

JACOBI

$$Ax = b \iff (L+D+U)x = b$$

$$Dx = -(L+U)x + b$$

$$x = -\underbrace{D^{-1}(L+U)}_{B_J} x + D^{-1}b$$

$$B_J = -D^{-1}(L+U)$$

"IN PRACTICE"

$$Dx_{k+1} = -(L+U)x_k + b$$

"IN GENERAL"

$$x_i^{(k+1)} = \frac{1}{a_{ii}} \left[b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^n a_{ij} x_j^{(k)} \right]$$

EXAMPLE

$$2x_1 - x_2 = 1$$

$$-x_1 + 2x_2 = 1$$

$$2x_1^{(k+1)} = x_2^{(k)} + 1$$

$$2x_2^{(k+1)} = x_1^{(k)} + 1$$

$$\rightarrow x_1^{(k+1)} = \frac{1}{2}(x_2^{(k)} + 1)$$

$$x_2^{(k+1)} = \frac{1}{2}(x_1^{(k)} + 1)$$

GAUSS-SEIDEL

$$Ax = b \iff (L+D+U)x = b$$

$$(L+D)x = -Ux + b$$

$$x = -\underbrace{(L+D)^{-1}}_{B_{GS}} Ux + \dots + (L+D)^{-1}b$$

B_{GS}

$$B_{GS} = -(L+D)^{-1}U$$

"IN PRACTICE"

$$(L+D)x_{k+1} = -Ux_k + b$$

"IN GENERAL"

$$x_i^{(k+1)} = \frac{1}{a_{ii}} \left[b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^n a_{ij} x_j^{(k)} \right]$$

G-S w/ ACCELERATION PARAMETER

$$\rightarrow (L+D)x_{k+1} = -Ux_k + b$$

$$Dx_{k+1} = Dx_k - (Lx_{k+1} + (D+U)x_k - b)$$

"IN PRACTICE"

$$Dx_{k+1} = Dx_k - U(Lx_{k+1} + (D+U)x_k - b)$$

$$(wL+D)x_{k+1} = Dx_k - U(D+U)x_k + wb$$

$$(wL+D)x_{k+1} = ((1-w)D - wU)x_k + wb$$

NOW, REVERTING TO NOTATION OF PREVIOUS DERIVATIONS,

$$x = \underbrace{(wL+D)^{-1}}_{B_w} ((1-w)D - wU)x + (wL+D)^{-1}wb$$

B_w

$$B_w = (wL+D)^{-1}((1-w)D - wU)$$

GAUSS-SEIDEL

$$(L+D)x_{k+1} = -Ux_k + b$$