HOMEWORK 6

PHYS 428

28 JULY 2017

$\frac{1}{u}$

| $n$ | $h$ | $\mu_n$ | $y(1)-\mu_n$ | $(y(1)-\mu_n)/h$ |
|---|---|---|---|---|
| 1 | 0.1 | 1.2 | 4.237 | 12 |
| 2 | 0.1 | 1.44 | 3.997 | 14.4 |

$\mu_1 = \mu_0 + h2\mu_0 \quad (h=0.1)$

$\mu_1 = 1 + 2h \quad \ldots \mu_1 = 1 + 2(0.1)$

$= 1 + 0.2$

$= 1.2$

$\mu_2 = \mu_1 + h2\mu_1$

$= (1+2h) + 2h(1+2h)$

$= (1+2h)^2 \quad \ldots \mu_2 = (1.2)^2 = 1.44$

$\longrightarrow \mu_n = (1+2h)^n$

$y(1) - \mu_1 = 2e - 1.2 = \alpha$

$\approx 4.237$

$y(1) - \mu_2 = 2e - 1.44 = \beta$

$\approx 3.997$

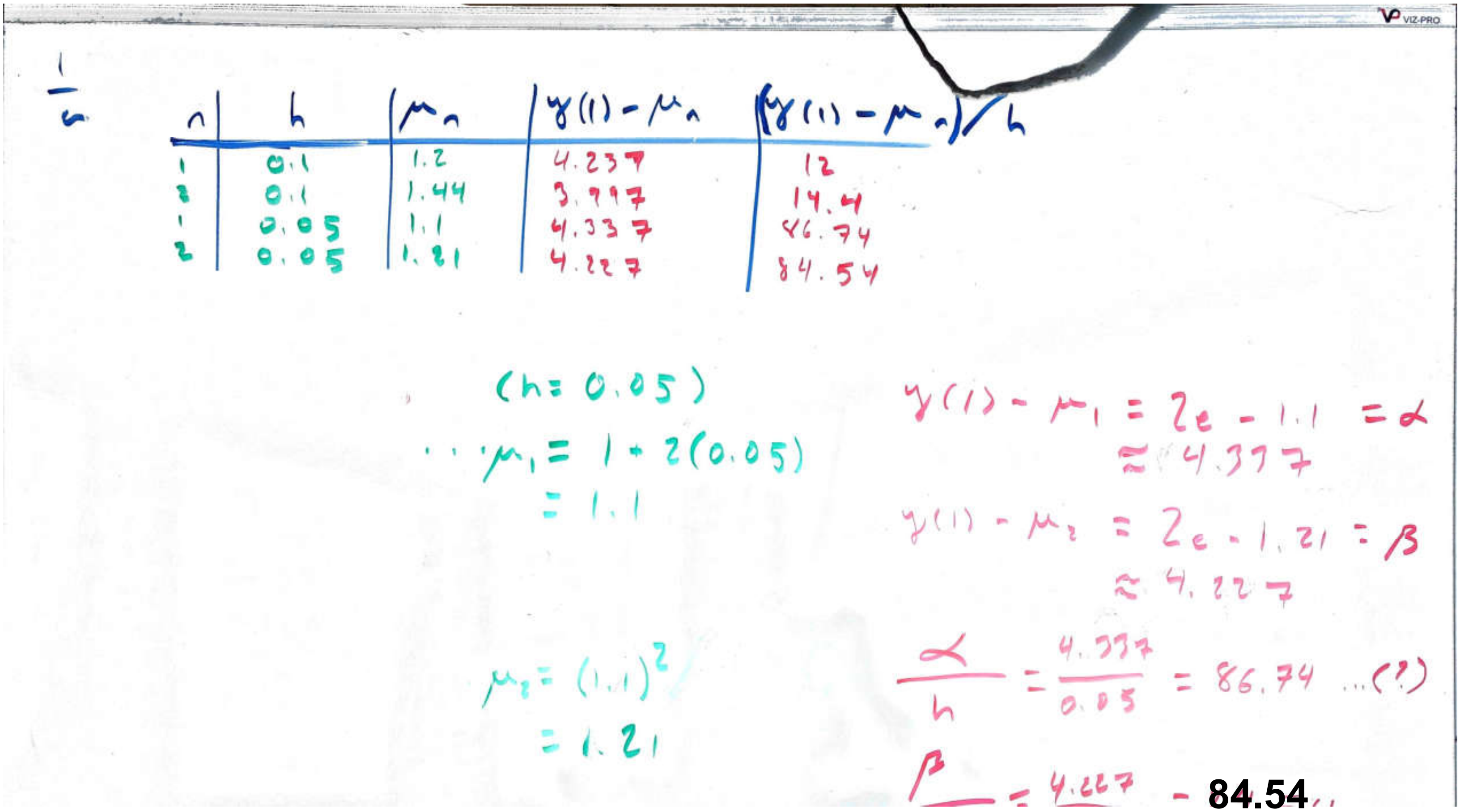
$\frac{\alpha}{h} = \frac{1.2}{0.1} = 12$

$\frac{\beta}{h} = \frac{1.44}{0.1} = 14.4$

$n = \frac{t_n - t_0}{h}$

$= \frac{1-0}{0.1}$

$= 10 \ldots$



$\frac{1}{\omega}$

| $n$ | $h$ | $\mu_n$ | $y(1) - \mu_n$ | $(y(1) - \mu_n)/h$ |
|---|---|---|---|---|
| 1 | 0.1 | 1.2 | 4.237 | 12 |
| 2 | 0.1 | 1.44 | 3.997 | 14.4 |
| 1 | 0.05 | 1.1 | 4.337 | 86.74 |
| 2 | 0.05 | 1.21 | 4.227 | 84.54 |

$(h = 0.05)$

$\therefore \mu_1 = 1 + 2(0.05)$

$= 1.1$

$\mu_2 = (1.1)^2$

$= 1.21$

$y(1) - \mu_1 = 2e - 1.1 = \alpha$

$\approx 4.337$

$y(1) - \mu_2 = 2e - 1.21 = \beta$

$\approx 4.227$

$\frac{\alpha}{h} = \frac{4.337}{0.05} = 86.74$ ...(?)

$\frac{\beta}{h} = \frac{4.227}{0.05} = 84.54$

$\frac{1}{n}$

| $n$ | $h$ | $\mu_n$ | $y(1) - \mu_n$ | $(y(1) - \mu_n)/h$ |
|---|---|---|---|---|
| 1 | 0.1 | 1.2 | 4.237 | 12 |
| 2 | 0.1 | 1.44 | 3.997 | 14.4 |
| 1 | 0.05 | 1.1 | 4.337 | 46.74 |
| 2 | 0.05 | 1.21 | 4.227 | 84.54 |
| 1 | 0.001 | 1.002 | 4.435 | 4.435e3 |
| 2 | 0.001 | 1.004 | 4.433 | 4.433e3 |

$(h = 0.001)$

$$\mu_1 = 1 + 2(0.001)$$
$$= 1.002$$

$$\mu_2 = (1.002)^2$$
$$\approx 1.004$$

$$y(1) - \mu_1 = 2e - 1.002 = \alpha$$
$$\approx 4.435$$

$$y(1) - \mu_2 = 2e - 1.004 = \beta$$
$$\approx 4.433$$

$$\frac{\alpha}{h} = \frac{4.435}{0.001} = 4.435 \times 10^3$$

$$\frac{\beta}{h} = \frac{4.433}{0.001} = 4.433 \times 10^3$$

W/ A
SMALLER
STEP
SIZE
LESS
ACCURATE APPROXIMATIONS
COME OUT. THIS SEEMS
COUNTER-INTUITIVE
I THINK I DID SOME → EARLIER STEP
WRONG...

$$n = \frac{t_n - t_0}{h}$$
$$= \frac{1 - 0}{0.1}$$
$$= 10 \ldots$$

$\frac{1}{b}$

| $n$ | $h$ | $\mu_n$ | $y(1) - \mu_n$ | $(y(1) - \mu_n)/(h)$ |
|---|---|---|---|---|
| 1 | 0.1 | 1.160 | $\approx 4.277$ | 42.77 |
| 2 | 0.1 | 1.346 | $\approx 4.091$ | 40.91 |
| 1 | 0.05 | | | |
| 2 | 0.05 | | | |
| 1 | 0.001 | | | |
| 2 | 0.001 | | | |

[redacted]

$= 1$

$h = 0.1$

$\mu_{n+1} = \mu_n\left(1 + h + \frac{h}{2} + h^2\right)$

$\mu_0 = 1$

$\mu_1 = \mu_0\left(1 + 0.1 + \frac{0.1}{2} + 0.1^2\right)$

$= 1.160$ ✓

$\mu_2 = \mu_1(1 + 0.1 + \dots)$

$= 1.346$

$y(1) = 2e \approx 5.437 \dots$

$y(1) - \mu_1 = 2e - 1.160 \approx \underset{= \alpha}{4.277}$

$y(1) - \mu_2 = 2e - 1.346 \approx \underset{= \beta}{4.091}$

$\longrightarrow \frac{\alpha}{h} = \frac{4.277}{0.1} = 42.77$

$\frac{\beta}{h} = \frac{4.091}{0.1} = 40.91$

LESS MORE

[redacted]

(A.E.)

$\frac{1}{b}$

| $n$ | $h$ | $\mu_n$ | $y(1) - \mu_n$ | $(y(1) - \mu_n)/h$ |
|---|---|---|---|---|
| 1 | 0.1 | 1.160 | 4.277 | 42.77 |
| 2 | 0.1 | 1.346 | 4.091 | 40.91 |
| 1 | 0.05 | 1.078 | 4.359 | 87.18 |
| 2 | 0.05 | 1.162 | 4.275 | 85.50 |
| 1 | 0.001 | | | |
| 2 | 0.001 | | | |

[redacted]

= 1

$h = 0.05$

$\mu_{n+1} = \mu_n(1 + h + \frac{h}{2} + h^2)$

$\mu_0 = 1$

$\mu_1 = \mu_0(1 + 0.05 + \frac{0.05}{2} + 0.05^2)$

$= 1.078$

$\mu_2 = \mu_1(1 + \cdots)$

$= 1.078(1 + \cdots)$

$= 1.078^2$

$= 1.162$

$y(1) = 2e \approx 5.437 \ldots$

$y(1) - \mu_1 = 2e - 1.078 = 4.35$

$y(1) - \mu_2 = 2e - 1.162 = 4.27$

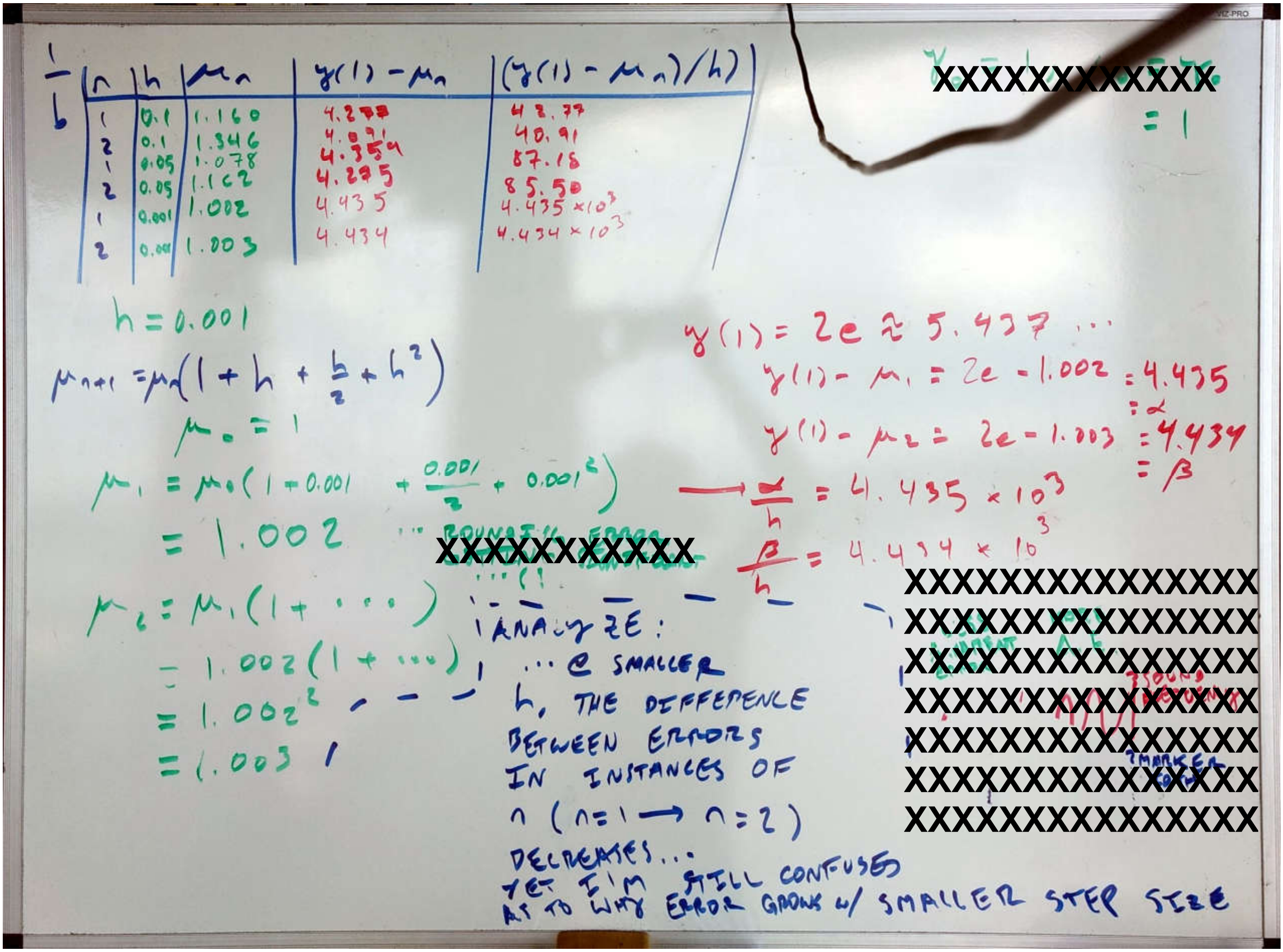
$\longrightarrow \frac{\alpha}{h} = \frac{4.359}{0.05} = 87.18$

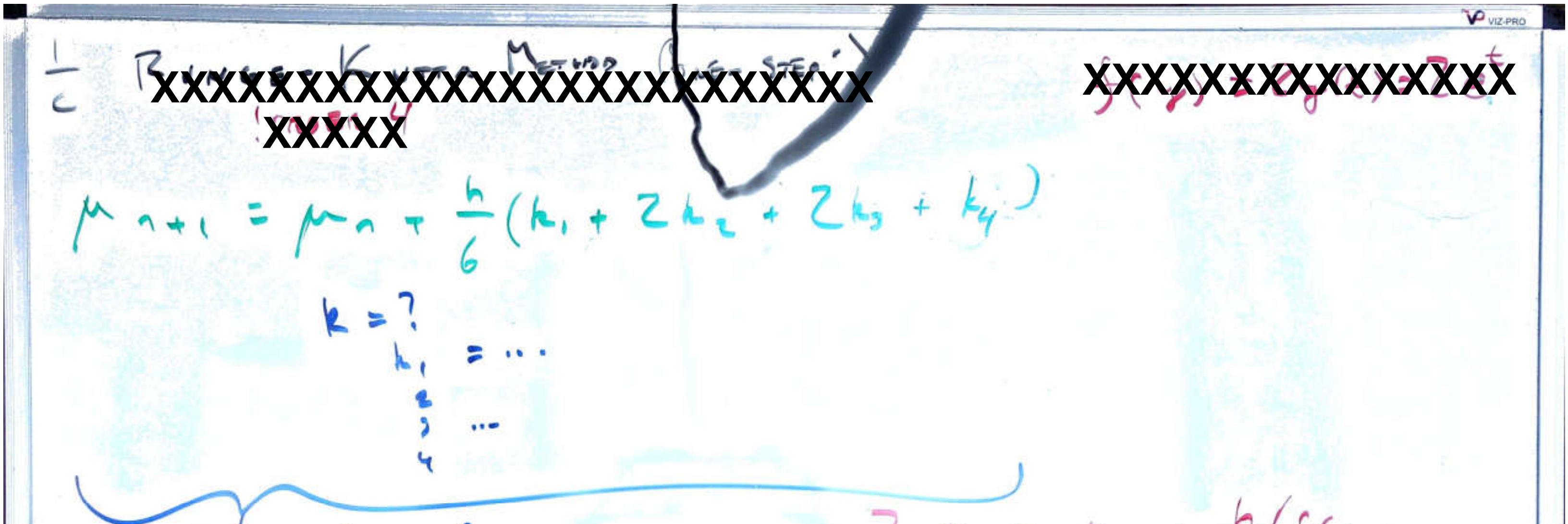
$\frac{\beta}{h} = \frac{4.275}{0.05} = 85.50$

LESS MORE

[redacted]

(A.E.)





$\frac{1}{c}$ [redacted]

[redacted]

[redacted]

$$\mu_{n+1} = \mu_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$k = ?$

$k_1 = \ldots$

$2$

$3 \quad \ldots$

$4$

$k_1 = f(\mu_n) = \ldots$

$k_2 = f(\mu_n + \frac{h}{2} k_1) =$

$k_3 = f(\mu_n + \frac{h}{2} k_2) =$

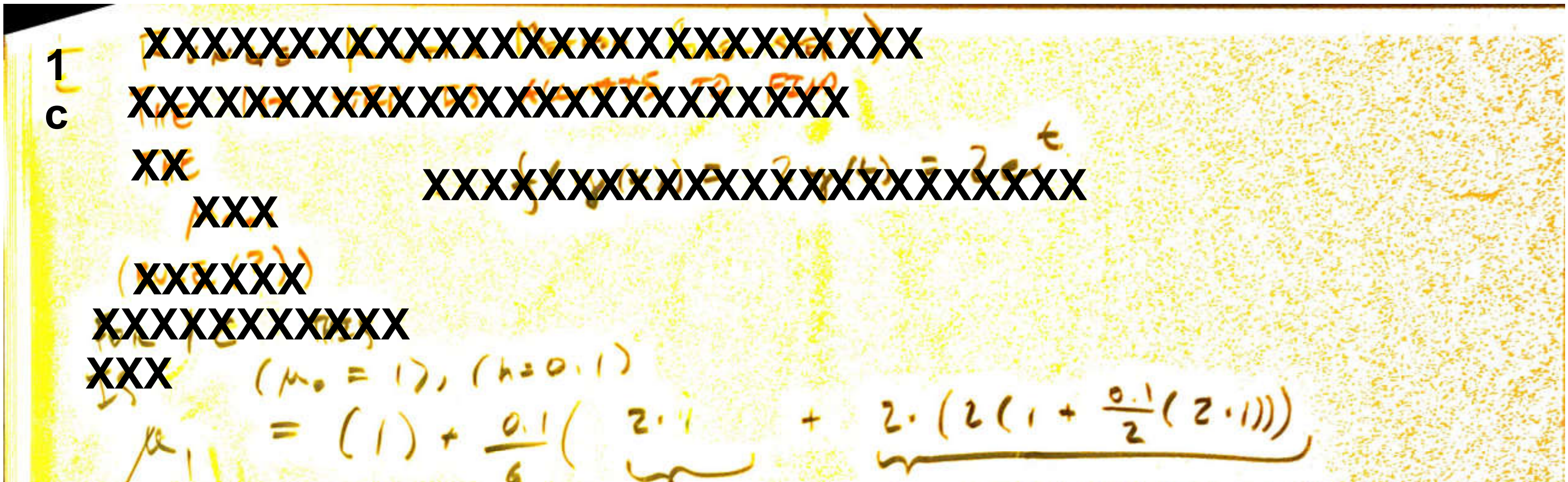
$k_4 = f(\mu_n + hk_3) =$

$$\mu_1 = \mu_0 + \frac{h}{6}(\underbrace{f(\mu_0)}_{k_1} + \ldots$$

$$\ldots + 2 \cdot \underbrace{(f(\mu_0 + \frac{h}{2} f(\mu_0))}_{k_2} + \ldots$$

$$\ldots + 2 \underbrace{(f(\mu_0 + \frac{h}{2}(f(\mu_0 + \frac{h}{2} f(\mu_0)))}_{k_3}$$

$$\ldots + \underbrace{f(\mu_0 + h\underbrace{((f(\mu_0 + \frac{h}{2}(f(\mu_0 + \frac{h}{2} f(\mu_0)))}_{k_3}}_{k_4}$$



1

c

[redacted]

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[redacted]

[redacted]

[redacted]

[redacted]

$(\mu_0 = 1), (h = 0.1)$

$$\mu_1 = (1) + \frac{0.1}{6}\Big( \underbrace{2\cdot 1}_{k_1} + \underbrace{2\cdot(2(1+\frac{0.1}{2}(2\cdot 1)))}_{k_2}$$

$$\overbrace{2\cdot(2(1}$$

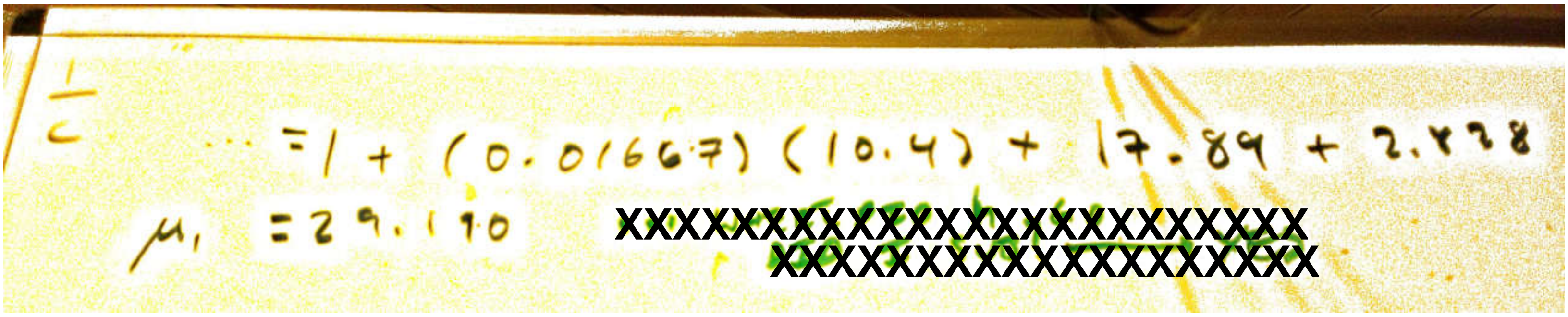
$$+ 2\cdot(2\cdot(1+\frac{0.1}{2}(2\cdot(2\cdot(1+\frac{0.1}{2}(2\cdot 1))))))$$

$$+ 2\cdot(1+0.1((2\cdot(1+\frac{0.1}{2}(2\cdot(1+\frac{0.1}{2}(2\cdot 1)))))))$$

$$= 1 + (0.01667)(2 + 8.4)$$

$$+ 2\cdot(2\cdot(1+(0.01667)\cdot 4.4$$

$$+ 2\cdot(1+0.1(2\cdot(1+0.01667\cdot 2.067)))$$

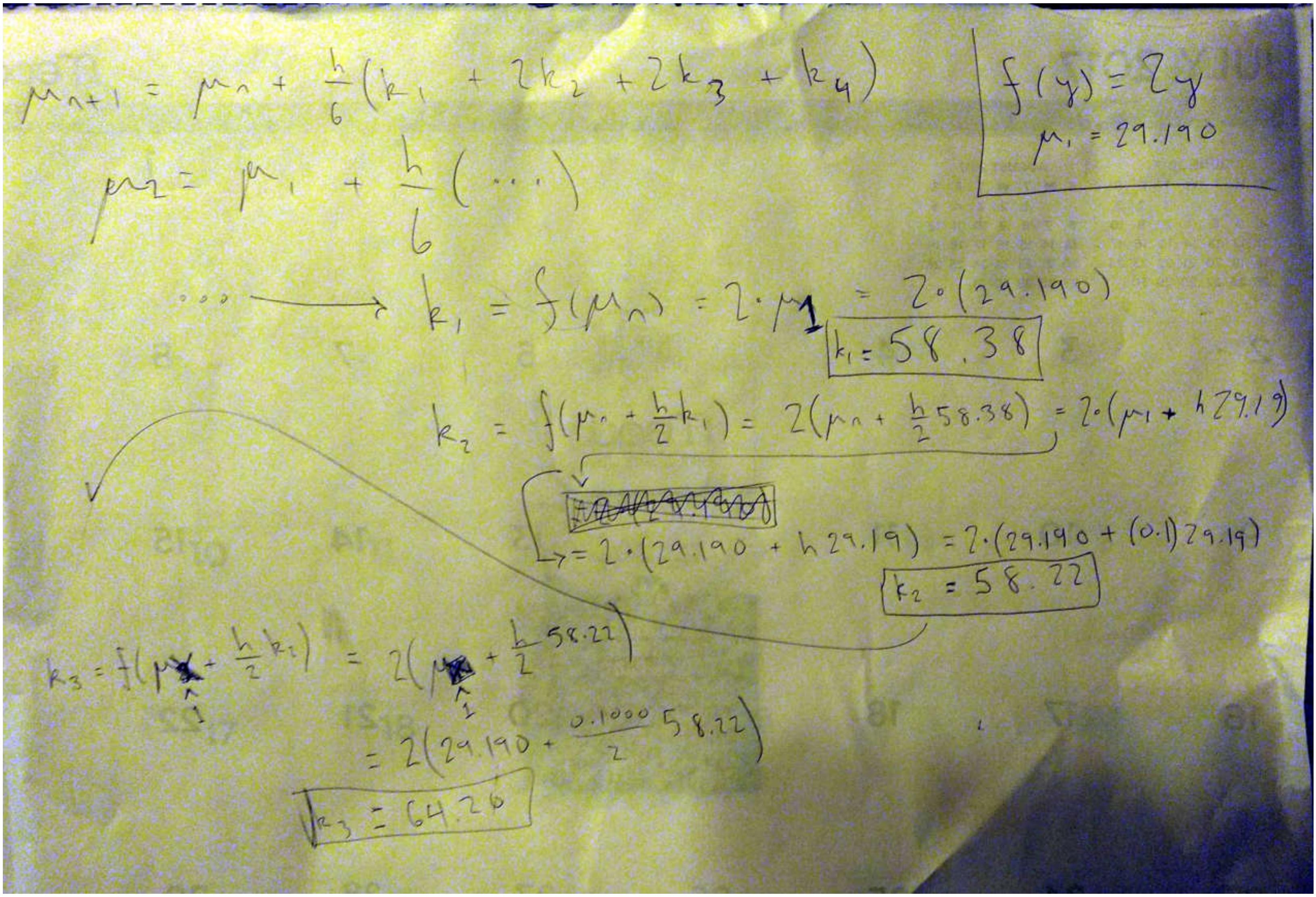


$\frac{1}{c}$

$\cdots = 1 + (0.01667)(10.4) + 17.89 + 2.838$

$\mu_1 = 29.190$ [redacted]

**1**
**C**



$$\boxed{\begin{array}{l} f(y) = 2y \\ \mu_1 = 29.190 \end{array}}$$

$$\mu_{n+1} = \mu_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$\mu_2 = \mu_1 + \frac{h}{6}(\ldots)$$

$$\ldots \longrightarrow k_1 = f(\mu_n) = 2\cdot\mu_1 = 2\cdot(29.190)$$

$$\boxed{k_1 = 58.38}$$

$$k_2 = f(\mu_n + \frac{h}{2}k_1) = 2(\mu_n + \frac{h}{2}58.38) = 2\cdot(\mu_1 + h\,29.19)$$

~~[illegible]~~

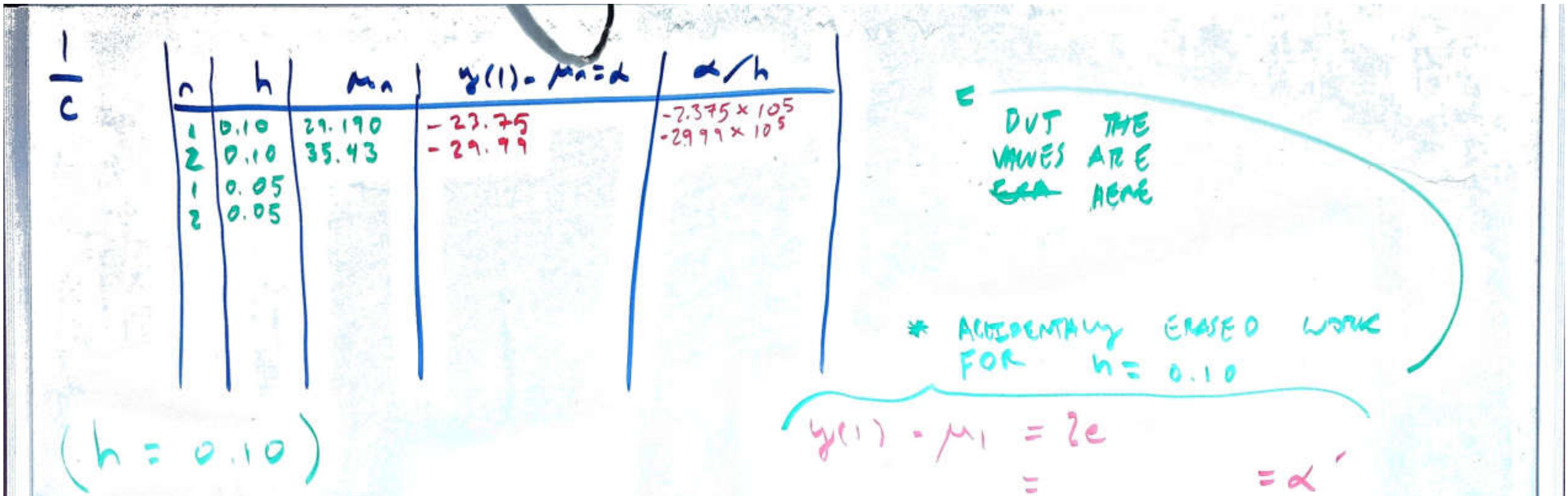
$$= 2\cdot(29.190 + h\,29.19) = 2\cdot(29.190 + (0.1)29.19)$$

$$\boxed{k_2 = 58.22}$$

$$k_3 = f(\underset{\hat{1}}{\mu_{\cancel{n}}} + \frac{h}{2}k_2) = 2(\underset{\hat{1}}{\mu_{\cancel{n}}} + \frac{h}{2}58.22)$$

$$= 2(29.190 + \frac{0.1000}{2}58.22)$$

$$\boxed{k_3 = 64.26}$$



$\frac{1}{c}$

| $n$ | $h$ | $\mu_n$ | $y(1) - \mu_n = \alpha$ | $\alpha / h$ |
|---|---|---|---|---|
| 1 | 0.10 | 29.190 | −23.75 | $-2.375 \times 10^5$ |
| 2 | 0.10 | 35.43 | −29.99 | $-2999 \times 10^5$ |
| 1 | 0.05 | | | |
| 2 | 0.05 | | | |

BUT THE VALUES ARE ~~ERA~~ HERE

* ACCIDENTALLY ERASED WORK FOR $h = 0.10$

$(h = 0.10)$

$$y(1) - \mu_1 = 2e$$
$$= \qquad = \alpha'$$
$$y(1) - \mu_2 = 2e -$$
$$= \qquad = \beta$$
$$\frac{\alpha'}{h^4} = \qquad =$$
$$\frac{\beta}{h^4} = \qquad =$$

**(k's on last page)**

$\frac{1}{c}$

| $n$ | $h$ | $\mu_n$ | $y(1) - \mu_n = \alpha$ | $\alpha / h$ |
|---|---|---|---|---|
| 1 | 0.10 | 29.190 | $-23.75$ | $-2.375 \times 105$ |
| 2 | 0.10 | 35.43 | $-29.99$ | $-2999 \times 10^5$ |
| 1 | 0.05 | | | |
| 2 | 0.05 | | | |

[redacted]

[redacted]

[redacted]

$(h = 0.05)$

$$\mu_1 = \mu_0 + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$\longrightarrow k_1 = f(\mu_0)$$

$$= \boxed{2 \cdot 1 = k_1}$$

$$k_2 = f(\mu_0 + \frac{h}{2}k_1) = 2\left(1 + \frac{0.05}{2}(2)\right)$$

$$\boxed{k_2 = 2.1}$$

$$y(1) - \mu_1 = 2e$$

$$= \qquad = \alpha'$$

$$y(1) - \mu_2 = 2e -$$

$$= \qquad = \beta$$

$$\frac{\alpha'}{h^4} = \qquad =$$

$$\frac{\beta}{h^4} = \qquad =$$

One-step Methods (?)

Euler's

More One-step

Runge-Kutta 4th Order

$k_1 = f(\mu_0)$

$k_2 = f(\mu_0 + \frac{h}{2}k_1)$

$\frac{1}{c}$

| $n$ | $h$ | $\mu_n$ | $y(1) - \mu_n = \alpha$ | $\alpha / h$ |
|---|---|---|---|---|
| 1 | 0.10 | 29.190 | $-23.75$ | $-2.375 \times 105$ |
| 2 | 0.10 | 35.43 | $-29.99$ | $-2.999 \times 10^5$ |
| 1 | 0.05 | | | |
| 2 | 0.05 | | | |

$f(y) = 2y$

BUT THE VALUES ARE ~~ERA~~ HERE

* ACCIDENTALLY ERASED WORK FOR $h = 0.10$

$(h = 0.05)$

$$\mu_1 = \mu_0 + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_3 = f\left(\mu_0 + \frac{h}{2} k_2\right)$$

$$= 2\left(1 + \frac{0.05}{2} 2.1\right) = \boxed{2.105 = k_3}$$

$$k_4 = f(\mu_0 + h k_3)$$

$$= 2(1 + 0.05 \cdot 2.105)$$

$$\boxed{k_4 = 2.2105}$$

$$y(1) - \mu_1 = 2e$$

$$= \qquad = \alpha'$$

$$y(1) - \mu_2 = 2e -$$

$$= \qquad = \beta$$

$$\frac{\alpha'}{h^4} = \qquad =$$

$$\frac{\beta}{h^4} = \qquad =$$

$\frac{1}{c}$

| $n$ | $h$ | $\mu_n$ | $y(1) - \mu_n = \alpha$ | $\alpha / h$ |
|---|---|---|---|---|
| 1 | 0.10 | 29.190 | -23.75 | $-2.375 \times 105$ |
| 2 | 0.10 | 35.43 | -29.99 | $-2999 \times 10^5$ |
| 1 | 0.05 | 1.105 | | |
| 2 | 0.05 | | | |

$f(y) = 2y$

OUT THE VALUES ARE ~~[illegible]~~ HERE

* ACCIDENTALLY ERASED WORK FOR $h = 0.10$

$(h = 0.05)$

$$\mu_1 = \mu_0 + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 1 + \frac{0.05}{6}(2 + 2\cdot 2.1 + 2\cdot 2.105 + 2.2105)$$

$$\boxed{\mu_1 = 1.105}$$

$$y(1) - \mu_1 = 2e$$

$$= \qquad = \alpha'$$

$$y(1) - \mu_2 = 2e -$$

$$= \qquad = \beta$$

$$\frac{\alpha'}{h^4} = \qquad =$$

$$\frac{\beta}{h^4} = \qquad =$$

$\frac{1}{c}$

| $n$ | $h$ | $\mu_n$ | $y(1) - \mu_n = \alpha$ | $\alpha / h$ |
|---|---|---|---|---|
| 1 | 0.10 | 29.190 | -23.75 | $-2.375 \times 105$ |
| 2 | 0.10 | 35.43 | -29.99 | $-2999 \times 10^5$ |
| 1 | 0.05 | 1.105 | | |
| 2 | 0.05 | | | |

$f(\ldots) = 2y$

BUT THE VALUES ARE HERE

* ACCIDENTALLY ERASED WORK FOR $h = 0.10$

$(h = 0.05)$

$$\mu_2 = \mu_1 + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$\longrightarrow k_1 = f(\mu_1) = 2 \cdot 1.105$$

$$\boxed{k_1 = 2.21}$$

$$k_2 = f(\mu_1 + \frac{h}{2}k_1) = 2 \cdot \left(1.105 + \frac{0.05}{2} 2.21\right)$$

$$\boxed{k_2 = 2.321}$$

$$y(1) - \mu_1 = 2e$$

$$= \qquad = \alpha'$$

$$y(1) - \mu_2 = 2e -$$

$$= \qquad = \beta$$

$$\frac{\alpha'}{h^4} = \qquad =$$

$$\frac{\beta}{h^4} = \qquad =$$

$\frac{1}{c}$

| $n$ | $h$ | $\mu_n$ | $y(1) - \mu_n = \alpha$ | $\alpha / h$ |
|---|---|---|---|---|
| 1 | 0.10 | 29.190 | -23.75 | $-2.375 \times 105$ |
| 2 | 0.10 | 35.43 | -29.99 | $-2999 \times 10^5$ |
| 1 | 0.05 | 1.105 | | |
| 2 | 0.05 | | | |

$f(y) = 2y$

E

DUT THE
VAUES ARE
~~ERA~~ HERE

* ACCIDENTALLY ERASED WORK
FOR $h = 0.10$

$y(1) - \mu_1 = 2e$

$=$ $= \alpha'$

$y(1) - \mu_2 = 2e -$

$=$ $= \beta$

$\frac{\alpha'}{h^4} =$ $=$

$\frac{\beta}{h^4} =$ $=$

$(h = 0.05)$

$$\mu_2 = \mu_1 + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$\rightarrow k_3 = f\left(\mu_1 + \frac{h}{2} k_2\right)$$

$$= 2 \cdot \left(1.105 + \frac{0.05}{2} 2.321\right)$$

$$\boxed{k_3 = 2.326}$$

$$k_4 = f(\mu_1 + h k_3)$$

$$= 2 \cdot (1.105 + 0.05 \cdot 2.326)$$

$$\boxed{k_4 = 2.21}$$

$\frac{1}{2}$

| $n$ | $h$ | $\mu_n$ | $y(1) - \mu_n = \alpha$ | $\alpha / h$ |
|---|---|---|---|---|
| 1 | 0.10 | 29.190 | -23.75 | $-2.375 \times 105$ |
| 2 | 0.10 | 35.43 | -29.99 | $-2999 \times 10^5$ |
| 1 | 0.05 | 1.105 | 4.332 | $6.931 \times 10^5$ |
| 2 | 0.05 | 1.219 | 4.218 | $6.748 \times 10^5$ |

$f(y) = 2y$

* ACCIDENTALLY ERASED WORK FOR $h = 0.10$

BUT THE VALUES ARE ~~ERA~~ HERE

(h = 0.05)

$$\mu_2 = \mu_1 + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 1.105 + \frac{0.05}{6}\Big(2.21 + 2 * 2.321 + \cdots$$

$$\cdots + 2 * 2.326 + \cdots$$

$$\cdots + 2.210\Big)$$

$$\boxed{\mu_2 = 1.219}$$

$$y(1) - \mu_1 = 2e - 1.105$$

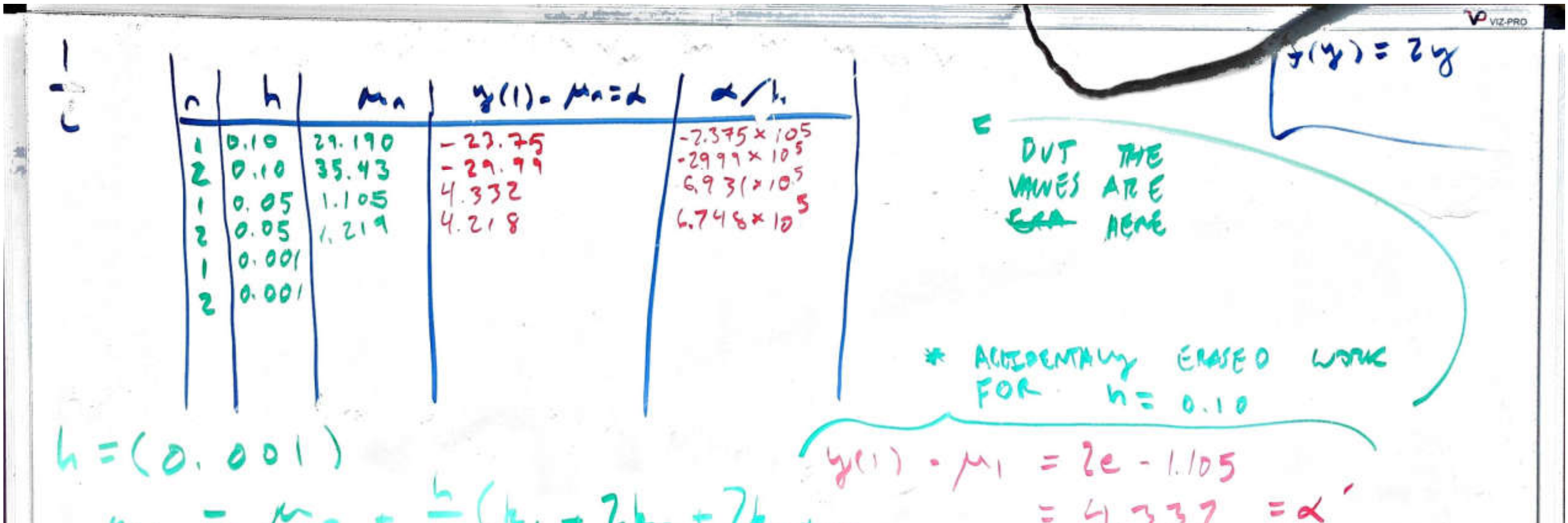
$$= 4.332 = \alpha'$$

$$y(1) - \mu_2 = 2e - 1.219$$

$$= 4.218 = \beta$$

$$\frac{\alpha'}{h^4} = \frac{4.332}{0.05^4} = 6931 \times 10^5$$

$$\frac{\beta}{h^4} = \frac{4.218}{0.05^4} = 6.748 \times 10^5$$



$\frac{1}{e}$

| $n$ | $h$ | $\mu_n$ | $y(1) - \mu_n = \alpha$ | $\alpha / h_1$ |
|---|---|---|---|---|
| 1 | 0.10 | 29.190 | −23.75 | $-2.375 \times 105$ |
| 2 | 0.10 | 35.43 | −29.99 | $-2999 \times 10^5$ |
| 1 | 0.05 | 1.105 | 4.332 | $6.931 \times 10^5$ |
| 2 | 0.05 | 1.219 | 4.218 | $6.748 \times 10^5$ |
| 1 | 0.001 | | | |
| 2 | 0.001 | | | |

$f(y) = 2y$

E

BUT THE
VALUES ARE
~~ERA~~ HERE

* ACCIDENTALLY ERASED WORK
FOR $h = 0.10$

$h = (0.001)$

$$\mu_1 = \mu_0 + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + \dots$$
$$\dots + k_4)$$

$$\longrightarrow k_1 = f(\mu_0) = \boxed{2.1 = k_1}$$

$$k_2 = f\left(\mu_0 + \frac{h}{2}k_1\right)$$
$$= 2\cdot\left(1 + \frac{0.001}{2}2\right)$$
$$\boxed{k_2 = 2.002}$$

$$y(1) - \mu_1 = 2e - 1.105$$
$$= 4.332 = \alpha'$$
$$y(1) - \mu_2 = 2e - 1.219$$
$$= 4.218 = \beta$$

$$\frac{\alpha'}{h^4} = \frac{4.332}{0.05^4} = 6.931 \times 10^5$$

$$\frac{\beta}{h^4} = \frac{4.218}{0.05^4} = 6.748 \times 10^5$$

| $n$ | $h$ | $\mu_n$ | $y(1) - \mu_n = \alpha$ | $\alpha / h$ |
|---|---|---|---|---|
| 1 | 0.10 | 29.190 | -23.75 | $-2.375 \times 10^5$ |
| 2 | 0.10 | 35.43 | -29.99 | $-2999 \times 10^5$ |
| 1 | 0.05 | 1.105 | 4.332 | $6.931 \times 10^5$ |
| 2 | 0.05 | 1.219 | 4.218 | $6.748 \times 10^5$ |
| 1 | 0.001 | | | |
| 2 | 0.001 | | | |

$f(y) = 2y$

E

OUT THE
VALUES ARE
~~ERA~~ HERE

* ACCIDENTALLY ERASED WORK
FOR $h = 0.10$

$h = (0.001)$

$$k_3 = f\left(\mu_0 + \frac{h}{2} k_2\right)$$

$$= 2 \cdot \left(1 + \frac{0.001}{2}(2.002)\right)$$

$$k_3 = 2.002$$

$$k_4 = f(\mu_0 + h \cdot k_3)$$

$$= 2 \cdot (1 + 0.001 \cdot 2.002)$$

$$\boxed{k_4 = 2.004}$$

$$y(1) - \mu_1 = 2e - 1.105$$

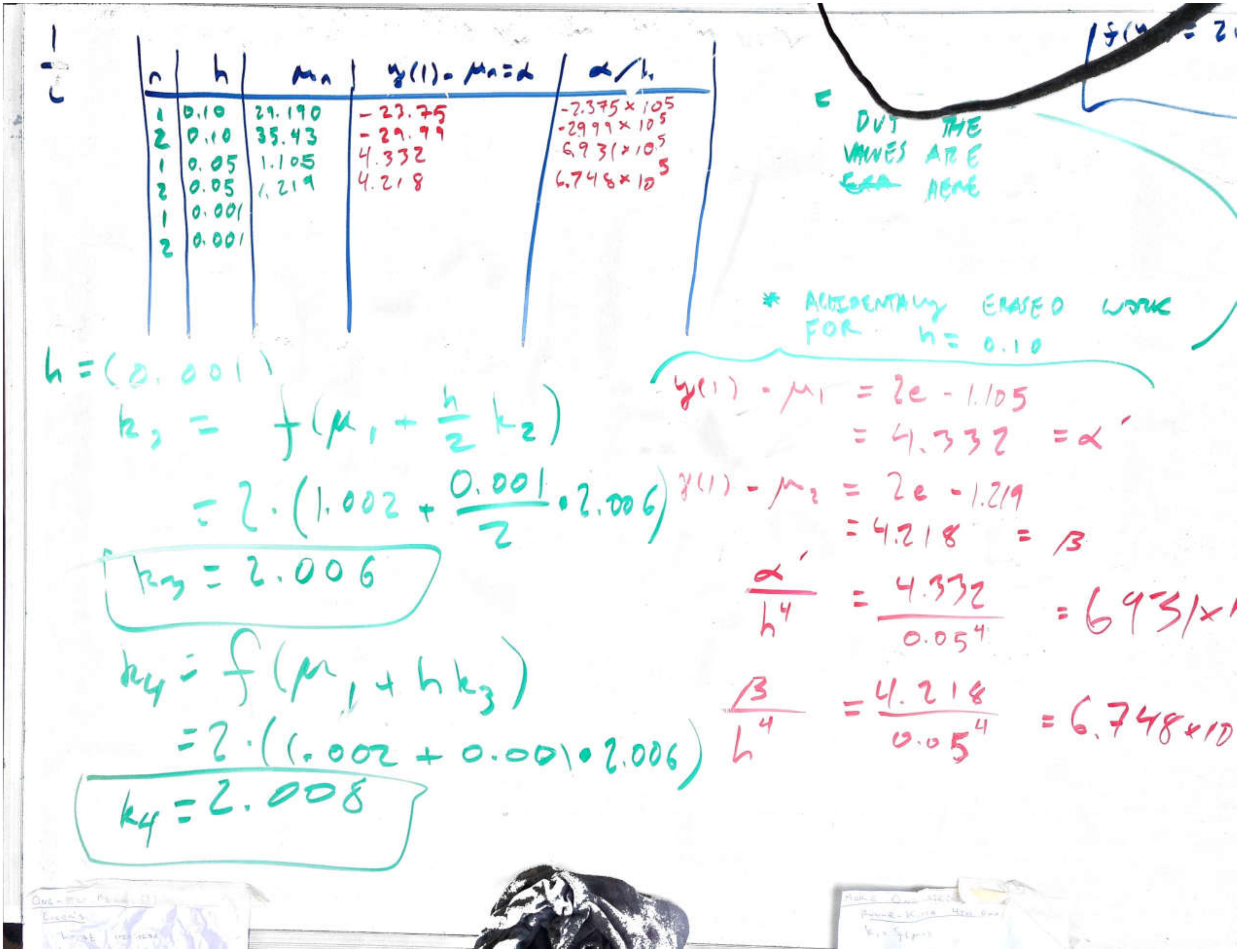
$$= 4.332 = \alpha'$$

$$y(1) - \mu_2 = 2e - 1.219$$

$$= 4.218 = \beta$$

$$\frac{\alpha'}{h^4} = \frac{4.332}{0.05^4} = 6931 \times 10^5$$

$$\frac{\beta}{h^4} = \frac{4.218}{0.05^4} = 6.748 \times 10^5$$



$\frac{1}{L}$

| n | h | $\mu_n$ | $y(1) - \mu_n = \alpha$ | $\alpha / h$ |
|---|---|---|---|---|
| 1 | 0.10 | 29.190 | -23.75 | $-2.375 \times 10^5$ |
| 2 | 0.10 | 35.43 | -29.99 | $-2999 \times 10^5$ |
| 1 | 0.05 | 1.105 | 4.332 | $6.931 \times 10^5$ |
| 2 | 0.05 | 1.219 | 4.218 | $6.748 \times 10^5$ |
| 1 | 0.001 | | | |
| 2 | 0.001 | | | |

f(y = 2

E

DUT THE
VALUES ARE
[illegible] HERE

* ACCIDENTALLY ERASED WORK
FOR h = 0.10

$$y(1) - \mu_1 = 2e - 1.105$$
$$= 4.332 = \alpha'$$
$$y(1) - \mu_2 = 2e - 1.219$$
$$= 4.218 = \beta$$
$$\frac{\alpha'}{h^4} = \frac{4.332}{0.05^4} = 6931 \times 1$$
$$\frac{\beta}{h^4} = \frac{4.218}{0.05^4} = 6.748 \times 10$$

$h = (0.001)$

$$k_3 = f(\mu_1 + \frac{h}{2} k_2)$$
$$= 2 \cdot (1.002 + \frac{0.001}{2} \cdot 2.006)$$
$$\boxed{k_3 = 2.006}$$
$$k_4 = f(\mu_1 + h k_3)$$
$$= 2 \cdot (1.002 + 0.001 \cdot 2.006)$$
$$\boxed{k_4 = 2.008}$$

$\frac{1}{2}$

$f(y) = 2y$

| $n$ | $h$ | $\mu_n$ | $y(1) - \mu_n = \alpha$ | $\alpha / h_1$ |
|---|---|---|---|---|
| 1 | 0.10 | 29.190 | −23.75 | $-2.375 \times 10^5$ |
| 2 | 0.10 | 35.43 | −29.99 | $-2999 \times 10^5$ |
| 1 | 0.05 | 1.105 | 4.332 | $6.931 \times 10^5$ |
| 2 | 0.05 | 1.219 | 4.218 | $6.748 \times 10^5$ |
| 1 | 0.001 | | | |
| 2 | 0.001 | | | |

OUT THE VALUES ARE ~~ERA~~ HERE

* ACCIDENTALLY ERASED WORK FOR h = 0.10

$h = (0.001)$

$$\mu_2 = \mu_1 + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$\longrightarrow k_1 = f(\mu_1) = 2 \cdot 1.002$$

$$\boxed{k_1 = 2.004}$$

$$k_2 = f\left(\mu_1 + \frac{h}{2} k_1\right)$$

$$= 2 \cdot \left(1.002 + \frac{0.001}{2} \cdot 2.004\right)$$

$$\boxed{k_2 = 2.006}$$

$$y(1) - \mu_1 = 2e - 1.105$$

$$= 4.332 = \alpha'$$

$$y(1) - \mu_2 = 2e - 1.219$$

$$= 4.218 = \beta$$

$$\frac{\alpha'}{h^4} = \frac{4.332}{0.05^4} = 6931 \times 10^5$$

$$\frac{\beta}{h^4} = \frac{4.218}{0.05^4} = 6.748 \times 10^5$$

$f(y) = 2y$

| $n$ | $h$ | $\mu_n$ | $y(1) - \mu_n = \alpha$ | $\alpha / h$ |
|---|---|---|---|---|
| 1 | 0.10 | 29.190 | -23.75 | $-2.375 \times 10^5$ |
| 2 | 0.10 | 35.43 | -29.99 | $-2999 \times 10^5$ |
| 1 | 0.05 | 1.105 | 4.332 | $6.931 \times 10^5$ |
| 2 | 0.05 | 1.219 | 4.218 | $6.748 \times 10^5$ |
| 1 | 0.001 | 1.002 | 4.435 | $4.435 \times 10^{12}$ |
| 2 | 0.001 | 1.004 | 4.433 | $4.433 \times 10^{12}$ |

* BUT THE VALUES ARE ~~ERA~~ HERE

* ACCIDENTALLY ERASED WORK FOR $h = 0.10$

$h = (0.001)$

$$\mu_2 = 1.002 + \frac{0.001}{6}(2.004 + \dots$$

$$\dots + 2 \cdot 2.006 + 2 \cdot 2.006 + \dots$$

$$\dots + 2.008)$$

$$\boxed{\mu_2 \cong 1.004}$$

$$y(1) - \mu_1 = 2e - 1.002$$
$$= 4.435 = \alpha'$$

$$y(1) - \mu_2 = 2e - 1.004$$
$$= 4.433 = \beta$$

$$\frac{\alpha'}{h^4} = \frac{4.435}{0.001^4} = 4.435 \times 10^{12}$$

$$\frac{\beta}{h^4} = \frac{4.433}{0.001^4} = 4.433 \times 10^{12}$$

→ ANALYZE: ... I STILL DON'T SEE WHY $\alpha / h$ GOES UP W/ $h$ DECREASING... IS THIS JUST BECAUSE WE ARE AT $n < 10$?

5

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 4 & -3 & 0 \\ 5 & 0 & 7 \end{bmatrix}$$

a USE PWR METHOD w/ INITIAL VECTOR,

$$\frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = v^{(0)}$$

& A CONVERGENCE TOLERANCE OF $5 \times 10^{-5}$ TO ESTIMATE DOMINANT EIGENVEC. & ASSOCIATED EIGVAL

$k = 1$

$$\omega = A v^{(0)}$$

$$= \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 4 & 5 \\ 4 & -3 & 0 \\ 5 & 0 & 7 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix} 10 \\ 1 \\ 12 \end{bmatrix}$$

$$v^{(1)} = \frac{\omega}{\|\omega\|_2} \; ; \; \|\omega\|_2 = \sqrt{\left(\frac{10}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{12}{\sqrt{3}}\right)^2} \approx 9.0370$$

$$\approx \frac{1}{9.0370 \cdot \sqrt{3}}\begin{bmatrix} 10 \\ 1 \\ 12 \end{bmatrix}$$

$$\approx 6.3889 \times 10^{-2}\begin{bmatrix} 10 \\ 1 \\ 12 \end{bmatrix}$$

$$\approx \begin{bmatrix} 6.3889 \times 10^{-1} \\ 6.3889 \times 10^{-2} \\ 7.6665 \times 10^{-1} \end{bmatrix}$$

[redacted]

## 5

SEE '5a.PY'

→ AT THE 8TH ITERATION, THE DIFFERENCE BETWEEN VALUES BETWEEN ITERATIONS REACHES $< 1 \times 10^{-5}$

FOR THE EIGENVALUE

FOR THE EIGENVECTOR, IT, DOESN'T CONVERGE UNTIL THE 11TH ITERATION

→ AT THE 8TH ITERATION THE EIGEN VECTOR ERROR IS

$$\approx \begin{bmatrix} \overset{\hat{v}}{0.54159} - \overset{v}{0.54152} \\ 0.29165 - 0.29139 \\ 0.78843 - 0.78858 \end{bmatrix} = \begin{bmatrix} 0.00007 \\ 0.00026 \\ 0.00015 \end{bmatrix}$$

5 SEE '5c.PY'

c THE DOMINANT EIGENVALUE & EIG. VEC. ARE APPROXIMATED

IN ONLY 4 ITERATIONS THE SAME RESULTS AS PREVIOUS METHODS ARE ACHIEVED