```
N@Sin[3 I + Pi / 2]
```

10.0677

```
N@Cos[3 I]
```

10.0677

```
bra7b[n_, x_] := Sum[ (1 / j) ^ (1 / 2) (2 x Cos[ x Log[n / j]] - Sin[x Log[n / j]]), {j, 1, n}]
bra7c[n_, x_] :=
 ((2 x Sin[ x Log[n]] + Cos[x Log[n]]) Sum[ (j) ^ (-1 / 2) Sin[x Log[j]], {j, 1, n}] +
   (2 x Cos[ x Log[n]] - Sin[x Log[n]]) Sum[ (j) ^ (-1 / 2) Cos[x Log[j]], {j, 1, n}])
bra8c[n_, x_] := (( x Sin[ x Log[n]] + (1 / 2) Cos[x Log[n]]) /
      ( x Cos[ x Log[n]] - (1 / 2) Sin[x Log[n]])
    Sum[ j ^ (-1 / 2) Sin[x Log[j]], {j, 1, n}] + Sum[ j ^ (-1 / 2) Cos[x Log[j]], {j, 1, n}])
bra8d[n_, x_] := Tan[x Log[n] + ArcCot[2 x]] Sum[ j ^ (-1 / 2) Sin[x Log[j]], {j, 1, n}] +
  Sum[ j ^ (-1 / 2) Cos[x Log[j]], {j, 1, n}]
bra8e[n_, x_] := Sum[ j ^ (-1 / 2) (Tan[x Log[n] + ArcCot[2 x]] Sin[x Log[j]] + Cos[x Log[j]]),
  {j, 1, n}]
bra8ex[n_, x_] := Sum[ j ^ (-1 / 2) (Sin[x Log[j]] + Cos[x Log[j]]), {j, 1, n}]
fs[n_, x_] := Tan[x Log[n] + ArcCot[2 x]]
```

```
bra8e[10 000, .9 I - .5 I + 30]
```

0.344309 + 0.503637 i

```
Zeta[30 I + .9]
```

0.344439 - 0.503704 i

```
Sin[x] / Cos[x]
```

Tan[x]

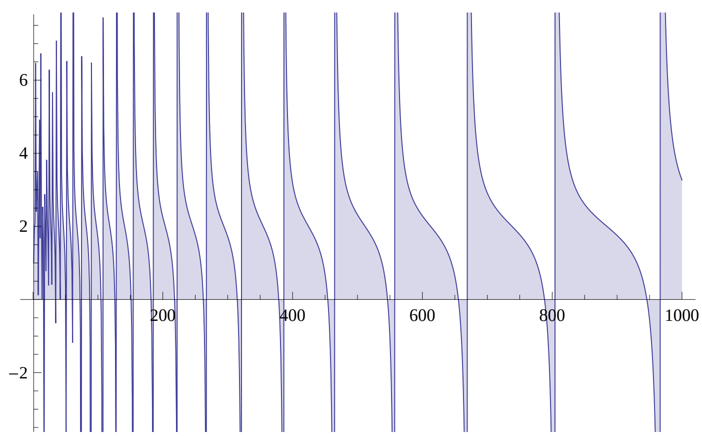
```
Cos[x] / -Sin[x]
```

-Cot[x]

```
ArcCot[2 N@Im@ZetaZero@1]
```

0.0353591

```
DiscretePlot[Re@bra8e[n, N@Im@ZetaZero@1 + 3], {n, 1, 1000}]
```




```
fs[n, N@Im@ZetaZero@3]
```

Tan[0.0199887 + 25.0109 Log[n]]

```
bra8es[n_, x_] := {Tan[x Log[n] + ArcCot[2 x]],
  Sum[ j^(-1 / 2) (Sin[x Log[j]]), {j, 1, n}], Sum[ j^(-1 / 2) (Cos[x Log[j]]), {j, 1, n}]}
```

```
bra8es[1000, 3 I - .5 I + 1000]
```

$\{-1.8733\times10^{-15}+1.\,i,\ 426052.-69231.6\,i,\ -69230.7-426052.\,i\}$

```
sl[n_, s_] := Sum[((1 - s) / s)^(1 / 2) (n / j)^s - ((1 - s) / s)^(-1 / 2) (n / j)^(1 - s), {j, 1, n}]
sl3[n_, s_] := Sum[((1 - (s + 1 / 2)) / (s + 1 / 2))^(1 / 2) (n / j)^(s + 1 / 2) -
   ((1 - (s + 1 / 2)) / (s + 1 / 2))^(-1 / 2) (n / j)^(1 - (s + 1 / 2)), {j, 1, n}]
sl4[n_, s_] := Sum[((1 / 2 - s) / (s + 1 / 2))^(1 / 2) (n / j)^(1 / 2 + s) -
   ((1 / 2 - s) / (s + 1 / 2))^(-1 / 2) (n / j)^(1 / 2 - s), {j, 1, n}]
sl5[n_, s_] := Sum[(n / j)^(1 / 2) (((1 / 2 - s) / (s + 1 / 2))^(1 / 2) (n / j)^s -
     ((1 / 2 - s) / (s + 1 / 2))^(-1 / 2) (n / j)^-s), {j, 1, n}]
sl6[n_, s_] := Sum[(n / j)^(1 / 2) (((1 / 2 - s) / (s + 1 / 2))^(1 / 2) (n / j)^s -
     (((1 / 2 - s) / (s + 1 / 2))^(1 / 2) (n / j)^s)^-1), {j, 1, n}]
```

```
sl6[100000, N@ZetaZero@10 - .5]
```

$7.59762\times10^{-14}-0.99995\,i$

```
(1 / 2 - s) / (1 / 2 + s)
```

$$\frac{\frac{1}{2}-s}{\frac{1}{2}+s}$$

```
((1 - (s + 1 / 2)) / (s + 1 / 2))
```

$$\frac{\frac{1}{2}-s}{\frac{1}{2}+s}$$

```
FullSimplify[a^1 - a^-1]
```

$$-\frac{1}{a}+a$$

```
Expand[
 (((1 / 2 - s) / (s + 1 / 2))^(1 / 2) (n / j)^s - (((1 / 2 - s) / (s + 1 / 2))^(1 / 2) (n / j)^s)^-1)]
```

$$\left(\frac{n}{j}\right)^s\sqrt{\frac{\frac{1}{2}-s}{\frac{1}{2}+s}}-\frac{\left(\frac{n}{j}\right)^{-s}\sqrt{\frac{\frac{1}{2}-s}{\frac{1}{2}+s}}}{2\left(\frac{1}{2}-s\right)}-\frac{\left(\frac{n}{j}\right)^{-s}s\sqrt{\frac{\frac{1}{2}-s}{\frac{1}{2}+s}}}{\frac{1}{2}-s}$$

```
FullSimplify[(1 / 2 - t I) / (1 / 2 + t I)^(1 / 2) - (1 / 2 - t I) / (1 / 2 + t I)^(-1 / 2)]
```

$$-\frac{(i+2t)^2}{2\sqrt{2+4\,i\,t}}$$

$$-\frac{(i+2t)^2}{2\sqrt{2+4it}} \ /.\ t \to \text{N@Im@ZetaZero@1}$$

-40.7853 + 34.1444 i

```
FullSimplify[(1/2 - t I) / (1/2 + t I)^(1/2) + (1/2 - t I) / (1/2 + t I)^(-1/2)]
```

$$\frac{3+4t(-i+t)}{2\sqrt{2+4it}} \ /.\ t \to \text{N@Im@ZetaZero@1}$$

35.7562 - 39.7375 i

```
sl7[n_, s_] := Sum[j^(-1/2) (((1/2 - s) / (s + 1/2))^(1/2) (n/j)^s -
      (((1/2 - s) / (s + 1/2))^(1/2) (n/j)^s)^-1), {j, 1, n}]
sl8[n_, s_, t_] := Sum[j^(-1/2) (((1/2 - s - t I) / (s + t I + 1/2))^(1/2) (n/j)^(s + t I) -
      (((1/2 - s - t I) / (s + t I + 1/2))^(1/2) (n/j)^(s + t I))^-1), {j, 1, n}]
sl9[n_, s_, t_] := Sum[j^(-1/2) (((1/2 - s - t I) / (s + t I + 1/2))^(1/2)
       (n/j)^s (n/j)^(t I) -
      (((1/2 - s - t I) / (s + t I + 1/2))^(-1/2) (n/j)^(-s) (n/j)^(-t I))), {j, 1, n}]
sl10[n_, s_, t_] := Sum[j^(-1/2) (((1/2 - s - t I) / (s + t I + 1/2))^(1/2) (n/j)^s
       (Cos[t Log[n/j]] + I Sin[t Log[n/j]]) - (((1/2 - s - t I) / (s + t I + 1/2))^(-1/2)
         (n/j)^(-s) (Cos[t Log[n/j]] - I Sin[t Log[n/j]]))), {j, 1, n}]
sl11[n_, s_, t_] := Sum[j^(-1/2) (((1/2 - s - t I) / (s + t I + 1/2))^(1/2)
         (n/j)^s (Cos[t Log[n/j]]) -
       (((1/2 - s - t I) / (s + t I + 1/2))^(-1/2) (n/j)^(-s) (Cos[t Log[n/j]]))), {j, 1, n}] +
   Sum[j^(-1/2) (((1/2 - s - t I) / (s + t I + 1/2))^(1/2) (n/j)^s (I Sin[t Log[n/j]]) -
       (((1/2 - s - t I) / (s + t I + 1/2))^(-1/2)
          (n/j)^(-s) (-I Sin[t Log[n/j]]))), {j, 1, n}]
ov[s_, t_] := ((1/2 - s - t I) / (s + t I + 1/2))^(1/2)
ot[s_, t_, n_, j_] := ((1/2 - s - t I) / (s + t I + 1/2))^(1/2) (n/j)^s
sl12[n_, s_, t_] := Sum[j^(-1/2)
     (ov[s, t] (n/j)^s (Cos[t Log[n/j]]) - (1/ov[s, t] (n/j)^-s (Cos[t Log[n/j]]))),
    {j, 1, n}] + Sum[j^(-1/2) (ov[s, t] (n/j)^s (I Sin[t Log[n/j]]) -
       (1/ov[s, t] (n/j)^(-s) (-I Sin[t Log[n/j]]))), {j, 1, n}]
sl13[n_, s_, t_] := Sum[j^(-1/2) (ot[s, t, n, j] (Cos[t Log[n/j]]) -
       (1/ot[s, t, n, j] (Cos[t Log[n/j]]))), {j, 1, n}] + Sum[
    j^(-1/2) (ot[s, t, n, j] (I Sin[t Log[n/j]]) - (1/ot[s, t, n, j] (-I Sin[t Log[n/j]]))),
    {j, 1, n}]
sl14[n_, s_, t_] := Sum[j^(-1/2)
     (ot[s, t, n, j] Cos[t Log[n/j]] - (1/ot[s, t, n, j] Cos[t Log[n/j]])), {j, 1, n}] +
   I Sum[j^(-1/2) (ot[s, t, n, j] Sin[t Log[n/j]] + (1/ot[s, t, n, j] Sin[t Log[n/j]])),
     {j, 1, n}]
sl15[n_, s_, t_] := Sum[j^(-1/2) ((ot[s, t, n, j] - 1/ot[s, t, n, j]) Cos[t Log[n/j]]), {j,
     1, n}] + I Sum[j^(-1/2) ((ot[s, t, n, j] + 1/ot[s, t, n, j]) Sin[t Log[n/j]]), {j, 1, n}]
sl16[n_, s_, t_] := Sum[j^(-1/2) ((ot[s, t, n, j] - 1/ot[s, t, n, j]) Cos[t Log[n/j]] +
      I ((ot[s, t, n, j] + 1/ot[s, t, n, j]) Sin[t Log[n/j]])), {j, 1, n}]
sl16a[n_, s_, t_] := Sum[j^(-1/2) ((ot[s, t, n, j] - 1/ot[s, t, n, j]) Cos[t Log[n/j]]),
   {j, 1, n}]
sl16b[n_, s_, t_] := Sum[j^(-1/2) (I ((ot[s, t, n, j] + 1/ot[s, t, n, j]) Sin[t Log[n/j]])),
   {j, 1, n}]
```

```
sl16[10 000, .1, N@Im@ZetaZero@4]
```

-0.0417406 + 0.234708 i

```
bb[s_, t_] := (1 / 2 - s - t I) / (1 / 2 + s + t I) ^ (1 / 2)
bb2[s_, t_] := (1 / 2 - s - t I) / (1 / 2 + s + t I) ^ (-1 / 2)
```

```
N[bb[0, 30] - bb2[0, 30]]
```

-120.953 + 111.272 i

```
N@2^(1 / 12)
```

1.05946

```
bo[n_, s_] :=
  (n^(1 - s)) / ((1 - s) n^s HarmonicNumber[n, s] - s n^(1 - s) HarmonicNumber[n, 1 - s])
```

```
bo[10 000 000, .7]
```

-0.00190374

```
Zeta[.7]
```

-2.77839

```
((1 / 2 - s - t I) / (s + t I + 1 / 2)) ^ (1 / 2) (n / j) ^s /. s → 0 /. t → N@Im@ZetaZero@1
```

0.0353518 - 0.999375 i

```
ot[0, N@Im@ZetaZero@1, 1, 1000] - 1 / ot[0, N@Im@ZetaZero@1, 1, 1000]
```

0. - 1.99875 i

```
N[Pi / 2]
```

1.5708

```
FullSimplify[((1 / 2 + s + t I) / (1 / 2 - s - t I)) ^ (-1 / 2)]
```

$$\frac{1}{\sqrt{-1+\frac{2}{1-2\,s-2\,i\,t}}}$$

```
ot[s_, t_, n_, j_] := ((1 / 2 - s - t I) / (s + t I + 1 / 2)) ^ (1 / 2) (n / j) ^s
```

$$\texttt{oxa[s\_, t\_, n\_, j\_] :=} \left(\frac{n}{j}\right)^s \sqrt{-1+\frac{2}{1+2\,s+2\,i\,t}}$$

$$\texttt{ox[s\_, t\_, n\_, j\_] :=} \left(\frac{n}{j}\right)^s \sqrt{-1+\frac{2\,(1+2\,s\;-\;2\,I\,t)}{(1+2\,s)^2+4\,t^2}}$$

```
sl16[n_, s_, t_] := Sum[j^(-1 / 2) ((ot[s, t, n, j] - 1 / ot[s, t, n, j]) Cos[t Log[n / j]] +
      I ((ot[s, t, n, j] + 1 / ot[s, t, n, j]) Sin[t Log[n / j]])), {j, 1, n}]
sl17[n_, s_, t_] := Sum[j^(-1 / 2) ((ox[s, t, n, j] - 1 / ox[s, t, n, j]) Cos[t Log[n / j]] +
      I ((ox[s, t, n, j] + 1 / ox[s, t, n, j]) Sin[t Log[n / j]])), {j, 1, n}]
```

```
sl17[10 000, 0, N@Im@ZetaZero@4]
```

0. - 0.00999865 i

```
sl16[10 000, 0, N@Im@ZetaZero@4]
```

$-3.64808\times10^{-16}-0.00999865\,i$

```
FullSimplify[((1 / 2 - s - t I) / (s + t I + 1 / 2)) ^ (1 / 2) (n / j) ^ s]
```

$$\left(\frac{n}{j}\right)^s\sqrt{-1+\frac{2}{1+2s+2\,i\,t}}$$

```
FullSimplify[Expand[(1 + 2 s + 2 I t) (1 + 2 s - 2 I t)]]
```

$(1+2s)^2+4t^2$

$$\text{FullSimplify}\left[\left(\frac{n}{j}\right)^s\sqrt{-1+\frac{2\,(1+2s-2\,I\,t)}{(1+2s)^2+4t^2}}\right]$$

$$\left(\frac{n}{j}\right)^s\sqrt{-1+\frac{2}{1+2s+2\,i\,t}}$$

```
FullSimplify[2 (1 + 2 s - 2 I t)]
```

$2+4s-4\,i\,t$

```
pl[n_, x_] := Sum[ j ^ (-1 / 2) (Cos[x Log[n] + ArcCot[2 x]] Cos[x Log[j]] +
      Sin[x Log[n] + ArcCot[2 x]] Sin[x Log[j]]), {j, 1, n}]
plb[n_, x_] := Sum[ j ^ (-1 / 2) (Cos[x Log[j]] + Tan[x Log[n] + ArcCot[2 x]] Sin[x Log[j]]),
   {j, 1, n}]
plc[n_, x_] := Sum[ j ^ (-1 / 2) (Cos[x Log[n] + ArcCot[2 x]] Cos[x Log[j]] +
      Sin[x Log[n] + ArcCot[2 x]] Sin[x Log[j]]), {j, 1, n}]
pld[n_, x_] := Cos[x Log[n] + ArcCot[2 x]] Sum[ j ^ (-1 / 2) Cos[x Log[j]], {j, 1, n}] +
   Sin[x Log[n] + ArcCot[2 x]] Sum[ j ^ (-1 / 2) Sin[x Log[j]], {j, 1, n}]
pldr[n_, x_, c_] := Cos[x Log[n] + ArcCot[2 x] + c] Sum[ j ^ (-1 / 2) Cos[x Log[j] + c], {j, 1, n}] +
   Sin[x Log[n] + ArcCot[2 x] + c] Sum[ j ^ (-1 / 2) Sin[x Log[j] + c], {j, 1, n}]
pldx[n_, x_] := (1 / 2) (E^ (I (x Log[n] + ArcCot[2 x])) + E^ (-I (x Log[n] + ArcCot[2 x])))
    Sum[ j ^ (-1 / 2) ((1 / 2) (E^ (I (x Log[j])) + E^ (-I (x Log[j])))), {j, 1, n}] +
   Sin[x Log[n] + ArcCot[2 x]] Sum[ j ^ (-1 / 2) Sin[x Log[j]], {j, 1, n}]
pldx2[n_, x_] := Sum[ j ^ (-1 / 2) (1 / 2)
     (E^ (I (x Log[n] + ArcCot[2 x])) + E^ (-I (x Log[n] + ArcCot[2 x])))
     ((1 / 2) (E^ (I (x Log[j])) + E^ (-I (x Log[j])))), {j, 1, n}] +
   Sin[x Log[n] + ArcCot[2 x]] Sum[ j ^ (-1 / 2) Sin[x Log[j]], {j, 1, n}]
pldr2[n_, x_, c_, d_] := Cos[x Log[n] + ArcCot[2 x] + c]
    Sum[ j ^ (-1 / 2) Cos[x Log[j] + c], {j, 1, n}] +
   Sin[x Log[n] + ArcCot[2 x] + d] Sum[ j ^ (-1 / 2) Sin[x Log[j] + d], {j, 1, n}]
pldr2a[n_, x_, c_, d_] := Cos[x Log[n] + ArcCot[2 x] + c]
   Sum[ j ^ (-1 / 2) Cos[x Log[j] + c], {j, 1, n}]
pldr2b[n_, x_, c_, d_] := Sin[x Log[n] + ArcCot[2 x] + d]
   Sum[ j ^ (-1 / 2) Sin[x Log[j] + d], {j, 1, n}]
```

```
pldr2[10 000, N@Im@ZetaZero@10, Pi / 2, 0]
```

0.887172

```
Cos[x Log[n] + ArcCot[2 x]] Sum[ j^(-1 / 2) Cos[x Log[j]], {j, 1, n}] +
 Sin[x Log[n] + ArcCot[2 x]] Sum[ j^(-1 / 2) Sin[x Log[j]], {j, 1, n}]
```

$$\mathrm{Cos}[\mathrm{ArcCot}[2\,x] + x\,\mathrm{Log}[n]] \sum_{j=1}^{n} \frac{\mathrm{Cos}[x\,\mathrm{Log}[j]]}{\sqrt{j}} + \mathrm{Sin}[\mathrm{ArcCot}[2\,x] + x\,\mathrm{Log}[n]] \sum_{j=1}^{n} \frac{\mathrm{Sin}[x\,\mathrm{Log}[j]]}{\sqrt{j}}$$

```
FullSimplify[j^(-1 / 2) (1 / 2) (E^(I (x Log[n] + ArcCot[2 x])) + E^(-I (x Log[n] + ArcCot[2 x])))
  ((1 / 2) (E^(I (x Log[j])) + E^(-I (x Log[j]))))]
```

$$\frac{1}{2}\, j^{-\frac{1}{2}-i\,x} \left(1 + j^{2\,i\,x}\right) \mathrm{Cos}[\mathrm{ArcCot}[2\,x] + x\,\mathrm{Log}[n]]$$

```
plc[n_, x_] := Sum[ j^(-1 / 2) (Cos[x Log[n] + ArcCot[2 x]] Cos[x Log[j]] +
      Sin[x Log[n] + ArcCot[2 x]] Sin[x Log[j]]), {j, 1, n}]
plc2[n_, x_] := Sum[ j^(-1 / 2)
    ((1 / 2) (Cos[x Log[n] + ArcCot[2 x] + x Log[j]] + Cos[x Log[n] + ArcCot[2 x] - x Log[j]]) +
      Sin[x Log[n] + ArcCot[2 x]] Sin[x Log[j]]), {j, 1, n}]
plc3[n_, x_] := Sum[ j^(-1 / 2) ((1 / 2) (Cos[x Log[n] + ArcCot[2 x] + x Log[j]] +
           Cos[x Log[n] + ArcCot[2 x] - x Log[j]]) + ((1 / 2) (Cos[x Log[n] + ArcCot[2 x] - x Log[j]] -
            Cos[x Log[n] + ArcCot[2 x] + x Log[j]]))), {j, 1, n}]
plc4[n_, x_] := Sum[ j^(-1 / 2) Cos[ArcCot[2 x] + x (-Log[j] + Log[n])], {j, 1, n}]
plc5[n_, x_] := Sum[Cos[ArcCot[2 x] + x  Log[n / j]] / j^(1 / 2), {j, 1, n}]
plc5a[n_, x_] :=
 (1 / Cos[ArcCot[2 x] + x  Log[n]]) Sum[Cos[ArcCot[2 x] + x  Log[n / j]] / j^(1 / 2), {j, 1, n}]
```

```
plc5a[1 000 000, N@Im@ZetaZero@20]
```

-0.000719073

```
plc5[1 000 000, N@Im@ZetaZero@20]
```

0.000499989

```
FullSimplify[
 ((1 / 2) (Cos[x Log[n] + ArcCot[2 x] + x Log[j]] + Cos[x Log[n] + ArcCot[2 x] - x Log[j]]) +
   ((1 / 2) (Cos[x Log[n] + ArcCot[2 x] - x Log[j]] - Cos[x Log[n] + ArcCot[2 x] + x Log[j]])))]
```

Cos[ArcCot[2 x] + x (-Log[j] + Log[n])]

```
Zeta[.8 + 10 I]
```

$1.44628 - 0.113501\,i$

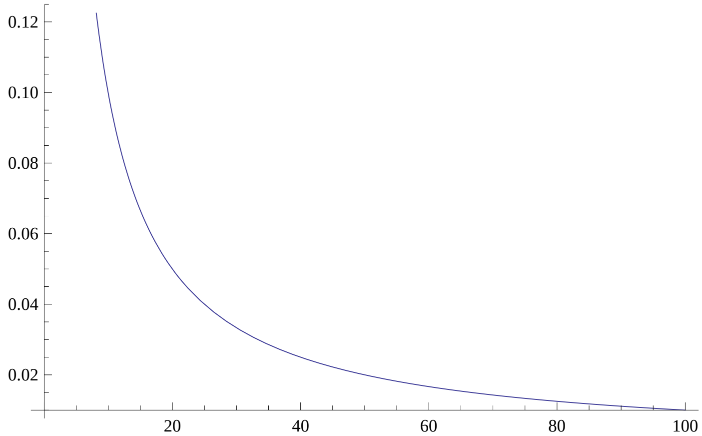
```
plc5a[1 000 000, .2 I + 20]
```

$0.554102 + 0.874923\,i$

```
Zeta[.7 + 20 I]
```

$0.557271 - 0.882999\,i$

```
Plot[Abs@ArcCot[n], {n, 0, 100}]
```



```
ArcCot[0]
```

$$\frac{\pi}{2}$$

```
plc5a[100 000, 1.5 I]
```

1.64493 + 0. i

```
FullSimplify[
 (1 / Cos[ArcCot[2 x] + x Log[n]]) Cos[ArcCot[2 x] + x Log[n / j]] / j^(1 / 2) /. x → 3 / 2 I]
```

$$\frac{\text{Cosh}\left[\text{ArcCoth}[3] - \frac{3}{2}\text{Log}\left[\frac{n}{j}\right]\right]\ \text{Sech}\left[\text{ArcCoth}[3] - \frac{3\,\text{Log}[n]}{2}\right]}{\sqrt{j}}$$

$$\text{Sum}\left[\frac{\text{Cosh}\left[\text{ArcCoth}[3] - \frac{3}{2}\text{Log}\left[\frac{n}{j}\right]\right]\ \text{Sech}\left[\text{ArcCoth}[3] - \frac{3\,\text{Log}[n]}{2}\right]}{\sqrt{j}}, \{j, 1, n\}\right]$$

$$\sum_{j=1}^{n}\frac{\text{Cosh}\left[\text{ArcCoth}[3] - \frac{3}{2}\text{Log}\left[\frac{n}{j}\right]\right]\ \text{Sech}\left[\text{ArcCoth}[3] - \frac{3\,\text{Log}[n]}{2}\right]}{\sqrt{j}}$$

```
plc5b[n_, x_] :=
 Sum[(Cos[ArcCot[2 x] + x Log[n / j]] / j^(1 / 2)) / Cos[ArcCot[2 x] + x Log[n]], {j, 1, n}]
plc5c[n_, x_] :=
 Table[(Cos[ArcCot[2 x] + x Log[n / j]] / j^(1 / 2)) / Cos[ArcCot[2 x] + x Log[n]], {j, 1, n}]
plc5d[n_, x_] := DiscretePlot[
  {Re[(Cos[ArcCot[2 x] + x Log[n / j]] / j^(1 / 2)) / Cos[ArcCot[2 x] + x Log[n]]],
   Im[(Cos[ArcCot[2 x] + x Log[n / j]] / j^(1 / 2)) / Cos[ArcCot[2 x] + x Log[n]]]}, {j, 1, n}]
plc5d2[n_, x_] := DiscretePlot[{Re[(Cos[ArcCot[2 x] + x Log[n / j]] / j^(1 / 2))],
   Im[(Cos[ArcCot[2 x] + x Log[n / j]] / j^(1 / 2))]}, {j, 1, n}]
```

```
plc5[n_, x_] := Sum[Cos[ArcCot[2 x] + x Log[n / j]] / j^(1 / 2), {j, 1, n}]
plc5w[n_, x_] := Sum[Cos[ArcCot[2 x] + x Log[n / j]] (n / j)^(1 / 2), {j, 1, n}]
plc5x[n_, x_] :=
 Sum[((1 / 2) (E^(I (x Log[n / j] + ArcCot[2 x])) + E^(-I (x Log[n / j] + ArcCot[2 x])))) /
   j^(1 / 2), {j, 1, n}]
plc5y[n_, x_] := Sum[((1 / 2) (E^(I (x Log[n / j])) E^(I ArcCot[2 x]) +
       E^(-I (x Log[n / j])) E^(-I (ArcCot[2 x])))) / j^(1 / 2), {j, 1, n}]
plc5z[n_, x_] := Sum[((n / j)^(1 / 2)) ((1 / 2) (E^(I (x Log[n / j])) E^(I ArcCot[2 x]) +
       E^(-I (x Log[n / j])) E^(-I (ArcCot[2 x])))), {j, 1, n}]
plc5z[n_, x_] := Sum[((n / j)^(1 / 2)) ((1 / 2) (E^(I (x Log[n / j])) E^(I ArcCot[2 x]) +
       E^(-I (x Log[n / j])) E^(-I (ArcCot[2 x])))), {j, 1, n}]
```

$$\text{plc5z2[n\_, x\_] := Sum}\left[\frac{1}{2} e^{-i\,\text{ArcCot}[2x]} \left(\frac{n}{j}\right)^{\frac{1}{2}-i x} + \frac{1}{2} e^{i\,\text{ArcCot}[2x]} \left(\frac{n}{j}\right)^{\frac{1}{2}+i x}, \{j, 1, n\}\right]$$

$$\text{plc5z3[n\_, x\_] := Sum}\left[\frac{1}{2\,n\text{^}(1/2)} \left(e^{-i\,\text{ArcCot}[2x]} \left(\frac{n}{j}\right)^{\frac{1}{2}-i x} + e^{i\,\text{ArcCot}[2x]} \left(\frac{n}{j}\right)^{\frac{1}{2}+i x}\right), \{j, 1, n\}\right]$$

$$\text{plc5z4[n\_, x\_] := Sum}\left[\frac{1}{2\,j\text{^}(1/2)} \left(e^{-i\,\text{ArcCot}[2x]} \left(\frac{n}{j}\right)^{-i x} + e^{i\,\text{ArcCot}[2x]} \left(\frac{n}{j}\right)^{i x}\right), \{j, 1, n\}\right]$$

$$\text{plc5z5[n\_, x\_] := Sum}\left[\frac{1}{2\,j\text{^}(1/2)} \left(e^{-i\,\text{ArcCot}[2x]} \left(\frac{n}{j}\right)^{-i x} + e^{i\,\text{ArcCot}[2x]} \left(\frac{n}{j}\right)^{i x}\right), \{j, 1, n\}\right]$$

$$\text{plc5z6[n\_, x\_] := Sum}\left[\frac{1}{2\,j\text{^}(1/2)} \left(e^{-i\,\text{ArcCot}[2x]} \left(\frac{n}{j}\right)^{-i x}\right), \{j, 1, n\}\right] + \text{Sum}\left[\frac{1}{2\,j\text{^}(1/2)} \left(e^{i\,\text{ArcCot}[2x]} \left(\frac{n}{j}\right)^{i x}\right), \{j, 1, n\}\right]$$

$$\text{plc5z7[n\_, x\_] := (1 / 2)}\left(e^{-i\,\text{ArcCot}[2x]}\, n^{-i x}\, \text{Sum}\left[\left(j^{-1/2+i x}\right), \{j, 1, n\}\right] + e^{i\,\text{ArcCot}[2x]}\, n^{i x}\, \text{Sum}\left[j^{-1/2-i x}, \{j, 1, n\}\right]\right)$$

```
plc5z7[10 000, N@Im@ZetaZero@20]
```

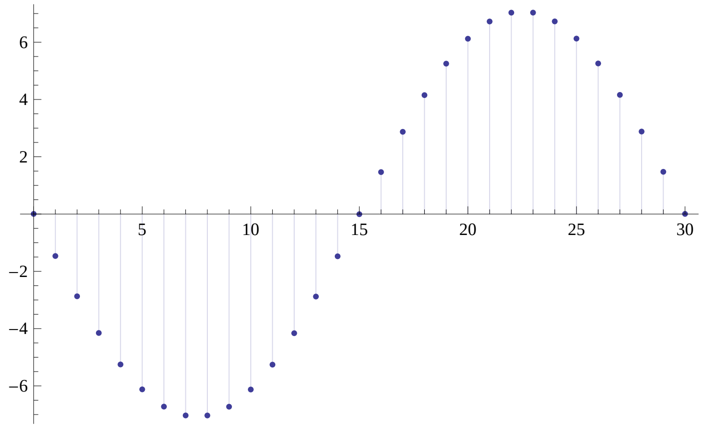
0.00499989 + 0. i

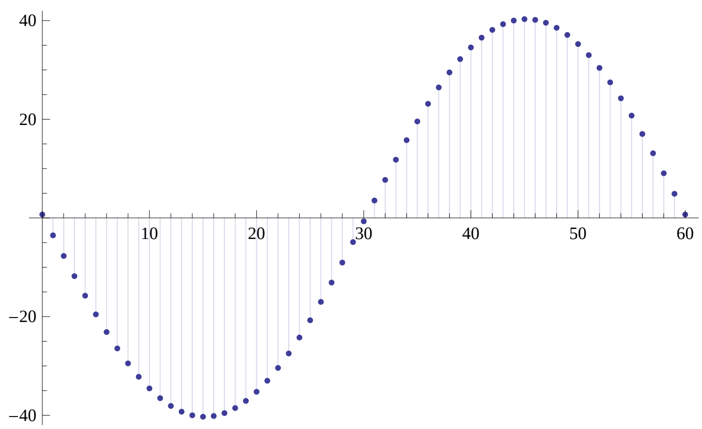
```
Expand[((n / j)^(1 / 2)) ((1 / 2)
    (E^(I (x Log[n / j])) E^(I ArcCot[2 x]) + E^(-I (x Log[n / j])) E^(-I (ArcCot[2 x]))))]
```

$$\frac{1}{2} e^{-i\,\text{ArcCot}[2x]} \left(\frac{n}{j}\right)^{\frac{1}{2}-i x} + \frac{1}{2} e^{i\,\text{ArcCot}[2x]} \left(\frac{n}{j}\right)^{\frac{1}{2}+i x}$$

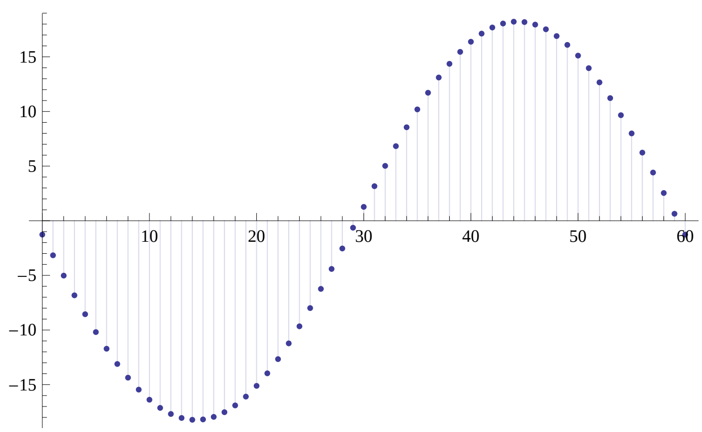
```
plc5f[n_, x_, c_] := Sum[Cos[ArcCot[2 x] + x Log[n] - x Log[j] + c] / j^(1 / 2), {j, 1, n}]
```

```
DiscretePlot[plc5f[10 000, N@Im@ZetaZero@1, j / 30 * 2 Pi], {j, 0, 30}]
```

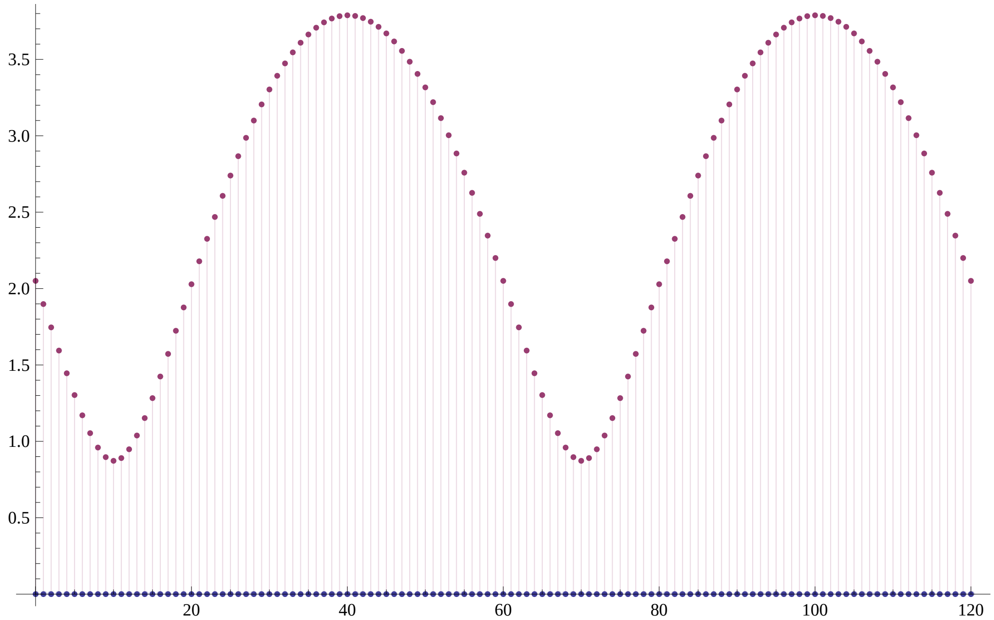





```
DiscretePlot[plc5f[30 000, 10, j / 60 * 2 Pi], {j, 0, 60}]
```



```
DiscretePlot[{0, Abs[plc5f[1000, 10 + .1 I, j / 120 * 2 Pi + .1 I]]}, {j, 0, 120}]
```



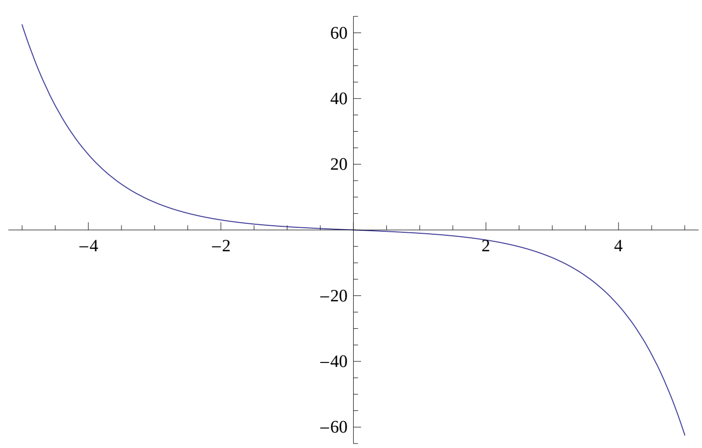
```
plc5f[2000, .1, 0]
```

1.05896

```
plc5fr[n_, x_, c_] := DiscretePlot[{Re@Cos[ArcCot[2 x] + x Log[n] - x Log[j] + c] / j^(1 / 2),
   Im@Cos[ArcCot[2 x] + x Log[n] - x Log[j] + c] / j^(1 / 2)}, {j, 1, n}]
plc5fr[10 000, N@Im@ZetaZero@4 + .3 I, 0]
```

Sum::div : Sum does not converge. ≫

```
Plot[Im[Cos[1 + x I]], {x, -5, 5}]
```



```
FullSimplify[D[Cos[ArcCot[2 x] + x Log[n] - x Log[j]] / j^(1 / 2), x]]
```

$$-\frac{\left(-\frac{2}{1+4\,x^2}-\mathrm{Log}[j]+\mathrm{Log}[n]\right)\mathrm{Sin}[\mathrm{ArcCot}[2\,x]+x\,(-\mathrm{Log}[j]+\mathrm{Log}[n])]}{\sqrt{j}}$$

```
s2[n_, x_] :=
 Sum[-1/Sqrt[j] (-2/(1 + 4 x^2) - Log[j] + Log[n]) Sin[ArcCot[2 x] + x (-Log[j] + Log[n])], {j, 1, n}]
```

$$\mathrm{Sum}\left[-\frac{1}{\sqrt{j}}\left(-\frac{2}{1+4\,x^2}-\mathrm{Log}[j]+\mathrm{Log}[n]\right)\mathrm{Sin}[\mathrm{ArcCot}[2\,x]+x\,(-\mathrm{Log}[j]+\mathrm{Log}[n])],\ \{j,1,n\}\right]$$

```
s2[10 000, N@Im@ZetaZero@3]
```

1.34591

```
ArcCot[2 x] /. x → N@Im@ZetaZero@1
```

0.0353591

```
Cos[ArcCot[2 t]]
```

$$\frac{1}{\sqrt{1+\frac{1}{4\,t^2}}}$$

```
plc5b[n_, x_] :=
 Sum[(Cos[ArcCot[2 x] + x Log[n / j]] / j^(1 / 2)) / Cos[ArcCot[2 x] + x Log[n]], {j, 1, n}]
plc5bz[n_, x_] :=
 Sum[N[(Cos[ArcCot[2 x] + x Log[n / j]] / j^(1 / 2)) / Cos[ArcCot[2 x] + x Log[n]]], {j, 1, n}]
```

```
plc5bz[100 000 000, 100.7 + .3 I]
```

1.13941 + 0.794154 i

```
Zeta[.8 + 100.7 I]
```

1.13939 - 0.794096 i

```
plc5r[n_, x_] := Sum[(n / j)^(1 / 2) (Cos[ArcCot[2 x] + x Log[n / j]]), {j, 1, n}]
plc5rf[n_, x_] := Table[(n / j)^(1 / 2) (Cos[x Log[n / j] + ArcCot[2 x]]), {j, 1, n}]
plc5rp[n_, x_, k_] := Sum[(n / j)^(1 / 2) (Cos[x Log[n / j] + ArcCot[2 x]]), {j, 1, k}]
```

```
Table[ Abs@plc5r[1000 j, N@Im@ZetaZero@1 + .1 I], {j, 1, 40}]
```

{1.69543, 4.07131, 4.26826, 6.27936, 7.42648, 6.43926, 7.09678, 9.892, 6.93413, 11.2195,
 9.09184, 11.7665, 11.8789, 9.3353, 12.9961, 14.3661, 12.1204, 12.8717, 15.889, 16.0257,
 13.4883, 12.7292, 15.6125, 18.2984, 18.495, 16.7483, 15.4597, 16.6229, 19.1843, 20.9845,
 20.9803, 19.3814, 17.4032, 16.7555, 18.2074, 20.7677, 23.019, 24.1251, 23.896, 22.6642}

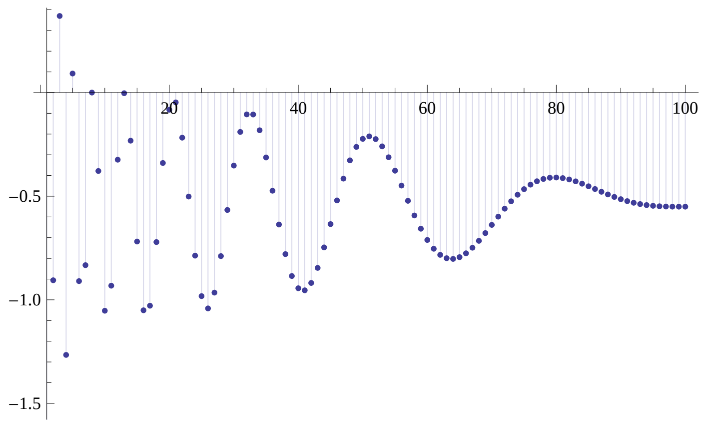
```
Table[ Abs@plc5r[1000 j, 1000 + .1 I], {j, 1, 40}]
```

{32.797, 50.6512, 67.14, 44.1667, 50.0057, 57.1352, 68.6563, 117.323, 80.3659, 131.248,
 86.9291, 140.95, 110.091, 141.078, 154.22, 145.282, 184.389, 164.341, 127.446, 171.996,
 164.79, 207.594, 183.741, 199.654, 203.698, 191.891, 192.733, 233.402, 249.151, 234.814,
 216.565, 191.492, 250.717, 207.788, 269.498, 268.63, 198.306, 287.412, 224.085, 212.268}

```
Table[ Abs@plc5r[1000 j, N@Im@ZetaZero@1], {j, 1, 40}]
```

{0.499687, 0.499687, 0.499687, 0.499687, 0.499687, 0.499687, 0.499687, 0.499687,
 0.499687, 0.499687, 0.499687, 0.499687, 0.499687, 0.499687, 0.499687, 0.499687,
 0.499687, 0.499687, 0.499687, 0.499687, 0.499687, 0.499687, 0.499687, 0.499687,
 0.499687, 0.499687, 0.499687, 0.499687, 0.499687, 0.499687, 0.499687, 0.499687,
 0.499687, 0.499687, 0.499687, 0.499687, 0.499687, 0.499687, 0.499687, 0.499687}

```
DiscretePlot[Im@plc5rp[100, N@Im@ZetaZero@1 + .1 I, j], {j, 1, 100}]
```




```
E^(I ArcCot[2 t]) /. t → .1
```

0.196116 + 0.980581 i

```
((t / I + 1 / 2) / (t / I - 1 / 2))^(1 / 2) /. t → .1
```

0.196116 + 0.980581 i

```
E^(-I ArcCot[2 t]) /. t → .1
```

0.196116 − 0.980581 i

```
((t / I - 1 / 2) / (t / I + 1 / 2))^(1 / 2) /. t → .1
```

0.196116 − 0.980581 i

```
plc5r[n_, x_] := Sum[(n / j)^(1 / 2) (Cos[ArcCot[2 x] + x Log[n / j]]), {j, 1, n}]
plc5rpo[n_, t_] := E^(I ArcCot[2 t]) / 2 n^(1 / 2 + I t) HarmonicNumber[n, 1 / 2 + I t] +
  E^(-I ArcCot[2 t]) / 2 n^(1 / 2 - I t) HarmonicNumber[n, 1 / 2 - I t]
plc5r[10 000, 15. + .1 I]
```

33.5606 − 66.4662 i

```
plc5rpo[10 000, 15. + .1 I]
```

33.5606 − 66.4662 i

```
FullSimplify[D[plc5rpo[n, t], n]]
```

$$\frac{1}{4} e^{-i \,\mathrm{ArcCot}[2 t]}\, n^{-\frac{1}{2}-i t} (1-2 i t) \left(\mathrm{HarmonicNumber}\left[n, \frac{1}{2}-i t\right] + n\,\mathrm{HurwitzZeta}\left[\frac{3}{2}-i t, 1+n\right] - n^{2 i t}\left(\mathrm{HarmonicNumber}\left[n, \frac{1}{2}+i t\right] + n\,\mathrm{HurwitzZeta}\left[\frac{3}{2}+i t, 1+n\right]\right)\right)$$

$$\text{at}[n\_, t\_] := \frac{1}{4} e^{-i\,\text{ArcCot}[2t]}\, n^{-\frac{1}{2}-i t}\, (1-2 i t) \left(\text{HarmonicNumber}\left[n, \frac{1}{2}-i t\right] + n\, \text{HurwitzZeta}\left[\frac{3}{2}-i t, 1+n\right] - n^{2 i t} \left(\text{HarmonicNumber}\left[n, \frac{1}{2}+i t\right] + n\, \text{HurwitzZeta}\left[\frac{3}{2}+i t, 1+n\right]\right)\right)$$

$$\text{at2}[n\_, t\_] := \frac{1}{4} e^{-i\,\text{ArcCot}[2t]}\, (1-2 i t) \left(n^{-\frac{1}{2}-i t} \left(\text{HarmonicNumber}\left[n, \frac{1}{2}-i t\right] + n\, \text{HurwitzZeta}\left[\frac{3}{2}-i t, 1+n\right]\right) - n^{-\frac{1}{2}+i t} \left(\text{HarmonicNumber}\left[n, \frac{1}{2}+i t\right] + n\, \text{HurwitzZeta}\left[\frac{3}{2}+i t, 1+n\right]\right)\right)$$

$$\text{at2a}[n\_, t\_] := \left(n^{-\frac{1}{2}-i t} \left(\text{HarmonicNumber}\left[n, \frac{1}{2}-i t\right] + n\, \text{HurwitzZeta}\left[\frac{3}{2}-i t, 1+n\right]\right) - n^{-\frac{1}{2}+i t} \left(\text{HarmonicNumber}\left[n, \frac{1}{2}+i t\right] + n\, \text{HurwitzZeta}\left[\frac{3}{2}+i t, 1+n\right]\right)\right)$$

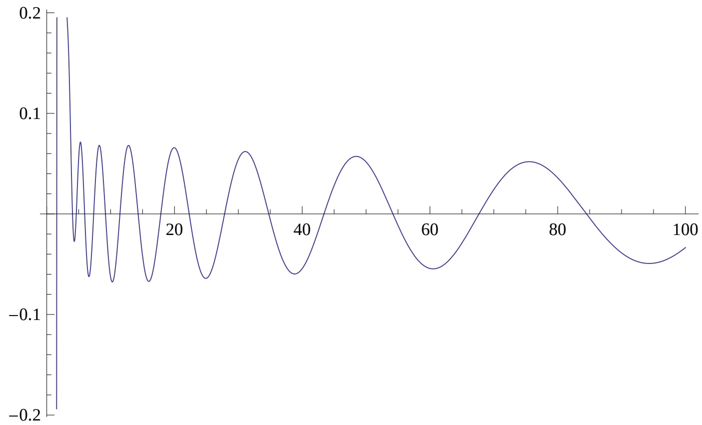
$$\text{at3}[n\_, t\_] := \left\{\frac{1}{4} e^{-i\,\text{ArcCot}[2t]},\ (1-2 i t),\ n^{-\frac{1}{2}-i t} \left(\text{HarmonicNumber}\left[n, \frac{1}{2}-i t\right] + n\, \text{HurwitzZeta}\left[\frac{3}{2}-i t, 1+n\right]\right),\ -n^{-\frac{1}{2}+i t} \left(\text{HarmonicNumber}\left[n, \frac{1}{2}+i t\right] + n\, \text{HurwitzZeta}\left[\frac{3}{2}+i t, 1+n\right]\right)\right\}$$

$$\text{at4}[n\_, t\_] := \left\{\frac{1}{4} e^{-i\,\text{ArcCot}[2t]},\ (1-2 i t),\ \left\{\left\{n^{-\frac{1}{2}-i t}, \text{HarmonicNumber}\left[n, \frac{1}{2}-i t\right], n\, \text{HurwitzZeta}\left[\frac{3}{2}-i t, 1+n\right]\right\}, \left\{-n^{-\frac{1}{2}+i t}, \text{HarmonicNumber}\left[n, \frac{1}{2}+i t\right], n\, \text{HurwitzZeta}\left[\frac{3}{2}+i t, 1+n\right]\right\}\right\}\right\}$$

$$\text{at4a}[n\_, t\_] := \left\{\frac{1}{4} e^{-i\,\text{ArcCot}[2t]},\ (1-2 i t),\ \left\{\left\{n^{-\frac{1}{2}-i t}, \text{HarmonicNumber}\left[n, \frac{1}{2}-i t\right] + n\, \text{HurwitzZeta}\left[\frac{3}{2}-i t, 1+n\right]\right\}, \left\{-n^{-\frac{1}{2}+i t}, \text{HarmonicNumber}\left[n, \frac{1}{2}+i t\right] + n\, \text{HurwitzZeta}\left[\frac{3}{2}+i t, 1+n\right]\right\}\right\}\right\}$$

$$\text{at4b}[n\_, t\_] := \left\{\frac{1}{4} e^{-i\,\text{ArcCot}[2t]},\ (1-2 i t),\ \left\{\left\{n^{-\frac{1}{2}-i t}\, \text{HarmonicNumber}\left[n, \frac{1}{2}-i t\right], n^{-\frac{1}{2}-i t}\, n\, \text{HurwitzZeta}\left[\frac{3}{2}-i t, 1+n\right]\right\}, \left\{-n^{-\frac{1}{2}+i t}\, \text{HarmonicNumber}\left[n, \frac{1}{2}+i t\right], -n^{-\frac{1}{2}+i t}\, n\, \text{HurwitzZeta}\left[\frac{3}{2}+i t, 1+n\right]\right\}\right\}\right\}$$

```
Plot[Re@at2[n, N@Im@ZetaZero@1 + .1 I], {n, 1, 100}]
```



$n^{-\frac{1}{2}-i t}\, n^{2 i t}$

$n^{-\frac{1}{2}+i t}$

```
at4a[10 000, N@Im@ZetaZero@1]
```

{0.249844 − 0.00883794 i, 1. − 28.2695 i, {{−0.00189316 + 0.00981916 i, −0.0946389 − 0.490859 i},
{0.00189316 + 0.00981916 i, −0.0946389 + 0.490859 i}}}

```
FullSimplify[D[plc5rpo[n, t], {n, 2}]]
```

$$\frac{1}{8} e^{-i \,\mathrm{ArcCot}[2 t]}\, n^{-\frac{3}{2}-i t} \left(-\left(1+4 t^2\right) \mathrm{HarmonicNumber}\left[n, \frac{1}{2}-i t\right] + (i+2 t) \left(-n^{2 i t} (i+2 t)\, \mathrm{HarmonicNumber}\left[n, \frac{1}{2}+i t\right] + n \left(-2 (i+2 t)\, \mathrm{HurwitzZeta}\left[\frac{3}{2}-i t, 1+n\right] + n (3 i+2 t)\, \mathrm{HurwitzZeta}\left[\frac{5}{2}-i t, 1+n\right] - n^{2 i t} \left((-2 i+4 t)\, \mathrm{HurwitzZeta}\left[\frac{3}{2}+i t, 1+n\right] + n (3 i-2 t)\, \mathrm{HurwitzZeta}\left[\frac{5}{2}+i t, 1+n\right]\right)\right)\right)\right)$$

```
E^(I ArcCot[2 t]) / 2 n^t HarmonicNumber[ n, t] /. n → 100 000 /. t → N@ZetaZero@1
```

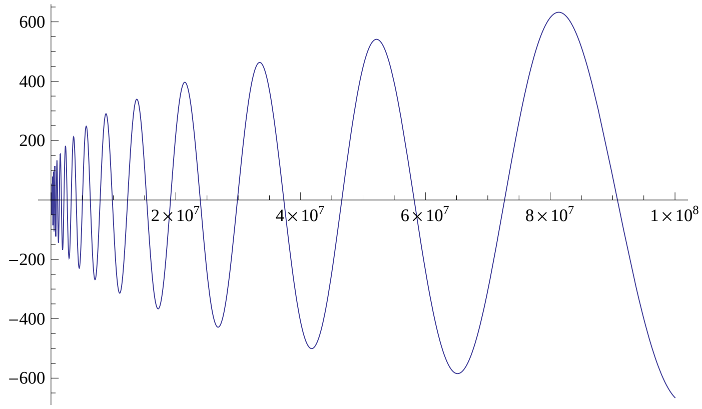
125.15 + 3660.23 i

```
plc5rpo[n_, t_] := E^(I ArcCot[2 t]) / 2 n^(1 / 2 + I t) HarmonicNumber[n, 1 / 2 + I t] +
  E^(-I ArcCot[2 t]) / 2 n^(1 / 2 - I t) HarmonicNumber[n, 1 / 2 - I t]
plc5rpos[n_, t_] := (1 / 2 - t) n^(1 / 2 + I t) HarmonicNumber[n, 1 / 2 + I t]
plc5rpos2[n_, s_] := Im[(s - 1) n^(-1 / 2 + s) HarmonicNumber[n, s]]
plc5rpos3[n_, s_] := Im[(s - 1) n^(-1 + s) HarmonicNumber[n, s]]
plc5rpos4[n_, s_] := Im[(s - 1) n^(-1 / 4 + s) HarmonicNumber[n, s]]
```

```
plc5rpos2[1 000 000 000, N@ZetaZero@1]
```

0.000223489

```
Plot[plc5rpos4[n, N@ZetaZero@1 + .1], {n, 1, 100 000 000}]
```




```
zets[n_, t_] := (E^(I ArcCot[2 t]) n^(1 / 2 + I t) HarmonicNumber[n, 1 / 2 + I t] +
    E^(-I ArcCot[2 t]) n^(1 / 2 - I t) HarmonicNumber[n, 1 / 2 - I t]) /
  ((2^(1 - (1 / 2 + t I)) Pi^-(1 / 2 + t I) Cos[ Pi (1 / 2 + t I) / 2] Gamma[(1 / 2 + t I)])
     E^(I ArcCot[2 t]) n^(1 / 2 + I t) + E^(-I ArcCot[2 t]) n^(1 / 2 - I t))
z2[n_, s_] := zets[n, Im[s] + Re[s] I - .5 I]
```

```
z2[100 000 000 000, 2.00000001]
```

1.64493 + 0. i

```
Zeta[.9]
```

−9.43011

```
FullSimplify[(E^(I ArcCot[2 t]) n^(1 / 2 + I t) HarmonicNumber[n, 1 / 2 + I t] +
    E^(-I ArcCot[2 t]) n^(1 / 2 - I t) HarmonicNumber[n, 1 / 2 - I t]) /
  ((2^(1 - (1 / 2 + t I)) Pi^-(1 / 2 + t I) Cos[ Pi (1 / 2 + t I) / 2] Gamma[(1 / 2 + t I)])
     E^(I ArcCot[2 t]) n^(1 / 2 + I t) + E^(-I ArcCot[2 t]) n^(1 / 2 - I t))]
```

$$\left(2^{i t}\pi^{\frac{1}{2}+i t}\left(\text{HarmonicNumber}\left[n,\frac{1}{2}-i t\right]+e^{2 i \,\text{ArcCot}[2 t]}n^{2 i t}\,\text{HarmonicNumber}\left[n,\frac{1}{2}+i t\right]\right)\right)\Big/$$
$$\left(2^{i t}\pi^{\frac{1}{2}+i t}+\sqrt{2}\,e^{2 i \,\text{ArcCot}[2 t]}n^{2 i t}\,\text{Cos}\left[\frac{1}{4}(\pi+2 i \pi t)\right]\text{Gamma}\left[\frac{1}{2}+i t\right]\right)$$

```
zets3[n_, t_] := (E^(1 / 2 + I ArcCot[2 t]) n^(1 / 2 + I t) HarmonicNumber[n, 1 / 2 + I t] +
    E^(1 / 2 - I ArcCot[2 t]) n^(1 / 2 - I t) HarmonicNumber[n, 1 / 2 - I t]) /
  ((2^(1 - (1 / 2 + t I)) Pi^-(1 / 2 + t I) Cos[ Pi (1 / 2 + t I) / 2] Gamma[(1 / 2 + t I)])
     E^(1 / 2 + I ArcCot[2 t]) n^(1 / 2 + I t) + E^(1 / 2 - I ArcCot[2 t]) n^(1 / 2 - I t))
z3[n_, s_] := zets3[n, Im[s] + Re[s] I - .5 I]
```

```
z3[100 000 000 000, .8 + I]
```

0.374874 + 0.886413 i

```
Zeta[.8 + I]
```

0.374874 − 0.886413 i

```
(E^(I ArcCot[2 t]) n^(1/2 + I t) HarmonicNumber[n, 1/2 + I t] +
   E^(-I ArcCot[2 t]) n^(1/2 - I t) HarmonicNumber[n, 1/2 - I t]) /
 ((2^(1 - (1/2 + t I)) Pi^-(1/2 + t I) Cos[Pi (1/2 + t I) / 2] Gamma[(1/2 + t I)])
    E^(I ArcCot[2 t]) n^(1/2 + I t) + E^(-I ArcCot[2 t]) n^(1/2 - I t))
```

$$\left(e^{-i\,\text{ArcCot}[2t]}\, n^{\frac{1}{2}-i t}\, \text{HarmonicNumber}\left[n, \frac{1}{2} - i t\right] + e^{i\,\text{ArcCot}[2t]}\, n^{\frac{1}{2}+i t}\, \text{HarmonicNumber}\left[n, \frac{1}{2} + i t\right]\right) \Big/ \left(e^{-i\,\text{ArcCot}[2t]}\, n^{\frac{1}{2}-i t} + 2^{\frac{1}{2}-i t}\, e^{i\,\text{ArcCot}[2t]}\, n^{\frac{1}{2}+i t}\, \pi^{-\frac{1}{2}-i t}\, \text{Cos}\left[\frac{1}{2}\pi\left(\frac{1}{2} + i t\right)\right] \text{Gamma}\left[\frac{1}{2} + i t\right]\right)$$

```
zets4[n_, t_] := (E^(1/2 + I ArcCot[2 t]) n^(1/2 + I t) HarmonicNumber[n, 1/2 + I t] +
    E^(1/2 - I ArcCot[2 t]) n^(1/2 - I t) HarmonicNumber[n, 1/2 - I t]) /
  ((2^(1 - (1/2 + t I)) Pi^-(1/2 + t I) Cos[Pi (1/2 + t I) / 2] Gamma[(1/2 + t I)])
     E^(1/2 + I ArcCot[2 t]) n^(1/2 + I t) + E^(1/2 - I ArcCot[2 t]) n^(1/2 - I t))
z4[n_, s_] := zets4[n, Im[s] + Re[s] I - .5 I]
```

```
z4[100 000 000 000, .8 + I]
```

0.374874 + 0.886413 i

```
(E^(1/2 + I ArcCot[2 t]) n^(1/2 + I t) HarmonicNumber[n, 1/2 + I t] +
   E^(1/2 - I ArcCot[2 t]) n^(1/2 - I t) HarmonicNumber[n, 1/2 - I t]) /
 ((2^(1 - (1/2 + t I)) Pi^-(1/2 + t I) Cos[Pi (1/2 + t I) / 2] Gamma[(1/2 + t I)])
    E^(1/2 + I ArcCot[2 t]) n^(1/2 + I t) + E^(1/2 - I ArcCot[2 t]) n^(1/2 - I t))
```

$$\left(e^{\frac{1}{2}-i\,\text{ArcCot}[2t]}\, n^{\frac{1}{2}-i t}\, \text{HarmonicNumber}\left[n, \frac{1}{2} - i t\right] + e^{\frac{1}{2}+i\,\text{ArcCot}[2t]}\, n^{\frac{1}{2}+i t}\, \text{HarmonicNumber}\left[n, \frac{1}{2} + i t\right]\right) \Big/ \left(e^{\frac{1}{2}-i\,\text{ArcCot}[2t]}\, n^{\frac{1}{2}-i t} + 2^{\frac{1}{2}-i t}\, e^{\frac{1}{2}+i\,\text{ArcCot}[2t]}\, n^{\frac{1}{2}+i t}\, \pi^{-\frac{1}{2}-i t}\, \text{Cos}\left[\frac{1}{2}\pi\left(\frac{1}{2} + i t\right)\right] \text{Gamma}\left[\frac{1}{2} + i t\right]\right)$$

```
zets5[n_, t_] := (E^(1/2 + I ArcCot[2 t]) E^((1/2 + I t) Log[n]) HarmonicNumber[n, 1/2 + I t] +
    E^(1/2 - I ArcCot[2 t]) E^((1/2 - I t) Log[n]) HarmonicNumber[n, 1/2 - I t]) /
  ((E^((1/2 - t I) Log[2]) E^((-1/2 - t I) Log[Pi]) Cos[Pi (1/2 + t I) / 2] Gamma[(1/2 + t I)])
     E^(1/2 + I ArcCot[2 t]) E^((1/2 + I t) Log[n]) +
    E^(1/2 - I ArcCot[2 t]) E^((1/2 - I t) Log[n]))
z5[n_, s_] := zets5[n, Im[s] + Re[s] I - .5 I]
```

```
z5[100 000 000 000 000 000 000, .6 + 30 I]
```

0.0222798 + 0.566553 i

```
Zeta[1.6 + 30 I]
```

0.725453 - 0.345898 i

```
1 - (1/2 + t I)
```

$$\frac{1}{2} - i t$$

```
zets6[n_, t_] := (E^(1 / 2 + I ArcCot[2 t]) E^((1 / 2 + I t) Log[n]) HarmonicNumber[n, 1 / 2 + I t] +
     E^(1 / 2 - I ArcCot[2 t]) E^((1 / 2 - I t) Log[n]) HarmonicNumber[n, 1 / 2 - I t]) /
   ((E^((1 / 2 - t I) Log[2]) E^((-1 / 2 - t I) Log[Pi])
        ((1 / 2) (E^(I (Pi (1 / 2 + t I) / 2)) + E^(-I (Pi (1 / 2 + t I) / 2)))) Gamma[(1 / 2 + t I)])
      E^(1 / 2 + I ArcCot[2 t]) E^((1 / 2 + I t) Log[n]) +
     E^(1 / 2 - I ArcCot[2 t]) E^((1 / 2 - I t) Log[n]))
z6[n_, s_] := zets6[n, Im[s] + Re[s] I - .5 I]

z6[100 000 000 000 000 000 000, .6 + 30 I]

0.0222798 + 0.566553 ⅈ

zets7[n_, t_] := Sum[(E^(1 / 2 + I ArcCot[2 t]) E^((1 / 2 + I t) Log[n / j]) +
      E^(1 / 2 - I ArcCot[2 t]) E^((1 / 2 - I t) Log[n / j])) /
    ((E^((1 / 2 - t I) Log[2]) E^((-1 / 2 - t I) Log[Pi])
         ((1 / 2) (E^(I (Pi (1 / 2 + t I) / 2)) + E^(-I (Pi (1 / 2 + t I) / 2)))) Gamma[(1 / 2 + t I)])
       E^(1 / 2 + I ArcCot[2 t]) E^((1 / 2 + I t) Log[n]) +
      E^(1 / 2 - I ArcCot[2 t]) E^((1 / 2 - I t) Log[n])), {j, 1, n}]
z7[n_, s_] := zets7[n, Im[s] + Re[s] I - .5 I]

z7[10 000, 1.6 + 30 I]

0.725453 + 0.345898 ⅈ

E^(-EulerGamma (1 / 2 + t I)) / (1 / 2 + t I)
 Product[ (1 + (1 / 2 + t I) / n)^-1 E^((1 / 2 + t I) / n), {n, 1, Infinity}]
```

$$\frac{e^{-\text{EulerGamma}\left(\frac{1}{2}+i\,t\right)+\frac{1}{2}\,\text{EulerGamma}\,(1+2\,i\,t)}\ \text{Gamma}\left[\frac{3}{2}+i\,t\right]}{\frac{1}{2}+i\,t}$$

```
zets8[n_, t_] := Sum[(E^(1 / 2 + I ArcCot[2 t]) E^((1 / 2 + I t) Log[n / j]) +
      E^(1 / 2 - I ArcCot[2 t]) E^((1 / 2 - I t) Log[n / j])) /
    ((E^((1 / 2 - t I) Log[2]) E^((-1 / 2 - t I) Log[Pi])
         ((1 / 2) (E^(I (Pi (1 / 2 + t I) / 2)) + E^(-I (Pi (1 / 2 + t I) / 2))))
         E^(-EulerGamma (1 / 2 + t I)) / (1 / 2 + t I)
         Product[ (1 + (1 / 2 + t I) / k)^-1 E^((1 / 2 + t I) / k), {k, 1, Infinity}])
       E^(1 / 2 + I ArcCot[2 t]) E^((1 / 2 + I t) Log[n]) +
      E^(1 / 2 - I ArcCot[2 t]) E^((1 / 2 - I t) Log[n])), {j, 1, n}]
z8[n_, s_] := zets8[n, Im[s] + Re[s] I - .5 I]

z8[10 000, 1.6 + 30 I]

0.725453 + 0.345898 ⅈ

zets9[n_, t_] := Sum[(E^(1 / 2 + I ArcCot[2 t]) E^((1 / 2 + I t) Log[n / j]) +
      E^(1 / 2 - I ArcCot[2 t]) E^((1 / 2 - I t) Log[n / j])) /
    ((E^((1 / 2 - t I) Log[2]) E^((-1 / 2 - t I) Log[Pi])
         ((1 / 2) (E^(I (Pi (1 / 2 + t I) / 2)) + E^(-I (Pi (1 / 2 + t I) / 2))))
         E^(-EulerGamma (1 / 2 + t I)) / (1 / 2 + t I)
         Product[ (1 + (1 / 2 + t I) / k)^-1 E^((1 / 2 + t I) / k), {k, 1, Infinity}])
       E^(1 / 2 + I ArcCot[2 t]) E^((1 / 2 + I t) Log[n]) +
      E^(1 / 2 - I ArcCot[2 t]) E^((1 / 2 - I t) Log[n])), {j, 1, n}]
z9[n_, s_] := zets9[n, Im[s] + Re[s] I - .5 I]

z9[10 000, 1.6 + 30 I]

0.725453 + 0.345898 ⅈ
```

```
be[t_] := (1 + t / n) ^-1 E^(t / n)
bo[t_] := E^(-EulerGamma t ) / t
FullSimplify[
 Pi^(1 / 2 - s) bo[s / 2] / bo[(1 - s) / 2] Product[be[s / 2] / be[(1 - s) / 2], {n, 1, Infinity}]]
```

$(2\pi)^{-s}\,\mathrm{Csc}\left[\frac{\pi s}{2}\right]\,\mathrm{Gamma}[s]\,\mathrm{Sin}[\pi s]$

```
Pi^(1 / 2 - s) Gamma[s / 2] / Gamma[(1 - s) / 2] /. s → .3
```

3.07154

$(2\pi)^{-s}\,\mathrm{Csc}\left[\frac{\pi s}{2}\right]\,\mathrm{Gamma}[s]\,\mathrm{Sin}[\pi s]$ /. s → .3

3.07154

$(2\pi)^{-s}$ Gamma[s] Sin[π s] / Sin[Pi s / 2] /. s → .3

3.07154

FullSimplify[Sin[π s] / Sin[Pi s / 2]]

$2\,\mathrm{Cos}\left[\frac{\pi s}{2}\right]$

```
plc5e[n_, x_] := Sum[j^(-1 / 2) (Cos[ArcCot[2 x] + x Log[n / j]]), {j, 1, n}]
Aa[n_, x_] := Sum[j^(-1 / 2), {j, 1, Floor[n]}]
plc5e2[n_, x_] := (Cos[ArcCot[2 x] + x Log[n / n]]) Aa[n, x] -
```

$$\mathrm{Integrate}\left[\mathrm{Aa}[j, x]\left(\frac{x\,\mathrm{Sin}\left[\mathrm{ArcCot}[2x] + x\,\mathrm{Log}\left[\frac{n}{j}\right]\right]}{j}\right), \{j, 1, n\}\right]$$

$$\mathrm{plc5e3}[n\_, x\_] := (\mathrm{Cos}[\mathrm{ArcCot}[2x] + x\,\mathrm{Log}[n/n]])\,\mathrm{HarmonicNumber}\left[\mathrm{Floor}[n], \frac{1}{2}\right] -$$
$$\mathrm{Integrate}\left[\mathrm{HarmonicNumber}\left[\mathrm{Floor}[j], \frac{1}{2}\right]\left(\frac{x\,\mathrm{Sin}\left[\mathrm{ArcCot}[2x] + x\,\mathrm{Log}\left[\frac{n}{j}\right]\right]}{j}\right), \{j, 1, n\}\right]$$

$$\mathrm{plc5e3a}[n\_, x\_] := \left\{(\mathrm{Cos}[\mathrm{ArcCot}[2x] + x\,\mathrm{Log}[n/n]])\,\mathrm{HarmonicNumber}\left[\mathrm{Floor}[n], \frac{1}{2}\right],\right.$$
$$\left.-\mathrm{Integrate}\left[\mathrm{HarmonicNumber}\left[\mathrm{Floor}[j], \frac{1}{2}\right]\left(\frac{x\,\mathrm{Sin}\left[\mathrm{ArcCot}[2x] + x\,\mathrm{Log}\left[\frac{n}{j}\right]\right]}{j}\right), \{j, 1, n\}\right]\right\}$$

$$\mathrm{plc5e3b}[n\_, x\_] := \left\{(\mathrm{Cos}[\mathrm{ArcCot}[2x] + x\,\mathrm{Log}[n/n]])\,\mathrm{HarmonicNumber}\left[\mathrm{Floor}[n], \frac{1}{2}\right],\right.$$
$$-\mathrm{Sum}\left[\mathrm{HarmonicNumber}\left[\mathrm{Floor}[n2 - 1], \frac{1}{2}\right]\right.$$

$$\texttt{Integrate}\left[\left(\frac{x\,\mathrm{Sin}\left[\mathrm{ArcCot}[2\,x]+x\,\mathrm{Log}\left[\frac{n}{j}\right]\right]}{j}\right],\ \{j,\ n2-1,\ n2\}\right],\ \{n2,\ 2,\ n\}\Big]\Big\}$$

$$\texttt{plc5e3c0}[n\_,\ x\_] := (\mathrm{Cos}[\mathrm{ArcCot}[2\,x]+x\,\mathrm{Log}[n/n]])\,\mathrm{HarmonicNumber}\left[\mathrm{Floor}[n],\ \frac{1}{2}\right] - \mathrm{Sum}\left[\mathrm{HarmonicNumber}\left[n2-1,\ \frac{1}{2}\right]\left(-\frac{1}{1+4\,x^2}\sqrt{4+\frac{1}{x^2}}\ x\left(2\,x\,\mathrm{Cos}\left[x\,\mathrm{Log}\left[\frac{n}{-1+n2}\right]\right] - 2\,x\,\mathrm{Cos}\left[x\,\mathrm{Log}\left[\frac{n}{n2}\right]\right] - \mathrm{Sin}\left[x\,\mathrm{Log}\left[\frac{n}{-1+n2}\right]\right] + \mathrm{Sin}\left[x\,\mathrm{Log}\left[\frac{n}{n2}\right]\right]\right)\right),\ \{n2,\ 2,\ n\}\right]$$

$$\texttt{plc5e3c}[n\_,\ x\_] := \left\{(\mathrm{Cos}[\mathrm{ArcCot}[2\,x]+x\,\mathrm{Log}[n/n]])\,\mathrm{HarmonicNumber}\left[\mathrm{Floor}[n],\ \frac{1}{2}\right],\ -\mathrm{Sum}\left[\mathrm{N}\left[\mathrm{HarmonicNumber}\left[n2-1,\ \frac{1}{2}\right]\left(-\frac{1}{1+4\,x^2}\sqrt{4+\frac{1}{x^2}}\ x\left(2\,x\,\mathrm{Cos}\left[x\,\mathrm{Log}\left[\frac{n}{-1+n2}\right]\right] - 2\,x\,\mathrm{Cos}\left[x\,\mathrm{Log}\left[\frac{n}{n2}\right]\right] - \mathrm{Sin}\left[x\,\mathrm{Log}\left[\frac{n}{-1+n2}\right]\right] + \mathrm{Sin}\left[x\,\mathrm{Log}\left[\frac{n}{n2}\right]\right]\right)\right)\right],\ \{n2,\ 2,\ n\}\right]\right\}$$

$$\texttt{plc5e3d}[n\_,\ x\_] := \left\{\frac{2}{\sqrt{4+\frac{1}{x^2}}}\,\mathrm{HarmonicNumber}\left[\mathrm{Floor}[n],\ \frac{1}{2}\right],\ -\mathrm{Sum}\left[\mathrm{HarmonicNumber}\left[n2-1,\ \frac{1}{2}\right]\left(-\frac{1}{1+4\,x^2}\sqrt{4+\frac{1}{x^2}}\ x\left(2\,x\,\mathrm{Cos}\left[x\,\mathrm{Log}\left[\frac{n}{-1+n2}\right]\right] - 2\,x\,\mathrm{Cos}\left[x\,\mathrm{Log}\left[\frac{n}{n2}\right]\right] - \mathrm{Sin}\left[x\,\mathrm{Log}\left[\frac{n}{-1+n2}\right]\right] + \mathrm{Sin}\left[x\,\mathrm{Log}\left[\frac{n}{n2}\right]\right]\right)\right),\ \{n2,\ 2,\ n\}\right]\right\}$$

$$\texttt{plc5e3e}[n\_,\ x\_] := \left\{\frac{2}{\sqrt{4+\frac{1}{x^2}}}\,\mathrm{HarmonicNumber}\left[\mathrm{Floor}[n],\ \frac{1}{2}\right],\ -\mathrm{Sum}\left[\mathrm{HarmonicNumber}\left[n2-1,\ \frac{1}{2}\right]\left(-\mathrm{Cos}\left[\mathrm{ArcCot}[2\,x]+x\,\mathrm{Log}\left[\frac{n}{-1+n2}\right]\right] + \mathrm{Cos}\left[\mathrm{ArcCot}[2\,x]+x\,\mathrm{Log}\left[\frac{n}{n2}\right]\right]\right),\ \{n2,\ 2,\ n\}\right]\right\}$$

plc5e2[100, .3]

1.29208

plc5e3[100, .3]

1.29208

plc5e3a[100, .3]

{9.56427, -8.27219}

plc5e3a[100, .3]

{9.56427, -8.27219}

D[(Cos[ArcCot[2 x] + x Log[n / j]]), j]

$$\frac{x \, \mathrm{Sin}\left[\mathrm{ArcCot}[2\,x] + x \, \mathrm{Log}\left[\frac{n}{j}\right]\right]}{j}$$

Integrate$\left[\frac{x \, \mathrm{Sin}\left[\mathrm{ArcCot}[2\,x] + x \, \mathrm{Log}\left[\frac{n}{j}\right]\right]}{j}, j\right]$

$$\frac{2\,x\,\mathrm{Cos}\left[x\,\mathrm{Log}\left[\frac{n}{j}\right]\right] - \mathrm{Sin}\left[x\,\mathrm{Log}\left[\frac{n}{j}\right]\right]}{\sqrt{4 + \frac{1}{x^2}}\;x}$$

plc5e[100, .3]

1.29208

FullSimplify[Sum[j ^ (-1 / 2), {j, 1, Floor[n]}]]

$$\mathrm{HarmonicNumber}\left[\mathrm{Floor}[n], \frac{1}{2}\right]$$

plc5e3[100, N@Im@ZetaZero@1]

Integrate::mpwc :
Integrate was unable to convert Floor[j] to Piecewise because the required number 1000 of piecewise cases sought exceeds the internal limit $MaxPiecewiseCases = 100. ≫

$$61.7624 - \int_1^{1000} \frac{1}{j} 14.1347 \, \mathrm{HarmonicNumber}\left[\mathrm{Floor}[j], \frac{1}{2}\right] \mathrm{Sin}\left[0.0353591 + 14.1347 \, \mathrm{Log}\left[\frac{1000}{j}\right]\right] dj$$

Integrate$\left[\left(\frac{x \, \mathrm{Sin}\left[\mathrm{ArcCot}[2\,x] + x \, \mathrm{Log}\left[\frac{n}{j}\right]\right]}{j}\right), \{j, n, n+1\}\right]$

$$\mathrm{ConditionalExpression}\left[\frac{2\,x\,\left(-1 + \mathrm{Cos}\left[x\,\mathrm{Log}\left[\frac{n}{1+n}\right]\right]\right) - \mathrm{Sin}\left[x\,\mathrm{Log}\left[\frac{n}{1+n}\right]\right]}{\sqrt{4 + \frac{1}{x^2}}\;x}, \mathrm{Re}[n] > 0 \,||\, \mathrm{Re}[n] \le -1 \,||\, n \notin \mathrm{Reals}\right]$$

FullSimplify$\left[\frac{2\,x\,\left(-1 + \mathrm{Cos}\left[x\,\mathrm{Log}\left[\frac{n}{1+n}\right]\right]\right) - \mathrm{Sin}\left[x\,\mathrm{Log}\left[\frac{n}{1+n}\right]\right]}{\sqrt{4 + \frac{1}{x^2}}\;x}\right]$

$$\frac{2\,x\,\left(-1 + \mathrm{Cos}\left[x\,\mathrm{Log}\left[\frac{n}{1+n}\right]\right]\right) - \mathrm{Sin}\left[x\,\mathrm{Log}\left[\frac{n}{1+n}\right]\right]}{\sqrt{4 + \frac{1}{x^2}}\;x}$$

$$\text{Integrate}\left[\left(\frac{x\,\text{Sin}\left[\text{ArcCot}[2\,x]+x\,\text{Log}\left[\frac{n}{j}\right]\right]}{j}\right),\{j,n-1,n\}\right]$$

$$\text{ConditionalExpression}\left[\frac{2\,x-2\,x\,\text{Cos}\left[x\,\text{Log}\left[\frac{n}{-1+n}\right]\right]+\text{Sin}\left[x\,\text{Log}\left[\frac{n}{-1+n}\right]\right]}{\sqrt{4+\frac{1}{x^2}}\,x},\ \text{Re}[n]\geq 1\,||\,\text{Re}[n]\leq 0\,||\,n\notin\text{Reals}\right]$$

$$\text{Integrate}\left[\left(\frac{x\,\text{Sin}\left[\text{ArcCot}[2\,x]+x\,\text{Log}\left[\frac{n}{j}\right]\right]}{j}\right),\{j,n2-1,n2\}\right]$$

$$\text{ConditionalExpression}\left[-\frac{1}{1+4\,x^2}\sqrt{4+\frac{1}{x^2}}\,x\left(2\,x\,\text{Cos}\left[x\,\text{Log}\left[\frac{n}{-1+n2}\right]\right]-2\,x\,\text{Cos}\left[x\,\text{Log}\left[\frac{n}{n2}\right]\right]-\text{Sin}\left[x\,\text{Log}\left[\frac{n}{-1+n2}\right]\right]+\text{Sin}\left[x\,\text{Log}\left[\frac{n}{n2}\right]\right]\right),\ \text{Re}[n2]\geq 1\,||\,\text{Re}[n2]\leq 0\,||\,n2\notin\text{Reals}\right]$$

$$\text{FullSimplify}\left[-\frac{1}{1+4\,x^2}\sqrt{4+\frac{1}{x^2}}\,x\left(2\,x\,\text{Cos}\left[x\,\text{Log}\left[\frac{n}{-1+n2}\right]\right]-2\,x\,\text{Cos}\left[x\,\text{Log}\left[\frac{n}{n2}\right]\right]-\text{Sin}\left[x\,\text{Log}\left[\frac{n}{-1+n2}\right]\right]+\text{Sin}\left[x\,\text{Log}\left[\frac{n}{n2}\right]\right]\right)\right]\ /.\ n2\to a$$

$$\frac{1}{1+4\,x^2}\sqrt{4+\frac{1}{x^2}}\,x\left(-2\,x\,\text{Cos}\left[x\,\text{Log}\left[\frac{n}{-1+a}\right]\right]+2\,x\,\text{Cos}\left[x\,\text{Log}\left[\frac{n}{a}\right]\right]+\text{Sin}\left[x\,\text{Log}\left[\frac{n}{-1+a}\right]\right]-\text{Sin}\left[x\,\text{Log}\left[\frac{n}{a}\right]\right]\right)$$

$$\text{FullSimplify}[(\text{Cos}[\text{ArcCot}[2\,x]])]$$

$$\frac{2}{\sqrt{4+\frac{1}{x^2}}}$$

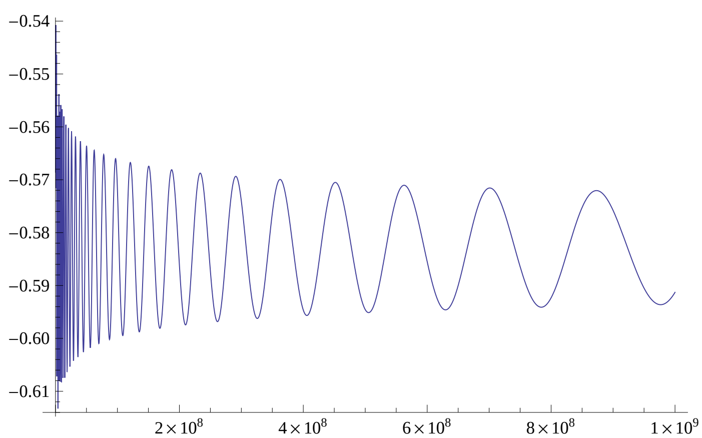
$$((\text{Cos}[\text{ArcCot}[2\,x]+x\ \text{Log}[n/j]])\ /.\ j\to n2)-((\text{Cos}[\text{ArcCot}[2\,x]+x\ \text{Log}[n/j]])\ /.\ j\to(n2-1))$$

$$-\text{Cos}\left[\text{ArcCot}[2\,x]+x\,\text{Log}\left[\frac{n}{-1+n2}\right]\right]+\text{Cos}\left[\text{ArcCot}[2\,x]+x\,\text{Log}\left[\frac{n}{n2}\right]\right]$$

$$\text{FullSimplify}\left[-\text{Cos}\left[\text{ArcCot}[2\,x]+x\,\text{Log}\left[\frac{n}{-1+n2}\right]\right]+\text{Cos}\left[\text{ArcCot}[2\,x]+x\,\text{Log}\left[\frac{n}{n2}\right]\right]\right]$$

$$-\text{Cos}\left[\text{ArcCot}[2\,x]+x\,\text{Log}\left[\frac{n}{-1+n2}\right]\right]+\text{Cos}\left[\text{ArcCot}[2\,x]+x\,\text{Log}\left[\frac{n}{n2}\right]\right]$$

```
tc[n_, t_, j_] :=
 j^(-1/2) Cos[ t Log[n] - t Log[j] + ArcCot[ 2 t]] / Cos[ t Log[n]  + ArcCot[ 2 t]]
tn[n_, t_] := Sum[ N[tc[n, t, j]], {j, 1, n}]
tl[t_] := Table[ Limit[ tc[n, t, j], n → Infinity], {j, 1, 30}]
tl2[t_] := Sum[ Limit[ tc[n, t, j], n → Infinity], {j, 1, 100}]
tco[n_, t_, j_] := j^(-1/2) (Cos[ t Log[j]] + Tan[t Log[n] + ArcCot[2 t]] Sin[t Log[j]])
tcoa[n_, t_, j_] := {j^(-1/2), Cos[ t Log[j]], Tan[t Log[n] + ArcCot[2 t]], Sin[t Log[j]]}
tco2[t_] := Sum[ Limit[ tc[n, t, j], n → Infinity], {j, 1, 100}]
tcr[n_, t_] := Sum[ j^(-1/2) (Cos[ t Log[j]] + I Sin[t Log[j]]), {j, 1, n}]
tcs[n_, t_] :=
 Sum[ j^(-1/2) (Cos[ t Log[j]] + Tan[t Log[n] + ArcCot[2 t]] Sin[t Log[j]]), {j, 1, n}]
tcs2[n_, t_] := Sum[
  j^(-1/2) (Cos[ t Log[j]] + I Sin[t Log[j]] + ( Tan[t Log[n] + ArcCot[2 t]] - I) Sin[t Log[j]]),
  {j, 1, n}]
tcs3[n_, t_] := Sum[ j^(-1/2) (Cos[ t Log[j]] + I Sin[t Log[j]]), {j, 1, n}] -
  Sum[ j^(-1/2) ((I - Tan[t Log[n] + ArcCot[2 t]]) Sin[t Log[j]]), {j, 1, n}]
tcs4[n_, t_] := Sum[ j^(-1/2 + ⅈ t), {j, 1, n}] -
  (I - Tan[t Log[n] + ArcCot[2 t]]) Sum[ j^(-1/2) Sin[t Log[j]], {j, 1, n}]
tcs5[n_, t_] := Sum[ j^(-1/2 + ⅈ t), {j, 1, n}] - (I - Tan[t Log[n] + ArcCot[2 t]])
   Sum[ j^(-1/2) ((1/(2 I)) (j^(I t) - j^(-I t))), {j, 1, n}]
tcs6[n_, t_] := Sum[ j^(-1/2 + ⅈ t), {j, 1, n}] - 1/2 (1 + ⅈ Tan[ArcCot[2 t] + t Log[n]])
   Sum[ j^(I t - 1/2) - j^(-I t - 1/2), {j, 1, n}]
Plot[Re@ tc[n, 14.3 + .1 I, 2], {n, 0, 1000000000}]
```




```
j^(-1/2) Cos[ t Log[n] - t Log[j] + ArcCot[ 2 t]] / Cos[ t Log[n]  + ArcCot[ 2 t]] /. j → 2 /.
 t → N@Im@ZetaZero@1 + .3 I
```

$$\frac{1}{\sqrt{2}}\,\mathrm{Cos}[(9.7621 + 0.208694\, i) - (14.1347 + 0.3\, i)\,\mathrm{Log}[n]]\;\mathrm{Sech}[(0.000749512 + 0.0353432\, i) - (0.3 - 14.1347\, i)\,\mathrm{Log}[n]]$$

```
Limit[1/Sqrt[2]
  Cos[(9.7621016365162` + 0.2086936658426518` I) - (14.134725141734695` + 0.3` I) Log[n]]
   Sech[(0.0007495116746682122` + 0.03534324346697683` I) -
     (0.3` - 14.134725141734695` I) Log[n]], n → Infinity]
```

−0.534926 − 0.209121 ⅈ

```
Table[
 j^(-1/2) Cos[ t Log[n] - t Log[j] + ArcCot[ 2 t]] / Cos[ t Log[n] + ArcCot[ 2 t]] /. j → 2 /.
  t → N@Im@ZetaZero@1 + .3 I
Limit[j^(-1/2) Cos[ t Log[n] - t Log[j] + ArcCot[ 2 t]] / Cos[ t Log[n] + ArcCot[ 2 t]] /. j → 2,
 n → Infinity]
```

$$\frac{e^{-2\,i\,\text{Interval}[\{0,\pi\}]+2\,i\,\text{Interval}[\{0,\pi\}]}}{\sqrt{2}}$$

$$\frac{e^{-t\,\text{Arg}[j]-2\,i\,\text{Interval}[\{0,\pi\}]}\left(e^{2\,t\,\text{Arg}[j]+2\,i\,\text{Interval}[\{0,\pi\}]}+e^{2\,i\,\text{Interval}[\{0,\pi\}]}\right)}{2\sqrt{j}}$$

$$\frac{e^{-t\,\text{Arg}[j]+2\,i\,\text{Interval}[\{-\pi,0\}]}\left(e^{2\,t\,\text{Arg}[j]+2\,i\,\text{Interval}[\{0,\pi\}]}+e^{2\,i\,\text{Interval}[\{0,\pi\}]}\right)}{2\sqrt{j}}$$

```
Limit[j^(-1/2) Cos[ t Log[n] - t Log[j] + ArcCot[ 2 t]], n → Infinity]
```

$$\frac{e^{-t\,\text{Arg}[j]}\left(e^{2\,t\,\text{Arg}[j]+2\,i\,\text{Interval}[\{0,\pi\}]}+e^{2\,i\,\text{Interval}[\{0,\pi\}]}\right)}{2\sqrt{j}}$$

```
N[j^(-1/2) Cos[ t Log[n] - t Log[j] + ArcCot[ 2 t]] / Cos[ t Log[n] + ArcCot[ 2 t]] /. j → 2 /.
  t → Im@ZetaZero@1 + .01 I]
```

0.707107 Cos[(9.76209 + 0.00695647 ⅈ) − (14.1347 + 0.01 ⅈ) Log[n]]
 Sech[(0.0000249949 + 0.0353591 ⅈ) − (0.01 − 14.1347 ⅈ) Log[n]]

```
Limit[0.7071067811865475`
  Cos[(9.762085766172962` + 0.006956466737293951` I) - (14.134725141734695` + 0.01` I) Log[n]]
  Sech[(0.000024994931694497765` + 0.03535911381021402` I) -
    (0.01` - 14.134725141734695` I) Log[n]], n → Infinity]
```

−0.654022 − 0.25568 ⅈ

```
tl[N@Im@ZetaZero@1 + .1 I] // TableForm
```

```
-0.614468 - 0.240217 i
-0.508982 + 0.0922935 i
0.319867 + 0.295211 i
-0.27654 - 0.26169 i
0.334924 + 0.0655546 i
-0.223487 + 0.216461 i
-0.125633 - 0.258235 i
0.250544 - 0.0939514 i
0.107063 + 0.22723 i
-0.186835 + 0.146183 i
-0.190053 - 0.120735 i
0.0270831 - 0.212886 i
0.189323 - 0.0793228 i
0.164906 + 0.107672 i
0.0151653 + 0.188857 i
-0.128068 + 0.130294 i
-0.17652 - 0.00245482 i
-0.121721 - 0.119965 i
-0.0112023 - 0.165344 i
0.0937728 - 0.130801 i
0.14992 - 0.044944 i
0.143818 + 0.0503971 i
0.0877786 + 0.119842 i
0.00799293 + 0.144735 i
-0.0677805 + 0.124306 i
-0.118851 + 0.0709431 i
-0.135388 + 0.00326279 i
-0.118112 - 0.060281 i
-0.0754648 - 0.105775 i
```

```
tn[100, .3 I + 30]
```

```
0.212089 + 0.463681 i
```

```
Zeta[.8 + 30 I]
```

```
0.252252 - 0.525921 i
```

```
tl2[.3 I + 30]
```

```
-0.740908 + 0.442076 i
```

```
tco2[.3 I + 30]
```

```
$Aborted
```

```
tc[n, x, 1]
```

```
1
```

```
1 + -0.7409083225112658` + 0.4420760093125105` i
```

```
0.259092 + 0.442076 i
```

```
Limit[j^(-1/2) Cos[ t Log[n] - t Log[j] + ArcCot[ 2 t]] / Cos[ t Log[n]  + ArcCot[ 2 t]],
 n → Infinity]
```

$$\frac{e^{-t\,\mathrm{Arg}[j]-2\,i\,\mathrm{Interval}[\{0,\pi\}]}\left(e^{2\,t\,\mathrm{Arg}[j]+2\,i\,\mathrm{Interval}[\{0,\pi\}]}+e^{2\,i\,\mathrm{Interval}[\{0,\pi\}]}\right)}{2\sqrt{j}}$$

```
Limit[j^(-1/2) Cos[ t Log[n] - t Log[j]] / Cos[ t Log[n] ] /. j → 2 /. t → 10 + .1 I, n → Infinity]
```

0.525903 + 0.398374 i

```
FullSimplify[tco[n, 10 + .1 I, 2]]
```

(0.565003 - 0.0296187 i) +
 (0.0391004 - 0.427992 i) Tanh[(0.000498704 + 0.0499534 i) - (0.1 - 10. i) Log[n]]

```
Limit[ (0.565003262515136` - 0.029618747630025505` i) +
  (0.039100442139108245` - 0.4279923225174692` i) Tanh[
    (0.0004987035366082857` + 0.049953421122248966` i) - (0.1` - 10.` i) Log[n]], n → Infinity]
```

0.525903 + 0.398374 i

```
Limit[tco[n, x, 3], n → Infinity]
```

$$\frac{e^{-2\,i\,\mathrm{Interval}[\{0,\pi\}]+2\,i\,\mathrm{Interval}[\{0,\pi\}]}}{\sqrt{3}}$$

```
Limit[Tan[t Log[n] + ArcCot[2 t]] /. t → 10 + .6 I, n → Infinity]
```

0. + 1. i

```
tcoa[10 000 000 000 000 000, 10 + .1 I, 2]
```

$\left\{\frac{1}{\sqrt{2}}, 0.799035 - 0.0418872\,i, 0.00123206 + 1.00028\,i, 0.605273 + 0.0552964\,i\right\}$

```
tcr[100 000, .45 I]
```

16.1392 + 0. i

```
Zeta[.95 + 3 I]
```

0.619329 - 0.105393 i

```
tcs4[100 000, .45 I + 3]
```

0.619307 + 0.105384 i

```
tcs6[10 000, .45 I + 3]
```

0.619129 + 0.105481 i

```
N[Tan[t Log[n] + ArcCot[2 t]] /. t → .2 I + 10] /. n → 100 000 000 000
```

-0.0000610936 + 1.00005 i

```
ac[n_, t_] := (t Sin[t Log[n]] + (1/2) Cos[t Log[n]]) / (t Cos[t Log[n]] - (1/2) Sin[t Log[n]])
ac2[n_, t_] := Tan[t Log[n] + ArcCot[2 t]]
```

```
ac[100, .3 I + 10]
```

-0.119026 + 1.05256 i

```
ac2[100, .3 I + 10]
```

$-0.119026 + 1.05256\,i$

```
Limit[ ac[n, .3 I + 10], n → Infinity]
```

$0. + 1.\,i$

```
FullSimplify[j ^ (-1 / 2) (Cos[ t Log[j]] + I Sin[t Log[j]])]
```

$j^{-\frac{1}{2}+i t}$

```
FullSimplify[j ^ (-1 / 2) ((I - Tan[t Log[n] + ArcCot[2 t]]) Sin[t Log[j]])]
```

$$-\frac{\text{Sin}[t\,\text{Log}[j]]\,(-i + \text{Tan}[\text{ArcCot}[2\,t] + t\,\text{Log}[n]])}{\sqrt{j}}$$

```
FullSimplify[j ^ (-1 / 2) ((j ^ (I t) - j ^ (-I t)))]
```

$j^{-\frac{1}{2}-i t}\left(-1 + j^{2 i t}\right)$

```
FullSimplify[(1 / (2 I)) (I - Tan[t Log[n] + ArcCot[2 t]])]
```

$$\frac{1}{2}\,(1 + i\,\text{Tan}[\text{ArcCot}[2\,t] + t\,\text{Log}[n]])$$

$$\text{tcs6}[n\_, t\_] := \text{Sum}\left[j^{-\frac{1}{2}+i t}, \{j, 1, n\}\right] - \frac{1}{2}\,(1 + i\,\text{Tan}[\text{ArcCot}[2\,t] + t\,\text{Log}[n]])\,\text{Sum}[\,j \wedge (I\,t - 1/2) - j \wedge (-I\,t - 1/2), \{j, 1, n\}]$$

$$\text{tcs7}[n\_, t\_] := \text{HarmonicNumber}\left[n, \frac{1}{2} - i\,t\right] - \frac{1}{2}\left(\text{HarmonicNumber}\left[n, \frac{1}{2} - i\,t\right] - \text{HarmonicNumber}\left[n, \frac{1}{2} + i\,t\right]\right)(1 + i\,\text{Tan}[\text{ArcCot}[2\,t] + t\,\text{Log}[n]])$$

$$\text{tcs8}[n\_, s\_] := \frac{n\,s\,(\text{HarmonicNumber}[n, 1 - s] - \text{HarmonicNumber}[n, s])}{n^{2 s}\,(-1 + s) + n\,s} + \text{HarmonicNumber}[n, s]$$

$$\text{tcs9}[n\_, s\_] := \frac{n\,s\,\text{HarmonicNumber}[n, 1 - s]}{n^{2 s}\,(-1 + s) + n\,s} - \frac{n\,s\,\text{HarmonicNumber}[n, s]}{n^{2 s}\,(-1 + s) + n\,s} + \text{HarmonicNumber}[n, s]$$

$$\text{tcs10}[n\_, s\_] := \frac{\text{HarmonicNumber}[n, 1 - s]}{n^{2 s-1}\,(-1 + s)/s + 1} - \frac{\text{HarmonicNumber}[n, s]}{n^{2 s-1}\,(-1 + s)/s + 1} + \text{HarmonicNumber}[n, s]$$

$$\text{tcs11}[n\_, s\_] := \left\{\frac{\text{HarmonicNumber}[n, 1 - s]}{n^{2 s-1}\,(-1 + s)/s + 1}, -\frac{\text{HarmonicNumber}[n, s]}{n^{2 s-1}\,(-1 + s)/s + 1}, \text{HarmonicNumber}[n, s]\right\}$$

$$\text{FullSimplify}\left[\text{Sum}\left[j^{-\frac{1}{2}+i t}, \{j, 1, n\}\right] - \frac{1}{2}\,(1 + i\,\text{Tan}[\text{ArcCot}[2\,t] + t\,\text{Log}[n]])\,\text{Sum}[\,j \wedge (I\,t - 1/2) - j \wedge (-I\,t - 1/2), \{j, 1, n\}]\right]$$

$$\text{HarmonicNumber}\left[n, \frac{1}{2} - i\,t\right] - \frac{1}{2}\left(\text{HarmonicNumber}\left[n, \frac{1}{2} - i\,t\right] - \text{HarmonicNumber}\left[n, \frac{1}{2} + i\,t\right]\right)(1 + i\,\text{Tan}[\text{ArcCot}[2\,t] + t\,\text{Log}[n]])$$

```
tcs7[10 000, .45 I + 3]
```

$0.619129 + 0.105481\,i$

```
tcs6[10 000, .45 I + 3]
```

0.619129 + 0.105481 i

```
FullSimplify[tcs7[n, (t - 1 / 2) I] /. t → s]
```

$$\frac{n\,s\,(\text{HarmonicNumber}[n, 1-s] - \text{HarmonicNumber}[n, s])}{n^{2 s}\,(-1+s) + n\,s} + \text{HarmonicNumber}[n, s]$$

```
tcs11[1 000 000 000 000 000 000 000, .7 + 3 I]
```

{-400 118. - 527 131. i, -0.00235389 + 0.00130609 i, 400 119. + 527 131. i}

```
Zeta[.7 + 3 I]
```

0.571252 - 0.0923229 i

$$\text{FullSimplify}\left[\frac{n\,s\,\text{HarmonicNumber}[n, s]}{n^{2 s}\,(-1+s) + n\,s}\right]$$

$$\frac{n\,s\,\text{HarmonicNumber}[n, s]}{n^{2 s}\,(-1+s) + n\,s}$$

$$\text{FullSimplify}\left[\frac{\text{HarmonicNumber}[n, s]}{n^{2 s-1}\,(-1+s)\,/\,s + 1} + \text{HarmonicNumber}[n, s]\right]$$

$$\left(1 + \frac{n\,s}{n^{2 s}\,(-1+s) + n\,s}\right) \text{HarmonicNumber}[n, s]$$

```
tcs10[1 000 000 000 000 000 000 000, .7 + 3 I]
```

0.571252 - 0.0923229 i

```
zz[n_, x_] :=
 Sum[(Cos[ArcCot[2 x] + x Log[n / j]] / j ^ (1 / 2)) / Cos[ArcCot[2 x] + x Log[n]], {j, 1, n}]
zza[n_, s_] := Sum[(Cos[ArcCot[I (2 s - 1)] + I ( s - 1 / 2) Log[n / j]] / j ^ (1 / 2)) /
   Cos[ArcCot[I (2 s - 1)] + I ( s - 1 / 2) Log[n]], {j, 1, n}]
zzb[n_, s_] := Sum[(Cosh[ArcCoth[1 - 2 s] + ( s - 1 / 2) Log[n / j]] / j ^ (1 / 2)) /
   Cosh[ ArcCoth[1 - 2 s] + ( s - 1 / 2) Log[n]], {j, 1, n}]
zz2[n_, x_] := Sum[(Cos[ArcCot[2 x] + x Log[n / j] + x Log[Pi] + Log[ Gamma[1 / 2 - x / 2]] -
        Log[Gamma[1 / 4 + x / 2]]] / j ^ (1 / 2)) / Cos[ArcCot[2 x] + x Log[n] +
      x Log[Pi] + Log[ Gamma[1 / 2 - x / 2]] - Log[Gamma[1 / 4 + x / 2]]], {j, 1, n}]
```

```
zzb[10 000, 2.]
```

1.64493

```
zz[10 000, .2 I + 1]
```

0.295264 + 0.801349 i

```
Zeta[.7 + 1 I]
```

0.284305 - 0.841353 i

```
((.2 I + 1) + 1 / 2 I) / I
```

0.7 - 1. i

```
-(.2 I + 1) I + 1/2
```

0.7 - 1. ⅈ

```
a / I
```

-ⅈ a

```
-(.7 + I) I + 1/2
```

1.5 - 0.7 ⅈ

```
-(s - 1/2) / I
```

-1. + 0.2 ⅈ

```
Expand[-(s - 1/2) / I]
```

$$-\frac{i}{2} + i\,s$$

```
FullSimplify[(Cos[ArcCot[2 (I s - I/2)] + (I s - I/2) Log[n/j]] / j^(1/2)) /
  Cos[ArcCot[2 (I s - I/2)] + (I s - I/2) Log[n]]]
```

$$\frac{1}{\sqrt{j}}\,\mathrm{Cosh}\left[\mathrm{ArcCoth}[1-2\,s]+\frac{1}{2}\,(-1+2\,s)\,\mathrm{Log}\left[\frac{n}{j}\right]\right]\,\mathrm{Sech}\left[\mathrm{ArcCoth}[1-2\,s]-\frac{\mathrm{Log}[n]}{2}+s\,\mathrm{Log}[n]\right]$$

```
ArcCot[I x]
```

-ⅈ ArcCoth[x]

```
Cos[-I ArcCoth[(2 s - 1)] + I ( s - 1/2) Log[n/j]]
```

$$\mathrm{Cosh}\left[\mathrm{ArcCoth}[1-2\,s]+\left(-\frac{1}{2}+s\right)\,\mathrm{Log}\left[\frac{n}{j}\right]\right]$$

```
Cos[-I ArcCoth[(2 s - 1)] + I ( s - 1/2) Log[n]]
```

$$\mathrm{Cosh}\left[\mathrm{ArcCoth}[1-2\,s]+\left(-\frac{1}{2}+s\right)\,\mathrm{Log}[n]\right]$$

```
zzb[n_, s_] := Sum[(Cosh[ArcCoth[1 - 2 s] + ( s - 1/2) Log[n/j]] / j^(1/2)) /
   Cosh[ ArcCoth[1 - 2 s] + ( s - 1/2) Log[n]], {j, 1, n}]
zzc[n_, s_] := Sum[(Cosh[ArcCoth[1 - 2 s] + ( s - 1/2) Log[n/j]] / j^(1/2)) /
   Cosh[ ArcCoth[1 - 2 s] + ( s - 1/2) Log[n] + (s - 1/2) Log[Pi] +
     Log[Gamma[1/4 - (s - 1/2) / 2]] - Log[Gamma[1/4 + (s - 1/2) / 2]]], {j, 1, n}]
zzd[n_, s_] := Sum[(Cosh[ArcCoth[1 - 2 s] + ( s - 1/2) Log[n/j]] / j^(1/2)) /
   Cosh[ ArcCoth[1 - 2 s] + ( s - 1/2) Log[Pi n] + Log[Gamma[(1 - s) / 2]] - Log[Gamma[s/2]]], {j,
   1, n}]
```

```
zzd[10 000, .2 + 3 I]
```

0.476013 - 0.0545908 ⅈ

```
Zeta[.2 + 3 I]
```

0.475964 - 0.0546585 ⅈ

```
FullSimplify[Cosh[ ArcCoth[1 - 2 s] + ( s - 1/2) Log[n] + (s - 1/2) Log[Pi] +
   Log[Gamma[1/4 - (s - 1/2) / 2]] - Log[Gamma[1/4 + (s - 1/2) / 2]]]]
```

$$\mathrm{Cosh}\left[\mathrm{ArcCoth}[1-2\,s]+\left(-\frac{1}{2}+s\right)\,\mathrm{Log}[n\,\pi]+\mathrm{Log}\left[\mathrm{Gamma}\left[\frac{1-s}{2}\right]\right]-\mathrm{Log}\left[\mathrm{Gamma}\left[\frac{s}{2}\right]\right]\right]$$

```
FullSimplify[(Cosh[ArcCoth[1 - 2 s] + (s - 1 / 2) Log[n / j]] / j^(1 / 2)) /
  Cosh[ ArcCoth[1 - 2 s] + (s - 1 / 2) Log[Pi n] + Log[Gamma[(1 - s) / 2]] - Log[Gamma[s / 2]]]]
```

$$\frac{1}{\sqrt{j}}\,\text{Cosh}\left[\text{ArcCoth}[1-2\,s]+\left(-\frac{1}{2}+s\right)\text{Log}\left[\frac{n}{j}\right]\right]$$

$$\text{Sech}\left[\text{ArcCoth}[1-2\,s]+\left(-\frac{1}{2}+s\right)\text{Log}[n\,\pi]+\text{Log}\left[\text{Gamma}\left[\frac{1-s}{2}\right]\right]-\text{Log}\left[\text{Gamma}\left[\frac{s}{2}\right]\right]\right]$$

```
FullSimplify[(Cosh[ArcCoth[1 - 2 s] + (s - 1 / 2) Log[n / j]] / j^(1 / 2)) /
  Cosh[ ArcCoth[1 - 2 s] + (s - 1 / 2) Log[ n]]]
```

$$\frac{1}{\sqrt{j}}\,\text{Cosh}\left[\text{ArcCoth}[1-2\,s]+\left(-\frac{1}{2}+s\right)\text{Log}\left[\frac{n}{j}\right]\right]\text{Sech}\left[\text{ArcCoth}[1-2\,s]+\left(-\frac{1}{2}+s\right)\text{Log}[n]\right]$$

```
j = 1 + x^2
j - 1 = x^2
sqrt (j - 1) = x
```

```
FullSimplify[Sin[ArcCot[(j - 1)^(1 / 2)]]] /. j → 3
```

$$\frac{1}{\sqrt{3}}$$

```
zz[n_, x_] :=
 Sum[(Cos[ArcCot[2 x] + x Log[n / j]] / j^(1 / 2)) / Cos[ArcCot[2 x] + x Log[n]], {j, 1, n}]
zzo[n_, x_] := Sum[Sin[ArcCot[(j - 1)^(1 / 2)]]
   Cos[ArcCot[2 x] + x Log[n / j]] / Cos[ArcCot[2 x] + x Log[n]], {j, 1, n}]
zzo2[n_, x_] := (1 / 2) Sum[(Sin[ArcCot[(j - 1)^(1 / 2)] + ArcCot[2 x] + x Log[n / j]] +
      Sin[ArcCot[(j - 1)^(1 / 2)] - ArcCot[2 x] - x Log[n / j]]) /
    Cos[ArcCot[2 x] + x Log[n]], {j, 1, n}]
zzo3[n_, x_] := (1 / 2) Sum[(Sin[ArcCot[(j - 1)^(1 / 2)] + ArcCot[2 x] + x Log[n / j]] +
      Sin[ArcCot[(j - 1)^(1 / 2)] - ArcCot[2 x] - x Log[n / j]])
    Sec[ArcCot[2 x] + x Log[n]], {j, 1, n}]
zzo4[n_, x_] := {(1 / 2) Sum[(Sin[ArcCot[(j - 1)^(1 / 2)] + ArcCot[2 x] + x Log[n] - x Log[j]])
     Sec[ArcCot[2 x] + x Log[n]], {j, 1, n}],
  (1 / 2) Sum[(Sin[ArcCot[(j - 1)^(1 / 2)] - ArcCot[2 x] - x Log[n] + x Log[j]])
     Sec[ArcCot[2 x] + x Log[n]], {j, 1, n}]}
zzo5[n_, x_] := (1 / 2) Sec[ArcCot[2 x] + x Log[n]]
  (Sum[Sin[ArcCot[(j - 1)^(1 / 2)] + ArcCot[2 x] + x Log[n] - x Log[j]] , {j, 1, n}] +
    Sum[Sin[ArcCot[(j - 1)^(1 / 2)] - ArcCot[2 x] - x Log[n] + x Log[j]], {j, 1, n}])
zzo5a[n_, x_] := {Sum[Sin[ArcCot[(j - 1)^(1 / 2)] + ArcCot[2 x] + x Log[n] - x Log[j]] ,
   {j, 1, n}], Sum[Sin[ArcCot[(j - 1)^(1 / 2)] - ArcCot[2 x] - x Log[n] + x Log[j]], {j, 1, n}]}
zzo5b[n_, x_] := {Sum[Sin[Pi - ArcTan[(j - 1)^(1 / 2)] - ArcTan[2 x] + x Log[n] - x Log[j]] ,
   {j, 1, n}], Sum[Sin[-ArcTan[(j - 1)^(1 / 2)] + ArcTan[2 x] - x Log[n] + x Log[j]], {j, 1, n}]}
zzo5c[n_, x_] := {-Sum[Sin[-ArcTan[(j - 1)^(1 / 2)] - ArcTan[2 x] + x Log[n] - x Log[j]] , {j,
     1, n}], Sum[Sin[-ArcTan[(j - 1)^(1 / 2)] + ArcTan[2 x] - x Log[n] + x Log[j]], {j, 1, n}]}
zzo5d[n_, x_] := {Sum[Sin[ArcTan[(j - 1)^(1 / 2)] + ArcTan[2 x] - x Log[n] + x Log[j]] ,
   {j, 1, n}], Sum[Sin[-ArcTan[(j - 1)^(1 / 2)] + ArcTan[2 x] - x Log[n] + x Log[j]], {j, 1, n}]}
zzo5da[n_, x_] := (1 / 2) Sec[Pi / 2 - ArcTan[2 x] + x Log[n]]
  (Sum[Sin[ArcTan[(j - 1)^(1 / 2)] + ArcTan[2 x] - x Log[n] + x Log[j]] , {j, 1, n}] +
    Sum[Sin[-ArcTan[(j - 1)^(1 / 2)] + ArcTan[2 x] - x Log[n] + x Log[j]], {j, 1, n}])
zzo5e[n_, x_] := {Sum[Sin[ArcTan[(j - 1)^(1 / 2)] + ArcTan[2 x] - x Log[n] + x Log[j]] ,
   {j, 1, n}], Sum[Sin[-ArcTan[(j - 1)^(1 / 2)] + ArcTan[2 x] - x Log[n] + x Log[j]], {j, 1, n}]}
zzo5f[n_, x_] := Sum[Sin[ArcTan[(j - 1)^(1 / 2)] + ArcTan[2 x] - x Log[n] + x Log[j]] +
   Sin[-ArcTan[(j - 1)^(1 / 2)] + ArcTan[2 x] - x Log[n] + x Log[j]], {j, 1, n}]
```

```
zzo5f[10 000, N@Im@ZetaZero@1]
```

0.00999375

```
Zeta[.8 + 30 I]
```

0.252252 - 0.525921 i

```
{(1 / 2) Sum[(Sin[ArcCot[(j - 1)^(1 / 2)] + ArcCot[2 x] + x Log[n] - x Log[j]])
    Sec[ArcCot[2 x] + x Log[n]], {j, 1, n}],
 (1 / 2) Sum[(Sin[ArcCot[(j - 1)^(1 / 2)] - ArcCot[2 x] - x Log[n] + x Log[j]])
    Sec[ArcCot[2 x] + x Log[n]], {j, 1, n}]}
```

$$\left\{\frac{1}{2}\sum_{j=1}^{n}\mathrm{Sec}[\mathrm{ArcCot}[2\,x]+x\,\mathrm{Log}[n]]\,\mathrm{Sin}\left[\mathrm{ArcCot}\left[\sqrt{-1+j}\right]+\mathrm{ArcCot}[2\,x]-x\,\mathrm{Log}[j]+x\,\mathrm{Log}[n]\right],\right.$$
$$\left.\frac{1}{2}\sum_{j=1}^{n}\mathrm{Sec}[\mathrm{ArcCot}[2\,x]+x\,\mathrm{Log}[n]]\,\mathrm{Sin}\left[\mathrm{ArcCot}\left[\sqrt{-1+j}\right]-\mathrm{ArcCot}[2\,x]+x\,\mathrm{Log}[j]-x\,\mathrm{Log}[n]\right]\right\}$$

```
N@ArcCot[(j - 1) ^ (1 / 2)] /. j → 1
```

1.5708

```
{Sum[Sin[ArcCot[(j - 1) ^ (1 / 2)] + ArcCot[2 x] + x Log[n] - x Log[j]] , {j, 1, n}],
 Sum[Sin[ArcCot[(j - 1) ^ (1 / 2)] - ArcCot[2 x] - x Log[n] + x Log[j]], {j, 1, n}]}
```

$$\left\{\sum_{j=1}^{n}\mathrm{Sin}\left[\mathrm{ArcCot}\left[\sqrt{-1+j}\right]+\mathrm{ArcCot}[2\,x]-x\,\mathrm{Log}[j]+x\,\mathrm{Log}[n]\right],\right.$$
$$\left.\sum_{j=1}^{n}\mathrm{Sin}\left[\mathrm{ArcCot}\left[\sqrt{-1+j}\right]-\mathrm{ArcCot}[2\,x]+x\,\mathrm{Log}[j]-x\,\mathrm{Log}[n]\right]\right\}$$

```
ArcTan[(j - 1) ^ (1 / 2)]
```

$$\mathrm{ArcTan}\left[\sqrt{-1+j}\right]$$

```
ArcTan[(j - 1) ^ (1 / 2)] + ArcTan[2 x] - x Log[n] +
 x Log[j] + -ArcTan[(j - 1) ^ (1 / 2)] + ArcTan[2 x] - x Log[n] + x Log[j]
```

```
Expand[(2 ArcTan[2 x] + 2 x Log[j] - 2 x Log[n]) / 2]
```

$$\mathrm{ArcTan}[2\,x]+x\,\mathrm{Log}[j]-x\,\mathrm{Log}[n]$$

```
(ArcTan[(j - 1) ^ (1 / 2)] + ArcTan[2 x] - x Log[n] + x Log[j] -
   (-ArcTan[(j - 1) ^ (1 / 2)] + ArcTan[2 x] - x Log[n] + x Log[j])) / 2
```

$$\mathbf{Cos}\left[\mathbf{ArcTan}\left[\sqrt{-1+j}\right]\right]$$

$$\frac{1}{\sqrt{j}}$$

```
Sin[ArcCot[(j - 1) ^ (1 / 2)]] /. j → 3
```

$$\frac{1}{\sqrt{3}}$$

```
Sin[Pi / 2 - ArcTan[(j - 1) ^ (1 / 2)]] /. j → 3
```

$$\frac{1}{\sqrt{3}}$$

```
Cos[ArcTan[(j - 1) ^ (1 / 2)]] /. j → 3
```

$$\frac{1}{\sqrt{3}}$$

```
ach[n_, x_, b_] := Sum[b^(-j/2) Cos[ x Log[n] - j Log[b] + ArcCot[2 x]], {j, 1, 23}]

ach[10 000 000, N@Im@ZetaZero@2, 5]

-0.311633

Log[2., 10 000 000]

23.2535

zt[n_, x_] := Sum[(Sin[ArcTan[2 x] - x Log[n / j]] / j^(1 / 2)), {j, 1, n}]
ztz[n_, x_] :=
 Sum[(Sin[ArcTan[2 x] - x Log[n / j]] / j^(1 / 2)) / Sin[ArcTan[2 x] - x Log[n]], {j, 1, n}]
ztz2[n_, x_] :=
 Sum[((Sin[ArcTan[2 x] - x Log[n]] Cos[x Log[j]] + Cos[ArcTan[2 x] - x Log[n]] Sin[x Log[j]]) /
     j^(1 / 2)) / Sin[ArcTan[2 x] - x Log[n]], {j, 1, n}]
ztz3[n_, x_] := Sum[Cos[x Log[j]] / j^(1 / 2), {j, 1, n}] +
  Sum[((Cos[ArcTan[2 x] - x Log[n]] Sin[x Log[j]]) / j^(1 / 2)) /
    Sin[ArcTan[2 x] - x Log[n]], {j, 1, n}]
ztz4[n_, x_] := Sum[(Sin[ArcSin[2 x / (4 x^2 + 1)^(1 / 2)] - x Log[n / j]] / j^(1 / 2)) /
   Sin[ArcSin[2 x / (4 x^2 + 1)^(1 / 2)] - x Log[n]], {j, 1, n}]

ztz4[10 000, N@Im@ZetaZero@1]

-0.0323453

N@ArcTan[140 000]

1.57079

Sin[ArcSin[2 x / (4 x^2 + 1)^(1 / 2)]]
```

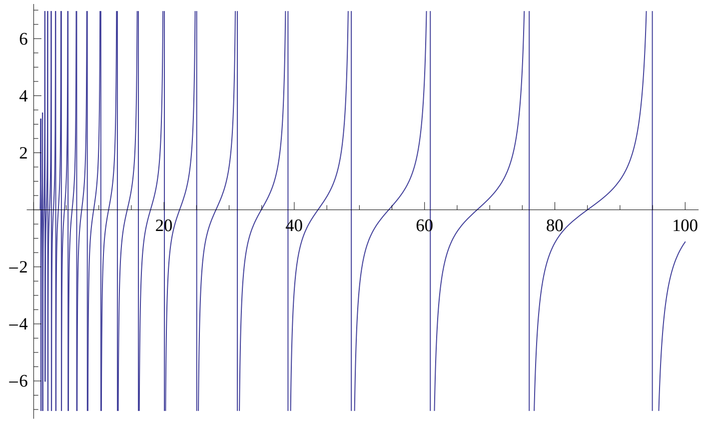
$$\frac{2\,x}{\sqrt{1+4\,x^2}}$$

```
Tan[t Log[n] + Pi / 2 - ArcTan[2 t]]

Cot[ArcTan[2 t] - t Log[n]]

Tan[ArcCot[2 t]]
```

$$\frac{1}{2\,t}$$

```
ezt[n_, x_] :=
 Sum[j^(-1/2) (Cos[x Log[j]] + Tan[x Log[n] + ArcCot[2 x]] Sin[x Log[j]]), {j, 1, n}]
ezt2[n_, x_] := Sum[j^(-1/2)
   (Cos[x Log[j]] + (Sin[(x Log[n] + ArcCot[2 x]) 2] / (1 + Cos[(x Log[n] + ArcCot[2 x]) 2]))
      Sin[x Log[j]]), {j, 1, n}]
ezt3[n_, x_] := Sum[j^(-1/2) ((1 + Cos[(x Log[n] + ArcCot[2 x]) 2]) Cos[x Log[j]] +
     (Sin[(x Log[n] + ArcCot[2 x]) 2]) Sin[x Log[j]]), {j, 1, n}]
ezt4[n_, x_] := Sum[j^(-1/2) (Cos[x Log[j]] + (Cos[(x Log[n] + ArcCot[2 x]) 2]) Cos[x Log[j]] +
     (Sin[(x Log[n] + ArcCot[2 x]) 2]) Sin[x Log[j]]), {j, 1, n}]
ezt5[n_, x_] := Sum[j^(-1/2) (Cos[x Log[j]] + Cos[2 (x Log[n] + ArcCot[2 x]) + x Log[j]] / 2 +
     Cos[2 (x Log[n] + ArcCot[2 x]) - x Log[j]] / 2 + Cos[2 (x Log[n] + ArcCot[2 x]) - x Log[j]] /
      2 - Cos[2 (x Log[n] + ArcCot[2 x]) + x Log[j]] / 2), {j, 1, n}]
ezt6[n_, x_] := Sum[j^(-1/2) (Cos[x Log[j]] + Cos[x Log[j] - 2 ArcCot[2 x] - 2 x Log[n]]),
  {j, 1, n}]
ezt6a[n_, x_] := {Sum[j^(-1/2) Cos[x Log[j]], {j, 1, n}],
  Sum[j^(-1/2) (Cos[x Log[j] - 2 (ArcCot[2 x] + x Log[n])]), {j, 1, n}]}
ezt7[n_, x_] := Sum[j^(-1/2) (Cos[x Log[j]] + Cos[x Log[j / n^2] - 2 ArcCot[2 x]]), {j, 1, n}]

ezt7[10 000, N@Im@ZetaZero@1]

-0.00154389

Zeta[.8 + 30 I]

0.252252 - 0.525921 i

Tan[13.7]

2.13984

Sin[13.7 × 2] / (1 + Cos[13.7 × 2])

2.13984

Cos[2 (x Log[n] + ArcCot[2 x]) + x Log[j]] / 2 + Cos[2 (x Log[n] + ArcCot[2 x]) - x Log[j]] / 2 +
 Cos[2 (x Log[n] + ArcCot[2 x]) - x Log[j]] / 2 - Cos[2 (x Log[n] + ArcCot[2 x]) + x Log[j]] / 2

Cos[x Log[j] - 2 (ArcCot[2 x] + x Log[n])]

Sum[j^(-1/2) Cos[14. Log[j]], {j, 1, Infinity}]

$Aborted

Sin[N@Im@ZetaZero@1 Log[600 000]]

-0.423671

ap[n_, x_] := Tan[x Log[n] + ArcCot[2 x]]
```

```
Plot[ap[n, N@Im@ZetaZero@1], {n, 1, 100}]
```




```
ext[n_, x_] := Sum[j^(-1/2)
    (((1/2) (j^(I x) + j^(-I x))) + Tan[x Log[n] + ArcCot[2 x]] Sin[x Log[j]]), {j, 1, n}]
ext2[n_, x_] := Sum[j^(-1/2) (((1/2) (j^(I x) + j^(-I x))) +
      Tan[x Log[n] + ArcCot[2 x]] ((1/(2 I)) (j^(I x) - j^(-I x)))), {j, 1, n}]
ext3[n_, x_] := Sum[(((1/2) (j^(-1/2 + I x) + j^(-1/2 - I x))) +
     Tan[x Log[n] + ArcCot[2 x]] ((1/(2 I)) (j^(-1/2 + I x) - j^(-1/2 - I x)))), {j, 1, n}]
ext4[n_, x_] := (1/2) (HarmonicNumber[n, 1/2 - I x] + HarmonicNumber[n, 1/2 + I x]) +
   Sum[(Tan[x Log[n] + ArcCot[2 x]] ((1/(2 I)) (j^(-1/2 + I x) - j^(-1/2 - I x)))), {j, 1, n}]
ext5[n_, x_] := (1/2) (HarmonicNumber[n, 1/2 - I x] + HarmonicNumber[n, 1/2 + I x]) +
   (Tan[x Log[n] + ArcCot[2 x]]
     ((1/(2 I)) (HarmonicNumber[n, (1/2 - I x)] - HarmonicNumber[n, (1/2 + I x)])))
```

$$\text{ext6[n\_, x\_]} := \frac{1}{2}\left(\text{HarmonicNumber}\left[n, \frac{1}{2} - i\, x\right] (1 - i\, \text{Tan}[\text{ArcCot}[2\, x] + x\, \text{Log}[n]]) + \text{HarmonicNumber}\left[n, \frac{1}{2} + i\, x\right] (1 + i\, \text{Tan}[\text{ArcCot}[2\, x] + x\, \text{Log}[n]])\right)$$

$$\text{ext6a[n\_, x\_]} := \left\{\frac{1}{2}\, \text{HarmonicNumber}\left[n, \frac{1}{2} - i\, x\right], +\frac{1}{2}\, \text{HarmonicNumber}\left[n, \frac{1}{2} + i\, x\right], -\frac{1}{2}\, i\, \text{HarmonicNumber}\left[n, \frac{1}{2} - i\, x\right] \text{Tan}[\text{ArcCot}[2\, x] + x\, \text{Log}[n]], +\frac{1}{2}\, i\, \text{HarmonicNumber}\left[n, \frac{1}{2} + i\, x\right] \text{Tan}[\text{ArcCot}[2\, x] + x\, \text{Log}[n]]\right\}$$

```
ext6a[100 000 000 000, N@Im@ZetaZero@1]
```

{-1068.46 - 11 128. i, -1068.46 + 11 128. i, 1068.46 - 102.589 i, 1068.46 + 102.589 i}

```
Zeta[.4 + N@Im@ZetaZero@1 I]
```

-0.0814815 - 0.013674 i

```
FullSimplify[(1/2) (HarmonicNumber[n, 1/2 - I x] + HarmonicNumber[n, 1/2 + I x]) +
   (Tan[x Log[n] + ArcCot[2 x]]
     ((1/(2 I)) (HarmonicNumber[n, (1/2 - I x)] - HarmonicNumber[n, (1/2 + I x)])))]
```

$$\frac{1}{2}\left(\text{HarmonicNumber}\left[n, \frac{1}{2} - i\, x\right] (1 - i\, \text{Tan}[\text{ArcCot}[2\, x] + x\, \text{Log}[n]]) + \text{HarmonicNumber}\left[n, \frac{1}{2} + i\, x\right] (1 + i\, \text{Tan}[\text{ArcCot}[2\, x] + x\, \text{Log}[n]])\right)$$

```
Expand[1/2 (HarmonicNumber[n, 1/2 - ⅈ x] (1 - ⅈ Tan[ArcCot[2 x] + x Log[n]]) +
    HarmonicNumber[n, 1/2 + ⅈ x] (1 + ⅈ Tan[ArcCot[2 x] + x Log[n]]))]
```

$$\frac{1}{2}\,\text{HarmonicNumber}\left[n, \frac{1}{2} - i\,x\right] + \frac{1}{2}\,\text{HarmonicNumber}\left[n, \frac{1}{2} + i\,x\right] - \frac{1}{2}\,i\,\text{HarmonicNumber}\left[n, \frac{1}{2} - i\,x\right]\,\text{Tan}[\text{ArcCot}[2\,x] + x\,\text{Log}[n]] + \frac{1}{2}\,i\,\text{HarmonicNumber}\left[n, \frac{1}{2} + i\,x\right]\,\text{Tan}[\text{ArcCot}[2\,x] + x\,\text{Log}[n]]$$

```
ext6[100 000 000 000 000 000 000, 10. + .1 I]
```

1.50989 + 0.115354 ⅈ

```
Zeta[.6 + 10 I]
```

1.50992 - 0.115339 ⅈ

```
xx[n_, x_] :=
 Sum[(Cos[ArcCot[2 x] + x Log[n / j]] / j^(1 / 2)) / Cos[ArcCot[2 x] + x Log[n]], {j, 1, n}]
xx2[n_, x_] := Sum[(Cos[ArcCot[2 x / I] + x / I Log[n / j]] / j^(1 / 2)) /
   Cos[ArcCot[2 x / I] + x / I Log[n]], {j, 1, n}]
xx3[n_, x_] := Sum[(Cosh[ArcCoth[2 x] - x Log[n / j]] / j^(1 / 2))
   Sech[ArcCoth[2 x] - x Log[n]], {j, 1, n}]
xx4[n_, x_] := Sum[N[(Cosh[x Log[n / j] - ArcCoth[2 x]] / j^(1 / 2))
    Sech[x Log[n] - ArcCoth[2 x]]], {j, 1, n}]
xx4a[n_, x_] := Sum[N[(Cosh[x Log[n / j] - ((1 / 2) Log[(2 x + 1) / (2 x - 1)])] / j^(1 / 2))
    Sech[x Log[n] - ((1 / 2) Log[(2 x + 1) / (2 x - 1)])]], {j, 1, n}]
xx5[n_, x_] := Sum[(Cosh[x Log[n / j] - ArcCoth[2 x]] / j^(1 / 2)), {j, 1, n}]
xx6[n_, x_] :=
 Sum[(Cosh[x Log[n / j] - ((1 / 2) Log[(2 x + 1) / (2 x - 1)])] / j^(1 / 2)), {j, 1, n}]
xx7[n_, x_] := Sum[(Cosh[x Log[n / j] - ArcCoth[2 x]] (n / j)^(1 / 2)), {j, 1, n}]
```

```
xx7[1000, N@ZetaZero@1 - 1 / 2 + .1]
```

-1.09019 - 1.29846 ⅈ

```
Zeta[.77 + 190 I]
```

1.75954 + 0.976522 ⅈ

```
ArcCot[2 x I]
```

-ⅈ ArcCoth[2 x]

```
Cos[I x]
```

Cosh[x]

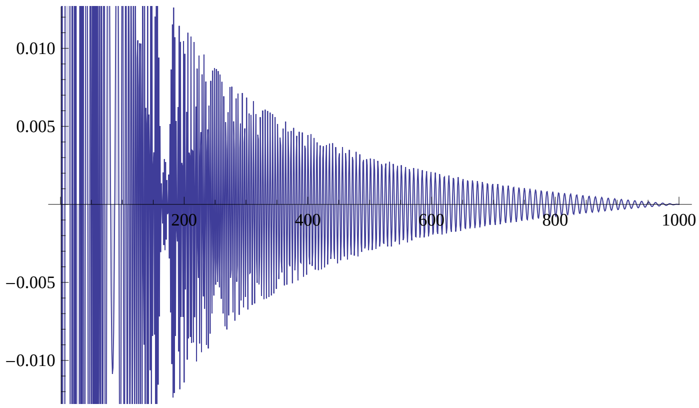
```
(.3 I + 30) / I
```

0.3 - 30. ⅈ

```
(Cos[ArcCot[2 x / I] + x / I Log[n / j]] / j^(1 / 2)) / Cos[ArcCot[2 x / I] + x / I Log[n]]
```

$$\frac{\text{Cosh}\left[\text{ArcCoth}[2\,x] - x\,\text{Log}\left[\frac{n}{j}\right]\right]\,\text{Sech}[\text{ArcCoth}[2\,x] - x\,\text{Log}[n]]}{\sqrt{j}}$$

```
DiscretePlot[
 Im[ Cosh[(sss = .1 + 530 I) Log[1000 / j] + ArcCoth[2 (sss)]] / j^(1 / 2)], {j, 1, 1000}]
```




```
x Log[n / j] - ((1 / 2) Log[(2 x + 1) / (2 x - 1)])
```

```
Log[(n / j)^x / (((2 x + 1) / (2 x - 1))^(1 / 2))]
```

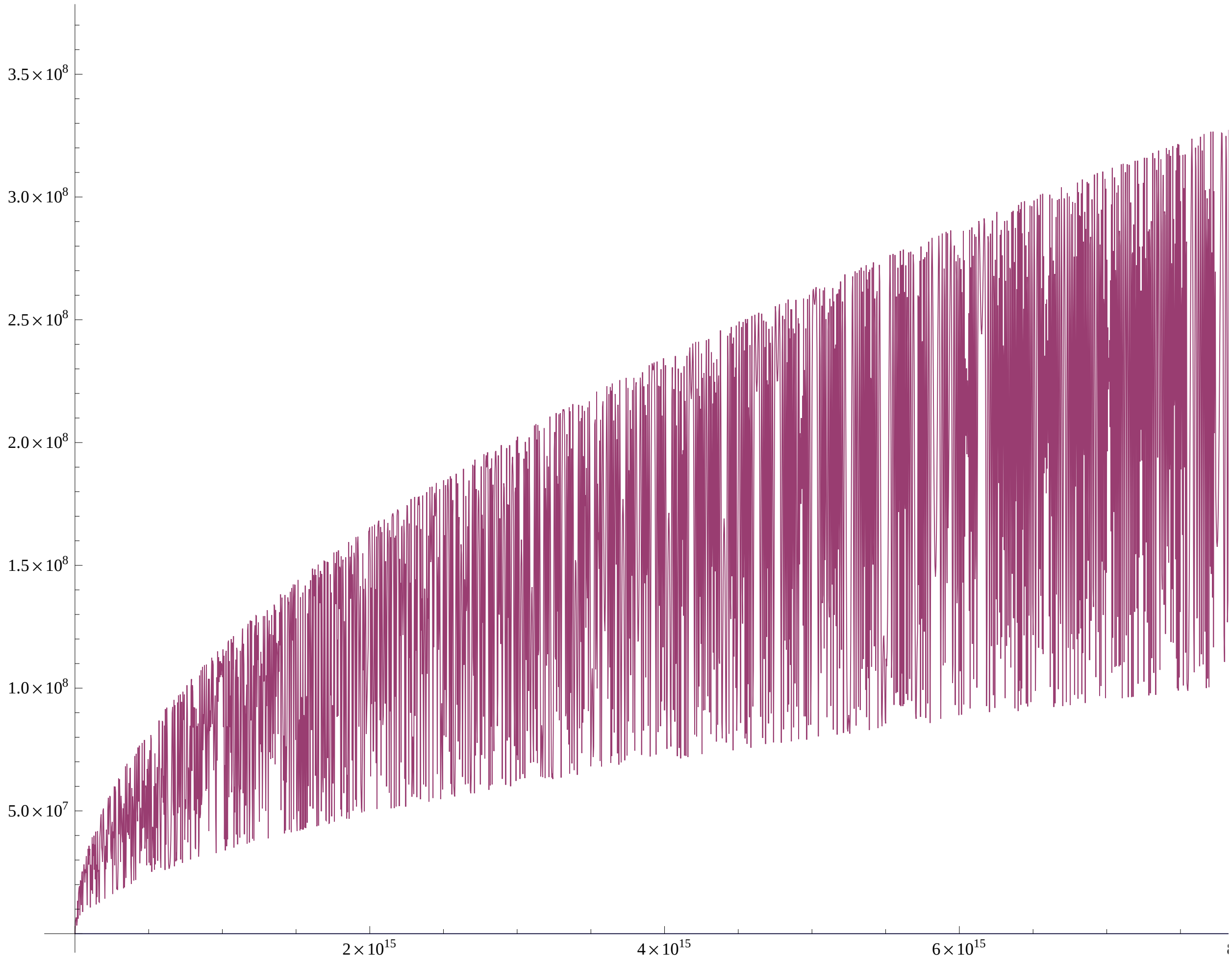
$$\text{Log}\left[\frac{\left(\frac{n}{j}\right)^x}{\sqrt{\frac{1+2\,x}{-1+2\,x}}}\right]$$

```
Log[(n / j)^x (((2 x + 1) / (2 x - 1))^(-1 / 2))]
```

$$\text{Log}\left[\frac{\left(\frac{n}{j}\right)^x}{\sqrt{\frac{1+2\,x}{-1+2\,x}}}\right]$$

```
xx8[n_, x_] :=
 Sum[(E^(x Log[n / j] - ArcCoth[2 x]) + E^(-(x Log[n / j] - ArcCoth[2 x]))) / 2 ((n / j) ^ (1 / 2)),
  {j, 1, n}]
xx9[n_, x_] := Sum[(E^(x Log[n / j]) E^-ArcCoth[2 x] + E^(-x Log[n / j]) E^ArcCoth[2 x]) / 2
   ((n / j) ^ (1 / 2)), {j, 1, n}]
xx10[n_, x_] := Sum[((n / j) ^x E^-ArcCoth[2 x] + (n / j) ^-x E^ArcCoth[2 x]) / 2 ((n / j) ^ (1 / 2)),
  {j, 1, n}]
xx11[n_, x_] := Sum[((n / j) ^ (x + 1 / 2) E^-ArcCoth[2 x] + (n / j) ^ (-x + 1 / 2) E^ArcCoth[2 x]) / 2,
  {j, 1, n}]
xx12[n_, x_] := E^-ArcCoth[2 x] / 2 Sum[((n / j) ^ (x + 1 / 2) ), {j, 1, n}] +
  E^ArcCoth[2 x] / 2 Sum[((n / j) ^ (-x + 1 / 2) ), {j, 1, n}]
xx13[n_, x_] := E^-ArcCoth[2 x] / 2 n^ (x + 1 / 2) Sum[j^ (-x - 1 / 2) , {j, 1, n}] +
  E^ArcCoth[2 x] / 2 n^ (-x + 1 / 2) Sum[j^ (x - 1 / 2) , {j, 1, n}]
xx14[n_, x_] := E^-ArcCoth[2 x] / 2 n^ (x + 1 / 2) HarmonicNumber[n, x + 1 / 2] +
  E^ArcCoth[2 x] / 2 n^ (-x + 1 / 2) HarmonicNumber[n, -x + 1 / 2]
xx15[n_, x_] := E^-ArcCoth[2 x] / 2 n^ (x + 1 / 2) (Zeta[x + 1 / 2] - Zeta[x + 1 / 2, n + 1]) +
  E^ArcCoth[2 x] / 2 n^ (-x + 1 / 2) (Zeta[-x + 1 / 2] - Zeta[-x + 1 / 2, n + 1])
xx16[n_, x_] := E^-ArcCoth[2 x] / 2 n^ (1 / 2 + x) (Zeta[1 / 2 + x]) +
  E^ArcCoth[2 x] / 2 n^ (1 / 2 - x) (Zeta[1 / 2 - x])

xx14[10 000 000, .2 + 10 I]

-35 099.5 - 47 114.8 i

xx16[10 000 000, .2 + 10 I]

-35 100. - 47 114.8 i
```

```
Plot[{-100, Abs[xx14[n, 140000 ⅈ + .01]]}, {n, 1, 10000000000000000}]
```



```
Plot[{-100, Abs[xx16[n, 140 000 i + .01]]}, {n, 1, 10 000 000 000 000 000}]
```