

1. a

$$\sin \frac{\pi}{2} x = \frac{\pi}{2} x - \frac{\pi^3}{8} \frac{x^3}{3!} + \frac{\pi^5}{32} \frac{x^5}{5!} - \frac{\pi^7}{256} \frac{x^7}{7!} + \dots$$

1. b

$$R_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} (x-a)^{n+1}$$

$$R_7(x) = \frac{f^{(8)}(\xi)}{8!} (x-a)^8, \quad f^{(8)}(x) = -\frac{\pi^8}{512} \sin\left(\frac{\pi}{2} x\right)$$

$$R_7(x) = -\frac{\pi^8}{512} \frac{\sin\left(\frac{\pi}{2} \xi\right)}{8!} x^8$$

1. c

$$\text{WE WANT } R_n(x) \leq 10^{-5}$$

$$\text{SAY } x = 0.1, \quad \xi = 0.001$$

$$\text{THEN } R_7(x) = -\frac{\pi^8}{512} \frac{\sin\left(\frac{\pi}{2} 0.001\right)}{8!} (0.1)^8 \approx 7.220 \times 10^{-15} < 10^{-5}$$

SO $n=7$ WOULD BE
ACCURATE ENOUGH