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$$\begin{bmatrix} a & b & 1 & 0 \\ c & d & 0 & 1 \end{bmatrix} \xrightarrow{R_1 \leftarrow \frac{1}{a}R_1} \begin{bmatrix} 1 & b/a & 1/a & 0 \\ c & d & 0 & 1 \end{bmatrix}$$

$$\Big\downarrow R_2 \leftarrow R_2 - cR_1$$

$$\begin{bmatrix} 1 & b/a & 1/a & 0 \\ 0 & 1 & -\dfrac{cb}{a\,d-\frac{cb}{a}} & \dfrac{1}{d-\frac{cb}{a}} \end{bmatrix} \xleftarrow{R_2 \leftarrow \frac{1}{d-\frac{cb}{a}}R_2} \begin{bmatrix} 1 & b/a & 1/a & 0 \\ 0 & d-\dfrac{cb}{a} & -\dfrac{c}{a} & 1 \end{bmatrix}$$

$$\left\{ -\frac{cb}{a}\frac{1}{d-\frac{cb}{a}} = -\frac{cb}{ad-\frac{abc}{a}} = \frac{cb}{ad-bc} \right\}$$

$$\begin{bmatrix} 1 & b/a & 1/a & 0 \\ 0 & 1 & \dfrac{cb}{ad-bc} & \dfrac{1}{d-\frac{cb}{a}} \end{bmatrix} \xrightarrow{R_1 \leftarrow R_1 - \frac{b}{a}R_2} \begin{bmatrix} 1 & 0 & \dfrac{1}{a} - \dfrac{b}{a}\dfrac{cb}{ad-bc} & -\dfrac{b}{a}\left(d-\frac{cb}{a}\right)^{-1} \\ 0 & 1 & \dfrac{cb}{ad-bc} & \left(d-\frac{cb}{a}\right)^{-1} \end{bmatrix}$$