PROVE THAT $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ HAS NO LU DECOMPOSITION

→ ONE MIGHT ASSUME SUCH A LU DECOMP. EXISTS AND SHOW THAT THIS BRINGS A CONTRADICTION

$$LU = \begin{bmatrix} a & 0 \\ b & c \end{bmatrix} \begin{bmatrix} x & y \\ 0 & z \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} ax & ay \\ bx & by + cz \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} ax & ay \\ 0 & by - bx + cz \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 - bx & 1 - bx \end{bmatrix}$$

$$= \begin{bmatrix} ax & ay \\ 0 & by - bx + cz \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 - bx & 1 - bx \end{bmatrix}$$

$$ax\alpha + ay\gamma = \emptyset$$

$$(by - bx + cz)\gamma = 1 - bx$$

$$ax\beta + ay\delta = 1$$

$$(by - bx + cz)\delta = 1 - bx$$

EXAMPLE: 3×3 CASE

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -m_{21} & 1 & 0 \\ -m_{31} & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ m_{21} & 0 & 0 \\ m_{31} & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ m_{21} & 0 & 0 \\ m_{31} & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ m_{21} \\ m_{31} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$