8/b FOR A 2×2

SHOW

GAUSS-SIEDEL

DOES THE SAME,

$\|B_{GS}\|_\infty \geq 1$

$$B_{GS} = -(L+D)^{-1}U$$

$$= -\begin{bmatrix} \frac{1}{a_{11}} & \emptyset \\ -\frac{a_{21}}{a_{11}a_{22}} & \frac{1}{a_{22}} \end{bmatrix} \cdot \begin{bmatrix} 0 & a_{22} \\ 0 & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} \emptyset & \frac{a_{22}}{a_{11}} \\ \emptyset & -\frac{a_{21}}{a_{11}} \end{bmatrix} = \begin{bmatrix} \emptyset & -\frac{a_{22}}{a_{11}} \\ \emptyset & \frac{a_{21}}{a_{11}} \end{bmatrix}$$

$$L + D = \begin{bmatrix} 0 & 0 & \cdots \\ a_{12} & 0 & \cdots \\ \vdots & & \ddots \\ a_{1,n} & \cdots & a_{n,n-1}\ 0 \end{bmatrix} + \begin{bmatrix} a_{11} & 0 & \cdots \\ 0 & a_{22} & \\ \vdots & & \ddots \\ \emptyset & & 0\ a_{nn} \end{bmatrix} = \begin{bmatrix} a_{11} & & & \\ & a_{22} & & \\ & & \ddots & \\ & & a_{n-1,n-1} & 0 \\ & & a_{n,n-1} & a_{nn} \end{bmatrix}$$

$$\rightarrow (L+D)^{-1}_{2\times 2} = \left(\begin{bmatrix} a_{11} & 0 \\ a_{21} & a_{22} \end{bmatrix}\right)^{-1} = \begin{bmatrix} a_{22} & \emptyset \\ -a_{21} & a_{11} \end{bmatrix} \cdot \frac{1}{a_{11}a_{22}}$$

$$= \begin{bmatrix} \frac{1}{a_{11}} & \emptyset \\ \frac{a_{21}}{a_{11}a_{22}} & \frac{1}{a_{22}} \end{bmatrix}$$