Gains From Joint Operation of Multiple Reservoir Systems

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Synergistic gains are defined as the gains in benefits due to joint operation of a system of reservoirs in excess of the benefits from optimal individual operation. These gains are a result of both the deterministic (regional) and the stochastic diversity of flows into each of the reservoirs. They are captured by the use of flexible reservoir operating rules which base release decisions for a given reservoir on the state of the entire system, not just on the state of that reservoir. One possible operating rule is formulated and is demonstrated to be highly effective in capturing synergistic gains. A hypothetical design problem of sizing three water supply reservoirs in parallel is solved by three methods. The first method assumes that no synergistic gains are achievable. The second and third methods recognize the existence of such gains. The method recommended here incorporates the operating rule into the design process and makes use of the observation that the system yield is a function of system capacity.

Water resources engineers and hydrologists have long recognized that the benefits derived from the joint operation of a system of reservoirs may exceed the sum of the benefits from the independent operation of each of the reservoirs. Independent operation implies that decisions about releases from one reservoir are not based on the state of any other reservoir. Joint operation implies that decisions about releases from one reservoir depend not only on the state of that reservoir but also on the states of the other reservoirs in the system.

Recognition of this principle is found in studies of operating policies for multiple reservoir systems [*Maass et al.*, 1962; *Roefs and Bodin*, 1970; *LeClerc and Marks*, 1973; *Mejía et al.*, 1974]. In fact, *Mejía et al.* [1974] evaluate the gain in benefits that can be achieved by using a joint operating rule as compared to an individual operating rule, and the gains are found to be substantial. It appears, however, that prior to the present study no one has identified the upper bound on these gains. Knowledge of this upper bound should prove to be very useful to the water resources engineer responsible for developing the operating policy for a system of reservoirs. By comparing simulated yields under any proposed policy to this upper bound, it is apparent whether or not there is much room for improvement in the operating policy. *James and Lee* [1971] and *Nyak and Arora* [1971] have taken the principle of synergistic gains into account in reservoir sizing. However, it is not clear that any of the methods suggested in the literature are capable of fully exploiting the potential gains from joint operation.

While some of the papers mentioned above considered multiple use reservoir operation, this study will deal with a method for designing a system of water supply reservoirs in parallel which is intended to exploit the potential gains resulting from joint operation. It is computationally straightforward and, in comparison with the method proposed by *James and Lee* [1971, pp. 292–293] or with a method which assumes individual reservoir operation, will result in equal or lower cost and substantially different system design.

In the remainder of the paper the benefits from joint operation under a water supply objective will be defined. One example of an effective joint operating rule will be presented, and a given design problem will be solved by three different methods: the 'independence method,' the method proposed by James and Lee, and the method proposed by this paper.

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SYNERGISM

In the case of reservoirs designed for water supply purposes, benefits are a nondecreasing function of the safe yield (a yield which can be delivered with a certain very high reliability) delivered by the system of reservoirs. In this paper the gain in benefits due to joint operation of a system of reservoirs, in excess of the benefits from optimal individual operations, is called 'synergistic gain,' and it is generally measured as a yield in cubic feet per second.

Throughout this paper the following case will be considered. There are three reservoirs on three different streams. From each reservoir it is possible to make a release (draft) to a pipeline or channel connected to the demand point. The amount of this draft is a decision variable which cannot exceed the total amount of water stored in the reservoir. Each reservoir has an ungated spillway. Water that is spilled is assumed to be wasted, i.e., there are no benefits or disbenefits associated with this spilled water.

Synergistic gains arise as a result of the diversity of flows in the several streams being used in the water supply system. This diversity can be divided into two portions: a deterministic portion (due to differences in climate) and a stochastic portion (due to differences in weather). The deterministic portion is the result of the differences in the average annual cycle of flows in one stream as compared to the others. The deterministic portion of synergistic gains is captured by employing an operating policy which drafts more from a reservoir in a season when its inflow is relatively high in comparison to that of the other reservoirs and drafts less from a reservoir in a season when its inflow is relatively low in comparison to that of the others. This type of synergism is captured in the methods of *Nyak and Arora* [1971] and *LeClerc and Marks* [1973]. The stochastic portion of the synergistic gain arises from the fact that streamflows on any two streams are not perfectly correlated with each other, even after removal of their annual cycles. The stochastic portion is captured by taking a larger than normal draft from a reservoir that is more nearly full and a less than normal draft from a reservoir that is more nearly empty.

In practice, synergistic gains are captured by employing an operating policy that attempts to minimize the spilling of water from any reservoir while there is capacity available elsewhere in the reservoir system. This is best accomplished by using operating rules such as the space rule proposed in *Maass et al.* [1962]. In general, any rule which maintains proportional amounts of empty space in each of the reservoirs will capture synergistic gains. The difference between the various rules one might consider lies only in the definition of proportional. A variation on the space rule was used in this study and is described in a later portion of this paper.

SAFE YIELD OF INDEPENDENTLY OPERATED RESERVOIRS

This study utilizes 5 years of mean daily flows from three streams in the Baltimore, Maryland, area. The geographical centers of the watersheds are separated by no more than 20 mi., and all are in the Piedmont region of Maryland. The small separation would suggest that the deterministic component of any synergistic gain should be small. The streams are Gwynns Falls at Villa Nova, Maryland (drainage area 32.5 mi²); Western Run at Western Run, Maryland (drainage area 59.8 mi²); and Little Gunpowder Falls at Laurel Brook, Maryland (drainage area 36.1 mi²). None of these streams are subject to regulation. Henceforth these three streams shall be called A, B, and C, respectively. The period of record used for this study runs from March 1, 1963, to February 29, 1968. This period includes the water year of lowest flow (1966) for each of the three streams in at least the past 30 years.

It should be noted that what is called safe yield throughout this paper means only that the yield specified will be delivered if future droughts are no worse than the drought within the period of record used. This method of establishing a safe yield may be inadequate in its simplicity, but the particular choice of procedure for establishing a capacity–safe yield curve is not vital to the method proposed in this study.

For each of the streams, computer simulations were run to establish capacity–safe yield curves. On each of the streams seven different capacities were simulated. In each case the reservoir was assumed to be full as of March 1, 1963, and the safe yield was then determined to be the largest draft which could be supplied every day for the 5-year period and for which the reservoir completely refilled after the time of maximum drawdown. The capacity–safe yield curves are given in Figure 1.

It should be noted that these simulations do not account for storage above the spillway elevation or for evaporation. Furthermore, a constant demand for water rather than a time-varying demand pattern is assumed. Each of these factors should be included in actual design studies, but their absence here does not affect the validity of the methods presented here, since the same simplifications are used consistently throughout the study.

SAFE YIELD OF A SYSTEM OF JOINTLY OPERATED RESERVOIRS

In order to design a system of water supply reservoirs it is necessary to be able to evaluate the safe yield of a proposed system. Since the safe yield of a system depends on the operating policy used for that system, it is necessary to develop an operating policy and the means with which to evaluate its effectiveness. Maximization of the safe yield of the system is clearly the primary goal, but in order to evaluate the effectiveness of an operating policy it is useful to know the upper bound on yield for any system so that it is possible to identify effective policies. The upper bound on system yield is the limit of available synergistic gains, and the measure of effectiveness of a policy is the degree to which it captures these gains.

The determination of the upper bound on the possible synergistic gains, given a particular set of reservoirs, is a straightforward procedure which does not appear to have been presented elsewhere in the water resources literature. Since synergistic gains are only lost by having some reservoirs spill water while others are not full, a fictitious system was simulated from which no synergism could be lost. This is a system in which all three streams flow into one large reservoir. The safe yield of such a reservoir represents an upper bound on the joint safe yield of any set of three reservoirs on the three streams the sum of whose capacities equals the capacity of the large fictitious reservoir.

Simulations were done to determine safe yield for such a fictitious reservoir by using the sum of the flows of streams A, B, and C as daily inflows to this reservoir. By using several capacities in simulations for this fictitious reservoir, upper bounds on system safe yields for particular system capacities were developed. These results are shown as the system capacity–system safe yield upper bound curve in Figure 2. If an operating rule is found for systems of three reservoirs which produces a yield equal (or nearly equal) to this upper bound, then no better operating rule need be found for the purposes of this study. The one rule tried in this study is found to be very successful; i.e., this rule makes it possible to operate the three reservoirs on the three streams almost as if they were one

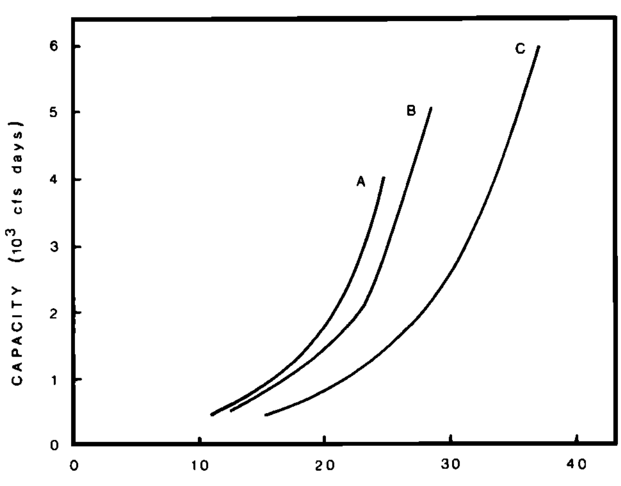

Fig. 1. Individual reservoir capacity–safe yield curves for streams A, B, and C.

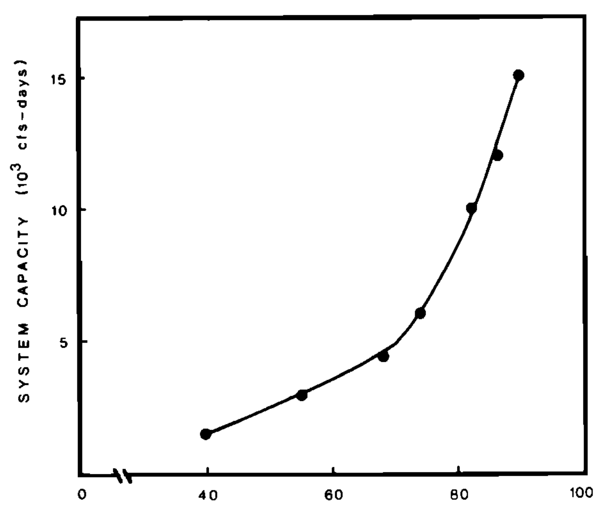

Fig. 2. System capacity–system yield curve based on simulations of a fictitious reservoir which receives inflows from all three streams.

reservoir with all three streams flowing in and with capacity equal to the sum of the three capacities.

THE OPERATING RULE

The particular operating rule used in this study was originally devised by us to be used in a situation similar to the one described by *LeClerc and Marks* [1973]. The situation is that of reservoirs in parallel, constructed primarily to deliver a given amount of water to a municipality and secondarily to provide a recreational service. In order to best provide this recreational service it is considered desirable to keep water levels in the reservoirs high (freeboards low) and to keep the differences between freeboards in the several reservoirs small. The operating rule employed in the present study is specifically chosen because it will simultaneously capture synergistic gains and maintain small and nearly equal amounts of freeboard in the three reservoirs. (The operating rule used by LeClerc and Marks captures only the deterministic portion of the synergism of the system.) There are many other operating rules that will capture the synergism equally well, or even better, and the choice of which operating rule to use should be based on the other objectives of the reservoir system.

The question of the appropriate time step to use for the operating rule is of great importance. Attempts to gain synergism from joint operation with a rule that makes monthly decisions were unsuccessful, but when daily time increments were used it was possible to show significant gains. The short time step allows the rule to 'react' to daily or weekly changes in the relative sizes of inflows. The short time step also results in a relatively low variance in flow predictions for upcoming periods, which is vital for a good operating rule.

In practice the operating rule used is this: Release only as much water from the system as is needed to meet the demand. Allocate the drafts among the three reservoirs in such a fashion that the predicted minimum freeboard at the end of the day is maximized. This is accomplished with the draft allocation method, which is given in the appendix of this paper.

Simulations of the entire 5-year record for the three reservoirs were run for several combinations of reservoir sizes. The direct allocation method is applied for each day in the 5-year period, and for each day a continuity accounting is carried out by adding inflows and subtracting draft and spill from each reservoir. The flow predictor used in the direct allocation method in the simulation was exceedingly simple. It predicted that today's inflow will equal yesterday's inflow. There are, to be sure, other predictors with lower variance, and one might want to consider using better predictors for operating, but even with this simple predictor the operating rule works very well.

Simulations of both the individual and the joint operating policies used in this study assume that the demand will always be satisfied (no shortages). In all simulations discussed in this paper the demand is, in fact, always satisfied. For real operation it is necessary to use a cascade of policies. The first policy sets the system draft for a relatively long time period (a month or more). This policy would be a hedging rule, to diminish draft in anticipation of major shortage. One could apply an implicit stochastic procedure (such as the method proposed by *Young* [1967]) with system storage as the state variable and system draft as the decision variable. The system draft T so determined then becomes a constraint for the daily draft allocation method described above. An effective overall policy requires a long time step for hedging decisions and a short time step for allocation decisions.

RESULTS OF SIMULATIONS USING THE OPERATING RULE

The results of one example simulation will be discussed in detail, and the results of the others will be given in Table 1. The sizes of the three reservoirs were chosen (arbitrarily) to be as follows: stream A, 4000 ft^3 d/s; stream B, 1000 ft^3 d/s; and stream C, 1000 ft^3 d/s. Their safe yields when they are operated individually are 24.9, 21.3, and 17.5 ft^3/s, respectively. If these reservoirs were operated independently, their combined safe yield would have been 63.7 ft^3/s (the sum of the three yields). If the three streams were diverted into one reservoir of 6000 ft^3 d/s capacity (equal to the sum of three capacities mentioned above), a safe yield of 73.6 ft^3/s could be realized. This means that the maximum possible synergistic gain is 9.9 ft^3/s, or 15.5%. Operating the three reservoir system jointly by the rule described above results in a safe yield of 73.4 ft^3/s. In other words, 98% (9.7/9.9 × 100) of the possible synergistic gains are captured by using this rule.

The results, shown in Table 1, indicate that for some choices of reservoir sizes the gains to be realized from joint operation may be substantial. These results do not provide any particular guidance in choosing reservoir sizes except for the following fact: Almost any combination of three reservoirs whose capacities sum to a given value will have very nearly equal system safe yields under joint operation. The qualification, 'almost,' refers to systems in which there is a very wide disparity in reservoir sizes, such as case 3 in Table 1. This fact proves to be very useful in the development of the system design method proposed below. Further tests of this hypothesis are war-

TABLE 1. Results of Simulations of a Joint Operating Policy

Reservoir Capacities, ft^3 d/s: A	B	C	Sums of Capacities, ft^3 d/s	Sums of Independent Yields, ft^3/s	Safe Yield Achieved in Joint Operation, ft^3/s	Maximum Potential Yield,* ft^3/s	Synergistic Gain Achieved, %	Potential Gain Actually Captured, %
4000	1000	1000	6000	63.7	73.4	73.6	15.5	98
2000	2000	2000	6000	72.1	73.6	73.6	2.1	>97†
500	5000	500	6000	58.8	73.1	73.6	25.2	97
4000	2000	4000	10,000	79.6	82.6	82.7	3.7	97
4000	1000	5000	10,000	74.4	82.5	82.7	10.9	98
4000	5000	1000	10,000	77.6	82.6	82.7	6.4	98
4000	6000	5000	15,000	89.0	91.1	91.1	2.4	>98†

*Safe yield from routing combined flows through one large reservoir of capacity equal to the sum of the three capacities.

†The potential gains are quite small in these cases; the shortfall from achieving the full potential gains is less than the rounding error in the calculation, 0.05 ft^3/s.

ranted, particularly in cases of greater hydrologic diversity.

The calculation of the daily allocation of drafts by the direct allocation method, the continuity accounting, and calculation of new surface areas and freeboards takes an average of 0.015 s of CPU time on the IBM 7094 computer. The simulation of a full 5-year period of operation for the three reservoirs took an average of 28.9 s.

To the best of our knowledge this result, that a multiple reservoir pattern may be represented as a single reservoir, has not been presented before. The following discussion will demonstrate how this result can be used in sizing a system of water supply reservoirs. It leads not only to a more streamlined method of design but also to significant savings in the cost of a multiple reservoir system designed to meet a given demand.

THREE METHODS OF DESIGNING A SYSTEM OF WATER SUPPLY RESERVOIRS IN PARALLEL

Three methods of designing a system of water supply reservoirs in parallel are applied to the current system in order to find the least-cost system design which will supply each of the following system yields: 89.4, 84.6, 80.9, 72.0 and 66.2 ft^3/s. From these designs the cost–system yield curve for each method is drawn for comparison. Method 1 assumes independent operation of reservoirs and does not recognize the existence of synergism. Method 2 is proposed by *James and Lee* [1971], and while it assumes joint operation and recognizes synergism, it does not utilize the relationship of system capacity and system yield mentioned above. Method 3, the method proposed in this study, assumes joint operation, recognizes synergism, and makes use of the relationship of system capacity and system yield.

The following relationships were defined for this example problem, which is to be solved by each method. Cost-reservoir capacity curves (Figure 3) were hypothesized for each of the three streams. They follow the general shape suggested in the literature [*Maass et al.*, 1962; *Hufschmidt and Fiering*, 1966]. The capacity-yield curves (Figure 1) are then combined with the cost-capacity curves to form the cost-yield curves for each of the three streams (Figure 4).

Method 1

By using the method of LaGrange multipliers it is possible to use these cost-yield curves to choose the least-cost system of reservoirs to supply any given system demand where the greatest possible draft from the system of reservoirs is made equal to the sum of the safe yields of three reservoirs. The problem may be formulated as follows:

$$\text{Min } Z = C_1(y_1) + C_2(y_2) + C_3(y_3) \tag{1}$$

subject to

$$\sum_{j=1}^{3} y_j = T \tag{2}$$

where y_j is the yield from reservoir j (ft^3/s), C_j is the cost of reservoir j (\$) and is a function of y_j (these functions $C_j(y_j)$ are seen graphically in Figure 4), and T is the total system demand (ft^3/s). A method for solving this problem with dynamic programing was demonstrated by *Wathne et al.* [1975]. To solve by LaGrange multipliers,

$$\text{Min } L = C_1(y_1) + C_2(y_2) + C_3(y_3) - \lambda\left(\sum_{j=1}^{3} y_j - T\right) \tag{3}$$

where L is the LaGrangian and λ is the LaGrange multiplier. Take the partial derivatives of L and set each of the partial derivatives of L equal to zero and solve.

$$\begin{aligned} \frac{\partial L}{\partial y_1} &= \frac{\partial C_1}{\partial y_1} - \lambda = 0 & \frac{\partial L}{\partial y_2} &= \frac{\partial C_2}{\partial y_2} - \lambda = 0 \\ \frac{\partial L}{\partial y_3} &= \frac{\partial C_3}{\partial y_3} - \lambda = 0 & \frac{\partial L}{\partial \lambda} &= -\sum_{j=1}^{3} y_j + T = 0 \end{aligned} \tag{4}$$

$$\begin{aligned} \frac{\partial C_1}{\partial y_1} &= \lambda & \frac{\partial C_2}{\partial y_2} &= \lambda \\ \frac{\partial C_3}{\partial y_3} &= \lambda & \sum_{j=1}^{3} y_j &= T \end{aligned} \tag{5}$$

which yields the conclusion that for minimum total costs, the marginal costs, with respect to yield, for each reservoir must be equal.

This problem can be solved graphically by using Figure 4, which is a graph of $C_j(y_j)$. The method is that of searching for a slope λ of each curve at which

$$\sum_{j=1}^{3} y_j = T \tag{6}$$

A more convenient method of solution is to use a graph of $\partial C_j/\partial y_j$ versus y_j, pick a value of λ and sum the values of y_j,

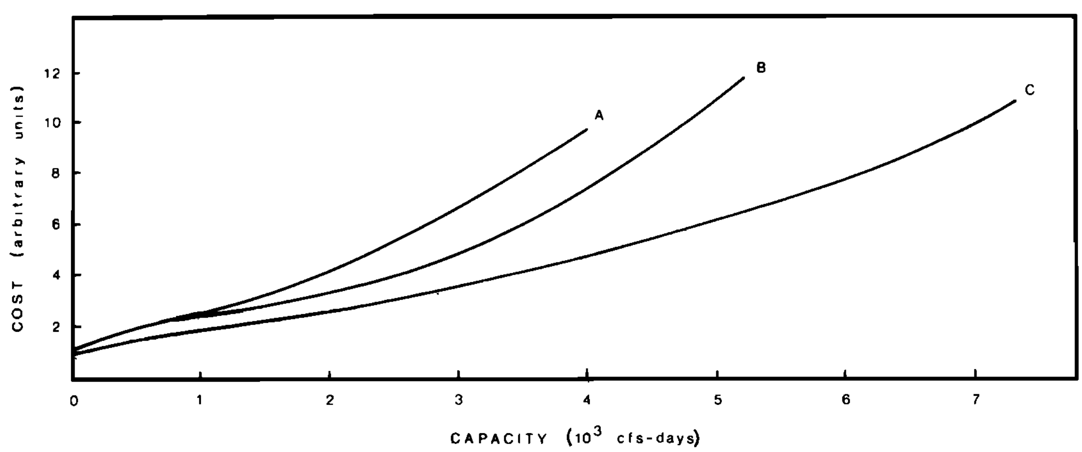

Fig. 3. Hypothetical cost-capacity curves for reservoirs A, B, and C.

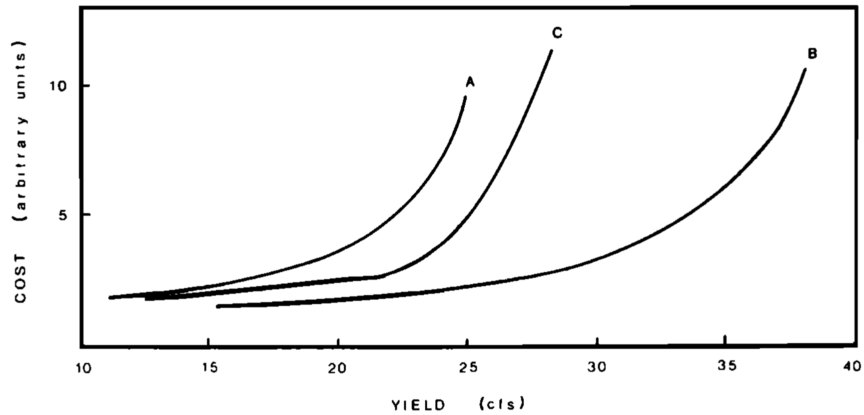

Fig. 4. Cost-yield curves for reservoirs A, B, and C based on Figure 3 and Figure 1.

and repeat until $\sum y_j = T$. By solving this problem repetitively for various values of T a cost curve is constructed (curve 1, Figure 5). The reservoir sizes and costs for each value of T are listed in Table 2. Because of synergistic gains these designs are not least-cost designs for the given value of T, since it is possible to achieve a larger system yield by employing an effective joint operating rule.

Method 2

The method described by *James and Lee* [1971, pp. 292–293] does take advantage of synergism but not in a systematic way and not in a way which is guaranteed to be optimal. Their method proceeds much as the previous method. Select a tentative system draft, select the least-cost system of reservoirs to achieve that tentative draft (use (5) above). Then search for a good operating policy for these reservoirs and simulate their operation. The safe yield achieved in the simulation becomes the system yield, and it is this yield which is plotted against the sum of the reservoir costs to form the system cost curve (Figure 5, curve 2), for which corresponding sizes and costs are listed in Table 2. As compared to method 1, the cost curve is shifted to the left; i.e., for any given yield the total cost is lower when designed by this method.

James and Lee do not mention methods for finding the best operating rule and do not mention the existence of an upper bound on system yield. This means that the operating policy chosen may be better than an individual operating policy but may not be the maximum possible. James and Lee provide no guidance to the development of operating policies; for equality of comparison we supplement their methodology with the joint operating policy described above in the direct allocation method. This policy is used in conjunction with James and Lee's method to constitute method 2 of this study.

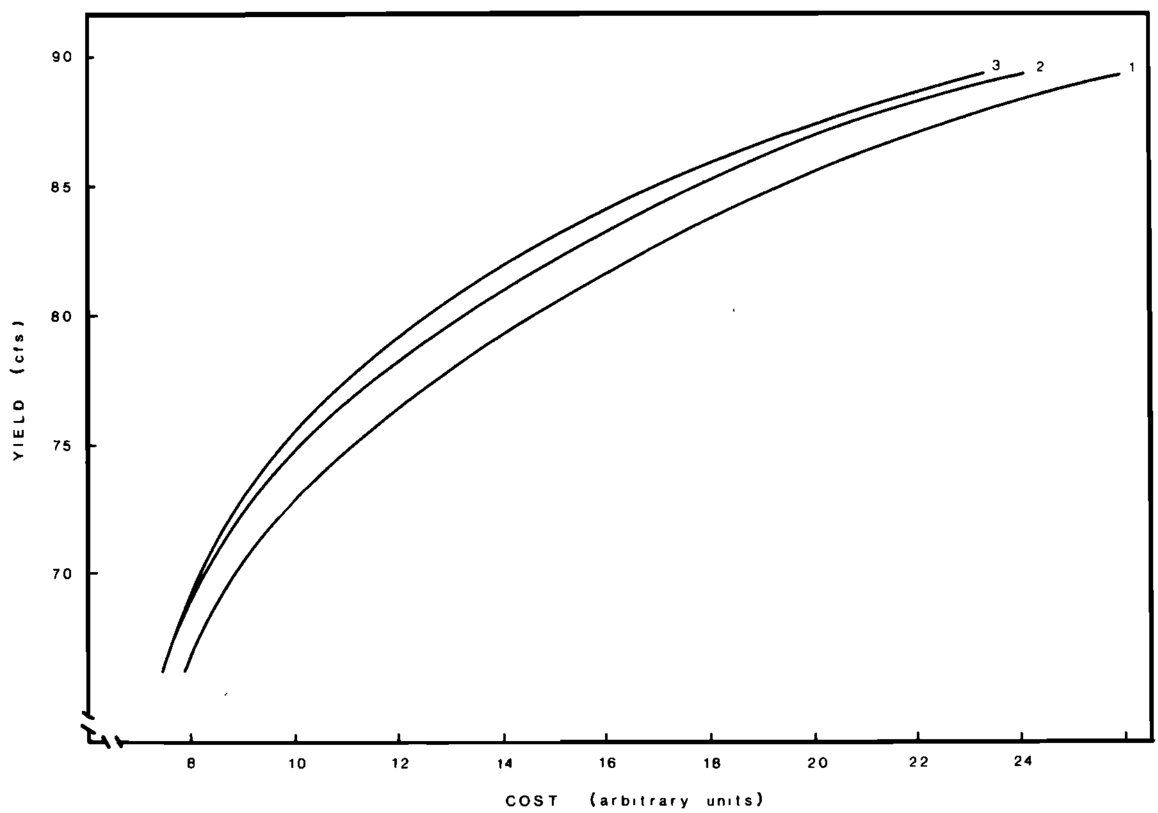

Fig. 5. Yield-cost curves for methods 1, 2, and 3 based on Table 2.

TABLE 2. Reservoir Sizes and Costs for Three Design Methods

Draft T, ft^3/s	Method	Total Cost	Capacity, ft^3 d/s A	B	C
89.4	1	26.0	3750	6600	4400
89.4	2	24.1	3580	6380	4140
89.4	3	23.4	3200	7000	3800
84.6	1	18.8	2950	6050	2970
84.6	2	17.1	2750	5825	2570
84.6	3	16.6	1976	6384	2780
80.9	1	15.4	2700	4530	2630
80.9	2	14.2	2600	4238	2287
80.9	3	13.3	1360	5650	2115
72.0	1	9.6	1465	2740	1980
72.0	2	8.8	1365	2370	1890
72.0	3	8.7	940	2945	1740
66.2	1	7.9	1000	1980	1770
66.2	2	7.4	880	1830	1630
66.2	3	7.4	800	1940	1600

Method 3

The method proposed in this study is based on our conclusion that within reasonable limits any combination of three reservoirs whose capacities sum to the same total capacity has nearly the same maximum system yield. The problem therefore reduces to one of finding the least-cost combination of capacities whose sum is equal to the required total capacity to achieve the desired system yield, since nearly all of the synergistic gains could be captured by the proposed joint operating policy (see Table 1). Figure 2 can be used to determine the system capacity necessary to achieve the desired system safe yield.

This design problem can be stated as follows:

$$\text{Max } Z = C_1(x_1) + C_2(x_2) + C_3(x_3) \tag{7}$$

subject to

$$\sum_{j=1}^{3} x_j = X \tag{8}$$

where x_j is the capacity of reservoir j, $C_j(x_j)$ is the cost-capacity function for reservoir j (from Figure 3), and X is the total capacity for which the upper bound on system yield is T (from Figure 2). Recognizing that the mathematical structure of this problem is identical to the problem stated in (1) and (2), we proceed directly to the solution: The marginal cost, with respect to capacity, for each of the reservoirs must be equal, and the capacities must sum to X. Check that these reservoir sizes will yield T when operated as a system; if not, establish what their maximum yield is and call this T. Then plot the sum of the costs of these three reservoirs against T as the system cost curve (Figure 5, curve 3). Alter T and repeat to develop more points on the curve.

The designs using this method result in system costs less than or equal to the cost obtained by using method 2 and everywhere less than that obtained by method 1.

SUMMARY AND CONCLUSIONS

Synergistic gain was operationally defined as the safe yield resulting from the joint operation of a system of water supply reservoirs in parallel minus the sum of the safe yields of the individual reservoirs. Synergistic gains arise as a result of both the deterministic and stochastic diversity of flows into the reservoirs, and these gains are generally captured by employing flexible system operating policies. One such policy was described and was demonstrated to be capable of achieving synergistic gains very nearly equal to the theoretical upper bound on synergistic gains.

Three different methods of sizing a system of three reservoirs were used. The first method selects sizes that would be the most cost effective if no synergistic gains were achievable. The second method recognizes the existence of synergism but does not necessarily exploit it fully in reservoir sizing. The third (the proposed method) recognizes synergism and makes use of the observation that system yield is a function of system capacity, and it optimally allocates the required capacity among the three reservoirs.

For all levels of system yields studied, the proposed method resulted in costs which were lower than those of the first method and were less than or equal to those of the second method. The proposed method is computationally more straightforward than the second method and resulted in sizing decisions that differed substantially from those in method 1 or 2.

Our experience with this study has led us to make several observations which we believe to be applicable to a much wider range of multiple reservoir problems (uses other than water supply alone and arrangements other than parallel). These are in need of verification and should prove to be fruitful avenues for future research.

1. A reservoir design algorithm should include the optimization of the system operating rule. The optimization of the system operating rule is facilitated by first establishing an upper bound on performance. One way of evaluating this upper bound is with a one-reservoir model for a multireservoir system. The convenience of being able to think of a multireservoir system as if it were one reservoir (being always fully cognizant of the extent to which the multireservoir system must fall short of single-reservoir performance) may serve to make some complex problems far easier to deal with.

2. Synergistic gains are captured by utilizing flexible operating policies. The ability to capture these gains is limited by physical constraints (such as the size of pipelines or canals that run from the reservoir to the demand point) and by the oper-

ator's ability to predict and adjust to future flows. A simulation model which attempts to simulate the capturing of synergistic gains by a given operating rule appears to work well with short time increments (e.g., 1 day), since it can be flexible, have good predicting ability, and mimic real time operations. There is a need for more study of the bias introduced by the use of long time steps in water resource systems models and for development of methods of compensating for this bias.

3. The individual reservoir cost-yield curves or capacity-yield curves by themselves may be irrelevant information in multiple reservoir design problems. What is more appropriate is a curve that relates costs to the service rendered by that reservoir. When a reservoir is part of a multireservoir system, its individual yield is a very poor surrogate measure of the service it provides. The system capacity-system yield curve is an excellent description of system behavior, since within reasonable limits any combination of three reservoirs whose capacities sum to the same total capacity has nearly the same maximum safe yield. Therefore individual reservoir capacity is a very good (but not perfect) measure of the service the reservoir provides. For this reason, the system capacity-system yield curve and the individual reservoir cost-capacity curves are relevant information for this design problem.

4. The method proposed in this study can be applied to the sizing of reservoirs to be added to an existing reservoir system. For example, in calculating the increase in safe yield that will result from the adding of a new reservoir one should not use the safe yield of that reservoir but rather the incremental increase in system safe yield as calculated from a simulation of the new system under a joint operating policy.

5. An important issue for future research is the tradeoff between the synergistic gains and the increased costs of a geographically extensive system. The increased costs of a geographically extensive system may include engineering costs (such as pipelines and pumps), costs of administration, and the political costs of building reservoirs in areas populated by persons other than the primary beneficiaries of the water supply system.

APPENDIX

The following definitions are necessary for the draft allocation method:

- j an index of reservoirs $j = 1, 2, 3$;
- D_j draft from reservoir j in the upcoming day, a decision variable, ft³;
- V_j volume of water in reservoir j at the beginning of the day, ft³;
- F_j freeboard in reservoir j at the beginning of the day, ft;
- S_j water surface area in reservoir j at the beginning of the day, ft²;
- T total draft required from all three reservoirs in the upcoming day, exogenously determined, ft³;
- I_j inflow to reservoir j in upcoming day (unknown at the beginning of the day and therefore not used in the draft allocation method but used in the continuity accounting procedure in the simulation), ft³;
- $\hat{I}_j$ predicted inflow to reservoir j in the upcoming day, ft³;
- B_j tentative approximate predicted freeboard for the end of the day, reservoir j (if $\hat{I}_j = I_j$ and $D_j = 0$, B_j will be very close to the exact freeboard since surface area changes very little in any given day; B_j can be greater than, less than, or equal to zero), ft;
- β_j approximate predicted freeboard for the end of the day, reservoir j, a decision variable (β_j may be greater than, less than, or equal to zero), ft;
- β the minimum $\beta_j, j = 1, 2, 3$, ft;
- $\Delta\beta$ a small increment of freeboard (can be a variable), ft;
- X_j capacity of reservoir j used only in the accounting procedure, not in direct allocation method, ft.

The draft allocation method consists of the following procedure:

1. From the state of the reservoirs determine $F_j, S_j, V_j, j = 1, 2, 3$, and from some exogenous procedures determine $\hat{I}_j, j = 1, 2, 3$, and T.
2. Compute $B_j, j = 1, 2, 3$: $B_j = F_j - (\hat{I}_j/S_j)$.
3. Compute β: $\beta = \text{Min}_j[B_j]$.
4. Increment β by $\Delta\beta$.
5. Compute $\beta_j, j = 1, 2, 3$: $\beta_j = \text{Max}\ \{B_j, \beta\}$.
6. Compute $D_j, j = 1, 2, 3$: $D_j = \hat{I}_j - F_jS_j + \beta_jS_j$.
7. Check if $D_j > V_j, j = 1, 2, 3$. If yes, change D_j such that $D_j = V_j$. If no, continue.
8. Check if $\sum_{j=1}^{3} D_j \geq T$. If yes, go to step 9. If no and if $\sum_{j=1}^{3} D_j$ is unchanged from the previous iteration, go to step 9. Otherwise go to step 4.
9. Accept $D_j, j = 1, 2, 3$ as the day's drafts for each of the three reservoirs. Stop.

Note that D_j represents a firm commitment of a draft; it can be delivered with certainty. Shortfalls ($\sum_{j=1}^{3} D_j < T$) will only occur when $\sum_{j=1}^{3} V_j < T$. The continuity accounting procedure used in the simulation along with the direct allocation method computes the volume at the beginning of the next day V_j' according to

$$V_j' = \text{Min}\ (V_j + I_j - D_j, X_j) \qquad j = 1, 2, 3$$

Given an assumed geometry for the reservoirs, the new values F_j' and S_j' are determined from this V_j'.

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REFERENCES

Hufschmidt, M. F., and M. B. Fiering, *Simulation Techniques for Design of Water-Resource Systems*, Harvard University Press, Cambridge, Mass., 1966.

James, L. D., and R. R. Lee, *Economics of Water Resources Planning*, McGraw-Hill, New York, 1971.

LeClerc, G., and D. H. Marks, Determination of the discharge policy for existing reservoir networks under differing objectives, *Water Resour. Res.*, *9*(5), 1155-1165, 1973.

Maass, A., et al., *Design of Water Resource Systems*, Harvard University Press, Cambridge, Mass., 1962.

Mejía, J. M., P. Egli, and A. LeClerc, Evaluating multireservoir operating rules, *Water Resour. Res.*, *10*(6), 1090–1098, 1974.

Nayak, S. C., and S. R. Arora, Optimal capacities for a multireservoir system using the linear decision rule, *Water Resour. Res.*, *7*(3), 485-498, 1971.

Roefs, T. G., and L. D. Bodin, Multireservoir operation studies, *Water Resour. Res.*, *6*(2), 410–420, 1970.

Wathne, M., C. S. ReVelle, and J. C. Liebman, Optimal capacities of water supply reservoirs in series and parallel, *Water Resour. Bull.*, *11*(3), 536-545, 1975.

Young, G. K., Finding reservoir operating rules, *J. Hydraul. Div. Amer. Soc. Civil Eng.*, *93*(HY6), 297-321, 1967.

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