

DAE8th Problem 2.9

Given:

2.9. A computer program has produced the following output for a hypothesis-testing problem:

```
Difference in sample means: 2.35
Degrees of freedom: 18
Standard error of the difference in sample means: ?
Test statistic:  $t_0 = 2.01$ 
P-value: 0.0298
```

- (a) What is the missing value for the standard error?
- (b) Is this a two-sided or a one-sided test?
- (c) If $\alpha = 0.05$, what are your conclusions?
- (d) Find a 90% two-sided CI on the difference in means.

Solution:

a)

The test statistic is $t_0 = (\mu_0 - \mu_1) / \text{stderr}$

Which implies that the $\text{stderr} = t_0 / (\mu_0 - \mu_1) = 2.01 / 2.35 = 0.8553$

b) Using $1 - \text{tcdf}(2.01, 18) = 0.0298$, which is the p-value, hence it is a one sided test.

c) a one sided test is performed by comparing the test statistic to a reference value of the t-distribution with $\alpha = 0.05$ and $df = 18$, which is $t_{\text{ref}} = 1.7341$

as $t_0 > t_{\text{ref}}$, we shall **REJECT H_0** .

c) here we need to construct the following inequality

$$\Delta - t_{\alpha/2, df} * \text{stderr} \leq \Delta \leq \Delta + t_{\alpha/2, df} * \text{stderr}$$

Where Δ is the difference in the sample means and the stderr as above. We need to realize that α should be 0.1 instead of 0.05 and therefore we need to find $t_{0.05, 18}$, which is 1.7341. The 90% confidence interval for the difference in sample means is therefore

$$\text{CI: } 2.35 - 1.7341 * 0.8553 \leq 2.35 \leq 2.35 + 1.7341 * 0.8553$$

Or more compactly

$$\text{CI: } 0.8668 \leq 2.35 \leq 3.8332$$