3
a

$f(x) = e^x$

KNOWN AT

$x_0 = 0$

$x_1 = 1$

$x_2 = 2$

$x_3 = 4$

WRITE THE INTERPOLATING POLY IN LAGRANGE FORM

$$P_n(x) = \sum_{k=0}^{n} f(x_k)\, l_k(x) = \sum_{k=0}^{3} e^{x_k}\, l_k(x)$$

$$= e^{x_0} l_0 + e^{x_1} l_1 + e^{x_2} l_2 + e^{x_3} l_3$$

$$l_k = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{x - x_i}{x_k - x_i}$$

$$l_0 = \prod_{\substack{i=0 \\ i \neq k}}^{3} \frac{x - x_i}{x_0 - x_i} = \frac{x - x_1}{x_0 - x_1} \cdot \frac{x - x_2}{x_0 - x_2} \cdot \ldots$$

$$\ldots \cdot \frac{x - x_3}{x_0 - x_3}$$

$$= \frac{x-1}{0-1} \cdot \frac{x-2}{0-2} \cdot \frac{x-4}{0-4}$$

$$l_0 = -\frac{1}{8}(x-1)(x-2)(x-4)$$

$$l_1 = \prod^{3} \frac{x - x_i}{x_1 - x_i} = \frac{x - x_0}{x_1 - x_0} \cdot \frac{x - x_2}{x_1 - x_2} \cdot \frac{x - x_3}{x_1 - x_3} = \frac{x - 0}{\underset{1}{1 - 0}} \cdot \frac{x - 2}{\underset{-1}{1 - 2}} \cdot \frac{x - 4}{\underset{-3}{1 - 4}}$$

$$l_1 = \frac{1}{3} x(x-2)(x-4)$$