

DAE8 Problem 2.7

Given:

2.7. Suppose that we are testing $H_0 : \mu_1 = \mu_2$ versus $H_0 : \mu_1 > \mu_2$ where the two sample sizes are $n_1 = n_2 = 12$. Both sample variances are unknown but assumed equal. Find bounds on the P -value for the following observed values of the test statistic.

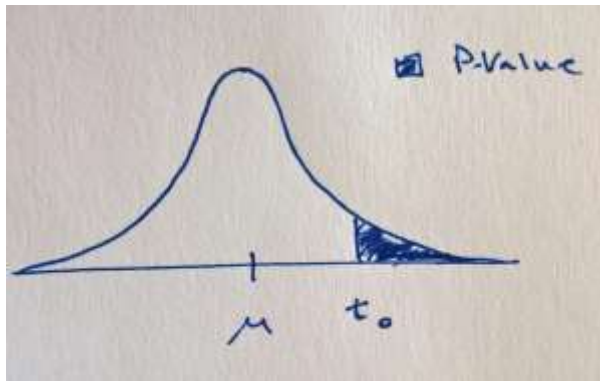
(a) $t_0 = 2.31$ (b) $t_0 = 3.60$ (c) $t_0 = 1.95$ (d) $t_0 = 2.19$

Solution:

Unknown sample variances, assumed equal \Rightarrow t-distribution with dF number of degrees of freedom

Number of degrees of freedom, $dF = n_1 + n_2 - 2 = 12 + 12 - 2 = 24 - 2 = 22$

The test is one sided and therefore the P -value is the probability of greater values than t_0 (see figure)



The P -value is therefore, $P = P(t > t_0) = 1 - \text{tCDF}(t_0, dF)$

Smaller values are more likely under H_1 .

The test statistic is computed as $t_0 = (\mu_1 - \mu_2) / (S_p \cdot \sqrt{1/n_1 + 1/n_2})$

where $S_p^2 = [(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2] / dF$

And $S^2 = \sum_{i=1, n} (y_i - \bar{y})^2 / (n - 1)$

The following MATLAB code computes the P -values

```
t0=[2.31, 3.6, 1.95, 2.19]';
n1=12;
n2=n1;
dF=n1+n2-2;
P=1-tcdf(t0, dF)
P = [0.0153, 0.0008, 0.0320, 0.0197]';
```