HOMEWORK 4

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PHYS 428

14 JULY 2017

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$$f(x+s) \approx f(x) + J_f(x)\, s$$

$$f(X_1) = \begin{bmatrix} 1/4 + 1/4 \\ \emptyset \end{bmatrix} = \begin{bmatrix} 1/2 \\ \emptyset \end{bmatrix}$$

$$f(x) = \begin{bmatrix} x^2 + y^2 \\ x^2 - y^2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$J_f(x) = \begin{bmatrix} 2x & 2y \\ 2x & -2y \end{bmatrix}$$

$$X_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$f(X_0) = \begin{bmatrix} 1 + 1 \\ 1 - 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$J_f(X_0) = \begin{bmatrix} 2 & 2 \\ 2 & -2 \end{bmatrix}$$

$$J_f(X_0) \cdot s_0 = -f(x_0)$$

$$s_0 = J_f(X_0)^{-1} \cdot -f(X_0)$$

$$= \frac{1}{8} \begin{bmatrix} -2 & -2 \\ -2 & 2 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1/2 \\ -1/2 \end{bmatrix}$$

$$X_1 = X_0 - s_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} -1/2 \\ -1/2 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

3 a

$$P_n(x) = \sum_{k=0}^{n} f(x_k)\, l_k(x)$$

x_0	x_1	x_2	x_3
0	1	2	3

$$P_3(x) = f(x_0)\, l_0(x) + \dots + f(x_4)\, l_4(x)$$

$$= f(x_0) \cdot \frac{x - x_1}{x_0 - x_1} \cdot \dots \cdot \frac{x - x_3}{x_0 - x_3} + \dots + f(x_3) \cdot \frac{x - x_0}{x_3 - x_0} \cdot \dots \cdot \frac{x - x_2}{x_3 - x_2}$$

$$f(x_k) = e^{x_k}$$

$$P_3(x) = e^{x_0} \cdot \frac{x-1}{0-1} \cdot \frac{x-2}{0-2} \cdot \frac{x-3}{0-3} + e^{x_1} \cdot \frac{x-0}{1-0} \cdot \frac{x-2}{1-2} \cdot \frac{x-3}{1-3} + \dots$$

$$\dots + e^{x_2} \cdot \frac{x-0}{2-0} \cdot \frac{x-1}{2-1} \cdot \frac{x-3}{2-3} + e^{x_3} \cdot \frac{x-0}{3-0} \cdot \frac{x-1}{3-1} \cdot \frac{x-2}{3-2}$$

2 1 −1 −2

$$= -e^{x_0} \frac{1}{6}(x-1)(x-2)(x-3) + e^{x_1} \frac{1}{2} x (x-2)(x-3) + \dots$$

$$\dots - e^{x_2} \frac{1}{2} x (x-1)(x-3) + e^{x_3} \frac{1}{6} x (x-1)(x-2)$$

$$= -e^{x_0} \frac{1}{6}(x-1)(x^2-5x+6) + e^{x_1} \frac{1}{2} x (x^2-5x+6) + \dots$$

$$\dots - e^{x_2} \frac{1}{2} x (x^2-4x+3) + e^{x_3} \frac{1}{6} x (x^2-3x+2)$$

$\frac{3}{a}$

$$P_4(x) = -\frac{1}{6}e^{x_0}(x^3 - 6x^2 + 11x - 6) + \frac{1}{2}e^{x_1}(x^3 - 5x^2 + 6x) + \cdots$$

$$\cdots - \frac{1}{2}e^{x_2}(x^3 - 4x^2 + \underset{3x}{\cancel{7x}}) + \frac{1}{6}e^{x_3}(x^3 - 3x^2 + \underset{2x}{\cancel{7x}})$$

$$= x^3\left(-\frac{1}{6}e^{x_0} + \frac{1}{2}e^{x_1} - \frac{1}{2}e^{x_2} + \frac{1}{6}e^{x_3}\right) + \cdots$$

$$\cdots + x^2\left(e^{x_0} - \frac{5}{2}e^{x_1} + 2e^{x_2} - \frac{1}{2}e^{x_3}\right) + \cdots$$

$$\cdots + x\left(-\frac{11}{6}e^{x_0} + 3e^{x_1} - \frac{\overset{3}{\cancel{1}}}{2}e^{x_2} + \frac{1}{\underset{3}{\cancel{6}}}e^{x_3}\right) + \cdots$$

$$\cdots + e^{x_0}$$

$$\boxed{P_4(x) \approx 0.8455\,x^3 - 1.060\,x^2 + 1.933\,x + 1}$$

EITHER THIS $P_4(x)$ OR THE NEXT IN 3b ARE INCORRECT ... OR BOTH... BUT THEY SHOULD BE EQUAL

3b

$x_0 = 0$

$x_1 = 1$

$x_2 = 2$

$x_3 = 3$

$$
\begin{array}{llll}
e^{x_0} & & & \\
e^{x_1} & > e^{x_1} - e^{x_0} & & \\
e^{x_2} & > e^{x_2} - e^{x_1} & > \frac{1}{2}\left(e^{x_2} - 2e^{x_1} + e^{x_0}\right) & \\
e^{x_3} & > e^{x_3} - e^{x_2} & > \frac{1}{2}\left(e^{x_3} - 2e^{x_2} + e^{x_1}\right) & > \frac{1}{6}\left(e^{x_3} - 3e^{x_2} + 3e^{x_1} - e^{x_0}\right)
\end{array}
$$

$$P_n(x) = e^{x_0} + \left(e^{x_1} - e^{x_0}\right)(x - x_0) + \frac{1}{2}\left(e^{x_2} - 2e^{x_1} + e^{x_0}\right)(x - x_0)(x - x_1) +$$

$$\ldots + \frac{1}{6}\left(e^{x_3} - 3e^{x_2} + 3e^{x_1} - e^{x_0}\right)(x - x_0)(x - x_1)(x - x_2)$$

$\frac{3}{6}$

$$P_4(x) \approx 1 + 1.718\,x + 1.477\,(x^2 - x) + \cdots$$

$$\cdots + 5.073\,(x^3 - 3x^2 + 2x)$$

$$\boxed{P_4(x) \approx 0.8455\,x^3 - 13.74\,x^2 + 10.39\,x + 1}$$

4

$x_0 = -2$ $\quad f[x_0] = -5$

$x_1 = -1$ $\quad f[x_1] = 1$

$x_2 = 0$ $\quad f[x_2] = 1$

$x_3 = 1$ $\quad f[x_3] = 1$

$x_4 = 2$ $\quad f[x_4] = 7$

$x_5 = 3$ $\quad f[x_5] = 25$

$f[x_0, x_1]$

$f[x_1, x_2]$

$f[x_2, x_3]$

$f[x_3, x_4]$

$f[x_4, x_5]$

$f[x_0, x_1, x_2]$

$f[x_1, x_2, x_3]$

$f[x_2, x_3, x_4]$

$f[x_3, x_4, x_5]$

...

...

$f[x_0, x_1, x_2, x_3]$

$f[x_1, x_2, x_3, x_4]$

$f[x_2, x_3, x_4, x_5]$

$f[x_0, x_1, x_2, x_3, x_4]$

$f[x_1, x_2, x_3, x_4, x_5]$

$f[x_0, \ldots, x_5]$

4

(x_n): (-2), (-1), (0), (1), (2), (3)

$f[\cdot,\cdot]$

$$\frac{1+5}{-1+2} = 6$$

$$\frac{1-1}{0+1} = 0$$

$$\frac{1-1}{1-0} = 0$$

$$\frac{7-1}{2-1} = 6$$

$$\frac{25-7}{3-2} = 18$$

$f[\cdot,\cdot,\cdot]$

$$\frac{0-6}{0+2} = \frac{-6}{2} = -3$$

$$\cancel{0} = 0$$

$$\frac{6-0}{2-0} = 3$$

$$\frac{18-6}{3-1} = \frac{12}{2} = 6$$

$f[\cdot,\cdot,\cdot,\cdot]$

$$\frac{0+3}{1+2} = 1$$

$$\frac{3-0}{2+1} = 1$$

$$\frac{6-3}{3-0} = 1$$

$f[\cdot,\cdot,\cdot,\cdot,\cdot]$

$$\cancel{\frac{1-1}{\cdots}} \to \boxed{0}$$

$(a_4 = 0)$

$$\cancel{\frac{1-1}{\cdots}} \to 0$$

$$\cdots \; f[x_0,\ldots,x_5] = \boxed{0}$$

$(a_5 = 0)$

FROM THE TABLE,

$\Longrightarrow n = 3$ ($a_4 = 0$, $a_5 = 0$ FOR $P_n(x) = a_0 + a_1(x - x_0) + \ldots$)

$\underline{5}$ $\sum^{n} l_k(x) = 1$

$$P_n(x) = \sum_{k=\emptyset}^{n} f(x_k)\, l_k(x)$$

$$f(x) = P_n(x) + \frac{f^{(n+1)}(\xi_n)}{(n+1)!}(x-x_0)(x-x_1)\ldots(x-x_n)$$

IF $f(x) = 1 \rightarrow f \equiv 1$, $f^{(n+1)}(\xi_n) = \emptyset$

$$1 = \sum_{k=\emptyset}^{n} 1 \cdot l_k(x) + \emptyset$$

$$\sum_{k=\emptyset}^{n} l_k(x) = \emptyset \qquad \therefore$$