Firm Debt-relief in Financial Downturn

Latest Version Available Here

Matt D'Urso *

October 16, 2024

Abstract

I study the aggregate implications of firm-specific debt-relief in a dynamic, stochastic general equilibrium model. Firms face persistent idiosyncratic risk and financial frictions that give rise to an endogenous distribution over capital, debt, and productivity, while leading to capital misallocation and life-cycle growth effects. This creates a role for policy: government can borrow on behalf of firms that are financially constrained in order to provide debt-relief. Compared to no policy in a recession, targeting firm-level excess returns to capital investment improves aggregate outcomes by $0.2\% - 0.7\%$, with investment improving by 3%, and the half-life of capital's recovery being reached 2 years earlier. It also ensures policy funds will be used for investment, not kept as retained earnings. However, this may not be a readily observable policy target, so I consider targets based on firm age and size, two easily measured variables that correlate with excess returns. Both improve outcomes over an untargeted policy, but not as much as the excess returns policy. Finally, once the government has amassed debt in order to fund policy, it is better to gradually pay it off, rather than to sharply raise taxes and pay down debt quickly.

*The author would like to thank their advisor, Julia K. Thomas, for her generous guidance and support, as well as Aubhik Khan and Kyle Dempsey for their invaluable tutelage. The author would also like to thank Ben Lidofsky and Rohan Shah for their invaluable tutelage. All errors are my own.
Department of Economics, The Ohio State University. Email: durso.18@osu.edu

I Introduction

Does firm-targeted debt-relief dampen financial downturn? Around the time of economic downturn, the idea of debt-relief, or "bailouts," becomes more common[1] in political and economic discourse. The policies are usually idiosyncratic, varying in size, scope, and may also allow for changes in market structures. In recent U.S. history, debt relief policies have been implemented during the Great Recession and the COVID-19 pandemic. Motivation for policy during the Great Recession was driven by concerns of larger firms, specifically the "Big 3[2]" in U.S. Auto, laying off workers and a negative wealth effect spreading throughout the economy. On the other hand, the Paycheck Protection Program (PPP) established during the pandemic was chiefly concerned with small business, with the main target being firms with fewer than 500 employees[3]. Besides determining which side of the size distribution to target, it remains to be seen if size targets provide the best readily observable variable to approximate the most efficient policy.

To answer the question, I build a dynamic stochastic general equilibrium model with persistent firm-level heterogeneity and financial frictions. The heterogeneity creates differences in optimal investment decision rules, while financial frictions can constrain some of these decisions by limiting firms' access to debt markets. The result is a dynamic distribution of firms, which provides a breadth of firm-level variables to target for policy analysis. I then use this model as a quantitative laboratory for policy experiments to study which debt-relief policy targets lead to the greatest improvements in aggregate outcomes. I find that debt-relief policy that targets firm-level excess returns to capital investment provides the best results when comparing recession troughs across policy regimes. Calibrated to the U.S. economy, most aggregate variables are $0.3-0.7\%$ closer to steady state values under this policy, while investment is 3% closer, compared to a scenario without relief. Consequently, the aggregate

[1] Blau et al. (2013) measure this with two approximations for political engagement, lobbying and connections maintained.

[2] General Motors, Ford Motor Company and Chrysler, at the time.

[3] The second of three "draws" targeted 300 employees. Other financial targets were also implemented such as limiting net worth and hedge funds.

capital stock reaches its no policy half-life[4] 2 years sooner than the no policy regime. Excess returns targeting inherently builds in a condition ensuring that relief resources are used for investment and are not kept as retained earnings, addressing concerns of debt-relief not reaching its intended location (Li (2021), Autor et al. (2022)). However, these benefits are not without costs. In my model, the government issues bonds to pay for debt-relief and, eventually, must raise taxes to pay off public debt, potentially inducing its own downturn.

Realizing that excess returns may be an measurably difficult target, I consider two more directly observable variables: size-based and age-based targeting, along with comparisons to an untargeted policy where all indebted firms are eligible. I find that age targeting provides better outcomes per dollar spent than (both large and small firm) size targeting, and both outperform untargeted policy. However, neither dampens a credit driven recession as much as targeting excess returns, as this policy can zero-in on the capital inefficiencies present.

In the model environment, firms invest in their capital stock through cash-on-hand and issuing bonds. However, they face financial frictions in the form of a collateralized borrowing and a lower bound on dividends. The collateral constraint limits firms' access to debt, and therefore investment, while the dividends restriction gives the collateral constraint its 'bite.' If a firm cannot afford to finance investment and they cannot provide enough collateral for a loan, this firm may not make up the difference by issuing sufficiently negative dividends[5]. This modeling choice allows me to capture some of the effects described in the corporate finance literature on the preference of debt financing over equity financing[6] in a tractable manner, while still focusing a quantitatively robust analysis on debt channels.

Even in normal economic times, these conditions slow investment and lead to market inefficiencies. Financial shocks exacerbate these inefficiencies, leading to a deterioration of the aggregate capital stock. The recovery path is a function of the distribution of firms,

[4]The level of capital corresponding to the point where it is halfway back to its steady state value, during a recession without relief policy.

[5]See Bianchi (2016) as an example of setting a lower bound on dividends as a way to balance tractability with quantitative robustness.

[6]Shleifer and Vishny's 1997 research survey and Brav et al.'s 2005 managerial survey show that some firms will choose external capital or even give up positive NPV investments in order to not decrease dividends.

which is inherently a slow moving object, and if capital requires "*time-to-build*," the recovery is slowed further, as firms are not able to update their capital immediately.

These aforementioned conditions provide a role for policy. Debt-relief that relaxes a firm's budget constraint affords them more investment, which dampens the impact of credit shocks, potentially reducing the severity and the duration of a recession. In the model, policy is conducted by a government, which issues bonds in order to fund debt-relief. Taxes are then raised in order to pay off its debt. Here, the government's role is that of one in Woodford (1990), to borrow on behalf of agents that are financially constrained. The benefit of extended borrowing capacity depends on to whom it is extended. Thus, it is important to consider these potential targets to discover if targeted policy is worth the cost of higher public debt.

Borrowing from the PPP, I model an alternative policy based on employment share eligibility, setting a threshold that corresponds to 500 employees as the target. Smaller firms are more likely to be farther from their ideal capital level than larger firms and thus face higher marginal returns to investment. On the other hand, larger firms are responsible for a greater share of aggregate economic activity than smaller firms[7].

Age-based targeting is based on firms' life-cycles. It takes time for young firms to grow and accumulate capital to reach their optimal capital level. The farther away they are from their optimal capital level, the higher their return on investment will be, as their marginal product is higher. These firms should be investing more[8]. The financial frictions in the model that limit access to debt, and thus investment, will slow down firm growth, endogenously producing in these life-cycle characteristics. I consider an age target of 5 years, formally

[7] Banz (1981) find that firm size is negatively correlated with risk adjusted returns; much work has been done discussing this correlation and whether or not causality exists (Berk, 1997; Asness et al., 2018). Gertler and Gilchrist (1994) state "While size per se may not be a direct determinant, it is strongly correlated with the primitive factors that do matter," suggesting policy would be better off targeting smaller firms. Crouzet and Mehrotra (2020) conclude that cyclical behavior in large firms firms may have implications for aggregate fluctuations, while small firms' behavior may not. This, along with he positively skewed employment distribution in the United States shown in the Business Dynamics Statistics (BDS) data may suggest the opposite targeting is appropriate.

[8] Faff et al. (2016) find that on average, younger firms invest more and also carry more debt, despite facing higher spreads (Amin et al., 2023) than older firms, indicating returns may be higher as well.

borrowing this definition from Goetz and Stinson's (2021) Census report. They define a young firm as "firms with positive employment for five years or less"[9].

Specificity of targets must also be considered. Concentrating public funds on important variables sounds appealing at fist, but in a decreasing returns to scale environment, keeping the marginal value of a "*bailout-unit*" high may require less specific targets. Furthermore, keeping resources concentrated on a small subset of firms may lead to misallocation in and of itself. If firms receive more resources than what would be required for optimal investment, the rest will may be stored as savings when it could have been used as investment by another firm. Moving to the other extreme and eliminating targets all together can lessen these issues, but at the cost of foregoing focusing on important variables. Thus an untargeted policy is needed for comparison.

The remainder of the paper is prepared as follows. Section II provides a review of the literature. Section III describes the model and provides a brief analysis. Section IV begins the quantitative exercise with calibrations. Section V discusses efficient policy, while section VI discusses implementable alternatives. Section VII concludes.

II Literature Review

There is a large literature on fiscal policy improving macroeconomic outcomes. Bianchi (2016) studies a model of systemic and idiosyncratic debt-relief, where systemic policy is a function of only the aggregate state, while idiosyncratic policy takes firm-level variables as arguments, as well. Debt-relief is intended to alleviate working capital constraints, a la Jermann and Quadrini (2012). Under an idiosyncratic policy regime, the potential for debt-relief enters into the firm's decision rules, creating a potential moral hazard problem. Firms are incentivized to over-borrow and may be unable to cover the cash flow mismatch between their outlays and revenue next period. In this setting, systemic policy outperforms idiosyncratic policy.

[9] Babina et al. (2019) find similar employment dynamics between firms of age 5 and firms of ages 4 and 6

This paper goes beyond the identical firm framework, bringing more depth to idiosyncratic policy. Indeed, I find the opposite result; targeted policy outperforms untargeted policy. Given the non-degenerate distribution of firms and the specific targets tested, most eligible firms are already at (or near) their borrowing limit anyway. Thus even after the policy takes effect, they are not responsible for more debt than before.

In this spirit, I turn to Buera et al. (2013). Agents are given the choice to be laborers or entrepreneurs and the government targets high productivity, low wealth agents with a free capital policy. While there are short run benefits of subsidizing productive entrepreneurs, in the long run that productivity will eventually decay. Long run costs accrue from policymakers being unable to adjust the original subsidy policies (for a myriad of potential reasons) and end up funding unproductive entrepreneurs. I depart from Buera et al. in a few ways. First, this work concerns itself with potential stimulus during recession. Policy is intended to counteract the effects of failing credit markets. Although many well-intended policies have inefficient inertia, I study those that exist only for a short period, as in the motivating examples. Moreover, these policies do not distribute free capital. If a firm receives debt-relief, the still must pay for investment. Related to this, the interest rate will endogenously change with the aggregate state, rather than being set by a large open economy as in the two aforementioned papers.

Jo and Senga (2019) study fiscal responses to credit market imperfections. As they point out, there are important general equilibrium effects to consider intrinsic in extended access to debt, and therefore capital. Increased factor prices make investment more difficult for untargeted firms. Moreover, these prices effect entry and continuation decisions that are made on extensive margins. The general equilibrium effect may dominate the benefits of policy depending on the idiosyncratic shock process assumed. While their paper focuses on long-run resource allocation through steady state comparisons, this paper will abstract away from endogenous entry and exit and focus on transitional dynamics.

As mentioned previously, in this environment, and as in Woodford (1990), the government's role is to extend its access to debt markets to firms that are financially constrained. Holmström and Tirole (1998) expand this idea in the presence of liquidity shocks. Angeletos et al. (2023) solve, non-linearly, for optimal dynamics of tax and public debt in the presence of collateral constraints. In order to focus more on the relevant asset distribution and gains from targeting, optimal debt levels will be left to future work. I do, however, corroborate their finding that tax smoothing, from slower debt repayment, provides smoother consumption over time.

Finally, this work leans on literature studying financial frictions, namely collateralized borrowing as in Khan and Thomas (2013). The collateralized borrowing allows me to replicate important firm life-cycle aspects that are relevant for policy analysis. Young firms grow as their ability to borrow rises[10]. Including persistent, yet uncertain, idiosyncratic productivity shocks means optimal (unconstrained) capital choices will vary across firms, and change dynamically, creating a richer distribution of firms over excess returns. This allows me to explore different dimensions of policy targeting.

III Model

III.a Firms

The model economy is populated with a unit mass of firms that use labor, n, and predetermined capital, k, to produce a homogeneous consumption good through a decreasing returns to scale, twice differentiable, production function, $y = z\varepsilon f(k,n)$. Each firms' productivity is subject to both aggregate, z, and idiosyncratic, ε, shocks. I assume both z and ε follow their own respective Markov processes, such that $z \in \mathbf{Z} \equiv \{z_1 \dots z_{N_z}\}$, where $\Pr(z' = z_g \mid z = z_f) = \pi^z_{f,g}$ and $\varepsilon \in \mathbf{E} \equiv \{\varepsilon_1, \dots \varepsilon_{N_\varepsilon}\}$, where $\Pr(\varepsilon' = \varepsilon_j \mid \varepsilon = \varepsilon_i) = \pi^\varepsilon_{i,j}$. $\sum_{g=1}^{N_z} \boldsymbol{\pi}^\varepsilon_{f,g} = 1$ and $\sum_{j=1}^{N_\varepsilon} \boldsymbol{\pi}^z_{i,j} = 1$ both hold.

[10] See Albuquerque and Hopenhayn (2004) for results driven by limited enforceability, Clementi and Hopenhayn (2006) for private information, and Bernanke and Gertler (1989) for agency costs.

In order to fund investments, i, for next period's capital stock, k', firms use their revenues and issue a risk-free bond, b', and b represents their current financial assets[11]. Thus, a given firm, (k, b, ε), is defined by its current capital stock, $k \in \mathbf{K} \subset \mathbf{R}_+$, debt level, $b \in \mathbf{B} \subset \mathbf{R}$, and idiosyncratic shock realization. They face collateralized borrowing constraints and may not borrow more than a given fraction, ζ, of their current capital stock[12] each period. Collateralized borrowing is a source of aggregate uncertainty that captures the health of financial markets. A higher ζ reflects lenders' positive beliefs in the market, thus they are willing to lend more to firms. On the other hand, a lower ζ reflects just the opposite. The parameter $\zeta \in \boldsymbol{\zeta} \equiv \{\zeta_1, \ldots, \zeta_{N_\zeta}\}$ is stochastic and follows its own Markov process, where $\Pr(\zeta' = \zeta_k \mid \zeta = \zeta_h) = \pi^{\zeta}_{h,k}$ and $\sum_{k=1}^{N_\zeta} \boldsymbol{\pi}^{\zeta}_{h,k} = 1$.

As firms age, their borrowing capacity increases with capital accumulation. In order to prevent the whole economy from outgrowing financial conditions, each period a fraction, π_e, of firms are exogenously forced to exit the market permanently at the end of a period. An exiting firm is replaced by an entrant with a capital stock of k_0, a total debt value of b_0, and idiosyncratic productivity, ε_0 drawn from the ergodic distribution implied by π_{ij}, from an infinite pool of potential entrants to ensure the mass of firms will not change.

At the start of each period, firms are made aware of their idiosyncratic, ε, and aggregate productivity, z, the financial state, ζ, and whether or not they will exit the market. There is no penalty for exiting the market; firms know if they will exit and, if so, simply optimize their labor choice given their remaining capital stock and productivity, produce output, and settle any outstanding financial obligations, which had an upper bound given from last period's financial state. They do not borrow or invest. This timing is crucial, as it insures all debt is risk-free.

Along with government debt, θ, and government debt repayment rate, ϕ, to be described in the section below, I can now characterize the state of the economy. The distribution of

[11] The convention used is $b > 0$ represents positive bonds issued (*in debt*) and $b < 0$ represents net savings.

[12] Many papers study future capital Kiyotaki and Moore (1997), or earnings-based Lian and Ma (2021) collateral. This will not meaningfully change the analysis, subject to a properly calibrated ζ.

firms over (k, b, ε) is described by the probability measure μ defined on the Borel algebra, $\mathcal{S}$, generated by the open subsets of the product space, $\mathbf{S} = \mathbf{K} \times \mathbf{B} \times \mathbf{E}$. The distribution of firms evolves over time according to the mapping, Γ, from the current aggregate state, $\mu' = \Gamma(z, \zeta, \theta, \mu)$. This process is known to all agents in the economy. For expositional ease, let the stochastic components of the aggregate state be $s = (z, \zeta)$ and its transition probabilities be $\pi^s_{l,m} = \Pr(s' = (z, \zeta)_m \mid s = (z, \zeta)_l)$, where each $\pi^s_{l,m}$ is derived from the transition probabilities $\pi^\zeta_{h,k}$ and $\pi^z_{f,g}$. Let the shocks realized in the aggregate state s_l be z_f and ζ_h, for each $l = \{1, \ldots, N_s\}$, where $N_s = N_z N_\zeta$. Finally, let S represent the entire aggregate state, $S_l = (s_l, \theta, \phi, \mu)$[13].

III.a.1 The Firms' Problem

With the aggregate state described, I now begin to characterize a given firm's problem. Their cash-on-hand is standard, with the addition of a payroll tax, $\tau(S)$ and debt-relief, $g(\Theta; S)$. Capital depreciates at rate δ and takes 1 period to build. Therefore, their cash-on-hand is their output, less taxed wage bill, plus undepreciated capital stock, minus debt obligations:

$$x(k, b, \varepsilon_i; S_l) = z_l \varepsilon_i F(k, n) - \boldsymbol{(1 + \tau(S_l))} w(S_l) n + (1 - \delta) k - \boldsymbol{(1 - g(\Theta; S_l))} b \qquad (1)$$

where $(1 - g(\Theta; S))$ is the fraction of debt the firm is still responsible for and Θ is a vector of firm-level variables that may be targeted for policy. This style of debt relief, where the government pays for a fraction of firm debt is akin to that of Bianchi (2016).

When entering a period, the firm does not yet know if it will be forced to exit or be allowed to continue. Before making any decisions, its value is then given by $v_0(k, b, \varepsilon, S)$, the probability-weighted sum of exiting with cash-on-hand, or continuing on to the next period

[13] Hereafter, I suppress the indices for the current exogenous aggregate state l and idiosyncratic productivity, i, except where necessary

and receiving continuation value, $v(k, b, \varepsilon, S)$:

$$v_0(k, b, \varepsilon_i; S_l) = \pi_e x(k, b, \varepsilon_i; S_l) + (1 - \pi_e) v(k, b, \varepsilon_i; S_l) \tag{2}$$

Once the period starts and firms are made aware of their continuation status, they choose their dividend payments, D, to borrow or save b', at the loan discount factor $q(S)$, and capital for next period, k', while not exceeding their collateral constraint of $b' \leq \zeta_l k$. The familiar law of motion for capital is $k' = i + (1 - \delta)k$. Taking as given the transition conditions of the aggregate and idiosyncratic states, the continuation value is given by:

$$v(k, b, \varepsilon_i; S_l) = \max_{k', b', D} \left[D + \sum_{m=1}^{N_s} \pi^s_{l,m} d_m(S_l) \sum_{j=1}^{N_\varepsilon} \pi^\varepsilon_{i,j} v_0(k', b', \varepsilon_j; S'_m) \right] \tag{3}$$

subject to:

$$\begin{aligned} &\underline{D} \leq D \leq x(k, b, \varepsilon_i; S_l) + q(S_l)b' - k' \\ &b' \leq \zeta_l k \\ &\mu' = \Gamma(S_l) \end{aligned} \tag{4}$$

where $d_m(S_l)$ is the stochastic discount factor on the next period's value of the firm, $\underline{D}$ is the lower bound on dividends paid out to the household, and $q(S)$ is the risk-free loan discount factor.

Notice there are no inter-temporal considerations for labor demand and the wage bill is paid out of production, meaning all firms with the same (k, ε) will have the same labor demand. Thus, the firm's labor decision rule is $g_N(k, \varepsilon, S)$. Following from this, a firm's output is $y(k, \varepsilon, S)$. Capital and debt decisions rules require arguments from the full individual state; they are $g_K(k, b, \varepsilon, S)$ and $g_B(k, b, \varepsilon, S)$, respectively.

III.b Government

The government is endowed with two sets of resources: revenue from a payroll tax, and funds borrowed, θ', from the household at the risk-free rate[14]. It uses these to pay for government expenditures, $G(S)$, public debt, θ, and private debt-relief, $T(\Theta;S)$, which is the sum of all relief provided:

$$T(\Theta;S) = \int_{\mathbf{S}} g(\Theta;S) b \mu(d[k \times b \times \varepsilon]) \tag{5}$$

Bonds issued by the government are either repaid through tax revenues, or rolled over to the next period at the risk-free rate. Public debt is a type of consol debt, where the speed at which it is paid is governed by the parameter ϕ, the fraction of debt paid. Conditional on making a payment, $(1-\phi)$ is the fraction of debt rolled over to next period. Finally, expenditures, $G(S)$, are *not* valued by households, *nor* are they used in production. The government's budget constraint is:

$$\tau(S)w(S) \int_{\mathbf{S}} [g_N(k,\varepsilon;S)] \mu(d[k \times b \times \varepsilon]) + q(S)\theta' \geq G(S) + \theta + T(\Theta;S) \tag{6}$$

where θ' is determined by:

$$\theta' = \begin{cases} (1-\phi)\theta & \textit{make payment} \\ \frac{1}{q(S)}\big(G(S) + \theta + T(\Theta;S) - \tau(\phi,S)w(S)N(S)\big) & \textit{no payment} \end{cases} \tag{7}$$

The functionality of this set up is that it pins down a singular path of taxes over time for a given ϕ. This allows for the analysis of different repayment horizons, providing a link to previous literature on tax smoothing.

[14] I assume the sovereign cannot default.

III.c Household

There is a representative household endowed with ownership of the firms and a unit of time that may be divided between labor and leisure. Excess revenues are transferred to the household in a lump-sum dividend payment. The household supplies labor to the firms in exchange for a wage, transfers resources intertemporally by purchasing bonds, κ', and shares, λ', and chooses consumption, c, to maximize its lifetime value, $W(\lambda, \kappa; S)$. Note in this set up, government and corporate bonds are perfect substitutes for the household. They are protected from idiosyncratic shocks, yet, tautologically, face aggregate risks due to the aggregate uncertainty in production and the financial state. The household discounts the future at rate $\beta \in (0, 1)$ per period and maximizes its periodic utility over consumption and leisure, $U(c, 1-n)$, subject to their budget constraint. The full household problem[15] is therefore:

$$W(\lambda, \kappa; S) = \max_{c, n^h, \lambda', \kappa'} \left[U(c, n^h) + \beta \sum_{m=1}^{N_s} \pi^s_{l,m} W(\lambda', \kappa'; S'_m) \right] \tag{8}$$

subject to:

$$\begin{gathered} c + q(S)\kappa' + \int \rho_1(k', b', \varepsilon'; S_l)\lambda'(d[k' \times b' \times \varepsilon') \leq \\ w(S)n^h + \kappa + \int \rho_0(k, b, \varepsilon; S_l)\lambda(d[k \times b \times \varepsilon]) \\ \mu' = \Gamma(S) \end{gathered} \tag{9}$$

where $\rho_1(k', b', \varepsilon'; S_l)$ is ex-dividend price of a share and $\rho_0(k, b, \varepsilon; S_l)$ is dividend-inclusive value of a share. Let $h^c(\lambda, \kappa; S_l)$ be the household's decision rule for consumption, $h^n(\lambda, \kappa; S_l)$ be the decision rule for labor hours, and $h^\kappa(\lambda, \kappa; S_l)$ be the decision rule for bonds. Finally, let $h^\lambda(k', b', \varepsilon', \lambda, \kappa; S_l)$ be the decision rule for shares in firms with next period values of k' capital, b' debt, and ε' idiosyncratic productivity[16].

[15]The household has access to a complete set of state-contingent claims. These assets are in zero net supply in equilibrium; for simplicity I do not model them here.

[16]The household can choose shares of these firm types in equilibrium, since it knows the transition probabilities of the aggregate and idiosyncratic states and the law of large numbers applies.

III.d Recursive Equilibrium

A *recursive competitive equilibrium* is a set of functions:

$$\left(w, q, \{d_m\}_{m=1}^{N_s}, \rho_0, \rho_1, v_0, g_N, g_K, g_B, g_D, W, h^c, h^N, h^\lambda, h^\kappa\right)$$

that solve the firms' and household's problems and clear the asset, labor and output markets, such that:

(i) v_0 solves (2-3) subject to (4), g_N is the decision rule for exiting firms, and (g_N, g_K, g_B, g_D) are the decision rules for continuing firms

(ii) The government's budget constraint (6), subject to (7) is satisfied

(iii) W solves (8) subject to (9) and $(h^c, h^N, h^\lambda, h^\kappa)$ are the household's decision rules

(iv) The shares market clears: $h^\lambda(k', b', \varepsilon_j, \lambda, \kappa; S) = \mu'(k', b', \varepsilon_j; S)$ for $(k', b', \varepsilon_j) \in \mathbf{S}$

(v) The labor market clears: $h^N(\lambda, \kappa; S) = \int_{\mathbf{S}} [g_N(k, \varepsilon; S)] \mu(d[k \times b \times \varepsilon])$

(vi) The goods market clears: $C(\lambda, \kappa; S_l) = \int_{\mathbf{S}} \Big[z\varepsilon F(k, n(k, \varepsilon; S)) - (1-\pi_e)\big(g_K(k, b, \varepsilon; S) - (1-\delta)k\big) + \pi_e\big((1-\delta)k - k_0\big)\Big] \mu(d[k \times b \times \varepsilon]) - G$

(vii) $\forall (A, \varepsilon_j) \in \mathcal{S}$ defines Γ, where $\chi(k_0) = \{1 \text{ if } (k_0, 0) \in A; 0 \text{ otherwise}\}$

$\mu'(A, \varepsilon_j) = (1-\pi_e) \int_{\{(k,b,\varepsilon_i) | (g^K(k,b,\varepsilon_i;s,\mu), g^B(k,b,\varepsilon_i;s,\mu)) \in A\}} \pi_{ij} \mu\left(d\left[k \times b \times \varepsilon_i\right]\right) + \pi_e \chi\left(k_0\right) H(\varepsilon_j)$

(viii) The bond market clears through Walras's Law: $h^\kappa(\lambda, \kappa; S) = \int_{\mathbf{S}} [g_B(k, b, \varepsilon; S)] \mu(d[k \times b \times \varepsilon]) + \theta$

III.e Prices

I begin my analysis by deriving a set of optimality conditions from the household's problem that may be subsumed into the firms' problem. Let C and N represent the optimal household choices of consumption and labor hours, respectively. Furthermore, let C'_m and N'_m be the optimal choices of consumption and labor next period when the aggregate state is S'_m. Then the real wage (10) is the marginal rate of substitution between leisure and consumption. Since the household owns the firms, the firms' stochastic discount factor (11) of future value is the household's marginal rate of substitution between consumption across states. The loan discount factor (12), is the conditional expectation of the marginal rate of substitution between consumption across states [17].

$$w(S) = \frac{D_2 U(C, 1-N)}{D_1 U(C, 1-N)} \tag{10}$$

$$d_m(S) = \beta \frac{D_1 U\left(C'_m, 1-N'_m\right)}{D_1 U(C, 1-N)} \tag{11}$$

$$q(S) = \beta \sum_{m=1}^{N_s} \pi^s_{l,m} \frac{D_1 U(C'_m, 1-N'_m)}{D_1 U(C, 1-N)} \tag{12}$$

Finally, let $p(S)$ be the household's marginal valuation of output. Again, since the household owns the firms, I can multiply the firms' value (3) by $p(S)$ to price all (current and future) output in units of marginal utility without altering decision rules. Letting $V_0 = p(S)v_0$ and $V = p(S)v$, this changes equations (2-3, 10-12) such that:

$$V(k, b, \varepsilon_i; S_l) = \max_{k', b', D} \left[p(S)D + \beta \sum_{m=1}^{N_s} \pi^s_{l,m} \sum_{j=1}^{N_\varepsilon} \pi^\varepsilon_{i,j} V_0(k', b', \varepsilon_j; S'_m) \right] \tag{13}$$

$$w(S) = \frac{D_2 U(C, 1-N)}{p(S)} \tag{14}$$

$$d_m(S) = \beta \frac{D_1 U\left(C'_m, 1-N'_m\right)}{p(S)} \tag{15}$$

$$q(S) = \beta \sum_{m=1}^{N_s} \pi^s_{l,m} \frac{D_1 U(C'_m, 1-N'_m)}{p(S)} \tag{16}$$

where:

$$p(S) = D_1 U(C, 1-N) \tag{17}$$

[17]The risk-free interest rate on bonds is: $r = \frac{1}{q(S)} - 1$.

III.f Allocations

A firm's labor choice is static. It chooses $n^*(k,\varepsilon) = z\varepsilon D_2 F(k,n) = (1+\tau)w$. Output is then $y(k,\varepsilon) = z\varepsilon F(k, z\varepsilon D_2 F(k,n))$, which can be used to define cash-on-hand in (1).

Turning from choices involving only static variables to those that involve dynamic ones, I divide the analysis into two groups of firms to illustrate the potential for policy to improve capital allocations. The first group can reach their optimal capital level unimpeded by the collateral constraint. Let ${k'}^*(\varepsilon) = \text{argmax}_{k'} V(k,b,\varepsilon;S)$ for this group, which I will also use as a reference point for the next group. The second group cannot post sufficient collateral to afford optimal investment.

Firms unimpeded by their collateral invest so $\frac{\beta}{p(S)}\big(\mathbb{E}_{\pi^s}\,\mathbb{E}_{\pi^\varepsilon}\, D_1 V_0(k',b',\varepsilon_j;S'_m)\big) = 1$. The expected marginal discounted (recall $p(S'_m)$ is embedded in V_0) value of an extra unit of capital equals its price, 1 output unit. Applying the Benveniste and Scheinkman condition, this is:

$$\frac{\beta}{p(S)}\Bigg[\sum_{m=1}^{N_s}\pi^s_{l,m}p(S'_m)\sum_{j=1}^{N_\varepsilon}\pi^\varepsilon_{i,j}\Big(z_m\varepsilon_j D_1 F\big({k'}^*(\varepsilon), n^*({k'}^*(\varepsilon),\varepsilon_j)\big) \\ + \big[1-\big(1+\tau(S')\big)w(S')\big]D_1 n^*({k'}^*(\varepsilon),\varepsilon_j) + (1-\delta)\Big)\Bigg] = 1 \tag{18}$$

These firms have no excess returns to capital investment. Any debt-relief policy will produce no extra output. Another way to see this is that neither the collateral constraint, nor debt enter into this optimality condition. Any relief will change neither labor, nor capital, and output will remain the same. Their choice of debt is then $b' \in \Big[\frac{\underline{D}+{k'}^*(\varepsilon)-x(k,b,\varepsilon;S)}{q(S)}, \zeta_l k\Big]$ from the conditions in (4), to be discussed further below.

Moving to firms bound by their collateral, I take advantage of the monotonically increasing value function in k to simplify the problem. Firms will borrow until $b' = \zeta k$ and invest such that:

$$k'(k,b,\varepsilon) = x(k,b,\varepsilon) + q(S)\zeta k - \underline{D} < {k'}^*(\varepsilon) \tag{19}$$

The strict inequality is the center of the misallocation problem that policy would seek to correct. These firms cannot reach their desired capital level and are unable to hire as much labor or produce as much output next period, compared to an environment without these financial frictions. Moreover, given decreasing returns to scale production, these firms have a higher shadow price on their collateral and higher returns on potential investment than firms without binding constraints. Again using the properties of the value function and going back to (18), substituting in $k'(k, b, \varepsilon)$ for $k'^*(\varepsilon)$ returns:

$$\frac{\beta}{p(S)} \Bigg[\sum_{m=1}^{N_s} \pi^s_{l,m} p(S'_m) \sum_{j=1}^{N_\varepsilon} \pi^\varepsilon_{i,j} \Big(z_m \varepsilon_j D_1 F\big(k'(k,b,\varepsilon), n^*(k'(k,b,\varepsilon), \varepsilon_j)\big) + \big[1 - \big(1 + \tau(S')\big) w(S')\big] D_1 n^*(k'(k,b,\varepsilon), \varepsilon_j) + (1-\delta) \Big) \Bigg] > 1 \tag{20}$$

Through (19), it is easy to see how a credit shock exacerbates these conditions, as the more $k'^*(\varepsilon) - k'(k, b, \varepsilon)$ increases, the more the inefficiencies grow. However, this is where the role for policy exists. Debt-relief policy that increases a firm's cash-on-hand can shrink, or even eliminate, the magnitude of the inequality in (19), as well as the difference between (20) and (18). Thus policy may increase output, investment, and labor hours compared to the same scenario without policy.

There is still the matter of issuing dividends and the unconstrained firms' debt. From here out, I assume a *zero-dividends* policy, meaning $\underline{D} = D = 0$. Firms use as much of their retained earnings as needed to invest. Those that need to borrow for optimal investment, but are unconstrained in doing so, will borrow to make up the difference. Funds leftover after investment will be used as savings. Constrained firms take this $\underline{D}$ as given in (19). Given market clearing, the household is indifferent to the firm paying dividends, or retaining any resources leftover after investment as savings; this policy assigns the decision rule to the latter. Therefore, this is an optimal dividend policy for all firms[18].

[18] The (weak) optimality of the zero-dividend policy should not be confused with multiplicity of equilibria, which does not exist in this model. This is a result of the firm's value function being linear in D and b.

Finally, I set all steady state quantities in the government's budget to 0. Since this paper is concerned with policy responses to shocks, the level of these steady state variables will have little to no impact on the insights derived from the results and are just a calibration exercise.

IV Computations

IV.a Calibration

The model has an annual frequency. The common, DRS production function is Cobb-Douglas: $z\varepsilon F(k,n) = z\varepsilon k^{\alpha} n^{v}$. Entrant firms' capital supply is a fixed at χ percent of the steady state aggregate stock: $k_0 = \chi \int k\tilde{\mu}(d[k \times b \times \varepsilon])$. Entrant debt, b_0, is assumed to be 0, as in Khan and Thomas (2013). Household utility, $U(c, 1-n^h) = \ln(c) + \psi(1-n^h)$, is the result of Hansen (1985) - Rogerson (1988) indivisible labor.

The household discount factor, β is set to 0.96, to imply a 4% risk-free real interest rate, as found in Gomme et al. (2011). The parameter governing labor's share of output, ν is set to 0.60 to match Cooley et al. (1995). I set the depreciation rate, δ, to imply an average investment-to-capital rate of 0.069, which matches the the BEA Fixed Asset Tables data. The chance of being exogenously forced to exit the market, π_e, is 8.5%, matching the average rate in the BDS Data Tables from 1993 to 2007. The parameter governing the marginal utility of leisure, ψ, is set to 2.15, so that in equilibrium, $1/3$ of the household's time endowment is spend on market labor. χ is set to 0.124, so that entrant firms enter the economy with a capital stock of 12.4% of the aggregate level, which implies that entrants' share of employment is 2.1%, to match the 2.3% figure in the BDS. Following Khan and Thomas (2013)), in a constant returns to scale setting, $\chi = 0.10$ makes it so that an entrant firm's labor would be 0.10 of the aggregate level, lining up with Davis and Haltiwanger (1992).

For the remainder of the article, I assume no aggregate uncertainty. The parameter governing the economy-wide collateral constraint, ζ, will change exogenously to trace transition paths under perfect foresight assumptions. The collateral constraint will take on two values, ζ_0 and ζ_l, representing normal financial times and a credit crunch, respectively. Aggregate TFP will remain at its steady state level, $\bar{z} = 1.0$. I assume that idiosyncratic productivity follows an AR(1) log-normal process, $\log \varepsilon' = \rho_\varepsilon \log \varepsilon + \eta'$, with $\eta' \sim N\left(0, \sigma^2_{\eta_\varepsilon}\right)$. The process is discretized using the Tauchen (1986) algorithm and 7 values so that $N_\varepsilon = 7$.

The next parameters are jointly calibrated to match the following moments. The parameter governing capital's share of output, α is set to 0.27 to target a capital output ratio of 2.23, from the BEA Fixed Asset Tables 1954 - 2002. The persistence of idiosyncratic productivity shocks, ρ_ε, is set at 0.640 to target a mean investment rate of 0.122. The standard deviation of idiosyncratic productivity shocks, $\sigma_{\eta\varepsilon}$, is 0.078 to target a standard deviation of investment rate of 0.337. Both of these values come from Cooper and Haltiwanger (2006). During normal economic times, the collateral constraint will be $\zeta_o = 1.11$, to target a debt-to-asset ratio of 0.372, as found from 1954-2006 nonfarm, nonfinancial businesses in the Flow of Funds. Finally, during a credit crunch, this value changes to $\zeta_l = 0.641$ in order to facilitate a peak-to-trough decrease in debt of about 26%. Khan and Thomas (2013) report this GDP deflated real lending decrease in their data from 2008Q4 - 2011Q4. Calibrated parameters are reported on Table 1, along with their closest related data target.

Figure (1) plots the distribution of firms with the median level of idiosyncratic productivity arising from the deterministic steady state. Notice the concentration of firms have relatively low capital and relatively high leverage ($\frac{b}{k}$). This combination increases the probability of being unable to make optimal capital investments due to financial restrictions, thus increasing excess returns in the economy, as the next figures show.

IV.b Steady State Inefficiencies

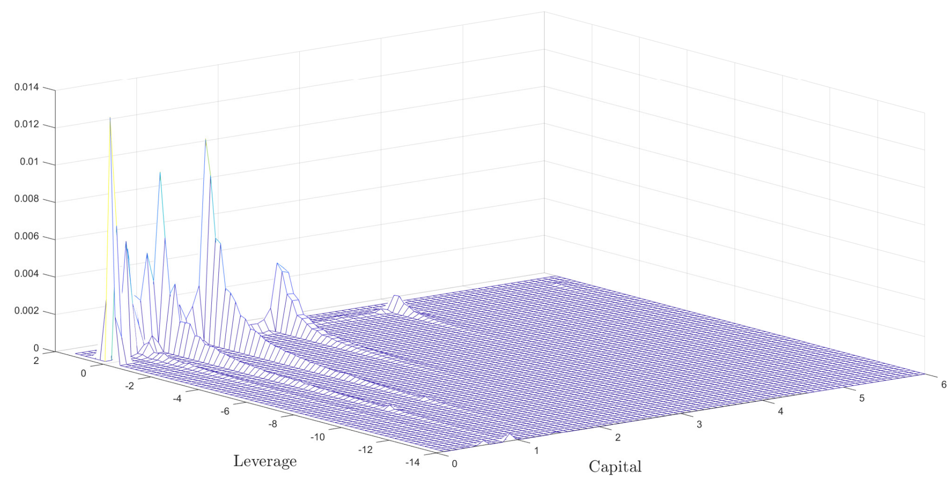
As described in Section III.f, there are inefficiencies present in the model before any shocks. Figure (2) shows mean ratio of a firm's capital choice to their optimal capital choice,

Table 1: Parameters

	Parameter		Target		Model
β	discount factor	$= 0.96$	real interest rate	$= 4.0\%$	4.16%
ν	labor share	$= 0.600$	labor share	$= 0.600$	0.600
χ	entrant K share	$= 0.124$	entrant N share	$= 0.023$	0.021
π_e	exit rate	$= 0.085$	exit rate	$= 0.085$	0.085
δ	depreciation	$= 0.069$	investment/capital	$= 0.069$	0.069
ψ	leisure preference	$= 2.150$	labor hours	$= 0.333$	0.332
α	capital share	$= 0.270$	capital/output	$= 2.230$	2.287
ρ_ε	persistence (idio)	$= 0.640$	mean i/k	$= 0.122$	0.130
η_ε	stand dev (idio)	$= 0.078$	stand dev i/k	$= 0.337$	0.449
ζ_o	collateral fraction	$= 1.110$	debt/assets	$= 0.372$	0.371
ζ_l	credit crunch	$= 0.641$	decrease in debt	$= 26.0\%$	25.95%

Note: Individually calibrated variables listed in top section. Jointly calibrated variables listed in bottom section.

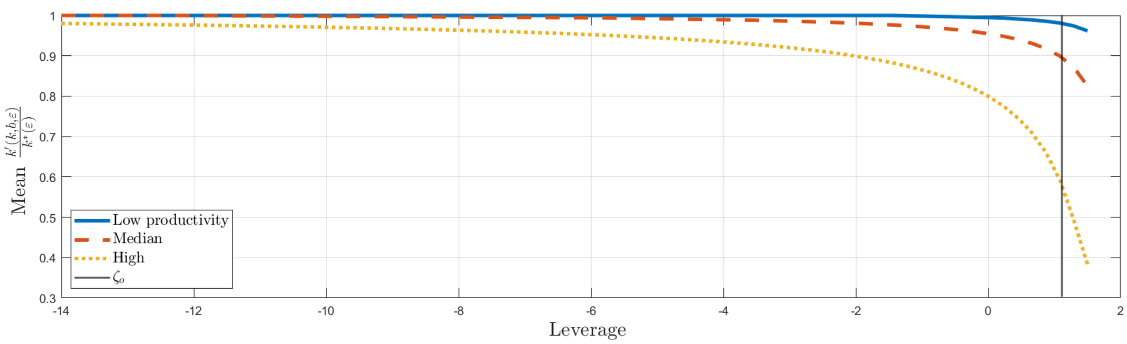
Figure 1: Steady State Distribution of Firms



Note: Distribution of firms with median level of idiosyncratic productivity in steady state. Most mass is found in relatively low capital and relatively high leverage areas of the state space. This increases the changes of a binding collateral constraint.

plotted over leverage for low, median, and high productivity firms. In a perfectly optimal environment, all firms would be able to reach their target capital, which would be shown as a horizontal line at $y = 1$ since $k'(k, b, \varepsilon) = k'^{*}(\varepsilon)$. However, given the financial frictions present, higher leveraged firms have relatively more debt in their budget constraint, or less capital to offer as collateral, hamstringing potential investment. Increasing leverage moves this ratio further from 1, representing increased inefficiencies; the distance to optimal capital increases at an increasing rate as leverage moves to and beyond ζ_o. This inefficiency is also increasing in firm-level productivity since higher ε values increase the marginal product of capital, raising the optimal capital choice. All else equal, higher productivity firms thus need to invest more than lower productivity firms, meaning they also need to borrow more.

Figure 2: Mean $\frac{k'(k,b,\varepsilon)}{k^*(\varepsilon)}$ by Leverage in Steady State



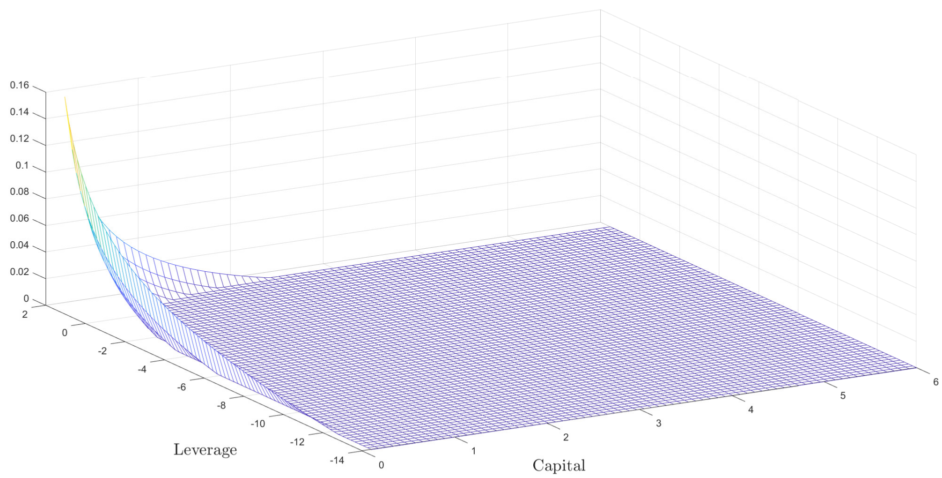
Note: As firms become more leveraged ($\frac{b}{k}$ increases), the distance from their optimal capital choice increases. Plotted for low, median, and high idiosyncratic productivity out of $N_\varepsilon = 7$.

Figure (3) paints a similar story displaying the amount of excess returns to capital investment arising from firms' decision rules in the steady state. From the parameterization of the model, (18) becomes:

$$\frac{\alpha}{1-\nu}\frac{\beta}{p(S)}\left(\sum_{m=1}^{N_s}\pi^s_{l,m}p(S'_m)\sum_{j=1}^{N_\varepsilon}\pi^\varepsilon_{i,j}k'^{\frac{\alpha+\nu-1}{1-\nu}}\left[z_m\varepsilon_j\left(\frac{\nu z_m\varepsilon_j}{\big(1+\tau(S)\big)w(S)}\right)^{\frac{\nu}{1-\nu}}\right.\right.$$
$$\left.\left.-\big(1+\tau(S)\big)w(S)\left(\frac{\nu z_m\varepsilon_j}{\big(1+\tau(S)\big)w(S)}\right)^{\frac{1}{1-\nu}}\right]+(1-\delta)\right)=1 \tag{21}$$

As firms move away from their optimal capital choice, the marginal benefit of capital investment increases and the Left Hand Side of the equation grows bigger than 1, the marginal cost of a unit of capital. Figure (3) displays the quantitative implications of the inefficiencies inherent in the model. At the same time, it also displays the potential quantitative benefit of policy. Policy that targets firms with the highest excess returns will naturally be the policy with the best returns per dollar spent.

Figure 3: Excess Returns in Steady State



Note: Excess returns in the steady state implied by decision rules for firms median idiosyncratic productivity. Firms with higher excess returns are further from their optimal capital stock because the marginal value of capital investment is still higher than its cost, 1.

V Efficient Policy in a Credit Crunch

A credit crunch in the model is a fall in the collateral parameter ζ_o to ζ_l, limiting the amount a firm can borrow and thus cutting their investment. This then makes it harder for firms to post collateral, creating a cycle as the capital stock deteriorates. The credit shock starts at the first date in the simulation, $t = 1$, and lasts for 3 periods. This agrees with Khan and Thomas (2013), as they use data from Reinhart and Rogoff (2009) to back out the average length of banking crisis in post-war advanced economies and find it to be 3.2 years. Following that, let ρ_l be the probability of leaving a crisis, such that ζ_l returns to ζ_o at a rate of $(1 - \rho_l) = 0.3125$ per period. The impulse response to a credit shock without any policy is labeled as "No Policy" in all figures.

V.a Excess Returns Policy

As discussed previously, optimal policy will look to minimize the distance between the marginal benefit and marginal cost of capital investment. This can be done by directly targeting firms with the highest excess returns. Moreover, the target should start with the highest excess return value and provide resources to that firm until their returns are equal to that of the second highest firm. Thereafter, it should provide relief to both those firms until their returns are equal to that of the third highest firm, and continue on in that manner until the policy funds are exhausted, or all excess returns equal 0.

This policy design has another added benefit; it naturally establishes a threshold for *eligible debt*. Some firms have debt that would be inefficient to be relieved. Consider a firm that has some positive level of debt, but not enough for the collateral constraint to be binding. This firm can reach its optimal investment level, so any debt-relief resources would just be held as savings by the firm (or passed on as dividends to shareholders in other specifications). Relief leads to no extra output, but still must be paid for by future tax increases. Thus optimal policy would set a threshold of eligible debt such that any relief payments being made will be used for investment by the firm. This then avoids concerns of funds not reaching their intended destination, a concern that has been raised for the PPP (Li (2021), Autor et al. (2022)). This eligible debt threshold is:

$$\max\Big\{\min\big[\frac{k'^{*}(\varepsilon) - x(k,b,\varepsilon)}{q(S)}, b\big], 0\Big\} \tag{22}$$

This equation makes sure that the firm in question has debt to be relieved in the first place. Beyond that, the relief policy will only pay out to the firm until their cash-on-hand becomes sufficient to pay for their optimal investment level. At that point the relief turns off and the firm will not again become eligible, even if there is remaining $b > 0$. By following the excess returns policy program, this threshold is naturally enforced.

V.a.1 Policy Parameters

Debt-relief will only be paid out on impact date. All policies tested will have a net present resource cost of 3.5% of steady state GDP, $\overline{Y}$. This is taken from the U.S. SBA Forgiveness

Platform Lender Submission Metrics, which estimates as of October 23, 2022, total PPP loan forgiveness was 3.46%[19] of GDP. The constant resource transfer across all policies helps ensure that this is a fair experiment and one policy is not given more resources than another.

I test multiple government debt repayment plans, captured by the value of the parameter, ϕ. Reported are $\phi = 0.20, 0.10,$ and 0. These indicate that in every period from the start of the repayment plan, the government will pay 20%, 10%, or 0% of the remaining public debt. Under the 0% repayment plan, the government never makes a payment on its debt. This is not intended to be a policy recommendation, but to serve as a comparison to observe the effects of the policy without the downward pressure from agents expecting higher taxes in the future needed to repay debt. Finally, repayments start in period $t = 11$, when the aggregate capital decline has passed its half-life. This is to avoid raising taxes in the heart of a recession. Table 2 lists these parameters.

Table 2: Excess Returns Policy Parameters

Parameter		Value
Resource Cost	$\frac{T(\Theta;S)}{\bar{Y}}$	3.5%[1]
Credit Shock Dates	t	1-3
Shock Recovery Rate	1-ρ_l	0.3125
Relief Date	t	1
Repayment (θ) Start	t	11[2]
Repayment (θ) Fractions Tested	ϕ	$0.20, 0.10, 0$

Note: [1]The SBA lists Total Forgiveness Paid at \$755.7B, 3.46% of Real GDP in Q3 2022.
[2]Half-life of aggregate capital decline.

The process of solving for these transition paths involves two simultaneous guesses on the time vectors of consumption and taxation. The appendix provides a general overview of this method, with details on updating the taxation vector.

[19]The SBA lists Total Forgiveness Paid at \$755.7B and Real GDP in Q3 2022 was \$21.851T in Chained 2017 Dollars (FRED).

V.b Policy Results

Figure (4) shows the impulse responses of aggregate variables to a credit shock with excess returns-targeted policy[20]. There are two main components to the policy, debt-relief and repayment, implied by the second fall in aggregates. The debt-relief leads to increases in most aggregate variables of approximately $0.3\% - 0.7\%$ compared to the no policy alternative at their respective troughs. Arising from the nature of the policy, investment has the biggest difference of 3% and the aggregate capital stock decline reaches its no policy half-life level 2 years sooner under this policy. Given the largest improvements are to investment, there are relatively less resources in the economy available for consumption, which saw the smallest through increases of 0.2%.

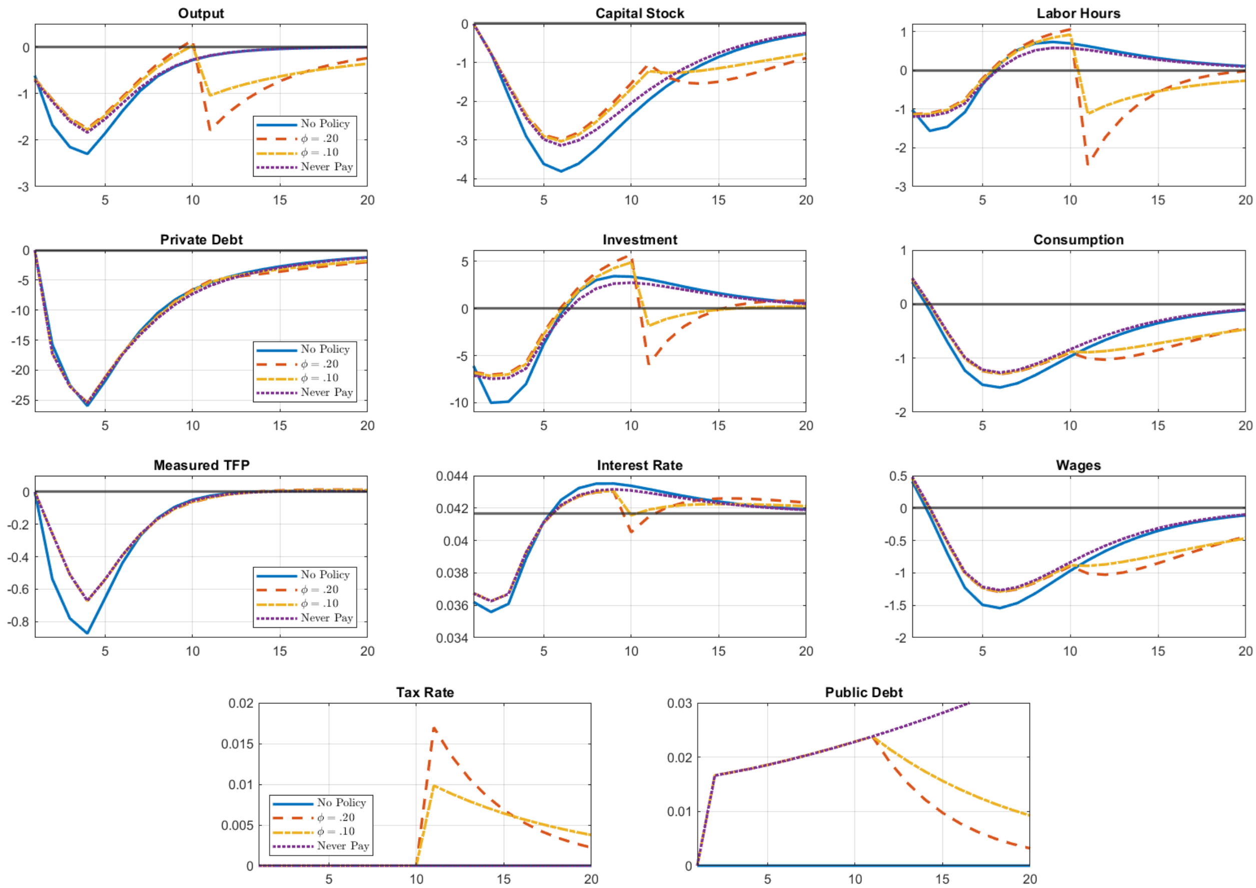
Figure (5) demonstrates the mechanism at hand. Plotted are excess returns implied by decision rules for firms with median idiosyncratic productivity on the impact date of the shock under this policy. As shown, there are more excess returns overall compared to Figure (3), however the top has been lowered and flattened[21]. The policy loosened the budget constraint of firms with the most to gain from investing in the marginal capital unit, leading to a surge in investment. Private debt remains mostly the same because firms are only using these extra resources for investment and not to pay off debt (by (22)). This is also shown in Figure (5) by none of the equalized values returning to 0. Under policy, the slow consumption decline to the trough keeps the interest rate higher, and relatively higher consumption keeps wages higher, as well.

Repayment horizons deserve specific consideration. Debt repayment can begin early in the recession, at which point taxes must be raised during economic downturn, or after most of the recovery has been completed. However, by that point, public debt keeps accruing

[20] One could solve for this as the policy is described; start with the highest returns, provide relief until equal to the next highest, repeat. A quicker way is to guess an equalization point on Figure (5) and back out the amount of relief necessary for indebted firms with returns above that point, then check if the implied policy uses the amount of aggregate resources required.

[21] There is a slight spike in the low capital, just negative debt region. These are small firms that happen to be net-savers, which renders them ineligible for debt-relief.

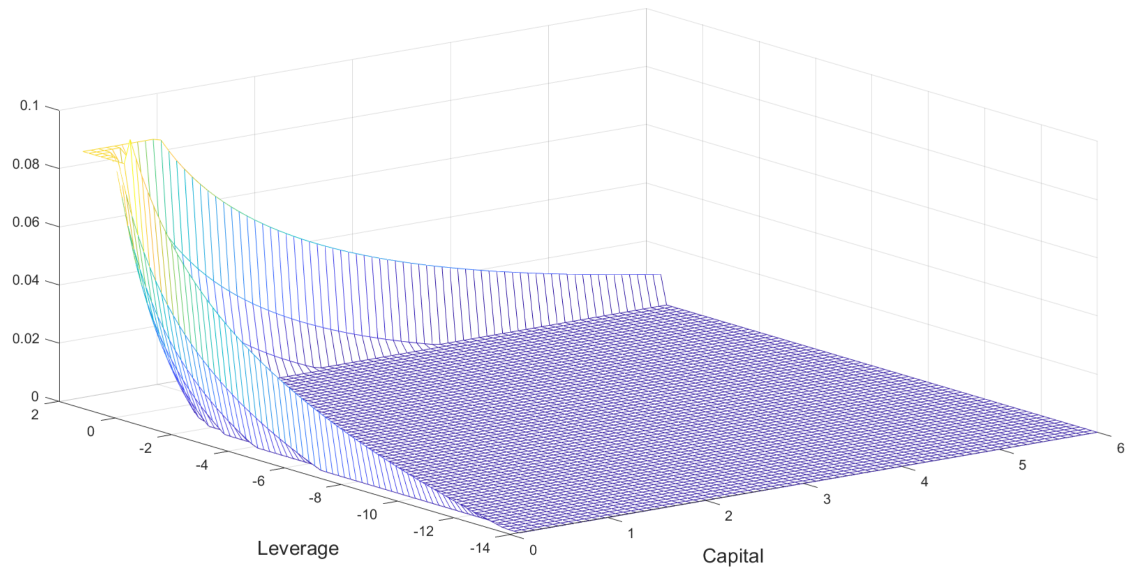
Figure 4: Excess Return-targeted Policy



Note: Impulse responses to a credit shock. The y-axis measures percent deviation from steady state values except for taxes, public debt, and the interest rate.

interest, increasing the amount of taxes needed to pay it. For illustrative purposes, I chose a repayment start date when the capital decay had already passed its half-life. The rise in taxes to pay for this policy induces another downturn, albeit one of smaller magnitude than the credit crunch when repayment is slower. The larger the share of public debt repaid each period, the higher taxes must be raised to pay off that amount. Even though paying off 10% of debt each period requires taxes to remain elevated for longer than paying off 20%, the path of taxes is smoother, leading to a smoother consumption path as well. The knowledge of future tax increases does not appear to have much of an effect at the trough of a recession. However, the no-repayment path does begin to deviate from the repayment paths more as they approach the date of the tax increases.

Figure 5: Excess Returns Under Policy on Impact Date



Note: More firms have higher excess returns in a recession, but the peak has decreased and evened out. The policy targets firms with the most to gain from the marginal capital unit, giving it the highest return per dollar spent. Small spike due to small firms that are net-savers; savers excluded from debt-relief.

VI Readily Observable Policy Targets

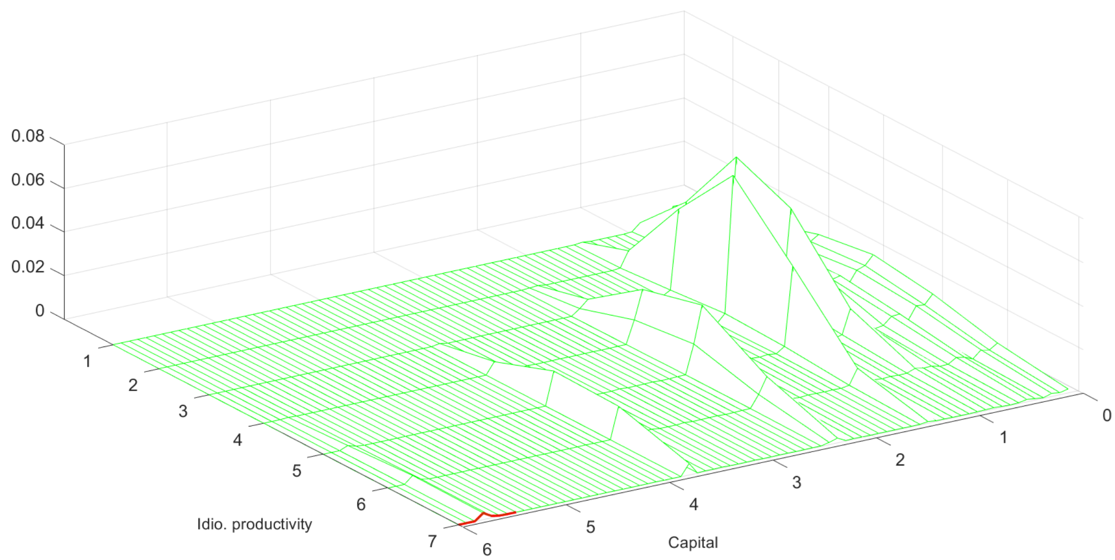
Identifying excess returns across the state space of an economy may not be feasible for policymakers, even if obtaining such information isn't cost prohibitive. I consider two readily observable policy targets: firm size and age. Results are compared to an untargeted policy at the end of the section.

VI.a PPP-based Employment Targeting

Since we are concerned with capital allocative efficiency, a good proxy for excess returns may be firm size. It was previously discussed how firm size may be correlated with excess returns as small firms may be further away from their optimal capital level, and thus face higher marginal returns.

Borrowing from the PPP targets, I set my firm size threshold at 500 employees. According to the BDS, from 2002 - 2007, 99.59% of firms would meet this criteria. In order to map this number to the model, I identify a $\bar{k}(\varepsilon)$ such that: $\mu_{PPP} \equiv \int \mu(k \leq \bar{k}(\varepsilon), b, \varepsilon) = 0.9959$, where $\forall k \in \mathbf{K}, \forall \varepsilon \in \mathbf{E}$, $n(k \leq \bar{k}(\varepsilon), \varepsilon) < n(k > \bar{k}(\varepsilon), \varepsilon)$. This ensures that the 99.59% of the distribution with the lowest labor demand has been targeted. It is impossible to rule out the possibility of a tie, meaning the identical firms may end up on opposite sides of the threshold. In this scenario, I randomly assign enough mass to be included from the set of identical firms on threshold to meet the 99.59% requirement, then exclude the rest. Figure (6) displays a mapping of included firms in green and excluded firms in red plotted in the (k, ε) space (recall $n^*(k, \varepsilon)$ is not a function of debt). Excluded firms all have the highest level of idiosyncratic productivity; this is shown as only a line on ε_7 in the figure.

Figure 6: PPP Inclusion



Note: Firms included under PPP employment targeting in green; firms excluded in red. Excluded firms all have the highest level of idiosyncratic productivity, all on the red line. Firm distribution plotted in the (k, ε) space.

VI.b Age-based Targeting

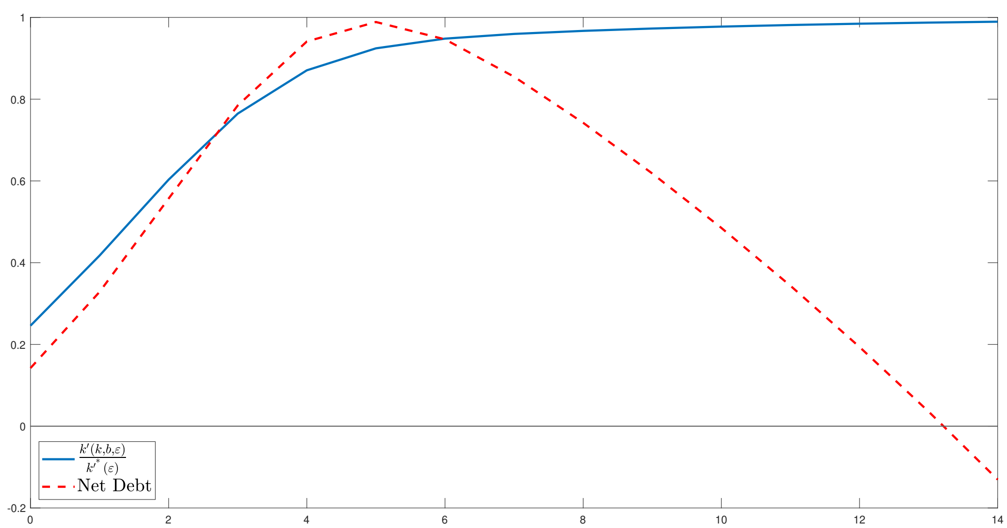
Beyond employment targeting, another proxy for excess returns may be firm age. In an environment absent financial frictions, entering firms would be able to immediately reach their optimal capital stock and excess returns across the economy would be 0. Given financial frictions, there is a slow growth process to $k'^*(\varepsilon)$ as firms age. This process of aging generates firm life-cycle results in the model, as young firms carry inefficiently low levels of capital.

However, this also means that there is a relationship between age and excess returns, and thus potential gains from age-targeted policy.

In order to model age-based policy, I create another individual state variable, a, for age. The individual state space for a firm is now (k, b, ε, a). I set an absorbing state for the maximum age of a firm at $a = 27$, as less than 1% of the distribution survives to that age. Outside of introducing age-based policy, this does not change any decision rules, rather it simply divides existing decision rules into age groups.

Figure (7) shows the average relative distance to optimal capital, $k'^{*}(\varepsilon)$, and average net debt for a cohort of firms as it ages. As firms age, they have more output and retained earnings to use for investment. Firms do not reach 90% of their optimal capital level until age 6 and do not fully reach their optimal capital level until around age 14. Consequently, debt continues to grow on average until age 5 and firms do not become net savers until age 13.

Figure 7: Cohort Mean $\frac{k'(k,b,\varepsilon)}{k'^{*}(\varepsilon)}$ and Debt in Steady State



Note: Average relative distance to optimal capital, $k'^{*}(\varepsilon)$ as firm cohort ages. Firms slowly accumulate capital to reach their desired level. Debt continues to grow on average until age 5 and firms do not become net savers until age 13.

VI.c Alternative Policy Results

These alternative policy targets will have the same aggregate resource cost of 3.5% of steady state output, be evaluated at $\phi = 0.10$, and have the same repayment start time as before of $t = 11$. In order to keep the resource cost constant, the fraction of debt relieved per firm will be different across policies. For the PPP targeted policy, 3.40% of firm debt will be relieved and for the age-based policy, this value is 15.31%. Since the PPP targets encompass more firms, it follows that relief per firm will be lower. The untargeted policy pays 3.38% of firm debt. Table 3 lists these policy parameters.

Table 3: Alternative Policy Parameters

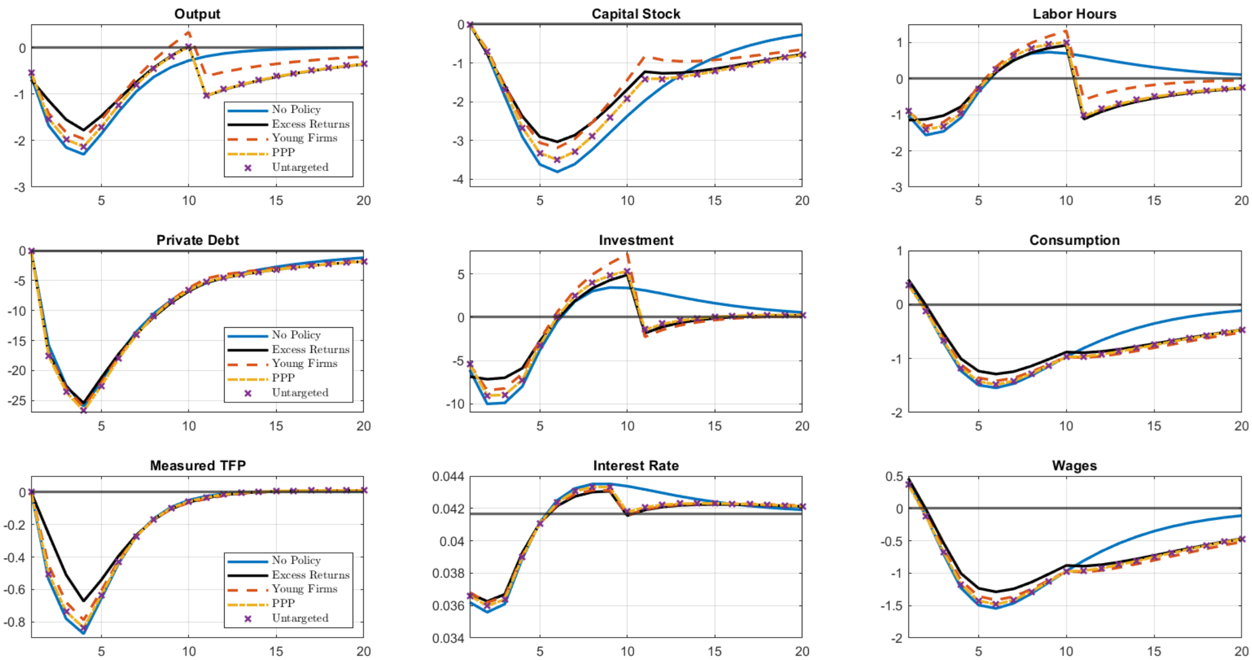
Parameter		Value
Resource Cost	$\frac{T(\Theta;S)}{\overline{Y}}$	3.5%[1]
Credit Shock Dates	t	1-3
Shock Recovery Rate	$1\text{-}\rho_l$	0.3125
Relief Date	t	1
Repayment (θ) Start	t	11[2]
Repayment (θ) Fractions Tested	ϕ	0.10
PPP Size Target	μ_{PPP}	0.9959
Age Target	a	5
Fraction of b paid (PPP)	$g(\overline{k}(\varepsilon), b; S)$	0.0340
Fraction of b paid (age)	$g(b, a; S)$	0.1531
Fraction of b paid (untargeted)	$g(b; S)$	0.0338

Note: Parameters in top bin are common across all policies, including excess returns-targeted policy. Different policies will relieve different fractions of firm debt to keep aggregate resource cost constant across experiments.

As might be expected, neither policy outperforms the excess returns targets. The main effect of targeting those firms with the highest returns to investment will lead to the highest returns per dollar spent. Moreover, there is a secondary effect from the design of the excess returns policy. Nothing in the alternative policies can enforce the eligible debt threshold (22). Indebted firms that do not need debt-relief to reach their optimal capital level are receiving public funds in these scenarios. There is some merit to this, consider if a second recession were to hit, these firms have a savings buffer to fall back on and keep investment up. However, this may be unlikely in advanced economies and is a less efficient use of funds in this experiment.

The age-based target leads to better results than the PPP employment target. The effects of building up a capital supply and retaining earnings are beneficial to all firms. On the other hand, some firms are efficiently small. A firm with low idiosyncratic productivity understands that this value is persistent, so their optimal value of $k'^{*}(k, b, \varepsilon)$ is lower than that of the median firm. They are more likely to be closer to affording their optimal investment, so their returns are also smaller. The untargeted policy still improves aggregate outcomes, though less than targeted policies. Figure (8) displays these results.

Figure 8: Young Firm & PPP Targets



Note: Impulse responses to a credit shock. The y-axis measures percent deviation from steady state values except for the interest rate. Public debt and taxes are omitted as they are the same across all policies.

The appendix (A.b) contains results for alternative size targets. I select a employment share threshold, $\leq 52.89\%$, such that decreasing the share of firms eligible increases the debt-relieved per firm to match that of the young firm policy, 15.31%. This cutoff is close to the population share of the smallest firm size bin, 0.55, in the BDS data from 1978-2006. I also test $\geq 52.89\%$ to see if targeting firms with a larger share of aggregate economic activity is worthwhile. Tightening the PPP target in this way leads to minor aggregate improvements, suggesting this policy may have been too loose. Using this new cutoff to target large firms leads to no noticeable improvements, though this result may be do to the idiosyncratic shock choice. A different process to account for the long right tail in the unconditional firm size distribution may be required for a more accurate answer.

VII Concluding Remarks

I develop a dynamic stochastic general equilibrium model where persistently heterogeneous firms face financial frictions and calibrate it to U.S. data. These frictions limit firm's access to debt, and thus investment, providing a role for debt-relief policy in a recession. I use this model as a quantitative laboratory to study the aggregate effects of firm-targeted debt relief.

The ideal policy would target the firm with the highest marginal value of capital investment over its cost and relieve their debt until their excess returns are equal to the second highest firm, then proceed in that manner. Policy that targets firms' excess returns to capital investment move aggregate investment at the trough of a recession by 3% closer to its steady state value, compared to no policy at all. The improvements in investment cause the aggregate capital supply's recovery to reach its half-life value under no policy 2 years faster, as well. Other aggregates improve by $0.2 - 0.7\%$. Understanding that excess returns may not be a readily observable target, I also consider size and age-based targets. Holding aggregate debt repayment costs constant, I find that a young firm (≤ 5 years old) target outperforms the size target of ≤ 500 employees set by the PPP, and others chosen to equalize debt paid per firm. Though these targeted policies outperform the untargeted policy, neither can match the ideal policy of excess returns targeting.

The government takes on a role similar to Woodford (1990); it borrows on behalf of firms that cannot. However, when it comes time to repay public debt, it may be more appealing to pay it down smoothly, meaning to pay a smaller fraction of debt each period to avoid sharp tax increases and a tax-induced recession.

Alternative observables that correlate with excess returns are a potential area for new work. Different tax structure can be studied to prevent the second dip in aggregates seen in the impulse responses. Endogenous entry and exit decision to allow for debt-relief of entrants can be studied as a comparison to the young firms target. I leave these ideas to future research.

References

R. Albuquerque and H. A. Hopenhayn. Optimal lending contracts and firm dynamics. *The Review of Economic Studies*, 71(2):285–315, 2004.

A. Amin, B. Bowler, M. M. Hasan, G. J. Lobo, and J. Tresl. Firm life cycle and cost of debt. *Journal of Banking & Finance*, 154:106971, 2023.

G.-M. Angeletos, F. Collard, and H. Dellas. Public debt as private liquidity: Optimal policy. *Journal of Political Economy*, 131(11):3233–3264, 2023.

C. Asness, A. Frazzini, R. Israel, T. J. Moskowitz, and L. H. Pedersen. Size matters, if you control your junk. *Journal of Financial Economics*, 129(3):479–509, 2018.

D. Autor, D. Cho, L. D. Crane, M. Goldar, B. Lutz, J. Montes, W. B. Peterman, D. Ratner, D. Villar, and A. Yildirmaz. The $800 billion paycheck protection program: where did the money go and why did it go there? *Journal of Economic Perspectives*, 36(2):55–80, 2022.

T. Babina, W. Ma, C. Moser, P. Ouimet, and R. Zarutskie. Pay, employment, and dynamics of young firms. *Kenan Institute of Private Enterprise Research Paper*, (19-25), 2019.

R. W. Banz. The relationship between return and market value of common stocks. *Journal of financial economics*, 9(1):3–18, 1981.

J. B. Berk. Does size really matter? *Financial Analysts Journal*, 53(5):12–18, 1997.

B. Bernanke and M. Gertler. Agency costs, net worth and business fluctuations. *American Economic Review*, 79(1):14–31, 1989.

J. Bianchi. Efficient bailouts? *American Economic Review*, 106(12):3607–3659, 2016.

B. M. Blau, T. J. Brough, and D. W. Thomas. Corporate lobbying, political connections, and the bailout of banks. *Journal of Banking & Finance*, 37(8):3007–3017, 2013.

A. Brav, J. R. Graham, C. R. Harvey, and R. Michaely. Payout policy in the 21st century. *Journal of financial economics*, 77(3):483–527, 2005.

F. J. Buera, B. Moll, and Y. Shin. Well-intended policies. *Review of Economic Dynamics*, 16(1):216–230, 2013.

G. L. Clementi and H. A. Hopenhayn. A theory of financing constraints and firm dynamics. *The Quarterly Journal of Economics*, 121(1):229–265, 2006.

T. F. Cooley, E. C. Prescott, et al. Economic growth and business cycles. *Frontiers of business cycle research*, 1:1–38, 1995.

R. W. Cooper and J. C. Haltiwanger. On the nature of capital adjustment costs. *The Review of Economic Studies*, 73(3):611–633, 2006.

N. Crouzet and N. R. Mehrotra. Small and large firms over the business cycle. *American Economic Review*, 110(11):3549–3601, 2020.

S. J. Davis and J. Haltiwanger. Gross job creation, gross job destruction, and employment reallocation. *The Quarterly Journal of Economics*, 107(3):819–863, 1992.

R. Faff, W. C. Kwok, E. J. Podolski, and G. Wong. Do corporate policies follow a life-cycle? *Journal of Banking & Finance*, 69:95–107, 2016.

M. Gertler and S. Gilchrist. Monetary policy, business cycles, and the behavior of small manufacturing firms. *The Quarterly Journal of Economics*, 109(2):309–340, 1994.

C. Goetz and M. Stinson. The business dynamics statistics: Describing the evolution of the us economy from 1978-2019. Technical Report 21-33, Center for Economic Studies, 2021.

P. Gomme, B. Ravikumar, and P. Rupert. The return to capital and the business cycle. *Review of Economic Dynamics*, 14(2):262–278, 2011.

G. D. Hansen. Indivisible labor and the business cycle. *Journal of monetary Economics*, 16 (3):309–327, 1985.

B. Holmström and J. Tirole. Private and public supply of liquidity. *Journal of political Economy*, 106(1):1–40, 1998.

U. Jermann and V. Quadrini. Macroeconomic effects of financial shocks. *American Economic Review*, 102(1):238–271, 2012.

I. H. Jo and T. Senga. Aggregate consequences of credit subsidy policies: Firm dynamics and misallocation. *Review of Economic Dynamics*, 32:68–93, 2019.

A. Khan and J. K. Thomas. Credit shocks and aggregate fluctuations in an economy with production heterogeneity. *Journal of Political Economy*, 121(6):1055–1107, 2013.

N. Kiyotaki and J. Moore. Credit cycles. *Journal of political economy*, 105(2):211–248, 1997.

M. Li. Did the small business administration's covid-19 assistance go to the hard hit firms and bring the desired relief? *Journal of Economics and Business*, 115:105969, 2021.

C. Lian and Y. Ma. Anatomy of corporate borrowing constraints. *The Quarterly Journal of Economics*, 136(1):229–291, 2021.

C. M. Reinhart and K. S. Rogoff. *This time is different: Eight centuries of financial folly*. princeton university press, 2009.

R. Rogerson. Indivisible labor, lotteries and equilibrium. *Journal of monetary Economics*, 21(1):3–16, 1988.

A. Shleifer and R. W. Vishny. A survey of corporate governance. *The journal of finance*, 52 (2):737–783, 1997.

G. Tauchen. Finite state markov-chain approximations to univariate and vector autoregressions. *Economics letters*, 20(2):177–181, 1986.

M. Woodford. Public debt as private liquidity. *The American Economic Review*, 80(2): 382–388, 1990.

A Appendix

A.a Determining Transition Paths

1. Guess a vector of $\{\hat{\tau}\}_0^{T+1}$ and $\{\hat{C}\}_0^{T+1}$ (from this point forward, all variables are time-vectors)
   - τ' is needed for k' decision
   - $\hat{C}$ implies a $w(S)$, $q(S)$, and $p(S)$
2. Back-solve decision rules from date T
3. Forward-solve the distribution (and find aggregates) for each t
4. Back out $\tilde{C}$ implied by aggregate resource constraint
5. Back out $\tilde{\tau}$ implied by following:

   Define: $Balance \equiv \hat{\tau}(S)w(S)N(S) + q(S)\theta' - \overline{G} - \theta - T(\Theta, S)$

   Then, $\hat{\tau}(S)w(S)N(S) = Balance + q(S)\theta' - \overline{G} - \theta - T(\Theta, S)$

   Define: $\Delta\hat{\tau}(S)$ as change in $\hat{\tau}(S)$ such that:

   $(\hat{\tau}(S) + \Delta\hat{\tau}(S))w(S)N(S) = \overline{G} + \theta + T(\Theta, S) - q(S)\theta'$

   – This is the increase in $\hat{\tau}(S)$ needed to set $Balance = 0$

   Then: $\Delta\hat{\tau}(S) = \left(\left(\overline{G} + \theta + T(\Theta, S) - q(S)\theta'\right)\left(\frac{1}{w(S)N(S)}\right)\right) - \hat{\tau}(S)$

   So, $\tilde{\tau} = (\hat{\tau}(S) + \Delta\hat{\tau}(S))$
6. Check guess, set $\{\tilde{\tau}\}_0^{T+1} = \{\hat{\tau}\}_0^{T+1}$ and $\{\tilde{C}\}_0^{T+1} = \{\hat{C}\}_0^{T+1}$, and repeat.

A.b Alternative Size Parameters

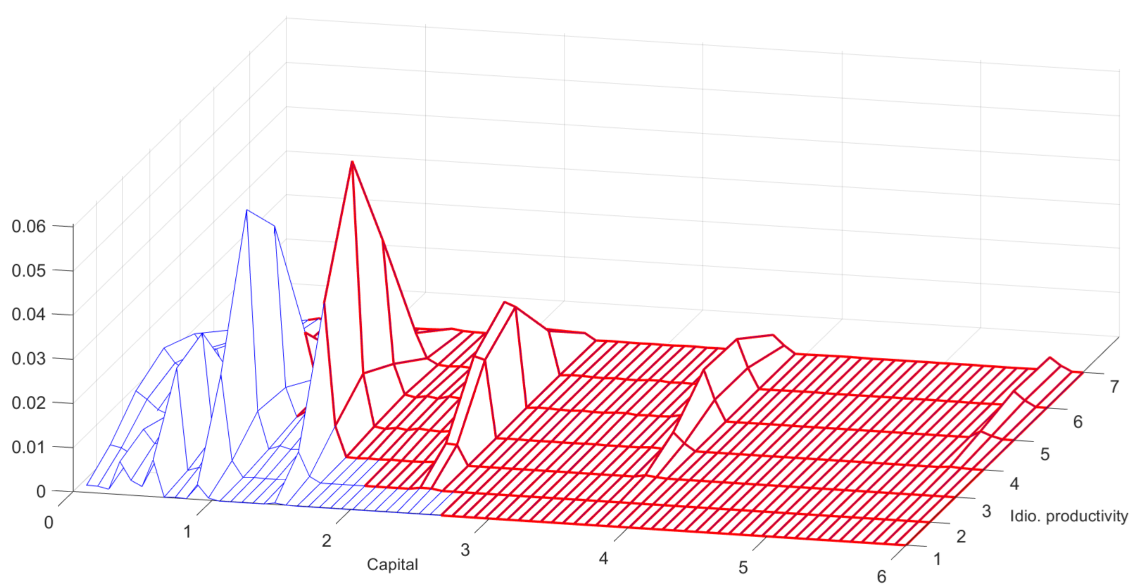
Tightening the PPP target such that the fraction of debt paid per firm is equal to the small firms target leads to minor aggregate improvements. Using this cutoff to target large firms leads to no noticeable improvements. The procedure is the same, but instead of searching for a $\overline{k}(\varepsilon)$ for a μ_{PPP} given by the data, I search for a $k_s(\varepsilon)$ and $k_L(\varepsilon)$ for small and large firms for a μ_{lim}, implied to set $g(b, S) = g(k_s(\varepsilon), b; S)$. This cutoff, 0.53, is very similar to the population share of the smallest firm size bin, 0.55, in the BDS data from 1978-2006. Given the endogenous differences across firms, relief per firm will be different for firms on either side of μ_{lim}.

Table 4: Alternative Size Targets

Parameter		Value
Resource Cost	$\frac{T(\Theta;S)}{\overline{Y}}$	3.5%
Credit Shock Dates	t	1-3
Shock Recovery Rate	1-ρ_l	0.3125
Relief Date	t	1
Repayment (θ) Start	t	11
Repayment (θ) Fractions Tested	ϕ	0.10
PPP Size Target	μ_{PPP}	0.9959
Small/Large Cutoff	μ_{lim}	0.5289
Age Target	5	
Fraction of b paid (PPP)	$g(\overline{k}(\varepsilon), b; S)$	0.0340
Fraction of b paid (age)	$g(b, S)$	0.1531
Fraction of b paid (small)	$g(k_s(\varepsilon), b; S)$	0.1531
Fraction of b paid (large)	$g(k_L(\varepsilon), b; S)$	0.0422

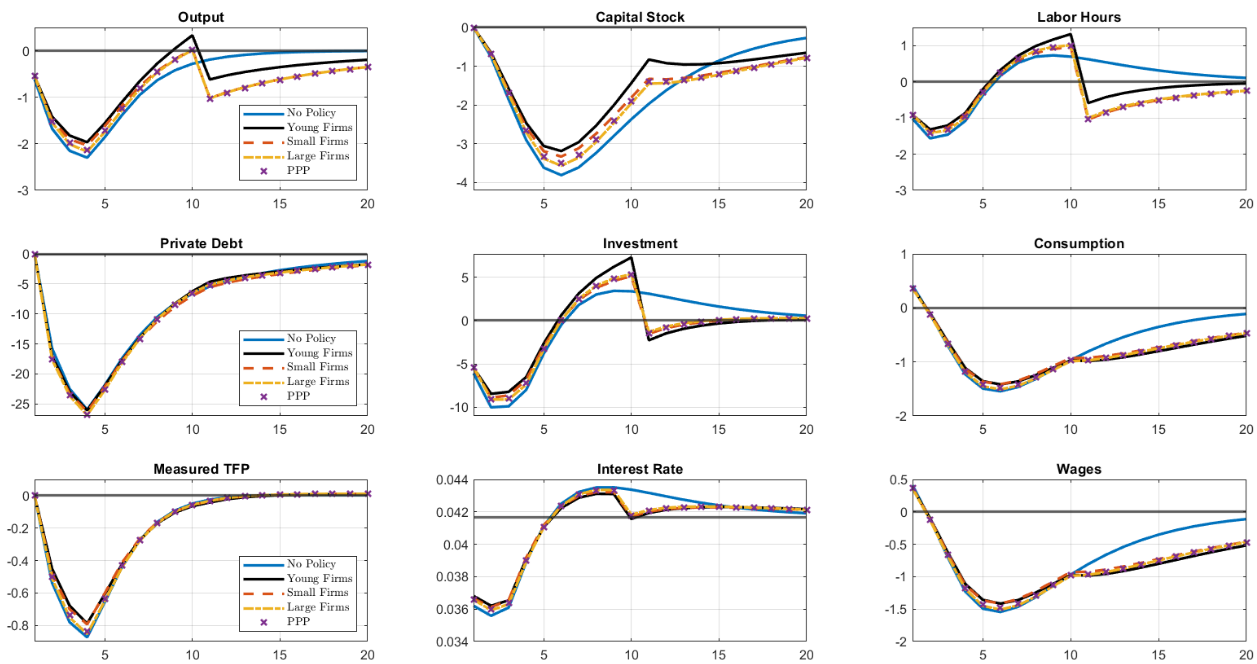
Note: Parameters in top bin are common across all policies, including excess returns-targeted policy. Different size targets are presented to see the results of an equal fraction of debt paid per firm. A test of large firm targeting is studied as well.

Figure 9: Alternative Size Policy Inclusion



Note: Firms included under new small threshold in blue; firms included in new large threshold in red. Firm distribution plotted in the (k, ε) space.

Figure 10: Alternative Size Targets



Note: Impulse responses to a credit shock. The y-axis measures percent deviation from steady state values except for the interest rate. Public debt and taxes are excluded as they are the same across all policies.