HW 4

1 $\begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 10 \end{bmatrix}$, OPTIMAL SOLUTION IS $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

a SOLVE BY "SUCCESSIVE OVER-RELAXATION" w/ OPTIMAL PARAMETER $\omega = 2 \big/ \left(1 + \sqrt{7/8}\right)$,

LET $x^{(\emptyset)} = \begin{bmatrix} \emptyset \\ \emptyset \\ \emptyset \end{bmatrix}$

EXAMPLE:

$$\left.\begin{aligned} 2x_1 - x_2 &= 1 \\ -x_1 + 2x_2 &= 1 \end{aligned}\right\} \longrightarrow \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

GNRLLY,

$$D x_{k+1} = D x_k - \omega \left(L x_{k+1} + (D+U) x_k - b \right)$$

IN THIS CASE,

$D = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, $U = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$, $L = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$

$$2x_1^{(k+1)} = 2x_1^{(k)} - \omega\left(2x_1^{(k)} - x_2^{(k)} - 1\right)$$

$$2x_2^{(k+1)} = 2x_2^{(k)} - \omega\left(-x_1^{(k+1)} + 2x_2^{(k)} - 1\right)$$

$$\begin{bmatrix} 2 & \emptyset \\ -\omega & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{k+1} = \begin{bmatrix} 2(1-\omega) & \omega \\ \emptyset & 2(1-\omega) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_k + \omega \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$B_\omega = \begin{bmatrix} 2 & \emptyset \\ -\omega & 2 \end{bmatrix}^{-1} \begin{bmatrix} 2(1-\omega) & \omega \\ \emptyset & 2(1-\omega) \end{bmatrix} = \begin{bmatrix} 1-\omega & \omega/2 \\ \frac{\omega(1-\omega)}{2} & \frac{\omega^2}{4} + 1 - \omega \end{bmatrix} \ldots$$

WHAT IS B_ω?

HE WOULD NOT DEIGN TO LEAVE BEHIND HIM ANY COMMENTARY OR WRITING ON SUCH SUBJECTS; BUT REPUDIATING AS SORDID AND