

Firm Debt Relief in Financial Downturn

Matt D’Urso*

November 22, 2024

[Latest Version Available Here](#)

Abstract

Does firm-targeted debt relief dampen financial downturn? I study the stabilization effects of firm-specific debt relief in a DSGE model calibrated to U.S. data, including the skewed size distribution of firms. Firms face persistent idiosyncratic risk and financial frictions that give rise to an endogenous distribution over capital, debt, productivity, and age, while leading to capital misallocation and life-cycle effects. I model financial frictions by assuming collateralized borrowing. A shock to firms’ access to credit exacerbates misallocation, leading to a crisis similar to the Great Recession. This creates a role for policy: government borrows on behalf of financially constrained firms and provides debt relief, allowing firms to invest more. Policy that targets firms with the highest level of excess return to investment improves aggregates the most, reducing the drop in output by over 50%. Understanding that this is not a readily observable policy target, I consider firm size and age as alternative targets. The model is well-suited for this, as it produces an age-size distribution matching U.S. data. Surprisingly, I find that policy that targets medium size firms outperforms these other targets, though they all fall short of the excess return policy.

*The author would like to thank Julia Thomas, for her guidance and support, Aubhik Khan and Kyle Dempsey for their advice, and Gabriel Mihalache, Ben Lidofsky, Rohan Shah, and Soyoung Lee for helpful comments. All errors are my own. Department of Economics, The Ohio State University. URSO.18@osu.edu

I Introduction

Does firm-targeted debt relief dampen financial downturn? Around the time of economic downturn, the idea of debt relief, or “bailouts,” becomes more common in political and economic discourse.¹ These policies are usually idiosyncratic, varying in size, scope, and may also allow for changes in market structures. In recent U.S. history, debt relief policies have been implemented during the Great Recession and the COVID-19 pandemic. Motivation for policy during the Great Recession was driven by concerns of larger firms, specifically the “Big 3” in U.S. Auto, laying off workers and a negative wealth effect spreading throughout the economy.² Furthermore, larger firms are responsible for a greater share of aggregate economic activity than smaller firms.³ On the other hand, the Paycheck Protection Program (PPP) established during the pandemic was chiefly concerned with small business, with the main target being firms with fewer than 500 employees.⁴ Smaller firms are, on average, more likely to face higher marginal returns to investment, despite weaker capital market access.⁵ Besides identifying which end of the size distribution to focus resources on, it remains to be seen if policy that targets size leads to the best improvements in aggregates during financial downturn.

To answer the question, I build a dynamic stochastic general equilibrium model with persistent firm-level heterogeneity and financial frictions. The model is calibrated to match key moments in the United States economic data, including the heavily skewed unconditional

¹Blau et al. (2013) measure this with two approximations for political engagement, lobbying and connections maintained.

²General Motors, Ford Motor Company and Chrysler, at the time, were considered the “Big 3” U.S. automobile manufacturers.

³Crouzet and Mehrotra (2020) conclude that cyclical behavior in large firms may have implications for aggregate fluctuations, while small firms’ behavior may not. This, along with the positively skewed employment distribution in the United States shown in the Business Dynamics Statistics (BDS) data may suggest the opposite targeting is appropriate.

⁴The second of three “draws” targeted 300 employees. Other financial targets were also implemented such as limiting net worth and hedge funds.

⁵Banz (1981) find that firm size is negatively correlated with risk adjusted returns; much work has been done discussing this correlation and whether or not causality exists (Berk, 1997; Asness et al., 2018). Gertler and Gilchrist (1994) state “While size per se may not be a direct determinant, it is strongly correlated with the primitive factors that do matter,” suggesting policy would be better off targeting smaller firms.

size distribution of firms, as well as the age-size distribution. The heterogeneity in production creates differences in optimal investment decision rules, while financial frictions can constrain some of these decisions by limiting firms’ access to debt markets. The result is a dynamic distribution of firms, which provides a breadth of firm-level variables to target for policy analysis. I then use this model as a quantitative laboratory for policy experiments to study which debt relief policy targets lead to the greatest improvements in aggregate outcomes.

I model financial market frictions by assuming collateralized borrowing. Investment among firms with insufficient collateral is limited by their inability to borrow, leading to a misallocation of capital. Young, productive firms are not large enough to post sufficient collateral, nor do they have enough retained earnings to reach their efficient investment level, slowing their growth. The financial frictions in the model endogenously produce these life-cycle characteristics. A shock to firms’ access to credit exacerbates the misallocation already present in the model as growing firms have a stronger reliance on external financing. This leads to a deterioration of the aggregate capital stock. The recovery path is a function of the distribution of firms, which is inherently a slow moving object, and if capital requires “*time-to-build*,” the recovery is slowed further, as firms are not able to update their capital immediately. This shock generates a recession with features similar to that of the 2008 Financial Crisis: an initial rise in consumption, a steep fall in investment, and a slow initial fall in output.

Financial frictions interact with firm-level productivity, a stochastic process drawn from a Pareto distribution. While the slowed growth seen in young and small firms can also be reproduced in models using the more common log-normal distribution ([Khan and Thomas, 2013](#)) for idiosyncratic productivity, this model delivers something further: many large firms are inefficiently smaller than they should be. Though these firms are considered large by their employment share, a significant portion of them have still not reached their optimal investment level implied by the long right tail of the productivity process. Thus, the model is well suited to study size and age debt relief targets.

Debt relief that aims to decrease and equalize the highest levels of excess return to capital investment across firms yields the best results, cutting the drop in output from 2.25% below steady state to 1% percent, a reduction of over 50%, measured at the trough date of the recession under no-policy. Excess return to capital investment is defined as the expected lifetime marginal value of investing in capital today, minus its cost. Targeting this measure of inefficiency moves resources to far more productive locations, causing a boom in endogenous TFP. Understanding that this may be a difficult target to measure, I consider readily observable policy targets, firm size and age.

Using the model’s size distribution targeted to employment size bins found in the Business Dynamics Statistics, I divide firms in small, medium, and large size groups. Many policies in the United States and abroad focus on financial assistance to small and young firms.⁶ However, I find that debt relief policy that targets medium size firms provides the best results when comparing recession troughs across observable policy regimes.⁷ Measured at the trough date of the recession under no policy, most aggregate variables are 0.5% closer to steady state values, with investment being 2% closer, under the medium size firm policy. Consequently, the aggregate capital stock reaches its no policy half-life roughly 1 year sooner than the no policy regime.⁸ However, these benefits are not without costs. In the model, the government issues bonds to pay for debt relief and, eventually, must raise taxes to pay off public debt, leading to a slower recovery.

When considering firm-level investment inefficiency, it is surprising that small firm policy targets do not yield the best results. Indeed, small firms invest at a much more inefficient rate than medium size firms. However, these small firms account for only a small share of output and aggregate TFP. Contrast this with large firms that are responsible for a disproportionate share of output, but are nearing the end of their life-cycle growth. For large firms, debt relief

⁶Horvath and Lang (2021), Jo and Senga (2019), Guner et al. (2008)

⁷Firm size bins are defined by data from Census Business Dynamics Statistics, the quantitative details of which are discussed in the Calibration IV section.

⁸The level of capital corresponding to the point where it is halfway back to its steady state value, during a recession without relief policy.

policy is targeted to a group that is important for aggregate production, but the policy does not correct much as most of these firms are near optimal investment levels. Medium size firms are more important to aggregate production than small firms, while still being hindered in their life-cycle growth by financial conditions, more so than large firms. Policy resources can correct more investment inefficiency in a group that is important to aggregate output.

Age-based policy that targets firms age 0-5 is also considered and compared to an untargeted debt relief policy where all indebted firms are eligible.⁹ I find that only the medium size and age targets outperform the untargeted policy in improving aggregates in the heart of a financial crisis. The effect of decreasing returns to scale production is strong. Keeping the marginal value of policy resources higher across more firms boosts the impact of untargeted policy. However, there is a unique benefit to targeting young firms, giving firms the resources to grow faster increases labor demand, decreasing the amount of payroll taxes needed to cover government debt.

Specificity of targets must also be considered. Concentrating public funds on important variables sounds appealing at first, but in a decreasing returns to scale environment, keeping the marginal value of a “*bailout-unit*” high may require less specific targets. Furthermore, keeping resources concentrated on a small subset of firms may lead to misallocation in and of itself. If firms receive more resources than what would be required for optimal investment, the rest will may be stored as savings when it could have been used as investment by another firm. Moving to the other extreme and eliminating targets all together can lessen these issues, but at the cost of foregoing focusing on important variables. Thus an untargeted policy is needed for comparison.

Debt relief allows firms that suffer the most to access funds that the private sector will not provide, mitigating the effects of a financial crisis, providing a potential role for policy. Debt relief that relaxes a firm’s budget constraint affords them more investment, which

⁹I consider an age target of 5 years, formally borrowing this definition of a young firm from [Goetz and Stinson’s \(2021\)](#) Census report. They define a young firm as “firms with positive employment for five years or less.” Moreover, [Babina et al. \(2019\)](#) find similar employment dynamics between firms of age 5 and firms of ages 4 and 6.

dampens the impact of credit shocks, reducing the severity and the duration of a recession. In the model, policy is conducted by a government, which issues bonds in order to fund debt relief. Taxes are then raised in order to pay off its debt. Here, the government’s role is that of one in [Woodford \(1990\)](#), to borrow on behalf of agents that are financially constrained. The benefit of extended borrowing capacity depends on to whom it is extended. Thus, it is important to consider these potential targets to discover if targeted policy, or policy in general, is worth the cost of higher public debt.

In the model environment, firms invest in their capital stock through cash-on-hand and issuing bonds. Aside from collateral constraints, they also face a lower bound on dividends. The dividends restriction gives the collateral constraint its ‘bite.’ If a firm cannot afford to finance investment and they cannot provide enough collateral for a loan, this firm may not make up the difference by issuing sufficiently negative dividends.¹⁰ This modeling choice allows me to capture some of the effects described in the corporate finance literature on the preference of debt financing over equity financing in a tractable manner, while still focusing a quantitatively robust analysis on debt channels.¹¹

The remainder of the paper is prepared as follows. Section II provides a review of the literature. Section III describes the model and provides analysis. Section IV begins the quantitative exercise with calibrations. Section V quantifies measures of efficiency in the model, while section VI discusses policy to remedy the effects of a financial crisis. Section VII concludes.

II Literature Review

There is a large literature on fiscal policy improving macroeconomic outcomes. [Bianchi \(2016\)](#) studies a model of systemic and idiosyncratic debt relief, where systemic policy is

¹⁰See [Bianchi \(2016\)](#) as an example of setting a lower bound on dividends as a way to balance tractability with quantitative robustness.

¹¹[Shleifer and Vishny](#)’s 1997 research survey and [Brav et al.](#)’s 2005 managerial survey show that some firms will choose external capital or even give up positive NPV investments in order to not decrease dividends.

a function of only the aggregate state, while idiosyncratic policy takes firm-level variables as arguments, as well. Debt relief is intended to alleviate working capital constraints, a la [Jermann and Quadrini \(2012\)](#). Under an idiosyncratic policy regime, the potential for debt relief enters into the firm's decision rules, creating a potential moral hazard problem. Firms are incentivized to over-borrow and may be unable to cover the cash flow mismatch between their outlays and revenue next period. In this setting, systemic policy outperforms idiosyncratic policy.

This paper goes beyond the identical firm framework, bringing more depth to idiosyncratic policy. Indeed, I find the opposite result; targeted policy outperforms untargeted policy. Given the non-degenerate distribution of firms and the specific targets tested, most eligible firms are already at (or near) their borrowing limit anyway. Thus even after the policy takes effect, they are not responsible for more debt than before.

In this spirit, I turn to [Buera et al. \(2013\)](#). Agents are given the choice to be laborers or entrepreneurs and the government targets high productivity, low wealth agents with a free capital policy. While there are short run benefits of subsidizing productive entrepreneurs, in the long run that productivity will eventually decay. Long run costs accrue from policymakers being unable to adjust the original subsidy policies (for a myriad of potential reasons) and end up funding unproductive entrepreneurs. I depart from [Buera et al.](#) in a few ways. First, this work concerns itself with potential stimulus during recession. Policy is intended to counteract the effects of failing credit markets. Although many well-intended policies have inefficient inertia, I study those that exist only for a short period, as in the motivating examples. Moreover, these policies do not distribute free capital. If a firm receives debt relief, they still must pay for investment. Related to this, the interest rate will endogenously change with the aggregate state, rather than being set by a large open economy as in the two aforementioned papers.

[Jo and Senga \(2019\)](#) study fiscal responses to credit market imperfections. As they point out, there are important general equilibrium effects to consider intrinsic in extended access

to debt, and therefore capital. Increased factor prices make investment more difficult for untargeted firms. Moreover, these prices effect entry and continuation decisions that are made on extensive margins. The general equilibrium effect may dominate the benefits of policy depending on the idiosyncratic shock process assumed. While their paper focuses on long-run resource allocation through steady state comparisons, this paper will abstract away from endogenous entry and exit and focus on transitional dynamics.

As mentioned previously, in this environment, and as in [Woodford \(1990\)](#), the government's role is to extend its access to debt markets to firms that are financially constrained. [Holmström and Tirole \(1998\)](#) expand this idea in the presence of liquidity shocks. [Angeletos et al. \(2023\)](#) solve, non-linearly, for optimal dynamics of tax and public debt in the presence of collateral constraints. In order to focus more on the relevant asset distribution and gains from targeting, optimal debt levels will be left to future work. I do, however, corroborate their finding that tax smoothing, from slower debt repayment, provides smoother consumption over time.

Finally, this work leans on literature studying financial frictions, namely collateralized borrowing as in [Khan and Thomas \(2013\)](#). The collateralized borrowing allows me to replicate important firm life-cycle aspects that are relevant for policy analysis. Young firms grow as their ability to borrow rises.¹² Including persistent, yet uncertain, idiosyncratic productivity shocks means optimal (unconstrained) capital choices will vary across firms, and change dynamically, creating a richer distribution of firms over excess returns. This allows me to explore different dimensions of policy targeting.

¹²See [Albuquerque and Hopenhayn \(2004\)](#) for results driven by limited enforceability, [Clementi and Hopenhayn \(2006\)](#) for private information, and [Bernanke and Gertler \(1989\)](#) for agency costs.

III Model

III.a Firms

The model economy is populated with a unit mass of firms that use labor, n , and predetermined capital, k , to produce a homogeneous consumption good through a decreasing returns to scale, twice differentiable, production function, $y = z\varepsilon f(k, n)$. Each firms' productivity is subject to both aggregate, z , and idiosyncratic, ε , shocks. I assume both z and ε follow their own respective Markov processes, such that $z \in \mathbf{Z} \equiv \{z_1 \dots z_{N_z}\}$, where $\Pr(z' = z_g \mid z = z_f) = \pi_{f,g}^z$ and $\varepsilon \in \mathbf{E} \equiv \{\varepsilon_1, \dots, \varepsilon_{N_\varepsilon}\}$, where $\Pr(\varepsilon' = \varepsilon_j \mid \varepsilon = \varepsilon_i) = \pi_{i,j}^\varepsilon$. $\sum_{g=1}^{N_z} \pi_{f,g}^\varepsilon = 1$ and $\sum_{j=1}^{N_\varepsilon} \pi_{i,j}^z = 1$ both hold. I assume ε is drawn from a bounded Pareto distribution with bounds ε_L and ε_H , and shape η . Each period, there is a ρ_ε probability that firms keep their current value of ε . With probability $(1 - \rho_\varepsilon)$, they draw a new value from the ergodic Pareto distribution.

In order to fund investments, i , for next period's capital stock, k' , firms use their revenues and issue a risk-free bond, b' , and b represents their current financial assets.¹³ They face collateralized borrowing constraints and may not borrow more than a given fraction, ζ , of their current capital stock each period.¹⁴ Collateralized borrowing is a reduced form method of capturing the health of financial markets. A higher ζ reflects lenders' positive beliefs in the market, thus they are willing to lend more to firms. On the other hand, a lower ζ reflects just the opposite. The parameter $\zeta \in \mathbf{\zeta} \equiv \{\zeta_1, \dots, \zeta_{N_\zeta}\}$ is stochastic and follows its own Markov process, where $\Pr(\zeta' = \zeta_k \mid \zeta = \zeta_h) = \pi_{h,k}^\zeta$ and $\sum_{k=1}^{N_\zeta} \pi_{h,k}^\zeta = 1$.

As firms age, their age being denoted by a , their borrowing capacity increases with capital accumulation. In order to prevent the whole economy from outgrowing financial conditions, each period a fraction, $\pi_a(a)$, of firms are exogenously forced to exit the market permanently at the end of a period. To reflect the negative correlation between age and exit rates seen in

¹³The convention used is $b > 0$ represents positive bonds issued (*in debt*) and $b < 0$ represents net savings.

¹⁴Many papers study future capital [Kiyotaki and Moore \(1997\)](#), or earnings-based [Lian and Ma \(2021\)](#) collateral. This will not meaningfully change the analysis, subject to a properly calibrated ζ .

the BDS, this exit shock is a function of firm age. Thus, a given firm, (k, b, ε, a) , is defined by its current capital stock, $k \in \mathbf{K} \subset \mathbf{R}_+$, debt level, $b \in \mathbf{B} \subset \mathbf{R}$, idiosyncratic shock realization, and age, $a \in \mathbf{A} \subset \mathbf{R}_+$. An exiting firm is replaced by an entrant with a capital stock of k_0 , a total debt value of b_0 , and idiosyncratic productivity, ε_0 drawn from the ergodic distribution implied by π_{ij} , from an infinite pool of potential entrants to ensure the mass of firms will not change. The resulting entrant rate is π_e .

At the start of each period, firms are made aware of their idiosyncratic, ε , and aggregate productivity, z , the financial state, ζ , and whether or not they will exit the market. There is no penalty for exiting the market; firms know if they will exit and, if so, simply optimize their labor choice given their remaining capital stock and productivity, produce output, and settle any outstanding financial obligations, which had an upper bound given from last period's financial state. They do not borrow or invest. This timing is crucial, as it insures all debt is risk-free.

Along with government debt, θ , to be described in the section below, I can now characterize the state of the economy. The distribution of firms over (k, b, ε, a) is described by the probability measure μ defined on the Borel algebra, \mathcal{S} , generated by the open subsets of the product space, $\mathbf{S} = \mathbf{K} \times \mathbf{B} \times \mathbf{E} \times \mathbf{A}$. The distribution of firms evolves over time according to the mapping, Γ , from the current aggregate state, $\mu' = \Gamma(z, \zeta, \theta, \mu)$. This process is known to all agents in the economy. For expositional ease, let the stochastic components of the aggregate state be $s = (z, \zeta)$ and its transition probabilities be $\pi_{l,m}^s = \Pr(s' = (z, \zeta)_m \mid s = (z, \zeta)_l)$, where each $\pi_{l,m}^s$ is derived from the transition probabilities $\pi_{h,k}^\zeta$ and $\pi_{f,g}^z$. Let the shocks realized in the aggregate state s_l be z_f and ζ_h , for each $l = \{1, \dots, N_s\}$, where $N_s = N_z N_\zeta$. Finally, let S represent the entire aggregate state, $S_l = (s_l, \theta, \mu)$.¹⁵

¹⁵Hereafter, I suppress the indices for the current exogenous aggregate state l and idiosyncratic productivity, i , except where necessary

III.a.1 The Firms' Problem

With the aggregate state described, I now begin to characterize a given firm's problem. Their cash-on-hand is their output, less taxed wage bill, plus undepreciated capital stock, minus debt obligations, with the addition of a payroll tax, $\tau(S)$ and debt relief, $g(k, b, \varepsilon, a; S)$. Capital depreciates at rate δ and takes 1 period to build. Therefore, cash-on-hand is written as:

$$x(k, b, \varepsilon, a; S) = z\varepsilon F(k, n) - (1 + \tau(S))w(S)n + (1 - \delta)k - (1 - g(k, b, \varepsilon, a; S))b \quad (1)$$

where $g(k, b, \varepsilon, a)$ is the fraction of debt the firm is relieved of and $(1 - g(k, b, \varepsilon, a; S))$ is the fraction of debt the firm is still responsible for. This modeling of debt relief, where the government pays for a fraction of firm debt is akin to that of [Bianchi \(2016\)](#).

When entering a period, the firm does not yet know if it will be forced to exit or be allowed to continue. Before making any decisions, its value is then given by $v_0(k, b, \varepsilon, a; S)$, the probability-weighted sum of exiting with cash-on-hand, or continuing on to the next period and receiving continuation value, $v(k, b, \varepsilon, a; S)$:

$$v_0(k, b, \varepsilon_i, a; S) = \pi_d(a)x(k, b, \varepsilon; S) + (1 - \pi_d(a))v(k, b, \varepsilon; S) \quad (2)$$

Once the period starts and firms are made aware of their continuation status, they choose their dividend payments, D , to borrow or save b' , at the loan discount factor $q(S)$, and capital for next period, k' , while not exceeding their collateral constraint of $b' \leq \zeta k$. The law of motion for capital is $k' = i + (1 - \delta)k$. Taking as given the transition conditions of the idiosyncratic states, the continuation value is given by:

$$v(k, b, \varepsilon_i; S_l) = \max_{k', b', D} \left[D + \sum_{m=1}^{N_s} \pi_{l,m}^s d_m(S_l) \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^\varepsilon v_0(k', b', \varepsilon_j; S'_m) \right] \quad (3)$$

subject to:

$$\begin{aligned}
D &\leq x(k, b, \varepsilon_i; S_l) + q(S_l)b' - k' \\
b' &\leq \zeta_l k \\
\mu' &= \Gamma(S_l)
\end{aligned} \tag{4}$$

where $d_m(S_l)$ is the relative valuation of dividends next period, conditional on the current state, S_l , and the next period state, S'_m and $q(S)$ is the risk-free loan discount factor.

Notice there are no inter-temporal considerations for labor demand and the wage bill is paid out of production, meaning all firms with the same (k, ε) will have the same labor demand. Thus, the firm's labor decision rule is $g_N(k, \varepsilon; S)$. Following from this, a firm's output is $y(k, \varepsilon; S)$. As younger firms have higher risk of exit than older firms, firms of different ages discount the future differently. Thus capital and debt decisions rules require arguments from the full individual state; they are $g_K(k, b, \varepsilon, a; S)$ and $g_B(k, b, \varepsilon, a; S)$, respectively.

III.b Government

The government is endowed with two sets of resources: revenue from a payroll tax, and funds borrowed, θ' , from the household at the risk-free rate. It uses these to pay for government expenditures, \bar{G} , public debt, θ , and private debt relief, $T(k, b, \varepsilon_i, a; S)$, which is the sum of all relief provided:

$$T(k, b, \varepsilon, a; S) = \int_{\mathbf{s}} g(k, b, \varepsilon, a; S) b \mu(d[k \times b \times \varepsilon \times a]) \tag{5}$$

Bonds issued by the government are either repaid through tax revenues, or rolled over to the next period at the risk-free rate. Public debt is a type of consol debt, where the speed at which it is paid is governed by the parameter ϕ , the fraction of debt paid. Conditional on making a payment, $(1 - \phi)$ is the fraction of debt rolled over to next period. Finally,

expenditures, \overline{G} , are *not* valued by households, *nor* are they used in production. The government's budget constraint is:

$$\tau(S)w(S) \int_{\mathbf{S}} [g_N(k, \varepsilon; S)] \mu(d[k \times b \times \varepsilon \times a]) + q(S)\theta' \geq \overline{G} + \theta + T(k, b, \varepsilon, a; S) \quad (6)$$

where θ' is determined by:

$$\theta' = \begin{cases} (1 - \phi)\theta & \text{make payment} \\ \frac{1}{q(S)} (\overline{G} + \theta + T(k, b, \varepsilon, a; S) - \tau(S)w(S)N(S)) & \text{debt rollover} \end{cases} \quad (7)$$

The functionality of this set up is that it pins down a singular path of taxes over time for a given ϕ , as the lowest tax rate that satisfies the budget condition. Varying ϕ allows for the analysis of different repayment horizons.

III.c Household

There is a representative household endowed with ownership of the firms and a unit of time that may be divided between labor and leisure. The household supplies labor to the firms in exchange for a wage, w , transfers resources intertemporally by purchasing bonds, κ' , and shares, λ' , and chooses consumption, c , to maximize its lifetime value, $W(\lambda, \kappa; S)$. Note in this set up, government and corporate bonds are perfect substitutes for the household. The household discounts the future at rate $\beta \in (0, 1)$ per period and maximizes its periodic utility over consumption and leisure, $U(c, 1 - n)$, subject to their budget constraint. The full household problem is therefore: ¹⁶

$$W(\lambda, \kappa; S) = \max_{c, n^h, \lambda', \kappa'} \left[U(c, 1 - n^h) + \beta \sum_{m=1}^{N_s} \pi_{l,m}^s W(\lambda', \kappa'; S'_m) \right] \quad (8)$$

¹⁶The household has access to a complete set of state-contingent claims. These assets are in zero net supply in equilibrium; for simplicity I do not model them here.

subject to:

$$\begin{aligned}
c + q(S)\kappa' + \int \rho_1(k', b', \varepsilon'; S_l) \lambda'(d[k' \times b' \times \varepsilon' \times a']) &\leq \\
w(S)n^h + \kappa + \int \rho_0(k, b, \varepsilon; S_l) \lambda(d[k \times b \times \varepsilon \times a]) & \\
\mu' = \Gamma(S) &
\end{aligned} \tag{9}$$

where $\rho_1(k', b', \varepsilon', a'; S_l)$ is ex-dividend price of a share and $\rho_0(k, b, \varepsilon, a; S_l)$ is dividend-inclusive value of a share. Let $h^c(\lambda, \kappa; S)$ be the household's decision rule for consumption, $h^n(\lambda, \kappa; S)$ be the decision rule for labor hours, and $h^\kappa(\lambda, \kappa; S)$ be the decision rule for bonds. Finally, let $h^\lambda(k', b', \varepsilon', a', \lambda, \kappa; S)$ be the decision rule for shares in firms with next period values of k' capital, b' debt, and ε' idiosyncratic productivity, of age a .¹⁷

III.d Recursive Equilibrium

A *recursive competitive equilibrium* is a set of functions:

$$(w, q, \{d_m\}_{m=1}^{N_s}, \rho_0, \rho_1, v_0, g_N, g_K, g_B, g_D, W, h^c, h^N, h^\lambda, h^\kappa)$$

that solve the firms' and household's problems and clear the asset, labor and output markets, such that:

- (i) v_0 solves (2-3) subject to (4), g_N is the decision rule for exiting firms, and (g_N, g_K, g_B, g_D) are the decision rules for continuing firms
- (ii) The government's budget constraint (6), subject to (7) is satisfied
- (iii) W solves (8) subject to (9) and $(h^c, h^N, h^\lambda, h^\kappa)$ are the household's decision rules
- (iv) The share market clears: $h^\lambda(k', b', \varepsilon_j, a', \lambda, \kappa; S) = \mu'(k', b', \varepsilon_j, a'; S)$ for $(k', b', \varepsilon_j, a') \in \mathbf{S}$
- (v) The labor market clears: $h^N(\lambda, \kappa; S) = \int_{\mathbf{S}} [g_N(k, \varepsilon; S)] \mu(d[k \times b \times \varepsilon \times a])$

¹⁷The household can choose shares of these firm types in equilibrium, since it knows the transition probabilities of the aggregate and idiosyncratic states and the law of large numbers applies.

- (vi) The goods market clears: $C(\lambda, \kappa; S) = \int_{\mathbf{S}} \left[z\varepsilon F(k, n(k, \varepsilon, a; S)) - (1 - \pi_d(a)) (g_K(k, b, \varepsilon; S) - (1 - \delta)k) + \pi_e((1 - \delta)k - k_0) \right] \mu(d[k \times b \times \varepsilon]) - \bar{G}$
- (vii) $\forall (A, \varepsilon_j) \in \mathcal{S}$ defines Γ , where $\chi(k_0) = \{1 \text{ if } (k_0, 0) \in A; 0 \text{ otherwise}\}$
- $$\mu'(A, \varepsilon_j) = (1 - \pi_d(a)) \int_{\{(k, b, \varepsilon_i, a) | (g^K(k, b, \varepsilon_i, a; s, \mu), g^B(k, b, \varepsilon_i, a; s, \mu)) \in A\}} \pi_{ij} \mu(d[k \times b \times \varepsilon_i \times a]) + \pi_e \chi(k_0) H(\varepsilon_j)$$
- (viii) The bond market clears through Walras's Law: $h^\kappa(\lambda, \kappa; S) = \int_{\mathbf{S}} [g_B(k, b, \varepsilon, a; S)] \mu(d[k \times b \times \varepsilon \times a]) + \theta'$

III.e Prices

I begin my analysis by deriving a set of optimality conditions from the household's problem that may be subsumed into the firms' problem. Let C and N represent the optimal household choices of consumption and labor hours, respectively. Furthermore, let C' and N' be the optimal choices of consumption and labor next period when the aggregate state is S' . Then the real wage (10) is the marginal rate of substitution between leisure and consumption. Since the household owns the firms, the firms' stochastic discount factor (11) of future value is the household's marginal rate of substitution between consumption across states. The loan discount factor (12), is the conditional expectation of the marginal rate of substitution between consumption across states.¹⁸

$$w(S) = \frac{D_2 U(C, 1 - N)}{D_1 U(C, 1 - N)} \quad (10)$$

$$d(S) = \beta \frac{D_1 U(C'_m, 1 - N'_m)}{D_1 U(C, 1 - N)} \quad (11)$$

$$q(S) = \beta \sum_{m=1}^{N_\varepsilon} \pi_{l,m}^s \frac{D_1 U(C'_m, 1 - N'_m)}{D_1 U(C, 1 - N)} \quad (12)$$

Let $p(S)$ be the household's marginal valuation of output. Again, since the household

¹⁸The risk-free interest rate on bonds is: $r = \frac{1}{q(S)} - 1$.

owns the firms, I can multiply the firms' value (3) by $p(S)$ to price all (current and future) output in units of marginal utility without altering decision rules. Letting $V_0 = p(S)v_0$ and $V = p(S)v$, this changes equations (3, 10-12) such that:

$$V(k, b, \varepsilon_i; S_l) = \max_{k', b', D} \left[p(S)D + \beta \sum_{m=1}^{N_s} \pi_{l,m}^s \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^\varepsilon V_0(k', b', \varepsilon_j; S'_m) \right] \quad (13)$$

subject to (2) and (4)

$$w(S) = \frac{D_2 U(C, 1 - N)}{p(S)} \quad (14)$$

$$d_m(S) = \beta \frac{D_1 U(C'_m, 1 - N'_m)}{p(S)} \quad (15)$$

$$q(S) = \beta \sum_{m=1}^{N_s} \pi_{l,m}^s \frac{D_1 U(C'_m, 1 - N'_m)}{p(S)} \quad (16)$$

where:

$$p(S) = D_1 U(C, 1 - N) \quad (17)$$

Finally, I assume $\tau(S)$ will never be higher than what is required to balance the government's budget each period, meaning (6) always holds with equality. The solution for $\tau(S)$ requires solving a fixed point problem every period, the details of which are found in Appendix A.b.

III.f Allocations

A firm's labor choice is static. It chooses $n^*(k, \varepsilon) = z\varepsilon D_2 F(k, n) = (1 + \tau)w$. Output is then $y(k, \varepsilon) = z\varepsilon F(k, z\varepsilon D_2 F(k, n))$, which can be used to define cash-on-hand in (1).

Turning from choices involving only static variables to those that involve dynamic ones, I divide the analysis into two groups of firms to illustrate the potential for policy to improve capital allocations. The first group can reach their optimal capital level unimpeded by the collateral constraint. Let $k'^*(\varepsilon) = \arg\max_{k'} V(k, b, \varepsilon, a; S)$ for this group, which I will also use as a reference point for the next group. The second group cannot post sufficient collateral to afford optimal investment.

Firms unimpeded by their collateral invest so $\frac{\beta}{p(S)} (\mathbb{E}_{\pi^s} \mathbb{E}_{\pi^\varepsilon} D_1 V_0(k', b', \varepsilon_j, a'; S'_m)) = 1$. The expected marginal discounted (recall $p(S'_m)$ is embedded in V_0) value of an extra unit of capital equals its price, 1 output unit. Applying the Benveniste and Scheinkman condition, this is:

$$\begin{aligned} \frac{\beta}{p(S)} \left[\sum_{m=1}^{N_s} \pi_{l,m}^s p(S'_m) \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^\varepsilon \left(z_m \varepsilon_j D_1 F(k'^*(\varepsilon), n^*(k'^*(\varepsilon), \varepsilon_j)) \right. \right. \\ \left. \left. + z_m \varepsilon_j D_2 F(k'^*(\varepsilon), n^*(k'^*(\varepsilon), \varepsilon_j)) D_1 n^*(k'^*(\varepsilon), \varepsilon_j) \right. \right. \\ \left. \left. + (1 + \tau(S')) w(S') D_1 n^*(k'^*(\varepsilon), \varepsilon_j) + (1 - \delta) \right) \right] = 1 \end{aligned} \quad (18)$$

These firms have no excess returns to capital investment. Any debt relief policy will produce no extra output. Another way to see this is that neither the collateral constraint, nor debt enter into this optimality condition. Any relief will change neither labor, nor capital, and output will remain the same. Their choice of debt is then $b' \in \left[\frac{D + k'^*(\varepsilon) - x(k, b, \varepsilon, a; S)}{q(S)}, \zeta k \right]$ from the conditions in (4), to be discussed further below.

Moving to firms bound by their collateral, I take advantage of the monotonically increasing value function in k to simplify the problem. Firms will borrow until $b' = \zeta k$ and invest such that:

$$k'(k, b, \varepsilon, a) = x(k, b, \varepsilon, a) + q(S) \zeta k - D < k'^*(\varepsilon) \quad (19)$$

The strict inequality is the center of the misallocation problem that policy would seek to correct. These firms cannot reach their desired capital level and are unable to hire as much labor or produce as much output next period, compared to an environment without these financial frictions. Moreover, given decreasing returns to scale production, these firms have a higher shadow price on their collateral and higher returns on potential investment than firms without binding constraints. Again using the properties of the value function and going

back to (18), substituting in $k'(k, b, \varepsilon, a)$ for $k^*(\varepsilon)$ returns:

$$\begin{aligned} \frac{\beta}{p(S)} \left[\sum_{m=1}^{N_s} \pi_{l,m}^s p(S'_m) \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^\varepsilon \left(z_m \varepsilon_j D_1 F(k'(k, b, \varepsilon, a), n^*(k'(k, b, \varepsilon, a), \varepsilon_j)) \right. \right. \\ \left. \left. + z_m \varepsilon_j D_2 F(k'(k, b, \varepsilon, a), n^*(k^*(\varepsilon), \varepsilon_j)) D_1 n^*(k'(k, b, \varepsilon, a), \varepsilon_j) \right. \right. \\ \left. \left. + (1 + \tau(S')) w(S') D_1 n^*(k'(k, b, \varepsilon, a), \varepsilon_j) + (1 - \delta) \right) \right] > 1 \end{aligned} \quad (20)$$

Through (19), it is easy to see how a credit shock exacerbates these conditions, as the more $k^*(\varepsilon) - k'(k, b, \varepsilon, a)$ increases, the more the inefficiencies grow. However, this is where the role for policy exists. Debt relief policy that increases a firm's cash-on-hand can shrink, or even eliminate, the magnitude of the inequality in (19), as well as the difference between (20) and (18). Thus policy may increase output, investment, and labor hours compared to the same scenario without policy.

There is still the matter of issuing dividends and the unconstrained firms' debt. From here out, I assume a *zero-dividends* policy, $D = 0$. Firms use as much of their retained earnings as needed to invest. Those that need to borrow for optimal investment, but are unconstrained in doing so, will borrow to make up the difference. Funds leftover after investment will be used as savings. Constrained firms take $D = 0$ as given in (19). Given market clearing, the household is indifferent to the firm paying dividends, or retaining any resources leftover after investment as savings; this policy assigns the decision rule to the latter. Therefore, this is an optimal dividend policy for all firms.¹⁹

Finally, I set all steady state quantities in the government's budget to 0. Since this paper is concerned with policy responses to shocks, the level of these steady state variables will have little impact on the insights derived from the results.²⁰

¹⁹The (weak) optimality of the zero-dividend policy should not be confused with multiplicity of equilibria, which does not exist in this model. This is a result of the firm's value function being linear in D and b .

²⁰ \overline{G} remains in the current model formulation as a placeholder for future explorations of direct stimulus to aggregate demand.

IV Calibration

The model has an annual frequency. The common DRS production function is Cobb-Douglas: $z\varepsilon F(k, n) = z\varepsilon k^\alpha n^\nu$. Entrant firms' capital supply is fixed at χ percent of the steady state aggregate stock: $k_0 = \chi \int k \tilde{\mu}(d[k \times b \times \varepsilon \times a])$. Household utility, $U(c, 1 - n^h) = \ln(c) + \psi(1 - n^h)$, is the result of Hansen (1985) - Rogerson (1988) indivisible labor.

The household discount factor, β is set to 0.96, to imply a 4% risk-free real interest rate, as found in Gomme et al. (2011). The parameter governing the marginal utility of leisure, ψ , is set to 2.14, so that in equilibrium, 1/3 of the household's time endowment is spent on market labor. Labor's share of output, ν is set to 0.60 to match Cooley et al. (1995). I set the depreciation rate, δ , to imply an average investment-to-capital rate of 0.069, which matches the BEA Fixed Asset Tables data. χ is set to 0.208, so that the relative employment size of the average entrant is 0.260 of the average firm, matching that in the BDS from 1990-2006. Entrant firm leverage, $\frac{b_0}{k_0}$ is set to 0.40, to match the Kauffman Firm Survey. The entrant rate, π_e is set to 0.106, in order to keep the mass of firms constant at 1, given the vector of exit rates by age, $\pi_d(a)$, taken from the BDS (1990-2006), in Table 1 below.

Table 1: Exit Hazard by Age

Age:	1	2	3	4	5	6-10	11+
Hazard:	0.2478	0.1640	0.1356	0.1174	0.1062	0.0840	0.0655

Note: Exit rates taken from BDS (1990-2006).

The next parameters are jointly calibrated to match the following moments. The parameter governing capital's share of output, α is set to 0.28 to target a capital output ratio of 2.25, from the BEA Fixed Asset Tables 1954 - 2006. The probability of a firm maintaining its ε draw is 0.99, to target a standard deviation of investment rates across firms of 0.337, from Cooper and Haltiwanger (2006). During normal economic times, the collateral constraint will be $\zeta_o = 0.981$, to target a debt-to-asset ratio of 0.372, as found from 1954-2006 nonfarm, nonfinancial businesses in the Flow of Funds. Finally, during a credit crunch, this value changes to $\zeta_l = 0.647$ in order to facilitate a peak-to-trough decrease in debt of about 26%.

Khan and Thomas (2013) report this GDP deflated real lending decrease in Commercial and Industrial loans in their data from 2008Q4 - 2011Q4. Calibrated parameters are reported on Table 2, along with their closest related data target.

Table 2: Parameters

	Parameter		Target	Data	Model
β	discount factor	= 0.960	real interest rate	= 0.040	0.041
ψ	leisure preference	= 2.140	labor hours	= 0.333	0.332
ν	labor share	= 0.600	labor share	= 0.600	0.600
δ	depreciation	= 0.069	investment/capital	= 0.069	0.069
χ	fraction of entrant K	= 0.208	avg. n_0/N	= 0.260	0.260
π_e	entrant rate	= 0.106	firm mass	= 1.000	1.000
$\frac{b_0}{k_0}$	entrant leverage	= 0.400	entrant leverage	= 0.400	0.400
α	capital share	= 0.280	capital/output	= 2.250	2.305
ρ_ε	maintain ε	= 0.990	std dev. i/k	= 0.337	0.358
ζ_o	collateral fraction	= 0.981	debt/assets	= 0.372	0.372
ζ_l	credit crunch	= 0.647	decrease in debt	= 0.260	0.260

IV.a Size Distributions

The model captures the size and age-size distributions in the U.S. through the use of a bounded Pareto distribution for firms' idiosyncratic shock process, discretized using $N_\varepsilon = 7$ values. The process of defining firm sizes bins in the model comes from Jo (2024). First, I define the threshold for small, medium, and large firms to be 1-19, 20-499, and 500+ employees respectively. To map this to the model, I use each bin's corresponding employment share, calculated from the BDS (1990-2006), to identify thresholds, \underline{n} and \bar{n} in the model such that firms that account for an employment share $\leq \underline{n}$ are small, firms that account for an employment share $\in [\underline{n}, \bar{n}]$ are medium, and firms that account for an employment share $\geq \bar{n}$ are large. These thresholds are defined such that the employment share bins they create exactly match the BDS data.²¹ I then check the resulting population share in each bin and compare that to the data. Using the population share of each firm size as calibration

²¹Since a firm's (k, ε) state determines its labor demand, it is possible that the employment share given by a (k, ε) makes a given bin too large compared to the BDS data. In that case, I allocate enough mass to the smaller bin to match the data and place the rest in the next bin.

targets, I set the last remaining parameters to $\varepsilon_L = 0.497$, $\varepsilon_H = 0.937$, and $\eta = 5.5$ to target the unconditional size distribution. These size bins will be used as eligibility thresholds for size-based policy in Section VI.b.1. The results are reported on Table 3.

Table 3: Size Distribution

Employment Bins	Emp. Share	Data	Model
		Pop. Share	Pop. Share
Small (1-19)	0.201	0.885	0.880
Med. (20-499)	0.319	0.112	0.101
Large (500+)	0.480	0.003	0.019

Given χ is chosen to match the average entrant’s relative share of employment, the model generates a age-size distribution comparable to what is observed in the U.S. for firms age 0-5, as well as the population weighted mean of each age group. Results for ages 1-5 are untargeted and serve as model validation. They are reported below on Table 4.

Table 4: Model Generated Age-Size Distribution

Age:	0	1	2	3	4	5	Pop. Weighted Average
Data:	0.260	0.338	0.378	0.417	0.451	0.477	0.368
Model:	0.260	0.302	0.362	0.427	0.506	0.595	0.377

Note: Ages 1-5 are untargeted in the calibration and are a result of the model.

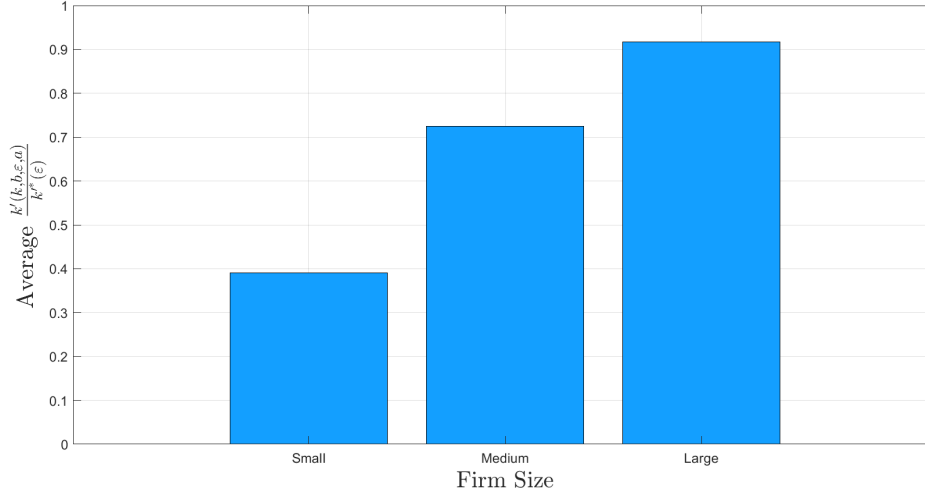
V Quantifying Inefficiency

V.a Steady State Inefficiencies

As described in Section III.f, there are inefficiencies present in the model before any shocks. Financial frictions limit investment and capital accumulation. Small firms do not have a large enough capital stock to post collateral for a loan, nor do they have enough retained earnings to afford their optimal investment level outright. This creates a wedge between their choice of $k'(k, b, \varepsilon, a)$ and the optimal k^* . Figure 1 shows the mean ratio of a firm’s capital choice to their optimal capital level, implied by their productivity, for different

size bins. In a perfectly optimal environment, all firms would be able to reach their target capital, which would be shown as all three bars at $y = 1$ since $k'(k, b, \varepsilon, a) = k^*(\varepsilon)$. However, given the financial frictions present, there is an inverse relationship between size and distance to optimal capital.

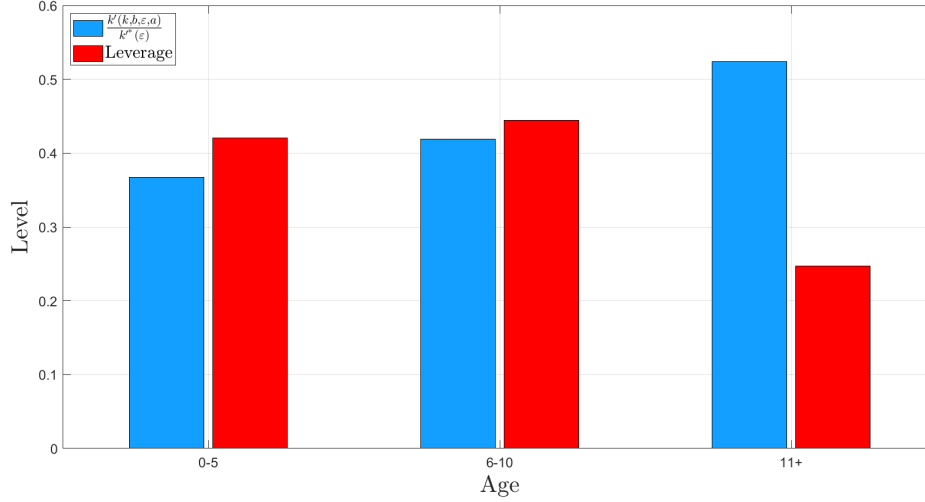
Figure 1: Average $\frac{k'(k, b, \varepsilon, a)}{k^*(\varepsilon)}$ Ratio by Firm Size



Note: Small firms do not have sufficient collateral or retained earnings to afford optimal investment, increasing the distance between their choice of capital and the optimal choice implied by their productivity level.

As evidenced by Table 4, firms grow as they age. They build up a stock of retained earnings to afford more capital investment. At first, this happens through external financing and taking on more debt. Building up their capital stock also allows them to post more collateral, giving them access to more external financing, further increasing their investment potential. As firms grow deeper into their life-cycle, they begin to deleverage as their retained earnings become larger, covering a greater share of investment. Thus, the older firms get, the more they are able to close the gap between their choice of capital, $k'(k, b, \varepsilon, a)$, and their efficient level, $k^*(\varepsilon)$. This is a slow process; on average, firms do not reach half of their optimal capital stock until age 11. This is in contrast to works that employ the commonly used log-normal distribution for idiosyncratic productivity shocks. For example, in [Khan and Thomas \(2013\)](#), the average firm's growth rate, measured by capital accumulation, approaches 0 at age 11. Figure 2 displays this trend.

Figure 2: Average $\frac{k'(k,b,\varepsilon,a)}{k^*(\varepsilon)}$ and Leverage across Ages



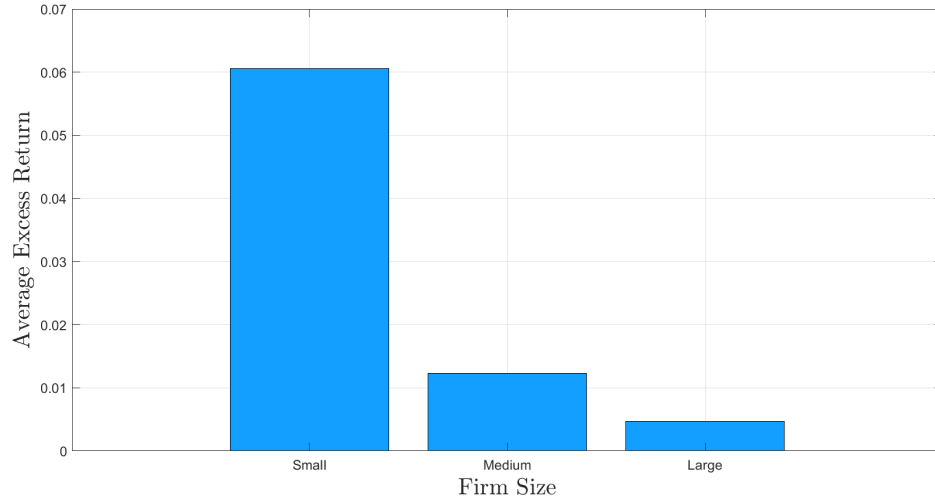
Note: As firms move through their life-cycle, they build up a stock of retained earnings to finance capital investment. They rely more on external financing earlier in their life-cycle, and deleverage as they age. Leverage is measured as $\frac{b}{k}$.

As firms move away from their optimal capital choice, their excess return to capital investment grows. From the parameterization of the model, (18) becomes:

$$\frac{\alpha}{1-\nu} \frac{\beta}{p(S)} \left(\sum_{m=1}^{N_s} \pi_{l,m}^s p(S'_m) \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^\varepsilon k'^{\frac{\alpha+\nu-1}{1-\nu}} \left[z_m \varepsilon_j \left(\frac{\nu z_m \varepsilon_j}{(1+\tau(S))w(S)} \right)^{\frac{\nu}{1-\nu}} - (1+\tau(S))w(S) \left(\frac{\nu z_m \varepsilon_j}{(1+\tau(S))w(S)} \right)^{\frac{1}{1-\nu}} \right] + (1-\delta) \right) = 1 \quad (21)$$

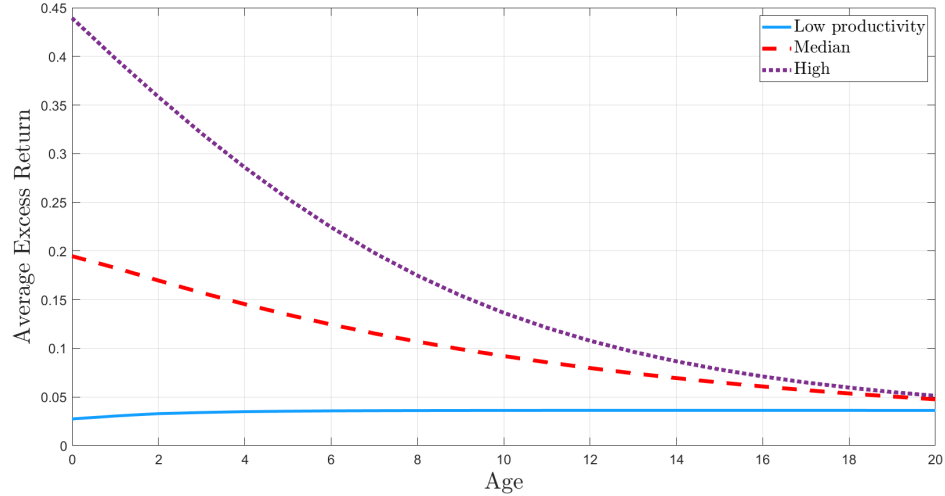
The further their capital choice is from the optimal level, the more the marginal lifetime benefit of capital investment increases and the Left Hand Side of the equation grows bigger than 1, the marginal cost of a unit of capital. On average, smaller and younger firms are farther from their optimal capital level than other firms, increasing their excess return to investment. Moreover, productive firms have a higher marginal product of capital, increasing their $k^*(\varepsilon)$, and thus their excess return on investment as well. These results highlight the concentration of inefficiency found in small firms, as well as young and productive firms. Figures 3 and 4 illustrate these results.

Figure 3: Average Excess Return to Investment by Size



Note: Larger firms are able to offer more collateral, or use higher retained earnings to finance investment. They are closer to their optimal investment, thus on average face lower excess returns.

Figure 4: Average Excess Return to Investment by Productivity across Ages



Note: More productive firms have a higher optimal level of capital and thus a higher level of optimal investment. This increases average excess return to investment. Moreover, as firms age, they accumulate more capital increasing their access to external financing, as well as their retained earnings.

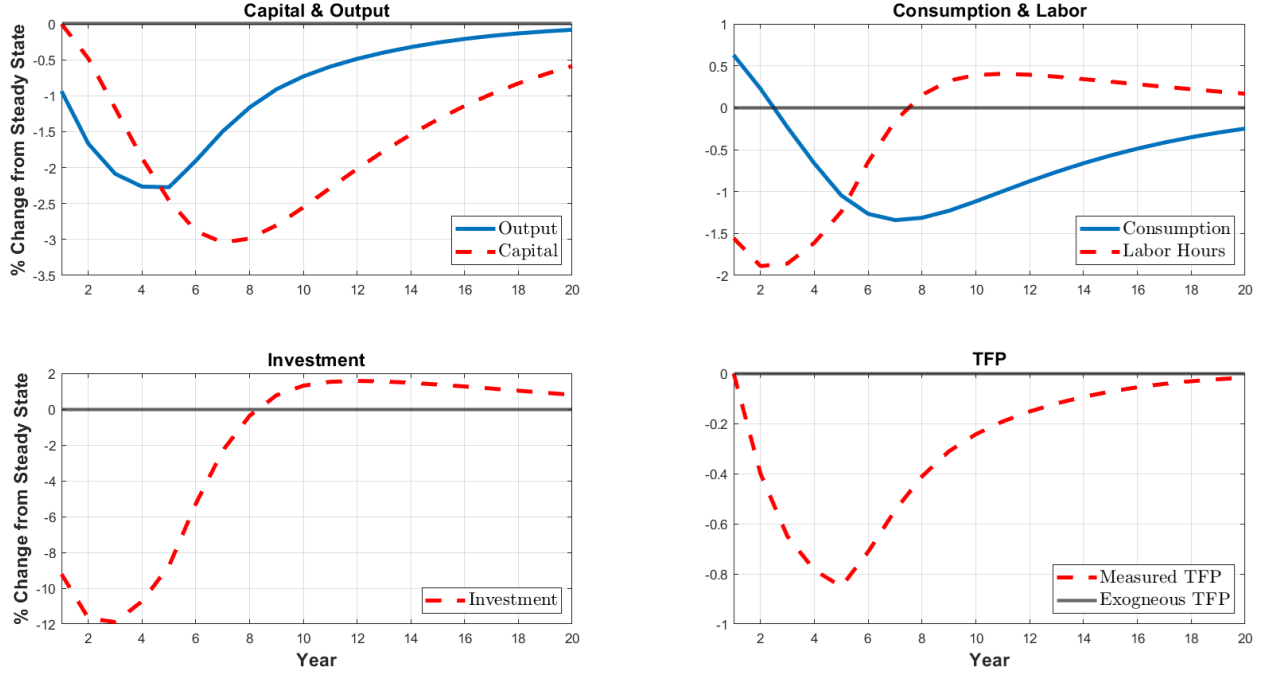
V.b Financial Crisis

I now begin the dynamic analysis of this study with a discussion on financial shocks without fiscal policy. The shock in the model is a credit crunch, an economy-wide contraction of credit, calibrated to match the 26% fall in debt discussed in the Calibration, Section IV.²²

²²Ivashina and Scharfstein (2010) find a large decrease in lending at the start of the Great Recession.

The shock in unexpected decrease in ζ_o to ζ_l and persist for 4 periods, then recovers at a rate of 31.25% per year, agreeing with [Khan and Thomas \(2013\)](#)'s calibration to the data presented in [Reinhart and Rogoff \(2009\)](#). Exogenous TFP does not change in this exercise.

Figure 5: Financial Crisis without Debt relief



Note: Model response to an unexpected credit shock of ζ_o falling to ζ_l . Shock persists to date 4 and recovers at rate 31.25% per year.

The response to the credit crunch shares defining characteristics with the Great Recession. There is an initial increase in consumption, as once the shock hits the household foresees lower returns to saving. Moreover, the fall in output is persistent as it takes time to rebuild the capital stock after the sharp downturn in investment. The persistent decline in endogenous TFP results from changes in the distribution of firms. Financial requirements have forced the majority of firms to become smaller, increasing misallocation of resources in the economy. Figure 5 displays these dynamics.

VI Debt Relief Policy

The purpose of debt relief in the model is to relax the budget constraint of firms to enable them to afford more investment, offsetting the impact of a credit crunch. The response to the credit conditions laid out is idiosyncratic, implying the impact of different policies will be as well. Given a fixed supply of policy funds, policymakers will be concerned with the best return on the resources spent and will focus resources accordingly.

Common across all policy experiments will be the total amount of debt relief paid out, $T(k, b, \varepsilon, a, S)$, to ensure that each policy has the same amount of resources being expended, as well as the government’s debt repayment start date and repayment fraction, ϕ , to ensure no exogenous variations in the tax rates. Repayment begins in period 7, the half-life point of the output recover. This avoids raising taxes in the heart of a recession. All policies will be in response to the credit shock described above in Section V.b and will occur on impact date. The total resource cost will be 4% of steady state output and starting at date $t = 7$, public debt will be paid down 5% per period, with the exception of one “free” policy shown to isolate the effect of taxation on the recovery path. Under this policy, the government rolls over its debt each period indefinitely. Resources are not being taken out of the aggregate resource constraint through taxes to pay for it. These parameters are reported on Table 5.

VI.a Targeting Inefficiency

The first policy experiment considered directly targets firm level excess return to capital investment. This policy identifies firms with the highest level of excess returns and issues debt relief to them until they match the firms with the second highest level of excess returns. The policy algorithm proceeds in that manner until funds are exhausted.²³

This policy design has another added benefit; it naturally establishes a threshold for

²³Computationally, one could implement this policy as described in the text. Another way to do so would be to realize there will be a level of excess returns that no indebted firm will exceed. A guess of this level implies a total resource cost for the government and this guess may be iterated on until convergence to the exogenously chosen resource cost.

eligible debt. Some firms have debt that would be inefficient to be relieved. Consider a firm that has some positive level of debt, but not enough for the collateral constraint to be binding. This firm can reach its optimal investment level, so any debt relief resources would just be held as savings by the firm (or passed on as dividends to shareholders in other specifications). Relief leads to no extra output, but still must be paid for by future tax increases. Thus optimal policy would set a threshold of eligible debt such that any relief payments being made will be used for investment by the firm. This then avoids concerns of funds not reaching their intended destination, a concern that has been raised for the Paycheck Protection Program (Li (2021), Autor et al. (2022)). This eligible debt threshold is:

$$\max\left\{\min\left[b, (k^*(\varepsilon) - x(k, b, \varepsilon, a) - q(S)\zeta k)\right], 0\right\} \quad (22)$$

This expression makes sure that the firm in question has debt to be relieved in the first place, $b > 0$. Beyond that, the relief policy will only pay out to the firm until their cash-on-hand becomes sufficient to pay for their optimal investment level. At that point the relief turns off and the firm will not again become eligible, even if there is remaining $b > 0$. By following the excess returns policy program, this threshold is naturally enforced.

Table 5: Excess Returns Policy Parameters

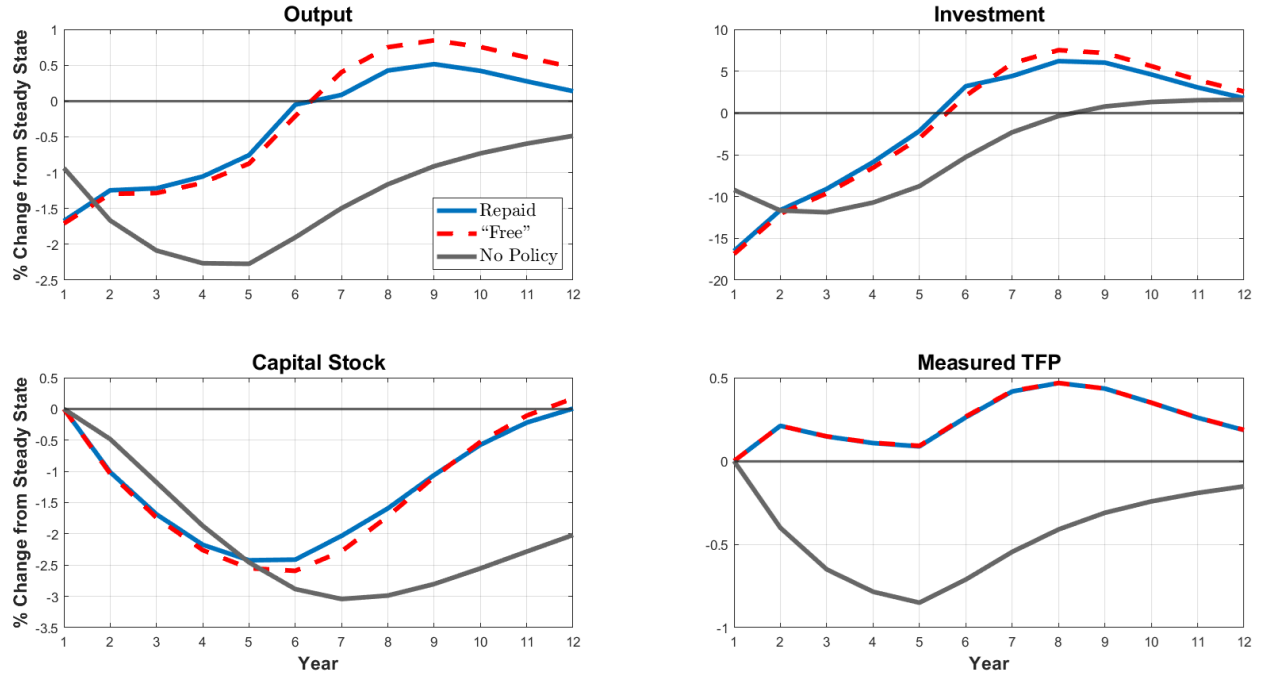
Parameter	Value	
Policy Cost	$\frac{T}{Y}$	4%
Credit Shock Dates	t	1-4
Shock Recovery Rate	ρ_ζ	0.313
Relief Date	t	1
Repayment (θ) Start	t	7
Repayment (θ) Fraction	ϕ	0.05

Note: These parameters will be common across all policy experiments, save for one “free” policy to illustrate the effect of taxation on recovery.

The process of solving for these transition paths involves two simultaneous guesses on the time vectors of consumption and taxation. The appendix provides a general overview of this method, with details on updating the taxation vector.

Figure 6 shows the impulse responses of aggregate variables to a credit shock with

Figure 6: Excess Return to Investment Policy Target



Note: Significant improvements with the excess return targeted policy. Policies relieves debt until the firms at the top of the excess returns distributions have this value equalized and lowered.

excess returns-targeted policy. There are two main components to the policy, debt relief and repayment, implied by the second fall in aggregates. The debt relief leads to major improvements in aggregates during a credit crisis. The output decline at the no policy trough date improves by over 50%. There is a massive initial improvement in endogenous TFP, which does not fall to steady state level even through the heart of the shock as resources are being moved to their most productive location. In fact, the policy may work too well. The household expects massive improvements in wealth such that they reduce their labor hours supplied at first, borrowing against their future expected income. Thus, there is a dip in initial output from the policy. Figure 10 in the appendix containing additional transition paths highlights this point further.

After the initial rise in TFP, it begins to fall again as credit markets are still in a poor state. There is then a second increase in TFP as the collateral parameter increases back to its steady state value.

Public debt repayment deserves specific consideration. Repayment can begin early in

the recession, at which point taxes must be raised during economic downturn, or after most of the recovery has been completed. However, by that point, public debt keeps accruing interest, increasing the amount of taxes needed to pay it. For illustrative purposes, I chose a repayment start date when the output decay had already passed its half-life. The rise in taxes to pay for this policy induces a slight downturn and slower recovery, which is more evident in the coming policy experiments. The larger the share of public debt repaid each period, the higher taxes must be raised to pay off that amount. The knowledge of future tax increases does not appear to have much of an effect at the trough of a recession. However, the no-repayment path does begin to deviate from the repayment paths more as they approach the date of the tax increases.

VI.b Readily Observable Targets

Identifying excess returns across the state space of an economy may not be feasible for policymakers, even if obtaining such information isn't cost prohibitive. I consider two readily observable policy targets: firm size and age. Results are compared to an untargeted policy at the end of the section.

VI.b.1 Size-based Targets

Since we are concerned with capital allocative efficiency, a good proxy for excess returns may be firm size. It was previously discussed how firm size may be correlated with excess returns as small firms may be further away from their optimal capital level, and thus face higher marginal returns.

I consider three sized-based policies targeted to small, medium, and large firms. These definitions can be found on on Table 3. As the total size of each policy will be held constant, and each targeted group is systematically different, the relief per firm must differ across polices. All indebted small firms will have 23.7% of their individual debt relieved, indebted medium size firms will have 14.5%, and indebted large firms will have 9.6%. The fractions

of firm's debt relieved for a given policy is reported on Table 6, where value common across policies are listed in the bottom panel.

Table 6: Size-based Policy Parameters

Parameter		Value
Small Target	μ_S	0.880
Medium Target	μ_M	0.101
Large Target	μ_L	0.019
Fraction of b paid (small)	$g(k, b, \varepsilon; S)$	0.237
Fraction of b paid (med)	$g(k, b, \varepsilon; S)$	0.145
Fraction of b paid (large)	$g(k, b, \varepsilon; S)$	0.096

Note: Different policies will relieve different fractions of firm debt to keep aggregate resource cost constant across experiments. $\mu_{S,M,L}$ represent the population share of firms implied by their employment share reported in Table 3.

Table 7 reports the response to a credit shock with a size-based policy. Given the results generated in Table 3, it is surprising that policy targeting medium size firms yields the best results rather than policy targeting small firms. Under the medium size policy, the trough of output remains a half percent closer to steady state and reaches its half life a year earlier when compared to the no policy scenario. The slower recovery due to the increase in taxes to pay for the policy is more noticeable here than in the excess return-based policy.

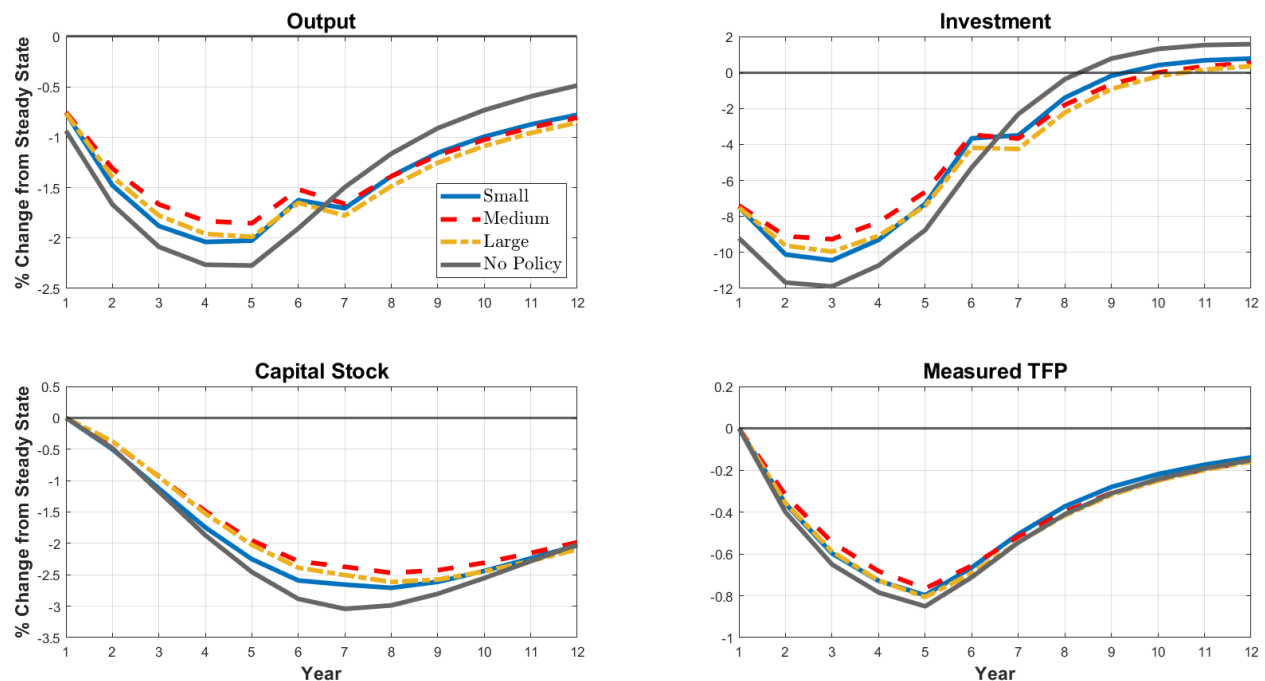
The benefit to targeting medium size firms depends on inefficiency at both the firm and economy-wide level. To understand this outcome, it is helpful to first discuss small and large firms. The benefit of targeting small firms is that resources can be allocated to firms facing higher levels of excess return to investment. However, these firms have a smaller impact on aggregate outcomes than large firms. Consider Table 3 again. Small firms make up 88% of the model population mass, yet are only responsible for a fifth of employment and output.²⁴ A drop in efficiency for 88% of the economy will only effect 20% of output and employment, and thus will have a relatively small impact on endogenous TFP. The inverse is true of larger firms. A sudden drop in efficiency for 2% of firms in the economy impacts almost half of output and employment. On the other hand, these large firms are more likely to have a

²⁴It can be shown that for a given firm i , both output share and employment share are $\frac{(\varepsilon_i k_i^\alpha)^{\frac{1}{1-\nu}}}{\int (\varepsilon k^\alpha)^{\frac{1}{1-\nu}} \mu(d[k \times \varepsilon])}$.

bigger capital stock and more retained earnings to see them through the crisis, so debt relief does not correct the same degree of misallocation when targeted to large firms.

Medium size firms are responsible for a larger share of output than smaller firms *and* face higher levels of excess return on average than large firms. These are firms more likely destined for the long right tail of the distribution, but have not yet outgrown their financial constraints. Debt relief can correct a greater incidence of misallocation compared to large firms, while targeting a group of firms that is relatively more important for aggregate outcomes than small firms.

Figure 7: Response to Size-targeted Policy



Note: Aggregates have the highest improvement under medium size policy targets. Medium size firms face higher excess returns than large firms and are responsible for a bigger portion of aggregate endogenous TFP than small firms.

VI.b.2 Age-based Targets

Beyond employment targeting, another proxy for excess returns may be firm age. In an environment absent financial frictions, entering firms would be able to immediately reach their optimal capital stock and excess return to investment across the economy would be 0.

Given financial frictions, there is a slow growth process to $k'^*(\varepsilon)$ as firms age. This process of aging generates firm life-cycle results in the model, as young firms carry inefficiently low levels of capital. However, this also means that there is a relationship between age and excess returns, and thus potential gains from age-targeted policy.

Age-based policy will target young firms age 0-5 years old, a definition taken from [Goetz and Stinson's \(2021\)](#) Census report. They define a young firm as “firms with positive employment for five years or less.” These firms will have 35.1% of their debt paid for. Policy parameters are listed on Table 7, along with the medium size firm target for reference.

Table 8 displays the response to a credit shock under a young firm targeted policy, along with the response under a policy targeting medium sized firms for comparison. Once again, the medium size firm target out performs other policy targets during the heart of the recession. However, the policy that targets young firms provides very similar results. This follows from Figure 4 as young firms, especially productive ones, face higher levels of excess return on investment. Moreover, this illustrates the distinction between young and small firms. Young firms may be small because they have not yet had enough time to accumulate retained earnings for investment and are still trying to grow. On the other hand, many small firms are purposefully small. They understand that their productivity is persistent and the probability of a significant increase in productivity is low.

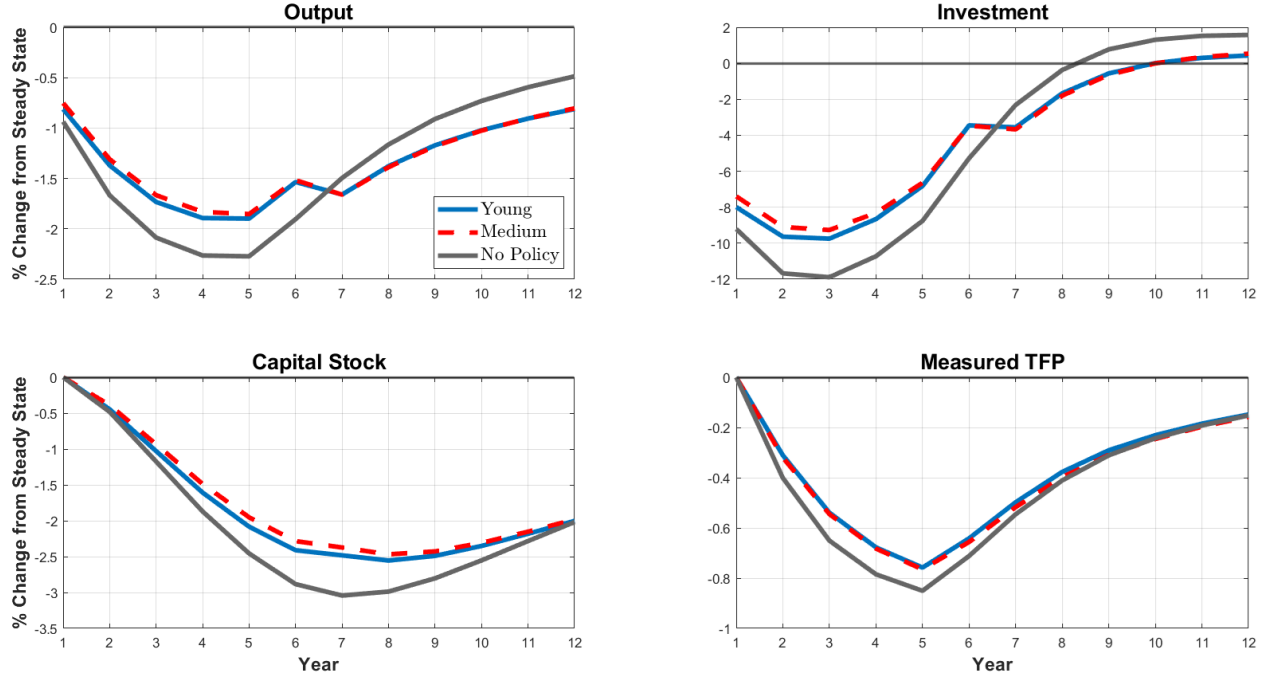
Weighing down the benefit to policy targeting young firms is their high exit rate, reported in Table 1. Firms that are receiving policy resources are systematically more likely to exit the economy than those not eligible for the policy. Ideally, policy would lower exit rates and improve results in that regard. However, these exit probabilities are taken from the BDS over 1990-2006. In a recession they will be higher than the values used here. It remains to be seen which effect would be stronger, and if one is stronger, will it be enough to significantly change the outcomes presented. I abstract from that discussion and leave it to future work.

Table 7: Age-based Policy Parameters

Parameter		Value
Age Target	a	[0-5]
Medium Target	μ_M	0.101
Fraction of b paid (young)	$g(k, b, \varepsilon, a; S)$	0.351
Fraction of b paid (medium)	$g(k, b, \varepsilon; S)$	0.145

Note: Age target is firms younger than 5 years old following the Census Report from [Goetz and Stinson \(2021\)](#).

Figure 8: Response to Age-targeted Policy



Note: Medium size targets lead to better aggregate results than age targets as well. Age targeting does lead to a lower tax burden in the recovery.

VI.b.3 Untargeted Policy

I consider one final policy structure, a policy without targets. All indebted firms are eligible to receive policy funds and will have 4.6% of their debt paid. Table 8 displays the parameters and Figure 9 displays the transition paths for this policy case.

Untargeted policy performs well compared to policies conditioned on other observable targets. Only the medium size policy leads to better outcomes in aggregate variables. The benefit to untargeted policy is that it takes advantage of decreasing returns to scale production technology. Concentrating resources on a subsection of the distribution does

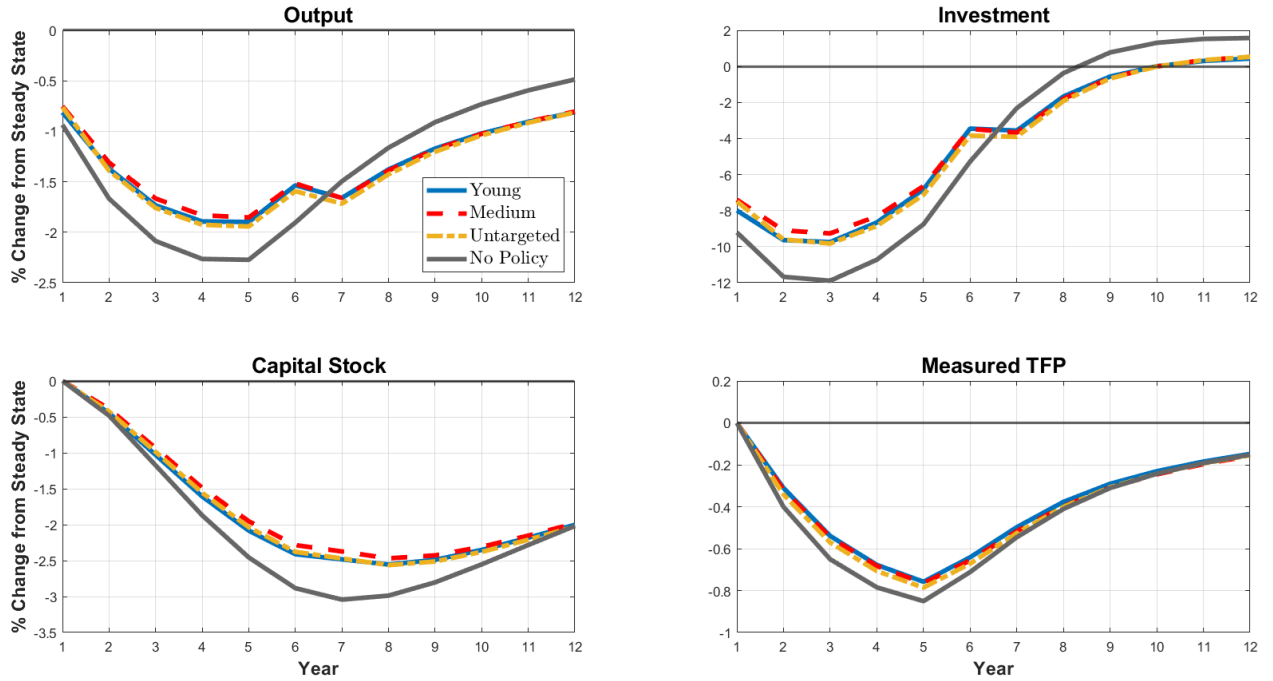
allow policymakers to focus on key variables, but decreases the marginal return to a “bailout unit” quickly. On the other hand, spreading the relief out to more firms keeps this marginal value higher over a larger population.

Table 8: Age-based Policy Parameters

Parameter		Value
Age Target	a	[0-5]
Medium Target	μ_M	0.101
Fraction of b paid (young)	$g(k, b, \varepsilon, a; S)$	0.351
Fraction of b paid (med)	$g(k, b, \varepsilon; S)$	0.145
Fraction of b paid (no target)	$g(b; S)$	0.046

Note: All indebted firms are eligible to receive relief under untargeted policy.

Figure 9: Response to Untargeted Policy



Note: Medium size targets lead to better aggregate results than no policy targets. However, having no policy targets for debt relief outperforms all other size targets.

VII Concluding Remarks

I develop a dynamic stochastic general equilibrium model where persistently heterogeneous firms face financial frictions and calibrate it to U.S. data. I use this model as a quantitative

laboratory to study the stabilization effects of firm-targeted debt relief in a financial crisis. I employ a bounded Pareto distribution to allow the model to capture the heavily skewed size distribution of firms seen in the United States. The model endogenously produces an age-size distribution of firms that closely matches that of the U.S. Financial frictions limit firm's access to debt, and thus investment, providing a role for debt relief policy in a financial downturn. The government takes on a role similar to [Woodford \(1990\)](#); it borrows on behalf of firms that cannot. However, when it comes time to repay public debt, the increased taxes lead to a slower recovery.

Policy that can decrease and equalize the highest levels of excess return to capital investment, the expected lifetime marginal value of capital investment minus its cost, yields the best results of those studied here. It allocates resources to more productive points in the distribution, dramatically cutting the fall in output by half at its no policy trough and drastically increasing endogenous TFP.

Understanding that this may not be a readily measurable policy target, I consider policies based on firm size and age, as these variables track well with excess returns. Policy that targets medium size firms performs the best out of these policies in terms of improving aggregates in the heart of a credit crunch. Output remains a half percent closer to its steady state value at the trough and reaches its half-life a year sooner. I also find that untargeted policy outperforms policy targeting small and large firms. This is due to widening the eligibility pool, keeping the marginal return of a debt relief unit higher across a larger population.

Alternative observables that correlate with excess returns are a potential area for new work. Different tax structure can be studied to prevent the second dip in aggregates seen in the impulse responses. Endogenous entry and exit decision to allow for debt relief of entrants can be studied as a comparison to the young firms target. I leave these ideas to future research.

References

- R. Albuquerque and H. A. Hopenhayn. Optimal lending contracts and firm dynamics. *The Review of Economic Studies*, 71(2):285–315, 2004.
- G.-M. Angeletos, F. Collard, and H. Dellas. Public debt as private liquidity: Optimal policy. *Journal of Political Economy*, 131(11):3233–3264, 2023.
- C. Asness, A. Frazzini, R. Israel, T. J. Moskowitz, and L. H. Pedersen. Size matters, if you control your junk. *Journal of Financial Economics*, 129(3):479–509, 2018.
- D. Autor, D. Cho, L. D. Crane, M. Goldar, B. Lutz, J. Montes, W. B. Peterman, D. Ratner, D. Villar, and A. Yildirmaz. The \$800 billion paycheck protection program: where did the money go and why did it go there? *Journal of Economic Perspectives*, 36(2):55–80, 2022.
- T. Babina, W. Ma, C. Moser, P. Ouimet, and R. Zarutskie. Pay, employment, and dynamics of young firms. *Kenan Institute of Private Enterprise Research Paper*, (19-25), 2019.
- R. W. Banz. The relationship between return and market value of common stocks. *Journal of financial economics*, 9(1):3–18, 1981.
- J. B. Berk. Does size really matter? *Financial Analysts Journal*, 53(5):12–18, 1997.
- B. Bernanke and M. Gertler. Agency costs, net worth and business fluctuations. *American Economic Review*, 79(1):14–31, 1989.
- J. Bianchi. Efficient bailouts? *American Economic Review*, 106(12):3607–3659, 2016.
- B. M. Blau, T. J. Brough, and D. W. Thomas. Corporate lobbying, political connections, and the bailout of banks. *Journal of Banking & Finance*, 37(8):3007–3017, 2013.
- A. Brav, J. R. Graham, C. R. Harvey, and R. Michaely. Payout policy in the 21st century. *Journal of financial economics*, 77(3):483–527, 2005.

- F. J. Buera, B. Moll, and Y. Shin. Well-intended policies. *Review of Economic Dynamics*, 16(1):216–230, 2013.
- G. L. Clementi and H. A. Hopenhayn. A theory of financing constraints and firm dynamics. *The Quarterly Journal of Economics*, 121(1):229–265, 2006.
- T. F. Cooley, E. C. Prescott, et al. Economic growth and business cycles. *Frontiers of business cycle research*, 1:1–38, 1995.
- R. W. Cooper and J. C. Haltiwanger. On the nature of capital adjustment costs. *The Review of Economic Studies*, 73(3):611–633, 2006.
- N. Crouzet and N. R. Mehrotra. Small and large firms over the business cycle. *American Economic Review*, 110(11):3549–3601, 2020.
- M. Gertler and S. Gilchrist. Monetary policy, business cycles, and the behavior of small manufacturing firms. *The Quarterly Journal of Economics*, 109(2):309–340, 1994.
- C. Goetz and M. Stinson. The business dynamics statistics: Describing the evolution of the us economy from 1978-2019. Technical Report 21-33, Center for Economic Studies, 2021.
- P. Gomme, B. Ravikumar, and P. Rupert. The return to capital and the business cycle. *Review of Economic Dynamics*, 14(2):262–278, 2011.
- N. Guner, G. Ventura, and Y. Xu. Macroeconomic implications of size-dependent policies. *Review of economic Dynamics*, 11(4):721–744, 2008.
- G. D. Hansen. Indivisible labor and the business cycle. *Journal of monetary Economics*, 16(3):309–327, 1985.
- B. Holmström and J. Tirole. Private and public supply of liquidity. *Journal of political Economy*, 106(1):1–40, 1998.

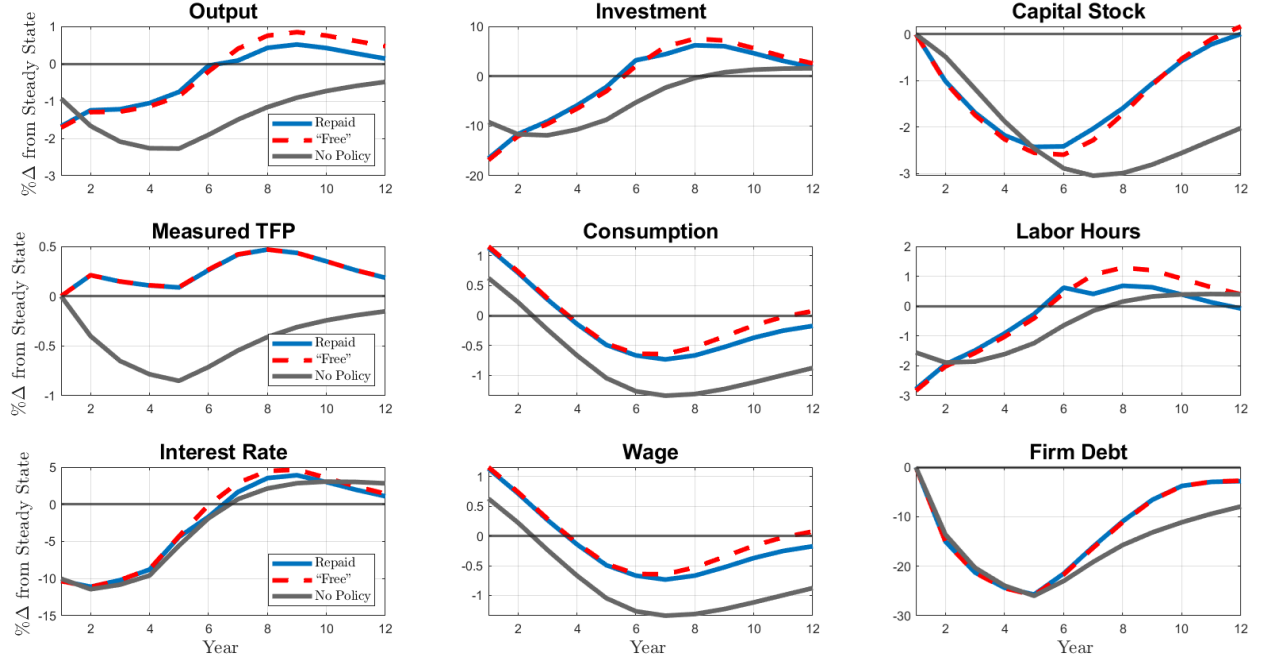
- A. Horvath and P. Lang. Do loan subsidies boost the real activity of small firms? *Journal of Banking & Finance*, 122:105988, 2021.
- V. Ivashina and D. Scharfstein. Bank lending during the financial crisis of 2008. *Journal of Financial economics*, 97(3):319–338, 2010.
- U. Jermann and V. Quadrini. Macroeconomic effects of financial shocks. *American Economic Review*, 102(1):238–271, 2012.
- I. H. Jo. Firm size and business cycles with credit shocks. *International Economic Review*, Forthcoming, 2024.
- I. H. Jo and T. Senga. Aggregate consequences of credit subsidy policies: Firm dynamics and misallocation. *Review of Economic Dynamics*, 32:68–93, 2019.
- A. Khan and J. K. Thomas. Credit shocks and aggregate fluctuations in an economy with production heterogeneity. *Journal of Political Economy*, 121(6):1055–1107, 2013.
- N. Kiyotaki and J. Moore. Credit cycles. *Journal of political economy*, 105(2):211–248, 1997.
- M. Li. Did the small business administration’s covid-19 assistance go to the hard hit firms and bring the desired relief? *Journal of Economics and Business*, 115:105969, 2021.
- C. Lian and Y. Ma. Anatomy of corporate borrowing constraints. *The Quarterly Journal of Economics*, 136(1):229–291, 2021.
- C. M. Reinhart and K. S. Rogoff. *This time is different: Eight centuries of financial folly*. princeton university press, 2009.
- R. Rogerson. Indivisible labor, lotteries and equilibrium. *Journal of monetary Economics*, 21(1):3–16, 1988.
- A. Shleifer and R. W. Vishny. A survey of corporate governance. *The journal of finance*, 52(2):737–783, 1997.

M. Woodford. Public debt as private liquidity. *The American Economic Review*, 80(2): 382–388, 1990.

A Appendix

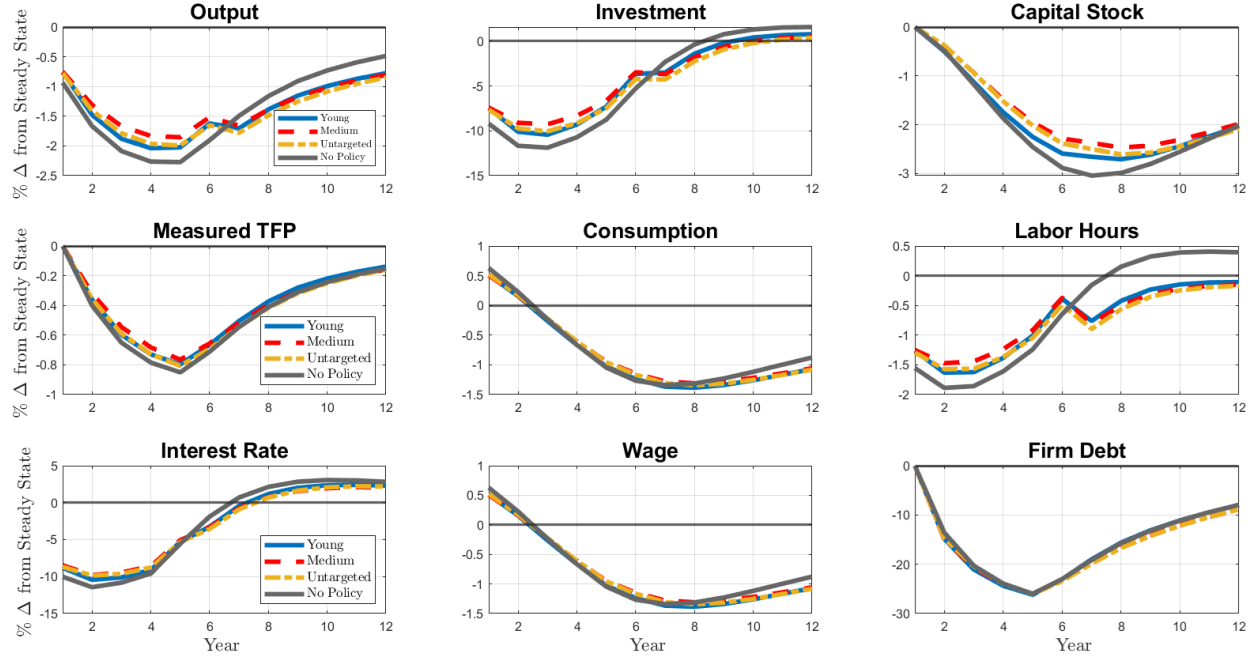
A.a Additional Figures

Figure 10: Response to Policy Targeted Excess Return to Investment



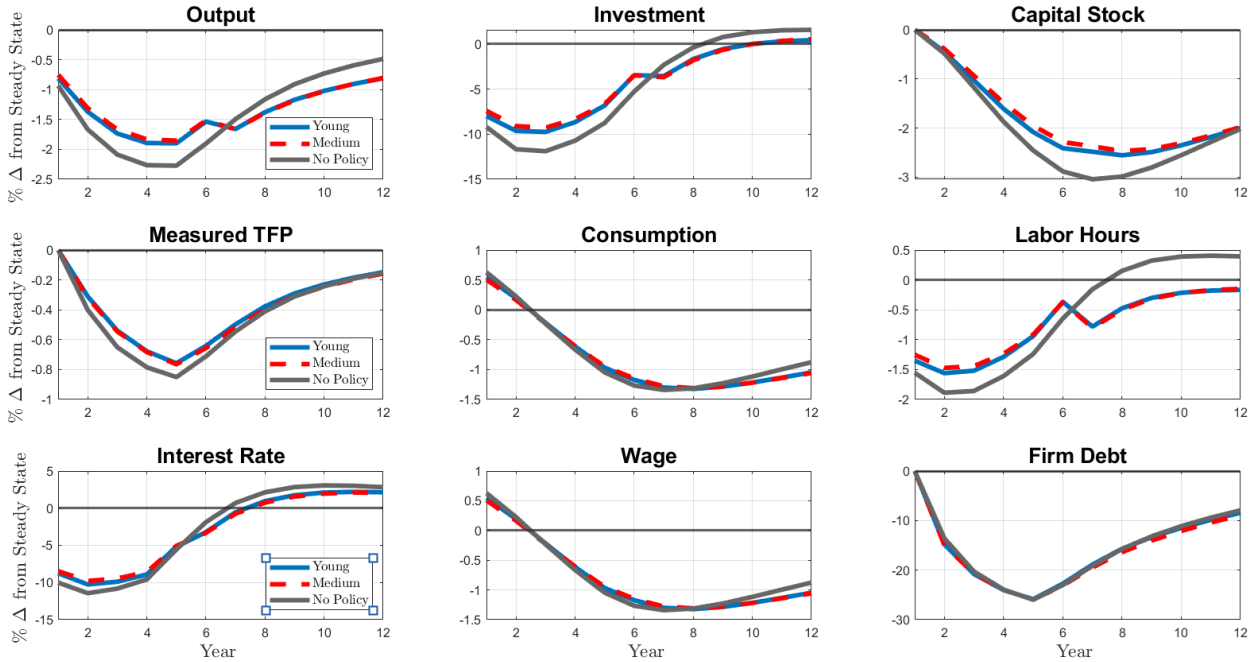
Note: Policy targets the firm(s) with the highest excess return to investment and provides debt relief until their excess return equals that of the firm with the second highest, and proceed in that fashion until all policy funds are spent. 'Free' policy indicates the policy under which the government continues to rollover its debt every period, never repaying it. Notice the wealth effect arising from such an effective policy; the household expects such an increase in their future wealth that they borrow against it, consuming more leisure, all the while maintaining high consumption.

Figure 11: Response to Size-targeted Policy



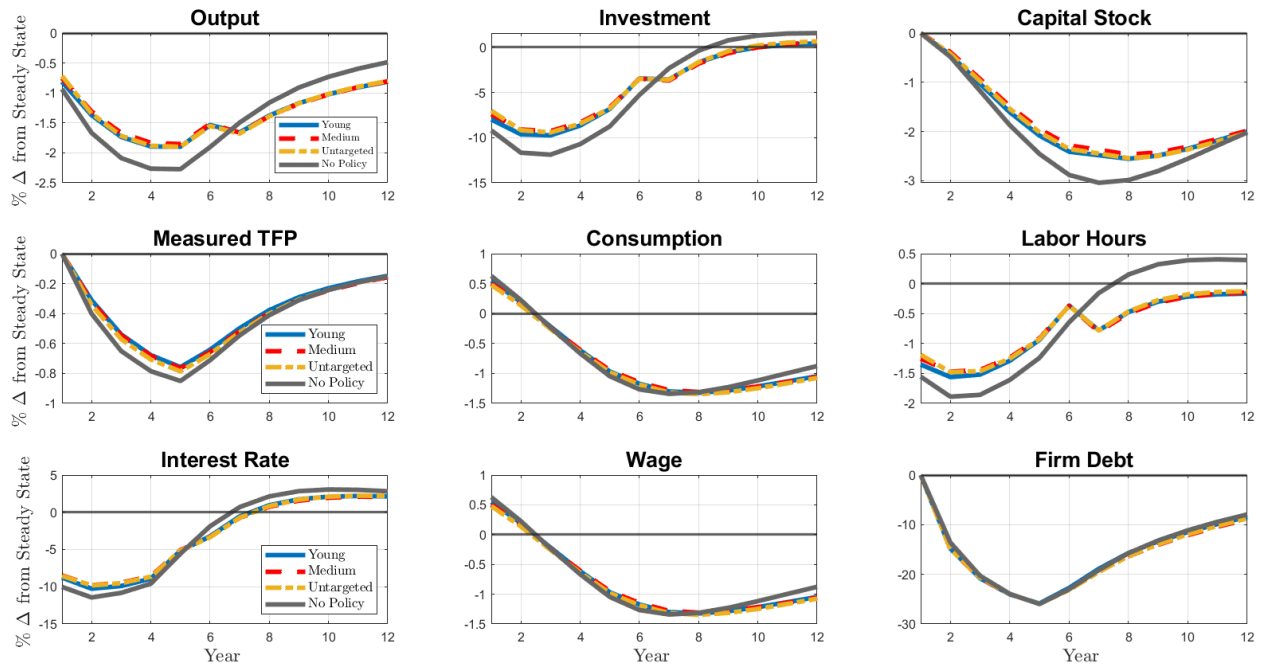
Note: Policy targets firms based on their size, indicated by Table 3.

Figure 12: Response to Age-targeted Policy



Note: Policy targets young firms (ages 0-5) compared to policy that targets medium size firms.

Figure 13: Response to Untargeted Policy



Note: Untargeted policy compared to policy that targets young medium size firms.

A.b Updating Price & Tax Vectors

1. Guess a vector of $\{\hat{\tau}\}_0^{T+1}$ and $\{\hat{C}\}_0^{T+1}$ (from this point forward, all variables are time-vectors)
 - τ' is needed for k' decision
 - \hat{C} implies a $w(S)$, $q(S)$, and $p(S)$
2. Back-solve decision rules from date T
3. Forward-solve the distribution (and find aggregates) for each t
4. Back out \tilde{C} implied by aggregate resource constraint
5. Back out $\tilde{\tau}$ implied by following:

Define: $Balance \equiv \hat{\tau}(S)w(S)N(S) + q(S)\theta' - \bar{G} - \theta - T(k, b, \varepsilon, a; S)$

Then, $\hat{\tau}(S)w(S)N(S) = Balance + q(S)\theta' - \bar{G} - \theta - T(k, b, \varepsilon, a; S)$

Define: $\Delta\hat{\tau}(S)$ as change in $\hat{\tau}(S)$ such that:

$$(\hat{\tau}(S) + \Delta\hat{\tau}(S))w(S)N(S) = \bar{G} + \theta + T(k, b, \varepsilon, a; S) - q(S)\theta'$$

– This is the increase in $\hat{\tau}(S)$ needed to set $Balance = 0$

Then: $\Delta\hat{\tau}(S) = \left((\bar{G} + \theta + T(k, b, \varepsilon, a; S) - q(S)\theta') \left(\frac{1}{w(S)N(S)} \right) \right) - \hat{\tau}(S)$

So, $\tilde{\tau} = (\hat{\tau}(S) + \Delta\hat{\tau}(S))$

6. Check guess, set $\{\tilde{\tau}\}_0^{T+1} = \{\hat{\tau}\}_0^{T+1}$ and $\{\tilde{C}\}_0^{T+1} = \{\hat{C}\}_0^{T+1}$, and repeat.