

$$y' = y$$

$$k_1 = f(u_n) = u_n$$

$$k_2 = f(u_n + hk_1) = u_n + hu_n = u_n(1+h)$$

$$\begin{aligned} u_{n+1} &= u_n + \frac{h}{2}(k_1 + k_2) = u_n + \frac{h}{2}(u_n + (1+h)u_n) \\ &= u_n \left(1 + h + \frac{h^2}{2}\right) \end{aligned}$$

$$= u_n + \frac{h}{2}u_n + \frac{h}{2}(1+h)u_n$$

$$= u_n + \frac{h}{2}u_n + \frac{h}{2}u_n + \frac{h^2}{2}u_n$$

$$y' = 2y$$

$$k_1 = f(u_n) = 2u_n$$

$$\begin{aligned} k_2 &= f(u_n + hk_1) = 2(u_n + hk_1) \\ &= 2(u_n + 2hu_n) \\ &= 2u_n(1+2h) \end{aligned}$$

$$\begin{aligned} u_{n+1} &= u_n + \frac{h}{2}(k_1 + k_2) = u_n + \frac{h}{2}(2u_n + 2u_n(1+2h)) \\ &= u_n + hu_n + hu_n(1+2h) \\ &= u_n + hu_n + hu_n + 2h^2u_n \\ &= u_n(1 + 2h + 2h^2) \end{aligned}$$