```
g[v_] := {Expand[Limit[ Sum[v[k, dx], {k, Log[dx + 1, x], Infinity}], dx → 0]],
  Expand[Limit[ Sum[v[k, dx], {k, 0, Log[dx + 1, x]}], dx → 0]],
  Expand[Limit[ Sum[ v[k, dx], {k, -Infinity, -Log[dx + 1, x]}], dx → 0]],
  Expand[Limit[ Sum[ v[k, dx], {k, -Log[dx + 1, x], 0}], dx → 0]],
  Expand[Limit[ Sum[ v[k, dx], {k, -Infinity, 0}], dx → 0]],
  Expand[Limit[ Sum[ v[k, dx], {k, 0, Infinity}], dx → 0]]}
```

```
r[k_, dx_] := dx (dx + 1) ^k; g[r]
```

$$\left\{-x,\ -1+x,\ \frac{1}{x},\ 1-\frac{1}{x},\ 1,\ -1\right\}$$

```
r[k_, dx_] := dx (dx + 1) ^-k; g[r]
```

$$\left\{\frac{1}{x},\ 1-\frac{1}{x},\ -x,\ -1+x,\ -1,\ 1\right\}$$

```
r[k_, dx_] := dx (dx + 1) ^(2 k); g[r]
```

$$\left\{-\frac{x^2}{2},\ -\frac{1}{2}+\frac{x^2}{2},\ \frac{1}{2x^2},\ \frac{1}{2}-\frac{1}{2x^2},\ \frac{1}{2},\ -\frac{1}{2}\right\}$$

```
r[k_, dx_] := dx (dx + 1) ^(-2 k); g[r]
```

$$\left\{\frac{1}{2x^2},\ \frac{1}{2}-\frac{1}{2x^2},\ -\frac{x^2}{2},\ -\frac{1}{2}+\frac{x^2}{2},\ -\frac{1}{2},\ \frac{1}{2}\right\}$$

```
r[k_, dx_] := dx (dx + 1) ^(3 k); g[r]
```

$$\left\{-\frac{x^3}{3},\ -\frac{1}{3}+\frac{x^3}{3},\ \frac{1}{3x^3},\ \frac{1}{3}-\frac{1}{3x^3},\ \frac{1}{3},\ -\frac{1}{3}\right\}$$

```
r[k_, dx_] := dx (dx + 1) ^(4 k); g[r]
```

$$\left\{-\frac{x^4}{4},\ -\frac{1}{4}+\frac{x^4}{4},\ \frac{1}{4x^4},\ \frac{1}{4}-\frac{1}{4x^4},\ \frac{1}{4},\ -\frac{1}{4}\right\}$$

```
r[k_, dx_] := dx (dx + 1) ^(s k); g[r]
```

$$\left\{-\frac{x^s}{s},\ -\frac{1}{s}+\frac{x^s}{s},\ \frac{x^{-s}}{s},\ \frac{1}{s}-\frac{x^{-s}}{s},\ \frac{1}{s},\ -\frac{1}{s}\right\}$$

```
{Expand[Limit[ Sum[ dx (dx + 1) ^(-s k), {k, 0, Log[dx + 1, x]}], dx → 0]],
 Expand[Integrate[E^(-s t), {t, 0, Log[x]}]]}
```

$$\left\{\frac{1}{s}-\frac{x^{-s}}{s},\ \frac{1}{s}-\frac{x^{-s}}{s}\right\}$$

```
{Integrate[E^(-s x), {x, 0, Infinity}],
 Expand[Limit[ Sum[ dx (dx + 1) ^(-s k), {k, 0, Infinity}], dx → 0]]}
```

$$\left\{\text{ConditionalExpression}\left[\frac{1}{s},\ \text{Re}[s]>0\right],\ \frac{1}{s}\right\}$$

```
{Expand[Limit[ Sum[ dx (dx + 1) ^(-s k), {k, Log[dx + 1, x], Infinity}], dx → 0]],
 Expand[Integrate[E^(-s t), {t, Log[x], Infinity}]]}
```

$$\left\{\frac{x^{-s}}{s},\ \text{ConditionalExpression}\left[\frac{x^{-s}}{s},\ \text{Re}[s]>0\right]\right\}$$

```
r[k_, dx_] := dx; g[r]
```

$$\left\{\text{Limit}\left[\sum_{k=\frac{\text{Log}[x]}{\text{Log}[1+dx]}}^{\infty} dx,\ dx\to 0\right],\ \text{Log}[x],\ \text{Limit}\left[\sum_{k=-\infty}^{-\frac{\text{Log}[x]}{\text{Log}[1+dx]}} dx,\ dx\to 0\right],\ \text{Log}[x],\ \text{Limit}\left[\sum_{k=-\infty}^{0} dx,\ dx\to 0\right],\ \text{Limit}\left[\sum_{k=0}^{\infty} dx,\ dx\to 0\right]\right\}$$

```
r[k_, dx_] := k dx^2; g[r]
```

$$\left\{\text{Limit}\left[\sum_{k=\frac{\text{Log}[x]}{\text{Log}[1+dx]}}^{\infty} dx^2\, k,\ dx\to 0\right],\ \frac{\text{Log}[x]^2}{2},\ \text{Limit}\left[\sum_{k=-\infty}^{-\frac{\text{Log}[x]}{\text{Log}[1+dx]}} dx^2\, k,\ dx\to 0\right],\ -\frac{1}{2}\text{Log}[x]^2,\ \text{Limit}\left[\sum_{k=-\infty}^{0} dx^2\, k,\ dx\to 0\right],\ \text{Limit}\left[\sum_{k=0}^{\infty} dx^2\, k,\ dx\to 0\right]\right\}$$

```
r[k_, dx_] := dx^1 (dx + 1) ^k k^0; g[r]
```

$$\left\{-x,\ -1+x,\ \frac{1}{x},\ 1-\frac{1}{x},\ 1,\ -1\right\}$$

```
r[k_, dx_] := dx^2 (dx + 1) ^k k^1; g[r]
```

$$\left\{x - x\,\text{Log}[x],\ 1 - x + x\,\text{Log}[x],\ -\frac{1}{x} - \frac{\text{Log}[x]}{x},\ -1 + \frac{1}{x} + \frac{\text{Log}[x]}{x},\ -1,\ 1\right\}$$

```
r[k_, dx_] := dx^3 (dx + 1) ^k k^2; g[r]
```

$$\left\{-2x + 2x\,\text{Log}[x] - x\,\text{Log}[x]^2,\ -2 + 2x - 2x\,\text{Log}[x] + x\,\text{Log}[x]^2,\ \frac{2}{x} + \frac{2\,\text{Log}[x]}{x} + \frac{\text{Log}[x]^2}{x},\ 2 - \frac{2}{x} - \frac{2\,\text{Log}[x]}{x} - \frac{\text{Log}[x]^2}{x},\ 2,\ -2\right\}$$

```
r[k_, dx_] := dx^4 (dx + 1) ^k k^3; g[r]
```

$$\left\{6x - 6x\,\text{Log}[x] + 3x\,\text{Log}[x]^2 - x\,\text{Log}[x]^3,\ 6 - 6x + 6x\,\text{Log}[x] - 3x\,\text{Log}[x]^2 + x\,\text{Log}[x]^3,\ -\frac{6}{x} - \frac{6\,\text{Log}[x]}{x} - \frac{3\,\text{Log}[x]^2}{x} - \frac{\text{Log}[x]^3}{x},\ -6 + \frac{6}{x} + \frac{6\,\text{Log}[x]}{x} + \frac{3\,\text{Log}[x]^2}{x} + \frac{\text{Log}[x]^3}{x},\ -6,\ 6\right\}$$

```
r[k_, dx_] := dx^s (dx + 1)^k k^(s - 1); g[r]
```

$$\left\{\text{Limit}\left[dx^s\, x\, \text{HurwitzLerchPhi}\left[1+dx, 1-s, \frac{\text{Log}[x]}{\text{Log}[1+dx]}\right], dx\to 0\right],\right.$$
$$\text{Limit}\left[-dx^s\left(x\,\text{LerchPhi}\left[1+dx, 1-s, 1+\frac{\text{Log}[x]}{\text{Log}[1+dx]}\right]+\right.\right.$$
$$\left.\left. dx\, x\, \text{LerchPhi}\left[1+dx, 1-s, 1+\frac{\text{Log}[x]}{\text{Log}[1+dx]}\right]-\text{PolyLog}[1-s, 1+dx]\right), dx\to 0\right],$$
$$\text{Limit}\left[-\frac{(-1)^s dx^s\, \text{HurwitzLerchPhi}\left[\frac{1}{1+dx}, 1-s, \frac{\text{Log}[x]}{\text{Log}[1+dx]}\right]}{x}, dx\to 0\right],$$
$$\text{Limit}\left[-\frac{dx^s\left(-\text{LerchPhi}\left[1+dx, 1-s, -\frac{\text{Log}[x]}{\text{Log}[1+dx]}\right]+x\,\text{PolyLog}[1-s, 1+dx]\right)}{x}, dx\to 0\right],$$
$$\text{Limit}\left[-(-1)^s dx^s\, \text{HurwitzLerchPhi}\left[\frac{1}{1+dx}, 1-s, 0\right], dx\to 0\right],$$
$$\left.\text{Limit}[dx^s\, \text{HurwitzLerchPhi}[1+dx, 1-s, 0], dx\to 0]\right\}$$

```
r[k_, dx_] := dx^1 (dx + 1)^(-2 k) k^0; g[r]
```

$$\left\{\frac{1}{2x^2}, \frac{1}{2}-\frac{1}{2x^2}, -\frac{x^2}{2}, -\frac{1}{2}+\frac{x^2}{2}, -\frac{1}{2}, \frac{1}{2}\right\}$$

```
r[k_, dx_] := dx^2 (dx + 1)^(-2 k) k^1; g[r]
```

$$\left\{\frac{1}{4x^2}+\frac{\text{Log}[x]}{2x^2}, \frac{1}{4}-\frac{1}{4x^2}-\frac{\text{Log}[x]}{2x^2}, -\frac{x^2}{4}+\frac{1}{2}x^2\,\text{Log}[x], -\frac{1}{4}+\frac{x^2}{4}-\frac{1}{2}x^2\,\text{Log}[x], -\frac{1}{4}, \frac{1}{4}\right\}$$

```
r[k_, dx_] := dx^3 (dx + 1)^(-2 k) k^2; g[r]
```

$$\left\{\frac{1}{4x^2}+\frac{\text{Log}[x]}{2x^2}+\frac{\text{Log}[x]^2}{2x^2}, \frac{1}{4}-\frac{1}{4x^2}-\frac{\text{Log}[x]}{2x^2}-\frac{\text{Log}[x]^2}{2x^2},\right.$$
$$\left.-\frac{x^2}{4}+\frac{1}{2}x^2\,\text{Log}[x]-\frac{1}{2}x^2\,\text{Log}[x]^2, -\frac{1}{4}+\frac{x^2}{4}-\frac{1}{2}x^2\,\text{Log}[x]+\frac{1}{2}x^2\,\text{Log}[x]^2, -\frac{1}{4}, \frac{1}{4}\right\}$$

```
r[k_, dx_] := dx^1 (dx + 1)^(2 k) k^0; g[r]
```

$$\left\{-\frac{x^2}{2}, -\frac{1}{2}+\frac{x^2}{2}, \frac{1}{2x^2}, \frac{1}{2}-\frac{1}{2x^2}, \frac{1}{2}, -\frac{1}{2}\right\}$$

```
r[k_, dx_] := dx^2 (dx + 1)^(2 k) k^1; g[r]
```

$$\left\{\frac{x^2}{4}-\frac{1}{2}x^2\,\text{Log}[x], \frac{1}{4}-\frac{x^2}{4}+\frac{1}{2}x^2\,\text{Log}[x], -\frac{1}{4x^2}-\frac{\text{Log}[x]}{2x^2}, -\frac{1}{4}+\frac{1}{4x^2}+\frac{\text{Log}[x]}{2x^2}, -\frac{1}{4}, \frac{1}{4}\right\}$$

```
r[k_, dx_] := dx^3 (dx + 1)^(2 k) k^2; g[r]
```

$$\left\{-\frac{x^2}{4}+\frac{1}{2}x^2\,\text{Log}[x]-\frac{1}{2}x^2\,\text{Log}[x]^2, -\frac{1}{4}+\frac{x^2}{4}-\frac{1}{2}x^2\,\text{Log}[x]+\frac{1}{2}x^2\,\text{Log}[x]^2,\right.$$
$$\left.\frac{1}{4x^2}+\frac{\text{Log}[x]}{2x^2}+\frac{\text{Log}[x]^2}{2x^2}, \frac{1}{4}-\frac{1}{4x^2}-\frac{\text{Log}[x]}{2x^2}-\frac{\text{Log}[x]^2}{2x^2}, \frac{1}{4}, -\frac{1}{4}\right\}$$