量统 23_ 期末大题完全解决方案 _ 答案

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1.

$$(1)E_1^{(1)} = H'_{11} = b, \quad E_2^{(1)} = H'_{22} = b, \quad E_1^{(2)} = \frac{|H'_{21}|^2}{E_1^0 - E_2^{(0)}} = \frac{a^2}{E_{01} - E_{02}}, \quad E_2^{(2)} = \frac{|H'_{12}|^2}{E_2^0 - E_1^{(0)}} = \frac{a^2}{E_{02} - E_{01}}$$

$$E_1 = E_{01} + b + \frac{a^2}{E_{01} - E_{02}}, \quad E_2 = E_{02} + b + \frac{a^2}{E_{02} - E_{01}}$$

$$(2)\begin{vmatrix} b-\lambda & a \\ a & b-\lambda \end{vmatrix} = 0, \quad \lambda_1 = b + a, \quad \lambda_2 = b - a, \quad E_1 = E_0 + b + a, \quad E_2 = E_0 + b - a$$

3.

$$(1)z = \sum g_i e^{-\beta\varepsilon_i} = e^{\alpha} \sum g_i e^{-\alpha-\beta\varepsilon_i}$$

$$\frac{\partial lnz}{\partial\beta} = \frac{1}{z}\frac{\partial z}{\partial\beta} = \frac{1}{z}\sum -g_i\varepsilon_i e^{-\beta\varepsilon_i} = \frac{e^{-\alpha}}{N}\sum -g_i\varepsilon_i e^{-\beta\varepsilon_i} = -\frac{1}{N}\sum \varepsilon_i g_i e^{-\alpha-\beta\varepsilon_i} = -\frac{1}{N}\sum \varepsilon_i n_i$$

$$\therefore \overline{E} = -N\frac{\partial lnz}{\partial\beta}$$

$$(2)d\overline{E} = \delta W + \delta Q, \quad \delta W = Ydy, \quad d\overline{E} = d\sum n_i\varepsilon_i = \sum n_i d\varepsilon_i + \sum \varepsilon_i dn_i$$

$$\because dW = \sum n_i d\varepsilon_i = \sum Y_k dy_k = \sum n_i \sum \frac{\partial\varepsilon_i}{\partial y_k}dy_k$$

$$\therefore Y_k = \sum n_i \frac{\partial\varepsilon_i}{\partial y_k} = \sum g_i e^{-\alpha-\beta\varepsilon_i}\frac{\partial\varepsilon_i}{\partial y_k} = -\frac{N}{z\beta}\frac{\partial z}{\partial y_k}$$

$$P = Y_k = -\frac{N}{\beta}\frac{\partial lnz}{\partial y_k} = \frac{N}{\beta}\frac{\partial lnz}{\partial V}$$

4.

$$(1)dE = \delta W + \delta Q, \quad \because \delta W = 0, \quad \therefore dE = \delta Q, C_V = \frac{\partial Q}{\partial T} = (\frac{\partial E}{\partial T})_V$$

$$(2)dF = -SdT - PdV, \quad \therefore P = -\frac{\partial F}{\partial V}$$

5.

$$\varepsilon_n = (n + \frac{1}{2})h\nu, \quad g_n = 1, \quad \theta_\nu = \frac{h\nu}{k}, x = \frac{\theta_\nu}{T} = \beta h\nu, \quad z^{\nu}(\beta) = \sum e^{-(n+\frac{1}{2})h\nu} = \frac{e^{-\frac{1}{2}\beta h\nu}}{1 - e^{-\frac{1}{2}\beta h\nu}}$$

$$\overline{E} = -N\frac{\partial lnz}{\partial\beta} = Nh\nu(\frac{1}{2} + \frac{1}{e^{\beta h\nu}} - 1)$$

7.

$$(1)W\{n_i\} = \prod \frac{g_i!}{n_i!(g_i-n_i)!} \approx = \frac{g_i^{g_i}}{n_i^{n_i}(g_i-n_i)^{g_i-n_i}}, \quad ln(W\{n_i\}) = \sum g_i lng_i - n_i lnn_i + (n_i - g_i)ln(g_i - n_i)$$

$$F = ln(W\{n_i\}) + \alpha(N - \sum n_i) + \beta(E - \sum n_i\varepsilon_i), \quad \frac{\partial F}{\partial n_i} = ln(\frac{g_i}{n_i} - 1) - \alpha - \beta\varepsilon_i = 0, \quad n_i = \frac{g_i}{e^{\alpha+\beta\varepsilon_i}+1}$$

$$(2)N = \int_0^{\mu_0} f(\varepsilon)g(\varepsilon)d\varepsilon = \int_0^{\mu_0} g(\varepsilon)d\varepsilon, \quad g(\varepsilon) = \frac{2\pi VJ}{h^3}(2m)^{\frac{3}{2}}\sqrt{\varepsilon}d\varepsilon$$

$$N = \frac{4\pi JV}{3h^3}(2m\mu_0)^{\frac{3}{2}}, \quad \mu_0 = \frac{h^2}{2m}(\frac{3N}{4\pi JV})^{\frac{2}{3}}, \quad E = \int_0^{\mu_0} \varepsilon f(\varepsilon)g(\varepsilon)d\varepsilon = \frac{3}{5}N\mu_0$$

8.

$$W\{n_i\} = \prod \frac{(n_i+g_i-1)!}{n_i!(g_i-1)!} \approx \frac{(n_i+g_i)^{n_i+g_i}}{n_i^{n_i}g_i^{g_i}}, \quad ln(W\{n_i\}) = \sum (n_i+g_i)ln(n_i+g_i) - n_i lnn_i - g_i lng_i$$

$$F = ln(W\{n_i\}) + \alpha(N - \sum n_i) + \beta(E - \sum n_i\varepsilon_i), \quad \frac{\partial F}{\partial n_i} = ln(\frac{g_i}{n_i} - 1) - \alpha - \beta\varepsilon_i = 0, \quad n_i = \frac{g_i}{e^{\alpha+\beta\varepsilon_i}+1} = \frac{g_i}{e^{\beta\varepsilon_i}+1}$$

9.

$$(1)\Omega(\varepsilon) = \int dxdydz \int dp_x dp_y dp_z = V\int_0^{2\pi} d\varphi \int_0^{\pi} sin\theta d\theta \int_0^{\sqrt{2m\varepsilon}} p^2 dp = \frac{4\pi}{3}V(2m\varepsilon)^{\frac{3}{2}}$$

$$g(\varepsilon)d\varepsilon = \frac{Jd\Omega(\varepsilon)}{h^3} = \frac{2\pi}{h^3}JV(2m)^{\frac{3}{2}}\sqrt{\varepsilon}d\varepsilon$$

$$(2)\Omega(\varepsilon) = \int dxdydz \int dp_x dp_y dp_z = V\int_0^{2\pi} d\varphi \int_0^{\pi} sin\theta d\theta \int_0^{\frac{\varepsilon}{c}} p^2 dp = \frac{4\pi}{3c^3}V\varepsilon^3$$

$$g(\varepsilon)d\varepsilon = \frac{Jd\Omega(\varepsilon)}{h^3} = \frac{8\pi}{c^3h^3}JV\varepsilon^3$$

10.

$$g(\varepsilon)d\varepsilon = \frac{2\pi}{h^3}V(2m)^{\frac{3}{2}}\sqrt{\varepsilon}d\varepsilon, \quad z = \int_0^{\infty} g(\varepsilon)e^{-\beta\varepsilon}d\varepsilon = \frac{V}{h^2}(\frac{2\pi m}{\beta})^{\frac{3}{2}}$$

$$\overline{E} = -N\frac{\partial lnz}{\partial\beta} = \frac{3}{2}NkT, \quad P = \frac{N}{\beta}\frac{\partial lnz}{\partial V} = \frac{NkT}{V} = \frac{2\overline{E}}{3V}$$

11.

$$\Omega(\varepsilon) = \int dxdy \int dp_x dp_y = S\int_0^{2\pi} d\theta = \int_0^{\sqrt{2m\varepsilon}} pdp = 2\pi Sm\varepsilon$$

$$g(\varepsilon)d\varepsilon = \frac{Jd\Omega(\varepsilon)}{h^2} = \frac{2\pi JSm}{h^2}, \quad N = \int_0^{\infty} \frac{2\pi JSm}{h^2(e^{-\beta\varepsilon}-1)} \to \infty$$

13.

$$\Omega(\varepsilon) = \int dxdydz \int dp_x dp_y dp_z = V\int_0^{2\pi} d\varphi \int_0^{\pi} sin\theta d\theta \int_0^{(\frac{\varepsilon}{\alpha})^{\frac{1}{l}}} p^2 dp = \frac{4\pi}{3}V(\frac{\varepsilon}{\alpha})^{\frac{3}{l}}$$

$$g(\varepsilon)d\varepsilon = \frac{Jd\Omega(\varepsilon)}{h^3} = \frac{4\pi JV\varepsilon^{\frac{3}{l}-1}}{h^3 l\alpha^{\frac{3}{l}}} = C_1 V\varepsilon^{\frac{3}{l}-1}$$

$$z = \int_0^{\infty} g(\varepsilon)e^{-\beta\varepsilon}d\varepsilon = C_1 V\int_0^{\infty} \varepsilon^{\frac{3}{l}-1}e^{-\beta\varepsilon}d\varepsilon$$

$$\overline{E} = -N\frac{\partial lnz}{\partial\beta} = \frac{N}{z}C_1 V\int_0^{\infty} \varepsilon^{\frac{3}{l}}e^{-\beta\varepsilon}d\varepsilon = \frac{N}{z}C_1 V\beta^{-\frac{l}{3}}\int_0^{\infty} x^{\frac{3}{l}}e^{-x}dx = \frac{N}{z}C_1 V\beta^{-\frac{l}{3}}\Gamma(\frac{3}{l}+1)$$

$$P = \frac{N}{\beta}\frac{\partial lnz}{\partial V} = \frac{N}{\beta z}C_1\int_0^{\infty} \varepsilon^{\frac{3}{l}-1}e^{-\beta\varepsilon} = \frac{N}{z}C_1\beta^{-\frac{1}{3}}\int_0^{\infty} x^{\frac{3}{l}-1}e^{-x}dx = \frac{N}{z}C_1\beta^{-\frac{1}{3}}\Gamma(\frac{3}{l})$$

$$\therefore P = \frac{l\overline{E}}{3V}$$

14.

$$\varepsilon_i = \varepsilon^t + \varepsilon_\alpha^l + U = \frac{p_x^2+p_y^2+p_z^2}{2m} + \varepsilon_\alpha^l + U, \quad g_i = \frac{g_\alpha^l}{h^3} dxdydzdp_xdp_ydp_z$$
$$n(p_x,p_y,p_z)dp_xdp_ydp_z = \frac{1}{h^3} e^{-\alpha - \frac{\beta}{2m}(p_x^2+p_y^2+p_z^2)} dp_xdp_ydp_z \int e^{-\beta U} dxdydz \sum_\alpha g_\alpha^l e^{-\beta \varepsilon_\alpha^l}$$
$$N = \frac{e^{-\alpha}}{h^3} \int e^{-\frac{\beta}{2m}(p_x^2+p_y^2+p_z^2)} dp_xdp_ydp_z \int e^{-\beta U} dxdydz \sum_\alpha g_\alpha^l e^{-\beta \varepsilon_\alpha^l}$$
$$n(p_x,p_y,p_z)dp_xdp_ydp_z = \frac{N}{(2\pi mkT)^{\frac{3}{2}}} e^{-\frac{p_x^2+p_y^2+p_z^2}{2mkT}} dp_xdp_ydp_z$$
$$n(v_x,v_y,v_z) = N(\frac{m}{2\pi kT})^{\frac{3}{2}} e^{-\frac{m}{2kT}(v_x^2+v_y^2+v_z^2)}$$

15.

$$\Omega(\varepsilon) = \int dxdydz \int dp_xdp_ydp_z = V\int_0^{2\pi} d\varphi \int_0^\pi sin\theta d\theta \int_0^{\frac{\varepsilon}{c}} p^2dp = \frac{4\pi V\varepsilon^2}{3c^2}$$
$$g(\varepsilon)d\varepsilon = \frac{J}{h^3} d\Omega(\varepsilon) = \frac{4\pi VJ\varepsilon^2}{c^3h^3} d\varepsilon = \frac{8\pi V\varepsilon^2}{c^3h^3} d\varepsilon$$
$$N = \int_0^{\mu_0} f(\varepsilon)g(\varepsilon)d\varepsilon = \int_0^{\mu_0} g(\varepsilon)d\varepsilon = \frac{8\pi V\mu_0^3}{3c^3h^3}, \quad \mu_0 = \frac{ch}{2}(\frac{3N}{\pi V})^{\frac{1}{3}}$$

16.

$$N = \frac{2\pi VJ}{h^3}(2m)^{\frac{3}{2}} \int_0^\infty \frac{\sqrt{\varepsilon}}{e^{\frac{\varepsilon}{kT_c}}-1} = 2.612J(\frac{2\pi mkT_c}{h^2})^{\frac{3}{2}}, \quad T_c = \frac{h^2}{2\pi mk}(\frac{n}{2.612J})^{\frac{2}{3}}$$

21.

$$(1)\varepsilon = h\nu = pc, \quad p = \frac{h\nu}{c}, \quad g(p)dp = \frac{8\pi V}{h^3}p^2dp, \quad g(\nu)d\nu = \frac{8\pi V\nu^2}{c^3}d\nu$$
$$n(\nu)d\nu = \frac{8\pi V\nu^2 d\nu}{c^3(e^{\frac{h\nu}{kT}}-1)}, \quad E(\nu,t)dt = \frac{8\pi Vh\nu^3 d\nu}{c^3(e^{\frac{h\nu}{kT}}-1)}$$
$$(2)N = \int_0^\infty n(\nu)d\nu = (272.7T)^3V$$

(3) 不守恒。

22.

$$g(p)dp = \frac{4\pi V}{h^2}pdp, \quad g(\nu)d\nu = \frac{4\pi V\nu d\nu}{c^2}, \quad n(\nu)d\nu = \frac{4\pi V\nu d\nu}{c^2(e^{\frac{h\nu}{kT}}-1)}$$
$$N = \int_0^\infty \frac{4\pi V}{c^2}\frac{\nu d\nu}{e^{\frac{h\nu}{kT}}-1} = \frac{4\pi V}{c^2}(\frac{kT}{h})^2 \int_0^\infty \frac{xdx}{e^x-1}$$

23.

$$\phi(\beta,y) = -\int_0^\infty \frac{8\pi V\varepsilon^2 d\varepsilon}{h^3c^3} ln(1-e^{-\beta\varepsilon}) = \frac{8\pi^5 V}{45\beta^3h^3c^3}, \quad \overline{E} = -\frac{d\phi}{d\beta} = bVT^4$$
$$b = \frac{8\pi^5 V}{15h^3c^3}, \quad P = \frac{1}{\beta}\frac{d\phi}{dV} = \frac{1}{3}bT^4 = \frac{\overline{E}}{3V}$$