$$\begin{bmatrix} 1 & 0 & 0 \\ 1/5 & 1 & 0 \\ 3/5 & 0 & 1 \end{bmatrix}\begin{bmatrix} -5 & 2 & -1 \\ 1 & 0 & 3 \\ 3 & 1 & 6 \end{bmatrix} = \begin{bmatrix} -5 & 2 & -1 \\ 0 & 2/5 & 14/5 \\ 0 & 11/5 & 27/5 \end{bmatrix} = M_1 P_1 A$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} -5 & 2 & -1 \\ 0 & 2/5 & 14/5 \\ 0 & 11/5 & 27/5 \end{bmatrix} = \begin{bmatrix} -5 & 2 & -1 \\ 0 & 11/5 & 27/5 \\ 0 & 2/5 & 14/5 \end{bmatrix} = P_2 M_1 P_1 A$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2/11 & 1 \end{bmatrix}\begin{bmatrix} -5 & 2 & -1 \\ 0 & 11/5 & 27/5 \\ 0 & 3/5 & 14/5 \end{bmatrix} = \begin{bmatrix} -5 & 2 & -1 \\ 0 & 11/5 & 27/5 \\ 0 & 0 & 20/11 \end{bmatrix} = M_2 P_2 M_1 P_1 A = U$$

$$\hat{L} U = A$$

$$\hat{L} = P_1^T L_1 P_2^T L_2 = \begin{bmatrix} 1 & 0 & 0 \\ 1/5 & 1 & 0 \\ 3/5 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2/11 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1/5 & 2/11 & 1 \\ -3/5 & 1 & 0 \end{bmatrix}$$

$$P = P_2 P_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \longrightarrow L = P\hat{L} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3/5 & 1 & 0 \\ -1/5 & 2/11 & 1 \end{bmatrix} = L$$

$$PA = LU \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} -5 & 2 & -1 \\ 1 & 0 & 3 \\ 3 & 1 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3/5 & 1 & 0 \\ -1/5 & 2/11 & 1 \end{bmatrix}\begin{bmatrix} -5 & 2 & -1 \\ 0 & 11/5 & 27/5 \\ 0 & 0 & 20/11 \end{bmatrix}$$

$$\begin{bmatrix} -5 & 2 & -1 \\ 3 & 1 & 6 \\ 1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} -5 & 2 & 1 \\ 3 & 1 & 6 \\ 1 & 0 & 3 \end{bmatrix} \quad \therefore$$