DAE8th Problem 2.8
Given:

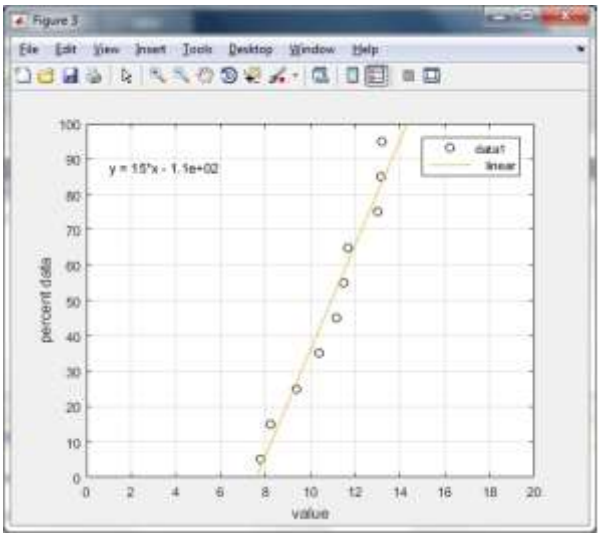
2.8. Consider the following sample data: 9.37, 13.04, 11.69, 8.21, 11.18, 10.41, 13.15, 11.51, 13.21, and 7.75. Is it reasonable to assume that this data is a sample from a normal distribution? Is there evidence to support a claim that the mean of the population is 10?

Solution

To investigate if the data was generated from a normal distribution, we generate a normality plot where order the data from smaller to larger according to index j
x=[x1,x2,…,xn]
x_sort={xj}
we set
yj=100(j-0.5)
and plot x against y. A normaly distributed data set should appear along a reasonably straight line.
The following code in MATLAB

```
x=[9.37,13.04, 11.69,8.21,11.18,10.41,13.15,11.51,13.21,7.75]';
n=length(x); %10
x_sort=sort(x,1,'ascend');
j=1:n;
y=100*(j-0.5)/n;
figure;
plot(x_sort,y,'ok');
grid on;
xlim([0,20]);
ylim([0,100]);
xlabel('value');
ylabel('percent data')
```

generates the normality plot [the basic fitting tool was used to fit a linear function to the data]

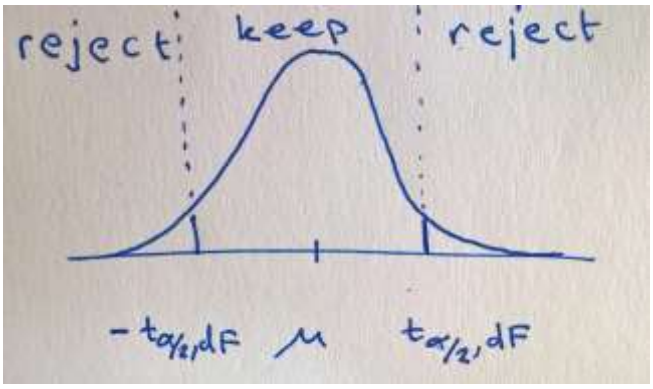


As the transformed data is reasonably linear, the normality assumption on the data holds.

The true variance is not known and therefore the t-distribution can be used as a reference when testing:

H0: $\mu=10$
H1: $\mu\neq10$

The number of degrees of freedom is dF =n-1=10-1=9.
The test statistic is computed as t0=(<y>-10)/(S/sqrt(n)).
The acceptable probability of falsely rejecting H0 is selected as alfa=0.05
Therefore the significance level of the test is 1-0.05=0.95
The test is double sided and H0 should be rejected if abs(t0)>$t_{alfa/2,dF}$ see figure



The following code in MATLAB generates statistics and the reference value from the t-distribution

```
y=[9.37,13.04, 11.69,8.21,11.18,10.41,13.15,11.51,13.21,7.75]';
n=length(y);
dF=n-1;
y_mean=mean(y);
y_ref=10;
alfa=0.05;
S=sqrt(sum((y-y_mean).^2)/dF); %NOTE: n-1 and NOT n
t0=(y_mean-y_ref)/(S/sqrt(n));
t_ref=tinv(alfa/2,dF);

if abs(t0)>abs(t_ref) %note abs since the test is double sided
    disp('reject H0!')
else
    disp('keep H0!');
end
```

Which outputs

```
t0 =

    1.5102


t_ref =

   -2.2622

keep H0!
```