## Competition:

$$\frac{dU_1}{dt} = r_1 U_1\left(1 - \frac{U_1 + \alpha U_2}{K_1}\right), \quad \frac{dU_2}{dt} = r_2 U_2\left(1 - \frac{U_2 + \beta U_1}{K_2}\right)$$

$r_i$ - growth rates
$K_i$ - carrying capacities
$\alpha$ - competition of 2 on 1
$\beta$ - competition of 1 on 2.

Typically, for species in the same niche, $\alpha\beta = 1$.

## Predator-Prey:

Lotka-Volterra:

$$\frac{dU}{dt} = \alpha U - \gamma UV, \quad \frac{dV}{dt} = e\gamma UV - \beta V$$

$\alpha$ - growth of $U$
$\gamma$ - predation coefficient
$e$ - growth of $V$
$\beta$ - natural death of $V$

More realistic:

Rosenzweig-MacArthur

$$\frac{dU}{dt} = rU\left(1 - \frac{U}{K}\right) - V\Phi(U), \quad \frac{dV}{dt} = eVI(\phi) - \beta V$$

Functional-response $\Phi(U)$:

Type I (Linear)

$$\Phi(U) = sU$$

Type II (Holling's disc equation):

$$\Phi(U) = \frac{sU}{1 + shU}$$

If we do this, might be a good idea to do linear functional responses first for simplicity.

s - attack/encounter rate
h - handling time / processing food time.

Everything upto here is from "Essential Mathematical Biology"

Three populations:

$$\frac{dU_1}{dt} = r_1 U_1 \left(1 - \frac{U_1 + \alpha U_2}{K_1}\right) - V\Phi_1(U_1)$$

$$\frac{dU_2}{dt} = r_2 U_2 \left(1 - \frac{U_2 + \beta U_1}{K_2}\right) - V\Phi_2(U_2)$$

$$\frac{dV}{dt} = e_1 V\Phi_1(U_1) + eV\Phi_2(U_2) - \beta V$$

Inspired by "Mathematical Models in Biology" pg 234

Not sure how to introduce competing predators.

Routh-Hurwitz is used to discover if these species can coexist.

## Routh-Hurwitz Criteria

Given the characteristic equation

$$\lambda^k + a_1\lambda^{k-1} + a_2\lambda^{k-2} + \dots + a_k = 0$$

define:

$$H_1 = (a_1),\quad H_2 = \begin{pmatrix} a_1 & 1 \\ a_3 & a_2 \end{pmatrix},\quad H_3 = \begin{pmatrix} a_1 & 1 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{pmatrix}$$

$$H_j = \begin{pmatrix} a_1 & 1 & 0 & 0 & \cdots & 0 \\ a_3 & a_2 & a_1 & 1 & \cdots & 0 \\ a_5 & a_4 & a_3 & a_2 & \cdots & 0 \\ a_{2j-1} & a_{2j-2} & a_{2j-3} & a_{2j-4} & \cdots & a_j \end{pmatrix},\quad H_k = \begin{pmatrix} a_1 & 1 & 0 & \cdots & 0 \\ a_3 & a_2 & a_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_k \end{pmatrix}$$

where the $(l,m)$ term in the matrix $H_j$ is

$a_{2l-m}$ for $0 < 2l-m < k$,

$1$ for $2l = m$,

$0$ for $2l < m$ or $2l > k+m$.

The steady-state $\hat{u}$ is stable iff the determinants of all determinants of all Hurwitz matrices are positive:

$$\det H_j > 0 \quad (j = 1, 2, \dots, k)$$

Not sure how to deal with 40%.

Introducing farmers:

$$\frac{dU_1}{dt} = r_1 U_1 \left(1 - \frac{U_1 + \alpha U_2}{K_1}\right) - V\Phi_1(U_1)$$

$$\frac{dU_2}{dt} = r_2 U_2 \left(1 - \frac{U_2 + \beta U_1}{K_2}\right) - V\Phi_2(U_2)$$

$$\frac{dV}{dt} = e_1 V\Phi_1(U_1) + e_2 V \Phi_2(U_2) - \beta V - \text{FARMERS}$$

Periodic removal => dirac delta impulses.

$$\delta_1 V\left(1 - \frac{V}{0.4V_0}\right) \sum_{n=1}^{\infty} \delta(t_n)$$

If $V = 0.4V_0$ then no foxes removed.

But if $V < 0.4V_0$ already then foxed introduced

BAD.