$$P_4 = x^4 + \alpha_{43} P_3 + \alpha_{42} P_2 + \alpha_{41} P_1 + \alpha_{40} P_0$$

$$= x^4 - \frac{\langle x^4, P_3 \rangle}{\|P_3\|^2} P_3 - \frac{\langle x^4, P_2 \rangle}{\|P_2\|^2} P_2 - \frac{\langle x^4, P_1 \rangle}{\|P_1\|^2} P_1 - \frac{\langle x^4, P_0 \rangle}{\|P_0\|^2} P_0$$

$$\frac{\langle x^4, P_3 \rangle}{\|P_3\|^2} P_3 \longrightarrow \langle x^4, P_3 \rangle = \int_{-1}^{1} x^4 \left(x^3 - \frac{3}{5}x\right) dx = \int_{-1}^{1} x^7 - \dots x^5 = 0$$

$$\longrightarrow P_4 = x^4 - \frac{\langle x^4, P_2 \rangle}{\|P_2\|^2} P_2 - \frac{\langle x^4, P_0 \rangle}{\|P_0\|^2} P_0$$

$$\longrightarrow \langle x^4, P_2 \rangle = \int_{-1}^{1} x^4 \cdot x^2 - x^4 \cdot \frac{1}{3}\, dx = 2\int_{0}^{1} x^6 - \frac{1}{3}x^4\, dx$$

$$\longrightarrow \|P_2\|^2 = \int_{-1}^{1} \left(x^2 - \frac{1}{3}\right)^2 dx \qquad = 2\left[\frac{1}{9} - \frac{1}{12}\right]$$

$$= 2\int_{0}^{1} x^4 - \frac{2}{3}x^2 + \frac{1}{9} \qquad = \frac{1}{36} = \frac{1}{42}$$

$$= 2\left[\frac{1}{5} - \frac{2}{6} + \frac{1}{9}\right] = -0.38$$