DSP 9/2/15

$\mathcal{F}\left(\sum_{n=-\infty}^{\infty} x_1[n]\right) = \sum \mathcal{F}(x)$ should know $\mathcal{F}(x'(t))$

$= \sum x[n] e^{jw} = X(e^{j\omega})$ $jw X(jw)$

but

$\mathcal{F}\left(\frac{x[n]}{y[n]}\right) \neq \frac{\mathcal{F}x[n]}{\mathcal{F}y[n]}$

<u>properties of the fourier transforme</u>:

<u>linearity</u> $y[n] = \alpha x_1[n] + \beta x_2[n]$ $\alpha, \beta \in \mathbb{C}$

$Y(e^{jw}) = \alpha X_1(e^{jw}) + \beta X_2(e^{jw})$

<u>parity</u> $x[n] \in \mathbb{R}$ $X(e^{jw}) \in \mathbb{C}$: twice as much data

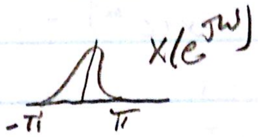
$Re(X(e^{jw})) \rightarrow$ even $Im(X(e^{jw})) \rightarrow$ odd function of

$|X(e^{jw})| \rightarrow$ even

$\measuredangle X(e^{jw}) \rightarrow$ odd $X(e^{jw}) = |X(e^{jw})| e^{j\measuredangle X(e^{jw})}$

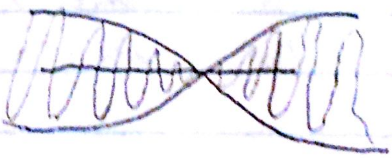
<u>time shifting</u> $y[n] \triangleq x[n-n_0]$ $Y(e^{jw}) = e^{-jwn_0} X(e^{jw})$

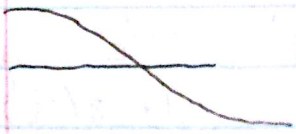
<u>modulation</u> $y[n] \triangleq x[n] e^{jw_0 n}$ w_0 is close to π



as $w \rightarrow \pi$,
make envelope

Amplitud modulation





$Y(e^{jw}) = X(e^{j(w-w_0)})$

differentiation in time : ?

differentiation in freq $j\frac{d}{d\omega}X(e^{j\omega}) = Y(e^{j\omega})$ $\quad y[n] = n\,x[n]$

parsivals theorm: $\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$

→ mag of C #

→ spectral energy dens.

convolution: if $z[n] \triangleq x[n] * y[n]$

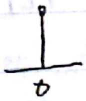
then $Z(e^{j\omega}) = X(e^{j\omega})\,Y(e^{j\omega})$ → range is always larger then the individuals

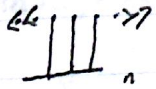
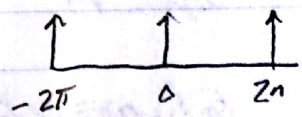
product. if $z[n] = x[n]\,y[n]$

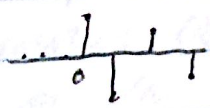
then $Z(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})\,Y(e^{j(\omega-\theta)})\,d\theta$

$$Z(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} Y(e^{j\theta})\,X(e^{j(\omega-\theta)})\,d\theta$$

Fourier transforms (expected)

1. $\delta[n] = \begin{cases} 1 & \text{if } n=0 \\ 0 & \text{else} \end{cases}$  <=> 

2. $v[n] = 1$ for all n  <=>  $V(e^{j\omega}) = 2\pi \sum \delta(\omega - 2$

3. $a^n u[n]$, $|a|<1$  <=> $\frac{1}{1-ae^{-j\omega}}$

4. $u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & \text{else} \end{cases}$ <=> $U(e^{j\omega}) = \underbrace{\frac{1}{1-e^{-j\omega}}}_{\text{jump @ } n=0} + \underbrace{\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)}_{\frac{1}{2} \text{ of } V[n]}$

Z transform

① consider $u[n]$ and Dirac impulses to justify the existance of $\mathcal{F}\{u[n]\}$

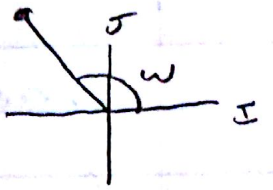
② is there a better transform?
↳ can fix $u[n]$ and compute a fourier trans
↳ introduce convergence factor

define: $\rho^n u[n]$ $\quad |\rho| < 1$

$$= \sum_{n=-\infty}^{\infty} \rho^n u[n] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} u[n] \Big[\underbrace{\tfrac{1}{\rho} e^{j\omega}}_{z}\Big]^{-n} = V[z]$$

$V[z]$ depends on $\angle z = \omega$
$|z| = \frac{1}{\rho}$



HOPE: z transform inherits all properties of F.T.

- need to go back to unit circle
- if $z = e^{j\omega}$ we dont need ρ this is a F.T.