

4b

$$\frac{df(x_0)}{dx} = N_2(h/9) + \frac{1}{2^2} K_3 h^3 + \dots$$

$$= N_2(h/9) + C_3 h^3 + C_4 h^4 + C_5 h^5 + \dots$$

$$= \underline{N_2(h/9)} + O(h^3)$$

SAY $\frac{f(x_0+h) - f(x_0)}{h} = N_0(h)$

THEN $N_2(h/9) = \frac{9N_1(h/3) - N_1(h/9)}{8}$

$$= \frac{1}{8} \left[9 \left(\frac{3N_0(h)}{2} - N_0(h/3) \right) - \left(\frac{3N_0(h/3)}{2} - N_0(h/9) \right) \right]$$

$$= \frac{9}{16} \left[3 \left(\frac{f(x_0+h)}{h} - \frac{f(x_0)}{h/3} \right) - \left(\frac{f(x_0+h/3)}{h/3} - \frac{f(x_0)}{h/9} \right) - \dots \right]$$

$$\dots - 3 \left(\frac{f(x_0+h/3)}{h/3} - \frac{f(x_0)}{h/9} \right) + \left(\frac{f(x_0+h/9)}{h/9} - \frac{f(x_0)}{h/9} \right) \right]$$