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$$y' = 2y, \quad y(0) = 1 = u_0$$

$$u_{n+1} = u_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$u_{n+1} = u_n \left( 1 + 2h + 2h^2 + \frac{4}{3}h^3 + \frac{2}{3}h^4 \right)$$

$$h = 0.1$$

$$u_1 = u_0 \left( 1 + 2 \cdot 0.1 + 2 \cdot 0.1^2 + \frac{4}{3} \cdot 0.1^3 + \frac{2}{3} \cdot 0.1^4 \right)$$

$$= 1.2214$$

$$u_2 = 1.2214^2 \approx 1.49181796$$

$$h = 0.05$$

$$u_1 = u_0 \left( 1 + 2 \cdot 0.05 + 2 \cdot 0.05^2 + \frac{4}{3} \cdot 0.05^3 + \frac{2}{3} \cdot 0.05^4 \right)$$

$$\approx 1.105170833$$

$$u_2 = u_1^2 \approx 1.221402571$$

$$h = 0.001$$

$$u_1 = u_0 \left( 1 + 2 \cdot 0.001 + 2 \cdot 0.001^2 + \frac{4}{3} \cdot 0.001^3 + \frac{2}{3} \cdot 0.001^4 \right)$$

$$\approx 1.002002001$$

$$u_2 = u_1^2 \approx 1.004008011$$