DEN5200

CONTROL SYSTEMS ANALYSIS AND DESIGN

Applying control systems theory to regulate the water level of a tank.

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Contents

Abstract

The dynamics of water filling a tank are analysed and modelled in order to apply control systems theory to regulate the water level in the tank. The equations governing the volume of water are linearized in order to create a transfer function which is then used to tune the parameters of the PID controller. The PID is tested on a nonlinear Simulink block in order to get accurate step response results. The method of manually controlling the pump voltage and two different PID controllers are applied using the CE117 trainer equipment in order to collect data real world data. The data is then compared to the results of the model revealing the nonlinearity of the system, the advantages of using a PID controller and the effect of changing PID parameters on a modelled and physical system.

Introduction

The aim of this laboratory report is to control the water level in a tank using manual and PID tuning in order to better understand how a system can be successfully controlled. Additionally, the aim is to understand how the control parameters of the PID relate to the step response when applied to a modelled system and to a real system. An analysis between the different responses of a modelled linear time invariant system and a real non linear system will be conducted. The PID controller is used in a closed negative feedback system to diminish the error in the system. The input consists in the error between the reference signal and the output signal, while the output is the control signal sent to the system. The PID acronym stands for "potential, integral, derivative", these three parameters change the way in which the whole system will behave when presented with a reference signal. The potential parameter decreases the rise time, but it increases instability and steady state error, the integral decreases steady-state error but increases rise time, while the derivative parameter reduces overshoot but increases rise time. Tuning these parameters in order to improve the performance of the system usually involves a trade-off between rise time, steady state error and overshoot. The following sections introduce the theoretical background and modelling of the system succeeded by tuning the PID parameters. The model step responses between the models and the experimental results will be compared. This includes: the bode plot editor model, the Matlab PID scope response, the real-world response and the manual water level control. This will involve in an analysis of the different characteristics of a control system such as rise time, overshoot, steady state error, stability and robustness. These parameters will give insight on how the transfer between model responses and real system response occurs.

Problem setup

The system that has been used to model the water is modelled using fluid mechanics and dynamics the derivation of which is beyond the scope of this report. However, the resulting equation that defines the change in volume dV(t) based on time is defined by the incoming flow $F_{in}(t)$, the constant k and the area of the base of the tank S:

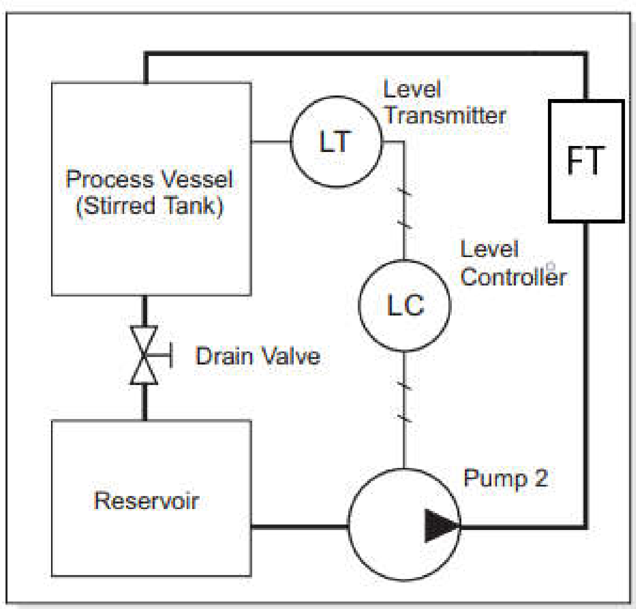
$$\frac{\mathrm{dV(t)}}{\mathrm{dt}} = Fin - k\sqrt{\mathrm{V(t)} \times \frac{1}{\mathrm{S}}} \qquad (1)$$

This equation is highly non-linear and must be linearized in order to model it as a transfer function G. By assuming that the area of the base S is 0.177 m^2 and by calculating the time constant of the system equal to the change in volume divide by the change in flow between two arbitrary heights then K is equal to 0.0006 and the time constant τ is equal to 140. With these two values, the transfer function that most accurately approximates the behaviour of the system is:

$$G(s) = \frac{1}{s+140} \qquad (2)$$

Equation 2 will work best when the height of the water in the cylinder is between 10.8 and 13.5 cm because these were the height values used to calculate the time constant value. Since the system is highly non-linear, the time constant will change at different heights causing the model to be inaccurate at difference height. To ensure the accuracy of the model, the experiment will be conducted at 10 cm which in the experimental setup occurs at the start of the heat exchanger in the CE117 water tank.

To achieve a closed loop system, the CE117 equipment is connected to the CE2000 controlling software where the pump voltage, the height of the water and the input flow of the water can be recorded and connected to a PID controller. Pump 2 is the pump that inputs water in the tank that can receive a range of voltages, it is connected to a reservoir from which the water is drained from the tank. LT stands for "level transmitter" and it is a parallel plate capacitor from which a controller converts the distance between plates to a voltage between 0 and 10 where at 0 the tank is empty and at 10 it is full. FT stands for "flow transmitter" and it represents the entering flow of water in the tank, its units are 1 L/minute per Volt where 0 volts represents no flow of water. The closed loop control system will therefore have as input the reference voltage for LT and the current voltage of LT, the



difference between the two values is then inserted in the PID controller and the system transfer function in order to calculate the required voltage for Pump 2.

The transfer function used for the PID control is equation 3, it is an “improper” transfer function because the order at the numerator is higher than the denominator. To build the PID block in Simulink, the fraction has been simplified to separate the Kd, Kp and Ki parameters.

$$\frac{K_d s^2+K_p s+K_i}{s} = K_d s + K_p + \frac{1}{s} K_i \quad (3)$$

Controller Design and Numerical Simulation

The complete closed loop Simulink system used as a model where the input is the LT voltage and the output is the LT voltage is shown below:

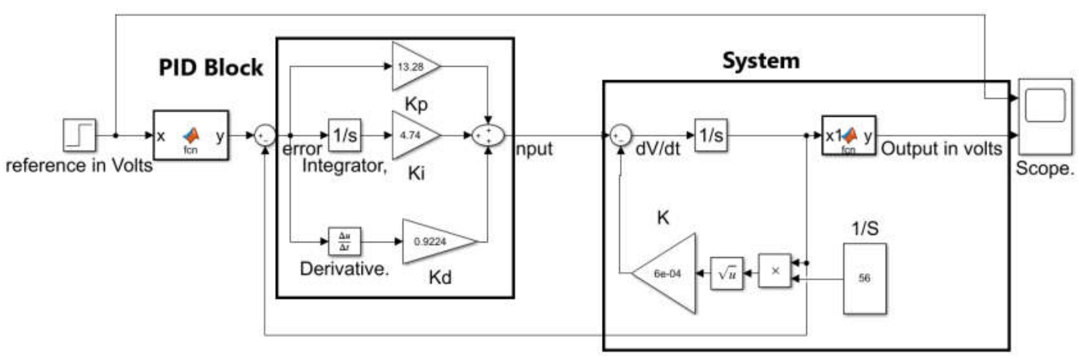


Figure 2: Simulink Complete System Model

Essentially, the “system” block in figure 2 simulates the height response of the water tank in order to receive the height voltage in the scope. In the CE117 the “system” portion of the controller would be replace by the actual water tank and LT sensor.

The controller was designed using the MATLAB app “control system designer” where the transfer function of the system G used was equation 2 and C was initially set 1. The system is a unit feedback system where the LT sensor voltage is not affected by any transfer function hence H is equal 1. To represent the PID controller transfer function, as shown in equation 3, an integrator and two real zeros were added to the closed loop bode plots. These values were moved around the bode plot diagram in order to achieve a stable and a satisfactory rise time, overshoot and steady state error in the step response. The robustness of the system was checked comparing it to the gain margin GM and phase margin PM used in the lectures where the requirements are PM > 45° and GM > 6 dB. After achieving satisfactory results, as shown in figure 3, the PID control parameters were exported to the Simulink block for further stability testing. The results of the app tuning and the scope results of the given PID controllers and the chosen PID controllers are shown in table 1 and 2, while the results scope graphs are shown in figure 4 and 5.

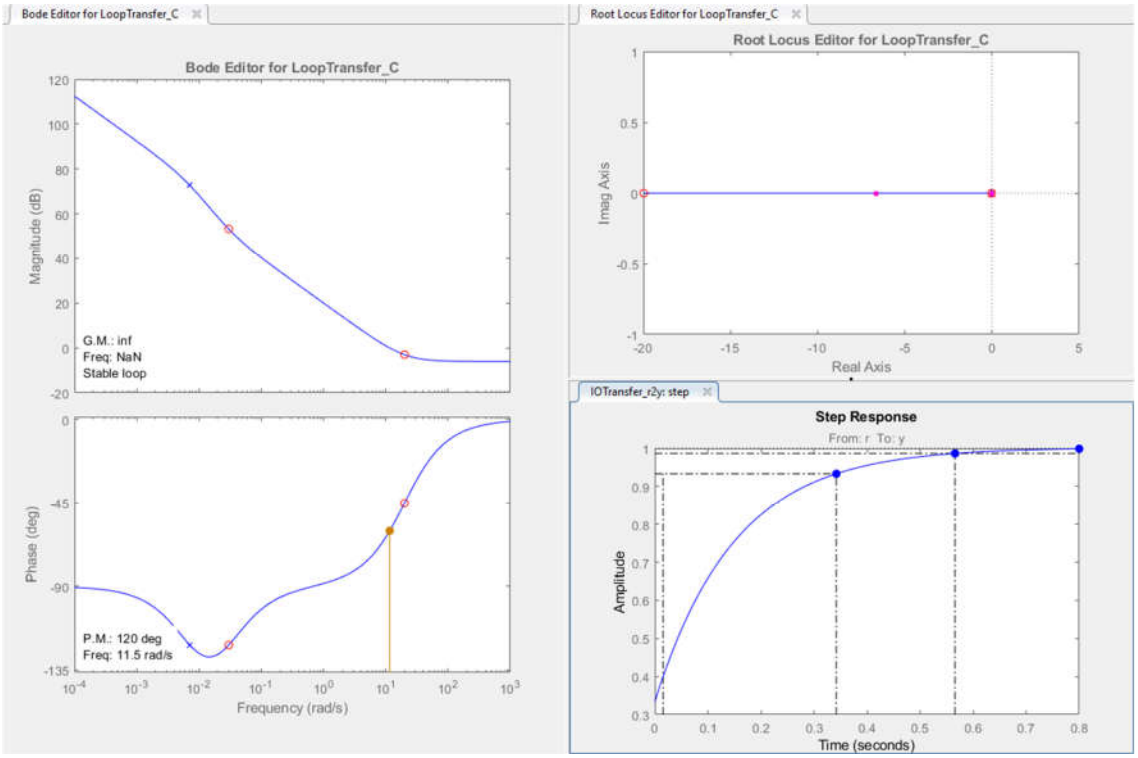


Figure 3: Final PID model where Kp = 10, Ki = 0.3, Kd = 0.5.

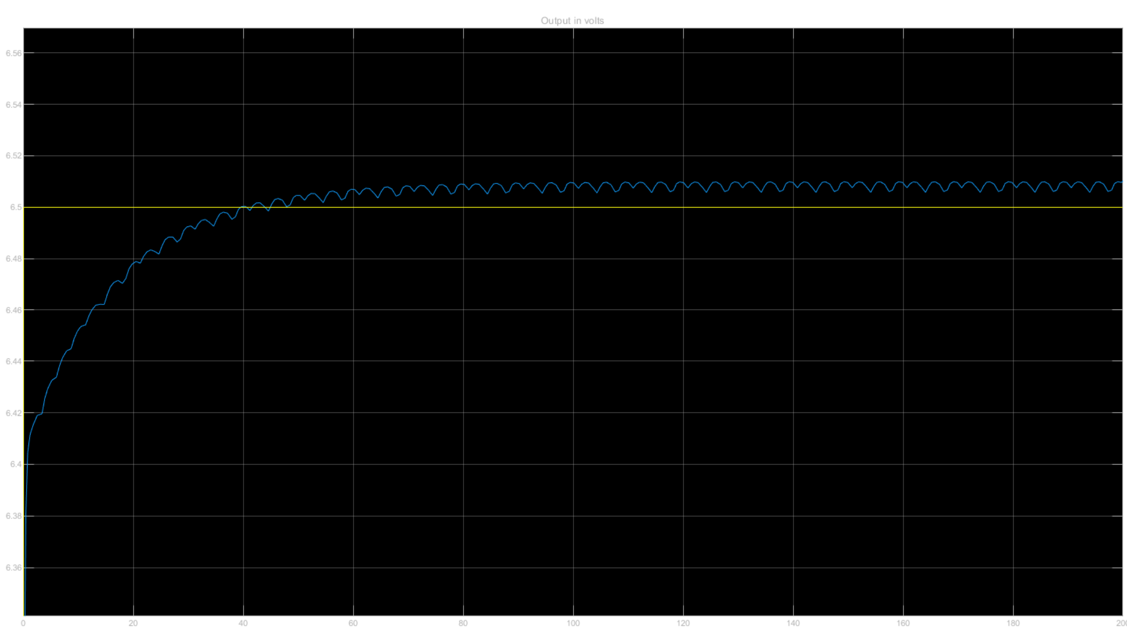


Figure 4: Given PID Values results for Kp = 5, Ki = 0.5, Kd = 0

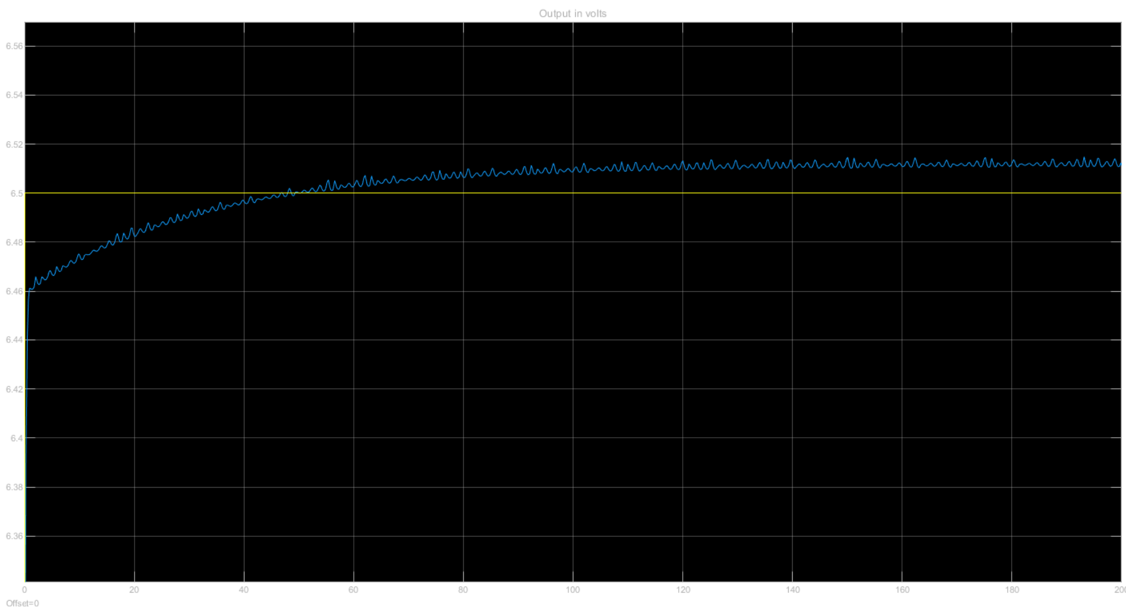


Figure 5: simulink scope results where Kp = 10, Ki = 0.3, Kd = 0.5

Table 1: results for Kp = 5, Ki = 0.5, Kd = 0

Given Values	
Modelling space	
Rise time	0.426
Overshoot	0.90%
Settling time	0.708
ess	1%
GM	inf.
PM	89.4
BW	5
Scope Test	
Rise Time	21
Settling time	35
ess	0.15%

Table 2: results for Kp = 10, Ki = 0.3, Kd = 0.5

Tuned values	
Modelling space	
Rise time	0.326
Overshoot	0.15%
Settling time	0.566
ess	1%
GM	inf.
PM	120
BW	11
Scope test	
Rise time	18
Settling time	27
ess	0.15%

By analysing figure 4 and 5, increasing Kd from 0 to 0.5 affects increases the frequency but reduces the magnitude of the oscillations in the output, by increasing Kd the transition is smoother and has less low frequency noise. Increasing Kd has also increased the settling time in the system from 18 seconds to 27 seconds. Increasing Kp doesn't seem to have affected the steady state value, however the rise time has decreased. Both PID values allow for a stable control system, however, the tuned PID values of 10, 0.3 and 0.5 allow for a more robust system since the phase margin is 30 degrees higher. The BW is also larger thus the system will be affected by a smaller range of noise frequencies.

Experimental Validation

By comparing the results of procedure 1 with the results of procedure 2, as shown in table 3 and figure 6 and 7, it is possible to understand the advantages of using a PID controller. By manually setting the voltage of the pump, it is impossible to directly and accurately control the height of the water level. The relationship between height and pump voltage needs to be calculated every time or intuitively determined. Thus, manually controlling requires trial and error and, since the system is highly non- linear, the results may change depending on the current height level and flow. With a controller, the input would be a required height rather than the pump voltage. The pump voltage would be determined by the error in the system and automatically increased or decreased. This behaviour is shown in figure 7 where the pump voltage graph is not a straight line but changes depending on the height level. Therefore, a PID controller allows for a more intuitive and automated method of controlling a system. Additionally, a PID controller decreases the time needed to achieve the desired level. By setting a single voltage level of the pump, the flow rate never changes, and the height will stabilize only after a long period of time. The rise time calculated is 450 seconds, while, to achieve a similar change in height, the rise time using a PID controller is 7.2 as shown in table 4. One downside of the PID controller is the overshoot and potential instability, for example, despite the height voltage being much larger than the pump voltage, by manually controlling the pump the overshoot is non-existent, while, using a PID controller the overshoot can vary depending on the tuning parameters. This is the trade-off between decreasing rise time and overshoot. In this case, the massive decrease in rise time from 450 to 7.2 seconds eclipses the small increase of 1.69% overshoot. A controller allows for a faster, more reliable and more intuitive method of controlling the water level of the tank.

By comparing the calculated and graphed time constants, they are close to the modelled one of 140 seconds. This is because the experiment was conducted within the height values that were used to model the system. However, because of random error and the non-linear nature of the system, these values are not the same. The calculated time constant and the graph time constant also differ, this may be because the formula used to calculate the time constant is derived from a linear model while the system is non-linear. This may have caused discrepancies between the two values.

Table 3: calculating the time constant

	Flow Rate (Volt)	**Flow Rate (L/minute)**	**Flow rate *q* (m^3/sec)**	**h(m)**
Level A	4.11	4.11	6.85E-05	1.08E-01
Level B	4.49	4.49	7.48E-05	1.50E-01
Difference	0.38	0.38	6.33E-06	4.20E-02
Area (m^2)	1.76E-02			
R (s/m^2)	6.63E+03			
τ (s)	116.72			
τ (graph)	163			
Rise time	450			
Overshoot	0%			
Settling time	94.8			

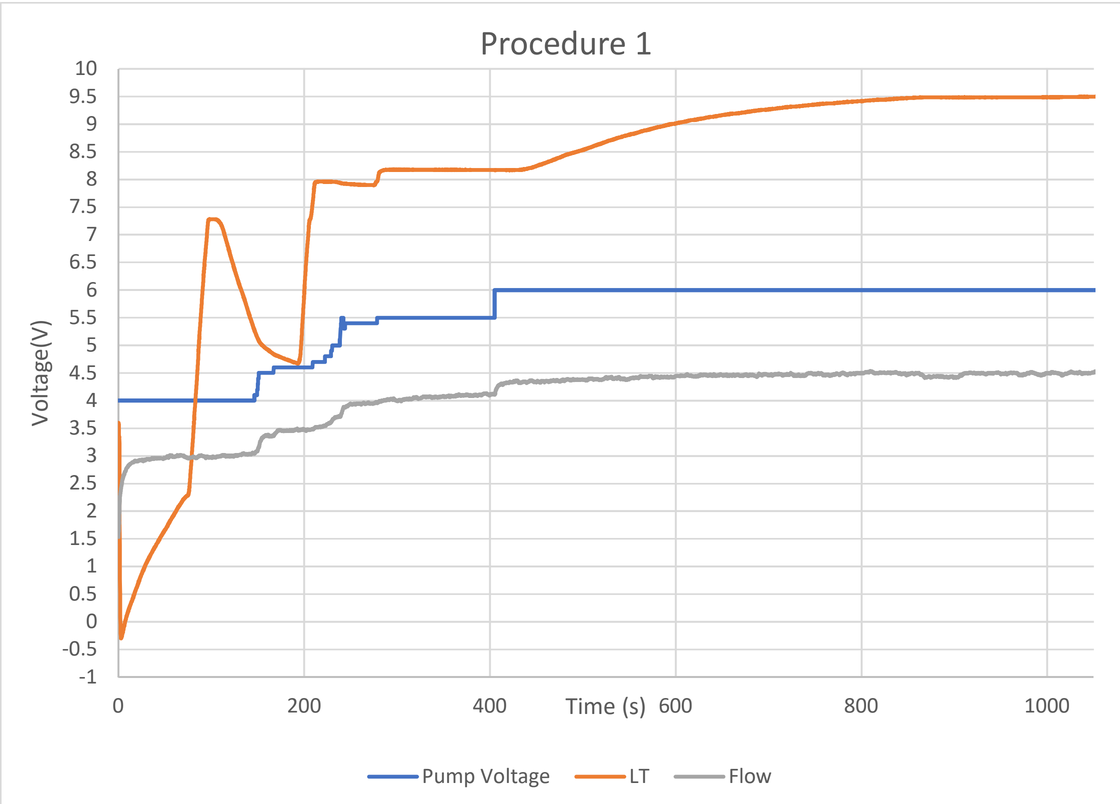


Figure 6: manual height level tuning graph

By analysing the responses between the simulation and physical model, there are some discrepancies caused by the high non-linearity of the physical system. There are already some differences between the control systems designer model and the Simulink block model. However, the changes between the scope model and the experimental results are shared between the two PID controllers, suggesting that this is a systematic error. The rise time is lower in the physical model, while the settling time is slightly higher. This could be due to a pump that has a different flow compared to the one in the model, this discrepancy caused the experiment to take a different time to reach its goal. The steady state error followed the model almost perfectly. While in the models there was no overshoot, in the experimental results there was a substantial overshoot. This may be caused by a different implementation of the PID controller in the CE2000 software.

The difference is rise time between the two PID controllers is as expected. The one with the largest Kp has a lower rise time but a larger overshoot. The higher Kp value in the chosen PID also causes the pump voltage to spike more rapidly than the other one. This can be seen from figure 7 and 8. The settling time of the chosen PID values is also increased, this is expected since the Kd value is larger and therefore it slows down the rate at which the voltage changes. The steady state error is the same in both experiments, this may be because the Ki value didn't change much between 0.3 and 0.5 and it thus had a negligible effect in changing the steady state error.

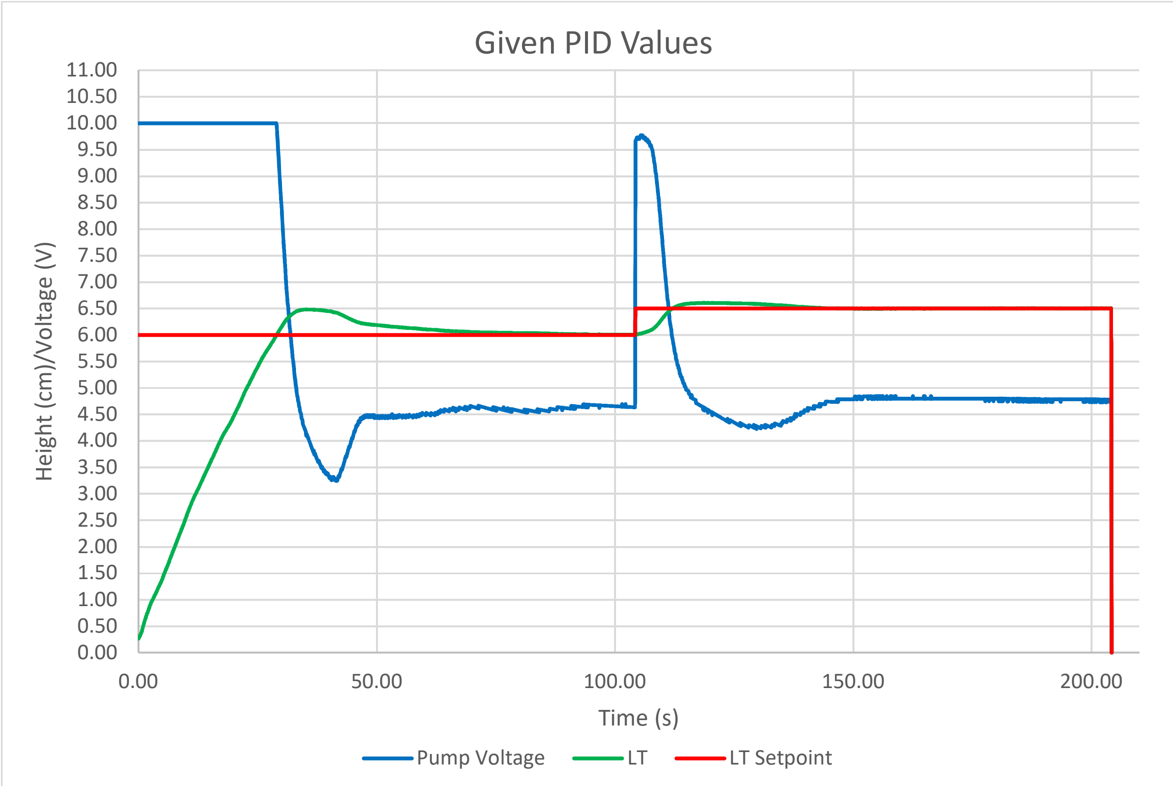


Figure 7

Table 4: results for Kp = 5, Ki = 0.5, Kd = 0

Given Values	
Modelling space	
Rise time	0.426
Overshoot	0.90%
Settling time	0.708
ess	1%
GM	inf.
PM	89.4
BW	5
Scope Test	
Rise Time	21
Settling time	35
ess	0.15%
Results 6 to 6.5 V	
Rise time	7.2
Overshoot	1.69%
Settling time	41.3
ess	0.15%

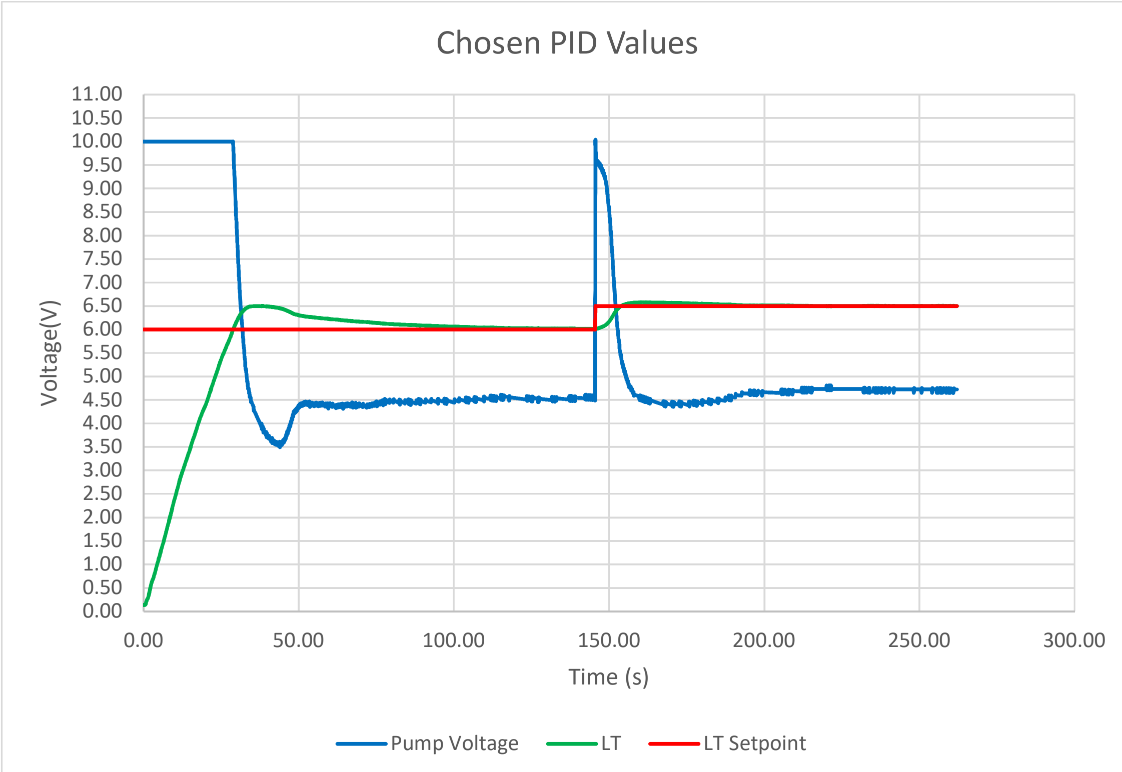


Figure 8

Table 5: results for Kp = 10, Ki = 0.3, Kd = 0.5

Tuned values	
Modelling space	
Rise time	0.326
Overshoot	0.15%
Settling time	0.566
ess	1%
GM	inf.
PM	120
BW	11
Scope test	
Rise time	18
Settling time	27
ess	0.15%
Results 6 to 6.5	
Rise time	7.35
Overshoot	1.23%
Settling time	64.35
ess	0.15%

Conclusion

The main aims of this lab were to understand the advantages of using a PID controller over manual control, how tuning the PID parameters affects the step response of the system. Additionally, the differences between a model and physical application of the PID controller were investigated. From the results it was clear that manual tuning was not the most efficient way of controlling the height of the water in the tank. Consequently, a PID controller allowed the decrease the time required to achieve a certain height and the water level reached the height in all the applications. Changing parameters of the PID controllers affects their performance, however the response in the modelling environment was slightly different to the response in the experimental environment. This is because of the different calibration of the experimental equipment and the high non-linearity of the system, where different height and different flows result in a changing time constant in the system. Overall, the PID controller performed well in both the real world scenario and in the modelling results leading to the conclusion that a well tuned and PID controller leads to a stable and reliable closed loop system despite its non-linearity.