HW 2

SOLVE THE TWO SYSTEMS OF LINEAR EQUATIONS:

$$A x_1 = e_1$$
$$A x_2 = e_2$$

WHERE e_1, e_2 ARE COLUMNS OF THE IDENTITY MATRIX

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\longrightarrow \det A = ad - bc$$

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} a & b & 1 & 0 \\ c & d & 0 & 1 \end{array}\right] \xrightarrow[=(2)]{(2)-c} \left[\begin{array}{cc|cc} a & b & 1 & 0 \\ 0 & d-c & -c & (1-c) \end{array}\right]$$

$$\downarrow (2)\cdot\frac{1}{d-c} = (2)$$

$$\left[\begin{array}{cc|cc} 1 & b/a & 1/a & 0 \\ 0 & 1 & \frac{-c}{d-c} & \frac{1-c}{d-c} \end{array}\right] \xleftarrow[=(1)]{(1)\cdot\frac{1}{a}} \left[\begin{array}{cc|cc} a & b & 1 & 0 \\ 0 & 1 & \frac{-c}{d-c} & \frac{1-c}{d-c} \end{array}\right]$$

$$\downarrow -b/a\cdot(2) + (1) = (1)$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 1/a - b/a & -b/a \\ 0 & 1 & -\frac{c}{d-c} & \frac{1-c}{d-c} \end{array}\right]$$

$$\underbrace{\qquad\qquad\qquad}_{A^{-1}}$$

IF THIS WERE DONE FOLLOWING BARANNYK'S NOTES, WOULD THE SOLUTION BE MORE CLEAR?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$a x_1 = 1$

$c x_1 = \emptyset$

$$A^{-1} = \begin{bmatrix} \frac{1-b}{a} & -\frac{b}{a} \\ -\frac{c}{d-c} & \frac{1-c}{d-c} \end{bmatrix} \xrightarrow[(2)\cdot a = (2)]{(1)\cdot a = (1)} \begin{bmatrix} 1-b & -b \\ -\frac{ca}{d-c} & \frac{a(1-c)}{d-c} \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x_1 & 0 \\ 0 & x_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$a \cdot x_1 = 1$; $b \cdot x_2 = 0$

$c \cdot x_1 = 0$; $d \cdot x_2 = 1$

$$\begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$$