5a

FROM MATLAB PROGRAM 'pmeth':

$\boxed{24}$ ITERATIONS NEEDED FOR CONVERGENCE OF EIGENVECTOR

$n_L = 22$ IS NUMBER OF ITERATIONS FOR EIGENVALUE CONVERGENCE

$n_c > n_L$ WHERE n_c IS NUMBER OF ITERATIONS FOR EIGENVECTOR

CONVERGENCE TOLERANCE VALUES ARE

$T_c \approx 3.1690 \times 10^{-5}$ $T_L \approx 1.6368 \times 10^{-5}$ WHERE T_c, T_L ARE FROM THE EIGENVECTOR CASE AND

$`T_c \approx 8.8066 \times 10^{-5}$ $`T_L \approx 4.5486 \times 10^{-5}$ $`T_c, `T_L$ ARE FROM THE EIGENVALUE CASE

```
%pmeth
%Matt Zeller
%12/3/2018
%PHYS 428
%This program finds eigenvector and dominant eigenvalue of matrix A
%v2 is the leading eigenvector, v1 follows, and so on
%tolVec is convergence tolerance for the vector, tolVal--the value
%c2 is leading 'convergence' as defined on page 280 of A Friendly
Introduction to
%Numerical Analysis--it is  "an estimate for the asymptotic rate of linear
convergence" of the sequence towards the dominant eigenvalue
%c1 follows c2
A = [1 4 5; 4 -3 0; 5 0 7];
v00 = ones(3,1);
v0 = ones(3,1);
v1 = ones(3,1);
v2 = ones(3,1);
v1 = (1/sqrt(3))*v1;
tolVec = 1;
tolVal = 1;
n = 0;
format long
%change tolVec to tolVal to evaluate convergence of the eigenvalue
while tolVec> 5*10^-5
   n=n+1;
   v2 = A*v1;
   c2 = (v2(3,1)-v1(3,1))/(v1(3,1)-v0(3,1));
   c1 = (v1(3,1)-v0(3,1))/(v0(3,1)-v00(3,1));
   tolVec = abs(c2-c1);
   tolVal = abs((v2(3,1)/v1(3,1))-(v0(3,1)/v00(3,1)));
   v00 = v0;
   v0 = v1;
   v1 = v2;
end
domEig = v2(3,1)/v1(3,1)
n
```

```
tolVec
tolVal
v2
```

```
domEig =

     1


n =

    24


tolVec =

     3.169036606109899e-05


tolVal =

     1.636784771186228e-05


v2 =

   1.0e+24 *

   0.662237996490559
   0.201428976555814
   1.050874143638659
```

*results for the case when eigenvalue convergence is used for convergence tolerance are listed in HW problem 5a. Alternatively, results can be reproduced by pmeth.m.

```
%inversepmethod
%Matt Zeller
%12/3/2018
%PHYS 428

%This program finds dominant eigenvalue of matrix A using
%inverse power method
A = [1 4 5; 4 -3 0; 5 0 7];
Ainv = inv(A);
v1 = (1/sqrt(3))*ones(3,1);
v2 = ones(3,1);
format long
disp(['n','    ','Estimate at n','     ','Reciprocal'])
disp(' ')
for n=1:10
   v2 = Ainv*v1;
   en = norm(v2,inf);
   disp([num2str(n),'    ',num2str(en),'                    ',num2str(1/en)])
   v2 = v2 / norm(v2,inf);
   v1 = v2;
end
disp(' ')
disp(' ')
disp(['The approximate eigenvalue of A nearest to q=1 is ',num2str(1/en),])
```

5b

n	Estimate at n	Reciprocal
1	0.33845	2.9547
2	0.88235	1.1333
3	1.0414	0.96026
4	1.0684	0.93595
5	1.0652	0.9388
6	1.0658	0.93828
7	1.0657	0.93835
8	1.0657	0.93834
9	1.0657	0.93834
10	1.0657	0.93834

The approximate eigenvalue of A nearest to q=1 is 0.93834

5c

RALEIGH QUOTIENT ITERATION METHOD CONVERGES AROUND $n = 10$, WHEREAS THE OTHER TWO METHODS CONVERGE AROUND $n = 20$

```
%inversepmethod
%Matt Zeller
%12/3/2018
%PHYS 428

%This program finds dominant eigenvalue of matrix A using
%Rayleigh Quotient
A = [1 4 5; 4 -3 0; 5 0 7];
v1 = (1/sqrt(3))*ones(3,1);
v2 = ones(3,1);
S = v1'*A/v1'*v1;
As = A-eye(3)*S;

disp(['n','    ','Estimate at n','                              ','Reciprocal'])
disp(' ')
format long
for n=1:20
   v2 = As'*v1;
   en = norm(v2,inf);
   disp([num2str(n),'    ',num2str(en,'%1.10e'),'                    ',num2str(1/en,'%↙
1.10e')])
   v2 = v2 / norm(v2,inf);
   v1 = v2;
end
disp(' ')
disp(' ')
disp(['The approximate dominant eigenvalue of A is ',num2str(1/en),])
disp(['The associated eigenvector is '])
disp(' ')
disp(v2)
```

```
>> raleigh

S =

   4.426352063787133
   4.426352063787133
   4.426352063787133

n    Estimate at n                  Reciprocal

1    7.0893163975e+00               1.4105732400e-01
2    8.0012810548e+00               1.2497998672e-01
3    9.7102721492e+00               1.0298372534e-01
4    8.8451560967e+00               1.1305622977e-01
5    9.2406199623e+00               1.0821784730e-01
6    9.0503974112e+00               1.1049238553e-01
7    9.1397218001e+00               1.0941252063e-01
8    9.0972759654e+00               1.0992301474e-01
9    9.1173272199e+00               1.0968126688e-01
10    9.1078262667e+00               1.0979568239e-01
11    9.1123207990e+00               1.0974152711e-01
12    9.1101926319e+00               1.0976716305e-01
13    9.1111997506e+00               1.0975502978e-01
14    9.1107229720e+00               1.0976077344e-01
15    9.1109486247e+00               1.0975805497e-01
16    9.1108418062e+00               1.0975934181e-01
17    9.1108923642e+00               1.0975873274e-01
18    9.1108684321e+00               1.0975902105e-01
19    9.1108797596e+00               1.0975888458e-01
20    9.1108743977e+00               1.0975894918e-01


The approximate dominant eigenvalue of A is 0.10976
The associated eigenvector is
```

```
0.036956950753800
1.000000000000000
0.377007561885549
```