7b

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots,$$

$$x \to x-h$$

$$a \to x$$

$$f(x-h) = f(x) + f'(x)(x-h-x) + \frac{f''(x)}{2!}(x-h-x)^2 + \dots$$

$$\dots + \frac{f'''(x)}{3!}(x-h-x)^3 + \dots$$

$$= f(x) - f'(x)h + \frac{f''(x)}{2!}h^2 + \dots$$

$$f'(x) = \frac{f(x) - f(x-h)}{h} + \frac{f''(x)}{2!}h + \dots$$

$$\dots - \frac{f'''(x)}{3!}h^2 + \dots$$

7b

$$f''(x) = \frac{2!}{h^2}\Big[f(x+h) - f(x) - f'(x)h + \cdots$$

$$\cdots - \frac{f'''(x)}{3!} h^3 + \cdots \Big]$$

$$= \frac{2!}{h^2}\Big[f(x+h) - f(x) - h\Big(\frac{f(x) - f(x-h)}{h} + \cdots$$

$$\cdots + \frac{f''(x)}{2!} h - \frac{f'''(x)}{3!} h^2 + \cdots \Big) + \cdots$$

$$\cdots + \Big(- \frac{f'''(x)}{3!} h^3 + \cdots \Big)\Big]$$

$$= 2\left[\frac{f(x+h) - 2f(x) + f(x-h)}{h^2} \right] + \cdots$$

$$\cdots + \frac{2!}{h^2}\Big[- \frac{f''(x)}{2!} h^2 + \frac{f'''(x)}{3!} h^3 + \cdots$$

$$\cdots - \frac{f'''(x)}{3!} h^3 - \frac{f^{(4)}(x)}{4!} h^4 - \frac{f^{(4)}(x)}{4!} h^4 + \cdots \Big]$$

7b

$$f''(x) = 2D_+D_-f(x) + 2\left[-\frac{f''(x)}{2!} - 2\frac{f^{(4)}(x)}{4!}h^2 + \dots\right]$$

$$D_+D_-f(x) = \frac{1}{2}f''(x) + \frac{1}{2}f''(x) + \frac{f^{(4)}(x)}{4!\cdot 2}h^2 + \dots$$

$$= f''(x) + O(h^2) \quad \therefore$$

ASYMPTOTIC ERROR CONSTANT

$$O(h^2) \approx C \cdot h$$

$$C = \frac{f^{(4)}(x)}{3!}$$