7 b

(i) $f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2$

$x \longrightarrow x - h$

$a \longrightarrow x$

$f(x-h) = f(x) + f'(x)(x-h-x) + \dots$

$\dots + \frac{f''(x)}{2!}(x-h-x)^2 + \dots$

$\dots + \frac{f'''(x)}{3!}(x-h-x)^3 + \dots$

$= f(x) - f'(x)h + \frac{f''(x)}{2!}h^2 - \dots$

$f'(x) = \frac{f(x) - f(x-h)}{h} + \frac{f''(x)}{2!}h - \dots$

$\dots - \frac{f'''(x)}{3!}h^2 + \dots$

7b

SOLVE FOR $f''(x)$ IN (1)
WHEN $x \to x+h$, $a \to x$

$$f''(x) = \frac{2!}{h^2}\Big[f(x+h) - f(x) - f'(x)h + \cdots$$

$$\cdots + \left(-\frac{f'''(x)}{3!}h^3 - \cdots\right)\Big]$$

$$= \frac{2!}{h^2}\Big[f(x+h) - f(x) - h\Big(\frac{f(x)-f(x-h)}{h} + \cdots$$

$$\cdots + \frac{f''(x)}{2!}h - \frac{f'''(x)}{3!}h^2 + \cdots\Big) + \cdots$$

$$\cdots + \left(-\frac{f'''(x)}{3!}h^3 - \cdots\right)\Big]$$

$$= 2\left[\frac{f(x+h) - 2f(x) + f(x-h)}{h^2}\right] + \cdots$$

$$\cdots + \frac{2!}{h^2}\Big[-\frac{f''(x)}{2!}h^2 + \frac{f'''(x)}{3!}h^3 - \cdots$$

$$\cdots - \frac{f'''(x)}{3!}h^3 - \frac{f^{(4)}(x)}{4!}h^4 - \cdots$$

$$\cdots - \frac{f^{(4)}(x)}{4!}h^4 + \cdots\Big]$$

7b

$$f''(x) = 2D_+D_-f(x) + 2\left[-\frac{f''(x)}{2!} - 2\frac{f^{(4)}(x)}{4!}h^2 - \dots\right]$$

$$D_+D_-f(x) = \frac{1}{2}f''(x) + \frac{1}{2}f''(x) + \frac{f^{(4)}(x)}{12}h^2 + \dots$$

$$= f''(x) + O(h^2) \quad \therefore$$