

# Firm Debt Relief in Financial Downturn

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# Can targeting debt relief improve stabilization in a financial crisis?

Firm debt relief has been implemented in the last two U.S. recessions

Policies vary on subsets of firms targeted:

- Paycheck Protection Program (2020-2021): Smaller firms
- Auto industry bailouts (2008-2009): “Big 3” in US auto

Differing economic support:

- Smaller, younger firms rely more on debt for investment (Faff et al., 2016)
- Large firms account for greater changes in aggregates (Crouzet & Mehrotra, 2020)

## **Can targeting debt relief improve stabilization in a financial crisis?**

- ▷ How should we build these targets?

# Answering the Question

General equilibrium model with firm heterogeneity and financial frictions

Heterogeneity:

- Analyze targeting different subsets of firms
  - Shape of distribution matters for aggregation outcomes
    - Unconditional size distribution matching U.S. firms
    - *Untargeted* age-size distribution of U.S. firms
- ▷ How: Persistent idiosyncratic productivity from bounded Pareto

Financial friction:

- Collateralized borrowing
  - Slow growth for young, small firms
  - Capital misallocation
- ▷ Crisis: Shock to collateral constraint

# Examined Debt Relief Policies

Debt relief policy where government pays a fraction of firm debt

- 3 targeted policies and 1 untargeted policy
- 1. Targeting largest excess return to investment
  - ▷ Expected discounted marginal benefit of capital investment minus cost
  - Effective policy, may not be readily available to policymakers
- 2. Size-targeted policy
  - ▷ Small, medium, and large firms
  - Measures readily available to policymakers; historical precedents
- 3. Age-targeted policy
  - ▷ Young, middle-age, mature firms
- 4. Untargeted: all indebted firms are eligible

# Preview of Results

1. Targeting firms with highest excess returns yields best results
  - Output trough 26% *less* severe (2.27% vs 1.67% fall from steady state)
2. Targeting medium size firms outperforms other size, age groups
  - More important to aggregate production over the course of a crisis than small firms
  - More hindered in their growth than large firms
3. Only excess return policy has substantial benefits over untargeted policy
  - Importance of diminishing marginal returns

# Literature

- Fiscal policy to alleviate financial downturn:
  - Bianchi (2016), Jeanne, & Korinek (2020), Elenev, Landvoigt & Van Nieuwerburgh (2022), Angeletos, Collard, & Dellas (2023)
  - ▷ **My contribution:** Value of policy resources varies over nontrivial distribution of firms
- Distributional effects of policy intervention:
  - Guner, Ventura, Xu (2008), Buera, Moll, & Shin (2013), Gourio and Roys (2014), Jo & Senga (2019)
  - ▷ **My contribution:** Extend distributional analysis to the topic of debt relief in a crisis
- Transmission of shocks from financial conditions:
  - Kiyotaki & Moore (1997), Jermann & Quadrini (2012), Khan & Thomas (2013), Jo (2024)
  - ▷ **My contribution:** Analyze how debt relief may improve such conditions

# Outline

1. Introduction
2. Model
  - 2.1 Firms
  - 2.2 Government
  - 2.3 Household
  - 2.4 Calibration
3. Investment decisions in steady state
4. Financial crisis and policy
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# Model Overview

## Agents

- Firms: DRS production with capital and labor
  - Collateralized debt limit:  $b' \leq \zeta k$
- Government: Collects payroll tax, borrows from HH, issues debt relief
- Representative household: Owns firms, supplies labor and loans

Aggregate state:  $(\zeta, z, \mu, \theta) = S$

- Exogenous: collateral constraint  $(\zeta)$ , TFP  $(z)$
- Endogenous: distribution of firms  $(\mu)$ , government debt  $(\theta)$



# Firms

Firm state  $(k, b, \varepsilon, a)$ : capital, debt, idiosyncratic productivity, age

- Production technology:  $z\varepsilon F(k, n)$
- Enter period with  $\varepsilon$ ; retain with probability  $\rho_\varepsilon$ 
  - Probability  $(1 - \rho_\varepsilon)$  draw new  $\varepsilon$  from bounded Pareto distribution
- Age dependent exit shock:  $\pi_d(a)$ 
  - Known before production
  - Considerations for age-based policies
- Intertemporal decisions on  $k'$  and  $b'$ 
  - $b' \leq \zeta k$

# Firm Budget Constraint & Debt Relief

Budget:

$$D \leq x(k, b, \varepsilon, a; S) - k' + q(S)b'$$

Cash:

$$x(k, b, \varepsilon, a; S) = z\varepsilon F(k, n) - (1 + \tau(S))w(S)n(k, \varepsilon; S) + \\ (1 - \delta)k - (1 - \mathcal{J}(b)g(k, b, \varepsilon, a; S))b$$

where:  $\mathcal{J}(b) = \begin{cases} 1 & \text{if } b > 0 \\ 0 & \text{if } b \leq 0 \end{cases}$

# Firm Problem

Start of Period Value:

$$V_0(k, b, \varepsilon, a; S_l) = \underbrace{\pi_d(a) x(k, b, \varepsilon_i, a; S_l)}_{\text{exiting value}} + (1 - \pi_d(a)) \underbrace{V(k, b, \varepsilon, a; S_l)}_{\text{continuation value}}$$

Continuation Value:

$$V(k, b, \varepsilon_i, a; S_l) = \max_{k', b', D} \left[ D + \sum_{m=1}^{N_s} \pi_{l,m}^s d_m(S_l) \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^\varepsilon V_0(k', b', \varepsilon_j, a'; S'_m) \right]$$

subject to:

$$0 \leq D \leq x(k, b, \varepsilon_i, a; S_l) + q(S_l)b' - k'$$

$$b' \leq \zeta k$$

$$\mu' = \Gamma(S_l)$$

## Government (1/2)

- Total current-period debt relief:  $T(k, b, \varepsilon, a; S)$
- Borrows:  $\theta'$  at risk-free rate:  $q(S)$
- Levies payroll tax:  $\tau(S)$  when paying outstanding debt obligations:  $\theta$

Budget constraint:

$$\tau(S)w(S)N(S) + q(S)\theta' \geq \theta + \underbrace{\int g(k, b, \varepsilon, a; S)b\mu(d[k \times b \times \varepsilon \times a])}_{= T(k, b, \varepsilon, a; S)}$$

where  $\tau$ ,  $\theta$ , and  $T$  are 0 in steady state

## Government (2/2)

Government debt relief to mitigate financial recession:

- Policy eligibility determined by firm state
- Initially funded by government debt

Evolution of public debt:

$$\theta' = \frac{1}{q(S)} \left( \theta + T(k, b, \varepsilon, a; S) - \tau(S)w(S)N(S) \right)$$

When repayment begins,  $\tau(S)$  must satisfy fiscal rule:

$$\theta' = (1 - \phi)\theta$$

- $\phi$  is the fraction of public debt paid per period

# Representative Household

- Period utility  $U(C, 1 - N)$ , and discount factor  $\beta \in (0, 1)$
- Supplies: labor for wage  $w(S)$ , loans at risk free rate  $q(S)^{-1}$
- Implied restrictions for equilibrium prices:
  - $w(S) = \frac{D_2 U(C, 1 - N)}{D_1 U(C, 1 - N)}$
  - $d_m(S_l) = \beta \frac{D_1 U(C'_m, 1 - N'_m)}{D_1 U(C, 1 - N)}$
  - $q(S_l) = \beta \sum_{m=1}^{N_s} \pi_{l,m}^s \frac{D_1 U(C'_m, 1 - N'_m)}{D_1 U(C, 1 - N)}$
- Equilibrium decision rules  $C = C(S)$ ,  $N = N(S)$

► HH Problem

## Annual Calibration (1/2)

$$U(C, 1 - N) = \ln(C) + \psi(1 - N) \quad z\varepsilon F(k, n) = z\varepsilon k^\alpha n^\nu$$

$$k_0 = \chi \int k \mu(d[k \times b \times \varepsilon \times a])$$

	Parameter		Target		Model
$\beta$	discount factor	= 0.960	real interest rate	= 0.040	0.041
$\psi$	leisure preference	= 2.140	labor hours	= 0.333	0.332
$\nu$	labor share	= 0.600	labor share	= 0.600	0.600
$\delta$	depreciation	= 0.069	investment/capital	= 0.069	0.069
$\frac{b_0}{k_0}$	entrant leverage	= 0.400	entrant leverage	= 0.400	0.400
$\alpha$	capital share	= 0.280	capital/output	= 2.250	2.305
$\rho_\varepsilon$	maintain $\varepsilon$	= 0.990	std dev. i/k	= 0.337	0.358
$\chi$	fraction of entrant $K$	= 0.208	avg. $n_0/N$	= 0.260	0.260
$\zeta_o$	collateral fraction	= 0.981	debt/assets	= 0.372	0.372

## Annual Calibration (2/2)

Size & Age/Size Distributions, Measured by Employment (Business Dynamics Statistics: 1990-2006)

Pareto bounds,  $[0.497, 0.937]$ , and shape,  $(5.5)$ , targeting unconditional size distribution

Employment Bins	Emp. Share	Pop. Share	
		BDS	Model
Small (1-19)	0.201	0.885	0.880
Med. (20-499)	0.319	0.112	0.101
Large (500+)	0.480	0.003	0.019

*Untargeted* age-size distribution:

Age	0	1	2	3	4	5	Avg.
BDS:	0.260	0.338	0.378	0.417	0.451	0.477	0.368
Model:	0.260	0.302	0.362	0.427	0.506	0.595	0.377



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# Constrained and Unconstrained Investment

Collateral constraint binds to varying degrees across firms

Non-binding:  $k'(k, b, \varepsilon, a; S) = k^*(\varepsilon; S)$

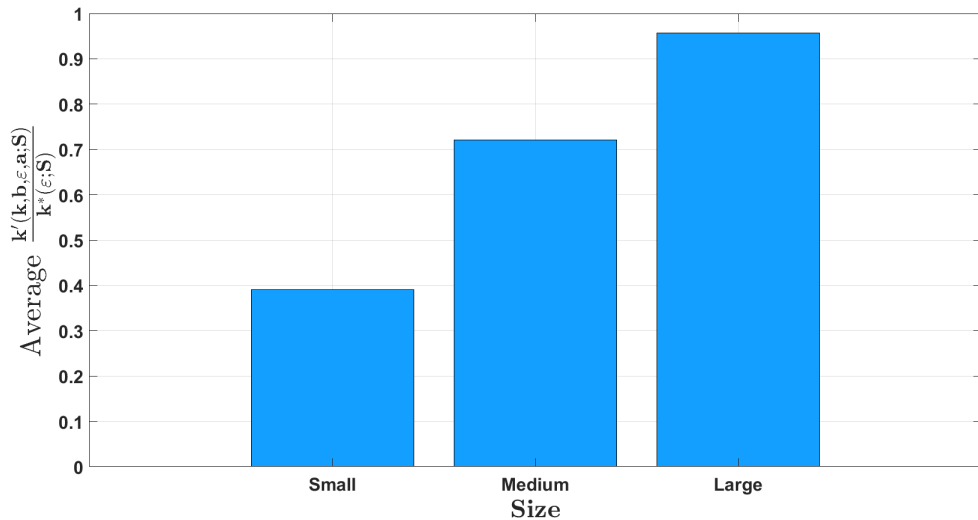
- Debt relief provides no extra investment
- Additional resources saved,  $b' = \frac{k^*(\varepsilon, S) - x(k, b, \varepsilon, a; S)}{q(S)}$ 
  - Does offer protection from future binding constraint

Binding:  $k'(k, b, \varepsilon, a; S) = x(k, b, \varepsilon, a) + q(S) \underbrace{\zeta k}_{b' = \zeta k} < k^*(\varepsilon; S)$

- Debt relief for these firms increases investment, reduces misallocation

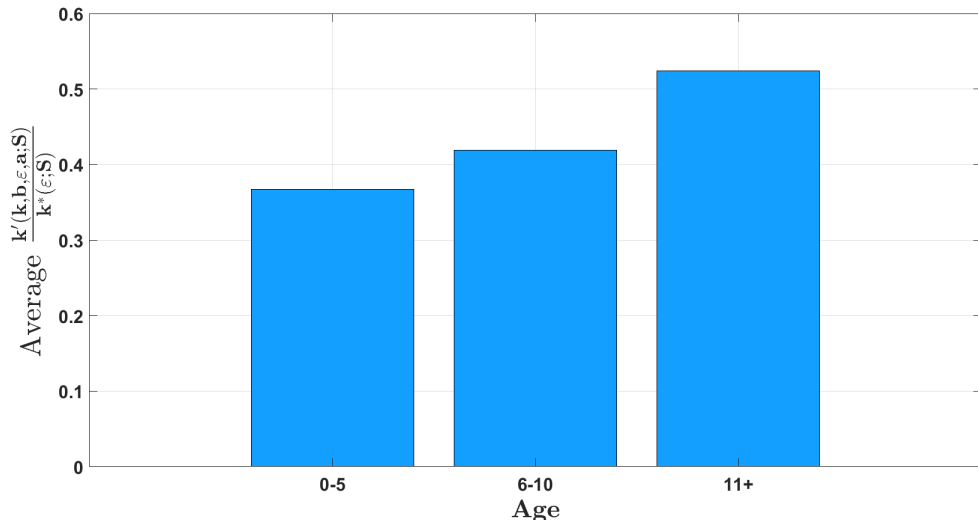
What do these firms look like?

# Firm Size and Capital Efficiency



- Fraction of adopted capital  $k'(k, b, \epsilon, a; S)$  over unconstrained capital  $k^*(\epsilon; S)$

## Firm Age and Capital Efficiency



- Fraction of adopted capital  $k'(k, b, \epsilon, a; S)$  over unconstrained capital  $k^*(\epsilon; S)$

# Measuring Investment Inefficiency

Excess return to investment: expected discounted marginal value of investing minus cost

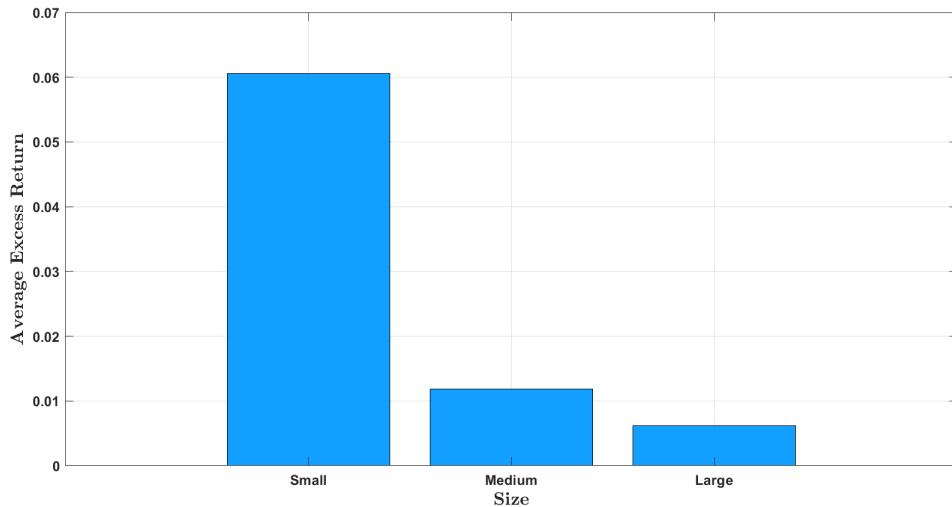
$$\mathbb{E}_{\pi^S} \mathbb{E}_{\pi^\varepsilon} \left[ d_m(S_l) \left( \frac{\partial \pi(k', b', \varepsilon_j, a'; S'_m)}{\partial k'} + (1 - \delta) \right) \right] - 1$$

where  $\pi$  is profit [▶ Parameterization](#)

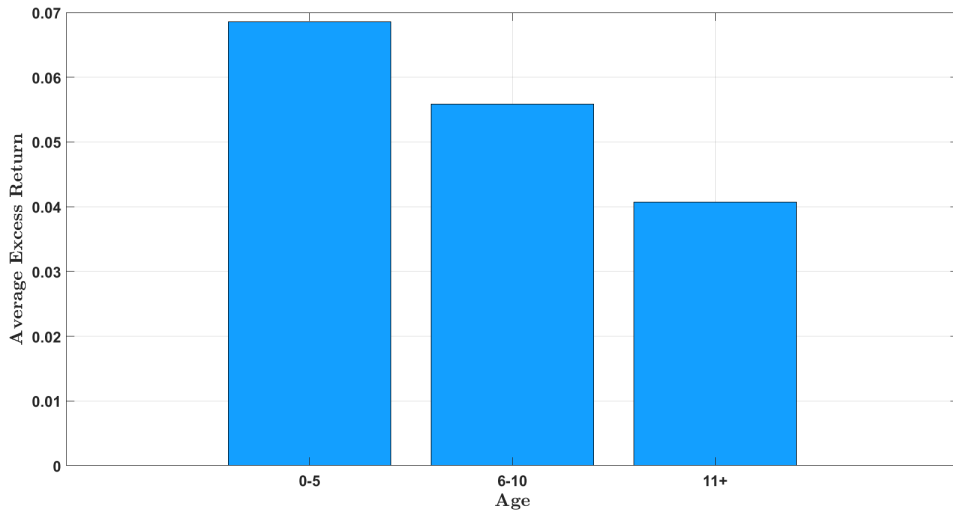
With efficient investment, excess return is 0

The further  $k'(k, b, \varepsilon, a, S)$  is from  $k^*(\varepsilon, S)$ , the higher the return

# Average Excess Return Across Firm Size



## Average Excess Return Across Ages



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# Credit Crisis without Relief Policy

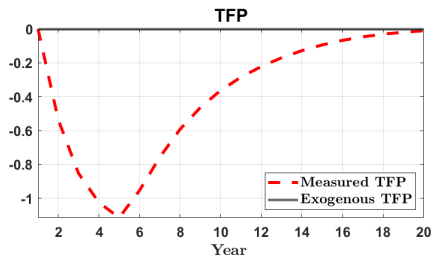
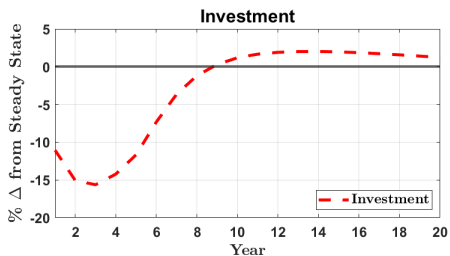
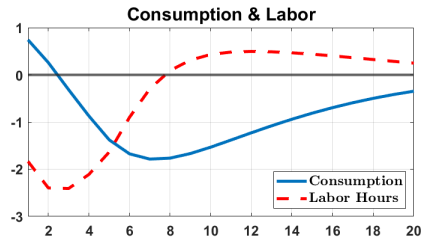
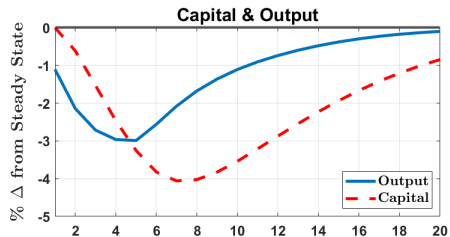
Fall in collateral parameter by 33%

- Supply shock in loanable funds market similar to 2008 (Duchin et al., 2010)
- Firms now require more capital to borrow as much as they would in steady state

Parameterization:

- Remains at low value for 4 periods, recovers 31.25%/year (Khan & Thomas, 2013)
- 26% fall in borrowing in the model, matching fall in C&I loans 2008-2011

# Response to Credit Crisis: No Policy



# Debt Relief in a Crisis

Policy pays fraction of outstanding debt

Common across all policies:

- Relief occurs on impact
- Total size of policy held constant at 4% of steady state output
  - PPP loan forgiveness was roughly 3.7% of U.S. real GDP by 2023
- Taxes increase in period 7 to pay public debt
  - Half-life of output recovery
  - Gov pays 5% of its debt per year

# Excess Return Policy

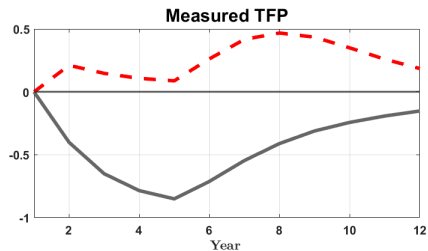
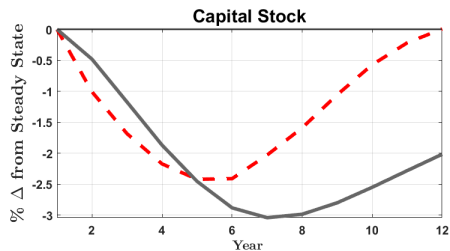
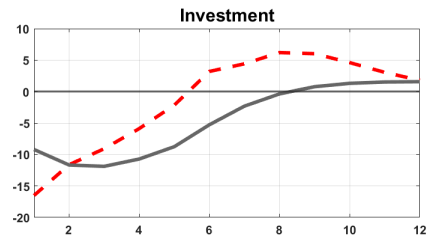
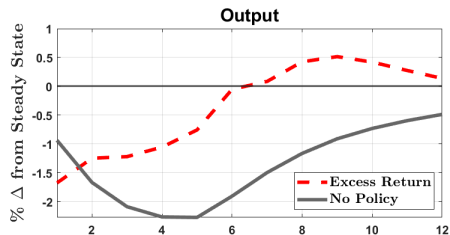
Reduce/equalize largest levels of excess return from top stair down

- Relieve debt of firm with greatest wedge to match 2nd greatest...
- Continue until policy funds exhausted

Does not relieve debt beyond what is necessary for investment

- All funds used for investment, not merely increasing share value
- Addresses concerns of debt-relief not reaching intended location (Li, 2021, Autor et al., 2022)

# Policy Targeting Highest Excess Return Levels



► Additional series

# Policy Targeting Small, Medium, Large firms

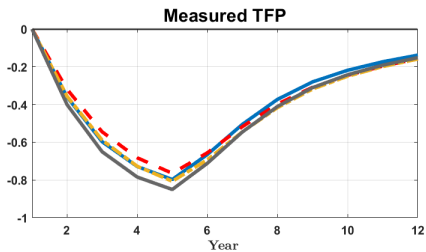
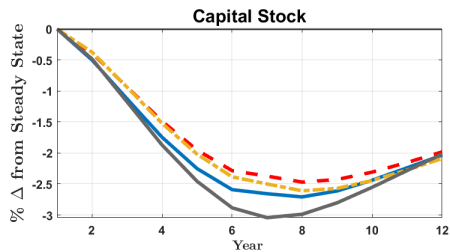
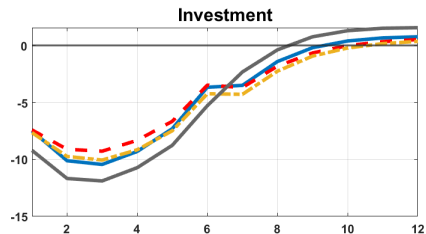
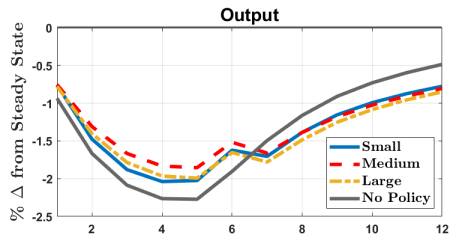
3 experiments for small, medium, large firm policy targets

- Smallest 88%, middle 10%, largest 2% of firms
- Eligibility based on model population shares [► Size Dist.](#)

In order to keep total size of policy constant, relief per firm must vary across policies:

- Fraction of  $b$  paid for **small** firms: 0.237
- Fraction of  $b$  paid for **medium** firms: 0.145
- Fraction of  $b$  paid for **large** firms: 0.096

# Policy Targeting Small, Medium, Large Firms



► Additional series

# Policy Targeting Young, Middle Age, Mature Firms

3 experiments for young, middle age, mature firm policy targets

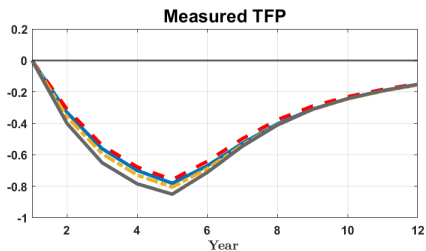
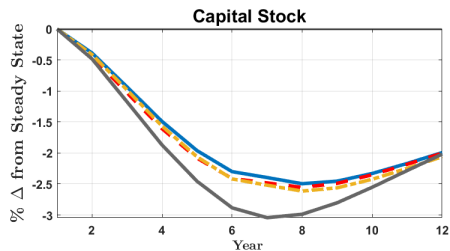
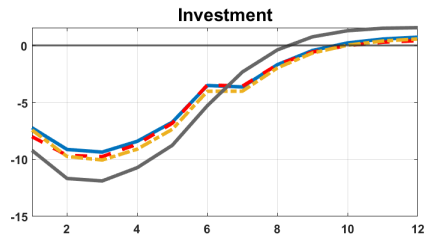
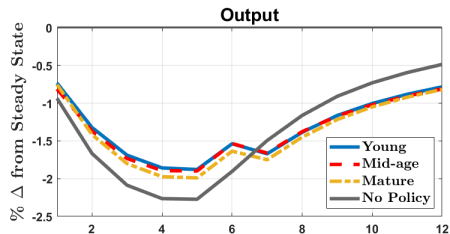
- Age bins:  $[0 - 5]$ ,  $[6 - 10]$ ,  $11+$

In order to keep total size of policy constant, relief per firm must vary across policies:

- Fraction of  $b$  paid for **young** firms: 0.352
- Fraction of  $b$  paid for **middle age** firms: 0.263
- Fraction of  $b$  paid for **mature** firms: 0.071



# Policy Targeting Young, Middle Age, Mature Firms



► Additional series

## Comparison to Untargeted Policy

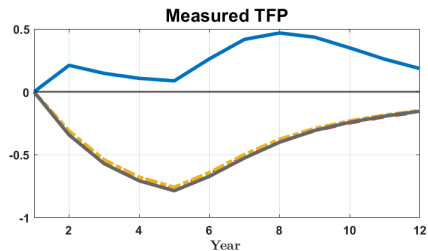
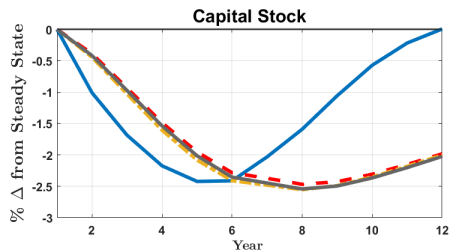
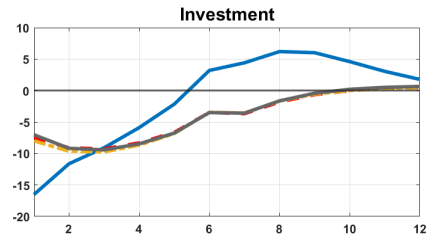
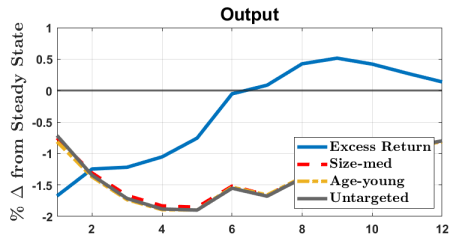
I consider one final policy: all indebted firms are eligible

- Fraction of total firm debt relieved: 0.047

Highlight effects of diminishing marginal returns

- Targeted policy can focus on key variables
- But, more concentrated policy → stronger diminishing marginal returns effects

# Untargeted Policy Compared to Best Alternative Policies



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# Conclusion

I develop a model of heterogeneous firms and financial frictions, matching the unconditional size, and age-size, distribution of firms in the U.S., to study firm targeted debt relief in a financial crisis

I consider policies targeting firms by excess return to investment, size, and age, as well as an untargeted policy

- Excess return policy outperforms all others
  - Fall in output diminished by 26% compared to no policy
- Among remaining policies, targeting medium size firms reduces fall in aggregates most
  - More likely to become large economic players than small firms
  - Further from their efficient investment than larger firms

Only excess return policy shows meaningful improvements over untargeted policy

- Demonstrates gains from targeting are possible, but conditioning on other readily observable variables must be considered

## Future Work

Endogenous entry/exit to study the implications of “cleansing” effects on debt relief

- Productive firms more likely to continue may boost effects of policy
- However, more firms overall exit than under current exit shock

Alternative tax structure to minimize slowed recovery when taxes increase

Varying arrival rate of new productivity shocks across firms

- Some firms may be more likely than others to retain their productivity
- The more persistent productivity is overall, the stricter targets should be to include productive firms

## **Appendix**

# Household Problem

Representative HH maximizes lifetime value,  $W(\lambda, \kappa; S)$

- Chooses: consumption,  $c$ , labor,  $N$ , shares,  $\lambda'$ , bond holdings,  $\kappa'$

$$W(\lambda, \kappa; S) = \max_{c, n, \lambda', \kappa'} [U(c, N) + \beta W(\lambda', \kappa'; S'_m)]$$

$$\text{st: } c + q(S_l)\kappa' + \int \rho_1(k', b', \varepsilon', a'; S'_m) \lambda'(d[k' \times b' \times \varepsilon' \times a']) \leq$$

$$w(S)n + \kappa + \int \rho_0(k, b, \varepsilon, a; S_l) \lambda(d[k \times b \times \varepsilon \times a])$$

- $\rho_1$  is ex-dividend price of a share,  $\rho_0$  is dividend-inclusive value of a share,  $\Gamma(\mu)$  is known

► back



# Market Clearing

$$\text{Capital : } K = \int k \mu(d[k \times b \times \varepsilon \times a])$$

$$\text{Labor : } N = \int n(k, \varepsilon) \mu(d[k \times b \times \varepsilon \times a])$$

$$\text{Output : } Y = \int z \varepsilon F(k, n(k, \varepsilon)) \mu(d[k \times b \times \varepsilon \times a])$$

$$\text{Firm-debt : } B = \int (b | b > 0) \mu(d[k \times b \times \varepsilon \times a])$$

$$\begin{aligned} \text{Consumption : } C = & Y - (1 - \pi_d(a)) (K' - (1 - \delta) k) \\ & + \pi_e ((1 - \delta) k - k_0) \end{aligned}$$

# Distribution

The distribution of firms is denoted by measure  $\mu$ , defined on the Borel algebra,  $\mathcal{S}$ , generated by the open subsets of the product space,  $\mathbf{S} = \mathbf{K} \times \mathbf{B} \times \mathbf{E} \times \mathbf{A}$ .

$\forall (A, \varepsilon_j) \in \mathcal{S}$  defines  $\Gamma$ , where  $\chi(k_0) = \{1 \text{ if } (k_0, 0) \in A; 0 \text{ otherwise}\}$

$$\begin{aligned} \mu'(A, \varepsilon_j) = & \\ (1 - \pi_d(a)) \int_{\{(k, b, \varepsilon_i, a) | (g^K(k, b, \varepsilon_i, a; s, \mu), g^B(k, b, \varepsilon_i, a; s, \mu)) \in A\}} & \pi_{ij} \mu(d[k \times b \times \varepsilon_i \times a]) \\ + \pi_e \chi(k_0) H(\varepsilon_j) & \end{aligned}$$

## Excess Return to Investment

Given the parameterization of the model, marginal benefit of capital investment should equal marginal cost of 1:

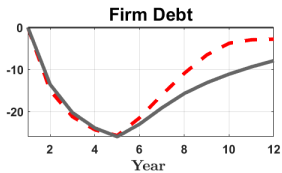
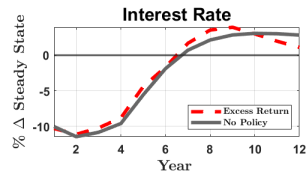
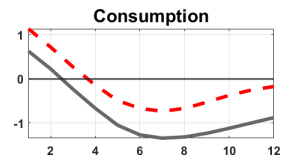
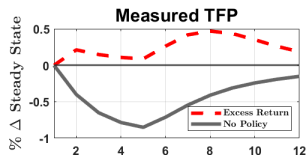
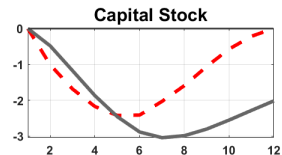
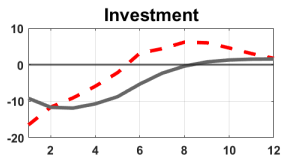
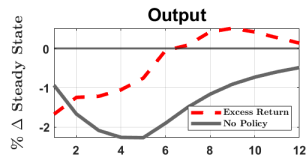
$$\frac{\alpha}{1-\nu} \frac{\beta}{p(S)} \left( \sum_{m=1}^{N_s} \pi_{l,m}^s p(S'_m) \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^\varepsilon k'^{\frac{\alpha+\nu-1}{1-\nu}} \left[ z_m \varepsilon_j \left( \frac{\nu z_m \varepsilon_j}{(1+\tau(S)) w(S)} \right)^{\frac{\nu}{1-\nu}} - (1+\tau(S)) w(S) \left( \frac{\nu z_m \varepsilon_j}{(1+\tau(S)) w(S)} \right)^{\frac{1}{1-\nu}} \right] + (1-\delta) \right) = 1$$

where,  $p(S) = \frac{\partial U(C, 1-N)}{\partial C}$

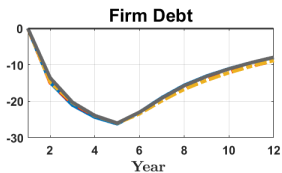
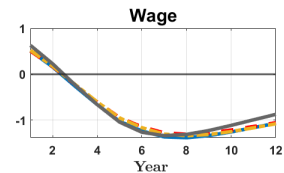
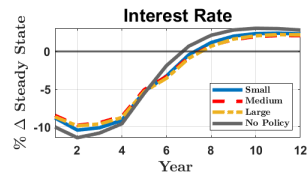
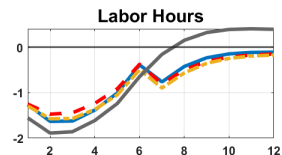
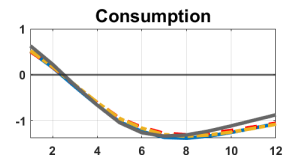
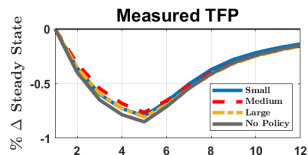
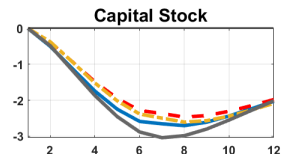
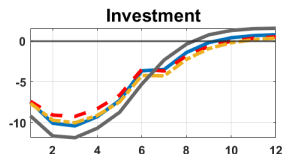
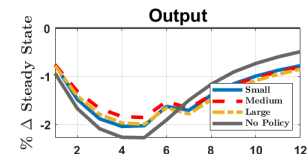
With insufficient investment, LHS > 1

[▶ back](#)

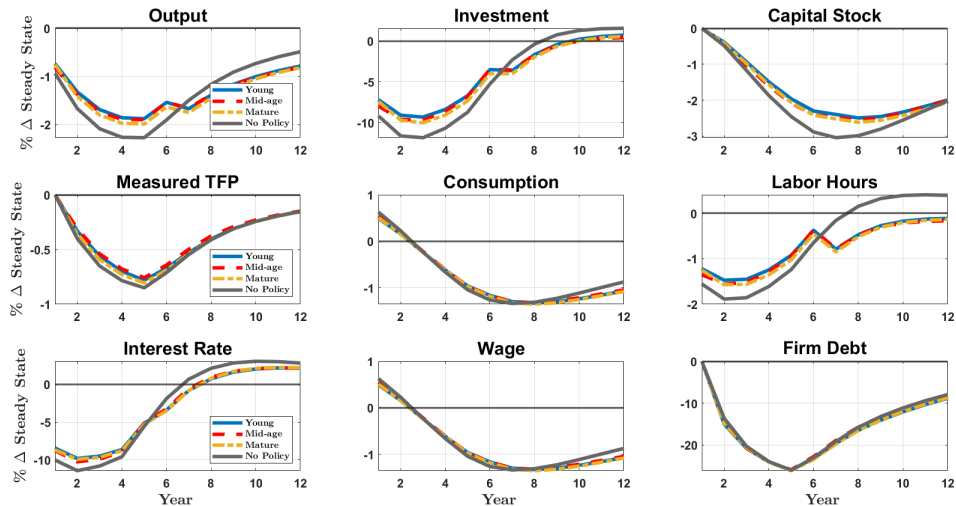
# Extra Series - Excess Return Target



# Extra Series - Size Targets

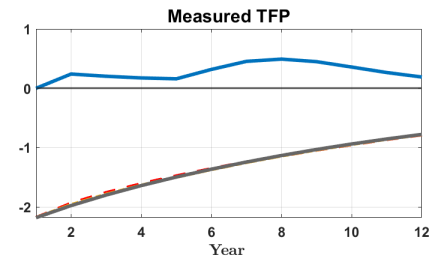
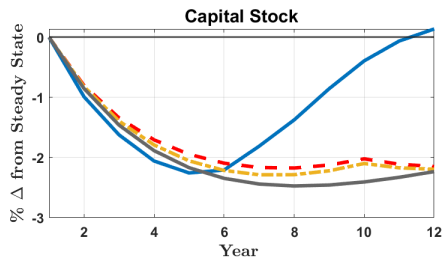
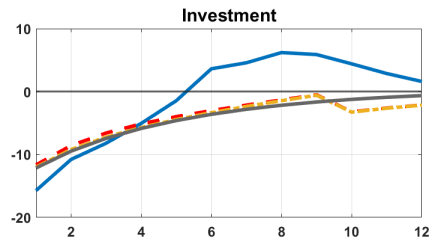
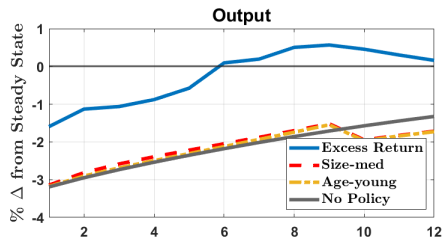


# Extra Series - Age Targets



► back

# Selected Policies Following TFP Shock



- 2% shock with persistence of 0.909

## Firm Exit Rates by Age (BDS, 1990-2006)

Age:	1	2	3	4	5	6-10	11+
$\pi_d(a)$ :	0.2478	0.1640	0.1356	0.1174	0.1062	0.0840	0.0655

Firm entrant rate = 10.6%

- Entrant rate selected to keep mass of firms at 1

[▶ back](#)



# Perfect Foresight Solution -1

1. Guess a vector of  $\{\hat{\tau}\}_0^{T+1}$  and  $\{\hat{C}\}_0^{T+1}$ 
  - $\tau'$  is needed for  $k'$  decision
  - $\hat{C}$  implies a  $w(S)$  and  $q(S)$
2. Back-solve decision rules from date  $T$
3. Forward-solve the distribution (and find aggregates) for each  $T$
4. Back out  $\tilde{C}$  implied by aggregate resource constraint
5. Back out  $\tilde{\tau}$  implied by following:

## Perfect Foresight Solution - 2

Define:  $Balance \equiv \hat{\tau}(S)w(S)N(S) + q(S)\theta' - \overline{G} - \theta - T(\Theta, S)$

Then,  $\hat{\tau}(S)w(S)N(S) = Balance + q(S)\theta' - \overline{G} - \theta - T(\Theta, S)$

Define:  $\Delta\hat{\tau}(S)$  as change in  $\hat{\tau}(S)$  such that:

$$(\hat{\tau}(S) + \Delta\hat{\tau}(S))w(S)N(S) = \overline{G} + \theta + T(\Theta, S) - q(S)\theta' \text{ and } \theta' = (1 - \phi)\theta$$

- This is the increase in  $\hat{\tau}(S)$  needed to set  $Balance = 0$ , while the fiscal rule is satisfied

Then:  $\Delta\hat{\tau}(S) = \left( (\overline{G} + \theta + T(\Theta, S) - q(S)\theta') \left( \frac{1}{w(S)N(S)} \right) \right) - \hat{\tau}(S)$

So,  $\tilde{\tau} = (\hat{\tau}(S) + \Delta\hat{\tau}(S))$

6. Check guess, set  $\{\tilde{\tau}\}_0^{T+1} = \{\hat{\tau}\}_0^{T+1}$  and  $\{\tilde{C}\}_0^{T+1} = \{\hat{C}\}_0^{T+1}$ , and repeat