HW 2

$$\cos\left(\tfrac{2}{\pi}x\right) = 1 - \frac{x^2}{2!}\left(\tfrac{2}{\pi}\right)^4 + \frac{x^4}{4!}\left(\tfrac{2}{\pi}\right)^8 + \cdots$$

$$= \sum_{n=0}^{\infty} (-1)^{3n} \frac{x^{2n}}{(2n)!}\left(\tfrac{2}{\pi}\right)^{4n} = 1 - \frac{x^2}{2!}\left(\tfrac{2}{\pi}\right)^4 + \frac{x^4}{4!}\left(\tfrac{2}{\pi}\right)^8 - \cdots$$

$$(-1)^{3n}\frac{x^{2n}}{(2n)!}\left(\tfrac{2}{\pi}\right)^{4n} = \frac{f^{(n-1)}(a)(x-a)^{n-1}}{(n-1)!} \quad \therefore R_n =$$

$$(-1)^{3n} x^{2n} \left(\tfrac{2}{\pi}\right)^{4n} = f^{(n-1)}(a)(x-a)^{n-1}$$

$$\frac{1}{(2n)!} = \frac{1}{(n-1)!}$$

$$\frac{f^{(n-1)}(a)(x-a)^{n-1}}{(n-1)!} = \frac{\left(\frac{d}{dx}\right)^{n-1}\left(\cos\left(\frac{2x}{\pi}\right)\right)\left(\frac{2x}{\pi}\right)^{n-1}}{(n-1)!}$$

$$\therefore \text{so } R_n = \frac{\left(\frac{d}{dx}\right)^n \cos\left(\frac{2}{\pi}\xi_n\right)\left(\frac{2x}{\pi}\right)^n}{n!}$$

$n=1$: $\dfrac{\left(\frac{d}{dx}\right)^0\left(\cos\left(\frac{2x}{\pi}\right)\right)\left(\frac{2x}{\pi}\right)^0}{0!}$

$= \cos\left(\frac{2}{\pi}(0)\right)$

$= 1$

$n=2$: $\dfrac{\left(\frac{d}{dx}\right)^1\left(\cos\left(\frac{2x}{\pi}\right)\right)\left(\frac{2x}{\pi}\right)^1}{1!}$

$= -\frac{2}{\pi}\sin\left(\frac{2x}{\pi}\right)\frac{2x}{\pi}$

$= -\frac{2}{\pi}\cdot\frac{2}{\pi}\sin(0)$

$= 0$

$n=3$: $\dfrac{-\left(\frac{2}{\pi}\right)^2\cos\left(\frac{2}{\pi}(0)\right)\left(\frac{2x}{\pi}\right)^2}{2!}$

$= -\left(\frac{2}{\pi}\right)^4 \frac{x^2}{2!}$