

Third and Fourth Sound in Liquid Helium II†

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This article discusses the possible existence of two hitherto undetected types of wave propagation in liquid helium II. Third sound is a surface wave of long wavelength on a liquid helium film during which the normal component remains stationary and the superfluid component oscillates parallel to the wall. To treat this properly it is necessary to consider temperature changes and evaporation from the surface of the film. Fourth sound may exist in narrow two-sided channels. The normal component again remains stationary and the superfluid component oscillates parallel to the wall, but the width of the channel must remain fixed and so there are oscillations in both total density and temperature.

1. INTRODUCTION

TO discuss wave propagation in liquid helium II, it is necessary to write down two separate hydrodynamical equations, one for the superfluid component and the other for the normal component. In first sound the two components move in the same direction in phase, and there is a first-order oscillation of the density but only a second-order oscillation of the temperature. In second sound the two components move in opposite directions out of phase, and the temperature oscillation is then first-order while the density oscillation is only second-order. Under the special circumstances to be considered in this article, there may exist an essentially different type of oscillation, during which the normal component remains stationary and the superfluid component alone oscillates.

2. SURFACE WAVES ON A LIQUID HELIUM FILM

Surface waves on bulk liquid helium were first discussed by Atkins¹ in order to explain the variation of surface tension with temperature. The phase velocity of such a wave is

$$v_p = \left(\frac{g\lambda}{2\pi} + \frac{2\pi\sigma}{\rho\lambda} \right)^{\frac{1}{2}}, \quad (1)$$

where λ is the wavelength, g is the acceleration due to gravity, σ is the surface tension, and ρ is the density of the liquid. The surface waves which affect the surface energy and hence the surface tension have such short wavelengths that the term involving g is unimportant.

Kuper² has suggested that these surface waves may be relevant to the critical velocity of superflow of a liquid helium film. According to Landau,³ when liquid helium II flows through a narrow channel the kinetic energy of flow of the superfluid component can be dissipated only by the creation at the wall of elementary

excitations of energy ϵ and momentum p , and this is not possible until the velocity of flow exceeds ϵ/p . The flow is therefore frictionless up to a critical velocity v_{crit} corresponding to the creation of those excitations for which $\epsilon/p (= v_{\text{crit}})$ is a minimum. Kuper suggests that the relevant excitations are quantized surface waves, or *ripples*. ϵ/p is then equal to the phase velocity as given by Eq. (1), and when $\lambda \sim 0.3$ cm it assumes a minimum value ~ 10 cm sec⁻¹, which is comparable with the experimental value of about 25 cm sec⁻¹ for the critical velocity of the film.

However, Atkins⁴ has pointed out that for a thin film the restoring force is not gravity but the forces which are responsible for the formation of the film, including the van der Waals forces of attraction between the helium atoms and the wall on which the film is formed. Also, it is important to use the formula for a surface wave on liquid of finite depth d equal to the film thickness:

$$v_p^2 = \left(\frac{f\lambda}{2\pi} + \frac{2\pi\sigma}{\rho\lambda} \right) \tanh\left(\frac{2\pi d}{\lambda}\right). \quad (2)$$

This is a minimum when λ is infinite and gives a critical velocity:

$$v_{\text{crit}}^2 = v_{p\infty}^2 = fd. \quad (3)$$

If f were due only to van der Waals forces, we would have

$$f = 3\alpha/d^4, \quad (4)$$

and the thickness d of the film at a height H above the surface of the bulk liquid would be given by

$$\alpha/d^3 = gH, \quad (5)$$

whence

$$v_{\text{crit}} = (3gH)^{\frac{1}{3}} \sim 50 \text{ cm sec}^{-1} \text{ if } H \sim 1 \text{ cm}. \quad (6)$$

This is again in good order of magnitude agreement with experiment. Actually there are other factors besides the van der Waals forces entering into the

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¹ K. R. Atkins, Can. J. Phys. **31**, 1165 (1953).

² C. G. Kuper, Physica **22**, 1291 (1956).

³ L. D. Landau, J. Exptl. Theoret. Phys. (U.S.S.R.) **5**, 71 (1941); **11**, 91 (1947).

⁴ K. R. Atkins, Physica **23**, 1143 (1957).

formation of the film,⁵⁻⁷ and the exact form of f is complicated and not completely understood, although its magnitude is probably not very different from the value arising from van der Waals forces alone.

Although the above arguments may be valid at 0°K, at a finite temperature it is necessary to use the two-fluid theory of liquid helium II and to write separate hydrodynamical equations for the normal and superfluid components. Arkhipov⁸ has attempted this, but has assumed, without explicitly stating his reasons, that $\text{div } \mathbf{v}_s = 0$ and $\text{div } \mathbf{v}_n = 0$. This assumption that the two components separately behave like incompressible fluids implies that there are no temperature gradients in the liquid $[(\partial/\partial t)(\rho S) + \rho S \text{div } \mathbf{v}_n = 0]$. In the analysis to be presented in this article, temperature gradients play an important role.

3. THIRD SOUND IN A LIQUID HELIUM FILM

We are interested in the case of wavelengths very long compared with the thickness of the film. This is analogous to the classical case of a long wave on water in a shallow channel, and it is well known that the oscillatory motion of an element of the fluid is mainly in a direction parallel to the bottom of the channel. We shall argue that the superfluid component oscillates in this fashion with its velocity \mathbf{v}_s almost parallel to the wall on which the film is formed. Because of its viscosity, the velocity v_{nx} of the normal component parallel to the wall is negligibly small compared with v_{sx} . (The z axis is perpendicular to the wall and the x axis is in the direction of propagation of the wave.) If a plane surface oscillates parallel to itself in the bulk liquid with angular frequency ω , the motion penetrates exponentially into the normal component with a characteristic "penetration depth" $(2\eta_n/\omega\rho_n)^{1/2}$. In the case of a film of thickness d , since the velocity of the normal component at the wall must be zero relative to the wall, it is clear that it is very difficult to make the normal component oscillate parallel to the wall if $d \ll (2\eta_n/\omega\rho_n)^{1/2}$. For the superfluid component we do not have to worry about viscosity and it is usually assumed that the tangential velocity at the wall can be finite. We shall in fact assume that the velocity of the superfluid component is independent of depth, although there is no direct experimental evidence for this.

To a first approximation, then, the normal component is at rest and there is a surface wave on the superfluid component, although there must be a small motion of the normal component in the z direction perpendicular to the wall to enable the rotons and

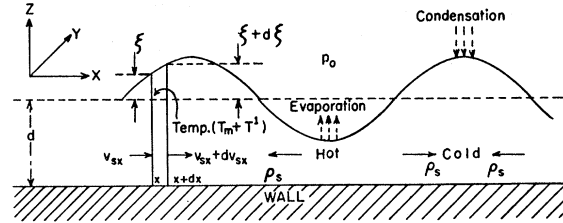


FIG. 1. Third sound in a liquid helium film.

phonons to distribute themselves uniformly throughout the depth of the film. At a peak of the wave an excess of superfluid has collected and the temperature at this point is lowered (see Fig. 1), while at a trough the temperature is raised, so that in addition to the pressure gradient present in the case of a classical liquid there is an additional restoring force due to the thermomechanical effect of the temperature gradient. Also, at the trough where the film is hot it will evaporate into the vapor phase, whereas at the peak the vapor will condense on to the film. There will be a flow of vapor from the troughs to the peaks, but we shall neglect the resulting small pressure gradients within the vapor and will assume that the pressure of the vapor on the surface of the film is everywhere equal to the value p_0 which it had in the absence of the surface wave.

In Fig. 1 the wave is propagating in the x direction and there is unit length of film in the y direction. At the point x the surface is raised a height ξ above its equilibrium position, the temperature is $T_m + T'$ where T_m is the mean temperature, and the velocity of the superfluid component is v_{sx} . The rate of evaporation of the film in g sec⁻¹ per unit area of surface is⁹

$$\frac{dm}{dt} = \gamma \left(\frac{M}{2\pi RT} \right)^{1/2} \left(\frac{dp}{dT} \right)_{v.p.e.} T' = KT', \quad (7)$$

where $(dp/dT)_{v.p.e.}$ is the slope of the vapor pressure curve and γ is very close to unity. Considering a slab of film of width dx , conservation of mass gives the equation

$$\rho_s d \frac{\partial v_{sx}}{\partial x} + \rho \frac{\partial \xi}{\partial t} + KT' = 0. \quad (8)$$

Considering the heat flow into the slab

$$\rho d C \frac{\partial T'}{\partial t} = \rho_s d \frac{\partial v_{sx}}{\partial x} S T - K L T'. \quad (9)$$

C , S , and L are the specific heat, entropy, and heat of vaporization per gram. The pressure at a point within the film has increased by

$$\delta p = \beta T' + \rho f \xi. \quad (10)$$

$\beta = (dp/dT)_{v.p.e.}$ is the slope of the vapor pressure curve.

⁹ Atkins, Rosenbaum, and Seki, Phys. Rev. **113**, 751 (1959).

⁵ K. R. Atkins, Can. J. Phys. **32**, 347 (1954).

⁶ S. Franchetti, Nuovo cimento **4**, 1504 (1956); **5**, 183 (1957); **5**, 1266 (1957).

⁷ K. R. Atkins, Progress in Low-Temperature Physics, edited by C. J. Gorter (North-Holland Publishing Company, Amsterdam, 1957), Vol. 2, p. 105.

⁸ R. G. Arkhipov, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 116 (1957) [translation: Soviet Phys. JETP **6**, 90 (1958)].

f is the symbol used previously to indicate the force acting on unit mass of helium at the surface of the film. (The exact significance of f may be complicated, but is best discussed in relation to a particular theory of the film.⁵⁻⁷) The hydrodynamical equation of motion of the superfluid component,

$$\frac{\partial \mathbf{v}_s}{\partial t} = -\frac{1}{\rho} \text{grad } p + S \text{grad } T, \quad (11)$$

becomes

$$\frac{\partial v_{sx}}{\partial t} = -f \frac{\partial \zeta}{\partial x} + \left(S - \frac{\beta}{\rho} \right) \frac{\partial T'}{\partial x}. \quad (12)$$

Second order terms have been consistently neglected.

A traveling wave solution

$$\zeta = \zeta_0 e^{i(\omega t - kx)} \quad (13)$$

plus similar expressions for T' and v_{sx} , is consistent with Eqs. (8), (9), and (12) if

$$\frac{\omega^2}{k^2} = \frac{\rho_s}{\rho} df + \frac{\rho_s}{\rho} ST \left[\left(S - \frac{\beta}{\rho} \right) - i \frac{Kf}{\rho\omega} \right] / \left[C - i \frac{KL}{\rho\omega d} \right]. \quad (14)$$

If we ignore evaporation effects by putting $K=0$, then

$$v_p^2 = \frac{\omega^2}{k^2} = \frac{\rho_s}{\rho} \left[df + \frac{ST(S - \beta/\rho)}{C} \right]. \quad (15)$$

The first term is reminiscent of Eq. (3), but above 1°K the second term is by far the larger and the temperature gradients are consequently very important. However, if we retain the evaporation effects with $\gamma=1$ in Eq. (7), then, for frequencies small compared with 10^3 sec^{-1} , the imaginary terms in the numerator and denominator of the last term of Eq. (14) are large compared with the real terms, and so the velocity of third sound is given by

$$u_3^2 \simeq \frac{\rho_s}{\rho} df \left[1 + \frac{TS}{L} \right]. \quad (16)$$

The second term in the bracket is small and the principal difference from Eq. (3) is the factor ρ_s/ρ .

If ζ_0 , T_0 , and v_0 are the amplitudes of the oscillatory parts of film thickness, temperature, and superfluid velocity, then it is easily shown from Eqs. (8), (9), and (12) that

$$\frac{v_0}{u_3} = \frac{\rho}{\rho_s} \frac{\zeta_0}{d}. \quad (17)$$

u_3 has the order of magnitude of the critical velocity of the film and therefore, except very near the λ -point where ρ/ρ_s becomes large, a large fractional change in film thickness is possible without the velocity of the superfluid component approaching its critical value.

Also,

$$\frac{T_0}{T} = -i \frac{\rho d S \omega \zeta_0}{KL d}. \quad (18)$$

At frequencies low enough to ensure that third sound is not appreciably attenuated, the temperature oscillation is immeasurably small and there is little hope that third sound could be detected by the type of receiver commonly used for second sound. The most hopeful method of observing third sound seems to be a direct observation of the oscillating film thickness by a suitable modification of the optical method developed by Jackson and his co-workers.¹⁰ We are currently developing such an approach. If third sound exists, its velocity should give information about the quantity f and hence about the factors affecting the formation of the film.

4. FOURTH SOUND IN A VERY NARROW CHANNEL

In a narrow two-sided channel the normal component is again constrained to remain at rest ($\mathbf{v}_n=0$), but, since the width of the channel remains constant, the oscillation of the superfluid component produces a first order change in density. The hydrodynamical equation of the superfluid component is

$$\frac{\partial v_{sx}}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + S \frac{\partial T}{\partial x}. \quad (19)$$

The equation expressing conservation of mass,

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n) = 0, \quad (20)$$

becomes

$$\frac{\partial \rho}{\partial t} + \rho_s \frac{\partial v_{sx}}{\partial x} = 0. \quad (21)$$

Since no entropy is transported by the flow of the superfluid component,

$$\frac{\partial}{\partial t} (\rho S) + \frac{2\chi}{T} T' = 0, \quad (22)$$

where $\chi T'$ is the heat flowing per second into unit area of the wall when the liquid is at an excess temperature T' . A possible solution of Eqs. (19), (21), and (22) is a traveling wave for which

$$\frac{\omega^2}{k^2} = \frac{\rho_s}{\rho} \frac{\partial p}{\partial \rho} + \frac{\rho_n}{\rho} \left(\frac{\rho_s}{\rho_n} \frac{TS^2}{C} \right) \left(1 - i \frac{2\chi}{\rho\omega d C} \right)^{-1} \quad (23)$$

$$= \frac{\rho_s}{\rho} u_1^2 + \frac{\rho_n}{\rho} u_2^2 \left(1 - i \frac{2\chi}{\rho\omega d C} \right)^{-1}, \quad (24)$$

¹⁰ E. J. Burge and L. C. Jackson, Proc. Roy. Soc. (London) A205, 270 (1951); L. C. Jackson and D. G. Henshaw, Phil. Mag. 44, 14 (1953); A. C. Ham and L. C. Jackson, Proc. Roy. Soc. (London) A240, 243 (1957).

where u_1 and u_2 are the velocities of first and second sound. At all temperatures the term in u_2^2 is small compared with the term in u_1^2 and so the attenuation per wavelength is never large. To an accuracy of better than 1%, the velocity of fourth sound is

$$u_4 \approx \left(\frac{\rho_s}{\rho} \right)^{\frac{1}{2}} u_1. \quad (25)$$

The amplitude of the temperature oscillation is given by

$$\frac{T_0}{T} = -\frac{S}{C} \left(1 - i \frac{2\chi}{\rho \omega d C} \right)^{-1} \frac{\rho_0}{\rho}. \quad (26)$$

At high frequencies, the term involving χ can be neglected and the temperature oscillation is a first-order effect as well as the density oscillation. At low frequencies, however, there is plenty of time for the heat to escape to the wall and the fractional change in temperature is small compared with the fractional

change in density. The amplitude of the oscillating part of the superfluid velocity is given by

$$v_0/u_4 = \rho_0/\rho_s. \quad (27)$$

Presumably fourth sound is strongly attenuated when v_0 exceeds the critical velocity for superflow through the channel. The critical velocity is much smaller than u_4 (except possibly in the immediate vicinity of the λ point) and so this interesting effect should occur for quite small fractional changes in density.*

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* *Note added in proof.*—Dr. R. H. Walmsley has pointed out that, in Eqs. (23) and (26), S should be replaced by $(S - \alpha u_1^2)$, where α is the isobaric coefficient of expansion. $(1 - \alpha u_1^2/S)$ is 1.6 at 2°K, 0 near 1.1°K, -12.5 at 0.8°K, and -7 near 0°K. Equation (25) therefore remains a good approximation. The temperature amplitude T_0 of Eq. (26) is somewhat modified. It is slightly increased above 1.1°K, becomes zero near 1.1°K, and changes phase with an increased magnitude below 1.1°K.

Some Properties of Green and Red-Green Luminescing CdS

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A series of electro-optical experiments at room temperature have been made with two types of high-purity single crystals of CdS. Those of type A, believed to have near-perfect stoichiometry, luminesce green (5145 Å) under 3650 Å ultraviolet irradiation, have low dark conductivity (2.5×10^{-11} mho cm^{-1} at 300°K), exhibit no photopeak, and have a relatively short carrier lifetime (about 4 microseconds or less). Those of type B, believed to have a sulfur deficiency, luminesce first strongly red (≈ 7200 Å), then strongly green (> 5145 Å) as the exciting radiation is increased in intensity. Simultaneously both emissions shift toward longer wavelengths. Also, as the red emission intensity begins to saturate, the intensity-rate change of conductivity abruptly changes. It is suggested that red luminescence is the result of a free hole capture by one of the electrons residing in a sulfur vacancy. This trapping sensitizes the crystal but at the expense of increasing carrier lifetime to millisecond magnitudes. Furthermore, some of these vacancies (~ 0.4 eV below the conduction band) act as donors to increase dark conductivity (about 10^{-9} mho cm^{-1} at 300°K).

1. INTRODUCTION

EVER since Lorenz¹ first grew synthetic single crystals of CdS, much has been written in the literature concerning this material's electro-optical properties.² One notes, however, that mention has rarely been made concerning green emission of a wavelength corresponding to direct recombination of free holes and electrons. Though green of a type has been analyzed by a few observers,^{3,4} such emissions appeared only when the crystals were cooled to low temperatures. Others^{5,6} have

shown that green electroluminescence can occur with voltage potentials approaching breakdown but the color has been sporadic and not too intense. In view of the notable lack of green emission from the crystals used by various researchers, some doubt arises as to the purity and stoichiometry of their samples and, hence, the accuracy of the data as truly representing intrinsic CdS.

Recently, the authors have been able to grow small quantities of spectrographically "pure" CdS crystals that show photoconductivity and luminescing properties related to what is believed to be a deficiency of sulfur.⁷ One class of these crystals, believed to be intrinsic CdS, is identified by its green-only luminescence at room temperature. A second class luminesces both red and

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² For review articles with excellent bibliographies, see R. H. Bube, Proc. Inst. Radio Engrs. **43**, 1836 (1955); A. Rose, Proc. Inst. Radio Engrs. **43**, 1850 (1955).

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⁴ J. Lambe, Phys. Rev. **98**, 985 (1955).

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⁶ G. Diemer, Philips Research Repts. **9**, 109 (1954).

⁷ A. A. Vuytsteke and Y. T. Sihvonen, Phys. Rev. **113**, 40 (1959).