HW 2

1

$$f(x+h) = \underbrace{f(x) + f'(x)h + \frac{f''(x)}{2!}h^2 + \cdots + \frac{f^{(n)}(x)}{n!}h^n}_{P_n(x)} + \underbrace{\frac{f^{(n+1)}(z)}{(n+1)!}h^{(n+1)}}_{R_n(x)}$$

$$\cos\left(\frac{\pi}{2}x + h\right) =$$

$$\frac{x^{n+1}}{(2n)!}\left(\frac{\pi}{2}\right)^{2n} \times \frac{(-1)^{n+1}\left(\frac{\pi}{2}\right)^{4(n-1)} x^{2(n-1)}}{2(n-1)!}$$

$n=1 \rightarrow \frac{1 \cdot 1}{1}$ $\quad n=2 \rightarrow \left(\frac{\pi}{2}\right)^4 \frac{x^2}{2!}$ $\quad n=3 \rightarrow \left(\frac{\pi}{2}\right)^8 \frac{x^4}{4!}$ $\quad n=4 \rightarrow \left(\frac{\pi}{2}\right)^{12} \frac{x^6}{6!}$

$$1 - \left(\frac{\pi}{2}\right)^4 \frac{x^2}{2!} + \left(\frac{\pi}{2}\right)^8 \frac{x^4}{4!} - \cdots$$

$$\cos\left(\frac{\pi}{2}x\right) = \cdots + (-1)^{n+1}\frac{\left(\frac{\pi}{2}\right)^{4(n-1)} x^{2(n-1)}}{2(n-1)!} + (-1)^n \frac{\left(\frac{\pi}{2}\right)^{4n} x^{2n}}{(2n)!}$$

$R_n(x)$

(c)

$$|R_n(x)| = \frac{\left(\frac{\pi}{2}\right)^{4n} |x|^{2n}}{2n!} < 10^{-5}$$

(b)

$[-1, 1]$

$$\rightarrow = \frac{\left(\frac{\pi}{2}\right)^{4n} |-1|^{2n}}{2n!} = \left(\frac{\pi}{2}\right)^{4n} \frac{1}{2n!} < 10^{-5} \rightarrow \frac{\left(\frac{\pi}{2}\right)^{4n}}{2n!} < 10^{-5} \rightarrow n! < \left(\frac{\pi}{2}\right)^{4n} 10^5$$

$$\rightarrow n \leq 10^5$$

IN $[-1, 1]$