LET $x = [x_1, \ldots, x_n]$

1

a

$\|x\|_1 \geq 0$ FOR $x \in \mathbb{R}^n$? (1)

$$\|x\|_1 = \sum_{i=1}^{n} |x_i| = |x_1| + |x_2| + \cdots + |x_n| \geq \emptyset \quad \therefore$$

$\|x\|_1 = \emptyset$ IFF $x = [x_1, \ldots, x_n] = \emptyset$? (1)

$$\|x\|_1 = \sum_{i=1}^{n} |x_i| = |x_1| + \cdots + |x_n| = \emptyset$$

NO VALUES CAN BE SUBTRACTED, SO ALL x_i OF x MUST BE ZERO, AND x MUST BE ZERO $\therefore$

b

$\|\lambda \cdot x\|_1 = |\lambda| \cdot \|x\|_1$ FOR $\lambda \in \mathbb{R}$ & $x \in \mathbb{R}^n$

$$\|\lambda \cdot x\|_1 = \sum_{i=1}^{n} |\lambda \cdot x_i| = |\lambda \cdot x_1| + \cdots + |\lambda \cdot x_n| = |\lambda| \cdot (|x_1| + \cdots + |x_n|) \quad \therefore$$

c

$\|x + y\|_1 \leq \|x\|_1 + \|y\|_1$ FOR $x \in \mathbb{R}^n$ & $y \in \mathbb{R}^n$

$$\|x + y\|_1 = \sum_{i=1}^{n} |x_i + y_i| = |x_1 + y_1| + \cdots + |x_n + y_n| \leq (|x_1| + |y_1|) + \cdots +$$

$\cdots + |x_n| \geq \emptyset \quad \therefore$

(1)

LL $\therefore$

$\mathbb{R}^n$

$\lambda \cdot x_n| = |\lambda| \cdot \left(|x_1| + \cdots + |x_n| \right) \quad \therefore$

$\mathbb{R}^n$

$| + \cdots + |x_n + y_n| \leq \left(|x_1| + |y_1| \right) + \cdots + \left(|x_n| + |y_n| \right) = \sum_{i=1}^{n} |x_i| + \sum_{i=1}^{n} |y_i| = \|x\|_1 + \|y\|_1$

LET $x = [x_1$

$\|x\|_\infty = \text{MAX} \{ |$

$\|x\|_\infty$ IS A VECTOR

PROOF:

1. $\|x\|_\infty \geq \emptyset$

   $\|x\|_\infty =$

2. $\|\alpha x\|_\infty \overset{\text{DEF}}{=} \text{MAX}$

3. $\|x + y\|_\infty = \text{MAX}$