

DAE 8th Problem 2.10

Given:

2.10. A computer program has produced the following output for a hypothesis-testing problem:

```
Difference in sample means: 11.5
Degrees of freedom: 24
Standard error of the difference in sample means: ?
Test statistic:  $t_0 = -1.88$ 
P-value: 0.0723
```

- (a) What is the missing value for the standard error?
- (b) Is this a two-sided or a one-sided test?
- (c) If $\alpha = 0.05$, what are your conclusions?
- (d) Find a 95% two-sided CI on the difference in means.

Solution:

See the solutions to problem 2.09 for more details

a) as $t_0 = D/\text{stderr}$, we find that $\text{stderr} = D/t_0 = 11.5/1.88 = 6.1170$

b) using `tinv` for the p-value 0.0723 and 24 degrees of freedom we find -1.5080, rather than -1.88. Further we find that $\text{tcdf}(-1.88, 24) = 0.0362 = 0.0723/2$ which implies that it is a two sided test.

Q: why is the test statistic negative? Doesn't that imply that the sample difference is negative?
I've assumed that the test is to see if it is different from zero...

c) We find t_{ref} by interrogating the t-distribution with significance level $1 - \alpha = 1 - 0.05/2 = 0.975$ and 24 degrees of freedom. In MATLAB we write

```
t_ref=tinv(0.975,24)
```

which returns 1.7109. Thus

```
abs(t0) = 1.7109 < abs(t_ref) = 2.0639
```

we shall KEEP H_0 !

c) here we need to construct the following inequality

$$\Delta - t_{\alpha/2, df} * \text{stderr} \leq \Delta \leq \Delta + t_{\alpha/2, df} * \text{stderr}$$

Where Δ is the difference in the sample means and the stderr as above. Here, α should be 0.05 and we need to find $t_{0.05/2, 24}$, which in MATLAB is

```
t_ref=tinv(0.975,24)
```

which returns 2.0639. The 90% confidence interval for the difference in sample means is therefore

$$\text{CI: } 11.5 - 2.0639 * 6.1170 \leq 11.5 \leq 11.5 + 2.0639 * 6.1170$$

Or more compactly

CI: $-1.1249 \leq 11.5 \leq 24.1249$