lecture 8/31/15

z- and laplace dont give physical intuition

<u>fourier:</u> if $x[n] \in \mathbb{C}$ is discrete then we formally define the fourier transform of x to be

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

problem: need conditions for $X(e^{j\omega})$ exists

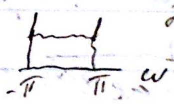
sufficient condition:

1. $\sum_{n=-\infty}^{\infty} |x[n]| < \infty$ → absolutely summable

2. finite energy $\sum_{n=-\infty}^{\infty} |x[n]|^2 < \infty$

$$\lim_{n\to\infty}\int_{-\pi}^{\pi}\Big|\underbrace{\sum_{n=-N}^{N} x[n]e^{-j\omega n}}_{\text{finite sum}} - X(e^{j\omega})\Big|^2 d\omega = 0$$

note: GIBBS



<u>inverse fourier:</u> given $X(e^{j\omega})$ we can reconstruct $x[n]$ using

$$X(e^{-j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

so,

$$e^{-j\omega n} = e^{-j(\omega+2\pi)n} = e^{-j\omega n}\underbrace{e^{-j2\pi n}}_{1}$$

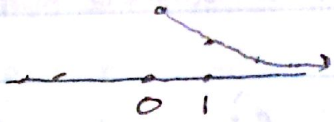
note frequency domain is continuous so,

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega \qquad \text{(synthesis)}$$

impulse response example:

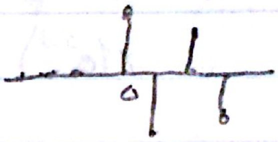
$h[n] = a^n u[n] \quad |a| < 1$

@ $a = 1/2$



$Y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$

@ $a = 0.9$



@$a=1/2$ $= \sum_{k=0}^{\infty} a^k x[n-k] = x[n] + \frac{1}{2} x[n-1] + \frac{1}{4} x[n-2]$

@$a=-0.9$ $= x[n] - 0.9\, x[n-1]$ $\longrightarrow$ high pass filter

averaging the past but exponentially forgetting the past

proof:

$H(e^{jw}) = \sum_{n=0}^{\infty} h[n] e^{-jwn} = \sum_{n=0}^{\infty} (a e^{-jw})^n = \frac{1}{1 - ae^{-jw}}$

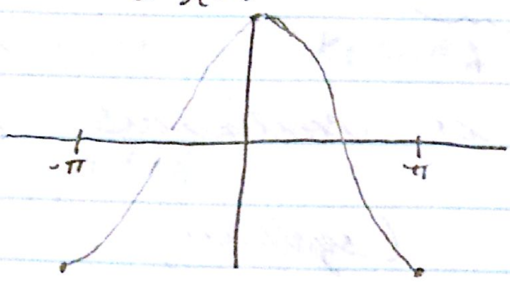
1. compute magnitude $H(e^{jw}) \cdot H(e^{jw})^*$

$= \frac{1}{(1-ae^{-jw})} \cdot \frac{1}{1 - a^* e^{jw}}$

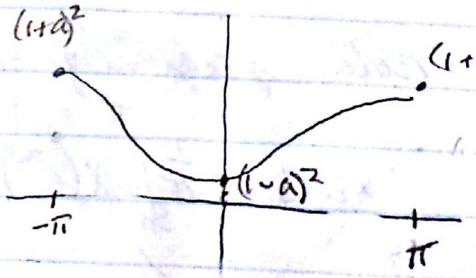
$= \frac{1}{1 + |a|^2 - 2a\cos(w)}$ if $a > 0$

$= \frac{1}{1 + a^2 - 2a\cos(w)}$ $\rightarrow$ mag is real so $H(e^{jw})$ is even

$\cos(w)$



$1 + a^2 - 2a\cos(w)$



high pass

$h(e^{jw})$ is low pass $1/$high pass

example 2: $x[n] = 1$ for all n

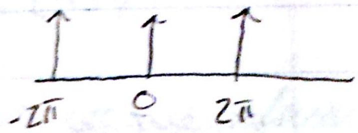
$$\Sigma |x[n]| = \infty$$
$$\Sigma |x[n]|^2 = \infty$$

claim: the fourier transform is a $\delta[n]$

proof 1:

$$X(e^{j\omega}) = 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$$

↑ needs to be periodic



$$= \frac{1}{2\pi} \cdot 2\pi \int_{-\pi}^{\pi} \delta(\omega) e^{j\omega} d\omega$$

$$= \int_{-\infty}^{\infty} \delta(\omega) e^{j\omega} d\omega = \int_{-\infty}^{\infty} \delta(\omega) d\omega = 1$$

proof 2: (informal) $x[n] = 1$ for all n

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} e^{-j\omega n} = 1 + \sum_{n=-\infty}^{-1} e^{-j\omega n} + \sum_{n=1}^{\infty} e^{-j\omega n}$$

$$= 1 + \underbrace{\sum_{n=1}^{\infty} e^{j\omega n} + \sum_{n=1}^{\infty} e^{-j\omega n}}_{\text{eulers}}$$

$$= 1 + 2 \sum_{n=1}^{\infty} \cos(n\omega)$$

@ $\omega = 0$ $X(e^{j\omega}) = \infty$

@ $\omega = 2\pi$ $X(e^{j\omega}) = \infty$ for $\omega = 2\pi k$, $X(e^{j\omega}) = \infty$

@ $\omega = \pi$ $X(e^{j\omega}) = ?$ $\sum_{n=1}^{\infty} = -1 + 1 - 1 + 1$

$0 < r < 1$ $\sum_{r=1}^{\infty} (-r)^n = -\frac{1}{2}$ so,

$1 + 2(-\frac{1}{2}) = 0$