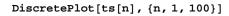
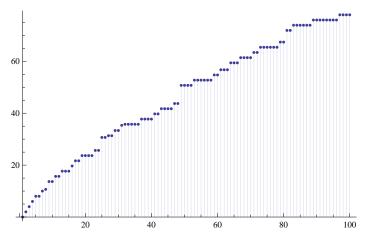
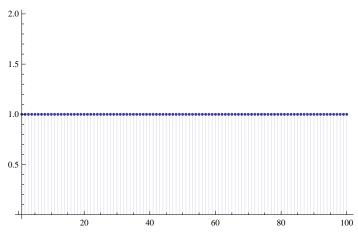
```
GCD[1000, 26]
2
Clear[gg, pp]
gg[n_{,d_{,k_{,j}}}] := gg[n,d,k] = Sum[GCD[j,d](1/k - gg[Floor[n/j],d,k+1]), \{j,2,n\}]
pp[n_{,k_{]}} := pp[n,k] = Sum[1/k-pp[Floor[n/j],k+1], \{j, 2, n\}]
DiscretePlot[gg[n, 3, 1] - pp[n, 1], \{n, 1, 100\}]
            ••••••
                                          100
Table[gg[n, 3, 1] - pp[n, 1], \{n, 1, 100\}]
                                                            8 8 8 8 8
8 8 8 8 8 8 8 8
                         8
                            8
                               8
                                  8
                                     8
                                        8
         8 8 8 8
                    8 8 8
                            8
                               8 8 8
                                        8
                                          8 8
 4
                 4
                    4
                            4
                                4
                                   4
                                        4
                                                4
            3
                                               3
Sum[GCD[j, k], {j, 1, 100}, {k, 1, 100 / j}]
725
Clear[g1]
g1[n_{,k_{-}}] := g1[n,k] = Sum[GCD[a,b]g1[Floor[n/(ab)],k-1],{a,1,n},{b,1,n/a}]
g1[n_, 0] := UnitStep[n-1]
g2[n_{,k_{]}} := Sum[(-1)^{(k-j)} Binomial[k, j] g1[n, j], {j, 0, k}]
lg[n_{-}] := Sum[(-1)^{(k+1)}/kg2[n,k], \{k, 1, Log2@n\}]
kk[n] := kk[n] = FullSimplify[MangoldtLambda[n] / Log[n]]
pr[n_{,s_{]}} := Sum[kk[j] j^s, {j, 2, n}]
ts[n_] := 2 pr[n, 0] + pr[n^(1/2), 1] - pr[n^(1/2), 0]
Table[\{n, \lg[2^n] - \lg[2^n - 1], \lg[3^n] - \lg[3^n - 1],
  lg[5^n] - lg[5^n - 1], lg[7^n] - lg[7^n - 1], {n, 1, 6}] // TableForm
$Aborted
```

```
\label{lem:table condition} Table[\{2^n, \ \lg[2^n] - \lg[2^n - 1] - 2\,kk[2^n]\}, \ \{n, 1, 10\}] \ // \ TableForm
4
            1
8
             0
16
32
             0
64
128
256
512
             0
            31
5
1024
Table \ [\{3^n,\ lg[3^n]-lg[3^n-1]-2\ kk[2^n]\},\ \{n,1,7\}]\ //\ Table Form
3
9
             2
27
81
             4
             0
243
             26
3
729
2187
Table[\{5^n, \ lg[5^n] - lg[5^n - 1]\}, \{n, 1, 6\}] \ // \ TableForm
25
               5
               \frac{\frac{2}{3}}{\frac{25}{2}}
125
625
3125
               125
15 625
Table \, [\, \{\, 7\,^{\wedge} n\,,\,\, \lg \, [\, 7\,^{\wedge} n\,]\,\, -\, \lg \, [\, 7\,^{\wedge} n\,-\, 1\,]\,\, \}\,,\,\, \{\, n\,,\,\, 1\,,\,\, 4\,\}\,] \,\,\, //\,\,\, Table \, Form
7
49
             7
             <u>2</u>
3
343
2401
```

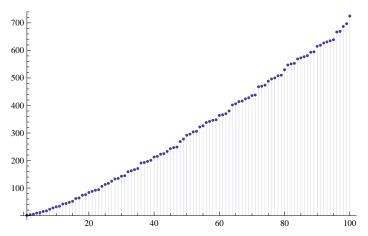




$\texttt{DiscretePlot[lg[n]-ts[n]+1, \{n, 1, 100\}]}$

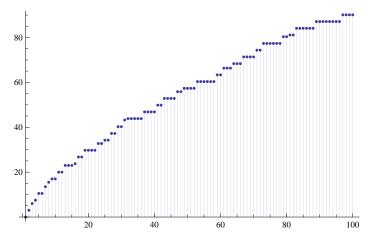


$DiscretePlot[g1[n, 1], \{n, 1, 100\}]$

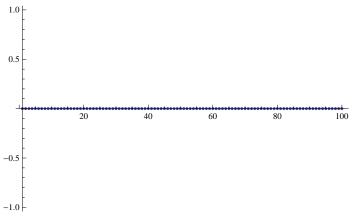


```
gh[n_] := Sum[1 \times 1 \mid MoebiusMu[m], \{j, 1, n\},
  \{k, 1, n/j\}, \{1, 1, (n/(jk))^(1/2)\}, \{m, 1, (n/(jk1^2))^(1/2)\}]
bin[z_{-}, k_{-}] := Product[z - j, {j, 0, k - 1}] / k!
FI[n_] := FactorInteger[n]; FI[1] := {}
dz[n_{-}, z_{-}] := dz[n, z] = Product[(-1)^p[[2]] Binomial[-z, p[[2]]], {p, FI[n]}]
gh2[n_] := Sum[dz[j, 2] l MoebiusMu[m],
  {j, 1, n}, {1, 1, (n/(j))^(1/2)}, {m, 1, (n/(j1^2))^(1/2)}
gh3[n_{\_}] := Sum[Abs[MoebiusMu[j]]1, \{j, 1, n\}, \{k, 1, n/j\}, \{l, 1, (n/(jk))^{(1/2)}\}]
gh4[n_] := Sum[dz[j, 2] EulerPhi[1], {j, 1, n}, {1, 1, (n/(j))^(1/2)}]
gh5[n_] := Sum[dz[j, 2] EulerPhi[l], \{1, 1, n^(1/2)\}, \{j, 1, n/1^2\}]
DiscretePlot[gh4[n], {n, 1, 100}]
700
600
500
400
300
200
100
                                                     100
DiscretePlot[g1[n, 1] - gh[n], {n, 1, 100}]
 1.0
0.5
                      40
                                                     100
-0.5
-1.0
Clear[g1]
g1[n_{,k]} := g1[n,k] =
  Sum[GCD[a, b, c]g1[Floor[n/(abc)], k-1], \{a, 1, n\}, \{b, 1, n/a\}, \{c, 1, n/(ab)\}]
g1[n_, 0] := UnitStep[n - 1]
g2[n_{,k_{]}} := Sum[(-1)^{(k-j)} Binomial[k, j] g1[n, j], {j, 0, k}]
lg[n_{-}] := Sum[(-1)^{(k+1)}/kg2[n,k], \{k, 1, Log2@n\}]
kk[n_{-}] := kk[n] = FullSimplify[MangoldtLambda[n] / Log[n]]
pr[n_, s_] := Sum[kk[j] j^s, {j, 2, n}]
ts[n_] := 3 pr[n, 0] + pr[n^(1/3), 1] - pr[n^(1/3), 0]
```

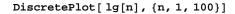
DiscretePlot[lg[n], {n, 1, 100}]

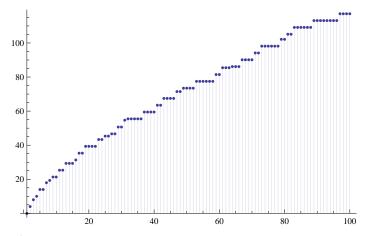


$DiscretePlot[lg[n] - ts[n], \{n, 1, 100\}]$

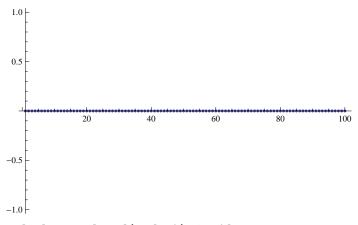


Clear[g1]





 $\texttt{DiscretePlot}[\ \lg[n]\ -\ \mathsf{ts}[n]\ ,\ \{n,\ 1,\ 100\}]$



```
\begin{split} & gg[n_{\_}] := Sum[\ GCD[j,n], \{j,1,n\}] \\ & gb[n_{\_}] := Sum[\ gg[j], \{j,1,n\}] \\ & ga[n_{\_}] := Sum[\ jk\ MoebiusMu[l], \{j,1,n\}, \{k,1,n/j\}, \ \{l,1,n/\ (jk)\}] \\ & gc[n_{\_}] := Sum[\ jEulerPhi[k], \{j,1,n\}, \{k,1,n/j\}] \end{split}
```

gb[100]

18 065

ga[100]

18 065

gc[100]

18065