1a $y' = 2y, \quad y(0) = 1 = u_0$

$u_{n+1} = u_n + hf(u_n)$

$f(y) = 2y, \quad y(t) = e^{2t}, \quad h = 0.1$

$u_1 = u_0 + hf(u_0)$

$= u_0(1 + 2h) = 1 \cdot (1 + 0.2) = 1.2$

$u_2 = u_1 + hf(u_1)$

$= u_1(1 + 2h) = 1.2 \cdot (1 + 0.2) = 1.44$

$h = 0.05$

$u_1 = 1 \cdot (1 + 0.1) = 1.1$

$u_2 = 1.1(1 + 0.1) = 1.21$

$h = 0.001$

$u_1 = 1(1 + 0.002) = 1.002$

$u_2 = 1.002(1 + 0.002) \approx 1.004$

```
%oneStepEuler
%Matt Zeller
%PHYS 428
%12/3/2018

h=0.001;
n=1/h
un=zeros(n,1);
j=1;
for i=0:h:1

    un(j,1)=(1+2*h)^j;
    j=j+1;

end
format long
un(n+1,1)
```

16 $y' = 2y$, $y(0) = y_0 = 1 = v_0$

$= f(y)$, $h = 0.1$

$k_1 = f(v_n)$

$k_2 = f(v_n + hk_1)$

$v_{n+1} = v_n + \frac{h}{2}(k_1 + k_2)$

$v_1 = v_0 + \frac{h}{2}(2v_0 + 2(v_0 + 2hv_0))$

$= v_0 + hv_0 + hv_0 + 2h^2 v_0$

$= v_0(1 + 2h + 2h^2)$

$= 1(1 + 2 \cdot 0.1 + 2 \cdot (0.1)^2) = 1.22$

$v_2 = v_1(1 + 2h + 2h^2)$

$= 1.22(1 + 2 \cdot 0.1 + 2 \cdot (0.1)^2) = 1.4884$

16 $y' = 2y$, $y(0) = y_0 = 1 = v_0$

$= f(y)$, $h = 0.05$

$v_1 = v_0(1 + 2h + 2h^2)$

$= (1 + 2\cdot(0.05) + 2\cdot(0.05)^2) = 1.105$

$v_2 = 1.105(1 + 2\cdot(0.05) + 2\cdot(0.05)^2 = 1.221025$

$h = 0.001$

$v_1 = (1 + 2\cdot(0.001) + 2\cdot(0.001)^2 \approx 1.002002$

$v_2 \approx 1.002002^2 \approx 1.004008$

```
%oneStepModEuler
%Matt Zeller
%PHYS 428
%12/3/2018

h=0.01;
n=1/h
un=zeros(n,1);
j=1;
un(1,1)=1;
for i=0:h:1
    un(j+1,1)=un(j,1)*(1+2*h+2*h^2);
    j=j+1;
end
format long
un(n+1,1)
```

1c $y' = 2y$, $y(0) = 1 = U_0$

$$U_{n+1} = U_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$

$$U_{n+1} = U_n\left(1 + 2h + 2h^2 + \frac{4}{3}h^3 + \frac{2}{3}h^4\right)$$

$h = 0.1$

$$U_1 = U_0\left(1 + 2\cdot 0.1 + 2\cdot 0.1^2 + \frac{4}{3}0.1^3 + \frac{2}{3}\cdot 0.1^4\right)$$

$$= 1.2214$$

$$U_2 = 1.2214^2 \approx 1.49181796$$

$h = 0.05$

$$U_1 = U_0\left(1 + 2\cdot 0.05 + 2\cdot 0.05^2 + \frac{4}{3}\cdot 0.05^3 + \frac{2}{3}\cdot 0.05^4\right)$$

$$\approx 1.1051\,70833$$

$$U_2 = U_1^2 \approx 1.221402571$$

$h = 0.001$

$$U_1 = U_0\left(1 + 2\cdot 0.001 + 2\cdot 0.001^2 + \frac{4}{3}\cdot 0.001^3 + \frac{2}{3}\cdot 0.001^4\right)$$

$$\approx 1.002002001$$

$$U_2 = U_1^2 \approx 1.004008011$$

```
%oneStepRungeOrderFour
%Matt Zeller
%PHYS 428
%12/3/2018

h=0.001;
n=1/h
un=zeros(n,1);
j=1;
un(1,1)=1;
for i=0:h:1
    un(j+1,1)=un(j,1)*(1+2*h+2*h^2+(4/3)*h^3+(2/3)*h^4);
    j=j+1;
end
format long
un(n+1,1)
```

1d $y' = 2y$, $y(0) = 1 = u_0$

$$u_{n+1} = \frac{h}{2}\left(3f(u_n) - f(u_{n-1})\right) + u_n$$

$$= \frac{h}{2}\left(6u_n - 2u_{n-1}\right) + u_n$$

$f(y) = 2y$, $y = e^{2x}$ EXACT SOLUTION IS e^2

$f(y_{n-1}) = 2y_{n-1} = 2e^2$

$$= \frac{h}{2}\left(6u_n - 2e^2\right) + u_n$$

$h = 0.1$

$$u_1 = \frac{0.1}{2}\left(6 - 2e^2\right) + 1 \approx 0.561094$$

$$u_2 = \frac{0.1}{2}\left(6 \cdot 0.561094 - 2\right) + 0.561094 \approx 0.629422$$

```
%oneStepAdamBashforth
%Matt Zeller
%PHYS 428
%12/3/2018

h=0.001;
n=1/h
un=zeros(n,1);
j=1;
un(1,1)=exp(2);
un(2,1)=1;
for i=0:h:1
    un(j+2,1)=(h/2)*(6*un(j+1,1)-2*un(j,1)) + un(j+1,1);
    j=j+1;
end
format long
un(n+1,1)
```