DSP 8/28/15

frequency domain discrete time:

(1) observe: complex exponentials $x[n] = e^{jwn}$ are "Eigen vectors" of LTI there exists a scalar $\lambda \in \mathbb{C}$ that does not depend on time "n" s.t. $T_x = \lambda x$ only 1

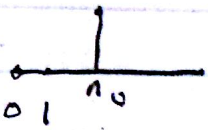
proof: $y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k] = \sum_{k=-\infty}^{\infty} x[k]\, e^{jw(n-k)}$

$$\underline{Y[n] = e^{jwn}} \sum_{k=-\infty}^{\infty} h[k] e^{-jwk} = H(e^{jw})$$

so, $\boxed{@\ w \quad Y[n] = H(e^{jw})\, e^{jwn}}$

example: LTI delay $Y[n] = x[n-n_0]$ $n_0 \in \mathbb{N}$

$h[n] = \delta[n-n_0]$

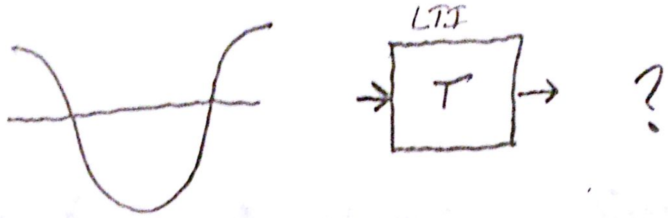


so,

$$H(e^{jw}) = \sum_{k=-\infty}^{\infty} h[k] e^{-jwk}$$

$$H(e^{jw}) = e^{-jn_0 w}$$

and $|H(e^{jw})| = 1$ the phase encodes dela



- output will Δ amplitud
- shift $= \phi + \angle H(e^{j\omega})$

sinusoidal response of LTI with a real impulse

$$x[n] = A\cos(\omega n + \phi) = \frac{A}{2}\left\{ e^{j(\omega n + \phi)} + e^{-j(\omega n + \phi)} \right\}$$

$$= \frac{A}{2}\left\{ e^{j\phi} e^{j\omega n} + e^{-j\phi} + e^{-j\omega n} \right\}$$

$$Tx[n] = \frac{A}{2} e^{j\phi}\, T(e^{j\omega n}) + \frac{A}{2} e^{-j\phi}\, T(e^{-j\omega n})$$

$$= \frac{A}{2} e^{j\phi}\, H(e^{j\omega}) e^{j\omega n} + A/2\, e^{-j\phi}\, H(e^{-j\omega n})\, e^{-j\omega n}$$

$$H(e^{-j\omega}) = \sum_{k=-\infty}^{\infty} h[k] e^{j\omega k} = \left[\sum_{k=-\infty}^{\infty} h[k] e^{-j\omega k}\right]^*$$

$$Tx[n] = \frac{A}{2} e^{j\phi}\, H(e^{j\omega}) e^{j\omega n} + \frac{A^*}{2^*} (e^{j\phi})^*\, H^*(e^{j\omega})\, (e^{j\omega n})^*$$

$$z + z^* = 2\,\mathrm{Re}(z)$$ so,

$$Tx[n] = \mathrm{Re}\left(A\, e^{j\phi}\, H(e^{j\omega})\, e^{j\omega n}\right)$$

$$H(e^{j\omega}) = |H(e^{j\omega})|\, e^{\angle H(e^{j\omega})}$$ so,

$$Tx[n] = A\, \mathrm{Re}\left(|H(e^{j\omega})|\, e^{j(\omega n + \phi + \angle H(e^{j\omega}))}\right)$$

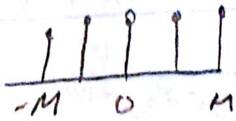
$$\boxed{y[n] = A\, |H(e^{j\omega})| \cos\left(\omega n + \phi + \angle H(e^{j\omega})\right)}$$

Scanned by CamScanner

frequency response "windowing"

1. moving average: $y[n] = \frac{1}{2M+1} \sum_{m=-M}^{M} x[n-m]$

$h[n] = \frac{1}{2M+1} \sum_{m=-M}^{M} \delta[n-m]$



"window"

"window" at zero is low pass filter

$H(e^{jw}) = \sum_{k=-\infty}^{\infty} h[k] e^{-jwk} = \frac{1}{2M+1} \sum_{k=-M}^{M} e^{-jwk}$

$H(e^{jw}) = \left(\frac{e^{jwM}}{2M+1}\right)\left(\frac{e^{-jw(2M+1)} - 1}{e^{-jw} - 1}\right) = \left(\frac{e^{-jw(M+1)} - e^{jwM}}{e^{-jw} - 1}\right)\left(\frac{1}{2M+\ldots}\right.$

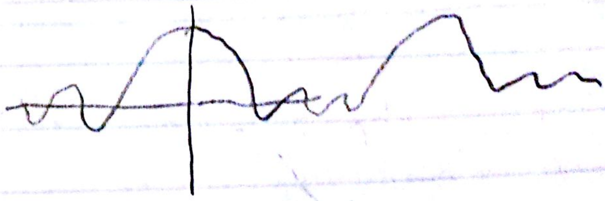
$= \left(\frac{1}{2M+1}\right)\left(\frac{e^{-jw/2}\left(e^{-jw(M+1/2)} - e^{jw(M+1/2)}\right)/2}{e^{-jw/2}\left(e^{-jw/2} - e^{jw/2}\right)/2}\right)$

$= \frac{1}{2M+1} \frac{\sin[(M+1/2)w]}{\sin(w/2)}$ .... when $w \simeq 0$

$\sin(w/2) \simeq w/2$

$$\boxed{H(e^{jw}) \simeq \frac{1}{2M+1} \frac{\sin((M+1/2)w)}{w/2}}$$

DIRICHLET KERNAL



2π periodic