An Overview of FMCW Systems in MATLAB

Kristen Parrish
Texas Instruments Inc.

An Overview of FMCW Systems in MATLAB

Kristen Parrish

I. Introduction

The basic FMCW system consists of a transmitter, receiver and mixer. A modulated signal is transmitted, received, and the transmitted and received signals are multiplied in the time domain and processed. My project proposal had several main points:

- Explain how FMCW radar is used to determine range and velocity information
- Mathematically derive three modulation schemes for FMCW systems
- MATLAB simulations for these three FMCW schemes, modeled ideally
- MATLAB simulations for the same three models, with effects of noise and phase error explored
- Document signal processing algorithms to improve results for FMCW systems
- Make comparisons to concepts learned in class (pulse compression, LFM chirped signals)

II. FMCW Basics

A. System overview

All FMCW systems use the same basic concept, and the three types (three different modulation schemes) only differ in the signal processing performed on the FFT. The process is shown graphically in Figure 1 and the system model is as follows:

1) Calculate transmitted signal
2) Calculate received signal
3) Mix signals (multiply in time domain)
4) Two sinusoidal terms are derived; filter out one
5) Perform FFT on filtered signal
6) Improved spectrum output with windowing, zero padding
7) Possibly perform additional post-processing

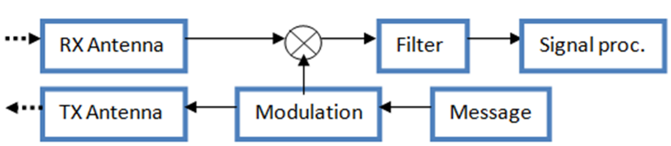

Fig. 1. Basic mechanics of FMCW.

B. Modulation derivations

By definition, frequency modulation is defined as

$$s_t = \cos(2\pi f_c t + \int_0^t f_{sig} d\tau) \qquad (1)$$

where f_c is the carrier frequency and f_{sig} is the signal that the carrier frequency is modulated with. The maximum or minimum difference between the modulated signal and the carrier frequency is $\pm\Delta f$. This equation demonstrates the transmitted frequency; the received frequency is delayed by $t_d = 2\Delta f R(t)/(cT)$, with $R(t) = \frac{R_0}{T/2} + vt$, and is also doppler shifted by $f_d = 2vf_c/c$, where T is the period of the modulation signal. Since the frequency modulated signal is periodic, the following analyses are performed for one period of the modulation.

Each of the following scenarios has this same basic modulation algorithm, but in the end all have different methods for deriving the range and velocity. Different scenarios work best for each modulation scheme, and so all three systems make use of different point scatters and have different system characteristics.

III. Sawtooth

Tx/Rx signals are, at any time t for $0 < t < T$,

$$\begin{aligned}
f_t(t) &= 2\Delta f(t - T/2) \\
f_r(t) &= 2\Delta f(t - t_d - T/2) + f_d \\
s_t(t) &= \cos(2\pi f_c t + 2\pi \int_0^t f_t d\tau) \\
&= \cos 2\pi(f_c t + \Delta f t^2 - \Delta f \cdot t \cdot T) \\
s_r(t) &= \cos 2\pi \left(f_c(t - t_d) + \Delta f t^2 - \Delta f \cdot t \cdot (T + t_d) + f_d t\right)
\end{aligned}$$

The transmitted and received signals are now mixed by multiplying in the time domain. By the trigonometric identity regarding the sum of cosines, the product of the two signals will have to distinct sinusoidal components. One of these will be at a frequency approximately twice the carrier frequency, which is not useful in signal processing. The other term is

$$\begin{aligned}
mixed(t) &= \frac{1}{2}\cos 2\pi(f_c t_d + (\Delta f t_d - f_d)t) \\
&= \frac{1}{2}\cos 2\pi(f_c t_d + (\frac{4\Delta f R}{Tc} - \frac{2f_c v}{c})t + \frac{2v}{c}t^2)
\end{aligned}$$

In the above expression, there are terms that are not time dependent (phase terms) and terms that are proportional to t^2 (time dependent phase terms), which are scene in the phase of the fourier transform of the signal. The terms that are proportional to t are seen in the spectrum of the signal. Of the time dependent terms, two are negligible. The frequency peak is then observed at

$$f = \frac{4\Delta f R}{Tc} + \frac{2f_c v}{c}$$

so that there is one peak containing both doppler and range information. This is verified in [1, 2].

A. Triangular

The triangular modulation analysis is similar to the sawtooth modulation, since the signal for $0 < t < T/2$ is like a sawtooth signal with half the period, and the signal for $T/2 < t < T$ is the negative of the signal for $0 < t < T/2$.

$$\begin{aligned} f_t(0 < t < T/2) &= 4\Delta f(t - T/4) \\ f_t(T/2 < t < T) &= 4\Delta f(t - t_d - T/4) + f_d \\ f_t(0 < t < T/2) &= 4\Delta f(-t - T/4) \\ f_t(T/2 < t < T) &= 4\Delta f(-t - t_d - T/4) + f_d \end{aligned}$$

When the signals are mixed, a low frequency and high frequency signal appear:

$$f_{up}(0 < t < T/2) = \frac{2f_c v}{c} - \frac{4\Delta f R}{Tc}$$
$$f_{dn}(T/2 < t < T) = \frac{2f_c v}{c} + \frac{4\Delta f R}{Tc}$$

so that two frequency terms show up in a spectrum of the return signal. We can use these frequencies then to solve for v and R. This is confirmed in [3, 4].

B. Sinusoidal

The sinusoidal modulation makes use of very different algorithms to extract range and doppler data [5]. Instead of looking at a linear chirping modulation signal with period T, the modulation signal is a cosine signal with frequency f_m. Again, Δf is the maximum frequency offset of the modulated signal compared to the carrier frequency.

$$\begin{aligned} f_t(t) &= \Delta f \cos(2\pi f_m t) \\ f_r(t) &= \Delta f \cos(2\pi f_m (t - t_d)) + \frac{f_d}{2} \end{aligned}$$

After modulation, the signals are written in terms of complex exponentials as follows, with ϕ_n an arbitrary phase:

$$\begin{aligned} s_t &= \exp\{j\Delta f/f_m \sin(2\pi f_m t) + \phi_1\} \\ s_r &= \exp\{j[-2\pi f_c t_d + \Delta f/f_m \sin(2\pi f_m (t - t_d)) + \phi_1]\} \end{aligned}$$

The mixed signal can be expanded using exponential notation, and low pass filtered. This results in

$$\begin{aligned} mixed(t) &= {s_t}^* \cdot s_r \\ &= exp\{j[2\pi f_d t + D\cos(2\pi/T(t - R/c) + \phi_2)]\} \\ &\cong J_0(D)\exp[j(2\pi f_d t + \phi_3)] \\ &\quad + J_1(D)\exp[j(2\pi f_d t + 2\pi f_m(t - R/c) + \phi_3)] \\ &\quad + J_1(D)\exp[j(2\pi f_d t - 2\pi f_m(t - R/c) + \phi_3)] \\ &\quad + other Bessel terms \\ D &= \frac{2\Delta f}{f_m}\sin(2\pi f_m R/c) \end{aligned}$$

There are many frequency peaks in the spectrum; we are only concerned with the center frequency (at the Doppler frequency f_d) and the upper and lower sidebands. These two side lobes differ in phase from the dominant frequency by $\pm 2\pi f_m R/c$.

IV. MATLAB SIMULATIONS

The simulations are comprised of the following parts:

- Problem and system definition (range, velocity, carrier frequency, etc)
- Transmitted signal (modulated carrier frequency)
- Recieved signal (Tranmissted signal delayed and doppler shifted)
- Mixed signal (product of transmitted and received signal)
- Hamming windowing, zero padding (x9)
- 1D FFT
- Additional FFT, sampling if necessary

I had expected that with my ideal, noiseless, distortionless systems, I should retrieve exactly the data I input in terms of range and velocity. However, I consistently noticed that the frequency peaks were consistently not exact, and especially the phase in the sinusoidal modulation simulation was off. I believe this is due to some phase jump caused by the cumulative summing command I used to integrate the signal for modulation. An illustration of this is shown in Figure 2, which shows a discontinuity in the curve smoothness at the end of the modulation period (1 ms).

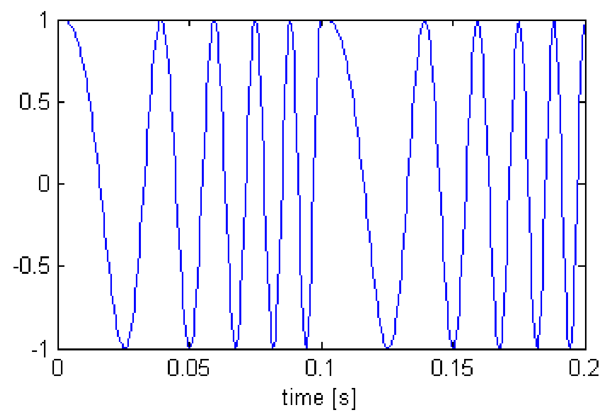

Fig. 2. Example of slope discontinuity in cumulative summing with MATLAB.

A. Sawtooth Modulation

1) Ideal simulation: The derivation above shows that simply taking the Fourier Transform will yield a single peak that contains both range and Doppler information (Figure 5). Therefore, additional signal processing is needed. A two dimensional signal processing method is demonstrated in [1, 6].

A system is simulated with two point scatters at 8 and 113 m from the target, moving 22 and 17 m/s away from the stationary radar, respectively. The carrier frequency $f_c = 77$ GHz and $\Delta f = 200$ MHz, with $T = 1ms$. The transmitted and received frequencies are shown in Figure 3. The difference in these frequencies is shown in Figure 4. Again, there is a frequency component dominated by high frequencies that is filtered out, thus only the difference is left.

2) 1D FFT: If the LPFed signal is filtered or undersampled, the high frequency spike will not appear in the spectrum. Thus there is one frequency peak in the signal for each point scatter, as derived above.

From the modulation derivation, for this scenario we should expect to see two frequency peaks at 21.96 kHz and 159.4 kHz, as in [1]. From a 1D FFT of the mixed signal, the spectrum exhibits frequency peaks at 21.99 kHz and 159.42 kHz, verifying that my model is working.

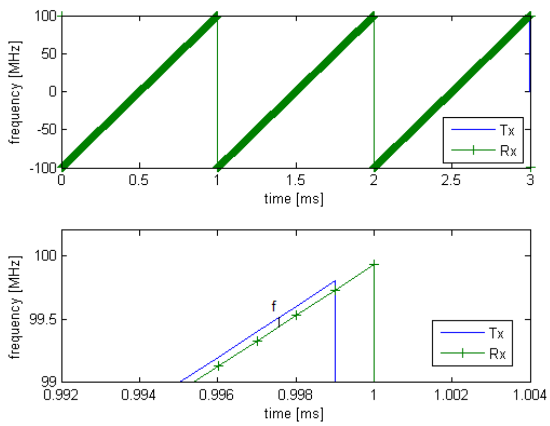

Fig. 3. Example of transmitted and received frequencies.

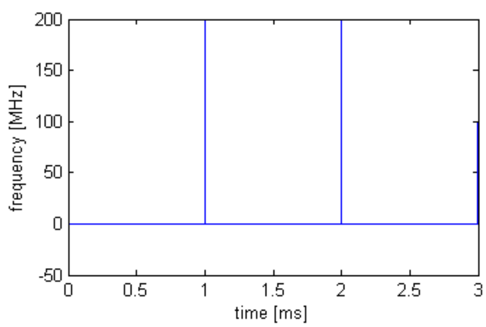

Fig. 4. Difference between transmitted and received frequencies. There are two different frequency components: a high frequency and a low frequency.

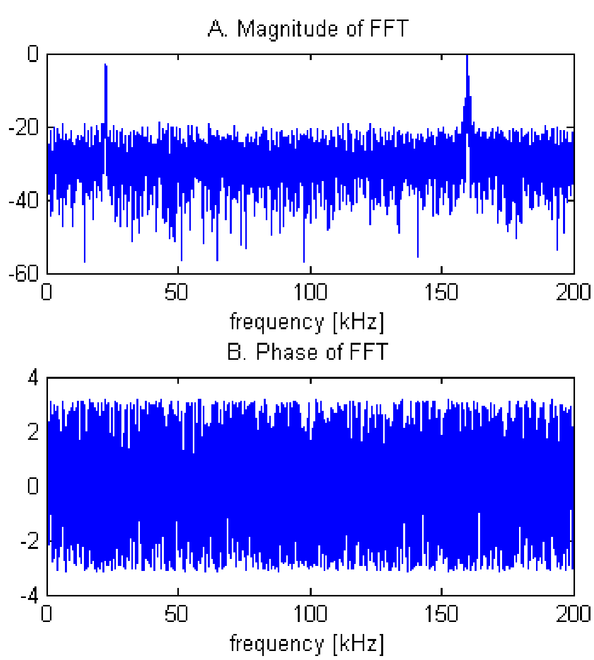

Fig. 5. 1D FFT of signal, for several modulation samples.

3) 2D FFT: As demonstrated in [6], a 2D FFT can be used to extract the range and doppler information from the spectrum. This is shown in Figure 6. The range axes are calibrated with the range and doppler axes calibrated based on the modulation derivation from above. The figure shown is based on 36 periods of modulation (36 ms). Note that the 20 dB dynamic range is consistent with the side lobe level from the 1D FFT. Also, the points aren't exactly at the appropriate range and doppler values. This is due to some 'drift' caused by the scatters physically moving. This is also evident in the 1D FFT - the width of the frequency peaks increases as the number of samples increases.

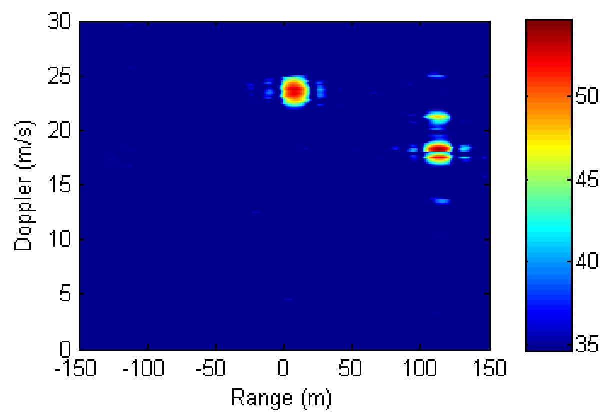

Fig. 6. 2D FFT of signal, for several modulation samples.

4) MTD: In [2], it is suggested that doppler information can be extracted with a sawtooth FMCW by using moving target doppler or moving target indicator technology. Similarly to what is implemented in pulsed radar, the rate of change of phase in the FFT of each modulation cycle is measured and used to calculate range. This is effectively the same as the 2D FFT performed above.

5) Effects of noise and phase error: White noise was added into the mixed signal in the time domain, to model what would happen in an RF mixer. Phase noise was also added in this step to model random phase error.

This noise and phase error does not affect the 1-D FFT other than raising the sidelobe level. This is to be expected, as the Fourier transform of white noise has a flat power spectral density, as shown in Figure 7. Thus, the 1D spectrum of the actual mixed signal, with a white noise signal included, is just the original signal with higher sidelobes. Thus, the MTD method is also not really affected, provided that the side lobes are still low enough that a frequency peak can be identified.

However, the 2D FFT is drastically affected by the addition of noise or phase error. With an addition of noise that decreases the side lobe level by approximately 2dB, the peaks in the 2D FFT are much less apparent, as indicated in Figures 8 and 9.

B. Triangular Modulation

1) Ideal simulation: For an ideal system, the FMCW system with triangular modulation needs no additional post processing, since the FFT of the signal yields two frequencies

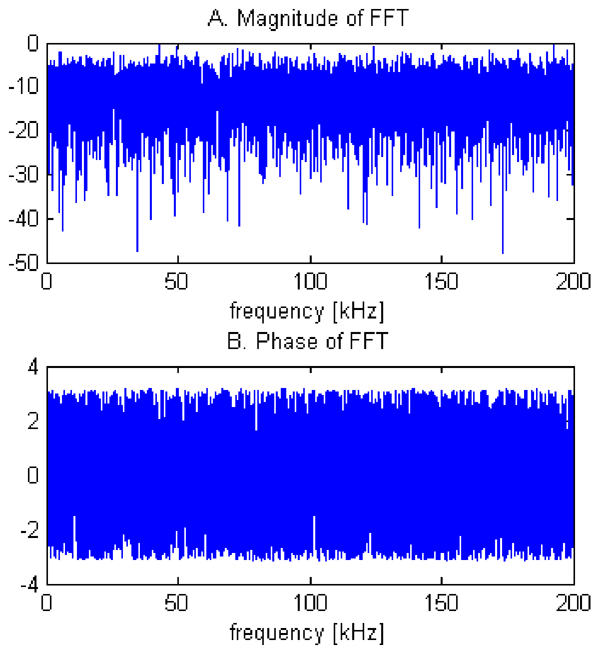

Fig. 7. 1D FFT of white noise.

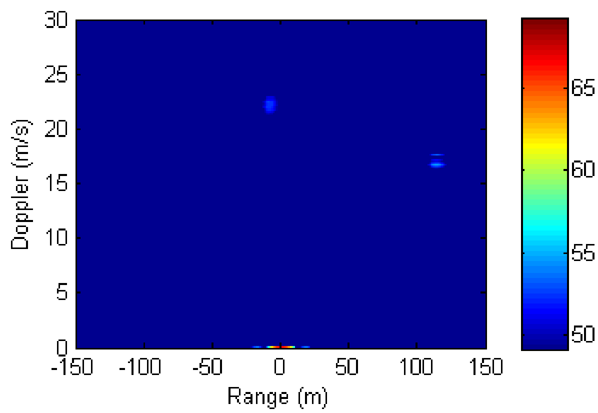

Fig. 8. 2D FFT of mixed signal, with white noise causing a 2dB SLL decrease in the 1D spectrum.

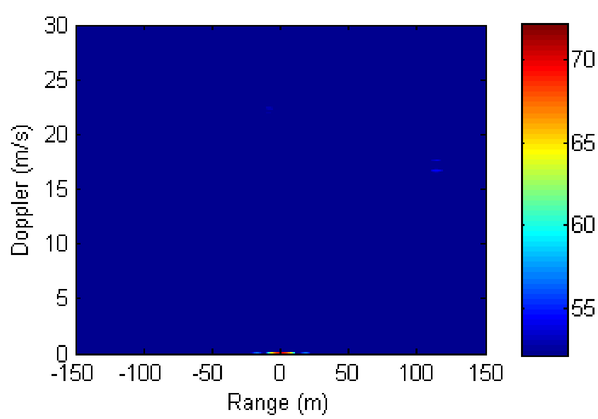

Fig. 9. 2D FFT of mixed signal, with white noise and phase error causing a 2dB SLL decrease in the 1D spectrum.

that can be used to calculate range and Doppler data.

The carrier frequency is $f_c = 77$ GHz and $\Delta f = 100$ MHz with $T = 1ms$. A system is simulated with one point scatter at a range of 15 m with a velocity of 87 m/s. The transmitted and received frequencies are shown in Figure 10. The low frequency components of the mixed signals is shown in Figure 11 - mixing acts to sift the terms into two sinusoidal terms; the other sinusoidal term is filtered out. Thus the frequency terms left are f_{up} and f_{dn}.

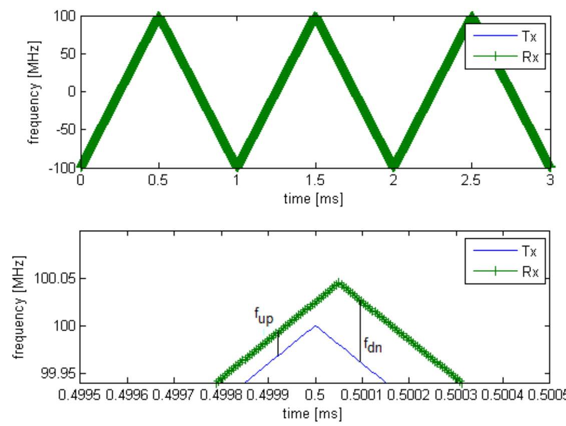

Fig. 10. Example of transmitted and received frequencies, for three samples.

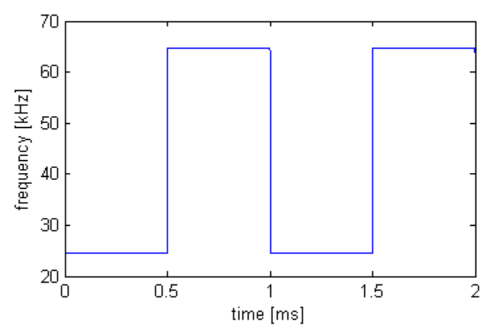

Fig. 11. Difference between transmitted and received frequencies. There are two different frequency components: a high frequency and a low frequency.

These two frequencies appear in the 1-D fourier transform (FFT) or the mixed signal. Based on the above modulation derivation, I expected peaks at 24.66 kHz and 64.66 kHz. From the actual spectrum, one peak is at 24.495 kHz, and the other at 64.61 kHz. From the above derivation, we can back solve for the range and velocity:

$$v = \frac{f_{up} + f_{dn}}{2} \frac{c}{2f_c} \cong 86.8m/s$$
$$R = \frac{f_{dn} - f_{up}}{2} \frac{cT}{2\Delta f} \cong 15.045m$$

With this modulation scheme, only one modulation sample is needed. A larger number of samples give a wider pulse width and is actually not helpful, so any system implementing this looks at each modulation period individually instead of accumulating the signal, like the sawtooth modulation systems must.

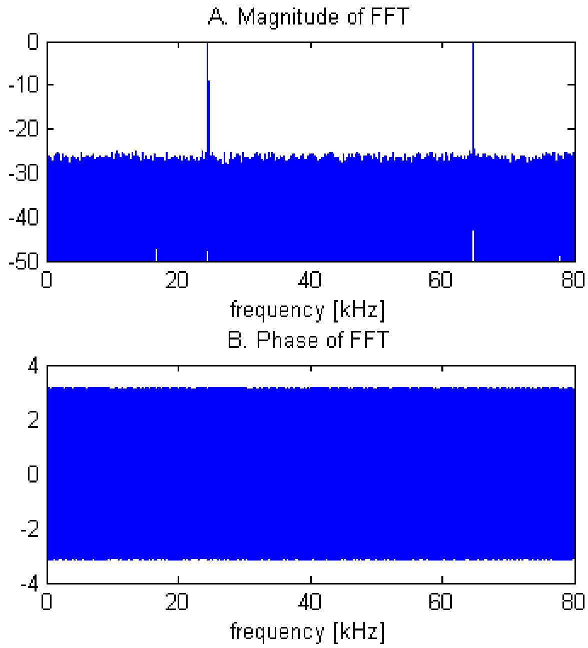

Fig. 12. 1D FFT of signal, for 1 sample.

2) Effects of noise and phase error: The same noise and phase error described above was added to the mixed signal. The addition of noise to the triangularly modulated signal has the same impact on the 1D FFT as the addition of noise to the sawtooth modulated signal; the impact is very little and the range and doppler information is not significantly distorted.

C. Sinusoidal modulation

1) Ideal simulation: The above derivation reveals that taking the Fourier transform of the mixed signal gives frequency peaks centered around a dominant peak at the doppler frequency. This is demonstrated by simulating a point scatter at $f_m = 8.26kHz$, $\Delta_f = 2kHz$, $f_c = 200MHz$, $R = 15km$, $v = 370.8m$.

The plot of the Fourier spectrum, Figure 13, shows three frequency peaks (others are filtered out by undersampling). From the spectral analysis,

$$f_d = 496.5Hz = \frac{2f_c}{c}v \rightarrow v = 372.375m/s$$

which is comparable to the input velocity of $370m/s$.

To extract range information, the phase is examined, as is implied by the derivation above. At the frequencies of the sidelobes, the value of the phase is extracted. The phase from Figure 14 was used for this calculation, since the zero-padding distorts the phase, and we don't need to lower the side lobe level. The difference between the phases of the two primary peaks (not the doppler peak) is related to the range as

$$\Delta\phi = 1.3942 = \frac{2\pi f_m R}{2c} \rightarrow R = 15.45km$$

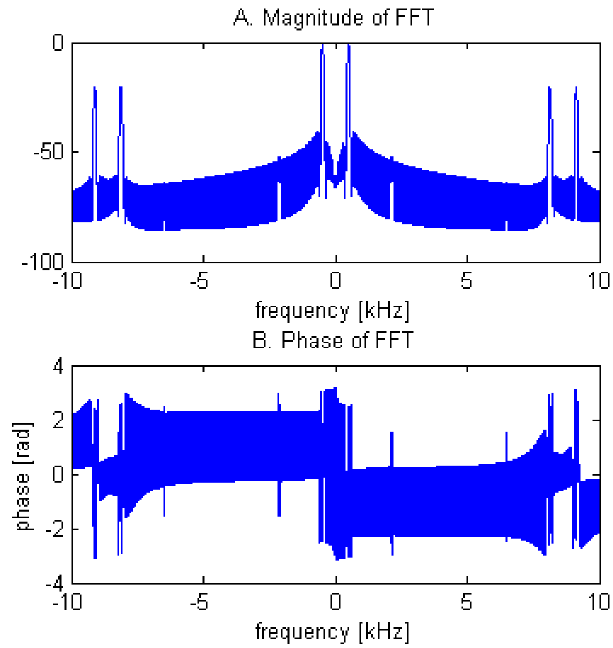

Fig. 13. 1D FFT of signal, for 1 sample, windowed and zeropadded.

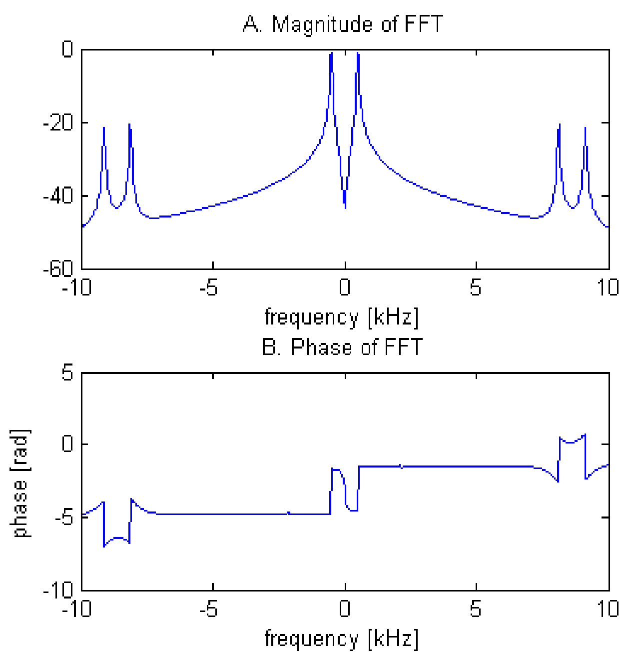

Fig. 14. 1D FFT of signal, for 1 sample, without windowing or zeropadding.

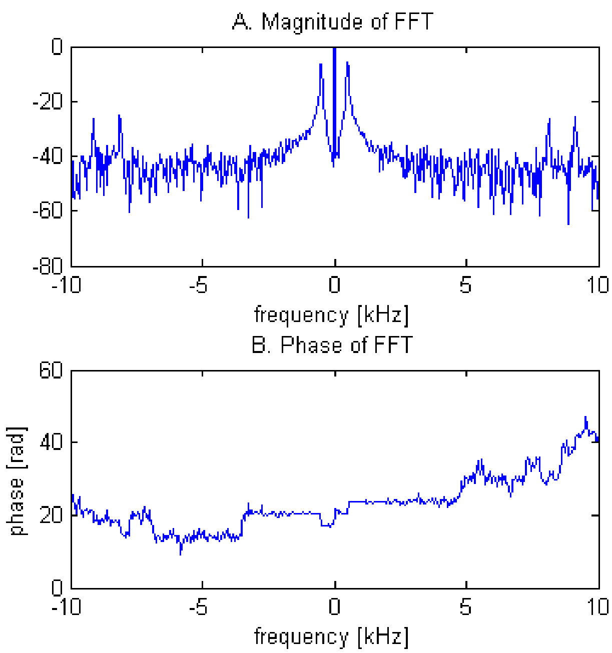

Fig. 15. 1D FFT of noisy signal, for 1 sample.

2) Effects of noise and phase error: The addition of noise greatly distorts the 1D FFT and seems to completely distort the phase. The noise level of the white noise added to the mixed signal is 1/3 the noise level added in to the 1D signal in the sawtooth simulation, but the negative effects on the two sideband peaks is much greater than that of the 1D signal, though this does not necessarily indicate a lower or higher sensitivity to noise. Doppler information can still be obtained; however, the phase is completely distorted.

D. Summary of Simulations

Based on these simulations, I was able to make some basic observations related to system performance and utility. Though the resulting spectrums I obtained were very dependent on sampling, windowing, and zero padding, these only affected my results in terms of FFT resolution (ie, I knew the frequency peak $\pm$ 100 Hz instead of $\pm$1 kHz, for example). I did not find this to be indicative of the actual radar resolution.

I found that the triangular modulation scheme seemed to be the easiest to implement and extract both range and doppler information from. This seems to be consistent with papers I've read. Incidentally, triangular modulation needs a smaller bandwidth to ensure linearity [7].

For a moving target, the sawtooth modulation scheme with a 1D FFT is not sufficient for extracting range and velocity information. However, if the targets were not moving, the target range would be easily extracted from this 1D spectrum. This explains why this sawtooth/upchirp/ramp modulation scheme is commonly used to measure snow pack/ground depth [8], and is not often used to extract doppler information [4].

Finally, sinusoidal modulation seemed to be most useful for extracting doppler information only. With any addition of noise, the phase was completely changed and so did not give an accurate extraction of range and doppler. However, due to the nature of the modulation scheme, the probabilty of false alarms is reduced [9].

V. FMCW Algorithms

A. Side-looking vehicle detection

The triangular modulation FMCW system can be used to detect car length (size) and speed [10]. The velocity of the car is calculated from the time that the vehicle spends in the 'sight' of the antenna, while the car length is classified based on the spectrum. A longer car has point scatters spaced farther apart, and the frequency peaks in the spectrum will also be spaced farther apart.

B. Digital signal processing

In [11], the analog signal (using triangular modulation) is digitized with an A/D converter to a video signal. A digital FFT is applied, and the signal is accumulated to lower the relative noise level. The spectrum is then examined to compute the range and doppler information. Additional, spectrum matching is used to eliminate clutter. A similar method is described in [12].

C. Phase-slope correction

In [13], a sawtooth modulation scheme was used to demonstrate a phase-slope signal processing algorithm. It is claimed that since it is possible to have highly linear ramping functions as modulated signals, this linearity can be used in adjusting the spectrum of the signal. The slope of the mixed signal is modified by multiplying the magnitude by a sum of negative and positive phase terms at each time point. This modified mixed signal is then windowed and an FFT is performed. The authors show that this algorithm reduce errors in range measurement by a maximum of 50% with the presented scenario.

D. Phase-error correction

In [14], a surface acoustic wave (SAW) reference delay line is used to compensate for phase error with sawtooth modulation systems. This technique is claimed to work for both system phase error (from non-linearities in the upramp) and oscillator phase error (from circuitry nonlinearity). A reference phase is produced from the transmitted and received signals that is proportional to the phase offset, and this is used to subtract out the existing phase error in the signal.

E. Matched filter bank

A large number of cross-correlation calculations is performed on each recieved signal mixed with the transmitted signal, with the most accurate recieved data being the one that matches the output of this calculation the closest. [5]. This calculation is applied to the FFT of the signal since applying to the time-domain signal is very computationally heavy.

VI. Conclusion

I completed the objectives set forth in my original proposal: I have analyzed and simulated three different modulation schemes for FMCW and drawn conclusions about these systems. My analysis and simulation results confirmed that phase error is a big issue in FMCW radar systems. I have also enclosed a brief survey of signal processing techniques that help improve the weaknesses of each technique.

References

[1] D. Barrick, N. Oceanic, and A. A. W. P. Lab, *FM/CW Radar Signals and Digital Processing.* Environmental Research Laboratories, 1973.

[2] A. Stove, "Linear FMCW radar techniques," in *IEEE Proceedings on Radar and Signal Processing*, vol. 139, no. 5, 1992, pp. 343–350.

[3] B. R. Mahafza, *Radar Systems Analysis and Processing Using MATLAB*, 3rd ed. CRC Press, 2009.

[4] H. Griffiths, "New ideas in FM radar," *Electronics & Communication Engineering Journal*, vol. 2, no. 5, pp. 185–194, 1990.

[5] B. Cantrell, H. Faust, A. Caul, and A. O'Brien, "New ranging algorithm for FM/CW radars," in *Radar Conference, 2001. Proceedings of the 2001 IEEE*, 2001, pp. 421–425.

[6] A. Wojtkiewicz, J. Misiurewicz, M. Nalecz, K. Jedrzejewski, and K. Kulpa, "Two-dimensional signal processing in FMCW radars," *Proc. XX KKTOiUE*, pp. 475–480.

[7] S. Piper, "FMCW linearizer bandwidth requirements," Mar 1991, pp. 142–146.

[8] Jon Holmgren and Matthew Sturm and Norbert E. Yankielun and Gary Koh, "Extensive measurements of snow depth using FM-CW radar," *Cold Regions Science and Technology*, vol. 27, no. 1, pp. 17 – 30, 1998.

[9] P. JEFFORD and M. HOWES, "Modulation schemes in low-cost microwave field sensors," *IEEE transactions on microwave theory and techniques*, vol. 31, no. 8, 1983.

[10] S. Park, T. Kim, S. Kang, and K. Koo, "A novel signal processing technique for vehicle detection radar," in *2003 IEEE MTT-S International Microwave Symposium Digest*, vol. 1, 2003.

[11] W. Chang, L. Huan, and L. Yubai, "A Practical FMCW Radar Signal Processing Method and Its System Implementation," in *ITS Telecommunications Proceedings, 2006 6th International Conference on*, 2006, pp. 1195–1199.

[12] A. Grzywacz, "Experimental investigations of digital signal processing techniques in an FMCW radar for naval application," in *Microwaves, Radar and Wireless Communications, 2002. MIKON-2002. 14th International Conference on*, vol. 3, 2002.

[13] T. Musch, "A High Precision 24 GHz FMCW-Radar using a Phase-Slope Signal Processing Algorithm," in *European Microwave Conference, 2002. 32nd*, 2002, pp. 1–4.

[14] M. Vossiek, T. Kerssenbrock, and P. Heide, "Signal Processing Methods For Millimetrewave FMCW Radar With High Distance And Doppler Resolution," in *27th European Microwave Conference and Exhibition*, vol. 2, 1997.