DAE 8th Problem 2.20

Given:

2.20. The shelf life of a carbonated beverage is of interest. Ten bottles are randomly selected and tested, and the following results are obtained:

Days	
108	138
124	163
124	159
106	134
115	139

(a) We would like to demonstrate that the mean shelf life exceeds 120 days. Set up appropriate hypotheses for investigating this claim.

(b) Test these hypotheses using $\alpha = 0.01$. What are your conclusions?

(c) Find the *P*-value for the test in part (b).

(d) Construct a 99 percent confidence interval on the mean shelf life.

Solution:

A) An Approriate set hypothesis' is

H0: $\mu > 120$

H1: $\mu \leq 120$

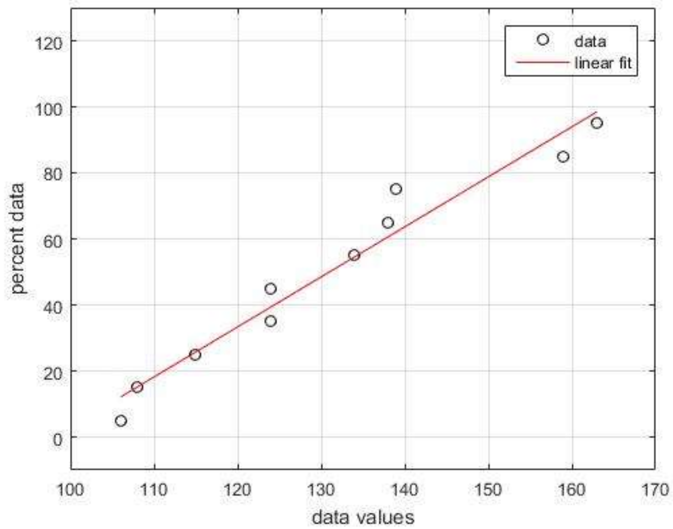
B) First test the normality assumption. In MATLAB this is achieved using

```
x=[108,124,124,106,115,138,163,159,134,139]';
n=length(x);
x_sort=sort(x,1,'ascend');
j=1:n;
y=100*(j-0.5)/n;
BETA=glmfit(x_sort,y(:));

figure;
plot(x_sort,y,'ok');
hold on;
plot(x_sort,BETA(1)+x_sort*BETA(2),'r');
legend('data','linear fit');
grid on;
xlim([100,170]);
```

```
xlabel('data values');
ylabel('percent data');
ylim([-10,130]);
```

which produces the following plot



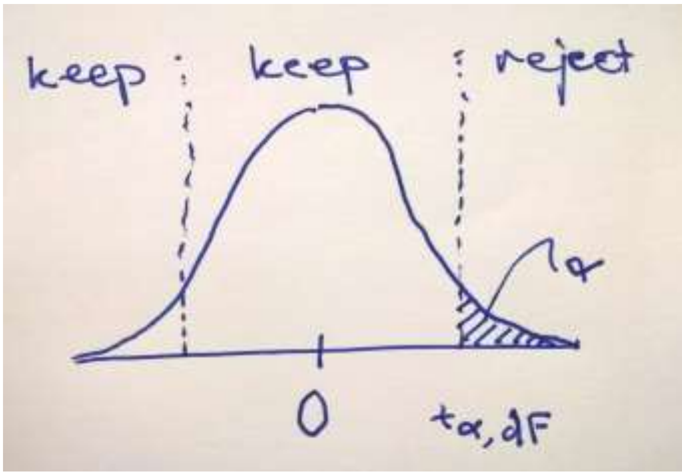
Clearly the normality assumption cannot be falsified. The variance is unknown and therefore the t-distribution should be used as a reference distribution. The statistic computed as

t0=(<y>-120)/(S/sqrt(n))

where <y> is the arithmetic mean, n is the number of samples and S is the sample standard deviation

S=sqrt(sum(i={1,n},(yi-<y>)^2)/(n-1))

This statistic is to be compared to a reference value using alfa=0.05 or equivalently a significance level of 1-alfa= 0.95, with n-1=10-1=9 degrees of freedom. As the test is one sided, the test consists of comparing which of t0 and t_ref is greatest (see figure).



In MATLAB code this consists of:

Which outputs

```
t0 =

    1.7798


tref =

    2.8214
```

Which implies that we cannot reject H0, so we keep it!
C) The P-value is found computing the compliment to the CDF of the t-distribution with 9 degrees of freedom for t0. In MATLAB this is `1-tcdf(t0,dF) which returns 0.0544`

D) The 99% confidence interval for the self life μ is

$<y>-t_{0.01/2,9}*S/sqrt(n) \le <y> \le <y>+t_{0.01/2,9}*S/sqrt(n)$
Which is

131- 3.2498*6.1806≤ 131 ≤ 131+ 3.2498* 6.1806

`Or more compactly`

`110.9143` ≤ 131 ≤ 151.0857