46/90

#1 10
2 10
3 8
4 not found
5 not found
6 4
7 7
8 7
9 not found

use divided difference table →

2 $f(\mathbf{v}) = \begin{bmatrix} x^2 + y^2 \\ x^2 - y^2 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$, $\mathbf{v}_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$f(\mathbf{v} + \mathbf{s}) \approx f(\mathbf{v}) + J_f(\mathbf{v})\mathbf{s}$

$J_f(\mathbf{v}) = \begin{bmatrix} 2x & 2y \\ 2x & -2y \end{bmatrix}$ ✓

$f(\mathbf{v}_0) = \begin{bmatrix} 1+1 \\ 1-1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$J_f(\mathbf{v}_0) = \begin{bmatrix} 2 & 2 \\ 2 & -2 \end{bmatrix}$ ✓

⋮

$\mathbf{s}_0 = -J_f^{-1}(\mathbf{v}_0) f(\mathbf{v}_0)$

do not invert in practice (expensive & may be inaccurate)

$= -\frac{1}{8}\begin{bmatrix} -2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$= -\frac{1}{8}\begin{bmatrix} -2 \\ -2 \end{bmatrix}$

but solve as a linear system

$\mathbf{s}_0 = \begin{bmatrix} -\frac{1}{4} \\ -\frac{1}{4} \end{bmatrix}$ ✓

$\mathbf{v}_1 = \mathbf{v}_0 + \mathbf{s}_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -\frac{1}{4} \\ -\frac{1}{4} \end{bmatrix} = \begin{bmatrix} 1.25 \\ 1.25 \end{bmatrix}$ ✓

10

3 $f(x) = e^x$, $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$ $n = 3$

$$P_3(x) = f(x_0)l_0(x) + f(x_1)l_1(x) + f(x_2)l_2(x) + f(x_3)l_3(x) \quad \checkmark$$

$$l_0(x) = \prod_{i=1}^{3} \frac{x - x_i}{x_0 - x_i} = \frac{x-x_1}{x_0-x_1}\frac{x-x_2}{x_0-x_2}\frac{x-x_3}{x_0-x_3} = \frac{x-1}{0-1}\frac{x-2}{0-2}\frac{x-3}{0-3}$$

$$= -\frac{1}{6}(x-1)(x^2-5x+6) = -\frac{1}{6}(x^3-6x^2+11x-6)$$

$$l_1(x) = \prod_{\substack{i=0 \\ i\neq 1}}^{3} \frac{x - x_i}{x_1 - x_i} = \frac{x-x_0}{x_1-x_0}\frac{x-x_2}{x_1-x_2}\frac{x-x_3}{x_1-x_3} = \frac{x-0}{1-0}\frac{x-2}{1-2}\frac{x-3}{1-3}$$

$$= \frac{1}{2}x(x^2-5x+6) = \frac{1}{2}(x^3-5x^3+6x) \quad \checkmark$$

$$l_2(x) = \prod_{\substack{i=0 \\ i\neq 1}}^{3} \frac{x - x_i}{x_2 - x_i} = \frac{x-x_0}{x_2-x_0}\frac{x-x_1}{x_2-x_1}\frac{x-x_3}{x_2-x_3} = \frac{x-0}{2-0}\frac{x-1}{2-1}\frac{x-3}{2-3}$$

$$= -\frac{1}{2}x(x^2-4x+3) = -\frac{1}{2}(x^3-4x^2+3x) \quad \checkmark$$

13

$$l_3(x) = \prod_{\substack{i=0 \\ i\neq 1}}^{3} \frac{x - x_i}{x_3 - x_i} = \frac{x - x_0}{x_3 - x_0}\,\frac{x - x_1}{x_3 - x_1}\,\frac{x - x_2}{x_3 - x_2} = \frac{x-0}{3-0}\,\frac{x-1}{3-1}\,\frac{x-2}{3-2}$$

$$= \frac{1}{6}\left(x^3 - 3x^2 + 2x\right) \checkmark$$

$$P_3(x) = e^0\left(-\frac{1}{6}\left(x^3 - 6x^2 + 12x - 6\right)\right) + \cdots$$

$$\cdots + e^1\left(\frac{1}{2}\left(x^3 - 5x^2 + 6x\right)\right) + \cdots \checkmark$$

$$\cdots - e^2\left(\frac{1}{2}\left(x^3 - 4x^2 + 3x\right)\right)$$

$$\approx \left[-\frac{1}{6}x^3 + x^2 - 2x + 1\right] + 1.359\left(x^3 - 5x^2 + 6x\right) + \cdots$$

$$\cdots - 3.695\left(x^3 - 4x^2 + 3x\right) \checkmark$$

3

$$P_3(x) \approx \left(-\frac{1}{6} + 1.359 - 3.695\right)x^3 + \cdots$$

$$\cdots + \left(1 - 5 \cdot 1.359 + 4 \cdot 3.695\right)x^2 + \cdots$$

$$\cdots + \left(-2 + 6 \cdot 1.359 - 3 \cdot 3.695\right)x^3 + \cdots$$

$$\cdots + 1$$

$$\boxed{P_3(x) \approx -2.5027x^3 + 8.985x^2 - 4.931x + 1}$$

?

✓

X

Correct approach though

3b

x_i	$f[\,.\,]$	$f[\,.\,,\,.\,]$	$f[\,.\,,\,.\,,\,.\,]$	...
0	e^0	$\frac{e^1-e^0}{1-0} = e-1$	$\frac{(e^2-e)-(e-1)}{2-0} = \frac{e^2-2e+1}{2}$	
1	e^1	$\frac{e^2-e^1}{2-1} = e^2-e$	$\frac{(e^3-e^2)-(e^2-e)}{3-1} = \frac{e^3-2e^2+e}{2}$	
2	e^2	$\frac{e^3-e^2}{3-2} = e^3-e^2$	... $f[\,.\,,\,.\,,\,.\,,\,.\,]$	
3	e^3		$\frac{1}{6}\left(e^3-2e^2+e-e^2+2e-1\right) = \ldots$	

$$\ldots = \frac{1}{6}\left(e^3-3e^2+3e-1\right)$$

3b

$$P_3(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \cdots$$

$$\cdots + a_3(x-x_0)(x-x_1)(x-x_2)$$

$$P_3(x) = e + (e-1)x + \frac{e^2-2e+1}{2}x(x-1) + \cdots$$

$$\cdots + \frac{1}{6}\left(e^3-3e^2+3e-1\right)x(x-1)(x-2)$$

$$\approx 0.8455\left(x^3-3x^2+2x\right) + 1.4762\left(x^2-x\right) + \cdots$$

$$\cdots + 1.7183x + 2.7183$$

$$\approx 0.8455x^3 + x^2(-3\cdot 0.8455 + 1.4762) + \cdots$$

$$\cdots + x(2\cdot 0.8455 - 1.4762 + 1.7183) + 2.7183$$

3b

$$P_3(x) \approx 0.8455x^3 - 1.06x^2 + 1.9331x + 2.7183$$

1

MY P_n FOR LAGRANGE FORM & NEWTON AREN'T EQUAL SO AT LEAST ONE IS INCORRECT. (neither)

3c

ASSUMING MY NEWTON'S FORM $P_3(x)$ IS CORRECT...

$P_3(1.5) = 6.0865$ $\quad e^{1.5} \approx 4.482$

VS.

$P_3(4) = 47.603$ $\quad e^4 \approx 54.598$ ok

$P_3(1.5)$ IS THE MORE ACCURATE APPROXIMATION IN TERMS OF ABSOLUTE ERROR, BUT $P_3(4)$ IS MORE ACCURATE IN TERMS OF RELATIVE ERROR.

#3d -?

8

8a HERMITE INTERPOLATION

1	0			
		1		
1	0	$\frac{3}{2}\ln 3$	$\frac{3}{4}\ln 3 - \frac{1}{2}$ $= b$	$\frac{1}{2}(1 - \ln 3) = a$ ✓
3	$3\ln 3$	$\ln 3 + 1$	$-\frac{1}{4}\ln 3 + \frac{1}{2}$	
3	$3\ln 3$			

$H(x) = a(x-1)^2(x-3) + b(x-1)^2 + (x-1)$ ✓

$H(1.5) = 0.59948$ ✓

HERMITE INTERPOLATING POLYNOMIAL IS MORE ACCURATE ✓

8b

$$f(x) = H(x) + e(x) \qquad (x-x_0)^2 \cdots (x-x_n)^2$$

$$= H(x) + \frac{(x-x_0)^2(x-x_1)^2}{(2n+2)!} f^{(2n+2)}(\xi)$$

$$\frac{(x-x_0)^2(x-x_1)^2}{(2n+2)!} f^{(2n+2)}(\xi) = f(x) - H(x) \qquad a < \xi < b$$

this is error need to analyze it

$$\frac{(x-1)^2(x-3)^2}{4!}\frac{2}{\xi^3} = f(x) - H(x) \quad \checkmark$$

$$\xi(x) = \left[\frac{1}{12}\frac{(x-1)^2(x-3)^2}{f(x)-H(x)}\right]^{1/3} \quad ?$$

SAY $a = 1, b = 3$

$1 < \xi(x) < 3$

$f(x) = x \ln x$

$n=1 \Rightarrow 2n+2 = 3$

$\Rightarrow$ need $f'''(\xi)$

$$\text{Error} = \frac{(x-1)^2(x-3)^2}{4!} \cdot \frac{2}{\xi^3} \leq \frac{(x-1)^2(x-3)^2}{12}$$

since $1 < \xi < 3 \Rightarrow \frac{1}{\xi^3} < 1$

can plot in Matlab to see error bounds

8b

$x = 1.5$

$$\xi_1(1.5) = \left[\frac{1}{12}\,\frac{(1.5-1)^2(1.5-3)^2}{f(1.5) - H(1.5)}\right]^{1/3}$$

$$\approx \left[\frac{1}{12}\,\frac{(0.25)(2.25)}{(1.5)(\ln(1.5) - (0.59948)}\right]^{1/3}$$

$$\approx 1.7518$$

?

7

$x_0 = 1$ z_0 -1

$x_1 = 2$ z_1 -1

z_2 -1

z_3 1

z_4 1

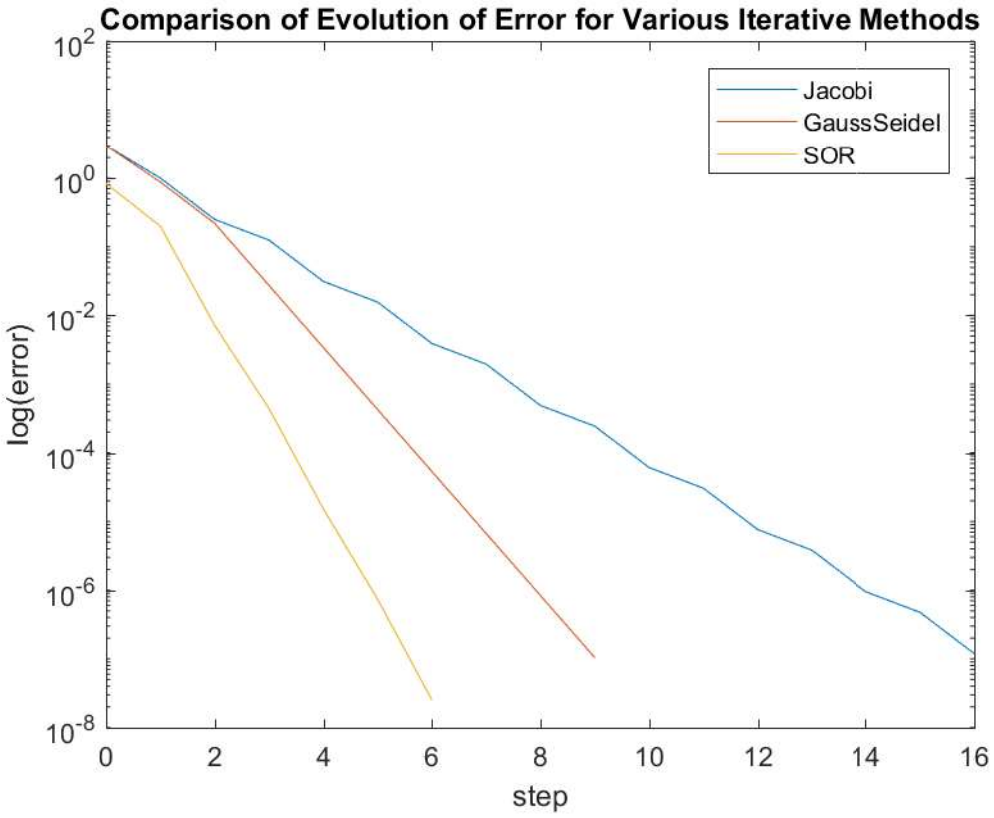
2	0	
2		0
$\frac{1+1}{2-1} = 2$	0	$-\frac{1}{2}$
	2	-1
-2		

$P(x) = -1 + 2(x-1) + \cdots$

$\cdots -\frac{1}{2}(x+1)^3(x-2)^2$

ANNIE
21 AUG?
2:20 PM

LCM H
22 Aug
2:00 PM


Comparison of Evolution of Error for Various Iterative Methods
Jacobi
GaussSeidel
SOR
log(error)
step
10^2
10^0
10^-2
10^-4
10^-6
10^-8
0
2
4
6
8
10
12
14
16

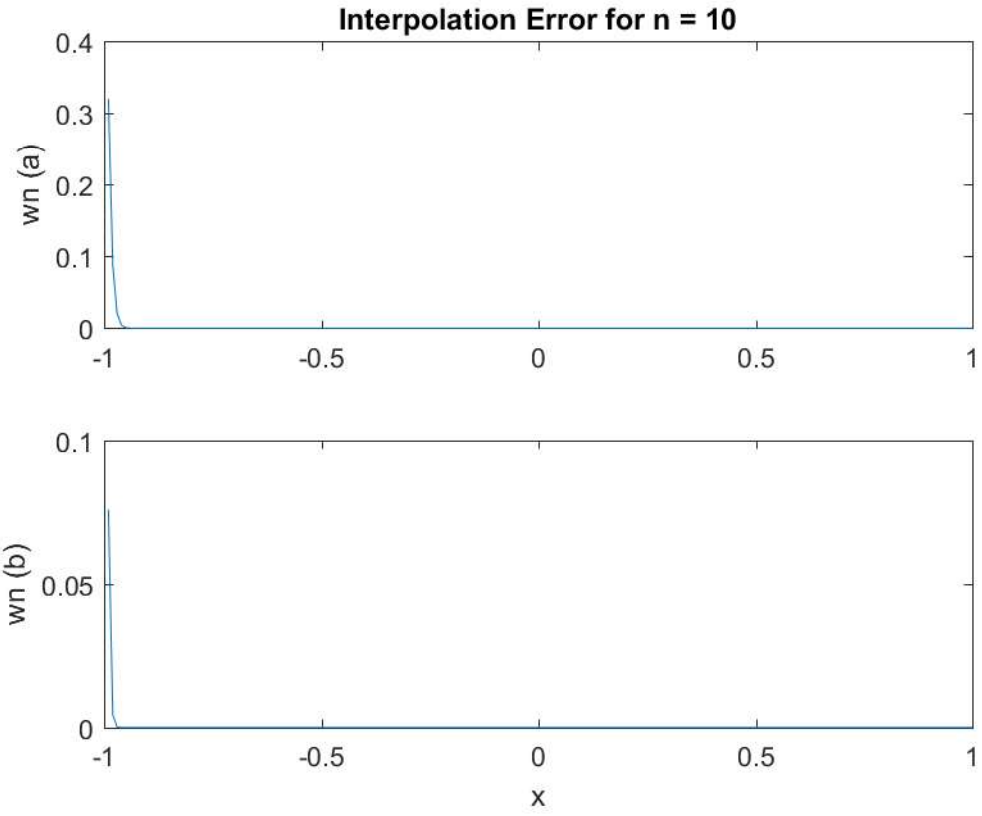
✓

Good job!

10

```
   iterMeth

n =

    17


xkPlus =

   1.0000e+00
   2.0000e+00
   3.0000e+00


ans =

  20×4 table

       N          cJ          cGS          cSOR
    _________    _________    _________    _________

    0.0000e+00       Inf          Inf          Inf
    1.0000e+00    3.0000e-01    2.7528e-01    2.7030e-01
    2.0000e+00    2.5000e-01    2.5000e-01    2.4618e-01
    3.0000e+00    5.0000e-01    1.2500e-01    3.4919e-02
    4.0000e+00    2.5000e-01    1.2500e-01    6.3685e-02
    5.0000e+00    5.0000e-01    1.2500e-01    3.4146e-02
    6.0000e+00    2.5000e-01    1.2500e-01    4.8894e-02
    7.0000e+00    5.0000e-01    1.2500e-01    3.3888e-02
    8.0000e+00    2.5000e-01    1.2500e-01    0.0000e+00
    9.0000e+00    5.0000e-01    1.2500e-01    0.0000e+00
    1.0000e+01    2.5000e-01    0.0000e+00    0.0000e+00
    1.1000e+01    5.0000e-01    0.0000e+00    0.0000e+00
    1.2000e+01    2.5000e-01    0.0000e+00    0.0000e+00
```

1.3000e+01	5.0000e-01	0.0000e+00	0.0000e+00
1.4000e+01	2.5000e-01	0.0000e+00	0.0000e+00
1.5000e+01	5.0000e-01	0.0000e+00	0.0000e+00
1.6000e+01	2.5000e-01	0.0000e+00	0.0000e+00
1.7000e+01	0.0000e+00	0.0000e+00	0.0000e+00
0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00



4

something is wrong possibly with indexing.

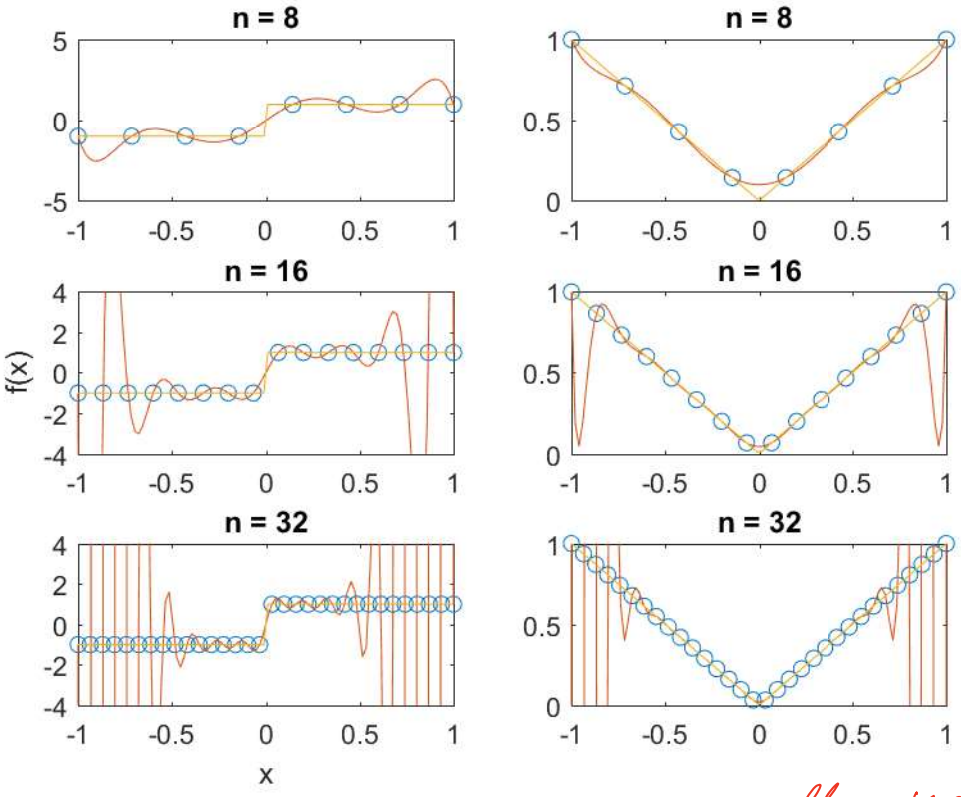
should be symmetric

Other values of n?

n=20, 30 ?

Discussion of results?

#7



(6)

equally spaced points are used?

Chebyshev points?

I was not able to run your programs for some reason. They look fine though.

— Discussion of your results?