DAE 8ed, Problem 2.4

Given:

Suppose that we are testing H_0 : $\mu = \mu_0$ versus H_1 : $\mu \neq \mu_0$. Calculate the P-value for the following observed values of the test statistic:

(a)
$$Z_0 = 2.35$$

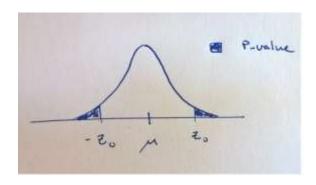
(a)
$$Z_0 = 2.35$$
 (b) $Z_0 = 1.60$ (c) $Z_0 = 2.15$

(d)
$$Z_0 = 1.89$$
 (e) $Z_0 = -0.90$

(e)
$$Z_0 = -0.90$$

Solution:

The Z statistic is from a standard normal distribution $N(\mu, sigma)$, where $\mu=0$ and sigma=1. The test is double sided as both values greater or smaller are possible under H1. The P-value is the probability for more extreme cases than ZO (see figure 01).



The distribution is symmetric around the mean (μ =0), with unit variance (sigma=1) and the P-value can therefore be described as twice the probability of more extreme cases than Z0, P(z>Z0), i.e.

$$P=2*P(z>Z0)$$

This probability in turn is derived from the CDF as

$$P(z>Z0)=1-P(z\leq Z0)=1-CDF(Z0)$$

The CDF of the normal distribution is determined by the error function, which does not have an analytical solution for the 1-D case. Using MATLAB/OCTAVE the following code will determine the Pvalues.

```
Z0=[2.35, 1.60, 2.15, 1.89, -0.90]'; %Vector cases A-E
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P=2*(1-normcdf(abs(Z0),0,1)) %Note the absolute value due to case e being negative

0.0188 0.1096 0.0316 0.0588 0.3681

Here the smaller values are more likely under H1.