N-Body simulation using Barnes-Hut

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Physics

Things To Investigate:

- Large Scale Structure
- Rotation Curves (DM)
- Colliding Galaxies
  - Barred Galaxies
- DM Subhalo

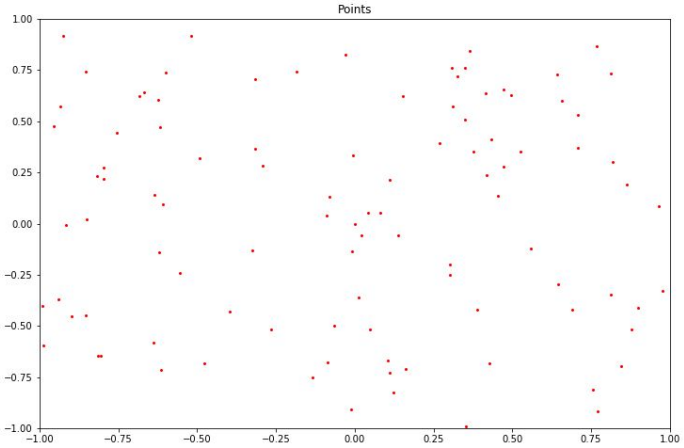
Algorithm

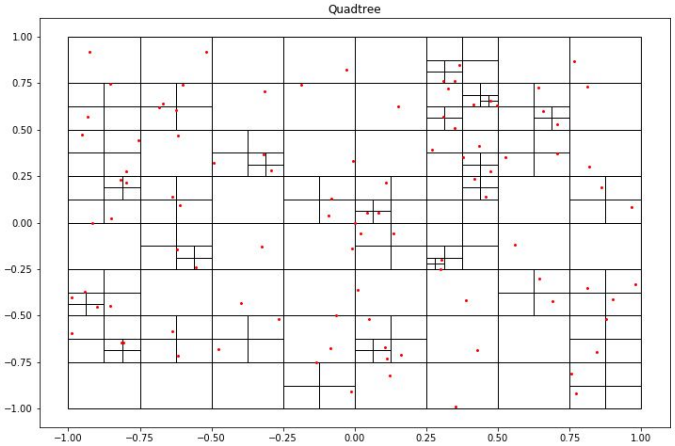
Brute Force

- Perfectly Accurate
- $O(n^2)$

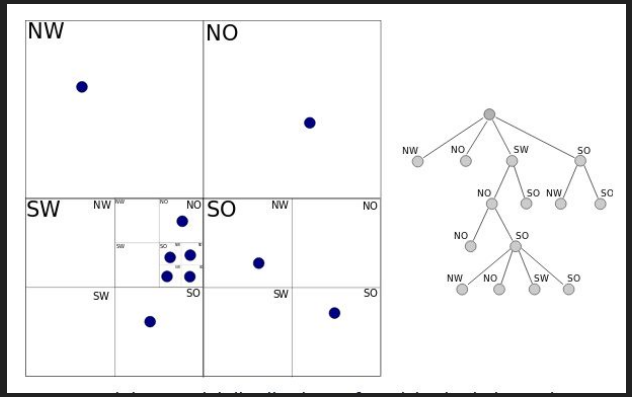
Barnes Hut

- Approximation
- O(n log n)





Barnes-Hut

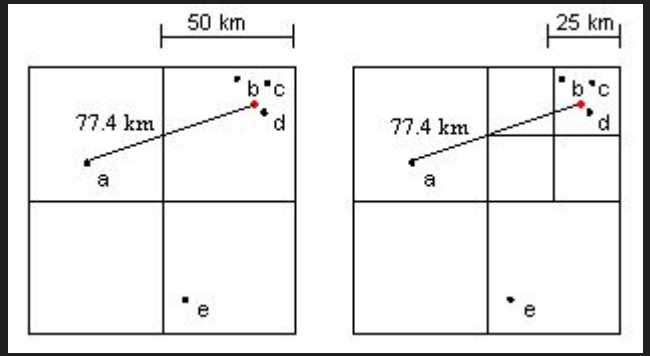


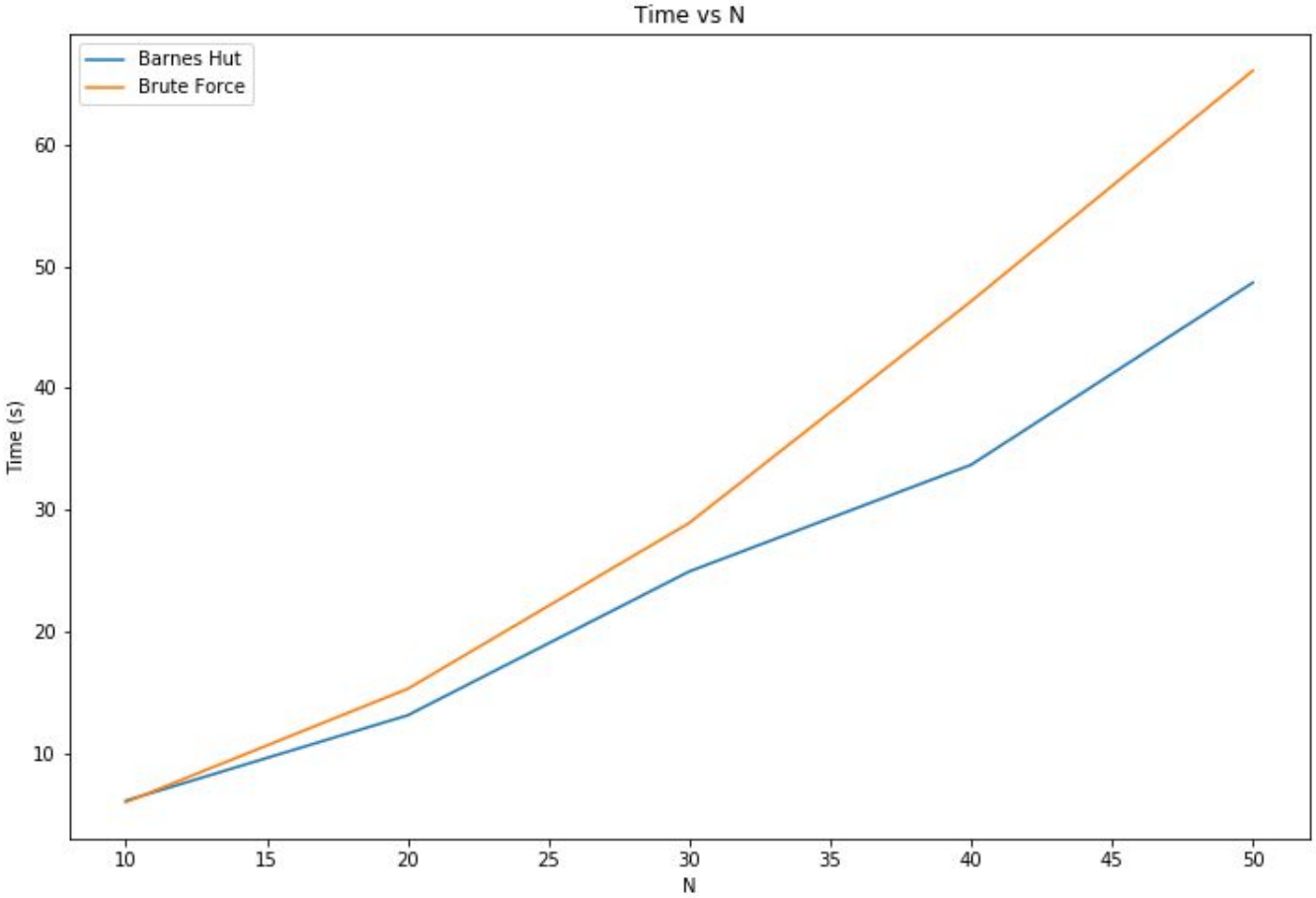
Tree Structure

- Recursively divide the region until every point has a separate node
- Every node stores: Points, COM, Mass, and size of node

Force Calculation:

- Traverse tree, find which nodes to use
- Require: $s/d < \theta$
  - s = sidelength of node
  - d = distance from point to COM of node
  - $\theta=0.5$
- Calculate total force on pt from used nodes




Time vs N
Barnes Hut
Brute Force
Time (s)
10
20
30
40
50
60
10
15
20
25
30
35
40
45
50
N

Rotational Curves

Stellar/Gas Velocity vs. Radial Distance

- $\Phi(r,0) = -2\pi G \int_0^\infty dk J_0(kr) \int_0^\infty dr r J_0(kr)\Sigma(r)$
- For exponential disk:
- $\Phi(r,0) = -\pi G\Sigma_0 r \left[I_0\left(\frac{r}{2h_r}\right) K_1\left(\frac{r}{2h_r}\right) - I_1\left(\frac{r}{2h_r}\right) K_0\left(\frac{r}{2h_r}\right) \right]$
- $\implies v(r)^2 = \pi G\Sigma_0 \frac{r^2}{h_r} \left[I_0\left(\frac{r}{2h_r}\right) K_0\left(\frac{r}{2h_r}\right) - I_1\left(\frac{r}{2h_r}\right) K_1\left(\frac{r}{2h_r}\right) \right]$
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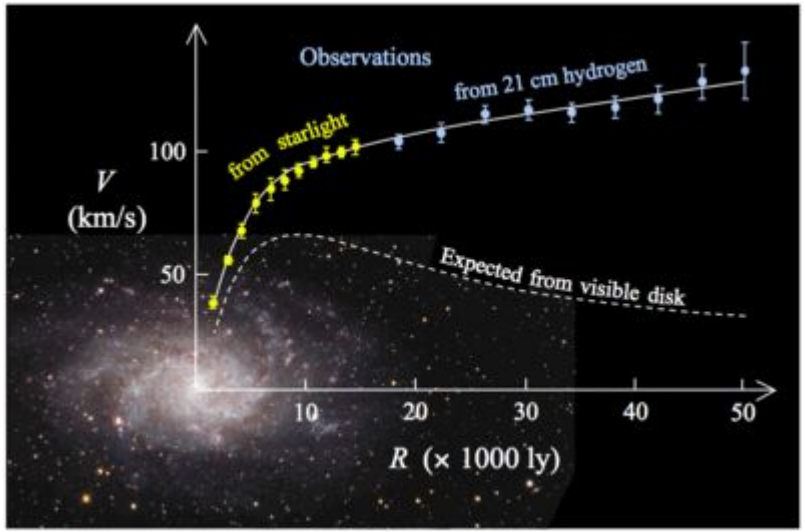

Dark Matter Subhalo

- Small halo orbiting inside potential well of parent halo
- Strong tidal forces
- Loses energy & ang. mom. From dynamical friction