2/b

ORDER OF CONVERGENCE

IN PYTHON IF I (2b.PY)

START $x_0 = -2.0$

$\{x_n\} \longrightarrow \alpha = 3^{1/3}$

&

$$r \approx \frac{\ln E_n}{\ln E_{n-1}} \approx 0.87$$ ← LINEAR CONVERGENCE

FOR $\{x_n\} = 26, 18, 12$

RATE OF CONVERGENCE

$$g(x_n) \approx g(\alpha) + g'(\alpha)(x_n - \alpha) + \frac{g''(\alpha)}{2!}(x_n - \alpha)^2$$

$$= \frac{2}{3}(3^{1/3}) + \frac{1}{(3^{1/3})^2} + \underbrace{\frac{(6/(3^{1/3})^4)}{2!}}_{\beta}(x_n^2 - 2\cdot 3^{1/3} x_n + 3^{2/3})$$

$$g'(x_n) \approx \beta\cdot(2x_n - 2\cdot 3^{1/3} + 3^{2/3})$$

$$|g'(\alpha)| \approx \beta(2\cdot 3^{1/3} - 2\cdot 3^{1/3} + 3^{2/3}) = \boxed{5.8 \approx k}$$