

# Exam 2, 2.16: GAUSSIAN ELIM. W/ PARTIAL PIVOTING

$$\begin{bmatrix} 1 & 0 & 0 \\ -1/4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 4 & 2 \\ 0 & 2 & 2 \\ 0 & 1 & 3/2 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 2 \\ 0 & 2 & 2 \\ 0 & 1 & 3/2 \end{bmatrix}$$

$M_1$   $P_1 A$

INTERCHANGE ROWS SO THAT THE LARGEST ENTRY IN SECOND COLUMN IS ON DIAGONAL

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 4 & 2 \\ 0 & 1 & 3/2 \\ 0 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 2 \\ 0 & 1 & 3/2 \\ 0 & 2 & 2 \end{bmatrix}$$

$P_2$   $M_2 P_1 A$

ANNIHILATE SUB DIA. ENTRIES IN 2ND COLUMN

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1/2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 4 & 2 \\ 0 & 1 & 3/2 \\ 0 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 2 \\ 0 & 1 & 3/2 \\ 0 & 0 & 1/2 \end{bmatrix}$$

$M_2$   $P_2 M_1 P_1 A$   $M_2 P_2 M_1 P_1 A$

$$-1 + 3/2 = -2/2 + 3/2 = 1/2$$

CAN NOW BACK-SUB TO SOLVE X

HERE L IS NOT LOWER TRIANGULAR BUT IT IS A PERMUTATION OF A LOWER TRI. MATRIX

THE EXPLICIT LU FACTORIZATION...

$$L = M^{-1} = (M_2 P_2 M_1 P_1)^{-1} = P_1^T L_1 P_2^T L_2$$

$$P_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_1^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_1 = \begin{bmatrix} 1 & 0 & 0 \\ -1/4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_1^{-1} = L_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1/4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_2^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1/2 & 1 \end{bmatrix}$$

$$M_2^{-1} = L_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1/2 & 1 \end{bmatrix}$$

$$L = P_1^T L_1 P_2^T L_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1/4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1/2 & 1 \end{bmatrix}$$

$$A = LU = \begin{bmatrix} 1/4 & 1/2 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} 4 & 4 & 2 \\ 0 & 1 & 3/2 \\ 0 & 0 & 1/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1/4 & 1 & 0 \\ 0 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 4 & 2 \\ 0 & 1 & 3/2 \\ 0 & 0 & 1/2 \end{bmatrix}$$

PHYSICS  
ORS IN  
ENTRY