HW 2

2 / b

$$g(x) = \frac{2}{3}x_n + \frac{1}{x_n^2}$$

$$g'(x) = \frac{2}{3} - \frac{2}{x_n^3} \qquad g'''(x) = -\frac{24}{x_n^5}$$

$$g''(x) = \frac{6}{x_n^4}$$

EXPAND IN TAYLOR SRS ABOUT

$$a = \alpha = 3^{1/3}$$

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots$$

$$g(x) \approx \left(\frac{2}{3}(3^{1/3}) + \frac{1}{(3^{1/3})^2}\right) + \underbrace{\left(\frac{2}{3} - \frac{2}{3}\right)}_{\to 0}(x - 3^{1/3}) + \cdots$$

$$\cdots + \left(\frac{6}{(3^{4/3})}\right)(x - 3^{1/3})^2 + \left(-\frac{24}{3^{5/3}}\right)(x - 3^{1/3})^3$$

$$g(x) \approx \frac{2}{3}\cdot 3^{1/3} + 3^{-2/3} + 6\cdot 3^{-4/3}(x - 3^{1/3})^2$$

$$= 3^{1/3} + 3^{1/3}(x - 3^{1/3})^2 \qquad (x-\alpha)^2 = x^2 - 2x\alpha + \alpha^2$$

TRUNC.

$$f'(a) \approx \frac{f(x) - f(a)}{(x-a)} - \frac{f''(a)}{2!}\frac{(x-a)^2}{(x-a)}$$

$$= g'(\alpha) \approx \frac{\left(\frac{2}{3}x_n + \frac{1}{x_n^2}\right) - \left(\frac{2}{3}3^{1/3} + 3^{-2/3}\right)}{(x_n - 3^{1/3})}$$

TRUNCATE FOR NOW

$$g'(x) \approx 2\cdot 3^{1/3}(x - 3^{1/2})\cdot 1$$

$$= 2\cdot 3^{1/3}(x - 3^{1/3})$$

NOW TRY $K \sim |g'(\alpha)|$

$$K \sim |2\cdot 3^{1/3}|$$