```
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_, z_] := Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}]
dz2[n_, z_, k_] := Sum[ dz[j, z] dz2[n / j, z, k - 1], {j, 2, n}];
dz2[n_, z_, 0] := UnitStep[n - 1]
dd[n_, z_, x_] := Sum[ Binomial[x, k] dz2[n, z, k], {k, 0, Log[2, n]}]
ddb[n_, z_, k_] := Sum[ dz[j, z] ddb[n / j, z, k - 1], {j, 1, n}];
ddb[n_, z_, 0] := UnitStep[n - 1]
Clear[D2, D2a]
D2[n_, k_] := D2[n, k] = Sum[ D2[Floor[n / j], k - 1], {j, 2, n}];
D2[n_, 0] := UnitStep[n - 1]
d2[n_, k_] := D2[n, k] - D2[n - 1, k]
Dz[n_, z_] := Sum[ bin[ z, k] D2[n, k], {k, 0, Log[2, n]}]
Ez[n_, t_] := Sum[ Dz[n, k] / k!, {k, 0, t}]
E2z[n_] := Sum[ D2[n, k] / k!, {k, 0, Log[2, n]}]
D2a[n_, x_, k_] := D2a[n, x, k] = Sum[d2[j, x] D2a[Floor[n / j], x, k - 1], {j, 2, n}];
D2a[n_, x_, 0] := UnitStep[n - 1]
Clear[K, P, pp, Pz, P1zz]
K[n_] := K[n] = FullSimplify[ MangoldtLambda[n] / Log[n]]
P[n_, k_] := P[n, k] = Sum[ K[j] P[Floor[n / j], k - 1], {j, 2, n}]
P[n_, 0] := UnitStep[n - 1]
pp[n_, k_] := P[n, k] - P[n - 1, k]
Pz[n_, z_, k_] := Pz[n, z, k] = Sum[ pp[j, z] Pz[Floor[n / j], z, k - 1], {j, 2, n}]
Pz[n_, z_, 0] := UnitStep[n - 1]
pz[n_, z_, k_] := Pz[n, z, k] - Pz[n - 1, z, k]
bin[z_, k_] := Product[z - j, {j, 0, k - 1}] / k!
P1z[ n_, z_] := Sum[ bin[z, k] P[n, k], {k, 0, Log[2, n]}]
p1z[n_, z_] := P1z[n, z] - P1z[n - 1, z]
P1zz[n_, z_, k_] := P1zz[n, z, k] = Sum[ p1z[j, z] P1zz[Floor[n / j], z, k - 1], {j, 2, n}]
P1zz[n_, z_, 0] := UnitStep[n - 1]
P1x[n_, z_, x_] := Sum[ Binomial[x, k] P1zz[n, z, k], {k, 0, Log[2, n]}]


dz2[100, 2, 2]

2612

dd[100, 3, 3]

68 494

dd[100, -1, -1]

100

ddb[100, 3, 3]

68 494

D2a[5000, 3, 3]

6478

D2[5000, 9]

6478
```

```
N@Log[(3^4)^5]
```

21.9722

```
N@(5 Log[(3^4)])
```

21.9722

```
N@(20 Log[(3)])
```

21.9722

```
Sum[ d2[j, 2] D2[100 / j, 3], {j, 2, 100}]
```

51

```
D2[100, 5]
```

51

```
D2a[100, 2, 3]
```

7

```
P[1000, 6]
```

$\frac{7547}{4}$

```
{Pz[1000, 6, 1], Pz[1000, 3, 2], Pz[1000, 2, 3], Pz[1000, 1, 6]}
```

$\left\{\frac{7547}{4}, \frac{7547}{4}, \frac{7547}{4}, \frac{7547}{4}\right\}$

```
{D2a[1000, 6, 1], D2a[1000, 3, 2], D2a[1000, 2, 3], D2a[1000, 1, 6]}
```

{5048, 5048, 5048, 5048}

```
pz[2^4 × 3 × 5 × 7, 2, 3]
```

180

```
pz[2^4 × 3 × 5 × 7, 1, 6]
```

180

```
pp[2^4 × 3 × 5 × 7, 6]
```

180

```
Sum[ pp[j, 3] pp[k, 3] pp[l, 3], {j, 2, 1000}, {k, 2, 1000 / j}, {l, 2, 1000 / (j k)}]
```

10

```
P[1000, 9]
```

10

```
mm[n_] := MoebiusMu[n]
1 - Sum[ mm[j], {j, 2, 30}] + Sum[ mm[j] mm[k], {j, 2, 30}, {k, 2, 30 / j}] -
 Sum[ mm[j] mm[k] mm[l], {j, 2, 30}, {k, 2, 30 / j}, {l, 2, 30 / (j k)}] +
 Sum[ mm[j] mm[k] mm[l] mm[m], {j, 2, 30}, {k, 2, 30 / j}, {l, 2, 30 / (j k)}, {m, 2, 30 / (j k l)}] -
 Sum[ mm[j] mm[k] mm[l] mm[m] mm[o], {j, 2, 30}, {k, 2, 30 / j},
  {l, 2, 30 / (j k)}, {m, 2, 30 / (j k l)}, {o, 2, 30 / (j k l m)}]
```

30

```
mm[n_] := 1 - Sum[ MoebiusMu[j] mm[n / j], {j, 2, n}]
m2[n_, k_] := k - Sum[ MoebiusMu[j] m2[n / j, k + 1], {j, 2, n}]
d2[n_, k_] := k - Sum[ dz[j, 2] d2[n / j, k + 1], {j, 2, n}]
mert[n_] := 1 - Sum[ mert[n / j], {j, 2, n}]
```

```
mm[100]
```

100

```
mert[100]
```

1

```
dd[100, 2, 1 / 2]
```

100

```
1 + 2 Sum[ dz[j, 1 / 2], {j, 2, 100}] + Sum[ dz[j, 1 / 2] dz[k, 1 / 2], {j, 2, 100}, {k, 2, 100 / j}]
```

100

```
1 + 2 Sum[ dz[j, 3 / 2], {j, 2, 100}] + Sum[ dz[j, 3 / 2] dz[k, 3 / 2], {j, 2, 100}, {k, 2, 100 / j}]
```

1471

```
2 * (1 / 2)
```

1

```
2 * 1 / 2
```

1

```
Series[ (x + 1) ^ -2, {x, 0, 20}]
```

$1 - 2x + 3x^2 - 4x^3 + 5x^4 - 6x^5 + 7x^6 - 8x^7 + 9x^8 - 10x^9 + 11x^{10} - 12x^{11} + 13x^{12} - 14x^{13} + 15x^{14} - 16x^{15} + 17x^{16} - 18x^{17} + 19x^{18} - 20x^{19} + 21x^{20} + O[x]^{21}$

```
m2[100, 1]
```

482

```
SeriesCoefficient[(x + 1) ^ (1 / 2), {x, 0, 0}]
```

1

```
dx[n_, z_, y_, k_] :=
 SeriesCoefficient[(x + 1) ^ (z), {x, 0, k}] + Sum[ dz[j, y] dx[n / j, z, y, k + 1], {j, 2, n}]
```

```
dx[100, 1 / 2, 2, 0]
```

100

```
Series[(x + 1) ^ (-1), {x, 0, 20}]
```

$1 - x + x^2 - x^3 + x^4 - x^5 + x^6 - x^7 + x^8 - x^9 + x^{10} - x^{11} + x^{12} - x^{13} + x^{14} - x^{15} + x^{16} - x^{17} + x^{18} - x^{19} + x^{20} + O[x]^{21}$

```
2 * 3 / 2
```

3

```
Sum[dz[j, 3], {j, 1, 100}]
```

1471

```
Sum[ dz[k, 2], {j, 1, 100}, {k, 1, 100 / j}]
```

1471

```
Sum[ dz[j, 1 / 2] dz[k, 1 / 2], {j, 1, 100}, {k, 1, 100 / j}]
```

100

```
Sum[ dz[j, 1 / 3] dz[k, 1 / 3] dz[l, 1 / 3], {j, 1, 100}, {k, 1, 100 / j}, {l, 1, 100 / (j k)}]
```

100

```
Sum[ dz[j, 3] dz[k, 1], {j, 1, 100}, {k, 1, 100 / j}]
```

3575

```
Sum[ (-1)^k (k + 1) (x^-1 - 1)^k, {k, 0, Infinity}]
```

x^2

```
Expand@p1z[210, z]
```

$-6 z + 11 z^2 - 6 z^3 + z^4$

```
Table[D[Expand@dz[210, z], {z, k}], {k, 0, 5}]
```

$\{z^4, 4 z^3, 12 z^2, 24 z, 24, 0\}$

```
dz[210, z]
```

z^4

```
P1zz[100, 2, 1]
```

$\frac{26561}{180}$

```
P1x[100, 2, 1 / 2] - 1
```

$\frac{428}{15}$

```
Expand[(x^2 - 1)^2]
```

$1 - 2 x^2 + x^4$

```
Sum[ ((Log[x] + 1)^-1 - 1)^k (-1)^k , {k, 0, Infinity}]
```

1 + Log[x]

```
Sum[ Binomial[z, k] (x - 2)^k, {k, 0, Infinity}]
```

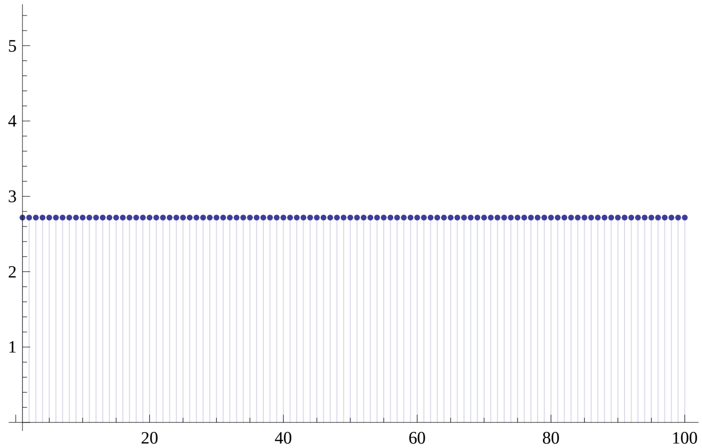
$(-1 + x)^z$

```
Clear[d3]
d3[n_, k_] := d3[n, k] = Sum[ d3[n / j, k - 1], {j, 3, n}]
d3[n_, 0] := UnitStep[n - 1]
d2z[n_, z_] := Sum[ bin[z, k] d3[n / 2^(z - k), k], {k, 0, 10}]
```

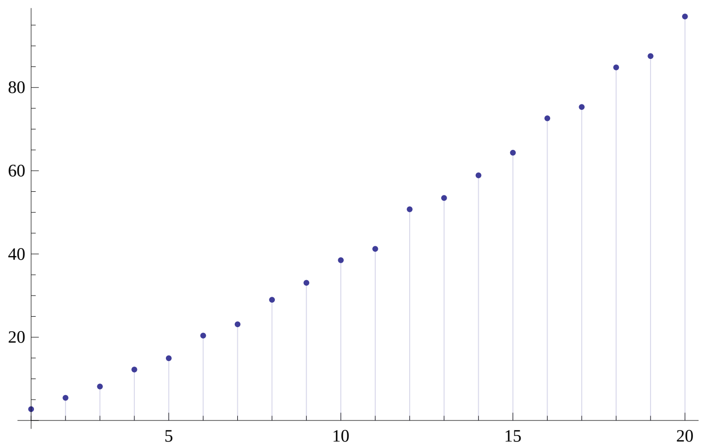
```
b2[100, 1] + b2[100, 0]

49

2 b2[50, 1] + 1

97

b2[100 / 4, 2] + 2 b2[100 / 4, 1] + 1

108

b2[100, 2]

186

b2[100, 2]

186

d2z[100, 1]

99

Table[ bin[1.5, k] d3[100 / 2^ (1.5 - k), k], {k, 0, 12}]

{1, 102., 114.375, -38.5, 16.2188, -5.70703, 1.41504, -0.188965, 0.0271912, 0., 0., 0., 0.}

Table[ bin[2.5, k] d3[100 / 2^ (2.5 - k), k], {k, 0, 12}]

{1, 82.5, 198.75, 45.9375, -4.14063, 0.539063, -0.0341797, 0.00244141, 0., 0., 0., 0., 0.}

Table[ bin[3.5, k] d3[100 / 2^ (3.5 - k), k], {k, 0, 12}]

{1, 52.5, 135.625, 50.3125, 2.46094, -0.0273438, 0., 0., 0., 0., 0., 0., 0.}

Table[ bin[3, k] d3[100 / 2^ (3 - k), k], {k, 0, 12}]

{1, 69, 183, 71, 0, 0, 0, 0, 0, 0, 0, 0, 0}

Table[ bin[3.3, k] d3[100 / 2^ (3.3 - k), k], {k, 0, 12}]

{1, 59.4, 159.39, 67.4245, 2.34341, -0.103603, 0., 0., 0., 0., 0., 0., 0.}

Table[ bin[4.3, k] d3[100 / 2^ (4.3 - k), k], {k, 0, 12}]

{1, 34.4, 70.95, 21.758, 1.76784, 0., 0., 0., 0., 0., 0., 0., 0.}

Table[ bin[.5, k] d3[100 / 2^ (.5 - k), k], {k, 0, 12}]

{1, 69.5, -99.75, 132.375, -128.633, 90.0156,
 -44.9736, 15.2432, -3.10281, 0.600052, -0.00927353, 0., 0.}

Table[Sum[ bin[z, k] d3[16 / 2^ (z - k), k], {k, 0, 12}], {z, .5, 4.5, .1}]

{6.95313, 9.01069, 10.1273, 11.6913, 13.5019, 15., 15.1797, 16.136, 16.8793, 18.9424,
 18.5859, 19.3264, 17.422, 19.168, 17.5015, 19., 15.0035, 13.848, 14.685, 15.64, 10.375,
 10.88, 8.695, 9.12, 6.8, 7., 4.1, 4.2, 4.3, 4.4, 1., 1., 1., 1., 1., 1., 0., 0., 0., 0., 0.}


Table[ bin[1.5, k] dd3[100 / 2^ (1.5 - k), k], {k, 0, 12}]

{dd3[35.3553, 0], 1.5 dd3[70.7107, 1], 0.375 dd3[141.421, 2], -0.0625 dd3[282.843, 3],
 0.0234375 dd3[565.685, 4], -0.0117188 dd3[1131.37, 5], 0.00683594 dd3[2262.74, 6],
 -0.00439453 dd3[4525.48, 7], 0.00302124 dd3[9050.97, 8], -0.00218201 dd3[18101.9, 9],
 0.00163651 dd3[36203.9, 10], -0.00126457 dd3[72407.7, 11], 0.00100112 dd3[144815., 12]}
```

```
Table[ bin[4.5, k] dd3[100 / 2^(4.5 - k), k], {k, 0, 12}]

{dd3[4.41942, 0], 4.5 dd3[8.83883, 1], 7.875 dd3[17.6777, 2],
 6.5625 dd3[35.3553, 3], 2.46094 dd3[70.7107, 4], 0.246094 dd3[141.421, 5],
 -0.0205078 dd3[282.843, 6], 0.00439453 dd3[565.685, 7], -0.00137329 dd3[1131.37, 8],
 0.000534058 dd3[2262.74, 9], -0.000240326 dd3[4525.48, 10],
 0.000120163 dd3[9050.97, 11], -0.0000650883 dd3[18101.9, 12]}

Table[ bin[.01, k] dd3[100 / 2^(.01 - k), k], {k, 0, 20}]

{dd3[99.3092, 0], 0.01 dd3[198.618, 1], -0.00495 dd3[397.237, 2], 0.0032835 dd3[794.474, 3],
 -0.00245442 dd3[1588.95, 4], 0.00195862 dd3[3177.9, 5], -0.00162892 dd3[6355.79, 6],
 0.00139389 dd3[12711.6, 7], -0.00121791 dd3[25423.2, 8], 0.00108124 dd3[50846.3, 9],
 -0.000972031 dd3[101693., 10], 0.000882781 dd3[203385., 11], -0.000808481 dd3[406771., 12],
 0.000745668 dd3[813541., 13], -0.000691873 dd3[1.62708 × 10^6, 14],
 0.000645287 dd3[3.25417 × 10^6, 15], -0.000604553 dd3[6.50833 × 10^6, 16],
 0.000568636 dd3[1.30167 × 10^7, 17], -0.000536729 dd3[2.60333 × 10^7, 18],
 0.000508198 dd3[5.20666 × 10^7, 19], -0.000482534 dd3[1.04133 × 10^8, 20]}


d3[1000, 6]

7

Floor[(1000 / 2^(4.5 - 10)) / 3^10]

0

If (n / 2^(z - k)) / 3^k ≥ 1, then keep a - goin'


Log[(a + 1) / a, n / a^z] /. {n → 100, z → 4.5, a → 3}

-1.17694

d3[100, 4]

1

Clear[da]
da[n_, k_, a_] := da[n, k, a] = Sum[ da[n / j, k - 1, a], {j, a, n}]
da[n_, 0, a_] := UnitStep[n - 1]
dap1z[n_, z_, a_] := Sum[ bin[z, k] da[n / a^(z - k), k, a + 1], {k, 0, 10}]
dap1zt[n_, z_, a_] := Table[ bin[z, k] da[n / a^(z - k), k, a + 1], {k, 0, 11}]

dap1zt[20, .5, 3]

{1, 15.5, -15.875, 15.75, -11.1719, 5.63281, -1.8457, 0.241699, -0.013092, 0., 0., 0.}

dap1zt[20, 1.5, 3]

{1, 12., 4.5, -0.625, 0.0234375, 0., 0., 0., 0., 0., 0., 0.}

dap1zt[20, -.5, 3]

$Aborted
```

```
DiscretePlot[Ez[n, 30] / E2z[n], {n, 1, 100}]
```




```
DiscretePlot[Ez[n, 30], {n, 1, 20}]
```




```
N@Ez[1, 30]
```

2.71828

```
E2z[5]
```

$$\frac{11}{2}$$