HW 2

GAUSSIAN ELIMINATION

$$Ax = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = b$$

$$M_1 A = \begin{bmatrix} 1 & 0 \\ -\frac{c}{a} & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ -\frac{c}{a} \cdot a + c & -\frac{c}{a} b + d \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & -\frac{c}{a} b + d \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - bc} \begin{bmatrix} \cdots \end{bmatrix} \xrightarrow{-\frac{c}{a} b + d} \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix} = A^{-1}$$

$$M_1 x = \begin{bmatrix} x_1 \\ -\frac{c}{a} x_1 + x_2 \end{bmatrix}, \quad M_1 b = \begin{bmatrix} e_1 \\ -\frac{c}{a} e_1 + e_2 \end{bmatrix}$$

$$Ux = \begin{bmatrix} a & b \\ 0 & -\frac{c}{a} b - d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} e_1 \\ -\frac{c}{a} e_1 + e_2 \end{bmatrix} = Mb = y$$