

```
ClearAll["Global`*"]
```

```
ClearAll::clloc: Cannot clear local variable j. >>
```

```
E2[n_, k_, b_] :=
  E2[n, k, b] = Sum[ E2[n / j, k - 1, b], {j, 2, n}] - b Sum[ E2[n / (b j), k - 1, b], {j, 1, n / b}];
E2[n_, 0, a_] := 1
D1[n_, k_, b_] := Sum[ Binomial[k + j - 1, k - 1] b^j
  Sum[FactorialPower[k, a] / a! E2[n / b^j, a, b], {a, 0, Log[If[b > 2, 2, b], n / b^j]}],
  {j, 0, Log[b, n]}]
M2[n_, b_] := Sum[ (-1)^k ( E2[n, k, b] - b E2[n / b, k, b]), {k, 0, Log[b, n]}]
P[n_, b_] := Sum[ ((-1)^(k + 1) E2[n, k, b] + b^k) / k, {k, 1, Log[b, n]}]
```

```
EL2[n_, k_, b_] :=
  Sum[ EL2[ n / j, k - 1, b], {j, 2, n}] - b Sum[ EL2[ n / (j b), k - 1, b], {j, 1, n / b}]
EL2[n_, 1, b_] := Sum[ Log[j], {j, 2, n}] - b Sum[ Log[j b], {j, 1, n / b}]
cheb[n_, b_] := Sum[ (-1)^(k + 1) EL2[n, k, b] + Log[b] b^k, {k, 1, Log[b, n]}]
```

```
N[{M2[100, 2], P[100, 2], cheb[100, 2]}]
```

```
{1., 28.5333, 94.0453}
```

```
et[n_, b_, a_] :=
  Binomial[a, 4] b^ (a - 4) Sum[ 1, {j, 2, n}, {k, 2, n / j}, {l, 2, n / (j k)}, {m, 2, n / (j k l)}] -
  Binomial[a, 3] b^ (a - 3)
  Sum[ 1, {j, 2, n}, {k, 2, n / j}, {l, 2, n / (j k)}, {m, 1, n / (j k l b)}] + Binomial[a, 2]
  b^ (a - 2) Sum[ 1, {j, 2, n}, {k, 2, n / j}, {l, 1, n / (j k b)}, {m, 1, n / (j k l b^2)}] -
  Binomial[a, 1] b^ (a - 1) Sum[ 1, {j, 2, n}, {k, 1, n / (b j)},
  {l, 1, n / (j k b^2)}, {m, 1, n / (j k l b^3)}] + Binomial[a, 0] b^a
  Sum[ 1, {j, 1, n / b}, {k, 1, n / (b^2 j)}, {l, 1, n / (j k b^3)}, {m, 1, n / (j k l b^4)}]
```

```
et[30, 3 / 2, 4]
```

```

$$-\frac{481}{16}$$

```

```
E2[30, 4, 3 / 2]
```

```

$$-\frac{481}{16}$$

```

```

es[n_, b_, a_] := Binomial[a, 2] b^ (a - 2) Sum[1, {j, 2, n}, {k, 2, n / j}] -
  Binomial[a, 1] b^ (a - 1) Sum[1, {j, 2, n}, {k, 1, n / (b j)}] +
  Binomial[a, 0] b^ a Sum[1, {j, 1, n / b}, {k, 1, n / (b^2 j)}]
es3[n_, b_, a_] := -3 b^1 Sum[1, {j, 2, n}, {k, 2, n / j}, {m, 1, n / (j k b)}] +
  3 b^2 Sum[1, {j, 2, n}, {k, 1, n / (b j)}, {m, 1, n / (j k b^2)}] -
  1 b^3 Sum[1, {j, 1, n / b}, {k, 1, n / (b^2 j)}, {m, 1, n / (j k b^3)}]
es4[n_, b_, a_] := 6 b^2 Sum[1, {j, 2, n}, {k, 2, n / j},
  {m, 1, n / (j k b)}, {l, 1, n / (j k m b^2)}] -
  4 b^3 Sum[1, {j, 2, n}, {k, 1, n / (b j)}, {m, 1, n / (j k b^2)}, {l, 1, n / (j k m b^3)}] +
  1 b^4 Sum[1, {j, 1, n / b}, {k, 1, n / (b^2 j)}, {m, 1, n / (j k b^3)}, {l, 1, n / (j k m b^4)}]
es5[n_, b_, a_] := -10 b^3 Sum[1, {j, 2, n}, {k, 2, n / j},
  {m, 1, n / (j k b)}, {l, 1, n / (j k m b^2)}, {s, 1, n / (j k m l b^3)}] +
  5 b^4 Sum[1, {j, 2, n}, {k, 1, n / (b j)}, {m, 1, n / (j k b^2)}, {l, 1, n / (j k m b^3)},
  {s, 1, n / (l j k m b^4)}] - 1 b^5 Sum[1, {j, 1, n / b}, {k, 1, n / (b^2 j)},
  {m, 1, n / (j k b^3)}, {l, 1, n / (j k m b^4)}, {s, 1, n / (j k m l b^5)}]

esa[n_, b_, a_] := {Binomial[a, 2] b^ (a - 2) Sum[1, {j, 2, n}, {k, 2, n / j}],
  -Binomial[a, 1] b^ (a - 1) Sum[1, {j, 2, n}, {k, 1, n / (b j)}],
  + Binomial[a, 0] b^ a Sum[1, {j, 1, n / b}, {k, 1, n / (b^2 j)}]}
es3a[n_, b_] := {-3 b^1 Sum[1, {j, 2, n}, {k, 2, n / j}, {m, 1, n / (j k b)}],
  3 b^2 Sum[1, {j, 2, n}, {k, 1, n / (b j)}, {m, 1, n / (j k b^2)}],
  - 1 b^3 Sum[1, {j, 1, n / b}, {k, 1, n / (b^2 j)}, {m, 1, n / (j k b^3)}]}
es4a[n_, b_] := {6 b^2 Sum[1, {j, 2, n}, {k, 2, n / j},
  {m, 1, n / (j k b)}, {l, 1, n / (j k m b^2)}], -4 b^3
  Sum[1, {j, 2, n}, {k, 1, n / (b j)}, {m, 1, n / (j k b^2)}, {l, 1, n / (j k m b^3)}], 1 b^4
  Sum[1, {j, 1, n / b}, {k, 1, n / (b^2 j)}, {m, 1, n / (j k b^3)}, {l, 1, n / (j k m b^4)}]}
es5a[n_, b_, a_] := {-10 b^3 Sum[1, {j, 2, n}, {k, 2, n / j},
  {m, 1, n / (j k b)}, {l, 1, n / (j k m b^2)}, {s, 1, n / (j k m l b^3)}],
  5 b^4 Sum[1, {j, 2, n}, {k, 1, n / (b j)}, {m, 1, n / (j k b^2)}, {l, 1, n / (j k m b^3)},
  {s, 1, n / (l j k m b^4)}], - 1 b^5 Sum[1, {j, 1, n / b}, {k, 1, n / (b^2 j)},
  {m, 1, n / (j k b^3)}, {l, 1, n / (j k m b^4)}, {s, 1, n / (j k m l b^5)}]}

es2a2[n_, b_] := {Sum[1, {j, 2, n}, {k, 2, n / j}],
  Sum[1, {j, 2, n}, {k, 1, n / (b j)}], Sum[1, {j, 1, n / b}, {k, 1, n / (b^2 j)}]}
es3a2[n_, b_] := {Sum[1, {j, 2, n}, {k, 2, n / j}, {m, 1, n / (j k b)}],
  Sum[1, {j, 2, n}, {k, 1, n / (b j)}, {m, 1, n / (j k b^2)}],
  Sum[1, {j, 1, n / b}, {k, 1, n / (b^2 j)}, {m, 1, n / (j k b^3)}]}
es4a2[n_, b_] := {Sum[1, {j, 2, n}, {k, 2, n / j}, {m, 1, n / (j k b)}, {l, 1, n / (j k m b^2)}],
  Sum[1, {j, 2, n}, {k, 1, n / (b j)}, {m, 1, n / (j k b^2)}, {l, 1, n / (j k m b^3)}],
  Sum[1, {j, 1, n / b}, {k, 1, n / (b^2 j)}, {m, 1, n / (j k b^3)}, {l, 1, n / (j k m b^4)}]}
es5a2[n_, b_] := {Sum[1, {j, 2, n}, {k, 2, n / j}, {m, 1, n / (j k b)}, {l, 1, n / (j k m b^2)},
  {s, 1, n / (j k m l b^3)}], Sum[1, {j, 2, n}, {k, 1, n / (b j)}, {m, 1, n / (j k b^2)},
  {l, 1, n / (j k m b^3)}, {s, 1, n / (l j k m b^4)}], Sum[1, {j, 1, n / b}, {k, 1, n / (b^2 j)},
  {m, 1, n / (j k b^3)}, {l, 1, n / (j k m b^4)}, {s, 1, n / (j k m l b^5)}]}

```

```
es[10, 3 / 2, 2]
```

```
2
```

```
E2[10, 2, 3 / 2]
```

```
2
```

es3[6, 3 / 2, 3]

$$-\frac{9}{8}$$

E2[6, 3, 3 / 2]

$$-\frac{9}{8}$$

es4[6, 3 / 2, 4]

$$\frac{81}{16}$$

E2[6, 4, 3 / 2]

$$\frac{81}{16}$$

N[**es2a2**[6, 1.1, 2]]

es2a2[6., 1.0000001]

{3, 5, 10}

N[**es3a2**[6, 1.000001]]

{1., 6., 16.}

N[**es4a2**[6, 1.000001]]

{1., 7., 23.}

es5a2[6, 1.000001, 2]

{1, 8, 31}

Binomial[p, 0]

1

```

es5a[n_, b_, a_] :=
{-10 b^3 Sum[1, {j, 2, n}, {k, 2, n/j}, {m, 1, n/(j k b)}, {l, 1, n/(j k m b^2)},
  {s, 1, n/(j k m l b^3)}], 5 b^4 Sum[1, {j, 2, n}, {k, 1, n/(b j)},
  {m, 1, n/(j k b^2)}, {l, 1, n/(j k m b^3)}, {s, 1, n/(l j k m b^4)}],
-1 b^5 Sum[1, {j, 1, n/b}, {k, 1, n/(b^2 j)}, {m, 1, n/(j k b^3)},
  {l, 1, n/(j k m b^4)}, {s, 1, n/(j k m l b^5)}]]

es2a2[n_, b_] := {Sum[1, {j, 2, n}, {k, 2, n/j}],
  Sum[1, {j, 2, n}, {k, 1, n/(b j)}], Sum[1, {j, 1, n/b}, {k, 1, n/(b^2 j)}]}
es3a2[n_, b_] := {Sum[1, {j, 2, n}, {k, 2, n/j}, {m, 1, n/(j k b)}],
  Sum[1, {j, 2, n}, {k, 1, n/(b j)}, {m, 1, n/(j k b^2)}],
  Sum[1, {j, 1, n/b}, {k, 1, n/(b^2 j)}, {m, 1, n/(j k b^3)}]}
es4a2[n_, b_] := {Sum[1, {j, 2, n}, {k, 2, n/j}, {m, 1, n/(j k b)}, {l, 1, n/(j k m b^2)}],
  Sum[1, {j, 2, n}, {k, 1, n/(b j)}, {m, 1, n/(j k b^2)}, {l, 1, n/(j k m b^3)}],
  Sum[1, {j, 1, n/b}, {k, 1, n/(b^2 j)}, {m, 1, n/(j k b^3)}, {l, 1, n/(j k m b^4)}]}
es5a2[n_, b_] := {Sum[1, {j, 2, n}, {k, 2, n/j}, {m, 1, n/(j k b)}, {l, 1, n/(j k m b^2)},
  {s, 1, n/(j k m l b^3)}], Sum[1, {j, 2, n}, {k, 1, n/(b j)}, {m, 1, n/(j k b^2)},
  {l, 1, n/(j k m b^3)}, {s, 1, n/(l j k m b^4)}], Sum[1, {j, 1, n/b}, {k, 1, n/(b^2 j)},
  {m, 1, n/(j k b^3)}, {l, 1, n/(j k m b^4)}, {s, 1, n/(j k m l b^5)}]}

ff[n_, k_, b_, r_] := Sum[ff[n/(j b), k-1, b, r], {j, 1, n}]; ff[n_, 0, b_, r_] := 1

es5a3[n_, b_] := {Sum[1, {j, 2, n}, {k, 2, n/j},
  {m, 1, n/(j k b)}, {l, 1, n/(j k m b^2)}, {s, 1, n/(j k m l b^3)}],
  Sum[1, {j, 2, n}, {k, 1, n/(b j)}, {m, 1, n/(j k b^2)}, {l, 1, n/(j k m b^3)},
  {s, 1, n/(l j k m b^4)}], Sum[1, {j, 1, n/b}, {k, 1, n/(b^2 j)},
  {m, 1, n/(j k b^3)}, {l, 1, n/(j k m b^4)}, {s, 1, n/(j k m l b^5)}]}

esp[n_, b_, p_] := {Sum[ff[n/(j k), p, b, 1], {j, 2, n}, {k, 2, n/j}],
  Sum[ff[n/j, p+1, b, 1], {j, 2, n}], ff[n, p+2, b, 1]}
esp2[n_, p_, b_] := (-1)^p
(b^(p-2) Binomial[p, 2] Sum[ff[n/(j k), p-2, b, 1], {j, 2, n}, {k, 2, n/j}] - b^(p-1)
  Binomial[p, 1] Sum[ff[n/j, p-1, b, 1], {j, 2, n}]) + b^p Binomial[p, 0] ff[n, p, b, 1])

esp3[n_, p_, b_] := (-1)^p
(b^(p-2) Binomial[p, 2] - b^(p-1) Binomial[p, 1] Sum[ff[n/j, p-1, b, 1], {j, 2, n}]) +
  b^p Binomial[p, 0] ff[n, p, b, 1])
esp3a[n_, p_, b_] := {b^(p-2) Binomial[p, 2], -b^(p-1) Binomial[p, 1]
  Sum[ff[n/j, p-1, b, 1], {j, 2, n}], b^p Binomial[p, 0] ff[n, p, b, 1]}
esp3b[n_, p_, b_] := {b^(p-2) ((p-1) (p) / 2),
  -b^(p-1) ((p+1) (p+2) - 2), b^p ((p+4) (p+3) / 2 - 5)}
esp3ab[n_, p_, b_] := {-b^(p-1) Binomial[p, 1] Sum[ff[n/j, p-1, b, 1], {j, 2, n}],
  b^p Binomial[p, 0] ff[n, p, b, 1], -b^(p-1) ((p+1) (p+2) - 2), b^p ((p+4) (p+3) / 2 - 5)}
esp3a2[n_, p_, b_] := {b^(p-2) ((p) (p-1) / 2),
  -b^(p-1) p Sum[ff[n/j, p-1, b, 1], {j, 2, n}], b^p ff[n, p, b, 1]}
esp3c[n_, p_, b_] := (-1)^(p)
(b^(p-2) ((p-1) (p) / 2) - b^(p-1) ((p+1) (p+2) - 2) + b^p ((p+4) (p+3) / 2 - 5))

Table[esp3a2[6, n, 1.01], {n, 1, 30}] // TableForm

```

0.	-5.	6.06
1.	-16.16	10.201
3.03	-18.3618	16.4848
6.1206	-28.8484	23.9339
10.303	-41.6242	32.5813
15.6091	-56.7545	42.4608
22.0712	-74.3064	53.6068
29.7226	-94.3479	66.0543
38.5969	-116.949	79.839
48.7286	-142.179	94.9975
60.1527	-170.112	111.567
72.9051	-200.82	129.585
87.0221	-234.38	149.09
102.541	-270.866	170.122
119.5	-310.358	192.721
137.937	-352.935	216.927
157.892	-398.677	242.782
179.405	-447.667	270.329
202.516	-499.99	299.611
227.268	-555.73	306.268
253.703	-589.352	337.675
281.864	-650.703	370.925
311.795	-715.712	406.064
343.542	-784.47	443.137
377.149	-857.071	482.194
412.664	-933.61	523.284
450.134	-1014.19	566.454
489.607	-1098.9	611.758
531.133	-1187.84	659.245
574.762	-1281.12	708.969

esp3a[6, 5, 1.00001]

{10.0003, -40.0016, 31.0016, 40, 31}

esp3b[6, 5, 1.00001]

{10.0003, -40.0016, 31.0016}

Binomial[5, 2]

10

Binomial[p, 2]

$$\frac{1}{2} (-1 + p) p$$

es5a3[6, 1.000001]

{1, 8, 31}

esp[6, 1.000001, 3]

{1, 8, 31}

N[es4a2[6, 1.000001]]

{1., 7., 23.}

```

FullSimplify[
  Expand[b^ (p - 2) ((p - 1) (p) / 2) - b^ (p - 1) ((p + 1) (p + 2) - 2) + b^p ((p + 4) (p + 3) / 2 - 5)]
  1
  - b^-2+p ((-1 + p) p - 2 b p (3 + p) + b^2 (2 + p (7 + p)))
  2
esp3c[n_, p_, b_] :=
  (-1) ^ (p) (b^ (p - 2) ((p - 1) (p) / 2) - b^ (p - 1) ((p + 1) (p + 2) - 2) + b^p ((p + 4) (p + 3) / 2 - 5))
esp3d[n_, p_, b_] :=
  (-1) ^ (p) (b^ (p - 2) ((p - 1) (p) / 2) - b^ (p - 1) ((p + 1) (p + 2) - 2) + b^p ((p + 4) (p + 3) / 2 - 5))

Table[ {1.01^n, esp3[6, n, 1.01], esp3c[6, n, aa = 1.01],
  esp3d[6, n, aa = 1.01], E2[6, n, aa]}, {n, 3, 30}] // TableForm

1.0303      -1.15302      -1.15302      -1.15302      -1.15302
1.0406      1.20606      1.20606      1.20606      1.20606
1.05101     -1.26016     -1.26016     -1.26016     -1.26016
1.06152     1.31532     1.31532     1.31532     1.31532
1.07214     -1.37157     -1.37157     -1.37157     -1.37157
1.08286     1.42891     1.42891     1.42891     1.42891
1.09369     -1.48737     -1.48737     -1.48737     -1.48737
1.10462     1.54697     1.54697     1.54697     1.54697
1.11567     -1.60772     -1.60772     -1.60772     -1.60772
1.12683     1.66964     1.66964     1.66964     1.66964
1.13809     -1.73274     -1.73274     -1.73274     -1.73274
1.14947     1.79706     1.79706     1.79706     1.79706
1.16097     -1.8626     -1.8626     -1.8626     -1.8626
1.17258     1.92939     1.92939     1.92939     1.92939
1.1843      -1.99745     -1.99745     -1.99745     -1.99745
1.19615     2.06679     2.06679     2.06679     2.06679
1.20811     -2.13743     -2.13743     -2.13743     20.8166
1.22019     -22.1944     2.2094      2.2094      1.96778
1.23239     -2.02648     -2.28272     -2.28272     -2.02648
1.24472     2.08628     2.35741     2.35741     2.08628
1.25716     -2.1472     -2.43348     -2.43348     -2.1472
1.26973     2.20925     2.51097     2.51097     2.20925
1.28243     -2.27245     -2.58988     -2.58988     -2.27245
1.29526     2.33682     2.67025     2.67025     2.33682
1.30821     -2.40238     -2.7521     -2.7521     -2.40238
1.32129     2.46915     2.83545     2.83545     2.46915
1.3345      -2.53714     -2.92031     -2.92031     -2.53714
1.34785     2.60638     3.00673     3.00673     2.60638

-1.00003

```

```
Table[ {esp3[6, n, 1.00001], (esp2[6, n, 1.00001]), E2[6, n, 1.00001]}, {n, 1, 10}] //
TableForm
```

```
$RecursionLimit::reclim: Recursion depth of 10000 exceeded. >>
```

```
$RecursionLimit::reclim: Recursion depth of 10000 exceeded. >>
```

```
-1.00006    -1.00006    -0.00005
-4.99996    -2.99996    3.0001
-1.00015    -7.00021    -1.00015
1.0002      1.0002      1.0002
-1.00025    -1.00025    -1.00025
1.0003      1.0003      1.0003
-1.00035    -1.00035    -1.00035
1.0004      1.0004      1.0004
-1.00045    -1.00045    -1.00045
1.0005      1.0005      1.0005
```

```
E2[6, 13, 1.000001]
```

```
-1.00007
```

```
esp3[6, 13, 1.000001]
```

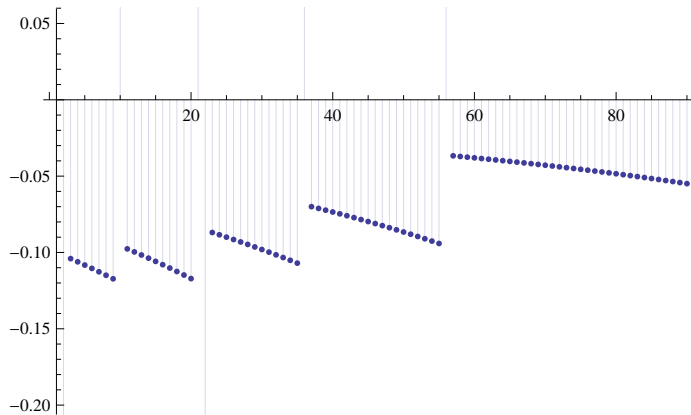
```
-1.00007
```

```
$RecursionLimit = 10 000
```

```
10 000
```

```
DiscretePlot[ ((-1)^(k+1) E2[n, k, b] + 1) / k, {k, 1, Log[b, n]}] /. {b -> 1.02, n -> 6}
```

```
DiscretePlot::iterb: Iterator {k, 1, Log[b, n]} does not have appropriate bounds. >>
```



```
esp3c[n_, p_, b_] :=
```

```
(-1)^(p) (b^(p-2) ((p-1) (p) / 2) - b^(p-1) ((p+1) (p+2) - 2) + b^p ((p+4) (p+3) / 2 - 5))
```

```
Sum[ ((-1)^(n-1) esp3c[6, n, 1.00001] + 1) / n, {n, 1, Log[1.00001, 1.2]}]
```

```
-1.00036
```

FullSimplify[Sum[((-1) ^ (n - 1) esp3c[6, n, b] + 1) / n, {n, 1, Log[b, 6 / 5]}]]

$$\frac{1}{10} \left(-7 + 10 \operatorname{HarmonicNumber} \left[\frac{\operatorname{Log} \left[\frac{6}{5} \right]}{\operatorname{Log}[b]} \right] + \right.$$

$$\left. 12 b \operatorname{LerchPhi} \left[b, 1, \frac{\operatorname{Log} \left[\frac{6b}{5} \right]}{\operatorname{Log}[b]} \right] + 10 \operatorname{Log}[1 - b] - \frac{6 (-1 + b) \operatorname{Log} \left[\frac{6}{5} \right]}{b \operatorname{Log}[b]} \right)$$

$$\operatorname{Limit} \left[\frac{1}{10} \left(-7 + 10 \operatorname{HarmonicNumber} \left[\frac{\operatorname{Log} \left[\frac{6}{5} \right]}{\operatorname{Log}[b]} \right] + 10 \operatorname{Log}[1 - b] - \frac{6 (-1 + b) \operatorname{Log} \left[\frac{6}{5} \right]}{b \operatorname{Log}[b]} \right), b \rightarrow 1 \right]$$

$$-\frac{7}{10} + \operatorname{EulerGamma} + i \pi - \frac{3}{5} \operatorname{Log} \left[\frac{6}{5} \right] + \operatorname{Log} \left[\operatorname{Log} \left[\frac{6}{5} \right] \right] + \operatorname{Limit} \left[\frac{6}{5} b \operatorname{LerchPhi} \left[b, 1, \frac{\operatorname{Log} \left[\frac{6b}{5} \right]}{\operatorname{Log}[b]} \right], b \rightarrow 1 \right]$$

$$-\frac{7}{10} + \operatorname{EulerGamma} + i \pi + \operatorname{Limit} \left[\frac{6}{5} b \operatorname{LerchPhi} \left[b, 1, \frac{\operatorname{Log} \left[\frac{6b}{5} \right]}{\operatorname{Log}[b]} \right], b \rightarrow 1 \right] - \frac{3}{5} \operatorname{Log} \left[\frac{6}{5} \right] + \operatorname{Log} \left[\operatorname{Log} \left[\frac{6}{5} \right] \right]$$

$$\operatorname{Limit} \left[\frac{1}{10} \left(12 b \operatorname{LerchPhi} \left[b, 1, \frac{\operatorname{Log} \left[\frac{6b}{5} \right]}{\operatorname{Log}[b]} \right] \right), b \rightarrow 1 \right]$$

$$\operatorname{Limit} \left[\frac{6}{5} b \operatorname{LerchPhi} \left[b, 1, \frac{\operatorname{Log} \left[\frac{6b}{5} \right]}{\operatorname{Log}[b]} \right], b \rightarrow 1 \right]$$

$$\mathbf{fe}[b_]:= -\frac{7}{10} + \operatorname{EulerGamma} + i \pi + \frac{6}{5} b \operatorname{LerchPhi} \left[b, 1, \frac{\operatorname{Log} \left[\frac{6b}{5} \right]}{\operatorname{Log}[b]} \right] - \frac{3}{5} \operatorname{Log} \left[\frac{6}{5} \right] + \operatorname{Log} \left[\operatorname{Log} \left[\frac{6}{5} \right] \right]$$

fe[1.00000001]

$$-1.00037 - 2.33621 \times 10^{-9} i$$

FullSimplify[Sum[((-1) ^ (n - 1) esp3c[6, n, b] + 1) / n, {n, 1, Log[b, 6 / 4]}] -

Sum[((-1) ^ (n - 1) esp3c[6, n, b] + 1) / n, {n, 1, Log[b, 6 / 5]}]]

$$\frac{1}{20} \left(-21 - 20 \operatorname{HarmonicNumber} \left[\frac{\operatorname{Log} \left[\frac{6}{5} \right]}{\operatorname{Log}[b]} \right] + 20 \operatorname{HarmonicNumber} \left[\frac{\operatorname{Log} \left[\frac{3}{2} \right]}{\operatorname{Log}[b]} \right] - \right.$$

$$\left. 24 b \operatorname{LerchPhi} \left[b, 1, \frac{\operatorname{Log} \left[\frac{6b}{5} \right]}{\operatorname{Log}[b]} \right] + 30 b \operatorname{LerchPhi} \left[b, 1, \frac{\operatorname{Log} \left[\frac{3b}{2} \right]}{\operatorname{Log}[b]} \right] - \frac{3 (-1 + b) \operatorname{Log} \left[\frac{1875}{512} \right]}{b \operatorname{Log}[b]} \right)$$

$$\operatorname{Limit} \left[\frac{1}{20} \left(-21 - 20 \operatorname{HarmonicNumber} \left[\frac{\operatorname{Log} \left[\frac{6}{5} \right]}{\operatorname{Log}[b]} \right] + 20 \operatorname{HarmonicNumber} \left[\frac{\operatorname{Log} \left[\frac{3}{2} \right]}{\operatorname{Log}[b]} \right] - \frac{3 (-1 + b) \operatorname{Log} \left[\frac{1875}{512} \right]}{b \operatorname{Log}[b]} \right), b \rightarrow 1 \right]$$

$$-\frac{21}{20} - \frac{3}{20} \operatorname{Log} \left[\frac{1875}{512} \right] - \operatorname{Log} \left[\operatorname{Log} \left[\frac{6}{5} \right] \right] + \operatorname{Log} \left[\operatorname{Log} \left[\frac{3}{2} \right] \right]$$

