Ex 3.1

$$x_1 + 10x_2 = 11.60 \quad (1)$$

$$x_1 + 15x_2 = 11.85 \quad (2)$$

$$x_1 + 20x_2 = 12.25 \quad (3)$$

a) $$\begin{bmatrix} 1 & 10 \\ 1 & 15 \\ 1 & 20 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \overset{!}{\cong} \begin{bmatrix} 11.60 \\ 11.85 \\ 12.25 \end{bmatrix}$$

b) $$\| \vec{A} \vec{x} \| \leq \| \vec{A} \| \cdot \| \vec{x} \|$$

$$A = \begin{bmatrix} 1 & 10 \\ 1 & 15 \\ 1 & 20 \end{bmatrix}, \quad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

SINCE $x_1, \vec{x}_2$ ARE UNKNOWN, $\vec{A}$ IS NOT, IN GENERAL, CONSISTENT

(1 & 2) $$\begin{bmatrix} 1 & 10 \\ 1 & 15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 11.60 \\ 11.85 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 11.10 \\ 0 & 1 & 0.05 \end{bmatrix} \quad \boxed{(x_1, x_2) = (11.10, 0.05)}$$

(1 & 3) $$\begin{bmatrix} 1 & 10 & 11.60 \\ 1 & 20 & 12.25 \end{bmatrix} \quad \boxed{(x_1, x_2) = (10.95, 0.065)}$$

$$\begin{bmatrix} 1 & 0 & 10.95 \\ 0 & 1 & 0.065 \end{bmatrix}$$

Ex 3.1

$$\left[\begin{array}{ccc} 1 & 15 & 11.85 \\ 1 & 20 & 12.25 \end{array}\right]$$

$$\left[\begin{array}{ccc} 1 & 0 & 10.65 \\ 0 & 1 & 0.08 \end{array}\right] \quad \boxed{(X_1, X_2) = (10.65, 0.08)}$$

$(X_1, X_2) = (10.95, 0.065)$

THE TRUE VALUE FOR (x_1, x_2) IS LIKELY BETWEEN THE UPPER & LOWER APPROXIMATION VALUES, SO (10.95, 0.065) ARE MOST LIKELY TO MINIMIZE ERROR

c) $\vec{A}^T \vec{A} \vec{x} = \vec{A}^T \vec{b}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 10 & 15 & 20 \end{bmatrix} \begin{bmatrix} 1 & 10 \\ 1 & 15 \\ 1 & 20 \end{bmatrix} \vec{x} = \begin{bmatrix} 1 & 1 & 1 \\ 10 & 15 & 20 \end{bmatrix} \begin{bmatrix} 11.60 \\ 11.85 \\ 12.25 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 45 \\ 45 & 725 \end{bmatrix} \vec{x} = \begin{bmatrix} 35.7 \\ 538.75 \end{bmatrix}$$

$$\left[\begin{array}{ccc} 1 & 0 & 10.925 \\ 0 & 1 & 0.065 \end{array}\right]$$

$$\boxed{\tilde{y}(t) = 10.925 + 0.065t}$$

$(X_1, X_2) = (10.95, 0.065)$ IS BETWEEN OTHER DATA POINTS, SO IT MAKES SENSE THAT $\tilde{y}(t)$ CONTAINS SIMILAR VALUES

Ex 3.5

$$\vec{A}^T \vec{r} \stackrel{!}{=} \emptyset$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \neq \emptyset$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix} \neq \emptyset$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix} = \emptyset$$

$\boxed{(C)}$

IF $\vec{x}$ IN LEAST SQUARES PROBLEM

$$\vec{A}\vec{x} \cong \vec{b}$$

IS TO BE A SOLUTION, $\vec{r} = \vec{b} - \vec{A}\vec{x}$ MUST BE ORTHOGONAL TO SPAN$(\vec{A})$, CONSEQUENTLY,

$$\vec{A}^T \vec{r} \stackrel{!}{=} \emptyset$$

We can now solve the upp... 1]

Ex 3.23

$$A = \begin{bmatrix} 1 & 1 \\ \varepsilon & 0 \\ 0 & \varepsilon \end{bmatrix} \; ; \; HA = ?$$

$$HA = H_2 H_1 A$$

$$\rightarrow H_1 a_1 = a_1 - \left(2 \frac{v_1^T a_1}{v_1^T v_1}\right) v_1$$

$$v_1 = a_1 - \alpha e_1 \; ; \; \alpha = \sqrt{1+\varepsilon^2} = 1$$

$$= \begin{bmatrix} 1 \\ \varepsilon \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ \varepsilon \\ 0 \end{bmatrix}$$

$$H_1 a_1 = \begin{bmatrix} 1 \\ \varepsilon \\ 0 \end{bmatrix} - \left(2 \frac{\begin{bmatrix} 2 & \varepsilon & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \varepsilon \\ 0 \end{bmatrix}}{\begin{bmatrix} 2 & \varepsilon & 0 \end{bmatrix} \begin{bmatrix} 2 \\ \varepsilon \\ 0 \end{bmatrix}}\right) \begin{bmatrix} 2 \\ \varepsilon \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ \varepsilon \\ 0 \end{bmatrix} - \left(2 \frac{2+\varepsilon}{4+\varepsilon^2}\right) \begin{bmatrix} 2 \\ \varepsilon \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ \varepsilon \\ 0 \end{bmatrix} - \begin{bmatrix} 2 \\ \varepsilon \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

$$H_1 a_2 = \begin{bmatrix} 1 \\ 0 \\ \varepsilon \end{bmatrix} - \left(2 \frac{\begin{bmatrix} 2 & \varepsilon & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ \varepsilon \end{bmatrix}}{\begin{bmatrix} 2 & \varepsilon & 0 \end{bmatrix} \begin{bmatrix} 2 \\ \varepsilon \\ 0 \end{bmatrix}}\right) \begin{bmatrix} 2 \\ \varepsilon \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ \varepsilon \end{bmatrix} - \left(2 \frac{2}{4+\varepsilon^2}\right) \begin{bmatrix} 2 \\ \varepsilon \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ \varepsilon \end{bmatrix} - \begin{bmatrix} 2 \\ \varepsilon \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -\varepsilon \\ \varepsilon \end{bmatrix}$$

Ex 3.23

$$H_1 A = \begin{bmatrix} -1 & -1 \\ 0 & -\varepsilon \\ 0 & \varepsilon \end{bmatrix}$$

$$\rightarrow H_2 a_2 = a_2 - \left(2 \frac{v_2^T a_2}{v_2^T v_2}\right) v_2$$

$$v_2 = \begin{bmatrix} 0 \\ -\varepsilon \\ \varepsilon \end{bmatrix} - \sqrt{2}\,\varepsilon \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -(1+\sqrt{2})\varepsilon \\ \varepsilon \end{bmatrix}$$

$$H_2 a_2 = \begin{bmatrix} -1 \\ -\varepsilon \\ \varepsilon \end{bmatrix} - \left(2 \frac{\begin{bmatrix} 0 & -(1+\sqrt{2})\varepsilon & \varepsilon \end{bmatrix} \begin{bmatrix} -1 \\ -\varepsilon \\ \varepsilon \end{bmatrix}}{\begin{bmatrix} 0 & -(1+\sqrt{2})\varepsilon & \varepsilon \end{bmatrix} \begin{bmatrix} 0 \\ -(1+\sqrt{2})\varepsilon \\ \varepsilon \end{bmatrix}} \right) \begin{bmatrix} 0 \\ -(1+\sqrt{2})\varepsilon \\ \varepsilon \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ -\varepsilon \\ \varepsilon \end{bmatrix} - \left(2 \frac{(2+\sqrt{2})\varepsilon}{(4+2\sqrt{2})\varepsilon^2} \right) \begin{bmatrix} 0 \\ -(1+\sqrt{2})\varepsilon \\ \varepsilon \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ -\varepsilon \\ \varepsilon \end{bmatrix} - \begin{bmatrix} 0 \\ -(1+\sqrt{2})\varepsilon \\ \varepsilon \end{bmatrix} = \begin{bmatrix} -1 \\ \sqrt{2}\,\varepsilon \\ 0 \end{bmatrix}$$

$$H_2 H_1 A = \begin{bmatrix} -1 & -1 \\ 0 & \sqrt{2}\,\varepsilon \\ 0 & \varepsilon \end{bmatrix} = \begin{bmatrix} R \\ 0 \end{bmatrix}$$

$$R = \begin{bmatrix} -1 & -1 \\ 0 & \sqrt{2}\,\varepsilon \end{bmatrix} \quad \text{QED}$$