PROVE THAT $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ HAS NO LU DECOMPOSITION

→ ONE MIGHT ASSUME SUCH A LU DECOMP. EXISTS AND SHOW THAT THIS BRINGS A CONTRADICTION

$$LU = \begin{bmatrix} a & 0 \\ b & c \end{bmatrix}\begin{bmatrix} x & y \\ 0 & z \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} ax & ay \\ bx & by + cz \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} ax & ay \\ 0 & by - bx + cz \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1-bx & 1-bx \end{bmatrix}$$

$$= \begin{bmatrix} ax & ay \\ 0 & by - bx + cz \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & \partial \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1-bx & 1-bx \end{bmatrix}$$

$$ax\alpha + ay\gamma = \emptyset$$
$$(by - bx + cz)\gamma = 1 - bx$$
$$ax\beta + ay\partial = 1$$
$$(by - bx + cz)\partial = 1 - bx$$

$$\varepsilon\gamma = \varepsilon\partial$$
$$\gamma = \partial =$$

$$E_{n-1} \cdot \dots \cdot E_2 E_1 A = U$$
$$E_1^{-1} E_2^{-1} \cdot \dots \cdot E_{n-1}^{-1} = L$$
$$LU = (E_{n-1} \cdot \dots$$

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LET $x = [x_1, \dots, x_n]^T = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ ← VECTOR

$$\|x\|_\infty = \text{MAX}\{|x_i|,\ i = 1, \dots, n\}$$

$\|x\|_\infty$ IS A VECTOR NORM

PROOF:

1. $\|x\|_\infty \geq \emptyset$

   $\|x\|_\infty = \emptyset \longleftrightarrow \text{MAX}|x_i| = \emptyset \longleftrightarrow |x_i| = \emptyset \longleftrightarrow x = \emptyset \;\therefore$

2. $\|\alpha x\|_\infty \overset{\text{DEF}}{=} \text{MAX}\{\underbrace{|\alpha x_i|}_{\text{REAL NUMBERS}},\ i = 1, \dots, n\} = |\alpha| \cdot \text{MAX}\{|x_i|,\ i = 1, \dots, n\}$

   $= |\alpha| \cdot \|x\|_\infty \;\therefore$

3. $\|x + y\|_\infty = \text{MAX}\{|x_i + y_i|,\ i = 1, \dots, n\} \leq \text{MAX}\{|x_i| + |y_i|,\ i = 1, \dots, n\}$

   $\leq \text{MAX}\{|x_i|,\ i = 1, \dots, n\}$

   $+ \text{MAX}\{|y_i|,\ i = 1, \dots, n\}$

   $= \|x\|_\infty + \|y\|_\infty \;\therefore$

$$U = (E_{n-1} \cdot \cdots \cdot E_1 A)(E_1^{-1} \cdot \cdots \cdot E_{n-1}^{-1})$$

1) $E_k^{-1} = (I - m_k e_k^T)^{-1} = I + m_k e_k^T$

2) $E_1^{-1} E_2^{-1} = I + m_1 e_1^T + m_2 e_2^T$

LU DECOMPOSITION

ASSUME $E_{n-1} \cdots E_2 E_1 A = U$ UPPER TRIANGULAR MATRIX

$(a_{kk}^{(k)} \neq \emptyset)$

$$E_k = \begin{bmatrix} 1 & \cdots & & & \emptyset \\ \vdots & \ddots & & & \\ & & 1 & & \\ & & -m_{k+1,k} & & \\ \emptyset & & \vdots & \ddots & \\ & & -m_{n,k} & & 1 \end{bmatrix} = I - m_k e_k^T$$

$$m_k = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ m_{k+1,k} \\ \vdots \\ m_{k,n} \end{bmatrix} \qquad e_k = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \leftarrow k^{th} \text{ ROW}$$

3) $$E_1^{-1} E_2^{-1} \cdot \cdots \cdot E_{n-1}^{-1} = \begin{bmatrix} 1 & & & & \emptyset \\ m_{21} & 1 & & & \\ m_{31} & m_{32} & \ddots & & \\ \vdots & \vdots & & \ddots & 1 \\ m_{n1} & m_{n2} & \cdots & m_{n,n-1} & 1 \end{bmatrix} = I + m_1 e_1^T + m_2 e_2^T + \cdots + m_{n-1} e_{n-1}^T = L$$

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$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$LU = (E_3 E_2 E_1 A)(E_1^{-1} E_2^{-1} E_3^{-1})$$

$$= E_1 A E_1^{-1}$$

$$= \begin{bmatrix} 1 & 0 \\ m & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -m & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ m & 1 \end{bmatrix} \begin{bmatrix} -m & 1 \\ 1-m & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -m & 1 \\ -m^2-m+1 & m+1 \end{bmatrix} \overset{!}{=} A = LU$$

$$\begin{bmatrix} -m & 1 \\ -m^2-m+1 & m+1 \end{bmatrix}\begin{bmatrix} x_{11} & x_{21} \\ x_{12} & x_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} -m & 1 \\ 0 & \cancel{m}+\cancel{1}-(-m^2-\cancel{m}+\cancel{1}) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1-(-m^2-m+1) & 1-(-m^2-m+1) \end{bmatrix}$$

$$\begin{bmatrix} -m & 1 \\ 0 & m^2 \end{bmatrix} =$$

$$-m\,x_{11} + x_{12} = \phi$$

$$m^2 x_{12} = 1$$

$$-m\,x_{21} + x_{22} = 1$$

$$m^2 x_{22} = 1$$

$$\boxed{x_{22} = 1/m^2}$$

$$\boxed{x_{12} = 1/m_2}$$

$$\boxed{x_{22} = x_{12}}$$

$$-m\,x_{11} + \frac{1}{m^2} = 0$$

$$-m\,x_{21} + \frac{1}{m^2} = 1$$

$$-m\,x_{11} = -\frac{1}{m^2}$$

$$-m\,x_{21} = 1 - \frac{1}{m^2}$$

$$x_{11} = \frac{1}{m^3}$$

$$x_{21} = -\frac{1}{m} + \frac{1}{m^3}$$

$$= -\frac{1}{m}\left(1 - \frac{1}{m^2}\right)$$