随机过程2020-2021期末

Deschain

2022 年 1 月 24 日

1.

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \end{bmatrix}$$

2.

$$P = \begin{bmatrix} \frac{3}{10} & \frac{1}{2} & \frac{1}{5} \\ \frac{3}{7} & \frac{19}{70} & \frac{3}{10} \\ \frac{3}{20} & \frac{21}{80} & \frac{47}{80} \end{bmatrix}, \quad \pi = \pi \cdot P, \quad \therefore \pi = [\frac{2}{7}, \frac{1}{3}, \frac{8}{21}]$$

3.

(a)

$$Z_n = Y(n) = Z(n-1) + Y(1), \quad \therefore Z_n \sim Markov$$

$$P(\sum_{k=0}^{N} X_k = m) = (\frac{1}{2})^N \binom{N}{\frac{m+N}{2}}$$

$$P(N(t) = N) = \frac{\lambda^N}{N!} e^{-\lambda}$$

$$P_{ij}(1) = \sum_{m=|j-i|}^{\infty} \frac{\lambda^m}{m!} e^{-\lambda} (\frac{1}{2})^m \binom{m}{\frac{m+j-i}{2}}$$

$\{Z_n\}$各状态相通，常返性相同，以下计算状态0的常返性。

$$P_{00}(n) = \sum_{n=0}^{\infty} \sum_{j=0}^{\infty} \frac{(n\lambda)^{2j}}{2j!} e^{-\lambda n} \binom{2j}{j} (\frac{1}{2})^{2j} = \sum_{j=0}^{\infty} \frac{\binom{2j}{j}}{(2j)!} \lambda^{2j} \sum_{n=0}^{\infty} n^{2j} e^{-\lambda n} \approx \sum_{j=0}^{\infty} \frac{\binom{2j}{j}}{(2j)!} \int_0^{\infty} x^{2j} e^{-\lambda x} dx$$

$$= \frac{1}{\lambda} \sum_{j=0}^{\infty} \binom{2j}{j} (\frac{1}{4})^j \approx \frac{1}{\lambda} \sum_{j=0}^{\infty} (\frac{1}{4})^j \frac{(\frac{2j}{e})^{2j} \sqrt{4\pi j}}{(\frac{j}{e})^{2j} 2\pi j} = \frac{1}{\lambda} \sum_{j=0}^{\infty} (\pi j)^{-\frac{1}{2}} \to \infty$$

$\therefore$是常返的。

4.

$$(a)P(T>t)=e^{-\lambda_A t}+(1-e^{-\lambda_A t})e^{\lambda_B(t-1)}=e^{-\lambda_A t}+e^{-\lambda_B(t-1)}-e^{-(\lambda_A+\lambda_B)t+\lambda_B},\quad t>1$$

$$F_T(t)=1+e^{-(\lambda_A+\lambda_B)t+\lambda_B}-e^{-\lambda_A t}-e^{-\lambda_B(t-1)},\quad t>1$$

$$f_T(t)=-(\lambda_A+\lambda_B)e^{-(\lambda_A+\lambda_B)t+\lambda_B}\lambda_A e^{-\lambda_A t}+\lambda_B e^{-\lambda_B(t-1)},\quad t>1$$

$$E(T)=\int_0^\infty tf_T(t)dt=e^{-\lambda_A}(\frac{1}{\lambda_A}-\frac{1}{\lambda_A+\lambda_B})+1+\frac{1}{\lambda_B}$$

$$(b)P(A+B=10)=\sum_{k=0}^{10}\frac{(2\lambda_A)^k\lambda_B^{10-k}}{k!(10-k)!}e^{-2\lambda_A-\lambda_B}$$

$$P(A=4,B=6)=\frac{(2\lambda_A)^4\lambda_B^6}{4!\times 6!}e^{-2\lambda_A-\lambda_B}$$

$$P(A=4|A+B=10)=\frac{\frac{(2\lambda_A)^4\lambda_B^6}{4!\times 6!}}{\sum\limits_{k=0}^{10}\frac{(2\lambda_A)^k\lambda_B^{10-k}}{k!(10-k)!}}$$

5.

(a)设每秒逃逸的粒子数为N，逃逸速率$P_1=(1-p)^k$

$$\therefore N(t)\sim Poisson(p_1\lambda),\quad \lambda=100,\quad E[N]=100(1-p)^k$$

(b)设总辐射强度为$Y(t)$，记$N(t)\sim Poisson(\lambda_1).\lambda_1=100(1-p)^kP_n(k)$，其中 $P_n(k)$是n个防护罩中有k个打开的概率。

$$Y(t)=\sum_{k=0}^{N(t)}V_k(t,t_0),\quad E[V_1(t,t_0)]=3e^{\mu(t-t_0)},\quad t\geq t_0$$

$$E[Y(t)]=\lambda_1\int_0^t 3e^{\mu(t-t_0)}dt_0=\frac{3\lambda_1}{\mu}(e^{\mu t}-1)=\frac{3\lambda_1}{\mu}(e^{10\mu}-1),\quad E[V_1^2(t,t_0)]=\frac{4}{3}e^{\mu(t-t_0)},\quad t\geq t_0$$

$$Var[Y(t)]=\lambda_1\int_0^t E[V_1^2(t,t_0)]dt_0=\lambda_1\int_0^t\frac{4}{3}e^{2\mu(t-t_0)}dt_0=\frac{2\lambda_1}{3\mu}(e^{20\mu}-1)$$