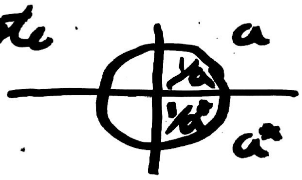
5.6.1 min phase + all pass

$$H_{MIN}(z) = H(z)\, H^*(1/z^*)\big|_{z=e^{jw}}$$

note



$$H(z) = H_{MIN}(z) \cdot H_{ap}(z)$$

prove: assume zero outside unit circle @ $1/c^*$
$|c| < 1$

$$H(z) = H_1(z)(z^{-1} - c^*) = \underbrace{H_1(z)(1 - cz^{-1})}_{H_{MIN}} \underbrace{\frac{z^{-1} - c^*}{1 - cz^{-1}}}_{H_{ap}}$$

5.6.2 ~

5.6.2 freq resp compensation of non min phase system

perfect compensation: $s_c[n] = s[n]$ and $H_c(z) = 1/H_d(z)$

→ assume H_d is stable and causal

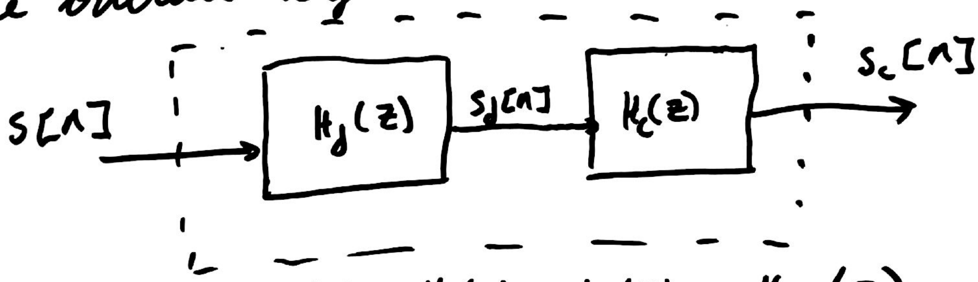
→ require H_c to be stable + causal

→ H_d is min phase

→ if H_d is a rational system function, we can make $H_{d\,min}$ from H_d

→ $H_d = H_{d\,min} H_{ap}$ and $H_c = 1/H_d$

→ the overall system function $s_c[n] = s[n]$



$G(z) = H_d(z)\ H_c(z) = H_{ap}(z)$

→ magnitude is exactly compensated

→ phase modified : $\angle H_{ap}(e^{j\omega})$

5.6.3 properties of minimum phase systems

→ relate min phase system to other system with same magnitude

→ unwrapped phase $\measuredangle H(e^{j\omega})$

→ $\measuredangle H(e^{j\omega}) = \measuredangle H_{MIN}(e^{j\omega}) + \measuredangle H_{ap}(e^{j\omega})$

→ unwrapped phase of all pass is negative from $0 \le \omega \le \pi$

→ reflecting zeros outside the unit circle causes the phase to become more negative

min phase lag function

→ make min phase lag more precice

→ $H(e^{j\omega}) > 0$ @ $\omega = 0$

→ note: if $h[n]$ is Re then $H(e^{j0})$ is Re

→ do this because $h[n]$ and $-h[n]$ have the same pole/zeros

→ multiplying by -1 alters the phase by π

Minimum group delay

→ for system w/ same magnitude

$$grd[H(e^{j\omega})] = grd[H_{MIN}] + grd[H_{ap}]$$

→ group delay for min phase is always less than grd. of non min phase

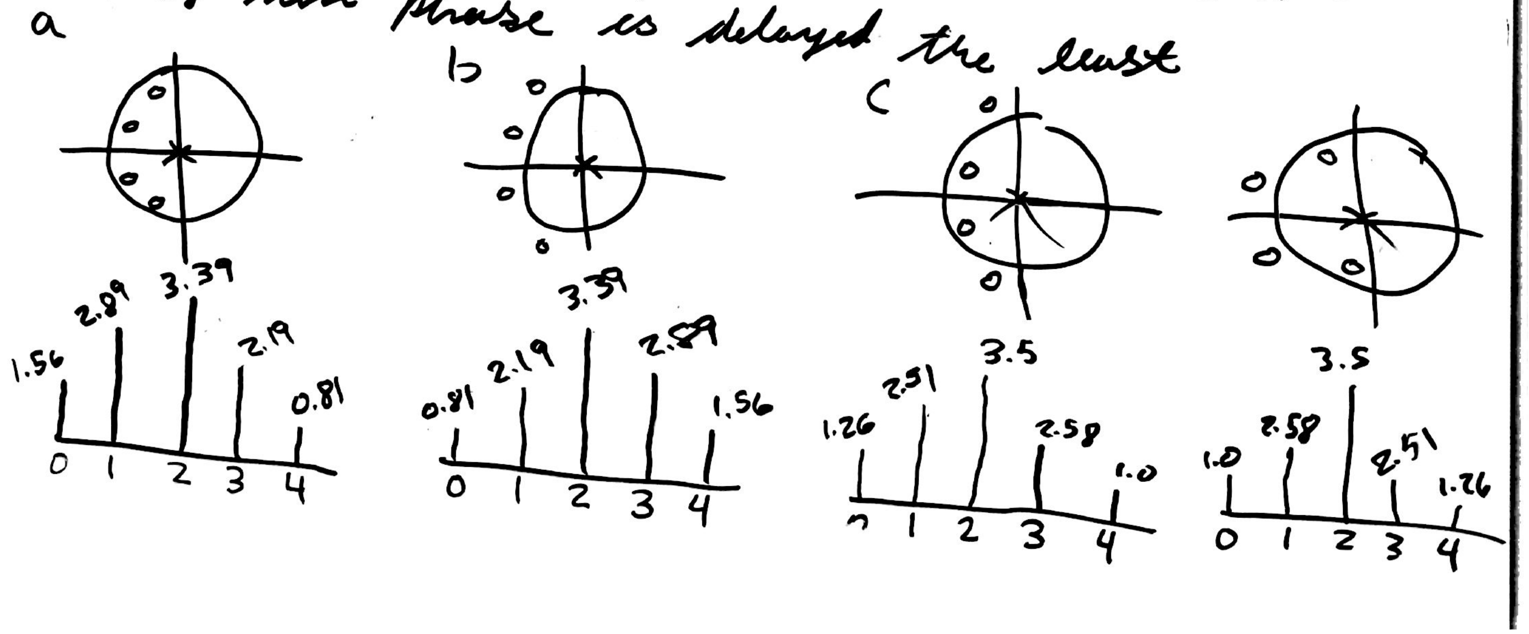
→ B.C. H_{ap} has positive grd always

→ rational system function (P/Z inside unit circle) has min group delay

min energy delay property

if $|H(e^{j\omega})| = |H_{MIN}(e^{j\omega})|$ then $|h[0]| \leq |h_{MIN}[0]|$

- since $|H(e^{j\omega})| = |H_{MIN}(e^{j\omega})|$ → systems have the same energy
- proof: PDF PG 350
- $$\sum_{m=0}^{n} |h[m]|^2 \leq \sum_{m=0}^{n} |h_{MIN}[m]|^2$$
- partial E of impulse response
- partial E of min phase is concentrated @ n=0
- E of min phase is delayed the least



5.1.1 freq response + phase + group delay

$|Y(e^{j\omega})| = |H(e^{j\omega})|\,|X(e^{j\omega})|$ ⟶ mag

$\measuredangle Y(e^{j\omega}) = \measuredangle H(e^{j\omega}) + \measuredangle X(e^{j\omega})$ ⟶ phase shift

- phase is not unique
  - ⟶ multiples of 2π
  - ⟶ atan obtains principle value
- <u>principle phase</u>: $ARG\{H(e^{j\omega})\} \in [-\pi, \pi]$
- $\measuredangle H(e^{j\omega}) = ARG\{H(e^{j\omega})\} + 2\pi r(\omega)$
- principle phase often has discontinuity @ 2π
- ARG ⟶ principle phase ⟶ wrapped phase
  - ⟶ in amp/phase representation (amp must be real)
    - ⟶ ARG can be unwrapped to a continuous phase curve ⟶ $arg\{H(e^{j\omega})\}$
- <u>Group delay</u>: $\tau(\omega) = grd\{H(e^{j\omega})\} = -\frac{d}{d\omega}\{arg\{H(e^{j\omega})\}\}$
  - ⟶ ARG and arg will have equal deriv. except for impulses @ discontinuities of ARG

    $-\frac{d}{d\omega}\{arg[H(e^{j\omega})]\} = -\frac{d}{d\omega}\{\measuredangle H(e^{j\omega})\}$ ⟶ if we ignore impulses caused by discontinuities @ 2π
  - ⟶ arg ⟶ continuous phase curve
  - ⟶ ARG ⟶ wrapped phase curve
  - ⟶ $\measuredangle H(e^{j\omega})$ ⟶ ambiguous phase

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5.7 linear systems w/ generalized linear phase

intro → in pass band want filters w/ → constant magnitude
→ zero phase

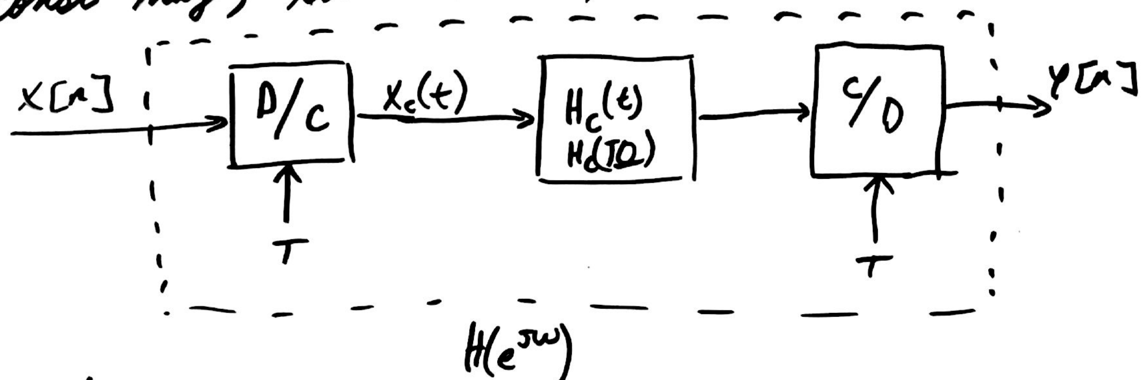
- → ϕ phase not possible in causal filter
- → linear phase w/ integer slope → time shift
- → non linear phase → effects shape of signal
- → this section looks at ideal time delay and linear phase by looking at systems w/ const. group delay

5.7.1 linear phase

ideal delay: $H_{id}(e^{j\omega}) = e^{-j\alpha\omega}$ $|\omega| < \pi$ and α is Re

→ const mag, linear phase, const group delay

→



how to think about non-integer delay in discrete time

$|H(e^{j\omega})| = 1$ $\quad \measuredangle H(e^{j\omega}) = -\omega\alpha$ $\quad \tau(\omega) = \alpha$

→ $IFT\{H(e^{j\omega})\} = \dfrac{\sin(\pi(n-\alpha))}{\pi(n-\alpha)}$

→ $Y[n] = X[n] * \dfrac{\sin(\omega_c(n-\alpha))}{\pi(n-\alpha)} = \sum_{k=-\infty}^{\infty} X[k] \dfrac{\sin(\omega_c(n-k-\alpha))}{\pi(n-k-\alpha)}$ $\quad \omega_c = \pi$

→ if $\alpha = n_d$ and n_d is an integer

$h_{id}[n] = \delta[n-n_d]$

$Y[n] = X[n-n_d]$

→ if α is not an integer, represent as sampled cont time signal

$Y_c(t) = X(nT - \alpha T)$ where T can be anything

→ α is + → time delay

→ α is − → time advance

note linear phase = const group delay

→ $H(e^{j\omega}) = |H(e^{j\omega})| e^{-j\alpha\omega} \quad |\omega| < \pi$

→ nonconstant mag, linear phase

$$x[n] \rightarrow \boxed{|H(e^{j\omega})|} \xrightarrow{w[n]} \boxed{e^{-j\alpha\omega}} \xrightarrow{y[n]}$$

→ $h_{lp}[n] = \dfrac{\sin(\omega_c(n-\alpha))}{\pi(n-\alpha)}$ $\qquad H_{lp}(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega} & |\omega| < \omega_c \\ 0 & \omega_c \le |\omega| < \pi \end{cases}$

→ ideal low pass w/ linear phase EX: pg 353

→ ILP:

if α is an integer $\alpha = n_d$

→ response is symmetric @ n_d

→ shift system to the left by n_d and the system is <u>even</u> and has zero phase → $h_{lp}[n] = \dfrac{\sin \omega_c n}{\pi n} = h_{lp}[-n]$

→ if α is a half integer,

$h_{lp}[2\alpha - n] = h_{lp}[n]$

→ not possible to obtain even function with zero phase

→ if 2α is an integer the response is symmetric about α

→ if 2α is not an integer, not symmetric

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5.7.2 general linear phase

has form $H(e^{jw}) = A(e^{jw}) e^{-j\alpha w + j\beta}$

- α and β are const.
- $A(e^{jw})$ is real

$\tau(\omega) = \alpha$

$\measuredangle H e^{jw} = \beta - \alpha w \quad 0 \le w < \pi$

---

constrain to const group delay

$$H(e^{jw}) = A(e^{jw}) e^{-j(\alpha w - \beta)} = A(e^{jw}) \cos(\beta - \alpha w) + j A(e^{jw}) \sin(\beta - \alpha w)$$

---

causal FIR system has general linear phase is,

1. $h[2\alpha - n] = h[n]$ → symmetric about α
   → α is integer → zero phase
   → α is half integer → non-zero phase

2. $h[2\alpha - n] = -h[n]$

3. impulse response has len $(M+1)$

$$h[n] = \begin{cases} h[M-n] & \text{for } 0 \le n \le M \\ 0 & \text{else} \end{cases}$$

→ $H(e^{jw}) = A(e^{jw}) e^{-jwM/2}$ where $A(e^{jw})$ is real periodic even

$$h[n] = \begin{cases} -h[M-n] & \text{for } 0 \le n \le M \\ 0 & \text{else} \end{cases}$$

→ $H(e^{jw}) = jA_o(e^{jw}) e^{-jwM/2} = A_o(e^{jw}) e^{-j\left(\frac{wM}{2} - \frac{\pi}{2}\right)}$

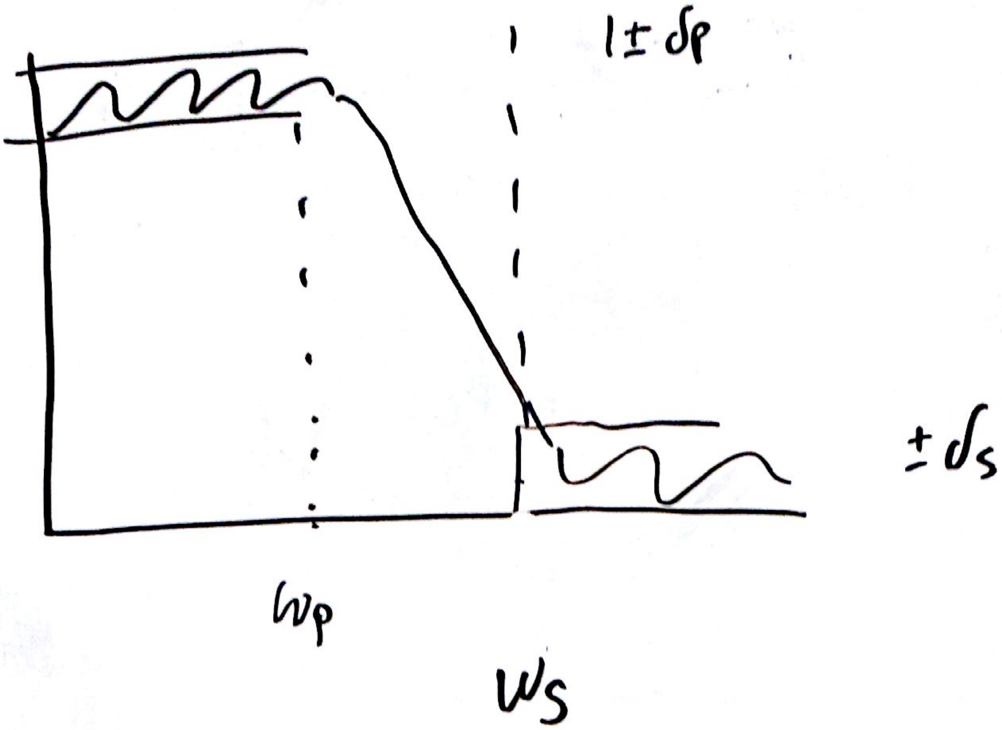
---

types of FIR linear phase systems

pg 358

5.7.4 relate FIR linear phase systems to min phase systems

7.1 filter spec.



discrete filter, continuous signal

$$H_{eff}(j\Omega) = \begin{cases} H(j\Omega) & \text{for } |\Omega| < \pi/T \\ 0 & \text{for } |\Omega| \geq \pi/T \end{cases}$$

use $\omega = \Omega T$

$$H(e^{j\omega}) = H_{eff}\left(j\frac{\omega}{T}\right) \text{ for } |\omega| < \pi$$

7.2 design of discrete IIR from continuous time filters

method : $h_c(t) \longrightarrow \boxed{C/D} \longrightarrow h[n]$ S.T. $h[n]$ meets specs.
T

+ C.T. IIR has closed form so D.T. representation is easy
+ approx methods in C.T. do not work if applied to D.T. directly
  B.C. → D.T. freq resp is periodic where C.T. is not

☆ use C.T. approximations for convenience <u>only</u>.

☆ D.T. filter design uses discrete time specifications

- → get specs for $h_c(t)$ by transforming specs for $h[n]$
- → take laplace T.F. of $h_c(t) \iff H_c(s)$
  - → methods in <u>Appendix B</u>
- → transform $h_c(t) \longrightarrow h[n]$
  $H_c(s) \longrightarrow H(z)$
  - → Im(s) maps to unit circle of z
  - → stable in C.T → stable in D.T.
    - → S-domain → poles left half of plane
    - → z-domain → poles in unit circle

7.2.1 impulse invarience

impulse invarience sampling the impulse response of C.T. filter

- useful if we want to sim. a C.T. system in D.T.
- keep time domain characteristics of C.T. filters in D.T
  - overshoot, energy compaction, controlled ripple

- method of obtaining D.T system whos freq resp is determined from C.T. system

- $$H(e^{j\omega}) = \sum_{k=0}^{\infty} H_c\left(j\left(\frac{2\pi k}{T_d} + \frac{j\Omega}{T_d}\right)\right)$$

  if $H_c(j\Omega) = 0$ for $|\Omega| \geq \frac{\pi}{T_d}$

  then
  $$H(e^{j\omega}) = \sum_{k=0}^{\infty} H_c\left(j\frac{\omega}{T_d}\right) \quad |\omega| \leq \pi$$

- $H_c(j\Omega)$ cannot be exactly Band limited but if it gets small @ ω close to π the aliasing is negligible

- $$h[n] = T_d\, h_c(nT_d) = \sum_{k=1}^{N} T_d A_k e^{s_k n T_d}\, u[n]$$
  $$= \sum_{k=1}^{N} T_d A_k (e^{s_k T_d})^n\, u[n]$$
  $$H(z) = \sum_{k=1}^{N} \frac{T_d A_k}{1 - e^{s_k T_d} z^{-1}}$$

- impulse invarience w/ a butterworth filter PG 529

7.2.2 bilinear transform

S-domain

$-\infty \leq \Omega \leq \infty$

Z domain

$-\pi \leq \omega \leq \pi$

* transform between D.T. and C.T. is non-linear

$$S = \frac{2}{T_d}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) \qquad z = \frac{1+(T_d/2)S}{1-(T_d/2)S} \quad \text{and } S = \sigma + j\omega$$

* T_d is of no consequence for impulse invar. or bilinear

* $\sigma < 0$ $|z| < 1$

  $\sigma > 0$ $|z| > 1$

  $\sigma = 0$ $|z| = 1$ → F.T. exists, sub $S = j\Omega$

→ $$z = \frac{1 + j\Omega T_d/2}{1 - j\Omega T_d/2}$$

→ sub $z = e^{j\omega}$

→ $$e^{j\omega} = \frac{1 + j\Omega T_d/2}{1 - j\Omega T_d/2}$$

→ $$S = \frac{2}{T_d}\left(\frac{1-e^{-j\omega}}{1+e^{-j\omega}}\right) = \frac{j2}{T_d}\tan(\omega/2)$$

→ $$\Omega = \frac{2}{T_d}\tan(\omega/2)$$

→ $$\omega = 2\,\mathrm{atan}(\Omega T_d/2)$$