```
ClearAll["Global`*"]
$RecursionLimit = 10000
10000
t[n_{k}] := Mod[n, k] - Mod[n-1, k]
f[n_{-}] := If[n = 1, 1, N[n^{-1}t[n, 5]]]
F[n_{-}, 0] := 1; F[n_{-}, k_{-}] := F[n, k] = Sum[f[j]F[Floor[n/j], k-1], {j, 1, n}]
F2[n_{j}, 0] := 1; F2[n_{j}, k_{j}] := F2[n, k] = Sum[f[j]F2[Floor[n/j], k-1], {j, 2, n}]
FtoF2[n_{,k_{]}} := Sum[(-1)^jBinomial[k, j] F[n, k-j], {j, 0, k}]
F2toF[n_{k}] := Sum[Binomial[k, j] F2[n, j], {j, 0, k}]
\mathtt{Fz} \left[ \mathtt{n}_{-}, \, \mathtt{z}_{-} \right] := \mathtt{Sum} \left[ \mathtt{FactorialPower} \left[ \mathtt{z}, \, \mathtt{a} \right] \, / \, \mathtt{a} \, ! \, \mathtt{F2} \left[ \mathtt{n}, \, \mathtt{a} \right], \, \left\{ \mathtt{a}, \, \mathtt{0}, \, \mathtt{Log} \left[ \mathtt{2}, \, \mathtt{n} \right] \right\} \right]
Fa[n_{,0}, a_{,1}] := 1; Fa[n_{,k_{,1}}, a_{,1}] := Fa[n, k, a] = Sum[f[j] Fa[n/j, k-1, a], {j, a, n}]
Fa2[n_{k_{a}}, k_{a}] := Fa2[n, k, a] = If[n < a^k, 0,
        \\ \text{Sum}[\texttt{Binomial}[k, j] \ f[a] \ ^j \ Fa2[Floor[n \ / \ a \ ^j], \ k \ - \ j, \ a \ + \ 1], \ \{j, \ 0, \ k\}]]; \ Fa2[n_, \ 0, \ a_] := 1 \\ \\ \text{Sum}[\texttt{Binomial}[k, j] \ f[a] \ ^j \ Fa2[n_, \ 0, \ a_] := 1 \\ \\ \text{Sum}[\texttt{Binomial}[k, j] \ f[a] \ ^j \ Fa2[n_, \ 0, \ a_] := 1 \\ \\ \text{Sum}[\texttt{Binomial}[k, j] \ f[a] \ ^j \ Fa2[n_, \ 0, \ a_] := 1 \\ \\ \text{Sum}[\texttt{Binomial}[k, j] \ f[a] \ ^j \ Fa2[n_, \ 0, \ a_] := 1 \\ \\ \text{Sum}[\texttt{Binomial}[k, j] \ f[a] \ ^j \ Fa2[n_, \ 0, \ a_] := 1 \\ \\ \text{Sum}[\texttt{Binomial}[k, j] \ f[a] \ ^j \ Fa2[n_, \ 0, \ a_] := 1 \\ \\ \text{Sum}[\texttt{Binomial}[k, j] \ f[a] \ ^j \ Fa2[n_, \ 0, \ a_] := 1 \\ \\ \text{Sum}[\texttt{Binomial}[k, j] \ f[a] \ ^j \ Fa2[n_, \ 0, \ a_] := 1 \\ \\ \text{Sum}[\texttt{Binomial}[k, j] \ f[a] \ ^j \ Fa2[n_, \ 0, \ a_] := 1 \\ \\ \text{Sum}[\texttt{Binomial}[k, j] \ f[a] \ ] \ f[a] \ [a] \ [a]
Fa3[n_{k_{-}}, k_{-}, a_{-}] := Fa3[n, k, a] = Sum[Binomial[k, j] f[m]^(k-j)
          \texttt{Fa3}[\texttt{Floor}[\texttt{n} \, / \, (\texttt{m} \, ^ \, (\texttt{k} \, - \, \texttt{j}) \, ) \, ] \, , \, \, \texttt{j}, \, \texttt{m} \, + \, \texttt{1} \, ] \, , \, \, \{\texttt{m}, \, \texttt{a}, \, \texttt{n} \, ^ \, (\texttt{1} \, / \, \texttt{k}) \, \} \, , \, \, \{\texttt{j}, \, \texttt{0}, \, \texttt{k} \, - \, \texttt{1}\} \, ]
Fa3[n_{,0,a_{,1}} := 1
fa[n_{,k_{,a}]} := Fa3[n, k, a] - Fa3[n-1, k, a]
Sum[fa[j, k-1, a] Fa3[Floor[n/j], 1, a], {j, a, t}] +
     Sum[fa[s, 1, a] fa[j, m, a] Fa3[Floor[n/(js)], k-m-1, a],
       {j, a, t}, {s, Floor[t/j] + 1, Floor[n/j]}, {m, 1, k - 2}
Lg[n_{-}] := Lg[n] = Sum[(-1)^(k+1)/kF2[n,k], \{k, 1, Log[2, n]\}]
lg[n] := Lg[n] - Lg[n-1]
LgF[n_{-}, 0] := 1; LgF[n_{-}, k_{-}] := LgF[n, k] = Sum[lg[j] LgF[Floor[n/j], k-1], {j, 2, n}]
Mb2[n] := Mb2[n] = Sum[(-1)^k F2[n,k], \{k, 1, Log[2, n]\}]
mb[n_] := Mb2[n] - Mb2[n-1]
MbF[n_{, 0}] := 1; MbF[n_{, k_{, 0}} := MbF[n, k] = Sum[mb[j] MbF[Floor[n/j], k-1], \{j, 2, n\}]
MbtoF[n_, z_] := Sum[(-1)^aFactorialPower[z+a-1, a]/a! MbF[n, a], {a, 0, Log[2, n]}]
FtoEta[n_, k_, b_] :=
  FtoEta[n, k, b] = Sum[(-1)^jBinomial[k, j]b^jF[n/b^j, k], \{j, 0, k\}]
EtaToF[n_, k_, b_] :=
  Sum[Binomial[k+j-1,k-1]b^{j}FtoEta[n/b^{j},k,b], \{j,0,Log[b,n]\}]
Table[n, N[F2[n, cc = 3]], N[FtoF2[n, cc]], Fa2[n, cc, 2], Fa3[n, cc, 2],
       Fa[n, cc, 2], Fa4[n, cc, 2, Floor[n^(1/2)]], {n, 1, 100}] // TableForm
                                                                                                       0
1
               Ο.
                                     0.
                                                           Ω
                                                                                 n
2
               0.
                                     0.
                                                           0
                                                                                 0
                                                                                                       0.
                                                                                                                             0.
3
               0.
                                     0.
                                                           0
                                                                                 0
                                                                                                       0.
                                                                                                                             0.
                                                           Ω
                                                                                 n
4
                                     0.
                                                                                                       0.
                                                                                                                             0.
               0.
5
               0.
                                     0.
                                                           0
                                                                                 0
                                                                                                       0.
                                                                                                                             0.
6
               0.
                                    0.
                                                           0
                                                                                 0
                                                                                                       0.
                                                                                                                             0.
7
                                    0.
                                                           0
                                                                                 0
                                                                                                       0.
                                                                                                                             0.
               0.
8
               -8.
                                    -8.
                                                          -8.
                                                                                 -8.
                                                                                                       -8.
                                                                                                                             -8.
9
               -8.
                                    -8.
                                                           -8.
                                                                                 -8.
                                                                                                       -8.
                                                                                                                             -8.
10
              -8.
                                    -8.
                                                           -8.
                                                                                 -8.
                                                                                                       -8.
                                                                                                                             -8.
11
              -8.
                                    -8.
                                                          -8.
                                                                                 -8.
                                                                                                       -8.
                                                                                                                             -8.
```

| | | | • | | | |
|----|-------------|-------------|-------------|-------------|---------------|-------|
| 12 | 28. | 28. | 28. | 28. | 28. | 28. |
| 13 | 28. | 28. | 28. | 28. | 28. | 28. |
| 14 | 28. | 28. | 28. | 28. | 28. | 28. |
| 15 | 28. | 28. | 28. | 28. | 28. | 28. |
| 16 | -20. | -20. | -20. | -20. | -20. | -20. |
| 17 | -20. | -20. | -20. | -20. | -20. | -20. |
| 18 | -74. | -74. | -74. | -74. | -74. | -74. |
| 19 | -74. | -74. | -74. | -74. | -74. | -74. |
| 20 | -14. | -14. | -14. | -14. | -14. | -14. |
| 21 | -14. | -14. | -14. | -14. | -14. | -14. |
| 22 | -14. | -14. | -14. | -14. | -14. | -14. |
| 23 | -14. | -14. | -14. | -14. | -14. | -14. |
| 24 | 58. | 58. | 58. | 58. | 58. | 58. |
| 25 | 58. | 58. | 58. | 58. | 58. | |
| | | | | | | 58. |
| 26 | 58. | 58. | 58. | 58. | 58. | 58. |
| 27 | 85. | 85. | 85. | 85. | 85. | 85. |
| 28 | 169. | 169. | 169. | 169. | 169. | 169. |
| 29 | 169. | 169. | 169. | 169. | 169. | 169. |
| 30 | -11. | -11. | -11. | -11. | -11. | -11. |
| 31 | -11. | -11. | -11. | -11. | -11. | -11. |
| 32 | -203. | -203. | -203. | -203. | -203. | -203. |
| 33 | -203. | -203. | -203. | -203. | -203. | -203. |
| 34 | -203. | -203. | -203. | -203. | -203. | -203. |
| 35 | -203. | -203. | -203. | -203. | -203. | -203. |
| 36 | 13. | 13. | 13. | 13. | 13. | 13. |
| 37 | 13. | 13. | 13. | 13. | 13. | 13. |
| 38 | 13. | 13. | 13. | 13. | 13. | 13. |
| 39 | 13. | 13. | 13. | 13. | 13. | 13. |
| 40 | 133. | 133. | 133. | 133. | 133. | 133. |
| 41 | 133. | 133. | 133. | 133. | 133. | 133. |
| 42 | -119. | -119. | -119. | -119. | -119. | -119. |
| 43 | -119. | -119. | -119. | -119. | -119. | -119. |
| 44 | 13. | 13. | 13. | 13. | 13. | 13. |
| 45 | 148. | 148. | 148. | 148. | 148. | 148. |
| 46 | 148. | 148. | 148. | 148. | 148. | 148. |
| 47 | 148. | 148. | 148. | 148. | 148. | 148. |
| 48 | 148. | 148. | 148. | 148. | 148. | 148. |
| 49 | 148. | 148. | 148. | 148. | 148. | 148. |
| 50 | | | | | | |
| 51 | -2. -2. | -2. -2. | -2. 2 | -2. -2. | -2. -2. | -2. |
| 52 | -2. 154. | -2. 154. | -2. | | | -2. |
| | | | 154. | 154. | 154. | 154. |
| 53 | 154. | 154. | 154. | 154. | 154. | 154. |
| 54 | -332. | -332. | -332. | -332. | -332. | -332. |
| 55 | -332. | -332. | -332. | -332. | -332. | -332. |
| 56 | -164. | -164. | -164. | -164. | -164. | -164. |
| 57 | -164. | -164. | -164. | -164. | -164. | -164. |
| 58 | -164. | -164. | -164. | -164. | -164. | -164. |
| 59 | -164. | -164. | -164. | -164. | -164. | -164. |
| 60 | 376. | 376. | 376. | 376. | 376. | 376. |
| 61 | 376. | 376. | 376. | 376. | 376. | 376. |
| 62 | 376. | 376. | 376. | 376. | 376. | 376. |
| 63 | 565. | 565. | 565. | 565. | 565. | 565. |
| 64 | -75. | -75. | -75. | -75. | - 75 . | -75. |
| 65 | -75. | -75. | -75. | -75. | - 75 . | -75. |
| 66 | -471. | -471. | -471. | -471. | -471. | -471. |
| 67 | -471. | -471. | -471. | -471. | -471. | -471. |
| | | | | | | |

84 85 945. 945. 945. 945. 945. 945. 86 945. 945. 945. 945. 945. 945. 87 945. 945. 945. 945. 945. 945. 88 1209. 1209. 1209. 1209. 1209. 1209. 89 1209. 1209. 1209. 1209. 1209. 1209.

91 -681. -681. -681. -681.-681. -681. 92 -405. -405.-405. -405.-405. -405. -405.-405.-405.-405. -405. 93 -405. 94 -405.-405.-405.-405.-405. -405.95 -405. -405.-405.-405.-405. -405.

-681.

-981. -981. -981. -981. 97 -981. -981. -981. -981. -981. -981. 98 -1275.-1275. -1275. -1275.-1275. -1275. 99 -978. -978. -978. -978. -978. -978. 100 -378. -378. -378. -378. -378. -378.

Table[$\{n, N[F[n, cc = 5]], N[F2toF[n, cc]],$

-681.

-681.

90

96

 $Fa2[n, cc, 1], Fz[n, cc], Fa4[n, cc, 1, Floor[n^(1/2)]],$ LgExp[n, cc], MbtoF[n, cc], EtaToF[n, cc, 2], $\{n, 1, 100\}$] // TableForm

-681.

1 1 1 1 1. 1. 1 1 1 2 -9. **-9.** -9. - 9. -9. -9. -9. -9. 3 6. 6. 6. 6. 6. 6. 6. 6. 4 26. 26. 26. 26. 26. 26. 26. 26. 5 51. 51. 51. 51. 51. 51. 51. 51. 6 -99. -99. -99. -99. -99. -99. -99. -99. 7 -64. -64.-64. -64.-64.-64.-64.-64.8 -24.-24.-24. -24.-24.-24.-24.-24.9 111. 111. 111. 111. 111. 111. 111. 111. 10 -139.-139. -139. -139. -139. -139. -139. -139. 11 -84.-84.-84.-84.-84.-84.-84.-84.12 216. 216. 216. 216. 216. 216. 216. 216. 13 281. 281. 281. 281. 281. 281. 281. 281. 14 -69. -69. -69. -69. -69. -69. -69. -69. 15 306. 306. 306. 306. 306. 306. 306. 306. 16 306. 306. 306. 306. 306. 306. 306. 306. 17 391. 391. 391. 391. 391. 391. 391. 391. 18 -959. -959. -959. -959. -959. -959. -959. -959. -864. -864. -864. -864. -864. 19 -864.-864. -864.

-681.

-981.

-681.

-981.

| 20 | -364. | -364. | -364. | -364. | -364. | -364. | -364. | -364. |
|-----|-----------------|-----------------|-----------------|---------|--------|--------|--------|--------|
| 21 | 161. | 161. | 161. | 161. | 161. | 161. | 161. | 161. |
| 22 | -389. | -389. | -389. | -389. | -389. | -389. | -389. | -389. |
| 23 | -274. | -274. | -274. | -274. | -274. | -274. | -274. | -274. |
| 24 | 326. | 326. | 326. | 326. | 326. | 326. | 326. | 326. |
| 25 | 701. | 701. | 701. | 701. | 701. | 701. | 701. | 701. |
| 26 | 51. | 51. | 51. | 51. | 51. | 51. | 51. | 51. |
| 27 | 996. | 996. | 996. | 996. | 996. | 996. | 996. | 996. |
| 28 | 1696. | 1696. | 1696. | 1696. | 1696. | 1696. | 1696. | 1696. |
| 29 | 1841. | 1841. | 1841. | 1841. | 1841. | 1841. | 1841. | 1841. |
| 30 | -1909. | -1909. | -1909. | -1909. | -1909. | -1909. | -1909. | -1909. |
| 31 | -1754. | -1754. | -1754. | -1754. | -1754. | -1754. | -1754. | -1754. |
| 32 | -1946. | -1946. | -1946. | -1946. | -1946. | -1946. | -1946. | -1946. |
| 33 | -1121. | -1121. | -1121. | -1121. | -1121. | -1121. | -1121. | -1121. |
| 34 | -1971. | -1971. | -1971. | -1971. | -1971. | -1971. | -1971. | -1971. |
| 35 | -1096. | -1096. | -1096. | -1096. | -1096. | -1096. | -1096. | -1096. |
| 36 | 1604. | 1604. | 1604. | 1604. | 1604. | 1604. | 1604. | 1604. |
| 37 | 1789. | 1789. | 1789. | 1789. | 1789. | 1789. | 1789. | 1789. |
| 38 | 839. | 839. | 839. | 839. | 839. | 839. | 839. | 839. |
| 39 | 1814. | 1814. | 1814. | 1814. | 1814. | 1814. | 1814. | 1814. |
| 40 | 2814. | | 2814. | | | 2814. | | |
| 41 | 3019. | 2814. | | 2814. | 2814. | | 2814. | 2814. |
| | - 2231. | 3019. | 3019. -2231. | 3019. | 3019. | 3019. | 3019. | 3019. |
| 42 | | -2231. | | - 2231. | -2231. | -2231. | -2231. | -2231. |
| 43 | -2016. | -2016. | -2016. | -2016. | -2016. | -2016. | -2016. | -2016. |
| 44 | -916. | -916. | -916. | -916. | -916. | -916. | -916. | -916. |
| 45 | 2459. | 2459. | 2459. | 2459. | 2459. | 2459. | 2459. | 2459. |
| 46 | 1309. | 1309. | 1309. | 1309. | 1309. | 1309. | 1309. | 1309. |
| 47 | 1544. | 1544. | 1544. | 1544. | 1544. | 1544. | 1544. | 1544. |
| 48 | 1544. | 1544. | 1544. | 1544. | 1544. | 1544. | 1544. | 1544. |
| 49 | 2279. | 2279. | 2279. | 2279. | 2279. | 2279. | 2279. | 2279. |
| 50 | -1471. | -1471. | -1471. | -1471. | -1471. | -1471. | -1471. | -1471. |
| 51 | -196. | -196. | -196. | -196. | -196. | -196. | -196. | -196. |
| 52 | 1104. | 1104. | 1104. | 1104. | 1104. | 1104. | 1104. | 1104. |
| 53 | 1369. | 1369. | 1369. | 1369. | 1369. | 1369. | 1369. | 1369. |
| 54 | -8081. | -8081. | -8081. | -8081. | -8081. | -8081. | -8081. | -8081. |
| 55 | -6706. | -6706. | -6706. | -6706. | -6706. | -6706. | -6706. | -6706. |
| 56 | -5306. | -5306. | -5306. | -5306. | -5306. | -5306. | -5306. | -5306. |
| 57 | -3881. | -3881. | -3881. | -3881. | -3881. | -3881. | -3881. | -3881. |
| 58 | -5331. | -5331. | -5331. | -5331. | -5331. | -5331. | -5331. | -5331. |
| 59 | -5036. | -5036. | -5036. | -5036. | -5036. | -5036. | -5036. | -5036. |
| 60 | 2464. | 2464. | 2464. | 2464. | 2464. | 2464. | 2464. | 2464. |
| 61 | 2769. | 2769. | 2769. | 2769. | 2769. | 2769. | 2769. | 2769. |
| 62 | 1219. | 1219. | 1219. | 1219. | 1219. | 1219. | 1219. | 1219. |
| 63 | 5944. | 5944. | 5944. | 5944. | 5944. | 5944. | 5944. | 5944. |
| 64 | 5304. | 5304. | 5304. | 5304. | 5304. | 5304. | 5304. | 5304. |
| 65 | 6929. | 6929. | 6929. | 6929. | 6929. | 6929. | 6929. | 6929. |
| 66 | -1321. | -1321. | -1321. | -1321. | -1321. | -1321. | -1321. | -1321. |
| 67 | -986. | -986. | -986. | -986. | -986. | -986. | -986. | -986. |
| 68 | 714. | 714. | 714. | 714. | 714. | 714. | 714. | 714. |
| 69 | 2439. | 2439. | | 2439. | | 2439. | 2439. | 2439. |
| 70 | -6311. | | | | | | | -6311. |
| 71 | - 5956 . | | | | | | -5956. | |
| 72 | - 556 . | -556. | | | | | | - 556. |
| 73 | - 191. | -191. | | | | | | - 191. |
| 74 | | | | | | | | |
| 75 | 3584. | -2041. 3584. | 3584. | 3584. | 3584. | 3584. | 3584. | 3584. |
| , 5 | 5501. | JJ0 1. | JJ0 1. | 5501. | JJ0 1. | 5501. | 5501. | JJ04. |

| 76 | 5484. | 5484. | 5484. | 5484. | 5484. | 5484. | 5484. | 5484. |
|-----|---------|---------|---------|---------|---------|---------|---------|--------|
| 77 | 7409. | 7409. | 7409. | 7409. | 7409. | 7409. | 7409. | 7409. |
| 78 | -2341. | -2341. | -2341. | -2341. | -2341. | -2341. | -2341. | -2341. |
| 79 | -1946. | -1946. | -1946. | -1946. | -1946. | -1946. | -1946. | -1946. |
| 80 | -1946. | -1946. | -1946. | -1946. | -1946. | -1946. | -1946. | -1946. |
| 81 | 3724. | 3724. | 3724. | 3724. | 3724. | 3724. | 3724. | 3724. |
| 82 | 1674. | 1674. | 1674. | 1674. | 1674. | 1674. | 1674. | 1674. |
| 83 | 2089. | 2089. | 2089. | 2089. | 2089. | 2089. | 2089. | 2089. |
| 84 | 12589. | 12589. | 12589. | 12589. | 12589. | 12589. | 12589. | 12589. |
| 85 | 14714. | 14714. | 14714. | 14714. | 14714. | 14714. | 14714. | 14714. |
| 86 | 12564. | 12564. | 12564. | 12564. | 12564. | 12564. | 12564. | 12564. |
| 87 | 14739. | 14739. | 14739. | 14739. | 14739. | 14739. | 14739. | 14739. |
| 88 | 16939. | 16939. | 16939. | 16939. | 16939. | 16939. | 16939. | 16939. |
| 89 | 17384. | 17384. | 17384. | 17384. | 17384. | 17384. | 17384. | 17384. |
| 90 | -16366. | -16366. | -16366. | -16366. | -16366. | -16366. | -16366. | -16366 |
| 91 | -14091. | -14091. | -14091. | -14091. | -14091. | -14091. | -14091. | -14091 |
| 92 | -11791. | -11791. | -11791. | -11791. | -11791. | -11791. | -11791. | -11791 |
| 93 | -9466. | -9466. | -9466. | -9466. | -9466. | -9466. | -9466. | -9466. |
| 94 | -11816. | -11816. | -11816. | -11816. | -11816. | -11816. | -11816. | -11816 |
| 95 | -9441. | -9441. | -9441. | -9441. | -9441. | -9441. | -9441. | -9441. |
| 96 | -12321. | -12321. | -12321. | -12321. | -12321. | -12321. | -12321. | -12321 |
| 97 | -11836. | -11836. | -11836. | -11836. | -11836. | -11836. | -11836. | -11836 |
| 98 | -19186. | -19186. | -19186. | -19186. | -19186. | -19186. | -19186. | -19186 |
| 99 | -11761. | -11761. | -11761. | -11761. | -11761. | -11761. | -11761. | -11761 |
| 100 | -4261. | -4261. | -4261. | -4261. | -4261. | -4261. | -4261. | -4261. |
| | | | | | | | | |