```
D2Cache[n_, k_, s_] :=
 Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1 / k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]

D2Fast[n_, k_] :=
 Sum[D2Cache[Floor[n / j], k - 1, 2], {j, Floor[n^(1 / 3)] + 1, Floor[n^(1 / 2)]}] +

  Sum[(Floor[n / j] - (Floor[n / (j + 1)])) D2Cache[j, k - 1, 2],
   {j, 1, n / Floor[n^(1 / 2)] - 1}] +

  Sum[d2cache[j, k - 1] (Floor[n / j] - 1), {j, 2, n^(1 / 3)}] +

  Sum[d2cache[j, m] D2Cache[n / (j s), k - m - 1, 2], {j, 2, n^(1 / 3)},
   {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[n / j]^(1 / 2)}, {m, 1, k - 2}] +

  Sum[(Sum[1, {m, Floor[n / (j (s + 1))] + 1, n / (j s)}])
    (Sum[d2cache[j, m] D2Cache[s, k - m - 1, 2], {m, 1, k - 2}]),
   {j, 2, n^(1 / 3)}, {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]

D2Fast[n_, 1] := Floor[n] - 1

LinnikSumFast[n_] := Sum[(-1)^(k + 1) / k D2Fast[n, k], {k, 1, Log[2, n]}]
RiePrimeCnt[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Table[{n, a = LinnikSumFast[n], b = RiePrimeCnt[n], a - b}, {n, 1, 100}] // TableForm
```

```
D2Cache[n_, k_, s_] :=
 Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1 / k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]

D2Fast[n_, k_] :=
 Sum[D2Cache[Floor[n / j], k - 1, 2], {j, Floor[n^(1 / 3)] + 1, Floor[n^(1 / 2)]}] +

  Sum[(Floor[n / j] - (Floor[n / (j + 1)])) D2Cache[j, k - 1, 2],
   {j, 1, n / Floor[n^(1 / 2)] - 1}] +

  Sum[d2cache[j, k - 1] (Floor[n / j] - 1), {j, 2, n^(1 / 3)}] +

  Sum[d2cache[j, m] D2Cache[n / (j s), k - m - 1, 2], {j, 2, Floor[n^(1 / 3)]},
   {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[n / j]^(1 / 2)}, {m, 1, k - 2}] +

  Sum[(Floor[n / (j s)] - Floor[n / (j (s + 1))])
    (Sum[d2cache[j, m] D2Cache[s, k - m - 1, 2], {m, 1, k - 2}]),
   {j, 2, Floor[n^(1 / 3)]}, {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]

D2Fast[n_, 1] := Floor[n] - 1

LinnikSumFast[n_] := Sum[(-1)^(k + 1) / k D2Fast[n, k], {k, 1, Log[2, n]}]
RiePrimeCnt[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Table[{n, a = LinnikSumFast[n], b = RiePrimeCnt[n], a - b}, {n, 1, 100}] // TableForm
```

```
D2Cache[n_, k_, s_] :=
 Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1 / k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]

D2Fast[n_, k_] :=
 Sum[D2Cache[Floor[n / j], k - 1, 2], {j, Floor[n^(1 / 3)] + 1, Floor[n^(1 / 2)]}] +

  Sum[(D2Cache[Floor[n / j], 1, 2] - D2Cache[Floor[n / (j + 1)], 1, 2]) D2Cache[j, k - 1, 2],
   {j, 1, n / Floor[n^(1 / 2)] - 1}] +

  Sum[d2cache[j, k - 1] D2Cache[Floor[n / j], 1, 2], {j, 2, n^(1 / 3)}] +

  Sum[d2cache[j, m] D2Cache[Floor[n / (j s)], k - m - 1, 2], {j, 2, Floor[n^(1 / 3)]},
   {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[Floor[n / j]^(1 / 2)]}, {m, 1, k - 2}] +

  Sum[(D2Cache[Floor[n / (j s)], 1, 2] - D2Cache[Floor[n / (j (s + 1))], 1, 2])
    (Sum[d2cache[j, m] D2Cache[s, k - m - 1, 2], {m, 1, k - 2}]),
   {j, 2, Floor[n^(1 / 3)]}, {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]

D2Fast[n_, 1] := Floor[n] - 1

LinnikSumFast[n_] := Sum[(-1)^(k + 1) / k D2Fast[n, k], {k, 1, Log[2, n]}]
RiePrimeCnt[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Table[{n, a = LinnikSumFast[n], b = RiePrimeCnt[n], a - b}, {n, 1, 100}] // TableForm
```

```
D2Cache[n_, k_, s_] :=
 Sum[Binomial[k, j] D2Cache[n / (m^(k - j)), j, m + 1], {m, s, n^(1 / k)}, {j, 0, k - 1}]
D2Cache[n_, 1, s_] := Floor[n] - s + 1; D2Cache[n_, 0, s_] := 1
d2cache[n_, k_] := D2Cache[n, k, 2] - D2Cache[n - 1, k, 2]
D2Cache[n_, k_] := D2Cache[n, k, 2]

D2Fast[n_, k_] :=
 Sum[d2cache[j, 1] D2Cache[Floor[n / j], k - 1], {j, Floor[n^(1 / 3)] + 1, Floor[n^(1 / 2)]}] +

  Sum[(D2Cache[Floor[n / r], 1] - D2Cache[Floor[n / (r + 1)], 1]) D2Cache[r, k - 1],
   {r, 1, n / Floor[n^(1 / 2)] - 1}] +

  Sum[d2cache[j, k - 1] D2Cache[Floor[n / j], 1], {j, 2, n^(1 / 3)}] +

  Sum[d2cache[j, m] D2Cache[Floor[n / (j s)], k - m - 1], {j, 2, Floor[n^(1 / 3)]},
   {s, Floor[Floor[n^(1 / 3)] / j] + 1, Floor[Floor[n / j]^(1 / 2)]}, {m, 1, k - 2}] +

  Sum[(D2Cache[Floor[n / (j s)], 1] - D2Cache[Floor[n / (j (s + 1))], 1])
    (Sum[d2cache[j, m] D2Cache[s, k - m - 1], {m, 1, k - 2}]),
   {j, 2, Floor[n^(1 / 3)]}, {s, 1, Floor[n / j] / Floor[Floor[n / j]^(1 / 2)] - 1}]

D2Fast[n_, 1] := Floor[n] - 1

LinnikSumFast[n_] := Sum[(-1)^(k + 1) / k D2Fast[n, k], {k, 1, Log[2, n]}]
RiePrimeCnt[n_] := Sum[PrimePi[n^(1 / j)] / j, {j, 1, Log[2, n]}]
Table[{n, a = LinnikSumFast[n], b = RiePrimeCnt[n], a - b}, {n, 1, 100}] // TableForm
```

1	0	0	0
2	1	1	0
3	2	2	0
4	$\frac{5}{2}$	$\frac{5}{2}$	0
5	$\frac{7}{2}$	$\frac{7}{2}$	0
6	$\frac{7}{2}$	$\frac{7}{2}$	0
7	$\frac{9}{2}$	$\frac{9}{2}$	0
8	$\frac{29}{6}$	$\frac{29}{6}$	0
9	$\frac{16}{3}$	$\frac{16}{3}$	0
10	$\frac{16}{3}$	$\frac{16}{3}$	0
11	$\frac{19}{3}$	$\frac{19}{3}$	0
12	$\frac{19}{3}$	$\frac{19}{3}$	0
13	$\frac{22}{3}$	$\frac{22}{3}$	0
14	$\frac{22}{3}$	$\frac{22}{3}$	0
15	$\frac{22}{3}$	$\frac{22}{3}$	0
16	$\frac{91}{12}$	$\frac{91}{12}$	0
17	$\frac{103}{12}$	$\frac{103}{12}$	0

18	$\frac{103}{12}$	$\frac{103}{12}$	0
19	$\frac{115}{12}$	$\frac{115}{12}$	0
20	$\frac{115}{12}$	$\frac{115}{12}$	0
21	$\frac{115}{12}$	$\frac{115}{12}$	0
22	$\frac{115}{12}$	$\frac{115}{12}$	0
23	$\frac{127}{12}$	$\frac{127}{12}$	0
24	$\frac{127}{12}$	$\frac{127}{12}$	0
25	$\frac{133}{12}$	$\frac{133}{12}$	0
26	$\frac{133}{12}$	$\frac{133}{12}$	0
27	$\frac{137}{12}$	$\frac{137}{12}$	0
28	$\frac{137}{12}$	$\frac{137}{12}$	0
29	$\frac{149}{12}$	$\frac{149}{12}$	0
30	$\frac{149}{12}$	$\frac{149}{12}$	0
31	$\frac{161}{12}$	$\frac{161}{12}$	0
32	$\frac{817}{60}$	$\frac{817}{60}$	0
33	$\frac{817}{60}$	$\frac{817}{60}$	0
34	$\frac{817}{60}$	$\frac{817}{60}$	0
35	$\frac{817}{60}$	$\frac{817}{60}$	0
36	$\frac{817}{60}$	$\frac{817}{60}$	0
37	$\frac{877}{60}$	$\frac{877}{60}$	0
38	$\frac{877}{60}$	$\frac{877}{60}$	0
39	$\frac{877}{60}$	$\frac{877}{60}$	0
40	$\frac{877}{60}$	$\frac{877}{60}$	0
41	$\frac{937}{60}$	$\frac{937}{60}$	0
42	$\frac{937}{60}$	$\frac{937}{60}$	0
43	$\frac{997}{60}$	$\frac{997}{60}$	0
44	$\frac{997}{60}$	$\frac{997}{60}$	0
45	$\frac{997}{60}$	$\frac{997}{60}$	0
46	$\frac{997}{60}$	$\frac{997}{60}$	0
47	$\frac{1057}{60}$	$\frac{1057}{60}$	0
48	$\frac{1057}{60}$	$\frac{1057}{60}$	0
49	$\frac{1087}{60}$	$\frac{1087}{60}$	0
50	$\frac{1087}{60}$	$\frac{1087}{60}$	0
51	$\frac{1087}{60}$	$\frac{1087}{60}$	0
52	$\frac{1087}{60}$	$\frac{1087}{60}$	0
53	$\frac{1147}{60}$	$\frac{1147}{60}$	0
54	$\frac{1147}{60}$	$\frac{1147}{60}$	0
55	$\frac{1147}{60}$	$\frac{1147}{60}$	0
56	$\frac{1147}{60}$	$\frac{1147}{60}$	0

57	$\frac{1147}{60}$	$\frac{1147}{60}$	0
58	$\frac{1147}{60}$	$\frac{1147}{60}$	0
59	$\frac{1207}{60}$	$\frac{1207}{60}$	0
60	$\frac{1207}{60}$	$\frac{1207}{60}$	0
61	$\frac{1267}{60}$	$\frac{1267}{60}$	0
62	$\frac{1267}{60}$	$\frac{1267}{60}$	0
63	$\frac{1267}{60}$	$\frac{1267}{60}$	0
64	$\frac{1277}{60}$	$\frac{1277}{60}$	0
65	$\frac{1277}{60}$	$\frac{1277}{60}$	0
66	$\frac{1277}{60}$	$\frac{1277}{60}$	0
67	$\frac{1337}{60}$	$\frac{1337}{60}$	0
68	$\frac{1337}{60}$	$\frac{1337}{60}$	0
69	$\frac{1337}{60}$	$\frac{1337}{60}$	0
70	$\frac{1337}{60}$	$\frac{1337}{60}$	0
71	$\frac{1397}{60}$	$\frac{1397}{60}$	0
72	$\frac{1397}{60}$	$\frac{1397}{60}$	0
73	$\frac{1457}{60}$	$\frac{1457}{60}$	0
74	$\frac{1457}{60}$	$\frac{1457}{60}$	0
75	$\frac{1457}{60}$	$\frac{1457}{60}$	0
76	$\frac{1457}{60}$	$\frac{1457}{60}$	0
77	$\frac{1457}{60}$	$\frac{1457}{60}$	0
78	$\frac{1457}{60}$	$\frac{1457}{60}$	0
79	$\frac{1517}{60}$	$\frac{1517}{60}$	0
80	$\frac{1517}{60}$	$\frac{1517}{60}$	0
81	$\frac{383}{15}$	$\frac{383}{15}$	0
82	$\frac{383}{15}$	$\frac{383}{15}$	0
83	$\frac{398}{15}$	$\frac{398}{15}$	0
84	$\frac{398}{15}$	$\frac{398}{15}$	0
85	$\frac{398}{15}$	$\frac{398}{15}$	0
86	$\frac{398}{15}$	$\frac{398}{15}$	0
87	$\frac{398}{15}$	$\frac{398}{15}$	0
88	$\frac{398}{15}$	$\frac{398}{15}$	0
89	$\frac{413}{15}$	$\frac{413}{15}$	0
90	$\frac{413}{15}$	$\frac{413}{15}$	0
91	$\frac{413}{15}$	$\frac{413}{15}$	0
92	$\frac{413}{15}$	$\frac{413}{15}$	0
93	$\frac{413}{15}$	$\frac{413}{15}$	0
94	$\frac{413}{15}$	$\frac{413}{15}$	0
95	$\frac{413}{15}$	$\frac{413}{15}$	0

96	$\frac{413}{15}$	$\frac{413}{15}$	0
97	$\frac{428}{15}$	$\frac{428}{15}$	0
98	$\frac{428}{15}$	$\frac{428}{15}$	0
99	$\frac{428}{15}$	$\frac{428}{15}$	0
100	$\frac{428}{15}$	$\frac{428}{15}$	0