$$(1-x^{1-s})\zeta(s)=\lim_{n\to\infty}\left(\sum_{j=1}^{\infty}\left(\frac{1}{j^s}-\frac{1}{(j+n\cdot x)^s}\right)-x^{1-s}\cdot\sum_{1\le j\le n}\frac{1}{j^s}\right),\Re(s)>0$$

and

$$(1-x^{1-s})\zeta(s)=\lim_{n\to\infty}\left(\sum_{j=1}^{n}\frac{1}{j^s}-x^{1-s}\cdot\sum_{j=1}^{\infty}\frac{1}{j^s}-\frac{1}{(j+n\cdot x^{-1})^s}\right),\Re(s)>0$$

Taking derivatives of different orders at x=1 for each of these gives different results.

For the kth derivative taken at x = 1 of the first one, the following identity results:

$$\zeta(s)=\lim_{n\to\infty}\sum_{j=1}^{n}\frac{1}{j^s}+n^k\cdot\left(\frac{s-1+k}{s-1}\right)\cdot\left(\zeta(s+k)-\sum_{j=1}^{n}\frac{1}{j^{s+k}}\right),\Re(s)>0$$

Could fractional derivatives expand this identity?

For the kth derivative taken at x = 1 of the second one, the following identity results:

$$\zeta(s)=\lim_{n\to\infty}\left(\sum_{j=1}^{n}\frac{1}{j^s}-\sum_{k=1}^{t}(-1)^k\cdot\binom{t}{k}\cdot\left(1+\frac{k}{s-1}\right)\cdot n^k\left(\zeta(s+k)-\sum_{j=1}^{n}\frac{1}{j^{s+k}}\right)\right),\Re(s)>-t+1$$

$$0=\lim_{n\to\infty} \zeta(s)-\sum_{j=1}^{n}\frac{1}{j^{s}}-n^{k}\cdot(1+\frac{k}{s-1})\cdot(\zeta(s+k)-\sum_{j=1}^{n}\frac{1}{j^{s+k}}), \Re(s)>0$$

$$0=\lim_{n\to\infty} n^{m}\cdot(1+\frac{m}{s-1})\cdot(\zeta(s+m)-\sum_{j=1}^{n}\frac{1}{j^{s+m}})-n^{k}\cdot(1+\frac{k}{s-1})\cdot(\zeta(s+k)-\sum_{j=1}^{n}\frac{1}{j^{s+k}}), \Re(s)>0$$

$$0=\lim_{n\to\infty} n^{m}\cdot(s-1+m)\cdot(\zeta(s+m)-\sum_{j=1}^{n}\frac{1}{j^{s+m}})-n^{k}\cdot(s-1+k)\cdot(\zeta(s+k)-\sum_{j=1}^{n}\frac{1}{j^{s+k}}), \Re(s)>0$$

Can s be 0 here? If not, why not? Just can't.

$$\zeta(s)-x^{1-s}\zeta(s)=\lim_{n\to\infty}\left(\sum_{j=1}^{\infty}\left(\frac{1}{j^{s}}-\frac{1}{(j+n\cdot x)^{s}}\right)-x^{1-s}\cdot\sum_{1\le j\le n}\frac{1}{j^{s}}\right),\Re(s)>0$$

Taking the derivative with respect to x, this is the same as starting with

$$-x^{1-s}\zeta(s)=\lim_{n\to\infty}\left(\sum_{j=1}^{\infty}\left(-(j+n\cdot x)^{-s}\right)-x^{1-s}\cdot\sum_{1\le j\le n}\frac{1}{j^{s}}\right),\Re(s)>0$$

Then, after derivative with respect to x,

$$\zeta(s)=\lim_{n\to\infty}\sum_{j=1}^{n}\frac{1}{j^{s}}+n^{k}\cdot\left(\frac{s-1+k}{s-1}\right)\cdot\left(\zeta(s+k)-\sum_{j=1}^{n}\frac{1}{j^{s+k}}\right),\Re(s)>0$$

$$(1-x^{1-s})\zeta(s)=\lim_{n\to\infty}(\sum_{j=1}^{\infty}(\frac{1}{j^s}-\frac{1}{(j+n\cdot x)^s})-x^{1-s}\cdot\sum_{1\le j\le n}\frac{1}{j^s}),\Re(s)>0$$

...

$$\zeta(s)=\lim_{n\to\infty}\sum_{j=1}^{n}\frac{1}{j^s}+n^k\cdot(\frac{s-1+k}{s-1})\cdot(\zeta(s+k)-\sum_{j=1}^{n}\frac{1}{j^{s+k}}),\Re(s)>0$$

...

$$\frac{\partial^z}{\partial x^z}(1-x^{1-s})\zeta(s)=(1-s)\frac{\Gamma(1-s)}{\Gamma(2-s-z)}\cdot x^{1-s-z}\cdot\zeta(s)$$

$$\frac{\partial^z}{\partial x^z}\sum_{j=1}^{\infty}(\frac{1}{j^s}-\frac{1}{(j+n\cdot x)^s})=-n^z\cdot\frac{\Gamma(1-s)}{\Gamma(1-s-z)}\cdot\sum_{j=1}^{\infty}(j+n\,x)^{-s-z}$$

$$\frac{\partial^z}{\partial x^z}-x^{1-s}\cdot\sum_{1\le j\le n}\frac{1}{j^s}=(1-s)\frac{\Gamma(1-s)}{\Gamma(2-s-z)}\cdot x^{1-s-z}\cdot\sum_{1\le j\le n}\frac{1}{j^s}$$

...

$$\zeta(s)=\lim_{n\to\infty}\sum_{1\le j\le n}\frac{1}{j^s}-((1-s)\frac{\Gamma(1-s)}{\Gamma(2-s-z)}\cdot x^{1-s-z})^{-1}\cdot n^z\cdot\frac{\Gamma(1-s)}{\Gamma(1-s-z)}\cdot\sum_{j=1}^{\infty}(j+n\,x)^{-s-z}$$

$$\zeta(s)=\lim_{n\to\infty}\sum_{1\le j\le n}\frac{1}{j^s}-\frac{1}{1-s}\cdot\frac{n^z}{x^{1-s-z}}\cdot\frac{\Gamma(2-s-z)}{\Gamma(1-s-z)}\cdot\sum_{j=1}^{\infty}(j+n\,x)^{-s-z}$$

$$\zeta(s)=\lim_{n\to\infty}\sum_{1\le j\le n}\frac{1}{j^s}-\frac{n^z}{x^{1-s-z}}\cdot\frac{1-s-z}{1-s}\cdot\sum_{j=1}^{\infty}(j+n\,x)^{-s-z}$$

$$\zeta(s)=\lim_{n\to\infty}\sum_{1\le j\le n}\frac{1}{j^s}-n^z\cdot(1-\frac{z}{1-s})\cdot\sum_{j=1}^{\infty}(j+n)^{-s-z}$$