DSP 9/11/15

PB 3.30 C

$x[n] = \cos(\frac{1}{2}\pi n)$ $n \in \mathbb{Z}$ $H(z) = \frac{1 - z^{-1}}{1 - \frac{1}{4}z^{-2}}$

$y[n] = A\cos(\pi/2\, n + \phi)$ $\Rightarrow$ always

if $H[n]$ is real, $A = |H(e^{j\pi/2})|$

and $e^{j\pi/2} = j$ $\phi = \mathrm{ATAN}(H(j))$

$e^{-j\pi/2} = -j$

$\frac{|1 + j|}{|1 + \frac{1}{4}j^2|} = \frac{\sqrt{2}}{5/4} = \frac{4\sqrt{2}}{5}$

LTI: freq in = freq out

$\angle \frac{1+j}{1+\frac{1}{4}j} = \angle(1+j) - \angle 5/4$

$\phi = \pi/4 - 0$

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today: inverse z-transform

next week: sampling theorem

midterm 1: through HW 3 everything before sampling

know: $a^n u[n] \overset{z}{\Leftrightarrow}$

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inverse z transform 3 ways:

1. general formula is similar to $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$

2. identify Taylor series expansion $z - \frac{z^2}{2} + \frac{z^3}{3} - \frac{z^4}{4} + \dots$

$= \log(1+z)$

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☆ 3. $H(z) = \dfrac{P(z)}{Q(z)}$ } polynomials

P.F. decomp $\quad X(z) = \dfrac{\sum_0^M b_k z^{*}}{\sum_0^N a_k z^{-k}} = \dfrac{z^{-M} \sum_0^M b_k z^{M-k}}{z^{-N} \sum_0^N a_k z^{N-k}}$

write as product (factorization of a polynomial $\delta^\circ M$)

$$X(z) = \frac{z^{-M} b_0 \prod_{k=1}^{M} (z - \overset{\downarrow \text{zeros}}{z_k})}{a_0 z^{-N} \prod_{k=1}^{N} (z - \underset{\searrow \text{poles}}{p_k})} = \frac{b_0 \prod_{k=1}^{M} (1 - z_k z^{-1})}{a_0 \prod_{k=1}^{N} (1 - p_k z^{-1})}$$

1. check if $H(z)$ is proper / rational

if $M \geq N$ (not proper) $\rightarrow$ do long division and collect poly in z^{-1}

$$\sum_{r=0}^{M-N} c_r z^{-r} \rightarrow \sum_{r0}^{M-N} c_r \delta[n-r]$$

xampl: $\quad X(z) = \dfrac{1 + 3z^{-1} + \frac{11}{6} z^{-2} + \frac{1}{3} z^{-3}}{1 + \frac{5}{6} z^{-1} + \frac{1}{6} z^{-2}} \qquad M = 3, \; N = 2$

want largest powers to match

$$+ 3z^{-1} + \frac{11}{6} z^{-2} + \frac{1}{3} z^{-3} = 2z^{-2}\left(1 + \frac{5}{6} z^{-1} + \frac{1}{6} + z^{-2}\right) \underbrace{+ 1 + z^{-1} + \frac{1}{6} z^{-2}}_{\text{remainder}}$$

$$z^{-1} + \frac{1}{6} z^{-2} = 1 + \frac{5}{6} z^{-1} + \frac{1}{6} z^{-2} + \frac{1}{6} z^{-1}$$

$$(z) = \underbrace{2z^{-1} + 1}_{\checkmark} + \frac{1/6\, z^{-1}}{1 + 5/6\, z^{-1} + 1/6\, z^{-2}}$$

if $N > M$ (proper) and if all poles are simple then

$$X(z) = \sum_{k=1}^{N} \frac{A_k}{1 - P_k z^{-1}} \qquad A_k = X(z)(1 - P_k z^{-1})\Big|_{z=P_k}$$

N ← # of poles

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if $X(z)$ has a pole of order > 1
P_i is an order of "D"

$$X(z) = \sum_{\substack{k \neq i \\ k=1}}^{N} \frac{A_k}{1 - P_k z^{-1}} + \sum_{q=1}^{D} \frac{B_q}{(1 - P_i z^{-1})^q}$$

← different

D = order $\qquad q$ = index $\qquad w = z^{-1}$

$$B_q = \frac{1}{(D-q)!\,(-P_k)^{(D-q)}} \frac{d^{D-q}}{dw^{D-q}}\left[(1 - P_i w)^D X(w^{-1})\right]$$