

Firm Debt Relief in Financial Downturn

Matt D'Urso

Ohio State University

January 19, 2025

Can targeting debt relief improve stabilization in a financial crisis?

Firm debt relief has been implemented in the last two U.S. recessions

Policies vary on subsets of firms targeted:

- Paycheck Protection Program (2020-2021): Smaller firms
- Auto industry bailouts (2008-2009): “Big 3” in US auto

Differing economic support:

- Smaller, younger firms rely more on debt for investment (Faff et al., 2016)
- Large firms account for greater changes in aggregates (Crouzet & Mehrotra, 2020)

Can targeting debt relief improve stabilization in a financial crisis?

- ▷ How should we build these targets?

Answering the Question

General equilibrium model with firm heterogeneity and financial frictions

Heterogeneity:

- Analyze targeting different subsets of firms
 - Shape of distribution matters for aggregation outcomes
 - Unconditional size distribution matching U.S. firms
 - *Untargeted* age-size distribution of U.S. firms
- ▷ How: Persistent idiosyncratic productivity from bounded Pareto

Financial friction:

- Collateralized borrowing
 - Slow growth for young, small firms
 - Capital misallocation
- ▷ Crisis: Shock to collateral constraint

Examined Debt Relief Policies

Debt relief policy where government pays a fraction of firm debt

- 3 targeted policies and 1 untargeted policy
1. Targeting largest excess return to investment
 - ▷ Expected discounted marginal benefit of capital investment minus cost
 - Effective policy, may not be readily available to policymakers
 2. Size-targeted policy
 - ▷ Small, medium, and large firms
 - Measures readily available to policymakers; historical precedents
 3. Age-targeted policy
 - ▷ Young, middle-age, mature firms
 4. Untargeted: all indebted firms are eligible

Preview of Results

1. Targeting firms with highest excess returns yields best results
 - Output trough 26% *less* severe (2.27% vs 1.67% fall from steady state)
2. Targeting medium size firms outperforms other size, age groups
 - More important to aggregate production over the course of a crisis than small firms
 - More hindered in their growth than large firms
3. Only excess return policy has substantial benefits over untargeted policy
 - Importance of diminishing marginal returns

Literature

- Fiscal policy to alleviate financial downturn:
 - Bianchi (2016), Jeanne, & Korinek (2020), Elenev, Landvoigt & Van Nieuwerburgh (2022), Angeletos, Collard, & Dellas (2023)
 - ▷ **My contribution:** Value of policy resources varies over nontrivial distribution of firms
- Distributional effects of policy intervention:
 - Guner, Ventura, Xu (2008), Buera, Moll, & Shin (2013), Gourio and Roys (2014), Jo & Senga (2019)
 - ▷ **My contribution:** Extend distributional analysis to the topic of debt relief in a crisis
- Transmission of shocks from financial conditions:
 - Kiyotaki & Moore (1997), Jermann & Quadrini (2012), Khan & Thomas (2013), Jo (2024)
 - ▷ **My contribution:** Analyze how debt relief may improve such conditions

Outline

1. Introduction
2. Model
 - 2.1 Firms
 - 2.2 Government
 - 2.3 Household
 - 2.4 Calibration
3. Investment decisions in steady state
4. Financial crisis and policy
5. Conclusion

Model Overview

Agents

- Firms: DRS production with capital and labor
 - Collateralized debt limit: $b' \leq \zeta k$
- Government: Collects payroll tax, borrows from HH, issues debt relief
- Representative household: Owns firms, supplies labor and loans

Aggregate state: $(\zeta, z, \mu, \theta) = S$

- Exogenous: collateral constraint (ζ) , TFP (z)
- Endogenous: distribution of firms (μ) , government debt (θ)

Firms

Firm state (k, b, ε, a) : capital, debt, idiosyncratic productivity, age

- Production technology: $z\varepsilon F(k, n)$
- Enter period with ε ; retain with probability ρ_ε
 - Probability $(1 - \rho_\varepsilon)$ draw new ε from bounded Pareto distribution
- Age dependent exit shock: $\pi_d(a)$
 - Known before production
 - Considerations for age-based policies
- Intertemporal decisions on k' and b'
 - $b' \leq \zeta k$

Firm Budget Constraint & Debt Relief

Budget:

$$D \leq x(k, b, \varepsilon, a; S) - k' + q(S)b'$$

Cash:

$$x(k, b, \varepsilon, a; S) = z\varepsilon F(k, n) - (1 + \tau(S))w(S)n(k, \varepsilon; S) + \\ (1 - \delta)k - (1 - \mathcal{J}(b)g(k, b, \varepsilon, a; S))b$$

where: $\mathcal{J}(b) = \begin{cases} 1 & \text{if } b > 0 \\ 0 & \text{if } b \leq 0 \end{cases}$

Firm Problem

Start of Period Value:

$$V_0(k, b, \varepsilon, a; S_l) = \underbrace{\pi_d(a) x(k, b, \varepsilon_i, a; S_l)}_{\text{exiting value}} + (1 - \pi_d(a)) \underbrace{V(k, b, \varepsilon, a; S_l)}_{\text{continuation value}}$$

Continuation Value:

$$V(k, b, \varepsilon_i, a; S_l) = \max_{k', b', D} \left[D + \sum_{m=1}^{N_s} \pi_{l,m}^s d_m(S_l) \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^\varepsilon V_0(k', b', \varepsilon_j, a'; S'_m) \right]$$

subject to:

$$0 \leq D \leq x(k, b, \varepsilon_i, a; S_l) + q(S_l)b' - k'$$

$$b' \leq \zeta k$$

$$\mu' = \Gamma(S_l)$$

Government (1/2)

- Total current-period debt relief: $T(k, b, \varepsilon, a; S)$
- Borrows: θ' at risk-free rate: $q(S)$
- Levies payroll tax: $\tau(S)$ when paying outstanding debt obligations: θ

Budget constraint:

$$\tau(S)w(S)N(S) + q(S)\theta' \geq \theta + \underbrace{\int g(k, b, \varepsilon, a; S)b\mu(d[k \times b \times \varepsilon \times a])}_{= T(k, b, \varepsilon, a; S)}$$

where τ , θ , and T are 0 in steady state

Government (2/2)

Government debt relief to mitigate financial recession:

- Policy eligibility determined by firm state
- Initially funded by government debt

Evolution of public debt:

$$\theta' = \frac{1}{q(S)} \left(\theta + T(k, b, \varepsilon, a; S) - \tau(S)w(S)N(S) \right)$$

When repayment begins, $\tau(S)$ must satisfy fiscal rule:

$$\theta' = (1 - \phi)\theta$$

- ϕ is the fraction of public debt paid per period

Representative Household

- Period utility $U(C, 1 - N)$, and discount factor $\beta \in (0, 1)$
- Supplies: labor for wage $w(S)$, loans at risk free rate $q(S)^{-1}$
- Implied restrictions for equilibrium prices:
 - $w(S) = \frac{D_2 U(C, 1 - N)}{D_1 U(C, 1 - N)}$
 - $d_m(S_l) = \beta \frac{D_1 U(C'_m, 1 - N'_m)}{D_1 U(C, 1 - N)}$
 - $q(S_l) = \beta \sum_{m=1}^{N_s} \pi_{l,m}^s \frac{D_1 U(C'_m, 1 - N'_m)}{D_1 U(C, 1 - N)}$
- Equilibrium decision rules $C = C(S)$, $N = N(S)$

► HH Problem

Annual Calibration (1/2)

$$U(C, 1 - N) = \ln(C) + \psi(1 - N) \quad z_{\varepsilon} F(k, n) = z_{\varepsilon} k^{\alpha} n^{\nu}$$

$$k_0 = \chi \int k \mu(d[k \times b \times \varepsilon \times a])$$

	Parameter		Target		Model
β	discount factor	= 0.960	real interest rate	= 0.040	0.041
ψ	leisure preference	= 2.140	labor hours	= 0.333	0.332
ν	labor share	= 0.600	labor share	= 0.600	0.600
δ	depreciation	= 0.069	investment/capital	= 0.069	0.069
$\frac{b_0}{k_0}$	entrant leverage	= 0.400	entrant leverage	= 0.400	0.400
α	capital share	= 0.280	capital/output	= 2.250	2.305
ρ_{ε}	maintain ε	= 0.990	std dev. i/k	= 0.337	0.358
χ	fraction of entrant K	= 0.208	avg. n_0/N	= 0.260	0.260
ζ_o	collateral fraction	= 0.981	debt/assets	= 0.372	0.372

Annual Calibration (2/2)

Size & Age/Size Distributions, Measured by Employment (Business Dynamics Statistics: 1990-2006)

Pareto bounds, $[0.497, 0.937]$, and shape, (5.5) , targeting unconditional size distribution

Employment Bins	Emp. Share	Pop. Share	
		BDS	Model
Small (1-19)	0.201	0.885	0.880
Med. (20-499)	0.319	0.112	0.101
Large (500+)	0.480	0.003	0.019

Untargeted age-size distribution:

Age	0	1	2	3	4	5	Avg.
BDS:	0.260	0.338	0.378	0.417	0.451	0.477	0.368
Model:	0.260	0.302	0.362	0.427	0.506	0.595	0.377

Outline

1. Introduction
2. Model
 - 2.1 Firms
 - 2.2 Government
 - 2.3 Household
 - 2.4 Calibration
3. Investment decisions in steady state
4. Financial crisis and policy
5. Conclusion

Constrained and Unconstrained Investment

Collateral constraint binds to varying degrees across firms

Non-binding: $k'(k, b, \varepsilon, a; S) = k^*(\varepsilon; S)$

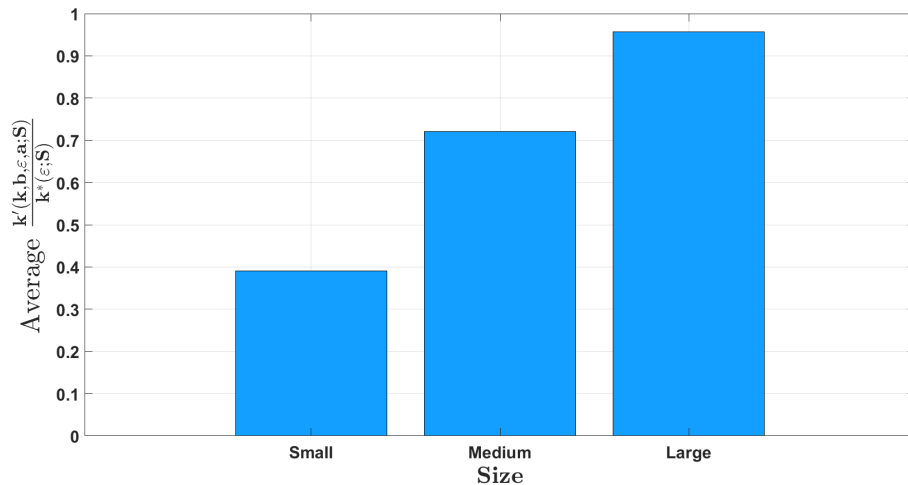
- Debt relief provides no extra investment
- Additional resources saved, $b' = \frac{k^*(\varepsilon, S) - x(k, b, \varepsilon, a; S)}{q(S)}$
 - Does offer protection from future binding constraint

Binding: $k'(k, b, \varepsilon, a; S) = x(k, b, \varepsilon, a) + q(S) \underbrace{\zeta k}_{b' = \zeta k} < k^*(\varepsilon; S)$

- Debt relief for these firms increases investment, reduces misallocation

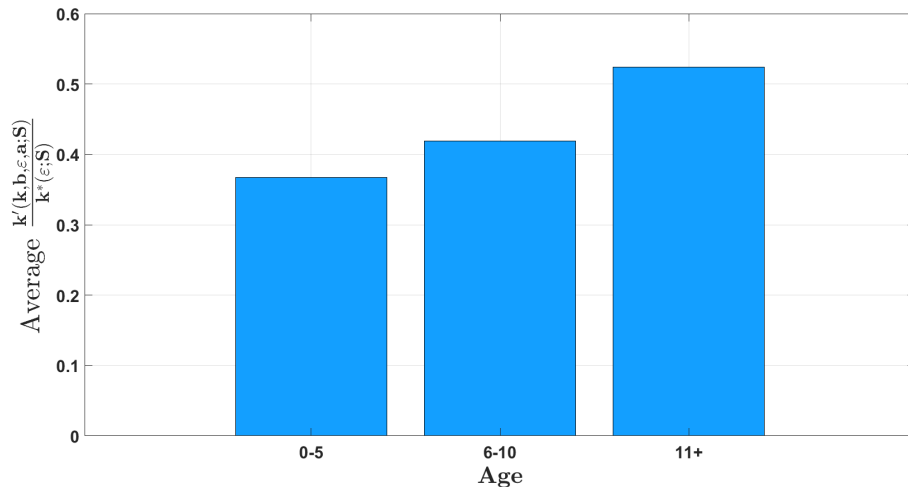
What do these firms look like?

Firm Size and Capital Efficiency



- Fraction of adopted capital $k'(k, b, \epsilon, a; S)$ over unconstrained capital $k^*(\epsilon; S)$

Firm Age and Capital Efficiency



- Fraction of adopted capital $k'(k, b, \epsilon, a; S)$ over unconstrained capital $k^*(\epsilon; S)$

Measuring Investment Inefficiency

Excess return to investment: expected discounted marginal value of investing minus cost

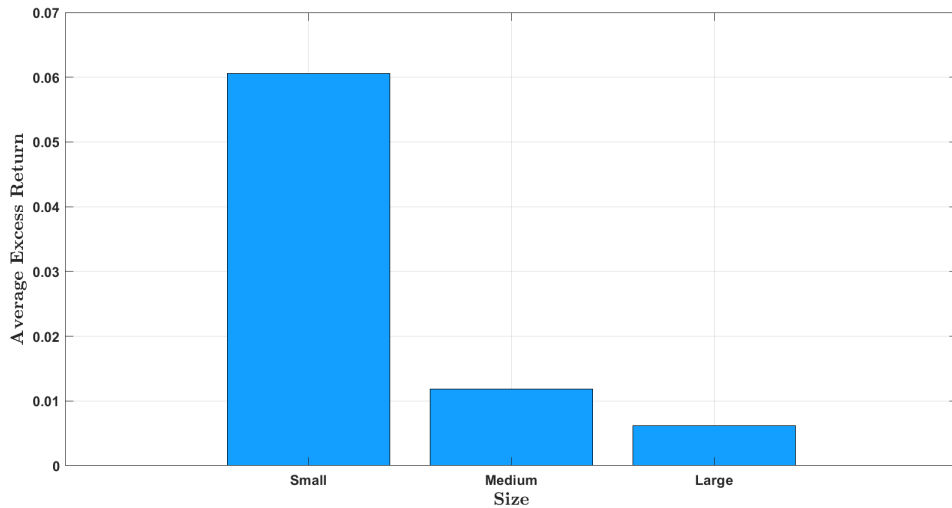
$$\mathbb{E}_{\pi^S} \mathbb{E}_{\pi^\varepsilon} \left[d_m(S_l) \left(\frac{\partial \pi(k', b', \varepsilon_j, a'; S'_m)}{\partial k'} + (1 - \delta) \right) \right] - 1$$

where π is profit [▶ Parameterization](#)

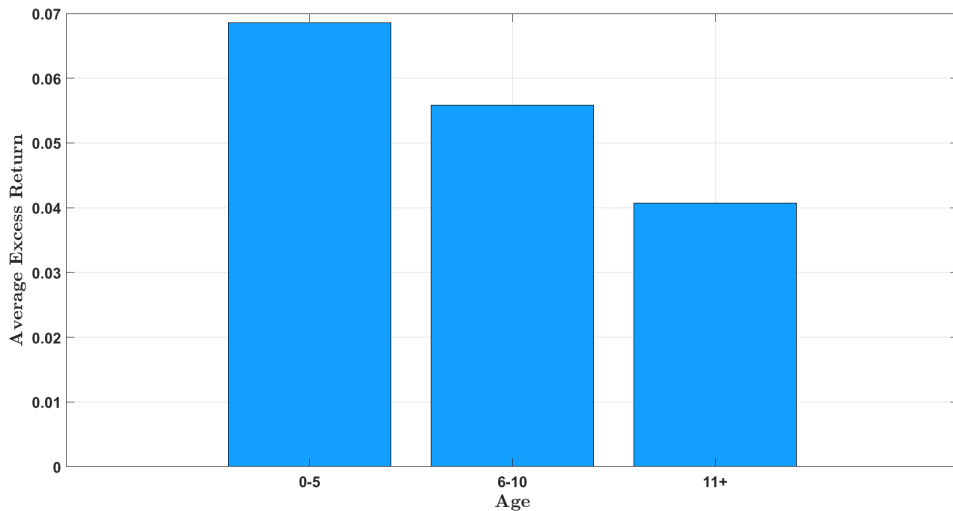
With efficient investment, excess return is 0

The further $k'(k, b, \varepsilon, a, S)$ is from $k^*(\varepsilon, S)$, the higher the return

Average Excess Return Across Firm Size



Average Excess Return Across Ages



Outline

1. Introduction
2. Model
 - 2.1 Firms
 - 2.2 Government
 - 2.3 Household
 - 2.4 Calibration
3. Investment decisions in steady state
4. Financial crisis and policy
5. Conclusion

Credit Crisis without Relief Policy

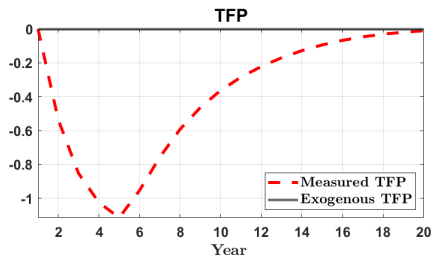
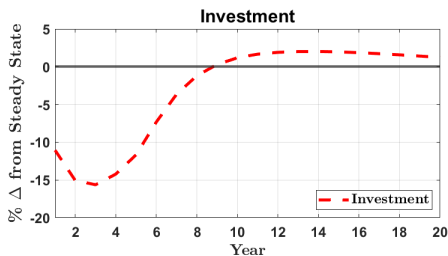
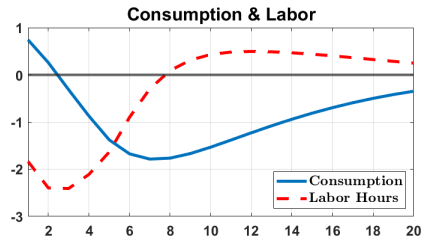
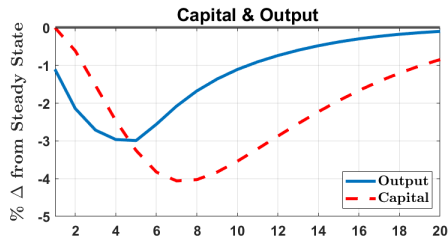
Fall in collateral parameter by 33%

- Supply shock in loanable funds market similar to 2008 (Duchin et al., 2010)
- Firms now require more capital to borrow as much as they would in steady state

Parameterization:

- Remains at low value for 4 periods, recovers 31.25%/year (Khan & Thomas, 2013)
- 26% fall in borrowing in the model, matching fall in C&I loans 2008-2011

Response to Credit Crisis: No Policy



Debt Relief in a Crisis

Policy pays fraction of outstanding debt

Common across all policies:

- Relief occurs on impact
- Total size of policy held constant at 4% of steady state output
 - PPP loan forgiveness was roughly 3.7% of U.S. real GDP by 2023
- Taxes increase in period 7 to pay public debt
 - Half-life of output recovery
 - Gov pays 5% of its debt per year

Excess Return Policy

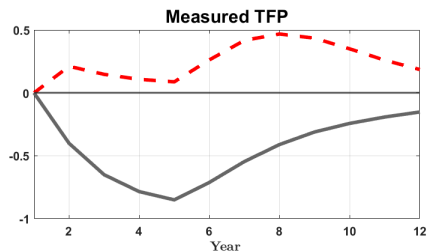
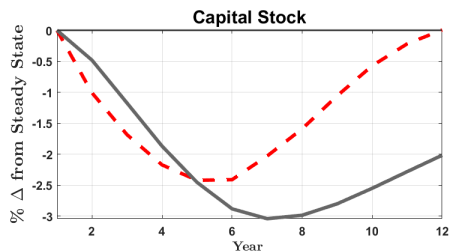
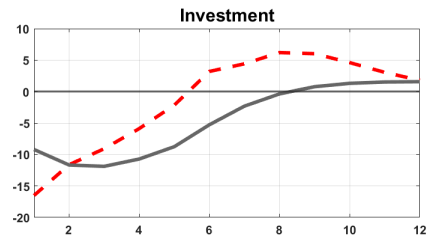
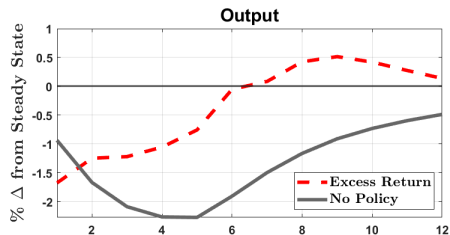
Reduce/equalize largest levels of excess return from top stair down

- Relieve debt of firm with greatest wedge to match 2nd greatest...
- Continue until policy funds exhausted

Does not relieve debt beyond what is necessary for investment

- All funds used for investment, not merely increasing share value
- Addresses concerns of debt-relief not reaching intended location (Li, 2021, Autor et al., 2022)

Policy Targeting Highest Excess Return Levels



► Additional series

Policy Targeting Small, Medium, Large firms

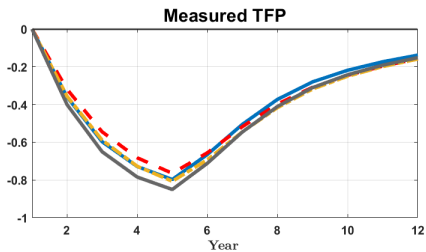
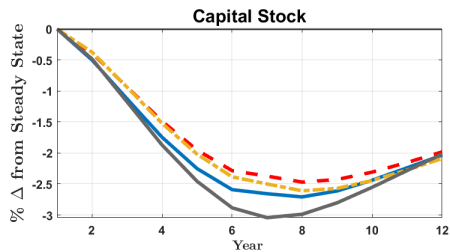
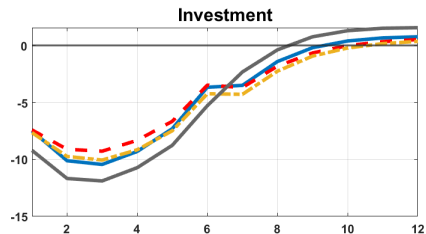
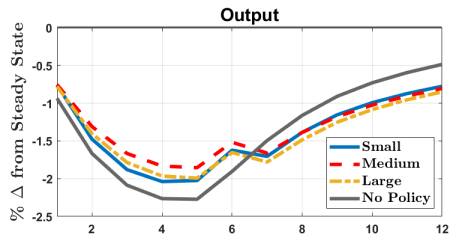
3 experiments for small, medium, large firm policy targets

- Smallest 88%, middle 10%, largest 2% of firms
- Eligibility based on model population shares [► Size Dist.](#)

In order to keep total size of policy constant, relief per firm must vary across policies:

- Fraction of b paid for **small** firms: 0.237
- Fraction of b paid for **medium** firms: 0.145
- Fraction of b paid for **large** firms: 0.096

Policy Targeting Small, Medium, Large Firms



► Additional series

Policy Targeting Young, Middle Age, Mature Firms

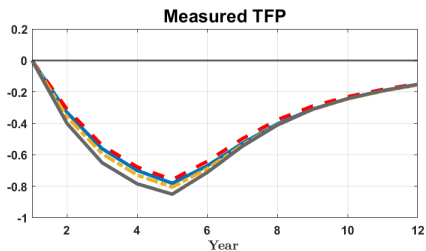
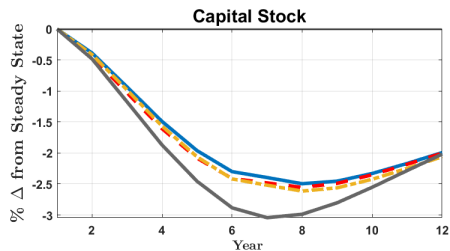
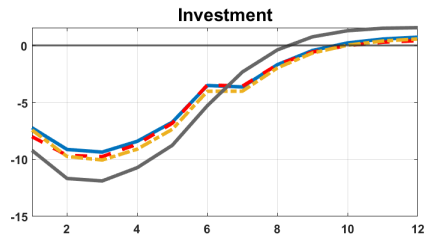
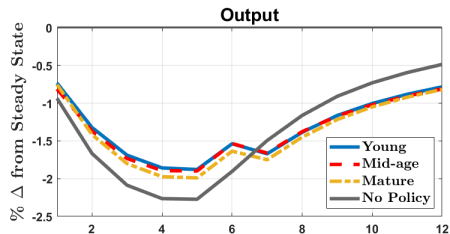
3 experiments for young, middle age, mature firm policy targets

- Age bins: $[0 - 5]$, $[6 - 10]$, $11+$

In order to keep total size of policy constant, relief per firm must vary across policies:

- Fraction of b paid for **young** firms: 0.352
- Fraction of b paid for **middle age** firms: 0.263
- Fraction of b paid for **mature** firms: 0.071

Policy Targeting Young, Middle Age, Mature Firms



► Additional series

Comparison to Untargeted Policy

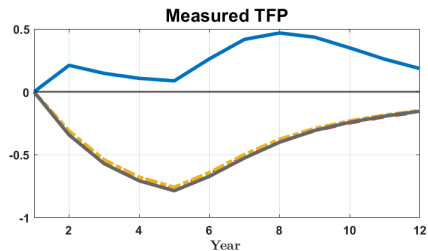
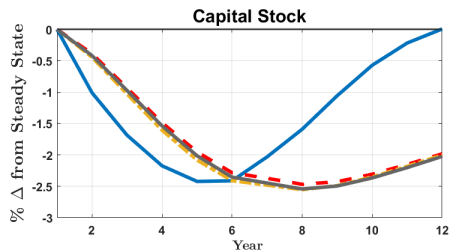
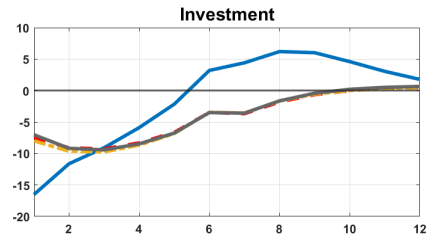
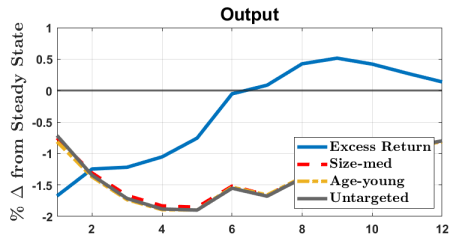
I consider one final policy: all indebted firms are eligible

- Fraction of total firm debt relieved: 0.047

Highlight effects of diminishing marginal returns

- Targeted policy can focus on key variables
- But, more concentrated policy → stronger diminishing marginal returns effects

Untargeted Policy Compared to Best Alternative Policies



Outline

1. Introduction
2. Model
 - 2.1 Firms
 - 2.2 Government
 - 2.3 Household
 - 2.4 Calibration
3. Investment decisions in steady state
4. Financial crisis and policy
5. Conclusion

Conclusion

I develop a model of heterogeneous firms and financial frictions, matching the unconditional size, and age-size, distribution of firms in the U.S., to study firm targeted debt relief in a financial crisis

I consider policies targeting firms by excess return to investment, size, and age, as well as an untargeted policy

- Excess return policy outperforms all others
 - Fall in output diminished by 26% compared to no policy
- Among remaining policies, targeting medium size firms reduces fall in aggregates most
 - More likely to become large economic players than small firms
 - Further from their efficient investment than larger firms

Only excess return policy shows meaningful improvements over untargeted policy

- Demonstrates gains from targeting are possible, but conditioning on other readily observable variables must be considered

Future Work

Endogenous entry/exit to study the implications of “cleansing” effects on debt relief

- Productive firms more likely to continue may boost effects of policy
- However, more firms overall exit than under current exit shock

Alternative tax structure to minimize slowed recovery when taxes increase

Varying arrival rate of new productivity shocks across firms

- Some firms may be more likely than others to retain their productivity
- The more persistent productivity is overall, the stricter targets should be to include productive firms

Appendix

Household Problem

Representative HH maximizes lifetime value, $W(\lambda, \kappa; S)$

- Chooses: consumption, c , labor, N , shares, λ' , bond holdings, κ'

$$W(\lambda, \kappa; S) = \max_{c, n, \lambda', \kappa'} [U(c, N) + \beta W(\lambda', \kappa'; S'_m)]$$

$$\text{st: } c + q(S_l)\kappa' + \int \rho_1(k', b', \varepsilon', a'; S'_m) \lambda'(d[k' \times b' \times \varepsilon' \times a']) \leq$$

$$w(S)n + \kappa + \int \rho_0(k, b, \varepsilon, a; S_l) \lambda(d[k \times b \times \varepsilon \times a])$$

- ρ_1 is ex-dividend price of a share, ρ_0 is dividend-inclusive value of a share, $\Gamma(\mu)$ is known

► back

Market Clearing

$$\text{Capital : } K = \int k \mu(d[k \times b \times \varepsilon \times a])$$

$$\text{Labor : } N = \int n(k, \varepsilon) \mu(d[k \times b \times \varepsilon \times a])$$

$$\text{Output : } Y = \int z \varepsilon F(k, n(k, \varepsilon)) \mu(d[k \times b \times \varepsilon \times a])$$

$$\text{Firm-debt : } B = \int (b | b > 0) \mu(d[k \times b \times \varepsilon \times a])$$

$$\begin{aligned} \text{Consumption : } C = & Y - (1 - \pi_d(a)) (K' - (1 - \delta) k) \\ & + \pi_e ((1 - \delta) k - k_0) \end{aligned}$$

Distribution

The distribution of firms is denoted by measure μ , defined on the Borel algebra, \mathcal{S} , generated by the open subsets of the product space, $\mathbf{S} = \mathbf{K} \times \mathbf{B} \times \mathbf{E} \times \mathbf{A}$.

$\forall (A, \varepsilon_j) \in \mathcal{S}$ defines Γ , where $\chi(k_0) = \{1 \text{ if } (k_0, 0) \in A; 0 \text{ otherwise}\}$

$$\begin{aligned} \mu'(A, \varepsilon_j) = & \\ (1 - \pi_d(a)) \int_{\{(k, b, \varepsilon_i, a) | (g^K(k, b, \varepsilon_i, a; s, \mu), g^B(k, b, \varepsilon_i, a; s, \mu)) \in A\}} & \pi_{ij} \mu(d[k \times b \times \varepsilon_i \times a]) \\ + \pi_e \chi(k_0) H(\varepsilon_j) & \end{aligned}$$

Excess Return to Investment

Given the parameterization of the model, marginal benefit of capital investment should equal marginal cost of 1:

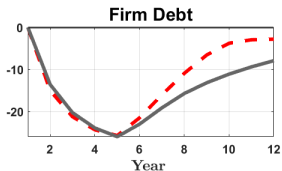
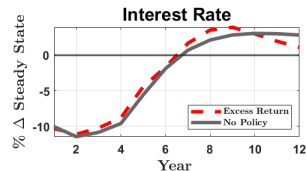
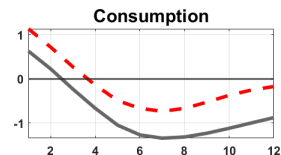
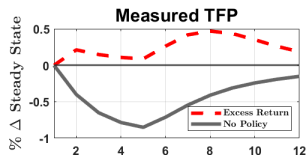
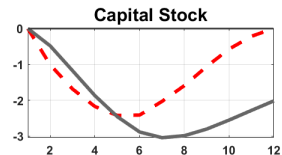
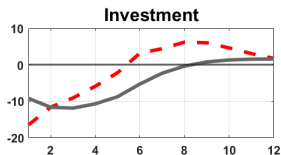
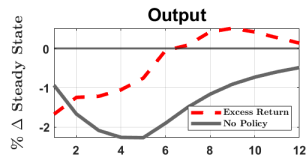
$$\frac{\alpha}{1-\nu} \frac{\beta}{p(S)} \left(\sum_{m=1}^{N_s} \pi_{l,m}^s p(S'_m) \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^\varepsilon k'^{\frac{\alpha+\nu-1}{1-\nu}} \left[z_m \varepsilon_j \left(\frac{\nu z_m \varepsilon_j}{(1+\tau(S)) w(S)} \right)^{\frac{\nu}{1-\nu}} - (1+\tau(S)) w(S) \left(\frac{\nu z_m \varepsilon_j}{(1+\tau(S)) w(S)} \right)^{\frac{1}{1-\nu}} \right] + (1-\delta) \right) = 1$$

where, $p(S) = \frac{\partial U(C, 1-N)}{\partial C}$

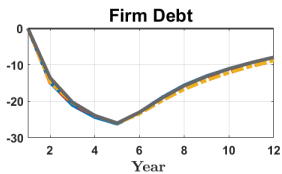
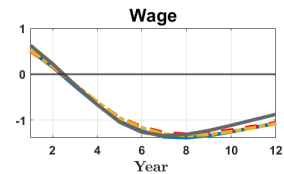
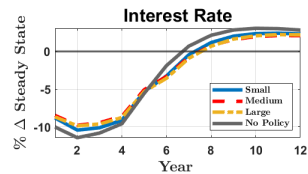
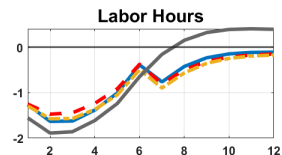
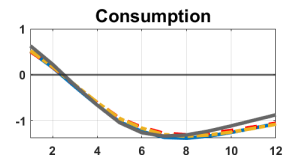
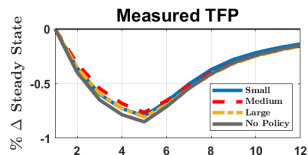
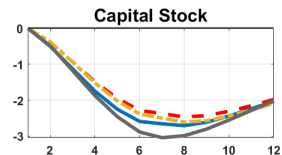
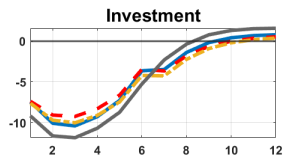
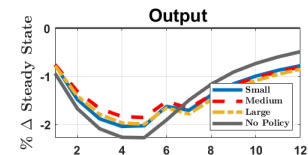
With insufficient investment, LHS > 1

[▶ back](#)

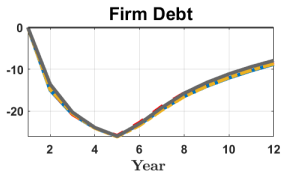
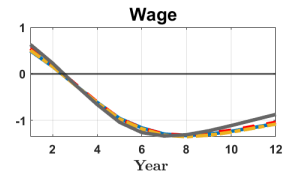
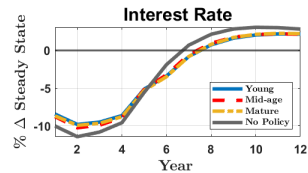
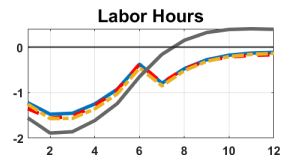
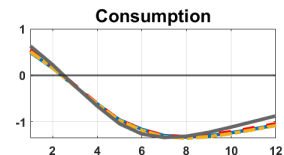
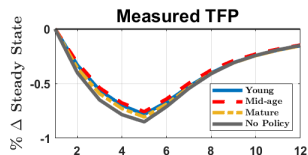
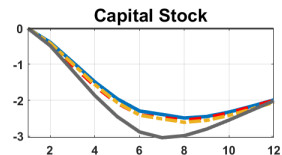
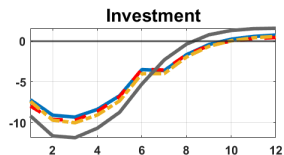
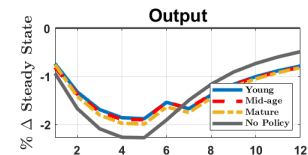
Extra Series - Excess Return Target



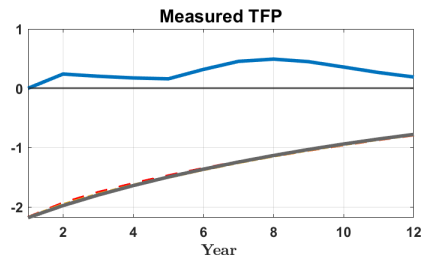
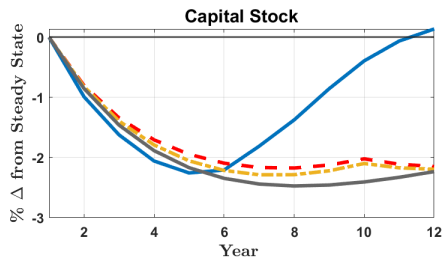
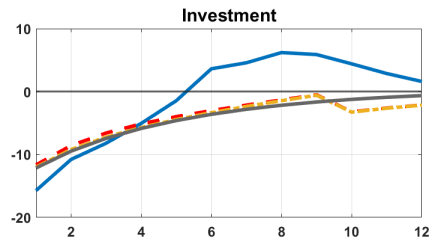
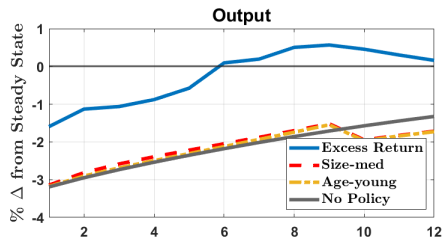
Extra Series - Size Targets



Extra Series - Age Targets



Selected Policies Following TFP Shock



- 2% shock with persistence of 0.909

Firm Exit Rates by Age (BDS, 1990-2006)

Age:	1	2	3	4	5	6-10	11+
$\pi_d(a)$:	0.2478	0.1640	0.1356	0.1174	0.1062	0.0840	0.0655

Firm entrant rate = 10.6%

- Entrant rate selected to keep mass of firms at 1

[▶ back](#)

Perfect Foresight Solution -1

1. Guess a vector of $\{\hat{\tau}\}_0^{T+1}$ and $\{\hat{C}\}_0^{T+1}$
 - τ' is needed for k' decision
 - \hat{C} implies a $w(S)$ and $q(S)$
2. Back-solve decision rules from date T
3. Forward-solve the distribution (and find aggregates) for each T
4. Back out \tilde{C} implied by aggregate resource constraint
5. Back out $\tilde{\tau}$ implied by following:

Perfect Foresight Solution - 2

Define: $Balance \equiv \hat{\tau}(S)w(S)N(S) + q(S)\theta' - \bar{G} - \theta - T(\Theta, S)$

Then, $\hat{\tau}(S)w(S)N(S) = Balance + q(S)\theta' - \bar{G} - \theta - T(\Theta, S)$

Define: $\Delta\hat{\tau}(S)$ as change in $\hat{\tau}(S)$ such that:

$$(\hat{\tau}(S) + \Delta\hat{\tau}(S))w(S)N(S) = \bar{G} + \theta + T(\Theta, S) - q(S)\theta'$$

- This is the increase in $\hat{\tau}(S)$ needed to set $Balance = 0$

Then: $\Delta\hat{\tau}(S) = \left((\bar{G} + \theta + T(\Theta, S) - q(S)\theta') \left(\frac{1}{w(S)N(S)} \right) \right) - \hat{\tau}(S)$

So, $\tilde{\tau} = (\hat{\tau}(S) + \Delta\hat{\tau}(S))$

6. Check guess, set $\{\tilde{\tau}\}_0^{T+1} = \{\hat{\tau}\}_0^{T+1}$ and $\{\tilde{C}\}_0^{T+1} = \{\hat{C}\}_0^{T+1}$, and repeat