DSP 8/26/15

Moving average → looking at "m" samples back in time

linear system → T is linear if for any seq. x and y
and any complex numbers α β
$T(\alpha x + \beta y) = \alpha T_x + \beta T_y$

NOTE: most systems are non-linear
1. linear systems allow us to build a spectral theory
2. gives a good first order approx

time invariance: $T(x[\cdot - n_0]) = T_x[\cdot - n_0]$

causal: a system T is causal if $T_x[n]$ depends only on the past values of x

memoryless: T is memoryless if $T_x[n]$ depends only on $x[n]$

looking at linear time invariant:

1. moving avg $\quad Y[n] = \frac{1}{M} \sum_{k=0}^{M-1} x[n-k]$
   - linear ✓ → proof $T(x_1 + x_2)$ vs $T(x_1) + T(x_2)$
   - time inv. ✓ → only depends on last M values

2. compressor / downsampler $\quad Y[n] = X[Mn]$
   - linear ✓
   - time inv. ✓

stability: input must be bounded
constant $A > 0$ S.T. $|x[n]| \le A$ for all n
then the output is also bounded $\exists B > 0$
S.T. $|T_x[n]| \le B$

Ex: $T_x[n] = x^2[n] \quad |x[n] \le A| \quad |T_x[n]| \le A^2 = B$ ✓ stable

$T_x[n] = n^2 x[n]$ unstable

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linear time invarient w/ impulse response

intro: instead of studying T, we can narrow down the analysis to a signal sequence. this seq. is the result to a seq

proof: $x[n] = \sum_{k=-\infty}^{\infty} x[n-k]\,\delta[k] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$

T: linear time invarient

$y[n] = T_x[n] = T(\sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$

so,

$x[n] = \sum_{k=-\infty}^{\infty} T(x[k]\,\delta[n-k]) = \sum_{k=-\infty}^{\infty} x[k]\,T(\delta[n-k]$

define the impulse response of T

$T\delta = n$

by time invarience, $T(\delta(n-k) = \underbrace{T\delta}_{h}[n-k]$

$h[n-k]$

$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h(n-k]$

conclusion: the output to a linear time inv system is given by the discrete time convolution

$$y[n] = x * h[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] = \sum_{k=-\infty}^{\infty} x[n-k]\,h[k]$$

<u>testing stability</u>:

remark: in cont time $h(t)$ $\int_{-\infty}^{\infty} |h(t)|\,d < \infty$

<u>discrete stable</u>: $\sum_{n-\infty} |h[n]| < \infty$ → impulse response must decay

<u>theorem</u>: A linear time invarient iff $\sum_{n-\infty}^{\infty} |h[n] < \infty$

stability proof

- sufficient condition:

if $\sum_{n=-\infty}^{\infty} |h[n]| < K$

consider bounded input $x[n] = A > 0$ $|x[n]| \leq A$

check output $B > 0$, $T_x[n] \leq B$

$$|T_x[n]| = \left|\sum_{k=-\infty}^{\infty} h[k]\, x[n-k]\right|$$
$$\leq \sum |h[k]||x[n-k]|$$
$$\leq A \sum_{k=-\infty}^{\infty} |h[k]| \leq AK = B$$

causal proof T is causal if $h[n] = 0$ if $n < 0$

$\sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$