JACOBI

$$Ax = b \longleftrightarrow (L+D+U)x = b$$

$$Dx = -(L+U)x + b$$

$$x = \underbrace{-D^{-1}(L+U)}_{B_J}x + D^{-1}b$$

$$B_J = -D^{-1}(L+U)$$

"In practice"

$$Dx_{k+1} = -(L+U)x_k + b$$

"In general"

$$x_i^{(k+1)} = \frac{1}{a_{ii}}\left[b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k)} - \sum_{j=i+1}^{n} a_{ij}x_j^{(k)}\right]$$

"Example"

$$2x_1 - x_2 = 1$$
$$-x_1 + 2x_2 = 1$$

$$2x_1^{(k+1)} = x_2^{(k)} + 1$$
$$2x_2^{(k+1)} = x_1^{(k)} + 1$$

$$\rightarrow x_1^{(k+1)} = \frac{1}{2}(x_2^{(k)} + 1)$$
$$x_2^{(k+1)} = \frac{1}{2}(x_1^{(k)} + 1)$$

GAUSS - SEIDEL

$$Ax = b \longleftrightarrow (L+D+U)x = b$$

$$(L+D)x = -Ux + b$$

$$x = \underbrace{-(L+D)^{-1}U}_{B_{GS}}x + \cdots$$
$$\cdots + (L+D)^{-1}b$$

$$B_{GS} = -(L+D)^{-1}U$$

"In practice"

$$(L+D)x_{k+1} = -Ux_k + b$$

"In general"

$$x_i^{(k+1)} = \frac{1}{a_{ii}}\left[b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}x_j^{(k)}\right]$$

G-S w/ ACCELERATION PARAMETER

$$(L+D)x_{k+1} = -Ux_k + b$$

$$Dx_{k+1} = Dx_k - (Lx_{k+1} + (D+U)x_k - b)$$

"In practice"

$$Dx_{k+1} = Dx_k - \omega(Lx_{k+1} + (D+U)x_k - b)$$

$$(\omega L + D)x_{k+1} = Dx_k - \omega(D+U)x_k + \omega b$$

$$(\omega L + D)x_{k+1} = ((1-\omega)D - \omega U)x_k + \omega b$$

Now, reverting to notation of previous derivations,

$$x = \underbrace{(\omega L + D)^{-1}((1-\omega)D - \omega U)}_{B_\omega}x + (\omega L + D)^{-1}\omega b$$

$$B_\omega = (\omega L + D)^{-1}((1-\omega)D - \omega U)$$