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THERE IS IN THE NOTES A THEOREM RELATING THE CONDITION NUMBER
OF A, RELATIVE RESIDUAL, & RELATIVE ERROR

CHECK THAT THIS HOLDS FOR

$A = \begin{bmatrix} 1.8969 & 0.8648 \\ 0.2161 & 0.1441 \end{bmatrix}$ $b = \begin{bmatrix} 0.8642 \\ 0.1440 \end{bmatrix}$

RESULTS APPROXIMATE,

$\tilde{x}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$x = \begin{pmatrix} 2 \\ .2 \end{pmatrix}$

$\tilde{x}_2 = \begin{pmatrix} 0.9911 \\ -0.4870 \end{pmatrix}$

THE THEOREM? →

LET A BE INVERTIBLE, $Ax = b$

x EXACT SOLUTION

$\tilde{x}$ APPROXIMATE SOLUTION

$e = x - \tilde{x}$ ERROR

$r = b - A\tilde{x}$ RESIDUAL

THEN $\dfrac{\|e\|}{\|x\|} \leq K(A) \cdot \dfrac{\|r\|}{\|b\|}$; $K(A)$ IS CONDITION NUMBER OF A

→ $K(A) = \|A\| \cdot \|A^{-1}\|$

$\dfrac{\|e_1\|_\infty}{\|x\|_\infty} = \dfrac{3}{2} \leq (3.279 \times 10^8) \cdot \dfrac{(0.0006)}{(0.8642)} = 2.277 \times 10^5$ LOOK FOR COND(...)

$\|x\|_\infty = 2$ $; K(A) = 3.279 \times 10^8$

$\|e_1\|_\infty = 3$

$\|e_2\|_\infty = 2.4870$

$\|r_1\|_\infty = 0.0006$ $\|r_2\|_\infty = 1 \times 10^{-5}$

$\|b\| = 0.8642$

THE THEOREM HOLDS

$\dfrac{\|e_2\|_\infty}{\|x\|_\infty} = \dfrac{2.4870}{2} \leq (3.279 \times 10^8) \dfrac{1 \times 10^{-5}}{0.8642}$

$= 3.794 \times 10^3$