Homework 4

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$$f(x+s) \approx f(x) + J_f(x)\, s$$

$$f(x) = \begin{bmatrix} x^2 + y^2 \\ x^2 - y^2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$J_f(x) = \begin{bmatrix} 2x & 2y \\ 2x & -2y \end{bmatrix}$$

$$X_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$f(X_0) = \begin{bmatrix} 1+1 \\ 1-1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$J_f(X_0) = \begin{bmatrix} 2 & 2 \\ 2 & -2 \end{bmatrix}$$

$$J_f(X_0) \cdot s_0 = -f(x_0)$$

$$s_0 = J_f(X_0)^{-1} \cdot -f(X_0)$$

$$= -\frac{1}{8}\begin{bmatrix} -2 & -2 \\ -2 & 2 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1/2 \\ -1/2 \end{bmatrix}$$

$$X_1 = X_0 - s_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} -1/2 \\ -1/2 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

$$f(X_1) = \begin{bmatrix} 1/4 + 1/4 \\ \varnothing \end{bmatrix} = \begin{bmatrix} 1/2 \\ \varnothing \end{bmatrix}$$

3
a

$f(x) = e^x$

KNOWN AT

$x_0 = 0$
$x_1 = 1$
$x_2 = 2$
$x_3 = 4$

WRITE THE INTERPOLATING POLY IN LAGRANGE FORM

$$P_n(x) = \sum_{k=\emptyset}^{n} f(x_k)\, l_k(x) = \sum_{k=\emptyset}^{3} e^{x_k}\, l_k(x)$$

$$= e^{x_0} l_0 + e^{x_1} l_1 + e^{x_2} l_2 + e^{x_3} l_3$$

$$l_k = \prod_{\substack{i=\emptyset \\ i \neq k}}^{n} \frac{x - x_i}{x_k - x_i}$$

$$l_0 = \prod_{\substack{i=\emptyset \\ i \neq k}}^{3} \frac{x - x_i}{x_0 - x_i} = \frac{x - x_1}{x_0 - x_1} \cdot \frac{x - x_2}{x_0 - x_2} \cdot \ldots$$

$$\ldots \cdot \frac{x - x_3}{x_0 - x_3}$$

$$= \frac{x-1}{\emptyset - 1} \cdot \frac{x-2}{\emptyset - 2} \cdot \frac{x-4}{\emptyset - 4}$$

$$l_0 = -\frac{1}{8}(x-1)(x-2)(x-4)$$

$$l_1 = \prod^{3} \frac{x - x_i}{x_1 - x_i} = \frac{x - x_0}{x_1 - x_0} \cdot \frac{x - x_2}{x_1 - x_2} \cdot \frac{x - x_3}{x_1 - x_3} = \frac{x - \emptyset}{\underset{1}{1 - \emptyset}} \cdot \frac{x-2}{\underset{-1}{1-2}} \cdot \frac{x-4}{\underset{-3}{1-4}}$$

$$l_1 = \frac{1}{3} x(x-2)(x-4)$$

$\frac{3}{a}$ $l_2 = \prod_{\ldots}^{3} \frac{x - x_i}{x_2 - x_i} = \frac{x - x_0}{x_2 - x_0} \frac{x - x_1}{x_2 - x_1} \frac{x - x_3}{x_2 - x_3}$

$$= \frac{x - \emptyset}{\underset{2}{2 - \emptyset}} \frac{x - 1}{\underset{1}{2 - 1}} \frac{x - 4}{\underset{-2}{2 - 4}}$$

$$l_2 = -\frac{1}{4} x(x-1)(x-4)$$

$$l_3 = \prod_{\ldots}^{3} \frac{x - x_i}{x_3 - x_i} = \frac{x - x_0}{x_3 - x_0} \frac{x - x_1}{x_3 - x_1} \frac{x - x_2}{x_3 - x_2}$$

$$= \frac{x - \emptyset}{\underset{4}{4 - 0}} \frac{x - 1}{\underset{3}{4 - 1}} \frac{x - 2}{\underset{2}{4 - 2}}$$

$$= \frac{1}{14} x(x-1)(x-2)$$

$$\longrightarrow \boxed{\begin{aligned} P_n(x) = & -\frac{1}{8} e^0 (x-1)(x-2)(x-4) + \cdots \\ & \cdots + \frac{1}{3} e^1 x(x-2)(x-4) + \cdots \\ & \cdots - \frac{1}{4} e^2 x(x-1)(x-4) + \cdots \\ & \cdots - \frac{1}{14} e^4 x(x-1)(x-2) \end{aligned}}$$

3b

x_i	$f[\cdot]$	$f[\cdot,\cdot]$	$f[\cdot,\cdot,\cdot]$	$f[\cdot,\cdot,\cdot,\cdot]$
0	e^0	$\frac{e^1 - e^0}{1 - 0}$	$\frac{4.67077 - 1.71828}{2 - \varnothing}$	$\frac{6.31124 - 1.47625}{4 - \varnothing}$
1	e^1	$\frac{e^2 - e^1}{2 - 1}$		
2	e^2	$\frac{e^4 - e^2}{4 - 2}$	$\frac{23.6045 - 4.67077}{4 - 1}$	
4	e^4			
		1.71828	1.47625	4.83499
		4.67077	6.31124	
		23.6045		

$\frac{3}{b}$

$$\longrightarrow P_3(x) = f[x_0] + f[x_0, x_1](x - x_0) + \cdots$$

$$\cdots + f[x_0, x_1, x_2](x - x_0)(x - x_1) + \cdots$$

$$\cdots + f[x_0, \ldots, x_3](x - x_0)(x - x_1)(x - x_2)$$

$$= 1 + 1.71828(x-1) + 1.47625(x-1)(x-e) + \cdots$$

$$\cdots + 4.83499(x-1)(x-e)(x-e^2)$$

$$P_3(x) = a_0 + (x - x_0)\Big(a_1 + (x - x_1)\big(a_2 + a_3(x - x_2)\big)\Big)$$

$$= \boxed{1 + (x-1)\Big(1.71828 + (x-e)\big(1.47625 + 4.83499(x-e^2)\big)\Big)}$$

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THESE ARE TAKEN FROM A POLYNOMIAL P(X) OF DEGREE ≤ 5

WHAT IS THE SPECIFIC DEGREE OF P(X)?

... DEG $P_n(x)$

x	-2	-1	0	1	2	3
$P_n(x)$	-5	1	1	1	7	25

IN ORDER FOR $P_n(x)$ TO CORRESPOND TO 6 DISCRETE DATUM,

$$P_n(x) \stackrel{!}{=} P_6(x) \quad (?)$$

$$\underbrace{\& \ \mathrm{DEG}(P_n) = n-1}$$

(GOT THIS FROM A TEXTBOOK, IN SECTION FOR LAGRANGE FORM...)