1.a $$\sin\frac{\pi}{2}x = \frac{\pi}{2}x - \frac{\pi^3}{8}\frac{x^3}{3!} + \frac{\pi^5}{32}\frac{x^5}{5!} - \frac{\pi^7}{256}\frac{x^7}{7!} + \cdots$$

1.b $$R_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-a)^{n+1}$$

$$R_7(x) = \frac{f^{(8)}(\xi)}{8!}(x-a)^8, \quad f^{(8)}(x) = -\frac{\pi^8}{512}\sin\left(\frac{\pi}{2}x\right)$$

$$R_7(x) = -\frac{\pi^8}{512}\frac{\sin\left(\frac{\pi}{2}\xi\right)}{8!}x^8$$

1.c WE WANT $R_n(x) \leq 10^{-5}$

SAY $x = 0.1$, $\xi = 0.001$

THEN $R_7(x) = -\frac{\pi^8}{512}\frac{\sin\left(\frac{\pi}{2}0.001\right)}{8!}(0.1)^8 \approx 7.220 \times 10^{-15} < 10^{-5}$

SO $n = 7$ WOULD BE ACCURATE ENOUGH