

3  
a

$$P_n(x) = \sum_{k=0}^n f(x_k) l_k(x)$$

$x_0$	$x_1$	$x_2$	$x_3$
0	1	2	3

$$P_4(x) = f(x_0) l_0(x) + \dots + f(x_4) l_4(x)$$

$$= f(x_0) \cdot \frac{x-x_1}{x_0-x_1} \cdot \dots \cdot \frac{x-x_4}{x_0-x_4} + \dots + f(x_4) \cdot \frac{x-x_0}{x_4-x_0} \cdot \dots \cdot \frac{x-x_3}{x_4-x_3}$$

$$f(x) = e^{x^2}$$

$$P_4(x) = e^{x_0^2} \cdot \frac{x-1}{0-1} \cdot \frac{x-2}{0-2} \cdot \frac{x-3}{0-3} + e^{x_1^2} \cdot \frac{x-0}{1-0} \cdot \frac{x-2}{1-2} \cdot \frac{x-3}{1-3} + \dots$$

$$\dots + e^{x_2^2} \cdot \frac{x-0}{2-0} \cdot \frac{x-1}{2-1} \cdot \frac{x-3}{2-3} + e^{x_3^2} \cdot \frac{x-0}{3-0} \cdot \frac{x-1}{3-1} \cdot \frac{x-2}{3-2}$$

$$= -e^{x_0^2} \frac{1}{6} (x-1)(x-2)(x-3) + e^{x_1^2} \frac{1}{2} x(x-2)(x-3) + \dots$$

$$\dots - e^{x_2^2} \frac{1}{2} x(x-1)(x-3) + e^{x_3^2} \frac{1}{6} x(x-1)(x-2)$$

$$= -e^{x_0^2} \frac{1}{6} (x-1)(x^2-5x+6) + e^{x_1^2} \frac{1}{2} x(x^2-5x+6) + \dots$$

$$\dots - e^{x_2^2} \frac{1}{2} x(x^2-4x+7) + e^{x_3^2} \frac{1}{6} x(x^2-3x+3)$$