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$$P_4 = x^4 + \underbrace{\frac{1}{0.38} \frac{1}{42} \left(x^2 - \frac{1}{2} \right)}_{\frac{1}{16} x^2 - \frac{1}{48}} - \frac{\langle x^4, P_0 \rangle}{\|P_0\|^2} P_0$$

$$\langle x^4, P_0 \rangle = \int_{-1}^1 x^4 \cdot 1 dx = 2 \int_0^1 x^4 dx = 2 \cdot \frac{1}{5} = \frac{2}{5}$$

$$\|P_0\|^2 = \langle 1, 1 \rangle = \int_{-1}^1 1 \cdot 1 dx = 2$$

$$P_4 = x^4 + \frac{1}{16} x^2 - \frac{1}{48} - \frac{\frac{2/5}{2} \cdot 1}{1}$$

$$P_4 = x^4 + \frac{1}{16} x^2 - 0.22$$