量统2015郭永期中

Deschain

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一、

1.(1)体系能量有确定值的状态。

(2)粒子在无穷远处出现的概率为0的状态。

(3)若$\psi(x)=\psi(-x)$，则是偶（正）宇称；若$\psi(x)=-\psi(-x)$，则是奇（负）宇称。

(4)能量为E的粒子有一定概率穿透高度为$U_0(U_0>E)$的势垒。

2.

$$E=\hbar\omega=h\nu,\lambda=\frac{h}{p}$$

3.

$$[a,a^\dagger]=\frac{m\omega}{2\hbar}([x,x]+\frac{i}{m\omega}([\hat{p}_x,x]-[x,\hat{p}_x])+\frac{i}{m^2\omega^2}[\hat{p}_x,\hat{p}_x])=\frac{m\omega}{2\hbar}\times\frac{i}{m\omega}\times 2i\hbar=1$$

4.

$$[\hat{p}_x,f(x)]\psi(x)=\hat{p}_x(f(x)\psi(x))-f(x)\hat{p}_x\psi(x)=-i\hbar\frac{\partial}{\partial x}(f(x)\psi(x))+i\hbar f(x)\frac{\partial}{\partial x}\psi(x)=-i\hbar(\frac{\partial}{\partial x}f(x))\psi(x)$$

$$[\hat{p}_x,f(x)]=-i\hbar\frac{\partial}{\partial x}f(x)$$

$$[\hat{L}_z,\hat{L}_+]=[\hat{L}_z,\hat{L}_x]+i[\hat{L}_z,\hat{L}_y]=i\hbar\hat{L}_y+\hbar\hat{L}_x=\hbar\hat{L}_+$$

$$[\hat{L}_z,\hat{L}_-]=[\hat{L}_z,\hat{L}_x]-i[\hat{L}_z,\hat{L}_y]=i\hbar\hat{L}_y-\hbar\hat{L}_x=-\hbar\hat{L}_-$$

5.设a_n对应的本征函数$|\psi_{a_n}>$,b_n对应的本征函数$|\psi_{b_n}>$，

$$<\psi_{a_n}|\psi_{b_n}>=<\psi_{b_n}|\psi_{a_n}>$$

$$[\hat{A},\hat{B}]\psi=\hat{A}\hat{B}\psi-\hat{B}\hat{A}\psi=\hat{A}\sum_i^\infty b_i\psi_{b_i}-\hat{B}\sum_k^\infty a_k\psi_{a_k}=\sum_i^\infty b_i\sum_j^\infty a_j\psi_{a_j,b_i}-\sum_k^\infty a_k\sum_l^\infty b_l\psi_{a_k,b_l}$$

$$\because P(a_n,b_n)=P(b_n,a_n)\therefore b_i=b_l,a_j=a_k(i=l,j=k)\therefore[\hat{A},\hat{B}]\psi=0,[\hat{A},\hat{B}]=0$$

6.记$\hat{L}_z$的本征态为$|m>$，

$$<m|\hat{L}_z|m>=m\hbar|m>$$

$$[\hat{L}_x,\hat{L}_y]|m>=i\hbar L_z|m>=i\hbar^2m|m>$$

$\therefore$在$\hat{L}_z$的本征态$|m>$中，$[\hat{L}_x,\hat{L}_y]=i\hbar^2m$

7.原式$=(\psi_n,(\hat{H}\hat{A}-\hat{A}\hat{H})\psi_n)=(\psi_n,\hat{H}\hat{A}\psi_n)-(\psi_n,\hat{A}\psi_n)E_n$

$\because\hat{H}$是线性Hermite算符 $\therefore(\psi_n,\hat{H}\hat{A}\psi_n)=(\hat{A}\psi_n,\hat{H}\psi_n)=E_n(\hat{A}\psi_n,\psi_n)$

$\because\hat{A}$是线性Hermite算符$\therefore(\hat{A}\psi_n,\psi_n)=(\psi_n,\hat{A}\psi_n)\therefore(\psi_n,[\hat{H},\hat{A}]\psi_n)=0$

二、

1.

$$B^\dagger = A^\dagger(AA^\dagger)A = A^\dagger(1-A^\dagger A)A = A^\dagger A - (A^\dagger)^\dagger A^\dagger = A^\dagger A = B$$

2.$\because B^2 = B \therefore B$有2个本征值$\lambda_0 = 1, \lambda_1 = 0$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}, \quad B^2 = A^\dagger A, \quad A^2 = 0$$

$$A = \begin{bmatrix} 0 & 0 \\ e^{i\delta} & 0 \end{bmatrix}$$

1　三、

1.基态：

$$E_0 = \frac{\pi^2\hbar^2}{ma^2}, \quad \psi_0 = \frac{2}{a}sin(\frac{\pi x_1}{a})sin(\frac{\pi x_2}{a}), \quad f_0 = 1$$

2.第一激发态：

$$E_1 = \frac{5\pi^2\hbar^2}{2ma^2}$$

$$\psi_{11} = \frac{2}{a}[sin(\frac{\pi x_1}{a})sin(\frac{2\pi x_2}{a}) - sin(\frac{\pi x_1}{a})sin(\frac{\pi x_2}{a})] \times \begin{cases} |10> \\ |11> \\ |1-1> \end{cases}$$

$$\psi_{12} = \frac{2}{a}[sin(\frac{\pi x_1}{a})sin(\frac{2\pi x_2}{a}) + sin(\frac{\pi x_1}{a})sin(\frac{\pi x_2}{a})]|00>$$

$$f_1 = 4$$

四、

$$\psi(r,\theta,\varphi) = \sqrt{\frac{8\pi}{3}}R(r)Y_{11}(\theta,\varphi) + \sqrt{\frac{4\pi}{3}}R(r)Y_{10}(\theta,\varphi)$$

$$(1)L^2 = 2\hbar^2, \quad P = 1$$

$$(2)P(L_z = 0) = \frac{1}{3}, P(L_z = \hbar) = \frac{2}{3}, \overline{L_z} = \frac{2}{3}\hbar$$

五、

$$\hat{\vec{J}} = \hat{\vec{S_1}} + \hat{\vec{S_2}}, \quad \hat{H} = A\hat{J}_z + \frac{B}{2}(\hat{J}^2 - \hat{S}_1^2 - \hat{S}_2^2)$$

$$(1)j = 0, m = 0, E_1 = -\frac{3}{4}B\hbar^2, \psi_1 = |00>$$

$$(2)j = 1, m = 1, E_2 = A\hbar + \frac{1}{4}B\hbar^2, \psi_2 = |11>$$

$$(3)j = 1, m = 0, E_3 = \frac{1}{4}B\hbar^2, \psi_3 = |10>$$

$$(4)j = 1, m = -1, E_4 = -A\hbar + \frac{1}{4}B\hbar^2, \psi_4 = |1-1>$$

2　六、

$$H' = -q\varepsilon x, \quad E_n^{(0)} = (n + \frac{1}{2})\hbar\omega$$

$$E_n^{(1)} =< n|-q\varepsilon x|n >= 0$$

$$H'_{n+1,n} =< n+1|-q\varepsilon x|n >= \sqrt{\frac{(n+1)\hbar}{2\mu\omega}}, \quad else \quad H'_{m,n} = 0$$

$$E_n^{(2)} = \frac{|H'_{n+1,n}|^2}{E_n^{(0)} - E_{n+1}^{(0)}} = -\frac{n+1}{2\mu\omega^2}$$

$$\psi_n = \frac{H'_{n+1,n}\psi_{n+1}^{(0)}}{E_n^{(0)} - E_{n+1}^{(0)}} = -\sqrt{\frac{n+1}{2\mu\hbar\omega^3}}|n+1>$$

$$\therefore \psi_n = |n> - \sqrt{\frac{n+1}{2\mu\omega^3}}|n+1>, \quad E_n = (n + \frac{1}{2})\hbar\omega - \frac{n+1}{2\mu\omega^2}$$