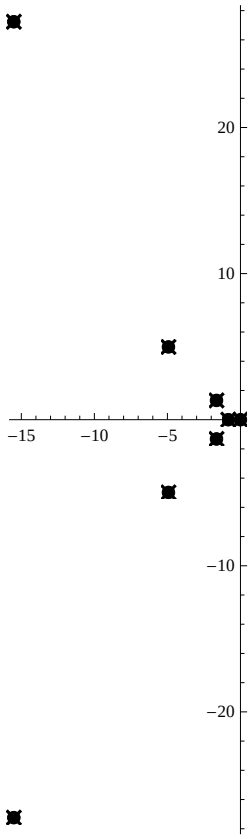
```
ClearAll["Global`*"]
f2[n_, k_, f_] := f2[n, k, f] = Sum[f2[j, k - 1, f] f2[n / j, 1, f], {j, Divisors[n]}];
f2[n_, 1, f_] := f[n]; f2[1, 1, f_] := 0; f2[n_, 0, f_] := 0; f2[1, 0, f_] := 1
ff[n_, z_, f_] := ff[n, z, f] = Sum[FactorialPower[z, a] / a! f2[n, a, f], {a, 0, Log[2, n]}]
F2[n_, k_ , f_] := F2[n, k, f] = Sum[ f[j] F2[ Floor[n / j], k - 1, f], {j, 2, n}];
F2[n_, 0, f_] := 1
bins[ z_, a_] := Product[ (z - k), {k, 0, a - 1}] / a!
F1[n_, z_, f_] := Expand[Sum[bins[z, a] F2[n, a, f], {a, 0, Log[2, n]}]]

r2[n_] := n
r[n_] := ff[n, 1, r2]
RootLocusPlot[1 / Expand[F1[400, x, r]], {k, 0, 1}, FeedbackType → None]
```




```
List @@ NRoots[F1[221, x, r] == 0, x][[All, 2]]
```

$$\{-31.331,\ -10.6336 - 6.07118\,i,\ -10.6336 + 6.07118\,i,\ -2.15108 - 1.86324\,i,\ -2.15108 + 1.86324\,i,\ -0.82962,\ -0.0199604\}$$

```
(2 E^(I Pi / 4)) List @@ NRoots[F1[331, x, r] == 0, x][[All, 2]]
```

$$\{-273.542 + 5.68434 \times 10^{-14}\,i,\ 0.500292 - 14.7331\,i,\ -0.874567 - 3.50624\,i,\ -1.3549 - 0.328372\,i,\ -1.3549 + 0.328372\,i,\ -0.000104393 + 1.28749 \times 10^{-19}\,i,\ -0.874567 + 3.50624\,i,\ 0.500292 + 14.7331\,i\}$$

```
vv := {-11.199685576035792` - 12.398224487807212` i,
  -11.199685576035792` + 12.398224487807212` i,
  -2.6719503346754907` - 1.8618449055430242` i, -2.6719503346754907` + 1.8618449055430242` i,
  -0.9338092178222006`, -0.03720467504094745`}
```

```
vv2 := {-31.33103586603707`, -10.63361495531753` - 6.07118055234119` i,
  -10.63361495531753` + 6.07118055234119` i, -2.1510767921924216` - 1.8632373382027279` i,
  -2.1510767921924216` + 1.8632373382027279` i,
  -0.8296202328515756`, -0.019960406091375695`}
```

```
Sum[-1 / (j^1), {j, vv2}]
```

```
52.0095 + 0. i
```

```
fo[z_] := Product[(1 - (z 3 E^(Pi / 8 I))) / j), {j, vv}]
```

```
fo[1]
```

```
550.816 + 1248.13 i
```

```
List @@ NRoots[F1[100, x, r] == 0, x][[All, 2]]
```

```
{-11.1997 - 12.3982 i, -11.1997 + 12.3982 i,
 -2.67195 - 1.86184 i, -2.67195 + 1.86184 i, -0.933809, -0.0372047}
```

```
List @@ NRoots[F1[100, x, r] == 0, x][[All, 2]]
```

```
{-9.21723 - 11.7841 i, -9.21723 + 11.7841 i,
 -2.20965 - 1.86061 i, -2.20965 + 1.86061 i, -0.695379, -0.000866226}
```

```
g2[n_, k_, s_, f_] :=
 g2[n, k, s, f] = Sum[g2[j, k - 1, s, f] g2[n / j, 1, s, f], {j, Divisors[n]}];
g2[n_, 1, s_, f_] := f[n, s]; g2[1, 1, s_, f_] := 0;
g2[n_, 0, s_, f_] := 0; g2[1, 0, s_, f_] := 1
gf[n_, z_, s_, f_] :=
 gf[n, z, s, f] = Sum[FactorialPower[z, a] / a! g2[n, a, s, f], {a, 0, Log[2, n]}]
G2[n_, k_, s_, f_] := G2[n, k, s, f] = Sum[f[j, s] G2[Floor[n / j], k - 1, s, f], {j, 2, n}];
G2[n_, 0, s_, f_] := 1
bins[z_, a_] := Product[(z - k), {k, 0, a - 1}] / a!
G1[n_, z_, s_, f_] := Expand[Sum[bins[z, a] G2[n, a, s, f], {a, 0, Log[2, n]}]]
```

```
rr2[n_, s_] := n^-s
rr[n_, s_] := gf[n, 1, s, rr2]
```

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. ≫

```
List @@ NRoots[G1[100, x, 0, rr] == 0, x][[All, 2]]
```

```
{-11.1997 - 12.3982 i, -11.1997 + 12.3982 i,
 -2.67195 - 1.86184 i, -2.67195 + 1.86184 i, -0.933809, -0.0372047}
```

```
Animate[RootLocusPlot[1 / Expand[G1[1000, x, bb , rr]],
  {k, 0, 1}, FeedbackType → None], {bb, -3, 3, .003}]
```