)] ^ 2 +
   12 (Sum [ Floor[x / (u^2 s)], {s, (u + 1), Floor[Floor[x / (u^2)] ^ (1 / 2)]}] +
       Sum[ Floor[x / ( u s^2)] - Floor[Floor[x / (u s)] ^ (1 / 2)]^2,
        {s, (u + 1), Floor[Floor[x / u] ^ (1 / 3)]}]) +
   24 Sum[ Floor[x / (u m s)], {s, u + 1, (x / u) ^ (1 / 3)}, {m, s + 1, (x / (u s)) ^ (1 / 2)}]
   , {u, 2, x^(1 / 4)}]

d4m5[1000]

13 952
```
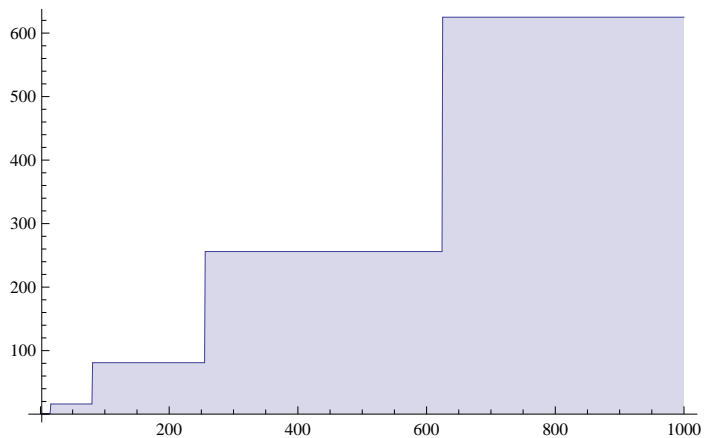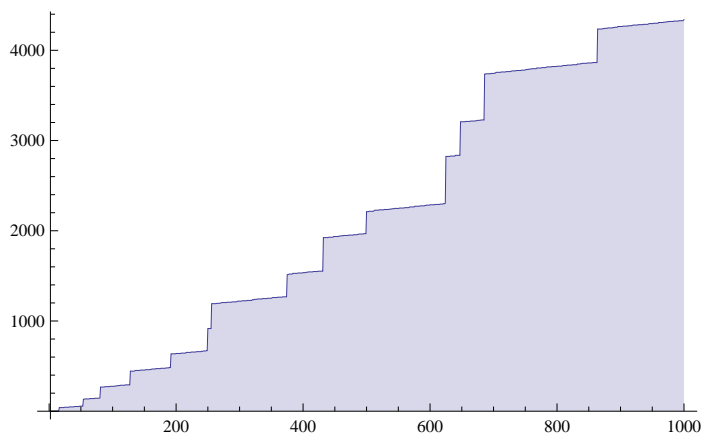
```
DiscretePlot[d4m5[x], {x, 2, 1000}]
```
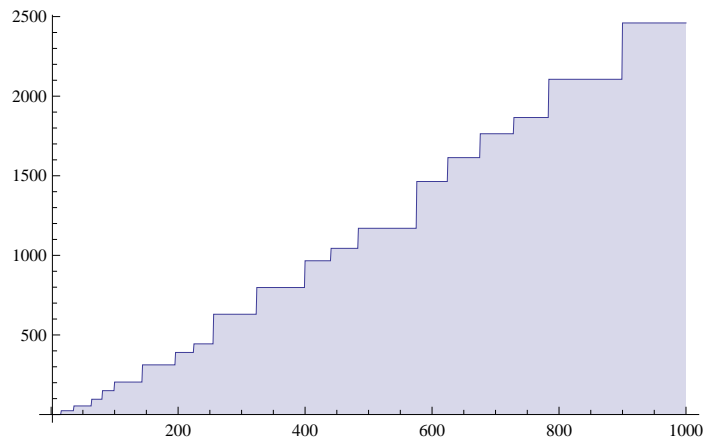


```
DDD[x_] := Floor[x^(1/4)]^4
```

```
DiscretePlot[DDD[x], {x, 2, 1000}]
```
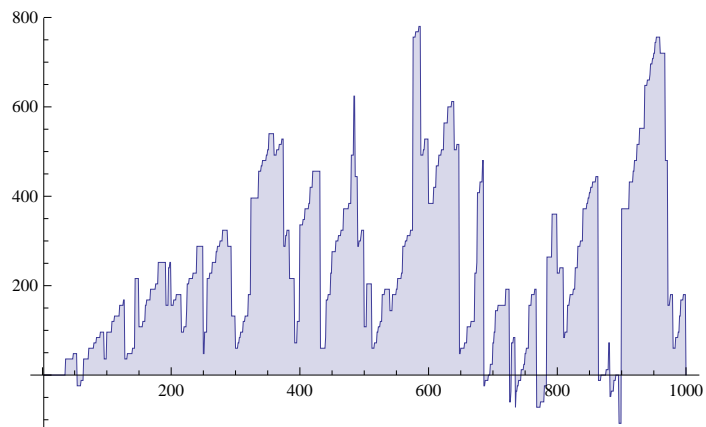


```
FFF[x_] := Sum[4 (Floor[x / (u^3)] + Floor[Floor[x / u]^(1/3)]^3), {u, 2, x^(1/4)}]
```

```
DiscretePlot[ FFF[x], {x, 2, 1000}]
```

```
GGG[x_ ] := Sum[6 Floor[ Floor[x / (u^2)] ^ (1 / 2)]^2, {u, 2, x^(1 / 4)}]
DiscretePlot[GGG[x], {x, 2, 1000}]
```
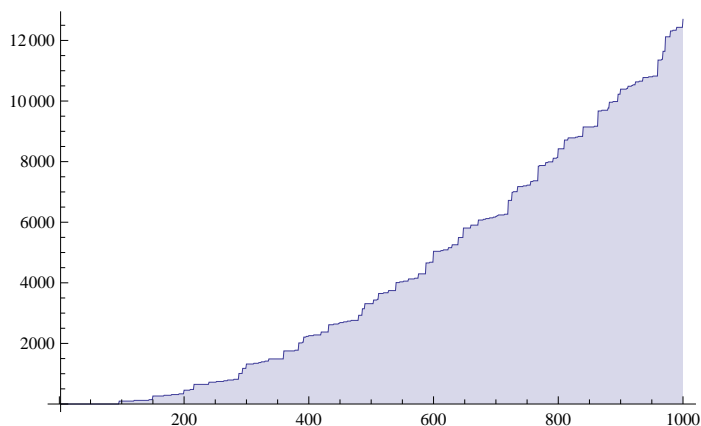


```
HHH[x_] := Sum[12 (Sum [ Floor[x / (u^2 s)], {s, (u + 1), Floor[Floor[x / (u^2)] ^ (1 / 2)]}] +
      Sum[ Floor[x / ( u s^2)] - Floor[Floor[x / (u s)] ^ (1 / 2)]^2,
       {s, (u + 1), Floor[Floor[x / u] ^ (1 / 3)]}]), {u, 2, x^(1 / 4)}]
DiscretePlot[HHH[x], {x, 2, 1000}]
```

```
III[x_] :=
 Sum[24 Sum[ Floor[x / (u m s)], {s, u + 1, (x / u) ^ (1 / 3)}, {m, s + 1, (x / (u s)) ^ (1 / 2)}],
   {u, 2, x^ (1 / 4)}]
DiscretePlot[III[x], {x, 2, 1000}]
```
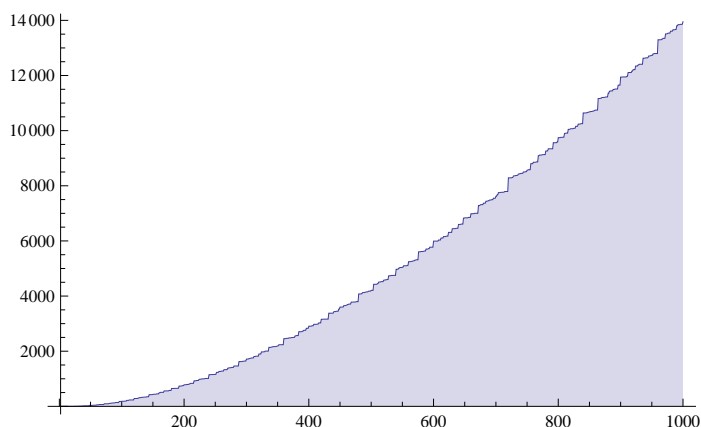


```
JJJ[x_] := 1 - DDD[x] + FFF[x] - GGG[x] + HHH[x] + III[x]
```

```
DiscretePlot[JJJ[x], {x, 2, 1000}]
```



```
DH[n_, k_, s_] := Sum[Binomial[k, k - j] DH[n / m^ j, k - j, m + 1], {m, s, n^ (1 / k)}, {j, 1, k}]
DH[n_, 0, s_] := 1
```

```
DH[100, 2, 1]
```

482

```
Sum[2 DH[100 / m, 1, m + 1] + 1, {m, 1, 100 ^ (1 / 2)}]
```

482

```
Sum[2 (Floor[100 / m] - m) + 1, {m, 1, 100 ^ (1 / 2)}]
```

482

```
Floor[100 ^ (1 / 2)] + Sum[2 Floor[100 / m] - 2 m, {m, 1, 100 ^ (1 / 2)}]
```

482

```
Floor[100 ^ (1 / 2)] - 2 ( Floor[100 ^ (1 / 2)] (Floor[100 ^ (1 / 2)] + 1)) / 2 +
 Sum[2 Floor[100 / m], {m, 1, 100 ^ (1 / 2)}]]
```

482

```
DP[n_] := Floor[n ^ (1 / 2)] -
   2 (Floor[n ^ (1 / 2)] (Floor[n ^ (1 / 2)] + 1)) / 2 + Sum[2 Floor[n / m], {m, 1, n ^ (1 / 2)}]
```

```
DP[100]
```

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```
Expand[Floor[n ^ (1 / 2)] - 2 (Floor[n ^ (1 / 2)] (Floor[n ^ (1 / 2)] + 1)) / 2]
```

$$-\text{Floor}\left[\sqrt{n}\right]^2$$

```
DH[n_, k_, s_] :=
 Sum[Sum[Binomial[k, k - j] DH[Floor[n / m^j], k - j, m + 1], {m, s, n ^ (1 / k)}], {j, 1, k}]
```

```
DH[n, 2, s]
```

$$1 + \sqrt{n} - s + \sum_{m=s}^{\sqrt{n}} 2 \left(-m + \text{Floor}\left[\frac{n}{m}\right]\right)$$

```
DH[n, 1, s]
```

$$1 + n - s$$

```
DH[n, 3, 1]
```

$Aborted

$$DH[n, 2, s] := 1 + \sqrt{n} - s + \sum_{m=s}^{\sqrt{n}} 2 \left(-m + \text{Floor}\left[\frac{n}{m}\right]\right)$$

$$DH[n, 1, s] := 1 + n - s$$

```
DH[n, 3, s]
```

$Aborted