

# Lesson 1: Week 9 - Notes 14 Levenshtein Distance

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## Week 9 - Levenshtein distance

depart\_leopard

paper\_pope

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## Useful links about Levenshtein distance

Levenshtein distance [https://en.wikipedia.org/wiki/Levenshtein\\_distance](https://en.wikipedia.org/wiki/Levenshtein_distance) Levenshtein Distance, in Three Flavors <https://people.cs.pitt.edu/~kirk/cs1501/Pruhs/Spring2006/assignments/editdistance/Levenshtein%20Distance.htm> Another Python version of Levenshtein distance [https://folk.idi.ntnu.no/mlh/hetland\\_org/coding/python/levenshtein.py](https://folk.idi.ntnu.no/mlh/hetland_org/coding/python/levenshtein.py) Levenshtein Distance [https://python-course.eu/levenshtein\\_distance.php](https://python-course.eu/levenshtein_distance.php)

## Week 9 - Levenshtein distance

levenshtein\_distance.py

# Levenshtein distance

Eric Martin, CSE, UNSW

COMP9021 Principles of Programming

When typing a sequence of letters  $w$  that is not recognised as a syntactically correct word, corrections can be suggested in the form of one or more syntactically correct words  $w'$  “close enough” to  $w$ , that is, such that  $w$  can be converted into  $w'$  at a “minimal cost”. The notion of *Levenshtein distance* offers a possible formalisation. One can convert a word  $w_1$  into a word  $w_2$  by successive applications of two transformations: *deletion* of an occurrence of a character and *insertion* of an occurrence of a character. For instance, one can convert DEPART into LEOPARD thanks to 2 deletions and 3 insertions, which can be achieved in many different ways, three of which are:

- DEPART -> EPART -> EPAR -> LEPAR -> LEOPAR -> LEOPARD
- DEPART -> LDEPART -> LDEOPART -> LDEOPARTD -> LEOPARTD -> LEOPARD
- DEPART -> EPART -> LEPART -> LEPAR -> LEOPARD -> LEOPARD

Associate a cost to each of both transformations. Denote the cost of a deletion by  $C_\delta$  and the cost of an insertion by  $C_\iota$ . We arbitrarily set both  $C_\delta$  and  $C_\iota$  to 1, but will still reason, design and implement in terms of arbitrary values. The cost of a conversion that involves  $n_\delta$  deletions and  $n_\iota$  insertions is then defined as  $n_\delta C_\delta + n_\iota C_\iota$ , so  $n_\delta + n_\iota$  with our choice of values for  $C_\delta$  and  $C_\iota$ . Given a word  $w$ , denote by  $|w|$  the length of  $w$ . Of course the cost of converting a word  $w_1$  into a word  $w_2$  is at most equal to  $|w_1|C_\delta + |w_2|C_\iota$ : it suffices to delete all occurrences of characters in  $w_1$  and insert all occurrences of characters in  $w_2$ . It is immediately verified that it is not possible to do any better if and only if  $w_1$  and  $w_2$  have no common character. We have seen that the cost of converting DEPART into LEOPARD is at most equal to  $2C_\delta + 3C_\iota$ , so 5 with our choice of values for  $C_\delta$  and  $C_\iota$ , and it is easy to verify that it is impossible to convert DEPART into LEOPARD at a lower cost.

We consider a third transformation: *substitution* of an occurrence of a character by an occurrence of another character, with an associated cost of  $C_\varsigma$ . Since a deletion and an insertion achieve the same effect as a substitution, it is sensible to set  $C_\varsigma$  to  $C_\delta + C_\iota$ , though other choices are legitimate. So DEPART can be converted into LEOPARD in new ways, still for a cost of 5 if  $C_\delta = 1$ ,  $C_\iota = 1$  and  $C_\varsigma = C_\delta + C_\iota$ , for instance with two substitutions and one insertion:

- DEPART -> LEPART -> LEPARD -> LEOPARD

The *Levenshtein distance* between two words  $w_1$  and  $w_2$  is the minimal cost of converting  $w_1$  into  $w_2$  using deletions, insertions and substitutions (allowing for substitutions makes no difference if  $C_\varsigma = C_\delta + C_\iota$ ): it is the (unique) integer of the form  $n_\delta C_\delta + n_\iota C_\iota + n_\varsigma C_\varsigma$  with (possibly nonunique)  $n_\delta$ ,  $n_\iota$  and  $n_\varsigma$  such that it is possible to convert  $w_1$  into  $w_2$  thanks to  $n_\delta$  deletions,  $n_\iota$  insertions and  $n_\varsigma$  substitutions.

We are interested in computing the Levenshtein distance  $d$  between two words  $w_1$  and  $w_2$ , and ignoring the order in which deletions, insertions and substitutions are applied, in finding all possible triples  $(S_\delta, S_\iota, S_\varsigma)$  where  $S_\delta$  is a set of occurrences of characters in  $w_1$ ,  $S_\iota$  is a set of occurrences of

characters in  $w_2$ , and  $S_\varsigma$  is a set of pairs consisting of an occurrence of a character in  $w_1$  and an occurrence of a character in  $w_2$ , such that  $w_1$  can be converted into  $w_2$  by applying deletion, insertion and substitution to all members of  $S_\delta$ ,  $S_\iota$  and  $S_\varsigma$ , respectively, for a cost of  $d$ . Taking DEPART and LEOPARD as an example again, and with  $C_\delta = 1$ ,  $C_\iota = 1$  and  $C_\varsigma = 2$ , the Levenshtein distance between both words is 5 and there are 9 such triples, which can conveniently be represented as follows (underscores in the second word are aligned with members of  $S_\delta$ , underscores in the first word are aligned with members of  $S_\iota$ , and different letters aligned in both words correspond to members of  $S_\varsigma$ ):

- $\_ \text{DE} \_ \text{PAR} \_ \text{T}$   
   $\text{L} \_ \text{EOPARD} \_$
- $\text{D} \_ \text{E} \_ \text{PAR} \_ \text{T}$   
   $\_ \text{LEOPARD} \_$
- $\text{DE} \_ \text{PAR} \_ \text{T}$   
   $\text{LEOPARD} \_$
- $\_ \text{DE} \_ \text{PART} \_$   
   $\text{L} \_ \text{EOPAR} \_ \text{D}$
- $\text{D} \_ \text{E} \_ \text{PART} \_$   
   $\_ \text{LEOPAR} \_ \text{D}$
- $\text{DE} \_ \text{PART} \_$   
   $\text{LEOPAR} \_ \text{D}$
- $\_ \text{DE} \_ \text{PART}$   
   $\text{L} \_ \text{EOPARD}$
- $\text{D} \_ \text{E} \_ \text{PART}$   
   $\_ \text{LEOPARD}$
- $\text{DE} \_ \text{PART}$   
   $\text{LEOPARD}$

There is only one way of minimal cost to convert a word  $w$  into the empty word: delete all occurrences of characters in  $w$ , for a cost of  $|w|C_\delta$ . Similarly, there is only one way of minimal cost to convert the empty word into a word  $w$ : insert all occurrences of characters in  $w$ , for a cost of  $|w|C_\iota$ . Consider two nonempty words  $w_1$  and  $w_2$ . Write  $w_1$  and  $w_2$  as  $w'_1c_1$  and  $w'_2c_2$  with  $c_1$  and  $c_2$  the last characters in  $w_1$  and  $w_2$ , respectively. In order to convert  $w_1$  into  $w_2$ , one can

- either convert  $w'_1$  into  $w_2$  and delete  $c_1$ ,
- or convert  $w_1$  into  $w'_2$  and insert  $c_2$ ,
- or convert  $w'_1$  into  $w'_2$  and in case  $c_1$  is distinct to  $c_2$ , substitute  $c_1$  by  $c_2$ .

Let  $v_\delta$  be  $C_\delta$  plus the minimal cost of converting  $w'_1$  into  $w_2$ . Let  $v_\iota$  be  $C_\iota$  plus the minimal cost of converting  $w_1$  into  $w'_2$ . Let  $v_\varsigma$  be the minimal cost of converting  $w'_1$  into  $w'_2$  in case  $c_1$  and  $c_2$  are the same letters, and  $C_\varsigma$  plus the minimal cost of converting  $w'_1$  into  $w'_2$  otherwise. Set  $v = \min(v_\delta, v_\iota, v_\varsigma)$ . Then  $v$  is the minimal cost of converting  $w_1$  into  $w_2$ . Moreover,

- if  $v = v_\delta$ , all ways of converting  $w'_1$  into  $w_2$  at minimal cost complemented with deleting  $c_1$ ,
- if  $v = v_\iota$ , all ways of converting  $w_1$  into  $w'_2$  at minimal cost complemented with inserting  $c_2$ ,
- if  $v = v_\varsigma$ , all ways of converting  $w'_1$  into  $w'_2$  at minimal cost complemented with substituting  $c_1$  by  $c_2$  in case  $c_1$  and  $c_2$  are different

make up all ways of converting  $w_1$  into  $w_2$  at a minimal cost.

Given two words  $w_1$  and  $w_2$ , consider the table with  $|w_1| + 1$  many columns and  $|w_2| + 1$  many

rows defined as follows.

- For all  $i \leq |w_1|$ , the  $(i + 1)$ -st column corresponds to the initial segment of  $w_1$  of length  $i$ .
- For all  $j \leq |w_2|$ , the  $(|w_2| - j + 1)$ -st row corresponds to the initial segment of  $w_2$  of length  $j$ .
- For all  $i \leq |w_1|$  and  $j \leq |w_2|$ , the element of the table at the intersection of the  $(i + 1)$ -st column and the  $(|w_2| - j + 1)$ -st row records the minimal cost of converting the initial segment of  $w_1$  of length  $i$  into the initial segment of  $w_2$  of length  $j$  as well as
  - if  $i > 0$ , a horizontal arrow in case the minimal cost of converting the initial segment of  $w_1$  of length  $i$  into the initial segment of  $w_2$  of length  $j$  is equal to  $C_\delta$  plus the minimal cost of converting the initial segment of  $w_1$  of length  $i - 1$  into the initial segment of  $w_2$  of length  $j$ ,
  - if  $j > 0$ , a vertical arrow in case the minimal cost of converting the initial segment of  $w_1$  of length  $i$  into the initial segment of  $w_2$  of length  $j$  is equal to  $C_\iota$  plus the minimal cost of converting the initial segment of  $w_1$  of length  $i$  into the initial segment of  $w_2$  of length  $j - 1$ ,
  - if  $i > 0$  and  $j > 0$ , a diagonal arrow in case the minimal cost of converting the initial segment of  $w_1$  of length  $i$  into the initial segment of  $w_2$  of length  $j$  is equal to the minimal cost of converting the initial segment of  $w_1$  of length  $i - 1$  into the initial segment of  $w_2$  of length  $j - 1$  in case the  $(i + 1)$ -st letter of  $w_1$  and the  $(j + 1)$ -st letter of  $w_2$  are the same, and to  $C_\varsigma$  plus the minimal cost of converting the initial segment of  $w_1$  of length  $i - 1$  into the initial segment of  $w_2$  of length  $j - 1$  otherwise.

For instance (with  $C_\delta = 1$ ,  $C_\iota = 1$  and  $C_\varsigma = 2$ ), if  $w_1$  is DEPART and  $w_2$  is LEOPARD then (ignoring the red colour) the table can be depicted as:

7	D	7 ↓	6 ↘	←	7 ↓	6 ↓	5 ↓	4 ↓	←	5 ↓				
6	R	6 ↓	←	7 ↓	6 ↓	5 ↓	4 ↓	3 ↘	←	4 ↓				
5	A	5 ↓	←	6 ↓	5 ↓	4 ↓	3 ↘	←	4	←	5			
4	P	4 ↓	←	5 ↓	4 ↓	3 ↘	←	4	←	5	←	6		
3	O	3 ↓	←	4 ↓	3 ↓	←	4 ↓	←	5 ↓	←	6 ↓	←	7 ↓	
2	E	2 ↓	←	3 ↓	2 ↘	←	3	←	4	←	5	←	6	
1	L	1 ↓	←	2 ↓	←	3 ↓	←	4 ↓	←	5 ↓	←	6 ↓	←	7 ↓
0	.	0 ↓	←	1 ↓	←	2 ↓	←	3 ↓	←	4 ↓	←	5 ↓	←	6 ↓
		.		D	E	P	A	R					T	
		0	1	2	3	4	5	6						

For another example (still with  $C_\delta = 1$ ,  $C_\iota = 1$  and  $C_\varsigma = 2$ ), if  $w_1$  is PAPER and  $w_2$  is POPE then (ignoring the red colour) the table can be depicted as:

4	E	4		3	←	4		3	↖	2	←	3
		↓		↓	↙	↓		↑	↖			
3	P	3		2	←	3		2	←	3	←	4
		↓	↙	↓	↙	↓	↖					
2	O	2		1	←	2	←	3	←	4	←	5
		↓		↓	↖	↑	↙	↑	↙	↑	↙	↑
1	P	1		0	←	1	←	2	←	3	←	4
		↓	↖				↙					
0	.	0	←	1	←	2	←	3	←	4	←	5
		.		P		A		P		E		R
		0	1	2	3	4	5					

The following observations derive from the previous considerations.

- For all  $i \leq |w_1|$ , the  $(i+1)$ -st column of the  $(|w_2|+1)$ -st row records  $i$  and a horizontal arrow.
- For all  $j \leq |w_2|$ , the  $(|w_2|-j+1)$ -st row of the first column records  $j$  and a vertical arrow.
- An element inside the table can be determined if the element to the left, the element below, and the element to the left and below have all been determined, hence it is possible to build the table by proceeding from the lower left corner up to the upper right corner.
- The Levenshtein distance between  $w_1$  and  $w_2$  is recorded in the table's top right corner.
- All possible ways of converting  $w_1$  into  $w_2$  at a minimal cost correspond to all paths from the lower left corner up to the upper right corner, which are best identified starting from the upper right corner, following the arrows left, down and diagonally; with the previous two table examples, there are 9 and 6 such paths, respectively, easily identified from the red arrows.

We will eventually define a class `Levenshtein_distance` but we first illustrate its functionality with tracing functions, using the DEPART -> LEOPARD example and with the suggested costs for deletion, insertion and substitution:

```
[1]: deletion_cost = 1
      insertion_cost = 1
      substitution_cost = 2

      word_1 = 'DEPART'
      word_2 = 'LEOPARD'
```

The table that has been described above is implemented as a list of lists, best viewed as a sequence of columns, read from left to right, each column being read from bottom to top, with DEPART and LEOPARD positioned as follows:

```
D
R
A
P
O
E
L
.
. D E P A R T
```

Each member of this list of lists is a pair whose first element is the minimal cost of converting the corresponding initial segment of DEPART with the corresponding initial segment of LEOPARD, and whose second element is a string containing a direction, that is:

- the character '/' if a diagonal move allows one to yield this minimal cost, thanks to a match or a substitution of the last characters of both initial segments;
- the character '-' if a horizontal move allows one to yield this minimal cost, thanks to a deletion of the last character of the initial segment of DEPART;
- the character '|' if a vertical move allows one to yield this minimal cost, thanks to an insertion of the last character of the initial segment of LEOPARD.

We also define a function to display the table, which to start with, stores costs of 0 and no move in all cells:

```
[2]: N_1 = len(word_1) + 1
N_2 = len(word_2) + 1
table = [[(0, []) for _ in range(N_2)] for _ in range(N_1)]

def represent(cell):
    return ''.join(x in cell[1] and x or ' ' for x in '/-|') + str(cell[0])

def display_table():
    for j in range(1, N_2):
        print(N_2 - j, word_2[N_2 - j - 1], end=' ')
        print(''.join(represent(table[i][j]) for i in range(N_1)))
    print('0 .', ' '.join(represent(table[i][0]) for i in range(N_1)))
    print('      .', ' '.join(word_1[i] for i in range(N_1 - 1)))
    print('      0', ' '.join(str(i) for i in range(1, N_1)))

display_table()
```

7 D	0	0	0	0	0	0	0
6 R	0	0	0	0	0	0	0
5 A	0	0	0	0	0	0	0
4 P	0	0	0	0	0	0	0
3 O	0	0	0	0	0	0	0
2 E	0	0	0	0	0	0	0
1 L	0	0	0	0	0	0	0
0 .	0	0	0	0	0	0	0
	.	D	E	P	A	R	T
	0	1	2	3	4	5	6

We have seen that it is straightforward to fill the first column and the last row of the table:

```
[3]: for i in range(1, N_1):
    table[i][0] = i, ['-']
    for j in range(1, N_2):
        table[0][j] = j, ['|']
```

```
display_table()
```

```

7 D |1 0 0 0 0 0 0
6 R |2 0 0 0 0 0 0
5 A |3 0 0 0 0 0 0
4 P |4 0 0 0 0 0 0
3 O |5 0 0 0 0 0 0
2 E |6 0 0 0 0 0 0
1 L |7 0 0 0 0 0 0
0 . 0 -1 -2 -3 -4 -5 -6
    . D E P A R T
    0 1 2 3 4 5 6

```

We then fill all cells of the table column by column, from left to right, from bottom to top for a given column:

```

[4]: d = {}
     for i in range(1, N_1):
         for j in range(1, N_2):
             d['-'] = table[i - 1][j][0] + deletion_cost
             d['|'] = table[i][j - 1][0] + insertion_cost
             d[' /'] = table[i - 1][j - 1][0] if word_1[i - 1] == word_2[j - 1] \
                 else table[i - 1][j - 1][0] + substitution_cost
             minimal_cost = min(d.values())
             table[i][j] = minimal_cost, [x for x in d if d[x] == minimal_cost]
     display_table()
     print()

```

```

7 D |1 /-|2 0 0 0 0 0
6 R |2 /-|3 0 0 0 0 0
5 A |3 /-|4 0 0 0 0 0
4 P |4 /-|5 0 0 0 0 0
3 O |5 /-|6 0 0 0 0 0
2 E |6 /-|7 0 0 0 0 0
1 L |7 / 6 0 0 0 0 0
0 . 0 -1 -2 -3 -4 -5 -6
    . D E P A R T
    0 1 2 3 4 5 6

```

```

7 D |1 /-|2 /-|3 0 0 0 0
6 R |2 /-|3 / 2 0 0 0 0
5 A |3 /-|4 |3 0 0 0 0
4 P |4 /-|5 |4 0 0 0 0
3 O |5 /-|6 |5 0 0 0 0
2 E |6 /-|7 |6 0 0 0 0
1 L |7 / 6 -|7 0 0 0 0
0 . 0 -1 -2 -3 -4 -5 -6
    . D E P A R T

```

	0	1	2	3	4	5	6
7 D	1 /- 2 /- 3 /- 4	0	0	0			
6 R	2 /- 3 / 2 - 3	0	0	0			
5 A	3 /- 4  3 /- 4	0	0	0			
4 P	4 /- 5  4 / 3	0	0	0			
3 O	5 /- 6  5  4	0	0	0			
2 E	6 /- 7  6  5	0	0	0			
1 L	7 / 6 - 7  6	0	0	0			
0 .	0 - 1 - 2 - 3 - 4 - 5 - 6						
	. D E P A R T						
	0 1 2 3 4 5 6						
7 D	1 /- 2 /- 3 /- 4 /- 5	0	0				
6 R	2 /- 3 / 2 - 3 - 4	0	0				
5 A	3 /- 4  3 /- 4 /- 5	0	0				
4 P	4 /- 5  4 / 3 - 4	0	0				
3 O	5 /- 6  5  4 / 3	0	0				
2 E	6 /- 7  6  5  4	0	0				
1 L	7 / 6 - 7  6  5	0	0				
0 .	0 - 1 - 2 - 3 - 4 - 5 - 6						
	. D E P A R T						
	0 1 2 3 4 5 6						
7 D	1 /- 2 /- 3 /- 4 /- 5 /- 6	0					
6 R	2 /- 3 / 2 - 3 - 4 - 5	0					
5 A	3 /- 4  3 /- 4 /- 5 /- 6	0					
4 P	4 /- 5  4 / 3 - 4 - 5	0					
3 O	5 /- 6  5  4 / 3 - 4	0					
2 E	6 /- 7  6  5  4 / 3	0					
1 L	7 / 6 - 7  6  5  4	0					
0 .	0 - 1 - 2 - 3 - 4 - 5 - 6						
	. D E P A R T						
	0 1 2 3 4 5 6						
7 D	1 /- 2 /- 3 /- 4 /- 5 /- 6 /- 7						
6 R	2 /- 3 / 2 - 3 - 4 - 5 - 6						
5 A	3 /- 4  3 /- 4 /- 5 /- 6 /- 7						
4 P	4 /- 5  4 / 3 - 4 - 5 - 6						
3 O	5 /- 6  5  4 / 3 - 4 - 5						
2 E	6 /- 7  6  5  4 / 3 - 4						
1 L	7 / 6 - 7  6  5  4 /- 5						
0 .	0 - 1 - 2 - 3 - 4 - 5 - 6						
	. D E P A R T						
	0 1 2 3 4 5 6						

The Levenshtein distance is to be found in the top right corner of the table:



```
[5]: def distance():
      return table[len(word_1)][len(word_2)][0]

distance()
```

[5]: 5

To compute all ways of converting DEPART into LEOPARD at a minimal cost, we define a generator function that operates on `backtraces`, the projection of `table` that for each cell, discards the first member (the cost) and keeps the second member (the directions). We trace execution, indicating whether a substitution (/), a match (\*), a deletion (-) or an insertion (|) is performed, indicating the row (j) and column (i) indexes of the cell where that match or transformation is done, and showing the result of the transformations done so far on both initial segments of DEPART and LEOPARD determined by that cell. It demonstrates how each of the nine paths that join the bottom left corner to the top right corner of the table are retrieved:

```
[6]: backtraces = [[table[i][j][1] for j in range(len(word_2) + 1)
                    ] for i in range(len(word_1) + 1)
                    ]

def compute_alignments(i, j):
    if i == j == 0:
        yield '', ''
    if '/' in backtraces[i][j]:
        for pair in compute_alignments(i - 1, j - 1):
            if word_1[i - 1] == word_2[j - 1]:
                print(f'{" " * i}* i = {i} j = {j} ',
                      pair[0] + word_1[i - 1], pair[1] + word_2[j - 1]
                      )
            else:
                print(f'{" " * i}/ i = {i} j = {j} ',
                      pair[0] + word_1[i - 1], pair[1] + word_2[j - 1]
                      )
            yield pair[0] + word_1[i - 1], pair[1] + word_2[j - 1]
    if '-' in backtraces[i][j]:
        for pair in compute_alignments(i - 1, j):
            print(f'{" " * i}- i = {i} j = {j} ', pair[0] + word_1[i - 1],
                  pair[1] + '_'
                  )
            yield pair[0] + word_1[i - 1], pair[1] + '_'
    if '|' in backtraces[i][j]:
        for pair in compute_alignments(i, j - 1):
            print(f'{" " * i}| i = {i} j = {j} ', pair[0] + '_',
                  pair[1] + word_2[j - 1]
                  )
            yield pair[0] + '_', pair[1] + word_2[j - 1]
```

```
list(compute_alignments(len(word_1), len(word_2)))
```

```

/ i = 1 j = 1 D L
* i = 2 j = 2 DE LE
| i = 2 j = 3 DE_ LEO
  * i = 3 j = 4 DE_P LEOP
    * i = 4 j = 5 DE_PA LEOPA
      * i = 5 j = 6 DE_PAR LEOPAR
        / i = 6 j = 7 DE_PART LEOPARD
| i = 0 j = 1 _ L
- i = 1 j = 1 _D L_
  * i = 2 j = 2 _DE L_E
    | i = 2 j = 3 _DE_ L_EO
      * i = 3 j = 4 _DE_P L_EOP
        * i = 4 j = 5 _DE_PA L_EOPA
          * i = 5 j = 6 _DE_PAR L_EOPAR
            / i = 6 j = 7 _DE_PART L_EOPARD
- i = 1 j = 0 D _
| i = 1 j = 1 D_ _L
  * i = 2 j = 2 D_E _LE
    | i = 2 j = 3 D_E_ _LEO
      * i = 3 j = 4 D_E_P _LEOP
        * i = 4 j = 5 D_E_PA _LEOPA
          * i = 5 j = 6 D_E_PAR _LEOPAR
            / i = 6 j = 7 D_E_PART _LEOPARD
/ i = 1 j = 1 D L
* i = 2 j = 2 DE LE
| i = 2 j = 3 DE_ LEO
  * i = 3 j = 4 DE_P LEOP
    * i = 4 j = 5 DE_PA LEOPA
      * i = 5 j = 6 DE_PAR LEOPAR
        | i = 5 j = 7 DE_PAR_ LEOPARD
          - i = 6 j = 7 DE_PAR_T LEOPARD_
| i = 0 j = 1 _ L
- i = 1 j = 1 _D L_
  * i = 2 j = 2 _DE L_E
    | i = 2 j = 3 _DE_ L_EO
      * i = 3 j = 4 _DE_P L_EOP
        * i = 4 j = 5 _DE_PA L_EOPA
          * i = 5 j = 6 _DE_PAR L_EOPAR
            | i = 5 j = 7 _DE_PAR_ L_EOPARD
              - i = 6 j = 7 _DE_PAR_T L_EOPARD_
- i = 1 j = 0 D _
| i = 1 j = 1 D_ _L
  * i = 2 j = 2 D_E _LE
    | i = 2 j = 3 D_E_ _LEO
      * i = 3 j = 4 D_E_P _LEOP

```

```

        * i = 4 j = 5 D_E_PA _LEOPA
        * i = 5 j = 6 D_E_PAR _LEOPAR
        | i = 5 j = 7 D_E_PAR_ _LEOPARD
        - i = 6 j = 7 D_E_PAR_T _LEOPARD_
/ i = 1 j = 1 D L
* i = 2 j = 2 DE LE
| i = 2 j = 3 DE_ LEO
* i = 3 j = 4 DE_P LEOP
* i = 4 j = 5 DE_PA LEOPA
* i = 5 j = 6 DE_PAR LEOPAR
- i = 6 j = 6 DE_PART LEOPAR_
| i = 6 j = 7 DE_PART_ LEOPAR_D
| i = 0 j = 1 _ L
- i = 1 j = 1 _D L_
* i = 2 j = 2 _DE L_E
| i = 2 j = 3 _DE_ L_EO
* i = 3 j = 4 _DE_P L_EOP
* i = 4 j = 5 _DE_PA L_EOPA
* i = 5 j = 6 _DE_PAR L_EOPAR
- i = 6 j = 6 _DE_PART L_EOPAR_
| i = 6 j = 7 _DE_PART_ L_EOPAR_D
- i = 1 j = 0 D _
| i = 1 j = 1 D_ _L
* i = 2 j = 2 D_E _LE
| i = 2 j = 3 D_E_ _LEO
* i = 3 j = 4 D_E_P _LEOP
* i = 4 j = 5 D_E_PA _LEOPA
* i = 5 j = 6 D_E_PAR _LEOPAR
- i = 6 j = 6 D_E_PART _LEOPAR_
| i = 6 j = 7 D_E_PART_ _LEOPAR_D

```

```

[6]: [('DE_PART', 'LEOPARD'),
      ('_DE_PART', 'L_EOPARD'),
      ('D_E_PART', '_LEOPARD'),
      ('DE_PAR_T', 'LEOPARD_'),
      ('_DE_PAR_T', 'L_EOPARD_'),
      ('D_E_PAR_T', '_LEOPARD_'),
      ('DE_PART_', 'LEOPAR_D'),
      ('_DE_PART_', 'L_EOPAR_D'),
      ('D_E_PART_', '_LEOPAR_D')]

```

Putting everything together in a class, with an extra display method, `display_all_aligned_pairs()`:

```

[7]: class Levenshtein_distance:
      def __init__(self, word_1, word_2, insertion_cost=1, deletion_cost=1,
                    substitution_cost=2
                    ):

```

```

self.word_1 = word_1
self.word_2 = word_2
self.insertion_cost = insertion_cost
self.deletion_cost = deletion_cost
self.substitution_cost = substitution_cost
self._table = self._get_distances_and_backtraces_table()
self._backtraces = [[self._table[i][j][1]
                      for j in range(len(self.word_2) + 1)
                      ] for i in range(len(self.word_1) + 1)
                    ]
self.aligned_pairs = self.get_aligned_pairs()

def _get_distances_and_backtraces_table(self):
    N_1 = len(self.word_1) + 1
    N_2 = len(self.word_2) + 1
    table = [[(0, []) for _ in range(N_2)] for _ in range(N_1)]
    for i in range(1, N_1):
        table[i][0] = i, ['-']
    for j in range(1, N_2):
        table[0][j] = j, ['|']
    d = {}
    for i in range(1, N_1):
        for j in range(1, N_2):
            d['-'] = table[i - 1][j][0] + self.deletion_cost
            d['|'] = table[i][j - 1][0] + self.insertion_cost
            d['/'] = table[i - 1][j - 1][0] \
                if self.word_1[i - 1] == self.word_2[j - 1] \
                else table[i - 1][j - 1][0] + self.substitution_cost
            minimal_cost = min(d.values())
            table[i][j] = minimal_cost, \
                [x for x in d if d[x] == minimal_cost]
    return table

def _compute_alignments(self, i, j):
    if i == j == 0:
        yield '', ''
    if '/' in self._backtraces[i][j]:
        for pair in self._compute_alignments(i - 1, j - 1):
            yield pair[0] + self.word_1[i - 1], \
                pair[1] + self.word_2[j - 1]
    if '-' in self._backtraces[i][j]:
        for pair in self._compute_alignments(i - 1, j):
            yield pair[0] + self.word_1[i - 1], pair[1] + '_'
    if '|' in self._backtraces[i][j]:
        for pair in self._compute_alignments(i, j - 1):
            yield pair[0] + '_', pair[1] + self.word_2[j - 1]

```

```

def distance(self):
    return self._table[len(self.word_1)][len(self.word_2)][0]

def get_aligned_pairs(self):
    return list(self._compute_alignments(len(self.word_1),
                                         len(self.word_2)
                                         )
              )

def display_all_aligned_pairs(self):
    print('\n\n'.join('\n'.join((pair[0], pair[1]))
                              for pair in self.aligned_pairs
                              )
          )

ld = Levenshtein_distance('DEPART', 'LEOPARD')
ld.distance()
ld.get_aligned_pairs()
ld.display_all_aligned_pairs()

```

[7]: 5

```

[7]: [('DE_PART', 'LEOPARD'),
      ('_DE_PART', 'L_EOPARD'),
      ('D_E_PART', '_LEOPARD'),
      ('DE_PAR_T', 'LEOPARD_'),
      ('_DE_PAR_T', 'L_EOPARD_'),
      ('D_E_PAR_T', '_LEOPARD_'),
      ('DE_PART_', 'LEOPAR_D'),
      ('_DE_PART_', 'L_EOPAR_D'),
      ('D_E_PART_', '_LEOPAR_D')]

```

```

DE_PART
LEOPARD

```

```

_DE_PART
L_EOPARD

```

```

D_E_PART
_LEOPARD

```

```

DE_PAR_T
LEOPARD_

```

```

_DE_PAR_T
L_EOPARD_

```

D\_E\_PAR\_T  
\_LEOPARD\_

DE\_PART\_  
LEOPAR\_D

\_DE\_PART\_  
L\_EOPAR\_D

D\_E\_PART\_  
\_LEOPAR\_D

7	D	7	6	← 7	6	5	4	← 5
		↓	↘	↓	↓	↓	↓	↘ ↓
6	R	6	← 7	6	5	4	3	← 4
		↓	↘ ↓	↓	↓	↓	↘	
5	A	5	← 6	5	4	3	← 4	← 5
		↓	↘ ↓	↓	↓	↘		
4	P	4	← 5	4	3	← 4	← 5	← 6
		↓	↘ ↓	↓	↘			
3	O	3	← 4	3	← 4	← 5	← 6	← 7
		↓	↘ ↓	↓	↘ ↓	↘ ↓	↘ ↓	↘ ↓
2	E	2	← 3	2	← 3	← 4	← 5	← 6
		↓	↘ ↓	↘				
1	L	1	← 2	← 3	← 4	← 5	← 6	← 7
		↓	↘ ↓	↘ ↓	↘ ↓	↘ ↓	↘ ↓	↘ ↓
0	.	0	← 1	← 2	← 3	← 4	← 5	← 6
		.	D	E	P	A	R	T
		0	1	2	3	4	5	6

4	<b>E</b>	4		3	←	4		3		<b>2</b>	←	<b>3</b>
		↓		↓	↙	↓		↑	↙			
3	<b>P</b>	3		2	←	3		<b>2</b>	←	3	←	4
		↓	↙	↓	↙	↓	↙					
2	<b>O</b>	2		<b>1</b>	←	<b>2</b>	←	3	←	4	←	5
		↓		↓	↙	↑	↙	↑	↙	↑	↙	↑
1	<b>P</b>	1		<b>0</b>	←	<b>1</b>	←	2	←	3	←	4
		↓	↙				↙					
0	<b>.</b>	<b>0</b>	←	1	←	2	←	3	←	4	←	5
		<b>.</b>		<b>P</b>		<b>A</b>		<b>P</b>		<b>E</b>		<b>R</b>
		0		1		2		3		4		5



2nd	w1	P	A	-	P	E	R	
	w2	P	-	O	P	E	-	cost
			d	i			d	3
3rd	w1	P	-	A	P	E	R	
	w2	P	O	-	P	E	-	cost
			i	d			d	3

order /-|

-	del	1	horiz
	ins	1	vert
/	sub or match	2 or 0	diag