Consider the kinematic unicycle model

$$\begin{aligned} \dot{x}_1 &= u_1 \cos(x_3) \\ \dot{x}_2 &= u_1 \sin(x_3) \\ \dot{x}_3 &= u_2 \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^3$ is the state and $u \in \mathbb{R}^2$ is the control input. A tight decomposition function for (1) is formed in the following way.

First, observe that the system

$$\dot{x} = \sin(w) \tag{2}$$

is mixed monotone with a tight decomposition function given by

$$d^{\sin}(w, \widehat{w}) := \begin{cases} \sin(w) & \text{if } (\cos(w), \cos(\widehat{w})) \succeq 0 \text{ and } |w - \widehat{w}| \leq \pi, \\ \sin(\widehat{w}) & \text{if } (\cos(w), \cos(\widehat{w})) \preceq 0 \text{ and } |w - \widehat{w}| \leq \pi, \\ \mathrm{sign}(w - \widehat{w}) & \text{if } |w - \widehat{w}| \geq 2\pi \\ \mathrm{sign}(w - \widehat{w}) & \text{if } \cos(w) \leq 0 \leq \cos(\widehat{w}) \text{ and } |w - \widehat{w}| \leq 2\pi, \\ \mathrm{sign}(w - \widehat{w}) & \text{if } \cos(w)\cos(\widehat{w}) \geq 0 \text{ and } \pi \leq |w - \widehat{w}| \leq 2\pi, \\ \min\{\sin(w), \sin(\widehat{w})\} & \text{if } w \leq \widehat{w} \text{ and } \cos(w) \geq 0 \geq \cos(\widehat{w}), \\ & \quad \text{and } |w - \widehat{w}| \leq 2\pi, \\ \max\{\sin(w), \sin(\widehat{w})\} & \text{if } w \geq \widehat{w} \text{ and } \cos(w) \geq 0 \geq \cos(\widehat{w}) \\ & \quad \text{and } |w - \widehat{w}| \leq 2\pi \end{cases} \tag{3}$$

and the system

$$\dot{x} = \cos(w) \tag{4}$$

is mixed monotone with a tight decomposition function given by

$$d^{\cos}(w, \widehat{w}) := \begin{cases} \cos(w) & \text{if } (\sin(w), \sin(\widehat{w})) \preceq 0 \text{ and } |w - \widehat{w}| \leq \pi, \\ \cos(\widehat{w}) & \text{if } (\sin(w), \sin(\widehat{w})) \succeq 0 \text{ and } |w - \widehat{w}| \leq \pi, \\ \mathrm{sign}(w - \widehat{w}) & \text{if } |w - \widehat{w}| \geq 2\pi \\ \mathrm{sign}(w - \widehat{w}) & \text{if } \sin(w) \geq 0 \geq \sin(\widehat{w}) \text{ and } |w - \widehat{w}| \leq 2\pi, \\ \mathrm{sign}(w - \widehat{w}) & \text{if } \sin(w)\sin(\widehat{w}) \geq 0 \text{ and } \pi \leq |w - \widehat{w}| \leq 2\pi, \\ \min\{\cos(w), \cos(\widehat{w})\} & \text{if } w \leq \widehat{w} \text{ and } \sin(w) \leq 0 \leq \sin(\widehat{w}) \\ & \quad \text{and } |w - \widehat{w}| \leq 2\pi, \\ \max\{\cos(w), \cos(\widehat{w})\} & \text{if } w \geq \widehat{w} \text{ and } \sin(w) \leq 0 \leq \sin(\widehat{w}) \\ & \quad \text{and } |w - \widehat{w}| \leq 2\pi. \end{cases} \tag{5}$$

Next observe that the system

$$\dot{x} = w_1 w_2 \tag{6}$$

is mixed monotone with a tight decomposition function given by

$$d^{w_1 w_2}(w, \widehat{w}) = \begin{cases} \min\{w_1 w_2,\ \widehat{w}_1 w_2,\ w_1 \widehat{w}_2,\ \widehat{w}_1 \widehat{w}_2\} & \text{if } w \preceq \widehat{w} \\ \max\{w_1 w_2,\ \widehat{w}_1 w_2,\ w_1 \widehat{w}_2,\ \widehat{w}_1 \widehat{w}_2\} & \text{if } \widehat{w} \preceq w. \end{cases} \tag{7}$$

Now a tight decomposition function for (1) is given by

$$d^{\mathrm{uni}}(x, u, \widehat{x}, \widehat{u}) = \begin{bmatrix} d^{w_1 w_2}\left(\begin{bmatrix} u_1 \\ d^{\cos}(x_3, \widehat{x}_3) \end{bmatrix}, \begin{bmatrix} \widehat{u}_1 \\ d^{\cos}(\widehat{x}_3, x_3) \end{bmatrix} \right) \\ d^{w_1 w_2}\left(\begin{bmatrix} u_1 \\ d^{\sin}(x_3, \widehat{x}_3) \end{bmatrix}, \begin{bmatrix} \widehat{u}_1 \\ d^{\sin}(\widehat{x}_3, x_3) \end{bmatrix} \right) \\ u_2 \end{bmatrix}. \tag{8}$$