Explaining the beta anomaly

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Table of Contents

LIST OF ABBREVIATIONS **II**

1 INTRODUCTION **1**

2 THE BETA ANOMALY **2**

2.1 CLASSICAL CAPITAL MARKET MODELS 3

2.1.1 Capital Asset Pricing Model *3*

2.1.2 Fama-French Three-Factor Model *4*

2.2 THE ANOMALY 5

3 EMPIRICAL ANALYSIS **8**

3.1 DATA AND METHODOLOGY 8

3.2 RESULTS 9

3.3 DISCUSSION 10

4 EXPLAINING THE BETA ANOMALY **12**

4.1 LEVERAGE CONSTRAINTS 12

4.2 BIASED BELIEFS 15

4.3 LOTTERY PREFERENCES 18

4.4 BENCHMARKING 21

4.5 OPERATING PERFORMANCE 24

4.6 INTEREST RATE 25

4.7 IDIOSYNCRATIC VOLATILITY 27

4.8 MISSPECIFICATIONS OF THE CAPM AND OTHER ISSUES 28

4.9 ARBITRAGE ACTIVITY 31

4.10 SYNOPSIS 32

5 CONCLUSION **32**

APPENDIX **34**

A FIGURES 34

Figure 1 *34*

B TABLES 35

Table 1 *35*

Table 2 *35*

Table 3 *36*

C LIST OF ALL STOCKS IN THE SAMPLE 37

D BIBLIOGRAPHY 40

EIDESSTATTLICHE ERKLÄRUNG **45**

List of Abbreviations

BAB	–	betting-against-beta
BE	–	book value of common equity
CAPM	–	Capital Asset Pricing Model
HML	–	high minus low
IPO	–	initial public offering
IVOL	–	idiosyncratic volatility
ME	–	market capitalization
MSCI	–	Morgan Stanley Capital International
SMB	–	small minus big
SML	–	Security Market Line

1 Introduction

At the heart of neoclassical asset pricing theory is a trade-off between risk and return. This trade-off is modelled in the Capital Asset Pricing Model (CAPM), which is attributed to Sharpe (1964), Lintner (1965) and Mossin (1966). The CAPM formalizes a positive relationship between expected returns and undiversifiable hence systematic risk as measured by beta. Due to its intuitive appeal, the CAPM has been of fundamental importance in finance ever since its creation and is taught in all basic finance lectures. However, contrary to this theoretical risk and return trade-off, there is compelling evidence that high beta stocks do not exhibit higher expected returns than low beta stocks. This phenomenon is known as the beta anomaly. Other names found in literature for the same phenomenon include low beta anomaly and betting-against-beta anomaly.

As it challenges the most fundamental principles of the capital market theory, Baker, Bradley and Wurgler (2011) call the beta anomaly possibly "the greatest anomaly in finance"[1]. In the framework of the CAPM, the anomaly is a mispricing that is highly persistent. Earliest evidence on the failure of the CAPM that is the beta anomaly comes from Friend and Blume (1970), Black (1972), Black, Jensen and Scholes (1972), Fama and MacBeth (1973) and Haugen and Heins (1975). Since then, numerous papers have examined the anomaly, trying to explain the empirically flat or even inversed relationship between beta and return. Frazzini and Pedersen (2014) analyze the influence of funding constraints and Jylhä (2017) investigates the effect of margin requirements. Antoniou, Doukas and Subrahmanyam (2015), Hong and Sraer (2016) and Jacobs (2017) relate the anomaly to various behavioral phenomena while Bali et al. (2017) focuses on lottery demand. Blitz (2014) and Baker, Bradley and Wurgler (2011) see the origin of the anomaly in issues arising from portfolio delegation. Fama and French (2016) observe the effect of firm-level profitability on expected returns whereas Muijsson, Fishwick and Satchell (2016) argue that interest rate movements cause the mispricing. Huang, Lou and Polk (2016) explore the influence of arbitrage activity. Liu, Stambaugh and Yuan (2016) find a significant relationship between idiosyncratic volatility and the beta anomaly and Schneider, Wagner and Zechner (2016) consider downside risk as the driver.

The purpose of this paper is twofold. First, the anomaly will be presented against the background of neoclassical models, i.e. the CAPM and the Fama-French three-factor model. The evidence on the existence of the beta anomaly from various studies as well as from this paper's data sample will be summarized. Second, the main objective of this

[1] Baker, Bradley and Wurgler (2011). p. 40.

paper is to give on overview of and discuss the numerous existing explanations of the anomaly. The explanatory power of the most commonly assumed drivers and the compatibility of different drivers will be investigated. Hence, this paper is a conglomeration of the different explanations of the anomaly but also aims at providing a complete picture of the origin of the anomaly. This is of great importance as insights will shed more light on the factors that can predict returns.

Related to the beta anomaly are the volatility anomaly and the idiosyncratic volatility anomaly, which, correspondingly to the beta anomaly, describe the flat or negative relationship between returns and the respective risk measure. In some studies, volatility anomaly is even used as an umbrella term for all three anomalies. Note that this paper deals with the beta anomaly while the volatility anomaly and the idiosyncratic volatility anomaly will not be explicitly investigated. Nevertheless, the volatility anomaly still is of relevance. Due to the high correlation between beta and total volatility[2], both anomalies are closely related and have similar possible drivers. Therefore, not all studies that are referred to in this paper explicitly examine the beta anomaly but rather try to explain the volatility anomaly. The idiosyncratic volatility anomaly too will reappear later when its relationship to the beta anomaly will be observed in detail.
Furthermore, this paper will exclusively examine the beta anomaly in stock markets although the beta and volatility anomalies translate to various other asset classes[3].

This paper will proceed as follows. Section 2 defines the anomaly and describes the evidence from previous studies on the anomaly. Section 3 presents the empirical analysis. Section 4 discusses existing explanations. Section 5 concludes.

2 The Beta Anomaly

This section will describe the beta anomaly and its empirical appearance as studied by other papers. The beta anomaly embodies the empirical failure of the CAPM. In the framework of the CAPM, the anomaly is a highly persistent mispricing. Because the anomaly is a systematic deviation from this classical capital market model, the CAPM will first be described. Then, the Fama-French three-factor model will be introduced as an extension of the CAPM since this model too is widely used. While, like the CAPM, it does not explain the beta anomaly, the three-factor model is specifically of relevance in this paper because it is commonly used to measure the strength of the anomaly. Moreover, extensions of the three-factor model with additional factors are used by various studies to

[2] Ibid. p. 47.
[3] E.g. Falkenstein (2010). pp. 98-134.

provide evidence for the validity of their explanation of the beta anomaly; therefore, factor model regressions will reappear throughout section 4. Hence, this section also aims at creating a basic understanding of factor models. Finally, the beta anomaly will be defined against the background of classical capital market models and the empirical evidence of the existence of the beta anomaly from various papers will be summarized.

2.1 Classical Capital Market Models

2.1.1 Capital Asset Pricing Model

A fundamental principle of classical capital market theory is the trade-off between risk and return when choosing an efficient portfolio. This principle dates back at least to Markowitz (1952) and is very intuitive given the general risk aversion of people. A risk averse investor will only accept a higher risk if the expected payoff increases accordingly, thus a positive relationship between risk and return is to be expected. Based on Markowitz' work, Sharpe (1964), Lintner (1965) and Mossin (1966) each independently formalized this risk-return relationship resulting in the Capital Asset Pricing Model (CAPM). However, not all risk is rewarded with a higher expected return in this model. In the CAPM, only such risk that is undiversifiable is priced. This undiversifiable risk derives from the correlation of a portfolio's returns with the returns on the market and is therefore measured via the portfolio's market sensitivity, called beta (β). The relationship between beta and the expected return is captured in the CAPM as:

$$E[r_i] = r_f + \beta_i \cdot \left(E[r_m] - r_f\right)$$

Here, $E[r_i]$ is the expected return of portfolio i, r_f is the risk-free rate and $E[r_m] - r_f$ is the expected excess return of the market, also known as the market risk premium. β_i is the aforementioned measurement of the relevant systematic risk, i.e. the sensitivity to market risk, defined as $\beta_i = \frac{cov\,(r_m, r_i)}{var(r_m)} = \rho_{im} \cdot \frac{\sigma_i}{\sigma_m}$. A security's beta is computed as the slope of the regression of its excess return on the excess market return. Thus, the market has a beta of 1.0 while betas of individual stocks can be significantly higher or lower. Graphically, the linear beta-expected return relationship is depicted as the Security Market Line (SML), which shows the expected return of any security or portfolio as a function of its systematic risk (Figure 1). The SML-slope is the market risk premium and the intercept is the risk-free rate.

The universal insights of the CAPM are based on a set of highly restrictive basic assumptions:

1. All investors have access to all information and are assumed to have homogeneous

expectations concerning the expected returns and their covariance matrix[4].

2. There is a risk-free rate at which all investors can lend or borrow unlimited amounts[5].

3. All investors are mean-variance optimizers following the portfolio selection model from Markowitz (1952)[6].

4. The market is frictionless, implying no transaction costs, no taxation of returns and no short-selling constraints[7].

Since its development, the CAPM has been at the core of finance teachings thanks to its simplicity and natural appeal. Its validity however is questioned by the beta anomaly as will be shown in section 2.3.

2.1.2 Fama-French Three-Factor Model

As an extension of the CAPM, Fama and French (1993) developed a factor model to explain the cross-section of returns more reliably. They enhance the CAPM by including premiums for small size and for a high book-to-market ratio, which are risk factors. Thus, their model regresses the excess stock returns on the excess return of the market, a size factor called SMB (small minus big) and a book-to-market factor called HML (high minus low). This translates into the following time-series regression model:

$$r_i - r_f = \alpha_i + b_i \cdot (r_m - r_f) + s_i \cdot SMB + h_i \cdot HML + e_i$$

e are the residuals, the intercept α captures abnormal returns not explained by the other factors and b, s and h are the coefficients of three factors. Note that while b_i from the Fama-French three-factor model resembles the CAPM-beta, they are not empirically identical. Due to correlation between the market and the factors SMB (0.32) or HML (-0.38), adding these factors to the regression compresses the betas towards 1.0[8]. To construct the factors SMB and HML, all stocks in the sample are sorted independently on size and book-to-market ratios. The relevant measure for size is a firm's market capitalization (ME) calculated as the number of shares times the stock price. Two size groups are generated (small and big) with the sample median as the breakpoint. The book-to-market ratios are computed as the book value of common equity divided by the market value (BE/ME). Stocks are sorted into three BE/ME groups (low, medium and high). This procedure results in six portfolios for which the value-weighted returns are calculated.

[4] Sharpe (1964). pp. 433-434.
[5] Ibid. p. 433.
[6] Sørensen (2015). p. 8.
[7] Ibid. p. 8.
[8] Fama and French (1993). p. 26.

Then, SMB and HML are defined as the differences between the return averages of the relevant portfolios[9]:

$$SMB = \frac{1}{3}[(small, low) + (small, medium) + (small, high)] - \frac{1}{3}[(big, low) + (big, medium) + (big, high)]$$

$$HML = \frac{1}{2}[(small, high) + (big, high)] - \frac{1}{2}[(small, low) + (big, low)]$$

The three-factor model too fails to explain the anomaly. However, factor models are of special interest because it is common practice to use the three-factor model or its extensions to analyze and explain observed returns and anomalies via regressions. In doing so, the regression intercept alpha illustrates the explanatory power of the factors used in the model as it captures the abnormal returns that are not explained by these factors. Hence, alphas indistinguishable from zero indicate that the factors used in the model sufficiently explain the observed returns.

2.2 The Anomaly

While the CAPM has theoretical appeal, it fails systematically to explain observed returns. This failure of the CAPM is the beta anomaly. The Fama-French model has some empirical success but still fails to explain the beta anomaly. The beta anomaly is the finding that, relative to the CAPM, low beta stocks exhibit positive abnormal returns and likewise, high beta stocks exhibit negative abnormal returns[10]. This means that returns on low (high) beta stocks are higher (lower) than classical capital market models suggest. The anomaly is very persistent and well reported. There is vast pervasive evidence for its existence in different markets across the globe with some of the earliest to detect the anomaly being Friend and Blume (1970), Black (1972), Black, Jensen and Scholes (1972), Fama and MacBeth (1973) and Haugen and Heins (1975).

Several characteristics of the beta-return relationship that translate into each other define the beta anomaly. These findings will first be described, then the empirical evidence will be explored.

First, the observed beta-return relationship is significantly flatter than postulated by the CAPM or in some cases even inversed. The difference in returns between high beta stocks

[9] Fama and French (1993). pp. 8-9.
[10] Bali et al. (2017). p. 1.

and low beta stocks is mostly statistically insignificant. This translates into a SML that is flat and has an intercept greater than r_f. Thus, the beta anomaly exists whenever the SML is flat. Second, risk-adjusted returns are significantly higher for low beta stocks than for high beta stocks regardless of which risk-adjustment measure is used. Third, the abnormal returns as captured by the intercept (α) of the regressions of excess stock returns on the excess market return and possibly other factors like SMB or HML are strictly positive for low beta stocks while high beta stocks exhibit negative alphas. Lastly, these findings transfer to specifically constructed portfolios: a high-minus-low-beta portfolio and a betting-against-beta (BAB) portfolio. The high-minus-low-beta portfolio goes long all stocks in the highest beta decile and short those in the lowest. The BAB portfolio has a short position in all stocks with a beta above the asset class median deleveraged to a beta of 1.0 and a long position in all stocks below the median leveraged to a beta of 1.0. Thus, the resulting portfolio is a zero-cost portfolio with a beta of 0. Obviously, the two portfolios capture opposite effects, the high-minus-low portfolio shows returns indistinguishable from zero and negative alphas whereas the BAB portfolio's returns, risk-adjusted returns and alphas are all positive.

As mentioned before, the empirical evidence on the anomaly is vast. Due to redundancy however, only results from selected studies will be presented here.
Early on, Friend and Blume (1970) provide evidence of the failure of the CAPM. They regress the performance measures from Sharpe, Treynor and Jensen each on the market-betas of random portfolios and find a highly significant negative relationship between each measure for risk-adjusted returns and the betas instead of the theoretically implicated flat relationship[11].

Black, Jensen and Scholes (1972) add on to these results by finding striking abnormal returns. By sorting the stocks from their sample into beta decile portfolios and then regressing the excess returns on the market excess returns, they find negative alphas for all five portfolios with a beta greater than 1.0 and positive alphas for all other portfolios[12]. Alphas decrease from 0.2012 for the lowest beta decile to -0.0829 for the highest decile, albeit only three of the ten alphas are statistically significant. Moreover, they compute the SML to be flatter than the CAPM indicates over the sample period from 1931 to 1965 (a regression slope of 0.0108 is found versus an average monthly market excess return of 0.0142) and they even find a negative slope for the last 8 years of that period[13]. These

[11] Friend and Blume (1970). pp. 565-566.
[12] Black, Jensen and Scholes (1972). p. 14.
[13] Ibid. p. 23.

results are partly confirmed by Fama and MacBeth (1973) who find an average slope over a similar timeframe that is smaller than the market excess return though this is not the case for all sub periods[14]. Correspondingly, they report an intercept greater than the risk-free rate of return. Decades later, Fama and French (1992) are unable to find a significant relationship between beta and returns; the observed returns are flat or even slightly decreasing as betas increase[15] leading to the conclusion that taking systematic risk seems to not be compensated at all. Interestingly, despite having detected the beta anomaly themselves, the Fama-French three-factor model fails to remedy the shortcomings of the CAPM with regards to the anomaly but instead is used in later studies to examine abnormal returns via the Fama-French alpha.

Recent studies support the earlier findings and show that the anomaly is still existent today. To illustrate the magnitude, \$1 invested into a portfolio including all stocks in the lowest beta quintile in 1968 grew to \$60.46 in 2008 whereas \$1 in the highest beta quintile portfolio grew to only \$3.77[16]. Thus, the highest beta portfolio underperformed the lowest beta portfolio by 964 percent and in real terms, the high beta portfolio could not even fully recover the \$1 invested. Frazzini and Pedersen (2014) provide further support. For US equities from 1926 to 2012, they observe declining alphas from the lowest to the highest beta decile. The CAPM alpha as well as three-, four- and five-factor alphas are all significantly positive for low beta deciles and negative alphas for the highest decile although only the three-factor alpha is statistically distinguishable from zero in the highest decile[17]. Meanwhile, the relationship between betas and returns is flat, the average monthly excess returns are 0.91% for the lowest decile and 0.97% for the highest[18]. Nevertheless, there are diverse findings on the SML slope. Instead of this flat but still positive slope, Bali et al. (2017) find some evidence for a negative slope. Monthly excess returns in their sample decrease from 0.69% to 0.35% albeit not monotonically and the resulting return of the high-minus-low-beta portfolio is -0.35%[19]. However, this return is statistically indifferent from zero (t-statistic -1.13), indicating a flat relationship. Hong and Sraer (2016) provide evidence for a time-varying relationship between betas and returns; in some periods, it is shown to be positive and in other periods an inverted U-shape is reported[20].

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[14] Fama and MacBeth (1973). p. 1973.
[15] Fama and French (1992). pp. 433, 440.
[16] Baker, Bradley and Wurgler (2011). p. 42.
[17] Frazzini and Pedersen (2014). p. 13.
[18] Ibid. p. 13.
[19] Bali et al. (2017). p. 8.
[20] Hong and Sraer (2016). p. 21.

While the beta-return relationship is flat, exchanging the returns with risk-adjusted returns results in a clearly negative relation. Frazzini and Pedersen (2014) compute annualized Sharpe ratios for the beta deciles, which decrease monotonically from 0.70 to 0.28 from the lowest to the highest decile[21]. Additionally, they confirm the decreasing alphas and Sharpe ratios for all 19 other MSCI developed countries[22], showing that the beta anomaly is a global phenomenon. Lastly, for the BAB portfolios, they find significantly positive abnormal returns in 13 of the 20 MSCI countries and positive Sharpe ratios in 19 of the 20 markets[23].

3 Empirical Analysis

3.1 Data and Methodology

The data in this paper is collected from two main sources. For the stocks and the market index, the monthly Total Return Index is retrieved from the Thomson Reuters data stream. Monthly returns are calculated as $r_{i,t} = \frac{Total\ Return\ Index_t - Total\ Return\ Index_{t-1}}{Total\ Return\ Index_{t-1}}$. The sample covers all months from December 1999 to November 2017. It includes all German stocks that have been a constituent of the MDAX at any point of time in the sample period, which leads to a total 148 stocks (see appendix). The proxy for the market portfolio is the MSCI Germany Index. All excess returns are above the 10-year German Government Bond yields. The annualized yields are obtained from the website of the Federal Reserve Bank of St. Louis[24]. Monthly risk-free rates are computed from the annualized yields as $r_f = (1 + annualized\ rate)^{\frac{1}{12}} - 1$. Betas are estimated from rolling regressions. At the stock-level, the monthly excess returns from the past twelve months are regressed on the contemporaneous excess market returns. At the beginning of each year, all actively traded stocks from the sample are sorted into ten beta-sorted portfolios based on the estimated pre-ranking betas. Thus, each portfolio comprises one beta decile, β_1 being the lowest beta decile portfolio and β_{10} the highest. On average, each portfolio contains ten stocks.

The decile portfolios are equally weighted. Post-ranking betas are estimated by regressing the excess portfolio returns from the entire sample period on the contemporaneous excess market returns. For portfolio volatilities, a one-year rolling standard deviation is used on

[21] Frazzini and Pedersen (2014). p. 13.
[22] Ibid. p. 10.
[23] Ibid. p. 10.
[24] Available from https://fred.stlouisfed.org/series/IRLTLT01DEM156N

the portfolio-level. Idiosyncratic volatilities are computed as the standard deviations of the residuals from regressions of the portfolio excess returns on the excess market return.

3.2 Results

Summary statistics for the decile portfolios are presented in Table 1. As intended by the construction[25], the average value of pre-ranking betas increase monotonically from the lowest to the highest beta portfolio. The betas for these two deciles are 0.06 and 1.58. Pre-ranking betas are a decent estimator for the post-ranking betas which is indicated by a high correlation between pre- and post-ranking betas (0.96). However, post-ranking betas are compressed toward 1.0. They increase from 0.60 to 1.15, albeit not monotonically.

The raw returns on the portfolios are a first indicator that the CAPM does not hold empirically in this sample. The relationship between betas and returns appears to be completely flat. β_{10} has the lowest average monthly returns at 0.40% and β_5 the highest at 1.53%, closely followed by β_1 (1.40%). By regressing the excess returns of the portfolios on the respective post-ranking betas, the SML slope and intercept can be observed (Table 2). Both are significantly different from the theoretically implied values. Following the CAPM, the intercept should be zero[26] and the slope equal to the market risk premium. On the contrary, the regression intercept is 1.18% which is statistically significant at the 10% level with a t-statistic of 2.28. Even stronger evidence is given by the beta coefficient. Its value is -0.43% although it is statistically indistinguishable from zero. Therefore, systematic risk holds no premium in the sample. This result is complemented by the R-squared value of this regression which shows that beta has no explanatory power regarding the expected returns. R-squared is as low as 0.06 and the adjusted R-squared is even slightly negative.

Further evidence for the beta anomaly in this sample is drawn from the risk-adjusted returns, which are also shown in Table 1. The Treynor-ratio, the Sharpe-ratio and Jensen's alpha are used as risk-adjusted performance measures. The Treynor-ratio intuitively lends the greatest support for the anomaly since betas increase across the portfolios by construction, which leads to decreasing beta-adjusted returns. These are declining from 1.44 for β_1 to -0.01 for β_{10} and the five lower beta deciles, except β_4, all exhibit higher beta-adjusted returns than the other five deciles. The results for the other two measures are less compelling. Jensen's alpha too is highest for the lowest decile and lowest for the highest

[25] Despite an issue that will be covered in section 3.3.
[26] The intercept should be zero and not equal to the risk-free rate because excess returns are used as the dependent variable.

decile. Across the other eight portfolios however, no clear pattern emerges. Similarly, the relationship between the Sharpe-ratio and beta appears to be flat. Values average at 0.1709 per month. β_3, β_6 and β_{10} have significantly lower than average Sharpe-ratios (all around 0.10) whereas only β_5 has significantly higher risk-adjusted returns, averaging at 0.2811. This indifference of Sharpe-ratios across the deciles is induced by a flat relationship between beta and the total volatilities in the sample. Across the ten portfolios the average standard deviations of returns show only little variation.

Lastly, the abnormal returns or alphas of the ten portfolios, measured as the intercepts of the regressions of the excess portfolio returns on the excess market returns, can be examined (Table 3). Consistent with the empirical findings by other papers presented in section 2, the alphas for the five lowest beta deciles are all positive with an average of 0.71% and four of them are statistically significant. β_5 delivers the highest alpha at 1.06% with a t-statistic of 3.88 indicating significance at the 1% level. β_1 closely follows with the second highest alpha of 0.98% with a t-statistic of 2.11. Among the five highest beta deciles, average alphas are smaller at 0.43% and the alphas of β_6, β_9 and β_{10} are statistically indistinguishable from zero. β_{10} also has the only reported negative alpha with -0.20%.

The relationship between return and beta, the alphas from regressions and the Treynor-ratio all hint at the existence of the beta anomaly in this sample. Not all findings, especially not the risk-adjusted performance measures, are consistent with the findings of related papers, but most of them indicate a flat relationship between beta and expected returns. While the beta anomaly is thus a bit fuzzy in this sample compared to empirical studies presented in section 2, clear deviations from the CAPM are still found. Therefore, the anomaly exists in this sample but its appearance is not as distinct due to issues that will be discussed in the next subsection.

3.3 Discussion

Several issues arise in this paper's data sample that could lead to a distortion of the results.

The sample size is small with only 148 stocks compared to other papers that typically include at least all U.S. stocks. Frazzini and Pedersen (2014), for example, include the data for over 50,000 stocks in their analysis[27]. Thus, it is possible that the results from the sample are not representative for the entire market. An identical effect can be induced by the small sample period. In most papers the covered period is longer[28]. However, this

[27] Frazzini and Pedersen (2014). p. 6.
[28] E.g. Frazzini and Pedersen (2014), Bali et al. (2017) and Hong and Sraer (2016).

should not affect the results too much since the sample period is still long enough to capture different market states and previous papers have also found valid results for similar lengths[29]. The small period also limits the length of the rolling windows to estimate the betas. Moreover, the choice of only MDAX constituents, which are mid-cap stocks, somewhat limits the extent of the anomaly in the sample. The beta anomaly is found to be strongest among small-cap stocks[30], none of which are included in the sample.

Additionally, an issue is caused by the infrequent rebalancing. Due to lack of computational power, the stocks are reassigned to the ten decile portfolios only at the beginning of each year as opposed to each month. Thereby, the variance of the betas in each portfolio increases greatly and while the pre-ranking betas increase monotonically across the deciles by construction at the beginning of each year, there is not a single year in which the pre-ranking betas increase monotonically at the end of the year. This compresses the average spread between the different portfolio betas and reduces the validity of the results. The difference in post-ranking betas between the lowest and highest decile is only 0.55, which is less than half of the spreads reported in Frazzini and Pedersen (2014) (1.18[31]) and Bali et al. (2017) (1.35[32]). Nonetheless, the total portfolio post-ranking betas still increase almost monotonically across the deciles so that the relationship between returns and betas can be observed. The indifference of total volatilities across the beta deciles is also possible caused by this infrequent sorting of the stocks into the portfolios.

Lastly, the estimation-technique for the betas can be questioned. Typically, when estimating betas via the regressions of the excess returns on the excess market returns, the contemporaneous market returns as well as five lags of the market returns are used and betas are computed as the sum of all coefficients[33]. However, there are other papers which also omit the lagged excess market returns[34] and the betas from this paper are, to some degree, robust to the alternative estimation of betas as $\beta_i = \rho_{i,m} \frac{\sigma_i}{\sigma_m}$ where $\rho_{i,m}$ is the correlation coefficient and σ_i and σ_m are the estimated volatilities of the asset and the market. Further precision to the beta estimates could be induced by using daily return data instead of monthly returns.

[29] E.g. Friend and Blume (1970).
[30] Lin and Jiang (2015). p. 20.
[31] Frazzini and Pedersen (2014). p. 13.
[32] Bali et al. (2017). p. 41.
[33] E.g. Hong and Sraer (2016). p. 19.
[34] E.g. Blitz (2014). p. 779 and Lin and Jiang (2015). p. 10.

4 Explaining the Beta Anomaly

Over the last decades, various theories have been developed regarding the reasons for the empirical failure of the CAPM that is the beta anomaly. The following subsections will each introduce possible drivers of the anomaly identified in other papers, present empirical evidence for the respective explanation and shortly discuss its validity. Questions that this section will try to answer are among others why there is such a high demand for high beta stocks that leads to their overpricing and why investors still prefer high beta stocks to leveraged investment in lower beta stocks despite their inferior risk-return trade-off. While some of the factors possess explanatory power on their own, the most compelling explanations for the anomaly are based on a combination of factors. Therefore, after discussing the drivers separately, they will be reconciled to give a complete picture of the anomaly. Moreover, not all studies on the beta anomaly are able to give a satisfying theoretical explanation for the mechanism through which the discussed factors cause the anomaly but only find the empirical proof for the effect of these factors. Also, note that limits to arbitrage in the sense of short-selling constraints play an important complementary part in some explanations but will not be explored individually.

The most common methods used in studies to prove the explanatory power of factors are bivariate portfolio analyses and factor model regressions. For the bivariate analyses, stocks are sorted into portfolios independently by beta and the possible driver of the anomaly. Explanatory power is then given if the beta anomaly disappears in the sense of increasing returns from lower to higher betas while the other factor remains constant. When using factor models, existing models such as the Fama-French three-factor model are augmented with an additional factor that captures a possible driver of the anomaly. Here, success is indicated by abnormal returns equal to zero meaning that the suggested model explains the flat SML. These two methods will reappear throughout this section.

This section will explore the effect of funding constraints, behavioral phenomena and lottery preferences, benchmarking, firm-level profitability, interest rate movements, idiosyncratic volatility and non-normally distributed returns on the beta anomaly.

4.1 Leverage Constraints

Leverage constraints can take different forms; any constraints on the funding limiting the multiple of a person's wealth that he can invest are meant by leverage constraints. Margin requirements are found to have a similar effect as leverage constraints[35]. To illustrate the

[35] Frazzini and Pedersen (2014). p. 4.

effect of constraints on investments, the constraints of an investor is described by m, then 1/m is the multiple of the person's wealth that can be invested. m = 0 expresses no funding constraint, m = 0.5 means the investor can invest twice his wealth into stocks and m = 1 means the investor cannot use any leverage[36].

Some of the earliest studies of the beta anomaly include leverage constraints as a possible explanation. Black (1972) and Friend and Blume (1970) mention the impact of the lack of a risk-free rate and unequal borrowing and lending rates on the SML. The non-existence of a risk-free rate theoretically flattens the Security Market Line by increasing the intercept[37] whereas a borrowing rate greater than the lending rate creates a bias only against risky portfolios[38]. The effect of leverage constraints is very intuitive when looking at the Capital Market Line. In theory, all rational investors hold the market portfolio and use leverage to adapt their portfolio to their personal risk aversion. If the use of leverage is prohibited, investors with low risk aversion who want to hold portfolios riskier than the market portfolio need to do so without leverage. The only possibility for them to increase the portfolio risk is therefore by selecting riskier assets. This creates a tilt towards high beta assets; the demand for high beta assets increases[39]. Consecutively, the price for these assets increases reducing their future returns, which causes the beta anomaly. Another effect of leverage constraints is that they prevent that the superior risk-return trade-off of low beta assets is arbitraged away[40].

Frazzini and Pedersen (2014) express the dependence of expected returns on the tightness of funding constraints in the following model:

$$E(r_i) = r_f + \psi + \beta_i \lambda$$ [41]

ψ measures the tightness of funding constraints and the risk premium λ is defined as $\lambda = E(r_m) - r_f - \psi$, the resulting alphas and Sharpe rations both decrease in beta. Thus, this model is consistent with the anomaly. If it holds empirically, this proves the explanatory power of leverage effects. However, the predictions derived from the model face issues empirically. When leverage constraints tighten, the contemporaneous returns of a BAB portfolio are expected to be negative as more people are driven into risky assets while future returns should increase[42]. To test this, the Frazzini and Pedersen use the TED

[36] Ibid. p. 4.
[37] Black (1972). pp. 446-452.
[38] Friend and Blume (1970). p. 569.
[39] Frazzini and Pedersen (2014). pp. 1-2 and Jylhä (2017). p. 6.
[40] Blitz and van Vliet (2007). p. 10.
[41] Frazzini and Pedersen (2014). p. 4.
[42] Ibid. p. 2.

spread, which is the spread between the Eurodollar and the Treasury bill rates, as a proxy for the funding tightness. In contradiction to their expectations, the TED spread negatively predicts both the contemporaneous and the future BAB return[43]. This casts some doubt on the explanatory power of leverage constraints but alternatively, the TED spread could just be an improper proxy for measuring the tightness of funding constraints[44].

Jylhä (2017) remedies this problem by relying on the active management of the minimum initial margin requirement by the Federal Reserve as a measure for funding constraints. The effect of the change in margin requirements on the SML is empirically noticeable, regressions of the slope and intercept on this measure and other control variables show a positive effect of margin requirements on the SML-intercept (0.024) and a negative effect on the SML-slope (-0.053)[45]. Both are statistically significant with t-statistics of 2.1 and -3.2. Thus, stricter margin requirements flatten the SML. The importance of margin requirements as a driver of the anomaly is underlined by the independence of the changes of the margin requirement from other macroeconomic factors[46] while the effect is also reported to not be driven by short-selling constraints[47].

Further support for the role of funding constraints is provided by Adrian, Etula and Muir (2014), who construct a leverage factor that resembles broker-dealer leverage. This leverage factor is correlated with other funding constraint proxies such as asset growth and the Baa-Aaa spread[48], and thus seems to successfully measure the tightness of leverage constraints. The leverage factor outperforms the Fama-French risk factors in trying to explain the beta anomaly which is expressed by a significantly higher adjusted R^2[49].

Convenient evidence can also be drawn from portfolio betas of unconstrained investors compared to those of those investors who are constrained in their use of leverage. By conventional wisdom, typical constrained investors are mutual funds, which cannot use more than 33% leverage according to the Investment Company Act of 1940[50], and individual investors[51]. For these groups, Frazzini and Pedersen (2014) report average portfolio ex-ante betas of 1.08 and 1.25 respectively[52]. Examples of unconstrained investors that use leverage extensively are private equity firms as well as Berkshire Hathaway, their

43 Ibid. p. 16.
44 Jylhä (2017). pp. 7-8.
45 Ibid. pp. 21-23.
46 Ibid. p. 14.
47 Ibid. pp. 30-32.
48 Adrian, Etula and Muir (2014). p. 2560.
49 Ibid. pp. 2571, 2577.
50 Baker, Bradley and Wurgler (2011). p. 46.
51 Frazzini and Pedersen (2014). p. 20.
52 Ibid. p. 19.

betas are 0.96 and 0.91[53]. Constrained investors indeed seem to systemically acquire higher beta assets.

Nevertheless, issues remain for this line of argumentation. Comparing the betas of unconstrained and constrained betas is difficult in the sense that the leverage constraints that investors face are partly unobservable[54] which leads to small sample sizes or the need to speculate on the use of leverage. And even though individual investors are considered very constrained as a whole[55], evidence from Iwasawa and Uchiyama (2014) suggests that their investments do not cause the beta anomaly[56]. Finally, leverage constraints deter from levered investments into low beta stocks but cannot fully explain the high demand for high beta stocks that leads to the mispricing.

4.2 Biased Beliefs

The beta anomaly is also linked to behavioral biases. These biases induce overreactions to high beta stocks leading to an overpricing and lower future returns.

A central measure with predictive power in this context is the investor sentiment[57]. A positive sentiment leads to overly optimistic views[58]. Thereby, periods with a positive sentiment are optimistic periods which attract an increased number of unsophisticated and overconfident investors causing increased noise trading[59]. This noise trading activity primarily affects high beta stocks because these are preferred by unsophisticated investors[60] which results in the mispricing. In pessimistic periods however, noise traders shun the stock market. Thus, Antoniou, Doukas and Subrahmanyam (2014) predict the CAPM to hold in pessimistic periods and a reversal of the risk-return relationship in optimistic periods[61]. To test this hypothesis, they use an index measuring sentiment that is constructed from proxies for the investors' propensity to invest in stocks[62]. Through this index, they can distinguish between optimistic and pessimistic periods. Indeed, the reported average monthly returns for the lowest and highest beta decile portfolios are 0.79% and 1.88% in pessimistic periods. In optimistic months, returns decrease from the lowest to

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[53] Ibid. p. 19.
[54] Ibid. p. 19.
[55] Ibid. p. 20.
[56] Iwasawa and Uchiyama (2014). pp. 67-70.
[57] Jacobs (2015). p. 73.
[58] Antoniou, Doukas and Subrahmanyam (2015). p. 1.
[59] Ibid. p. 6.
[60] Barber and Odean (2001). pp. 282-284.
[61] Antoniou, Doukas and Subrahmanyam (2015). p. 7.
[62] Ibid. p. 7.

the highest decile from 1.01% to -0.15%[63]. These results are complemented by regressions of the portfolio returns on the respective betas and other common risk factors. For pessimistic months, an upwards-sloping SML is found, beta coefficients range from 0.84 to 1.46 depending on which factors are included besides beta. On the contrary, the coefficients range from -0.65 to -0.88 in optimistic months indicating an inversion of the SML[64]. Jylhä (2017)[65] and Jakobs (2017) empirically confirm that a positive sentiment can flatten the SML and thus cause the beta anomaly. The latter uses the erroneous expectations of economic experts, up and down market states and local consumer confidence indices as proxies for sentiment[66].

In the next step in proving the theory that increased noise trading in optimistic periods causes the beta anomaly, Antoniou, Doukas and Subrahmanyam (2015) observe noise trading activity separately in periods with positive and negative sentiment trough three proxies. First, they find that the flows to mutual funds, through which unsophisticated investors participate in the stock market, increase by $22 billion during optimistic months compared to pessimistic months[67]. Second, analysts' earnings forecasts are more optimistic when the sentiment is high as measured by forecast errors[68]. Third, the probability of informed trading, meaning investments based on fundamental information, is generally higher for low beta stocks and in optimistic periods this probability further increases for low beta stocks while it decreases for high beta stocks[69]. All three proxies suggest that noise trading activity increases in optimistic periods. Their data also shows that when sentiment is positive, small investors are net buyers of high beta stocks and net sellers of low beta stocks[70]. This evidence suggests that the anomaly is induced by sentiment-driven noise trading.

An argumentation related to the sentiment-driven explanation is based on aggregate disagreement between optimists and pessimists. The CAPM assumption of homogeneous expectations clearly does not hold in reality as people disagree about the expected returns. This disagreement is amplified by beta because more uncertain outcomes from higher return variance leave more room for speculation on the future returns[71]. Additionally,

[63] Ibid. p. 35.
[64] Ibid. p. 36.
[65] Jylhä (2017). p. 23.
[66] Jakobs (2017). pp. 18-20.
[67] Antoniou, Doukas and Subrahmanyam (2015). p. 13.
[68] Ibid. p. 13-14.
[69] Ibid. p. 15.
[70] Ibid. p. 16.
[71] Baker, Bradley and Wurgler (2011). p. 2.

high beta stocks tend to attract overconfident investors, who make predictions with false precision[72] resulting in further dispersion of beliefs. With the existence of short-sales constraints, the consequence of disagreement is that only optimists hold high beta stocks in the equilibrium and set the price of these stocks because pessimists who would short such stocks are sidelined by the constraints[73]. This leads to an overvaluation of high beta stocks. Interestingly, the question why optimistic investors do not hold leveraged low beta portfolios instead of holding high beta assets is not answered directly with leverage constraints by Hong and Sraer (2016). Instead, they argue that in incomplete markets, where the number of assets is limited, idiosyncratic risk cannot be completely diminished via diversification and would be magnified by leveraging stocks; it poses an indirect leverage constraint[74]. In their theory, the SML is a function of the aggregate disagreement[75], the overpricing of high beta stocks increases with the aggregate disagreement, beta and the fraction of investors that cannot short assets[76]. Below a certain beta-cut-off, pessimists and optimists both hold the stocks long implying a positive slope. Beyond this cut-off, optimists still have long positions, pessimists however would want to short these stocks but only those who are unconstrained can do so. The resulting slope is negative, giving the SML an inverted U-shape with the cut-off level decreasing with higher disagreement and a higher fraction of constrained investors[77]. Empirically, this theory holds. In months with low disagreement, an upward sloping beta-return relationship is observed, whereas in high disagreement months this relationship has the predicted inverted U-shape[78]. Similarly, regressions including a coefficient capturing the concavity of the beta-return relationship show that a higher aggregate disagreement leads to a more concave SML[79].

Further biases amplify the effects of investor sentiment and aggregate disagreement. The representativeness bias is of relevance; high beta assets disproportionally account for stocks with the highest realized returns[80]. Investors acknowledge investments into small speculative stocks, e.g. Microsoft right after their IPO, as great investments but ignore similar stocks that failed. Thus, an irrational demand for high beta stocks is created. The effect of this bias is expected to be strongest in up markets when investor sentiment is

[72] Jakobs (2017). p.2.
[73] Ibid. p. 12.
[74] Hong and Sraer (2016). p. 3.
[75] Ibid. p. 3.
[76] Ibid. p. 7.
[77] Ibid. p. 3.
[78] Ibid. p. 21.
[79] Ibid. p. 22.
[80] Baker, Bradley and Wurgler (2011). p. 44.

high[81].

Additionally, the role of analysts can be observed. Attention grabbing assets, which are naturally very volatile assets, attract unsophisticated and overconfident investors. A good example is the current frenzy around Bitcoins. Attention for certain stocks results, among others, from extensive analyst coverage. Furthermore, analysts' earnings forecasts tend to be biased upwards and investors do not properly correct for this bias and overreact[82]. Especially for high beta stocks, the forecasts seem to be inflated more aggressively by analysts[83] leading to the overpricing of these stocks.

Jakobs (2017) also finds highly significant abnormal returns around global earnings announcements. While the abnormal return of low beta stocks increases by 2.5 basis points per announcement day, those of high beta stocks decrease by 34 basis points[84]. This supports the notion that the anomaly is directly related to earnings news.

However, the evidence on the role of analysts is ambiguous. Lin and Jiang (2015) report the anomaly to be strongest among high analyst coverage stocks[85]. Conversely, Antoniou, Doukas and Subrahmanyan (2015) find the anomaly only in stocks with low coverage[86]. A further discussion of the validity of behavioral explanations of the anomaly will occur at the end of the next subsection.

4.3 Lottery Preferences

Continuing the theme of behavioral biases from the previous subsection, a specific bias that is connected to the beta anomaly is the demand for lottery-like stocks. These stocks are defined by a high probability of large short-term returns[87]. One stock characteristic generating such a high probability for up-moves is a high market beta, thus lottery demand especially falls on high beta stocks[88]. This creates price pressure on high beta stocks leading to lower subsequent returns. Hence, investor preferences for lottery stocks can result in the anomaly.

Kumar (2009) intensively investigated lotteries in the stock market. Important findings are that lottery-like stocks are characterized by a low price, high idiosyncratic skewness and high idiosyncratic volatility[89]. The reasoning behind this is that lottery investors are

[81] Jakobs (2017). p. 18.
[82] Hsu, Kudoh and Yamada (2013). p. 30.
[83] Ibid. pp. 31,35.
[84] Jakobs (2017). p. 13.
[85] Lin and Jiang (2015). p. 20.
[86] Antoniou, Doukas and Subrahmanyam (2015). p. 20.
[87] Bali et al. (2017). p. 1.
[88] Ibid. p. 1.
[89] Kumar (2009). pp. 1899-1900.

looking for cheap bets which exhibit return distributions with the possibility of extreme returns[90]. Instead of diversifying their portfolios, these lottery investors choose only a few, volatile stocks to generate upside potential[91]. As a behavioral phenomenon, lottery demand is concentrated among individual investors whereas institutional investors do no prefer lottery stocks[92], which implies individual investors to be responsible for the beta anomaly. Kumar also provides preliminary evidence that lottery stocks underperform with a lower average annual return of more that 4%[93].

The main paper promoting lottery demand as the driver of the beta anomaly is Bali et al. (2017). Their proxy for lottery stocks is MAX, which is the average of the five highest daily returns in a month[94] but they also show all results to be robust against alternative definitions of MAX (averaging 1 to 4 days with the highest returns)[95]. This measure is strongly correlated with the characteristics of lottery stocks. From the lowest to the highest MAX decile, the average stock price decreases from $70.76 to $14.99, idiosyncratic volatility increases from 0.94 to 4.58 and the idiosyncratic skewness increases from -0.17 to 0.69[96].

First, they show that lottery demand-driven price pressure falls predominantly on high beta stocks. On average, MAX increases monotonically from 2.52 % in the lowest to 4.61% in the highest beta decile[97]. The average cross-sectional correlation between beta and MAX is 0.3 and varies from -0.03 to 0.84 per month[98].

Next, Bali et al. explore the effect of lottery demand on future returns. One-month-ahead excess returns across MAX deciles indicate the predicted relationship. From the second lowest to the highest decile the returns decrease monotonically from 1.00% to -0.40%, the exception to the monotony is the lowest decile in which the average one-month-ahead excess return is 0.74%[99]. Further evidence is drawn from a bivariate portfolio analysis. Returns are sorted on beta while controlling for MAX. Effectively, the beta anomaly is greatly diminished. In the average MAX decile, the high-minus-low beta portfolio's alpha is -0.14%, which is economically small and statistically insignificant with a t-statistic of -0.85%, and in each individual MAX decile, too, the alphas are indistinguishable from

[90] Ibid. pp. 1890-1891.
[91] Blitz and van Vliet (2007). p. 12.
[92] Ibid. pp. 1906, 1909.
[93] Ibid. p. 1926.
[94] Bali et al. (2017). p. 3.
[95] Ibid. p. 12.
[96] Ibid. pp. 24-25.
[97] Ibid. p. 33.
[98] Ibid. p. 19.
[99] Ibid. p. 34.

zero whereas they are significantly negative without the control for MAX[100]. Regression analyses support these findings: without MAX the beta coefficients are insignificant and range from 0.06 to 0.263. Only when MAX is included, a positive market risk premium is detected with statistically significant beta coefficients ranging from 0.265 to 0.470[101].

Moreover, Bali et al. augment existing factor models with FMAX, a factor constructed based on MAX with the factor-forming technique from Fama and French (1993) to capture lottery demand. Previously negative alphas for high-minus-low beta portfolios become indistinguishable from zero[102] and the average adjusted R-squared value of factor regressions explaining the BAB returns increase significantly from 0.22 to 0.47 when FMAX is included[103]. Lastly, compelling evidence is found when differentiating between months with high and months with low correlation between MAX and beta. Only subsequent to those months where lottery demand falls primarily on high beta stocks, i.e. this correlation is high, the anomaly is apparent. Then, average monthly returns and alphas of high-minus-low portfolios are both negative ($r = -0.68$ and $\alpha = -0.72$) and become indistinguishable from zero after including FMAX in the model[104]. This shows that lottery preferences are connected to the beta anomaly. But although an effect of lottery demand is apparent, it does not fully explain the anomaly as the high-minus-low portfolio alphas are still not positive after the control.

Support for the lottery demand-based explanation comes from Lin and Jiang (2015). By analyzing various firm characteristics, they find that high beta stocks have a significantly higher probability of a lottery-like payoff. In the lowest beta quintile, this probability is only 10.98% compared to 31.49% in the highest quintile[105]. In regressions, previously insignificant beta coefficients become significant and positive after controlling for firm characteristics connected to a jackpot payoff while, consistent with Bali et al. (2017), high-minus-low beta portfolio alphas are no longer negative[106]. This suggests that there is a premium on systematic risk which however is nullified by the contrary effect of lottery demand, leading to a flat SML

Difficulties of the lottery demand-based explanations and of those explanations based on other behavioral phenomena generally result from the implication of these theories that

[100] Ibid. p. 11.
[101] Ibid. pp. 13-14.
[102] Ibid. p. 15.
[103] Ibid. p. 17.
[104] Ibid. p. 20.
[105] Lin and Jiang (2015). p. 13.
[106] Ibid. pp. 16, 47.

individual investors drive the anomaly. Therefore, the anomaly should be stronger for stocks primarily held by individual investors. While Bali et al. (2017) find the anomaly to be strongest for stocks with low institutional ownership as captured by negative, economically large and highly significant alphas and returns of the high-minus-low portfolios when institutional ownership is low[107], Lin and Jiang (2015) report the anomaly to be stronger among those stocks with high institutional ownership after controlling for size[108], which cannot be reconciled with neither the lottery demand-driven nor other behavioral explanations. Moreover, Schneider, Wagner and Zechner (2016) argue that the anomaly can be better explained by skewness which is only partially captured by lottery demand proxies[109].

4.4 Benchmarking

Benchmarking is a standard practice for institutional investors such as pension funds or mutual funds[110] and is thus especially related to portfolio delegation. The performance of these investors is evaluated relative to a pre-defined benchmark and they usually receive their bonus only when they outperform the benchmark by a certain amount[111]. A typical contract holds a mandate to maximize the portfolios information ratio relative to the benchmark without the use of leverage[112]. The information ratio is the absolute return difference between the portfolio and the benchmark divided by the tracking error, which is the volatility of this return difference[113].

Consequently, agency issues arise between the portfolio managers and their clients. Although the clients should primarily care about maximizing their risk-adjusted returns, the mandate to beat the benchmark poses an incentive for managers to overpay high beta stocks as an easy means to increase the expected returns[114] and thereby the probability of them receiving the call option-like bonus[115]. The result is a flattening of the SML, i.e. the beta anomaly. Note that the managers are prevented from increasing their expected returns by leveraged investment in low beta stocks through leverage constraints[116]. In this framework, benchmarking also acts as a limit to arbitrage. It prevents institutional investors, who should be rational investors, from offsetting or exploiting the irrational demand

[107] Bali et al. (2017). p. 22.
[108] Lin and Jiang (2015). pp. 19-20.
[109] Schneider, Wagner and Zechner (2016). pp. 3, 26.
[110] Blitz (2014). p. 772.
[111] Ibid. p. 771.
[112] Baker, Bradley and Wurgler (2011). p. 45.
[113] Ibid. p. 45.
[114] Blitz (2014). pp. 771-772.
[115] Baker and Haugen (2012). p. 11.
[116] Blitz (2014). p. 772.

for high beta stocks created by individual investors[117] as the information ratio deters from overweighting low beta stocks and underweighting high beta stocks[118].

Moreover, a modification of the CAPM to capture the effects of benchmarking shows that the beta anomaly is a natural implication of relative performance measures. Due to the relevance of solely the risk and return relative to the benchmark, the market portfolio, which is the proxy for a benchmark here, becomes the risk-free alternative[119]:

$$r_i - E[r_m] = \beta_i(r_m - E[r_m]) + \varepsilon_i$$

ε is the idiosyncratic return. As expressed by the formula, beta now only explains short-term variations of the relative return $r_i - E[r_m]$; in the long-run however, no premium is associated with beta because $r_m - E[r_m]$ simply becomes zero[120]. In this model, tilting towards assets with betas higher or lower than the risk-free market portfolio is avoidable and hence poses an idiosyncratic risk, which is unpriced[121]. This means all assets have the same expected return and the SML is completely flat.

Furthermore, Blitz (2014) argues that the size and value factors from Fama and French (1993) are no risk factors but hold premiums for additional agency issues. Small size holds a premium because of client- or self-imposed[122] restrictions on the liquidity of stocks and additional operational costs for covering small-cap stocks[123]. Premiums for value stocks exist because investments in these are harder to justify ex-post in performance reviews and annual reports as compared to glamour stocks, which have a stronger past operating performance[124]. Also, glamour stocks are easier to sell to clients. Therefore, the agency-based model from the above equation is augmented with the size and value factors.

Empirically, this model suggested by Blitz (2014) outperforms the standard Fama-French three-factor model. Sorted on beta and size, the agency-based model's mean absolute alpha from factor regressions (0.06%) is less than half of the Fama-French alpha (0.16%)[125]. Moreover, only 4% of the alphas are statistically significant at the 5% level

117 Baker, Bradley and Wurgler (2011). p. 40.
118 Ibid. pp. 46-47.
119 Blitz (2014). p. 772.
120 Ibid. p. 775.
121 Ibid. p. 776.
122 Restriction can be self-imposed because to grow the firm's funds in the future, a larger scale is needed.
123 Ibid. p. 777.
124 Ibid. pp. 777-778.
125 Ibid. p. 787.

(44% in the Fama-French model) and the F-test cannot reject the agency-based model while rejecting the Fama-French model[126].

Nevertheless, this model itself does not explain why some studies report negative relationships between beta and return. The answer might lie in additional issues with portfolio delegation. Agency issues are also possible within the funds: Baker and Haugen (2012) believe portfolio managers wanting to impress their Chief Investment Officer to be one reason for an additional tilt towards high beta assets[127]. Moreover, tournament behavior between portfolio managers can reinforce the beta anomaly. The top performing funds receive by far the largest share of the total flows to funds[128] causing managers to stretch for additional returns via high beta assets[129].

To investigate the validity of the benchmarking explanation of the beta anomaly, the assets of institutional investors and the fraction of assets in institutional ownership must be examined. Because institutional investors are known to have mandates to beat benchmarks, they should primarily hold volatile assets and the beta anomaly should be strongest for stocks under their ownership if benchmarking is its main driver. In fact, institutional investors do hold stocks more volatile than the market[130], the average beta of mutual funds in the US in the 2000s was 1.10[131]. Consistent with this, Iwasawa and Uchiyama (2014) find institutional investors in the Japanese equity market to be focused on high beta stocks[132]. This is evidence that institutional investors are incentivized to invest in high beta stocks. However, the evidence on the correlation between the fraction of assets held by institutions and the strength of the anomaly is mixed. Lin and Jiang (2015) report the anomaly to be strongest among stocks with high institutional ownership[133]. On the contrary, Bali et al. (2017) find the anomaly to be prominent among stocks held by individuals[134] and Dutt and Humphery-Jenner (2013) note that benchmarking is an arguable explanation for developed markets but faces difficulties in emerging markets[135], where the anomaly is equally strong[136]. Additionally, Blitz (2014) shows that his agency-based model explains the beta anomaly early and late in his sample although the fraction of

[126] Ibid. p. 787.
[127] Baker and Haugen (2012). pp. 12-14.
[128] Sirri and Tufano (1998). p. 1598.
[129] Blitz (2014). p. 797 and Falkenstein (2010). p. 85.
[130] Baker and Haugen (2012). p. 13.
[131] Baker, Bradley and Wurgler (2011). p. 47.
[132] Iwasawa and Uchiyama (2014). pp. 62, 64, 66.
[133] Lin and Jiang (2015). pp. 19-20.
[134] Bali et al. (2017). p. 22.
[135] Dutt and Humphery-Jenner (2013). p. 1000.
[136] Baker and Haugen (2012). p. 7.

assets held by institutions was only high late in the sample[137]. While one possible explanation for these findings is that institutional ownership is an improper proxy for portfolio delegation because implicit forms, e.g. through private bankers, are not captured[138], another explanation is that benchmarking also translates to individual investors. Falkenstein (2010) formulates a model in which all agents are status-oriented and therefore have relative utility functions[139]. Then, wealth matters only in comparison to the wealth of others. As a result, the model from Blitz (2014) holds for all investors[140].

While the validity of the relative performance models that predict equal returns on all assets is questionable, benchmarking indeed poses an incentive for investors to overpay high beta stocks. Thus, enough investors focusing on their relative performance could explain the beta anomaly.

4.5 Operating Performance

Other studies present evidence hinting at the profitability of the underlying firms as a driver of the anomaly. Based on the observation that the operating profitability is negatively correlated with volatility[141], Novy-Marx (2016) finds that the returns on a defensive BAB-like strategy are explained by properly controlling for size, value and profitability. The alphas of CAPM-type and three-factor model regressions are 1.87% and 1.64% respectively, including profitability reduces the alpha to a statistically insignificant 0.11%[142]. This is a first indicator at the explanatory power of the operating performance. Similarly, Walkshäusl (2013) examines the validity of the Fama-French model augmented with a quality factor. He uses two different quality factors based on either the operating profit or on the stability of the firm's cash flows[143]. Both factors lead to identical results. The quality factors hold an average monthly premium of 0.33% and 0.40% and reduces the alphas in factor regressions from 0.98% to 0.71% and 0.73%[144]. The loadings on these profitability factors are positive for the low volatility quintiles and negative for the high volatility quintiles, thus the profitability factors capture the abnormal returns of low and high beta stocks and explain the deviation of the empirically observed

137 Blitz (2014). pp. 795-796.
138 Ibid. p. 796.
139 Falkenstein (2010). pp. 5, 72-77, 92.
140 Ibid. pp. 81-83, 93-93.
141 Novy-Marx (2016). p. 11, Walkshäusl (2013). p. 183 and Dutt-Humphery-Jenner (2013). p. 1007.
142 Novy-Marx (2016). p. 23.
143 Walkshäusl (2013). p. 183.
144 Ibid. p. 183.

SML from the one implied by the CAPM. This is evidence that there is a premium on high firm-level profitability and that profitability decreases in beta.

Further support comes from Fama and French (2016). They augment their three-factor model with a profitability and an investment factor. They justify the use of these two factors via the Dividend Discount Model, which indicates that higher expected profitability and lower expected investments by the firm both result in higher expected returns[145] and they show that the premiums for these two factors are not captured by the three-model[146]. Across 25 portfolios from a double sort on size and beta, this five-factor model produces a lower absolute average alpha (0.072 versus a three-factor alpha of 0.106)[147] and more importantly, only one of the 25 portfolio-alphas is economically large and statistically significant[148]. Consistent with Walkshäusl (2013), the factor loadings of the profitability and investment factors are positive for the four lowest beta quintiles and negative for the highest[149]. Thus, the five-factor model partly remedies the systematic failures of the CAPM and three-factor model with regards to the beta anomaly.

However, the channel through which profitability is related to the anomaly is somewhat unclear. Dutt and Humphery-Jenner (2013) argue that lower volatility firms have better access to capital, which in turn can be used for entrepreneurial projects resulting in surprise earnings[150]. This could partly justify the positive abnormal returns on low beta stocks. Nonetheless, the explanatory power of profitability seems limited. The alphas from Walkshäusl (2013) and Fama and French (2016) are only reduced by a small amount, the alphas are still significant on average when controlling for profitability in the study conducted by Walkshäusl (2015)[151] and the F-Test rejects the Fama-French five-factor model at the 5% level[152]. Therefore, return premiums from high firm-level profitability are only secondary in explaining the beta anomaly.

4.6 Interest Rate

Muijsson, Fishwick and Satchell (2016) developed a theory to explain the beta anomaly through interest rate movements caused by exogenous macroeconomic factors. In their

145 Fama and French (2016). p. 70.
146 Ibid. p. 81.
147 Ibid. p. 79.
148 Ibid. p. 84.
149 Ibid. p. 84.
150 Dutt and Humphery-Jenner (2013). p. 1003.
151 Walkshäusl (2013). p. 183.
152 Fama and French (2016). p. 76.

theory, the failure of the CAPM comes from not capturing changes of the interest rate. They argue that the beta anomaly appears whenever interest rates fall.

Interest rate movements are expected to influence low and high beta portfolios asymmetrically. While the authors suspect an increase of the expected return of low beta assets when interest rates fall, high beta assets experience decreasing returns simultaneously[153]. The opposite happens when interest rates rise. This Asymmetry is justified by generally higher gearing ratios of low beta firms[154] and by the notion that high beta portfolios are short bonds while low beta portfolios are long bonds in theory[155]. The CAPM also indicates this asymmetry:

$$E[r_i] = \beta_i E[r_m] + (1 - \beta_i) r_f$$

The risk-free rate affects the returns positively or negatively depending on whether beta is greater or smaller than 1 as the factor $(1 - \beta_i)$ becomes positive or negative. However, the effect of the changes in the interest rates are not properly modelled in the CAPM possibly due to incongruences between its one-periodic nature and time series data[156].

To test their hypothesis, Muijsson, Fishwick and Satchell use various factor model regressions in which they include the change of the interest rate or a dummy variable for a positive change as factors beside the excess market return[157]. The results are supportive of their theory. The coefficient of the change of the interest rate is economically large, highly significant and has the predicted sign for both high beta and low beta portfolios[158]. Similarly, regressions including the dummy show that positive changes affect the returns on low beta portfolios negatively and those on high beta portfolios positively. This is captured by the regression alphas. For low beta portfolios, the alpha is 0.77% when the interest rate falls and -0.159% when it rises. The respective values for high beta portfolios are -0.282% and 0.269%[159]. They also find that interest rate changes do not systematically affect beta but instead influence the intercept of the SML. These results suggest that falling interest rates directly lead to a flattening of the SML.

[153] Ibid. pp. 306-307.
[154] Ibid. pp. 306-307.
[155] Ibid. p. 310.
[156] Ibid. p. 310.
[157] Ibid. pp. 311-312.
[158] Ibid. p. 315.
[159] Ibid. p. 316.

4.7 Idiosyncratic Volatility

In subsection 4.2, idiosyncratic volatility (IVOL) was already briefly mentioned as limits to arbitrage preventing investors from levering low beta portfolios[160] instead of investing in high beta stocks. This already suggests that IVOL poses an incentive for investors with low risk aversion to invest in high beta stocks. This subsection will continue by exploring IVOL as the main driver of the beta anomaly.

Various studies confirm the existence of an IVOL anomaly[161], which is very similar to the beta anomaly in its appearance. IVOL is found to negatively predict returns; high IVOL produces negative alphas, especially among overpriced stocks, whereas alphas are positive when IVOL is low[162]. Moreover, certain papers find the IVOL anomaly to be correlated to the beta anomaly but stronger (in terms of the return differences between the most and least volatile assets)[163] which causes Liu, Stambaugh and Yuan (2016) to believe that IVOL is the hidden force behind the beta anomaly. They hypothesize that the observed negative abnormal returns of overpriced high beta stocks, which result in the beta anomaly, emerge from two underlying correlations: from the positive correlation between beta and IVOL and from the negative correlation between IVOL and alpha among overpriced stocks. Through IVOL as an intermediary, these correlations lead to the negative correlation between beta and abnormal returns among overpriced stocks. Thus, the observed beta anomaly is a side effect of the IVOL anomaly.

Liu, Stambaugh and Yuan (2016) construct a mispricing measure from 11 other mispricing anomalies to investigate whether certain stocks are over- or underpriced[164]. A double sort on this measure and beta reveals that while the highest beta decile holds more over- than underpriced stocks, the fraction of underpriced stocks in this decile is still substantial[165]. This questions beta-driven explanations of the anomaly since these imply that investors who choose to invest in high beta assets would invest in the overpriced assets and not the underpriced ones, which seems illogical. Instead, idiosyncratic risk is explored as the driver as it deters investors from correcting the occurring mispricing.

Only among overpriced stocks, the IVOL-alpha relationship is negative[166]. Liu, Stambaugh and Yuan also show that the correlation between beta and IVOL is positive with a

[160] Hong and Sraer (2016). p. 3.
[161] E.g. Li, Sullivan and Garcia-Feijóo (2014). p. 53 and Jacobs (2015). p. 71.
[162] Schneider, Wagner and Zechner (2016). pp. 14-15.
[163] Li, Sullivan and Garcia-Feijóo (2014). p. 57 and Liu, Stambaugh, Yuan (2016). p. 9.
[164] Liu, Stambaugh, Yuan (2016). p. 4.
[165] Ibid. pp. 7-8.
[166] Ibid. pp. 8-9.

value of 0.33. This is possibly due to stocks with a higher IVOL being more prone to sentiment driven disagreement which again is positively correlated with beta[167]. They report the IVOL anomaly to be three times as strong as the beta anomaly[168]. These findings lead to their conclusion that their theory holds: the beta anomaly arises from the beta-IVOL correlation and the existence of overpriced stocks with high IVOL[169]. In fact, controlling for the mispricing measure and IVOL completely diminishes the beta anomaly and, on the contrary, betas fail to explain the relationship between IVOL and alphas[170]. Lastly, Liu, Stambaugh and Yuan successfully show that the beta anomaly only exists in a time-series when both the correlation between beta and IVOL and the probability of stocks being overpriced is high, the latter being measured by investor sentiment. Then, the alpha on a high-minus-low beta portfolio is a highly statistically significant -1.16%. When only one of these factors is high, the alphas are still negative but smaller and less significant on average and when both factors are low, the alphas are positive[171].

Although this is conclusive evidence, other papers show that the beta anomaly matters beyond IVOL. Frazzini and Pedersen (2014) as well as Bali et al. (2017) include controls for IVOL. They too report the positive correlation between beta and IVOL[172] but show that the anomaly persists after the control[173]. In every IVOL decile, the alphas of BAB portfolios are positive and significant ranging from 0.39% to 0.59%[174]. Moreover, Schneider, Wagner and Zechner (2016) argue that the apparent effect of IVOL is merely a consequence of misspecifications of the CAPM causing abnormally seeming pricing errors among high beta stocks that are captured as idiosyncratic volatility[175].

4.8 Misspecifications of the CAPM and Other Issues

Lastly, the influence of model misspecifications of the CAPM and related issues on the beta anomaly will be examined.

Several issues concerning the betas are reported. Cheol (1994) argues that the true market betas are unobservable. The anomaly arises from using only the observable beta compo-

[167] Ibid. p. 9.
[168] Ibid. p. 9.
[169] Ibid. p. 10.
[170] Ibid. p. 11.
[171] Ibid. p. 13.
[172] Bali et al. (2017). p. 9.
[173] Ibid. pp. 12, 36.
[174] Frazzini and Pedersen (2014). Appendix B. p. 10.
[175] Schneider, Wagner and Zechner (2016). p. 14.

nent which is computed against a specific benchmark as the real market portfolio is unobservable[176]. Not including this unobservable beta component and the induced latent beta risk causes the seeming mispricing, while the observed relationship still can be consistent with the CAPM[177].

Bai et al. (2015) find that estimated betas on a rolling basis, as commonly used, are an improper proxy for the true market betas. While the estimated betas deliver a flat SML, the true betas are strongly positively related to the returns in their study[178]. Consistent with this, the correlation they observe between estimated betas and true betas ranges from being close to zero to strongly negative values depending on the sample[179].

Cederburg and O'Doherty (2016) connect the anomaly to the use of unconditional betas in the CAPM resulting in downward biased alphas[180]. A systematic relation between market weights and firm-level betas as well as a time-varying dispersion of betas causes this bias as the conditional betas are negatively correlated with both the equity premium and the volatility of the market[181].

Furthermore, certain assumptions of the CAPM are challenged, i.e. that returns are normally distributed, especially the skewness of returns seems to matter regarding the anomaly. In the early literature on the beta anomaly, Haugen and Heins (1975) already hint at the failure of the CAPM to include the skewness of returns[182]. Bucher and Wagner (2016) model equities as call options on the underlying firm's assets to explain the anomaly, thus stressing the role of call-option features of stocks namely non-linearity and skewness[183], and Schneider, Wagner and Zechner (2016) focus on the return skewness.

In the standard CAPM, beta captures upside risk as well as downside risk and both are rewarded with higher returns. However, this is not intuitive because these two risks are treated asymmetrically by investors. Upside risk is essentially upside potential and therefore desirable by investors whereas downside risk mostly is the relevant risk for investors[184]. The CAPM betas are shown to greatly underestimate the downside risk[185]. Downside risk is primarily related to negatively skewed returns[186], meaning that the effect of skewness on returns is not properly displayed in the CAPM. On this basis, Schneider,

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176 Cheol (1994). p. 331.
177 Ibid. p. 334, p. 337.
178 Bai et al. (2015). pp. 24-25.
179 Ibid. p. 25.
180 Cederburg and O'Doherty (2016). pp. 739-740.
181 Ibid. pp. 739-740, 764.
182 Haugen and Heins (1975). pp. 778-779.
183 Buchner and Wagner (2016). pp. 285, 287-288.
184 Chong, Jin and Phillips (2014). p. 3.
185 Ibid. p. 10.
186 Schneider, Wagner and Zechner (2016). pp. 10-11.

Wagner and Zechner (2016) examine the explanatory power of the skewness regarding the beta anomaly, finding that the higher the downside risk, the more the CAPM overestimates the expected returns. As a direct consequence, the realized returns of stocks with negative skewness, which are primarily high beta stocks, then seem abnormally low.

In their framework, negative skewness is mainly induced by downside risk from credit risk[187], which supports the finding that skewness becomes more negative as leverage increases[188]. The skewness of return means fat tails in the return distribution[189]. Return skewness is negatively correlated to beta; negative skewness is especially found for high-beta stocks[190]. Stocks with positively skewed returns hold a return premium, thereby the skewness positively predicts the returns. The excess return on the highest skewness portfolio is 1.54% with an alpha of 0.82% versus an excess return of 0.14% and an alpha of -0.54% for the lowest skewness decile[191]. Meanwhile, skewness helps explain the returns on BAB strategies. These strategies are highly dependent on the skewness of the stocks, excess returns are negative and alphas insignificant when skewness is high (positive) and returns are higher and alphas more significant as skewness decreases[192] showing that negative skewness strengthens the anomaly.

In support of the role of downside risk, Lin and Jiang (2015) show that stocks in the highest beta quintile have a probability of 25.3% to be distressed while for low beta stocks this value is only 7.83%[193]. This distress risk also appears to have explanatory power in regressions[194]. Moreover, Schneider, Wagner and Zechner (2016) find that skewness matters beyond the lottery characteristics of stocks investigated in section 4.3[195], which is interesting considering that Bali et al. (2017) report that the anomaly vanishes when controlling for lottery demand while it persists when controlling for either skewness or co-skewness[196]. Additionally, Falkenstein (2010) generally questions skewness as a measure of risk and documents inconsistencies in the relationship of skewness and returns across different asset classes[197].

[187] Ibid. pp. 10-11.
[188] Schneider, Wagner and Zechner (2016). p. 12, p. 24 and Buchner and Wagner (2016). p. 288.
[189] Schneider, Wagner and Zechner (2016). p. 9.
[190] Ibid. p. 12 and Bali et al. (2017). p. 9.
[191] Schneider, Wagner and Zechner (2016). p. 21.
[192] Ibid. p. 22.
[193] Lin and Jiang (2015). pp. 13, 45.
[194] Ibid. pp. 50-51.
[195] Schneider, Wagner and Zechner (2016). p. 26.
[196] Bali et al. (2017). p. 11.
[197] Falkenstein (2010). pp. 69-70.

4.9 Arbitrage Activity

This subsection is exempt from the rest of the section as it does not discuss an explanation of the origin of the beta anomaly. However, it is still of relevance since it examines a reason for the time-varying appearance of the anomaly.

Against this background, Huang, Lou and Polk (2016) investigate the relationship between arbitrage activity and the strength of the beta anomaly. According to them, the slope and intercept of the SML are depending on the time-varying beta arbitrage activity. By using the excess return co-movement of beta-arbitrage stocks relative to a benchmark as a proxy for arbitrage activity[198], they show that the SML is downwards sloping in the first six months when many arbitrageurs trade on the beta anomaly which reverses to a strongly positive slope in the third year[199]. Conversely, the SML is weakly upward sloping in the short-run when arbitrage activity is low but slopes downward in the long-run[200]. Analyzing the alphas of factor model regressions of beta arbitrage strategies, which are closely resembling the BAB portfolio, supports these results. The monthly alphas of 1.19% (in the first six months) and -1.04% (in the third year) after high activity compare to a short-term alpha indistinguishable from zero and 0.54% in the third year after low activity[201]. Thus, trading on the anomaly when arbitrage activity is low is unprofitable in the short-run but profitable in the long-run as the correction of the mispricing suffers from a severe delay. When trading is crowded however, the returns on BAB strategies are positive at first and the anomaly weakens as prices stabilize, but in the long-run an overcorrection of prices is observed, prices are destabilized by the high arbitrage activity and beta arbitrage trading leads to losses[202].

Huang, Lou and Polk reason this with a positive feedback loop created by arbitrage activity. They argue that initially successful bets on BAB portfolios increases the prices of low beta stocks and decreases the prices of high beta stocks. If the underlying firms have leverage, these price changes lead to an actual divergence of the betas, low betas fall further while high betas rise[203]. Trades on the anomaly send a signal that attracts increased bets against beta when beta arbitrage trading becomes crowded although the profitability has already decreased[204].

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198 Huang, Lou and Polk (2016). pp. 2, 14-15.
199 Ibid. pp. 22-23.
200 Ibid. pp. 22-23.
201 Ibid. pp. 16-17.
202 Ibid. p. 23.
203 Ibid. p. 23-24.
204 Ibid. p. 23-26.

4.10 Synopsis

Having explored the major possible drivers of the beta anomaly, it appears that, although several factors claim to do so, no single factor can sufficiently explain the beta anomaly free of any doubt. Rather, it is reasonable to assume that all or some of the effects explored coexist and explain the anomaly altogether.

As suggested by Baker, Bradley and Wurgler (2011), the anomaly could be attributed to two aggregated effects. First, an irrational demand for high-beta stocks causes the initial mispricing. Several behavioral phenomena, i.e. lottery preferences, sentiment, overconfidence and additional biases, contribute to this irrational demand as they drive individual investors into high beta stocks. Additionally, Leverage constraints and benchmark mandates play a role here as they shift the focus of institutional investors to high beta assets. The combination of these factors also reasons the mixed evidence on whether individual or institutional investors cause the anomaly.
The mispricing might be magnified further by hidden premiums for profitability or return skewness. This irrational demand causes investors to bid up the prices of high beta stocks, which lowers their future returns and flattens the SML.

Second, limits to arbitrage ensure the persistence of the anomaly. Here, major components are short-selling and leverage constraints. Benchmarking too can deter institutional investors from exploiting the anomaly. Moreover, idiosyncratic volatility in incomplete markets can substitute traditional leverage constraints as leveraging would amplify the idiosyncratic risk. Some papers also mention high transaction costs connected to high beta stocks that subsume the profitability of arbitraging the beta anomaly[205] and thus, successfully pose a barrier to arbitrage.

5 Conclusion

In this paper, it is shown by analyzing evidence from various studies that the CAPM does not hold empirically. Precisely, the suggested positive relationship between portfolio betas and expected returns is not observed. Low beta stocks exhibit positive abnormal returns and high beta stocks exhibit negative abnormal returns. This mispricing, known as the beta anomaly, results in a flat Security Market Line. These findings are supported by the data sample in this paper.

However, the CAPM is not be completely irrelevant. The existence of the beta anomaly merely highlights that the CAPM as a model is very restrictive and simplifies the capital

[205] Baker, Bradley and Wurgler (2011). p. 45 and Li, Sullivan and Garcia-Feijóo (2014). pp. 58-62.

market by ignoring the factors explored in this paper. Hence, the beta anomaly might not be anomalous but is merely a result from this omission. Under its assumptions, which are violated in real markets, the CAPM still holds. Furthermore, Jakobs (2017) notes that the anomaly exists within asset classes but it is not observable across different asset classes. Therefore, the CAPM is expected to be a better fit for asset allocation than for security selection.

Several possible explanations for the existence of the anomaly have been discussed. Leverage constraints stop investors from matching their risk aversion by leveraging or deleveraging the market portfolio and instead drive those with low risk aversion into high beta assets. High beta stocks also attract irrational demand from biased and overconfident investors, especially investors preferring lottery-like stocks tilt towards high beta stocks. Mandates to beat specific benchmarks are incentives for institutional investors to hold high beta stocks. Additionally, different averages regarding the profitability of low and high beta stocks, falling interest rates and negatively skewed returns strengthen the anomaly. The anomaly is likely to be attributed to a mixture of all these effects. Behavioral biases and benchmarking cause the overvaluation of high beta stocks and lower their future returns while limits of arbitrage from leveraging constraints, benchmarking, short-selling constraints, idiosyncratic volatility and transaction costs result in the persistence of this mispricing.

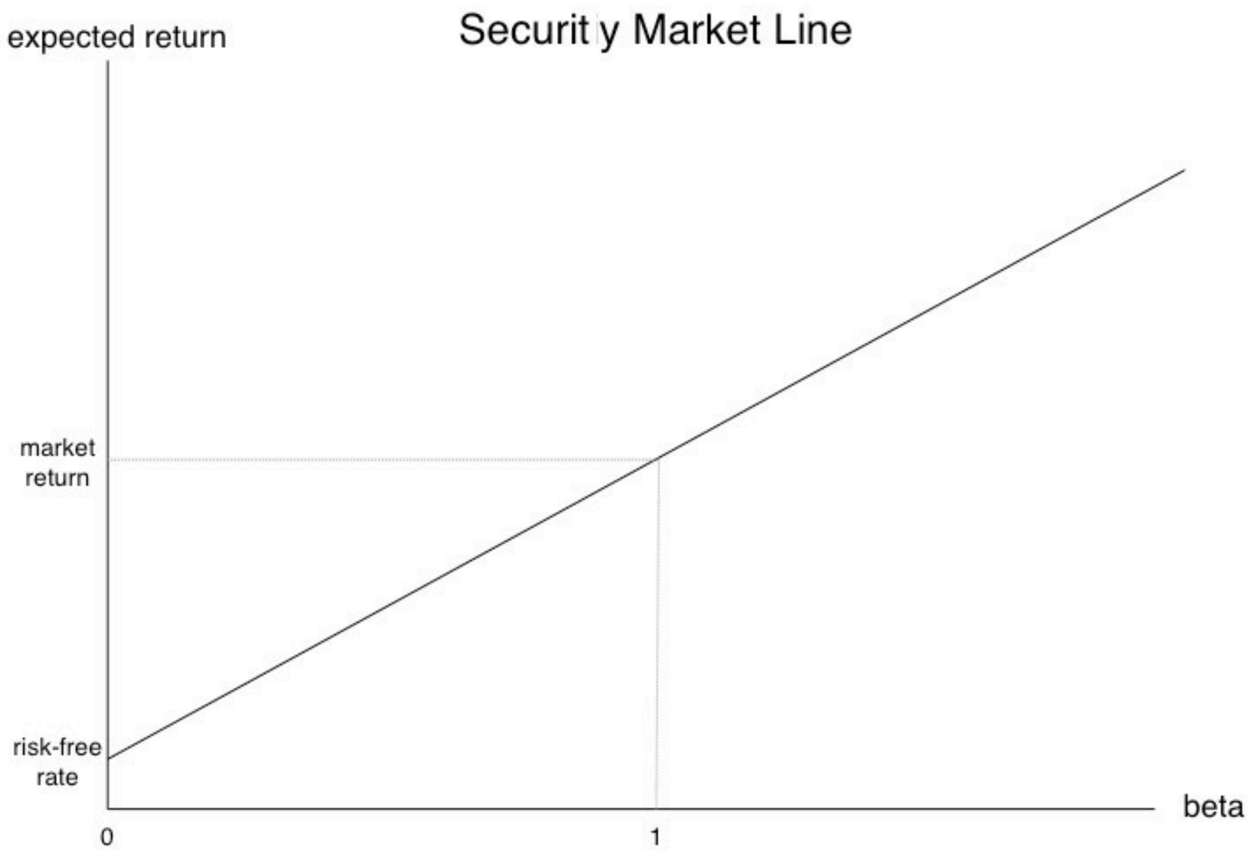
Considerable effort is needed to give further insights into the relevance of each factor regarding the anomaly. A data sample including all stocks from various markets is needed as a foundation to ensure the validity and representativeness of any results. For this entire sample, proxies for all factors discussed here would have to be constructed and then incorporated in a single comprehensive model to observe the significance of each factor and give a satisfying explanation for existence of the beta anomaly.

Appendix

A Figures

Figure 1

Theoretical Security Market Line according to the CAPM



B Tables

Table 1

Summary statistics for 10 beta-sorted portfolios

January 2001-November 2017. At the beginning of each year, all stocks are ranked in ascending order based on their estimated beta. The ranked stocks are assigned to one of the ten decile portfolios. All measures are monthly. The portfolios are equally-weighted. The ex-ante betas, the returns (r), the volatility (VOL), the Sharpe-ratio (Sharpe), the Treynor-ratio (Treynor) and Jensen's alpha (Jensen) are monthly averages across the entire sample period. Ex-ante portfolio betas are the mean of the stock-level betas. Post-ranking betas are the coefficients from regressing the excess portfolio returns on the excess market returns. The volatility is computed as the standard deviation on a one-year rolling-basis of the portfolio returns. The idiosyncratic volatility (IVOL) is computed as the standard deviation of the residuals from regressions of the excess portfolio returns on the excess market returns. VOL, IVOL, Jensen and r are shown in percent, betas, Sharpe and Treynor are presented as absolute values.

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}
Ex-ante β	0.06	0.34	0.49	0.60	0.72	0.82	0.92	1.03	1.35	1.58
Post-ranking β	0.60	0.56	0.62	0.65	0.77	0.83	0.88	0.85	1.08	1.15
r	1.40	0.84	0.73	1.23	1.53	0.70	1.13	1.33	1.28	0.40
VOL	6.79	4.33	5.20	5.03	5.56	6.04	5.78	6.14	8.05	8.02
IVOL	6.62	3.36	4.20	3.93	3.87	4.49	3.86	4.51	6.66	5.83
Sharpe	0.15	0.20	0.11	0.24	0.28	0.12	0.19	0.18	0.15	0.09
Treynor	1.44	0.17	0.05	-0.02	0.02	-0.02	0.01	0.00	0.01	-0.01
Jensen	1.15	0.25	0.19	0.81	0.79	0.13	0.55	0.48	0.42	-0.51

Table 2

Security Market Line estimate

This Table shows the results from regressing the excess portfolio returns on the respective post-ranking betas. Const. denotes the intercept of the regression. β coeff. shows the coefficient of the post-ranking betas. T-statistics are shown in parentheses. Statistical significance at the 10% level is indicated by *. The columns labeled R^2 and adj. R^2 present the R-squared and adjusted R^2 values of the regression.

const.	β coeff.	R^2	adj. R^2
0.0118*	-0.0043	0.0557	-0.0624
(2.28)	(-0.69)		

Table 3

CAPM-type regressions

This Table shows the intercepts (α) from regressing the excess portfolio returns on the contemporaneous excess market returns. Alphas are in percent. T-statistics are presented in parentheses. *, ** and *** means that the values are statistically different from zero at the 10%, 5% and 1% level of significance.

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}
α	0.98**	0.44*	0.30	0.79***	1.06***	0.20	0.61**	0.82**	0.70	-0.20
	(2.11)	(1.85)	(1.03)	(2.87)	(3.88)	(0.64)	(2.27)	(2.60)	(1.50)	(-0.50)

C List of All Stocks in the Sample

AAREAL BANK
AGIV REAL ESTATE
AIRBUS (FRA)
ALSTRIA OFFICE REIT
ALTANA DEAD - 28/08/10
ARCANDOR
AURUBIS
AVA DEAD - 10/02/06 - TOT RETURN IND
AWD HOLDING DEAD - 11/11/09
AXEL SPRINGER
BAADER BANK
BABCOCK BORSIG DEAD - 29/01/13
BAUER
BAYER.HYPO-UND-VBK. DEAD - 16/09/08
BAYWA
BEATE UHSE
BEIERSDORF
BERU DEAD - 02/10/09
BHW HOLDING DEAD - 13/02/08
BILFINGER BERGER
BOSS (HUGO)
BOSS (HUGO) PREF. DEAD - 18/06/12
BRENNTAG
BUDERUS DEAD - DELIST.03/08/04
CARGOLIFTER DEAD - 22/10/12
CECONOMY
CELANESE DEAD - 27/09/07
COMDIRECT BANK
CONTINENTAL
COVESTRO
CTS EVENTIM
DEGUSSA DEAD - 18/09/06
DEPFA BANK DEAD - TAKEOVER 27604H
DEPFA DT.PFANDBRIEF BK. DEAD
DEUTSCHE BOERSE
DEUTSCHE EUROSHOP
DEUTSCHE POSTBANK DEAD - 23/12/15
DEUTSCHE WOHNEN BR.SHS.
DEUTZ - TOT RETURN IND
DIEBOLD NIXDORF
DIS DT.INDUSTRIE SVS. DEAD - 20/06/08
DMG MORI
DOUGLAS HOLDING DEAD - 29/07/13
DRAEGERWERK PREF.
DT.PFANDBRIEFBANK
DUERR
DYCKERHOFF PREF. DEAD - 28/08/13
EDOB ABWICKLUNGS
ELRINGKLINGER
EPCOS DEAD - 27/10/09

ESCADA PREF. DEAD - CONV.TO 775055
EVONIK INDUSTRIES
FIELMANN
FRAPORT
FRESENIUS PREF.
FUCHS PETROLUB PREF.
GAGFAH DEAD - 04/07/17
GEA GROUP
GENERALI DTL.HLDG. DEAD - 12/05/14
GERRESHEIMER
GERRY WEBER INTL.
GFK DEAD - 20/10/17
GIGASET
GOLD-ZACK
GRAND CITY PROPERTIES
GSW IMMOBILIEN
HAMB.HAFEN UD.LOGISTIK
HANNOVER RUCK.
HEIDELBERGCEMENT
HEIDELB.DRUCKMASCHINEN
HELLA GMBH & KGAA
HOCHTIEF - TOT RETURN IND
HORNBACH HOLDING
HYPO REAL ESTATE HLDG. DEAD - 14/10/09
IKB DEUTSCHE INDSTRBK. DEAD - 26/01/17
INDUS HOLDING
INNOGY
IVG IMMOBILIEN DEAD - 15/08/14
KUKA
JENOPTIK
JUNGHEINRICH PREF.
K + S
KABEL DEUTSCHLAND HLDG.
KAMPS DEAD - DEAD - 09/04/04
KION GROUP
KLOECKNER & CO
KLOECKNER-WERKE
KOENIG & BAUER
KOLBENSCHMIDT PIERBURG DEAD - 02/10/07
KRONES
KRONES PREF. DEAD - CONV.ORD.686872
LANXESS
LEG IMMOBILIEN
LEONI
LOEWE DEAD - 09/10/14
LOGWIN
MAN
MANNHEIMER HOLDING DEAD - 14/03/13
MCKESSON EUROPE
MEDION
MERCK KGAA
METRO

MLP
MPC MUENCHMEYER CAP.K
MTU AERO ENGINES HLDG.
NORMA GROUP
OSRAM LICHT
PATRIZIA IMMOBILIEN
PFLEIDERER DEAD - 04/12/12
PHOENIX DEAD - MGERGER 929030
PRAKTIKER DEAD - 31/05/17
PROSIEBENSAT 1 MEDIA
PUMA
RATIONAL
RHEINMETALL
RHEINMETALL PREF. DEAD - EX INTO 929129
RHOEN-KLINIKUM
RHOEN-KLINIKUM PREF. DEAD - EX-INTO 307055
RTL GROUP
SALZGITTER
SCHAEFFLER
SCHWARZ PHARMA DEAD - 24/08/09
SGL CARBON
SIXT
SKY DEUTSCHLAND DEAD - 17/09/15
SOFTWARE N
STADA ARZNEI N
STEINHOFF INTL.HDG.(FRA)
STINNES DEAD - DEAD.12/05/03
STROEER
SUEDZUCKER
SYMRISE
TAG IMMOBILIEN
TALANX AKTGSF.
TECHEM DEAD - 03/03/09
TECIS HOLDING DEAD - DELIST.03/02/03
TEREX MT H&P SOL DEAD - 27/01/14
TOGNUM DEAD - 14/03/13
TUI
UNIPER SE
VONOVIA
VOSSLOH
WACKER CHEMIE
WCM BETS.-UND GRUNBSZ.
WEDECO WATER TECHNOLOGY DEAD - DELIST 05/07/05
WELLA PREF. DEAD - 13/11/07
ZALANDO
ZAPF CREATION

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Lehmann, Matthias Köln, 18.02.2018