

3a

$$f(x) = e^x, \quad x_0 = 0, x_1 = 1, x_2 = 2, x_3 = 3$$

$$p_3(x) = \sum_{k=0}^3 f(x_k) \cdot l_k(x) = f(x_0)l_0(x) + f(x_1)l_1(x) + \dots \\ \dots + f(x_2)l_2(x) + f(x_3)l_3(x)$$

$$l_k = \prod_{\substack{i=0 \\ i \neq k}}^3 \frac{x - x_i}{x_k - x_i}$$

$$l_0 = \prod_{i=1}^3 \frac{x - x_i}{x_0 - x_i} = \frac{x - x_1}{x_0 - x_1} \frac{x - x_2}{x_0 - x_2} \frac{x - x_3}{x_0 - x_3} = \frac{x - 1}{0 - 1} \frac{x - 2}{0 - 2} \frac{x - 3}{0 - 3} = \dots$$

$$\dots = 1 \cdot -\frac{1}{2} \cdot -\frac{1}{3} (x - 1)(x^2 - 5x + 6)$$

$$= \frac{1}{6} (x^3 - 5x^2 + 6x - x^2 + 5x - 6)$$

$$= -\frac{1}{6} (x^3 - 6x^2 + 11x - 6)$$