DSP lecture 9/4/15

z transform:

R.O.C: when the $\sum_{n=-\infty}^{\infty} x[n] z^{-n}$ converges then, z is within the region of convergence of the z transform of x

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \quad z \in R.O.C.(X)$$

note: if unit circle $\in$ ROC, we can recover the fourier transform of x by evaluating X(z) on the unit circle

$$z = e^{jw} \quad w \in [-\pi, \pi]$$

$$X(z)\Big|_{z=e^{jw}} = X(e^{jw})$$

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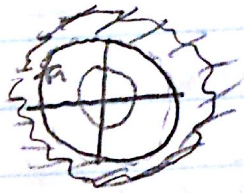
example: $x[n] = a^n u[n] \quad a \in \mathbb{C}$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

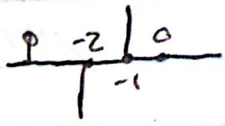
$$= \sum_{n=0}^{\infty} \left(\frac{a}{z}\right)^n \qquad \text{B.C} \quad a^n z^{-n} = \left(\frac{a}{z}\right)^n$$

$$X(z) = \frac{1}{1-\frac{a}{z}} \quad \text{iff } \left|\frac{a}{z}\right| < 1 \quad \text{or} \quad |a| < |z|$$

$$\boxed{X(z) = \frac{1}{1-\frac{a}{z}} = \frac{1}{1-az^{-1}}}$$



example: $x[n] = -a^n u[-n-1]$



$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=-\infty}^{-1} a^n z^{-n}$$

get rid of neg. indicies $(-n \to m)$

$$= - \sum_{m=1}^{\infty} a^{-m} z^{n}$$

$$= \sum_{m=1}^{\infty} \left(\frac{z}{a}\right)^m = -\frac{z}{a}\left(\frac{1-0}{1-\frac{z}{a}}\right) \quad \text{true iff } \left|\frac{z}{a}\right| < 1$$

note:

$$\sum_{n=n_0}^{n_1} r^n = r^{n_0}\left(\frac{r^{(n_1-n_0+1)}-1}{r-1}\right) = \frac{r^{n_1-1} - r^{n_0}}{r-1}$$

$$X(z) = \frac{z}{a}\,\frac{1}{\frac{z}{a}-1} = \frac{(z/a)}{z/a - 1}$$

$$\boxed{X(z) = \frac{1}{1-\frac{a}{z}} = \frac{1}{1-az^{-1}}}$$

same z transform

ROC is inside $\left|\frac{z}{a}\right| < 1$

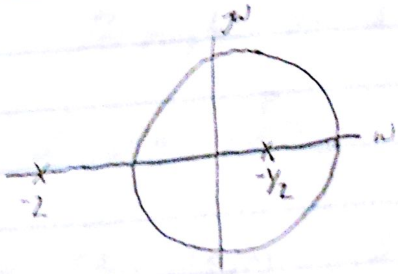
common z transforms + R.O.C

$x[n]$		$X(z)$	ROC
$\delta[n]$	$\Leftrightarrow$	1	$\mathbb{C}$
$u[n]$	$\Leftrightarrow$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$ (causal)
$a^n u[n]$	$\Leftrightarrow$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\Leftrightarrow$	$\frac{1}{1-az^{-1}}$	$\lvert a\rvert > \lvert z\rvert$ (anti causal)

properties of z transform and R.O.C

1. the ROC of any seq is always a ring $R_1 < |z| < R_2$
2. if $x[n]$ has a F.T. ROC contains unit circle
3. if $x[n]$ is finite, the ROC is all of $\mathbb{C}$
   ↳ You may need to remove 0
4. if there exists N_0 S.T. $x[n] = 0$ for all $n \leq N_0$ then the ROC has the form $|z| > R_1$
5. if there exists N_1 S.T. for all $n \geq N_1$ $x[n] = 0$ then the ROC has the form $|z| < R_2$

Example:



pole-zero plot $X(z)$

What is $x[n]$? → there are 3 ROC's
$|z| > 2$
$\frac{1}{2} < |z| < 2$
$|z| < \frac{1}{2}$

$$X(z) = \frac{\quad}{(1-\frac{1}{2}z^{-1})(1+2z^{-1})}$$