$\frac{3}{b}$

$$\longrightarrow P_3(x) = f[x_0] + f[x_0, x_1](x - x_0) + \cdots$$

$$\cdots + f[x_0, x_1, x_2](x - x_0)(x - x_1) + \cdots$$

$$\cdots + f[x_0, \ldots, x_3](x - x_0)(x - x_1)(x - x_2)$$

$$= 1 + 1.71828(x-1) + 1.47625(x-1)(x-e) + \cdots$$

$$\cdots + 4.83499(x-1)(x-e)(x-e^2)$$

$$P_3(x) = a_0 + (x - x_0)\Big(a_1 + (x - x_1)\big(a_2 + a_3(x - x_2)\big)\Big)$$

$$= \boxed{1 + (x-1)\Big(1.71828 + (x-e)\big(1.47625 + 4.83499(x-e^2)\big)\Big)}$$

$\frac{3}{b}$

x_i	$f[\cdot]$	$f[\cdot,\cdot]$	$f[\cdot,\cdot,\cdot]$	$f[\cdot,\cdot,\cdot,\cdot]$
0	e^0	$\frac{e^1 - e^0}{1 - 0}$	$\frac{4.67077 - 1.71828}{2 - 0}$	$\frac{6.31124 - 1.47625}{4 - 0}$
1	e^1	$\frac{e^2 - e^1}{2 - 1}$	$\frac{23.6045 - 4.67077}{4 - 1}$	
2	e^2	$\frac{e^4 - e^2}{4 - 2}$		
4	e^4			
		1.71828	1.47625	4.83499
		4.67077	6.31124	
		23.6045		

$\underline{3}$
a

$f(x) = e^x$

KNOWN AT

$x_0 = 0$
$x_1 = 1$
$x_2 = 2$
$x_3 = 4$

WRITE THE INTERPOLATING POLY IN LAGRANGE FORM

$$P_n(x) = \sum_{k=\emptyset}^{n} f(x_k)\, l_k(x) = \sum_{k=\emptyset}^{3} e^{x_k}\, l_k(x)$$

$$= e^{x_0} l_0 + e^{x_1} l_1 + e^{x_2} l_2 + e^{x_3} l_3$$

$$l_k = \prod_{\substack{i=\emptyset \\ i \neq k}}^{n} \frac{x - x_i}{x_k - x_i}$$

$$l_0 = \prod_{\substack{i=\emptyset \\ i \neq k}}^{3} \frac{x - x_i}{x_0 - x_i} = \frac{x - x_1}{x_0 - x_1} \cdot \frac{x - x_2}{x_0 - x_2} \cdot \ldots$$

$$\ldots \cdot \frac{x - x_3}{x_0 - x_3}$$

$$= \frac{x-1}{\emptyset - 1} \cdot \frac{x-2}{\emptyset - 2} \cdot \frac{x-4}{\emptyset - 4}$$

$$l_0 = -\frac{1}{8}(x-1)(x-2)(x-4)$$

$$l_1 = \prod^{3} \frac{x - x_i}{x_1 - x_i} = \frac{x - x_0}{x_1 - x_0} \cdot \frac{x - x_2}{x_1 - x_2} \cdot \frac{x - x_3}{x_1 - x_3} = \frac{x - \emptyset}{\underset{1}{1 - \emptyset}} \cdot \frac{x - 2}{\underset{-1}{1 - 2}} \cdot \frac{x - 4}{\underset{-3}{1 - 4}}$$

$$l_1 = \frac{1}{3} x(x-2)(x-4)$$

4

$$l_k(x) = \prod_{\substack{i=\emptyset \\ i \neq k}}^{n} \frac{x - x_i}{x_k - x_i}$$

x	-2	-1	0	1	2	3
$p(x)$	-5	1	1	1	7	25

BY THE UNIQUENESS THEOREM FROM LECTURE NOTES 21 (... $p(x) = q(x)$ ...),

$$l_k(x) = \prod_{\substack{i=\emptyset \\ i \neq k}}^{n} \frac{x - x_i}{x_k - x_i} = p(x) \quad \text{FOR SOME } n \leq 5,$$

$$x = \{x_0 = -2,\ x_1 = -1,\ x_2 = 0,\ x_3 = 1,\ x_4 = 2,\ x_5 = 3\}$$

$$p(x) = \{-5, 1, 1, 1, 7, 25\}$$

OTHERWISE $p(x)$ DOES NOT HAVE DEGREE n

$\longrightarrow n = \emptyset$

$$l_k(x) = \emptyset \neq p(x) \longrightarrow 1 \leq n \leq 5$$

$n = 1$

$$l_0(x) = \frac{x - x_1}{x_0 - x_1} = \frac{x + 1}{-2 + 1} = -x - 1$$

$\frac{3}{c}$

$P_{3_L}(x) = 0.8455\,x^3 - 1.060\,x^2 + 1.933\,x + 1$

$e^{1.5} \approx 0.8455(1.5)^3 - 1.060(1.5)^2 + 1.933(1.5) + 1$

≈ 4.368

$e^4 \approx 0.8455(4)^3 - \cdots$

≈ 45.88

$P_{3_N}(x) = 5.073\,x^3 - 13.74\,x^2 + 10.39\,x + 1$

$e^{1.5} \approx 5.073(1.5)^3 - 13.74(1.5)^2 + 10.39(1.5) + 1$

-30.92

≈ 2.791

I FOUND $\frac{13.74}{1.060} \approx 2\,\frac{10.39}{1.933} = 2\,\frac{a_{3N}}{a_{3L}}$

$\approx 2\,\frac{5.073}{0.8455} = 2\,\frac{a_{1N}}{a_{1N}}$

WHAT IF I...

$P_{3_N}(x) \longrightarrow P_{3_N}' = 5.073x^3 - \frac{13.74}{2}x^2 + 10.39x + 1$

NOW,

$e^{1.5} \approx 2.791 \longrightarrow 5.073(1.5)^3 - 6.87(1.5)^2 + 10.39(1.5) + 1$

$17.12 - 15.46 + 15.59 + 1$

(SHIT

3
a

$$P_4(x) = -\frac{1}{6}e^{x_0}(x^3 - 6x^2 + 11x - 6) + \frac{1}{2}e^{x_1}(x^3 - 5x^2 + 6x) + \cdots$$

$$\cdots - \frac{1}{2}e^{x_2}(x^3 - 4x^2 + 7x) + \frac{1}{6}e^{x_3}(x^3 - 3x^2 + 3x)$$

$$= x^3\left(-\frac{1}{6}e^{x_0} + \frac{1}{2}e^{x_1} - \frac{1}{2}e^{x_2} + \frac{1}{6}e^{x_3}\right) + \cdots$$

$$\cdots + x^2\left(e^{x_0} - \frac{5}{2}e^{x_1} + 2e^{x_2} - \frac{1}{2}e^{x_3}\right) + \cdots$$

$$\cdots + x\left(-\frac{11}{6}e^{x_0} + 3e^{x_1} - \frac{7}{2}e^{x_2} + \frac{1}{2}e^{x_3}\right) + \cdots$$

$$\cdots + e^{x_0}$$

$$\boxed{P_4(x) \approx -11.18x^3 - 1.060x^2 - 9.497x + 1}$$

EITHER THIS $P_4(x)$
OR THE NEXT
IN 3b ARE
INCORRECT ... OR BOTH...
BUT THEY SHOULD BE EQUAL

3/6

$$P_4(x) \approx 1 + 1.718x + 1.477(x^2 - x) + \cdots$$

$$\cdots + 5.073(x^3 - 3x^2 + 2x)$$

$$\boxed{P_4(x) \approx 5.073x^3 - 13.74x^2 + 10.39x + 1}$$

4

$l_2(x) = \frac{1}{2}(x^2 + 3x + 2)$

$l_2(x_0) = \frac{1}{2}((-2)^2 + 3(-2) + 2) = \frac{1}{2}(4 - 6 + 2) = \emptyset \neq P(x_0)$

$\longrightarrow 3 \leq n \leq 5$

$n = 3$

$l_0(x) = \frac{x - x_1}{x_0 - x_1} \cdot \frac{x - x_2}{x_0 - x_2} \cdot \frac{x - x_3}{x_0 - x_3} = \frac{x+1}{-2+1} \cdot \frac{x - \emptyset}{-2-\emptyset} \cdot \frac{x-1}{-2-1} = -\frac{1}{6}x(x+1)(x-1)$

$= -\frac{1}{6}(x^3 + x)$

$l_0(x_0) = -\frac{1}{6}((-2)^3 - 2) = -1 \neq P(x_0)$

$l_1(x) = \frac{x - x_0}{x_1 - x_0} \cdot \frac{x - x_2}{x_1 - x_2} \cdot \frac{x - x_3}{x_1 - x_3} = \frac{x+2}{-1+2} \cdot \frac{x - \emptyset}{-1-\emptyset} \cdot \frac{x-1}{-1-1} = \frac{1}{2}x(x+2)(x-1)$

$= \frac{1}{2}(x^3 + x^2 - 2x)$

$l_1(x_0) = \frac{1}{2}((-2)^3 + (-2)^2 - 2(-2)) = \emptyset \neq P(x_0)$

3b

$x_0 = 0$

$x_1 = 1$

$x_2 = 2$

$x_3 = 3$

$$\begin{array}{lllll}
e^{x_0} & & & & \\
 & > e^{x_1} - e^{x_0} & & & \\
e^{x_1} & & > \frac{1}{2}(e^{x_2} - 2e^{x_1} + e^{x_0}) & & \\
 & > e^{x_2} - e^{x_1} & & > \frac{1}{6}(e^{x_3} - 3e^{x_2} + 3e^{x_1} - e^{x_0}) & \\
e^{x_2} & & > \frac{1}{2}(e^{x_3} - 2e^{x_2} + e^{x_1}) & & \\
 & > e^{x_3} - e^{x_2} & & & \\
e^{x_3} & & & &
\end{array}$$

$$P_n(x) = e^{x_0} + (e^{x_1} - e^{x_0})(x - x_0) + \frac{1}{2}(e^{x_2} - 2e^{x_1} + e^{x_0})(x - x_0)(x - x_1) + \cdots$$

$$(e^{x_3} - 3e^{x_2} + 3e^{x_1} - e^{x_0})(x - x_0)(x - x_1)(x - x_2)$$

$$\approx 1 + 1.718x + 1.477(x^2 - 1) + \cdots$$

$$\cdots + 5.073(x^4 - 3x^3 + 3x)$$

$$\boxed{P_n(x) = 5.073x^4 - 15.22x^3 - 13.74x^2 + 16.94x + \cdots - 0.4770}$$

3
a

$$P_4(x) = -\frac{1}{6}e^{x_0}(x^3 - 6x^2 + 11x - 6) + \cdots$$

$$\cdots + \frac{1}{2}e^{x_1}(x^3 - 5x^2 + 6x) + \cdots$$

$$\cdots - \frac{1}{2}e^{x_2}(x^3 - 4x^2 + 7x) + \cdots$$

$$\cdots + \frac{1}{6}e^{x_3}(x^3 - 3x^2 + 3x)$$

$$= -\frac{1}{6}e^{x_0}x^3 + e^{x_0}x^2 - \frac{11}{6}e^{x_0}x + e^{x_0} + \cdots$$

$$\cdots + \frac{1}{2}e^{x_1}x^3 - \frac{5}{2}e^{x_1}x^2 + 3e^{x_1}x + \cdots$$

$$\cdots - \frac{1}{2}e^{x_2}x^3 + 2e^{x_2}x^2 - 7e^{x_2}x + \cdots$$

$$\cdots + \frac{1}{6}e^{x_3}x^3 - \frac{1}{2}e^{x_3}x^2 + \frac{1}{2}e^{x_3}x$$

3
a

$$P_n(x) = \sum_{k=0}^{n} f(x_k)\, l_k(x)$$

x_0	x_1	x_2	x_3
0	1	2	3

$$P_4(x) = f(x_0)\, l_0(x) + \cdots + f(x_4)\, l_4(x)$$

$$= f(x_0)\cdot \frac{x-x_1}{x_0-x_1}\cdot \cdots \cdot \frac{x-x_4}{x_0-x_4} + \cdots + f(x_4)\cdot \frac{x-x_0}{x_4-x_0}\cdot \cdots \cdot \frac{x-x_3}{x_4-x_3}$$

$$f(x_k) = e^{x_k}$$

$$P_4(x) = e^{x_0}\cdot \frac{x-1}{0-1}\cdot \frac{x-2}{0-2}\cdot \frac{x-3}{0-3} + e^{x_1}\cdot \frac{x-0}{1-0}\cdot \frac{x-2}{1-2}\cdot \frac{x-3}{1-3} + \cdots$$

$$\cdots + e^{x_2}\cdot \frac{x-0}{2-0}\cdot \frac{x-1}{2-1}\cdot \frac{x-3}{2-3} + e^{x_3}\cdot \frac{x-0}{3-0}\cdot \frac{x-1}{3-1}\cdot \frac{x-2}{3-2}$$

2 1 -1 -2

$$= -e^{x_0}\frac{1}{6}(x-1)(x-2)(x-3) + e^{x_1}\frac{1}{2}x(x-2)(x-3) + \cdots$$

$$\cdots - e^{x_3}\frac{1}{2}x(x-1)(x-3) + e^{x_3}\frac{1}{6}x(x-1)(x-2)$$

$$= -e^{x_0}\frac{1}{6}(x-1)(x^2-5x+6) + e^{x_1}\frac{1}{2}x(x^2-5x+6) + \cdots$$

$$\cdots - e^{x_3}\frac{1}{2}x(x^2-4x+7) + e^{x_3}\frac{1}{6}x(x^2-3x+3)$$

5

$$\sum^{n} \ell_k(x) = 1$$

$$P_n(x) = \sum_{k=\emptyset}^{n} f(x_k)\,\ell_k(x)$$

$$f(x) = P_n(x) + \frac{f^{(n+1)}(\xi_n)}{(n+1)!}(x-x_0)(x-x_1)\dots(x-x_n)$$

IF $f(x) = 1 \rightarrow f \equiv 1$, $f^{(n+1)}(\xi_n) = \emptyset$

$$1 = \sum_{k=\emptyset}^{n} 1 \cdot \ell_k(x) + \emptyset$$

$$\sum_{k=\emptyset}^{n} \ell_k(x) = \emptyset \quad \therefore$$

4

(x_n)	$f[\cdot,\cdot]$	$f[\cdot,\cdot,\cdot]$	$f[\cdot,\cdot,\cdot,\cdot]$	$f[\cdot,\cdot,\cdot,\cdot,\cdot]$
(-2)				
	$\frac{1+5}{-1+2} = 6$			
(-1)		$\frac{\emptyset - 6}{0+2} = \frac{-6}{2} = -3$		
	$\frac{1-1}{0+1} = \emptyset$		$\frac{\emptyset + 3}{1+2} = 1$	
		$\emptyset = \emptyset$		$\frac{1-1}{\dots} = \boxed{\emptyset}$ $(a_4 = \emptyset)$
$(\emptyset)$	$\frac{1-1}{1-0} = \emptyset$		$\frac{3-\emptyset}{2+1} = 1$	
(1)		$\frac{6-0}{2-0} = 3$		$\frac{1-1}{\dots} = \emptyset$
	$\frac{7-1}{2-1} = 6$		$\frac{6-3}{3-\emptyset} = 1$	
(2)		$\frac{18-6}{3-1} = \frac{12}{2} = 6$		
	$\frac{25-7}{3-2} = 18$			
(3)				

$\dots > f[x_0, \dots, x_5] = \boxed{\emptyset}$ $(a_5 = \emptyset)$

FROM THE TABLE,

$\Longrightarrow n = 3$ $\left(a_4 = \emptyset,\ a_5 = \emptyset \text{ FOR } p_n(x) = a_0 + a_1(x - x_0) + \dots\right)$

$\frac{3}{a}$

$$l_2 = \prod_{\dots}^{3} \frac{x - x_i}{x_2 - x_i} = \frac{x - x_0}{x_2 - x_0} \; \frac{x - x_1}{x_2 - x_1} \; \frac{x - x_3}{x_2 - x_3}$$

$$= \frac{x - 0}{\underset{2}{2 - 0}} \; \frac{x - 1}{\underset{1}{2 - 1}} \; \frac{x - 4}{\underset{-2}{2 - 4}}$$

$$l_2 = -\frac{1}{4} x (x-1)(x-4)$$

$$l_3 = \prod_{\dots}^{3} \frac{x - x_i}{x_3 - x_i} = \frac{x - x_0}{x_3 - x_0} \; \frac{x - x_1}{x_3 - x_1} \; \frac{x - x_2}{x_3 - x_2}$$

$$= \frac{x - 0}{\underset{4}{4 - 0}} \; \frac{x - 1}{\underset{3}{4 - 1}} \; \frac{x - 2}{\underset{2}{4 - 2}}$$

$$= \frac{1}{14} x (x-1)(x-2)$$

$$\boxed{\begin{aligned} P_n(x) &= -\frac{1}{8} e^0 (x-1)(x-2)(x-4) + \cdots \\ &\cdots + \frac{1}{3} e^1 x (x-2)(x-4) + \cdots \\ &\cdots - \frac{1}{4} e^2 x (x-1)(x-4) + \cdots \\ &\cdots - \frac{1}{14} e^4 x (x-1)(x-2) \end{aligned}}$$