6a

$$P_4 = x^4 + \alpha_{43}P_3 + \alpha_{42}P_2 + \alpha_{41}P_1 + \alpha_{40}P_0$$

$$= x^4 - \frac{\langle x^4, P_3\rangle}{\|P_3\|^2}P_3 - \frac{\langle x^4, P_2\rangle}{\|P_2\|^2}P_2 - \frac{\langle x^4, P_1\rangle}{\|P_1\|^2}P_1 - \frac{\langle x^4, P_0\rangle}{\|P_0\|^2}P_0$$

($\langle x^4, P_1\rangle \to 0$)

$$\langle x^4, P_0\rangle = \int_{-1}^{1} x^4\,dx = \frac{1}{5}x^5\Big|_{-1}^{1} = \frac{2}{5}$$

$$\alpha_{40} = -\frac{1}{5}$$

$$\langle x^4, P_2\rangle = \int_{-1}^{1} x^6\,dx - \frac{1}{3}\int_{-1}^{1} x^4\,dx = \frac{2}{7} - \frac{1}{3}\cdot\frac{2}{5} = \frac{16}{105}$$

$$\|P_2\|^2 = \int_{-1}^{1}\left(x^2 - \frac{1}{3}\right)^2 dx = \frac{2}{5} - \frac{4}{9} + \frac{2}{9} = \frac{8}{45}$$

$$\alpha_{42} = -\frac{16}{105}\cdot\frac{45}{8} = -\frac{90}{105} = -\frac{6}{7}$$