Exercise in the Camera Model

Henrik Aanæs, haa@imm.dtu.dk

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This is the second exercise in view geometry, related to Chapter 1 and the first part of Chapter 2 of my lecture notes. Firstly, there will be questions relation to radial distortion, and the camera model in general. This will be followed by questions on two view geometry, and epipolar geometry in particular.

1 Camera Model

Question 1. Following the approach from the first exercise project the 3D points from the `Box3D` function via a pinhole or projective camera where the rotation is given by the function `Rot(0.2,-0.3,0.1)`, which computes a rotation matrix from Euler angles. Furthermore

$$\mathbf{t} = \begin{bmatrix} 0.8866 \\ 0.5694 \\ 0.1911 \end{bmatrix} \quad , \quad f = 1000 \quad , \quad \Delta x = 300 \quad , \quad \Delta y = 200 \ .$$

The recommended axis is set via

```
axis equal
axis([0 640 0 480])
```

Question 2. Using the same pinhole camera as in Question 1, extend the model with radial distortion with $k_3 = -1e-6$ and $k_5 = 1e-12$. Project the `Box3D` points and compare to the results from Question 1.

Question 3. Try to set $k_5 = 0$, project and compare to the previous two questions.

2 Epipolar Geometry – Part 1

Question 4. Write a MatLab function `CrossOp(x)`, which takes a 3 vector and returns the 3×3 matrix corresponding to the cross product with that vector, c.f. Appendix A.5. Think of a way of checking that you function works correct, e.g. comparing to the matlab function `cross`, and perform the check. This is a way of validating your code.

Question 5. Set up two cameras `Cam1` and `Cam2` with internal parameters

$$\mathbf{A} = \begin{bmatrix} 100 & 0 & 300 \\ 0 & 1000 & 200 \\ 0 & 0 & 1 \end{bmatrix} \quad ,$$

The rotation of the fist camera, `Cam1`, is the identity matrix and the translation of this camera is zero. The second camera's, `Cam2`, rotation is given by `Rot(-0.1,0.1,0)`, and a translation vector of

$$\mathbf{t} = \begin{bmatrix} 0.2 \\ 2 \\ 0.1 \end{bmatrix} \quad .$$

Project the 3D point

$$Q = \begin{bmatrix} 1 \\ 0.5 \\ 4 \\ 1 \end{bmatrix}$$

To these two camera with the results denoted by $\mathbf{q}_1$ and $\mathbf{q}_2$ respectively. What is the coordinates of these two image points.

Question 6. Compute the fundamental matrix, of the cameras in Question 5.

Question 7. What is the epipolar line in camera two given the scenario in Question 5?

Question 8. With the scenario from the previous four questions is $\mathbf{q}_2$ located on the epipolar line from Question 7 – do the computations? Why must this be so?

3 Epipolar Geometry – Part 2

In this part you should work with the data from the file `TwoImageData.mat`. This file contains

- Two images `im1` and `im2`
- The internal parameters from both cameras `A`.
- The rotation matrices for the two cameras `R1` and `R2`.
- The translation vectors for the two cameras `T1` and `T2`.

Question 9. Let Q denotes 3D coordinates in the global coordinate system, and $\tilde{\text{Q}}$ 3D coordinates in the coordinate system of camera one, i.e.

$$\tilde{\mathbf{Q}} = \mathbf{R_1 Q} + \mathbf{t_1} \quad .$$

Perform the calculations showing that

$$\mathbf{Q} = \mathbf{R}_1^\mathbf{T}\tilde{\mathbf{Q}} - \mathbf{R}^\mathbf{T}\mathbf{t}_1 \quad .$$

Question 10. Do the calculations demonstrating that if cameras one and two work on coordinates in the coordinate system of camera one, then the camera models are given by

$$\mathbf{q_1} = \mathbf{A}[\mathbf{I}\ \mathbf{0}] \begin{bmatrix} \tilde{\mathbf{Q}} \\ 1 \end{bmatrix} \quad , \quad \mathbf{q_2} = \mathbf{A}[\tilde{\mathbf{R_2}}\ \tilde{\mathbf{t_2}}] \begin{bmatrix} \tilde{\mathbf{Q}} \\ 1 \end{bmatrix} \quad ,$$

with

$$\tilde{\mathbf{R_2}} = \mathbf{R_2}\mathbf{R}_1^\mathbf{T} \quad , \quad \tilde{\mathbf{t_2}} = \mathbf{t2} - \mathbf{R_2}\mathbf{R}_1^\mathbf{T}\mathbf{t_1} \quad .$$

Question 11. Compute the fundamental matrix between the two images.

Question 12. Make a MatLab function that

1. Displays the two images in two figures.
2. Lets you annotate an image point in the first image. Here the function `q1=[ginput(1) 1]'` is recommended.
3. Computes the corresponding epipolar line in the second image.
4. Draws this epipolar line in the second image. Here I have written the function `DrawImageLine(Rows,Cols,l)` to help you out.

Experiment with this function, validating that the epipolar lines go through the image point in image two corresponding to the image point you annotated in image one.

Question 13. Repeat Question 12. but where you annotate the image points in image *two* and observe the epipolar line in image *one*.

4 Homography — Photographing a Plane

In this section you will work with the points generated by the MatLab function
`PointsInPlane`

This function generates a bunch of points in a plane, plots a 2D figure of these and return the 3D points. These 3D points are given by

$$Q = aA + bB + C$$

where

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} \quad , \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad , \quad C = \begin{bmatrix} 1 \\ 0.5 \\ 5 \end{bmatrix} \quad .$$

Question 14. Project these 3D points with the camera described in Question 1.

Question 15. Compute the homography describing the transformation from the plane described by A, B, C to the image plane.

Question 16. Compute the homography describing the transformation from the image plane to the plane described by A, B, C. Apply this homography to the points in the image plane plot the result and confirm that you get the same, plot as that generated by
`PointsInPlane`