```
FactInteger[n_] := If[n == 1, {}, FactorInteger[n]]
d[n_, z_] := Product[1 / (j[[2]] !) Pochhammer[z, j[[2]]], {j, FactInteger[n]}]
Table[{n, d[n, -1], MoebiusMu[n]}, {n, 1, 100}] // TableForm
```

1	1	1
2	-1	-1
3	-1	-1
4	0	0
5	-1	-1
6	1	1
7	-1	-1
8	0	0
9	0	0
10	1	1
11	-1	-1
12	0	0
13	-1	-1
14	1	1
15	1	1
16	0	0
17	-1	-1
18	0	0
19	-1	-1
20	0	0
21	1	1
22	1	1
23	-1	-1
24	0	0
25	0	0
26	1	1
27	0	0
28	0	0
29	-1	-1
30	-1	-1
31	-1	-1
32	0	0
33	1	1
34	1	1
35	1	1
36	0	0
37	-1	-1
38	1	1
39	1	1
40	0	0
41	-1	-1
42	-1	-1
43	-1	-1
44	0	0
45	0	0
46	1	1
47	-1	-1
48	0	0
49	0	0
50	0	0
51	1	1

```
52      0       0
53     -1      -1
54      0       0
55      1       1
56      0       0
57      1       1
58      1       1
59     -1      -1
60      0       0
61     -1      -1
62      1       1
63      0       0
64      0       0
65      1       1
66     -1      -1
67     -1      -1
68      0       0
69      1       1
70     -1      -1
71     -1      -1
72      0       0
73     -1      -1
74      1       1
75      0       0
76      0       0
77      1       1
78     -1      -1
79     -1      -1
80      0       0
81      0       0
82      1       1
83     -1      -1
84      0       0
85      1       1
86      1       1
87      1       1
88      0       0
89     -1      -1
90      0       0
91      1       1
92      0       0
93      1       1
94      1       1
95      1       1
96      0       0
97     -1      -1
98      0       0
99      0       0
100     0       0
```

d[6, -1]

1

```
FactInteger[n_] := If[n == 1, {}, FactorInteger[n]];
d[n_, z_] := Product[1 / (j[[2]] !) Pochhammer[z, j[[2]]], {j, FactInteger[n]}];
Table[{n, d[n, -1], MoebiusMu[n]}, {n, 1, 100}] // TableForm
```

1	1	1
2	-1	-1
3	-1	-1
4	0	0
5	-1	-1
6	1	1
7	-1	-1
8	0	0
9	0	0
10	1	1
11	-1	-1
12	0	0
13	-1	-1
14	1	1
15	1	1
16	0	0
17	-1	-1
18	0	0
19	-1	-1
20	0	0
21	1	1
22	1	1
23	-1	-1
24	0	0
25	0	0
26	1	1
27	0	0
28	0	0
29	-1	-1
30	-1	-1
31	-1	-1
32	0	0
33	1	1
34	1	1
35	1	1
36	0	0
37	-1	-1
38	1	1
39	1	1
40	0	0
41	-1	-1
42	-1	-1
43	-1	-1
44	0	0
45	0	0
46	1	1
47	-1	-1
48	0	0
49	0	0
50	0	0
51	1	1

52	0	0
53	−1	−1
54	0	0
55	1	1
56	0	0
57	1	1
58	1	1
59	−1	−1
60	0	0
61	−1	−1
62	1	1
63	0	0
64	0	0
65	1	1
66	−1	−1
67	−1	−1
68	0	0
69	1	1
70	−1	−1
71	−1	−1
72	0	0
73	−1	−1
74	1	1
75	0	0
76	0	0
77	1	1
78	−1	−1
79	−1	−1
80	0	0
81	0	0
82	1	1
83	−1	−1
84	0	0
85	1	1
86	1	1
87	1	1
88	0	0
89	−1	−1
90	0	0
91	1	1
92	0	0
93	1	1
94	1	1
95	1	1
96	0	0
97	−1	−1
98	0	0
99	0	0
100	0	0

N[Log[Log[10]]]

0.834032

Integrate[1, {x, 1, n}, {y, 1, n / x}]

ConditionalExpression[1 + n (−1 + Log[n]), Re[n] ≥ 0 || n ∉ Reals]

```
Integrate[1, {x, 1, n}, {y, 1, n / x}, {z, 1, n / (x y)}]
```

$$\text{ConditionalExpression}\left[-1 + n + \frac{1}{2} n (-2 + \text{Log}[n]) \text{Log}[n], \text{Re}[n] \geq 0 \mid\mid n \notin \text{Reals}\right]$$

```
Integrate[1, {x, 1, n}, {y, 1, n / x}, {z, 1, n / (x y)}, {w, 1, n / (x y z)}]
```

$$\text{ConditionalExpression}\left[1 - n + \frac{1}{6} n \text{Log}[n] (6 + (-3 + \text{Log}[n]) \text{Log}[n]), \text{Re}[n] \geq 0 \mid\mid n \notin \text{Reals}\right]$$

```
JJ[n_, a_] := Sum[d^a MoebiusMu[n / d], {d, Divisors[n]}]
```

```
JJ[2, 0]
```

```
0
```

```
FactInteger[n_] := If[n == 1, {}, FactorInteger[n]]
d[n_, z_] := Product[1 / (j[[2]] !) Pochhammer[z, j[[2]]], {j, FactInteger[n]}]
ds[n_, k_] := Sum[ds[j, k - 1] ds[n / j, 1], {j, Divisors[n]}]
ds[n_, 1] := If[n < 2, 0, 1]
ds[n_, 0] := If[n == 1, 1, 0]
DS[n_, k_] := Sum[ds[j, k], {j, 2, n}]
DD[n_, k_] := Sum[d[j, k], {j, 1, n}]
```

```
DS[100, 2]
```

```
283
```

```
DD[100, 2]
```

```
482
```

```
DS[n_, k_] := Sum[DS[Floor[n / j], k - 1], {j, 2, n}]
DD[n_, k_] := Sum[DD[Floor[n / j], k - 1], {j, 1, n}]
DS[n_, 0] := 1
DD[n_, 0] := 1
TestDS[n_, k_] := Sum[(-1) ^ (k - j) Binomial[k, j] DD[n, j], {j, 0, k}]
TestDD[n_, k_] := Sum[Binomial[k, j] DS[n, j], {j, 0, k}]
```

```
TestDD[1000, 2]
```

```
7069
```

```
DD[1000, 2]
```

```
1000
```