2 $f(v) = \begin{bmatrix} x^2 + y^2 \\ x^2 - y^2 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}, \quad v_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$f(v+s) \approx f(v) + J_f(v)\, s$$

$$J_f(v) = \begin{bmatrix} 2x & 2y \\ 2x & -2y \end{bmatrix}$$

$$f(v_0) = \begin{bmatrix} 1 + 1 \\ 1 - 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$J_f(v_0) = \begin{bmatrix} 2 & 2 \\ 2 & -2 \end{bmatrix}$$

$\vdots$

$\vdots$

$$s_0 = - J_f^{-1}(v_0)\, f(v_0)$$

$$= \frac{1}{8}\begin{bmatrix} -2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

$$s_0 = \begin{bmatrix} -\frac{1}{4} \\ -\frac{1}{4} \end{bmatrix}$$

$$v_1 = v_0 + s_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} \frac{1}{4} \\ \frac{1}{4} \end{bmatrix} = \begin{bmatrix} 1.25 \\ 1.25 \end{bmatrix}$$

3 $f(x) = e^x$, $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$ $n = 3$

$$P_3(x) = f(x_0)\ell_0(x) + f(x_1)\ell_1(x) + f(x_2)\ell_2(x) + f(x_3)\ell_3(x)$$

$$\ell_0(x) = \prod_{i=1}^{3} \frac{x - x_i}{x_0 - x_i} = \frac{x - x_1}{x_0 - x_1}\,\frac{x - x_2}{x_0 - x_2}\,\frac{x - x_3}{x_0 - x_3} = \frac{x-1}{0-1}\,\frac{x-2}{0-2}\,\frac{x-3}{0-3}$$

$$= -\frac{1}{6}(x-1)(x^2 - 5x + 6) = -\frac{1}{6}(x^3 - 6x^2 + 12x - 6)$$

$$\ell_1(x) = \prod_{\substack{i=0\\ i\neq 1}}^{3} \frac{x - x_i}{x_1 - x_i} = \frac{x - x_0}{x_1 - x_0}\,\frac{x - x_2}{x_1 - x_2}\,\frac{x - x_3}{x_1 - x_3} = \frac{x-0}{1-0}\,\frac{x-2}{1-2}\,\frac{x-3}{1-3}$$

$$= \frac{1}{2}x(x^2 - 5x + 6) = \frac{1}{2}(x^3 - 5x^3 + 6x)$$

$$\ell_2(x) = \prod_{\substack{i=0\\ i\neq 1}}^{3} \frac{x - x_i}{x_2 - x_i} = \frac{x - x_0}{x_2 - x_0}\,\frac{x - x_1}{x_2 - x_1}\,\frac{x - x_3}{x_2 - x_3} = \frac{x-0}{2-0}\,\frac{x-1}{2-1}\,\frac{x-3}{2-3}$$

$$= -\frac{1}{2}x(x^2 - 4x + 3) = -\frac{1}{2}(x^3 - 4x^2 + 3x)$$

$\underline{3}$

$$l_3(x) = \prod_{\substack{i=0 \\ i\neq 1}}^{3} \frac{x-x_i}{x_3-x_i} = \frac{x-x_0}{x_3-x_0}\;\frac{x-x_1}{x_3-x_1}\;\frac{x-x_2}{x_3-x_2} = \frac{x-0}{3-0}\;\frac{x-1}{3-1}\;\frac{x-2}{3-2}$$

$$= \frac{1}{6}\left(x^3 - 3x^2 + 2x\right)$$

$$P_3(x) = e^0\left(-\frac{1}{6}\left(x^3 - 6x^2 + 12x - 6\right)\right) + \cdots$$

$$\cdots + e^1\left(\frac{1}{2}\left(x^3 - 5x^2 + 6x\right)\right) + \cdots$$

$$\cdots - e^2\left(\frac{1}{2}\left(x^3 - 4x^2 + 3x\right)\right)$$

$$\approx \left[-\frac{1}{6}x^3 + x^2 - 2x + 1\right] + 1.359\left(x^3 - 5x^2 + 6x\right) + \cdots$$

$$\cdots - 3.695\left(x^3 - 4x^2 + 3x\right)$$

3

$$P_3(x) \approx \left(-\frac{1}{6} + 1.359 - 3.695\right)x_3 + \cdots$$

$$\cdots + \left(1 - 5 \cdot 1.359 + 4 \cdot 3.695\right)x_2 + \cdots$$

$$\cdots + \left(-2 + 6 \cdot 1.359 - 3 \cdot 3.695\right)x_3 + \cdots$$

$$\cdots + 1$$

$$\boxed{P_3(x) \approx -2.5027x_3 + 8.985x_2 - 4.931x + 1}$$

3b

x_i	$f[\cdot]$	$f[\cdot,\cdot]$	$f[\cdot,\cdot,\cdot]$	$\cdots$
0	e^0	$\dfrac{e^1-e^0}{1-0} = e-1$	$\dfrac{(e^2-e)-(e-1)}{2-0} = \dfrac{e^2-2e+1}{2}$	
1	e^1	$\dfrac{e^2-e^1}{3-1} = e^2-e$	$\dfrac{(e^3-e^2)-(e^2-e)}{3-1} = \dfrac{e^3-2e^2+e}{2}$	
2	e^2	$\dfrac{e^3-e^2}{3-2} = e^3-e^2$	$\cdots f[\cdot,\cdot,\cdot,\cdot]$	
3	e^3			

$$\frac{1}{6}\left(e^3-2e^2+e-e^2+2e-1\right) = \cdots$$

$$\cdots = \frac{1}{6}\left(e^3-3e^2+3e-1\right)$$

3b

$$P_3(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \cdots$$

$$\cdots + a_3(x-x_0)(x-x_1)(x-x_2)$$

$$P_3(x) = e + (e-1)x + \frac{e^2-2e+1}{2}x(x-1) + \cdots$$

$$\cdots + \frac{1}{6}(e^3-3e^2+3e-1)x(x-1)(x-2)$$

$$\approx 0.8455(x^3 - 3x^2 + 2x) + 1.4762(x^2 - x) + \cdots$$

$$\cdots + 1.7183x + 2.7183$$

$$\approx 0.8455x^3 + x^2(-3 \cdot 0.8455 + 1.4762) + \cdots$$

$$\cdots + x(2 \cdot 0.8455 - 1.4762 + 1.7183) + 2.7183$$

3b

$$\boxed{P_3(x) \approx 0.8455x^3 - 1.06x^2 + 1.9331x + 2.7183}$$

MY P_n FOR LAGRANGE FORM & NEWTON AREN'T EQUAL SO AT LEAST ONE IS INCORRECT.

3c

ASSUMING MY NEWTON'S FORM $P_3(x)$ IS CORRECT...

$P_3(1.5) = 6.0865$ vs. $e^{1.5} \approx 4.482$

$P_3(4) = 47.603$ vs. $e^4 \approx 54.598$

$P_3(1.5)$ IS THE MORE ACCURATE APPROXIMATION IN TERMS OF ABSOLUTE ERROR, BUT $P_3(4)$ IS MORE ACCURATE IN TERMS OF RELATIVE ERROR.

8a HERMITE INTERPOLATION

1	0			
		1	$\frac{3}{4}\ln 3 - \frac{1}{2}$ $= b$	
1	0			$\frac{1}{2}(1 - \ln 3)$ $= a$
		$\frac{3}{2}\ln 3$	$-\frac{1}{4}\ln 3 + \frac{1}{2}$	
3	$3\ln 3$			
		$\ln 3 + 1$		
3	$3\ln 3$			

$$H(x) = a(x-1)^2(x-3) + b(x-1)^2 + (x-1)$$

$$H(1.5) = 0.59948$$

HERMITE INTERPOLATING POLYNOMIAL IS MORE ACCURATE

8b

$$f(x) = H(x) + e(x)$$

$$= H(x) + \frac{(x-x_0)^2(x-x_1)^2}{(2n+2)!} f^{(2n+2)}(\xi)$$

$$\frac{(x-x_0)^2(x-x_1)^2}{(2n+2)!} f^{(2n+2)}(\xi) = f(x) - H(x)$$

$$\frac{(x-1)^2(x-3)^2}{4!} \frac{2}{\xi^3} = f(x) - H(x)$$

$$\xi(x) = \left[\frac{1}{12} \frac{(x-1)^2(x-3)^2}{f(x)-H(x)}\right]^{1/3}$$

SAY $a = 1$, $b = 3$

$$1 < \xi(x) < 3$$

8b

$x = 1.5$

$$\mathscr{E}_4(1.5) = \left[\frac{1}{12}\,\frac{(1.5-1)^2(1.5-3)^2}{f(1.5)-H(1.5)}\right]^{1/3}$$

$$\approx \left[\frac{1}{12}\,\frac{(0.25)(2.25)}{(1.5)\ln(1.5)-(0.59948)}\right]^{1/3}$$

$$\approx 1.7518$$

$x_0 = 1$	z_0	-1			
$x_1 = 2$			2	0	
	z_1	-1			0
			2		
	z_2	-1		0	$-\frac{1}{2}$
			$\frac{1+1}{2-1} = 2$		
	z_3	1		2	-1
			-2		
	z_4	1			

ANNIE
21 AUG?
2:20 PM

LCMH
22 AUG
2:00 PM

$$P(x) = -1 + 2(x-1) + \cdots$$

$$\cdots - \frac{1}{2}(x-1)^3(x-2)^2$$